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## Doctoral Thesis

# Firm Heterogeneity, Country-level Asymmetry and the Structure of the Gains from Trade

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# Introduction

The seminal contribution by Melitz (2003) has paved the way for a large body of the literature to study international trade from a firm-level perspective. A crude summary of the findings of this strand of the literature: (i) only more productive firms can export, (ii) aggregate trade flows are attributable to a handful of firms which export many products to many destinations, (iii) these happy few exporters are more skill and capital intensive, pay higher wages, and expand after trade liberalization while non-exporters shrink (Melitz, 2003; Mayer and Ottaviano, 2007; Bernard et al., 2007). As stressed by Melitz (2003), the heterogeneous impact of trade on firms is a direct consequence of their initial heterogeneity in productivity.

Now if we extrapolate this conjecture to a World economy comprised of asymmetric countries, shall we expect trade liberalization to affect countries differently? which mechanism would explain such a heterogeneous impact of trade on asymmetric countries? There are four potential cross-country differences in: (i) the state of technology, (ii) market size, (iii) stringency of local standards, and (iv) demand structure. In an early extension of the Melitz (2003) model to the asymmetric case, Demidova (2008) focuses on technological asymmetry and shows that trade raises welfare in the technologically advanced country at the expense of a welfare loss for the laggard one. Demidova and Rodriguez-Clare (2013) propose a two-country version of the Melitz (2003) model which allows for a possible asymmetry in market size. In contrast to Demidova (2008), they find that unilateral trade liberalization is welfare improving for both partners.

This contradiction mainly stems from the absence of the Home market effect in Demidova and Rodriguez-Clare (2013), and mirrors thus a dependence of the nature of the impact of trade (on asymmetric countries) on the modeling strategy. This possible theoretical debate was absent in subsequent literature, which remains silent on the welfare implications of country-level asymmetry in market size under firm heterogeneity. Another aspect of asymmetry that received little attention in theoretical literature is the difference in the degree of stringency of local standards across countries. In deed, given the absence of a conventional modeling approach of non-tariff barriers, trade models have seldom examined the welfare implications of such asymmetry in local standards.

On the demand side, since earliest work by Linder (1961), cross-country difference in demand structure has received little attention in the literature. The Linder (1961) hypothesis predicts more trade between similar countries, with rich countries trading high-quality goods, and poor countries trading low-quality ones. The pervasiveness of horizontal differentiation in earlier variants of the Melitz (2003) model could be one of the reasons why asymmetry in demand has not been studied in great detail. Nevertheless, even when we abstract from quality, asymmetry on the demand side may arise when preferences are non-CES. Using quadratic preferences, Melitz and Ottaviano (2008) show that larger countries are characterized by a higher degree of price sensitivity, which implies a tougher competitive environment. This, in turn, induces a tougher selection into exporting and forces foreign exporters to charge lower markups to larger destinations.

By contrast, using a different class of preferences, Bertoletti, Etro, and Simonovska (2018) obtain an opposite result, whereby selection into exporting and export prices do not vary with a destination's population size. Instead, they solely depend on its per-capita income level. In particular, the authors show that as the elasticity of demand decreases with individual income under indirectly-separable preferences, rich markets are easier to penetrate and foreign exporters charge them higher markups, and thus higher prices. Simonovska (2015) provides empirical support for this theoretical prediction. She finds that an identical good is sold at a higher price on richer destinations.

Seen this way, the country-specific aspect of the demand elasticity has been implemented in these non-CES models for two main reasons. The first is to predict the impact of per-capita income and population size on the extensive margin of trade. The second is to provide a theoretical rationale for the empirically observed price discrimination. However, little has been said on the potential implications of these more realistic patterns of price sensitivity for income and size effects on the intensive margin of trade, and on whether it may give rise to a variable elasticity of trade margins to trade costs across countries. For instance, despite strong empirical evidence showing that per-capita income affects significantly price elasticities, (Simonovska, 2015; Faber and Fally, 2017; Handbury, 2019), whether this implies a stronger income effect on the intensive margin of trade, and potentially induces a country-specific elasticity of trade margins to trade costs, has not been explored yet in theoretical trade models.

Besides allowing for a possible cross-country difference in the degree of price sensitivity, non-CES preferences offer additional features of flexibility, under firm heterogeneity, which received more attention in the literature. For instance, the variability of markups across firms and the incompleteness of the pass-through it entails have been studied in more details. Under different alternatives to the CES, (Melitz and Ottaviano, 2008; Bertoletti, Etro, and Simonovska, 2018; Arkolakis et al., 2018; Mrázová and Neary, 2017) highlighted that more productive firms face less elastic demand, an so charge higher markups and do not fully pass on a cost reduction to consumers. Bertoletti, Etro, and Simonovska (2018) show both theoretically and empirically that the incompleteness of the pass-through significantly reduces the magnitude of welfare gains from trade.

The flexibility in markups under firm heterogeneity and non-CES preferences has also revived the debate on the existence of the "pro-competitive effect of trade". By considering two possible behaviours of the relative love for variety (RLV)[1] under directly-separable preferences, Zhelobodko et al. (2012) find that the pro-competitive effect of market size enlargement holds only under increasing RLV.[2] However, it turns into an "anti-competitive effect" in the opposite case. A recent work by Arkolakis et al. (2018), shows that pro-competitive reduction in domestic markups is either dominated by an increase in foreign markups when preferences are directly-separable, or both effects exactly cancel out when preferences are homothetic. This reveals that the existence of variable markups at the firm-level may dampen rather than magnify the gains from trade.

In this sense, in recent trade models incorporating firm heterogeneity and variable markups, the focus is squarely on restoring a theoretical role for the pro-competitive effect of trade, which appears to be elusive (Arkolakis et al., 2018; Fally, 2019). However, another theoretically appealing feature of such settings has received little attention: the demand elasticity is firm-specific. Besides giving rise to variable markups, this key property opens the door for a more realistic modeling of consumer behavior than allowed by homothetic CES preferences. In particular, in the absence of restrictions on demand curvature, consumers may be more, or less reactive to price variations of varieties supplied by more, or less productive firms. This induces more flexible patterns of allocation of additional export market shares upon trade liberalization, with important implications for the gains from trade. In spite of being theoretically appealing, the welfare implications of the firm-specific aspect of the demand elasticity has received little attention in recent theoretical work.

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[1]This corresponds to the elasticity of the inverse demand.

[2]This corresponds to the case where the price elasticity of demand is decreasing in individual consumption, as initially assumed by Krugman (1979).

The main objective of this dissertation is to address the aforementioned questions, which despite their theoretical appeal, received little attention in existing theoretical work in international trade, and are thus still open. The goal of this dissertation is thus threefold. The first consists in studying the welfare implications of standards liberalization under country-level asymmetry both in market size and stringency of local standards. The second is to examine both theoretically and empirically the income effect on trade margins, and on the degree of their sensitivity to trade costs. The third objective is to concentrate on the firm-specific aspect of the demand elasticity beyond the CES, and to examine the role it plays in determining the magnitude and the structure of the gains from trade.

Towards this goal, I embed alternative assumptions both on the demand and supply side in the canonical Melitz-Chaney model of international trade with heterogeneous firms (Melitz, 2003; Chaney, 2008). With the aid of these simple amendments, I propose different variants of this canonical model which are well-suited to address the question that is at stake in each chapter. In so doing, the current dissertation contributes to trade theory with heterogeneous firms along three lines.

In **Chapter 1**, I propose a version of the Melitz (2003) model for the case of three possibly asymmetric countries separated by non-tariff barriers. In the absence of a pre-established cost hierarchy to standards, this chapter covers two possible hierarchies. The first is "purely vertical" where compliance with foreign standards is costly only when they are more stringent than local ones. The second is "verti-zontal" in the sense that compliance is always costly regardless of whether foreign standards are less, or more stringent than local ones. The contribution of this chapter is twofold. First, I show that standards liberalization is welfare improving only when the cost hierarchy is "verti-zontal" and the trading partner is larger than the excluded country. Second, upon implementing more realistic assumptions on consumer behavior, I show that this result holds only when consumers' preference for better standards is relatively weak.

In **Chapter 2**, I propose a structural gravity model with heterogeneous firms, asymmetric countries and indirectly additive preferences nesting non-homotheticity as a general case and the CES as a homothetic exception. The contribution of this chapter is threefold. First, I show, both theoretically and empirically, that the intensive margin of trade increases only with per-capita income in general equilibrium, and that per-capita income dampens the sensitivity of trade margins to trade costs. Second, I highlight two new welfare channels: an additional selection effect occurring on the export market, and an increase in nominal wage in the liberalizing country. Third, the contribution of the current chapter to the gravity literature is a fully structural gravity equation that exhibits both inward and outward multilateral resistances, and additionally exhibits a variable elasticity of aggregate trade flows to fixed trade barriers under non-homothetic

preferences. Finally, aiming at obtaining general results without losing in tractability, the current chapter proposes a new method that I call "the Exponent Elasticity Method" (EEM). This simple method delivers tractable solutions in general equilibrium despite added flexibility in preferences.

**Chapter 3** considers a general yet tractable demand system encompassing directly- and indirectly-separable preferences, with homothetic CES as a common ground. An added flexibility of this demand system is that it allows for two alternative curvatures of demand. Beyond the CES, demand may be either "sub-convex": less convex than the CES, or "super-convex": more convex than the CES. Embedded in a general equilibrium trade model featuring standard assumptions on the supply side, this flexible demand system yields new comparative statics results and a wide range of predictions for the gains from trade, while illustrating existing ones in a simple and compact way.

The main finding of this chapter is that demand curvature plays a crucial role in driving comparative statics results, shaping the structure of the gains from trade as well as determining the magnitude of these gains, whereas the type of preferences affects only marginally the results. In particular, taking the CES as a boundary case, I show that when demand is sub-convex, selection into markets is more relaxed, the partitioning of firms by export status is more pronounced, net variety gains and gains from selection coexist, and gains from trade are smaller than those obtained under CES demand. I also emphasize that the type of preferences plays only a second-order role. For instance, under sub-convex demands, directly-separable preferences provide an upper bound for the gains from trade, while indirectly-separable preferences provide a lower bound. All these patterns are reversed when demand is super-convex.

# Chapter 1

# A Simple Model of Standards Liberalization

## 1.1   Introduction

Empirical research in international trade has overwhelmingly substantiated cross-country differences in stringency of local standards and its detrimental effect on bilateral trade flows (Chen and Mattoo, 2008; Otsuki, Wilson, and Sewadeh, 2001). Such asymmetry gave rise to a continuous process of standards harmonization mainly between developed and developing countries, with important implications for North-South and South-South trade. This has been empirically investigated by Disdier, Fontagné, and Cadot (2015) who show that standards harmonization fosters North-South trade, yet at the expense of reducing South-South trade. The authors emphasized that such scenario of standards liberalization is welfare improving for Northern and Southern participants, while it might induce a welfare loss for excluded Southern countries.

While this conjecture is empirically well established, it received little attention in theoretical literature. For instance, international trade models have seldom examined the welfare implications of standards liberalization mainly due to the absence of a pre-established cost hierarchy to standards, and of a conventional way to model compliance costs. The objective of the current chapter is threefold. First, I build on the baseline Melitz (2003) model to provide a theoretical framework that is well-suited for examining the welfare implications of standards liberalization. Second, I examine the welfare implications of standards liberalization under standard assumptions both on the demand and supply side. Third, upon incorporating more realistic assumptions on consumer behavior and cost compliance, I separately examine two different scenarios of standards liberalization; (i) an alignment of a country on the local standards of its partner; and (ii) standards harmonization, whereby initially different standards converge to an intermediate degree of stringency.

Towards this goal, I proceed in three steps. First, I consider the case of three countries that are asymmetric both in market size and stringency of local standards. Second, I propose two possible cost hierarchies to standards. The first is "purely vertical" where compliance with foreign standards is costly only when they are more stringent than local ones. The second is "verti-zontal" in the sense that compliance is always costly even when foreign standards are less stringent than local ones.[1] Third, I embed these simple amendments in the Melitz (2003) model, and incrementally introduce more realistic assumptions on the demand and supply side.

The current chapter highlights two novel results on the welfare implications of standards liberalization. First, I show that standards liberalization is welfare improving only when the cost hierarchy is "verti-zontal" and the trading partner is larger than the excluded country. Second, under more realistic assumptions, I show that standards liberalization occurring through harmonization or alignment, is welfare improving only when consumers' preference for better standards is relatively weak.

The remainder of this chapter is organized as follows. The next section spells out the model. Section 3 offers a characterization of the general equilibrium. In section 4, I study the welfare effect of standards liberalization. Section 5 offers a parsimonious extension which allows to study two different scenarios of standards harmonization under more plausible assumptions. The last section concludes. The main proofs are provided in Appendix A.

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[1]The term "verti-zontal" has been first introduced by Di Comite, Thisse, and Vandenbussche (2014) to describe a hybrid product differentiation regime. I find it useful to resort to their terminology in this asymmetric standards context where the term "verti-zontal" has the following meaning. Despite their vertical nature (one standards is more stringent than another), standards are horizontal in terms of the cost of compliance they entail. That is, they imply an identical cost of compliance.

## 1.2   The Model

Consider three countries indexed by $\iota=i,j,k$ and populated by $L_\iota$ identical agents, each of which supplies $E_\iota$ units of efficient labor. Using labor as a unique factor of production, each economy produces two goods. The first is horizontally differentiated and supplied as a continuum of varieties (indexed by $\omega \in \Omega$) which are produced by monopolistically competitive firms. The second is homogeneous and produced under perfect competition at a unit cost. Free labor mobility across sectors along with the latter assumption on the homogeneous good fix nominal wage to 1. Individual labor endowment $E_\iota$ is thus both income and expenditure. These countries may be asymmetric only in market size *(Y=EL)* and in stringency of local standards.

In each country, preferences of the representative consumer are represented with an indirect utility function that has an inter-sectoral Cobb-Douglas form:

$$V = ( \frac{E}{p_h} )^\alpha ( [ \int_{\omega \in \Omega} ( \frac{p_\omega}{E} )^{1-\sigma} d\omega ]^{\frac{1}{1-\sigma}} )^{1-\alpha}$$

where $\alpha \in ]0.1[$, $p_h$ is the price of the homogeneous good, and $\sigma > 1$ is the constant elasticity of substitution (CES) between varieties of the differentiated good. On the supply side, each country has an *endogenous* mass of entrants $M_\iota^e$, each of which aims to engage in monopolistic competition and to sell its own variety of the differentiated good conditionally on successful entry. For instance, upon paying the sunk fixed cost of entry $F_e$ (in efficiency labor units), firms draw their random productivity $\varphi$ from a cumulative distribution function $G(\varphi)$. As in Melitz (2003), CES demand along with the presence of fixed trade cost $(f_{ij} > f_{ii})$ guarantee the existence and uniqueness of the domestic and export productivity cutoffs, respectively given by $\varphi_{ii}^*$ and $\varphi_{ij}^*$.[2] At this stage, the partitioning of firms by export status $(\varphi_{ij}^* > \varphi_{ii}^*)$ is ensured by the fact that international trade involves higher fixed cost than does domestic trade $(f_{ij} > f_{ii})$.

*Non-Tariff barriers-* Let us assume that the stringency of local standards significantly varies across these countries. Such asymmetry implies that exporting a variety from an origin $i$ to a destination $j$ involves also a variable cost of compliance with country $j$'s local standards $c_{ij} \geq 1$.[3] For instance, any firm intending to export to a given country has to follow these steps: it first produces its variety according to local standards (to be eligible to sell on the domestic market),

---

[2]Recall that under the CES, the operating profit of a $\varphi$-productivity firm is given by $\pi^o(\varphi) = \frac{r(\varphi)}{\sigma}$. Given the exogeneity of $\sigma$ and the completeness of the pass-through it implies, the operating profit increases monotonically with firm productivity at a constant slope $(\sigma-1)$. As a result, the upward sloping profit line crosses the horizontal fixed cost line only once. This intersection yields the cutoff productivity level $\varphi^*$ at which the firm makes zero profit $(\pi(\varphi^*) = \pi^o(\varphi^*) - f = 0)$

[3]For expositional simplicity, the nature of the variable cost of compliance $c_{ij} \geq 1$ is assumed to be multiplicative.

adapts the fraction of its output that is destined to be exported to a foreign country to its specific requirements (which include conformity assessment, packaging, and labeling), and then exports with no risk of rejection at the border.

*Hierarchy of standards-* For the sake of generality, I consider two possible cases. The first where standards are "purely vertical": compliance with foreign standards is compulsory only if they are more stringent than local standards. Put differently, exporting a variety produced according to stringent standards to a destination where local standards are relatively less stringent does not require any additional cost of compliance as there is no risk of rejection at the border. In the second case, standards are assumed to be "verti-zontal": exporting always involves a variable cost of compliance with foreign standards regardless of whether they are more or less stringent that local standards. In other words, a variety that is initially produced according to very stringent standards should be adapted to foreign standards despite their relatively lower stringency, otherwise it can be rejected at the border.

Let $s_i > 0$ be a measure of stringency of standards in country $i$, the two possible hierarchies of standards can be summarized as follows:

$$Case(1): Vertical: c_{ij} \begin{cases} > 1 & \text{if } s_j > s_i \\ 1 & \text{otherwise} \end{cases}; \qquad Case(2): Vertizontal: c_{ij} > 1 \; \forall s_j, s_i$$

## 1.3   Characterization of the equilibrium

To start with, individual demand captured by a $\varphi$-productivity firm on its domestic market $j$ can be obtained using the Roy identity as follows:

$$x_{jj}(\varphi) = \beta E_j \frac{p_{jj}(\varphi)^{-\sigma}}{P_j^{1-\sigma}}; \; \beta = \frac{\sigma - 1}{\sigma - 1 + (\alpha/1 - \alpha)} < 1; \tag{1.1}$$

where $p_{jj}(\varphi) = \varphi^{-1} \frac{\sigma}{\sigma-1}$ is the profit maximizing price that firm $(\varphi)$ charges to domestic consumers, and $P_j$ is the price index in country $j$ given by

$$P_j^{1-\sigma} = M_j^e \int_{\varphi_{jj}^*}^{+\infty} p_{jj}(\varphi)^{1-\sigma} dG(\varphi) + M_i^e \int_{\varphi_{ij}^*}^{+\infty} p_{ij}(\varphi)^{1-\sigma} dG(\varphi) + M_k^e \int_{\varphi_{kj}^*}^{+\infty} p_{kj}(\varphi)^{1-\sigma} dG(\varphi), \tag{1.2}$$

where $M^e$ is the endogenous mass of entrants in each country, and $p_{ij}(\varphi) = c_{ij} \, p_{ii}(\varphi) = c_{ij} \, \varphi^{-1} \frac{\sigma}{\sigma-1}$ is the optimal price set by a $\varphi$-productivity exporter serving destination $j$ from origin $i$.[4] Firm-level domestic revenues are given by $r_{jj}(\varphi) = p_{jj}(\varphi) \, x_{jj}(\varphi) \, L_j$, and using the Lerner index[5], domestic operating profits can be written as:

$$\pi_{jj}^o(\varphi) = \frac{r_{jj}(\varphi)}{\sigma} \; ; \; r_{jj}(\varphi) = \beta \, (E_j L_j) \, (\frac{p_{jj}(\varphi)}{P_j})^{1-\sigma} \tag{1.3}$$

Following Melitz (2003), I use the zero cutoff profit condition which states that the least productive successful entrant on a given market (say, $j$) makes zero profits: $\pi_{jj}(\varphi_{jj}^*) = \pi_{jj}^o(\varphi_{jj}^*) - f_{jj} = 0$ and I obtain the following partial equilibrium expression of the domestic cutoff in country $j$:

$$(\varphi_{jj}^*)^{\sigma-1} = \kappa_1 \, \beta^{-1} \, \sigma \, f_{jj} \, Y_j^{-1} \, P_j^{1-\sigma}, \tag{1.4}$$

where $\kappa_1 = (\sigma/\sigma - 1)^{\sigma-1}$ is a constant and $Y_j = E_j L_j$ is country $j$'s market size.

Recall that the mass of entrants $M^e$ in the price index in equation (1.2) is endogenous. Using the free entry and the labor market clearing conditions, I solve for it as follows:

The free entry condition for firms in any country (say, $j$) equalizes average expected profits of entering the market to the sunk cost of entry, and is given by :

$$P(\varphi \geq \varphi_{jj}^*)[ \int_{\varphi_{jj}^*}^{+\infty} \pi_{jj}(\varphi) \frac{g(\varphi)}{P(\varphi \geq \varphi_{jj}^*)} d\varphi + P_{ji} \int_{\varphi_{ji}^*}^{+\infty} \pi_{ji}(\varphi) \frac{g(\varphi)}{P(\varphi \geq \varphi_{ji}^*)} d\varphi + P_{jk} \int_{\varphi_{jk}^*}^{+\infty} \pi_{jk}(\varphi) \frac{g(\varphi)}{P(\varphi \geq \varphi_{jk}^*)} d\varphi ] = F_e, \tag{1.5}$$

As in Melitz (2003), $P_{ji} = \frac{P(\varphi \geq \varphi_{ji}^*)}{P(\varphi \geq \varphi_{jj}^*)}$ and $P_{jk} = \frac{P(\varphi \geq \varphi_{jk}^*)}{P(\varphi \geq \varphi_{jk}^*)}$ stand respectively for the probability of exporting to destinations $i$ and $k$ from country $j$. Using the Lerner index and rearranging, the above free entry condition can be rewritten as:

$$\int_{\varphi_{jj}^*}^{+\infty} r_{jj}(\varphi) g(\varphi) d\varphi + \int_{\varphi_{ji}^*}^{+\infty} r_{ji}(\varphi) g(\varphi) d\varphi + \int_{\varphi_{jk}^*}^{+\infty} r_{jk}(\varphi) g(\varphi) d\varphi = \sigma \Lambda(.), \tag{1.6}$$

where $\Lambda(.) = [F_e + P(\varphi \geq \varphi_{jj}^*) f_{jj} + P(\varphi \geq \varphi_{ji}^*) f_{ji} + P(\varphi \geq \varphi_{jk}^*) f_{jk}]$.

---

[4]Similarly, $p_{kj}(\varphi) = c_{kj} \, p_{kk}(\varphi) = c_{kj} \, \varphi^{-1} \frac{\sigma}{\sigma-1}$

[5]$(\frac{p_{jj}(\varphi) - \varphi^{-1}}{p_{jj}(\varphi)} = \sigma^{-1})$

Next, let us look at the labor market clearing condition that equalizes total labor demand to total labor supply in country $j$. While all entrants (both successful and unsuccessful) incur the sunk entry cost $F_e$, only the successful amongst them use labor to start producing. Specifically, labor demand by a $\varphi$-productivity successful entrant $ld(\varphi)$ depends on its export status:

$$
ld(\varphi) = \begin{cases} [(q_{jj}(\varphi) * \varphi^{-1}) + f_{jj}], & \text{if } \varphi \geq \varphi_{jj}^* \\ [(q_{ji}(\varphi) * c_{ji}\varphi^{-1}) + f_{ji}] & \text{if } \varphi \geq \varphi_{ji}^* \\ [(q_{jk}(\varphi) * c_{jk}\varphi^{-1}) + f_{jk}] & \text{if } \varphi \geq \varphi_{jk}^* \end{cases} \tag{1.7}
$$

where $q_{jj}(\varphi) = x_{jj}(\varphi)L_j$ is the market demand captured by $\varphi$-productivity firm on the domestic market.[6] Using the optimal pricing rule, the labor demand per firm can be rewritten as follows:

$$
ld(\varphi) = \begin{cases} [(\frac{\sigma-1}{\sigma})r_{jj}(\varphi) + f_{jj}] & \text{if } \varphi \geq \varphi_{jj}^* \\ [(\frac{\sigma-1}{\sigma})r_{ji}(\varphi) + f_{ji}] & \text{if } \varphi \geq \varphi_{ji}^* \\ [(\frac{\sigma-1}{\sigma})r_{jk}(\varphi) + f_{jk}] & \text{if } \varphi \geq \varphi_{jk}^* \end{cases} \tag{1.8}
$$

Using firm labor demand from the above equation, the labor market clearing condition can be written as :

$$
(1-\alpha)E_jL_j = M_j^e[\Lambda(.) + (\frac{\sigma-1}{\sigma})(\int_{\varphi_{jj}^*}^{+\infty} r_{jj}(\varphi)g(\varphi)d\varphi + \int_{\varphi_{ji}^*}^{+\infty} r_{ji}(\varphi)g(\varphi)d\varphi + \int_{\varphi_{jk}^*}^{+\infty} r_{jk}(\varphi)g(\varphi)d\varphi)], \tag{1.9}
$$

By plugging the expected average revenues of successful entrants in $j$ from equation (1.6) into the above equation and rearranging, I obtain the mass of entrants in country $j$ :

$$
M_j^e = \frac{(1-\alpha)E_jL_j}{\sigma\Lambda(.)} \tag{1.10}
$$

Notice that the mass of entrants obtained above is *still endogenous* since $\Lambda(.)$[7] depends on three endogenous variables which are the cutoff productivity levels to serve the domestic market $j$ and the two export markets $i,k$ : $\varphi_{jj}^*$, $\varphi_{ji}^*$, and $\varphi_{jk}^*$. Following Feenstra (2010), in order to derive an equivalent expression of the mass of entrants that is *purely exogenous*, I start with specifying two

---

[6]Likewise, $q_{ji}(\varphi) = x_{ji}(\varphi)L_i$ and $q_{jk}(\varphi) = x_{jk}(\varphi)L_k$ stand respectively for the market demand a $\varphi$-productivity exporter reaps on destinations $i$, and $k$.

[7]Recall that $\Lambda = [F_e + P(\varphi \geq \varphi_{jj}^*)f_{jj} + P(\varphi \geq \varphi_{ji}^*)f_{ji} + P(\varphi \geq \varphi_{jk}^*)f_{jk}]$

useful assumptions.

First, let us assume that in all countries, firm productivity $\varphi$ is Pareto distributed over $(1, +\infty)$ with shape parameter $\theta$: $G(\varphi_0 < \varphi) = 1 - \varphi^{-\theta}$. The second is a direct implication of the free entry condition: since there are zero net profits at equilibrium, the total revenues reaped by monopolistically competitive firms must equate total payments to the labor force involved in the production of the differentiated good. With the aid of these two amendments, I obtain a *purely exogenous* equivalent of the mass of entrants previously derived in equation (1.10):[8]

$$M_j^e = \Psi Y_j \tag{1.11}$$

where $\Psi = \frac{(1-\alpha)(\sigma-1)}{\sigma\theta F_e}$. In line with Feenstra (2010) and as assumed by Chaney (2008), the general equilibrium[9] mass of entrants $M_j^e$ is proportional to country $j$'s market size $Y_j$.[10]

Now I can start solving for the general equilibrium price index in country $j$ and I proceed in three steps. First, as demand is isoelastic and firm productivity is drawn from a Pareto distribution that is unbounded above, I can easily solve for the integrals embodying the expected average prices set by domestic firms and exporters serving market $j$ in equation (1.2). Second, based on the partial equilibrium expression of the domestic cutoff $\varphi_{jj}^*$, I show that the presence of non-tariff barriers marginally tightens selection into exporting[11] by expressing the relative export cutoffs[12] as follows:

$$\begin{cases} \frac{\varphi_{ij}^*}{\varphi_{jj}^*} = c_{ij} \left(\frac{f_{ij}}{f_{jj}}\right)^{\frac{1}{\sigma-1}} > 1 \\ \frac{\varphi_{kj}^*}{\varphi_{jj}^*} = c_{kj} \left(\frac{f_{kj}}{f_{jj}}\right)^{\frac{1}{\sigma-1}} > 1 \end{cases} \tag{1.12}$$

Finally, upon solving for the integrals and using the above expressions of the relative export cutoffs along with the equilibrium mass of entrants from equation (1.11), I solve for the general equilibrium price index in country $j$:

$$P_j^{1-\sigma} = \kappa_2 \, \varphi_{jj}^{*\,\sigma-1-\theta} \left[ M_j^e + M_i^e \, c_{ij}^{-\theta} \left(\frac{f_{ij}}{f_{jj}}\right)^{1-\frac{\theta}{\sigma-1}} + M_k^e \, c_{kj}^{-\theta} \left(\frac{f_{kj}}{f_{jj}}\right)^{1-\frac{\theta}{\sigma-1}} \right] \tag{1.13}$$

---

[8]See Appendix A.1 for a detailed proof.

[9]Hereafter, **bold symbols** refer to the **general equilibrium expression** of the endogenous variable at question.

[10]Recall that $Y_j = E_j L_j$

[11]As mentioned in the first section, selection into exporting is initially dictated by relatively higher fixed costs of exporting: $f_{ij} > f_{ii}$

[12]As compared with the domestic cutoff in country $j$.

where $\kappa_2$ is a constant[13] and $M_i^e = \Psi Y_i$, $M_k^e = \Psi Y_k$ stand respectively for the equilibrium mass of entrants in countries $i, k$. Now by plugging the general equilibrium price index $P_j^{1-\sigma}$ in the partial equilibrium expression of the domestic cutoff in equation (1.4) and rearranging, I solve for the domestic cutoff in country $j$ in general equilibrium:

$$\varphi_{jj}^* = \kappa_3 \ f_{jj}^{\frac{1}{\theta}} \ Y_j^{-\frac{1}{\theta}} \ [M_j^e + M_i^e \ c_{ij}^{-\theta} \ (\frac{f_{ij}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}} \ + \ M_k^e \ c_{kj}^{-\theta} \ (\frac{f_{kj}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}}]^{\frac{1}{\theta}} \tag{1.14}$$

where $\kappa_3$ is a constant.[14]

It is worth mentioning that, as in Melitz (2003), the welfare effect of standards harmonization can be fully captured by the behavior of the productivity cutoff for domestic sellers $\varphi_{jj}^*$.
For instance, since firms make zero net profits at equilibrium, the real wage, $W_j = P_j^{-1}$, can be considered as a sufficient measure of welfare per capita in country $j$.[15] Then, by simply rearranging equation (1.4), it is readily verified that consumer welfare $W_j$ increases proportionally with the domestic cutoff $\varphi_{jj}^*$:[16]

$$W_j = P_j^{-1} = \kappa_4 \ (\frac{Y_j}{\sigma f_{jj}})^{\frac{1}{\sigma-1}} \ \varphi_{jj}^* \tag{1.15}$$

Finally, it is noteworthy to stress that given the purely exogenous nature of the equilibrium mass of entrants[17] $M^e$, standards liberalization would never imply a shift in the pattern of entry in the long run. As a result, the home market effect is ruled out despite the presence of an outside sector. The welfare analysis can be then simply carried on using equations (1.14) and (1.15).

## 1.4   Welfare implications of Standards Harmonization

As well documented in the literature, countries adopt different standards. Specifically, while rich countries adopt stringent national or regional standards, developing countries align on international standards which are relatively less stringent (Chen and Mattoo, 2008). As mentioned in the first section, this chapter covers two possible hierarchies of standards: *(i) a purely vertical hierarchy* where the additional cost of compliance with foreign standards is required only if they

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[13]$\kappa_2 = (\frac{\sigma}{\sigma-1})^{1-\sigma} \frac{\theta}{[\theta-(\sigma-1)]}$

[14]$\kappa_3 = (\frac{\sigma\theta}{\beta[\theta-(\sigma-1)]})^{\frac{1}{\theta}}$

[15]Recall that the nominal wage is exogenously pinned down by the outside sector and fixed to unity.

[16]$\kappa_4 = (\frac{\beta}{\kappa_1})^{1/(\sigma-1)}$

[17]See equations (1.11) and (1.13).

are more stringent than local ones; *(ii) a verti-zontal hierarchy* where the additional cost of compliance is always compulsory to avoid rejection regardless of whether foreign standards are more, or less stringent than those adopted locally.

In this section, I separately examine welfare implications of standards liberalization under each type of cost hierarchy. Then, I specify the conditions under which standards liberalization is welfare improving. Let us now consider a country pair *(j, k)* and assume that local standards in *k* are more stringent than in *j*: $s_k > s_j$. Then, I study the welfare implication of an alignment of country *j* on the local standards adopted in country *k*.

## Case 1: Purely Vertical standards

**Proposition 1.** *If the hierarchy of standards is purely vertical, the country that aligns on its partner's local standards experiences a welfare loss.*

**Proof.** See Appendix A.2

**Intuition:** Under this scenario of standards liberalization, local standards in country *j* become more stringent.[18] This immediately implies an increase in the cost of compliance for exporters in country *i* ($c_{ij}$), which in turn makes exporting from *i* to *j* more selective. The decrease in the mass of exporters [19] it entails makes competition in country *j* more relaxed, which leads to a decrease in the domestic cutoff $\varphi_{jj}^*$, and hence to a welfare loss.

## Case 2: Verti-zontal standards

**Proposition 2.** *Under verti-zontal standards, the country that aligns on its partner's local standards enjoys a welfare gain if and only if its partner is larger than the excluded country.*

**Proof.** See Appendix A.3

**Intuition:** The alignment of country *j* on local standards in country *k* implies a simultaneous decrease / increase in the cost of compliance for exporters serving market *j* from country *k* / country *i*. This translates into an increase in the number of varieties imported from country *k* and a decrease in the number of those imported from country *i*. As the equilibrium mass of entrants is proportional to market size, a relatively larger market size of the partner $k$[20] ensures

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[18]Since it has aligned on the standards of country *k* which are initially more stringent

[19]Notice that the mass of firms serving market *j* from country *k* remains unchanged. For instance, they never incur a cost of compliance ($c_{kj} = 1$) since country *k*'s standards are more stringent.

[20]As compared with excluded country *i*

then that the former effect dominates the latter. This net increase in the mass of firms competing on market $j$ reflects tougher competitive conditions and leads to an increase in the domestic cut-off $\varphi_{jj}^*$, and hence to a welfare gain.

## 1.5  Welfare Analysis: Towards a Parsimonious Extension

In this section, I build on the simple welfare analysis provided in Section.4 and I propose a parsimonious extension that is more realistic and more granular as compared to the previous one. The key idea I explore here is how a more realistic modeling of consumer behavior and of the cost of compliance with standards opens the door for a wide range of predictions for welfare gains from standards liberalization. Another major benefit of this new approach is that it paves the way for a more granular welfare analysis. Indeed, it allows to study two different scenarios of standards liberalization. The first consists in an alignment of a country on initially more stringent standards imposed by its partner, and can be called '"Standards Alignment". The second corresponds to "Standards Harmonization", whereby two countries whose standards, initially different in terms of stringency, converge to an intermediate level.

I proceed in three steps. First, I start with imposing three additional and plausible assumptions. Second, I show how these simple amendments induce only slight changes in the general equilibrium expression of the domestic cutoff, which guarantees then high tractability despite added complexity. Finally, I derive novel welfare predictions under each of the above mentioned scenarios.

### 1.5.1  More Realistic Assumptions on the Demand and the Supply side

**Assumption A1.** *In all countries, consumers perceive the stringency of standards as a signal of higher quality and have a preference for goods produced under more stringent standards. Specifically, Let us assume that such a preference for higher standards is captured by the exogenous parameter $\gamma \geq 0$ in the utility function described below:*

$$V = \left( \frac{E}{p_h} \right)^{\alpha} \left( \left[ \int_{\omega \in \Omega} E^{\sigma-1} \left( \frac{p_\omega}{s_\omega^\gamma} \right)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \right)^{1-\alpha}$$

**Assumption A2.** *For all domestic firms established in any given country, having the right to sell their varieties on the domestic market is conditional on compliance with local standards. This latter involves only a variable cost of compliance, denoted by "vc", that is strictly increasing in the stringency of local standards and given by: $vc_d = s_\omega^\delta$, with $\delta \geq 0$.*

**Assumption A3.** *For all firms established in any country (say, i), exporting to a foreign market (say, j) involves, not only a fixed cost ($f_{ij}$), but also, a variable cost of compliance with foreign standards. This latter is denoted by $vc_{i,j}$, and given by: $vc_{i,j} = (\frac{s_j}{s_i})^{\varepsilon_{i,j}}$,*

*where as before, $s_i > 0$ and $s_j > 0$ are positive measures of stringency of standards, respectively, in country i and country j, and $\varepsilon_{i,j}$ captures the nature of the cost hierarchy of standards, as follows:*

$$
\varepsilon_{i,j} \begin{cases} > 0 & \text{if } s_j > s_i \text{ for any cost hierarchy of standards} \\ = 0 & \text{if } s_j < s_i \text{ and the hierarchy is "purely vertical"} \\ < 0 & \text{if } s_j < s_i \text{ and the hierarchy is "verti-zontal"} \end{cases}
$$

## 1.5.2 Solving for the General Equilibrium Domestic Cutoff

As was the case for the preceding analysis, the domestic cutoff is a sufficient statistics for welfare analysis. In order to solve for the general equilibrium expression of this key variable, I proceed in two steps. To start with, I show how these additional assumptions induce changes in the pricing rules on the domestic and the export market as follows:

$$
\begin{cases} \forall \varphi \geq \varphi_{ii}^*, \ p_{ii}(\varphi) = \frac{\sigma}{\sigma-1} \varphi^{-1} s_i^{\delta} \\ \forall \varphi \geq \varphi_{ij}^*, \ p_{ij}(\varphi) = \frac{\sigma}{\sigma-1} \varphi^{-1} s_i^{\delta} (\frac{s_j}{s_i})^{\varepsilon_{i,j}} \end{cases}
$$

By taking into account consumers' preference for higher standards, as indicated in Assumption A1, and the above changes in the pricing rule, the initial partial equilibrium expression of the domestic cutoff, in country j, in equation (1.4) can be rewritten as:

$$
\varphi_{jj}^* = \kappa_0 \, s_j^{(\delta-\gamma)} \, f_{jj}^{\frac{1}{\sigma-1}} \, Y_j^{-\frac{1}{\sigma-1}} \, P_j^{-1} \tag{1.16}
$$

where $\kappa_0 = (\frac{\sigma \kappa_1}{\beta})^{\frac{1}{\sigma-1}}$ is a constant, and $f_{jj}$ is the fixed cost of accessing market $j$ for local firms. $Y_j$ and $P_j$ respectively denote aggregate expenditure and the partial equilibrium price index in country $j$. Here, the degree of stringency of standards in country $j$, $s_j$, arise as a new determinant of firm selection on this market in partial equilibrium. Now by taking Assumptions A1, A2, and A3 in due account, solving for the general equilibrium price index, and plugging its expression in equation (1.16) and rearranging yields the following solution for the domestic cutoff in general equilibrium:

$$\varphi_{jj}^* = \kappa_6 \ s_j^{(\delta-\gamma)} \ f_{jj}^{\frac{1}{\theta}} \ Y_j^{-\frac{1}{\theta}} \ [s_j^{\theta(\gamma-\delta)} \ Y_j + s_i^{\theta(\gamma-\delta)} \ Y_i \ (\frac{s_j}{s_i})^{-\theta\varepsilon_{i,j}} \ (\frac{f_{ij}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}} \ + \ s_k^{\theta(\gamma-\delta)} \ Y_k \ (\frac{s_j}{s_k})^{-\theta\varepsilon_{k,j}} \ (\frac{f_{kj}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}}]^{\frac{1}{\theta}}$$

$$(1.17)$$

where $\kappa_6$ is a constant.[21]

### 1.5.3 Welfare Implications of "Standards Alignment" vs "Standards Harmonization"

As in the preceding analysis, let us assume first that the degree of stringency of standards varies across countries. While local standards are the most stringent in country $k$, they are the least stringent in country $i$. Within these bounds, local standards in country $j$ have an intermediate degree of stringency: $s_k > s_j > s_i$. Let us also recall that these three countries are assumed to be asymmetric in size, with country $k$ is the largest and country $i$ the smallest: $Y_k > Y_j > Y_i$.

Now based on the above order of stringency of standards across countries and by invoking Assumption A3, the theoretically possibles signs of $\varepsilon_{i,j}$ and $\varepsilon_{k,j}$ can be summarized as follows:

| Cost Hierarchy of Standards | Purely-vertical | Verti-zontal |
|:---:|:---:|:---:|
| $\varepsilon_{i,j}$ | $> 0$ | $> 0$ |
| $\varepsilon_{k,j}$ | $0$ | $< 0$ |

Let us also briefly recall that $\delta$ is a new supply-side parameter, that is identical across countries and firms, and corresponds to the elasticity of marginal cost with respect to the degree of stringency of local standards in any country. Similarly, as stated in Assumption A1, $\gamma$ is a newly introduced demand shifter, that is identical across countries and firms, and captures consumers' preference for higher standards.

Now by using the above definitions of these additional parameters and inspecting the new general equilibrium expression of the domestic cutoff in equation (17), I obtain two novel welfare predictions clearly stated in the following propositions:

**Proposition 3.** *Under purely vertical standards, an alignment of country $j$ on the standards of country $k$ is welfare improving if and only if the elasticity of marginal costs with respect to local standards exceeds the degree of preference of consumers for higher standards: $\delta > \gamma$.*

---

[21]$\kappa_6 = ( \frac{(1-\alpha)(\sigma-1)}{\beta[\theta-(\sigma-1)]F_e} )^{\frac{1}{\theta}}$

**Proposition 4.** *Under verti-zontal standards, standards liberalization occurring through an alignment of country j on the standards of country k, or a harmonization of standards is welfare improving if and only if the elasticity of marginal costs with respect to local standards exceeds the degree of preference of consumers for higher standards: $\delta > \gamma$.*

# Conclusion

In this chapter, I combine cross-country differences in stringency of local standards with country-level asymmetry in market size in a Melitz-like framework. In the absence of cost hierarchy to standards, I consider two possibles cases where compliance can be costly or not depending on the difference between local and foreign standards. Under standard assumptions on the supply and the demand side, I show that standards liberalization is welfare improving only when the cost hierarchy is "verti-zontal" and the trading partner is larger than the excluded country. Then, upon implementing more realistic assumptions on consumer behavior and the cost of compliance with standards, I show that standards liberalization, occurring through alignment or harmonization, is welfare improving only when consumers exhibit a weak preference for better standards.

# Appendix A

## A.1 Deriving an exogenous equivalent of the equilibrium mass of entrants:

In order to isolate the endogenous component of the equilibrium expression of $M_j^e$ obtained in equation (1.10), this latter can be rewritten as follows:

$$M_j^e = \frac{(1-\alpha)E_j L_j}{\sigma[F_e + P(\varphi \geq \varphi_{jj}^*)f_{jj} + P(\varphi \geq \varphi_{ji}^*)f_{ji} + P(\varphi \geq \varphi_{jk}^*)f_{jk}]} \tag{1.18}$$

$$\Longleftrightarrow (1-\alpha)E_j L_j = \sigma[M_j^e F_e + Y_j]; \ Y_j = M_j f_{jj} + M_{ji} f_{ji} + M_{jk} f_{jk} \tag{1.19}$$

where $M_j = M_j^e P(\varphi \geq \varphi_{jj}^*)$ is the equilibrium mass of domestic firms in country $j$, and $M_{ji} = M_j^e P(\varphi \geq \varphi_{ji}^*)$, $M_{jk} = M_j^e P(\varphi \geq \varphi_{jk}^*)$ stand for the equilibrium mass of firms exporting to destinations $i$, and $k$ respectively.

Moreover, a straightforward implication of the free entry condition is that in any country (say, $j$), total equilibrium revenues of successful entrants equalize total payments to the labor force involved in the production of the differentiated good:

$$(1-\alpha)E_j L_j = M_j \int_{\varphi_{jj}^*}^{+\infty} r_{jj}(\varphi)\frac{g(\varphi)}{P(\varphi \geq \varphi_{jj}^*)}d\varphi + M_{ji} \int_{\varphi_{ji}^*}^{+\infty} r_{ji}(\varphi)\frac{g(\varphi)}{P(\varphi \geq \varphi_{ji}^*)}d\varphi + M_{jk} \int_{\varphi_{jk}^*}^{+\infty} r_{jk}(\varphi)\frac{g(\varphi)}{P(\varphi \geq \varphi_{jk}^*)}d\varphi \tag{1.20}$$

Recall that the zero cutoff profit condition on the domestic and exports markets implies that: $r_{jj}(\varphi_{jj}^*) = \sigma f_{jj}$, $r_{ji}(\varphi_{ji}^*) = \sigma f_{ji}$, and $r_{jk}(\varphi_{jk}^*) = \sigma f_{jk}$, by multiplying and dividing each term of the equation above with the respective cutoff revenue, it can be rewritten as follows:

$$\acute{R}_j = M_j \sigma f_{jj} \int_{\varphi_{jj}^*}^{+\infty} \frac{r_{jj}(\varphi)}{r_{jj}(\varphi_{jj}^*)}\mu_{jj}(\varphi)d\varphi + M_{ji}\sigma f_{ji} \int_{\varphi_{ji}^*}^{+\infty} \frac{r_{ji}(\varphi)}{r_{ji}(\varphi_{ji}^*)}\mu_{ji}(\varphi)d\varphi + M_{jk}\sigma f_{jk} \int_{\varphi_{jk}^*}^{+\infty} \frac{r_{jk}(\varphi)}{r_{jk}(\varphi_{jk}^*)}\mu_{jk}(\varphi)d\varphi \tag{1.21}$$

where $\acute{R}_j = (1-\alpha)E_jL_j$ and $\mu(\varphi)$ is equilibrium distribution.[22] As demand is CES and firm productivity is drawn from an unbounded Pareto distribution (with $\theta$ as a shape parameter), it readily verified that:

$$\int_{\varphi_{jj}^*}^{+\infty} \frac{r_{jj}(\varphi)}{r_{jj}(\varphi_{jj}^*)}\mu_{jj}(\varphi)d\varphi = \int_{\varphi_{ji}^*}^{+\infty} \frac{r_{ji}(\varphi)}{r_{ji}(\varphi_{ji}^*)}\mu_{ji}(\varphi)d\varphi = \int_{\varphi_{jk}^*}^{+\infty} \frac{r_{jk}(\varphi)}{r_{jk}(\varphi_{jk}^*)}\mu_{jk}(\varphi)d\varphi = \frac{\theta}{[\theta-(\sigma-1)]} \tag{1.22}$$

Now by plugging the above solution of the integrals in equation (1.21) and rearranging, I obtain the following exogenous equivalent of the endogenous component Y:

$$Y_j = M_jf_{jj} + M_{ji}f_{ji} + M_{jk}f_{jk} = (1-\alpha)E_jL_j\frac{[\theta-(\sigma-1)]}{\sigma\theta} \tag{1.23}$$

Finally, by plugging the above expression of $Y_j$ in equation (1.19) and rearranging, I obtain a purely exogenous equilibrium expression of the mass of entrants in country $j$:

$$M_j^e = \underbrace{\frac{(1-\alpha)(\sigma-1)}{\sigma\theta F_e}}_{\Psi} E_jL_j = \Psi Y_j \tag{1.24}$$

# A.2 Welfare effect of standards harmonization under a purely vertical hierarchy of standards:

An alignment of country $j$ on its partner $k$'s more stringent standards implies higher variable cost of compliance for firms serving market $j$ from the excluded country $i$. As a result, the mass of firms exporting from $i$ to $j$ decreases, and country $j$ experiences thus a welfare loss visible though a decline in its domestic cutoff. Using the general equilibrium expression of the domestic cutoff from equation (1.14), this can be shown as follows:

$$\varphi_{jj}^* = \kappa_3 \; f_{jj}^{\frac{1}{\theta}} \; Y_j^{-\frac{1}{\theta}} \; [M_j^e + M_i^e \, c_{ij}^{-\theta} \, (\frac{f_{ij}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}} + M_k^e \, c_{kj}^{-\theta} \, (\frac{f_{kj}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}}]^{\frac{1}{\theta}}$$

$$\varepsilon_{c_{ij}}^{\varphi_{jj}^*} = \frac{d\ln(\varphi_{jj}^*)}{d\ln(c_{ij})} = -\frac{\Delta_{ij}}{\Delta_j} < 0 \; ; \quad \begin{cases} \Delta_{ij} = M_i^e \, c_{ij}^{-\theta} \, (\frac{f_{ij}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}} \\ \Delta_j = [M_j^e + M_i^e \, c_{ij}^{-\theta} \, (\frac{f_{ij}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}} + M_k^e \, c_{kj}^{-\theta} \, (\frac{f_{kj}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}}] \end{cases} \tag{1.25}$$

---

[22] $\mu_{jj}(\varphi) = \frac{g(\varphi)}{P(\varphi \geq \varphi_{jj}^*)}$, $\mu_{ji}(\varphi) = \frac{g(\varphi)}{P(\varphi \geq \varphi_{ji}^*)}$, and $\mu_{jk}(\varphi) = \frac{g(\varphi)}{P(\varphi \geq \varphi_{jk}^*)}$.

# A.3 Welfare effect of standards harmonization under a purely verti-zontal hierarchy of standards:

Under this case, an alignment of country $j$ on its partner $k$'s more stringent standards implies a simultaneous decrease in the variable cost of compliance for firms serving market $j$ from partner $k$, and increase in this additional cost for exporters based in the excluded country $i$. Hence, this decision can be welfare improving for country $j$ only if its partner $k$ is larger than the excluded country $i$ so as the increase in the mass of varieties imported from the former outweighs the decrease in the mass of varieties imported from the latter. This can be easily shown by assuming initially symmetric trade costs[23] and computing the difference between these two elasticities:

$$|\varepsilon_{c_{kj}}^{\varphi_{jj}^*}| - |\varepsilon_{c_{ij}}^{\varphi_{jj}^*}| = \frac{\mathbb{T}\Psi}{\tilde{\Delta}_j}(Y_k - Y_i) > 0 \text{ iff } Y_k > Y_i, \tag{1.26}$$

where $\mathbb{T} = c_{xj}^{-\theta}(\frac{f_{xj}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}}$, $\Psi = \frac{(1-\alpha)(\sigma-1)}{\sigma\theta F_e}$, and $\tilde{\Delta}_j = [M_j^e + c_{xj}^{-\theta}(\frac{f_{xj}}{f_{jj}})^{1-\frac{\theta}{\sigma-1}}(M_i^e + M_k^e)]$ is the equivalent of $\Delta_j$ (in equation (1.25) ) with symmetric trade costs.

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[23]That is before standards harmonization, $c_{ij} = c_{kj} = c_{xj}$, and $f_{ij} = f_{kj} = f_{xj}$

# Chapter 2

# Structural Gravity under Size Asymmetry and Non-Homotheticity

## 2.1 Introduction

The Constant Elasticity of Substitution (CES) model of monopolistic competition has long been a solid foundation for seminal contributions in international trade theory (Krugman, 1980; Melitz, 2003; Anderson and Van Wincoop, 2003; Chaney, 2008). Yet it is fair to say that this model suffers from two major drawbacks. First, it imposes a constant demand elasticity, which mainly precludes pro-competitive reduction in domestic markups to occur upon trade liberalization. Second, such rigidity in preferences, not only, constrains per-capita income and population size to have an identical effect on trade margins, but also, restricts the sensitivity of trade margins to trade costs to be constant in a gravity context (Chaney, 2008).

In recent trade models incorporating asymmetry both in income and size at the country-level, firm heterogeneity in productivity levels and non-CES preferences (Arkolakis et al., 2018; Fally, 2019), the standard homothetic CES assumption has been mainly relaxed to allow for variable markups at the firm-level, so as to restore a theoretical role for the "pro-competitive effect of trade". In spite of deriving a gravity equation under flexible preferences and country-level asymmetry, these recent papers remain silent on the potential implications of these more realistic assumptions for income and size effects on trade margins, and on whether it may give rise to a variable elasticity of trade margins to trade costs across countries.

In contrast, this is what the current chapter mainly focuses on. Does the fact that per-capita income affects significantly price elasticities, as documented in recent micro-level studies, see e.g. (Simonovska, 2015; Faber and Fally, 2017; Handbury, 2019), imply stronger income effect on trade margins, and on their sensitivity to trade costs? Does the structure of welfare gains from unilateral trade liberalization depend on whether the trading partner is relatively large or small?

These are the two main questions that I address in the current chapter. I do so in the context of a new class of gravity models featuring monopolistic competition, firm-level heterogeneity, country-level asymmetry, unbounded Pareto distribution, indirectly-separable preferences, while taking the presence of variable and fixed trade barriers in due account.

Importantly, the family of preferences considered in this chapter offers a subtle nesting of the CES case as a homothetic benchmark, and a prominent non-homthetic alternative exhibiting an income-decreasing price elasticity. This latter allows then for a more realistic modeling of consumer behavior, whereby richer consumers are less price sensitive. Thus, by combining cross-country differences in income levels with such added flexibility in preferences, I can properly examine the following questions: does the dampening effect of per-capita income on the price elasticity induce a stronger income effect on trade margins in general equilibrium? does it imply that higher income level dampens the sensitivity of trade margins to trade costs?

The benefit of focusing on unbounded Pareto is twofold. First, it makes it possible to derive a gravity equation and gives rise to constant trade elasticity. This restricts then the theoretical focus on trade margins and allows for a solid empirical examination of the above questions. Second, this supply side restriction also gives rise to a constant uni-variate distribution of markups. Hence, in the welfare analysis, the focus is squarely on gains from selection as in Melitz (2003), and variety gains as in Krugman (1980).

The contribution of this chapter is threefold. First, I show, both theoretically and empirically, that the intensive margin of trade increases only with per-capita income in general equilibrium, and that per-capita income dampens the sensitivity of trade margins to trade costs. Second, I highlight two new welfare channels: an additional selection effect occurring on the export market, and an increase in nominal wage in the liberalizing country. Third, the contribution of the current chapter to the gravity literature is a fully structural gravity equation that exhibits both inward and outward multilateral resistances, and additionally exhibits a variable elasticity of aggregate trade flows to fixed trade barriers under non-homothetic preferences. Finally, aiming at obtaining general results without losing in tractability, the current chapter proposes a new method that I call "the Exponent Elasticity Method" (EEM). This simple method delivers tractable solutions in general equilibrium despite added flexibility in preferences.

The findings of the current chapter are related to a large number of theoretical and empirical papers in the international trade literature. Many authors examined income and size effects on bilateral trade flows under non-homothetic preferences. By introducing non-homotheticity in a Ricardian framework, Fieler (2011) finds that bilateral trade increases significantly with per-

capita income, whereas it remains unaffected by population size. Markusen (2013) derives an identical prediction in a Heckscher Ohlin framework. A more recent work by Bertoletti, Etro, and Simonovska (2018) have addressed this question under monopolistic competition with heterogeneous firms. They find that the extensive margin of trade (number of exporters) increases only with destination's per-capita income. Despite apparent similarity to these previous conclusions, here the key novelty is in the increased granularity of the analysis. In deed, I emphasize that the significant impact of per-capita on aggregate trade flows is mainly driven by its strong impact on the intensive margin of trade. I also show that per-capita income affects not only trade margins, but also determines the degree of their sensitivity to trade barriers, which is a novel result in this strand of the literature.

A large body of work has examined the welfare gains from trade under various classes of preferences; see e.g (Melitz, 2003; Melitz and Ottaviano, 2008; Melitz and Redding, 2015; Arkolakis et al., 2018; Feenstra, 2018; Fally, 2019). Modeling approaches and conclusions vary, but a common feature of the aforementioned papers is their overwhelming focus on gains from tougher selection on the domestic market, due to Melitz (2003). In contrast, the current chapter shows that this selection effect is not always operative. It occurs only if the trading partner is large enough compared to the World economy. Additionally, under this case, I highlight two new sources of gains from trade: a selection effect occurring on the export market, and increase in nominal wage in the liberalizing country.

Another related paper in the literature is by Melitz and Ottaviano (2008) who derive a structural gravity equation where the toughness of competition in the destination is jointly determined by its market size and a measure of market access and comparative advantage. Instead, in this chapter, the effects of per-capita income, population size and market access on trade margins are studied separately. In particular, the toughness of competition in the importing country and the exporter's ease of market access are solely captured by their respective multilateral resistance terms as in Anderson and Van Wincoop (2003). In this sense, the gravity equation I derive can be considered as an augmented version of this of Chaney (2008) in two respects. First, its structural aspect is reinforced as it exhibits both inward and outward multilateral resistances as in Anderson and Van Wincoop (2003). Second, it yields an income-decreasing elasticity of bilateral trade flows with respect to fixed trade barriers, when preferences are non-homothetic.

The rest of this chapter is organized as follows. In Section 2, I spell out the model and derive novel theoretical predictions both under non-homotheticity and market size asymmetry. Section 3 presents the empirical analysis and tests the validity of the novel results obtained under non-homotheticity. Section 4 concludes. Empirical results are provided in Appendix B. Appendix C provides the proofs for the main theoretical results, and explains the "EEM" method.

## 2.2  Theoretical Framework

### 2.2.1  Set up of the model

*Consumer preferences.–* I assume that consumer preferences are indirectly additive:

$$V = \int_{\omega \in \Omega} v(\frac{p_\omega}{w})\, d\omega, \text{ with } v'(\frac{p_\omega}{w}) < 0 \text{ and } v''(\frac{p_\omega}{w}) > 0 \qquad (2.1)$$

As stressed by Bertoletti and Etro (2016), a key property of this family of preferences is that the price elasticity of demand corresponds to the elasticity of the marginal sub-utility and is thus given by $\sigma(\frac{p}{w}) = -\frac{v''(\frac{p}{w})\frac{p}{w}}{v'(\frac{p}{w})} > 1$. This implies that preferences are always non-homothetic, except under the CES case where the price elasticity of demand ceases to vary with the price-income ratio.

In order to shed light on the sensitivity of the theoretical predictions of the model to the nature of preferences (non-homothetic vs CES) and to test their empirical validity, I cover two possible cases. A general and realistic non-homothetic case where $\sigma(\frac{p}{w})$ is increasing in the price-income ratio $(\frac{p}{w})$,[1] opposed to the rigid CES case where $\sigma$ is exogenous. Following Mrázová and Neary (2017), I use the elasticity of the second derivative of the sub-utility function $\varsigma(\frac{p}{w}) = -\frac{v'''(\frac{p}{w})\frac{p}{w}}{v''(\frac{p}{w})}$ as a unit-free measure of demand convexity and I a specify a subtle condition for both cases to be nested:[2]

$$\sigma'(\frac{p}{w}) = \begin{cases} > 0, & \text{if } \varsigma(\frac{p}{w}) < 1 + \sigma(\frac{p}{w}) \ (non-homothetic) \\ = 0, & \text{if } \varsigma(\frac{p}{w}) = 1 + \sigma(\frac{p}{w}) \ (homothetic : CES) \end{cases} \qquad (2.2)$$

*Asymmetric countries.–* Consider a World economy composed of N asymmetric countries that differ both in size and income levels. Let country $i$ be populated by $L_i$ identical agents, each supplying a unit of efficient labor. As each economy involves only one sector producing a differentiated good $k$, nominal wage $w_i$ is endogenous and corresponds to both per-capita income and

---

[1]Notice that only this alternative case is considered since the other theoretically possible alternative: ($\sigma$ increasing in income) does not seem to be plausible and requires additional conditions to guarantee weak convexity and avoid thus issues related to the existence of the equilibrium.

[2]Mrázová and Neary (2017) propose the elasticity of the slope of direct demand as a sufficient measure of demand convexity. Here, I simply apply this general definition to the case of indirectly-additive preferences.

individual expenditure on horizontally differentiated varieties of good $k$. I solve for the nominal wage in general equilibrium upon closing the model using the trade balance condition.[3]

*Identical Technology and Costly Trade.–* In all countries, firm productivity $\varphi$ is Pareto distributed over $[1, +\infty[$ with shape parameter $\theta$: $G(\varphi_0 < \varphi) = 1 - \varphi^{-\theta}$.[4] Any $\varphi$-productivity firm based in country $i$ and aiming to serve country $j$ must pay a fixed cost $w_i f_{ij}$ (where $f_{ij}$ is measured in efficiency labor units) and a variable trade cost that takes the form of an iceberg transport cost $\tau_{ij} > 1$. Domestic trade involves only an overhead production cost $f_{ii}$, $\tau_{ii} = 1$.

*Individual demand and optimal pricing rule.–* Using the Roy identity ($x = -\frac{\partial V}{\partial p} / \frac{\partial V}{\partial w}$), the individual demand a $\varphi$-productivity exporter from country $i$ captures on destination $j$ can be derived as follows:

$$x_{ij}(\varphi) = \frac{|v'(\frac{p_{ij}(\varphi)}{w_j})|}{|\eta_j|} \tag{2.3}$$

where $|\eta_j|$ is the price aggregator in country $j$ reflecting the toughness of competition on this market through the number of domestic and foreign firms competing on its market, as well as their average degree of price competitiveness, as shown below:

$$|\eta_j| = |\sum_{i=1}^{N} M_i^e \int_{\varphi_{ij}^*}^{+\infty} \frac{p_{ij}(\varphi)}{w_j} v'(\frac{p_{ij}(\varphi)}{w_j}) dG(\varphi)| \tag{2.4}$$

where $M_i^e$ is the endogenous mass of entrants in origin $i$ and $p_{ij}(\varphi)$ is the profit-maximizing export price charged by a $\varphi$-productivity exporter from origin $i$ to consumers in destination $j$:

$$p_{ij}(\varphi) = \begin{cases} \frac{w_i \tau_{ij}}{\varphi} m_{ij}(\varphi) & \text{preferences: non-homothetic} \\ \frac{w_i \tau_{ij}}{\varphi} m_{ij}(\varphi) & \text{preferences: homothetic CES} \end{cases}$$

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[3]I purposely abstract from including an outside sector pinning down wages so that general equilibrium effect on wages is not ruled out. Moreover, this ensures the absence of the Home market effect (HME) and thus simplifies the welfare analysis.

[4]Notice that under the general non-homothetic case, $\sigma(\frac{p}{w})$ is increasing in price and thus firm-specific since firms are heterogeneous. As the cutoff exporter (serving destination $j$ from origin $i$) is the least productive and charges the highest price, he faces relatively more elastic demand (than an average productivity exporter): $\sigma_{ij}^*(\frac{p_{ij}^*}{w_j}) > \tilde{\sigma}_{ij}(\frac{\tilde{p}_{ij}}{w_j})$. It is then sufficient to assume that $\theta - (\sigma_{ij}^* - 1) \in ]0, 1[ \; \forall i, j$ to ensure that productivity distribution of firms has a finite mean. However, under the CES, only this standard assumption: $\theta > (\sigma - 1)$ is needed as $\sigma$ is identical across firms.

where $m_{ij}(\varphi)$ is the markup set by a $\varphi$-productivity exporter from origin $i$ while serving market $j$, and is given by:

$$m_{ij}(\varphi) = \begin{cases} \dfrac{\sigma(\frac{p_{ij}}{w_j})}{\sigma(\frac{p_{ij}}{w_j})-1} & \text{preferences: non-homothetic} \\[2em] \dfrac{\sigma}{\sigma-1} & \text{preferences: homothetic CES} \end{cases}$$

*Incomplete pass-through and destination-specific pricing .–* The above expression of the pricing rule on the export market clearly indicates that the export price has four determinants: (i) nominal wage in the origin country $w_i$; (ii) the variable trade cost $\tau_{ij}$; (iii) nominal wage in the destination $w_j$; and (iv) the exporter's productivity level $\varphi$. Following the same order, let $\rho_1$, $\rho_2$, $\rho_3$, and $\rho_4$ respectively denote the elasticity of the export price with respect to each of its determinants:

$$\rho_1 = \frac{d\log p_{ij}(\varphi)}{d\log w_i}, \quad \rho_2 = \frac{d\log p_{ij}(\varphi)}{d\log \tau_{ij}}, \quad \rho_3 = \frac{d\log p_{ij}(\varphi)}{d\log w_j}, \quad \text{and } \rho_4 = \frac{d\log p_{ij}(\varphi)}{d\log \varphi}$$

Inspection of the export pricing rule clearly shows that nominal wage in the origin country $w_i$, the variable trade cost $\tau_{ij}$, and the productivity level of the exporting firm $\varphi$ enter the expression of the marginal cost in a multiplicative way. It follows then that $\rho_1 = \rho_2 = |\rho_4|$. Clearly, these three parameters capture the degree of completeness of the relative cost-price pass-through, and their value hinges on the nature of preferences:

$$\rho_1 = \rho_2 = |\rho_4| = \begin{cases} 1 + [\underbrace{\dfrac{d\log m_{ij}(\varphi)}{d\log \sigma_{ij}(\varphi)}}_{<0} \underbrace{\dfrac{d\log \sigma_{ij}(\varphi)}{d\log p_{ij}(\varphi)}}_{>0}] \; < 1 & \text{preferences: non-homothetic} \\[2em] 1 \text{ since } \sigma \perp p_{ij} & \text{preferences: homothetic CES} \end{cases}$$

Under the homothetic CES case, the demand elasticity is exogenous, this implies a constant markup, and thus a complete pass-through: $\rho_1 = \rho_2 = |\rho_4| = 1$. However, beyond the CES case, the demand elasticity increases with the price level, and thus decreases with firm productivity. This implies that more productive firms face lower demand and so, set higher markups and only partially pass-on their cost advantage to consumers. Hence, beyond the CES case, the relative pass-through is incomplete: $\rho_1 = \rho_2 = |\rho_4| < 1$.

Similarly, as the non-homothetic alternative allows the demand elasticity to decrease with per-capita income, it is then readily verified that the export price increases with the income level of the destination only under this case:

$$
\rho_3 = \begin{cases} \underbrace{[\dfrac{dlog\ m_{ij}(\varphi)}{dlog\ \sigma_{ij}(\varphi)}}_{<0}\ \underbrace{\dfrac{dlog\ \sigma_{ij}(\varphi)}{dlog\ w_j}}_{<0}] \ > 0 & \text{preferences: non-homothetic} \\ 0 \text{ since } \sigma \perp w_j & \text{preferences: homothetic CES} \end{cases}
$$

Three key properties of indirectly-additive preferences are worth emphasizing. First, it allows for a subtle nesting of the homothetic CES case and a non-homothetic alternative using a unique condition that is pinned down by the relationship between the elasticity and convexity of direct demand, the so-called "demand manifold" by Mrázová and Neary (2017). Second, under the non-homothetic case, the price elasticity of demand is allowed to increase with prices and decrease with per-capita income. Combined with heterogeneity at the firm-level and asymmetry at the country level, this added flexibility allows then for a more realistic modeling of consumer and firm behavior. That is, on any market, more productive firms set higher markups, as documented by De Loecker and Warzynski (2012), and at the world level, consumers in richest countries are the least price sensitive. Third, this class of preferences allows for added flexibility in both price and income effects while retaining the property that prices are summarized by a unique price aggregator, which is very convenient under monopolistic competition.

It is also worth noting that indirectly-separable preferences can be seen as an exception in this regard. For instance, all alternative classes of preferences have properties that are too restrictive in terms of income and price effects. Under directly-separable preferences, as in Arkolakis et al. (2018), the price elasticity varies across goods, yet the income effect remains very implicit. Melitz and Ottaviano (2008) work with quasi-linear preferences, which generates a price-increasing demand elasticity, but suppresses income effects. Comin, Lashkari, and Mestieri (2015) obtain flexible income effects using Non-homothetic CES preferences. Yet, this latter restricts price elasticities to be identical across goods.

Theoretically, it may yield more flexible price effects but at the price of more complexity, since this requires two price aggregators to fully characterize the demand system Fally (2018). The QMOR preferences used by Feenstra (2018), and the case of implicitly-additive preferences considered by Arkolakis et al. (2018) in a heterogeneous firms setting offer two examples of this complex case.

*Equilibrium conditions .–* Now let us recall that the mass of entrants $M_i^e$ in any origin $i$ is endogenous. Using the Free Entry (FE) and the Labor Market Clearing (LMC) conditions, I solve for it as follows :

The Free Entry condition states that in any country (say, $i$), average expected profits by entrant, conditional on successful entry, must equate the sunk cost of entry, and is given by :

$$P(\varphi \geq \varphi_{ii}^*) \left[ \int_{\varphi_{ii}^*}^{+\infty} \pi_{ii}(\varphi) \frac{g(\varphi)}{P(\varphi \geq \varphi_{ii}^*)} d\varphi + \sum_{j=1}^{(N-1)} P_{ij} \int_{\varphi_{ij}^*}^{+\infty} \pi_{ij}(\varphi) \frac{g(\varphi)}{P(\varphi \geq \varphi_{ij}^*)} d\varphi \right] = w_i F_e$$

(2.5)

where $P_{ij} = \frac{P(\varphi \geq \varphi_{ij}^*)}{P(\varphi \geq \varphi_{ii}^*)}$ is the probability of exporting from country $i$ to country $j$ as in Melitz, 2003. Domestic profits and revenues are, respectively, given by $\pi_{ii}(\varphi) = \frac{r_{ii}(\varphi)}{\sigma_{ii}(wi)} - w_i f_{ii}$, with $r_{ii}(\varphi) = p_{ii}(\varphi) x_{ii}(\varphi) L_i$, $\forall \varphi \geq \varphi_{ii}^*$. Likewise, export profits and revenues can be written as $\pi_{ij}(\varphi) = \frac{r_{ij}(\varphi)}{\sigma_{ij}(wj)} - w_i f_{ij}$, $r_{ij}(\varphi) = p_{ij}(\varphi) x_{ij}(\varphi) L_j$ $\forall \varphi \geq \varphi_{ij}^*$, using the pricing rule $p_{ij}(\varphi)$ and the individual demand $x_{ij}(\varphi)$ described in equation (2.3).[5]

Using the Lerner index and rearranging, the free entry condition boils down to :

$$\tilde{R}_i = \tilde{\sigma}_i^{\varpi}(\tilde{w}) \left[ w_i F_e + P(\varphi \geq \varphi_{ii}^*) w_i f_{ii} + \sum_{j=1}^{(N-1)} P(\varphi \geq \varphi_{ij}^*) w_i f_{ij} \right] \qquad (2.6)$$

where $\tilde{R}_i = \int_{\varphi_{ii}^*}^{+\infty} r_{ii}(\varphi) g(\varphi) d\varphi + \sum_{j=1}^{(N-1)} \int_{\varphi_{ij}^*}^{+\infty} r_{ij}(\varphi) g(\varphi) d\varphi$ stands for the expected average revenues of successful entrants in country $i$, and $\tilde{\sigma}_i^{\varpi}(\tilde{w}) = \sum_{j=1}^{N} (\frac{w_j L_j}{Y_\varpi}) \tilde{\sigma}_{ij}(w_j)$ is the weighted average

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[5]The expression of the operating profit $\pi_{ii}^o(\varphi) = \frac{r_{ii}(\varphi)}{\sigma_{ii}(wi)}$ is obtained using the Lerner index: $\frac{p_{ii}(\varphi) - (w_i/\varphi)}{p_{ii}(\varphi)} = \frac{1}{\sigma_{ii}(wi)}$. Importantly, notice that I always assume non-homotheticity and consider the CES as a homothetic exception. As a result, the price elasticity of demand faced by $\varphi$-productivity exporter from origin $i$ on market $j$ is expressed as a function of nominal wage in the destination $w_j$, not only for expositional simplicity, but also to put an emphasis on its destination specific aspect. The firm $(\varphi)$/origin$(w_i)$ and dyad$(\tau_{ij})$- specific aspects of $\sigma$ are recalled and put in use only when needed. The same choice of terminology applies to domestic firms facing $\sigma_{ii}(w_i)$ on the domestic market.

price elasticity of demand that a firm from origin $i$ expects to face while serving the world market.

Notice that $\tilde{\sigma}_{ij}(w_j) = \int_{\varphi_{ij}^*}^{+\infty} \sigma_{ij}(\frac{p_{ij}(\varphi)}{w_j}) g(\varphi) d\varphi$ is the expected price elasticity of demand to be faced by an average productivity exporter serving destination $j$ from origin $i$. It is also worth mentioning that $Y_\omega = \sum_{j=1}^N w_j L_j = \tilde{w} L_\omega$ is the World GDP, with $\tilde{w}$ is the average per-capita income at the World level and $L_\omega = \sum_{j=1}^N L_j$ is the World population.

The Labor Market Clearing condition (LMC) requires that total labor demand by entrants equates a country's labor endowment. While all entering firms (both successful and unsuccessful) incur the sunk entry cost $F_e$, only successful entrants use labor to start producing. In particular, labor demand by a $\varphi$-productivity successful entrant in any country (say, $i$), $ld(\varphi)$ depends on its export status:

$$\forall \varphi \geq \varphi_{ii}^*, \; ld(\varphi) = [(q_{ii}(\varphi) * \varphi^{-1}) + f_{ii}] + \sum_{j=1}^{(N-1)} [(q_{ij}(\varphi) * \tau_{ij}\varphi^{-1}) + f_{ij}] \, \mathcal{X}_{ij} \quad (2.7)$$

where $\mathcal{X}_{ij}$ is a dummy, equal to 1 if $\varphi \geq \varphi_{ij}^*$, and 0 otherwise. $q_{ii}(\varphi) = x_{ii}(\varphi)L_i$ is the market demand captured by $\varphi$-productivity firm on the domestic market. Likewise, $q_{ij}(\varphi) = x_{ij}(\varphi)L_j$ is the market demand a $\varphi$-productivity exporter reaps on destination $j$. Using the optimal pricing rule, the labor demand per successful entrant can be rewritten as follows:

$$ld(\varphi) = [w_i^{-1}(\frac{\sigma_{ii}(w_i)-1}{\sigma_{ii}(w_i)})r_{ii}(\varphi) + f_{ii}] + \sum_{j=1}^{(N-1)} [w_i^{-1}(\frac{\sigma_{ij}(w_j)-1}{\sigma_{ij}(w_j)})r_{ij}(\varphi) + f_{ij}] \, \mathcal{X}_{ij}$$
$$(2.8)$$

Using firm labor demand from the above equation, the labor market clearing condition can be simplified and written as :

$$L_i = M_i^e \left[ (F_e + P(\varphi \geq \varphi_{ii}^*)f_{ii} + \sum_{j=1}^{(N-1)} P(\varphi \geq \varphi_{ij}^*)f_{ij}) + w_i^{-1}\mu(\tilde{\sigma}_i^\omega)\tilde{R}_i \right] \quad (2.9)$$

where $\mu(\tilde{\sigma}_i^\omega) = \frac{\tilde{\sigma}_i^\omega(\tilde{w})-1}{\tilde{\sigma}_i^\omega(\tilde{w})}$ is the inverse of the markup that a successful entrant in country i would charge while serving the World market.[6]

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[6]Notice that $\mu$ boils down to $(\sigma - 1/\sigma)$ under the CES since $\sigma$ is exogenous.

By plugging the expected average revenues of successful entrants in $i$ from the free entry condition in equation (2.6) into the labor market condition in equation (2.9) and rearranging, I solve for the equilibrium mass of entrants in origin $i$ as follows:

$$M_i^e = \frac{w_i L_i}{\tilde{\sigma}_i^{\omega}(\tilde{w})\Psi_i} \qquad (2.10)$$

As assumed by Chaney (2008), the equilibrium mass of entrants is proportional to market size. Clearly, $M_i^e$ also decreases proportionally with the average level of price sensitivity at the World level $\sigma_i^{\omega}(\tilde{w})$, and importantly with $\Psi_i = [w_i F_e + P(\varphi \geq \varphi_{ii}^*) w_i f_{ii} + \sum_{j=1}^{(N-1)} P(\varphi \geq \varphi_{ij}^*) w_i f_{ij}]$ which reflects the degree of remoteness of origin $i$ from all potential destination markets in the World economy.

*Solving for the general equilibrium.*– In order to gain in generality without losing in tractability, I propose a new method that I call "the Exponent Elasticity Method" (EEM, hereafter). The objective of this simple method is to deliver tractable solutions in general equilibrium despite added flexibility in preferences. The starting point is the partial equilibrium expression of the price aggregator initially provided in equation (2.4). Using the above expression of the equilibrium mass of entrants, the partial equilibrium price aggregator can be rewritten as:

$$|\eta_j| = (w_j L_j)(\frac{w_j L_j}{Y_\omega})^{-1} \sum_{i=1}^{N} c_i^E (\frac{w_i L_i}{Y_\omega}) f_{ij}^{-1} \underbrace{\int_{\varphi_{ij}^*}^{+\infty} \frac{p_{ij}(\varphi)}{w_j} |v'(\frac{p_{ij}(\varphi)}{w_j})| dG(\varphi)}_{\mathbf{I}} \qquad (2.11)$$

where $c_i^E = [\sigma_i^{\omega}(\tilde{w}) w_i (\alpha_e + \alpha_{ii} P(\varphi \geq \varphi_{ii}^*) + P(\varphi \geq \varphi_{ij}^*) + \sum_{j=1}^{(N-2)} \alpha_{ik} P(\varphi \geq \varphi_{ij}^*))]^{-1}$ is a proxy for entry conditions in country $i$, $\alpha_e = \frac{F_e}{f_{ij}}$, $\alpha_{ii} = \frac{f_{ii}}{f_{ij}}$, and $\alpha_{ik} = \frac{f_{ik}}{f_{ij}}$.

Clearly, the mathematical challenge here is how to solve for the above integral ($\mathbf{I}$) without specifying a functional form of the sub-utility function. As is well known, this latter should exhibit a constant demand elasticity, which can be used then as a constant for integrating. Such simplicity is only possible under CES demand, which is the unique case where it is possible to solve for this integral. As the flexible family of preferences considered in this chapter encompasses the CES and a non-homothetic alternative allowing the demand elasticity to vary with prices and income levels, it is then impossible to solve for the above integral under such added flexibility in preferences.

Given the impossibility to solve for the integral in the current setting, the key idea that the EEM method proposes is to locally approximate the integral (**I**) around the equilibrium with a multiplicative equivalent which has a finite number of determinants, such as the exponent of each determinant embodies the elasticity of $[\frac{\bar{p}_{ij}(\varphi)}{w_j} \, |v'(\frac{\bar{p}_{ij}(\varphi)}{w_j})|\,]$ with respect to it. This requires a multi-step procedure that I expose in detail in a supplementary Appendix. With the aid of this simple method, I obtain a tractable solution for the price aggregator in general equilibrium:

$$|\eta_j| \equiv w_j^{(\frac{1+\epsilon_3^F}{1-\epsilon_7^F})} \; L_j^{(\frac{1+\epsilon_3^F}{1-\epsilon_7^F})} \; (\frac{w_j L_j}{Y_w})^{\frac{1}{(\epsilon_7^F-1)}} \; \Psi_j^{\frac{1}{(\epsilon_7^F-1)}} \tag{2.12}$$

where $\Psi_j = [\sum_{i=1}^{N}(\frac{w_i L_i}{Y_w})f_{ij}^{(\epsilon_5^F-1)}\tau_{ij}^{\epsilon_2^F}w_i^{\epsilon_1^F}]^{-1}$ is a reminiscent of the inward multilateral resistance term in Anderson and Van Wincoop (2003), and the explicit expressions of the above exponents are given by:

$$\begin{cases} \epsilon_1^F = \bar{\rho}_1(1 - \bar{\sigma}_{ij}(w_j)) + [\, [|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1) - \theta]\frac{[1-\rho_1^*(1-\sigma_{ij}^*(w_j))]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)}\,] < 0 \\[2mm] \epsilon_2^F = \bar{\rho}_2(\,1 - \bar{\sigma}_{ij}(w_j)\,) + [\,|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1) - \theta\,] = -\theta < 0 \\[2mm] \epsilon_3^F = (1 - \bar{\rho}_3)(\,\bar{\sigma}_{ij}(w_j) - 1\,) + [\,[\theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1)\,]\frac{[(1-\rho_3^*)(\sigma_{ij}^*(w_j)-1)+1+\delta^*]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)}\,] > 1 \\[2mm] \epsilon_5^F = -\frac{[\,\theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)\,]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} < 0 \\[2mm] \epsilon_7^F = -\frac{[\,\theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)\,]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} < 0 \end{cases}$$

$$\tag{2.13}$$

The multilateral resistance term $\Psi_j$ can be interpreted as a remoteness index since it reflects how far is destination $j$ from the rest of the world. Importantly, given the negative sign of $\epsilon_7^F$, $|\eta_j|$ is strictly decreasing in $\Psi_j$. This implies that the more remote is a destination, the fewer are the firms serving it and thus the more relaxed is competition on its market.[7]

Last, but not least, it worth noting that the "EEM" method delivers a unique solution for the general equilibrium price aggregator since it rests on a local approximation around a unique trade equilibrium. Nevertheless, such local approximation allows only to study the impact of small deviations from the equilibrium. Accordingly, I only examine the general equilibrium effects of small changes in per-capita income and in the variable trade cost in the next sections.

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[7]Regardless of the nature of preferences, a straightforward implication of Pareto distribution is that a change in the variable trade cost affects only the extensive margin, while the intensive margin remains silent. As a result, the toughness of competition is characterized solely with the number of competing firms, while the average price level is not taken into account.

### 2.2.2 Income vs Size effects on trade margins in general equilibrium

#### The intensive margin of trade

Using the general expression of firm-level export revenues, and this of the general equilibrium price aggregator from equation (2.12) and rearranging, I can solve for the intensive margin in general equilibrium as follows:

$$
r_{ij}(\varphi) \equiv w_i^{\epsilon_1} \, \tau_{ij}^{\epsilon_2} \, \varphi^{\epsilon_4} \, w_j^{\zeta-1} \, \underbrace{\left(\frac{w_j L_j}{Y_w}\right)^{\frac{1}{(1-\epsilon_7^F)}} \Psi_j^{\frac{1}{(1-\epsilon_7^F)}}}_{Chaney(2008)}
\tag{2.14}
$$

The exponents ($\epsilon_1$, $\epsilon_2$, $\epsilon_4$) respectively capture the elasticity of firm-level export revenues with respect to nominal wage in the origin country $w_i$, the variable trade cost $\tau_{ij}$, and firm productivity $\varphi$. Their expressions are respectively given by:

$\epsilon_1 = \rho_1 \, (1 - \sigma_{ij}(w_j))$ ;  $\epsilon_2 = \rho_2 \, (1 - \sigma_{ij}(w_j))$ ;  $\epsilon_4 = |\rho_4| \, (\sigma_{ij}(w_j) - 1)$.

While the expressions of these elasticities are quite standard, here the novel element is $\zeta$ which captures the gross positive income effect on firm-level export revenues in general equilibrium, and given by $\zeta = 1 + [\, (1 + \epsilon_3) - (\frac{1+\epsilon_3^F}{1-\epsilon_7^F}) \,]$, with $\epsilon_3 = (1 - \rho_3)(\, \sigma_{ij}(w_j) - 1\,)$.[8] Inspection of its expression immediately reveals that it boils down to 1 when preferences are CES. Hence, under the CES case, per-capita income $w_j$ and population size $L_j$ affect the intensive margin only through the RMS channel, exactly as in Chaney (2008). By contrast, under the non-homothetic alternative, $\zeta$ exceeds unity, allowing then per-capita income to have a stronger impact on the intensive margin in general equilibrium, as explained below.

It is also easy to notice that the remoteness of the destination $\Psi_j$ and its relative market size, (RMS, hereafter), $(\frac{w_j L_j}{Y_w})$ both affect positively the intensive margin and with the same magnitude. These two channels have already been highlighted in Chaney (2008). The focus here is on the different channels through which income and size affect separately the intensive margin. This is a theoretically clean way to highlight the novel implications of non-homotheticity, and thus compare the results with the Chaney (2008) benchmark.

*Size effect.–* The general equilibrium expression of the intensive margin in (2.14) clearly indicates that the RMS is the unique channel through which population size ($L_j$) can affect firm-level export revenues. For instance, it is readily verified that a population size enlargement implies

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[8]To avoid confusion, "income" refers to per-capita income ($w_j$), "size" to population size ($L_j$) and "market size" to their scalar ($w_j L_j$).

larger market demand- a proportional increase in the number of consumers-, but also, tougher competition - a proportional increase in the mass of domestic competitors.[9] Given their identical magnitude, these two opposite effects always cancel out under indirectly-separable preferences.

*Income effect.–* Contrary to population size, the magnitude of the income effect and the channels through which it affects the intensive margin crucially depend on the nature of preferences:[10]

$$\zeta = \begin{cases} > 1 & \text{if preferences are non-homothetic} \\ 1 & otherwise \end{cases} \tag{2.15}$$

When preferences are CES, $\sigma$ is exogenous and demand is iso-elastic, the gross positive income effect boils down to a proportional increase in individual demand ($\zeta$ equates unity, as indicated above). This positive impact on individual demand is then ruled out by a proportional increase in the mass of domestic competitors. Put differently, as the homothetic aspect of the CES implies that consumer preferences are unaffected by per-capita income, this latter acts only as a size parameter. Hence, under the CES case, the Chaney (2008) result is replicated, the RMS stands then as the unique channel through which income and size would have a positive and identical impact on the intensive margin.

By contrast, when preferences are non-homothetic, the price elasticity of demand is decreasing in income ($\sigma'_j(w_j) < 0$), any $\varphi$-productivity exporter charges then a higher markup to richer destinations where consumers are less price sensitive. In spite of charging higher prices, the exporting firm reaps larger individual demand for two reasons. First, as the export price increases less than proportionally with the destination's income level ($\rho_3 < 1, \forall\varphi$), the exported variety provides the consumer with a higher level of marginal sub-utility. This, in turn, increases individual demand for this variety. Second, and importantly, these two partial equilibrium effects (higher price and larger individual demand) prevail in general equilibrium as they dominate the total income effect on export revenues of all other active exporters (intensive) as well as infra-marginal ones (extensive).[11] Therefore, the gross positive income effect is more than proportional ($\zeta > 1$) and dominates thus the negative competition effect (-1), yielding thus a net positive income effect:

$$\zeta_n = \zeta - 1 \begin{cases} > 0 & \text{if preferences are non-homothetic} \\ = 0 & otherwise \end{cases} \tag{2.16}$$

---

[9]Recall that the mass of domestic entrants is proportional to market size as indicated in equation (2.10).
[10]See Appendix C.1 for a detailed proof.
[11]See Appendix C.1 for a detailed decomposition of the income effect in general equilibrium.

## The extensive margin of trade

Similarly, by recalling that the cutoff productivity exporter makes zero profits, and using the expression of firm-level profits and the general equilibrium price aggregator from equation (12), I can solve for the extensive margin in general equilibrium as follows:[12]

$$\varphi_{ij}^* \equiv w_i^{\left(\frac{1-\epsilon_1^*}{\epsilon_4^*}\right)} \, \tau_{ij} \, f_{ij}^{\frac{1}{\epsilon_4^*}} \, w_j^{\Delta_5} \, L_j^{\Delta_6} \, \underbrace{\left(\frac{w_j L_j}{Y_w}\right)^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}} \, \Psi_j^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}}}_{Chaney(2008)} \tag{2.17}$$

where $\Delta_5 = \frac{1}{\epsilon_4^*} \left[ \left(\frac{1+\epsilon_3^F}{1-\epsilon_7^F}\right) - (\epsilon_3^* + 1 + \delta^*) \right]$ and $\Delta_6 = \frac{1}{\epsilon_4^*} \left[ \left(\frac{1+\epsilon_6^F}{1-\epsilon_7^F}\right) - 1 \right]$, $\epsilon_6^F = \frac{[\,\theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)\,]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} > 0$

and $\delta^* = - \frac{dlog\,\sigma_{ij}^*(w_j)}{dlog\,w_j}$; $\epsilon_1^* = \rho_1^*\left(1 - \sigma_{ij}^*(w_j)\right)$; $\epsilon_3^* = (1 - \rho_3^*)\left(\sigma_{ij}^*(w_j) - 1\right)$; $\epsilon_4^* = |\rho_4^*|\left(\sigma_{ij}^*(w_j) - 1\right)$.

Equation (2.17) clearly indicates that while $\left(\frac{1-\epsilon_1^*}{\epsilon_4^*}\right)$ and $\left(\frac{1}{\epsilon_4^*}\right)$, respectively, correspond to the elasticity of the extensive margin with respect the nominal wage in the origin country $w_i$, and the fixed cost of exporting $f_{ij}$, here the two novel elements are $\Delta_5$, and $\Delta_6$. These latter respectively capture the direct income and size effects on the extensive margin in general equilibrium (excluding their common indirect effect channeled through the RMS).

Inspection of their respective expressions immediately reveals that while $\Delta_6$ is always equal to zero regardless of whether preferences are CES or non-homothetic, the sign of $\Delta_5$ hinges on the nature of preferences. This has an important implication for the income and size effects on the extensive margin in general equilibrium, as explained below.

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[12]See Appendix C.5 for a detailed explanation.

*Size effect.*– Regardless of the nature of preferences, it is readily verified that the unique channel through which population size $L_j$ could affect the toughness of firm selection into exporting is the RMS as highlighted in Chaney (2008). For instance, since $\epsilon_6^F = |\epsilon_7^F|$, $\Delta_6$ collapses always to zero whether preferences are CES or non-homothetic.

*Income effect.*– Contrary to population size, per-capita income can affect the extensive margin through an additional channel conditionally on non-homotheticity. Specifically, when preferences are non-homothetic, the price elasticity of demand is increasing in price and decreasing in income, which implies that $\Delta_5 < 0$.[13] A new "preference" channel arises then: *an increase in per-capita income makes selection into exporting less tough as infra-marginal exporters face henceforth a less elastic demand (as consumers become less price sensitive once they get richer).*

Nevertheless, under the CES case, $\sigma$ is identical across firms, $\Delta_5$ collapses to zero and the new "preference" channel is ruled out. As a result, per-capita income and population size act again interchangeably as size parameters. Their impact on the extensive margin is solely channeled through the RMS ($\Delta_5 = \Delta_6 = 0$). The Chaney (2008) result is thus replicated.

$$\Delta_5 \begin{cases} < 0 & \text{if preferences are non-homothetic} \\ = 0 & otherwise \end{cases} \tag{2.18}$$

### 2.2.3   Generalized Structural Gravity and Trade Elasticity

In this section, I derive a generalized structural gravity equation and I shed light on the extent to which the nature of preferences affects the sensitivity of trade flows to variations in trade barriers. Bilateral exports from origin $i$ to destination $j$ are given by:

$$X_{ij} = M_i^e \int_{\varphi_{ij}^*}^{+\infty} r_{ij}(\varphi) g(\varphi) d\varphi \tag{2.19}$$

Using individual demand from equation (2.3), the equilibrium mass of entrants in (2.10) and rearranging, bilateral exports can be rewritten as:

$$X_{ij} = [\tilde{\sigma}_i^{\varphi}(\tilde{w})]^{-1} \frac{w_i L_i}{\Psi_i} \frac{w_j L_j}{|\eta_j|} \underbrace{\int_{\varphi_{ij}^*}^{+\infty} \frac{p_{ij}(\varphi)}{w_j} |v'(\frac{p_{ij}(\varphi)}{w_j})| g(\varphi) d\varphi}_{\text{I}}, \tag{2.20}$$

---

[13]See Appendix C.2 for a detailed proof.

where $\sigma_i^{\varpi}(\tilde{w})$ is the average price elasticity of demand at the World level and $\Psi_i$ measures the degree of remoteness of origin $i$ from all potential destination markets in the World economy.[14] Notably, a straightforward implication of the free entry condition is that it allows the average degree of price sensitivity at the world level $\sigma_i^{\varpi}(\tilde{w})$ and remoteness from destination markets $\Psi_i$ to have a deterrent impact on entry in any origin $i$.

As mentioned in the previous section, the integral (**I**) is approximated with the aid of the "EEM" method. This latter delivers a tractable solution for the price aggregator in general equilibrium in equation (2.12). By plugging this general equilibrium expression in the bilateral trade equation above, I derive the following *Structural Gravity Equation*:

$$X_{ij} = \kappa_6 \left[\tilde{\sigma}_i^{\varpi}(\tilde{w})\right]^{-1} \frac{Y_i Y_j}{Y_\varpi} \frac{\Psi_j}{\Psi_i} \; w_i^{\epsilon_1^F} \; \tau_{ij}^{\epsilon_2^F} \; f_{ij}^{\epsilon_5^F}, \tag{2.21}$$

where $\epsilon_1^F = \bar{\rho}_1(1 - \bar{\sigma}_{ij}(w_j)) + \left[\; [\;|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1) - \theta\;]\; \frac{[1 - \rho_1^*(1 - \sigma_{ij}^*(w_j))]}{|\rho_4^*|(\sigma_{ij}^*(w_j) - 1)}\;\right] < 0,$

$\epsilon_2^F = \bar{\rho}_2(1 - \bar{\sigma}_{ij}(w_j)) + [\;|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1) - \theta\;] = -\theta < 0$ and $\epsilon_5^F = -\frac{[\;\theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1)\;]}{|\rho_4^*|(\sigma_{ij}^*(w_j) - 1)} < 0.$

This structural gravity equation can be considered as an augmented version of Chaney (2008)'s gravity equation in three respects. First, its structural aspect is reinforced since it exhibits, not only the inward multilateral resistance term ($\Psi_j$ measures the easiness of penetrating destination $j$), but also the outward multilateral resistance term ($\Psi_i$ captures the toughness of exporting from origin $i$) as in Anderson and Van Wincoop (2003). These multilateral resistance terms have opposite effects on bilateral trade and affect it through different margins. While the positive impact of the former $\Psi_j$ occurs at both margins (extensive and intensive)[15] as in Chaney (2008), the negative impact of the latter $\Psi_i$ occurs only at the extensive margin.

This effect is absent in the Chaney (2008) model since the free entry condition is not imposed and can be explained as follows: *the more remote is an origin i from destination markets (mainly due to its geographical location which entails higher fixed costs of exporting: high $f_{ij} \; \forall \; j$),[16] the lower are expected export profits, the fewer are entrants and thus exporters.[17]*

---

[14]$\Psi_i = [w_i F_e + P(\varphi \geq \varphi_{ii}^*) w_i f_{ii} + \sum_{j=1}^{(N-1)} P(\varphi \geq \varphi_{ij}^*) w_i f_{ij}]$, as indicated in equation (2.10)

[15]As competition is more relaxed in remote destinations, this, not only, offers easier entry conditions for prospective exporters, but also, allows successful ones to reap large market shares. Hence, the remoteness of the destination respectively magnifies the extensive and intensive margins of trade.

[16]I focus only on geography and abstract from the cost of labor in the origin $w_i$ because it also corresponds to individual expenditure on the differentiated good and thus has a positive impact on entry as well.

[17]For any given level of trade barriers and destination characteristics.

As in (Chaney, 2008; Melitz and Ottaviano, 2008; Arkolakis et al., 2018), regardless of the nature of preferences, unbounded Pareto distribution of firm productivity always gives rise to a constant trade elasticity $(\epsilon_2^F = -\theta)$. However, the second novelty here is that in spite of generating a constant elasticity of aggregate trade flows to the variable trade cost, it allows per-capita income to determine the degree of sensitivity of trade margins to variable trade cost variation when preferences are non-homothetic, as in (Mrázová and Neary, 2014; Carrère, Mrázová, and Neary, 2020). In particular, using the definition of the elasticity of bilateral trade to the variable trade cost $(\epsilon_2^F)$, its absolute value can be decomposed into an (intensive: firm-level export revenues) and (extensive: Number of exporting firms) component capturing the elasticity of each margin to the variable trade cost:

$$|\varepsilon_{\tau_{ij}}^{X_{ij}}| = |\epsilon_2^F| = \underbrace{\bar{\rho}_2( \bar{\sigma}_{ij}(w_j) - 1 )}_{\iota:intensive} + \underbrace{\left[ \theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1) \right]}_{\chi:extensive} = \theta \qquad (2.22)$$

As highlighted by Chaney (2008), $\sigma$ magnifies the sensitivity of the intensive margin, whereas it dampens the sensitivity of the extensive margin to a small change in the variable trade cost. As non-homotheticity implies that the price elasticity of demand is decreasing in income, a new theoretical prediction arises: *When preferences are indirectly additive and non-homothetic, higher per-capita income affects the sensitivity of trade margins to a small variation in the variable trade cost in two opposite ways: it dampens the sensitivity of the intensive margin, whereas it magnifies the sensitivity of the extensive margin.*

Third, another novelty of this structural gravity equation is that it allows for the fixed trade cost elasticity of bilateral trade flows to be either constant or variable depending on the nature of preferences. In contrast to the CES case where this elasticity is constant as in Chaney (2008), non-homotheticity breaks this constant link, yielding an income-decreasing fixed trade cost elasticity of bilateral trade. Using the expression of $(\epsilon_5^F)$, the absolute value of the elasticity of aggregate trade flows with respect to the fixed trade cost can be decomposed as follows:

$$|\varepsilon_{f_{ij}}^{X_{ij}}| = |\epsilon_5^F| = \underbrace{0}_{\iota:intensive} + \underbrace{\frac{\left[ \theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1) \right]}{|\rho_4^*|(\sigma_{ij}^*(w_j) - 1)}}_{\chi:extensive} = \begin{cases} \frac{\theta}{\sigma-1} - 1 \text{ (CES)} \\ \frac{\theta}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} - v \text{ (non-homothetic)} \end{cases}$$

$$(2.23)$$

where $v = \frac{|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} < 1$ is orthogonal to destination's per-capita income $w_j$ as the effects of income on both elasticities cancel out.

Under non-homotheticity, the fixed trade cost elasticity of bilateral trade is decreasing in destination's per-capita income: $\frac{d|\epsilon_5^F|}{dw_j} < 0$. The underlying economics are simple: an increase in destination's per-capita income ($w_j' > w_j$) lowers the export cutoff ($\varphi^{*'}_{ij} < \varphi^*_{ij}$), which in turn generates a more than proportional increase in the cutoff export price ($p^{*'}_{ij} > p^*_{ij}$).[18] As the price elasticity of demand is increasing in the price-income ratio, the "new" cutoff exporter faces then a more elastic demand ($\sigma^{*'}_{ij} > \sigma^*_{ij}$), which in turn lowers the fixed trade cost elasticity of bilateral trade ( $|\epsilon_5^F|' < |\epsilon_5^F|$ ). In deed, as rich destinations are the easiest to penetrate, the productivity level of infra-marginal exporters is the lowest, these latter capture then very small market shares upon a reduction in the fixed trade cost. Therefore, rich destinations are the least elastic to a variation in fixed trade barriers as long as they are the least selective.

Last, but not least, it is worth mentioning that it is necessary to assume that the liberalizing country $j$ is a small open economy. This additional assumption ensures that any variation in the variable or fixed cost of importing in destination $j$ ($\tau_{ij}$, $f_{ij}$ $\forall i$) has no impact on entry conditions in any source country $i$ ($\tilde{\sigma}_i^{\wp}(\tilde{w})$, $\Psi_i$).[19]

### 2.2.4 Solving for nominal wages in general equilibrium

In this short section, I close the model by solving for the nominal wage in destination $j$, $w_j$, using the trade balance condition (TB):

$$(TB)_j : \underbrace{\sum_{i=1}^{(N-1)} X_{ji}}_{j's\,exports} = \underbrace{\sum_{i=1}^{(N-1)} X_{ij}}_{j's\,imports} \tag{2.24}$$

By plugging bilateral trade flows from the structural gravity from equation (2.21) in the trade balance condition above and rearranging, I solve for the relative wage in destination $j$ as follows:

$$w_j = \left(\frac{\Psi_j}{\tilde{\Psi}_{Row}}\right)^{\frac{1}{(\epsilon_1^F-1)}} \Lambda^{\frac{1}{(\epsilon_1^F-1)}}, \tag{2.25}$$

where $\tilde{\Psi}_{Row} \equiv \sum_{i=1,i\neq j}^{(N-1)}(\frac{L_i}{L_{\wp}})\Psi_i$ is the weighted average degree of remoteness of all destinations in the World, excluding $j$ and $\Lambda$ is a general equilibrium object that is orthogonal to bilateral tariffs.[20]

---

[18]Due to a joint increase in the cutoff marginal cost and in markups when the destination gets richer.

[19]$\frac{d\tilde{\sigma}_i^{\wp}(\tilde{w})}{d\tau_{ij}} = \frac{d\tilde{\sigma}_i^{\wp}(d\tilde{w})}{df_{ij}} = \frac{d\Psi_i}{d\tau_{ij}} = \frac{d\Psi_i}{df_{ij}} = 0.$

[20]See Appendix C.3 for a detailed derivation.

To gain some intuition on this result, the general equilibrium expression of the relative wage can be interpreted as follows: the more a destination is relatively remote (compared to the World average), the lower is the relative wage of its workers. For instance, a remote destination is characterized by relaxed competition as fewer firms compete on its market. As a result, this destination is easy to penetrate for all origins and accumulates trade deficits. This, in turn, imposes a downward adjustment of nominal wage in this destination so as the price competitiveness of its exporters is boosted and trade balance is restored. With the aid of this intuitive reasoning, I highlight two new sources of welfare gains from trade that I present and explain in the next section.

### 2.2.5 Welfare Analysis

In order to define the structure of the welfare gains from trade, I identify the channels affecting consumer welfare in a destination $j$ upon it cuts tariffs on imports from a trading partner $i$.

I start with disentangling pure variety gains as in Krugman ([1980]) from gains from selection, due to Melitz ([2003]). Then, I highlight two new sources of welfare gains from unilateral trade liberalization: *(i) an additional gain from selection that occurs on the export market, (ii) an increase in nominal wage in the liberalizing country.* Importantly, I show that Melitz ([2003])'s selection effect and the two additional welfare channels highlighted above, are operative only when the trading partner $i$ is relatively large compared to the World economy. A mirror image of this result is that the liberalizing economy reaps only pure variety gains as in Krugman ([1980]) when the trading partner is relatively small.

Using the extensive margin in general equilibrium from equation (2.17), the productivity cutoff to serve destination $j$ from origin $i$, the export cutoff to serve it from any other origin $o$ and the domestic cutoff in $j$ can be respectively written as:

$$
\begin{cases}
\varphi_{ij}^* \equiv w_i^{\left(\frac{1-\epsilon_1^*}{\epsilon_4^*}\right)} \tau_{ij} f_{ij}^{\frac{1}{\epsilon_4^*}} w_j^{\Delta_5} L_j^{\Delta_6} \left(\frac{w_j L_j}{Y_w}\right)^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}} \Psi_j^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}}, \\
\varphi_{oj}^* \equiv w_o^{\left(\frac{1-\epsilon_1^*}{\epsilon_4^*}\right)} \tau_{oj} f_{oj}^{\frac{1}{\epsilon_4^*}} w_j^{\Delta_5} L_j^{\Delta_6} \left(\frac{w_j L_j}{Y_w}\right)^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}} \Psi_j^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}}, \\
\varphi_{jj}^* \equiv w_j^{\left(\frac{1-\epsilon_1^*}{\epsilon_4^*}\right)} f_{jj}^{\frac{1}{\epsilon_4^*}} w_j^{\Delta_5} L_j^{\Delta_6} \left(\frac{w_j L_j}{Y_w}\right)^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}} \Psi_j^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}}
\end{cases}
\tag{2.26}
$$

Let me now introduce a key parameter for the welfare analysis: $\xi_1$. This elasticity captures the potential impact of a small reduction in the variable cost of importing from an origin $i$ on the toughness of competition in the liberalizing country $j$. This "pro-competitive" effect[21] occurs

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[21]To avoid confusion, this term solely refers to an increase in the intensity of competition on market $j$.

only when the trading partner $i$ is relatively large compared to the World economy. As shown below, this is visible through changes in destination $j$'s remoteness index $\Psi_j$ depending on the relative market size of its partner $i$:[22]

$$\xi_1 = -\frac{d\log \Psi_j}{d\log \tau_{ij}} = \left(\frac{w_i L_i}{Y_\omega}\right) \underbrace{\epsilon_2^F}_{<0} f_{ij}^{(\epsilon_5^F - 1)} \tau_{ij}^{\epsilon_2^F} w_i^{\epsilon_1^F} \Psi_j^{-1} = \begin{cases} < 0 & \text{if } \left(\frac{w_i L_i}{Y_\omega}\right) >> 0^+ \\ 0 & \text{otherwise} \end{cases} \quad (2.27)$$

To gain some intuition on the dependence of the sign of $\xi_1$ on the GDP share of the origin $i$, consider these two opposite cases. When country $j$ unilaterally liberalizes trade with a relatively small partner $i$, it experiences a slight increase in the mass of firms competing on its market, with negligible impact on the toughness of competition ($\xi_1 = 0$). By contrast, when the trading partner $i$ is relatively large, the mass of competing firms increases tremendously and competition on the destination market $j$ becomes tougher ($\xi_1 < 0$). This "pro-competitive" effect has a mirror image: upon unilaterally liberalizing trade with a relatively large partner $i$, a destination $j$ becomes less remote from the World economy. However, when the favored partner is relatively small, the remoteness index of the destination remains unchanged.

Now I define the following elasticities so as each of which captures a specific welfare channel. Using the productivity cutoffs from equation (2.26) along with the wage equation in (2.25), their final expressions can be written as follows:

$$\begin{cases} |\xi_2| = \left| -\frac{d\log \varphi_{ij}^*}{d\log \tau_{ij}} \right| = 1 - \frac{\xi_1}{\epsilon_4^*(\epsilon_7^F - 1)} \leq 1 \\[2mm] \xi_3^d = -\frac{d\log \varphi_{jj}^*}{d\log \tau_{ij}} = \frac{\xi_1}{\epsilon_4^*(\epsilon_7^F - 1)} \geq 0 \\[2mm] \xi_4^o = -\frac{d\log \varphi_{oj}^*}{d\log \tau_{ij}} = \frac{\xi_1}{\epsilon_4^*(\epsilon_7^F - 1)} \geq 0 \\[2mm] \xi_5 = -\frac{d\log w_j}{d\log \tau_{ij}} = \frac{\xi_1}{(\epsilon_1^F - 1)} \geq 0 \end{cases} \quad (2.28)$$

where $|\xi_2|$ captures the increase in the mass of imported varieties from partner $i$, and thus reflects pure variety gains as in Krugman (1980). As for the selection effect highlighted by Melitz (2003), it is captured by $\xi_3^d$ which reflects the $\widetilde{exit}$ of the least productive domestic firms in the liberalizing country $j$.[23] Moreover, $\xi_4^o$ capture the $\widetilde{loss}$ of varieties imported from any other origin $o$. It is worth mentioning that the $\widetilde{loss}$ of imported varieties captured by $\xi_4^o$ mirrors the $\widetilde{exit}$ of

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[22]The expression of $\xi_1$ is obtained using the definition of $\Psi_j$ from equation (2.12).

[23]For sake of precision, "tilde" is used hereafter to indicate that the effect at question is potential since its existence crucially depends on the value of $\xi_1$, which in turn mirrors the relative market size of the trading partner $i$. Specifically, the welfare channel at question is considered as being potential as long as it is operative when ($\xi_1 < 0$), whereas it vanishes when ($\xi_1 = 0$).

the least productive firms serving destination $j$ from any origin $o$ due to tougher selection on the export market $j$, which leads to an $\widetilde{increase}$ in average export productivity. This additional gain from selection (on the export market) represents a new welfare channel and is complementary to the selection effect, due to Melitz (2003) which occurs on the domestic market.

Finally, $\xi_5$ captures the impact of unilateral trade liberalization on the nominal wage in the liberalizing country $w_j$. In particular, this effect is positive when the partner is relatively large ($\xi_1 < 0$), and thus can be considered as another new source of welfare gains from trade. However, this new welfare channel vanishes when the trading partner is relatively small ($\xi_1 = 0$), as it is the case for the selection effects (both on the domestic and the export markets) mentioned above.

Now using the elasticities in (2.28), the different sources of welfare gains from trade can be captured by the following summary statistic:

$$
\Gamma = \underbrace{\left[\, |\xi_2| - (\, \xi_3^d + \sum_{o=1,o\neq i,j}^{(N-2)} \xi_4^o)\,\right]}_{\Theta_v\,:Net\ Variety\ Effect} + \underbrace{\left[\, \xi_3^d + \sum_{o=1,o\neq i,j}^{(N-2)} \xi_4^o\,\right]}_{\Theta_s\,:Melitz(domestic)+Additional\ selection(export)} + \underbrace{\xi_5}_{impact\ on\ nominal\ wage}
$$

(2.29)

Importantly, in such a global equilibrium with asymmetric countries, it is insightful to stress that the relative size of the trading partner shapes the structure of welfare gains for the liberalizing country. For instance, it is readily verified from equation (2.27) that when its partner $i$ is relatively small ($\xi_1 = 0$), the liberalizing economy $j$ reaps only pure variety gains as in Krugman (1980). By contrast, when partner $i$ is relatively large ($\xi_1 < 0$), the magnitude of the gross variety gain[24] is reduced, the selection effect as in Melitz (2003) is operative, and two new sources of welfare gains arise: an additional selection effect on the export market and an increase in nominal wage in the liberalizing country. In order to shed more light on the mechanisms underlying these two different structures of welfare gains, I proceed to a two-cases analysis as follows:

### Case 1: the trading partner $i$ is relatively small

Under this case, the liberalizing economy reaps only small gains from variety as in Krugman (1980) despite firm heterogeneity and the presence of fixed costs. The absence of the gains from selection as in Melitz (2003) -along with the two new welfare channels mentioned above- is due to the fact that the arrival of few newly imported varieties from partner $i$ has a negligible impact on the intensity of competition on the destination market $j$ ($\xi_1 = 0$). As a result, neither the least

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[24]It refers to the increase in the mass of imported varieties from country $i$.

productive domestic firms in country $j$, nor the least productive exporters serving this destination from all other origins ($\forall o \neq i$) are forced to exit the market.

It is worth mentioning that despite the rigidity of labor supply in this one sector model, firm exit can only be caused by tougher competition on the final good market since unilateral trade liberalization by country $j$ does not entail any positive demand shock on its domestic labor market. Melitz and Ottaviano (2008) studied the welfare implications of unilateral trade liberalization using a linear demand system exhibiting a choke price that is decreasing in the number of competing firms. They find that, in the short run, the liberalizing economy gains always from selection[25] as in Melitz (2003) regardless of the size of the partner as an increase in the number of competing firms translates mechanically into tougher competitive conditions for domestic firms.

Moreover, the authors show that the liberalizing economy experiences a welfare loss in the long rue due to changes in the patterns of entry (Home Market Effect). By contrast, as I depart from a different theoretical setting involving only one sector, many asymmetric countries, indirectly additive preferences and fixed costs, I obtain different results. In particular, in the current chapter, the toughness of competition on a given destination $j$ is mainly captured by its remoteness index[26] $\Psi_j$ which varies significantly only when the the trading partner is relatively large. As a result, the selection effect [27] is not operative when the trading partner is relatively small and the liberalizing country reaps only pure variety gains in the short run.

As for the long run effects, in contrast to Melitz and Ottaviano (2008), the liberalizing economy does not experience a welfare loss since the Home Market Effect is not operative in the absence of a freely traded outside sector, as in Demidova and Rodriguez-Clare (2013). Finally, there is no significant[28] variation in nominal wages in the liberalizing country $j$, this can be explained as follows. Given the small relative size of partner $i$, the arrival of few newly imported varieties from this country has a negligible impact on destination $j$'s remoteness index ($\xi_1 = 0$). This implies that the degree of easiness of penetrating market $j$ from any other origin ($\forall o \neq i$) remains unchanged. Hence, there is no variation in country $j$'s imports from all other sources. This, in turn, implies that there is no necessary adjustment of nominal wage in country $j$ to restore trade balance.

Therefore, the liberalizing country enjoys only pure variety gains as in Krugman (1980) despite firm heterogeneity. Using indirectly additive preferences, Bertoletti, Etro, and Simonovska

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[25]It also enjoys net variety gains and a pro-competitive reduction in domestic markups.

[26]Along with the mass of domestic competitors.

[27]Both on the domestic market as in Melitz (2003) and on the export market (additional selection effect).

[28]Notice that when the partner is relatively small, the impact of unilateral trade liberalization on the remoteness index of the liberalizing country is negligible, which implies a negligible effect on nominal wages in this country.

(2018) find a similar result, yet the mechanism is different. Specifically, they abstract from fixed costs and highlight only pure variety gains since the choke price is orthogonal to the mass of competing firms under indirectly-additive preferences. Instead, I incorporate fixed costs in the model and I show that despite their presence, the liberalizing economy gains only from variety since the intensity of competition on its market remains unchanged when its partner is relatively small, and thus all other welfare channels are ruled out.

## Case 2: the trading partner $i$ is relatively large

Under this case, the arrival of a large number of newly imported varieties from partner $i$ makes competition tougher on the destination market $j$. This pro-competitive effect is mirrored by a significant decrease in destination $j$'s remoteness index upon reducing the variable cost of importing from partner $i$ ($\xi_1 < 0$). Importantly, this increase in the intensity of competition in the liberalizing economy has three major consequences that shape the structure of its welfare gains. First, it forces the least productive domestic firms in country $j$ to exit the market, gains from selection as in Melitz (2003) are thus recovered.

Second, due to tougher competitive conditions on market $j$, the least productive exporters serving this destination from all other origins ($\forall o \neq i$) are also forced to exit this export market. This additional selection effect occurring on the export market leads to an increase in average export productivity and can be then considered as a new source of welfare gain. Nevertheless, due to these two waves of firm exit, the liberalizing economy $j$ experiences a net variety loss for country $j$. For instance, the less than proportional increase in the mass of varieties imported from partner $i$ does not compensate the total variety loss implied by the exit of domestic firms and the least productive exporters from all other origins.[29]

Third, this variety loss reflects the fact that the liberalizing country $j$ is importing less from all other sources ($\forall o \neq i$) and accumulating trade surpluses. Hence, an upward adjustment of the nominal wage in country $j$ is needed so that its exporters become less price competitive, export less to all other countries, and thus trade balance is restored. This increase in the nominal wage in the liberalizing economy is the second new welfare channel highlighted in this chapter. In contrast to Demidova and Rodriguez-Clare (2013), the impact of unilateral trade liberalization on nominal wage in the liberalizing country is overturned. For instance, in their two-country model, unilateral trade liberalization implies a downward adjustment of nominal wage in the liberalizing economy to boost the price competitiveness of its exporters so that they export more and trade balance is restored.

---

[29]In spite of being relatively large, partner $i$'s GDP share can not exceed $\frac{1}{2}$.

Moreover, the authors emphasized that despite the decrease in nominal wage it entails, unilateral trade liberalization is welfare improving since the selection effect and the decrease in average price it implies dominate the negative impact on nominal wage. By contrast, I find that nominal wage and the domestic cutoff vary in the same direction: they both increase upon unilaterally liberalizing trade with a relatively large partner. Seen this way, the increase in nominal wage arises as an additional source of welfare gains. Importantly, it is complementary to Melitz (2003)'s selection effect on the domestic market along with the new selection effect on the export market, and the sum of these gains outweighs the previously mentioned net variety loss. Finally, the structure of welfare gains can be summarized as follows:

$$
\Gamma = \begin{cases} |\xi_2| = 1 & \text{if } i \text{ is relatively small} \\ \underbrace{\Theta_v}_{<0} + \underbrace{\Theta_s}_{>0} + \underbrace{\xi_5}_{>0} & \text{if } i \text{ is relatively large} \end{cases}
\tag{2.30}
$$

# 2.3 Empirical Analysis

The objective of the current empirical exercise is twofold. First, I separately estimate the impact of a destination $j$'s income level $w_j$ and population size $L_j$ on the number of firms based in any foreign country $i$ and able to sell varieties of any HS2-defined good $k$ on its market ($N_{ij,k}$: extensive margin of trade), and on the revenues each foreign exporter earns on its market ($r_{ij,k}$: intensive margin of trade).[30] Second, after classifying destinations into three income categories (low, middle and high-income), I investigate the role that a destination's per-capita income level plays in determining the degree of sensitivity of both trade margins to variable and fixed trade barriers.

Before proceeding, let us recall that the preceding theoretical exercise highlighted three novel predictions under non-homotheticity:

1. Only per-capita income has a significant positive impact on the intensive margin of trade in general equilibrium.

2. Per-capita income dampens the elasticity of the intensive margin with respect to the variable trade cost

3. Per-capita income dampens the elasticity of the extensive margin with respect to the fixed trade cost

Hence, what is at stake here is to check whether these three novel theoretical predictions, derived under non-homothetic preferences, are empirically relevant or not. As throughout the theoretical model, non-homotheticity has been proposed as a prominent alternative to the homothetic CES benchmark, the current empirical exercise can show then whether this theoretically appealing alternative outperforms the CES empirically, or not.

I proceed in four steps. First, I describe data sources. Second, I discuss the econometric challenges to be encountered during the current exercise and propose conventional solutions to meet them. Third, I propose two different types of estimation: the first using panel data, and the second is cross-sectional, and I spell out the corresponding econometric specification. Fourth, I provide a detailed interpretation of the empirical results, and draw a conclusion on the empirical relevance of non-homotheticity.

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[30]By multiplying both margins, I obtain the value of destination $j$'s total imports of good $k$ from origin $i$ $X_{ij,k}$ and estimate the impact of country $j$'s characteristics on its bilateral imports at the industry level.

### 2.3.1 Data Sources

Data on both margins of trade are retrieved from *the World Bank's Exporter Dynamics Database*. This dataset provides precise information on the number of exporters from origin *i* serving destination *j* in HS2-defined industry *k* in year *t* $N_{ij,k,t}$ (extensive margin) and the average revenue per exporter $r_{ij,k,t}$ (intensive margin) for 218 countries and 97 HS2-defined industries during the 1997-2014 period. Data on weighted average ad valorem equivalents of tariff protection at the HS2-industry level are collected from *MacMap-HS6* and *WITS*, respectively, provided by CEPII, and ITC (UNCTAD-WTO). As for data on time-invariant gravity variables such as physical distance, contiguity, common language and colonial ties, it is collected from *CEPII's GeoDist database*. Moreover, the *World Bank's World Development Indicators* reports data on country-specific variables such as per-capita income *w* and population size *L*. Finally, the *United Nations's Historical Classification database* provides a time-varying classification of all countries by income categories.

### 2.3.2 Econometric Challenges and Solutions

As stressed by Yotov et al. (2016), estimating the structural gravity model is challenging as it is subject to numerous econometric issues that should be addressed properly. Specifically, to obtain econometrically sound estimates of the parameters of interest, I have to address the following issues: presence of zero trade flows; heteroskedasticity in trade data; endogeneity of the trade policy variable; gradual adjustment to trade policy changes; and unobservable multilateral resistance terms.

I start with focusing on the endogeneity of trade policy variables, and I show that it is readily taken in due account by invoking a common key property of the data sets on ad valorem equivalents of tariff protection used in the current exercise. For instance, the authors of MacMap-HS6 clearly state that ad valorem equivalents (AVEs, hereafter) and weighting schemes are not computed country by country, but instead using reference groups of countries. These latter are built as a result of a clustering procedure based on trade openness and GDP per capita, and are designed as large groups of countries sharing similar trade-relevant characteristics. As a result, the use of this methodology limits the direct influence of country-specific protection, since protection patterns differ significantly across countries in each group. This, in turn, minimizes endogeneity while computing and aggregating protection (Guimbard et al., 2012). The authors also emphasize that ITC (UNCTAD-WTO) methodology is also based on reference groups, while it differs only in terms of weighting schemes.

Then, following Yotov et al. (2016), I meet the remaining challenges as follows. In order to take into account the information contained in zero trade flows and to control for heteroscedasticity of

trade data, I use the Poisson Pseudo Maximum Likelihood (PPML) estimator recommended by Silva and Tenreyro (2006).[31] In line with Trefler (2004) who severely criticized trade estimations performed with panel data pooled over consecutive years, Olivero and Yotov (2012) proved that it produces suspicious estimates of trade elasticity. To avoid such a critique, I use only the years: 1999, 2002, 2005, 2008, 2011 and 2014, which is comparable to the 3-year intervals in Trefler (2004). Finally, time-varying, directional (source and destination), country-sector-specific dummies control for the multilateral resistance terms ($\Psi_{i,k}$; $\Psi_{j,k}$) and expenditures ($Y_{i,k}$; $Y_{j,k}$) at the industry $k$ level.

### 2.3.3   Econometric Specifications

### A. Panel Data Estimation

**1. Income vs Size Effects on Trade Margins**

$$T_{ij,k,t} = exp[\beta_0 + \beta_1 lnLAG\_TARIFF_{ij,k,t} + \beta_2 lnINCOME_{j,t} + \beta_3 lnSIZE_{j,t} +$$

$$\beta_4 lnMRT_{j,k,t} + \nu_{i,k,t} + \gamma_{ij}] + \epsilon_{ij,k,t}$$

Here $T_{ij,k,t}$ is a trade covariate which refers to each trade margin (intensive: $r_{ij,k,t}$; extensive: $N_{ij,k,t}$), as well as their scalar ($X_{ij,k,t}$) which corresponds to the value of bilateral trade in commodity $k$ between partners $i$ and $j$ in year $t$. Moreover, $lnLAG\_TARIFF_{ij,k,t}$ is the first lag of the logarithm of the variable trade cost. $lnINCOME_{j,t}$, $lnSIZE_{j,t}$ and $lnMRT_{j,k,t}$ correspond to the logarithm of destination $j$'s per-capita income, population size and multilateral resistance term, respectively. $\nu_{i,k,t}$ encompasses the time varying and sector-specific source country dummy variables that account for the (log of) outward multilateral resistances and total shipments. $\gamma_{ij}$ captures the country-pair fixed effects which absorb time-invariant components of trade cost (such as, distance, contiguity, common language and colonial links).

Notably, as multilateral resistances are not directly observable, I construct the inward multilateral resistance term based on the theoretically-derived remoteness index of destination $j$ ($\Psi_j$) as follows:[32]

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[31]I also estimate and report OLS estimates to allow for an immediate comparison with PPML estimates.

[32]See equation (2.12).

$$MRT_{j,k,t} = \sum_{i=1}^{N} \left(\frac{X_{i,k,t}}{X_{\wp,k,t}}\right) distance_{ij} \tag{2.31}$$

where $X_{\wp,k,t} = \sum_{i=1}^{N} X_{i,k,t}$ is the value of total World exports of good $k$ in year $t$. $MRT_{j,k,t}$ reflects then destination $j$'s geographical remoteness form World's best exporters of good $k$. Finally, $\epsilon_{ij,k,t}$ is an error term.

## 2. Income effect on the elasticity of the intensive margin with respect to the variable trade cost

$$r_{ij,k,t} = exp[\beta_0 + \beta_1 IC1_{j,t}^c * lnLAG\_TARIFF_{ij,k,t} + \beta_2 IC2_{j,t}^c * lnLAG\_TARIFF_{ij,k,t} +$$

$$\beta_3 IC3_{j,t}^c * lnLAG\_TARIFF_{ij,k,t} + \beta_4 lnDIST_{ij} + \beta_5 BRDR_{ij} + \beta_6 LANG_{ij} + \beta_7 CLNY_{ij} + v_{i,k,t} +$$

$$\mu_{j,k,t}] + \epsilon_{ij,k,t}$$

Here $r_{ij,k,t}$ corresponds to the intensive margin of trade. To capture the gradual reactivity of this trade margin to the variable trade cost depending on the income level of the destination, I interact the lagged tariff with the following dummies. $IC1_{j,t}^c$, $IC2_{j,t}^c$ and $IC3_{j,t}^c$ indicate whether the destination $j$ is classified as a low-income, middle-income or high-income country in year $t$, respectively.

Specifically, I classify destinations by income categories using two methods. The first consists in resorting to the distribution of GDP per capita. Low-income destinations are destinations with a GDP per capita below the 25th percentile of the distribution, while middle-income destinations are those with GDP per capita between the 25th and 75th percentile of the distribution. High-income destinations are those whose GDP per capita exceeds the 75th percentile. The second simply adopts the United Nations's historical classification. Notice that the superscript (c) of income category dummies indicates the classification method in use, such as (a) refers to the first method and (b) to the second. $lnDIST_{ij}$ is the logarithm of bilateral distance. $BRDR_{ij}$, $LANG_{ij}$ and $CLNY_{ij}$ are indicator variables that capture the presence of contiguous borders, common language and colonial ties, respectively. $v_{i,k,t}$ and $\mu_{j,k,t}$ denote the directional, time-varying country-sector specific fixed effects, which account for the multilateral resistances and market size, respectively, on the exporter and on the importer side. Importantly, due to the high dimension of these fixed effects, pairwise fixed effects are absent from this specification, so as to obtain a reasonable magnitude of the coefficients of interest ($\beta_1$;$\beta_2$;$\beta_3$). The above specification includes then a standard set of time-invariant gravity variables such as: distance, contiguity, common language and colonial links. Finally, $\epsilon_{ij,k,t}$ is an error term.

**3. Income effect on the elasticity of the extensive margin with respect to the fixed trade cost**

Similarly, in order to test empirically the income effect on the sensitivity of the extensive margin to fixed trade barriers, I adopt the following specification:

$$N_{ij,k,t} = exp[\beta_0 + \beta_1 lnLAG\_TARIFF_{ij,k,t} + \beta_2 IC1^c_{j,t} * lnDIST_{ij} + \beta_3 IC2^c_{j,t} * lnDIST_{ij}$$

$$+ \beta_4 IC3^c_{j,t} * lnDIST_{ij} + \beta_5 BRDR_{ij} + \beta_6 LANG_{ij} + \beta_7 CLNY_{ij} + v_{i,k,t} + \mu_{j,k,t}] + \epsilon_{ij,k,t}$$

## B. Cross-Sectional Estimation

This estimation draws heavily on the three specifications used in the preceding analysis, with only two amendments. First, in the first specification, the pair fixed effect is now substituted with a standard set of time-invariant gravity variables, including distance, contiguity, common language and colonial links. Sencond, in the three preceding specifications, the time subscript $t$ is now dropped. The three regressions are run for each of the following years: 2002, 2005, and 2008.

## 2.3.4   Interpretation of the Gravity Estimation Results

Inspection of the results obtained with Panel data estimation reveals that the non-homothetic alternative always outperforms the CES empirically. First, CES preferences imposes a similarity between per-capita income and population size both in terms of the channels through which they affect trade margins in general equilibrium, and the magnitude of their effects. It fails then to explain why while destination's per-capita income $w_j$ has a positive and significant impact on the intensive margin and thus on bilateral trade, its population size has no significant impact on it, as shown in table A, Appendix B.

By contrast, non-homotheticity allows individual income to determine the degree of price sensitivity of the consumer and explains this empirical result as follows: *as per-capita income dampens the price elasticity of demand, exporters from any origin country charge higher markups and thus sets higher export prices for richer destinations. This positive price effect is accompanied by a positive effect on the volume of firm-level exports. In deed, since the export price increases less than proportionally with destination's per-capita income, exported varieties provide foreign consumer with a higher marginal sub-utility, which induces an increase in individual demand for foreign varieties despite their high price. These two positive partial-equilibrium income effects persists in general equilibrium and dominate the negative impact of tougher domestic competition. This generates then a net positive income effect under non-homotheticity in general equilibrium.*

Second, results in table (B) validate the second novel theoretical prediction under non-homotheticity: *the elasticity of the intensive margin with respect to the variable trade cost is decreasing in destination's per-capita income since richer consumers are less price sensitive.* However, the CES-based prediction is in stark contrast with this empirical result as long as the degree of price sensitivity is assumed to exogenous in a CES world. Moreover, results in table (C) validate the third novel theoretical prediction: *the elasticity of the extensive margin with respect to the fixed trade cost is decreasing in destination's per-capita income since richer destinations are easier to penetrate, which implies relatively low productivity level of infra-marginal exporters. As a result, they capture small export market shares upon a reduction in fixed trade barriers. This is reflected by the mild reaction of the extensive margin in rich destinations.*

These three novel theoretical predictions are also empirically validated by the cross-sectional estimation (Tables D1, D2, D3, E1, E2, E3, F1, F2, and F3), with only one slight change compared to the preceding results: the positive significant impact of destination's population size on the intensive margin is restored.

## 2.4   Conclusion

The current chapter embedded indirectly-additive preferences in a gravity model featuring standard assumptions on the supply side, while taking country-level asymmetry and the presence of both variable and fixed trade barriers in due account. By combining cross-country difference in per-capita income levels with flexibility in preferences, the model generates three novel theoretical predictions. First, the intensive margin of trade increases only with a destionation's per-capita income in general equilibrium. Sencond, higher income dampens the sensitivity of the intensive margin to the variable trade cost. Third, higher income also dampens the sensitivity of the extensive margin to the fixed trade cost. With the aid of a structural gravity estimation, this chapter proves the empirical validity of these novel theoretical predictions obtained under non-homotheticity. In this sense, the current chapter provides an alternative to the homothetic CES which is both theoretically more appealing and empirically more relevant.

On the other hand, the current chapter examines the role that country-level asymmetry in market size plays in shaping the structure of welfare gains from unilateral trade liberalization. In particular, I show that when the trading partner is relatively small, unilateral trade liberalization delivers only pure variety gains as in Krugman (1980) despite firm heterogeneity and the presence of fixed costs. In contrast, when this latter is relatively large, Melitz (2003)'s selection effect on domestic firms is restored, and two new welfare channels arise: an additional selection effect occurring on the export market, and an increase in nominal wage in the liberalizing country. Finally, the contribution of this chapter to the gravity literature is a structural gravity equation that can be considered as an augmented version of this of Chaney (2008) in two respects. First, its structural aspect is reinforced as it exhibits both inward and outward multilateral resistances as in Anderson and Van Wincoop (2003). Second, it yields an income-decreasing elasticity of bilateral trade flows with respect to fixed trade barriers, when preferences are non-homothetic.

# Appendix B

**Table A. Destination's income vs size effects on trade margins**

|  | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| VARIABLES | X | N | r | ln_X | ln_N | ln_r |
| ln_ LAG_TARIFF | -2.012*** | -0.915*** | -0.948*** | -1.421*** | -0.468*** | -0.953*** |
|  | (0.296) | (0.106) | (0.288) | (0.106) | (0.0406) | (0.0824) |
| ln_INCOME | 0.682*** | 0.138 | 0.360*** | 0.511*** | 0.169*** | 0.342*** |
|  | (0.0952) | (0.0866) | (0.0910) | (0.0467) | (0.0273) | (0.0410) |
| ln_SIZE | 0.353 | 0.534*** | -0.200 | 0.519*** | 0.286*** | 0.233** |
|  | (0.266) | (0.134) | (0.303) | (0.128) | (0.0576) | (0.114) |
| ln_MRT | 0.556*** | 0.131*** | 0.293*** | 0.263*** | 0.103*** | 0.160*** |
|  | (0.0531) | (0.0108) | (0.0354) | (0.0138) | (0.00658) | (0.00964) |
|  |  |  |  |  |  |  |
| Observations | 219,444 | 219,444 | 219,444 | 176,338 | 176,338 | 176,338 |
| R-squared |  |  |  | 0.635 | 0.787 | 0.564 |
| pair FE | Yes | Yes | Yes | Yes | Yes | Yes |
| origin-sector-year FE | Yes | Yes | Yes | Yes | Yes | Yes |
|  |  |  |  |  |  |  |
| Zeros included | Yes | Yes | Yes | No | No | No |
| Estimator | PPML | PPML | PPML | OLS | OLS | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes | Yes | Yes |

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

**Table B. Income-decreasing elasticity of the intensive margin to the variable trade cost**

| VARIABLES | (1) r | (2) ln_r | (3) r | (4) ln_r |
|---|---|---|---|---|
| $IC1^a * ln\_LAG\_TARIFF$ | -2.610*** | -1.948*** | | |
| | (0.420) | (0.326) | | |
| $IC2^a * ln\_LAG\_TARIFF$ | -1.820*** | -1.666*** | | |
| | (0.384) | (0.234) | | |
| $IC3^a * ln\_LAG\_TARIFF$ | -1.654*** | -0.920*** | | |
| | (0.299) | (0.239) | | |
| ln_DIST | -0.481*** | -0.624*** | -0.477*** | -0.623*** |
| | (0.0285) | (0.0239) | (0.0284) | (0.0239) |
| BRDR | 0.177*** | 0.311*** | 0.176*** | 0.309*** |
| | (0.0632) | (0.0666) | (0.0633) | (0.0666) |
| LANG | 0.0255 | 0.151*** | 0.0141 | 0.147*** |
| | (0.0625) | (0.0445) | (0.0625) | (0.0444) |
| CLNY | 0.158** | 0.0787 | 0.165** | 0.0810 |
| | (0.0741) | (0.0740) | (0.0745) | (0.0738) |
| $IC1^b * ln\_LAG\_TARIFF$ | | | -2.543*** | -1.951*** |
| | | | (0.627) | (0.435) |
| $IC2^b * ln\_LAG\_TARIFF$ | | | -2.863*** | -2.040*** |
| | | | (0.269) | (0.256) |
| $IC3^b * ln\_LAG\_TARIFF$ | | | -0.915*** | -0.774*** |
| | | | (0.250) | (0.211) |
| | | | | |
| Observations | 217,938 | 177,312 | 217,938 | 177,312 |
| R-squared | 0.898 | 0.687 | 0.898 | 0.687 |
| destination-sector-year FE | Yes | Yes | Yes | Yes |
| origin-sector-year FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table C. Income-decreasing elasticity of the extensive margin to fixed trade cost**

| VARIABLES | (1)<br>N | (2)<br>ln_N | (3)<br>N | (4)<br>ln_N |
|---|---|---|---|---|
| ln_ LAG_TARIFF | -1.053*** | -0.641*** | -0.994*** | -0.612*** |
|  | (0.241) | (0.138) | (0.248) | (0.139) |
| $IC1^a * ln\_DIST$ | -1.255*** | -1.008*** |  |  |
|  | (0.0862) | (0.0439) |  |  |
| $IC2^a * ln\_DIST$ | -1.040*** | -0.804*** |  |  |
|  | (0.0342) | (0.0329) |  |  |
| $IC3^a * ln\_DIST$ | -0.999*** | -0.777*** |  |  |
|  | (0.0474) | (0.0461) |  |  |
| BRDR | 0.257*** | 0.229*** | 0.259*** | 0.229*** |
|  | (0.0815) | (0.0824) | (0.0816) | (0.0840) |
| LANG | 0.971*** | 0.552*** | 0.949*** | 0.545*** |
|  | (0.101) | (0.0537) | (0.102) | (0.0543) |
| CLNY | 0.919*** | 0.582*** | 0.933*** | 0.579*** |
|  | (0.111) | (0.0944) | (0.111) | (0.0960) |
| $IC1^b * ln\_DIST$ |  |  | -1.240*** | -1.034*** |
|  |  |  | (0.121) | (0.0613) |
| $IC2^b * ln\_DIST$ |  |  | -1.125*** | -0.849*** |
|  |  |  | (0.0381) | (0.0357) |
| $IC3^b * ln\_DIST$ |  |  | -0.973*** | -0.761*** |
|  |  |  | (0.0437) | (0.0418) |
|  |  |  |  |  |
| Observations | 217,938 | 177,312 | 217,938 | 177,312 |
| R-squared | 0.880 | 0.766 | 0.880 | 0.766 |
| destination-sector-year FE | Yes | Yes | Yes | Yes |
| origin-sector-year FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

**Table D1. Destination's income vs size effects on trade margins (Year: 2002)**

| VARIABLES | (1)<br>X | (2)<br>N | (3)<br>r | (4)<br>ln_X | (5)<br>ln_N | (6)<br>ln_r |
|---|---|---|---|---|---|---|
| ln_LAG_TARIFF | -1.303*** | -0.357 | -0.558 | -1.734*** | -0.539*** | -1.194*** |
| | (0.482) | (0.359) | (0.566) | (0.318) | (0.201) | (0.193) |
| ln_INCOME | 0.725*** | 0.290*** | 0.379*** | 0.463*** | 0.207*** | 0.256*** |
| | (0.0385) | (0.0393) | (0.0371) | (0.0294) | (0.0233) | (0.0198) |
| ln_SIZE | 0.644*** | 0.262*** | 0.332*** | 0.514*** | 0.186*** | 0.328*** |
| | (0.0393) | (0.0317) | (0.0821) | (0.0277) | (0.0221) | (0.0173) |
| ln_MRT | 0.318*** | 0.0786*** | 0.305*** | 0.109*** | 0.0187 | 0.0907*** |
| | (0.0779) | (0.0301) | (0.111) | (0.0242) | (0.0162) | (0.0179) |
| ln_DIST | -1.112*** | -1.020*** | -0.318*** | -1.171*** | -0.619*** | -0.553*** |
| | (0.0818) | (0.0774) | (0.0685) | (0.0589) | (0.0507) | (0.0396) |
| BRDR | 0.274* | 0.194 | 0.145 | 0.501*** | 0.298** | 0.202** |
| | (0.145) | (0.215) | (0.182) | (0.164) | (0.145) | (0.0950) |
| LANG | 0.203 | 0.718*** | -0.0642 | 0.642*** | 0.473*** | 0.169** |
| | (0.144) | (0.164) | (0.125) | (0.116) | (0.0899) | (0.0660) |
| CLNY | 0.424** | 0.731** | 0.230 | 0.599** | 0.609*** | -0.0103 |
| | (0.196) | (0.305) | (0.234) | (0.271) | (0.211) | (0.152) |
| Constant | 4.927*** | 3.851*** | 4.183*** | 8.256*** | 2.041*** | 6.215*** |
| | (0.952) | (0.908) | (1.013) | (0.617) | (0.481) | (0.367) |
| | | | | | | |
| Observations | 30,794 | 30,794 | 30,794 | 23,822 | 23,822 | 23,822 |
| R-squared | | | | 0.524 | 0.566 | 0.481 |
| origin-sector FE | Yes | Yes | Yes | Yes | Yes | Yes |
| | | | | | | |
| Zeros included | Yes | Yes | Yes | No | No | No |
| Estimator | PPML | PPML | PPML | OLS | OLS | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes | Yes | Yes |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table D2. Destination's income vs size effects on trade margins (Year: 2005)**

| VARIABLES | (1) X | (2) N | (3) r | (4) ln_X | (5) ln_N | (6) ln_r |
|---|---|---|---|---|---|---|
| ln_LAG_TARIFF | -2.703*** | -1.019*** | -1.379*** | -1.532*** | -0.598*** | -0.934*** |
| | (0.616) | (0.367) | (0.400) | (0.247) | (0.148) | (0.154) |
| ln_INCOME | 0.654*** | 0.230*** | 0.363*** | 0.408*** | 0.207*** | 0.201*** |
| | (0.0464) | (0.0655) | (0.0408) | (0.0250) | (0.0186) | (0.0159) |
| ln_SIZE | 0.647*** | 0.241*** | 0.373*** | 0.496*** | 0.185*** | 0.310*** |
| | (0.0347) | (0.0369) | (0.0362) | (0.0251) | (0.0182) | (0.0134) |
| ln_MRT | 0.839*** | 0.257*** | 0.267** | 0.264*** | 0.0780*** | 0.186*** |
| | (0.116) | (0.0603) | (0.107) | (0.0315) | (0.0194) | (0.0198) |
| ln_DIST | -1.121*** | -1.069*** | -0.174* | -1.238*** | -0.650*** | -0.588*** |
| | (0.0777) | (0.0732) | (0.0891) | (0.0531) | (0.0415) | (0.0300) |
| BRDR | 0.122 | 0.0551 | 0.201 | 0.388*** | 0.238** | 0.150* |
| | (0.191) | (0.194) | (0.247) | (0.139) | (0.110) | (0.0882) |
| LANG | -0.0237 | 0.640*** | -0.172 | 0.533*** | 0.404*** | 0.130** |
| | (0.125) | (0.129) | (0.128) | (0.0873) | (0.0614) | (0.0534) |
| CLNY | 0.951*** | 1.297*** | 0.559** | 0.783*** | 0.727*** | 0.0561 |
| | (0.213) | (0.375) | (0.247) | (0.200) | (0.174) | (0.135) |
| Constant | 1.613 | 3.508*** | 3.890*** | 8.042*** | 1.654*** | 6.388*** |
| | (1.133) | (0.833) | (0.868) | (0.535) | (0.377) | (0.327) |
| | | | | | | |
| Observations | 53,395 | 53,395 | 53,395 | 40,304 | 40,304 | 40,304 |
| R-squared | | | | 0.517 | 0.545 | 0.485 |
| origin-sector FE | Yes | Yes | Yes | Yes | Yes | Yes |
| | | | | | | |
| Zeros included | Yes | Yes | Yes | No | No | No |
| Estimator | PPML | PPML | PPML | OLS | OLS | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes | Yes | Yes |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table D3. Destination's income vs size effects on trade margins (Year: 2008)**

| VARIABLES | X | N | r | ln_X | ln_N | ln_r |
|---|---|---|---|---|---|---|
| ln_LAG_TARIFF | -3.154*** | -1.356*** | -1.124** | -1.902*** | -0.759*** | -1.143*** |
| | (0.600) | (0.389) | (0.482) | (0.203) | (0.122) | (0.129) |
| ln_INCOME | 0.611*** | 0.226*** | 0.355*** | 0.362*** | 0.206*** | 0.155*** |
| | (0.0381) | (0.0317) | (0.0288) | (0.0199) | (0.0136) | (0.0124) |
| ln_SIZE | 0.647*** | 0.288*** | 0.327*** | 0.472*** | 0.209*** | 0.263*** |
| | (0.0283) | (0.0253) | (0.0315) | (0.0194) | (0.0128) | (0.0107) |
| ln_MRT | 0.653*** | 0.210*** | 0.364*** | 0.304*** | 0.0867*** | 0.217*** |
| | (0.0933) | (0.0495) | (0.0668) | (0.0330) | (0.0185) | (0.0196) |
| ln_DIST | -1.062*** | -0.972*** | -0.369*** | -1.199*** | -0.657*** | -0.542*** |
| | (0.0553) | (0.0587) | (0.0458) | (0.0419) | (0.0292) | (0.0228) |
| BRDR | 0.557*** | 0.419*** | 0.0464 | 0.712*** | 0.451*** | 0.261*** |
| | (0.136) | (0.149) | (0.141) | (0.121) | (0.0937) | (0.0696) |
| LANG | 0.0858 | 0.843*** | -0.179** | 0.552*** | 0.455*** | 0.0972** |
| | (0.136) | (0.118) | (0.0848) | (0.0712) | (0.0508) | (0.0442) |
| CLNY | 0.563*** | 0.879*** | 0.0412 | 0.565*** | 0.516*** | 0.0489 |
| | (0.159) | (0.227) | (0.136) | (0.137) | (0.104) | (0.0789) |
| Constant | 3.546*** | 2.590*** | 5.556*** | 8.545*** | 1.246*** | 7.299*** |
| | (0.908) | (0.768) | (0.648) | (0.447) | (0.307) | (0.252) |
| | | | | | | |
| Observations | 82,241 | 82,241 | 82,241 | 62,208 | 62,208 | 62,208 |
| R-squared | | | | 0.531 | 0.588 | 0.491 |
| origin-sector FE | Yes | Yes | Yes | Yes | Yes | Yes |
| | | | | | | |
| Zeros included | Yes | Yes | Yes | No | No | No |
| Estimator | PPML | PPML | PPML | OLS | OLS | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes | Yes | Yes |

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table E1. Income-decreasing elasticity of the intensive margin to the variable trade cost (Year: 2002)**

| VARIABLES | (1)<br>r | (2)<br>ln_r | (3)<br>r | (4)<br>ln_r |
|---|---|---|---|---|
| $IC1^a * ln\_LAG\_TARIFF$ | -2.289*** | -2.898*** | | |
| | (0.887) | (0.983) | | |
| $IC2^a * ln\_LAG\_TARIFF$ | -3.132*** | -1.407** | | |
| | (0.514) | (0.560) | | |
| $IC3^a * ln\_LAG\_TARIFF$ | -3.436*** | -1.617 | | |
| | (0.862) | (1.296) | | |
| ln_DIST | -0.612*** | -0.729*** | -0.611*** | -0.726*** |
| | (0.0537) | (0.0466) | (0.0533) | (0.0470) |
| BRDR | 0.233* | 0.178 | 0.239* | 0.205 |
| | (0.129) | (0.125) | (0.128) | (0.129) |
| LANG | 0.145 | 0.145* | 0.146 | 0.143* |
| | (0.105) | (0.0802) | (0.105) | (0.0803) |
| CLNY | 0.0274 | 0.154 | 0.0257 | 0.146 |
| | (0.189) | (0.146) | (0.188) | (0.147) |
| $IC1^b * ln\_LAG\_TARIFF$ | | | -0.209 | -0.984 |
| | | | (1.251) | (1.186) |
| $IC2^b * ln\_LAG\_TARIFF$ | | | -3.026*** | -2.037*** |
| | | | (0.670) | (0.775) |
| $IC3^b * ln\_LAG\_TARIFF$ | | | -3.334*** | -1.805*** |
| | | | (0.551) | (0.651) |
| | | | | |
| Observations | 28,920 | 23,945 | 28,920 | 23,945 |
| R-squared | 0.892 | 0.732 | 0.892 | 0.732 |
| destination-sector FE | Yes | Yes | Yes | Yes |
| origin-sector FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table E2. Income-decreasing elasticity of the intensive margin to the variable trade cost (Year: 2005)**

| VARIABLES | (1) r | (2) ln_r | (3) r | (4) ln_r |
|---|---|---|---|---|
| $IC1^a * ln\_LAG\_TARIFF$ | -3.169*** | -1.648*** | | |
| | (0.829) | (0.556) | | |
| $IC2^a * ln\_LAG\_TARIFF$ | -1.449*** | -1.291*** | | |
| | (0.492) | (0.380) | | |
| $IC3^a * ln\_LAG\_TARIFF$ | -1.516*** | -0.679* | | |
| | (0.386) | (0.390) | | |
| Distance | -0.494*** | -0.688*** | -0.495*** | -0.688*** |
| | (0.0447) | (0.0336) | (0.0447) | (0.0337) |
| contiguity | 0.399*** | 0.217** | 0.387*** | 0.210** |
| | (0.101) | (0.0932) | (0.102) | (0.0934) |
| common language | 0.150* | 0.184*** | 0.143* | 0.178*** |
| | (0.0817) | (0.0647) | (0.0820) | (0.0647) |
| colonial links | 0.160 | -0.00573 | 0.160 | -0.000731 |
| | (0.120) | (0.133) | (0.120) | (0.132) |
| $IC1^b * ln\_LAG\_TARIFF$ | | | -3.138** | -2.321*** |
| | | | (1.234) | (0.628) |
| $IC2^b * ln\_LAG\_TARIFF$ | | | -2.154*** | -1.665*** |
| | | | (0.537) | (0.493) |
| $IC3^b * ln\_LAG\_TARIFF$ | | | -1.196*** | -0.620** |
| | | | (0.405) | (0.300) |
| | | | | |
| Observations | 51,466 | 40,629 | 51,466 | 40,629 |
| R-squared | 0.946 | 0.673 | 0.946 | 0.674 |
| destination-sector FE | Yes | Yes | Yes | Yes |
| origin-sector FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table E3. Income-decreasing elasticity of the intensive margin to the variable trade cost (Year: 2008)**

| VARIABLES | (1) r | (2) ln_r | (3) r | (4) ln_r |
|---|---|---|---|---|
| $IC1^a * ln\_LAG\_TARIFF$ | -3.508*** | -1.257*** | | |
| | (0.716) | (0.423) | | |
| $IC2^a * ln\_LAG\_TARIFF$ | -2.736*** | -2.588*** | | |
| | (0.433) | (0.343) | | |
| $IC3^a * ln\_LAG\_TARIFF$ | -1.855*** | -1.097*** | | |
| | (0.448) | (0.305) | | |
| Distance | -0.472*** | -0.597*** | -0.472*** | -0.596*** |
| | (0.0342) | (0.0247) | (0.0342) | (0.0248) |
| contiguity | 0.142* | 0.324*** | 0.141* | 0.324*** |
| | (0.0776) | (0.0661) | (0.0775) | (0.0658) |
| common language | 0.0780 | 0.109** | 0.0773 | 0.107** |
| | (0.0702) | (0.0504) | (0.0702) | (0.0503) |
| colonial links | 0.115 | 0.0548 | 0.116 | 0.0566 |
| | (0.0924) | (0.0799) | (0.0923) | (0.0797) |
| $IC1^b * ln\_LAG\_TARIFF$ | | | -3.782*** | -1.304** |
| | | | (0.966) | (0.544) |
| $IC2^b * ln\_LAG\_TARIFF$ | | | -2.820*** | -2.706*** |
| | | | (0.439) | (0.353) |
| $IC3^b * ln\_LAG\_TARIFF$ | | | -1.892*** | -0.899*** |
| | | | (0.403) | (0.276) |
| | | | | |
| Observations | 80,837 | 63,010 | 80,837 | 63,010 |
| R-squared | 0.829 | 0.644 | 0.829 | 0.644 |
| destination-sector FE | Yes | Yes | Yes | Yes |
| origin-sector FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table F1. Income-decreasing elasticity of the extensive margin to fixed trade cost (Year: 2002)**

| VARIABLES | (1) N | (2) ln_N | (3) N | (4) ln_N |
|---|---|---|---|---|
| ln_ LAG_TARIFF | -0.728** | -0.177 | -0.506 | -0.164 |
| | (0.359) | (0.346) | (0.385) | (0.369) |
| $IC1^a * ln\_DIST$ | -1.251*** | -1.058*** | | |
| | (0.148) | (0.0626) | | |
| $IC2^a * ln\_DIST$ | -0.934*** | -0.789*** | | |
| | (0.0489) | (0.0554) | | |
| $IC3^a * ln\_DIST$ | -1.232*** | -0.855*** | | |
| | (0.113) | (0.159) | | |
| BRDR | 0.233* | 0.0647 | 0.204 | 0.0622 |
| | (0.137) | (0.144) | (0.140) | (0.154) |
| LANG | 0.627*** | 0.456*** | 0.579*** | 0.450*** |
| | (0.117) | (0.0819) | (0.101) | (0.0818) |
| CLNY | 1.058*** | 0.632*** | 1.074*** | 0.606*** |
| | (0.148) | (0.164) | (0.154) | (0.171) |
| $IC1^b * ln\_DIST$ | | | -1.167*** | -1.033*** |
| | | | (0.198) | (0.0927) |
| $IC2^b * ln\_DIST$ | | | -1.180*** | -0.904*** |
| | | | (0.0666) | (0.0673) |
| $IC3^b * ln\_DIST$ | | | -0.958*** | -0.780*** |
| | | | (0.0538) | (0.0670) |
| | | | | |
| Observations | 28,920 | 23,945 | 28,920 | 23,945 |
| R-squared | 0.910 | 0.813 | 0.906 | 0.811 |
| destination-sector FE | Yes | Yes | Yes | Yes |
| origin-sector FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table F2. Income-decreasing elasticity of the extensive margin to fixed trade cost (Year: 2005)**

| VARIABLES | (1) N | (2) ln_N | (3) N | (4) ln_N |
|---|---|---|---|---|
| ln_ LAG_TARIFF | -1.000*** | -0.607*** | -0.951*** | -0.556*** |
| | (0.242) | (0.184) | (0.248) | (0.191) |
| $IC1^a * ln\_DIST$ | -1.235*** | -0.996*** | | |
| | (0.0813) | (0.0500) | | |
| $IC2^a * ln\_DIST$ | -1.001*** | -0.789*** | | |
| | (0.0445) | (0.0413) | | |
| $IC3^a * ln\_DIST$ | -1.107*** | -0.836*** | | |
| | (0.0554) | (0.0642) | | |
| BRDR | 0.138 | 0.0837 | 0.0971 | 0.0482 |
| | (0.105) | (0.101) | (0.109) | (0.105) |
| LANG | 0.722*** | 0.453*** | 0.689*** | 0.434*** |
| | (0.0805) | (0.0589) | (0.0836) | (0.0588) |
| CLNY | 1.135*** | 0.629*** | 1.135*** | 0.629*** |
| | (0.122) | (0.125) | (0.124) | (0.127) |
| $IC1^b * ln\_DIST$ | | | -1.295*** | -1.117*** |
| | | | (0.129) | (0.0705) |
| $IC2^b * ln\_DIST$ | | | -1.151*** | -0.879*** |
| | | | (0.0448) | (0.0427) |
| $IC3^b * ln\_DIST$ | | | -1.005*** | -0.749*** |
| | | | (0.0487) | (0.0545) |
| | | | | |
| Observations | 51,466 | 40,629 | 51,466 | 40,629 |
| R-squared | 0.891 | 0.754 | 0.888 | 0.755 |
| destination-sector FE | Yes | Yes | Yes | Yes |
| origin-sector FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

**Table F3. Income-decreasing elasticity of the extensive margin to fixed trade cost (Year: 2008)**

| VARIABLES | (1) N | (2) ln_N | (3) N | (4) ln_N |
|---|---|---|---|---|
| ln_ LAG_TARIFF | -1.325*** | -0.782*** | -1.248*** | -0.777*** |
| | (0.379) | (0.199) | (0.381) | (0.197) |
| $IC1^a * ln\_DIST$ | -1.322*** | -0.993*** | | |
| | (0.0948) | (0.0534) | | |
| $IC2^a * ln\_DIST$ | -1.109*** | -0.810*** | | |
| | (0.0413) | (0.0371) | | |
| $IC3^a * ln\_DIST$ | -0.978*** | -0.759*** | | |
| | (0.0533) | (0.0401) | | |
| BRDR | 0.310*** | 0.337*** | 0.334*** | 0.352*** |
| | (0.0890) | (0.0893) | (0.0882) | (0.0910) |
| LANG | 1.039*** | 0.584*** | 1.025*** | 0.574*** |
| | (0.123) | (0.0594) | (0.122) | (0.0595) |
| CLNY | 0.892*** | 0.513*** | 0.919*** | 0.517*** |
| | (0.122) | (0.0938) | (0.121) | (0.0946) |
| $IC1^b * ln\_DIST$ | | | -1.239*** | -0.956*** |
| | | | (0.111) | (0.0639) |
| $IC2^b * ln\_DIST$ | | | -1.158*** | -0.840*** |
| | | | (0.0458) | (0.0368) |
| $IC3^b * ln\_DIST$ | | | -0.997*** | -0.764*** |
| | | | (0.0493) | (0.0410) |
| | | | | |
| Observations | 80,837 | 63,010 | 80,837 | 63,010 |
| R-squared | 0.865 | 0.742 | 0.864 | 0.741 |
| destination-sector FE | Yes | Yes | Yes | Yes |
| origin-sector FE | Yes | Yes | Yes | Yes |
| Income classification method | a | a | b | b |
| Zeros included | Yes | No | Yes | No |
| Estimator | PPML | OLS | PPML | OLS |
| pairwise clustering | Yes | Yes | Yes | Yes |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

# Appendix C

## C.1 Gross positive income effect on the intensive margin:

This effect is captured by $\zeta$, which is given by: $\zeta = 1 + [(1 + \epsilon_3) - (\frac{1+\epsilon_3^F}{1-\epsilon_7^F})]$.

I show that this gross effect exceeds unity only under non-homotheticity as follows. First, when preferences are homothetic (CES case), it is readily verified that $(1 + \epsilon_3) = (\frac{1+\epsilon_3^F}{1-\epsilon_7^F}) = \sigma$. Hence, $\zeta$ collapses to 1.

Second, when preferences are non-homothetic, I need to rewrite $(\frac{1+\epsilon_3^F}{1-\epsilon_7^F})$ as follows, so as to decompose the general equilibrium effect of income into an "intensive" (impact on export revenues of active exporters) and an "extensive" (impact on infra-marginal exporters) components as follows:

Denote by $\Delta_1 = (1 + \epsilon_3) = [\rho_3 + (1 - \rho_3)\sigma_{ij}(w_j)]$, which capture the impact of income variation on the export revenues of the firm $(\varphi)$ at question[33] Likewise, $\Delta_2 = (\frac{1+\epsilon_3^F}{1-\epsilon_7^F}) = \Delta_3 + \Delta_4$ such as:

- $\Delta_3 = \Delta_{31} \Delta_{32} = \Delta_{31} [\bar{\rho}_3 + (1 - \bar{\rho}_3)\bar{\sigma}_{ij}(w_j)]$ embodies the income effect on the "intensive" margin, with $\Delta_{31} = \frac{|\rho_4^*| * (\sigma_{ij}^*(w_j)-1)}{\theta + |\rho_4^*|(\sigma_{ij}^*(w_j)-1) - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)} < 1$ since $\theta > |\rho_4^*|(\sigma_{ij}^*(w_j) - 1)$, and $|\rho_4^*|(\sigma_{ij}^*(w_j) - 1) > |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j) - 1)$ as $\sigma$ is increasing in price.

- $\Delta_4 = \Delta_{41} \Delta_{42} = \Delta_{41} [(1 - \rho_3^*)(\sigma_{ij}^*(w_j) - 1) + 1 + \delta^*]$ embodies the income effect on the "extensive" margin, with $\Delta_{41} = \frac{\theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)}{\theta + |\rho_4^*|(\sigma_{ij}^*(w_j)-1) - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)} \approx 0$.

Therefore, $\zeta = 1 + [\underbrace{\Delta_1}_{>1} - [\underbrace{\Delta_{31}}_{<1} \Delta_{32} + \underbrace{\Delta_{41}}_{\approx 0} \Delta_{42}]] > 1$.

---

[33]The absence of superscript indicates that a random exporter is at question.

# C.2 direct[34] income effect on the extensive margin

This effect is captured by $\Delta_5$, which can be written, using the $\Delta$s defined above, as :

$\Delta_5 = \frac{1}{\epsilon_4^*}[\Delta_2 - \Delta_{42}]$ and its sign crucially depends on the nature of preferences:

- Under the CES, it easy to verify that $\Delta_2 = \Delta_{42} = \sigma$. Hence, $\Delta_5$ collapses to 0.

- Under non-homotheticity, $\Delta_2 - \Delta_{42} = \underbrace{\Delta_{31}}_{<1} \Delta_{32} - \underbrace{(1 - \Delta_{41})}_{\approx 1} \Delta_{42} < 0$

since $\Delta_{32} = [\bar{\rho}_3 + (1 - \bar{\rho}_3)\bar{\sigma}_{ij}(w_j)] < \Delta_{42} = [(1 - \rho_3^*)(\sigma_{ij}^*(w_j) - 1) + 1 + \delta^*]$, as $\sigma_{ij}^*(w_j) > \bar{\sigma}_{ij}(w_j)$.

# C.3 Solving for nominal wage in the liberalizing country $j$

I solve for the nominal wage in destination $j$ $w_j$ using the trade balance condition (TB):

$$(TB)_j : \underbrace{\sum_{i=1}^{(N-1)} X_{ji}}_{j's\,exports} = \underbrace{\sum_{i=1}^{(N-1)} X_{ij}}_{j's\,imports} \tag{2.32}$$

Using equation (2.21), bilateral trade flows can be written as:

$$\begin{cases} X_{ji} = \kappa_6 \, [\tilde{\sigma}_j^{\omega}(\tilde{w})]^{-1} \, \frac{Y_j Y_i}{Y_{\omega}} \, \frac{\Psi_i}{\Psi_j} \, w_j^{\epsilon_1^F} \, \tau_{ji}^{\epsilon_2^F} \, f_{ji}^{\epsilon_5^F} \\ X_{ij} = \kappa_6 \, [\tilde{\sigma}_i^{\omega}(\tilde{w})]^{-1} \, \frac{Y_i Y_j}{Y_{\omega}} \, \frac{\Psi_j}{\Psi_i} \, w_i^{\epsilon_1^F} \, \tau_{ij}^{\epsilon_2^F} \, f_{ij}^{\epsilon_5^F} \end{cases} \tag{2.33}$$

By plugging bilateral trade flows in the trade balance condition above, normalizing nominal wage in all other countries to unity $w_i = 1 \,\forall i \neq j$, along with simplifying by $[\tilde{\sigma}_j^{\omega}(\tilde{w})]^{-1}$ and $L_j$, and rearranging, I obtain the following general equilibrium expression of nominal wage in country $j$:

$$w_j = (\frac{\Psi_j}{\tilde{\Psi}_{Row}})^{\frac{1}{(\epsilon_1^F - 1)}} \Lambda^{\frac{1}{(\epsilon_1^F - 1)}}, \tag{2.34}$$

---

[34]Putting aside the common (RMS) channel.

where $\tilde{\Psi}_{Row} \equiv \sum_{i=1,i\neq j}^{(N-1)}(\frac{L_i}{L_\omega})\Psi_i$ is the weighted average degree of remoteness of all destinations in the World, excluding $j$, and $\Lambda$ is a general equilibrium object that is orthogonal to bilateral tariffs, as shown below:

$$\Lambda = \frac{\acute{\Psi}_j}{\Phi_j^{OUT}}\frac{(\Phi_j^{IN})^{-1}}{\acute{\Psi}_{Row}}, \quad \begin{cases} (\Phi_j^{IN})^{-1} = \sum_{i=1,i\neq j}^{(N-1)}(\frac{L_i}{L_\omega})\tau_{ij}^{\epsilon_2^F} f_{ij}^{\epsilon_5^F} \\ \Phi_j^{OUT} = \sum_{i=1,i\neq j}^{(N-1)}(\frac{L_i}{L_\omega})\tau_{ji}^{\epsilon_2^F} f_{ji}^{\epsilon_5^F} \end{cases} \tag{2.35}$$

Notice that $\acute{\Psi}_j = \frac{\Psi_j}{w_j}$ refers to the remoteness index of country $j$ (as an origin) from all other destinations, derived in equation (2.10), adjusted by its domestic cost of labor (and thus orthogonal to $w_j$). Moreover, $\Phi_j^{OUT}$ is an equivalent measure of the country $j$'s outward multilateral resistance term. As they are both decreasing in bilateral tariffs, the ratio $\frac{\acute{\Psi}_j}{\Phi_j^{OUT}}$ is thus orthogonal to $\tau_{ji}$. Similarly, $\acute{\Psi}_{Row}= \sum_{i=1,i\neq j}^{(N-1)}(\frac{L_i}{L_\omega})\Psi_i$ is the weighted average outward multilateral resistance term of all other countries. Moreover, $(\Phi_j^{IN})^{-1}$ is an inverse measure of country $j$'s inward multilateral resistance term. Hence, as they are both decreasing in bilateral tariffs, their ratio is thus orthogonal to $\tau_{ij}$.

## C.4 The "EEM" method: a detailed explanation

### C.4.1 Objective of the "EEM" method

As stressed in the main text, added flexibility in preferences raises tractability issues. Thus, in order to gain in flexibility without losing in tractability, I resort to a new and simple method that I call the "Exponent Elasticity Method" (EEM, hereafter). The objective of this simple method is to deliver a tractable solution for the general equilibrium price aggregator despite added flexibility in preferences. The starting point is the partial equilibrium expression of the price aggregator initially provided in equation (2.11):

$$|\eta_j| = (w_j L_j)(\frac{w_j L_j}{Y_\omega})^{-1}\sum_{i=1}^{N} c_i^E(\frac{w_i L_i}{Y_\omega})f_{ij}^{-1}\underbrace{\int_{\varphi_{ij}^*}^{+\infty} \frac{p_{ij}(\varphi)}{w_j}|v'(\frac{p_{ij}(\varphi)}{w_j})|\,dG(\varphi)}_{\mathbf{I}}$$

where $c_i^E = [\,\sigma_i^\omega(\tilde{w})\,w_i\,(\alpha_e + \alpha_{ii}\,P(\varphi \geq \varphi_{ii}^*) + P(\varphi \geq \varphi_{ij}^*) + \sum_{j=1}^{(N-2)}\alpha_{ik}\,P(\varphi \geq \varphi_{ij}^*))\,]^{-1}$ is a proxy for entry conditions in country $i$, $\alpha_e = \frac{F_e}{f_{ij}}$, $\alpha_{ii} = \frac{f_{ii}}{f_{ij}}$, and $\alpha_{ik} = \frac{f_{ik}}{f_{ij}}$.

Clearly, the mathematical challenge here consists in solving for the above integral (**I**) without specifying a functional form of the sub-utility function. Assuming that the productivity distribution is unbounded Pareto is necessary, but not sufficient for the above integral to be solved. Indeed, this further requires that the sub-utility function should exhibit a constant demand elasticity which, in turn, can be used as a constant for integrating. Such simplicity is only possible under homothetic CES preferences, which is the unique case where it is possible to solve for this integral. Since the flexible family of preferences considered in this chapter encompasses the homothetic CES and a non-homothetic alternative allowing the demand elasticity to vary with prices and income levels, it is then impossible to solve for integral (**I**) under such added flexibility in preferences.

Given the impossibility to solve for the integral in the current setting, the key idea that the "EEM" method proposes is to locally approximate the integral (**I**) around the equilibrium with a multiplicative equivalent which has a finite number of determinants, such as the exponent of each determinant embodies the elasticity of [ $\frac{\bar{p}_{ij}(\varphi)}{w_j} |v'(\frac{\bar{p}_{ij}(\varphi)}{w_j})|$ ] with respect to it. This requires a five-step procedure that I explain in detail as follows.

## C.4.2 A Five-step procedure:

### Step 1. Rewrite the integral (I) using unbounded Pareto:

By invoking this assumption, it is readily verified that that $P(\varphi \geq \varphi_{ij}^*) = \varphi_{ij}^{*-\theta}$. Using this, integral (**I**) can be rewritten as:

$$\mathbf{I} = \varphi_{ij}^{*-\theta} \underbrace{\int_{\varphi_{ij}^*}^{+\infty} \frac{p_{ij}(\varphi)}{w_j}|v'(\frac{p_{ij}(\varphi)}{w_j})| \gamma_{ij}(\varphi) \, d\varphi}_{\mathbf{I_0}} \qquad (2.36)$$

where $\gamma_{ij}(\varphi) = \frac{g(\varphi)}{P(\varphi \geq \varphi_{ij}^*)}$ is the productivity distribution conditional on successful penetration of country $j$'s market for firms based in country $i$. Clearly, the unique difference between the initial integral (**I**) and new integral (**I$_0$**) is that this latter is expressed using the conditional productivity distribution $\gamma_{ij}(\varphi)$.

### Step 2. Approximate integral (I$_0$) with a multiplicative equivalent:

Now by recalling that under indirectly-separable preferences, the price elasticity of demand is equal to the elasticity of the marginal sub-utility: $\sigma(\frac{p}{w}) = -\frac{v''(\frac{p}{w})\frac{p}{w}}{v'(\frac{p}{w})} > 1$, and that the elasticity

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of the export price with respect to each of its determinants is given by: $\rho_1 = \frac{dlog\ p_{ij}(\varphi)}{dlog\ w_i}$, $\rho_2 = \frac{dlog\ p_{ij}(\varphi)}{dlog\ \tau_{ij}}$, $\rho_3 = \frac{dlog\ p_{ij}(\varphi)}{dlog\ w_j}$, and $\rho_4 = \frac{dlog\ p_{ij}(\varphi)}{dlog\ \varphi}$, the integral ($\mathbf{I}_0$) can be locally approximated as follows:

$$\mathbf{I}_0 \equiv w_i^{\bar{\rho}_1(1-\bar{\sigma}_{ij}(w_j))}\ \tau_{ij}^{\bar{\rho}_2(1-\bar{\sigma}_{ij}(w_j))}\ w_j^{(1-\bar{\rho}_3)(\bar{\sigma}_{ij}(w_j)-1)}\ \bar{\varphi}_{ij}(\varphi_{ij}^*)^{|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)} \qquad (2.37)$$

Since operating profits are monotonically increasing in productivity, and exporting from country $i$ to country $j$ involves not only, a variable trade cost $\tau_{ij}$, but also a fixed cost $f_{ij}$, the equilibrium export cutoff $\varphi_{ij}^*$ exists and is unique. This, in turn, ensures that this local approximation (around the trade equilibrium) delivers a unique multiplicative equivalent to integral $\mathbf{I}_0$.

## Step 3. Obtain a final expression of integral (I) using this of ($\mathbf{I}_0$) and unbounded Pareto:

Let us now recall that unbounded Pareto distribution gives rise to constant mean-to-min ratio: $\bar{\varphi}_{ij} = \frac{\theta}{\theta-1}\ \varphi_{ij}^*$. By plugging the multiplicative equivalent of integral ($\mathbf{I}_0$) from equation (2.37) into the initial expression of integral ($\mathbf{I}$) in equation (2.36) and invoking this practical property of unbounded Pareto, I obtain the following multiplicative equivalent for integral ($\mathbf{I}$):

$$\mathbf{I} \equiv \kappa\ w_i^{\bar{\rho}_1(1-\bar{\sigma}_{ij}(w_j))}\ \tau_{ij}^{\bar{\rho}_2(1-\bar{\sigma}_{ij}(w_j))}\ w_j^{(1-\bar{\rho}_3)(\bar{\sigma}_{ij}(w_j)-1)}\ \varphi_{ij}^{*[\ |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)-\theta\ ]} \qquad (2.38)$$

where $\kappa = (\frac{\theta}{\theta-1})^{|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1)}$ is a constant.

## Step 4. Approximate the partial equilibrium export cutoff $\varphi_{ij}^*$ with an explicit multiplicative equivalent:

Let us first recall that the export cutoff $\varphi_{ij}^*$ is endogenous and is defined as the implicit solution of the zero profit condition on the export market: $\pi_{ij}(\varphi_{ij}^*)=0$

$$\varphi_{ij}^*:\ [p_{ij}^*(\varphi_{ij}^*) - w_i\tau_{ij}(\varphi_{ij}^*)^{-1}]\frac{|v'(\frac{p_{ij}^*(\varphi_{ij}^*)}{w_j})|}{|\eta_j|}L_j - w_if_{ij} = 0 \qquad (2.39)$$

Now by isolating the firm-specific component of the operating profit on the left hand-side, approximating it in a multiplicative way, as in equation (2.38), and rearranging, I obtain the following explicit equivalent of the partial equilibrium export cutoff:

$$\varphi_{ij}^* \equiv \tau_{ij} \ w_i^{\frac{[1-\rho_1^*(1-\sigma_{ij}^*(w_j))]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)}} \ w_j^{-\frac{[(1-\rho_3^*)(\sigma_{ij}^*(w_j)-1)+1+\delta^*]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)}} \ f_{ij}^{\frac{1}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)}} \ L_j^{-\frac{1}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)}} \ |\eta_j|^{\frac{1}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)}}$$

(2.40)

where the superscript (*) refers to the firm at the export cutoff, and $\delta^* = - \frac{dlog \ \sigma_{ij}^*(w_j)}{dlog \ w_j}$ is the absolute value of the elasticity of the price elasticity of demand, faced by the cutoff exporter in country $i$, with respect to per-capita income in destination $j$.

### Step 5. Solving for the general equilibrium price aggregator $|\eta_j|$ :

Now by plugging the explicit equivalent of the export cutoff from equation (2.40) in the expression of integral (**I**) in equation (2.38), this latter can be expressed solely as a function of ($w_i$, $\tau_{ij}$, $w_j$, $f_{ij}$, $L_j$, $|\eta_j|$ ). Then, by plugging this latter in the partial equilibrium expression of the price aggregator given by equation (2.11), and rearranging, I obtain the following general equilibrium expression of the price aggregator:

$$|\eta_j| \equiv w_j^{(\frac{1+\epsilon_3^F}{1-\epsilon_7^F})} \ L_j^{(\frac{1+\epsilon_3^F}{1-\epsilon_7^F})} \ (\frac{w_j L_j}{Y_w})^{\frac{1}{(\epsilon_7^F-1)}} \ \Psi_j^{\frac{1}{(\epsilon_7^F-1)}}$$

(2.41)

where $\Psi_j = [\sum_{i=1}^{N}(\frac{w_i L_i}{Y_w})f_{ij}^{(\epsilon_5^F-1)}\tau_{ij}^{\epsilon_2^F}w_i^{\epsilon_1^F}]^{-1}$, and the explicit expressions of the above exponents are given by:

$$\begin{cases} \epsilon_1^F = \bar{\rho}_1(1-\bar{\sigma}_{ij}(w_j)) + [ \ [|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1) - \theta] \ \frac{[1-\rho_1^*(1-\sigma_{ij}^*(w_j))]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} \ ] < 0 \\ \epsilon_2^F = \bar{\rho}_2( \ 1-\bar{\sigma}_{ij}(w_j) \ ) + [ \ |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1) - \theta \ ] = -\theta < 0 \\ \epsilon_3^F = (1-\bar{\rho}_3)( \ \bar{\sigma}_{ij}(w_j)-1 \ ) + [ \ [ \ \theta - |\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1) \ ] \ \frac{[(1-\rho_3^*)(\sigma_{ij}^*(w_j)-1)+1+\delta^*]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} \ ] > 1 \\ \epsilon_5^F = -\frac{[ \ \theta-|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1) \ ]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} < 0 \\ \epsilon_7^F = -\frac{[ \ \theta-|\bar{\rho}_4|(\bar{\sigma}_{ij}(w_j)-1) \ ]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} < 0 \end{cases}$$

(2.42)

# C.5 Solving for trade margins in general equilibrium

Let us start with writing an approximate equivalent of firm-level export revenues (intensive margin) in partial equilibrium:

$$r_{ij}(\varphi) \equiv w_i^{\epsilon_1} \ \tau_{ij}^{\epsilon_2} \ w_j^{(\epsilon_3+1)} \ \varphi^{\epsilon_4} \ L_j \ |\eta_j|^{-1} \tag{2.43}$$

where the exponents ($\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4$) are respectively given by:
$\epsilon_1 = \rho_1 \ ( \ 1 - \sigma_{ij}(w_j) \ )$; $\epsilon_2 = \rho_2 \ ( \ 1 - \sigma_{ij}(w_j) \ )$; $\epsilon_3 = (1 - \rho_3) \ ( \ \sigma_{ij}(w_j) - 1 \ )$; $\epsilon_4 = |\rho_4| \ ( \ \sigma_{ij}(w_j) - 1 \ )$.

Now by plugging the general equilibrium price aggregator from equation (2.41) in the above expression and rearranging, I solve for the intensive margin of trade in general equilibrium:

$$r_{ij}(\varphi) \equiv w_i^{\epsilon_1} \ \tau_{ij}^{\epsilon_2} \ \varphi^{\epsilon_4} \ w_j^{\zeta-1} \ \underbrace{\left(\frac{w_j L_j}{Y_w}\right)^{\frac{1}{(1-\epsilon_7^F)}} \Psi_j^{\frac{1}{(1-\epsilon_7^F)}}}_{Chaney(2008)} \tag{2.44}$$

where $\zeta = 1 + [ \ (1 + \epsilon_3) - (\frac{1+\epsilon_3^F}{1-\epsilon_7^F}) \ ]$ captures the gross positive income effect on firm-level export revenues.

Similarly, by plugging the general equilibrium price aggregator from equation (2.41) in the explicit partial equilibrium expression of the export cutoff, I can solve for the extensive margin of trade in general equilibrium as follows:

$$\varphi_{ij}^* \equiv w_i^{\left(\frac{1-\epsilon_1^*}{\epsilon_4^*}\right)} \ \tau_{ij} \ f_{ij}^{\frac{1}{\epsilon_4^*}} \ w_j^{\Delta_5} \ L_j^{\Delta_6} \ \underbrace{\left(\frac{w_j L_j}{Y_w}\right)^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}} \Psi_j^{\frac{1}{\epsilon_4^*(\epsilon_7^F-1)}}}_{Chaney(2008)} \tag{2.45}$$

where $\Delta_5 = \frac{1}{\epsilon_4^*} \ [ \ (\frac{1+\epsilon_3^F}{1-\epsilon_7^F}) - (\epsilon_3^* + 1 + \delta^*) \ ]$ and $\Delta_6 = \frac{1}{\epsilon_4^*} \ [ \ (\frac{1+\epsilon_6^F}{1-\epsilon_7^F}) - 1 \ ]$, $\epsilon_6^F = \frac{[ \ \theta - |\bar\rho_4|(\bar\sigma_{ij}(w_j)-1) \ ]}{|\rho_4^*|(\sigma_{ij}^*(w_j)-1)} > 0$

and $\delta^* = - \frac{dlog \ \sigma_{ij}^*(w_j)}{dlog \ w_j}$; $\epsilon_1^* = \rho_1^*( \ 1 - \sigma_{ij}^*(w_j) \ )$; $\epsilon_3^* = (1 - \rho_3^*)( \ \sigma_{ij}^*(w_j) - 1 \ )$; $\epsilon_4^* = |\rho_4^*| \ ( \ \sigma_{ij}^*(w_j) - 1 \ )$.

# Chapter 3

# International Trade under Monopolistic Competition beyond the CES

## 3.1 Introduction

Recent empirical research in international trade has revealed that the price elasticity of demand varies significantly across firms. This central observation stands in stark contradiction with CES demand which imposes a constant demand elasticity under monopolistic competition. This suggests that it is crucial to depart from the homothetic CES to examine gains from trade under more realistic patterns of price sensitivity. In theoretical trade literature, transition to non-CES preferences took place gradually. While in earlier departures from the CES, a large body of work has focused on specific types of preferences, only few recent papers have proposed more general demand systems encompassing prominent alternatives to the CES case.

However, increased generality raises tractability issues, which in turn requires some concessions. Accordingly, a large body of work always impose a restriction on the curvature of demand: they generally assume that demand is "sub-convex". They also abstract from fixed costs of accessing markets. Instead, they resort to assuming the existence of a choke price to ensure self-selection of firms into markets, see e.g. (Melitz and Ottaviano, 2008; Bertoletti, Etro, and Simonovska, 2018; Arkolakis et al., 2018; Feenstra, 2018; Fally, 2019). Alternatively, other papers squarely focus on a specific type of preferences, while keeping demand curvature unrestricted and taking fixed costs in due account (Zhelobodko et al., 2012; Mrázová and Neary, 2017). Both existing modeling approaches exhibit the same limitations for welfare analysis. First, they imply that at least one channel of welfare gains from trade is ruled out. Second, they preclude the welfare implications of the curvature of demand and those of the type pf preferences to be studied in a common framework.

In contrast, this is what the current chapter aims for. Towards this goal, I propose a theoretical framework combining standards assumptions on the supply side with a flexible and restriction-free demand system. In particular, the supply side is identical to (Melitz, 2003; Chaney, 2008; Arkolakis, Costinot, and Rodriguez-Clare, 2012; Arkolakis et al., 2018; Fally, 2019), and incorporates monopolistic competition, firm heterogeneity and Pareto distribution of firm productivity. However, here the novel aspect is that I consider a flexible demand system which encompasses two commonly used families of preferences (directly- and indirectly-separable), and nests two alternative curvatures of demand (beyond the CES, demand can be either "sub-convex" or "super-convex"). By imposing standard restrictions on the supply side, the modeling approach proposed in this chapter offers then a theoretically clean way to examine the welfare implications of these alternative assumptions on the curvature of demand and the nature of preferences.

The goal of this chapter is to examine three major questions in trade theory with heterogeneous firms under more realistic consumer behavior than allowed by CES preferences. First, does demand curvature play a role in determining the toughness of firm selection and the degree of their partitioning by export status? Second, under which demand conditions, net variety gains and gains from selection coexist in general equilibrium? Third, to which extent the curvature of demand and the type of preferences determine the magnitude of the gains from trade?

The main finding of this chapter is that demand curvature plays a crucial role in driving comparative statics results, shaping the structure of the gains from trade as well as determining the magnitude of these gains, whereas the type of preferences affects only marginally the results. In particular, taking the CES as a boundary case, I show that when demand is sub-convex, selection into markets is more relaxed, the partitioning of firms by export status is more pronounced, net variety gains and gains from selection coexist, and gains from trade are smaller than those obtained under CES demand. I also emphasize that the type of preferences plays only a second-order role. For instance, under sub-convex demands, directly-separable preferences provide an upper bound for the gains from trade, while indirectly-separable preferences provide a lower bound. All these patterns are reversed when demand is super-convex.

The rest of this chapter is organized as follows. Section 2 describes in detail the general demand system considered in the current chapter. Section 3 offers a simple characterization of demand curvature. Section 4 illustrates novel comparative statics results. Section 5 examines the gains from trade and highlights novel welfare implications of demand curvature. The last section concludes. Appendix D provides the proofs for the main results as well as a detailed explanation of the "EEM" method.

## 3.2  A Flexible Demand System

This section describes the generalized Gorman-Pollak demand system considered by Fally (2019) and recalls sufficient conditions under which such demand system is integrable, following Fally (2018). It also proposes a simple and useful parameterization that allows for a subtle nesting of directly- and indirectly-separable preferences.

### 3.2.1  Generalized Gorman-Pollak Demand

Consider a representative consumer whose income $w$ is entirely spent on a set of varieties, denoted by $\Omega$. For each variety $\omega \in \Omega$, suppose that demand is determined by its price $p_\omega$, consumer income $w$ and an aggregator $\Lambda$ :

$$x_\omega = Q(\Lambda)\, D_\omega\big(V(\Lambda)\, \frac{p_\omega}{w}\big), \qquad (3.1)$$

where $\Lambda = \Lambda(p, w)$ is itself a scalar function off all prices and income, homogeneous of degree zero in $(p, w)$. $\Lambda$ is implicitly determined by the budget constraint, i.e. it is the implicit solution of :

$$\int_{\omega \in \Omega} p_\omega\, Q(\Lambda)\, D_\omega\big(V(\Lambda)\, \frac{p_\omega}{w}\big)\, d\omega = w \qquad (3.2)$$

### 3.2.2  Conditions for integrability

Integrability conditions can be defined as regularity restrictions that are sufficient to ensure that a demand system can be derived from a rational utility maximizing consumption behavior (Fally, 2018).[1]  Following Fally (2018), the generalized Gorman-Pollak demand system is integrable under the following conditions:

1. $D_\omega$ is differentiable and sufficiently downward slopping and elastic, i.e. $\sigma_\omega = -\frac{d\log D_\omega}{d\log p_\omega} > 1$.

2. Q and V are differentiable and $[\varepsilon_V\, \sigma_\omega - \varepsilon_Q]$ has the same sign for all $\Lambda$ and $\frac{p_\omega}{w}$.

3. For any set of normalized prices $\frac{p_\omega}{w}$, equation (1) admits a solution in $\Lambda$,

   where $\varepsilon_Q$ and $\varepsilon_V$ denote the elasticity of Q and V with respect to $\Lambda$.

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[1]See Fally (2018) for further details on integrability conditions.

### 3.2.3 A Useful Parameterization

In order to nest indirectly- and directly-separable preferences in a simple way, I propose the following parameterization: $Q(\Lambda)=\Lambda^{-\beta}$ and $V(\Lambda)=\Lambda^{\alpha}$. It is without loss of generality to assume that $\alpha$ and $\beta$ are both dummies (whose values can be either 0 or 1), such as the case ($\alpha=0$ and $\beta=1$) corresponds to indirectly-separable preferences, while directly-separable preferences correspond to ($\alpha=1$ and $\beta=0$). This implies that the difference $[\varepsilon_V \sigma_\omega - \varepsilon_Q]$ is positive under both cases, which is sufficient to ensure integrability.

# 3.3 Characterization of Demand Curvature

At this stage, the flexibility of the demand system described above is reflected in how departing from homothetic CES preferences allows the demand elasticity to vary either with normalized prices when preferences are indirectly-separable, or with consumption levels when preferences are directly-separable. However, such flexibility raises the following question: under which conditions the demand elasticity increases, decreases, or ceases to vary with normalized prices or individual consumption?

This section aims at addressing this question, and by doing so it completes the characterization of the demand side of the model. Towards this goal, the current section draws heavily on Mrázová and Neary (2017). For instance, it adopts their approach that they call "a firm's eye view of demand".

### 3.3.1 A simple Measure of Demand Curvature

Following Mrázová and Neary (2017), the starting point is the fact that a monopolistically competitive firm takes the demand function it perceives as given. As this approach is partial-equilibrium by definition, it is more convenient to express the Gorman-Pollak demand in equation (1) solely as a function of the price: $x(\Lambda, \frac{p}{w}) \equiv x(D(p)) \equiv x(p)$.[2]
Let us recall that the price elasticity of demand is given by:

$$\sigma = -\frac{d\log x(p)}{d\log p} = -\frac{d\log D(p)}{d\log p} > 0$$

As in Mrázová and Neary (2017), I measure the convexity of demand using the elasticity of the slope of direct demand :

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[2]I drop the variety subscript for expositional simplicity. Since consumer income and the aggregator are taken as given, they are dropped from this simplified demand function, so as to concentrate on the relationship between $x$ and $p$.

$$\zeta = -\frac{d \log D'(p)}{d \log p}$$

Now in order to measure demand curvature in a simple and unit-free way, I work with the "superelasticity" of Kimball (1995), defined as the elasticity with respect to price of the elasticity of demand:

$$S = \frac{d \log \sigma}{d \log p} = (1 + \sigma) - \zeta$$

Clearly, the sign of the "superelasticity" S is pinned down by the relationship between the elasticity $\sigma$ and the convexity $\zeta$ of the direct demand function, the so-called "demand manifold" by Mrázová and Neary (2017). Interestingly, the "superelasticity", due to Kimball (1995), can be considered as a sufficient statistic for demand curvature. That is, its sign clearly indicates whether demand is CES, sub-convex, or super-convex :

$$\text{demand is} \begin{cases} \text{sub-convex} & \text{if } S > 0 \\ CES & \text{if } S = 0 \\ \text{super-convex} & \text{if } S < 0 \end{cases}$$

A peculiar property of CES demand is that it exhibits an exogenous demand elasticity, which is reflected by zero "superelasticity". It is then convenient to take the CES case as a benchmark to characterize both alternative curvatures in a simple way. Following Mrázová and Neary (2017), a demand function is locally sub-convex if for the same level of demand elasticity $\sigma$, it exhibits a lower degree of convexity $\zeta$ as compared to the CES case. Similarly, a demand function is super-convex if it is more convex than the CES at a given level of demand elasticity.

Panel A. The admissible region in $(\zeta, \sigma)$ space    Panel B. The Super- and Sub-convex regions
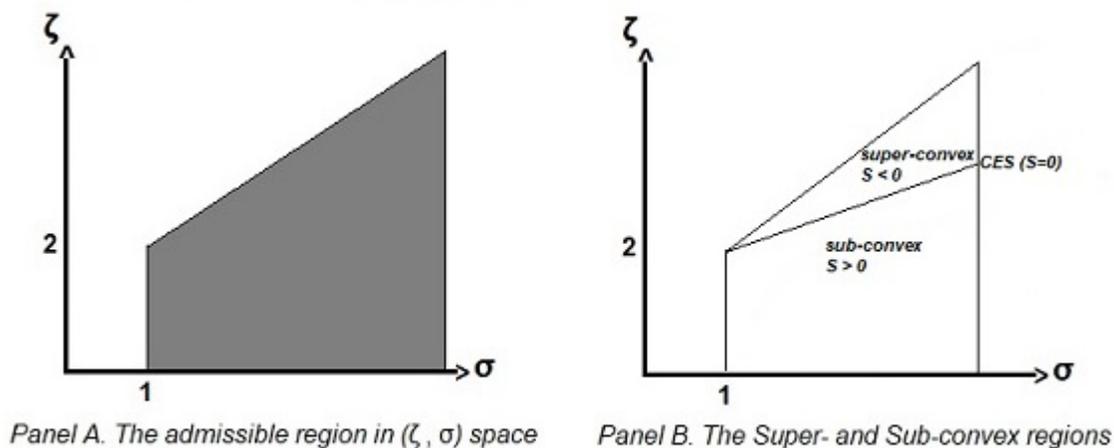
It is now possible to graphically illustrate these three possible cases of demand curvature in a simple and compact way in the $(\zeta, \sigma)$ space. Before proceeding, I resort to the first- and second-order conditions of profit maximization to impose restrictions on the values of $\sigma$ and $\zeta$, as in Mrázová and Neary (2017).

As indicated in the beginning of this section, I consider a monopolistically competitive firm that takes the direct demand function it perceives as given and maximizes its profit accordingly. From the first-order condition, a positive price-cost margin implies that the price elasticity of demand must be greater than one:

$$(p - \varphi^{-1})x' + x = 0 \Rightarrow \sigma > 1 \tag{3.3}$$

where $\varphi^{-1}$ is the marginal cost of a $\varphi$-productivity firm.

From the second-order condition, decreasing marginal revenue requires that the degree of demand convexity $\zeta$ must be smaller than twice the demand elasticity:

$$2x' + (p - \varphi^{-1})x'' < 0 \Rightarrow \zeta < 2\sigma \tag{3.4}$$

As in Mrázová and Neary (2017), the above restrictions imply an admissible region in $(\zeta, \sigma)$ space, as shown by the shaded region in Figure 1, panel A.[3]

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[3]Ideally, one would borrow an upper bound estimate of the demand elasticity from the empirical literature, and thus concentrate on a more realistic part of the admissible region.

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As illustrated in Figure 1, panel B, the CES line (whose equation is given by $\zeta = 1+\sigma$) divides the admissible region in two. While points located above the CES line correspond to super-convex demands, points below the CES boundary correspond to the case of sub-convex demands. Within the sub-convex region, the "superelasticity" is positive (S > 0), which implies that the demand elasticity increases in price (or, equivalently, decreases with consumption) if and only if demand is sub-convex.

In contrast, the super-convex region is characterized with a negative "superelasticity" (S < 0). This implies that the demand elasticity decreases in price (or, equivalently, increases with consumption) when demand is super-convex. Clearly, CES demand is a boundary case under which the demand elasticity does not vary with price or consumption levels, as reflected by the zero "superelasticity" (S=0). Hence, moving along the CES line only changes the value of the elasticity of demand while preserving its exogenous nature.

## 3.4    Illustrating Comparative Statics Results

### 3.4.1    Variable markups and Relative pass-through

Again, following Mrázová and Neary (2017), the starting point of the analysis is the fact that a monopolistically competitive firm producing a variety $\omega$ at a $\varphi^{-1}$ marginal cost takes the price aggregator $\Lambda$ as given. Whether it perceives the partial equilibrium demand function in its direct or inverse form, the first-order condition of profit maximization in equation (3.3) yields a unique optimal pricing rule:

$$p(\varphi) = \varphi^{-1} \, m(\varphi), \tag{3.5}$$

where $m(\varphi) = \frac{\sigma(\varphi)}{\sigma(\varphi)-1}$ is the markup set by the $\varphi$-productivity firm.

Let us now denote by $\eta(\varphi) = -\frac{dlog\ p(\varphi)}{dlog\ \varphi}$ the absolute value of the elasticity of price with respect to firm productivity :

$$\eta(\varphi) \equiv 1 + \frac{dlog\ m(\varphi)}{dlog\ \sigma(\varphi)} \, S$$

The above expression clearly shows how demand curvature governs the degree of completeness of the relative pass-through.[4] Under the CES case, demand elasticity is exogenous ($\forall \varphi$, $\sigma(\varphi) = \sigma$), the "superelasticity" of demand S collapses to zero, and so $\eta$ is fixed to unity : $\eta = 1$. Departing from the CES benchmark allows then for a positive or negative deviation of $\eta(\varphi)$ from unity that is pinned down by the sign of the "superelasticity" S :

$$
\eta(\varphi) \begin{cases} < 1 & \text{if } S > 0 \\ = 1 & \text{if } S = 0 \\ > 1 & \text{if } S < 0 \end{cases}
$$

Therefore, when demand is sub-convex, a higher productivity, which other things equal implies lower price (or equivalently, higher individual consumption), is associated with a lower demand elasticity and so, a higher markup, implying less than 100 percent pass-through. By contrast, when demand is super-convex, a higher productivity is associated with a higher demand elasticity and so, a lower markup, implying more than 100 percent pass-through.

### 3.4.2 Demand Curvature, Firm Selection, and Partitioning of Firms

In order to examine the role that demand curvature plays in determining the toughness of firm selection and the partitioning of firms by export status,[5] I proceed in two steps.

First, I show how demand curvature determines the nature of the elasticity of a firm's operating profit with respect to its productivity; whether it is constant, increasing or decreasing with firm productivity. Then, I graphically illustrate the results in a compact way and infer new implications of demand curvature for firm selection and the partitioning of firms by export status.

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[4]Notice that the final expression of $\eta(\varphi)$ has been simplified using the fact that an increase in productivity, other things equal, must lower a firm's price (and equivalently, increases individual consumption of the variety it supplies).

[5]This concept initially introduced by Melitz (2003) refers to the fact that exporting is more selective than serving the domestic market, and so only more productive firms export. This implies then that exporters are on average more productive than non-exporters.

Panel A. Variable vs Constant Elasticity of Operating Profits

Panel B. Demand Curvature, Selection and Firm Partitioning

## 1. Constant vs Variable productivity elasticity of operating profits

Again, as in Mrázová and Neary (2017), the starting point is the fact that a $\varphi$ productivity firm, which engages in monopolistic competition, takes the demand function as given and maximizes its profit accordingly. Its operating profit can be written in an approximate way:[6]

$$\pi^o(\varphi) = \frac{p(\varphi)\, x(\varphi)\, L}{\sigma(\varphi)} \equiv \frac{p(\varphi)^{1-\sigma(\varphi)}}{\sigma(\varphi)}\, L \equiv \varphi^{e(\varphi)}\, L, \tag{3.6}$$

where $e(\varphi) = \frac{d\log \pi^o(\varphi)}{d\log \varphi} = \eta(\varphi)[\sigma(\varphi) - 1 + S] > 0$.

It is readily verified that the operating profit is always monotonically increasing in firm productivity regardless of the curvature of demand. However, this latter plays a critical role in determining whether the pace at which the logarithm of operating profits increases with this of firm productivity is constant, or variable.

Clearly, the CES case is very special: as the demand elasticity is exogenous ($\forall \varphi$, $\sigma(\varphi) = \sigma$), the "superelasticity" collapses to zero (S=0), and the relative pass-through is complete ($\forall \varphi$, $\eta(\varphi) = 1$). This yields a constant elasticity of operating profits with respect to firm productivity (both in logarithms): $\log \pi^o(\varphi)$ always increases with $\log \varphi$ at a constant pace regardless of the firm's productivity level ($\forall \varphi$, $e(\varphi) = \sigma - 1$). This is illustrated by the upward sloping "CES" line in Figure 2, panel A.

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[6]Notice that the price aggregator $\Lambda$ and individual income $w$ are absent in this simplified version of the Gorman-Pollak demand function described in equation (3.1). In deed, as these latter are assumed to be taken as given, and the focus is squarely on the relationship between operating profits and firm productivity, I abstract from both of them in the above expression for expositional simplicity.

Therefore, departing from CES demand allows for a variable elasticity of operating profits: $e(\varphi)$ may then increase or decrease with firm productivity $\varphi$. This implies that the pace at which $\log \pi^o(\varphi)$ increases with $\log \varphi$ may be faster or slower, as compared to the CES benchmark, depending on demand curvature.

To show this in a simple way, let us define E as the productivity elasticity of the elasticity of operating profits:

$$E = \frac{dlog\, e(\varphi)}{dlog\, \varphi} = \frac{dlog\, \eta(\varphi)}{dlog\, \varphi} + \frac{dlog\, [\sigma(\varphi) - 1 + S]}{dlog\, \sigma(\varphi)} \frac{dlog\, \sigma(\varphi)}{dlog\, p(\varphi)} \frac{dlog\, p(\varphi)}{dlog\, \varphi} \tag{3.7}$$

After simplification, the final expression of E boils down to:

$$E \equiv -\eta S \begin{cases} < 0 & \text{if demand is sub-convex} \\ = 0 & \text{if demand is CES} \\ > 0 & \text{if demand is super-convex} \end{cases} \tag{3.8}$$

This reveals that $\log \pi^o(\varphi)$ is convex/concave in $\log \varphi$ when demand is super-convex/sub-convex, as shown in Figure 2, panel A. Here, the CES case arises again as a boundary for this comparative statics result. Visibly, under super-convex demands, a movement from left to right along the horizontal axis implies a relatively faster movement (as compared with the CES case) along the $\log \pi^o(\varphi)$ curve. By contrast, the same movement along the horizontal axis generates a relatively slower movement along the $\log \pi^o(\varphi)$ curve when demand is sub-convex.

The underlying economics are simple: when demand is super-convex, the demand elasticity increases in firm productivity, and this has three implications for firm profits. First, more productive firms set lower markups, and so a higher productivity induces a more than proportional reduction in price ($\forall \varphi, \eta(\varphi) > 1$). This reveals that under super-convex demands, a firm's initial level of price competitiveness (given by its productivity level) is magnified by lower markups. Second, consumers are more reactive to price variations of varieties supplied by more productive firms. Third, the markup rate ($\frac{1}{\sigma(\varphi)}$) is lower for more productive firms. Combination of the first two effects clearly shows that the super-convex aspect of demand magnifies the sensitivity of firm revenues to firm productivity. As illustrated in Figure 2, panel A, this generates a relatively faster response of operating profits to firm productivity,[7] despite the fact that higher productivity implies lower markup rate.

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[7]Both in logarithmic terms.

These patterns are reversed when demand is sub-convex: more productive firms face lower demand elasticity, and so they set higher markups and enjoy higher markup rates ($\frac{1}{\sigma(\varphi)}$). Nevertheless, facing less elastic demand implies that consumers are less sensitive to price variations of the varieties they supply. On top of that, setting higher markups dampens the initial level of price competitiveness of these firms. Combination of these last two effects reveals that the sub-convex aspect of demand dampens the sensitivity of firm revenues to firm productivity. Given the dominance of the revenues effect, this immediately implies a relatively slower response of operating profits to firm productivity,[8] as shown in panel A of Figure 2.

Finally, the linearity of the profile of (the logarithm of) operating profits across firms under CES demand clearly shows that this latter is a boundary case. For instance, the CES is a special case where the demand elasticity does not vary with firm productivity. This peculiar property of CES demand has three implications: (i) a firm's level of price competitiveness is solely pinned down by its productivity level; (ii) the elasticity of firm revenues to firm productivity is constant ($\sigma$ - 1); and (iii) the markup rate is identical across firms ($\forall \varphi$, $\frac{1}{\sigma(\varphi)} = \frac{1}{\sigma}$). Such rigidities immediately ensure that the CES delivers an intermediate outcome.

## 2. Novel Implications of Demand Curvature for Firm Selection and Firm Partitioning

### 2.1 Demand Curvature and Firm Selection into the Domestic Market

As illustrated in Figure 2, panel B, for any given level of fixed cost of accessing the domestic market $f$, super-convex demands provide an upper bound for the domestic productivity cutoff $\varphi_d^*$,[9] whereas sub-convex demands provide a lower bound. Within these bounds, CES demand delivers an intermediate outcome: $\varphi_d^*(sub - convex) < \varphi_d^*(CES) < \varphi_d^*(super - convex)$. This reveals that firm selection is the toughest when demand is super-convex, whereas it is the easiest when demand is sub-convex. Within these two polar cases, the CES yields an intermediate degree of firm selection.

The economic force behind this (partial-equilibrium) result can be explained as follows. When demand is super-convex, the initial level of price competitiveness of more productive firms (implied by their initially high productivity levels) is magnified by lower markups. In addition to that, consumers are more sensitive to price variations of varieties supplied by this category of firms. Hence, as compared with the CES benchmark, the super-convex aspect of demand reinforces the allocation of larger market shares to more productive firms. This induces then a

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[8]Both in logarithmic terms.

[9]According to Melitz (2003), the domestic productivity cutoff corresponds to the productivity level required to make at least zero profits and successfully enter the domestic market.

relatively tougher competitive environment for low productivity firms. As compared to the CES case, setting higher markups makes these less productive firms even less price competitive. This implies then additional difficulty in capturing enough market shares to successfully enter the market, which is reflected by higher domestic cutoff under super-convex demands. All these patterns are reversed when demand is sub-convex.

## 2.2 Demand Curvature and Partitioning of Firms by Export Status

Now let us consider a simple case where the World is comprised of many symmetric countries, and accessing a foreign market via exporting involves only a fixed cost $f_x$. In the absence of variable trade costs,[10] I must assume that $f_x > f$, to ensure that firms are partitioned by export status as in Melitz (2003). That is, among successful entrants (firms with productivity $\varphi \geq \varphi_d^*$), only more productive firms export (a subset of firms with productivity $\varphi \geq \varphi_x^* > \varphi_d^*$).

Such partitioning of firms is quite standard in heterogeneous firms models. However, the novel idea I explore here is how demand curvature determines the degree of this partitioning of firms by export status. As illustrated in Figure 2, panel B, while the distance between the export and the domestic cutoffs $[\varphi_x^* - \varphi_d^*]$ is the smallest when demand is super-convex; it is the largest when demand is sub-convex. CES demand, again delivers an intermediate result:

$$[\varphi_x^* - \varphi_d^*] \text{ (super-convex)} < [\varphi_x^* - \varphi_d^*] \text{ (CES)} < [\varphi_x^* - \varphi_d^*] \text{ (sub-convex)}$$

The underlying economics are simple: when demand is super-convex, selection is relatively tougher (as compared to the CES benchmark) and only (relatively)[11] more productive firms successfully enter the domestic market. Hence, a relatively large subset of these very productive firms can export. Put differently, these firms are enough productive to successfully enter the domestic market despite tougher competitive conditions implied by super-convex demands. It follows then that a large fringe of these firms is enough price competitive to penetrate the export market. This reveals then that, as compared with the CES benchmark, the partitioning of firms by export status is less pronounced when demand is super-convex. This result is reversed when demand is sub-convex.

Finally, it is worth noting that these two novel comparative statics results are partial-equilibrium by definition.[12] Yet, the intuitive explanation for both results provides the basis for understand-

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[10]Here, I abstract from variable trade costs for expositional simplicity. However, they will be taken in due account in the general equilibrium trade model that I spell out in the next section.

[11]This refers to an immediate comparison with the CES benchmark.

[12]Thus far, I have worked with Mrázová and Neary (2017)'s "firm's eye view of demand" which is a partial equilibrium approach, as stated by the authors.

ing the general equilibrium behavior. For instance, in Section 5.4, I will show that both results hold in general equilibrium, and that they have crucial implications for the gains from trade.

# 3.5   Monopolistic Competition with Heterogeneous Firms under Generalized Demands

To illustrate the usefulness of the theoretical approach I have developed in previous sections, I turn next to apply it to a canonical model of international trade, a one-sector, one-factor, multi-country, general equilibrium model of monopolistic competition, where firms are heterogeneous in productivity levels, and countries are symmetric and separated by symmetric trade barriers, as in Melitz (2003).

More specifically, I assume throughout the model that the World economy is comprised of $N$ symmetric countries which share the same level of labor endowment $L$ and the same wage $w$. This latter is normalized to one ($w$=1) by choice of labor as numéraire. Each economy involves one sector supplying a continuum of horizontally differentiated varieties using labor as a unique production factor. Labor is immobile across countries, and serving foreign markets is only possible via exporting, and this involves both variable and fixed trade costs. Following Melitz (2003), I consider three possible scenarios of higher exposure to trade: (i) a (small) decrease in the variable trade cost; (ii) a (small) decrease in the fixed trade cost; and (iii) a (small) increase in the number of trading countries in the World economy.

I begin with a brief exposition of the supply-side of the model in **Section 5.1**. This latter draws heavily on Melitz (2003), with only one additional assumption: firm productivity is Pareto distributed. In **Section 5.2**, I first embed the general demand system (described in **Section I**) in the model. Then, I derive firm-level variables and spell out the equilibrium conditions. Then, in **Section 5.3**, I propose a new and simple method and show how it delivers tractable solutions in general equilibrium despite increased generality on the demand-side.

In **Section 5.4**, I show that the new comparative statics results (discussed in **Section 4.2**) hold in general equilibrium. In particular, I emphasize that demand curvature, by governing these comparative statics at the industry level, plays a critical role in determining the magnitude and the structure of the gains from higher exposure to trade. However, the type of preferences has only a second-order importance from a welfare standpoint. Finally, **Section 5.5** proposes a more granular analysis of the gains from trade. It examines three different scenarios of trade liberalization, and provides a firm-level explanation for the main result of this chapter.

### 3.5.1 Supply

Each country has an endogenous mass $M_e$ of monopolistically competitive firms that incur a sunk fixed cost $f_e$ to enter the market. Firms then endogenously enter up to the point at which aggregate profits net of the fixed entry cost, $f_e$, are zero. As in Melitz (2003), upon entry, firms draw their initial productivity level $\varphi$ from a common distribution g($\varphi$). This latter has positive support over [1, +∞] and has a continuous cumulative distribution G($\varphi$). I assume that G($\varphi$) is Pareto with the same shape parameter $\theta > 0$ around the World:

**Assumption A1 [Unbounded Pareto]** $\forall \varphi \in [1, +\infty]$, $G(\varphi) = 1 - \varphi^{-\theta}$, with $\theta > 0$.

Thus far, the above assumption is the unique restriction that I have imposed in the current model.[13] Specifically, I concentrate on the case where the Pareto distribution is unbounded above. Far from being a minor technical detail, this specific feature of the productivity distribution has three main benefits which are worth emphasizing.

First, this unique restriction on the supply side is sufficient to greatly simplify the analysis, while keeping the demand system very flexible and unrestricted. In particular, unbounded Pareto is the central assumption on which rests the simple method that I propose to obtain tractable solutions in general equilibrium under general demands. Second, as is well known, unbounded Pareto is a common distributional assumption in models of monopolistic competition featuring firm-level heterogeneity and CES preferences. Hence, imposing Assumption A1 ensures that the novel results highlighted in the current chapter are solely attributable to alternative assumptions about the curvature of demand and the type of preferences.

Third, in more recent trade models with heterogeneous firms incorporating non-CES preferences, some authors work with bounded Pareto distribution, which has important implications for the gains from trade. As demonstrated by Feenstra (2018), bounded Pareto is a sufficient condition for gains from (i) selection, (ii) variety, and (iii) reduction in domestic markups to coexist in general equilibrium. Hence, assuming instead that the Pareto distribution is unbounded above opens the door for a purely demand-driven condition for the coexistence of these gains in general equilibrium.

Accordingly, I will be able to properly address the following questions: under such standard assumptions on the supply side, what are the novel implications of the flexible demand system

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[13]Pareto distribution of firm productivity is obviously the most common assumption in models of monopolistic competition incorporating firm-level heterogeneity in productivity levels (Chaney, 2008; Melitz and Ottaviano, 2008; Feenstra, 2010; Feenstra, 2018; Arkolakis, Costinot, and Rodriguez-Clare, 2012; Arkolakis et al., 2018; Fally, 2019).

considered in this chapter for the gains from trade? What matters more from a welfare standpoint: the curvature of demand or the type of preferences? Clearly, this is a theoretically clean way to highlight the novel welfare predictions that can be derived by solely departing from the homothetic CES benchmark.

## 3.5.2 Trade Equilibrium

In this section, I characterize the trade equilibrium for arbitrary values of trade costs. I proceed in three steps. I first show how the general demand system, introduced in **Section 2**, shapes firm-level variables. Using these latter, I write then the equilibrium conditions more explicitly. Finally, I introduce a new and simple method that I call the "Exponent Elasticity Method" (EEM, hereafter), and I show how it delivers tractable solutions in general equilibrium under general demands.

### 1. Firm-level Variables

Following Melitz (2003), I assume that each firm must incur an overhead production cost $f$ (in labor units) to start producing for the domestic market. Serving foreign markets is only possible via exporting, and is more costly than operating on the domestic market. For instance, exporting involves two types of costs: a fixed cost of accessing foreign markets $f_x$, and a variable trade cost $\tau$ modeled in the standard iceberg formulation, whereby $\tau > 1$ units of a good must be shipped for 1 unit to arrive at destination.

Accordingly, for a firm with productivity $\varphi$, the constant marginal cost of serving the domestic, and export markets are respectively given by $\varphi^{-1}$ and $\tau \varphi^{-1}$. The first-order condition of profit maximization from equation (3.3) implies that a firm's pricing rule on these respective markets is given by:

$$
\begin{cases}
p_d(\varphi) = \varphi^{-1} \frac{\sigma(\varphi)}{\sigma(\varphi)-1} \\
p_x(\varphi) = \tau \varphi^{-1} \frac{\sigma(\varphi)}{\sigma(\varphi)-1}
\end{cases}
\tag{3.9}
$$

The above expressions of firm-level markups $\frac{\sigma(\varphi)}{\sigma(\varphi)-1}$ stem from a combination of firm-level heterogeneity in productivity levels on the supply side and flexibility in preferences on the demand side.

As stressed in **Section 3**, in the current setting, the demand elasticity always varies with firm productivity $\sigma(\varphi)$ except under the CES case which imposes a constant demand elasticity.[14] As to whether the demand elasticity increases or decreases with firm productivity, this hinges on the curvature of demand. As shown in **Section 3**, when demand is sub-convex, the price elasticity of demand is decreasing in firm productivity, and thus more productive firms charge higher markups as they face less elastic demand. In contrast, when demand is super-convex, the price elasticity of demand is increasing in firm productivity, and thus more productive firms charge lower markups since they face more elastic demand.

Now by rewriting the generalized Gorman-Pollak demand function in equation (3.1) more explicitly using the parameterization (described in **Section 2.3**), along with invoking the symmetry assumption at the country level and rearranging, the revenues earned from domestic sales and export sales to a given country can be, respectively, written as:[15]

$$
\begin{cases}
r_d(\varphi) = p_d(\varphi)^{1-\sigma(\varphi)}\, P^{a(\varphi)}\, L \\
r_x(\varphi) = p_x(\varphi)^{1-\sigma(\varphi)}\, P^{a(\varphi)}\, L
\end{cases}
\tag{3.10}
$$

where $L$ and $P$ denote the aggregate expenditure and the partial equilibrium price index in every country, and $a(\varphi) = \frac{\beta + \alpha\,\sigma(\varphi)}{\beta + \alpha\,\bar{\sigma}}$. Then, with the aid of the Lerner index,[16] I can simply express operating profits as revenues divided by the price elasticity of demand, as is standard in the literature:

$$
\begin{cases}
\pi_d^o(\varphi) = \frac{r_d(\varphi)}{\sigma(\varphi)} \\
\pi_x^o(\varphi) = \frac{r_x(\varphi)}{\sigma(\varphi)}
\end{cases}
\tag{3.11}
$$

Finally, taking into account the presence of fixed costs, domestic and export profits can be, respectively, written as:

$$
\begin{cases}
\pi_d(\varphi) = \pi_d^o(\varphi) - f \\
\pi_x(\varphi) = \pi_x^o(\varphi) - f_x
\end{cases}
\tag{3.12}
$$

---

[14]As previously mentioned in **Section 3**, beyond the CES case, the demand elasticity may vary with individual consumption (under directly-separable preferences) or with price levels (under indirectly-separable preferences). Since both variables are pinned down by firm productivity $\varphi$ at equilibrium, it is then both useful and meaningful to write the demand elasticity as a function of firm productivity.

[15]See Appendix D.1 for more details.

[16]As is well known, the Lerner index stems from the first-order condition of profit maximization, and implies that the markup rate is inversely related to the demand elasticity: $\frac{p(\varphi)-\varphi^{-1}}{p(\varphi)} = \sigma(\varphi)^{-1}$.

Two key implications of the symmetry assumption are worth emphasizing. First, by ensuring that all markets are identical both in terms of size $L$ and intensity of competition (captured by the price index $P$), it implies that the difference between domestic and export profits is solely driven by the presence of trade frictions. Second, the symmetry assumption also ensures that all countries share the same average demand elasticity $\bar{\sigma}$. I can thus concentrate on the firm-specific aspect of the demand elasticity and examine its implications in general equilibrium.

## 2. Equilibrium Conditions

As in Melitz (2003), since operating profits are monotonically increasing in productivity, the presence of fixed costs of accessing domestic and foreign markets, $f$ and $f_x$, implies the existence of two productivity cutoffs. The first is the domestic productivity cutoff, denoted by $\varphi_d^*$, and defined as the minimum productivity level required to make non-negative profits on the domestic market. The second is the export productivity cutoff $\varphi_x^*$ such that among successful entrants in any country, only those with a productivity level of at least $\varphi_x^*$ find it profitable to export.

By their definition, the domestic cutoff must then satisfy the zero profit condition on the domestic market (ZPCD): $\pi_d(\varphi_d^*) = 0$. Similarly, the export cutoff must satisfy the zero profit condition on the export market (ZPCX): $\pi_x(\varphi_x^*) = 0$. Using these two equilibrium conditions, the cutoff levels can be identified implicitly by:

$$
\begin{cases}
(ZPCD)\ \varphi_d^* : \sigma_d^*(\varphi_d^*)^{-1}\, p_d^*(\varphi)^{1-\sigma_d^*(\varphi_d^*)}\, P^{a_d^*(\varphi_d^*)}\, L = f \\
(ZPCX)\ \varphi_x^* : \sigma_x^*(\varphi_x^*)^{-1}\, p_x^*(\varphi)^{1-\sigma_x^*(\varphi_x^*)}\, P^{a_x^*(\varphi_x^*)}\, L = f_x
\end{cases}
\tag{3.13}
$$

where $P$ is the price index in any country and is given by:

$$
P = M_e^{-1}\left[\int_{\varphi_d^*}^{+\infty} p_d(\varphi)^{1-\sigma(\varphi)} g(\varphi)d\varphi \;+\; (N-1)\int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} g(\varphi)d\varphi\right]^{-1}
\tag{3.14}
$$

As previously mentioned in **Section 4.2**, for any arbitrary values of the per-unit trade cost $\tau$, assuming that the fixed cost of exporting is larger than the fixed cost of accessing the domestic market, $f_x > f$, ensures that exporting is always a more selective activity: $\forall \tau \geq 1$, $\varphi_x^* > \varphi_d^*$. This partitioning implies then that exporters are on average more productive than firms serving only the domestic market.

Upon sinking the fixed entry cost $f_e$, all entering firms expect positive profits. Yet, as mentioned above, only successful entrants on a given market (either domestic, or export) earn positive profits. This, in turn, implies that the average profit at the industry level is positive. Following Melitz (2003), the average profit, net of the sunk entry cost, must be set to zero to ensure a bounded mass of entrants at equilibrium. This requires imposing the Free Entry condition (FE). This equilibrium condition equalizes the average expected profit conditional on successful entry to the sunk entry cost $f_e$:

$$[1 - G(\varphi_d^*)] \int_{\varphi_d^*}^{+\infty} \pi_d(\varphi)\mu_d(\varphi)d\varphi + (N-1)[1 - G(\varphi_x^*)] \int_{\varphi_x^*}^{+\infty} \pi_x(\varphi)\mu_x(\varphi)d\varphi = f_e$$

(3.15)

where $\mu_d(\varphi)$ and $\mu_x(\varphi)$ correspond to the productivity distribution conditionally on successful entry, respectively, on the domestic, and the export market:

$$\begin{cases} \mu_d(\varphi) = \frac{g(\varphi)}{[1-G(\varphi_d^*)]} & \forall \, \varphi \geq \varphi_d^* \\ \mu_x(\varphi) = \frac{g(\varphi)}{[1-G(\varphi_x^*)]} & \forall \, \varphi \geq \varphi_x^* \end{cases}$$

Finally, the last equilibrium condition is the Labor Market Clearing condition (LMC). This latter ensures that in any country, total labor demand equates total labor supply:

$$M_e \left[ f_e + [1 - G(\varphi_d^*)] \int_{\varphi_d^*}^{+\infty} l_d(\varphi)\mu_d(\varphi)d\varphi + (N-1)[1 - G(\varphi_x^*)] \int_{\varphi_x^*}^{+\infty} l_x(\varphi)\mu_x(\varphi)d\varphi \right] = L$$

(3.16)

where $l_d(\varphi)$ and $l_x(\varphi)$ correspond, respectively, to the amount of labor used by a $\varphi$-productivity firm to serve the domestic, and the export market:

$$\begin{cases} l_d(\varphi) = \frac{\sigma(\varphi)-1}{\sigma(\varphi)} r_d(\varphi) + f & \forall \, \varphi \geq \varphi_d^* \\ l_x(\varphi) = \frac{\sigma(\varphi)-1}{\sigma(\varphi)} r_x(\varphi) + f_x & \forall \, \varphi \geq \varphi_x^* \end{cases}$$

To summarize, there are 4 equilibrium conditions: the Labor Market Clearing condition (LMC), the Free Entry condition (FE), and two Zero Cutoff Profit conditions: (ZPCD) and (ZPCX). By imposing the symmetry assumption at the country level, along with normalizing wage to unity (by choice of labor as numéraire), the set of unknown equilibrium variables is reduced to 4: the mass of entrants $M_e$, the price index $P$, and the two productivity cutoffs $\varphi_d^*$ and $\varphi_x^*$.

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# 3. Solving for the General Equilibrium

I start with solving for the equilibrium mass of entrants $M_e$ using the Free Entry (FE) and the Labor Market Clearing (LMC) conditions:

$$M_e = \frac{L}{\bar{\sigma}\left[ f_e + [1 - G(\varphi_d^*)] f + (N - 1)[1 - G(\varphi_x^*)] f_x \right]} \tag{3.17}$$

where $\bar{\sigma} = \frac{1}{N} \int_{\varphi_d^*}^{+\infty} \sigma(\varphi)\, g(\varphi) d\varphi + \frac{N-1}{N} \int_{\varphi_x^*}^{+\infty} \sigma(\varphi)\, g(\varphi) d\varphi$ is the weighted average demand elasticity to be faced by a successful entrant while serving the World market.

Since increased generality raises tractability issues, I resort to a new and simple method that I call "the Exponent Elasticity Method" (EEM, hereafter). The objective of this method is to deliver a tractable solution for the general equilibrium price index despite added flexibility on the demand side. The starting point is the partial equilibrium price index in equation (3.14):

$$P = M_e^{-1} \big[ \underbrace{\int_{\varphi_d^*}^{+\infty} p_d(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_d} + (N-1) \underbrace{\int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_x} \big]^{-1}$$

Clearly, the mathematical challenge consists in solving for both integrals ($I_d$, $I_x$) without assuming that the demand elasticity is identical across firms ($\forall \varphi, \sigma(\varphi) = \sigma$) and then using this latter as a constant for integrating. Such simplicity is only possible under CES demand, which is the unique case where it is possible to solve for these integrals. As the general demand system considered in this chapter encompasses the CES and more flexible alternatives allowing the demand elasticity to vary across firms, it is then impossible to solve for these integrals under general demands.

Given the impossibility to solve for these integrals in the current setting, the key idea that the EEM method proposes is to locally approximate both integrals ($I_d$, $I_x$) around the equilibrium with a multiplicative equivalent which has a finite number of determinants, such as the exponent of each determinant embodies the elasticity of the average price with respect to it.[17] This requires a multi-step procedure that I expose in detail in Appendix D.2. By implementing this simple method, I obtain a tractable solution for the general equilibrium price index:

---

[17]To be precise, this corresponds to the average price to the power of $(1 - \bar{\sigma})$: $\bar{p}_d^{1-\bar{\sigma}_d}$ and $\bar{p}_x^{1-\bar{\sigma}_x}$.

$$P \equiv c_E^{[1 + \varepsilon_{I_x}(P)]^{-1}} \; L^{-[\frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)}]} \; T(\tau, f_x, N)^{-[1 + \varepsilon_{I_x}(P)]^{-1}} \tag{3.18}$$

where $c_E = \bar{\sigma} \, [ \, f_e + [1 - G(\varphi_d^*)] \, f + (N-1)[1 - G(\varphi_x^*)] \, f_x \, ]$, and $T(\tau, f_x, N) = N \, \tau^{-\theta} \, f_x^{- \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}}$ is an index of exposure to trade. That is, an increase in an economy's exposure to trade occurring through a decrease in variable or fixed trade costs, or an increase in the number of trading countries in the World economy, implies a higher level of $T(\tau, f_x, N)$. As shown in Appendix D.2, both elasticities $\varepsilon_{I_x}(L)$ and $\varepsilon_{I_x}(P)$ take a simple form, and are respectively given by:

$$\begin{cases} \varepsilon_{I_x}(L) = \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} > 0 \\[2mm] \varepsilon_{I_x}(P) = a_x^*(\varphi_x^*) \, \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} > 0 \end{cases} \tag{3.19}$$

Given the positive sign of $\varepsilon_{I_x}(P)$, it is then readily verified that higher exposure to trade induces tougher competition on the domestic market of any economy by lowering its price index. Now by plugging the above general equilibrium expression of the price index in the zero profit conditions in equation (3.13) and rearranging, I obtain the following expressions of the domestic and the export productivity cutoffs, in general equilibrium :

$$\varphi_d^* \equiv f^{\varepsilon_{\varphi_d^*}(f)} \; c_E^{\gamma_d^*} \; L^{\varepsilon_{\varphi_d^*}(L)} \; T(\tau, f_x, N)^{\varepsilon_{\varphi_d^*}(T)} \tag{3.20}$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_d^*}(f) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \\[2mm] \gamma_d^* = -\frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \, [1 + \varepsilon_{I_x}(P)]^{-1} \\[2mm] \varepsilon_{\varphi_d^*}(L) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \, [ \, a_d^*(\varphi_d^*) \, \frac{1 + \varepsilon_{I_x}(L)}{1 + \varepsilon_{I_x}(P)} - 1 \, ] \\[2mm] \varepsilon_{\varphi_d^*}(T) = \frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \, [1 + \varepsilon_{I_x}(P)]^{-1} \end{cases}$$

Similarly, the general equilibrium export cutoff can be written as:

$$\varphi_x^* \equiv \tau \, f_x^{\varepsilon_{\varphi_x^*}(f_x)} \; c_E^{\gamma_x^*} \; L^{\varepsilon_{\varphi_x^*}(L)} \; T(\tau, f_x, N)^{\varepsilon_{\varphi_x^*}(T)} \tag{3.21}$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_x^*}(f_x) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} \\ \gamma_x^* = -\frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} \left[1 + \varepsilon_{I_x}(P)\right]^{-1} \\ \varepsilon_{\varphi_x^*}(L) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} \left[a_x^*(\varphi_x^*)\frac{1+\varepsilon_{I_x}(L)}{1+\varepsilon_{I_x}(P)} - 1\right] \\ \varepsilon_{\varphi_x^*}(T) = \frac{a_x^*(\varphi_x^*)}{\eta_d^*[\sigma_x^*(\varphi_x^*)-1+S]} \left[1 + \varepsilon_{I_x}(P)\right]^{-1} \end{cases}$$

Finally, I can solve for the general equilibrium operating profits on the domestic and the export markets by plugging the general equilibrium expression of the price index in their respective partial equilibrium expression provided in equation (3.11):

$$\begin{cases} \forall \varphi \geq \varphi_d^*, \ \pi_d^o(\varphi) = c_E^{\gamma(\varphi)} \ \varphi^{\eta(\varphi)[\sigma(\varphi)-1+S]} \ L^{\varepsilon_{\pi^o(\varphi)}(L)} \ T(\tau, f_x, N)^{\varepsilon_{\pi^o(\varphi)}(T)} \\ \forall \varphi \geq \varphi_x^*, \ \pi_x^o(\varphi) = c_E^{\gamma(\varphi)} \ \varphi^{\eta(\varphi)[\sigma(\varphi)-1+S]} \ \tau^{\eta(\varphi)[1-\sigma(\varphi)-S]} \ L^{\varepsilon_{\pi^o(\varphi)}(L)} \ T(\tau, f_x, N)^{\varepsilon_{\pi^o(\varphi)}(T)} \end{cases}$$

(3.22)

where $\gamma(\varphi) = a(\varphi) \left[1 + \varepsilon_{I_x}(P)\right]^{-1}$, $\varepsilon_{\pi^o(\varphi)}(L) = 1 - a(\varphi) \left[\frac{1+\varepsilon_{I_x}(L)}{1+\varepsilon_{I_x}(P)}\right]$, and finally, $\varepsilon_{\pi^o(\varphi)}(T)$ is given by $\varepsilon_{\pi^o(\varphi)}(T) = -a(\varphi) \left[1 + \varepsilon_{I_x}(P)\right]^{-1}$.

### 3.5.3   Welfare Analysis

In the previous section, I have solved for the main equilibrium variables ($M_e$, $P$, $\varphi_d^*$, $\varphi_x^*$). Now I use their general equilibrium expressions to solve for two key variables for welfare analysis: the total mass of firms competing in a single country, $M$, and the weighted average productivity of these firms, $\bar{\varphi}$. I proceed as follows.

Let $M_d$ denote the equilibrium mass of domestic firms. Using the equilibrium mass of entrants $M_e$, and the general equilibrium expression of the domestic cutoff $\varphi_d^*$, $M_d$ can be then written as: $M_d = M_e \left[1 - G(\varphi_d^*)\right]$. Similarly, the equilibrium mass of exporting firms in any country is given by $M_x = M_e \left[1 - G(\varphi_x^*)\right]$. The total mass of firms competing in any country (or, alternatively the total mass of available varieties in any country) is then given by $M = M_d + (N-1) M_x$.

As in Melitz (2003), the weighted average productivity of all firms (both domestic and foreign exporters) competing in a single country can be written as:

$$\bar{\varphi} = \frac{M_d}{M_W} \int_{\varphi_d^*}^{+\infty} \varphi \, \mu_d(\varphi) d\varphi + \frac{(N-1)M_x}{M_W} \int_{\varphi_x^*}^{+\infty} \varphi \, \mu_x(\varphi) d\varphi \qquad (3.23)$$

where $M_W = N M_e$ is the total mass of entrants at the World level, and can be thought of as a proxy for the size of the World market. By solving for the above integrals, and using the general equilibrium expressions of the domestic and the export cutoffs and rearranging, $\bar{\varphi}$ can be then written as a function of the domestic cutoff:

$$\bar{\varphi} = \varphi_d^* \underbrace{\Psi(.)}_{>1} \qquad (3.24)$$

where $\Psi(.)$ is larger than one and can be considered as constant since Assumption A1 implies a constant mean-to-min ratio $\frac{\bar{\varphi}}{\varphi_d^*}$.[18]

Since Free Entry implies that there are zero net profits at equilibrium, the real wage, $P^{-1}$, is then a sufficient measure for welfare per worker $W$ in this setting:

$$W = P^{-1} = M\, \bar{p}(\bar{\varphi})^{1-\bar{\sigma}(\bar{\varphi})} \equiv M\, \bar{\varphi}^{\,\bar{\eta}[\bar{\sigma}(\bar{\varphi})-1]} \qquad (3.25)$$

Now by plugging the final expression of $\bar{\varphi}$ from equation (3.24) in the above expression and simplifying,[19] I can write welfare per capita $W$ as a function of solely the total number of available varieties $M$, and the domestic cutoff $\varphi_d^*$:

$$W \equiv M\, \varphi_d^{*\,\bar{\eta}[\bar{\sigma}(\bar{\varphi})-1]} \qquad (3.26)$$

It is now worth noting that all the variables that I have solved for in general equilibrium mainly depend on the trade exposure index $T(\tau, f_x, N)$. Hence, trade liberalization can be generically modeled as an increase in $T(\tau, f_x, N)$, which simply reflects that an economy is more exposed to trade. Following Melitz (2003), I can then go more granular and separately examine three different mechanisms that lead to an increase in the exposure of an economy to trade. As previously mentioned, these scenarios include: (i) a small reduction in the variable trade cost $\tau$; (ii) a small decrease in the fixed cost of exporting $f_x$ ; and (iii) a small increase in the number of trading countries $N$.

### 1. Measuring the Gains From Trade

The final expression of welfare per capita $W$ in equation (3.26) clearly shows that consumer welfare is more sensitive to changes in the domestic cutoff $\varphi_d^*$ than to variations of the total mass of available varieties $M$. Moreover, inspection of the general equilibrium expression of the domestic

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[18]As is well known, this a straightforward implication of unbounded Pareto. For expositional clarity, the explicit expression of $\Psi(.)$ is relegated to Appendix D.4

[19]Visibly, the simplification consists simply in dropping the constant $\Psi(.)$ from the final expression of welfare per worker in equation (3.26).

cutoff immediately reveals that an increase in the exposure to trade, higher $T(\tau, f_x, N)$, induces an increase in the the productivity cutoff for domestic firms $\varphi_d^*$. This ensures then that higher exposure to trade, occurring through any of the aforementioned mechanisms, always generates a welfare gain. In other words, by inducing tougher selection on the domestic market, trade is always welfare improving even if this selection effect may lead to a net decrease in the total mass of available varieties.

Based on this standard result, due to Melitz (2003), I can use the elasticity of the domestic cutoff with respect to the trade exposure index as a sufficient measure of the magnitude of the gains from trade (GFT, hereafter):[20]

$$GFT = \varepsilon_{\varphi_d^*}(T(\tau, f_x, N)) = \frac{dlog\ \varphi_d^*}{dlog\ T(\tau, f_x, N)} > 0 \qquad (3.27)$$

Then, when the welfare analysis gets more granular, the magnitude of the gains from higher exposure to trade occurring under each specific scenario is simply given by: [21]

$$\begin{cases} GFT(\tau^-) = -\varepsilon_{\varphi_d^*}(\tau) = \frac{dlog\ \varphi_d^*}{dlog\ T(\tau, f_x, N)} \left(- \frac{dlog\ T(\tau, f_x, N)}{dlog\ \tau}\right) \\ GFT(f_x^-) = -\varepsilon_{\varphi_d^*}(f_x) = \frac{dlog\ \varphi_d^*}{dlog\ T(\tau, f_x, N)} \left(- \frac{dlog\ T(\tau, f_x, N)}{dlog\ f_x}\right) \\ GFT(N^+) = \varepsilon_{\varphi_d^*}(N) = \frac{dlog\ \varphi_d^*}{dlog\ T(\tau, f_x, N)} \left(\frac{dlog\ T(\tau, f_x, N)}{dlog\ N}\right) \end{cases} \qquad (3.28)$$

While the above measures are quite standard, here the novel idea I explore is how alternative assumptions about the curvature of demand and the nature of preferences affect the magnitude of the gains from trade. Does this latter mainly hinge on the curvature of demand or the type of preferences? Departing from the CES benchmark, under which alternative assumptions about preferences and demand, gains from trade are smaller or larger than those obtained under the CES? The first objective of the current chapter is to address these two questions. This is what I do in **Sections 5.4 and 5.5** .

Before that, I provide first a detailed exposition of the sources of gains from trade that are theoretically possible both in closely related literature and in the current chapter. Then, I recall the second objective of this chapter, which consists in separately examining the novel implications of the curvature of demand and the nature of preferences for the coexistence of these sources of welfare gains from trade in general equilibrium.

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[20]Notice that I use the same notation as in **Section 2**, whereby $\varepsilon_Y(x) = \frac{dlog\ Y}{dlog\ x}$ is the elasticity of Y with respect to x.

[21]Notice that $GFT(\tau^-)$, $GFT(f_x^-)$, and $GFT(N^+)$ reflect three different scenarios where higher exposure to trade occurs, respectively, through (i) a small reduction in the variable trade cost $(\tau^-)$; (ii) a small reduction in the fixed trade cost $(f_x^-)$; and (ii) a small increase in the number of trading countries $(N^+)$

## 2. Sources of Welfare Gains From Trade

### 2.1 Potential Sources under Monopolistic Competition with Heterogeneous Firms

Under monopolistic competition, the potential sources of welfare gains from trade are threefold. First, consumers have access to a wider range of products, as newly imported varieties become available on the domestic market. This first source is emphasized by Krugman (1980), and can be referred to as a "gross variety gain". The second gain arises in a setting with heterogeneous firms as in Melitz (2003) and can be explained as follows. As trade reallocates market shares from domestic firms to relatively more productive exporters, the least productive domestic firms are forced to exit the market, which leads to an increase in (weighted) average productivity at the industry level. This welfare channel highlighted by Melitz (2003) can be thought of as a "gain from selection", or equivalently called Melitz (2003) 's "selection effect". The third source consists in a reduction in the markups charged by domestic firms due to import competition. This is so-called "pro-competitive effect of trade" is due to Krugman (1979).

### 2.2 Active Sources in Previous Trade Models with Heterogeneous Firms and CES preferences

Trade models incorporating firm heterogeneity and CES utility have overwhelmingly substantiated that gains from trade solely stem from the "selection effect", due to Melitz (2003). The absence of the two other welfare channels is caused by the rigidity of CES preferences. First, under monopolistic competition, CES utility imposes constant markups, which precludes then the "pro-competitive effect of trade" to occur. Second, as demonstrated by Feenstra (2010), the consumer's gross gain from newly imported varieties exactly cancels out with the loss of domestic varieties (due to firm exit) when preferences are CES. Thus, trade yields zero net gains from variety.[22]

### 2.3 Active Sources in More Recent Trade Models with Heterogeneous Firms and Non-CES preferences

In more recent trade models incorporating firm heterogeneity and non-CES preferences, identifying the welfare channel(s) that is (are) operative in general equilibrium is more complex and

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[22]Feenstra (2010) derives this result using also unbounded Pareto distribution of firm productivity on the supply side. More recent work by Melitz and Redding (2015) emphasizes that trade yields only gains from selection under CES demand regardless of whether the Pareto distribution is bounded or unbounded above. This ensures then that Feenstra (2010)'s result is solely driven by the rigidity of CES preferences.

requires more subtle distinctions between two different modeling approaches.

On the one hand, a large body of work (Melitz and Ottaviano, 2008; Bertoletti, Etro, and Simonovska, 2018; Feenstra, 2018; Arkolakis et al., 2018; Fally, 2019) abstracted from fixed costs, restricted demand to be sub-convex, and assumed that it exhibits a choke price to ensure selection into both domestic and export markets.[23] A key implication of this modeling approach is that the type of preferences determines whether a welfare channel is operative or not, and thus shapes the structure of the gains from trade. Arkolakis et al. (2018) focus on directly-separable as well as homothetic preferences (excluding the CES case). They abstract from variety gains given the absence of fixed costs in their setting and emphasize that trade generates only gains from selection, as in Melitz (2003). In particular, they show that the "pro-competitive effect of trade" does not represent an additional source of gains from trade. In fact, upon trade liberalization, the reduction in domestic markups is either dominated by the increase in foreign markups when preferences are directly-separable, or exactly offset by higher foreign markups when preferences are homothetic. Using indirectly-separable preferences, Bertoletti, Etro, and Simonovska (2018) find a different result. They show that trade liberalization yields only pure variety gains as in Krugman (1980) despite firm heterogeneity in productivity levels.

On the other hand, only few papers went beyond the CES using a different modeling approach (Mrázová and Neary, 2017; Zhelobodko et al., 2012). Specifically, while allowing demand to be either sub-convex or super-convex, both papers concentrate on directly-separable preferences.[24] Moreover, both papers assume that trade is frictionless and model globalization as an increase in the size of the World market.[25] The last important detail is that in the absence of a choke price on the demand side, both papers incorporate fixed costs to ensure selection into the domestic market. Given the absence of trade frictions, there is no selection into exporting in both papers, and so all active firms serve the World market.

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[23]Imposing a choke price is a necessary restriction to ensure firm selection in the absence of fixed costs. It also rules out the CES case in papers considering general demand systems, such as Arkolakis et al. (2018) and Fally (2019).

[24]Notice that Mrázová and Neary (2017) and Zhelobodko et al. (2012) characterize the behavior of the demand elasticity in the same way, yet using a different terminology. In deed, Zhelobodko et al. (2012) use the concept of RLV (the Relative Love for Variety) to measure the elasticity of the inverse demand. This latter corresponds to the elasticity of the marginal sub-utility with respect to consumption levels. An increasing RLV implies then that the price elasticity of demand decreases in consumption, which corresponds to the case of "sub-convex demand" in the terminology of Mrázová and Neary (2017). Similarly, a decreasing RLV in Zhelobodko et al. (2012) is equivalent to the case that Mrázová and Neary (2017) call "super-convex demand".

[25]To be precise, while Zhelobodko et al. (2012) study the impact of an enlargement in market size of a given trading country ($L$), Mrázová and Neary (2017) examine the impact of an increase in the number of trading countries at the World level ($N$). Under cross-country symmetry in both papers, this is then equivalent to focusing on the intensive margin of the World market size in the former paper, and to concentrating on the extensive margin in the latter.

This modeling approach has three implications for the gains from trade that are worth noting. First, the variety effect occurs only on "cutoff" domestic varieties. In particular, globalization may induce a decrease or an increase in the number of domestic varieties in any market. Second, Melitz (2003)'s selection effect is operative only when demand is sub-convex (Mrázová and Neary, 2017; Zhelobodko et al., 2012).[26] Third, when the selection effect of trade occurs, it is fully driven by the pro-competitive reduction in markups of less efficient firms (Zhelobodko et al., 2012). This implies that the magnitude of gains from selection is governed by this of the pro-competitive effect of trade. Nevertheless, this decrease in markups of less productive firms (occurring under sub-convex demands) can not be considered as an additional gain from trade because it exactly cancels out with the increase in markups set by more productive firms (Mrázová and Neary, 2017).

In short, all the aforementioned papers convey the same message: regardless of the type of preferences and whether demand is CES or sub-convex, trade liberalization delivers only gains from selection as in Melitz (2003).[27] In this regard, the recent work by Feenstra (2018) where he restores a theoretical role for the pro-competitive effect of trade and gains from variety can be seen as an exception in the literature. In particular, Feenstra (2018) shows that the three welfare channels (that are theoretically possible under monopolistic competition and firm heterogeneity) can be simultaneously operative if and only if firm productivity is drawn from a bounded Pareto distribution.

### 2.4 Active Sources of Welfare Gains in the Current Chapter

In this chapter, I propose a different modeling approach which combines standard assumptions on the supply side with a flexible demand system, while taking variable and fixed trade barriers in due account. The main objective of this approach is to identify a demand-based condition which is sufficient to restore a theoretical role for these three sources of welfare gains from trade. Despite apparent similarity to Feenstra (2018), the main difference is in the nature of the condition under which these three welfare channels are simultaneously operative in general equilibrium. This theoretically possible result can be hereafter referred to as a state of "coexistence of gains from trade". As hinted to in the previous paragraph, in Feenstra (2018), the coexistence of gains from selection, variety, and domestic markup reduction in general equilibrium hinges on a feature of the supply side: productivity distribution must be Pareto and bounded above.

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[26]As shown by Mrázová and Neary (2017), globalization encourages entry of less efficient firms when demand is super-convex. Similarly, Zhelobodko et al. (2012) find that market size enlargement triggers entry of less productive firms under decreasing RLV.

[27]While retaining Bertoletti, Etro, and Simonovska (2018) as an exception in this regard.

In contrast, in the current model, the distribution is Pareto, but unbounded above, which immediately rules out the supply side effect in Feenstra (2018). Hence, here the novel aspect is that the coexistence of these three sources of gains from trade solely hinges on demand conditions.

### 3. A Sufficient Statistic for Coexistence of Gains From Trade

As stressed in the two previous subsections, Melitz (2003)'s selection effect is always operative in the current model. In fact, the general equilibrium expression of the domestic cutoff clearly shows that for any curvature of demand and type of preferences, higher exposure to trade always constrains less efficient domestic firms to exit the market. While ensuring that trade always delivers welfare gains as in Melitz (2003), this result indicates that trade liberalization has countervailing effects on the total mass of available varieties. On the one hand, higher exposure to trade leads to an increase in the mass of imported varieties. On the other hand, it forces the least productive firms to exit the market, and leads then to a reduction in the mass of domestic varieties. The sign of the net variety effect (NVE, hereafter) will thus depend on a horse race between the initial gross variety gain from newly imported varieties as in Krugman (1980), and the loss from disappearing domestic varieties due to firm exit as in Melitz (2003). Therefore, the NVE can be simply measured as follows:

$$NVE = \varepsilon_{M_m}(T(\tau, f_x, N)) - \varepsilon_{M_d}(T(\tau, f_x, N)) \tag{3.29}$$

where $M_m = (N-1)M_x$ is the total mass of imported varieties in any trading country, and $\varepsilon_{M_m}(T(\tau, f_x, N))$, $\varepsilon_{M_d}(T(\tau, f_x, N))$ are, respectively, the elasticities of the equilibrium mass of imported varieties, and the equilibrium mass of domestic firms with respect to the trade exposure index.[28] Inspection of the above expression clearly indicates that net variety gains and gains from selection coexist if and only if the net variety effect of trade is strictly positive: $NVE > 0$.

Departing from the CES benchmark, the flexible demand system considered in this chapter allows for the demand elasticity to be firm-specific, which implies the existence of variable markups in the current setting. This raises then the following question: Is it theoretically possible that the three sources of gains from trade coexist in the current model? Put differently, under which condition(s) the coexistence of net variety gains and gains from selection can be accompanied by a pro-competitive reduction in domestic markups?

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[28]Notice that the general equilibrium export cutoff (embodied in $M_m$) is written as a function of $\tau$, $f_x$ and $T(\tau, f_x, N)$, it is then more convenient to separately examine the net variety effect of trade under each of the three scenarios of trade liberalization considered in this chapter.

As I will show in the next two sections (**5.4** and **5.5**), the current chapter provides a clear and gradual response to this question. Departing from the homothetic CES as a boundary case where trade yields a zero net variety effect (NVE=0),[29] I show that the sign of the NVE is solely pinned down by the curvature of demand. That is, whether gains from selection and net variety coexist in general equilibrium ($NVE > 0$) or not ($NVE < 0$) crucially depends on whether demand is sub-convex or super-convex. Then, once the coexistence of these two gains is ensured, I show that whether they are accompanied by the pro-competitive effect on domestic markups or not, depends on whether preferences are directly- or indirectly-separable.

As mentioned earlier in this section, the objective of this chapter is twofold. First, I separately examine the role that the curvature of demand and the type of preferences play in determining the magnitude of the gains from trade. Second, I characterize sufficient conditions for the three sources of welfare gains to coexist in general equilibrium. This is what I focus on next.

### 3.5.4 Demand Curvature and Gains from Higher Exposure to Trade

This section shows how demand curvature plays a crucial role in determining the magnitude of the gains from higher exposure to trade. It also provides an industry-level explanation for this novel result. To do so, I proceed in three steps.

First, I derive a general formula for the gains from increased exposure to trade occurring through any of the three aforementioned scenarios of trade liberalization. This simple formula is presented and interpreted in **Theorem 1**. Second, I derive two novel results showing how demand curvature determines, not only, the degree of toughness of firm selection on any market, but also, the degree of their partitioning by export status. These two general equilibrium results are respectively presented in **Theorem 2**, and **Theorem 3**. Finally, I connect these three theorems and I provide an industry-level explanation for the new welfare result, whereby demand curvature, by governing the toughness of firm selection and the degree of their partitioning, plays a critical role in determining the gains from trade.

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[29] As I will show next, under CES demand, I replicate Feenstra ([2010])'s result, whereby trade yields zero net gains from variety (NVE=0) and generates thus only gains from selection.

# 1. A Simple Welfare Formula

**Theorem 1** *Under Generalized Gorman-Pollak demand, unbounded Pareto distribution of firm productivity and country-level symmetry, welfare gains from increased exposure to trade, occurring through any mechanism of trade liberalization, are given by*

$$GFT(T) = \frac{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*)} \frac{1}{\theta + \Delta_x} > 0$$

where $\theta$ is the shape parameter of the Pareto distribution, S is the "superelasticity" of Kimball (1995), and $\Delta_x = [\,\eta_x^*(\sigma_x^*(\varphi_x^*) - 1) - a_x^*(\varphi_x^*)\,\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1) + S\,]$ reflects the curvature of demand. $a_d^*(\varphi_d^*)$ and $a_x^*(\varphi_x^*)$, are respectively given by $a_d^*(\varphi_d^*) = \frac{\beta + \alpha\,\sigma_d^*(\varphi_d^*)}{\beta + \alpha\,\bar{\sigma}}$, $a_x^*(\varphi_x^*) = \frac{\beta + \alpha\,\sigma_x^*(\varphi_x^*)}{\beta + \alpha\,\bar{\sigma}}$. Finally, $\alpha$ and $\beta$ are dummies capturing the type of preferences as described in **Section 2**.

Clearly, the above formula offers a parsimonious generalization of previous welfare formulas derived by (Arkolakis, Costinot, and Rodriguez-Clare, 2012; Arkolakis et al., 2018; Fally, 2019) in two respects. First, it encompasses three theoretically possible scenarios of trade liberalization. For instance, it clearly shows that regardless of whether higher exposure to trade occurs through a small reduction in variable or fixed trade costs, or a small increase in the number of trading countries, the welfare gains it delivers always take the same simple form. Second, as compared with Arkolakis et al. (2018), and Fally (2019), here the novel aspect of this formula is that it allows, not only, the type of preferences, but also, the curvature of demand to play a role in determining the magnitude of the gains from trade.

By recalling that under the CES case, the demand elasticity is identical across firms ($\forall \varphi, \sigma(\varphi) = \sigma$), the pass-through is complete ($\eta=1$) and the "superelasticity" is equal to zero (S=0), and so $\Delta_x$ boils down to zero ($\Delta_x=0$), it follows by inspection that Arkolakis, Costinot, and Rodriguez-Clare (2012)'s result is replicated under the CES case: ($GFT = \frac{1}{\theta}$). Beyond the CES, it is clear that increased generality of the current demand system allows, not only, the curvature of demand, but also, the type of preferences to play a role in determining the magnitude of the gains from trade. Yet, as in the current section the focus is squarely on the role of demand curvature, let us now examine its novel implications for the gains from trade.

When demand is sub-convex ($S > 0$), the demand elasticity decreases in firm productivity. This implies that the firm at the domestic cutoff faces a higher elasticity than the firm at the export cutoff: $\sigma_d^*(\varphi_d^*) > \sigma_x^*(\varphi_x^*)$. This latter faces then a higher demand elasticity than the average productivity exporter: $\sigma_x^*(\varphi_x^*) > \bar{\sigma}_x(\bar{\varphi}_x)$, and so $\Delta_x$ is strictly positive ($\Delta_x > 0$).

Inspection of the new welfare formula immediately reveals that sub-convex demands yield smaller gains from trade as compared with the CES benchmark (where $\Delta_x$=0). This result is reversed when demand is super-convex ($S < 0$): now the firm at the domestic cutoff faces the lowest demand elasticity, and the cutoff exporter faces a lower demand elasticity than the average productivity exporter: $\sigma_d^*(\varphi_d^*) < \sigma_x^*(\varphi_x^*) < \bar{\sigma}_x(\bar{\varphi}_x)$. This yields a strictly negative value of $\Delta_x$ ($\Delta_x < 0$). This, in turn, immediately implies that super-convex demands deliver larger gains from trade as compared with the CES benchmark.

## 2. General Equilibrium Implications of Demand Curvature for Firm Selection and Partitioning of Firms by Export Status

In **Section 4**, I highlighted two novel comparative statics results showing how demand curvature governs the toughness of firm selection as well as the degree of the partitioning of firms by export status. I also provided a partial equilibrium explanation for both results and emphasized that this latter sets the scene for understanding the general equilibrium behavior. Now I show that both results hold in general equilibrium, as clearly stated in the two following theorems:[30]

**Theorem 2** *Under Generalized Gorman-Pollak demand, unbounded Pareto distribution of firm productivity, country-level symmetry and the presence of fixed costs of accessing markets, super-convex demands provide an upper bound for the degree of toughness of firm selection on any market in general equilibrium, while sub-convex demands provide a lower bound. Within these bounds, CES demand delivers an intermediate outcome.*

$$\begin{cases} \forall f > 0, \ \varphi_d^* \ (sub-convex) \ < \ \varphi_d^* \ (CES) \ < \ \varphi_d^* \ (super-convex) \\ \forall f > f_x, \ \varphi_x^* \ (sub-convex) \ < \ \varphi_x^* \ (CES) \ < \ \varphi_x^* \ (super-convex) \end{cases}$$

**Theorem 3** *Under Generalized Gorman-Pollak demand, unbounded Pareto distribution of firm productivity, country-level symmetry and the presence of fixed costs of accessing markets, super-convex demands provide a lower bound for the degree of partitioning of firms by export status in general equilibrium, while sub-convex demands provide an upper bound. Within these bounds, CES demand delivers an intermediate outcome.*

$$\forall f > f_x, \ \frac{\varphi_x^*}{\varphi_d^*} \ (super-convex) \ < \ \frac{\varphi_x^*}{\varphi_d^*} \ (CES) \ < \ \frac{\varphi_x^*}{\varphi_d^*} \ (super-convex)$$

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[30]Proofs of **Theorem 2**, and **Theorem 3** are respectively provided in sections D.5 and D.6 of Appendix D.

## 3. An Industry-level Explanation for the Welfare Implications of Demand Curvature

Based on the partial equilibrium explanation provided in **Section 4.2**, here I emphasize that there is a common economic force driving **Theorem 2** and **Theorem 3**. Then, I show that it is by governing these comparative statics results, that demand curvature determines the magnitude of the gains from higher exposure to trade.

As mentioned in **Section 4.2**, departing from the CES benchmark, when demand is super-convex, selection is relatively tougher and only very productive firms successfully enter the domestic market. Hence, a relatively large subset of these very productive firms can export. In other words, these firms are enough productive to successfully enter the domestic market despite tougher competitive conditions implied by super-convex demands. It follows then that a large fringe of these firms is enough price competitive to penetrate the export market. This reveals then that under super-convex demands, the partitioning of firms by export status is less pronounced as compared with the CES case. This result is reversed when demand is sub-convex.

Importantly, these novel comparative statics results have crucial implications for the gains from trade, which can be explained as follows. Departing from the CES case, when demand is super-convex, successful entrants are relatively fewer and more productive since selection is relatively tougher under this curvature of demand. In addition to that, the fact that the partitioning of firms is relatively less pronounced under this case clearly indicates that a relatively large fringe of active firms export. Therefore, when demand is super-convex, the import competition effect is magnified since domestic firms in any trading country face a relatively fiercer competition from a relatively large number of exporters that are on average relatively more productive (as compared to the CES benchmark). This, in turn, induces a relatively stronger selection effect of trade, forcing then a relatively larger fringe of domestic firms to exit the market. Therefore, by magnifying Melitz (2003)'s selection effect, super-convex demands deliver larger gains from trade than those obtained under CES demand. These patterns are reversed when demand is sub-convex.

### 3.5.5    A More Granular Analysis of the Gains From Trade

As compared with the preceding analysis, this section goes more granular in two respects. First, I separately examine three different scenarios of trade liberalization, including a (small) decrease in either the variable or fixed trade cost, and a (small) increase in the number of trading countries. Second, under each scenario, I provide a more granular explanation where added flexibility in both firm and consumer behaviors plays a critical role.

I will show that increases in the exposure to trade occurring through any of these scenarios will generate very similar results. In all cases, while demand curvature plays a crucial role in determining the magnitude and the structure of welfare gains from trade, the type of preferences has only a second-order importance from a welfare standpoint.

In particular, I show that sub-convex demands provide a lower bound for the gains from trade and super-convex demands provide a higher bound. Within these bounds, CES demand delivers an intermediate outcome. Under sub-convex demands, directly-separable preferences delivers larger gains than those obtained with indirectly-separable preferences. When demand demand is super-convex, this order is reversed. As for the structure of welfare gains from trade, I will show that net variety gains and gains from selection coexist if and only if demand is sub-convex. Under this case, only when preferences are directly-separable, the pro-competitive effect of trade is restored, and thus the three sources of gains from trade coexist.

### 1. A Small Decrease in the Variable Trade Cost

In this subsection, I examine the gains from higher exposure to trade occurring through a small reduction in the variable trade cost $\tau$. I proceed in three steps. First, I show how demand curvature plays a first-order role in determining the magnitude of the gains from trade, while the type of preferences only marginally affects the result. Second, I characterize a sufficient condition for coexistence of gains from selection, net variety gains, and pro-competitive reduction in domestic markups. Third, I provide a finer explanation at the firm-level for these novel results.

### 1.1 Magnitude of the Gains From Trade

For a general demand system which encompasses the CES and two alternative types of preferences (directly- and indirectly-separable) and curvatures of demand (sub-convex and super-convex), I show that gains from a small reduction in the variable trade cost take a simple form:

$$GFT(\tau^-) = -\varepsilon_{\varphi_d^*}(\tau) = \frac{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*)\,\theta}{a_x^*(\varphi_x^*)\,\theta + \Delta_x} \qquad (3.30)$$

where $\theta$ is the shape parameter of the Pareto distribution, S is the "superelasticity" of Kimball (1995), and $\Delta_x = [\,\eta_x^*(\sigma_x^*(\varphi_x^*) - 1) - a_x^*(\varphi_x^*)\,\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1) + S\,]$ reflects the curvature of demand. $a_d^*(\varphi_d^*)$ and $a_x^*(\varphi_x^*)$, are respectively given by $a_d^*(\varphi_d^*) = \frac{\beta + \alpha\,\sigma_d^*(\varphi_d^*)}{\beta + \alpha\,\bar{\sigma}}$, $a_x^*(\varphi_x^*) = \frac{\beta + \alpha\,\sigma_x^*(\varphi_x^*)}{\beta + \alpha\,\bar{\sigma}}$, where $\alpha$ and $\beta$ are exogenous parameters capturing the type of preferences as described in **Section 2**.

Clearly, the above expression offers a parsimonious generalization of Feenstra (2010)'s result where he shows that under CES demand, a small reduction in the variable trade cost leads to a proportional increase in the domestic cutoff. By recalling that under the CES case, the demand elasticity is identical across firms ($\forall \varphi, \sigma(\varphi) = \sigma$), the pass-through is complete ($\eta$=1) and the "superelasticity" is equal to zero (S=0), and so $\Delta_x$ boils down to zero ($\Delta_x$=0), it follows by inspection that Feenstra (2010)'s result is replicated under the CES case. Beyond the CES, it is clear that added flexibility of the current demand system allows, not only, the curvature of demand, but also, the type of preferences to play a role in determining the magnitude of the gains from trade. The welfare implications of these prominent alternatives to the CES case can be studied separately as follows.

### The Role of Demand Curvature

When demand is sub-convex ($S > 0$), the demand elasticity decreases in firm productivity. This implies that the firm at the domestic cutoff faces a higher elasticity than the firm at the export cutoff: $\sigma_d^*(\varphi_d^*) > \sigma_x^*(\varphi_x^*)$. This latter faces then a higher demand elasticity than the average productivity exporter: $\sigma_x^*(\varphi_x^*) > \bar{\sigma}_x(\bar{\varphi}_x)$. Thus, $\Delta_x$ is strictly positive ($\Delta_x > 0$), which immediately implies smaller gains from trade as compared with the CES benchmark (where $\Delta_x$=0).

This result is reversed when demand is super-convex ($S < 0$): now the firm at the domestic cutoff faces the lowest demand elasticity, and the cutoff exporter faces a lower demand elasticity than the average productivity exporter: $\sigma_d^*(\varphi_d^*) < \sigma_x^*(\varphi_x^*) < \bar{\sigma}_x(\bar{\varphi}_x)$. This yields a strictly negative value of $\Delta_x$ ($\Delta_x < 0$), and immediately implies larger gains from trade as compared with the CES benchmark.

When preferences are indirectly-separable (including the CES case), $a_d^*=a_x^*=1$.[31] However, when preferences are directly-separable and non-CES, $a_d^*$ and $a_x^*$ may be higher or lower than 1 depending on demand curvature. In fact, $a_d^*$ and $a_x^*$ respectively capture the relative demand elasticity faced by firms at the domestic, and the export cutoffs. Such firms are the least productive among all active firms in their respective markets and so, when demand is sub-convex, they face a relatively higher demand elasticity: $a_d^* > 1$ and $a_x^* > 1$. In contrast, when demand is super-convex, the demand elasticity increases in firm productivity and so, both cutoff firms face a relatively lower elasticity: $a_d^* < 1$ and $a_x^* < 1$.

Inspection of the simple expression of the gains from trade in in equation (3.30) reveals that increases less than proportionally with $a_d^*$.[32] As mentioned above, this latter positively deviates from unity only when preferences are directly-separable and demand is sub-convex. Hence, under sub-convex demands, directly-separable preferences yield higher gains from trade than indirectly-separable preferences. This order is reversed when demand is super-convex: $a_d^*$ negatively deviates from from unity under directly-separable preferences. This latter delivers then lower gains from trade than indirectly-separable preferences under super-convex demands.

The above results clearly show that while demand curvature plays a critical role in determining the magnitude of the gains from trade, the type of preferences plays only a second-order role. This latter has only a marginal impact on the magnitude of the gains that is initially pinned down by the curvature of demand. This novel finding of the current chapter can be graphically illustrated in Figure 3.

---

[31] As indicated in **Section 2**, indirectly-separable preferences correspond to the case where $\beta = 1$ and $\alpha = 0$. This immediately implies that parameter (a) is identical across all firms and fixed to unity under this class of preferences: $\forall \varphi, a(\varphi) = 1$.

[32] Since both of $a_d^*$ and $a_x^*$ simultaneously deviate from unity and in the same direction, when preferences are directly-separable and non-CES.
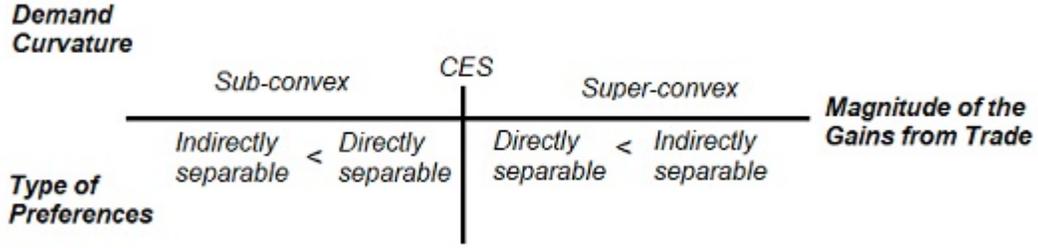
## 1.2 Structure of the Gains From Trade

Using the standard measure proposed in equation (3.29) and the general equilibrium expressions of the domestic and export cutoffs, the net variety effect of a small reduction in the variable trade cost can be simply written as:

$$NVE(\tau^-) \equiv 1 - |\varepsilon_{\varphi_d^*}(\tau)| = 1 - \frac{\eta_x^*[\sigma_x^*(\varphi_x^*) - 1 + S]}{\eta_d^*[\sigma_d^*(\varphi_d^*) - 1 + S]} \frac{a_d^*(\varphi_d^*)\,\theta}{a_x^*(\varphi_x^*)\,\theta + \Delta_x} \qquad (3.31)$$

where $\theta$ is the shape parameter of the Pareto distribution, S is the "superelasticity" of Kimball (1995), and $\Delta_x = \left[\, \eta_x^*(\sigma_x^*(\varphi_x^*) - 1) - a_x^*(\varphi_x^*)\,\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x) - 1) + S\,\right]$ reflects the curvature of demand. $a_d^*(\varphi_d^*)$ and $a_x^*(\varphi_x^*)$, are respectively given by $a_d^*(\varphi_d^*) = \frac{\beta + \alpha\,\sigma_d^*(\varphi_d^*)}{\beta + \alpha\,\bar{\sigma}}$, $a_x^*(\varphi_x^*) = \frac{\beta + \alpha\,\sigma_x^*(\varphi_x^*)}{\beta + \alpha\,\bar{\sigma}}$, where $\alpha$ and $\beta$ are exogenous parameters capturing the type of preferences as described in **Section 2**.

Under the CES case, I can immediately replicate Feenstra (2010)'s result. In the current setting, the CES is a unique exception where the demand elasticity is identical across firms ($\forall \varphi, \sigma(\varphi) = \sigma$), the pass-through is complete ($\eta$=1) and the "superelasticity" is equal to zero (S=0). Thus, $\Delta_x$ boils down to zero ($\Delta_x$=0), and so $|\varepsilon_{\varphi_d^*}(\tau)|$ equates unity: $|\varepsilon_{\varphi_d^*}(\tau)|$ = 1. Hence, under CES demand, a small reduction in the variable trade cost yields zero net gains from variety, as in Feenstra (2010). In this sense, the CES can be considered as a boundary case where gross gains from newly imported varieties exactly cancel out with disappearing domestic varieties due to firm exit.

Inspection of the above expression reveals that beyond the CES case, the NVE may be strictly positive or negative depending on demand curvature. When demand is sub-convex, the demand elasticity is decreasing in firm productivity and so, the firm at the domestic cutoff faces the highest demand elasticity: $\sigma_d^*(\varphi_d^*) > \sigma_x^*(\varphi_x^*) > \bar{\sigma}_x(\bar{\varphi}_x)$. This implies a strictly positive value of $\Delta_x$ ($\Delta_x > 0$) and so, a less than proportional increase in the domestic cutoff upon a small

reduction in the variable trade cost: $|\varepsilon_{\varphi_d^*}(\tau)| < 1$. This, in turn, immediately yields a positive net variety effect (NVE>0). This result is reversed when demand is super-convex: the firm at the domestic cutoff faces the lowest demand elasticity: $\sigma_d^*(\varphi_d^*) < \sigma_x^*(\varphi_x^*) < \bar{\sigma}_x(\bar{\varphi}_x)$. This implies a strictly negative value of $\Delta_x$ ($\Delta_x < 0$) and so, a more than proportional increase in the domestic cutoff upon a small reduction in the variable trade cost: $|\varepsilon_{\varphi_d^*}(\tau)| > 1$. This, in turn, immediately yields a negative net variety effect (NVE<0).

Therefore, regardless of whether preferences are directly- or indirectly-separable, gains from selection and net variety gains coexist in general equilibrium if and only if demand is sub-convex. As was the case for the magnitude of the gains from trade, the type of preferences has only a marginal effect on the structure of the gains from trade. This what I discuss next.

Now by simply invoking peculiar properties of both families of preferences, I can easily specify the additional condition for the pro-competitive effect of trade to be operative. When preferences are indirectly-separable, the demand elasticity is invariant to changes in the intensity of competition (Bertoletti and Etro, 2017; Bertoletti, Etro, and Simonovska, 2018). This peculiar property precludes any adjustment in domestic markups upon trade liberalization. In contrast, when preferences are directly-separable, the demand elasticity varies with consumption level, and may thus increase or decrease with changes in the intensity of competition depending on the curvature of demand.

As is well known, when preferences are directly-separable and demand is sub-convex, the demand elasticity decreases with the consumption level. As higher exposure to trade lowers demand for domestic varieties, domestic firms face then a higher demand elasticity and are thus forced to reduce their markups. By contrast, under this class of preferences, when demand is super-convex, trade induces an increase in domestic markups (Mrázová and Neary, 2017; Zhelobodko et al., 2012).Therefore, an increase in the exposure of an economy to trade, occurring through a decrease in the variable trade cost, delivers gains from: (i) selection; (ii) a net increase in product variety; and (iii) a pro-competitive reduction in domestic markups, if and only demand is sub-convex and preferences are directly-separable.

## 1.3 A Finer Explanation at the Firm-level

In order to provide a finer explanation for the results highlighted above, I proceed in three steps. I first study the effect of a small reduction in the variable trade cost on the profile of profits across exporting firms. Then, I show how this initial effect on export profits is transmitted to purely domestic firms through the competition channel. Finally, I connect the last two effects to show how the added flexibility in consumer and firm behaviors gives rise to these novel welfare predictions.

The Impact of Trade on the Profile of Operating Profits across Active Exporters

The impact of a small reduction in the variable trade cost $\tau$ on operating profits of any active exporter ($\forall \varphi \geq \varphi_x^*$) can be derived using the absolute value of the elasticity of its general equilibrium expression with respect to $\tau$. Using the general equilibrium expression of operating profits on the export market in equation (3.22), this elasticity can be simply written as:

$$\forall \varphi \geq \varphi_x^*, \quad \varepsilon_{\pi_x^o(\varphi)}(\tau^-) = \underbrace{\eta(\varphi)[\sigma(\varphi) - 1 + S]}_{I^+} - \underbrace{\frac{a(\varphi)\,\theta}{[1 + \varepsilon_{I_x}(P)]}}_{C^-} \quad (3.32)$$

The above expression shows that the net outcome depends on a horse race between two opposite effects. The first is positive, given by ($I^+$), and can be called "the initial positive effect". That is, a reduction in the variable trade cost makes any active exporter more price competitive, and thus raises its revenues and operating profits at the initial level of competition. The second is negative, given by ($C^-$), and can be thought of as a "competition effect". In deed, this reduction in the variable trade cost makes every active exporter more price competitive and may induce an increase in the number of exporters.[33] This, in turn, leads to an increase in the intensity of competition on the export market.

As shown below in Table 1, the net effect is always heterogeneous across active exporters, except under the CES case where it boils down to zero for all exporters. As to whether this net effect is strictly positive or negative for the least or the most productive exporters, I show that this crucially depends on demand curvature.

---

[33]Inspection of the general equilibrium expression of the export cutoff clearly indicates that a reduction in the variable trade cost always leads to a proportional increase in the mass of exporters. Yet, this standard result holds only when the instantaneous general equilibrium effect (channeled through the trade exposure index) is ignored.

**Figure 4. Impact of Trade on Active Exporters and Domestic Firms under Sub-convex Demands.**



Panel A. Impact of Trade on Active Exporters                Panel B. Impact of Trade on Domestic Firms
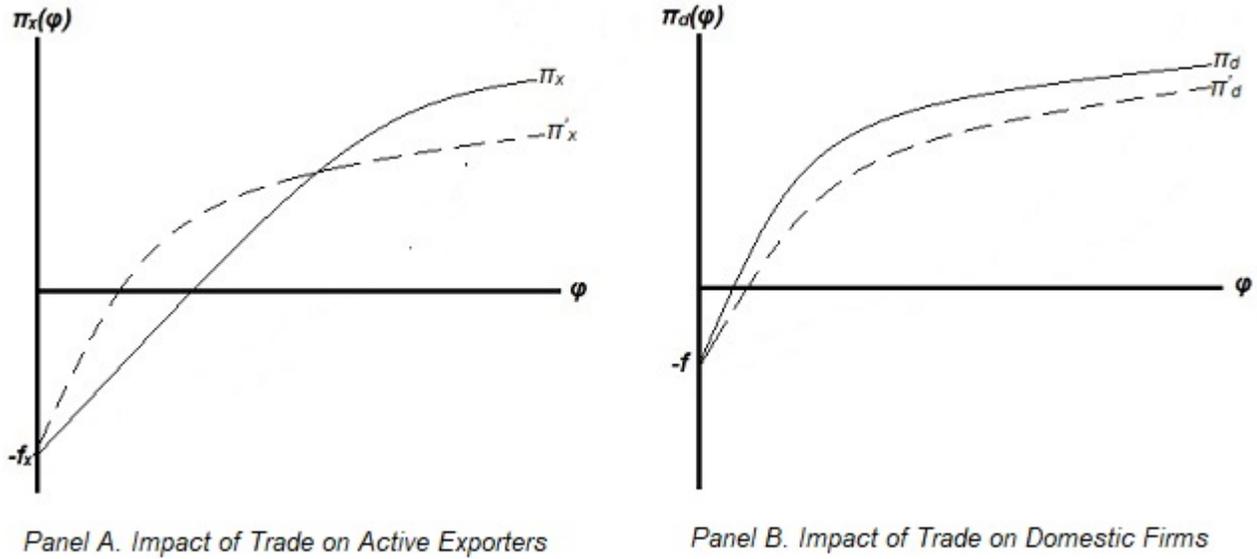
Table 1. Heterogeneous vs Identical Impact of Trade on Active Exporters

| Demand | Sub-convex | CES | Super-convex |
|---|---|---|---|
| Least productive exporters: $\varphi \to \varphi_x^*$ | $\varepsilon_{\pi_x^o(\varphi)}(\tau^-) > 0$ | $\varepsilon_{\pi_x^o(\varphi)}(\tau^-) = 0$ | $\varepsilon_{\pi_x^o(\varphi)}(\tau^-) < 0$ |
| Most productive exporters: $\varphi \to +\infty$ | $\varepsilon_{\pi_x^o(\varphi)}(\tau^-) < 0$ | $\varepsilon_{\pi_x^o(\varphi)}(\tau^-) = 0$ | $\varepsilon_{\pi_x^o(\varphi)}(\tau^-) > 0$ |

Regardless of the type of preferences, upon a small reduction in the variable trade cost $\tau$, operating profits rise for the least productive exporters, whereas they fall for the most productive ones when demand is sub-convex. This is illustrated in Figure 4, panel A, where the solid locus $\pi_x$ denotes the initial profile of export profits across firms, while the dashed locus $\pi_x'$ denotes the post-variable trade cost reduction profile when demand is sub-convex. By contrast, when demand is super-convex, this result is reversed. As illustrated in Figure 5, panel A, the outcome exhibits a strong "Matthew Effect": while the most productive exporters (who are initially the most profitable) experience a net increase in their operating profits, the least productive exporters (who are initially the least profitable) experience a net decrease in their operating profits.[34]

---

[34]In the words of Mrázová and Neary (2017), the "Matthew Effect" refers to the case where "to those who have, more shall be given". It is also worth noting that this effect occurs under sub-convex demands in Mrázová and Neary (2017). By contrast, in the current chapter, the "Matthew Effect" occurs when demand is super-convex. I will explain why we obtain the same result under opposite demand curvatures in the next subsection.

**Figure 5. Impact of Trade on Active Exporters and Domestic Firms under Super-convex Demands.**



Panel A. Impact of Trade on Active Exporters      Panel B. Impact of Trade on Domestic Firms
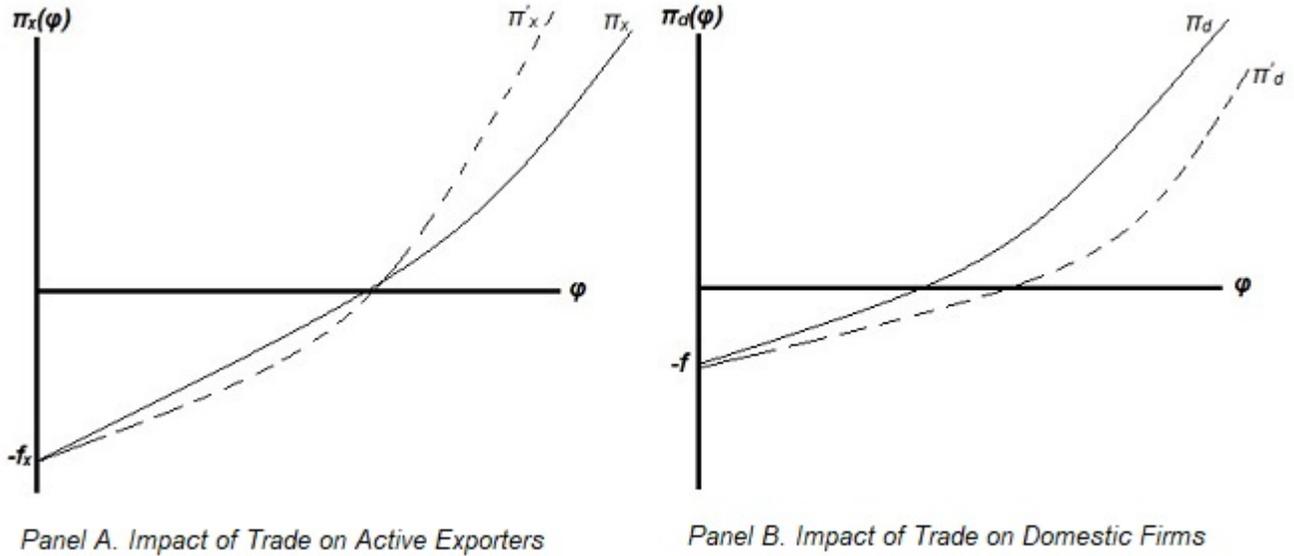
**Proposition 1** *Whether preferences are directly- or indirectly-separable, the impact of a small reduction in the variable trade cost on the profile of operating profits across active exporters crucially depends on demand curvature. When demand is sub-convex, profits rise for less productive exporters, whereas those of more productive ones fall. By contrast, when demand is super-convex, profits rise for more productive exporters, whereas those of less productive exporters fall. CES demand is a boundary case where the profits of all exporters remain unchanged regardless of their productivity level.*

The economic intuition behind this result can be explained as follows. When demand is sub-convex, the least/most productive exporters face the highest/lowest demand elasticity. This implies that consumers react the most/least to price variations of varieties supplied by the least/most productive exporters. It follows then that upon a small reduction in the variable trade cost, the initial positive effect $(I^+)$ is so magnified for less productive exporters that it dominates the competition effect $(C^-)$. Hence, profits rise for less productive exporters. Conversely, for more productive exporters, the initial positive effect $(I^+)$ is too mild to offset the competition effect $(C^-)$, so their operating profits fall.

These results are reversed when demand is super-convex: now consumers react the most to price variations of varieties sold by the most productive exporters, and the least to price variations of those supplied by the least productive exporters. Hence, the initial positive effect $(I^+)$ dominates for more productive exporters, so their operating profits rise.
In contrast, for less productive exporters, the initial positive effect $(I^+)$ is not strong enough to offset the competition effect $(C^-)$, so their operating profits fall.

## Curvature of Demand vs Type of Preferences: A Detailed Discussion

Clearly, the result highlighted above is solely attributable to added flexibility on the demand side in the current model. In deed, incorporating both cases of sub-convex and super-convex demands opens the door for a more realistic modeling of consumer behavior than allowed by the homothetic CES. The added flexibility here is that consumers may exhibit either a weak or strong price sensitivity to the cheapest or the most expensive varieties. This, in turn, allows the general equilibrium effect of trade to be heterogeneous across exporters beyond the CES case.

Here, again the type of preferences has minor implications for this result. Specifically, the nature of preferences solely determines whether the competition effect ($C^-$) is identical across exporters or firm-specific. As previously mentioned, when preferences are indirectly-separable, (a) is always equal to one ($\forall \varphi, a(\varphi) = 1$), which clearly indicates that the competition effect is identical across exporters under this class of preferences.

By contrast, when preferences are directly-separable and non-CES, $a(\varphi)$ may be either strictly higher or lower than one depending on demand curvature and the productivity level of the firm at question. That is, under sub-convex demands, $a(\varphi)$ is strictly higher than one for the less productive exporters, and strictly lower than one for more productive exporters ($\forall \varphi \to \varphi_x^*$, $a(\varphi) > 1$; $\forall \varphi \to +\infty$, $a(\varphi) < 1$).[35] This order is reversed under super-convex demands ($\forall \varphi \to \varphi_x^*$, $a(\varphi) < 1$; $\forall \varphi \to +\infty$, $a(\varphi) > 1$). Now I can easily verify that even when the competition effect is firm-specific, the novel result highlighted in Proposition 1 always holds with the aid of the following example:

Under sub-convex demands, and directly-separable preferences, even though the competition effect is magnified for the least productive exporters, these latter experience a net increase in their operating profits, as highlighted in Proposition 1. This immediately reveals that the initial positive effect ($I^+$) always dominates the competition effect ($C^-$). Hence, it is the magnitude of the initial positive effect ($I^+$) that pins down the sign of the net effect of trade on export profits, as reflected by Proposition 1. Finally, as demand curvature governs the magnitude of the initial positive effect ($I^+$), I can then conclude that demand curvature plays a first-order role in driving this result, while the type of preferences has only a second-order importance in this regard.

## Impact of Trade on Purely Domestic Firms

As illustrated in panel B of Figures 4 and 5, trade liberalization always induces an increase in the domestic cutoff, which reflects that increased exposure to trade forces the least productive domestic firms to exit the market. While this is a standard result due to Melitz (2003), the novelty here is twofold.

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[35]Given the definition of demand curvature and the expression of $a(\varphi)$, this follows by inspection.

First, demand curvature determines the nature of the competitive effect of trade. Inspection of Proposition 1 reveals that whether demand is CES or not plays a critical role. When they are, operating profits of all active exporters remain unchanged, and thus additional export market shares are entirely reapt by infra-marginal exporters. Increased labor demand by these new exporters bids up the real wage and forces the the least productive firms to exit, exactly as in Melitz (2003). Hence, as stressed by Melitz (2003), the rigidity of the CES constrains the competitive effect of trade to occur only on the labor market. It precludes then another important and more intuitive channel for the competitive effect of trade, which operates through increases in the intensity of competition on the final good market.[36]

However, beyond CES demand, higher exposure to trade always increases profits of one category of active exporters. Whether this latter corresponds to the most or the least productive exporters, this crucially depends on whether demand is sub-convex or super-convex. This immediately ensures that under both alternatives, the competitive effect of trade operates through an increase in the intensity of competition on the final good market.

Second, demand curvature governs the magnitude of the competitive effect of trade. As hinted to in the previous paragraph, demand curvature determines which category of active exporters reaps additional market shares upon trade liberalization. By doing so, it immediately pins down the magnitude of firm exit. For instance, as highlighted in Proposition 1, when demand is super-convex, additional export market shares are reapt by the most productive exporters. Since this category of exporters has initially large market shares, their increase (upon variable trade cost reduction) leads then to a sharp increase in the intensity of competition. This, in turn, forces a large fringe of domestic firms to exit the market.

In contrast, when demand is sub-convex, additional export market shares are reapt by the least productive exporters. Their market shares are initially small, and so their increase induces a slight increase in the intensity of competition. Hence, only a small fringe of domestic firms is forced to exit the market. The CES is then a special case where additional export market shares are reapt by new exporters. Only under CES demand, these latter face the average demand elasticity ($\forall \varphi$, $\sigma(\varphi) = \bar{\sigma} = \sigma$). Hence, the magnitude of firm exit under CES demand can be considered as an intermediate outcome.

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[36]Melitz and Ottaviano (2008) restored a theoretical role for this channel using quadratic preferences, which are non-additive.

## 2. A small Decrease in the Fixed Trade Cost

Now I study the case where higher exposure to trade occurs through a small reduction in the fixed cost of exporting $f_x$. As in the previous case, I separately examine the welfare implications of the curvature of demand and the type of preferences. Using the standards measures proposed in **Section 5.3**, the magnitude of the gains from trade (GFT) and the net variety effect (NVE) which captures their structure are respectively given by:

$$
\begin{cases}
GFT(f_x^-) = -\varepsilon_{\varphi_d^*}(f_x) = \dfrac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \dfrac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*)} \dfrac{1}{\theta+\Delta_x} \\[4mm]
NVE(f_x^-) \equiv \dfrac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} - |\varepsilon_{\varphi_d^*}(f_x)| = \dfrac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} - \dfrac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \dfrac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*)} \dfrac{1}{\theta+\Delta_x}
\end{cases}
$$

$$(3.33)$$

Inspection of the above expressions clearly shows that a small decrease in the fixed trade cost induces identical results to those described for the small reduction in the variable trade cost.

The only difference here is in the theoretical mechanism driving the result. In deed, under this scenario of trade liberalization, such small reduction in the fixed cost of exporting encourages entry of infra-marginal firms to the export market. As to whether they capture small or large market shares, this crucially depends on demand curvature. When demand is super-convex, these new exporters face the lowest demand elasticity, their relatively low productivity level is thus a mild disadvantage under this case and so, they capture large market shares. This, in turn, magnifies the import competition effect. This induces then a strong selection effect of trade, forcing a large fringe of domestic firms to exit the market. These patterns are reversed when demand is sub-convex. As it imposes the demand elasticity to be identical across firms, CES demand is clearly a boundary case.

Therefore, by magnifying Melitz (2003)'s selection effect, super-convex demands provides an upper bound for the gains from trade. As before, sub-convex demands provide a lower bound, while the CES delivers an intermediate outcome. As for the structure of the gains from trade, it is readily verified that, as before, gains from selection and net variety gains coexist only under sub-convex demands. Additionally, under this curvature of demand, when preferences are directly-separable, the pro-competitive effect of trade on domestic markups is operative, and thus the three sources of welfare gains from trade coexist.

# 3. A small Increase in the Number of Trading Countries $N$

Similarly, I can now investigate the case where higher exposure to trade occurs through a small increase in the number of trading countries at the World level. As in the previous two cases, the objective is to separately examine the welfare implications of the curvature of demand and the type of preferences. As before, using the standards measures proposed in **Section 5.3**, the magnitude of the gains from trade (GFT) and the net variety effect (NVE) which captures their structure are respectively given by:

$$\begin{cases} GFT(N^+) = \varepsilon_{\varphi_d^*}(N) = \frac{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*)} \frac{1}{\theta+\Delta_x} \\ NVE(N^+) \equiv 1 - \varepsilon_{\varphi_d^*}(N) = 1 - \frac{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \frac{a_d^*(\varphi_d^*)}{a_x^*(\varphi_x^*)} \frac{1}{\theta+\Delta_x} \end{cases} \tag{3.34}$$

Inspection of the above expressions clearly shows that a small increase in the number of trading countries yields identical results to those described for the two previous scenarios.

The only difference here is in the theoretical mechanism underlying the result. In deed, under this scenario of trade liberalization, such small increase in the number of countries implies that domestic firms in any existing country face an additional competition from exporters in these newly trading countries. As to whether this competition effect is strong or mild , this is fully governed by demand curvature. When demand is super-convex, additional export market shares are reapt by the most productive exporters in these newly trading countries. This, in turn, magnifies the import competition effect, which induces then a strong selection effect of trade, forcing a large fringe of domestic firms to exit the market. These patterns are reversed when demand is sub-convex. As it imposes the demand elasticity to be identical across exporters, CES demand is clearly a boundary case.

Hence, as before, by magnifying Melitz (2003)'s selection effect, super-convex demands provides an upper bound for the gains from trade. Sub-convex demands provide a lower bound, while the CES delivers an intermediate outcome. As for the structure of the gains from trade, it is readily verified that, as before, gains from selection and net variety gains coexist only under sub-convex demands. Additionally, under this curvature of demand, when preferences are directly-separable, the pro-competitive effect of trade on domestic markups is restored. Therefore, these three sources of welfare gains from trade coexist if and only if demand is sub-convex and preferences are directly-separable.

# 3.6   Conclusion

This chapter develops a general yet tractable theoretical framework which combines standard assumptions on the supply side with a flexible demand system, while taking variable and fixed trade barriers in due account. The current model is then well-suited to examine the welfare implications of different scenarios of trade liberalization under general demand conditions. The novelty here is that it is possible to separately examine the implications of the curvature of demand and the type of preferences for the gains from trade. The key finding of this chapter is that while demand curvature plays a crucial role in driving comparative statics results and determining the structure and the magnitude of the gains from trade, the type of preferences has only a second-order importance from a welfare standpoint. A key message of this chapter is that rather than assuming a specific type of preferences, more precise estimates of the curvature of demand are necessary to answer comparative statics questions, and to quantify the gains from trade.

# Appendix D

## D.1 Deriving firm-level revenues in partial equilibrium

Let us start with recalling that the price aggregator is the implicit solution of the following equation:

$$\int_{\omega \in \Omega} p_\omega \, Q(\Lambda) \, D_\omega \left( V(\Lambda) \, \frac{p_\omega}{w} \right) d\omega = w \qquad (3.35)$$

Now using the parameterization of functions $Q(\Lambda)$ and $V(\Lambda)$: $Q(\Lambda)=\Lambda^{-\beta}$, $V(\Lambda)=\Lambda^{\alpha}$, and recalling that the elasticity of each function with respect to its determinant is given by: $\varepsilon_Q = -\beta$, $\varepsilon_V = \alpha$, and $\varepsilon_{D_\omega} = -\sigma_\omega > 1$, along with normalizing wage to unity ($w=1$) by choice of labor as numéraire, and rearranging yields:

$$\Lambda = \left[ \int_{\omega \in \Omega} p_\omega^{1-\sigma_\omega} \, d\omega \right]^{\frac{1}{(\beta+\alpha\bar{\sigma})}} \qquad (3.36)$$

where $\bar{\sigma}$ is the average demand elasticity at the industry level, $\alpha$ and $\beta$ are both dummies capturing the type of preferences, such as the case ($\alpha=0$ and $\beta=1$) corresponds to indirectly-separable preferences, while directly-separable preferences correspond to ($\alpha=1$ and $\beta=0$).

Now let us denote by $P(\Lambda) = \Lambda^{-(\beta+\alpha\,\bar{\sigma})} = \left[ \int_{\omega \in \Omega} p_\omega^{1-\sigma_\omega} \, d\omega \right]^{-1}$ the conventional price index. Then, with the aid of the parameterization along with the above definition of the conventional price index $P$, the Gorman-Pollak demand function described in equation (3.1) (in the main text) boils down to $x_\omega = p_\omega^{-\sigma_\omega} \, P^{a_\omega}$, where $a_\omega = \frac{\beta+\alpha\,\sigma_\omega}{\beta+\alpha\,\bar{\sigma}}$. By assuming that any $\omega$ variety is supplied by a $\varphi$ productivity firm, I can work throughout with $\varphi$ as a firm subscript: $\sigma_\omega = \sigma(\varphi)$; $a_\omega = a(\varphi)$. Finally, by recalling that consumers are identical and a firm's market demand is given by $q(\varphi) = x(\varphi) \, L$, firm revenues boil down to: $r(\varphi) = p(\varphi) \, q(\varphi) = p(\varphi)^{1-\sigma(\varphi)} \, P^{a(\varphi)} \, L$.

## D.2 The "Exponent Elasticity Method" (EEM)

As stressed in the main text, increased generality raises tractability issues. Thus, in order to gain in generality without losing in tractability, I resort to a new and simple method that I call the "Exponent Elasticity Method" (EEM, hereafter) which delivers a tractable solution for the general equilibrium price index despite added flexibility in preferences. The starting point is the partial equilibrium expression of the price index provided in equation (3.14) in the main text:

$$P = M_e^{-1} \Big[ \underbrace{\int_{\varphi_d^*}^{+\infty} p_d(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_d} + (N-1) \underbrace{\int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} g(\varphi) d\varphi}_{I_x} \Big]^{-1}$$

Clearly, the mathematical challenge consists in solving for both integrals ($I_d$, $I_x$) without assuming that the demand elasticity is identical across firms ($\forall \varphi$, $\sigma(\varphi) = \sigma$) and then using this latter as a constant for integrating. Such simplicity is only possible under CES demand, which is the unique case where it is possible to solve for these integrals. As the general demand system considered in this chapter encompasses the CES and more flexible alternatives allowing the demand elasticity to vary across firms, it is then impossible to solve for these integrals under general demands.

Given the impossibility to solve for these integrals in the current setting, the key idea that the EEM method proposes is to locally approximate both integrals ($I_d$, $I_x$) around the equilibrium with a multiplicative equivalent which has a finite number of determinants, such as the exponent of each determinant embodies the elasticity of the average price with respect to it.[37] This requires a five-step procedure that I explain in detail as follows.

### The "EEM" method: A Five-step Procedure

### Step 1. Rewrite the integral ($I_x$) using unbounded Pareto:

By invoking this assumption, it is readily verified that $[1 - G(\varphi_x^*)] = \varphi_x^{*-\theta}$. Using this, integral $I_x$ can be rewritten as:

$$I_x = \varphi_x^{*-\theta} \underbrace{\int_{\varphi_x^*}^{+\infty} p_x(\varphi)^{1-\sigma(\varphi)} \mu_x(\varphi) d\varphi}_{I_{x_0}} \tag{3.37}$$

---

[37]To be precise, this corresponds to the average price to the power of $(1-\bar{\sigma})$: $\bar{p}_d^{1-\bar{\sigma}_d}$ and $\bar{p}_x^{1-\bar{\sigma}_x}$.

where $\mu_x(\varphi) = \frac{g(\varphi)}{[1-G(\varphi_x^*)]}$ is the productivity distribution conditional on successful penetration of the export market. Clearly, the unique difference between the initial integral $I_x$ and the new integral $I_{x_0}$ is that this latter is expressed using the conditional productivity distribution $\mu_x(\varphi)$.

## Step 2. Approximate integral $I_{x_0}$ with a multiplicative equivalent:

Now by recalling that $\eta(\varphi) = -\frac{dlog\ p(\varphi)}{dlog\ \varphi}$ is our measure of the relative cost-price pass-through and that the variable trade cost $\tau$ is multiplicative by definition, the integral $I_{x_0}$ can be locally approximated as follows:

$$I_{x_0} \equiv \tau^{\bar{\eta}_x(1-\bar{\sigma}_x(\bar{\varphi}_x))}\ \ \bar{\varphi}_x(\varphi_x^*)^{\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)} \tag{3.38}$$

Since operating profits are monotonically increasing in productivity, and exporting involves not only, a variable trade cost $\tau_{ij}$, but also a fixed cost $f_x$, the equilibrium export cutoff $\varphi_x^*$ exists and is unique. This, in turn, ensures that this local approximation (around the trade equilibrium) delivers a unique multiplicative equivalent to integral $I_{x_0}$.

## Step 3. Obtain a final expression of integral $I_x$ using this of $I_{x_0}$ and unbounded Pareto:

Let us now recall that unbounded Pareto distribution gives rise to constant mean-to-min ratio: $\bar{\varphi}_x = \frac{\theta}{\theta-1}\ \varphi_x^*$. By plugging the multiplicative equivalent of integral $I_{x_0}$ from equation (3.38) into the initial expression of integral $I_x$ in equation (3.37) and invoking this practical property of unbounded Pareto, I obtain the following multiplicative equivalent for integral $I_x$:

$$I_x \equiv \ \ \kappa\ \ \tau^{\bar{\eta}_x(1-\bar{\sigma}_x(\bar{\varphi}_x))}\ \ \varphi_x^{*[\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)-\theta]} \tag{3.39}$$

where $\kappa = (\frac{\theta}{\theta-1})^{\bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)}$ is a constant.

## Step 4. Approximate the partial equilibrium export cutoff $\varphi_x^*$ with an explicit multiplicative equivalent:

Let us first recall that the export cutoff $\varphi_x^*$ is endogenous and defined as the implicit solution of the zero profit condition on the export market (ZPCX): $\pi_x(\varphi_x^*)=0$, described in equation (3.13) in the main text:

$$(ZPCX) \; \varphi_x^* : \sigma_x^*(\varphi_x^*)^{-1} \, p_x^*(\varphi)^{1-\sigma_x^*(\varphi_x^*)} \, P^{a_x^*(\varphi_x^*)} \, L = f_x \qquad (3.40)$$

Now by isolating the firm-specific component of the operating profit on the left hand-side, approximating it in a multiplicative way, as in equation (3.38), and rearranging, I obtain the following explicit equivalent of the partial equilibrium export cutoff:

$$\varphi_x^* \equiv \tau \; f_x^{\frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}} \; L_j^{-\frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}} \; P^{-\frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}} \qquad (3.41)$$

## Step 5. Solving for the general equilibrium price aggregator $P$ :

Now by plugging the explicit equivalent of the export cutoff from equation (3.41) in the expression of integral $I_x$ in equation (3.39), this latter can be expressed solely as a function of $(\tau, f_x, L, P)$.

Then, by applying the same procedure for integral $I_d$, and plugging the final expressions of both integrals in the partial equilibrium price index given by equation (3.14) in the main text, and rearranging, I obtain a tractable solution for the general equilibrium price index:

$$P \equiv c_E^{[1+\varepsilon_{I_x}(P)]^{-1}} \; L^{-[\frac{1+\varepsilon_{I_x}(L)}{1+\varepsilon_{I_x}(P)}]} \; T(\tau, f_x, N)^{-[1+\varepsilon_{I_x}(P)]^{-1}} \qquad (3.42)$$

where $c_E = \bar{\sigma} \, [\, f_e + [1 - G(\varphi_d^*)] \, f + (N-1)[1 - G(\varphi_x^*)] \, f_x \,]$ captures entry conditions in every country,

$T(\tau, f_x, N) = N \, \tau^{-\theta} \, f_x^{-\frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]}}$ is an index of exposure to trade, and both elasticities $\varepsilon_{I_x}(L)$ and $\varepsilon_{I_x}(P)$ take a simple form, and are respectively given by:

$$\begin{cases} \varepsilon_{I_x}(L) = \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} > 0 \\ \varepsilon_{I_x}(P) = a_x^*(\varphi_x^*) \frac{[\theta - \bar{\eta}_x(\bar{\sigma}_x(\bar{\varphi}_x)-1)]}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} > 0 \end{cases} \qquad (3.43)$$

## D.3 Solving for the domestic and export cutoffs in general equilibrium

By plugging the general equilibrium price index from equation (3.42) in the explicit partial equilibrium expression of the export cutoff in equation (3.41), and applying the same procedure for the domestic cutoff, I can solve for their general equilibrium expressions, respectively, as follows:

$$\varphi_x^* \equiv \tau \; f_x^{\varepsilon_{\varphi_x^*}(f_x)} \; c_E^{\gamma_x^*} \; L^{\varepsilon_{\varphi_x^*}(L)} \; T(\tau, f_x, N)^{\varepsilon_{\varphi_x^*}(T)} \qquad (3.44)$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_x^*}(f_x) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} \\ \gamma_x^* = -\frac{a_x^*(\varphi_x^*)}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} \left[ 1 + \varepsilon_{I_x}(P) \right]^{-1} \\ \varepsilon_{\varphi_x^*}(L) = \frac{1}{\eta_x^*[\sigma_x^*(\varphi_x^*)-1+S]} \left[ a_x^*(\varphi_x^*) \frac{1+\varepsilon_{I_x}(L)}{1+\varepsilon_{I_x}(P)} - 1 \right] \\ \varepsilon_{\varphi_x^*}(T) = \frac{a_x^*(\varphi_x^*)}{\eta_d^*[\sigma_x^*(\varphi_x^*)-1+S]} \left[ 1 + \varepsilon_{I_x}(P) \right]^{-1} \end{cases}$$

Similarly, the general equilibrium domestic cutoff can be written as:

$$\varphi_d^* \equiv f^{\varepsilon_{\varphi_d^*}(f)} \; c_E^{\gamma_d^*} \; L^{\varepsilon_{\varphi_d^*}(L)} \; T(\tau, f_x, N)^{\varepsilon_{\varphi_d^*}(T)} \qquad (3.45)$$

where the above exponents are respectively given by:

$$\begin{cases} \varepsilon_{\varphi_d^*}(f) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \\ \gamma_d^* = -\frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \left[ 1 + \varepsilon_{I_x}(P) \right]^{-1} \\ \varepsilon_{\varphi_d^*}(L) = \frac{1}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \left[ a_d^*(\varphi_d^*) \frac{1+\varepsilon_{I_x}(L)}{1+\varepsilon_{I_x}(P)} - 1 \right] \\ \varepsilon_{\varphi_d^*}(T) = \frac{a_d^*(\varphi_d^*)}{\eta_d^*[\sigma_d^*(\varphi_d^*)-1+S]} \left[ 1 + \varepsilon_{I_x}(P) \right]^{-1} \end{cases}$$

## D.4 Solving for the weighted average productivity at the industry level in general equilibrium

As mentioned in the main text, the weighted average productivity of all firms (both domestic and foreign exporters) competing in a single country can be written as:

$$\bar{\varphi} = \frac{M_d}{M_W} \int_{\varphi_d^*}^{+\infty} \varphi \, \mu_d(\varphi) d\varphi + \frac{(N-1)M_x}{M_W} \int_{\varphi_x^*}^{+\infty} \varphi \, \mu_x(\varphi) d\varphi \qquad (3.46)$$

where $M_W = NM_e$ is the total mass of entrants at the World level, and can be thought of as a proxy for the size of the World market. By solving for the above integrals, $\bar{\varphi}$ can be then written as a function of the domestic and the export cutoffs:

$$\bar{\varphi} = \frac{M_d}{M_W} \left(\frac{\theta}{\theta-1}\right) \varphi_d^* + \frac{(N-1)M_x}{M_W} \left(\frac{\theta}{\theta-1}\right) \varphi_x^* \qquad (3.47)$$

Now using the general equilibrium expressions of the export and domestic cutoffs, respectively, from equations (3.44) and (3.45), the ratio $\frac{\varphi_x^*}{\varphi_d^*}$ can be written as:

$$\frac{\varphi_x^*}{\varphi_d^*} = \tau \; \left(\frac{f_x^{\varepsilon_{\varphi_x^*}(f_x)}}{f^{\varepsilon_{\varphi_d^*}(f)}}\right) \; c_E^{\Delta_{x,d}(c_E)} \; L^{\Delta_{x,d}(L)} \; T^{\Delta_{x,d}(T)} \qquad (3.48)$$

where $\Delta_{x,d}(c_E) = \gamma_x^* - \gamma_d^*$, $\Delta_{x,d}(L) = \varepsilon_{\varphi_x^*}(L) - \varepsilon_{\varphi_d^*}(L)$, and $\Delta_{x,d}(T) = \varepsilon_{\varphi_x^*}(T) - \varepsilon_{\varphi_d^*}(T)$.
Using the above relationship between the export and the domestic cutoff, the weighted average productivity in equation (3.47) can be rewritten as follows:

$$\bar{\varphi} = \varphi_d^* \, \Psi(.) \qquad (3.49)$$

where $\Psi(.) = \left[ \frac{M_d}{M_W} \left(\frac{\theta}{\theta-1}\right) + \frac{(N-1)M_x}{M_W} \left(\frac{\theta}{\theta-1}\right) \tau \left(\frac{f_x^{\varepsilon_{\varphi_x^*}(f_x)}}{f^{\varepsilon_{\varphi_d^*}(f)}}\right) c_E^{\Delta_{x,d}(c_E)} L^{\Delta_{x,d}(L)} T^{\Delta_{x,d}(T)} \right] > 1.$

## D.5 Proof of Theorem 2

**Theorem 2** states that on any market (domestic, or export), super-convex demands provide an upper bound for the productivity cutoff in general equilibrium, while sub-convex demands provide a lower bound. Within these bounds, the CES delivers an intermediate result. This can be shown using the general equilibrium expression of the domestic cutoff in equation (3.45) (or, equivalently this of the general equilibrium export cutoff) as follows:
Let $\varphi_d^*(super)$, and $\varphi_d^*(sub)$ denote the domestic cutoffs, respectively, under super-convex demands, and sub-convex demands. Using equation (11), the ratio $\frac{\varphi_d^*[super]}{\varphi_d^*[sub]}$ can be written as:

$$\frac{\varphi_d^*[super]}{\varphi_d^*[sub]} = f^{\Delta_1} \; c_E^{\Delta_2} \; L^{\Delta_3} \; T^{\Delta_4} \tag{3.50}$$

Using the expressions of the elasticity of the general equilibrium domestic cutoff with respect to each of its determinants provided in equation (3.45), $\Delta_1$, $\Delta_2$, $\Delta_3$, and $\Delta_4$ are respectively given by:

$$\begin{cases} \Delta_1 = \varepsilon_{\varphi_d^*}(f)[super] - \varepsilon_{\varphi_d^*}(f)[sub] \\ \Delta_2 = \gamma_d^*[super] - \gamma_d^*[sub] \\ \Delta_3 = \varepsilon_{\varphi_d^*}(L)[super] - \varepsilon_{\varphi_d^*}(L)[sub] \\ \Delta_4 = \varepsilon_{\varphi_d^*}(T)[super] - \varepsilon_{\varphi_d^*}(T)[sub] \end{cases}$$

Finally, inspecting the expressions of the above elasticities, and recalling that the firm at the cutoff faces the lowest demand elasticity under super-convex demands, whereas it faces the highest demand elasticity under sub-convex demands, immediately reveals that $\frac{\varphi_d^*[super]}{\varphi_d^*[sub]} > 1$. Since under CES demand, the cutoff productivity firm faces the average demand elasticity, this ensures then that the CES delivers an intermediate result:
$\varphi_d^*[sub-convex] < \varphi_d^*[CES] < \varphi_d^*[super-convex]$.

## D.6 Proof of Theorem 3

**Theorem 3** states that super-convex demands provide a lower bound for the degree of partitioning of firms by export status in general equilibrium, while sub-convex demands provide an upper bound. Within these bounds, the CES delivers an intermediate result. This can be shown using the general equilibrium expression of the ratio $\frac{\varphi_x^*}{\varphi_d^*}$ in equation (3.48), as follows:

$$\frac{\varphi_x^*}{\varphi_d^*} = \underbrace{(\frac{f_x^{\varepsilon_{\varphi_x^*}(f_x)}}{f^{\varepsilon_{\varphi_d^*}(f)}})}_{(1)} \; \tau \; c_E^{\Delta_{x,d}(c_E)} \; L^{\Delta_{x,d}(L)} \; T^{\Delta_{x,d}(T)} \tag{3.51}$$

Since entry conditions $c_E$, market size $L$, and the degree of exposure to trade $T$ are identical across countries, the partitioning of firms by export status is mainly driven by the presence of variable and fixed trade barriers. Hence, inspecting the above ratio while restricting our focus on its first component (1), and recalling that under super-convex demands, the firm at the domestic cutoff faces a lower demand elasticity than the firm at the export cutoff and that this order

is reversed under sub-convex demands, immediately reveals that $\frac{\varphi_x^*}{\varphi_d^*}$[super-convex] $<$ $\frac{\varphi_x^*}{\varphi_d^*}$[sub-convex]. Finally, recalling that under CES demands both cutoff firms face the same demand elasticity $\sigma_d^* = \sigma_x^* = \sigma$ ensures that the CES delivers an intermediate outcome, and thus completes the proof.

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