



Morphogenesis of complex networks : an application in urban growth

Michele Tirico

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Pour obtenir le diplôme de doctorat

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Préparée au sein de l'Université Le Havre Normandie

Morphogenesis of complex networks. An application in urban growth

**Présentée et soutenue par
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Introduction

At the beginning of human society, humans lived in small and nomadic groups and extracted what they needed to survive from nature. Around 10 000 years ago in Near East, first cereal cultivation and animal domestication were introduced; these changes in the way that humans live coincide with the beginning of the Neolithic period¹. Humans understood that the aggregation in a community can help them to solve practical problems and increment their possibilities to survive. They were motivated by individual needs to accommodate their requirements and made individual choices. The behaviour of people is impacted by what they have around them, and they develop different forms of aggregation. Actually, technical innovations and the agricultural revolution led to sedentism and the emergence of first settlements which tended to have a small population, often only a few families. Gradually, the size of these communities incremented, some of them were fused in a bigger one and a variety of human communities of various sizes formed. The results of the human organization are that they modify land: cities are the quintessence of these changes [203].

Cities are the place for the association and the interaction of humans to ensure the individual satisfaction of needs. Cities are characterized by overlapping relations between numerous abstract or physical urban actors, which generate elaborate spatio-temporal dynamics, often irreversible, discontinuous, and unpredictable. Considering a city under this perspective means to suppose that a city could be considered as a system of interacting and heterogeneous elements. A city can be compared with many other systems and as we will observe in this work, there are some characteristics in common between urban systems and other kinds of natural and artificial systems.

Cities have been the subject of many studies in a large number of fields, several approaches have been used and many questions have been answered. We choose here to focus on a fundamental aspect of cities, the study of essential processes that are behind the formation, the evolution, and the maintaining of them. To give a contribution to this aspect of cities, we tackle the problem through the complexity theory point of view and we assume that cities might be considered as complex systems.

Morphogenesis (from the Greek "*morphê*", shape, and "*genesis*", creation, literally, "beginning of the shape") is the biological process that causes an

¹Agriculture started to develop in different regions at different times, hence there is not a single date for the beginning of the Neolithic period. Anyway, early artefacts that prove the socio-technological revolution are founded in Near East.

organism to develop its shape². In a system, elements aggregate in different ways, organize themselves, cluster, stimulate or reduce the formation of new elements: to put it in a verb, they form. Several mechanisms control the formation of a system and they are summarized in a process that allows its formation over time. Morphogenesis is the ensemble of dynamics underlying the development process by which the system acquires the shape and functions of its parts. Hence, to understand the functioning of a system, it is crucial to study the functional role of the form and understand if the form is the cause or the effect (often both) of the organization.

Understanding a shape (its morphology) often demands that we understand the process of its formation (its morphogenesis). The first step towards understanding is to make the reality intelligible, considering the object to study encoded in a rigorous formalism. For instance, the form of a real object that is approximatively a circular ring can be encoded in a mathematical formalism (a circle). As such, the object can be described by the identification of a centre and a diameter. An equation and a subspace of the Cartesian plane correspond to the object. In this way, we obtain a factual knowledge (also known as declarative knowledge) of the real object and the achievement of it consists in a representation of information, that can be classified, summarized, and interpreted. Declarative knowledge involves knowing-that something is; in this way, we capture information about the object (for instance its central symmetry). However, in this manner, we obtain some information which is fundamental to understand the object, but it is not enough to understand its form: in other words, we capture its main characteristics but we have probably not enough information about the process of formation. We also need the know-how about the object, the way that it forms, to obtain a procedural knowledge of it. The study of morphogenesis means to understand the characteristics of the object and investigate the creation and the evolving processes. Modelling the object and simulating the evolution in time, integrating both kinds of knowledge, are a way to study morphogenesis.

Motivations

Since early human settlements, urbanization³ has described a process that has continuously increased, with brusque variations as for instance the quick growth since the middle of 19th century until nowadays. In the last two centuries, a drastic population increment characterizes human society and it is expected to increase by 2 billion persons in the next 30 years [287]. According to the 2018 Revision of World Urbanization Prospects [288], more than half of the population lives in urban areas and it is expected that the proportion will reach the 68% by 2050. Today there are 23 megacities (with a population of more than 10 million) and the number is estimated to be 43 in 2030. The sizes of cities have a large consequence on nature and resources because they consume 60% of the world's energy and for their functioning,

²www.en.wikipedia.org, visited on 20/08/2020.

³Urbanization is the process that describes the tendency of people to aggregate in urban areas. It is measured by the fraction of individuals living in urban areas.

they produce around 80% of the Earth’s greenhouse gases [128]⁴. Urban sprawl is a form of urban growth that happens through an unsustainable urbanization. It contributes to climate change mainly in terms of the loss of uncontaminated lands and increment of traffic-related emissions [156], which damages people’s health and natural environment. Due to the behaviour of humans, land surface, flora, fauna, atmosphere, and oceans are irreversibly altered. The neologism Anthropocene, popularized in 2000 by the Nobel Prize-winning Paul J. Crutzen and Eugene F. Stoermer [89], proposed for the first time in 2008 as a formal geological epoch, highlights that we are now living in a time when the Earth is hopelessly shaped by humankind. The United Nations has designed that a way to ensure a future for new generations is the sustainable urbanization and to take better account of overpopulation and climate issues.

Billions of natural and human events compose the morphogenetic process of cities and the way that they are combined often produces the emergence of new and unpredictable dynamics. Understanding how the city was formed (and will evolve) and the effect of political decisions can have to the urban growth are important to ensure sustainable development of human settlements. As such, realistic models of urban growth have high relevance for urban planning, energy management, and transportation investments. The definition of adequate frameworks able to capture the heterogeneity of aspects that play into the formation of urban areas is crucial to understand the urban growth and make realistic forecasting, useful to help political decision-makers to define a sustainable strategy.

Contribution

We start with a basic consideration: a city is a large number of abstract or physical components, located in a portion of the land, often related in an elaborate manner. Individuals, economic and policy actors, promoters, etc. produce continuous internal evolution and modification of their components, their quantity, their behaviours, and their adaptation to changes. They make decisions autonomously and they often tend to maximize their gain and minimize their efforts. They can be influenced by exogenous aspects and often they are not conscious that they constitute a whole (the city), because often they are not motivated by global objectives. Some of these elements tend to cluster, producing an overlapping of spatio-temporal patterns. Segregation phenomena (the tendency of individuals to form groups), the specialization of sub-areas in productive and economic areas, gradients of population density are basic examples of spatial patterns.

From a physical point of view, dynamics can be explained by forces; the latter can be internal (a part of the city is acting on other parts) and external (exogenous aspects are acting to a part of the city). The first ones spontaneously form when elements of the city organize themselves and the second ones exist because a city is not isolated (nature constrains the evolution of the city, the exchanges with other cities). These forces traversed the city, ensuring the maintaining, the functioning, and the individuality of the

⁴It is possible that these scenarios will be affected by the Covid-19 pandemic.

city. They constrain the creation of a city, indicating the direction and sub-areas where the city grows: forces, therefore, steer the creation of physical objects over the land, becoming a fundamental mechanism of urban morphogenesis. As a trace over the land, these forces produce many changes in nature. Buildings, streets (and in general transportation and infrastructure networks), modifications of orography, rivers, and lakes, etc. are the physical results of forces inside the urban system. These new physical elements (the urban fabric) also contribute to the internal dynamics and the organization of the components of the urban system. Among these physical elements, we will focus on street networks because they represent a major organizational component of the urban systems. Streets are the backbone of the city, the structural support of human activities, the physical witness of the evolution of the urban area. Understanding street network evolution may reveal important information about the functioning of cities and their growth.

Street networks, and by extension urban fabric, retro-act to the constituent elements that had produced them, driving new and evolving organizations. The characteristics of the streets may motivate people to use the infrastructure. The density of buildings may influence the attractiveness of a neighbourhood and that has an effect on economic or policy actors on their choices. Political decisions incentivise (or not) some kind of plans which may increase the sprawl. These are basic observations that motivate us to consider urban fabric as the result of internal and contrasting forces that are produced by many heterogeneous elements also influenced by their own results.

Turing [285] suggested that some patterns we observe in life arise from a biochemical dynamic of elements named morphogens. The cells would be influenced by these morphogens and under this influence would differentiate and structure themselves. The patterns that we see as the pigmentation in animals are the reflection of clusters and inhomogeneities of the underlying biochemical organization. Morphogens interact and move, gradients of concentration form, and impact the way that cells structure. An evident analogy can be observed between this vision and our thoughts about cities: a substrate of morphogens (population, economical actors, etc.) evolves and the organization that they (eventually) find, leads to the creation of new physical elements (urban fabric); here we focus on one of them, the street network and its morphogenesis.

These basic observations allow us to suppose that cities respect principles of complexity theory. In many systems such as urban systems, spatial aspects (as the distance between entities, the relative position inside the system, the way that they are spatially arranged, etc.) of elements affect the manner that they interact with others. Moreover, a system like a city is also open because it needs to exchange with the outside to ensure its identity and its functioning. Elements are immersed in a spatial environment, which represents a source of energy, matter, and information for the system. These exchanges and these external aspects affect the system locally, meaning that elements are conditioned in a region of space. For example, the spatial environment of an urban system contains information about the orography

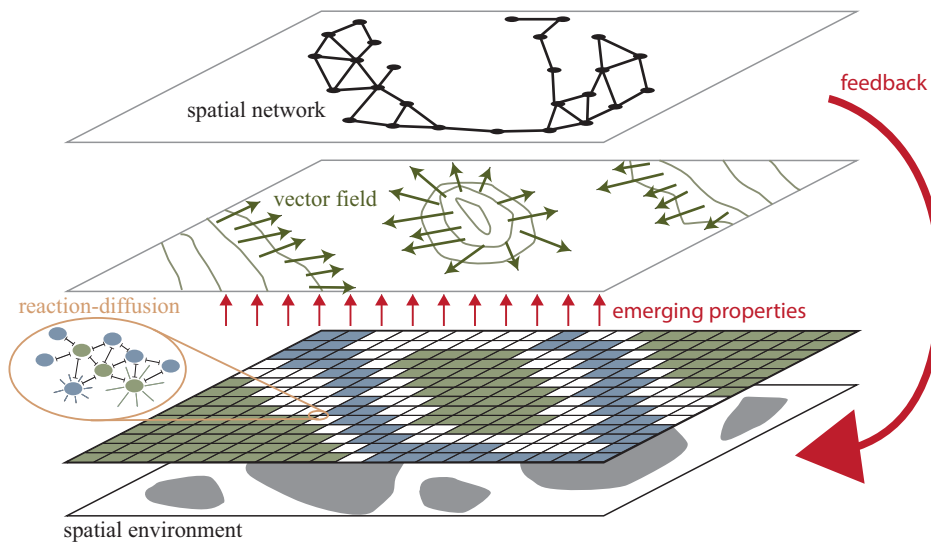


Figure 1
The concept. Morphogens govern the formation of a network which in turn retro-acts to their behaviour.

of the territory, natural or artificial constraints, influences of other urban systems. The spatial environment contains organizational factors of the system.

To simulate the pattern formation, we consider two kinds of morphogens (activators and inhibitors). They are spatially located, they reach, self-produce and move. Three key ingredients are required for the formation of patterns: the positive feedback of the activator (the autocatalysis), which results in a self-production of the activators, the negative feedback of the inhibitors, which suppresses the growth of the activators and the different moving speeds of morphogens. In two dimensional Euclidean space the system exhibits (under some particular circumstances) a wealthy variety of unpredictable patterns, including stable spots, moving spots, self-replication spots, waves, mazes, chaotic patterns, etc. The evolution of the concentration of morphogens in space over time can be simulated by the coupling of two stabilizing dynamics: the reaction (which describes the microscopic interactions, productions, degradations of those elements in a region) and the diffusion (which describes the motion of morphogens from regions of high concentration to regions of low concentration). As time passes, concentrations are at equilibrium or develop complicate and unpredictable structures through breaking, splitting, elongating, propagating, etc.

When a spatio-temporal pattern (stable or not) forms, elements find a dynamic equilibrium: contrasting forces reduce the tendency of morphogens to disperse, and they cluster in a region. A dynamic vector field is a representation of these forces. It is the *trait d'union* between the morphogens and the network, representing the causal relations between internal dynamics and emerging forms. Finally, the network retro-acts to its constitutive components. The feedback is a mechanism that may incentivise the creation or inhibit the form. It is the crucial aspect that breaks the direct causality between the actions to the responses of the system, leading to the complex behaviour of the system, unexpected trajectories and emerging properties.

Formally, the framework that we are going to expose is based on these dynamics, as schematically described in fig. 1:

- **Morphogens interactions**

A layer inspired by the cellular automata theory will simulate the microscopic interactions and the movement of morphogens at a high level of abstraction. The evolution of the concentration of morphogens is inspired by the Gray-Scott system of differential equations.

- **Emerging properties drive the form**

A dynamic vector field, computed from the morphogens' layer, constraints a spatial network (graph theory gives valid support to encode it).

- **A feedback mechanism controls the form**

The network became a source of events, it affects the morphogens. Positive or negative feedbacks locally increase or reduce the organization of morphogens, favouring or inhibiting the morphogenesis.

The whole system is immersed in a spatial environment. To model that, the framework contains an environment layer, a set of values associated with a regular grid.

Objectives

The models and the framework that we are going to expose in this work are based on basic principles of complex theories and they aim to simulate the morphogenesis of a specific class of spatial complex systems that appears under the form of a network. This thesis attempts to tackle the modelling problems of the morphogenesis of complex networks by considering that the network is spatialized and located in an environment which constraints it and it is constrained by the network. The constraints are forces. We are aware that to reach this goal we need to try to evaluate our results. This is based on measuring the sensibility of the model to initial parameters and the evaluation of trajectories. As far as possible, under these aspects, we want to reveal the effect of each basic mechanism.

Besides these preliminary internal evaluations, we tackle the challenge of an external evaluation, comparing our results to real case studies. This is a first attempt which must be discussed with experts of the domain. We focus on urban systems, we observe some characteristic properties, and we analyse the street network of France department cities via measures from graph theory and fractal theory. Finally, we propose an application in a real context, in order to simulate some scenarios and evaluate the effect of initial parameters sets to a real urban system.

Many systems have common properties with urban systems, hence we can investigate their formation keeping main traits in common. Motivated by the aim to be as general as possible and led by parsimonious principles, through this work, our focus is on common dynamics among different complex systems. We will describe essential mechanisms that seem to be in common with other complex systems and we will define in what way an urban system is different. With our approach, we want to capture at once essential morphogenic processes and analyse and validate our results. Finally,

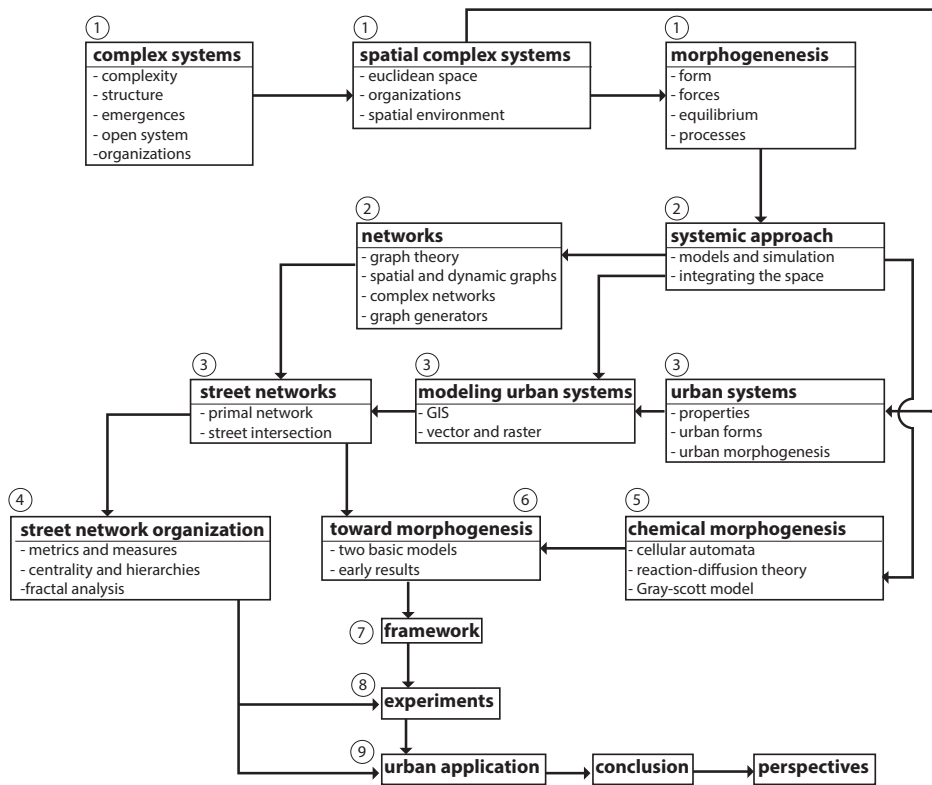


Figure 2
Thesis outline. Boxes indicate main concepts and number in circle indicate chapters.

this approach allows us to integrate specific and more sophisticated mechanisms (related to the application) and geo-referenced information about urban systems.

Outline

Due to the multidisciplinary aspect of our approach, it is hard to define a straight path to expose the main aspects of our work. Our choice is not a linear document. The work is decomposed in some bricks, linked to a few other bricks. Each of them can contain a state of the art and our contribution; each chapter can contain more than one brick. The outline is schematically represented on fig. 2.

We start with chapter 1, where we expose general principles of system theory, in which circumstances some systems could be considered complex and why it is important to have a focus on spatial aspects to understand how they form. In chapter 2, we observe that the result of the organization of systems can be represented as a network; here we expose why graph theory is a useful tool to bring out their properties and simulates the evolution of the form. In chapter 3, we study cities under the complex system perspective, and we propose some methodological methods to simulate and measure street networks. Chapter 4 is dedicated to the study of street networks and why graph measure, centrality theory, and fractal theory can outline the organizations of these systems. To this end, we analyse French department cities and we compare their properties to six theoretical graphs. In chapter 5, the reader finds a chemical approach to investigate and simulate morphogenesis: fundamental traits of the reaction-diffusion theory are

exposed and, inspired by Cellular Automaton theory, we present a way to simulate and analyse emerging patterns. Chapter 6 addresses the problem of modelling spatial systems: here we present two approaches and we make some considerations about the strategy to conceptualize a general framework. These ideas are integrated into the framework described in chapter 7: here we summarize the main characteristics and give the way that one could implement it. Chapter 8 shows the potentiality of our approach, proposing how to simulate the morphogenesis of a spatial system. We investigate the sensibility of the framework to initial parameters, the effect of dynamics to the results and we compare in a first approach our results to French department cities. Chapter 9 is devoted to an urban application: here we propose how to integrate real aspects from reality in our framework and the capacity of it to simulate different scenarios. We end this study (chapter conclusion) summarizing new knowledge that this work has produced, discussing what seems to be interesting, and giving different perspectives.

Chapter 1

Morphogenesis of complex systems

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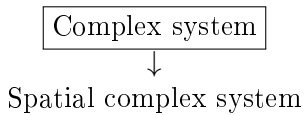
Keywords complex systems, organization, emergence, form, morphogenesis, feedback, forces.

"Everything is related to everything else, but near things are more related than distant things."

Waldo Rudolph Tobler [283]

This chapter introduces the context of this work, exposing the notion of system (section 1.1) and under which conditions a system shows a complex behaviour. It then summarizes reasons why spatial aspects are crucial in some systems (section 1.2), in what manner these aspects affect the way that elements interact and how they organize themselves. The chapter ends exposing in which way complex system forms itself (section 1.3).

1.1 Systems



Many natural and artificial phenomena are due to distinct and interacting elements (hereafter also called parts or entities). These elements can be atoms, people, natural or artificial objects, galaxies, etc. In these cases, we refer to chemical systems, social systems, biological systems, systems of galaxies, etc.

"A system can be defined as a set of elements standing in interrelations. Interrelation means that elements p , stand in relations R , so that the behaviour of an element p in R is different from its behaviour in another relation R' . If the behaviours in R and R' are not different, there is no interaction, and the elements behave independently with respect to the relations R and R' ."

Ludwig von Bertalanffy [42]

The concept of a system is already present in the work of Sadi Carnot [68] when he laid the foundations of thermodynamics, but the idea to study elements and their interactions as a system was explored in the 1940s in biology (by Ludwig von Bertalanffy [42]), in information theory (by Norbert Wiener [299] and Claude Shannon [264]), and in psychology (by William Ross Ashby [14]). System theory is a way to study an object in terms of the relationships and interdependencies among its parts. From these seminal works, the principles of system theory have been defined and thanks to an intense epistemological work [72, 200, 247] (see [214] and references therein) the notion of complexity was integrated. System theory is also an interdisciplinary approach, which would give a global framework for explaining phenomena in a wide range of fields of knowledge.

Different definitions of system exist and no one is globally agreed upon; it seemed interesting the von Bertalanffy's point of view, because he focus on the fact that a system is a set of elements in relation/interaction. Elements exchange, interact and behave in accordance to a subset of the system. Interaction is a kind of action that occurs as two or more objects have an effect

upon one another¹, establishing a relation in a given time period. Studying how parts interact is the fundamental key to understand the behaviour of the system because, under some conditions, parts organize themselves and the system displays properties, regularities, forms and order. Elements of a system may have similar behaviour and similar properties or not in a given time period: in these cases, the system is composed of different kinds of elements, with different characteristics or behaviours, hence a different way to affect other elements. For instance, a colony of ants is a biological system where some individuals have social role, size and behaviour different from others [245]. In such a system we have a specialization: one queen, some reproductive males and many workers. In some fish shoaling, at a given time, a limited number of individuals is in possession of particular information: they can lead the migration of a large number of individuals [242]. Rather than oppose, different kinds of individuals combine themselves to produce a collective behaviour [271].

Many systems co-exist at the same time: if we consider a system, we distinguish it and its environment. This last represent *"l'ensemble des éléments n'appartenant pas au système dont l'état est susceptible d'affecter (ou d'être affecté par) le système"*²(Le Moigne [175]). Many biological, natural or physic systems exchange information, energy, and matter with the environment. For living systems, it is a condition to stay alive (for instance an embryo which absorbs matter from its amniotic liquid). In a narrow sense, it is also difficult to conceive a system that never has exchanges with the environment in which it is defined; in other words, a system is rarely isolated³. Open system refers to a system that interacts with its environment, whereas closed systems do not. A closed system is considered to be isolated, in which no exchanges are required to ensure its existence. In the following, we refer to systems which exchange energy, matter or informations with their environments.

1.1.1 From systems to complex systems

There exist some systems where their characteristics cannot be predicted only by the characteristics and behaviour of their elements. Moreover, a focus on the interactions is not enough to understand how these characteristics emerge. In other words, the characteristics of the system cannot be reduced to the juxtaposition of the characteristics of each element and their interactions considered in isolation. In many cases, to understand a system we must consider it as a whole.

A complex⁴ system is characterized by some properties:

¹www.en.wikipedia.org/wiki/Interaction, visited on 24/07/2020.

²*"The set of elements which are not contained in the system. The state of these elements could impact (or could be impacted) by the system."*

³No experience has been reported of an ideally isolated system. The only example known (supposed) is the universe which is considered for the moment as an isolated system. This remains a postulated to be demonstrated.

⁴The term complex is ubiquitous in many research fields and different meanings in different contexts. For instance, in information theory, the complexity of description [164](the minimal amount of computational resources needed to describe a given ob-

- The high cardinality of the set of (possibly heterogeneous) elements and the consistent number of interactions between them.
- Local relations/interactions that make the system decentralized (there is no global control or global information).
- A continuous flow of energy, information, or matter from the environment contributes to the functioning of the system and its organization. The stop of the flow may produce the disintegration of the system.
- The emergence of some macroscopic properties, which are not characteristic of elements but of the system.
- Emergent properties feed back to elements.
- The system is in a state and it tries to maintain its state. Depending on the disturbances, it can leave its state to join another. The state is generally close to a critical point.
- Elements could eventually evolve too, changing their behaviour or their characteristics. That contributes to the adaptability of the systems.

Some authors propose other properties. De Rosnay [247] enlarge the complex system notion by a theological aspect: *"un système est un ensemble d'éléments en interaction dynamique organisés en fonction d'un but"*⁵. Maturana and Varela [195] emphasize the need for the formation of a membrane, that defines an outside and an inside where the system produces itself⁶. Per Bak [17] claims that systems are in a critical state, which can suddenly evolve and fall inside new states.

The coexistence of these properties contributes to the emergence of unexpected properties, forms, organizations, behaviours, degrees of adaptation, etc. The amount of information associated with the global features of the system is greater than the sum of the information associated with each of the constituents⁷. The approaches based on complex systemic help to pick up phenomena by avoiding simplifications which reduce and decompose the knowledge in a set of dissociated elements, hiding the connection between them [202]. Following this, the study of physical phenomena has undergone a deep transformation that can be considered as a paradigm shift⁸. To better understand the notion of complexity we can observe in what a complex system is different from other kinds of systems.

ject). In computer science, computational complexity measures how many operations are needed to perform some computational task. In this dissertation, we refer to the complexity of systems.

⁵A system is a set of interacting elements organized to achieve a goal.

⁶The formation of a membrane does not mean that the system becomes close. It still exchanges with the environment through the membrane.

⁷Aristotle.

⁸Kuhn [167] says *"By paradigms we mean the whole body of the universally recognized scientific accomplishments, which have, for some time, provided a model of the set of problems and solutions that are acceptable to those who work within a given research field"*. A paradigm shift indicates a fundamental change in the approach to study a phenomenon (in the concepts and in experimental practices).

A complex system does not mean a complicated system. A complicated system refers merely to systems made up of many interrelated parts. These relations can be not easy to study, but each of them can be studied and explained without considering the whole system [200]. An aeroplane is a complicated system. It is composed of a large number of components and a large number of these parts need to control a single aspect of an aeroplane. However, the whole aeroplane is the simple assembly of its parts.

It is not easy to make distinction between chaos and complexity. Chaotic systems are deterministic system where trajectories over the time diverge substantially. Chaos models consider few (digital) variables and study their evolution; from this, some characteristics behaviours are deduced. Therefore chaos does not deal with an environment and it is concerned with the dynamics of few parameters values while complex systems concern in the same time the structure and the dynamic of systems and the interactions which compose them, including their environment. Roughly, chaotic models investigate closed systems rather than complex system models investigate open systems. Nevertheless different elements of interactions between the system and the environment can be sometimes included in chaos models.

Complexity is not the opposite of simplicity: in many cases, behind non-evident and organized events there are simple mechanisms of interactions. In a complex system, elements may interact in a simple way, but they yield to dynamics that are difficult to understand.

Organizations and structures

The notion of the system refers to the whole, the functioning of all elements. The notion of interaction refers to the connection between elements. The notion of the organization refers to the way that elements are related to the whole, the way that the collection of elements becomes a system.

*"L'organisation est la propriété d'un système capable à la fois de maintenir et de se maintenir, et de relier et de se relier, et de produire et de se produire"*⁹

Edgar Morin [201]

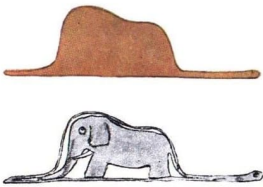
The organization is the result of interactions between entities that generate and maintain order inside the system. It binds elements in relationships, ensures the autonomy of the system, reinforces the condition of existence of the system, making an island of order in disorder. Morin [200] has argued that a more accurate and inclusive term is self-eco-re-organization:

- **self**: points to the capability of the system to self-regulate, it adjusts its properties and maintains an organizational persistence (the endogenous character of the system). Self-organization implies a high

⁹Organization is the attribute of a system that is at once capable of maintenance and self-maintenance, of interconnecting different elements as well as holding itself together and of producing and producing itself.

degree of decentralization of the system: there is no global control and the system autonomously adapts itself if some conditions change. Single elements neither need explicit organizational prescription nor have access to general information on the organizational design ruling the system. The self-organization can be considered as the opposite of a hierarchically designed organization.

- **eco**: points to the necessity of exchanges with the environment (the transformation of the system due to the impact of exogenous aspects). We cannot conceive the autonomy of the system without considering the environment in which the system exists. The system needs a dependency on the external to ensure its independence. An intense dependency of the system to its environment is observed in such systems named *dissipative* [122], where the formation of some organizations in the system is ensured by the consumption/dispersion of energy or matter. The state of the system is maintained by these exchanges which allow it to form in particular organized structures. The disorder is evacuated outside the system and the energy, the matter, and the information are consumed [232].
- **re**: indicates the ongoing dynamic, the regenerative nature of the organization (the continuous transformation of the system). The most popular example comes from nature: a living system is composed of billions of cells which form the organism. During its life, the system renews constantly its cells and the organism evolves.



The organization expresses at the same time the action (to organize) and the result (an organization). Organization and system are connected, one does not have sense without the other¹⁰. The essence of the organization of its parts is the structure of the system, which ensures the identity of the system. If the organization of a system changes, it loses its identity, it disintegrates.

The structure is *"the actual components [elements] plus the actual relations that take place between them while realizing it as a particular composite unity [system] characterized by a particular organization"* (Maturana [196]). *"Une structure qui ne serait plus entendu comme un invariant et moins encore comme une loi expliquant le comportement du système"*¹¹ (Le Moigne [176]). The structure is the manner in which the system is actually made by elements: therefore the system exists only if the identity of the system is ensured by the organization. The three actions (maintain-interconnect-produce), which coexist and are interdependent, define the three main features of the organization (eco-self-re). These latter are related, and their coexistence begets the system. The structure of a system is more static than its organization: it evolves in the time but some aspects, typical of the corresponding system, not change. The structure of a system can change

¹⁰ *"L'organisation, la chose organisée, le produit de cette organisation et l'organisant sont inséparables"* (the organization and its result, the organized system, are indivisible, Poul Valéry [289])

¹¹ *"The organization is either not considered as invariant or general law of the system."*

without it losing its identity if the organization is ensured; if the organization of a system is not conserved through its structural changes, the system disintegrates. The conservation of the organization is the fundamental condition to ensure the existence of the system over time.

To characterize these concepts we point out the relations between organizations, structure, and time. Two events in relations (the cause and the effect, the interaction, and the emergence) can be considered classified in relation to time. If the effect and the cause are located in same time scale¹², the effect is synchronic to the cause; otherwise, it is diachronic. The first one is often related to the organization and how elements behave and interact with each other and with the environment. The second one can be considered as a relation that has an effect on the structural changes of the system. Let us show an example of from urban context. Traditionally, a city is composed of many individuals that move, interact, exchange with nature, and try to accommodate their personal needs. These actions can be considered on the same time scale and they are the basis of the organization of the city. In a long time period, these actions are having an effect on structural aspects of the city too. The manner that people behave affects physical elements of the city, which conditions the accessibility of sub-areas, the formation of new lifestyles, the requirement of residential areas, the specializations of neighbours, and the segregation of communities. These changes are diachronic to the causes, produce structural changes¹³ on the system. Roughly, synchronic effects entail how elements are related and diachronic effects entail how systems form.

1.1.2 Feedback

Roughly, feedback is a circular process of influence where outputs have an effect on inputs. This mechanism plays a major role in the functioning of a complex system. Systemic loops can act in different ways, depending on the nature of the feedback:

- **Positive feedback**

It is a catalysis function that increases some properties, creating a cascade of change in a direction. It amplifies the deviations from the stable state, the system is self-powered. In broad outline, A produces more of B which in turn produces more of A . Positive feedback tends to lead to instability and produces growth, oscillation, or chaotic behaviours. Positive feedback can reinforce an initial change in a system (due to the environment or a bifurcation) in the same direction as the initial deviation. A good example of positive feedback is blood clotting. Once a vessel is damaged, platelets start to cling to the injured site and release chemical substances that attract more platelets. The mechanism continues until a clot is formed.

¹²Time scale cannot be detached from the observer.

¹³Which also affects the way that people behave (the effects feed back to the organization).

- **Negative feedback**

It is a stabilizing function which decreases some properties. While positive feedbacks increase the phenomena, negative feedbacks limit it, providing opposite reactions to the actions. Negative feedback tends to boost equilibrium and reduces the effects of perturbations. With negative feedback, the system tends to a stable state, because the internal mechanism that produces instabilities is inhibited by the feedback. Mammalian regulation of blood pressure is an example. When the pressure increases, signals are sent from the blood vessels to the brain, which in turn sends signals to the heart and the heart rate slows up. Similarly, when the pressure drops, the brain sends signals to the heart and the heart rate speeds up. The mechanism ensures constant pressure in the vascular system. It activates when the pressure is out of the optimal range.

Positive and negative feedbacks must not be considered as "good" or "bad" effects. They should be considered as mechanisms which incentivize or inhibit processes. These mechanisms are a necessary condition for a complex behaviour but not sufficient¹⁴. Positive feedbacks can work in the opposite direction of negative ones, regulating together the system, and leading to new organizations. Large scale structures can diachronically emerge from small-scale synchronic interactions. These structures then influence the behaviour of elements via cross-scale feedbacks. Elements create their environment and then they are in turn influenced by it. In urban context, culture, religion and social norms, created by humans, in turn influence humans. These emerging properties retro-acts to societies and cities. The dynamic is a cross-scale feedback.

1.1.3 Adaptability

In many cases a system shows characteristic of adaptability. Adaptability refers to the ability of systems to maintain their organization when something changes. Thus, the adaptability of the system is its capacity to maintain its global behaviour under an exogenous (for instance, the presence of disturbances from the environment) or internal (for instance, sudden evolution of the behaviour of some elements) change. In particular, a complex adaptive system is one whose collective behaviour exhibits adaptation and elements learn as they interact [144].

Disturbances can even make the system more robust, by shifting its trajectories into a new attraction basin. In that sense, a system is intrinsically adaptive: it maintains its basic organization in spite of continuous changes in its environment [145]. The adaptability is a property of many biological [160] and artificial [193] systems. Two concepts come from this capacity.

¹⁴For instance, the fly-ball governor is a component of many mechanical systems. It controls the speed of an engine. Two balls are attached to a frame and as their rotational speed increases, they swing outward and close the inlet valve for steam. That causes the engine to slow down. When the rotational speed decreases, they open the inlet valve of steam. This negative feedback mechanism does not produce self-organization, it just controls the speed of rotation of an engine.

The first is the robustness, that indicate the capacity of the system to accomplish its task and work after the failure of some elements. The second is named the resilience of the system, defined as the capacity to conserve its functions after some changes. For example, a bird flock is robust because even if some of them die during migration, it is yet able to go to its destination. The same system is resilient, if it disperses at first time and reassembles after it is attacked by a falcon.

1.1.4 Emergences

There exist a relation between micro-dynamics (interactions) and macro-outcomes (emergence). An emergence (from Latin *emergo*, to arise) is a novel think of the system: compared to features of each part of the system, an emergence gives a novel and not trivial¹⁵ information about the system [200]. An emergent system property arises from interactions between elements of the system; elements do not themselves display this new system property and the property cannot merely be deduced by examining elements and their behaviours [247].

Define the term emergence is not so simple because it is a term used in different ways both in science and philosophy.

Let:

- A be a set of interacting entities whose behaviours are defined by a theory or a model T ,
- B be a global phenomenon linked to all or a subset of the entities.

If the phenomenon cannot be described by T or composition of T and requires another theory or model T' , then there is emergence.

Definition 1:
Emergence

There are different conceptions of what counts as emergent depending on how novelty is understood in the manner that it is related to the functioning of the system [41]. Chalmers [73] drew the distinction between weak and strong emergence. This distinction can be incorporated into the distinction between epistemological and ontological emergence proposed by O'Connor and Wong [213]. Both kinds of emergence are important because they contribute to the understanding of the system. The weak (epistemic) emergence only exists in the eyes of the outside observer. It is related to his observation, the theory, or the model, and it consists simply of changing the level of the observer's gaze from the parts to the whole. Meanwhile, the strong (ontological) emergence undergoes the observer's perception. The global phenomenon feeds back on the components of the assembly or the observed subset. Chalmers says to us that strong emergence has radical consequences, affecting the whole system and the behaviour of elements. The global phenomenon feeds back on the components of the assembly or the observed subset.

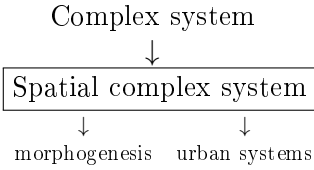
¹⁵ A piece of not trivial information is one that cannot be derived with different axioms (defined *a priori*).

Summary 1:
Complex systems

In a complex system, a large number of lower ordered elements organizes and displays a high degree of order. A complex system cannot be divided by simply examining its parts because the elements' collective behavior is not a juxtaposition of their individual behaviours. While the organization is seen as a dynamic capacity of the system to exist and evolve, and the structure is seen as the diachronic result of these dynamics, the emergence can be understood as a global feature of a system arising from local interactions between elements and this last with the environment. We do not take emergence as a given unexplainable property of the system, but we take them as something which arises from the system, deserves an explanation and gives an explanation about the system.

How common properties emerge from a mass of interactions between heterogeneous elements poses different problems: how can we understand how those elements influence others? How many elements contribute to the definition of characteristics of each element? To try to answer these questions an attempt in our context to spatial aspects may be useful.

1.2 Spatial systems



Studying a phenomenon implies building a representation of a part of the world that is important for our understanding. Studying a system means collecting and encoding in formalism all important information about elements, relations, and the environment. The selection of this information and the correct consideration of them is crucial to improve our knowledge. Spatial information is often useful to characterize systems.

Euclidean space of dimension d is a space in which postulates and axioms of Euclidean geometry apply. Let be $\mathbb{R}$ the set of real numbers, the Cartesian product $\mathbb{R}^d, d = 1, 2, 3^{16}$ defines the point as the tuple $X = (x_1, \dots, x_d)$ of d coordinates. In 2 dimensional Euclidean space, the distance $dist(X_1, X_2)$ between two tuples of coordinates X_1 and X_2 is given by the distance formula based on the Pythagorean theorem. Here we focus on Euclidean space and hereafter the word space always refers to it.

Spatial aspects are ubiquitous in many human and natural phenomena and they are a pillar into geographic studies whose aim explains reality, taking into account the spatial arrangement of objects, phenomena, individuals, etc. [133]. In archaeology, they are helpful to understand the historical evolution of human settlements: introducing the spatial analysis, researchers improve knowledge not only considering the properties of artefacts but also investigating their spatial arrangements [6]. In sociology, human behaviour was typically observed disregarding spatial aspects. However, the Chicago ecological [138] school proposes to overcome this lack, investigating human behaviour from a spatial perspective. Human relationships are often affected by spatial constraints and they are a function of distance: the near-

¹⁶Hilbert space generalizes the Euclidean space. It extends principle of Euclidean space for an infinite number of dimensions.

ness incentives the formation of social relations and reinforces human relationships [178]. In economical studies, location theory is based on the idea that economic actors (for example, retail activities) are arranged in space following some optimal-distribution laws, in accordance with the metric distance. The ice-cream seller problem (formally defined in the game theory by the Hotelling model [148]) shows the importance of the distance between these actors to understand why they choose their location. Indeed, in anisotropic space, two antagonist entities prefer to stay close in order to maximize their economic basins. Spatial aspects influence trade in economic [150], structures in many transportation systems (streets, cargo ship systems and grid networks) [23] and spread in contagious diseases [206]. Proximity influences electrical activity in human brain [62] and resistance to external forces in granular materials [216]. Consider space and spatial information (like the relative position of elements and their metric distance) are therefore crucial to understand several systems.

1.2.1 Spatial elements

Spatial systems are so named because the position of elements affects their likelihood of interacting. Basically, the relations between a system and Euclidean space is defined through characteristics of elements.

Each element within a spatial system are associated with a tuple of coordinates $X \in \mathbb{R}^d$; in this manner, the element is spatially located. The location of the element may vary over time.

Definition 2:
spatial element

A spatial configuration is defined by a space, in our case Euclidean space, a set of spatial objects with their positions and their relationships at a given time. The way that a spatial system exchanges with its environment may be also related to spatial aspects. It is defined by a vector space of real numbers, with a scalar product. That makes possible to measure distances and angles. That means information, matter, and energy are locally exchanged with the system in a limited region of space. The same external aspects can be related to space or not: for instance, the temperature of the environment of a living system may be constant or in gradient. The temperature affects locally the system.

1.2.2 Spatial interactions

A spatial interaction is an action between at least two spatial elements of a system sufficiently close to interact. Spatial interactions depend on the relative location and/or the arrangement of them. Interactions are various in the manner and in time: competition for limited space, segregation, the propensity to aggregate etc.

The neighbourhood of an element p is the set of elements $N(p)$ which interact with it. In a spatial system, a simple kind of neighbourhood is based on distance: $N(p) = \{q : \text{dist}(p, q) < \delta\}$. Elements interact only with elements

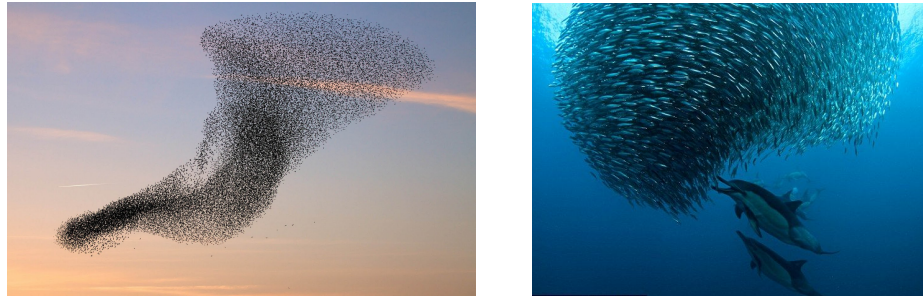


Figure 1.1

Spatial interactions. A bird flock (left) and a fish school (right). Source: publicly available on the web.

in its neighbourhood and the interaction depends on geometric aspects (i.e. the distance and the visibility).

The evolution of a flock bird is a basic example that helps us to explain those interactions. A flock bird model concerning decentralized the movement of individuals [243]. In this model, the collective behaviour of the system is the result of three simple spatial rules that regulate the movement of each element:

- **Separation:** to prevent any collisions, individuals move away from other elements.
- **Alignment:** individuals align their direction to the global direction of their neighbours.
- **Cohesion:** individuals move toward of the barycentre of its neighbourhood.

Similar rules can also explain the collective behaviour of fish schools [271]. This example illustrates that the spatial configuration (distance between elements and arrangement of neighbourhood of each element) of the system affects the behaviour of individuals within the system. An emergent global movement of the whole system emerges (fig. 1.1).

1.2.3 Organizations in spatial complex systems

Extensive properties increase with the size of their supporting elements, while intensive properties are independent of the size. For instance, in an urban system, the rent (intensive property) of household (elements) is spatially located and in a region of space, we observe the average rent (extensive property) that could define a spatial pattern (spatial emergence).

In a spatial system, the order appears under different kinds of organization; one of them is the tendency of elements to dynamically arranges with sufficient regularity, forming spatial patterns. At a specific moment, such repeated properties may refer to shape, orientation, connectedness, density, or distribution. Regular geometries, gradients of concentrations, symmetries, etc. are tangible evidence that the system is spatially organized. These regularities can also refer to properties of entities: the formation of different areas in which these properties are located is one of the reasons why a system is able to ensure its persistence. We think for example of the

specialization of a living system such as an organ. Patterns appear during the formation of bacteria veil, snowflakes, and animal coats (fig. 1.2).

Many system are composed of elements that have approximately similar behaviour and characteristic. However, it appears that those elements do not have the same importance into the system: some of them are more important than others, and the elements are organized in a hierarchical manner¹⁷. Hierarchy is not an intrinsic characteristic of the elements, but a result, an emerging property. Hence the hierarchical organization is a spontaneous manner of elements to organize themselves. Leaf systems (where the midrib of leaves connects to veins which in turn are connected to smaller veins and so on, fig. 1.3, left) and street networks (where we can observe an

¹⁷In this work, the term hierarchy has taken on a generalized meaning, divorced from its original meaning of a vertical authority structure

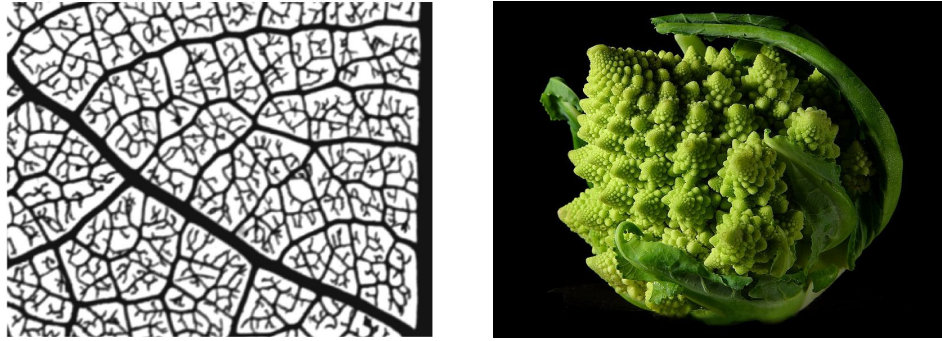


Figure 1.2

Spatial patterns in nature. Colonies of bacteria (top, source: [35]), the skin of the clownfish amphiprion percula (middle left) and the panthera onca (middle right, source: publicly available on the web) and snowflakes photographed by Wilson A. Bentley (bottom, source: www.snowflakebentley.com)

Figure 1.3

Hierarchical and scale-free organization. Hierarchical organization of venation of a dicotyledon *Gloeospermum* leaf, arranged in a network-like way (left, source [49]) and scale-invariance organization of the *broccolo romanesco* (right, source: publicly available on the web).



inequality of elements to capture main transportation flow, see chapters 3 and 4) are examples. This inequality is also important in the functioning of the system because characterizes the way that it evolves and the manner that it responds to external disturbance (e.g. in a city, issues on the fundamental transportations axis produces more effects on the transport that in less important streets).

In a system, the scale-invariance organization describes situations where the essential structure, properties and/or dynamics remain unchanged (i.e. invariant) when considering the system at different scale¹⁸. This means that a property holds regardless of the scale; this also explains why power law appears in this circumstance¹⁹. In a spatial system, the scale-invariance organization can manifest in different ways; one of them is the manner that the system fills space. The scale invariance often results in self-similarity (anisotropy scale invariance) and self-affinity (scale invariance with dominant directions) of their parts or a subpart of the system. We will detailed these aspects in the next (section 4.4.1). Self-similar properties manifest themselves as a morphogenetic mechanism of some spatial systems (e.g. the *broccolo romanesco*, fig. 1.3, right): repetitive and scaled structures naturally appear as these systems grow.

Summary 2: spatial complex system

Spatial complex systems refer to complex system where spatial aspects are crucial into the definition of relations/interactions between its constitutive elements and the exchanges with its environment. The way that elements organize themselves can be observed via the formation of spatial patterns, the repetitive of same properties at different scales and the different importance that a part of the system has compared to the rest of the system.

¹⁸The scale is a set of numbers, amounts, words, etc., used to measure or compare levels of something; the set could refer to temporal, topological, spatial measurements.

¹⁹We must point out that in real situations, the invariance hardly covers all scales. We will discuss that in section 4.4.

1.3 Complex systems morphogenesis

*"Si la question de la priorité de l'œuf sur la poule ou de la poule sur l'œuf vous embarrasse, c'est que vous supposez que les animaux ont été originellement ce qu'ils sont à présent"*²⁰

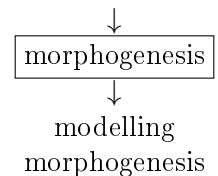
Denis Diderot

Historically, the first declination of the term *form* defines it as the geometry of contour of a physical object, located in space, like a silhouette observable from an external point of view. The form is "the *shape* or appearance of something"²¹ and it is related to the observer. Under this point of view, there is an intrinsic dichotomy between how the object appears and how the object functioning. The separation between shape and content cannot allow investigating how the object forms. This definition is not helpful to investigate the morphogenesis, the combination of processes that provides the formation of the object.

The work of D'Arcy Thompson can be considered as an early tentative to overcome this dichotomy. In *On Growth and Form* [91], collecting and comparing a great amount of vegetal, animal, and immaterial shapes, he suggests that bio-mathematical principles define shapes in nature (fig. 1.4). Analogies between animals, common features of plants, recurrent geometries in rocks may be expressed in a rigorous geometrical way. He found the notion of species in a geometrical manner, suggesting that some structural aspects are invariant during the evolution of forms. Thompson pointed out (in biology, but he also suggested in nonliving world) that the form is not a static thing but arises from growth: *"Everything is what it is because it got that way"*.

An important contribution was given by the Gestalt²² school [162]. it provided a new definition of form, defining it as the physical result of internal relationships between parts of the object. Applied to system theory, the form is the combination of the shape and the organization. Hence, morphogenetic processes (fig. 1.5) are not a simple collection of mechanisms that

spatial complex system



²⁰ "You are embarrassed about the chicken or the egg dilemma because you suppose that animals are always the same."

²¹Source: online Cambridge Dictionary (www.dictionary.cambridge.org), visited on 10/07/2020.

²²The German term *gestalt* means form.

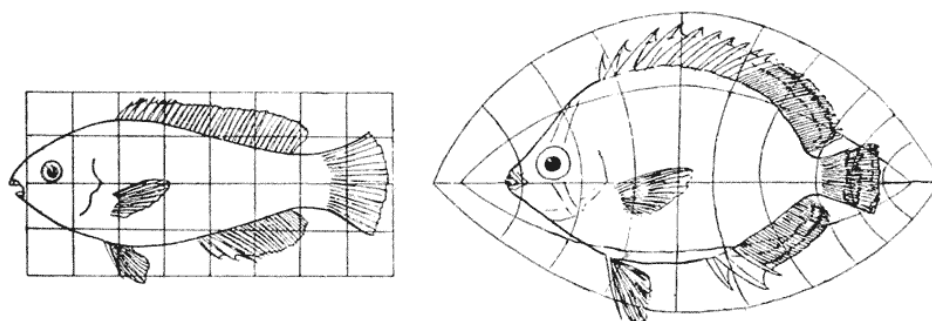
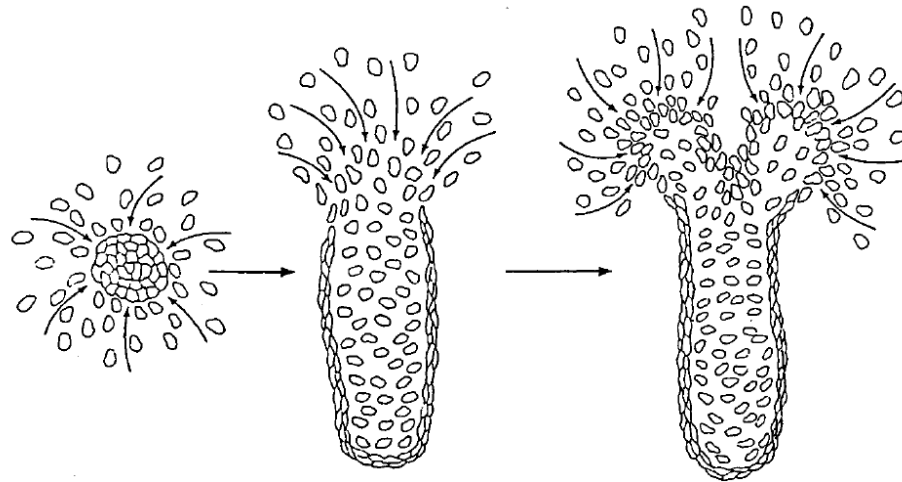


Figure 1.4
Morphogenesis according to D'arcy. Deforming the initial form of a Scaroid fish (left) we can obtain a Pomacanthus (right). Source [91]

Figure 1.5

Schematic illustration of the cell condensation process shows the growth of a cartilage. Cells aggregate initially into a central focus (left). Development of the cartilaginous elements restrict cell recruitment to the distal end of the condensation (centre). When conditions are appropriate the aggregation undergoes a Y-bifurcation (right). Source [266].



define the shape of the system, they are the constitutive dynamics that ensure the being of the system.

The notion of form resumes fundamental aspects of a system. Studying how a system form means studying how constitutive elements are related, how they organize themselves, exploring how internal mechanisms led to the formation of its structural proprieties and the shape.

1.3.1 Forms and feedback

As said before, feedbacks are crucial to the functioning of systems and it cannot be disregarded during its formation. The basic distinction between positive and negative feedback acts into the morphogenetic process. Positive feedback destabilizing the system, applying an effect within the system. When the effect concerns the size of the system, positive feedbacks foster the growth. Conversely, negative feedback stabilizes the system, reducing an effect with the system. The system tends to stabilizing its size or destroying its form. Both processes coexist in a complex system and the dominance of one of them produces the formation or the stabilization/disintegration of structured forms. When the system is dominated by positive feedbacks, we have morphogenesis, literally the creation of the system's current form. Otherwise, the negative feedbacks are more relevant than positive ones and we have morphostasis, literally the maintain of the system's current form. The coexistence of these antagonist mechanisms produces hierarchies, specializes a part of the system, structures, and heterogeneities, characterizing the form of the system. In chapter 5 we will expose how the formation of patterns in a reaction-diffusion system is a good example of these contrasting processes.

When we consider the time, effects can be synchronic or diachronic. For the first one, we are interested in a given moment of time and the effect of the process on the self-organization and the form. For the second, we analyse the evolution through time. Depending on the scale, the effects are not instantaneous, but over a period of time. Urban sprawl is an example of a diachronic effect. New lifestyles and the massive increment of private

mobility requests have produced since the 50s the formation of sparse and discontinuous suburbs. To connect these areas, streets are built and cities become more and more adapt to the car mobility that pedestrian and public mobility. These forms of urbanization retro-acts, incrementing the tendency of people to prefer private transport. At the individual level, the process is synchronic.

1.3.2 Forms and equilibriums

The essence of the notion of equilibrium is twofold. First, the concept of equilibrium is relative because it depends on the time and on the scale of observation. A system could appear stable at a macro-scale (liquid in a glass) and unstable at the micro-scale (molecules of the liquid are in agitation). The body of a child does not change in a day but at the end of 20 years, the body is that of an adult. Second, depending on the system considered, different kinds of equilibrium could co-exist (i.e. we have a chemical equilibrium, a physical equilibrium, a hydro-thermic equilibrium). A system can reach more than one kind of equilibrium. For instance, a cell in a hypertonic solution could be in a quiescent state and exchanges matter with the environment via an osmosis process.

Claim that a system is in equilibrium is a delicate task. For our purposes, we make a basic distinction:

- **At equilibrium**

Stable and resilient forms. For this kind of system at least one kind of equilibrium, at a fixed time-space scale is reached. Often, the form is difficult to perturb because a feedbacks lead the system to a morphostasis.

- **Out of equilibrium**²³

Those forms have not yet reached equilibrium and are still evolving. Once equilibrium is reached, elements can still be renewed or exchanged [86].

- **Far from equilibrium**

Local equilibria are in a critical state, the system suddenly evolves and falls (by bifurcation) in a new state [17, 171, 293].

1.3.3 Forms and forces

As we said above, a system exchanges with the environment: a flow of energy goes into the system, the system consuming inputs, forming organized forms. Internal forces are activated by these dynamics; these forces lead the forms.

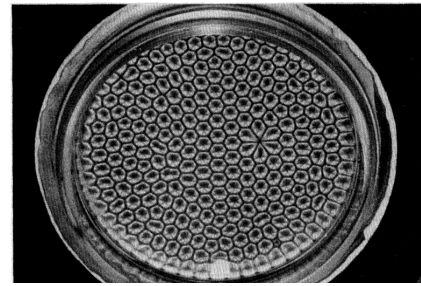
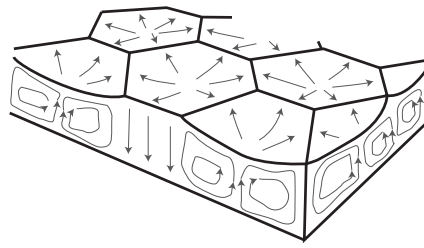
The Rayleigh–Bénard convection [36] is an example (fig. 1.6). We have a thin layer of liquid²⁴. At the beginning, the top and the bottom plane have

²³also named not yet at equilibrium.

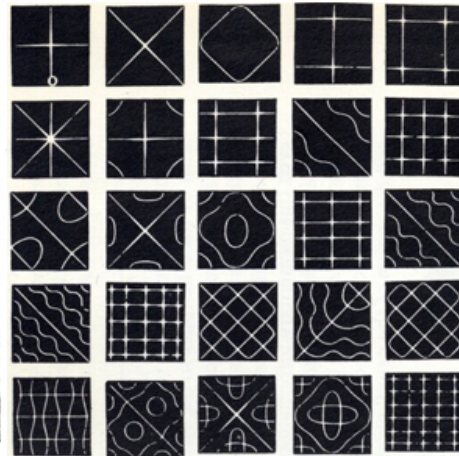
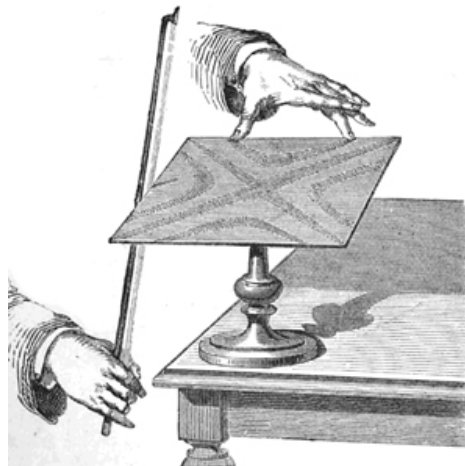
²⁴The height of the layer is small compared to the surface.

Figure 1.6

The Rayleigh-Bérnard experiment. The schematic representation of atoms' flow (left) and the spontaneous formation of hexagonal patterns in silicone oil (right, source: [166]).

**Figure 1.7**

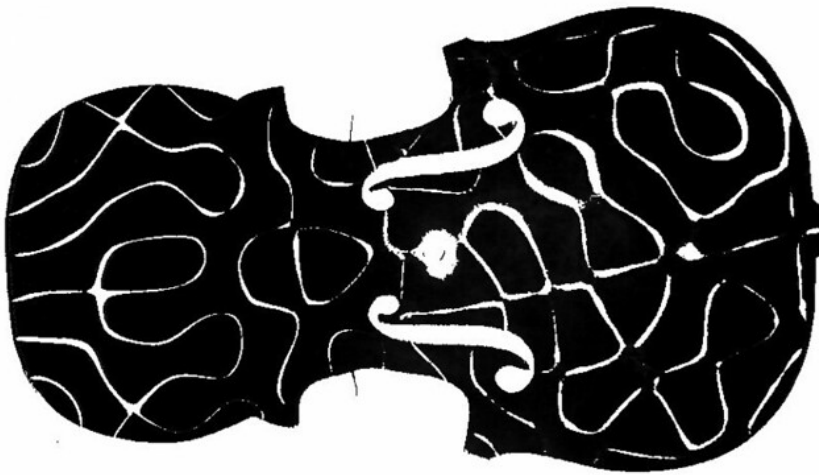
The Chladni's experiment and some representative patterns. Source: publicly available on the web.



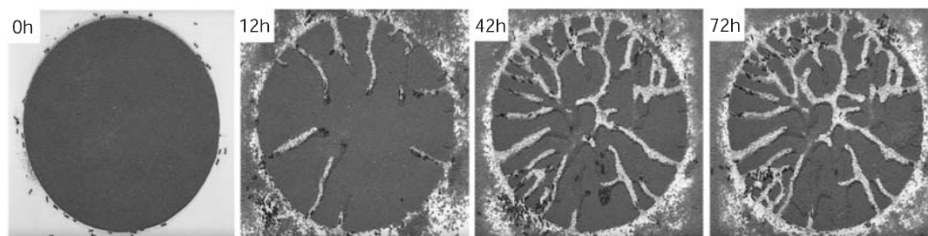
the same temperature, the system tends toward the thermal equilibrium. The layer is exposed to a gradient of temperature: on one face the temperature is higher than the other face. The cool liquid at the surface tries to sink to the heated side and vice-versa. These two opposite movements cannot take place at the same time in the same region, a form of self coordination spontaneously arise. As moved by a self-regulated vector field, flows of ascendant and descendent atoms move. Regular hexagonal cells appear on both face of the layer and disappear when the gradient of temperature expires.

The Chladni experiment [77] is another good example of forces that governs morphogenesis. In 1787, he found that if an edge of a metal plate, covered by fine sand, is bowed with a violin bow, the powder gathers into geometrical forms (fig. 1.7). The plate is crossed by a series of contractions, dilatations, and oscillations which impact the position of grains as a vector field. The formation of patterns depends on the vibration of the plate: the intensity and the frequency of the vibrations govern the spatial organization of the sand grains.

The last two examples show that the formation of forms can be governed by the internal or exogenous forces. Elements of the system find a new equilibrium (far from equilibrium for the first example, at equilibrium for the second).

**Figure 1.8**

The Chladni's experiment performed on the bottom of a violin. Source: publicly available on the web.

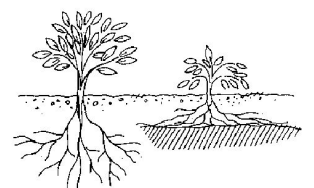
**Figure 1.9**

The formation of ant galleries. Source: [61]

1.3.4 Forms and environment

In some cases the environment represents a physical constraint for the formation of the system: the location of obstacles impacts the ants trails [221], the orography is a constraint for cities (section 3.2), the irregular characteristics of the soil impact the branching of plant roots. Spatial constraints affect the organizations of such systems; it is not the cause of the genesis of forms but it is able to impact the residual form. Figure 1.8 depicts that resulting forms from the Chladni experiment (see section 1.3.3) are impacted by the geometry of the surface.

The environment can be modified by the system. An ant nest is an intricate piece of architecture with a maze of interconnected passages. During the creation of galleries, there is no central control, the form emerges as a result of local and decentralized interactions of ants. The authors in [61] disperse 200 *Messor sancta* ants around a sand disk and observe the formation of several branched and converging galleries (fig. 1.9). The organization of ants impacts the environment: as the negative of the organization of the system, the environment evolves as an intricate series of galleries. A similar process could be observed in urban systems, because people modify the land and build cities. We detailed that in chapter 3.



To understand the form of a system we must investigate its morphogenetic mechanisms. Forms depend on the equilibrium state at which the system is. It can be guided by internal and/or external forces and tend to adapt to internal and/or external requirements. A system and its environment are interwoven, therefore the morphogenesis of

Summary 3:
morphogenesis

the system also depends on exogenous factors. Feedback mechanisms increase and maintain the order and the tail of the system, incentivise its constitution, lead the system to new organizations, or destroy the system.

Chapter 2

Morphogenesis of spatial complex networks

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Keywords systemic approach, spatial complex networks, network morphogenesis, geometric and dynamic graph, graph generators.

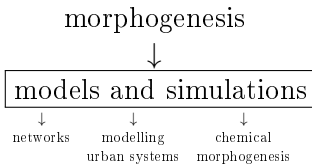
"Confusing a model with reality would be like going to a restaurant and eat the menu"

Arthur Bloch¹

The previous chapter gives a general description of complex systems, the role of spatial aspects in some of them, and how these last influence the morphogenesis. Many complex systems are structured as networks. Complex network research represent the intersection between complexity theory and networks.

The chapter starts with an overview an overview of the systemic approach, model and simulation (section 2.1). Then we point out that for a specific class of networks (called spatial network), spatial aspects play a fundamental role into the definition of their properties and their behaviour. A focus on these aspects is required to study their morphogenetic processes. To this end, we introduce some notions from the graph theory, a pillar of discrete mathematics (section 2.2). We show how to consider notions of space and time in a graph, some specific characteristics of graphs embedded in a plane. We end this chapter with a review of geometrical graph generators and empirical observations in real spatial networks (section 2.3).

2.1 Models and simulations



The difficulty to understand a lot of systems and the incapacity to decompose the behaviours in their essential mechanisms had imposed an epistemological shift. The difficulty is not just in the formalization of the system. It is also in the scientific approach: to encompass the complexity, we need to shift from an approach that avoids the ambiguity, contradiction, and inaccuracies to move forward integration of incertitude, chaos, and fuzzy. From the analytic (that aim to completely control phenomena, measuring all aspects and reduces it in a few fundamental parts) to the systemic approach (which is based on the incapacity to understand the system as the sum of information). The systemic approach does not abandon fundamental principles of science; broking a part of Descartes' *scientific method*², it proposes that a system must be best understood in a whole contest, rather than in isolation [173].

According to Morin [202], to understand the complexity we need to replace:

- the principle of reduction (that consists of knowing elements from only the knowledge of their characteristics) by the distinction principle (that conceives the relation between the whole and the elements).
- the principle of disjunction (that consists of isolating and separating disciplines) by the conjunction principle (that overcome the hermetic of cognitive fields, integrating knowledge).

¹As a joke, the author of Murphy's law named it as the Golomb's Law on mathematical model.

²Components of a phenomenon can be analysed independently and the addition of them is the whole phenomenon.

- the deterministic principle by a principle that integrates the uncertainty, the incapacity to all predict, and the difficulties to validate cause-effect relations between micro-relations and macro-properties.

The systemic approach is "*une manière d'entrer dans la complexité*"³ (Donnadieu and Karsky [94]); It can be decomposed in three steps [247]:

- **Investigating reality:**
the observation and the description of main aspects of the system (properties and dynamics).
- **Modelling the system:**
the operative encoding in a rigorous formalism of all information that may be useful to answer our question.
- **Simulating the evolution:**
the study of the evolution of the model over time.

The result is a collection of information that may be useful to improve our understanding of the studied system. To increase our understanding, we should complete the study of the system as a "loop": observe reality, capture its elements and main evolutive mechanisms, define a model, simulate its evolutions, compare results to reality, and modify if necessary the model and so on.

2.1.1 Investigating reality

The first step of the systemic approach can be decomposed; "*la triangulation systémique*" (the systemic triangulation, fig. 2.1) [176] integrates:

1. **The structural aspect:**
it concerns how the system is composed: its essential properties, the network of interactions, the arrangement of elements, the configuration of the system.
2. **The functional aspect:**
it is relative to what the system does and its role in the environment.
3. **The dynamic aspect:**
it concerns the evolutionary nature of the system, the processes of formation and evolution of the system.

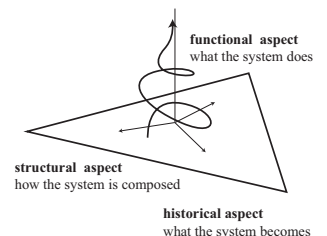


Figure 2.1
The systemic triangulation.

The systemic triangulation develops by combining these three aspects. More exactly, "*on se déplace d'un aspect à un autre au cours d'un processus en hélice qui permet, à chaque passage, de gagner en approfondissement et en compréhension, mais sans que jamais on puisse croire que l'on a épuisé cette compréhension.*"⁴ (Donnadieu and Karsky [94]).

³ "A way for us to understand complexity".

⁴ "We move from one aspect to another during a spiral process that allows each passage, to gain both in depth and in comprehension, but without ever letting one believe that one has reached the end for this comprehension".

2.1.2 Modelling the system

Scientists make and improve theories to understand many phenomena⁵. Often the way to construct theories is iterative: the object is studied from different points of view and the observer tries to answer some questions. From this perspective, the key role of a model is to answer observer's⁶ questions about something.

"To an observer B, an object A is a model of an object A to the extent that B can use A* to answer questions that interest him about A."*

Marvin Minsky [198]

Artificial and natural phenomena can be investigated through scientific models. The capacity of a model to improve our understanding depends on its intelligibility. The capacity of a model to explicate (and make intelligible) an object does not be related to the idea that it is able to reproduce the phenomenon. When we study a complex system, a model may reproduce a complex behaviour but maybe it is not intelligible, making the model useless to understand the phenomenon.

It is impractical and might bear to no relevant details resume all aspect of the studied system and we need to establish main aspects (a model represents in all cases a part of reality). The modeller chooses the main aspects to integrate into the model. The choice depends on which part of the system he is interested in, trying to condense in a rigorous framework the essentiality of the studying phenomenon. A model is in all cases a simplification, it is perfectible; experiences and observations are fundamental to improve the model and our capacity to model. A model is always a conceptualization of the object, hence we cannot be sure that it is useful⁷.

In a positive vision, where we can concept models that are able to answer all questions with a mathematical encoding, we can concepts explicative models. In this way, we try to explicate properties of objects, validate hypothesis, describe reality or develops technological devices⁸. A second vision consist to concept a model to predict the future. In this way, models are concept to describe the phenomenon. They are concept to make scenarios and often they replay questions under the form "what if.". Changing parameters of the model, we can explore trajectories and study causality between initial configurations and final predictions.

Le Moigne [176] summarizes the actions that the modeller makes when he concepts a model. Modelling is:

⁵Roughly, theorisation is the science that aims to describe a phenomena with explications and information, and relations between their state variables.

⁶"If there is no other, there will be no I. If there is no I, there will be none to make distinctions" (Chuang-tsu).

⁷"All models are wrong, but some are useful" (Box [55])

⁸Under this vision, the model should validate hypothesis: "a simplified version of reality, built in order to demonstrate certain of the properties of reality" (Haggett [133])

*"Action d'élaboration et de construction intentionnelle, par composition de symboles, de modèles susceptibles de rendre intelligible un phénomène perçu complexe, et d'amplifier le raisonnement de l'acteur projetant une intervention délibérée au sein du phénomène; raisonnement visant notamment à anticiper les conséquences de ces projets d'actions possibles."*⁹

Jean-Louis Le Moigne [176]

2.1.3 Simulating the morphogenesis

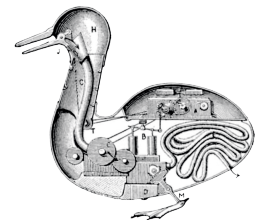
A complex system model can be made for different goals: e.g. to investigate the evolution of the system, to understand the internal mechanisms of the system, to describe mechanisms of interactions as clear as possible, to study the causality between events. A model may be useful to study the evolution of the system, simulating scenarios. Simulation is a dynamic reproduction of evolutive mechanisms of a system; a simulation requires a model. While the model represents the system itself, the simulation represents the evolutive processes of the system over time.

In order to consider the time, the framework which allows us to simulate the evolution of the system should integrate aspects about the way that the time moves forwards. We can consider the time in two manners: continuous (we can observe the model at an infinite number of moments) or discrete (we observe the system over an ordered sequence of time steps).

A model is a representation of something. It does not contain all aspects of the system, but rather only those that the modeler judge relevant. Our purpose is to study the morphogenesis of complex systems; to simulate it, we must avoid two traps:

- **The problem of the analogy:**
a model can carefully reproduce forms and dynamics, without considering real mechanisms of formation. In this case, the system is structured by dynamics that do not respect reality. The shape may be similar but not the construction (therefore the form too).
- **The problem of the completeness:**
A useful model may not integrate all information concerning the system. The challenge is to capture the only fundamental aspects useful to our task and to show that they are sufficient and necessary. The model must integrate the main morphogenetic mechanisms.

To investigate the morphogenesis of complex systems and capture dominants' morphogenetic processes, it is crucial to define the well-adapted level of abstraction. The model should on one hand consider features of the



The Vaucanson's duck.

⁹ "The elaboration and the deliberate construction thorough symbols of intelligible models, that increase the degree of intelligibility of the modeller; to model means to go forward effects of the possible initial conditions."

system which lead the form, and on the other hand to not flood it with microscopic details.

2.1.4 Integrating the space

As mentioned above (section 1.2), spatial aspects play an important role in the functioning of many systems. It is so self-evident that a model that addresses these systems must integrate spatial information. In practice, we need a rigorous formalism that allows us to encode the following information:

- **spatial localization:**
to each element of the systems is assigned an attribute (a tuple X of coordinates $x_1, \dots, x_d$) that allows us to localize the element in Euclidean space $\mathbb{R}^d$, $d = 1, 2, 3$.
- **Spatial neighbours:**
in general, elements of systems locally interact (with a limited number of neighbours); in a spatial system, to identify the neighbourhood of an element, we can refer to metric distance.
- **spatial interactions:**
the way that elements interact with each other depends on spatial aspects. It may depend on the metric distance¹⁰ and/or spatial arrangement of their neighbours.

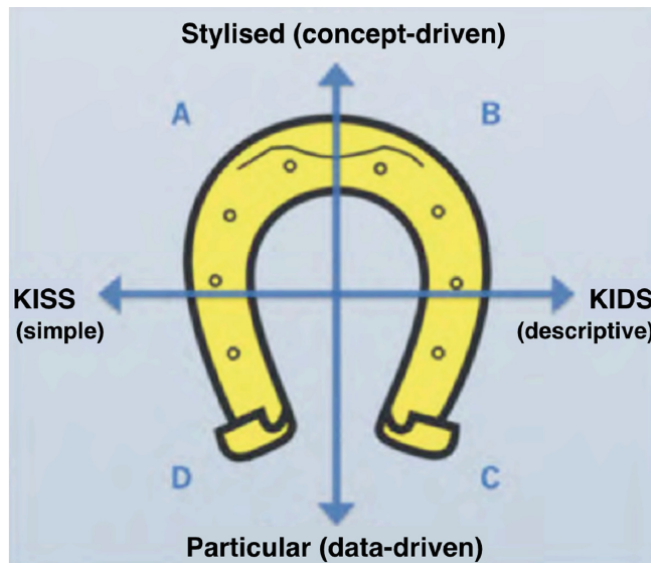
In section 2.2 and in chapter 5 we will show that graphs and cellular automata, among different useful formalisms, allows us to encode spatial information.

2.1.5 How to concept a spatial model

Nowadays, there exist many spatial models with different characteristics and objectives [255]. Due to mathematical, scientific, and computational contributions, in the last years, the diversity of models increases [291]. Banos and Sanders [19] propose a taxonomy of spatial models, crossing two (continuous) axes:

- **The KISS-KIDS axis:**
it corresponds to the degree of simplicity of the model. The extremes correspond to the KISS (Keep It Simple, Stupid) approach [15], which try to use simpler mechanisms to model reality, and the KIDS (Keep It Descriptive, Stupid) approach [96], which try to define the model as descriptive as possible.
- **The particular-stylised axis:**
it corresponds to the degree of abstraction of the model. Stylised models aim to reproduce more general mechanisms. They are less

¹⁰For instance, this is the case of gravitation models in geography, see section 3.1.

**Figure 2.2**

The "horseshoe": a template to read and concept models in geography. Source: [19].

related to the specific study cases. Otherwise, particular models integrate many details of the study cases and require a large amount of information. The model is less general and fits a limited number of sites.

The resulting template (fig. 2.2) is a way to categorize spatial models and also a way to suppose that a model is "useful" (if it is over the horseshoe) at the same time.

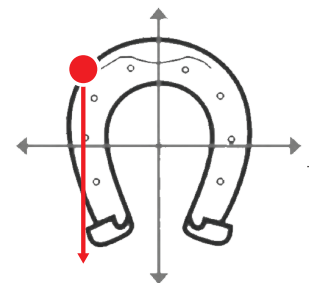
The particular-stylised axis correspond to the choice of the modeller to prefer "*une organisation spatiale précise, observée en un lieu donné à un moment donné, ou une organisation-type, simplifiée, que l'on observe de manière répétée dans le temps et/ou l'espace*"¹¹ (Banos and Sanders [19]). The KISS-KIDS axis embodies the degree of abstraction of the model. The modeller can integrate a large number of processes (that may produce a model difficult to handle but probably more realistic) or a few processes (to tends to a model as minimal as possible) In this last case, the resulting model is easier to handle and shows more direct relations between- parameters and emergences, trajectories, and outputs.

There are two reasons to prefer the top-left quadrant [291]: the first one is epistemological (we should be able to understand what the model do, avoiding the "black box" trap) and the second one is ontological (it is not necessary to use parameters that do not contribute to the understanding of the model. This is the principle of the parsimony¹²). A complicated model is unintelligible, the simplest model which reproduces the system is preferred and the top-left quadrant in fig. 2.2 may be a good starting point for the future implementations.

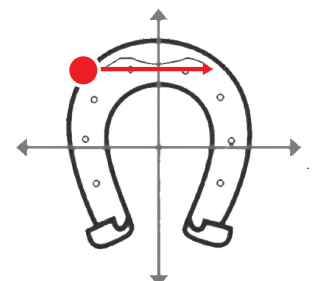
A model with few mechanisms/parameters probably is easier to control and we can understand the causal relations between inputs and outputs. More-

¹¹ A given spatial organization, observed at a given spatio-temporal configuration, or a stylized organization, observed over several spatio-temporal configurations.

¹² Also known as Occam's razor, who states that "*the simplest solution is most likely the right one*".

**Figure 2.3**

Integrating more spatial information.

**Figure 2.4**

Integrating more morphogenetic mechanisms.

over, due to its flexibility, a model concept in such a way can be implemented it in a second step. To do that, we can follow two directions:

- **through the vertical axis:**
incrementing inputs and information, making the model more data-driven (fig. 2.3).
- **through the horizontal axis:**
implementing processes, making the model more descriptive (fig. 2.4).

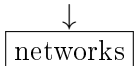
According to the template in fig. 2.2, to investigate morphogenesis of complex systems, we should define a general and initial framework placed in the quadrant *A*; this initial model can be implemented, incrementing the level of details of its evolutive mechanism (going forward a KIDS model) or increasing the number of details from the specific study cases (going forward a particular model).

Summary 4:
model and simulation

A model is always an approximation of reality. To investigate phenomena we should observe it and capture fundamental aspects. To not bear to no relevant details, our approach must be led by a parsimonious principle. With this principle, we can focus on computational aspects of models, we have an understanding about its behaviour and we can implement it at a latter time.

2.2 Networks

models and simulations



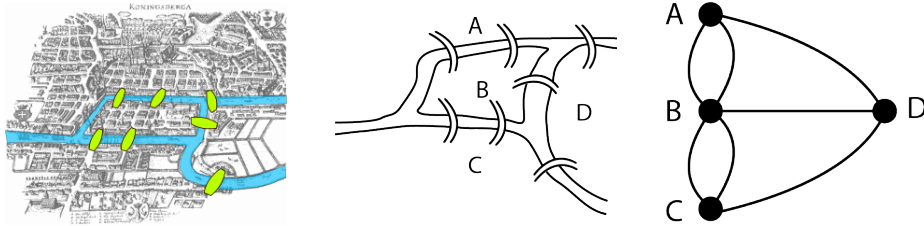
street network

Networks represent a wide range of natural and artificial systems [46, 172, 209]; they are composed by nodes in relations by links and catches the system organization. Networks often hide the wishes to represent the system with points and oriented or not oriented links connecting them. This easily comprehensible graphic representation offers obvious facilities such as the representation of a discrete universe but often masks its complexity. It is necessary to study them not weakly but deeply.

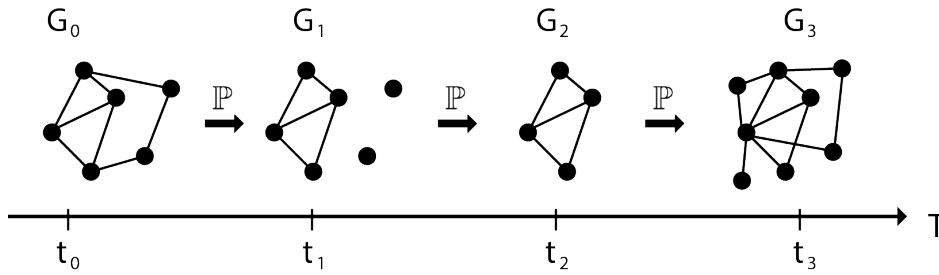
2.2.1 Graph representation

While a network may be thought close to reality, a graph is a mathematical representation of elements and their relationships. Graph theory is the fundamental branch of mathematics that studies the theoretical aspects of networks. In a graph (network), edges (links) connect vertices (nodes). The graph formalism synthesizes essential information and quantitatively brings out characteristics of networks.

The first application of graph theory to solve a practical problem from reality was done by Euler (fig. 2.5). The city centre of Königsberg (now named Kaliningrad, Russia) is located in the Pregel river whose crest includes an island. In Euler's time, seven bridges across the river; a popular problem in 1736 was to find a path that cross once the seven bridges of Königsberg and came back to the starting point. The Euler's solution was to draw the

**Figure 2.5**

The seven bridges of Königsberg problem. The map of Königsberg, (left, source: www.wikipedia.org); its schematic representation (centre); the corresponding graph, where dots represent sides and lines are the bridges (right).

**Figure 2.6**

The evolution of a dynamic graph.

elements that he had judged important to solve the problem as a graph: he defines four vertices, which represent sides of the river, and seven edges, which represent the connection between two distinct sides. Under this representation, Euler disregards irrelevant information about the geometry of the city and keeps only the connections between lands. Via the graph representation, he observes that the solution of the problem depends on the number of incident edges of each vertex, the degree. The possibility of a path through the graph, traversing each edge exactly once, is that the graph has zero or two vertices of odd degree. He demonstrates in a rigorous way that there was not such a path in the graph obtained from the Königsberg city¹³. The graph theory was born.

Graph theory has been applied by scientists to represent, analyse, and simulate a large number of systems. Several useful master-books had synthesized main definitions [79, 93, 286].

A graph $G = (V, E)$ consists of two finite sets:

- $V = \{v_1, v_2, \dots, v_n\}$: the not empty vertex set V of the graph, composed by vertices. The set size is denoted by $|V| = n$;
- $E = \{e_1, e_2, \dots, e_m\}$: the eventually empty edge set, defined by the function $\mathbb{E} : V \times V \rightarrow E$, that assigns relationships between vertices. The set size is denoted by $|E| = m$;

Definition 3:
graph

In frame 1 we resume main aspects of a graph.

Frame 1:
Graphs

The edge $e = (v_1, v_2) \in E$ is a pair of vertices $v_1, v_2 \in V$ (called adjacent or neighbour vertices) and it is incident to vertices v_1 and v_2 . An undirected graph is one in which the edge set is composed of unordered pairs of vertices $(v_1, v_2) = (v_2, v_1)$; otherwise, the edge set is composed of pairs of ordered vertices and the graph is directed.

The neighbourhood $N(v) = \{u : (v, u) \in E\}$, $N(v) \subset V$ of the vertex v consists of the set of vertices neighbour to v . The degree k of vertex v , hence noted as $k(v)$, is the number of edges incident on v or equivalently, the size of the neighbourhood set $k(v) = |N(v)| \in \mathbb{N}$.

If more than one edge connects the same pair of vertices, these edges are denoted parallel and the graph is called multigraph. Otherwise the graph is called single, there are no loops (an edge which joins a vertex to itself) and no parallel edges.

A path between two vertices v_1 and v_n in a graph, noted as $p(v_1, v_n)$, is a finite not empty ordered sequence of distinct vertices $v_1, v_2, \dots, v_n$ such that, given $i \in \mathbb{N}^*$, $i < n : v_i \in V$ and $e_{i,i+1} = (v_i, v_{i+1}) \in E$. If $v_1 = v_n$ the path is a cycle.

A weighted graph is a labelled graph (labels are assigned to vertices or edges) all of whose are positive or zero real numbers. More specifically, a vertex-weighted graph has weights on its vertices and an edge-weighted graph has weights on its edges. In edge-weighted graph, a function $L \times E \rightarrow \mathbb{R}_+$ which maps a positive or zero real numbers in the tuple $L = (l_1, l_2, \dots, l_m)$ to each edges in E (we noted as $l(e)$ the real number assigned to the edge e), the length of the path is $len(v_1, v_n) = \sum_{i=1}^n l_i$. The weighted shortest path distance $len_{min}(v_1, v_n)$ is the path that minimize $len(v_1, v_n)$; we noted as $len(G) \in \mathbb{R}_+$ the sum of weights of edges.

The sub-graph $G' = (V', E')$ of a graph $G = (V, E)$ is a graph where sets V' and E' are a subset of V and E respectively. The partial graph $P = (V, E')$ of the graph $G = (V, E)$ is a graph where the set E' is a subset of E . A graph is called connected if there exist a path between each pairs of vertices. Two vertices are also called connected if exist a path which links them. Given a vertex v of a graph G , the set $S(v)$ contains all vertices connected to the vertex v . The sub-graph of G induced by the set $S(v)$ is called connected component of v .

A tree is a connected graph where exist exactly one path between two distinct vertices. If we remove an edge, the graph became de-connected: the resulting graph became a forest of at least two trees. If we add an edge, we create a cycle and the graph is no longer a tree. A tree of n vertices has $n - 1$ edges. The weighted minimum spanning tree $MST(G)$ is a the tree that minimize $len(MST(G))$. In other words, the weighted $MST(G)$ is the tree and a partial graph of G . Finally, a vertex in a tree with degree 1 is called leaf.

2.2.2 Dynamic graphs

The capacity to represent a system as a graph is often increased with time aspects [69, 146]. Systems reorganize themselves in time. Due to interactions between their entities, their properties evolve, the number of nodes increases or decreases, and even the manner that they interact may vary. To formalize temporal aspects of a network, we define:

A discrete dynamic graph [225] $G(t)$ is defined by the triplet $(T, G_0, \mathbb{P})$:

- the initial sequence of discrete time steps $T = 0, 1, 2, \dots, t$.
- the starting graph $G_0 = (V_0, E_0)$.
- the process $\mathbb{P} : T \times G \rightarrow G$, a function that defines $G_t \xrightarrow{\mathbb{P}} G_{t+1}$.

Definition 4:
dynamic graph

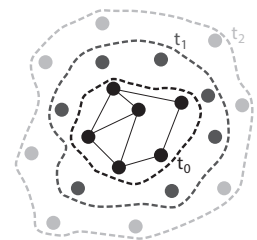
We note the static graph at time step $t \in T$ as $G_t = (V_t, E_t) \subseteq G(t)$. A dynamic graph¹⁴ is a sequence of static graphs, defined for each time step. The process $\mathbb{P}$ defines the relation between two graphs, it modifies the vertex set and/or the edge set. Several aspects can define the relation between two graphs at two time steps. For instance, we note as $\mathbb{P}(V)$ a process that is defined by the vertex set. According to our terminology, in a preferential attachment model [20], where the evolution of the graph is governed by the degree k and a rate of growth r , the evolving process can be noted as $\mathbb{P}(k, r)$.

In several real situations, we observe that complex systems grow, not just evolve. For instance, an ant nest is composed of a set of galleries and the number of elements increases (or stays stable) over time. Observing the experiment in fig. 1.9, ants dig new galleries and existing ones never been destroyed, are getting longer and bifurcate. As a sedimentary process, the shape of the nest grows and the existing parts does not disappear. The corresponding dynamic graph, that captures the morphogenesis of the nest, is composed of increasing sets of elements. Formally, we define:

A growing graph is a dynamic graph where $V_t \subseteq V_{t+1}$ for each $t \in T$.

Definition 5:
growing graph

To define a growing graph we do not consider the edge set. Keeping the previous example about the morphogenesis of an ant nest, the starting point of a new gallery might be the middle of an existing one. In a graph representation, the oldest edge was removed and substituted by new edges, and the new vertex has degree 3. Streets networks, vascular networks, leaf vein networks, and crack networks often grow in this manner.

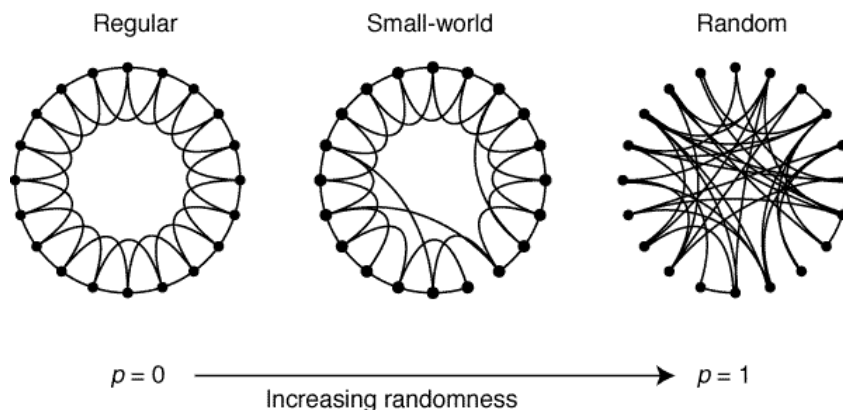


¹³Mathematically speaking, the graph is not Eulerian because there was not an Eulerian path.

¹⁴Hereby, we indicate a dynamic graph as a discrete dynamic graph.

Figure 2.7

The Watts and Strogatz model. Let be a regular graph dispose as a ring (left), the Watts' procedure consists randomly rewire each edge of the regular graph with probability p such that self-connections and duplicate edges are excluded. Source [296].



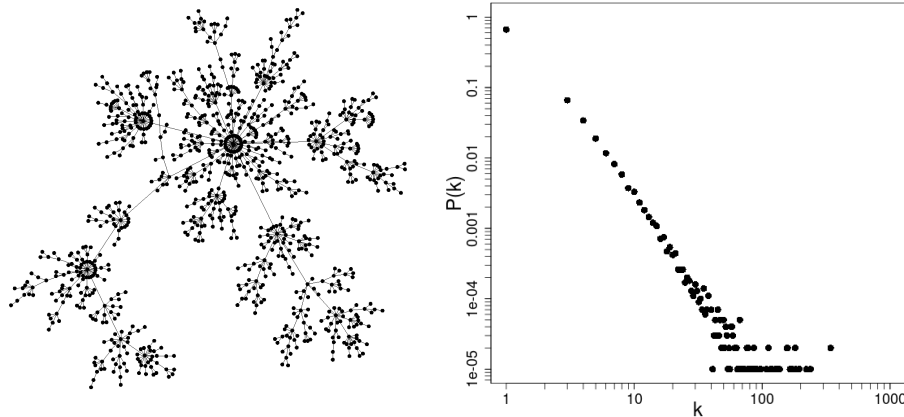
2.2.3 Complex networks

Since the Euler solution, graph theory gives a useful way to solve many problems and to study many real phenomena. Few examples: the four colour problem (find the minimum number of colours to map adjacent regions in such a way neighbouring regions has different colours), the cops and robber problem (one group of individuals attempts to track down members of another group), the Euclidean Steiner problem (given a set of point in the space, find how to connect points minimizing the sum of lines length), the maximum flow problem (find the maximum amount of flow that a pipeline would allow to flow from a source to a sink). In addition to the development of mathematical graph theory into a substantial body of knowledge, graphs are now massively applied in social science. For instance, social studies address problems of connectivity (how individuals are connected to others) [33] and centrality (which individuals are best connected to others) [109]. Moreover, graphs formalism was helpful to study interactions in a group of individuals [295].

Despite the evident contribution that graph theory was been able to give in theoretical and applicative fields, the real capacity of the graph formalism to capture properties of interacting elements was not really observed. Most networks are complex [46, 172, 209]: they are composed by a large amount of nodes, which interacts in a not evident way. They dynamically organize themselves, defining emerging properties which cannot be detected investigating each node via classical graph measures. Hence, in the last three decades, we observe a substantial new movement of interest, with the focus shifting away from the study of small, regular and theoretical graphs to consideration of large, heterogeneous and dynamic real networks.

The growth of interest about researchers in network formalism was promoted through to the apparition of a few fundamental papers and their massive application to observe real-world systems. Watts and Strogatz [296] observed that between regular (a graph where each vertex has the same degree) [79] and random (in which connections are defined by some random processes)[98] graphs, there is an intermediate type of graphs which can represent a large number of complex networks. These networks show a small-world behaviour¹⁵ (fig. 2.7). A relevant property regards the degree

¹⁵In simple terms, the small-world concept describes the fact that despite they are

**Figure 2.8**

The Barabasi and Albert model. A scale-free graph of 1000 vertices (left) and the probability distribution of degree of a graph with 100000 vertices (right). The Barabasi and Albert algorithm [20] is composed by two operations: the growth (a number of vertices was created at every time step) and the preferential attachment (the probability that new nodes will be connected to a node depends to the the degree of this last).

of nodes: in many networks, the degree distribution $P(k)$, defined as the probability of a node v has degree $k \in \mathbb{N}$, significantly deviate from the constant value of regular graph or the Poisson distribution of random graphs. Several empirical results show that for many large networks the degree is distributed as a power-law [20] and the scale-free characteristic impacts its own growth [95] (fig. 2.8). In many cases, the degree distribution exhibits a hierarchical tail. In many cases, the tail is even scale-free. Complex networks show no trivial topology: for this kind of network, topology is neither fully random nor fully regular. They often show sub-structures, clusters, and patterns. The representation as a graph of several systems is useful but many classical measures and their analyses are not enough to capture how they are organized.

The interest of many researchers shifted away from the analysis of static and homogeneous graphs (and properties of their elements) to the consideration of properties of dynamic and heterogeneous networks. The study of these systems as a network has produced new results, which is the backbone of complex systems. However, a class of networks are composed by elements embedded in space and the way that they are in relation depends on spatial aspects. This is the case of spatial networks.

Summary 5:
networks and graphs

2.3 Spatial networks

Many complex network applications focused on the characterization of topological and temporal aspects, while spatial aspects are marginally considered or neglected. For these studies, a network is the representation of an abstract system, where the position of elements is not considered and their interactions are not related to spatial aspects. Social networks [284], transportation networks [117], cargo ship networks [158], neural networks [62], biological networks [222], Internet [117], airline networks [131], crack

composed of a large number of elements, in most cases there exist a relatively small number of short paths between any pairs of nodes.

networks [51] are some representation of systems structured as a network embedded on Euclidean space (fig. 2.9). For these networks, geometry is relevant and topology does not contain all information needed to understand their functioning [23]. In the following, we will develop some elements of graph theory that will be useful to consider spatial dependencies in networks and we will show some examples.

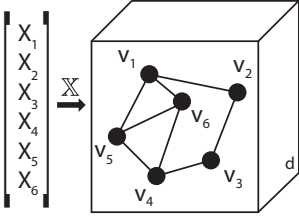
2.3.1 Geometric graphs

Geometric graph theory is a sub-field of graph theory, concerned graphs embedded in Euclidean space.

Definition 6:
geometric graph

A geometric graph $G(x) = (V(x), E)$ is defined, in a d dimension euclidean space $\mathbb{R}^d$, by:

- the graph $G = (V, E)$.
- the function $\mathbb{X} : X \times V \rightarrow \mathbb{R}^d, d = 1, 2, 3$ maps to each vertex $v_i \in V$ a tuple of real numbers $X_i = (x_1, \dots, x_d), x_d \in \mathbb{R}_+$.

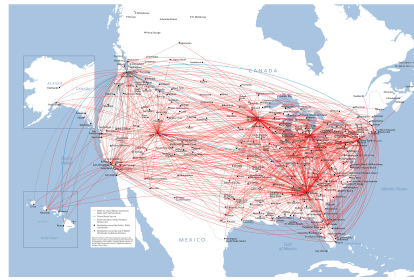


In the following we will consider $d = 2$. Let us show an example. A broad representation of an airline network is a set of nodes (the airports) connected by a set of links (the scheduled fly) [131]. The system can be represented as a 2-dimensional geometric graph, where the vertices represent airports and weighted edges between every pair of airports connected by a scheduled flight (fig. 2.9a). The weight of edges could be the cost of the fly, the time of the fly, the travel distance, etc. (the information that gives the attributes are or geometric or not). Under this representation, new information can be captured. They complete topological information. A graph representation without spatial information may be helpful to answer: which are the well-connected airport? The spatial graph representation integrates the possibility also to answer to question as: is the geographical position an aspect that contributes to the connectivity of an airport? To sum up, new information and new questions bring out by the integration of spatial information, which in turn helps us to improve our understanding of reality.

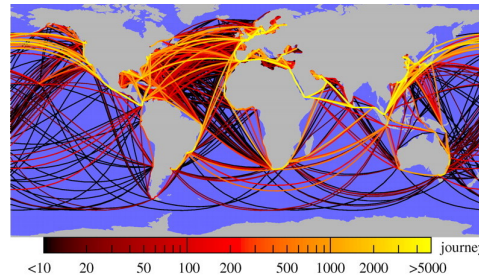
A geometric graph can also be dynamic: in this case we named as geometric dynamic graph $G(t, x)$ the triplet $(T, G_0(x), \mathbb{P})$.

Figure 2.9

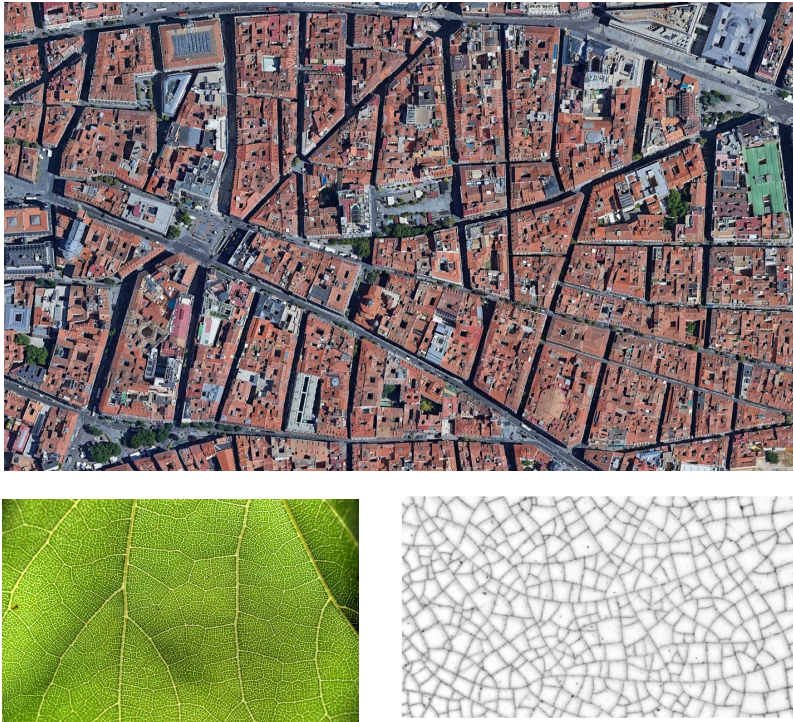
Two geometric graph applications. The network of Delta Air Lines (fig. 2.9a, source: www.delta.com); the study of the importance of cargo ship trips with centrality measure (fig. 2.9b, source: [158], see section 4.3).



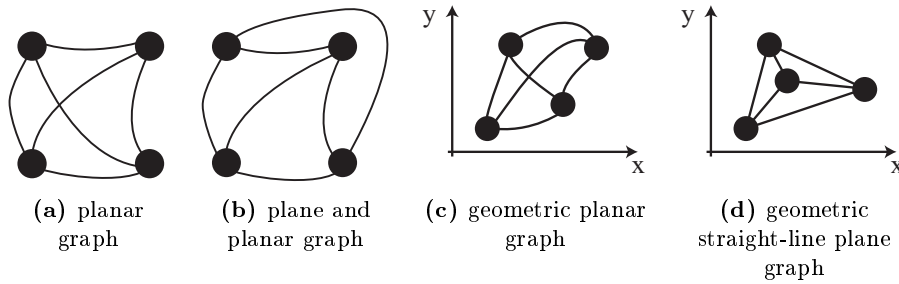
(a) The Delta airline network.



(b) Centrality in a cargo ship network.

**Figure 2.10**

Three theoretic planar graph applications. The city centre of Madrid (top, source Google Earth); the venation network of grape leaves (bottom left, source: publicly available on the internet); the crack pattern in the glaze of a ceramic plate (bottom right, source [50])

**Figure 2.11**

Geometric straight-line plane graph. A planar graph, the corresponding plane graph, a geometric graph and a geometric planar graph where edges are drawn as straight-line segment.

2.3.2 Geometric planar graphs

The geometric graph representation is a helpful tool to study many systems. In a 2-dimensional space, under this representations, edges can overlap and cross. However, in some real systems, we rarely observe that. Leaf networks, crack networks, street networks are examples of systems that seem to be embedded in 2-dimensional space and where rarely edges intersect (fig. 2.10).

We need a rigorous formalism to class and study these networks (fig. 2.11). A graph is said planar if it can be drawn in the plane so that its edges intersect only their end vertices (fig. 2.11a). Such a drawing of a planar graph is called plane graph (fig. 2.11b) and it can be regarded as an isomorphism of the planar graph. The study of planar graphs necessarily involves topological aspects. A plane graph is not necessary geometric, because vertices are not embedded in the space: the fact that vertices are drawn is just a representation and not a property of them. Geometrical plane graphs are plane graphs embedded in 2-dimensional Euclidean space, and any pair of edges meet only at their end vertices.

In a geometric plane graph, the way that edges are drawn is not related to spatial aspects: we can draw edges in an infinite way and they respect the geometrical condition fig. 2.11c. Thus, the previous definitions are not enough for our study. We define as the straight-line graph the geometric graph where the edges are drawn as a straight-line and the length of the segment corresponds to the euclidean distance. Corollary to this definition is the rectilinear crossing number $rcr(G)$, defined as the minimum number of crossing edges in a straight-line graph¹⁶.

Combining the rectilinear crossing number and the straight-line graph we obtain a rigorous representation of several spatial systems (fig. 2.11d).

Definition 7:
geometric straight-line
plane graph

The geometric straight-line plane graph $G(x) = (V(x), E(x))$ is the straight-line graph embedded in 2-dimensional Euclidean space where $rcr(G) = 0$.

Hereby, graphs that respect definition 7 are called planar graph and the planarity condition of a graph means that the graph is a geometric straight-line plane graph.

2.3.3 Geometrical planar graph generators

The simple question addressed here is: what we can say about $\mathbb{P}$? This section proposes six theoretic planar graph generators that are interesting for the study of spatial systems in the sense that they provide basic graphs to compare with spatial networks (fig. 2.12). They also provide initial information about the formalization of the morphogenic process $\mathbb{P}$.

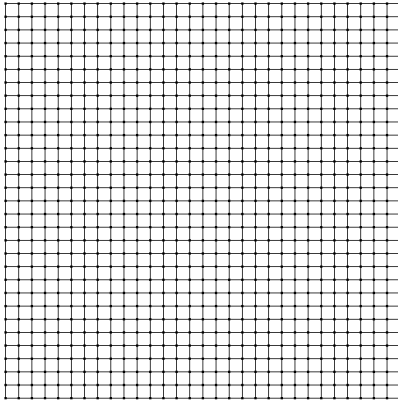
The lattice square graph is a grid graph embedded in 2-dimensional space. It is planar by construction and edges can be drawn as a straight-line segment (fig. 2.12a). Lattice graphs and regular graphs are similar because in the lattice graph the degree of vertices is 4 for all vertices excepted boards of the lattice.

An interesting class of planar graph generators can be deduced when n vertices are randomly placed in the space and connected by some rules. It exists different manners to define how vertices are connected. we propose five simple methods that create planar graphs. All the methods starts with a vertex set $V(x) = \{v_1, v_2, \dots, v_n\}$.

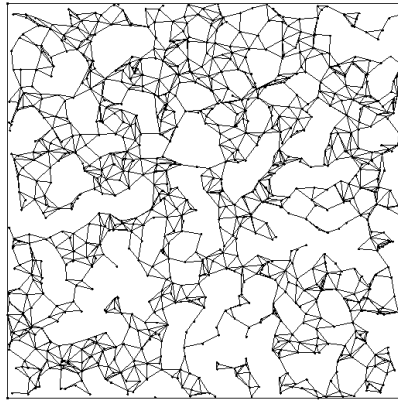
- **The deterministic planar random graph:**

let $\delta \in \mathbb{R}_+$ be a parameter. The procedure consists in randomly selecting each pairs of vertices $(v_i, v_j), i \neq j : dist(v_i, v_j) < \delta$, adding an undirected straight-line edge $e = (v_i, v_j)$ such that crossing edges are excluded (fig. 2.12b).

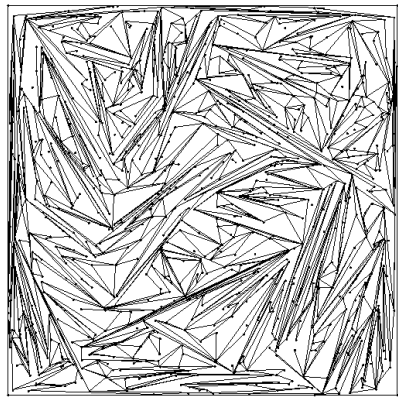
¹⁶Estimating the $rcr(G)$ is a difficult problem [114] and it has several practical applications: for instance, the design of a grid power network where crossing pipelines can cause potential failures. In this case, the problem consists to find the straight-line graph that minimizes the $rcr(G)$.



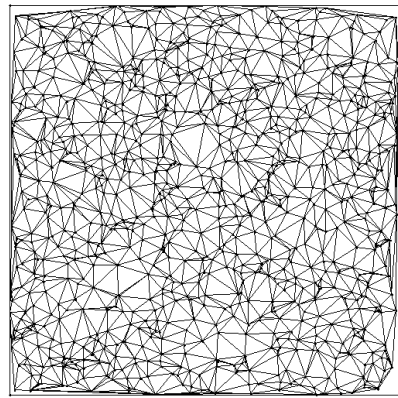
(a) The grid graph



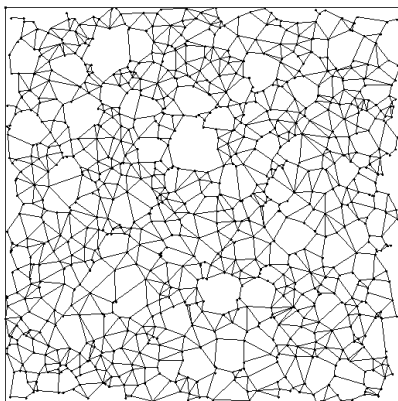
(b) The deterministic graph



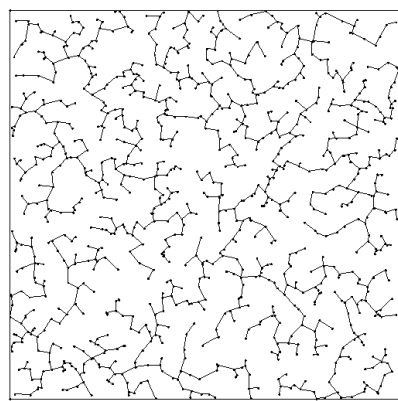
(c) The probabilistic graph



(d) The Delaunay graph



(e) The Gabriel graph



(f) The Euclidean tree

Figure 2.12

Six theoretic planar graphs. Each graph is composed by 1000 vertices. Vertices in figs. 2.12b to 2.12f are spatially arranged as the same manner.

- **The probabilistic planar random graph:**
let $p \in [0, 1]$ be a parameter. The procedure consists in randomly selecting each pairs of vertices $(v_i, v_j), i \neq j$, adding an undirected edge $e = (v_i, v_j)$ such that crossing edges are excluded (fig. 2.12c).
- **The Delaunay graph:**
the Delaunay triangulation of a set of n vertices embedded in $\mathbb{R}^2$ is a triangulation¹⁷ such that no vertices are inside the circumcircle¹⁸ of each triangle. The Delaunay graph is the dual of the Voronoi tessellation. A direct construction of the Delaunay graph is possible. Let v_i, v_j and v_k be three vertices in V . The procedure consists in adding the edge (v_i, v_j) if no other vertices are inside the circumcircle (fig. 2.12d).
- **The Gabriel graph:**
let $\delta \in R^+$ be a geometric parameter. The procedure consists in randomly selecting each pairs of vertices $(v_i, v_j), i \neq j$ and add an edge if the circle with the radius $r = \text{dist}(v_i, v_j)/2$ that passes by v_i and v_j does not contains a vertex in $V(x)$. The Gabriel graph is a sub-graph of the Delaunay graph [194] (fig. 2.12e).
- **The Euclidean spanning graph:**
the simplest procedure to build it is to run the Kruskal's algorithm [79] to the Delaunay graph (fig. 2.12f).

Here we have presented the procedures to build six geometric planar graphs. In chapter 4 we will show main characteristics of these graphs.

2.3.4 Morphogenesis of spatial complex networks

In the following, we resume some processes behind the formation of spatial networks and how they can be modelled in a dynamic geometric graph.

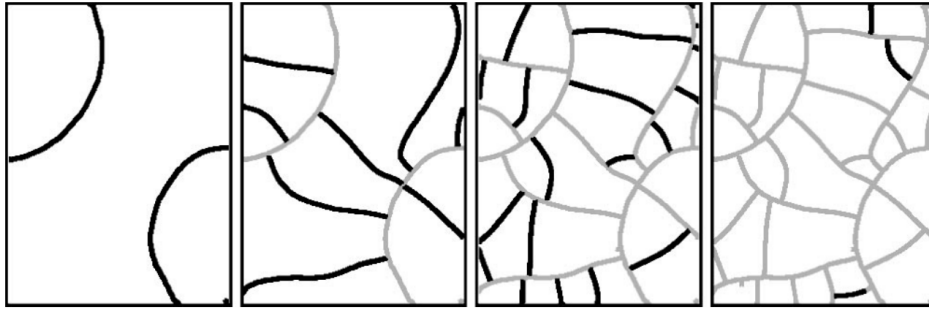
Local interactions and decentralization

In last decades, following the concept that the growth of spatial networks is subject to certain optimization processes [117], various authors proposed models that aim to minimize metrics [130], reproduce emergent characteristics [27] or find compromise between antagonist properties [56]. Optimization has been shown to be a driving force for the growth of urban systems. For example, the geometrical optimization of global elements of street networks used in [84] reproduces a coherent and realistic growth of cities. Using a global optimization process, interesting works are developed in order to reproduce the evolution of some biological structures, as insect nests [290] and also Mammalia bones [223].

Although global approaches are useful to model the evolution of spatial networks, in many spatial systems, local interactions are predominant and behind the organization of its elements. Thus, in a spatial network, the

¹⁷Triangulation is a division of a planar object into a set of triangles.

¹⁸The circumcircle of a triangle is the only circle which passing by all its vertices.

**Figure 2.13**

The evolution of a fracture pattern. From left to right, figures show the same surface at four different moments. In each images, the black line is the newly appeared fracture. Source [52]

formation of stable structures is often a consequence of elementary local interactions and depends on the configuration of its neighbourhood [210, 270]. Rarely they are the result of central controls. They can be impacted by external factors but their own structures are governed by internal mechanisms.

Planar graph generators with a focus on local interactions are proposed in urban studies: Barthélemy et al. [25] describe a model of street networks growth with local optimization of intersections. In the model proposed by Rui et al. [251], a competition of nodes added step-by-step reproduce a self-regulated street network growth. Despite ant paths look like a structure that would require some kind of central coordination, a local biochemical mechanism (the stigmergy) of positive or negative reinforcements of trails govern the moving of insects [271]. This mechanism of spatial network formation, observed in several ant species [222] and even in humans [139], does not require an external control and a global information about system configuration. Through a local-driven interaction approach, the network grows spatially and temporally in a coherent way: for instance, the model proposed in [1], generates a connected network that represents an abstraction of street networks.

Coherency

We observe that the generation of disconnected sub-graphs is not in many cases an appropriate representation of growth for many spatial network applications. Each part of a vascular network, a street network, an ant nest network, a venation leaves network is connected to the rest of the network. The process of formation of fractures in the glaze of ceramics is a sequence of adding lines or elongating existing lines [52]. During the fracture formation, existing lines are not impacted and the structure grows with the addition of new lines [221](fig. 2.13). The oldest fractures divide space into subspaces which are recursively subdivided by new fractures. The hierarchies in fracture patterns [51] are related to the sequence of cracks: the principal lines decomposed the surface and the following fracture has an impact on the existing fractures. This kind of structure can be detected not only in fracture patterns but also in other network-like patterns, as street networks and leaf networks [221] (fig. 2.10).

We can define their growth as follow:

Definition 8:
coherent growth

A growing graph $G(t) = (V(t), E(t))$ grows in a coherent way if at each time step, all vertices are connected. If the graph is composed by two or more components, each of them must respects the previous condition. Corollary: the number of components never increases over time.

Mixing topological and geometrical aspects

Processes behind the morphogenesis of spatial networks do not disregard topology aspects. They are mixed with geometric aspects. For instance, in a street network, we observe that the number of incident streets in an intersection rarely exceeds six. Spatial constraints restrict the appearance of large degrees. They also imply the tendency of a limited number of hubs and clusters. The topology of the corresponding street network is therefore impacted by geometrical aspects. Xie et al. [304] proposed a model that mixes topology and geometry. They show that different kinds of planar graphs modifying few geometric and topological parameters.

Networks and environment

Exogenous aspects can impact network formation. The evolution of human infrastructures is tightly related to economical, social, and technical aspects [116]. For instance, the spatial distribution of airports not only depends on endogenous aspects (the distance between them, the required number of scheduled flights, etc.) but also some other external aspects (the network of cities which overlaps the airline network, socio-economical requirements of the community, etc.). Similarly, the spatial formation of the street network is not only the results of some interaction rules of the elements which govern the formation of the network; some external aspects, such as the orography or the imposition of some policy or economic agents.

Many models integrate exogenous aspects. In an hybrid-model, authors in [16, 199] propose to couple a cellular automaton layer (section 5.2) to a growing spatial network in order to simulate the co-evolution between roads and build-up. The model was generalized and applied in a real scenario as a useful decision-maker tool by authors in [237]. Also authors in [275], [1] and [261] simulate the urban growth integrate integrating different aspects in a multilayer model.

Summary 6:
networks

If we consider a system composed of entities, the interactions define a network. Networks capture the essentiality of their structural properties, relationships between elements, and evolutive processes. Among them, there exists a class of networks (called spatial networks) where elements are embedded in Euclidean space, edges also indicate a geometrical relation and the structural properties depend on spatial aspects. Graph theory allows us to encode temporal and spatial aspects in a rigorous formalism and we had observed that both have a crucial role in their formation.

Chapter 3

Urban systems and morphogenesis of street networks

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Keywords cities and urban systems, urban forms, urban morphogenesis, street network representation, GIS.

"Se ti dico che la città a cui tende il mio viaggio è discontinua nello spazio e nel tempo, ora più rada ora più densa, tu non devi credere che si possa smettere di cercarla"¹.

Italo Calvino

In previous chapters, we introduced essential notions of complex system theory (chapter 1) and we observed that spatial information plays an important role into the functioning of some systems. The elements of those systems, immersed in an environment, are located in Euclidean space and interactions are impacted by metric aspects. The network of interactions/relations of those systems can be represented as a graph (chapter 2).

In this chapter, we will observe that cities present the aforementioned characteristics and we will focus on an important part of them: the street network. In section 3.1 we will show that under which circumstances a city can be considered under the complexity perspective. Then we will introduce the main principles behind street network morphogenesis (section 3.2) and our proposal for modelling street networks (section 3.3).

3.1 Urban systems

spatial complex system



urban system



modelling urban
systems

Geographical studies are a wide field of knowledge that focuses on natural and anthropological phenomena with a relevant account of space information. One of the challenges of geography is to understand how human society use and transform the earth to accommodate their requirements. The way that people interact with the earth (and people interact with one another) cannot be studied within only one classical category of studies. These studies require the integration of different approaches from different disciplines. Sociology, biology, the science of the earth, and technology, therefore, contribute to the definition of the field of geography. A *trait d'union* between them is the space and its effects on the relations between elements. From classical paradigms of observations, in which the earth was described in terms of nomenclatures, locations, and toponyms, geographical studies now consider the relations human/earth more complicated than initially supposed.

The importance of early geographical studies was that they contributed to solve different human problems. For example, maps of earth were useful to prepare wars, manage lands, control resources, plan colonial conquests or optimize land use. Quantitative geography aims to integrate the descriptive and regional approach of classic (and qualitative) geography, in order to explain and model the earth and the impacts of humans on it [132]. The increment of computing power and the introduction of some notions from other sciences (e.g. self-organization, emergence, and complexity) [57, 92] have contributed to this new way to study geography. An important effect is observable in urban studies [31], a branch of geographical studies.

¹ "If I tell you that the city toward which my journey tends is discontinuous in space and time, now scattered, now more condensed, you must not believe the search for it can stop."

The study of cities² embodies the quintessence of the meeting between people and earth³. Cities are the location where individuals interact and accommodate their requirements to live, producing socio-economical effects and altering the environment. The city is the space of aggregation, dialogue, integration, and exchange for the majority of people. Geographers, urbanists, architects, sociologists, anthropologists, focus on the effects of these phenomena of aggregation, interaction, and transformation. Buildings, streets, squares, public and private gardens, gathering places are among the physical elements of these studies.

The quantitative study of cities begins with the assumption that the functioning of cities and their main characteristics can be described using concepts from classical physics (force, mass and energy). Christaller, in his central places theory [78], describes the specializations of cities and their subparts with postulates from economic equilibriums. Gravitational models are used to study inter-urban migration [306] and socio-economic interactions between cities or their neighbourhoods [7, 151]. Hierarchies, specializations, equilibriums, and exchanges are described in their main traits via these approaches. However, these approaches often treat cities without an adequate attention to temporal aspect. These approaches often considered cities like predictable and explicable object, ignoring questions of uncertainty [111]. Rational and exhaustive approaches behind classical studies can drive to an oversimplification of the complexity of cities [300]. Complexity theory provides a comprehensive framework to study urban dynamics, it can build stronger connections between qualitative and quantitative urban disciplines [230]. For this reasons we will study street network morphogenesis from a complexity science perspective.

3.1.1 Cities as systems

The term complexity is polysemous and often viewed from the perspective of a particular discipline (chapter 1). A general agreement can be find into the principle that certain large systems are characterized by decentralized interactions of their many constituent parts. Within a given environment, parts of the system often are affect by exogenous aspects. They exchange with the environment energy, matters and information. These dynamics contribute to functioning of the system and its identity. The way that parts are in relations, the effect of exogenous factors to the system and feedbacks loops produce complex effects, and the emergence of unexpected properties. These general principles can be observed in cities and complex system theory has become a popular framework for investigating and describing cities.

Jacobs [152], described the self-organization of main North American cities, focusing on their socio-functional aspects. She suggested that cities are close to natural systems, where a plethora of functions and inputs play to define their organization (named the "*urban organized complexity*"). Despite this work is mainly descriptive and gives qualitative and stylized in-

²The word "city" is used throughout in the general acception of the urban area, including its suburbs.

³"*Cities are about connecting people*" (Batty [32]).

formation about urban dynamics, it represents an important step for the study of cities under the perspective of complex system. Forrester [104] postulated that flows of matter and people between sub-parts of cities can be considered as interactions inside an urban system. Systems of differential equations model the evolution of these exchanges and he suggest that economical actors regulate those flows. Decentralized approaches used in segregations models suggest that complex patterns emerge from individual choices [258]. In chapter 2 we have briefly observed that complex networks theory offer an interesting perspective for better understanding cities. The emblematic statement "*cities as systems within systems of cities*" of Berry [40] embodies that a city is at the same time a set of interacting elements and an element in interaction with other cities. These considerations are quantitatively confirmed by the Simpop models proposed by Sanders et al. [254]: here the interactions between cities at a macro-scale (regional or national) drive to the emergence of hierarchies and social, economical and functional specializations.

Urban systems can be observed from economic, cultural, anthropological or technical points of view. Cities cannot be wrapped up in only one taxonomic description. Dynamics of this kind of system cannot be reduced to what a single discipline has to offer. In this heterogeneous and fragmented context, complex system theory becomes useful, tying together different fields.

Cities are not a complicated system, where a large number of elements can be decomposed, investigated and then recomposed to explicate the organization [236]. Cities are complex, because unexpected behaviours and properties arise from a large number of interacting elements and relations between overlapping sub-parts. Different parts of urban systems are interconnected and, to understand them, we cannot investigate them independently. Cities are not decomposable in simpler and simpler sub-parts, which can be recomposed as a series of interconnected sets⁴ [4].

3.1.2 Levels of observation

Urban systems can be observed at different levels of organization [233]. In a simplified schematization, the degree of aggregation also corresponds to the scale of observation of the system.

- **Basic elements**

At a microscopic scale of observation, the basic elements correspond to highly disaggregated elements such as individuals. At this level, we can ask questions about human behaviour, e.g. evacuation in emergency situation, the impact of daily trip to the congestion of transportation axis...

- **City as a whole**

At an intermediate level of observation, elements of city can be considered aggregated. Socio-economical dynamics can be associated with regions of space. At this level, we can study aggregated quantities as population, pollution or economical factor distribution, for example.

⁴ "*A city is not a tree*" (Alexander [4]).

- **System of cities**

The maximal aggregated level corresponds to a system of urban systems [40]. In this context, we can study interactions and exchanges between cities.

To give an appropriate definition of levels of aggregation is more delicate because the separation between them is not always easy to determine. Especially an intermediate level (e.g. the definition of neighbourhoods in cities), it is not easy to capture limits because each part could be considered as an autonomous system. Despite the conceptual definition of three levels of aggregation is more or less clear, it is often difficult to apply in real situations.

3.1.3 Urban forms

In section 1.3 we introduced the notion of morphogenesis, distinguishing two words: form and shape. Urban studies also distinguish between these words; in fact, we have a distinction between urban fabric (shape) and urban form.

Urban fabric refers to the arrangements, geometry, and aesthetic features of elements of cities. Several spatial features of elements are useful: geometrical proportions, distance, visibility, textures of surfaces, colours, etc. Four fundamental morphological elements compose urban fabric: building, lots (also named plots or parcels), streets and public spaces (e.g. places and gardens) [85]⁵. These elements constantly transform through time. Buildings are renovated, streets are prolonged and connected and parcels split and merged. These elements have a shape and are in spatial relationships: buildings are defined and are shaped by open space around them, streets are used by people, and serve as transport media. Parcels defines the visibility of buildings and their edges shape the streets. Urban forms can therefore be investigated under a historical point of view because the physical properties of urban fabric conserve traces of transformations and replacements [66].

Urban morphology refers to the study of the formation and the transformation of physical elements of the city, from its formative years to its subsequent transformations. To this end, urban morphologists integrate abstract aspects of cities that are related to the formation of physical elements of cities. The general agreement of urban morphologists is that cities can be studied via the observation of the evolution in time of their physical elements.

Despite the evident contribution of morphological studies to describe cities, these approaches were often contested. These approaches cannot completely explain internal mechanisms of urban morphogenesis. "*L'objet forme urbaine ne peut pas être saisi que par une démarche transversale, pluridisciplinaire*"⁶ (Roncayolo [246]). The contribution of different disciplines is necessary to understand how cities evolve. "*La forme urbaine est un concept*

⁵We can combine those elements; for instance, a block is a set of adjacent parcels surrounded by streets.

⁶"The study of the urban form should favour cross-disciplinary approaches."

polymorphe et polysémique, témoignant de sa complexité"⁷ (Fusco [110]). The morphogenesis of urban systems cannot be studied only considering the historical evolution of its physical elements. The emancipation to the exclusivity of the historical vision is necessary to integrate the complexity of phenomena and the plurality of disciplines in the study of morphogenesis. The signification of history is important but it cannot be considered as a method or a discipline [220]. The morphogenesis of urban systems should be tackled by merging several points of view.

Due to the impossibility to define in a unique way urban forms, there exist different ways of formalization; in frame 2 we resume main approaches (called *registres de la forme urbaine*) proposed by Lévy [181].

3.1.4 Urban organizations

In many spatial systems, we observe that some properties or quantities (e.g. mass, density, intensity) do not scale linearly with geometrical measures (e.g. length, area). For instance, the relationship between the number of cities and their size is hierarchical: for a given area, there are many more villages than bigger cities [212]. A hierarchical distribution appears between some characteristics of cities and the population size: the relation between population and many urban phenomena (e.g. number of crimes or traffic) or economical aspects (average income per habitant) is rarely linear [44]. The building density of a city does not decrease proportionally/linearly from the city centre to the periphery, but the rate is scale-invariant with the population [179]. These characteristics are observed in statistical distributions of some properties. They indicate that urban systems organize themselves in hierarchical structures. We are interested in the different levels of importance of elements of the system and their capacity to affect the behaviour and the adaptability of the whole system. Sections 4.2.6 and 4.3 will detail these aspects with a focus on street networks.

Due to the absence of a dominant scale of observation, in many spatial systems, patterns repeat at different scales; that means we cannot associate these properties to a metric scale. This characteristic is observable in urban systems too. For instance, we observe a scale-invariance organization of building footprint [278, 281] and street networks [12, 272]. Cavailhes et al. [71] have observed that urban fabric and economic dynamics are related by scale-free laws. Section 4.4 will detail this aspect, focussing on street networks.

Patterns are ubiquitous in spatial systems. With regard to urban systems, Alexander [5] observes the existence of rules which articulating the inextricable relationships between elements, their functions, and their role in society. No individual part is complete of itself, so each part needs other parts: a street is incomplete without a place, a place only makes sense in the context of street networks.

Alexander [4] suggests two kinds of spatial organizations:

⁷ "The concept of urban form is polymorph and polysemic, attesting to its complexity."

The complexity of urban systems provides different conceptualizations of their forms, depending on the point of observation. According to Lévy [181], urban forms could be observed as:

- **layout form**[185, 268]:
The study of forms via their plasticity (size, volume, shape) and their perception (textures, colours, materials). This approach is useful to study the culture and the folklore of the society, the symbolism of urban elements, and the navigability of streets.
- **social form**[152, 246]:
The study of social, ethnological, and demographical aspects, activity, and public functions of cities. The study of public space as places, public buildings, and streets plays an important role.
- **bioclimatic form** [63]:
The study of the impact of natural aspect to the formation of the urban fabric. This point of view has been carried out in the last decades, on the environmental costs generated by urban transformation processes, focusing attention on soil consumption and the effects the urban sprawl on the quality of life.
- **planning form** [143, 174]:
The study of the urban form focusing on the degree of planning (geometrical or organic plan, orthogonal, or radio-centric plan) in relation with its centralities (mono-centric or poly-centric cities).
- **tissue form** [65, 70, 85]:
The study of the relations between physical elements of cities in historical and evolutive perspective. Pioneer morphological studies were carried out by the three independent schools: Italian (Muratori, Caniggia, Rossi, Aynomino), French (Castex, Depaule, Panerai) and English (Conzen). This classical point of view is based on three fundamental principles (the geometry, the scale of observation and the historical evolution). These studies show that a relation between socio-economical events and urban morphologies can be observed from a historical perspective.

Frame 2:

registers of urban form

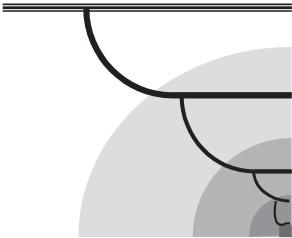
- The organization of socio-technical functions (e.g. the spatial distribution of humans and activities).
- The organization of urban fabric (e.g. the geometric relationship between streets and built-up).

The inequality of spatial distribution of social housing [239], the tendency of commercial activity to locate in places where flux of transportation is relevant [229], the social segregation [184] are few examples of the first type of spatial organizations. The urban sprawl [59] and the morphological patterns of geometrical relations between elements of urban fabric [10] are examples of the spatial organization of physical elements.

3.1.5 Streets networks

The substantive street⁸ came from the Latin *sternere* which means pave: paving a part of the space provides a distinction between the paved and the uncontaminated part and also gives a function of the paved one.

Streets are the result of the human interactions. They are fundamental for the formation of settlements and they are resilient to human changing. Streets, defining relationships between settlements and land, connecting area considered important by humans [5], impacting the lifestyle of people. The straightness of streets is correlated with actual displacements of people [142]. The accessibility of urban areas and human behaviours are in relationships with geometrical characteristics of the street network [118]. The street network is the main transportation infrastructure, its geometry impacts relation between people [185]. Indirectly, street configuration has an effect to the house price [302], the spatial arrangements of urban functions [215] and the quality of life [253].



A street network is composed of several streets with different levels of importance. For instance, a hierarchy of streets can be observed according to their speed limits: few major roads and a majority of capillary streets. This hierarchy has also a physical effect on urban forms: streets with high capacity are also wider and with more lanes than secondary ones. Even people's behaviour is impacted by this hierarchy: assuming that a user wants to reach his destination as quickly as possible, he prefers to move to major streets, and stay on faster streets allowed as much as possible. Street typologies are associated with the spatial scale of the area that they serve. The properties of streets impact their individual choice which in turn has an effect on properties of urban forms and on the organization of the urban system. In other words, the streets and the other elements of urban fabrics co-evolve.

If we consider the street network as streets connected by junctions, this in itself does not constitute a complex system or even a complex network. Junctions have more or less similar degree of connectivity (between 3 and 4), very much like random and regular networks. Conversely, street networks are complex if we consider the process that had build them, the functions that they host within the scope of functioning of the whole urban system and the dynamics that they support.

Summary 7: Urban systems

With regard to the scale of observations, an urban system is a collection of heterogeneous elements. Some of them constitute the urban fabric (buildings, parcels, streets and lots) and the result of interactions/relations of people with nature. To understand how these elements are arranged and how they evolve in the time we must inves-

⁸The English vocabulary makes a distinction between roads and streets: the first ones generally concern the most paved transportation way, which connect cities, and the second ones are typically in cities, contoured by buildings. Due to the impossibility to practically make an exact distinction in an evolutive context (sometimes a road may become a street), some inconsistencies in the database, and the incapacity to discriminate that in all situations, in the next we do not distinguish between these terms.

investigate internal processes and all kinds of relations between them: in other words, study the urban form. Streets are subject to the hierarchy principle: there are streets with more or less central importance. These hierarchies structure the whole urban system.

3.2 Morphogenesis of urban systems

Urban morphogenesis is a dynamical process in which the organization of urban elements and the formation of structures observed at a macroscopic scale emerge as a result of microscopic interactions and cross-level feedbacks (see section 1.3). The morphogenesis of urban systems concerns several aspects and it is a hard task to resume all of them; the following sections will provide a description of some specific morphogenetic processes that concern urban systems. Here we focus on street networks.

3.2.1 Morphogenesis of European cities

The spontaneous formations of early settlements coincides with the beginnings of human socialization [203]. The primordial needs to survive, exchange informations and find resources was the first form of organization. It occurred in the absence of an explicit design. In European cities⁹ [38], the process of urbanization tends initially to occur along the *percorso ma-*

⁹While in European cities the public space is a fundamental space of relations and the forms are the consequence of unconditioned mobility, in Islamic cities interactions are conditioned by social segregation constraints for family groups and the space is partitioned to increase the isolation of family-controlled sub-areas. Consequently, urban fabrics have different patterns: in the former public space has a fully permeable structure, while the latter have a highly hierarchical structure.

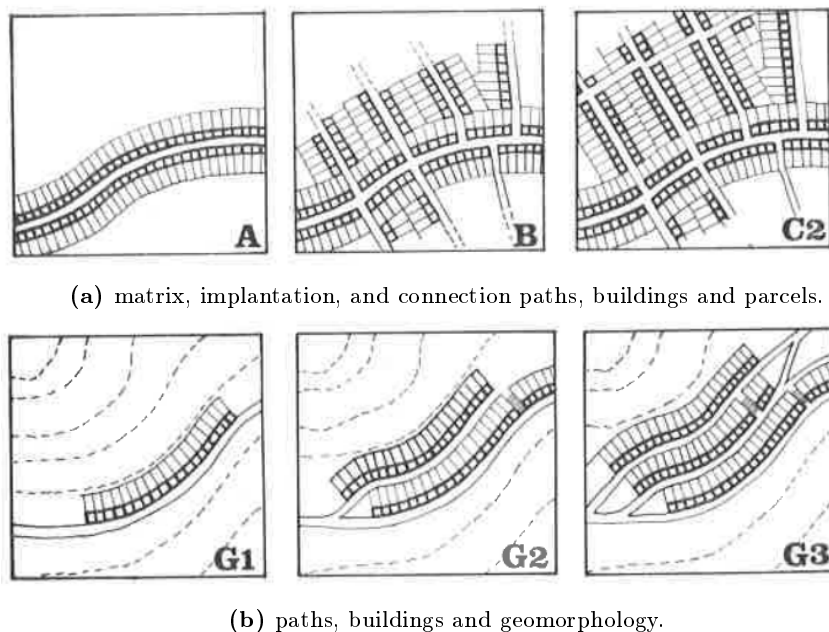


Figure 3.1
Morphogenesis of European cities. Source: [66].

trice (the matrix path), a linear path linking important places¹⁰. These paths also connect early settlements and sources of matter. The geometry is strongly impacted by the orography and seeks to connect two places in a way that reduce the length of the path. The urbanization process tends initially to occur along a quasi linear path linking places. Matrix paths are considered important by the community. Parcels which delimit the land around buildings (often an house), in many cases with rectangular-shaped footprint, with the shortest edge on the path, are disposed closely along the matrix path. The frontage of parcels becomes discontinuously broke, nucleating new paths, called *percorso d'impianto* (the implantation path), which are generally perpendicular to matrix paths. This led to the development of *percorso di collegamento* (the connection path) to connect implantation paths, thus closing off the block. The need for interactions between people is therefore accommodated through gradual adjustments of the existing tissue, responding not so much to issues of global coherence, but only current and individual demands (fig. 3.1a).

The geomorphology of the site plays an important role during the urban morphogenesis, by imposing a specific pattern of connections between places. The structuring role of environmental features is evident for geographical areas which do not allows the human implantation. In these cases the environmental characteristics have an organizational influence, they act as a physical barrier. Rivers and mountains constraint the development of the settlement. In the case of settlements developing along hill crests, a minimal resistance principle imposes the minimization of the energy cost of mobility (fig. 3.1b).

3.2.2 Planned versus spontaneous cities

Planned cities are supposed that they were entirely built in a short span of time: the main infrastructures of a city (or a suburb) are placed in the same time, respecting a global vision of a planner. Planned street patterns seem "optimal" regarding its topology and its geometry. For instance, the square grid is optimal in the sense that it reduces the mean distance between two points of the city and favorites a regular distribution of buildings [262]. Another example is the radial layout, used when the goal of the people is not to go uniformly from a point to another but to reach a central point as quickly as they can. Most of time it is not the case. Often cities adapt their form and evolve following external actions or new internal requirements. Local structures overlap existing structures and physical elements extend spatially as much as the city growths in terms of population and economy. Agents add sequentially streets to the current city's layout. Each addition is locally optimal at a given time and for the agent that makes it. If a city is spontaneous, it is the result of local non-concerted interactions that trying to optimize and adapt in a short period of time the accessibility of a suburb.

Forms in spontaneous cities are unanticipated. Urban forms are heterogeneous but ordered, self-similar, and often grow spontaneously. Planned

¹⁰Below, we use the terminology used by Caniggia in [66].

cities are often an exception and they remain such (planned) for short periods of time. Hence, for the sake of universality, it is useful to consider settlements that underwent non-designed development.

3.2.3 Effects of self-organization in street network layout

Self-organized cities are based on the assumption that people are not motivated by some global principles of spatial or community order when making their individual choices. Their main purpose is to accommodate their actual needs with the minimization of energy employed. In many cases there is no reason why humans should take more effort than needed to reach an aim [307].

An interesting effect of the self-regulated organization of people and the spontaneous formation of urban fabric concerns the typology of street intersections (fig. 3.2) [24]. The morphogenesis provides the formation of a new street (implantation and connection paths) from an existing street (matrix or implantation path): at a given time, a new street builds and the intersection typically has a T form. These new streets are not defined by global and centralized actions, but they are spontaneous and optimal locally. Moreover, a consistent part of the paths was build to connect an interesting area to the rest of the settlement: it results that a large number of these paths ends in a cul-de-sac. The resulting network of streets is composed of a large number of T intersections and dead ends. In the primal representation of the street network (see section 3.3.5), the graph is composed of a large number of vertices with degree 3 and 1. The *ville haute* of Le Havre is a good example (fig. 3.2, right).

In several planned cities, rather than developing via sedimentation of urban elements over a long period of time, streets are laid out on a more or less regular orthogonal grid¹¹. Typically, for these urban forms, a large master plan was defined: often we observe the main axis, parcels are reshaped and a part of the built-up is destroyed or transformed. Although this kind of urbanization is often a way to colonize new continents (the majority of North American cities, but also in India and South America), it can also

¹¹A regular grid is not the only "planned" urban configuration: for instance, in the growth of the suburb of North American cities, with the aim to reduce the accessibility, tree-like and sinuous streets have been planned.

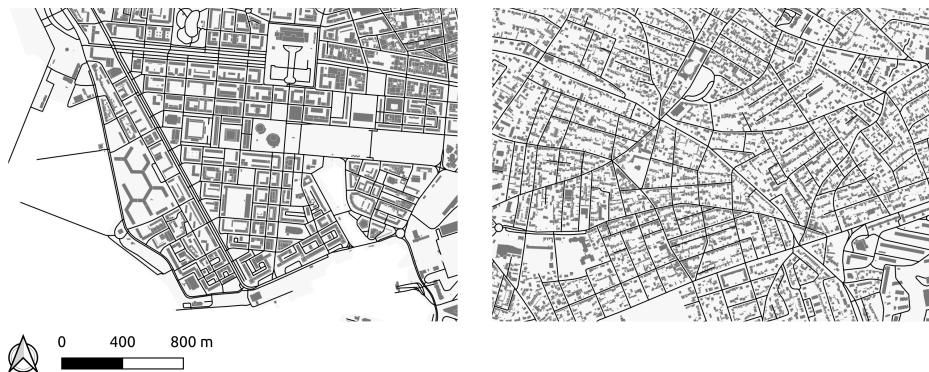


Figure 3.2
Urban forms in Le Havre. The spontaneous formation of the *ville haute* (right) versus the planning *ville basse* (left).

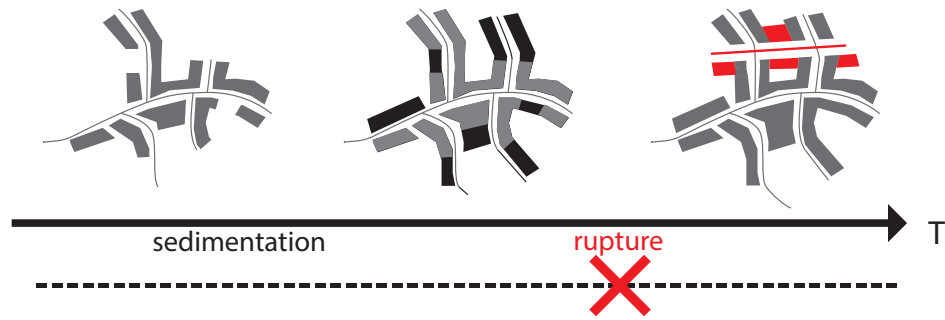


Figure 3.3
Sedimentation and rupture
during the urban morphogenesis.

be observed in European cities (Barcelona, Turin, Nice, etc.). Practical reasons motivate this kind of urbanization (accessibility, navigability, good orientation to the wind, etc.). Examples of planned configurations can be founded in different periods: for instance, Siracusa in Ancient Greece, Cuneo in the Roman Period, but also 18th centuries planning of Barcelona). The result is that for planned areas we can observe that streets orthogonally intersect, and the corresponding primal representation of the street network contains many vertices with degree 4. We can observe that into the *ville basse* of Le Havre (fig. 3.2, left).

3.2.4 Sedimentation and ruptures

Often the physicality and the functions of urban fabric elements (streets, build-up, gardens, squares, etc.) do not vary for long periods of time. Thanks to the persistence of these elements, we observe a sedimentary process behind the formation of the tissue: existing physical elements conserve previous size and evolve thanks to a process of juxtaposition. This idea is present in the *processo tipologico* (typological process, the transformation of basic type) of Caniggia [66], the *burgage cycle* (consolidation of the block) of Cozen [85] and the distinction between *elementi primari* (primary elements, structures which never change) and *elementi secondari* (secondary elements, residential area) of Rossi [248]. A similar process was described also in leaf venation [49, 221] formation and in the sequence of cracks in ceramic surfaces [52].

Although the evolution of urban forms is continuous most of time, brusque ruptures also appear during the formation process (fig. 3.3). Ruptures concern the sedimentation of physical elements, socio-technical function changes and new human usage. At significant historical moments, cities changed in strong relationship with social, cultural or natural changes. These events caused a discontinuity of the urban tissue, defining distinct urban forms. The reconstruction of Le Havre city centre after the Second World War, the Haussmann's renovation of Paris in the 19th century, the re-organization of London after the Great Fire in 1666 are few examples of ruptures in the process of sedimentation.

Summary 8: Urban morphogenesis

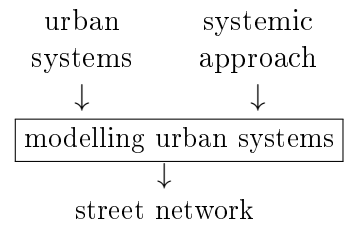
The city favours the proximity and its formation is constrained by a set of local forces resulting from social, economic, political, but also topographical characteristics. This context generates and surrounds

local interactions and it is a source of self-organization. We cannot sum it up with an overall design intent. External factors may be an organizational factor, defining and amplifying individual choices, but they are not the reason why organizations emerge. System features cannot be traced back to the choices or aims of a single parameter. In general, cities evolve continuously via a sedimentary process or via an imposition of a layout. Discontinuity moments appear during the spatial formation of the urban fabric. The topology and the geometry of streets are strongly impacted by these dynamics. Different forms appear as an overlapping of different processes.

3.3 Modelling urban systems

The first step of modelling any urban system consists of identifying the basic elements that give an appropriate representation of the phenomenon that interests us [255]. The spatial dimension plays a fundamental role (section 1.2) [235]: we should tackle the difficulty to locate physical elements and take into account the effects of space in the interaction between elements. To do that, we need a flexible framework that allows us to represent physical elements in Euclidean space, together with their geometry and all other (unspatialized) associated data [37, 57]. Finally, we have to consider the possibility that these objects can change over time.

Geographical Information System (GIS) can be useful for our purposes. It is an information system specializing in the input, storage, manipulation, analysis, and reporting of spatially related information [53]. The fundamental capability of GIS is the ability to relate spatial informations to unspatialized informations (fig. 3.4). Different kinds of non-spatial data can be represented in different forms. GIS provides a way to collect, select, visualize, analyse, and share geographical data. Each element, identified by a unique identifier (the *ID*), and graphically represented in a map, has an associated set of features, stored in a database. For instance, the physical location and geometrical characteristics of a hospital are drawn in a 2-dimensional map and the entire element is related to the actual use of the building: the number of patients, the number of surgeries per month, the postcode address, etc. Among the wide range of possible applications, GIS can be used to study cities.



3.3.1 Vector and raster representation

GIS integrates two representations (fig. 3.5) of elements in a map:

- **The grid-based representation (raster)**

A 2-dimensional array decomposes the space into a set of cells. Although many shapes are possible (e.g. hexagons, triangles), the square is the most commonly used. Cells are typically similar in size, localized by a tuple of coordinates $X = (x, y)$ and identified by an *ID*. This last maps the cell to a record in a database. A raster may contain

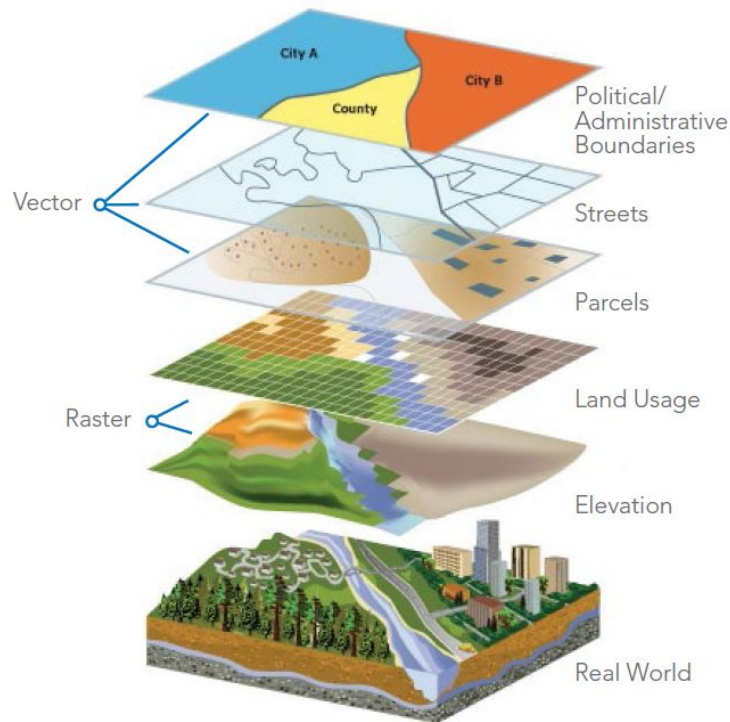


Figure 3.4
Geographical Information System (GIS). Source: www.esri.com.

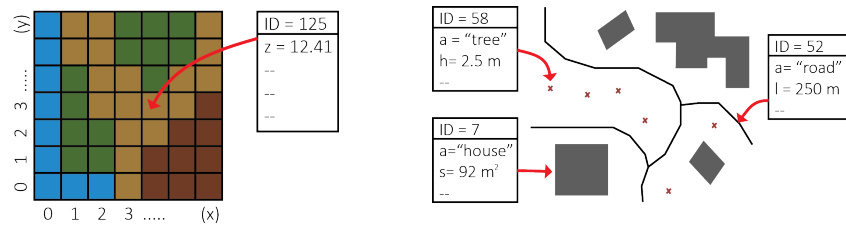


Figure 3.5
Raster (left) and vector (right) representation.

different kinds of information: the land usage of a rural area (descriptive information), the average household income of a neighborhood (quantitative information), the average elevation of the cell (the z coordinate of the cell), etc.

- **The line-based representation (vector)**

Here space is considered as much as possible continuous, that gives (in theory) an infinite set of coordinates. A vector representation is composed of three geometrical elements: the point, the straight line, and the polyline. An ID is assigned to each element. Differences between these elements are about geometrical features that characterize them. In a 2-dimensional space, a couple of coordinates $X = (x, y)$ for a point, two couples for the extremities of a straight line, a sequence of segments for the polyline (a closed polyline is a polygon). For instance, we can indicate the position of a tree in a forest with a point, a straight portion of a street with a line, a river with a polyline, and the footprint of a school building with a polygon.

3.3.2 Levels of aggregation and representation

Urban systems can be represented at different levels of observation (section 3.1.1); to each level corresponds a different method of representation. Elements that compose the urban system can have zero (points), one (lines), two (surfaces) or three (solids) dimensions. Moreover, the dimension of the element depends on the scale of observation and the same physical object can be represented in different ways. For example, at high level of aggregation a system of cities could be represented as set of points (eventually connected by segments). At an intermediate level, a city can be represented as a set of polygons (the footprints of buildings, squares, etc.), points (trees, light poles, etc.) and polylines (streets, rivers, etc.). At the lower level, points can represent individuals. As we mentioned before, the choice depends on the studied phenomenon. For instance, represent individuals may be useful to investigate social phenomena and the effect of urban fabric to the way that they interact while represents relations between cities modelled as points may be useful to investigate economical relationships between them. Moreover, technological limits can eventually have an effect to our choices: for instance, in a raster representation, the cell size determines how coarse or fine the information in the raster will be (fig. 3.6). In several cases, the vectorial representation is more adapted (the next section concerns the street network case).

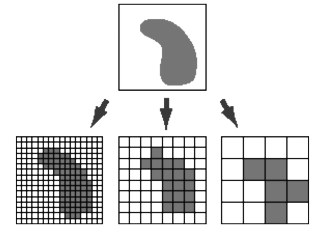
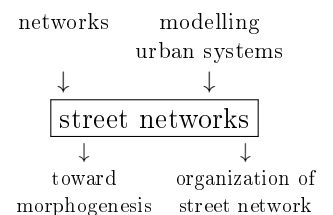


Figure 3.6
The granularity of informations in a raster representation

3.3.3 Modelling street networks

Starting within the fields of transport and infrastructure engineering [115, 134], street networks have commonly been treated as sets of more or less homogeneous physical elements with a prevalent direction. These elements connect locations and intersect at junctions. Streets could be integrated into a GIS framework in a raster or a vectorial representation. Representing streets via raster implicates some problems because the size of cells determines the quality of representation. In fig. 3.7, we capture an aerial photo of the Saint François suburb (Le Havre) and we overlap a regular grid. The image shows that the quality of the raster representation depends on the size of cells, vector representation is scale-independent.

A raster representation is typically obtained by a satellite orthophoto; the results could be impacted by overlapping elements: for instance, via raster representation, we cannot identify underground streets and streets under projected vegetation. Otherwise, vector representation is generally well adapted to represent street networks. Moreover, the latter representation implies an evident correspondence with graph theory. In order to apply methods and analysis from graph theory, the street network must be represented as a graph. There are two ways to do that: the primal and the dual representation [112].



3.3.4 The dual representation

The dual approach represents the streets as an undirected graph, where vertices are a portion of streets and edges are intersections between them [227] (frame 3 and fig. 3.8).

Dual representation may seem a bit odd but provides several advantages in analysing the topology of street networks [249]. Street networks in dual representation show properties similar to many other unspecialized complex networks [227]. Actually, often street networks, with significant size [67] and represented with the dual approach, present scale-free vertex degree distribution and high values of the average clustering coefficient¹². Moreover, small-world proprieties have been observed [154] and, characteristics of these networks had been analyzed with centrality indicators (see section 4.3) [88].

Despite the contribution of these approaches to investigate structural properties of urban fabrics, several inconsistencies were been found [241]. For us, the most important inconvenience is the absence of spatial aspects. The dual representation of a street network is a graph without relations with Euclidean space: vertices are not located and edges do not indicate geometrical information. Moreover, the terminology may be misleading: the dual representation should not be confounded with the dual of a graph¹³.

¹²Also known as the global clustering coefficient, it represents the tendency of two connected nodes to be both connected to a third node (in other words, nodes cluster together) [83].

¹³A plane graph divides the 2-dimensional space into regions called faces. In the dual graph, faces become vertices and adjacent faces become edges. (pas sure)

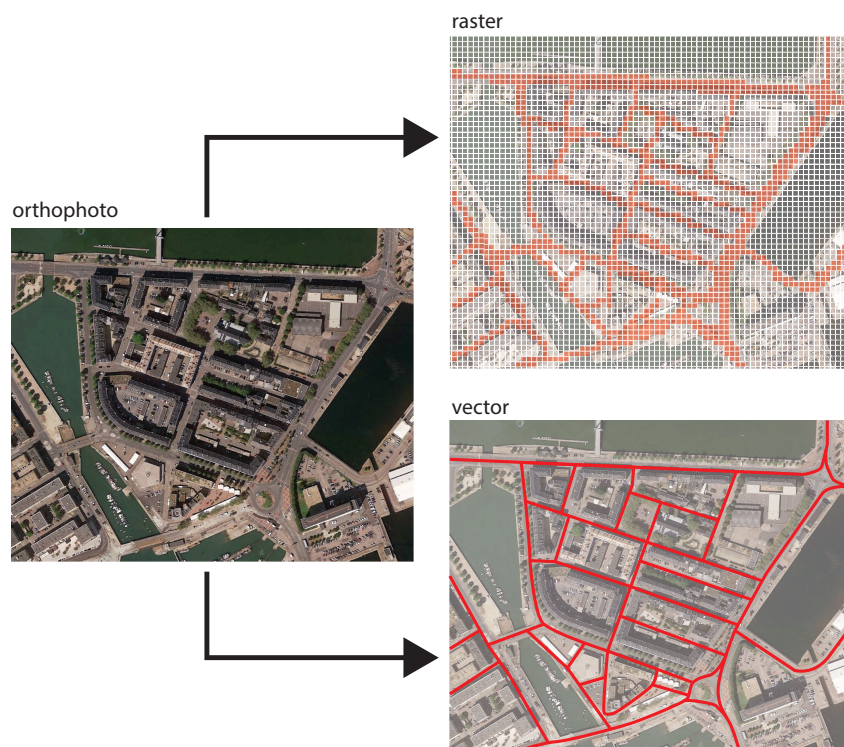


Figure 3.7
Representation of Saint François neighbourhood, Le Havre. The orthophoto (left), the raster (top right) and the vector (bottom right) representation.

Many approaches to model street networks in dual representation have been proposed, each of them offers insights into diverse aspects of the urban system (fig. 3.8).

- **The cognitive approach** [90, 141, 142]

Observing the city from a pedestrian point of view, Space Syntax theory proposes to decompose the accessible space of human settlements as a series of straight lines. These last, representing the longest visibility lines of walkers, define the Space Syntax axial map. Each line is a vertex in the corresponding graph and if two lines intersect, there is an edge between the corresponding vertices.

- **The odonym approach** [154, 157]

Streets are grouped by their name. Vertices correspond to streets with the same name.

- **The geometric approach** [84, 169, 227]

Streets are grouped by geometrical reasons. Two continuous segments of streets are two distinct vertices that are connected by an edge if the segments form an angle bigger than a fixed threshold.

Frame 3:

the dual representation

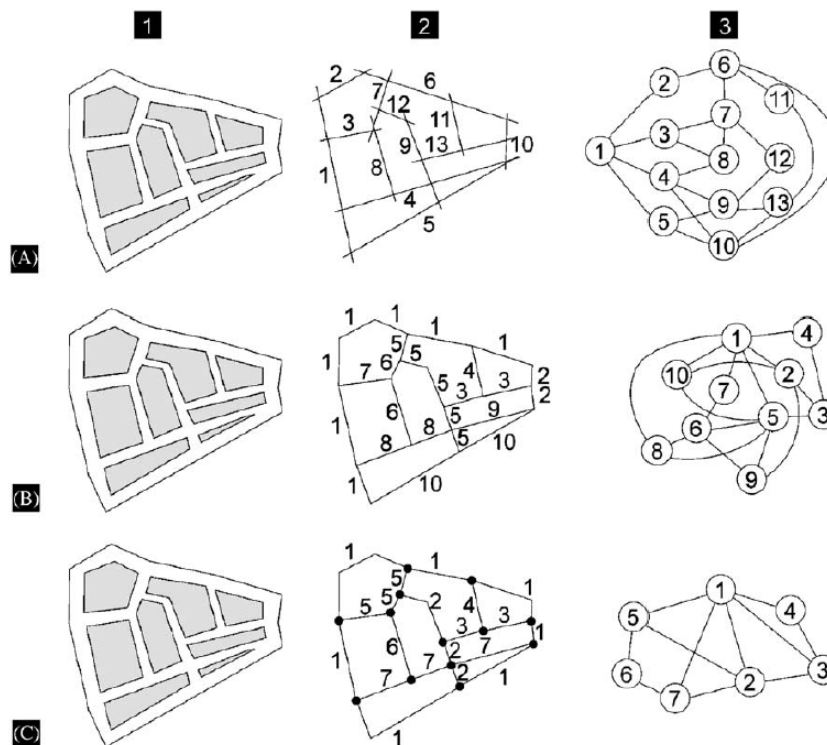


Figure 3.8

The dual representation. Row 1 is a fictive settlement, row 2 the identification of vertices, and row 3 the dual graph. Line (A) is the cognitive approach, line (B) the odonym approach, and line (C) the geometric approach. Source: [227].

3.3.5 The primal representation

The primal representation of a street network is a spatial graph $G(x)$ where edges are segments of streets and relevant points of streets are vertices. These last are spatially embedded in two or three-dimensional Euclidean



Figure 3.9

The building footprint (grey regions) and the street network (black lines and dots) of Ragusa city-centre, Italy.

space and edges are eventually labelled by some features. These last can be related to geometrical aspects¹⁴ (the length of the street, the width of the street, etc.) or not (characteristics of the pavement, the number of walkers per hour, etc.).

Topological and metrical aspects of graphs are fundamental to capture essential characteristics of street networks. While some graphs show similar topological features, they actually appear different: the primal representation of the street network of Ragusa town (Italy) is an explicative example (fig. 3.9). Comparing the West and the Est sub-area, we observe similar topological features, because both sub-graphs have a large number of vertices with four incident edges. However, comparing streets in the Est-West direction, vertices in the West side are closer than vertices in the Est side: this simple example shows the crucial role of metric features to characterize street networks.

3.3.6 Some problems to construct the primal graph

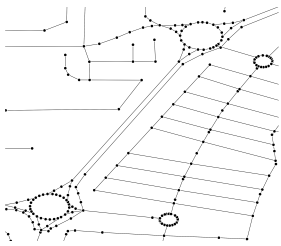


Figure 3.10

The primal representation of traffic circles in Le Havre

Streets are often stored as polylines in a GIS; in order to assure an adequate geometrical approximation, polylines are composed of a large number of segments (fig. 3.10). A street can be seen as a chain of segments, and an edge corresponds to each segment in the spatial graph. Hence, the corresponding node set V of the spatial graph $G(x)$ contains a large number of vertices with degree 2. These last are artefacts, an element that only serves to approximate the geometry of the street. Statistical information about the spatial distribution of elements of the street network is strongly impacted because this representation is not able to bring out main elements.

Several methods were proposed to reduce these effects; the goal is to construct a graph that contains the most important information as the topology and the geometry. A way to tackle these practical problems is to simplify

¹⁴Geometrical information that cannot be inferred from the graph.

the spatial graph, merging close vertices: when the distance between two vertices is less than a threshold, they are merged and incident edges of both become incident to a new vertex. This approach is based on a simple method. Some inconsistencies do not assure the adequate representation of the streets. The arbitrary threshold is not adapted to a sparse edge length distribution, typically observed in the streets network (see chapter 4); depending on the threshold, two close but not intersecting streets could merge; a loss of information results because the length of edges becomes less representative of the actual length of streets; finally, there is no guarantee that the vertex degree distribution of the simplified graph is not peaked in 2.

A second approach is given by the Ramer–Douglas–Peucker algorithm [238]: in its general form, it downsamples a polyline to a similar polyline with fewer points (fig. 3.11). The procedure divides the polyline and removes points far from the approximate polyline. A threshold (a fit criterium) defines the maximum distance of the starting polyline to the approximating polyline. Like the previous method, this one also requires an arbitrary parameter and does not assure the preservation of geometrical features. The vertex degree distribution could be perturbed by the presence of many vertices with degree 2.

To resume, we have to define a rigorous method to identify the main elements of street networks, disregarding elements only used to approximate streets and guarantee its geometrical and topological features.

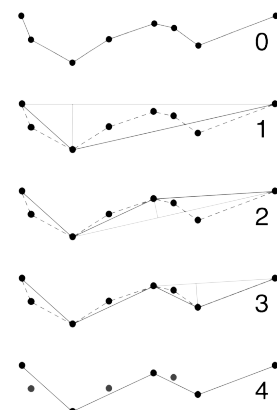


Figure 3.11
The Ramer–Douglas–Peucker procedure.

3.3.7 The street intersection network

In order to overcome these problems, our approach focuses on the most important elements of a street network: intersections¹⁵ and dead ends¹⁶. Let $G = (V, E)$ be a generic graph; a positive weight $l(e)$ is associated to each edge e . Let $V(2)$ be the subset of vertices of degree 2, $V(1)$ the set of dead ends, and $V(+)$ the set of intersections. The algorithm 1 removes all vertices in $V(2)$ and possibly transforms the initial graph in a multigraph fig. 3.12. The algorithm replaces each vertex v of degree 2 and its incident edges e_1 and e_2 by a new edge e_3 , connecting the two neighbours of v . The weight of e_3 is the sum of weights of e_1 and e_2 . The procedure stops when $V(2) = \emptyset$. The algorithm can be applied to graphs $G(x)$ embedded in Euclidean space: in this case, the weight of edges can be computed as the Euclidean distance between the corresponding pair of vertices. The corresponding graph remains spatially embedded and the sum of attributes $l(e)$ does not change.

When we apply the algorithm 1 to the primal representation of a street network, we obtain a street intersection network. It can be defined as:

The street intersection network, noted as $SIN(G)$, is a geometric graph where vertices, located in a plane, are the dead ends or the intersections

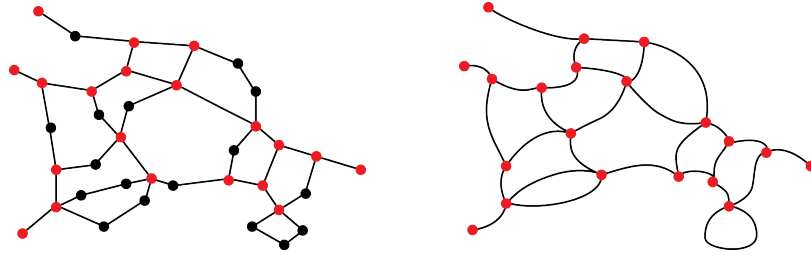
Definition 9:
street intersection network

¹⁵ An intersection is an at-grade junction where three or more segments of streets meet.

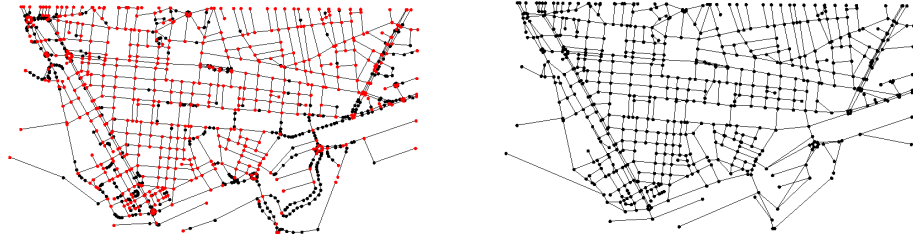
¹⁶ Also called *cul-de-sac*, is the part of the street that is closed at one end.

Figure 3.12

A graph before (left) and after (right) the application of the algorithm 1. Black dots are vertices with degree 2, red dots are all other vertices and segments indicate edges.

**Figure 3.13**

The street intersection network of Le Havre city centre. Algorithm 1 transforms the primal representation of the street network (left) in a graph without vertices with degree 2 (right).



Algorithm 1: Building the street intersection network

```

input :  $G = (V, E)$  graph, edges have a generic attribute  $l$ 
output: the graph  $G$  without vertices with degree 2
begin
  while  $V(2) \neq \emptyset$  do
     $v \leftarrow$  vertex in  $V(2)$ 
     $e_1 = (v, v_1), e_2 = (v, v_2) \leftarrow$  incident edges of  $v$ 
     $e_3 \leftarrow (v_1, v_2)$  /* add a new edge */
     $l(e_3) \leftarrow l(e_1) + l(e_2)$  /* set attribute to the new edge */
     $E \leftarrow E + e_3 - \{e_1, e_2\}$ 
     $V \leftarrow V - v$ 
  end while

```

of streets and the edges are weighted by the length of the streets.

The algorithm removes all vertices need to approximate the geometry of streets (fig. 3.13). The graph conserves the geometric properties of the initial network. The shape of edges is lost, but we keep their lengths.

Summary 9:
modelling urban
systems

The raster and the vector representation are both useful to model elements that compose urban systems. However, to model streets, the vector representation shows many advantages: the most important one is that allows us to represent them as a graph. The street intersection network is a primal representation that conserves information about both geometry and the topology of the street network.

Chapter 4

Outlining organization of spatial networks

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Keywords spatial network measures, betweenness centrality, fractal analysis, robustness.

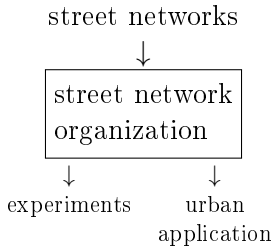
"I really don't trust statistics much. A man with his head in a hot oven and his feet in a freezer has statistically an average body temperature".

Charles Bukowski

Previous chapters exposed basic notions of complexity theory (chapter 1) and the capacity of a systemic approach to model and simulate morphogenesis of spatial complex networks (chapter 2). The street networks are an important component of urban systems and we propose to study them as spatial graphs (chapter 3).

The chapter starts with an introduction of network analysis and the description of our dataset (section 4.1). We present here some measures, specially adapted to the study of spatial complex networks and, more specifically, to street networks. Selected measures came from graph theory (sections 4.2 and 4.3) and fractal theory (section 4.4). We apply them on a dataset composed of the graph representation of street networks of French department cities, the Le Havre metropolitan area (in the following called CODAH), and six theoretic planar graphs (previously described in section 2.3.3).

4.1 Studying complex spatial networks



Graph measures are essential as a rigorous resource in many complex system studies, including characterization, representation, classification, and comparison. A large literature had been able to develop useful measures (see the survey [83] and references therein), the average vertex degree, the diameter, the clustering coefficient, average shortest path length for example. These measures are used to characterize evolving processes [211], the robustness [82], the resilience to external events [3], find communities [105] and so on. However, they cannot completely reveal properties of spatial graphs. For example, cluster coefficient and assortativity present limits when the vertex degree distribution is in a short range (this is the case of street networks, where degree is often between 1 and 6). These measure are mainly focussed on topological aspects of graphs, without taking into account geometry and location of elements. In other words, with those measures the spatialization of the network is ignored [23].

Geographers were ones of the earlier researchers that seek to answer the question of how to choose the most appropriate measures with special consideration of space. From early applications in the 60s on spatial networks, geographers proposed a number of indicators from graph theory to characterize urban streets and highway networks [115, 134, 159]. However, these earlier approaches do not consider the wealth of information contained in spatial networks: the complex network concept was not yet integrated and technological limits did not allow researchers to investigate heterogeneous spatial networks as in present days [48].

To investigate street networks, we should consider spatial information, choosing measures that integrate the notion of space. The first step is to build a dataset that contains this kind of information.

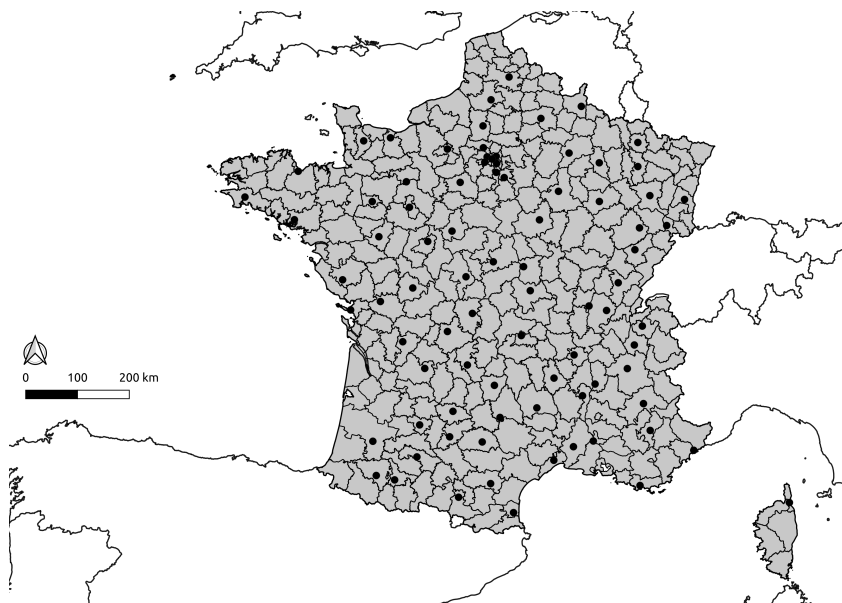


Figure 4.1
France departments cities.

4.1.1 Geographical informations

A geographical data is a piece of information about a spatialized object. It is collected into a dataset. To build our geographical dataset, we define for each studied area a connected 2-dimensional space that aims to represent the urban area. To do that, we use French administrative boundaries, selecting the corresponding municipality administrative level. We are aware of the imperfect representation of urban areas with administrative boundaries. In fact, due to sub-urbanization, long-distance commuting, sprawl, the limits of cities are often fuzzy¹. Moreover, these problems are related to the level of observation/representation that we choose to investigate the phenomena. In last decades, researchers tackled the challenge to give a rigorous procedure to define urban boundaries, measuring demographics [11], analyzing urban growth [187], using fractal measures [278], clustering measures on street networks [192] and land use [179]. We are also aware of possible effects of boundary definition in network measures [120, 244].

We prefer in this early evaluation of our models to give an unique and empirical definition of urban area via administrative limits, in order to give a global comparison of our results to reality. We do not exclude in future works to go more deeply to improve our methodology in boundary limits definition.

4.1.2 French department cities

Since 2016, France is composed of 18 regions and 101 departments (96 in Europe and 5 in the oversea area); we focus on the European cities (fig. 4.1).

¹In most cases, it is difficult to distinguish a city from a network of cities, like in large conurbations or densely connected regions (e.g. the Ruhr area in Germany or the Emilia axis in Italy).



Figure 4.2
Le Havre metropolitan area.

The GIS database is open free²; we use the python library *OSMnx*³ [47] to collect them. Geographical information are in a *ESRI shapefile* format (*.shp*): the corresponding adjacency list was obtained with the python package *Networkx*⁴ and then converted in a *.dgs* format. Finally, the corresponding street intersection network was created and analysed implementing the *Java* library *GraphStream*⁵ [226].

4.1.3 The Le Havre metropolitan area (CODAH)

The CODAH⁶, centered on the city of Le Havre, is an administrative area located in the estuary of the Seine river. It includes 17 cities and around 275 000 people. It extends in an area of 20 000 hectares. The length of its street network is around 220 Kilometres (fig. 4.2). As regards urban forms, the CODAH shows a large variety of case study: productive activities (industries and commercial port), high urbanized areas and a constellation of secondary settlements. We can find urban spontaneous (*la ville haute*) and well planned (*la ville basse*) urban areas. The corresponding street network was obtained from the platform *OpenStreetMap*.

²<https://openstreetmap.org>

³<https://geoffboeing.com/2016/11/osmnx-python-street-networks>

⁴<https://networkx.github.io>

⁵<https://graphstream-project.org>

⁶<http://www.lehavreseinemetropole.fr>

4.1.4 Theoretic planar graph

Six geometric planar graphs were used to compare the urban dataset (French cities and CODAH) to theoretic graphs. To this end, we applied the six procedures exposed in section 2.3.3 to a set of vertices in euclidean space $\mathbb{R}^2$ of 10×10 units. The grid graph is composed of a set $|V| = 941$ vertices. The 1000 vertices of the other graphs are scattered over a plane in the same random manner. The probabilistic graph is obtained with a probability of $p = 0.5$ and the deterministic graph with a geometric threshold of $\delta = 0.5$ units.

Our dataset is composed by 103 geometric planar graphs: 96 French department street networks, the CODAH street network and six theoretic planar graphs. We remove vertices with degree 2 with the algorithm 1 (section 3.3.7) before to analyse them.

Summary 10:
the dataset

4.2 Global properties of spatial networks

In this section we get interested in topological and metrical measures able to characterize street networks and discriminate them from classical planar graphs (see frame 1 for the notation). In what follows, we consider as the mathematical representation of a generic street intersection network a geometric planar graph $G(x)$. We assign the attributes $l(e) \in R_+$ to the edges of each graph; they indicate the length of streets.

4.2.1 Degree

The probability of finding vertices with degree $k \in \mathbb{N}$ is $P(k) = V(k)/|V|$ (where $V(k)$ is the number of vertices with degree k and $|V|$ is the total number of vertices). The distribution degree $P(k)$ is usually an interesting measure for many complex networks, such as it is observed in scale-free networks [20]. Even in spatial networks, an ample fluctuation of $P(k)$ can be observed (for instance, in airline networks [21]); however, if spatial constraints are relevant (e.g. an insect galleries network), and even more if vertices are embedded in a plane (e.g. a leaf network), the $P(k)$ is not in a broad range [221] and the distribution is peaked around 3 [60]. This is the case for the street networks in its primal representation (section 3.3.5), where the degree distribution of nodes have a fast decay and rarely exceeds 6 [60].

The first basic topological measure is the average vertex degree, defined as:

$$\langle k \rangle = \frac{2 |E|}{|V|} \quad (4.1)$$

The eq. (4.1) measures the mean of the number of streets that emanate from each end-nodes or intersection. We expect to find $2 < \langle k \rangle < 4$, where the lower value characterizes a tree ⁷ and the upper value a square grid.

⁷A tree of n vertices has $n - 1$ edges.

4.2.2 Connectivity

Connectivity indicators are used to characterize transportation networks [115, 134, 303]; the gamma index $\gamma \in [0, 1]$ is a basic measure of connectivity for a graph. It is expressed by the ratio between the number of edges $|E|$ and the maximum number of possible edges in the graph $|E|_{max}$. For a planar graph, it is defined as:

$$\gamma = \frac{|E|}{3(|V| - 2)} \quad (4.2)$$

The values of the gamma index are in $[0, 1]$ and a higher values indicates graphs close to the complete graph.

4.2.3 Planning versus self-organized cities

The degree distribution $P(k)$ of street intersection network often shows an abundance of vertices with $k = 1$ and $k = 3$ in spontaneous⁸ areas than well planned area, where $P(k)$ is peaked around $k = 4$ (see section 3.2.3). For these reasons in a recent paper [84], Courtat et al. proposed the topological measure organic ratio *org* as:

$$org = \frac{|V(1)| + |V(3)|}{|V|} \quad (4.3)$$

The organic ratio $org \in [0, 1]$ discriminates planned street networks ($org \simeq 0$) from self-organized ones ($org \simeq 1$).

4.2.4 Tree versus complete graph

The meshedness coefficient $mes \in [0, 1]$ [60, 67], also named α index, corresponds to the number of cycles present in the graph compared to the maximum number of possible cycles.

$$mes = \frac{|E| - |V| + 1}{2|V| - 5} \quad (4.4)$$

The meshedness coefficient is 0 if the graph is a tree and 1 if the graph is a complete graph.

4.2.5 Size and cost

The sum of length of streets:

$$l(G) = \sum_{e \in E} l(e) \quad (4.5)$$

is not useful. We can have a look about the sum of lengths of edges normalized by the the total number of edges:

$$\langle l(G) \rangle = \frac{l(G)}{|E|} \quad (4.6)$$

⁸Courtat et al. [84] used the term "organic".

Average street length, in urban form analysis, is an useful tool to measure size and density of block [262]. Comparing $\langle l(G) \rangle$ to the the length of the corresponding minimum spanning tree:

$$cost = \frac{l(G)}{l(MST(G))} \quad (4.7)$$

we obtain the cost⁹, a measure of accessibility and a basic information about the robustness¹⁰ of the street network. From the accessibility point of view, a tree is not an efficient network.

4.2.6 Robustness

The study of the robustness of complex systems is an important field. It reveals the adaptability of the system against failures or attacks (section 1.1.3); many complex networks display an high degree of robustness [3, 147]. Robustness was observed in animal organizations [61, 222], streets [60] and infrastructures [54]. A street network has a high level of resistance to failures: in most cases, in the corresponding graph representation, there are several paths between two distinct vertices, hence it is difficult to lose the connectivity between them. In this field, we can find studies that aim to correlate robustness and other indicators like the closeness [193] and the efficiency [60]. The analysis of robustness is important in many practical problems: a high degree of robustness ensures the navigability of the street network even in case of failures. In the urban context, failures are the interruption of a street due to natural cataclysms, planning, or maintenance of infrastructures, and public events. Hence the study of adequate measures of robustness is important for the study of urban systems, urban planners and public administrators.

In a graph, the robustness can be measured by studying how it becomes fragmented as an increasing number of vertices is removed. A measure of fragmentation is the size of the vertex set of the giant component of the graph. The vertex removal can take place randomly. The robustness $rob \in [0, 1]$ is conventionally measured [60, 61, 193] by the values of fraction of vertices required for the giant graph to reach the value $0.5|V|$. The value of rob is usually an averaged value obtained for a consistent number of experiments (100 experiments in our analysis).

4.2.7 Global properties of street networks

The goal of this section is to give advice about the values of the indicators previously described. To this end, we analyse the street intersection network of 96 French department cities, the CODAH and the six geometric planar graphs. Table 4.1 is the statistical summary of street networks and table 4.2 shows values of indicators resumes theoretic planar graph. We can infer that

⁹The cost measure can also be computed in a topological way: a topological cost can be defined as the ratio between the number of edges of G and the number of edges in the corresponding $MST(G)$.

¹⁰A tree is not a robust graph. We discuss this point next.

	$ V $	$\langle k \rangle$	$ E $	$l(G)(km)$	$\langle l(G) \rangle (m)$	γ	org	mes	$cost$	rob
min	1039	2.397	1354	12.426	3.713	0.4	0.698	0.083	0.657	0.15
max	42187	3.04	58498	350.847	15.194	0.507	0.948	0.171	0.849	0.29
range	41148	0.643	57144	338.421	11.481	0.107	0.25	0.088	0.192	0.14
median	5343	2.783	7560.5	49.706	7.166	0.464	0.855	0.141	0.747	0.22
mean	7603.667	2.781	10713.156	68.498	7.825	0.464	0.848	0.14	0.748	0.219
std.dev	7451.318	0.113	10682.791	61.856	2.361	0.019	0.051	0.015	0.036	0.026
coef.var	0.98	0.04	0.997	0.903	0.302	0.04	0.061	0.107	0.049	0.12

Table 4.1

Summary of characteristics of street intersection network of the 96 French department cities. Number of vertices $|V|$, average vertex degree $\langle k \rangle$, number of edges $|E|$, total length of edges $l(G)$, average edge length $\langle l(G) \rangle$, gamma index γ , organic ratio org , meshedness coefficient mes , cost $cost$ and robustness rob .

graph	$\langle k \rangle$	γ	org	mes	$cost$	rob
euclideanTree	1.998	0.334	0.470	0.000	1.000	0.06
probabilistic	2.640	0.441	0.400	0.122	0.682	0.33
deterministic	4.355	0.727	0.234	0.271	0.496	0.33
gabriel	3.876	0.647	0.313	0.243	0.520	0.36
grid	3.863	0.645	0.120	0.242	0.521	0.40
delaunay	5.940	0.992	0.017	0.332	0.332	0.46

Table 4.2

Characteristics of six geometric planar graphs. Average vertex degree $\langle k \rangle$, gamma index γ , organic ratio org , meshedness coefficient mes , cost $cost$ and robustness rob .

the values of the vertex set and the edge set of street intersection networks are in an ample range. The coefficient of variation¹¹ and the standard deviation of $|V|$ and $|E|$ are high. The γ , org , mes , $cost$, and rob indicators of cities do not display an ample range of values. Finally, few graphs show values of average degree that exceed 3, low values of mes and high values org ; we can state that the street intersection networks have many tree-like structures, many dead ends, and more vertices with 3 than vertices with degree 4 or more. Values suggest that the majority of French cities have not been planned, and they are the results of a spontaneous process of formation. These characteristics are scarcely observable in basic geometric planar graphs; actually, we observe a large dispersion of values.

Despite the probabilistic graph seems to have values of some indicators close to street intersection networks (especially $\langle k \rangle$, γ , mes , $cost$, and rob), the degree suggests that its structural properties are incompatible with our geographical data set. The degree distribution is exponential for the probabilistic graph (fig. 4.3), while it is peaked for CODAH. The probability $P(k)$ to find a vertex with degree k for theoretic planar graph is broader than the CODAH, where we observe many vertices with degree around 6 and also a node with degree 26 (the probabilistic graph). Edge length distributions (fig. 4.4) seem exponential.

Results allow us to suppose that behind the formation of French street

¹¹In statistics, the coefficient of variation is a measure of the dispersion of independent variables; it is defined as $\sigma^* = \sigma/\mu \in [0, 1]$, the ratio of the standard deviation to the mean of a distribution. High values of σ^* suggest values highly dispersed.

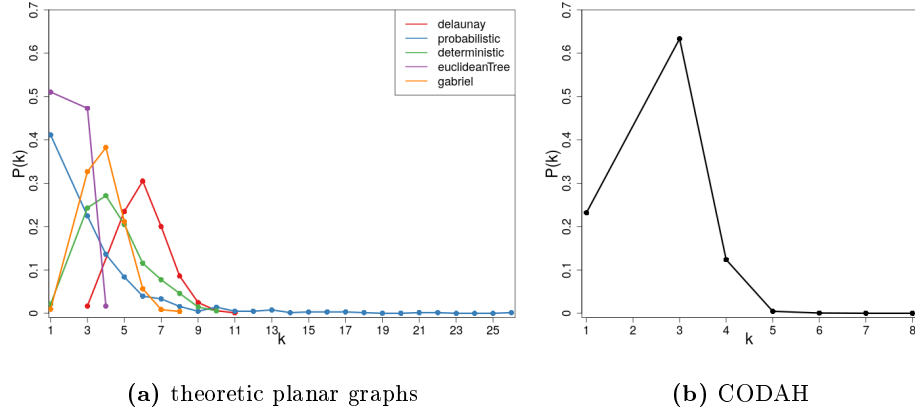


Figure 4.3
The probability degree distribution of theoretic planar graphs and the CODAH.

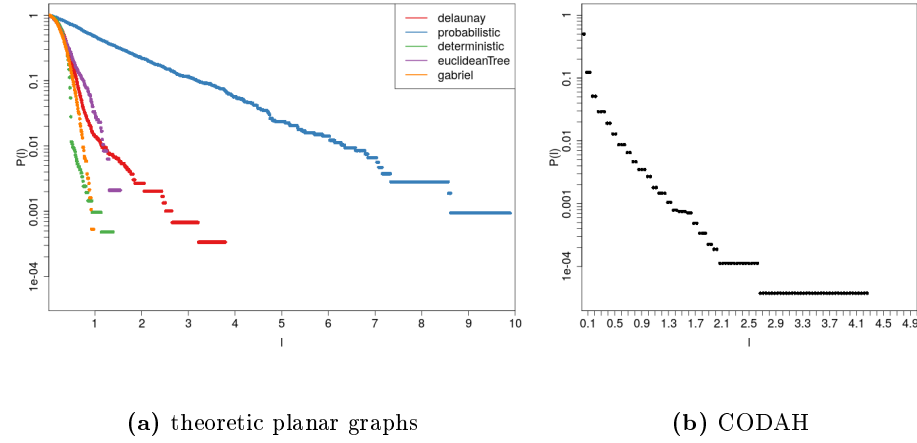


Figure 4.4
The cumulative edge length distribution of theoretic planar graphs and the CODAH.

networks, there are some common processes; these processes are different to processes used to generate our basic geometrical planar graphs. Moreover, the size of graphs does not impact the topological features of street networks, thus similar properties can be detected in cities with different sizes.

The scatter plot in fig. 4.5 compares the organic ratio and the meshedness coefficient of French department street network¹². While there are many tree structures (low values of *mes*), there are also fewer planning structures (low values of *org*). Results suggest that these measures are related but we have not sufficient information to ensure a correlation.

In fig. 4.6 we plot the linear correlation between the total edge length and the size of corresponding vertex set. Values fit the regression line with a coefficient of determination $R^2 = 0.896$ ¹³. We also observe that the majority of cities are in the first third of the plot: the size of the vertex set of three cities exceeds average values (which also corresponds to red points in fig. 4.5) and this is also correlated to the total length of edges.

Figure 4.7 reports the robustness of street networks and the six theoretic planar graphs. As we expected, the lower values of robustness are observed

¹²We do not plot points of theoretic planar graphs because they are too far.

¹³In statistics, the coefficient of determination $R^2 \in [0, 1]$ is a measure that indicates how well the data fit the regression line. High values of R^2 suggest a good approximation.

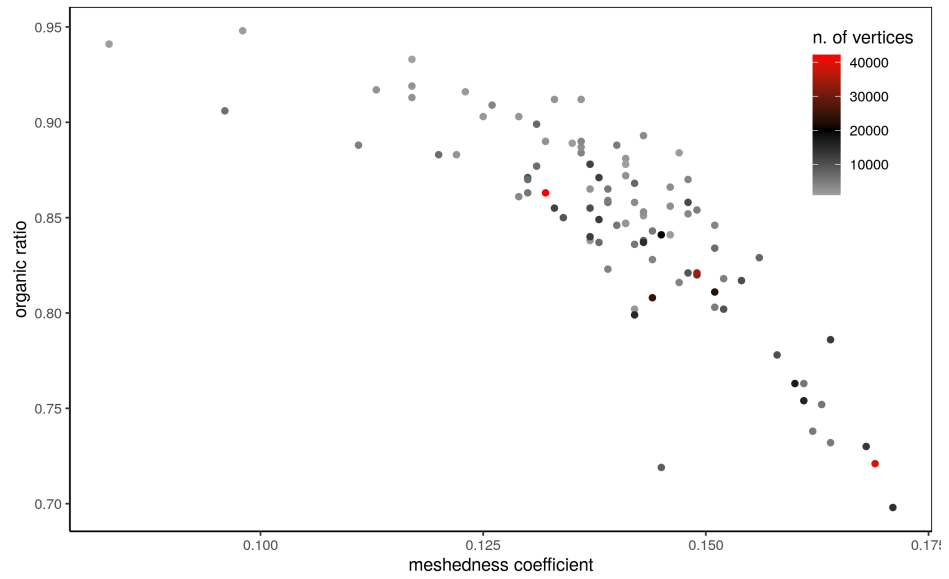


Figure 4.5
Organic ratio *org* versus
meshedness coefficient *mes*.

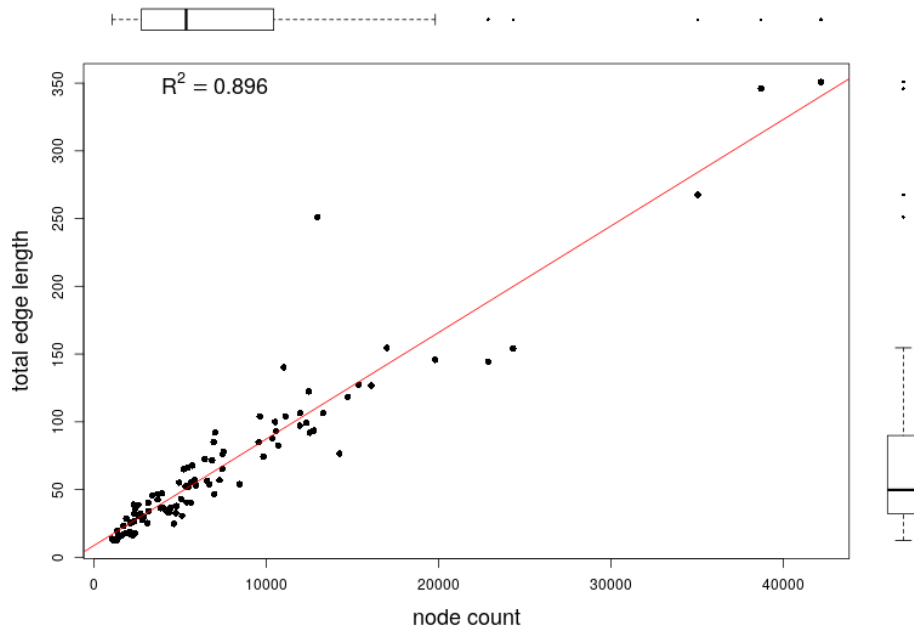
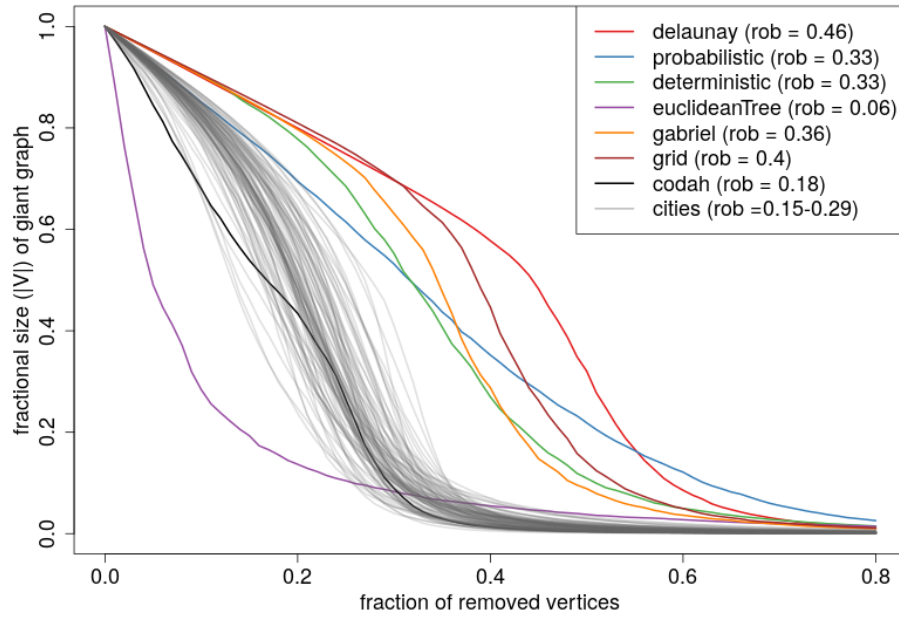
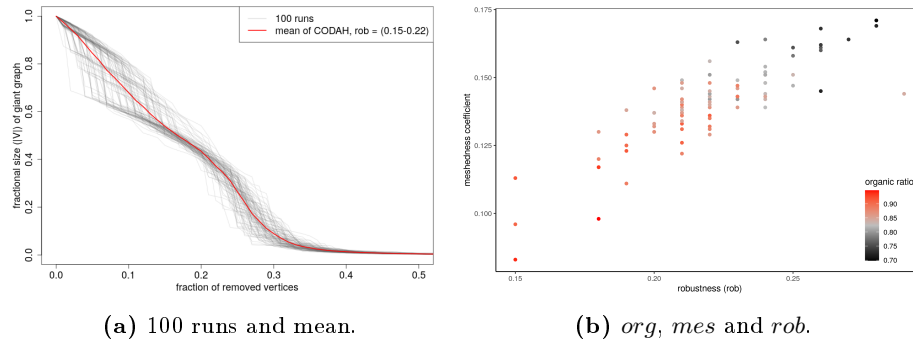


Figure 4.6
Number of vertices $|V|$ ver-
sus the total length edge
 $l(G)$. Box plots represent the
medians, interquartiles (the
 25^{th} and the 75^{th} percentile)
and potential outlier (values
more than the 1.5 of the 75^{th}
or less than the 1.5 of the 5^{th}
percentile) of $|N|$ (horizontal
axes) and $l(G)$ (vertical axes)
of French department cities.

in the euclidean tree graph: each removed vertex represents an important failure because it de-compose the graph in at least two components. All other theoretic planar graph are more robust than real street networks (see the values of *rob* and the shape of the distribution). Figure 4.8a plots the average (red line) of 100 experiments (black lines) and the range of values of *rob*. Finally we observe some relations between *rob*, *org* and *mes* indicators of French cities (fig. 4.8b). Robustness decreases if cities are like trees (low values of *mes*) and increase if graphs contain many vertices with $k \neq 1, 3$ (low values of *org*).

**Figure 4.7**

Robustness of French department cities and six basic graphs.

**(a)** 100 runs and mean.**(b)** *org*, *mes* and *rob*.**Figure 4.8**

Robustness of CODAH and French department cities. in fig. 4.8a, the average robustness curve; in fig. 4.8b, the relation between *rob*, *org* and *mes* indicators.

Graph theory proposes indicators useful to characterize geometric planar graphs. French department and the CODAH street networks show similar global properties. These last are incompatible with theoretic planar graphs.

Summary 11:
global properties of
spatial networks

4.3 Hierarchies and centrality

Social groups can be seen as networks of interactions [295]. Bavelas [33] observed that a central location in a social group corresponds to a certain degree of freedom and power of the individuals. The importance of an individual in a system depends on how it is related to the rest of the network: some individuals are more central than others [108].

Centrality theory allows us to identify relative relationships between elements in a graph, building a scale of importance [109]. Centrality analysis was largely used in the last decades to characterize networks and centrality

measures seek to quantify the importance of vertices¹⁴ in relation to others. Centrality is used not only in social network analysis, but also to study the web [217], the interactions between proteins [153] and in infrastructures. In this last field, centrality theory was introduced by Space Syntax [142] (section 3.3.3) and it becomes an important tool to study primal [228] and dual [227] representation of street networks. Centrality theory was applied to study spatial organization of urban areas [87, 112] (fig. 4.9), to characterize spatial distribution of retails [215], and to measure the rate of the urban growth [24, 257, 270]. Centrality indicators seem to be correlated to transportation informations [113, 170] and robustness [193].

Many centralities characterize the importance of a node or a link in a network. The degree and the closeness centrality measure how elements are "near (close) to others", the betweenness centrality measures the tendency of an element to be "between others", the Eigenvector, the Katz and the Page rank centrality measure the influence of a node by a score.

¹⁴Similar measures are also applied to characterize edges.

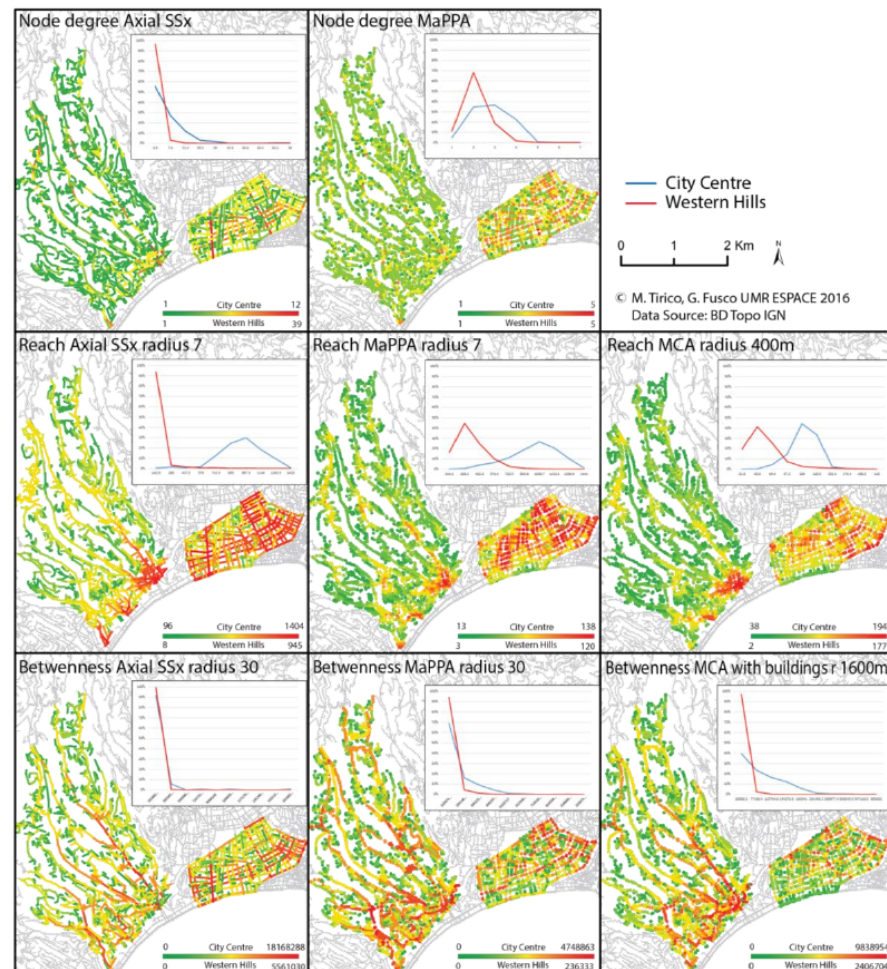
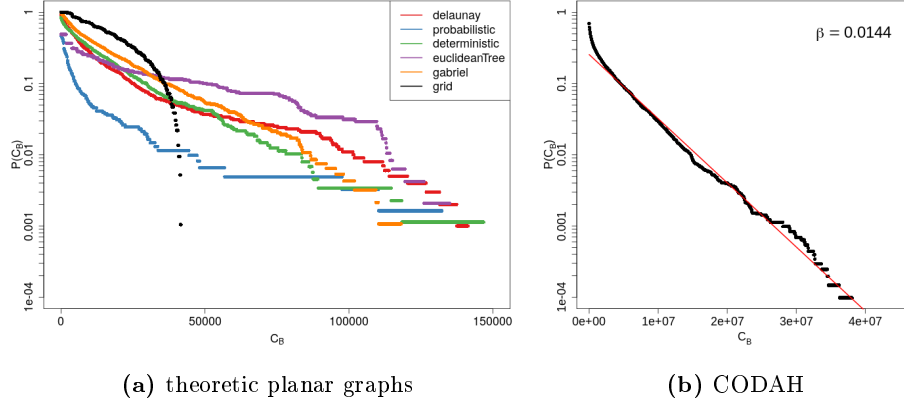


Figure 4.9

Centrality measures, applied to the street network of Nice (France), quantitatively discriminate tree-like and planned urban forms. Source [112].

**Figure 4.10**

The cumulative distribution of the betweenness centrality of 6 theoretic planar graphs (fig. 4.10a) and the CODAH (fig. 4.10b).

4.3.1 The betweenness centrality

The betweenness centrality evaluates the centrality of a vertex (or an edge) considering the number of shortest path that pass through. High values of betweenness centrality for a vertex, as well as indicating the property of it to be crossed by an high number of shortest paths, is also an indicator of network robustness. Networks with many vertices with high values of betweenness centrality are less robust and more susceptible to become inefficient in case of failures of part of them [147]. In general, betweenness centrality scale with the degree, but in spatial networks, due the absence of hubs and the compactness the degree distribution, they are not equivalent. The betweenness centrality captures important aspects of the structure of the networks, bringing out the the hierarchy of importance of elements.

The betweenness centrality is computed comparing the shortest paths which traversed the vertex and to all shortest paths in the connected graph [22]. Formally, the betweenness centrality of a vertex v is defined as:

$$C_B(v) = \sum_{s \neq t \neq v} \frac{\sigma(s, t|v)}{\sigma(s, t)} \quad (4.8)$$

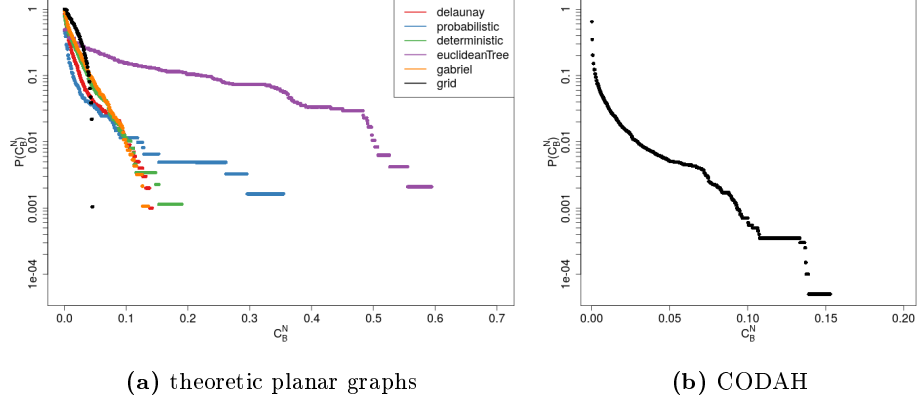
where $\sigma(s, t|v)$ is the number of shortest paths going from s to t through v and $\sigma(s, t)$ is the number of topological shortest paths going from s to t . At least one shortest path might exist for a pair of vertices s and t . Equation (4.8) can be normalized as :

$$C_B^N(v) = \frac{2}{(|V| - 1)(|V| - 2)} C_B(v) \quad (4.9)$$

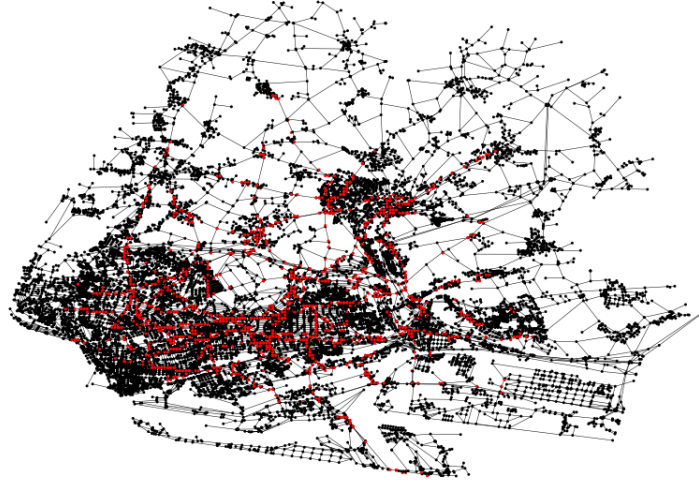
where $|V|$ is the number of vertices of the corresponding component of v . The normalization guaranties $C_B^N(v) \in [0, 1]$. The betweenness of an end point is zero and we expect to find 1 in star networks, high values in tree graphs and in general in graphs with a broad degree distribution.

4.3.2 The statistical distribution of betweenness centrality

The study of the function which fits independent observations is a fundamental topics in many statistic researchers [80]. In a scale free network, the

**Figure 4.11**

The cumulative distribution of the normalized betweenness centrality of six theoretic planar graphs (fig. 4.11a) and the CODAH graph (fig. 4.11b).

**Figure 4.12**

Spatial distribution of the normalized betweenness centrality of the CODAH. Red dots correspond to vertices with $C_B^N > \max(C_B^N)/10$.

betweenness centrality probability distribution, as its corresponding degree distribution, obeys a power-law: $P(C) \sim C^{-\beta}$ [21, 22, 123], where the exponent $\beta \in]2, \infty[$. In street networks, despite authors in [170] proposed that the betweenness centrality distribution is power-law like, researchers find an agreement that betweenness centrality exhibits an exponential behaviour: $P(C) \sim e^{-C/\beta}$ [87, 228].

These studies had showed that the fitted equation of betweenness centrality distribution depends to the essential characteristics of urban form. The statistical distribution and is related to the degree of planning [24]: the distribution of betweenness centrality in self-organized cities is broader than planned cities [228] (see also the third row of fig. 4.9). These observations seem corroborated in [87], where authors proposed to fit betweenness centrality distribution of planned cities with a Gaussian function $P(C) \sim e^{-C^2/2\sigma^2}$ and self-organized cities with exponential function $P(C) \sim e^{-C/\beta}$. Moreover, authors in [251] validate their street network model generator observing that the betweenness centrality distributions fits an exponential curves.

Figure 4.10 reports the cumulative distributions of C_B of our six theoretic planar graph and the CODAH. Only the distribution of the probabilistic

graph fits an exponential distribution, resembling the shape of the distribution of CODAH. The fitting equation confirms the general result previously mentioned; actually, the distribution is single scale and fits the exponential equation $P(C) \sim e^{-C/0.0144}$. The normalized distributions of theoretic planar graph and CODAH are reported in fig. 4.11. Here we focus our attention on the range of expected values of betweenness centrality: our results suggest that only three theoretic planar graphs (Gabriel, deterministic and Delaunay) can be compared to the CODAH graph. However, we cannot find an appropriate fitting equation for CODAH: we suggest that it is due to the heterogeneity of topological characteristic of corresponding graph.

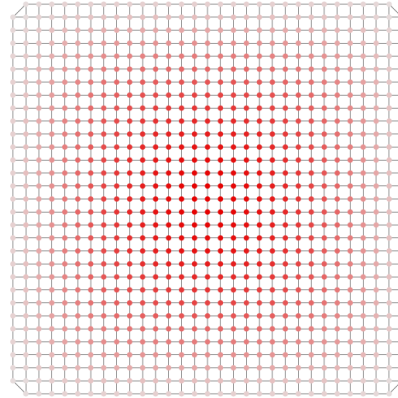
4.3.3 The spatial distribution of betweenness centrality

Figure 4.12 depicts the spatial distribution of the betweenness centrality of the CODAH. We observe the tendency of shortest paths to traverse few central nodes, defining main flow axes (in Le Havre city centre) and crucial nodes between suburbs (Montivilliers and Octeville-sur-Mer) to the north border of Le Havre. The intersections of Le Havre city centre are high connected (where we find a majority of four-way intersections) and this area shows lower values of betweenness centrality. It indicates that the main transportation axes do not traverse this area. The betweenness centrality nicely captures the main mobility axes, discriminating them to secondary routes.

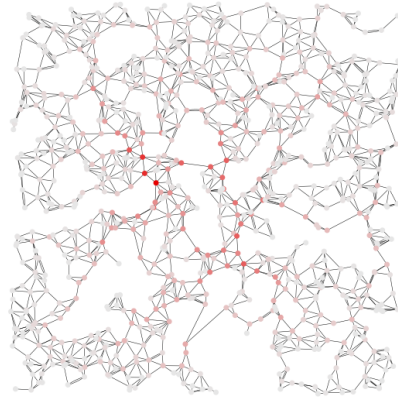
As shown in fig. 4.13, the spatial distributions of the normalized betweenness centrality of the six theoretic planar graphs exhibit different trends and it depends on the structural properties of these graphs. The regularity of the grid graph also impacts the betweenness centrality, identifying a progressive increment of values from the boards to the centroid of the graph. In the probabilistic graph, vertices with high connectivity capture the majority of shortest paths. A similar effect can be observed in the tree graph: the effect is also due to a large number of leaves (where $C_B(v) = 0$). The Gabriel graph is a sub-graph of the Delaunay graph, the vertices with high values of C_B^N does therefore not correspond.

Measure from centrality theory are widely applied to study street networks. The study of betweenness centrality is a way to investigate hierarchies of elements in spatial complex networks. We can investigate hierarchies observing the statistical and spatial distribution of values computed for each vertices of the graph. Values computed for three theoretic planar graph are similar to ones computed for CODAH.

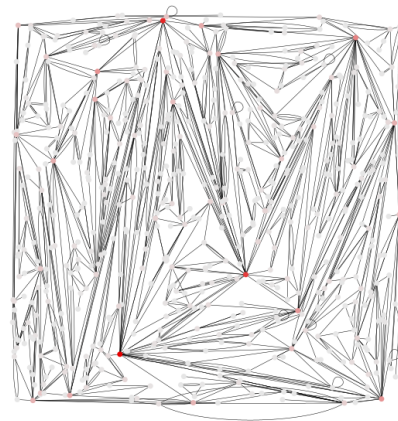
Summary 12:
hierarchical
organization



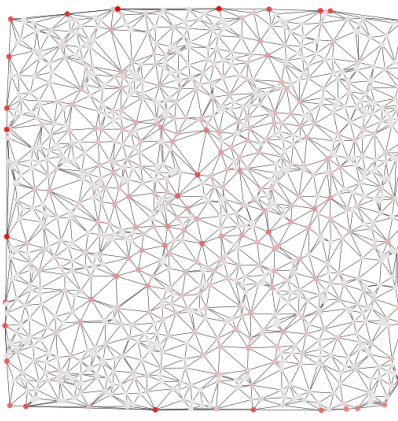
(a) Grid graph



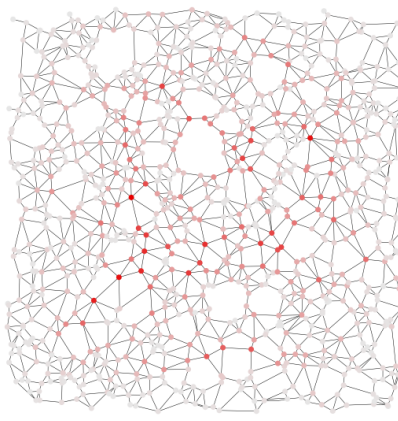
(b) The deterministic graph



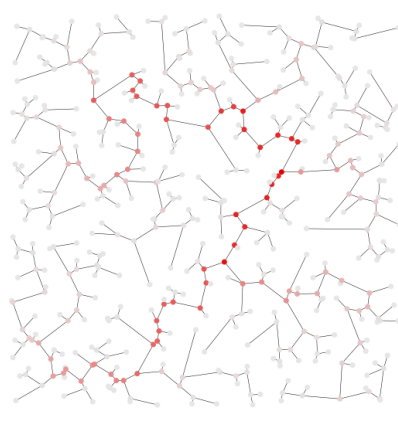
(c) The probabilistic graph



(d) The Delaunay graph



(e) The Gabriel graph



(f) The Euclidean tree

Figure 4.13
The spatial distribution of the normalized betweenness centrality of theoretic planar graphs.

4.4 Scale invariance and fractal theory

"People live at different scales."

Pierre Frankhauser

Many systems show similar properties at different scale of observation (section 1.2.3). These properties are therefore independent to the classical idea of dimension. When the scale of observation is a metric measure¹⁵, a physical object that shows a scale-invariant shape is called geometric fractal¹⁶.

More precisely, a fractal (from the Latin *fractus*, broken) is a rough or fragmented geometric shape that can be subdivided into parts, each of them is a reduced/size copy of the whole [100]. A fractal is composed of repeating spatial patterns and/or the elements are arranged in such a way the appearance of the system does not change at different scales of observation. These properties are not proportional and cannot be detected via classical geometrical metrics such as length, area, or density. Fractal analysis is the tool that investigates these characteristics; the measurement of the coastal lines and its generalization in cartography was an early example of its application [188]. The masterpieces *The Fractal Geometry of Nature* of Mandelbrot [189] structured the fractal theory and collect a large number of natural example.

In general mathematical terms, if we use P to describe the property of any systems which vary with the scale of observation S , a scale-invariant system can be described by a power law: $P \approx S^D$. P may correspond to a measure of concentration, and the scale S may correspond to a linear distance. The exponent D quantifies how much the scale-free effect is evident in the relation. It escapes to natural numbers and became a positive decimal number.

4.4.1 Growing fractals

There exists several kinds of processes that allow the creation of a fractal object¹⁷. A growing fractal is a mathematical object that is created by recursive processes. Growing fractals are produced by some rigorous and deterministic rules [224], applied to their own outputs.

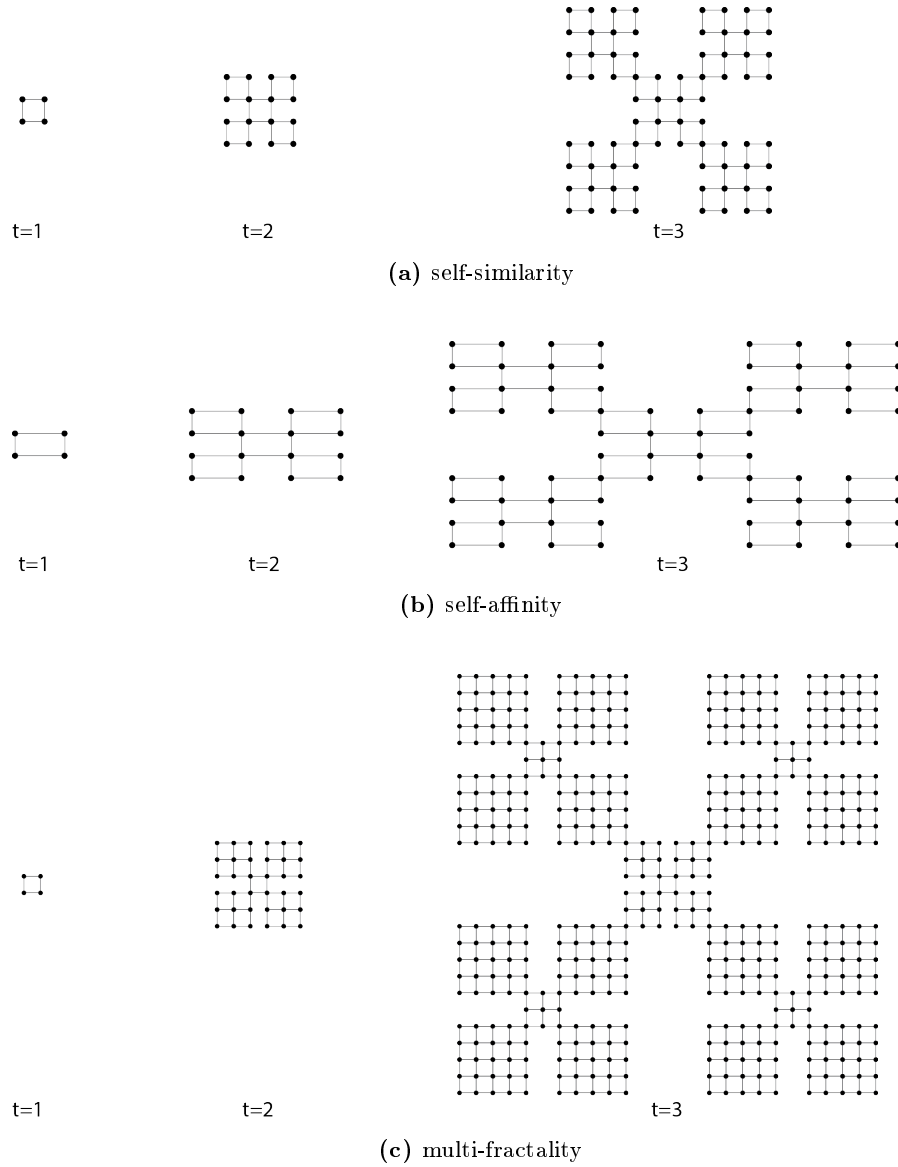
Growing fractals can be classified with regard to the dominant direction of the growth [101] (fig. 4.14). When no dominant directions dominate the recursive process, the growing fractal is self-similar and the shape is approximately isotropy (fig. 4.14a). Otherwise, self-affine fractals (fig. 4.14b) are the result of anisotropic transformations by which their images are invariant. The different dimension measures do not necessarily coincide.

Strictly speaking, all real objects cannot be considered as (mathematical) fractals. Indeed, a fractal must exhibit scale-invariant properties over an

¹⁵There are many different scales than can be used in fractal analysis (e.g. a temporal scale), but we will focus here on geometric scale.

¹⁶In the next, the term fractal will refer to geometric fractal.

¹⁷For instance, fractal generative processes are strange attractors, L-systems, random stochastic fractals.

**Figure 4.14**

Three steps of three growing geometrical graphs. Their growth is self-similar (fig. 4.14a), self-affine (fig. 4.14b) and multi-fractal (fig. 4.14c).

infinity of scales. Even in his pioneer works, Mandelbrot [190] suggests that in nature, most of fractal objects do not display this precision. The rate of growth may vary during the formation. A mono-fractal object is characterized by one rate of growth. In the case of the rate are not constant, the evolutive process generates a multi-fractal [292] (fig. 4.14c).

4.4.2 The Hausdorff-Besicovitch fractal dimension

In addition to show the possibility to investigate scale-invariance organization of systems via fractal analysis, Mandelbrot also systematizes and organizes the concept of space-independent theoretical objects introduced by Hausdorff [137] and Besicovitch [43]. Mathematically speaking, the Hausdorff-Besicovitch dimension of a fractal strictly exceeds its topological dimension D_t ¹⁸; defining a non-integer value. The relationships between the

¹⁸A point has topological dimension $D_t = 0$, a line has $D_t = 1$, a surface has $D_t = 2$.

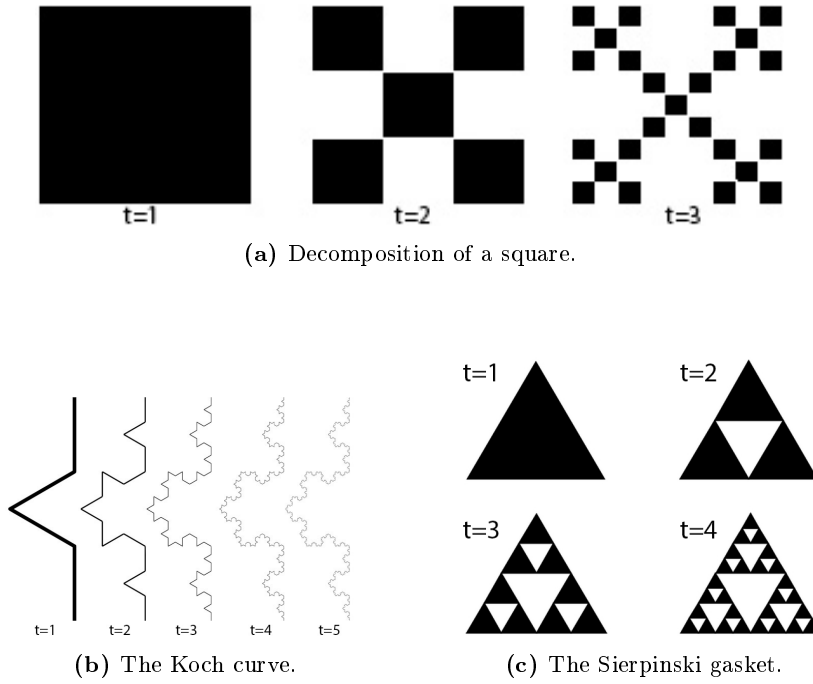


Figure 4.15
Three generative fractal processes.

number of elements N of an object and the scale factor S , raised to the dimension D is $N = S^D$, and the estimated value of the Hausdorff-Besicovitch dimension is:

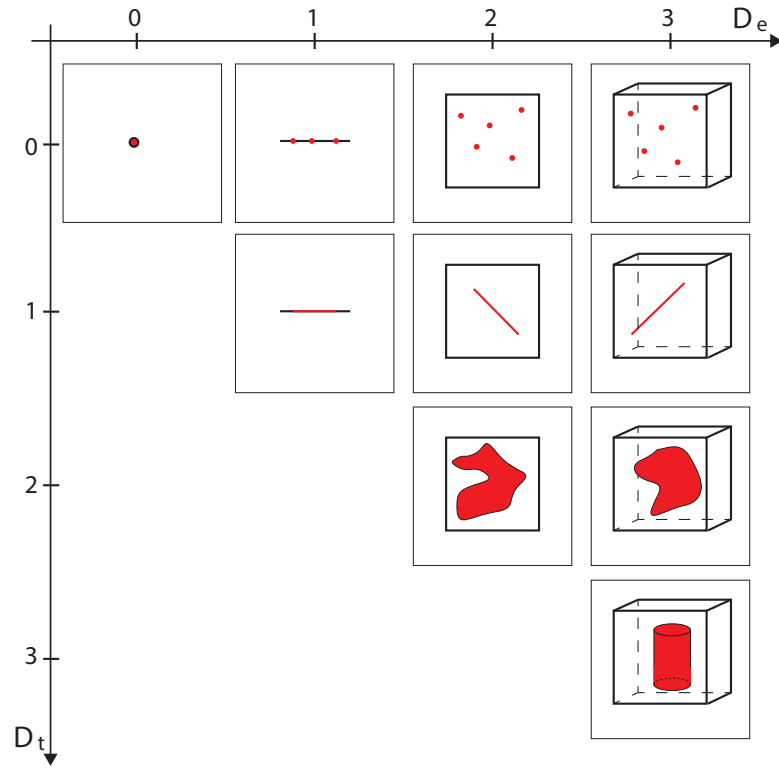
$$D = \frac{\log N}{\log S} \quad (4.10)$$

Let us explain a few examples (fig. 4.15). In fig. 4.15a the initial pattern is composed of $N = 1$ square; after the first iteration, the square is reduced by a factor of $S = 3$ and the pattern becomes composed of $N = 5$ squares. The same process could be applied to each square for an infinite number of steps. According to eq. (4.10), $3^D = 5$, thus $D = \log 5 / \log 3 = 1.465$.

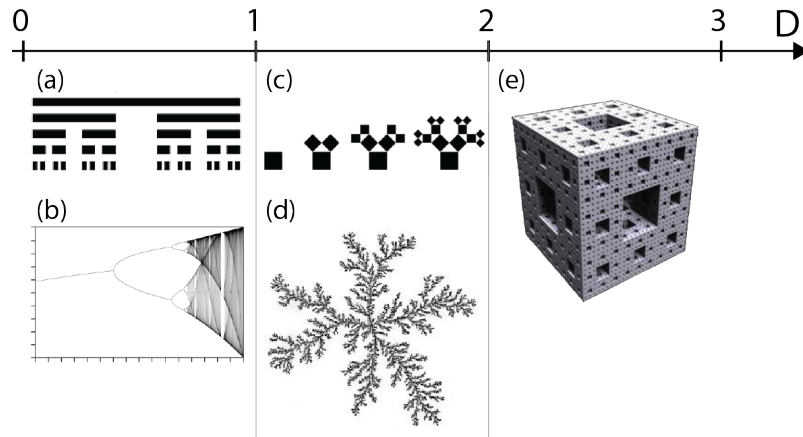
If a recursive process magnifies a segment by a factor $S = 3$, erasing the middle third section of each segment and replacing it by two other segments of equal length, we obtain the Koch's curve [161] (fig. 4.15b). According to eq. (4.10), $3^D = 4$, thus $D = \log 4 / \log 3 = 1.262$.

The Sierpinski gasket [267] (fig. 4.15c) is constructed from an equilateral triangle. At each step, each triangle is subdivided in four triangles and the central triangle is removed. The gasket has fractal Hausdorff-Besicovitch dimension $D = \log 3 / \log 2 = 1.585$.

The fractal dimension of an object is greater than its topological dimension D_t but less than the euclidean dimension D_e of the embedding space in which the object exists (fig. 4.16a). D_t and D_e are the lower and the upper limits of D : for instance, in 2-dimensional space ($D_e = 2$), the fractal dimension of a point set is limited in $0 < D \leq 2$ and, for a set of lines, $1 < D \leq 2$ (fig. 4.16b).



(a) Euclidean and topological dimension.



(b) Examples of fractals.

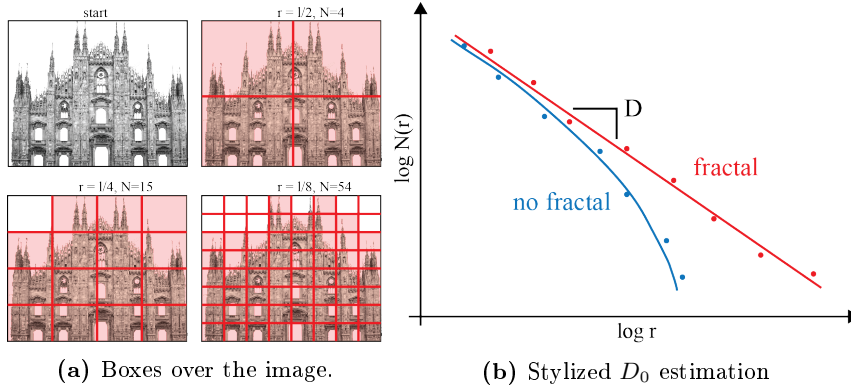
Figure 4.16

Lower and the upper limits of fractal dimension. In fig. 4.16a, euclidean d (abscissa) and topological d_t (ordinate) dimension. In fig. 4.16b, five fractal objects: the Cantor set (a), the logistic equation (b), the Pythagoras tree (c), the DLA tree (d), the Menger sponge (e).

4.4.3 The capacity dimension

An operative limit of the Hausdorff-Besicovitch dimension D is the continuity range of scales where it is defined. If this requirement is relaxed, euclidean $\mathbb{R}^2$ space becomes discrete; this last could be considered as a grid-square lattice, and squares as a set of regular boxes. The Hausdorff-Besicovitch dimension becomes the capacity dimension D_0 (also called box-counting dimension) and $D = D_0$ for self-similar and mono-fractal object [99]. It is defined as:

$$D_0 = \lim_{r \rightarrow 0} \frac{\log N(r)}{\log r} \quad (4.11)$$

**Figure 4.17**

Estimating the capacity dimension with the box-counting algorithm. Boxes overlaid on the image of the cathedral of Milan and a log-log plot suggest the estimate of the dimension.

where $N(r)$ is the minimum number of non-empty boxes of size r to cover the object (fig. 4.17a). In the event that a linear relationship between the logarithm of $N(r)$ and the logarithm of r exists, measures at corresponding size fit the equation $N(r) \approx r^{-D_0}$; in a log/log plot, D_0 is the slope of the fit line (fig. 4.17b).

The box-counting algorithm can be applied for all geometric objects. The capacity dimension D_0 of a geometric graph $G(x)$ is given by the relation between the minimum number of boxes to include vertices and the size of the box [269]. The measure disregards the number of vertices contained in the boxes, thus the box-counting algorithm does not provide adequate information about the spatial distribution of vertices. Finally, the capacity estimation disregards the local differences of its properties: two geometric graphs could have the same capacity dimension while their configurations look like different.

4.4.4 The multi-fractal analysis

Distinguishing scale-free properties in a spatial system is not so easier as geometric mathematical objects. The Hausdorff-Besicovitch estimation and its direct derivation, the box-counting algorithm, are useful tools to investigate the self-similarity of idealized (mathematical) objects, but they present some lack for the estimation of self-affinity and multi-fractality. The capacity estimation ignores spatial relations between elements. It does not take into account the spatial properties of the system as the density and the distance between elements.

The Renyi fractal dimension estimation [140] can help us to improve our understanding because it considers the density of properties. Let r be a size region in d dimensional euclidean space, $P(r)$ is a probability density function. This latter can be described in a spectrum of moments [135]. The generalized fractal dimension D_q of order $q \in \mathbb{R}$ is defined as:

$$D_q = \frac{1}{q-1} \lim_{r \rightarrow 0} \frac{\log \sum_i N(r) P_i(r)^q}{\log r}, q \neq 1 \quad (4.12)$$

Equation (4.12) is not defined for $q = 1$ but it was heuristically shown the existence of the limit from $q \rightarrow 1$ [140]. For an uniform fractal, $P(r)$ is constant and D_q does not vary with q ($D_q = D = D_0$). Otherwise,

eq. (4.12) suggests that D_q decreases when q increases. The multi-fractal estimation also respects the upper (euclidean) and the lower (topological) limits of the Hausdorff-Besicovitch dimension ($D_t < D_q < D_e$).

The capacity dimension estimation is a specific case of multi-fractal analysis. According to eq. (4.12) we can infer the capacity dimension D_0 , the information dimension D_1 and the correlation dimension D_2 . To obtain D_0 , we consider the number of not-empty boxes, for D_1 the density of elements in a region (box), and for D_2 the euclidean distance between elements. In the following, we describe three popular algorithms used to compute fractal dimensions D_0 , D_1 , D_2 of a set of points embedded in 2-dimensional space. These approaches can also be extended to investigate the spatial distribution of vertices of a geometric graph $G(x)$, thus the graph representation of a street intersection network.

The information dimension

When $q = 1$, eq. (4.12) becomes the information dimension:

$$D_1 = \lim_{r \rightarrow 0} \frac{S(r)}{\log r} \quad (4.13)$$

where $S(r) = -\sum_i P_i \log P_i$ is the Shannon's entropy [263] of the number of vertices P_i contained in a box of size r . In information theory, the Shannon entropy is a classical diversity index, and it gives a measure of the degree of unevenness of a distribution and quantifies the uncertainty about a random property of the system. The eq. (4.13) measures the probability to find the same concentration of elements of vertices at different scales.

The correlation dimension

When $q = 2$, eq. (4.12) becomes the correlation dimension:

$$D_2 = \lim_{r \rightarrow 0} \frac{C(r)}{\log r} \quad (4.14)$$

where:

$$C(r) = \sum_i P_i^2 \quad (4.15)$$

is the correlation sum, the probability P_i to find two vertices connected within the box i [168]. It can be applied in a continuous media. In a discrete media (a set of vertices in the space), we can use the Grassberger-Procaccia algorithm [126]. The eq. (4.15) becomes:

$$C(r) = \frac{\sum_i \sum_j \Theta(r - \text{dist}(X_i, X_j))}{\frac{1}{2}N(N-1)} \quad (4.16)$$

where:

$$\Theta(r - \text{dist}(X_i, X_j)) = \begin{cases} 1 & \text{if } \text{dist}(X_i, X_j) \leq r \\ 0 & \text{if } \text{dist}(X_i, X_j) > r \end{cases} \quad (4.17)$$

is the Heaviside step function and $dist(X_i, X_j)$ is the euclidean distance between a pair of vertices located at $X = (X_i, X_j)$. The correlation dimension gives a more accurate description of the graph than previous dimensions because it also considers euclidean distances between vertices.

4.4.5 Fractals in urban studies

Geographers and urban researchers are attracted by fractal theory [125, 277]. Fractal theory were applied to investigate three main aspects of cities:

- **Spatial distribution of properties and characteristics:**
in this vein, fractal analysis is an useful tool to study population distribution [8] and land-use patterns [182].
- **Urban fabrics:**
classical (geometric) measures allow us to study the contours, the arrangement, and the geometry [252] of elements that composed urban fabric (section 3.1.3). With fractal measures, the urban fabric can be also studied without a dominant scale of observation but in a spectrum of scale [106, 107]. Fractal analysis was applied to characterize limits of urban areas [28] [278], to describe the building footprint [64] and the geometry of streets [272, 279], and to evaluate spatial relations between green space and building covering [107].
- **Urban growth:**
for instance, fractal measure are used to study cities at different moments [74, 204] and to evaluate the ecological impact of urban growth [276].

Multi-fractal approach in urban studies

Multi-fractal analysis has found numerous applications such as the study of financial market [155], the recognition of image patterns [76], and even in music [186] or cognitive researches [191]. Since pioneer applications of fractal analysis in urban studies [29, 107], it is well recognize that also street network can be investigated with a large spectrum of measures [75]. It is applied to investigate the historical evolution of the London street intersection network [204] and Beijing urban fabric [74].

During the formation of urban forms, more than one single process of generation coexists: fractals appear discontinuously, urban elements cover the space in an anisotropic way and spatial distribution of elements hardly changes linearly. A single fractal indicator may not be always sufficiently to describe the scale invariance of an urban system. The generalization of fractal concepts through the introduction of a spectrum of measures allows us to power the classical fractal analysis. With this approach, we can compare cities with fractal that growth with different rates [8].

Algorithm 2: Estimate fractal dimension of street intersection network

```

input : the street intersection network
output: kind of fractality
begin
   $D_0 \leftarrow$  compute the capacity dimension
  if measures fit linear regression then
     $D_1 \leftarrow$  compute the information dimension
     $D_2 \leftarrow$  compute the correlation dimension
    if  $D_0 \approx D_1 \approx D_2$  then
      | the network is global mono-fractal
    else
      | the network is global multi-fractal
    else
      | the network could be fractal only locally (or not)
  end if

```

Local and global fractal dimension in urban studies

A fractal is a mathematical object that cannot find complete correspondence in reality. What we can expect from a multi-fractal analysis of urban forms? In geographical studies, the fractal measure is not expected to be constant [124]. In urban studies, measures of fractal dimension vary in space [39]. As for all real fractals, also for cities, the same scale-invariant properties appear over a limited range of scales [265]. Fractal dimension estimates of the whole study area a subareas of it are often different [280]. Moreover, we can observe different scale invariances in a given range: the values of fractal dimension D are not constant over the range and they exhibit local variation [277].

For these reasons, defining under which conditions (and about what properties) a city will be expected as a fractal is still a pending question [234]. We can consider that some properties of urban systems are scale-invariant over a large scale, while others may be that in a few scales. Consider a large spectrum is reasonable in the study of urban fabric [28], especially when we investigate the street intersection network [204] in a multi-fractal perspective [74].

In order to define a general approach (algorithm 2) to investigate the scale invariance organization of street intersection networks, we need to answer if it shows global fractal behaviour. To this end, the box-counting analysis is enough to detect the kind of fractality of the urban system: the first step is to compute the capacity dimension D_0 via the box-counting method and to observe the statistical distribution. In the case where data fit a linear equation and $1 < D_0 < 2^{19}$, the network can be treated as a global fractal. The information dimension D_1 and the correlation dimension D_2 can be computed. At this step, we can judge whether or not the network can be treated as a multi-fractal. If $D_0 \approx D_1 \approx D_2$ the network is mono-fractal, otherwise we can study the network as a multi-fractal object.

¹⁹To evaluate that we estimate the coefficient of determination R^2

Multi-fractal analysis of street intersection network

In this section, the analysis of scale-invariance organization of street intersection network is mainly based on a few emblematic French department cities (fig. 4.21) and the CODAH.

Sub-areas of CODAH appear different (figs. 4.21c and 4.22). Here we focus on the whole urban area, the rebuilt area (fig. 4.22b) and the Montivilliers town (fig. 4.22b). The capacity dimension D_0 estimation (fig. 4.18) suggests that only the whole street intersection network present a scale-free distribution. This is confirmed by the fitness of the regression line of capacity dimension distribution (see the coefficient of determination R^2).

Distributions of capacity dimension estimation of cities of Tarbes (fig. 4.21b) and Marseille (fig. 4.21a) appear different (fig. 4.19a). Only the distribution of Marseille fits the regression line with a reasonable approximation. This allows us to suppose that the street intersection network of Tarbes does not have global fractal properties. For cities with a low value of R^2 , the information and the correlation dimension are not studied. In the case where data fit the regression line, the fractal property of the street intersection network can be considered global. In fig. 4.19b we compare the capacity dimension of CODAH (fig. 4.21c) and Paris (fig. 4.21d). Both distribution fits a linear equation ($R^2 > 0.99$) with a different slope (D_0). Both cities can be considered global fractals. To evaluate if they are also multi-fractal, we compute the information and the correlation dimensions. On one hand, the D_0 of Paris (fig. 4.21d) is significantly different to D_1 and D_2 ; these measures suggest that the street intersection network of Paris is multi-fractal. On the other hand, results of CODAH (fig. 4.21c) suggest that the corresponding street intersection network should be investigated as mono-fractal.

The existence of a global fractal behaviour does not mean that we cannot have local variations. Despite values of capacity estimation of CODAH fit

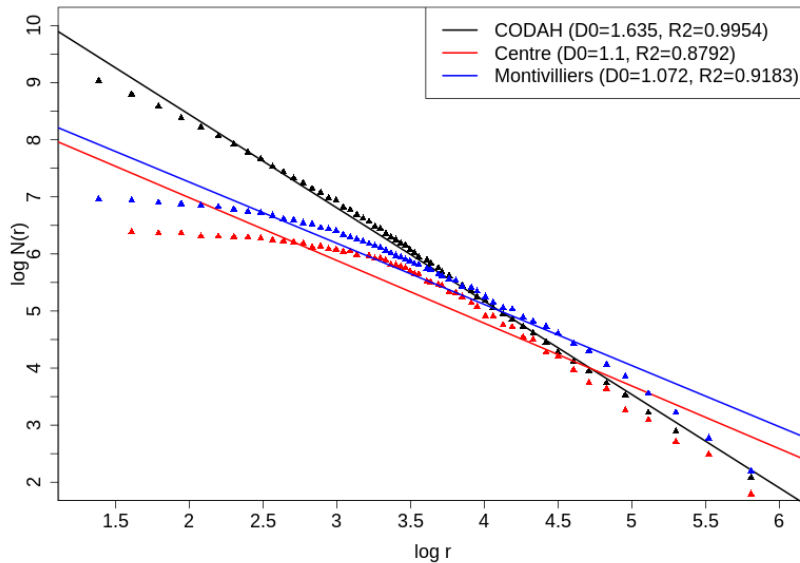
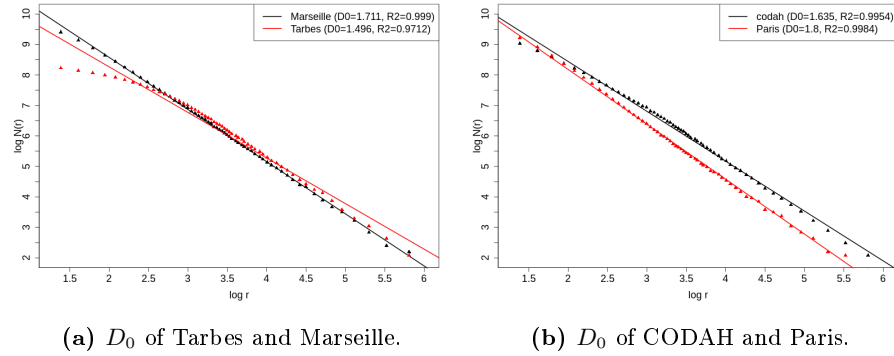


Figure 4.18

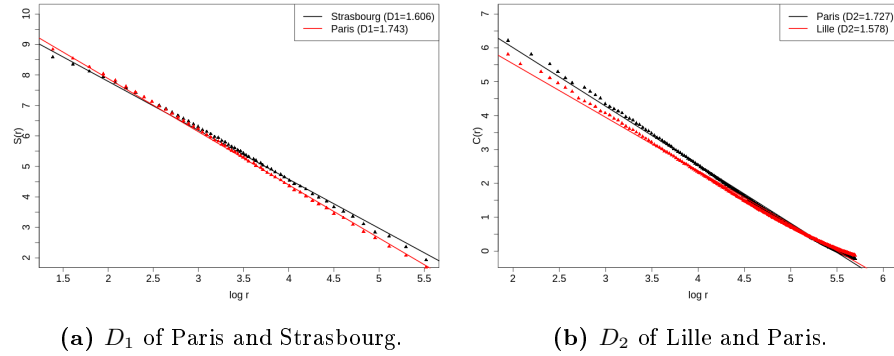
The capacity dimension D_0 of the CODAH and two sub-areas (the rebuilt area of Le Havre and Montivilliers). The logarithm of the number of not-empty boxes $N(r)$ versus the the logarithm of the normalized size r .

Figure 4.19

The capacity dimension D_0 of Tarbes, Marseille, CODAH and Paris. The logarithm of the number of not-empty boxes $N(r)$ versus the logarithm of the normalized size r .

**Figure 4.20**

The information D_1 Strasbourg and Paris and the capacity dimension D_2 of Paris and Lille. The entropy of not-empty boxes $S(r)$ versus the logarithm of the normalized size r . The logarithm of the correlation sum $C(r)$ versus the logarithm of the length r .

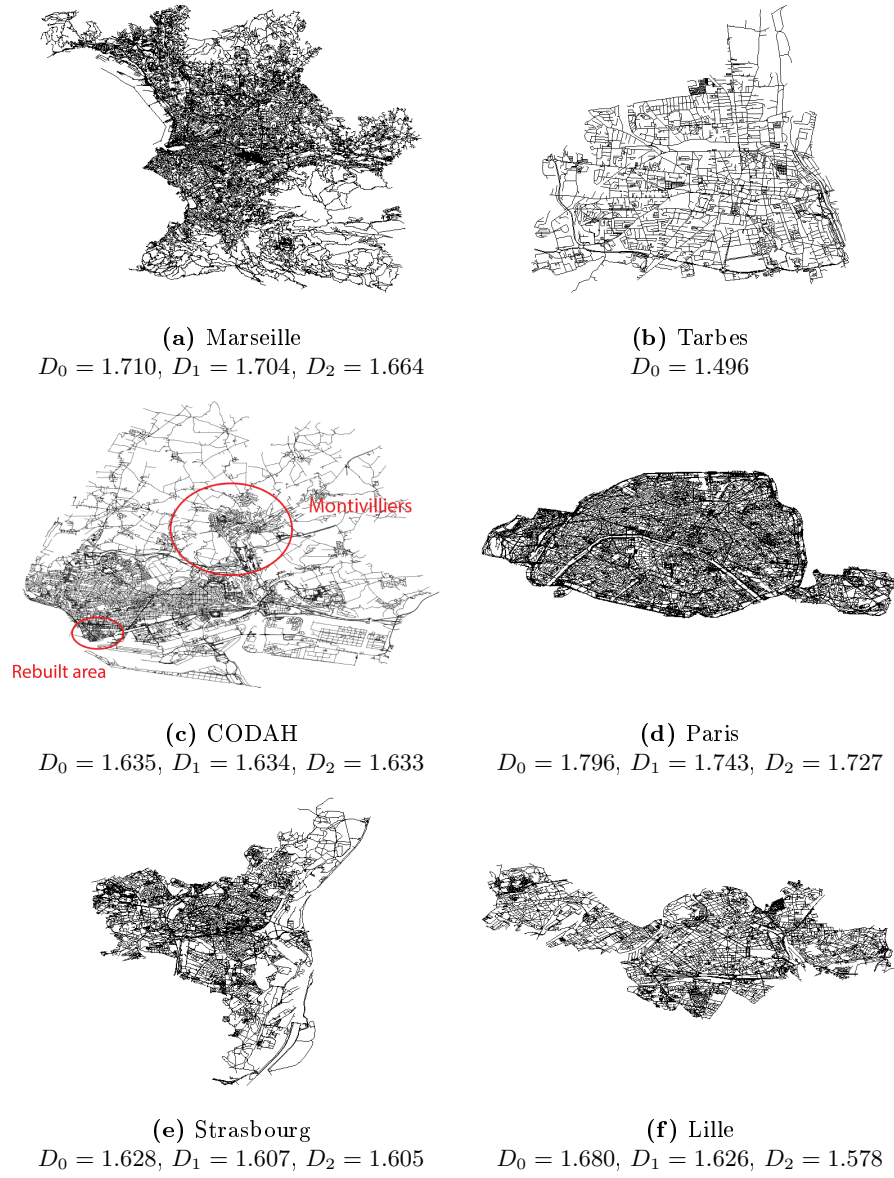


the regression line, they also shows local variations. More precisely, when we study the capacity dimension of CODAH, it seems that values fits two lines. That confirms the remarks of Tannier and Pumain [277] about the regression lines. The study of multi-fractality of CODAH at a local scale may reveal interesting and more detailed results. We are aware of limits of the proposed measure. We do not exclude to improve our methods in future works, defining procedures that detects adequate scales, local variations, or different fitting lines. Our goal is to present applicable measures to real cases and to our simulations and to compare results.

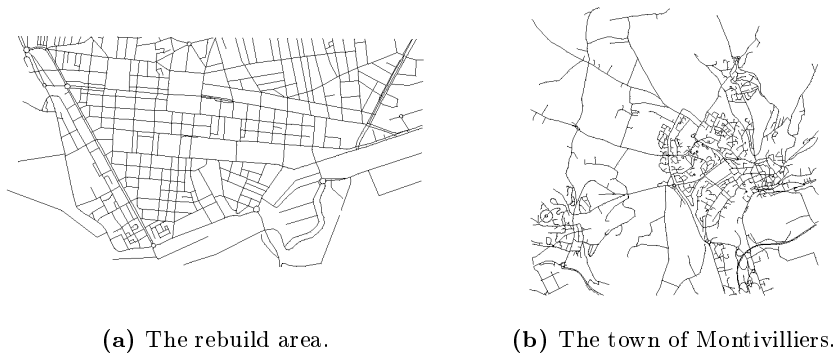
As stated above, the drawback of the capacity estimation is that it is a purely geometric notion and that is not sensitive to the density of elements. The information dimension D_1 (fig. 4.20a) of Strasbourg (fig. 4.21e) and Paris (fig. 4.21d) present a distribution of vertices more uniform over the city. Otherwise, Strasbourg presents sub-urban areas less dense than others: this effect is captured by the low value of D_1 . The fig. 4.20b depicts the correlation dimension D_2 distribution of Lille (fig. 4.21f) and Paris (fig. 4.21d). The lower value obtained for Lille indicates a sparse distribution of vertices; otherwise, Paris presents a more compact and ordered structure.

Summary 13:
scale-invariance
organization

The fractal theory provides a structured approach to characterize the spatial arrangement of vertices of street networks. Real objects (as street networks) can be fractal local or global, mono or multi fractal. Different estimations allow us to study street intersection network over a range of scales of observation.

**Figure 4.21**

Six emblematic street intersection network and the corresponding fractal dimension.

**Figure 4.22**

Two sub-areas of CODAH.

Chapter 5

Morphogenesis through the reaction diffusion theory

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Keywords Reaction diffusion theory, Turing model, Cellular Automata, Gray-Scott model.

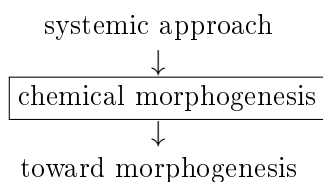
*"My life seemed to be a series of events and accidents.
Yet when I look back, I see a pattern".*

Benoit B. Mandelbrot

In chapter 1 we observed that, in some circumstances, from the the decentralized behaviour of elements of a system at the microscopic level of observation arises unexpected properties. Those properties does not characterize the element, they are the results of their organization. At a high level of observation, we can observe the formation of patterns.

In this chapter we will focus on the pattern formation and on methods to simulate it. We start with the basic notions of physical chemistry and the reaction-diffusion theory (section 5.1). We will describe basic dynamics and we will concentrate our attention on Gray-Scott differential equation system. In section 5.2, inspired by cellular automata, we will formalize a way to simulate the formation of spatial patterns in a discrete media.

5.1 Morphogenesis and physical chemistry



In chemistry a solution is a mixture of at least two substances (also named compounds). The concentration s of a substance S in a solution is the relative quantity of it in a unit of volume. It ranges in $[0, 1]$.

We can find similarities with spatial systems. The spatial concentration of an element is the ratio of the abundance of elements with similar properties to the quantity of all elements in a region of space. The notion of spatial concentration is distinguished from the spatial distribution because the first is a relative measure and the second an absolute measure.

5.1.1 Dynamic equilibriums

The chemical equilibrium is the condition in which concentrations' compounds in a solution do not change in a defined time period. Internal physical (e.g. forces and exchanges) and chemical (e.g. reaction and autocatalysis) balance each other so there are no change. Some global properties can appear in a chemical system without the properties of the reactants, the product of the reaction, the nature of interaction have changed. The change is the spatial distribution of the interactions. Thus, say that a solution is in a dynamic equilibrium does not mean that there are not evolutive processes inside the solution. It means that concentrations do not change in a fixed period of time.

The formation and the transformation of compounds often involve a chemical reaction. During these processes, one or more compounds (the reactants) are converted to one or more compounds (the products).

In a majority of chemical systems, we cannot observe spatial patterns. The second law of thermodynamics affirms that, in a closed system, the order

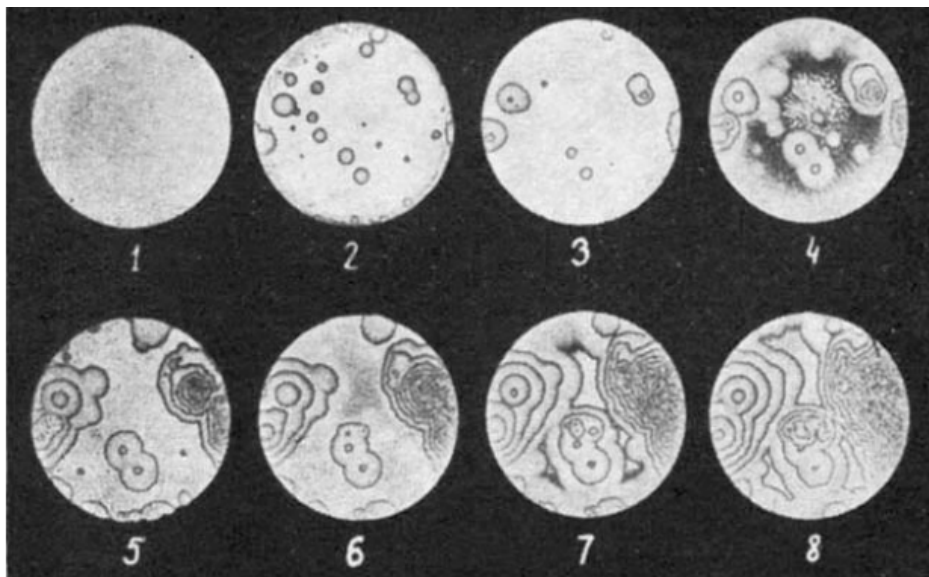
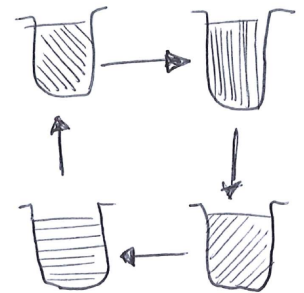


Figure 5.1
The Belousov-Zhabotinsky reaction. Source [305]

progressively decreases and disappears¹. The diffusion reduces the order of molecules, and gradients of concentration go to a homogeneous concentration. The initial order of two separated and not reactive compounds, due to the diffusion, disappears.

The clock reaction² is a chemical process that spontaneously exhibits temporal periodicity. It illustrates that homogeneous concentrations of at least two compounds can generate instabilities [231]. During the chemical process, the concentrations of compounds oscillate in the solution and the system is not at the chemical equilibrium. Belousov [34], while he was studying the Krebs' cycle³, noted a periodic change of concentrations in a homogeneous solution. Twenty years later, Zhabotinsky observed that in these kinds of reactions, the formation of oscillations is not only temporal but also spatial. Figures 5.1 and 5.2 depict the Belousov-Zhabotinsky reaction, the propagation of a spatial organization in a clock reaction with a series of sixteen photos⁴. The propagation of the concentrations in the form of bans or waves is due to the different speeds of the reaction. After the reaction depleted the material, the gradients of concentration disappear due to the diffusion of them and space disorganization takes place. As Glansdorff and Prigogine [122] affirmed, the Belousov-Zhabotinsky reaction is an example of a dissipative process. Organizations appear while energy and matter dissipates. Organizations disappear after the matter is exhausted.



Belousov and Zhabotinsky suggested that the different speeds of reacts are the key to understand the formation of gradients of concentration. To understand the diffusion phenomena of a compound in a solution we should consider it at two different scales of observation. At the microscopic scale, compounds are unstable and move in the solution in a not predictable manner. The chemical and the physical properties of compounds contribute

¹It implies that the total quantity of useful energy is constantly decreasing.

²Also named periodic or oscillator reaction.

³The Krebs' cycle is a series of chemical reactions used by all aerobic organisms to release stored energy.

⁴In fig. 5.1 we select first eight photos.

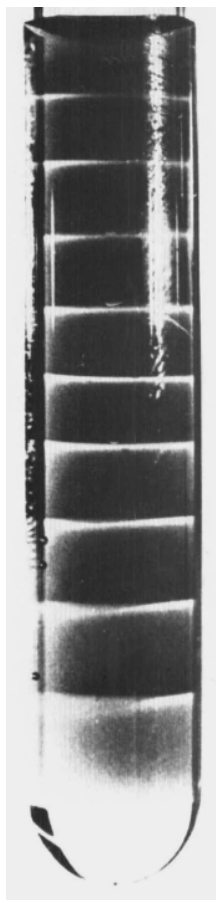


Figure 5.2
The bands formation in a Belousov-Zhabotinsky reaction. Source: [97].

to the speed of the movement, which in turn defines the position of compounds. At the macroscopic scale, the flow of moving compounds tends to increase the order. The different speeds of reacts, the catalytic mechanisms, and the concentration differences generate organizations.

5.1.2 The reaction-diffusion theory

The reaction-diffusion theory aims to explain the pattern formation in nature by the coupling of two physico-chemical processes. The study of the autocatalysis [183], the oscillation of reactions [34], the propagation of dominant genes [103, 163], and the embryos' formation [240] mark the beginning of the theory (see [129] and the references therein). These studies are supported by models (see section 5.1.3).

A contribution in this field was done by Alan Mathison Turing [285]. He wanted to answer a simple question: how life might develop from a stable and homogeneous mass of cells? He wanted to understand, at least at a simplified and abstract level, how a mass of identical cells, symmetrically arranged, could break the symmetry, differentiate themselves, generating the wealth of natural forms. His idea is that a locally asymptotically stable equilibrium can be destabilized by adding diffusion, and it is the combination of reaction and diffusion that creates shapes, results of heterogeneous concentrations. He suggested that in a living system, some elements are able to drive the form; he named these elements morphogens⁵. The neologism indicates an element form-producer, that contributes to the formation and the maintaining of the system. In a system, there exists different type of elements and some of them may be the source of formation and the growth of a living system. These morphogens not only move and interact with each other but also self regulate their own production. The second fundamental idea of Turing was indeed to couple the diffusion with an autocatalytic reaction.

According to Turing, the laws of physical chemistry are sufficient to explain many aspects of morphogenesis. The reaction-diffusion theory may apply to the study of biological phenomena [13, 136, 205]. From early applications of reaction-diffusion theory, the authors has compared mathematical models to biological patterns [206, 207]. In [177] authors show that the formation of bans of calcium in a cytosol solution can be stylized by a reaction-diffusion mechanism. Tabony and Job [273, 274] had investigated (*in vitro*) the self-organization of microtubules, suggesting that those biochemical systems are dissipative and governed by reactive and diffusive mechanisms. Microtubules spontaneously form in an initial homogeneous media [121].

A consensual and global explication of pattern formation in biology with reaction-diffusion models is not yet accepted and the goal of this work is not to contribute in this field. Anyway, as regards our studies, this theory respects the basic principles of complexity and reaction-diffusion models

⁵Today in biology, a morphogen is a gene that encodes a protein or lipid whose function is to produce a concentration gradient of a specific molecule. This concentration's role is to give position information and induce another cell to differentiate.

simulate the morphogenesis.

5.1.3 The Turing model

Reaction-diffusion models (also named Turing's models) simulate the spatio-temporal evolution of two or more kind of morphogens. They are based on a reaction-diffusion mechanism. These models provide a way of creating patterns where no patterns were initially present. The concentrations of two kind of morphogens A and B change at each position x in a continuous time t . These evolutions are expressed by differential equation system⁶:

$$\begin{cases} \frac{\partial a}{\partial t} = D_a \nabla^2 a + F(a, b) \\ \frac{\partial b}{\partial t} = D_b \nabla^2 b + G(a, b) \end{cases} \quad (5.1)$$

where D_a, D_b are diffusion rates, ∇ is the Laplacian operator (see section 5.1.5), $a = a(x, t)$ and $b = b(x, t)$ are the concentration of two chemical substances, functions of position x and time t , $F(a, b)$ and $G(a, b)$ are the reaction functions. Turing [285], pointed out that these functions integrate two processes: the autocatalytic production and the degradation of concentration. In his paper [285], he proposed and given a mathematical resolution of a linear formalization of functions $F(a, b)$ and $G(a, b)$:

$$\begin{cases} \frac{\partial a}{\partial t} = D_a \nabla^2 a + f_1 a + f_2 b + f_3 \\ \frac{\partial b}{\partial t} = D_b \nabla^2 b + g_1 a + g_2 b + g_3 \end{cases} \quad (5.2)$$

where the production and degradation are linearly controlled by their own concentrations. For easy reading, eq. (5.2) may be re-written in matrix notation:

$$\frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} D_a & 0 \\ 0 & D_b \end{pmatrix} \begin{pmatrix} \nabla^2 a \\ \nabla^2 b \end{pmatrix} + \begin{pmatrix} f_1 & f_2 \\ g_1 & g_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = D \begin{pmatrix} \nabla^2 a \\ \nabla^2 b \end{pmatrix} + J \begin{pmatrix} a \\ b \end{pmatrix} \quad (5.3)$$

where, constants f_3 and g_3 could be omitted. The study of stability shows that there exists a solution if conditions:

$$\begin{cases} \text{tr}(J) = f_1 + g_2 < 0 \\ \det(J) = f_1 g_2 - f_2 g_1 > 0 \end{cases} \quad (5.4)$$

are respected. To do that the signs should be:

$$\begin{pmatrix} + & + \\ - & - \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} + & - \\ + & - \end{pmatrix}$$

Observing the signs, we can infer that a necessary condition to have instability is that there is some kind of antagonism between the production

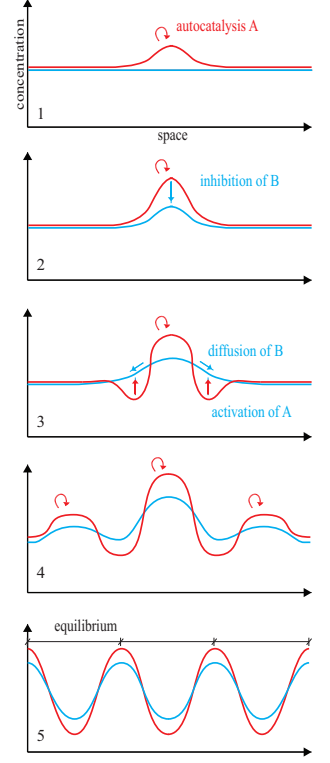


Figure 5.3

The stylized representation of the pattern formation in an activator-inhibitor system.

⁶A dynamical system is a model in which a function, or a system of functions, describes the time dependence of particles in space. He must not be confused with the definition of systems used in this dissertation.

and the degradation of morphogens. For interested readers, they can find a complete presentation of stability analysis in [206].

A solution of the eq. (5.2) was proposed by authors in [165]: they reports that the system produces six different propagations of concentration (fig. 5.4), but only the last two present organizations (cases V and VI in fig. 5.5). The case V reproduces the oscillation of waves, which propagate from an initial area where compounds are not in equilibrium.

Waves shape in spiral patterns and the mathematical model reproduces a phenomenon similar to the empirical observations of Zhabotinsky in [305]. Case VI represents the situation in which Turing patterns appear. The domain of parameters is called the pattern formation space (or Turing space).

The reaction at the same time increases the concentration a of the compound A and decreases the concentration b of the compound B (fig. 5.3). Thanks to a located perturbation, where a in a located area is greater than b , the auto-catalytic reaction increments the speed of the reaction, generating the production of a and increment the delta between a and b in the located area. The different speeds of the diffusion of the two compounds contribute to the oscillation of the reaction: B , which diffuses faster than A , $b > a$ around the initial perturbation⁷. The production of A and B decreases, the reaction becomes less important and the system going to an apparent equilibrium. However, around the initial perpetuated point, concentrations a and b are different, providing new auto-catalytic reactions. The system lost its initial homogeneous state, new reactions destabilize the equilibrium and the diffusion allows the formation of dynamic patterns. The symmetry is break and the homogeneity is lost: the model, in a stylized and simplified way, answers the Turing's question.

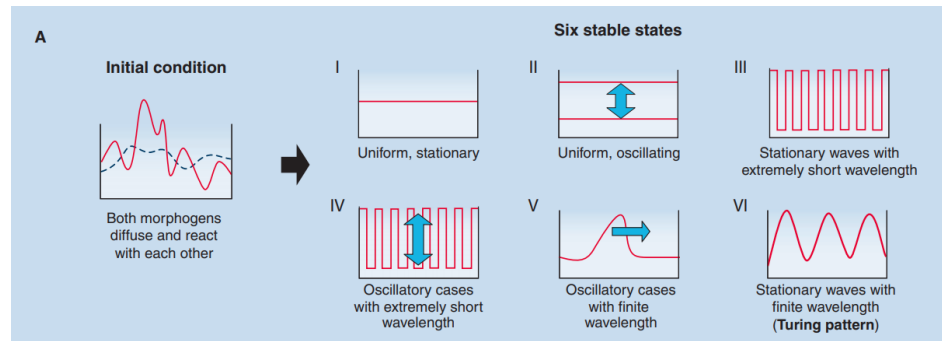


Figure 5.4
Six interesting cases described by Turing. Source: [165].

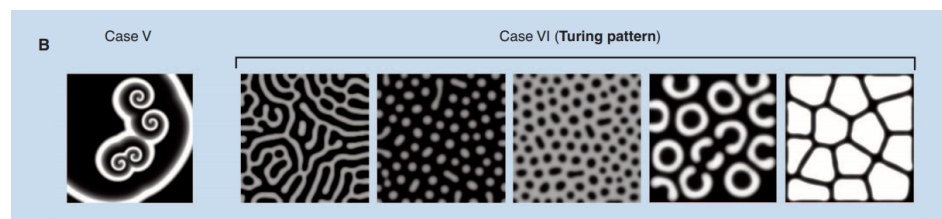


Figure 5.5
The cases V and VI of a reaction-diffusion. Source: [165].

The most popular reaction functions used in reaction-diffusion models:

The Schnakenberg model [259] : The Lengyel-Epstein model [180]:

$$\begin{cases} F(a, b) = f_1 - f_2 + f_3 \frac{a^2}{b} \\ G(a, b) = g_1 - g_2 \frac{a^2}{b} \end{cases} \quad (5.5) \quad \begin{cases} F(a, b) = f_1 - a - \frac{4ab}{1+a^2} \\ G(a, b) = g_1 \left(a - \frac{ab}{1+a^2} \right) \end{cases} \quad (5.8)$$

The Sel'kov model [260]:

$$\begin{cases} F(a, b) = 1 - ab^{f_1} \\ G(a, b) = g_1 b(ab^{f_1-1} - 1) \end{cases} \quad (5.6) \quad \begin{cases} F(a, b) = f_1 - f_2 a - H(a, b) \\ G(a, b) = g_3 - g_4 b - H(a, b) \end{cases} \quad (5.9)$$

The Gierer-Meinhardt model [119]:

$$\begin{cases} F(a, b) = f_1 \frac{a^2}{b} - f_2 a + f_3 \\ G(a, b) = f_1 a^2 - g_1 b + g_2 \end{cases} \quad (5.7) \quad \text{where} \quad H(a, b) = (h_1 ab)/(h_2 + h_3 a + h_4 a^2) \quad (5.10)$$

Frame 4:
Reaction-diffusion
models

5.1.4 The reaction

Functions $F(a, b)$ and $G(a, b)$ in eq. (5.1) characterize the kinetics of chemical reaction. In the Turing formalization, they are linear. In frame 4 we report the most popular reaction functions. The initial Turing concept inspired therefore many other formalizations of these ideas. An example is the Schnakenberg model [259] and the Sel'kov model [260], where the autocatalytic production is quadratic and the reduction is independent of the concentration of compounds. Another field of models are called activator-inhibitor models (fig. 5.6): the basic idea of the kinetic in these models is that the compound A catalyzes its own production but also the production of B . At the same time, B inhibits the production of A . B diffuses faster than A . The Gray-Scott [127] model (see eq. (5.14) in section 5.1.7), the Gierer-Meinhardt [119], the Murray-Thomas [205] model and Lengyel-Epstein [180] model are examples of this kind of kinetics.

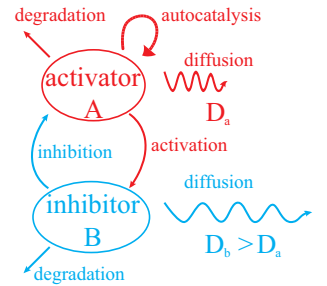


Figure 5.6
Stylized representation of an
activator-inhibitor system.

5.1.5 The Laplacian operator

The classical approach⁸ of diffusion is based on the principle of conservation of mass: the Fick's laws [102]. It affirms that the flow J of a compound C ⁹ is proportional to the gradient of the concentration c of the compound. In

⁷We obtain the same if B diffuses slower than A .

⁸This approach of diffusion is only applicable to a dilute media, where the concentration is not constant.

⁹Here we refer to diffusion in physical chemistry, but the flow could be an exchange of material, information, energy, individuals, etc.

one dimension:

$$J \propto -\frac{\partial c}{\partial x} \Rightarrow J = D \frac{\partial c}{\partial x} \quad (5.11)$$

where $c = c(x, t)$ is a function of time and space and D is the diffusion coefficient or diffusivity of the compound c ; the minus sign indicates that the flux is from a high to a low concentration (fig. 5.7). If the conservation of matter is ensured¹⁰ and the rate D is constant¹¹, the eq. (5.11) becomes the classical diffusion equation:

$$\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x} = D \frac{\partial^2 c}{\partial x^2} \quad (5.12)$$

For two or more dimensions, assuming that c is continuous and derivable, applying the divergence theorem the last equation becomes:

$$\frac{\partial c}{\partial t} = D \nabla^2 c \quad (5.13)$$

where ∇^2 is a Laplacian operator, which indicates the diffusion of matter in more than one direction. Modelling the Laplacian operator is a fundamental task in a reaction-diffusion model and each different way has an impact into resulting patterns.

5.1.6 The diffusion drives the instability

Turing pointed out that the different diffusion rates (D_a and D_b) are crucial to have spatial patterns. If we do not have diffusion ($D_a = D_b = 0$) in eq. (5.1), the concentrations a and b tend to a linearly stable uniform steady state. Otherwise, in presence of diffusion, the formation of patterns is driven by the ratio $d = D_a/D_b \neq 1$. For this reason, the study of the dynamic of the system are often studied on observing the behaviour of the system in relation to the rate of diffusion. A reaction-diffusion system exhibits a Turing instability (to underline the role of the diffusion, it is also named diffusion-driven instability) if the homogeneous steady state is stable to small perturbations in the absence of diffusion but unstable to small spatial perturbations when diffusion is present.

5.1.7 The Gray-Scott model

The Turing concept opens a new field of experiments and several variants of the Turing model was proposed. Among these proposition, we focus on the Gray-Scott model. Essentially, the reaction of two morphogens A, B consists of 3 mechanisms (fig. 5.8):

- $A + 2B \rightarrow 3B$
a morphogen A reacts with two morphogens B and A transforms into a morphogen B .

¹⁰The rate of change of the amount of material in a region is equal to the rate of flow across the boundary.

¹¹For the sake of simplicity, we focus on situations where D is not space and time dependent; however, we are aware of an arising of modelling situations where D can vary in the space and in time.

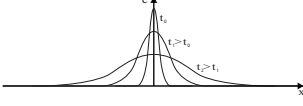
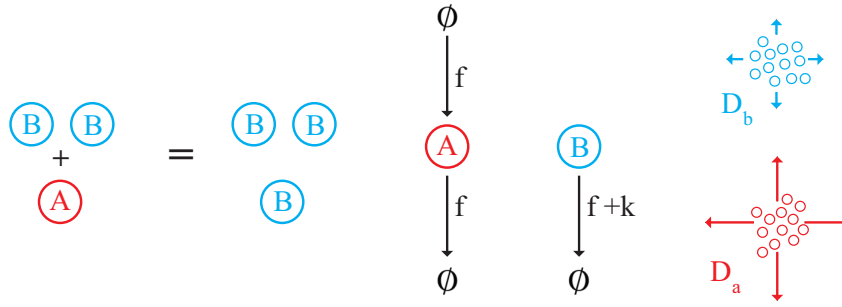


Figure 5.7
The stylized representation of concentration in the time.

**Figure 5.8**

Schematic representation of Gray-Scott model. The reaction of three morphogens and the different rate of diffusion in the Gray-Scott model.

- $\emptyset \xrightarrow{f} A \xrightarrow{f} \emptyset$
morphogens A are added or removed with the "feed" rate f .
- $B \xrightarrow{f+k} \emptyset$
the concentration of morphogens B decays with a rate $(f + k)$, where k is the a "kill" rate; the sum is larger than the one controlling the decay of A .

After rescaling and including a diffusion rate, the evolution in a continuous media may be written as:

$$\begin{cases} \frac{\partial a}{\partial t} = D_a \nabla^2 a - ab^2 + f(1 - a) \\ \frac{\partial b}{\partial t} = D_b \nabla^2 b + ab^2 - (f + k)b \end{cases} \quad (5.14)$$

where D_a, D_b are diffusion rates, ∇^2 the Laplacian operator, $a \in [0, 1], b \in [0, 1]$ concentrations of morphogens A and B , f and k are the feed and the kill rate.

A reaction diffusion model is composed by two distinct mechanisms, which are coupled together. These last are modelled by systems of differential equations and, depending to the parameter set and the initial conditions, the model could show (or not) some patterns. We consider a reaction-diffusion system as the representation of some mechanisms of interactions that are represented by reaction equations and are related to real phenomena. The diffusion simulates the movement of these elements. Remarks: (1) the production and the degradation of morphogens are necessary but not sufficiency conditions to have the pattern formation; (2) morphogens must move with different speeds.

Summary 14:

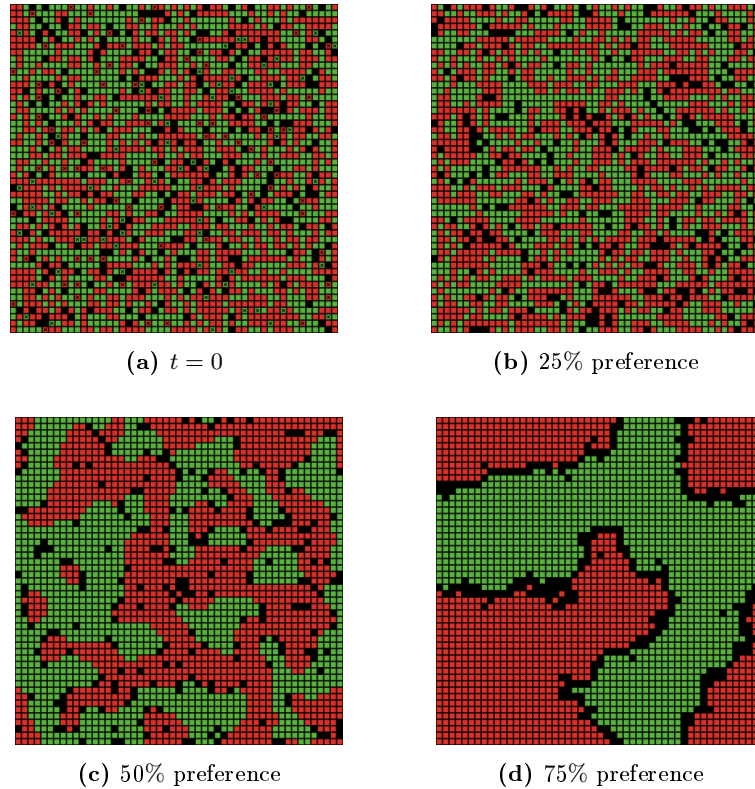
The reaction-diffusion theory

5.2 Simulating the chemical morphogenesis

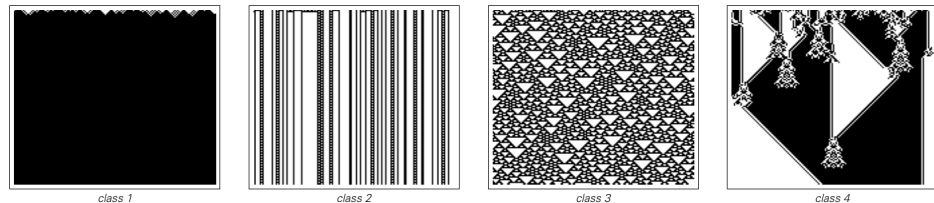
Dynamic reaction-diffusion systems are currently modelled by differential equations which can allow us to determine the global (macroscopic) behaviour, by integrating them. We work with this representation in a continuous spatio-temporal space. To model interactions and displacements, we need to locate temporal and computable data. We need a deterministic rewrite of the dynamic system. It should evolve over discrete time and

Figure 5.9

The Schelling's model [258]. A 2 dimensional array is initialized with 10% of cells black (empty cells) and the remaining cells red or green cells (two ethnic groups, fig. 5.9a). The transition function consists to simultaneously exchange red and green cells to the black one. Simulation stops if the neighbourhood of each red and green cells contains at least one (fig. 5.9b), two (fig. 5.9c) or three (fig. 5.9d) cells like it.

**Figure 5.10**

The Wolfram classification. Source [301].



discrete space. Cellular automata allow simulation of discretized physical systems and are therefore a possible choice¹².

A cellular automata is defined by:

- **The dimension d :**
it is the integer which defines the dimension space $\mathbb{N}^d$ in which the automaton operates. Elements of the space are called cells.
- **The state set S :**
it is the finite space which defines the different states that a cell can take.
- **The neighbourhood N :**
it is the set of cells assigned to each cell. There are several definitions of neighbourhood (see frame 5 and fig. 5.11).
- **The transition function F :**
let T be a discrete time series, the function F is the set of rules

¹²Finite differences is another possible choice.

The neighbourhood of a cell can be defined in different ways. Figure 5.11 shows two most frequently used neighbourhoods in two dimensional space. Let $n = (i_0, j_0)$ be a cell, the Von Neumann neighbourhood (fig. 5.11a) is:

$$N_{vn}^r(n) = \{(i, j) : |i - i_0| + |j - j_0| \leq r\} \quad (5.15)$$

and the Moore neighbourhood (fig. 5.11b) is:

$$N_m^r(n) = \{(i, j) : |i - i_0| \leq r, |j - j_0| \leq r\} \quad (5.16)$$

where $r \in \mathbb{N}$ is the radius and $i, j \in \mathbb{N}$ are the positions of the cell.

Frame 5:

The neighbourhood

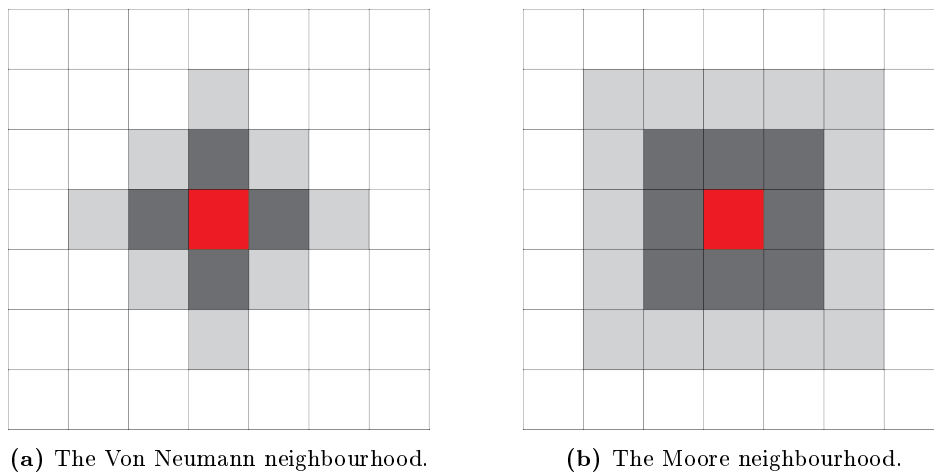


Figure 5.11

The Von Neumann and the Moore neighbourhood of a cell in 2-dimensional space. Red indicates the core cell, dark gray and light gray indicates the neighbourhoods of $r = 1$ and $r = 2$ respectively.

that define the transition from the configuration at time step t to the configuration at time step $t + 1$.

Cellular automata were originally introduced by Von Neumann and Ulam in 1940s as a representation of biological systems [208]. The capacity of cellular automata is that they are able to produce in an intuitive framework a variety of complex situations with simple transition rules. The wealthy of possible configurations of the 2-dimensional cellular automata "game of life" of Conway and the intricate patterns generate by the 256 transition rules (for a 1-dimension array) by Wolfram [301] are a simple example of the capacity of cellular automata to simulate behaviours of complex systems (frame 6).

If we associate to each cell a coordinate in a Euclidean space $\mathbb{R}^d, d = 2, 3$, a cellular automata becomes a mathematical idealization of a spatial system. The application of a cellular automata, declined as a regular lattice mapped to geographical information, in geography may provide a better understanding of human and natural phenomena [37]. Applications in geography of cellular automata are: the study of urban land-use dynamics [298], urban development [30] and social segregation [258]. The Schelling's model [258] is probably the most known application of cellular automata in the study of spatial segregation of ethnic groups. A regular grid, three states (green, red, and black), and a simple "what-if" rule (prefer cells in your state). The

Frame 6:
The Wolfram
classification of cellular
automaton

The classification of behaviour of cellular automaton is still a pending question. However, the classical proposition of Wolfram [301] is an useful *Ariadne's thread* in many cases (fig. 5.10). In a one-dimensional array of cells with two possible states (black or white), the Wolfram classification identifies four behaviours. Ilachinski [149] summarizes the different states described by Wolfram as follow:

- **Class 1:**
nearly all initial patterns evolve quickly into a stable, homogeneous state. Any randomness in the initial pattern disappears.
- **Class 2:**
nearly all initial patterns evolve into stable or oscillating structures. Some of the randomness in the initial pattern may filter out, but some remain. Local changes in the initial pattern tend to remain local.
- **Class 3:**
nearly all initial patterns evolve in a pseudo-random or chaotic manner. Any stable structures that appear are quickly destroyed by the surrounding noise. Local changes to the initial pattern tend to spread indefinitely.
- **Class 4:**
nearly all initial patterns evolve into structures that interact in complex and interesting ways, with the formation of local structures that are able to survive for long periods of time.

simulation shows that behind the spatial formation of discriminatory patterns there are intuitive and local processes of iteration (fig. 5.9). Despite it does not take into account socio-economic factors or discriminative processes, it is one more powered social segregation model [18]. The Shelling's model suggests that a city (space) may be highly segregated (patterns) if people (cells) prefer to stay among people (neighbourhood) similar (state) to them.

5.2.1 The morphogen layer

A compound is a specific kind of chemical element; several kinds of elements are present in a chemical solution and they are arranged in the space. The concentration of a compound is the number of elements in the unity of space. Therefore, we have two different levels of observation (fig. 5.12): (1) a lower level, which corresponds a lower level of aggregation, each element (of different compounds) are arranged in the Euclidean space; (2) the upper level, which corresponds to a greater level of aggregation, the Euclidean space is observed as a series of cells, which overlaps a set of elements. We can therefore deduce a concentration of each kind of compound in each cell. Inspired by cellular automata theory and the Gray-Scott model, we formally

define the morphogen layer in 2-dimensional space¹³ as follows:

A morphogen layer $L_m(t)$ is defined by:

- **The space L :**

it is a subspace of $\mathbb{N}^d$, where the dimension space is $d = 2$. Cells are noted as (n_1, n_2) .

- **The state set S :**

to each cell is assigned the couple of states (a, b) $a \in [0, 1]$, $b \in [0, 1]$, $a + b = 1$, which represents the concentration of morphogens A and B at each time step t .

- **The neighbourhood N :**

to each cell is assigned the Moore neighbourhood N_m^r , where $r = 1$ is the radius of diffusion.

- **The transition function F :**

Inspired by eq. (5.14), the state of a cell (a, b) at time step $t + 1$ is defined by:

$$\begin{cases} a(x, y, t + 1) = a(x, y, t) + D_a \nabla^2 a - a(x, y, t)b(x, y, t)^2 \\ \quad \quad \quad + f(1 - a(x, y, t)) \\ b(x, y, t + 1) = b(x, y, t) + D_b \nabla^2 b + a(x, y, t)b(x, y, t)^2 \\ \quad \quad \quad - (f + k)b(x, y, t) \end{cases} \quad (5.17)$$

where D_a and D_b are the diffusion parameters, ∇^2 the Laplacian operator and k and f are the feed and kill parameters. The Laplacian operator can be re-written as:

$$\nabla^2 m = D \sum_{i,j=-r}^r m(x + i + r, y + j + r, t) k_{i,j} \quad (5.18)$$

where $m = a$ or $m = b$, and the $k_{i,j}$ is an element of the bisymmetric matrix $K = (2r + 1 \times 2r + 1)$:

$$K = \begin{bmatrix} k_{i-r,j-r} & k_{i-r,j-r+1} & \dots & k_{i-r,j} & \dots & k_{i-r,j+r} \\ k_{i-r+1,j-r} & k_{i-r+1,j-r+1} & \dots & k_{i-r+1,j} & \dots & k_{i-r+1,j+r} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{i,j-r} & k_{i,j-r+1} & \dots & -1 & \dots & k_{i,j+r} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ k_{i-r,j+r} & k_{i-r+1,j+r} & \dots & k_{i+r,j} & \dots & k_{i+r,j+r} \end{bmatrix} \quad (5.19)$$

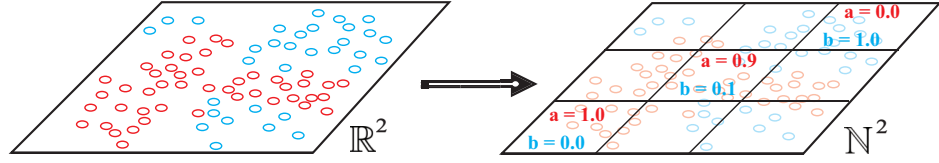
To ensure the equilibrium of mass (section 5.1.5), $\sum_{i,j=-r}^r a_{i+r,j+r} = 0$.

Definition 10:
morphogen layer

¹³The definition 10 can be easily adapted for cellular automata in three or more dimensions.

Figure 5.12

The spatial distribution of morphogens A, B in the Euclidean space $\mathbb{R}^2$ and the associated representation as a concentration state a, b in $\mathbb{N}^2$.



The following sections will explore the behaviour of the morphogen layer. They will focus on the effect of diffusive parameters, the kernel of diffusion the kill and the feed parameter. The simulations will be compared with studies that had explored the behaviour of a system of differential equations. Our goal is to validate our approach and describe the behaviour of the Gray-Scott in a discrete media.

5.2.2 Effects of diffusion

The diffusion rate plays a crucial role to the evolution of concentrations of morphogens: it depends on the value of diffusion parameters D_a, D_b , the ratio $d = D_a/D_b \neq 1$ and the kernel matrix K .

In order to evaluate the effect of the kernel into the pattern formation, we make a few basic experiments. Depending on values $k_{i,j}$, and the radius r , a matrix can cause a wide range of effects to the diffusion rate. For $r = 1$, the corresponding kernel K is a matrix 3×3 . We named the Moore kernel with radius $r = 1$ the matrix:

$$\begin{bmatrix} 0.125 & 0.125 & 0.125 \\ 0.125 & -1 & 0.125 \\ 0.125 & 0.125 & 0.125 \end{bmatrix} \quad (5.20)$$

and the Von Neumann kernel with radius $r=1$ the matrix:

$$\begin{bmatrix} 0 & 0.25 & 0 \\ 0.25 & -1 & 0.25 \\ 0 & 0.25 & 0 \end{bmatrix} \quad (5.21)$$

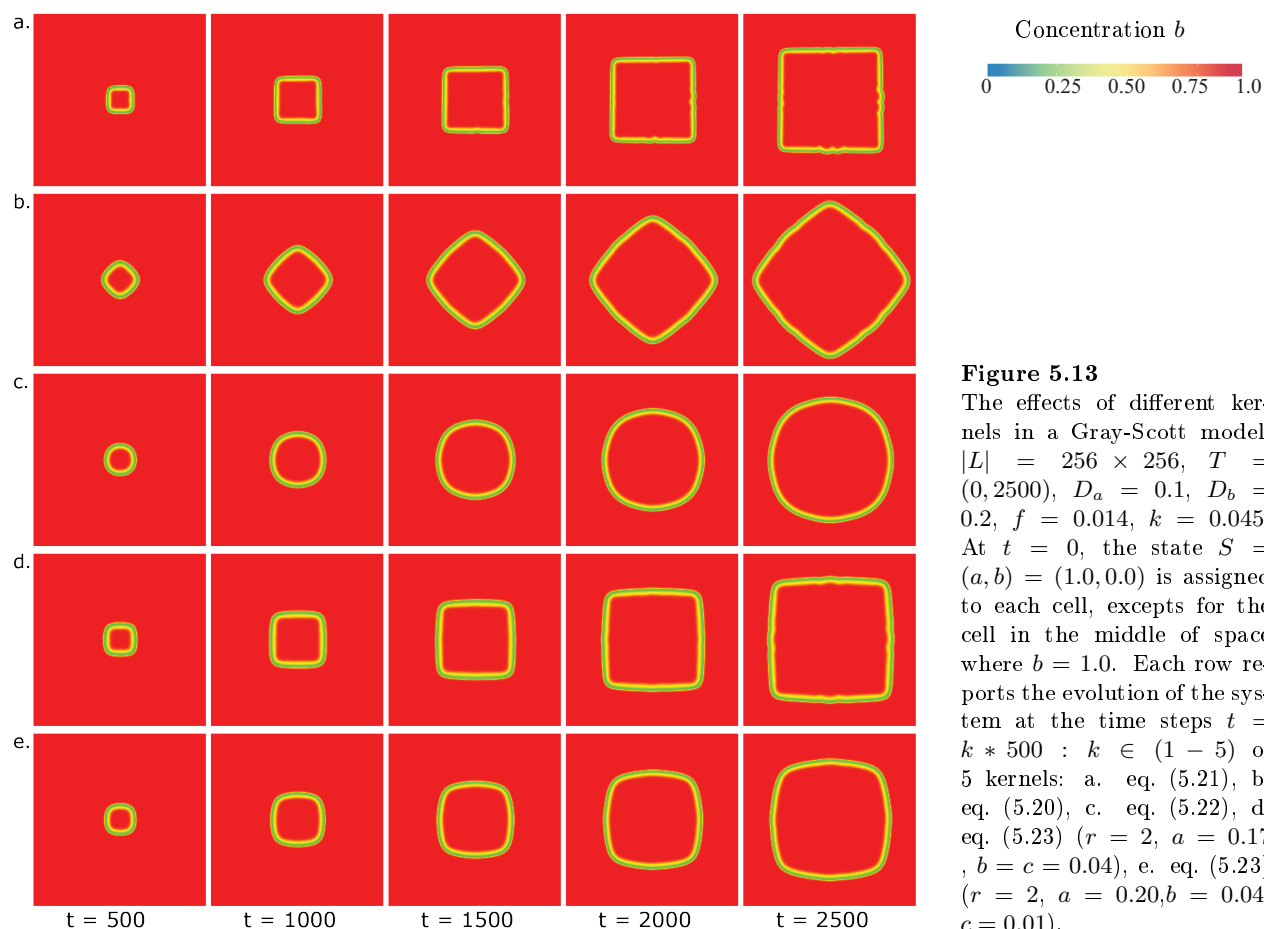
In order to approximate derivatives of the Laplacian operator, we smooth edges:

$$\begin{bmatrix} 0.05 & 0.20 & 0.05 \\ 0.20 & -1 & 0.20 \\ 0.05 & 0.20 & 0.05 \end{bmatrix} \quad (5.22)$$

or we consider a radius $r = 2$ with a progressive smooth:

$$\begin{bmatrix} 0 & 0 & c & 0 & 0 \\ 0 & b & a & b & 0 \\ c & a & -1 & a & c \\ 0 & b & a & b & 0 \\ 0 & 0 & c & 0 & 0 \end{bmatrix} \quad (5.23)$$

Figure 5.13 depicts effects of different kernel. The same initial configuration is perturbed in the middle; each row shows the evolution of the system as

**Figure 5.13**

The effects of different kernels in a Gray-Scott model. $|L| = 256 \times 256$, $T = (0, 2500)$, $D_a = 0.1$, $D_b = 0.2$, $f = 0.014$, $k = 0.045$. At $t = 0$, the state $S = (a, b) = (1.0, 0.0)$ is assigned to each cell, excepts for the cell in the middle of space where $b = 1.0$. Each row reports the evolution of the system at the time steps $t = k * 500 : k \in (1 - 5)$ of 5 kernels: a. eq. (5.21), b. eq. (5.20), c. eq. (5.22), d. eq. (5.23) ($r = 2$, $a = 0.17$, $b = c = 0.04$), e. eq. (5.23) ($r = 2$, $a = 0.20$, $b = 0.04$, $c = 0.01$).

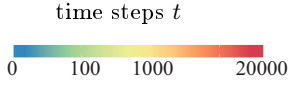
the propagation of a wave. We observe that the kernel in eq. (5.22) approximates a central symmetric. It seems that it approximates the propagation of a wave in an anisotropic space (fig. 5.13c).

Heat-maps in fig. 5.14 reports 100×100 simulations of the same initial morphogen layer. Simulations were stopped when the concentrations become stable. Red areas correspond to simulations where after 20000 steps they did not reach the equilibrium. Only simulations with a low value of the ratio $d = D_a/D_b$ show the formation of patterns; moreover, we observe in all heat-map a bifurcation curve that separates simulations where patterns appear to which where the initial gradient of concentration falls into a homogeneous state.

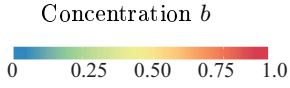
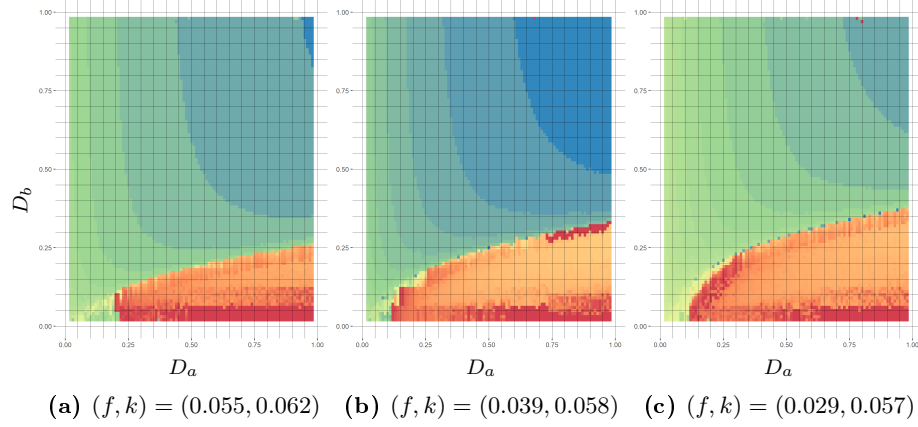
Finally, fig. 5.15 reports the effect of the values of the diffusion parameters into the model. Despite the two simulations was computed for the same initial configuration and the ratio d is the same for both cases, they produce two different patterns. It seems that the size of spots is a function to absolute values of D_a and D_b .

5.2.3 Effects of feed and kill parameters

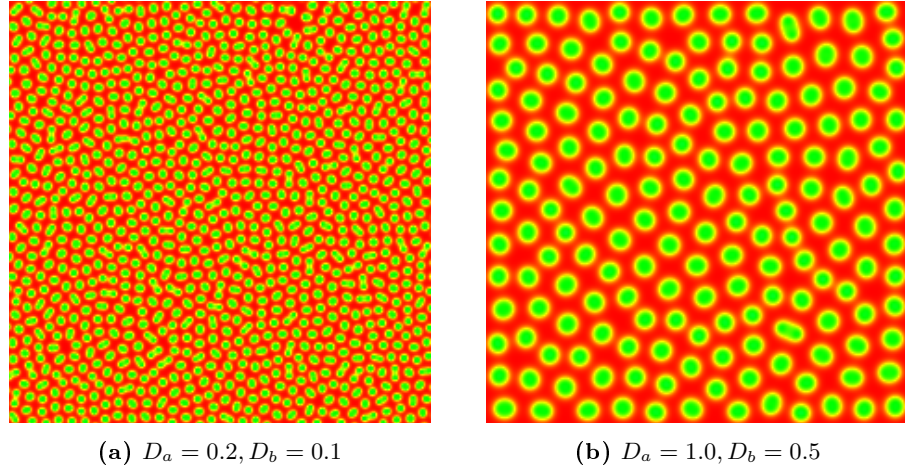
Simulations of the Gray-Scott model can lead to very different patterns using slightly different parameters. Compared to the majority of reaction-

**Figure 5.14**

The Effect of ratio $d = D_a/D_b$ in a Gray-Scott model. $|L| = 128 \times 128$. At $t = 0$, the state $S = (a, b) = (1.0, 0.0)$ is assigned to each cell, excepts for 100 cells, randomly selected, perturbed with $b = 1.0$.

**Figure 5.15**

The effects of different values of diffusion parameters in a Gray-Scott model. $|L| = 256 \times 256$, $T = (0 - 2500)$. At $t = 0$, the state $S = (a, b) = (1.0, 0.0)$ is assigned to each cell, excepts for 100 cells, randomly selected, perturbed with $b = 1.0$.



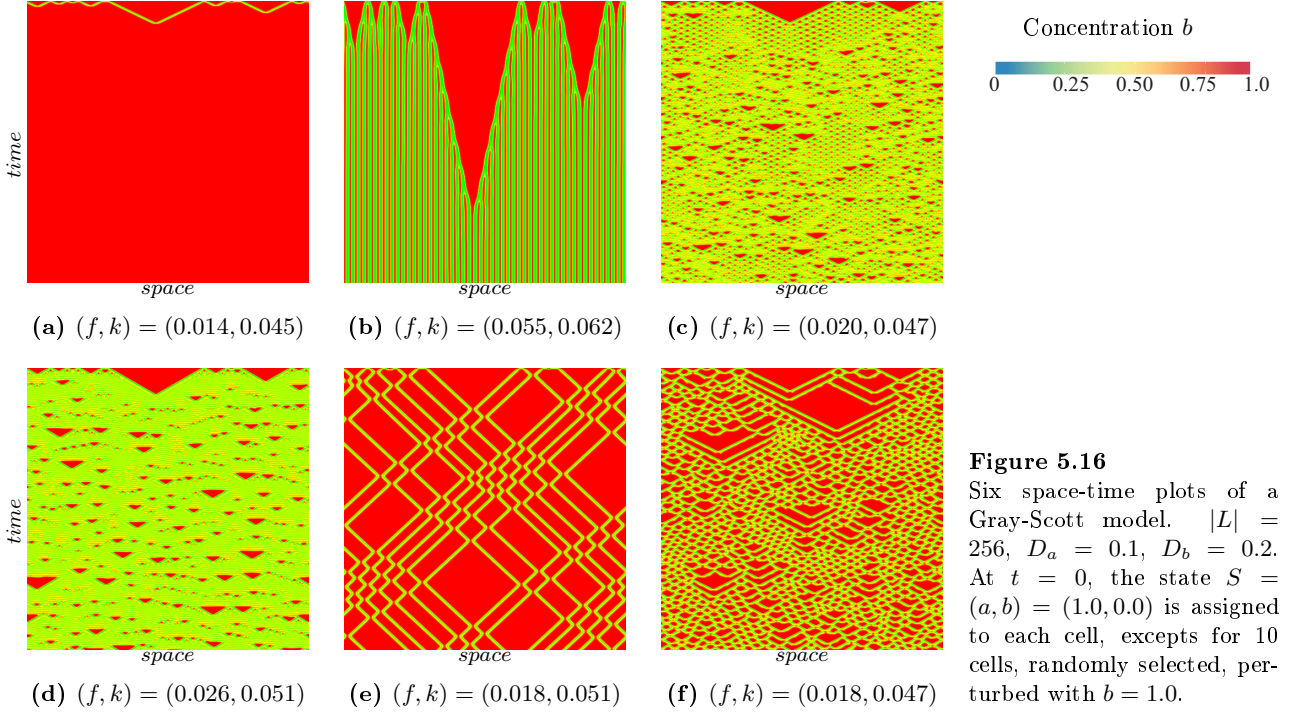
diffusion models, where production and degradation of morphogens are modelled by linear equations, the Gray-Scott model, due to the quadratic autocatalytic reaction and parameters f and k , leads to much more complicated dynamics. The Turing space is commonly defined for feed and the kill parameters.

The morphogen layer in one-dimensional space

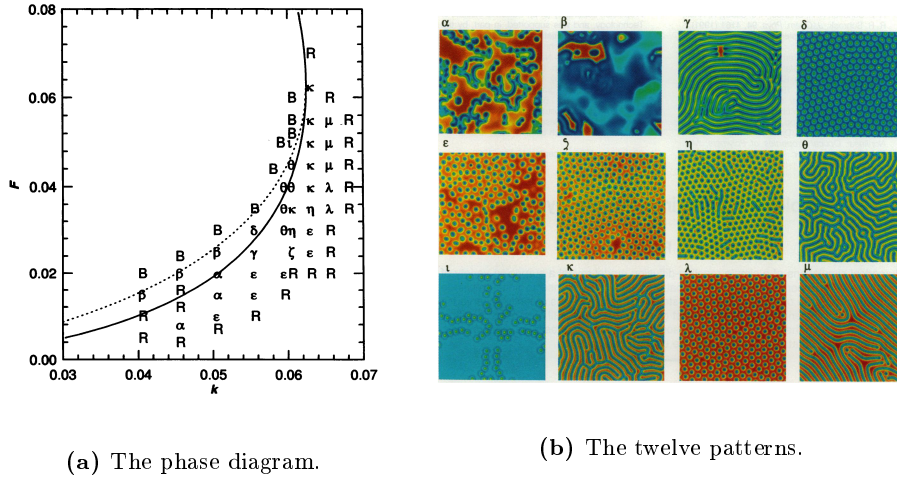
The morphogen layer can easily adapted in a one dimension space. It suffices to set different values of f and k and perturb the initial state of cells $(a, b = 1.0, 0.0)$ with a pulse ($b = 1.0$) to obtain a wealthy set of patterns (fig. 5.16). These patterns display several visual analogies with the patterns obtained by Wolfram with his cellular automata models (fig. 5.10).

The stability analysis

The study of the behaviour of the Gray-Scott model can be also done for a domain of dimensional 2. Here we report some results in the continuous domain and then the correspondences in the discrete domain. For more details in continuous media see [2, 197, 207, 297].

**Figure 5.16**

Six space-time plots of a Gray-Scott model. $|L| = 256$, $D_a = 0.1$, $D_b = 0.2$. At $t = 0$, the state $S = (a, b) = (1.0, 0.0)$ is assigned to each cell, excepts for 10 cells, randomly selected, perturbed with $b = 1.0$.

**Figure 5.17**

The Pearson classification. Section 5.2.3 depicts the (f, k) phase diagram. Latin letter R and B denote the red and the blue steady state, Greek letters some patterns reports on the right panel. Colours varying from red ($a \approx 1$) to blue ($a < 0.2$); yellow is in the middle. Section 5.2.3 depicts twelve emblematic patterns. Source: [218].

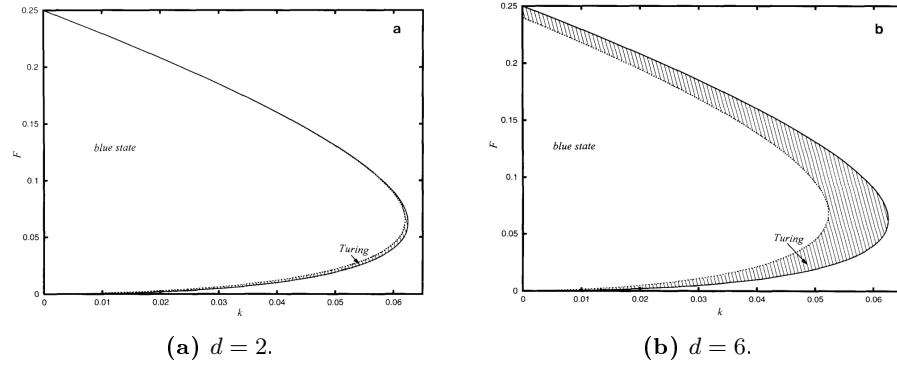
The stability analysis for Gray-Scott system shows three main solutions of eq. (5.14). Neglecting for a moment the diffusion, the homogeneous steady state solution, referred as the red state (section 5.2.3), where $(a_r, b_r) = (1.0, 0.0)$ is always linearly stable for positive values of parameters (f, k) . Under the condition that $\alpha = 1 - 4(f + k)^2 > 0$, we also have two additional homogeneous steady states solutions. The blue state :

$$(a_b, b_b) = \left(\frac{1 - \sqrt{\alpha}}{2}, \frac{f(1 + \sqrt{\alpha})}{2(f + k)} \right) \quad (5.24)$$

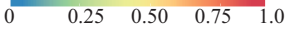
and the intermediate state:

$$(a_i, b_i) = \left(\frac{1 + \sqrt{\alpha}}{2}, \frac{f(1 - \sqrt{\alpha})}{2(f + k)} \right) \quad (5.25)$$

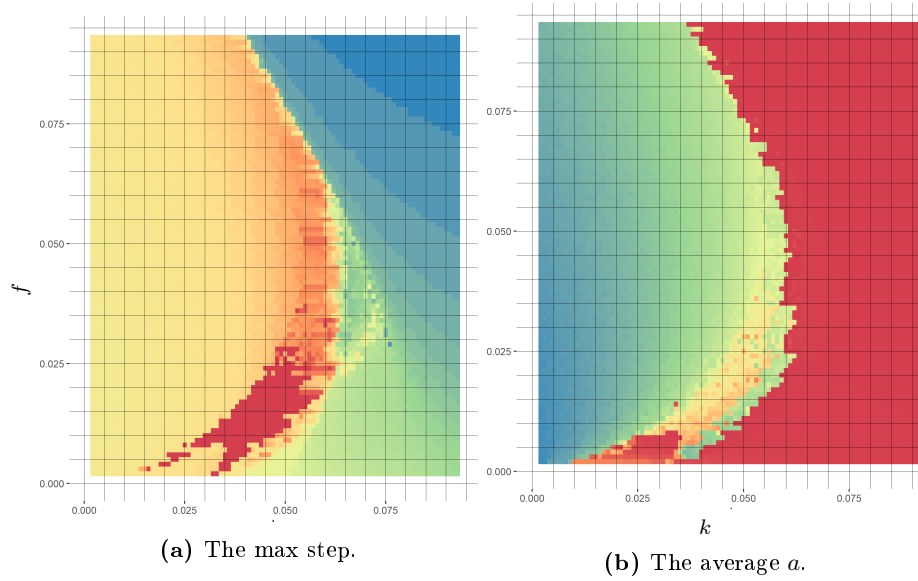
The latter couple indicates the Turing space. The blue state is a homoge-

**Figure 5.18**

The Turing space in the Gray-Scott model for two ratio $d = D_a/D_b$. Source: [197].

Figure 5.19a: time step t Figure 5.19b: conc. b **Figure 5.19**

Heat map of last step of simulation (in fig. 5.19a, colours are in logarithmic scale) and the average concentration a (fig. 5.19a). 90×90 simulations performed on a space of size $|L| = 256 \times 256$, $D_a = 0.1$, $D_b = 0.2$. At $t = 0$, the state $S = (a, b) = (1.0, 0.0)$ is assigned to each cell, excepts for the cell in the middle of space where $b = 1.0$.

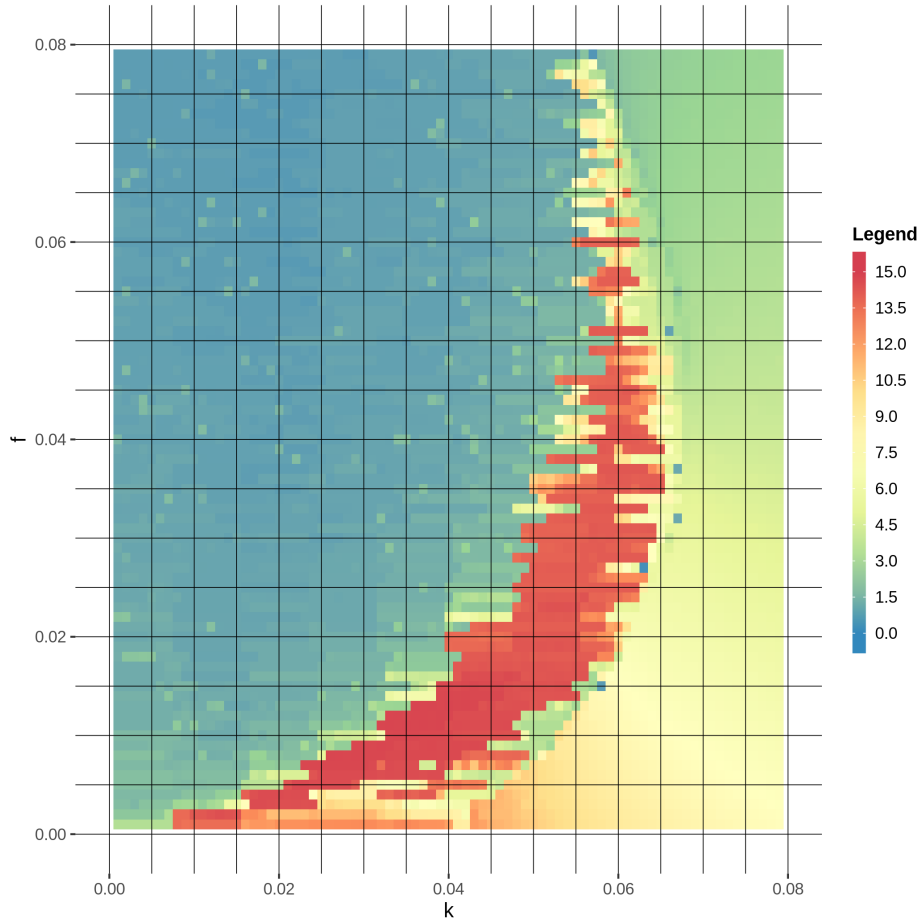


neous state where no gradients of concentrations appear, but the concentrations (a, b) varying as a function of f and k . Otherwise, the red state is the state where the concentration is $(a_r, b_r) = (1.0, 0.0)$ for all values of that not respect the condition $\alpha > 0$. The stability analysis shows that, if the conditions $\alpha > 0$, $f > 0$, $k > 0$ are respected, the intermediate state is not at equilibrium while the blue state is at equilibrium. In a (f, k) phase diagram (fig. 5.17), the blue and the intermediate states appear as two regions separated by the saddle-node curve:

$$f(k) = \frac{1 - 8k \pm \sqrt{1 - 16k}}{8} \quad (5.26)$$

On the saddle-node bifurcation curve we observe the equilibrium point in $(f_e, k_e) = (0.0625, 0.0625)$: for this couple and for all possible spatial configuration of morphogens, no perturbations is able to spark the stable state.

Now, taking into account the diffusion, the study of stability confirms the existence of three states and it gives more information about the changing of state from the blue to the intermediate state. With the diffusion, the intermediate state corresponds to the Turing space in which it is possible to observe the formation of patterns. The Turing space, in a Gray-Scott is identified in a (f, k) phase diagram as the region in vicinity of the curve

**Figure 5.20**

Heat map of Shannon entropy computed for the statistical distribution of a . 90×90 simulations performed on a space of size $|L| = 256 \times 256$, $D_a = 0.1$, $D_b = 0.2$. At $t = 0$, the state $S = (a, b) = (1.0, 0.0)$ is assigned to each cell, excepts for the cell in the middle of space where $b = 1.0$.

described by the eq. (5.26). It is identified by the equation:

$$[d(f + k) - (b_0^2 + f)]^2 - 4d(f + k)(b_0^2 - f) = 0 \quad (5.27)$$

where $d = D_a/D_b$ is the diffusive ratio and b_0 is the initial concentration of B . The (f, k) phase diagram (fig. 5.18) confirms the importance of the ratio d as a component involved in the stability. If the diffusion ratio d increases, the upper limit moves to the lower limits, the Turing space reduces and the blue state increases its importance.

Pattern classification

In the proximity of bifurcation, a wealth variety of patterns may appear. These can be observed by setting the initial state of the system to the red state and adding a finite perturbation of $(a, b) = (0.5, 0.25)$. Pearson [218] produces a systematic classification of patterns that arising in a Gray-Scott; he describes 12 types different patterns [218].

Figure 5.19 plots basic indicators computed at the end of each simulation. More precisely, fig. 5.19a plots the heat map of the number of steps before that the morphogen layer reaches a stable state and fig. 5.19b the average concentration of A at the final step. If the Gray-Scott morphogen layer does not become stable, we stop the simulation at the step $t_{max} = 20000$. As we

expected, we observe the red, the blue, and the intermediate state. The result finds correspondence with the stability analysis reported for continuous media. In the blue state region, from the perturbing zone, the propagation is wave-like; here the simulations stop before t_{max} because the state of cells does not vary and the gradient of concentration disappears. In the red state region, after few steps, the perturbation suddenly vanishes. The intermediate area seems to be divided in two macro areas: for $f \lesssim 0.030$, the morphogen layer is far from the equilibrium (section 1.3.2), chaotic and unstable patterns appears. For values $f \gtrsim 0.030$, morphogen layer reaches a stable state at different time step; this effect indicate that the system is out of equilibrium. Different types of patterns appear at the end of the simulations: i.e. spots, worms, and waves.

The formation of patterns in a morphogen layer can be observed from a statistical point of view. *Lato sensu*, a pattern can be interpreted as an heterogeneous statistical distribution of morphogens. Hence, statistical indicators of diversity are useful to capture the behaviour of the system. To this end, we compute the Shannon entropy [263] of the statistical concentration a as:

$$H(a) = \sum_{c \in L} a(c) \log_2 a(c) \quad (5.28)$$

and we plot results in a heat-map in fig. 5.20. Due to the impossibility to observe gradients of concentration at the final step, red and blue states show lower values of $H(a)$. The intermediate state brings out because the formation of patterns (stable or unstable) produces heterogeneous values in a broad range.

Summary 15:
The Gray-Scott model

The Gray-Scott model is a reaction-diffusion model that was widely studied in the last 40 years. A cellular automata formalism has been used to simulate it in a discrete media. We had found several correspondences between results from these two approaches.

Chapter 6

Toward the morphogenesis of complex networks

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Keywords graph model, complex networks model, vector and seeds, creation and death of seeds, handle intersections.

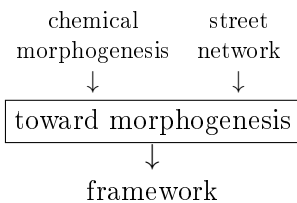
*"Le hasard, toujours indispensable, n'est jamais seul et n'explique pas tout. Il faut qu'il y ait la rencontre entre l'aléa et une potentialité organisatrice"*¹

Edgar Morin [202]

In chapter 2 we observed that many complex systems (chapter 1) are structured as networks and we focused on such of them that spatial aspects affect their properties and the way that they form. Graph theory provides a useful tool to formalize these systems: we can model them, measure their properties (chapter 4), and simulate their formation.

The goal of this chapter is to simulate spatial network formations. To this end, we will describe two models, based on two approaches (section 6.1). The first one (section 6.2) is largely inspired by geometric planar graph generators and the second one introduces the complexity (section 6.3). Resulting graphs have several traits in common. The chapter ends with a discussion about these models (section 6.4). These observations will be the basis for the framework exposed in chapter 7.

6.1 Modelling spatial networks



When you look at the skeleton of a leaf, you discover a complex hierarchical pattern; it shows regularities and structures. Space and time are imprinted/written in the structure. The spatialization of elements reveals keys of genesis of these structures. The leaf blades are located in space and they are not directed by a central command.

In unplanned cities, new streets are made to connect the existing network to a sub part of the land. When a street connects, an intersection of at least three branches forms. The street ends with a cul-de-sac or intersecting an existing element of the network. After a while, the end-point may become the origin of a new branching or an elongation of the street. The processes behind these events are often decentralized and they are constrained by spatial aspects. The result is a sedimentation and addition of new elements to the existing network. The growth is coherent because at each moment new elements are connected to the rest of the network.

A few ants move and build a nest, the end part of plant roots dig the ground and bifurcate, cracks and unplanned streets are the results of branching and elongations. While some elements of the system operatively make a network, some others move, interact, bring goods etc. These elements are spatially constrained and they constitute a system.

These basic observations inspired our thinks. We suppose that there exists some moving elements of the system that are leaded by internal and external forces and that builds a spatial network. Their movement and their interactions are completely decentralized. We named them seeds (fig. 6.1).

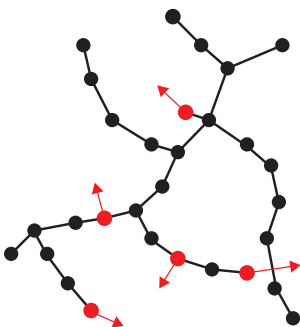


Figure 6.1
Seeds, vectors and the network.

¹ "The change is necessary but it is not able to all explain. We need to mix it with the organization".

6.1.1 Seeds, vectors and dynamics

The path that a seed makes in a continuous spacetime environment can be represented with a line. This representation can be discretized in a discrete media: in a sequence of time steps, the elements is located in a point of the space and the trip that it has made is a straight line. We suppose that the movement of seeds are due to external or internal reasons. To model these actions, we suppose that a displacement vector relates the position of the seed at two time steps. The branching and the end-points observed in reality can be considered as the creation and the death of these elements.

Formally we define:

A seed $s(t)$ is a virtual vertex that move in euclidean space $\mathbb{R}^2$ and builds a growing planar graph $G(x, t)$ in a discrete time series $T = 1, 2, 3...$. At time step t , only one vertex $v \in V_t$ is associated to the seed.

Definition 11:
Seed

The relation between positions of the seed at time t and at $t + 1$ is defined by a displacement vector $\vec{v}$. It is noted as $\vec{v}(s) = (i, \theta)$, where $i \in \mathbb{R}_+$ is the magnitude and $\theta \in [0, 2\pi]$ is the direction of the vector. The set of seeds at time step t is noted as S_t . A process $\mathbb{P}(S)$ governs the formation of the graph $G(x, t)$ in such a way $G_t \xrightarrow{\mathbb{P}(S)} G_{t+1}$. It is the result of three kinds of events:

- **The movement of seeds:**
it is formally defined by a displacement vector $\vec{v}(s)$.
- **The death and the creation of seeds:**
at each time step, the size of the seeds set S changes because new seeds born (over existing vertices) or existing seeds die. The creation of new seeds produces the branching and the death provides the formations of end-vertices or intersections. The graph grows in a coherent way, because new seeds born over an existing vertices (section 2.3.4).
- **Building the graph:**
the movements of the seeds are bounded by the geometry of the graph. During the displacement of a seed, it might become close to existing vertices or edges, or its path might intersect an existing edge. The handling of these situations ensures the planarity of the geometric graph. In the next we will detailed two methods.

The results is a dynamic graph embedded in $\mathbb{R}^2$ (definition 4, page 39, definition 6, page 42), which respect the planarity condition (definition 7, page 44) and that grows in a coherent way (definition 5, page 39).

6.1.2 Approaches

We want to formalize the following dynamics: an evolving set of seeds, moved by displacement vectors, builds a geometric planar graph. To this

end we follow two approaches. The first approach (section 6.2) is a basic model. We construct a geometric planar graph and we fix geometric rules (angle and length). These rules are noisy by a random process in order to create heterogeneity in the appearance of the growing graph. It is a random planar graph without structure. The graph genesis is isolated (in the meaning of isolated system). The second model (section 6.3) introduces complexity. The genesis of the graph is open, because the construction of the graph is influenced by reaction diffusion.

6.2 The geometric planar graph generator

In many spatial networks, we observe that the elongation tends to respect the straight direction and the branching appears with regularity and apparently not in accordance with a deterministic reason. The basic idea of this first model is that the formation of a graph is led by biased randomness.

We assume that:

- the future position of the seed depends on the actual position and the previous position,
- the displacement vector is biased by a probability in such a way the seed moves approximatively in a straight line,
- the creation and death of seeds respect a probability,
- when the seed intersects the existing graph, it dies and a new connection is created.

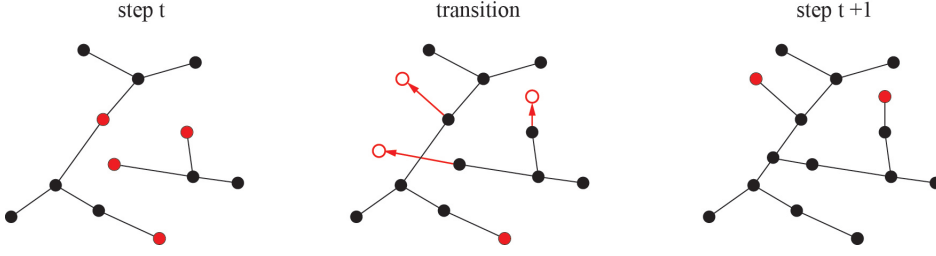
6.2.1 The movement of seeds

A random walk is a mathematical formalism where a stochastic process (biased or not) allows to explore space (topological or Euclidean) using a set of particles. Introduced by Pearson by an iconic question in 1905² [219], and for the first time applied in botany [58], random walk was widely used to model many phenomena in biology [81], chemistry [9] in social networks analysis [256] and urban studies [45].

If a particle explores a topological space (a lattice or a graph) in discrete time steps, a stochastic process provides the probability to move from a cell (or a vertex) to one of its neighbours. In euclidean space, the walk of the particle is a sequence of locations. These positions can be represented as vertices of a geometric graph embedded in the space and the path of particles as edges. In this sense, random walk provides a formalism to simulate the formation of a path graph³.

² "A man starts from a point O and walks 1 yards in a straight line; he then turns through any angle whatever and walks another 1 yards in a second straight line. He repeats this process n times. I require the probability that after these n stretches he is at a distance between r and $r + S_r$ from his starting point, O ".

³A path graph on n vertices is a tree where $V = \{v_1, v_2, \dots, v_n\}$ is the vertex set and $E = \{(v_1, v_2), (v_2, v_3), \dots, (v_{n-1}, v_n)\}$ is the edge set.

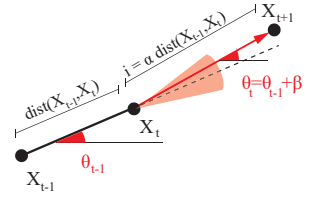
**Figure 6.3**

Interactions between seeds and the graph in the geometric planar graph generator.

The movement of a seed is inspired by the walk of a particle in euclidean space. The process is Markovian⁴, hence the position of a seed s at time $t + 1$ depends on the position of the seed s at time t . The vector at time step t defines the position at time step $t + 1$ and it is a function of the actual position and the position of the seed at time step $t - 1$. Moreover, we suppose that the magnitude and the direction are affected by a random influence:

$$\vec{v}_t(s) : \begin{cases} i_t = \alpha \text{dist}(X_{t-1}, X_t) \\ \theta_t = \beta + \theta_{t-1} \end{cases} \quad (6.1)$$

where $\alpha \in \mathbb{R}_+$ and $\beta \in [-\pi, +\pi]$ are two free parameters (fig. 6.2). If α and β change at each time step in a random way, and if we connect the vertex created at t with the vertex created at $t - 1$, the seed builds a path graph of t vertices and $t - 1$ edges.

**Figure 6.2**

The vector associated to a seed in the geometric planar graph generator.

6.2.2 Creation and death of seeds

A Bernoulli trial (or binomial trial) is a random experiments with two possible outcomes: 0 or 1. The experiment can be eventually biased by a parameter $p \in [0, 1]$ and used to define an action in a simulation:

$$\text{if } \text{random}(0, 1) < p \text{ then do something} \quad (6.2)$$

A Bernoulli trial can be used to obtain a pseudo-random binomial output biased by a probability. It can simulate aspects in a system that are not completely predictable. Here it is used to control the formation and the degradation of seeds. More precisely, we have two kinds of events that impact the seed set S . For each vertex $v \in V_t$, we add a new seed with a probability $p_c \in [0, 1]$ and we remove each existing seed with a probability $p_d \in [0, 1]$.

6.2.3 Building the graph

At time step t the seed s located in X_t overlaps the vertex v and, according to the vector $\vec{v}_t(s)$, its potential position is X_{t+1} . Before moving in X_{t+1} , we check if the potential line $\overline{X_t X_{t+1}}$ intersects an existing edge $e \in E_t$. If not, we move the seed at new position X_{t+1} , we create a new vertex v in X_{t+1} and a new edge between this latter and the vertex in X_t . Otherwise, we remove the seed, we add a new vertex v_x at the intersection X_x between

⁴Roughly, in a sequence of events, the probability of each event depends only on the state reached in the previous event.

Algorithm 3: A step in the geometric planar graph generator

```

input :  $S(t) = \{s_1, \dots, s_n\}$ , seed set at time step  $t$ 
          $G(x, t)$  the geometric planar graph at  $t$ 
output:  $S(t)$  and  $G(x, t)$  at  $t + 1$ 
begin
  foreach vertex  $v \in V$  do                                /* add new seeds */
    if  $\text{random}(0, 1) < p_c$  then
       $s \leftarrow$  new seed at  $v$ 
       $S \leftarrow S + s$ 
  foreach seed  $s \in S$  do
    if  $\text{random}(0, 1) < p_d$  then                                /* remove seeds */
       $S \leftarrow S - s$ 
    else
       $\vec{v} \leftarrow$  vector to apply at seed  $s$                 /* see eq. 6.1 */
       $X_t \leftarrow$  position of seed  $s$ 
       $v_t \leftarrow$  corresponding vertex of seed  $s$ 
       $X_{t+1} \leftarrow X_t + \vec{v}$  /* compute future position of  $s$  */
      if line  $\overline{X_t X_{t+1}}$  does not intersect edge  $e \in E$  then
        move the seed to  $s(X_t)$ 
         $V \leftarrow V + v(X_{t+1})$ 
         $E \leftarrow E + (v_t, v(X_{t+1}))$ 
      else
         $X_x \leftarrow$  coordinate of intersection
         $V \leftarrow V + v(X_x)$  /* create new vertex at  $X_x$  */
         $E \leftarrow E + \{(v_x, v_1), (v_x, v_2), (v_x, v_t)\} - (v_1, v_2)$ 
         $S \leftarrow S - s$ 

```

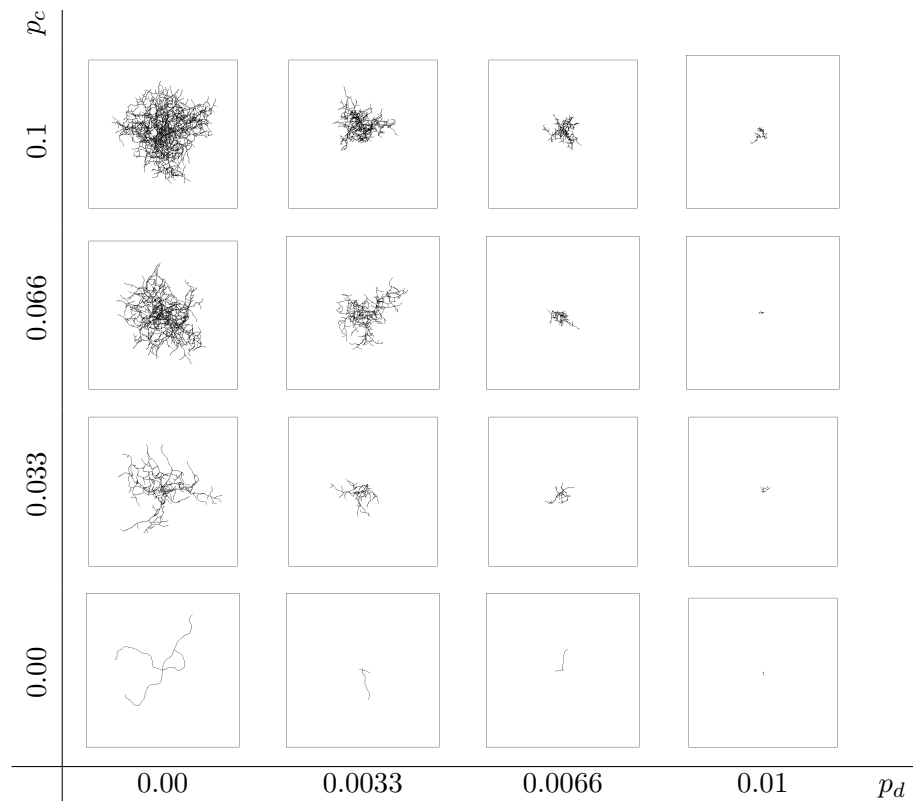
the line $\overline{X_t X_{t+1}}$ and the edge (v_1, v_2) , we remove the edge (v_1, v_2) and we add three new edges: (v_x, v_1) , (v_x, v_2) and (v_x, v_t) . The result is a growing geometric graph (fig. 6.3); it grows coherently and it respects at each time step the planarity condition.

6.2.4 Simulation and results

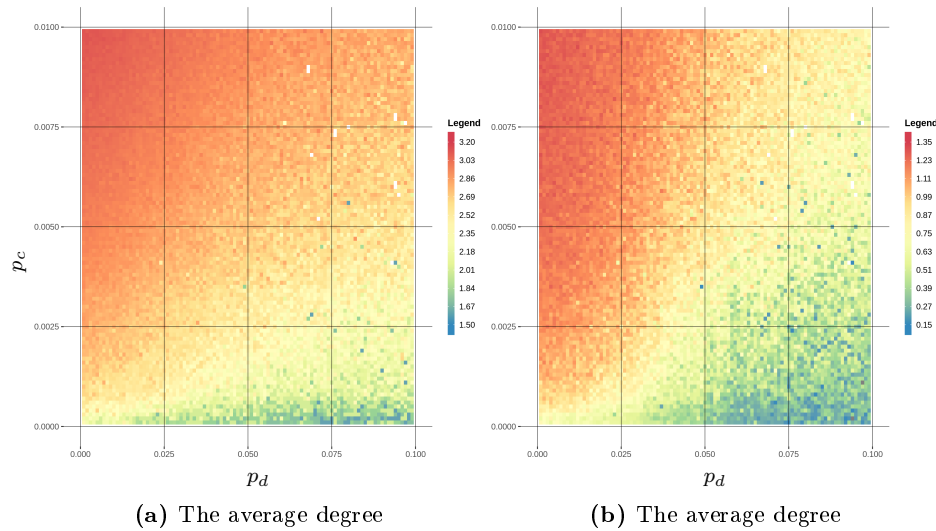
The simulation starts with a star graph of 5 vertices⁵ in the middle of space; a seed is assigned to each leaf (see algorithm 3). In accordance to eq. (6.1), values assigned to each seed are randomly chosen in a range $\alpha \in [0.9, 1.1]$ and $\beta \in [-0.05\pi, +0.05\pi]$.

Figure 6.4 depicts the graph of 16 simulations at time step $t = 200$, where probabilities $p_d = k \times 0.0033$, $p_c = k \times 0.033$, $k = \{0, 1, 2, 3\}$. The model is able to reproduce different types of growth with a few parameters and different graphs appear with different values of p_d and p_c . As we expected, for high values of p_d , the graph grows very slowly because seeds readily disappear after few steps. Otherwise, for high values of p_c , many seeds (proportionally to the size of the graph) born. The resulting graph fills

⁵The model needs at least a vertex to start; no seeds are required.

**Figure 6.4**

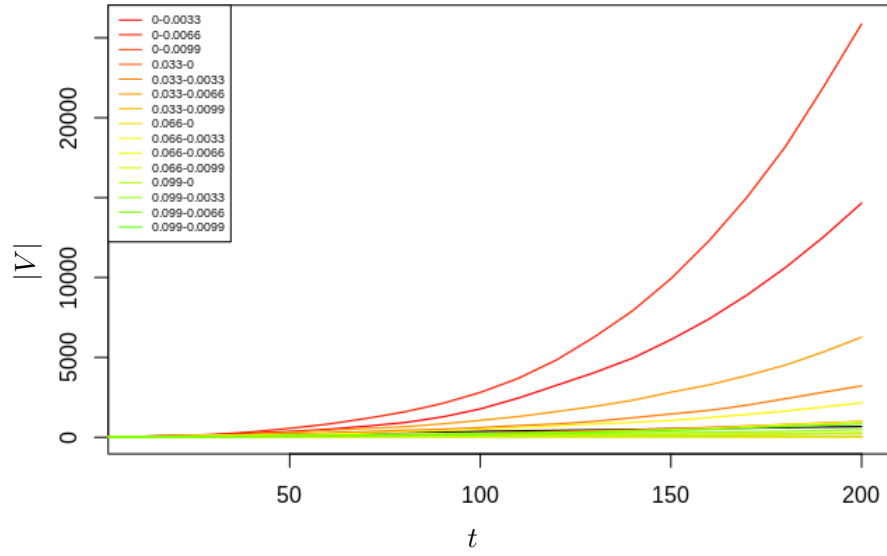
Simulations of the geometric planar graph generator at time step $t = 200$ for various constant values of p_c and p_d .

**Figure 6.5**

Heat maps at time step $t = 200$ of the number of average degree (fig. 6.5a) and fractal dimension D_0 (fig. 6.5b) .

rapidly the space.

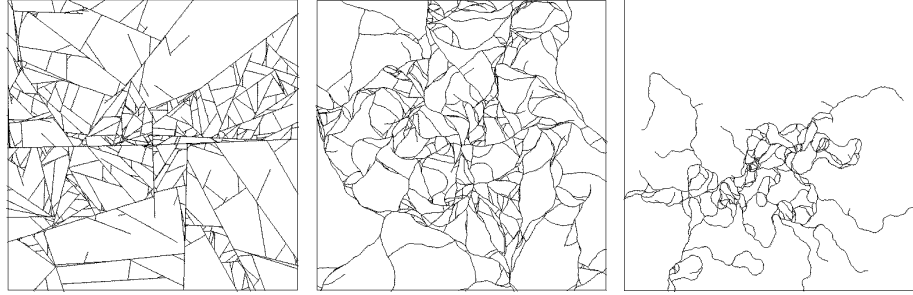
We investigate resulting graphs with metrics exposed in chapter 4. Before to analyse them, we remove vertices with degree 2 (see algorithm 1), in order to capture main properties of graphs. Starting from the same initial configuration, we run 100×100 simulations. Probability p_d varies between 0 and 0.1 and probability p_c varies between 0 and 0.01. Different behaviour of the model are observed during the simulation. Trajectories depends to the parameters p_c and p_d . For high values of p_c , the average degree (fig. 6.5a) is ranging around 3, which corresponds to what is observed in many street networks (section 4.2.7). However, the study of fractal dimension (fig. 6.5b)

**Figure 6.6**

The evolution of the number of vertices, calculated for 16 graphs depicted in fig. 6.4.

Figure 6.7

Three graphs generated with the geometric planar graph generator. The same initial parameter set and varying the parameter β ($p_c = 0.001, p_d = 0.005, \beta = 0$ (left), $\beta = 0.05\pi$ (centre), $\beta = 0.1\pi$ (right)).



suggests that a global scale-invariance organization organization cannot be observed for these graphs.

The effect of p_c on the growth rate is confirmed by the evolution of graphs over the time. The rate of growth of the vertex set $|V|$ (fig. 6.6) suggests that, while for lower values of p_c the graph grows linearly with the time, for high values of p_c the growth rate seems to fit an exponential curve. Finally, the parameter β affects the growth of graph (fig. 6.7): for high values of β , it is more probable that a seed turns around and, intersecting its own path and dies.

Summary 16:

The geometric planar graph generator

The chance and geometrical rules control the movement, the creation and the death of a set of seeds. A geometric graph is the results of their trips. The trips of the seeds and their interactions with the graph define the shape of a growing geometric graph. The growth is always coherent. The topology of resulting graphs seems to be similar to street networks but no structures and no invariance of scale are observed: the graph fills the space homogeneously.

6.3 The reaction-diffusion planar graph generator

Turing [285] suggested that behind the formation and the specialization of cells in an living system there exists a set of spatially located interacting elements (section 5.1.2). Due to their crucial role into the morphogenesis of the system he calls them morphogens. A basic reaction-diffusion system considers two kinds of morphogens: an activator, which produces a positive feedback, self reinforcing its own production, and an inhibitor, which moves slowly and suppresses the growth of the other one. The coupling of these competing mechanisms arises an emergent phenomenon: the formation of a wealthy of spatio-temporal patterns (e.g. spots, strips, waves, chaotic patterns).

Morphogens group together and a gradient of concentration appears. As noted in section 1.1.4, emergences can roughly be defined as new properties of the system resulting from local and decentralized interactions of elements. As Chalmers [73] suggests, emergences can be classified in weak (a new property related to the understanding of the observer) and strong (a new property that has also a role into the functioning of the system). Our proposition is: those emerging patterns have an important role in the functioning of the system, leading the morphogenesis of spatial networks.

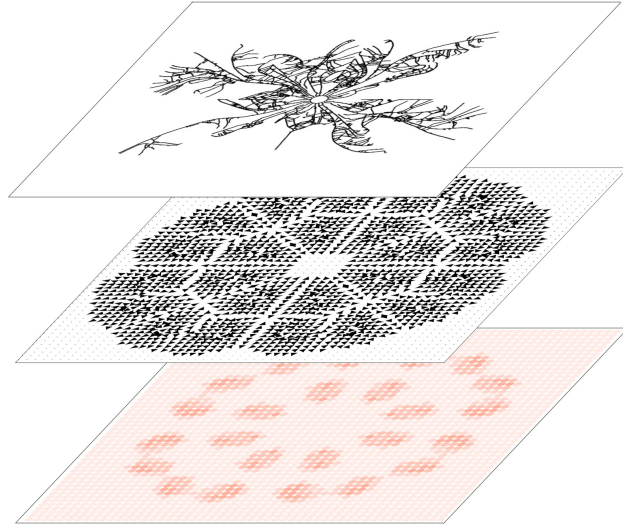
In a reaction diffusion model, no particular concentrations can exist without others close concentrations. All group of morphogens contrasts with another group of morphogens. No individual group is complete of itself, so each group needs other groups. At the periphery of groups, forces (attractive or repulsive) allow similar kinds of morphogens to stay gathered, ensuring the emergence of sub-areas with the same concentration of morphogens. These forces guide the formation of spatial networks. The causal relation between the morphogenetic substrate and the emerging network is ensured by internal forces.

These basic dynamics can be transposed in a model. They are also similar to dynamics inside the geometric planar graph generator: forces move seeds and build the graph. The main difference is about the way that vectors are defined: here the gradient of concentration of morphogen becomes a set of attractive or repulsive forces. We complete the model with the following assumptions:

- the future position of the seed depends on the current position and the concentration of morphogens,
- the vector field is computed with the concentration of morphogen B ,
- geometry and the topology of the graph and the concentration of morphogen B control the death and the generation of seeds.

6.3.1 A multilayer model

We model these dynamics with a multilayer model, composed of three interdependent layers:

**Figure 6.8**

The three interdependent layers of the reaction-diffusion planar graph generator: the morphogen layer $L(t)$ (bottom), dynamic vector field $\vec{L}(t, b)$ (centre) and the growing geometric graph $G(x, t)$ (top).

- a morphogen layer $L(t)$ models the reaction-diffusion of morphogens A and B ,
- a dynamic vector field $\vec{L}(t)$ for the morphogen B , which controls the position of a set of seeds $S(t)$,
- a growing geometric graph $G(x, t)$ [282] (fig. 6.8).

6.3.2 The reaction diffusion

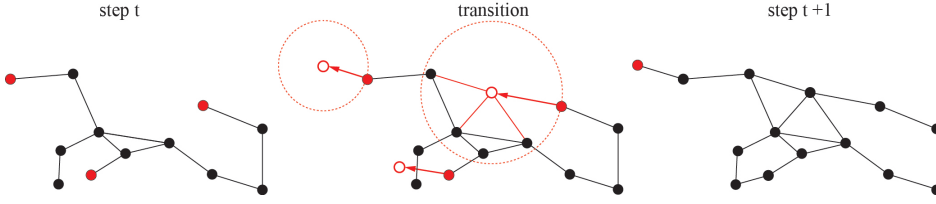
We briefly recall the reaction-diffusion dynamic. In the first layer, two morphogens A and B interact and diffuse in Euclidean space. At the microscopic scale, A catalyses its own production and also the production of B . At the same time, B inhibits the production of A . B diffuses faster than A . In a region of space, the concentrations of A and B are noted a and b respectively. At the macroscopic scale, the evolution of interactions and the diffusion of morphogens can yield patterns of concentration. A way to model the evolution of the morphogen A and B is to define a morphogen layer $L_m(t)$ (definition 10, page 109) where the cell state (a, b) , $a, b \in [0, 1]$ represents the concentration of morphogens and the evolution of concentrations follows the Gray-Scott transition state (section 5.1.7).

6.3.3 The movement of seeds

Accordingly to our assumptions, in a discrete representation, we compute a dynamic vector field $\vec{L}(t)$ for the morphogen B . We consider the concentration b of the cell in $X \in \mathbb{Z}^2$ as a mass and we suppose that a force of attraction (or repulsion) exists between each cell⁶. The Newton's law describes the force $\vec{F}$ between two masses m_1 and m_2 as:

$$\vec{F} = G \frac{m_1 m_2}{r^2} \quad (6.3)$$

⁶As the same, the concentration could be considered as an electric charge. In this case, we obtain an electrical field.

**Figure 6.9**

Interactions between seeds and the graph in the reaction-diffusion planar graph generator.

where $G = 6.674 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ is the gravitational constant and r the euclidean distance between masses. According to eq. (6.3), at each cell on layer L is associated the vector:

$$\vec{b}(X) = \sum_{Y \in L} \frac{b(X)b(Y)}{dist(X, Y)^2} \hat{XY} \quad (6.4)$$

where X and Y are cells in the morphogen layer $L_m(t)$, $dist(X, Y)$ is the euclidean distance between centres of cells located in X and Y , $\hat{XY}$ is the relative unit vector.

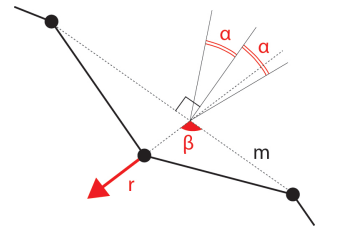
The second layer is completed by an evolving set of moving seeds. During the simulation, the vector field is applied to the moving seeds which follow the evolution of the gradient of morphogen concentration. We apply the sum of four nearest vectors to each seed. The moving seeds build the last layer of the model, the geometric planar graph.

6.3.4 Spatial interactions and generations of seeds

The dynamic vector field affects the positions of moving seeds, which generates the graph. However, the morphogenesis of the graph is not only governed by the reaction-diffusion mechanism. During the simulation, the graph grows and its configuration impacts its own morphogenesis.

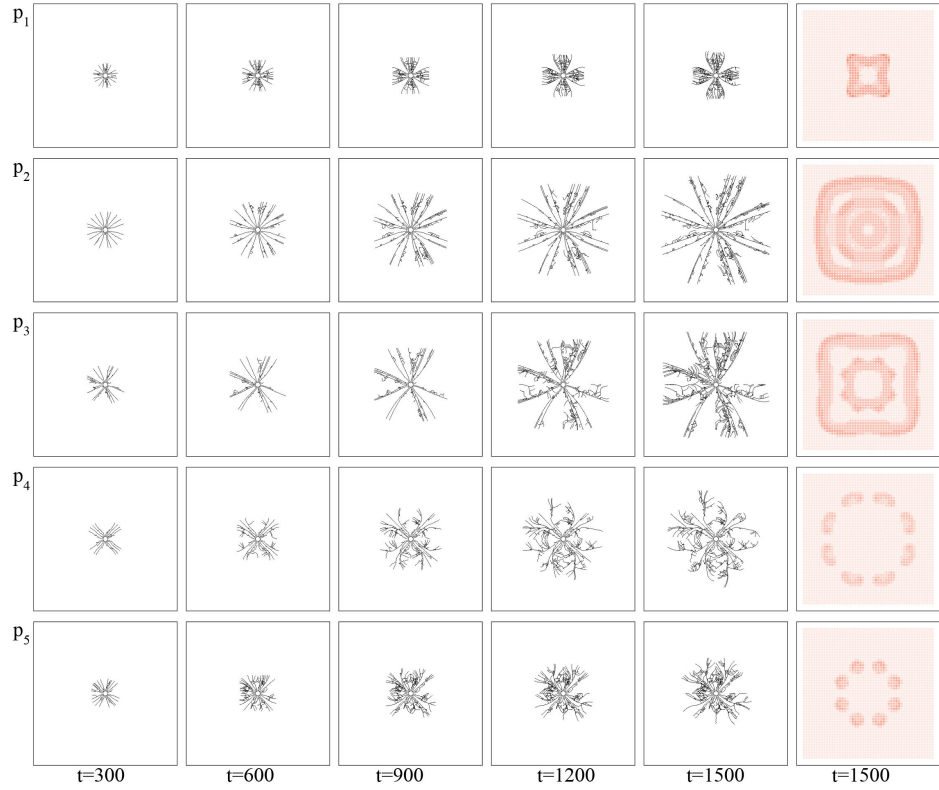
At each seed corresponds a unique vertex in the growing graph. At each time step, we apply the corresponding vector $\vec{r}$ to each moving seed and calculate its future position X_f as X_t moved by $\vec{r}$. In this way we take into account the evolution of morphogen concentration near the seeds. Concerning the seed movement, several cases can occur; given u a node at position X_t and v a future node at position X_f :

- if the potential edge between u and v crosses another edge, we do not create node v , we connect u to the nearest node, and remove the seed,
- otherwise we create node v and the edges between u and v .
 - If there are no other nodes in a radius $dist(X_t, X_f)$ around v we keep the seed,
 - otherwise we connect v with all other nodes in radius $dist(X_t, X_f)$ and we remove the seed. (fig. 6.9)

**Figure 6.10**

The seed creation in the reaction-diffusion planar graph generator.

Seeds are not only removed, but also created during the simulation. We create a seed at a node u of degree two if the corresponding vector $\vec{r}$ is almost

**Figure 6.11**

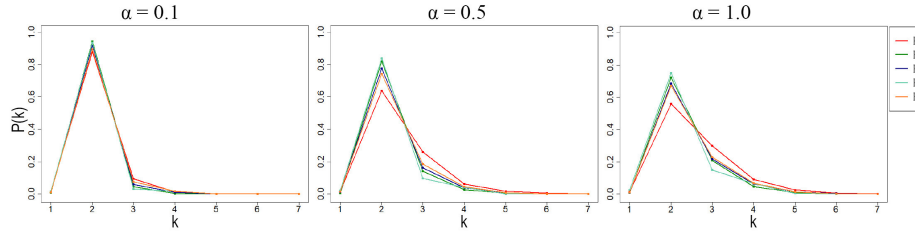
Simulations of five growing graphs and relative pattern, with $\alpha = 1.0^\circ$. Each row shows the corresponding pattern at final time step $t = 1500$ (last column) and the corresponding growing graph every 300 time steps.

perpendicular to the line passing between the two neighbours of u (fig. 6.10). The parameter α is the maximal allowed deviation from the right angle. The aim of the last mechanism is to consider the evolution during the simulation of the reaction-diffusion layer, in order to handle the generation of new seeds. The vector field evolves during the simulation and the corresponding nearest vector of the vertex in the growing graph changes magnitude and direction: each vertex in the graph can be a potential source of seeds as result of combination of the vector field and the network geometry.

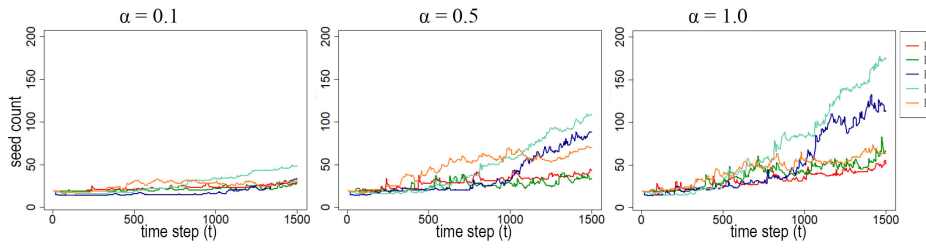
Thanks to this methodology we obtain graphs which grow coherently, embedded in two-dimensional space and where edges do not cross one another. The simulation starts with the creation of the three layers and ends when the moving seed set becomes empty or after a predefined number of steps. At each time step, we update the reaction-diffusion layer and compute the corresponding vector field. The graph evolves thanks to the addition of new seeds and the movement of seeds, respecting local rules of interaction. Algorithm 4 summarizes the simulation.

6.3.5 Simulation and results

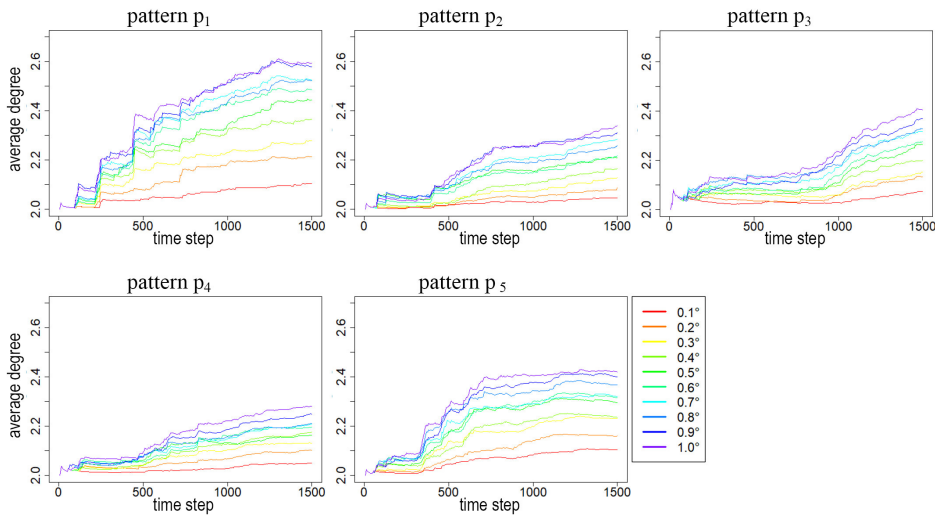
The reaction diffusion layer consists of a lattice of 50×50 unit cells. The start concentration of morphogens is the homogeneous state $a = 1.0$ and $b = 0.0$, with a small localized perturbing pulse ($b = 1.0$), a given number of moving seeds and associated nodes in the middle of the space. Diffusion parameters are $D_a = 0.1$, $D_b = 0.2$, feed and kill parameter set (f, k) is chosen in order to obtain some classical patterns: p_1 (0.055 , 0.062), p_2

**Figure 6.12**

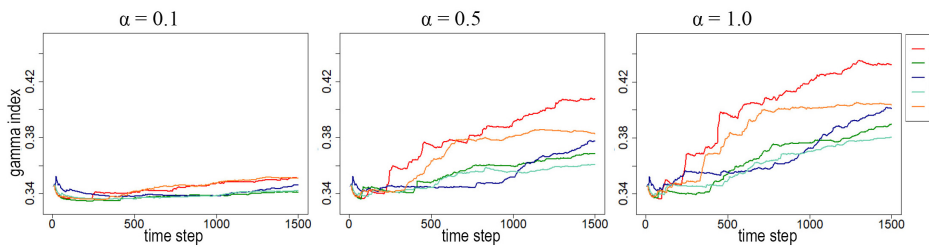
The degree distribution of five growing graphs calculated for all patterns and for three values of $\alpha = [0.1^\circ, 0.5^\circ, 1.0^\circ]$.

**Figure 6.13**

The evolution of number of moving seeds, calculated for all patterns and for three values of $\alpha = [0.1^\circ, 0.5^\circ, 1.0^\circ]$.

**Figure 6.14**

The evolution of the average degree calculated for all patterns and for all values of $\alpha = [0.1^\circ, 0.2^\circ, \dots, 1.0^\circ]$.

**Figure 6.15**

The evolution of gamma index computed for all patterns and for three values of $\alpha = [0.1^\circ, 0.5^\circ, 1.0^\circ]$.

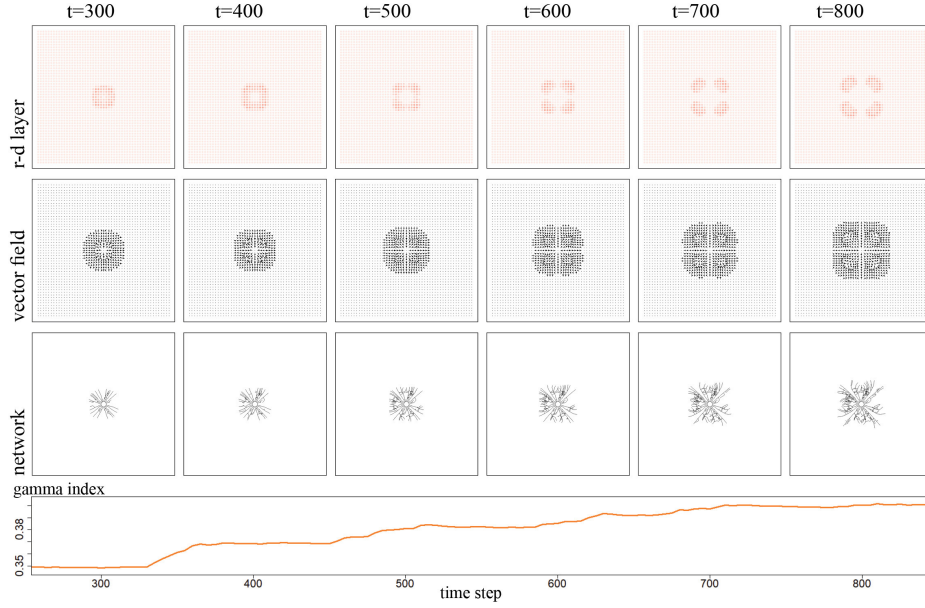
Algorithm 4: A step in the reaction-diffusion planar graph generator.

```

input :  $L(t)$  the morphogen layer at time step  $t$ 
          $\vec{L}(t)$  the dynamic vector field computed for  $B$  at  $t$ 
          $S(t)$ , seed set at  $t$ 
          $G(x, t)$  the geometric planar graph at  $t$ 
output:  $L(t)$ ,  $\vec{L}(t)$ ,  $S(t)$  and  $G(x, t)$  at  $t + 1$ 
begin
  update the reaction-diffusion layer  $L(t)$  /* See section 5.1.7 */
  update the vector field  $\vec{L}(t)$  /* See section 6.3.3 */
  foreach vertex  $v \in V(x)$  with degree 2 do /* add seeds */
     $\vec{r} \leftarrow$  sum of nearest vectors
     $\vec{m} \leftarrow$  vector between the two neighbours of  $v$ 
     $\beta \leftarrow$  angle between  $\vec{r}$  and  $\vec{m}$ 
    if  $|\beta \% 2\pi| \leq \pi/2 \pm \alpha$  then
       $s \leftarrow$  a new seed at  $v$ 
       $S \leftarrow S + s$ 
  foreach  $s \in S$  do
     $v \leftarrow$  vertex where the seed  $s$  is located
     $X_s \leftarrow$  current position of seed  $s$ 
     $\vec{r} \leftarrow$  the sum of nearest vectors
     $X_f \leftarrow X_s$  moved by  $\vec{r}$  /* future position of seed  $s$  */
    if line  $\overline{X_s X_f}$  intersects edge segments then
       $u \leftarrow$  nearest vertex from  $s$ 
       $E \leftarrow E + (u, v)$  /* Add edge */
       $S \leftarrow S - s$  /* Remove seed */
    else
       $u \leftarrow$  new vertex at  $X_f$ 
       $V \leftarrow V + u$ 
       $E \leftarrow E + (u, v)$  /* Connect the vertex to the
        previous one */
      Move seed  $s$  to  $X_f$ 
       $N \leftarrow$  set of vertices within a distance  $dist(X_t, X_f)$  from
         $X_f$ 
      if  $N \neq \emptyset$  then
        foreach vertex  $v(x_t) \in N$  do /* Connect vertex to
          nearest vertices */
          if line  $\overline{X_t X_f}$  does not intersect edge  $e \in E$  then
             $E \leftarrow E + (v(x_t), v)$ 
           $S \leftarrow S - s$ 

```

(0.039, 0.058), p_3 (0.029, 0.057), p_4 (0.014, 0.054), p_5 (0.025, 0.060) [2]. At the centre of each cell in the reaction diffusion layer we compute the vector $\vec{b}$ following the eq. (6.4). For each seed, the corresponding vector $\vec{r}$ is the sum of the four nearest vectors. For each pattern p , we varied the free parameter

**Figure 6.16**

Mitosis of pattern p_5 . Snapshots (from step $t = 300$ to $t = 800$, computed each 100 steps) of the three layers (the first three rows) and the corresponding evolution of γ index (the fourth row).

$\alpha = [0.1^\circ, 0.2^\circ, \dots, 1.0^\circ]$ and we obtained fifty growing networks⁷.

In this section, we study some properties of all growing graphs generated by the model with few typical topological metrics largely applied in spatial graph analysis [60] (chapter 4). Our interest focuses on three aspects: investigating patterns effects on growing networks, evaluating the evolution of graph topology and studying the impact of the α parameter.

In fig. 6.11 we show growing networks every 300 steps, corresponding to different reaction-diffusion patterns with the relative concentration of morphogen B at step $t = 1500$; each growing graph is obtained with $\alpha = 1.0^\circ$. We observe that growing networks strongly depend on related reaction-diffusion patterns. See section 4.2 for more informations about measures used below.

In most planar graph applications, the degree of the vertices is comprised between 1 and 7 section 4.2. In our case study, degree distribution follows a fast decay from $k = 2$ to $k = 6$, typically observed in street, leaf and ant gallery networks (fig. 6.12). In addition, the high probability to generate a new seed, suggested by the frequency $P(2)$, is not actually correlated to real generation of new seeds. During the simulation, only few vectors related to vertices with $k = 2$ permits to generate a new seed. Figure 6.13 depicts the evolution of the number of seeds for three values of $\alpha = [0.1^\circ, 0.5^\circ, 1.0^\circ]$.

In fig. 6.14 we show the evolution of the average degree $\langle k \rangle = 2|E|/|V|$ for all patterns for all values of the parameter α . As expected, the parameter α and the pattern play crucial role on the evolution of the average degree

⁷Videos of 3 layers are computed for $\alpha = 1.0^\circ$ until step $t = 2500$, saved every 25th iterations; playback is 3 frames per second.

Pattern p_1 : <https://youtu.be/2izGpD2XU0w>

Pattern p_2 : <https://youtu.be/IwG3oSewSpI>

Pattern p_3 : <https://youtu.be/ceQVYPadENY>

Pattern p_4 : <https://youtu.be/LMn6vv9dy7Q>

Pattern p_5 : <https://youtu.be/vMiAC5rZpzs>

$\langle k \rangle$. The parameter α impacts the connectivity and a sudden variation is detected at the same time step for each graph. Results suggest that the evolution of morphogens concentration governs the growth of the network and the parameter α amplifies graph average degree.

In order to characterize the evolution of graph density, we calculate the gamma index γ . In fig. 6.15 we observe ample fluctuations during the simulation. Like for the average degree $\langle k \rangle$ evolution, we observe non monotonic curves for all patterns, as a consequence of an unstable process. Due to sudden changes of morphogen concentration b , graphs grows discontinuously. For instance, the mitosis process (observed in pattern p_5) increases the number of unstable spots (fig. 6.16). In the reaction diffusion layer, this process causes the creation of new seeds (fig. 6.10). The network suddenly grows and the γ index shows the rise of the graph density. This clearly demonstrates that growing network mechanisms proposed in this model consider at all times the evolution of morphogens concentration and the spatial properties of the network.

Summary 17:

The reaction-diffusion planar graph generator

A reaction-diffusion system, applied to a regular grid, governs a dynamic vector field, which impacts a set of moving seeds: the result is a growing geometric graph embedded in $\mathbb{R}^2$. Topology and geometry of the graph, and the concentration of morphogens fully control the growth of the graph.

6.4 Toward the morphogenesis

In this chapter, we have exposed 2 models able to generate graphs embedded in two-dimensional euclidean space. These models are based on the same dynamics (seeds, moved by displacement vectors, generate a geometric graph) but underlying processes are based on different approaches. The first model can be considered as a model in the field of classical geometric graph generators while the second one introduces the complexity.

Both models are controlled by a few parameters. The geometric planar graph generator integrates four parameters, that represent the range in which a random event might occur: parameters α and β biased the vectors and parameters p_c and p_d biased the creation and the death of seeds. Varying the probability to create or remove seeds, different growths are observed and we had studied the sensibility of the model. A combination of these parameters (observed in a $p_c - p_d$ diagram, fig. 6.5) seems to generate interesting graphs, which shows characteristics similar to street networks studied in chapter 4. In the reaction-diffusion planar graph generator, the network growth is governed by the parameter α (concerning the generation of new seeds) and reaction-diffusion parameters (D_a, D_b, f, k) . Some combinations of these parameters (the Turing space, see section 5.1.3) produce the formation of some characteristic patterns and we had observed that each of them generates graphs structured in different manners. This model generates interesting graphs too; moreover, compared to the first model, graphs fill the space less homogeneously, forming structures. These latter

depend on reaction-diffusion layer. These characteristics are close to those observed in street networks.

Steps of simulation are similar (see algorithms 3 and 4). We firstly create new seeds, with the binomial trial (geometric planar graph generator), or comparing the graph geometry to the vector field (reaction-diffusion planar graph generator). Then, we compute the future positions and we handle the intersection between the path of the seed and edges (geometric planar graph generator) or closer vertices (reaction-diffusion planar graph generator). The main difference is about the mechanism that led to the death of seeds. We do not consider this possibility in the reaction-diffusion planar graph generator because it does not have an impact on the graph formation: if a seed does not be moved by a vector, the topology and the geometry of the graph do not change.

Simple local rules ensure the connectedness of the graph over time. We had defined two methods that handle interactions between the seeds and the graph elements (only edges in the first model, edges and vertices in the second one); we had observed that both respect the planarity condition and the growth is coherent.

These preliminary results leave several open questions that require further investigation. One of them is about the definition of different levels of abstraction. For instance, in urban studies, they may represent the sprawl of a single city at an urban area level or the densification of connections between cities at a regional level. Our approaches integrate few essential physical dynamics and permit to investigate elementary mechanisms of spatial graph generation.

We had also observed inconsistencies and lacks. These early results will be completed in the next chapter by other morphogenetic mechanisms, in order to take into account the feedback of the graph to the internal morphogenetic constituents. In a complex system, the network plays a crucial role in the spatial evolution of forces acting on its development, it feeds back and it affects its own growth. Moreover, both models do not integrate exogenous effects on the functioning of the system; for the sake of simplicity, we preferred to not consider exogenous effects. The growth of the system is closed in the first model and open in the second one (in this last graphs grow only under the influence of the reaction-diffusion). The reaction-diffusion planar graph generator is design as multilayer model; it can be easily completed by other layers and we will see that one of them may represent the effect of a exogenous dynamics to the evolution of the system. The framework proposed in the next chapter aims to overtake these limits, keeping peculiarity of both models.

Chapter 7

The framework

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Keywords Morphogenesis of spatial networks, framework, convolution, components and dynamics.

"Essentially, all models are wrong, but some are useful"

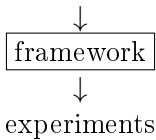
George E. P. Box

In the previous chapters, we observed that a spatial complex system is a system composed of a large number of spatial elements. Those elements locally interact and they behave constrained by an environment. This latter surrounds the system and exchanges with him (chapter 1). The result is that spatial elements self-organize, the system becomes a whole, new properties emerge and feed back to the systems, controlling, creating, and maintaining the evolving forms. Our focus is on some specific spatial complex systems which integrate elements organized as a spatial network (chapter 2).

In this chapter, we present our main contribution: a framework that simulates the morphogenesis of spatial complex systems. Our contribution is inspired by models and observations in chapters 5 and 6. We will extend them by the formalization of the environment of the system and a cross-layer feedback mechanism. We will expose the main principles and how they can be implemented. Experiments and applications with a detailed formalization of these principles will take place in chapters 8 and 9. The reader will find a general description of the framework (section 7.1) and the formalization of its components (section 7.2), and a discussion about the system dynamics (section 7.3).

7.1 General framework description

toward morphogenesis



The framework simulates, at an high level of abstraction, the evolution of two kinds of elements. They are called morphogens, because, when they organizes themselves, they controls the morphogenesis of a spatial network. Those elements react each other and diffuse in the space. Their influences to the network are modelled with a dynamic vector field. The formation of the network feeds back to the morphogens. The whole system is immersed in an environment.

7.1.1 Organization of morphogens

Usually, spatial complex systems are composed by heterogeneous elements. Under particular circumstances, some of these elements, due to their decentralized interactions, group together. A gradient of concentration (pattern) forms, and it impacts the form of the system, influencing the formation of overlying structures. Following Turing's idea, these elements are called morphogens, because they affect the form of the whole system. Morphogens move in the space and interact. Considering morphogens at a high level of abstraction, we may observe in particular conditions (the Turing space) the formation of spatio-temporal patterns. The evolution of their concentration can be described by two essential mechanisms: reaction and diffusion. In the proposed framework, those dynamics are modelled with a morphogen layer $L_m(t)$ (definition 10, page 109).

A system can be rarely considered isolated: if we focus on a system, its environment is all that can influence it (section 1.1). Moreover, due to the crucial role that spatial aspects play on the functioning of the system, effects from the environment are in many cases spatially located too. That means exchanges appear in a specific region of the space; they locally change their properties and locally contribute to the functioning of the system (section 1.3.4). We will model those influences with an environment layer L_e (definition 12, page 138).

7.1.2 Gradients govern the network formation

The evolution of the network is controlled by the gradient of the concentration of morphogens. The observable patterns are not only a new property of the system, but they are also important for the functioning of the system (section 1.1.4). These patterns control the form of emerging structures: in our case, a spatial network. To do that, we suppose that the positions of elements, called seeds (definition 11, page 119), are controlled by the concentration of morphogens. The environment can contribute to the definition of the position of seeds. We model them with a dynamic vector field $\vec{L}(t)$ and an evolving set of seeds $S(t)$.

Seeds move in space and their displacements create the network. Moreover, during the simulation, the cardinality of the seeds set evolves: the concentration of morphogens does not only control the positions of the seeds but also their creation and their destruction. The environment can also impact seeds' behaviour: as a constraint, an inhibitor, or a stimulus to the network formation. The environment contributes to the creation, the degradation, and the displacement of seeds. Finally, due to the impossibility to completely determine the effect of morphogens and the environment to the network, we introduce a random component in the movement, the creation, and the death of seeds.

7.1.3 The feedback mechanism

Finally, we close the loop with a cross-level feedback (section 1.1.2): the morphogenesis of the networks influences elements which controlled its growth, the morphogens. The creation of elements of the network is an event that locally impacts the behaviour of morphogens. In this work, we will explore different impacts: we will locally modify the diffusivity, the concentration and the rate of creation and death of morphogens. In this way, we are able to simulate positive and negative feedbacks. Finally, we assume that the creation of a node may produce two kinds of feedback, synchronic or diachronic. In the first case, the node directly modifies the concentration of the morphogens in the region around it, and in the second one, the node modifies some characteristics of the environment. Operatively, we locally modify the concentrations a and b of the cell in the morphogen layer $L_m(t)$ or the state S_e of the cell in the environment layer L_e .

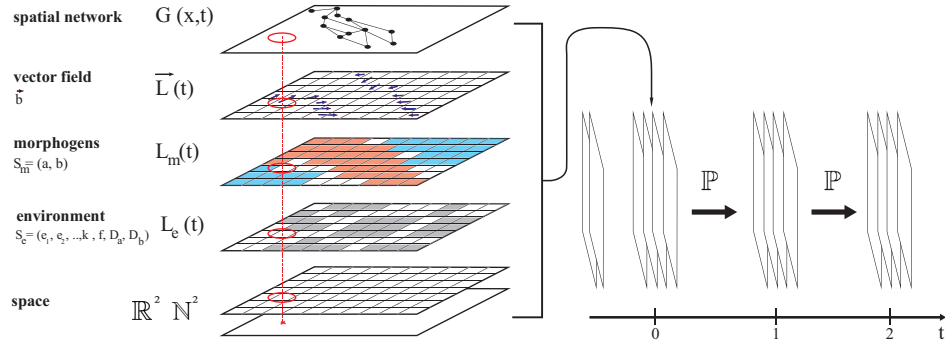


Figure 7.1
A schematic representation of the evolution in time of the framework.

7.2 Components of the framework

The framework (fig. 7.1) is composed of:

- the space $\mathbb{R}^2$ and $\mathbb{N}^2$,
- the environment layer $L_e(t)$, which models local effects of exogenous aspects to the system,
- the morphogen layer $L_m(t)$, which models the evolution of concentration of morphogens,
- the dynamic vector field $\vec{L}(t)$, which models the effect in the emerging gradient of morphogen concentration to the position of an evolving set of seeds $S(t)$,
- and the geometric graph $G(x,t)$.

7.2.1 Space $\mathbb{R}^2$ and $\mathbb{N}^2$, and time T

The layers of the framework are spatially and temporally linked: Euclidean space $\mathbb{R}^2$ and a regular square grid $\mathbb{N}^2$ allow the location of morphogens, cells, seeds, vectors, vertices, and edges, and the evolution of dynamic layers is computed for a sequence of time steps $T = 0, 1, 2, \dots, t$. The sequence of time steps T is the same associated to all components of the model, so that the network and the layers evolve synchronously.

7.2.2 The environment layer $L_e(t)$

An environment surrounds the system and represents all aspects that might affect its evolution. It is modelled with the following layer:

Definition 12:
the environment layer

The environment layer $L_e(t)$ of the framework is defined by:

- $L_e(t)$ is a subspace of $\mathbb{N}^2$. Element of space are noted as c_e and they are mapped to a couple of coordinates in $\mathbb{R}^2$.
- To each cell is assigned a state set $S_e = (e_1, e_2, \dots, D_a, D_b, f, k)$ of real values.

Values $(e_1, e_2, \dots)$ indicate the contribution of the environment to the organization of the seeds and (D_a, D_b, f, k) are parameters used into the associated morphogen layer $L_m(t)$. The environment layer can evolve over time under the effect of external factors and feedbacks from the geometric graph.

7.2.3 The morphogen layer $L_m(t)$

The morphogen layer $L_m(t)$ is defined in section 5.2.1. It is a regular square lattice, where the state of cells synchronously evolve with a transition function inspired by the Gray-Scott model (section 5.1.7). The state of a cell $S_m = (a, b)$ represents the concentrations of morphogens A and B .

7.2.4 The vector field $\vec{L}(t)$

The dynamic vector field $\vec{L}(t)$ controls the displacement of the seeds. It is composed by a set of vectors. Each vector is assigned to a cell of the space $\mathbb{N}^2$ and it is computed from the concentration of the morphogen B^1 . The vector field could be combined with a second vector field computed considering a property $e \in S_e$ of the environment. We will show in section 8.1.4 that a second vector field can integrate some exogenous aspects. In chapter 9 we will integrate in the framework the orography of an urban system).

Convolution

In our work, convolution consists to compute the vector field $\vec{L}_m(t)$ from the morphogen layer $L_m(t)$ at the time step t (fig. 7.2). We are inspired by convolution in image processing. In the framework, the convolution is a function that maps a vector to each cell: $L_m(t) \times K \times N \rightarrow \vec{L}_m(t)$, where $L_m(t)$ is the morphogen layer at time step t , K is a kernel, N is the neighbourhood associated to each cell, and $\vec{L}_m(t)$ is the resulting vector field. The kernel is a 3×3 bisymmetric matrix of real values in $[0, 1]$. The dynamic vector field is computed at each time step of the simulation.

¹Choosing B is arbitrary, because in each cell $a + b = 1$. If we chose A , the magnitude of vectors does not change but the direction is inverted.

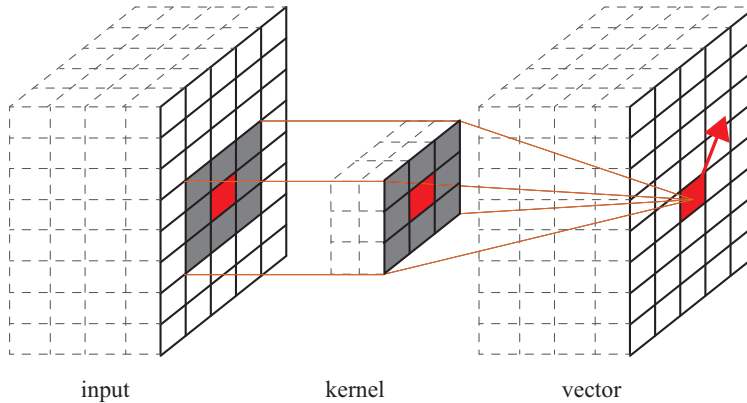


Figure 7.2

The convolution. A vector is computed for each cell (red). The direction and the magnitude of the vector is a function of the concentration b of the cells, the concentration of its neighbours (grey), and a kernel matrix. The methods could be applied in 3-dimensional space.

The vector $\vec{b} = (b_x, b_y)$ associated to the cell $c = (x, y)$ computed for the concentration b may be written as:

$$\vec{b} : \begin{cases} b_x = \sum_{i,j=-r}^r (i)(b(x, y) - k_{i,j} b(x + i, y + j)) \\ b_y = \sum_{i,j=-r}^r (j)(b(x, y) - k_{i,j} b(x + i, y + j)) \end{cases} \quad (7.1)$$

where $k_{i,j}$ are the elements of the matrix K . In our model, the convolution kernel and the diffusion kernel (eq. (5.22)) are identical. As we noted for the diffusion, the eq. (5.22), ensures a good approximation of continuous spatial effects into a discrete space. The vector $\vec{b}(t)$ obtained by eq. (7.1) is added to a random vector $\vec{r}(t)$ at time step t to define the displacement of the seed (see section 7.3.2).

7.2.5 The spatial complex network $G(t, x)$

A set $S(t)$ (eventually empty) of seeds (definition 11, page 119) evolves and builds the network. The seeds are moved by the vector field $\vec{L}(t)$ and during the simulation new seeds could be added or removed (see section 7.3.4). The results of the displacement of seeds is represented by a growing planar graph (definition 7, page 44, definition 5, page 39), which grows in a coherent way (definition 8, page 48)

7.3 System dynamics

The configuration of the framework at each time step depends on the configuration of the framework at the previous step. Each component of the model updates synchronously. Algorithm 5 describes a step of the simulation. In the next section we will detail it.

7.3.1 Reaction-diffusion mechanism

The displacement of morphogens and their interactions, at a lower level of observation, is simulated at a high level with a reaction-diffusion system (section 5.2). The Gray-Scott mechanism simulates the evolution of concentrations a and b during the time. Parameters (D_a, D_b, f, k) used to compute the concentrations are embedded in the state set D_e of the corresponding cell in the environment layer $L_e(t)$.

7.3.2 Building the network

When a spatial patterns emerges in $L_m(t)$, it affects the behaviour of seeds. A seed $s \in S$ is associated to a coordinate $X_t \in \mathbb{R}^2$ and a vertex $v_t \in V$ at time step t . A displacement vector $\vec{v}(s)$ moves the seed to the new position X_{t+1} . A new vertex v_{t+1} is created at the new position of the seed at X_{t+1} . An edge, drawn as a straight line, is created to connect vertices v_t and v_{t+1} .

Algorithm 5: A step in the framework.

```

input : space  $\mathbb{R}^2$  and  $\mathbb{N}^2$ 
          $L_e(t)$  the environment layer at time step  $t$ 
          $L_m(t)$  the morphogens layer at  $t$ 
          $\vec{L}(t)$  the vector field at  $t$ 
          $S(t)$  the seed set at  $t$ 
          $G(x, t)$  the geometric planar graph at  $t$ 
output:  $L_e(t)$ ,  $L_m(t)$ ,  $\vec{L}(t)$ ,  $S(t)$  and  $G(x, t)$  at  $t + 1$ 
begin
  test and eventually create new seeds
  update  $L_m(t)$  and  $\vec{L}$ 
  foreach seed  $s \in S$  do
    if  $s$  is not removed then
      compute potential displacement of  $s$ 
      if no intersections then
        move  $s$  and add an edge and a vertex to  $G(x, t)$ 
        compute feedback
      else
        handle intersections and remove  $s$ 

```

At each time step, we compute the new position of the seed as the sum of the vector $\vec{b}$ and a random vector $\vec{r}$. The methods are described in section 6.2.1. The magnitude and the direction of the random vector $\vec{r}$ depends on the previous position and the direction of the movement of the seed at the previous step. The displacement of the seed is similar to the displacement in the previous step but diverges by a random quantity (eq. (6.1)). The introduction of randomness makes it possible to reproduce what we may observe in real networks: a straight direction with fluctuations.

Once the random vector and vector field are computed, we sum them up. Our methodology allows us to sum a third vector $\vec{e}$, which represents an exogenous factor that affects the movement of the seed and it is spatially represented by the value $e \in S_e$ (fig. 7.3).

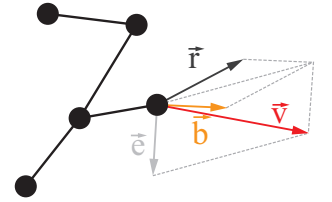


Figure 7.3
The sum of vectors.

7.3.3 Handling intersections

Among the aspects that affect the seeds, the characteristics of the graph play an important role: during its trip, a seed interacts with vertices and edges around it. In chapter 6 we exposed two methods to handle cases where seeds are close to vertices or when their trips cross an edge. We summarize them.

- **Handling only intersections** (see the geometric planar graph generator, section 6.2):
when the seed crosses an existing edge, it is removed, a new vertex is added in the point of intersection and it is connected to the seed vertex.

- **Handling also the neighbourhood** (see the reaction-diffusion planar graph generator, section 6.3):
here we consider vertices and edges close to the seed. When an element of the graph is detected by the seed, it is removed and its corresponding vertex is connected to close elements.

Both methods respect the idea that interactions appear between close elements, preserve the coherence of the network, and ensure the planarity of the graph. However, the two methods produce graphs with differences in topology and in geometry. Through the first method, the topology of the network is not strongly impacted, because interactions just create vertices with degree 3. The paths of the seeds are respected. The second method has an effect on the vertex degree distribution, because, due to the spatial arrangement of vertices, many of them can be connected to the seed vertex. To conclude, both methods respect our assumptions and one can choose in accordance with the actual case study.

7.3.4 Creation and death of seeds

During the simulation, seeds can be removed or added (over an existing vertex). This eventuality is biased by a binomial trial. We start with the method used for the geometric planar graph generator (section 6.2.2). In this model, at each time step and for each existing seed, if a random value in the range $[0, 1]$ is less than a fixed threshold in the range $[0, 1]$, we remove the seed. As the same, we test all existing vertices and we eventually add a new seed. Although this method contributes to the creation of graphs with different characteristics (fig. 6.4), it also shows some limits. In the simulations we do not observe relevant structures. Graphs fill $\mathbb{R}^2$ homogeneously and vertices are not arranged in a scale-free way. This method does not consider exogenous aspects of the network.

A way to combine the topology of the graph with the concentration of morphogens to generate a new seed was exposed in the reaction-diffusion planar graph generator (section 6.3.4): a new seed was added considering the degree of vertices, the geometry of edges, and the vector field. Exogenous aspects of the systems are not considered and the creation and the death of seeds only depend on deterministic aspects.

We are aware of these limits and we combine in the framework the advantages of methods used in the two models in chapter 6. We combine the topological and geometric aspects of the graph, the characteristics of the environment, the concentration of morphogens, and uncertainty.

In the framework, for each existing vertex, we add a seed if:

$$\text{random}(0, 1) < p_c \text{ function}(V(x), S_e, S_m) \quad (7.2)$$

and, we remove existing seed if:

$$\text{random}(0, 1) < p_d \text{ function}(S_e, S_m) \quad (7.3)$$

We do not consider the characteristics of the network when seeds are removed because this aspect is considered during the interactions between

these latter and the existing graph (see section 7.3.3). Many different implementations of these functions are possible. We based the following implementation on previous observations (section 2.3.4). Spatial complex networks often form as a combination of the topological, geometric and exogenous aspects. In this work, we experiment with the following case. Let ρ be the concentration of a morphogen in the cell which contains a vertex v . The degree of v is noted $k(v)$. We create a new seed if over the vertex if:

$$\text{random}(0, 1) < \frac{p_c \rho}{k(v)} \quad (7.4)$$

and we remove an existing seed if:

$$\text{random}(0, 1) < p_d(1 - \rho) \quad (7.5)$$

where $p_c, p_d \in [0, 1]$ are two parameters. In this way, the same concentration ρ controls at the same time the creation and the death of seeds in the cell. Moreover, the creation of a seed is inversely proportional to the degree of the associated vertex. That take into account the scarcely probability to have high values of degree. In the next chapter we will study the impact of the choice between concentration a or b to drive the life-cycle of the seeds.

7.3.5 The feedback

The feedback mechanisms are very present in many real systems and ubiquitous in complex systems, especially in living systems (section 1.1.2). Basically, some chains of interactions loop on themselves. Each element of the loop acts on itself positively or negatively. This is a circular causality, in which the elements of the loop are both cause and effect.

Feedback does not only controls the functioning of the system, it may be an important morphogenetic process (section 1.3.1). In many systems, at the same time, different feedback mechanisms co-exist. Some of them support the growth of the system (positive feedbacks) while others limit the growth (negative feedbacks). When positive feedbacks dominate negative feedbacks, we have morphogenesis, otherwise we have morphostasis. The overlapping of these processes characterizes the system, defining complex hierarchical patterns, emerging properties and structures. Finally, the feedback can act synchronically (the effect of the cause are in a the same period of time) or diachronically (the effect and the cause are not in the same period of time).

Feedback effects are present in the proposed framework. They are at the level of the morphogens, where the autocatalysis of the morphogens A and the inhibition of B can be seen as positive and negative feedbacks. We also integrate a cross-level feedback. The basic idea is that the pattern formation at the morphogen layer governs the form of the network, which in turn feeds back to the morphogen layer. The framework allows us to consider several aspects of the network as a source of feedback. They can be implemented to explore different aspects of a system. In this work, we are interested on the morphogenesis of the network; we therefore prefer to focus on generative events. There exist many possible events that can be

considered. We will focus only on the creation of a node, which probably represents the more intuitive and representative event among them.

The effect of the event is considered "local": it locally affects morphogens (in other words, the event acts to the morphogens in the vicinity of it). The behaviour of morphogens synchronically changes, the concentration of them evolves and they (eventually) find a new manner to organize themselves. According to the basic characteristics of the Gray-Scott model (section 5.1.7), the event may locally produce a change into one of the following aspects:

- The diffusivity of the morphogens (parameters D_a and D_b).
- The rate of creation and death of morphogens (parameters f and k).
- The density of morphogens in a region (concentrations a and b).

The following implementations are based on observations about the pattern formation in a discrete Gray-Scott model (sections 5.2.2 and 5.2.3). Operatively, the feedback can be implemented with the two following methods:

- In the first method, we suppose that the feedback happens through the environment layer L_e . We recall that to each cell $c_e(X) \in L_e$ located in X is assigned the state set $S_e = (D_a, D_b, f, k)$ and during the simulation, the morphogen layer $L_m(t)$ updates the state $S_m = (a, b)$ of the cell $c_m(X)$ according to the state S_e . When a new node is created, we change one or more values in the corresponding cell. The changes in the network produce a diachronic effect on the organization of the morphogens: morphogens are not directly impacted by the change but they will find a new configuration.
- In the second method, we suppose that the creation of a new node affects directly the concentration of morphogens in the corresponding cell. Hence, when it happens, we locally change the concentration of a or b . The event and the effect are synchronic.

In chapter 8, we will detail our methods and we will present early results (with and without feedbacks). We will also discuss how the feedback impacts the morphogenesis in our framework.

Summary 18:
The framework

In this chapter we presented a general framework that simulates the morphogenesis of a spatial network. The system is driven by three dominant dynamics: reaction-diffusion mechanism, emergence of a dynamic network and cross-layer feedbacks. Many questions, such as parameter values or implementation choices, are left open. We will discuss them in the following chapter.

Chapter 8

Experiments

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Keywords Experiments, effects of feedbacks, model behaviour.

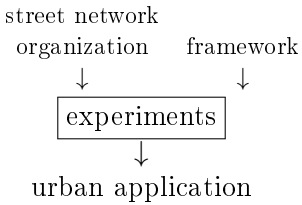
"We can only see a short distance ahead, but we can see plenty there that needs to be done"

Alan Turing

The proposed framework (chapter 7), inspired by complex systems principles (chapter 1), aims to simulate the morphogenesis of complex spatial networks (chapter 2). It is a multilayer model: the first layer is inspired by cellular automata theory and the Gray-Scott model and it simulates the pattern formation through a reaction-diffusion mechanism (chapter 5). Unexpected patterns constraint the form of a spatial network (chapter 2) with a dynamic vector field in the second layer. The network (the third layer) feeds back to morphogens. The fourth layer completes the model, it aims to simulate exogenous aspects that can locally impact the morphogenesis. The proposed framework can be found applications in different fields; here our main focus is on the morphogenesis of street networks (chapter 3).

Firstly, we will test the sensibility of the framework to parameters and the effect of basic dynamics without the feedback of the graph to the morphogens (section 8.1). Then, we will expose the crucial role that cross-layer feedbacks have in the morphogenetic processes (section 8.2). We will compare our networks to theoretic planar graph and French department street networks. To quantitatively characterize them, we will use measures described in chapter 5. We discuss our results in section 8.3.

8.1 Experiments without feedbacks



We choice to investigate the proposed framework with an incremental approach: in order to answer few basic questions about its behaviour, we start by studying a basic initial configuration which do not integrate the cross-layer feedback. In this way we can investigate effects of a few initial parameter settings to resulting graphs. We are beginning our evaluation, making four experiments; each of them would evaluate:

- **no.fb.1:**
the growth rate of graph with different rates of creation and death of seeds. To do that, we study the sensibility of the framework in a (p_c, p_d) diagram.
- **no.fb.2:**
the relations between the characteristics of the graphs and the Turing space (f and k parameters).
- **no.fb.3:**
the characteristics of graphs when the growth is driven by the morphogens A or the morphogens B .
- **no.fb.4:**
the effect of different kinds of vector fields to the graph. We will combine vectors computed from the concentration $\vec{b}$, random vectors $(\vec{r})$ and vectors from a the state e of the environment layer L_e ($\vec{e}$).

experiment	initial configuration ^a				to evaluate
	f, k	p_c, p_d	ρ	$\vec{L}$	
<i>no.fb.1</i>	static RD ^b	$p_c, p_d \in [0, 1]$	b	$\vec{b}$	the impact of p_c and p_d
<i>no.fb.2</i>	(1) in Turing space ^c (2) 3 couples ^d	(1) 3 couples ^e (2) (0.001, 0.003)	b	$\vec{b}$	the impact of f and k
<i>no.fb.3</i>	(0.030, 0.062)	(0.01, 0.03)	a or b	$\vec{a}$ or $\vec{b}$	the effect of ρ
<i>no.fb.4</i>	no RD	(0.003, 0.001)	b	$\vec{b} + \vec{e}$	the effect of a 2 nd vector field

^a $|\mathbb{N}^2| = 2^8 \times 2^8$, concentration cells are homogeneous ($a, b = 1.0, 0.0$) with a perturbation ($b = 1.0$), $D_a = 1.0$, $D_b = 0.5$, $t_{max} = 3000$.

^b Experiments are computed with 2 static configurations of L_m : the patterns *solitons* and *holes* after 4000 steps are obtained with a perturbation in the middle ($b = 1.0$) and with parameters $(f, k) = (0.030, 0.062)$ and $(f, k) = (0.039, 0.058)$.

^c parameters f, k used in the simulation allows the formation of patterns (they are in the Turing space, section 5.1.3). Operatively, we select the couples that were the Shannon entropy at the final step is $H(a) > 5$ (fig. 5.20).

^d with a perturbation $b = 1.0$, the parameters allow the formation of *holes* $(f, k) = (0.039, 0.058)$, *moving spots* $(f, k) = (0.014, 0.054)$ and *solitons* $(f, k) = (0.030, 0.062)$ respectively.

^e $(p_c, p_d) = \{(0.055, 0.005), (0.035, 0.010), (0.065, 0.025)\}$.

Table 8.1
Experiments without the feedback.

To this end, we perform our simulations using the following experimental setting:

Space and time The framework is defined in $\mathbb{N}^2$ with $2^8 \times 2^8$ cells and the size of a cell is 1 unit. All simulations stop after 3000 steps of simulation.

The Morphogens' layer $L_m(t)$ Every cell is assigned with two states $a, b \in [0, 1]$, and the evolution in a discrete time series is defined by a Gray-Scott mechanism. As has been said in section 5.2.3, in an f, k diagram we can identify some zones that produce a specific dynamic (fig. 8.1). We noted that there exists the blue zone and the red zone, where no patterns can be observed. Between these zones, we have the Turing space. Two other zones exist, an unstable area where patterns evolve in time (dynamic far from equilibrium) and a stable area where morphogens find an equilibrium (dynamic out of equilibrium). Moreover there exists a point, named *equilibrium* $(f, k) = (0.0625, 0.0625)$ where no kind of perturbation is able to activate the formation of a pattern (dynamic at equilibrium). Figure 8.3 depicts the evolution of four pattern formations, named *holes* $(f, k) = (0.039, 0.058)$, *mazes* $(f, k) = (0.029, 0.057)$, *moving spots* $(f, k) = (0.014, 0.054)$ and *solitons* $(f, k) = (0.030, 0.062)$ (fig. 8.2). Diffusivity is the same in all simulation, $(D_a, D_b) = (1.0, 0.5)$.

The environment layer L_e In the following experiments we consider an environment layer (static) L_e and we set to each cell a state $S_e = (f, k, D_a, D_b)$ in order to obtain patterns in fig. 8.3. Only for the experiment *no.fb.4* (section 8.1.4), we add discrete values $e_1 \in [0, 1]$ to investigate the effect of a second vector field to the evolving graph.

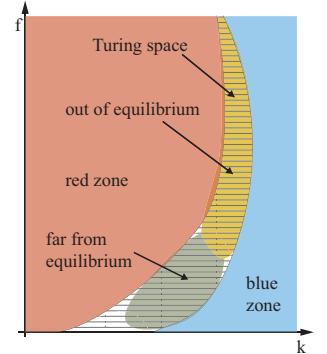


Figure 8.1
The stylized representation of states of Gray-Scott model.

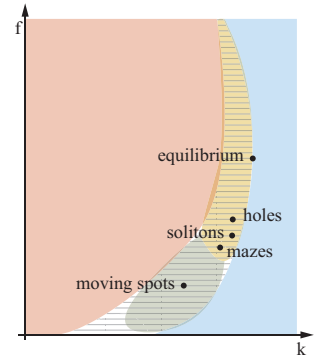
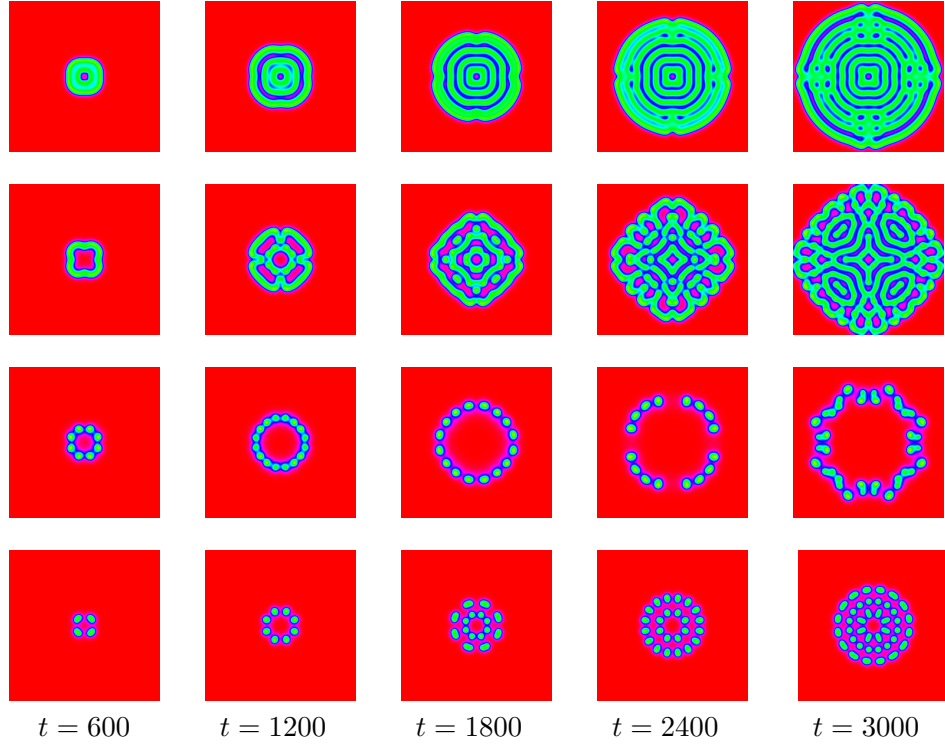
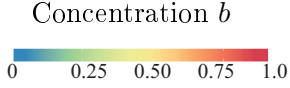
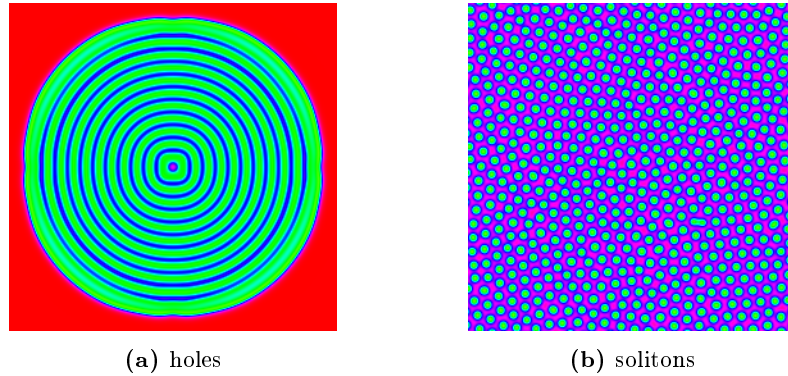
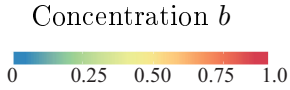


Figure 8.2
The stylized representation of patterns in the $f - k$ diagram.

**Figure 8.3**

The evolution of 4 patterns in a Gray-Scott model: *holes* $(f, k) = (0.039, 0.058)$, *mazes* $(f, k) = (0.029, 0.057)$, *moving spots* $(f, k) = (0.014, 0.054)$ and *solitons* $(f, k) = (0.030, 0.062)$. Each simulation is initialized with an homogenous state $(a, b) = (1.0, 0.0)$ perturbed in the middle of space ($b = 1.0$).

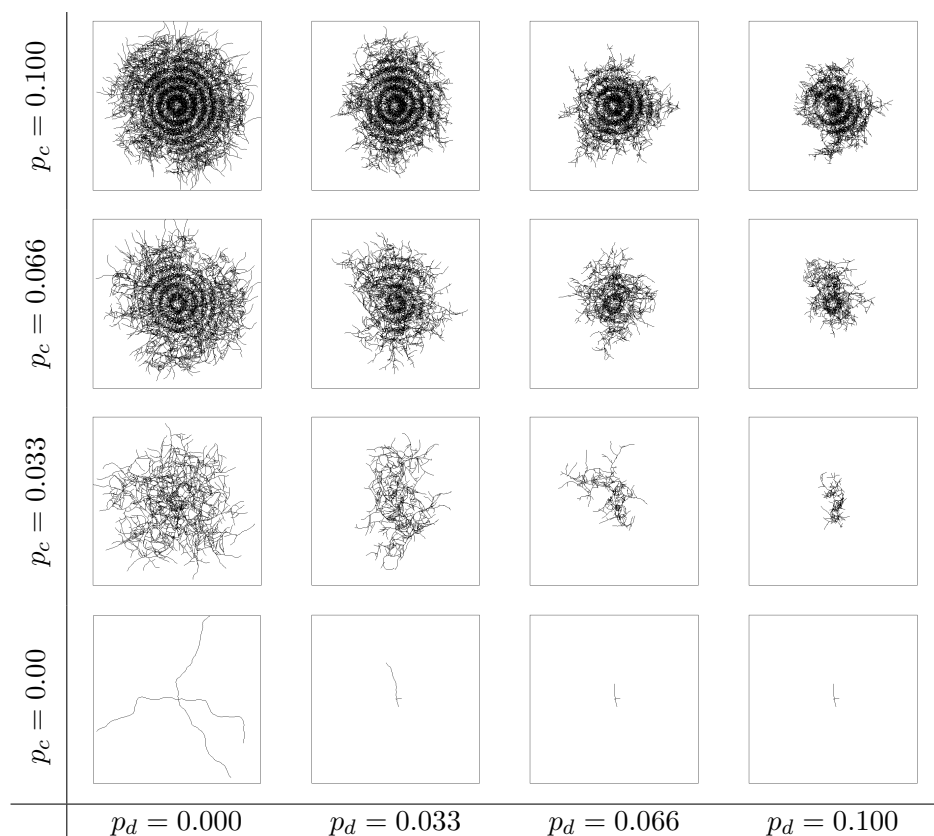
**Figure 8.4**

Experiment *no.fb.1*. Configuration at time step $t = 4000$ of two homogeneous states perturbed in the middle with two different patterns (holes and solitons).

Vector field $\vec{L}$ We assign to every cells a vector $\vec{b}$, obtained with the concentration of morphogens B and computed with the convolution method (section 7.2.4).

Intersections We only consider the case where the displacement of the seed intersects an existing edge (section 7.3.3). We handle this situation with the methodology exposed in section 6.2: if the trip of a seed intersects an existing edge, we remove the seed, we add a new vertex at the intersection, and we connect the vertex where the seed born to the new vertex. The growth of resulting networks are coherent during the simulation, and the planarity condition is respected.

Creation and death of seeds We use in $\rho = b$ in eqs. (7.4) and (7.5) for experiments *no.fb.1*, *no.fb.2* and *no.fb.4*. In experiment *no.fb.3*, $\rho = a$.

**Figure 8.5**

Experiment *no.fb.1*. Simulations with the holes static configuration for various constant values of p_c and p_d .

Graph analysis In order to capture some essential properties of graphs, we apply the algorithm 1 before analysing them.

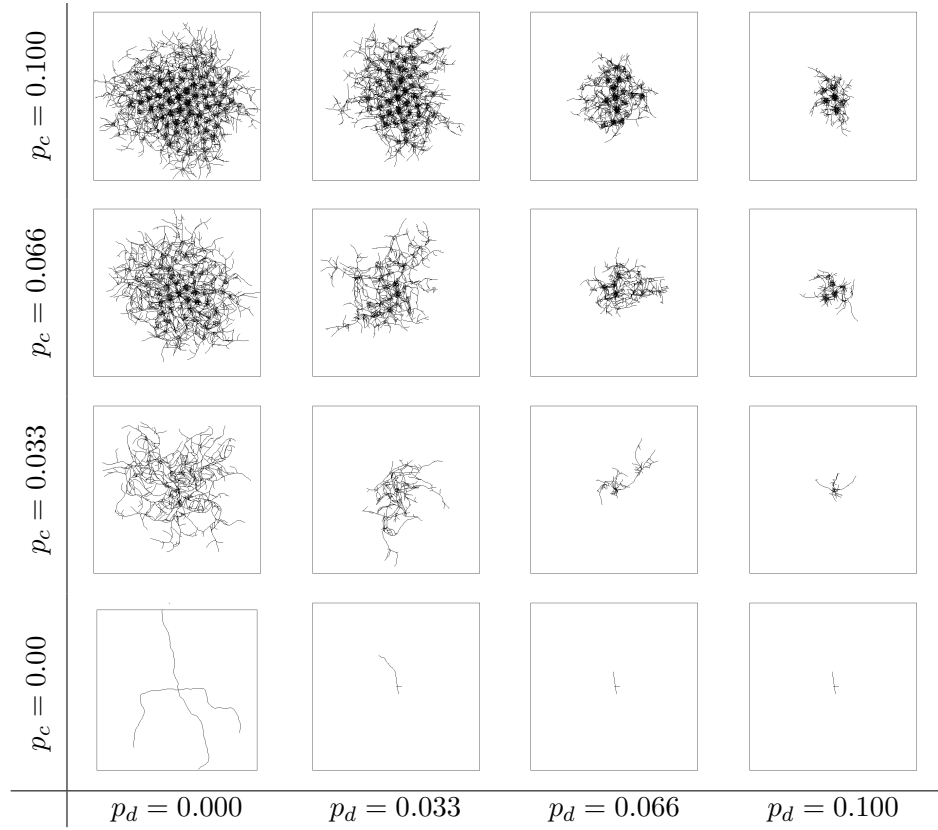
In table 8.1 we summarize the experimental settings of the proposed experiments.

8.1.1 Experiment *no.fb.1*: graphs and Turing space

The goal of this first experiment is twofold: to evaluate if a static configuration of morphogens influences the formation of structures in the graph and to evaluate if parameters p_c and p_d impact the rate of growth of the graph. We build two emblematic patterns (*holes* and *solitons*, fig. 8.4, rows 1 and 4 in fig. 8.3). To make the first pattern, we perturb the middle of space of an homogeneous state $(a, b) = (1.0, 0.0)$ with a pulse ($b = 1.0$). To make the second one, we perturb the 10% of the homogeneous state with a pulse. We capture the pattern at the time step $t = 4000$. At this moment, we initialize the simulations with a star graph of 5 vertices in the middle of space and we stop them after 3000 steps. Figures 8.5 and 8.6 depict the graphs obtained for 16 couples of parameters p_c and p_d . We observe the important role of these parameters to the formation of the graph: as we observed for graphs obtained with the geometric planar graph generator (section 6.2, fig. 6.4), high values of p_c cause a rapid growth, mitigated by p_d . The static concentration layers cause a new effect: a spatial correlation between regions where b is high and region with a high number of vertices. The lower values of p_c , combined with the effect of the vector field, minimize

Figure 8.6

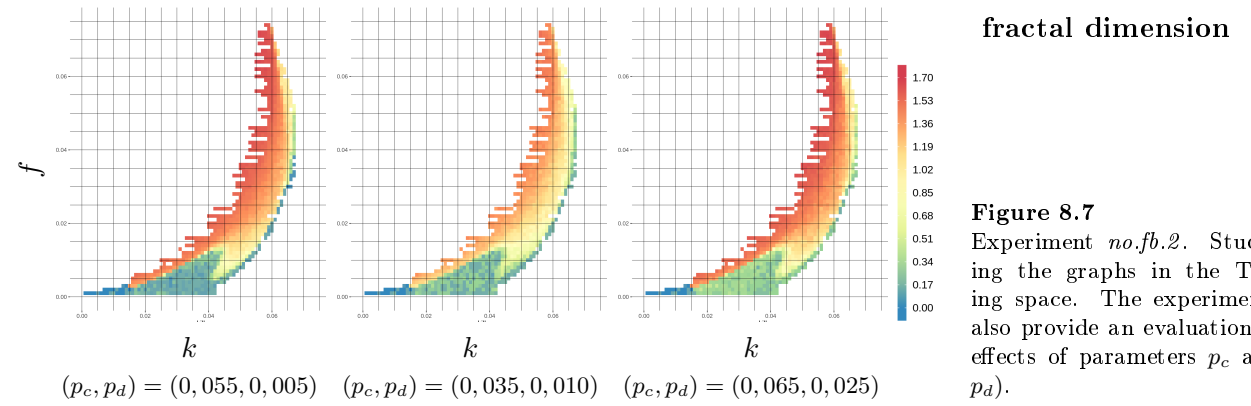
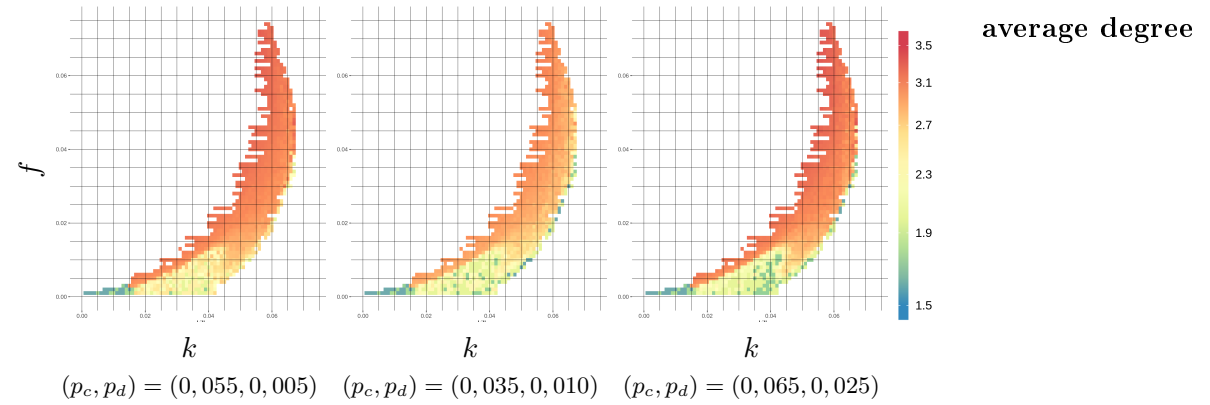
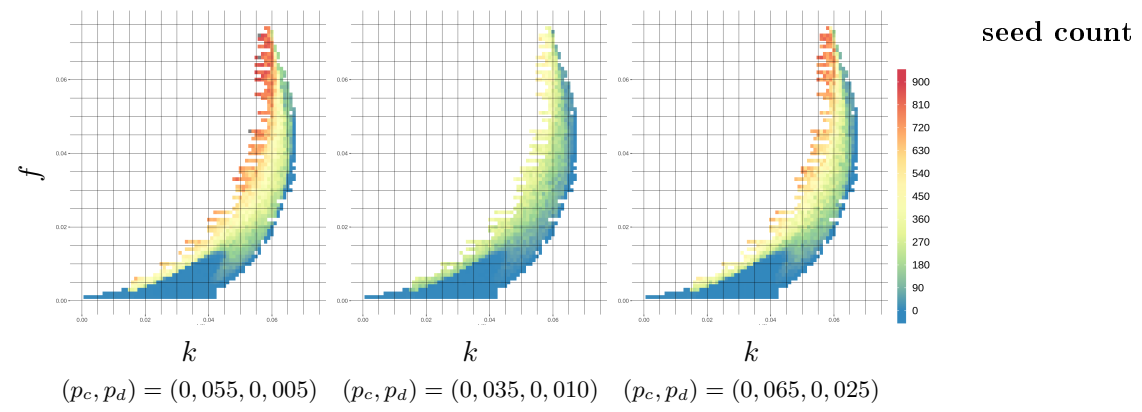
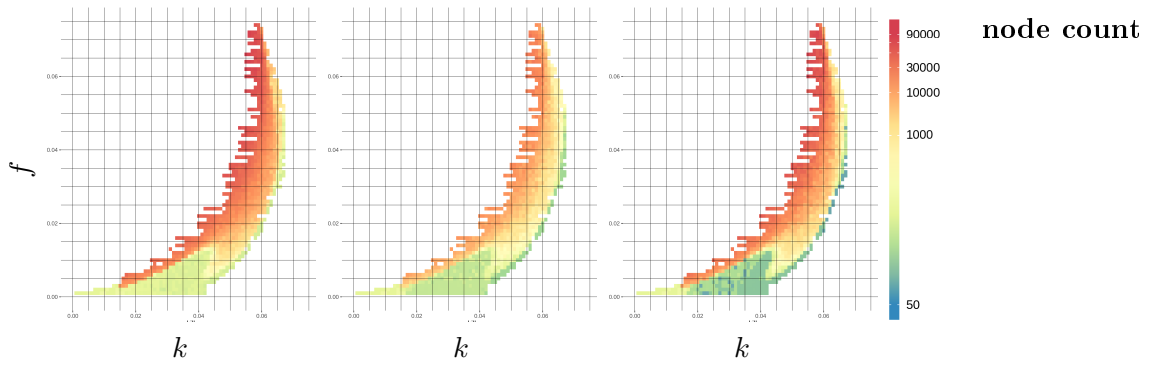
Experiment *no.fb.1*. Simulations with the solitons static configuration for various constant values of p_c and p_d .



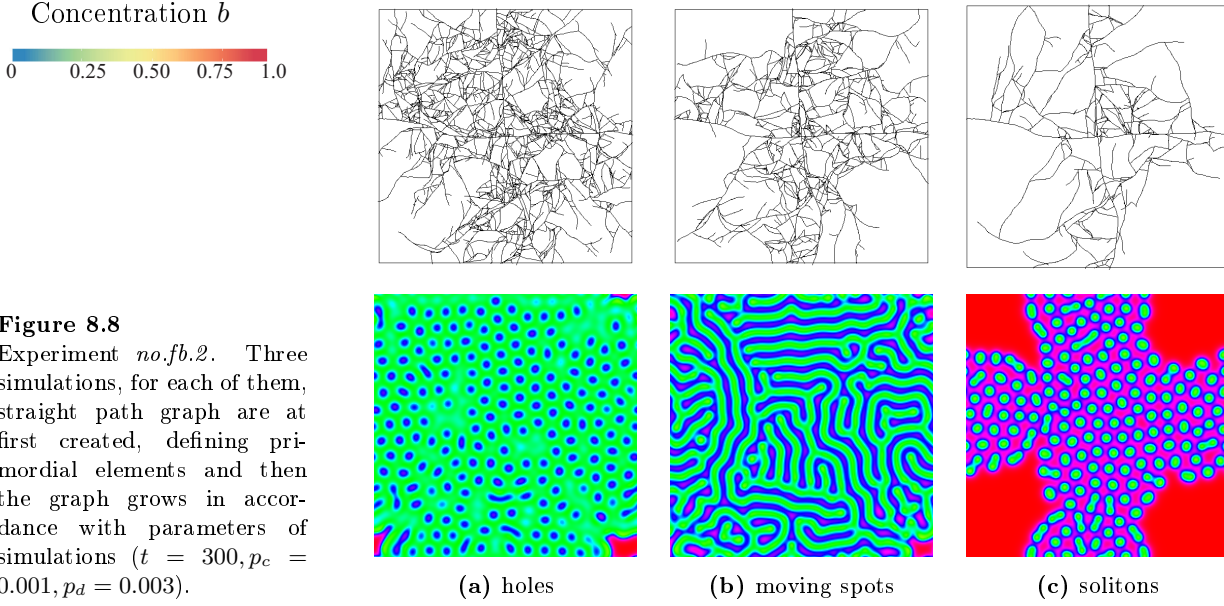
the possibility of seeds to go through regions with a high concentration of B . These results suggest that the coupling between the graph and a layer of concentrations allows us to go beyond the main limits of the geometric planar graph generator (absence of structures and regularity in the rate of growth).

8.1.2 Experiment *no.fb.2*: effect of parameters f and k

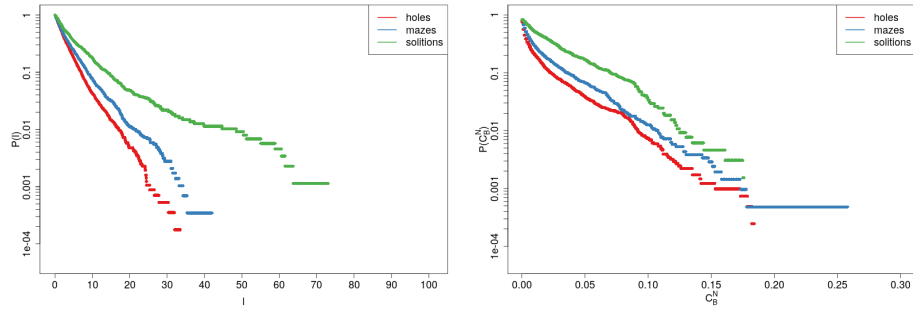
The experiment reported in fig. 8.7 consists to evaluate the effects of different evolving patterns (and incidentally the parameters p_c and p_d) to the evolution of the graph. To this end, we perturb an homogeneous concentration $(a, b) = (1.0, 0.0)$ of morphogens with a pulse ($b = 1.0$) in the middle of space. The initial graph is a star graph of 5 vertices and it is located over the initial perturbation. As we observed in section 5.2.3, the Gray-Scott dynamic can generate stable or unstable patterns only for some initial configurations of parameters f and k (the Turing space, section 5.1.3); the Shannon entropy is an indicator able to detect these the Turing space in a f, k diagram (fig. 5.20) because high values of it suggests that concentration are heterogeneous. In those experiments, we had previously selected all couples of f, k which had produced values of Shannon entropy higher than 5 in the experiment reported in fig. 5.20. We simulate the evolution of the system for those couple f, k and we observe the behaviour for three couple of growth rates, $(p_c, p_d) = \{(0.055, 0.005), (0.035, 0.010), (0.065, 0.025)\}$. Figure 8.7 reports the size of the vertex set, the number of seeds, the fractal dimension, and the average degree of the graph at step 3000. We observe

**Figure 8.7**

Experiment *no.fb.2*. Studying the graphs in the Turing space. The experiments also provide an evaluation of effects of parameters p_c and p_d .

**Figure 8.8**

Experiment *no.fb.2*. Three simulations, for each of them, straight path graph are at first created, defining primordial elements and then the graph grows in accordance with parameters of simulations ($t = 300, p_c = 0.001, p_d = 0.003$).

**Figure 8.9**

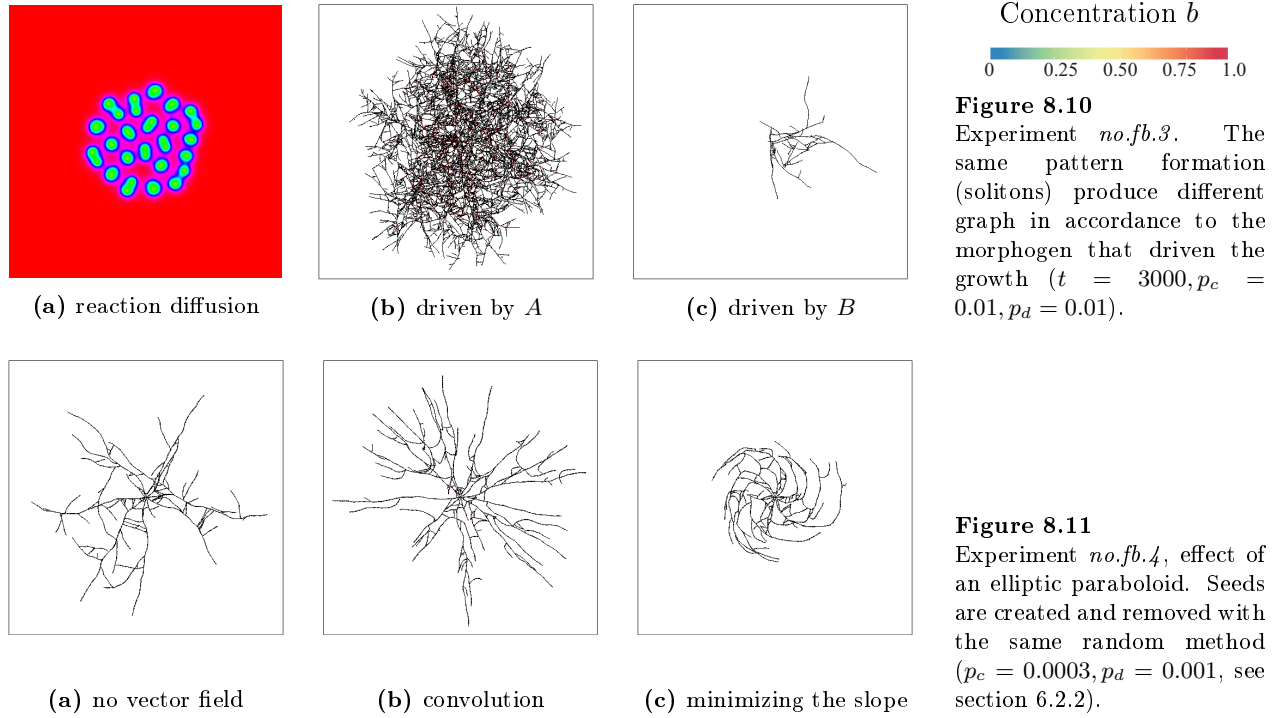
Experiment *no.fb.2*. The cumulative length edge distribution (left) and the cumulative distribution of betweenness centrality of 3 patterns (right).

that the formation of the graph is impacted by the kind of dynamic of the reaction-diffusion layer. For lower values of f ($f \lesssim 0.03$), the layer produces far from equilibrium patterns¹ (section 1.3.2): the graph grows discontinuously and the number of seeds is less than simulations with high values of f . When patterns are far from equilibrium, small graphs generate, the average degree is less than 2.5 and vertices are not arranged in a scale-free way. For high values of f , the patterns are not yet at equilibrium² and the resulting graphs are composed of more vertices. Values (p_c, p_d) reduce or increase these effects, but they do not govern those dynamics.

A study of few characteristic patterns All previous experiments have been made with an initial star graph and only one perturbation in the morphogen layer. Here we want to assess the behaviour of the framework when the the initial graph is not a star graph. In this experiment, three simulations (where parameters f and k should make holes, mazes or solitons patterns) are initialized with two path graphs which cross in the middle of space; 10% of initial vertices perturbs the homogeneous state $(a, b) = (1.0, 0.0)$ with a pulse $b = 1.0$. Figure 8.8 shows that the graph coupled

¹e.g., chaotic patterns and moving spots.

²e.g. solitons, mazes and holes



with the solitons dynamic grows slower than the graph coupled with holes dynamic: at the end of the simulation, it is composed of fewer elements. As we have observed before, the creation of new seeds is strongly impacted by the concentration of b and in solitons morphogens B cluster in many small spots. Moreover, each spot is stable, when it appears it does not move: hence the trip of a seed rarely passes through a spot. Although the growth rate of the graphs is different for these patterns, we observe different aspects in common (table 8.2 and fig. 8.9). Moreover, compared these last with results in section 4.2, we observe that the main characteristics of these graphs are close to the French department street intersection network. We suppose that at least in the situation represented by this experiment, the framework builds graphs that might be compared with real street networks.

8.1.3 Experiment *no.fb.3*: growth driven by morphogens A or B

In previous experiments, we studied the growth of the graphs where the rate of the growth is driven by the concentration b . For those experiments, the

<i>pattern</i>	$ V $	$ E $	$\langle k \rangle$	γ	<i>org</i>	<i>mes</i>	<i>cost</i>
solitons	650	877	2.698	0.451	0.975	0.13	0.772
mazes	2075	2887	2.783	0.464	0.961	0.141	0.777
holes	4073	5676	2.787	0.465	0.959	0.141	0.775

Table 8.2

Experiment *no.fb.2*. Characteristics of three graphs obtained with 3 emblematic patterns (*holes*, *mazes* and *solitons*, fig. 8.3). Number of vertices $|V|$, number of edges $|E|$, average degree $\langle k \rangle$, gamma index γ , organic ratio *org*, meshedness coefficient *mes* and cost *cost*.

probability that a seed was added is high when the corresponding vertex is located in a region with many morphogens B . In experiment *no.fb.3* we compare graphs driven by a and b . Figure 8.10a shows the configuration of an homogeneous concentration perturbed in the middle of the space after 3000 steps; fig. 8.10b and fig. 8.10c show the configuration of 2 graphs at the final step. Both simulations are defined by the same parameter set; the only difference is about which morphogen drives the creation and the degradation of seeds. In the first case, a seed was added when it is close to many morphogens B and removed when it is close to many morphogens A ($\rho = b$ in eqs. (7.4) and (7.5)); in the second case, $\rho = a$. In the latter case, we observe an uncontrolled growth of the graph and it is also difficult to find a correlation between the evolution of the morphogen layer and the graph. Otherwise, when the creation of seeds is driven by the concentration b , the graph grows slowly and respects the evolution of the vector field.

8.1.4 Experiment *no.fb.4*: combining vector fields

The methods used to model the effect of a gradient of concentration to the graph is based on the assumption that the seeds are repulsed by a high concentration of morphogens (experiment *no.fb.3* suggests to prefer the morphogens B). However, in some applications, the movement of the seeds cannot be only driven by repulsion or attractive forces. An example is exposed in section 3.2.1, where we report the impact of the orography during the street formation. The formation of a new street is driven by a principle of minimization of effort. The creation of the street follows the minimal slope as a way to minimize the effort of people during their moving or to minimize the cost of civil engineering work.

According to these observations, we propose to simulate this effect with a vector to apply to the seed that follows the minimal slope of the gradient (fig. 8.11). We discretize the equation of an elliptic paraboloid ($z = x^2/a + y^2/b$) in a environment layer (definition 12, page 138) and we add the value $e_1 = z$ to the state of each cell in the L_e . We locate the vertex of the paraboloid in the middle of space and we initialize the simulation with a star graph of 5 vertices. The method to add and remove seeds is the same; it is used in the geometric planar graph generator (see section 6.2.2). In section 8.1, the movement of seeds is controlled by the displacing vector by $\vec{v}(s) = \vec{r}$. In section 8.1 the displacement vector for each seed is $\vec{v}(s) = \vec{r} + \vec{e}$. In section 8.1, the vector $\vec{e}$ is computed with the convolution method and in section 8.1 the vector follows the minimal slope. This basic comparison is an important test to evaluate a way to integrate the effect of orography on the growth of an urban system (chapter 9). It also suggests that the framework can be easily integrated by a different vector field, allowing applications in many contexts.

8.1.5 Experiments without feedback: discussion

The goal of previous experiments is to give early information about the effects of the parameter sets to the behaviour of the proposal framework.

This last allows many implementations and many parameters driven the results to different trajectories. We choose to separate our evaluation in different experiments and we investigate the framework before integrating cross-level feedbacks. This approach is incremental and experiments gave fundamental information for the next step of implementation.

We observed that the growth rate is controlled by parameters p_c and p_d , and the coupling with a layer of morphogens ensures the formation of structures. The experiment *no.fb.1* shows that even in a static configuration of morphogens the growth rate is controlled by parameters p_c, p_d and clusters of vertices form over cells with high density of morphogens (figs. 8.5 and 8.6).

When we consider the morphogens' layer as a dynamic layer (experiment *no.fb.2*), the resulting graphs are related to the patterns in the morphogen layer. To study that, we plot in f, k diagrams main characteristics of graphs. We observe a correlation between the pattern formation and resulting graphs (fig. 8.7). The effect of parameters f and k to resulting graphs shows a correlation between the kind of dynamics inside the morphogens' layer and the networks. When we select some specific patterns (fig. 8.8), we observe that our networks and street networks are similar (table 8.2 and fig. 8.9, see section 4.2 and table 4.1).

Experiment *no.fb.3* shows that it is easy to define which morphogen should drive the graph. The growth rate of the graphs and the pattern formation are correlated when we choice the morphogens B (fig. 8.10).

Experiment *no.fb.4* shows that we can integrate different vector fields into the framework. Each vector field can be dynamic or static, and different effects on the movement of the seeds can be design (fig. 8.11).

Basic principles of complexity are integrated into the framework to study the formation of a spatial network. Elements are not led by centralized control, a continuous flow traverses the system (parameter f and k), external aspects can be integrated, all graphs respect the planarity condition and they grow in a coherent way. Resulting graphs seem to have properties similar to street networks. Once that, we have this background of informations, we can integrate the feedback of the network to the substrate layers.

Summary 19:
Basic experiments

8.2 Experiments including feedbacks

In previous chapters, we have reported that in many complex systems forms can be seen as the result of antagonist feedbacks. Especially in living systems, forms are the result of the balance between positive and negative feedbacks. Roughly speaking, the first ones tend to increment the size of the system while the second ones tend to stabilize or destroy the system. We have morphogenesis when positive feedbacks are dominant, otherwise we have morphostasis (sections 1.1.2 and 1.3.1).

In the next experiments, we will integrate cross-level feedbacks in our simulations. Our methods are based on characteristics of Gray-Scott models

exp	f, k ^a				init. config. ^b		feedback		
	equi.	mazes	mov.sp.	sol.	D_a, D_b	f, k	a, b	D_a, D_b	f, k
pos.fk	-	x	x	x	1.0, 0.5	0.04, 0.08 ^c	-	-	a couple (f, k)
pos.diff	x	x	-	x	1.0, 0.5	a couple (f, k)	-	1, 0.5	-
pos.morp	x	x	-	x	1.0, 1.0	a couple (f, k)	1, 1	-	-
neg.fk	-	x	x	x	1.0, 0.5	a couple (f, k)	-	-	0.04, 0.08 ^c
neg.diff	-	x	x	x	1.0, 0.5	a couple (f, k)	-	1, 1	-
neg.morp	-	-	-	-	1.0, 0.5	a couple (f, k)	0, 0	-	-
no.fb	-	x	x	x	1.0, 0.5	a couple (f, k)	-	-	-

^a we test four couples of (f, k) : *equilibrium* $(f, k) = (0.0625, 0.0625)$, *mazes* $(f, k) = (0.029, 0.057)$, *moving spots* $(f, k) = (0.014, 0.054)$, *solitons* $(f, k) = (0.030, 0.062)$. However, we observe interesting structures in three cases.

^b $|\mathbb{N}| = 2^9 \times 2^9$, concentration cells are homogeneous ($a, b = 1.0, 0.0$), each simulations stop after 5000 steps, $(p_c, p_d) = (0.01, 0.02)$ and morphogenesis is driven by B ($\rho = b$).

^c the red state (section 5.2.3).

Table 8.3
Experiments with feedback.

and can be applied with or without the environment layer L_e . As for previous experiments, the way that we conceptualize our experiments is always incremental. In order to understand each process, we prefer to integrate into following experiments one feedback mechanism at a time.

Following sections will complete the section 7.3.5. Here we propose six methods. Each experiment (table 8.3) is computed for four couples (f, k) . Each of them should make in a classical configuration *solitons*, *equilibrium*³, *moving spots* and *mazes* patterns. The triggered event is the creation of a new node and it will locally affect parameters (f, k) , diffusivity (D_a, D_b) or the concentration b . Our table of experiments is composed of 24 simulations with feedbacks and 4 experiments without feedbacks. We make latter experiments in order to compare different situations and quantitatively measure feedback effects. Among those simulation, we select 15 with feedback (the only ones where we observe the formation of interesting structures, see next sections) and 3 without feedback. Each simulation are computed in a space $\mathbb{N}^2$ of $2^9 \times 2^9$ cells; they stop after 5000 steps, parameters $(p_c, p_d) = (0.01, 0.02)$ and morphogenesis is driven by B . For the sake of visibility, figs. 8.12 to 8.16 depict the configuration after 3000 steps in a $\mathbb{N}^2$ of $2^8 \times 2^8$ cells.

In sections bellow, we describe our simulations (how we have obtained our graphs) and we propose a visual descriptions of results (sections 8.2.1 and 8.2.2). Then we quantitatively measure the characteristics of the graphs and we compare resulting networks to French department street networks (section 8.2.3). Finally, we discuss our results (section 8.3).

³When $(f, k) = (0.0625, 0.0625)$, no perturbations are able to spark the spatial distribution of morphogens (section 5.2.3).

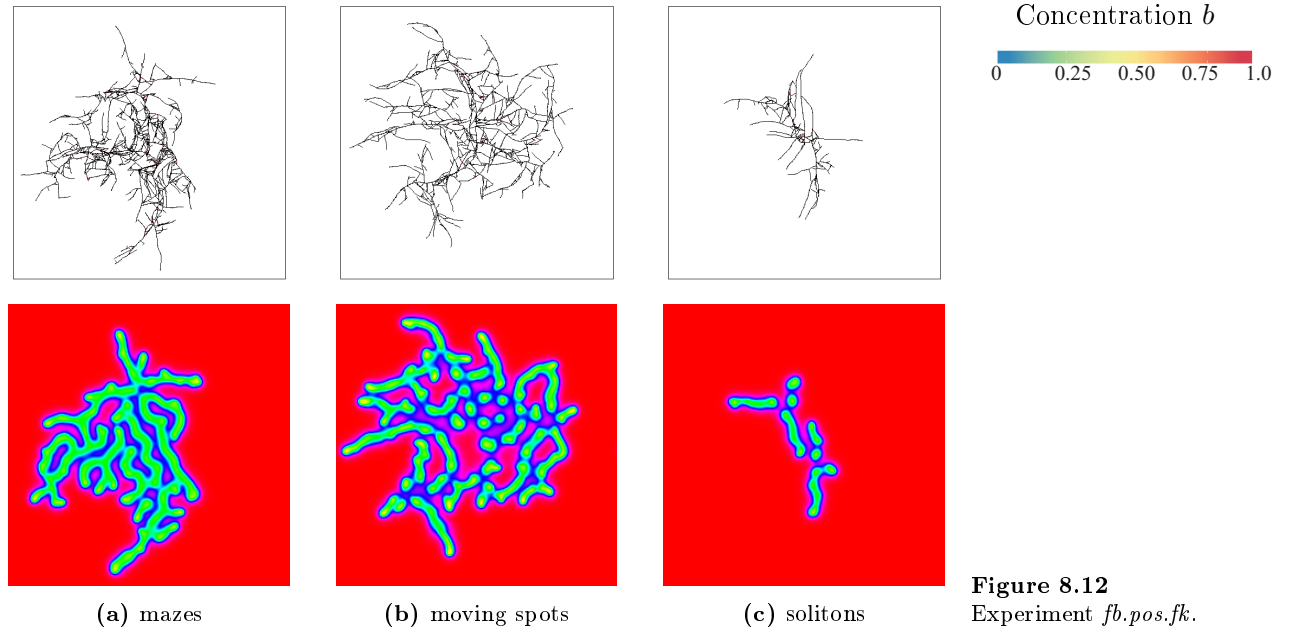


Figure 8.12
Experiment *fb.pos.fk*.

8.2.1 Positive feedbacks

Among morphogenetic mechanisms that regulate a system, there exist positive feedbacks: they are processes that stimulates the growth of the system. In our context, the creation of a node favours the capacity of morphogens to self-organize, creating a better configuration of the world or injecting morphogens. To well appreciate this mechanism, we suppose that in the configuration of the morphogen layer cannot allow the formation of pattern. This sterile configuration can be obtained in different manners. We cannot have patterns if the rates of diffusion are $D_a = D_b$, if the parameters (f, k) are out of the Turing space or if the concentrations are homogeneous $(a, b) = (1.0, 0.0)$ (no gradients and no perturbations).

The creation of a new node locally changes something. In the next three experiments, the creation of a new node imply the change of the parameters f and k (experiment *fb.pos.fk*), the diffusivity D_a and D_b (experiment *fb.pos.diff*) or the concentration b (experiment *fb.pos.morp*). Simulations bellow were initialized in a homogenous state $(a, b) = (1.0, 0.0)$, with a star graph of five vertices and a perturbation ($b = 1.0$) in the middle of space.

Experiment *fb.pos.fk*: incentivise the growth changing f and k

We assign the state $S_e = (D_a, D_b, f, k) = (1.0, 0.5, 0.04, 0.08)$ to each cell c_e in the spatial environment L_e . This configuration corresponds to the red state, no perturbations are able to disrupt the homogeneous state. We start the simulation. If a new node is created, we modify the state of the corresponding cell c_e in order to allow the formation of patterns. To do that, f and k are in the Turing space. From this change to the end of the simulation the cell is in a situation that allows the catalysis of B . Clearly, to have a pattern more than a cell should be in this configuration. After a while, in the cell and in its neighbourhood, a cluster of morphogens B

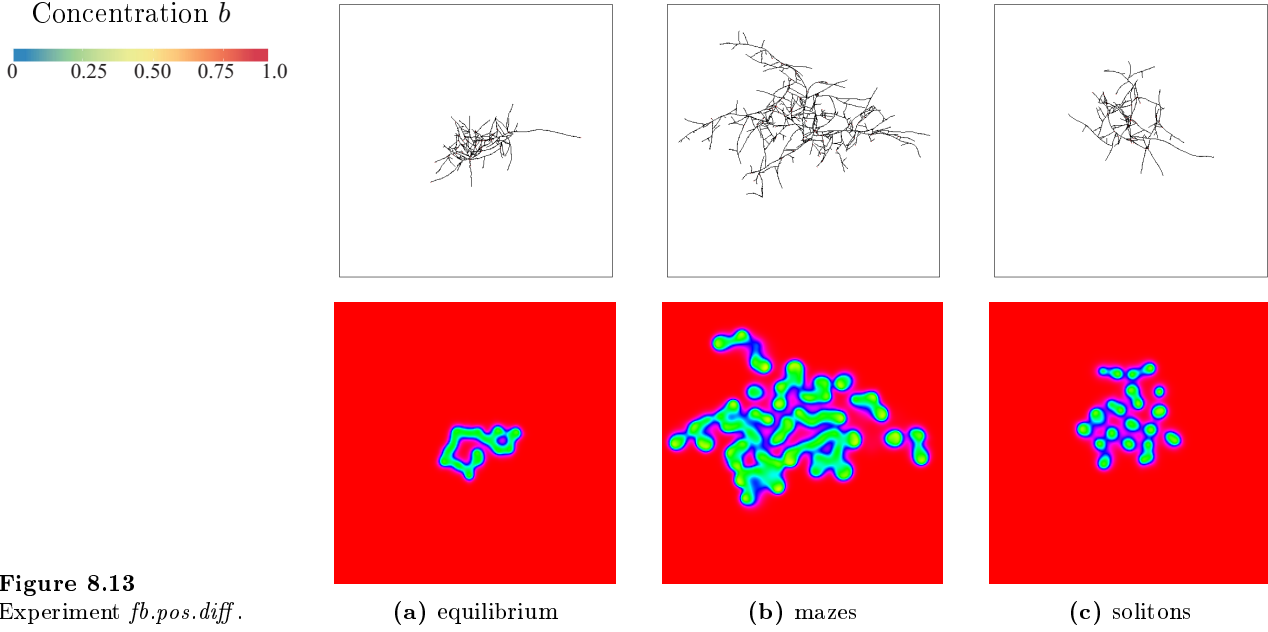


Figure 8.13
Experiment *fb.pos.diff*.

spontaneously clusters, gradients of concentrations generates vectors, new seeds born (due to the high concentration b), and the network grows. Seeds explore space, new nodes born and we observe in the morphogen layer new patterns, never seen in a classical configuration.

We report 3 cases: each of them may produce the formation of *mazes*, *moving spots* or *solitons* (fig. 8.12). Due to the rapidity of propagation of morphogens B , in the *mazes* experiment (fig. 8.12a) we observe that morphogens B follow and push seeds. The high concentrations of b along the trip of seeds facilitate the formation of new branching. The continuity of morphogens cannot be observed also in *moving spot* experiment (fig. 8.12b). Here, clusters of morphogens split in several spots. Due to their instabilities, morphogens B stay close for a while and they form different clusters. The network is sparse and less compact. The *soliton* experiment is the slower one, the graph is composed by few elements and morphogens tend to separate in spots (fig. 8.12c). We also test the equilibrium pattern: here, due to its high stability, even the feedback is not able to stimulate the formation of a graph.

Experiment *fb.pos.diff*: incentivise the growth changing the diffusivity

In this experiments (fig. 8.13), we assign to each cell $c_e \in L_e$ the same rate of diffusivity: $D_a = D_b = 1.0$. As we have observed in chapter 5, a different speed of morphogens is a necessary condition to have patterns. In this configuration, no perturbations are able to spark the homogeneous state and all inhomogeneous spatial distribution of morphogens go toward the homogenous state in a few steps. In the following experiments, the creation of a new node locally changes the diffusivity of morphogens; when new nodes born, we set $D_b = 0.5$ in the corresponding cell $c_e \in L_e$. In this situation, morphogens group together. New patterns diachronically appear

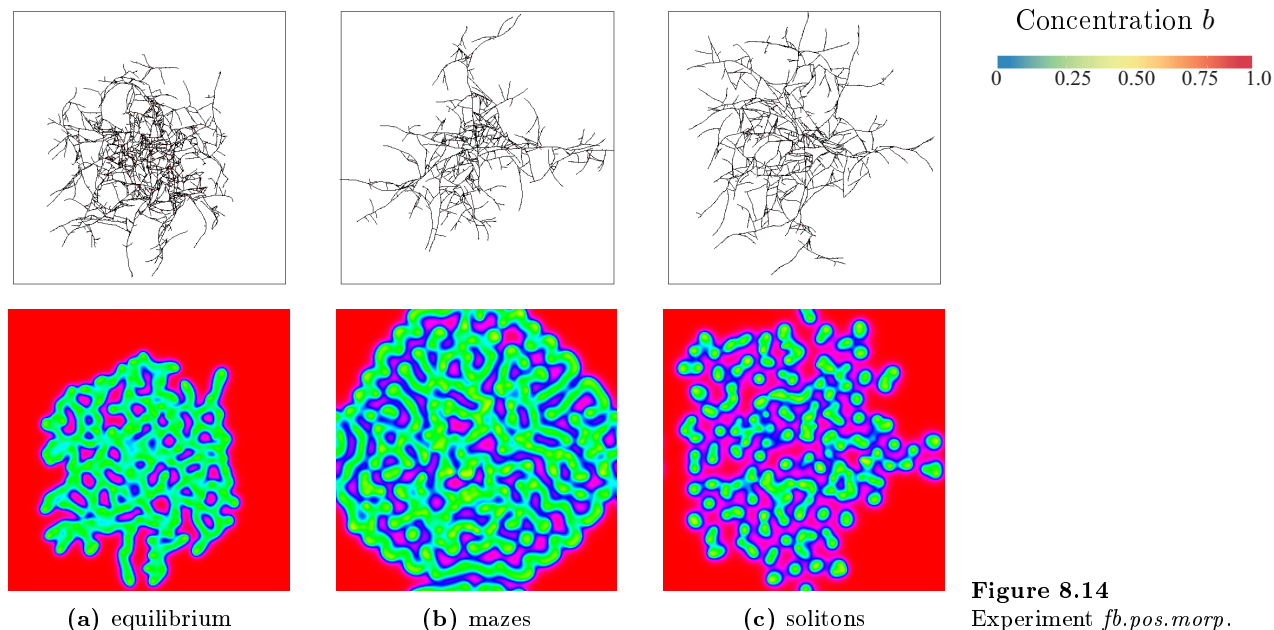


Figure 8.14
Experiment *fb.pos.morp*.

and the growth of the network stimulates its own growth.

Figure 8.13 reports three simulations. Even into the *equilibrium* experiment the stability is broken (fig. 8.13a). In this situation, the graph grows a bit and the number of morphogens B increase. In the *mazes* experiment, we observe a high degree of instability: regions that have previously reached a high concentration b suddenly evolve (fig. 8.13b). In *solitons* experiment, spots appear over the cell that was activate by the graph; spots are unstable (fig. 8.13c). Finally, due to the high instability of patterns, no organizations are observed in the *moving spot* simulation.

Experiment *fb.pos.morp*: incentivise the growth injecting morphogens B

This experiment (fig. 8.14) is a bit different from previous ones because the creation of a new node does not produce a changing into the state of the corresponding cell of the environment; it directly modifies the concentration of morphogens B . When a new vertex is created, we set the concentration $b = 1$ in the corresponding cell in L_m . The trigged event and the effect on the morphogen layer are synchronic and direct. The feedback and the autocatalysis of B go toward the same "direction", producing an explosive growth of the system.

Here we report three simulations (fig. 8.14). In *equilibrium* experiment (fig. 8.14a) the formation of the network produces the entering of an important quantity of B . The equilibrium experiment makes the graph with the largest vertex set of our simulations. In *mazes* experiment (fig. 8.14b), when a seed passes through stable concentrations, these last become unstable and morphogens re-organize themselves. In *soliton* experiment (fig. 8.14c) we observe that morphogens do not organize themselves in the classical way. No stable and regular spots form. No relevant differences between the simulations with the feedback to simulations without are observed in *moving spot*

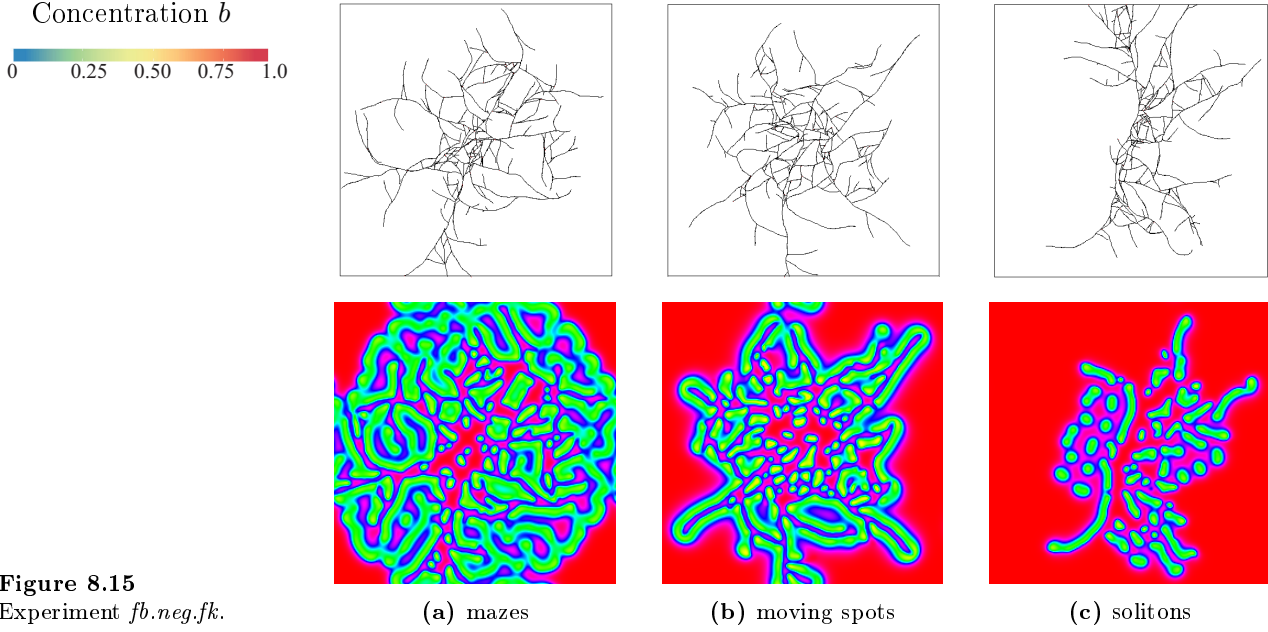


Figure 8.15
Experiment *fb.neg.fk*.

experiment. We do not observe substantial differences with experiments without when we add this kind of feedback in *moving spot* experiment.

8.2.2 Negative feedbacks

In many complex systems, negative feedbacks stabilize structures by counteracting the effects of positive feedback. We propose three experiments based on feedback mechanisms that reduce/inhibit the growth of the network. Compared to the positive feedback experiments, we inverse our initial configurations and our actions. More precisely, while in positive feedback experiments the initial configuration is in a sterile condition, in negative feedback experiment the initial configuration is a classical configuration. While for the first experiments the feedback stimulating the growth methods, in the second ones the feedbacks discourage the growth.

We propose three negative feedback mechanisms: the creation of a new node locally modifies parameters (f, k) , the diffusivity (D_a, D_b) or the concentration b . Simulations start with a star network of 5 nodes and a perturbation $b = 1.0$ in the middle of the space. The morphogen layer starts in a homogeneous state $(a, b) = (1.0, 0.0)$; the diffusion rates are $(D_a, D_b) = (1.0, 0.5)$. Parameters f and k allow the formation of emblematic patterns (*equilibrium*, *mazes*, *moving spots* or *solitons*, fig. 8.3) in a classical condition.

Experiment *fb.neg.fk*: reduce the growth changing f and k

When a new vertex is created, parameters f and k of the corresponding cells $c_e \in L_e$ are set in the red state, $(f, k) = (0.04, 0.08)$. The configuration is out of the Turing space. While the network grows, more and more cells become in this sterile condition; during the simulation, the networks become more and more constrained by sterile areas. Morphogens B seem to bypass the graph. Like a physical limit, a barrier defines a changing of behaviour

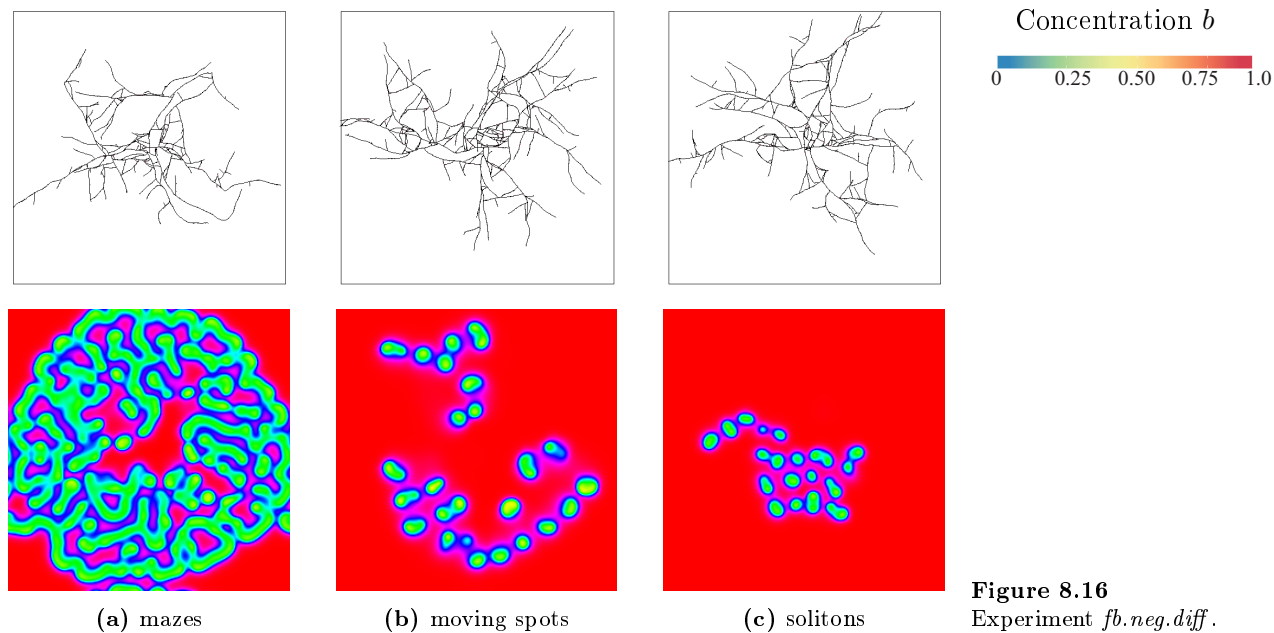


Figure 8.16
Experiment *fb.neg.diff*.

of morphogens: on one side no organizations can form, and on the other side, morphogens create patterns according to the initial f, k parameter set.

We focus on three simulation (fig. 8.15). The creation of barriers and enclaves of morphogens are more pronounced into the *moving spots* (fig. 8.15b) and *mazes* (fig. 8.15a) experiment, and less pronounced into *soliton* experiment (fig. 8.15c): this situation is caused by the difference in the speeding up of morphogens to fill the space⁴. In *solitons* experiment, we observe a continuous edge of morphogens between the network and the rest of space (where morphogens tend to organize in a classical manner). In all simulations, we observe the formation of homogeneous concentrations of B bordered by the network: these sub-areas are stable, the graph and morphogens seem to be in an equilibrium state. An equilibrium state is also observed into the *moving spots* experiment. This behaviour is unexpected and it diverges to classical simulation with this couple: moving spots patterns are far from equilibrium.

Experiment *fb.neg.diff*: reduce the growth changing the diffusivity

In this experiment (fig. 8.16), when a new node born, we set the diffusion of the corresponding cell as $D_a = D_b = 0.5$. Around these cells, the graph grows and no clusters form. Otherwise, in regions not reached by the graph, a pattern formation is observed in accordance with the correspond dynamic (*mazes*, *moving spots* and *solitons*). Comparing to the previous experiment, morphogens are not able to identify a defined contour around the graph.

⁴Remark: the speeding of the morphogens is not only due to the diffusion rate but also to the autocatalysis.

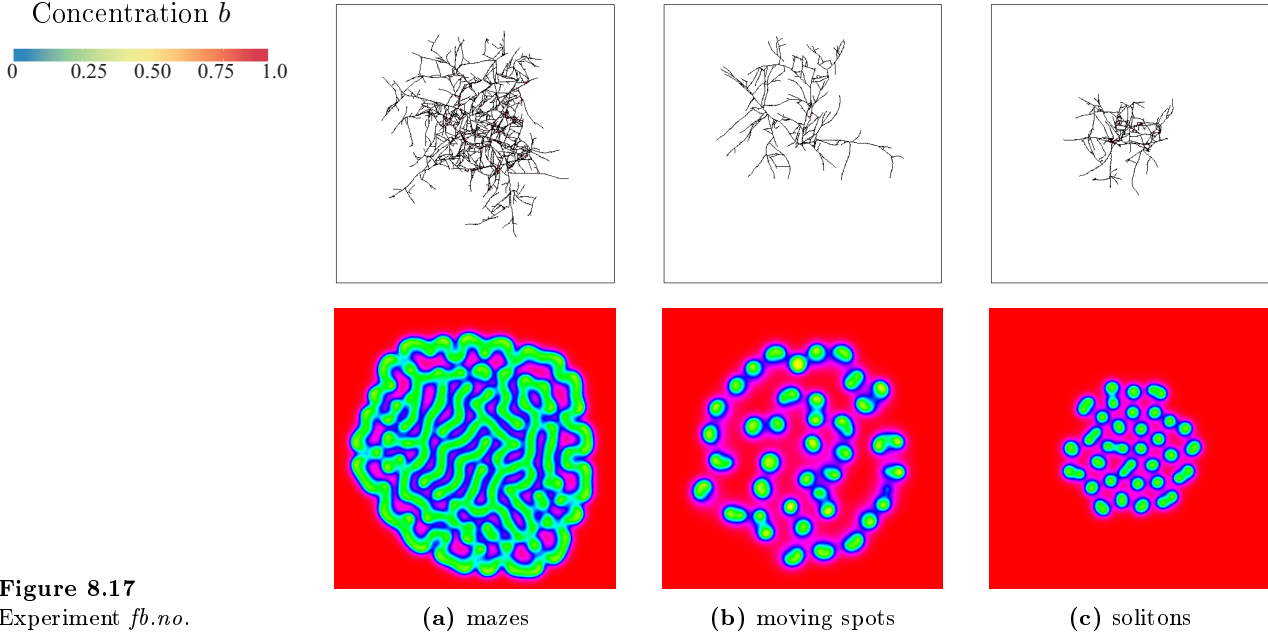


Figure 8.17
Experiment *fb.no.*

Experiment *fb.neg.morp*: reduce the growth injecting morphogens B

We test the case where the creation of a new nodes instantaneously modify the concentration b into the corresponding cell. Due to the rapidity of diffusion, when that happens, a consistent quantity of morphogens B moves from cells around the modified cell to it. The network is not able to perturb the evolution of morphogens. Resulting graphs are roughly the same that we can make without the feedback.

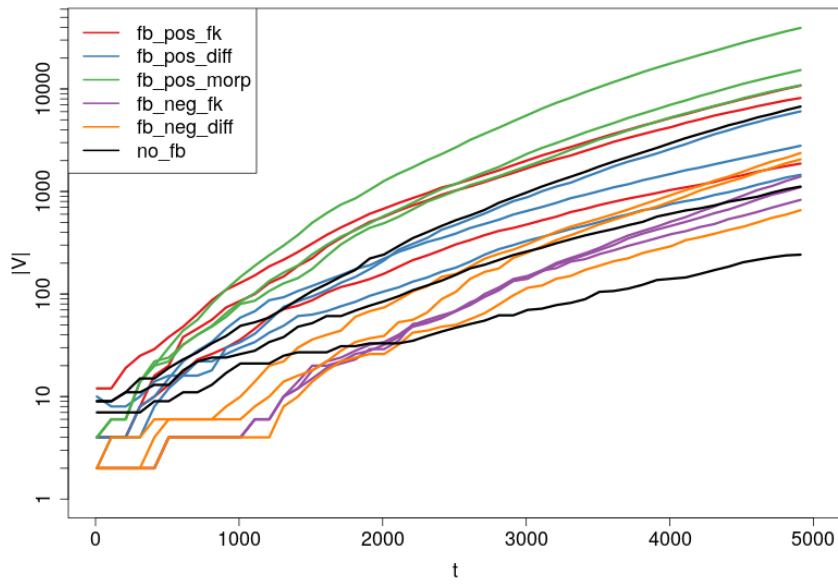
Experiment *fb.no*: a benchmark

The goal of previous experiments is to test the impact of different feedback mechanisms on the morphogenesis of spatial networks. So we judge crucial compare those networks with network obtained without feedbacks. To this end, we build three networks with the same initial configurations of previous experiments (fig. 8.17). Each of them are driven by *mazes* (fig. 8.17a), *moving spots* (fig. 8.17b) and *solitons* (fig. 8.17c) pattern formation. In order to break the central symmetry, we perturb the middle of space with values of concentrations b in a range $(0.95 - 1.0)$.

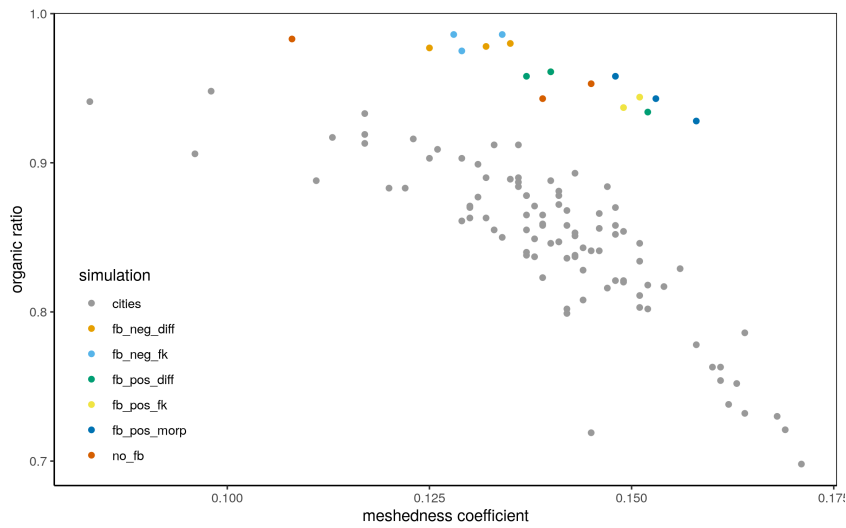
8.2.3 Experiments including feedbacks: results

Resulting networks are evaluated through indicators detailed in chapter 4. In the next, networks described in previous section will be compared with the French department streets networks.

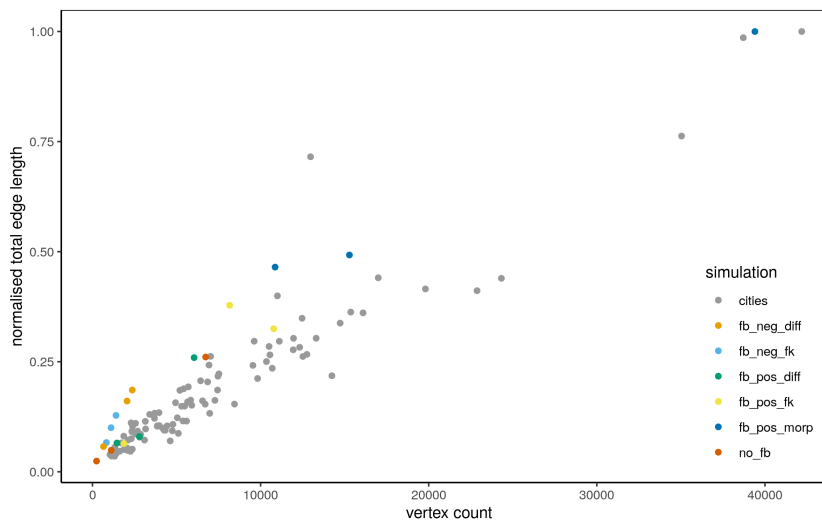
The feedback has an impact on the rate of growth of networks: the negative feedbacks stabilize the growth of the networks while the positive feedbacks incentivise the growth. Figure 8.18 depicts that first networks grow faster than second ones. Rates of growths are correlated with the kind of feedback.

**Figure 8.18**

The evolution of feedback simulations. Network obtained with a positive feedback grow faster than network with negative feedback.

**Figure 8.19**

Feedback simulations and French department cities in a *mes – org* scatter-plot.

**Figure 8.20**

Relation between the number of vertices $|V|$ and the normalized total length edge $l(G)$.

experiment	pattern	$ V $	$ E $	$\langle k \rangle$	γ	<i>org</i>	<i>mes</i>	<i>cost</i>
<i>fb.pos.fk</i>	movingSpots	8162	11332	2.777	0.463	0.961	0.14	0.768
<i>fb.pos.fk</i>	mazes	10778	15421	2.862	0.477	0.944	0.151	0.753
<i>fb.pos.fk</i>	solitons	1872	2662	2.844	0.475	0.937	0.149	0.789
<i>fb.pos.diff</i>	equilibrium	2792	4013	2.875	0.479	0.934	0.152	0.763
<i>fb.pos.diff</i>	mazes	6046	8399	2.778	0.463	0.961	0.14	0.783
<i>fb.pos.diff</i>	solitons	1453	1999	2.752	0.459	0.958	0.137	0.793
<i>fb.pos.morp</i>	equilibrium	39404	57558	2.921	0.487	0.928	0.158	0.736
<i>fb.pos.morp</i>	mazes	15284	22033	2.883	0.481	0.943	0.153	0.743
<i>fb.pos.morp</i>	solitons	10863	15435	2.842	0.474	0.958	0.148	0.757
<i>fb.neg.fk</i>	movingSpots	828	1128	2.725	0.455	0.986	0.134	0.803
<i>fb.neg.fk</i>	mazes	1103	1486	2.694	0.45	0.975	0.129	0.814
<i>fb.neg.fk</i>	solitons	1399	1877	2.683	0.448	0.986	0.128	0.807
<i>fb.neg.diff</i>	movingSpots	2057	2819	2.741	0.457	0.98	0.135	0.795
<i>fb.neg.diff</i>	mazes	657	873	2.658	0.444	0.977	0.125	0.811
<i>fb.neg.diff</i>	solitons	2369	3215	2.714	0.453	0.978	0.132	0.797
<i>fb.no</i>	movingSpots	242	307	2.537	0.426	0.983	0.108	0.877
<i>fb.no</i>	mazes	6731	9477	2.816	0.469	0.953	0.145	0.766
<i>fb.no</i>	solitons	1116	1543	2.765	0.462	0.943	0.139	0.804

Table 8.4

Characteristics of networks obtained with and without feedbacks. Number of vertices $|V|$, number of edges $|E|$, average degree $\langle k \rangle$, the gamma index γ , organic ratio *org*, meshedness coefficient *mes* and cost *cost*.

$SIN(G)$	$ V $	$\langle k \rangle$	$ E $	$l(G)(km)$	$\langle l(G) \rangle (m)$	γ	<i>org</i>	<i>mes</i>	<i>cost</i>
Ajaccio	2383	2.585	3080	36.367	14.472	0.431	0.917	0.113	0.814
Grenoble	14242	3.04	21648	76.556	3.713	0.507	0.698	0.171	0.675
Lyon	16094	2.948	23725	126.674	5.659	0.491	0.754	0.161	0.694
Paris	38711	3.022	58498	345.916	6.135	0.504	0.721	0.169	0.657
Privas	1320	2.397	1582	12.426	9.229	0.4	0.941	0.083	0.848
Rouen	5840	2.948	8609	57.091	7.024	0.492	0.763	0.161	0.703
Troyes	5384	2.958	7962	40.503	5.628	0.493	0.738	0.162	0.706
Versailles	5656	2.967	8390	55.728	7.047	0.495	0.752	0.163	0.699
CODAH	15120	2.706	20456	156.923	7.671	0.451	0.853	0.130	0.774

Table 8.5

Characteristics of street intersection networks of 8 emblematic French department cities and the CODAH. Number of vertices $|V|$, average degree $\langle k \rangle$, number of edges $|E|$, total length of edges $l(G)$, average edge length $\langle l(G) \rangle$, the gamma index γ , organic ratio *org*, meshedness coefficient *mes* and cost *cost*.

Table 8.4 resumes main characteristics of our networks; values are close to empirical values from French department street networks. Table 8.5 reports a few emblematic examples. Different initial configurations produce a number of networks with a different size. These preliminaries results show that our approach generates networks that have several global characteristics close to street networks.

We observe in a *org* – *mes* plot (fig. 8.19) that values from our simulation

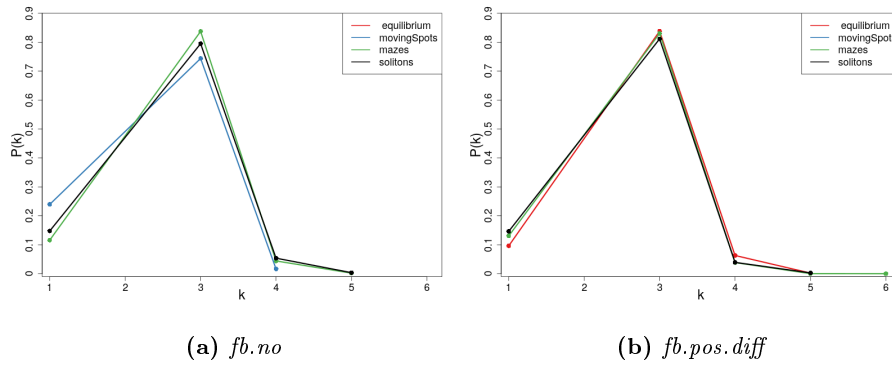


Figure 8.21
The degree distribution of feedback simulations.

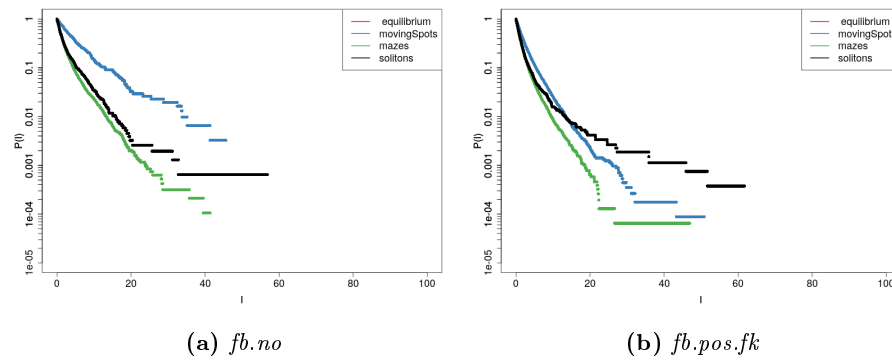


Figure 8.22
The cumulative length edge distribution of feedback simulations.

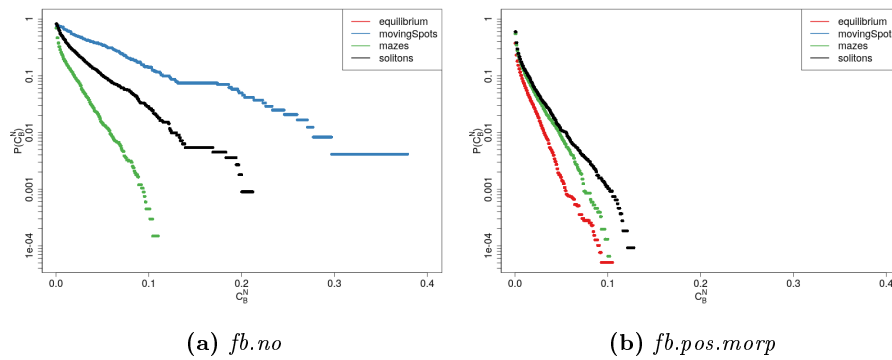
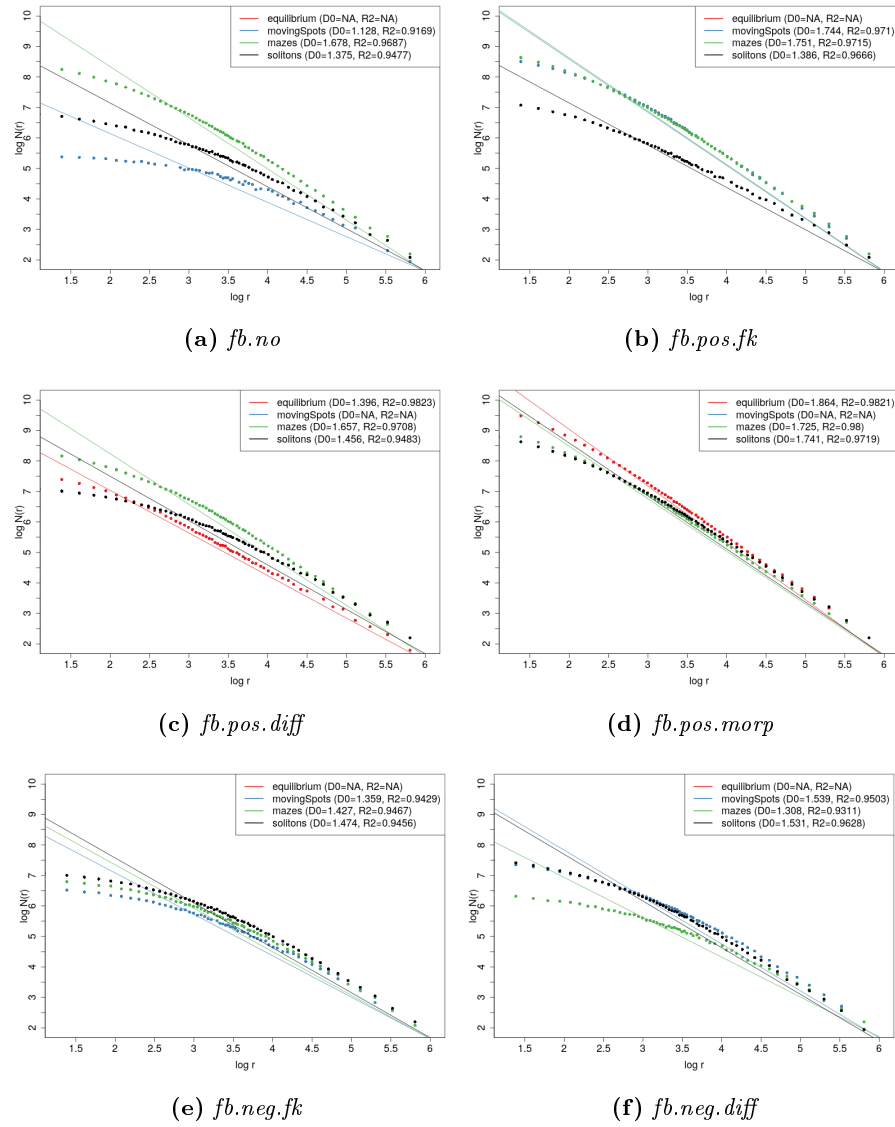


Figure 8.23
The betweenness distribution of feedback simulations.

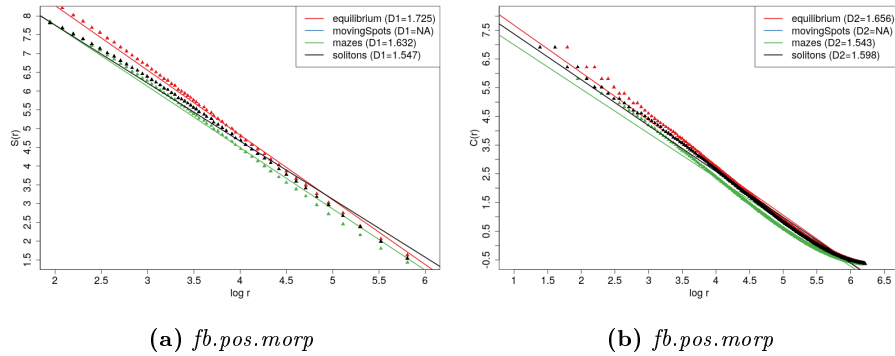
and values from French department streets are located in the same region. These elements confirm the capacity of our approach to generating graphs with global characteristics close to French cities. However, values of *mes* are higher in our networks than in cities, hence the framework produces networks with more tree structures than the real networks. That is due to our completely decentralized approach: graphs are not planned hence the number of vertices with degree 4 is less than real cities.

The normalized total length of edges and the number of vertices are plotted with data measured over different cities (fig. 8.20). We observe that the linear relation between these basic values corresponds to what we have previously observed (fig. 4.6).

We do not observe relevant differences between the vertex degree statistical distribution (fig. 8.21) of graphs obtained without feedbacks (fig. 8.21a) to

**Figure 8.24**

The capacity dimension D_0 estimation of 18 experiments.

**Figure 8.25**

The information (left) and the correlation (right) estimation of *fb.pos.morp*

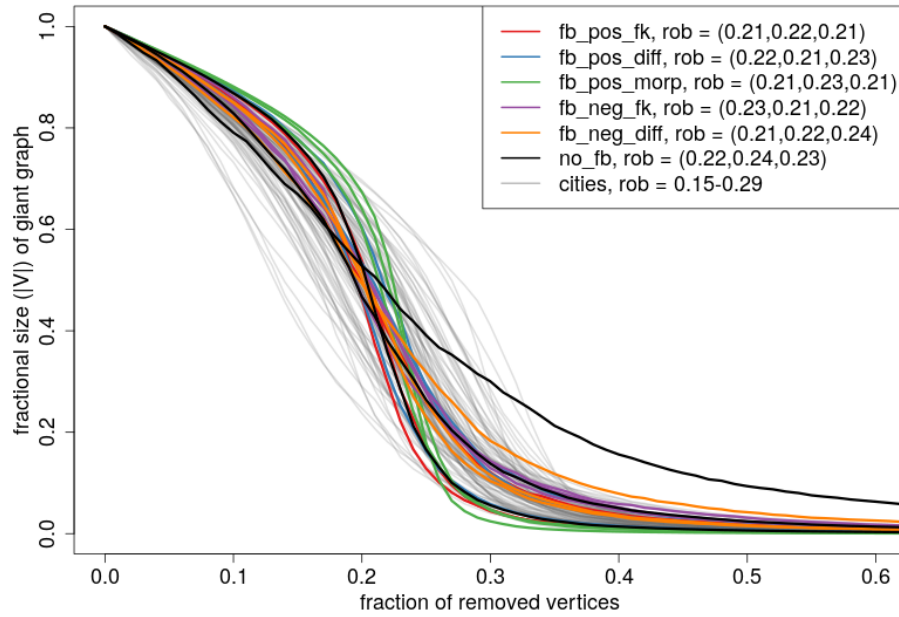


Figure 8.26
Robustness of feedback experiments.

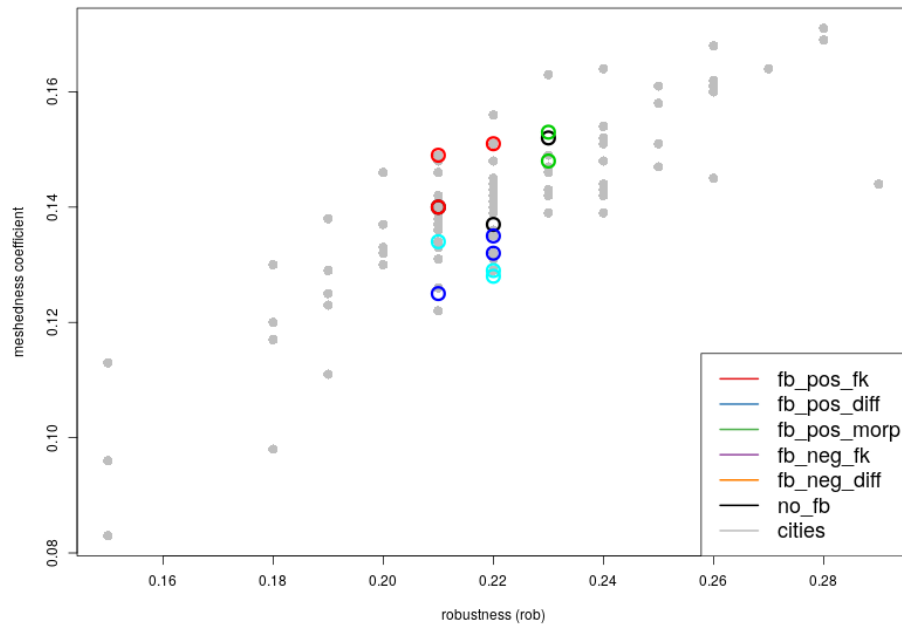


Figure 8.27
Robustness and meshedness coefficient of feedback experiments and French department cities

graphs obtained with feedback (fig. 8.21b). As the same, the edge length distribution is similar in all simulation (fig. 8.22). All these distribution seems similar to distributions of CODAH (section 4.2.7 and fig. 4.3b).

Different emerging properties characterize spatial systems. One of them is that a few elements are more important than the majority of others (section 1.2.3). Street networks are strongly hierarchized (section 4.3): there are a few nodes that capture the majority of shortest paths. This property is observed in our graphs (fig. 8.23) and can be computed through the study of

the betweenness centrality (section 4.3.1). In simulations without feedbacks (fig. 8.23a), the betweenness centrality statistical distribution is less hierachized than distributions of graphs obtained with feedbacks (fig. 8.23b). Distributions obtained with experiment *fb.pos.morp* (fig. 8.23b) are similar to those obtained with the analysis of CODAH (fig. 4.11b).

The capacity dimension D_0 indicates that we cannot observe scale-invariance organization (even locally) in networks without feedbacks (fig. 8.24, to study scale-invariance organization we follow the algorithm 2, page 92). Values hardly fit a linear regression (see low values of the coefficient of determination R^2) and values of D_0 are low. A scale-invariance organization is observed for graphs computed with feedbacks; this characteristic is more or less evident in all simulations and it can be observed on a large scale. Information and correlation dimensions confirm that networks are self-similar even when we consider the density in a region of vertices or their mutual distance (fig. 8.25).

Finally, we study the robustness of the networks (section 4.2.6). We observe that values of *rob* are in a compact range but the way that robustness decreases are different (fig. 8.26). Curves are similar for each simulation, except for networks obtained without feedback. Figure 8.27 compares our networks to French street networks: our networks show the same relation between of meshedness coefficient and robustness of French cities.

8.3 Discussion

We started our evaluation disregarding the feedback mechanism. In such simulations, the rate of growth of the network is related to emerging patterns into the morphogen layer. For patterns far from equilibrium (patterns which reach an equilibrium after a while, section 1.3.2), networks grow slower than networks drive by patterns out from equilibrium (patterns which never reach an equilibrium): the high instability of the first kind of patterns reduces the growth rate of the networks. The first pattern formation makes networks with a lower value of the average degree and the fractal dimension than networks obtained with the second dynamic. We also investigate the effect of parameters p_c and p_d which control the rate of growth of the network via the creation and the extinction of seeds. As we expected, the growth rate seems to be correlated to parameters p_c and p_d . These preliminary results suggest that our parameters affect the growth and each of them has a direct (and trivial) effect. The global behaviour of the framework is predictable⁵.

As regards the feedbacks, we note new interesting behaviours. Positive feedbacks incentivise the growth of the network while negative feedbacks reduce it. Combining these mechanisms to patterns formation, we observe that the growth does not correspond with what we had previously observed. Patterns out of equilibrium (e.g. the *moving spots* pattern) generated graphs denser and bigger than graphs obtained by patterns far from equilibrium. The equilibrium pattern (where no perturbations can vary the actual con-

⁵By predictable, we mean that we can determine how the system is going to behave qualitatively when we vary one of the parameters.

figurations) suddenly changes the configuration, producing the graphs with the bigger vertex set of our simulations. The feedback mechanism leads morphogens to a new way to organize themselves; combined to the fact that these changes are not homogeneous in the space, mixed patterns arise.

Comparing networks obtained by the same pattern formation, they have a rate of growth different: this confirms that the growth is not only governed by patterns but also by the feedback. The combination of those morphogenetic mechanisms produces a wealth of possibilities that must be deeply investigated. Our results suggest that with the feedback mechanisms, the framework simulates morphogenesis of systems with a complex behaviour.

In this work, we observed that spatial networks (and especially street networks) show some peculiarities: elements are arranged in a hierarchical manner, the absence of a characteristic scale of observation, and the tendency to stay connected in the case of a failure of a part of the network. Graph and fractal theories help us to quantitatively measure those properties. We compared our networks to real networks. Global characteristics do not vary too much: spatial aspects affect the structure, the topology, and the geometry of graphs. The graphs obtained without feedback mechanisms have characteristics close to those observed in the street networks. They show distributions of degree and length of edges close to those observed on cities, as well as the proportion of tree shapes and regular sub-parts. The feedback plays a role in other aspects of the network. First of all, we observe hierarchies more evident in these simulations: betweenness centrality indicates that there are a few vertices more important than others. Moreover, the formation of the network is more linked to the organization of morphogens: the symmetry is never respected and the graph fills the space in an inhomogeneous manner. This behaviour is captured by a fractal analysis; this analysis also brings out that the positive feedback produces networks more self-similar than networks where negative feedback dominates. Without feedback, our networks do not show consistent hierarchical organizations of elements of the network and a low degree of self-similarity.

Chapter 9

An urban application

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Keywords An urban application, Fécamp.

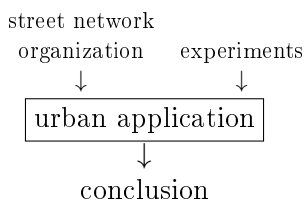
*"Potrei dirti di quanti gradini sono le vie fatte a scale, di che sesto gli archi dei porticati, di quali lamine di zinco sono ricoperti i tetti; ma so già che sarebbe come non dirti nulla. Non di questo é fatta la città, ma di relazioni tra le misure del suo spazio e gli avvenimenti del suo passato."*¹

Italo Calvino

In chapter 1 we exposed the complexity system theory, the reasons by which it is useful to study some real phenomena and basic notions. We observed that many systems are structured as networks and can be represented as geometric graphs (chapter 2). Cities can be studied as systems and streets as spatial complex networks (chapter 3). The main characteristics of those networks have been quantitatively measured (chapter 4). Inspired by chemistry, cellular automata theory, and reaction-diffusion theory, we have formalized in chapter 5 a way to simulate the evolution of concentration in a discrete space of moving elements (called morphogens). In chapter 6 we make two basic models that aim to simulate the network formation. Taking into account those experiences, we formalized a general framework (chapter 7) and we carried out experiments (chapter 8). We finally have all the ingredients to simulate the morphogenesis of street networks.

The goal of this chapter is to give an early and not exhaustive understanding of an urban application of the proposed framework. To this end, we make several oversimplifications that aim to capture some intuitive phenomena observed in urban systems. Several questions remain open and would need further investigation (chapter conclusion). In this chapter, we suggest some directions, we explore the potentiality of our framework, and we discuss preliminary results. In section 9.1 we describe elements that had motivated our approaches, in section 9.2.1 we describe the details of the application and we discuss our results. In section 9.3 we discuss future applications and we make some basic observations about morphogens in urban systems.

9.1 General model description



Street networks are a structural component of urban systems by allowing the movement of people and matter. They support human activities and form complex spatial systems as cities. The form of street networks reflects the organization of the urban system and its enlargement could approximate urban growth. The formation and the persistence of streets are therefore the results of continuous interactions and/or movement of parts of the city. Those dynamics might be constrained and influenced by exogenous factors. Through this work, we have observed that some structural components of an urban system govern the growth of the streets which in turn affect their functioning. An oversimplification is to consider the formation of the

¹ "I could tell you how many steps make up the streets rising like stairways, and the degree of the arcades' curves, and what kind of zinc scales cover the roofs; but I already know this would be the same as telling you nothing. The city does not consist of this, but of relationships between the measurements of its space and the events of its past".

street networks governed by a substrate of former-producer elements (morphogens). The street network influence in turn the substrate, conditioning the formation of complex structures.

It is difficult to consider an urban system isolated: it exchanges, it is constrained and it is influenced by exogenous aspects (the nature, other urban systems, socio-economical decisions). Those influences may append locally: some features of the environment of the urban system can be considered spatialized and they impacted the system in a region of space. If organizations emerge, the environment can locally influence them.

Different natural and artificial aspects may affect the street network formation. The orography of the land is one of them. For social, technological, and economical reasons, streets are build accommodating the climbs, minimizing the efforts and reducing the slope. Moreover, there exists some areas of the surface of earth which represent for urban system a constraint. Lakes, rivers, heavy climbs are three examples; it is highly unlikely that in these areas settlements form. The spatial configuration of the land become an organizational factor for the streets, influencing its, form and its functioning.

The environment also indicates areas that encourage, discourage, or even prevent the growth of the city. This can be increased by the policy of the administration to restrict the number of permission to build in a natural area, or to fix with a low value the expected density of build-up. These policies have an effect on the probability that an area becomes more or less used by humans and covered by artificial objects.

9.2 The urban application

The following urban application is based on the proposed framework exposed in chapter 7. Here we recall its main features. A reaction-diffusion layer simulates the evolution of the two kind of morphogens. Under some specific conditions, patterns form: the gradient of concentration of morphogens controls the creation, the dead and the movement of a set of moving elements. These elements (called seeds) are moved by a dynamic vector field (computed from the concentration of morphogens) and they build a spatial complex network. This last feeds back to the morphogen layer: when a new node born, some characteristics of the corresponding region of space change. An environment surround the system. Feedbacks may locally modify its characteristics and it may locally impact the growth of the network.

In an urban context, morphogens represent two kind of interacting actors (see section 9.2.1 for a few examples) which control the development of the street network. We complete those dynamics with the following hypothesis:

- the formation of new streets encourage morphogen to cluster around the street network,
- the administrative policy is to discourage the urban growth in natural areas,

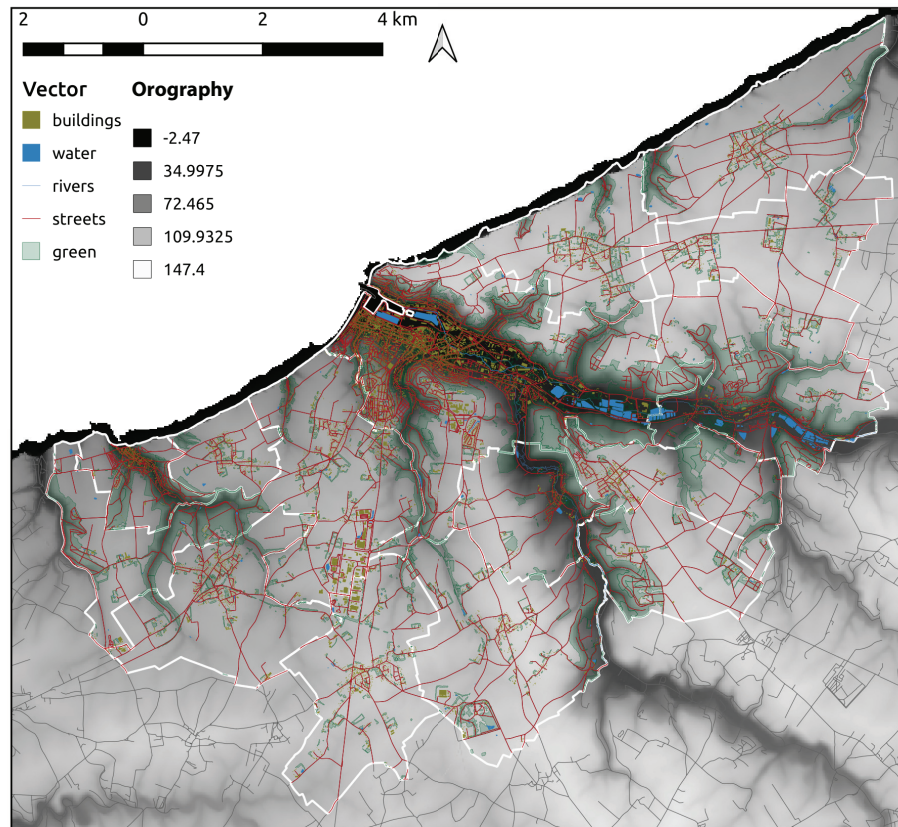


Figure 9.1
The Fécamp study area.

- streets develop by minimizing the climb,
- the growth rate of the network is mitigated by the building density.

These assumptions are an oversimplification of a possible real situation. We do not exclude in future works to make different assumptions in order to evaluate different scenarios.

9.2.1 The case study: the urban area of Fécamp

Fécamp is a town in the Seine-Maritime department (in the Normandy region, northern France) of around 20 000 inhabitants (in 2009). It is situated in the valley of the Valmont river². The study area is composed by the town of Fécamp and its neighbouring municipalities. This area is an emblematic example of different urban forms heavily impacted by natural constraints (fig. 9.1). Informations (spatially referenced) about the case study are obtained from the BD TOPO 2019 of IGN³. The dataset includes geometries of the urban fabric, the orography, and the land use.

9.2.2 Components

The framework is composed of:

²<https://fr.wikipedia.org/wiki/Fécamp>, visited on 20/08/2020.

³*Institut national de l'information géographique et forestière*, www.ign.fr

Space $\mathbb{R}^2$ and $\mathbb{N}^2$, and time T . The space sizes are $|\mathbb{N}| = 2^9 \times 2^9$ and $|\mathbb{R}| = 512 \times 512$ units; simulations are carried for 3000 steps.

The environment layer $L_e(t)$. Each cell $c_e \in L_e$ is associated to a state $S_e = (d, g, h, s, o, D_a, D_b, f, k)$ (definition 12, page 138), where:

- $d = [0, 1]$ is the density of buildings, computed as the percentage of coverage foot-print of buildings.
- $g = [0, 1]$ is the rate of coverage of vegetation.
- $h = \{0, 1\}$ represents hydrological limits. We assign to each cell that overcome a river or a lake the state $h = 1.0$, otherwise we assign $h = 0.0$.
- $s = \{0, 1\}$ indicate the administrative limits of the study area. The state of cell that covers the study area is $s = 0.0$, otherwise $s = 1.0$.
- $z = [0, 1]$ indicates the altitude of the region of the land. In order to have to each cell a state in the same interval of morphogens layer, z are normalized by the range of values. In this way, morphogens and the orography contributions are comparable.
- D_a, D_b, f, k are parameters of the morphogen layer.

The morphogen layer $L_m(t)$. Each cell $c_m \in L_m$ is assigned to the state $S_m = (a, b)$, $a, b \in [0, 1]$ which represents the concentration of morphogens A and B . The evolution of concentrations is defined by the Gray-Scott transition function (definition 10, page 109).

The vector field $\vec{L} = \vec{L}_e(o) + \vec{L}_m(b)$. The vector assigned to each cell is the sum of two distinct vectors: the vector from the concentration b and the z -coordinate. The first vector is obtained with the convolution procedure (section 7.2.4) and the second one follow the direction of minimization of climb (see the experiment *no.fb.4*, section 8.1.4).

The evolving seed set $S(t)$. We initialize all simulations with an empty set S of seeds (definition 11, page 119).

The street network $G(t, x)$. The initial geometric graph $G(x, t)$ is the primal representation (section 3.3.5) of the street network of Fécamp, where vertices represent intersections, end-nodes (cul-de-sacs) or approximate the geometry of the street. At the end of each simulation, we remove vertices with degree 2 (algorithm 1, section 3.3.7).

9.2.3 Simulations, mechanisms and parameters

A step of simulation of the framework is reported in algorithm 5 (section 7.3). Each simulation starts with the initialization of the layers and the

pattern	$ V $	$ E $	$\langle k \rangle$	γ	<i>org</i>	<i>mes</i>	<i>cost</i>	<i>rob</i>
Equilibrium	4888	6600	2.7	0.45	0.912	0.13	0.79	0.20
MovingSpots	11364	15195	2.674	0.446	0.94	0.126	0.803	0.16
Mazes	9688	13164	2.718	0.453	0.931	0.132	0.795	0.17
Solitons	5245	7092	2.704	0.451	0.914	0.13	0.792	0.19
Init	3243	4290	2.646	0.441	0.896	0.122	0.798	0.21

Table 9.1

Characteristics of the Fécamp street network (initial configuration and after four simulations). Number of vertices $|V|$, number of edges $|E|$, average degree $\langle k \rangle$, gamma index γ , organic ratio *org*, meshedness coefficient *mes*, cost *cost* and robustness *rob*.

encoding of geographical information of Fécamp into the framework. For each cell $c_e \in L_e$ over a node, we set parameters $(f, k) \in F_e$ in such way to allows the formation of a pattern (*equilibrium* $(f, k) = (0.0625, 0.0625)$, *mazes* $(f, k) = (0.029, 0.057)$, *moving spots* $(f, k) = (0.014, 0.054)$ and *solitons* $(f, k) = (0.030, 0.062)$, see fig. 8.3), otherwise we set cells in red state $(f, k) = (0.04, 0.08)$. We randomly perturb 10% of cells in L_m with a pulse $b = 1.0$ and we start the simulation when L_m has reached the equilibrium state.

At each time step, a new seed can be assigned to a vertex v if:

$$\text{random}(0, 1) < p_c \left(\frac{b - g - d - h - s}{k(v)} \right) \quad (9.1)$$

and each seed can be removed if:

$$\text{random}(0, 1) < p_d ((1 - b) + g + h + s) \quad (9.2)$$

where $p_c = 0.01, p_d = 0.02$ are two free parameters (section 7.3.4).

Finally, for each step of the simulation, we compute the cross-level feedback. We suppose that the formation of a new portion of streets produce a positive and diachronic feedback, favouring the morphogens to group over the corresponding cell. To do that, we use the methodology exposed in the experiment *fb.pos.fk* (section 8.2.1). When a new node born, the values of feed and kill rates of the cell below $c_e \in L_e$ update. The cell leaves the red state and parameters f and k are updated to allow the formation of one of aforementioned patterns.

9.2.4 Results

As discussed in section 8.3), the framework builds networks with characteristics similar to street networks. In this section, we devote our efforts towards studying whether our framework is also able to conserve the main global characteristics of a real street network after some simulations. Moreover, we would study if the framework is able to make different scenarios from different parameter settings. These last might indicate different socio-economical exogenous dynamics or policy decisions.

Table 9.1 reports global characteristics of four resulting networks. We observe that different growth rates and the size of the vertices set increases

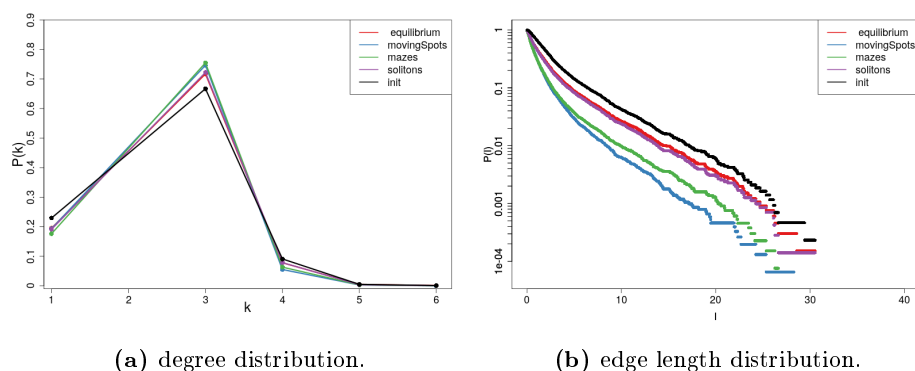


Figure 9.2
Distribution of vertex degree and length edge of Fécamp. Each panel compare the initial configuration and the configuration at the end of simulation.

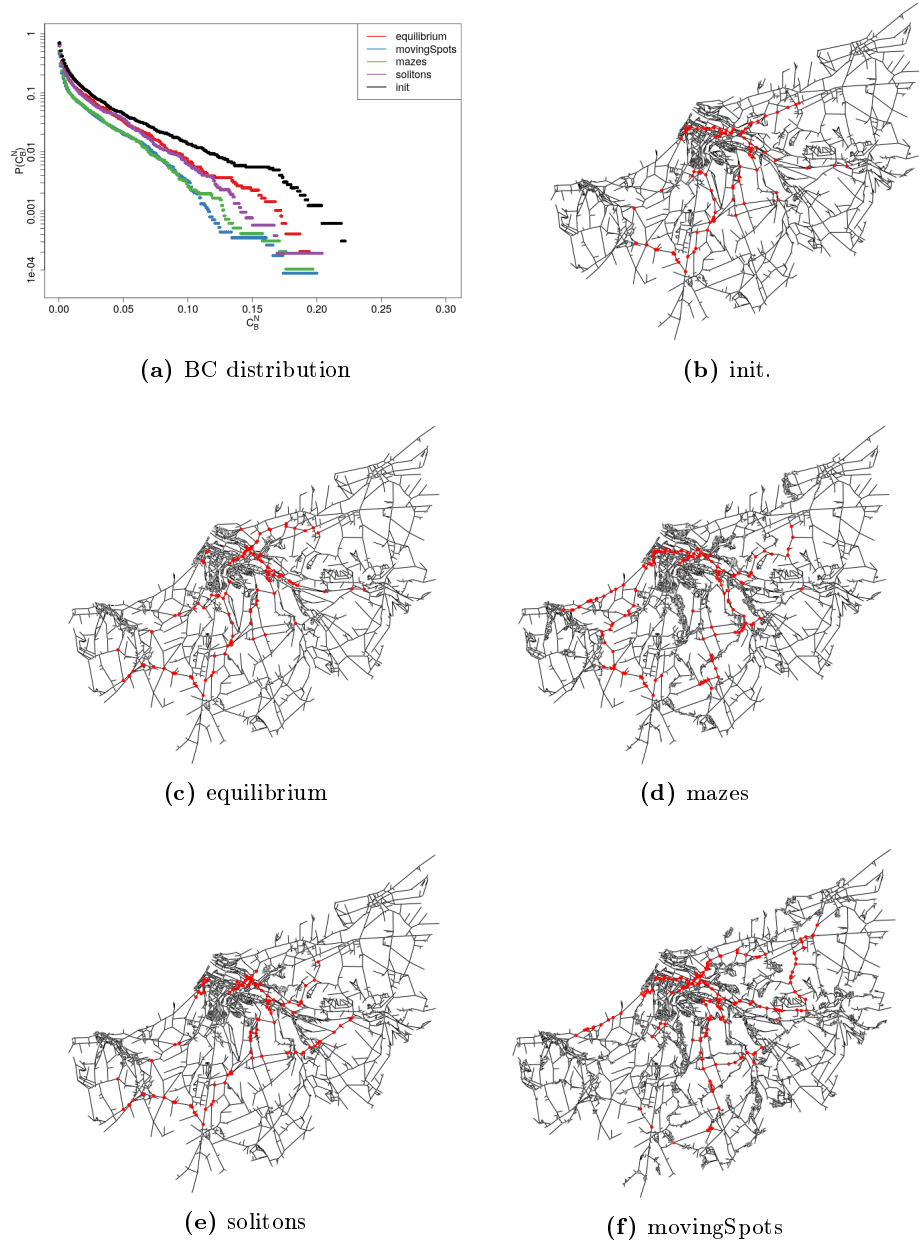
between 150% (*equilibrium*) and 350% (*moving spots*). Each new network conserves the main characteristics of the initial network with few variations. We observe the values of the average degree, gamma index, organic ratio, the meshedness coefficient increase, robustness decrease and cost does not substantially vary.

The increment of the organic ratio is related to the evolution of the degree distribution (fig. 9.2a). After simulations, the relative number of vertices with degree 3 increases, and the relative number of vertices with $k = 4$ decreases. Our approach does not consider a central control (often behind the formation of planning cities); global characteristics of resulting networks are similar to what we have observed in real self-organized street networks.

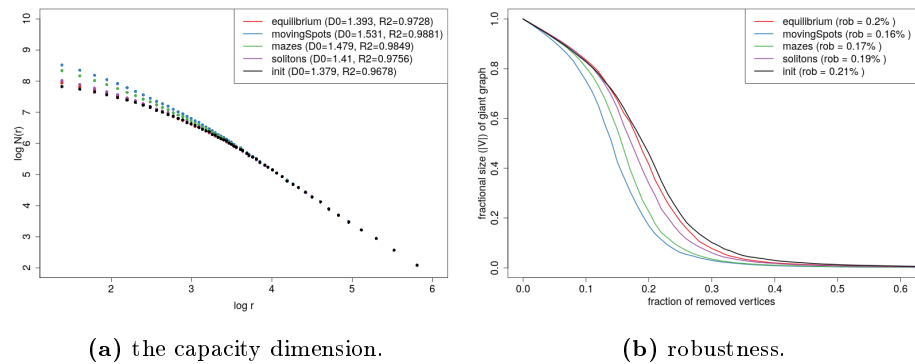
The geometry of the network was impacted by the simulation: we observe an increment of hierarchies on the edge length distribution (fig. 9.2b). The same effect is observed in the distribution of the normalized betweenness centrality (fig. 9.3a). We also note that the spatial distribution of more important vertices varies in each simulation, denoting that the basic morphogenetic process has an effect on shortest paths (fig. 9.3).

The study of the capacity dimension (fig. 9.4a) of the initial networks suggests that the street network of the study area is locally fractal (a portion of values fit the regression line). After each simulation, values that are not close to the regression line (points computed for the bigger boxes) became closer to the line. The graph obtained from the *moving spots* simulation becomes fractal in a large range of scales.

While the self-similarity increases the robustness decreases (fig. 9.4b). This effect is also correlated with properties measured by the other indicators: the increment of the meshedness coefficient suggests an increment of tree-like structures. These last are generally less robust than graphs with a homogeneous distribution of degree. Despite this variation, the robustness is still in the range of values observed in French street networks ($rob = (0.15 - 0.29)$ in real street networks, fig. 4.7, $rob = (0.16 - 0.20)$ in our networks). Moreover, the betweenness centrality distribution indicates that graphs after the simulation contain more important vertices than the initial graphs. These vertices capture an increasing number of paths, suggesting that the graph is more fragile in the case of dysfunctions.

**Figure 9.3**

The effect of the growth to the spatial distribution of betweenness centrality (red dots indicates values that exceeded $0.3 \max(C_b^N)$).

**Figure 9.4**

Capacity dimension and robustness of Fécamp.

9.3 Morphogens and city components

Through this work, we had been driven by principles of generality. We had preferred to identify basic dynamics behind different complex networks and try to simulate their morphogenesis. The objectives we had is to compare our model with field data and see if it is able to integrate this information. To go further, other parameters must be taken into account and the processes must be considered in a more heterogeneous way.

At a microscopic scale, people live nearby the street network and use it to navigate the city; at a macroscopic scale, the density population is related to streets. An high correlation was found between vertex centrality (section 4.3) and population density [250, 294]. The spatial relation between the density of population and the street network is not in all situations ensured; it is still unpredictable, rarely synchronic, and often affected by a large amount of socio-economical and natural factors. For instances, models proposed in [26, 251] simulate the simultaneous evolution of street networks and density population. These basic observations allow us to suppose that the morphogenesis of the street network and the evolution of population density are related.

A simplification is to consider, at a microscopic scale, humans as elements of the system: they interact with each other, attempt to accommodate their individual needs, and move. Moreover, we can suppose that the population density is related to the evolution of urban fabric: in other words, we can suppose that it exists a relation between the concentration of people in a region of space and the existence of buildings and streets. In this field, population could be considered for the street network as a morphogen because it affects the network growth.

In an urban area, people are not the only elements that contribute to the functioning of the system. Economical factors or political actors coexists with people, bias the behaviour of individuals, contributing to the functioning of the urban system. These aspects can be also related to space: e.g. the prices and the attractiveness of some neighbourhoods, the policy of the governments to incentivise the growth of a suburb. We can consider these elements as morphogens. They interact with people and they are spatially located. The reaction-diffusion theory might be a way to model the evolution of an urban (socio-economic) actor (morphogen A) and population (morphogen B).

On one hand, the population may generate positive feedback to its own density, and on the other hand, the street biases the spontaneous organization of the people, defining the accessibility of an area. A well-connected suburb is also accessible; this characteristic is also viewed as something that increases the interest of people in the suburb. People choose to move in these areas and the population density locally increases. Positive feedback of the street network to the population density could be observed under some conditions and in some circumstances. New streets are positive factors leading to subsequent increases in population, while the additional population density is a factor in subsequently deploying streets. In this chapter, we have discussed only the case of positive feedback. We are aware that there exist

several cases where the street could be considered as a negative factor for the growth of a suburbs (e.g., high-speed roads often reduces the attractiveness of a suburbs, influencing the rent value, the quality of air, etc..). Further investigations will focus on these mechanisms, exploring the morphogenesis as the complex co-existence of positive and negative feedbacks.

To explore possibilities that our framework has in an urban context, future studies shall focus on the understanding of which component of an urban system can be considered as a morphogen. Here we suggest that the interaction between the population and economical actors can have a crucial role in the formation of street networks. We need deeper studies to validate this hypothesis.

Summary 20:
An urban application

In order to make a practical example of the framework and validate it in a real situations, we simulated the street network growth and we compared our results to the initial configuration of the cities. The simulation of Fécamp street network growth suggests that the framework is able to produce different scenarios. The framework allows to consider several characteristics of real case studies, like the build-up, green areas and policies decisions. This application suggests that the framework may be helpful both to investigate urban growth and to support decisions of urban planners.

Conclusion

The problem addressed by this work concerns the morphogenesis of spatial complex networks. Our contribution is to model such phenomenon, defining a general framework that might find application in several real case studies. Our approach, dominated by a principle of parsimony, does not focus on specific details of the study case. It is interested on computational aspects of morphogenesis. According to complex system theory, we designed a framework based on three fundamental dynamics: the decentralized interactions of a heterogeneous set of morphogens, the emergence of unexpected properties that force the formation of a spatial network and the feedback of this latter to the way that morphogens organize themselves. The dynamics that led morphogens are inspired by reaction-diffusion theory; the evolution of concentration of morphogens have been simulated over a regular grid. Under certain conditions, concentrations patterns spontaneously form. We transform these patterns in two actions: (1) the concentration controls the growth rate of the network; (2) the gradient of concentration is transformed in a dynamic vector field which constrains the spatial arrangement of elements of the network. The evolving network feeds back to the morphogens, stimulating or reducing their capacity to organize themselves. We are interested in those open systems where the position and the distance influence the way that elements interact. The system is surrounded by an environment that locally sways it.

Lots of questions arise. Can we use the same explanatory approach to understand why and how objects (physical or not) develop, construct and transform? We do not try to answer the first question. We have not tackle the problem from the point of view of researchers in applicative fields (geographers). We approach the second question as computer scientists, with the desire to describe computable processes for which we can study trajectories. Our explanatory approach is based on causality, form and emergence. In our work we tackle the problem of the level of abstraction, of the observer and of emergence. Thus, when we only consider the morphogen layer, the emergence of patterns is unpredictable but the dynamics are predictable: the study of the pattern formation has exposed that a specific combination of initial parameters produces a specific dynamic in the morphogen layer. We started our evaluation disregarding the feedback of the network to the morphogens. In these simulations, the global behaviour of the system seems to be controlled by morphogens: the growth rate depends to the pattern formation (a far from equilibrium dynamic makes graphs with less vertices than an out of equilibrium dynamic) and it is biased by parameters p_c and

p_d . The morphogenesis of the network is driven by pattern formation, suggesting that there exists a causal relation between the dynamics inside the morphogen layer and the morphogenesis of the spatial network.

When we add cross-level feedbacks, they lead our simulations to unexpected and interesting behaviours. Combining the effect of the creation of the network to the patterns formation, we observe that the growth does not correspond with what we had previously observed: pattern formation in these simulations do not drive alone the growth of the network. The morphogenesis becomes an elaborate combination between feedbacks and morphogens dynamics. The causal relation between resulting forms (the network) and the substrate causes (pattern formation) cannot be observed. We observe unexpected, asymmetric and mixed patterns in the morphogens' layer and the networks show properties closer to street networks than networks obtained without feedbacks. Compared with experiments without feedbacks, where parameters f and k dominate the growth, in these simulations the morphogenesis is a complex overlapping of rate of growths, pattern formation and cross level feedbacks.

Real networks that we addressed in this work show some specific characteristics; we had measured different global properties: the elements are arranged in a hierarchical manner, the absence of a characteristic scale of observation, and the capacity of the system to adapt to failures of part of it. Graph and fractal theories help us to quantitatively measure these characteristics; we compared our results to six theoretical geometrical graphs and French department street networks. Measures proposed in this work are not able to completely characterize street networks. They are mainly focused on global characteristics, disregarding local variations. They identify dominant properties, allowing the definition of applicable measures, distinguishing our networks to real networks and theoretic graphs. These measures are also useful to characterize our simulations: we notice that indicators computed on graphs obtained with the framework are close to indicators computed on street networks of French department cities. Finally, the application of our framework in a real case study (Fécamp town) shows that the changing of a basic dynamics leads to different increments of self-similarity, hierarchies and different decrements of robustness. Thanks to the integration of some illustrative urban aspects (build-up, green areas, natural constraints and policy decisions) the framework produces different scenarios that may be useful for urban planners.

In this work we investigated the growth of a city with a systemic approach and we had made some assumptions which respect at same time real dynamics and complex system principles. The global and general correspondences between our networks and real street networks suggest that a city can be investigated under this point of view and essential mechanisms here modelled can be behind the morphogenesis of cities. However, as we had previously observed, these evidences cannot be completely ensured that our approach is exhaustive: we need more deeper investigations because we judge correlation between our results and our analysis a condition not sufficient to state that. Our results still are reassuring, they structure our knowledge and they set a basis for future studies.

Perspectives

The perspectives of this work are various and both in theoretical and applicative fields. They can be structured into three (connected) axes: (1) the implementation of the framework with more sophisticated dynamics, (2) the increase of the capacity to compare results with reality, (3) and the study of different real case study.

Many questions arise about our framework. This work suggests that the form of a system is at the same time the result and the cause of the functioning of the system and we would like to explore these processes under this point of view. Feedbacks are crucial to ensure the form in many real context and we suggest to deeper investigate this aspect through the proposal framework. The morphogenesis may be the result of opposing feedbacks, which stimulate and inhibit the evolution of forms at the same time: from antagonist mechanism arise unsuspected trajectories and morphogenesis might come out from this dichotomy. It seems an interesting scientific challenge to try to explore those dynamics.

The evolution of concentration of morphogens and the integration of the spatial environment was obtained with a regular square grid. This method has two important advantages: it is easy to implement and find an adequate correlation with many geographical data set. However, this approach, due to its intrinsic regularity, may not be an appropriate way to model elaborate dynamics of complex systems. Decomposing the space in an irregular way (e.g. with an Voronoi tessellation) or modelling morphogens with an agent-based system could be interesting fields to explore.

Other questions are about the network and what it would represent. Interesting perspectives open when we do not consider euclidean space as a dominant aspect in the functioning of the system. Different notions of "local" and "distance" can be defined for unspatialized networks: in this way the approach becomes more generalist and the framework might find application in more contexts. Moreover, spatial networks considered in this work, are also transportations systems, where links and nodes are traversed by flows of matters, energies, individuals, informations. It seems that the consideration of this aspect may drive to new understandings about morphogenesis. The geometric graph representation used in this work becomes incomplete to integrate these aspects and we could explore the possibility to integrate other notions from temporal networks to our formalism.

In this work we had briefly tackle two aspects of a scientific method: the observation (and the analysis) of reality and the critical evaluation of our results. On the one side, we aim to improve the analysis of real study cases, that will make a consistent background of informations about real phenomena. To this end, we judge important to increment our capacity to analyse street networks, with the investigation of improved approaches that are able not only to discriminate street networks to basic geometric graph, but also to deeper characterize each real situation. Our basic results suggest that it is interesting investigate the robustness and the scale invariance of networks. On the other side, we might investigate the behaviour space of the framework, in order to give a more exhaustive advice about it sensibility

to parameter set, a better exploration of trajectories, and eventually finding characteristic patterns. These explorations are necessary to be confident on the approach and they represent an important step forward an in-depth assessment. Genetic algorithms may be used to target one specific scenario or dynamic. The calibration of the framework may be useful in a decision making context, in order to help urban planners.

The study of urban growth requires to take into account a narrower coexistence of different human and economical phenomena, and the simulation of the co-evolution of different urban components. Clearly this demands a deep knowledge of urban dynamics. Our framework may be also applied to investigate morphogenesis of other study cases (e.g. vascular networks, ant nests and leaf venation networks). This will represent at once un interesting challenge (about the data set construction and the application in a three dimensional space) and an external validation of our approach.

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Abstract

The characteristics, functions and morphogenesis processes of a large number of complex spatial networks are influenced by the position and the geometry of their constituent elements. In this work, we address the computational aspects of the morphogenesis of complex networks by proposing a general model, simulating their formation. The networks are generated under the influence of constraints expressed through a vector field that is determined using a reaction-diffusion system. We use a Gray-Scott model to produce a wide variety of dynamic patterns. The resulting vector field controls the geometry and the growth rate of the constructed network that feeds back the reaction-diffusion process. A study was carried out on the influence of the patterns and feedback processes on the structure of the obtained networks using measures from graph theory and multi-fractality theory. A process of validation and evaluation of the model's behaviour was carried out and applied by comparing the networks obtained to largest French cities and the most relevant geometric planar graphs.

Keywords complex networks, complex systems, morphogenesis, spatial networks, reaction-diffusion systems, urban growth models, graph generator, fractal theory.

Résumé

Les caractéristiques, les fonctionnements et les processus de morphogénèse d'un grand nombre de réseaux spatio-complexes sont influencés par la position et la géométrie de leurs éléments constitutifs. Nous abordons, dans ce travail, les aspects computationnels de la morphogénèse de réseaux complexes, en proposant un modèle général, capable de simuler leur formation. Les réseaux sont générés sous l'influence de contraintes qui s'expriment par l'intermédiaire d'un champ vectoriel qui est déterminé à l'aide d'un système de réaction-diffusion. Nous utilisons un modèle de Gray-Scott produisant une grande variété de motifs dynamiques. Le champ vectoriel obtenu contrôle la géométrie et le taux de croissance du réseau construit qui rétroagit sur le processus de réaction-diffusion. Une étude a été réalisée sur l'influence des motifs et des processus de rétroaction sur la structure des réseaux obtenus en s'appuyant sur des mesures de réseaux complexes et de multi-fractalités. Une démarche de validation et d'évaluation du comportement du modèle a été effectuée et appliquée en comparant les réseaux obtenus à ceux structurant les villes françaises les plus importantes en taille et les plus connues graphes géométriques planaires.

Mots clés réseaux complexes, systèmes complexes, morphogénèse, réseaux spatiaux, systèmes de réaction-diffusion, modèles de croissance urbain, générateur de graphes, théorie des fractales.

Riassunto

Le caratteristiche, il funzionamento e i processi di morfogenesi delle reti spaziali complesse sono influenzate dalla posizione e dalla geometria dei suoi elementi costitutivi. Questo lavoro si concentra sugli aspetti computazionali della morfogenesi di queste reti, proponendo un modello generale e flessibile capace di simulare la loro formazione. La generazione delle reti è controllata attraverso un campo vettoriale dinamico intermediario, determinato da un sistema di reazione-diffusione. Il modello di Gray-Scott è stato utilizzato per produrre un'ampia varietà di motivi dinamici. Il campo vettoriale ottenuto controlla la geometria e il tasso di crescita della rete. Quest'ultima a sua volta retroagisce sul processo di reazione-diffusione. Sono stati studiati gli effetti dei motivi e dei processi di retroazione sulle strutture emergenti utilizzando misure derivate dalla teoria dei grafi e dalla teoria dei multi-frattali. Con l'obiettivo di validare e valutare il comportamento del modello, le reti ottenute sono state confrontate con le più grandi città Francesi e con una serie di grafi planari.

Parole chiave reti complesse, sistemi complessi, morfogenesi, reti spaziali, sistemi di reazione diffusione, modelli di crescita urbana, generatore di grafi, teoria dei frattali.