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Rostislav Haliplii

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PARIS 1 PANTHEON-SORBONNE
UNIVERSITY



Hedging in Alternative Markets

PhD Student:
Rostislav Haliplii

Under the guidance of:

Emeritus Professor Dominique GUEGAN
Dr. Marius-Cristian FRUNZA

October 18, 2020

I dedicate my PhD thesis to my mother

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Preface

Ben's Mr. Market allegory may seem out-of-date in today's investment world, in which most professionals and academicians talk of efficient markets, dynamic hedging and betas. Their interest in such matters is understandable, since techniques shrouded in mystery clearly have value to the purveyor of investment advice. After all, what witch doctor has ever achieved fame and fortune by simply advising 'Take two aspirins'?

Warren Buffett, *American Investors*

The COVID pandemic has changed the world. It also changed the financial markets. Traditional financial markets, including equities, forex or interest rates, show irrational and counterintuitive behaviours. The boundary between the traditional and alternative markets is murky. Therefore, investment professionals need to reassess the way risk is apprehended and managed.

Alternative markets are a permanent source of innovation for quantitative analysts and risk managers. Cryptocurrencies, commodities or exotic currencies display particular features, poorly covered by classic modelling techniques. Therefore, studying the dynamics of alternative assets is relevant in the current environment marked by market dislocation. The modelling approaches and the risk management strategies used in the sphere of alternative investments might play a crucial role in the traditional markets.

The primary motivation of this thesis was to study the hedging strategies in illiquid markets encompassing oil distillates and cryptocurrencies. Based on my previous professional experience in energy trading, I initiated my research journey by exploring the oil distillates markets, encompassing kerosene, gasoil or regrade. The oil market is extensively covered by academic research, but the literature concerning the oil distillates is relatively scarce. The critical problem is the way companies hedge their market risk related to price fluctuations of oil distillates. An international airliner, for instance, needs to manage the market risk of

the kerosene price carefully.

“ Whilst oil futures market is liquid, oil distillates exhibit switching in liquidity regimes, depending on the market condition and the horizon. Moreover, markets like kerosene or gasoil display a considerable level of geographical fragmentation, thereby leading to complex operational issue for risk managers. Therefore, companies may need to hedge their exposure with proxy-instruments (ie. Crude Brent futures), thereby generating basis risk. The initial goal was to study the structural risks related to proxy-hedging when the two markets (ie. kerosene and oil markets) exhibit different behaviours. The crucial problem is the difference in liquidity, maturity and efficiency between the instrument that a risk manager wants to hedge and the hedging instrument. What is happening when we hedge a financial instrument exhibiting low liquidity and efficiency with an instrument which is more liquid and more efficient?

To answer this question, one needs to analyze how two markets apparently related, like oil and oil distillates, can grow apart. The difference in behaviour goes back to their cradle. Assessing such difference would require an anthropological research rather than an econometric analysis.

“ Over the timespan of my thesis, we witnessed the rise of a new asset class that has many things in common with commodities: crypto-currencies. Interestingly, I remarked that despite the existence of a few dozen significant crypto-currencies, they do not exhibit similar market features. Bitcoin is from far the most popular and liquid crypto-currency, while the others (Altcoin) are less appealing to investors. The difference between Bitcoin and Altcoins in terms of efficiency, liquidity and structure became more relevant over time. Thus, I found an overarching perspective between the oil-oil distillates relationships and Bitcoin-Altcoin relationship. Consequently, I pursued studying these markets and analyzing the way proxy-hedging function in the world of cryptocurrencies. The study of crypto-currencies in this context can bring valuable learnings for the oil distillates markets and not only.

A common topic in modelling both crypto-currencies returns and oil distillates is the usage of non-Gaussian frameworks. For modelling assets' returns, I used distributions from the generalized hyperbolic family and for the volatility I employed the GARCK or GARCH-like formalism.

Risk managers from various walks of the investment profession assess the efficiency of a hedging strategy in terms of expected profit and loss over a period of time. The closer the expectation is to zero, the better the hedge. This judgement might work well in traditional markets. In markets that change structure and liquidity, looking only at the expectation

profit/loss can lead to biased choices. I show that is crucial to assess not only the expectation but the full distribution of the profit and loss, focusing on the nature of the tails. Moreover, in a good strategy, the behaviour of the hedging instrument should be similar to that of the hedged instrument. Thus, using empirical distribution benchmark techniques provides with a fully-fledged picture upon the effectiveness of a hedging strategy. Distribution forecasting gives a useful insight, whether a hedging approach is a passive or a proactive risk management tool.

Rostislav Halpili

Summary and Synthesis

*Remember, that time is money. He that can earn ten shillings a day by his labor, and goes abroad, or sits idle, one half of that day, though he spends but sixpence during his diversion or idleness, ought not to reckon that the only expense; he has really spent, or rather thrown away, five shillings besides. **Benjamin Franklin** ([Franklin (1750)])*

An alternative investment is a financial instrument that does not fall into one of the traditional investment categories. Alternative markets are a fairly recent asset class, and they present an out of the box perspective upon the financial markets exploring the financial universe beyond traditional investments like equity, bonds, currency. Private equity or venture capital, hedge funds, real property, commodities, and tangible assets are all examples of alternative investments. Over the recent years, alternative investment encompasses hedge funds, cryptocurrencies, art, wine, precious stones, collection cars sports athletes, sports bets, weather, biodiversity, archaeology, vintage: (ie. Stamps, old coins, old letters, books), game trophies. Most alternative markets display scarce liquidity and are not regulated or have a weak regulatory framework. Understanding and analyzing alternative markets requires techniques used at the fringe of finance.

In the eyes of a layperson, financial markets are a very recent invention and began to play an essential role in the real economy only over the last few decades. In reality, financial markets go back to the times of Ancient Rome or on the trails of the Silk Road in the ancient Sumerian civilization.

The first form of currency was the grains lent by the monarch to peasants to grow crops. At harvest time grains were returned to the lender with an additional amount, accounting for taxes. In this way, the interest rate appeared concomitantly to monies. Commodities and currencies are intrinsically linked, and their historical evolution is conflated. The trading of physical commodities constituted the starting point of finance. When money was created as an abstract concept, exchanging commodities for funds triggered the development of financial markets.

Since the 1970s, we observed two phenomena in the financial world. On the one hand, currencies lost their intrinsic values once the gold standard was abandoned. The value of a currency reflects markets' confidence in the emitting central bank.

On the other hand, we witness the so-called financialization of commodities markets. Sophisticated derivatives were underwritten on the commodities markets and constituted a significant part of investors portfolios held for diversification purposes. Moreover, the end-users of a commodity are not anymore the primary purchasers of that commodity, but the speculators, who are looking to generate profits from the market fluctuations

Indeed, central banks and big financial institutions took control over and have a monopoly over transactions related to fiat currencies and also have a significant stake in the commodities markets. The inflow of liquidity in the commodities markets and in particular in the oil markets changed their structure profoundly. The arrival of investors' monies in commodities was not heterogeneous. Some markets like oil or precious metals, attracted more liquidity, while others, including oil distillates or agricultural commodities, attracted less. Needless to say, that this dissymmetry generates many opportunities for speculators but also many issues for the end-users, who face critical challenges in managing the price risk. It is the case of industrial companies exposed to oil distillates markets. This thesis explores, amongst other things, this problem and underlines the issues generated by asymmetries across the various markets.

The previous economic crisis, marked by Lehman's bankruptcy, showed that under certain market circumstances, money could lose their value. The public discovered negative interest rates and learnt that lenders might need to pay debtors when you lend them money. With negative interest rates, central banks put in place a sort of tax on liquidity, a fee that cash owners need to pay to hold their funds in a bank.

The current COVID crisis brought in March 2020 a new paradigm shift: physical assets with negative prices. For the first time in history, an asset traded at a negative price. The primary U.S. oil contract closed at a negative price, plummeting to -37.63 USD for a barrel. Interestingly, oil is one of those assets having a well-defined fundamental value, related to the marginal cost of extraction and the marginal consumption value.

When its trading range tested the upper limit in May 2008, the barrel reaching 150 USD, it was hardly conceivable to see the price below zero. Analysts pointed out that it is just

a technicality, but this event is redefining what the term asset means. The sale of an asset that requires the owner to pay the acquirer, and not the opposite, reshapes the concept of ownership. The only item that requires its possessor to pay upon disposal is a penalty or a tax note.

The standard features for all assets that give them the status of financial instruments are value, transferability and time-dynamic. A negative price means that the asset carries no value and that the transferability becomes irrelevant. In the context of fiat currencies, the definition of an asset requires to be reconsidered.

The crucial role of fiat currencies and the extensive stake central banks have in the real economy started to raise a lot of questions to politicians and policymakers. At the beginning of the pandemic outbreak, Chinese financial entities owned as of 2019 over 1.10 trillion USD of U.S. debt, accounting for 26 % of the U.S. debt held by foreign countries and representing more than 5% of the total U.S. outstanding debt. Moreover, the trade deficit with China is 419 billion USD, accounting for 47% of the overall U.S. deficit in goods.

Thus, China was not only America's most prominent banker but also the leading supplier of goods. Therefore, the severe COVID crisis that hit the Chinese economy hurts ineluctably U.S. growth, but also the position of the U.S. dollar. The Coronavirus outbreak represents an event comparable with the 1918 Spanish influenza pandemic, and the consequences upon the U.S. economy and U.S. currency are disruptive.

The current pandemic and the money printing policy of central banks weakened the trust of the public in fiat currencies. We may be witnessing the beginning of the end of the current financial system, and people are looking to an alternative. The increasing lack of confidence in the banking system culminating with Lehman defaults, the perspective of deposit holder to lose their economies and the destruction of many silos of the economy as a result of COVID undermines the credibility of fiat currencies.

Bitcoin and cryptocurrencies appeared as a solution to these problems. They bypass not only the financial system but also the governmental power backing the financial system. For these reasons, Bitcoin does represent more than a simple currency. For the Bitcoin believers, which look with trustless eyes to the leading global fiat currency, the current events are a sign that the financial system is changing. The nature of cryptocurrencies and in particular of Bitcoin is analyzed in this thesis.

Negative asset prices mean that possessing an asset is automatically implying a tax or penalty for its owner. If the negative prices expand, in the same way, the negative interest rates did, the world may witness an economic overhaul, implying a possible growing role for crypto-currencies.

Investing in alternative assets generates irrational investors behaviour which can build up over time and create bubbles. Bitcoin had few bubble events, and at one time it was expected that the leading cryptocurrency to reach six digits value.

Bitcoin did not break the dream level of 100k USD and will most likely not reach it in the foreseeable future. Nevertheless, even the most prominent critics of cryptos did not expect Bitcoin to survive and to trade at a price seven times higher than in it did in 2014. Since the beginning, critics have preached that regulation, taxation, and many other menaces will hammer once and for all the last nail in the coffin of cryptos. But, recent events show that those views are far from becoming a reality, and investors would need another decade to understand the nature of Bitcoin and crypto-assets in general. The new decade starts in an environment of global political torment, which will push investors to look for safe harbour investments. In this thesis, I investigated how crypto-currencies investors do behave and how their market expectation impacts their investment decisions.

Cryptocurrencies and commodities have a lot of similar issues, and both hedgers and investors in these markets face common challenges. I analyzed these topics in my thesis research, and I used advanced econometric and modelling tools to describe them appropriately.

Alternative financial markets are encompassing the spectrum of instruments that go beyond the boundaries of traditional markets (ie, stocks, bonds, currencies). Traditional financial markets are assumed to be efficient, a term introduced by [Fama (1970)]. In an efficient market, the price is an unbiased estimate of the fair value of the traded instrument, and the price evolution oscillates around the fair value, the swings following a random variable.

The research making the object of this thesis focuses on two alternative markets: crypto-currencies and oil-distillates. Most alternative markets are far from being efficient, and this generates a lot of challenges in terms of modelling. Models based on Gaussian distributions are still the most popular choice for quantitative financial analysts and are implemented even in markets which are far from being efficient. A sound modelling framework for alternative assets should start from non-Gaussian distribution. Therefore, throughout this thesis, the

overarching theme for all simulations and estimations is the use of generalized hyperbolic distributions. This approach has a two-edged justification. On the one hand, it is critical to developing a fully-fledged quantitative framework beyond the Gaussian universe, thereby testing the performance of the new model in real-life situations. On the other hand, the markets making the object of this research (oil distillates and crypto-currencies) have neither the fundamentals nor the empirical behaviour that could justify traditional modelling.

In the world of traditional asset classes, contingencies' valuations and hedging strategies are governed by the iconic recipe of Black and Scholes ([Black and Scholes (1973)]). The biggest flaw of this formalism, highlighted since Mandelbrot's seminal publication ([Mandelbrot (1966)]) is its reliance on the Gaussian framework. The feasibility of this formalism, when applied to alternative markets, is hindered by the incompatibility with the empirical features of such assets. Hedging strategies in alternative markets, encompassing oil distillates and crypto-currencies requires a more complex approach and should incorporate not only the price dynamic but also the liquidity structure and the market dislocation. Needless to say, that hedging Bitcoin exposure cannot be based on a straightforward Black and Scholes formalism and should focus on covering tail events. Classic hedging assessment studies the profit and loss time series and whether this distribution has a zero average. In our proposed formalism, we do not focus on the expected outcome of the hedging strategy, but on the distribution and its tails. In alternative markets, it is not enough to show that hedging a position generates a zero-sum outcome but to ensure that the hedging instrument can forecast in terms of density the target instrument.

This study is particularly relevant in the context of the current market environment marked by the pandemic outbreak, social unrest, economic depression and political turmoil. In these conditions, even traditional market exhibit features of alternative markets. This change in behaviour is due to a decoupling between assets valuation and their fundamentals. For example, the stock market had a sharp recovery after the pandemic outbreak, despite the fact the real economy is experiencing a severe contraction. Therefore, the stock markets evolve in a territory with no connections to any fundamentals, thereby making it similar from this point of view to crypto-currencies.

A common feature of oil products and crypto-currencies (ie. Bitcoin) is that both are created through a process called mining. The process consists of extracting them from a given environment. In the case of oil, the process involves drilling the physical reserves. The profitability of this process is related to the difference between the oil market price and the cost of drilling operations. Mining Bitcoin does not imply a physical extraction, but solving a complex encryption puzzle that requires significant computational resources.

Mining Bitcoin is profitable if the market price of the leading cryptocurrency is higher than the cost of computing resources needed for solving the cryptographic puzzle.

For both oil products and crypto-currencies, the mining/drilling process induces a lot of unique features that impact their market efficiency. Can a tradable mined resource constitute an efficient market?

In the case of an asset traded in a classic, efficient market, all players have homogeneous access to all available information and the ability to buy and sell a fraction of the available asset. For instance, if a traditional currency faces a massive and sudden depreciation, the central bank will try to mitigate the issues by buying back currencies or altering the interest rates. If a share observes significant market fluctuations, its issuer or market-maker can implement a buyback strategy, thereby stabilizing the price. Obviously, in the case of oil or crypto-currency these actions are not applicable. Drilling oil and mining Bitcoin creates an asymmetry amongst “investors” due to the fact that not all drillers/miners have access to the same tools. Thus, some players have more advantages than others given the feature of their equipment. Those having more powerful drilling/mining tools have a comparative advantage in the price discovery. A second asymmetry stems from the difference in information between the market player involved in drilling/mining and those who are pure speculators ([Frunza (2015)]). For all these reasons, before going further in advance modelling, it is important to study the mining process, especially for crypto-currencies.

Chapter 1 corresponds to the working paper: “To Mine or Not to Mine? The Bitcoin Mining Paradox” [Halipili *et al.* (2020b)]. It explores the profitability of Bitcoin mining using the real options theory. The research addresses the problem of a Bitcoin miner and proposes a model that simulates the fundamental mining reward in order to predict the mining difficulty, evaluate the hardware efficiency and measure the likelihood of breakeven on the initial investment.

Bitcoin miners who set up mining operations face many economic uncertainties, such as high price volatility or increasing mining difficulty, both impacting the profitability and the payback of the initial investment. The most common valuation tool is Net Present Value (NPV) and the valuation of mining or drilling operations makes no exception. Investors make projections of the Bitcoin price and assess the value of the farm based on these projections. However, such valuations are deterministic and maybe not adapted to a situation where the Bitcoin price is very volatile. The fundamental inadequacy of the NPV approach and other discounted cash flow approaches to capital budgeting are that they ignore, or cannot correctly capture, management’s flexibility to adapt and revise later decisions (i.e., review its implicit operating strategy). The traditional NPV approach, in particular, makes implicit

assumptions concerning an “expected scenario” of cash flows and presumes management’s commitment to a certain “operating strategy”. The real options theory introduced by [Trigeorgis (1996)] and [Myers (1977)] incorporates in the valuation of business the value of the various options that the management has. I implement a real options theory-based model to assess the profitability of a mining operation. The model provides trigger prices determining the right actions for managing the operation.

The results of the research show that Bitcoin mining activity has transformed from fast-payback investment scheme nourished by the hype and social euphoria, to more of a utility business. Second, the econometric results based on Granger’s test show that variations of Bitcoin price have a delayed or no impact on the mining difficulty. This proves that miners exhibit irrational behaviour when it comes to adjusting their business strategy in different economic cycles of Bitcoin.

Moreover, the results show that investing in the latest mining technology is not as profitable and sustainable as before the market crash in December 2017. Miners’ irrational behaviour fueled the continuous exponential increase in mining difficulty, albeit low prices of Bitcoin.

When market behave irrationally, all conditions are met for bubbles to appear. One of the first bubbles documented in history was the *Tulipmania* that took place in 1636-1637. Joseph de la Vega, a philosopher that lived throughout that period, wrote in his book *Confusion of Confusion* ([de la Vega (1668)]): *What really matters is an awareness of how greed and fear can drive rational people to behave in strange ways when they gather in the marketplace..* Bubbles are a trademark of alternative markets. When the information is heterogeneously spread amongst investors, and the price does not work well enough as an information aggregation, the transfer of data from informed to the less informed is taking place at a cost. If the information inefficiency persists, the cost is paid continuously by investors, thereby inflicting a sharp rise in price. Bitcoin experienced such an episode in 2017 when its price raise from a few thousands to 20,000 dollars in only a few months. 2020 brought an explosive start for Bitcoin, and for most of Altcoin¹. Many were those believing that we could witness another 2017 pattern with prices reaching the maximum historical level. 2020 is not 2017 for many reasons. First, 2017 was a bubble created by a mass-hysteria around crypto-currencies and the belief that Bitcoin may be the new Holy Grail that could free the slaves of the post-modern capitalism. Second, Bitcoin became more transparent in terms of available information, and it is not anymore an obscure instrument as it was back in 2017. Last, but not least the current Bitcoin market is more mature and better crystallized. The different segments of liquidity are stable, and there is a solid base of crypto-investors participating in the price discovery process. To repeat the 2017 exponential raise, a fresh

¹Altcoins are the other cryptocurrencies launched after the success of Bitcoin

inflow of liquidity and a new generation of long-term investors would be needed. But, in reality, those investors would be more than simple Bitcoin holders, they should be believers capable of spreading the word and changing the traditional views about fiat money.

Bubbles are also related to fraud in financial markets. Bubbles occur in many cases when security is the object of manipulation or a hoax. Penny stock scams, microcap fraud and pump and dump schemes have things in common with a 'bubble' phenomenon, as in all these cases the price of a security gets inflated far beyond its fundamental 'fair value', and this inflation is accelerated by other investors who will buy the security thereby boosting the exponential rise of the price. The common points of manipulation and bubbles are also mentioned in the literature. [Zhao (2014)] studied the unusual and puzzling stock price performance of USEC Inc., a company specialized in producing enriched uranium for nuclear plants. In July 2013 the stock price surged as much as ten times during merely sixteen trading days without apparent value-changing information being released. The bubbles that occurred for several minor crypto-currencies had alleged ties to market manipulation attempts.

The research presented in chapter 2 was submitted to the 14th International Conference on Computational and Financial Econometrics (CFE 2020) and is published as a working paper [Halipii *et al.* (2020a)]. An initial study ran in parallel was published as in peer-reviewed conference proceedings: "Bubbles on Bitcoin Price: The Bitcoin Rush" [Guegan *et al.* (2020c)]. The chapter explores the occurrence and the timing of bubbles in the top 50 crypto-currencies.

This study assesses the presence of bubble effects in this market with customized tests able to detect the timing of various bubbles. The Sup-Augmented Dickey-Fuller and the Generalized Sup-Augmented Dickey-Fuller tests were applied for each to the full history of exchanges rates relative to the US dollars. I analyze the evolution of a representative sample of crypto-currencies over time, encompassing both high and low liquidity coins. The obtained results support our initial intuition underlining two main reasons for bubbles: the investor rush in the initial day of the coin culminating with the 2017 Bitcoin bubble and the various momentum linked to idiosyncratic factors for multiple coins. Several crypto-currencies prices had episodes of rapid inflation in 2017 related to the Bitcoin bubble, and a few emerging coins saw their prices pumped by speculative actions.

After analyzing the source of inefficiencies on crypto-currencies and oil distillates market and the irrational behaviour of the investors on these markets, enough intelligence is gathered to allow the study of appropriate hedging strategies.

The research presented in chapter 3 is published as a working paper: "Proxy-Hedging of Bitcoin Exposures With Altcoins" [Guegan *et al.* (2020a)]. It explores the topic of proxy

hedging in the crypto-currencies market with a focus on Bitcoin. The research addresses the problem of a Bitcoin investor or a Bitcoin miner that hedges its price risk with proxy coins. As such, if a Bitcoin miners want to cover their Bitcoin price risk since the volumes exchanged on this market may be thin, they might use one of the 'proxy-hedge' options described earlier. However, choosing the right one means making a trade-off between market liquidity and basis risk. As candidates for proxy-hedging Bitcoin exposure, this study focuses on Ethereum, Bitcoin Cash and Bitcoin Satoshi Vision (Bitcoin SV). Ethereum is the second coin in terms of capitalization, while Bitcoin Cash and Bitcoin SV are Bitcoin forks meaning that they are aimed to follow the Bitcoin price closely.

This study focuses on finding the most useful proxy hedge instrument for the Bitcoin-USD market. Due to its particularities, this market does not exhibit the same features as traditional financial markets do. In appearance, it seems very related to other Altcoins (alternative coins), but in reality, it exhibits unusual volatility clustering effects. This behaviour has a direct impact on the hedging strategies of business exposed to crypto-currencies, including the hedge funds, mining farms or ICO projects. I explored the econometric features of Bitcoin and other Altcoins and underline the need for fat tail distributions and volatility clustering models.

The problem is studied in two steps: first the various econometric models with fat tails are explored in relation with the returns of daily time series and second the proxy hedging is test based on density forecasts methods using the results of the first step.

In order to capture the leptokurtic distribution shape of daily returns of crypto-currencies' prices and overpass the limitations the classic Gaussian models, I considered the following set of candidate distributions from the generalized hyperbolic family: t-Student, Log-Normal and Normal Inverse Gaussian (NIG). They retained my attention for their capacity to take in heavy account tails and their straightforward estimation. The fittings are compared based upon the Bayesian Information Criterion (BIC). The results show that NIG distribution exhibits the best fit for the Bitcoin daily returns, similar results being found for the other Altcoins.

The current literature relative to hedging in crypto-currencies market focuses mainly on the risks related to level forecasting when using a proxy-hedge but ignores the density forecasting completely. The main issue with proxy-hedging is the fact that markets have a different depth. On the one hand, a shock in the Bitcoin market might not be fully reflected in the Altcoin prices. On the other hand, a small variation in the Bitcoin price may generate a shock in the Bitcoin forks (Bitcoin Cash and Bitcoin SV) due to the difference in market structures. Moreover, the Altcoin currencies are easier to manipulate than the Bitcoin market given the lower level of information, and the lower number of active traders. The differences in the distribution features also generate the basis risk of proxy-hedges using

both plain or derivatives based strategies, thereby underlying the need for testing the density forecasting ability.

For testing the proxy-hedging with Ethereum, Bitcoin Cash or Bitcoin SV, a trader exposed to Bitcoin price risk should assess the density forecasting capacity of an econometric risk model. Thus, a model estimated on Bitcoin Cash or Bitcoin SV returns should be tested in terms of density forecasting on the Bitcoin prices.

I reviewed the different density forecasting techniques starting with [Diebold and Mariano (2002)] who introduced in the early 1990s seminal tests of the null hypothesis of no difference in the accuracy of two competing forecasts. For assessing the density forecasting in proxy hedging, I used a test proposed by [Gneiting and Ranjan (2011)] that develops the weighting approach of [Amisano and Giacomini (2007)] but avoids counter intuitive inferences.

The historical backtesting shows that Ethereum was a poor proxy-hedging candidate for Bitcoin before 2017, due to the fact that the last went through a bubble during that period while Ethereum did not. Bitcoin and Bitcoin Cash distributions should not be very different in theory, as partially confirmed by the backtesting. Around 2019, the extreme events on Bitcoin are not followed by similar moves in Bitcoin Cash, thereby supporting the claims that Bitcoin Cash is targeted by speculators, due to the fact that has lower liquidity and transparency than Bitcoin. Bitcoins Satoshi Vision shows significant differences with Bitcoin, especially in the tails region after December 2019. This is explained by the fact that there were some price manipulation attempts on this Altcoin, based on unverified rumours spread amongst market participants about the existence of a Bitcoin SV *whale*.

The approach used to test the hedging strategy for crypto-currencies is leveraged for oil distillates. Despite being a completely different market, hedging oil distillates exposures face the same types of challenges as hedging crypto-currencies positions. Companies exposed to jet fuel price risk prefer to hedge their exposure using crude oil or Gasoil contracts even if jet fuel future contracts are also available because the liquidity on jet fuel is very thin. If an Asian airline company wants to cover its jet fuel price risk, since the volumes exchanged on this market are thin, it might use one of the 'proxy-hedge' options including ICE Brent oil, ICE LS Gasoil and Singapore Gasoil.

The research exposed in this chapter 4 was presented at the 2017 IPAG conference in Nice, France and was published in peer-reviewed conference proceedings: "Impact of Contagion on Proxy-Hedging in Jet-Fuel Markets" [Guegan *et al.* (2020b)]. It explores the topic of proxy hedging in middle distillates market with a focus on jet fuel. The research addresses the problem of a refinery or an airline company that hedges its jet fuel price risk with proxy instruments, including Brent futures and gasoil futures. It focuses on finding the

most useful proxy hedge instrument for the Singapore spot market. Due to its particularities, this market does not exhibit the same features as traditional financial markets do. In appearance, it seems very related to the oil market, but in reality, it exhibits insufficient liquidity and shows unusual volatility clustering effects. This behaviour, as well as the potential contagion effects, have a direct impact on the hedging strategies of refineries, airline companies and jet fuel traders.

The problem is studied in two steps: first, the various econometric models with fat tails and volatility clustering are explored in relation with the returns of daily time series and second the proxy hedging is a test based on density forecasts methods using the results for the first step.

The results from the first part show that NIG distribution, APARCH specifications of the volatility dynamics capture in an appropriate manner the behaviour of jet fuel, brent and gasoil prices. Also, GARCH switching regimes models are good candidates for analyzing the markets that might exhibit thin liquidity.

The second part shows that the NIG model fitted on the Singapore Gasoil as a proxy has the best density forecasting abilities from the considered choices. The main finding of this paper is that a trader exposed to jet fuel price risk might think he has different hedging alternatives in terms of markets, where in reality from a risk management perspective, the alternatives could exhibit similar behaviour in term of density forecasting capability. Contagion impacts the proxy-hedging negatively primarily when the behaviour of jet fuel and its proxy-hedging are decoupling at the same time, thereby leaving the trader with limited options. The results show that Singapore Gasoil Futures contract is the best candidate for hedging the Singapore Jet Fuel spot price.

Chapter 5 discuss the potential future directions of research based on the findings of this thesis. Expanding the scope of real option theory models to other fields of finance is one of the foreseeable axis of research. Testing for bubbles can be implemented to other assets types and markets, thereby constituting an additional direction of development for my work.

Another future direction for my research is the consideration of liquidity on various exchanges, as trading crypto-currencies and oil distillates usually involve different brokerage fees and liquidity across other exchanges. Another direction is the consideration of transaction costs in the Gneiting test score function, as future trading contracts usually involve brokerage fees and liquidity across different product maturities. This leads to addressing the problem of dimensionality, as it would be necessary to consider a technique such as approximate dynamic programming to produce a hedging policy that reflects such costs.

Chapter 1

To mine or not to mine? The Bitcoin Mining Paradox

1

*Bitcoin is mostly about anonymous transactions, and I don't think over time that's a good way to go. I'm a huge believe in digital currency... but doing it on an anonymous basis I think that leads to some abuses, so I'm not involved in Bitcoin. **Bill Gates***

Abstract *The aim of this chapter is to study the profitability of Bitcoin mining, using the real options theory. The main factors driving the marginal Bitcoin mining profitability are the Bitcoin price, the hashrate, the predictability of mining difficulty and the hardware efficiency. We propose a real options model that simulates the fundamental mining reward and measures the likelihood of breakeven on initial investment and explores also the relationship between the Bitcoin price and the mining difficulty in different economic cycles. Some of our findings questions the rationality of miner's decisions and attempts to measure their impact on the economics of Bitcoin. Our results show that after the 2017 bubble Bitcoin, miners had an irrational behavior and did not adjust their strategy based on the price levels.*

1.1 Introduction

On 3rd January 2009, Satoshi Nakamoto created the first Bitcoin by generating the first block of the chain hashing using his personal computer. Since then, from a necessary activity to sustain the blockchain network operation, the Bitcoin mining has become a new type of business with a constantly increasing interest. However, miners who set up mining operations face many economic uncertainty, such as high price volatility or increasing mining difficulty, both impacting the profitability and the payback of the initial investment.

Bitcoin hit the peak of its popularity towards the end of 2017 when its exchange rate with the US dollar rallied to almost 20,000 on some exchanges. Currently, Bitcoin is still the most popular crypto-currency. It's economics involves various market participants such as long term investors, short term speculators and miners. The first category of crypto-currency

¹This chapter corresponds to the working paper: "To Mine or Not to Mine? The Bitcoin Mining Paradox", Rostislav H. Guegan D., Frunza M. 2020, [Haliplii *et al.* (2020b)].

investors have a "buy and hold" strategy and expect that the value of their portfolio will experience a strong growth. The second category attempt to benefit both from anticipating the market movements. Miners however, rely heavily on the predictability of the cash-flows, as they have sunk costs encompassing the amortization of the hardware equipment and the electricity and other maintenance costs paid in fiat currency (like USD). Mining Bitcoin became in the recent years a fully-fledged business. Owners of mining businesses, called "farms" buy or rent huge infrastructures of computing capacities that generate cryptocurrencies and cover the operating costs.

The study and analysis of the crypto-currency mining is however a relatively new field. [Cocco and Marchesi (2016)] proposed a model which simulates the mining process and the Bitcoin transactions, by implementing a mechanism for Bitcoin's price discovery, and specific behaviors for each typology of trader. [Courtois *et al.* (2014)] also considered the economics of mining. They addressed the fundamental uncertainty in mining operations such as high volatility.

The profitability of Bitcoin mining is a topic which was not properly studied and many investors lost significant amount of money in building mining operations that failed only after few months. Our researches focuses on assessing the profitability of a mining farm and reviews the various valuation methods. We use the real options theory (ROT) to develop a comprehensive framework for the valuation of Bitcoin mining business. Such an approach incorporates the various options embedded in the value of a mining firm. This chapter shows that Bitcoin mining farms owners and more broadly Bitcoin miners manage irrationally their operations with respect to price movements. They make decisions too late and in their own detriment, fact which is driven by false expectations and irrational decision making.

This chapter aims to enrich the scarce literature on the economics of Bitcoin mining and attempts to estimate a realistic solution in predicting mining rewards. This chapter is organized as follows:

Section 1.2 introduces the concept of Bitcoin mining difficulty.

Section 1.3 explores the econometric relationship between the Bitcoin prices and mining difficulty.

Section 1.4 presents the valuation methodology framework for a mining farm based on the real option theory.

Section 1.5 presents the optimal management decisions given by this framework.

Section 1.6 concludes.

1.2 Background: Understanding Bitcoin Mining Difficulty

Bitcoin is a decentralized peer-to-peer digital cash system ([Lee (2015)]). The Bitcoin protocol uses a mathematical equation that adds blocks to a chain of transactions known as a blockchain. Each block uses a hash code from the previous block to timestamp the newly added block.

Blocks are added to the blockchain every 10 minutes via miners who compete against each other to figure out a mathematical equation (SHA-256) whose answer must begin with four zeroes. The process requires extensive computer processing power, which equates to electrical usage. The first miner to discover a suitable solution to the equation receives an award of 6.5 BTC (since 09/07/2016). Once this solution is found, a new block is added and validated into the blockchain. The measure showing how many computations are required to validate a new block within 10 minutes, and consequently to earn the mining rewards, is called mining difficulty.

Bitcoin is designed to adjust its mining difficulty every 2,016 blocks. If a block is found every 10 minutes (as it was intended initially for even emission) finding 2016 blocks will take exactly 2 weeks. Therefore, difficulty is changed every 2,016 blocks (approximately 14 days) based on the amount of computing power deployed to the network. This ensures that the block production interval at the next period remains constant at around every 10 minutes. When there are fewer machines competing to solve math problems to earn the next payout of newly created Bitcoin, difficulty falls; when there are more computers in the game, it rises ([Frunza (2015)]).

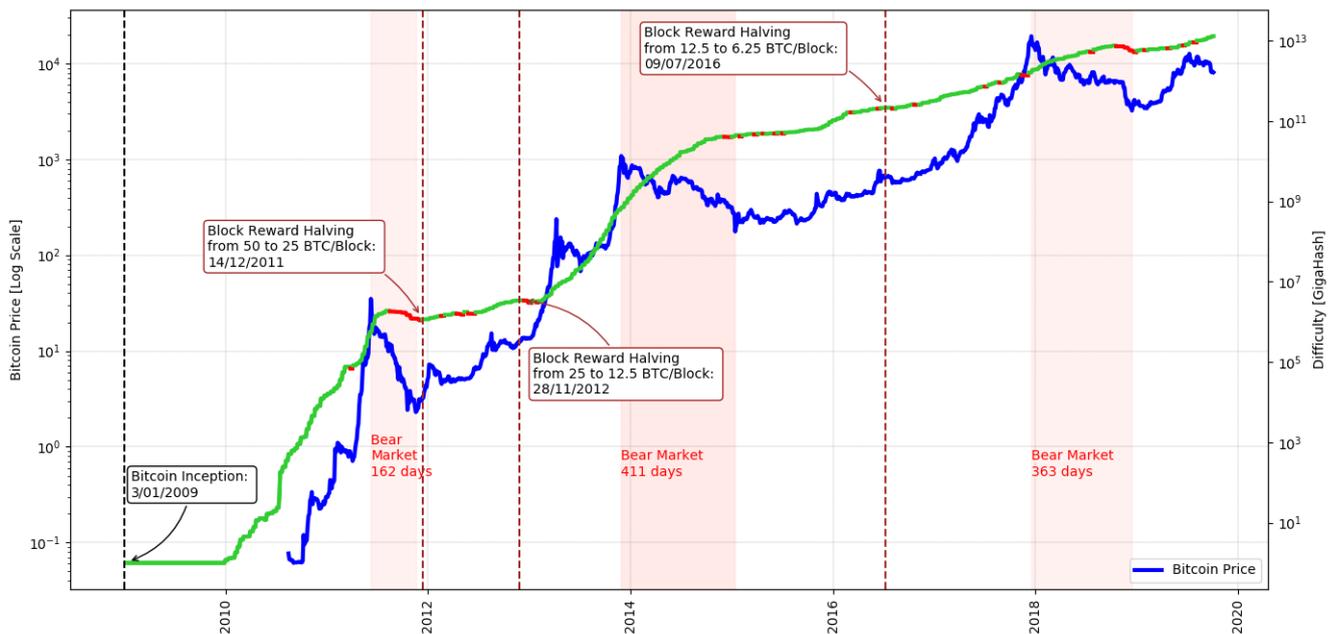


Fig. 1.1 Evolution of Bitcoin price and mining difficulty between 3-Jan-2009 and 5-Oct-2019

1.2.1 Relationship between price and mining difficulty

Figure 1.1 shows the historical evolution of Bitcoin's price in relationship to the the mining difficulty² between January 2009 and October 2019. Table 1.1 shows the major changes in price for Bitcoin, the corresponding ranges in difficulty as well as the lag between the variation in price and the variation in difficulty.

Table 1.1 Major price changes, corresponding changes in difficulty and the lag between the variation in price and the variation in mining difficulty between 3-Jan-2009 and 5-Oct-2019

Date	Price	Change %	Date	Difficulty	Change %	Lag (Days)
2009-01-03	0	-	2009-01-03	0	-	0
2011-06-10	35	100.0%	2011-08-15	1.89E+06	100.0%	66
2011-11-19	2.3	-93.4%	2011-12-10	1.09E+06	-42.3%	21
2013-11-29	1083.9	47026.1%	2014-12-03	4.03E+10	3694724.5%	369
2015-01-14	176.5	-83.7%	2014-12-30	3.95E+10	-2.1%	-15
2017-12-17	19271.25	10818.6%	2018-10-04	7.45E+12	18781.0%	291
2018-12-15	3276.30	-83.0%	2018-12-19	5.11E+12	-31.4%	4

The evolution of Bitcoin's price and Bitcoin's mining difficulty depicted in Figure 1.1 and Table 1.1 leads to the following observations:

- Every four years, Bitcoin's block reward (earned by miners who successfully validate new blocks in the Bitcoin blockchain) is halved. First halving occurred in 2012 from 50 BTC to 25 BTC and then another to 12.5 BTC in 2016. The next halving event which will drop the block reward to 6.25 BTC is estimated to happen in May of 2020.
- There were three significant bear markets since Bitcoin's inception. Each of them had a different lifetime, however all three corrections were followed by a correction in the mining difficulty. In all three bear markets, the price of Bitcoin dropped by 80-90% on average, while the difficulty adjusted more conservatively.
- The difficulty tends to grow exponentially, at a faster pace than the price of Bitcoin. It also tends to drop less violently and for less extended periods of time.
- The difficulty adjustment speed decreased over time, while the time lag increased from 2 months (66 days) to 1 year (369 and 291 days). This can be probably explained by the fact that before 2014, the largest majority of miners were using heterogeneous mining hardware varying from central processing units (CPUs) to small scale graphics processing units (GPUs) mining farms. They were usually mining from their homes or garages. Those miners exhibited more prompt reactions to drops in Bitcoin price, since their initial investment costs were insignificant so that they could switch of their mining operation at any time.

²All data used in this chapter has been sourced from Blockchain.info, a benchmark web site which displays detailed information about all transactions and Bitcoin blocks.

- After 2014, with the appearance of hardware based on application-specific integrated circuits (ASICs), Bitcoin mining became a large scale institutional activity with high entry and operational costs. As a result, large miners were more reluctant to turn off their farms in the expectation that the price will rebound sooner or later. This change in miners behavior, has clearly impacted the cyclical economics of the Bitcoin, and does not necessarily mean the miners were more rational before 2014. Figure 1.2 focuses on the evolution of the Bitcoin price and Bitcoin’s mining difficulty between 21-Sep-2014 and 5-Oct-2019.

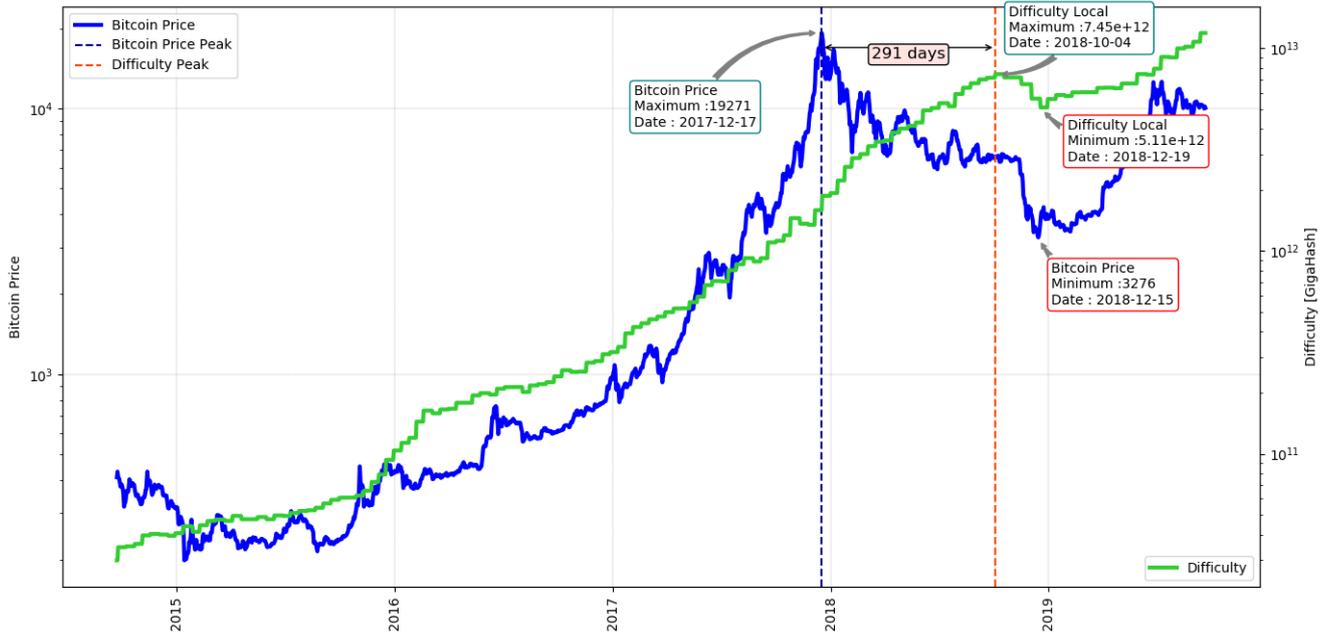


Fig. 1.2 Evolution of the Bitcoin price and mining difficulty between 21-Sep-2014 and 5-Oct-2019

As explained in the previous paragraph, Bitcoins are created each time a miner validates a new block. The rate of block creation is adjusted every 2016 blocks to aim for a constant two week adjustment period (equivalent to 6 per hour.) The number of Bitcoins generated per block is set to decrease geometrically, with a 50% reduction every 210,000 blocks, or approximately four years. The result is that the number of Bitcoins in existence will not exceed slightly less than 21 million. This decreasing-supply algorithm was chosen because it approximates the rate at which commodities like gold are mined. The daily mining rewards expressed in Bitcoin and US dollars are calculated as follows:

$$R_t^{BTC} = \frac{H_t^{hash/s} * B_t^{BTC} * (1 - C_t\%) * S}{(D_t^{hash} * 2^{32})} \quad (1.1)$$

$$E_t^{USD} = \frac{P_{ASIC}^{Watt} * E_{kWh}^{USD} * 24}{1000} \quad (1.2)$$

$$R_t^{USD} = R_t^{BTC} * BTC_t^{USD} - E_t^{USD} \quad (1.3)$$

where :

- R_t^{BTC} is the daily mining reward in Bitcoins.
- E_t^{USD} is the daily electricity cost in US dollars.
- R_t^{USD} is the daily mining reward in US dollars.
- $H_t^{hash/s}$ is the Hashrate (hashes / second); The hash rate is a general measure of the processing power of the Bitcoin network. It is a measure of how many times the network can attempt to solve the cryptographic puzzle every second.
- D_t is the mining Difficulty.
- C_t is the Mining Pool fee in percents.
- B_t is the block reward, which is equal to the number of Bitcoins a miner gets if he/she successfully mines a block of the currency.
- S is the Numbers of seconds in one day ($60 * 60 * 24 = 86400$).
- P_{ASIC}^{Watt} is the power of an ASIC antminer.
- E_{kWh}^{USD} is the electricity cost in US dollars per kWh.
- BTC_t^{USD} is the daily Bitcoin price in US dollars.

1.3 Econometric analysis of the relationship between price and difficulty

In the light of the mechanism of Bitcoin mining it is crucial to analyse the causality between Bitcoin's price and the mining difficulty. Analyzing causality, in the Granger sense ([Granger (1988)]), involves testing whether lagged information on Bitcoin's price provides any statistically significant information about the mining difficulty. Intuitively one would expect to observe that the Bitcoin price behavior determines the dynamic of the mining difficulty. Decreasing Bitcoin prices should determine a decrease in difficulty and an increase in price would lead to an increase in difficulty.

We conduct the Granger causality test to analyse our previous intuition. In Table 1.2 we summarize the result of our test: the null hypothesis is that Bitcoin's Price do not Granger-cause the Difficulty. The null hypothesis is rejected at 5% significance level indicating that Bitcoin's price behaviour influence the mining difficulty. Table 1.3 shows the results of the Granger causality test assuming the null hypothesis is that the 'Difficulty' does not Granger-cause Bitcoin's Price. The level of p-value for different lag values indicate that the null hypothesis is not rejecting.

Figure 1.3 shows the evolution of Bitcoin's price and mining difficulty between 21-Sep-2014 and 21-Sep-2019. A straightforward way to model the difficulty is the linear regression:

$$\log(Difficulty_t) = \beta * t + \alpha + \epsilon_t \quad (1.4)$$

Table 1.2 Results of the Granger causality test: The null hypothesis is that Bitcoin's Price do not Granger-cause the Difficulty.

F-statistic	p-value	lag
16.8869	0.0000	1
5.3269	0.0001	5
6.1954	0.0000	10
3.5976	0.0000	15
2.6585	0.0000	25

Table 1.3 Results of the Granger causality test: The null hypothesis is that the 'Difficulty' does not Granger-cause Bitcoin's Price.

F-statistic	p-value	lag
0.2045	0.6512	1
2.0027	0.0754	5
1.7688	0.0613	10
1.4557	0.1137	15
1.3568	0.1119	25

where, β is the slope, α is the intercept and ϵ_t are the residuals.

The results of the linear regression are presented in Table 1.4 exhibiting an adjusted R^2 of 0.977. The evolution of Bitcoin's price, mining difficulty between 21-Sep-2014 and 21-Sep-2019 and the linear fit are depicted in Figure 1.3.

Table 1.4 Regression result of the linear regression model. The adjusted R^2 is 0.977

Parameter	Estimate	Std. Deviation	p-value
α	-0.3382	0.6512	0.000
β	2.0027	0.0754	0.000

Nevertheless a better option would be to fit an ARIMA(p,d,q) model (Box Jenkins, 1960) defined as follows:

$$y'_t = c + \phi_1 y'_{t-1} + \dots + \phi_p y'_{t-p} + \theta_1 \epsilon_{t-1} + \dots + \theta_q \epsilon_{t-q} + \epsilon_t \quad (1.5)$$

where y'_t is the differenced series of log mining difficulty ($\log(Difficulty_t)$) (it may have been differenced more than once). The variables on the right hand side include both lagged values of y_t and lagged errors.

Based on the AIC criteria, the best ARIMA model is specified for p=3, d=1 and q=3, the parameters being presented in Table 1.5. The equation (3) describing the mining reward can be rewritten as:

$$R_t^{BTC} = \frac{H_t^{hash/s} * B_t^{BTC} * (1 - C_t^{\%}) * S}{(e^{y_t} * 2^{32})} \quad (1.6)$$

where y_t follows the ARIMA process estimated above.

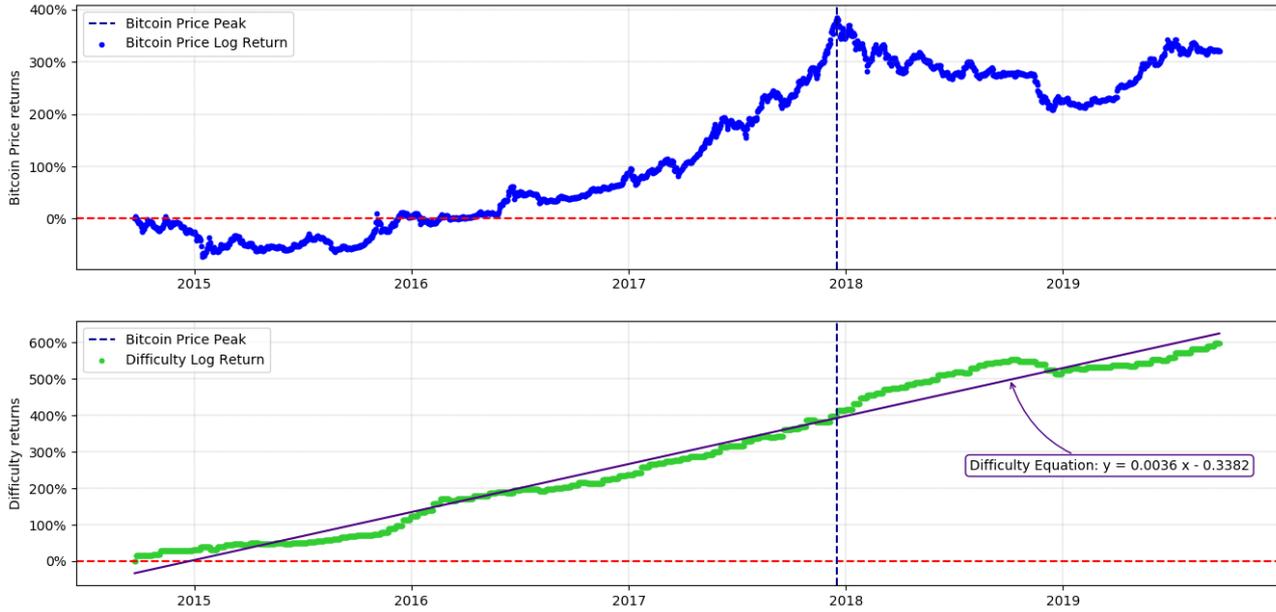


Fig. 1.3 Log returns of Bitcoin price and mining difficulty between 21-Sep-2014 and 21-Sep-2019

Table 1.5 ARIMA model estimation results

Parameter	Value	Std error	Z-test	p-value	Confidence interval [0.025 , 0.975]
c	0.0033	0.000	7.993	0.000	[0.002 , 0.004]
ϕ_1	0.8067	0.036	22.317	0.000	[0.736 , 0.878]
ϕ_2	0.7314	0.064	11.492	0.000	[0.607 , 0.856]
ϕ_3	-0.9512	0.035	-26.821	0.000	[-1.021 , -0.882]
θ_1	-0.8467	0.039	-21.647	0.000	[-0.923 , -0.770]
θ_2	-0.7351	0.070	-10.452	0.000	[-0.873 , -0.597]
θ_3	0.9601	0.039	24.534	0.000	[0.883 , 1.037]

1.4 Mining Profitability Modeling

1.4.1 Valuation with real options theory

The most common valuation tool is Net Present Value (NPV) and the valuation of Bitcoin farm makes no exception. Investors make projections of the Bitcoin price and assess the value of the farm based on these projections. However, such valuations are deterministic and may be not adapted to a situation where the Bitcoin price is very volatile.

The basic inadequacy of the NPV approach and other discounted cash flow approaches to capital budgeting is that they ignore, or cannot properly capture, management's flexibility to adapt and revise later decisions (i.e., review its implicit operating strategy). The traditional NPV approach, in particular, makes implicit assumptions concerning an "expected scenario" of cash flows and presumes management's commitment to a certain "operating strategy". The real options theory introduced by [Trigeorgis (1996)] and [Myers (1977)] incorporates in the valuation of a business the value of the various options that the management has.

The list of investment options available for a manager of a Bitcoin mining farm are:

- (1) **The option to defer investment:** enables management to defer investment and benefit from the uncertainty about Bitcoin prices during this period.
- (2) **Time-to-build option:** allows to stop a step-by-step investment in the farm. A low price level would stop a new investment.
- (3) **Option to abandon:** allows management to shut down the farm or to sell the mining equipment (ASICs), if BTC price are low.
- (4) **Option to switch:** Bitcoin mining operations can be designed to switch to another crypto-currency depending on profitability.
- (5) **Growth option:** Expand the operations and reinvest the generated Bitcoin in new equipment.
- (6) **Multiple interacting options:** It represents combinations of real options

Compared to real option theory, standard discounted cash flow techniques will tend to understate the option value attached to growing profitable lines of business and lead to sub-optimal business decisions.

1.4.2 Valuation of a Bitcoin mining farm

Based on a framework developed by [Morck *et al.* (1989)] the valuation of a Bitcoin mining farm depends on the Bitcoin price and the inventory of generated Bitcoin. The formalism can be described as following:

$$\frac{dS}{S} = \mu_S dt + \sigma_S dB_S \quad (1.7)$$

$$\frac{dI}{I} = (\mu_I - q(S, t, I))dt + \sigma_I dB_I \quad (1.8)$$

where

- S is the BTC price,
- μ_S is the Brownian trend
- and σ_S the Bitcoin volatility.
- I is the stock of generated Bitcoins ,
- μ_I is the speed of generating Bitcoins,
- q is the quantity of sold Bitcoin,
- and σ_I the empiric variability of Bitcoins stock.

The speed of generating BTC is proportional to the mining reward and can be expressed as:

$$\mu_I = \gamma \cdot R_t^{BTC} = \gamma \cdot \frac{H_t^{hash/s} * B_t^{BTC} * (1 - C_t\%) * S}{(e^{y_t} * 2^{32})} \quad (1.9)$$

The proportion γ depends on the feature of the operational features of the farm, accounting for effective percentage of day an equipment is fully operational.

The after tax profit of the company is :

$$f(S, I, t) = (1 - \pi_{corp})(S \cdot q(S, t, I) - A(q, t) - \lambda \cdot I) \quad (1.10)$$

where

- $f(S, I, t)$ is the net cash flow of the farm at moment t
- π_{corp} local company taxation
- $A(q, t)$ is the costs of transaction when selling "q" BTC
- λ is the mining cost
- $V(S, I, t)$ is the farm value

Applying Ito to the value of company $V(t)$ in regards to the stochastic process $S(t)$ and $I(t)$ we obtain:

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial I} dI + \frac{\partial V}{\partial S} dS + 0.5 \frac{\partial^2 V}{\partial I^2} (dI)^2 + 0.5 \frac{\partial^2 V}{\partial S^2} (dS)^2 + \frac{\partial^2 V}{\partial S \partial I} (dS dI)$$

Injecting the Ito development in the Bellman equation we obtain:

$$\begin{aligned} r \cdot V(t) \cdot dt &= f(t) \cdot dt + \mathbf{E}(dV) \\ r \cdot V(t) \cdot dt &= f(t) \cdot dt + V_t dt + V_I \cdot (\gamma \cdot R_t - q) \cdot dt + V_S \cdot \mu_S \cdot dt \\ &\quad + 0.5 V_{SS} \sigma_S^2 \cdot dt + 0.5 V_{II} \sigma_I^2 \cdot dt + \rho_{IS} V_{IS} \sigma_S \sigma_I \cdot dt \\ 0 &= f(t) - rV(t) + V_t + V_I \cdot (\gamma \cdot R_t - q) + V_S \cdot \mu_S \\ &\quad + 0.5 V_{SS} \sigma_S^2 + 0.5 V_{II} \sigma_I^2 + \rho_{IS} V_{IS} \sigma_S \sigma_I \end{aligned}$$

Introducing the after tax cash-flows in the Bellman equation the equations becomes:

$$\begin{aligned} 0 &= (1 - \pi_{corp})(S \cdot q(S, t, I) - A(q, t) - \lambda \cdot I(t)) - r \cdot V \\ &\quad + V_t + V_I \cdot (\gamma \cdot R_t - q) + V_S \cdot \mu_S + 0.5 V_{SS} \sigma_S^2 + 0.5 V_{II} \sigma_I^2 + \rho_{IS} V_{IS} \sigma_S \sigma_I \end{aligned}$$

Thus a Bitcoin mining farm manager needs to find the maximum rate q of sold Bitcoins with respect to the above equation.

$$\exists q : 0 = \underbrace{\max}_{q \in (0, q_{max})} [(1 - \pi_{corp})(S \cdot q(S, t, I) - A(q, t) - \lambda \cdot I(t)) - r \cdot V + V_t \quad (1.11)$$

$$+ V_I \cdot (\gamma \cdot R_t - q) + V_S \cdot \mu_S + 0.5 V_{SS} \sigma_S^2 + 0.5 V_{II} \sigma_I^2 + \rho_{IS} V_{IS} \sigma_S \sigma_I] \quad (1.12)$$

with the boundary conditions : $V(S, I, t = T) = 0$, $V(S = 0, I, t < T) = 0$, $\lim_{S \rightarrow \infty} \frac{\partial V(S, I, t)}{\partial S} \propto I$

When solving the above equation, the dynamic of the farm valuations depends on the level of BTC price $S(t)$ with two reference prices S_a and S_r . S_a denotes the Bitcoin price

that would trigger abandoning and decommissioning the operation. S_r represents the Bitcoin price that would trigger further investing in the farm. Depending on these two, the value of the farm and the corresponding management decisions are:

$$\mathbf{V}_t = \begin{cases} V^a(t) & \text{if } S(t) \leq S_a \\ & \text{immediately sell Bitcoin stock and } \mathbf{abandon} \text{ mining} \\ V^k(t) & \text{if } S_a < S(t) \leq S_r \\ & \mathbf{keep} \text{ mining and balance the Bitcoin stock} \\ V^r(t) & \text{if } S(t) > S_r \\ & \text{immediately } \mathbf{reinvest} \text{ all Bitcoin in new mining equipment} \end{cases}$$

1.5 Application: Optimal mining decision

Before applying the framework introduced above to the valuation of a Bitcoin farm, few considerations about the Bitcoin mining hardware are necessary.

Hash calculations to mine Bitcoin have been getting more and more complex, and consequently the mining hardware evolved to adapt to this increasing difficulty. Bitcoin mining difficulty increased significantly since 2017 as a result of added hash power on the network. Bitcoin network difficulty is adjusted to compensate for increased hash power in order to ensure block times remain consistent at around ten minutes.

In 2015, Bitcoin miners saw the beginning of a considerable rise in network hash power, primarily due to the introduction of Bitmain's Antminer line. Antminer utilized specially designed application specific integrated chips (ASIC) that were thousands of times better at completing the SHA-256 algorithm Bitcoin's proof-of-work system uses. The evolution of different type of ASIC mining hardware between 2014 to 2019 are presented in Table 3.2.

Table 1.7 exhibits the cost of the kWh across few countries, underlining that Asian countries and URSS republics have a net advantage in term of electricity cost compared to developed countries

Since China is the country with the the largest Bitcoin mining operations, the electricity price which is considered in further simulations is 0,08 per kWh. We proceed to solve the stochastic optimization problem described in equation 1.11 through a numerical simulation approach. The simulation of the value and profitability of a Bitcoin mining farm encompasses the following steps:

- (1) Difficulty and mining reward simulation
- (2) Bitcoin price simulation
- (3) Profitability simulation of the mining farm and optimal decisions

We apply this process to three types of mining equipments: Antminter S7, Antminter

Table 1.6 ASIC AntMiner Hardware from 2014 to 2019.

Product	Release Date	Mhash/s	Mhash/J	Power Watts	Price	Currently Shipping
Antminer S1	2014-03-11	180,000	500	360	299	Discontinued
Antminer S2	2014-05-24	1,000,000	900	1100	2259	Discontinued
Antminer S3	2014-08-06	441,000	1300	340	382	Discontinued
Antminer S4	2014-10-01	2,000,000	1429	1400	1400	Discontinued
Antminer S5	2015-02-04	1,155,000	1957	590	370	Discontinued
Antminer S5+	2015-09-16	7,722,000	2247	3436	2307	No
Antminer S7	2015-09-16	4,860,000	2666	1210	1900	No
Antminer S9	2017-11-01	14,000,000	5833	1372	2400	Yes
Antminer S9se	2019-07-01	16,000,000	8000	1280	2400	Yes
Antminer S11	2018-11-01	20,500,000	7500	1530	2300	Yes
Antminer S15	2018-12-01	28,000,000	8000	1596	2350	Yes
Antminer S17	2019-04-01	53,000,000	16500	2200	2544	Yes
Antminer S17e	2019-11-01	64,000,000	17000	2250	2880	Yes
Antminer T17	2019-05-01	38,000,000	9500	2200	1096	Yes
Antminer T17e	2019-11-01	53,000,000	16500	2915	1529	Yes

Table 1.7 **Electricity price by country (USD/kWh)**: Countries from South East Asia and ex-URSS block have low prices. The only developed countries with similar levels are Canada and United States. European Union has generally high prices.

Country	US cents/kWh	Country	US cents/kWh
Kirgizstan	2	South Africa	12
Ukraine	3	Israel	15
Uzbekistan	4.95	Hong Kong	18
Russia	5	France	19.39
Thailand	6	United Kingdom	20
Pakistan	7	Singapore	21.53
Dubai	7.62	Japan	22
Vietnam	8	Sweden	27.1
China	8	Italy	28.39
Indonesia	8.75	Netherlands	28.89
Canada	9	Australia	30
India	10	Spain	30
Malaysia	10	Belgium	30.46
United States	11	Germany	31.41
Taiwan	12	Denmark	40.38

S9 and Antminter S11.

1.5.1 *Difficulty simulation*

The mining reward is linked to the mining difficulty in equation 1.3. The projection of the mining difficulty is based on the ARIMA model described in equation 1.5. Figure 1.4 shows the projection of the mining reward for the Antminter S7. The green curve represents the observed historical mining reward and the blue curve represent the projected reward based on the projected difficulty via the ARIMA model and on the features of the Antminter S7.

Figure 1.5 and Figure 1.6 show the same projections for Antminer S9 and Antminer S11 respectively.

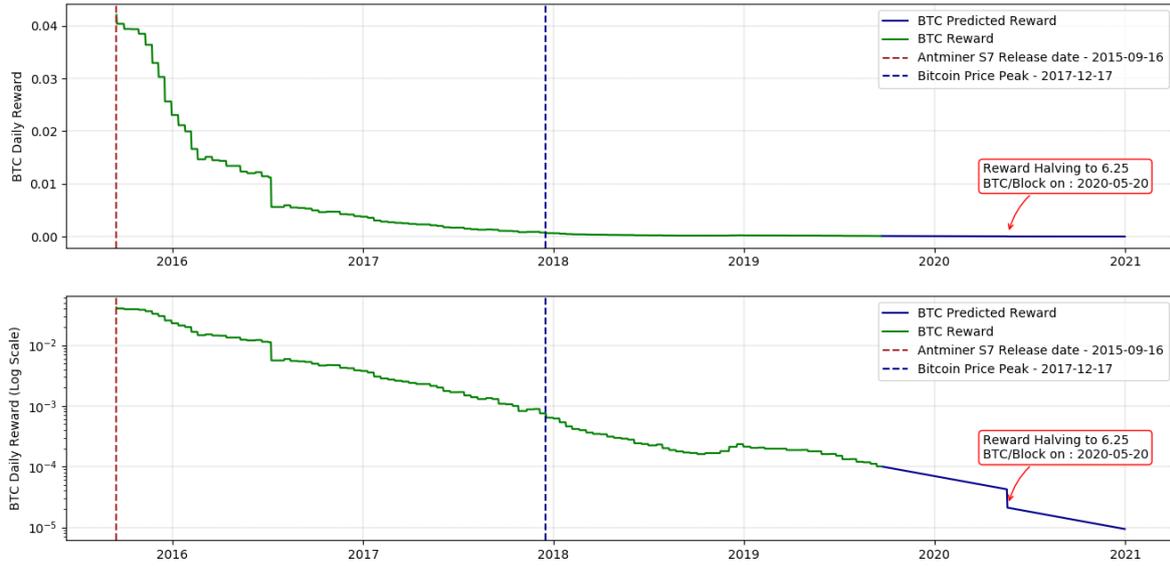


Fig. 1.4 Projection of the Daily Mining Reward (BTC) for Antminer S7

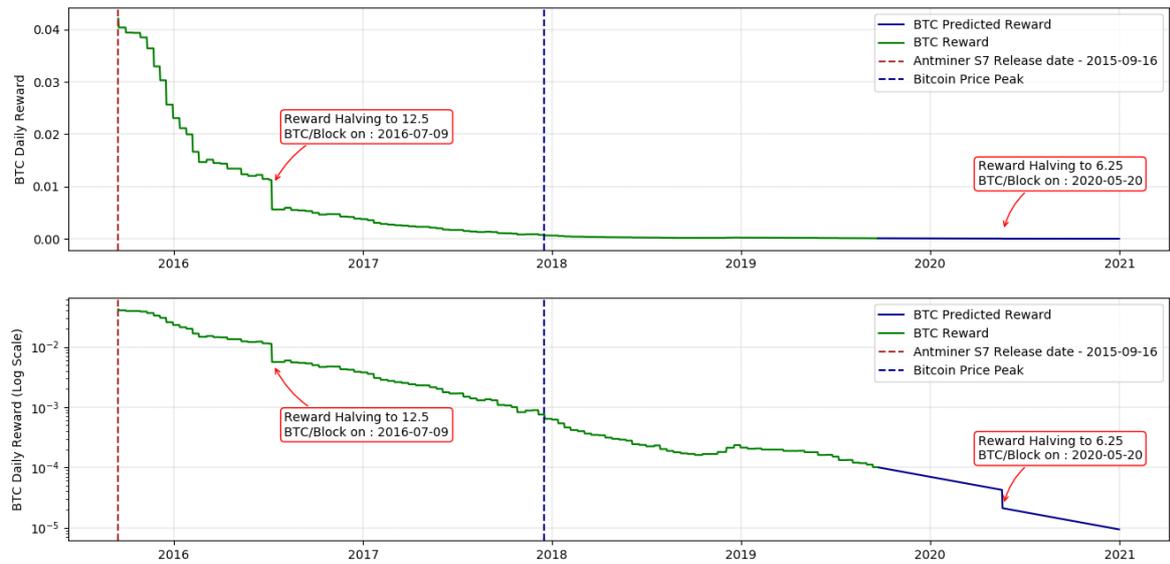


Fig. 1.5 Projection of the Daily Mining Reward (BTC) for Antminer S9

1.5.2 Bitcoin Price Simulation using Bootstrap Monte Carlo technique

The Bitcoin price simulation is based on bootstrapping the expected distribution from the observed distribution of historical returns of the daily spot prices. we used the bootstrapping approach in order to project the futures Bitcoin prices via an empirical distribution closer to the historical density. Bitcoin does not follow a Gaussian distribution ([Frunza (2015)]), thereby a historical approach being preferred .

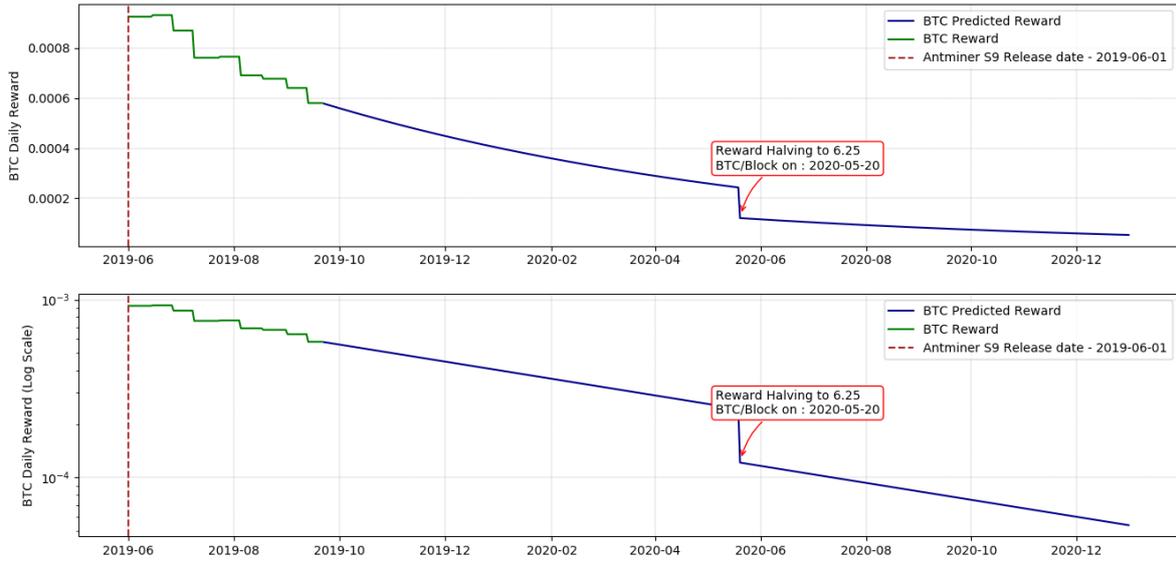


Fig. 1.6 Projection of the Daily Mining Reward (BTC) for Antminter S11

The observed historical distribution of returns is considered for the period starting from January 2017 until October 2019. The reason of taking this period is because it captures the rise and the crash of the cryptocurrency market benchmark. The period for which the simulation is done in this chapter is one year.

Below are outlined the list of steps required before simulating the Bitcoin price using the Bootstrap Monte Carlo method:

- (1) Calculate the daily log returns from the historical Bitcoin prices S_t :

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \quad (1.13)$$

- (2) Calculate the average return μ as the average of the total range of N historical log returns:

$$\mu = \sum_{i=1}^N \frac{r_i}{N} \quad (1.14)$$

- (3) Calculate the volatility of returns σ as the population standard deviation of the historical log returns:

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (r_i - \mu)^2}{N}} \quad (1.15)$$

- (4) Calculate the shift which is required to ensure that mean of simulated distribution of returns is the risk neutral expectation of the return over the recommended holding period:

$$Shift_t = E[R_t^{rn}] - \mu N - 0.5\sigma^2 N \quad (1.16)$$

where $E[R_t^{rn}]$ is the risk neutral expectation of the return over the corresponding holding period.

Once the pre-simulation steps are completed, next is the simulation stage. The minimum number of recommended simulations is $N_{sim} = 10000$. For each simulation from $j = 1$ to N_{sim} , the following steps are done:

- (1) Select a vector of random returns r_i from the pre-calculated vector of log returns for each time period within the holding period resulting into the vector of returns ordered and indexed from 1 to M. This is first done by randomly selecting an index number from the range of 1 to M index numbers for each time period within the recommended holding period and then constructing a vector of returns by looking up returns corresponding to each randomly selected index number. The same index number may be used more than once in the same simulation.
- (2) Calculate the cumulative sum of selected random returns for the selected period:

$$Rcum_t^j = \sum_{i=1}^M r_i \quad (1.17)$$

- (3) Simulate the path using the price $Psim_t^j$ of the underlying asset at each observation period t_i until the end of the holding period:

$$Ssim_t^j = S_0 e^{(Rcum_t^j + Shift_t)} \quad (1.18)$$

Bitcoin simulated price paths are shown in Figure 1.7.

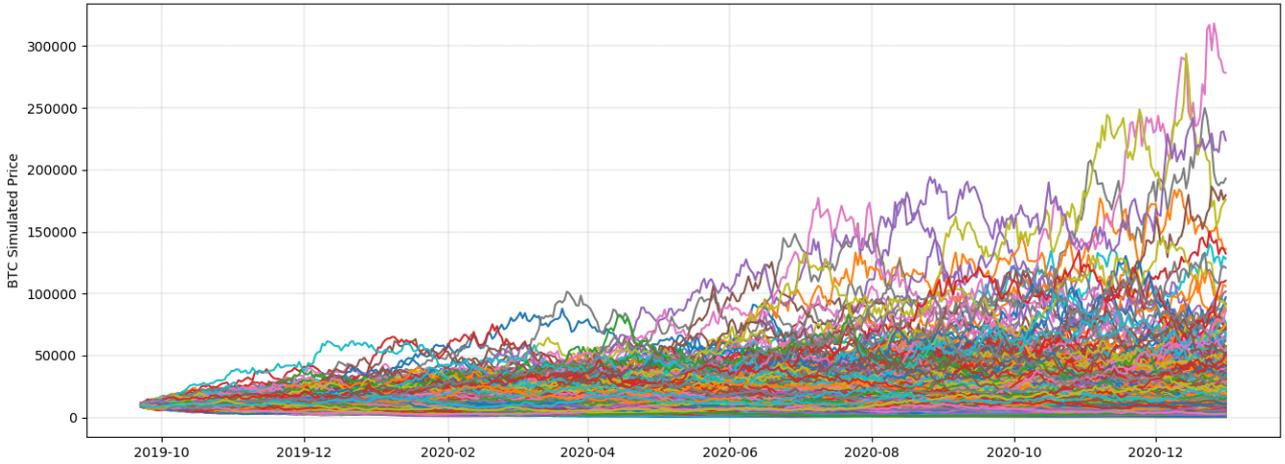


Fig. 1.7 Bitcoin Simulated Price Paths

Once the simulation is complete, three scenario paths are chosen:

- The *pessimistic* scenario shall be the path at the 10th percentile;
- The *average* scenario shall be the path at the 50th percentile;
- The *optimistic* scenario shall be the path at the 90th percentile.

Figure 1.8 shows the distribution of Bootstrap Monte Carlo simulation paths and how the three scenarii are chosen.

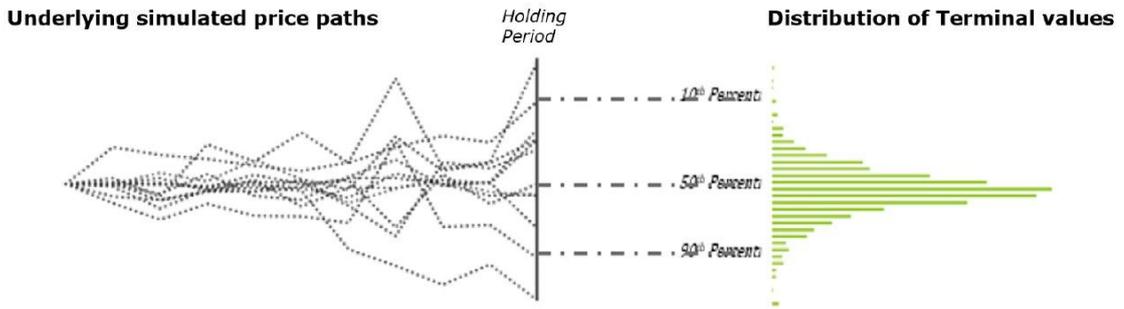


Fig. 1.8 Bootstrap Monte Carlo Scenario paths

Percentiles should be applied to the distribution of terminal prices at holding period (in our case one year) simulated under Bootstrapped Monte Carlo methodology. No discounting from recommended holding period to the valuation date is required.

1.5.3 Profitability simulation of the mining farm and optimal decisions

When simulating the trajectories for Bitcoin price the profitability is estimated. For each trajectory the optimal decision is concluded: reinvest, continue and abandon. For each type of decision and for each type of equipment the average prices S_a and S_r are estimated.

Figure 1.9 shows the evolution of the mining profitability estimated in US dollar for Antminer S7. The profitability reaches a peak towards the end of 2017 and becomes negative in the beginning of 2018. For the three considered scenarii of the Bitcoin price after September 2019 the operation is not profitable

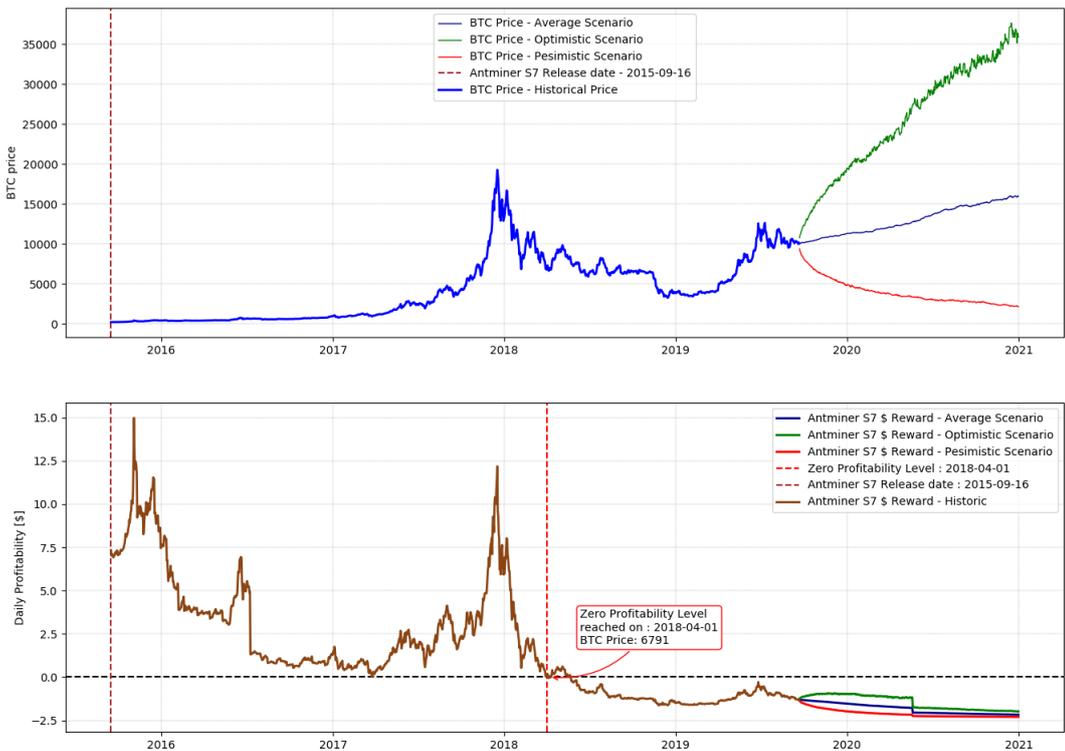


Fig. 1.9 Mining profitability Simulation estimated in US dollar for Antminer S7

Figure 1.10 shows the evolution of the mining profitability estimated in US dollar for Antminer S9. The profitability reaches a peak towards the end of 2017 and becomes negative in the last quarter of 2018. Under the optimistic Bitcoin price scenario the Antminer S9 is slightly profitable until May 2020. Under the other two scenarii the operation is not profitable.

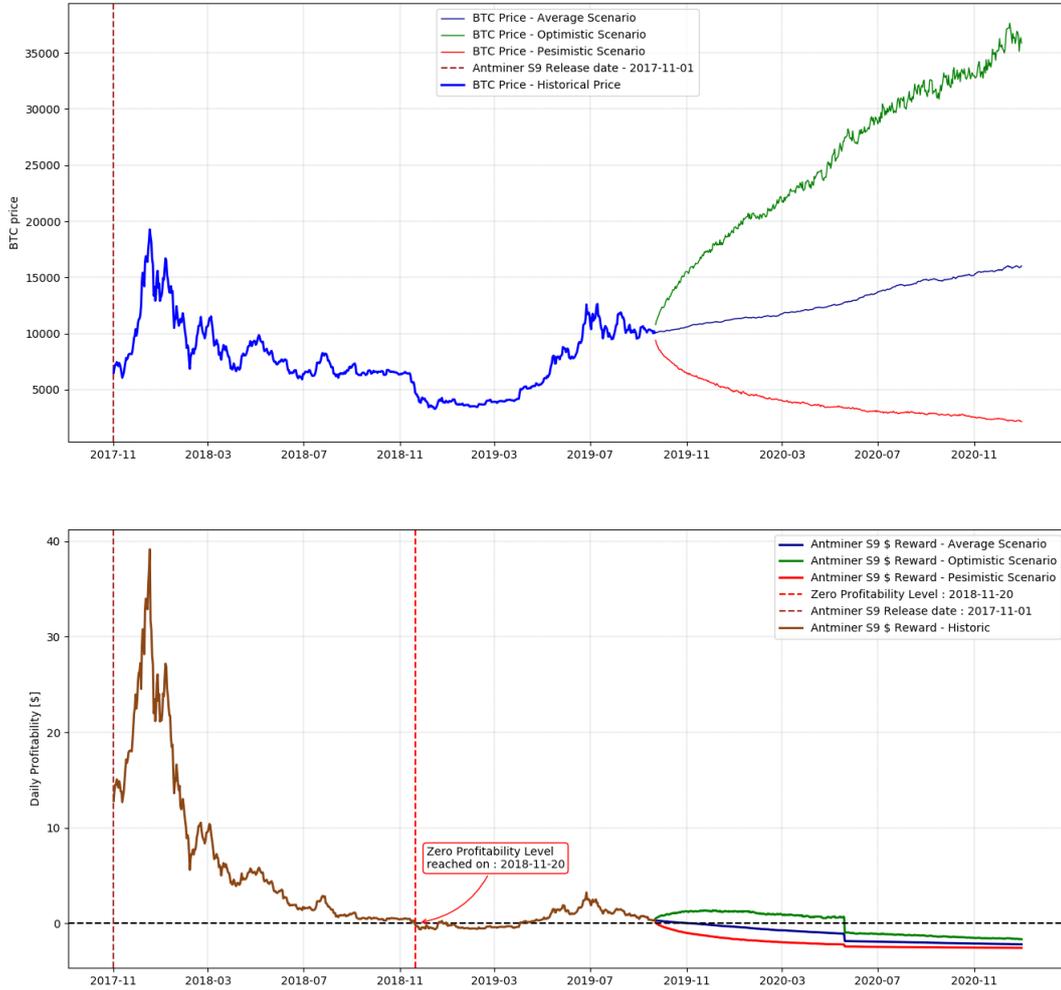


Fig. 1.10 Mining profitability Simulation estimated in US dollar for Antminer S9

Figure 1.11 shows the evolution of the mining profitability estimated in US dollar for Antminer S11. The profitability reaches a peak towards the July 2019 and becomes negative in the last quarter of 2019. Under the optimistic Bitcoin price scenario the Antminer S9 is slightly profitable until May 2020. Under the other two scenarii the operation is not profitable.

1.5.3.1 Mining equipment payback likelihood

If we account for the price of the equipment and the daily profitability it is easy to determine the payback time of the equipment and the cumulative profitability. We assume here γ is equal to one, meaning that the equipment is operating continuously without any break.

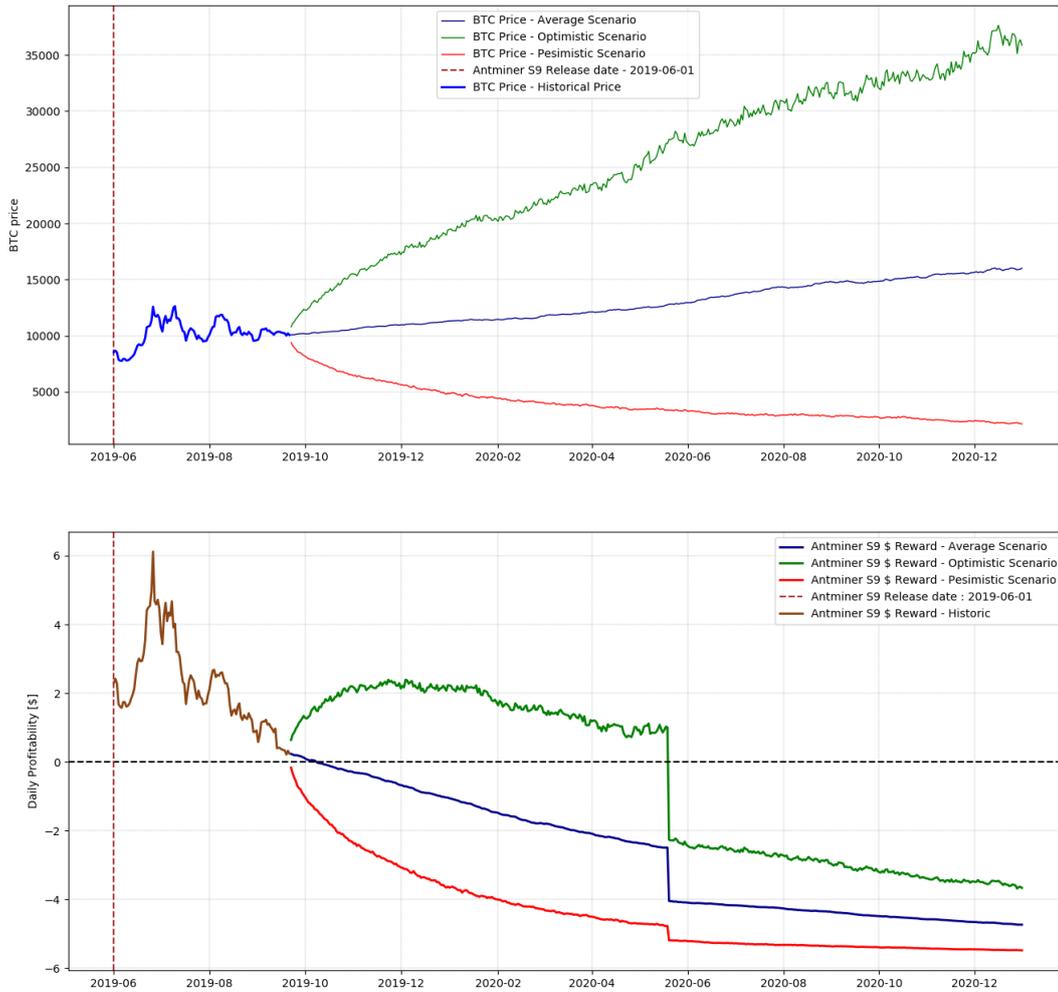


Fig. 1.11 Mining profitability Simulation estimated in US dollars for Antminer S11

Figure 1.12 shows the cumulative mining profitability for the Antminer S7. The payback period is 5 months. The cumulative profitability reaches its peak in May 2019. Under the optimistic and average price scenarios, the equipment remains profitable in the long run. Under the pessimistic scenario, the equipment burns the cumulative profits in the long run. Figure 1.13 shows the cumulative mining profitability for the Antminer S9. The payback period is 6 months. The cumulative profitability reaches its peak in May 2018. Under the three price scenarios, the equipment remains profitable in the long run.

Figure 1.14 shows the cumulative mining profitability for the Antminer S11. The equipment does not payback. The simulated cumulative profitability reaches its peak in May 2020. Under the three price scenarios, the equipment is not profitable in the long run.

Table 1.8 shows the abandon price S_a for the three types of mining equipment

Table 1.8 Abandon price S_a for the three types of mining equipment

Mining Equipment	Release Date	Breakeven Date	Abandon Date	Abandon price S_a
Antminer S7	2015-09-16	2016-09-03	2018-05-22	8224
Antminer S9	2017-11-01	2018-04-06	2019-04-11	5126
Antminer S11	2018-11-01	Never	2020-09-02	11508

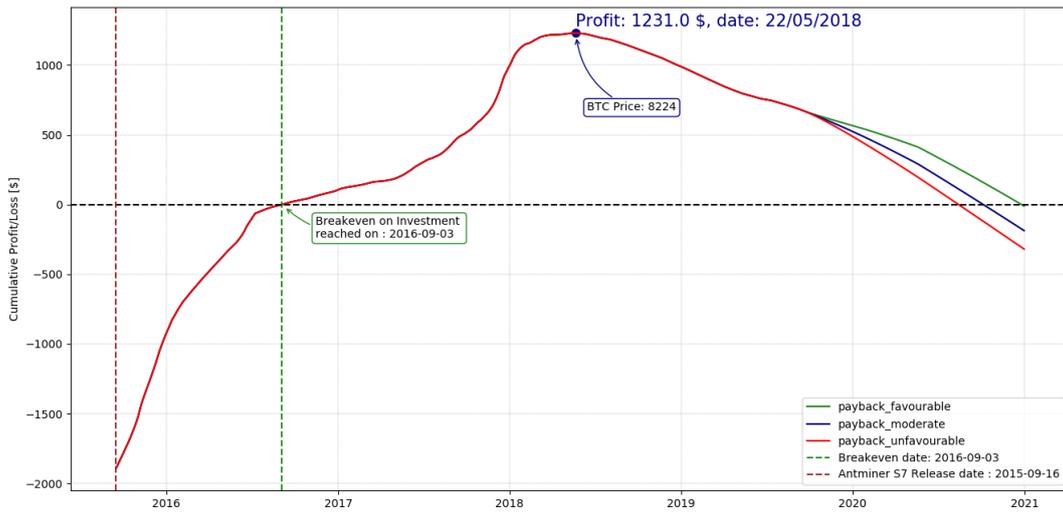


Fig. 1.12 Cumulative mining profitability for Antminer S7

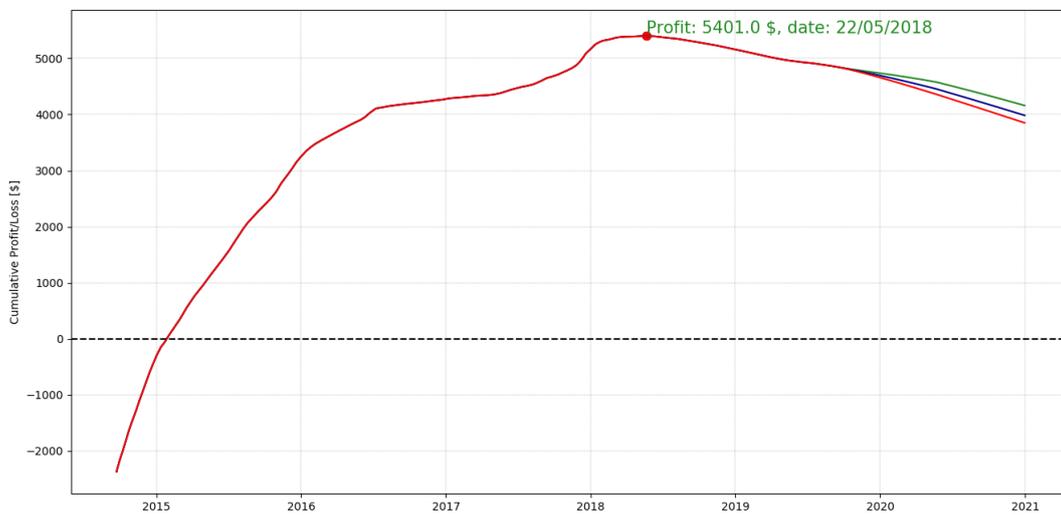


Fig. 1.13 Cumulative mining profitability for Antminer S9

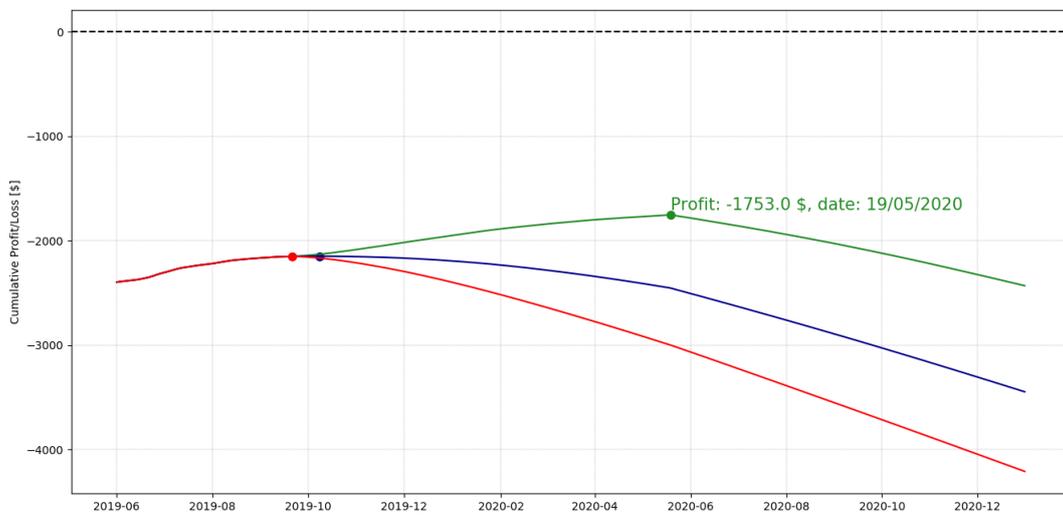


Fig. 1.14 Cumulative mining profitability for Antminer S11

1.6 Conclusions

This chapter explores the profitability of Bitcoin mining using the real options theory. The research addresses the problem of a Bitcoin miner and proposes a model that simulates the fundamental mining reward in order to predict the mining difficulty, evaluate the hardware efficiency and measure the likelihood of breakeven on initial investment. First, our findings conclude that bitcoin mining activity has transformed from fast-payback investment scheme nourished by the hype and social euphoria, to more of an utility business. Second, results show that variations of Bitcoin price have a delayed or no impact on the mining difficulty. This proves that miners exhibit an irrational behavior when it comes to adjusting their business strategy in different economic cycles of Bitcoin. Moreover, the results show that investing in latest ASIC miners is not as profitable and sustainable as before the market crash in December 2017, provided the continuous exponential increase in mining difficulty albeit low prices of Bitcoin. Last but not least, the most popular crypto-centric question today is how is the next BTC block reward going to affect Bitcoin price, however this is not the subject of this research.

Chapter 2

Bubbles on Altcoins: Rush versus Manipulation

1

*What really matters is an awareness of how greed and fear can drive rational people to behave in strange ways when they gather in the marketplace. **Joseph de la Vega**, *Confusion of Confusion* [de la Vega (1668)]*

Abstract *The aim of this chapter is to explore the bubble effects on different crypto-currencies. Bubbles are generated by investors' urge to step-in a promising market and by price pumping trades. The main goal of this study is to assess the presence of bubble effects in this market with customized tests able to detect the timing of various bubbles. We analyze the evolution of a representative sample crypto-currencies over time encompassing both high and low liquidity coins. The results show that several crypto-currencies prices had episodes of rapid inflation in 2017 related to the Bitcoin bubble and a few emerging coins saw their prices pumped by speculative actions.*

2.1 Introduction

When in 1971 Nixon administration liberated the US dollar from the Breton Woods' covenant which implied a monetary mass backed by gold, many economist predicted the beginning of country's economic decline. Nixon's idea that dollar is backed by confidence, remained one of America's fundamental doctrine. And yet investors had appetite for a currency backed by gold and the opportunity came with the Internet era in the early 2000 when digital gold currencies made surface. Most of those second generation digital currencies like iGolder, gbullion and e-gold, were in fact electronic money backed by one ounce of gold which were stored for a fee. Their legacy was short as the companies that ran those currencies were either shut down by the Federal Government for various crimes or faded away die to heavy regulatory burdens ([Frunza (2015)]).

A new revolution in digital currencies took place on 3rd January 2009, when Satoshi

¹The research presented in this chapter was submitted to the 14th International Conference on Computational and Financial Econometrics (CFE 2020) and is published as a working paper [Haliplii *et al.* (2020a)]. An initial study ran in parallel was published as in peer-reviewed conference proceedings: "Bubbles on Bitcoin Price: The Bitcoin Rush" Haliplii R., Guegan D. 2019 [Guegan *et al.* (2020c)].

Nakamoto created the first Bitcoin by generating the first block of the chain hashing using his personal computer. Since then, from a necessary activity to sustain the blockchain network operation, the Bitcoin mining has become a new type of business with a constantly increasing interest

Despite a negative and reluctant reception from part of the public opinion, cryptocurrencies are without any doubt the main financial innovation, since the credit derivatives. Many libertarian economics see this new 'virtual' currency as the new Holy Grail of a 21st century global economy trapped in a long recovery post-crisis scenario. Its advocates pledge for its advantages as sources of progress in the electronic economy and also from democratizing the global trade and the access to currencies. Bitcoin, from far the most popular made surface in 2013 (Figure 2.2) when its exchange rate with the US dollar rallied from almost nothing to 1,000 dollars for one Bitcoin, thereby being most likely the first virtual financial bubble.

Bitcoin hit the peak of its popularity in the end of 2017 when its exchange rate with the US dollar rallied on some exchanges to almost 20,000 US dollars. It suddenly became a mainstream subject of portfolio diversification and a potential effective hedge against systemic risks for traditional financial assets. At some point, Bitcoin was even attributed the status of digital gold. [Dyhrberg (2016)]

Bitcoin did not break the dream level of 100k USD and will most likely not reach it in the foreseeable future. Nevertheless, even the biggest critics of cryptos did not expect that Bitcoin to survive and to trade at a price 7 times higher than in it did in 2014. Those critics have preached that regulation, taxation, and many other menaces will hammer once and for all the last nail in the coffin of cryptos. But, it seems that those views are far from becoming a reality and both critics and sycophants and advocates would need another decade to understand the nature of Bitcoin and crypto-assets in general. The new decade starts in an environment of global political torment, which will push investors to look for safe harbor investments. And this might be in the long run the main catalyst of cryptocurrencies.

After Bitcoin a myriad of crypto-currencies saw the light with different range of success. Some of them followed Bitcoin's pattern, while some had their own direction. Altcoins traded on thinner liquidity compared to Bitcoin and the younger coins were in many cases targeted by market pumpers, aiming to extract easy profits.

The main motivation however behind this study is to assess the sharp explosions in cryptocurrencies prices that occurred between 2014 and 2020. The research aims to identify the coins that followed the 2017 Bitcoin rush and those that exhibit bubbles due to market manipulation.

This research enriches the literature related to financial bubbles and explores the sudden

price inflation on a relevant sample of crypto-currencies. The main results are around the testing for the presence and the timing of bubbles in crypto-currencies prices in USD. The chapter is organized as follows:

Section 2.2 discusses from a qualitative point of view the bubble phenomena in financial markets,

Section 2.3 describes the methodology for bubble testing introduced by [Phillips *et al.* (2013)],

Section 2.4 explains the particularities of bubbles related to investors's rush and to market pumping,

Section 2.5 presents the results of bubbles tests,

Section 2.6 concludes.

2.2 Bubbles on financial markets

The development of the stock market in Amsterdam in the early seventeen century, brought the first historical mentions about bubbles ([Frunza (2015)]). In 1668 Joseph de la Vega wrote the book " *Confusion of Confusions*" ([de la Vega (1668)]) which is the earliest detailed recollection describing the operations of modern stock market and the mechanism of bubbles. The presentation of the various mechanisms of bubble creation which included margin trading, short selling, manipulation were presented as a dialog between a merchant, a shareholder, and a philosopher ([Held (2006)]). De la Vega's book was written between two episodes that marked the early financial markets: the *tulip mania* and the South Sea company bubble.

Tulip market in the Netherlands along with rice market in Japanese Dojima were the first futures markets. Tulips were brought in Holland in the late sixteen century and the tulip bulbs were traded in the main Dutch cities. With a shrinkage in supply due to a disease that affected the tulips the price started to increase and *tulip mania* erupted and peaked during the winter of 1636-37, when some bulbs were reportedly valuing more than precious metals. In February 1637, tulip bulb contract prices collapsed and the trade stopped, generating massive losses for many of the speculators.

South Sea Company had a small trading business since the early 1700 with Spanish colonies. When the war between Spain and Holland started in 1718, the company developed a scheme which converted government debt in South Sea stock, thereby reducing Dutch governments' debt. At that moment a bubble kicked off and the South Sea Co. created many investing vehicles and in parallel many companies got listed in order to profit from the bubble. Thus more than 50 companies rose by 100% to 800% in less than a year and then lost nearly all of their gains within two months ([Frehen *et al.* (2013)]). Directors of South

Sea Company used illegal methods like bribery and misrepresentation in order to inflate the stock. Also South Sea stock bubble was a fertile ground of propagation for many other stock frauds. In September 1720, the bubble exploded and prices dropped massively in only few days, thereby exposing the fraud behind the South Sea Company. Figure 2.1 shows the evolution of the stock price for South Sea Company and other associated vehicles as well as its influence on other stock prices like the Old East India Company and Bank of England.

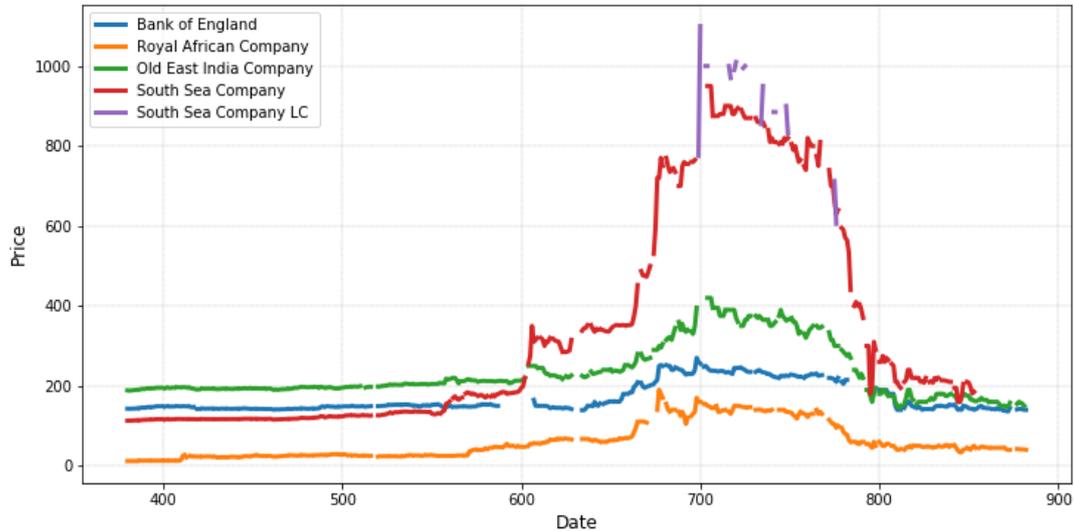


Fig. 2.1 South Sea bubble: Evolution of the stock price from South Sea company and other associated vehicles as well as its influence on other stock prices like the Old East India Company and Bank of England (Source [Frehen *et al.* (2013)])

Having fair conditions and homogeneous information for all players of financial markets it is only a prerequisite for achieving efficiency in price discovery. Even in the ideal case when traders and investors have the same information about the “*dice box*”, there are many other factors that bring inequality and favor one or other player². First is the access to liquidity, as a small number of investors are able to inject significant amount of funds in a very short time window in a market. Second is the structural role in a market of a player; many small investors tend to follow the actions of a larger or more reputed investor. Also a high number of small investors can synchronize their actions in order to distort the price signal and thereby creating a mini-bubble; this action can appear as a genuine price evolution to other players.

The business culture plays also a role, which becomes more crucial as markets become more globalized. The theory of efficient market hypothesis is often studied and probably was developed in the spirit of markets from developed countries, like the United States which

²The most fundamental principle of all in gambling is simply equal conditions, e.g. of opponents, of bystanders, of money, of situation, of the dice box, and of the die itself. To the extent to which you depart from that equality, if it is in your opponents favour, you are a fool, and if in your own, you are unjust. Girolamo Cardano, *Liber De Ludo Aleae (Book on Games of Chance)*([Cardano (2006)])

has one of the most transparent economies. Nevertheless in some emergent markets, like crypto-currencies the investment culture is less transparent, many listed currencies having scarce information available to investors.

2.3 Testing for bubbles

Since the South Sea Company frenzy in the early eighteenth century, financial markets faced many bubbles and as many crashes, the Black Tuesday from 1929 being one of the most dramatic one.

The features of financial bubbles are explored the academic literature. [Zhao (2014)] studied the unusual and puzzling stock price performance of USEC Inc., a company specialized in producing enriched uranium for nuclear plants. In July 2013 the stock price surged as much as ten times during merely sixteen trading days without apparent value-changing information being released and the hypothesis of market manipulation and speculative bubbles are analyzed.

[GENG and LU (2014)] studied bubble-creating stock attacks, an interesting form of market fraud which is a mixture of manipulation and speculative bubble, in which speculators implicitly coordinate to pump up the stock price without any significant fundamental news in order exploit behavioral-biased investors. The research provided empirical evidence in the Chinese stock market underlining that stocks with low mutual fund ownership and stocks with high average purchase costs of existing shareholders are more likely to be attacked.

[Johansen *et al.* (1999)] presented a synthesis of all the available empirical evidence in the light of recent theoretical developments for the existence of characteristic log-periodic signatures of growing bubbles in a variety of markets.

Few straightforward methods for testing a market for bubble are proposed by the recent works of Peter Phillips ([Phillips *et al.* (2013)] and [Phillips *et al.* (2011)]). These approaches come with enhanced versions of the Augmented Dickey-Fuller (ADF) test ([Dickey and Fuller (1979)] and [Said and Dickey (1984)]): Sup ADF test and Generalized Sup-ADF test.

The testing procedure for the Augmented Dickey-Fuller test for a unit root in timeseries is based on the model:

$$y_t = \alpha + \beta y_{t-1} + \gamma_1 \Delta y_{t-1} + \dots + \gamma_p \Delta y_{t-p} + \varepsilon_t, \quad (2.1)$$

where p is the lag order and $\varepsilon_t \propto N(0, \sigma_t)$.

[Phillips *et al.* (2013)] improved the basic version of the ADF test with recursive approach that involving a rolling window ADF style regression implementation. If the rolling window regression sample starts from the r_1^{th} fraction of the total sample and ends at the r_2^{th} fraction of the sample, where $r_2 = r_1 + r_w$ and r_w is the fractional window size of the regression. The

empirical regression model can then be written as :

$$y_t = \alpha_{r_1, r_2} + \beta_{r_1, r_2} y_{t-1} + \gamma_{r_1, r_2}^1 \Delta y_{t-1} + \dots + \gamma_{r_1, r_2}^p \Delta y_{t-p} + \varepsilon_t, \quad (2.2)$$

where α_{r_1, r_2} is the intercept, β_{r_1, r_2} the coefficient on a time trend and p the lag order of the autoregressive process computed on the window $r_1 T, r_2 T$. Under this circumstances the unit root null hypothesis is $\mathbf{H}_0 : \beta = 1$ and the explosive root right-tailed alternative hypothesis is $\mathbf{H}_a : \beta > 1$. The ADF statistic (based on this regression is denoted by $ADF_{r_1}^{r_2}$ [Phillips *et al.* (2011)] where ADF_0^1 is the ADF statistics for the full sample. Right sided unit root tests are informative about explosive or submartingale behavior³ in the timeseries and can be used in speculative bubble detection.

The Sup-ADF test introduced in ([Phillips *et al.* (2011)]) for single bubble detection is searching for the maximum value of the test, for all forward looking the windows on given sample. The window size r_w varies from the smallest sample window noted r_0 to 1. In terms of the formalism in equation (2.2) the starting point r_1 is 0; and the end point so r_2 is chosen such as the statistic $ADF_0^{r_2}$ is maximized, fact that can be written as

$$SADF(r_0) = \underbrace{\sup}_{r_2 \in [r_0, 1]} ADF_0^{r_2} \quad (2.3)$$

A step further improvement of the Sup-ADF test is the Generalized Sup-ADF (GSADF) leveraging the idea of repeatedly running the ADF test regression on subsamples of the data in a recursive fashion. Thus, in addition to varying the end point of the regression r_2 from r_0 to 1, the GSADF test allows the starting point r_1 in to change 0 to $r_2 - r_0$. The GSADF statistic searches for the biggest ADF statistic over all possible starting points and all possible window lengths.

$$GSADF(r_0) = \underbrace{\sup}_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \quad (2.4)$$

The bottom line of this test is to search for period where the prices exhibit consistently exponentially increasing trajectories.

2.4 Mechanisms of Bubbles

Two main mechanisms of bubble creation on crypto-currencies markets are studied in the following sections. The first mechanism concerns the rapid price inflation related to a sudden rush of investors to invest in a coin. This rush is explained by the psychological behavior of fear of not missing out. The fear of not missing a good investment opportunity push investors to irrational behavior. Thus a sudden inflow of liquidity leads to bubble formation.

The second mechanism concerns a group of speculators trying to extract profit from pumping the price. The spread of over-optimistic news about a coin leads the layperson to

³It should be recall that a discrete-time submartingale is a price time series y_1, y_2, y_3, \dots is satisfying $\mathbf{E}[y_{n+1} | \Phi_n] \geq y_n$, Φ_n being the filtration with all information at moment when the price is y_n

invest and create momentum. Ultimately, the speculator sells when the price touches its peak.

2.4.1 *Bubbles and investors' rush*

In a previous study [Guegan and Frunza (2018)] showed that Bitcoin price went through multiple bubbles, the most relevant being the one that took place in 2017. The cryptocurrency world generated a Bitcoin rush, translated not only in a high number of new comers in the crypto-world, but also in a bubble of Bitcoin's value. As of 2017, the total number of Bitcoin wallets users raised to around 10 millions compared to only 80 thousands in early 2013.

Table 2.1 **Testing for bubbles:** Sup-Augmented Dickey-Fuller and Generalized Sup-Augmented Dickey-Fuller have both statistics above the 95 % critical value thereby rejecting the null hypothesis of a no bubble episode in the considered Bitcoin USD times series

Test name	Statistic	Critical value (95%)
Sup ADF test	27.56	0.99
Generalized Sup ADF test	27.56	1.92

The assessment of the formation and propagation of a bubble in markets can be exert through many ways. The results obtained from applying this bubble detection approach to the Bitcoin-US dollar rate daily returns is exhibited in Table 2.1. The critical values for a 95 % confidence level are the asymptotic values communicated in [Phillips *et al.* (2013)].

Figure 2.2 shows the evolution over time for the statistics of two tests and indicates the corresponding timing of the bubbles. The Sup-Augmented Dickey-Fuller test indicates three bubbles, two during 2013 and one in 2017. The Generalized Sup-Augmented Dickey-Fuller test indicates same results.

Bubbles are periods when markets change dramatically their features and also give a positive and test biased signal to behavioral investors. During bubble periods market prices are far away from fundamentals and investors may take irrational decisions. Bitcoin is no exception.

[Guegan and Frunza (2018)] identified the episodes of Bitcoin bubble resulting from the Phillips test (Table 2.2). The main bubble episodes were in November 2013 - January 2014, when Bitcoin peaked at USD 1,151, and May 2017 - January 2018, when Bitcoin peaked at USD 19,498.68.

2.4.2 *Bubbles and fraud on the market*

Bubbles are also related to fraud on financial markets. Bubbles occur in many cases when a security is the object of a manipulation or a fraud. Penny stock scams, microcap fraud and pump and dump schemes have things in common with a 'bubble' phenomenon, as in all

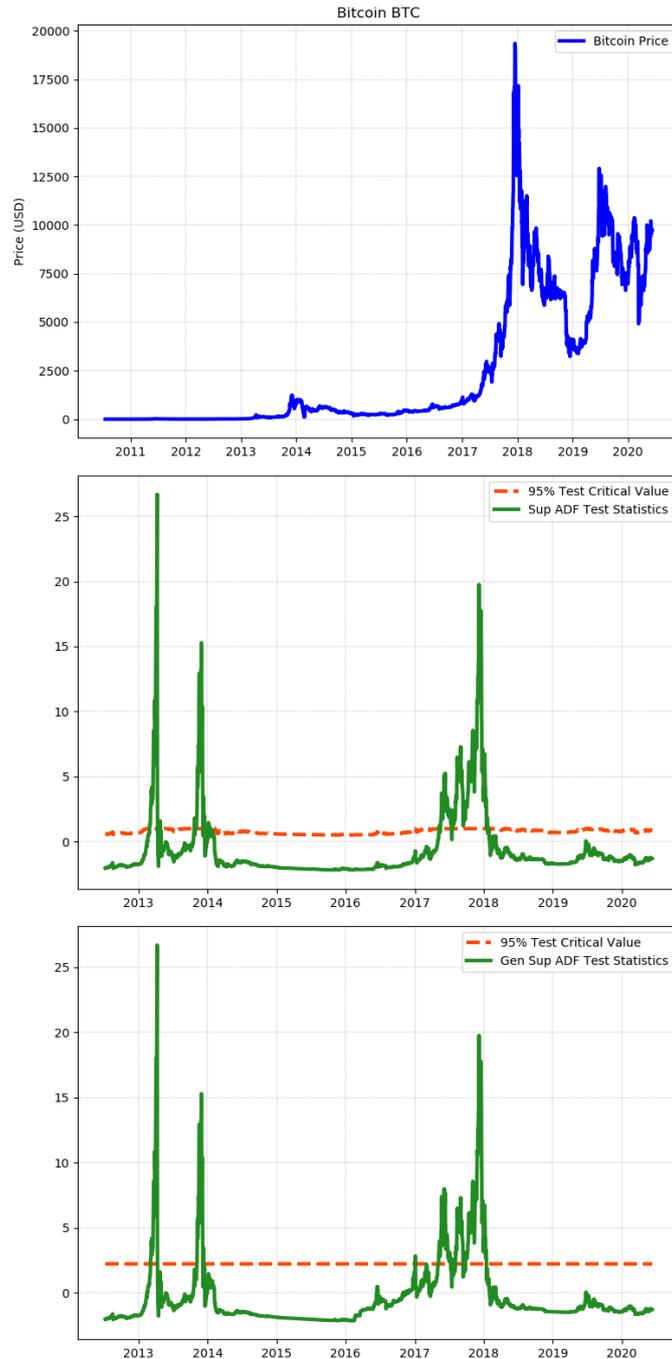


Fig. 2.2 **Bubble detection tests on Bitcoin prices: The Sup-Augmented Dickey-Fuller test indicates two bubbles during 2013 and during 2017. The Generalized sup augmented Dickey-Fuller test indicates same results, two bubbles during 2013 and one during 2017.**

these cases the price of a security get inflated far beyond its fundamental 'fair value', and this inflation is accelerated by other investors which will buy the security thereby boosting the exponential rise of the price. The common points of manipulation and bubbles are mentioned also in the literature. [Zhao (2014)]studied the unusual and puzzling stock price performance of USEC Inc., a company specialized in producing enriched uranium for nuclear plants. In July 2013 the stock price surged as much as ten times during merely sixteen trading days without apparent value-changing information being released.

Table 2.2 **Timeline of Bitcoin bubbles:** The main bubble episodes are in November 2013 - January 2014 and May 2017 - January 2018

Bubble timeline	Initial Price	Peak Price
2011-02-01 / 2011-02-18	USD 0.95	USD 1.1
2011-04-23 / 2011-06-13	USD 1.7	USD 35.00
2013-03-05 / 2013-03-05	USD 40.04	USD 237.00
2013-11-06 / 2014-01-11	USD 258.23	USD 1,151.00
2017-05-07/ 2018-01-21	USD 1,560.41	USD 19,498.68

Few other examples are presented below to illustrate this phenomenon.

A first example is the *Halliburton vs Fund* case which started in 2002 and went through various phases until late 2014. Halliburton is a Texas based provider of diverse products and services mainly in the oil and gas industry.

In the original case⁴ Halliburton was charged for issuing a series of false and misleading statements materially changing its revenue recognition policy that recognized revenue on claims and change orders relating to cost overruns which its clients had not approved. The misrepresentations artificially inflated Halliburton's reported revenues and earnings, thereby artificially inflating the price of Halliburton securities. On May 28, 2002, after the close of the market, Halliburton issued a press release announcing that the SEC is conducting an investigation into its accounting for cost overruns and its market price dropped by 3.3% in one day on extremely heavy trading volume.

Figure 2.3 exhibits the results of the density forecast test and bubble testing. The density forecast test which benchmarks the forecasting capacity of Normal versus Normal Inverse Gaussian distributions shows that during the class period the stock price returns were characterized by heavy tails. Sup-Augmented Dickey-Fuller and Generalized Sup-ADF (GSADF) tests indicate that a bubble has occurred during the class period, but also in the post-class period.

A second example is Aegerion Pharmaceuticals, which is a biopharmaceutical company founded in 2005 and listed on NASDAQ, engaged in the development and commercialization of novel therapeutics to treat debilitating and fatal rare diseases. Its main product is Juxtapid capsules, an adjunct to a low-fat diet and other lipid-lowering treatments.

A class action was filed against Aegerion¹¹ because they allegedly made false and/or misleading statements, as well as failed to disclose important adverse facts about marketing its drugs in violation of the FDCA. The rise of the stock started in early 2013 based on the marketing announcements for the new drugs. On November 8, 2013, news reports revealed that the Company received an FDA warning letter about the fact that the new drug Juxtapid

⁴<http://securities.stanford.edu/filings-case.html?id=102452>

¹¹BODNER vs AEGERION PHARMACEUTICALS
http://securities.stanford.edu/filings-documents/1051/AEGR00_01/2014115_f01c_14CV10105.pdf

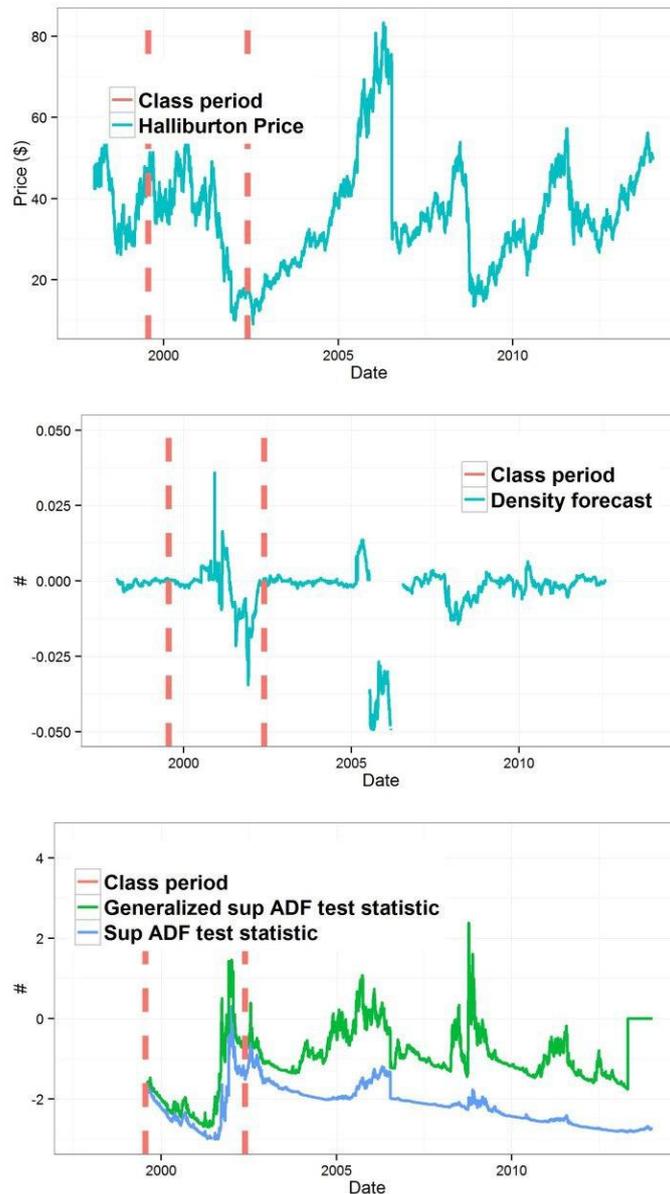


Fig. 2.3 **Halliburton case:** **First chart-** Evolution of the Halliburton stock price. **Second chart:** Density forecast benchmarks the forecasting capacity test normal versus normal inverse Gaussian. The test shows that during the class period the stock price returns were characterized by heavy tails. The same behavior is observed during the 2008 crisis. **Third chart-** Bubble tests Sup-Augmented Dickey-Fuller and Generalized Sup-ADF tests indicate that a price explosion occurred during the class period, but also in the post-class period.

was mislabeled and its distribution violated the law.

Figure 2.4 indicates that the two procedures Sup-ADF and Generalized Sup-ADF applied to the Aegerion stock confirm that a bubble occurred during the class action period between May and November 2013. On January 10, 2014, the Company received a subpoena from the U.S. Department of Justice requesting documents regarding its marketing and sale of Juxtapid. Aegerion shares declined \$7.98 per share.

A third example is that of Galena Biopharma, a biotechnology firm concerned by a class action. Figure 2.5 displays the application of the bubble detection test for the litigation

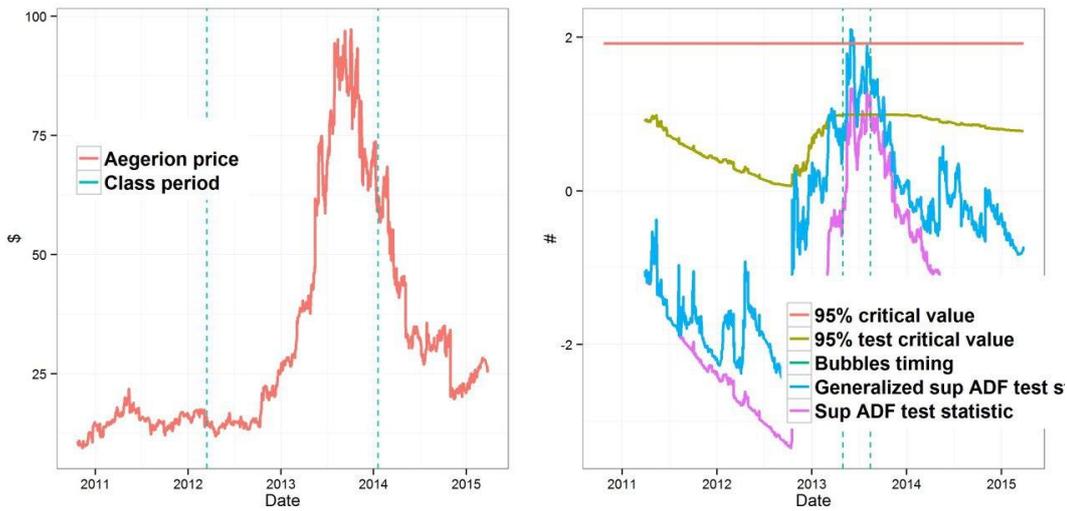


Fig. 2.4 **Aegerion case:** The two procedures Sup-ADF and GSADF applied to the Aegerion stock confirms that a bubble occurs during the class action period between May and November 2013.

case, involving Galena Biopharma, which allegedly misrepresented few facts of its activity, thereby inflating the price of its stock⁵. Both tests confirm the presence of a bubble during the alleged class period.

2.5 Application to top 50 Altcoins

This study extends the bubble testing applied to Bitcoin prices to the top 50 coins in terms of liquidity. Table 2.3 depicts a snapshot of top 50 coins in terms of liquidity as of 15th of June 2020.

Rank	Name	Symbol	Price (\$)	MarketCap (Mln \$)	MarketCap (%)	MarketCap (Cumul%)	Volume (Mln \$)	Volume (%)	Volume (Cumul%)
1	Bitcoin	BTC	9423.8	173433	64.66	64.66	17718	27.78	27.78
2	Ethereum	ETH	237.9	26488	9.88	74.54	7198	11.28	39.06
3	Tether	USDT	1.0	9193	3.43	77.97	22098	34.64	73.70
4	XRP	XRP	0.19	8496	3.17	81.14	1054	1.65	75.35
5	Bitcoin Cash	BCH	240.4	4433	1.65	82.79	1268	1.99	77.34
6	Bitcoin SV	BSV	182.0	3356	1.25	84.04	904	1.42	78.76
7	Litecoin	LTC	44.7	2905	1.08	85.12	1783	2.79	81.55
8	Binance Coin	BNB	16.6	2595	0.97	86.09	206	0.32	81.87
9	EOS	EOS	2.5	2419	0.90	86.99	1415	2.22	84.09
10	Cardano	ADA	0.07	2029	0.76	87.75	168	0.26	84.35
11	Tezos	XTZ	2.6	1962	0.73	88.48	78	0.12	84.47
12	Crypto.com Coin	CRO	0.10	1875	0.70	89.18	43	0.07	84.54
13	Stellar	XLM	0.07	1496	0.56	89.74	397	0.62	85.16
14	Chainlink	LINK	4.07	1427	0.53	90.27	267	0.42	85.58
15	UNUS SED LEO	LEO	1.24	1241	0.46	90.73	15	0.02	85.60
16	Monero	XMR	65.86	1159	0.43	91.16	54	0.08	85.68
17	TRON	TRX	0.01	1107	0.41	91.57	1379	2.16	87.84
18	Huobi Token	HT	4.2	942	0.35	91.92	82	0.13	87.97
19	Neo	NEO	10.9	772	0.29	92.21	362	0.57	88.54

⁵Michael E. Deering, et al. v. Galena Biopharma, Inc., et al. <http://securities.stanford.edu/filings-case.html?id=105188>

20	Ethereum Classic	ETC	6.4	750	0.28	92.49	657	1.03	89.57
21	USD Coin	USDC	1.0	734	0.27	92.76	246	0.39	89.96
22	Dash	DASH	73.3	700	0.26	93.02	399	0.63	90.59
24	Maker	MKR	553.8	557	0.21	93.46	20	0.03	90.64
25	Cosmos	ATOM	2.8	548	0.20	93.66	106	0.17	90.81
26	VeChain	VET	0.009	523	0.20	93.86	256	0.40	91.21
27	HedgeTrade	HEDG	1.7	507	0.19	94.05	1	0.00	91.21
28	Zcash	ZEC	51.7	485	0.18	94.23	219	0.34	91.55
29	NEM	XEM	0.04	406	0.15	94.38	12	0.02	91.57
30	Ontology	ONT	0.56	397	0.15	94.53	83	0.13	91.70
31	Basic Attention Token	BAT	0.23	340	0.13	94.66	80	0.12	91.82
32	Dogecoin	DOGE	0.002	314	0.12	94.78	187	0.29	92.11
33	OKB	OKB	5.16	310	0.12	94.90	93	0.15	92.26
34	DigiByte	DGB	0.02	299	0.11	95.01	30	0.05	92.31
35	FTX Token	FTT	2.87	271	0.10	95.11	2	0.00	92.31
36	Paxos Standard	PAX	1.00	245	0.09	95.20	183	0.29	92.60
37	0x	ZRX	0.36	239	0.09	95.29	69	0.11	92.71
38	Zilliqa	ZIL	0.02	232	0.09	95.38	113	0.18	92.89
39	OMG Network	OMG	1.64	231	0.09	95.47	112	0.18	93.07
40	Kyber Network	KNC	1.2	218	0.08	95.55	88	0.14	93.21
41	THETA	THETA	0.2	217	0.08	95.63	55	0.09	93.30
42	Synthetix Network Token	SNX	1.12	212	0.08	95.71	2	0.00	93.30
43	Decred	DCR	16.7	195	0.07	95.78	89	0.14	93.44
44	Hedera Hashgraph	HBAR	0.04	189	0.07	95.85	6	0.01	93.45
45	ICON	ICX	0.33	184	0.07	95.92	29	0.05	93.50
46	Algorand	ALGO	0.23	181	0.07	95.99	20	0.03	93.53
47	Qtum	QTUM	1.76	171	0.06	96.05	323	0.51	94.04
48	Augur	REP	15.02	165	0.06	96.11	21	0.03	94.07
49	Binance USD	BUSD	1.001	164	0.06	96.17	51	0.08	94.15
50	Enjin Coin	ENJ	0.18	156	0.06	96.23	11	0.02	94.17

Table 2.3: Snapshot of Top 50 coins ranked by liquidity as of 15th of June 2020

The Sup-ADF test and the Generalized Sup-ADF are applied for each coin to the full history of exchanges rates relative to the US dollars. The results are presented in Table 2.4 and indicate the maximal of the statistics for each test and the critical value. The test outcome indicates whether a bubble occurred or not in the history of that coin.

Coin name	Coin abr.	Sup ADF test			Generalized Sup ADF test		
		Statistic	Critical value	Bubble	Statistic	Critical value	Bubble
Bitcoin	BTC	26.7	0.99	YES	26.7	2.21	YES
Ethereum	ETH	14.77	0.99	YES	14.77	2.21	YES
XRP	XRP	21.33	0.99	YES	21.33	2.21	YES
Crypto.com	CRO	2.48	0.99	YES	4.34	2.21	YES
Tether	USDT	-11.5	0.01	NO	-4.06	2.21	NO
Bitcoin Cash	BCH	-1.67	0.71	NO	0.22	2.21	NO
Homeros	HMR	-1.51	0.76	NO	-0.65	2.21	NO
Chainlink	LINK	6.18	0.99	YES	7.88	2.21	YES
PlayFuel	PLF	-0.61	0.97	NO	-0.18	2.21	NO
Aion	AION	-2.21	0.49	NO	0.82	2.21	NO
Hyperion	HYN	13.47	0.99	YES	14.05	2.21	YES
NewYork Exchange	NYE	0.37	0.99	NO	0.55	2.21	NO

Bitcoin SV	BSV	0.81	0.99	NO	3.59	2.21	YES
Litecoin	LTC	12.95	0.99	YES	13.61	2.21	YES
EOS	EOS	0.8	0.99	NO	0.98	2.21	NO
CTAGtoken	CTAG	-0.44	0.98	NO	1.05	2.21	NO
Binance Coin	BNB	0.57	0.99	NO	2.99	2.21	YES
Elamachain	ELAMA	0.62	0.99	NO	0.93	2.21	NO
Gaps Chain	GAPS	-0.01	0.99	NO	2.34	2.21	YES
Tezos	XTZ	-2.5	0.36	NO	4.58	2.21	YES
Huobi Token	HT	-0.63	0.98	NO	1.55	2.21	NO
Cardano	ADA	-1.43	0.81	NO	0.56	2.21	NO
HedgeTrade	HEDG	0.08	0.99	NO	2.26	2.21	YES
Okex	OKB	0.03	0.99	NO	3.85	2.21	YES
Stellar	XLM	23.27	0.99	YES	29.93	2.21	YES
SENSO	SENSO	-0.9	0.94	NO	0.55	2.21	NO
Monero	XMR	10.88	0.99	YES	10.88	2.21	YES
NEO	NEO	7.48	0.99	YES	7.48	2.21	YES
TRON	TRX	-2.38	0.41	NO	-0.47	2.21	NO
FTX Token	FTT	-1.47	0.8	NO	2.37	2.21	YES
Tap	XTP	-1.75	0.68	NO	0.4	2.21	NO
LEO Token	LEO	-0.36	0.99	NO	0.23	2.21	NO
Ethereum Classic	ETC	7.2	0.99	YES	7.7	2.21	YES
USD Coin	USDC	-4.66	0.01	NO	-2.48	2.21	NO
Dash	DASH	12.35	0.99	YES	13.19	2.21	YES
IOTA	MIOTA	-0.97	0.94	NO	-0.7	2.21	NO
Maker	MKR	-7.7	0.01	NO	-0.95	2.21	NO
Cosmos	ATOM	-0.57	0.98	NO	0.87	2.21	NO
ZCash	ZEC	-99.84	0.01	NO	1.96	2.21	NO
ZB	ZB	-0.52	0.98	NO	3	2.21	YES
VeChain	VET	-1.3	0.87	NO	-0.54	2.21	NO
NEM	XEM	19.57	0.99	YES	19.57	2.21	YES
BitMax Token	BTMX	-0.97	0.94	NO	1.33	2.21	NO
Ontology	ONT	-1.08	0.92	NO	-0.14	2.21	NO
Basic Attention Token	BAT	3.8	0.99	YES	3.8	2.21	YES
Quant	QNT	3.75	0.99	YES	4.24	2.21	YES
0x	ZRX	-0.97	0.94	NO	-0.51	2.21	NO
Reserve Rights Token	RSR	0.11	0.99	NO	3.88	2.21	YES
Dogecoin	DOGE	6.17	0.99	YES	11.82	2.21	YES
BitTorrent	BTT	1.49	0.99	YES	3.3	2.21	YES

Table 2.4: Results of the Sup augmented Dickey-Fuller test and the Generalized sup augmented Dickey-Fuller applied for each to the full history of exchanges rates relative to the US dollars.

The results support our initial intuition underlining two main reasons for bubbles: the investor rush in the initial day of the coin culminating with the 2017 Bitcoin bubble and the various momentum linked to idiosyncratic factors for various coins.

Most bubbles including those observed on Ether (Figure 2.13), Ripple (Figure 2.14), Litecoin (Figure 2.16), Stellar (Figure 2.17), Monero (Figure 2.18) Ethereum classic (Figure 2.20), NEO (Figure 2.19), Dash (Figure 2.21), NEM (Figure 2.22), Basic Attention Token (Figure 2.23) and Dogecoin (Figure 2.25) conflated with the Bitcoin rush from 2017 (Figure 2.2).

The other bubbles identified in several minor coins are explained by idiosyncratic factors described below, aiming to underline that alleged manipulation might have took place.

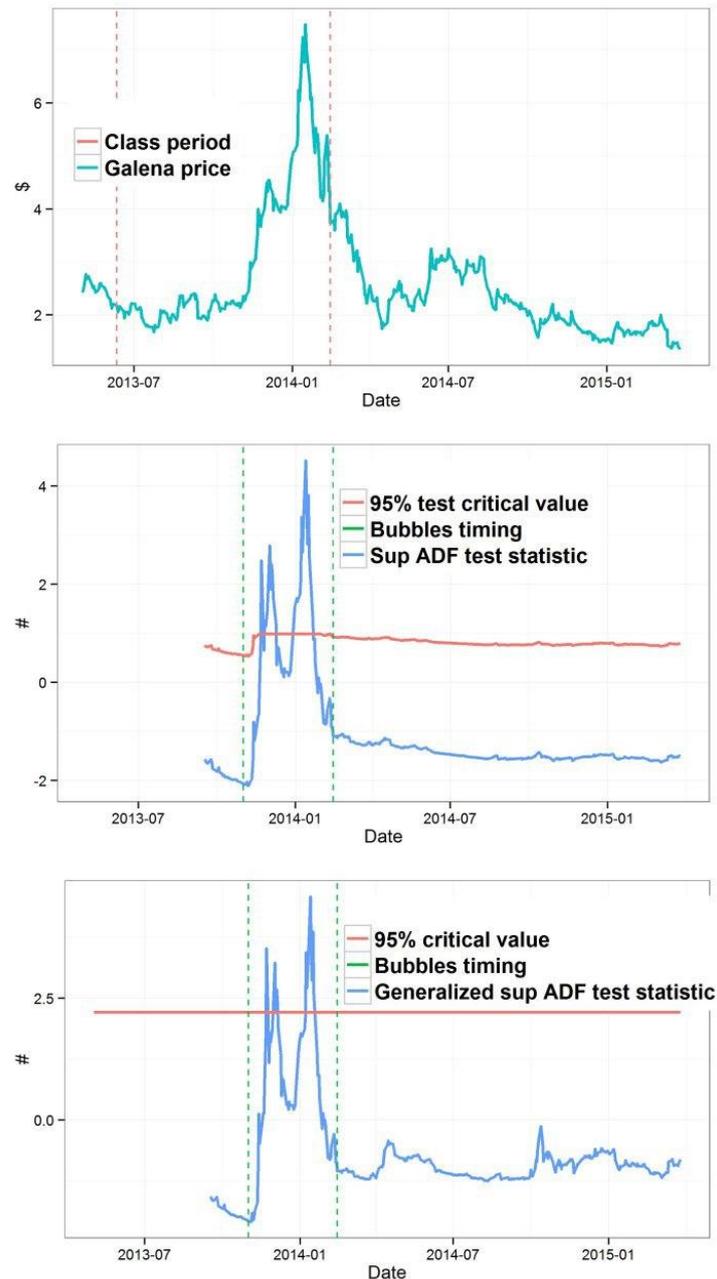


Fig. 2.5 **Galena Biopharma bubble.** The first graph shows the evolution of the stock price , the second shows the Sup-ADF test and the last graph shows GSADF test. Both tests confirm the presence of a bubble during the alleged class period.

2.5.1 *Bitcoin Satoshi Vision*

Bitcoin Satoshi Vision (BSV) moved into positive territory since mid-December 2019 with an increase of over 80 percent. BSV seemed to be linked to a recent document submitted to US court by, Craig Wright, the self-proclaimed inventor of Bitcoin. The Australian cryptographer may have seemingly controlled an investment trust holding 10 billion USD in Bitcoin. Speculators implied that he would also detain a significant amount in Bitcoin forks, thereby boosting the prices of Bitcoin Cash and Satoshi Vision. Figure 2.6 depicts the test results implying a bubble in early 2020.

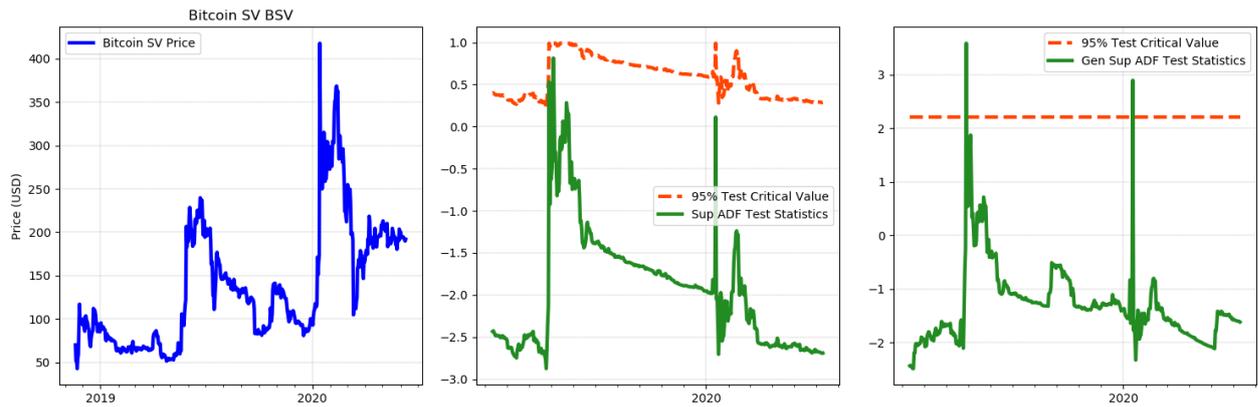


Fig. 2.6 Bubble detection tests on Bitcoin Satoshi Vision(BSV) prices

In February 2020, Bitcoin SV lost massively altitude after a one month rally accounting for a substantial 320% increase that pushed the price above 415 USD. But, this bubble was highly speculative and it seems that the price was seriously pumped by few speculators that spread the news that a certain Craig Wright may sit on a big stash of Bitcoin. Craig Wright was caught in an ongoing court battle in the United States and as a consequence, many documents were revealed to the public.

Furthermore, the self-proclaimed Bitcoin inventor, Satoshi Nakamoto's alter ego confirmed a few days later through official sources that they neither possess nor even claim to possess the access to the 8 billion USD of Bitcoin mined in the Bitcoin's early days. The announcement brought havoc on the BSV market and the price plunged by over 40% falling below 250 USD in only two days.

2.5.2 Tezos

Tezos is one of the altcoins that had a good start in 2020 despite the global market contraction. Tezos continued its expansion since the beginning of the year and increased its value by 95%. Tezos is one of the Altcoins that does not operate through a proof of work but through a proof of stake. Its founders claim that their blockchain has been facilitating the development of new security tokens and tokenization of projects lately. Figure 2.7 shows that a bubble occurred due to a big inflow of liquidity which is a fertile ground for "pump and dump" manipulation.

2.5.3 BitTorrent Token

BitTorrent Token (BTT) gained six times the price of its initial coin offering (ICO), which occurred on the 28th of January 2019⁶. Figure 2.8 shows the bubble that occurred shortly after the ICO took place. The price dropped over 2019 after the ICO rush.

⁶<https://www.coindesk.com/bittorrent-token-is-already-nearly-6-times-its-ico-price>

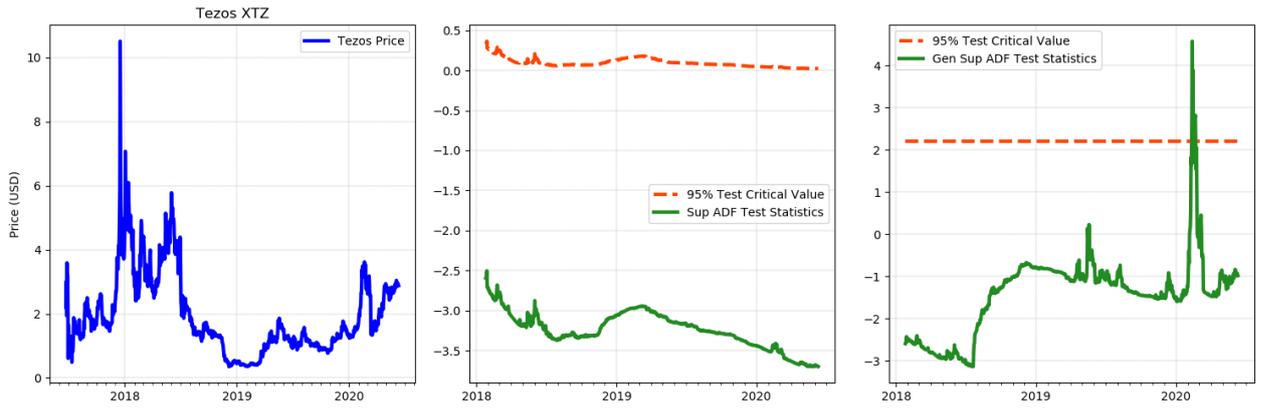


Fig. 2.7 Bubble detection tests on Tezos prices

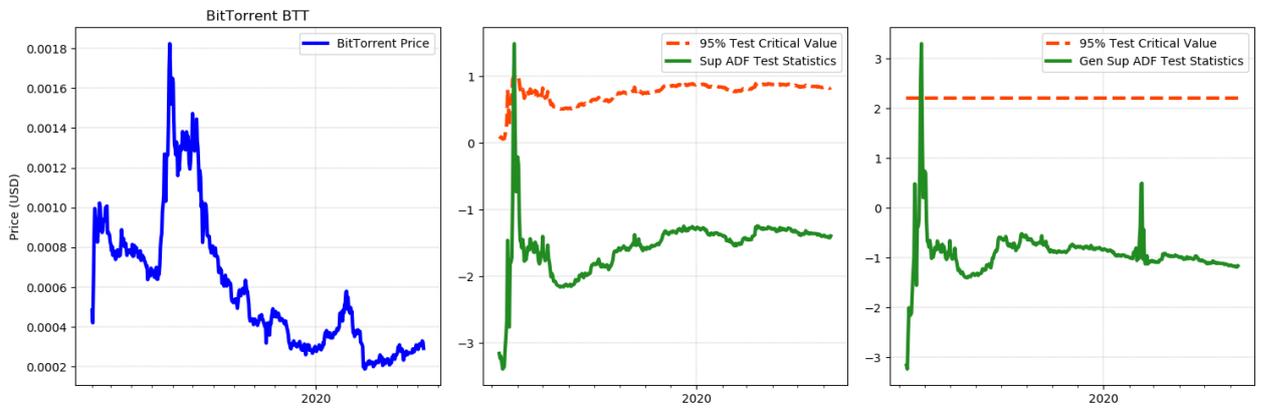


Fig. 2.8 Bubble detection tests on BitTorrent prices

2.5.4 *OKEx*

OKEx went through a bubble in 2020 (Figure 2.9). The exchange trading platform promoting its own coin was accused of market manipulation and fraud for the way it forced early settlement of Bitcoin Cash futures. The settlement happened the day before Bitcoin Cash split into two different currencies, and some traders claimed they lost thousands as a result. OKEx denied the wrongdoing.

2.5.5 *Binance*

Binance coin had big gains in 2019 (Figure 2.10), bubbling to its highest price in more than 15 months as anticipation surrounding Binance's platform latest developments. Binance was one of the platforms promoting ICOs in 2019, at the time when investors faded away due to suspicious firm rising money via this tool. It became a pioneer of IEOs (Initial Exchange Offerings), which are similar to old-school ICOs but the tokens/coins are offered and operated directly by the exchange.

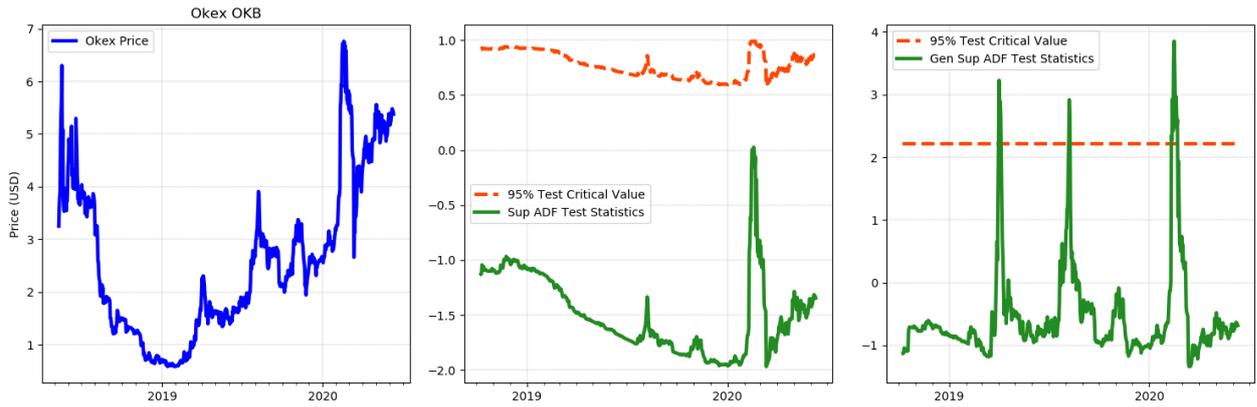


Fig. 2.9 Bubble detection tests on OKEx prices

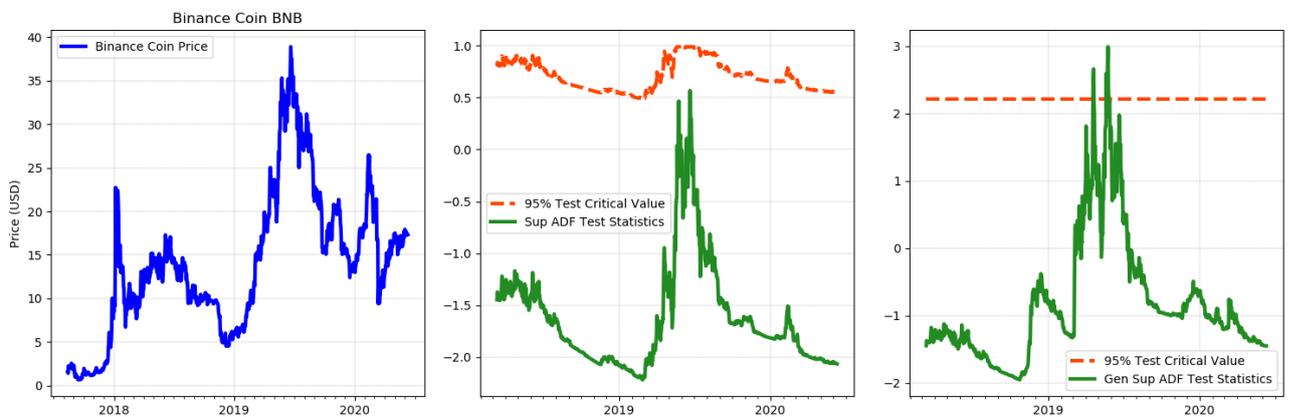


Fig. 2.10 Bubble detection tests on Binance prices

2.5.6 *Link*

LINK went through a bubble in 2019 (Figure 2.11) explained by the growing awareness of the importance of Chainlink for any smart contract that requires secure and reliable middleware to connect it to real world data. Chainlink had few interesting partnerships with big companies such as Google, Oracle, and SWIFT. Nevertheless, there was not enough fundamentals to explain the sudden price increase.

2.5.7 *Crypto.com*

CRO had a short bubble episode in 2020 (Figure 2.12). There were not much news behind this increase, but Wirecard, the supplier of cryptocurrency debit cards for Crypto.com, went through real problems earlier this year. The auditor of the company Ernst & Young discovered that \$2 billion were missing from company’s treasury and may be related to fraudulent transactions. This potentially explained the price collapse which followed.

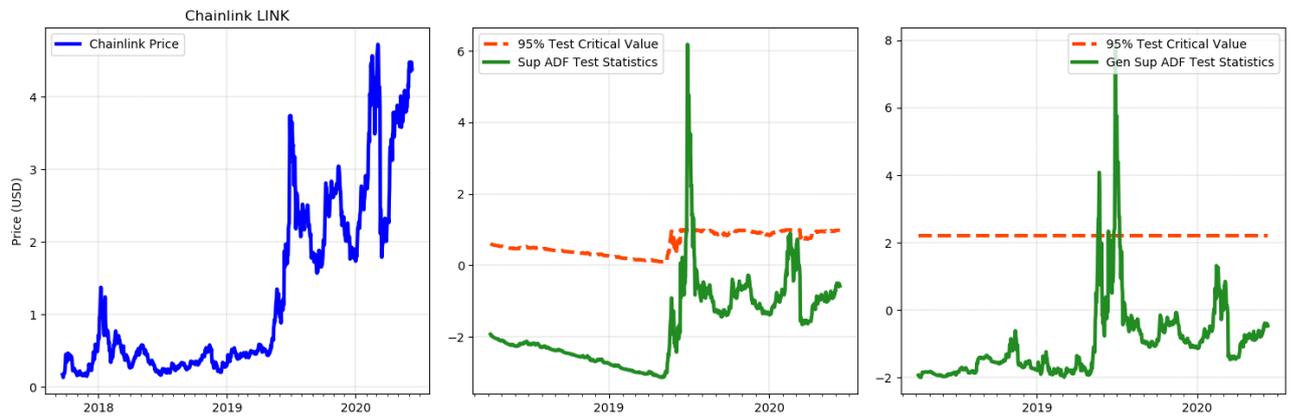


Fig. 2.11 Bubble detection tests on Link prices

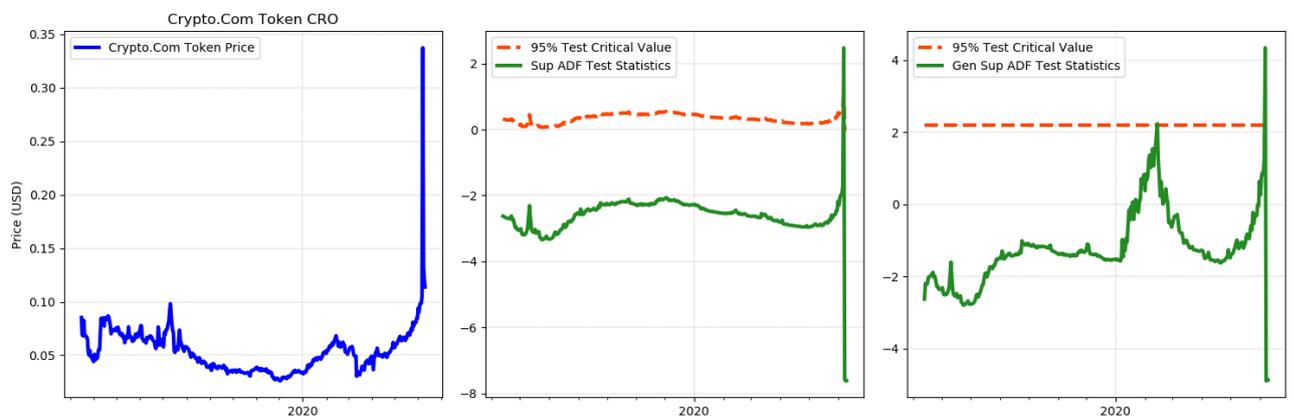


Fig. 2.12 Bubble detection tests on Crypto.com prices

2.6 Conclusions

This chapter explores the occurrence and the timing of bubbles in the top 50 cryptocurrencies. The Sup-Augmented Dickey-Fuller and the Generalized Sup-Augmented Dickey-Fuller tests were applied for each to the full history of exchanges rates relative to the US dollars. The obtained results support our initial intuition underlining two main reasons for bubbles: the investor rush in the initial day of the coin culminating with the 2017 Bitcoin bubble and the various momentum linked to idiosyncratic factors for various coins.

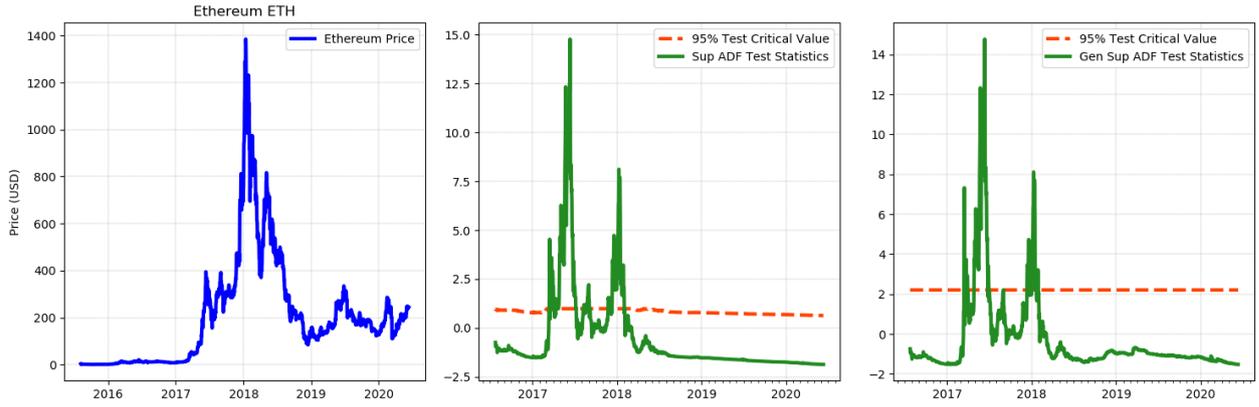


Fig. 2.13 ETH

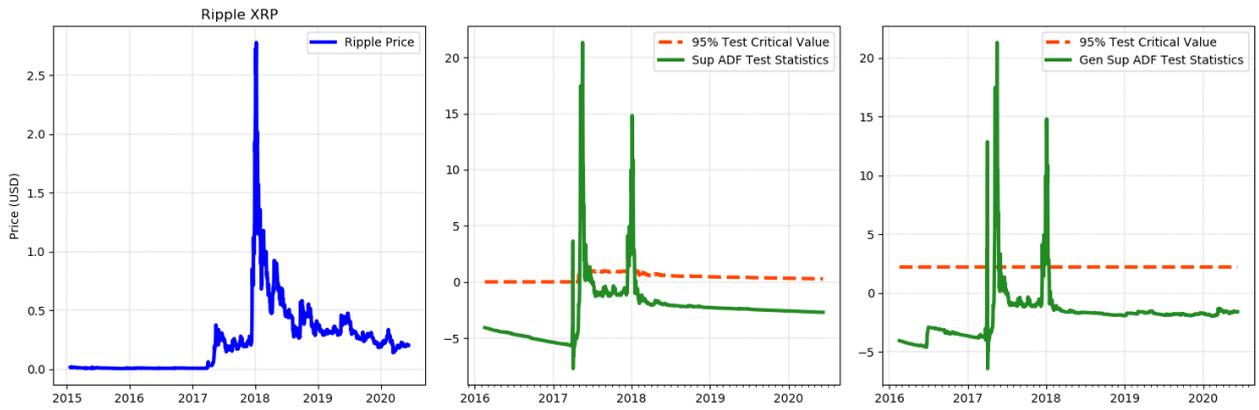


Fig. 2.14 XRP

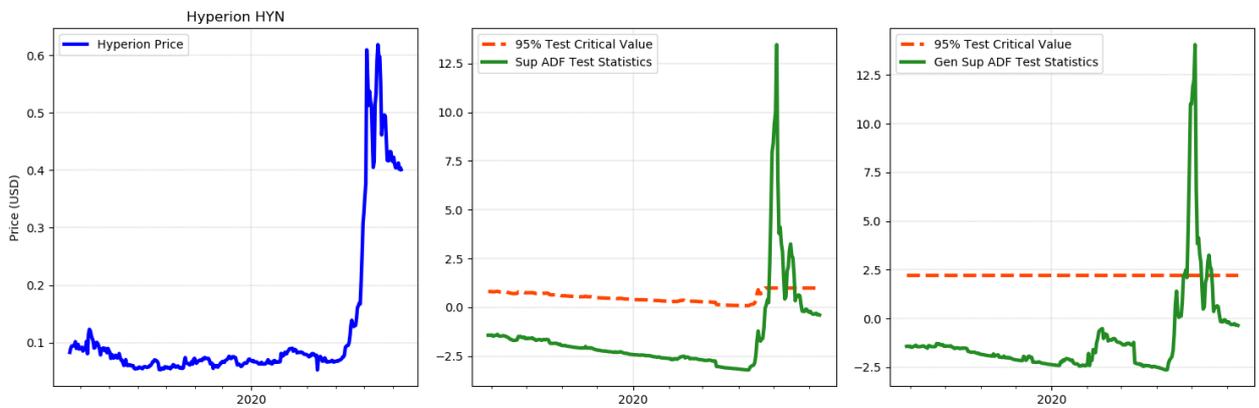


Fig. 2.15 HYN

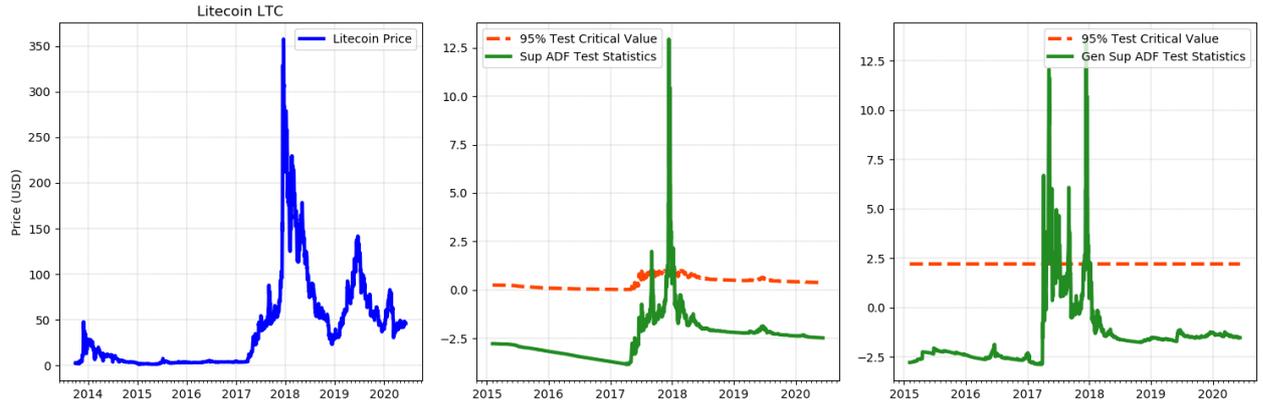


Fig. 2.16 LTC

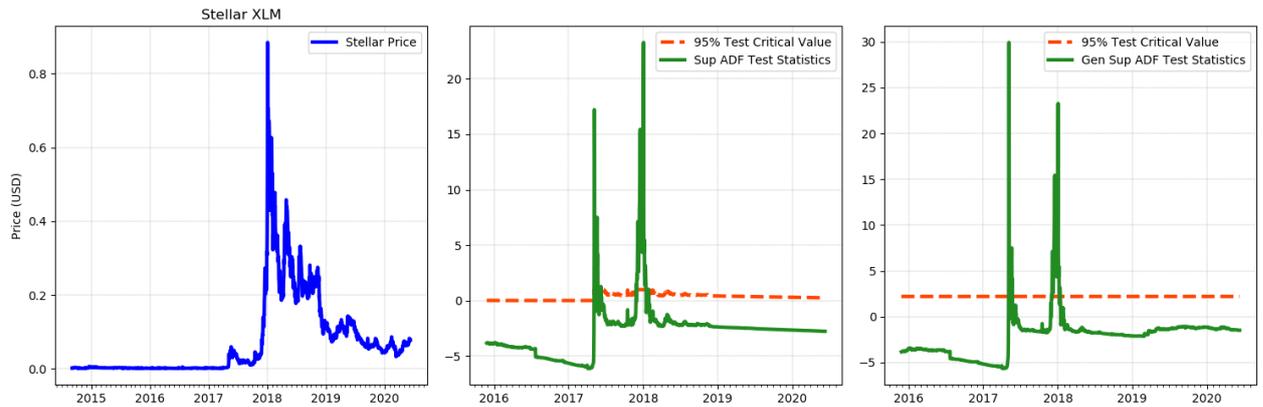


Fig. 2.17 XLM

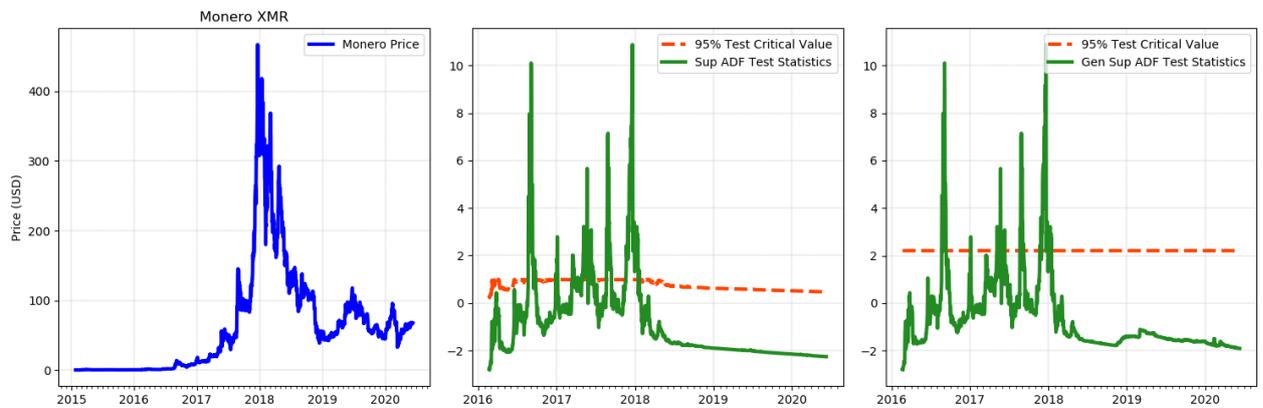


Fig. 2.18 XMR

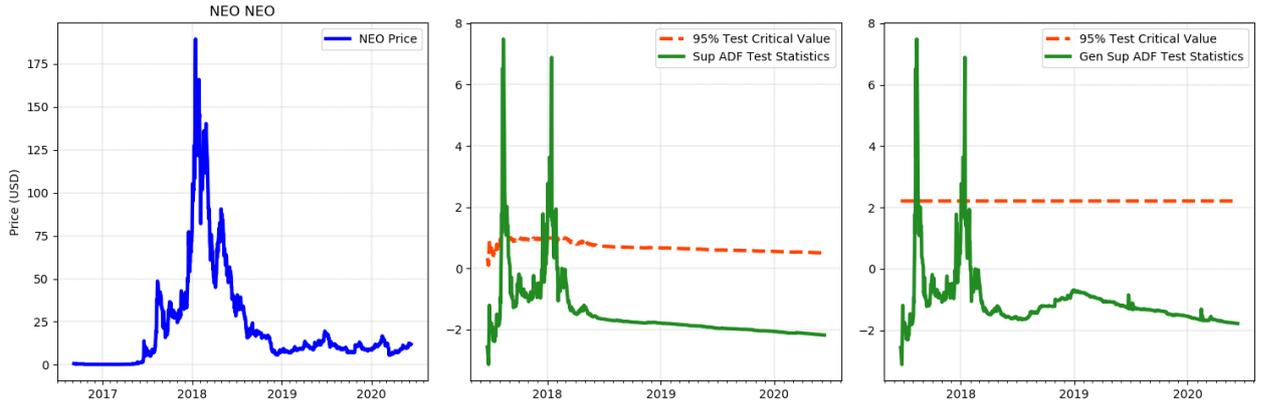


Fig. 2.19 NEO

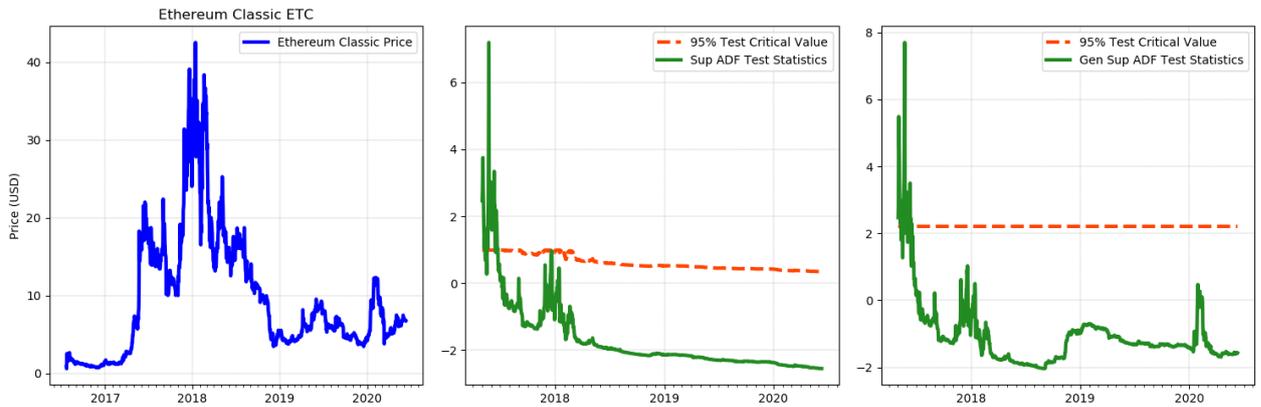


Fig. 2.20 ETC

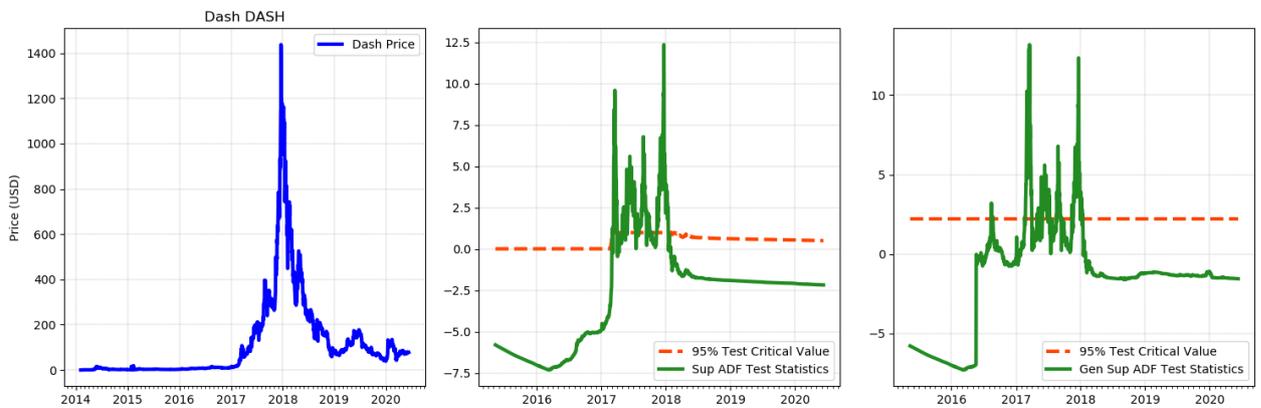


Fig. 2.21 DASH

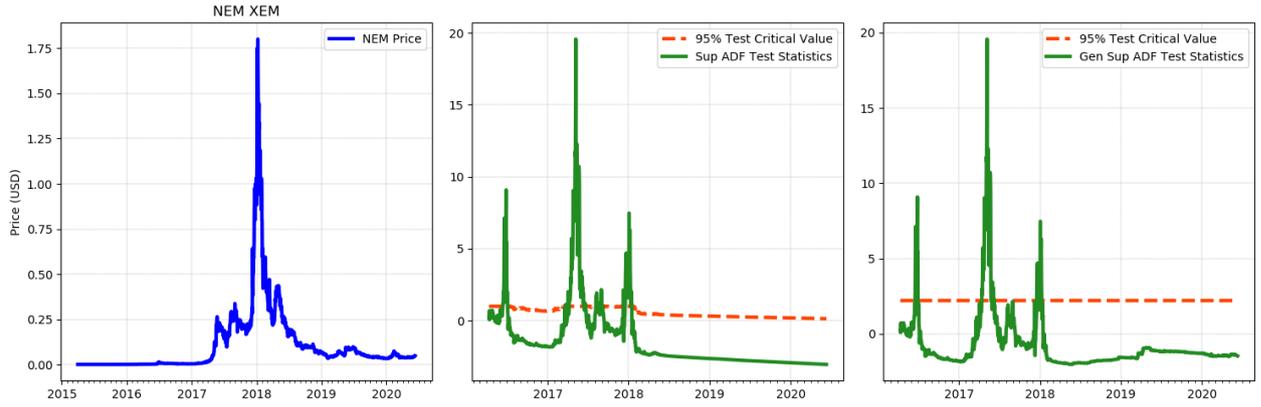


Fig. 2.22 XEM

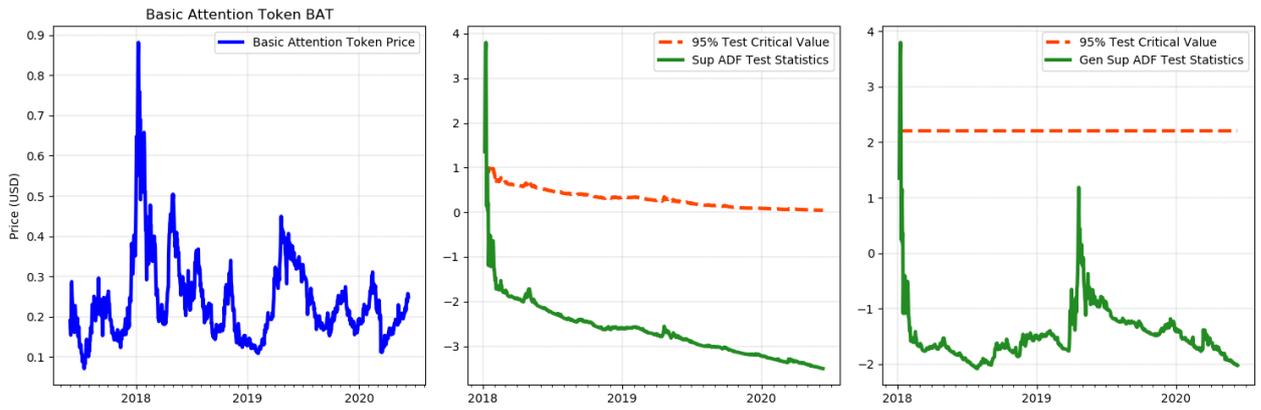


Fig. 2.23 BAT

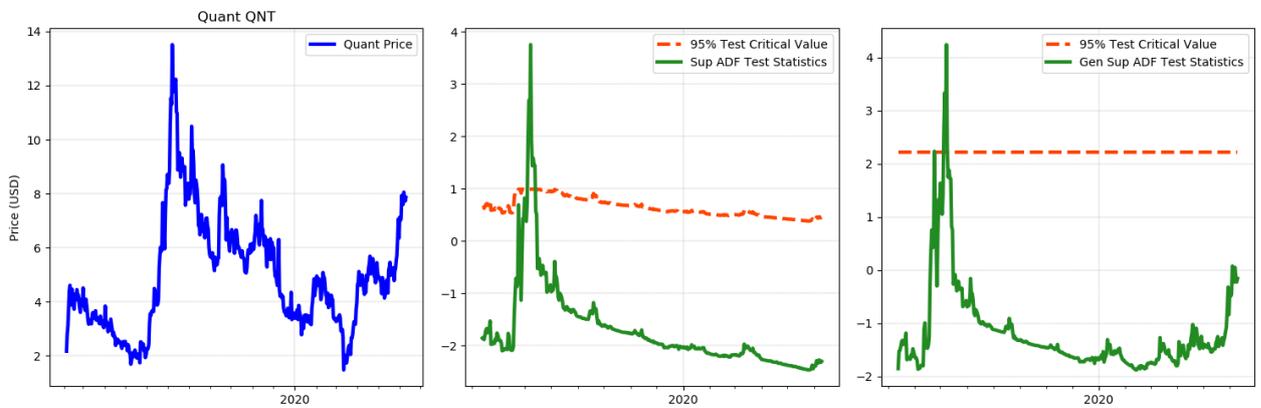


Fig. 2.24 QNT

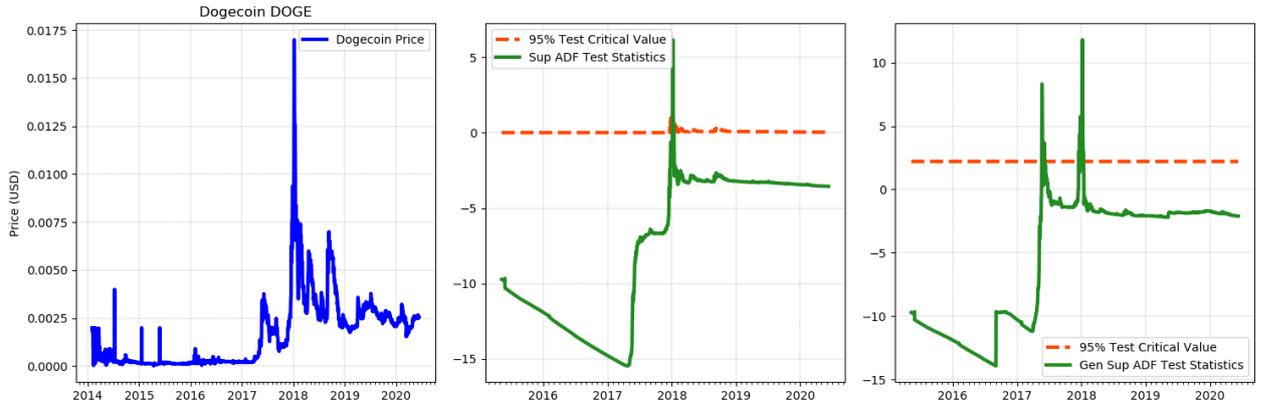


Fig. 2.25 DOGE

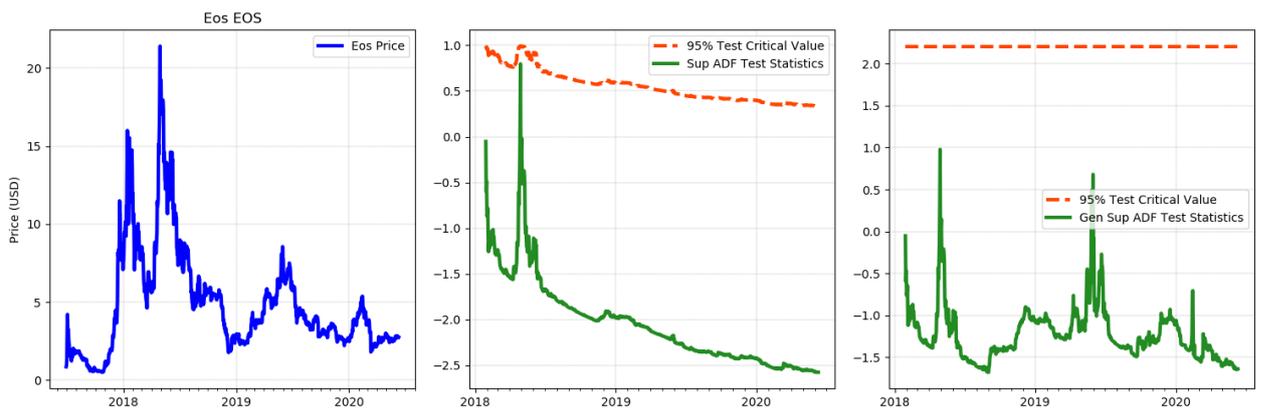


Fig. 2.26 EOS

Chapter 3

Proxy-hedging of Bitcoin exposures with Altcoins

1

It's not the employer who pays the wages. Employers only handle the money.

*It's the customer who pays the wages. **Henry Ford***

Abstract *This chapter explores the associated risks with hedging in crypto-currencies markets. It focuses on finding the most effective proxy hedge instrument for the Bitcoin-USD market. Due to its particularities, this market does not exhibit the same features as traditional financial markets do. In appearance it seems very related to other altcoins (alternative coins), but in reality it exhibits unusual volatility clustering effects. This behaviour has a direct impact on the hedging strategies of business exposed to crypto-currencies, including the hedge funds, mining farms or ICO projects. The paper explores the econometric features of Bitcoin and other Altcoins and underlines the need of fat tail distributions and volatility clustering models. Also it examines the density forecasting capacity of various proxy hedge instruments including Bitcoin, Bitcoin Cash and Ether exchange rates.*

3.1 Introduction

Bitcoin hit the peak of its popularity at the end of 2017 (Figure 3.4) when its exchange rate with the US dollar rallied on some exchanges to almost 20,000 US dollars. It suddenly became a mainstream subject of portfolio diversification and a potential effective hedge against systemic risks for traditional financial assets. At some point, Bitcoin has even attributed the status of digital gold ([Dyhrberg (2016)]).

The emergence of blockchain technology and the low correlation of cryptocurrencies with traditional financial markets has been a popular subject of debate in the academic landscape. [Dong and Weiguo (2014)] attempted to address these roles: whether Bitcoin is a real currency, and what its financial features are. Using daily data of the exchange rates quoted from the world primary Bitcoin dealers since the inception of Bitcoin and the spot market exchange rates, he calculated the triangle arbitrage asset price to decompose

¹The research presented in this chapter is published as a working paper: "Proxy-Hedging of Bitcoin Exposures With Altcoins", Guegan D., Halipili R. and Frunza M. 2020, [Guegan *et al.* (2020a)].

the features of this currency. The results suggest significant liquidity discount of Bitcoin and risk premium as a financial asset to British Pound Sterling (2.46%) and Chinese Yuan (0.3%). However, Bitcoin, as investment objectives instead of a currency unit, is associated with excess risk and low returns. Such poor performance discourages investors to spend Bitcoin as currency and to pursue the arbitrage profit.

[Pieters and Vivanco (2015)] examines the implied Bitcoin exchange rates between 18 currencies and the US dollar to official spot exchange rates, and find that they can vary greatly. He showed that this difference could be partially explained by government intervention in the formal markets. This finding implies that the bitcoin exchange rate can be used to evaluate the accuracy of the official exchange rate data, and provides a method to approximate the time trend of the black market exchange rate if such data is not available.

[Dyhrberg (2016)] set out to explore the hedging capabilities of Bitcoin by applying the asymmetric GARCH methodology used in analysis of gold returns. The results show that Bitcoin can be used as a hedge against stocks in the Financial Times Stock Exchange Index. Besides, Bitcoin can be used as a hedge against the American dollar in the short-term. Bitcoin thereby exhibits some of the same hedging abilities as gold and can be included in the variety of tools available to market analysts to hedge market-specific risk.

Nevertheless, following the launch of Bitcoin futures on CME in December 2017, the cryptocurrency market suffered a massive crash which has significantly undermined investors credibility the future investment potential of Bitcoin as an investment asset. In this sense, [Corbet and Vigne (2018)] concluded in their study that the introduction of Bitcoin futures has significantly increased the volatility in the spot market and that overall, they are not a useful hedging tool.

In a nutshell, most of the reviews published on the Bitcoin subject either explored its hedging potential or debated its future as a mainstream currency unit. Few however focused on the hedging strategies and challenges of this highly volatile asset.

This paper aims to enrich the scarce literature on the economics of Bitcoin and to explore the different challenges of Bitcoin proxy-hedging with different Altcoins. The main contribution of this research consists in showing that despite having various proxy-hedging alternatives on the Altcoin markets, Bitcoin cannot be fully hedged because the density forecasting capabilities of Altcoins are inadequate especially in the tail regions. The paper is organized as follows :

- **Section 2** explains the necessity and challenges of Bitcoin proxy-hedging.
- **Section 3** explores the econometric features of top cryptocurrencies, underlining the need of fat tailed models ,
- **Section 4** depicts the density forecasting methodologies and shows how can they be used to assess the proxy-hedging efficacy,

- **Section 5** presents the backtesting results of proxy-hedging with Altcoins
- **Section 6** concludes.

3.2 Cryptocurrency hedging

3.2.1 Why to hedge?

Most cryptocurrency investors prefer a "buy and hold" strategy and expecting that someday their portfolio would "go to the moon". Some market participants have to hedge their crypto-exposure for the following reasons:

- *Miners* need to pay their electricity and other costs in fiat currency (like USD), so they may wish to have more predictability on their returns. They could sell their crypto directly whenever they successfully mine new coins. However, it may be more beneficial for them to pool their cryptos and sell in one larger batch periodically to negotiate better fees and reduce individual transfer costs.
- *Initial Coin Offering (ICO)* projects usually incur costs in USD and require more predictable cash flows. However, they do not want to be seen selling out their tokens or crypto, as it could be a negative sign to investors/token holders. Hence they could opt to proxy-hedge some of the risks to ensure more predictable cash flow.
- *Hedge Funds* may employ strategies based on a return relative to Bitcoin. In this case, they would overlay their portfolio with a BTC equivalent hedge so that the returns they get are relative to BTC's performance.
- *Whales* or individuals who own large amounts of the digital currency cannot quickly sell significant amounts of cryptocurrency due to tight liquidity. Also, such a big sale might spread market fear. Hence an alternative is to proxy-hedge the position while slowly reducing the overall exposure in the long run.

3.2.2 How to hedge?

A significant challenge in "proxy hedging" ([Viken and Thorsrud (2014)]) consists in finding the proxy instrument which minimizes basis risk and hedge volatility. Below we introduce the most popular methods of hedging the crypto-exposure:

- *Futures*

Similar to traditional financial markets, there are cryptocurrencies futures which a trader can use to (proxy)-hedge out her position, as shown in Figure 3.1. Currently, the crypto market comes with two types of futures:

- *CME, CBOE* Monthly Futures: are traded in USD and settle in USD, the minimum contract size being 1 BTC on CBOE and 5 BTC on CME.
- *Bitmex* Quarterly Futures: are traded in USD, but settled in BTC, contract sizes

start from 1 USD, margin requirements are less strict than on regulated futures exchanges. However, the exchange default risk is higher.

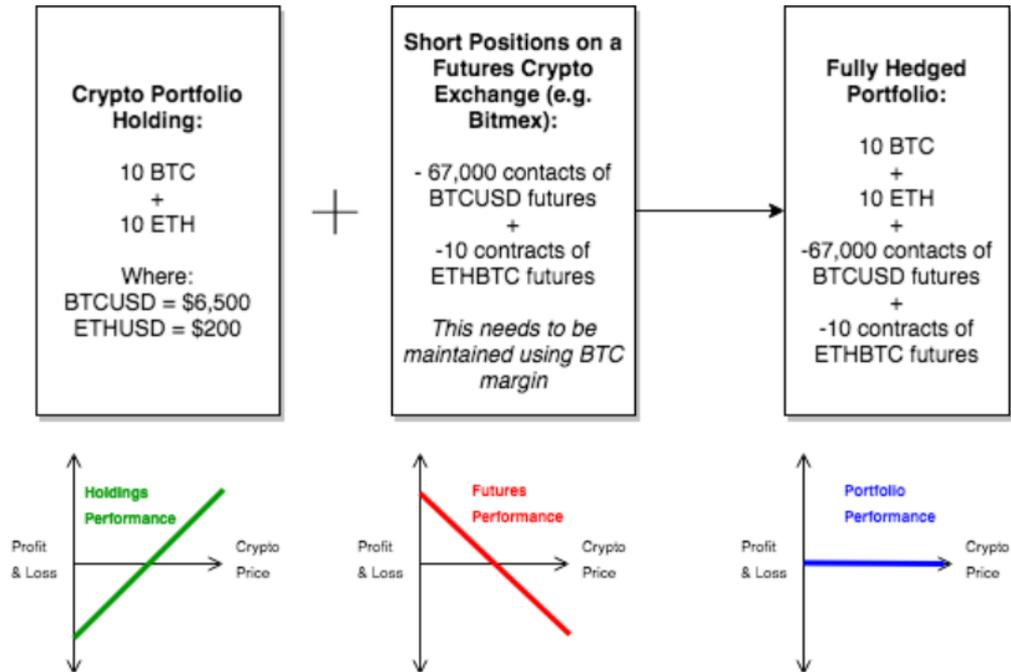


Fig. 3.1 Hedging Mechanism using Futures

Using futures for proxy-hedging may require lower daily margin funding than short-selling and may be profitable in certain market conditions (i.e. backwardation). In practice, it is an ineffective risk hedge ([Corbet and Vigne (2018)]) as futures markets are not liquid and are limited to a small number of significant coins (ie. Bitcoin, Ethereum, Ripple) and provide an additional layer of risk which is the basis risk.

- *Options*

Options in the cryptocurrency space are relatively new and limited in volume. The only exchanges that offer it are currently *Deribit* and *Bitmex*. Hedging with options can be pretty complicated, and there are multiple ways a trader can build the desired payoff. Below is one of the most straightforward ways to hedge out the downside risk by selling put options (Figure 3.2).

One of the main benefits of using options is the diverse payout profiles. Buying put options to hedge means limited downside with unlimited upside. The caveat of options in cryptocurrency space is the fact they are relatively expensive due to the volatile nature of the market. Therefore, one would have to pay a significant premium to have this optionality/payout structure, which makes this hedging solution not sufficient either. When proxy-hedging Bitcoin exposure with Altcoin options (or vice-versa) it is necessary to understand the relationship between the densities of the two coins returns,

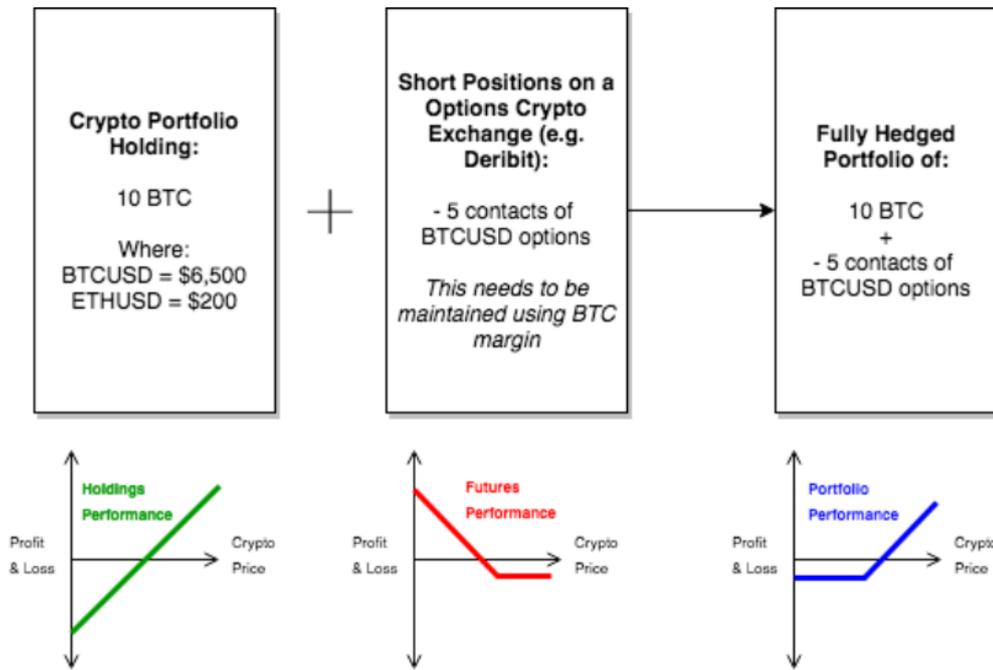


Fig. 3.2 Hedging Mechanism using Options

especially in the region of tails.

- *Perpetual Swaps*

Perpetual swaps (perpetuals) have recently grown in popularity as more and more crypto exchanges have started to offer them. Their use is very similar to that of inverse futures, the main differences being a periodic funding rate (usually 8 hours) and no expiry date. The key reasons to use this instrument are the same as for futures. Perpetual swaps are prices with short-term funding rate, thereby tracking closers the underlying prices than the futures for most cases. However, this makes the actual hedging cost less predictable (since funding rates are re-adjusted every 8 hours).

- *Short Selling*

The most popular and straightforward hedging method is to short-sell the crypto that one is looking to hedge (Figure 3.3).

Short selling as a proxy-hedging strategy is best suited for investors carrying a diverse range of cryptos that want to hedge using few coins for their entire portfolio. Compared to simple selling, short-selling implies an additional margin funding cost to the transaction fee. It also requires constant portfolio monitoring as various events common to crypto markets like exchange defaults, coin de-listings, airdrops and forks can break the hedge.

Nevertheless, it is the most convenient way to proxy-hedge in practice as all the funds are kept in the crypto wallet; it works on a broader variety of coins and is easy to implement.

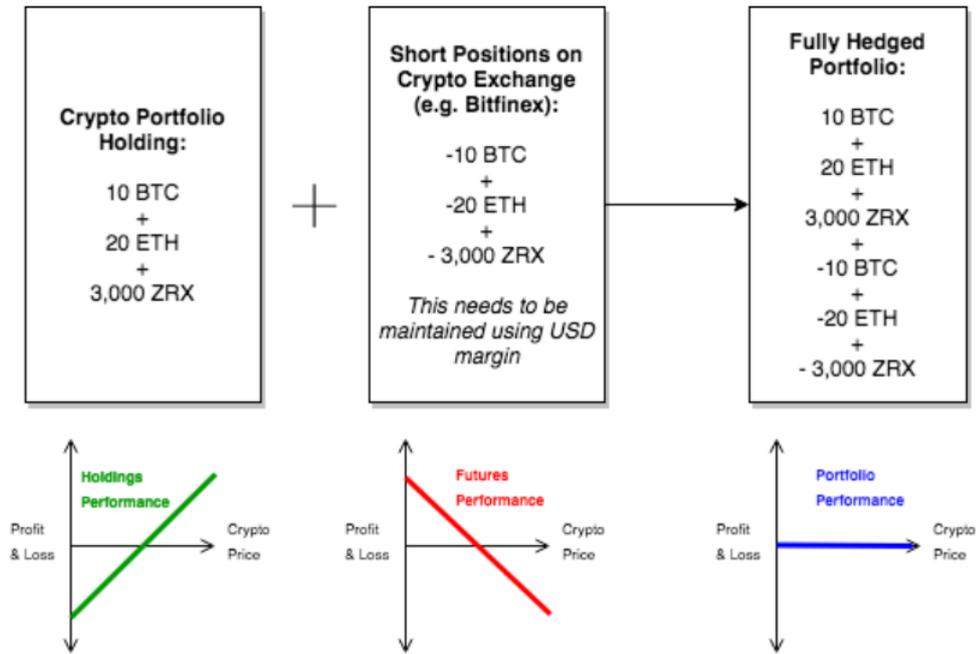


Fig. 3.3 Hedging Mechanism using Short-Selling

Therefore, we consider short-selling as being at the moment the most practical way to proxy-hedge and going forward, we will only focus on this method in our econometric analysis.

3.3 Econometric modelling of Bitcoin and related Altcoins

3.3.1 Dataset presentation

As emphasized earlier, the final goal of this article is to find the best proxy-hedge for Bitcoin. Figures 3.4 and 3.5 show the evolution and performance of top 30 cryptocurrencies by market capitalization for the past three years as of 30th of January 2020.

The historical timeframe between 1st of January 2017 and 30th of January 2020 exhibits the period of market "hype" and more importantly of the crash of Bitcoin, period where its effective hedging was vital. As of 30th of January 2020, there were almost 3200 different coins, however as Table 3.1 shows, almost 91% of the traded volumes are concentrated in top 30 coins.

Table 3.2 synthesizes the summary statistics over the considered dataset.

Despite Bitcoin being considered the benchmark digital currency while all other cryptocurrencies are highly correlated to it, Figure 3.6 shows that every major coin exhibits its own behaviour due to idiosyncratic events. In particular, dollar-indexed stable coins like Tether(USDT), USD Coin (USDC) and True USD (TUSD) oscillate around the unity value and cannot be considered as an effective Bitcoin proxy-hedge.

Table 3.3 also demonstrates that during and following the market crash, some coins

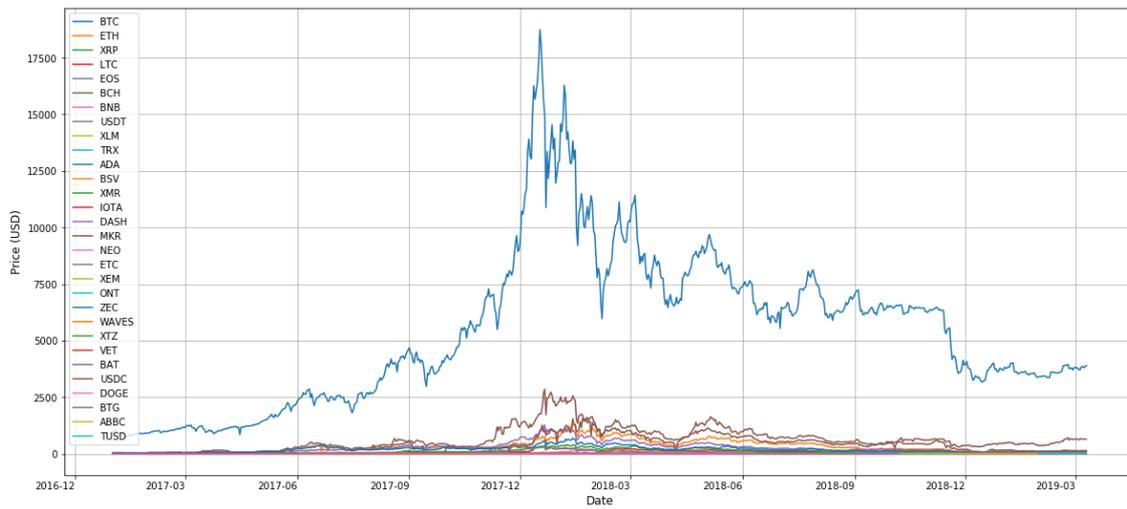


Fig. 3.4 Top 30 Cryptocurrencies (1-Jan-2017 to 30-Jan-2020)

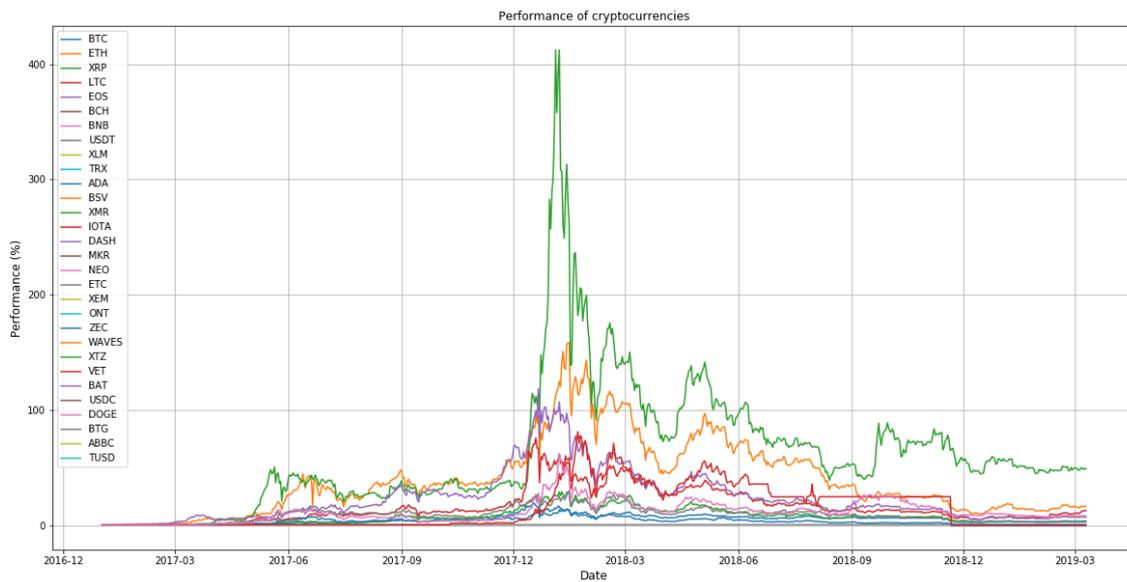


Fig. 3.5 Top 30 Cryptocurrencies Performance (1-Jan-2017 to 30-Jan-2020)

dropped more significantly, while others recovered better than Bitcoin.

3.3.2 Distribution Fitting Results

In order to capture the leptokurtic distribution shape of our dataset and overpass the limitations of using the classic Gaussian models, we consider the following set of candidate distributions: t-Student, Log Normal and Normal Inverse Gaussian (NIG), which retained our attention for their capacity to take in account heavy tails. The results of the statistical estimation are exhibited in the following tables. The 95% confidence intervals are computed through bootstrap approaches. The fittings are compared based upon the Bayesian Information Criterion (BIC)². The results synthesized in Table 3.4. NIG distribution exhibits

²In our formalism, the higher, the absolute value of the BIC, the better the fit is.

Table 3.1 Top 30 Cryptocurrencies by Market Capitalization as of 30-Jan-2019

Rank	Name	Symbol	Price (\$)	Market Cap (Mln \$)	Market Cap (%)	Market Cap (Cum%)	Volume (Mln \$)	Volume (%)	Volume (Cum%)
1	Bitcoin	BTC	3952.06	69483	51.57%	51.57%	9643	31.38%	31.38%
2	Ethereum	ETH	137.083	14419	10.70%	62.27%	4551	14.81%	46.19%
3	XRP	XRP	0.31459	13034	9.67%	71.94%	624	2.03%	48.22%
4	Litecoin	LTC	57.2077	3480	2.58%	74.52%	1772	5.77%	53.99%
5	EOS	EOS	3.73707	3387	2.51%	77.03%	1371	4.46%	58.45%
6	Bitcoin Cash	BCH	133.050	2350	1.74%	78.77%	271	0.88%	59.33%
7	Binance Coin	BNB	14.3875	2031	1.51%	80.28%	142	0.46%	59.79%
8	Tether	USDT	1.00906	2012	1.49%	81.77%	7866	25.60%	85.39%
9	Stellar	XLM	0.09931	1908	1.42%	83.19%	198	0.64%	86.03%
10	TRON	TRX	0.02296	1531	1.14%	84.33%	146	0.48%	86.51%
11	Bitcoin SV	BSV	66.8084	1180	0.88%	85.21%	73	0.24%	86.75%
12	Cardano	ADA	0.04551	1180	0.88%	86.09%	30	0.10%	86.85%
13	Monero	XMR	50.8195	856	0.64%	86.73%	64	0.21%	87.06%
14	IOTA	MIOTA	0.28276	786	0.58%	87.31%	7	0.02%	87.08%
15	Dash	DASH	83.8067	728	0.54%	87.85%	247	0.80%	87.88%
16	Maker	MKR	648.528	649	0.48%	88.33%	1	0.00%	87.88%
17	NEO	NEO	8.85259	575	0.43%	88.76%	290	0.94%	88.82%
18	Ethereum Classic	ETC	4.38412	477	0.35%	89.11%	225	0.73%	89.55%
19	NEM	XEM	0.04462	402	0.30%	89.41%	18	0.06%	89.61%
20	Ontology	ONT	0.95814	311	0.23%	89.64%	34	0.11%	89.72%
21	Zcash	ZEC	51.0892	310	0.23%	89.87%	227	0.74%	90.46%
22	Waves	WAVES	2.77641	278	0.21%	90.08%	14	0.05%	90.51%
23	Tezos	XTZ	0.45639	277	0.21%	90.29%	7	0.02%	90.53%
24	VeChain	VET	0.00471	261	0.19%	90.48%	12	0.04%	90.57%
25	Basic Atten- tion Token	BAT	0.20069	249	0.18%	90.66%	29	0.10%	90.67%
26	USD Coin	USDC	1.02142	242	0.18%	90.84%	21	0.07%	90.74%
27	Dogecoin	DOGE	0.00200	238	0.18%	91.02%	15	0.05%	90.79%
28	Bitcoin Gold	BTG	12.63500	220	0.16%	91.18%	10	0.03%	90.82%
29	ABBC Coin	ABBC	0.51611	220	0.16%	91.34%	42	0.14%	90.96%
30	TrueUSD	TUSD	1.01521	206	0.15%	91.49%	33	0.11%	91.07%

the best fit for the Bitcoin daily returns. The same result is found for the other Altcoins.

Crypto-currencies are inefficient and incomplete financial markets, and thus it is challenging to find the hedging instrument that perfectly mirrors a given price risk. Bitcoin investors and miners will to hedge only partially their risk. Mostly, they do this by purchasing Altcoins including Ether, Bitcoin Cash or Bitcoin Satoshi Vision. The prices of these Altcoins with Bitcoin are correlated in the long run, however, in the short term, price co-movements are asynchronous. This erratic relationship defines the basis risk, which is the financial risk that occurred when the chosen "proxy-hedge" does not entirely offset the price risk of the main underlying asset ([Kamara and Siegel (1987)], [Ankirchner and

Table 3.2 Summary Statistics of daily returns for Top 30 Crypto-currencies

Symbol	Start Date	End Date	Minimum	Maximum	Mean	StDev	Skewness	Kurtosis
BTC	2017-01-01	2019-03-10	-0.3228	0.3267	0.0017	0.0506	-0.7496	8.3724
ETH	2017-01-01	2019-03-10	-0.8489	0.7999	0.0035	0.0798	-0.7102	30.4464
XRP	2017-01-01	2019-03-10	-0.9647	1.2040	0.0048	0.1046	0.7350	38.7022
LTC	2017-01-01	2019-03-10	-0.5887	0.4777	0.0032	0.0727	-0.1188	12.7054
EOS	2017-01-01	2019-03-10	-1.1289	0.9723	0.0029	0.1061	-0.0598	38.3374
BCH	2017-01-01	2019-03-10	-0.5094	0.5054	-0.0009	0.0912	-0.0431	7.3804
BNB	2017-01-01	2019-03-10	-1.5827	1.6892	0.0042	0.1315	0.2050	84.7638
USDT	2017-01-01	2019-03-10	-0.6931	0.7491	-0.0000	0.0401	1.4298	261.5700
XLM	2017-01-01	2019-03-10	-0.6118	0.6608	0.0046	0.1021	0.3966	11.1839
TRX	2017-01-01	2019-03-10	-0.6936	0.6870	0.0049	0.1097	0.7059	10.4172
ADA	2017-01-01	2019-03-10	-0.5108	0.3399	-0.0051	0.0793	-0.5803	6.9861
BSV	2017-01-01	2019-03-10	-0.5065	0.4243	0.0000	0.0947	-0.2404	11.5619
XMR	2017-01-01	2019-03-10	-0.6504	0.4637	0.0016	0.0708	-0.8721	13.5493
IOTA	2017-01-01	2019-03-10	-0.6489	0.8565	-0.0018	0.0743	3.1070	84.8318
DASH	2017-01-01	2019-03-10	-0.4488	0.2977	0.0025	0.0677	-0.5283	5.6660
MKR	2017-01-01	2019-03-10	-2.6458	2.3598	0.0032	0.2114	-0.9602	80.0224
NEO	2017-01-01	2019-03-10	-0.6442	0.4312	-0.0003	0.0922	-0.9444	11.4443
ETC	2017-01-01	2019-03-10	-0.6439	0.3768	0.0014	0.0770	-1.1926	11.8493
XEM	2017-01-01	2019-03-10	-0.8578	0.4344	-0.0021	0.0823	-1.5754	22.1302
ONT	2017-01-01	2019-03-10	-0.5433	0.6417	-0.0012	0.0884	0.5855	11.9189
ZEC	2017-01-01	2019-03-10	-0.7284	0.6853	0.0001	0.0831	-0.0806	20.8247
WAVES	2017-01-01	2019-03-10	-0.3676	0.2723	-0.0005	0.0699	-0.3063	4.0116
XTZ	2017-01-01	2019-03-10	-1.0347	0.4895	-0.0006	0.1055	-2.9225	25.3973
VET	2017-01-01	2019-03-10	-6.0600	0.7984	-0.0036	0.2261	-23.5288	632.2901
BAT	2017-01-01	2019-03-10	-4.8249	4.5526	-0.0007	0.3212	-1.2287	176.0286
USDC	2017-01-01	2019-03-10	-0.1725	0.1725	0.0001	0.0487	-0.1309	2.4681
DOGE	2017-01-01	2019-03-10	-0.5562	0.3992	0.0026	0.0786	-0.2229	8.4396
BTG	2017-01-01	2019-03-10	-0.5523	0.4610	-0.0029	0.0889	-0.6931	9.9278
ABBC	2017-01-01	2019-03-10	-1.4281	1.1117	0.0128	0.2050	-1.0744	21.0627
TUSD	2017-01-01	2019-03-10	-0.0100	0.0100	0.0002	0.0042	0.1700	3.2310

Imkeller (2011)).

This is clearly seen from the 30 days rolling correlation plot in Figure 3.9 and 3.10, which shows that despite the "obviously" highly correlated dynamics of the spot prices, there is basis risk when hedging Bitcoin portfolios and this is mainly explained by exchange liquidity, market structure and time factors.

As such, if a Bitcoin miner wants to cover her Bitcoin price risk since the volumes exchanged on this market maybe thin, she might use one of the 'proxy-hedge' options described earlier. However, choosing the right one means making a trade-off between market liquidity and basis risk.

The current literature focuses mainly on the risks related to level forecasting when using a proxy-hedge but ignores the density forecasting completely. The main issue with proxy-hedging is the fact that markets have a different depth. On the one hand, a shock in the

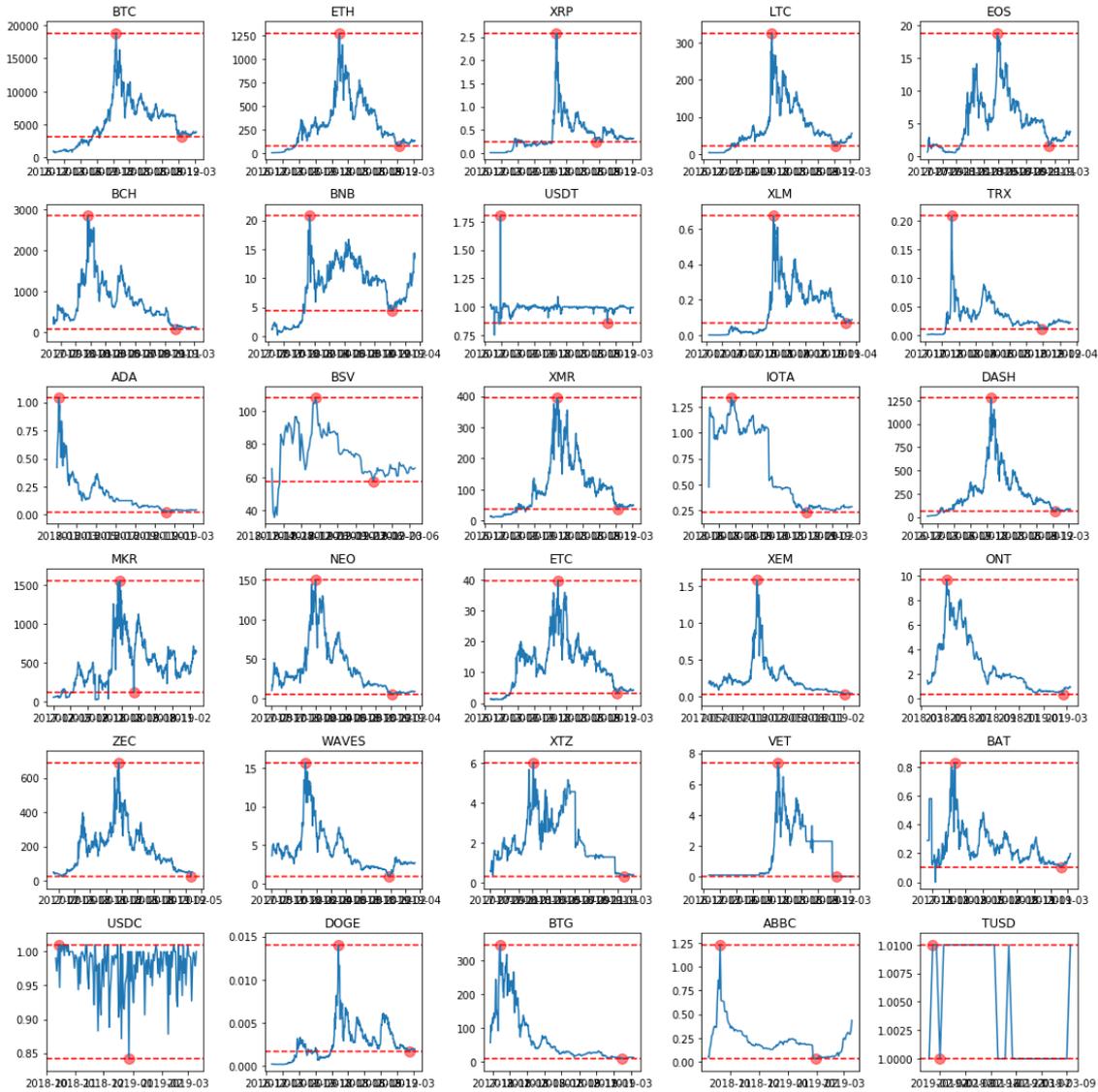


Fig. 3.6 Top 30 Crypto Currencies

Bitcoin market might not be fully reflected in the Altcoin prices. On the other hand, a small variation in the Bitcoin price may generate a shock in the Bitcoin forks (Bitcoin Cash and Bitcoin SV) due to the difference in market structures. Moreover, the Altcoin currencies are easier to manipulate than the Bitcoin market given the lower level of information, and the lower number of active traders. The differences in the distribution features also generate the basis risk of proxy-hedges using both plain or derivatives based strategies, thereby underlying the need for testing the density forecasting ability.

From the panel of crypto-currencies presented in Table 3.2 the present study focuses on Ethereum, Bitcoin Cash and Bitcoin Satoshi Vision. Ethereum is the second coin in term of capitalization, while Bitcoin Cash and Bitcoin SV are Bitcoin forks meaning that they are aimed to follow closely the Bitcoin price. For testing the proxy-hedging with Ethereum, Bitcoin Cash or Bitcoin SV, a trader exposed to Bitcoin price risk should assess the density forecasting capacity of an econometric risk model. Thus, a model estimated on Bitcoin Cash

Table 3.3 Top 30 Cryptocurrencies Recovery after Crash as of 10-03-2019

Symbol	Max Date	Min Date	Max Price	Min Price	Percentage Drop (%)	Recovery (%)
BTC	2017-12-17	2018-12-15	18750.91	3169.53	83.1%	4.67%
ETH	2018-01-15	2018-12-15	1271.07	82.05	93.54%	4.46%
XRP	2018-01-04	2018-08-14	2.5800	0.2468	90.43%	2.73%
LTC	2017-12-19	2018-12-07	324.37	22.36	93.11%	11.25%
EOS	2018-04-29	2018-12-07	18.74	1.55	91.73%	12.57%
BCH	2017-12-21	2018-12-15	2857.30	73.83	97.42%	2.03%
BNB	2018-01-13	2018-12-08	20.79	4.34	79.12%	58.6%
USDT	2017-02-27	2018-10-15	1.8000	0.8612	52.16%	14.16%
XLM	2018-01-07	2019-02-06	0.6720	0.06744	89.96%	3.58%
TRX	2018-01-05	2018-11-25	0.2083	0.01103	94.7%	5.87%
ADA	2018-01-04	2018-12-07	1.0400	0.02609	97.49%	1.78%
BSV	2018-12-23	2019-02-06	107.61	57.60	46.47%	15.76%
XMR	2018-01-09	2018-12-15	394.01	37.84	90.4%	3.14%
IOTA	2018-07-24	2018-12-15	1.3400	0.2352	82.45%	4.59%
DASH	2017-12-21	2018-12-15	1276.00	57.80	95.47%	1.93%
MKR	2018-01-21	2018-04-07	1563.14	115.49	92.61%	36.2%
NEO	2018-01-29	2018-12-07	149.98	5.46	96.36%	2.3%
ETC	2018-01-15	2018-12-07	39.81	3.28	91.76%	2.68%
XEM	2018-01-07	2019-02-07	1.500	0.032	97.95%	0.72%
ONT	2018-05-03	2019-02-19	9.620	0.363	96.23%	6.15%
ZEC	2018-01-13	2019-03-04	690.16	24.360	96.47%	3.9%
WAVES	2017-12-19	2018-11-25	15.70	0.92	94.14%	12.04%
XTZ	2018-01-04	2019-01-28	6.0200	0.3219	94.65%	1.53%
VET	2018-01-22	2018-12-12	7.4100	0.00278	99.96%	0.03%
BAT	2018-01-21	2019-02-06	0.8300	0.1013	87.8%	13.41%
USDC	2018-10-14	2018-12-28	1.0100	0.8416	16.67%	94.06%
DOGE	2018-01-07	2019-02-07	0.0140	0.0017	87.71%	1.99%
BTG	2017-11-27	2019-01-28	346.75	8.15	97.65%	1.31%
ABBC	2018-10-21	2019-01-31	1.230	0.032	97.38%	33.75%
TUSD	2019-01-31	2019-02-02	1.010	1.000	0.99%	100.0%

Table 3.4 Distribution Fitting for Bitcoin Spot Log Returns. NIG distribution exhibits the best fit.

Gaussian		Student		LogNormal		NIG	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	0.002 [0.000 0.000]	μ	0.006 [0.000 0.000]	μ	0.002 [0.000 0.000]	μ	0.215 [0.000 0.000]
σ	0.051 [0.045, 0.056]	σ	0.028 [0.024, 0.027]	σ	-24.635 [0.024, 0.027]	α	-0.047 [0.023, 0.026]
		ν	1.902 [3.4, 3.6]	β	24.636 [1.061, 1.081]	β	0.007 [0.11, 0.12]
						δ	0.023 [0.442, 0.852]
BIC	-1724.58		-1760.64		-1724.82		-1764.04

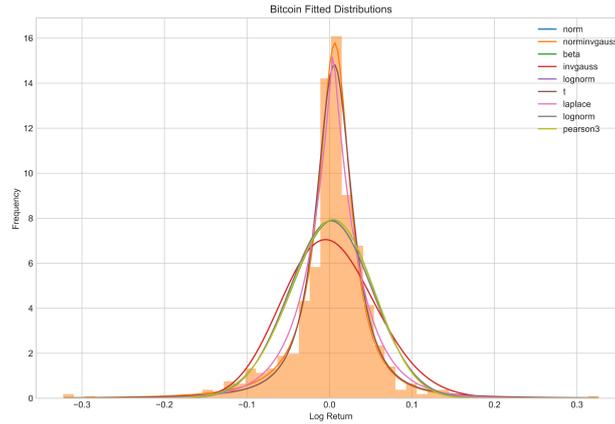


Fig. 3.7 Bitcoin Log Returns - Fitted Distributions

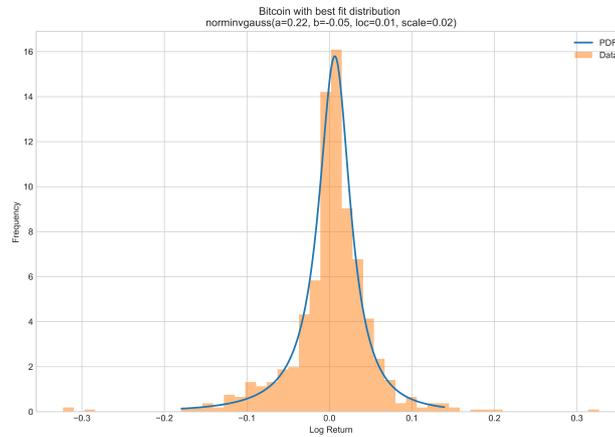


Fig. 3.8 Bitcoin Log Returns - Best Fitted Distribution

or Bitcoin SV returns should be tested in terms of density forecasting on the Bitcoin prices.

3.4 Forecasting densities

3.4.1 *Model-Free Forecasts*

In the early 1990s [Diebold and Mariano (2002)] introduced a seminal tests of the null hypothesis of no difference in the accuracy of two competing forecasts. Compared to other approaches that appeared is based directly on predictive performance, and we entertain a wide class of accuracy measures. More specifically the Diebold-Martino test presented in the Annexe does not make any reference to the underlying model of the forecast, but only to its outcomes.

3.4.2 *Vuong's test for comparing two distributions*

[Vuong (1989)] introduced an approach for comparing two densities for characterizing a time series. The density forecast in a time series context, is based on an **in the sample fit/out**

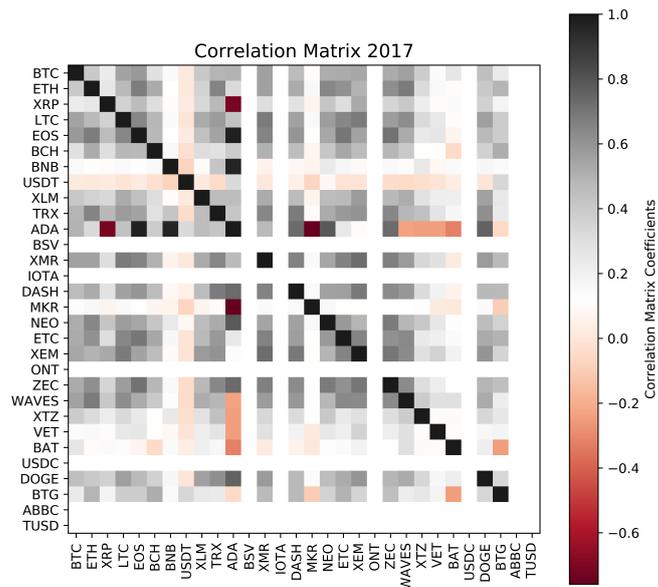


Fig. 3.9 Heatmap of daily Correlations from 1-Jan-2017 to 1-Jan-2018 for the main coins

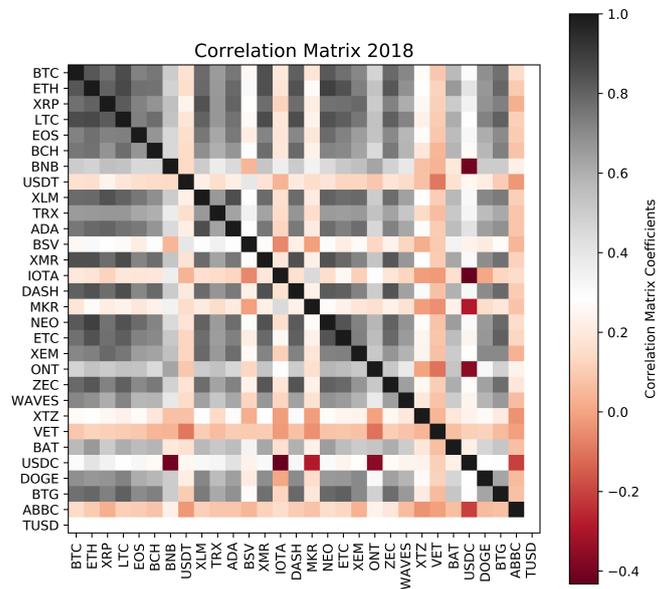


Fig. 3.10 Heatmap of daily Correlations from 1-Jan-2018 to 1-Jan-2020 for the main coins

of sample forecasts(See Annexe).

3.4.3 Weighted logarithmic scoring test

Generalizing the comparison framework introduced by Vuong and assuming a loss function $S(f,y)$ depending on the density forecast f and the realization y of the future observations. In a seminal work [Amisano and Giacomini (2007)] proposed a formal out-of-sample tests for ranking competing density forecasts that are valid under very general conditions. The score proposed is :

$$S(f, y) = -\log f(y) \quad (3.1)$$

The test is useful for comparing weighted averages of the logarithmic scores, thereby allowing for a greater weight on particular regions of the distribution. The weighted score is :

$$S_w^f(f_t, y_t) = w(y_t) \cdot S(f_t, y_t) = w^*\left(\frac{y_t - \mu_{y_t}}{\sigma_{y_t}}\right) S(f_t, y_t) \quad (3.2)$$

where $\mu_{y_t} = \frac{1}{m} \sum_{i=1}^m y_i$, $\sigma_{y_t} = \frac{1}{m} \sum_{i=1}^m y_i^2 - \left[\frac{1}{m} \sum_{i=1}^m y_i\right]^2$ and following weighting functions are proposed for that exercise:

$$w^*(x) = \begin{cases} \phi(x) & \text{for center of distribution} \\ 1 - \phi(x)/\phi(0) & \text{for tails of distribution} \\ \Phi(x) & \text{for right tail} \\ 1 - \Phi(x) & \text{for left tail} \end{cases} \quad (3.3)$$

The Giacomini-Amisano test is resumed as the expected difference of the score fro the two models f and g depending on parameters θ and γ , normalized by the test's standard deviation.

$$Z_n = \frac{\mathbf{E}_t(S_w^f(f_t, y_t|\theta) - S_w^g(g_t, y_t|\gamma))}{\omega_n} \quad (3.4)$$

where the weighted likelihood ratio is defined for a given function $w(\bullet)$ and two alternative conditional densities forecasts f and g , on the sample y_i as following:

$$W\Lambda_{n,i} = -w(y_i)(\log(f(y_i, \hat{\theta}_m)) - \log(g(y_i, \hat{\gamma}_m))), \quad (3.5)$$

where $i = m + 1, m + 2, \dots, m + n - k$ and $\hat{\theta}_m$ and $\hat{\gamma}_m$ are the parameters of models f and g , estimated on the sample y_{i^*} with $i^* = 1, \dots, m$. This statistic is defined as:

$$Z_n = \frac{\mathbf{E}(W\Lambda_{n,i})}{\sqrt{n\tilde{\omega}_n}} \quad (3.6)$$

where $\mathbf{E}(W\Lambda_{m,n,i}) = n^{-1} \sum_{i=m+1}^{m+n-k} W\Lambda_{m,i}$ and $\tilde{\omega}_n$ is a consistent estimator of the asymptotic variance ω_n computed as :

$$\tilde{\omega}_n = \frac{1}{n} \sum_{j=-k+1}^{k+1} \sum_{j=m}^{m+n-|j|} W\Lambda_{n,j} W\Lambda_{n,j+h} \quad (3.7)$$

where k is the number of time steps ahead for which the forecast is made. The limit of this statistic for a sample sufficiently large is the normal standard distribution $N(0,1)$. A level α test rejects the null hypothesis of equal performance of forecasts f and g whenever $Z > z_{\alpha/2}$, where $z_{\alpha/2}$ is the $(1 - z_{\alpha/2})$ -quantile of a standard normal distribution. In case of rejection, one could choose g if $W\Lambda_{m,n,i}$ is positive and f if $W\Lambda_{m,n,i}$ is negative.

Giacomini & Amisano has many of the features of Diebold Mariano test including the way the variance of the test is computed. As explained in recent paper [Diebold (2012)] the Diebold-Mariano test was intended for comparing forecasts in a model free perspective. The Diebold-Mariano test was not intended for comparing models. Much of the large ensuing literature, uses Diebold-Marino type tests for comparing models, in pseudo-out-of-sample environments. Sure these test are still useful tools for comparing models but they do have pitfalls and can provide counterintuitive results as pointed by [Diks *et al.* (2011)] and [Gneiting and Ranjan (2011)].

3.4.4 Gneiting test

This section describes the technique for reaching the main goal of this paper, the testing in terms of density forecasting of proxy-hedging strategies. In a recent paper [Gneiting and Ranjan (2011)] proposed a test that develops the weighting approach of [Amisano and Giacomini (2007)] but avoids counter intuitive inferences. We use this test for assessing the density forecasting in proxy hedging.

Gneiting's test aims to built a proper score with the respect of the above definition based on appropriately weighted versions of the continuous ranked probability score(CPRS). For any density function $f(y)$ with a cumulative distribution function $F(z) = \int_{-\infty}^z f(y)dy$ the continuous ranked probability score is then defined as

$$CPRS(F, y) = \int_{-\infty}^{\infty} PS(F(r), 1(y \leq r)) dr \quad (3.8)$$

where

$$PS(F(r), 1(y \leq r)) = (1(y \leq r) - F(r))^2 \quad (3.9)$$

is the Brier probability score for the probability forecast $F_t(r) = \int_{-\infty}^r f(y)dy$ of the event $y \leq r$

The weighted probability score described by [Matheson and Winkler (1976)] and [Gneiting and Raftery (2007)] is written as :

$$S^w(f, y) = - \int_{-\infty}^{\infty} PS(F(r), 1(y \leq r)) \cdot w_r(r) dr = - \int_{-\infty}^{\infty} (1(y \leq r) - F(r))^2 \cdot w_r(r) dr \quad (3.10)$$

where the weighting function $w_r(r)$ applies to the forms presented in equation 4.7. In a discrete form the above score can be approximated by assuming an I steps equidistant discretization of a target region with the boundaries y_l, y_u

$$S_f^w(f, y) = \frac{y_u - y_l}{I - 1} \sum_{i=1}^I w(y_i) PS(F(y_i), I(y \leq y_i)) \quad (3.11)$$

$y_i = y_l + i \frac{y_u - y_l}{I}$ The test based on the following statistic which is leveraged from the Amisano-Giacomini test :

$$Z_n = \frac{E(S_f^w(f, y) - S_f^w(g, y))}{\widehat{\omega}_n} \quad (3.12)$$

where

$$E_t(S_f^w(f, y)) = \frac{1}{n - k + 1} \sum_{t=m}^{m+n-k} S(f_{t+k}, y_{t+k}) \quad (3.13)$$

$$E_t(S_f^w(g, y)) = \frac{1}{n - k + 1} \sum_{t=m}^{m+n-k} S(g_{t+k}, y_{t+k}) \quad (3.14)$$

and $\widehat{\omega}_n$ is an estimate of $var(\sqrt{n}(E_t(S_f^w(f, y)) - E_t(S_f^w(g, y))))$

3.5 Backtesting results of proxy-hedging

Density forecasting techniques provide with insightful information for risk assessment purposes especially in the commodities markets dominated by non-Gaussian behavior and volatility clustering ([Frunza and Guégan (2013)]).

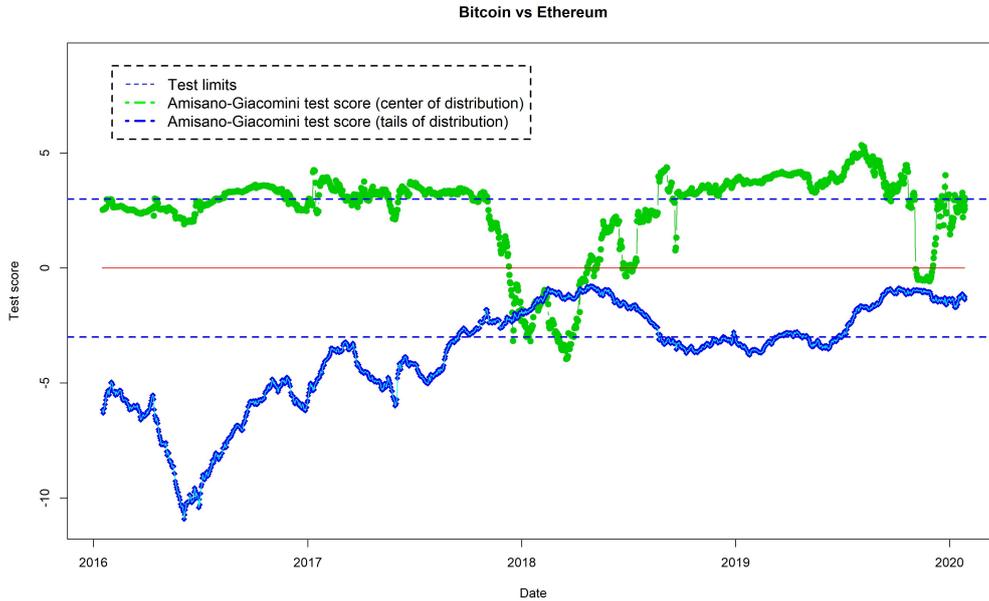


Fig. 3.11 Evolution of the Amisano-Giacomini Test Score for NIG model for Ethereum

Following a similar approach as that introduced by [Guegan *et al.* (2018)] and based on the specifications of the Amisano-Giacomini and Gneiting tests presented above we built a testing process for the proxy hedging strategies. The full dataset contains the daily prices of Bitcoin, Ethereum, Bitcoin Cash and Bitcoin SV between 01/08/2015 and 30/01/2020. The testing process has the following steps:

- (1) A model **M1**(NIG model) is estimated on the daily returns of the proxy (Ethereum, Bitcoin Cash or Bitcoin SV). The data set contains a (*sample*) the first 250 consecutive days of the considered full sample.
- (2) A model **M2**(NIG model) is estimated on the daily returns of the Bitcoin prices. The data (*sample*) contains a window of 250 consecutive days, which starts at the same moment as the previous sample dataset.
- (3) The Amisano-Giacomini test and Gneiting test scores are computed for comparing the model M1 estimated on the proxy with model M2 estimated on the actual Bitcoin returns. The aim of this step is to compare the density of the two NIG models corresponding to Bitcoin returns and to the proxy returns. The core of the distribution and the tails are compared with appropriate weighting functions.
- (4) The *sample* window is rolled over with one day. Steps 1-3 are repeated until the end of the full sample
- (5) Time series of Amisano-Giacomini test and Gneiting test scores are built.

The previous sections underlined that NIG distribution exhibits good fitting features for all the coins studied in this article. Therefore we will consider the NIG for both M1 and M2 models. Therefore the test score will assess the power of the density fitted on the proxy to forecast the Bitcoin distributions feature.

Figure 3.11 shows the evolution of the Amisano-Giacomini testing Score for NIG model, where as the proxy hedge is realized with Ethereum. The test was implemented considering weighting for the core of the distribution and for the tails. Until December 2017 the test score with tails weighting rejects at 99% confidence level the null hypothesis that the model fitted on the proxy is similar to the model fitted on the Bitcoin. After December 2017 both scores (tails and core of the distributions) moved closer to the confidence region thereby not rejecting the null hypothesis. In fact in 2017 as showed by [Guegan and Frunza (2018)] Bitcoin went through a bubble, while Ethereum did not. Therefore, the Ethereum's behavior drifted away from Bitcoin's pattern, thereby explaining the difference in the distributions underlined by the Amisano-Giacomini test.

Figure 3.12 shows the evolution of the Gneiting testing Score for NIG model, where as the proxy hedge is realized with Ethereum. The results are in line with those from the previous test. Ethereum is a poor proxy-hedging candidate before 2017, but the differences between Ethereum's and Bitcoin's distributions attenuate during 2018.

Figure 3.13 shows the evolution of the Amisano-Giacomini testing Score for NIG model, where as the proxy hedge is realized with Bitcoin Cash. Bitcoin Cash is a mirror coin of Bitcoin aimed to be more liquid with lower transaction costs. In theory, the distribution of their returns should not be very different. This is partially confirmed by this test. Around 2019, the score for the tails rejects at 99% confidence level the null hypothesis that the

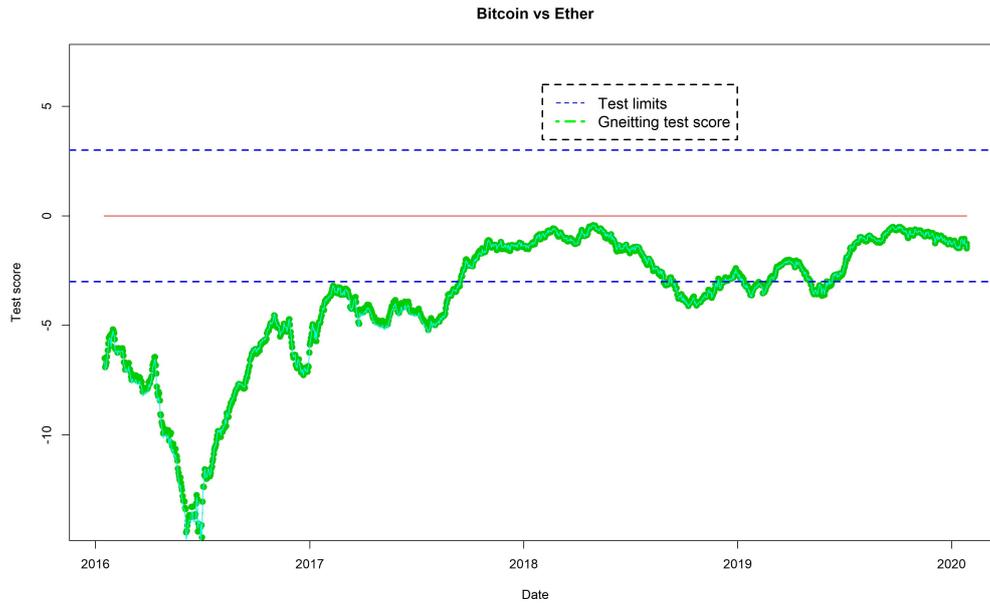


Fig. 3.12 Evolution of the Gneitting Test Score for NIG model for Ethereum

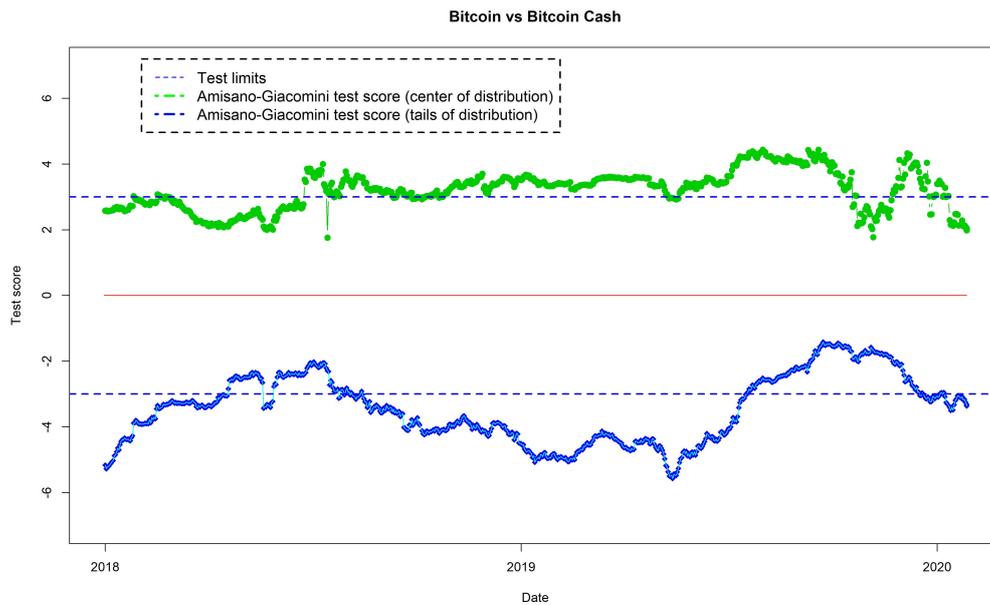


Fig. 3.13 Evolution of the Amisano-Giacomini Test Score for NIG model for Bitcoin Cash

model fitted on the Bitcoin Cash is similar on tails to the model fitted on Bitcoin prices' return. It means that the extreme events on Bitcoin are not followed by similar moves in Bitcoin Cash. This finding supports critiques claiming that Bitcoin Cash is targeted by speculators, due to the fact that has lower liquidity and transparency than Bitcoin.

Figure 3.14 shows the evolution of the Gneitting testing Score for NIG model, where as the proxy hedge is realized with Bitcoin Cash. The results reinforce the findings of the previous test, showing that in 2019 Bitcoin Cash's behavior deviates from Bitcoin's behavior.

Figure 3.15 shows the evolution of the Amisano-Giacomini testing Score for NIG model

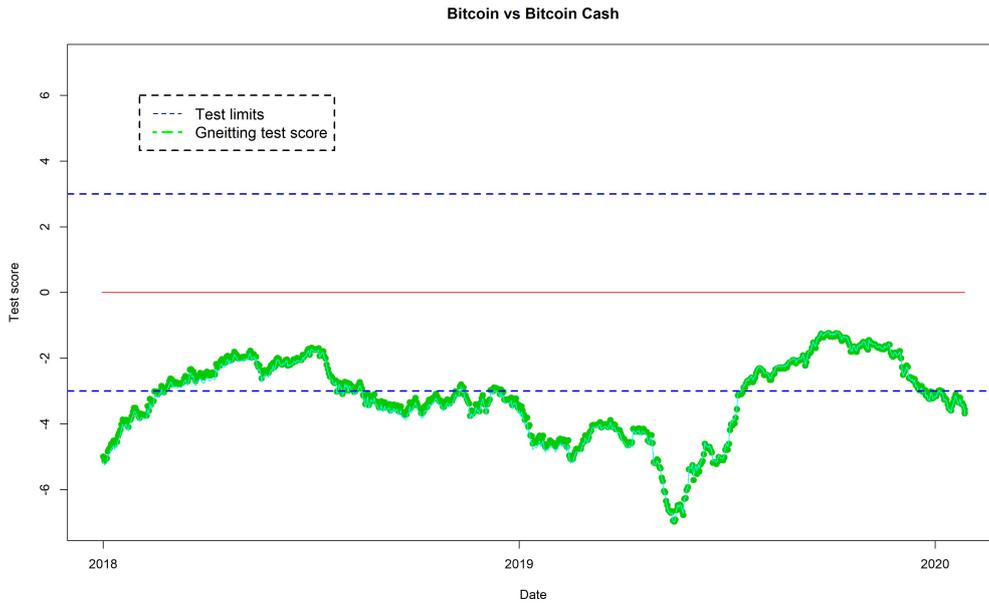


Fig. 3.14 Evolution of the Gneitting Test Score for NIG model for Bitcoin Cash

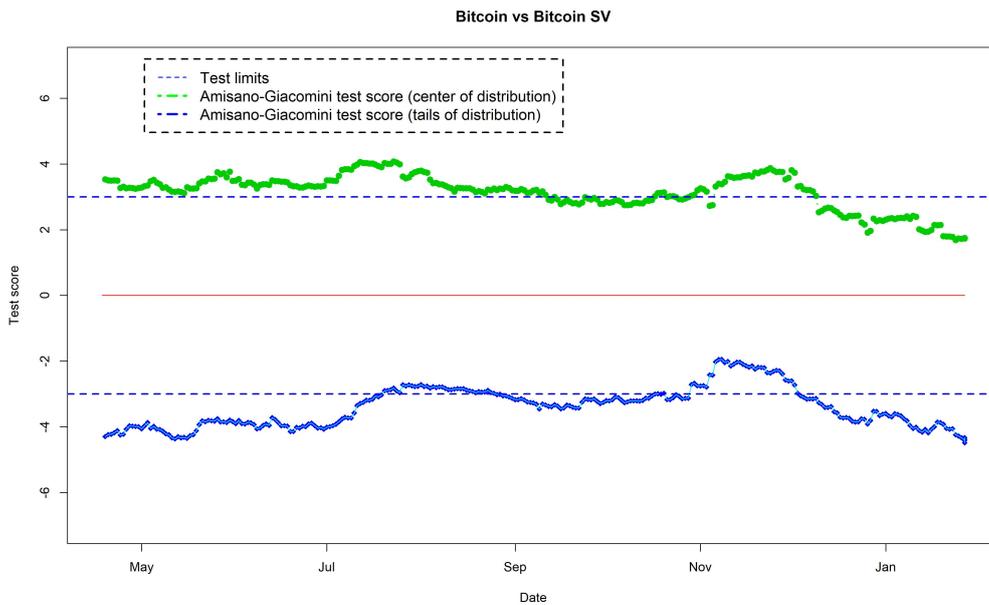


Fig. 3.15 Evolution of the Amisano-Giacomini Test Score for NIG model for Bitcoin SV

and Figure 3.16 shows the evolution of the Gneitting testing Score for NIG model, where as the proxy hedge is realized with Bitcoin SV. Between May and November 2019 Bitcoin SV's density is not different from that of Bitcoin. Significant differences start to occur especially on the tails region after December 2019. This is explained by the fact that there were some price manipulation attempt on this Altcoin, based on unverified rumors spread amongst market participants about the existence of a Bitcoin SV *whale*.

Crypto-currencies and especially Altcoins can be targeted by manipulators attempting to distort the price pattern for example with "pump and dump" techniques ([Frunza (2015)]).

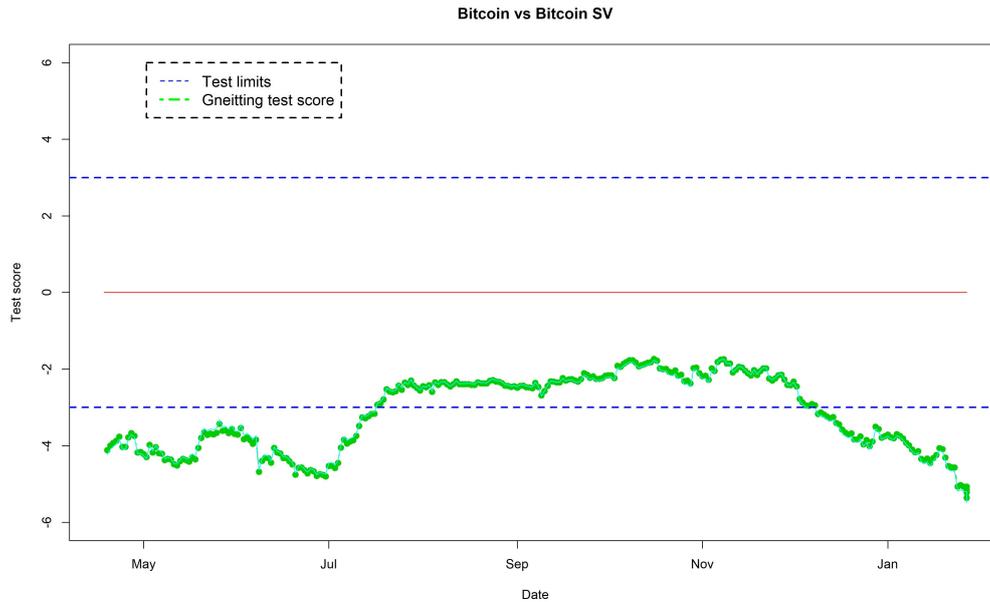


Fig. 3.16 Evolution of the Gneiting Test Score for NIG model for Bitcoin SV

Therefore, they can exhibit irrational behaviors in relation to Bitcoin's evolution.

The three charts (Figure 3.12, Figure 3.14 and Figure 3.16) and the testing scores exhibit a positive trend after December 2017 due to the fact that the NIG model increases its forecasting capability in terms of density. The forecasting test specified above is implemented over rolling windows, hence the newer data is included in the test gradually. Thus, prior to 2018 there were limited alternatives for proxy-hedging and Ethereum was a poor candidate. After 2018, all three coins (Ethereum, Bitcoin Cash, Bitcoin SV) shows behavior closer to Bitcoin's pattern. Nevertheless, there are periods when the differences between the proxy density and the Bitcoin density become more pronounced. This fact is explained by the difference in market structure and by the fact that Bitcoin Cash and Bitcoin SV are easier subjects to market manipulation.

Bitcoin and the three Altcoins studied in this paper are different in term of returns time series, but have on certain temporal windows structural similarities in term of density features. Thus a crypto-currency investor could be tempted to interpret that he has different hedging alternatives in term of markets, but from a risk management perspective, the three Altcoins could exhibit a similar behavior under certain circumstances, thereby hindering the panel of eligible hedging instruments

3.6 Conclusions

This paper explores the topic of proxy hedging in crypto-currencies market with a focus on Bitcoin. The research addresses the problem of a Bitcoin investor or a Bitcoin miner that hedges its price risk with proxy coins including Ethereum, Bitcoin Cash and Bitcoin SV. The problem is studied in two steps: first the various econometric models with fat tails are

explored in relation with the returns of daily time series and second the proxy hedging is test based on density forecasts methods using the results for the first step.

The historical backtesting shows that Ethereum was a poor proxy-hedging candidate for Bitcoin before 2017, due to the fact that the last went through a bubble during that period while Ethereum did not.

Bitcoin and Bitcoin Cash distributions should not be very different in theory, as partially confirmed by the backtesting. Around 2019, the extreme events on Bitcoin are not followed by similar moves in Bitcoin Cash, thereby supporting the claims that Bitcoin Cash is targeted by speculators, due to the fact that has lower liquidity and transparency than Bitcoin.

Bitcoins Satoshi Vision shows significant differences with Bitcoin especially in the tails region after December 2019. This is explained by the fact that there were some price manipulation attempt on this Altcoin, based on unverified rumors spread amongst market participants about the existence of a Bitcoin SV *whale*.

A future direction for our research is the consideration of liquidity on various exchanges, as trading crypto-currencies usually involves different brokerage fees and liquidity across different exchanges. This leads to addressing the problem of dimensionality, as it would be necessary to consider a technique such as approximate dynamic programming to produce a hedging policy that reflects such costs.

Chapter 4

Impact of contagion on proxy-hedging in jet-fuel markets

1

In 1859 the human race discovered a huge treasure chest in its basement. This was oil and gas, a fantastically cheap and easily available source of energy. We did, or at least some of us did, what anybody does who discovers a treasure in the basement - live it up, and we have been spending this treasure with great enjoyment

Kenneth E. Boulding, *English-American economist*

Abstract *The aim of this research² is to explore the risk associated with hedging in jet fuel markets. It focuses on finding the most effective proxy hedge instrument for the Singapore spot market. Due to its particularities, this market does not exhibit the same features as traditional financial markets do. In appearance it seems very related to the oil market, but in reality it exhibits insufficient liquidity and shows unusual volatility clustering effects. This behavior as well as the potential contagion effects have a direct impact on the hedging strategies of refineries, airline companies and jet fuel traders. This study explores the econometric features of the jet fuel price and underlines the need of fat tail distributions and volatility clustering models. Also it examines the density forecasting capacity of various proxy hedge instruments including kerosene, crude and gasoil futures. The results show that Singapore Gasoil Futures contract is the best candidate for hedging the Singapore Jet Fuel spot price.*

4.1 Introduction

An extensive literature covers the economy of oil markets, but less attention is given to the oil distillates and particularly to the jet fuel market. The lack of efficiency in oil and middle distillates markets was pointed previously by the academic literature ([Balbás *et al.* (2008)], [Kanamura *et al.* (2010)], [Roncoroni *et al.* (2015)]). Oil distillates markets are by their nature dependent on the oil market behavior, but also are exposed to specific risks linked

¹The research exposed in this chapter was presented at the 2017 IPAG conference in Nice, France and was published in peer-reviewed conference proceedings: "Impact of Contagion on Proxy-Hedging in Jet-Fuel Market", Haliplii R., Guegan D., Frunza M. 2020 [Guegan *et al.* (2020b)]

²Paper presented at the 2017 IPAG conference in Nice, France

to the changes in the supply/demand equilibrium for those products. Therefore in this research we intend to explore the modeling based on non-Gaussian distribution, volatility clustering and regime switching. Furthermore we address the problem of contagion between the fuel-related markets and focus on benchmarking the densities of these markets over various periods of time.

The main motivation however behind this study is to address the challenges faced by a company trading illiquid refined products such as jet fuel and providing it with optimal solutions with regards to their proxy hedging. [Nascimento and Powell (2008)] modeled the jet fuel price using two-factor model to allow mean-reversion in the short-term and proposed oil future contracts for tackling the hedging problem. [Adams and Gerner (2012)] investigated the effect of the maturity on the cross-hedging performance of jet-fuel within an Error Correction model. They evaluated the performance of several oil forwards contracts including WTI, Brent, Gasoil and heating oil to manage jet-fuel spot price exposure. Their results highlight that the standard approach in the literature to use crude oil as a cross hedge for jet fuel is not optimal for time horizons of three months or less. By contrast, for short hedging horizons their results indicate that gasoil forwards contracts represent the highest cross hedging efficiency for jet-fuel spot price exposure, while for maturities of more than three months, the predominance of gasoil diminishes in comparison to WTI and Brent.

[Clark *et al.* (2003)] have attempted to test for the most effective cross hedging instrument for the Singapore jet fuel spot market, using regression techniques. Their research concludes that for the period February 1997 to August 2001, Heating Oil futures contract gives best in sample results. Nevertheless, after correcting for serial correlation, their out of sample results proved to be weak for all regression models and ambiguous with respect to the heating oil contract.

This chapter aims to enrich the scarce literature on the economics of oil distillates and attempts to estimate a good model capturing the dynamics of jet fuel futures. In contrast to level forecasting regression and co-integration models used in previously mentioned papers, our research provides a different approach for testing proxy-hedging based on density forecasting. The main finding of this research is that a trader exposed to jet fuel price risk might think he has different hedging alternatives in term of markets, where in reality from a risk management perspective, the alternatives could exhibit a similar behavior in term of density forecasting capability. The contagion between markets reduces considerably the range of possible options for a hedger. Also the contagion effect can create basis risk management issues, especially in times of high volatility and scarce liquidity. The chapter is organized as follows:

Section 4.2 explores the econometric features of oil middle distillates refined products (including gasoil and jet fuel),

Section 4.3 explains the challenges of jet fuel proxy-hedging as well as the associated basis risk,

Section 4.4 assesses the density forecasting methodologies (including probability forecasting Gneiting Test [Gneiting and Ranjan (2011)]),

Section 4.5 presents the results of the ability of more liquid traded products such as Brent Crude and Gasoil returns to forecast density the jet fuel market,

Section 4.6 concludes.

4.2 Econometric modeling of the Singapore jet fuel and related oil distillates

The first part of this research is dedicated to the econometric study of the Singapore jet fuel and related oil distillates prices. Our aim is not to find the "true" model that would explain the behavior of these commodities, but to propose a benchmark from different models commonly used to describe financial assets. Based on the historical time series, few models are estimated with the objectives to capture volatility clustering. Clustering in volatility is another ubiquitous feature observed in returns. Few models from the GARCH universe allow to capture this phenomena emphasizing the various particularities of the return series. Thus, we explore the following models :

- Models without volatility clustering, but with non Gaussian innovations (NIG, t-Student, Asymmetric Student)
- Models with volatility clustering and Gaussian innovations (GARCH, eGARCH, iGARCH, GJR-GARCH, APARCH)
- Models with volatility clustering and non-Gaussian innovations
- Markov Regime Switching GARCH models

4.2.1 Dataset presentation

As emphasized earlier, the final goal of this article is to assess the risk of a refinery or airline company that hedges its exposure to illiquid petroleum products such as jet fuel. There are two primary futures contracts which are commonly used for jet fuel hedging: brent crude and gasoil. These contracts serve as the primary benchmarks across the world. In addition, there are many other contracts (futures, crack futures, swaps and options) available for jet fuel hedging, most of which are tied to one of the major, global trading hubs of Singapore, US Gulf Coast (Houston/New Orleans) and NW Europe/ARA (Amsterdam, Rotterdam and Antwerp).

For this purpose, we consider ICE Brent Crude, ICE Low Sulphur Gasoil and Singapore 50ppm Gasoil Futures for our proxy analysis. We also consider the Singapore Jet Kerosene (Platts) vs. Gasoil (Platts) Futures differential (called Regrade), often used in jet fuel

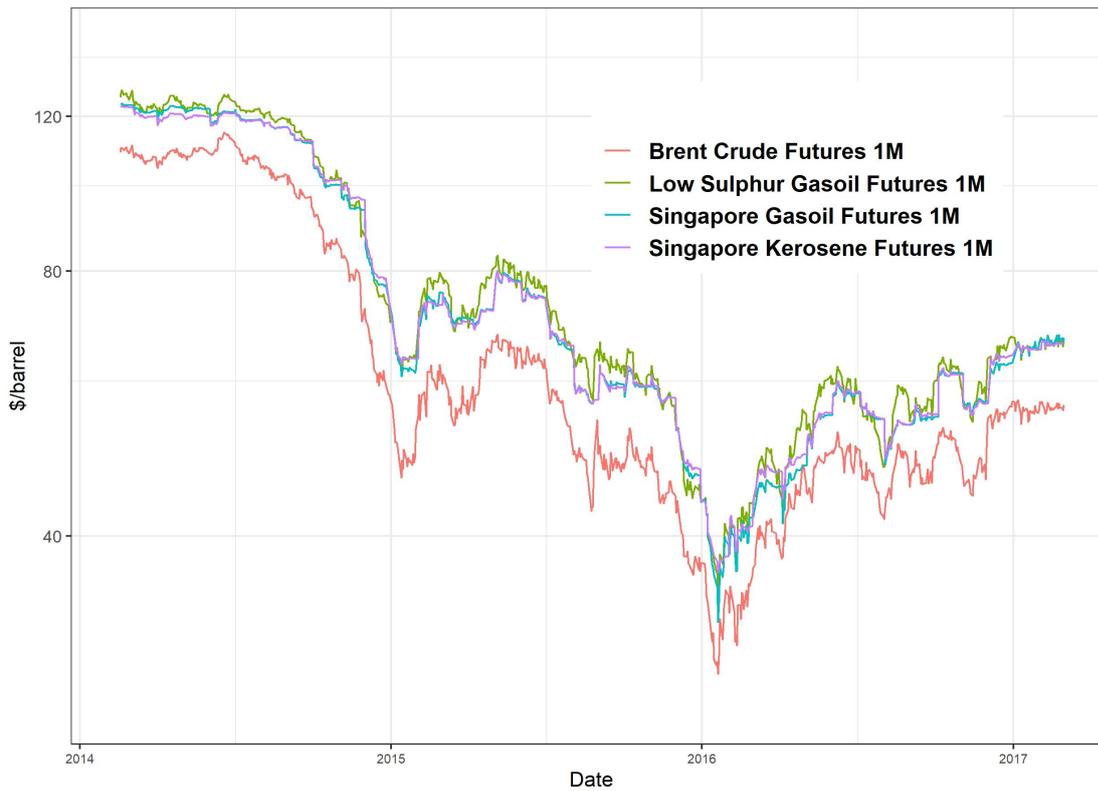


Fig. 4.1 Evolution of the front month futures of Singapore Jet Kerosene , ICE Brent Crude, ICE Low Sulphur Gasoil and Singapore 50ppm Gasoil (USD/bbl)

hedging. As for the jet fuel, there are three reference futures contracts for each geographical hub: Platts CIF NWE, USGC Jet 54 and Singapore Jet FOB. For our analysis, we consider the jet fuel contract traded in Singapore. Figure 4.1³ presents the evolution of the above mentioned front month futures contract quoted in USD/barrel.

The Table 4.1 synthesizes the summary statistics over the considered dataset. ICE Brent and ICE Low Sulphur Gasoil exhibit a higher volatility compared to the other three series. We notice that compared to the highly liquid Brent and LS Gasoil futures, Singapore Jet Fuel/Kerosene, Singapore Gasoil and Regrade exhibit a considerably higher kurtosis values which implies the need of heavy tailed distributions for modeling purposes.

Figures 4.2, 4.3, 4.4, 4.5 and 4.7 show the historical prices for Brent, LS Gasoil, Jet Kerosene and Regrade futures for the most liquid maturities of the curve.

4.2.2 Generalized Hyperbolic models

A recent modeling technique introduced here permits both skewness and kurtosis in the assets returns. Indeed, these features are not accounted for in the previous modelings. Following the works of [Eberlein and Prause (2002)] and [Barndorff-Nielsen (1977)] done on financial assets, we calibrate the class of Generalized Hyperbolic distributions to our data

³The ICE Low Sulphur Gasoil contract, quoted in USD per metric tones on the exchange, has been converted here to USD/bbl using a scale conversion factor of 7.45 used in the industry.

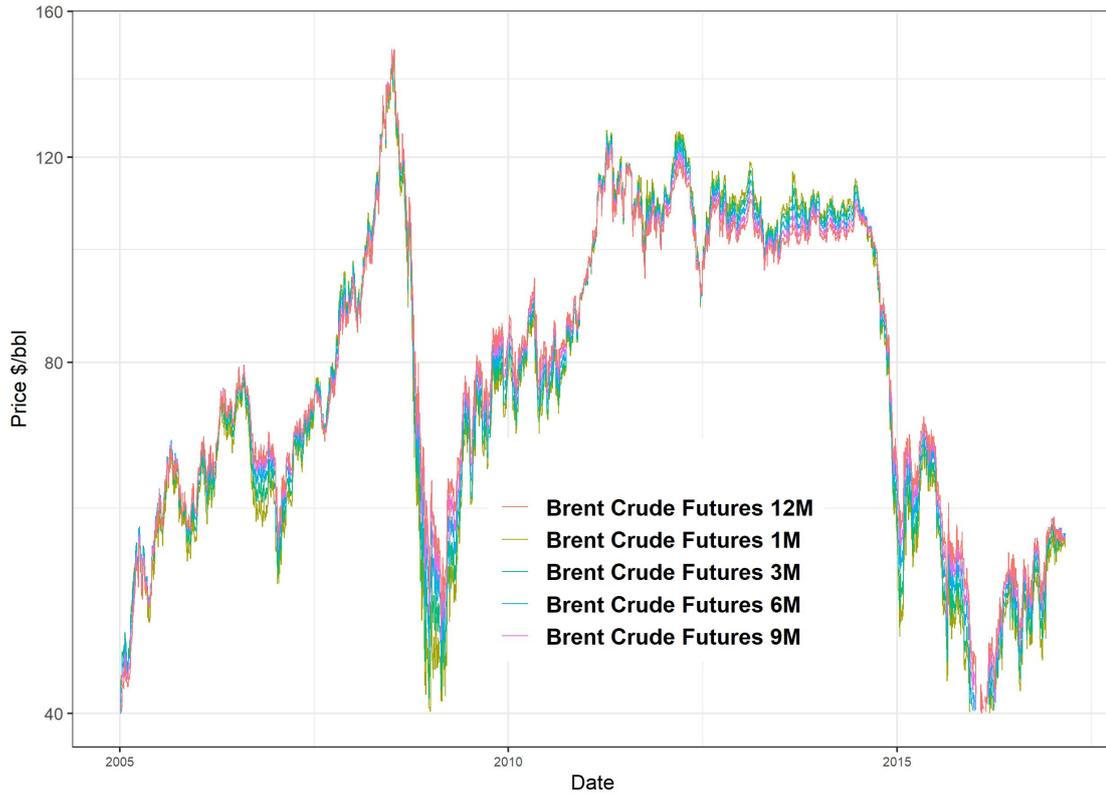


Fig. 4.2 Evolution of Brent Crude Futures price listed on ICE for the following maturities: 1M, 3M, 6M, 9M and 12M

Table 4.1 Summary Statistics. ICE Brent and ICE Low Sulphur Gasoil exhibit a higher volatility compared to the other three series. Regrade and Singapore Kersone have a more pronounced Kurtosis.

<i>Underlying</i>	<i>Mean</i>	<i>Volatility</i>	<i>Skewness</i>	<i>Kurtosis</i>
Regrade	-0.003	0.287	0.803	34.157
Singapore Gasoil	-0.001	0.289	-0.374	15.039
Singapore Kerosene	-0.001	0.265	-0.338	22.417
ICE Brent	-0.001	0.390	0.248	5.134
ICE LS gasoil	-0.001	0.328	0.597	6.862

sets. This very flexible class of distributions (definition recalled in Annexe 1) is able to capture heavy tails and asymmetry. It is characterized by five parameters with a parameter which permits very specific shapes. The four other parameters are linked in an easy way with the first four moments of the distribution.

4.2.2.1 *Distributions Fit Results*

In the view of hedging in illiquid markets we begin to fit the best model in term of AIC(BIC). In order to add leptokurtic distribution shapes of our datasets and overpass the limitations of using the classic Gaussian modeling framework, we consider the following set of candidate distributions: t-Student, Asymmetric Student(ASTD) and Normal Inverse Gaussian (NIG),

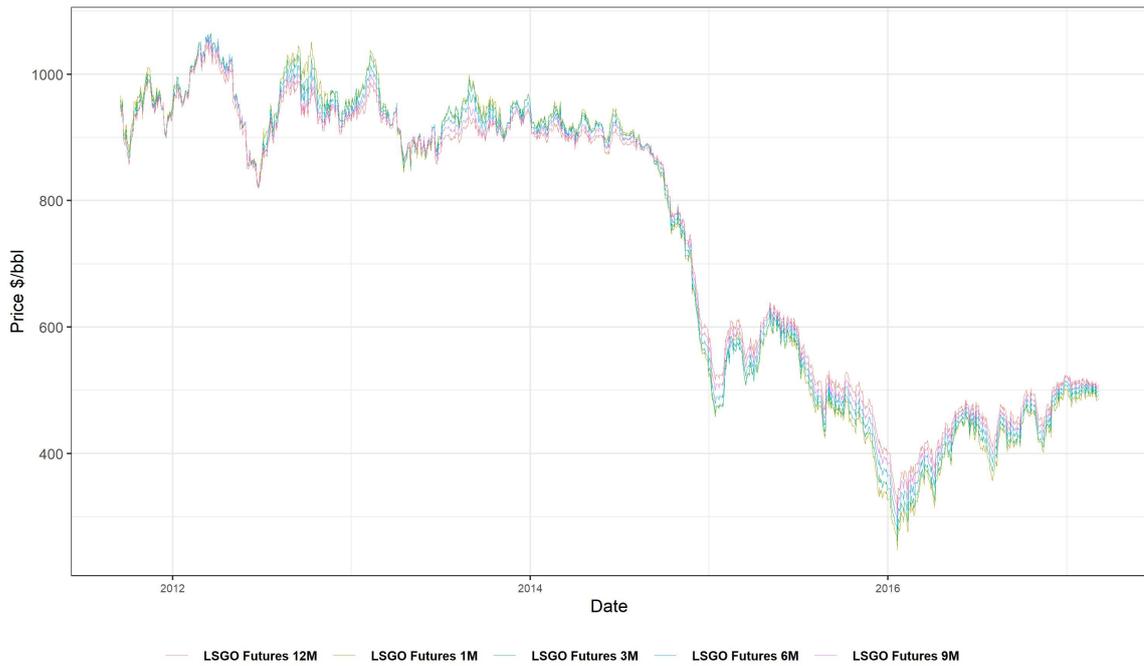


Fig. 4.3 Evolution of Low Sulphur Gasoil Futures price listed on ICE for the following maturities: 1M, 3M, 6M, 9M and 12M

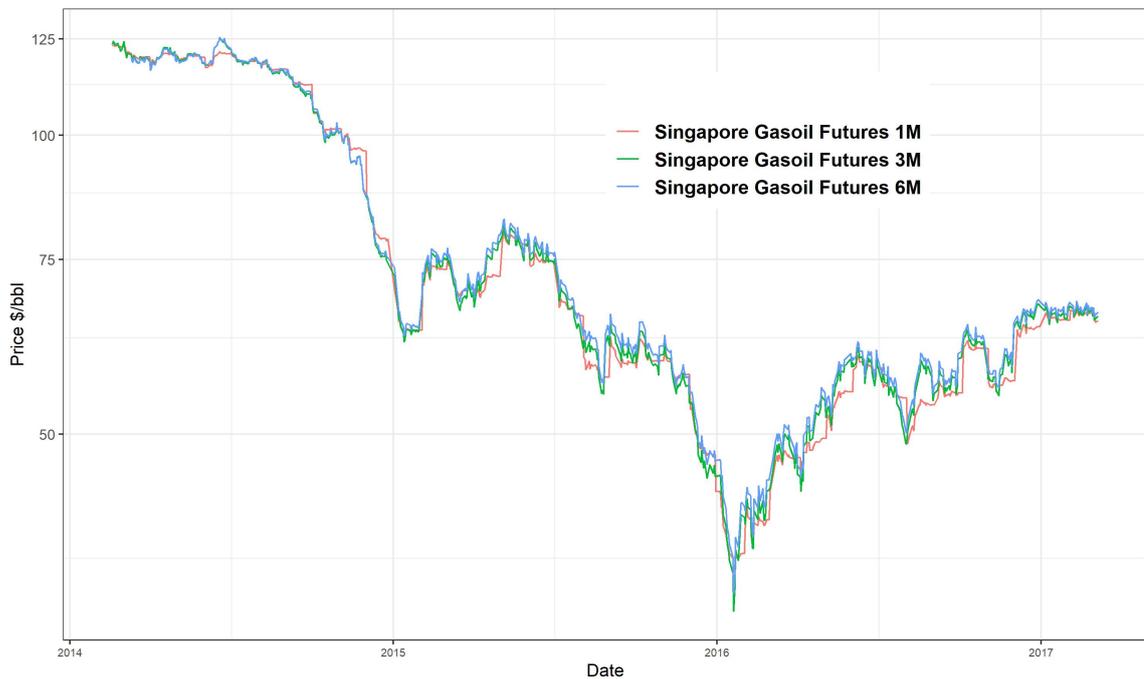


Fig. 4.4 Evolution of Gasoil 0.5% (Platts) Futures price listed on Singapore exchange for the following maturities: 1M, 3M and 6M

which retained our attention for their capability to take in account heavy tails. The results of the statistical estimation are provided in Table 4.2, Table 4.3, Table 4.4, Table 4.5 and Table 4.6. The 95% confidence intervals are compute through bootstrap approaches. The fittings

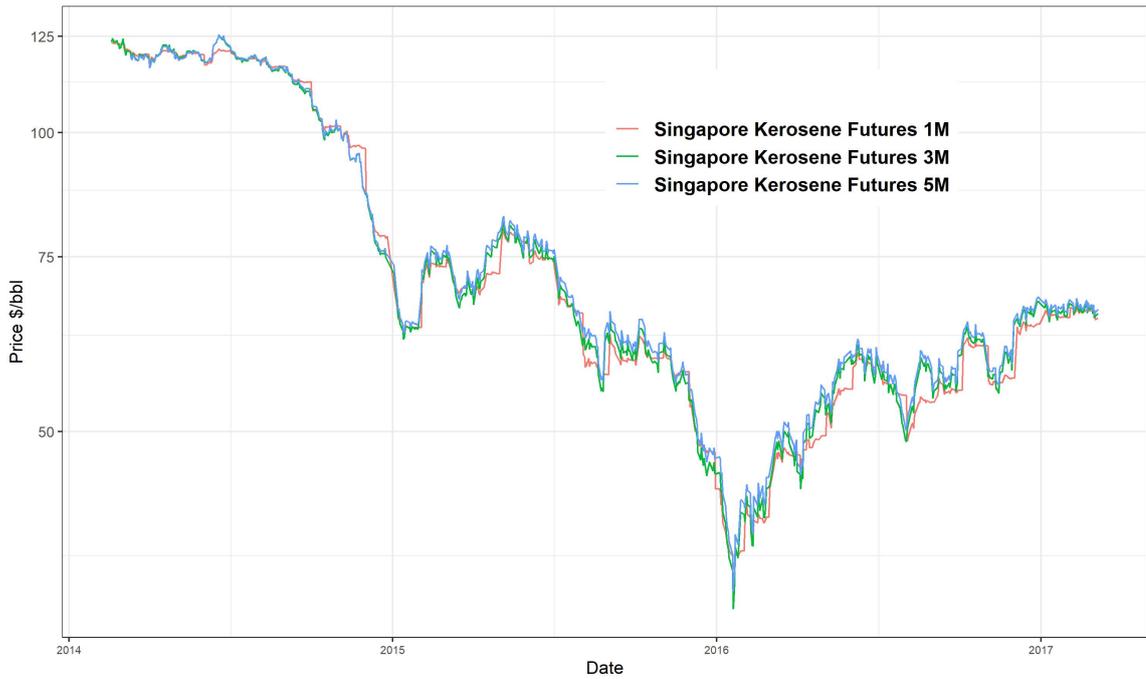


Fig. 4.5 Evolution of Jet Kerosene (Platts) Futures price listed on Singapore exchange for the following maturities: 1M, 3M and 5M

are compared based upon the Bayesian Information Criterion (BIC)⁴. NIG and Student distributions exhibit the best fits for the jet fuel returns as well as for the proxy-hedging candidates.

Table 4.2 Distribution Fitting for ICE Brent Front Month Futures returns. NIG and Student distribution exhibit the best fits in regards of the BIC criteria.

<i>Gaussian</i>		<i>Student</i>		<i>ASTD</i>		<i>NIG</i>	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	0.000 [0.000 0.000]	μ	-0.001 [0.000 0.000]	μ	0.000 [0.000 0.000]	μ	-0.000 [0.000 0.000]
σ	0.025 [0.023, 0.026]	σ	0.026 [0.024, 0.027]	σ	0.026 [0.024, 0.027]	α	0.025 [0.023, 0.026]
		ν	3.529 [3.4, 3.6]	β	1.071 [1.061, 1.081]	β	0.115 [0.11, 0.12]
				ν	3.550 [3.440,3.660]	δ	0.676 [0.442, 0.852]
BIC	-1724.58		-1760.64		-1724.82		-1764.04

4.2.3 Volatility models

Typical Gaussian flat volatility failed to provide with conspicuous valuations for contingencies and also underestimated the risk measures. The dynamic volatility models add value also for testing hedging strategies as, the traditional flat volatiles model tend to underesti-

⁴In our formalism the higher the absolute value of the BIC, the better the fit is.

Table 4.3 Distribution Fitting for ICE LS Gasoil Front Month Futures daily returns. NIG and Student distributions exhibit the best fits in regards of the BIC criteria.

<i>Gaussian</i>		<i>Student</i>		<i>ASTD</i>		<i>NIG</i>	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	-0.000 [-0.230 ,0.220]	μ	-0.001 [-0.203,0.323]	μ	-0.001 [-0.101,0.14]	μ	0.000 [-0.120,0.122]
σ	0.021 [0.019,0.023]	σ	0.028 [0.024,0.033]	σ	0.028 [0.025, 0.032]	α	0.021 [0.019,0.022]
		ν	2.468 [2.325, 2.514]	β	1.001 [0.952, 1.053]	β	0.064 [0.042, 0.086]
				ν	2.469 [2.221,2.66]	δ	0.349 [0.247, 0.424]
BIC	-1856.41		-1922.24		-1880.75		-1923.88

Table 4.4 Distribution Fitting for Singapore Jet Fuel/Kerosene Front Month Futures daily return. Student and NIG distributions exhibit the best fits in regards of the BIC criteria.

<i>Gaussian</i>		<i>Student</i>		<i>ASTD</i>		<i>NIG</i>	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	-0.000 [-0.210 ,0.215]	μ	0.000 [-0.213,0.223]	μ	-0.000 [-0.11,0.12]	μ	0.000 [-0.110,0.132]
σ	0.017 [0.014,0.023]	σ	0.066 [0.028,0.088]	σ	0.066 [0.038,0.091]	α	0.012 [0.002,0.021]
		ν	2.010 [1.968,2.13]	β	0.938 [0.842,1.201]	β	-0.112 [-0.154,-0.049]
				ν	2.010 [1.840,1.260]	δ	0.100 [0.042, 0.152]
BIC	-2011.99		-2260.43		-2192.86		-2245.14

Table 4.5 Distribution Fitting for Singapore Gasoil Front Month Futures returns. Student and NIG distributions exhibit the best fits in regards of the BIC criteria.

<i>Gaussian</i>		<i>Student</i>		<i>ASTD</i>		<i>NIG</i>	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	0.000 [-0.20 ,0.205]	μ	0.000 [-0.23,0.22]	μ	-0.000 [-0.112,0.122]	μ	0.000 [-0.110, 0.101]
σ	0.018 [0.015,0.021]	σ	0.082 [0.059,0.118]	σ	0.082 [0.041,0.128]	α	0.015 [0.09,0.021]
		ν	2.010 [1.64,2.48]	β	0.958 [0.847,1.131]	β	-0.079 [-0.104,-0.045]
				ν	2.010 [1.740,2.460]	δ	0.100 [0.042, 0.152]
BIC	-1949.60		-2151.21		-2082.55		-2141.86

Table 4.6 Distribution Fitting for Regrade Front Month Futures returns. Student and NIG distributions exhibit the best fits in regards of the BIC criteria.

Gaussian		Student		ASTD		NIG	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	-0.000 [-0.22 ,0.205]	μ	-0.003 [-0.28,0.2]	μ	-0.005 [-0.112, 0.122]	μ	-0.001 [-0.10, 0.111]
σ	0.287 [0.265,0.312]	σ	0.911 [0.88,1.08]	σ	0.911 [0.85,1.02]	α	0.181 [0.184,0.221]
		ν	2.010 [1.842, 2.268]	β	0.983 [0.652,1.201]	β	0.038 [0.018,0.054]
				ν	2.010 [1.540,2.560]	δ	0.100 [0.042, 0.152]
BIC	128.943		-177.201		-117.776		-156.712

mate the clustering effect. Thus, for this purpose we consider the GARCH-type models. The GARCH process introduced by [Bollerslev (1987)] and its different variations have gained increasing prominence for modelling financial asset over the last decade. The GARCH process presents three particular features compared to other modellings. First it assumes that the present conditional variances is linearly linked to the past conditional variances and to past market squared return. Second for an accurate calibration GARCH requires large datasets. Third the models transfers through volatility pastern the risk premium of the underlings price. The classic GARCH framework bring obviously significant improvements in term of econometric description compared to the classic Gaussian model, and yet Bollerslev's GARCH remains still under an assumption of normally distributed innovations.

Further under the framework described by [Bollerslev (1987)], ϵ_t follows a GARCH(1,1) process is

$$\epsilon_t | \phi_{t-1} \propto N(0, h_t) \text{ or } z_t \propto N(0, 1) \quad (4.1)$$

$$h_t = \alpha_0 + \alpha_1 \cdot \epsilon_{t-1}^2 + \beta_1 \cdot h_{t-1} \quad (4.2)$$

where ϕ_t is the corresponding σ -algebra generated by the previous an present information; The unconditional variance is $h_0 = \frac{\alpha_0}{(1-\alpha_1-\beta_1)}$. GARCH model assumes that the conditional variance is a linear function of past squared disturbances and the past conditional variance, genuinely making h_t ϕ_t -predictable.

Generalizing the above given definition a GARCH(p,q) follows

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_q \epsilon_{t-q}^2 + \beta_1 h_{t-1} + \dots + \beta_p h_{t-p}^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}$$

where $p \geq 0, q \geq 0, \alpha_0 > 0, \alpha_i > 0, i=1, \dots, q; \beta_i \geq 0, i=1, \dots, p.$ and $\sum_{i=1}^q \alpha_i + \sum_{i=1}^p \beta_i < 1$

5.

⁵For insuring the covariance stationarity of the GARCH(p,q) it is imposed that the persistence is inferior to the unity

In order to mitigate the existence of significant kurtosis and skewness effects assets returns an extension of the GARCH model could be the introduction of non-Gaussian (Generalized Hyperbolic) innovations, with the parametrization introduced in the previous section:

$$z_t \propto GH(\lambda; \alpha; \beta; \mu; \delta) \text{ or} \quad (4.3)$$

$$\epsilon_t | \phi_{t-1} \propto GH\left(\lambda; \frac{\alpha}{\sqrt{h_t}}; \frac{\beta}{\sqrt{h_t}}; \mu\sqrt{h_t}; \delta\sqrt{h_t}\right) \quad (4.4)$$

$$h_t = \alpha_0 + \alpha_1 \cdot \epsilon_{t-1} + \beta_1 \cdot h_{t-1} \quad (4.5)$$

GARCH diffusion presents in term of pricing three particular features compared to other modellings. First the GARCH derivatives prices depends of risk premium embedded in the underlying asset. Second the GARCH pricing model is non-Markovian and is an interesting alternative for markets with serial dependency. Third the GARCH models might explain some valuation biases of out-of the money options, associated with classic models.

We estimated through max-likelihood method the volatility models presented above. Table 4.7 exhibits the results of fitting of GARCH-type model with normal, Student and NIG innovations for Singapore Jet Fuel daily returns. The models with NIG innovation show a superior fitting performance in terms of BIC /AIC. APARCH model fits better than the rest of the GARCH family for all three innovation types. The APARCH with NIG innovations and with a power factor (δ) of 0.94 exhibits the best features, underlining the leverage effects in volatility.

4.2.3.1 Markov Regime Switching GARCH models

Despite adding value for modeling assets with leptokurtic behavior single regime GARCH models, fail to capture time of a transition between a low risk and high risk regime. An alternative was introduced by [Haas *et al.* (2004)] with the switching regime GARCH model detailed in the below formula.

$$h_t = \begin{cases} \alpha_0^1 + \alpha_1^1 \cdot \epsilon_{t-1}^2 + \beta_1^1 \cdot h_{t-1}; \\ \alpha_0^2 + \alpha_1^2 \cdot \epsilon_{t-1}^2 + \beta_1^2 \cdot h_{t-1}; \end{cases} \quad (4.6)$$

Middle distillates markets are particularly concerned by this feature due to the variation in liquidity. Thus one volatility regime can correspond to thin liquidity conditions while another to appropriate levels of liquidity.

The results of fitting the switching GARCH model for the underlyings studied in this chapter are exhibited in Table 4.8. The occurrence of two distinct states with statistically significant probability of transition is confirmed for Singapore Gasoil and Regrade. The particularity of these two underlyings is the fact that they trade on thinner liquidity than

Table 4.7 Fitting of GARCH-type model with normal, Student and NIG innovations for Singapore Jet Fuel daily returns

Model	Normal				STD				NIG			
	Param.	Est.	Std. Error	p-val	Param.	Est.	Std. Error	p-val	Param.	Est.	Std. Error	p-val
GARCH	ω	0.000	0.000	0.000	ω	0.000	0.000	0.007	ω	0.000	0.000	0.001
	α_1	0.002	0.000	0.000	α_1	0.347	0.061	0.000	α_1	0.307	0.055	0.000
	β_1	0.994	0.000	0.000	β_1	0.652	0.064	0.000	β_1	0.692	0.043	0.000
					ν	2.314	0.074	0.000	α	-0.228	0.047	0.000
BIC/AIC	-5.351	-5.333			-6.714	-6.690			-6.770	-6.739		
eGARCH	ω	-0.040	0.004	0.000	ω	-0.904	0.265	0.001	ω	-0.642	0.201	0.001
	α_1	-0.061	0.018	0.001	α_1	-0.004	0.040	0.911	α_1	-0.005	0.066	-
	β_1	0.994	0.000	0.000	β_1	0.895	0.027	0.000	β_1	0.907	0.026	0.000
	γ_1	0.033	0.013	0.011	γ_1	0.558	0.057	0.000	γ_1	0.886	0.259	0.001
					ν	2.100	0.037	0.000	α	-0.264	0.049	0.000
BIC/AIC	-5.424	-5.399			-6.677	-6.646			-6.776	-6.740		
iGARCH	ω	0.000	0.000	0.900	ω	0.000	0.000	0.000	ω	0.000	0.000	0.000
	α_1	0.000	0.001	0.995	α_1	0.348	0.042	0.000	α_1	0.308	0.039	0.000
	β_1	1.000			β_1				β_1	0.692		
					ν	2.313	0.040	0.000	α	-0.229	0.046	0.000
BIC/AIC	-5.355	-5.343			-6.717	-6.698			-6.773	-6.748		
gjrGARCH	ω	0.000	0.000	0.010	ω	0.000	0.000	0.071	ω	0.000	0.000	0.001
	α_1	0.000	0.005	1.000	α_1	0.314	0.089	0.000	α_1	0.274	0.066	0.000
	β_1	0.980	0.002	0.000	β_1	0.649	0.098	0.000	β_1	0.690	0.044	0.000
	γ_1	0.032	0.010	0.001	γ_1	0.073	0.127	0.566	γ_1	0.081	0.109	0.459
				ν	2.318	0.097	0.000	α	-0.231	0.047	0.000	
BIC/AIC	-5.406	-5.382			-6.712	-6.681			-6.768	-6.731		
APARCH	ω	0.001	0.000	0.000	ω	0.000	0.000	0.520	ω	0.003	0.005	0.573
	α_1	0.007	0.002	0.001	α_1	1.000	0.335	0.003	α_1	1.000	0.229	0.000
	β_1	0.991	0.000	0.000	β_1	0.636	0.040	0.000	β_1	0.670	0.054	0.000
	γ_1	1.000	0.001	0.000	γ_1	0.093	0.080	0.245	γ_1	0.081	0.109	0.455
	δ	0.432	0.038	0.000	δ	1.646	0.319	0.000	δ	0.939	0.365	0.010
					ν	2.128	0.053	0.000	α	-0.302	0.049	0.000
BIC/AIC	-5.538	-5.507			-6.748	-6.711			-6.833	-6.790		

the other three markets considered in this study. This finding confirms our initial assumption and is a valuable learning when testing the risk related to proxy hedging.

Table 4.8 Switching Regime GARCH models fitting for ICE Brent, ICE Low Sulphur Gasoil, Singapore Gasoil, Jet Fuel/Kerosene and Regrade

	State1			State2			P_1	P_2
	α_{01}	α_{11}	β_1	α_{02}	α_{12}	β_2		
Regrade	0.0008	0.1677	0.3855	0.1053	0.0266	0.7489	0.8670	0.8546
Sing GO	0.0001	0.0001	0.0001	0.0001	0.0001	0.9484	0.9532	0.2853
Kerosene	0.0001	0.0423	0.0001	0.0008	0.0001	0.7934	0.9442	0.9145
Brent	0.0001	0.2862	0.5219	0.0005	0.0016	0.8411	0.9673	1.0000
LSGO	0.0001	0.0004	0.0001	0.0001	0.0863	0.7308	0.9946	0.0000

4.3 Proxy hedging

In an incomplete financial market, it is hardly ever possible to find the hedging instrument that perfectly mirrors a given price risk. Most of refiners and airline companies attempt to hedge around 80% of their jet fuel exposure ([Adams and Gerner (2012)]). In large part, they do this by purchasing futures contracts on crude oil, the feedstock for producing jet fuel, or other oil derivative products such as heating oil, used in USA, and gasoil in Europe. The regional prices of these commodities with jet fuel are correlated in the long run, however in the short term, price co-movements are asynchronous. This erratic relationship defines the basis risk, which is the financial risk occurred when the chosen "proxy-hedge" does not entirely offset the price risk of the main underlying asset ([Kamara and Siegel (1987)], [Ankirchner and Imkeller (2011)]).

This is clearly seen from the 1 Month rolling correlation plot in Figure 4.6, which shows that despite the 'obviously' highly correlated dynamics of the spot prices exhibited earlier in Figure 4.1, there exists basis risk when hedging in oil markets and this is mainly explained by product, location and time factors.

Another representation of the basis risk is exhibited in Figure 4.7 which shows the evolution of the price difference between Singapore Gasoil and Jet Kerosene, known also as Regrade. All this is to show that even when a proxy instrument appears to be a highly correlated instrument, the basis risk associated with it undermines the effectiveness of the proxy-hedge as it exacerbates the cash flow volatility that the hedge is designed to reduce.

A major challenge in "proxy hedging" ([Viken and Thorsrud (2014)]) consists in finding the proxy instrument which minimizes basis risk and hedge volatility. As such, an Asian company willing to cover its long jet fuel exposure has the following options:

- *Jet Fuel direct hedge*: sell Singapore Jet Kerosene Futures or first line swap

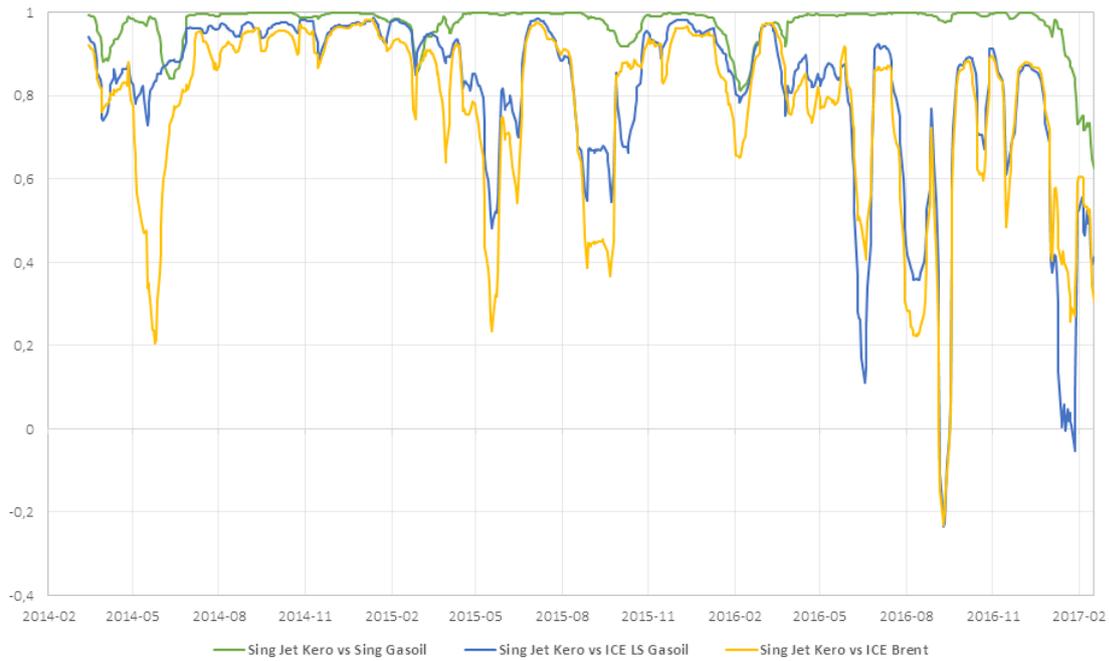


Fig. 4.6 1M Rolling Correlation of Front Month Futures

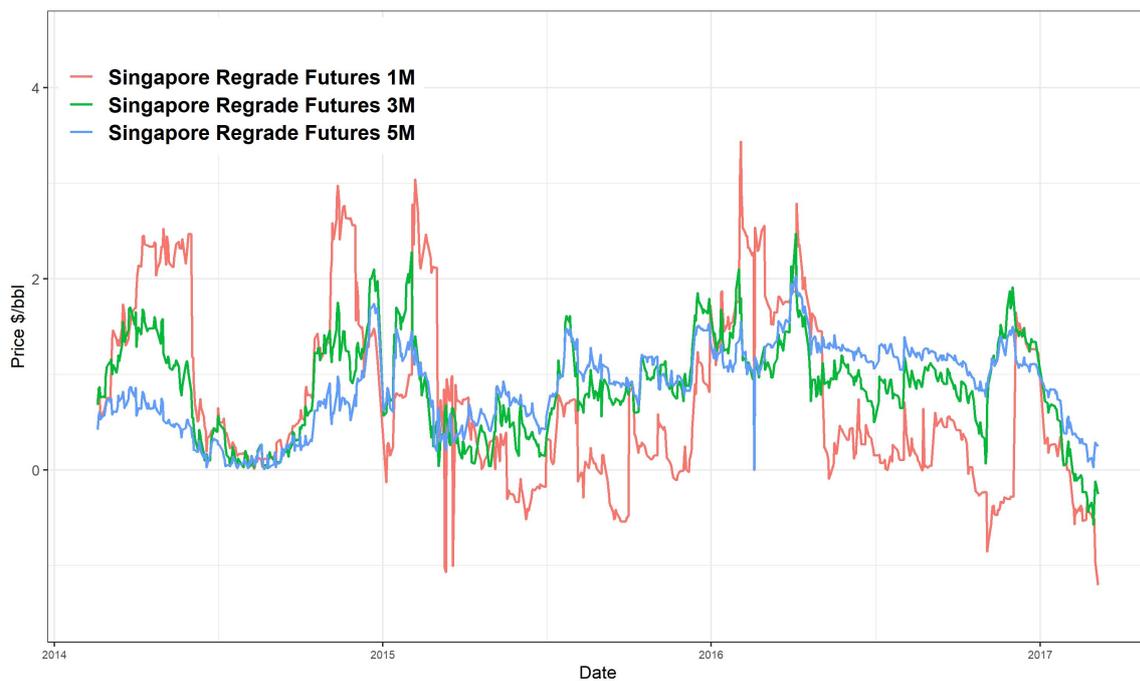


Fig. 4.7 Evolution of Singapore Regrade Futures price

- *Crude 'proxy hedge'*: sell ICE Brent Crude Futures or first line swap
- *Gasoil 'proxy hedge'*: sell ICE LS Gasoil or Singapore 0.5% Gasoil Futures
- *Basis risk 'slice & dice proxy hedge'*: hedge jet fuel price components opportunistically (Firstly hedge crude oil component, secondly Brent/Gasoil crack, and lastly Regrade).

Companies exposed to jet fuel price risk prefer to hedge their exposure using crude oil or

Gasoil contracts even if jet fuel future contracts are also available because the liquidity on jet fuel is very thin. Figure 4.8 exhibits volumes of the considered dataset on a logarithmic scale. We notice that ICE Brent Crude oil and LS Gasoil futures are significantly more liquid than the Singapore Kerosene and Gasoil 0.5% futures. If jet fuel contracts were available at the same "cost" as crude oil contracts, then clearly this would be a better alternative.

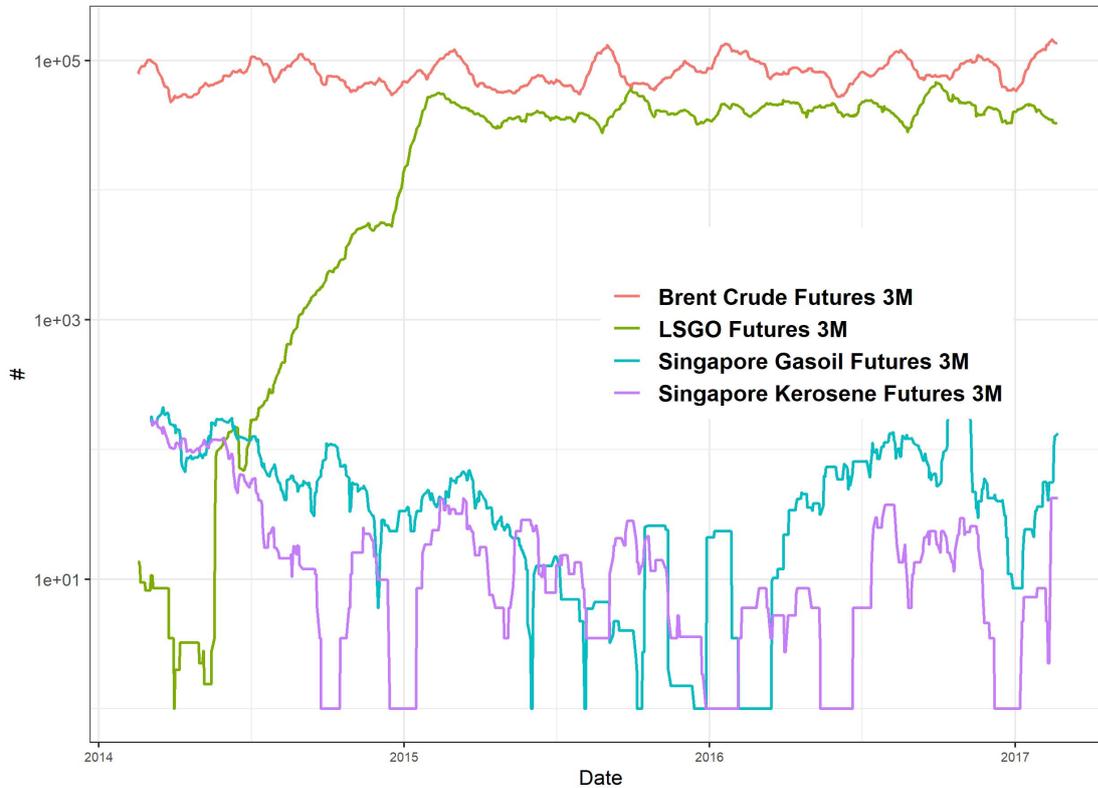


Fig. 4.8 ICE Brent, ICE LS Gasoil, Singapore Gasoil and Jet Fuel Futures Liquidity

As such, if an Asian airline company wants to cover its jet fuel price risk, since the volumes exchanged on this market are thin, it might use one of the 'proxy-hedge' options described earlier. However, choosing the right one means making a trade-off between liquidity and basis.

The current literature focuses mainly on the risks related to level forecasting when using a proxy-hedge, but ignores completely the density forecasting. The main issue with proxy hedging is the fact that markets have different depth. On one hand a shock in the Brent market might not be fully reflected in the Jet fuel market. On the other hand a small variation in the Brent Market might generate a shock in the Jet fuel market due to difference in liquidity. The basis risk of proxy hedge using both plain or derivatives based strategies is generated also by the differences in the distribution features thereby underlying the need of testing the density forecasting ability.

For testing the proxy-hedging with Brent, gasoil or regrade, a trader exposed to jet fuel price risk should assess the density forecasting capacity of an econometric risk model. Thus

a model estimated on Brent or Gasoil returns should be tested in terms of density forecasting on the jet fuel prices. Furthermore these test would be a good indicator of how the proxy hedging is affected by contagion, mainly in periods were regime switches are observed.

4.4 Forecasting densities

4.4.1 Gneiting test

This section describes the technique for reaching the main goal of this study, the testing in terms of density forecasting of proxy-hedging strategies. In a recent paper [Gneiting and Ranjan (2011)] proposed a test that develops the weighting approach of [Amisano and Giacomini (2007)] but avoids counter intuitive inferences. We use this test for assessing the density forecasting in proxy hedging.

Gneiting's test aims to built a proper score with the respect of the above definition based on appropriately weighted versions of the continuous ranked probability score(CPRS). For any density function $f(y)$ with a cumulative distribution function $F(z) = \int_{-\infty}^z f(y)dy$ the continuous ranked probability score is then defined as

$$CPRS(F, y) = \int_{-\infty}^{\infty} PS(F(r), 1(y \leq r)) dr \quad (4.7)$$

where

$$PS(F(r), 1(y \leq r)) = (1(y \leq r) - F(r))^2 \quad (4.8)$$

is the Brier probability score for the probability forecast $F_t(r) = \int_{-\infty}^r f(y)dy$ of the event $y \leq r$

The weighted probability score described by [Matheson and Winkler (1976)] and [Gneiting and Raftery (2007)] is written as :

$$S^w(f, y) = - \int_{-\infty}^{\infty} PS(F(r), 1(y \leq r)) \cdot w_r(r) dr = - \int_{-\infty}^{\infty} (1(y \leq r) - F(r))^2 \cdot w_r(r) dr \quad (4.9)$$

where the weighting function $w_r(r)$ applies to the forms presented in equation 4.7. In a discrete form the above score can be approximated by assuming an I steps equidistant discretization of a target region with the boundaries y_l, y_u

$$S_f^w(f, y) = \frac{y_u - y_l}{I - 1} \sum_{i=1}^I w(y_i) PS(F(y_i), I(y \leq y_i)) \quad (4.10)$$

$y_i = y_l + i \frac{y_u - y_l}{I}$ The test based on the following statistic which is leveraged from the Amisano-Giacomini test :

$$Z_n = \frac{E(S_f^w(f, y) - S_f^w(g, y))}{\widehat{\omega}_n} \quad (4.11)$$

where

$$E_t(S_f^w(f, y)) = \frac{1}{n - k + 1} \sum_{t=m}^{m+n-k} S(f_{t+k}, y_{t+k}) \quad (4.12)$$

$$E_t(S_f^w(g, y)) = \frac{1}{n - k + 1} \sum_{t=m}^{m+n-k} S(g_{t+k}, y_{t+k}) \quad (4.13)$$

and $\widehat{\omega}_n$ is an estimate of $var(\sqrt{n}(E_t(S_f^w(f, y)) - E_t(S_f^w(g, y)))$

4.5 Backtesting results of proxy-hedging

Density forecasting techniques provide with insightful information for risk assessment purposes especially in the commodities markets dominated by non-Gaussian behavior and volatility clustering ([Frunza and Guégan (2013)]).

Thus based on the specifications of the Gneitting test presented above we built a testing process for the proxy hedging strategies. The full dataset contains the daily prices of Jetfuel, ICE Brent, ICE gasoil or Singapore Gasoil between 01/01/2014 and 01/03/2017. The testing process has the following steps:

- (1) A model (M1) is estimated on the daily returns of the proxy (ICE Brent, ICE gasoil or Singapore Gasoil). The data set contains a (*out of sample*) the first 250 consecutive days of the considered full sample.
- (2) A model (M2) is estimated on the daily returns of the jet fuel prices. The data set (*in sample*) contains a window of 250 consecutive days, which starts immediately after the end of the out of sample dataset.
- (3) The Gneitting test score is computed for comparing the model M1 estimated *out of sample* on the proxy with model M2 estimated on the actual *in sample* jet fuel returns
- (4) The *out of sample* window is rolled over with one day and same is for the *in sample* window. Steps 1-3 are repeated until the end of the full sample
- (5) A time series of Gneitting test scores is built.

The previous sections underlined that NIG distribution exhibits good fitting features for all the underlying studied in this article. Therefore we will consider the NIG model for both *out of sample* and *in sample*. Therefore the test score will assess the power of the density fitted on the proxy to forecast the jet fuel distribution feature.

Figure 4.9 shows the evolution of the testing Score for NIG model, where as the proxy hedge is realized with Singapore Gasoil. Until July 2016 the score rejects at 99% confidence level the null hypothesis that the model fitted on the proxy is similar to the model fitted on the jet fuel and in fact the proxy provides with better results. After July 2016 the score enter in the confidence region thereby not rejecting the null hypothesis. Towards 2017 the NIG model fitted on proxy losses gradually from its forecasting capacity but remains close to the confidence region

Figure 4.10 shows the evolution of the testing Score for NIG model, where as the proxy hedge is realized with ICE Low Sulphur Gasoil. Until July 2016 the score does not rejects at 99% confidence level the null hypothesis that the model fitted on the proxy is similar to the model fitted on the jet fuel. After July 2016 the NIG model fitted on Low Sulphur Gasoil proxy has lost its forecasting capacity and became inappropriate.

Figure 4.11 shows the evolution of the testing Score for NIG model, where as the proxy

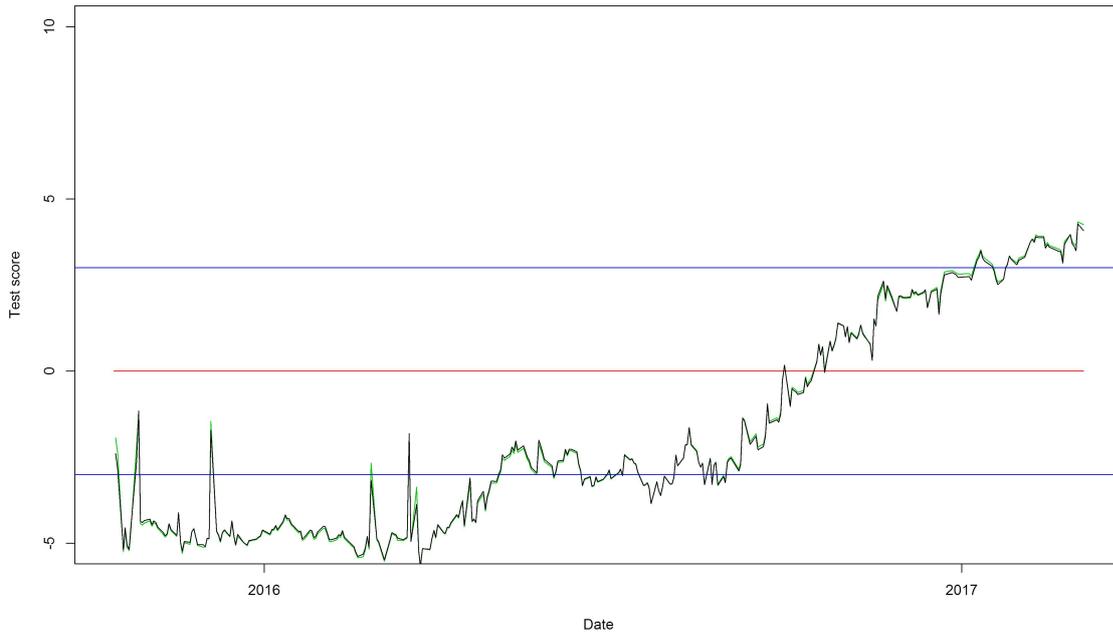


Fig. 4.9 Evolution of the Gneiting Test Score for NIG model with Singapore Gasoil. The horizontal solid lines are the boundaries out of which test's null hypothesis is rejected



Fig. 4.10 Evolution of the Gneiting Test Score for NIG model and ICE Low Sulphur Gasoil. The horizontal solid lines are the boundaries out of which test's null hypothesis is rejected

hedge is realized with ICE Brent. Until July 2016 the score does not reject at 99% confidence level the null hypothesis that the model fitted on the proxy is similar to the model

fitted on the jet fuel. After July 2016 the NIG model fitted on Brent returns has lost massively its forecasting ability and became inappropriate.



Fig. 4.11 Evolution of the Gneiting Test Score for NIG model with ICE Brent. The horizontal solid lines are the boundaries out of which test's null hypothesis is rejected

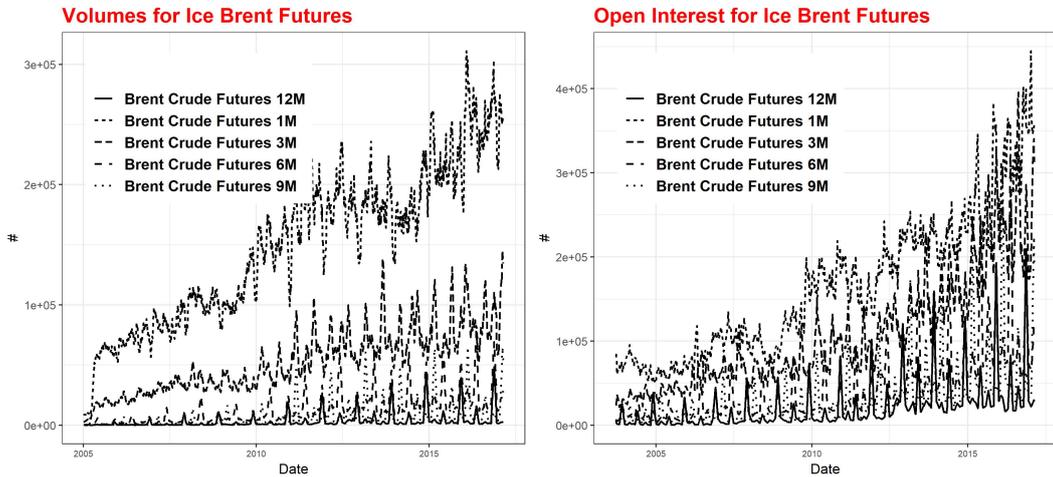


Fig. 4.12 Left: Volume Ice Brent Futures. Right: Open Interest for Ice Brent Futures

The three charts (Figure 4.11, Figure 4.10 and Figure 4.9) and of the testing scores exhibit a positive trend after July 2016 due to the fact that the NIG model losses its forecasting capability in terms of density. The forecasting test specified above is implemented over rolling windows, hence the newer data is included in the test gradually. Thus the test score does not have a jump in 2016 but shows a continuous trend. Moreover the

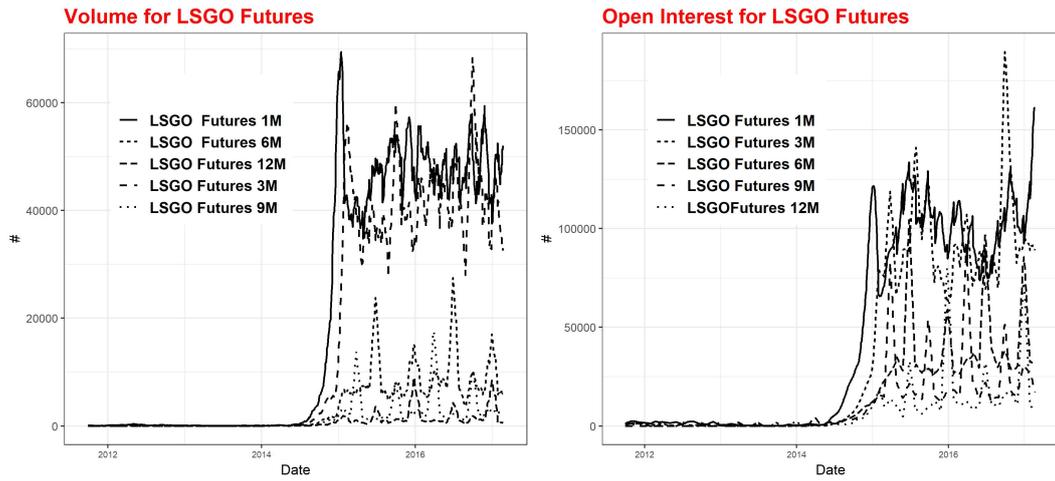


Fig. 4.13 Left: Volume for ICE Low Sulphur Gasoil Futures. Right: Open Interest for ICE Low Sulphur Gasoil Futures

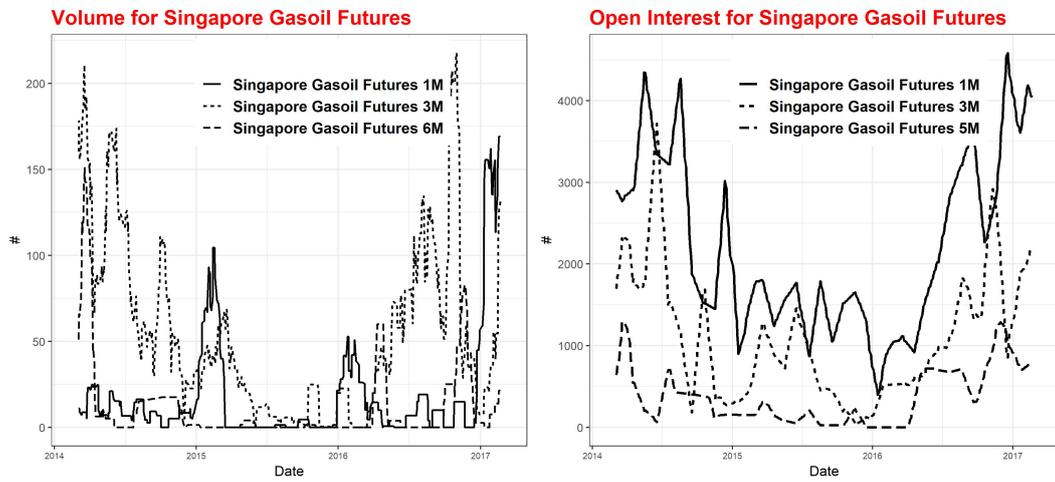


Fig. 4.14 Left: Volume for Singapore Gasoil 0.5% (Platts)Futures. Right: Open Interest for Singapore Gasoil 0.5% (Platts)Futures

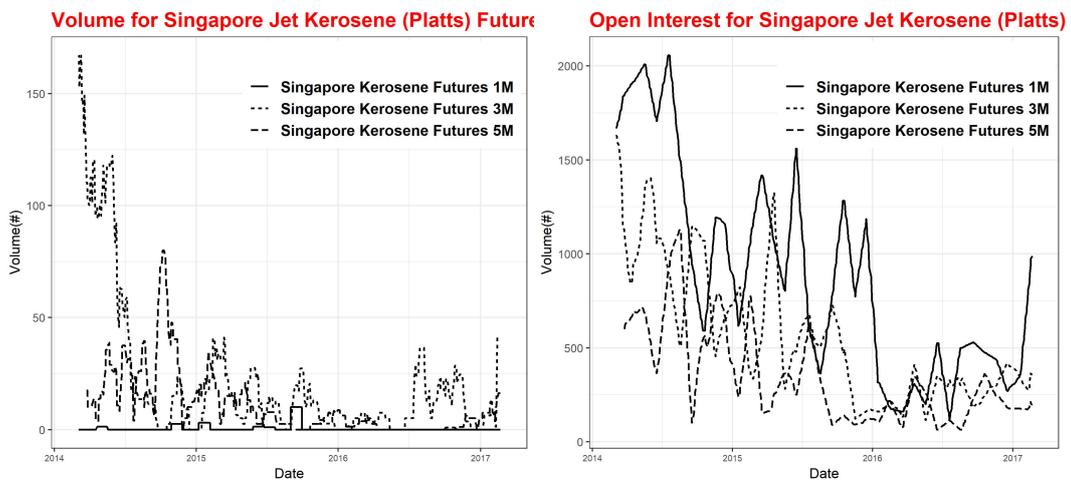


Fig. 4.15 Left: Volume for Singapore Jet Kerosene (Platts)Futures. Right: Open Interest for Singapore Jet Kerosene (Platts)Futures

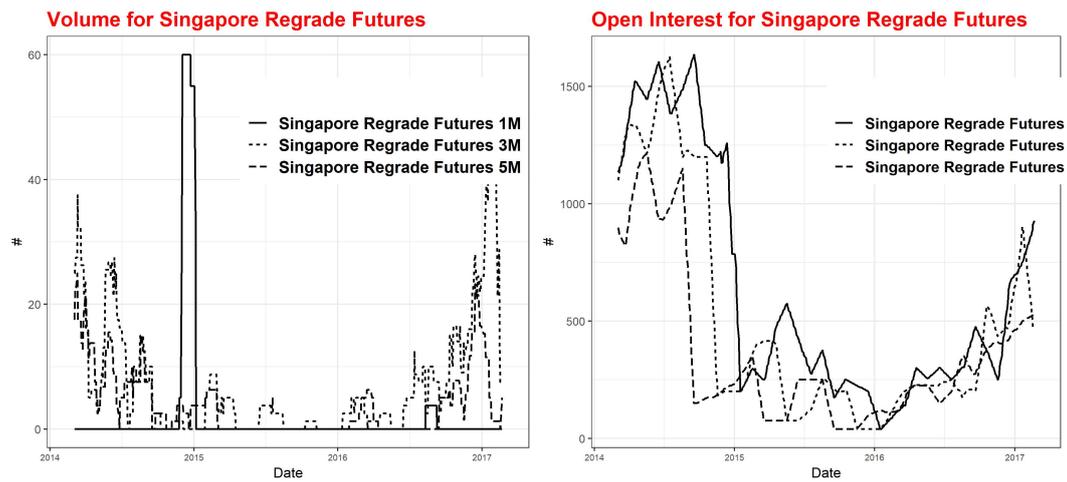


Fig. 4.16 Left: Volume Singapore Regrade Futures. Right: Open Interest for Singapore Regrade Futures

contagion of market coupling related to the extreme events, contributes to the stability of the hedging process with proxies. Nevertheless if the contagion of market decoupling persists during regime changes, the hedging can become less stable especially in condition of high volatility. Interestingly in the studied case the contagion effect diminishes concomitantly the interdependence for all four markets after July 2016, Brent, Jetfuel, Singapore and Ice Gasoil, thereby leaving the hedger with very limited options in terms of basis risk management.

The Singapore and London gasoil markets are different in term of returns time series, but have structural similarities in term of density features. Thus a trader (ie. airliner) could be tempted to interpret that he has different hedging alternatives in term of markets, but from a risk management perspective, the three studied alternatives exhibit a similar behavior. Changes in behavioral regimes can alter the proxy-hedging process especially due to the fact that contagion impact the coupling and the decoupling of the four markets in similar ways.

4.6 Conclusions

This chapter explores the topic of proxy hedging in middle distillates market with a focus on jet fuel. The research addresses the problem of a refinery or an airline company that hedges its jet fuel price risk with proxy instruments including Brent futures and gasoil futures. The problem is studied in two steps: first the various econometric models with fat tails and volatility clustering are explored in relation with the returns of daily time series and second the proxy hedging is tested based on density forecasts methods using the results for the first step.

The results from the first part show that NIG distribution, APARCH specifications of the volatility dynamics capture in an appropriate manner the behavior of jet fuel, Brent and gasoil prices. Also GARCH switching regimes models are good candidates for analyzing the markets that might exhibit thin liquidity.

The second part shows that the NIG model fitted on the Singapore Gasoil as proxy has the best density forecasting abilities from the considered choices. The main finding of this study is that a trader exposed to jet fuel price risk might think he has different hedging alternatives in term of markets, where in reality from a risk management perspective, the alternatives could exhibit a similar behavior in term of density forecasting capability. Contagion impacts negatively the proxy-hedging especially when the behavior of jetfuel and its proxy-hedging are decoupling at the same time, thereby leaving the trader with limited options.

A future direction for our research is the consideration of transaction costs in the Gneiting test score function, as trading future contracts usually involves brokerage fees and liquidity across different product maturities. This leads to addressing the problem of dimensionality, as it would be necessary to consider a technique such as approximate dynamic programming to produce a hedging policy that reflects such costs.

Chapter 5

Outlook

Life can only be understood backwards; but it must be lived forwards. Søren Kierkegaard, Danish philosopher,

Abstract *The aim of this chapter is to resume the conclusion of the thesis and to expose the future direction of development for the current research.*

The results presented in this thesis constitute a solid starting point for an entire new area of research concerning the alternative markets. With the development of crypto-currencies and the increasing demand for new investment opportunities, the alternative markets begin to play a key role in the global financial universe.

As shown in the previous chapters, alternative markets require a different modeling approach due to their unique particularities. Moreover, a paradigm shift from the classical theories of financial market governed by Fama's formalism is necessary, to propose efficient tools for those operating on alternative markets. Valuation and hedging in these markets might require a fresh new perspective on most formalism used in finance. For instance, the valuation of an instrument is defined unanimously as an expectation. If the price of an assets has a quasi-stable time trajectory, defining value as an expectation would make sense. But, in many cases the assets follow patterns characterized by spikes or jumps and in such condition the relevance of the expectation as a valuation metric is significantly low.

I showed that in the case of hedging in alternative market, the traditional formalism of measuring the performance of a hedging strategy based on the expected value of the deviations between the target and the hedge is not sound. The use of density forecasting approaches is better fit to assess the efficiency of hedging strategies in alternative markets. This area of research merits a particular attention and can be expanded in future studies.

Net Present Value is a common tool for valuing companies and projects. This formalism is not robust when dealing with mining projects (commodities or crypto-currencies). In fact when the generated cash-flows of a project are indexed on a stochastic process, NVP

generates inconsistent results. Real Option theories is a a sound alternative that provides with appropriate results. Expanding the scope of real option theory models to other fields of finance is one of the foreseeable axis of research.

Bubbles are generated by investors' urge to step-in a promising market and by price pumping trades. In this thesis I showed the presence of bubble effects in crypto-currencies with customized tests able to detect the timing of various bubbles. I analyzed the evolution of a representative sample crypto-currencies over time encompassing both high and low liquidity coins. The results showed that several crypto-currencies prices had episodes of rapid inflation in 2017 related to the Bitcoin bubble and a few emerging coins saw their prices pumped by speculative actions. Testing for bubbles can be implemented to other assets types and markets, thereby constituting an additional direction of development for my work.

Another future direction for my research is the consideration of liquidity on various exchanges, as trading crypto-currencies and oil distillates usually involves different brokerage fees and liquidity across different exchanges. Another direction is the consideration of transaction costs in the Gneiting test score function, as trading future contracts usually involves brokerage fees and liquidity across different product maturities. This leads to addressing the problem of dimensionality, as it would be necessary to consider a technique such as approximate dynamic programming to produce a hedging policy that reflects such costs.

Last but not the least, I believe that ethical issues when investing in alternative markets should be studied in a future work. The democratization of personal finance and investment brought a lot of individual investors outside the scope of traditional markets. Thus, member of the public have access to alternative investments including crypto-currencies, commodities or illiquid assets. These markets are very complex even for professionals of investment, thereby bringing the questions of how ethical is proposing an alternative instrument to individual investors. Such markets are inefficient by nature and the information is not spread homogeneously amongst market players. Members of the public are amongst those deprived of premium information and face of scarcity of data when they are investing in alternative markets. The ethics of investing in a alternative market is a stringent point that needs to be addressed in the near future.

Annexes

In this annex we recall some econometric tools used in several parts of this document.

5.1 Diebold's test

Diebold's approach is straightforward and considers to series of forecast (y_t^1) and (y_t^2) for the real time series y_t for $t \in \{1, 2, \dots, T\}$. The associated prediction errors e^1 and e^2 with the associated loss functions $g(y_t, y_t^1) = g(e_t^1)$ and $g(y_t, y_t^2) = g(e_t^2)$. The difference between the loss functions for the two forecast is $d_t = g(e_t^1) - g(e_t^2)$ with an empirical expected value.

$$\bar{d} = \frac{1}{T} \sum_1^T g(e_t^1) - g(e_t^2) \quad (5.1)$$

The null and alternative hypothesis of equal forecast accuracy for two forecasts is resumed as:

- H0: $\mathbf{E}(g(e_{it})) = \mathbf{E}(g(e_{jt}))$, or $E(d_t) = 0$
- Ha $\mathbf{E}(g(e_{it})) \neq \mathbf{E}(g(e_{jt}))$, or $E(d_t) \neq 0$

The null hypothesis is that the two forecasts have the same accuracy. The alternative hypothesis is that the two forecasts have different levels of accuracy.

With the notations $\mu = E(d_t)$ and $f_d(0) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} E(d_t - \mu)(d_t - k) - \mu$

the statistics for the null hypothesis H_0 is:

$$\sqrt{T}(\bar{d} - \mu) \rightarrow \mathbf{N}(0, 2\pi f_d(0)) \quad (5.2)$$

$$\frac{\sqrt{T}(\bar{d} - \mu)}{2\pi f_d(0)} \rightarrow \mathbf{N}(0, 1) \quad (5.3)$$

and can be rearranged as:

$$S_{DM} = \frac{\bar{d}}{2\pi \hat{f}_d(0)} \rightarrow N(0, 1) \quad (5.4)$$

where $\hat{f}_d(0)$ is a consistent estimator for $f_d(0)$ and S_{DM} denotes the Diebold-Marino statistic

Under the null hypothesis, the test statistics Diebold-Marino is asymptotically $\mathbf{N}(0, 1)$ distributed. The null hypothesis of no difference will be rejected if the computed Diebold-Marino statistic falls outside the range of $-z_\alpha/2$ to $z_\alpha/2$, that is if $|S_{DM}| > z_\alpha/2$, where $z_\alpha/2$ is the upper (or positive) z-value from the standard normal table corresponding to half of the desired a level of the test.

5.2 Vuong's test

This framework is implemented on the basis of a rolling window of size $m+k$ consisting of the past m observations which are used to fit a density and k time steps ahead observations for the forecast. Let's suppose that X_1, \dots, X_T is a random process which can be partitioned as $X_t = (Y_t, \mathbf{Z}_t)$ where Y_t is the variable of interest of size m and \mathbf{Z}_t is a vector of forecasts of size k .

The size of the total sample is $T = m + n + k$. At each moment t with $t = \{m, \dots, m + n\}$, two density forecasts based on the distribution f and g can be compared¹. In Vuong's framework, the forecasts are produced based on measurable functions of the data in the rolling estimation window and the test uses the likelihood ratio test for non-nested hypotheses, the differences being based on the Kullback-Leibler information criterion which measure the distance between a given distribution and the true distributions. The likelihood ratio can be written as:

$$\begin{aligned} LR_n(\hat{\theta}, \hat{\gamma}) &= L(\hat{\theta}) - L(\hat{\gamma}) \\ &= \sum_{i=1}^n \log(F(Y_i|Z_i, \hat{\theta})) - \log(G(Y_i|Z_i, \hat{\gamma})) \end{aligned}$$

The Vuong test can be summarized as following:

- H_0 The null hypothesis assuming that the two distribution f and g are equal

$$\mathbf{E} \left[\log \frac{f(Y_i|Z_i, \hat{\theta})}{g(Y_i|Z_i, \hat{\gamma})} \right] = 0 \quad (5.5)$$

- H_{af} the alternative hypothesis stating that $f(\theta)$ is a better choice than $g(\gamma)$

$$\mathbf{E} \left[\log \frac{f(Y_i|Z_i, \hat{\theta})}{g(Y_i|Z_i, \hat{\gamma})} \right] > 0 \quad (5.6)$$

- H_{ag} the alternative hypothesis stating that $G(\gamma)$ is a better choice than $F(\theta)$

$$\mathbf{E} \left[\log \frac{f(Y_i|Z_i, \hat{\theta})}{g(Y_i|Z_i, \hat{\gamma})} \right] < 0 \quad (5.7)$$

For testing the null hypothesis the following statistic is computed:

$$\frac{LR_n(\theta, \gamma)}{\sqrt{n \cdot \hat{\omega}_n}} \rightarrow N(0, 1) \quad (5.8)$$

where assuming the notation $d_i = \log(f(Y_i|Z_i, \hat{\theta})) - \log(g(Y_i|Z_i, \hat{\gamma}))$,

$$\hat{\omega}_n = \frac{1}{n} \sum_{t=1}^n [d_i]^2 - \left[\frac{1}{n} \sum_{t=1}^n d_i \right]^2 \quad (5.9)$$

¹This notation are kept for the rest of the chapter

5.3 Generalized Hyperbolic distributions

This brief review of the Generalized Hyperbolic distribution functions focuses on the Normal Inverse Gaussian function. The generic form of a Generalized Hyperbolic model is:

$$f(x; \lambda; \chi; \psi; \mu; \sigma; \gamma) = \frac{(\sqrt{\psi\chi})^{-\lambda} \psi^\lambda (\psi + \frac{\gamma^2}{\sigma^2})^{0.5-\lambda}}{\sqrt{2\pi\sigma} K_\lambda(\sqrt{\psi\chi})} \times \frac{K_{\lambda-0.5}(\sqrt{(\chi + \frac{(x-\mu)^2}{\sigma^2})(\psi + \frac{\gamma^2}{\sigma^2})}) e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi + \frac{(x-\mu)^2}{\sigma^2})(\psi + \frac{\gamma^2}{\sigma^2})})^{\lambda-0.5}},$$

where $K_\lambda(x)$ is the modified Bessel function of the third kind:

$$K_\lambda(x) = \frac{1}{2} \int_0^\infty y^{\lambda-1} e^{-\frac{x}{2}(y+y^{-1})} dy. \quad (5.10)$$

With properly chosen parameters, this distribution reduces to the following distributions:

- (1) $\lambda = 1$: hyperbolic distribution
- (2) $\lambda = -1/2$: NIG distribution
- (3) $\lambda = 1$ and $\xi \rightarrow 0$: Normal distribution
- (4) $\lambda = 1$ and $\xi \rightarrow 1$: Symmetric and asymmetric Laplace distribution
- (5) $\lambda = 1$ and $\chi \rightarrow \pm\xi$: Inverse Gaussian distribution
- (6) $\lambda = 1$ and $|\chi| \rightarrow 1$: Exponential distribution
- (7) $-\infty < \lambda < -2$: Asymmetric Student
- (8) $-\infty < \lambda < -2$ and $\beta = 0$: Symmetric Student
- (9) $\gamma = 0$ and $0 < \lambda < \infty$: Asymmetric Normal Gamma distribution

Among the Generalized Hyperbolic family, the Normal Inverse Gaussian distribution can be obtained by setting $\lambda = -\frac{1}{2}$ in the previous equation. Thus:

$$f(x; -\frac{1}{2}; \chi; \psi; \mu; \sigma; \gamma) = \frac{\chi^{\frac{1}{2}} (\psi + \frac{\gamma^2}{\sigma^2})}{\pi \sigma e^{\sqrt{-\psi\chi}}} \times \frac{K_1(\sqrt{(\chi + \frac{(x-\mu)^2}{\sigma^2})(\psi + \frac{\gamma^2}{\sigma^2})}) e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi + \frac{(x-\mu)^2}{\sigma^2})(\psi + \frac{\gamma^2}{\sigma^2})})}.$$

By changing the variables of the previous equation $c = \frac{1}{\sigma^2}$; $\beta = \frac{\gamma}{\sigma^2}$; $\delta = \sqrt{\frac{\chi}{c}}$; $\alpha = \sqrt{\frac{\psi}{\sigma^2} + \beta^2}$ we obtain a more popular representation, and the density of the $NIG(\alpha, \beta, \mu, \delta)$ distribution is equal to:

$$f_{NIG}(x; \alpha; \beta; \mu; \delta) = \frac{\delta \alpha \cdot \exp(\delta\gamma + \beta(x - \mu))}{\pi \cdot \sqrt{\delta^2 + (x - \mu)^2}} K_1(\alpha \sqrt{\delta^2 + (x - \mu)^2}).$$

The moments (mean, variance, skewness and kurtosis) are respectively equal to:

$$\begin{aligned} E(X) &= \mu + \delta \frac{\beta}{\gamma} \\ V(X) &= \delta \frac{\alpha^2}{\gamma^3} \\ S(X) &= 3 \frac{\beta}{\alpha \cdot \sqrt{\delta\gamma}} \\ E(X) &= 3 + 3(1 + 4(\frac{\beta}{\alpha})^2) \frac{1}{\delta\gamma} \end{aligned}$$

Thus, the NIG distribution allows for behavior characterized by heavy tails and strong asymmetries, depending on the parameters α , β and δ .

5.4 GARCH models

Few popular variations of the GARCH model include:

- *The integrated GARCH (IGARCH) model* The integrated GARCH model ([Engle and Bollerslev (1986)]) assumes that the persistence is one. Omitted structural breaks should be assessed before using an iGARCH model.

$$\epsilon_t | \phi_{t-1} \propto N(0, h_t) \text{ or } z_t \propto N(0, 1) \quad (5.11)$$

$$h_t = \alpha_0 + (1 - \beta_1) \cdot \epsilon_{t-1}^2 + \beta_1 \cdot h_{t-1} \quad (5.12)$$

- *The Glosten-Jagannathan-Runkle(GJR-GARCH) model* introduced by [Glosten *et al.* (1993)] adds asymmetry in the volatility process:

$$h_t = \alpha_0 + (\alpha_1 + c \cdot I_{t-1}) \cdot \epsilon_{t-1}^2 + \beta_1 \cdot h_{t-1} \quad (5.13)$$

where

$$I_{t-1} = \begin{cases} 0 & \epsilon_{t-1} \geq 0 \\ 1 & \epsilon_{t-1} < 0 \end{cases}$$

- *The exponential GARCH (EGARCH) model* introduced by [Nelson (1991)] aims to capture asymmetric reaction of volatility to the positive and negative information about the market. Volatility of the EGARCH model, which is measured by the conditional variance is an explicit multiplicative function of lagged innovations.

$$\log h_t = \alpha_t + \sum_{i=1}^{\infty} \beta_i g(Z_{t-k}) \quad (5.14)$$

where the function g is defined as $g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$, $g(Z_t)$ having a zero mean $E[g(Z_t)] = 0$. No restriction are imposed in this version of the GARCH model. EGARCH can also assess whether the shocks in variance are persistent or not.

- *The Asymmetric Power GARCH model (APARCH)* introduced by [Ding *et al.* (1993)] accounts for leverage effect and also the fact that the sample autocorrelation of absolute returns is higher than that of squared returns([Reider (2009)]).

$$h_t^{0.5 \cdot \zeta} = \alpha_0 + \sum_{i=1}^q \alpha_i (|\epsilon_{t-i}^\zeta| - \gamma_i \cdot \epsilon_{t-i})^\zeta + \sum_{i=1}^p \beta_i h_{t-i}^{0.5 \cdot \zeta} \quad (5.15)$$

It can be notice that equation 5.15 with $\zeta=2$ and $\gamma_i = 0$ matches the classic GARCH model with Gaussian innovations.

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