



**HAL**  
open science

# Three Essays in Financial Networks and Shock Propagation

Jonas Heipertz

► **To cite this version:**

Jonas Heipertz. Three Essays in Financial Networks and Shock Propagation. Economics and Finance. École des hautes études en sciences sociales (EHESS), 2019. English. NNT : 2019EHES0106 . tel-03168297

**HAL Id: tel-03168297**

**<https://theses.hal.science/tel-03168297>**

Submitted on 12 Mar 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

École des Hautes Études en Sciences Sociales



École doctorale de l'EHESS

UMR Paris Jourdan Sciences Économiques (UMR 8545)

## PhD Thesis

Submitted to the EHESS for the degree of Doctor of Philosophy in Economics

Analysis and Policy in Economics

Jonas Heipertz

## Three Essays in Financial Networks and Shock Propagation

**Advisor:** Romain Rancière

**Co-Advisor:** Amine Ouazad

Defended at the Paris School of Economics on December 2, 2019

### Jury:

|                   |                      |   |
|-------------------|----------------------|---|
| <b>President</b>  | Thomas Chaney        | Professor of Economics at Sciences Po                           |
| <b>Reviewers</b>  | Isabelle Méjean      | Professor of Economics at CREST-École Polytechnique             |
|                   | Guillaume Vuillemeys | Associate Professor of Finance at HEC Paris                     |
| <b>Examiner</b>   | Gilles Saint-Paul    | Professor of Economics at PSE and ENS                           |
| <b>Co-Advisor</b> | Amine Ouazad         | Professor of Economics at HEC Montreal                          |
| <b>Advisor</b>    | Romain Rancière      | Professor of Economics at the University of Southern California |

École des Hautes Études en Sciences Sociales



École doctorale de l'EHESS

UMR Paris Jourdan Sciences Économiques (UMR 8545)

## Thèse de Doctorat

Pour l'obtention du grade de docteur en Sciences Économiques à l'EHESS

Analyse et Politique Économiques

Jonas Heipertz

## Trois Essais sur les Réseaux Financiers et la Propagation de Chocs

**Directeur:** Romain Rancière

**Co-Directeur:** Amine Ouazad

Soutenue à la Paris School of Economics le 2 décembre, 2019

### Jury:

|                     |                      |   |
|---------------------|----------------------|---|
| <b>Président</b>    | Thomas Chaney        | Professor of Economics at Sciences Po                           |
| <b>Rapporteurs</b>  | Isabelle Méjean      | Professor of Economics at CREST-École Polytechnique             |
|                     | Guillaume Vuillemeay | Associate Professor of Finance at HEC Paris                     |
| <b>Examineur</b>    | Gilles Saint-Paul    | Professor of Economics at PSE and ENS                           |
| <b>Co-Directeur</b> | Amine Ouazad         | Professor of Economics at HEC Montreal                          |
| <b>Directeur</b>    | Romain Rancière      | Professor of Economics at the University of Southern California |

# Acknowledgments

First of all, I would like to thank my advisor, Romain Rancière. I am deeply grateful for the trust he has put in me and his continuous support during this long journey of my PhD - at the beginning by accepting me as his student and later also as a co-author. Through our frequent exchanges, I could learn immensely from him and obtained many fruitful impulses for my work. His generosity in sharing ideas and opening many doors have brought me to where I am now. I am very glad to have met him, professionally as well as personally.

I am also greatly indebted to Amine Ouazad, my co-advisor, who inspired me an intense taste for structural econometrics. I fondly remember and enjoyed the long days at PSE and Montreal when I had the chance to work together with him on our paper. He was and is a great teacher and I owe him many of the modeling and structural estimation skills that I have now. His feedback was decisive for the development and maturing of my research. I am very glad to have worked with him and hope that our collaborations will continue in the future.

I would also like to express my sincere gratitude to the members of the jury, Thomas Chaney, Isabelle Méjean, Gilles Saint-Paul, and Guillaume Vuillemeys for having accepted to read and evaluate my work. I especially thank my two referees Isabelle Méjean and Guillaume Vuillemeys. They had very little time to write their reports and I highly appreciate their patience, their thoughts, and their time. I am very grateful to Isabelle Méjean whose feedback on previous versions of my work was very influential for the first chapter of my thesis.

Next, I would like to thank Natacha Valla, on whom I could always count on. I first met Natacha at the CEPII where we started our joint work with the French securities holdings statistics. Since

then, Natacha has followed my research and provided important insights from the policy world. I owe her extremely valuable professional experiences and encounters and still enjoy working with her on a common project.

My other co-authors also deserve a special mention. I had the chance to work with Ana Maria Santacreu, Matthieu Bussière, Laurent Ferrara and Menzie Chinn. All of them have shared their knowledge with me and allowed me to develop competences in very diverse fields. Especially, I would like to thank Ana Maria, who invited me for research visits at the Saint-Louis FED.

Furthermore, I would like to thank all the researchers that I had the chance to meet in the institutions I worked. Among them, I will remember the discussions with the macro-finance group at the CEPII, with the SEMSI team at the Banque de France, and with economists at the EIB. I am also grateful to Gilles Saint-Paul and Nicolas Dromel for having entrusted me with the writing of the newsletter of the SCOR-PSE chair. Reading and digesting research frontier papers in the field of macro-economic risks for a wider audience has perfectly complemented my research.

The empirical work of this thesis could not have been possible without the Open Data Room of the Banque de France. Especially, I would like to acknowledge help from Christian Pfister and Zahira El Barni.

Last but not least, I thank my family, my friends, and Suzanne, the love of my life, who has supported me throughout these long years.

# Abstract

Financial inter-dependencies are since the financial crisis at the forefront of macroeconomic research and policy making. The world had painfully learned how small and localized events can travel through the global financial systems with huge repercussions for the real economy. Since then, many studies have analyzed the propagation properties of *given* financial exposure networks. Each day, however, large amounts of financial assets are traded and financial institutions' balance-sheets change in response to new information, regulation or monetary policy. Changes in exposures crucially affect the transmission of shocks. This thesis develops fully-fledged theoretical general equilibrium frameworks that show how financial networks emerge *endogenously* from trade in financial assets between heterogeneous institutions. I use a large multitude of micro and macro-level datasets including confidential data from the Banque de France on financial institutions' balance-sheets and securities holdings to structurally identify risk-preferences, institutions' beliefs about the distribution of future financial asset returns, and the specific constraints that drive financial network formation. The thesis also derives an explicit firm-level link of financial networks to an economy's productive structure.

Chapter 1 of the thesis shows how firm-level productivity shocks propagate through financial networks. If firms need external funds to finance capital expenditure, banks create linkages between them that go beyond their input-output relationships. These links affect aggregate output. The chapter builds a multi-sector input-output production model of heterogeneous firms that are financed by heterogeneous leverage targeting banks. Banks are themselves connected through bilateral cross-holdings. In the financial market equilibrium, endogenous financial asset prices intro-

duce a new propagation channel of productivity shocks. Structural parameters such as bank-level leverage constraints determine the strength of this channel and one statistic, the *financial network multiplier*, is sufficient to capture it. I use a large dis-aggregated confidential French dataset on matched firm-bank relationships through corporate bond investments in the 2011-2015 period to estimate the model. The model can be used to study macro-prudential regulation and monetary policy.

Chapter 2 uses bank- and instrument-level data on asset holdings and liabilities to identify and estimate a general equilibrium model of trade in financial instruments shaping an endogenous network of interlinked banks' balance-sheets. Bilateral ties are formed as each bank selects the size and the diversification of its assets and liabilities. Shocks propagate due to the response, rather than the size, of bilateral ties to such shocks. This general equilibrium propagation of shocks reveals a financial network where the strength of a tie is determined by the elasticity of substitution between financial assets. General equilibrium analysis predicts the propagation of real, financial and policy shocks. The network's shape adjusts endogenously in response to shocks, to either amplify or mitigate partial equilibrium shocks and the network exhibits key theoretical properties: (i) more connected networks lead to less amplification of partial equilibrium shocks, (ii) the influence of a bank's equity is independent of the size of its holdings; (iii) more risk-averse banks are more diversified, lowering their own volatility but increasing their influence on other banks.

The general equilibrium based network model is structurally estimated on dis-aggregated data for the universe of French banks. The chapter shows that the endogenous change in the network matters two to three times more than the initial network of cross-holdings for the transmission of shocks. The estimated network is used to assess the effects of the ECB's quantitative easing policy on asset prices, balance sheets, individual bank distress risk, and network systemic-ness.

Finally, chapter 3 concludes the thesis with a more aggregated sector-level analysis of shock propagation. The chapter first studies how the sharp deterioration of the net external portfolio position of France between 2008 and 2014 was driven by sectoral patterns such as the banking sector retrenchment and the increase in foreign liabilities of the public and corporate sectors, but

was mitigated by the expansion of domestic and foreign asset portfolios of insurance companies. It provides a network representation of the links between domestic sectors and the rest of the world, and documents their evolution between 2008 and 2014. Sectoral shock propagation through inter-sectoral security holdings is studied in a balance-sheet contagion model. The estimation of the model using generalized method of moments shows that the financial sectors of the economy (banking, mutual fund, and insurance sector) are affected by balance-sheet contagion.

**JEL Codes:** D57, D85, G12, D4, D5, E5, E44, F30, G11, G20, G21

**Keywords:** Endogenous Networks, Shock Propagation, Asset Trade, Asset and Liability Management, General Equilibrium, Real-Financial Linkages, Micro-Origin of Aggregate Fluctuations



# Résumé

Depuis la crise financière, les interdépendances entre les institutions financières font partie intégrante de l'analyse et de la politique économique. Le monde entier a appris douloureusement comment des chocs relativement petits et localisés (comme la crise des subprimes aux États-Unis) peuvent se propager par le système financier à travers le monde et générer des conséquences importantes pour l'économie réelle. Depuis lors, les analyses académiques ont cherché à comprendre les caractéristiques du système financier qui déterminent le risque systémique. Le point de départ de la majorité de ces études est de considérer la structure du réseau financier comme étant donnée. Cependant, chaque jour des quantités importantes d'instruments financiers sont échangées. Cette structure change donc. Les agents économiques ajustent leurs bilans – par exemple à la réception de nouvelles informations, lors de l'introduction d'une nouvelle régulation financière, ou encore en fonction de la politique monétaire. Cette thèse démontre que ces ajustements endogènes jouent un rôle clé pour la transmission de chocs par les réseaux financiers.

Sur le plan théorique, la thèse construit des modèles d'équilibre général qui permettent de comprendre la formation endogène des réseaux. En effet, le réseau est le résultat d'échanges d'instruments financiers entre les institutions financières. Les analyses empiriques exploitent quant-à-elles plusieurs bases de données micro- et macro-économiques pour mesurer l'importance des mécanismes théoriques. La thèse bénéficie particulièrement des bases confidentielles et détaillées de la Banque de France sur les détentions de titres ainsi que les bilans des banques françaises pour identifier les paramètres structurels du modèle.

Le premier chapitre fait le lien avec l'économie réelle. Il montre comment les chocs de pro-

ductivité au niveau d'une entreprise peuvent se propager par le réseau financier : deux entreprises peuvent être liées par leur financement externe même si elles n'ont pas ou peu de liens par leur activité réelle. Ces liens peuvent avoir des conséquences macro-économiques. Le chapitre construit un modèle théorique multi-sectoriel de la production avec des entreprises hétérogènes qui financent une partie de leurs dépenses en émettant des titres. Ces titres de créances sont achetés par les banques qui sont contraintes de respecter un certain ratio de capitalisation. A l'équilibre, les chocs de productivité se propagent par les réseaux financiers (le multiplicateur financier). Le modèle est estimé avec des données désagrégées sur les liens entre entreprises et banques par les titres de créances. Le modèle pourra également se prêter à l'étude de la régulation macro-prudentielle et la politique monétaire.

Le deuxième chapitre de la thèse exploite les données désagrégées sur les avoirs et les engagements des banques. Elles permettent d'identifier et estimer un modèle d'équilibre général d'échanges d'instruments financiers. Les banques choisissent la taille et la diversification de leurs bilans. Ainsi, des liens bilatéraux entre banques se créent constituant un réseau financier endogène. La propagation de chocs est déterminée, non pas par le montant des investissements mais par les ajustements des bilans bancaires face à ces chocs. En effet, c'est l'élasticité de substitution entre instruments financiers qui est déterminante. Le réseau change d'une manière endogène en fonction d'un choc, soit pour l'amplifier d'avantage, soit pour l'atténuer. Le réseau a les caractéristiques clés suivantes : (i) un réseau plus connecté implique moins d'amplification, (ii) l'influence d'une banque est indépendante de la taille de ses positions, (iii) les banques qui sont plus averses au risque réussissent à diminuer leur risque, mais accroissent leur influence sur les autres banques du réseau. Sur le plan empirique, le chapitre montre que la réponse endogène du réseau est deux à trois fois plus importante pour la propagation de chocs que le réseau préexistant. Le modèle estimé permet d'évaluer les effets des programmes d'achat de titres mis en place par la Banque Centrale Européenne (BCE) sur les prix d'actifs, les bilans des banques, le risque d'insolvabilité des banques, et le risque systémique.

Le troisième chapitre conclut la thèse avec une analyse plus agrégée. Il montre d'abord que des

évolutions sectorielles, comme l'expansion de l'endettement extérieur du secteur public français, ont été à l'origine de la détérioration de la position extérieure nette de la France entre 2008 et 2014. Il fournit une représentation de réseau des liens entre les secteurs institutionnels domestiques avec l'étranger et documente leurs évolutions sur la période. La propagation des chocs sectoriels à travers les détentions de titres est étudiée dans un modèle de contagion qui est estimé grâce à la méthode de moments généralisés.

**Codes JEL:** D57, D85, G12, D4, D5, E5, E44, F30, G11, G20, G21

**Mots-clés:** Réseaux financiers, Propagation de chocs, Echange d'actifs, Gestion actif/passif, Equilibre général, Liens réels-financiers, Origine micro-économique de fluctuations agrégées

# Contents

|  |           |
|--|-----------|
| <b>Acknowledgments</b>                                     | <b>1</b>  |
| <b>Abstract</b>  | <b>3</b>  |
| <b>Résumé</b>  | <b>6</b>  |
| <b>1 Introduction</b>                                      | <b>17</b> |
| <b>2 Firms, Financial Networks, Aggregate Fluctuations</b> | <b>23</b> |
| 2.1 Introduction . . . . .                                 | 23        |
| 2.2 The Model . . . . .                                    | 28        |
| 2.2.1 Household . . . . .                                  | 28        |
| 2.2.2 Firms . . . . .                                      | 29        |
| 2.2.3 Banks . . . . .                                      | 33        |
| 2.2.4 Equilibrium . . . . .                                | 35        |
| 2.3 The Financial Network Multiplier . . . . .             | 38        |
| 2.4 Estimation . . . . .                                   | 40        |
| 2.4.1 Data . . . . .                                       | 42        |
| 2.4.1.1 Firms . . . . .                                    | 42        |
| 2.4.1.2 Banks . . . . .                                    | 43        |
| 2.4.2 Empirical Strategy . . . . .                         | 45        |
| 2.4.2.1 Substitution and Factor Elasticities . . . . .     | 46        |

|  |           |
|--|-----------|
| <i>CONTENTS</i>  | 10        |
| 2.4.2.2 Productivity Shocks . . . . .                                      | 47        |
| 2.4.2.3 Banks' Structural Parameters . . . . .                             | 48        |
| 2.5 Equilibrium Results . . . . .  | 48        |
| 2.5.1 Financial Spill-Overs . . . . .                                      | 50        |
| 2.5.2 Importance of the Financial Network . . . . .                        | 51        |
| 2.5.3 Which firms are most influential? . . . . .                          | 51        |
| 2.6 Conclusion . . . . .   | 52        |
| <b>3 Endogenous Financial Networks</b>                                     | <b>53</b> |
| 3.1 Introduction . . . . .   | 53        |
| 3.2 The Financial Network . . . . .  | 61        |
| 3.2.1 Balance-Sheet Diversification and Size . . . . .                     | 62        |
| 3.2.2 Market Equilibrium with Trade Costs . . . . .                        | 64        |
| 3.2.3 Shock Propagation Through the Financial Network . . . . .            | 65        |
| 3.2.4 Network Structure: Amplification or Mitigation . . . . .             | 69        |
| 3.2.5 Measuring the Systemicness of Institutions . . . . .                 | 70        |
| 3.3 Structural Estimation . . . . .  | 72        |
| 3.3.1 Parameterization: Demand, Trade Costs, Beliefs . . . . .             | 73        |
| 3.3.2 Model Identification: Intuitions . . . . .                           | 74        |
| 3.3.3 From Return Beliefs to Net-Demands: A Dynamic Factor Model . . . . . | 75        |
| 3.3.4 From Net-Demands to Return Beliefs: Identification . . . . .         | 78        |
| 3.3.5 Estimation Procedure . . . . .                                       | 78        |
| 3.4 Data . . . . .   | 79        |
| 3.5 Structural Parameters . . . . .  | 83        |
| 3.5.1 Return Beliefs . . . . .   | 83        |
| 3.5.2 Risk-aversion . . . . .  | 87        |
| 3.5.3 Trade costs . . . . .  | 88        |
| 3.6 The Network in General Equilibrium . . . . .                           | 89        |

|  |            |
|--|------------|
| <i>CONTENTS</i>  | 11         |
| 3.6.1 Network Structure . . . . .  | 89         |
| 3.6.2 Structural Policy Evaluation: the case of ECB Quantitative Easing . . . . .  | 90         |
| 3.7 Conclusion . . . . .   | 94         |
| <b>4 Domestic and External Sectoral Portfolios</b>                                 | <b>96</b>  |
| 4.1 Introduction . . . . .   | 96         |
| 4.2 Data Description . . . . .   | 100        |
| 4.3 Domestic and External Sectoral Portfolios . . . . .                            | 102        |
| 4.3.1 Constructing Sectoral Portfolios . . . . .                                   | 102        |
| 4.3.2 Stylized Facts on French Domestic and External Sectoral Portfolios . . . . . | 104        |
| 4.3.2.1 External Portfolios . . . . .  | 104        |
| 4.3.2.2 External Cross Sectoral Portfolios . . . . .                               | 107        |
| 4.3.2.3 Domestic versus Foreign Portfolios . . . . .                               | 108        |
| 4.3.3 The Network Structure of Domestic and External Portfolios . . . . .          | 109        |
| 4.4 An Estimated Model of Sectoral Balance-Sheet Contagion . . . . .               | 111        |
| 4.4.1 The Model . . . . .  | 112        |
| 4.4.1.1 Asset Demand and Supply . . . . .  | 113        |
| 4.4.1.2 Sectoral Balance-Sheet Equilibrium . . . . .                               | 114        |
| 4.4.1.3 Balance-Sheet Contagion . . . . .  | 116        |
| 4.4.2 Identification and Estimation . . . . .                                      | 117        |
| 4.4.2.1 Moment Conditions . . . . .  | 118        |
| 4.4.2.2 Two-Step GMM . . . . .   | 119        |
| 4.4.3 Results . . . . .  | 119        |
| 4.5 Conclusion . . . . .   | 122        |
| <b>Bibliography</b>  | <b>123</b> |
| <b>A Appendix of Chapter 2</b>   | <b>129</b> |
| A.1 Figures and Tables . . . . .   | 130        |

|          |   |            |
|----------|---|------------|
| A.2      | Proofs . . . . .                                      | 137        |
| A.2.1    | Lemma 2.1: Firms' Marginal Costs . . . . .            | 137        |
| A.2.2    | Lemma 2.2: Within-Sector Sales Distribution . . . . . | 137        |
| A.2.3    | Lemma 2.3: Firms' Debt Supply . . . . .               | 138        |
| A.2.4    | Lemma 2.4: Leverage Targeting . . . . .               | 139        |
| A.2.5    | Proposition 2.1: Aggregate Output . . . . .           | 139        |
| A.2.6    | Proposition 2.3: Financial Multiplier . . . . .       | 141        |
| <b>B</b> | <b>Appendix of Chapter 3</b>                          | <b>143</b> |
| B.1      | Figures & Tables . . . . .                            | 144        |
| B.2      | Theory . . . . .                                      | 162        |
| B.2.1    | Proofs: General Equilibrium Model . . . . .           | 162        |
| B.2.2    | Proofs of Identification . . . . .                    | 163        |
| B.3      | Data . . . . .  | 168        |
| B.3.1    | Estimation of Returns . . . . .                       | 168        |
| B.3.2    | Imputation of the Real Asset . . . . .                | 170        |
| <b>C</b> | <b>Appendix of Chapter 4</b>                          | <b>171</b> |
| C.1      | Figures . . . . .                                     | 172        |
| C.2      | Tables . . . . .                                      | 180        |

# List of Figures

|     |  |     |
|-----|--|-----|
| 2.1 | Input-Output and Banking Network for France in 2011. . . . .                                 | 25  |
| 2.2 | Inter-period Timing and Cash Flows . . . . .   | 30  |
| 2.3 | Bank $b$ 's Balance-Sheet . . . . .  | 33  |
| 2.4 | Growing Debt of French Corporates . . . . .  | 41  |
| 2.5 | Evolution of the French Banking Sector's Total Assets . . . . .                              | 44  |
| 4.1 | Sector $i$ 's Asset and Liability Allocation . . . . .                                       | 113 |
| A.1 | Validation of Productivity Shock Estimation . . . . .  | 133 |
| A.2 | Total Influence and its Components . . . . .   | 135 |
| A.3 | Average Influence by Sector: IO versus Financial Network Multiplier . . . . .                | 135 |
| B.1 | The Equilibrium Mean-Variance Frontier with Disagreement About Stochastic Pay-offs . . . . . | 144 |
| B.2 | The Equilibrium Relationship between a bank's Influence and its Risk Aversion . . . . .      | 145 |
| B.3 | Banks' Estimated Return-Beliefs Factors and Macro-Variables . . . . .                        | 146 |
| B.4 | Comparing Identified Beliefs to Actual Returns . . . . .                                     | 147 |
| B.5 | Comparing Identified Beliefs to Actual Returns (Non-Financial Sector) . . . . .              | 148 |
| B.6 | The Network of Cross Holdings . . . . .  | 149 |
| B.7 | The Sensitivity of Interbank Cross Holdings to Price Changes . . . . .                       | 150 |
| B.8 | Linkages through Primitive Instruments . . . . .   | 151 |
| B.9 | The Correlation of Total Assets with Network Influence and Sensitivity . . . . .             | 152 |



|  |     |
|--|-----|
| B.10 ECB Shock and Network Structure: Impact of Monetary Policy on Influence and Sensitivity . . . . .         | 153 |
| B.11 Banks' Estimated Return-Beliefs Factors and Macro-Variables . . . . .                                     | 160 |
| B.12 ECB Shock and Bank Size . . . . .   | 161 |
| C.1 International Investment Position of France from 2008Q1 to 2015Q3. . . . .                                 | 172 |
| C.2 Change of the International Investment Position of France due to Valuations from 2008Q1 to 2015Q3. . . . . | 173 |
| C.3 Network of Domestic Sectors in 2008Q1 and 2014Q1. . . . .  | 174 |
| C.4 Network of Domestic Sectors and the Rest of the World in 2008Q1 and 2014Q1. . . . .                        | 175 |
| C.5 Network of Domestic Sectors, the rest of the Eurozone, and the Non-Eurozone in 2008Q1 and 2014Q1. . . . .  | 176 |
| C.6 Evolution of Balance-Sheet Contagion Effects from 2008Q1 to 2014Q1. . . . .                                | 177 |
| C.7 Balance-Sheet Contagion Effects in 2008Q1 and 2014Q1. . . . .  | 178 |

# List of Tables

|     |  |     |
|-----|--|-----|
| A.1 | Firms' Size and Bond Financing by Sector . . . . .   | 130 |
| A.2 | Banks' Size and Corporate Debt Holdings . . . . .  | 131 |
| A.3 | Estimated Total Factor Productivity and Elasticities . . . . .                                 | 132 |
| A.4 | Equilibrium Influence Decomposition: The Financial Multiplier . . . . .                        | 134 |
| A.5 | Who is Influent and through which Channel? . . . . .   | 136 |
| B.1 | How do Banks Form their Beliefs? How Beliefs about Returns Respond to Realizations . . . . .   | 154 |
| B.2 | Estimated Trade Costs by Financial Instrument . . . . .  | 155 |
| B.3 | Simulation of a PSPP Shock, Bond Yields, Corporate Spread, and the Equity Premium . . . . .    | 156 |
| B.4 | Banks' Heterogeneous Response to the ECB's PSPP Shock . . . . .                                | 157 |
| B.5 | Banks' Heterogeneous Response to the ECB's PSPP Shock – Initial Characteristics                | 158 |
| B.6 | Risk Aversion and Return Belief Uncertainty . . . . .  | 159 |
| C.1 | External Portfolios of French Sectors in 2008Q1 and 2014Q1: Total, Equity, and Debt. . . . .   | 180 |
| C.2 | External Portfolios of French Sectors in 2008Q1 and 2014Q1: Change Over Time.                  | 181 |
| C.3 | External Portfolios of French Sectors in 2008Q1 and 2014Q1: Eurozone and Non-Eurozone. . . . . | 181 |

|     |   |     |
|-----|---|-----|
| C.4 | External Portfolios of French Sectors in 2008Q1 and 2014Q1: GIPS and Non-GIPS. . . . .  | 182 |
| C.5 | External Portfolios of French Sectors in 2008Q1 and 2014Q1: Change Over Time Due To Valuation (Exchange Rate and Price), and Flows. . . . . | 183 |
| C.6 | External Portfolios of French Sectors in 2008Q1 and 2014Q1: Eurozone Sectors. . . . .   | 184 |
| C.7 | Portfolios of French Sectors in 2008Q1 and 2014Q1: Domestic, Foreign, and Consolidated. . . . .   | 185 |
| C.8 | Portfolios of French Sectors in 2008Q1 and 2014Q1: Share of External Positions. . . . .   | 186 |
| C.9 | Balance-Sheet Contagion Model: Estimation Results. . . . .  | 186 |

# Chapter 1

## Introduction

I started working on shock propagation in financial networks during a time when the world economy was still recovering from the global financial crisis. The world had painfully learned how small and localized events can travel through the global financial system and generate huge repercussions for the real economy. This made me very curious about how to fully capture, analyze, and most importantly measure the complex channels through which such propagation happens. Which institutions contribute most to spreading shocks? How can individually rational behavior of a financial institution generate risks for the whole system? During that time, I worked at the French research center CEPII (Centre d'Etudes Prospectives et d'Informations Internationales) and had access to PROTIDE, the comprehensive securities holdings statistics collected by the Banque de France. This database contains close to the totality of securities holdings of French residents and is a unique tool to address these questions.

Financial inter-dependencies are since the financial crisis at the forefront of macroeconomic research and policy making. Many papers analyze the propagation properties of financial exposure networks (Eisenberg & Noe (2001*a*), Elliott, Golub & Jackson (2014)). One key finding of this literature is that more inter-connectedness can lead to more diversification but it may also increase the risk of default cascades depending on the particular shape of the network. Most studies consider bilateral exposures between financial institutions as given. Each day, however, large amounts

of financial assets are traded and financial institutions' balance-sheets change in response to new information, regulation or monetary policy. This could lead to an underestimation of shock propagation. Indeed, a seminal paper, Elliott et al. (2014) concludes that "*A fully endogenous study of the network of cross-holdings and of asset holdings is a natural next step*" (p. 3148). Thus, to analyze financial stability, institution systemicness, or shock propagation, it is crucial to take the endogenous response of financial networks to changing environments into account. This is what I address in Chapter 2 of the thesis "The Transmission of Shocks in Endogenous Financial Networks: A Structural Approach".

In all three chapters of the thesis, I approach shock propagation as an equilibrium phenomenon. Through trade in financial assets between heterogeneous institutions, a shock to one specific market or one specific institution generates a partial market disequilibrium. Prices have to adjust. Due to the endogenous response of asset holdings and liability issuance to price changes, the initial shock can affect even seemingly unrelated markets or institutions. A key theoretical object in all three papers is therefore the elasticity of substitution between financial assets. While the elasticity of substitution is central in workhorse models of the industrial organization literature (Berry, Levinsohn & Pakes 1995), it has been so far absent from the theoretical financial network literature which focuses primarily on the number of links in incomplete networks or the size of exposures. A common objective of all three chapters is to develop theoretical frameworks that can be structurally identified and estimated. To do so, very dis-aggregated data such as institution-level securities holdings as well as balance-sheet data is used.

The most aggregated analysis of shock propagation is done in Chapter 3, which represents the start of my work in this field. The chapter first traces descriptively how the large deterioration of the French net external asset position resulted from trade in financial assets undertaken by different sectors of the economy (banking, mutual funds, insurance, corporate, household and public sector) as well as valuation changes experienced by the corresponding sectoral portfolios. An estimated model of sectoral balance-sheet contagion exposes the role played by network linkages, in the form of domestic and foreign security cross-holdings, in the transmission of real and financial shocks.

The estimated model explores how changes in the value of a sector's assets propagate through the re-pricing of its liabilities which are held as assets by other sectors. The model builds on (Kojien & Yogo 2019) who show how asset demand can be represented using a logit model, where portfolio shares depend on asset prices and characteristics. This provides some micro-foundations for balance-sheet contagion across sectors. However, a full general equilibrium model of asset trade leading to the derivation of an endogenous network of assets and institutions is beyond the scope of this paper.

Chapter 2 of the thesis, "The Transmission of Shocks in Endogenous Financial Networks: A Structural Approach", develops such a fully-fledged general equilibrium model. It uses bank- and instrument-level data on asset holdings and liabilities to identify and estimate a general equilibrium model of trade in financial instruments shaping an endogenous network of interlinked banks' balance-sheets. Bilateral ties are formed as each bank selects the size and the diversification of its assets and liabilities. Shocks propagate due to the response, rather than the size, of bilateral ties to such shocks. General equilibrium analysis predicts the propagation of real, financial and policy shocks. The network's shape adjusts endogenously to either amplify or mitigate shocks and has key theoretical properties. For example, akin to Elliott et al. (2014), more connected networks lead to less amplification of partial equilibrium shocks. Also, more risk-averse banks are more diversified, lowering their own volatility but increasing their influence on other banks. The general equilibrium based network model is structurally estimated on dis-aggregated data for the universe of French banks. The endogenous change in the network matters two to three times more than the initial network of cross-holdings for the transmission of shocks. The estimated network is used to assess the effects of the European Central Bank's quantitative easing policy on asset prices, balance sheets, individual bank distress risk, and network systemic-ness.

A key challenge for the estimation of the model is that banks' belief about the future distribution of returns is fundamentally unobserved. However, if banks use dynamic factor models to forecast returns, a common assumption in empirical asset pricing, beliefs are shown to be revealed by the way how banks adjust their balance-sheets positions over time. The variation in observable

net demands for financial instruments identifies substantial heterogeneity (in risk-aversion, beliefs, trading costs) between financial institutions. By backing out beliefs in a framework with heterogeneous agents, the model follows the precedent set by Ross (2015) in the case of a representative agent

In chapter 1 “Heterogeneous Firms, Financial Networks, Aggregate Fluctuations”, I put the implications of financial networks for the real economy at the center of the analysis. Indeed, firms can be connected through the financial sector beyond of what is implied by their input-output relationships. This paper shows how firm-level productivity shocks propagate through financial networks when firms need external funds to finance their capital expenditure. The chapter integrates a multi-sector input-output production economy of heterogeneous firms with a network of inter-linked heterogeneous banks that target leverage ( Greenwood, Landier & Thesmar (2015)). The joint analysis of both, real and financial linkages allows to distinguish shock propagation through real channels (Acemoglu, Carvalho, Ozdaglar & Tahbaz-Salehi (2012)) from propagation through the banking network. I show that a financial multiplier exists that captures how firm-level productivity shocks travel through the financial network and eventually affect aggregate output. Structural parameters determine the strength of this channel and one statistic is sufficient to capture it. I use dis-aggregated confidential French data on matched firm-bank relationships through corporate bond investments in the 2011 - 2015 period to estimate the model. The model can be used to study macro-prudential regulation and monetary policy.

Besides my focus on financial networks and shock propagation, I have also worked on topics of monetary policy and international finance during my PhD. First, in “Managing Macroeconomic Fluctuations with Flexible Exchange Rate Targeting”, I study in a DSGE model how central banks in small open economies may use their exchange rate to stabilize macro-economic fluctuations and increase welfare. A precondition is that the central bank has sufficient foreign reserves available to intervene in the foreign exchange markets. The effectiveness of the exchange rate depends on the economy’s share of imports and the elasticity of trade. Second, I revisit the famous uncovered interest parity condition before and after the financial crisis. “The New Fama Puzzle” shows that

the relation between interest differentials and future currency depreciation has reversed after 2008. A higher interest rate differential between two countries no longer seems to predict an appreciation of the high interest rate currency. This is the “old” Fama puzzle. Instead - post crisis - higher interest rate differentials predicted much larger currency appreciations than the one-to-one relationship that would follow in a risk-neutral environment with efficient market.

A key contribution of my thesis to the study of shock propagation in financial networks is to introduce general equilibrium frameworks that can be structurally estimated. The general equilibrium perspective provides a close link of the literature on financial asset demand and portfolio choice with the literature on financial networks. This approach offers a wide range of insights into the complex process and stages of propagation. Indeed, shock propagation can be interpreted as the process of how partial market disequilibria are cleared through partial equilibrium price adjustments. This process, in turn, is driven by the fundamentals underlying market participants’ demand and supply decisions. Chapter 2 develops the tools to reveal such fundamentals from the data. Under general assumptions, the variation in observable balance-sheet data maps into structural parameters such as risk preferences, beliefs about future returns, and costs encountered when trading specific financial assets. Chapter 1 contributes a very explicit link of how propagation of shocks in financial networks may affect firms and aggregate output.

The research conducted in this thesis, provides several exciting avenues for future research. First, a key advantage of deriving financial network propagation from first principles is that counterfactual analysis is possible. For example, Chapter 2 shows how the European Central Banks’ quantitative easing programs affected not only asset prices but also the shape of the financial network itself. However, also implications of macro-prudential policy such as minimum capital requirements can be assessed. Such policies might make a specific institution safer, but can have unintended consequences through the endogenous change of the financial network topology. Regarding the financial multiplier, counterfactual analysis can also be used to understand in great detail how financial regulation affects real output. Second, the frameworks developed in this thesis are well-suited to study how different types of financial institutions affect financial stability



and eventually the real economy. For example, research (for example Chodorow-Reich, Ghent & Haddad (2016)) and current debates in central banking stress the importance of non-bank financial intermediaries such as insurance companies for financial stability and the financing of the real economy. Long-term investors, such as insurances, might contribute to more financial stability by isolating specific financial assets from transitory market movements. Third, the high dis-aggregation of the analyses conducted in this paper potentially allows to derive much more targeted policy interventions than used so far by central banks or policy makers. For example, it would be very interesting to study in more detail how corporate bond purchases - such as performed by the European Central Bank as of June 2016 - could be targeted to have the largest impact on the real economy. The financial network multiplier could provide guidance in this respect.

## Chapter 2

# Heterogeneous Firms, Financial Networks, and Aggregate Fluctuations

### 2.1 Introduction

The financial sector can be an engine of growth but also amplify shocks and cause long-lasting recessions. This has been painfully demonstrated by the global financial crisis. Financial institutions play this central role because they are strongly involved in many stages of firms' production processes. Indeed, many firms rely on external financing sources to cover their capital expenditure. This not only makes firms dependent on financial institutions but also exposes financial institutions to credit risks. For example, a firm-level productivity shock to a car producer not only affects suppliers but also the risk exposure of its lenders and might even pose a threat to financial stability.<sup>1</sup> If the affected financial institution is broadly invested in different sectors of the economy, a productivity shock to one firm in its portfolio can even affect firms that are distant or seemingly unrelated

---

<sup>1</sup>Too high corporate debt risk exposure has been a real issue in France since the financial crisis. In May 2018 surging corporate debt has led the French High Council for Financial Stability to impose a 5%-of-capital exposure limit on lending of the countries six biggest banks (BNP Paribas, Société Générale, Crédit Agricole, Credit Mutuel, BPCE, and La Banque Postale) to the most highly indebted corporates at the time (Air France KLM, Suez, Airbus, EDF, and Casino). See coverage of this move in the international press (accessed as of November 8, 2019): Ballooning French corporate debt rattles regulators, but not lenders (Reuters, December 21, 2017) and France sets limit for bank exposure to corporate debt (Reuters, May 11, 2018).

in terms of their input-output relationship. Two questions are addressed in this paper: First, how does this financial multiplier of productivity shocks depend on structural parameters, the shape of the financial network, and the position of the firm in the input-output network? Second, how important is the financial multiplier empirically for an economy's aggregate output?

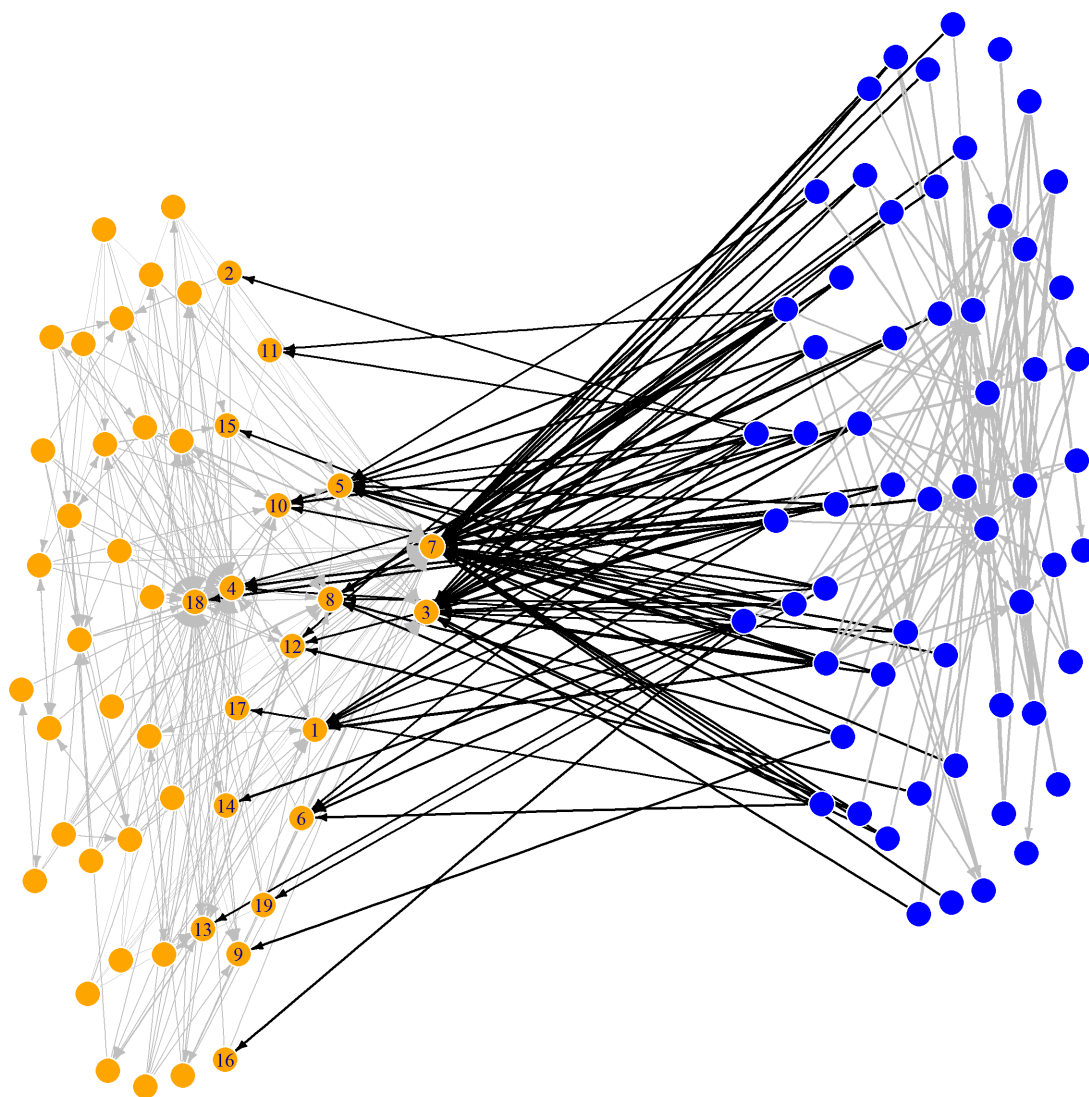
Answering these questions requires detailed data on banks' exposures to individual firms. Figure 2.1 presents such data for the case of France. It shows that corporate bond holdings of 155 French banks and their bilateral exposures in terms of equity shares and debt securities are highly concentrated. Only a subset of banks invest in corporate bonds, and only a subset of sectors issue significant amounts. Among the sectors that receive most bond financing through banks are electricity, gas, steam and air conditioning supply (3), telecommunications (5), legal and accounting services, head offices (7), and warehousing and support activities for transportation (8). For example, a shock to a telecommunications firm could affect warehousing firms through their indirect financial connections. Since firms and banks respond endogenously to productivity shocks, answering the questions of this paper also requires to specify the optimal behavior of heterogeneous firms on product markets and financial markets as well as banks' balance-sheet management. The empirical identification of the financial multiplier requires to allow for shock propagation through the input-output production network (see Acemoglu et al. (2012)) to separately identify additional propagation through the financial markets.

In this paper, I build a multi-sector model of heterogeneous firms augmented with heterogeneous leverage targeting banks to study the financial multiplier propagation channel of productivity shocks. To the best of my knowledge, this is the first paper to integrate input-output linkages between firms with a financial network between heterogeneous banks. Data requirements may explain this gap. I use confidential data from the Banque de France from regulatory filings on a large number of firms, a large number of banks, and their linkages to estimate firm-level productivity shocks and the structural parameters of the model. Financial equilibrium prices are obtained numerically by applying Brouwer's fixed point theorem for over 1,400 financial assets that are traded between firms and banks and serve as a propagation vector of productivity shocks. I find that the

financial network multiplier is empirically relevant. Depending on the banks' costs of deviating from a bank-specific leverage target, it captures positive or possibly negative spillovers to other firms in the economy.

Figure 2.1: Input-Output and Banking Network for France in 2011.

*This figure shows the French sectoral input-output production network (left), banks' cross-holdings of debt and equity securities (right), and linkages between banks and firms through corporate debt markets in 2011.*



*Note: Orange nodes correspond to 50 real sectors (ISIC-2 classification) which are connected through input-output production relations. Blue nodes correspond to French banks which are connected through equity and debt securities cross-holdings. Banks' investments into corporate debt securities are symbolized by black arrows. For confidentiality reasons, banks are aggregated into groups of three in consecutive sizes. Data sources: PROTIDE, M-SITMENS, and FIBEN.*

In the model, firms are connected to the financial sector because they face a cash-in-advance

constraint to finance capital expenditure (see Rajan & Zingales (1998)). This external financing constraint is a crucial ingredient for the mechanism of the model as it requires firms to issue costly debt before production. Thus, firms' marginal costs depend on the costs of funds, which are determined in the financial market equilibrium. Since firms within a sector produce differentiated varieties and face downward-sloping demand, firms pass on their financing costs to product prices which ultimately affects aggregate output. Firms are linked to banks because banks invest in corporate debt and re-balance their portfolio in response to productivity shocks. This is the key mechanism of the paper. Indeed, banks are assumed to target a given leverage ratio (following Greenwood et al. (2015)). However, since deviations from target leverage are costly (see Zhou, Tan, Faff & Zhu (2016)), banks face a trade-off between maximizing their value of equity and keeping leverage close to target. In response to a change in their asset value, banks may therefore either increase or decrease their balance-sheet size. If banks are allowed to let their leverage fluctuate sufficiently, productivity shocks tend to generate positive spillovers (see Banerjee & Feinstein (2019))

I shows that one statistics is sufficient to capture the effect of firms' productivity shocks on aggregate output akin to the *influence* measure proposed by Acemoglu et al. (2012). In contrast to pure production networks, influence in my model features an additional term. I call this term the *financial network multiplier* of productivity shocks. It captures the total impact of changes in the financial market equilibrium induced by a firm-level productivity shock. As a financial counterpart to the Leontieff inverse in production networks, a key role in this term is played by the price-impact or co-liquidity matrix of trade (see also Heipertz, Ouazad & Ranci ere (2019), Koijen & Yogo (2019)). This matrix captures the general equilibrium response of asset prices and is given by the inverse of the semi-elasticity of excess market demand with respect prices. It, in turn, depends on the structural parameters of banks' balance-sheet management and firms' debt supply.

**Related Literature:** The paper is related to the literature on the micro-origins of aggregate fluctuations. This literature studies the conditions under which micro-economic shocks show-up in

macroeconomic aggregates. In particular, the granular origins hypothesis emphasizes that if firms' size distribution is fat-tailed, idiosyncratic shocks to large firms do not wash-out, but may lead to aggregate fluctuations (Gabaix 2011). Following the granular origins hypothesis, a theoretical and empirical literature shows how aggregate fluctuations can also arise from micro-economic shocks due to input-output production relations (Acemoglu et al. 2012, di Giovanni, Levchenko & Mejean 2014, Grassi & Carvalho 2019, Grassi 2018). My paper shares with this literature the aim to explain aggregate fluctuations by micro-economic shocks, however, it departs from it by shifting the focus to the contribution of the financial network to the propagation of shocks. In this literature, the paper most closely related is Bigio & La'O (2019) who build on Acemoglu et al. (2012) (for example, allowing for decreasing returns to scale) and introduce exogenous sector-level frictions. They show how sectoral distortions including exogenous financial frictions propagate through the production network. Different to them, the financial friction in my paper, however, depends on financial asset prices that are determined endogeneously in the financial market equilibrium. As such, the financial friction in my paper introduces an additional, new channel for the propagation of shocks.

The paper also relates to the literature on the real effects of bank lending that uses matched firm-bank data. In common with this literature, I analyze the role of banks for the real economy. In contrast to this literature, however, I focus on shocks originating in the real economy. Jiménez, Mian, Peydró & Saurina (2019) uses the Spanish matched credit register to study the effect of credit booms on firms' production and allow for general equilibrium adjustments. Two other examples using matched firm-bank data are Amiti & Weinstein (2018) and Alfaro, Garcia-Santana & Moral-Benito (2018). They measure the importance of banks' credit supply for investment, output, and employment.

**Structure of the paper:** In Section 2.2 I present the model and solve for the equilibrium and Section 2.3 derives the main theoretical result of the paper, the financial network multiplier. In Section 2.4, I present the data and explain the empirical strategy and present estimated structural parameters and firm-level productivity shocks. Section 2.5 presents the main empirical results, the

estimated financial network multiplier using French data. Section 2.6 concludes.

## 2.2 The Model

This section describes the structure of the economy. There are three types of agents. First, a representative household consumes and supplies labor to firms. Second, firms belong to different sectors, hire labor, and source inputs for production from other firms in the economy. Input-output relationships between firms constitute a *production network*. Firms also have to finance a fixed share of their capital expenditure by debt. Third, banks provide funds to firms by investing into firms' debt. Banks also issue liabilities that can be bought by other banks. Bilateral exposures between banks constitute a *financial network*.

In each period, firms are subject to idiosyncratic total factor productivity shocks. Since the model is static, I drop the time index in the following.

### 2.2.1 Household

The representative household has preferences  $U(C, L)$  over a composite consumption good  $C$  and hours worked  $L$ . The composite consumption good is a Cobb-Douglas aggregation of  $N$  sector goods  $C = \eta \prod_{s=1}^N C_s^{\gamma_s}$ , where  $C_k$  is the amount of good  $k$  consumed by the household and  $\eta$  is a normalization constant.  $\gamma_k$  is the share of good  $k$  in the household's consumption expenditure. The price of good  $k$  and the price index of the composite consumption good are  $P_k$  and  $P = \prod_{s=1}^N P_s^{\gamma_s}$ , respectively.

Each good  $k$  further is made of a number  $N_k$  of varieties. Varieties are aggregated with a constant elasticity of substitution  $\varepsilon_k$ , such that  $C_k = \left( \sum_{f=1}^{N_k} C(k, f)^{\frac{\varepsilon_k-1}{\varepsilon_k}} \right)^{\frac{\varepsilon_k}{\varepsilon_k-1}}$  and the household consumes the amount  $C(k, i)$  of sector  $k$ 's variety  $i$ . The price of good  $k$  is an index of variety prices and satisfies  $P_k = \left( \sum_{f=1}^{N_k} P(k, f)^{1-\varepsilon_k} \right)^{\frac{1}{1-\varepsilon_k}}$ .

The household receives a wage  $h$  for working as well as profits for owning firms.

### 2.2.2 Firms

Firms of the same sector use the same production technology to produce their varieties. Since firms face downward-sloping demand, they have market power and set prices. Firms are heterogeneous in total factor productivity and external financing needs. There are  $N_k$  firms in sector  $k$  and  $N$  sectors in the economy.

**Production Technology:** A firm  $i$  of sector  $k$  uses labor  $L(k, i)$ , intermediate inputs  $X(k, i)$ , and capital  $K(k, i)$  to produce  $Y(k, i)$  units of its variety with Cobb-Douglas production function,

$$Y(k, i) = \eta_k \widetilde{Z}(k, i) X(k, i)^{\alpha_k} K(k, i)^{\beta_k} L(k, i)^{1-\alpha_k-\beta_k}, \quad (2.1)$$

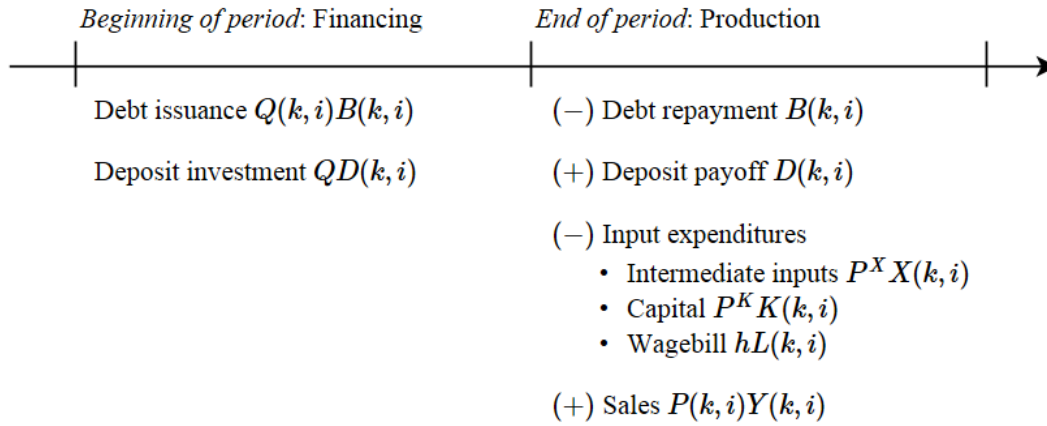
where  $\widetilde{Z}(k, i)$  is the stochastic level of total factor productivity and  $\eta_k$  is a normalization constant. The share of intermediate inputs and capital are denoted  $\alpha_k$  and  $\beta_k$ , respectively.

Firms of sector  $k$  require a share  $w_{kl}^X$  of sector  $l$ 's goods as intermediate input. That is  $X(k, i) = \prod_{s=1}^N X(k, i, s)^{w_{ks}^X}$  is an aggregation of sector goods and  $X(k, i, l)$  is a composite of sector  $l$ 's varieties. Similarly, also capital is a composite of sector goods,  $K(k, i) = \prod_{s=1}^N K(k, i, s)^{w_{ks}^K}$ , where  $w_{kl}^K$  is the share of sector  $l$ 's good in sector  $k$ 's capital good. Price indices of sector  $k$ 's intermediate inputs and capital satisfy  $P_k^X = \prod_{s=1}^N P_k^{w_{ks}^X}$  and  $P_k^K = \prod_{s=1}^N P_k^{w_{ks}^K}$ , respectively. Further, firms can substitute between sector  $l$ 's varieties with the same elasticity of substitution  $\varepsilon_k$  as households.  $X(k, i, l, f)$  and  $K(k, i, l, f)$  denote the amounts of sector  $l$ 's variety  $f$  used as intermediate input and capital, respectively, in the production of variety  $i$  of sector  $k$ .

There is no fundamental difference between the role of intermediate inputs and capital in the firm's production technology: Both inputs are chosen at the same point in time and enter current period's production. I distinguish between the two, however, because their financing differs as becomes clear following.



Figure 2.2: Inter-period Timing and Cash Flows



**Input Financing:** Firms need to finance a part of their capital expenditure externally. As argued by Rajan & Zingales (1998) external financing needs depend on factors such as industry or age. For example, drugs and pharmaceutical firms typically need more outside funds than firms in the textile industry. To capture this heterogeneity, I assume that firm  $i$  of sector  $k$  needs sufficient *cash-in-advance* to finance an exogenous share  $S(k,i) \in [0, 1)$  of its capital expenditure.

To acquire cash, firms access financial markets before production. At the beginning of the period, firm  $i$  of sector  $k$  issues a number  $B(k,i)$  of debt contracts at price  $Q(k,i)$ . The proceeds from bond issuance are held in the form of deposits  $D(k,i)$  which cost  $Q$ . At the end of the period, production takes place both, debt contracts and deposits, pay off 1 unit of currency. The cash in advance constraint requires that the payoff from deposit holdings covers an exogenous fraction of capital expenditure,  $D(k,i) \geq S(k,i)P^K K(k,i)$ . Remaining capital expenditure is financed by internal operating cash flows. Intermediate input expenditure  $P^X X(k,i)$  and the wage-bill  $hL(k,i)$ , on the other hand, are fully financed by the firm's operating cash flows. Figure 2.2 summarizes the inter-period timing of the firm's decisions and cash flows.

Firms' marginal costs not only depend on the wage and other sectors' prices (due to input-output production relationships), but also on the price of debt (due to the debt-financing of capital expenditures).

**Lemma 2.1. Capital Surcharge.** Firm  $i$  of sector  $k$  incurs costs  $\Lambda(k,i)$  to produce the marginal

unit of its variety:

$$\Lambda(k, i) \equiv \widetilde{Z(k, i)}^{-1} \chi(k, i) \prod_{s=1}^N P_s^{\alpha_k w_{ks}^X + \beta_k w_{ks}^K} h^{1-\alpha_k-\beta_k}, \quad (2.2)$$

where  $\chi(k, i) \equiv \left(1 - S(k, i) \left(1 - \frac{Q^D}{Q(k, i)}\right)\right)^{\beta_k} \geq 1$ .<sup>2</sup> Thus, debt financing introduces a surcharge on capital expenses into marginal costs.<sup>3</sup> Profit maximization under Dixit and Stiglitz (1977) monopolistic competition implies a constant mark-up pricing rule  $P(k, i) = \mathcal{M}_k \Lambda(k, i)$ , where  $\mathcal{M}_k \equiv \frac{\varepsilon_k}{\varepsilon_k - 1}$ . Sector  $k$ 's good price is  $P_k = \mathcal{M}_k \Lambda_k$ , where  $\Lambda_k \equiv \left(\sum_{f=1}^{N_k} \Lambda(k, i)^{1-\varepsilon_k}\right)^{\frac{1}{1-\varepsilon_k}}$  are sector  $k$ 's marginal costs.

*Proof.* See Appendix A.2.1. □

Lemma 2.1 shows that debt financing introduces a capital surcharge. The more a firm relies on debt to finance capital expenditure  $S(k, i) \rightarrow 1$ , the larger the additional expense  $\chi(k, i) \rightarrow Q^D/Q(k, i)$ . Vice versa, the surcharge disappears  $\chi(k, i) \rightarrow 1$ , if  $S(k, i) \rightarrow 0$ . Furthermore, the larger the share  $\beta_k$  of capital in production, the larger the influence of debt prices on marginal costs. Marginal costs decrease if the price of debt increases, and vice versa.

**Monopolistic Competition:** Firm  $i$  of sector  $k$  faces downward-sloping demand  $Y(k, i) = (P(k, i)/P_k)^{-\varepsilon_k} Y_k$  for its variety, where  $Y_k$  is sector  $k$ 's gross output. Downward-sloping demand endows firms with monopolistic power and variety prices in sector  $k$  are set at a mark-up  $\mathcal{M}_k$  over marginal costs. The more substitutable sector  $k$ 's varieties are (the higher  $\varepsilon_k$ ), the lower is the mark-up.

Since a firm's debt price enters marginal costs, it also affects its variety price, and eventually the sector good price,  $P_k$  (see Lemma 2.1). The sector good price, in turn, affects demand for other firms of sector  $k$ . For example, an increase of the bond price of Peugeot (e.g. due to the ECB's corporate bond purchase program) would decrease Peugeot's marginal costs. With constant mark-up pricing Peugeot's cars become cheaper, while the relative price of Renault's cars increases.

<sup>2</sup>This is the case as long as issuing debt is costly to the firm, i.e.  $\frac{1}{Q(k, i)} \cdot \frac{1}{R} > 1$ .

<sup>3</sup>Indeed, marginal costs can be rewritten as  $\Lambda(k, i) = \widetilde{Z(k, i)}^{-1} (P^X)^{\alpha_k} (\chi(k, i)^{1/\beta_k} P^K)^{\beta_k} h^{1-\alpha_k-\beta_k}$  where  $\chi(k, i)^{1/\beta_k} P^K$  is the contribution of capital expenses to the cost of producing the marginal unit of output.

Debt prices change relative variety prices and lead to demand re-balancing within the same sector. Ceteris paribus, Peugeot produces more and Renault produces less. This example also illustrates that changes in debt prices can even affect firms that do not issue *any* debt.<sup>4</sup>

Lemma 2.2 formalizes how firms' sales depend on debt prices due to monopolistic competition.

**Lemma 2.2. *Within-Sector Sales Distribution.*** *Firm  $(k, i)$ 's sales are a share of sector  $k$ 's aggregate sales,*

$$P(k, i)Y(k, i) = \zeta(k, i)P_k Y_k, \quad (2.3)$$

with  $\zeta(k, i) \equiv \left( \frac{\widetilde{Z(k, i)}}{Z_k} / \frac{\chi(k, i)}{\chi_k} \right)^{\varepsilon_k - 1} \in (0, 1)$ ,  $Z_k$  sector  $k$ 's productivity, and  $\chi_k$  its capital surcharge (defined as the productivity-weighted power mean of firms' capital surcharges). The within-sector sales distribution is determined by relative (i) productivity, and (ii) financing costs.

*Proof.* See Appendix A.2.2. □

**Debt Supply:** To produce gross output  $P(k, i)Y(k, i)$ , firm  $i$  of sector  $k$  demand inputs such that it spends constant fractions  $\alpha_k$ ,  $\beta_k$ , and  $1 - \alpha_k - \beta_k$  of total expenditure on intermediate inputs, capital, and labor. Under Cobb-Douglals technology total expenditure is given by the product  $\Lambda(k, i)Y(k, i)$ , because marginal costs equal average costs. Due to monopolistic competition, total expenditure is a constant fraction of sales  $\mathcal{M}_k^{-1}P(k, i)Y(k, i)$ . Since total sales are determined by the firms' productivity and debt prices (see Lemma 2.2), so is total expenditure and input demand. Finally, external financing needs imply that the supply of debt contracts depends on firms' productivity and debt prices.

**Lemma 2.3. *Firms' Debt Supply.*** *From  $(k, i)$ 's balance-sheet identity at the beginning of the period and financing constraint, debt supply is*

$$B(k, i) = \beta_k \cdot \frac{Q^D}{Q(k, i)} S(k, i) \chi(k, i)^{-1/\beta_k} \mathcal{M}_k^{-1} \zeta(k, i) P_k Y_k. \quad (2.4)$$

---

<sup>4</sup>In this market structure of monopolistic competition, firms do not internalize the effect of their pricing decisions on sectoral prices. Grassi (2019) provides a model with Bertrand or Cournot competition (but without debt financing) between firms of the same sector.

Deposit demand is  $D(k, i) = B(k, i) \cdot Q(k, i) / Q^d$ . Both functions are homogeneous of degree zero in financial asset prices.

*Proof.* See Appendix A.2.3. □

Before characterizing the equilibrium in variety and labor markets in Section 2.2.4, I describe next how banks allocate their resources between the financial assets in this economy.

### 2.2.3 Banks

There are  $N^B$  banks which are indexed by  $b$ . Each bank finances its portfolio by an exogenous number  $\overline{E}(b)$  of shares, an exogenous number of deposits  $\overline{D}(b)$ , and by issuing  $B(b)$  debt contracts. Banks choose their balance-sheet at the beginning of the period. As in Greenwood et al. (2015) and consistent with empirical evidence provided for example by Adrian & Shin (2010) banks target a given leverage ratio and adjust their balance-sheet size to achieve it.

Banks are heterogeneous in their equity endowments, their leverage target  $\tau^*(b)$ , and their portfolio allocation.

Figure 2.3: Bank  $b$ 's Balance-Sheet

| Assets |  | Liabilities  |
|--------|--|--|
| Banks  | Equity, $Q_b^{E'} E_d(b)$<br>Debt, $Q_b^{B'} B_d(b)$<br>Deposits, $Q^D D(b)$ | Equity, $Q^E(b) \overline{E}(b)$<br>Debt, $Q^B(b) B(b)$<br>Deposits, $Q^D \overline{D}(b)$ |
| Firms  | Debt, $Q_{ki}^{B'} B_{ki}(b)$  |  |
| Rest   | $Q^o O(b, r)$  |  |

**Banks' Balance-Sheet Diversification:** With equity, debt and deposit prices noted  $Q^E(b)$ ,  $Q^B(b)$ , and  $Q^D$ , respectively, bank  $b$ 's total value is  $A(b) \equiv Q^E(b) \overline{E}(b) + Q^D \overline{D}(b) + Q^B(b) B(b)$ . Bank  $b$  invests into a rich set of financial assets: First, bank  $b$  holds  $E(b, d)$  units of other bank  $d$ 's equity and  $B(b, d)$  of debt. Deposits are homogeneous across banks, such that  $D(b)$  denotes the

units of a generic deposit held by banks  $b$ .<sup>5</sup> Such cross-holdings of banks' liabilities constitute the financial network. Second, banks invest into debt contracts issued by firms  $B(b, k, i)$ . These investments constitute the link between firms and banks. Third, bank  $b$  also invests into an outside asset  $O(b, r)$  which is issued by the rest of the world  $r$  at price  $Q^O$ . Similar to Kojien & Yogo (2019), the outside asset captures items on banks' balance-sheets that are not part of before-mentioned assets (here for example: loans, government debt, or investments abroad). Figure 2.3 illustrates bank  $b$ 's balance-sheet.

How do banks allocate their portfolio across financial assets? Following a seminal literature on financial networks (e.g. Eisenberg & Noe (2001*b*), Elliott et al. (2014)) I take banks' portfolio allocation as given.<sup>6</sup> It is summarized by the  $N^B \times (3N^B + \sum_{s=1}^N N_s + 1)$ -size matrix  $\Omega$ , whose  $b$ th row,  $\omega'(b)$ , contains the shares of bank  $b$ 's portfolio invested into financial assets. Financial assets are ordered as in Figure 2.3, such that the first  $3N^B$  columns contain banks' equity, debt, and deposits.

**Endogenous Balance-Sheet Size:** Each bank  $b$  is also characterized by an exogenous leverage target  $\tau^*(b)$ . Leverage is defined as the value of debt and deposits over the value of equity,  $\tau(b) \equiv (Q^D \overline{D(b)} + Q^B(b)B(b)) / Q^E(b)\overline{E(b)}$ . If the value of banks' assets increase, actual leverage decreases because the value of equity increases *ceteris paribus*. To restore leverage to target, banks issue liabilities, buy assets, and scale up their balance-sheet. Vice versa, if the value of banks' assets decreases, equity value decreases, and leverage goes up. In this case, the bank sells assets, pays back debt, and scales down its balances-sheet. I assume that banks pay a quadratic costs for deviating from their leverage target  $\frac{1}{2}c(b)(\tau(b) - \tau^*(b))$ .<sup>7</sup> Since the equity value at the

---

<sup>5</sup>Thus, linkages of banks through the interbank deposit market are summarized in this single financial asset. This is done in order to match the fact that bilateral linkages in the interbank deposit market are unobserved. I do, however, observe each banks' aggregate amounts held and issued.

<sup>6</sup>Recent papers such as Farboodi (2017) and Heipertz, Ouazad and Rancière (2019) show that endogeneous portfolio choice contributes largely to our understanding of financial networks and the propagation of shocks. The assumption of fixed portfolio weights, however, provides a useful benchmark for the question addressed in this paper and simplifies the analysis considerably.

<sup>7</sup>Through the implementation of the Basel III accords by the Capital Requirement Directive IV (CRD) and the Capital Requirement Regulations (CRR), supervisors have received far-reaching powers to sanction banks that breach regulatory requirements such as minimum capital ratios. Regulatory fines can amount up to 10% of financial institutions' annual turnover (see Noyer, 2013).

end of the period increases with the balance-sheet size, banks face a trade-off between maximizing equity value and keeping leverage at target. The bank chooses its balance-sheet size and its debt issuance in order to optimize this trade-off.

**Lemma 2.4. *Leverage Targeting.*** *Under leverage targeting bank  $b$  supplies debt such that*

$$\underbrace{\tau(b) - \tau^*(b)}_{\text{Leverage Deviation}} = \frac{1}{c(b)} \underbrace{\left( \text{diag } Q^{-1} \omega(b) - \frac{1}{Q^B} \right)}_{\text{Return on Equity}} Q^E. \quad (2.5)$$

*If the cost of deviating from target goes to infinity  $c(b) \rightarrow \infty$ , the bank ensures  $\tau(b) = \tau^*(b)$ . Bond supply is homogeneous of degree zero in prices. Asset demand follows from the balance-sheet identity as  $A(b) \cdot \text{diag } Q^{-1} \cdot \omega(b)$ .*

*Proof.* See Appendix A.2.4. □

Banks' endogenous balance-sheet size implies that cross-price demand elasticities can be positive (in contrast to standard demand systems). To understand this, imagine that deviation costs are very high and that the corporate debt price  $Q(k, i)$  increases. First, the fall in the price increases the bank's return on equity (see Lemma 2.4). The bank has an incentive to issue more debt and invest more into its portfolio (the standard negative substitution effect applies). Second, however, the decrease of the corporate debt price also increases leverage because the value of today's assets decreases. If the cost of deviating from target leverage is very high, the bank has to scale down its balance-sheet. It pays back debt and sells assets (fire sales). As a result, demand for all financial assets decreases in proportion of their portfolio weight.

## 2.2.4 Equilibrium

There are three types of markets in the economy, markets for varieties, labor, and financial assets. In this section, I first show that aggregate output and firms' sales can be solved analytically as a function of financial asset prices. In the second step, the financial markets equilibrium is obtained numerically.

**Proposition 2.1. Aggregate Output.** *For given financial asset prices and normalized labor supply  $L = 1$ , variety and labor market clearings imply*

$$\log Y = \eta_Y + \gamma_k' (I - \mathcal{W})^{-1} (\log Z_k - \log \chi_k), \quad (2.6)$$

where the production network is  $\mathcal{W} \equiv (\text{diag}(\alpha_k)W^X - \text{diag}(\beta_k)W^K) \text{diag}(\mathcal{M}_k)^{-1}$  and  $\eta_Y$  is a constant. The  $k$ -size vectors  $Z_k$  and  $\chi_k$  collect sectoral productivity and capital surcharges, respectively. With a normalized final good price  $P = 1$ , sectoral sales are  $P_k Y_k = (I - \mathcal{W}')^{-1} \gamma_k \cdot \eta_{PY} Y$ , where  $\eta_{PY}$  is a constant. Firm  $(k, i)$ 's sales and bond supply follow from Equation 2.6 and Lemmas 2.2 and 2.3.

*Proof.* See Appendix A.2.5. □

Proposition 2.1 shows how productivity shocks propagate through the production network in partial equilibrium (fixed financial asset prices). The propagation of both (i) productivity shock and (ii) capital surcharges is determined by the Leontieff inverse  $(I - \mathcal{W})^{-1}$  which captures all direct and indirect consumption of sectoral goods. Note that the case of no external financing needs is nested in the Equation 2.6: If  $S(k, i) = 0 \forall k, i$ ,  $\chi_k = 1$  and aggregate output is determined by productivity shocks alone. With external financing needs, aggregate output is also affected by financial asset prices through firms' capital surcharges.<sup>8</sup>

In general equilibrium financial asset prices adjust. Following a positive productivity shock, firm  $(k, i)$ 's reduces its variety price (due to the constant market-up rule). It produces more because demand is elastic, demands more inputs, and issues additional debt to meet external financing needs. Thus, the productivity shock generates excess supply of firm  $(k, i)$ 's debt. It also generates excess demand for debt of other sector  $k$  firms who produce less and shrink their debt supply (Lemma 2.2). To clear partial disequilibria in the debt markets, the price of firm  $(k, i)$ 's debt has to decrease and the debt price of other sector  $k$  firms increases. In response to changes in debt prices, however, banks' asset return and leverage changes. Banks adjust their balance-sheet size to re-

---

<sup>8</sup>Bigio and La'O (2019) derive a similar expression for the propagation of exogenous sector-level distortions.

optimize their trade-off between return and leverage target. Resulting changes in banks' demand for and supply leads to new partial disequilibria in financial markets and to subsequent rounds of banks' balance-sheet size adjustments. Therefore, the initial shock to  $(k, i)$ 's productivity may propagate through financial markets even to firms that are very distant in terms of their input-output relations.

**Rest of the World:** To capture trade in financial assets with other agents of the economy, for example insurances, funds, or foreign entities, I introduce a *catch-all* rest of the world sector. Indeed, this is important in the empirical part of the paper in order to close the system of balance-sheets and clear financial markets. Rest of world's asset demand and supply is obtained as the residual necessary to clear markets and is assumed to be exogenous. While this assumption is strong, the analysis does not change if the rest of the world would be further dis-aggregated or would also seek to optimize its balance-sheet. In this case, the propagation of firms' productivity shocks through financial markets would likely be amplified. The reason is that productivity shock would spread to a larger number of financial markets. Nonetheless, I believe that the empirical results obtained in Section 2.4 provide a useful benchmark for the extent to which banks' leverage targeting contributes to the propagation of firms' productivity shocks.

Financial markets are assumed to be fully competitive and the equilibrium is defined as follows.

**Definition 2.1. Financial Market Equilibrium.** *The financial market excess demand function is the  $J \equiv 2N^B + 2 + \sum_{s=1}^N N_s$ -size column vector  $\Delta(Q; \theta)$  that collects (i) banks' demand and supply of  $N^B$  equity shares,  $N^B$  debt contracts, and 1 bank deposit, (ii) firms' demand for the bank deposit and supply of  $\sum_{s=1}^N N_s$  debt contracts, and (iii) rest of the world's demand for all financial assets and supply of 1 outside asset. The competitive financial market equilibrium is a  $J$ -size price vector  $Q^*$  such that*

$$\Delta(Q^*; \widetilde{Z}_{1i}, \dots, \widetilde{Z}_{Ni}) = 0. \quad (2.7)$$

I use Brouwer's fixed point theorem to proof existence and compute the equilibrium. All conditions for its application are met. First, financial asset demand and supply functions are homogeneous of degree zero and continuous in financial asset prices. Therefore, market excess demand is



homogeneous of degree zero and continuous as well. Second, Walras' law is satisfied because all balance-sheet identities hold in equilibrium. Numerically, the fixed point is obtained by constructing a transition function that updates the price vector in each step proportionally to the current level of excess demand in each market.

**Proposition 2.2.** *There exists an equilibrium price vector  $Q^* \in \mathbb{R}^J$ .*

*Proof.* The proof follows standard arguments as Mas-Colell, Whinston, Green et al. (1995a).  $\square$

## 2.3 The Financial Network Multiplier

The model predicts how productivity shocks propagate through the economy. In addition to propagation via product prices in the *IO*-network, productivity shocks also transmit through adjustments in firms' debt prices in the financial network. In this section, I show that one statistic is sufficient to characterize a firm's systemic importance for the economy. Following Acemoglu et al. (2012), this statistic is called *influence*  $\mathcal{I}_{ki}$ ; I call its financial component the *financial multiplier*.

To see this, I obtain the total differentiation of aggregate output (Equation 2.6) with respect to the vector of log productivities. Due to the capital surcharge, a financial channel appears:

$$d \log Y = \underbrace{\gamma_k' (I - \mathcal{W})^{-1} \mathcal{E}_{Z_{ki}}^{Z_k} d \log Z_{ki}}_{\text{Real}} - \underbrace{\gamma_k' (I - \mathcal{W})^{-1} \left( \mathcal{E}_{Z_{ki}}^{\chi_k} d \log Z_{ki} + \mathcal{E}_{Q^r}^{\chi_k} d \log Q^r \right)}_{\text{Financial}}, \quad (2.8)$$

where  $\mathcal{E}$  denotes an elasticity matrix of the upper-script vector with respect to the subscript vector. The capital surcharge only depends on financial assets that are demanded or supplied by (real) firms. To see this more clearly, I partition the price vector  $Q$  into two parts:  $Q^r \equiv (Q^D, Q_{ki})$  collects the deposit price and corporate debt prices, and  $Q^f \equiv (Q_b^B, Q_b^E, Q^O)$  the rest.

The financial channel captures two effects: First, productivity shocks change the relative productivity of firms within each sector. Due to monopolistic competition, the within-sector sales distribution changes in response (see Lemma 2.2) and with it the importance of firm-level capital surcharges. As a result, sector-level capital surcharges change as well. This is the first term of the

financial channel. Second, financial asset corporate debt supply changes (Lemma 2.3). Financial asset prices have to change in response to clear financial markets. This effect is captured in the second term of the financial channel.

The response of financial asset prices depends on the price-elasticity of market excess demand. Indeed, total differentiation of the financial market equilibrium yields  $d \log Q = - (\partial \Delta_{\log Q})^{-1} \partial \Delta_{\log Z_{ki}} d \log Z_{ki}$ , where  $\partial \Delta_{\log Q}$  denotes the  $J \times J$ -size semi-elasticity matrix of market excess demand with respect to prices.<sup>9</sup> The inverse of this matrix is the price impact of trade: the lower the elasticity, the larger the price response. It is this price-impact matrix that shapes the propagation of shocks across financial markets.

From the formula for the inverse of a  $2 \times 2$  block matrix, the change in  $Q^r$  that restores equilibrium is:

$$d \log Q^r = (I - \mathcal{F})^{-1} \left( \partial \Delta_{\log Q^r}^r \right)^{-1} \Delta_{\log Z_{ki}}^r d \log Z_{ki}. \quad (2.9)$$

The matrix  $\mathcal{F}$  is key. Indeed, it has the interpretation of an adjacency matrix of financial assets demanded or supplied by firms (here: deposits and debt contracts) and is, thus, the financial counterpart of the  $IO$  matrix  $\mathcal{W}$ :

$$\mathcal{F} \equiv \left( \partial \Delta_{\log Q^r}^r \right)^{-1} \partial \Delta_{\log Q^f}^r \left( \partial \Delta_{\log Q^f}^f \right)^{-1} \partial \Delta_{\log Q^r}^f. \quad (2.10)$$

It captures how price changes propagate through the financial network. To understand this matrix, we can break down the propagation into a sequence of price adjustments by developing the geometric sum  $(I - \mathcal{F})^{-1} = I + \mathcal{F} + \mathcal{F}^2 + \dots$ : A productivity shock  $d \log Z_{ki}$  first generates a partial disequilibrium  $\Delta_{\log Z_{ki}}^r$ . It then implies a price change in markets directly affected by  $\left( \partial \Delta_{\log Q^r}^r \right)^{-1}$ . The change in corporate debt prices and the deposit price induces banks to rebalance their balance-sheet. Excess demand in other markets appears  $\partial \Delta_{\log Q^r}^f$  which is cleared by a price change  $\left( \partial \Delta_{\log Q^f}^f \right)^{-1}$ . In turn, the price change in other financial markets leads to re-balancing and excess demand for debt issued by firms  $\partial \Delta_{\log Q^r}^f$ , which again must be cleared  $\left( \partial \Delta_{\log Q^r}^r \right)^{-1}$ .

---

<sup>9</sup> $\partial \Delta_{\log Z_{ki}}$  is defined equivalently

This closes a full cycle of financial asset price adjustments and is repeated infinitely until markets returned to equilibrium.

Equation 2.9 permits fully solving for the financial channel of propagation. Proposition 2.3 contains the main theoretical result of the paper.

**Proposition 2.3. *Financial Network Multiplier.*** *The effect of firm-level productivity shocks on aggregate output is given by the  $\sum_{s=1}^N N_s$ -size influence vector  $\mathcal{I}_{ki}$ ,*

$$d \log Y = \mathcal{I}_{ki}' d \log Z_{ki}. \quad (2.11)$$

*Influence is additive in a real and a financial component: The IO-influence is  $\mathcal{I}_{ki}^{R'} = \gamma_k' (I - \mathcal{W})^{-1} \mathcal{E}_{Z_{ki}}^{Z_k}$  (akin to Acemoglu et al. (2012)). The financial network multiplier is*

$$\mathcal{I}_{ki}^{F'} = -\gamma_k' (I - \mathcal{W})^{-1} \left( \mathcal{E}_{Z_{ki}}^{\chi_k} + \mathcal{E}_{Q^r}^{\chi_k} (I - \mathcal{F})^{-1} \left( \partial \Delta_{\log Q^r}^r \right)^{-1} \Delta_{\log Z_{ki}}^r \right). \quad (2.12)$$

*Closed form expressions for the elasticities of sectoral productivity and capital surcharge are given in the appendix.*

*Proof.* See Appendix A.2.6. □

Influence converges to its counterpart in pure production networks ( $\mathcal{I}_{ki}^{F'} \rightarrow 0$ ) if external financing needs disappear  $S(k, i) \rightarrow 0 \forall (k, i)$ .

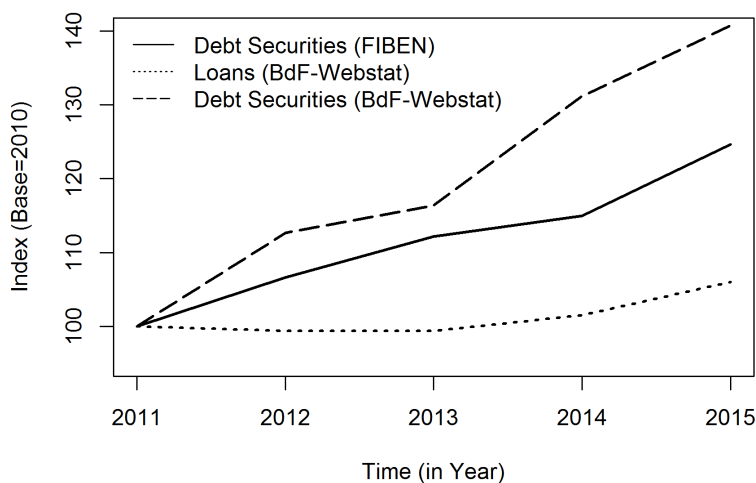
## 2.4 Estimation

I estimate the model with French data for the 2011-2015 period to measure how the financial multiplier affects the propagation of productivity shocks. In this section, I describe the data used, the empirical strategy to estimate firm-level productivity shocks and the model's parameters, and present the estimation results.

The 2011-2015 period in France is particularly interesting to study the role of debt financing for the real economy and the banking sector, because debt issuance by French firms has increased considerably ever since the global financial crisis. This is in stark contrast to other Euro-Zone countries and generated French regulators' concern about too high corporate indebtedness.

Figure 2.4: Growing Debt of French Corporates

This figure shows indices (base=2011) of French corporate debt securities and loans outstanding over the 2011-2015 period in nominal values.



Note: The series "Loans (BdF-Webstat)" corresponds to Banque de France's Webstat series *ENDET.Q.FR.S.N.LE.F4.S11.A1.S12.2.N.E.Z*. The series "Debt Securities (BdF-Webstat)" is obtained from the difference between total non-financial debt (Webstat series *ENDET.Q.FR.S.N.LE.FEND.S11.A1.S.2.N.E.Z*) and the former. The series "Debt Securities (FIBEN)" is the aggregation of corporate regulatory tax filing (French: "Liasse Fiscale") items *DS—Emprunts obligataires convertibles* and *DT—Autres emprunts obligataires* of all firms included in the database.

Debt security issuance has increased by a multiple of the increase of corporate borrowing through loans. This is likely due to ultra-accommodative monetary policy and bond-purchase programs of the ECB pushing down interest rates in this period. As shown in Figure 2.4 and measured as the aggregation of bonds (convertible and non-convertible) issued by firms included in the firm-level database FIBEN, it increased by +24.6% versus +6.0% for loans over the period. In amounts, total debt securities stood at 232.68 billion EUR in 2011 and surged to 289.98 billion EUR in 2015. The difference of the measure obtained from FIBEN to the aggregate measure from Banque de France's Webstats can be explained by the (i) the narrower definition of debt securities applied, and (ii) the selection of firms included in the firm-level data (see Section 2.4.1).

Thus, the French context provides a relevant testing ground of this paper's mechanism of pro-

ductivity shock propagation through corporate bond markets and banks' leverage targeting.

### 2.4.1 Data

The estimation of the model poses three key data challenge: First, firm-level data is necessary to estimate firm-level productivity shocks. To analyze propagation across the national economy, this data has to cover a large fraction of firms that are active in different sectors. Second, bank-firm and bank-bank linkages have to be observed in the form of a specific bank's holding of a specific firm's or another bank's debt, respectively. Third, banks' balance-sheets have to be available in order to calculate leverage and portfolio shares as well as to close the system of balance-sheets.

All three challenges are addressed by merging three firm- and bank-level datasets collected by the Banque de France.

#### 2.4.1.1 Firms

For annual firms' balance-sheets from 2011 to 2015, I rely on the *Fichier bancaire des Entreprises* (FIBEN). This dataset is constructed from regulatory tax filings (*Liasse Fiscale*) and includes all French firms with a turnover of at least 750,000 euros. It contains detailed balance-sheet and profit and loss statements, which include measures of sales (*itemFR*—operating revenue), capital stock (*itemBJ*—value of fixed assets), expenditure on intermediate inputs (*itemFU*—raw material costs and *itemFW*—other and external expenses) and labor costs (*itemFY*—salaries and *itemFZ*—social contributions). The number of employees is available as well (*itemYP*). I use the value of outstanding convertible and non-convertible bonds (*itemDS* and *itemDT*) to measure firms's external financing through bonds.

All firms are identified by the French company identification number SIREN and are classified in industries following the *Nomenclature d'activités française* which I map into the International Standard Industrial Classification (ISIC) 2-digit level classification. This is done in order to merge firms with the sectoral input-output table for the French economy. I obtain this table from the World Input-Output Database (WIOD). Following Bigio and La'O (2019), I exclude the financial,

insurance, real estate, rental, and leasing sectors (sectors *K64 – K66* and *L68*) from the dataset on firms and from the *IO*-production network. In particular, I consider purchases of these sectors recorded in the original *IO*-table as final use, and the commodity share in other sectors' production as part of the capital share of production. The reason for this is that the production function in Equation 2.1 should capture the physical process of production and not the distribution of value added through financial instruments.

Table A.1 provides descriptive statistics of firms by ISIC group level.<sup>10</sup> There are on average a total of 199,538 firms per year in the data. The average firm sells about 11.6 million EUR worth of output and has total assets of about 20.8 million EUR. The subset of bond-issuing firm is smaller, but firms are considerably larger. On average 1,359 firms per year issue bonds. The average bond-issuing firm sells 197.4 million EUR worth of goods and possess assets valued at 1,098.8 million EUR. The sector that relies most strongly on bonds is the transportation and storage sector.

The estimation of firm-level productivity shocks requires to deflate nominal values of firms' gross output value and input expenditure. Since price indices are sector-specific, I acquire industry-level deflators of intermediate inputs, gross fixed capital formation, and gross output from the EU Klems database. Furthermore, since hours worked per employee might vary across sector, I obtain a proxy of average hours per worker and sector also from EU Klems.

#### 2.4.1.2 Banks

Bank-level data comes from two sources. First, banks' balance-sheets stem from financial institutions' regulatory filings of the *M\_SITMENS* form. This form is part of the reporting system SURFI (*Système Unifié des Reporting Financiers*) used by the Banque de France for the compilation of the French monetary and financial statistics.<sup>11</sup> All French-resident banks (including French-resident subsidiaries of banks headquartered in other European countries) that have more than either (i) 2 billion euros worth of total assets under management, (ii) 500 million euros worth of loans out-

---

<sup>10</sup>In the empirical estimation, however, I use the 2-digit level industry classification which implies a total number of 50 sectors.

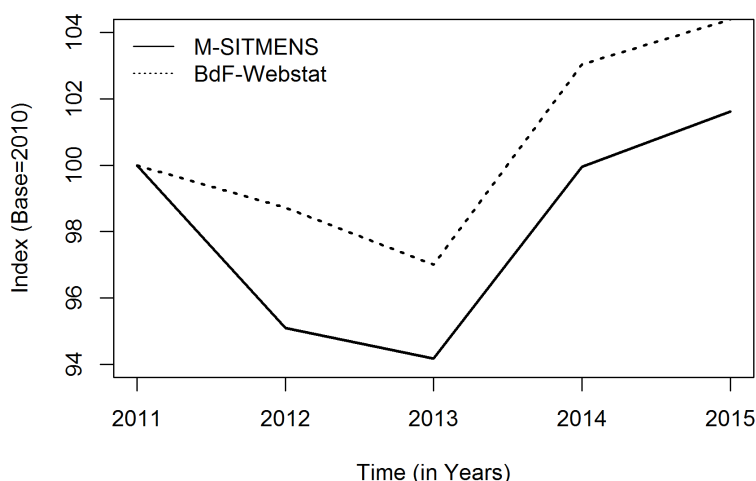
<sup>11</sup>In application of the current regulatory framework, Regulation (EU) No 1071/2013 of September 24, 2013 concerning the balance sheet of the monetary financial institutions sector (ECB/2013/33).

standing to resident households, or (iii) 500 million euros worth of leasing contracts outstanding to resident non-financial corporations or 200 millions outstanding to resident households, are obliged to file the form. Balance-sheets of banks that are part of a banking network of regional banks such as *Crédit Agricole* and *Crédit Mutuel* are consolidated at the group level.<sup>12</sup> In particular, I obtain banks' total assets (*itemS0H\_0010*) and equity book value (the sum of *itemS05\_0390*, *itemS05\_0400*, *itemS05\_0530*, and *itemS05\_0620*) from this dataset. The aggregation of banks' total assets from this data mirrors well the evolution of the French banking sector size as measured by the French financial accounts (Figure 2.5). In amounts, bank-level aggregated total assets stood at 7.04 trillion EUR in 2011 (versus 8.93 trillion EUR).

Banks are identified by the French bank identification code CIB (*Code interbancaire*).

Figure 2.5: Evolution of the French Banking Sector's Total Assets

*This figure shows indices (base=2011) of total assets of the French banking sector over the 2011-2015 period in nominal values.*



*Note: The series "BdF-Webstat" corresponds to Banque de France's Webstat series Q.N.FR.W0.S12K.S1.N.A.LE.F.\_Z.\_Z.XDC.\_T.S.V.N.\_T from the French quarterly financial accounts. The series "M-SITMENS" is the aggregation of total assets (*itemS0H\_0010*) of French banks included in the bank-level database.*

Second, holdings of securities by bank stem from the quarterly PROTIDE survey (Production de statistiques de Titres en Détention) among French resident custodians and financial institu-

<sup>12</sup>Also other financial institutions than banks that meet these criteria file the *M\_SITMENS* form. The list as of November 8, 2019 of filing institutions is available online: Filing institutions MSITMENS (Banque de France, January 2019).

tions.<sup>13</sup> The data is collected on a security-by-security basis and contains the totality of securities holdings as they are reported in the French financial accounts. Importantly, it contains issuer identifiers, i.e. the SIREN in the case of a non-financial corporation and the CIB in the case of a financial institution. This mapping is crucial to generate the bank-firm linkages.

I merge securities holdings from PROTIDE with balance-sheets from M\_SITMENS. In the resulting bank-level dataset, there are between 155-174 banks per year. In 2015, the average bank has 41.1 billion EUR worth of total assets, but the distribution of bank size is very concentrated, with a median of 8.8 billion EUR (see Table A.2) and a 99th percentile of 678.1 billion EUR. The size distribution is relatively stable over time. The average French bank invests 0.24%-0.50% into French corporate debt securities depending on the year and some banks hold considerably more (95th percentile is 1.48% in 2015). While this still seems small as a share of the balance-sheet, it does amount to an important share of banks' capital. In 2015, the average exposure of banks' equity to corporate debt was  $0.5/6.88 = 7.27\%$ . Since corporate debt investments are highly concentrated in a few large companies, even small shares can imply large risk exposure to single corporates. Furthermore, as shown in Columns 10 and 11 of Table A.1, banks hold a considerable share of corporate bond markets depending on the sector.

The average capital ratio fluctuates between 5.88% and 7.14% over the sample period.

## 2.4.2 Empirical Strategy

Identifying the model means finding the structural parameters that determine the household's preferences ( $\gamma_k$ ), firms' technology ( $\alpha_k, \beta_k, \varepsilon_k, w_{ks}^X, w_{ks}^K$ ), productivity shocks  $Z_{ki}$ , financing constraint  $S_{ki}$ , and banks' balance-sheet management ( $\tau_b^*, c_b, \Omega, \overline{D_b}, \overline{E_b}$ ). Sector-level parameters are obtained for a total of 50 sectors at the 2-digit *ISIC* industry level. I present estimation results averaged at the *ISIC* group level (see Table A.3).

---

<sup>13</sup>The list as of November 8, 2019 of filing institutions is available online: Filing institutions PROTIDE (Banque de France, September 2018)



### 2.4.2.1 Substitution and Factor Elasticities

To identify the elasticities of substitution and sectoral factor elasticities, I follow Osotimehin (2019) and aggregate firms' first-order conditions to the sector-level. Once aggregated to the sector-level, firm-level heterogeneity disappears in some key equations. First, I obtain the elasticity of substitution  $\varepsilon_k$  from aggregating profits of sector  $k$  firms. Cobb-Douglas production technology at the firm level implies that profits are a function of price and constant marginal costs,  $\Pi(k, i) = (P(k, i) - \Lambda(k, i))Y(k, i)$ . With the pricing rule, profits are simply a constant fraction of sales  $\Pi(k, i) = (1 - \mathcal{M}_k^{-1})P(k, i)Y(k, i)$ . Aggregation yields sectoral profits as a function of sectoral sales

$$\Pi_k = (1 - \mathcal{M}_k^{-1})P_k Y_k. \quad (2.13)$$

I measure profits before tax and sales (as operating revenue) in *FIBEN* and find  $\varepsilon_k = \frac{1}{T} \sum_{t=1}^T \frac{P_{kt} Y_{kt}}{\Pi_{kt}}$  as the average over the period.

Second, I aggregate firms' optimal labor demand  $hL(k, i) = (1 - \alpha_k - \beta_k) \cdot Y(k, i)\Lambda(k, i)$  and intermediate goods input demand  $P^X X(k, i) = \alpha_k \cdot Y(k, i)\Lambda(k, i)$  and obtain with the pricing rule:

$$hL_k = (1 - \alpha_k - \beta_k) \mathcal{M}_k^{-1} P_k Y_k, \quad (2.14)$$

$$P^X X_k = \alpha_k \cdot \mathcal{M}_k^{-1} P_k Y_k. \quad (2.15)$$

Factor elasticities  $(1 - \alpha_k - \beta_k) \mathcal{M}_k^{-1} = \frac{1}{T} \sum_{t=1}^T \frac{h_t L_{kt}}{P_{kt} Y_{kt}}$  and  $\alpha_k \mathcal{M}_k^{-1} = \frac{1}{T} \sum_{t=1}^T \frac{P_t^X X_{kt}}{P_{kt} Y_{kt}}$  can be backed out from these equation taking the average over the period. To measure labor costs, I use the total wage-bill (salaries plus social contributions), intermediate input expenditures are obtained as the sum of raw material costs and other expenses.

Elasticity estimates are shown in columns 1 – 3 of Table A.3. Compared to the literature (see Imbs & Méjean (2015)), I find rather high values for the elasticity of substitution  $\varepsilon_k$  and it varies greatly over the sectors. This seems to indicate that profits have been rather low compared to sales over the 2011 to 2015 period. The implication of high values of substitution between varieties in the model is that small marginal cost differences between firms lead to large differences in

firm-level sales. Estimated factor elasticities are consistent with values obtained in the literature. For example, Halpern, Koren & Szeidl (2015) find, using a semi-parametric approach, a factor elasticity for intermediate inputs of 0.752 and a labor share of 0.198. My estimates for France are 0.67 and 0.19, respectively.

Consumption, intermediate input, and capital shares are directly obtained for France from the World-Input-Output Database. Data is available from 2011 to 2014 and I average over yearly data to find these parameters.

### 2.4.2.2 Productivity Shocks

Firm-level productivity is given by the Solow residual,  $\widetilde{Z}(k, i) = \frac{Y(k, i)}{\eta_k X(k, i)^{\alpha_k} K(k, i)^{\beta_k} L(k, i)^{1-\alpha_k-\beta_k}}$ . In the data, output, intermediate inputs, and capital are measured in euro amounts. Since firms of the same sector use the same composite intermediate input and capital good, they also face the same price-index,  $P^X$  and  $P^K$ . I use industry price deflators from EU Klems database to deflate the raw material costs and other expenses, and the book value of fixed asset (Lopez-Garcia, di Mauro & Altomonte (2014)). Labor is measured by the number of workers multiplied by a proxy of average hours worked in each sector constructed from EU Klems data. Since firm-level prices are unobservable, firms' CES demand can be used to back-out output as  $Y(k, i) = (P(k, i)/P_k Y_k)^{\frac{\varepsilon_k}{\varepsilon_k-1}} Y_k$ . Sector-level output is deflated by the gross output industry price deflator.

I then calculate firm-level productivity as

$$\widetilde{Z}(k, i) = \frac{(P(k, i)/P_k Y_k)^{\frac{\varepsilon_k}{\varepsilon_k-1}} Y_k}{\eta_k X(k, i)^{\alpha_k} K(k, i)^{\beta_k} L(k, i)^{1-\alpha_k-\beta_k}}. \quad (2.16)$$

The estimated distribution of firm-level log productivity varies across sectors (Table A.3). While the average construction firm (sector group  $F$ ) is most productive 0.13, it is closely followed by the arts entertainment, and recreation sector ( $R$ ) with 0.11, and transport and storage ( $H$ ) with 0.10. The average manufacturing firm ( $C$ ) has a productivity of 0.09. The largest dispersion of productivities is obtained for firms active in professional, scientific, and technical activities ( $M$ ). Aggregation to the sector-level somewhat changes this ranking. The manufacturing sector is with

0.94 more productive than construction (0.65) and the  $M$  sector grouping is the most productive.

Figure A.1 panel (a) shows that the distribution of productivity in the manufacturing sector is centered around a higher value than the one of the total economy. This is consistent with stylized facts. The dynamics of country-level aggregate productivity are broadly consistent with the estimates obtained for France by EU Klems (panel (b)).

### 2.4.2.3 Banks' Structural Parameters

To find banks' structural parameters I rely on the detailed data provided by the Banque de France on balance-sheets and bank-level security-level holdings. In particular, since I assume that portfolio weights are fixed (following a large literature in financial networks), I can read the matrix of portfolio weights  $\Omega$  and the book values of equity and other liabilities directly from the data. Further, I assume that target leverage is the prevailing leverage in the data (see Table A.2 for distribution moments of leverage over the sample period). This follows Greenwood et al. (2015). For the cost of leverage deviation, I calibrate the same value for all banks of 0.1 which is consistent with Zhou et al. (2016).

## 2.5 Equilibrium Results

This section presents the equilibrium results obtained from solving numerically for the financial market equilibrium. Indeed, to measure the importance of the financial network multiplier for the propagation of productivity shocks, the equilibrium financial asset prices have to be known (Equation 2.7). To do so I use the transition function  $T : \mathbb{R}^J \rightarrow \mathbb{R}^J$  that maps a candidate price vector  $\mathbf{Q}$  into a new candidate price vector to clear excess demand.<sup>14</sup> The resulting equilibrium

<sup>14</sup>I use the following transition function, but other transition functions are possible as well:

$$T(\mathbf{Q}) = \frac{1}{\sum_{j=1}^J Q_j + \max\{0, \Delta_j(\mathbf{Q}; \cdot)\}} (\mathbf{Q} + \max\{0, \Delta(\mathbf{Q}; \cdot)\}).$$

In each step, this transition function increases the price for markets that are in excess demand only. Since prices are on a unit simplex this implies that prices in excess supply markets decrease proportionally. Since Walras law is satisfied, this procedure yields a market equilibrium.

solves the financial market equilibrium at  $1e^{-6}$  tolerance. I then obtain the Jacobian of market excess demand with respect to log financial prices  $\log Q$  and with respect to log productivity  $\log Z_{ki}$  numerically.<sup>15</sup> Together with the closed form expressions in the proof of Proposition A.2.6 in the appendix, all terms can be calculated at market equilibrium  $Q^*$ . Following, I present equilibrium results for the year of 2011 where the number of total assets is  $J = 1,456$ . My goal is to measure the importance of the financial network multiplier for the propagation of productivity shocks.

Recall that from Proposition 2.3, a firm's total influence is additive in an IO and a financial component. I use a variance decomposition across firms to understand which component is most important for a firm's overall influence.

$$\text{Var}(\mathcal{I}_{ki}) = \text{Var}(\mathcal{I}_{ki}^R) + \text{Var}(\mathcal{I}_{ki}^{F \text{Composition}}) + \text{Var}(\mathcal{I}_{ki}^{F \text{Asset Price}}) + \mathbb{C}, \quad (2.17)$$

where I further decomposed the financial network multiplier into a compositional effect<sup>16</sup> and a financial asset price change effect<sup>17</sup>.

The influence of a firm on aggregate output through asset prices is considerable (Table A.4). Across all sectors, the share of the cross-firm standard deviation of the asset-price component of the financial network multiplier is 0.53. It is lower than firms' influence through the production network (which has a standard deviation share of 1.01). For some sectors, however, influence through debt prices explains more of the variation of total influence. For example, electricity, gas, steam and air conditioning supply (sector group  $D$ ) firms' influence varies more due to financial market propagation than due the propagation through the production network. This is consistent with the fact that in this sector (containing French major electricity provider EDF) firms rely heavily on bond financing (see Table A.1). Figure A.2 show the overall relation between total influence and its components across sectors. In terms of the absolute magnitude, Figure A.3 shows that higher influence through the input-output production network does not necessarily mean higher influence

<sup>15</sup>Though, in principal, a closed form solution is available. I use the "numDeriv" R package to obtain the derivatives.

<sup>16</sup>The compositional effect is:  $\mathcal{I}_{ki}^{F \text{Composition}} = \gamma_k' (I - \mathcal{W})^{-1} \mathcal{E}_{Z_{ki}}^{\mathcal{X}_k}$ .

<sup>17</sup>The asset price change effect is  $\mathcal{I}_{ki}^{F \text{Asset Price}} = \mathcal{E}_{Q^r}^{\mathcal{X}_k} (I - \mathcal{F})^{-1} \left( \partial \Delta_{\log Q^r}^r \right)^{-1} \Delta_{\log Z_{ki}}^r$ .

through financial asset markets, and vice versa.<sup>18</sup>

### 2.5.1 Financial Spill-Overs

The financial network multiplier is mostly positive. Interestingly, however, for some firms it is negative and the effect of a productivity shock to these firms on aggregate output is damped by financial markets. The reason for this result is that substitution elasticities in the financial network are not necessarily negative. A price decrease in one corporate debt market does not necessarily lead to more demand for other firms' corporate bonds. Indeed, this is the case if leverage deviation costs are too high. To understand this result, imagine a positive firm-level productivity shock. The firm-level productivity shock decreases firms' marginal costs and the firm passes it on the lower variety prices. Since demand is elastic within a sector, the firm increases production to satisfy higher demand. To produce more, the firm needs more inputs but faces a financing constraint for capital. It therefore issues more bonds. The productivity shock generates a positive bond supply shock and therefore excess supply on the market for the firms' corporate debt.

As a result, the firms' corporate debt price needs to decrease which increases its return. Due to the return increase, banks have an incentive to scale up their balance-sheet and invest more. However, the decrease in the corporate debt price also increases banks' leverage. It depends on the strength of both channels, if the productivity shock to the firm leads to positive or negative spillovers to other financial markets.

$$\underbrace{\uparrow \tau(b) - \tau^*(b)}_{\text{Leverage Deviation}} = \frac{1}{c(b)} \underbrace{\left( \text{diag } Q^{-1} \omega(b) - \frac{1}{Q^B} \right)}_{\text{Return on Equity}} Q^E \uparrow \uparrow \quad (2.18)$$

If leverage deviation costs are low, the bank has room to scale up its balance-sheet and demands more of other firms' corporate debt. This is the case depicted in Equation 2.18. This increases

<sup>18</sup>The composition effect of the financial network multiplier is negative on average, because a productivity shock increases the weight of a given firm in sectoral sales and therefore also the relative importance of the firms' capital surcharge. Since the capital surcharge is always smaller than one (its log is smaller than 0), the composition effect is negative.

corporate debt prices for all firms that banks are invested in. Higher corporate debt prices alleviate financing constraints of other firms in the economy who pass on their decrease in marginal costs to their variety prices and, therefore, to downstream firms and eventually the household and increased aggregate output. On the other hand, if leverage deviation costs are very high, the bank has to scale down its balance-sheet which initiates a fire-sale spiral that crashes corporate debt prices, increases marginal costs, and eventually decreases aggregate output.

It is important not only to measure the influence of a firm through its production network, but also through the endogenous change in asset prices.

## 2.5.2 Importance of the Financial Network

The propagation of corporate debt price changes through the financial network crucially depends on the matrix  $\mathcal{F}$ . It is a function of banks' elasticities of substitution between financial assets. As in Heipertz et al. (2019) this matrix is key to understand the propagation of shocks from market to market and bank to bank. Also here the cost of leverage deviation is crucial for its shape. Since banks are heavily invested in each others' liabilities (inter-bank-deposits, loans but also equity and debt securities), leverage targeting has important effects on other banks. This is shown by Greenwood et al. (2015). Through these connections a change in the debt price of one firm can impact all banks in the financial network.

## 2.5.3 Which firms are most influential?

In order to understand which firm propagates more through which channel, I run four OLS regressions of the firms' influence on its log productivity, its equilibrium price, and its financing share. The results (Table A.5) Consistent with the theory, firms with a higher productivity level have a higher influence through the IO network, since high productivity firms produce a high share of sectoral sales. High productivity firms also have a high influence through the financial network. This is the case because a shock to those firms generate a large bond supply shift and leads therefore to a larger increases in banks' balance-sheets and as a result in debt prices of other firms (see

column 4). Interestingly, firms with a high equilibrium debt price are less influential. Indeed, high-debt-price firms are relatively less financially constrained in equilibrium compared to other firms. In response to a productivity shock, only a small number of bonds has to be issued to finance capital expenditures and financial asset prices adjust less.

## 2.6 Conclusion

In this paper, I have shown how productivity shocks propagate through endogenous financial asset prices and effect real output. The influence of a firm on aggregate output through the financial markets can be summarized by one sufficient statistic, i.e. the financial network multiplier. The paper contributes to the literature on the network-origins of aggregate fluctuations a new propagation mechanism that comes from the endogenous response of financial asset prices and their effect on the marginal costs of firms.

This paper opens several exciting avenues for further research. First, the financial network multiplier allows to study propagation in much greater detail. Indeed, through counterfactual analysis, the effect of *one* specific financial institutions' capital ratio on real firms can be assessed. The explicit link between real fluctuations and financial asset prices also allows future research to introduce different financial intermediaries such as insurances. Indeed, recent papers such as Chodorow-Reich et al. (2016) argue that insurance companies as long-term investors contribute to more financial stability by isolating specific financial assets from transitory price movements.

Counterfactual analysis can be used to understand in great detail how, for example, regulatory changes (such as increased leverage requirements) affects real output. The estimated framework is also ideally suited to study the effect of large scale asset purchase programs such as the corporate bond purchase programs (CSPP) implemented by the European Central Bank as of June 2016. It permits to study targeted purchases of debt securities which following the financial network multiplier statistic have the largest influence on aggregate output.

## Chapter 3

# The Transmission of Shocks in Endogenous Financial Networks: A Structural Approach

*“It’s not a bank large enough to cause systemic crisis,”* said Lorenzo Codogno, a former chief economist at Italy’s treasury who operates LC Macro Advisors, a consultancy in London. *“But,”* he added, *“we have seen that even small banks can cause huge problems.”* – E.C.B. Takes Reins of Italian Bank to Prevent Wider Crisis, New York Times, January 2, 2019.

### 3.1 Introduction

Does financial network structure matter for financial stability? A seminal theoretical literature (Acemoglu, Ozdaglar & Tahbaz-Salehi 2015, Elliott et al. 2014, Gabrales, Gottardi & Vega-Redondo 2017) suggests that contractual links between banks’ balance sheets shape a network structure that affects aggregate stability above and beyond what is captured by standard aggregate and bank-level prudential ratios. Elliott et al. (2014) shows that a moderate increase in diversification increases network connectivity and *amplifies* the propagation of defaults, while more complete diversification makes financial networks more resilient to bilateral default transmission. On the empirical side, a key challenge is the development of identifiable and micro-founded endogenous



network formation models that (i) allow for the endogenous emergence *and* transformation of financial ties, (ii) match the large heterogeneity of institution- and security-level holdings, (iii) test the empirical relevance of theoretical network mechanisms.

This paper addresses these three challenges. First, it puts forward a model of heterogeneous banks and securities where financial linkages form endogenously, shaping a network of interlinked banks' balance-sheets through trade in securities. The network is shaped by bank-specific beliefs about future payoffs, combined with their risk-aversion and cost of equity; as well as by the cost of trading financial instruments. Individual banks' decisions to form bilateral ties makes a network of banks emerge out of a structural general equilibrium model of trade in financial assets and liabilities. Second, beliefs about payoffs, risk aversions, and costs of equity are structurally estimated<sup>1</sup> using disaggregated balance sheet information on assets and liabilities across a number of financial instruments for the entire universe of the 303 French banks. Third, estimated parameters enable a comparative statics prediction of the network response to financial shocks such as the ECB's quantitative easing programs. In particular, this large policy shock affects network topology, with consequences for asset prices, banks' distance to insolvency and their influence on key parts of the economy.

The microfounded model is an equilibrium in incomplete markets and incomplete information, where heterogeneous banks agree to disagree about instruments' future payoffs and cannot fully learn other banks' beliefs from the observation of traded prices. Under such market and information structure, trade occurs in contrast with the typical no-trade result (Merton 1987, Milgrom & Stokey 1982). At general equilibrium, banks form bilateral ties with each other by either buying assets or supplying liabilities. Trading occurs, and ties form, for two reasons: first, because banks are heterogenous in their beliefs, and second, for risk-sharing purposes as they differ in risk aversion.

The network emerges from its ability to turn partial equilibrium shocks into general equilib-

---

<sup>1</sup>In that sense, the paper follows the literature sparked by Ross (2015), which aims at identifying the market's subjective probabilities of future events from prices. Here we back out each institution's forecast from the time variation of demands.

rium impacts. Indeed, partial equilibrium shocks propagate through ties by causing potentially costly shifts in bank-level diversification of assets and liabilities. Trade costs limit banks' ability to diversify. The propagation of shocks in general equilibrium provides the micro-foundation of a weighted and directed network of banks. For each bank pair, the weights represent the total equilibrium adjustment in one bank's equity returns with respect to the other bank's equity price. The adjacency matrix represents the general equilibrium response to the partial equilibrium change in equity return for the vector of banks. The combination of banks' instrument-level rebalancing of assets and liabilities, and each market's adjustment of prices to achieve *partial* equilibrium, implies a sequence of propagation of shocks that brings markets back to a *general* equilibrium. This general equilibrium framework yields a network, which is based on the final impact of a change in the equity price of a bank on the equity prices of the other banks.

Such network is amenable to typical measures of bank influence on, and bank sensitivity to, network shocks. We derive measures of banks' influence and sensitivity that capture the chain sequence of balance sheet rebalancings; such complex propagation mechanisms can be measured with a simple transform of the network's adjacency matrix of general equilibrium equity responses. Any parameter change or shock also alters the structure of the network itself, resulting in time-varying shifts in influence and sensitivity, and thus shifts in the propagation of future shocks.

The influence of a bank may not be driven by its size: the strength of network links crucially depends on the elasticity of *substitution* between financial instruments rather than on the size of holdings, or the size of the balance sheets. Indeed, we highlight the presence of small banks (in terms of equity or total assets) that have a sizable influence on other banks. The elasticity of substitution is central in workhorse models of the industrial organization literature (Berry et al. 1995), yet has been so far absent in both the theoretical financial network literature which focuses primarily on the number of links in incomplete networks, and in policy discussion which focuses on the size of the banks and that of asset holdings to detect sources of systemic risk.

Theoretical results reveal an important tension between the bank's balance-sheet size, the bank's decision to diversify, and the effects of such diversification on the system-wide fragility.

In particular, we show while the more risk-averse banks reduce the variance of their returns by diversifying their portfolio, they increase their influence in the network. An increase in a bank's risk-aversion increases both the influence and the sensitivity of its equity. This novel finding which derives from the endogeneity of the network to individual asset demand and supply decisions runs in vast contrast with two common "*rules of thumb*": first, the largest banks may not be the most systemic ones; and second, portfolio diversification may increase influence and systemicness rather than reduce it.

The financial network's key topological characteristics are pinned down by the trade costs, risk aversions, and beliefs of the underlying banks. This approach enables an understanding of the evolution of the financial network over time in terms of the evolution of structural parameters. It also leads to out-of-sample predictions of the impact of policy shifts on asset prices, net demands, banks' balance-sheet diversification. Such counterfactual analysis captures both network propagation and network transformation. The network changes as banks' measures of influence and sensitivity shift in response to shocks. This paper is to the best of our knowledge the first to provide a direct structural link between net financial asset trade in general equilibrium, network structure, and network effects.

The model is identified under the framework that each bank is using its own factor model of return forecasting. Such factors explain banks' asset and liability diversification. Under rather standard assumptions, a key result is that the mapping from the factor model of return beliefs to the factor model of net holdings is one-to-one: from the structural parameters (beliefs, risk aversion) into the optimal portfolio of assets and liabilities. The model is thus identified by inverting such mapping. This means mapping available time-varying security holdings from banks' balance sheets into structural parameters. Identifying the factor model of net demands yields the identification of the factor model used to predict returns, as well as each institution's risk aversion and cost of equity. The correlation structure of factors pins down the first two moments and comoments of return beliefs. Finally, security-specific trade costs are identified by the autocorrelation of net holdings, over and above what is predicted by the autocorrelation of return beliefs. This process

overall identifies the financial network implied by the partial and general equilibrium response to any change in the model's parameters.

The model is estimated on disaggregated information for the universe of French banks. The banking sector is disaggregated at the individual bank level for 303 banks using balance-sheet data that includes cross-holdings of each bank's equity and detailed instrument holdings. Ties of banks with other sectors are observed using a quarterly dataset of who-to-whom sector-level and instrument-level information for seven sectors of the French economy and the rest of the world. Sectoral balance sheets provide the bilateral ties of 355 financial instruments, which includes traded securities, non-traded financial assets (e.g. loans), and real assets. Connecting banks with other sectors of the economy is key for our general equilibrium approach. Banks are indeed connected through other instruments than their equities. This requires solving for the equilibrium of instrument markets. Such approach account for indirect links that reflect their network interactions with other sectors of the economy.

The model's estimation starts by estimating each bank's factor model. A small number of factors is able to explain the bulk of the variance in net demands. The factors exhibit a close correspondence with drivers of the *Global cycle* and the *Eurozone cycle* that correlate with US and Euro policy rates, global GDP growth and global trade growth, the VIX measure of implied global risk aversion, or interest rates on government securities for the countries that were subject to the sovereign crisis of 2011 (Greece, Ireland, Italy, Portugal and Spain). Return beliefs are consistent with realized returns: they explain up to 39 percent of the variance of ex-post returns depending on the sector and instrument. The factor structure on return beliefs is rather similar across sectors, up to a few changes in the ordering of the factors (e.g banking being more sensitive to short-run factors and insurance to long run factors). Return beliefs are generally in line with ex-post returns. They tend to be correlated across sectors and especially so for the banking, mutual funds, and corporate sectors. The belief model fails, however, to predict returns in one key instance: during the 2007-2008 crisis when some institutional sectors made counter-cyclical net asset purchases, i.e. buying assets whose returns are declining. Such misperception right before the financial crisis is

akin to Gennaioli & Shleifer (2018). Such crisis led to a broad reassessment of beliefs, yet not all banks reacted in the same way: more risk-averse banks (but not larger and more leveraged banks) revised their beliefs more aggressively following these forecasting errors.

The estimation of the model sheds light on the relative importance of each of the financial network components in the propagation of shocks. Valuation effects through the initial network of cross-equity holdings among banks play a much minor role in the transmission of shocks than the endogenous rebalancing of cross-equity holdings and the endogenous change of holdings of non-equity instruments. For this reason, the transmission of shocks in the financial sector happens more strongly *between* banking groups than *within* banking groups.

The paper simulates the propagation of the ECB's quantitative easing policy: how such propagation affects asset prices, banks' balance sheets, and the shape of the network itself. One quarter of quantitative easing in the Euro area leads to a reduction of the French long term government yield by 44 bps along with a flattening of the yield curve and an increase in the equity premium. Banks rebalanced their balance sheet holdings from debt to equity, and, in parallel, increased (decreased) their equity (bond) liabilities, and increased their distance to insolvency (according to the first order variation of Atkeson, Eisfeldt & Weill (2017) in general equilibrium). Effects are strongly heterogeneous across banks. Less risk averse and more influential banks reshuffle their balance sheets less. A very novel result is that the influence of banks in the financial network affects their portfolio rebalancing beyond the role of initial balance sheets and risk aversion. The financial network becomes less fragile and systemic after the policy shock: mean influence and mean sensitivity goes down. Such reduction is larger for more influent and/or more sensitive banks but, importantly, is uncorrelated with their asset size.

The paper contributes to the theoretical and the empirical literatures. First, the paper follows the theoretical research program laid out by Elliott et al. (2014), who stated that "a fully endogenous study of the network of cross-holdings and of asset holdings is a natural next step." The paper endogenizes the network structure by using a general equilibrium model of asset trade. Second, the paper estimates the network structure using disaggregated data on bank's balance-sheets. This

builds a bridge between the empirical literature on asset demand among heterogeneous institutions (Kojien & Yogo 2019, Miranda-Agrippino & Rey 2015), extending it to endogenous asset *supply* and asset demand, and the theoretical network literature.

The paper contributes to the theory literature by providing a model that sits on a continuum between non-tradable contracts and costlessly tradable securities. The higher the trade costs, the more persistent the bilateral ties formed between institutions. The financial network forms endogenously as linkages are constrained by financial instruments' trade costs, yet securities can be traded on the secondary market. Highly tradable instruments (e.g. listed equity, with lower trade costs) lead to more volatile links and lower price-elasticities. A periphery of fairly static, hard-to-trade instruments, forms around a set of highly tradable instruments. As such our network differs with networks with non-tradable bilateral contracts in which ties remain as initially formed until a default occurs, as in Acemoglu et al. (2015). The two types of networks (trade-based v. contract-based) differ both in terms of network formation and in terms of network propagation. Attempts to endogenize the second type of network rely on network formation games as in Gabrales et al. (2017) and in Farboodi (2014), while endogenizing the first type of network relies on a general equilibrium model of asset trade as in this paper. Propagation in bi-lateral networks occurs when a negative shock to a debtor party makes him unable to honor a contract with a creditor, which in turn lack the funds needed to deliver on its on its obligations to third parties. By contrast, propagation in our network results from prices changes and balance sheet reshuffling and can occur even across seemingly unrelated assets and across seemingly unrelated institutions. Our paper provides micro-level foundations to the propagation mechanism among vulnerable banks analyzed by Greenwood et al. (2015).

Our network provides a novel perspective on insolvency risk due to the propagation of shocks through the financial network structure (Acemoglu et al. 2015, Elliott et al. 2014, Farboodi 2014, Gabrales et al. 2017, Greenwood et al. 2015). Indeed, the paper suggests that network structure either amplifies or mitigates insolvency risk due to shocks, which can be simply measured by the ratio of the general equilibrium response to the partial equilibrium shock. The measure of

conductance proposed by Cheeger (1969) suggests that a higher conductance, i.e. a more tightly connected weighted network, exhibits less amplification. This is reminiscent of Elliott et al. (2014): beyond some threshold, a more diversified network reduces default contagion. Furthermore, by estimating the response of banks' entire balance sheets to shocks, we can measure how the financial network amplifies or mitigates default risk – as measured by the distance to insolvency – even in absence of a realized default. Such a measure is especially relevant for regulators that would like to assess the risk of capital shortfalls and the potential need for recapitalization or forced deleveraging in a bank. In that dimension, our paper proposes both micro-foundations and empirical estimation.

By using a general equilibrium model with heterogeneous agents, this paper follows a finance tradition that goes back to Merton (1987) and includes seminal contributions such as Berrada (2006), Gandhi & Serrano-Padial (2015), Coimbra & Rey (2017). The contribution of the paper to this literature is to show how observable net demands for financial instruments identify a model with substantial heterogeneity (in risk-aversion, beliefs, trading costs). By backing out beliefs in a framework with heterogeneous agents, the model follows the precedent set by Ross (2015) in the case of a representative agent. The paper shares with Kojen & Yogo (2019) the objective of a structural model that simultaneously matches asset demands and imposes market clearing but our approach differs in two dimensions. First, we simultaneously model the demand and supply of assets, which is key to derive the network of linkages between bank's balance sheets. Second, rather than estimating a discrete choice model to understand portfolio shares, we are using the observed structure and *size* of net demands for financial instruments to reveal the factor structure of beliefs about asset returns and explain institution size and leverage.

The paper contributes to the literature on the evaluation of the effects of shocks on both individual banks and the stability of the financial system by providing an empirical framework that is (i) immune from the Lucas' critique, (ii) jointly takes into account the propagation of shocks in the network and the transformation of the network, (iii) yields bank-level prediction of balance-sheets, asset prices, and network-based measures of financial fragility. The paper addresses these 3 challenges by bridging the gap between the theoretical literature on the effect of monetary policy

on risk-taking by heterogeneous banks (Coimbra & Rey 2017), and the empirical literature. Our structural approach complements the reduced-form empirical approaches of Dell’Arricia, Laeven & Suarez (2017) for the US, and of Kojien & Yogo (2019) for the Euro Area, on the risk-exposure and asset prices consequences of large-scale asset purchases. It does so by micro founding the demand and price responses to shocks affecting bank’s balance-sheets and by using the tools of network analysis to characterize the evolution of financial fragility.

The paper proceeds as follows. Section 3.2 describes the endogenous diversification of assets and liabilities of each bank at market equilibrium. The Section makes the financial network emerge endogenously from such institution-level arbitrage. Section 3.3 introduces the identification and estimation of return beliefs using the time varying cross-correlation of security holdings. Section 3.4 presents the data. Section 3.5 takes the model to the data and estimates beliefs and risk aversions. Section 3.6 uses such estimates to simulate the impact of quantitative easing interventions by the European Central Bank on key asset prices, balance-sheets, and network topology.

## 3.2 The Financial Network

This Section presents the model of endogenous network formation between firms who trade in both liquid and illiquid assets. The relative illiquidity of a subset of financial instruments leads to a persistent yet evolving network in which firms’ balance-sheet are connected by both liquid assets and by costly-to-trade assets. Firms form ties between firms by either purchasing assets or supplying liabilities. Trading occurs, and ties form, for two reasons: first, as firms are heterogeneous in their beliefs, and second, for risk-sharing purposes. Yet, trading costs, due to transaction costs, lack of market clearing mechanisms (over-the-counter securities), or other drivers, limit the ability of firms to engage in the full set of profitable trades. This leads to a network structure whose ability to respond to shocks is affected by the magnitude of trading costs.

This Section presents such constrained general equilibrium framework, which provides the micro-level foundations for the network of banks. The network of banks measures the response



of firms' equities to partial equilibrium shocks. Such partial equilibrium shocks propagate as they cause a re-diversification of assets and liabilities. Key measures summarize respectively each firm's *influence* and *exposure*, which respectively characterize a firm's systemicness and financial fragility.

### 3.2.1 Balance-Sheet Diversification and Size

Firms  $i = 1, 2, \dots, N$  trade  $j = 1, \dots, J$  financial instruments over time periods indexed by  $t$ . In such period  $t$ , financial instrument  $j$  delivers an observable payoff  $\pi_{jt}$  and has a market price  $p_{jt}$ .

In period  $t$ , firm  $i$ 's belief about the *payoff* of financial instrument  $j$  in  $t + 1$  is represented by a firm- and instrument-specific random variable  $\tilde{\pi}_{ijt+1}$ . While the beliefs about end-of-period returns are firm-specific (each firm estimates a forecast of returns), the asset price  $p_{jt}$  is public information. In that sense, the law of one price implies that the uncertainty lies in the projection of future dividends and future security values.

$$\tilde{r}_{ijt} = \frac{\tilde{\pi}_{ijt}}{p_{jt}} = \frac{\mathbb{E} \left[ \widetilde{p_{jt+1}} + \widetilde{d_{jt+1}} \mid \Omega_{it} \right]}{p_{jt}} \quad (3.1)$$

where  $\widetilde{p_{jt+1}}$  is the price in period  $t + 1$ ,  $\widetilde{d_{jt+1}}$  the dividend in period  $t + 1$  and  $\Omega_{it}$  the information set of firm  $i$  in period  $t$ . Firm  $i$ 's beliefs about returns are represented in vector form as  $\tilde{\mathbf{r}}_{it} = \tilde{\boldsymbol{\pi}}_{it} / \mathbf{p}_t$ .

Each firm  $i$  forms such beliefs based on (i) on a firm-specific information set and (ii) a firm-specific forecasting model.<sup>2</sup> Model and information are not directly observable by the econometrician. The firm's joint distribution of instrument-level returns  $\tilde{\mathbf{r}}$  is a continuous function of  $\mathbb{R}^J$  noted  $f_i(\tilde{\mathbf{r}})$ .

Firm  $i$  seeks to maximize the return on its equity  $E_{it}$  by raising funds on the market, through the emission of liabilities, and investing the total sum of the capital and these raised funds in financial instruments. The firm's trading activity is represented by a level of gross demand  $D_{ijt}$  and gross supply  $S_{ijt}$  (in value) that is both security- and firm-specific. The demand in value is the product

<sup>2</sup>Both the information set and the model are parameterized and estimated in Section 3.3.3.

of its price and its quantity:  $D_{ijt} = p_{jt} \cdot z_{ijt}^D$ ; and similarly  $S_{ijt} = p_{jt} \cdot z_{ijt}^S$ .

Each firm's equity is part of the set of instruments. We write  $e(i) \in \{1, 2, \dots, J\}$  the function that maps firm  $i$ 's equity into the set of instruments. Hence  $E_{it} = p_{e(i)t} \cdot z_{ie(i)t}^S$ . Firm  $i$ 's number of shares  $z_{ie(i)t}^S$  is fixed, while its price  $p_{e(i)t}$  is free to adjust to market demand and supply. Each additional unit of total demand  $\mathbf{D}'_{it} \mathbf{1}_J = \sum_j D_{ijt}$  for assets beyond initial capital  $E_{it}$  requires raising a corresponding additional unit of liability  $\mathbf{S}'_{it} \mathbf{1}_J = \sum_j S_{ijt}$ , with  $\mathbf{1}_J$  the  $J$ -column vector of ones. The firm's balance sheet satisfies the usual equality of assets and liabilities:

$$\mathbf{D}'_{it} \mathbf{1}_J = \mathbf{S}'_{it} \mathbf{1}_J + E_{it} \quad (3.2)$$

which simplifies to  $\Delta'_{it} \mathbf{1}_J = E_{it}$  when defining the net demand vector as  $\Delta_{it} = \mathbf{D}_{it} - \mathbf{S}_{it}$ . Hence the net demand in period  $t$ , together with firm  $i$ 's current capital  $E_{it}$ , makes up period  $t$ 's balance sheet and asset-liability diversification; the firm achieves such diversification by trading  $\Delta_{it} - \Delta_{it-1} \in \mathbb{R}^J$ .

Firm  $i$  faces a trading cost  $\gamma_{ij} \geq 0$  per quantity traded for adjusting its net demand  $\Delta_{ijt}$  for security  $j$  from  $\Delta_{ijt-1}$  to  $\Delta_{ijt}$ . When the trading cost  $\gamma_{ij} = 0$ , the firm's holdings of security  $j$  are not constrained by its past holdings.<sup>3</sup> When  $\gamma_{ij} \rightarrow \infty$  the firm's current holdings of such illiquid assets are entirely pinned down by its previous holdings.

The firm's risk appetite is represented by a utility function  $u(\cdot)$ , a function of its net income. Noting  $\Delta_{it}$  firm  $i$ 's stacked vector of net demands in period  $t$ , the firm maximizes:

$$\begin{aligned} \max_{\Delta_{it} \in \mathbb{R}^J} & \int u_i(\Delta'_{it} \tilde{\mathbf{r}}) f_i(\tilde{\mathbf{r}}) d\tilde{\mathbf{r}} - \|\gamma_i / \mathbf{p} \cdot (\Delta_{it} - \Delta_{it-1})\|^2 \\ \text{s.t.} & \quad \Delta'_{it} \mathbf{1}_J = E_{it}, \end{aligned} \quad (3.3)$$

where the vector of trading costs  $\gamma_i / \mathbf{p}$  is per unit of security traded. Such optimization program yields a vector  $\Delta_{it}(\mathbf{p}_t)$  of net demands as a function of the price vector  $\mathbf{p}_t$  given the joint distribution of its beliefs about future payoffs, its initial equity, and its initial balance sheet diversification  $\Delta_{it-1}$ . Utility functions  $u_i$  are increasing, continuous, and concave. The cost of equity  $\lambda_i$  is the Lagrange

<sup>3</sup>Such trading costs are structurally identified in Section 3.5.

multiplier of the funding constraint  $\Delta'_i \mathbf{1}_J = E_{it}$ .

The term  $\|\gamma_i/\mathbf{p} \cdot (\Delta_{it} - \Delta_{it-1})\|^2$  captures the firm's balance sheet inertia: although the firm aims at changing demand and supply according to shifts in the distribution of return beliefs  $f_i$ , it might not be able to achieve a full adjustment of its balance-sheet from one quarter to the next. Firms are highly specialized in trading and managing certain classes of financial instruments, and therefore vary in their ability to respond to a change in beliefs about returns of a particular instrument. Here  $\cdot$  (resp.,  $/$ ) is the term by term vector product (resp., term by term division) and  $\|\cdot\|^2$  is a norm on trading activity in  $\mathbb{R}^J$  that measures the magnitude of trading activity.

### 3.2.2 Market Equilibrium with Trade Costs

Given each firm's net demand for each financial instrument  $i$  as a function of (i) the price vector, (ii) return beliefs, (iii) initial equity, (iv) initial balance sheet, this subsection establishes the existence and properties of the market equilibrium in  $t$  with trading costs.

Two key ingredients enable trade in contrast with Milgrom & Stokey (1982): first, markets are not complete as the set of instruments does not include the full set of firm-specific contingent claims. Second, each firm's model, that maps private information into forecasts of returns, is not common knowledge, which mirrors the framework of Lintner's (1969). Hence observing the firm's demand for assets does not fully reveal its information. Firms trade (i) for risk-sharing purposes as their risk preferences  $u_i$  differ, and (ii) as they 'agree to disagree' each holding beliefs  $\tilde{\mathbf{r}}_i$  about stochastic returns.

Market equilibrium in  $t$  is defined as follows. For any instrument  $j$  that is also the equity of a firm  $i$ ,  $j = e(i)$ , the net demand of instrument  $j$  is  $\sum_{i'=1}^N \Delta_{i'e(i)t} - E_{it}$ ; otherwise it is simply  $\sum_{i'=1}^N \Delta_{i'jt}$ . Denote the stacked  $J$ -vector of firms' equity by  $\mathbf{E}_t$ , equal to  $E_{it}$  when  $j = e(i)$  and 0 otherwise.

**Definition 3.1. (Equilibrium)** *An equilibrium in period  $t$  is a  $J$ -vector of prices for each financial*

instrument  $\mathbf{p}_t^*$  that clears each of the  $J$  instrument markets:

$$\sum_{i=1}^N \Delta_{it} (p_{1t}^*, p_{2t}^*, \dots, p_{Jt}^*) = \mathbf{E}_t (p_{1t}^*, p_{2t}^*, \dots, p_{Jt}^*), \quad (3.4)$$

where  $\Delta_{it} (\mathbf{p}_t^*)$  and  $\mathbf{E}_t (\mathbf{p}_t^*)$  are  $J$ -column vectors.

**Proposition 3.1. (Existence and Local Uniqueness of Equilibrium)** *There exists an equilibrium price vector  $\mathbf{p}_t^* \in \mathbb{R}^J$ . Such equilibrium is locally unique almost surely.*

*Proof.* See Appendix B.2.1 for existence based on standard arguments. Finite trade costs do not affect the existence proof as the trade cost term  $\|\gamma_i/\mathbf{p} \cdot (\Delta_{it} - \Delta_{it-1})\|^2$  is homogeneous of degree zero. Intuitively, such trade costs simply lower the price elasticity of demand. Local uniqueness follows from a result by Sard (1942) and used in Debreu (1970). As demand is a continuous function of beliefs, utility functions, and equity, the set of critical points of the Jacobian has measure zero. □

### 3.2.3 Shock Propagation Through the Financial Network

The financial network predicts how partial equilibrium shocks to a limited set of equity or non-equity instruments affect other banks' value by propagating through balance sheets whose shifts in diversification is potentially limited by trade costs. The framework will suggest that with large trade costs  $\gamma \rightarrow \infty$ , the network is akin to a fixed network of equity cross-holdings and of holdings of primitive instruments, including debt. With finite trade costs, the network ties' sensitivity to prices is a key mechanism that explains the amplification or the mitigation of shocks.

The model is flexible and can accommodate a large variety of shocks such as either (i) a shock to one firm's equity value, which affects the equity value of all other firms in the economy, (ii) monetary policy interventions that shift the demand for debt instruments or (iii) shifts in the market beliefs about a subset of financial instruments, e.g. driven by external factors such as market volatility or forecasts of fundamentals. In each case, the financial network multiplies the

initial, partial equilibrium shock, and yields a predicted general equilibrium impact for each of the financial securities.

Each of the three types (i)–(iii) of partial equilibrium shocks can be represented by a shock  $d\theta$  to a parameter  $\theta$  that affects net demands. If, for instance,  $d\theta$  is a shock to the market value of the equity of firm  $k$ , then  $d\theta = dE_k$  and the partial equilibrium shift in net demand is  $\partial\Delta_k/\partial\theta \cdot d\theta$ .

Firm  $i$ 's general equilibrium equity value shift is driven by shifts in its assets and liabilities' market values, written compactly as:

$$\frac{dE_i}{d\theta} = \mathbf{1}' \frac{d}{d\theta} \Delta_i \quad (3.5)$$

as a consequence of the funding constraint  $E_i = \mathbf{1}'\Delta_i$  (Section 3.2.1).

The total impact of the shock  $d\theta$  on the vector  $\Delta_i \in \mathbb{R}^J$  of net demands of each firm  $i = 1, 2, \dots, N$  is the sum of a partial equilibrium term and a general equilibrium term. The total derivative of net demand w.r.t. the shock expands according to the chain rule as:

$$\frac{d\Delta_i}{d\theta} = \frac{\partial\Delta_i}{\partial \log \mathbf{p}} \frac{d \log \mathbf{p}}{d\theta} + \frac{\partial\Delta_i}{\partial \theta}, \quad (3.6)$$

where  $\frac{\partial\Delta_i}{\partial \theta} \in \mathbb{R}^J$  is the partial equilibrium rebalancing of assets and liabilities at constant prices;  $\frac{\partial\Delta_i}{\partial \log \mathbf{p}} \in M_{J,J}(\mathbb{R})$  is the sensitivity of net demand to the prices of financial instruments, a consequence of banks' rebalancing of their balance sheet in response to price changes.  $\frac{d \log \mathbf{p}}{d\theta} \in \mathbb{R}^J$  is the market-wide shift in the price vector, common to all institutions as the law of one price holds.

The set of instruments is split into *primitive* instruments and *equity* instruments,  $\mathbf{p}' = ((\mathbf{p}^p)' (\mathbf{p}^e)')$  and  $\Delta_i' = ((\Delta_i^p)' (\Delta_i^e)')$ . The equity instruments play a special role as they pin down the market value of the firm. Thus:

$$\frac{dE_i}{d\theta} = \mathbf{1}' \frac{\partial\Delta_i}{\partial \theta} + \mathbf{1}' \begin{pmatrix} \frac{\partial\Delta_i^p}{\partial \log \mathbf{p}^p} & \frac{\partial\Delta_i^p}{\partial \log \mathbf{p}^e} \\ \frac{\partial\Delta_i^e}{\partial \log \mathbf{p}^p} & \frac{\partial\Delta_i^e}{\partial \log \mathbf{p}^e} \end{pmatrix} \cdot \begin{pmatrix} \frac{d \log \mathbf{p}^p}{d\theta} \\ \frac{d \log \mathbf{p}^e}{d\theta} \end{pmatrix}, \quad (3.7)$$

and  $\mathbf{1}' \frac{\partial\Delta_i}{\partial \theta}$  is the partial equilibrium shift in firm  $i$ 's value, simply noted  $\mathbf{pe}$ . With a perfectly inelastic

supply of equity shares,<sup>4</sup> the fluctuation in the market value of equity is driven solely by the change in the market price of equity. This implies that  $\frac{d \log \mathbf{p}^e}{d\theta} = (\text{diag} \mathbf{E})^{-1} \frac{d \mathbf{E}}{d\theta}$ . In this expression, the presence of the equity values  $E_i$  and  $\mathbf{E}$  on both sides will provide us with the equilibrium response of firms' values to the shock  $d\theta$ . Such response will depend on equity cross-holdings as  $\mathbf{1}'_i \Delta_i^e$  is the row vector of bank  $i$ 's holdings of other banks' equity.

Such cross-holdings of bank equity by firm  $i$  respond to the vector of equity prices  $\log \mathbf{p}^e$ . The impact of log prices on the value of cross-holdings is driven by two terms. The first term is the mechanical impact of prices on the value of the holdings, keeping the number of shares held constant. The second term captures the endogenous response of the number of bank shares held  $\mathbf{z}_i^e$ , to price changes.

$$\mathbf{1}'_e \frac{\partial \Delta_i^e}{\partial \log \mathbf{p}^e} = \mathbf{1}'_e \frac{\partial (\mathbf{p}^e \cdot \mathbf{z}_i^e)}{\partial \log \mathbf{p}^e} = \underbrace{\mathbf{1}'_e \text{diag} \Delta_i^e}_{\text{Valuation effect}} + \underbrace{\mathbf{1}'_e \left[ (\mathbf{p}^e \mathbf{1}'_e) \cdot \frac{\partial \mathbf{z}_i^e}{\partial \log \mathbf{p}^e} \right]}_{\text{Rebalancing effect}} \quad (3.8)$$

where  $\cdot$  is the term by term vector product. This is simply noted  $H_i + \partial H_i / \partial \log \mathbf{p}^e$ , the sum of a pure *valuation effect* (changes in prices at given holdings), and a pure *rebalancing effect* (changes in equity holdings at given prices). The first term  $H_i$  is the row vector of holdings of other banks' equity (in value) as in Elliott et al. (2014). The second term, noted  $\partial H_i / \partial \log \mathbf{p}^e$  is due to bank  $i$ 's response of holdings to the price change.

The response of each firm  $i$ 's market value in equation 3.7 can now be stacked into an equity vector  $\mathbf{E} = \{E_i\}_i$ .<sup>5</sup> This provides a familiar formula. This formula expresses the response of firms' value to a shock  $d\theta$  in terms of a network of holdings  $H$  and a partial equilibrium shock  $\mathbf{p}^e$ .

<sup>4</sup>While this allows for stock splits, an extension accommodates the case of public offerings and stock buy-backs.

<sup>5</sup>Throughout the paper, the notation  $\{X_i\}_i = X$  means that a series of row vectors  $i = 1, 2, \dots, N$  is stacked into a squared matrix of size  $N$ .

Stacking the scalars  $\frac{dE_i}{d\theta}$  into a column vector,

$$\frac{d\mathbf{E}}{d\theta} = \underbrace{\left[ \mathbb{I} - \left( H + \frac{\partial H}{\partial \log \mathbf{p}^e} + \left\{ \frac{\partial (\mathbf{1}'_p \Delta_i^p)}{\partial \log \mathbf{p}^e} \right\}_i \right) (\text{diag} \mathbf{E})^{-1} \right]^{-1}}_{[\mathbb{I} - \mathcal{A}]^{-1}} \left[ \mathbf{pe} + \left\{ \frac{\partial \mathbf{1}'_p \Delta_i^p}{\partial \log \mathbf{p}^p} + \frac{\partial \mathbf{1}'_e \Delta_i^e}{\partial \log \mathbf{p}^p} \right\}_i \frac{d \log \mathbf{p}^p}{d\theta} \right] \quad (3.9)$$

where the first term can be simply written  $[\mathbb{I} - \mathcal{A}]^{-1} \mathbf{pe}$ .

$\mathcal{A}$  is the adjacency matrix of this paper's financial network. It is the sum of three terms: the network  $H (\text{diag} \mathbf{E})^{-1}$  of equity cross holdings (due to the *valuation effect* described above); the network  $\frac{\partial H}{\partial \log \mathbf{p}^e} (\text{diag} \mathbf{E})^{-1}$  of the response of equity cross holdings (due to the *rebalancing effect* described above); and a new additional term  $\left\{ \frac{\partial (\mathbf{1}'_p \Delta_i^p)}{\partial \log \mathbf{p}^e} \right\}_i (\text{diag} \mathbf{E})^{-1}$ , due to the response of *non-equity ties* between banks to the shocks.

Such adjacency matrix  $\mathcal{A}$  is equal to Elliott et al.'s (2014) adjacency matrix of cross holdings  $H (\text{diag} \mathbf{E})^{-1}$  in the specific case where (i) the number of bank shares held by any other bank is kept constant, (ii) there is no interbank linkage through other instruments than equity. Then,

**Definition 3.2. (Endogenous Financial Network)** *The financial network is a weighted and directed graph  $\mathcal{G} = (V, E)$  whose vertices  $V$  are the banks  $\{1, 2, \dots, N\}$ . There is an edge  $(i, i', a_{ii'}) \in E$  from bank  $i$  to bank  $i'$  if a partial equilibrium shock in the value (assets–liabilities) of bank  $i$  affects the market price of the equity of bank  $i'$ . The weight  $a_{ii'}$  of the edge  $(i, i')$  measures the magnitude and the sign of the impact. The adjacency matrix of such financial network is  $\mathcal{A}$ . The general equilibrium impact of a partial equilibrium shock on banks' values is:*

$$\frac{d\mathbf{E}}{d\theta} = [\mathbb{I} - \mathcal{A}]^{-1} \left[ \mathbf{pe} + \left\{ \frac{\partial \mathbf{1}'_p \Delta_i^p}{\partial \log \mathbf{p}^p} + \frac{\partial \mathbf{1}'_e \Delta_i^e}{\partial \log \mathbf{p}^p} \right\}_i \frac{d \log \mathbf{p}^p}{d\theta} \right], \quad (3.10)$$

When the number of equity and non-equity instruments held by each bank is fixed (no rebalancing effect), and when the price of non-equity instruments is fixed, the network is simply the network of cross-holdings,  $\mathcal{A} = H (\text{diag} \mathbf{E})^{-1}$ , noted  $C$  in Elliott et al.'s (2014), is the matrix of cross-holdings as a proportion of each bank's market capitalization. In general, Equation (3.9) shows that the

propagation of shocks is affected by two terms in addition to  $C$ :

$$\mathcal{A} \equiv \left( H + \frac{\partial H}{\partial \log \mathbf{p}^e} + \left\{ \frac{\partial (\mathbf{1}'_p \Delta_i^p)}{\partial \log \mathbf{p}^e} \right\}_i \right) (\text{diag } \mathbf{E})^{-1}, \quad (3.11)$$

where  $\frac{\partial H}{\partial \log \mathbf{p}^e} (\text{diag } \mathbf{E})^{-1}$  is due to the rebalancing effect, i.e. the network of cross-holdings' responses to equity price changes; and  $\left\{ \mathbf{1}'_p \frac{\partial \Delta_i^p}{\partial \log \mathbf{p}^e} \right\}_i$  is the set of non-equity links between banks.

When trade costs are large  $\gamma_i \rightarrow +\infty$ , the network  $\mathcal{A}$  is pinned down by the initial conditions  $H$ ; while low trade costs  $\gamma_i \simeq 0$  imply a larger role of beliefs ('expectations' in Krugman (1991)) than initial conditions in shock propagation.

### 3.2.4 Network Structure: Amplification or Mitigation

Whether a partial equilibrium shock is amplified or mitigated depends on the structure of the financial network. Noting  $\|\mathbf{pe}\|$  the magnitude of the partial equilibrium shock,<sup>6</sup> the ratio of the magnitude of the general equilibrium shock  $\left\| \frac{d\mathbf{E}}{d\theta} \right\|$  to the partial equilibrium shock  $\|\mathbf{pe}\|$  is a measure of the magnification of the initial partial equilibrium shock. When  $\left\| \frac{d\mathbf{E}}{d\theta} \right\| / \|\mathbf{pe}\| > 1$  the partial equilibrium shock is amplified, while in the opposite case the shock is mitigated. The maximum of such ratio is noted  $\alpha$ :

$$\alpha = \max \frac{\left\| \frac{d\mathbf{E}}{d\theta} \right\|}{\|\mathbf{pe}\|}, \quad (3.12)$$

where the maximum is taken over all possible partial equilibrium shocks  $\mathbf{pe}$  in  $\mathbb{R}^N$ . It is equal to the norm of the inverse of the Laplacian  $(\mathbb{I} - \mathcal{A})^{-1}$  of the graph  $\mathcal{G}$ . A result by Cheeger (1969) implies that  $\alpha$  has an upper bound equal to a measure of the network's connectedness. Hence network shock amplification is related to the graph's topology.

We proceed as follows. Given a partition of the financial network  $\mathcal{G}$  into two subgraphs  $G$  and  $\bar{G}$ , the *conductance* of the cut  $(G, \bar{G})$  measures the connection between subgraph  $G$  and subgraph

<sup>6</sup>Results of propositions 3.2 and 3.3 are independent of the specific norm. Such results apply for any norm on the Euclidean space  $\mathbb{R}^N$  with dimension  $N$  equal to the number of institutions.



$\bar{G}$ :

$$\varphi(G) = \frac{\sum_{j \in G, j' \in \bar{G}} a_{jj'}}{\min(a(G), a(\bar{G}))} \quad (3.13)$$

where  $a(G) = \sum_{j \in G} \sum_{j' \in V} a_{jj'}$ . The Cheeger constant  $h_G$  is the minimum of the conductance  $\varphi(G)$  over all subgraphs  $G$  of  $\mathcal{G}$ .

$$h_G = \min \varphi(G) \quad (3.14)$$

Then, Cheeger (1969) shows that the second smallest eigenvalue of the Laplacian is bounded below by  $h_G^2/2$ . Given that the maximum of  $\alpha$  over all potential partial equilibrium shocks is the norm of the inverse of the Laplacian, we obtain the following proposition:

**Proposition 3.2.** *(Network Conductance and Maximum Propagation of Shocks) In a financial network with a higher conductance (more 'tightly' connected) shocks are less magnified. Precisely,*

$$\frac{1}{2h_G} \leq \alpha \leq \frac{2}{h_G^2}. \quad (3.15)$$

*In such inequality,  $\frac{1}{2h_G} \leq \frac{2}{h_G^2}$  as the degree of each vertex of the network is 1, which implies that  $\varphi(G) \leq 1$ , and  $h_G \leq 1$ .*

*Proof.* This result is proven by Cheeger (1969). □

The estimation of the Cheeger constant is challenging as the minimum is taken over all possible subgraphs of  $\mathcal{G}$ , of which there are  $2^J - 1$ . A common approach to this issue is to use a spectral cut (Spielman 2007): given the eigenvector corresponding to the second smallest eigenvalue of the laplacian, securities are sorted into each of the subgraph according to the sign of their value in the eigenvector. Spielman (2007) shows that this leads to a cut corresponding to the minimum 3.14.

### 3.2.5 Measuring the Systemicness of Institutions

While the previous analysis focuses on the economy-wide propagation of partial equilibrium shocks, we focus here on individual securities. This will allow us to measure the position of a firm's equity in the network of financial instruments.

Two simple measures characterize (i) the *influence* and (ii) the *sensitivity* of a bank to a partial equilibrium shock. Influence is a measure of the systemicness bank, sensitivity measures its fragility. Influence typically differs from sensitivity: institutions at the fringes of the financial network exhibit high sensitivity but negligible influence.

**Definition 3.3. (*Influence and Sensitivity*)** *The vector of influences of each bank is*<sup>7</sup>

$$\mathbf{Influence}'_t = \mathbf{1}' [\mathbb{I} - \mathcal{A}_t]^{-1}, \quad (3.16)$$

*measuring the general equilibrium change in equity value of the total financial system caused by a unit increase of a bank's partial equilibrium shock. Influence is akin to the centrality measure presented in Bonacich (1987). The sensitivity vector is*

$$\mathbf{Sensitivity}_t = [\mathbb{I} - \mathcal{A}_t]^{-1} \mathbf{1}, \quad (3.17)$$

*measuring the impact on each bank of a unit increase in value of all bank equities.*

The structure of the financial network provides an upper bound on the average influence and sensitivities. The following proposition suggests that a “more connected” network, in the sense of Cheeger’s conductance, will also have smaller influences and sensitivities.

**Proposition 3.3. (*Magnification*)** *An upper bound for the sum of the magnitudes of influences (resp., sensitivities) is the number  $\alpha$  that measures the amplification or mitigation of shocks:*

$$\sum_{i=1}^N |\mathbf{Influence}_{it}| \leq \alpha \leq \frac{2}{h_G^2}, \quad (3.18)$$

*with the same inequality for the sum  $\sum_{i=1}^N |\mathbf{Sensitivity}_{it}|$ .*

*Proof.* Endow the space  $\mathbb{R}^N$  with the  $L^1$  norm  $\|(x_1, \dots, x_N)\| = \sum_{i=1}^N |x_i|$ . Then:  $\|\mathbf{Influence}_t\| \leq$

---

<sup>7</sup>One potential issue is that the influence vector includes the security itself. Thus, an alternative definition excludes each instrument from the influence vector. This yields similar results in the empirical sections of this paper. A similar comment applies to the sensitivity vector.

$\frac{1}{N} \left\| (\mathbb{I} - \mathcal{A})^{-1} \right\| \|\mathbf{e}\| = \alpha$ . The inequality  $\alpha \leq \frac{2}{h_G^2}$  comes from proposition 3.2. Given that the spectrum of a matrix is identical to the spectrum of its transpose, the same inequality holds for the sensitivities.  $\square$

Influence and sensitivity measures are typically uncorrelated with the size of holdings. In particular, a bank's holdings can be small but exhibit a strong elasticity with respect to the returns of other securities. Appendix Figure B.9 plots network influence against the size of holdings using estimates provided in Section 3.4, and finds no systematic correlation between the two metrics.

Figure B.2 illustrates an equilibrium tension between the firm-level need for diversification, and the financial connections that such diversification implies. In particular, while the more risk averse firms *reduce* the variance of their returns by diversifying their portfolio, they *increase* their influence in the network. This is illustrated by two simulations of market equilibrium, one with a uniform distribution of risk aversions, over  $[1, 10]$ ; and one with a uniform distribution of risk aversions over  $[1, 15]$ . While the distribution of risk aversions is uniform, the distribution of balance sheet sizes is such that the least risk-averse firms have balance sheet sizes about 2.5 times the balance sheet size of firms with a risk aversion above 4 (Subfigure (b)). Subfigures (a) and (c) illustrate the trade-off between firm-level diversification and network-level influence: firms with the lowest risk aversion have the equity with the greatest influence (as defined in equation (3.16)), and the greatest impact of their equity on their immediate connections (Subfigure (c)).

### 3.3 Structural Estimation

Structural estimation of the model in a parametric context requires (i) a specification of banks' utility functions  $u_i$ , i.e. risk preferences, (ii) a specification of institutions' belief formation model and data. The first point is addressed by using CARA utility with trade costs. The CARA setup is equivalent to maximizing expected returns under a value-at-risk constraint. In this case the importance of the value-at-risk constraint is measured by the risk-aversion parameter  $\rho_i$  (Coimbra & Rey 2017). Point (ii) is addressed by considering that each institution builds a specific factor

model of returns.

### 3.3.1 Parameterization: Demand, Trade Costs, Beliefs

Each firm trades off the expected return and the expected variance of such stochastic net income. The relative importance of such variance for firm  $i$  is noted  $\rho_i$ .<sup>8</sup> Hence firm  $i$  chooses its net demand of instruments by maximizing

$$\begin{aligned} \operatorname{argmax}_{\Delta_{it}} Q(\Delta_{it}) &\equiv \Delta_{it}' \boldsymbol{\mu}_{it} - \frac{1}{2} \rho_i \Delta_{it}' \boldsymbol{\Sigma}_{it} \Delta_{it} - \|\boldsymbol{\gamma}_i / \mathbf{p} \cdot (\Delta_{it} - \Delta_{it-1})\|^2 \\ \text{s.t. } \Delta_{it}' \mathbf{1}_J &= E_{it} \end{aligned} \quad (3.19)$$

where  $\boldsymbol{\mu}_{it} \equiv \mathbb{E}[\tilde{r}_{it}] \equiv \mathbb{E}[\tilde{r}_t | \Omega_{it}]$  is the  $J$ -vector of mean return beliefs and  $\boldsymbol{\Sigma}_{it} \equiv \mathbb{V}\text{ar}[\tilde{r}_{it}] \equiv \mathbb{V}\text{ar}[\tilde{r}_t | \Omega_{it}]$  is the  $J$ -square matrix of the variance-covariance of return beliefs. Stochastic beliefs about returns are not multi-collinear and have strictly positive variances, in other words the variance-covariance of beliefs  $\mathbb{V}\text{ar}[\tilde{\mathbf{r}}_{it}]$  is symmetric, positive-definite.

The following proposition provides a closed-form expression for net demands with firm-specific beliefs and trade costs:

**Proposition 3.4. (CARA Firm Demand with Trade Costs)** *Each firm  $i$ 's net demand for instruments depends on the first two moments of its return beliefs, its risk aversion, its cost of capital, and its trade costs. Formally, the  $J$ -vector of net demands is*

$$\Delta_{it} = (\Gamma_i + \rho_i \boldsymbol{\Sigma}_{it})^{-1} (\boldsymbol{\mu}_{it} - \eta_{it} \mathbf{1} + \Gamma_i \Delta_{it-1}), \quad (3.20)$$

where  $\eta_{it}$  is the cost of equity, i.e. the impact of a marginal relaxation of the equity constraint on the mean-variance objective,

$$\eta_{it} = \frac{\mathbf{1}' (\Gamma_i + \rho_i \boldsymbol{\Sigma}_{it})^{-1} \boldsymbol{\mu}_{it} - E_{it} + \mathbf{1}' \Gamma_i \Delta_{it-1}}{\mathbf{1}' (\Gamma_i + \rho_i \boldsymbol{\Sigma}_{it})^{-1} \mathbf{1}}, \quad (3.21)$$

---

<sup>8</sup>This mean-variance goal for a firm formally corresponds to the concept of absolute risk aversion in the context of household choice under Gaussian return uncertainty with Constant Absolute Risk Aversion (CARA).

and  $\Gamma_i \equiv \text{diag } \gamma_i$  is the  $J \times J$  diagonal matrix of trade costs.

*Proof.* See Appendix B.2.1. □

### Mean-Variance Frontier at Market Equilibrium

Equilibrium prices clear the market of each security  $j$  and reflect the underlying trade-off between risk and return of each individual institution  $i$ . Thus the equilibrium market price  $p_i$  should reflect the distribution of beliefs  $\tilde{\pi}_{ij}$  about payoffs.

This is what Figure B.1 illustrates in a market simulation with  $J = 100$  instruments and  $N = 3$  banks. When institutions have common beliefs, figure B.1 shows that the risk-return frontier describes a typical mean-variance frontier (Cochrane 2009) with no convex hull. Equilibrium prices are obtained by a contraction mapping derived from the general equilibrium fixed point in prices. Each point of these simulated graphs is a security. Different shades and point types are for each of the 3 banks. With common beliefs, the mean return and risk converge to a parabola, where securities with an average positive (resp., negative) covariance with other securities' payoffs lie in the upper part (resp., lower part) of the parabola.

When institutions disagree about the mean and variance of payoffs, or when institutions face constraints on their demand or supply of securities, securities typically lie inside the convex hull of the mean-variance frontier. The slope of the fitted line of the upper part of the parabola is an increasing function of the bank's risk aversion.

### 3.3.2 Model Identification: Intuitions

Knowing firms' beliefs, risk aversion, and equity levels is key to the estimation of the sensitivity of network ties to shocks (network adjacency matrix of equation (3.11)).

Indeed, bilateral ties emerge endogenously from banks' beliefs about stochastic returns (means, variances, and correlations), and from their risk aversion and equity level. In turn, the *observation* of time-varying financial ties and equity levels identifies the time-variation of banks' stochastic beliefs and their risk aversion.

For instance, in a simple case with only two assets, a negative correlation between the demand  $\Delta_{ijt}$  for listed equities and the demand  $\Delta_{ij't}$  of long term debt reveals the belief that the return  $\tilde{r}_{ijt}$  on listed equities and the return  $\tilde{r}_{ij't}$  on long term debt are negatively correlated. The relationship between beliefs about correlations  $\text{Corr}(\tilde{r}_{ijt}, \tilde{r}_{ij't})$  and the magnitude of time-variation in holdings identifies the firm's risk aversion  $\rho_i$ .

Each firm is characterized by (i) beliefs about  $J(J-1)/2$  correlations and  $J$  means in each time period  $t$ , (ii) a constant risk aversion parameter  $\rho$ , and (iii) an equity level  $E_t$ . For each firm  $i$ , we observe (a) assets and liabilities  $\Delta_{ijt}$  for each of the  $J$  instruments in each time period  $t$  and (b) their equity level  $E_t$ . Identifying beliefs and risk aversion requires a set of assumptions on the time-varying dynamics of first- and second-order moments. The literature has provided a range of approaches to model time-varying correlation structures (Engle & Kelly 2012, Harvey 2010, Harvey & Thiele 2016). In this paper, each bank forecasts returns using a firm-specific factor model, which provides the variance-covariance structure of the forecast returns.

The following sections show that (i) assuming a firm-specific factor structure in stochastic returns implies a firm-specific factor structure in net demands, and (ii) the relationship from the factor structure of returns to the factor structure of net demands can be *inverted* to identify each firm's beliefs about returns' means, variances, and correlations. Both factors and loadings are specific to each bank  $i$ , reflecting the fact that firms have heterogeneous beliefs about both the factors that price assets *and* about the comovement of prices with factors.

Trade costs are identified by estimating the impact of the previous period's holdings on current holdings. We lighten the exposition by presenting identification with no trade cost, and present identification with such costs in Appendix Proposition B.7.

### 3.3.3 From Return Beliefs to Net-Demands: A Dynamic Factor Model

Recent empirical asset pricing literature (Miranda-Agrippino & Rey 2015, Kojien & Yogo 2019) suggests that unobservable and persistent factors drive asset returns. Similarly, in this paper, each firm uses a factor model turning public and private information into a forecast of the joint distribu-

tion of returns.

Data used in this paper includes up to 355 financial instruments (Section 3.4). Given such level of disaggregation, financial instruments are grouped into natural classes, e.g. listed and unlisted equities, long-term and short-term debt. Within each instrument class, the firm forms beliefs about the joint distribution of financial instruments' returns; the firm also forms beliefs about the joint distribution of returns across instrument classes. For the sake of simplicity, we present here an approach without grouping by instrument class.

Stochastic return beliefs about instruments follow a factor structure:

$$\tilde{\mathbf{r}}_{it} = \boldsymbol{\varphi}_i + \Lambda_i \mathbf{f}_{it+1} + \boldsymbol{\varepsilon}_{it}, \quad \Sigma_{\boldsymbol{\varepsilon}i} \equiv \mathbb{V}\text{ar}[\boldsymbol{\varepsilon}_{it}], \quad (\text{FSRB})$$

where there are two sources of uncertainty: first, the variance of the forecast factor  $\mathbf{f}_{it+1}|\mathbf{f}_{it}$  measures the uncertainty coming from firm  $i$ 's factor structure; second,  $\boldsymbol{\varepsilon}_{it}$  measures firm  $i$ 's idiosyncratic uncertainty about returns. Each factor  $\mathbf{f}_{it+1}$  follows an autoregressive process:

$$\mathbf{f}_{it+1} = \boldsymbol{\phi}_i + \Phi_i \mathbf{f}_{it} + \mathbf{u}_{it+1}, \quad \Sigma_{\mathbf{u}i} \equiv \mathbb{V}\text{ar}[\mathbf{u}_{it+1}], \quad (\text{ARF})$$

where  $\boldsymbol{\phi}_i$  is a  $K \times 1$  vector of factor constants,  $\Phi_i$  is the  $K$ -diagonal matrix of autoregressive coefficients.  $\mathbf{u}_{it+1}$  are the innovations to factors with variance covariance matrix  $\Sigma_{\mathbf{u}i}$ . Thus  $(\boldsymbol{\phi}_i, \Phi_i, \Sigma_{\mathbf{u}i})$  describes the autoregressive dynamic of factors.

Overall, firms form beliefs in two steps. First, firms forecast the value of the factors  $\mathbf{f}_{it+1}$ ; second, firms use the loadings  $(\boldsymbol{\varphi}_i, \Lambda_i, \Sigma_{\boldsymbol{\varepsilon}i})$  to forecast the mean and the variance-covariance of returns.

Proposition 3.5 below shows that when firms use a dynamic factor model to forecast return beliefs, then their net demands also follow a dynamic factor model. Indeed, the factor structure of net demands is:

$$\Delta_{it} = \mathbf{c}_i + L_i \mathbf{f}_{it} + \boldsymbol{\xi}_{it}, \quad \Sigma_{\boldsymbol{\xi}i} \equiv \mathbb{V}\text{ar}[\boldsymbol{\xi}_{it+1}], \quad (\text{FSND})$$

where the vector of factors  $\mathbf{f}_{it}$  follows the same autoregressive dynamic (ARF). The vector of constants  $\mathbf{c}_i$ , the net-demand loadings  $L_i$ , and the time-varying term  $\xi_{it}$  are a function of the risk aversion  $\rho_i$ , the constant  $\varphi_i$  and the loadings  $\Lambda_i$ :

$$\mathbf{c}_i = \frac{1}{\rho_i} [\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}]^{-1} (\varphi_i + \Lambda_i \phi_i), \quad L_i = \frac{1}{\rho_i} [\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}]^{-1} \Lambda_i \Phi_i, \quad (3.22)$$

and the residual:

$$\xi_{it} = -\frac{1}{\rho_i} [\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}]^{-1} \eta_{it} \mathbf{1}. \quad (3.23)$$

Here the  $\eta_{it}$  is the cost of equity of equation (3.20).

The factors  $\mathbf{f}_{it}$ , whose autoregressive dynamic is described by equation (ARF), are the same in the factor structure of returns (equation (FSRB)) and in the factor structure of net demands (equation (FSND)). What differs across the two factor structures are the constants and the loadings. Yet constant and loadings of *returns* map into the constant and loadings of *net demands*. This result is summed up in the following proposition:

**Proposition 3.5. (Implied Net-Demand Dynamic Factor Structure)** *There exists a mapping from the factor structure (FSRB) of return beliefs to the factor structure (FSND) of net demands, noted  $\delta$ :*

$$\delta : (\rho_i, \varphi_i, \Lambda_i) \mapsto (\mathbf{c}_i, L_i), \quad (3.24)$$

The mapping is from  $\mathbb{R} \times \mathbb{R}^J \times \mathcal{L}$  to  $\mathbb{R}^J \times M_{J,K}$ .

The set  $\mathcal{L}$  is the set of loadings  $\{\Lambda_i \in M_{J,K} \mid \mathbf{1}' (\Lambda_i \odot \Lambda_i) \mathbf{1} = K\}$  that sum to  $K$ . Hence  $\dim(\mathbb{R}^J \times M_{J,K}) = \dim(\mathbb{R} \times \mathbb{R}^J \times \mathcal{L})$ . In this  $K$  is the number of factors,  $J$  the number of instruments, and  $M_{p,q}$  the set of real matrices of dimension  $p \times q$ .

*Proof.* See Appendix B.2.1. □



### 3.3.4 From Net-Demands to Return Beliefs: Identification

The estimation of the dynamic factor model of net-demands and of the autoregressive process of the factors yields for each firm  $i$ : (i) the autoregressive dynamic of factors  $(\phi_i, \Phi_i, \Sigma_{ui})$ , (ii) the constant and loadings  $(\mathbf{c}_i, L_i)$  of net-demands, and (iii) the residuals  $\xi_{it}$ . The model is identified by finding the factor structure of return beliefs  $(\varphi_i, \Lambda_i, \Sigma_{\epsilon i})$  that matches these reduced-form estimates. This is equivalent to the inversion of the mapping  $\delta$ .

The autoregressive dynamics  $(\phi_i, \Phi_i, \Sigma_{ui})$  of the factors of return beliefs is identical to the autoregressive dynamic of the factors for net demands. For the constants and loadings of return beliefs, cost of equity, and risk aversion the proof is more elaborate. The following proposition shows that  $\delta$  is invertible:

**Proposition 3.6. (*Identification of Return Beliefs and Risk Aversion*)** *The mapping  $\delta$  from the factor structure of return beliefs to the factor structure of net demands can be inverted.  $(\rho_i, \varphi_i, \Lambda_i) = \delta^{-1}(\mathbf{c}_i, L_i)$  identifies (i) risk aversion  $\rho_i$ , (ii) the constant  $\varphi_i$  and the loadings  $\Lambda_i$  of the return belief factor structure.*

*Proof.* The closed-form expression for the inverse  $\delta^{-1}$  is presented in Appendix B.2.1. The intuition of the proof is that the loadings on return beliefs are scaled equivalents of the loadings on net demands. The scaling is a function of risk aversion and the variance-covariance of return loadings. □

This key proposition allows us to conclude the identification of the model without trade costs.

Appendix Proposition B.7 provides the identification procedure for the case with finite trade costs, where the source of identifying variation is the correlation between past and current holdings.

### 3.3.5 Estimation Procedure

The model's estimation proceeds as follows. The estimation yields the paper's financial network in step 5 below:

1. The factor model for net demand is estimated for each bank. This yields estimated factors  $\hat{\mathbf{f}}_{it}$  in each period, loadings  $\hat{\mathbf{L}}_i$ , the constants  $\hat{\mathbf{c}}_i$ , the residual  $\hat{\xi}_{it}$ .
2. The autoregressive process for the unobserved factors  $\hat{\mathbf{f}}_{it+1}|\hat{\mathbf{f}}_{it}$  is estimated. In this paper we use a first-order autoregressive process. This yields the constant  $\phi_i$ , the autoregressive coefficients  $\Phi_i$ , and the variance-covariance matrix  $\Sigma_{ui}$  of innovations.
3. Using the inverse of the mapping  $\delta$  from reduced form factor structure  $(\mathbf{c}_i, L_i)$  to the structural parameters we estimate the risk aversion, as well as the constant, loadings, and variance-covariance of the factor structure of return beliefs  $(\rho_i, \varphi_i, \Lambda_i)$ . The variance-covariance  $\Sigma_{ei}$  is identified from  $\Sigma_{\xi_i}$  and  $\Sigma_{ui}$  using equation (3.23).
4. These parameter pin down the firm's net demand  $\Delta_{it}$  for any price vector  $\mathbf{p}_t$  according to proposition 3.4.
5. The financial network is the matrix  $\mathcal{A}_t$  obtained by summing the network of holdings (observed), the response of equity holdings to equity prices (estimated), and the response of non-equity holdings to equity prices (Definition 3.2).

The identification of the model with trade costs follows the same steps: the factor model of step 1 is replaced by an augmented factor model (Bernanke, Boivin & Eliasziw 2005, Fan, Ke & Liao 2016), where the factor estimation (FSND) includes the lagged net demand  $\Delta_{it-1}$ . The mapping  $\delta$  thus obtained is presented in Proposition B.7 of the Appendix.

The next section estimates beliefs, risk aversions, and the financial network using a comprehensive longitudinal balance-sheet data with assets and liabilities of all French banks.

### 3.4 Data

The theoretical model brings about at least three key empirical questions: (i) whether the network is relevant for the transmission of financial shocks, (ii) which components of the network matter most: the network of equity cross holdings (due to the pure *valuation effect*), the network of responses

of equity cross holdings to equity price changes (due to the pure *rebalancing effect*), and/or the network of *non-equity ties* between banks, (iii) whether shocks affect the network structure itself.

Answering these empirical questions leads to a number of data challenges. First, measuring institution-level equity cross-holdings for both banks and non-banks, with longitudinal time variation. Second, measuring banks' balance sheets and the non-equity links between them, again in the longitudinal dimension. Third, measuring the response of the financial instruments (loans, debt) of other sectors (household sector, non-financial corporate sector, rest of the world) to shocks (e.g. ECB interventions). These three challenges are addressed by merging the financial instruments of individual banks' balance sheets with other banks' balance sheets and with the balance sheets of other sectors.

### **Equity Cross-Holdings**

For banks' balance sheets and cross-equity ownership at the bank level, we rely on the Bureau Van Dijk's collection of detailed data for the period 2013–2018 on cross-equity ownerships, built from annual reports. The 2013-2018 is particularly relevant as it covers the period of the ECB's Public Sector Purchase Program. For France, the Bureau Van Dijk bank data are a longitudinal panel of 294 commercial banks, followed over 24 quarters, where balance sheets are reported at a disaggregated level (so-called C1, C2, and U1, U2 consolidation levels) that shows both time-varying parent-subsidary relationships as well as time-varying ownership of equity across groups. For instance, HSBC France fully owns HSBC Real Estate Leasing France as well as HSBC Factoring France and HSBC SFH. The latter subsidiary is specialized in mortgage lending activities. Such parent-subsidary relationships evolve over time: while Edmond de Rothschild France is fully owned by Edmond de Rothschild in March 2013, the 100% share ownership declines to 86% in March 2016. The longitudinal equity panel also reports holdings of shares across groups, such as the 35% ownership of La Banque Postale Financement (a subsidiary specialized in consumer credit) by Société Générale. This enables an estimation of the net demand  $\Delta_i^e$  of banks for equity instruments. Hence such longitudinal holdings data enables the construction of the

$C_t = H_t(\text{diag}E_t)^{-1}$  network adjacency matrix in each quarter from  $t = 2013Q1$  to  $t = 2018Q4$ .

Across time, such data reveals that *within*-banking group equity holdings are more stable than *across*-banking group holdings. This can be observed in the data when forming groups based on a 40% ownership cutoff. Within groups, the standard deviation of share ownership changes is about 3.1 percentage points smaller than the same standard deviation for across-group share ownership changes (7.5 ppt). This suggests that the *rebalancing effect* may be much more relevant to the transmission of shocks *across groups*, while the *valuation effect* (yielding the network of cross holdings, noted  $C$  in definition 3.2) may be more quantitatively relevant *within groups*. Section 3.6 below quantifies the importance of the three key network transmission mechanisms shaping the network of definition 3.2.

### Who-to-Whom Sectoral Data

This paper's general equilibrium modeling suggests that shocks propagate through non-equity links across banks (e.g. loans, debt, deposits); and through links between banks and other sectors (e.g. the non-financial corporate). This requires building a data set of sectors, where the financial sector's assets and liabilities is the sum of all banks' balance sheets. Each bank balance sheet data obtained through the Bureau Van Dijk provides itemized holdings of securities, where securities are sorted into holdings within the financial sector and holdings outside the sector. For instance, such balance sheets include the EUR amount of deposits from the banking sector, and from other sectors such as the household sector. This enables an estimation of the net demand  $\Delta_i^p$  of the net demand of banks  $i$  for 20 primitive non-equity instruments: one real asset and 19 categories of financial instruments including currency, deposits, securities (stocks, debt, fund shares), loans (short-term, long-term), entitlements (insurance, pension), and derivatives.

For other sectors than the financial sector, we rely on the detailed security-holdings statistics collected by the *Banque de France* at quarterly frequency through the *PROTIDE* survey. One key feature of this data set is that it provides who-to-whom holdings, in contrast with U.S. Flow of Funds data. The six sectors are 1) banking, which includes the central bank, 2) insurance,

3) mutual funds, 4) the corporate sector, 5) household, including non-profit institutions, and 6) the public sector.

We map the full asset- and liability-side of banks' balance sheets by accounting category into the same instrument categories as available in the sectoral accounts (European System of Accounts 2010). These categories are debt securities (short-term F.31 and long-term F.32), equity (listed F.511 and unlisted F.512), fund shares (money market F.521 and non-money market F.522), currency (F.21), deposits (F.22), loans (short-term F.41 and long-term F.42), derivatives (F.7), insurance policies (F.6), other accounts (F.8).

Three facts assess the validity of the longitudinal who-to-whom-sector-bank matched data set. First, we check that the total assets of all banks, minus the interbank cross-equity holdings, match the total assets of the financial sector. Such check reveals that cumulative assets under management of all banks included in the final dataset amount on average (over time) to approximately 80% of the aggregate banking sector balance-sheet as observed in the sectoral accounts data. The remaining 20% are largely explained by the fact that the banking sector definition applied here also includes money-market funds, owing to the fact that liabilities of money-market funds exhibit similar characteristics as deposits offered by monetary financial institutions. Assets under management display a typical 'superstar' distribution, as predicted by the model's simulation displayed in Figure B.2. The model's simulations indeed predict the endogenous emergence of a set of large banks. A similar pattern is observed in the data, the 90th percentile holds on average 77%, and the 99% percentile on average 28% of total banking assets. Second, we check that the total sum of bank holdings of specific securities outside the financial sector matches the total EUR holdings of the financial sector of this security. Third, banks' demand for assets  $\Delta_{it} = \Delta_{it}(p_{1t}^*, p_{2t}^*, \dots, p_{Jt}^*; \theta)$  aggregates up with similar properties as a sector-level CARA demand where the sector-level risk aversion is the harmonic mean of individual banks' risk aversions.

### Estimating Realized Returns

A key test of the model's estimates of beliefs is to compare banks' beliefs about returns to their realizations, for each instrument. In order to obtain the first and second moment of returns, we construct time series of returns (i) due to changes in prices and (ii) due to payoffs, e.g. dividend income.

Returns due to valuation changes can be derived from information on the amount in Euro of stocks outstanding and valuation changes by financial instrument and sector. While the data does not include changes in the number of instruments, a key insight is that, while different banks and sectors change their holdings differently, they experience the same price shifts. Appendix B.3.1 shows how a panel regression with institution and instrument fixed effects can identify returns using the variation of balance-sheet positions and instruments' income.

Returns due to payoffs on financial instruments are constructed from information recorded in the income accounts on different types of income received and paid by sector. Types of income are dividends, interest payments, investment income attributable to mutual fund shareholders, insurance policy holders, and investment income payable on pension entitlements.

## 3.5 Structural Parameters

The procedure described in section 3.3 enables to estimate the structural parameters of the model: beliefs about future returns, risk-aversion and trading costs. The first step of the procedure is the estimation of the dynamic factor model.

### 3.5.1 Return Beliefs

**Dynamic Factors and Macroeconomic Variables** The estimation of the dynamic factor model used to construct return beliefs enables both the factors and the factor loadings to vary across institutional sectors. The estimation results reveal however that the factors are rather similar across sectors with some variation in their order of importance for explaining the variance of net demand.

Figure B.3, left panel, plots the time series of the first three factors for the banking sector with shaded area corresponding to the Global Financial Crises (2007-2008) and the European Debt Crisis (2011-2013). These first three factors explain 41% of the variance in net assets demand. Figure B.3, left panel, right panel, plot the same factors alongside with macroeconomic variables which exhibit the strongest co-movement with each of the factor. The first and second factor captures remarkably well the Global Financial Crisis and the European Debt Crisis. The first factor co-moves with World GDP growth and captures the global cycle and the global crisis. The second factor co-moves tightly with the average spread with the average interest rate of the GIIPS countries (Greece, Ireland, Italy, Portugal, Spain). Given the exposure of the French banking sector the Euro Sovereign Debt crisis in Southern Europe, it is very reassuring to see it captured by the second factor. The third factor co-moves with the Euro Area GDP growth. Overall the first three factors used to explain net asset demand by the banking sector display a time series profile which captures well either the 2007-2008 financial crisis or the 2011-2013 Eurozone sovereign crisis and correlate well with variables capturing the global cycle, the Euro cycle, and sovereign stress in the GIIPS countries. The net demands of the other sectors are roughly explained by the same set of factors but their importance tends to change. The insurance and corporate sector net asset demands areas explained more by the Global and Euro growth cycles and less so my sovereign stress in the GIIPS countries.

**Return Beliefs and Ex-post Returns** The estimated factor model for return beliefs, described in Section 3.3.4, yields for each sector and each instrument, a one-quarter-ahead return forecast. Figure B.4 and Figure B.5 plot alongside the time series of realized ex-post returns the time series of corresponding return beliefs, the 95<sup>th</sup> percent confidence forecast interval band, for the financial sectors and the real sectors, and for each of the following financial instruments: Bonds (Short-Term and Long-Term), Stocks (Listed and Unlisted), Mutual Fund Shares (Money Market and Non-Money Market).

The R-square of an OLS regression of ex-post returns on ex-ante return beliefs and a constant

is reported on the right-hand upper corner of each plot. In many cases, the ex-ante return beliefs predict well ex-post returns. The corporate sector ex-ante return beliefs explain 31 percent of the variance of ex-post stock returns. The household sector ex-ante return beliefs explain 35 percent of the variance of the Non Money Market Mutual Funds ex-post returns. In both cases, the time series of return belief tracks very well the asset crash of 2007-2008, and the subsequent rebound. Other good predicting performance include the prediction of unlisted stock returns by the banking sector and the household sector, the prediction of short-term bonds return by the insurance sector, the prediction of the return to mutual funds by the corporate sector, the household sector, and the mutual fund sector itself. In several instances however, the model either does not predict ex-post return or more puzzlingly its predictions negatively correlate with ex-post returns. We shall notice that this feature is mostly driven by the 2007-2008 crisis. A potential explanation is that several institutional sectors during that period had to increase their purchases of assets even if their returns were declining. Since beliefs are implicitly derived from net demands, those counter-cyclical purchases can drive the negative correlation between return beliefs and ex-post returns. This is the case for the banking sector which hoarded short-term liquid assets during the crisis, for the public sector who bought large stock shares to recapitalize the banking sector and the automobile sector during the crisis, and for the insurance sector which increased considerably its asset holdings during the crisis.<sup>9</sup>

Comparing beliefs across sector reveal that while there is substantial disagreement in beliefs at each point in time, the return beliefs of the banking sector and the mutual fund sectors display a strong comovement. The correlation between the return beliefs of the banking sector and those of the mutual fund sectors are high for most financial instruments: listed stocks (0.54), unlisted stocks (0.46), non money-market mutual fund share (0.34), short-term debt (0.40). There are however episodes in which the return beliefs differ substantially. For example, the mutual fund sector exhibited much more pessimistic beliefs during the 2007-2008 crisis about the return to stocks and to debt securities, while the banking sector became more pessimist on the returns to

---

<sup>9</sup>The increasing size the insurance sector balance sheet is document in HRV. Its counter-cyclical purchases during the crisis can be, in part, explained by government moral suasion as a way to facilitate the banking sector deleveraging.



stocks and mutual fund shares during the sovereign crisis of 2011-2012. One interpretation is that the bank bailout of 2007-2008 avoided the need for banks to engage in massive fire-sales (with deep discount prices), while the mutual fund sector faced large withdrawals from customers and had to engage in such fire-sales. On the opposite, banks were suffering from significant liquidity or solvency stress during the sovereign debt crisis, due to their exposure to GIIPS debt, and therefore faced a pressure to sell-off rapidly other assets that the mutual fund sector did not experience then. The comparison of beliefs among the three real sectors reveal a sharp contrast between the belief returns of the corporate and household that typically comove, and that of the public sector that often displays counter-cyclical beliefs, and especially so during the 2007-2008 crisis. This is consistent with the role played by government in providing bailout and in the debt financing of large public sector deficits during the crisis.

**Bank-Specific Return Beliefs** In order to obtain bank's specific return beliefs, we regress net-demands of each bank for each financial instrument on the net-demand factors estimated for the banking sector computed using sectoral data. The time-variation in net-demands for financial instruments and factors identifies the net-demands loadings. The underlying assumption is that each bank forms belief using the same return-belief factors as the banking sector as a whole but with bank-specific loadings. From that point, onwards, we can apply the procedure described in and obtain all the bank-specific structural parameters according to the procedure described here 3.3.5.

#### **Belief Updating and Bank's heterogeneity.**

We can assess the heterogeneity of the return beliefs' model across banks by looking at how they differentially incorporate information from realized returns in updating their return belief. We do so by regressing the difference between realized return and bank's return belief on the same variable lagged – by itself and interacted with bank's characteristics – controlling for time, instrument, and bank fixed effects.

Table B.1, column 1, shows that the differential between realized returns and return belief is mean reverting suggesting that, on average, banks do update their belief in order to minimize their forecast error about realized returns. There is, however, considerable heterogeneity in this correction mechanism. Using our model estimates of risk-aversion, we show that more risk-averse banks are *more aggressively* updating their belief than less risk-averse banks (Column 2). Risk-aversion is therefore associated with banks being more Bayesian in using the realized information, and implicitly less contrarian, that is less likely to withstand beliefs that are not (yet) being validated by realized returns. By contrast, banks that are more leveraged, or larger, are *less aggressively* correcting their beliefs. This result, which stands when controlling for risk aversion, suggests a possible overconfidence bias of larger and more leveraged banks in the long run validity of their return belief model.

In short it is highly reassuring to observe that the pattern of belief updating observed in the data is largely consistent with priors on which type of banks are more or less likely to follow a Bayesian updating behavior.

### 3.5.2 Risk-aversion

Section 3.3.4 suggests that the mapping between the factor structure of net demand and that of the return beliefs identifies beliefs about all instruments' first and second-order moments (variances and covariances), up to a global constant of overall volatility  $\sigma_{\epsilon}^2$ . Specifically the model identifies the product  $\rho\sigma_{\epsilon}^2$ , that is the degree of risk-aversion multiplied by the residual variance of the belief return model. We recover the degree of risk-aversion as follows. We match the overall residual variance of the return belief model to the overall residual variance of ex-post returns, that is the fraction of the variance of the ex-post returns that is not explained by the factors.<sup>10</sup> Table B.6, upper panel, report risk-aversion estimates, one by sector, obtained by matching variances. As in typical in the equity risk premium literature (Ang 2014, Ait Sahalia & Lo 2000, Ross 2015), this

<sup>10</sup>Note that here we are only scaling the *residual variance* of the belief model. Therefore the residual variance could be of the same order of magnitude of that of ex-post return and yet the variance of beliefs conditional on factors could be sensibly smaller than that of ex-post returns.

approach generates estimates ranging from 8.9 to 21.7, depending on the institutional sector, and which are within the range of estimates of the literature.<sup>11</sup>

Sector-level risk aversion estimates are also ranked in a natural way: a higher estimated risk-aversion in financial sectors subject to capital requirement (banks, insurance) than for the mutual funds sector or the corporate sector.

Bank-level estimates display substantial heterogeneity, and estimates that the disaggregation of the banking sector is key in lowering risk aversion estimates. The mean equal to 4.25, while the median is 0.25, with a standard deviation of 7.7. The large degree of heterogeneity in risk-aversion is expected given differences in size at given return-belief and cost of equity. These differences in risk aversion generate substantial heterogeneity in the response to monetary policy shocks (Section 3.6.2).

### 3.5.3 Trade costs

When trade costs are high, the bank's current balance sheet constrains its future diversification choices. Hence the identification of trade costs is obtained by estimating the coefficient of the regression of future holdings on past holdings (with controls). Trade costs and beliefs are separately identified, even when beliefs are autocorrelated. Indeed, an instrument for which a bank's beliefs are highly volatile but holdings are persistent is likely to have high trade costs. Hence, intuitively, trade costs are identified by the volatility of net-demand over and above the volatility of holdings explained by return beliefs.

High trade costs imply persistent linkages between banks in the financial network. Table B.2 displays the estimated trade costs. Specifically, it shows the average estimated cost (in cents) by asset class, and by bank. Granting a long-term (resp., short-term) loan to non-banks entails an average cost of 2.04¢ (resp., 1.12¢). Exchanging deposits or short-term debt securities issued by banks

---

<sup>11</sup>While imposing the matching of belief returns to ex-post returns is a way to discipline belief formation, one cannot rule out that beliefs are substantially more volatile than ex-post returns as papers have suggested, e.g. Shleifer & Summers (1990). We thus test the robustness of our results to letting the variance of return beliefs to be a multiple of the variance of ex-post returns (Table B.6, bottom three panels); in this case estimated risk-aversion parameters are substantially smaller.

is virtually costless ( $0.07\text{¢}$  and  $0.02\text{¢}$ , respectively) suggesting that the past volume of a bank's local deposits may not be constraining its investment choices (Jayaratne & Morgan 2000). Financial instruments (debt securities, equity shares, and loans) issued by banks trade at a lower cost than those issued by non-banks, which include shadow banks and non-financial corporations.<sup>12</sup>

## 3.6 The Network in General Equilibrium

### 3.6.1 Network Structure

Section 3.2.3 suggests that the network structure of bank interlinkages turns partial equilibrium shocks to banks' values into general equilibrium shocks that affect the market prices of equity shares. As such, shocks to the demand of a specific financial instrument, e.g. debt, have three different propagation mechanisms that affects banks' equity values and balance sheet diversification.

First, a shock propagates by affecting the value of equity shares at given holdings (the *valuation* effect), a phenomenon that propagates through the network of initial cross holdings of equities as in Elliott et al. (2014). Figure B.6 plots such network for the 4th quarter of 2013. The figure makes apparent the partition of the French banking sector into a number of banking groups formed by a holding company surrounded by a myriad of subsidiaries. This propagation mechanism can therefore not explain the impact of isolated shocks to one banking group or subsidiaries affecting the balance sheet of banks in *other* groups. Hence other sources of network propagation, i.e. the terms from equation (3.11), need to be factored in.

Second, shocks affect the diversification of holdings of bank equity: this phenomenon is accounted for by equation (3.10)'s second term. Figure B.7 plots this network. Formally, it depicts the sensitivity of the network of cross-holdings to changes in equity prices. In sharp contrast with Figure B.6, the network structure that emerges from these two additional components includes many important links *between* banking groups, and thus contributes to the propagation of

---

<sup>12</sup>Chernenko & Sunderam (2014) and Chortareas, Girardone & Ventouri (2011) present evidence of market frictions experienced by banks and shadow banks.

shocks throughout the banking sector. Note also that Société Générale appears now at the center of the *between-groups* links but did not display any special feature when only the network of cross-holdings was considered.

Third, shocks affect the bank-to-bank links due to non-equity instruments. Figure B.8 depicts bank linkages due to non-equity links. The graph suggests, as for the previous network mechanism, that intergroup linkages are substantially more important than within-group linkages. Notice that in each graph the color coding is specific to the mechanism, and cannot be easily compared across network graphs.

Hence the table below assesses the relative importance of each propagation mechanism. It indeed presents the standard deviation of the elements of each of the three network components of the adjacency matrix  $\mathcal{A}$ . The table suggests that the second and third network components are 2-3 times more empirically relevant than the network of cross-equity holdings. The second and third terms are of approximately the same magnitude.

|      | Cross-Equity Holdings           | Endogenous Response of Ties  | Non-Equity Links  |
|------|---------------------------------|--|---|
|      | $H(\text{diag}\mathbf{E})^{-1}$ | $\partial H / \partial \log \mathbf{p}^e (\text{diag}\mathbf{E})^{-1}$ | $\left\{ \partial(\mathbf{1}'_p \Delta_i^p) / \partial \log \mathbf{p}^e \right\}_i (\text{diag}\mathbf{E})^{-1}$ |
| S.D. | 0.608                           | 1.566  | 1.575   |

Finally, a shock changes the structure of the network itself ( $d\mathcal{A}/d\theta$ ): the position of banks in the network (their influence and sensitivity, defined in Section 3.2.5) shifts in a way that either mitigates or amplifies the shock (Section 3.2.4). This mechanism is estimated in the case of the ECB's quantitative easing at the end of the next section.

### 3.6.2 Structural Policy Evaluation: the case of ECB Quantitative Easing

In this section, we use our structural model to assess the effects on bank balance-sheet and on the financial network of a large-scale quantitative easing policy through which a central bank expands its balance-sheet by purchasing securities while increasing its monetary base. Precisely, we sim-

ulate a demand shock for government bonds which corresponds to the cumulative net purchases over a quarter during the first phase of the Extended Asset Program (March 2015 to March 2016) of the European Central Bank. The quarterly demand shock (Euro 180 bn.) corresponds to three times the monthly purchases (Euro60 bn.) For simplicity, we choose for the shock to replicate the Public Sector Purchase Program (PSPP) and thus ignore the purchase of non-government bonds. We choose to simulate the shock one year before the actual shock occur so that the estimated return beliefs used in the simulation are not contaminated by the actual shock.

Table B.3, Panel (a) summarizes the calibration of the shock which follows the design of actual ECB policy: (i) asset purchases are made in proportion to country's share in the capital of the ECB, which implies that 20 percent (80 percent) of bonds purchased were French Government bonds (Other Euro Area Government bonds); (ii) the proportion of short and long-term government bonds are made such so as to keep constant the outstanding share of each type of debt. The calibration of the shock thus implies a initial impact 4 times bigger for the rest of the Euro Area government bond market than for the French market.

**Impact on yields and spreads** Table B.3, Panel (b) reports the effects of the QE shock on asset prices in partial and in general equilibrium on French government bonds, corporate bonds (spread) and equity (premium). In partial equilibrium, the yield on French government debt decreases to clear the additional demand by the Eurosystem. The yield on long-term French debt decreases substantially more ( $-7.4$  bps) than the one on short-term French debt ( $-1.0$  bps) implying a flattening of the yield curve (decrease in the term premium). Since in partial equilibrium only the prices of government debt change, the equity premium increases. The general equilibrium effect is much stronger than the partial equilibrium one with a decline in long-term (short-term) bond yield of  $-43.9$  bps ( $-37.4$ ).<sup>13</sup> The price of the corporate bonds and that of equity increase sharply in general equilibrium so that the term premium and equity premium remain the same. In short, the calibrated QE shock, despite a modest partial equilibrium impact, has a strong impact on govern-

<sup>13</sup>Network structure matters for general equilibrium price responses as the comparative statics on prices  $d \log \mathbf{p} / d\theta$  depend on the adjacency matrix of the network graph.

ment bond yields in general equilibrium and a very large pass-through into the price of other assets (corporate bonds, equity).

Even if we do not use any information on quantities, prices or beliefs during the period of the policy shock, we obtain structural estimates for the effects on yield which are reassuringly of the same order magnitude to those obtained from reduced-form regressions using actual data for the period of the shock. Andrade, Breckenfelder, De Fiore, Karadi & Tristani (2016) report a median impact of 43 bps on 10-year government bonds based on a meta-study of 24 studies. Their own estimates based on 2-days event windows around the program announcement and around the program implementation suggests an effect of 13 bps after the announcement and an additional 14 basis points after the implementation. Kojien, Koulischer, Nguyen & Yogo (2018) using an IV approach, exploiting the predictability of purchases found an effect ranging from 2 bps to 60 bps depending on country and maturity.

**Impact on banks' balance sheets** Figure B.4 summarizes the effects of the ECB shock on various balance sheet measures for individual banks. Figure B.4 plots the distributions of the log-change in these various measures between the 25<sup>th</sup> and the 75<sup>th</sup> percentiles and report the mean and various quantile of the distribution of log changes below each distributional plot. The median log-change in the size of banks' balance sheets is close to zero but this hides considerable heterogeneity, with a subset of banks experiencing a substantial reduction in balance-sheet size (at the 25<sup>th</sup> percentile, the reduction is  $-2.1$  percent). On the asset side, the most critical change regards debt holdings, which is not surprising given the nature of the shock. The median change in debt holdings is  $-0.77$  percent but there is substantial heterogeneity: at the 25<sup>th</sup> percentile, banks reduce their holding by  $-6.0$  percent. The equity holdings change very little except at tails of the distribution resulting in an increase in the mean equity equity holdings of  $+10.6$  percent. On the liability side, there is little change in the equity position of banks except at the upper tail (95<sup>th</sup> quantile) where banks increase their equity position by 9.2 percent. There is however a substantial decrease in debt liabilities resulting in a mean reduction of  $-7.4$  percent, with 25 banks experi-

encing a reduction of more than 3.9 percent. Altogether the ECB shock results in a reduction in balance sheet size through a simultaneous reduction in both debt holdings and debt liabilities for a sizable fraction of the banks.

The bottom panel plots asset volatility and distance to insolvency (Atkeson et al. 2017). The distance to insolvency increases for almost all banks even if the magnitude of the change is small. The majority of banks experience a reduction of asset volatility – the median bank experiences a decline in volatility of  $-0.2$  percentage point, but there is substantial heterogeneity across banks with the 25<sup>th</sup> percentile experiencing a reduction of  $-1.1$  percent and 95<sup>th</sup> percentile an increase of 1.4 percent.

Table B.5 explores how banks differ in their response to the ECB quantitative easing shocks, that is how the rebalancing of their portfolio of bonds and equities varies with their initial portfolio, their risk-aversion, as well as in their degree of influence and sensitivity in the network. There is a considerable literature on the heterogeneity of the response of banks to monetary policy shocks as a function of their exposure to interest rate risk and other shocks (Flannery & James 1984, Landier, Sraer & Thesmar 2013, Dell’Arricia et al. 2017). A key unsolved question we address here is whether the *position* of banks in the financial network, as captured here by their influence and sensitivity, is a key source of heterogeneity.

The ECB shock increases the equity-premium (+7.23 bps) which should lead to a rebalancing of the portfolio towards equity and away from bonds. Indeed the constant terms on Table B.5 reveals a reduction in debt holdings and an increase in equity holdings. There is however substantial heterogeneity in the magnitude of such response. First, banks with initially large debt equity holdings reduce their holdings by less, and those with initially large equity holdings increase their holdings by less. Second, rebalancing is more muted for more risk-averse banks: there is both a lower increase in their equity holdings and a lower decline of bond holdings than other banks. Third, network-based heterogeneity does significantly matter: banks that are more influent also rebalance significantly less between bonds and equities. By contrast, the degree of sensitivity does not correlate significantly with changes in bonds and equity holdings.



**Impact on network systemicness** The effects of the ECB policy shocks on the network and its systemicness can be measured through changes in influence and sensitivity for each of the 306 banks. In order to compute such changes, we recompute those network measures in the new market equilibrium following the sovereign asset purchases by the ECB, and compare them to the initial estimates. Summary statistics for changes in influence and sensitivity suggest substantial shifts in banks' network positions. The ECB shock causes a mean reduction of influence and sensitivity of 0.45 and 0.27 percentage points respectively following the quarterly ECB sovereign purchases (Euro 180 bn). Since the later shock represents only about 8.2 percent of the total ECB program (Euro 2200 bn.), we shall expect for the overall program to have a sizable reduction in systemicness. Furthermore, the effects are extremely skewed with large changes happening on the left tail of the distribution. 10 (5) percent of banks experience of reduction of their influence of more than 1.74 ppt (3.87 ppt). Figure B.10 shows a very strong negative correlation between the changes of the influence and sensitivity measure and their initial level, that is, the banks that were more sensitive and more influential show a larger reduction in their systemicness than the others. If the objective of the ECB program was to reduce the systemic risk stemming from banks that were either a potential source of fragility to the network (because of their influence) or at great risk of being subject to risk (because of their sensitivity), our results show that it is actually what does happen after the policy intervention. By contrast, Appendix Figure B.12 displays no relationship between changes in influences and sensitivity and the size of the banks.

### 3.7 Conclusion

This paper shows how a structural general equilibrium model of asset trade micro-founds and estimates an endogenous network of banks. Such endogenization of the financial network through asset trade enables a novel connection between general equilibrium analysis – the amplification of partial equilibrium shocks in general equilibrium – and topological measures of network stability derived from Cheeger (1969).

Shocks propagate through this network and cause an endogenous transformation of the structure of the network in the response to the shock. In the endogenous network, the initial network of cross-holdings and the size of banks matter far less for shock transmissions than the rebalancing of banks' asset and liability portfolios induced by the shock.

While only using net demands as an input, the estimated network can be estimated and used to simulate the effect of a large policy shock such as ECB quantitative easing. Such shock reduces government bonds yield, increases the distance to insolvency for banks, and reduce the systemicness and fragility of the financial network.

The current development of the literature on endogenous network has been mostly based on a game-theoretical approach (e.g. Farboodi (2014)). This paper shows the theoretical and empirical promises of an approach based on general equilibrium models that can be structurally identified by using the wealth of granular data on the full structure of firms' balance sheets.

# Chapter 4

## Domestic and External Sectoral Portfolios: Network Structure and Balance-Sheet Contagion

### 4.1 Introduction

Large shocks, such as the 2007-2008 financial crisis, can trigger sizable changes in countries' external portfolios, and in the dynamics of their net foreign assets positions. Such aggregate changes result from trade in financial assets undertaken by different sectors of the economy (banking, mutual funds, insurance, corporate, household and public sector) as well as valuation changes experienced by the corresponding sectoral portfolios. Using a unique security-level database covering the universe of asset and liability positions, valuation changes and flows of domestic and foreign security portfolios of French sectors between 2008 and 2014, this paper documents how sectoral portfolio patterns shape the dynamics of a country's external portfolio investment position. An estimated model of balance-sheet contagion exposes the role played by network linkages, in the form of domestic and foreign security cross-holdings, in the transmission of real and financial shocks.

While there is a vast literature on cross-border bank holdings and flows, there is so far limited

evidence on international asset and liability holdings for other sectors of the economy. Data limitations might explain this gap. For example, the Coordinated Portfolio Investment Survey (CPIS) of the IMF only reports security holdings at the sector-level for some countries. The survey does not report any information on sector-level liabilities, nor any information on flows and valuations. In addition, the CPIS does not integrate external and domestic portfolios.

The ability to trace the institutional sector of the holding and issuing entity at the security-level makes the data uniquely suitable for the analysis of balance-sheet contagion at the sectoral level. We refer to balance-sheet contagion in a broad sense as a phenomenon where a shock to one sector also affects balance-sheets of other sectors through bilateral exposures (Kiyotaki & Moore (2002)). For example, the value of A's claim on B depends on the value of B's claims on C, who itself might have a claim on A. In focusing on balance-sheet contagion, the paper relates to a large literature on shock propagation through networks of firms and banks. For example, in the seminal paper by Eisenberg and Noe (2001), banks hold debt claims on each other and an outside asset. A sufficiently large shock to a bank's outside asset wipes out its equity and induces a default on its debt pro-rata, therefore affecting creditor banks.

The paper has three objectives. First, we document in a number of stylized facts the evolution of domestic and foreign portfolio assets and liabilities of French sectors between 2008 and 2014. Second, we analyze the network structure of cross-holdings among domestic sectors and the foreign sector. Third, we propose and structurally estimate a model of balance-sheet transmission through bilateral asset and liability positions. The model allows identifying the critical financial linkages that play a disproportionate role in the propagation of domestic and foreign sectoral shocks.

The main results are as follows: First, the net external portfolio position of France over the 2008-2014 period evolves from a positive net external position of 4.7 percent of GDP to a large negative net position of  $-35.7$  percent of GDP. This large deterioration ( $-40.4$  ppts) resulted mostly from the deterioration of the net external position of the public sector ( $-24.1$  ppts), the banking sector ( $-15.1$  ppts), and the corporate sector ( $-7.9$  ppts), and was only partly compensated by an improvement of the net external position of the insurance sector ( $+7.6$  ppts). The

deterioration of the net external portfolio position came entirely from the external liability side of the aggregate balance-sheet, the external asset side remained constant (at about 100 percent of GDP), a reduction of the external assets of the banking sector ( $-9.8$  ppts) being compensated by an increase in the external assets of the insurance sector, and of mutual funds. The deterioration was three times larger vis-à-vis non-Eurozone countries ( $-30.7$  ppts) than vis-à-vis the Eurozone ( $-9.7$  ppts), and five times larger for public sector liabilities ( $-19.8$  vs.  $-4.3$  ppts). Within the Eurozone, the French sectors, and chiefly the banking sector, exhibited a large retrenchment from GIPS countries ( $-11.3$  ppts).

The analysis of cross-sectoral international portfolios reveals that the exposure of French sectors to Eurozone sectors changed dramatically over the period: a large reduction of the exposure to the Eurozone banking sector ( $-4.3$  ppts) and Eurozone corporate sector ( $-9.6$  ppts) was mirrored by a large increase in the exposure to the Eurozone mutual fund and insurance sector ( $+16.0$  ppts).

Second, we discuss the joint evolution of the domestic and external sectoral portfolios. The banking sector exhibited a severe retrenchment with an amelioration of  $+11.4$  ppts of its net domestic position coinciding with a  $-15.1$  ppts deterioration of its net external position. The insurance sector increased significantly its net position in both domestic ( $+15.0$  ppts) and external assets ( $+7.6$  ppts). Consolidating the domestic and external sectoral net position reveals that the insurance sector is the largest net security creditor ( $+89.9$  percent of GDP), while the public sector ( $-77.0$  percent of GDP) and the corporate sector ( $-62.9$  percent of GDP) are the largest net security debtors.

Third, we provide a network representation of bilateral asset and liability positions of sectors. Between 2008Q1 and 2014Q1, the banking sector substituted public sector securities with corporate securities and its liabilities became increasingly held by the rest of the world. The insurance sector ended up playing a much larger role in the financing of the economy through securities by increasing its holdings of securities issued by all sectors.

Fourth, the estimation of our balance-sheet contagion model shows that only the financial sectors of the economy propagate shocks on their security asset position, i.e. the banking sector, the

insurance sector, and mutual funds. That is the impact of portfolio returns on the price of real sector securities through balance-sheet mechanisms comes out as insignificant. This is consistent with the intuition that the price of securities issued by the real sector is mostly driven by real shocks rather than financial contagion. We also show that consistent with the evolution of inter-sectoral linkages, the exposure of the banking and insurance sectors to public sector shocks has increased over the 2008 to 2014 period. All financial sectors are vulnerable to foreign sector shocks, consistent with their substantial external asset positions.

The descriptive section of the paper contributes to the literature on cross-sector balance-sheet exposures. Castrén & Kavonius (2009) use flow-of-funds data for the Eurozone and resort to entropy methods to estimate bilateral sectoral linkages due to data limitations. We do not have such limitations since our database allows us to observe inter-sectoral exposures in the data. Our main contribution is to integrate the analysis of domestic and international sectoral portfolios. Galstyan & Velic (2018) also focus on international sectoral portfolios. Their analysis covers a cross-section of countries but is constrained by the limitations of the CPIS data as discussed in Section 4.2.

The estimated model of balance-sheet contagion explores how changes in the value of a sector's assets propagate through the re-pricing of its liabilities which are held as assets by other sectors. The estimated model makes three contributions to the literature. First, we show how balance-sheet contagion can be structurally derived and identified from an equilibrium model of asset demand and supply building on Kojen & Yogo (2019). We can, therefore, relate the magnitude of contagion to the sensitivity of demand and supply schedules to changes in security returns. Second, compared to Castrén & Kavonius (2009), we obtain inter-sectoral linkages directly from the data and estimate the sector-specific link between the price of assets and liabilities from the data. Third, we provide estimates of sectors' vulnerability to shocks originating in any part of the financial system rather than in the banking sector only.

The model builds on Kojen & Yogo (2019) who show how asset demand can be represented using a logit model, where portfolio shares depend on asset prices and characteristics. We ex-

tend this model to incorporate both demand and supply decisions, and further derive and estimate implied balance-sheet contagion. While such model provides some microfoundations for balance-sheet contagion across sectors through asset demand and supply responses to changes in returns, a full general equilibrium model of asset trade leading to the derivation of an endogenous network of assets and institutions is beyond the scope of the paper and can be found in Heipertz et al. (2019).

Other models test other channels of network contagion. Shin (2008) constructs a model of balance-sheet contagion incorporating value-at-risk constraints. Contagion in Greenwood et al. (2015) works through the price impact of fire-sales. Allen & Gale (2000) focus on the interbank market and analyze how a financial crisis in one region spreads to other regions analyzing how different network structures affect the resilience or proneness of the network to contagion (see also Acemoglu et al. (2012)).

## 4.2 Data Description

We use data on French portfolio investments collected by the Banque de France at quarterly frequency since 2008 through the exhaustive survey PROTIDE (PROduction de statistiques de Titres en DÉtention) among French resident custodians and financial institutions. Therefore, foreign asset portfolios also include holdings by French resident entities of liabilities issued by their foreign affiliates. Indeed, it contains the totality of portfolio investments, as they appear in the French international investment position and balance of payments statistics. The latter are constructed through aggregation of the security-level data contained in PROTIDE. Security accounts held by French residential households in non-residential custodial accounts are not recorded.<sup>1</sup>

The data is collected security-by-security and provides a wide range of information on the security itself, the sector of the holder and the issuer. At the most dis-aggregated level it distinguishes a security in the portfolio of a specific institutional sector. The security holdings data has

---

<sup>1</sup>This omission, however, applies to many statistics on residential financial transactions. It might partly be addressed by the recent aggregation of national securities holding statistics by the ECB into the Securities Holding Statistics (SHS) database.

been complemented by the security issuance survey of the Banque de France in order to obtain the complete coverage of French resident liabilities. Indeed, this is crucial since PROTIDE only records security holdings and does therefore not cover securities issued by French residents that are held by non-resident financial institutions or custodians.

The dataset is unique in the landscape of the literature on capital flows for at least two reasons: First, it provides a complete whom-to-whom mapping of portfolio assets and liabilities between domestic sectors and between domestic sectors and the rest of the world from 2008Q1 until today. The information on foreign portfolio investments by domestic residents indicates the type of the security ( $\tau$ ), the sector of the French holder ( $k$ ), and the country ( $n$ ) and sector of the issuer ( $j$ ). Information on holdings of domestic assets by foreign residents includes the type of the security ( $\tau$ ), and the sector of the French issuer ( $j$ ). In addition, it includes for the Eurozone the country ( $n$ ) and sector of the security holder ( $k$ ). Thus, an asset or liability security-level position can be expressed as  $a_{\tau jkn}$  and  $l_{\tau jkn}$  in which the country subscript  $n$  is dropped for domestic securities. To the best of our knowledge, this paper is the first to use such comprehensive portfolio cross-holdings of security assets and liabilities to analyze domestic and international financial linkages over the 2008-2014 period. Second, the data has a high degree of dis-aggregation (i.e. security-by-security at holding sector level) and covers all sectors of the economy.

Cross-border portfolio investments represent about one-third of the total international investment positions of France, followed by other investments, and foreign direct investments (Figure C.1 (a)). Even more significant is the role of portfolio investments as a key driver of the net foreign asset position of France over the period (Figure C.1 (b)). Between 2008 and 2014, one observes a large trend increase in the net negative position in portfolio investments and a large trend decrease in the net negative position in other investments. The increase in net portfolio liabilities exceeds the decrease in net liabilities of other investments, the difference explaining the deterioration of the overall net international investment position.

The breakdown of the change of the net foreign asset position into flows and valuation shows that flows dominate the dynamics and clearly confirms the substitution of other investments by



portfolio investments as the driver of the change in the net position. However, portfolio investment valuation still plays a sizable role for the aggregate foreign asset and liability positions (Figure C.2). This is consistent with the fact that other investments (of which a large part are bank loans) exhibit smaller valuation effects than securities do. Aggregate valuation effects, therefore, reflect, to a large extent, portfolio valuation effects.

## 4.3 Domestic and External Sectoral Portfolios

The objective of this section is first to describe the construction of domestic and external sectoral portfolios and then to show how the analysis of these portfolios helps to understand, for the period 2008-2014, (i) the deterioration of the net external portfolio position of France, (ii) the joint evolution of domestic and external sectoral portfolios, and (iii) the evolution of the financial network structure implied by bilateral asset and liability positions.

### 4.3.1 Constructing Sectoral Portfolios

Sectoral portfolios are obtained by aggregating security-level asset and liability positions of each sector. The underlying data are the security-level PROTIDE data which report, at quarterly frequency, the issuer and the holder of a security, and allows to decompose the change of asset and liability positions into flows and valuations. We consider the following six institutional sectors defined in the European System of National and Regional Accounts (ESA 2010):

1. Banks and money market funds referred to as the *banking sector*
2. Other financial corporations, which are (mostly) mutual funds and thus referred to as *mutual funds*<sup>2</sup>

---

<sup>2</sup>The sector *other financial corporations* comprises all institutional units whose principal activity is the production of financial services for the market other than banks, money-market funds, insurance corporations or pension funds. The largest subcategory of this sector are non-money market investment funds (OPCVM in French), that is collective investment schemes issuing fund shares that are not close substitutes to deposits (contrary to money-market funds) and might invest in long-term financial assets or real estate. It also includes other financial intermediaries (e.g. security and derivative dealers, leasing and factoring corporations), financial auxiliaries (e.g. brokers and consultants) and captive financial institutions and money lenders.

3. Non-financial corporations referred to as the *corporate sector*
4. The *insurance sector*
5. Households and non-profit institutions serving households referred to as the *household sector*
6. The *public sector*

For each sector, we report the portfolio assets ( $A_t$ ) and liabilities ( $L_t$ ) in equity ( $E_t$ ) and debt ( $B_t$ ). The portfolio of each sector is divided between the domestic portfolio (domestic assets and liabilities held by other domestic agents), and the foreign portfolio (indexed by \*). Net asset positions are obtained as the difference between assets and liabilities.

By construction, the sum of domestic assets equals the sum of domestic liabilities,

$$\sum_{i=1}^6 (B_{it}^A + E_{it}^A) = \sum_{i=1}^6 (B_{it}^L + E_{it}^L),$$

which does not prevent each individual sector to have a positive or negative net domestic position. The sum of net foreign asset positions of each sector equals the country's net external portfolio ( $NEP_t$ ):

$$NEP_t = \sum_{i=1}^6 NEP_{it} = \sum_{j=1}^6 (B_{it}^{A*} + E_{it}^{A*}) - \sum_{j=1}^6 (B_{it}^{L*} + E_{it}^{L*}).$$

Further, the evolution of any position between two periods can be decomposed into new flows and changes in the market value of existing positions. The change in the net foreign position of sector  $i$  can thus be written as

$$NEP_{it} - NEP_{it-1} = F_{it} + \Delta Val(NEP_{it-1}),$$

where  $F_{it}$  measures the net new flows during period  $t$  and  $\Delta Val(NEP_{it-1})$  measures the change in valuation of the portfolio hold at time  $t - 1$ . The latter can be further decomposed into a price effect and an exchange rate effect.

## 4.3.2 Stylized Facts on French Domestic and External Sectoral Portfolios

### 4.3.2.1 External Portfolios

Table C.1 decomposes external assets and liabilities in percent of French GDP of each sector in 2008Q1 and 2014Q1. Table C.1, Panel A aggregates debt and equity positions which are disaggregated in Panel B and Panel C. Changes in assets and liabilities between 2008Q1 and 2014Q1 are measured in percentage point (ppt) changes of their share of GDP. The analysis of sectoral portfolios allows to understand the role played by each sector in the large deterioration of the external portfolio position of France from a net positive position in 2008Q1 (+4.7 percent of GDP) to a large net negative position (−35.7 percent of GDP) in 2014Q1:

*Stylized Fact 1 [Net External Position]: The sharp deterioration of the net external position between 2008Q1 and 2014Q1 (−40.4 ppts) is explained by the deterioration of the external position of: the public sector (−24.1 ppts), the banking sector (−15.1 ppts), and the corporate sector (−7.9 ppts). The insurance sector is the only sector whose net external position improved (+7.6 ppts).*

Between 2008Q1 and 2014Q1, the external asset position barely changed from 100.3 to 101.3 percent of GDP while the external liability position deteriorated sharply from 95.6 to 137.0 percent of GDP, thus explaining entirely the deterioration of the net external position. Underlying those aggregates are large re-shufflings in sectoral positions both on the asset and the liability side.

*Stylized Fact 2 [External Assets and Liabilities]: The stability in aggregate external asset portfolio between 2008Q1 and 2014Q1 has been associated with a sharp reduction in the external assets of the banking sector (−9.3 ppts) largely compensated by an increase in the external assets of the insurance sector (+6.3 ppts) and of mutual funds (+1.5 ppts). The deterioration of the net external position (Stylized Fact 1) is due to a deterioration in the liability positions.*

Breaking down positions between debt and equity (Table C.1, Panel A and Panel B) reveals that the net external position in 2008Q1 was positive in debt (+12.8 percent of GDP) and negative

in equity (−8.1 percent of GDP). By 2014Q1, the net external position in equity barely changed (−6.6 percent of GDP), but the net position in debt became strongly negative (−29.1 percent of GDP). This implies that the sharp deterioration in the net external portfolio (−40.4 percent of GDP) can be explained by that of the debt liability position. The underlying sectoral patterns can be summarized as:

*Stylized Fact 3 [Debt versus Equity Portfolios]: The deterioration in the net external position (−40.4 ppts) is mostly explained by the increase of debt liabilities (+39.2 ppts) and is mostly driven by the increased debt liabilities of the public sector (+24.2 ppts), the banking sector (+6.5 ppts), and the corporate sector (+6.5 ppts). On the asset side, banks reduce sharply their external debt holdings (−9.8 ppts), while the insurance sector increased them significantly (+4.9 ppts).*

Table C.2 summarizes the stylized facts described above by showing the contribution of each sector and each balance-sheet item to the change in the net external portfolio position of France between 2008Q1 and 2014Q1.

Table C.3 contrasts the role of Eurozone and non-Eurozone countries in external portfolios. The net position of France vis-à-vis the Eurozone deteriorated from 23.3 to 13.6 percent of GDP. Vis-à-vis non-Eurozone countries, the net position of France deteriorated three times more, from −18.6 to −49.3 percent of GDP. The related sectoral patterns can be summarized as follows:

*Stylized Fact 4 [Eurozone versus ROW]: The deterioration of the net portfolio position vis-à-vis the non-Eurozone countries has been three times larger than vis-à-vis the Eurozone countries (−30.7 ppts vs. −9.7 ppts). The banking sector shifted from being a net portfolio creditor to becoming a net debtor vis-à-vis non-Eurozone countries. The deterioration in the public sector liability position has been about five times larger vis-à-vis the non-Eurozone countries than vis-à-vis Eurozone countries (−19.8 ppts vs. −4.3 ppts).*

Table C.4 decomposes the Eurozone asset portfolios further into assets held on GIPS countries (Greece, Italy, Portugal, Spain) versus asset held on non-GIPS countries.<sup>3</sup> As expected the

---

<sup>3</sup>For such decomposition, the information contained in PROTIDE is restricted on the asset side.

reduction in the exposure to GIPS countries has been massive between 2008Q1 and 2014Q1, from 23.5 to 12.2 percent of GDP, and especially so in the banking sector (from 10.5 to 4.7 percent of GDP). By contrast the external asset position towards non-GIPS countries has increased slightly from 39.2 to 43.4 percent of GDP, with the largest contribution coming from the insurance sector (+3.2 ppts).

*Stylized Fact 5 [GIPS versus Non-GIPS]: The asset exposure of French sectors to Eurozone countries exhibited a retrenchment from GIPS countries (−11.3 ppts), with a large contribution of the banking sector (−5.9 ppts), and a more modest expansion towards the non-GIPS countries (+4.2 ppts).*

Table C.5, Panel A decomposes the evolution of international sectoral portfolios between 2008Q1 and 2014Q1 between *flows and valuations*. Panel B and Panel C provide the same decomposition for debt and equity portfolios. Overall, exchange rate fluctuations play a modest role compared to changes in asset prices and portfolio flows. As previously discussed, most of the change in the country's external net portfolio position came from the debt liability side, and since the valuation losses on the liability side were, to a large extent, compensated by valuation gains on the asset side, net flows contributed seven times more to the change in the net portfolio position than net valuation effects (−36.2 ppts vs. −5.1 ppts). The relative role of flows and valuations varies however substantially across sectors as summarized below.

*Stylized Fact 6 [Valuations versus Flows]: Valuation effects account for about 12 percent of the change in the net external portfolio position between 2008Q1 and 2014Q1. The increase in the assets portfolio of the insurance sector is balanced between flows and valuation effects while the banking sector mostly experienced negative net asset flows. Valuation effects account for 15 to 20 percent of the deterioration of the liability position of the public sector, corporate sector, and the banking sector. Net valuation effects are about equally divided between equity and debt portfolios.*

### 4.3.2.2 External Cross Sectoral Portfolios

Table C.6 describes the asset and liability linkages between French sectors and the Eurozone sectors. The table shows how much a French sector is exposed to each of the four Eurozone sectors, i.e. the banking sector, the corporate and household sector, the mutual funds and insurance sector, and the public sector.

Looking at the total assets combined of French sectors reveals a striking fact: while the overall exposure stays nearly constant, the distribution of that exposure across the different Eurozone sectors changes dramatically between 2008Q1 and 2014Q1: the exposure of French sectors to the Eurozone banking sector was reduced from 16.6 to 12.2 percent of GDP, and that to the Eurozone corporate and household sectors was halved from 20.3 to 10.7 percent of GDP; meanwhile the exposure to the mutual funds and insurance sectors more than tripled from 6.7 percent of 22.7 percent of GDP. Turning to specific French sectors, the French banking sector reduced its exposure to the Eurozone banking sector substantially (from 7.7 to 4.7 percent of GDP), but it reduced exposure to the Eurozone corporate and household sector even more (from 7.9 to 1.3 percent of GDP). The insurance sector expanded its asset position in Eurozone mutual funds and insurances (from 0.9 to 7.5 percent of GDP).

On the liability side, French sectors combined increased their liabilities to Eurozone sectors from 39.1 to 51.5 percent of GDP, an increase which is almost entirely due to larger holdings of French assets by the Eurozone banking sector.

*Stylized Fact 7 [Eurozone Sectors]: During the period 2008Q1-2014Q1, the exposure of French sectors to Eurozone sectors changed dramatically: a large reduction of the exposure to the Eurozone banking sector (−4.3 ppts) and Eurozone corporate and household sector (−9.6 ppts) was mirrored by a large increase in the exposure to the Eurozone mutual funds and insurance sector (+16.0 ppts). The increase in French liabilities towards the Eurozone (+12.3 ppts) has been almost entirely absorbed by the Eurozone banking sector.*

### 4.3.2.3 Domestic versus Foreign Portfolios

Table C.7 contrasts the evolution of the domestic and the foreign portfolio between 2008Q1 and 2014Q1. Panel A presents the domestic positions, Panel B the external positions, and Panel C the consolidated positions. While the aggregate domestic portfolio is balanced, by construction, there are internal imbalances at the sector-level as well as external and consolidated imbalances. The comparison of the domestic, foreign and consolidated portfolios across sectors can be summarized as follows:

*Stylized Fact 8 [Domestic and Foreign Portfolios]: The banking sector improves its domestic net portfolio position (+11.4 ppts) while its foreign net portfolio position deteriorates (−15.1 ppts), as a result of a shift of its liabilities from foreign to domestic, and of its assets from foreign to domestic. The banking sector’s consolidated net position deteriorated modestly (−3.8 ppts). The public sector increased its external liabilities significantly more than its domestic liabilities (−24.2 ppts vs. −10.4 ppts). The insurance sector increased its domestic assets (+13.2 ppts) significantly more than its foreign assets (+6.3 ppts).*

Consolidating the domestic and external net positions (Panel C) reveals that in 2014Q1 the insurance sector is the largest net security creditor (+89.9 percent of GDP) while the public sector (−77.0 percent of GDP) and the corporate sector (−62.9 percent of GDP) are the largest net debtors.

By looking at the share of foreign assets in total portfolios in 2008Q1 and 2014Q1 (Table C.8), one can assess the relative role of *portfolio growth* vs. *portfolio re-balancing*, as discussed by Kraay & Ventura (2000). An implication of portfolio growth is that the share of foreign assets in the portfolio must remain roughly constant over time. Interestingly, the portfolio growth hypothesis works very well for the aggregate assets position - a constant share of about 37 percent of foreign assets over total assets - despite large heterogeneity across sectors, with the retrenchment of the banking sector being compensated by the outwards expansion of other sectors, mostly the

household sector. On the liability side, the aggregate portfolio liability position became increasingly international, reflecting re-balancing towards foreign liabilities in all sectors. These findings can be summarized as follows:

*Stylized Fact 9 [Portfolio Growth versus Re-balancing]: The share of the aggregate foreign asset portfolio remained constant between 2008Q1 and 2014Q1 despite large heterogeneity across sectors, with the banking sector exhibiting retrenchment, and the household sector foreign expansion. On the asset side, the evolution of the portfolio of the insurance sector and that of mutual funds are consistent with the portfolio growth hypothesis. The share of foreign liabilities in total liabilities increased sharply over the period and all sectors re-balanced their liabilities towards international liabilities. Only the insurance sector exhibits a behavior consistent with portfolio growth on the liability side.*

### 4.3.3 The Network Structure of Domestic and External Portfolios

The cross-sectoral asset and liability positions between domestic sectors, and between domestic sectors and the rest of the world form a weighted network whose weights are determined by the size of cross-sectoral exposures. Figure C.3 graphs the network of domestic sectors. An arrow pointing from sector  $i$  to sector  $j$  indicates the amount of liabilities issued by sector  $j$  which are held in the asset portfolio of sector  $i$ . Cross-holdings are normalized by GDP and positions amounting to less than 1 percent of GDP are omitted to keep the graph easily readable. Each node represents a sector and the size of each node is proportional to the sum of total security assets of the sector. The position of sectors in the graph is based on the number of links (exceeding one percent of GDP) through which a sector is connected with other sectors. The intensity of cross-sector links is captured by the thickness of the arrows.

Figure C.3 compares the domestic network in 2008Q1 and 2014Q1. From Table C.7, we know that the total of domestic security assets positions (which by construction is also the total of domestic security liabilities) barely changes (from 167.1 to 170.8 percent of GDP), but the network graph reveals that the *distribution of domestic cross-sectoral asset and liability position*



*did change substantially.* A large fraction of the increase in the asset position of the insurance sector (from 44.9 to 58.1 percent of GDP) corresponds to a change in its holdings of public sector securities (+6.2 ppts). The banking sector slightly increased its domestic asset position (from 38.8 to 43.2 percent of GDP) as a result of a reduction in its holdings of corporate sector securities (from 5.6 to 3.5 percent of GDP), and an *increase* in its holdings of public sector securities (from 7.9 to 12.9 percent of GDP). The banking sector also reduced its liability positions (from 56.1 to 49.1 percent of GDP), which is reflected in the reduction of the holdings of banking sector securities in the portfolio of corporations, households, and mutual funds. Finally, the government reduces its holding of securities issued by corporations (through privatization of government's assets) and households reduce their holdings of mutual funds liabilities. These findings can be summarized as follows:

*Stylized Fact 10 [Domestic Network]: The banking and insurance sectors increase their holdings of public sector securities. The banking sector did so by reducing its holdings of corporate sector securities and the insurance sector by increasing its overall asset position. The reduction of banking sector liabilities has been associated with a reduction in the holdings of banking sector liabilities by corporations, mutual funds, and households.*

Figure C.4 adds the rest of the world (ROW) as an additional node to the network, and thus the network graph maps both external and domestic sector portfolios. In 2008Q1, the ROW accounted for 37.5 percent of French security assets, and 36.4 percent of French security liabilities. By 2014Q1, the foreign share of assets was about the same, but that of foreign liabilities increased to 44.5 percent of GDP. The graph shows the reversal in the banking sector net external portfolio through a sharp reduction in external assets and a sharp increase in external liabilities. Securities issued by the French corporate sector became also increasingly held by the ROW. Indeed, the liabilities of the corporate sector held in domestic portfolios remain constant at about 47 percent of GDP, while corporate liabilities held in foreign portfolios increased from 30.8 to 38.8 percent of GDP. Figure C.3 breaks down the ROW between Eurozone and non-Eurozone countries. The

increase in foreign liabilities of the most externally indebted French sectors (banking sector, corporate sector, and public sector) has been disproportionately tilted towards non-Eurozone countries, which thus experienced an increase in their holdings of French assets by 29.1 ppts vs. 12.3 ppts for Eurozone countries. These findings can be summarized as follows:

*Stylized Fact 11 [International Network]: Between 2008Q1 and 2014Q1, France experienced an increase in its international financial integration driven by the increase in liability exposure of the banking sector, the corporate sector, and the public sector. Such international financial integration has been disproportionately tilted towards liability exposure to non-Eurozone countries. The insurance sector is the only sector that significantly increased its foreign assets.*

## 4.4 An Estimated Model of Sectoral Balance-Sheet Contagion

In this section, we put forward and estimate a simple equilibrium model of balance-sheet contagion. The central idea is that the value of liabilities issued by a sector depends on the value of assets held on the asset side of the sector's balance-sheet. The transmission of value changes from the asset to the liability side of the balance-sheet is called *balance-sheet contagion*.

From a structural point of view, such balance-sheet contagion is, in fact, the result of demand and supply decisions taken by market participants and a price discovery mechanism that leads to market clearing. Indeed, the steps from a change in the value of sector's assets to a change in the value of its liabilities are as follows: *First*, a change in the value of assets induces the sector to rebalance its asset portfolio and to adjust its liabilities. This, *second*, generates partial disequilibria on asset markets and therefore leads to partial price changes, which, *third*, induce further portfolio rebalancings and liability adjustments by other sectors until eventually, a new market clearing price vector emerges. In equilibrium, there is thus a specific comovement of the value of a sector's assets and liabilities. A change in the value of assets of one sector can, through equilibrium price changes, propagate to all sectors of the economy.

Fundamentally, the degree of balance-sheet contagion depends on the shape of sectoral demand

and supply function for financial assets, and in particular on the sensitivity of sectors' demand and supply of a given asset to price changes. Demand and supply are in turn determined by sector-specific balance-sheet management objectives, return beliefs, and constraints. Here, instead of deriving optimal demand and supply schedules from balance-sheet optimization problems, we build on Koijen & Yogo (2019) and model asset demand and supply directly as functions of asset characteristics.

In this framework, balance-sheet contagion has a simple form and is captured by a function of (i) sector-specific balance-sheet contagion coefficient and (ii) the network of bilateral sectoral exposures.

#### 4.4.1 The Model

There is a set of domestic institutional sectors indexed by  $i \in \{1, 2, \dots, I\}$ . The set of assets available for investment to sector  $i$  is noted by  $\mathcal{P}_i^A$ . It comprises securities issued by domestic sectors indexed by  $j(i')$ ,  $i' \in \{1, 2, \dots, I\}$ , and securities that are issued by foreign entities. The foreign portfolio of each sector is different and is summarized by one bundled *sector-specific foreign asset* denoted by  $f(i)$ . It thus allows to capture the heterogeneity of returns on foreign asset portfolios across sectors highlighted by Galstyan & Velic (2018) for the case of Germany. Similarly, all other types of instruments that are not part of portfolio investments, such as loans, cash or real assets are bundled into a *sector-specific outside asset* noted  $o(i)$ . Each sector  $i$  can also raise funds through issuing either securities denoted by  $j(i)$  or other financial instruments which are again bundled into the outside asset  $o(i)$ . The set of assets available to sector  $i$  for raising funds is denoted  $\mathcal{P}_i^L$ .

Figure 4.1: Sector  $i$ 's Asset and Liability Allocation

|                                     | Assets              | Liabilities         |
|-------------------------------------|---------------------|---------------------|
| Domestic Securities                 | $\omega_{i1t}^A$    |                     |
|                                     | $\omega_{ij(i)t}^A$ | $\omega_{ij(i)t}^L$ |
|                                     | ...                 |                     |
|                                     | $\omega_{iJt}^A$    |                     |
| Foreign Portfolio Investments       | $\omega_{if(i)t}^A$ |                     |
| Other Assets, e.g. loans, cash, ... | $\omega_{io(i)t}^A$ | $\omega_{io(i)t}^L$ |
|                                     | $= 1$               | $= 1$               |

Figure 4.1 summarizes notations and illustrates the complete allocation of sector  $i$ 's assets and liabilities across the different assets, where  $\omega_{ikt}^s$  is the share of asset  $k$  in sector  $i$ 's assets ( $s = A$ ) or liabilities ( $s = L$ ) in period  $t$ .<sup>4</sup>

In the following, we model equilibrium returns on the domestic securities issued by the different sectors, taking as given returns on foreign portfolio investments and outside assets.

#### 4.4.1.1 Asset Demand and Supply

We build on the approach followed by Kojien & Yogo (2019) and model a sector's optimal portfolio allocation as a logit model of asset characteristics. Compared to the mean-variance case, where the optimal allocation depends on the full variance-covariance matrix of returns, this might seem an over-simplification. However, when it is additionally assumed that (i) returns follow a factor structure, *and* that (ii) the factor loadings depend on asset characteristics, the optimal mean-variance allocation and the characteristics-based logit model can be shown to be approximately equivalent. In this case, asset characteristics are sufficient to capture expected returns and their variance-covariance.

With  $s \in \{A, L\}$  noting assets or liabilities, the share of sector  $i$ 's balance-sheet invested in (or

<sup>4</sup>The share of assets invested in own liabilities is generally different from zero, because entities within the sector can invest in securities issued by an entity that is part of the same sector.

raised through) asset  $k$  in period  $t$  is modeled as

$$\omega_{ikt}^s = \frac{\exp(b_{i0}^s + b_i^s \log P_{kt} + \log U_{ikt}^s)}{1 + \sum_{l \in \mathcal{P}_i^s} \exp(b_{i0}^s + b_i^s \log P_{lt} + \log U_{ilt}^s)}, \quad (4.1)$$

where  $b_{i0}^s$  and  $b_i^s$  are sector-specific intercept and coefficient on the log-price,  $\log P_{kt}$ , respectively, that determine the shape of sector  $i$ 's demand and supply function. Asset characteristics other than the price that generate latent demand or supply for asset  $k$  are captured by the term  $\log U_{ikt}^s$ . Equation 4.1 and the budget constraint imply that the share of the outside asset is  $\omega_{io(i)t}^s = 1/1 + \sum_{l \in \mathcal{P}_i^s} \exp(b_{i0}^s + b_i^s \log P_{lt} + \log U_{ilt}^s)$  and we derive the growth factors of balance-sheet shares from period  $t-1$  to  $t$  as

$$\frac{\omega_{ikt}^s}{\omega_{ikt-1}^s} = \left( \frac{P_{kt}}{P_{kt-1}} \right)^{b_i^s} \cdot \frac{U_{kt}^s}{U_{kt-1}^s} \cdot \frac{\omega_{io(i)t}^s}{\omega_{io(i)t-1}^s}. \quad (4.2)$$

Taking logs of Equation 4.2 and using a first-order approximation, the growth rate of the share of asset  $k$  in sector  $i$ 's balance-sheet,

$$z_{ikt}^s \approx b_i^s r_{kt} + \Delta u_{ikt}^s + z_{o(i)t}^s, \quad (4.3)$$

is approximately linear in the contemporaneous return on asset  $k$ ,  $r_{kt}$ , the log-difference of latent demand (or supply),  $\Delta u_{ikt}^s$ , and the growth rate of the share of the outside asset,  $z_{o(i)t}^s$ . The coefficient on the log-price,  $b_i^s$ , is therefore simply the *sensitivity of balance-sheet share growth rates to contemporaneous returns*.

#### 4.4.1.2 Sectoral Balance-Sheet Equilibrium

Sectoral balance-sheet identities are used to determine the equilibrium vector of returns:

$$A_{it} = L_{it}. \quad (4.4)$$

Now, using the linear expression for the growth rate of asset shares from Equation 4.3, sectoral balance-sheet identities can be decomposed as

$$\sum_{l \in \mathcal{P}_i^s} \omega_{ilt-1}^A \left[ b_i^A r_{lt} + \Delta u_{ilt}^A \right] + z_{o(i)t}^A = \omega_{ij(i)t-1}^L \left[ b_i^L r_{j(i)t} + \Delta u_{ij(i)t}^L \right] + z_{o(i)t}^L. \quad (4.5)$$

Returns on domestic securities must thus move to equalize asset- and liability side of the balance-sheet given the balance-sheet allocation in  $t - 1$ , and exogenous changes in the allocation of the rest of the balance-sheet. Stacking balance-sheet identities (one for each domestic sector) in matrix notation, the  $I$ -column vector of equilibrium returns on domestic securities is shown to be

$$\mathbf{r}_t = \underbrace{\left( I - \text{diag}(\beta) \tilde{\Omega}_{t-1} \right)^{-1}}_{\equiv C(\beta)_{t-1}} \cdot \left( \underbrace{\text{diag}(\alpha) \tilde{z}_{ot}}_{(i) \text{ Outside Assets}} + \underbrace{\text{diag}(\beta) \text{diag}(\tilde{\omega}_{f,t-1}) \mathbf{r}_{ft}}_{(ii) \text{ Foreign Assets}} + \underbrace{\boldsymbol{\varepsilon}_t}_{(iii) \text{ Shocks}} \right), \quad (4.6)$$

where *tilde* denotes normalization by the shares of a sector's liabilities raised through securities,  $\omega_{ij(i)t-1}^L$ . Equilibrium returns thus crucially depend on the ratio  $\beta_i \equiv \frac{b_i^A}{b_i^L}$ , that we call *balance-sheet contagion coefficient*, which is the relative sensitivity of sector  $i$ 's asset demand over asset supply to contemporaneous returns. Importantly, given that  $\beta_i$  is a ratio, its value is unrestricted and, in particular, can be either smaller or larger than one. Further, we defined  $\alpha_i \equiv 1/b_i^L$ , the vector  $\tilde{z}_{ot} \equiv \tilde{z}_{o(i)t}^A - \tilde{z}_{o(i)t}^L$  as the asset-liability difference in growth rates of the share of outside assets in sectors' balance-sheets, and  $\boldsymbol{\varepsilon}_t$  the vector of shocks that contains sectors' latent demand and supply for domestic securities and foreign portfolio investments.

As shown in Equation 4.6, equilibrium returns are determined by the endogenous reaction of balance-sheet shares of domestic securities to exogenous changes in the allocation of the rest of the balance-sheet, i.e. exogenous changes in (i) the outside asset share, (ii) the return on foreign portfolio investments, and (iii) the net of latent demand and supply. The matrix  $C(\beta)_t$ , called *contagion matrix*, captures how exogenous changes in sectoral balance-sheets propagate to changes in equilibrium returns.

### 4.4.1.3 Balance-Sheet Contagion

To illustrate how an exogenous change in sectoral balance-sheets propagates to equilibrium returns, consider a shock  $d\boldsymbol{\varepsilon}_t$ . Total differentiation of Equation 4.6,

$$d\mathbf{r}_t = C(\boldsymbol{\beta})_{t-1} \cdot d\boldsymbol{\varepsilon}_t, \quad (4.7)$$

shows that the contagion matrix,  $C(\boldsymbol{\beta})_{t-1}$ , maps sectoral shocks to changes in equilibrium returns. The rows of the contagion matrix correspond to returns on securities issued by a sector and the columns to the sectors in which the shock originated. The elements of the contagion matrix are sector-specific multipliers that measure the strength of the full transmission of sectoral shocks through the network.

To understand how shocks propagate through sectoral balance-sheets, we decompose the full return change (Equation 4.7) into a sequence of partial return adjustments

$$d\mathbf{r}_t = \underbrace{d\boldsymbol{\varepsilon}_t}_{d\mathbf{r}_t|_1} + \underbrace{\text{diag}(\boldsymbol{\beta}) \tilde{\boldsymbol{\Omega}}_{t-1} d\mathbf{r}_t|_1}_{d\mathbf{r}_t|_2} + \underbrace{\left(\text{diag}(\boldsymbol{\beta}) \tilde{\boldsymbol{\Omega}}_{t-1}\right)^2 d\boldsymbol{\varepsilon}_t + \dots}_{d\mathbf{r}_t|_3}, \quad (4.8)$$

where Equation 4.8 follows from expanding the contagion matrix as infinite sum  $C(\boldsymbol{\beta})_{t-1} = \sum_{p=0}^{\infty} \left(\text{diag}(\boldsymbol{\beta}) \tilde{\boldsymbol{\Omega}}_{t-1}\right)^p$ . Under this sequential representation, in each step returns move such as to revalue a sector's liabilities to match the value of its assets and restore the balance-sheet identity. In particular, the chain from the shock to the full change in returns is as follows: First, the shock leads to a discrepancy between sectors' assets and liabilities of magnitude  $d\boldsymbol{\varepsilon}_t$ . In order to restore balance-sheet identities, returns thus initially move one-for-one,  $d\mathbf{r}_t|_1 = d\boldsymbol{\varepsilon}_t$ , with the shock in order to revalue liabilities and match assets. This initial partial change of returns, however, leads to rebalancings of demand and supply and value changes on the asset side, which again generate balance-sheet disequilibria of magnitude  $\text{diag}(\boldsymbol{\beta}) \tilde{\boldsymbol{\Omega}}_{t-1} d\mathbf{r}_t|_1$ . These, in turn, require a *second* change in returns,  $d\mathbf{r}_t|_2$ , and so forth until the shock is fully propagated and a new balance-sheet equilibrium is reached.

The role of sectors in the propagation of shocks depends on the balance-sheet contagion coefficients  $\beta$  and the time-varying matrix of normalized asset shares  $\tilde{\Omega}_{t-1}$ . Indeed, ceteris paribus, the role of a given sector  $i$  for shock propagation increases, the stronger the sensitivity of its asset shares to returns ( $b_i^A$  high) and the weaker the reaction of its liability shares to returns ( $b_i^L$  low). Further, the more a sector is exposed to a given asset  $k$  (the higher the asset share  $\omega_{ikt-1}^A$ ), the stronger the effect on the value of sectoral assets for a given change in returns. Similarly, the lower the share of securities in total sectoral liabilities (the lower the liability share  $\omega_{ij(i)-1}^L$ ), the more returns must move in order to revalue liabilities and restore balance-sheet equilibria.

#### 4.4.2 Identification and Estimation

The model as specified in Equation 4.6 can be identified and estimated through two-step GMM. For the identification of the parameters of the model, we need to make assumptions on the distribution of the residual  $\varepsilon_t$ . In particular, we assume that residuals have zero-mean,

$$\mathbb{E}[\varepsilon_t] = \mathbf{0}, \quad (4.9)$$

and are uncorrelated across sectors. We do, however, allow for heteroscedasticity across sectors, such that

$$\text{Var}[\varepsilon_t] \equiv \Sigma_\varepsilon = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_S). \quad (4.10)$$

The assumptions on the distribution of the  $I$ -column vector of residuals provide  $I + I(I + 1)/2$  moment conditions, corresponding to  $I$  first order moments and  $I(I + 1)/2$  second order moments from the variance-covariance matrix of the sectoral shocks. There are  $3I$  parameters, corresponding to the  $I$ -column vectors  $\alpha$  and  $\beta$ , and the number of variances in the variance-covariance matrix of  $\varepsilon_t$ . The model is exactly identified if there are as many moment conditions as parameters. The model is over-identified if the number of sectors  $I$  is larger than 3.

The baseline estimation is performed until the assumption of no correlation of residuals across sectors. We complete this baseline by allowing for various common shock structures: a global



shock that affects all the sectors simultaneously, a financial shock that affects all the financial sectors of the economy (banking, mutual funds, insurance), and a real shock that affects the real sectors of the economy (corporate and public). A global shock with variance  $\sigma_g$ , for example, changes the variance-covariance matrix to

$$\Sigma_{\mathcal{E}}^g = \begin{pmatrix} \sigma_g + \sigma_1 & \sigma_g & \sigma_g & \sigma_g & \sigma_g \\ \sigma_g & \sigma_g + \sigma_2 & \sigma_g & \sigma_g & \sigma_g \\ \sigma_g & \sigma_g & \sigma_g + \sigma_3 & \sigma_g & \sigma_g \\ \sigma_g & \sigma_g & \sigma_g & \sigma_g + \sigma_4 & \sigma_g \\ \sigma_g & \sigma_g & \sigma_g & \sigma_g & \sigma_g + \sigma_5 \end{pmatrix}. \quad (4.11)$$

Similarly, the financial and the real shock introduce covariance between the financial and real sectors, respectively.

#### 4.4.2.1 Moment Conditions

Equation 4.9 implies first-order moments,

$$m_1(\alpha, \beta) \equiv \mathbb{E} \left[ \mathbf{r}_t - \text{diag}(\beta) \left( \tilde{\Omega}_{t-1} \mathbf{r}_t - \text{diag}(\tilde{\omega}_{ft-1}) \mathbf{r}_{ft} \right) - \text{diag}(\alpha) \tilde{z}_{ot} \right], \quad (4.12)$$

and second order moments,

$$M_2(\alpha, \beta, \Sigma_{\mathcal{E}}) \equiv \mathbb{E} \left[ \left( \mathbf{r}_t - \text{diag}(\beta) \left( \tilde{\Omega}_{t-1} \mathbf{r}_t - \text{diag}(\tilde{\omega}_{ft-1}) \mathbf{r}_{ft} \right) - \text{diag}(\alpha) \tilde{z}_{ot} \right) \cdot \left( \mathbf{r}_t - \text{diag}(\beta) \left( \tilde{\Omega}_{t-1} \mathbf{r}_t - \text{diag}(\tilde{\omega}_{ft-1}) \mathbf{r}_{ft} \right) - \text{diag}(\alpha) \tilde{z}_{ot} \right)' \right] - \Sigma_{\mathcal{E}}. \quad (4.13)$$

We stack the  $I(I+1)/2$  independent second-order moment conditions into the vector  $m_2(\alpha, \beta, \Sigma_{\mathcal{E}})$  and define the vector of empirical moment conditions as  $\bar{m}(\alpha, \beta, \Sigma_{\mathcal{E}})' \equiv (\bar{m}'_1, \bar{m}'_2)'$ .

#### 4.4.2.2 Two-Step GMM

In the first step, we obtain the minimizer of the sum of squared empirical moment conditions

$$(\widehat{\alpha}, \widehat{\beta}, \widehat{\Sigma_\varepsilon})_{GMM1} = \operatorname{argmin}_{\alpha, \beta, \Sigma_\varepsilon} \{ \bar{m}(\alpha, \beta, \Sigma_\varepsilon) \cdot I \cdot \bar{m}(\alpha, \beta, \Sigma_\varepsilon)' \}.$$

In the second step, the inverse of the estimate of the variance-covariance matrix of moment conditions obtained in the first step,  $S_T = \frac{1}{T} \sum_{t=1}^T \widehat{m}_t|_{GMM1} \widehat{m}_t|_{GMM1}'$ , is used as a weighting matrix for the empirical moments and final estimates are obtained as

$$(\widehat{\alpha}, \widehat{\beta}, \widehat{\Sigma_\varepsilon})_{GMM2} = \operatorname{argmin}_{\alpha, \beta, \Sigma_\varepsilon} \{ \bar{m}(\alpha, \beta, \Sigma_\varepsilon) \cdot S_T^{-1} \cdot \bar{m}(\alpha, \beta, \Sigma_\varepsilon)' \}.$$

The variance-covariance matrix of the estimates results from the gradient of the empirical moment conditions and the updated estimate of the variance-covariance matrix of moments.

### 4.4.3 Results

The estimates of the model's parameters are presented in Table C.9. First, balance-sheet contagion coefficients are significantly different from zero only for the financial sectors, i.e. for the banking sector, the insurance sector, and for mutual funds, while they are statistically insignificant for the real sectors. This result is consistent with our prior that decisions on the balance-sheet structure of the real sectors (corporate and public sector) are much more driven by factors such as demand for goods and services or the general macroeconomic outlook, rather than the return on the security portfolio. Further, public sector investments might rather be motivated by stabilization concerns and issuance of government bonds by public spending.

Second, the contagion coefficients for the financial sectors are all positive, which is consistent with the intuition that the value of securities issued by the financial sectors increases with the return of the securities held on the asset side. Third, we find that the balance-sheet contagion coefficient is highest for mutual funds (1.56), followed by banks (0.71) and insurances (0.06),

which is consistent with the mutual funds and banking sector managing portfolios more actively, while insurances follow long-term passive investment strategies.

These results are robust to the introduction of common shocks across sectors and the magnitude of the coefficient are very similar, with at most a slight decline. Indeed, as shown in columns (2)-(4) of Table C.9, allowing for a global shock (common to all sectors) or a real shock decrease the estimates slightly. Allowing for a shock common to all financial sectors leaves balance-sheet contagion coefficients virtually unchanged.

We now turn to how balance-sheet contagion varies over time. As shown in Equation 4.7, the multipliers mapping shocks to equilibrium returns are given by the contagion matrix  $C(\beta)_t$ . Figure C.7 therefore shows the time-series of the contagion matrix for banks, insurances and mutual funds. Returns on securities issued by the corporate sector and the public sector are not impacted by balance-sheet contagion, since balance-sheet contagion coefficients are insignificant. We make the following observations: *First*, shocks to the banking sector and mutual funds on the return of own securities are amplified through balance-sheet contagion with a stable factor of approximately 1.2 to 1.3, while the impact of shocks to the insurance sector on own securities is roughly unity. The reason for this is the very low exposure of sectoral portfolios to the insurance sector and the fact that insurances are financed to a very small extent by security issuance. *Second*, in line with the re-balancing of sectoral portfolios towards public sector securities (Figure C.3), transmission of public sector shocks to other sectors increases over the sample period: Insurance sector returns (2008Q1: 0.15; 2014Q1: 0.76) and banking sector returns (2008Q1: 0.09; 2014Q1: 0.14) become more sensitive to public sector shocks. Mutual funds securities remain relatively resilient to public sector shocks, given the small exposure of mutual funds to public sector securities (on average 4.26%).

Turning to shocks on the return of foreign securities held by domestic sectors, we generally find a large effect on financial sector returns, because all financial sectors have large foreign asset positions (banking: 46.4%, insurance: 38.7%, and mutual funds: 50.4%). Therefore, a one ppt shock to only the return of foreign securities held by the banking sector, for example, has a large

*initial* impact on the value of total banking assets. The *full* contagion effect of this shock through the network will be such that the banking sector is the most resilient to shocks on the return of its foreign security position with a contagion factor of 0.49 in 2008Q1 and further reduced to 0.44 in 2014Q1 (in line with the retrenchment of the banking sector's foreign asset portfolio discussed in Section 4.3). Mutual funds achieve a reduction in the effect of a foreign portfolio shock to the return of its securities from 1.37 in 2008Q1 to 1.11 in 2014Q1.

The most dramatic change over the period regards the exposure of the insurance sector to foreign portfolio risk. A one ppt shock to the return of the foreign portfolio of insurances implies a change of 0.69 ppt of the return on insurance securities in 2008Q1, but a 2.26 ppt change in 2014Q1. This result comes from two sources: First, the insurance sector expands sharply its external asset portfolio in the post-crisis phase from 26.8 percent of GDP in 2008Q1 to 33.1 percent of GDP in 2014Q1. Second, the share of securities in the financing mix of the insurance sector decreased drastically from 4.5 percent of GDP in 2008Q1 to 1.3 percent of GDP in 2014Q1. Consequently, with a lower volume of securities on the liability side, a given shock to the value of assets requires, *ceteris paribus*, a larger change in the return on the issued securities to restore the balance-sheet identity.<sup>5</sup>

Shocks to the corporate sector and mutual funds transmit mostly to the insurance and the mutual fund sector, while the banking sector appears relatively resilient. Insurance sector shocks are confined to the insurance sector itself and do not transmit through the network. This again is due to the small share of securities in the liabilities of the insurances as well as therefore also the small share in other sectors' asset portfolios. Figure C.7 offers an alternative representation of the same results comparing how sectoral shocks affect the returns on financial sectors' securities in 2008Q1 and 2014Q1.

---

<sup>5</sup>Recall that because the residency principle applies to our data, the rest of the world shock also includes shocks to foreign affiliates.

## 4.5 Conclusion

In this paper, we traced the evolution of domestic and external sectoral portfolios using a unique security-level database on positions, valuation changes and flows of all domestic and foreign portfolios of French sectors between 2008Q1 and 2014Q1. We generate stylized facts on their contributions to the dynamics of the French net external portfolio position and on bilateral linkages between sectors through security portfolios. The model developed in Section 4.4 shows how balance-sheet contagion can be structurally derived from a balance-sheet equilibrium where sectoral asset demand and supply for securities are functions of endogenously determined returns on domestic securities. The model is estimated and used to measure the degree and evolution of balance-sheet contagion between French institutional sectors. With a similar access to detailed whom-to-whom databases on holdings and issuance of securities, the methodology put forward in this paper could be easily applied to measure contagion through security inter-linkages across countries.

# Bibliography

- Acemoglu, D., Carvalho, V. M., Ozdaglar, A. & Tahbaz-Salehi, A. (2012), ‘The Network Origins of Aggregate Fluctuations’, *Econometrica* **80(5)**, 1977–2016.
- Acemoglu, D., Ozdaglar, A. & Tahbaz-Salehi, A. (2015), ‘Systemic risk and stability in financial networks’, *American Economic Review* **105**.
- Adrian, T. & Shin, H. (2010), ‘Liquidity and leverage’, *Journal of Financial Intermediation* .
- Ait Sahalia, Y. & Lo, A. W. (2000), ‘Nonparametric risk management and implied risk aversion’, *Journal of Econometrics* **94**.
- Alfaro, L., Garcia-Santana, M. & Moral-Benito, E. (2018), ‘On the Direct and Indirect Real Effects of Credit Supply Shocks’, *Harvard Business School Working Paper, No. 18-052* .
- Allen, F. & Gale, D. (2000), ‘Financial Contagion’, *Journal of Political Economy* .
- Amiti, M. & Weinstein, E. D. (2018), ‘How Much Do Idiosyncratic Bank Shocks Affect Investment? Evidence from Matched Bank-Firm Loan Data’, *Journal of Political* .
- Andrade, P., Breckenfelder, J., De Fiore, F., Karadi, P. & Tristani, O. (2016), ‘The ECB’s Asset Purchase Programme: An Early Assessment’, *Mimeo* .
- Ang, A. (2014), ‘Asset management: A systematic approach to factor investing’, *OUP Catalogue, Oxford University Press* .

- Atkeson, A. G., Eisfeldt, A. L. & Weill, P.-O. (2017), 'Measuring the financial soundness of us firms, 1926–2012', *Research in Economics* **71**(3), 613–635.
- Banerjee, T. & Feinstein, Z. (2019), 'Price Mediated Contagion through Capital Ratio Requirements', *Cornell University Working Paper*, *arXiv:1910.12130v1* .
- Bernanke, B., Boivin, J. & Elias, P. (2005), 'Measuring the effects of monetary policy: a factor-augmented vector autoregressive (favar) approach', *Quarterly Journal of Economics* **120**(1).
- Berrada, T. (2006), 'Incomplete information, heterogeneity and asset pricing', *Journal of Financial Econometrics* **4**.
- Berry, S., Levinsohn, J. & Pakes, A. (1995), 'Automobile Prices in Market Equilibrium', *Econometrica* .
- Bigio, S. & La'O, J. (2019), 'Distortions in Production Networks', *NBER Working Paper No. 22212* .
- Bonacich, P. (1987), 'Power and Centrality: A Family of Measures', *American Journal of Sociology* **92**(5).
- Castrén, O. & Kavonius, I. (2009), 'Balance Sheet Interlinkages and Macro-Financial Risk Analysis in the Euro-Area', *European Central Bank, Working Paper Series N. 1124* .
- Cheeger, J. (1969), 'A lower bound for the smallest eigenvalue of the laplacian', *Proceedings of the Princeton conference in honor of Professor S. Bochner* .
- Chernenko, S. & Sunderam, A. (2014), 'Frictions in Shadow Banking: Evidence from the Lending Behavior of Money Market Mutual Funds', *Review of Financial Studies* **27**(6).
- Chodorow-Reich, G., Ghent, A. C. & Haddad, V. (2016), 'Asset Insulators', *2nd Annual Financial Institutions, Regulation and Corporate Governance Conference* .

- Chortareas, G., Girardone, C. & Ventouri, A. (2011), 'Financial Frictions, Bank Efficiency and Risk: Evidence from the Eurozone', *Journal of Business Finance and Accounting* **28(1-2)**.
- Cochrane, J. (2009), 'Asset Pricing', *Princeton University Press* .
- Coimbra, N. & Rey, H. (2017), 'Financial Cycles with Heterogeneous Intermediaries', *NBER Working Paper No. 23245* .
- Debreu, e. a. (1970), 'Economies with a finite set of equilibria', *Econometrica* **38(3)**.
- Dell'Arricia, G., Laeven, L. & Suarez, G. (2017), 'Bank Leverage and Monetary Policy's Risk Taking Channel: Evidence from the United States', *Journal of Finance* **72**.
- di Giovanni, J., Levchenko, A. A. & Mejean, I. (2014), 'Firms, Destinations, and Aggregate Fluctuations', *Econometrica* **82(4)**, 1303–1340.
- Eisenberg, L. & Noe, T. H. (2001a), 'Systemic risk in financial systems', *Management Science* **47**.
- Eisenberg, L. & Noe, T. H. (2001b), 'Systemic Risk in Financial Systems', *Management Science* **47(2)**, 236–249.
- Elliott, M., Golub, B. & Jackson, M. O. (2014), 'Financial Networks and Contagion', *American Economic Review* **104(10)**, 3115–3153.
- Engle, R. & Kelly, B. (2012), 'Dynamic Equicorrelation', *Journal of Business and Economic Statistics* **30(2)**.
- Fan, J., Ke, Y. & Liao, Y. (2016), 'Augmented factor models with applications to validating market risk factors and forecasting bond risk premia', *Available at SSRN 2753404* .
- Farboodi, M. (2014), 'Intermediation and Voluntary Exposure to Counterparty Risk', *Available at SSRN: <https://ssrn.com/abstract=2535900>* .
- Flannery, M. J. & James, C. M. (1984), 'The effect of interest rate changes on the common stock returns of financial institutions', *The Journal of Finance* **39(4)**.



- Gabaix, X. (2011), 'The Granular Origins of Aggregate Fluctuations', *Econometrica* **79(3)**, 733–772.
- Gabrales, A., Gottardi, P. & Vega-Redondo, F. (2017), 'Risk sharing and contagion in networks', *The Review of Financial Studies* **30**.
- Galstyan, V. & Velic, A. (2018), 'International Investment Patterns: The Case of German Sectors', *Open Economies Review* .
- Gandhi, A. & Serrano-Padial, R. (2015), 'Does belief heterogeneity explain asset prices: The case of the longshot bias', *Review of Economic Studies* **82**.
- Gennaioli, N. & Shleifer, A. (2018), 'A crisis of beliefs: Investor psychology and financial fragility', *Princeton University Press* .
- Grassi, B. (2018), 'IO in I-O: Size, Industrial Organization, and the Input-Output Network Make a Firm Structurally Important'.
- Grassi, B. & Carvalho, V. M. (2019), 'Large Firm Dynamics and the Business Cycle', *American Economic Review* .
- Greenwood, R., Landier, A. & Thesmar, D. (2015), 'Vulnerable Banks', *Journal of Financial Economics* **115(3)**, 471–485.
- Halpern, L., Koren, M. & Szeidl, A. (2015), 'Imported Inputs and Productivity', *American Economic Review* .
- Harvey (2010), 'Tracking a Changing Copula', *Journal of Empirical Finance* **17(3)**.
- Harvey, A. & Thiele, S. (2016), 'Testing Against Changing Correlation', *Journal of Empirical Finance* **38**.
- Heipertz, J., Ouazad, A. & Ranci ere, R. (2019), 'The Transmission of Shocks in Endogenous Financial Networks: A Structural Approach', *NBER Working Paper No. 26049* .

- Imbs, J. & Méjean, I. (2015), 'Elasticity Optimism', *American Economic Journal: Macroeconomics* .
- Jayarathne, J. & Morgan, D. (2000), 'Capital Market Frictions and Deposit Constraints at Banks', *Journal of Money, Credit and Banking* **32(1)**.
- Jiménez, G., Mian, A., Peydró, J.-L. & Saurina, J. (2019), 'The Real Effects of the Bank Lending Channel', *Journal of Monetary Economics* .
- Kiyotaki, N. & Moore, J. (2002), 'Balance-Sheet Contagion', *The American Economic Review* .
- Koijen, R. S., Koulischer, F., Nguyen, B. & Yogo, M. (2018), 'Inspecting the Mechanism of Quantitative Easing in the Euro Area', *Banque de France Working Paper No. 601* .
- Koijen, R. S. & Yogo, M. (2019), 'A Demand System Approach to Asset Pricing', *Journal of Political Economy* .
- Kraay, A. & Ventura, J. (2000), 'Current Accounts in Debtor and Creditor Countries', *Quarterly Journal of Economics* **115(4)**.
- Krugman, P. (1991), 'History versus expectations', *The Quarterly Journal of Economics* **106(2)**.
- Landier, A., Sraer, D. & Thesmar, D. (2013), 'Banks exposure to interest rate risk and the transmission of monetary policy', *NBER Working Paper No. 18857* .
- Lintner, J. (1969), 'The aggregation of investor's diverse judgments and preferences in purely competitive security markets', *Journal of Financial and Quantitative Analysis* **4**.
- Lopez-Garcia, P., di Mauro, F. & Altomonte, C. (2014), 'Micro-Based Evidence of EU competitiveness, The COMPNET Database', *Working Paper Series No 1634* .
- Mas-Colell, A., Whinston, M. D., Green, J. R. et al. (1995a), *Microeconomic Theory*, Vol. 1, Oxford University Press New York.

- Mas-Colell, A., Whinston, M. D., Green, J. R. et al. (1995b), *Microeconomic theory*, Vol. 1, Oxford university press New York.
- Merton, R. C. (1987), 'A simple model of capital market equilibrium with incomplete information', *The journal of finance* **42**(3), 483–510.
- Milgrom, P. & Stokey, N. (1982), 'Information, trade and common knowledge', *Journal of Economic Theory* **26**.
- Miranda-Agrippino, S. & Rey, H. (2015), 'World asset markets and the global financial cycle', *NBER Working Paper No. 21722* .
- Osootimehin, S. (2019), 'Aggregate Productivity and the Allocation of Resources over the Business cycle', *Review of Economic Dynamics* .
- Rajan, R. G. & Zingales, L. (1998), 'Financial dependence and growth', *American Economic Review* .
- Ross, S. (2015), 'The Recovery Theorem', *Journal of Finance* .
- Sard, A. (1942), 'The measure of the critical values of differentiable maps', *Bulletin of the American Mathematical Society* **48**(12).
- Shin, H. (2008), 'Risk and Liquidity in a System Context', *Journal of Financial Intermediation* .
- Shleifer, A. & Summers, L. H. (1990), 'The noise trader approach to finance', *Journal of Economic Perspectives* **4**(2).
- Spielman, D. (2007), 'Spectral graph theory and its applications', *Foundations of Computer Science, 48th Annual IEEE Symposium* .
- Zhou, Q., Tan, K. J. K., Faff, R. & Zhu, Y. (2016), 'Deviation from Target Capital Structure, Cost of Equity and Speed of Adjustment', *Journal of Corporate Finance* .

# **Appendix A**

## **Appendix of Chapter 2**

## A.1 Figures and Tables

Table A.1: Firms' Size and Bond Financing by Sector

This table provides descriptive statistics on firm size and bond financing by sector. Columns (2 – 4) and columns (5 – 7) contain the average number of firms per year, the average of sales (measured as total operating revenues), and the average of total assets for all firms in the sample and the subset of bond-issuing firms, respectively. The importance of bond financing is measured by the share of bonds in total liabilities (Columns 8 and 9) in 2011 and 2015. The importance of French banks as external fund providers through bonds is measured by the share of banks' corporate bond holdings in the total value outstanding (Columns 10 and 11). Firm-level data stems from FIBEN; bank-level data from PROTIDE.

|       | All Firms |              |               | Bond-Issuing Firms |              |               |                       |      |                     |      |
|-------|-----------|--------------|---------------|--------------------|--------------|---------------|-----------------------|------|---------------------|------|
|       | <i>N</i>  | <i>Sales</i> | <i>Assets</i> | <i>N</i>           | <i>Sales</i> | <i>Assets</i> | <i>Bonds / Assets</i> |      | <i>Inv. / Bonds</i> |      |
|       |           |              |               |                    |              |               | 2011                  | 2015 | 2011                | 2015 |
| A     | 3,504     | 3.9          | 10.0          | 21                 | 51.5         | 553.3         | 0.1                   | 0.1  | 0.0                 | 0.0  |
| B     | 800       | 30.3         | 60.7          | 4                  | 10.3         | 21.9          | 8.2                   | 7.4  | 0.4                 | 0.4  |
| C     | 34,328    | 25.0         | 29.7          | 318                | 85.8         | 322.9         | 6.3                   | 7.1  | 3.6                 | 19.5 |
| D     | 331       | 383.8        | 1654.0        | 10                 | 9073.0       | 37,603.8      | 9.7                   | 12.9 | 3.5                 | 3.2  |
| E     | 1,743     | 18.2         | 37.8          | 11                 | 9.9          | 21.1          | 2.9                   | 5.7  | 0.0                 | 0.2  |
| F     | 33,117    | 5.3          | 5.0           | 61                 | 17.1         | 32.5          | 10.7                  | 8.2  | 0.0                 | 8.6  |
| G     | 60,333    | 6.6          | 4.6           | 293                | 29.1         | 25.0          | 9.8                   | 3.9  | 7.6                 | 3.3  |
| H     | 10,684    | 17.9         | 39.6          | 43                 | 1,948.2      | 6,507.6       | 27.2                  | 27.9 | 2.8                 | 2.3  |
| I     | 12,460    | 3.3          | 4.7           | 80                 | 23.2         | 120.1         | 18.1                  | 16.6 | 19.4                | 7.1  |
| J     | 6,757     | 22.6         | 56.7          | 128                | 307.3        | 1,549.3       | 17.1                  | 17.5 | 5.2                 | 4.3  |
| M     | 15,071    | 9.2          | 63.6          | 244                | 72.5         | 2,154.0       | 17.0                  | 20.2 | 4.9                 | 4.5  |
| N     | 9,798     | 8.6          | 10.0          | 42                 | 21.9         | 71.4          | 9.4                   | 27.5 | 41.0                | 0.2  |
| P     | 1,502     | 4.5          | 8.1           | 11                 | 47.6         | 205.2         | 9.3                   | 5.5  | 59.1                | 0.0  |
| Q     | 5,849     | 7.8          | 10.6          | 78                 | 23.9         | 63.4          | 19.4                  | 21.7 | 23.8                | 35.9 |
| R     | 3,256     | 11.8         | 9.3           | 16                 | 15.6         | 25.8          | 12.7                  | 14.8 | 0.0                 | 0.0  |
| Total | 199,538   | 11.6         | 20.8          | 1,359              | 197.4        | 1,098.8       | 16.3                  | 18.5 | 4.3                 | 4.1  |

Note: ISIC rev. 4 classification: A. Agriculture, forestry and fishing; B. Mining and quarrying; C. Manufacturing; D. Electricity, gas, steam and air conditioning supply; E. Water supply; sewerage, waste mngmt and remediation activities; F. Construction; G. Wholesale and retail trade; repair of motor vehicles and motorcycles; H. Transportation and storage; I. Accommodation and food service activities; J. Information and communication; M. Professional, scientific and technical activities; N. Administrative and support service activities; P. Education; Q. Human health and social work activities; R. Arts, entertainment and recreation; S. Other service activities.

†: in million EUR

‡: in %

Table A.2: Banks' Size and Corporate Debt Holdings

This table provides descriptive statistics on the aggregate banking sector (first panel), followed by bank-level statistics (second panel). Total banking sector assets as measured by Banque de France's Webstats are compared to total assets as aggregated from the bank-level data used in the paper. Bank-level descriptive statistics include the number of banks in the sample, the average and quantiles of the bank size distribution, banks' corporate debt securities investments as share of their total assets, and banks' capital ratio. Bank-level data stems from the merged dataset of *M\_SITMENS* and *PROTIDE*.

|               |                                      |                   | 2011    | 2012    | 2013    | 2014    | 2015    |
|---------------|--------------------------------------|-------------------|---------|---------|---------|---------|---------|
| <i>Sector</i> | <i>Total Assets*</i>                 | <i>Webstats</i>   | 8925.30 | 8810.99 | 8658.43 | 9196.18 | 9317.21 |
|               |                                      | <i>Aggregated</i> | 7036.96 | 6692.05 | 6627.49 | 7033.44 | 7150.56 |
|               | <i>Corp. Debt (%)</i>                | <i>Webstats</i>   | 0.33    | 0.36    | 0.43    | 0.35    | 0.41    |
| <i>Banks</i>  | <i>N</i>                             |                   | 155     | 159     | 151     | 149     | 174     |
|               | <i>Total Assets*</i>                 | <i>Mean</i>       | 45.40   | 42.09   | 43.89   | 47.20   | 41.10   |
|               |                                      | <i>p10</i>        | 0.37    | 0.41    | 0.70    | 0.78    | 0.23    |
|               |                                      | <i>p50</i>        | 9.39    | 9.35    | 10.43   | 10.53   | 8.77    |
|               |                                      | <i>p90</i>        | 96.71   | 71.00   | 80.65   | 91.89   | 61.04   |
|               |                                      | <i>p95</i>        | 211.27  | 206.35  | 220.32  | 240.60  | 184.11  |
|               |                                      | <i>p99</i>        | 828.60  | 728.24  | 737.71  | 824.96  | 678.11  |
|               | <i>Corp. Debt (%)<sup>‡</sup></i>    | <i>Mean</i>       | 0.24    | 0.24    | 0.33    | 0.31    | 0.50    |
|               |                                      | <i>p50</i>        | 0.02    | 0.01    | 0.03    | 0.02    | 0.00    |
|               |                                      | <i>p90</i>        | 0.46    | 0.44    | 0.68    | 0.69    | 0.69    |
|               |                                      | <i>p95</i>        | 0.80    | 0.55    | 1.15    | 0.85    | 1.48    |
|               | <i>Capital Ratio (%)<sup>†</sup></i> | <i>Mean</i>       | 7.14    | 6.49    | 5.88    | 5.68    | 6.88    |
|               |                                      | <i>p10</i>        | 3.19    | 3.10    | 2.86    | 2.84    | 2.70    |
|               |                                      | <i>p50</i>        | 7.12    | 6.50    | 5.83    | 5.65    | 6.87    |
|               |                                      | <i>p90</i>        | 8.01    | 7.75    | 7.34    | 7.54    | 7.72    |
|               |                                      | <i>p95</i>        | 10.44   | 9.93    | 10.21   | 9.21    | 11.05   |

\*: in billion EUR

†: the capital ratio is calculated as the ratio of equity book value (the sum of *itemS05\_0390*, *itemS05\_0400*, *itemS05\_0530*, and *itemS05\_0620* in *M\_SITMENS*) over total asset value (*itemS0H\_0010*).

‡: the lower quantiles of the corporate debt share distribution is suppressed because it is equal to zero.

Table A.3: Estimated Total Factor Productivity and Elasticities

*This table presents the summary statistics aggregated at the ISIC group-level of estimated production factor-elasticities, substitution elasticities, and firm-level log total factor productivity shocks over the 2011-2015 period.*

|   | Elasticities |           |              |            | log Z(k, i) |      |       | log Z <sub>k</sub> * |      |
|---|--------------|-----------|--------------|------------|-------------|------|-------|----------------------|------|
|   | $\alpha_k$   | $\beta_k$ | $\epsilon_k$ | $\gamma_k$ | Mean        | SD   | p50   | Mean                 | SD   |
| A | 0.58         | 0.16      | 19.17        | 0.02       | 0.02        | 0.12 | 0.01  | 0.58                 | 0.01 |
| B | 0.74         | 0.14      | 7.64         | 0.00       | 0.06        | 0.11 | 0.06  | 0.92                 | 0.02 |
| C | 0.67         | 0.14      | 25.15        | 0.15       | 0.09        | 0.11 | 0.10  | 0.94                 | 0.01 |
| D | 0.74         | 0.14      | 11.81        | 0.03       | 0.09        | 0.12 | 0.09  | 0.56                 | 0.02 |
| E | 0.65         | 0.15      | 14.43        | 0.01       | 0.07        | 0.12 | 0.08  | 0.55                 | 0.01 |
| F | 0.69         | 0.06      | 19.82        | 0.01       | 0.13        | 0.10 | 0.15  | 0.65                 | 0.00 |
| G | 0.14         | 0.77      | 40.96        | 0.20       | 0.07        | 0.13 | 0.08  | 0.55                 | 0.00 |
| H | 0.62         | 0.08      | 26.29        | 0.04       | 0.10        | 0.12 | 0.12  | 1.08                 | 0.08 |
| I | 0.52         | 0.14      | 15.26        | 0.08       | 0.00        | 0.13 | -0.01 | 0.67                 | 0.01 |
| J | 0.53         | 0.19      | 15.03        | 0.04       | 0.09        | 0.13 | 0.11  | 0.63                 | 0.02 |
| M | 0.57         | 0.13      | 14.66        | 0.02       | 0.07        | 0.14 | 0.09  | 1.27                 | 0.01 |
| N | 0.42         | 0.16      | 20.37        | 0.01       | 0.07        | 0.13 | 0.08  | 0.53                 | 0.00 |
| P | 0.48         | 0.04      | 17.76        | 0.12       | 0.04        | 0.13 | 0.05  | 0.44                 | 0.01 |
| Q | 0.38         | 0.08      | 12.15        | 0.22       | 0.09        | 0.12 | 0.11  | 0.76                 | 0.01 |
| R | 0.70         | 0.10      | 12.15        | 0.07       | 0.11        | 0.11 | 0.13  | 0.72                 | 0.01 |

Note: ISIC rev. 4 classification: A. Agriculture, forestry and fishing; B. Mining and quarrying; C. Manufacturing; D. Electricity, gas, steam and air conditioning supply; E. Water supply; sewerage, waste mngmt and remediation activities; F. Construction; G. Wholesale and retail trade; repair of motor vehicles and motorcycles; H. Transportation and storage; I. Accommodation and food service activities; J. Information and communication; M. Professional, scientific and technical activities; N. Administrative and support service activities; P. Education; Q. Human health and social work activities; R. Arts, entertainment and recreation; S. Other service activities.

\*Sector-level productivity is:  $Z_k = \left( \sum_{f=1}^{N_k} Z(k, f)^{\epsilon_k - 1} \right)^{\frac{1}{\epsilon_k - 1}}$ .

Figure A.1: Validation of Productivity Shock Estimation

This figure shows how estimated firm-level productivity shocks relate to stylized facts and existing productivity estimates. Subfigure A.1a show the distribution of estimated firm-level productivity for the total French economy and the manufacturing sector across the 2011-2015 period. Productivity shock are standardized by their yearly mean and standard deviation. Subfigure A.1b compares the evolution of estimated aggregate productivity given by  $Z = \prod_s^N Z_s^{\gamma_s}$  with the EU Klems estimate of aggregate French productivity.

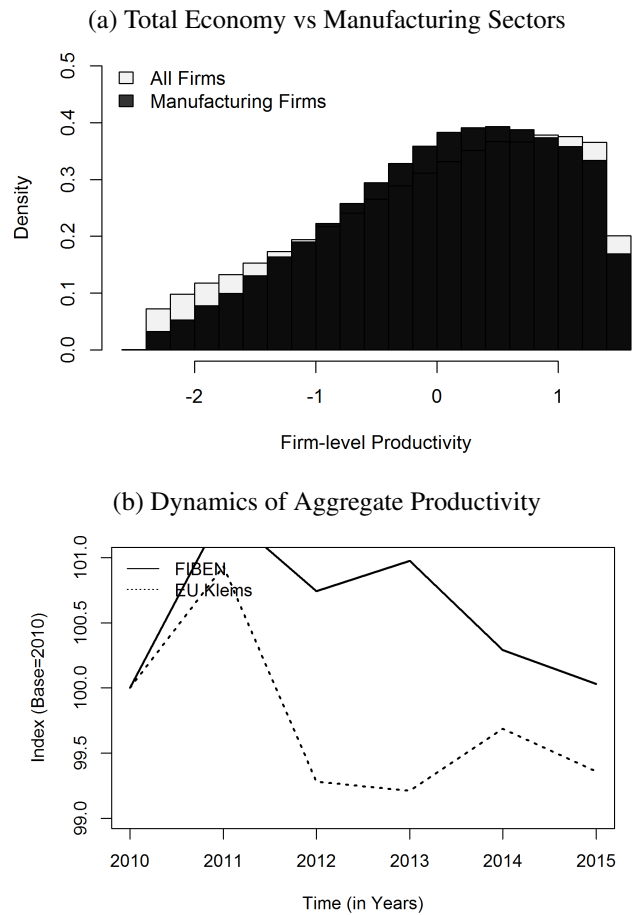




Table A.4: Equilibrium Influence Decomposition: The Financial Multiplier

This table shows the decomposition of equilibrium influence following Equation 2.12 into the IO component and the financial multiplier for the year 2011. All numbers are multiplied by a factor of 1000 and terms have been winsorized at the 1th and the 99th percentile.

|       | $\mathcal{I}_{ki}$ |                            | $\mathcal{I}_{ki}^R$ |                                   | $\mathcal{I}_{ki}^F$ |                                   |               |                                   |
|-------|--------------------|----------------------------|----------------------|-----------------------------------|----------------------|-----------------------------------|---------------|-----------------------------------|
|       | Mean               | $\sigma(\mathcal{I}_{ki})$ | Mean                 | $\sigma/\sigma(\mathcal{I}_{ki})$ | Composition*         |                                   | Asset Prices† |                                   |
|       | Mean               | $\sigma(\mathcal{I}_{ki})$ | Mean                 | $\sigma/\sigma(\mathcal{I}_{ki})$ | Mean                 | $\sigma/\sigma(\mathcal{I}_{ki})$ | Mean          | $\sigma/\sigma(\mathcal{I}_{ki})$ |
| A     | 0.065              | 0.131                      | 0.059                | 0.78                              | -0.023               | 0.35                              | 0.029         | 0.52                              |
| B     | 0.017              | 0.013                      | 0.016                | 0.88                              | -0.005               | 0.44                              | 0.006         | 0.68                              |
| C     | 0.032              | 0.058                      | 0.041                | 1.29                              | -0.012               | 0.42                              | 0.010         | 0.40                              |
| D     | 0.032              | 0.021                      | 0.023                | 0.54                              | -0.010               | 0.54                              | 0.019         | 1.07                              |
| E     | 0.019              | 0.023                      | 0.021                | 0.93                              | -0.004               | 0.50                              | 0.003         | 0.14                              |
| F     | 0.006              | 0.005                      | 0.005                | 0.72                              | -0.001               | 0.28                              | 0.002         | 0.71                              |
| G     | 0.002              | 0.004                      | 0.003                | 1.01                              | -0.002               | 0.72                              | 0.001         | 0.35                              |
| H     | 0.144              | 0.113                      | 0.143                | 0.98                              | -0.031               | 0.50                              | 0.032         | 0.47                              |
| I     | 0.000              | 0.000                      | 0.000                | 1.12                              | 0.000                | 0.40                              | 0.000         | 0.40                              |
| J     | 0.051              | 0.053                      | 0.059                | 1.01                              | -0.031               | 0.69                              | 0.024         | 0.55                              |
| M     | 0.037              | 0.074                      | 0.031                | 0.99                              | -0.006               | 0.23                              | 0.022         | 0.66                              |
| N     | 0.011              | 0.016                      | 0.011                | 0.87                              | -0.004               | 0.53                              | 0.004         | 0.43                              |
| P     | 0.059              | 0.091                      | 0.028                | 0.29                              | -0.006               | 0.12                              | 0.037         | 0.96                              |
| Q     | 0.007              | 0.015                      | 0.003                | 0.30                              | -0.001               | 0.15                              | 0.004         | 0.96                              |
| R     | 0.195              | 0.145                      | 0.154                | 0.72                              | -0.040               | 0.36                              | 0.074         | 0.66                              |
| Total | 0.027              | 0.063                      | 0.028                | 1.01                              | -0.009               | 0.37                              | 0.011         | 0.53                              |

Note: ISIC rev. 4 classification: A. Agriculture, forestry and fishing; B. Mining and quarrying; C. Manufacturing; D. Electricity, gas, steam and air conditioning supply; E. Water supply; sewerage, waste mngmt and remediation activities; F. Construction; G. Wholesale and retail trade; repair of motor vehicles and motorcycles; H. Transportation and storage; I. Accommodation and food service activities; J. Information and communication; M. Professional, scientific and technical activities; N. Administrative and support service activities; P. Education; Q. Human health and social work activities; R. Arts, entertainment and recreation; S. Other service activities.

\*: The composition effect corresponds to  $-\gamma_k' (I - \mathcal{W})^{-1} \mathcal{E}_{Z_{ki}}^k$ .

†: The asset price effect corresponds to  $-\gamma_k' (I - \mathcal{W})^{-1} \mathcal{E}_{Q^r}^k (I - \mathcal{F})^{-1} (\partial \Delta_{\log Q^r}^r)^{-1} \Delta_{\log Z_{ki}}^r$ .

Figure A.2: Total Influence and its Components

This figure provides the relation between total influence with its input-output component and the financial multiplier, respectively. Results refer to 2011 equilibrium.

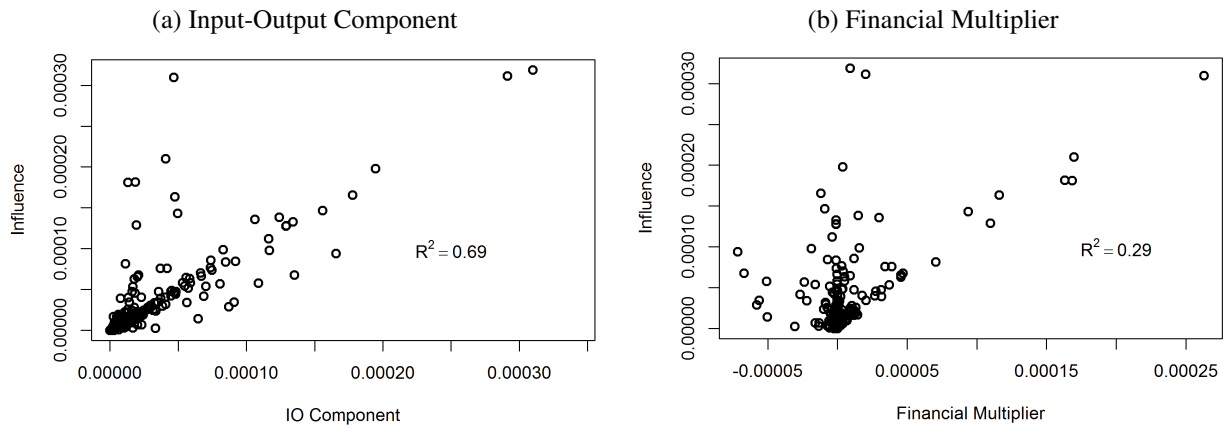


Figure A.3: Average Influence by Sector: IO versus Financial Network Multiplier

This figure plots the average influence of a sector through the asset price component of the financial network multiplier (y – axis) against its influence through the IO network.

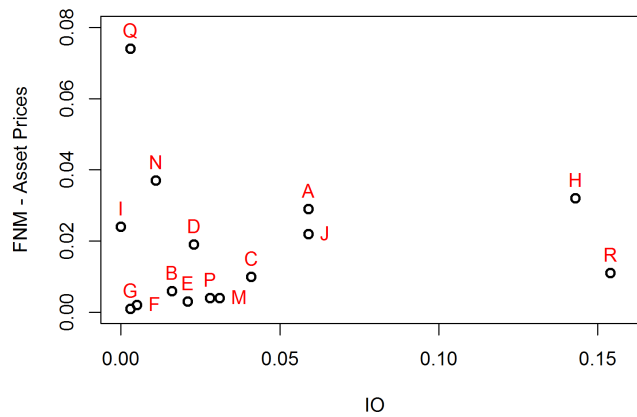


Table A.5: Who is Influent and through which Channel?

This table shows results from an OLS regression of the different components of influence on (i) the equilibrium debt price, (ii) productivity, and (iii) the financing share. Results are for the year 2011.

|                                 | Dependent variable:         |                             |                      |                                  |
|---------------------------------|-----------------------------|-----------------------------|----------------------|----------------------------------|
|                                 | $\mathcal{I}_{ki}^R$<br>(1) | $\mathcal{I}_{ki}^F$<br>(2) | Composition*<br>(3)  | Asset Prices <sup>†</sup><br>(4) |
| Equilibrium Price $\log Q_{ki}$ | -0.049*<br>(0.029)          | -0.170***<br>(0.031)        | 0.150***<br>(0.029)  | -0.230***<br>(0.029)             |
| Productivity, $\log Z_{ki}$     | 0.320***<br>(0.028)         | -0.019<br>(0.030)           | -0.260***<br>(0.028) | 0.190***<br>(0.028)              |
| Financing Share, $S_{ki}$       | 0.006**<br>(0.002)          | 0.006**<br>(0.003)          | -0.017***<br>(0.002) | 0.017***<br>(0.002)              |
| Constant                        | -0.034<br>(0.029)           | -0.038<br>(0.031)           | 0.110***<br>(0.029)  | -0.100***<br>(0.029)             |
| N                               | 1,394                       | 1,364                       | 1,386                | 1,386                            |
| Adjusted R <sup>2</sup>         | 0.110                       | 0.022                       | 0.110                | 0.110                            |
| F-Stat                          | 57.000***                   | 11.000***                   | 61.000***            | 59.000***                        |

Significance: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

\*: The composition effect corresponds to  $-\gamma_k' (I - \mathcal{W})^{-1} \mathcal{E}_{Z_{ki}}^k$ .

†: The asset price effect corresponds to  $-\gamma_k' (I - \mathcal{W})^{-1} \mathcal{E}_{Q^r}^k (I - \mathcal{F})^{-1} \left( \partial \Delta_{\log Q^r}^r \right)^{-1} \Delta_{\log Z_{ki}}^r$ .

## A.2 Proofs

### A.2.1 Lemma 2.1: Firms' Marginal Costs

Each firm minimizes costs to produce a given level of output. Firm  $(k, i)$  additionally faces a cash-in-advance constraint in form of a requirement to have sufficient deposits  $D(k, i)$  available when production takes place to cover a fraction  $S(k, i)$  of capital expenditures. Deposits can only be bought at the beginning of the period and the firm issues bonds to do so. Bonds have to be paid back at the end of the period. With cash-flows as illustrated in Figure 2.2,  $(k, i)$ 's cost minimization problem is

$$\begin{aligned} \text{Cost}(k, i) = \min \quad & P^X X(k, i) + P^K \cdot K(k, i) + hL(k, i) - D(k, i) + B(k, i) \\ \text{s.t.} \quad & Y(k, i) \geq \overline{Y(k, i)}, \\ & S(k, i) \cdot P^K \cdot K(k, i) \leq D(k, i), \\ & Q^D D(k, i) \leq Q(k, i)B(k, i). \end{aligned}$$

First order conditions yield that input expenditures are:

$$\begin{aligned} X(k, i) &= \alpha_k \cdot \frac{\text{Cost}(k, i)}{P^X} \\ K(k, i) &= \beta_k \cdot \frac{\text{Cost}(k, i)}{\chi(k, i)^{1/\beta_k} P^K} \\ L(k, i) &= (1 - \alpha_k - \beta_k) \cdot \frac{\text{Cost}(k, i)}{h} \end{aligned}$$

where the capital surcharge is defined by  $\chi(k, i) \equiv \left(1 - S(k, i) \left(1 - \frac{Q^D}{Q(k, i)}\right)\right)^{\beta_k}$ . Minimized costs are  $\text{Cost}(k, i) = Y(k, i)\Lambda(k, i)$ , where  $\Lambda(k, i)$  is the Lagrange multiplier of the output constraint  $Y(k, i) \geq \overline{Y(k, i)}$ . Thus, marginal costs are:

$$\Lambda(k, i) \equiv Z(k, i)^{-1} \chi(k, i)^{\beta_k} (P^X)^{\alpha_k} (P^K)^{\beta_k} h^{1-\alpha_k-\beta_k}.$$

Plugging in the definitions of the intermediate input and capital bundle price index yields Equation 2.2 in the text.

### A.2.2 Lemma 2.2: Within-Sector Sales Distribution

Under monopolistic competition firms face downward-sloping demand for their variety. Demand comes from households and other firms of the economy. Households maximization of aggregate consumption yields demand for  $(k, i)$ 's variety equal to  $C(k, i) = \left(\frac{P(k, i)}{P_k}\right)^{-\varepsilon_k} C_k$ . Other firms demand the variety as intermediate input  $X(s, f, k, i) =$

$\left(\frac{P(k,i)}{P_k}\right)^{-\varepsilon_k} X(s, f, k)$  and capital  $K(s, f, k, i) = \left(\frac{P(k,i)}{P_k}\right)^{-\varepsilon_k} K(s, f, k)$  for production. Then:

$$Y^d(k, i) = \left(\frac{P(k, i)}{P_k}\right)^{-\varepsilon_k} (C_k + X(s, f, k) + K(s, f, k))$$

is aggregate demand for  $(k, i)$ . Market clearing in the sector good market gives  $Y_k = C_k + X(s, f, k) + K(s, f, k)$ . Subject demand, the firm sets the price to solve  $\max_{P(k,i)} P(k,i)Y(k,i) - \Lambda(k,i)Y(k,i)$ . This yields the standard constant mark-up pricing rule:

$$P(k, i) = \mathcal{M}_k \Lambda(k, i).$$

The markup is defined as  $\mathcal{M}_k \equiv \frac{\varepsilon_k}{\varepsilon_k - 1}$ . From firm-level prices, the price of the sector good follows  $P_k = \mathcal{M}_k \left(\sum_{f=1}^{N_k} \Lambda(k, f)^{1-\varepsilon_k}\right)^{\frac{1}{1-\varepsilon_k}}$ .

This defines sectoral marginal costs  $\Lambda_k = \left(\sum_{f=1}^{N_k} \Lambda(k, f)^{1-\varepsilon_k}\right)^{\frac{1}{1-\varepsilon_k}}$ . To solve for  $(k, i)$  sales in terms of sectoral sales, I plug the variety price and the sectoral price into demand. This yields

$$P(k, i)Y(k, i) = \frac{(Z(k, i)\chi(k, i)^{-1})^{\varepsilon_k - 1}}{\sum_{f=1}^{N_k} (Z(k, f)\chi(k, f)^{-1})^{\varepsilon_k - 1}} P_k Y_k.$$

To obtain  $\zeta(k, i)$  in the Lemma, I find sector  $k$ 's productivity as  $Z_k = \left(\sum_{f=1}^{N_k} Z(k, f)^{\varepsilon_k - 1}\right)^{\frac{1}{\varepsilon_k - 1}}$  and sectoral capital surcharge by expanding the ratio with  $Z_k^{\varepsilon_k - 1}$  such that:

$$\zeta(k, i) = \left(\frac{Z(k, i)}{Z_k}\right)^{\varepsilon_k - 1} \frac{(\chi(k, i)^{-1})^{\varepsilon_k - 1}}{\sum_{f=1}^{N_k} \left(\frac{Z(k, f)}{Z_k} \chi(k, f)^{-1}\right)^{\varepsilon_k - 1}} = \left(\frac{Z(k, i)}{Z_k} / \frac{\chi(k, i)}{\chi_k}\right)^{\varepsilon_k - 1}.$$

The inverse of the sectoral capital surcharge is defined as  $\chi_k^{-1} \equiv \left(\sum_{f=1}^{N_k} \left(\frac{Z(k, f)}{Z_k} \chi(k, f)^{-1}\right)^{\varepsilon_k - 1}\right)^{\frac{1}{\varepsilon_k - 1}}$ , a productivity-share weighted power mean of order  $\varepsilon_k - 1$ .

### A.2.3 Lemma 2.3: Firms' Debt Supply

With the first order condition on capital demand from the cost minimization, we have  $P^K K(k, i) = \beta_k \cdot \frac{\Lambda(k, i)Y(k, i)}{\chi(k, i)^{1/\beta_k}}$ . Using the pricing rule, capital expenditure is a fraction of sales, and with the result from Lemma 2.2, a fraction of sectoral sales

$$P^K K(k, i) = \beta_k \cdot \chi(k, i)^{-1/\beta_k} \mathcal{M}_k^{-1} P(k, i)Y(k, i) = \beta_k \cdot \chi(k, i)^{-1/\beta_k} \mathcal{M}_k^{-1} \zeta(k, i) P_k Y_k.$$

Using the cash-in-advance financing constraint  $D(k, i) = S(k, i)P^k K(k, i)$  and the budget constraint at the beginning of the period  $Q^D D(k, i) = Q(k, i)B(k, i)$  debt supply as a function of financial asset prices and sectoral sales is:

$$B(k, i) = \beta_k \cdot \frac{Q^D}{Q(k, i)} S(k, i) \chi(k, i)^{-1/\beta_k} \mathcal{M}_k^{-1} \zeta(k, i) P_k Y_k.$$

This is equation 2.4 in the text.

## A.2.4 Lemma 2.4: Leverage Targeting

Banks seek to maximize the value of their equity at the end of the period given by  $R^E(b)Q^E \overline{E(b)}$  and to minimize the deviation of their actual leverage  $\tau(b)$  from target leverage  $\tau^*(b)$ . The value of equity at the end of the period is equal to the value of assets at the end of the period minus the value of debt, i.e.  $\omega(b)' \frac{1}{Q} A(b) - \frac{1}{Q^B(b)} Q^B(b) B(b) - \frac{1}{Q^D} Q^D \overline{D(b)}$ . And since total assets value is  $A(b) = Q^D \overline{D(b)} + Q^B(b) B(b) + Q^E(b) \overline{E(b)}$ , the maximization problem is

$$\begin{aligned} & \max \omega(b)' \frac{1}{Q} \left( Q^D \overline{D(b)} + Q^B(b) B(b) + Q^E(b) \overline{E(b)} \right) - B(b) - \overline{D(b)} - \frac{1}{2} c(b) (\tau(b) - \tau(b)^*)^2 \\ \Leftrightarrow & \max \omega(b)' \frac{1}{Q} Q^B(b) B(b) - B(b) - \frac{1}{2} c(b) (\tau(b) - \tau(b)^*)^2. \end{aligned}$$

With leverage  $\tau(b)$  defined as the value of debt over the value of equity at the beginning of the period, the first order condition follows as in the text. Solving for debt  $B(k, i)$  provides banks' endogenous debt supply.

## A.2.5 Proposition 2.1: Aggregate Output

To derive aggregate log output as a function of financial asset prices and shocks, start with the clearing conditions for variety markets:

$$P(k, i)Y(k, i) = P(k, i)C(k, i) + \sum_{s=1}^N \sum_{f=1}^{N_s} P(k, i)X(s, f, k, i) + P(k, i)K(s, f, k, i).$$

With demand functions for intermediate inputs  $X(s, f, k, i)$  and capital  $K(s, f, k, i)$  (see Section A.2.2), demand can be written as a function of aggregate consumption and sectoral sales

$$P(k, i)Y(k, i) = \left( \frac{P(k, i)}{P_k} \right)^{1-\varepsilon_k} \left( \gamma_k P \cdot C + \sum_{s=1}^N (\alpha_k w_{sk}^X + w_k^K \beta_k) \mathcal{M}_s^{-1} P_s Y_s \right).$$

Here I used the fact that  $\sum_{f=1}^{N_s} \zeta(s, f) = 1$ . Summing over all firms of the same sector yields

$$P_k Y_k = \gamma_k P \cdot C + \sum_{s=1}^N (\alpha_k w_{sk}^X + w_k^K \beta_k) M_s^{-1} P_s Y_s.$$

and in vector form sectoral sales can be solved as a function of the Leontieff inverse and consumption:

$$P_k Y_k = (I - \mathcal{W}')^{-1} \gamma_k \cdot P \cdot C$$

Firms' capital surcharges do not impact the distribution of sectoral sales. Aggregation of sectoral sales yields total sales  $PY = \mathbf{1}'(I - \mathcal{W}')^{-1} \gamma_k \cdot P \cdot C$ . Then I use the definition of sectoral prices to solve for the equilibrium wage  $h$ . In particular,

$$P_k = \mathcal{M}_k Z_k^{-1} \chi_k \prod_{s=1}^N P_s^{\alpha_k w_{sk}^X + w_k^K \beta_k} h^{1 - \alpha_k - \beta_k},$$

with sectoral productivity  $Z_k$  and sectoral capital surcharge defined in Section A.2.2. Taking the log of each sectoral price and stacking the result into a matrix equation gives the sectoral prices as a function of productivity, capital surcharge and wage,

$$\log P_k = (I - \mathcal{W}')^{-1} \log(\mathcal{M}_k Z_k^{-1} \chi_k) + \log h \cdot (I - \mathcal{W}')^{-1} (1 - \alpha_k - \beta_k).$$

Premultiplying with the row-vector  $\gamma_k'$  gives the log real wage, since  $\gamma_k' (I - \mathcal{W}')^{-1} (1 - \alpha_k - \beta_k) = 1$ .

$$\log h - \log P = -\gamma_k' (I - \mathcal{W}')^{-1} \log(\mathcal{M}_k Z_k^{-1} \chi_k).$$

Together with aggregate profits  $\pi = (1 - \mathcal{M}_k^{-1})' Y_k P_k$  the household budget constraint implies that consumption expenditure equals the sum of profits and labor income

$$PC = hL + (1 - \mathcal{M}_k^{-1})' Y_k P_k.$$

Normalizing labor supply to  $L = 1$  and using the fact that sectoral shares are a function of consumption expenditures from the market clearing conditions, yields consumption expenditure as a function of the wage.

$$PC = \left(1 - (1 - \mathcal{M}_k^{-1})' (I - \mathcal{W}')^{-1} \gamma_k\right)^{-1} \cdot h.$$

Taking logs yields and using the real wage, I obtain aggregate log consumption:

$$\log C = -\log \left(1 - (1 - \mathcal{M}_k^{-1})' (I - \mathcal{W}')^{-1} \gamma_k\right) - \gamma_k' (I - \mathcal{W}')^{-1} \log(\mathcal{M}_k Z_k^{-1} \chi_k).$$

Finally, aggregate output follows from taking logs of aggregate sales  $Y = \mathbf{1}'(I - \mathcal{W}')^{-1}\gamma_k C$  such that equation 2.1 follows:

$$\log Y = \eta_Y + \gamma_k' (I - \mathcal{W}')^{-1} \log (Z_k \chi_k^{-1}).$$

where  $\eta_Y \equiv \log (\mathbf{1}'(I - \mathcal{W}')^{-1}\gamma_k) - \log (1 - (1 - \mathcal{M}_k^{-1})'(I - \mathcal{W}')^{-1}\gamma_k) - \gamma_k' (I - \mathcal{W}')^{-1} \log (\mathcal{M}_k)$  is a constant. I also define  $\eta_{PY} \equiv \mathbf{1}'(I - \mathcal{W}')^{-1}\gamma_k$  which relates aggregate output to aggregate consumption  $Y = \eta_{PY} C$ .

## A.2.6 Proposition 2.3: Financial Multiplier

The financial multiplier (see Equation 2.12) depends on two elasticities of the sectoral capital surcharges: First, with respect to firm productivities  $\varepsilon_{Z_{ki}}^{\chi_k}$ , second with respect to the deposit and corporate debt prices  $\varepsilon_{Q^r}^{\chi_k}$ . It also depends on the elasticity of sectoral productivity to firm productivity  $\varepsilon_{Z_{ki}}^{\chi_k}$ . Their expressions are obtained following.

Starting with  $\varepsilon_{Z_{ki}}^{\chi_k}$ . The derivative of the sector  $k$ 's log capital surcharge (as defined in A.2.2) with respect to productivities of firms that are not part of sector  $k$  is zero. Taking the log gives  $\log \chi_k = -\frac{1}{\varepsilon_k - 1} \log \left( \sum_{s=1}^{N_k} (Z(k, f) \chi(k, f)^{-1})^{\varepsilon_k - 1} \right) + \frac{1}{\varepsilon_k - 1} \log \left( \sum_{s=1}^{N_k} Z(k, f)^{\varepsilon_k - 1} \right)$  and the derivative with respect to  $\log Z(k, i)$  gives

$$\varepsilon_{Z(k, i)}^{\chi_k} \equiv \frac{d \log \chi_k}{d \log Z(k, i)} = -\zeta(k, i) + \left( \frac{Z(k, i)}{Z_k} \right)^{\varepsilon_k - 1}.$$

Thus, the matrix  $\varepsilon_{Z_{ki}}^{\chi_k}$  is  $N \times \sum_{s=1}^N N_s$ -size block-diagonal:

$$\varepsilon_{Z_{ki}}^{\chi_k} = \begin{pmatrix} \varepsilon_{Z(1,1)}^{\chi_1} & \varepsilon_{Z(1,2)}^{\chi_1} & \cdots & \varepsilon_{Z(1,N_1)}^{\chi_1} & 0 & 0 & \cdots \\ 0 & 0 & \cdots & 0 & \varepsilon_{Z(2,1)}^{\chi_2} & \varepsilon_{Z(2,2)}^{\chi_2} & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \ddots \end{pmatrix}.$$

To obtain elasticities  $\varepsilon_{Q^r}^{\chi_k}$ , note that sector  $k$ 's capital surcharge does not depend on corporate debt prices of other sectors' firms. Their only common component is the deposit price. Total differentiation of sector  $k$ 's capital surcharge with respect to firm  $(k, i)$  capital surcharge gives:  $d \log \chi_k = \zeta(k, i) d \log \chi(k, i)$ . Firm  $(k, i)$ 's capital surcharge changes in response to its own log debt price by  $\frac{d \log \chi(k, i)}{d \log Q(k, i)} = -\beta_k S(k, i) \frac{Q^D}{Q(k, i)} \chi(k, i)^{\frac{1}{\beta_k}}$  and the deposit price by  $\frac{d \log \chi(k, i)}{d \log Q^D} = \beta_k S(k, i) \frac{Q^D}{Q(k, i)} \chi(k, i)^{\frac{1}{\beta_k}}$ . Therefore:

$$\varepsilon_{Q^D}^{\chi_k} = \beta_k \sum_{f=1}^{N_k} \zeta(k, f) S(k, i) \frac{Q^D}{Q(k, i)} \chi(k, i)^{\frac{1}{\beta_k}} \quad \text{and} \quad \varepsilon_{Q(k, i)}^{\chi_k} = -\beta_k \zeta(k, i) S(k, i) \frac{Q^D}{Q(k, i)} \chi(k, i)^{\frac{1}{\beta_k}},$$



and the elasticity matrix  $\mathcal{E}_{Q^r}^{\mathcal{X}^k}$  is of size  $N \times (1 + \sum_{s=1}^N N_s)$  and looks like this:

$$\mathcal{E}_{Q^r}^{\mathcal{X}^k} = \begin{pmatrix} \varepsilon_{Q^D}^{\mathcal{X}^1} & \varepsilon_{Q(1,1)}^{\mathcal{X}^1} & \cdots & \varepsilon_{Q(1,N_s)}^{\mathcal{X}^1} & 0 & 0 & \cdots \\ \varepsilon_{Q^D}^{\mathcal{X}^2} & 0 & \cdots & 0 & \varepsilon_{Q(2,1)}^{\mathcal{X}^2} & \varepsilon_{Q(2,2)}^{\mathcal{X}^2} & \cdots \\ \varepsilon_{Q^D}^{\mathcal{X}^3} & 0 & \cdots & 0 & 0 & 0 & \ddots \end{pmatrix}.$$

The elasticity matrix  $\mathcal{E}_{Z_{ki}}^{Z^k}$  has the same form as  $\mathcal{E}_{Z_{ki}}^{\mathcal{X}^k}$  with  $\varepsilon_{Z(k,i)}^{Z^k} = \left(\frac{Z(k,i)}{Z_k}\right)^{\varepsilon_k - 1}$ .

# **Appendix B**

## **Appendix of Chapter 3**

## B.1 Figures & Tables

Figure B.1: The Equilibrium Mean-Variance Frontier with Disagreement About Stochastic Payoffs

Prices converge to an equilibrium vector price vector  $\mathbf{p}^*$  ( $\Delta(\mathbf{p}^*) = 0$ ) that trades-off return and risk. With all firms agreeing about the distribution of stochastic payoffs, expectations and variances of returns  $\tilde{\mathbf{r}}_i = \tilde{\pi}_i/\mathbf{p}$  describe a mean-variance frontier. For securities that are positively correlated with the average portfolio, equilibrium means and variances of returns sit on an upward sloping 1-dimensional curve. Securities that are negatively correlated with the average portfolio sit on the lower part of the mean-variance parabola. When firms disagree about payoffs, the mean-variance curve has points in its convex hull.

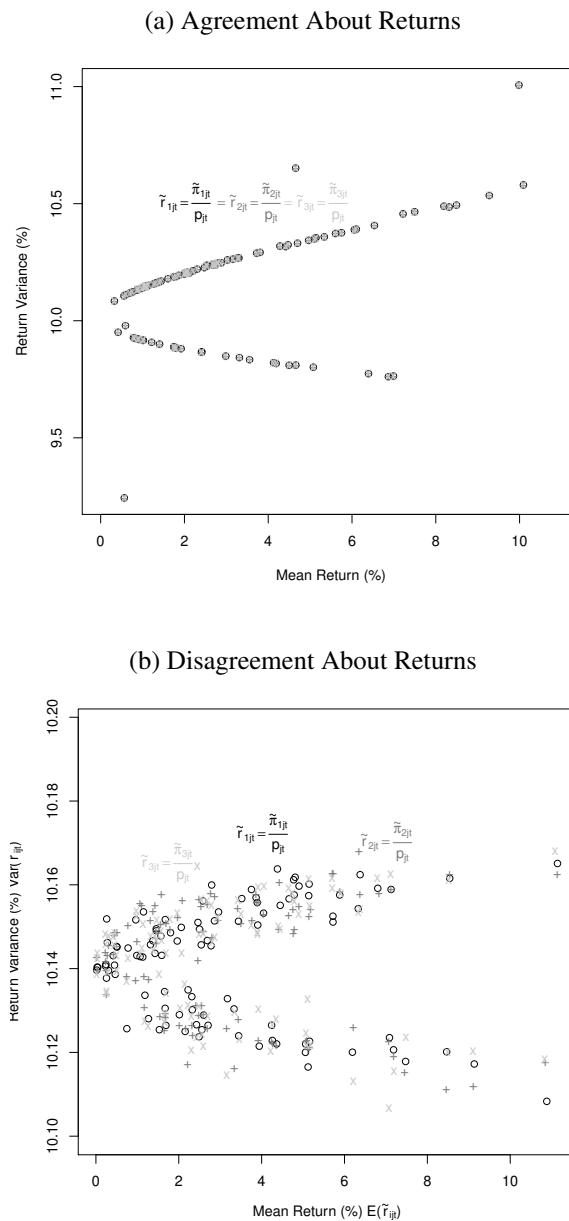
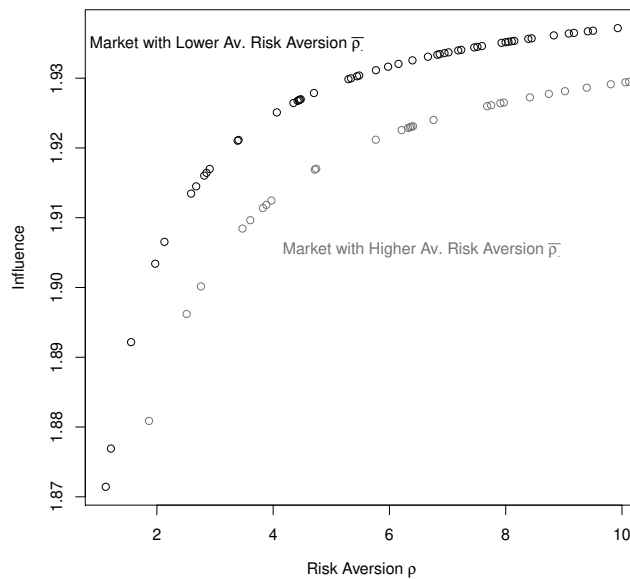


Figure B.2: The Equilibrium Relationship between a bank’s Influence and its Risk Aversion

These figures present the equilibrium relationship (i) between each bank’s risk aversion  $\rho_i$  and its influence in the financial network, defined in Section 3.2.5; (ii) between each bank’s risk aversion  $\rho_i$  and its balance sheet size. The first market simulation (black points) has a lower average risk aversion: risk aversions are drawn from a uniform distribution over  $[1, 10]$ . The second market simulation (grey points) has a higher average risk aversion: they are drawn from a uniform distribution over  $[1, 15]$

(a) Influence of a Firm’s Equity and Its Risk Aversion



(b) Balance Sheet Size and Risk Aversion

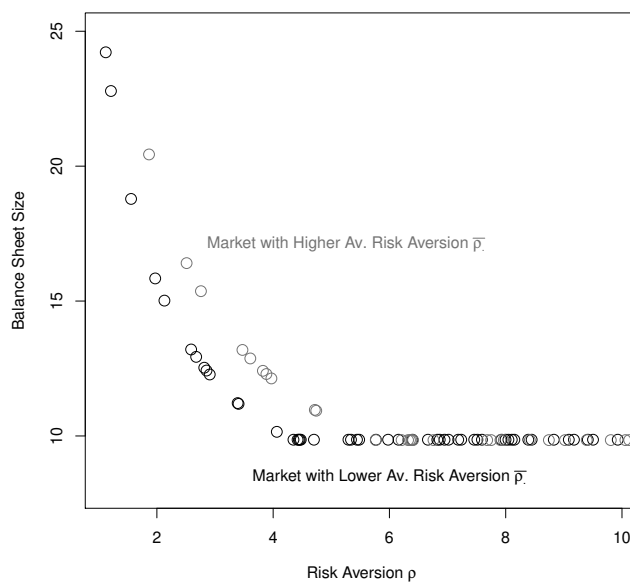


Figure B.3: Banks' Estimated Return-Beliefs Factors and Macro-Variables

This figure shows the first three factors of return-beliefs and net-demands of the banking sector. Shaded areas correspond to the Global Financial Crisis (2007Q3-2008Q4) and the European Debt Crisis (2011Q4-2013Q1), respectively. Subgraphs (b), (d), and (f) relate standardized factors to standardized observable macro-variables.

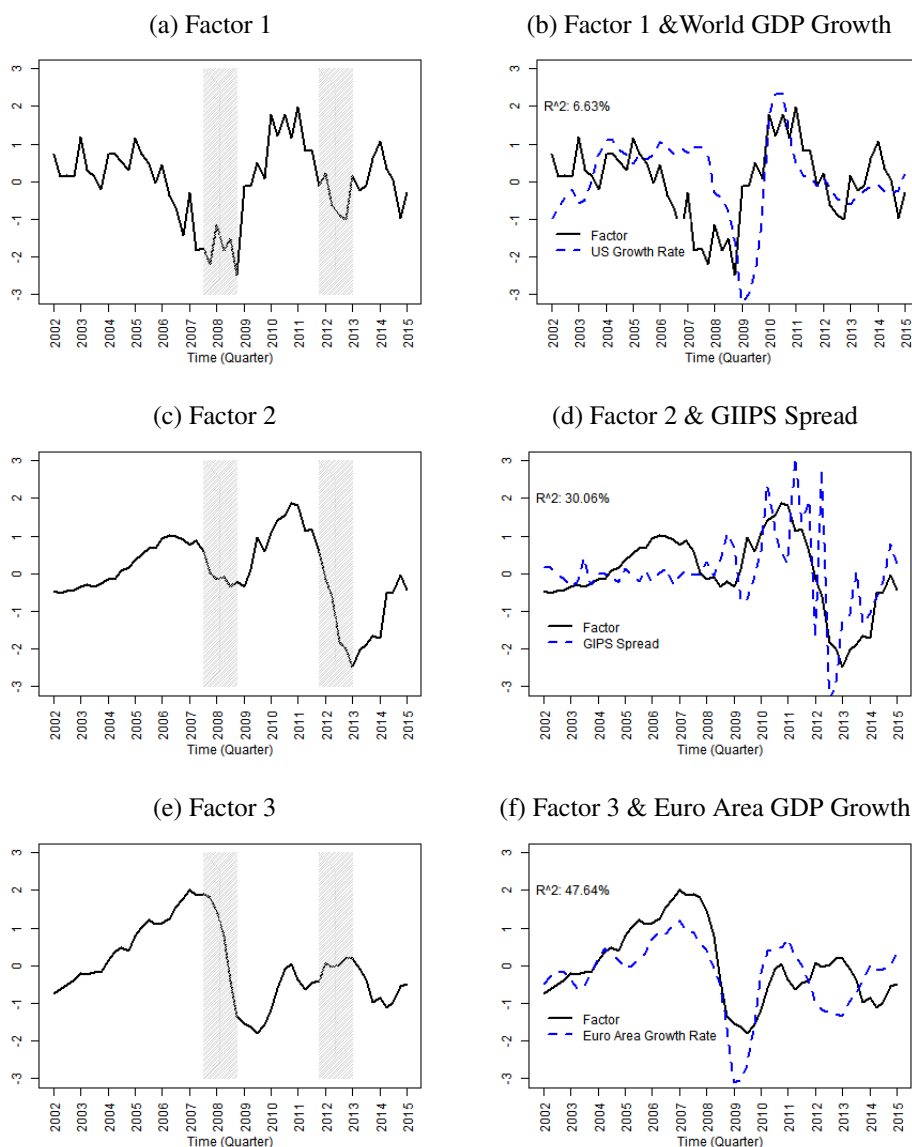


Figure B.4: Comparing Identified Beliefs to Actual Returns

*These plots compare average sector-level beliefs about returns with actual returns.*

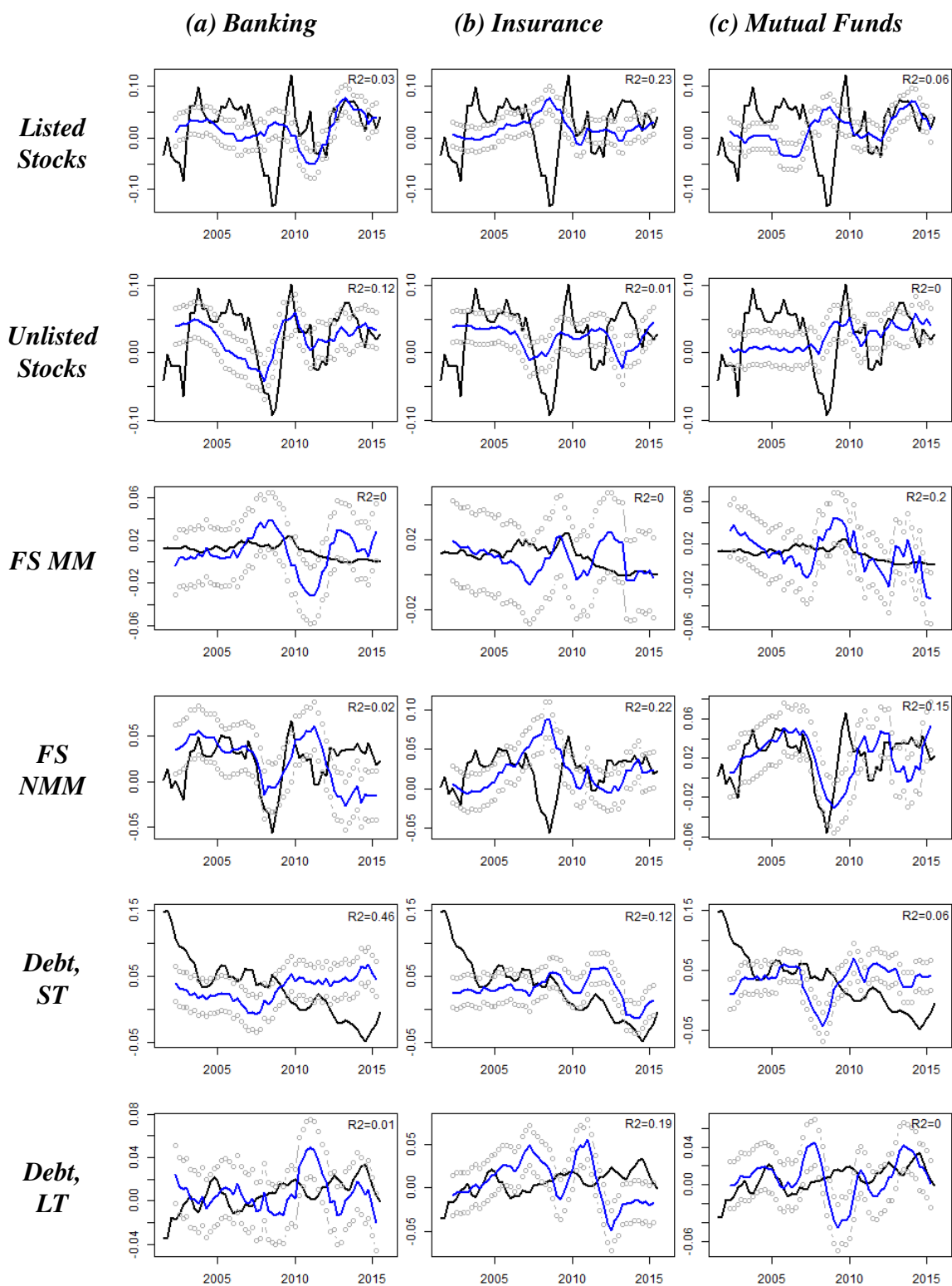


Figure B.5: Comparing Identified Beliefs to Actual Returns (Non-Financial Sector)

The expected return belief is drawn in blue with 95% confidence bands based on the sectors return uncertainty. Ex-post returns are drawn in black.

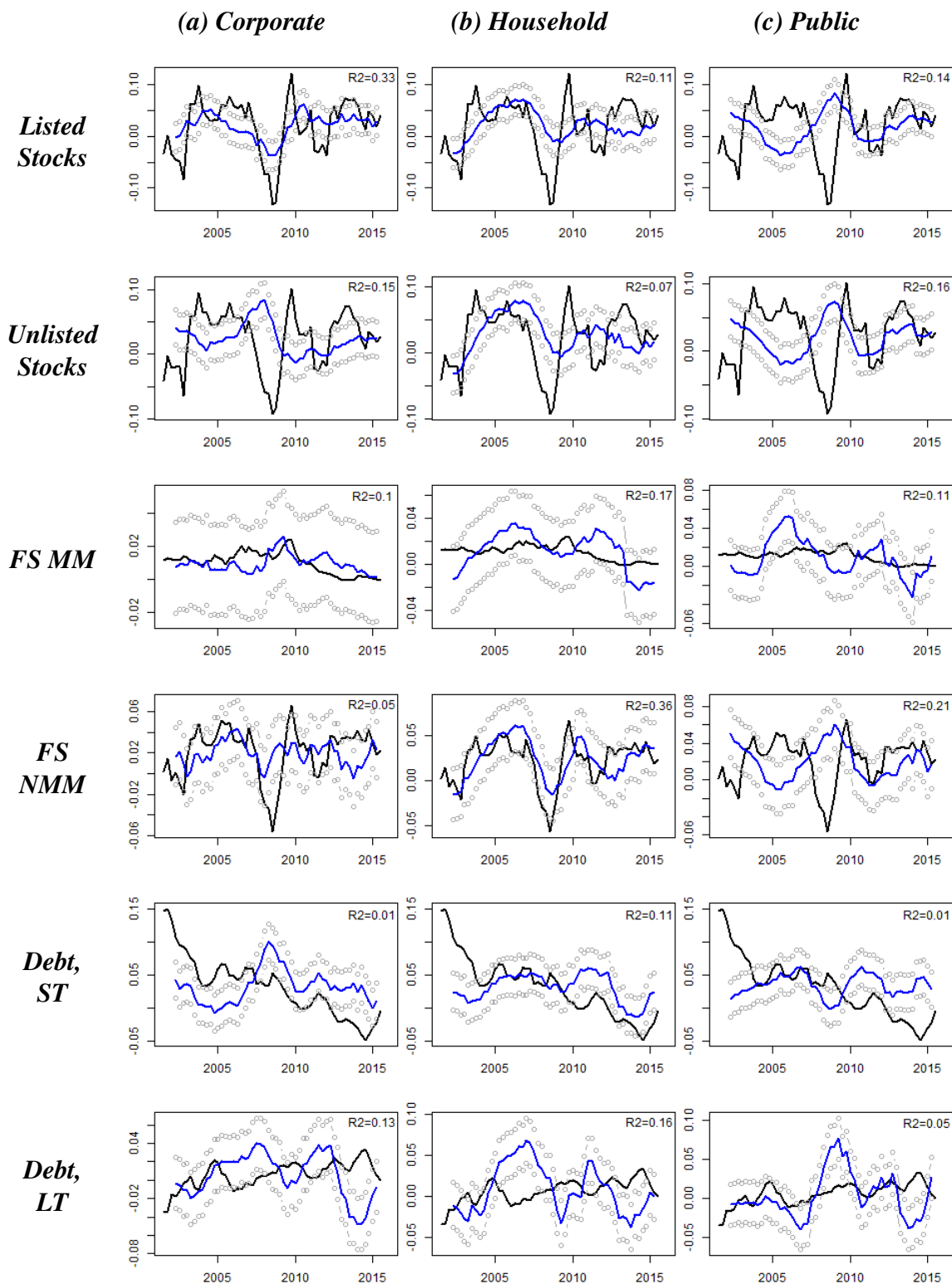
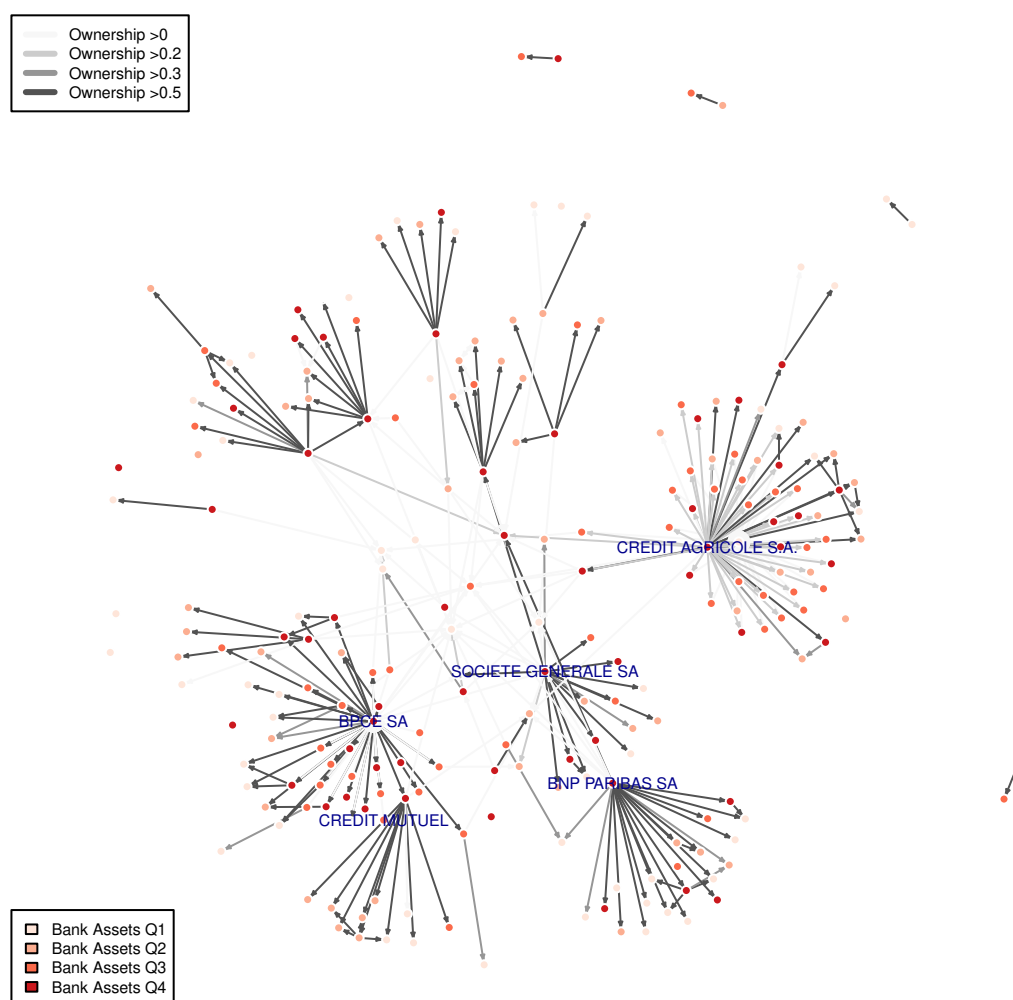


Figure B.6: The Network of Cross Holdings

This figure depicts the network of cross holdings  $C$  (Equation 3.11 of Definition 3.2) for the 4th quarter of 2013. This network does not incorporate the response  $\frac{\partial H}{\partial \log \mathbf{p}^e} (\text{diag} \mathbf{E})^{-1}$  of the holdings to equity prices and the interbank links going through primitive instruments  $\left\{ \mathbf{1}'_p \frac{\partial \Delta_i^p}{\partial \log \mathbf{p}^e} \right\}_i (\text{diag} \mathbf{E})^{-1}$ . As such it represents the network of cross holdings introduced by Elliott et al. (2014).

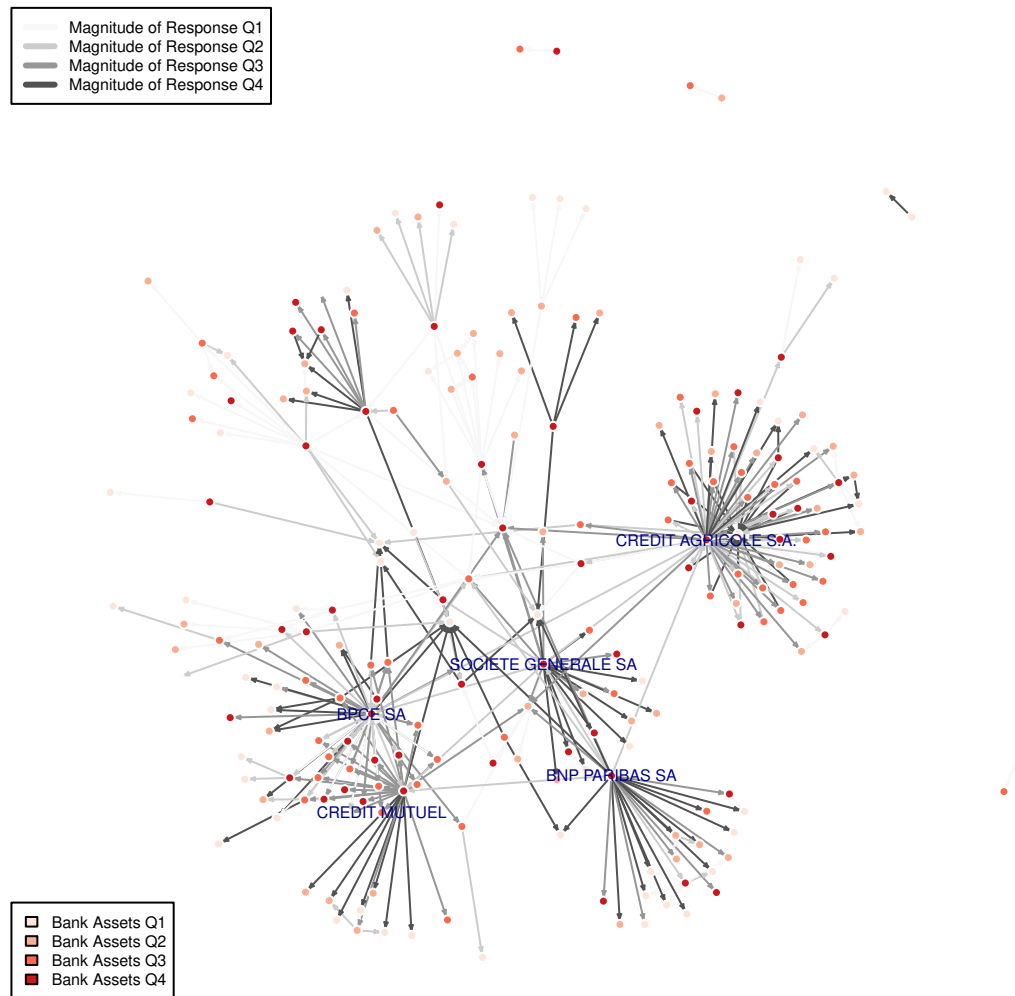


The directed edges indicate ownership. Darker edges correspond to larger share ownership. The vertices are colored according to the total assets of each bank.



Figure B.7: The Sensitivity of Interbank Cross Holdings to Price Changes

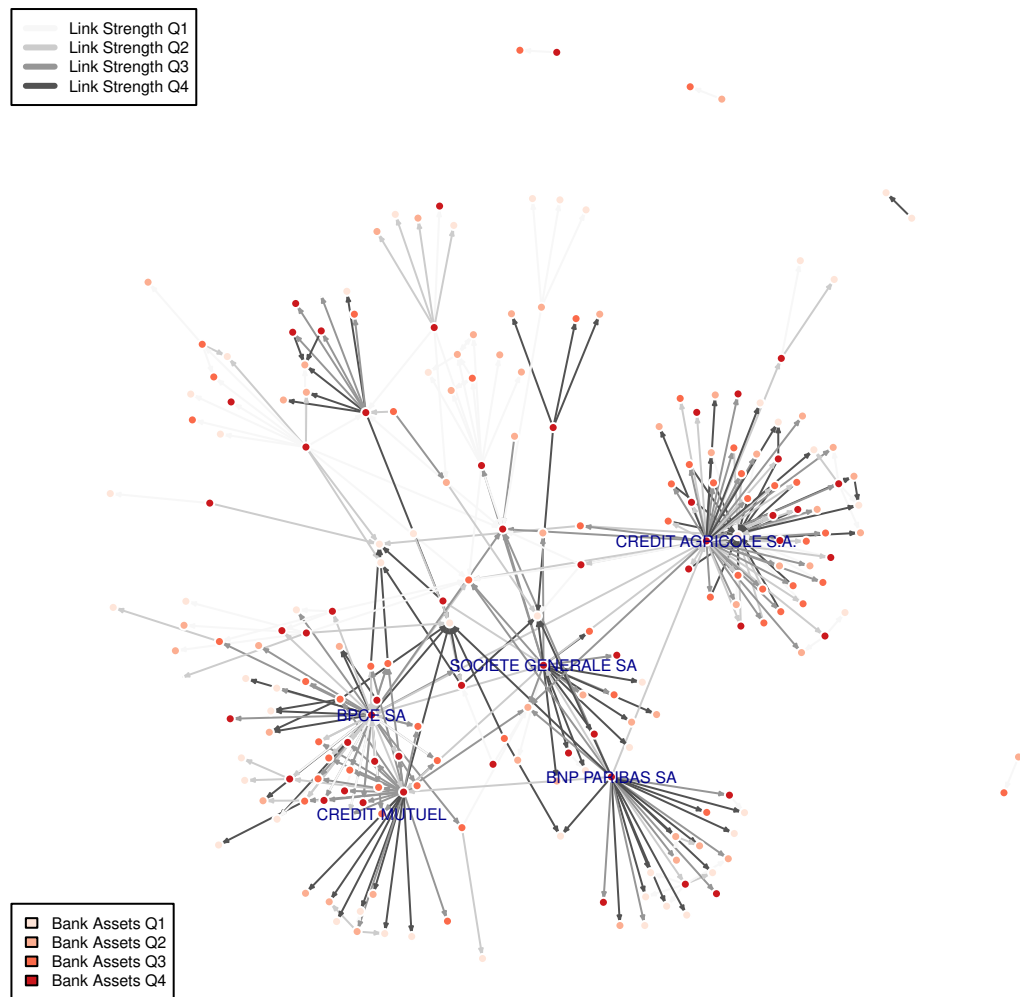
This figure depicts the second component of network structure: the second term  $\frac{\partial H}{\partial \log p^e} (\text{diag } \mathbf{E})^{-1}$  of Equation 3.11 of Definition 3.2. This is, for each bank pair, the sensitivity of the equity cross holdings for the 4th quarter of 2013 to the equity price.



The directed edges indicate the sign of the impact. Darker edges correspond to larger impact. The vertices are colored according to the total assets of each bank.

Figure B.8: Linkages through Primitive Instruments

This network represents the third component  $\left\{ \frac{\partial(\mathbf{1}'_p \Delta_i^p)}{\partial \log \mathbf{p}^e} \right\}_i$  in the expression of the network definition 3.2. This figure depicts the interbank linkages due to primitive instruments (Equation 3.11) for the 4th quarter of 2015.



The directed edges indicate the sign of the impact. Darker edges correspond to larger impact. The vertices are colored according to the total assets of each bank.

Figure B.9: The Correlation of Total Assets with Network Influence and Sensitivity

The top (resp., bottom) figure relates network influence (resp., sensitivity) to bank size. See Section 3.2 for the definition of these network measures.

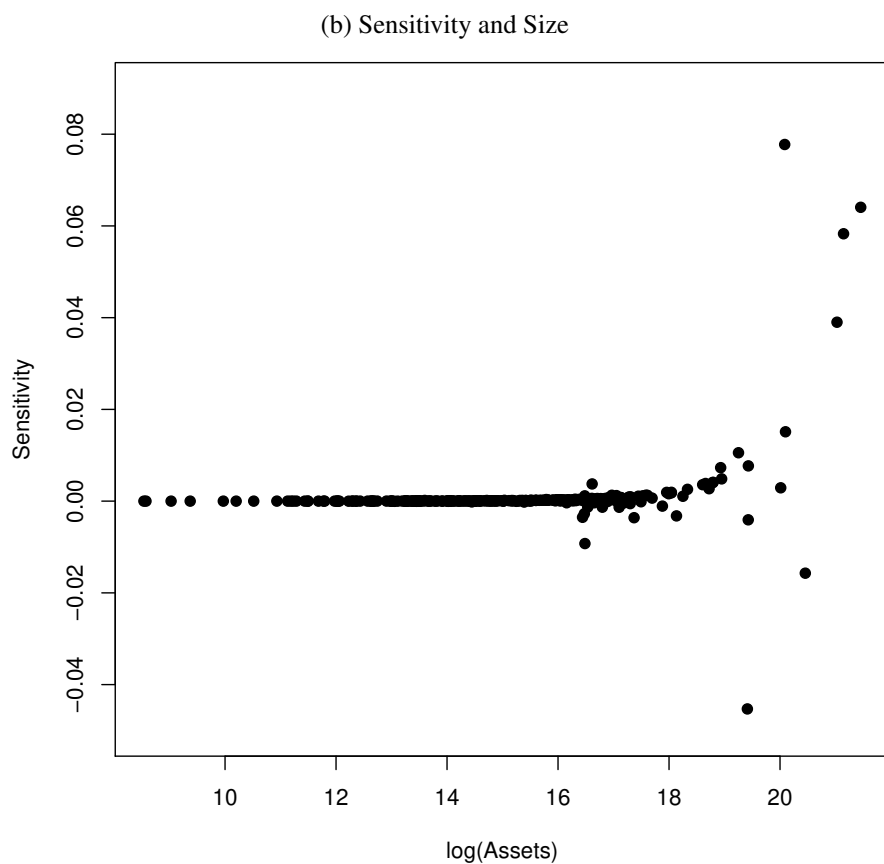
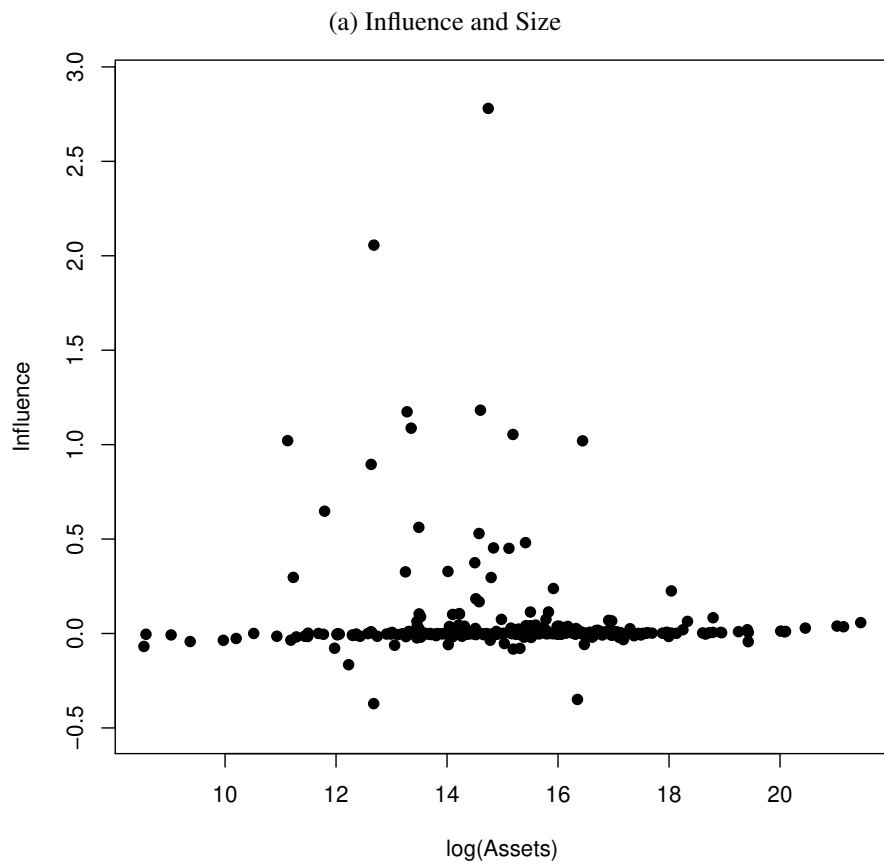
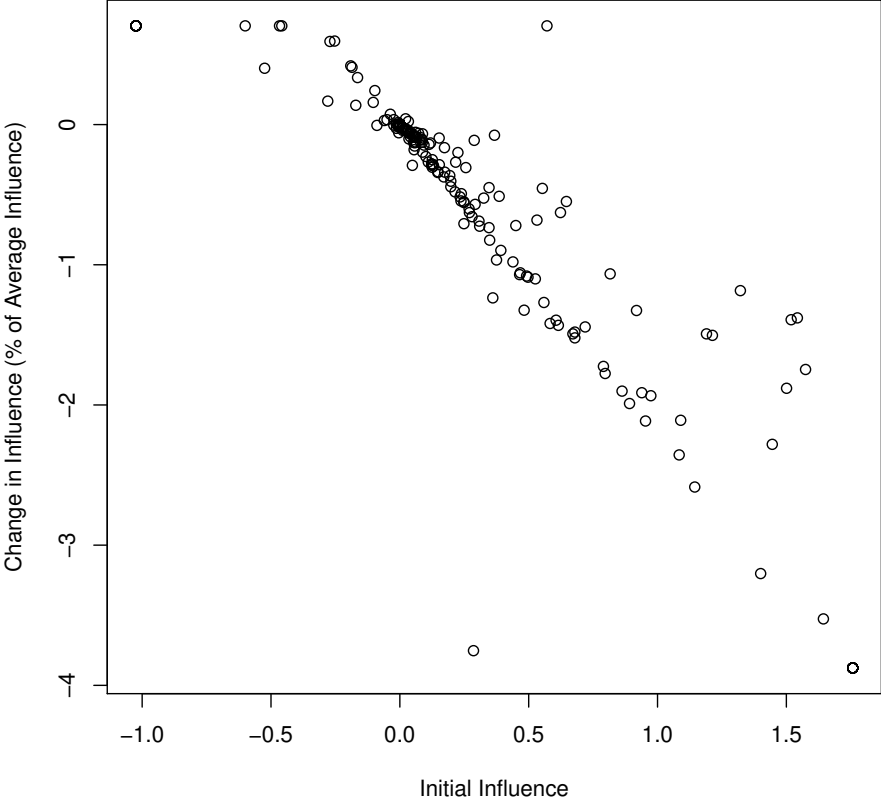


Figure B.10: ECB Shock and Network Structure: Impact of Monetary Policy on Influence and Sensitivity

These two figures present the impact of the ECB shock on the influence and the sensitivity of banks. The impact is a general equilibrium change, i.e. after the series of balance sheet re-diversification leading to the equilibrium price vector.

(a) Impact on Firm Influence in General Equilibrium



(b) Impact on Firm Sensitivity in General Equilibrium

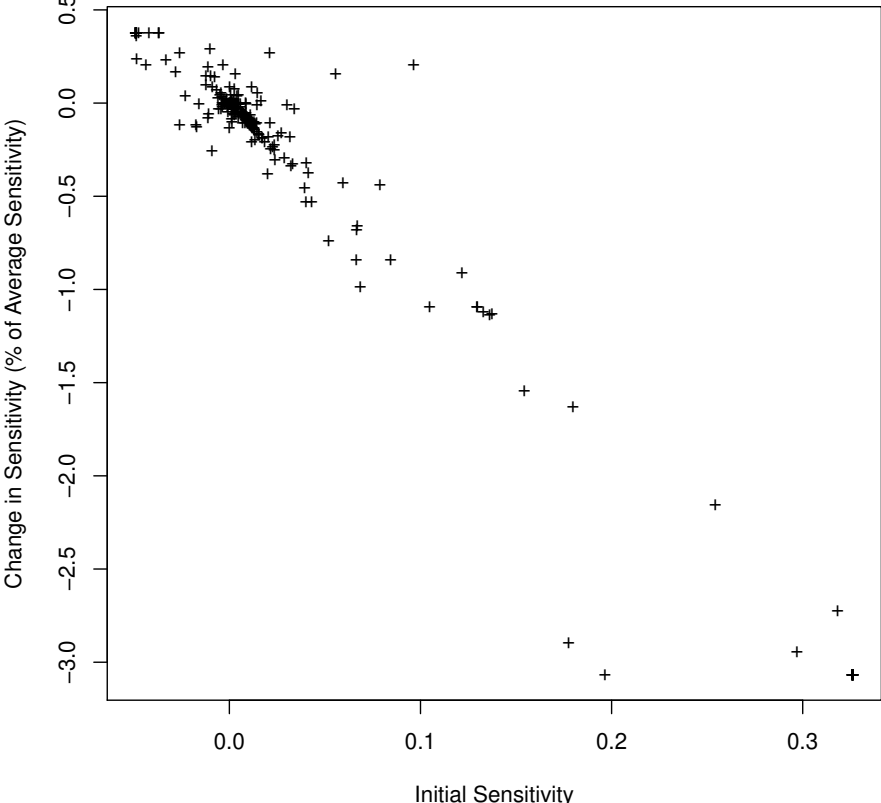


Table B.1: How do Banks Form their Beliefs? How Beliefs about Returns Respond to Realizations

*This table shows results of panel regressions of the difference between realized returns and beliefs about mean returns on its lag and banks' characteristics.*

|  | Dependent variable:<br>Realized Return <i>minus</i> Bank's Belief in $t$ |                      |                     |                     |
|--|--|----------------------|---------------------|---------------------|
|  | (1)  | (2)                  | (3)                 | (4)                 |
| Realized Return <i>minus</i><br>Bank's Belief in $t - 1$ | 0.846***<br>(0.004)  | 0.906***<br>(0.014)  | 0.323***<br>(0.083) | 0.908***<br>(0.011) |
| × Log Risk Aversion                                      |  | -0.022***<br>(0.005) |                     |                     |
| × Leverage   |  |                      | 0.554***<br>(0.088) |                     |
| × Log Size   |  |                      |                     | 0.015***<br>(0.002) |
| <i>Fixed Effects</i>                                     |  |                      |                     |                     |
| Time   | Yes  | Yes                  | Yes                 | Yes                 |
| Bank   | Yes  | Yes                  | Yes                 | Yes                 |
| Instrument   | Yes  | Yes                  | Yes                 | Yes                 |
| Adjusted $R^2$   | 0.757  | 0.757                | 0.758               | 0.757               |
| F Statistic  | 404.715  | 402.474              | 403.06              | 403.083             |
| Observations   | 18,802   | 18,802               | 18,802              | 18,802              |

Table B.2: Estimated Trade Costs by Financial Instrument

*This table shows the average cost (in euro-cents, ¢) of trading one unit of a financial instrument, i.e. deposits, short- and long-term debt securities and loans, fund shares, and equity shares issued by banks and non-banks. The average is taken by financial instrument category; as well as for banks and non-banks. The vector of trade costs  $\gamma_i$  is identified by the persistence of banks' net-demands in excess of what can be explained by the time-variation of return-beliefs (in equation B.16 in the Appendix B.2.1,  $G_i$  is the autocorrelation of holdings).*

|                        |           | N     | Mean    | Std.  | t-Stat <sup>†</sup> | p-Value |
|------------------------|-----------|-------|---------|-------|---------------------|---------|
| <i>Deposits</i>        |           | 749   | 0.066 ¢ | 0.010 | 1.896               | 0.029   |
| <i>Debt Securities</i> |           |       |         |       |                     |         |
| Long-Term              | Banks     | 291   | 0.108 ¢ | 0.013 | 1.446               | 0.075   |
|                        | Non-Banks | 1,384 | 0.301 ¢ | 0.047 | 2.391               | 0.008   |
| Short-Term             | Banks     | 304   | 0.022 ¢ | 0.002 | 1.557               | 0.060   |
|                        | Non-Banks | 1,112 | 0.235 ¢ | 0.040 | 1.984               | 0.024   |
| <i>Equity</i>          |           |       |         |       |                     |         |
| Fundshares             |           | 831   | 0.241 ¢ | 0.053 | 1.313               | 0.095   |
| Shares                 | Banks     | 3,373 | 0.250 ¢ | 0.037 | 3.969               | 0.000   |
|                        | Non-Banks | 1,662 | 0.604 ¢ | 0.102 | 2.413               | 0.008   |
| <i>Loans</i>           |           |       |         |       |                     |         |
| Long-Term              | Banks     | 302   | 0.103 ¢ | 0.015 | 1.166               | 0.122   |
|                        | Non-Banks | 1,958 | 2.045 ¢ | 0.444 | 2.038               | 0.021   |
| Short-Term             | Banks     | 287   | 0.189 ¢ | 0.024 | 1.323               | 0.093   |
|                        | Non-Banks | 1,961 | 1.119 ¢ | 0.217 | 2.280               | 0.011   |

*N*: number of instruments  $\times$  institution pairs in each asset class. <sup>†</sup>: the null hypothesis tests whether the trade cost is significantly different from zero.

Table B.3: Simulation of a PSPP Shock, Bond Yields, Corporate Spread, and the Equity Premium

*This table shows the calibration (Panel (a)) of an increase in the ECB's Public Sector Purchase Program (PSPP) by 180 bil. Eur in 2014Q1 and the response of selected asset prices (Panel (b))*

## (a) Calibration of the ECB's PSPP Purchase Shock

|                                 |            | ECB Capital Key <sup>†</sup><br>(%) | Share Outstanding<br>(%) | Net Demand Shock<br>(bil. Eur) |
|---------------------------------|------------|-------------------------------------|--------------------------|--------------------------------|
| French Government Debt          | Long Term  | 20.00                               | 88.05                    | 31.70                          |
|                                 | Short Term | 20.00                               | 11.95                    | 4.30                           |
| Other Euro Area Government Debt | Long Term  | 80.00                               | 86.82                    | 13.18                          |
|                                 | Short Term | 80.00                               | 125.02                   | 18.98                          |
| Deposits                        |            |                                     |                          | -180.00                        |

## (b) Response of Selected Asset Prices

|                              |            | Effect (in bps)                  |                     |
|------------------------------|------------|----------------------------------|---------------------|
|                              |            | Partial Equilibrium <sup>‡</sup> | General Equilibrium |
| French Government Bond Yield | Long Term  | -7.43                            | -43.90              |
|                              | Short Term | -1.00                            | -37.43              |
| French Corporate Spread      | Long Term  | 7.43                             | 7.56                |
|                              | Short Term | 1.00                             | 1.08                |
| Term Premium                 |            | -6.42                            | -6.46               |
| Equity Premium               |            | 7.43                             | 7.23                |

<sup>†</sup>: The ECB capital is paid in by the national central banks of all EU Member States. Each country's share is calculated using a key based on its population and gross domestic product.

<sup>‡</sup>: In partial equilibrium asset prices only change in markets that are directly impacted by the PSPP shock. For example, since the equity premium is calculated as the difference between the return on equity and the yield on long-term government debt, it moves in the same magnitude as the latter.

Table B.4: Banks' Heterogeneous Response to the ECB's PSPP Shock

This table shows the general equilibrium change of banks' balance-sheets to an increase in the ECB's Public Sector Purchase Program (PSPP) by 180 bil. Eur in 2014Q1. Density plots depict the kernel density estimation of the empirical distribution of the general equilibrium change between the first and third quartile. Distance to insolvency (DI) from Atkeson et al. (2017) is calculated as the the drop in asset value that would render the firm insolvent, measured in units of the firm's asset standard deviation.

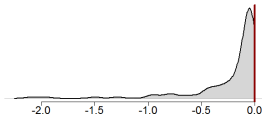
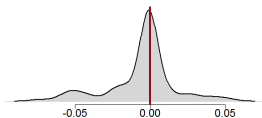
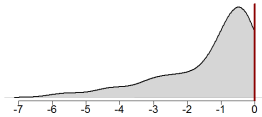
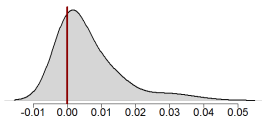
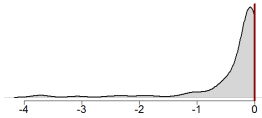
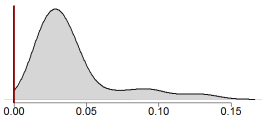
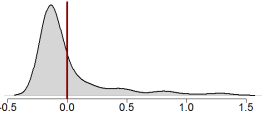
| GE-Initial in:      | Mean  | Q75-<br>Q25 | Quantile |         |       |       |       |       |
|---------------------|---|-------------|----------|---------|-------|-------|-------|-------|
|                     |   |             | 5%       | 25%     | 50%   | 75%   | 95%   |       |
| Size (%)            |    | -6.25       | 2.11     | -35.76  | -2.12 | -0.08 | -0.01 | -0.00 |
| <i>Assets</i>       |   |             |          |         |       |       |       |       |
| Equity (%)          |   | 10.67       | 0.14     | -16.61  | -0.08 | -0.00 | 0.07  | 32.30 |
| Debt (%)            |  | -23.45      | 5.89     | -131.92 | -6.02 | -0.77 | -0.13 | -0.01 |
| <i>Liabilities</i>  |   |             |          |         |       |       |       |       |
| Equity (%)          |  | 0.66        | 0.04     | -1.50   | 0.00  | 0.00  | 0.04  | 9.17  |
| Debt (%)            |  | -7.43       | 3.85     | -42.14  | -3.86 | -0.09 | -0.02 | -0.01 |
| DI ( $\times 100$ ) |  | 0.55        | 0.13     | 0.00    | 0.01  | 0.03  | 0.14  | 1.75  |
| Asset Vol. (ppts)   |  | 5.09        | 1.60     | -1.10   | -0.19 | -0.12 | 1.41  | 36.36 |



Table B.5: Banks' Heterogeneous Response to the ECB's PSPP Shock – Initial Characteristics

*This table shows results from an OLS regression of banks' balance-sheet responses to an increase in the ECB's Public Sector Purchase Program (PSPP) by 180 bil. Eur in 2014Q1 on banks' risk aversion (columns (1) and (4)), cost of equity (columns (2) and (5)), and the initial balance-sheet (columns (5) and (6)). Estimated log-risk-aversion has a sample average of 0.6, a standard deviation of 3.3, and an interquartile range of 7.1.*

|                     | (1)<br>$\Delta \log \text{ Bonds}$ | (2)<br>$\Delta \log \text{ Bonds}$ | (3)<br>$\Delta \log \text{ Bonds}$ | (4)<br>$\Delta \log \text{ Equity}$ | (5)<br>$\Delta \log \text{ Equity}$ | (6)<br>$\Delta \log \text{ Equity}$ |
|---------------------|------------------------------------|------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| Initial Influence   |                                    | 0.087**<br>(0.039)                 |                                    |                                     | -0.011*<br>(0.006)                  |                                     |
| log Risk Aversion   | 0.122***<br>(0.023)                | 0.121***<br>(0.022)                | 0.118***<br>(0.020)                | -0.008***<br>(0.003)                | -0.008***<br>(0.002)                | -0.009***<br>(0.003)                |
| log Bonds           | 0.075***<br>(0.022)                | 0.074***<br>(0.022)                | 0.079***<br>(0.024)                |                                     |                                     |                                     |
| log Size            | -0.013<br>(0.021)                  | -0.004<br>(0.021)                  | -0.014<br>(0.021)                  | 0.001<br>(0.002)                    | 0.001<br>(0.002)                    | 0.001<br>(0.002)                    |
| Initial Sensitivity |                                    |                                    | -0.545<br>(0.607)                  |                                     |                                     | -0.045<br>(0.034)                   |
| log Equity          |                                    |                                    |                                    | -0.008***<br>(0.0026)               | -0.009***<br>(0.003)                | -0.007***<br>(0.002)                |
| Constant            | -0.228***<br>(0.068)               | -0.265***<br>(0.073)               | -0.197***<br>(0.061)               | 0.014*<br>(0.007)                   | 0.015*<br>(0.008)                   | 0.017**<br>(0.009)                  |
| Observations        | 155                                | 155                                | 155                                | 154                                 | 154                                 | 154                                 |
| R-squared           | 0.418                              | 0.425                              | 0.429                              | 0.277                               | 0.29                                | 0.287                               |

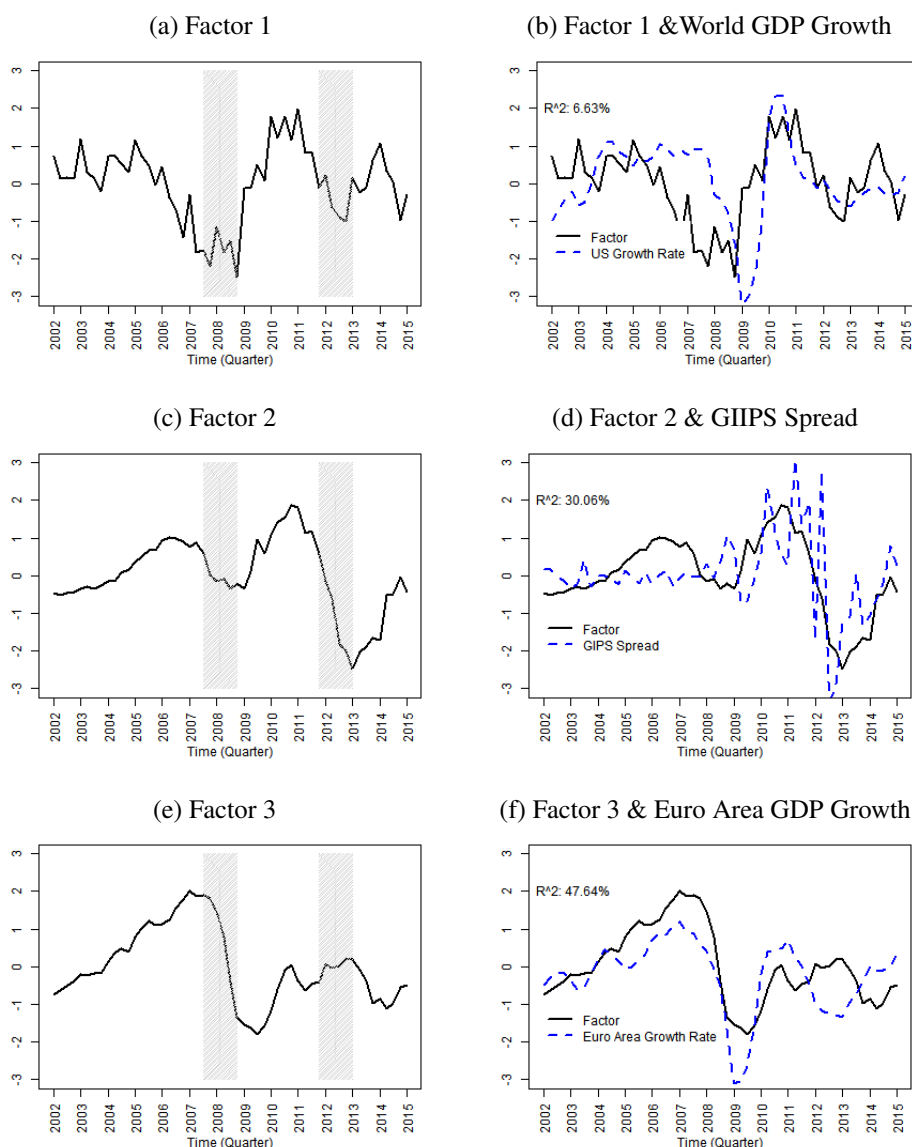
Table B.6: Risk Aversion and Return Belief Uncertainty

*This table presents the estimates of risk aversion and the standard deviation of return beliefs for different values of the calibration ratio  $\delta$  of economy-wide return beliefs over economy-wide ex-post returns.*

|              |                     | <i>Banking</i> | <i>Insurance</i> | <i>Mutual Funds</i> | <i>Corporate</i> | <i>Household</i> | <i>Public</i> | <i>RoW</i> |
|--------------|---------------------|----------------|------------------|---------------------|------------------|------------------|---------------|------------|
| $\delta = 1$ | $\rho$              | 21.69          | 23.97            | 8.94                | 12.02            | 13.94            | 9.053         | 12.046     |
|              | $\sigma_{\epsilon}$ | 0.01           | 0.008            | 0.009               | 0.0111           | 0.0118           | 0.0104        | 0.009      |
| $\delta = 2$ | $\rho$              | 14.18          | 16.96            | 7.56                | 8.42             | 12.12            | 6.36          | 9.22       |
|              | $\sigma_{\epsilon}$ | 0.0183         | 0.0156           | 0.0130              | 0.0227           | 0.0160           | 0.0211        | 0.0539     |
| $\delta = 4$ | $\rho$              | 8.73           | 11.99            | 5.52                | 6.01             | 8.66             | 4.76          | 8.74       |
|              | $\sigma_{\epsilon}$ | 0.0357         | 0.0312           | 0.0272              | 0.0446           | 0.0326           | 0.0376        | 0.0604     |
| $\delta = 6$ | $\rho$              | 4.67           | 7.58             | 3.45                | 3.86             | 5.57             | 2.99          | 6.78       |
|              | $\sigma_{\epsilon}$ | 0.0898         | 0.0779           | 0.0721              | 0.1082           | 0.0817           | 0.1055        | 0.1024     |

Figure B.11: Banks' Estimated Return-Beliefs Factors and Macro-Variables

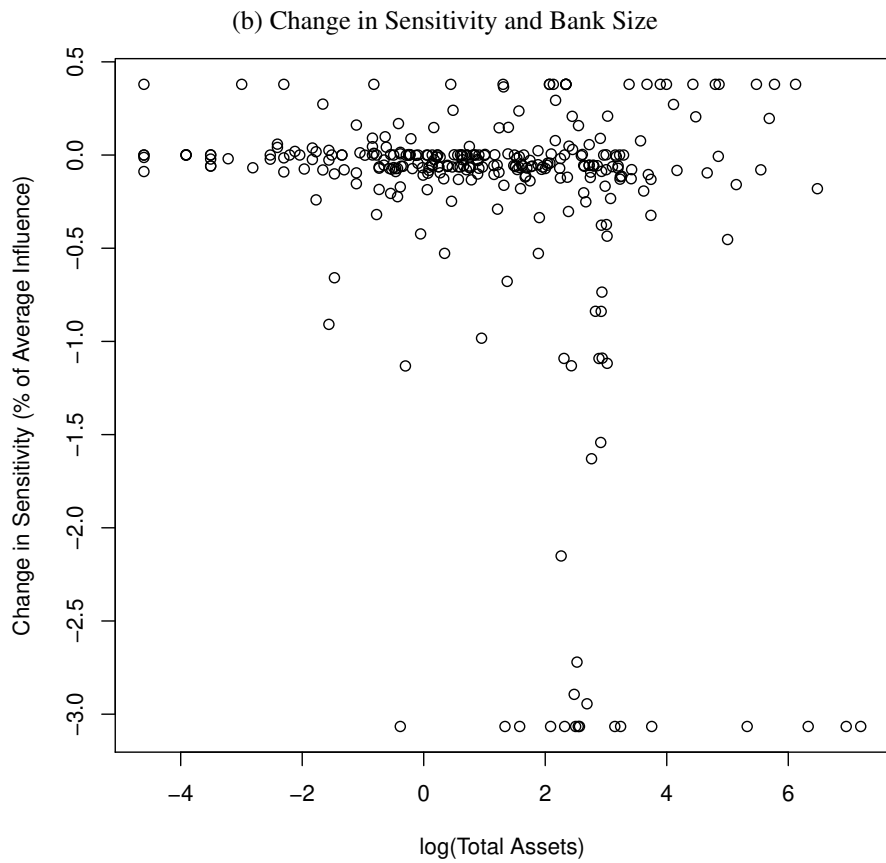
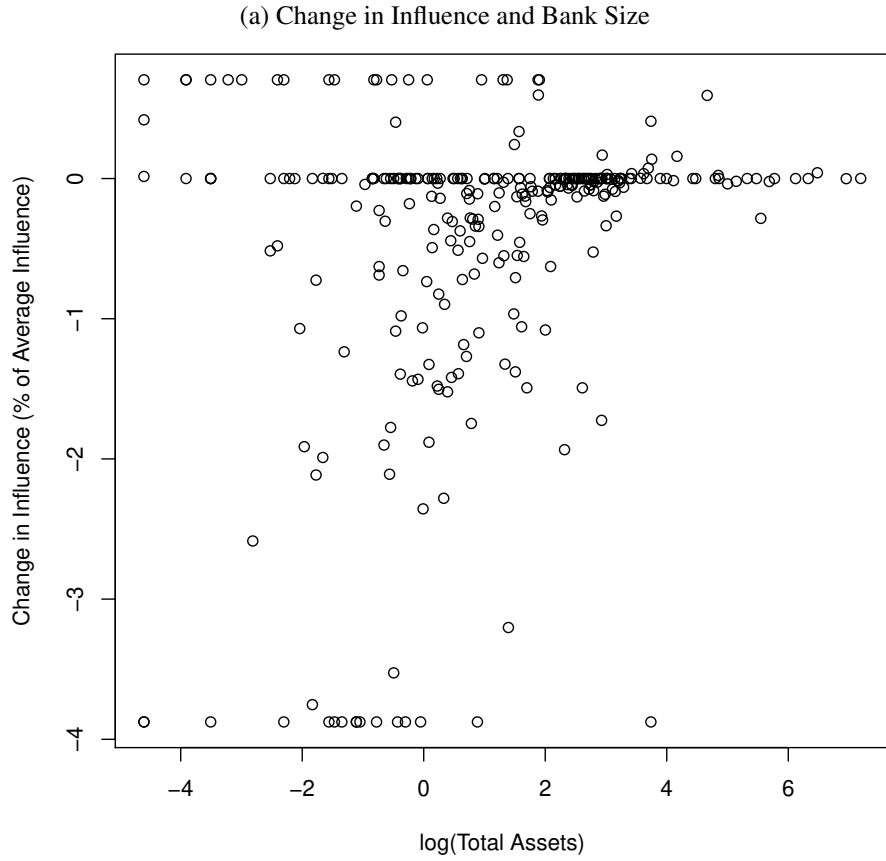
This figure shows the first three factors of return-beliefs and net-demands of the banking sector. Shaded areas correspond to the Global Financial Crisis (2007Q3-2008Q4) and the European Debt Crisis (2011Q4-2013Q1), respectively. Subgraphs (b), (d), and (f) relate standardized factors to standardized observable macro-variables.



Source of Macro-Variables: IMF WEO database and IFS.

Figure B.12: ECB Shock and Bank Size

*These two figures present the impact of the ECB shock on the influence and the sensitivity of banks, against their size (total assets).*



## B.2 Theory

### B.2.1 Proofs: General Equilibrium Model

#### Proposition B.1. (Existence of Equilibrium)

*Proof.* For clarity of exposition and without loss of generality, we omit the subscript  $t$  in the notations for this proof. Write  $\Delta(\mathbf{p}) = \sum_{i=1}^N \Delta_i(p_1, p_2, \dots, p_J) - \mathbf{E}(p_1, p_2, \dots, p_J)$ . Throughout the paper,  $\Delta(\mathbf{p})$  is the value of net demand. We thus write  $\mathbf{z}(\mathbf{p})$  the net demand in units of financial instruments, and thus  $\Delta(\mathbf{p}) = \mathbf{p} \odot \mathbf{z}(\mathbf{p})$  is the term by term product of the price vector and the net demand vector in units.

An equilibrium price vector is a  $J$ -vector  $\mathbf{p}$  such that  $\mathbf{z}(\mathbf{p}) = \mathbf{0}$ .

Following Mas-Colell, Whinston, Green et al. (1995b), we need to show that the aggregate net demand curve  $\mathbf{p} \in \mathbb{R}_{+*}^J \mapsto \mathbf{z}(\mathbf{p}) \in \mathbb{R}^J$  satisfies the following properties:

1.  $\mathbf{z}(\cdot)$  is continuous.
2.  $\mathbf{z}(\cdot)$  is homogenous of degree zero.
3.  $\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = 0$  for all  $\mathbf{p}$ , i.e. Walras law is satisfied.
4. There is an  $s > 0$  such that  $z_j(\mathbf{p}) > -s$  for all financial instruments  $j = 1, 2, \dots, J$  and for all price vectors  $\mathbf{p}$ .
5. If  $\mathbf{p}^n \rightarrow \mathbf{p}$ , where  $\mathbf{p} \neq \mathbf{0}$  and  $p_j = 0$  for some  $j$ , then

$$\max \{z_1(\mathbf{p}^n), \dots, z_J(\mathbf{p}^n)\} \rightarrow \infty$$

The continuity of  $\mathbf{z}(\cdot)$  over  $\mathbb{R}_{+*}^J$  is established as both the joint distribution of returns  $f_i$  and the utility function  $u_i$  of each firm  $i$  are continuous functions of  $\mathbf{p}$  at each point of  $\mathbb{R}^{+*J}$ .

The homogeneity of  $\mathbf{z}(\cdot)$  is established by writing the first-order condition of program (3.3)

with  $\|\cdot\|$  the Euclidean norm:

$$\int u'_i(\Delta'_{it}\tilde{\mathbf{r}}) p_{jt} \tilde{r}_{ijt} f_i(\tilde{\mathbf{r}}) d\tilde{\mathbf{r}} - 2(\gamma_j/p_j)^2 p_{jt} \cdot (z_{ijt} - z_{ijt-1}) + \lambda_i \Delta_{ijt} = 0,$$

where  $\lambda_i$  is the Lagrange multiplier of the funding constraint, which is homogeneous of degree 0. The first two terms are also homogeneous of degree 0, which establishes the homogeneity of degree 0 of  $z$ .

Walras law follows from the sum of the funding constraints, as

$$\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) = \sum_{j=1}^J \sum_{i=1}^N (\Delta_{ij}(\mathbf{p}) - \mathbf{1}(j(i) = j) E_i(\mathbf{p})) = 0. \quad (\text{B.1})$$

On point 4.: the existence of a lower bound for net demands  $\mathbf{z}(\mathbf{p})$  follows from the funding constraint (3.2). Indeed, assume that there is no  $s$  such that  $z_j(\mathbf{p}) > -s$  for all financial instruments  $j = 1, 2, \dots, J$  and for all price vectors  $\mathbf{p}$ . Then we can build a sequence  $(s^k, \mathbf{p}^k)$  such that  $s^k \rightarrow \infty$  as  $k \rightarrow \infty$ , and for each  $k$ , there is a  $j'(k)$  in  $\{1, 2, \dots, J\}$  for which  $-s^k > z_{j'(k)}(\mathbf{p}^k)$ . Given the funding constraint, this implies that there will be a similar sequence for which demand will go to infinity. Formally, there is a sequence  $(\sigma^k, \boldsymbol{\pi}^k)$  such that  $\sigma^k \rightarrow \infty$  as  $k \rightarrow \infty$ , and for each  $k$ , there is a  $j''(k)$  in  $\{1, 2, \dots, J\}$  for which  $z_{j''(k)}(\boldsymbol{\pi}^k) > \sigma^k$ . This, however, implies that the variance of the firm's portfolio diverges to infinity as  $k \rightarrow \infty$ , which cannot be a solution to the optimization program (3.19). The last point follows from the fact that if asset  $j$ 's return diverges to infinity,  $\mu_j \rightarrow \infty$ , one of the components of net demand will diverge to infinity. Given that properties 1–5 are satisfied, an equilibrium price vector  $\mathbf{p}^*$  exists with  $\mathbf{z}(\mathbf{p}^*) = 0$ .  $\square$

## B.2.2 Proofs of Identification

### Proposition B.5. (Implied Net Demand Dynamic Factor Model)

*Proof.* Without trade costs, CARA net-demand are

$$\Delta_{it} = \frac{1}{\rho_i} \Sigma_{it}^{-1} (\boldsymbol{\mu}_{it} - \eta_{it} \mathbf{1}). \quad (\text{B.2})$$

Using the factor structure of return beliefs (FSRB), the moments of stochastic return beliefs are parametrized as

$$\boldsymbol{\mu}_{it} = \boldsymbol{\varphi}_i + \Lambda_i (\boldsymbol{\phi}_i + \Phi_i \mathbf{f}_{it}), \quad (\text{B.3})$$

$$\Sigma_{it} = \Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}, \quad (\text{B.4})$$

such that net demands are

$$\Delta_{it} = \frac{1}{\rho_i} (\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i})^{-1} (\boldsymbol{\varphi}_i + \Lambda_i (\boldsymbol{\phi}_i + \Phi_i \mathbf{f}_{it}) - \eta_{it} \mathbf{1}). \quad (\text{B.5})$$

Then, net demand has the factor structure

$$\Delta_{it} = \mathbf{c}_i + L_i \mathbf{f}_{it} + \boldsymbol{\xi}_{it}, \quad (\text{B.6})$$

where constants, loadings, and residual of the net demand factor model are defined as

$$\begin{aligned} \mathbf{c}_i &\equiv \frac{1}{\rho_i} [\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}]^{-1} (\boldsymbol{\varphi}_i + \Lambda_i \boldsymbol{\phi}_i) \\ L_i &\equiv \frac{1}{\rho_i} [\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}]^{-1} \Lambda_i \Phi_i \\ \boldsymbol{\xi}_{it} &\equiv -\frac{1}{\rho_i} [\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}]^{-1} \mathbf{1} \eta_{it}. \end{aligned}$$

□

### Proposition B.6. (Identification of Return Beliefs and Risk Aversion)

*Proof.* Because all three objects  $\Sigma_{\varepsilon i}, \Sigma_{ui}, \Phi_i$  are known parameters in this mapping, we introduce scaled return belief loadings  $\tilde{\Lambda}_i \equiv \Sigma_{\varepsilon i}^{-1/2} \Lambda_i \Sigma_{ui}^{1/2}$ , and scaled net-demand loadings  $\tilde{L}_i \equiv \Sigma_{\varepsilon i}^{-1/2} L_i \Phi_i^{-1} \Sigma_{ui}^{1/2}$

to simplify derivations. Then, scaled belief loadings and risk aversion need to satisfy (dropping the firm index):

$$\frac{1}{\rho} \left( I + \tilde{\Lambda} \tilde{\Lambda}' \right)^{-1} \tilde{\Lambda} = \tilde{L}. \quad (\text{B.7})$$

The invertibility of the mapping  $g$  is proven in two steps: *First*, we show that for a given risk aversion  $\rho$ ,  $\tilde{\Lambda}$  can be obtained by applying the method of undetermined coefficients and solving a quadratic matrix equation. *Second*,  $\rho$  is determined using the fact that  $\Lambda$  must be in the domain of the mapping  $g$ , i.e. a matrix of factor loadings satisfying the property that the sum of its squared elements is equal to  $K$ .

We start by writing scaled return belief loadings,  $\tilde{\Lambda}$ , as a matrix product of scaled net-demand loadings and an unknown square-matrix,

$$\tilde{\Lambda} = \tilde{L} \cdot \left( \rho^{-1} I - \tilde{\Lambda}' \tilde{L} \right)^{-1}. \quad (\text{B.8})$$

Equation (B.8) leads us to conjecture that the solution for  $\tilde{\Lambda}$  is of the form,

$$\tilde{\Lambda} = \tilde{L} \cdot X, \quad (\text{B.9})$$

where  $X$  is an undetermined square-matrix of coefficients. From equations (B.8) and B.9 it follows that

$$X = \left( \rho^{-1} I - X' \tilde{L}' \tilde{L} \right)^{-1}. \quad (\text{B.10})$$

Because equation (B.10) is symmetric, it must be that  $X = X'$  and equation (B.10) can be written as the quadratic matrix equation

$$\underbrace{I}_{\mathcal{A}} \cdot X^{-2} - \underbrace{\rho^{-1} I}_{\mathcal{B}} \cdot X^{-1} + \underbrace{\tilde{L}' \tilde{L}}_{\mathcal{C}} = 0, \quad (\text{B.11})$$



which has, if (i)  $\mathcal{A} = I$ , (ii)  $\mathcal{B}\mathcal{C} = \mathcal{C}\mathcal{B}$ , and (iii)  $\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}$  has a square-root, the solution

$$X^*(\rho) = \left( \frac{1}{2} \left[ \rho^{-1}I + \left( \rho^{-2}I - 4\tilde{L}'\tilde{L} \right)^{1/2} \right] \right)^{-1}. \quad (\text{B.12})$$

Conditions (i) and (ii) are easily checked to be satisfied. Condition (iii) is satisfied as long as  $\rho$  is small enough for  $\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}$  to be positive semi-definite. A positive semi-definite matrix has a unique positive semi-definite square root. While it is irrelevant for the variance of return beliefs, which square root is chosen, the natural solution is the positive semi-definite square root. Indeed, the positive semi-definite square root will preserve the direction of factors as identified by the factor model on net-demands. Choosing another square root would make it necessary to change the direction of factors.

With  $X^*(\rho)$ , risk aversion  $\rho$  follows from the predicate of the domain of the mapping  $g$ . The sum of squared elements of  $\Lambda$  must be equal to the number of factors,  $K$ .

$$\mathbf{1}'(\Lambda \circ \Lambda)\mathbf{1} = \mathbf{1}' \left( \left( \Sigma_{\varepsilon}^{1/2} \tilde{L} X^*(\rho) \Sigma_u^{-1/2} \right) \odot \left( \Sigma_{\varepsilon}^{1/2} \tilde{L} X^*(\rho) \Sigma_u^{-1/2} \right) \right) \mathbf{1} = K. \quad (\text{B.13})$$

□

**Proposition B.7. (Identification with Trade Costs)** *The optimal net demand schedule of firm  $i$  in the presence of trade costs can be written as*

$$\Delta_{it} = \mathbf{c}_i + L_i \mathbf{f}_{it} + \xi_{it} + G_i \Delta_{it-1}, \quad (\text{B.14})$$

where the constant vector  $C_i$ , the loadings  $L_i$ , the residual  $\xi_{it}$ , and  $G_i$  the transformed trade costs are functions of the unobserved factor structure of return beliefs, the coefficients of the auto-

regressive processes of underlying factors, the trade costs and risk preference:

$$\begin{cases} \mathbf{c}_i &= \frac{1}{\rho_i} \left( \frac{1}{\rho_i} \Gamma_i + \Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i} \right)^{-1} (\boldsymbol{\varphi}_i + \Lambda_i \boldsymbol{\phi}_i) \\ L_i &= \frac{1}{\rho_i} \left( \frac{1}{\rho_i} \Gamma_i + \Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i} \right)^{-1} \Lambda_i \boldsymbol{\Phi}_i \\ G_i &= \frac{1}{\rho_i} \left( \frac{1}{\rho_i} \Gamma_i + \Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i} \right)^{-1} \Gamma_i \\ \xi_{it} &= - \frac{1}{\rho_i} \left( \frac{1}{\rho_i} \Gamma_i + \Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i} \right)^{-1} \eta_{it} \mathbf{1} \end{cases} \quad (\text{B.15})$$

The identification of structural parameters in the presence of trade costs is similar to the identification without them. The additional reduced form estimate  $G_i$  is used to express trade costs in terms of structural parameters and observations,

$$\Gamma_i = \rho_i (\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i}) G_i (I - G_i)^{-1}. \quad (\text{B.16})$$

Plugging into the expression of net-demand factor loadings,  $L_i$ , yields

$$(I - G_i)^{-1} L_i = \frac{1}{\rho_i} (\Lambda_i \Sigma_{ui} \Lambda_i' + \Sigma_{\varepsilon i})^{-1} \Lambda_i \boldsymbol{\Phi}_i. \quad (\text{B.17})$$

Thus the mapping  $\delta$  in the text (Equation 3.24) is modified to

$$\check{\delta} : (\rho_i, \boldsymbol{\varphi}_i, \Lambda_i, \Gamma_i) \mapsto (\mathbf{c}_i, L_i, G_i). \quad (\text{B.18})$$

The identification proceeds by inverting the mapping (see Proposition 3.6) using scaled net-demand loadings  $\check{L}_i \equiv \Sigma_{\varepsilon i}^{-1/2} (I - G_i)^{-1} L_i \boldsymbol{\Phi}_i^{-1} \Sigma_{ui}^{1/2}$ .

## B.3 Data

### B.3.1 Estimation of Returns

There are two challenges for deriving returns on financial instrument categories from sectoral account data.

1. Returns are specific to the sectoral investment position. The sectoral accounts provide information on aggregates of financial instruments of the same type. No information exists on sectoral portfolio allocations across financial instruments of the same type. Returns realized on financial instruments of the same type therefore vary across sector, due to differences in inter-type allocations.
2. Pay-offs are recorded only at the sectoral level as income received or paid in the income accounts. Therefore dividends, interest payments and investment income cannot be directly attributed to financial instruments.

We use the variation of investment positions across sectors and financial instruments, the variation of valuation changes across sectors and financial instruments and the variation of income received or paid across sectors to estimate subsequently: (i) the return due to valuation changes and (ii) the return due to payoffs. Financial instruments are indexed by  $j = 1, \dots, J$  and sectors are indexed by  $i = 1, \dots, I$ . Furthermore,  $P_t^D$  and  $P_t^S$  are matrices of size  $I \times J$  that collect the demand and supply by  $I$  sectors for  $J$  financial instruments, respectively. The rate of return due to valuation changes is denoted by  $g_{jt+1}$  and the rate of return due to payoffs is denoted by  $\pi_{jt+1}$ .

**Assumption B.1. (Separate Estimation of Returns and Shifts in Demand)** *The return on asset  $j$  demanded or supplied by sector  $i$  have an asset-specific component and a component specific to the financial position  $f \in \{D, S\}$  of sector  $i$ .*

$$\pi_{ijt+1}^f = \pi_{jt+1} + u_{it+1}^f, \quad f \in \{D, S\} \quad (\text{B.19})$$

$$g_{ijt+1}^f = g_{jt+1} + v_{it+1}^f, \quad f \in \{D, S\} \quad (\text{B.20})$$

where  $D$  denotes the demand/asset side of the balance-sheet and  $S$  the supply/liability side of the balance-sheet. We assume that the component specific to the financial position  $f \in \{D, S\}$  of sector  $i$  is iid distributed with mean zero.

Collecting the pay-offs and valuation gains received or payed on  $J$  assets by  $I$  sectors in the matrices  $\Pi_t^f$  and  $G_t^f$  of dimension  $I \times J$ , respectively, Assumption B.1 implies that the return in amount of currency units realized can be written in matrix notation as

$$\Pi_t^f = P_t^f \text{diag}(\pi_{t+1}) + \text{diag}(u_{t+1}^f) P_t^f, \quad f \in \{D, S\} \quad (\text{B.21})$$

$$G_t^f = P_t^f \text{diag}(g_{t+1}) + \text{diag}(v_{t+1}^f) P_t^f, \quad f \in \{D, S\} \quad (\text{B.22})$$

As argued above, not all returns realized by sectors on financial instruments are fully observable. For example, only the sum of interest payments received by sector  $i$  is observable, but not interest payments received on loans and coupon payments received on debt securities separately. Indeed, there are matrices  $O^\Pi$  and  $O^G$  of dimension  $J \times K$  that map unobservable return components to observable functions of returns, such that we can transform equations B.21B.22 to

$$\Pi_t^f O^\Pi = \left[ P_t^f \text{diag}(\pi_{t+1}) + \text{diag}(u_{t+1}^f) P_t^f \right] O^\Pi, \quad f \in \{D, S\} \quad (\text{B.23})$$

$$G_t^f O^G = \left[ P_t^f \text{diag}(g_{t+1}) + \text{diag}(v_{t+1}^f) P_t^f \right] O^G, \quad f \in \{D, S\} \quad (\text{B.24})$$

Note that if all returns were observable,  $O^\Pi = O^G = I$ . Finally, we estimate  $g_{jt+1}$ ,  $\pi_{jt+1}$  such as to minimize the sum of squared residuals. This method simplifies to weighted Ordinary Least Squares when we assume additionally that the sector and financial position specific components of returns are uncorrelated.

### **B.3.2 Imputation of the Real Asset**

The estimation of the model requires information on complete balance-sheets, i.e. stock and change (valuation + flows + other changes) for both financial and non-financial assets. These data are available in the following form:

1. Financial asset stocks and changes: at quarterly frequency from quarterly sectoral accounts from EUROSTAT. The datasets are: `nasq_10_f_bs`, `nasq_10_f_gl`, `nasq_10_f_tr`, `nasq_10_f_oc`.
2. Non-financial asset stocks: at annual frequency from both EUROSTAT (`nasa_10_nfa`) and INSEE.
3. Non-financial asset changes to due flows: at quarterly frequency from EUROSTAT (`nasq_10_nf_tr`), but strangely not from INSEE.
4. Non-financial asset changes due to valuation, flows and other changes: at annual frequency from INSEE, but not from EUROSTAT.

Quarterly stocks and changes of the financial part of sectoral balance-sheet are directly obtained from the sectoral financial accounts from EUROSTAT. The non-financial part of the balance-sheet are obtained in three steps:

1. We aggregate quarterly flows from EUROSTAT to annual flows and calculate the share of the annual flow that takes place in each quarter. For example we will find that in 2008.1 30% of the 2008 annual flow took place, etc.
2. We take annual changes (valuation + flows + other changes) from INSEE and apply the inter-annual breakdown for flows obtained in Step 1 to all types of changes.
3. We take the annual stocks from INSEE and build up the quarterly stocks from the quarterly series of changes obtained in Step 2.

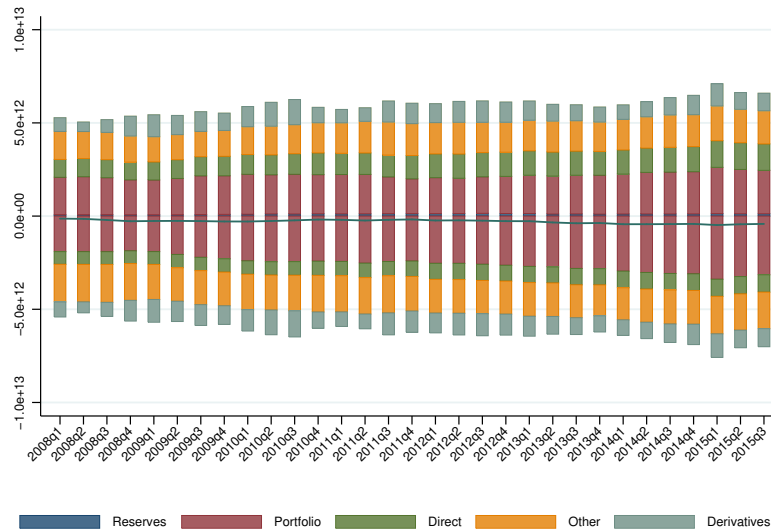
# **Appendix C**

## **Appendix of Chapter 4**

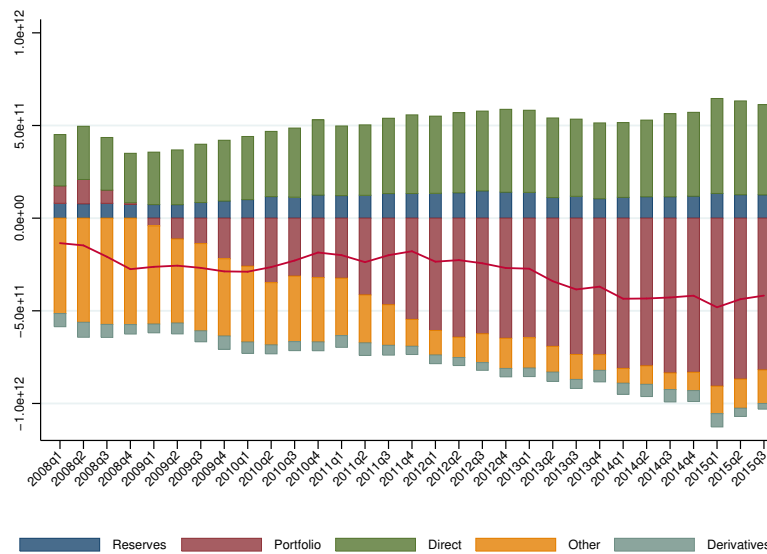
## C.1 Figures

Figure C.1: International Investment Position of France from 2008Q1 to 2015Q3.

This figure shows the evolution of the breakdown of gross (Panel (a)) and net (Panel (b)) external assets and liabilities of French residents into the five categories: reserves, portfolio investments, direct investments, other investments, and derivatives. Amounts are in euros.



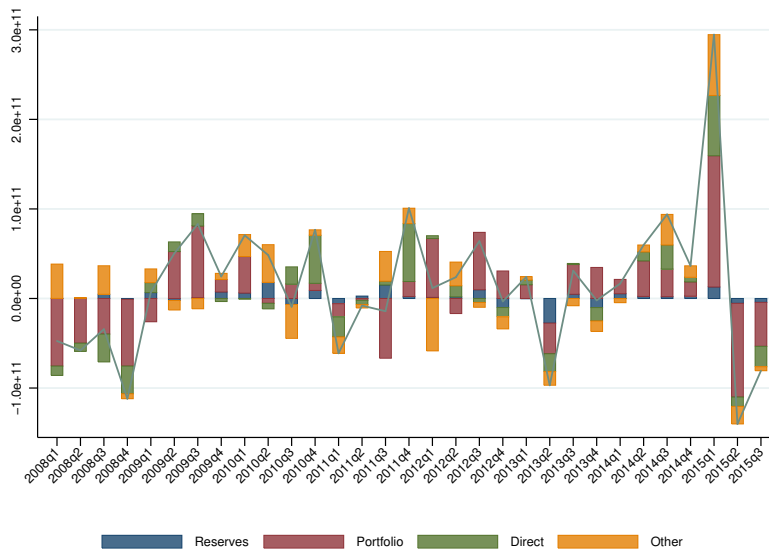
(a) Gross External Asset and Liability Positions



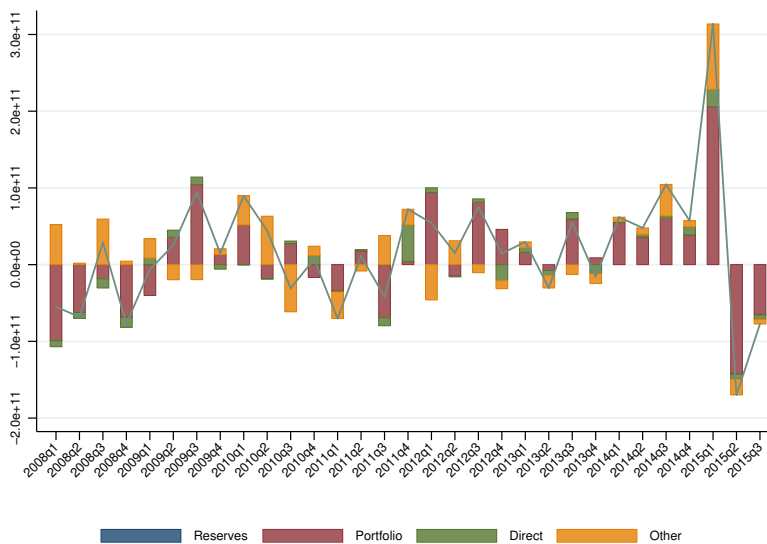
(b) Net (Asset minus Liability) External Position

Figure C.2: Change of the International Investment Position of France due to Valuations from 2008Q1 to 2015Q3.

*This figure shows the evolution of the change of the external asset (Panel (a)) and liability (Panel (b)) positions of French residents due to valuation effects for reserves, portfolio investments, direct investments and other investments. Amounts are in euros.*



(a) Valuation of External Assets

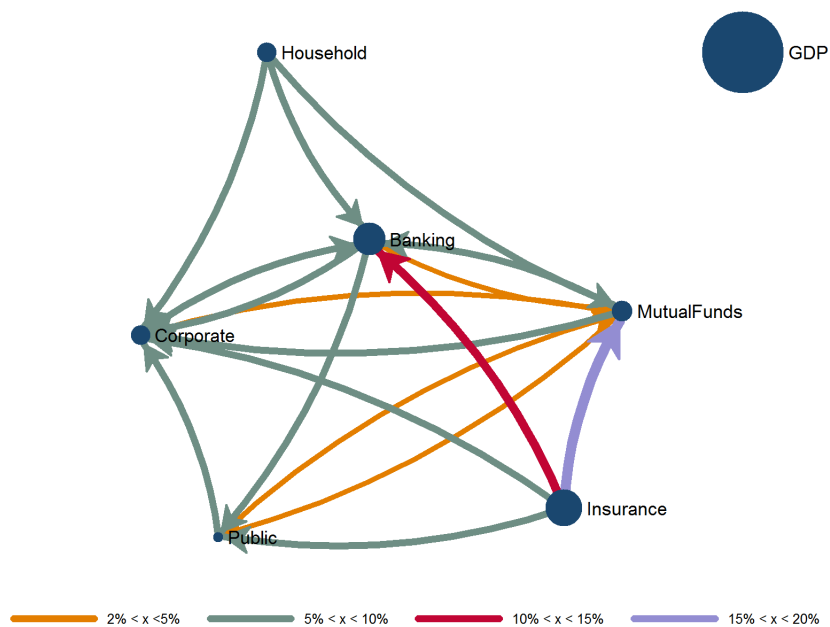


(b) Valuation of External Liabilities

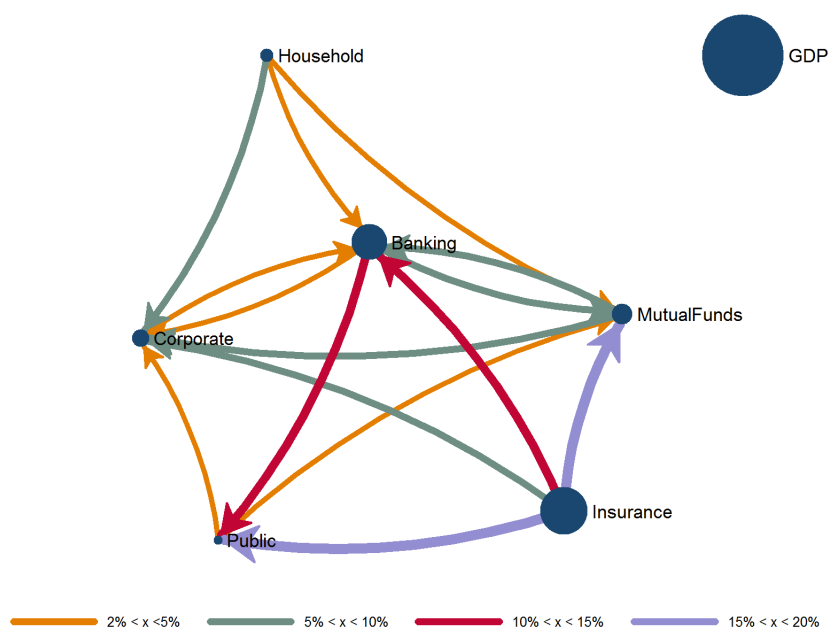


Figure C.3: Network of Domestic Sectors in 2008Q1 and 2014Q1.

*This figure shows bilateral exposures between the domestic institutional sectors of France in percent of annual French GDP. Each node represents an institutional sector. The size of the node is proportional to the sector's total assets. A parting arrow illustrates a sector's exposure to liabilities of the arriving sector. The size of the arrow is proportional to the bilateral exposure. A node of the size of French GDP is plotted at the top-right of the Figure to provide a reference for the scale of the nodes.*



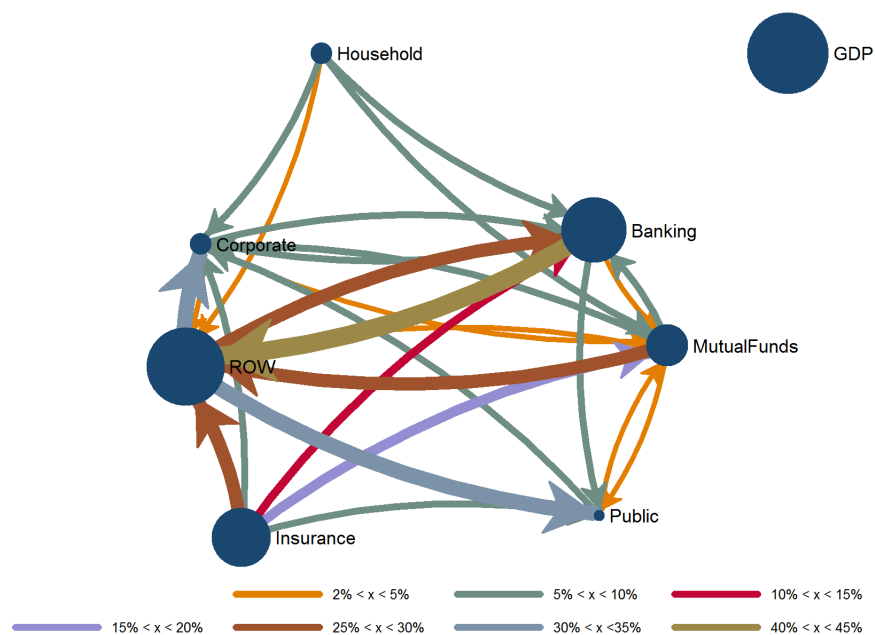
(a) 2008Q1



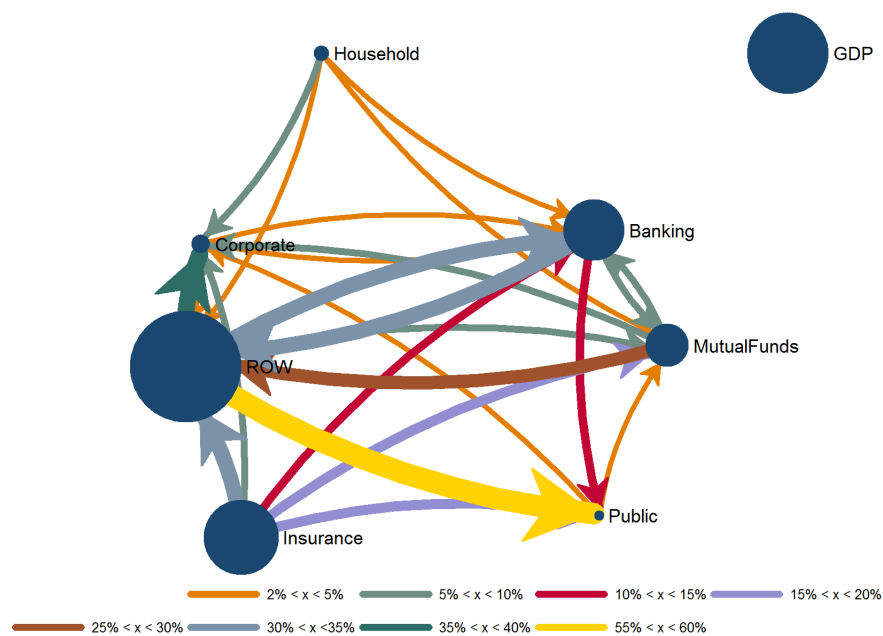
(b) 2014Q1

Figure C.4: Network of Domestic Sectors and the Rest of the World in 2008Q1 and 2014Q1.

This figure shows bilateral exposures between the domestic institutional sectors of France and the rest of the world in percent of annual French GDP. Each node represents an institutional sector. The size of the node is proportional to the sector's total assets. A parting arrow illustrates a sector's exposure to liabilities of the arriving sector. The size of the arrow is proportional to the bilateral exposure.



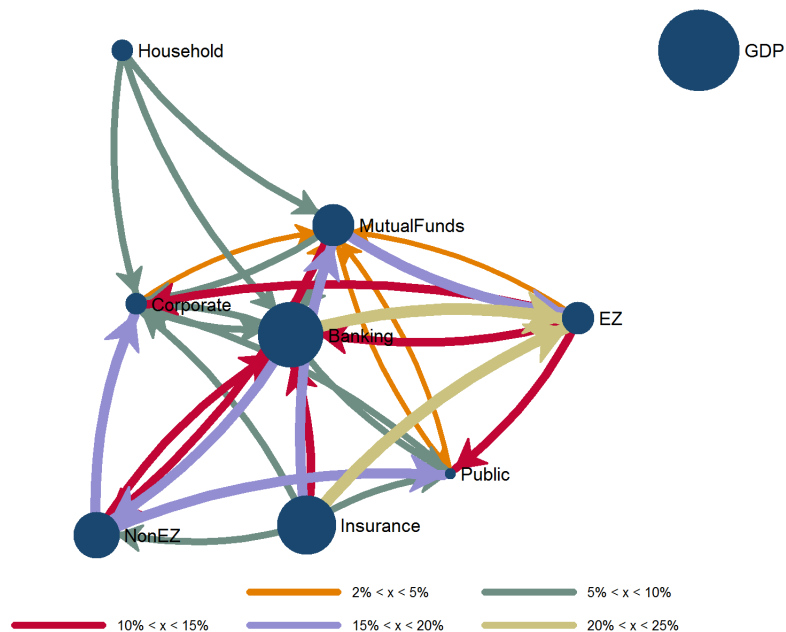
(a) 2008Q1



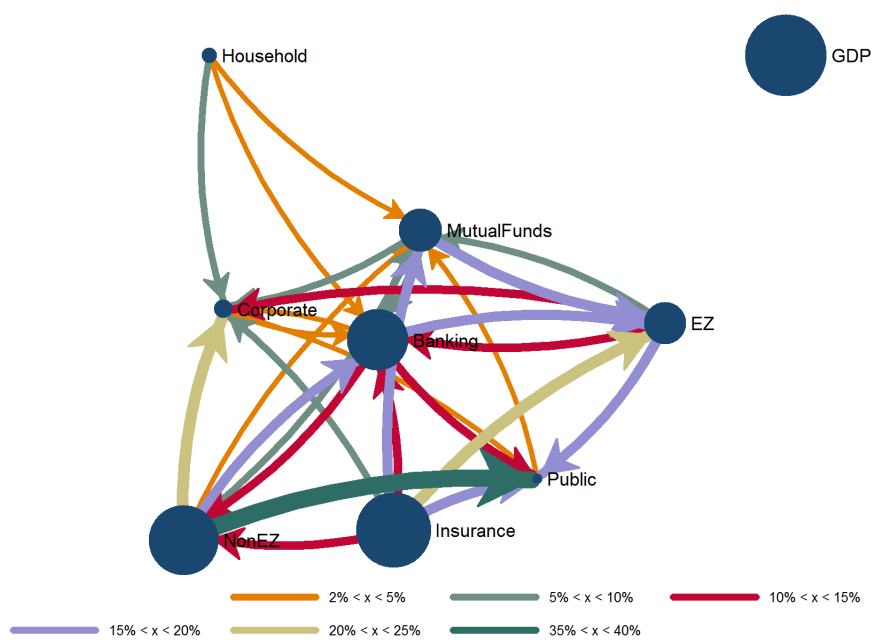
(b) 2014Q1

Figure C.5: Network of Domestic Sectors, the rest of the Eurozone, and the Non-Eurozone in 2008Q1 and 2014Q1.

This figure shows bilateral exposures between the domestic institutional sectors of France, the rest of the Eurozone (excluding France), and the Non-Eurozone in percent of annual French GDP. Each node represents an institutional sector. The size of the node is proportional to the sector's total assets. A parting arrow illustrates a sector's exposure to liabilities of the arriving sector. The size of the arrow is proportional to the bilateral exposure.



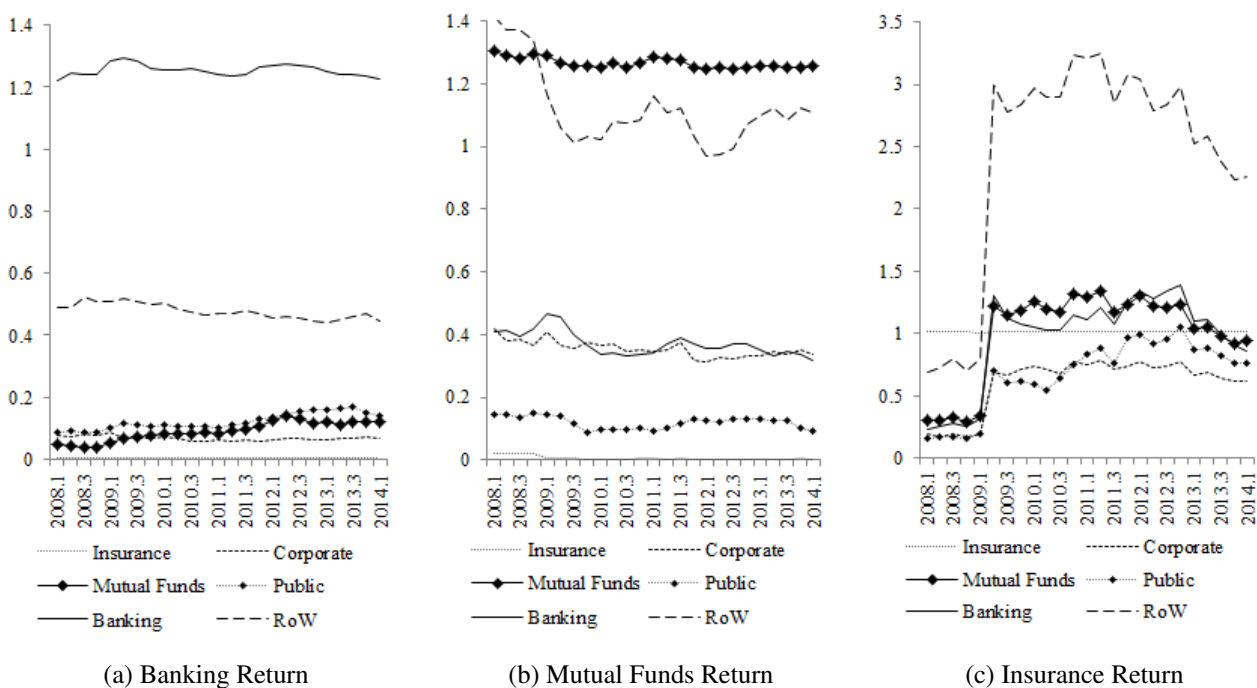
(a) 2008Q1



(b) 2014Q1

Figure C.6: Evolution of Balance-Sheet Contagion Effects from 2008Q1 to 2014Q1.

This figure shows the effect of a sector-specific shock on returns after full propagation. In particular, Panel (a) shows how the return on securities issued by the banking sector depend on a 1 ppt shock originating in different sectors of the economy. It is given by the rows of the contagion matrix  $C(\beta)_t$ , which varies strongly over time due to changes in the network of bilateral sectoral exposures.



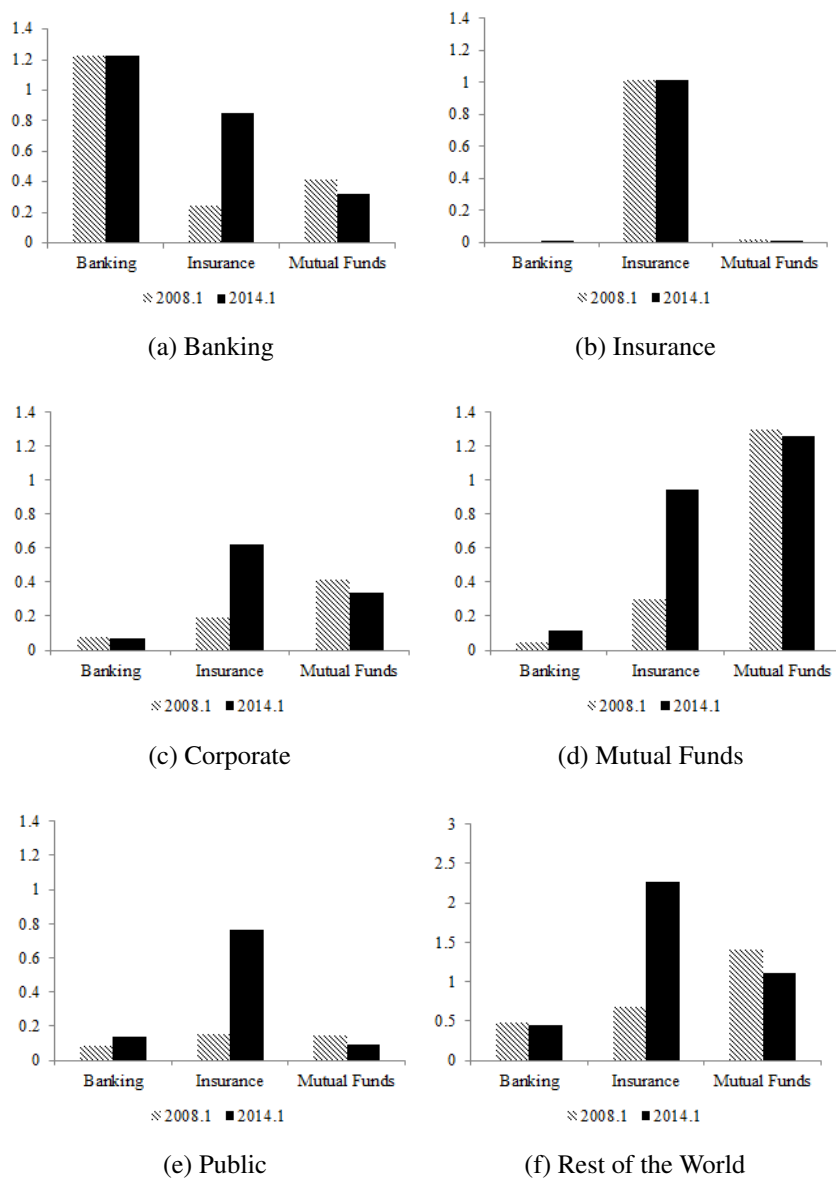
(a) Banking Return

(b) Mutual Funds Return

(c) Insurance Return

Figure C.7: Balance-Sheet Contagion Effects in 2008Q1 and 2014Q1.

Each panel compares the effect on returns on different securities of a shock on a specific sector. For example, Panel (a) shows the effect on the return of securities issued by the banking sector, the insurance sector and mutual fund sector of a 1ppt shock on the Banking sector in 2008Q1 and 2014Q1.





## C.2 Tables

Table C.1: External Portfolios of French Sectors in 2008Q1 and 2014Q1: Total, Equity, and Debt.

*This table decomposes the external asset (A), liability (L) and net ( $\Delta$ , Asset-Liability) portfolio positions of French sectors (Panel A) into equity (Panel B) and debt (Panel C) portfolios. Numbers are in percent of French GDP.*

|                        | 2008Q1 |       |          | 2014Q1 |        |          |
|------------------------|--------|-------|----------|--------|--------|----------|
|                        | A      | L     | $\Delta$ | A      | L      | $\Delta$ |
| <i>Panel A: Total</i>  |        |       |          |        |        |          |
| Banking                | 41.14  | 25.50 | 15.64    | 31.88  | 31.35  | 0.52     |
| Mutual Funds           | 26.85  | 4.96  | 21.89    | 28.37  | 9.61   | 18.76    |
| Insurance              | 26.83  | 1.91  | 24.92    | 33.11  | 0.57   | 32.54    |
| Corporate              | 2.40   | 30.80 | -28.40   | 2.57   | 38.87  | -36.30   |
| Household              | 2.61   | 0.00  | 2.61     | 2.99   | 0.00   | 2.99     |
| Public                 | 1.40   | 32.40 | -31.00   | 1.50   | 56.57  | -55.07   |
| Total                  | 100.26 | 95.57 | 4.70     | 101.31 | 136.97 | -35.66   |
| <i>Panel B: Equity</i> |        |       |          |        |        |          |
| Banking                | 4.49   | 4.14  | 0.35     | 5.04   | 3.50   | 1.55     |
| Mutual Funds           | 14.80  | 3.93  | 10.87    | 15.27  | 6.33   | 8.94     |
| Insurance              | 2.41   | 1.35  | 1.06     | 3.81   | 0.17   | 3.64     |
| Corporate              | 1.08   | 22.92 | -21.84   | 1.34   | 24.54  | -23.19   |
| Household              | 1.34   | 0.00  | 1.34     | 1.37   | 0.00   | 1.37     |
| Public                 | 0.71   | 0.00  | 0.71     | 0.74   | 0.00   | 0.74     |
| Total                  | 24.22  | 32.34 | -8.12    | 27.95  | 34.54  | -6.59    |
| <i>Panel C: Debt</i>   |        |       |          |        |        |          |
| Banking                | 36.65  | 21.36 | 15.29    | 26.83  | 27.86  | -1.02    |
| Mutual Funds           | 12.05  | 1.03  | 11.02    | 13.10  | 3.28   | 9.82     |
| Insurance              | 24.42  | 0.56  | 23.86    | 29.30  | 0.40   | 28.91    |
| Corporate              | 1.32   | 7.88  | -6.56    | 1.22   | 14.33  | -13.11   |
| Household              | 1.27   | 0.00  | 1.27     | 1.62   | 0.00   | 1.62     |
| Public                 | 0.69   | 32.40 | -31.71   | 0.75   | 56.57  | -55.82   |
| Total                  | 76.04  | 63.23 | 12.81    | 73.37  | 102.43 | -29.06   |

Table C.2: External Portfolios of French Sectors in 2008Q1 and 2014Q1: Change Over Time.

*This table summarizes the change between 2008Q1 and 2014Q1 in external asset (A), liability (L) and net ( $\Delta$ , Asset-Liability) portfolio positions of French sectors as exhibited in Table C.2. Numbers are in percentage points of French GDP. Changes smaller than 1 ppts are set to zero.*

|              | Debt |       |          | Equity |      |          | Total    |
|--------------|------|-------|----------|--------|------|----------|----------|
|              | A    | L     | $\Delta$ | A      | L    | $\Delta$ | $\Delta$ |
| Banking      | -9.8 | +6.5  | -16.3    | 0.0    | 0.0  | +1.2     | -15.1    |
| Mutual Funds | +1.1 | +2.3  | -1.2     | 0.0    | +2.4 | -1.9     | -3.1     |
| Insurance    | +4.9 | 0.0   | +5.1     | +1.4   | -1.2 | +2.6     | +7.6     |
| Corporate    | 0.0  | +6.5  | -6.6     | 0.0    | +1.6 | -1.4     | -7.9     |
| Household    | 0.0  | 0.0   | 0.0      | 0.0    | 0.0  | 0.0      | 0.0      |
| Public       | 0.0  | +24.2 | -24.1    | 0.0    | 0.0  | 0.0      | -24.1    |
| Total        | -2.7 | +39.2 | -41.9    | +3.7   | +2.2 | +1.5     | -40.3    |

Table C.3: External Portfolios of French Sectors in 2008Q1 and 2014Q1: Eurozone and Non-Eurozone.

*This table decomposes the external asset (A), liability (L) and net ( $\Delta$ , Asset-Liability) portfolio positions of French sectors into Eurozone (Panel A) and Non-Eurozone (Panel B) portfolios. Numbers are in percent of French GDP.*

|                              | 2008Q1 |       |          | 2014Q1 |       |          |
|------------------------------|--------|-------|----------|--------|-------|----------|
|                              | A      | L     | $\Delta$ | A      | L     | $\Delta$ |
| <i>Panel A: Eurozone</i>     |        |       |          |        |       |          |
| Banking                      | 22.90  | 11.56 | 11.34    | 18.49  | 13.64 | 4.85     |
| Mutual Funds                 | 15.54  | 3.23  | 12.31    | 18.94  | 6.30  | 12.64    |
| Insurance                    | 20.42  | 0.48  | 19.94    | 22.56  | 0.25  | 22.31    |
| Corporate                    | 1.69   | 11.23 | -9.54    | 1.98   | 14.25 | -12.27   |
| Household                    | 1.32   | 0.00  | 1.32     | 1.50   | 0.00  | 1.50     |
| Public                       | 0.88   | 12.64 | -11.76   | 0.97   | 17.02 | -16.05   |
| Total                        | 62.41  | 39.13 | 23.28    | 65.09  | 51.46 | 13.62    |
| <i>Panel B: Non-Eurozone</i> |        |       |          |        |       |          |
| Banking                      | 18.24  | 13.94 | 4.30     | 13.39  | 17.71 | -4.32    |
| Mutual Funds                 | 11.31  | 1.73  | 9.58     | 9.43   | 3.31  | 6.12     |
| Insurance                    | 6.41   | 1.43  | 4.98     | 10.55  | 0.32  | 10.24    |
| Corporate                    | 0.71   | 19.57 | -18.86   | 0.59   | 24.62 | -24.03   |
| Household                    | 1.28   | 0.00  | 1.28     | 1.49   | 0.00  | 1.49     |
| Public                       | 0.53   | 19.76 | -19.24   | 0.52   | 39.55 | -39.02   |
| Total                        | 37.85  | 56.44 | -18.58   | 36.23  | 85.51 | -49.28   |



Table C.4: External Portfolios of French Sectors in 2008Q1 and 2014Q1: GIPS and Non-GIPS.

*This table decomposes the external asset portfolio positions of French sectors into GIPS and Non-GIPS portfolios. Numbers are in percent of French GDP. GIPS comprises Greece, Italy, Portugal and Spain.*

|              | GIPS   |        |        | Non-GIPS |        |        |
|--------------|--------|--------|--------|----------|--------|--------|
|              | 2008Q1 | 2014Q1 | Change | 2008Q1   | 2014Q1 | Change |
| Banking      | 10.54  | 4.67   | -5.87  | 12.34    | 10.99  | -1.35  |
| Mutual Funds | 3.88   | 2.20   | -1.68  | 11.65    | 13.61  | 1.96   |
| Insurance    | 8.36   | 4.56   | -3.80  | 12.05    | 15.24  | 3.19   |
| Corporate    | 0.38   | 0.44   | 0.06   | 1.31     | 1.37   | 0.06   |
| Household    | 0.11   | 0.30   | 0.19   | 1.23     | 1.41   | 0.18   |
| Public       | 0.26   | 0.07   | -0.19  | 0.59     | 0.79   | 0.20   |
| Total        | 23.53  | 12.24  | -11.29 | 39.17    | 43.41  | 4.24   |

Table C.5: External Portfolios of French Sectors in 2008Q1 and 2014Q1: Change Over Time Due To Valuation (Exchange Rate and Price), and Flows.

*This table decomposes the change between 2008Q1 and 2014Q1 of external asset (A), liability (L) and net ( $\Delta$ , Asset-Liability) portfolio positions (total, equity, and debt) of French sectors into valuation due to exchange rate movements (FX), due to price movements, and flows. Numbers are in percent of French GDP.*

|                        | Assets |       |        | Liabilities |       |       | Net   |       |        |
|------------------------|--------|-------|--------|-------------|-------|-------|-------|-------|--------|
|                        | FX     | Price | Flow   | FX          | Price | Flow  | FX    | Price | Flow   |
| <i>Panel A: Total</i>  |        |       |        |             |       |       |       |       |        |
| Banking                | 0.87   | 0.44  | -9.00  | 0.58        | 1.62  | 4.82  | 0.29  | -1.18 | -13.82 |
| Mutual Funds           | 0.61   | -0.63 | 2.65   | 0.06        | 0.95  | 2.45  | 0.54  | -1.58 | 0.20   |
| Insurance              | 0.07   | 3.03  | 4.85   | 0.01        | -0.38 | 0.38  | 0.06  | 3.41  | 4.47   |
| Corporate              | 0.02   | 0.03  | 0.29   | 0.28        | 2.14  | 7.67  | -0.26 | -2.11 | -7.37  |
| Household              | 0.07   | 0.04  | 0.47   | 0.00        | 0.00  | 0.00  | 0.07  | 0.04  | 0.47   |
| Public                 | 0.06   | 0.06  | 0.07   | 0.16        | 3.89  | 21.96 | -0.10 | -3.84 | -21.89 |
| Total                  | 1.64   | 3.10  | 1.06   | 1.09        | 8.22  | 37.27 | 0.55  | -5.12 | -36.21 |
| <i>Panel B: Equity</i> |        |       |        |             |       |       |       |       |        |
| Banking                | 0.06   | -0.75 | 1.08   | 0.01        | -0.49 | 0.17  | 0.06  | -0.26 | 0.91   |
| Mutual Funds           | 0.46   | -1.25 | 2.01   | 0.01        | 0.67  | 0.90  | 0.45  | -1.92 | 1.11   |
| Insurance              | 0.05   | 0.00  | 1.60   | 0.00        | -0.32 | 0.02  | 0.05  | 0.32  | 1.58   |
| Corporate              | 0.01   | -0.04 | 0.38   | 0.00        | 0.87  | 2.40  | 0.01  | -0.92 | -2.02  |
| Household              | 0.07   | 0.00  | 0.08   | 0.00        | 0.00  | 0.00  | 0.07  | 0.00  | 0.08   |
| Public                 | 0.04   | -0.04 | 0.08   | 0.00        | 0.00  | 0.00  | 0.04  | -0.04 | 0.08   |
| Total                  | 0.67   | -1.96 | 6.08   | 0.01        | 0.73  | 3.50  | 0.66  | -2.70 | 2.58   |
| <i>Panel C: Debt</i>   |        |       |        |             |       |       |       |       |        |
| Banking                | 0.81   | 1.18  | -10.08 | 0.57        | 2.10  | 4.64  | 0.23  | -0.92 | -14.72 |
| Mutual Funds           | 0.15   | 0.62  | 0.64   | 0.06        | 0.28  | 1.55  | 0.09  | 0.34  | -0.91  |
| Insurance              | 0.01   | 3.03  | 3.25   | 0.01        | -0.06 | 0.36  | 0.01  | 3.09  | 2.89   |
| Corporate              | 0.00   | 0.07  | -0.09  | 0.28        | 1.27  | 5.26  | -0.27 | -1.19 | -5.35  |
| Household              | 0.00   | 0.05  | 0.39   | 0.00        | 0.00  | 0.00  | 0.00  | 0.05  | 0.39   |
| Public sector          | 0.02   | 0.10  | -0.01  | 0.16        | 3.89  | 21.96 | -0.15 | -3.79 | -21.97 |
| Total                  | 0.96   | 5.06  | -5.01  | 1.07        | 7.48  | 33.78 | -0.11 | -2.42 | -38.79 |

Table C.6: External Portfolios of French Sectors in 2008Q1 and 2014Q1: Eurozone Sectors.

*This table decomposes the external asset and liability portfolio positions of French sectors into portfolios with four institutional sectors of the rest of the eurozone, i.e. banking, corporate and household, mutual funds and insurance, as well as the public sector. Numbers are in percent of French GDP.*

|                        | Assets  |                   |                    |        |       | Liabilities |                   |                    |        |       |
|------------------------|---------|-------------------|--------------------|--------|-------|-------------|-------------------|--------------------|--------|-------|
|                        | Banking | Corp. & Household | Mutual F. & Insur. | Public | Total | Banking     | Corp. & Household | Mutual F. & Insur. | Public | Total |
| <i>Panel A: 2008Q1</i> |         |                   |                    |        |       |             |                   |                    |        |       |
| Banking                | 7.73    | 7.88              | 0.75               | 6.52   | 22.88 | 11.03       | 0.37              | 0.17               | 0.00   | 11.57 |
| Mutual Funds           | 2.60    | 6.02              | 4.07               | 2.84   | 15.53 | 2.60        | 0.22              | 0.40               | 0.01   | 3.23  |
| Insurance              | 5.50    | 4.88              | 0.86               | 9.17   | 20.41 | 0.46        | 0.01              | 0.01               | 0.00   | 0.48  |
| Corporate              | 0.26    | 0.71              | 0.47               | 0.25   | 1.69  | 9.63        | 1.13              | 0.46               | 0.00   | 11.22 |
| Household              | 0.30    | 0.52              | 0.47               | 0.05   | 1.34  | 0.00        | 0.00              | 0.00               | 0.00   | 0.00  |
| Public                 | 0.18    | 0.32              | 0.06               | 0.29   | 0.85  | 12.43       | 0.00              | 0.20               | 0.00   | 12.63 |
| Total                  | 16.57   | 20.33             | 6.68               | 19.12  | 62.7  | 36.15       | 1.73              | 1.24               | 0.01   | 39.13 |
| <i>Panel B: 2014Q1</i> |         |                   |                    |        |       |             |                   |                    |        |       |
| Banking                | 4.67    | 1.26              | 4.94               | 7.62   | 18.49 | 12.53       | 0.49              | 0.63               | 0.00   | 13.65 |
| Mutual Funds           | 2.20    | 5.30              | 7.90               | 3.55   | 18.95 | 5.35        | 0.16              | 0.79               | 0.00   | 6.30  |
| Insurance              | 4.56    | 3.29              | 7.51               | 7.20   | 22.56 | 0.23        | 0.00              | 0.01               | 0.00   | 0.24  |
| Corporate              | 0.44    | 0.34              | 1.02               | 0.18   | 1.98  | 12.2        | 1.38              | 0.67               | 0.00   | 14.25 |
| Household              | 0.30    | 0.40              | 0.78               | 0.03   | 1.51  | 0.00        | 0.00              | 0.00               | 0.00   | 0.00  |
| Public                 | 0.07    | 0.15              | 0.53               | 0.23   | 0.98  | 15.55       | 0.03              | 1.45               | 0.00   | 17.03 |
| Total                  | 12.24   | 10.74             | 22.68              | 18.81  | 64.47 | 45.86       | 2.06              | 3.55               | 0.00   | 51.47 |

Table C.7: Portfolios of French Sectors in 2008Q1 and 2014Q1: Domestic, Foreign, and Consolidated.

*This table presents asset (A), liability (L) and net ( $\Delta$ , Asset-Liability) positions in domestic (Panel A), foreign (Panel B), and consolidated (Panel C) portfolios of French sectors. Numbers are in percent of French GDP.*

|                              | 2008Q1 |        |          | 2014Q1 |        |          |
|------------------------------|--------|--------|----------|--------|--------|----------|
|                              | A      | L      | $\Delta$ | A      | L      | $\Delta$ |
| <i>Panel A: Domestic</i>     |        |        |          |        |        |          |
| Banking                      | 38.82  | 56.11  | -17.28   | 43.22  | 49.13  | -5.91    |
| Mutual Funds                 | 24.30  | 39.42  | -15.12   | 24.40  | 42.21  | -17.81   |
| Insurance                    | 44.94  | 2.59   | 42.34    | 58.10  | 0.73   | 57.37    |
| Corporate                    | 23.48  | 47.59  | -24.12   | 20.32  | 46.89  | -26.57   |
| Household                    | 23.47  | 0.00   | 23.47    | 15.26  | 0.00   | 15.26    |
| Public                       | 11.78  | 21.42  | -9.63    | 9.87   | 31.82  | -21.95   |
| Total                        | 167.13 | 167.13 | 0.00     | 170.77 | 170.77 | 0.00     |
| <i>Panel B: Foreign</i>      |        |        |          |        |        |          |
| Banking                      | 41.14  | 25.50  | 15.64    | 31.88  | 31.35  | 0.52     |
| Mutual Funds                 | 26.85  | 4.96   | 21.89    | 28.37  | 9.61   | 18.76    |
| Insurance                    | 26.83  | 1.91   | 24.92    | 33.11  | 0.57   | 32.54    |
| Corporate                    | 2.40   | 30.80  | -28.40   | 2.57   | 38.87  | -36.30   |
| Household                    | 2.61   | 0.00   | 2.61     | 2.99   | 0.00   | 2.99     |
| Public                       | 1.40   | 32.40  | -31.00   | 1.50   | 56.57  | -55.07   |
| Total                        | 100.26 | 95.57  | 4.70     | 101.31 | 136.97 | -35.66   |
| <i>Panel C: Consolidated</i> |        |        |          |        |        |          |
| Banking                      | 79.96  | 81.61  | -1.64    | 75.09  | 80.48  | -5.39    |
| Mutual Funds                 | 51.15  | 44.38  | 6.77     | 52.77  | 51.82  | 0.95     |
| Insurance                    | 71.77  | 4.50   | 67.27    | 91.21  | 1.30   | 89.91    |
| Corporate                    | 25.87  | 78.39  | -52.52   | 22.88  | 85.76  | -62.87   |
| Household                    | 26.07  | 0.00   | 26.07    | 18.25  | 0.00   | 18.25    |
| Public                       | 13.18  | 53.82  | -40.64   | 11.37  | 88.39  | -77.02   |
| Total                        | 267.4  | 262.70 | 4.70     | 272.09 | 307.75 | -35.66   |

Table C.8: Portfolios of French Sectors in 2008Q1 and 2014Q1: Share of External Positions.

*This table shows the share of external positions in asset and liability portfolios of French sectors in 2008Q1, 2014Q1, and the change over time. Numbers are in percent of the total sectoral portfolio.*

|              | Assets |        |        | Liabilities |        |        |
|--------------|--------|--------|--------|-------------|--------|--------|
|              | 2008Q1 | 2014Q1 | Change | 2008Q1      | 2014Q1 | Change |
| Banking      | 51.45  | 42.45  | -9.00  | 31.25       | 38.96  | 7.71   |
| Mutual Funds | 52.49  | 53.76  | 1.27   | 11.18       | 18.54  | 7.37   |
| Insurance    | 37.39  | 36.30  | -1.08  | 42.39       | 43.85  | 1.46   |
| Corporate    | 9.27   | 11.21  | 1.95   | 39.29       | 45.33  | 6.04   |
| Household    | 9.99   | 16.40  | 6.41   | 0.00        | 0.00   | 0.00   |
| Public       | 10.63  | 13.17  | 2.54   | 60.21       | 64.00  | 3.79   |
| Total        | 37.50  | 37.24  | -0.26  | 36.38       | 44.51  | 8.13   |

Table C.9: Balance-Sheet Contagion Model: Estimation Results.

*This table presents the GMM estimates of the structural parameters of the balance-sheet contagion model, i.e. the balance-sheet contagion parameters  $\beta$  and the variance of sectoral shocks  $\sigma_1^2, \dots, \sigma_5^2$  for different common shock structures: Column (1) shows the estimation results assuming no common shock; results for a shock common to all sectors (global), common to financial sectors only (financial), and common to real sectors (real) are shown in columns (2), (3), and (4), respectively. Significance is noted "\*\*\*\*" at 1%, "\*\*\*" at 5%, and "\*" at 10%. Standard errors are reported in brackets.*

|                       | (1)                |                       | (2)                |                       | (3)                |                        | (4)                |                       |
|-----------------------|--------------------|-----------------------|--------------------|-----------------------|--------------------|------------------------|--------------------|-----------------------|
|                       | $\beta$            | $\sigma^2$            | $\beta$            | $\sigma^2$            | $\beta$            | $\sigma^2$             | $\beta$            | $\sigma^2$            |
| Banking               | 0.71***<br>(0.247) | 9.2e-5***<br>(3.0e-5) | 0.55**<br>(0.283)  | 7.1e-5*<br>(5.0e-5)   | 0.73***<br>(0.297) | 1.0e-4**<br>(6.0e-5)   | 0.58***<br>(0.233) | 2.9e-5***<br>(3.0e-5) |
| Insurance             | 0.06***<br>(0.016) | 3.1e-3***<br>(9.6e-4) | 0.05***<br>(0.017) | 3.1e-3**<br>(1.51e-3) | 0.06***<br>(0.017) | 3.7e-3***<br>(1.17e-3) | 0.06*<br>(0.036)   | 3.9e-3<br>(1.45e-3)   |
| Mutual Funds          | 1.56***<br>(0.269) | 1.7e-4<br>(1.7e-4)    | 1.49***<br>(0.243) | 1.1e-4<br>(1.3e-4)    | 1.52***<br>(0.22)  | 1.3e-4<br>(1.3e-4)     | 1.48***<br>(0.167) | 1.0e-4<br>(1.2e-4)    |
| Corporate             | 0.86<br>(33.884)   | 8.5e-3<br>(0.158)     | -0.46<br>(57.645)  | 1.2e-2<br>(0.339)     | 0.11<br>(56.720)   | 1.3e-2<br>(0.324)      | 0.20<br>(77.776)   | 1.8e-2<br>(0.414)     |
| Public                | -0.41<br>(0.911)   | 2.6e-4***<br>(5.0e-5) | -0.33<br>(0.877)   | 2.5e-4***<br>(5.0e-5) | -0.35<br>(0.957)   | 2.6e-4***<br>(5.0e-5)  | -0.02<br>(1.277)   | 6.1e-4<br>(0.004)     |
| Common Shock Variance |                    |                       |                    |                       |                    |                        |                    |                       |
| Global                | No                 |                       |                    | 4.1e-6<br>(1.0e-5)    | No                 |                        | No                 |                       |
| Financial             | No                 |                       | No                 |                       | 2.6e-6<br>(1.7e-5) |                        | No                 |                       |
| Real                  | No                 |                       | No                 |                       | No                 |                        | 3.5e-4<br>(0.004)  |                       |