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Discipline : Analyse et Politique Économiques

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Essays on Asymmetric Information and Regulation

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À ma grand-mère, Danielle

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RÉSUMÉ

Cette thèse est constituée de trois essais indépendants, chacun traitant d'un problème théorique particulier relatif aux questions d'asymétrie d'information ou de régulation.

Le premier chapitre s'intéresse à l'analyse des mécanismes efficaces de dissolution de partenariats en présence de contraintes financières. Ces mécanismes sont susceptibles de s'appliquer à de nombreuses situations telles que, par exemple, la dissolution des *joint ventures*, les procédures de faillite, les divorces ou encore la redistribution des terres agricoles. Ce travail propose une condition nécessaire et suffisante d'existence des mécanismes dits de «premier rang» lorsque les partenaires sont initialement contraints financièrement. Il est également montré comment mettre en œuvre ces mécanismes par le biais d'une enchère. L'existence de ces mécanismes de premier rang est étroitement liée à la répartition initiale des droits de propriétés ainsi qu'à celle des ressources financières entre les partenaires. Les répartitions homogènes des droits de propriétés initiaux et des ressources financières sont en général préférables. Lorsque les mécanismes de premier rang ne sont pas réalisables, ce travail propose également une caractérisation des mécanismes de second rang.

Le second chapitre étudie la régulation optimale d'entreprises dont l'activité peut provoquer des dommages à des tiers. Le cœur de l'analyse consiste à déterminer s'il est plus efficace d'attribuer les pénalités financières, en cas d'accident avéré, au propriétaire de l'entreprise ou bien à son employé en charge de veiller à la sûreté de l'activité. Dès lors que l'employé est sujet à des contraintes financières, il est montré que la régulation optimale consiste à attribuer la totalité des pénalités financières au propriétaire de l'entreprise, sans quoi, le niveau des mesures de précaution est sous-optimal. Si, au contraire, le niveau des mesures de précaution dépend des actions jointes de deux agents plutôt que d'un seul, tous deux contraints financièrement, alors la régulation optimale consiste à attribuer les

pénalités de manière égale aux deux agents.

Le troisième chapitre traite de la régulation optimale des aéroports. Il met en lumière l'importance de réguler de façon jointe les prix des services aéronautiques, ceux des services commerciaux ainsi que le niveau des investissements dans l'infrastructure aéroportuaire. Les régulations traditionnellement utilisées y sont analysées et il est montré que celles-ci ne sont généralement pas optimales. En particulier, elles ne semblent pas permettre de fournir des incitations à investir suffisantes aux aéroports. Une méthode est proposée pour corriger ces régulations. Les conséquences de la séparation verticale entre un aéroport et les compagnies aériennes ainsi que l'observabilité des niveaux d'investissement sur la régulation optimale sont également analysées.

Mot-clés : *Asymétrie d'information, Régulation, Mécanismes incitatifs, Sélection Adverse, Aléa Moral, Partenariat, Contraintes Financières, Aéroports.*

SUMMARY

This thesis consists of three self-contained essays, each dealing with a particular theoretical issue in asymmetric information problems and regulation.

The first chapter is devoted to the analysis of partnership dissolution mechanisms when partners face limited financial resources. Those mechanisms cover a large range of applications such as termination of joint ventures, bankruptcy procedures, divorce or, land reallocation. The work presents an existence condition of first-best dissolution mechanisms with financially-constrained partners as well as a methodology to implement those mechanisms in practice. It is shown that a well-balanced distribution of initial ownership rights and financial resources among partners is crucial for the existence of the first-best solution - more extreme distributions generally lead to a failure of negotiations on efficient dissolution. Second-best mechanisms are also investigated as well as generalizations to richer settings.

The second chapter considers the optimal regulation of firms whose activity may be harmful to society. The analysis focuses on determining whether monetary punishments should target more the owner of the firm or the employee in charge of taking precautionary measures. When the employee has limited financial resources, the analysis shows that monetary punishments should be imposed solely on the owner of the firm if the regulator wants the firm to take appropriate precautionary measures. An extension to the case of two financially-constrained individuals jointly in charge of safety shows that monetary punishments should be evenly distributed among the two injurers.

The third chapter addresses the issue of regulating airports. It sheds light on the importance of jointly regulating the price of aeronautical services, the one for commercial services offered at the airport as well as the level of investment chosen by the airport. An investigation of traditional regulations shows that they generally fail to implement

the optimal regulation as they do not provide sufficient incentives to invest. A corrective regulation is proposed to tackle this issue. The consequences of the nature of the airport-airline relationship and the observability of investment on the optimal regulation are also investigated.

Keywords: *Asymmetric Information, Regulation, Mechanism Design, Adverse Selection, Moral Hazard, Partnership, Financial Constraints, Airport regulation.*

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GENERAL INTRODUCTION

Organizations and firms are complex entities in which several individuals have their own characteristics, knowledge, and goals. Formal or informal contracts among their members govern the nature of their relationship, assign responsibilities, and provide incentives to fulfill the purpose of the organization. A fundamental question is whether those organizations are adequately designed to achieve efficiency and, if not, how can they be improved? For economists, the problem is twofold. On the one hand, a wide range of inefficiencies arise internally within the firm due to asymmetric information, allocation of ownership, bargaining power and authority, financial constraints and capital structure, performance measurement, diverging priorities and goals to mention just a few. On the other hand, it is most often the case that organizations or their members pursue goals that differ from those promoted by economists. Generally, economists tend to focus on what is best for *society* and not necessarily for the interest of organizations themselves. For instance, a pharmaceutical firm might want to withhold a drug patent as long as possible to enjoy a monopoly position while it would be socially desirable – from the point of view of the economist – to authorize other producers to distribute a generic drug. Environmental protection is another important concern for society that firms, absent any regulation, are likely to disregard. Hence, as economists, we have not only to understand how organizations work and what are their sources of inefficiencies but we also have to determine what goals should be promoted and how to make organizations comply with them.

Far from the neoclassical theory that viewed firms solely as a production function combining inputs such as capital and labor to produce output, recent theories consider organizations and firms as a collection of individuals with dispersed information, often conflicting interests and generally some degree of complementarity. One of the most important breakthroughs in the understanding of organizations is undoubtedly the

paradigm of asymmetric information and conflicting interests which led to what has been coined “The Theory of Incentives”. Even in an organization with a well-defined hierarchy of authority such as a principal-agent relationship, it is impossible to perfectly control and monitor the agent as long as they detain private information relevant to the organization. Think of how difficult it might be for a landlord to assess the effort of an agent in charge of the management of their property or for the State to evaluate the amount of subsidy that should receive an airport without being able to truly learn its cost structure. Those two examples correspond to the two types of private information considered by The Theory of Incentives: moral hazard (or hidden action) and adverse selection (or hidden knowledge). The private information forces the principal to design incentive schemes so as to elicit the agent’s information which requires to concede *information rent* to the agent. The problem with those frictions is not only that the agent enjoys larger payoffs at the expense of the principal, but mainly that it introduces distortions in decisions, output or actions of the organization. Other elements such as limited liability, risk aversion, performance measurement issues add up to this information problem and create further distortions. More complex environments in which several individuals have private information and must take a collective decision are often even more challenging. Hopefully, a wide range of theoretical models and empirical findings have allowed a better understanding of organizational design in the last forty years.

As mentioned earlier, economists not only endeavor to understand how organizations work but they also try to design solutions to make them function more effectively or more in accordance with society’s preferences. The difficulty of such a task is that a *designer* also faces informational constraints – sometimes even more severe than those faced by members of the organization – alongside administrative and legal constraints. The intervention of a designer cannot simply rule out the very existence of private information within organizations nor can it force its revelation. The designer must then resort to methods similar to those used within organizations. Administrative and legal constraints may further limit the scope of intervention through codes of regulation and tort or civil law.

The work presented in this thesis is to be viewed in this context. It is constituted by three independent papers, yet each deals with problems related to the economics of

organizations and investigates how they can be improved. The first chapter investigates the design of efficient dissolution mechanisms in partnerships with various initial ownership structures and limited financial resources. The second chapter is concerned with the optimal distribution of fines within a firm whose activity may cause an accident while some of its members may have insufficient financial resources to pay their share of fines. The third chapter studies the optimal design of airport regulation and its implementation. I briefly introduce each of these chapters below to provide the reader with an overview of the topics studied as well as their main results.

In the first chapter, entitled “Partnership Dissolution with Cash-Constrained Agents”, I investigate the design of trading mechanisms to efficiently reallocate an asset within a group of agents – a partnership – when they face limited financial resources. The study of efficient trading mechanisms goes back to the seminal work of [Myerson and Satterthwaite \(1983\)](#) who consider bilateral trade between a buyer and a seller who have both private information. Their setting is very general and encompasses many important problems, yet they show that ex post efficient trade is impossible. It is worth noticing that, contrary to the standard principal-agent problem in which the contracting frictions comes from the separation of who has the bargaining power (the principal) and who has the information (the agent), neither the buyer nor the seller has bargaining power in a formal way as trade is organized by a benevolent designer. Their impossibility result, therefore, seems that it is only due to the presence of two-sided asymmetric information. [Cramton, Gibbons, and Klemperer \(1987\)](#) generalize this trading problem to any number of agents – that they call *partners* – and assume that those agents could have initial ownership rights in the asset to be traded. The interpretation of their trading mechanism goes beyond the simple buyer-seller story and encompasses many more applications such as termination of joint ventures, divorces, bankruptcy procedures, or land reallocation among farmers. More importantly, they show that ex post efficient trade is, in fact, possible if initial ownership rights are evenly distributed among traders. Their result therefore reveals that the problem lies in the combination between multi-sided asymmetric information and initial allocation of property rights – the Myerson and Satterthwaite’s trading problem simply being an extreme case in which the seller has full initial ownership. Those findings clearly establish that the famous *Coase Theorem* ([Coase, 1960](#)), which stipulates that decentralized trade is efficient regardless of the initial allocation of property rights, is invalidated in the presence

of asymmetric information. The main lesson of [Cramton, Gibbons, and Klemperer \(1987\)](#) is therefore that successful negotiations to reallocate the asset among partners rely on the fact that no party is initially given excessive bargaining power through ownership rights. In my work, however, I argue that those equal-share ownership structures might not be desirable in the presence of cash-constrained agents. Instead, heavily financially constrained partners should be initially given larger ownership shares. Allocating initial ownership not only allocates bargaining power but it also affects the future volumes of trade. It is therefore better to reduce as much as possible the volume of trade for partners with low financial resources so as to minimize the share they might have to buy out to the others. The main economic insight is that achieving efficient reallocation of resources requires that liquid and illiquid assets are appropriately balanced among traders. I derive a necessary and sufficient condition for the feasibility of ex post efficient, interim incentive compatible, interim individually rational, ex post budget balance and ex post cash-constrained mechanisms. I propose a construction of those mechanisms as well as a way to implement it through a simple auction. I show that cash-constrained mechanisms are able to dissolve some partnerships that [Cramton, Gibbons, and Klemperer \(1987\)](#)'s mechanism fails to dissolve in the presence of limited cash resources. When the initial distribution of ownership and financial resources is such that efficient trading mechanisms do not exist, I investigate second-best mechanisms. I show that the final allocation of ownership should be distorted in favor of wealthy partners to overcome the problem of limited financial resources. Finally, my work extends to more general settings than partnership dissolution problems: It solves the general mechanism design problem of allocating private resources to cash-constrained agents with type-dependent utility outside options. For instance, the framework applies to privatization problems in which the state wants to allocate an initially publicly-owned asset (road, railway, real estate) to a private firm among a pool of candidates. The allocation mechanism that I propose takes into account the fact that each candidate has outside opportunities and limited cash resources.

In the second chapter, entitled "Optimal Structure of Penalties with Judgment-Proof Injurers", I study the problem of regulating a firm that may cause an accident as a result of its activity. It is common that production may result in causing damage such as an accidental spill of toxic waste, harmful air pollution or causes illness to consumers. For

instance, construction workers could be exposed to harmful substances such as asbestos during the course of their normal duties or untreated spills that could leak and result in soil contamination. This does not mean that those activities should be forbidden per se but that their inherent risk should be taken into account seriously. To this end, it is generally the case that there exists a set of precautionary measures, controls or technologies to avoid or at least reduce the likelihood that harmful events occur. The main problem, however, is that those who have the ability to take precautionary measures are often not directly –or only partially– concerned by the damage their activity may cause. In a perfect world with well-defined property rights, the Coase theorem would apply so that decentralized bargaining between potential injurers and potential victims would lead to an efficient provision of precautionary measures by the former. In practice, unfortunately, it is more often the case that such decentralized markets do not exist, either because potential victims are not even aware of the risk or lack the ability to take action against potential injurers. Following the seminal work of [Becker \(1968\)](#), economists have developed a wide range of measures to regulate those situations. One of them is *ex post regulation* and consists of imposing monetary penalties (or fines) on injurers whenever a harmful event is detected. This should, in principle, provide potential injurers with incentives to take appropriate precautionary measures. One of the central questions is how to efficiently design those penalties, namely, how large should be the monetary sanctions and how they should be allocated if there is more than one injurer. This question is even more important when injurers are part of a firm in which the internal organization suffers from asymmetric information problems. The seminal works of [Newman and Wright \(1990\)](#) and [Segerson and Tietenberg \(1992\)](#) have shown that in principal-agent firms with moral hazard only the total amount of penalties imposed on the firm matters. In other words, the allocation of legal responsibilities within the firm is irrelevant as the private contract between the principal and their agent can undo any distribution of penalties. In this chapter, I challenge the view that the allocation of penalties within the firm is neutral. For that matter, I rely on the assumption that some injurers may be judgment proof, that is, they may have insufficient assets to pay for the fines imposed on them. I examine two interesting cases: (i) A simple moral hazard problem in which only one worker (agent) is in charge of preventing the damage on behalf of an employer (principal) and, (ii) a double-sided moral hazard problem in which the joint actions of two firms determine

safety measures. In the first scenario, if the worker may be judgment proof, I show that the optimal regulation requires to fully target the employer. In the second scenario, if both firms can be judgment proof, the optimal regulation must target each firm equally. Those findings contrast with the result of the regulation literature, the so-called “Equivalence Principle” (see [Segerson and Tietenberg \(1992\)](#)), that states that only the total amount of fines matters whereas their distribution among tortfeasors is inconsequential. My result stems from the way I model the presence of limited financial resources. Unlike previous works, I do not assume that the contract between potential injurers must include ex post limited liability constraints. Instead, I assume that at the contracting stage, the players’ wealth is unknown and ex post transfers potentially unbounded. When uncertainty is resolved, players only pay what they can afford. I show that players therefore become strategic in concealing profits from the regulation. In the two-firm case, for instance, firms optimally choose to transfer profits to the less targeted entity in case of accident. Equal distribution of fines minimizes firms’ ability to conceal profits and therefore maximizes their incentives to exert safety care.

The third chapter, co-authored with David Martimort and Jérôme Pouyet, is entitled “How to Regulate Modern Airports?”. Modern airports have become complex structures carrying out many activities in the same place. In addition to their traditional core business – aeronautical services to passengers – it is by now well-established that non-aeronautical services, hereinafter referred to as “commercial activities”, account for a significant share of the airports’ total revenues. Those commercial activities are, for instance, retailing services in the airport, car parking or office rental to mention just a few. Moreover, airports are also responsible for investing in their infrastructure in order to enhance the overall quality for passengers and to relieve congestion in the facilities. Finally, airports do not directly supply aeronautical services to passengers but they supply access to their facilities to airlines which, in turn, supply aeronautical services to passengers. We can therefore consider that the relationship between airports and airline is non vertically integrated. In this work, we investigate the design of the optimal regulation policy and its implementation in this context. To this end, we consider that prices of both aeronautical services and commercial activities as well as investment decisions must be jointly regulated. The specificity of our analysis also lies in the following important features that characterize modern airports. First, we consider that an airport and an airline

are not vertically integrated. The airport can charge a price to provide an airline with an “essential facility” (runway, terminal buildings, navigational services) but the price of aeronautical services is set by the airline. Second, we take into consideration the fact that aeronautical services and commercial activities carried out at the airport exhibit a specific kind of complementarity: Demand for commercial activities is conditional upon the fact that a consumer is in the airport, that is, the fact that a consumer has a positive demand for aeronautical services but not the reverse. Third, we assume that the airport has to exert some (observable or not observable) investment decision in infrastructure in order to increase the value for aeronautical services to consumers. Our results are as follows. First, we characterize the first-best solution for the regulator when she can freely choose prices of aeronautical and commercial services as well as the level of investment made by the airport. We show that the first-best prices relies on a Ramsey-Boiteux pricing rule and are both above their associated social marginal cost. We point out that some of the traditional regulations implemented are likely to fail in the context of modern airports. For instance, we show that the use of a price-cap regulation for both aeronautical and commercial services successfully implements the optimal regulation for these prices but fails to provide enough investment incentives to the airport. We propose subsidy-penalty policy on investment decision to supplement the price-cap regulation and fully implement the first-best regulation. Our analysis also contributes to the debate of whether revenues generated from commercial activities should be included in the price-cap formula (single-till *v.s.* dual till): We show that this choice is irrelevant and has no impact on the optimal regulation. We then investigate how the nature of the relationship between the airport and an airline affects the optimal regulation. Our results point out that when the airport can offer a contract with both a fixed access charge and a unit price for providing one unit of aeronautical services to the airline, the regulator can implement the optimal regulation as if the airport and the airline were a vertically integrated structure. However, when the airport can only offer a unit price for aeronautical services, the optimal regulation is modified leading to an increase in the price of aeronautical services to passengers. This change stems from the double marginalization problem that occurs along the supply chain. Finally, we look at the case of unobservable investment decision. In that case, we solve the moral hazard problem between the regulator and the airport. Compared to case where investment decisions are observable, we show that the regulator must set higher

prices for both aeronautical and commercial activities to give the airport proper incentives to invest in the infrastructure. The optimal level of investment is, however, lower than in the observable case.

1. PARTNERSHIP DISSOLUTION WITH CASH-CONSTRAINED AGENTS ¹

Abstract

When partnerships come to an end, partners must find a way to efficiently reallocate the commonly owned assets to those who value them the most. This requires that the aforementioned members possess enough financial resources to buy out the others' shares. I investigate ex post efficient partnership dissolution when agents are ex post cash constrained. I derive necessary and sufficient conditions for ex post efficient partnership dissolution with Bayesian (resp. dominant strategy) incentive compatible, interim individually rational, ex post (resp. ex ante) budget balanced and ex post cash-constrained mechanisms. Ex post efficient dissolution is more likely to be feasible when agents with low (resp. large) cash resources own more (resp. less) initial ownership rights. Furthermore, I propose a simple auction to implement the optimal mechanism. Finally, I investigate second-best mechanisms when cash constraints are such that ex post efficient dissolution is not attainable.

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JEL Classification: D02, D23, D40, D44, D82, C72.

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1.1. INTRODUCTION

The Coase theorem stipulates that when transaction costs are sufficiently low, negotiations will always lead to an efficient outcome regardless of the initial allocation of ownership rights. Unfortunately, however, assuming something as simple as asymmetric information means that those transaction costs are no longer negligible. This is illustrated by the major contribution of [Myerson and Satterthwaite \(1983\)](#) who consider trade between a seller (the owner) and a buyer with bilateral asymmetric information. Their striking result is that no mechanism can achieve ex post efficient trade. [Cramton, Gibbons, and Klemperer \(1987\)](#) – henceforth CGK – show that the [Myerson and Satterthwaite \(1983\)](#) impossibility result becomes a possibility result if agents initially own equal (or close to equal) shares. The economic insight is that ownership rights give agents bargaining power in negotiations. When this bargaining power is excessive - the seller has “monopoly” power over the good in [Myerson and Satterthwaite \(1983\)](#) – negotiations fail. Equal-share ownership, on the contrary, is enough to curb the bargaining power of each agent and restore trade efficiency.

However, allocating ownership rights not only allocates bargaining power but it also determines the volume of trade. If efficiency requires one agent to buyout the shares of $(n - 1)$ other agents, equal-share ownership will require the transfer of a fraction $(n - 1)/n$ of all shares. It therefore means that the buyer will have to assume large cash payments to compensate the sellers. If some agents are financially constrained, such payments may be unfeasible when traded volumes are excessive and equal-share ownership might not be desirable anymore.

In this paper, I consider limited cash resources as another source of inefficiency in those trading problems. I investigate how trading mechanisms should be constructed and which ownership structures allow for efficient trade when agents are cash constrained. Formally, I build on the partnership dissolution model, first initiated by CGK, in which I consider agents with (possibly asymmetric) cash resources. In this framework, each partner initially owns shares of common assets. Dissolution simply means that following some event (disagreement, natural termination, bankruptcy), the commonly owned assets must be reallocated, that is, each partner will buy or sell their share to others. The framework therefore applies to various economic problems such as divorce, inheritance,

termination of joint-ventures, privatizations. Examples of applications are covered in more detail below.

Although the presence of cash – or budget – constraints seems to be a reasonable claim, little is still known about the design of trading mechanisms with cash-constrained agents. In auction design, earlier contributions of [Laffont and Robert \(1996\)](#), [Che and Gale \(1998, 2006\)](#) and [Maskin \(2000\)](#) have investigated cash constraints in standard auction settings. More recently, several authors have recognized limited cash resources as one of the gaps that limits the implementation of theory into practice.¹ In partnership problems, limited cash resources are directly linked to the initial distribution of ownership. Buying out a partner with large initial entitlements requires the ability to raise enough money to compensate them. In privatization of public-private partnerships or in spectrum allocations, cash constraints matter as the value of traded assets is often worth millions.² In divorce, inheritance or joint-venture problems, it is likely that agents have limited access to credit markets. Cash resources also create some kind of bargaining power: Partners with large cash resources could take advantage of very cash-constrained partners.

APPLICATIONS. I now present some applications that can be addressed within the partnership dissolution framework.

(i) *Divorce, inheritance*: Marriage or civil union represent, among other aspects, the joint ownership of some assets and the pursuance of a common goal. If dissolution occurs, *i.e.* divorce, the partners ought to agree on the reallocation of the family home, cars, and other possessions. Limited financial resources, especially compared to the market value of the family home, may complicate the process of finding an agreement on who should be the final owner. Alternatively, the assets to be traded may have more sentimental value than market value (*e.g.* the inherited childhood home) so that it may be difficult to use it as a collateral to borrow from a bank. Along the same lines, heirs of the deceased's property (real estate, business, debts) may want to reallocate the inheritance differently to what they had initially been entitled by the testament.

¹See, among others, [Dobzinski, Lavi, and Nisan \(2012\)](#), [Bichler and Goeree \(2017\)](#), [Carbajal and Mu'alem \(2018\)](#) and [Baisa \(2018\)](#). Two other important recent contributions to auction design with cash constraints are [Pai and Vohra \(2014\)](#) and [Boulatov and Severinov \(2019\)](#).

²[Cramton \(1995\)](#) argues that cash constraints have played a major role in the Nationwide Narrowband PCS Auction in 1994. Some bidders had likely dropped out from the auction because of limited resources although they had large valuations for the good.

(ii) *Joint ventures*: Business associates, joint ventures or venture capital firms are often governed by partnership law. For instance, in biotechnology and high-technology sectors, it is common that strong-potential young firms with low financial resources decide to rely on alliances with larger firms to compensate for the lack of complementary assets and liquidity (Aghion and Tirole, 1994, Lerner and Merges, 1998, Aghion, Bolton, and Tirole, 2004). Interestingly, Aghion, Bolton, and Tirole (2004) point out that not only can dissolution be triggered by dispute or unsuccessful results but it may also be due to the very nature of this form of partnership. Indeed, Aghion, Bolton, and Tirole (2004) report that those partnerships are generally temporary by nature and the young firms eventually seek other sources of funding requiring an exit from the partnership. As recognized by both Aghion and Tirole (1994) and Lerner and Merges (1998), the presence of cash constraints for the young firm generates inefficiencies in investment decisions as well as in allocation of ownership.

(iii) *Bankruptcy procedures*: Wolfstetter (2002) mentions that some bankruptcy procedures can be seen as a partnership dissolution problem. One example of a bankruptcy procedure, a cash auction, is given by Aghion, Hart, and Moore (1994): All remaining assets of the bankrupt firm are simply sold in an auction to the highest bidder. Some bidders may be former owners (with positive ownership rights in the firm) and other may be outsiders (with null ownership rights). Aghion, Hart, and Moore (1994) believe that a cash auction would be the “ideal bankruptcy procedure” (p. 855) in the absence of the difficulty to raise enough cash to buy the firm at its true value. They argue that cash constraints will likely result in a lack of competition in the auction and the firm would then be sold at a low price. This stresses the importance of designing mechanisms that directly take cash constraints into consideration.

(iv) *Land reallocation*: Land reallocation problems may also be challenged by the partnership framework. Che and Cho (2011) report the inefficiencies of the initial land allocation in the Oklahoma Land Rush in the late 19th century. Therefore, reallocation of those lands required to take into consideration the initial ownership structure induced by the first allocation. More recently, Loertscher and Wasser (2019) mention that land reallocation will be a major challenge in China. Starting in 1978, several reforms occurred in China to give household land use rights to farmers and then secure household land transfer rights

(2002, 2007, 2008) while the land is still collectively owned by villages.³ Participation by farmers in reallocating their land must then be voluntary and compensated by monetary transfers. The State Council of China believes that agricultural modernization in China will occur through the reallocation of the use of land from traditional farmers to a new generation of farmers (professional farmers or dragonhead enterprises; see [Zhang \(2018\)](#)). Traditional farmlands are considered too small and inefficient and reallocating them to larger and more skilled producers would help the modernization of agriculture in China.

CONTRIBUTION. The main feature of a partnership model is that participation constraints in the dissolution mechanism depend on the initial allocation of ownership rights among partners. Partners with relatively more initial shares have a higher claim and their participation in the mechanism is harder to ensure, making the initial ownership structure a determinant condition for optimal dissolution. In their seminal paper, CGK answer this problem by designing ex post efficient, Bayesian incentive compatible, interim individually rational and ex post budget balanced mechanisms. Their main finding consists in characterizing the initial allocations of property rights among partners that allow for ex post efficient dissolution. They show that equal-share partnerships can always be ex post efficiently dissolved whereas partnerships with excessive concentration of ownership are less likely to be dissolved efficiently. In particular, in a two-agent partnership with extreme ownership – one agent owns the whole asset –, their model collapses to the one of [Myerson and Satterthwaite \(1983\)](#) which proves the impossibility of ex post efficient dissolution in extreme ownership partnerships.

One of the main simplifying assumptions in CGK, however, is that all partners are always endowed with enough money so that they are able to pay for the monetary transfers proposed by the dissolution mechanism. While this assumption may seem innocuous in many contexts, I argue that it is not the case in the partnership dissolution problem. First, one of the purposes of a partnership generally consists in sharing the burden of acquiring costly assets such as firm premises, industrial equipment, computer hardware and software for businesses, or real estate, cars and household appliances for a couple. In some business partnerships, one agent provides the physical capital while the other one provides the human capital.⁴ It is also possible that dissolution occurs

³See [Ma, Wessler, Heerink, Qu, et al. \(2013\)](#) for a detailed chronology of land reforms in China.

⁴For instance, [Landeo and Spier \(2014\)](#) cite the *Haley v. Talcott* case. In, 2001, the two of them started a

precisely when partners face financial difficulties. That is, some partnerships may be, by essence, constituted by cash-constrained agents.⁵ Second, from both the theoretical and practical point of view, introducing cash constraints in the partnership dissolution model of CGK requires nontrivial changes in the design of monetary transfers. For instance, ex post monetary transfers in cash-constrained mechanisms must be bounded and must also satisfy the exact same conditions as in CGK at the interim level; namely incentive compatibility and individual rationality. Constructing those transfers therefore requires to find ex post transfers with a lower range but with the exact same marginals when projected at the interim level. In CGK, and in many other works on partnership dissolution, the absence of cash constraints is implicitly used and greatly simplifies the construction of dissolution mechanisms but they may also induce unreasonably high transfers for some states of the world.

In this paper, I first investigate the possibility of ex post efficient partnership dissolution when partners are ex post cash constrained, that is, when they have an upper bound on the payments they can make to other partners to buy out their shares. I derive necessary and sufficient conditions for ex post efficient partnership dissolution with Bayesian (resp. dominant strategy) incentive compatible, interim individually rational, ex post (resp. ex ante) budget balanced and ex post cash-constrained mechanisms. While the necessary and sufficient conditions for ex post efficient dissolution end up being quite a natural generalization of the results of CGK, the construction of the mechanism transfer function requires some extra work. I fully characterize these conditions and I show that the equal-share partnership is no longer the initial ownership structure that ensures feasibility of ex post efficient dissolution. Instead, partners who are initially relatively more (resp. less) cash-constrained than others must receive relatively more (resp. less) initial ownership rights. Intuitively, the more cash-constrained a partner is, the higher the utility they would receive in the mechanism (as they cannot be asked to pay much but they could still receive the asset) relatively to less cash-constrained partners. Thus, a very cash-constrained partner with few initial ownership rights will always be willing to participate (low maximal monetary transfers and low utility if they refuse the mechanism). It follows

restaurant in Delaware, Talcott provided the capital while Haley was supposed to manage the restaurant without salary for the first year.

⁵Some companies are specialized in providing short-term financial resources to partners facing a dissolution, see the *Shotgun fund*, for instance.

that giving more initial ownership rights to these cash-constrained partners does not change their participation decision but it implies that other less cash-constrained partners receive less initial ownership rights, which reduces their claim. This result sheds light on a new link between liquid and illiquid assets in partnership regardless of prior investment decisions. It is worth noting that the asymmetry in cash constraints among partners drives the result that *optimal* initial ownership rights must also be asymmetric. If partners all have the same exact cash resources, the equal-share partnership is still *optimal* as in CGK.

Focusing on equal-share and equal-cash-resources partnerships, I characterize the minimal amount of cash each partner must hold so that ex post efficient dissolution is achievable. Interestingly, the minimal amount of cash resources each partner must possess is increasing in the number of partners and converges to the maximal possible value of the asset when the number of partners becomes large. For instance, if a four-agent partnership with equal-share owns an asset worth 1 million and valuations are uniformly distributed on the unit interval, each agent must possess 0.8 million for ex post efficient dissolution to be possible. This result stresses the importance of taking cash constraints into account as they appear to be quite restrictive.

Interestingly, the standard equivalence theorem between Bayesian and dominant strategy implementation is robust to the addition of cash constraints. That is, any ex post efficient, interim individually rational and ex post cash-constrained mechanism can be implemented in dominant strategies with ex ante budget balance or with bayesian incentive compatibility with ex post budget balance. Moreover, transfers in both cases can be interim equivalent for all agents, that is, one can ensure the same interim utilities for all agents.⁶ It also appears that there is an equivalence between ex post cash constraints and interim cash constraints. More precisely, I show that relaxing cash constraints from the ex post level to the interim level does not weaken the conditions for ex post efficient dissolution.

As the general mechanism design formulation is often difficult to apply, I propose a simple bidding game that implements the ex post efficient dissolution mechanisms.

⁶This equivalence has first been studied by [Mookherjee and Reichelstein \(1992\)](#) and [Makowski and Mezzetti \(1994\)](#) for the same classes of mechanisms but without cash constraints. [Kosmopoulou \(1999\)](#) proves the equivalence when replacing interim individual rationality constraints by ex post ones.

In this bidding game, partners receive/pay an upfront transfer and then simply submit bids in an all-pay auction. It is constructed such that the bidding strategy of a partner is increasing in their valuation so that the highest bidder (the winner) is also the partner with the highest valuation. The upfront payment ensures interim individual rationality, budget balance and cash constraints. This bidding game can replace the one proposed by CGK, which fails to satisfy cash constraints when some or all partners have low cash resources.

Finally, I investigate second-best mechanisms to characterize the optimal allocation of final ownership rights when cash constraints are such that the first-best allocation is not attainable. First, I characterize all incentive compatible, interim individually rational and cash-constrained mechanisms for any possible allocation rule. Second, building on [Lu and Robert \(2001\)](#), [Loertscher and Wasser \(2019\)](#) and [Boulatov and Severinov \(2019\)](#), I show that solving the problem of maximizing a weighted sum of ex ante gains from trade and the revenue collected on agents requires allocating the asset to the agent(s) with the highest *ironed virtual valuation*. As in [Lu and Robert \(2001\)](#), [Loertscher and Wasser \(2019\)](#), the solution of maximizing ex ante gains from trade subject to incentive compatibility, interim individually rationality, cash constraints and budget balance is simply a particular solution to the previous problem for a specific weight.

LITERATURE REVIEW. Several contributions related to cash constraints can be found in the auction literature. [Laffont and Robert \(1996\)](#) characterize optimal auctions under independent valuations and symmetric cash-constrained agents. [Maskin \(2000\)](#) also examines constrained efficiency in auctions with symmetrical cash constraints. [Malakhov and Vohra \(2008\)](#) restrict the analysis to two-agent problems but assume only one agent is cash-constrained. More recently, [Boulatov and Severinov \(2019\)](#) propose a complete characterization of optimal auctions with asymmetrically cash-constrained bidders. However, adding cash constraints in a partnership problem is different than in the auction settings, namely, the budget balance requirement in partnership problems ties all partners monetary transfers together which makes the construction of those monetary transfers a challenge in itself.

Another strand of the literature considers agents whose valuations *and* cash resources are private information. [Che and Gale \(1998, 2006\)](#) compare the performances of standard

auctions in that case. [Che and Gale \(2000\)](#) study the optimal pricing of a seller facing a cash-constrained buyer who has private information over their valuation and “budget”. [Pai and Vohra \(2014\)](#) derive optimal auctions when both valuations and budget are private information. Assuming that cash resources are private information is an interesting feature, however, as it creates a multidimensional incentive compatibility problem, I will consider only commonly known budget in the present paper.

Finally, it is worth noting that ex post cash constraints are quite different from the ex post individual rationality requirement that has been extensively studied in the literature with [Gresik \(1991\)](#), [Makowski and Mezzetti \(1994\)](#), [Kosmopoulou \(1999\)](#) and [Galavotti, Muto, and Oyama \(2011\)](#) among others. The ex post individual rationality constraints, sometimes called “budget constraints” or “ex post regret-free”, require that agents do not have negative net utility ex post. On the contrary, *pure* ex post cash constraints ignore the utility a partner derives from the share of ownership they receive in the dissolution mechanism. In a mechanism where a good is traded among several agents, the ex post participation constraints are generally harder to satisfy for the agents who receive nothing as they do not enjoy utility from consumption. As for the ex post cash constraints, they are generally harder to satisfy for the agent who receives the highest quantity of the good, as they must pay higher prices (due the monotonicity of the allocation rule under incentive compatibility) and that the utility generated from the consumption is ignored. In other words, ex post cash constraints assume that the ability to pay of an agent in a mechanism cannot be contingent on what they receive in the mechanism.

ORGANIZATION OF THE PAPER. In Section [1.2](#), I present the theoretical framework for studying partnership dissolution mechanisms. Section [1.3](#) gives necessary and sufficient conditions to achieve ex post efficiency with cash-constrained agents. Section [1.4](#) provides characterization results of these conditions. Section [1.5](#) proposes a bidding game that replicates the mechanism through an all-pay auction. Section [1.6](#) second-best mechanisms. Finally, Section [1.7](#) proposes some extensions.

1.2. THEORETICAL FRAMEWORK

Consider a finite number of risk-neutral agents $n \geq 2$ indexed by $i \in N := \{1, \dots, n\}$. Each agent $i \in N$ initially owns a share $r_i \in [0, 1]$ of a perfectly divisible asset, where $\sum_{i \in N} r_i = 1$.⁷ Each agent $i \in N$ has private information over their valuation v_i for the asset. Valuations are independently distributed according to a commonly known cumulative distribution function F with support $V := [\underline{v}, \bar{v}] \subseteq \mathbb{R}_+$ and density function f .⁸ Further assume that F is absolutely continuous. Let $v := (v_1, \dots, v_n) \in V^n$ and $r := (r_1, \dots, r_n) \in \Delta^{n-1}$ denote the vectors of valuations and initial shares, respectively. This defines the standard partnership framework as first studied by CGK.

The additional assumption I require concerns agents' cash resources. Each agent $i \in N$ is endowed with some amount of cash $l_i \in \mathbb{R}_+$. This amount represents the upper bound on payments that agent i can be requested to make in the mechanism. The source of these cash constraints is not explicitly modeled here and each l_i is considered to be exogenously determined and publicly known at the beginning of the game.⁹ Let $l = (l_1, \dots, l_n) \in \mathbb{R}_+^n$ denote the vector of agents' cash resources.

Dissolving the partnership consists in reallocating the commonly owned asset to the agents who value it the most. As valuations for the asset are private information to the agents, dissolution requires to make them reveal their valuations. By the Revelation Principle, the analysis can be restricted to the search of direct revealing mechanisms in which each agent's optimal strategy consists in *truthfully* revealing their valuation. Such mechanisms will be refer to as *dissolution mechanisms*.

In a dissolution mechanism, each agent reports their valuation v_i and then receives an allocation of the asset $s_i(v)$ and a monetary transfer $t_i(v)$, both depending on the vector of all reports $v \in V^n$. Let $s(v) := (s_1(v), \dots, s_n(v))$ denote the *allocation rule*, where $s_i : V^n \rightarrow [0, 1]$ such that $\sum_{i \in N} s_i(v) = 1$ for all $v \in V^n$, and $t(v) := (t_1(v), \dots, t_n(v))$

⁷The requirement that $\sum_{i \in N} r_i = 1$ is not necessary to derive the condition under which ex post efficient dissolution is feasible. I chose to impose it to fit the partnership dissolution story of CGK. In Section 1.7, I provide an extension and examples in which property rights can take many other forms.

⁸In Section 1.7, I show that the main results can easily be extended to asymmetric distributions of independent valuations.

⁹Those limited cash resources can be the result of different financial situations of the agent after considering their personal wealth, borrowing capacities, debts or limited liability.

denote the *transfer rule* where $t_i : V^n \rightarrow \mathbb{R}$. By convention, the couple (s, t) represents a dissolution mechanism implementing allocation rule s with transfers t .

ILLUSTRATING EXAMPLE. To illustrate the theoretical framework, consider the following example.¹⁰ A pharmaceutical firm, say partner 1, and a R&D firm, say partner 2, decide to form a partnership to develop a new pharmaceutical drug. Initial ownership, r_1 and r_2 , represent claims on final output, that is, shares that each firm has the right to retain on the final value of the partnership. It is common that pharmaceutical firms own shares of small R&D firms to whom they provide capital and liquidity. Valuations, v_1 and v_2 , represent perceived potential cash flows from exploiting the drug. Finally, l_1 and l_2 corresponds to each firm's financial resources (cash holdings, borrowing capacities). Once the drug has been developed, the two firms negotiate the rights to exploit it. Either the pharmaceutical buys out the R&D firm and sell the drug on the market, or the R&D firm obtains full ownership and try to sell it to another pharmaceutical company. The dissolution mechanism will (i) make each firm truthfully report their valuation so that it is possible to allocate the drug to the one with the highest valuation; (ii) determine associated monetary transfers to compensate the partner who relinquishes their claim on the product.

UTILITY. The utility function of agent i is assumed to be linear in ownership shares and separable in money. Hence, agent i has utility $v_i \alpha_i + \beta_i$ when they own a share of the asset α_i and has an amount of money of β_i . Therefore, participation in a dissolution mechanism (s, t) gives agent i utility (net of initial ownership rights):

$$u_i(v) := v_i(s_i(v) - r_i) + t_i(v).$$

By convention, when a function is evaluated at a vector (v_i, v_{-i}) it is implicitly assumed that the argument are still ordered by the agents' indices, where $v_{-i} \in V^{n-1}$ is the vector of all agents' valuations except the one of agent i . For instance, $s_i(v_i, v_{-i}) = s_i(v_1, v_2, \dots, v_n)$.

As valuations are private information, each agent considers their interim utility, *i.e.* their utility averaging over all other agents' valuations (given that they all report truthfully).

¹⁰The example is inspired by [Minehart and Neeman \(1999\)](#).

Let $U_i(v_i)$ be agent i 's interim utility, that is,

$$U_i(v_i) := v_i(S_i(v_i) - r_i) + T_i(v_i),$$

where $S_i(v_i) := \mathbb{E}_{-i}s_i(v)$, $T_i(v_i) := \mathbb{E}_{-i}t_i(v)$ and where \mathbb{E}_{-i} is the expectation operator over all valuations except v_i .

INCENTIVE COMPATIBILITY. The first property required on a dissolution mechanism is that it induces information revelation, thereafter called *incentive compatibility*. Two standard notions of incentive compatibility will be considered separately: (i) *interim* incentive compatibility (IIC), and (ii) *ex post* incentive compatibility (EPIC). Formally, IIC and EPIC are defined as follows.

Definition 1 *A dissolution mechanism (s, t) is interim incentive compatible (IIC) if for all $i \in N$, $v_i \in V$ and $\tilde{v}_i \in V$,*

$$U_i(v_i) \geq v_i(S_i(\tilde{v}_i) - r_i) + T_i(\tilde{v}_i).$$

Definition 2 *A dissolution mechanism (s, t) is ex post incentive compatible (EPIC) if for all $i \in N$, $v_i \in V$, $\tilde{v}_i \in V$ and $v_{-i} \in V^{n-1}$,*

$$u_i(v_i, v_{-i}) \geq v_i(s_i(\tilde{v}_i, v_{-i}) - r_i) + t_i(\tilde{v}_i, v_{-i}).$$

Definitions 1 (resp. 2) simply defines dissolution mechanisms (s, t) such that truth-telling is a Bayesian Nash (resp. dominant strategy) equilibrium. Notice that if a dissolution mechanism (s, t) is EPIC then it is also IIC, but the reverse is not necessarily true. Both IIC and EPIC will be investigated in the first-best analysis whereas I will restrict to IIC for the analysis of second-best mechanisms.

INDIVIDUAL RATIONALITY. The second property of a dissolution mechanism is that participation is voluntary. Following CGK, I require that dissolution mechanisms are interim individual rational (IIR). Given that utilities are defined net of the initial ownership shares, IR is defined as follows.

Definition 3 *A dissolution mechanism (s, t) is interim individually rational (IIR) if for all $i \in N$ and $v_i \in V$,*

$$U_i(v_i) \geq 0.$$

Notice that IIR depends on r_i for each agent i . The higher r_i the more difficult it is to satisfy $U_i(v_i) = v_i(S_i(v_i) - r_i) + T_i(v_i) \geq 0$ for a given mechanism (s, t) . This is the main feature of partnership problems: The initial distribution of ownership shares has a direct impact on the feasibility of dissolution mechanisms through the constraints it imposes on each agent's minimal claim.¹¹

BUDGET BALANCE. A dissolution mechanism (s, t) is said to be budget balanced when no subsidy is required to implement the allocation rule s . Two notions of budget balance are considered: (i) *ex post* budget balance (EPBB), and (ii) *ex ante* budget balance (EABB). Formally, these two standard notions write:

Definition 4 A dissolution mechanism (s, t) is *ex post budget balanced* (EPBB) if for all $v \in V^N$,

$$\sum_{i \in N} t_i(v) = 0.$$

Definition 5 A dissolution mechanism (s, t) is *ex ante budget balanced* (EABB) if

$$\mathbb{E} \left[\sum_{i \in N} t_i(v) \right] = 0.$$

EPBB implies that for every profile of valuation $v \in V^N$, the transfers required to implement the allocation rule s cancel out between agents. EABB, however, only requires transfers to cancel out *on average*. Therefore, EPBB is, of course, a stronger requirement than EABB. As it will be shown below, the only advantage of relaxing budget balance from the *ex post* level (EPBB) to the *ex ante* one (EABB) consists in being able to impose incentive compatibility at the *ex post* level (EPIC) rather than at the interim level (IIC).

CASH-CONSTRAINED MECHANISMS. Finally, I require that dissolution mechanisms satisfy *ex post* cash constraints, that is, no agent can be required to pay more than their cash resources.

¹¹See Section 1.7 for an extension of more general interim individual rationality constraints. I show that the framework is not limited to partnership dissolution problems and can be easily extended to study the problem of optimally allocating a good to agents with type-dependent outside options.

Definition 6 A dissolution mechanism (s, t) is *ex post cash-constrained (EPCC)* if for all $i \in N$ and $v \in V^n$,

$$t_i(v) \geq -l_i.$$

The cash-constrained requirement is imposed at the ex post level and therefore assumes the most extreme form of cash constraints. As it will be shown later, ex post cash constraints are equivalent to interim cash constraints so that relaxing the ex post requirement to the interim level has no benefit.

1.3. EX POST EFFICIENT DISSOLUTION

I start by investigating *ex post efficient dissolution mechanisms* or, equivalently, *first-best* dissolution mechanisms. I provide necessary and sufficient conditions for the existence of such mechanisms when agents are cash-constrained. Two types of dissolution mechanisms are investigated. First, I will consider dissolution mechanisms simultaneously satisfying IIC, IIR, EPBB and EPCC, thereafter referred to as *Bayesian mechanisms*. Second, I will consider dissolution mechanisms simultaneously satisfying EPIC, IIR, EABB and EPCC, thereafter referred to as *dominant strategy mechanisms*. These two classes of mechanisms have been widely studied and one of the most important result in the literature is that they are *equivalent* in various environments.¹² Not surprisingly, the equivalence between Bayesian mechanism and dominant strategy mechanisms extends to an environment with ex post cash constraints.

EX POST EFFICIENT ALLOCATION RULE. Ex post efficiency requires that the allocation rule maximizes the gains from trade for every realization of valuations $v \in V^n$. Let s^* denote the ex post efficient allocation rule, it must satisfy for all $v \in V^n$

$$s^*(v) \in \arg \max_{s \in \Delta^{n-1}} \sum_{i \in N} v_i s_i(v).$$

The solution to this linear problem simply requires to allocate full ownership rights to the agent with the highest valuation. In case of tie between two or more agents (*i.e.*, they have the same valuation) assume, without loss of generality, that the agent with the lowest

¹²The word *equivalent* is stressed here as this notion has to be carefully defined and may vary across different equivalence theorems. See [Manelli and Vincent \(2010\)](#) on that matter.

index is allocated the whole asset.¹³ Therefore, the ex post efficient allocation rule for agent i can simply be written as

$$s_i^*(v) = \begin{cases} 1 & \text{if } \rho(v) = i \\ 0 & \text{if } \rho(v) \neq i, \end{cases} \quad (1.1)$$

where $\rho(v) := \min \{j \in N \mid j \in \arg \max_i v_i\}$, so that ties are always broken in favor of the agent with the lowest index.

1.3.1. Groves Mechanisms

To derive the main condition for ex post efficient dissolution of partnerships, I rely on the methodology derived by [Makowski and Mezzetti \(1994\)](#). They show that every mechanism satisfying both ex post efficiency (EF) and incentive compatibility (either interim or ex post) must be a Groves mechanism, and can be fully characterized by a specific transfer function defined up to a constant.

Let $g(v) := \sum_{i \in N} v_i s_i^*(v)$ denote the maximum gains from trade at $v \in V^n$ and let the transfer function writes

$$t_i^*(v) = g(v) - v_i s_i(v) - h_i(v), \quad (1.2)$$

for some function $h_i : V^n \rightarrow \mathbb{R}$ and all $i \in N$. The first two terms of this function, namely $g(v) - v_i s_i(v)$, ensure that the mechanism implements the ex post efficient allocation rule by inducing revelation. The last term, $-h_i(v)$, is an arbitrary function whose purpose is to collect money back from agent without distorting their incentives to reveal information. The following proposition details the required properties of the function h_i .

Proposition 1 ([Makowski and Mezzetti \(1994\)](#)) *The dissolution mechanism (s^*, t^*) is*

- a. *EF and IIC if and only if t^* satisfies (1.2) and $\mathbb{E}_{-i} h_i(v_i, v_{-i}) = \mathbb{E}_{-i} h_i(v'_i, v_{-i}) = H_i$ for all $v_i, v'_i \in V$ and $H_i \in \mathbb{R}$ is a constant;*
- b. *EF and EPIC if and only if t^* satisfies (1.2) and $h_i(v_i, v_{-i}) = h_i(v'_i, v_{-i})$ for all $v_i, v'_i \in V$.*

¹³As $F(\cdot)$ is assumed to be absolutely continuous, ties occur with probability zero and thus can be ignored in the design of the ex post efficient mechanism. However, this will no longer be the case in the second-best analysis.

Proof. See [Makowski and Mezzetti \(1994\)](#). ■

I now turn to the existence of ex post efficient Bayesian and dominant strategy dissolution mechanisms.

1.3.2. *The Existence Condition*

Proposition 1 ensures that a dissolution mechanism (s^*, t^*) satisfies EF and IIC (resp. EPIC) if transfers follows (1.2) and the function $h_i(\cdot)$ is independent of v_i on average over v_{-i} (resp. independent of v_i). Therefore, investigating ex post efficient Bayesian or dominant strategy mechanisms can be done by directly imposing all other requirements (individual rationality, budget balance and cash constraints) on t^* . I restrict the presentation of the argument to ex post efficient Bayesian mechanisms. The case of dominant strategy mechanisms is almost exactly the same and therefore relegated to Appendix A.

BAYESIAN MECHANISMS. Take a dissolution mechanism (s^*, t^*) satisfying EF and IIC (*i.e.* t^* satisfies Proposition 1.a). Imposing EPBB requires $\sum_{i \in N} t_i^*(v) = 0$ for all $v \in V^n$ or, equivalently,

$$\sum_{i \in N} h_i(v) = (n-1)g(v), \text{ for all } v \in V^n. \quad (1.3)$$

The term $(n-1)g(v)$ can be interpreted as the ex post deficit generated by an EF-IIC mechanism. EPBB then implies that the $h_i(\cdot)$ functions are designed to absorb this deficit while satisfying the requirement of Proposition 1.a. At the ex ante stage, EPBB requires (taking expectations over all v on both sides of equation (1.3))

$$\sum_{i \in N} H_i = (n-1)G, \quad (1.4)$$

where $H_i := \mathbb{E}h_i(v) = \mathbb{E}_{-i}h_i(v)$ and $G := \mathbb{E}g(v)$.

At the same time, imposing IIR on (s^*, t^*) requires that $U_i(v_i) = v_i(S_i^*(v_i) - r_i) + T_i^*(v_i) \geq 0$ for all $v_i \in V$ and $i \in N$. Replacing $T_i^*(v_i) := \mathbb{E}_{-i}t_i^*(v)$ by its expression (given by taking expectations \mathbb{E}_{-i} of equation (1.2)) gives $\mathbb{E}_{-i}g(v) - H_i - v_i r_i \geq 0$. Rearranging,

IIR requires that

$$H_i \leq \mathbb{E}_{-i} g(v) - v_i r_i, \text{ for all } v_i \in V, i \in N.$$

Define,

$$C(r_i) := \inf_{v_i} \{ \mathbb{E}_{-i} g(v) - v_i r_i \}. \quad (1.5)$$

Then IIR be can rewritten as

$$H_i \leq C(r_i) \text{ for all } i \in N. \quad (1.6)$$

Equation (1.6) simply defines $C(r_i)$ as the maximal amount of money that can be collected on agent i at the interim stage without violating IIR. Notice that $C(r_i)$ is a decreasing function of r_i . This reflects that initial shares provide bargaining power to their owner: Higher initial shares allows an agent to claim a larger part of the gains from trade.

Finally, imposing EPCC on (s^*, t^*) gives $t_i^*(v) = g(v) - v_i s_i^*(v) - h_i(v) \geq -l_i$ for all $v \in V^n$ and $i \in N$. This implies that at the interim stage (taking expectation over all v_{-i}):

$$H_i \leq \mathbb{E}_{-i} [g(v) - v_i s_i^*(v)] + l_i, \text{ for all } v_i \in V, i \in N.$$

Straightforward computations give that $\min_{v_i} \mathbb{E}_{-i} [g(v) - v_i s_i^*(v)] = 0$ at $v_i = \bar{v}$ and thus the above equation simply rewrites:

$$H_i \leq l_i, \text{ for all } v_i \in V, i \in N. \quad (1.7)$$

Equation (1.7) yields the maximal amount of money that can be collected on agent i at the interim stage due to the presence of cash constraints.

THE DISSOLUTION CONDITION. It appears that the existence of ex post efficient Bayesian dissolution mechanisms simply relies on whether it is possible to collect enough money from agents to cover the ex ante deficit $(n - 1)G$ given the upper bounds $C(r_i)$ and l_i implied by IIR and EPCC, respectively. I now state the existence condition of ex post efficient Bayesian dissolution mechanisms.

Theorem 1 *An EF, IIC, IIR, EPBB and EPCC dissolution mechanism exists if and only if*

$$\sum_{i \in N} \min\{C(r_i), l_i\} \geq (n-1)G. \quad (1.8)$$

Proof. (Necessity) Equations (1.6) and (1.7) are both necessary conditions for IIR and EPCC, respectively. Combining the two equations gives $H_i \leq \min\{C(r_i), l_i\}$ for all $i \in N$. Summing over $i \in N$ gives $\sum_{i \in N} H_i \leq \sum_{i \in N} \min\{C(r_i), l_i\}$. Finally, using equation (1.4) (implied by EPBB) yields $(n-1)G \leq \sum_{i \in N} \min\{C(r_i), l_i\}$ which concludes the necessity part.

(Sufficiency) To prove sufficiency, I explicitly construct a transfer function that satisfies all requirements when (1.8) holds. Let $t_i^C(v) = g(v) - v_i s_i^*(v) - h_i^C(v)$ where $h_i^C(v)$ is defined as follows:

$$h_i^C(v) := \frac{n-1}{n} \left[g(v) + \frac{1}{n-1} \sum_{j \neq i} s_j^*(v) \psi(v_j) - s_i^*(v) \psi(v_i) \right] + \phi_i, \quad (1.9)$$

where

$$\psi(v_k) := \frac{\int_{\underline{v}}^{v_k} F(x)^n dx}{F(v_k)^n}, \quad (1.10)$$

and $\phi_i \in \mathbb{R}$ is a constant. I further discuss the choice of the transfer function after the proof.

EF and IIC. The transfer function $t_i^C(v)$ writes as equation (1.2). Therefore, if $h_i^C(v)$ satisfies Proposition 1.a then the mechanism (s^*, t^C) is EF and IIC. Standard computations (see Appendix A) give

$$\mathbb{E}_{-i} h_i^C(v) = \frac{n-1}{n} G + \phi_i.$$

Hence, $\mathbb{E}_{-i} h_i^C(v)$ does not depend on v_i and satisfies Proposition 1.a. Let $H_i^C := \mathbb{E}_{-i} h_i^C(v)$. The mechanism (s^*, t^C) is EF and IIC.

EPBB. The dissolution mechanism (s^*, t^C) is EPBB if it satisfies $\sum_{i \in N} t_i^C(v) = 0$ for all

$v \in V^N$. Notice that,

$$\begin{aligned} \sum_{i \in N} t_i^C(v) &= (n-1)g(v) - \frac{n-1}{n} \left[ng(v) + \frac{1}{n-1} \sum_{i \in N} \sum_{j \neq i} s_j^*(v_j) \psi(v_j) - \sum_{i \in N} s_i^*(v) \psi(v_i) \right] - \sum_{i \in N} \phi_i \\ &= - \sum_{i \in N} \phi_i. \end{aligned}$$

EPBB is therefore equivalent to $\sum_{i \in N} \phi_i = 0$.

IIR. Recall from equation (1.6) that IIR requires $H_i^C \leq C(r_i)$. Hence, (s^*, t^C) satisfies IIR if

$$H_i^C = \frac{n-1}{n} G + \phi_i \leq C(r_i), \text{ for all } i \in N.$$

EPCC. Finally, EPCC requires that $t_i^C(v) \geq -l_i$ for all $i \in N$, $v \in V^n$, or, equivalently, $\min_{v \in V^N} t_i^C(v) \geq -l_i$ for all $i \in N$. Notice that

$$t_i^C(v) = \begin{cases} -\frac{n-1}{n} [v_i - \psi(v_i)] - \phi_i & \text{if } \rho(v) = i \\ \frac{1}{n} [v_j - \psi(v_j)] - \phi_i & \text{if } \rho(v) = j \neq i. \end{cases}$$

The following lemma is useful to determine the minimum of t_i^C .

Lemma 1 For all $k \in N$, $[v_k - \psi(v_k)]$ is nonnegative and increasing in $v_k \in V$.

Proof. See Appendix A. ■

From Lemma 1 it is clear that the minimum of t_i^C is attained when $\rho(v) = i$ and $v_i = \bar{v}$. Therefore,

$$\begin{aligned} \min_{v \in V^n} t_i^C(v) &= -\frac{n-1}{n} [\bar{v} - \psi(\bar{v})] - \phi_i \\ &= -\frac{n-1}{n} G - \phi_i. \end{aligned} \tag{1.11}$$

EPCC is then equivalent to $-\frac{n-1}{n} G - \phi_i \geq -l_i$ for all $i \in N$.

Combining IIR and EPCC yields the following condition

$$\phi_i \leq \min\{C(r_i), l_i\} - \frac{n-1}{n} G.$$

Then, for each $i \in N$, let the constant be

$$\phi_i = \min\{C(r_i), l_i\} - \frac{1}{n} \sum_{j \in N} \min\{C(r_j), l_j\}.$$

It is straightforward that $\sum_{i \in N} \phi_i = 0$ so that EPBB holds for t_i^C . Furthermore, if condition (1.8) holds, i.e. $\sum_{j \in N} \min\{C(r_j), l_j\} \geq (n-1)G$, it is immediate that $\phi_i \leq \min\{C(r_i), l_i\} - \frac{n-1}{n}G$ so that IIR and EPCC also hold for t_i^C . ■

Remark. In the absence of cash constraints, i.e. when the l_i 's are sufficiently large for each $i \in N$ so that $\min\{C(r_i), l_i\} = C(r_i)$ for all $i \in N$ and $r \in \Delta^{n-1}$, condition (1.8) simply rewrites

$$\sum_{i \in N} C(r_i) \geq (n-1)G.$$

It can easily be shown that this condition is simply equation (D) in CGK (Theorem 1, p. 619). Without cash constraints, the feasibility of ex post efficient dissolution mechanisms relies only upon the sum of maximal collectible charges due to individual rationality constraints. As shown by CGK, there always exists initial ownership structures such that ex post efficient dissolution is feasible (and in particular, equal-share partnerships). With cash constraints, however, ex post efficient dissolution might be impossible regardless of the distribution of ownership and cash resources. See Section 1.4 for a detailed analysis on that matter.

ON THE TRANSFER FUNCTION. The major difficulty in establishing Theorem 1 lies in the construction of the transfer function t_i^C to prove the sufficiency of condition (1.8). Unfortunately, the transfer function proposed by CGK – that I will denote t_i^{CGK} thereafter – cannot be used to prove the sufficiency part of Theorem 1. Investigating t_i^{CGK} allows for a better understanding of the construction of cash-constrained mechanisms. Furthermore, the transfer function t_i^{CGK} is not peculiar to the work of CGK and it is commonly used in

the mechanism design literature.¹⁴ Formally, t_i^{CGK} writes

$$t_i^{\text{CGK}}(v) = c_i - \int_{\underline{v}}^{v_i} x dF(x)^{n-1} + \frac{1}{n-1} \sum_{j \neq i} \int_{\underline{v}}^{v_j} x dF(x)^{n-1},$$

where $c_i \in \mathbb{R}$ is a constant and $\sum_{i \in N} c_i = 0$ (see CGK, p. 628). Imposing EPCC requires that $t_i^{\text{CGK}}(v) \geq -l_i$ for all $i \in N$ and $v \in V^n$. It is immediate that $\min_{v \in V} t_i^{\text{CGK}}(v) = c_i - \int_{\underline{v}}^{\bar{v}} x dF(x)^{n-1}$, that is when $v_i = \bar{v}$ and $v_j = \underline{v}$ for all $j \neq i$. Straightforward computations yield that t_i^{CGK} is EPCC if

$$c_i - \frac{n-1}{n} G - \int_{\underline{v}}^{\bar{v}} x [1 - F(x)] dF(x)^{n-1} \geq -l_i. \quad (1.12)$$

Applying a similar reasoning to that of the proof of Theorem 1, it is easy to show that a mechanism (s^*, t_i^{CGK}) is EF, IIC, IIR, EPBB and EPCC if and only if

$$\sum_{i \in N} \min \left\{ C(r_i), l_i - \int_{\underline{v}}^{\bar{v}} x [1 - F(x)] dF(x)^{n-1} \right\} \geq (n-1)G. \quad (1.13)$$

This condition is undoubtedly more restrictive than condition (1.8).¹⁵ This is due to the fact that the minimum of the variable part of t_i^{CGK} (*i.e.* ignoring the constant c_i) is lower than the minimum of the variable part of t_i^{C} (*i.e.* ignoring the constant ϕ_i). It is therefore interesting to notice that what matters for EPCC is the minimum of the *variable part* of the transfer function. If the variable part of the transfer function is too low, adding a higher constant to shift the function up to satisfy EPCC does not help as this would contradict EPBB.

Therefore, it is necessary that t_i^{C} has a lower span than t_i^{CGK} to satisfy EPCC. Notice that IIC requires that both transfer functions must be equal at the interim level (up to a constant), that is $\mathbb{E}_{-i} t_i^{\text{CGK}}(v) = \mathbb{E}_{-i} t_i^{\text{C}}(v) + \text{cst}$ for some constant.¹⁶ The problem of constructing t_i^{C} is then a problem of constructing multivariate random variables with

¹⁴To the best of my knowledge, this function has first been introduced by [d'Aspremont and Gérard-Varet \(1979\)](#). It can also be found in [Lu and Robert \(2001\)](#), [Ledyard and Palfrey \(2007\)](#) and [Segal and Whinston \(2011\)](#) among others.

¹⁵See Section 1.5 for an example in which the dissolution mechanism proposed by CGK does not allow for efficient dissolution whereas it is possible with my mechanism.

¹⁶This is a consequence of Proposition 1 and one of the most important features of incentive compatibility. See [Myerson \(1981\)](#), [Makowski and Mezzetti \(1994\)](#) and [Williams \(1999\)](#).

given marginals (IIC), bounded support (EPCC) and such that the sum of all is zero (EPBB). In this paper, I have constructed the function t_i^C by adapting the one proposed in an unpublished paper of [Dudek, Kim, and Ledyard \(1995\)](#) who study ex post individually rational Bayesian mechanisms with no initial endowments. In future works, it would be interesting to fully characterize the space of transfer functions that satisfy EPCC.

DOMINANT STRATEGY MECHANISMS. As mentioned earlier, the same dissolution condition applies to dominant strategy mechanisms.

Theorem 2 *An EF, EPIC, IIR, EABB and EPCC dissolution mechanism exists **if and only if***

$$\sum_{i \in N} \min\{C(r_i), l_i\} \geq (n-1)G. \quad (1.14)$$

Proof. See Appendix A. ■

Condition (1.14) is exactly the same as condition (1.8). This implies that when it is possible to ex post efficiently dissolve a partnership with a Bayesian mechanism then it is also possible to do it with a dominant strategy, and reciprocally. The equivalence, however, goes further than that as discussed below.

1.3.3. The Equivalence Theorem

From Theorem 1 and Theorem 2 it is then clear that when it is possible to implement ex post efficient dissolution with a Bayesian mechanism then ex post efficient dissolution can also be implemented with a dominant strategy mechanism, and *vice versa*. Yet, the following results give a much stronger equivalence between the two classes of mechanisms.

Theorem 3 *If (s^*, \tilde{t}) is an EF, EPIC, IIR, EABB and EPCC dissolution mechanism, then there exists a t such that*

1. (s^*, t) is an EF, IIC, IIR, EPBB and EPCC dissolution mechanism;
2. $\mathbb{E}_{-i} t_i(v_i, v_{-i}) = \mathbb{E}_{-i} \tilde{t}_i(v_i, v_{-i})$ for all $i \in N, v_i \in V$.

Proof. See Appendix A. ■

The converse is also true.

Theorem 4 *If (s^*, t) is an EF, IIC, IIR, EPBB and EPCC dissolution mechanism, then there exists a \tilde{t} such that*

1. (s^*, \tilde{t}) is an EF, EPIC, IIR, EABB and EPCC dissolution mechanism;
2. $\mathbb{E}_{-i} t_i(v_i, v_{-i}) = \mathbb{E}_{-i} \tilde{t}_i(v_i, v_{-i})$ for all $i \in N, v_i \in V$.

Proof. See Appendix A. ■

The additional feature of the equivalence theorem relies on the equivalence between the interim transfers. Thus, Theorem 3 and 4 state that one can alternatively choose to implement ex post efficient dissolution with Bayesian or dominant strategy mechanisms *and* offer the same interim transfers and utilities to every agent. It means that any final distribution of welfare among agents that is attainable in Bayesian mechanisms is also attainable in dominant strategy mechanisms.

1.3.4. Interim Cash Constraints

So far, I have assumed the strictest requirement for cash constraints, namely, ex post cash constraints. An important question is whether relaxing the requirement from the ex post to the interim level helps relaxing the dissolution condition (1.8). Formally, the constraints would become $\mathbb{E}_{-i} t_i(v) \equiv T_i(v_i) \geq -l_i$ for all $v_i \in V$. This constraint therefore requires that each agent, when privately informed about their type, thinks that they will have enough cash in expectations. This is a softer budget constraint than ex post cash constraints as it may occur that agents have to pay more than l_i at the end of the game. The following result provides an important insight about interim cash constraints.

Proposition 2 *An EF, IIC, IIR, EPBB and EPCC exists (s, t) if and only if there exists an EF, IIC, IIR, EPBB and **interim cash-constrained** mechanism (s, \tilde{t}) such that $\mathbb{E}_{-i} t_i(v_i, v_{-i}) = \mathbb{E}_{-i} \tilde{t}_i(v_i, v_{-i})$ for all $i \in N$ and $v_i \in V$.*

Proof. See Appendix A.

Proposition 2 shows that the existence of dissolution mechanisms with interim cash constraints is equivalent to the existence of dissolution mechanisms with ex post cash constraints. Therefore, relaxing cash constraints to the interim level has no advantages compared to the ex post level. This result straightforwardly applies to dominant strategy mechanisms.

1.4. FIRST-BEST CHARACTERIZATION RESULTS

The condition that must hold to implement ex post efficient dissolution (equation (1.8)) depends both on the initial ownership structure r and on the cash resources l . It is then natural to investigate the set of partnerships that can be efficiently dissolved when r and l vary.

1.4.1. Optimal Initial Ownership Structures

For a given distribution of cash resources $l \in \mathbb{R}_+^n$, I first characterize the initial ownership structures that maximize the contributions that can be collected on agents $\sum_{i \in N} \min\{C(r_i), l_i\}$. Initial ownership structures $r^*(l) \in \arg \max_{r \in \Delta^{n-1}} \sum_{i \in N} \min\{C(r_i), l_i\}$ are said to be *optimal*. It may obviously be the case that even the optimal initial ownership structures do not allow for ex post efficient dissolution if cash resources are low for some agents.¹⁷

Recall that $C(r_i) = \inf_{v_i} \{\mathbb{E}_{-i} g(v) - v_i r_i\}$. Let $y = \max_{j \neq i} v_j$, then

$$\begin{aligned} \mathbb{E}_{-i} g(v) - v_i r_i &= v_i \mathbb{E}_{-i} \mathbb{1}\{v_i > y\} + \mathbb{E}_{-i} y \mathbb{1}\{v_i < y\} - v_i r_i \\ &= v_i F(v_i)^{n-1} + \int_{v_i}^{\bar{v}} y dF(y)^{n-1} - v_i r_i. \end{aligned}$$

Differentiating this expression with respect to v_i , the first-order condition gives $F(v_i^*(r_i))^{n-1} =$

¹⁷This happens when $\sum_{i \in N} \min\{C(r_i^*(l)), l_i\} < (n-1)G$. It is then possible to characterize the minimal subsidy that would be necessary to satisfy the ex post efficient dissolution condition (1.8).

r_i , where $v_i^*(r_i)$ is said to be the *worst-off type* of agent i .¹⁸ Therefore,

$$C(r_i) = \int_{v_i^*(r_i)}^{\bar{v}} y dF(y)^{n-1}, \quad (1.15)$$

which is continuous and differentiable in r_i . The Envelope Theorem directly gives $C'(r_i) = -v_i^*(r_i) \leq 0$ and $C''(r_i) = -\frac{\partial v_i^*}{\partial r_i}(r_i) < 0$ so that $C(r_i)$ is both decreasing and concave in r_i . Notice also that $C(0) = \int_{\underline{v}}^{\bar{v}} y dF(y)^{n-1}$ and $C(1) = 0$.

It is useful to introduce the following notation. Let $\tilde{r}_i \in [0, 1]$ be such that $C(\tilde{r}_i) = l_i$ when $l_i \leq C(0)$ and let $\tilde{r}_i = 0$ if $l_i > C(0)$. This threshold is such that cash constraints are more restrictive than the individual rationality constraint when $r_i < \tilde{r}_i$ and the opposite when $r_i \geq \tilde{r}_i$. As \tilde{r}_i is decreasing in l_i , a higher \tilde{r}_i indicates that cash constraints are more restrictive for agent i .

The characterization results depend both on the total amount of available cash resources and on its distribution over agents. Assume, without loss of generality, that $l_1 \geq \dots \geq l_n$ so that $\tilde{r}_1 \leq \dots \leq \tilde{r}_n$. Consider first the case in which cash constraints are not too severe, that is when $\sum_{i \in N} \tilde{r}_i \leq 1$.

Proposition 3 *Assume $\sum_{i \in N} \tilde{r}_i \leq 1$, then the optimal distribution of property rights $r^* \in \Delta^{n-1}$ is as follows:*

- a. *If $\tilde{r}_i \leq \frac{1}{n}$ for all $i \in N$, then $r^* = (\frac{1}{n}, \dots, \frac{1}{n})$;*
- b. *If $\tilde{r}_i > \frac{1}{n}$ for some $i \in N$, then $r^* = (\hat{r}, \hat{r}, \dots, \hat{r}, \tilde{r}_p, \tilde{r}_{p+1}, \dots, \tilde{r}_n)$ where $\hat{r} = \frac{1 - \sum_{j \geq p} \tilde{r}_j}{p-1}$ for some $p \in N$ such that $\max_{i < p} \tilde{r}_i < \hat{r} \leq \min_{j \geq p} \tilde{r}_j$.*

Proof. See Appendix A. ■

Proposition 3.a is simply CGK main result (Proposition 1, p.621). When each agent is endowed with enough cash, *i.e.* $\tilde{r}_i \leq \frac{1}{n}$, then the equal-share ownership structure is optimal. However, as soon as at least one agent's cash resources go below some threshold,

¹⁸The second-order derivative immediately writes $(n-1)f(v_i)F(v_i)^{n-2} \geq 0$ so that the first-order condition characterizes a minimum.

i.e. $\tilde{r}_i > \frac{1}{n}$ for some $i \in N$, Proposition 3.b implies that the optimal ownership structure allocates more initial ownership rights to more cash-constrained agents.

To illustrate Proposition 3.b, consider a two-agent partnership in which agent 1 has large cash resources so that $\tilde{r}_1 = 0$ and agent 2 is heavily cash constrained so that $\tilde{r}_2 \in [\frac{1}{2}, 1)$. It is clear that starting from any $r_2 < \tilde{r}_2$, and in particular $r_2 = \frac{1}{2}$, it would be possible to strictly increase $\sum_{i=1,2} \min\{C(r_i), l_i\} = C(r_1) + l_2$ by increasing r_2 up to \tilde{r}_2 as $C(\cdot)$ is a decreasing function and $\min\{C(r_2), l_2\} = l_2$ is unchanged for all $r_2 \leq \tilde{r}_2$. In other words, it is innocuous to give more initial ownership rights to heavily cash-constrained agent as they are already limited by their cash resources but it allows to give less initial ownership rights to less cash-constrained agents and then collect more from them.

When cash constraints are more severe, for some or all agents, so that $\sum_{i \in N} \tilde{r}_i > 1$, the structure of the optimal initial ownership structure can be characterized as follows.

Proposition 4 *Assume $\sum_{i \in N} \tilde{r}_i \geq 1$, then the optimal distribution of property rights $r^* \in \Delta^{n-1}$ is such that $r_i^* \leq \tilde{r}_i$ for all $i \in N$ and $\sum_{i \in N} \min\{C(r_i^*), l_i\} = \sum_{i \in N} \min\{C(0), l_i\}$.*

Proof. First, notice the following upper bound, $\sum_{i \in N} \min\{C(r_i), l_i\} \leq \sum_{i \in N} \min\{C(0), l_i\}$ for all $r \in \Delta^{n-1}$. For every $i \in N$, let $r_i \leq \tilde{r}_i$ which is always possible as $\sum_{i \in N} r_i = 1 \leq \sum_{i \in N} \tilde{r}_i$. Then $\min\{C(r_i), l_i\} = \min\{C(0), l_i\}$ for all $i \in N$ and $\sum_{i \in N} \min\{C(r_i), l_i\} = \sum_{i \in N} \min\{C(0), l_i\}$. To conclude, it is clear that choosing any $r_i > \tilde{r}_i$ would decrease $\sum_{i \in N} \min\{C(r_i), l_i\}$. ■

When $\sum_{i \in N} \tilde{r}_i \geq 1$, optimal initial ownership structures may or may not have a clear characterization. Consider for instance the case in which all agents $i < p$ for some $p \in N \setminus \{1\}$ have large cash resources so that $\tilde{r}_i = 0$ for all $i < p$ and all agents $i \geq p$ have low cash resources so that $\tilde{r}_i > 0$ and $\sum_{i \in N} \tilde{r}_i \geq 1$. Then, Proposition 4 gives that all agents $i < p$ must have $r_i^* = 0$ and for $i \geq p$, the r_i^* must be such that $\sum_{i \geq p} r_i^* = 1$. Therefore, initial ownership goes only to agents with few cash resources and this resembles Proposition 3.b. Consider now a case in which for instance $\frac{1}{2} = \tilde{r}_1 = \tilde{r}_2 \leq \tilde{r}_3 \leq \dots \leq \tilde{r}_n$. Then choosing $r_1^* = r_2^* = \frac{1}{2}$ and $r_i^* = 0$ for all $i \geq 3$ is optimal. Hence, at some point, cash constraints are so severe that optimal initial ownership structures have no clear structure other than that of Proposition 4.

1.4.2. Minimal Cash Resources To Dissolve Equal-Share Partnerships

One of the main results of CGK states that equal-share partnership can always be ex post efficiently dissolved. [de Frutos and Kittsteiner \(2008\)](#) report that one-half to two-third of two-agent partnerships exhibit equal-share ownership. It is then interesting to investigate how cash constraints mitigate this finding.

Proposition 5 *Assume $l_i := \underline{l}$ for all $i \in N$. Then, every equal-share partnership is dissolvable for any absolutely continuous cumulative distribution function $F(\cdot)$ if and only if*

$$\underline{l} \geq \frac{n-1}{n}G. \quad (1.16)$$

Furthermore, for any $F(\cdot)$, \underline{l} is increasing in n and converges to \bar{v} when n goes to infinity.

Proof. From CGK (Proposition 1), an equal-share partnership is always dissolvable in the absence of cash constraints, that is, $\sum_{i \in N} C(\frac{1}{n}) \geq (n-1)G$. Equation (1.16) immediately follows from the dissolution condition $\sum_{i \in N} \min\{C(\frac{1}{n}), \underline{l}\} \geq (n-1)G$.

Recall that $G = \mathbb{E}[\max_{j \in N} v_j] = \int_{\underline{v}}^{\bar{v}} y dF(y)^n$. Differentiating the right-hand side of (1.16) with respect to n gives $\frac{1}{n^2}G + \frac{n-1}{n} \frac{\partial G}{\partial n} \geq 0$ as G is increasing in n . Finally, if $n \rightarrow \infty$ then $G \rightarrow \bar{v}$ and so does $\frac{n-1}{n}G$. ■

Equation (1.16) simply states that in equal-share-equal-cash partnerships, each agents must be endowed with a fraction $\frac{1}{n}$ of the total ex ante expected deficit generated by a Groves mechanism. By construction, this ex ante deficit becomes larger as the number of agents increases as the probability of $\max_{i \in N} v_i$ increases (which must be distributed to $(n-1)$ agents, see Section 1.3) and so do the minimal cash resources.

Finally, the result that $\underline{l} \rightarrow \bar{v}$ when $n \rightarrow +\infty$ suggests that cash constraints are likely to be a major concern as the number of agents becomes large. In many cases, it seems reasonable to think that one of the main purposes of forming a partnership is precisely to split the burden of a costly investment between partners because of initial cash constraints. Hence, it seems unlikely that every agent possesses the maximal value of the asset in cash when dissolution occurs.

1.4.3. Optimal Organization of Cash and Ownership

So far, I have only investigated the optimal allocation initial ownership for fixed distributions of cash resources or minimal cash requirements for equal-share partnerships. To assess the importance of cash constraints in the feasibility of ex post efficient dissolution mechanisms, it is also useful to investigate what would be the best joint initial allocations of ownership and cash resources.

Consider that both $r \in \Delta^{n-1}$ and $l \in \mathbb{R}_+^n$ are allowed to vary. An *optimal* organization of the partnership (r^*, l^*) solves

$$(r^*, l^*) \in \arg \max_{\hat{r}, \hat{l}} \sum_{i \in N} \min\{C(\hat{r}_i), \hat{l}_i\},$$

subject to $r \in \Delta^{n-1}$ and $\sum_{i \in N} l_i = L$, where $L \in \mathbb{R}_+$ is exogenous. In general, there is no unique solution to this problem and I will solely focus on the maximal attainable value for collectible charges $\sum_{i \in N} \min\{C(r_i^*), l_i^*\}$ at an arbitrary optimal solution.

Recall that, in the absence of cash constraints, the equal-share partnership is always ex post efficiently dissolvable. As a result, when one is allowed to perfectly choose r and l simultaneously, the only reason ex post efficiency would fail would come from insufficient total cash resources. The following propositions derive the maximal collectible charge depending on total cash resources L .

Proposition 6 *Assume $L \geq \sum_{i \in N} C(1/n)$, then the maximal collectible charge is*

$$\sum_{i \in N} \min\{C(r_i^*), l_i^*\} = \sum_{i \in N} C(1/n).$$

The optimal ownership structure is unique and is $r_i^ = \frac{1}{n}$ for all $i \in N$ (equal-share). Any distribution of cash resources satisfying $l_i \geq C(1/n)$ for all $i \in N$ is optimal.*

Proof. First notice that for any (r, l) , $\sum_{i \in N} \min\{C(r_i), l_i\} \leq \sum_{i \in N} C(r_i) \leq \sum_{i \in N} C(1/n)$, where the first inequality is by definition of the min function and the second one comes from optimality of equal-share ownership in the absence of cash constraints. Then, simply let $r_i^* = \frac{1}{n}$ for all $i \in N$ and choose any l^* such that $l_i^* \geq C(1/n)$ for all $i \in N$. This

last inequality can always be satisfied as $L \geq \sum_{i \in N} C(1/n)$. It immediately follows that $\sum_{i \in N} \min\{C(r_i^*), l_i^*\} = \sum_{i \in N} C(1/n)$ which is necessarily the maximum. ■

Proposition 6 simply establishes the optimality of equal-share partnerships when cash resources are large enough and can be freely allocated among partners. The following corollary is directly linked to the result of CGK.

Corollary 1 *Assume $L \geq \sum_{i \in N} C(1/n)$ and that it is possible to simultaneously choose r and l , then ex post efficient dissolution mechanism are always feasible.*

Proof. From Proposition 6, $\sum_{i \in N} \min\{C(r_i^*), l_i^*\} = \sum_{i \in N} C(1/n)$ and equal-share partnership are always dissolvable as $\sum_{i \in N} C(1/n) > (n-1)G$ (CGK, 1987). ■

When total cash resources are large enough and can be freely allocated among partners the problem simply collapses to the CGK problem without cash constraints. The next proposition establishes the impact of more severe cash constraints on the total collectible charge.

Proposition 7 *Assume $L < \sum_{i \in N} C(1/n)$, then the maximal collectible charge is*

$$\sum_{i \in N} \min\{C(r_i^*), l_i^*\} = \sum_{i \in N} l_i,$$

that is, the maximal collectible charge is entirely defined by the total amount of cash resources.

Proof. Notice that $\sum_{i \in N} \min\{C(r_i), l_i\} \leq \sum_{i \in N} l_i$ by definition of the min function. If there exists an organization (r^*, l^*) such that $C(r_i^*) \geq l_i^*$ for all $i \in N$ then $\sum_{i \in N} \min\{C(r_i^*), l_i^*\} = \sum_{i \in N} l_i^* = L$ would attain its maximal value. From the assumption of the proposition, $L < \sum_{i \in N} C(1/n)$, and continuity of $\sum_{i \in N} C(r_i)$, it is straightforward that there must exist a nonempty subset $R \in \Delta^{n-1}$ such that $\sum_{i \in N} C(\tilde{r}_i) > L$ for all $\tilde{r} \in R$. Take an arbitrary $r^* \in R$, then it is always possible to find an l^* such that $C(r_i^*) \geq l_i^*$ for all $i \in N$. Hence $\sum_{i \in N} \min\{C(r_i^*), l_i^*\} = \sum_{i \in N} l_i^* = L$. ■

When the total amount of cash resources are such that $L < \sum_{i \in N} C(1/n)$, the maximal collectible charge is simply bounded above by the total amount of amount cash resources L . The optimal organization therefore allocates initial cash resources so that the maximal

collectible is equal to the total amount of amount cash resources. The feasibility of ex post efficient dissolution mechanism is therefore characterized as follows.

Corollary 2 *Assume $L < \sum_{i \in N} C(1/n)$ and that it is possible to simultaneously choose r and l , then ex post efficient dissolution is feasible if and only if $L > (n - 1)G$.*

Proof. Immediate from Proposition 7. ■

From Corollary 2, one can assess how much cash resources affect the feasibility of ex post efficient dissolution mechanisms. Whenever $L < (n - 1)G$, ex post efficient dissolution mechanisms would require to produce an ex post deficit of $(n - 1)G - L$ to ensure all other constraints. This means that, even if ownership and cash can be perfectly allocated initially, the total amount of cash resources must still cover the ex ante cost of implementing a Groves mechanism.

1.4.4. Some Examples

To illustrate Propositions 3, 4 and 5, assume that the asset value is between 0 and 1 million and valuations are uniformly distributed over support $[0, 1]$.

EXAMPLE 1. Let $n = 3$ and assume $\tilde{r} = (0, 0.4, 0.5)$. This happens when cash resources are approximately $(l_1, l_2, l_3) \approx (0.66, 0.49, 0.43)$. Assuming equal-share ownership gives $\sum_{i=1,2,3} \min\{C(\frac{1}{3}), l_i\} = C(\frac{1}{3}) + l_2 + l_3 \approx 1.47$ as $C(\frac{1}{3}) \approx 0.54 > l_2 > l_3$. Given that $(n - 1)G = \frac{3}{2}$, it follows that the equal-share ownership structure does not allow for ex post efficient dissolution.

Instead, as $\sum_{i \in N} \tilde{r}_i = 0.9$ and $\frac{1}{3} < \tilde{r}_2 < \tilde{r}_3$, Proposition 3.b gives that the optimal ownership structure writes $\tilde{r}^* = (0.1, 0.4, 0.5)$. As $\sum_{i=1,2,3} \min\{C(\frac{1}{3}), l_i\} \approx 1.57 > \frac{3}{2}$, the optimal ownership structures allows for ex post efficient dissolution. Ownership rights are then inversely proportional to cash resources and the optimal ownership structure is quite asymmetric.

EXAMPLE 2. Let $n = 3$ and assume $\tilde{r} = (0.3, 0.4, 0.45)$. This corresponds to $(l_1, l_2, l_3) \approx (0.56, 0.50, 0.47)$. Notice that $\sum_{i=1,2,3} \tilde{r}_i = 1.15 > 1$ so that Proposition 4 applies and thus $r_i^* \leq \tilde{r}_i$ for $i = 1, 2, 3$. As $C(0) = \frac{2}{3} > l_1 > l_2 > l_3$, it follows that $\sum_{i=1,2,3} \min\{C(r_i), l_i\} =$

$l_1 + l_2 + l_3 \approx 1.52 > \frac{3}{2}$ and the partnership can be ex post efficiently dissolved. Notice that choosing for instance $r^* = (0.3, 0.3, 0.4)$ or $r^* = (0.3, 0.25, 0.45)$ both give $r_i^* \leq \tilde{r}_i$ for all $i \in N$ and achieve the same outcome. However, agent 2 receives less initial ownership rights than agent 1 whereas the former has larger cash resources than the latter. This illustrates that optimal ownership structures might not always exhibit an inversely proportional relationship between cash and ownership rights when cash resources are severe, *i.e.* $\sum_{i \in N} \tilde{r}_i \geq 1$.

EXAMPLE 3. Assume equal-share ownership, $r_i = \frac{1}{n}$ for all $i \in N$ and symmetric cash resources $l_i = \tilde{l}$ for all $i \in N$. According to Proposition 5, ex post efficient dissolution is possible if and only if $\tilde{l} \geq \frac{n-1}{n}G = \frac{n-1}{n+1}$. Hence, when $n = 2, 3, 4, 5$ and 6 , the minimal cash resources are respectively $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}$ and $\frac{5}{7}$. A partnership with 5 agents for instance, each partner must possess two-third of a million to achieve ex post efficient dissolution.

1.5. IMPLEMENTATION: A SIMPLE AUCTION

The ex post efficient dissolution mechanism presented in Section 1.3 is appealing for its convenient mathematical properties which greatly simplifies the analysis of such mechanisms. It is, however, less appealing for the practitioner as it requires the setup of a game in which each agent reports their valuation and is communicated the transfer function $t_i^C(v)$.

EX POST EFFICIENT BIDDING GAME. I propose a simple auction that replicates the ex post efficient dissolution mechanism whenever the ex post efficient dissolution condition (1.8) holds. Each agent proposes a bid and the highest bidder receives full ownership of the asset. The auction is designed such that the bidding strategies are increasing in the agents' valuations so that the highest bidder coincides with the agent with the highest valuation. Each agent also receives side payments conditional on their share of ownership rights and cash resources to ensure IIR, EPBB and EPCC.

Let $b := (b_1, \dots, b_n) \in \mathbb{R}_+^n$ denote the vector of bids. In the ex post efficient bidding game each agent receives a side payment $\phi_i(r, l)$ and pays a price $p_i(b_1, \dots, b_n)$.

Theorem 5 *A bidding game with prices*

$$p_i(b_1, \dots, b_n) := \begin{cases} (n-1) \left[b_i + \frac{1}{n} \underline{v} \right] & \text{if } b_i \geq \max_k b_k \\ - \left[b_j + \frac{1}{n} \underline{v} \right] & \text{if } b_j \geq \max_k b_k, \end{cases}$$

and side-payments

$$\phi_i(r, l) := \frac{1}{n} \sum_{j \in N} \min\{C(r_j), l_j\} - \min\{C(r_i), l_i\},$$

efficiently dissolves any dissolvable partnership with cash-constrained agents.

Proof. Let b_i be player i 's strategy and $b(v_j)$ be the bidding strategy of player $j \neq i$ with valuation v_j . Agent i 's interim expected utility (omitting side payments) when bidding b_i writes

$$\begin{aligned} U_i(b_i; v_i) &:= \left[v_i - (n-1) \left(b_i + \frac{1}{n} \underline{v} \right) \right] \mathbb{E}_{-i} \mathbb{1}\{b_i > \max_{k \neq i} b(v_k)\} \\ &+ \sum_{j \neq i} \mathbb{E}_{-i} \left[\mathbb{1}\{b(v_j) > b_i\} \mathbb{1}\{b(v_j) > \max_{k \neq i, j} b(v_k)\} \left[b(v_j) + \frac{1}{n} \underline{v} \right] \right]. \end{aligned}$$

Solving for strictly increasing symmetric Bayesian equilibrium, the bidding strategy of the $j \neq i$ players, $b(v_j)$, is strictly increasing and then invertible. Notice that $\mathbb{1}\{b_i > \max_{k \neq i} b(v_k)\} = \mathbb{1}\{b^{-1}(b_i) > \max_{k \neq i} v_k\}$ and $\mathbb{1}\{b(v_j) > \max_{k \neq i} b(v_k)\} = \mathbb{1}\{v_j > \max_{k \neq i} v_k\}$. It follows that player i 's interim expected utility rewrites

$$U_i(b_i; v_i) = \left[v_i - (n-1) \left(b_i + \frac{1}{n} \underline{v} \right) \right] Z(b^{-1}(b_i)) + \int_{b^{-1}(b_i)}^{\bar{v}} \left[b(v_j) + \frac{1}{n} \underline{v} \right] dZ(v_j),$$

where $Z := F^{n-1}$. Let $z = Z'$, differentiating $U(b_i; v_i)$ with respect to b_i and simplifying using $\frac{\partial b^{-1}}{\partial b_i}(b_i) = \frac{1}{b'(b^{-1}(b_i))}$ gives

$$\frac{\partial U_i}{\partial b_i}(b_i; v_i) = -(n-1)Z(b^{-1}(b_i)) + \frac{z(b^{-1}(b_i))}{b'(b^{-1}(b_i))} \left[v_i - nb_i - \underline{v} \right].$$

At equilibrium, $b(v_i)$ must be such that $\frac{\partial U_i}{\partial b_i}(b(v_i); v_i) = 0$. Therefore, $b(v_i)$ must solve

$$-(n-1)Z(v_i) + \frac{z(v_i)}{b'(v_i)} [v_i - nb(v_i) - \underline{v}] = 0.$$

It is easy to show that $b(v_i) := \int_{\underline{v}}^{v_i} \frac{\int_{\underline{v}}^t F(s)^n ds}{F(t)^{n+1}} f(t) dt$ solves this first-order differential equation and is strictly increasing in v_i .¹⁹ It follows that at the Bayesian equilibrium, player i pays a price

$$p_i(b(v_1), \dots, b(v_n)) = \begin{cases} (n-1) \left[\int_{\underline{v}}^{v_i} \frac{\int_{\underline{v}}^t F(s)^n ds}{F(t)^{n+1}} f(t) dt + \frac{1}{n} \underline{v} \right] & \text{if } b_i \geq \max_k b_k \\ - \left[\int_{\underline{v}}^{v_j} \frac{\int_{\underline{v}}^t F(s)^n ds}{F(t)^{n+1}} f(t) dt + \frac{1}{n} \underline{v} \right] & \text{if } b_j \geq \max_k b_k. \end{cases} \quad (1.17)$$

It can easily be proven that $p_i(b(v_1), \dots, b(v_n))$ together with $\phi_i(r, l)$ exactly replicates the transfer rule of Theorem 1, $t_i^C(v)$, for all $v \in V^n$ and all $i \in N$. The bidding game is thus EF (as $b(\cdot)$ is increasing, the bidder with the highest valuation gets full ownership), IIR, EPBB and EPCC as it reproduces the transfer rule of Theorem 1. ■

Theorem 5 can be interpreted as follows. The agent with the highest bid, $b_k = \max_{i \in N} b_i$, receives full ownership of the asset and gives an amount of money equal to b_k to all other agents $j \in N \setminus \{k\}$. Unconditional on bids, each agent receives a transfer $\phi_i(r, l)$.

Furthermore, there is no need for an outside party to advance money before running the auction. In practice, the auctioneer could simply announce to each agent their side payment $\phi_i(r, l)$, run the auction and then compute total payment for each agent, $\phi_i(r, l) - p_i(b_1, \dots, b_n)$ from the submitted bids. Agents with a $\phi_i(r, l) - p_i(b_1, \dots, b_n) < 0$ pay the auctioneer who then redistribute this amount of money to the agents with $\phi_i(r, l) - p_i(b_1, \dots, b_n) \geq 0$. As the auction is budget balanced, this can always be done without outside funding.

¹⁹The first-order condition is also sufficient. Notice that $b(v_i) = \int_{\underline{v}}^{v_i} \psi(t) \frac{f(t)}{F(t)} dt$ where $\psi(t)$ is defined by equation (1.10). Then $b'(v_i) = \psi(v_i) \frac{f(v_i)}{F(v_i)} = (v_i - nb(v_i) - \underline{v}) \frac{f(v_i)}{F(v_i)}$, where the second equality stems from Lemma 1. Hence, after simplifications, $\frac{\partial U_i}{\partial b_i}(b_i; v_i) = (n-1)Z(b^{-1}(b_i)) \left[-1 + \frac{v_i - nb_i - \underline{v}}{b^{-1}(b_i) - nb_i - \underline{v}} \right]$. It follows that $\frac{\partial U_i}{\partial b_i}(b_i; v_i) \geq 0$ (resp. ≤ 0) when $b_i \leq b(v_i)$ (resp. $\geq b(v_i)$) for any $v_i \in V$.

COMPARISON WITH CGK: UNIFORM AND SYMMETRIC EXAMPLE. It is interesting to investigate the properties of the bidding game proposed in Theorem 5 with the one proposed by CGK (Theorem 2, p. 620). In CGK, the bidding game has prices $p_i^{\text{CGK}}(\mathbf{b}) = b_i - \frac{1}{n-1} \sum_{j \neq i} b_j$ and side payments $c_i^{\text{CGK}}(\mathbf{r}) = \int_{\underline{v}}^{v_i^*} x dF(x)^{n-1} - \frac{1}{n} \sum_{j \in N} \int_{\underline{v}}^{v_j^*} x dF(x)^{n-1}$ where v_i^* is defined as in Section 1.4.1. The equilibrium bidding strategy in CGK is given by $b^{\text{CGK}}(v_i) = \int_{\underline{v}}^{v_i} u dF(u)^{n-1}$.

For simplicity, assume that valuations are uniformly distributed over support $[0, 1]$, and consider an equal-share-equal-cash partnership, *i.e.* $r_i = \frac{1}{n}$ and $l_i = \tilde{l}$ for all $i \in N$. It immediately follows that $\phi_i(\mathbf{r}, \mathbf{l}) = c_i^{\text{CGK}}(\mathbf{r}) = 0$ for all $i \in N$, that is, side payments are zero for all agents in both bidding games. In the cash-constrained auction, the maximal price is obtained by maximizing equation (1.17), which is the same as minimizing t_i^{C} (ignoring the constant term) as the auction replicates the dissolution mechanism. Therefore, from equation (1.11), $\max_{\mathbf{v} \in V^n} p_i(\mathbf{b}) = \frac{n-1}{n} G = \frac{n-1}{n+1}$ as $G = \frac{n}{n+1}$ in the uniform case. Alternatively, using the LHS of equation (1.12) (again ignoring the constant) gives that $\max_{\mathbf{v} \in V^n} p_i^{\text{CGK}}(\mathbf{b}) = \frac{n-1}{n}$. Hence, it is clear that $\max_{\mathbf{v} \in V^n} p_i^{\text{CGK}}(\mathbf{b}) > \max_{\mathbf{v} \in V^n} p_i(\mathbf{b})$ so that the maximal price in CGK auction is strictly higher than the one in the cash-constrained auction.

Given that side payments are null in both auctions, EPCC requires that the maximal price never exceeds the agents' financial resources \tilde{l} . Let $n = 2$, then $\max_{\mathbf{v} \in V^n} p_i^{\text{CGK}}(\mathbf{b}) = \frac{1}{2}$ and $\max_{\mathbf{v} \in V^n} p_i^{\text{C}}(\mathbf{b}) = \frac{1}{3}$. It follows that any of the two auctions ex post efficiently dissolves this symmetric partnership for $\tilde{l} \geq \frac{1}{2}$ and none dissolves it if $\tilde{l} < \frac{1}{3}$ (this indeed violates condition (1.8)). However, only the cash-constrained auction ex post efficiently dissolves this partnership for $\tilde{l} \in [\frac{1}{3}, \frac{1}{2})$ as CGK auction requires agents to pay prices that may exceed their financial resources. This particular example illustrates the discrepancy between the necessary and sufficient condition (1.8) and the sufficient dissolution condition with CGK mechanism, condition (1.13).

1.6. SECOND-BEST DISSOLUTION MECHANISMS

When condition (1.8) is not satisfied, it is not possible to simultaneously ensure an ex post efficient allocation rule and all the required constraints. Therefore, the problem consists

in finding a new mechanism whose allocation rule simultaneously maximizes the ex ante surplus and satisfies all the constraints. These mechanisms are referred to as *second-best mechanisms*.

One of the most difficult challenges when investigating second-best mechanisms comes from the participation and cash constraints. Usually, the structure of the problems studied in the literature are such that those types of constraints can be replaced by the one for the lowest/highest type independently of the mechanism in question. In partnership models, however, the worst-off types are endogenously determined by the mechanism and must be defined simultaneously to the allocation rule.

Second-best mechanisms in partnership models have mainly been studied by [Lu and Robert \(2001\)](#) and [Loertscher and Wasser \(2019\)](#) who consider Bayesian mechanisms without cash constraints. I build on these two papers and on [Boulatov and Severinov \(2019\)](#) who investigate the design of optimal auctions in the presence of cash-constrained bidders.

METHODOLOGY AND ASSUMPTIONS. I investigate dissolution mechanisms satisfying IIC, IIR, EPCC but I relax EPBB to EABB as imposing EPBB had proved too difficult. I conjecture, however, that adding the EPBB requirement can directly be derived using the present work and only requires to construct an appropriate transfer function. I hope that future work will address this issue.

The first-best analysis shows that implementing the ex post efficient allocation rule requires that enough money can be collected on agents to cover the cost of imposing incentive compatibility. The same logic applies to any other allocation rule. Following the methodology of [Lu and Robert \(2001\)](#) and [Loertscher and Wasser \(2019\)](#), I start by investigating allocation rules that maximize a linear combination of the expected surplus and the expected revenue that can be collected on agents by imposing IIC, IIR and EPCC but ignoring EABB. This allows to characterize optimal second-best mechanisms for any possible budget deficit.²⁰ It then appears that solving the second-best problem including

²⁰It may sometimes not be desirable to strictly impose budget balance. For instance, if the dissolution problem is run by a public authority, it may be willing to achieve a more efficient outcome at the expense of covering a strictly positive deficit.

EABB is just a particular solution to the previous problem for well-chosen weights on expected surplus and collected revenue.

Then, for any weight $\lambda \in [0, 1]$, the objective function writes,

$$W_\lambda := (1 - \lambda) \sum_{i \in N} \mathbb{E}[v_i(s_i(v) - r_i)] + \lambda \sum_{i \in N} \mathbb{E}[-t_i(v)], \quad (1.18)$$

where the first term is the expected surplus and the second term is the expected revenue. I introduce the following two notations. For any $\lambda \in [0, 1]$, let

$$\alpha(v_i | \lambda) = v_i - \lambda \frac{1 - F(v_i)}{f(v_i)} \quad \text{and} \quad \beta(v_i | \lambda) = v_i + \lambda \frac{F(v_i)}{f(v_i)},$$

where $\alpha(\cdot | \lambda)$ and $\beta(\cdot | \lambda)$ are referred to as *buyer's virtual valuation* and *seller's virtual valuation*, respectively (see [Lu and Robert \(2001\)](#)). Notice that for any $v_i \in (\underline{v}, \bar{v})$ and $\lambda \in [0, 1]$, $\alpha(v_i | \lambda) < v_i < \beta(v_i | \lambda)$. The following assumptions are made on these functions.

Assumption 1 *For any $\lambda \in [0, 1]$, the virtual valuations $\alpha(v_i | \lambda)$ and $\beta(v_i | \lambda)$ are both strictly increasing in v_i .*

This assumption is a regularity assumption on the distribution function $F(\cdot)$ to avoid bunching due to nonregular distribution functions.²¹ It is standard in the literature (see [Myerson \(1981\)](#)) and weaker than imposing increasing hazard rate.

Assumption 2 *Assume that $f(\cdot)$ is nonincreasing.*

This assumption is not necessary as suggested by [Boulatov and Severinov \(2019\)](#) but it is imposed in the present work as it greatly simplifies the analysis.

1.6.1. Characterization Of General IIC, IIR And EPCC Mechanisms

The analysis of dissolution mechanisms is no longer restricted to the ex post efficient allocation rule s^* defined in Section 1.3. Therefore, Proposition 1 does not apply here and

²¹However, second-best mechanisms will generically involve bunching due to the initial allocation of ownership rights and to the cash constraints. This assumption on the distribution of valuation therefore only rules out bunching due to nonregular distribution functions.

it is necessary to characterize dissolution mechanisms satisfying IIC, IIR and EPCC for any possible allocation rule.

INCENTIVE COMPATIBILITY. Take any allocation rule $s(v) = (s_1(v), \dots, s_n(v))$ such that $s_i(v) \in [0, 1]$ for all $i \in N$, $v \in V^n$ and $\sum_{i \in N} s_i(v) = 1$ for all $v \in V^n$. The following standard characterization of IIC mechanisms applies (see [Myerson \(1981\)](#)).

Lemma 2 *A dissolution mechanism (s, t) is IIC if and only if*

$$S_i \text{ is nondecreasing for all } i \in N, \quad (\text{IC1})$$

$$U_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i(x) - r_i) dx \quad \text{for all } v_i, v_i^* \in V. \quad (\text{IC2})$$

Proof. The proof is standard and thus omitted (see [Myerson \(1981\)](#), [Lu and Robert \(2001\)](#)). ■

The set of dissolution mechanisms satisfying IIC must then be such that the interim allocation rule $S_i := \mathbb{E}_{-i} s_i$ is nondecreasing and the interim utility of each agent must satisfy (IC2). Lemma 2 also implies that U_i is absolutely continuous. Recall that, by definition, $U_i(v_i) = v_i(S_i(v_i) - r_i) + T_i(v_i)$ so that (IC2) indirectly defines the interim transfers as follows

$$T_i(v_i) = T_i(v_i^*) - \int_{v_i^*}^{v_i} x dS_i(x) \quad \text{for all } v_i, v_i^* \in V, \quad (1.19)$$

which is decreasing in v_i .

INDIVIDUAL RATIONALITY. Imposing IIR requires that $U_i(v_i) \geq 0$ for all $i \in N$ and $v_i \in V$ where $U_i(v_i)$ is defined by (IC2). Let $v_i^* \in \arg \min_{\tilde{v}_i \in V} U_i(\tilde{v}_i)$ denote a *worst-off* type for agent i , then IIR can be characterized as follows (see CGK and [Lu and Robert \(2001\)](#)).

Lemma 3 *A dissolution mechanism (s, t) is IIC and IIR if and only if it satisfies Lemma 2 and for every $i \in N$,*

$$U_i(v_i^*) \geq 0, \quad (\text{IR1})$$

where v_i^* denotes a worst-off type of agent i and

$$v_i^* \in V^*(S_i) := \{v_i \mid S_i(x) \leq r_i, \forall x < v_i; S_i(y) \geq r_i, \forall y > v_i\}. \quad (\text{IR2})$$

Proof. The proof is the same as in [Lu and Robert \(2001\)](#). ■

(IR1) simply states that the continuum of constraints $U_i(v_i) \geq 0$ can be replaced by imposing IIR only on the set of worst-off types $V^*(S_i)$ and (IR2) defines this set. Notice that (i) the set of worst-off types is endogenously determined by the interim allocation rule S_i and, (ii) this set contains all agents who are expected to be neither a buyer nor a seller.

CASH CONSTRAINTS. Finally, imposing EPCC requires that $t_i(v) \geq -l_i$ for all $i \in N$ and $v \in V^n$. As first noted by [Laffont and Robert \(1996\)](#), it is possible to set a mechanism (s, t) where $t_i(v)$ depends only on agent i 's private information so that $t_i(v) = T_i(v_i)$ for all $v \in V^n$. It affects neither the objective function W_λ nor IIC nor IIR as they all depend only on the interim transfers $T_i(v_i)$. Therefore, without loss of generality, EPCC is satisfied by requiring $T_i(v_i) \geq -l_i$ for all $i \in N, v_i \in V$.

Following [Boulatov and Severinov \(2019\)](#), define

$$\bar{m}_i := \inf \{v_i \in V \mid T_i(v_i) = T_i(\bar{v})\}. \quad (1.20)$$

Then, any dissolution mechanism (s, t) for which $\bar{m}_i < \bar{v}$ is such that $T_i(v_i) = T_i(\bar{v})$ for all $v_i \in [\bar{m}_i, \bar{v}]$ as IIR imposes that $T_i(v_i)$ is nondecreasing in v_i . As S_i is nondecreasing it follows from (1.19) that imposing $T_i(v_i) = T_i(\bar{v})$ for all $v_i \in [\bar{m}_i, \bar{v}]$ requires that $S_i(v_i)$ is constant over $[\bar{m}_i, \bar{v}]$. Then, without loss of generality, $S_i(v_i) = S_i(\bar{m}_i)$ for all $v_i \in [\bar{m}_i, \bar{v}]$ must hold for interim transfer to be constant.

EPCC can therefore be replaced with the following two conditions for all $i \in N$

$$T_i(\bar{m}_i) \geq -l_i, \quad (\text{CC1})$$

$$S_i(v_i) = S_i(\bar{m}_i) \quad \text{for all } v_i \in [\bar{m}_i, \bar{v}]. \quad (\text{CC2})$$

(CC1) ensures that all transfers are lower or equal to each agent's cash resources while

(CC2) is necessary for interim transfers to stay constant when $v_i \in [\bar{m}_i, \bar{v}]$.

Notice that (CC1) can also be expressed in terms of interim utility, that is, $T_i(\bar{m}_i) = U_i(\bar{m}_i) - \bar{m}_i(S_i(\bar{m}_i) - r_i) \geq -l_i$. Replacing $U_i(\bar{m}_i)$ by (IC2) evaluated at \bar{m}_i gives that (CC1) rewrites

$$U_i(v_i^*) \geq \bar{m}_i(S_i(\bar{m}_i) - r_i) - \int_{v_i^*}^{\bar{m}_i} (S_i(x) - r_i) dx - l_i, \quad (\text{CC1})$$

so that (CC1) is expressed only in terms of the interim utility of a worst-off type.

1.6.2. The Second-Best Optimization Program

The problem therefore consists in maximizing W_λ subject to (IC1), (IC2), (IR1), (IR2), (CC1), (CC2) and the two resource constraints $s_i(v) \in [0, 1]$ for all $v_i \in V$ and $\sum_{i \in N} s_i(v) = 0$ for all $v \in V^n$. First, notice that the transfer function only enters the objective function but can be completely removed from the problem by directly imposing (IC2) on W_λ . Rewrite,

$$W_\lambda = \sum_{i \in N} \int_V \left\{ (1 - \lambda)v_i(S_i(v_i) - r_i) - \lambda T_i(v_i) \right\} dF(v_i).$$

(IC2) gives that $T_i(v_i) = U_i(v_i^*) + \int_{v_i^*}^{v_i} (S_i(x) - r_i) dx - v_i[S_i(v_i) - r_i]$. Plugging this expression in the objective function, integrating by parts and rearranging yields

$$W_\lambda = \sum_{i \in N} \int_V (S_i(v_i) - r_i) \left[v_i + \mathbb{1}\{v_i \leq v_i^*\} \lambda \frac{F(v_i)}{f(v_i)} - \mathbb{1}\{v_i \geq v_i^*\} \lambda \frac{1 - F(v_i)}{f(v_i)} \right] dF(v_i) - \lambda \sum_{i \in N} U_i(v_i^*).$$

Now I want to impose (CC2) directly on the objective function. For that matter, first consider the following result.

Lemma 4 *For any IIC, IIR and EPCC mechanism with $l \in (\mathbb{R}_+^*)^n$, if $\bar{m}_i < \bar{v}$ then $\bar{m}_i \geq \sup V^*(S_i)$.*

Proof. As $\bar{m}_i < \bar{v}$ then $T_i(\bar{m}_i) = -l_i$ and then $U_i(\bar{m}_i) = \bar{m}_i(S_i(\bar{m}_i) - r_i) - l_i$. IIR implies that $U_i(\bar{m}_i) = \bar{m}_i(S_i(\bar{m}_i) - r_i) - l_i \geq 0$ which immediately requires that $S_i(\bar{m}_i) > r_i$. By

definition of $V^*(S_i)$, $S_i(v_i) \leq r_i$ for all $v_i \leq \inf V^*(S_i)$ and $S_i(v_i) = r_i$ for all v_i in the interior of $V^*(S_i)$. It follows that $\bar{m}_i \geq \sup V^*(S_i)$. ■

As lemma 4 ensures that $\bar{m}_i \geq \sup V^*(S_i)$, plugging (CC2) into the objective function and using the definitions of $\beta(v_i | \lambda)$ and $\alpha(v_i | \lambda)$ yields

$$W_\lambda(s, \mathbf{U}, \bar{\mathbf{m}}) := \sum_{i \in \mathbf{N}} \int_{\underline{v}}^{\bar{m}_i} (S_i(v_i) - r_i) \left[\mathbb{1}\{v_i \leq v_i^*\} \beta(v_i | \lambda) + \mathbb{1}\{v_i \geq v_i^*\} \alpha(v_i | \lambda) \right] dF(v_i) \\ + \sum_{i \in \mathbf{N}} \int_{\bar{m}_i}^{\bar{v}} (S_i(\bar{m}_i) - r_i) \alpha(v_i | \lambda) dF(v_i) - \lambda \sum_{i \in \mathbf{N}} U_i(v_i^*),$$

where $W_\lambda(s, \mathbf{U}, \bar{\mathbf{m}})$ defines the objective as a function of the allocation rule s , the interim utilities of the worst-off types $\mathbf{U} = (U_1(v_1^*), \dots, U_n(v_n^*))$ and the thresholds $\bar{\mathbf{m}} := (\bar{m}_1, \dots, \bar{m}_n)$.

THE RELAXED PROBLEM. As it is standard in the literature, consider a relaxed problem in which (IC1) and (IR2) are ignored. It will be proven later that the relaxed problem satisfies those constraints. Let $\chi := (\chi_1, \dots, \chi_n) \in \mathbb{R}_+^n$ and $\tau := (\tau_1, \dots, \tau_n) \in \mathbb{R}_+^n$ denote the Lagrange multipliers associated with (IR1) and (CC1), respectively. Then, the relaxed problem is given by the following Lagrangian

$$\mathcal{L} := W_\lambda(s, \mathbf{U}, \bar{\mathbf{m}}) + \sum_{i \in \mathbf{N}} \chi_i U_i(v_i^*) + \sum_{i \in \mathbf{N}} \tau_i \left(U_i(v_i^*) - \bar{m}_i (S_i(\bar{m}_i) - r_i) \right) + \int_{v_i^*}^{\bar{m}_i} (S_i(x) - r_i) dx + l_i.$$

A little algebra shows that the Lagrangian can be rewritten

$$\mathcal{L} = \sum_{i \in \mathbf{N}} \int_{\underline{v}}^{\bar{m}_i} (S_i(v_i) - r_i) \left[\mathbb{1}\{v_i \leq v_i^*\} \beta(v_i | \lambda) + \mathbb{1}\{v_i \geq v_i^*\} \left[\alpha(v_i | \lambda) + \frac{\tau_i}{f(v_i)} \right] \right] dF(v_i) \\ + \sum_{i \in \mathbf{N}} \int_{\bar{m}_i}^{\bar{v}} (S_i(\bar{m}_i) - r_i) \left[\alpha(v_i | \lambda) - \frac{\tau_i \bar{m}_i}{1 - F(\bar{m}_i)} \right] dF(v_i) + \sum_{i \in \mathbf{N}} (\chi_i + \tau_i - \lambda) U_i(v_i^*) + \sum_{i \in \mathbf{N}} \tau_i l_i.$$

Notice that

$$\int_{\bar{m}_i}^{\bar{v}} (S_i(\bar{m}_i) - r_i) \alpha(v_i | \lambda) dF(v_i) = \int_{\bar{m}_i}^{\bar{v}} (S_i(\bar{m}_i) - r_i) \mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] dF(v_i), \quad (1.21)$$

Then, define the *virtual valuation* of agent i as

$$\Gamma_i(v_i | v_i^*, \bar{m}_i, \lambda) := \begin{cases} \beta(v_i | \lambda) & \text{if } v_i \in [\underline{v}, v_i^*) \\ v_i^* & \text{if } v_i = v_i^* \\ \alpha(v_i | \lambda) + \frac{\tau_i}{f(v_i)} & \text{if } v_i \in (v_i^*, \bar{m}_i) \\ \mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \frac{\tau_i \bar{m}_i}{1 - F(\bar{m}_i)} & \text{if } v_i \geq [\bar{m}_i, \bar{v}]. \end{cases}$$

Using the fact that $S_i(v_i) = \int_{V^{n-1}} s_i(v) \prod_{j \neq i} dF(v_j)$ and the definition of Γ_i , the Lagrangian simply rewrites

$$\mathcal{L} = \int_{V^n} (s_i(v) - r_i) \Gamma_i(v_i | v_i^*, \bar{m}_i, \lambda) \prod_{k \in N} dF(v_k) + \sum_{i \in N} (\chi_i + \tau_i - \lambda) U_i(v_i^*) + \sum_{i \in N} \tau_i l_i.$$

Notice that Γ_i is strictly increasing over $[\underline{v}, v_i^*)$ and (v_i^*, \bar{m}_i) and constant over $[\bar{m}_i, \bar{v}]$. Yet, the virtual valuation Γ_i has a downward discontinuous jump at v_i^* and it is not clear how it behaves at \bar{m}_i . Therefore, it is not possible to directly solve the problem by pointwise maximizing \mathcal{L} as it would violate the monotonicity of S_i at $v_i = v_i^*$.

To ensure that (IC1) is satisfied, I first replace the virtual valuation Γ_i by an ironed virtual valuation around v_i^* (see [Myerson \(1981\)](#), [Lu and Robert \(2001\)](#), [Loertscher and Wasser \(2019\)](#)). Formally, for any given λ , define $\bar{x}_i \in V$ such that $\beta(\bar{x}_i | \lambda) = \alpha(\bar{m}_i | \lambda) + \frac{\tau_i}{f(\bar{m}_i)}$. Now, for any $x_i \in [\underline{v}, \bar{x}_i]$ let y_i be such that

$$\alpha(y_i | \lambda) + \frac{\tau_i}{f(y_i)} = \beta(x_i | \lambda). \quad (1.22)$$

Then, define

$$\delta_i(v_i | x_i, \bar{m}_i, \lambda) := \begin{cases} \Gamma_i(v_i | x_i, \bar{m}_i, \lambda) & \text{if } v_i \notin [x_i, y_i] \\ \beta(x_i | \lambda) & \text{if } v_i \in [x_i, y_i]. \end{cases}$$

The function $\delta_i(\cdot | \cdot)$ is referred to as the *ironed virtual valuation* of agent i . It coincides with the virtual valuation Γ_i everywhere but on $[x_i, y_i]$ where it is constant. By definition of x_i and y_i , $\delta_i(\cdot | \cdot)$ is therefore increasing and continuous on $[\underline{v}, \bar{m}_i]$.

The methodology is as follows. First, I replace Γ_i by δ_i in the Lagrangian of the

relaxed problem and solve for s_i in the new problem by pointwise maximization. Building on [Boulatov and Severinov \(2019\)](#), I show that δ_i is nondecreasing at $v_i = \bar{m}_i$ so that, combined with the definition of δ_i for $v_i \notin [x_i, y_i]$, the solution s_i is increasing in v_i and so does S_i , satisfying (IC1). Second, I prove that the solution to the problem with ironed virtual valuation δ_i also solves the problem with virtual valuation Γ_i .

For some $x_i \in [\underline{v}, \bar{x}_i]$, consider the problem of maximizing the following Lagrangian

$$\hat{\mathcal{L}} := \int_{\mathcal{V}^n} (s_i(v) - r_i) \delta_i(v_i | x_i, \bar{m}_i, \lambda) \prod_{k \in \mathbb{N}} dF(v_k) + \sum_{i \in \mathbb{N}} (\chi_i + \tau_i - \lambda) \mathcal{U}_i(x_i) + \sum_{i \in \mathbb{N}} \tau_i l_i.$$

Pointwise maximization with respect to s_i gives

$$s_i(v_i, v_{-i}) = \begin{cases} 1 & \text{if } \delta_i(v_i | x_i, \bar{m}_i, \lambda) > \max_{j \neq i} \delta_j(v_j | x_j, \bar{m}_j, \lambda) \\ \in [0, 1] & \text{if } \delta_i(v_i | x_i, \bar{m}_i, \lambda) = \max_{j \neq i} \delta_j(v_j | x_j, \bar{m}_j, \lambda) \\ 0 & \text{if } \delta_i(v_i | x_i, \bar{m}_i, \lambda) < \max_{j \neq i} \delta_j(v_j | x_j, \bar{m}_j, \lambda), \end{cases} \quad (1.23)$$

as the problem is linear in s_i , full ownership goes to the agent with the highest ironed virtual valuation δ_i . If two or more agents have the highest valuation, the final distribution of ownership among those agents does not affect optimality. Yet, contrary to the first-best mechanism, ties may occur with positive probability (due to bunching regions) so that the way the mechanism breaks ties among agents will now affect IIR. As in [Lu and Robert \(2001\)](#) and [Boulatov and Severinov \(2019\)](#), the design of tie-breaking rules becomes an important element of the mechanism.

Using equation (1.23), the Lagrangian $\hat{\mathcal{L}}$ rewrites

$$\hat{\mathcal{L}} = \int_{\mathcal{V}^n} \left\{ \max_{i \in \mathbb{N}} \delta_i(v_i | x_i, \bar{m}_i, \lambda) - \sum_{i \in \mathbb{N}} r_i \delta_i(v_i | x_i, \bar{m}_i, \lambda) \right\} \prod_{k \in \mathbb{N}} dF(v_k) + \sum_{i \in \mathbb{N}} (\chi_i + \tau_i - \lambda) \mathcal{U}_i(x_i) + \sum_{i \in \mathbb{N}} \tau_i l_i.$$

The following result is a generalization of [Boulatov and Severinov \(2019\)](#), Theorem 1 (p. 15). Define first,

$$\delta_i^-(\bar{m}_i | x_i, \bar{m}_i, \lambda) = \lim_{v_i \uparrow \bar{m}_i} \delta_i(\bar{m}_i | x_i, \bar{m}_i, \lambda) = \alpha(\bar{m}_i | \lambda) + \frac{\tau_i}{f(\bar{m}_i)},$$

then,

Theorem 6 *A solution to the maximization of $\hat{\mathcal{L}}$ is such that*

1. *For all $i \in \mathbb{N}$ such that $\bar{m}_i \leq \bar{m}_j$ for some $j \neq i$, $\delta_i(v_i | x_i, \bar{m}_i, \lambda)$ is continuous at $v_i = \bar{m}_i$, that is,*

$$\tau_i = \frac{\left(\mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \alpha(\bar{m}_i | \lambda) \right) (1 - F(\bar{m}_i)) f(\bar{m}_i)}{1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)}. \quad (1.24)$$

2. *If it exists, agent $z_1 \in \mathbb{N}$ such that $\max_{i \neq z_1} \bar{m}_i < \bar{m}_{z_1} < \bar{v}$ then $\delta_{z_1}(\bar{m}_{z_1} | x_{z_1}, \bar{m}_{z_1}, \lambda) > \delta_{z_1}^-(\bar{m}_{z_1} | x_{z_1}, \bar{m}_{z_1}, \lambda) = \max_{i \neq z_1} \delta_i(\bar{m}_i | x_i, \bar{m}_i, \lambda)$. Hence, we have*

$$\alpha(\bar{m}_{z_1} | \lambda) + \frac{\tau_{z_1}}{f(\bar{m}_{z_1})} = \max_{i \neq z_1} \frac{\int_{\bar{m}_i}^{\bar{v}} \alpha(v_i | \lambda) dF(v_i) + \bar{m}_i f(\bar{m}_i) \alpha(\bar{m}_i | \lambda)}{1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)}. \quad (1.25)$$

Proof. See Appendix B. ■

Although complex, Theorem 6 gives one important simple result: At $v_i = \bar{m}_i$, the ironed virtual valuation $\delta_i(\cdot | \cdot)$ is either continuous or has a discontinuous upward jump. Hence, combined with the fact that by definition $\delta_i(\cdot | \cdot)$ is increasing on $[\underline{v}, \bar{m}_i]$, the ironed virtual valuation is therefore nondecreasing for all $v_i \in V$. It follows that the allocation rule that maximizes $\hat{\mathcal{L}}$, equation (1.23), is nondecreasing in v_i . Thus, S_i is also nondecreasing in v_i and pointwise maximization of $\hat{\mathcal{L}}$ satisfies (IC1). Then the following holds.

Theorem 7 *If there exists an $x := (x_1, \dots, x_n) \in \times_{i \in \mathbb{N}} [\underline{v}, \bar{x}_i]$ and a solution to the following problem,*

$$\begin{aligned} (A) \quad \max_{s, \bar{m}, U, \tau, \chi} \quad & \hat{\mathcal{L}} = \int_{V^n} \sum_{i \in \mathbb{N}} (s_i(v) - r_i) \delta_i(v_i | x_i, \bar{m}_i, \lambda) \prod_{k \in \mathbb{N}} dF(v_k) \\ & + \sum_{i \in \mathbb{N}} (\chi_i + \tau_i - \lambda) U_i(x_i) + \sum_{i \in \mathbb{N}} \tau_i l_i \\ \text{s.t.} \quad & s_i(v) \geq 0 \quad \forall i \in \mathbb{N}, v \in V^n \\ & \sum_{j \in \mathbb{N}} s_j(v) = 1, \end{aligned}$$

(B) For all $i \in N$, $S_i(v_i) = r_i$ for $v_i \in [x_i, y_i]$.

Then if $(s^*, U^*, \bar{m}^*, \tau^*, \chi^*)$ satisfies (A) and (B) it also solves the full problem.

Proof. See Appendix B.

From Theorem 7, directly maximizing $\hat{\mathcal{L}}$ subject to the resource constraints is sufficient to obtain the solution to the general problem. So, the second-best solution amounts to give the final ownership to the agent with the highest ironed virtual valuation characterized by $\delta_i(v_i | \cdot)$. Those ironed virtual valuations are nondecreasing in v_i so that higher valuations give (weakly) better chances to receive final ownership. In case of tie (in terms of ironed virtual valuation), however, the second-best solution can lead to a situation in which more than one agent receives final ownership shares.

Yet, it remains to determine to characterize more precisely the ironed virtual valuations that depend on the endogenously determined cut-off types \bar{m} and the Lagrange multipliers τ associated with (CC1). The following three corollaries derived from Theorem 6 help better characterize the ironed virtual valuations. The proofs are relegated to Appendix B.

Corollary 3 At optimum, τ_i is decreasing in \bar{m}_i for all $i \in N$.

Corollary 4 For all $i \in N$ such that $\delta_i^-(\bar{m}_i | x_i, \bar{m}_i, \lambda) = \delta_i(\bar{m}_i | x_i, \bar{m}_i, \lambda)$, $\delta_i(\bar{m}_i)$ is increasing in \bar{m}_i .

Corollary 5 There is a bijection between $\bar{m} = (\bar{m}_1, \dots, \bar{m}_n)$ and $\tau = (\tau_1, \dots, \tau_n)$ according to equations (1.24) and (1.25).

To illustrate the intuition of these results, take two agents with $\bar{m}_i \leq \bar{m}_j$, then $\tau_i \geq \tau_j$ from Corollary 3. From Corollary 4, $\delta_i(\bar{m}_i | x_i, \bar{m}_i, \lambda) \leq \delta_j(\bar{m}_j | x_j, \bar{m}_j, \lambda)$. Loosely speaking, agent j is given an advantage over agent i in terms of ironed virtual valuation for large valuations as $\delta_i(\bar{m}_i | x_i, \bar{m}_i, \lambda) \leq \delta_j(\bar{m}_j | x_j, \bar{m}_j, \lambda)$ but this advantage is compensated by a disadvantage for middle range valuations as $\tau_i \geq \tau_j$ implies that $\alpha(u | \lambda) + \frac{\tau_i}{f(u)} \geq \alpha(u | \lambda) + \frac{\tau_j}{f(u)}$ for some $u \in V$.

Corollary 5 shows that the relationship between the cut-off types \bar{m} and the Lagrange multipliers τ is a one-to-one relationship. It follows that determining the optimal values

of \bar{m} (resp. τ) uniquely determines the optimal values of τ (resp. \bar{m}) therefore greatly simplifying the problem.

Moreover, differentiating $\hat{\mathcal{L}}$ with respect to $U_i(x_i)$ gives $\chi_i + \tau_i - \lambda = 0$. Hence, for any $\lambda > 0$ (*i.e.* nonnegative weight on collected revenue), it is clear that χ_i and τ_i cannot be simultaneously null, that is, either the individual rationality or the cash constraint (or both) is binding. As $\chi_i = \lambda - \tau_i$ for all $i \in N$ it is straightforward that if $\bar{m}_i \leq \bar{m}_j$ then $\chi_i \leq \chi_j$ as $\tau_i \geq \tau_j$. If some agent j with cut-off \bar{m}_j gets some rent at optimum, *i.e.* $U_j(x_j) > 0$, all agents i with a lower cut-off $\bar{m}_i \leq \bar{m}_j$ must also get some rent, *i.e.* $U_i(x_i) > 0$ as $\chi_i \leq \chi_j = 0$ implies $\chi_i = 0$. Therefore, there is a clear relationship between the cut-off value \bar{m}_i and the presence of rents.

In order to obtain a more detailed characterization of the second-best solution, the next subsection investigates the case of bilateral equal-share partnerships.

1.6.3. Bilateral Equal-Share Partnerships

Consider a partnership with $n = 2$ whose partners are denoted by i and j . To focus on the problem of cash-constraints, assume equal distribution of ownership shares $r_1 = r_2 = \frac{1}{2}$.

From Theorem 7, a solution to the second-best problem must be such that $S_i(v_i) = r_i$ for all $v_i \in [x_i, y_i]$ and $S_j(v_j) = r_j$ for all $v_j \in [x_j, y_j]$. In particular, $S_i(x_i) = S_j(x_j) = \frac{1}{2}$. Assume $x_i < x_j$, then $\delta_i(u) = \delta_j(u)$ for all $u \in [\underline{v}, x_i]$. It follows that $S_i(u) = S_j(u)$ for all $u \in [\underline{v}, x_i]$. But then as $S_i(x_i) = S_j(x_j) = \frac{1}{2}$ it must be that $S_j(w) = \frac{1}{2}$ for all $w \in [x_i, x_j]$ as S_j must be nondecreasing. Yet, for $S_j(w)$ to be constant on $w \in [x_i, x_j]$ it is necessary that $\delta_j(w)$ is also constant, however, $x_i < x_j$ implies that $\delta_j(w)$ is strictly increasing on $w \in [x_i, x_j]$. Hence, $x_i < x_j$ is not possible. The exact same reasoning applies for $x_i > x_j$. Therefore, the only possible solution is $x_i = x_j =: x^*$.

Now, it is possible to characterize the relationship between the cash resources l_i and l_j with the threshold values \bar{m}_i and \bar{m}_j .

Lemma 5 *If $l_i > l_j$ then $\bar{m}_i \geq \bar{m}_j$.*

Proof. Assume that $l_i > l_j$ but assume instead that $\bar{m}_i < \bar{m}_j$. Then, $\tau_i > \tau_j$ and $y_i < y_j$ from the definition of y_i , equation (1.22). Moreover $\tau_i > \tau_j$ implies that $\tau_i > 0$ and thus (CC1) is binding for agent i . Moreover, $\tau_i > \tau_j$ implies that $\delta_i(u) \geq \delta_j(u)$ for all $u \in [y_i, \bar{m}_i]$ and thus $S_i(u) \geq S_j(u)$ for all $u \in [y_i, \bar{m}_i]$. From Corollary 4, $\delta_i(\bar{m}_i) < \delta_j(\bar{m}_j)$ and then $S_i(\bar{m}_i) \leq S_j(\bar{m}_j)$. Then, as (CC1) must be satisfied for agent j . Notice that as $\bar{m}_i < \bar{m}_j$ it must be also be that $\chi_i < \chi_j$ so that $\chi_j > 0$ and thus $U_j(\chi_j) = 0$. Therefore, (CC1) for agent j writes

$$\begin{aligned} l_j &\geq \bar{m}_j \left(S_j(\bar{m}_j) - \frac{1}{2} \right) - \int_{y_j}^{\bar{m}_j} \left(S_j(u) - \frac{1}{2} \right) du \\ &= \bar{m}_i \left(S_j(\bar{m}_j) - \frac{1}{2} \right) + \int_{\bar{m}_i}^{\bar{m}_j} \left(S_j(\bar{m}_j) - S_j(u) \right) du - \int_{y_j}^{\bar{m}_i} \left(S_j(u) - \frac{1}{2} \right) du \\ &\geq \bar{m}_i \left(S_i(\bar{m}_i) - \frac{1}{2} \right) - \int_{y_j}^{\bar{m}_i} \left(S_j(u) - \frac{1}{2} \right) du, \end{aligned}$$

where the third line stems from the fact that $S_i(\bar{m}_i) \leq S_j(\bar{m}_j)$ and, $\int_{\bar{m}_i}^{\bar{m}_j} \left(S_j(\bar{m}_j) - S_j(u) \right) du \geq 0$ as $\bar{m}_i < \bar{m}_j$ and $S_j(\bar{m}_j) \geq S_j(u)$ for all $u \in [\bar{m}_i, \bar{m}_j]$.

Assume first that $\bar{m}_i \leq y_j$, then $\int_{y_j}^{\bar{m}_i} \left(S_j(u) - \frac{1}{2} \right) du = 0$ as $S_j(u) = \frac{1}{2}$ for all $u \in [\bar{m}_i, y_j] \subset [x^*, y_j]$. Assume now that $\bar{m}_i > y_j$, then $\int_{y_j}^{\bar{m}_i} \left(S_j(u) - \frac{1}{2} \right) du \leq \int_{y_j}^{\bar{m}_i} \left(S_i(u) - \frac{1}{2} \right) du$ as $S_i(u) \geq S_j(u)$ for all $u \in [y_j, \bar{m}_i]$ and $S_i(u) - \frac{1}{2} \geq 0$ for all $u \in [y_i, y_j]$. Hence, in both cases,

$$\begin{aligned} l_j &\geq \bar{m}_i \left(S_i(\bar{m}_i) - \frac{1}{2} \right) - \int_{y_j}^{\bar{m}_i} \left(S_j(u) - \frac{1}{2} \right) du \geq \bar{m}_i \left(S_i(\bar{m}_i) - \frac{1}{2} \right) - \int_{y_i}^{\bar{m}_i} \left(S_i(u) - \frac{1}{2} \right) du \\ &= U_i(\chi_i) + l_i, \end{aligned}$$

where the equality holds as (CC1) binds for agent i . From (IR), $U_i(\chi_i) \geq 0$ so that the above result implies that $l_i \leq l_j$, contradicting the initial assumption that $l_i > l_j$. Therefore, $l_i > l_j$ implies that $\bar{m}_i \geq \bar{m}_j$. ■

Assume now, that $l_2 > l_1$. It follows from Lemma 5 that $\bar{m}_1 \leq \bar{m}_2$ and thus $\tau_1 \geq \tau_2$. Figure 1.1 illustrates the different possible shapes for the ironed virtual valuations of each agent where it is assumed, without loss of generality, that $V = [0, 1]$ to simplify the exposition.

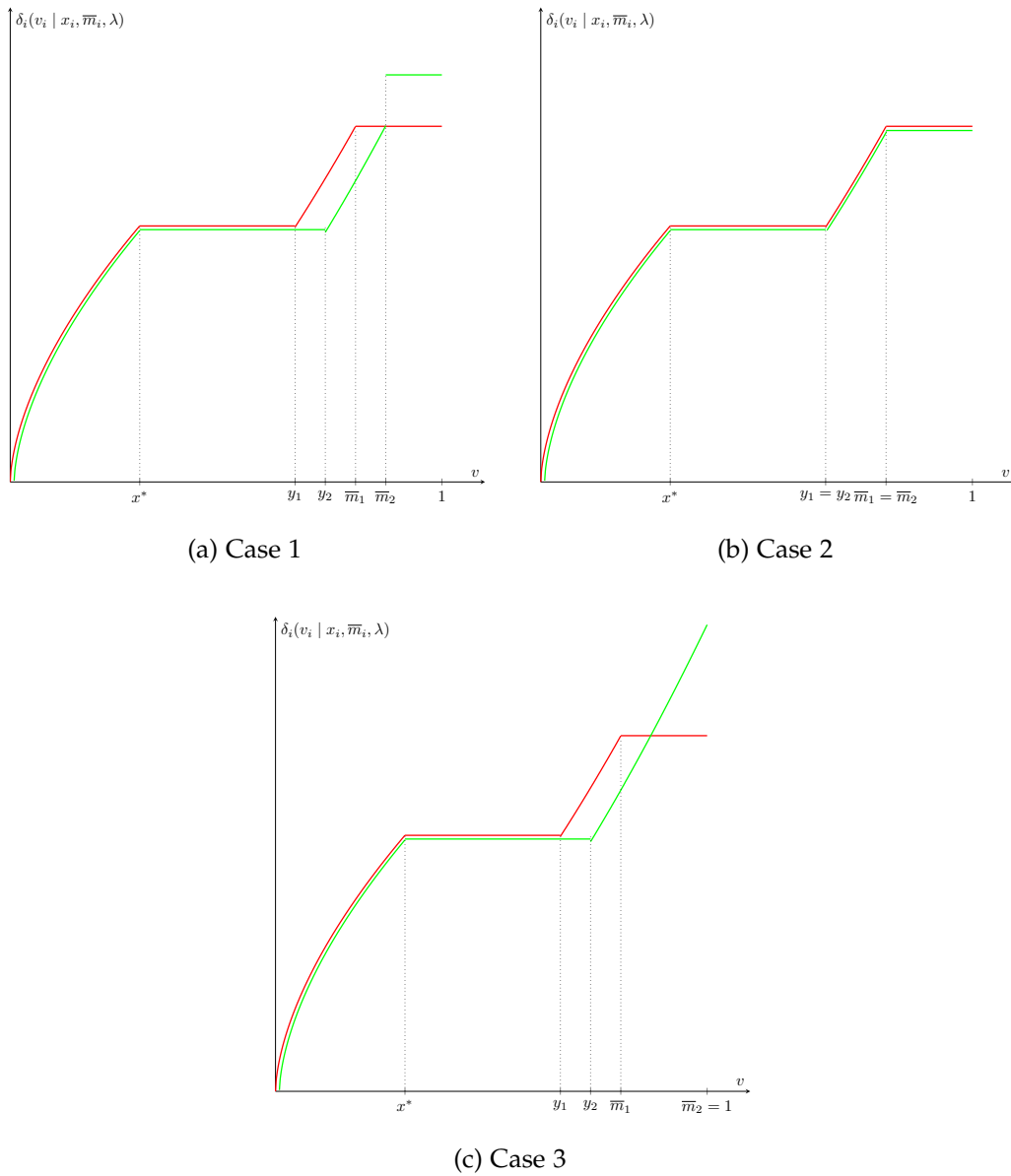


Figure 1.1: Ironed virtual valuations for agent 1 (red) and 2 (green) when $l_1 < l_2$.

Case 1 illustrates the case in which both agents are cash-constrained but l_1 and l_2 are far apart. In that case, agent 2 is defined as in equation (1.25) of Theorem 6 and agent 1 by equation (1.24). Therefore agent 1 has a continuous ironed virtual valuation at \bar{m}_1 while agent 2's ironed virtual valuation jumps at \bar{m}_2 . Both agents are similar for low and medium valuations (below y_1) but then agent 1 is advantaged over $[y_1, \bar{m}_1]$ and agent 2 is advantaged over $[\bar{m}_1, \bar{v}]$.

In case 2, the two agents share the same cut-off $\bar{m}_1 = \bar{m}_2$. This happens whenever they are both cash-constrained and the difference between l_1 and l_2 is small. As $\bar{m}_1 = \bar{m}_2$ it follows that τ_1 and τ_2 are defined by equation (1.24) so that both ironed virtual valuations are continuous at $\bar{m}_1 = \bar{m}_2$.

Case 3 occurs when agent 1 is cash-constrained while agent 2 is almost not cash-constrained. In that case, agent 2's ironed virtual valuation is not distorted for large valuations but agent 2 is still disadvantaged over agent 1 for some valuations above y_1 .

1.6.4. Budget Balanced Second-Best Mechanisms

Now, assume that the goal is to maximize the ex ante surplus subject to incentive compatibility, individual rationality, cash constraints *and* budget balance as in Section 1.3.²² This problem can be written as

$$\max_{s,t} \sum_{i \in N} \mathbb{E} \left[v_i(s_i(v) - r_i) \right],$$

subject to incentive compatibility, individual rationality, cash constraints and $-\sum_{i \in N} \mathbb{E} [t_i(v)] \geq K$. Where $K \in \mathbb{R}_+$ is a given budget limit that should not be exceeded by the transfers made to the agents. Denote by $\lambda \geq 0$, the Lagrange multiplier associated with the budget balance constraint so that the problem rewrites,

$$\max_{s,t} \sum_{i \in N} \mathbb{E} \left[v_i(s_i(v) - r_i) \right] + \lambda \left(-\sum_{i \in N} \mathbb{E} [t_i(v)] - K \right),$$

²²Although, as previously said, I restrict the analysis to ex ante budget balanced mechanisms rather than ex post budget balanced due to the complexity of the latter.

subject to incentive compatibility, individual rationality and cash constraints. Notice that maximizing this objective is equivalent to maximize the following objective

$$\max_{s,t} (1 + \lambda)W_{\frac{\lambda}{1+\lambda}}(s, t) - \lambda K,$$

subject to incentive compatibility, individual rationality and cash constraints and where W_ω is given by equation (1.18), *i.e.* the objective function of the problem solved in Section 1.6, where λ is replaced by $\frac{\lambda}{1+\lambda}$ and it is clear that $\frac{\lambda}{1+\lambda} \in [0, 1)$.

Therefore, maximizing the ex ante surplus subject to incentive compatibility, individual rationality, cash constraints *and* budget balancedness is equivalent to solve the second-best problem derived in Subsection 1.6.2 for a well chosen λ .

1.7. EXTENSIONS

1.7.1. Other Ownership Structures

The ex post efficient dissolution condition, the Equivalence Theorem and the auction (Theorems 1, 2, 3, 4 and 5) actually apply to more general settings than the partnership problem discussed above. The interpretation of the setting as a partnership problem lies in the assumption that for all $i \in N$, $r_i \in [0, 1]$ and $\sum_{i \in N} r_i = 1$, that is, each partner initially owns a share of an asset which is then redistributed to the one with the highest valuation.

Although crucial for the characterization of optimal initial ownership structures in Section (1.4) and their interpretation, this assumption plays a very limited role in the derivation of Theorems 1-5. Coming back to Section 1.3, recall that IIR can be written as $H_i \leq C(r_i)$ where $C(r_i) := \inf_{v_i} \{ \mathbb{E}_{-i} g(v) - v_i r_i \}$. From there on, Theorems 1-5 can be written only using the definition of $C(r_i)$ without any particular reference to the particular outside options $v_i r_i$.

Consider the following. If agent $i \in N$ refuses to participate in the mechanism, then it receives $\varphi_i(v; \theta, l) \in [0, \bar{v}]$ where φ_i is assumed to be concave in v_i and $\theta \in \mathbb{R}_+^n$ is some vector of parameters where $\theta_i \in \mathbb{R}_+$ is associated with agent i . Notice that φ_i may depend on all valuations, on the cash resources and on some other parameters defining the agents.

Setting $\varphi_i(v; \theta, l) := v_i \theta_i$ with $\sum_{i \in N} \theta_i = 1$ simply replicates the ownership structure investigated in the rest of the paper. In the general case, IIR writes

$$v_i S_i^*(v_i) + T_i^*(v_i) - \mathbb{E}_{-i} \varphi_i(v; \theta, l) \geq 0, \text{ for all } v_i \in V, i \in N.$$

Replacing $T_i^*(v_i) := \mathbb{E}_{-i} t_i^*(v)$ where $t_i^*(v)$ is given by equation (1.2), IIR rewrites

$$H_i \leq \mathbb{E}_{-i} g(v) - \mathbb{E}_{-i} \varphi_i(v; \theta, l), \text{ for all } v_i \in V, i \in N.$$

Define for each $i \in N$,

$$Z_i(\theta, l) := \inf_{v_i \in V} \{ \mathbb{E}_{-i} g(v) - \mathbb{E}_{-i} \varphi_i(v; \theta, l) \},$$

then IIR rewrites

$$H_i \leq Z_i(\theta, l), \text{ for all } i \in N.$$

The function $Z_i(\cdot)$ serves the same role as $C(\cdot)$ in Section 1.3. It defines the maximal amount that can be collected on agent i without violating their interim individual rationality constraint. Hence, $v_i^* \in \arg \min_{v_i} \{ \mathbb{E}_{-i} g(v) - \mathbb{E}_{-i} \varphi_i(v; \theta, l) \}$ is agent i 's worst-off type which is characterized by the first-order condition of the minimization problem: $F(v_i^*)^{n-1} = \frac{\partial}{\partial v_i} \mathbb{E}_{-i} \varphi_i(v; \theta, l)$.²³

The condition for the existence of ex post efficient dissolution (with Bayesian or Dominant Strategy mechanisms) then simply rewrites

$$\sum_{i \in N} \min \{ Z_i(\theta, l), l_i \} \geq B.$$

The equivalence theorems still hold and the cash-constrained auction (Section 1.5) still applies by modifying the side payments accordingly. Therefore, the dissolution condition applies to more general settings than the ones with outside options of the form $v_i r_i$.

²³The first-order condition is necessary and sufficient. Indeed the second-order derivative of $\mathbb{E}_{-i} g(v) - \mathbb{E}_{-i} \varphi_i(v; \theta, l)$ immediately writes $(n-1)f(v_i)F(v_i)^{n-2} - \frac{\partial^2}{\partial v_i^2} \mathbb{E}_{-i} \varphi_i(v; \theta, l) < 0$ as $\varphi_i(v; \theta, l)$ is assumed to be concave in v_i .

EXAMPLE 1: ALLOCATION OF A PRIVATE GOOD. The simplest and most general extension of the partnership dissolution framework is that of the allocation of a private good to agents with type-dependent outside opportunities. Assume the state wants to privatize a publicly-owned asset (road, spectrum, ...). A group of n candidate firms is considered. None of the firms has initial ownership rights in the publicly-owned asset but each has some outside opportunities that likely depend on their ability to efficiently run a business. Applying the partnership dissolution framework with general outside opportunities $\varphi_i(\cdot)$ directly provides the mechanism that efficiently allocates the asset to the most efficient firm. Those kinds of allocation problems generally involve highly valued assets and require large monetary transfers to get control rights. As previously showed, the mechanism I propose performs better than that of CGK in the presence of limited cash resources and could be used to help small firms with low cash resources to compete against larger well-established incumbents.

EXAMPLE 2: SILENT PARTNERS. This example is inspired by [Ornelas and Turner \(2007\)](#). Assume n partners jointly own a business where each of them can claim a share $r_i \in [0, 1]$ (and $\sum_{i \in N} r_i = 1$) of total output value. Each partner's valuation $v_i \in V$ represents their ability to run the business and the business value is given by the valuation of the partner in charge, say partner 1. In this setting, it is assumed that partner 1 has claims r_1 on total output value but is also given *control rights* over the business. This framework provides a way to unbundle *control rights* from *ownership rights* on the value of the output.

Let (with a slight abuse of notations) $\varphi_1 := v_1 r_1$ and $\varphi_j := \mathbb{E}_{-j} v_1 r_j$ for all $j \neq 1$. The difference with the standard case is that if agent $i \in N$ refuses to participate in the mechanism, the business continues as usual, that is, it is still worth v_1 .

Assume now that the partners consider dissolving their partnership and let $Z_i(r_i)$ be partner i 's maximal payment before refusing to participate in dissolution. Then $Z_1(r_1) = C(r_1)$ as partner 1 exhibits exactly the same outside option as in the previous analysis. On the contrary, for each $j \neq 1$, $Z_j(r_j) = \inf_{v_j} \{ \mathbb{E}_{-j} g(v) - r_j \mathbb{E}_{-j} v_1 \} = \int_{\underline{v}}^{\bar{v}} x dF(x)^{n-1} - r_j \int_{\underline{v}}^{\bar{v}} x dF(x)$ where it is easy to show that the worst-off type is $v_j^* = \underline{v}$ for all $j \neq 1$. The dissolution condition therefore writes $\sum_{i \in N} \min\{Z_i(r_i), l_i\} \geq B$.

Let $n = 3$, $F(v_i) = v_i$ and $V = [0, 1]$. Then $Z_1(r_1) = 2/3 - (2/3)r_1^{3/2}$ and $Z_j(r_j) =$

$2/3 - (1/2)r_j$ for $j \neq 1$. In the absence of cash constraints, dissolution is possible if and only if $r_1 \leq 9/16$ and the remaining shares can be distributed arbitrarily between the silent partners. Furthermore, the extreme ownership structure where $r_1 = 0$ is also dissolvable. With cash-constraints, however, those extreme ownership structures where r_1 is close to zero will generally prevent ex post efficient dissolution. Assume for instance that partner 1 has $l_1 = 1/2$ while the other two partners are not cash-constrained. Then, ex post efficient dissolution is impossible whenever $r_1 < 1/3$. Cash constraints somehow prevents extreme ownership structures to be desirable in some cases.

1.7.2. *Asymmetric Distributions of Valuations*

Most of the results concerning ex post efficient dissolution mechanisms naturally extends to asymmetric distribution of the agents' valuations. More precisely, Theorems 1, 2, 3 and 4 still hold with only slight modifications of the dissolution condition for the first two. I give the main elements of the proof for Bayesian mechanisms which mainly consists in the construction of the transfer function to satisfy EPCC. The case of Dominant Strategy mechanisms can easily be obtained using this transfer function and the proof of Theorem 2 and is therefore omitted.

Extending the analysis to asymmetric distributions of valuations is important as many applications of the partnership dissolution framework can be better represented that way. In the divorce problem for instance, the asset that has to be reallocated might be the family house of one of the spouses. The family house might have a low market value but a large sentimental value for the spouse who has spent their childhood in the family house. Therefore it is more likely that this spouse has a larger valuation for the family house than the other one, *i.e.* distributions of valuations are likely to be asymmetric. In the biotechnology sector, large well-established firms partner with small young firms to develop new research. It is more likely that large firms have higher valuations (as they are more capable of using the results of the research) than small firms. The question is therefore to understand how asymmetric distributions of valuations and limited cash resource interplay and affect the dissolution condition.

THE DISSOLUTION CONDITION. Consider the same setting as the one described in Section 1.2 except for the distribution of valuations. Assume that partner $i \in N$ has valuation $v_i \in V$

where each v_i is independently distributed from an absolutely continuous cumulative distribution function F_i . Let $f_i = F_i'$ be the probability distribution function for v_i . For convenience for any $x \in V$, let $\mathbf{G}(x) := \prod_{k \in N} F_k(x)$, $\mathbf{G}_i(x) := \prod_{k \neq i} F_k(x)$ and $\mathbf{G}_{ij}(x) := \prod_{k \neq i, j} F_k(x)$ denote the distributions of the maximum of all valuations, all valuations except v_i and all valuations except v_i and v_j , respectively.

Proposition 1 holds with asymmetric distributions of valuations (see [Makowski and Mezzetti \(1994\)](#)), then the transfer function of partner $i \in N$ must write $t_i(v) = g(v) - v_i s_i^*(v) - h_i(v)$. Recall that the ex ante cost of implementing a Groves mechanism writes $G := \mathbb{E}g(v) = \int_V x d\mathbf{G}(x)$ as in the case of symmetric distributions of valuations. The main difference concerns the upper bound of how much can be taken from partner i without violating their individual rationality constraints. In Section 1.2, this upper bound was defined by $C(r_i) = \inf_{v_i} \{\mathbb{E}_{-i}g(v) - v_i r_i\}$ so that two partners with the same initial ownership shares $r_i = r_j$ had the same upper bounds $C(r_i) = C(r_j)$. With asymmetric distributions of valuations two agents with the same initial ownership shares may have different upper bounds. Formally, simply let $\mathcal{C}_i(r_i) := \inf_{v_i} \{\mathbb{E}_{-i}g(v) - v_i r_i\}$ denote this upper bound in the asymmetric case. The only difference is that now each partner may have a different function $\mathcal{C}_i(\cdot)$ depending on their cumulative distribution function F_i . These new pieces of notations are sufficient to state the result extending the analysis to asymmetric distributions of valuations.

Proposition 8 *An EF, IIC (resp. EPIC), IIR, EPBB (resp. EABB) and EPCC dissolution mechanism exists if and only if*

$$\sum_{i \in N} \min\{\mathcal{C}_i(r_i), l_i\} \geq (n-1)G. \quad (1.26)$$

Proof. I provide a sketch of the main elements of the proof for Bayesian mechanisms. See Appendix C for further details. The “only if” part (necessity) is obtained exactly as in the proof of Theorem 1: Simply replace $C(r_i)$ by $\mathcal{C}_i(r_i)$ in equation (1.6) and then combining it with (1.7) and (1.4) it gives (1.26).

The “if” part (sufficiency) requires the construction of an appropriate transfer function. Again, using the transfer function proposed by [Dudek, Kim, and Ledyard \(1995\)](#) is helpful

for the cash-constrained case. Consider the following:

$$t_i(\mathbf{v}) := \begin{cases} - \int_{\underline{v}}^{v_i} \sum_{k \neq i} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_k(x)}{F_k(x)} dx - \frac{n-1}{n} \underline{v} - \phi_i & \text{if } \rho(\mathbf{v}) = i \\ \int_{\underline{v}}^{v_j} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_i(x)}{F_i(x)} dx + \frac{1}{n} \underline{v} - \phi_i & \text{if } \rho(\mathbf{v}) = j \neq i, \end{cases} \quad (1.27)$$

where $\rho(\mathbf{v})$ is defined as in Section 1.3 and $\phi_i \in \mathbb{R}$ is a constant.

It is immediate to see from (1.27) that EPBB is equivalent to $\sum_{i \in \mathbb{N}} \phi_i = 0$. It is also EF as it allocates all the ownership shares to the partner with the highest valuation. Moreover, it is clear that the minimum of this function is attained when $\rho(\mathbf{v}) = i$ and $v_i = \bar{v}$, that is

$$\min_{\mathbf{v} \in \mathbb{V}^n} t_i(\mathbf{v}) = \phi_i - \int_{\underline{v}}^{\bar{v}} \sum_{k \neq i} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_k(x)}{F_k(x)} dx - \frac{n-1}{n} \underline{v}.$$

A little algebra on the integral term (see Appendix C for detailed computations) shows that EPCC can be written as

$$\phi_i \leq \int_{\underline{v}}^{\bar{v}} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_i(x)}{F_i(x)} dx + \frac{1}{n} \underline{v} - G + l_i. \quad (1.28)$$

Computing the interim transfer $T_i(v_i) := \mathbb{E}_{-i} t_i(\mathbf{v})$ gives

$$T_i(v_i) = \int_{\underline{v}}^{\bar{v}} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_i(x)}{F_i(x)} dx + \frac{1}{n} \underline{v} - G + \int_{v_i}^{\bar{v}} x d\mathbf{G}_i(x) - \phi_i,$$

Detailed computations of $T_i(v_i)$ and proof that the mechanism is IIC can be found in Appendix C. The characterization of $C(r_i) = \int_{v_i^*(r_i)}^{\bar{v}} x dF(x)^{n-1}$ given by equation (1.15) naturally extends to asymmetric distributions of valuations as follows: $\mathcal{C}_i(r_i) = \int_{v_i^*(r_i)}^{\bar{v}} x d\mathbf{G}_i(x)$, where $v_i^*(r_i)$ is the worst-off type that solves $\inf_{v_i} \{ \mathbb{E}_{-i} g(\mathbf{v}) - v_i r_i \}$ which is given by $\mathbf{G}_i(v_i^*(r_i)) = r_i$. As in CGK, satisfying IIR is equivalent to $T_i(v_i^*(r_i)) \geq 0$. Therefore, using the expression of $\mathcal{C}_i(r_i)$, IIR can be written as

$$\phi_i \leq \int_{\underline{v}}^{\bar{v}} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_i(x)}{F_i(x)} dx + \frac{1}{n} \underline{v} - G + \mathcal{C}_i(r_i). \quad (1.29)$$

Combining equations (1.28) and (1.29) and simplifying gives

$$\phi_i \leq \min\{C_i(r_i), l_i\} + \int_{\underline{v}}^{\bar{v}} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_i(x)}{F_i(x)} dx + \frac{1}{n} \underline{v} - G. \quad (1.30)$$

Simply let

$$\phi_i = \min\{C_i(r_i), l_i\} + \int_{\underline{v}}^{\bar{v}} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_i(x)}{F_i(x)} dx + \frac{1}{n} \underline{v} - G - \frac{1}{n} \left[\sum_{j \in N} \min\{C_j(r_j), l_j\} - (n-1)G \right].$$

Given that equation (1.26) is satisfied, it is then immediate that ϕ_i satisfies equation (1.30) so that the mechanism is IIR and EPCC. For EPBB, it is easy to show that $\sum_i \phi_i = 0$ holds after noticing that $\sum_{i \in N} \int_{\underline{v}}^{\bar{v}} \frac{\int_{\underline{v}}^x \mathbf{G}(y) dy}{\mathbf{G}(x)} \frac{f_i(x)}{F_i(x)} dx = G - \underline{v}$ where the result is obtained by integration by parts. ■

SOME CHARACTERIZATION RESULTS. Providing a full characterization of the dissolution condition with asymmetric distributions of valuations and asymmetric cash resources is not very insightful. Many subcases can occur depending on the various initial conditions for distributions of valuations and for cash resources. Instead, I provide particular characterization results to illustrate some of the effects of asymmetric distributions of valuations on optimal initial ownership structures.

In order to make meaningful and easy comparisons between the partners, assume that distributions of valuations can be ranked according to first-order stochastic dominance as follows: For all $x \in V$, $F_1(x) \leq F_2(x) \leq \dots \leq F_n(x)$. It follows that for any $i, j \in N$ with $i < j$, partner i is more likely to have a higher valuation than partner j . For any $i < j$ and $x \in V$, notice that $\mathbf{G}_i(x) = \mathbf{G}_j(x) \frac{F_j(x)}{F_i(x)} \geq \mathbf{G}_j(x)$ as $F_i(x) \leq F_j(x)$. Given that for any $k \in N$ and $\hat{r} \in [0, 1]$, $\mathbf{G}_k(v_k^*(\hat{r})) = \hat{r}$ then for any i, j with $i < j$, $v_i^*(\hat{r}) \leq v_j^*(\hat{r})$ as $\mathbf{G}_i(x) \geq \mathbf{G}_j(x)$ and $\mathbf{G}_k(x)$ is increasing for any $x \in V$. It follows that for any i, j with $i < j$ and any $\hat{r} \in [0, 1]$,

$\mathcal{C}_i(\hat{r}) \leq \mathcal{C}_j(\hat{r})$ as

$$\begin{aligned} \mathcal{C}_i(\hat{r}) - \mathcal{C}_j(\hat{r}) &= \int_{v_i^*(\hat{r})}^{\bar{v}} x d\mathbf{G}_i(x) - \int_{v_j^*(\hat{r})}^{\bar{v}} x d\mathbf{G}_j(x) \\ &= v_j^*(\hat{r})\mathbf{G}_j(v_j^*(\hat{r})) - v_i^*(\hat{r})\mathbf{G}_i(v_i^*(\hat{r})) - \int_{v_i^*(\hat{r})}^{\bar{v}} \mathbf{G}_i(x) dx + \int_{v_j^*(\hat{r})}^{\bar{v}} \mathbf{G}_j(x) dx \\ &= \int_{v_j^*(\hat{r})}^{\bar{v}} [\mathbf{G}_j(x) - \mathbf{G}_i(x)] dx - \int_{v_i^*(\hat{r})}^{v_j^*(\hat{r})} [\mathbf{G}_i(x) - \hat{r}] dx \leq 0, \end{aligned}$$

where the second line is obtained after integrating the two terms by parts and the third line uses the fact that $\mathbf{G}_i(v_i^*(\hat{r})) = \mathbf{G}_j(v_j^*(\hat{r})) = \hat{r}$. The inequality follows from $\mathbf{G}_i(x) \geq \mathbf{G}_j(x)$ for all $x \in V$, $v_i^*(\hat{r}) \leq v_j^*(\hat{r})$ and $\mathbf{G}_i(x) \geq \hat{r}$ for $x \in [v_i^*(\hat{r}), v_j^*(\hat{r})]$ so that the first is always nonpositive and the second one always nonnegative.

Consider now the optimal ownership structure $r^* \in \arg \max_{r \in \Delta^{n-1}} \sum_{i \in N} \mathcal{C}_i(r_i)$ in the absence of cash constraints. Recall that from the Envelope Theorem, $\mathcal{C}'_k(r_k) = -v_k^*(r_k)$, then optimality conditions implies $\mathcal{C}'_i(r_i^*) = \mathcal{C}'_j(r_j^*)$ for all i, j , which is equivalent to $v_i^*(r_i^*) = v_j^*(r_j^*)$. As $v_i^*(r_i^*) \leq v_j^*(r_j^*)$ and $v_k^*(\cdot)$ is increasing for all $k \in N$, it follows that the initial ownership structure maximizing $\sum_{i \in N} \mathcal{C}_i(r_i)$ must satisfy $r_1^* \geq r_2^* \geq \dots \geq r_n^*$. This result is the one obtained by [Figuroa and Skreta \(2012\)](#), Corollary 1.

Intuitively, $\mathcal{C}_i(\hat{r}) \leq \mathcal{C}_j(\hat{r})$ for $i < j$ means that less money can be collected on partners whose valuations are more likely to be high as they are more likely to have a higher initial outside option. Optimality conditions reveal that it is better to give more initial ownership rights to those partners as already few money can be collected on them and fewer initial ownership rights to partners who are more likely to have low valuations in order to collect larger amount of money from them. This feature is reminiscent of the results of [Figuroa and Skreta \(2012\)](#) who characterize optimal ownership structures in partnership dissolution problems with asymmetric distributions of valuations. They show that if asymmetries in distributions of valuations are quite important, the optimal ownership structures might be very extreme. Limited cash resources, however, may mitigate those extreme ownership structures as it is illustrated in the following example.

A TWO-AGENT EXAMPLE. Take the case of a large pharmaceutical firm i and a small R&D firm j forming an alliance to develop a new drug. Ownership shares r_i and $r_j := 1 - r_i$

represent initial claims on the output generated with the new drug. It is reasonable to think that the pharmaceutical firm has a higher potential than the R&D firm to distribute the drug once it has been developed. This idea is simply modeled by assuming that the pharmaceutical firm is more likely to have a higher valuation for the drug than the R&D firm. Then, let $F_i(x) \leq F_j(x)$ for all $x \in V$. Furthermore, the small R&D firm can be assumed to be financially constrained while I assume that the large firm is not for convenience. Let $l_i = +\infty$ and $l_j < +\infty$ and define $r_i^* \in \arg \max_{r_i \in [0,1]} C_i(r_i) + C_j(1 - r_i)$.

Consider first the case in which $l_j \geq C_j(1 - r_i^*)$. Then, it is clear that r_i^* is also the solution to $\max_{\hat{r}_i} C_i(\hat{r}_i) + \min\{C_j(1 - \hat{r}_i), l_j\}$. Previous computations show that $r_i^* > r_j^*$, *i.e.* the pharmaceutical firm should have more initial ownership rights as it is more likely to be the final owner of the drug. If asymmetries in distributions of valuations are large, then it can become optimal to initially give a very large share of ownership rights to the pharmaceutical firm.

But now consider the case in which $l_j < C_j(1 - r_i^*)$. It is clear that r_i^* is not the optimal ownership structure anymore. Indeed, take \hat{r}_i such that $C_j(1 - \hat{r}_i) = l_j$. As $C_j(\cdot)$ is a decreasing function this implies that $\hat{r}_i < r_i^*$ and as $C_i(\cdot)$ is also a decreasing function it implies that $C_i(\hat{r}_i) > C_i(r_i^*)$. It follows that $C_i(\hat{r}_i) + C_j(1 - \hat{r}_i) = C_i(\hat{r}_i) + l_j > C_i(r_i^*) + l_j$. If it were beneficial to further decrease \hat{r}_i then the left derivative of $C_i(r_i) + C_j(1 - r_i)$ at $r_i = \hat{r}_i$ would be negative, *i.e.*, $C_i'(\hat{r}_i) - C_j'(1 - \hat{r}_i) < 0$. But from optimality conditions recall that $C_i'(r_i^*) = C_j'(1 - r_i^*)$ and thus $C_i'(\hat{r}_i) > C_i'(r_i^*) = C_j'(1 - r_i^*) > C_j'(1 - \hat{r}_i)$ as $C_i'(\cdot)$ and $C_j'(\cdot)$ are both decreasing. Therefore, the left derivative of the objective at $r_i = \hat{r}_i$ is nonnegative and \hat{r}_i is the solution to the problem when $l_j < C_j(1 - r_i^*)$.

In the latter case, $\hat{r}_i < r_i^*$, that is, the distortion in initial ownership rights due to asymmetries in distributions of valuations is mitigated by the presence of cash constraints. Giving more initial ownership shares to the pharmaceutical firm is good as it is likely that it will efficiently be the final owner of the drug but at the same time it could make impossible an efficient buyout by the financially-constrained R&D firm.

1.7.3. Partnership Formation Game

Disposable financial resources are not exogenous to the partnership problem. Partners initially invest some of their financial resources to contribute to the partnership assets and receive cash flows from the day-to-day operations of the partnership. As a result, disposable financial resources of a partner at the dissolution stage is most likely to depend on their the initial ownership stakes. I consider now the following problem.

An exogenous number of agents $n \geq 2$ decide to form a partnership which requires and initial investment of $X \in \mathbb{R}_+$ and future value of $v_i \sim F(v_i)$ for partner i . In the first stage, partners choose an initial ownership structure $r = (r_1, \dots, r_n) \in \Delta^{n-1}$ which fully determines the allocation of initial investment and agree on an ex post efficient dissolution mechanism. That is, partner i with initial ownership share r_i has to invest $r_i X$ of their own financial resources. The disposable financial resources of partner i after their investment $r_i X$ is denoted by $l_i(r_i X)$, where $l_i(0)$ represents their initial financial resources prior to the investment. In the second stage, each partner privately learns their valuation v_i and dissolution occurs according to the previous agreement. In the dissolution mechanism, ex post cash constraints are determined by the partners disposable financial resources $l_i(r_i X)$.

I impose some structure on the initial asset value and disposable financial resources.

Assumption 3 *The initial investment X is lower than the expected maximal valuation $G = \mathbb{E}[\max_{i \in N} v_i]$, that is $X < G$.*

This assumption simply ensures that forming the partnership is ex ante beneficial. Indeed, assuming that ex post efficient dissolution is feasible, the expected value of the partnership is G as the asset is finally allocated to the partner with the highest valuation. Initial investment is X so that the partnership is ex ante beneficial when $G - X \geq 0$.

The relationship between the initial investment and the final value of the partnership could be equivalently formulated as follows. Each partner has a final valuation of the asset $v_i = X + w_i$ where $w_i \sim F(y - X)$ with support $y \in [\underline{v} - X, \bar{v} - X]$, that is, each partner has a final valuation for the asset that can be either lower or greater than the initial investment cost. With this formulation $G = \mathbb{E}[\max_{i \in N} X + w_i] = X + \mathbb{E}[\max_{i \in N} w_i]$.

Therefore, assuming $G > X$ implies that $\mathbb{E}[\max_{i \in \mathbb{N}} w_i] > 0$, that is, in expectation there is at least one partner who values the asset more than the initial cost at the end of the partnership. The next assumption defines the structure of disposable financial resources.

Assumption 4 *After the initial investment at cost $r_i X$, the amount of disposable financial resources, denoted by $l_i(r_i X)$, is a function $l_i : [0, X] \rightarrow \mathbb{R}_+$ such that for all $i \in \mathbb{N}$*

- $l_i(r_i X)$ is decreasing, and twice continuously differentiable;
- $\sum_{i \in \mathbb{N}} l_i(0) \geq X$, $l_i(0) < G$ and $l'_i(0) \leq -1$.

The strongest requirement on $l_i(\cdot)$ is the assumption that it is monotonically decreasing in invested capital. On the one hand, it is intuitive that after investing in the asset, each partner has less financial resources at their disposal.²⁴ On the other hand, it is generally the case that partners receive operating cash flows proportional to their ownership shares in the partnership so that a larger initial contribution to the asset gives rights to a larger share of operating cash flows. By assuming that $l_i(r_i X)$ is a decreasing function of the initial contribution, I therefore assume that the initial investment of each partner always outweighs the additional financial resources provided by operating cash flows. Concavity of $l_i(\cdot)$ implies that as the initial contribution increases, it is more and more costly in terms of disposable financial resources. Intuitively, this means that the marginal cost of finding additional financing capacities is increasing.

The assumption that $\sum_{i \in \mathbb{N}} l_i(0) \geq X$ simply ensures that partners have enough total initial financial resources to purchase the asset at cost X . Then, $l_i(0) < G$ implies that each partner's initial financial resources never exceeds the expected value of the partnership. This assumption rules out very wealthy partners for whom the partnership potential value is low compared to their financial capabilities. Finally, $l'_i(0) \leq -1$ implies that investing one euro in the asset decreases future disposable financial resources by more than one euro.

For instance, disposable financial resources could take the linear form $l_i(r_i X) = \alpha_i - r_i X$ where $\alpha_i \leq G$ represents the initial financial resources of partner i before investment.

²⁴Those resources do not necessary represent cash resources, they can be borrowing capacities as well. Then, one can think that after borrowing a large amount of money to finance the initial investment, the ability to raise further amounts of money is more and more difficult.

DISSOLUTION STAGE. Assume partners have already chosen how to allocate initial contribution in the first stage so that the initial ownership structure $r \in \Delta^{n-1}$ is given. Disposable financial resources are determined by $l_i(r_i X)$ for each partner $i \in N$. Following Section 1.3, ex post efficient dissolution is feasible if and only if

$$\sum_{i \in N} \min\{C(r_i), l_i(r_i X)\} \geq (n-1)G. \quad (1.31)$$

The formation game only affects the distribution of initial ownership shares and financial resources which must satisfy a very similar necessary and sufficient for an ex post efficient dissolution mechanism to exist.

THE FORMATION STAGE. In the first stage, partners must agree on (i) an allocation of investments to purchase the asset at cost X and (ii) an allocation of surplus in the ex post efficient dissolution mechanism.

First, let us define the expected utility of dissolution for each partner. From Proposition 1, the ex post utility of partner i in ex post efficient dissolution mechanism must satisfy $u_i(v) = g(v) - h_i(v)$. Let $h_i(v) = \frac{n-1}{n}G + \phi_i(r_i)$ as in Section 1.3 so that ex ante utility of partner i writes as $U_i(r_i) := \frac{1}{n}G - \phi_i(r_i)$. The first term, $\frac{1}{n}G$, is the expected surplus partner i can derive from obtaining final ownership at the dissolution stage.²⁵ The second term, $\phi_i(r_i)$, is the constant transfer (independent of the realization of valuations) partner i receives in the mechanism. As in Section 1.3, this transfer must satisfy the following conditions

$$\begin{aligned} \sum_{i \in N} \phi_i(r_i) &= 0, \\ \phi_i(r_i) &\leq \min\{C(r_i), l_i(r_i X)\} - \frac{n-1}{n}G, \text{ for all } i \in N, \end{aligned}$$

where the first condition ensures ex post budget balance and the second condition ensures both interim individual rationality and ex post cash constraints. Any vector $\phi := (\phi_1(r_1), \dots, \phi_n(r_n))$ satisfying these two conditions will be called feasible.

Now let $W_i(r_i) := -r_i X + U_i(r_i)$ denote the expected utility of agent i before signing

²⁵Here $1/n$ is the ex ante probability that partner i has the highest valuation as valuation are iid.

the partnership agreement. Agent i is willing to participate in the partnership if $W_i(r_i) \geq 0$, that is, if their expected utility in the dissolution mechanism exceeds their initial investment. A partnership agreement is therefore a pair (r, ϕ) where $r \in \Delta^{n-1}$ is the initial ownership structure (and thus initial contribution to the investment) and $\phi \in \mathbb{R}^n$ is the vector of constant transfers in the dissolution mechanism. I assume that agents behave cooperatively to determine the partnership agreement (r, ϕ) , that is, they choose an initial ownership structure r and future constant transfers ϕ such that all agents are willing to participate ex ante and so that ex post efficient dissolution is feasible.

Ex ante participation writes $U_i(r_i) \geq r_i X$ for all $i \in N$. Using $U_i(r_i) := \frac{1}{n}G - \phi_i(r_i)$, it rewrites as $\frac{1}{n}G - \phi_i(r_i) \geq r_i X$. Interestingly, summing this condition over all $i \in N$ gives the following necessary condition: $G \geq X$, which is guaranteed by Assumption 3. For each agent $i \in N$, this condition also requires that $\phi_i(r_i) \leq \frac{1}{n}G - r_i X$. The following result greatly simplifies the analysis.

Proposition 9 *Given Assumption 4, the existence of an ex post efficient mechanism guarantees ex ante participation constraints in the formation game for each agent $i \in N$.*

Proof. Assume (r, ϕ) is such that 1.31 holds. Then, it must be the case that, for all $i \in N$, $\phi_i(r_i) \leq l_i(r_i X) - \frac{n-1}{n}G$ so that $U(r_i) \geq G - l_i(r_i X)$. Notice that $G > r_i X + l_i(r_i X)$ for all $r_i \in [0, 1]$ as under Assumption 4, $r_i X + l_i(r_i X)$ is decreasing in r_i from $l'_i(r_i X) \leq -1$ and $l_i(0) < G$. It immediately follows that $U(r_i) \geq G - l_i(r_i X) > r_i X$, i.e., that the ex ante participation constraint to the formation game is satisfied. ■

Intuitively, the ex post efficient dissolution mechanism guarantees to each agent a utility level that depends on their disposable financial resources. As those resources decrease at a higher rate than the initial cost of investment, the ex post constraints are stronger than the ex ante ones. As a result, existence of an ex post efficient dissolution mechanism is a sufficient condition for ex ante participation. The result of Proposition 9 relies heavily on Assumption 4 and especially on the fact that disposable financial resources are decreasing at a larger rate than one and that initial financial resources are less than G .

EFFICIENT INITIAL ALLOCATION. Let us consider what would be the initial allocations of ownership so that all agents are willing to participate and ex post efficient disso-

lution is feasible. First, assume that $l_i(\frac{1}{n}X) \geq \frac{n-1}{n}G$. Then one possible efficient initial allocation is simply $r_i = \frac{1}{n}$ for all $i \in N$, that is, an equal-share partnership. Indeed, what can be collected on agent at the dissolution stage is given by (1.31) writes $\sum_{i \in N} \min\{C(\frac{1}{n}), l_i(\frac{1}{n}X)\} \geq \sum_{i \in N} \frac{n-1}{n}G = (n-1)G$ as $C(\frac{1}{n}) > \frac{n-1}{n}G$.

For instance, let $n = 2$ and assume valuations are uniformly distributed on the unit interval. Suppose also that $l_i(r_iX) = \alpha_i - r_iX$ where $\alpha_i < G$. What are the conditions on initial financial resources that guarantee the possibility of forming an equal-share partnership? Straightforward computations gives that $G = \frac{2}{3}$ so that $l_i(\frac{1}{n}X) \geq \frac{n-1}{n}G$ is equivalent to $\alpha_i \geq \frac{1}{3} + \frac{1}{2}X$. When the initial investment is null, i.e., $X = 0$, each partner must have at least $\frac{1}{3} = \frac{1}{2}G$ of financial resources which is reminiscent of previous characterization results. When $X > 0$, initial required financial resources are increasing in X .

Another interesting question is how should initial ownership rights be allocated when one partner is relatively more financially constrained than the other one? In Section 1.4, partners with low cash resources were supposed to receive relatively more initial ownership rights. But this was not taking cash constraints as given. In the partnership formation game, giving more ownership rights to cash-constrained partners also means that they will have to incur a larger initial investment. Consider a two-agent partnership in which partner i has large cash resources but partner j is heavily cash constrained so that at the equal-share partnership $\sum_{i \in N} \min\{C(\frac{1}{n}), l_i(\frac{1}{n}X)\} = C(1/2) + l_j(\frac{1}{2}X) < G$. Further assume that $l_i(r_iX) + l_j(r_jX) > G$ with $l_k(r_kX) = \alpha_k - r_kX$. Consider an increase in partner i initial share (with respect to equal-share ownership). If it is possible to find $r_i \geq 1/2$ such that $C(r_i) > l_i(r_iX)$ and $C(r_j) > l_j(r_jX)$ then the partnership is ex post efficient dissolvable. This will typically occur if, for partner i , $C(\cdot)$ decreases at a lower rate than $l_i(\cdot)$ and the reverse for partner j . Given assumptions on initial financial resources, this is consistent with partner i suffering from an increase of financing cost at increasing marginal rate. The result is ambiguous and is likely to depend on the relative rate of change of financial resources and minimal collectible fee.

1.7.4. *Financing Dissolution with Outside Parties*

The previous analysis shows that feasibility of ex post efficient dissolution relies on a “well-balanced” distribution of ownership and cash resources among partners. Problems occur when some partners have too large ownership shares or too few cash resources – then the budget balance condition prevents poor partners to compensate those with large ownership shares. In this section, I investigate two extensions in which the partnership has recourse to an outside party to make ex post efficient dissolution feasible.

EXTERNAL BUYER. Given the nature of the problem, a natural remedy to the non-feasibility of ex post efficient dissolution mechanisms would be to introduce a new potential buyer of the partnership asset with no initial ownership and large cash resources, such as a bank or a private fund. Contrary to current partners, this new potential buyer would not have to be compensated for the value of already owned ownership shares and would have large enough cash resources to help to ensure the budget balance condition. In this section, I investigate to what extent this additional potential buyer can restore feasibility of ex post efficient dissolution mechanisms.

Consider a partnership with a set of partners $N = \{1, \dots, n\}$, an initial ownership structure $r \in \Delta^{n-1}$ and cash resources $l \in \mathbb{R}_+^n$. To simplify the exposition, let $C_{[n]}(r_i) := C(r_i)$ be the maximal collectible fee on partner $i \in N$ due to interim individual rationality when reallocation occurs among the n partners only and where $C(r_i)$ is defined as in (1.15). Let $G_{[n]} := \mathbb{E}[\max_{i \in N} v_i]$ be the expectation of the maximal valuation when there are n partners. Further assume that $\sum_{i \in N} \min\{C_{[n]}(r_i), l_i\} = \sum_{i \in N} l_i < (n-1)G_{[n]}$, that is, ex post efficient dissolution is not feasible for this partnership due to heavy cash constraints.

Suppose that a new potential buyer, denoted by $n+1$, with no initial ownership ($r_{n+1} = 0$) and unlimited cash resources ($l_{n+1} = +\infty$) now participates in the dissolution mechanism. This potential buyer has valuation $v_i \sim F(v_i)$ for the asset. Under this assumption, the final allocation of the asset is no more limited to original members of the partnership as it can now be sold to an outside party. The potential buyer can therefore be modeled as being the $n+1$ partner with no initial ownership and unlimited cash resources. Therefore, ex post efficient dissolution will allocate final ownership to the

agent with highest valuation among the set $N \cup \{n+1\}$. Let $C_{[n+1]}(r_i)$ denote the maximal collectible fee on partner $i \in N \cup \{n+1\}$ and $G_{[n+1]} := \mathbb{E}[\max_{i \in N \cup \{n+1\}} v_i]$ the expectation of the maximal valuation with $n+1$ partners. Feasibility of ex post efficient dissolution requires that the following holds:

$$\sum_{i \in N \cup \{n+1\}} \min\{C_{[n+1]}(r_i), l_i\} \geq nG_{[n+1]}.$$

Equivalently, this condition can be rewritten as

$$\sum_{i \in N} \min\{C_{[n+1]}(r_i), l_i\} + C_{[n+1]}(0) \geq nG_{[n+1]}. \quad (1.32)$$

For the n original partners, the addition of a potential buyer with no change in the initial ownership structure directly leads to an increase in the collectible fee due to IIR, that is, $C_{[n+1]}(r_i) > C_{[n]}(r_i)$.²⁶ Moreover, there is now an additional collectible fee $C_{[n+1]}(0)$ due to the presence of the potential buyer. This fee is the highest possible as $C_{[n+1]}(r_i)$ is decreasing in r_i . Overall, the addition of the potential buyer strictly increases the left-hand side of (1.32). However, the right-hand side of (1.32) also increases with the participation of the potential buyer as $G_{[n+1]} \geq G_{[n]}$. This occurs as the cost of implementing an incentive compatible mechanism increases in the number of participants. Therefore, the question is whether the increase of collectible fees on original partners and the potential buyer outweighs the increase in the cost of implementing an incentive compatible mechanism. The following proposition shows that, unfortunately, it is not the case.

Proposition 10 *Let $(N, r, l) \in \{1, \dots, n\} \times \Delta^{n-1} \times \mathbb{R}_+^n$ be a partnership such that ex post efficient dissolution is not feasible due to cash constraints, i.e., $\sum_{i \in N} \min\{C_{[n]}(r_i), l_i\} = \sum_{i \in N} l_i < (n-1)G_{[n]}$. Then, considering the participation of an additional potential buyer $n+1$ with no initial ownership share $r_{n+1} = 0$ and unlimited cash resources $l_{n+1} = +\infty$ never allows for ex post efficient dissolution.*

Proof. First, notice from (1.15) that $C_{[n+1]}(0) = G_{[n]}$. It follows that $C_{[n+1]}(r_i) \leq G_{[n]}$ as $C_{[n+1]}(r_i)$ is decreasing in r_i . Therefore, $\sum_{i \in N} \min\{C_{[n+1]}(r_i), l_i\} = \sum_{i \in N} l_i$ so that

²⁶Recall that $C_{[n]}(r_i) = \inf_{v_i} \{\mathbb{E}_{-i} g(v) - v_i r_i\}$. Using the Envelope Theorem, it is immediate that $C_{[n]}(r_i)$ is increasing in n .

collectible fees on original partners do not increase with the addition of the new buyer. Then total collectible fees on the $n + 1$ partners writes $\sum_{i \in N} l_i + C_{[n+1]}(0) = \sum_{i \in N} l_i + G_{[n]} < nG_{[n]} < nG_{[n+1]}$, that is, the partnership with the $n + 1$ new potential buyer is not feasible either. ■

The addition of the potential buyer is not enough to compensate for the lack of financial resources in the original partnership with n agents. Although this result could be mitigated by the fact that cash constraints are not that severe initially, the fact that the potential buyer is assumed to have no ownership and unlimited financial resources make the result of Proposition 10 quite strong. In other words, it means that the addition of a potential new buyer can be beneficial only if cash constraints are not very severe so that the usefulness of doing is likely to be limited.

COALITION OF A PARTNER WITH A PRIVATE FUND. Consider now another possibility for partners to finance their participation in ex post efficient dissolution mechanisms. Assume n partners have initial ownership structure $r \in \Delta^{n-1}$ and cash resources $l \in \mathbb{R}_+^n$. Suppose that ex post efficient dissolution is not feasible due to very important cash constraints for one partner, say partner n . Formally, assume $\sum_{i \in N} \min\{C(r_i), l_i\} < (n - 1)G$ but $\sum_{i < n} \min\{C(r_i), l_i\} + C(r_n) \geq (n - 1)G$, that is, non-feasibility of ex post efficient dissolution is due to partner n low cash resources.

Assume that a private fund proposes to partner n to team up so that if n receives final ownership, the private fund will receive a share $1 - \alpha \in [0, 1]$ of the asset. The private fund has unlimited amount of financial resources. I model the coalition between partner n and the private fund as a single agent. The set of partners therefore becomes $\tilde{N} = \{1, \dots, n - 1, \tilde{n}\}$ where \tilde{n} denotes the coalition between partner n and the fund. Initial ownership shares and cash resources are unchanged for partners $i < \tilde{n}$. The coalition inherits initial ownership shares of partner n , i.e., $r_{\tilde{n}} = r_n$ whereas it has now unlimited cash resources $l_{\tilde{n}} = +\infty$. The valuation of the asset in the coalition is given by $v_{\tilde{n}} \sim F_{\tilde{n}}(v_{\tilde{n}} \mid \alpha)$ where I assume that $F_{\tilde{n}}(v_{\tilde{n}} \mid 0) = F(v_{\tilde{n}})$ and $F_{\tilde{n}}(v_{\tilde{n}} \mid \alpha') \geq F_{\tilde{n}}(v_{\tilde{n}} \mid \alpha) \geq F(v_{\tilde{n}})$ for all $\alpha' < \alpha < 1$. In other words, the more partner n shares the asset with the private fund, the less likely it is that the coalition has a high valuation (in the first-order stochastic dominance sense).²⁷ I also assume that the ex post efficient mechanism takes into account

²⁷It seems reasonable to assume that the private fund has generally a lower valuation for the asset than

the loss induced by the sharing of the asset by partner n and the private fund. Hence, ex post efficiency considers valuation $(v_1, \dots, v_{n-1}, v_{\tilde{n}})$ to reallocate ownership.

This problem can therefore be seen as a partnership problem with n partners, initial ownership structure $r = (r_1, \dots, r_n)$ and cash resources $l = (l_1, \dots, l_{n-1}, +\infty)$. Valuations are drawn from $F(v_i)$ for all $i < n$ and from $F_{\tilde{n}}(v_{\tilde{n}} \mid \alpha)$ for the coalition. As shown in Section 1.7, the dissolution condition can easily be extended to asymmetric distribution of valuations. From Proposition 8, ex post efficient dissolution is feasible if and only if $\sum_{i \in N} \min\{\mathcal{C}_i(r_i), l_i\} \geq (n-1)G$ where $\mathcal{C}_i(r_i) = \mathcal{C}_j(r_j)$ for all $i < n, j < n$ and $\mathcal{C}_n(x) \geq \mathcal{C}_i(x)$ for all $i < n$ and $x \in [0, 1]$. Following the same reasoning as in Section 1.7, $\mathcal{C}_i(r_i) \leq C(r_i)$ for all r_i so that less can be collected on partners $i < n$.

Whether the private fund allows for ex post efficient dissolution is ambiguous. On the one hand, the private fund relaxes the cash constraint and increases the collectible fees on partner \tilde{n} compared to partner n . On the other hand, the coalition also implies that collectible fees on partners $i < n$ decrease as $\mathcal{C}_i(r_i) \leq C(r_i)$. The overall effect is highly dependent on the actual distribution of valuations and initial distribution of ownership and cash resources. However, the following proposition shows that the coalition may help to implement the first-best dissolution mechanism if the private fund's share is low enough.

Proposition 11 *There exists a threshold $\hat{\alpha} \in [0, 1)$ such that ex post efficient dissolution is feasible for any coalition between partner n and a private fund for all $\alpha > \hat{\alpha}$.*

Proof. Take α arbitrarily close to 1. It follows that all valuations (including that of partner \tilde{n}) are drawn from $F(\cdot)$ as $F_{\tilde{n}}(\cdot \mid \alpha)$ converges in distribution to $F(\cdot)$. Then, we have approximately $\mathcal{C}_i(x) = \mathcal{C}_n(x) = C(x)$ for all $i < n$ and all $x \in [0, 1]$. As the private fund relaxes the cash constraint for partner n , the total collectible fee approximately writes $\sum_{i < n} \min\{C(r_i), l_i\} + C(r_n) \geq (n-1)G$ so that ex post efficient dissolution is feasible for α arbitrarily close to 1. Hence, there must exist a threshold $\hat{\alpha} < 1$ such that for all $\alpha > \hat{\alpha}$, this condition holds. ■

partners themselves as the private fund lacks the fundamentals skills to run the business as good as partners do.

Proposition 11 therefore states that the coalition between a private fund and the partner with low cash resources can be beneficial as long as the private fund's share of final ownership remains reasonably low. Although I have not investigated the participation constraint of the private fund, the result of Proposition 11 indicates that the limited share of ownership granted to the private fund may hinder its willingness to participate. As the size of final ownership claimed by the private fund is likely to increase in the amount of financial resources it provides to the coalition, the benefits of the coalition are, once again, likely to be restricted to cases in which cash constraints are not too severe.

1.8. CONCLUSION

In this paper, I study partnership dissolution problems with cash-constrained agents. This framework applies to various economic settings such as divorces, terminations of joint ventures, bankruptcy procedures or land reallocation. Relying on the mechanism design literature, I construct dissolution mechanisms that perform well even in the presence of cash-constrained agents. I derive necessary and sufficient conditions for ex post efficient partnership dissolution with interim (resp. ex post) incentive compatible, interim individually rational, ex post (resp. ex ante) budget balance and ex post cash-constrained mechanisms. I show that the dissolution condition is a generalization of the condition found in CGK. Interestingly, when partners have asymmetric cash constraints, the equal-share partnership is no more the *optimal* initial ownership structure (as found by CGK). Instead, the *optimal* initial ownership structure allocates relatively more (resp. less) property rights to more (resp. less) cash-constrained partners. This result sheds light on the role of the distributions of liquid and illiquid assets in organizations.

I show that the standard equivalence between Bayesian and dominant strategy mechanisms remains valid under the assumption of cash-constrained partners. This result indicates that both classes of mechanisms can be equivalently implemented and that there is no new restrictions due to the presence of cash-constraints.

I propose a simple "cash-constrained" auction to implement the ex post efficient dissolution mechanisms. It simply consists in asking agents to submit bids to an auctioneer who finally allocates ownership rights to the highest bidder. Prices and side payments

are designed to satisfy all the desired properties of the dissolution mechanism. I further show that the cash-constrained auction allows to dissolve some partnerships with cash-constrained agents that CGK's auction would fail to dissolve.

Finally, I investigate second-best mechanisms whose objective function is a convex combination of the expected surplus and the expected collected revenues. I characterize the set of interim incentive compatible, interim individually rational and ex post cash-constrained mechanism for any allocation rule. I show that the problem can be solved using a relaxed problem. The solution involves (i) ironing of the virtual valuations of the partners and, (ii) favors heavily cash-constrained agents towards less cash-constrained agents for medium-range valuation and (iii) favors less cash-constrained agents towards heavily cash-constrained agents for high-range valuations. Imposing a budget balance condition can be simply done by solving the above problem for a well-chosen weight of the convex combination the expected surplus and the expected collected revenues.

APPENDIX A

Additional computations for Theorem 1. To prove that t_i^C is IIC, Proposition 1 requires that $\mathbb{E}_{-i} h_i^C(v) = H_i$ where $H_i \in \mathbb{R}$ is a constant. Taking expectations \mathbb{E}_{-i} over equation (1.9) gives

$$\begin{aligned} \mathbb{E}_{-i} h_i^C(v) &= \frac{n-1}{n} \left[\mathbb{E}_{-i} g(v) + \frac{1}{n-1} \sum_{j \neq i} \mathbb{E}_{-i} s_j(v) \psi(v_j) - \psi(v_i) \mathbb{E}_{-i} s_i(v) \right] + \phi_i \\ &= \frac{n-1}{n} \left[v_i F(v_i)^{n-1} + \int_{v_i}^{\bar{v}} x dF(x)^{n-1} + \int_{v_i}^{\bar{v}} \psi(v_j) F(v_j)^{n-2} dF(v_j) - \psi(v_i) F(v_i)^{n-1} \right] + \phi_i. \end{aligned}$$

Replacing $\psi(\cdot)$ by its expression and integrating the third term by part gives

$$\begin{aligned} \mathbb{E}_{-i} h_i^C(v) &= \frac{n-1}{n} \left[v_i F(v_i)^{n-1} + \int_{v_i}^{\bar{v}} x dF(x)^{n-1} + \left(- \int_{\underline{v}}^{\bar{v}} F(x)^n dx + \frac{\int_{\underline{v}}^{v_i} F(x)^n dx}{F(v_i)} + \int_{v_i}^{\bar{v}} F(x)^{n-1} dx \right) \right. \\ &\quad \left. - \frac{\int_{\underline{v}}^{v_i} F(x)^n dx}{F(v_i)} \right] + \phi_i \\ &= \frac{n-1}{n} \left[v_i F(v_i)^{n-1} + \int_{v_i}^{\bar{v}} x dF(x)^{n-1} - \int_{\underline{v}}^{\bar{v}} F(x)^n dx + \int_{v_i}^{\bar{v}} F(x)^{n-1} dx \right] + \phi_i. \end{aligned}$$

Notice that, by integration by parts

$$\int_{v_i}^{\bar{v}} F(x)^{n-1} dx = \bar{v} - v_i F(v_i)^{n-1} - \int_{v_i}^{\bar{v}} x dF(x)^{n-1}.$$

Plugging this expression into $\mathbb{E}_{-i} h_i^C(v)$ gives

$$\begin{aligned}\mathbb{E}_{-i} h_i(v) &= \frac{n-1}{n} \left[\bar{v} - \int_{\underline{v}}^{\bar{v}} F(s)^n ds \right] - \phi_i \\ &= \frac{n-1}{n} G - \phi_i,\end{aligned}$$

which concludes the proof. \blacksquare

Proof of Lemma 1. Notice that,

$$\begin{aligned}\int_{\underline{v}}^{v_k} \frac{\int_{\underline{v}}^t F(s)^n ds}{F(t)^{n+1}} f(t) dt &= - \left[\frac{\int_{\underline{v}}^t F(s)^n ds}{nF(t)^n} \right]_{\underline{v}}^{v_k} + \frac{1}{n} \int_{\underline{v}}^{v_k} dt \\ &= \frac{1}{n} \left[v_k - \frac{\int_{\underline{v}}^{v_k} F(s)^n ds}{F(v_k)^n} - \underline{v} \right] \\ &= \frac{1}{n} [v_k - \psi(v_k) - \underline{v}],\end{aligned}$$

where the first line stems from integration by parts and the second line is obtained using L'Hôpital's rule,

$$i.e., \lim_{t \rightarrow \underline{v}} \frac{\int_{\underline{v}}^t F(s)^n ds}{nF(t)^n} = \lim_{t \rightarrow \underline{v}} \frac{F(t)}{nf(t)} = 0.$$

Then, $v_k - \psi(v_k) = n \int_{\underline{v}}^{v_k} \frac{\int_{\underline{v}}^t F(s)^n ds}{F(t)^{n+1}} f(t) dt + \underline{v}$ from which it immediately follows that $[v_k - \psi(v_k)]$ is nonnegative and increasing in $v_k \in V$. \blacksquare

Proof of Theorem 2. (Necessity) Take a dissolution mechanism (s^*, t^*) satisfying EF and EPIC, *i.e.*, satisfying Proposition 1.b. Then, $t_i^*(v) = g(v) - v_i s_i^*(v) - h_i(v)$ with $h_i(v)$ is constant in v_i . Define $H_i := \mathbb{E}_{-i} h_i(v)$, it is then straightforward to see that equations (1.6) and (1.7) necessary in Bayesian mechanisms are also necessary for dominant strategy mechanisms. As shown in the proof of Theorem 1, the necessity of equations (1.6) and (1.7) implies the necessity of $\sum_{i \in N} \min\{C(r_i), l_i\} \geq (n-1)G$ which is (1.14).

(Sufficiency) Consider the following transfer function for agent i : $t_i^*(v) = g(v) - v_i s_i^*(v) - \Phi_i$ where $\Phi_i \in \mathbb{R}$ is a constant. From Proposition 1.b., $t_i^*(v)$ is EF and EPIC. EABB requires $\mathbb{E} [\sum_i t_i^*(v)] = 0$, which is equivalent to $\sum_i \Phi_i = (n-1)G$. EPCC requires that $t_i^*(v) = g(v) - v_i s_i^*(v) - \Phi_i \geq -l_i$ which is equivalent to $\Phi_i \leq l_i + \sum_{j \neq i} v_j s_j^*(v)$ for all $v \in V^n$. Thus, the most restrictive case gives $\Phi_i \leq l_i$. Finally, IIR requires

$$\mathbb{E}_{-i} v_i (s_i^*(v) - r_i) + \mathbb{E}_{-i} t_i^*(v_i) = \mathbb{E}_{-i} g(v) - \Phi_i - v_i r_i \geq 0 \quad \text{for all } v_i \in V,$$

or equivalently that $\Phi_i \leq C(r_i)$ with $C(r_i) := \inf_{v_i} \{\mathbb{E}_{-i} g(v) - v_i r_i\}$. Aggregating IIR and EPCC, *i.e.* $\Phi_i \leq l_i$ and $\Phi_i \leq C(r_i)$, respectively, gives $\Phi_i \leq \min\{C(r_i), l_i\}$ for all $i \in N$.

Take for instance, $\Phi_i = \min\{C(r_i), l_i\} - \frac{\sum_{k \in N} \min\{C(r_k), l_k\} - (n-1)G}{n}$. It is immediate that EABB is satisfied as $\sum_{i \in N} \Phi_i = (n-1)G$. Moreover, as condition (1.14) is satisfied, *i.e.* $\sum_i \min\{C(r_i), l_i\} \geq (n-1)G$, it is clear that $\Phi_i \leq \min\{C(r_i), l_i\}$ therefore satisfying IIR and EPCC as well. ■

Proof of Theorem 3. The dissolution mechanism (s^*, \tilde{t}) is EF and EPIC then, from Proposition 1.b., $\tilde{t}_i(v) = g(v) - v_i s_i^*(v) - k_i(v_{-i})$ for some function k_i independent of v_i . Let $K_i := \mathbb{E}_{-i} k_i(v_{-i})$. From EABB, $\mathbb{E} \sum_{i \in N} \tilde{t}_i(v) = (n-1)G - \sum_{i \in N} K_i = 0$. Take another dissolution mechanism (s^*, t) with

$$t_i(v) = g(v) - v_i s_i^*(v) - h_i(v) + \frac{n-1}{n}G - \phi_i - K_i,$$

where $h_i(v)$ is defined by equation (1.9) for which $\mathbb{E}_{-i} h_i(v_{-i}) = \frac{n-1}{n}G - \phi_i$ and $\sum_{i \in N} h_i(v) = (n-1)g(v) - \sum_{i \in N} \phi_i$. Then, it is immediate that (s^*, t) is EF and IIC. From (1.11), $\min_v g(v) - v_i s_i(v) - h_i(v) = -\frac{n-1}{n}G + \phi_i$ and then $\min_v t_i(v) = -K_i$. As \tilde{t} is EPCC, $\tilde{t}_i(v) = g(v) - v_i s_i^*(v) - k_i(v_{-i}) \geq -l_i$ for all $i \in N$, $v \in V^n$ which implies that $\mathbb{E}_{-i}[g(v) - v_i s_i^*(v)] - K_i \geq -l_i$ for all $i \in N$, $v_i \in V$ and then that $-K_i \geq -l_i$ given that the infimum of $\mathbb{E}_{-i}[g(v) - v_i s_i^*(v)]$ is zero. It follows that $\min_v t_i(v) = -K_i \geq -l_i$ so that t is also EPCC. Straightforward computations shows that $\sum_{i \in N} t_i(v) = 0$ as $\sum_{i \in N} K_i = (n-1)G$. Finally, notice that $\mathbb{E}_{-i} t_i(v) = \mathbb{E}_{-i}[g(v) - v_i s_i^*(v)] - K_i = \mathbb{E}_{-i} \tilde{t}_i(v)$ implying that (s^*, t) is payoff equivalent to (s^*, \tilde{t}) at the interim stage and thus also IIR. ■

Proof of Theorem 4. The dissolution mechanism (s^*, t) is EF and IIC then, from Proposition 1.a., $t_i(v) = g(v) - v_i s_i^*(v) - h_i(v)$ where $\mathbb{E}_{-i} h_i(v) =: H_i$. Take another dissolution mechanism (s^*, \tilde{t}) where

$$\begin{aligned} \tilde{t}_i(v) &= t_i(v) + h_i(v) - H_i \\ &= g(v) - v_i s_i^*(v) - H_i. \end{aligned}$$

Then, from Proposition 1.b., \tilde{t} is EF and EPIC. From EPCC, $t_i(v) = g(v) - v_i s_i^*(v) - h_i(v) \geq -l_i$ for all $i \in N$, $v \in V^n$ which implies that $\mathbb{E}_{-i}[g(v) - v_i s_i^*(v) - H_i] \geq -l_i$ for all $i \in N$, $v_i \in V$. This is equivalent to $-H_i \geq -l_i$ for all $i \in N$. Then notice that $\min_v \tilde{t}_i(v) = -H_i \geq -l_i$ such that \tilde{t} is also EPCC. Computing $\mathbb{E} \sum_{i \in N} \tilde{t}_i(v) = \mathbb{E} \sum_{i \in N} t_i(v) = 0$ and thus \tilde{t} is EABB. Finally, as $\mathbb{E}_{-i} \tilde{t}_i(v) = \mathbb{E}_{-i} t_i(v)$ for all $i \in N$, $v_i \in V$, then (s^*, \tilde{t}) is interim payoff equivalent to (s^*, t) and also IIR. ■

Proof of Proposition 2. (Only if) Assume there exists an EF, IIC, IIR, EPBB and EPCC mechanism (s, t) . From EPCC we have $t_i(v) \geq -l_i$ for all $i \in N$, $v \in V^n$. Then, simply let $\tilde{t}_i(v) = t_i(v)$ for all i, v and it is immediate that $\mathbb{E}_{-i} \tilde{t}_i(v_i, v_{-i}) \geq -l_i$ for all i, v . The mechanism (s, \tilde{t}) therefore satisfies all the properties of the mechanism (s, t) and also satisfies interim cash constraints.

(If) Assume there exists an EF, IIC, IIR, EPBB and *interim* cash-constrained mechanism (s, \tilde{t}) . As (s, \tilde{t}) is IIC, the transfer rule is Groves in expectations and can be written as $\tilde{t}_i(v) = g(v) - v_i s_i(v) - h_i(v)$ with $\mathbb{E}_{-i} h_i(v) =: H_i$ for all i, v_i . The interim cash constraints then writes $\mathbb{E}_{-i} \tilde{t}_i(v_i, v_{-i}) \geq -l_i$ for all $i \in N$,

$v_i \in V$ or equivalently

$$G_i(v_i) - \mathbb{E}_{-i} v_i s_i(v) - H_i \geq -l_i \quad \text{for all } i, v_i.$$

This inequality is equivalent to $H_i \leq \inf_{v_i} \{G_i(v_i) - \mathbb{E}_{-i} v_i s_i(v)\} + l_i$ for all $i \in N$. As the infimum is zero, we simply have $H_i \leq l_i$ for all $i \in N$. Remark that this is the same necessary condition to satisfy the cash constraints as in the proof of Theorem 1, *i.e.*, it is the same necessary condition than for a mechanism with EPCC. As the other conditions (EF, IIR and EPBB) are the same we can say that we have a mechanism (s, \tilde{t}) only if $\sum_i \min\{C_i, l_i\} \geq (n-1)G$. But from Theorem 1, we know that this condition is sufficient to construct a mechanism (s, t) satisfying EPCC. ■

Proof of Proposition 3. Starting with Proposition 3.a., assume that $\tilde{r}_i \leq \frac{1}{n}$ for all $i \in N$. Notice that $\max_{r \in \Delta^{n-1}} \sum_{i \in N} C(r_i) = \sum_{i \in N} C(\frac{1}{n})$ and thus for all $r \in \Delta^{n-1}$, $\sum_{i \in N} \min\{C(r_i), l_i\} \leq \sum_{i \in N} C(\frac{1}{n})$. It is then clear that choosing $r_i^* = \frac{1}{n}$ for all $i \in N$ is such that for each $i \in N$, $\min\{C(r_i^*), l_i\} = C(r_i^*) = C(\frac{1}{n})$ provided that $r_i^* \geq \tilde{r}_i$ for all $i \in N$. Hence, $\sum_{i \in N} \min\{C(r_i), l_i\} = \sum_{i \in N} C(\frac{1}{n})$ which is the upper bound.

Consider now Proposition 3.b., *i.e.* assume that $\tilde{r}_i > \frac{1}{n}$ for some $i \in N$. Define $\mathcal{L}(r, \lambda) = \sum_{i \in N} \min\{C(r_i), l_i\} + \lambda(\sum_{i \in N} r_i - 1)$ where $\lambda \in \mathbb{R}$ is the Lagrange multiplier associated with the constraint $\sum_{i \in N} r_i = 1$. Notice that $\sum_{i \in N} \min\{C(r_i), l_i\}$ is concave as $C(r_i)$ is concave for each $i \in N$ and differentiable everywhere except at $r_i = \tilde{r}_i$. Let $\delta_{r_i} \mathcal{L}(r, \lambda)$ denote the superdifferential of the Lagrangian in r_i , then

$$\delta_{r_i} \mathcal{L}(r, \lambda) = \lambda + \begin{cases} 0 & \text{if } r_i < \tilde{r}_i \\ [C'(\tilde{r}_i), 0] & \text{if } r_i = \tilde{r}_i \\ C'(r_i) & \text{if } r_i > \tilde{r}_i. \end{cases}$$

The necessary optimality condition writes $0 \in \delta_{r_i} \mathcal{L}(r, \lambda)$ for all $i \in N$. First, assume that there is at least one $r_j^* < \tilde{r}_j$. Then $\lambda = 0$ and $r_i > \tilde{r}_i$ is impossible as it is impossible to have $C'(r_i) = 0$ with $r_i > \tilde{r}_i$ (indeed $C'(r_i) = 0$ only occurs when $v = 0$ and $r_i = 0$). But then, if all $r_i^* \leq \tilde{r}_i$ with one strict inequality at least, it follows that $\sum_{i \in N} r_i^* < \sum_{i \in N} \tilde{r}_i \leq 1$ which is also impossible. Therefore, it is necessary that $r_i \geq \tilde{r}_i$ for all $i \in N$. Assume now that $r_i > \tilde{r}_i$ for all $i \in N$. Then, the necessary optimality condition implies that $\lambda + C'(r_i^*) = 0$ for all $i \in N$. But then it follows that $r_i^* = \frac{1}{n}$ for all $i \in N$ which is impossible as some $\tilde{r}_i > \frac{1}{n}$ contradicting that $r_i^* > \tilde{r}_i$ for all $i \in N$.

Hence, the solution must be such $r_i^* \geq \tilde{r}_i$ for all $i \in N$ with at least one equality. Let $\mathcal{A} := \{i \in N \mid r_i^* > \tilde{r}_i\}$ and $\mathcal{B} := \{j \in N \mid r_j^* = \tilde{r}_j\}$. Then, for all $i \in \mathcal{A}$, $\lambda + C'(r_i^*) = 0$ implies that $\lambda > 0$ and $r_i^* = r_k^*$ for any two $i, k \in \mathcal{A}$. For any $i \in \mathcal{A}$, and let $r_i^* = \hat{r}$ with $\hat{r} := \frac{1 - \sum_{j \in \mathcal{B}} \tilde{r}_j}{|\mathcal{A}|}$. As by assumption $\tilde{r}_1 \leq \dots \leq \tilde{r}_n$ and for all $i \in \mathcal{A}$ it is necessary that $\hat{r} > \tilde{r}_i$, it is possible to rewrite $\mathcal{A} := \{i \in N \mid i < p\}$ and $\mathcal{B} := \{j \in N \mid j \geq p\}$ for some $p \in N \setminus \{1\}$ and $\hat{r} = \frac{1 - \sum_{j \geq p} \tilde{r}_j}{p-1}$. It is also necessary that $\hat{r} \leq \tilde{r}_j$ for all $j \in \mathcal{B}$. The solution therefore writes $r^* = (\hat{r}, \hat{r}, \dots, \hat{r}, \tilde{r}_p, \tilde{r}_{p+1}, \dots, \tilde{r}_n)$ and $\max_{i < p} \tilde{r}_i < \hat{r} \leq \min_{j \geq p} \tilde{r}_j$. ■

APPENDIX B

Proof of Theorem 6. The right derivative of $\delta_i(v_i | \cdot)$ writes:

$$\frac{\partial_+ \delta_i(v_i | \cdot)}{\partial \bar{m}_i} = \begin{cases} 0 & \text{if } v_i < \bar{m}_i \\ \frac{f(\bar{m}_i)}{1-F(\bar{m}_i)} \left[\frac{1}{1-F(\bar{m}_i)} \int_{\bar{m}_i}^{\bar{v}} \alpha(v_i | \lambda) dF(v_i) - \alpha(\bar{m}_i | \lambda) - \frac{\tau_i}{f(\bar{m}_i)} - \frac{\tau_i \bar{m}_i}{1-F(\bar{m}_i)} \right] & \text{if } v_i \geq \bar{m}_i, \end{cases}$$

then we simply have

$$\frac{\partial_+ \delta_i(v_i | \cdot)}{\partial \bar{m}_i} = \begin{cases} 0 & \text{if } v_i < \bar{m}_i \\ \frac{f(\bar{m}_i)}{1-F(\bar{m}_i)} [\delta_i(\bar{m}_i | x_i, \bar{m}_i, \lambda) - \delta_i^-(\bar{m}_i | x_i, \bar{m}_i, \lambda)] & \text{if } v_i \geq \bar{m}_i. \end{cases}$$

Now, taking the right derivative of $\hat{\mathcal{L}}$ at \bar{m}_i gives

$$\begin{aligned} \frac{\partial_+ \hat{\mathcal{L}}}{\partial \bar{m}_i} &= \int_{V^n} \frac{\partial_+ \max_{i \in N} \delta_i(v_i | \cdot)}{\partial \bar{m}_i} dF(v) - r_i \int_V \frac{\partial_+ \delta_i(v_i | \cdot)}{\partial \bar{m}_i} dF(v_i) \\ &\quad + f(\bar{m}_i) \int_{V^{n-1}} \left(\max_{j \neq i} \{ \delta_i^-(\bar{m}_i | \cdot), \max_{j \neq i} \delta_j(v_j | \cdot) \} - \max_{j \neq i} \{ \delta_i(\bar{m}_i | \cdot), \max_{j \neq i} \delta_j(v_j | \cdot) \} \right) dF(v_{-i}) \\ &\quad - r_i \left(\delta_i^-(\bar{m}_i | \cdot) - \delta_i(\bar{m}_i | \cdot) \right) f(\bar{m}_i). \end{aligned}$$

Notice that the terms that depend on r_i cancel out. Thus we obtain

$$\begin{aligned} \frac{\partial_+ \hat{\mathcal{L}}}{\partial \bar{m}_i} &= \int_{V^n} \frac{\partial_+ \max_{i \in N} \delta_i(v_i | \cdot)}{\partial \bar{m}_i} dF(v) \\ &\quad + f(\bar{m}_i) \int_{V^{n-1}} \left(\max_{j \neq i} \{ \delta_i^-(\bar{m}_i | \cdot), \max_{j \neq i} \delta_j(v_j | \cdot) \} - \max_{j \neq i} \{ \delta_i(\bar{m}_i | \cdot), \max_{j \neq i} \delta_j(v_j | \cdot) \} \right) dF(v_{-i}). \end{aligned}$$

First, assume that $\delta_i^-(\bar{m}_i | \cdot) > \delta_i(\bar{m}_i | \cdot)$. This is equivalent to say that

$$\tau_i > \frac{\left(\mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \alpha(\bar{m}_i | \lambda) \right) (1 - F(\bar{m}_i)) f(\bar{m}_i)}{1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)}.$$

Using the fact that $\partial_+ \delta_j(v_j | \cdot) / \partial \bar{m}_i = 0$ for all $j \neq i$ and the expression of the right derivative of $\delta_i(v_i | \cdot)$, the first term of $\frac{\partial_+ \hat{\mathcal{L}}}{\partial \bar{m}_i}$ rewrites

$$\begin{aligned} \int_{V^n} \frac{\partial_+ \max_{i \in N} \delta_i(v_i | \cdot)}{\partial \bar{m}_i} dF(v) &= \\ \int_{v \in V^n \mid v_i \in [\bar{m}_i, \bar{v}], \max_{j \neq i} \delta_j(v_j | \cdot) < \delta_i(\bar{m}_i | \cdot)} \frac{f(\bar{m}_i)}{1 - F(\bar{m}_i)} (\delta_i(\bar{m}_i | \cdot) - \delta_i^-(\bar{m}_i | \cdot)) dF(v) &= \\ f(\bar{m}_i) \int_{v_{-i} \in V^{n-1} \mid \max_{j \neq i} \delta_j(v_j | \cdot) < \delta_i(\bar{m}_i | \cdot)} (\delta_i(\bar{m}_i | \cdot) - \delta_i^-(\bar{m}_i | \cdot)) dF_{-i}(v_{-i}). \end{aligned}$$

As for the second term, it rewrites

$$\begin{aligned} & f(\bar{\mathbf{m}}_i) \int_{\mathbf{V}^{n-1}} \left(\max \{ \delta_i^-(\bar{\mathbf{m}}_i | \cdot), \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot) \} - \max \{ \delta_i(\bar{\mathbf{m}}_i | \cdot), \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot) \} \right) dF(\mathbf{v}_{-i}) = \\ & f(\bar{\mathbf{m}}_i) \int_{\mathbf{V}_{-i} \in \mathbf{V}^{n-1} \mid \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot) < \delta_i(\bar{\mathbf{m}}_i | \cdot)} (\delta_i^-(\bar{\mathbf{m}}_i | \cdot) - \delta_i(\bar{\mathbf{m}}_i | \cdot)) dF(\mathbf{v}_{-i}) \\ & + f(\bar{\mathbf{m}}_i) \int_{\mathbf{V}_{-i} \in \mathbf{V}^{n-1} \mid \delta_i(\bar{\mathbf{m}}_i | \cdot) \leq \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot) < \delta_i^-(\bar{\mathbf{m}}_i | \cdot)} (\delta_i^-(\bar{\mathbf{m}}_i | \cdot) - \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot)) dF(\mathbf{v}_{-i}). \end{aligned}$$

As the first term of $\frac{\partial_+ \hat{\mathcal{L}}}{\partial \bar{\mathbf{m}}_i}$ cancels out with the first term of the above equation we finally get

$$\frac{\partial_+ \hat{\mathcal{L}}}{\partial \bar{\mathbf{m}}_i} = f(\bar{\mathbf{m}}_i) \int_{\mathbf{V}_{-i} \in \mathbf{V}^{n-1} \mid \delta_i(\bar{\mathbf{m}}_i | \cdot) \leq \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot) < \delta_i^-(\bar{\mathbf{m}}_i | \cdot)} (\delta_i^-(\bar{\mathbf{m}}_i | \cdot) - \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot)) dF(\mathbf{v}_{-i}).$$

Therefore, this expression is exactly the same as in [Boulatov and Severinov \(2019\)](#). It follows that is it not possible that $\delta_i^-(\bar{\mathbf{m}}_i | \cdot) > \delta_i(\bar{\mathbf{m}}_i | \cdot)$ and that the set $A_i(\cdot) := \{\mathbf{v}_{-i} \in \mathbf{V}^{n-1} \mid \delta_i(\bar{\mathbf{m}}_i | \cdot) \leq \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot) < \delta_i^-(\bar{\mathbf{m}}_i | \cdot)\}$ has positive measure as it would imply that $\frac{\partial_+ \hat{\mathcal{L}}}{\partial \bar{\mathbf{m}}_i} > 0$, contradicting the optimality of $\bar{\mathbf{m}}_i$.

The same reasoning as in [Boulatov and Severinov \(2019\)](#) can be done to prove that it is not possible to have $\delta_i^-(\bar{\mathbf{m}}_i | \cdot) < \delta_i(\bar{\mathbf{m}}_i | \cdot)$ and the set $B_i(\cdot) := \{\mathbf{v}_{-i} \in \mathbf{V}^{n-1} \mid \delta_i^-(\bar{\mathbf{m}}_i | \cdot) < \max_{j \neq i} \delta_j(\mathbf{v}_j | \cdot) \leq \delta_i(\bar{\mathbf{m}}_i | \cdot)\}$ has a positive measure.

Let $\delta_i(\bar{\mathbf{m}}_i | \cdot)$ be replaced by $\delta_i(\bar{\mathbf{m}}_i)$ for convenience of the following lemmas.

Lemma 6 *Assume that for some i and j , $\delta_i^-(\bar{\mathbf{m}}_i) \leq \delta_j^-(\bar{\mathbf{m}}_j)$. Then $\delta_i^-(\bar{\mathbf{m}}_i) \leq \delta_i(\bar{\mathbf{m}}_i)$.*

Proof. Assume that $\delta_i^-(\bar{\mathbf{m}}_i) \leq \delta_j^-(\bar{\mathbf{m}}_j)$ for some i and j but $\delta_i(\bar{\mathbf{m}}_i) < \delta_i^-(\bar{\mathbf{m}}_i)$. Then, as for any $k \in \mathbf{N}$, $\delta_k(\underline{\mathbf{v}}) = \underline{\mathbf{v}}$ we must have $\delta_i^-(\bar{\mathbf{m}}_i) > \underline{\mathbf{v}}$. Then, for all $k \in \mathbf{N}$ there exists a $\tilde{\mathbf{v}}_k \in (0, \bar{\mathbf{m}}_k]$ such that $\delta_k(\mathbf{v}_k) < \delta_i^-(\bar{\mathbf{m}}_i)$ for all $\mathbf{v}_k \in [\underline{\mathbf{v}}, \tilde{\mathbf{v}}_k]$.

At the same time, as $\underline{\mathbf{v}} \leq \delta_i(\bar{\mathbf{m}}_i) < \delta_i^-(\bar{\mathbf{m}}_i) \leq \delta_j^-(\bar{\mathbf{m}}_j)$ and $\delta_j(\underline{\mathbf{v}}) = \underline{\mathbf{v}}$, then, by continuity of $\delta_j(\mathbf{v}_j)$ there must exist some $\tilde{\mathbf{v}}_j$ such that $\delta_j(\tilde{\mathbf{v}}_j) = \delta_i^-(\bar{\mathbf{m}}_i)$ and a $\mathbf{b}_j > 0$ (that can be made arbitrarily small) such that $\delta_i(\bar{\mathbf{m}}_i) < \delta_j(\mathbf{v}_j) < \delta_i^-(\bar{\mathbf{m}}_i)$ for all $\mathbf{v}_j \in (\tilde{\mathbf{v}}_j - \mathbf{b}_j, \tilde{\mathbf{v}}_j)$. Hence, for all $\mathbf{v}_j \in (\tilde{\mathbf{v}}_j - \mathbf{b}_j, \tilde{\mathbf{v}}_j)$ and all $\mathbf{v}_k \in [\underline{\mathbf{v}}, \tilde{\mathbf{v}}_k]$ for all $k \neq i, j$ we have $\delta_i(\bar{\mathbf{m}}_i) < \max\{\delta_j(\mathbf{v}_j), \max_{k \neq i, j} \delta_k(\mathbf{v}_k)\} < \delta_i^-(\bar{\mathbf{m}}_i)$. As both $(\tilde{\mathbf{v}}_j - \mathbf{b}_j, \tilde{\mathbf{v}}_j)$ and $[\underline{\mathbf{v}}, \tilde{\mathbf{v}}_k]$ have non-zero measure, it follows that $A_i(\cdot)$ has positive measure which contradicts the optimality of $\bar{\mathbf{m}}_i$. Thus, we must have $\delta_i^-(\bar{\mathbf{m}}_i) \leq \delta_i(\bar{\mathbf{m}}_i)$. \blacksquare

Lemma 7 *Let $\mathbf{h}_1 \in \arg \max_{j \in \mathbf{N}} \delta_j^-(\bar{\mathbf{m}}_j)$. Assume either (i) $\delta_i^-(\bar{\mathbf{m}}_i) < \delta_{\mathbf{h}_1}^-(\bar{\mathbf{m}}_{\mathbf{h}_1})$ or (ii) $\delta_i^-(\bar{\mathbf{m}}_i) = \delta_{\mathbf{h}_1}^-(\bar{\mathbf{m}}_{\mathbf{h}_1})$ and $\delta_i(\bar{\mathbf{m}}_i) \leq \delta_{\mathbf{h}_1}(\bar{\mathbf{m}}_{\mathbf{h}_1})$. Then $\delta_i(\bar{\mathbf{m}}_i) \leq \delta_i^-(\bar{\mathbf{m}}_i)$.*

Proof. Case (i). Assume (i) but suppose that $\delta_i^-(\bar{\mathbf{m}}_i) < \delta_i(\bar{\mathbf{m}}_i)$. Then, as $\delta_k(\underline{\mathbf{v}}) = \underline{\mathbf{v}}$ for all $k \in \mathbf{N}$ we must have $\delta_i(\bar{\mathbf{m}}_i) > \underline{\mathbf{v}}$. For all $k \neq i, \mathbf{h}_1$ is it clear that there exists $\tilde{\mathbf{v}}_k \in (\underline{\mathbf{v}}, \bar{\mathbf{m}}_k]$ such that $\delta_k(\mathbf{v}_k) < \delta_i(\bar{\mathbf{m}}_i)$ for all $\mathbf{v}_k \in [\underline{\mathbf{v}}, \tilde{\mathbf{v}}_k]$.

Additionally either $\underline{\mathbf{v}} \leq \delta_i^-(\bar{\mathbf{m}}_i) < \delta_{\mathbf{h}_1}^-(\bar{\mathbf{m}}_{\mathbf{h}_1}) \leq \delta_i(\bar{\mathbf{m}}_i)$ or $\underline{\mathbf{v}} \leq \delta_i^-(\bar{\mathbf{m}}_i) < \delta_i(\bar{\mathbf{m}}_i) < \delta_{\mathbf{h}_1}^-(\bar{\mathbf{m}}_{\mathbf{h}_1})$. In both cases, there must exist a $\tilde{\mathbf{m}}_{\mathbf{h}_1} \in (\underline{\mathbf{v}}, \bar{\mathbf{m}}_{\mathbf{h}_1}]$ such that $\delta_{\mathbf{h}_1}(\mathbf{v}_{\mathbf{h}_1}) < \delta_i(\bar{\mathbf{m}}_i)$ for all $\mathbf{v}_{\mathbf{h}_1} \in [\underline{\mathbf{v}}, \tilde{\mathbf{m}}_{\mathbf{h}_1}]$. As $\delta_i^-(\bar{\mathbf{m}}_i) <$

$\delta_i(\bar{m}_i)$ there must be a $b_{h_1} > 0$ (that can be made arbitrarily small) such that $\delta_i^-(\bar{m}_i) < \delta_{h_1}(v_{h_1}) < \delta_i(\bar{m}_i)$ for all $v_{h_1} \in (\tilde{m}_{h_1} - b_{h_1}, \tilde{m}_{h_1})$.

It immediately follows that for all $k \neq i, h_1$, $v_k \in [\underline{v}, \tilde{v}_k)$ and for all $v_{h_1} \in (\tilde{m}_{h_1} - b_{h_1}, \tilde{m}_{h_1})$, $\delta_i(\bar{m}_i) < \max\{\delta_{h_1}^-(v_{h_1}), \max_{k \neq i, h_1} \delta_k(v_k)\} < \delta_i(\bar{m}_i)$, i.e., the set $B_i(\cdot)$ has a positive measure, which contradicts the optimality of \bar{m}_i .

Case (ii). Assume now (ii) and $\delta_i^-(\bar{m}_i) < \delta_i(\bar{m}_i)$. This would imply $\delta_i^-(\bar{m}_i) = \delta_{h_1}^-(\bar{m}_{h_1}) < \delta_i(\bar{m}_i) \leq \delta_{h_1}(\bar{m}_{h_1})$. It follows that $\delta_i(\bar{m}_i) > \underline{v}$ and also that the set $\{v_k \mid \delta_k(v_k) < \delta_{h_1}(\bar{m}_{h_1})\}$ includes $[\underline{v}, \bar{m}_k)$ for all $k \neq i, h_1$ as by definition $\delta_i^-(\bar{m}_i) = \max_{j \in N} \delta_j^-(\bar{m}_j) < \delta_i(\bar{m}_i) \leq \delta_{h_1}(\bar{m}_{h_1})$.

Additionally, we have the $\{v_i \mid \delta_{h_1}^-(\bar{m}_{h_1}) < \delta_i(v_i) \leq \delta_{h_1}(\bar{m}_{h_1})\} = [\bar{m}_i, \bar{v}]$ which has positive measure as $\bar{m}_i < \bar{v}$ (given that we have a jump in $\delta_i(\cdot)$).

Therefore, for all $v_i \in [\bar{m}_i, \bar{v}]$ and $v_k \in [\underline{v}, \bar{m}_k)$ for $k \neq i, h_1$, we have that $\delta_{h_1}^-(\bar{m}_{h_1}) < \max\{\delta_i(v_i), \max_{k \neq i, h_1} \delta_k(v_k)\} \leq \delta_{h_1}(\bar{m}_{h_1})$ and thus the set $B_{h_1}(\cdot)$ has positive measure which contradicts the optimality of \bar{m}_{h_1} . ■

Lemma 8 *Let $h_1 \in \arg \max_{j \in N} \delta_j^-(\bar{m}_j)$ and assume that for $i \neq h_1$, $\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i) < \delta_{h_1}^-(\bar{m}_{h_1})$ then we must have $\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i) < \delta_{h_1}(\bar{m}_{h_1})$.*

Proof. Assume instead that $\delta_{h_1}(\bar{m}_{h_1}) \leq \delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i) < \delta_{h_1}^-(\bar{m}_{h_1})$. It follows that $\delta_{h_1}^-(\bar{m}_{h_1}) > \underline{v}$. Hence, as for all $k \in N$, $\delta_k(\underline{v}) = \underline{v}$, there exists a $\tilde{v}_k \in (\underline{v}, \bar{m}_k]$ such that $\delta_k(v_k) < \delta_{h_1}^-(\bar{m}_{h_1})$ for all $v_k \in [\underline{v}, \tilde{v}_k)$. Then, as $\underline{v} \leq \delta_{h_1}(\bar{m}_{h_1}) \leq \delta_i(\bar{m}_i) < \delta_{h_1}^-(\bar{m}_{h_1})$, there must exist a $\tilde{m}_i \in (\underline{v}, \bar{m}_i]$ and a $b_i > 0$ such that $\delta_{h_1}(\bar{m}_{h_1}) \leq \delta_i(v_i) < \delta_{h_1}^-(\bar{m}_{h_1})$ for all $v_i \in (\tilde{m}_i - b_i, \tilde{m}_i)$. Then, the set A_{h_1} would have positive measure and this would contradict the optimality of \bar{m}_{h_1} .

Lemma 9 *There exists a unique $z_1 \in \arg \max_{j \in N} \delta_j^-(\bar{m}_j)$ and $z_1 \in \arg \max_{j \in H_1} \delta_j(\bar{m}_j)$ such that for all $i \neq z_1$ we have*

$$\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i).$$

and either

$$\min\{\delta_{z_1}^-(\bar{m}_{z_1}), \delta_{z_1}(\bar{m}_{z_1})\} > \max_{i \neq z_1} \delta_i(\bar{m}_i) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i)$$

or

$$\delta_{z_1}(\bar{m}_{z_1}) \geq \delta_{z_1}^-(\bar{m}_{z_1}) = \max_{i \neq z_1} \delta_i(\bar{m}_i) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i).$$

Proof. Let $H_1 = \{h_1 \in N \mid h_1 = \arg \max_{j \in N} \delta_j^-(\bar{m}_j)\}$, i.e., the set of agents for which $\delta_j^-(\bar{m}_j)$ is the maximum. Let $Z_1 = \{z_1 \in H_1 \mid z_1 = \arg \max_{j \in N} \delta_j(\bar{m}_j)\}$, i.e., the set of agents in H_1 for which $\delta_j(\bar{m}_j)$ is the maximum.

Then, for all $i \notin H_1$ we must have $\delta_i^-(\bar{m}_i) < \delta_{h_1}^-(\bar{m}_{h_1})$ and both Lemma 6 and Lemma 7 (through condition (i)) apply. Hence $\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i)$ for all $i \notin H_1$.

Now consider $i \in H_1$.

1. Case $|H_1| > 1$. Clearly, for all $i \in H_1$, Lemma 6 applies and thus $\delta_i^-(\bar{m}_i) \leq \delta_i(\bar{m}_i)$ for all $i \in H_1$.

(a) Case $|Z_1| > 1$ then all $i \in H_1 \cap \bar{Z}_1$ have $\delta_i^-(\bar{m}_i) < \delta_j(\bar{m}_j)$ for $j \in Z_1$ and all $k, j \in Z_1$ have $\delta_k^-(\bar{m}_k) < \delta_j(\bar{m}_j)$. Thus Lemma 7 (part (ii)) applies for all $i \in H_1 \cap \bar{Z}_1$ and all $i \in Z_1$. Hence $\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i)$ for all $i \in N$.

(b) If $|Z_1| = 1$, then Lemma 7 (part (ii)) applies to all $i \in H_1 \cap \bar{Z}_1$ but not to $z_1 \in Z_1 = \{z_1\}$.

2. Case $|H_1| = 1$. Then it is immediate that $|Z_1| = 1$ as well. Then Lemma 7 (part (ii)) applies to all $i \in H_1 \cap \bar{Z}_1$ but not to $z_1 \in Z_1 = \{z_1\}$.

Then either $|Z_1| > 1$ and $\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i)$ for all $i \in N$ or $|Z_1| = 1$ and $\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i)$ for all $i \neq z_1$. By definition of Z_1 , when $|Z_1| = 1$, $\delta_{z_1}(\bar{m}_{z_1}) > \max_{i \neq z_1} \delta_i(\bar{m}_i) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i)$. As $z_1 \in H_1$ we also have that $\delta_{z_1}^-(\bar{m}_{z_1}) \geq \max_{i \neq z_1} \delta_i(\bar{m}_i) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i)$. Therefore, we must have $\min\{\delta_{z_1}^-(\bar{m}_{z_1}), \delta_{z_1}(\bar{m}_{z_1})\} \geq \max_{i \neq z_1} \delta_i^-(\bar{m}_i) = \max_{i \neq z_1} \delta_i(\bar{m}_i)$.

The only reason we could have an equality in the above inequality is when $\delta_{z_1}^-(\bar{m}_{z_1}) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i)$ as we have shown that $\delta_{z_1}(\bar{m}_{z_1}) > \max_{i \neq z_1} \delta_i^-(\bar{m}_i)$. But if we assume $\delta_{z_1}^-(\bar{m}_{z_1}) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i)$ then it means that Lemma 6 applies and thus we must have $\delta_{z_1}^-(\bar{m}_{z_1}) \leq \delta_{z_1}(\bar{m}_{z_1})$.

Hence either we have $\min\{\delta_{z_1}^-(\bar{m}_{z_1}), \delta_{z_1}(\bar{m}_{z_1})\} > \max_{i \neq z_1} \delta_i^-(\bar{m}_i) = \max_{i \neq z_1} \delta_i(\bar{m}_i)$ or we have $\max_{i \neq z_1} \delta_i(\bar{m}_i) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i) = \delta_{z_1}^-(\bar{m}_{z_1}) \leq \delta_{z_1}(\bar{m}_{z_1})$. ■

Lemma 10 *There exists $\bar{m}_{z_1} > \bar{m}_j$ for all $j \neq z_1$ if and only if either for all $j \neq z_1$, $\min\{\delta_{z_1}^-(\bar{m}_{z_1}), \delta_{z_1}(\bar{m}_{z_1})\} > \delta_j^-(\bar{m}_j) = \delta_j(\bar{m}_j)$ or $\delta_{z_1}(\bar{m}_{z_1}) > \delta_{z_1}^-(\bar{m}_{z_1}) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i) = \max_{i \neq z_1} \delta_i(\bar{m}_i)$.*

Proof. (Only if) Assume $\bar{m}_p > \bar{m}_j$ for all $j \neq p$. First, let us show that this implies that $p \in \arg \max_{j \in N} \delta_j^-(\bar{m}_j)$. Suppose the contrary, that is, $p \notin H_1$. Then by Lemma 6 and 7 we must have $\delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p)$ and thus $\delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p) < \delta_i^-(\bar{m}_i)$ for some $i \in H_1$. From Lemma 8 it must then also be the case that $\delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p) < \delta_i(\bar{m}_i)$ for $i \in H_1$. But then $\min\{\delta_i^-(\bar{m}_i), \delta_i(\bar{m}_i)\} > \delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p)$. For any j , let $\Delta_j(\bar{m}_j) := \frac{\int_{\bar{m}_j}^{\bar{v}} \alpha(v|\lambda) dF(v_j) + \bar{m}_j f(\bar{m}_j) \alpha(\bar{m}_j|\lambda)}{1 - F(\bar{m}_j) + \bar{m}_j f(\bar{m}_j)}$, i.e. the value of $\delta_i(\bar{m}_i)$ when it is continuous at \bar{m}_i . Notice that when $\min\{\delta_i^-(\bar{m}_i), \delta_i(\bar{m}_i)\} = \delta_i^-(\bar{m}_i)$ then we must have $\tau_i \leq \hat{\tau}_i$ and then it follows that $\delta_i^-(\bar{m}_i) \leq \Delta_i(\bar{m}_i)$. Hence we must have $\Delta_i(\bar{m}_i) \geq \delta_i^-(\bar{m}_i) > \Delta_p(\bar{m}_p) = \delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p)$. However, from Lemma ??, $\Delta_j(\bar{m}_j)$ is increasing in \bar{m}_j and thus this would imply that $\bar{m}_i > \bar{m}_p$, a contradiction. Now when $\min\{\delta_i^-(\bar{m}_i), \delta_i(\bar{m}_i)\} = \delta_i(\bar{m}_i)$ we must have $\tau_i \geq \hat{\tau}_i$ and thus $\delta_i(\bar{m}_i) \leq \Delta_i(\bar{m}_i)$. This would then imply $\Delta_i(\bar{m}_i) > \Delta_p(\bar{m}_p)$ and thus $\bar{m}_i > \bar{m}_p$, again a contradiction. Hence, if $\bar{m}_p > \bar{m}_j$, we must have $p \in H_1$.

From now on, we know that $p \in H_1$, that is, $\delta_p^-(\bar{m}_p) \geq \delta_j^-(\bar{m}_j)$ for all $j \in N$. Let us consider the following subcases.

1. Assume $p \notin Z_1$.

(a) If $|Z_1| = 1$ then from Lemma 9, there is a unique $i \in Z_1$ such that $\min\{\delta_i^-(\bar{m}_i), \delta_i(\bar{m}_i)\} \geq \delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p)$. But then, if $\delta_i(\bar{m}_i) \geq \delta_i^-(\bar{m}_i)$ we must have $\tau_i \leq \hat{\tau}_i$, $\Delta_i(\bar{m}_i) \geq \delta_i^-(\bar{m}_i) \geq \delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p) = \Delta_p(\bar{m}_p)$ and thus $\bar{m}_i \geq \bar{m}_p$ which is a contradiction. If otherwise $\delta_i(\bar{m}_i) \leq \delta_i^-(\bar{m}_i)$ then we must have $\Delta_i(\bar{m}_i) \geq \delta_i(\bar{m}_i) \geq \delta_p^-(\bar{m}_p) = \delta_p(\bar{m}_p) = \Delta_p(\bar{m}_p)$. Which also leads to the contradiction that $\bar{m}_i \geq \bar{m}_p$. Then, if $p \notin Z_1$ we cannot have $|Z_1| = 1$.

- (b) If $|Z_1| > 1$ then we have $\delta_j^- = \delta_j$ for all $j \in N$. As $p \in H_1$ and $|Z_1| > 1$ we must have $|H_1| > 1$ and then there exists a $i \in H_1$, $i \neq p$ such that $\delta_p^- = \delta_p = \delta_i^- = \delta_i$. It directly follows that $\Delta_p = \Delta_i$ and thus $\bar{m}_p = \bar{m}_i$, which is a contradiction. Therefore, we must have $p \in Z_1$.

2. Assume now that $p \in Z_1$.

- (a) If $|H_1| = 1$ then $\delta_p^- > \delta_j^-$ for all $j \neq p$. We also have $|Z_1| = 1$ and thus $\delta_p > \delta_j$ for all $j \neq p$. As $|H_1| = 1$, then $H_1 = \{p\}$ and thus for all $j \neq p$, $\delta_j^- = \delta_j$. Hence $\min\{\delta_p^-, \delta_p\} > \delta_j^- = \delta_j$ for all $j \neq p$, which is the first condition of the Lemma.
- (b) If $|H_1| > 1$ and $|Z_1| = 1$. Then $\delta_p > \delta_j$ for all $j \neq p$. As $p \in H_1$ and $|H_1| > 1$, then from Lemma 6 we must have $\delta_p^- \leq \delta_p$. But as $|H_1| > 1$, there must exist a $i \in H_1$, $i \neq p$ such that $\delta_i^- = \delta_p^-$. As $i \in H_1$ but $i \notin Z_1$ then $\delta_i^- = \delta_i$. It follows that we must have $\delta_p > \delta_p^- = \delta_i^- = \delta_i$ which is the second condition of the Lemma.
- (c) If $|H_1| > 1$ and $|Z_1| > 1$. Then we must have $\delta_j^- = \delta_j$ for all $j \in N$. Then take any $i \in Z_1$, $i \neq p$ which exists as $|Z_1| > 1$, it follows that $\delta_p = \delta_p^- = \delta_i = \delta_i^-$. However, this implies that $\Delta_p = \Delta_i$ and thus that $\bar{m}_p = \bar{m}_i$ which is a contradiction.

Then, assuming $\bar{m}_p > \bar{m}_i$ implies that for any $j \neq p$, either $\min\{\delta_p^-, \delta_p\} > \delta_j^- = \delta_j$ or $\delta_p > \delta_p^- = \delta_j^- = \delta_j$ which concludes the proof of the *only if* statement.

(If) For any $j \neq p$ we have $\delta_j^- = \delta_j = \Delta_j$. Assume first that $\min\{\delta_p^-, \delta_p\} > \delta_j^- = \delta_j$ for some $j \neq p$. If $\delta_p \geq \delta_p^-$ then we must have $\tau_p \leq \hat{\tau}_p$ and thus $\Delta_p \geq \delta_p^- > \delta_j^- = \delta_j = \Delta_j$. This implies that $\bar{m}_p > \bar{m}_j$. Same logic applies when $\delta_p \leq \delta_p^-$ as this implies that $\tau_p \geq \hat{\tau}_p$, $\Delta_p \geq \delta_p > \delta_j^- = \delta_j = \Delta_j$ and thus $\bar{m}_p > \bar{m}_j$. Assume now that for some $j \neq p$, $\delta_p > \delta_p^- = \delta_j^- = \delta_j$. Then, $\tau_p < \hat{\tau}_p$ and $\Delta_p > \delta_p^- = \delta_j^- = \delta_j = \Delta_j$, implying $\bar{m}_p > \bar{m}_j$. ■

Lemma 11 *Let $z_1 \in \arg \max_{j \in N} \delta_j^-(\bar{m}_j)$, for any $j \neq z_1$ it is not possible that $\min\{\delta_{z_1}^-(\bar{m}_{z_1}), \delta_{z_1}(\bar{m}_{z_1})\} > \delta_j^-(\bar{m}_j) = \delta_j(\bar{m}_j)$. Therefore, only the case $\delta_{z_1}(\bar{m}_{z_1}) > \delta_{z_1}^-(\bar{m}_{z_1}) = \max_{i \neq z_1} \delta_i^-(\bar{m}_i) = \max_{i \neq z_1} \delta_i(\bar{m}_i)$ is possible.*

Proof. Assume that there is an \bar{m}_{z_1} such that $\min\{\delta_{z_1}^-(\bar{m}_{z_1}), \delta_{z_1}(\bar{m}_{z_1})\} > \delta_j^-(\bar{m}_j) = \delta_j(\bar{m}_j)$ for all $j \neq z_1$. Notice that for any $k \in N$, $\delta_k(\underline{v}) = \underline{v}$ and $\min\{\delta_{z_1}^-(\bar{m}_{z_1}), \delta_{z_1}(\bar{m}_{z_1})\} > \bar{v}$. So, there must exist a $\hat{m}_{z_1} < \bar{m}_{z_1}$ such that for all $v_{z_1} \in [\hat{m}_{z_1}, \bar{m}_{z_1})$ we have $\max_{i \neq z_1} \delta_i(\bar{m}_i) < \delta_{z_1}(v_{z_1}) \leq \delta_{z_1}(\bar{m}_{z_1})$. But then, for all those $v_{z_1} \in [\hat{m}_{z_1}, \bar{m}_{z_1})$, we have $\delta_{z_1}(v_{z_1}) > \max_{i \neq z_1} \max_{v_i \in V} \delta_i(v_i)$ and thus $S_{z_1}(v_{z_1}) = 1$ for all $v_{z_1} \in [\hat{m}_{z_1}, \bar{m}_{z_1})$. It follows that $T_{z_1}(v_{z_1}) = T_{z_1}(\bar{v})$. But then, this means that \bar{m}_{z_1} is not the smallest value in V before transfers become constant due to cash-constraints. Therefore, there must exist a \hat{m}_{z_1} such that $\delta_{z_1}^-(\hat{m}_{z_1}) = \max_{i \neq z_1} \delta_i(\bar{m}_i)$. ■

Proof of Theorem 7. Assume that for a given $x^* = (x_1^*, \dots, x_n^*)$, the vector $\Theta^* := (s^*, U^*, \bar{m}^*, \tau^*, \chi^*)$ solves (A) and satisfies (B). Then, from (A) we must have that $s_i^*(v)$ writes

$$s_i^*(v_i, v_{-i}) = \begin{cases} 1 & \text{if } \delta_i(v_i | x_i^*, \bar{m}_i^*, \lambda) > \max_{j \neq i} \delta_j(v_j | x_j^*, \bar{m}_j^*, \lambda) \\ \text{something in } [0, 1] & \text{if } \delta_i(v_i | x_i^*, \bar{m}_i^*, \lambda) = \max_{j \neq i} \delta_j(v_j | x_j^*, \bar{m}_j^*, \lambda) \\ 0 & \text{if } \delta_i(v_i | x_i^*, \bar{m}_i^*, \lambda) < \max_{j \neq i} \delta_j(v_j | x_j^*, \bar{m}_j^*, \lambda) \end{cases}$$

As shown previously, \bar{m}^* is such that for all $i \in \mathbb{N}$, $\delta_i(v_i | x_i^*, \bar{m}_i^*, \lambda)$ is nondecreasing w.r.t. v_i . It follows that $S_i(v_i)$ is also nondecreasing in v_i . From (B), $V^*(S_i^*) = [x_i^*, y_i^*]$ for all $i \in \mathbb{N}$ and thus s_i^* satisfies all constraints of the original problem.

Notice also that for any $v_i^* \in V^*(S_i^*) = [x_i^*, y_i^*]$

$$\begin{aligned} \mathbb{E} \left[\sum_{i \in \mathbb{N}} (S_i^*(v_i) - r_i) \delta_i(v_i | x_i^*, \bar{m}_i^*, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \chi_i^* - \tau_i^*) U_i(x_i^*) + \sum_{i \in \mathbb{N}} \tau_i^* l_i &= \\ \mathbb{E} \left[\sum_{i \in \mathbb{N}} (S_i^*(v_i) - r_i) \Gamma_i(v_i | v_i^*, \bar{m}_i^*, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \chi_i^* - \tau_i^*) U_i(v_i^*) + \sum_{i \in \mathbb{N}} \tau_i^* l_i. & \quad (1.33) \end{aligned}$$

This equality stems from two facts.

(i) We have that $\delta_i(v_i | x_i^*, \bar{m}_i^*, \lambda) = \Gamma_i(v_i | v_i^*, \bar{m}_i^*, \lambda)$ for all $v_i \notin [x_i^*, y_i^*]$ and $v_i^* \in [x_i^*, y_i^*]$ by definition of $\delta_i(\cdot)$ so that the expectation is the same on both sides for $v_i \notin [x_i^*, y_i^*]$. For $v_i \in [x_i^*, y_i^*]$, δ_i and Γ_i differ, but at the same time we have that $S_i^*(v_i) = r_i$ as s^* satisfies (B). Thus the expectation is the same on both sides.

(ii) As $V^*(S_i^*) = [x_i^*, y_i^*]$ it is clear that $U_i(v_i^*) = U_i(x_i^*)$.

Now, by definition of Θ^* , we must have that for all $\hat{\Theta} := (\hat{s}, \hat{U}, \hat{m}, \hat{\tau}, \hat{\chi})$

$$\begin{aligned} \mathbb{E} \left[\sum_{i \in \mathbb{N}} (S_i^*(v_i) - r_i) \delta_i(v_i | x_i^*, \bar{m}_i^*, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \chi_i^* - \tau_i^*) U_i(x_i^*) + \sum_{i \in \mathbb{N}} \tau_i^* l_i &\geq \\ \mathbb{E} \left[\sum_{i \in \mathbb{N}} (\hat{S}_i(v_i) - r_i) \delta_i(v_i | x_i^*, \hat{m}_i, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \hat{\chi}_i - \hat{\tau}_i) \hat{U}_i + \sum_{i \in \mathbb{N}} \hat{\tau}_i l_i. & \quad (1.34) \end{aligned}$$

At the same time we also have that for any $\hat{v}_i \in V^*(\hat{S}_i)$

$$\begin{aligned} \mathbb{E} \left[\sum_{i \in \mathbb{N}} (\hat{S}_i(v_i) - r_i) \delta_i(v_i | x_i^*, \hat{m}_i, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \hat{\chi}_i - \hat{\tau}_i) \hat{U}_i + \sum_{i \in \mathbb{N}} \hat{\tau}_i l_i &\geq \\ \mathbb{E} \left[\sum_{i \in \mathbb{N}} (\hat{S}_i(v_i) - r_i) \Gamma_i(v_i | \hat{v}_i, \hat{m}_i, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \hat{\chi}_i - \hat{\tau}_i) \hat{U}_i + \sum_{i \in \mathbb{N}} \hat{\tau}_i l_i, & \quad (1.35) \end{aligned}$$

as for all $v_i < \hat{v}_i$ we have $\delta_i(v_i | x_i^*, \hat{m}_i, \lambda) \leq \Gamma_i(v_i | \hat{v}_i, \hat{m}_i, \lambda)$ and $\hat{S}_i(v_i) - r_i \leq 0$ and for all $v_i > \hat{v}_i$ we have $\delta_i(v_i | x_i^*, \hat{m}_i, \lambda) \geq \Gamma_i(v_i | \hat{v}_i, \hat{m}_i, \lambda)$ and $\hat{S}_i(v_i) - r_i \geq 0$.²⁸ Hence, for all $v_i \in V$ we have $(\hat{S}_i(v_i) - r_i) \delta_i(v_i | x_i^*, \hat{m}_i, \lambda) \geq (\hat{S}_i(v_i) - r_i) \Gamma_i(v_i | \hat{v}_i, \hat{m}_i, \lambda)$.

²⁸ $\hat{S}_i(v_i) - r_i \leq 0$ (≥ 0) for all $v_i < \hat{v}_i$ ($v_i > \hat{v}_i$) directly stems from the fact that $\hat{v}_i \in V^*(\hat{S}_i)$.

But then combining equation (1.33), (1.34) and (1.35) we get that for all $\hat{\theta}$ and $\hat{v}_i \in V^*(\hat{S}_i)$

$$\begin{aligned} \mathbb{E} \left[\sum_{i \in \mathbb{N}} (S_i^*(v_i) - r_i) \Gamma_i(v_i | v_i^*, \bar{m}_i^*, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \chi_i^* - \tau_i^*) U_i(v_i^*) + \sum_{i \in \mathbb{N}} \tau_i^* l_i &\geq \\ \mathbb{E} \left[\sum_{i \in \mathbb{N}} (\hat{S}_i(v_i) - r_i) \Gamma_i(v_i | \hat{v}_i, \hat{m}_i, \lambda) \right] - \sum_{i \in \mathbb{N}} (\lambda - \hat{\chi}_i - \hat{\tau}_i) \hat{U}_i + \sum_{i \in \mathbb{N}} \hat{\tau}_i l_i, &\quad (1.36) \end{aligned}$$

which directly means that θ^* is also the maximum of the original problem. \blacksquare

Proof of Corollary 3. Assume first that τ_i is defined by equation (1.24). It can easily be rewritten as

$$\hat{\tau}_i = \frac{f(\bar{m}_i) \int_{\bar{m}_i}^{\bar{v}} \alpha(v_i | \lambda) dF(v_i) - \alpha(\bar{m}_i | \lambda) (1 - F(\bar{m}_i)) f(\bar{m}_i)}{1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)}.$$

Differentiating w.r.t. to \bar{m}_i gives the following condition for the numerator (after factorization by $(1 - F(\bar{m}_i))$ which is positive)

$$\begin{aligned} \left(f'(\bar{m}_i) \mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \alpha'(\bar{m}_i | \lambda) f(\bar{m}_i) - \alpha(\bar{m}_i | \lambda) f'(\bar{m}_i) \right) \left(1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i) \right) \\ - \bar{m}_i f'(\bar{m}_i) \left(\mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \alpha(\bar{m}_i | \lambda) \right) f(\bar{m}_i) \leq 0, \end{aligned}$$

which reduces to

$$\begin{aligned} - \alpha'(\bar{m}_i | \lambda) f(\bar{m}_i) \left(1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i) \right) \\ + f'(\bar{m}_i) (1 - F(\bar{m}_i)) \left[\mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \alpha(\bar{m}_i | \lambda) \right] \leq 0. \end{aligned}$$

The above inequality holds as α' is positive, f is nonincreasing and $\mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \alpha(\bar{m}_i | \lambda) \geq 0$. Then, τ_i is decreasing in \bar{m}_i .

Now assume instead that τ_{z_1} is defined by equation (1.25). For a given \bar{m}_{-z_1} the right-hand side of (1.25) is constant. Fix τ_{z_1} and assume \bar{m}_{z_1} increases. Then $\alpha(\bar{m}_{z_1} | \lambda) + \frac{\tau_{z_1}}{f(\bar{m}_{z_1})}$ increases as $\alpha(v_i | \lambda)$ is increasing in v_i and f is nonincreasing. Hence, to maintain the equality defined by equation (1.25) it is necessary that τ_{z_1} decreases. \blacksquare

Proof of Corollary 4. Assume that for $i \in N$, $\delta_i^-(\bar{m}_i) = \delta_i(\bar{m}_i)$. This implies that τ_i is defined by equation (1.24). Therefore,

$$\begin{aligned} \delta_i(\bar{m}_i) &= \mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \frac{\tau_i \bar{m}_i}{1 - F(\bar{m}_i)} \\ &= \mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \bar{m}_i \frac{(\mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] - \alpha(\bar{m}_i | \lambda)) f(\bar{m}_i)}{1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)} \\ &= \frac{(1 - F(\bar{m}_i)) \mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] + \bar{m}_i f(\bar{m}_i) \alpha(\bar{m}_i | \lambda)}{1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)} \\ &= \frac{\int_{\bar{m}_i}^{\bar{v}} \alpha(v_i | \lambda) dF(v_i) + \bar{m}_i f(\bar{m}_i) \alpha(\bar{m}_i | \lambda)}{1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)}. \end{aligned}$$

Differentiating this expression w.r.t. \bar{m}_i gives the following numerator

$$\begin{aligned} &\bar{m}_i f(\bar{m}_i) \alpha'(\bar{m}_i | \lambda) [1 - F(\bar{m}_i) + \bar{m}_i f(\bar{m}_i)] + \\ &(1 - F(\bar{m}_i)) \bar{m}_i f'(\bar{m}_i) \left[\alpha(\bar{m}_i | \lambda) - \frac{1}{1 - F(\bar{m}_i)} \int_{\bar{m}_i}^{\bar{v}} \alpha(v_i | \lambda) dF(v_i) \right]. \end{aligned} \quad (1.37)$$

Given the assumption that f is nonincreasing, that $\alpha(\cdot)$ is increasing and that $\alpha(\bar{m}_i | \lambda) - \frac{1}{1 - F(\bar{m}_i)} \int_{\bar{m}_i}^{\bar{v}} \alpha(v_i | \lambda) dF(v_i) = \alpha(\bar{m}_i | \lambda) - \mathbb{E}[\alpha(v_i | \lambda) | v_i \geq \bar{m}_i] \leq 0$ then (1.37) is positive and thus $\delta_i(\bar{m}_i)$ is increasing in \bar{m}_i whenever $\delta_i(\bar{m}_i)$ is continuous at \bar{m}_i . ■

Proof of Corollary 5. (i) Take any $\bar{m} \in \times_{i \in N} (y_i, \bar{v}]$. Then for all $\bar{m}_i \leq \bar{m}_j$ for some j , τ_i is uniquely defined by (1.24). If it exists, $\bar{m}_{z_1} > \max_{i \neq z_1} \bar{m}_i$ defines τ_{z_1} from equation (1.25).

(ii) Now take any τ . Notice first that for all $i \in N$, \bar{m}_i is decreasing in τ_i . Indeed, for any \bar{m}_i satisfying (1.24), the RHS is decreasing in \bar{m}_i . Now for \bar{m}_{z_1} satisfying (1.25), it is clear that the RHS does not depend on τ_{z_1} nor \bar{m}_{z_1} . Then, if τ_{z_1} increases, the only way to satisfy the equality is that \bar{m}_{z_1} decreases as $\alpha(v_{z_1} | \lambda) + \frac{\tau_{z_1}}{f(v_{z_1})}$ is increasing in v_{z_1} .

Then, for any i, j such that $\tau_i \geq \tau_j$ we must have $\bar{m}_i \leq \bar{m}_j$. It follows that \bar{m}_i is uniquely defined by (1.24) as the RHS is decreasing in \bar{m}_i . Finally, if there exists a $\tau_{z_1} < \min_{i \neq z_1} \tau_i$ then we must have $\max_{i \neq z_1} \bar{m}_i < \bar{m}_{z_1}$ and \bar{m}_{z_1} solves (1.25). The RHS of (1.25) depends only on $\min_{i \neq z_1} \tau_i$ and thus there exists a unique \bar{m}_{z_1} that solves (1.25) for a given τ_{z_1} given that $\alpha(v_{z_1} | \lambda) + \frac{\tau_{z_1}}{f(v_{z_1})}$ is increasing in v_{z_1} . ■

2. OPTIMAL STRUCTURE OF PENALTIES WITH JUDGMENT-PROOF INJURERS ¹

Abstract

I characterize the optimal regulation of a firm constituted by potential judgment-proof agents. I investigate two cases: (i) A principal hires an agent to undertake a prevention effort on their behalf; (ii) Two agents are jointly responsible of undertaking a prevention effort. In both cases, agents are in charge of exerting an unobservable level of safety care to reduce the probability of an accident that may occur due to the firm risky activity. Agents are called judgment proof when their final wealth is not enough to pay for the monetary penalties imposed by the regulator. I show that the standard Equivalence Theorem, stating that the distribution of penalties among injurers is irrelevant, does not hold in this context. Instead, in a principal-agent firm, the optimal regulation requires to fully target the principal if the agent can be subject to judgment proofness. In a two-agent firm, the optimal regulation consists in an almost equal sharing of penalties among agents.

Keywords: *Moral Hazard, Regulation, Limited Liability, Judgment Proofness.*

JEL Classification: K13, K32, G33, D86.

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2.1. INTRODUCTION

When a firm's activity may cause an accident to outside parties, how should be designed penalties to induce the firm to undertake enough safety measures, that is, what should be the optimal total amount of penalties and how should it be apportioned among the firm's members?

The earlier works of [Newman and Wright \(1990\)](#) and [Segerson and Tietenberg \(1992\)](#) suggest that only the total amount of penalties matters and not their allocation within the firm. The main argument relies on the existence of private transactions within the firm that can undo any allocation of responsibilities coming from the regulation authority. This result, known as the Equivalence Principle, had a significant influence on many works in the economic literature of tort and environmental law.

This result relies on the strong assumption that the private contract between injurers always satisfies each party's solvency towards the potential payment of fines. However, when injurers may be judgment proof (see [Shavell \(1986\)](#)), that is, financially insolvent when facing the penalties imposed by the regulation authority, it may be optimal not to provide them with additional resources to limit corporate liability. For instance, [Ringleb and Wiggins \(1990\)](#), empirically find that large firms prefer to buy inputs whose manufacture is risky to small firms with few assets. Others establish special subsidiaries or product manufacturing by contract with dedicated small-scale specialized producers. Those choices are deliberately made so as to shield the large firm's assets from potential liability in case of accident. When considering situations of long-term or large-scale hazards such as environmental degradation (hazardous waste, water pollution) or workers exposition to harmful substances (asbestos, radiation, vinyl chloride) the amount of damages are likely to be large and therefore exceed some of the injurers' financial resources if no (or small) corporate compensation exists.

In this paper, I consider a situation in which the Equivalence Principle breaks down. Under the assumption that some injurers may be judgment-proof and that the private contract between them does not guarantee solvency towards the payment of fines, the allocative role of penalties is restored. I consider a firm whose activity is risky and may cause an harm to third parties. To reduce the probability of accident, the firm has to exert

costly precautionary care. I then investigate how should penalties be optimally allocated among injurers in the cases of simple and double moral hazard.

First, I consider a principal-agent firm in which the principal hires and agent to undertake an effort of prevention. The agent's effort is unobservable and the problem is a moral hazard one. Therefore, parties can contract only on outcome realizations (accident or no accident). At the contracting stage, I assume that the agent's financial resources are a random variable so that parties are unsure about the agent's ability to make monetary transfers when the outcome is realized. In the absence of regulation, the principal has no incentive to induce the agent to exert effort as the harm does not affect her. Thus, I introduce a regulation authority who can impose *ex post* penalties on the principal and on the agent when an accident occurs. Obviously, as the agent has limited resources, he may not be able to pay for the penalties imposed on him.

In the literature, it is usually assumed that transfers from the agent to both the regulation authority and the principal are designed *ex ante* such that they never exceed the agent's resources *ex post*. I depart from this modeling by allowing unbounded transfers in the first place, which will be truncated *ex post* in case of insolvency of the agent (or equivalently "judgment-proofness"). Indeed, if one assumes that the private transaction between the principal and the agent is generally not observed by the regulation authority, then there is no reason for the principal to ensure the agent's ability to pay for the penalties. This assumption is also supported by empirical evidence ([Ringleb and Wiggins \(1990\)](#)) that shows that large firms intentionally choose to leave risky manufacturing to small firms with few assets.

For a given regulation policy, I derive the equilibrium contract between the principal and the agent when the former has all the bargaining power. I find that the equilibrium effort level of prevention decreases as the share of penalties imposed on the agent increases. This results stems from the fact that imposing a larger share of penalties on an *ex post* potentially insolvent agent acts as an *ex ante* decrease in the total amount of penalties imposed on the principal-agent relationship. It follows that if the regulation authority wants the firm to exert an efficient precautionary care level, it must impose the penalties on the principal only. Targeting the agent is always detrimental to the provision of effort of prevention.

I also investigate the design of the optimal regulation when the bargaining power varies inside the firm. I show that when the principal has most of the bargaining power, it is still optimal to impose all penalties on her. However, when the agent has most of the bargaining power, the previous optimal regulation may lead to an excess of precautionary care (with respect to the first-best level). In that case, the regulation must be such that the total expected fine paid by the firm decreases. Finally, I investigate whether we should authorize the principal to give rewards to the agent in case of accident. It appears that if the agent has most of the bargaining power, allowing the principal to reward the agent in case of accident leads to the first-best level of precautionary care.

Second, I investigate a situation in which two agents are responsible for exerting an unobservable effort of prevention and may both be potentially insolvable *ex post*. The regulation authority now faces a multiple tortfeasors problem where each injurer can be judgment-proof. The contracting problem between the two agents is now described as a double moral hazard problem in a partnership: In the first stage, agents sign a binding agreement to maximize their joint profits and, in the second stage they simultaneously choose their effort levels (similar to [Cooper and Ross \(1985\)](#)). In this problem, the sharing of profits among agents not only plays the traditional role of incentive provision in the partnership (due to moral hazard) but also the role of revenue concealment from the regulation authority.

When agents have symmetric initial resources distributions, the solution to this problem shows that agents sign a contract such that the profits in case of accident go to the agent who is the least targeted by the regulation policy. This allows the partnership to escape as much as possible from the penalties and thus provide very low effort provision. This result resembles firms' strategies to create insolvent subsidiaries to escape from paying fines.

The optimal regulation policy of the partnership consists in an equal-sharing of the penalties among the two agents. Any other allocation of penalties results in a decrease in effort provision. Notice that when agents have asymmetric initial resources distribution, the optimal allocation of penalties is centered around the equal-sharing allocation while being adjusted to target more the agent with higher average initial resources.

The paper is organized as follows. In Section 2, I present the model and the standard framework. In section 3, I develop the principal-agent problem with a judgment-proof agent. Section 4 investigates the double moral hazard problem with two judgment-proof agents. Section 6 concludes.

2.2. THE STANDARD FRAMEWORK

A firm undertakes a project that may cause an accident harming third parties. The owner of the firm (the principal) hires a worker (the agent) to run the firm on her behalf. Both of them are assumed to be risk-neutral. The firm's activity generates a certain surplus $\Pi \geq 0$ that accrues to the principal but also causes an environmental harm D with probability $1 - e$. The agent is responsible for preventing the harm by exerting effort $e \in [0, 1]$ at personal cost $\psi(e)$. For the problem to be well-behaved, I assume $\psi', \psi'', \psi''' > 0$, $\psi'(0) = 0$ and $\psi'(1) = +\infty$ so that effort solution is always interior.² The agent's effort is unobservable to both the principal and the regulator.

REGULATION. It is assumed that only third parties suffer from the harm D , leaving the firm with no natural incentives to prevent the accident. Therefore, there is room for a regulation authority (the regulator) to act in favor of third parties. Throughout the paper I assume that only ex post regulation is available to the regulator, that is, the regulator can impose fines on the firm's parties only after an harm occurred. A regulatory policy is a couple $(\alpha, F) \in [0, 1] \times [0, D]$, where F is the total amount of the fine imposed on the firm and α (resp. $1 - \alpha$) is the share of the fine charged on the agent (resp. principal).³ I also assume that the regulator's objective is to make the probability of accident as close as possible to its first-best level.⁴ This last assumption, in addition to greatly simplify the analysis, allows me to distinguish the judgment-proof problem from other considerations.

PRIVATE CONTRACT. Due to the moral hazard problem, the principal cannot directly offer a contract contingent on the effort level exerted by the agent. Instead, she offers transfers to the agent conditional on the occurrence of an accident. Let us denote by $t_N \in \mathbb{R}$

²See for instance Chapter 5 of [Laffont and Martimort \(2002\)](#).

³I assume that the total fine cannot exceed total harm D caused by the firm as it is usually the case in the literature.

⁴This differs from the literature in which the regulator takes into account both the harm to the third parties and the profit of the firm.

and $t_A \in \mathbb{R}$ those transfers where the subscripts N and A stand for “no accident” and “accident”, respectively. Typically, the principal will offer $t_N \geq 0$ and $t_A \leq 0$ so that t_N and t_A act as a reward and a punishment, respectively. Therefore, the principal has the ability to punish the agent by the means of private transactions as well as the regulator has the ability to punish the agent (and the principal) by the means of the regulation policy.

THE STANDARD MODEL WITH LIMITED LIABILITY. To see how the results depart from the case with a judgment-proof agent it is useful to briefly expose the standard analysis.

For this section only, I will assume the standard limited liability constraints on transfers, that is, $t_N \geq -l$ and $t_A \geq -l + \alpha F$ where $l \in \mathbb{R}_+$ is the agent’s cash/financial resources. Those ex post constraints ensure that the agent is always endowed with enough resources to honor both the private transfer (t_A) and the regulatory transfer (αF). The limited liability constraint in case of accident $t_A \geq -l + \alpha F$ implies, *de facto*, that the principal provides the agent with the necessary resources in the private transaction. As shown below, this requirement trivially implies that the regulator’s choice of distribution of penalties is irrelevant.

Notice that the amount of resources l may not necessarily represent the full range of the agent’s resources but may represent a reasonable lower bound on the “collectible assets” that can easily be observable and seized when an accident occurs. I will therefore abstract from the situation in which the agent can engage in strategies to hide the real value of his assets.⁵

The timing of the game is as follows. First the regulator publicly commits to a regulatory policy (α, F) . Second the principal offers a contract (t_N, t_A) to the agent. Third, the agent chooses his effort level e .

For a given regulation policy (α, F) and private contract (t_N, t_A) the agent’s expected utility is given by

$$U^B = et_N + (1 - e)[t_A - \alpha F] - \psi(e),$$

⁵Hiriart and Martimort (2006) follow the same approach.

and the expected profit of the principal writes

$$V^B = \Pi - et_N - (1 - e)[t_A + (1 - \alpha)F].$$

To induce a particular level of effort e , the principal must choose (t_N, t_A) such that the agent has the proper incentives to do so. Relying on the *first-order approach*, I replace the set of the agent's incentive constraints by the first-order condition of his utility maximization problem with respect to effort:

$$t_N - t_A + \alpha F = \psi'(e), \quad (2.1)$$

which is a necessary condition when the effort level is interior.⁶ This equation reveals that the agent's incentives to exert effort depends on monetary incentives from both private transactions and regulation policy.

The principal must also ensure the participation of the agent ($U^B \geq 0$) and accounts for ex post limited liability. Her maximization problem then writes

$$\max_{\{e, t_N, t_A\}} V^B = \Pi - [et_N + (1 - e)t_A] - (1 - e)(1 - \alpha)F,$$

subject to equation (2.1), $U^B \geq 0$, $t_N \geq -l$ and $t_A \geq -l + \alpha F$ for all $l \in [0, \bar{l}]$.

Examining equation (2.1) immediately reveals that the ex post limited liability constraint on t_N is always satisfied when the one on t_A holds. It is therefore possible to drop condition $t_N \geq -l$ from the principal's problem.

Using (2.1), the agent's expected utility becomes

$$U^B = e\psi'(e) - \psi(e) + t_A - \alpha F = R(e) + t_A - \alpha F, \quad (2.2)$$

where $R(e) := e\psi'(e) - \psi(e)$ is nonnegative, increasing and convex in e . Using (2.1) and (2.2) it is useful to rewrite the principal's objective in terms in the plane (e, U^B) as follows

$$V^B = \Pi - \psi(e) - (1 - e)F - U^B. \quad (2.3)$$

⁶From the initial assumption it also a sufficient condition for the agent's problem. Indeed, from convexity of ψ the agent's maximization problem is concave in e . Moreover, ψ' is an increasing function of e and (2.1) defines a unique solution for any given t_N , t_A and αF .

This formulation clearly shows that the principal's objective takes into account the total amount of the fine and must also leave some rent to the agent. The problem of the principal rewrites

$$\begin{aligned} \max_{\{e, U^B\}} \quad & \Pi - \psi(e) - (1 - e)F - U^B \\ \text{s.t.} \quad & U^B \geq 0 \\ & U^B \geq R(e) - l, \end{aligned}$$

where the first constraint is the agent's participation constraints and the second one the limited liability constraints in the plane (e, U^B) .

From linearity of U^B it is clear that at least one constraint must bind at the optimum. Let us focus on the case where only the limited liability constraint binds, $U^B = R(e) - l$.⁷ Substituting the binding constraints in the principal's objective gives

$$\max_e \quad \Pi - e\psi'(e) - (1 - e)F.$$

Notice that the principal's maximization problem is independent of α and that the principal takes into account the entire value of the fine F . This leads us to the following statement.

Proposition 12 (Equivalence Principle ⁸) *In a principal-agent relationship with moral hazard, when the principal has to ensure the agent's ability to pay the regulator in any states of the world, the way the regulator allocates responsibilities within the firm does not affect the equilibrium level of effort to prevent the harm to occur.*

This result is not new and generally known as the Equivalence principle. It states that, even in the presence of moral hazard and agent's limited liability, the regulator cannot increase the level of effort by targeting more the principal or the agent.

The equilibrium level of effort, e^B solves the following first-order condition:

$$\psi'(e^B) + e^B\psi''(e^B) = F. \quad (2.4)$$

⁷The case where $U^B = 0$ at optimum implies that the level of effort attains the first best, *i.e.* $\psi'(e^{FB}) = F$. This occurs when the level of liability l is large enough, namely, when $l \geq R(e^{FB})$. This case presents no interest in the analysis as the moral hazard problem entails no distortion. Therefore, assume $l < R(e^{FB})$.

As the left-hand side of the equation is increasing in e , the optimal regulation policy sets $F = D$ according to the *maximum punishment principle*. The α can take any value in $[0, 1]$ without changing the optimal level of effort nor the distribution of revenues between the principal and the agent.

2.3. JUDGMENT-PROOF AGENT

In this section, I develop the problem of designing a regulatory policy (α, F) for a judgment-proof agent, that is, when he may be unable to pay his share of the fine for some levels of wealth. I argue that the formal treatment of limited liability constraints in the regulation literature is generally too restrictive as they require the principal to fully ensure the agent's ability to pay his share of the penalty.

In the standard treatment of moral hazard problem, limited liability constraints on transfers formally put a limit on the monetary punishment the agent may have to pay to the principal. When the relationship generates only private benefits and no negative externality (such as an accident), the limited liability constraints only applies on the transfers between the principal and the agent and do not involve a third party such as a regulator. In that case, those constraints reasonably assume that the principal must offer a contract whose transfers do not exceed some lower bound.

In the presence of a regulator, however, the assumption of ex post limited liability means that the principal must ensure the agent's ability to pay both private transfers and potential fines to the regulator. This would require (i) that the principal is legally bound to ensure agent's liability in any scenario and (ii) sufficient observability of the private contract that takes place within the firm. When at least one of these conditions fails it is unlikely that the principal will ensure agent's liability towards the regulator. Indeed, assume that the principal does not provide the agent with enough cash to pay for the fine for some realizations of the agent's wealth. When the accident occurs, the agent may simply not be able to pay for the entire fine and will pay at most with his disposable cash. The fact that the agent cannot pay for punishments coming from the regulator does not hurt the principal. On the contrary, the agent's inability to pay the fine to the regulator reduces the total amount of the firm paid by the firm making the principal better off.

To model a judgment-proof agent, I do not assume ex post limited liability constraints as in the previous section. In other words, the private contract offered by the principal does not need to provide the agent with some minimal level of resources in case of accident. Instead, the principal can impose unlimited punishment on the agent. Obviously, as the agent is still resource-constrained, he will never pay above his level of resources and his transfer will be naturally bounded below. It is then necessary to modify the agent's expected utility accordingly.

AGENT'S PAYOFF. Assume that an accident occurs. For a given level of resources $l \in \mathbb{R}_+$, the agent now faces the private transfer $t_A < 0$ to the principal and the regulatory transfer αF to the regulator. If $t_A - \alpha F$ is larger than the agent's resource $-l$, the agent can fully pay the principal and the regulator. On the contrary, if $t_A - \alpha F < -l$ then the agent has not enough financial resources to cover both transfers. In this section, I assume that the regulator has the ability to collect money before the principal so that when $t_A - \alpha F < -l$, the agent pays the regulator first and gives the remaining resources, if any, to the principal. Let $\tilde{m} := \tilde{c}_P(t_A, \alpha, F; l) - \tilde{c}_R(\alpha, F; l)$ denote the transfer the agent receives when an accident occurs where $\tilde{c}_P(\cdot)$ and $\tilde{c}_R(\cdot)$ denote the agent's transfers to the principal and the regulator, respectively. Formally,

$$\begin{aligned}\tilde{c}_P(t_A, \alpha, F; l) &= \mathbb{1}_{\{l \geq \alpha F - t_A\}} t_A - \mathbb{1}_{\{l \in [\alpha F, \alpha F - t_A]\}} (l - \alpha F), \\ \tilde{c}_R(\alpha, F; l) &= \mathbb{1}_{\{l \geq \alpha F\}} \alpha F + \mathbb{1}_{\{l \leq \alpha F\}} l.\end{aligned}$$

Notice that the transfer to the regulator, $\tilde{c}_R(\cdot)$ is independent of t_A . This stems from the fact that the regulator has priority over the agent's wealth so that the principal cannot reduce the agent's payment to the regulator by punishing him more through t_A .

In order to greatly simplify the mathematical reasoning I now make the following assumption.

Assumption 5 *The agent's wealth l is a random variable drawn from an absolutely continuous cumulative distribution function $H(l)$ over the support $[0, \bar{l}]$ where $\bar{l} > D$ so that the maximal agent's wealth exceeds the monetary damage caused by an accident. The realization of the agent's wealth is known to all players only at the end of the game.*

As all players now view l as a random variable, they will evaluate their payoff by taking expectations of $\tilde{c}_P(\cdot)$ and $\tilde{c}_R(\cdot)$ over l . Define $c_P(t_A, \alpha, F) := \mathbb{E}_l \tilde{c}_P(t_A, \alpha, F; l)$ and $c_R(\alpha, F) := \mathbb{E}_l \tilde{c}_R(\alpha, F; l)$. This makes the analysis easier as now $c_P(t_A, \alpha, F)$ and $c_R(\alpha, F)$ are differentiable in each argument. No particular conditions are imposed on $H(\cdot)$ so that the randomness assumption of l is quite mild and could simply represent that there is always a small uncertainty about the agent's wealth when the regulator decides to enforce penalties. Furthermore, I show below that the framework with a judgment-proof agent and random wealth can be equivalently rewritten in a model with certain wealth that resembles the model presented in section 3.2.

Formally, the total expected transfer of the agent in case of accident is given by

$$m(t_A, \alpha, F) = \int_{\alpha F - t_A}^{\bar{l}} (t_A - \alpha F) dH(l) + \int_0^{\alpha F - t_A} (-l) dH(l). \quad (2.5)$$

Taking partial derivatives of $m(t_A, \alpha, F)$ with respect to t_A and α gives:

$$\begin{aligned} \frac{\partial m(t_A, \alpha, F)}{\partial t_A} &= \int_{\alpha F - t_A}^{\bar{l}} dH(l) \geq 0, \\ \frac{\partial m(t_A, \alpha, F)}{\partial \alpha} &= - \int_{\alpha F - t_A}^{\bar{l}} F dH(l) \leq 0. \end{aligned}$$

Thus, the transfer of the agent is nondecreasing in t_A and nonincreasing in α . More precisely, $m(t_A, \alpha)$ increases in t_A when t_A is high enough, but as soon as t_A becomes too low the agent's payment becomes flat and equal to $-l$. The same thing happens with α : the agent's transfer is decreasing in α as long as α is not too high and then becomes flat when the regulator asks too much money.

It is now possible to write the agent's expected utility as

$$U = et_N + (1 - e) [c_P(t_A, \alpha, F) - c_R(\alpha, F)] - \psi(e),$$

where the only difference with the standard framework is that t_A is replaced by $c_P(t_A, \alpha, F) - c_R(\alpha, F)$. For a given private contract (t_N, t_A) the induced level of effort e is given by

$$t_N - [c_P(t_A, \alpha, F) - c_R(\alpha, F)] = \psi'(e). \quad (2.6)$$

Therefore, when the principal offers t_A in case of accident, the agent now considers $c_P(t_A, \alpha, F) - c_R(\alpha, F)$ rather than directly t_A . Using (2.6), the agent's expected utility becomes

$$U = R(e) + c_P(t_A, \alpha, F) - c_R(\alpha, F) \quad (2.7)$$

where $R(e) = e\psi'(e) - \psi(e) \geq 0$ is defined as in section 3.2. Equation (2.7) shows that when the principal wants to induce level of effort e , she has to give a positive rent $R(e)$ to the agent. Then the principal can extract this rent through the side-payment $c_P(t_A, \alpha, F) - c_R(\alpha, F)$. Notice that for any $t_A \leq \alpha F - \bar{l}$, $c_P(t_A, \alpha, F) - c_R(\alpha, F)$ is bounded below by $-\mathbb{E}[l]$. That is, even when the principal sets a very low t_A , she cannot extract more than $\mathbb{E}[l]$ from the agent through the side-payment.

Again, when the principal sets t_A she only expects to receive $c_P(t_A, \alpha, F)$ from the agent. Thus, her expected profit is given by

$$V = \Pi - et_N - (1 - e)c_P(t_A, \alpha, F) - (1 - e)(1 - \alpha)F.$$

Notice that

$$\begin{aligned} \frac{\partial c_P(t_A, \alpha, F)}{\partial t_A} &= \int_{\alpha F - t_A}^{\bar{l}} dH(l) \geq 0, \\ \frac{\partial c_P(t_A, \alpha, F)}{\partial \alpha} &= \int_{\alpha F}^{\alpha F - t_A} FdH(l) \geq 0. \end{aligned}$$

Naturally, the more the principal increases the punishment in case of accident (reduces t_A) the more she can expect to collect on the agent. More interestingly, let us investigate what happens for the principal when the regulator targets the agent more, *i.e.* when α increases. Two effects are at play: On the one hand, as the agent becomes more targeted by the regulation, the principal expects to collect less on the agent as the regulator has priority over the agent's wealth which is illustrated by $\partial c_P(\cdot)/\partial \alpha \leq 0$. On the other hand, an increase in α increases the principal's expected payoff through a decrease in his share of the fines. The overall effect, however, is positive on the principal's expected payoff.⁹

⁹This can be easily seen by differentiating the principal's expected payoff in case of accident with respect to α . Indeed, $\frac{\partial}{\partial \alpha}(-c_P(t_A, \alpha, F) - (1 - \alpha)F) = -\int_{\alpha F}^{\alpha F - t_A} FdH(l) + F \geq 0$.

Using (2.6) and (2.7), it is useful to rewrite the principal's profit as

$$V = \Pi - \psi(e) - (1 - e)[c_R(\alpha, F) + (1 - \alpha)F] - U. \quad (2.8)$$

It is instructive to compare the objective of the principal with a judgment-proof injurer with the one obtained in the standard framework, namely, equation (2.3). The principal still receives benefits from production Π , has to pay $\psi(e)$ as if she were exerting effort herself and leaves a rent U to the agent. However, the principal considers the threat of the penalty differently. Notice that $c_R(\alpha, F) + (1 - \alpha)F \leq \alpha F + (1 - \alpha)F = F$, that is, the principal does not consider the total amount of fines F as some of it is imposed on a potentially insolvent agent. The assumption of judgment-proofness on the agent's side formalizes the idea that potential insolvency of an injurer creates a discrepancy between the sanction and the way it is perceived by the firm.

When the regulator targets more the agent (α increases), the firm then faces an overall lower sanction than F . At the same time, the burden of payments rests more upon the agent whose participation must be ensured.

The principal's problem now consists in maximizing her objective subject to the agent's participation. It is worth to stress once again that the principal does not ensure a limited liability constraints on transfers so that t_A is here unconstrained. For the sake of clarity, let us first consider the following formulation of the principal's problem

$$\begin{aligned} \max_{e, t_A} \quad & V = \Pi - et_N - (1 - e)c_P(t_A, \alpha, F) - (1 - e)(1 - \alpha)F \\ \text{s.t.} \quad & U = R(e) + c_P(t_A, \alpha, F) - c_R(\alpha, F) \geq 0. \end{aligned}$$

As in section 3.2, let us make a change of variable so that the principal chooses e and U instead of e and t_A . However, this requires to take into account that even if t_A is unbounded below, $c_P(t_A, \alpha, F) - c_R(\alpha, F)$ is bounded by $-\mathbb{E}[l]$. This limits how much the principal can take collect on the agent through the side-payment. Using (2.7), this condition can be written as $c_P(t_A, \alpha, F) - c_R(\alpha, F) = U - R(e) \geq -\mathbb{E}[l]$. Notice that this constraint resembles the standard limited liability constraint of section 3.2. The principal's

problem rewrites

$$\begin{aligned}
 (\text{JP}) : \max_{e, \mathcal{U}} & \quad \Pi - \psi(e) - (1 - e) [c_R(\alpha, F) + (1 - \alpha)F] - \mathcal{U} \\
 \text{s.t.} & \quad \mathcal{U} \geq 0 \\
 & \quad \mathcal{U} \geq R(e) - \mathbb{E}[l].
 \end{aligned}$$

The following proposition summarizes the solution to the optimization problem.

Proposition 13 *Assume the agent may be judgment proof for some realizations of his wealth. Then, the equilibrium effort level, $e(\alpha, F)$, induced by the principal is nondecreasing in α . More precisely,*

- For $\alpha \in [0, \tilde{\alpha}_1)$, $c_P(t_A, \alpha, F) - c_R(\alpha, F) = -\mathbb{E}[l]$, the agent has a positive utility, $\mathcal{U} > 0$, and the optimal effort level is given by

$$\psi'(e) + e\psi''(e) = c_R(\alpha, F) + (1 - \alpha)F. \quad (2.9)$$

- For $\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}_2]$, $c_P(t_A, \alpha, F) - c_R(\alpha, F) = -\mathbb{E}[l]$ and the optimal effort level is given by the binding participation constraint of the agent, $\mathcal{U} = 0$,

$$e\psi'(e) - \psi(e) = \mathbb{E}[l]. \quad (2.10)$$

- For $\alpha \in (\tilde{\alpha}_2, 1]$, $c_P(t_A, \alpha, F) - c_R(\alpha, F) > -\mathbb{E}[l]$, the agent's participation constraint is binding, $\mathcal{U} = 0$, and the optimal effort level is given by

$$\psi'(e) = c_R(\alpha, F) + (1 - \alpha)F. \quad (2.11)$$

Proof. The proof and the characterization of thresholds $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are in the appendix.

■

DISTRIBUTION OF PENALTIES. When the agent may be judgment proof, the equilibrium solution crucially depends on the distribution of liabilities α and $1 - \alpha$ within the firm. Proposition 13 states that the optimal effort level $e(\alpha, F)$ is nonincreasing in α , that is, the firm exerts less and less precautionary effort to prevent an accident as the regulator

increases the liability of the agent to pay for the damage. The intuitive explanation for this result is as follows. From the agent's point of view, only the size of the punishment matters and not to whom it is due. Therefore, whether the monetary punishment comes from the principal or the regulator is irrelevant for the agent's decision to exert the precautionary effort. From the principal's point of view, however, a shift in regulation that targets more the agent induces a lower expected total fine on the firm through the agent's inability to pay in some states of the world. As a result, the principal perceives the fine less and less as a threat and does not want to induce a high effort level.

Therefore, the Equivalence Principle does not hold anymore in this context. In other words, the structure of penalties designed by the regulator affects the equilibrium level of effort chosen by the firm. This naturally raises the question of determining the optimal regulatory policy (α, F) with a judgment-proof agent. In the simple case in which the regulator is only concerned about making the probability of accident as close as possible to its first-best level, the optimal distribution of fines is as follows.

Corollary 6 *With a judgment-proof agent, the optimal regulatory policy consists in targeting only the principal of the firm, that is, $\alpha = 0$ so that the principal faces the total amount of the fine F .*

This result contrasts with [Segerson and Tietenberg \(1992\)](#). As soon as the agent may be unable to pay the regulator in some states of the world, the Equivalence Principle fails and the distribution of penalties within the firm is no more neutral. Considering the standard formulation presented in section 3.2, this result should not be surprising at all. Indeed, the ex post limited liability constraint $t_A \geq -l + \alpha F$ in the standard model artificially assumes that the principal must ensure the agent's ability to pay his share fines for any value of α . This implicitly amounts to saying that the whole burden of penalties lies on the principal, which is equivalent to consider that $\alpha = 0$ in the judgment-proof case.

It is worth noting that when the regulator chooses $\alpha = 0$, the agent's transfer to the regulator is naturally $c_R(0, F) = 0$. In that case, Proposition 13 gives that the equilibrium effort level is uniquely defined by $\psi'(e) + e\psi''(e) = c_R(0, F) + F = F$.¹⁰ This effort level is the same as the one obtained in section 3.2 (equation (2.4)) with the standard

¹⁰To see that the effort level is uniquely defined by the equation, simply notice that $\frac{\partial}{\partial e}(\psi'(e) + e\psi''(e)) = 2\psi''(e) + e\psi'''(e) > 0$ as ψ'' , $\psi''' > 0$ by assumption.

limited liability constraints. However, as soon as $\alpha > 0$, the equilibrium effort level in the judgment proof case decreases.

EQUILIBRIUM CHARACTERIZATION. The equilibrium characterization along the value of α is also worth to investigate. When the principal is mostly targeted, *i.e.* $\alpha \in [0, \tilde{\alpha}_1)$, the equilibrium effort level is given by (2.9). As the left-hand side of the equation is strictly decreasing in α , so is the equilibrium level of effort. Moreover, the principal chooses t_A such that $c_P(t_A, \alpha, F) - c_R(\alpha, F) = -\mathbb{E}[U]$ in order to extract as much as possible from the agent through the side-payment. Even if the principal is not as concerned by the threat of the sanction as in the standard framework, she is still inducing a quite large effort level which forces her to leave a positive rent to the agent, $U > 0$. Indeed, to induce this effort level, she must leave a rent $R(e)$ to the agent that is greater than what she can extract from the agent through the side-payment. When $\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}_2]$, the effort level is given by equation (2.10), which is simply the binding participation constraint of the agent. The principal still chooses t_A so as to extract all the rent from the agent. She can therefore implement the effort level at no informational cost. The equilibrium effort level is constant over the region $\alpha \in [\tilde{\alpha}_1, \tilde{\alpha}_2]$ as the principal is still concerned by the threat of sanction and does not face the trade-off between increasing the effort level and minimizing the rent left to the agent. However, when the agent becomes mainly targeted, *i.e.* when $\alpha \in (\tilde{\alpha}_2, 1]$, the principal is almost not anymore concerned by the threat of the sanction and the effort level decreases once again with respect to α . As effort level becomes low, the rent $R(e)$ left to the agent becomes low as well and the principal chooses higher values of t_A so that the agent still wants to participate. Therefore, we can also see that the more the principal is targeted by the regulation the larger the punishment t_A she imposes on the agent.

Consider finally the case where the regulator mainly targets the agent, that is, $\alpha \in (\tilde{\alpha}_2, 1]$. Recall that the first-best solution to the moral hazard problem is defined by $\psi'(e^{FB}) = F$. Then, it is clear that it resembles equation (2.11) that determines the equilibrium level of effort for $\alpha \in (\tilde{\alpha}_2, 1]$. When the agent is mainly targeted, his incentives to exert effort comes mainly from the regulator and that from the principal decreases as she is less and less concerned by the risk of accident. Equation (2.11) resembles the first-best solution as the agent internalizes the risk of accident but the effort level is much lower as the fine the agent's expect to pay is less and less important.

TOTAL AMOUNT OF PENALTIES. The second choice of the regulator is the total amount of fine F that is imposed on the firm. Recall that $F \in [0, D]$ as tort law generally precludes fines to exceed monetary damages caused by the accident. The optimal level of fine is defined as follows.

Corollary 7 *When the regulator optimally allocates fines within the firm ($\alpha = 0$), the optimal level of fine that maximizes the equilibrium agent's effort is $F = D$, that is, the maximum punishment principle applies with a judgment-proof agent.*

If Corollary 6 challenges the view that allocation of penalties within the firm is relevant when the agent may be judgment-proof, Corollary 7 re-establishes a very well known result: the so-called Becker's maximum punishment principle. This last result is not surprising: potential judgment proofness of the agent reduces the expected total fine perceived by the principal. Thus, in essence, Corollary 6 tells us that fully targeting the principal is the only way to maximize the "perceived" total fine which is a maximum punishment principle in itself. Intuitively, choosing the maximal amount of total fines ensures that the firm internalizes the risk of accident at most. In particular, when $\alpha = 0$ and $F = D$, the effort level is given by $\psi'(e) + e\psi''(e) = D$ which corresponds to the equilibrium effort in the standard case (when the distribution of fines does not matter) when the regulator chooses the optimal regulation. It is also interesting to notice that choosing $F = D$ is optimal even when $\alpha > 0$.

UNCERTAINTY OF PENALTIES. So far, I have assumed that the regulation was nonrandom so that the firm can perfectly anticipate the amount of fines and its distribution between the principal and the agent. In practice, however, it is likely that there is some uncertainty about the exact amount and who will be accountable for it. The analysis of the judgment-proof case shows that what matters for regulation is how the firm perceives the threat of paying fines. As soon as the fine is not at its maximum or if there is a chance that the agent cannot fully pay his share, the threat perceived by the firm becomes lower. As a result, any uncertainty in the total amount of penalties or about its distribution will makes the firm less concerned about the accident. Whenever possible, the regulator should announce and commit to a certain regulation policy to ensure that the firm better internalizes the risk of accident.

AN EQUIVALENT FORMULATION. The judgment-proof problem can be simply rewritten in a formulation very similar to the standard model with ex post limited liability constraints. This reformulation is useful to interpret and compare the judgment-proof problem with the standard formulation as well as providing a simpler and more tractable model.

Simply consider the following expected payoffs and limited liability constraints:

$$\begin{aligned} U^E &= et_N + (1 - e)[t_A - c_R(\alpha, F)] - \psi(e) \\ V^E &= \Pi - et_N - (1 - e)[t_A + (1 - \alpha)F] \\ t_A &\geq -\mathbb{E}[l] + c_R(\alpha, F) \end{aligned}$$

where the only difference with the standard model of section 3.2 is that the agent's payment to the regulator is $c_R(\alpha, F)$ instead of αF and l replaced by $\mathbb{E}[l]$. The incentive constraints immediately writes $t_N - [t_A - c_R(\alpha, F)] = \psi'(e)$. Therefore, $U^E = R(e) + t_A - c_R(\alpha, F)$ and the limited liability constraint can be rewritten as $U^E - R(e) + c_R(\alpha, F) \geq -\mathbb{E}[l] + c_R(\alpha, F) \Leftrightarrow U^E \geq R(e) - \mathbb{E}[l]$. The participation constraint of the agent still writes $U^E \geq 0$ and the principal's expected payoff is $V^E = \Pi - \psi(e) - (1 - e)[c_R(\alpha, F) + (1 - \alpha)F] - U$. The principal's problem therefore writes exactly as problem (JP) and the equilibrium effort level is characterized by proposition 13.

This equivalent formulation shows that it is as if ex post limited liability constraints in the standard framework were replaced by interim limited liability constraints. In other words, the private contract ensures that the agent has enough resources to pay the regulator but only in average and not ex post. This suggests that even if the principal is legally bound on the size of the punishment is limited the nonneutrality of the distribution of fines still hold when the principal is constrained on the size of punishment.

Notice also that the judgment-proof problem is not due to the randomness of the agent's wealth. If instead, we assume that l is certain – as in the standard framework – then it is easy to see that the agent's expected utility defined by (2.7) and the principal's expected profit defined by (2.8) can be accommodated by simply replacing $c_p(t_a, \alpha, F)$ and $c_R(\alpha, F)$ by $\tilde{c}_p(t_a, \alpha, F; l)$ and $\tilde{c}_R(\alpha, F; l)$. The principal's maximization problem writes as

follows:

$$\begin{aligned} \max_{e,U} \quad & \Pi - \psi(e) - (1 - e) [\tilde{c}_R(\alpha, F; l) + (1 - \alpha)F] - U \\ \text{s.t.} \quad & U \geq 0 \\ & U \geq R(e) - l. \end{aligned}$$

It is easy to see that this problem is very similar to the (JP) problem. Proposition 13 applies with the minor change of replacing $c_R(\alpha, F)$ by $\tilde{c}_R(\alpha, F; l)$ and an appropriate change in threshold values $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$. Consider for instance the case in which $U = R(e) - l > 0$ at equilibrium. Then the equilibrium effort level is given by $e\psi''(e) + \psi'(e) = \tilde{c}_R(\alpha, F; l) + (1 - \alpha)F$. Notice that,

$$\frac{\partial}{\partial \alpha} (\tilde{c}_R(\alpha, F; l) + (1 - \alpha)F) = \begin{cases} 0 & \text{if } l \geq \alpha F \\ -F & \text{if } l < \alpha F, \end{cases}$$

so that the equilibrium effort level $e(\alpha)$ is constant and at its second-best level as long as $l \geq \alpha F$ but starts decreasing as soon as $l < \alpha F$, or equivalently when the agent's share of fines is large enough. Although the certainty of the agent's wealth makes the problem less "continuous" in α , it does not change the qualitative result that the optimal regulation should concentrate fines mainly on the principal. This shows that the judgment-proof problem exists even in the absence of uncertainty about the amount of financial resources the agent possesses.

LIMITS OF THE OPTIMAL REGULATION. Although the optimal regulation with a judgment-proof agent seems to be very clear, it can be difficult to implement as it is for various reasons. First, there might be legal restriction on the choice of the distribution of liabilities within the firm. The question of whether the principal can be vicariously liable for the acts of the agent is likely to depend on the nature of their relationship. Vicarious liability generally applies when the agent commits negligent acts in the course of employment. In the case of the present paper, the agent can be seen as an employee or an independent contractor whose responsibility is to ensure some level of precautionary care on behalf of the principal. Although the principal is responsible to incentivize the agent, the latter can still be seen as responsible for his choice of precautionary care. Unobservability of the

effort level or the precise private contract between the two makes it difficult to assess each injurer's responsibility in the accident.

Second, if the regulator decides to target only the principal two problems can arise: (i) The principal may simply not engage in production as she expects that the benefits from production Π do not cover the expected fines and costs of precaution. In other words, the principal's participation constraint must also be taken into account in the design of the optimal regulation. Or (ii) If the fine is large, the principal may also have insufficient financial resources to cover the whole payment by herself. In that case, also relying on the agent's wealth may improve the amount that the regulator is able to collect.

DISTRIBUTION OF REVENUES WITHIN THE FIRM. One interesting aspect of the framework with a judgment-proof agent is that α also affects the distribution of revenues within the firm. Let $e = e(\alpha, F)$ the equilibrium effort level defined by proposition 13. First, the agent's expected utility is $U = R(e(\alpha, F)) - \mathbb{E}[l] > 0$ for all $\alpha \in [0, \hat{\alpha}_1)$ and then $U = 0$ for all $\alpha \in (\hat{\alpha}_1, 1]$. On the principal's side, let $V(\alpha)$ denote the value function of program (JP). From the envelope theorem, $V'(\alpha) = -(1 - e) \frac{\partial}{\partial \alpha} (c_R(\alpha, F) + (1 - \alpha)F) > 0$ so that the principal's expected payoff is increasing in α . Therefore, an increase in α benefits the principal but decreases the rent of the agent. It may also appear surprising that the optimal regulation $(\alpha, F) = (0, D)$ is such that the agent gets the highest possible rent although he is the one who may cause the accident by taking too little precautionary care. In fact, leaving the agent with a high rent is the best possible way to incentivize him to maximize the effort level.

2.3.1. *Participation of the Principal*

As mentioned above, when the regulator imposes the optimal regulation $(\alpha, F) = (0, D)$ it may occur that the principal makes negative profits $V < 0$. Even if the regulator wants to avoid as much as possible the occurrence of an accident, it does not mean that the economic activity must be prohibited. Ensuring the participation of the principal is therefore important if the regulator values the production of the firm (though not modeled here).

For simplicity, first assume that the regulator sets $F = D$ by default and can only use α

as an instrument policy. It follows that the optimal regulation $(\alpha, F) = (0, D)$ ensures the principal's participation as long as

$$\Pi - \psi(e^*) - (1 - e^*)[c_R(0, D) + (1 - \alpha)D] - R(e^*) + \mathbb{E}[l] \geq 0,$$

where e^* solves equation (2.9) for $\alpha = 0$ and $F = D$. This condition can be violated when Π or $\mathbb{E}[l]$ are low. If this occurs, assuming that the regulator only considers $\alpha \in [0, \tilde{\alpha}_1]$, the new optimal choice of allocation of responsibilities, α^V within the firm must solve

$$\Pi - \psi(e(\alpha^V, D)) - (1 - e(\alpha^V, D))[c_R(\alpha^V, D) + (1 - \alpha)D] - R(e(\alpha^V, D)) + \mathbb{E}[l] = 0$$

This is indeed possible as the profit of the principal is increasing in α so that there exists a $\alpha^V > 0$ such that the principal's profit is nonnegative.

A more complete analysis of the optimal regulation subject to the principal's participation would allow for change in both α and F with respect to Corollaries 6 and 7. Let $\Omega(\alpha, F) := c_R(\alpha, F) + (1 - \alpha)F$ and $V(\Omega(\alpha, F))$ denote the total amount of fines imposed on the firm and the value function of problem (JP), respectively. From the Envelope Theorem, it is straightforward to see that the principal's expected payoff decreases in $\Omega(\cdot)$ as $\partial V(\Omega(\alpha, F))/\partial \Omega = -(1 - e(\alpha, F)) < 0$. Furthermore, proposition 13 states that the equilibrium effort level is (weakly) increasing in $\Omega(\cdot)$. Therefore, to ensure the principal's participation constraint and maximize the equilibrium effort level, the regulator must choose Ω such that

When both α and F are close to 0 and D , respectively, the equilibrium is given by Proposition 13 (a). Therefore, when the regulator seeks to maximize precautionary care it is equivalent to choose α and F so that

$$\begin{aligned} \max_{\Omega \in [\mathbb{E}[l], D]} \quad & \Omega \\ \text{s.t.} \quad & V(\Omega) \geq 0 \end{aligned}$$

It is clear that the constraint must bind so that the optimal choice of Ω satisfies $V(\Omega^*) = 0$. Uniqueness of Ω^* is guaranteed by $V'(\Omega) < 0$. Then, the regulator must choose (α^*, F^*) so that $c_R(\alpha^*, F^*) + (1 - \alpha^*)F^* = \Omega^*$. Observe, however, that the choice of (α^*, F^*) is not

unique so that the optimal regulation can be achieved with various combinations of the regulation instruments. More precisely, if (α^*, F^*) implements Ω^* then it is also possible to find $(\hat{\alpha}, \hat{F})$ with $\hat{\alpha} > \alpha^*$ and $\hat{F} > F^*$, that is, another regulation scheme in which the agent is more targeted but the total amount of fines imposed on the firm increases.¹¹

Proposition 14 *When the optimal regulation $(\alpha^*, F^*) = (0, D)$ is such that the principal does not want to participate ex ante, the regulation policy (α, F) must be such that the total perceived fine solves $V(\Omega(\alpha, F)) = 0$, that is, the principal's expected profit is null. The choice of (α, F) is nonunique.*

Participation of the principal crucially depends on the benefits of the productive activity Π and on the agent's expected wealth $\mathbb{E}[l]$. Indeed, let $e = e(\Omega)$, then the optimal regulation subject to the principal's participation sets Ω to solve

$$\Pi - \psi(e(\Omega)) - (1 - e(\Omega))\Omega - R(e(\Omega)) + \mathbb{E}[l] = 0.$$

Totally differentiating this expression yields $\frac{d\Omega}{d\Pi} = \frac{d\Omega}{d\mathbb{E}[l]} = -\frac{1}{V'(\Omega)} > 0$. Hence, an increase in either the productive activity of the expected agent's wealth allows the regulator to set a higher total perceived fine Ω which, in turn, leads to higher level of precautionary care. Quite intuitively, it seems therefore easier to regulate profitable businesses with wealthy members rather than small returns activities with very financially constrained agents.

2.3.2. Observability of the Private Contract: Negligence Rule

So far, I have assumed that the private contract was not observable to the Regulator, or, equivalently, that the Regulator was not using the private contract as a tool to provide the firm with incentives to take precautionary care. I only considered the "strict liability" rule, that is, the firm is subject to fines when an accident occurs, regardless of the implemented effort level.

Assume now that the court can observe the private contract and infer the equilibrium effort level from the incentive scheme offered by the principal. This kind of regulation is generally referred to as a "negligence rule". The regulator defines a negligence standard

¹¹Naturally, I assume that $\Omega^* < \Omega(0, D)$ and $(\alpha^*, F^*) \in (0, 1) \times (0, D)$ so that there exists $\hat{\alpha} \in (\alpha^*, 1]$ and $\hat{F} \in (F^*, D]$.

(an effort level), and the firm is subject to fines only if both an accident occurs and the negligence standard is not met.

Suppose that the regulator sets a negligence standard $e^S \in [0, 1]$ and a regulatory policy (α, F) . When an accident occurs, the regulator investigates the firm and observes the private contract from which is deduced the equilibrium effort level e^* . If the equilibrium effort level is equal or higher than the negligence standard, no fine is imposed on the firm. If, however, the effort level is lower than the negligence standard, the regulatory policy (α, F) applies.

Hence, following the analysis of [Demougin and Fluet \(1999\)](#), it is clear that when the firm does not comply with the negligence standard, it faces the exact same trade-off than in the strict liability rule, i.e., problem (JP). Therefore, it is still optimal to fully target the principal when the negligence standard is not met.

The regulator can then set the negligence standard to the first-best level of effort $e^S = e^{FB}$. The principal will therefore have to choose either to comply and induce the first-best level of effort or to choose not to comply and face problem (JP). As in Proposition 4 of [Demougin and Fluet \(1999\)](#), it is clear that such a negligence rule always weakly improve the equilibrium effort level.

2.3.3. *Extension of Liability to the Principal*

Assume that if ex post the agent has not enough wealth to pay his share of fines αF , then the Regulator can seize the amount remaining due to the principal with probability $\gamma \in [0, 1]$. This practice is commonly referred to as extended liability. I now consider whether extension of liability to the principal can help mitigate the judgment-proof problem.

First, notice that the utility of the agent is still defined by equation (2.7). However, the principal's expected profit now writes

$$V^J = \Pi - et_N - (1 - e)c_P(t_A, \alpha, F) - (1 - e)[(1 - \alpha)F + \gamma(\alpha F - c_R(\alpha, F))],$$

where $\alpha F - c_R(\alpha, F) \leq 0$ is the expected unpaid amount of fines by the agent. Once again using equation (2.7), the principal's expected profit rewrites as:

$$V^J = \Pi - \psi(e) - (1 - e)[(1 - \alpha)F + c_R(\alpha, F) + \gamma(\alpha F - c_R(\alpha, F))] - U.$$

Notice that for $\gamma = 0$, the problem is identical to the (JP) problem for any (α, F) and for $\gamma = 1$ it is as if the Regulator fully targets the principal, that is, $(\alpha, F) = (0, F)$. The principal's maximization problem writes $\max_{e, U} V^J$ subject to $U \geq 0$ and $U \geq R(e) - \mathbb{E}[U]$. This problem differs from (JP) only through the additional payment of $\gamma(\alpha F - c_R(\alpha, F))$ in case an accident occurs in the principal's expected profit. Therefore, Proposition 13 directly applies by adding this additional payments in the right-hand side of (2.9) and (2.11) and an appropriate change in threshold levels. For instance, when α is low enough, the equilibrium effort level is given by

$$e\psi''(e) + \psi(e) = (1 - \alpha)F + c_R(\alpha, F) + \gamma(\alpha F - c_R(\alpha, F))$$

It is clear that having $\gamma > 0$ strictly increases the equilibrium effort level, that is, extending liability to the principal mitigates the judgment-proof problem. Immediate computations shows that the right-hand side of this equation decreases in α for $\gamma < 1$ and is constant in α for $\gamma = 1$. Hence, the optimal regulation policy still consists in fully targeting the principal ($\alpha = 0$) for $\gamma < 1$. When $\gamma = 1$, the Regulator can fully pass on any unpaid fines by the agent to the principal and the structure of penalties becomes irrelevant.

2.3.4. *Nonmonetary Sanction: Jail sentences*

Other penalties than monetary sanctions can be considered such as jail sentences or reputation losses for instance. It seems intuitive that nonmonetary sanctions can be used as a way to mitigate the judgment-proof problem.

Segerson and Tietenberg (1992) consider the joint use of monetary and nonmonetary sanctions when the firm's assets are insufficient or when actual fines are below efficient levels for other legal reasons. They assume that members of the firm can be incarcerated in addition to the payment of fines when an accident occurs. Jail sentences generates disutility for individuals directly from the years spent in jail and indirectly also from

reputational losses while it also generates a social cost of incarceration. They find that efficiency of nonmonetary sanctions crucially hinges on large reputational losses for the members of the firm.

As they also rely on the assumption that the firm can partially escape paying the fines levied on them, their results suggest that the same type of effect would apply in the judgment-proof case. Therefore, imposing nonmonetary sanctions such as jail sentences - especially on the agent - would help mitigate the judgment-proof problem as well.

2.4. BARGAINING POWER AND OPTIMAL REGULATION

So far, I have assumed that the principal had all the bargaining power in the choice of the private contract. It seems important to investigate the importance of this assumption for the choice of the equilibrium effort level. The results for the optimal regulation suggest that imposing the total amount of the fine on the principal forces her to fully internalize the sanction. But this result holds because the principal is the one with the bargaining power in the relationship. Indeed, if the agent has more bargaining power in the private relationship, he will try to impose his terms to the principal and will obviously benefit from a heavily targeted principal.

To investigate this issue, assume that $b \in [0, 1]$ and $1 - b$ denote the bargaining power of the principal and the agent, respectively. Let us now assume that the principal and the agent have the following objective

$$bV + (1 - b)U = b \left[\Pi - et_N - (1 - e)c_P(t_A, \alpha, F) - (1 - e)(1 - \alpha)F \right] \\ + (1 - b) \left[et_N + (1 - e) \left[c_P(t_A, \alpha, F) - c_R(\alpha, F) \right] - \psi(e) \right]$$

The agent's local incentive constraint is still characterized by (2.7). Substituting it into the objective of the coalition and expressing everything in the plane (e, U) as before, the objective can be rewritten as

$$T := (\Pi - \psi(e) - (1 - e) \left[c_R(\alpha, F) + (1 - \alpha)F \right] - U) + \beta U,$$

where $\beta := \frac{1-b}{b}$ represents the agent's relative bargaining power.¹² When the agent has bargaining power, he will try to extract rent from the principal. However, I still assume that the principal cannot reward the agent in case of accident, that is, $t_A \leq 0$, so that there is an upper bound on how much rent the agent can extract from the principal. Formally, if $t_A \leq 0$ then $c_P(t_A, \alpha, F) \leq 0$. This constraint can be rewritten as $c_P(t_A, \alpha, F) = U - R(e) + c_R(\alpha, F) \geq 0$. The maximization problem writes

$$\begin{aligned} \max_{e, U} \quad & (\Pi - \psi(e) - (1 - e)[c_R(\alpha, F) + (1 - \alpha)F] - U) + \beta U \\ \text{s.t.} \quad & U \geq 0 \\ & U \geq R(e) - \mathbb{E}[U] \\ & U \leq R(e) - c_R(\alpha, F) \\ & \Pi - \psi(e) - (1 - e)[c_R(\alpha, F) + (1 - \alpha)F] - U \geq 0, \end{aligned}$$

Notice that the value of β plays a crucial part in the solution of this problem. Let us first consider the case $\beta \in [0, 1)$. It immediately follows that U enters the objective negatively. Therefore, as in the case in which the principal has all the bargaining power (special case $\beta = 0$ here), at least one of the agent's constraint must bind and the principal's participation constraint is relaxed as U decreases.

Let μ and ν be the Lagrange multipliers associated with the two first constraints. Ignoring the principal's participation constraint, the first-order conditions of the problem write¹³

$$\begin{aligned} \psi'(e) + \nu e \psi''(e) &= c_R(\alpha, F) + (1 - \alpha)F \\ \mu + \nu &= 1 - \beta. \end{aligned}$$

For simplicity let us focus on the case where α is low (similarly to Proposition 13, case (a)). Only the second constraint binds and thus $\mu = 0$. It follows that the equilibrium effort level is given by $\psi'(e) + (1 - \beta)e\psi''(e) = c_R(\alpha, F) + (1 - \alpha)F$.

¹²After plugging (2.7) into T and changing variables the actual objective of the coalition is bT . As it is equivalent to maximize bT and T , I choose the latter for convenience.

¹³As long as Π is large enough, the principal's participation constraint will not be a problem here as she has most of the bargaining power.

Proposition 15 *When the principal is dominant, $\beta < 1$, the equilibrium effort level is increasing in β and the optimal regulation policy still satisfies $(\alpha, F) = (0, D)$.*

This result simply confirms results obtained in section 2.3 when the principal has most of the bargaining power. It shows, however, that an increase in the agent's bargaining power mitigates the judgment-proof problem as the equilibrium effort level increases in β .

When $\beta > 1$, the agent has most of the bargaining power and U now enters the objective positively. It is therefore clear that either the third or the fourth constraint is now binding. For simplicity, I assume that $\Pi > D$ so that the participation constraint of the principal is never binding (as shown below). Let $U = R(e) - c_R(\alpha, F)$, then maximizing the objective with respect to e gives the following first-order condition¹⁴

$$\psi'(e) + (1 - \beta)e\psi''(e) = c_R(\alpha, F) + (1 - \alpha)F.$$

Notice that when $\beta = 1$ and $\alpha = 0$, the equilibrium effort level solves $\psi'(e) = F$. Hence, if the regulator sets $F = D$, it is possible to achieve the first-best effort level (characterized by $\psi'(e) = D$). This occurs as $\beta = 1$ is equivalent to $b = 1/2$, that is, the principal-agent coalition puts the same weights on each member's payoff. However, as equilibrium effort level is increasing β it follows that if $\beta > 1$ and $(\alpha, F) = (0, D)$ then the equilibrium effort level is higher than the first-best one. The following proposition summarizes those results.

Proposition 16 *When the agent is dominant, $\beta \geq 1$, the equilibrium effort level is increasing in β . If the regulator chooses $(\alpha, F) = (0, D)$, the equilibrium effort level attains the first-best level for $\beta = 1$ but exceeds the first-best level when $\beta > 1$.*

When the agent is *dominant*, setting the regulation policy $(\alpha, F) = (0, D)$ might lead to an excessive precautionary effort level. The intuition behind this finding is the following. The *dominant* agent tries to extract as much rent as possible from the principal through increases in payments t_N and t_A . However, recall that only punishments are available in case of accident ($t_A \leq 0$) so that once the agent has set $t_A = 0$ (from the binding constraint $U = R(e) - c_R(\alpha, F)$), the only to extract more rent from the principal is through an increase in t_N , the payment in the absence of accident. This makes the agent even more

¹⁴The second-order condition requires that $-\psi''(e) - (1 - \beta)[\psi''(e) + e\psi'''(e)] \leq 0$ or, equivalently, that β is not too large. To ensure that the effort level is always interior, I assume that β is such that the second-order condition always hold.

incentivized that no accident occurs and in that situation the effort level becomes higher than the first-best one. The optimal regulation in that case might therefore surprisingly be milder and a decrease in the total amount of perceived fines $c_R(\alpha, F) + (1 - \alpha)F$ becomes desirable. Once again, several combinations of (α, F) can achieve a lower total amount of perceived fines.

REWARDS IN CASE OF ACCIDENT. So far, I have assumed that the principal could not offer rewards ($t_A \geq 0$) in case of accident but only a punishment ($t_A \leq 0$). Although it may be difficult to regulate private contracts between a principal and an agent, it may be possible to make sure that an agent does not receive bonuses when accident occurs. This legal restriction can be imposed for ethical reasons or following the reasoning of incentive theory. Furthermore, when the principal has all the bargaining power, it is intuitive that she would not offer rewards to the agent as it would both reduce incentive provision and the possibility of extracting rent from him.

When the agent is dominant, however, we have seen that the private contracts is such that $t_A = 0$ when only punishments are available. This suggests that a dominant agent would like to set positive t_A if possible. Assume now that rewards in case of accident, *i.e.* $t_A \geq 0$, are allowed. The agent's payoff in case of accident can still be written as $m(t_A, \alpha, F) = c_P(t_A, \alpha, F) - c_R(\alpha, F)$ so that neither his expected utility nor his incentive constraint changes. For the principal, however, the payoff in case of accident simply becomes t_A (instead of $c_P(t_A, \alpha, F)$) as the reward she pays the agent is independent from the regulation policy. Her expected payoff therefore rewrites $V^R = \Pi - et_N - (1 - e)t_A - (1 - e)(1 - \alpha)F$.

For simplicity, assume that the agent has all the bargaining power. It follows that the maximization problem writes

$$\begin{aligned} \max_{e, t_A} \quad & R(e) + m(t_A, \alpha, F) \\ \text{s.t.} \quad & \Pi - e\psi'(e) - (1 - e)(1 - \alpha)F - em(t_A, \alpha, F) - (1 - e)t_A \geq 0, \end{aligned}$$

where it is clear that we can ignore the agent's participation constraint. As $\frac{\partial m(t_A, \alpha, F)}{\partial t_A} \geq 0$, the objective is increasing in t_A whereas the principal's profit is decreasing in t_A . Then, the principal's participation constraint is binding. Notice that if $t_A \geq \alpha F$, *i.e.* in case of accident

the agent receives a reward that covers his share of fines, then $m(t_A, \alpha, F) = t_A - \alpha F$. Assume that the equilibrium t_A is greater than αF , then it follows that the binding participation constraint of the principal gives $t_A = \Pi - e\psi'(e) - (1 - e)(1 - \alpha)F + e\alpha F$. Plugging this expression into the objective reduces the problem to $\max_e \Pi - \psi(e) - (1 - e)F$. This program is simply the social objective and it yields the first-best level of effort $\psi'(e) = F$.

This result, although not surprising, stresses two important insights for the regulation.¹⁵ First, this finding shows that the allocation of penalties between the principal and the agent is irrelevant when the agent has all the bargaining power. Intuition could have suggested that members with more bargaining power should be more targeted but this is not the case. Second, it shows that rewards should be allowed in case of accident. When they are, a fully dominant agent can extract the whole surplus from the principal and the first-best level of effort is attained. This contrasts with the result of proposition 16 in which only punishments are allowed and the agent chooses an excessively large effort level.

2.5. TWO-SIDED MORAL HAZARD

In many situations, the responsibility of preventing an accident lies with more than one agent. The probability of accident may then depend upon the action of several agents and it is not possible to hire a single agent in charge of safety. In that case, what should be the optimal targeting policy if some of the tortfeasors are judgment-proof? Agents may differ in their wealth characteristics and cost of effort. Should the regulation change with respect to agents' efficiency and wealth?

To capture the fact that the probability of accident depends on more than one agent, I now model a two-agent partnership with double-sided moral hazard. Assume that each partner receives a non-contractible benefit b only when no accident occurs whereas they receive $\Pi < b$ independently of the accident. Therefore, their contract consists in choosing a sharing of Π for the two state of the world. The environmental harm is denoted

¹⁵As in standard principal-agent models, giving the bargaining power to the informed party makes the moral hazard problem disappear as the rent extraction-efficiency trade-off vanishes.

by $D < \Pi$. Agent 1 and agent 2 exert efforts $e \in [0, 1]$ and $a \in [0, 1]$, respectively. The probability of accident is determined by $(1 - p(e, a))$ where $p(e, a)$ is the joint production function. Individual cost functions of effort are $\psi(e)$ and $C(a)$ that are both increasing and convex functions. To obtain a tractable model I further assume the following specific functional forms: $p(e, a) = e + a$, $\psi(e) = \gamma_1 \frac{e^2}{2}$ and $C(a) = \gamma_2 \frac{a^2}{2}$.

As in the one-sided moral hazard case, the regulator chooses the level of total fine $F \in [0, D]$ and a distribution of fines (α_1, α_2) among the two agents, where $\alpha_1 + \alpha_2 = 1$ and $\alpha_i F$ is the share of the fine imposed on agent $i = 1, 2$. For simplicity, I will assume that $F = D$ so that the regulator only has to determine the distribution of fines.

Each agent $i = 1, 2$ has a liability l_i , uniformly distributed over $[0, \bar{l}_i]$. Neither the regulator nor the agents know the value of l_1 and l_2 until the end of the game. The final wealth of agent i is given by the realization of his liability and his share of the total profit Π . An agent $i = 1, 2$ is said to be judgment proof when his final wealth is lower than the amount of the fine $\alpha_i D$ he has to pay to the regulator. I will further assume that $D > \max\{E[l_1], E[l_2]\}$ so that in expectation, none of the agent has enough resources to pay the fine in full.

The timing of the game is the following. First, the regulator announces an ex post regulation policy (D, α) . Second, the agents observe the regulation policy and contract upon profit. The contracting stage is close to [Cooper and Ross \(1985\)](#). Agents play a two-stage game in which they first agree on a binding contract with respect to their respective share of profit in the two possible states of the world (“no accident” and “accident”). In the second stage, taking the terms of the contract as given, they simultaneously choose their effort level e and a .

The design of the private transactions among the two agents has a particular interest in the context of judgment proofness. Here, I assume that agents agree on a contract $(t_N, t_A) \in [0, \Pi]^2$ where t_k is the share of agent 1 in state $k = N, A$ and thus $\Pi - t_k$ is the share of agent 2 in state $k = N, A$. As usual in double moral hazard problems, the sharing of the profit ensures distribution of incentives in the partnership. With judgment-proof agents, however, the sharing of profit plays the additional role of concealing revenue to the regulation authority.

Following the same line as the previous principal-agent model, the potential agents' inability to pay the regulator modify their ex post transfer in case of accident. Let $m_i(t_A)$ denote the transfer of agent $i = 1, 2$ in case of accident. Again, this transfer incorporates both the private and the public transactions. For uniformly distributed level of liability $l_i \in [0, \bar{l}_i]$, I define

$$m_1(t_A) := \int_{\alpha_1 D - t_A}^{\bar{l}_1} \frac{(t_A - \alpha_1 D)}{\bar{l}_1} dl + \int_0^{\alpha_1 D - t_A} \frac{(-l)}{\bar{l}_1} dl,$$

$$m_2(t_A) := \int_{\alpha_2 D - \Pi + t_A}^{\bar{l}_2} \frac{(\Pi - t_A - \alpha_2 D)}{\bar{l}_2} dl + \int_0^{\alpha_2 D - \Pi + t_A} \frac{(-l)}{\bar{l}_2} dl.$$

Naturally, the payoff of agent 1 is increasing in t_A while the one of agent 2 is decreasing in t_A . Let $M(t_A) := m_1(t_A) + m_2(t_A)$ be the total profit of the partnership when an accident occurs. Notice that it depends upon t_A , that is, the way profit is split up in case of accident. This stems directly from the assumption that agent may be judgment proof: internal distribution of profits matter now.

I solve the perfect Bayesian equilibrium of the contracting game by backward induction. For a given contract $(t_N, t_A) \in [0, \Pi]^2$, agents' utility functions write

$$U_1 = p(e, a)(b + t_N) + [1 - p(e, a)]m_1(t_A) - \psi(e),$$

$$U_2 = p(e, a)(b + \Pi - t_N) + [1 - p(e, a)]m_2(t_A) - C(a).$$

At the second stage of the contracting game, agents simultaneously choose their effort level. Differentiating the utility of each agent with respect to own effort and equating to zero gives the two incentive constraints:

$$b + t_N - m_1(t_A) = \gamma_1 e, \quad (2.12)$$

$$b + \Pi - t_N - m_2(t_A) = \gamma_2 a. \quad (2.13)$$

Given that functions $m_i(\cdot)$ are monotonic, the equilibrium effort levels e and a are uniquely defined by a contract (t_N, t_A) in the subgame. For simplicity, I ignore both agents' participation constraints. This approach would therefore fit with a situation in which agents are already engaged in production and cannot decide to quit ex ante like

for instance if agents run an established nuclear power plant that cannot be stopped overnight.¹⁶

Therefore, assume agents choose a sharing of profit and effort levels to maximize joint profits as follows

$$\max_{\{e, a, t_N, t_A\}} (e + a)(2b + \Pi) + (1 - (e + a))M(t_A) - \psi(e) - C(a)$$

subject to constraints (2.12), (2.13) and $(t_N, t_A) \in [0, \Pi]^2$.

Notice that t_A enters directly into the objective function of the firm. Usually, without judgment-proofness, profit sharing only serves as a way to distribute incentives within the firm and affects joint profits only indirectly through changes in equilibrium effort level. In the judgment-proof case, profit sharing also directly affects profits in case of accident as shifting monetary resources from one agent to the other also serves as a way of concealing profits to the regulator. For intermediate values of α_1 , the total profit of the firms in case of accident is a U-shape function of t_A . The minimal total profit in case of accident is attained for intermediate values of t_A and is exactly $\Pi - D$ for $t_A \in [\alpha_1 D, \Pi - D + \alpha_1 D]$. In other words, for intermediate values of t_A , the firm pays the whole fine whereas it can increase total profit by shifting resources more extremely to one or another agent.

Consider first the case of an interior solution in the sense $(t_N, t_A) \in (0, \Pi)^2$. Plugging (2.12) into (2.13), the first-order conditions with respect to e , a and t_A write:

$$2b + \Pi - M(t_A) - \gamma_1 e - \lambda \gamma_1 = 0 \quad (2.14)$$

$$2b + \Pi - M(t_A) - \gamma_2 a - \lambda \gamma_2 = 0 \quad (2.15)$$

$$(1 - (e + a) - \lambda)M'(t_A) = 0. \quad (2.16)$$

where λ is the Lagrange multiplier associated with (2.13). Analyzing those first-order conditions (details in the appendix) shows that there exists a local maximum for which

¹⁶In the one-sided moral hazard case, the agent can be seen as an employee or an independent contractor who is in charge of take precautionary care on behalf of the principal. In that case, it is crucial to take into account his participation constraint as he may simply refuse to take part in a risky activity.

$M'(t_A^I) = 0$ so that $t_A^I \in [\alpha_1 D, \Pi - D + \alpha_1 D]$ and

$$e^I = \frac{\gamma_2(2b + D)}{\gamma_1(\gamma_1 + \gamma_2)}$$

$$a^I = \frac{\gamma_1(2b + D)}{\gamma_2(\gamma_1 + \gamma_2)}$$

Notice that only t_A^I depends upon α_1 whereas e^I and a^I only depend upon D and marginal costs of efforts. More importantly, as the solution requires $M'(t_A^I) = 0$, it means that total profits of the firm are at their lowest possible value, namely $M(t_A^I) = \Pi - D$. Therefore, the firm faces the whole amount of fines and agents have to provide quite high effort levels.¹⁷

Intuition suggests that this candidate is, in some cases, the worst possible scenario for the firm as it has to pay the fine in full and exert effort levels accordingly. As mentioned above, this candidate to the maximization problem is only a local maximum and it may not be a global one.

This is indeed the case when the regulator heavily targets one agent. Let us assume that α_1 is close to zero, that is, agent 1 is almost not targeted by the regulation while agent 2 faces almost the whole fine D . Then the following may arise.¹⁸

Proposition 17 *When the regulation heavily targets agent 1 (resp. agent 2), the equilibrium contract may exhibit an extreme sharing out of the profit, namely, $t_N = t_A = \Pi$ (resp. $t_N = t_A = 0$).*

That is, when an accident occurs, the two-agent partnership secures profit by giving it to the least targeted agent. Notice that when $t_A = \Pi$ and α_1 , then $M(t_A) = \Pi - \alpha_1 D - \mathbb{E}[l_2] > \Pi - D$ if α_1 is small enough as $D > \mathbb{E}[l_2]$. As the firm is able to secure profits in case of accident by moving resources to the least targeted agent, it will also exert a lower total

¹⁷As a way of comparison, the first-best levels of effort solve $\max_{e,a} (e+a)(2b+\Pi) + (1-(e+a))(\Pi-D) - \psi(e) - C(a)$ and thus they write $e^{FB} = (2b+D)/\gamma_1$ and $a^{FB} = (2b+D)/\gamma_2$. Therefore $e^I = \frac{\gamma_2}{\gamma_1+\gamma_2} e^{FB} < e^{FB}$ and $a^I = \frac{\gamma_1}{\gamma_1+\gamma_2} a^{FB} < a^{FB}$. When $\gamma_1 = \gamma_2$, the total level of effort $e^I + a^I$ is exactly twice as less as the first-best total of effort $e^{FB} + a^{FB}$.

¹⁸Whether the interior equilibrium is a global maximum crucially depends on the size of D . For low D , the interior is a global maximum, whereas it is always dominated by extreme sharing when D becomes higher.

effort level as the threat of an accident is also lower. In the case where $t_N = t_A = \Pi$, equilibrium effort levels are given by

$$e = \frac{b + \alpha_1 D}{\gamma_1},$$

$$a = \frac{b + \mathbb{E}[l_2]}{\gamma_2}.$$

The interpretation is as follows. Each agent has natural incentives to exert effort as $b > 0$ is obtained only when no accident occurs. Agent 1 has all the contractible profit Π whether an accident occurs or not so that an increase in α_1 gives him additional incentives to exert effort. Agent 2, however, receives nothing whether an accident occurs or not so that his incentives to exert effort are unchanged with respect to α_1 . However, agent 2 faces a large share of the fine (as α_1 is small) and therefore expects to pay $\mathbb{E}[l_2]$ if an accident occurs.

For larger α_1 , the extreme equilibrium $t_N = t_A = \Pi$ may not hold anymore. In that case, we have $t_A = \Pi$ but $t_N < \Pi$ so that $e = \frac{\gamma_2(2b + \alpha_1 D + \mathbb{E}[l_2])}{\gamma_1(\gamma_1 + \gamma_2)}$ and $a = \frac{\gamma_1(2b + \alpha_1 D + \mathbb{E}[l_2])}{\gamma_2(\gamma_1 + \gamma_2)}$. By symmetry, when α_1 is close to 1, we have $t_N = t_A = 0$, $e = \frac{b + \mathbb{E}[l_1]}{\gamma_1}$ and $a = \frac{b + (1 - \alpha_1)D}{\gamma_2}$ and for lower α_1 , $t_A = 0$, $t_N > 0$, $e = \frac{\gamma_2(2b + (1 - \alpha_1)D + \mathbb{E}[l_1])}{\gamma_1(\gamma_1 + \gamma_2)}$ and $a = \frac{\gamma_1(2b + (1 - \alpha_1)D + \mathbb{E}[l_1])}{\gamma_2(\gamma_1 + \gamma_2)}$.

When agents both share the same marginal cost of effort, that is, $\gamma_1 = \gamma_2$, the optimal regulation takes a very simple form.

Proposition 18 *When $\gamma_1 = \gamma_2$, the optimal regulation (minimize probability of accident) requires:*

$$\alpha_1^* = \frac{1}{2} + \frac{\mathbb{E}[l_1] - \mathbb{E}[l_2]}{2D}.$$

Thus, the optimal regulation policy is centered around 1/2 and targets more the agent with more liability. More importantly, this optimal regulation is unique and thus the way it is designed matter for incentivizing agents to exert effort.

Two important things stems from Proposition 18. First, the Equivalence Principle fails to apply due to the judgment-proofness possibility. Agents anticipate the regulation and write contracts accordingly. Second, in a double moral hazard setting, the optimal regulation allocates the total fines evenly among injurers. This is in sharp contrast with the optimal regulation in the case of one-sided moral hazard in which targeting the principal only was optimal.

2.6. CONCLUSION

In this paper, I have proposed a theoretical foundation for the optimal distribution of penalties among several potential injurers. Assuming that injurers have limited financial resources and therefore sometimes declared judgment proof, I show that the usual Equivalence Principle does not hold anymore. On the contrary, both in the principal-agent firm and in the two-agent partnership firm, the optimal regulation distributes fines toward the injurers with available cash resources. On their side, firms anticipate the regulation and try to prevent paying the fines as much as possible. This requires the optimal contract to solve a trade-off between allocating incentives (to avoid the accident) and sharing profits in case of accident to avoid paying penalties. My result stems from relaxing the modeling assumption that the individuals must contract *ex ante* to avoid *ex post* insolvency. Instead, I assume that there is no need to contract *ex ante* on that matter as insolvency simply implies not paying the fines.

APPENDIX

Proof of Proposition 13. Let λ and μ be the Lagrange multipliers associated with the participation constraint $U \geq 0$ and the constraint $U \geq R(e) - \mathbb{E}[l]$, respectively. First-order conditions write

$$\begin{aligned}\mu e \psi''(e) + \psi'(e) &= c_R(\alpha, F) + (1 - \alpha)F \\ \lambda + \mu &= 1.\end{aligned}$$

It is clear that $\lambda = \mu = 0$ is impossible. Consider first that $\lambda = 0$, so that $\mu = 1$. It follows that the equilibrium effort level e_1 is given by

$$e_1 \psi''(e_1) + \psi'(e_1) = c_R(\alpha, F) + (1 - \alpha)F,$$

which is equation (2.9). Let $e_1(\alpha)$ be the implicit solution to this equation. Notice that the LHS is strictly increasing in e and the RHS is strictly decreasing in α so that $e_1(\alpha)$ is decreasing in α . As $\mu = 1$, the second constraint is binding so that $U = R(e_1(\alpha)) - \mathbb{E}[l]$. This holds as long as the agent's participation constraint is also satisfied, that is $U = R(e_1(\alpha)) - \mathbb{E}[l] \geq 0$. As $e_1(\alpha)$ decreases in α , there exists a threshold $\hat{\alpha}_1$ such that $U = R(e_1(\hat{\alpha}_1)) - \mathbb{E}[l] = 0$. The principal sets $t_A = \alpha F - \hat{l}$ to extract as much rent as possible from the agent.

Consider now the case where both $\mu > 0$ and $\lambda > 0$. Complementary slackness implies that $U = R(e) - \mathbb{E}[l] = 0$. For this case, the effort level $e_2 = e_1(\hat{\alpha}_1)$ is constant with respect to α . Notice that the

first-order condition with respect to e gives that $\mu = \frac{c_R(\alpha, F) + (1 - \alpha)F - \psi'(e_2)}{e_2 \psi''(e_2)}$. It is easy to see that for $\alpha = \hat{\alpha}_1$ we have $c_R(\hat{\alpha}_1, F) + (1 - \alpha)F = e_2 \psi''(e_2) + \psi'(e_2) > \psi'(e_2)$ so that $\mu > 0$. This solution is then valid until α reaches the second threshold $\hat{\alpha}_2$ defined by $c_R(\hat{\alpha}_2, F) + (1 - \alpha)F = \psi'(e_2)$.

Finally, when $\mu = 0$ it implies $\lambda = 1$ and $U = 0$. The equilibrium effort level is defined by $\psi'(e_3) = c_R(\alpha, F) + (1 - \alpha)F$ and is decreasing in α . Recall that $U = R(e) + c_P(t_A, \alpha, F) - c_R(\alpha, F)$ so that $U = 0$ requires that $c_P(t_A, \alpha, F) - c_R(\alpha, F) > -\mathbb{E}[l]$ in that situation as $R(e_3(\alpha)) < R(e_2) = \mathbb{E}[l]$ for all $\alpha > \hat{\alpha}_2$. ■

Interior Solution for the Two-Sided Moral Hazard Problem

Equation (2.16) directly rewrites $M'(\underline{t}) [1 - (e + a) - \lambda] = 0$.

• Let us assume that $M'(\underline{t}) \neq 0$. Thus, we must have $1 - (e + a) - \lambda = 0$. The bordered Hessian of the problem writes:

$$H = \begin{bmatrix} 0 & -\gamma_1 & -\gamma_2 & -M'(\underline{t}) \\ -\gamma_1 & -\gamma_1 & 0 & -M'(\underline{t}) \\ -\gamma_2 & 0 & -\gamma_2 & -M'(\underline{t}) \\ -M'(\underline{t}) & -M'(\underline{t}) & -M'(\underline{t}) & 0 \end{bmatrix}$$

Using [Sydsæter, Hammond, Seierstad, and Strom \(2008\)](#) we have to compute two determinants:

$$B_2 := \begin{vmatrix} 0 & -\gamma_1 & -\gamma_2 \\ -\gamma_1 & -\gamma_1 & 0 \\ -\gamma_2 & 0 & -\gamma_2 \end{vmatrix} = \gamma_1 \gamma_2 (\gamma_1 + \gamma_2) > 0$$

$$B_3 := \begin{vmatrix} 0 & -\gamma_1 & -\gamma_2 & -M'(\underline{t}) \\ -\gamma_1 & -\gamma_1 & 0 & -M'(\underline{t}) \\ -\gamma_2 & 0 & -\gamma_2 & -M'(\underline{t}) \\ -M'(\underline{t}) & -M'(\underline{t}) & -M'(\underline{t}) & 0 \end{vmatrix} = [M'(\underline{t})]^2 [\gamma_1^2 + \gamma_1 \gamma_2 + \gamma_2^2] > 0$$

These determinants are neither both negative (local min) nor of alternate sign (local max). Thus, this stationary point is a saddle point.

• Let us assume that $M'(\underline{t}) = 0$. Solving $M'(\underline{t}) = 0$ for \underline{t} we get:

$$\int_{\alpha_1 D - \underline{t}}^{\bar{l}_1} dH(l) - \int_{\alpha_2 D - \Pi + \underline{t}}^{\bar{l}_2} dH(l) = 0$$

Then, the solutions \underline{t}^F lie in the interval $[\alpha_1 D, \Pi - D + \alpha_1 D]$. We simply obtain that ¹⁹

$$M(\underline{t}^F) = \Pi - D.$$

In other words, if this were a solution to the partnership problem, the agents would pay all the damages without trying to escape and would exert maximal levels of efforts.

Equation (2.14) and (2.15) write

$$2b + D - \lambda\gamma_1 = \gamma_1 e,$$

$$2b + D - \lambda\gamma_2 = \gamma_2 a.$$

Summing this two constraints and plugging this into the constraint gives:

$$\lambda = \frac{2b + D}{(\gamma_1 + \gamma_2)},$$

and then

$$e = \frac{\gamma_2(2b + D)}{\gamma_1(\gamma_1 + \gamma_2)}$$

$$a = \frac{\gamma_1((2b + D))}{\gamma_2(\gamma_1 + \gamma_2)}$$

The partnership's total revenue at this equilibrium writes:

$$V^F = \Pi - D + \frac{\gamma_1^2 + \gamma_1\gamma_2 + \gamma_2^2}{2\gamma_1\gamma_2(\gamma_1 + \gamma_2)}(2b + D)D^2.$$

Let L^* be the Hessian of the Lagrangian at the candidate solution:

$$L^* := \begin{bmatrix} -\gamma_1 & 0 & 0 \\ 0 & -\gamma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From [Luenberger and Ye \(1984\)](#) we have a local maximum if $\mathbf{y}^T L^* \mathbf{y} < 0$ for all $\mathbf{y} \in M = \{\mathbf{y} = (y_1, y_2, y_3) : (-\gamma_1, -\gamma_2, 0)\mathbf{y} = 0\}$. Here, we have

$$\mathbf{y}^T L^* \mathbf{y} = -\gamma_1 y_1^2 - \gamma_2 y_2^2 < 0.$$

Then we have a local maximum.

¹⁹Be careful, here other \underline{t} might solve $M'(\underline{t}) = 0$. Check what happens.

3. HOW TO REGULATE MODERN AIRPORTS?

(with David Martimort and Jérôme Pouyet)

Abstract

In addition to the provision of aeronautical services, modern airports are also in charge of providing commercial services to passengers as well as investing in their infrastructure. We investigate the optimal regulation of airports and its implementation. Optimal prices for aeronautical services and commercial services follow a Ramsey-Boiteux pricing rule. Traditional price-cap regulations of aeronautical and commercial activities fail to implement the optimal regulation as they do not provide sufficient incentives to invest. A successful implementation must rely on a price-cap formula supplemented by a subsidy policy specific to the regulation of investment. The choice between a single-till and a dual-till approach does not change the optimal regulation. We also investigate the consequences of the nature of the airport-airline relationship and of the observability of investment on the optimal regulation.

Keywords: *Airports, Price-cap Regulation, Non-Aeronautical Revenues, Vertical Separation.*

JEL Classification: L51, L93, L22.

3.1. INTRODUCTION

Until the 1980s, most airports were publicly-owned companies. Airport privatization began with the privatization of seven airports in the UK, and notably three in the London area (Heathrow, Gatwick, and Stansted). They were sold in 1987 by the UK government to a pure privately-owned company, British Airports Authority. Since then, many airports have been privatized in Europe (Copenhagen, Vienna, Rome for instance) as well as in New Zealand and Australia.¹ New waves of privatizations also occurred in Asia and South America more recently.² According to the Airports Council International, over 40% of European Airports were partially or fully privatized in 2016 and those account for about three-quarters of total passenger traffic in Europe. Privatization not only occurs through the transfer of ownership to private actors but also through the transfer of management. Many airports are publicly owned but privately operated as it is mainly the case in the US. Others such as Athens, Hamburg and Rome are partially for-profit entities in which private investors have stakes but limited to a minority interest. [Gillen \(2011\)](#) argues that minority private interests have often been sufficient to cause a change in management attitude and leads the airport to develop commercial value. The motive for privatization has generally been to improve operational efficiency and access to private sector financing and investment. [Vogel \(2006\)](#) presents evidence that privatized European airports exhibits better financial performance and are more cost-efficient than publicly-owned airports. [Tretheway \(2001\)](#) argues that privatization allows airports to raise equity capital and finance investments when they are needed. Publicly-owned airports, on the contrary, generally suffer from government priorities and a rigid budget process. Airport privatization also seems to foster strategic investment where public-owned entities were unable to create value. Although benefits from the privatization of ownership or orientation to a for-profit privately-managed airport seem to largely improve airport's cost-efficiency, adaptability and investment incentives it has also raised concerns among practitioners and economists. Unregulated privately-owned airports, however, are likely to pursue profit maximization as they operate on a relatively captive market of both passengers and airlines through their control of "an essential facility"

¹See for instance [Oum, Zhang, and Zhang \(2004\)](#) and [Oum, Adler, and Yu \(2006\)](#).

²See [Hooper \(2002\)](#) for airport privatization in Asia.

(runways, terminal buildings, navigational services). The traditional view is that airports behave as monopolies and almost every privatization was accompanied by some sort of price regulation (Tretheway, 2001). For instance, Bel and Fageda (2010) find evidence that nonregulated airports charge higher aeronautical charges than regulated ones. Non-aeronautical services such as commercial activities (retailing services, car parking, office rental) are also a major source of revenues. By now, they represent 40% of total airport revenues (see ACI (2017)) and their magnitude raises further concerns about the risk that airports may abuse market power.

Various forms of regulations have been adopted to tackle this issue although price-cap, rate-of-return, and light-handed regulations have been predominant. Price-cap regulation has been widely used in Europe (UK, France, Spain, Germany, for instance) and consists in setting a cap – reevaluated every three or five years by the regulator – on the price that the airport can charge for providing services.³ Although rate-of-return regulation has been criticized for its complexity and inefficiencies by several authors, it is still used by airports in Geneva, Zurich, Athens, Amsterdam and others (Reynolds, Sean, Veronese, and von Hinten Reed, 2018).⁴ An important question to be addressed by regulators concerns the scope of the regulation. Should it be solely used to set prices of aeronautical services or should it also be extended to prices of commercial activities? Should revenues generated by commercial activities be included in the computations of the price-cap formula? This last point is generally referred to as the problem of single-till versus dual-till regulation. Some have argued that price-cap regulation tends to be overcomplicated and as the presence of commercial activities drives airports to set relatively low aeronautical prices, it might be better to avoid unnecessary regulation (Beesley, 1999, Starkie, 2001). Other critics against price-cap regulation suggest that imposing low charges for aeronautical services lowers airports' incentives to invest in the infrastructure.⁵ Overall, there is still a lively debate around the various distortive effects of single and dual-till approaches in

³See Oum and Fu (2009) and Reynolds, Sean, Veronese, and von Hinten Reed (2018) for more detailed information.

⁴Tretheway (2001) and Kunz and Niemeier (2000) argue that rate-of-return is inefficient and tends to be complex.

⁵Investing in the infrastructure helps to reduce congestion or attract richer passengers and therefore can lead to an increase in passengers' willingness to pay. Under a price-cap regulation, an airport can imperfectly raise prices to capture this additional willingness to pay and thus does not fully internalize the benefits from investing in the infrastructure. On that matter see Czerny (2006) and Oum, Zhang, and Zhang (2004).

airport regulation.⁶

The difficulty of establishing the shape and scope of regulation for modern airports lies in their complexity. As mentioned above, airports carry out commercial services (or concessions) to passengers in addition to their core business of providing aeronautical services to airlines and passengers. The specific complementarity between the demands of those two activities plays a crucial role in the airport pricing decisions as we shall see below. Furthermore, airports are in charge of investment decisions in their infrastructure to improve the overall quality for passengers and to relieve congestion in the facilities. Finally, airports do not directly provide aeronautical services to passengers but instead charge airlines so that they can access the infrastructure. The final price of aeronautical services paid by passengers is therefore charged by the airlines and not the airport. Therefore, regulating prices for aeronautical services ultimately depends on the nature of the vertical relationship between the airport and airlines.⁷

In this paper, we investigate the design of the optimal airport regulation policy and its implementation in various scenarios. We allow for the possibility that prices of both aeronautical services and commercial activities as well as investment decisions can be jointly regulated. We assume that the airport sets the intermediary price for aeronautical services to the airline as well as the price of commercial activities. The airline sets the final price for aeronautical services to passengers. To take into account the specific complementarity between demands of aeronautical and commercial services, we assume that the consumption of commercial services is restricted to passengers. We further assume that the decision to consume aeronautical services is independent of a consumer's valuation for commercial services, that is, consumers decide to fly solely based on their valuation for aeronautical services.⁸ The airport is also responsible for investing in the

⁶Oum, Zhang, and Zhang (2004) provide empirical evidence on the distortions caused by single and dual till regulation; Czerny (2006) shows analytically that single-till generally dominates the dual-till approach but that none of them gives proper incentives for investments. Malavolti (2016) and Ivaldi, Sokullu, and Toru (2015) investigate this question by considering the airport as a platform in a two-sided market environment.

⁷We call "modern", airports which satisfy those main characteristics. Namely, airports that are in charge of activities outside their core business of aeronautical services, which are responsible for investments and that are in vertical relationships with airlines. Gillen (2011) uses a close definition to ours.

⁸Other authors have made similar albeit not equivalent assumptions about demand complementarity. For instance, Czerny (2006) assumes that consumption of commercial services is restricted to passengers but he also assumes that consumers decide to fly by taking into account the sum of the surplus generated by aeronautical and commercial activities. As a result, a positive surplus generated by the consumption of

infrastructure and we model the benefits of investment by assuming that it increases the passengers' value for aeronautical services. We investigate the case of observable and unobservable (moral hazard) investment decisions. The airport-airline relationship is vertically separated and we study two types of contracts. (i) The airport may offer a fixed price along with a unit price for providing aeronautical services to the airline and; (ii) only a unit price.

Our results are as follows. First, we characterize the optimal solution for the regulator when she can freely choose the prices of aeronautical and commercial services as well as the level of investment made by the airport. We show that optimal prices obey a Ramsey-Boiteux pricing rule and are both above their associated social marginal cost. We find that the presence of commercial activities only partially mitigates the unregulated airport's incentives to raise prices of aeronautical prices. As for the price of commercial services, we find that an unregulated airport would charge the monopoly price on commercial services as passengers are captive once they have entered the airport.

Assuming first observability of investment decisions and vertical integration between the airline and the airport, we investigate the implementation of the optimal regulation. We show that traditional regulations are likely to fail in the context of modern airports. For instance, price-cap regulation for both aeronautical and commercial services successfully implements Ramsey-Boiteux prices but fails to provide enough investment incentives to the airport. We propose a subsidy-penalty policy on investment decisions to supplement the price-cap regulation and fully implement the optimal regulation. Our analysis also contributes to the debate of whether revenues generated from commercial activities should be included in the price-cap formula (single-till *v.s.* dual till): We show that this choice is irrelevant and has no impact on the optimal regulation in the context of observable investment decisions.

Relaxing the assumption of vertical integration between the airport and the airline, our results point out that, when the airport can offer a contract with both a fixed access charge and a unit price for providing aeronautical services to the airline, the regulator

commercial service can compensate for a negative surplus on aeronautical services. Although it may be true for some passengers (business) or when several airports are available for the same aeronautical services, we believe that our assumption better reflects the consumer decision to fly.

can implement the optimal regulation as if the airport and the airline were a vertically integrated structure. However, when the airport can only offer a unit price for aeronautical services, the optimal regulation is modified and leads to an increase in the price of aeronautical services to passengers. In the absence of the fixed access charge, the airport is unable to fully extract the rent from the airline and the increase in the price of aeronautical services is necessary to satisfy the airport break-even condition. Interestingly, neither the optimal price for commercial services nor the rule to define the optimal level of investment decision is modified in that case.

Finally, we look at the case of unobservable investment decisions (moral hazard). Compared to the case where investment decisions are observable, we show that the regulator must set higher prices for both aeronautical and commercial activities to give the airport proper incentives to invest in the infrastructure. Intuitively, rising prices helps the airport to better appropriate the additional surplus generated by investing in the infrastructure and therefore provides incentives to do so. The optimal level of investment is, however, lower than in the observable case as the regulator wants to avoid an excessive price increase.

RELATED LITERATURE. Most of the recent literature on airport regulation takes into account the interdependence of aeronautical and non-aeronautical services. Whether prices of each activity should be regulated and whether revenues from non-aeronautical services should cover a portion of airports' costs are among the most debated issues. Following [Beesley \(1999\)](#), [Starkie \(2001\)](#) conjectures that the interdependence of aeronautical and non-aeronautical services suffices to temper airports' abuse of market power and therefore advocates abolition of price-cap regulation. However, several contributions (including [Zhang and Zhang \(2003\)](#); [Oum, Zhang, and Zhang \(2004\)](#); [Zhang, Fu, and Yang \(2010\)](#); [Yang and Zhang \(2011\)](#)) have shown that the presence of commercial activities only partially mitigates the incentives to set excessive prices of aeronautical services for unregulated airports. These studies build on the same interdependence assumption which was firstly introduced in a formal model by [Zhang and Zhang \(1997\)](#). Similarly to our approach, they assume that commercial activities do not affect individuals' decisions to fly and that their consumption is restricted to passengers. Unlike our model, however, their relationship is not micro-founded. [Czerny \(2006\)](#), on the other hand, assumes

that individuals take into account the consumption of commercial services when they decide whether to fly. Under this assumption, he shows that price-cap regulation can implement optimal prices only if both aeronautical and commercial services are regulated. Concerning revenues generated from commercial services, both theoretical and empirical works tend to favor the single-till regime at non-congested airports (Zhang and Zhang (1997); Czerny (2006); Yang and Zhang (2011); Czerny, Shi, and Zhang (2016)) whereas the dual-till regime seems preferable at congested airports (Oum, Zhang, and Zhang, 2004, Lu and Pagliari, 2004, Yang and Zhang, 2011). Another recent strand of the literature, such as Ivaldi, Sokullu, and Toru (2015), Malavolti (2016) and Malavolti and Marty (2017) investigates airport regulation by adopting a two-sided market perspective. Finally, vertical relationships and arrangements between airports and airlines have notably been investigated by Zhang, Fu, and Yang (2010), Fu, Homsombat, and Oum (2011) and Yang, Zhang, and Fu (2015).

The paper is organized as follows. Section 3.2 presents the model relying on the complementarity between aeronautical services and commercial activities. Section 3.3 derives the optimal regulation for prices of aeronautical services and commercial activities and for the level of investment when the airport and the airline are vertically integrated. Section 3.4 shows that the implementation of the optimal regulation of prices and investments fails when one only relies on price-cap regulation and that regulating investments through a marginal subsidy on invested capital is necessary. Section 3.5 investigates the consequences of the vertical separation between the airport and the airline on the optimal regulation policy. Section 3.6 examines how the optimal regulation is modified when investments are non-verifiable and we show that prices of aeronautical services and commercial activities are both impacted. All proofs are in the Appendix.

3.2. THE MODEL

Consider an airport providing both aeronautical and commercial services. We assume that the airport possesses market power in both activities. For convenience, we assume that there is only one airline using this airport.

We consider that revenues from commercial activities are conditional upon revenues

from aeronautical services. Indeed, passengers may want to consume commercial services once they entered the airport. It seems, however, unlikely that a consumer not interested in aeronautical services has a demand for the airport's commercial activities. Therefore, commercial activities are complementary to aeronautical services, but the reverse is not true.

The recent literature on airport-airline relationships has emphasized the importance of the relationship between aeronautical services and commercial activities. However, its nature differs from one paper to another. For instance, [Czerny \(2006\)](#) assumes that an individual decides to fly if its surplus from the consumption of aeronautical services plus the consumption of commercial activities is positive. This assumption implies that an individual who gets a negative surplus from flying would still want to travel if the positive rent from commercial services were to compensate for the negative rent from flying. [Ivaldi, Sokullu, and Toru \(2015\)](#) and [Malavolti \(2016\)](#) consider airports as two-sided platforms arguing that passengers are attracted by both aeronautical services and commercial activities whereas airlines prefer airports that are attractive to passengers. Although it may be true in some cases, airlines generally face severe competition to access airport facilities and airports tend to have significant market power.⁹ As a result, the airport may not take into account the airlines' side. Finally, [Oum, Zhang, and Zhang \(2004\)](#) propose a relationship between aeronautical services and commercial activities similar to what we are suggesting. Unlike our model, however, their relationship is not micro-founded.

DEMAND FOR AERONAUTICAL SERVICES. Suppose there is a continuum of individuals with valuation \tilde{v} for aeronautical services, where \tilde{v} is drawn from a cumulative distribution $F(\cdot, e)$ on $V = [0, \bar{v}]$, with a strictly positive density $f(\cdot, e)$ and no mass point. Note that both the cumulative distribution and the density of \tilde{v} depend upon the amount of investment e of the airport, as we will see below. Hereafter, an individual who decides to consume aeronautical services will be referred to as "a passenger."

INVESTMENT IN AIRPORT INFRASTRUCTURE. The airport can invest an amount $e \in \mathbb{R}_+$ to enhance the quality of its infrastructure or to relieve congestion. Investing in airport

⁹See [Oum and Fu \(2009\)](#) for a discussion on airports' market power and airline market structure.

infrastructure directly leads to an increase in consumer demand for aeronautical services. As a result, commercial activities also indirectly benefit from the investment.

This feature appears in the specification of the cumulative distribution function $F(\cdot, e)$ of consumers' valuations for aeronautical services. We assume that $F(\cdot, \hat{e})$ first-order stochastically dominates $F(\cdot, e)$ for any $\hat{e} > e$, that is, $F(v, \hat{e}) \leq F(v, e)$ for any $v \in V$ and $\hat{e} > e$. Intuitively, higher investment levels make higher valuations for aeronautical services more likely. Commercial activities are also positively affected by investment decisions as for given prices p and p_0 , $D(p, \hat{e}) \geq D(p, e)$ so that $D_0(p, p_0, \hat{e}) \geq D_0(p, p_0, e)$ for any $\hat{e} > e$.¹⁰

DEMAND FOR COMMERCIAL SERVICES. We assume that only passengers can consume commercial services. Therefore, once a passenger is in the airport, we suppose that he has a valuation \tilde{v}_0 for commercial services, where \tilde{v}_0 is drawn from a cumulative distribution $G(\cdot)$ on $V_0 = [0, \bar{v}_0]$, with a strictly positive density $g(\cdot)$ and no mass point.¹¹

TOTAL DEMAND. Let p and p_0 denote unit prices of aeronautical and commercial services, respectively. The airport sets p_0 whereas the airline sets p . Following our assumption of unidirectional complementarity between aeronautical and commercial services, the indirect utility of a consumer can be expressed as:

$$\max\{\tilde{v} - p, 0\} + \mathbb{1}_{\{\tilde{v} - p \geq 0\}} \max\{\tilde{v}_0 - p_0, 0\}.$$

¹⁰Under this assumption, the level of congestion can be captured by changing the specification of the cumulative distribution function. A congested airport with CDF $\tilde{F}(v, e)$ is such that $\tilde{F}(v, 0) \geq F(v, 0)$ for all $v \in V$, where $F(v, e)$ is the CDF of a less congested airport. In other words, in the absence of investment, a less congested airport has a demand of a *lesser* quality than a less congested airport. The CDF can also be adjusted to reflect the marginal utility of investment at congested and less congested airports: For instance, if $\frac{\partial \tilde{F}}{\partial e}(v, e) \leq \frac{\partial F}{\partial e}(v, e)$ for all $v \in V$ and $e \in [0, 1]$, one euro of investment in the congested airport enhances its demands relatively more than the investment of one euro in the less congested airport.

¹¹We assume that investment in the airport infrastructure does not directly affect the demand for commercial services. This assumption could be relaxed by assuming that total investment is $e + e_0$ where e and e_0 are investment levels specific to aeronautical and commercial services, respectively. Demand for commercial services could then be written as $G(v_0, e_0)$, social marginal cost becomes $c_s(p_0, e_0)$ and price elasticity of commercial services writes $\zeta(p_0, e_0)$. Optimal prices and investment level in aeronautical services follow equations (3.9), (3.10) and (3.11) with the slight change in social marginal cost and price elasticity of commercial services. An additional first-order condition defines the optimal level of investment in commercial services: $-(p_0 - c_0) \frac{\partial G}{\partial e_0}(p_0, e_0) - \frac{1}{1+\lambda} \int_{p_0}^{\bar{v}} \frac{\partial G}{\partial e_0}(v_0, e_0) dG(v_0, e_0) = 1$. An increase in the investment level specific to commercials services, decreases the social marginal cost of aeronautical services and therefore decreases the optimal price of aeronautical services.

The indicator function $\mathbb{1}$ captures the unidirectional complementarity between aeronautical services and commercial activities. This formulation implies that: (i) only passengers (consumers with $\tilde{v} - p \geq 0$) can benefit from the consumption of commercial services, and (ii) when choosing whether to fly or not, consumers only take into account the surplus they can derive from consumption of aeronautical services.¹²

From our assumption of unidirectional complementarity, the demand for aeronautical services depends only upon the price p set by the airline, that is:

$$D(p, e) = \Pr\{\tilde{v} \geq p|e\} = 1 - F(p, e).$$

The price elasticity of this demand is denoted by:

$$\varepsilon(p, e) = -\frac{pD'(p, e)}{D(p, e)} = \frac{pf(p, e)}{1 - F(p, e)}.$$

Although we do not explicitly model competition in the airline industry, this feature can be captured through the elasticity of demand. For a given price p and level of investment e , the more competitive the airline industry is, the more elastic demand is.¹³

Demand for commercial activities, however, depends upon both the price of aeronautical services p and the price of commercial activities p_0 . For a given p_0 , a passenger buys commercial services with probability $1 - G(p_0)$. Demand for commercial activities is obtained by taking the unidirectional complementarity $\mathbb{1}_{\{\tilde{v}-p \geq 0\}}$ into account and integrating over V :

$$D_0(p, p_0, e) = \int_V \mathbb{1}_{\{\tilde{v}-p \geq 0\}} [1 - G(p_0)] dF(v, e) = D(p, e) [1 - G(p_0)].$$

¹²By way of comparison, the indirect utility of a consumer in Czerny (2006) can be written as $\mathbb{1}_{\{\tilde{v}-p+\tilde{v}_0-p_0 \geq 0\}} [\tilde{v} - p + \max\{\tilde{v}_0 - p_0, 0\}]$ in our model. Therefore, it satisfies (i) as consumers can enjoy commercial services only if they decide to fly. But it does not satisfy (ii) as consumers can decide to fly even if they get a negative surplus from aeronautical services as long as it is compensated by a positive surplus from consumption of commercial services.

¹³Competition can be captured by changing the specification of the cumulative distribution function F . Indeed, assume that each consumer has an outside option for aeronautical services whose net valuation is a random variable \tilde{w} drawn from a cumulative distribution $H(\cdot)$, with density $h(\cdot)$. The probability that the consumer is willing to pay p for aeronautical services, or equivalently, the residual demand, is as follows: $\tilde{D}(p, e) = \Pr\{\tilde{v} - p \geq \tilde{w}\} = \int_0^{\tilde{v}} H(v - p) f(v, e) dv$. Therefore, the elasticity of demand can be expressed as: $\tilde{\varepsilon}(p, e) = -\frac{p\tilde{D}'(p, e)}{\tilde{D}(p, e)} = \frac{p \int_0^{\tilde{v}} h(v-p) f(v, e) dv}{1 - \int_0^{\tilde{v}} H(v-p) f(v, e) dv}$.

The elasticity of demand with respect to price can be expressed as a function of p_0 only:

$$\zeta(p, p_0) \equiv \zeta(p_0) = -\frac{p_0 \frac{\partial D_0}{\partial p_0}(p, p_0, e)}{D_0(p, p_0, e)} = \frac{p_0 g(p_0)}{1 - G(p_0)}.$$

To ensure the quasi-concavity of optimization problems and monotonicity of equilibrium prices we make the following assumptions.

Assumption 6 *Monotone hazard rate property:*

$$\frac{d}{dv} \left(\frac{1 - F(v, e)}{f(v, e)} \right) \leq 0 \quad \forall v \in V \text{ and } \frac{d}{dv_0} \left(\frac{1 - G(v_0)}{g(v_0)} \right) \leq 0 \quad \forall v_0 \in V_0.$$

Economically, these assumptions guarantee that the elasticities for aeronautical services and commercial activities are both increasing in the price of the service concerned.

Assumption 7 *Consumer demand for aeronautical services increases in the amount of investment at an increasing marginal rate:*

$$\frac{\partial F}{\partial e}(v, e) \leq 0, \quad \frac{\partial^2 F}{\partial e^2}(v, e) \geq 0, \quad \forall v \in V.$$

The first part of this assumption guarantees that increasing the level of investment increases the demand for aeronautical services (in the first-order stochastic demand sense). The second ensures the concavity of optimization program and the validity of the first-order approach in the case of non-observable investments. Economically, it means that additional investments are less and less effective to increase the demand of aeronautical services.

Costs. We assume that aeronautical and commercial services are produced at constant marginal costs $c \in \mathbb{R}_+$ and $c_0 \in \mathbb{R}_+$, respectively. Hence, the cost structure exhibits neither economies of scope nor economies of scale in our model. However, we can still interpret the investment e as a fixed cost for setting up airport infrastructure. For future reference, let $c_{pr}(p_0) = c - (p_0 - c_0)(1 - G(p_0))$ denote the private marginal cost of production. This cost takes into account the marginal cost of aeronautical services minus the profits generated by commercial activities.

3.3. BENCHMARK

We now present the first-best solution to the regulator's problem. To this end, the regulator must be able to (i) freely set prices for aeronautical services and commercial activities (p, p_0) and (ii) suggest an amount of investment e to the airport. Note that this last point implicitly requires observability of investment decisions e . As the regulator controls both p and p_0 , we can consider the airport-airline relationship as if it were vertically integrated. Finally, we assume that the regulator aims to maximize social welfare subject to the profit of the integrated structure (airport-airline) being nonnegative.

Relying on the results of the first-best solution, we aim to shed light on the debate around airport regulation. For instance, several authors argue that demand complementarity between aeronautical and commercial services favorably mitigates the airport's incentives to excessively raise prices. The presence of commercial services indeed alleviates the airport's incentives to increase the price for aeronautical services, but the effect is only partial. Worst still, we find that an unregulated airport sets the monopoly price for commercial activities unconditional upon the competitiveness of aeronautical services. Those findings directly stem from our assumption of unidirectional complementarity between aeronautical services and commercial activities.

The present section is organized as follows. First, we briefly present the problem of an unregulated airport. Then, we derive the optimal regulation policy for prices of aeronautical and commercial services and we examine how our results might shed some light on the debate about airport regulation. Finally, we investigate the optimal investment rule when e is contractible.

3.3.1. *The Unregulated Airport*

Let us first consider the simple case in which the airport can freely choose the prices of both aeronautical and commercial services as well as investment in the infrastructure. We assume that the airport and the airline are vertically integrated so that their joint profit writes:

$$\mathcal{P}\mathcal{R}(p, p_0, e) = [1 - F(p, e)] [p - c + (p_0 - c_0)(1 - G(p_0))] - e.$$

This expression contains the airline's profit $(1 - F(p, e))(p - c)$ and the airport's profit (net of investment costs) $(1 - F(p, e))(p_0 - c_0)(1 - G(p_0)) - e$. In the absence of regulation, the airport chooses (p, p_0, e) to maximize $\mathcal{PR}(p, p_0, e)$. The solution to this problem is as follows.

Proposition 19 *The unregulated airport sets prices:*

$$\frac{p^m - c_{pr}(p_0^m)}{p^m} = \frac{1}{\epsilon(p^m, e^m)}, \quad (3.1)$$

$$\frac{p_0^m - c_0}{p^m} = \frac{1}{\zeta(p_0^m)}, \quad (3.2)$$

and investment decisions solve:

$$-\frac{\partial F}{\partial e}(p^m, e^m)(p^m - c_{pr}) = 1. \quad (3.3)$$

Notice that the price of aeronautical services depends on the private marginal cost of production, c_{pr} , and not only on the marginal cost to produce aeronautical services c . As $c_{pr} < c$, it follows that p is lower than if no commercial services were carried out at the airport and thus the unregulated airport takes into account the complementarity between services. As we will see below, however, the unregulated airport only partially internalizes the complementarity between services and sets an excessive price for aeronautical services. From (3.2), it is clear that the airport sets the monopoly price for commercial services and the demand for aeronautical services plays no role in that.

3.3.2. Optimal Price Regulation

Let us now turn to the optimal regulation for prices of aeronautical and commercial services. First, we still assume that the airport and the airline are vertically integrated. Additionally, we allow the regulator to provide public subsidies T to the integrated structure in order to ensure the nonnegativity of their joint profit. For convenience, define:

$$\mathcal{PR}(p, p_0, e, T) = \mathcal{PR}(p, p_0, e) + T. \quad (3.4)$$

Second, let us define consumer surplus. It is simply given by the sum of the surplus generated by aeronautical services and the surplus generated by commercial services.

Again, the unidirectional complementarity must be taken into account, that is, the surplus generated by commercial activities is conditional upon the consumption of aeronautical services. Therefore, we obtain:

$$\mathcal{CS}(p, p_0, e) = \int_{v \geq p} (v - p) dF(v, e) + (1 - F(p, e)) \int_{v_0 \geq p_0} (v_0 - p_0) dG(v_0). \quad (3.5)$$

Finally, we can define social welfare as the sum of consumer surplus and the profit of the integrated structure, net of the social cost of public subsidies:

$$SW(p, p_0, e, T) = \mathcal{CS}(p, p_0, e) + \mathcal{PR}(p, p_0, e, T) - (1 + \lambda)T, \quad (3.6)$$

where the parameter $\lambda > 0$ stands for the cost of public funds.¹⁴ The optimization problem of the regulator writes as follows:

$$\begin{aligned} (\mathcal{OS}) : \quad & \max_{(p, p_0, e, T)} SW(p, p_0, e, T) \\ & \text{s.t. } \mathcal{PR}(p, p_0, e, T) \geq 0. \end{aligned} \quad (3.7)$$

Before deriving optimal prices, we should stress that the social marginal cost of aeronautical services must take into account the consumer surplus and the profit generated by commercial activities arising from the demand for aeronautical services. Consequently, the social marginal cost of aeronautical services is both lower than the marginal cost of production, c , and lower than the private cost of production of the integrated structure. Formally, recall that the private cost of production writes as: $c_{pr}(p_0) = c - (p_0 - c_0)(1 - G(p_0))$. This is the cost of production of aeronautical services minus the profits generated by commercial activities. However, this cost does not include the social costs and benefits, and, therefore, we define the social marginal cost of aeronautical services as:

$$c_s(p_0) = c_{pr}(p_0) - \frac{s_0(p_0)}{1 + \lambda} < c, \quad (3.8)$$

where $s_0(p_0) = \int_{v_0 \geq p_0} (v_0 - p_0) dG(v_0)$ denotes the consumer surplus generated by commercial activities. The social marginal cost of aeronautical services is lower than the associated private cost as it incorporates the consumer surplus derived from commercial

¹⁴In developed countries, this cost is about 0.3. See [Oum, Waters, and Yong \(1992\)](#).

activities, weighted by the cost of public funds.

Proposition 20 *The socially optimal regulatory policy of aeronautical services and commercial activities relies on a Ramsey-Boiteux pricing rule. Optimal prices write as:*

$$\frac{p^{rb} - c_s(p_0^{rb})}{p^{rb}} = \frac{\lambda}{1 + \lambda} \frac{1}{\varepsilon(p^{rb}, e^{rb})} > 0, \quad (3.9)$$

$$\frac{p_0^{rb} - c_0}{p_0^{rb}} = \frac{\lambda}{1 + \lambda} \frac{1}{\zeta(p_0^{rb})} > 0, \quad (3.10)$$

and are both greater than their associated social marginal cost.

Notice that the optimal price for aeronautical services relies on the associated social marginal cost of production $c_s(\cdot)$ and not only on the private marginal cost as in the unregulated case. This result is intuitive as the difference between the two costs is precisely the consumer surplus generated by commercial activities, weighted by the cost of public funds, which the unregulated airport obviously ignores.

In light of the above results, we are able to revisit some *common wisdom* of the debate about airport regulation.¹⁵

Common Wisdom 1 *For aeronautical services and commercial activities, prices must equal marginal costs.*

On the contrary, our results on the optimal regulation suggest that both prices must be greater than their associated social marginal cost. For aeronautical services, pricing at marginal cost would ignore additional profits and surplus that could be derived from an increase in passengers also consuming commercial services. However, it is not optimal either to price aeronautical and commercial services exactly at their social marginal cost as the cost of public funds is positive. The regulator wants to avoid giving too large a subsidy to ensure the break-even condition and therefore prefers, to a certain extent, that the airport makes some profits on each activity.

¹⁵We draw upon Gillen (2011). He offers a presentation of the evolution of airport governance and reports that entities such as the ICAO (International Civil Aviation organization) and IATA (International Air Transport Association) have traditionally considered that pricing at marginal cost is efficient, that pricing below marginal cost indicates predation or that high price-cost margins indicates market power.

Common Wisdom 2 *The price of aeronautical services must cover the marginal cost of production.*

From Proposition 20, we see that the price of aeronautical services does not necessarily cover the marginal cost of production. Indeed, the price of aeronautical services must be greater than the social marginal cost of this activity, which includes revenues and consumer surplus generated by commercial activities.

The regulator's choice to increase p^{rb} is a trade-off between an increase in the profits of the integrated structure motivated by the nonnegativity constraint and a decrease in the consumer surplus generated by aeronautical services and commercial activities. Indeed $p^{rb} - c_s(p_0) = \frac{\lambda}{1+\lambda} \frac{p^{rb}}{\epsilon(p^{rb}, e^{rb})}$ and:

$$p^{rb} - c_{pr}(p_0^{rb}) = \frac{\lambda}{1+\lambda} \frac{p^{rb}}{\epsilon(p^{rb}, e^{rb})} - \frac{s_0(p_0^{rb})}{1+\lambda}.$$

When the demand for aeronautical services is very elastic or when the consumer surplus generated by commercial activities is significant, it is possible that $p^{rb} - c_{pr}(p_0) < 0$. In that case, it is clear that the profit of the integrated structure is negative without a subsidy. This suggests that, even with a positive cost of public funds, the regulator may want to price aeronautical services below marginal cost and subsidize the integrated structure so as it breaks even. Notably, when commercial services generate a high consumer surplus, the regulator is more likely to set the price of aeronautical services below the marginal cost of production and must, therefore, provide a higher subsidy to ensure that the integrated structure breaks even.

Small regional airports and big international hubs do not face the same level of demand and competition. Therefore, their ability to derive revenue from aeronautical and commercial services is likely to vary according to their size. Some regional airports may face more competition than larger hubs so that price elasticity for aeronautical services is larger for small airports. In that case, our optimal regulation would tend to have to subsidize more small airports as the elasticity of demand would require lower aeronautical prices. Small airports could also be modeled by assuming that their fixed costs are larger relative to the demand they face. In our case, optimal prices and investment levels are be

independent of additional fixed cost so that airport's profits would be inversely related to those fixed cost, and larger subsidies would be needed for the airport to break-even.

Common Wisdom 3 *The intensity of competition can be assessed through the profits generated by commercial activities.*

Competition does not affect profit margins. From Proposition 20, the price-cost margins $\frac{p^{rb}-c_s}{p^{rb}}$ and $\frac{p_0^{rb}-c_0}{p_0^{rb}}$ depend upon the inverse of the elasticity of aeronautical services and commercial activities, respectively. Therefore, more competition in the airline industry increases the elasticity of demand for aeronautical services and decreases the price-cost margin of the airline.

However, the price-cost margin of commercial activities is not affected by this change in competition in the airline industry. Again, this stems directly from the unidirectional externality between aeronautical services and commercial activities. Once the consumer is in the airport, the airport enjoys monopoly power on commercial services, regardless of the intensity of competition between airlines. For the same reasons, equivalent commercial activities outside the airport do not compete with commercial activities supplied by the airport.

3.3.3. Optimal Level of Investment

Let us now turn to the optimal regulation of investment. We assume here that the investment level e is observable and thus contractible. We investigate the unobservable case in Section 3.6.

Proposition 21 *The optimal level of investment e^{rb} solves:*

$$-\frac{\partial F}{\partial e}(p^{rb}, e^{rb}) (p^{rb} - c_s(p_0^{rb})) - \frac{1}{1+\lambda} \int_{v \geq p^{rb}} \frac{\partial F}{\partial e}(v, e^{rb}) dv = 1. \quad (3.11)$$

When Assumption 7 holds, a marginal increase in the level of investment e directly leads to an increase in the demand for aeronautical services. Note that the first term in the left-hand side of (3.11) can be decomposed as:

$$-\frac{\partial F}{\partial e}(p^{rb}, e^{rb}) (p^{rb} - c) - \frac{\partial F}{\partial e}(p^{rb}, e^{rb}) (c - c_s(p_0^{rb})).$$

When e increases, $-\frac{\partial F}{\partial e}(p^{rb}, e^{rb})(p^{rb} - c)$ represents the increase of the airline's profit on the last unit sold whose sign is *a priori* ambiguous. The term $-\frac{\partial F}{\partial e}(p^{rb}, e^{rb})(c - c_s(p_0^{rb}))$ captures the positive benefit of an increase in e on the consumer surplus generated from commercial activities. Finally, $\frac{1}{1+\lambda} \int_{v \geq p^{rb}} \frac{\partial F}{\partial e}(v, e^{rb})$ represents the positive impact on the surplus of supra-marginal consumers, that is, consumers who would still buy aeronautical services even if their price were to increase. When the airport invests in quality, the mass of these supra-marginal consumers increases.

3.4. FAILURES OF TRADITIONAL REGULATIONS

We now turn to the question of implementing the optimal regulation. We investigate the traditional use of price-cap regulation and show that it fails to provide incentives for implementing the optimal level of investment. The sole use of a price-cap regulation only allows implementation of prices satisfying equations (3.9) and (3.10), but for a suboptimal level of investment $e \neq e^{rb}$. We then show that an additional policy must supplement the price-cap regulation. We also investigate the traditional rate-of-return regulation and dual-till versus single-till approaches.

3.4.1. Implementation of Optimal Prices

Let \bar{p} denote the price-cap chosen by the regulator. The airport is free to choose any combinations of prices and level of investment (p, p_0, e) as long as the following condition is satisfied:

$$w p + w_0 p_0 \leq \bar{p}, \quad (3.12)$$

where the coefficients (w, w_0) are also chosen by the regulator.¹⁶

We still consider that the airport-airline pair is vertically integrated. Hence, the

¹⁶This "global price-cap" formula differs from the usual price-cap formula used by regulation authorities. Czerny (2006) reports that, in practice, only aeronautical services are subject to the price-cap formula. However, it is easy to see that if we assume that the price-cap formula only applies to aeronautical services, the airport will choose the monopoly price for commercial services which is strictly higher than the Ramsey-Boiteux price defined by (3.10). Hence, restricting the price-cap formula only to aeronautical services would surely lead to a failure of the implementation of optimal prices and investment level. For more on the global price-cap formula, see Laffont and Tirole (1993) and Laffont and Tirole (2000) who advocate its use.

integrated structure faces the following optimization problem:

$$\begin{aligned} \max_{(p, p_0, e)} \quad & [1 - F(p, e)] [p - c + (p_0 - c_0)(1 - G(p_0))] - e \\ \text{s.t.} \quad & wp + w_0 p_0 \leq \bar{p}. \end{aligned}$$

Let μ denote the Lagrange multiplier of the price-cap constraint (3.12). First-order conditions with respect to p and p_0 write as follows:

$$\frac{\hat{p} - c_{pr}}{\hat{p}} = \frac{1}{\varepsilon(\hat{p}, \hat{e})} - \frac{\mu w}{\hat{p} f(\hat{p}, \hat{e})'} \quad (3.13)$$

$$\frac{\hat{p}_0 - c_0}{\hat{p}_0} = \frac{1}{\zeta(\hat{p}_0)} - \frac{\mu w_0}{\hat{p}_0 (1 - F(\hat{p}, \hat{e})) g(\hat{p}_0)'} \quad (3.14)$$

and the optimal level of investment is given by:

$$-\frac{\partial F}{\partial \hat{e}}(\hat{p}, \hat{e}) (\hat{p} - c_{pr}(\hat{p}_0)) = 1. \quad (3.15)$$

Notice first that even if prices were set at their optimal regulated levels $\hat{p} = p^{rb}$ and $\hat{p}_0 = p_0^{rb}$, condition (3.15) would differ from (3.11). It follows that even if price-cap regulation successfully implements optimal prices, it surely fails to implement an optimal investment level.

Assume for the moment that the integrated structure chooses \hat{e} according to (3.15) and let us implement optimal prices for this particular level of investment. That is, let us implement prices that satisfy equations (3.9) and (3.10) for $e = \hat{e}$.

Proposition 22 *The following choice of (w, w_0, \bar{p}) :*

$$\begin{aligned} w^*(\hat{e}) &= 1 - F(p^{rb}, \hat{e}) + s_0(p_0^{rb})f(p^{rb}, \hat{e}), \\ w_0^*(\hat{e}) &= (1 - F(p^{rb}, \hat{e}))(1 - G(p_0^{rb})), \\ \bar{p}^*(\hat{e}) &= w^*(\hat{e})p^{rb} + w_0^*(\hat{e})p_0^{rb}, \end{aligned}$$

implements Ramsey-Boiteux prices defined by (3.9) and (3.10) for the investment level $e = \hat{e}$.

Therefore, a traditional price-cap regulation does not provide any incentives to achieve the optimal level of investment. It is therefore not sufficient to implement the optimal regula-

tion. Price-cap regulation can, however, achieve the Ramsey-Boiteux prices corresponding to the particular investment level $e = \hat{e}$.

These results are summarized in the next proposition.

Proposition 23 *A price-cap regulation alone can implement Ramsey-Boiteux pricing only for the equilibrium level of investment $e = \hat{e}$ defined by equation (3.15). This level of investment, however, differs from the optimal one defined by (3.11) so that the optimal regulation cannot be implemented with the sole use of price-cap regulation.*

This suggests that airports subject to price-cap regulation still have weak incentives to invest. Ramsey-Boiteux pricing can be achieved by the price-cap formula but differs from the optimal regulation prices as they apply to an airport choosing $\hat{e} \neq e^{rb}$.¹⁷ Successful implementation of the optimal regulation policy must then also take into account the implementation of investment levels.

3.4.2. Regulation of Investment Decisions

We now examine the implementation of the optimal level of investment given by equation (3.11). We first investigate the traditional rate-of-return regulation and show that it is prone to over-investment from the airport. We then propose a subsidy-penalty regulation of investment to supplement the previous price-cap regulation in order to implement the optimal regulation policy.

RATE-OF-RETURN REGULATION. Under a rate-of-return regulation, the integrated structure freely chooses the level of investment e for which it has a guaranteed rate of s on invested capital.¹⁸ Formally, for given (p, p_0) , the optimization problem of the integrated structure is the following:

$$\begin{aligned} \max_e \quad & [1 - F(p, e)] [p - c + (p_0 - c_0)(1 - G(p_0))] - e \\ \text{s.t.} \quad & [1 - F(p, e)] [p - c + (p_0 - c_0)(1 - G(p_0))] \leq s \cdot e, \end{aligned} \quad (3.16)$$

¹⁷The sole use of a price-cap regulation fails to achieve optimal prices and level of investment as equation (3.9) depends upon e so that if $e \neq e^{rb}$ it follows that $p^{rb}(\hat{e}) \neq p^{rb}(e^{rb})$. However, the price of commercial activities is independent of e and is therefore at its optimal level.

¹⁸So far, we have normalized, without loss of generality, the cost of capital to 1. All results can be easily generalized to a cost of capital $r > 0$.

where constraint (3.16) stands for the rate-of-return constraint with a maximum guaranteed rate s chosen by the regulator.

When the rate of return s is greater than the cost of capital, which is equal to 1 in our case, the rate-of-return constraint is binding at the optimum. Therefore, the optimization problem of the integrated structure can be simplified to:

$$\max_e (s - 1)e.$$

This clearly indicates that the integrated structure has an incentive to over-invest as soon as the rate-of-return regulation guarantees a rate s greater than the cost of capital. This well-known effect is commonly referred to as the [Averch and Johnson \(1962\)](#) effect.

Proposition 24 *Rate-of-return regulation gives the airport incentives to over-invest even when prices are set at their socially optimal levels.*

SUBSIDY-PENALTY POLICY. To achieve the optimal regulation of investment, we propose the following solution. The regulator offers a subsidy when the level of investment is below the optimal level of investment e^{rb} and imposes a penalty in case of overinvestment. Let s denote the marginal subsidy-penalty. The optimization problem of the integrated structure becomes:

$$\max_e [1 - F(p, e)] [p - c + (p_0 - c_0)(1 - G(p_0))] - e - s (e - e^{rb}),$$

where $-s(e - e^{rb})$ represents the amount of the subsidy when $e < e^{rb}$ and the amount of the penalty when $e > e^{rb}$.

The first-order condition of this problem with respect to e writes as:

$$-\frac{\partial F}{\partial e}(p, e) [p - c + (p_0 - c_0)(1 - G(p_0))] - s = 1. \quad (3.17)$$

It suffices now to set the optimal marginal subsidy s^{rb} equal to:

$$s^{rb} = \frac{1}{1 + \lambda} \int_{p^{rb}}^{\bar{v}} \frac{\partial F}{\partial e}(v, e^{rb}) dv. \quad (3.18)$$

and to choose the price-cap and weights $(\bar{p}^*(e^{rb}), w^*(e^{rb}), w_0^*(e^{rb}))$ to ensure that first-order conditions (3.13), (3.14) and (3.17) yield optimal prices (p^{rb}, p_0^{rb}) and the optimal level of investment e^{rb} defined by equations (3.9), (3.10) and (3.11), respectively.

Proposition 25 *The optimal regulation of prices and level of investment can be implemented by combining a price-cap regulation of aeronautical and commercial prices and a subsidy-penalty policy on investment.*

We insist on the fact that any price-cap regulation must go along with a marginal subsidy-penalty on invested capital policy that in turn requires that prices are set at their socially optimal levels to achieve the optimal level of investment. Omitting one dimension of this policy affects both prices and level of investments. In any case, a rate-of-return regulation leads to over-investment and should be avoided.

Last, observe that the optimal marginal subsidy s^{rb} given by (3.18) is exactly equal to the positive impact of investment on supra-marginal consumers, which is not internalized by an unregulated airport.

3.4.3. Single-till Versus Dual-till Regulation

Any form of regulation requires non-negativity of the airport's revenues. As revenues from commercial activities are now significant for modern airports, it raises the question of whether we should include revenues generated from commercial activities into the price-cap formula. This question is commonly referred to as the single-till versus the dual-till approach. The first one includes commercial revenues into the airport's total revenues to compute the price-cap formula, whereas the second does not.

Single-till and dual-till approaches have been widely discussed in the literature and the conclusions differ from one paper to another. For instance, [Beesley \(1999\)](#) is one of the first to attack the single-till approach but he also recognizes the difficulties to adopt a dual-till approach in practice. On the contrary, [Starkie \(2001\)](#) supports the dual-till approach, arguing that it could reduce some distortions on aeronautical prices and on investment incentives induced by the single-till approach.¹⁹ In this section, we address

¹⁹[Oum, Zhang, and Zhang \(2004\)](#) are in line with [Starkie \(2001\)](#) while [Lu and Pagliari \(2004\)](#) and [Czerny \(2006\)](#) argue that the single-till approach dominates the dual-till one. See [Czerny \(2006\)](#) for a more detailed discussion.

this question and find that, when investments are verifiable, the single-till approach and the dual-till are equivalent so that the distinction between the two of them is irrelevant.

Observe that, so far, we have implicitly assumed a single-till regime for the price-cap regulation, that is, revenues from aeronautical services as well as revenues from commercial services were included in the break-even condition. Let us now assume a dual-till regime wherein the revenues from commercial activities must cover a fraction αe of the investment and those from aeronautical services must cover a fraction $(1 - \alpha)e$, where $\alpha \in [0, 1]$. Notice that the cases for which $\alpha < 1$ correspond to an hybrid-till regime while $\alpha = 1$ corresponds to a pure dual-till regime. This specification implicitly assumes that the investment level is verifiable. The first budget constraint associated with the revenues generated by commercial activities writes as:

$$(1 - F(p, e))(1 - G(p_0))(p_0 - c_0) - \alpha e \geq 0. \quad (3.19)$$

Likewise, revenues generated by aeronautical services must cover the other fraction of investments $(1 - \alpha)e$ and the following budget constraint must hold:

$$(1 - F(p, e))(p - c) - (1 - \alpha)e \geq 0. \quad (3.20)$$

The problem of the regulator is then as follows:

$$\begin{aligned} \max_{(p, p_0, e, \alpha)} \quad & \mathcal{SW}(p, p_0, e) \\ \text{s.t.} \quad & (1 - F(p, e))(1 - G(p_0))(p_0 - c_0) - \alpha e \geq 0 \\ & (1 - F(p, e))(p - c) - (1 - \alpha)e \geq 0. \end{aligned} \quad (3.21)$$

When the level of investment e is verifiable, the regulator is able to choose how to allocate the investment cost on each budget constraint by setting α . The allocation of the investment cost will depend upon which constraint is “more likely” to bind, that is, the constraint which is the hardest to satisfy. At the optimum, the opportunity cost of allocating one additional unit of investment must be the same for the two budget constraints. We then obtain the following result.

Proposition 26 *When investments are verifiable and the airport and the airline are vertically*

integrated, the optimal regulation of aeronautical services, commercial activities and investment under a dual-till approach is identical to the one obtained in Proposition 20.

Notice that one can also interpret this result in the following way: when investment is verifiable, the optimal regulation does not depend upon the choice of a single-till regime or a dual-till regime.

3.5. VERTICAL SEPARATION

In this section, we relax the assumption that the airport and the airline are vertically integrated. This relationship is now characterized by a contract between the two entities. The airport provides aeronautical services to the airline (landing rights, aircraft parking areas, airport taxiways, passenger facilities) and the airline is responsible for charging prices to consumers. Investment decisions, however, are still carried out by the airport.

We investigate how the optimal regulation is modified for two types of contracts between the airport and the airline. In both cases, we assume that the airport can offer a contract specifying a unit price for aeronautical services, but may or may not be able to set a fixed access charge to its facilities. For simplicity, we also assume that the airport has all the bargaining power in the relationship, i.e., the airport makes a take-it-or-leave-it offer to the airline.²⁰

3.5.1. Fixed Access Charge

First, we consider that the airport can set both a fixed access charge A and a unit price w for aeronautical services. When the airline sets p , demand for aeronautical services is $1 - F(p, e)$ so that it has to buy the same quantity of aeronautical services to the airport at unit price w . Formally, the airline solves:

$$\max_p (p - w - c)(1 - F(p, e)) - A.$$

²⁰The assumption that the airport has all the bargaining power can be justified in environments in which airlines are engaged in fierce competition to access to the airport facilities. Moreover, airlines may have limited access to close substitutes although this idea may not hold for low-cost carriers. Major airports may also have significant market power over dominant carriers due to the non-substitutability and to the cost of moving away from major hubs. See [Gillen, Oum, and Tretheway \(1988a\)](#) and [Oum and Fu \(2009\)](#) for detailed discussions.

Notice that the price p set by the airline depends upon the level of investment e chosen by the airport. The first-order condition of this problem writes as follows:

$$p = w + c + \frac{1 - F(p, e)}{f(p, e)}. \quad (3.22)$$

When Assumption 6 holds, there is a one-to-one correspondence between w and p .²¹ It follows that choosing the level of investment e and the unit price w uniquely determines the price to passengers p set by the airline for aeronautical services. Let $P(w, e)$ denote the solution to the airline first-order condition (3.22).

Once the unit price w has been set, the airport must decide the level of the fixed access charge A . As the airport has all the bargaining power, the fixed access charge is chosen so as to extract all the profit of the airline, that is:

$$A = (P(w, e) - w - c)(1 - F(P(w, e), e)) = \frac{(1 - F(P(w, e), e))^2}{f(P(w, e), e)}. \quad (3.23)$$

From equation (3.23), and assuming that the airport can receive a lump-sum subsidy T , the airport's profit writes as:

$$[1 - F(P(w, e), e)] [P(w, e) - c + (p_0 - c_0)(1 - G(p_0))] - e + T. \quad (3.24)$$

This profit is equivalent to the integrated structure's profits $\mathcal{PR}(P(w, e), p_0, e, T)$ defined by equation (3.4). The only difference is that p is replaced by $P(w, e)$, the price chosen by the airline for a given unit price w and a level of investment e .

As in Section 3.3, assume that the regulator can choose the level of investment e and the price of commercial activities p_0 . It is not anymore possible to directly choose the final price of aeronautical service p , but if we assume that the regulator can choose the unit price w , then the following holds.

Proposition 27 *When the airport and the airline are not vertically integrated, but the airport can set both a fixed access charge A and a unit price w for the access to its facilities, the optimal regulation is achieved in the same way as described in Proposition 20 and Proposition 21.*

²¹Indeed, let $p(w, e)$ be an implicit solution of (3.22). Differentiating (3.22) on both sides with respect to w gives $\frac{\partial p}{\partial w}(w, e) = (1 - \frac{\partial}{\partial p}(\frac{1 - F(p(w, e), e)}{f(p(w, e), e)}))^{-1} > 0$ as the denominator is positive when Assumption 6 holds.

Therefore, the optimal regulation is unchanged when contracts between the airport and the airline allow for both a unit price and a fixed access charge. As in the two-part tariff literature, a contract of the type (A, w) allows the airport to extract all the profit of the airline with A and w is set to maximize the profit of the replicated integrated structure. As we will see below, the fixed access charge is crucial for the result of Proposition 27.

3.5.2. No Fixed Access Charge

We investigate the same vertical relationship between the airport and the airline when the airport cannot use a fixed access charge A . In that case, the unit price w plays two roles: (i) it collects profit on the airline for each unit of aeronautical services provided and (ii) it determines the final price p and therefore how many units are eventually sold. Unfortunately, this distorts both prices as well as the investment level.

The objective of the airline is the same as in the case with a fixed access charge and $P(w, e)$ is still set according to equation (3.22). The airport, however, cannot extract the profit of the airline through a fixed charge and must at least break even, that is:

$$[1 - F(P(w, e), e)] [w + (p_0 - c_0)(1 - G(p_0))] - e \geq 0. \quad (3.25)$$

Due to the absence of fixed access charge, the only available tools to satisfy the previous equation are the unit price w and the level of investment e . We then obtain the following result.

Proposition 28 *When the airport cannot charge a fixed access charge A to the airline, the optimal regulation of aeronautical services and commercial activities leads to the following prices and level of investment:*

- *The price of aeronautical services is greater than the Ramsey-Boiteux price:*

$$\frac{p^o - c_s(p_0^o)}{p^o} = \frac{\lambda}{1 + \lambda} \frac{1}{\varepsilon(p^o, e^o)} \left(1 + \frac{1}{\frac{\partial P}{\partial w}(w^o, e^o)} \right) > \frac{\lambda}{1 + \lambda} \frac{1}{\varepsilon(p^o, e^o)}, \quad (3.26)$$

$$\frac{p_0^o - c_0}{p_0^o} = \frac{\lambda}{1 + \lambda} \frac{1}{\zeta(p_0^o)} > 0. \quad (3.27)$$

- The optimal level of investment e^o satisfies:

$$-\frac{\partial F}{\partial e}(p^o, e^o)(p^o - c_s(p_0^o)) - \frac{1}{1 + \lambda} \int_{p^o}^{\bar{v}} \frac{\partial F}{\partial e}(v, e^o) dv = 1. \quad (3.28)$$

The airport cannot extract the airline profit through the fixed access charge A and must therefore use the unit price w as an instrument to satisfy the break-even condition (3.25). The unit price is distorted upward and the price of aeronautical services $P(w, e)$ set by the airline increases above its previous level. This effect is the standard double marginalization problem. The vertical relationship between the airport and the airline and the absence of a fixed access charge raises the cost paid by the airline to access the airport facilities. The airline passes through this increase to the price of aeronautical services to consumers.

By contrast, the price of commercial activities p_0 is unchanged and stays at the Ramsey-Boiteux price level $p_0^o = p_r^{rb}$. Interestingly, the optimal regulation rule for the investment is the same as in the integrated structure case. Hence, we obtain that the choice of the optimal level of investment is orthogonal to the choice of the optimal level of prices even when the vertical relationship between the airport and the airline is imperfect. However, it is important to recall that the level of investment e^o differs from the benchmark level e^{rb} due to the change in prices (p, p_0) induced by the distortions. Only the rule to choose the optimal level of investment is unchanged.

It is worth noting that the distortion of the optimal regulation due to the absence of access charge is unambiguously detrimental to social welfare. Double marginalization creates a loss even when the regulation authority can impose the unit price for aeronautical services set by the airport. Propositions 27 and 28 therefore suggest that forbidding the use of two-part tariffs in airport-airline relationships is detrimental to social welfare when the airport is regulated. As reported by [Fu, Homsombat, and Oum \(2011\)](#), price discrimination is generally prohibited, notably by the The International Air Transport Association ([IATA, 1997](#)). Our results provide an argument in favor of reconsidering those guidelines.

3.6. NON-VERIFIABLE INVESTMENTS

So far, we have assumed that the level of investment e was verifiable and hence contractible. After determining the optimal level of investment, the regulator was able to implement it by using a marginal subsidy-penalty on invested capital as we suggested in Section 3.4.

However, in many cases, the level of investment is fully contractible due to intangible investment opportunities or difficulties in measuring investment returns. In this section, we investigate how prices and the level of investment are affected by the non-verifiability of investments.

The airport chooses e to maximize its profits and the regulator now faces the following incentive constraint:

$$e \in \arg \max_{\tilde{e}} (1 - F(p, \tilde{e}))(p - c + (p_0 - c_0)(1 - G(p_0))) - \tilde{e}.$$

In other words, the set of feasible levels of investment is restricted to the set of e that maximize the airport's profit for each value of p and p_0 . When Assumption 7 holds, the first-order condition to this problem is a sufficient condition and writes as:

$$-\frac{\partial F}{\partial e}(p, e)[p - c + (p_0 - c_0)(1 - G(p_0))] = 1. \quad (3.29)$$

Contrary to the optimal investment rule (see equation (3.11)), the airport only perceives the private cost when choosing its optimal level of investment, that is, the cost of supplying aeronautical services to the airline minus the profit on commercial activities. This private cost ignores the social benefits on consumer surplus generated by both aeronautical services and commercial activities.

Let μ be the Lagrange multiplier associated with the incentive constraint (3.29). When prices are set at their Ramsey-Boiteux levels (see Proposition 20), if the following holds:

$$-\frac{\partial F}{\partial e}(p^{rb}, e^{rb})[p^{rb} - c + (p_0^{rb} - c_0^{rb})(1 - G(p_0^{rb}))] > 1, \quad (3.30)$$

then this implies that Ramsey-Boiteux prices as defined by Proposition 20 do not provide enough incentives to invest in the infrastructure.

Proposition 29 *Suppose that the level of investment is non-verifiable and that $\mu > 0$ (i.e., equation (3.30)) holds). The social optimum obtained by regulating both prices of aeronautical services and commercial activities has the following properties.*

- *Prices are greater than Ramsey-Boiteux prices:*

$$\frac{p^{nv} - c_s(p_0^{nv})}{p^{nv}} = \frac{\lambda}{1 + \lambda} \frac{1}{\varepsilon(p^{nv}, e^{nv})} - \frac{\mu}{1 + \lambda} \frac{\frac{\partial F}{\partial e} - \frac{\frac{\partial^2 F}{\partial e \partial p}}{\frac{\partial F}{\partial e}}}{p^{nv} f(p^{nv}, e^{nv})}, \quad (3.31)$$

$$\frac{p_0^{nv} - c_0}{p_0^{nv}} = \frac{\lambda}{1 + \lambda} \frac{1}{\zeta(p_0^{nv})} - \frac{\mu}{1 + \lambda} \frac{\frac{\partial F}{\partial e} [(1 - G(p_0^{nv}) - (p_0^{nv} - c_0)g(p_0^{nv}))]}{p_0^{nv} g(p^{nv}, e^{nv})(1 - F(p^{nv}, e^{nv}))}}. \quad (3.32)$$

- *Conditional upon the above price levels, the optimal level of investment induced by the incentive constraint e^{nv} is lower than the optimal level of investment when e is verifiable:*

$$\begin{aligned} -\frac{\partial F}{\partial e}(p^{nv}, e^{nv}) (p^{nv} - c_s(p_0^{nv})) - \frac{1}{1 + \lambda} \int_{p^{rb}}^{\bar{v}} \frac{\partial F}{\partial e}(v, e^{rb}) dv \\ = 1 - \frac{\mu}{1 + \lambda} \frac{\frac{\partial^2 F}{\partial e^2}}{\frac{\partial F}{\partial e}}(p^{nv}, e^{nv}) > 1. \end{aligned} \quad (3.33)$$

When the investment level is non-verifiable, the airport lacks incentives to invest at the Ramsey-Boiteux price levels because, for these prices, the airport's margin is too low and investing to create demand is not interesting enough. Therefore, in order to increase the airport's incentives to invest, the regulator sets higher prices so that the airport's margin increases when investment increases. However, the level of investment e^{nv} is lower than the optimal level of investment. This results from the trade-off between increasing the price of aeronautical services to support higher investment levels and keeping the aeronautical price low enough to increase consumer surplus on both aeronautical and commercial activities.

To summarize, non-verifiability of investment induces (i) higher prices for both aeronautical and commercial services (ii) a level of investment lower than the optimal one in the verifiable case.

3.7. CONCLUDING REMARKS

Modern airports are often privately-owned entities in charge of several activities. Besides their core business, aeronautical services, airports also offer commercial services to passengers. The magnitude of revenues generated by commercial services and the captivity of passengers once they are in the airport has raised the question of extending the regulation to these activities in addition to that of aeronautical services. Another important policy issue has been the choice between the use of single-till or dual-till regulation, that is, whether revenues generated by commercial services should cover airports' investments.

This paper models an airport as an entity in charge of providing aeronautical and commercial services as well as investing in its infrastructure. We assume that individuals decide whether to fly uniquely based on the surplus they can derive from the consumption of aeronautical services. In addition, commercial services are available only to individuals who have decided to fly. We also suppose that the airport can invest in the infrastructure to relieve congestion or increase the quality of services. As a result, higher investment in the infrastructure increases the demand for aeronautical services.

The optimal regulation concerns both prices of aeronautical and commercial services as well as investment decisions. Optimal prices follow a Ramsey-Boiteux pricing rule and are both above their social marginal cost which takes into account the demand complementarity between aeronautical and commercial services.

We investigate a traditional price-cap regulation in a single-till regime and show that its sole use fails to implement both optimal prices and level of investment. This result stems from the fact that the price-cap does not provide enough incentive to invest in the infrastructure. To implement the optimal regulation, the price-cap regulation must be supplemented by a subsidy policy on the level of investment. We also show that the optimal regulation is unchanged in a dual-till regime in which the regulator can choose the fraction of investments that must be covered by commercial revenues.

We point out that the nature of the airport-airline relationship may affect the design of the optimal regulation. When two-part tariffs are available, we show that the optimal

regulation is unchanged with respect to the case in which the airport and the airline are vertically integrated. When the airport can only charge a per-unit price, however, we find that the optimal price of aeronautical services must increase due to a standard double marginalization problem. Those findings shed light on the benefits of allowing some form of price discrimination in airport-airline relationships.

Finally, we consider the optimal regulation when investment decisions are unobservable to the regulator. We find that prices of both aeronautical and commercial services must increase to give the airport proper incentives to invest in its infrastructure.

APPENDIX

Proof of Proposition 19. The airport freely chooses p , p_0 and e so as to maximize $\mathcal{P}\mathcal{R}(p, p_0, e)$. First-order conditions of the problem write as:

$$\begin{aligned} -f(p, e)[p - c + (p_0 - c_0)(1 - G(p_0))] + 1 - F(p, e) &= 0, \\ (1 - F(p, e))[1 - G(p_0) - (p_0 - c_0)g(p_0)] &= 0, \\ -\frac{\partial F}{\partial e}(p, e)[p - c + (p_0 - c_0)(1 - G(p_0))] - 1 &= 0. \end{aligned}$$

Rearranging each equation and using $c_{pr}(p_0) = c - (p_0 - c_0)(1 - G(p_0))$ in the first and third equations, we obtain:

$$\begin{aligned} p - c_{pr}(p_0) &= \frac{1 - F(p, e)}{f(p, e)}, \\ p_0 - c_0 &= \frac{1 - G(p_0)}{g(p_0)}, \\ -\frac{\partial F}{\partial e}(p, e)[p - c_{pr}(p_0)] &= 1. \end{aligned}$$

Dividing the first two equations by p and p_0 , respectively, gives equations (3.1) and (3.2). The last equation already corresponds to equation (3.3). ■

Proof of Proposition 20. Consider the maximization problem of the regulator. Notice that the objective simply rewrites as:

$$SW(p, p_0, e, T) = CS(p, p_0, e) + \mathcal{P}\mathcal{R}(p, p_0, e) - \lambda T.$$

As the cost of public funds is strictly positive, the regulator sets T as low as possible until the participation constraint of the airport-airline entity binds, i.e., $T = -\mathcal{P}\mathcal{R}(p, p_0, e)$. Then the problem rewrites as:

$$\max_{p, p_0, e} \mathcal{C}\mathcal{S}(p, p_0, e) + (1 + \lambda)\mathcal{P}\mathcal{R}(p, p_0, e). \quad (3.34)$$

First-order conditions with respect to p and p_0 give:

$$\begin{aligned} -(1 - F(p, e)) - fs_0(p_0) + (1 + \lambda)(-f(p - c_{pr}(p_0)) + 1 - F(p, e)) &= 0, \\ -(1 - F(p, e))(1 - G(p_0)) + (1 + \lambda)(1 - F(p, e))(1 - G(p_0) - g(p_0)(p_0 - c_0)) &= 0, \end{aligned}$$

where recall that $s_0(p_0) = \int_{v_0 \geq p_0} (v_0 - p_0) dG(v_0)$ and $c_{pr}(p_0) = c - (p_0 - c_0)(1 - G(p_0))$. Rearranging we obtain:

$$\begin{aligned} p - c_s(p_0) &= \frac{\lambda}{1 + \lambda} \frac{1 - F(p, e)}{f(p, e)}, \\ p_0 - c_0 &= \frac{\lambda}{1 + \lambda} \frac{1 - G(p_0)}{g(p_0)}. \end{aligned}$$

Dividing by p and p_0 , respectively, yields equations (3.9) and (3.10) of Proposition 20. ■

Proof of Proposition 21. Consider again problem (3.34). The first-order condition with respect to e writes as:

$$\int_p^{\bar{v}} (v - p) \frac{\partial f}{\partial e}(v, e) dv - \frac{\partial F}{\partial e}(v, e) \int_{p_0}^{\bar{v}_0} (v_0 - p_0) dG(v_0) + (1 + \lambda) \left[-\frac{\partial F}{\partial e}(v, e)(p - c_{pr}(p_0)) - 1 \right] = 0.$$

By integration by parts, the first term of this equation rewrites as:

$$\begin{aligned} \int_p^{\bar{v}} (v - p) \frac{\partial f}{\partial e}(v, e) dv &= \left[(v - p) \frac{\partial F}{\partial e}(v, e) \right]_p^{\bar{v}} - \int_p^{\bar{v}} \frac{\partial F}{\partial e}(v, e) dv \\ &= - \int_p^{\bar{v}} \frac{\partial F}{\partial e}(v, e) dv. \end{aligned}$$

Using this result and dividing both sides by $(1 + \lambda)$, the first-order condition with respect to e rewrites as:

$$-\frac{\partial F}{\partial e}(v, e) \left[p - c_{pr}(p_0) + \frac{1}{1 + \lambda} \int_{p_0}^{\bar{v}_0} (v_0 - p_0) dG(v_0) \right] - \frac{1}{1 + \lambda} \int_p^{\bar{v}} \frac{\partial F}{\partial e}(v, e) dv = 1.$$

Recall that $c_s(p_0) = c_{pr}(p_0) - \frac{s_0(p_0)}{1 + \lambda}$, where $s_0(p_0) = \int_{p_0}^{\bar{v}_0} (v_0 - p_0) dG(v_0)$, then we obtain equation (3.11). ■

Proof of Proposition 22. Assume that the regulator chooses (\bar{p}^*, w^*, w_0^*) and recall that $c_s(p_0) = c_{pr}(p_0) - \frac{s_0(p_0)}{1 + \lambda}$. Then equation (3.13) rewrites as:

$$\frac{\hat{p} - c_s(\hat{p}_0)}{\hat{p}} = \frac{1}{\varepsilon(\hat{p}, \hat{e})} + \frac{s_0(\hat{p}_0)}{(1 + \lambda)\hat{p}} - \frac{\mu w^*}{pf(\hat{p}, \hat{e})}.$$

Assume that the airport chooses $\hat{p} = p^{rb}$ and $\hat{p}_0 = p_0^{rb}$ then the left-hand side of the last equation is given by (3.9) at $e = \hat{e}$ and we obtain:

$$\begin{aligned}\mu &= \frac{p^{rb} f(p^{rb}, \hat{e})}{(1+\lambda)w^*} \left[\frac{1}{\varepsilon(p^{rb}, \hat{e})} + \frac{s_0(p_0^{rb})}{p^{rb}} \right] \\ &= \frac{p^{rb} f(p^{rb}, \hat{e})}{(1+\lambda)w^*} \left[1 - F(p^{rb}, \hat{e}) + s_0(p_0^{rb}) f(p^{rb}, \hat{e}) \right] \\ &= \frac{1}{1+\lambda} > 0,\end{aligned}$$

so that the constraint is binding. It is straightforward to check that that plugging $\mu = 1/(1+\lambda)$ into the right-hand side of equation (3.14) evaluated at $\hat{p} = p^{rb}$ and $\hat{p}_0 = p_0^{rb}$ gives equation (3.10) for $e = \hat{e}$. ■

Proof of Proposition 23. Assume that there exists a combination of (w, w_0, \bar{p}) such that prices are set exactly at $p = r^{rb}$ and $p_0 = p_0^{rb}$, that is, optimal prices as defined in Proposition 20. Evaluating equation (3.15) at those prices clearly shows that the airport's choice of level of investment will differ from the optimal one as defined by equation (3.11). ■

Proof of Proposition 24. Immediate from the text. ■

Proof of Proposition 25. Immediate from the text. ■

Proof of Proposition 26. Let λ_1 and λ_2 be the Lagrange multipliers associated with the first and the second constraints, respectively. The Lagrangian of the problem therefore writes as:

$$\mathcal{L} = SW(p, p_0, e) + \lambda_1 [(1 - F(p, e))(1 - G(p_0))(p_0 - c_0) - \alpha e] + \lambda_2 [(1 - F(p, e))(p - c) - (1 - \alpha)e].$$

The first-order condition with respect to α immediately gives that $\lambda_1 = \lambda_2$. Let us define $\lambda = \lambda_1 = \lambda_2$. It is immediate that the Lagrangian rewrites as:

$$\mathcal{L} = \mathcal{CS}(p, p_0, e) + (1 + \lambda)\mathcal{PR}(p, p_0, e),$$

so that the problem is equivalent to the regulator's problem under a single-till approach. ■

Proof of Proposition 27. As the regulator can freely choose the price of commercial activities and the level of investment, assume they are set to their optimal level, that is, $p_0 = p_0^{rb}$ and $e = e^{rb}$ as defined by equations (3.10) and (3.11). Then, assume the regulator chooses the following unit price of aeronautical services charged by the airport:

$$w^{rb} = c_s(p_0^{rb}) - c - \frac{1}{1+\lambda} \frac{1 - F(p^{rb}, e^{rb})}{f(p^{rb}, e^{rb})}.$$

The airline chooses the price of aeronautical services according to equation (3.22). When $w = w^{rb}$, it solves

$$p = c + \left(c_s(p_0^{rb}) - c - \frac{1}{1+\lambda} \frac{1 - F(p^{rb}, e^{rb})}{f(p^{rb}, e^{rb})} \right) + \frac{1 - F(p, e^{rb})}{f(p, e^{rb})},$$

or, rearranging,

$$p - c_s(p_0^{rb}) = \frac{1 - F(p, e^{rb})}{f(p, e^{rb})} - \frac{1}{1+\lambda} \frac{1 - F(p^{rb}, e^{rb})}{f(p^{rb}, e^{rb})}.$$

It is clear that choosing $p = p^{rb}$ is a solution to this equation and from Assumption 7, it is unique. ■

Proof of Proposition 28. The Lagrangian of the problem writes as follows:

$$\begin{aligned} \mathcal{L}(w, p_0, e) &= \mathcal{CS}(P(w, e), p_0, e) + \mathcal{PR}(P(w, e), p_0, e) \\ &\quad + \lambda \left((1 - F(P(w, e), e)) [w + (p_0 - c_0)(1 - G(p_0))] - e \right). \end{aligned}$$

To simplify the notation, let $P = P(w, e)$. The first-order condition with respect to w writes as:

$$-\frac{\partial P}{\partial w} f(P, e) \left[P - c_{pr}(p_0) + s_0(p_0) \right] + \lambda \left(-\frac{\partial P}{\partial w} f(P, e) [w + (p_0 - c_0)(1 - G(p_0))] + 1 - F(P, e) \right) = 0.$$

Using equation (3.22) and rearranging gives equation (3.26). The first-order conditions with respect to p_0 and e immediately give equations (3.27) and (3.28). ■

Proof of Proposition 29. The Lagrangian of the problem writes as follows:

$$\mathcal{L} = \mathcal{CS}(p, p_0, e) + (1 + \lambda) \mathcal{PR}(p, p_0, e) + \mu \left(-\frac{\partial F}{\partial e}(p, e) [p - c + (p_0 - c_0)(1 - G(p_0))] - 1 \right).$$

The first-order condition with respect to p writes as:

$$-(1 + \lambda) f(p, e) (p - c_s(p_0)) + \mu \left(-\frac{\partial F}{\partial e}(p, e) - \frac{\partial^2 F}{\partial e \partial p}(p, e) [p - c_{pr}(p_0)] \right) + \lambda (1 - F(p, e)) = 0.$$

Using the incentive constraint, we obtain that $p - c_{pr}(p_0) = -1 / \frac{\partial F}{\partial e}(p, e)$. Plugging this into the above first-order condition, rearranging and dividing by p on both sides gives equation (3.31). From Assumption 7, both $\frac{\partial F}{\partial e}(p, e)$ and $\frac{\partial^2 F}{\partial e \partial p}(p, e)$ are negative so that the price of aeronautical services with non-verifiable investment decisions is greater than Ramsey-Boiteux prices defined by equation (3.9).

Now, the first-order condition with respect to p_0 writes as:

$$\begin{aligned} &-(1 + \lambda) g(p_0) (p_0 - c_0) (1 - F(p, e)) + \lambda (1 - F(p, e)) (1 - G(p_0)) \\ &\quad + \mu \left(-\frac{\partial F}{\partial e}(p, e) [1 - G(p_0) - g(p_0) (p_0 - c_0)] \right) = 0. \end{aligned}$$

Rearranging and dividing on both sides by p_0 gives equation (3.32). Whenever $p_0^{nv} < p_0^m$, the monopoly price of commercial services defined in equation (3.2), we must have that $1 - G(p_0^{nv}) - g(p_0^{nv})(p_0^{nv} - c_0) > 0$ from concavity of the integrated structure's profit.

Finally, following the proof of Proposition 20, the first-order condition with respect to e gives:

$$-\frac{\partial F}{\partial e}(v, e) \left[p - c_{pr}(p_0) + \int_{p_0}^{\bar{v}_0} (v_0 - p_0) dG(v_0) \right] - \frac{1}{1 + \lambda} \int_p^{\bar{v}} \frac{\partial F}{\partial e}(v, e) dv = 1 + \frac{\mu}{1 + \lambda} \frac{\partial^2 F}{\partial e^2}(v, e) [p - c_{pr}(p_0)].$$

Using once again the fact that $p - c_{pr}(p_0) = -1/\frac{\partial F}{\partial e}$ immediately yields equation (3.33). ■

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