Choix d'innovation

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Dedication

This thesis is dedicated to my mother. I came to Paris to feel closer to her.

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Introduction

The goal of this introduction will be to give the reader a general initiation as to how economics treats property rights and how this relates to intellectual property. Most of the discussion is aimed at summarizing the relevant literature but some commentary is original. Section 0.1 delves into the roots of the debate and the modern legal language way of discussing such rights. Section 2.4.4 aims to give some brief definitions of the kind of efficiencies economists concern themselves with, and to discuss how these notions apply to the Coase theorem and property rights. Dynamic aspects of property rights and incomplete contracts are elaborated in section 2.3.1. Finally some comments on economic arguments of intellectual property are reviewed in section 0.4.

0.1 The origins on the debate about private property

The roots of the debate about property rights originate from Ancient Greece through its two most revered philosophers, Plato and Aristotle. Plato's most famous work, the Republic, is a treatise on an idealized society, one that has managed to halt to a minimum its own deterioration from the perfect form. Plato's view of property rights is purely instrumental in that it is something that will help maintain the ideal society from deteriorating. His conception of ownership is as an important source of corruption that creates clannish self-interest and considers the panacea of this influence to be the abolition of private property. Aristotle takes a stand against Plato, his former teacher, in being one of the first defenders of private property. In "Politics", Aristotle reasons that without private property people would interfere in each other's affairs without being motivated by love. He views the act of waiving your rights to property against an individual as a way to be virtuous. Consequently, a limitation of this right would limit the ability to be virtuous. The debate between Aristotle and Plato has echoed for millennia, with various philosophers taking different sides of this debate. For instance, Hegel defended property rights based on his theory of person-hood,

stating that people are defined by their will and the only way to manifest their will is through physical objects. Philosophers even had views on intellectual property. Hume for instance, claimed that property should be placed on only goods which are scarce[1].

Perhaps the most influential modern non-economic normative view of property is John Locke's theory of 'homesteading' [2]. Locke's view of property rights is as a method of linking a person who is creating value to that value. This is done by mixing one's labor with the object or land which makes the physical object inseparable from its founder. The view is often rather vague as it does not distinguish between the different amounts of value added and the scope of homesteading.

Economics has always focused not on the origins of property, but on its effects. Using this lens, perhaps the most famous critic of the Lockean theory of property was Karl Marx who claimed the opposite, that private property is the means by which workers become alienated from their labor. The logic behind this is rather simple: if an employee adds a number of hours' worth of labor, he will necessarily be compensated less than that number of hours' worth by the property owner otherwise there would be no way of making profit, hence exploitation. This is one of the first views of property which focused on its dynamics. Specifically here, the dynamics on wealth inequality. However these kinds of interpretations have been superseded as value has been associated not with only inputs but by the tastes of agents and the relative scarcity of resources. Similarly, profit could be entirely explained by other factors such as the relative advantage firms have in information, whether it be an edge in production, taste, impulses of consumers etc. This does not entail that property is disconnected from value, merely that value is not caused by labor, though it is correlated.

Perhaps the first fully prescriptive system of property was articulated by Henry George [3]. His system aims to reduce some of the dynamics described by Marx. George devised a system where property is temporarily allocated to the highest bidder. What is ingenious about the modern version of the Georgist scheme is that it aims to eliminate land rents by making tenants bid for their own rents. This creates a system where people will only earn their labor rent and not the land rent of value. The most known response to this is the view of Hayek[4]. In this view the function of property is not homogeneous across individuals and making ownership temporary is prescriptive in not only the system of property but also in what agents should pursue. For instance, if an agent uses land to pursue non-monetary goals, the tendency will be for that agent to be replaced by an agent who pursues monetary

[1] see Plant (1934a) for views of various philosophers
[2] Locke (2014)
[3] George (1973)
[4] von Hayek (1991)

goals since those goals will be aligned much closer with the ability to bid. In other words, the effect of the property system is not simply to allocate goods efficiently but to allow agents to discover their own goals.

0.1.1 What is a property right?

A tentative answer to this question is[5]: property is simply the default contract. That is, if people do not agree on a contract, property is what is taken as the baseline. 'Property as the default' view is simple enough: Person A can contract with person B that person B will not touch or use item z without A's permission. This in fact, requires no property right at all. What does require a property right is that all other people will also not be able to do with z as they please. If agents could all simultaneously consent or if there were solely two agents who could contract, there would be no need for property rights. Indeed, property rights rely on the inability to contract or simply the costliness to contracting with all agents simultaneously. This basic reasoning is the motivation behind incomplete contracting (which is discussed in section 0.3.1).

0.1.2 The language of property rights

The most basic method of discussing property rights is by using jurist language[6]. Discussions of rights are separated into different hierarchies but in most applications only two levels are needed.

First order rights, which describe the direct rights an agent possesses, are usually the positive rights to act on an asset or to exclude another agent. The right to use is called a "privilege" and the right to exclude is called a "claim". These rights are zero sum in the sense that if all agents have a "privilege" then no agent has a claim. On the other hand if at least one agent, agent B, does not have a "privilege", then agent B has a "duty" and there is some set of agents, A, who either individually or collectively (perhaps democratically) have a claim against B.

Second order rights dictate the use of first order rights. For instance, when one talks of "power" this is in reference to the right to transfer, waive or annul "claim" and "privilege" rights. For instance, the right to change who can use the property is a second order right. One can also speak of "immunity", which means that one has the right for his "claim" or "privilege"

[5]The presentation borrows from Munzer (1990)
[6]See Hohfeld (1917)

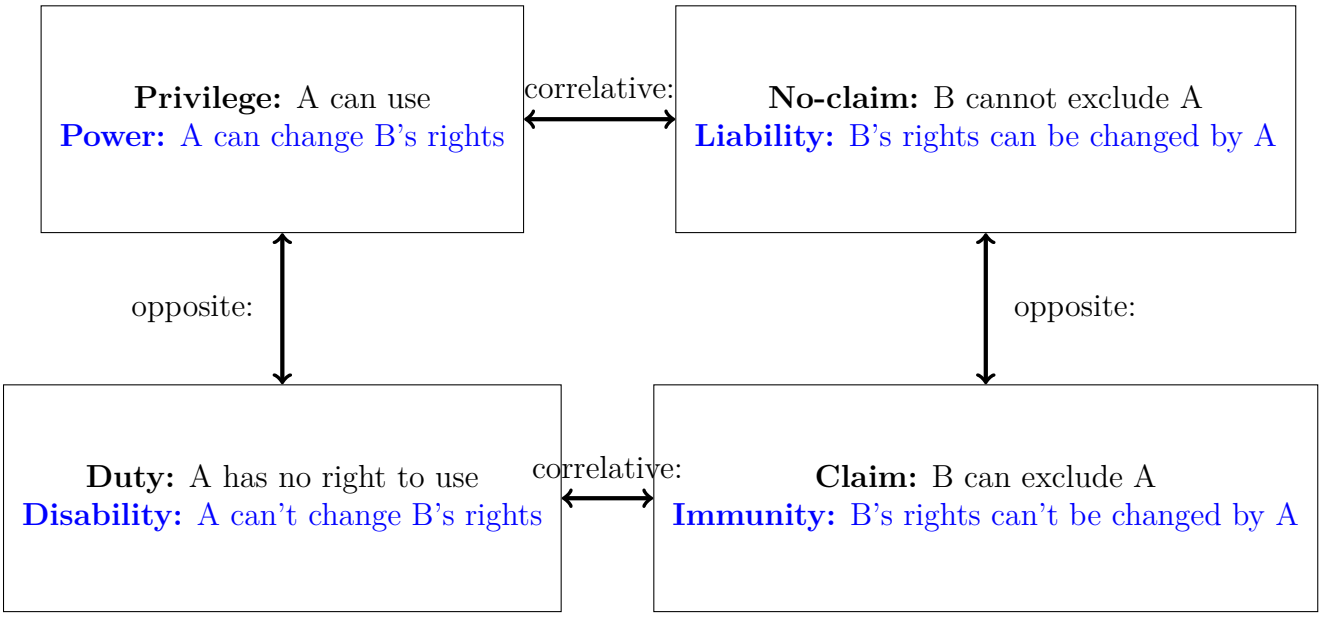


Figure 1: First order rights, Second order rights

to not be affected by others. Second order rights are about how first order rights can be changed. The possibility space of first order rights is increasing in power and decreasing in immunity. Second order rights may also have the feature of circularity; Agent A may have power over B; B may have power over C; and C may have power over A.

Notice that if an agent has power over an object, this entails the ability to control someone's first order rights. Both first order and second order rights may be under negotiation in contracts. The arrangements that can legally emerge are much narrower without power. However, second order right do not entail the right to destroy or abandon an object. The right to destroy or abandon requires *infinite order* rights. This is because the destruction and abandonment of an asset implies that all other agents use and power rights on the asset are violated. Even if all the power rights were centralized on a single agent, this still does not entail the right to destroy or abandon. This is due to the fact that we also have to consider who has the rights to change power (a third order right). And this reasoning can be applied recursively, hence the right to destroy and abandon entails infinite order rights[7].

The contractual possibility of first order rights depends on the distribution of claims or privileges. If all agents have privilege rights on an asset then this naturally entails that the only contracts agents can draw are either committing to using or not using the asset. If

[7]For an interesting analysis of the right to destroy/abandon see, Strahilevitz (2005), Strahilevitz (2009)

on the other hand an agent has claim rights on an asset, then that agent can also contract the exclusion of other agents from using that asset. If cost is independent of the number of agents one is contracting with, then there is no advantage to uniting claim rights upon a single individual. However, if approaching each agent is costly then it may be advantageous to allocate claims to a single agent. While the contractual possibility space is entirely available in all cases, the property right regime can achieve the same contractual space with fewer parties. Similarly, the types of arrangements possible (corporations, partnerships, non-profits, licenses, bailments, non-voting common stock, trusts, agencies, employee-employer relationships, marriages, etc) entirely depend on second order rights. With this in mind, we clarify how some property right paradigms fit into this conceptual framework.

To clarify ideas it is useful to know how this taxonomy matches with traditional economic ideas. For instance, clearly if there is a law that requires property owner A to allow access to B, this implies that agent B has a privilege of use, and implies that A has a no-claim. Similarly, this implies that B has immunity and A has a disability. A price control is a limitation on what price one can sell their good for. As such, it is a "power" limitation in the sense that without a price control A could transfer the asset on wider terms.

The above puts a heavy emphasis on 'use' and 'exclusion', however the notion of 'use' in the case of land is a broad term that encompasses numerous rights that are separable. The additional rights that can be constructed from 'use' are: access, the right to freely move within that territory; management, the right to control the internal organization of the land; withdrawal, the right to extract things from the land; alienation, the right to sell or lease[8]. Land ownership specifically has been summarized by the simple hierarchical relationship corresponding to the five rights, authorized entrant $\in$ authorized user $\in$ claimant $\in$ proprietor $\in$ owner (each level of the hierarchy adds a right). [9]

The two natural limits to the jural taxonomy are when only the state has second order rights and sovereignty. One possibility for the absence of a second order right is that all agents have privileges, this is termed *Open access* (open sea and atmosphere or explicit prevention of exclusion zones). Alternatively, the absence of second order rights could be when the state allocates claim or privilege rights to a specific group. This could look like a king choosing vassals or democracy selecting managers. *Sovereignty*, on the other, hand implies that someone has infinite order rights. If someone has the capacity to make someone else an owner, this can only be represented by an infinite recursion. However, the specific scope of ownership will depend on the regulations in place. The scope of ownership has

[8] Ostrom and Hess (2010)
[9] See Schlager and Ostrom (1992)

often been articulated as "the right to do with your property as you wish as long as nobody else is harmed by it". However, such definitions are problematic as the notion of harm can be interpreted in a variety of ways. A simple solution to this looseness is to revise the definition to "the ability to use ones property in any way one wishes as long as the *physical* characteristics of others property is not affected" [10]. Notice that this definition is not free from interpretation because the notion of 'physical' is not clear (for example: a change in air quality). However once the notion is adopted it creates an objective measurable standard that is open to external verification. We define the word *intrusive* in this physical sense.

[10] Alchian (1965)

Private property is often a term used to describe some kind of constrained sovereignty. While both sovereignty and private property imply infinite order rights on the set of rights granted, the set of rights granted by private property is smaller. Private property does not entail that the owner has higher order rights on all possible uses. If we imagine the three sets below, private property implies simply that $B \cap C$. Private property is sovereignty in the special case where $B \cap C = C$. These relationships are summarized in figure 2.

$$A := \{\text{Possible uses of an asset}\}$$
$$B := \{\text{Non-intrusive uses}\}$$
$$C := \{\text{Uses on which privilege is granted}\}$$

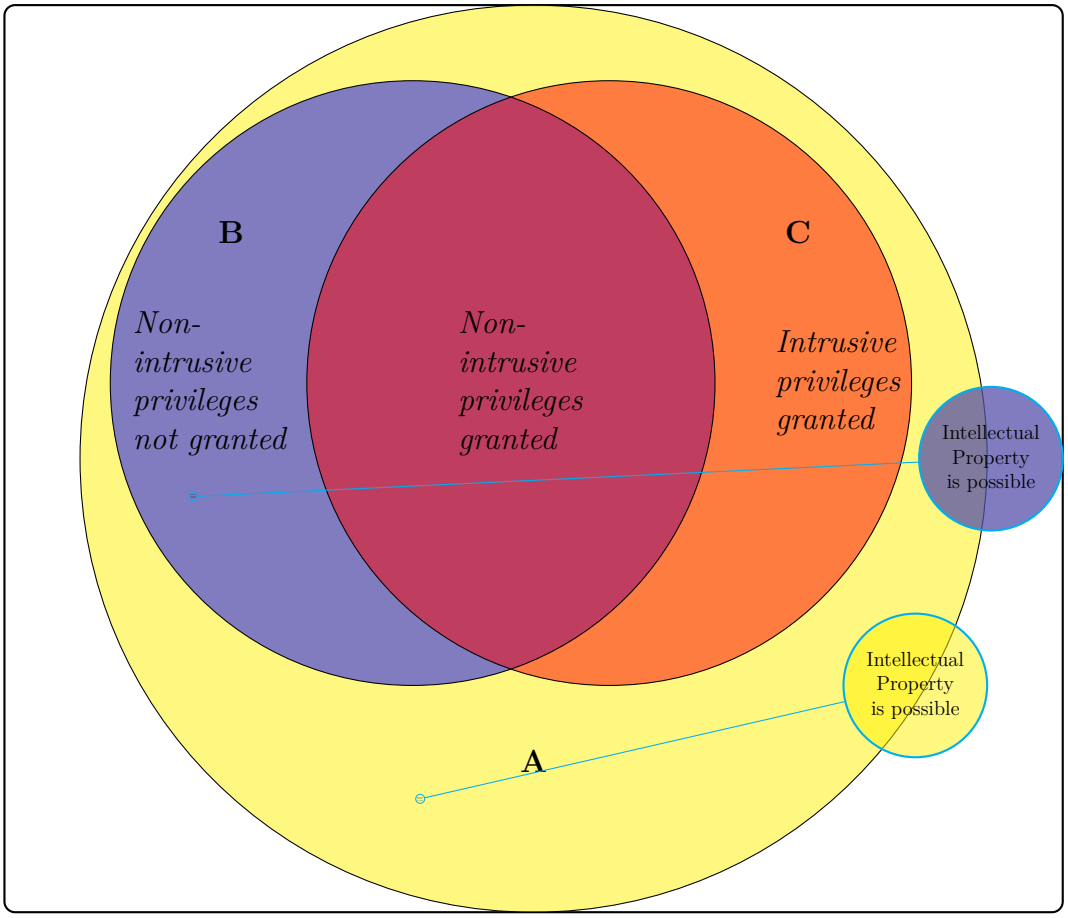


Figure 2: Intellectual property is only possible if a privilege is not granted

Communal/Public property on the other hand gives a subset of agents in society the right

to use but not the right to exclude. Power is also given to a subset of agents. For example, this may be some community that aims to allocate fishing or hunting rights. However, since alienation or transfer is limited, this generally implies higher order rights are in the hands of the state. The distinction between communal and public ownership is how the government chooses to exercise its higher order rights. If they are exercised for the sake of public servants, such as military reservation, then it is called "public" and if not we call it "communal" [11]. The word "communal" usually also implies that there is some kind of privilege that agents have over the land. In the limit, as the set of agents who can exclude is empty, communal property becomes *open access property*. In general, any regime with weak claim rights is a first come first serve type of property and agents operating in such a regime ignore the cost of use [12].

There are other types of regimes which are less commonly discussed such as the *Georgian system of property*. The Georgian system of property temporarily grants first and second order rights to individuals for a given period of time conditional on the individuals having the highest bid. Since ownership is temporary, and no permanent right is possible, this implies no higher rights than third order rights. Additionally, agents no longer have the right to transfer ownership since the decision to keep ownership temporary and conditional on winning the auction is ultimately given to the state.

One way of presenting economics is as being the analysis of exchange. Adam Smith's two books are said to be about two notions of such an exchange; personal exchange and commercial exchange. These two notions correspond closely to the hierarchy of rights. To give a gift or to share something is within the sphere of personal exchange, as such agents need only have first order rights. Commercial exchange on the other hand is about impersonal exchange. Such exchange is conditional, and subsequently depends on, the ability to contract. The ability to contract on assets requires higher order rights. Hence, the degree to which a society can become a commercial society depends on how higher order rights are distributed.

The language presented is especially interesting for the analysis of intellectual property. Whilst each physical property can be seen as a list of rights and the matching of those rights to individuals or groups of individuals as described above, the concept of intellectual property is qualitatively different. The notion of intellectual property is a limitation of the first order rights of "privilege" on physical property. That is, if one has intellectual property

[11] For a discussion of using common property as a policy tool see: Ciriacy-Wantrup and Bishop (1975)

[12] Alchian and Demsetz (1973) mentions how the Canadian government in 1970 set an upper limit to the number of seals to be clubbed which caused speed of hunting to be the competitive trait leading to over-hunting

on the concept of a wooden chair, this is in fact the limitation on the use rights of all owners of wood. Alternatively, there are instances where the law permits copying but not commercialization. In these instances, the first order rights are not affected, but higher order rights are curbed since one cannot contract on the specific use of the asset.

What is especially peculiar with intellectual property is that no privileges are granted. Suppose that agent A owns a physical asset and agent B owns an intellectual asset on a use of A's asset. This in fact means that neither agent has the right to use A's physical asset in that way. It is giving both agents veto power over the use of an asset. Instead for the the asset to be used in that specific way, both parties must consent.

In this first segment we have seen the roots of the debate about property rights and how modern jurist language represents this schism. We have attempted to demarcate between private property and sovereignty by appealing to the scope of higher order rights granted. In this preliminary method of articulating property, it can be seen that, prima facie, sovereignty is may be incompatible with intellectual property. This is because the former entails that the agent has rights on all the uses of an asset, whilst intellectual property limits which uses are allocated rights.

0.2 The static economics of property rights

Is the change in allocation of rights substantive? In other words, is this whole exercise just a redefinition which does not imply any changes in resulting outcomes? In theory, public and private property can both pursue the same kind of goals, such as profits and charity. In practice however, once the incentives of the agents are taken into account, the theory of property rights becomes descriptive. A price control is an instance of a reduction of higher order rights. It is in effect, a limitation on the conditions of transfer an owner makes. A simple example of the effects of a price control can be illustrated by an agent renting out an apartment. If the price is artificially low, the agent will prefer childless/petless adults to avoid noise or damage to his property. In other words, the specific regime can lead to differences in both what kind of investments are undertaken, and who ends up using an asset. This difference is said to be causal in that the regime is a sufficient condition for the emergence of a pattern. Indeed, a prominent explanation for the rise in productivity since ancient times is the shift from common property to private property[13].

Economists reason the effect of the transition from common to private ownership into three

[13] see Anderson and Hill (1983) and North and Thomas (1973)

subcategories. The most common argument for the creation of private property is rent dissipation: since no agent owns the resource before an action is taken, the agents engage in an unproductive race to capture the resource before other agents. The 'unproductiveness' of this race is due to the fact that the asset is scarce and renewable. If the asset was renewable but not scarce, then there would be enough of it regardless of how much consumed. If it was scarce, and not renewable, then its the early consumption does not harm its *total* quantity. The second reason is that there are high transaction costs (this notion is explained in section 0.2.1) to enforcement in a commune. Finally, the third is the incentive to work that is diminished[14].

Much of economics treats law as merely an instrument to utility maximization. The distinctions of each branch of law (tort, contract, family, property, etc.) create their own rules that individually and independently increase utility and efficiency. Discussion of property rights can be broken down into four distinct questions:

- What are the assets that property rights protect?
- From whom is the property protected?
- What is the content of property protection?(what rights are granted to the beneficiaries)
- What is the enforcement mechanism by which property is to be protected?

The economics way of answering these questions generally leans on two kinds of efficiency: allocative efficiency and investment efficiency. Allocative efficiency means either to allocate the asset to the agent who values it the most or to the agent who has the lowest cost to operate it. Both of these notions require a static concept of value. Value in economics is usually broken down into two components, market value and subjective value. These notions are important in that the whole framework of analyzing property rights leans on their interaction. The paragraph below briefly analyzes the interaction of these notions. One of the principle understandings in economics is the deduction of the market value of an asset through the description of the agents subjective values. Note that a positive market value does not imply that exchange occurs; indeed subjective value is the key to the whole framework of the optimal allocation of property rights. When discussing numerous independent assets, the above logic holds. However when the utility of assets is not independent, additional notions

[14]Works on rent dissipation:Dasgupta and Heal (1979) Gordon (1954) Cheung (1970) Schaefer (1957) Scott (1955) Clark (1990). Works on transaction costs: Coase (1960) Demsetz (1983). For the incentive to work see: North (1990)

enter into the framework. Independence of assets simply means that each asset is to be valued separately and does not depend on whether other assets are acquired with them.

Identical and strictly positive subjective value for all agents

If an asset has an identical positive value for all agents, then it has a positive market value but no exchange occurs.

Identical and null value for all agents

If an asset has zero subjective value for all agents, then it also has zero market value because nobody is willing to buy it.

Identical and negative value for all agents

In this case, the market value is negative and the allocation of property rights means, "who will be targeted to receive this asset". In such a case, there is a demand for the right to abandon or destroy. The decision whether to force the ownership of the asset on someone should depend on whether the asset is best left abandoned or destroyed. If the optimal use of the asset is its destruction then ownership should be forced. If its optimal use is abandonment, then no property right is necessary. Of course there may also be a situation where one requires someone to own something without giving that person the right to abandon or destroy.

Variable and weakly positive value for all agents

Suppose now that variance is introduced into the mix. If agents have differential positive value for the asset, then a positive market value exists and an exchange shall occur unless the highest value user is one who is allocated the property.

Variable and weakly negative value for all agents

Similarly, if all agents have a differential negative value of the good then there is still a market value to it unless the highest value user owns it. This is because if anyone other than the highest value user owns the good, they would be willing to pay to transfer their ownership to

the highest value user. In this case, they would consider the lowest cost alternative between subsidizing the highest value user to ownership versus destroying it or abandoning it.

Variable, positive and negative value

Some difficulty arises when we mix the cases. For instance if the distribution of subjective value includes both positive and negative values, then clearly if transaction costs are zero, then there will be trade-off if the good is given to anyone but the highest value user.

0.2.1 Coasian paradigm

A *transaction cost* can be defined as the cost of accessing the market value. So by definition, if an agent owns an item at equilibrium and has a lower valuation of it than the market value of the object, this must be because of the transaction cost. In other words, the broad category of transaction costs can include psychological, institutional, physical factors etc. Anything that prevents an entailment of the form "if this individual owns it then this individual has the highest value". From the point of view of efficiency (to be defined in the next paragraph), the question of making destruction or abandonment illegal becomes relatively more important as transaction costs increase due to the risk of over-destruction or over-abandonment. A liquidity constraint (also known as a *pecuniary externality*) is also a sort of transaction cost: if agents cannot buy a good which has a market value lower than their subjective value then there is a reason for allocational inefficiency. Similarly, if an agent does not know the market price or is ethically against using the market mechanism, these are both types of transaction costs. There are many things in society which are either naturally or legally inalienable (kidneys, votes, future labor, historically important assets, etc), and to the extent that inalienable endowments exist, these can be interpreted as exorbitantly high transaction costs. From the framework examined above, a transaction cost is usually a function of a lack of higher order rights. One can only transact on the rights they have.

The notion of efficiency in economics has a static and a dynamic dimension. Static efficiency is usually termed *allocationally* efficient. This simply means that the set of actions which maximize the sum of agents values is taken[15]. When the question being posed is related to ownership of an asset, allocational efficiency simply means that an asset is owned by its highest subjective value user. It must be noticed that this contrasts with the much stricter notion of Pareto efficiency which says that a state is efficient if it is not possible to improve

[15]This kind of efficiency is also known as Kaldor Hicks' efficiency

someone's situation without reducing someone else's value.

The dynamic notion of efficiency used in economics is *investment* efficiency. This conception focuses on growth. The idea behind investment efficiency is that the allocation that results will lead to the highest amount of growth and hence, eventually, the highest long run payoff. The two notions are sometimes in conflict in that static efficiency is not necessarily good for growth. The interaction between these two ultimately depends on the discount rates of agents. When the agents do not discount the future and reason intergenerationally, the two are perfectly compatible.

The Coase theorem is fundamentally about static efficiency. The theorem states that if transaction costs are zero, the result of the market interactions is allocationally efficient. This can also be interpreted from an action standpoint to say that agents will interact among themselves in such a way to maximize the total payoffs. If, on the other hand, there are non-zero transaction costs, we can only discuss constrained efficiency in the sense "amongst those who entered the market, the one with the highest subjective value will receive it". Certain readings of Coase interpret the theorem as implying that in a zero transaction cost world there would be no firms; such a reading, however, depends on not having gains from specialization[16].

The Coase theorem is of direct relevance to most analysis of externalities. Externality is often a poorly defined concept[17]. One temptation is to define it as effects on non-consenting parties, however, this is too large of a conception since competition is all about negative externalities between firms. Instead, externalities are best defined as effects on non-consenting parties *which do not pass through the market mechanism.* The theorem was initially framed with externalities in mind. Perhaps its most counter-intuitive result is that it implies that externalities become internalized if there are sufficiently low transaction costs. In other words, agents individual private costs will be equal to the social cost.

The theorem also describes the kind of effects the legal system can have. For instance in a situation where there is an infringer and the owner of the property that is being infringed. If the owner has full claim rights (veto capacity) on his property, and others can only use it with his permission, this is called a property rule. If on the other hand there is a fixed or court determined cost associated with infringement this is called a liability rule. The theorem states that when transaction costs are sufficiently low, both liability and property rules will result in an allocationally efficient outcome. This has sprouted a rich literature on the choice

[16] See Demsetz (2011)

[17] for details about why it is a poorly defined concept see Cheung (1970)

of legal rules as a function of transaction costs[18]. For instance, the liability rule may be preferred due to: the holdout problem[19]; free riders; accident situations; if the infringer is better informed; if the infringer has less liquidity; etc. Alternatively, if transaction costs are deemed to be sufficiently low, the legal rules can be chosen for criteria other than allocational efficiency, for example, distributional considerations.

In this section we have briefly discussed some notions of efficiency and transaction costs. The notion of value was articulated in an attempt to make the link between subjective value and market value. This link is important for reasons that will be explored in the next section. We have clarified the difference between the static and dynamic notion of efficiency and how they relate to each other. Finally, have seen that in a static world, efficiency can be achieved by appealing to the Coase theorem whose main condition is a sufficiently low transaction cost. The Coasian setup will be used in chapter 2 of this thesis, the buyout that will occur is in effect a coasian bargain.

[18]Theoretical: Calabresi and Melamed (1972) Empirical: Kaplow and Shavell (1995)

[19]The holdout problem is distinct from the holdup problem in that the holdout means that agents will not reveal their true value, whilst the holdup problem means that no investment will take place

0.3 The Dynamic creation of property rights

Once we introduce time into the picture, a few things become more complicated. Time may create new property in one of two ways. Either because the actual material goods have increased, or because new information has led to an increase in property (for example, the discovery of existing assets). New property creates questions about how to allocate property that previously had no owner. In other words, time gives rise to questions about property allocation before it exists, *ex-ante*, and how property is allocated after it exists, *ex-post*.

There are ex-ante rules one could adopt that solve property future allocation problems. For instance, if all surface area is fully allocated, then new physical property will just be allocated to whoever owns the surface on which it is discovered. Full geographical rights in this manner give rise to questions of volume rights, such as air or underground. This shape of the rights expansion has different implications. If land property, taken as a base, expands into the sky via a square cone, reverse square cone or rectangle shape this can determine operating costs of underground facilities or the cost of flying overhead. For instance if there are reverse square cone rights extending underground this will inevitably mean that some agents will own the same underground space. Any regime of full ex-ante ownership must fully specify the lengths of these cones. The relationships are summarized in table 2.

	Vertical Rights	**Cone rights**	**Reverse Cone rights**
Air Space	Unallocated	Unallocated	Full allocation
Underground	Overlapping	Full allocation	Overlapping

Table 1: Results from volume rights law

Ex-ante fully allocating surface area rights is difficult mainly because agents are often not interested in allocating property before it has a value. Instead, property rights emerge naturally as the value of assets increases, as there will be demand to create rights[20]. An ex-ante regime of property can apply to both physical and biological property. For instance, if a piece of land is found to contain oil, said oil would go to its owner. Similarly, for organisms, if a pet is owned, one usually owns its offspring.

Consider an asset that creates new assets and is owned ex ante. There are three cases to consider: 1) the case where creation is independent of use; 2) the case where the creation is increasing in use; and 3) the case where creation is decreasing in use. Each case shall be considered each in turn.

[20] for details about the emergence of property rights see Alchian and Demsetz (1973)

If the production of future assets is independent of usage then the owner need only consider the demand side of the market. If, on the other, hand the production of future assets increases with use, the tendency will be for use to be maximized. Finally, if the generation of future assets decreases with use, then the optimal extraction rate will depend on the discount rate of the owner. In the third case the concepts of allocational and investment efficiency depend on the discount rate, which naturally leads to following the question: Whose discount factor should be used? Presumably, the owner's discount rate will always play a role however if she intends to sell the asset to others, then the owner will also take into account the discount rate of others. Once the interaction between the agents is well specified it is possible to claim that the owner will have the incentive to harvest whatever resource is in question at the optimal rate. For the case of fisheries this just means the owner will tend to calculate the optimal rate of fishing per period. If the demand for fish is more or less constant per period, this harvest rate will correspond to the long run maximum number of fish.

The cost of ex-ante allocation is an important factor in determining the regime that will be adopted. For animals there are times when ex-ante allocation can be cheap (branding, collars, microchips, etc), and times when it can be expensive (fish, birds, etc). If it is difficult to create ex-ante allocation then there will be effects which depend on the ex-post regime adopted.

The basic problem of dynamic property rights is conditionality. That is, property that is only allocated conditional on some effort. A potential normative role for the economist is to judge if the effort in question is desirable. It seems clear that if the effort is investment in some socially desirable good, then the effect of the conditionality is positive. However, conditionality can also cause negative effects.

Consider the case where elk move between properties. The ex-ante ownership of the animals would result in Coasian bargaining. If the elk owner has a sufficiently positive value in owning the elk, a few things can occur. If the elk trespass has a low cost to the neighbor, then the free roam will be accepted, perhaps with payment to the neighbor. If the trespassing cost to the neighbor is large, then the resulting outcome may be to build a fence. If both the neighbor and the owner benefit greatly from the presence of the elk, then they may jointly undertake investments to improve the quality of life or reproduction rate of the elk, perhaps in the form of a sanctuary. Notice that the result is dependent on the cost/benefit structure stemming from the agents, and the specific environment they find themselves in.

If it is costly to create an ex-ante allocation on animals, a number of ex-post conditional property regimes may arise, each with its own effects. If the animals are only owned conditionally on being on the land, this creates incentives for fencing as long as the wild animal

has either a positive subjective value or a market value. If the animal is only owned when killed, then this creates an incentive to kill it. If land is lost by reason of having elk on it (re-possessed), then this creates an incentive to evict or hide the elk. In other words, the conditionality of property rights can have a plethora of effects. Notice that the fence may emerge in both the ex-ante ownership and the conditional "on land" regime, however in the latter case, the presence of the fence is not necessarily allocationally efficient.

In the case of public property rights, often the conditionality is on geography. For instance, if some property's fruits are shared based on a certain geographical specification, this incentivizes entering the geographical area in question. In a sense, the only way to sell one's share in the property is to move. This often has the effect of involuntary dilution of one's share due to new entrants. In the case of private property, a similar dilution may occur in relation to stock ownership. Nevertheless, this dilution is usually for an associated sum with the idea of increasing the value of the shares held by investors by more than their dilution.

When production plays a role, property is best attributed to the people who are responsible for the production. This could be because they have knowledge of how to use it, or because they have some characteristics, such as risk bearing ability, which would create higher productivity.

Conditionality can shed new light on the normative theories of property rights. For instance, Locke's theory that something is owned conditionally on mixing one's labor with it, whilst a moral theory, from the point of view of economics, has descriptive content in the sense that such a property rights paradigm incentivizes people to combine their labor with objects that can be appropriated. From the value-maximization point of view this is not necessarily efficient relative to ex-ante ownership because this creates an over-incentive for labor instead of output. However, in a world where there is too much uncertainty about the output of investments and a general unforeseeability, a simple heuristic in the form of labor mixing may be better than no heuristic.

A specific case of this conditionality is labor. Firms decide to reward employees based on their productivity. The implicit assumption being that agents put in effort as a function of the compensation that will be made conditional on that effort. For instance, if there is a set of agents, a set of assets, and each agent can only work on a single asset, then it is simple to show that more production will be achieved in the case where agents own a higher fraction of the assets they work on than if their ownership was more distributed. This basic logic has led to the development of the modern theory of the firm due to Hart and Moore (more on this below).

The idea that property is granted conditional on some actions is a simple way to frame numerous concepts in economics. The questions of allocation and investment efficiencies are both dependent on the conditions under which new property is distributed. The *tragedy of the commons* is a notion where, due to common ownership of a good, agents over-use that good. This incentive to "over-use" emerges naturally because the deterioration of the good from overusing is shared among all owners, whilst the benefits from use are focused.

A illustrative example of the concept of the tragedy of the commons is over-fishing. The issue with fishing is generally that all agents have the privilege of fishing the fish without actually owning the fish. The specific conditionality is that the fish are *only owned once fished* out of the water, which creates an incentive to over-fish. The tragedy of the commons appears when rights are granted conditionally, the severity of the tragedy increasing as the rights granted increase. To illustrate this, we need only note that if agents possess the right to eat a fish conditionally on fishing it, then only the agents whose subjective value is greater than their cost will fish. In this scenario, over-fishing will occur purely as a function of population. If, on the other hand higher order rights are given, then agents will be incentivized to fish until the market value equals their costs: this leads to strictly higher demand than the first order rights case. In other words, giving higher order values to the agents will incentivize them to not only fish for themselves but also for the rest of the community. Note, however, that this outcome may not be allocationally efficient since some agents may have a lower cost of fishing. In general, we may say that if there are full unconditional property rights, and the transaction costs are low, the allocational and efficient outcomes are achieved. On the other hand, conditional property rights give rise to overuse (relative to the unconditional case) and no property rights gives rise to under-usage.

So far the discussion has assumed that numerous effort levels are possible but that the conditionality to earn property is binary. That is, if the agent puts in more than some critical threshold then he gets the property. However, the same reasoning holds for more continuous assets such as monetary compensation. It is possible to analyze the connection between the effort level (input) and the distribution of goods (output). In the incomplete contracting approach, the new property is created conditional on some ex-ante effort, but distributed as a function of ex-post bargaining power. While in the static Coasian view, the ex-post distribution does not affect decisions, in an incomplete contract world the ex-post distribution can change decisions. This basic tension leads to some general results on private property from the point of view of incomplete contracts.

0.3.1 Incomplete contracting

The general motivation for property rights in the incomplete contract literature is that property rights allow for investments to be undertaken before a contract is signed. There is a need to give negotiating power because ex-post, the other party has no reason to compensate for more than the value added to the transaction. In other words, other parties have no need to compensate agents for their fixed costs which were undertaken before the contract was signed.

What stops the contracting from being done ex-ante? There are two commonly given reasons: either contracting ex-ante is not profitable, or because the future states cannot be described (Hart and Moore, 1999). The foundations of incomplete contracting have often been criticized on the ground that firms can contract on outcomes instead of states, and this can be equivalent to the first best contracts[21]. This contractual incompleteness is part of a setup for a larger problem in economics, the *hold-up* problem, which says that if agents cannot use their sunk costs in the first period to negotiate in the second period, they will always under-invest.

The justifications often go very far to explain something that can have quite a common sense foundation. Why can agents not contract ex-ante? A simple answer could be that the agents are not agents ex-ante. For instance, if we imagine an individual throughout their life, some of their choices will be decided by those around them, either because of the cultural atmosphere or because they are not capable of making decisions. For example, in a family structure, a parent may wish to invest for their child but they cannot contract long term on the child's behalf (this would be a form of slavery). Instead, the parent can optimize ex-ante investments for their children without committing them to long term contracts.

Incomplete contracts imply a number of things about the theory of the firm. The theory of the firm is often framed as being about whether to outsource or in-source production. The problem with in-sourcing is that agents will be less motivated to put in effort because the benefits of the effort will be split; the problem with outsourcing is that the agents' ex-ante investments cannot be recovered later on. If we imagine an employee within a firm that is choosing between projects which bring in profits, she will prefer projects where the firm can observe that she is the source of the profits. If, on the other hand she was independent, she would instead choose projects which would maximize her profits. This reasoning has been used by economists to explain why innovation often occurs in small firms rather than large

[21] see Maskin (2002) and Maskin and Tirole (1999b)

ones[22].

The mechanics of the incomplete contracting model are simple. Agents may make investments that are only useful if used with something else. For example, an agent may learn to code but not have a computer. In a first period, firms can undertake ex-ante unobservable investments. In the later period they can try to sign a contract with some other party who owns the asset which then makes the investment useful. The problem is that the investment has already been undertaken by the time they are negotiating with the other party. The other party has no reason to compensate them based on their past investment. This then leads to firms underinvesting. If there is only one firm that can undertake the investment, then the solution is simple: that firm needs only to buy ex-ante the asset from the other party, and then invest, therefore reaping the profit on its own.

However, if both firms undertake investments, the firm which does not own the asset will always underinvest[23]. The solution to this larger problem is algorithmic in nature, and thus requires more time periods: Firm 1, is given the asset, and firm 2 is given the right to buy the asset at a certain price[24]. Firm 1 has the incentive to invest because otherwise the second firm will not buy it and firm 2 has the incentive to invest because otherwise it will not recuperate its purchase price.

This same logic can be applied through re-negotiation: instead of buying the asset that has been worked on, the parties' just bargain after the investment. However, all the bargaining power has to be given to the party that invested. This sequential logic has limitations if there is uncertainty about what the optimum investment is[25]. Note that the ownership of the asset itself is not the causal factor. What is important is that a firm that is not needed does not have veto power. Veto power in the language of rights implies that an agent can stop another agent from using or deciding who will use an asset.

More generally, the incomplete contracts model has a few conclusions: 1) If only a single agent can make asset specific investments, then investment efficiency prescribes that only this agent should own all the assets; 2) All assets should be controlled by a single person at a time (not neccesarily the same agent); and 3) No more than a single agent should have veto power over an asset[26]. An explicit assumption of this model is that assets only have

[22] see Holmstrom (1989)

[23] There is no notion of equal ownership possible because the asset is indivisible.

[24] for a first best this price should be the value of the asset after the second firm has also invested

[25] for designing the option contract see, Nöldeke and Schmidt (1998), for sequential investments with complementary assets see, Zhang and Zhang (2014),Bessen and Maskin (2009) , for the breakdown of conditional contracts see, Maskin and Tirole (1999a)

[26] This is found in Hart and Moore (1990)

value to the ultimate coalition that ends up using them[27]. Conceptually, we can imagine the effort as flowing towards three different components. The three components are, either the effort flows directly into the agent putting in the effort (human capital), either into an asset (physical capital), or into another party. The question of private property has to do with how many parties should have veto power over the use of the asset, and how should the veto power be distributed. In general, the presence of the veto power is a disruptive force. Consequently it is best to give it to the party whose non-participation would already be most disruptive or whose participation is already necessary. [28]

Specifically, if the effort(s) flows directly into either the agent(s) putting in the effort or directly into the asset, it is best to minimize veto power. When the effort flows directly into its own agent, then it is best if no veto power exists at all. If, on the other hand, the effort flows directly into the asset, then it is best if the asset is given to a single agent, namely, the agent who is the most productive with the asset[29]. Finally, if effort flows into other agents (perhaps we can imagine agents funding each others education), then it is no longer optimal to minimize veto. If only one agent exerts the effort then he should own the asset. If both agents exert effort that flow into the other, then both agents should have veto power [30].

Some additional results from the incomplete contracts literature are highlighted below. Agents can also endogenously decide between them who will own the asset. This will depend on their relative marginal contributions to the asset, and their ex-post bargaining power. If there are liquidity constraints, both may prefer a third party to own the asset. The framework can also be used to discuss the narrow incentives of the integrated firm, where there will be tension between pursuing commercial payoffs and scientific research. In the context of innovation, the incomplete contracts framework implies that for ex-ante contracts to be less restrictive, a larger amount of liability is required in order to weed out bad inventors. If there is also asymmetric information between the two parties, it has been shown that joint ownership with veto power is optimal, as this induces parties to share their information[31];

The setup of veto power given to unnecessary members is especially suited to analyzing intellectual property. The kind of situation described, where a party is in a coalition for the sole reason that they hold ownership in an asset is, in fact, the norm in intellectual property regimes. Whilst most literature on contracts assumes that the value of a contract is described

[27]While this may be true in physical property, it is probably false for intellectual property.

[28]the original model was only intended for human capital, created by Hart and Moore (1990) which builds on the work of Grossman and Hart (1986)

[29]For efforts flowing to assets, see Schmitz (2013), Gattai and Natale (2016), Schmitz (2017).

[30]see Hamada (2011)

[31]see respectively, Aghion and Tirole (1994),(Lerner and Malmendier, 2010),(Anton and Yao, 1994), (Rosenkranz and Schmitz, 1999)

by the market value, the abstraction in the coalitional incomplete contracts allows an analysis of subjective value. To illustrate the point, suppose there are two transactions occurring in different places by different parties. If the values of the transactions are based on market value then the values are not necessarily independent (perhaps both transactions are about a Star Wars production). However, if the value of the transaction is based on subjective value, the value created by each transactions is independent. In other words, if subjective value is used, the value created by one coalition does not depend on the value created by the second coalition.

The conditions for this to be the case are simple. If either of the two transactions include an end consumer then the value of the transactions are independent. While if both transactions are intermediary this entails a risk that the first transaction reduces profits of the second[32]. The independent case can be stated simply: it is not because one agent figured out how to use his assets better first that the second agent will be less happy about discovering the same method.

Depreciation in use is the most natural way to conceptualize the quantitative differences between physical and intellectual capital in the subjective value paradigm. In a model where assets are used sequentially in different transactions, one can articulate the difference between physical capital and intellectual capital. Transactions that occur later and only use a physical asset will lose value due to the wearing out of the asset; whilst transactions that use intellectual capital do not lose value with use. In a value-as-profit model, intellectual property would also depreciate based on the assumption that knowledge leaks out and becomes less valuable, but in a subjective value framework, this logic does not apply.

In this section we have discussed how the creation of new property creates problems for dynamic efficiency and how some ex-ante allocation reduces the problem. We have also seen how the inability to allocate property ex-ante creates a need to articulate the distribution of goods ex-post. We have seen how the incomplete contracts framework can be used to motivate property ownership by minimizing veto power. This section also related to the contribution of chapter 2, which is entirely within the incomplete contracts framework.

[32]For example Anton and Yao (1994) use the profit notion of value and not the subjective notion of value because they assume that as knowledge leaks, eventually the asset becomes worthless

0.4 Intellectual Property

The term "intellectual property" encompasses three primary notions: Patents, Copyright and Trademarks. Patents grant monopoly novel inventions, copyrights grant exclusive rights over original works, and trademarks protect a specific image or logo with which companies identify themselves. Ownership of a patent and trademark is granted upon successful registration of the same whilst copyrights are innate in the works. When economists discuss intellectual property, they usually only encompass patents and copyrights. The presence of trademarks is uncontroversially considered as a positive in economics, as it helps consumers identify the origin of specific products. From the point of view of the producer, trademarks give them the ability to monetize their reputation (in legal jargon, goodwill).

Intellectual property is subject to a unique asymmetry in legal enforcement. In a property trespassing dispute, where the trespasser is found to be in the right (without mention of his idiosyncratic attributes or circumstances), the effect would be free access to the property by everyone. The total value of this free access would be naturally bounded due to the scarcity of the property (what economists call rivalry). Only so much space can be occupied and there can only be so many resources to extract. On the other hand, if the asset is an intellectual asset, and the court denies the intellectual property, the potential user base is virtually boundless due to the non-degradability of a process. In other words there is a structural asymmetry in patent law, where patent validations are private goods and patent negations are public goods. This means that unless the public goods problem is overcome, the regime may consistently not take into account the proper costs and benefits of intellectual property.

The general goal of intellectual monopoly protection is to increase innovation. The purpose is not to protect inventors, rather protecting inventors is simply a means to and end, innovation. In other words, if it can be shown empirically that intellectual property law inhibits innovation, the legal system should theoretically abolish the rights altogether. Much of the problem with this stance is that the legal system imposes a system and asks for evidence that it is not necessary, when in most traditions of common law, the approach is the opposite, the legal system will intervene if there is evidence to support a certain claim. This implies that there is a shift in the burden of evidence: the intellectual property rights approach gives a solution and then asks for evidence that the problem does not exist.

Economists often group patents and copyrights because the arguments around market structure that are used to justify both these kinds of rights are essentially identical. To balance incentives/dead-weight loss of intellectual property, economists focus on the ratio of the cost of creation to the cost of copying. If the cost of development is immense but so is the cost

of copying, then there is no need for intellectual monopoly since an entrant cannot come in to reap the profits. Similarly, the case where the cost of copying and the cost of innovating are both low also implies little need for protection.

The justification for intellectual property has been changing, but they can be separated into two categories: ex-ante and ex-post. Ex-ante reasons, that is, to justify what would occur by the presence of protection *before* an asset exists, are about creating the incentives so that the asset/innovation will be created. Ex-post reasons on the other hand are justifications for protection *after* the asset is created. Ex-post justifications ensure that the asset is optimally used, in both an investment efficiency sense and an allocational efficiency sense[33].

A rather serious issue of ex-post enforcement of intellectual property rights is that firms have an incentive to push their product. The number of reasons or functions that agents may have to purchase a product can also be important. If the reasons they have to purchase a product are numerous, then it is ambiguous what giving each firm a monopoly on a single product can do. Perhaps, one possibility is that firms will simply differentiate to specialize into different functions. If the product has relatively few dimensions on which agents wish to evaluate it, such as a medical drug where the relevant evaluative dimension is something of the form "expected health as a function of time", the product pushing can be unambiguously negative. A firm may indeed push an inferior product because it is unable to sell a superior product (for this to occur agents need simply to not be perfectly informed on product quality[34]). A similar effect may occur for future innovations, where inferior technologies will be researched because vertical control is guaranteed, and no hold-up problem will occur.

The origin of copyright in England is particularly interesting because it originated the debate on intellectual property. Stationers' Company, a publishing house, petitioned parliament for the first copyright law in 1643 making a number of arguments, the most economically important being: 1) Books are a luxury good and demand for it is elastic (firms cannot raise the price without losing money), therefore it cannot harm the public; 2) A monopoly would create a safer environment for sales and increase both the number of books and increase their sales; 3) Protection would reward the book authors. After almost two centuries of copyrights, it was noticed that there was a divergence between American book prices and English book prices, so the arguments in parliament changed. The new emphasis was on how poorly publishers could predict the sales of books. The poor ability to forecast future sales implied a need for higher prices so that the publishers could recuperate their costs. Interestingly this argument, prima facie, does not justify the divergence in prices between

[33]for a discussion of ex-ante versus ex-post justifications Lemleyt (2004)
[34]For a good introduction to these kinds of problems see: Spiegler (2011)

US and UK prices, and as Plant points out, the firms likely understated their ability to predict sales[35].

The static deadweight loss that occurs due to intellectual monopoly is easy to illustrate. A book publisher can produce any number of books, and there may be demand for books up to the cost of producing them. The monopoly producer of books does not in fact have the incentive to produce books until the price collapses to the marginal cost because it would reduce the price for all the books sold, even the ones sold to high value users. So a firm will choose some intermediate price between the marginal cost and the highest value of consumer. The loss in welfare in the state with the fewer books at higher price relative to the state with more books at lower price is what economists call deadweight loss.

There are market reactions that reduce the deadweight loss but this comes at the expense of the firms profits. For instance, if a book can be read and then has no value for an individual once read and that individual can re-sell it, then this represents competition which can cause a price decrease (In theory this can even cause a price decrease down to the cost of re-selling it). Even if the consumers don't sell the book(either because they like to keep them or because it is illegal) the problem could still be alleviated if the product is a durable good. If a good is durable, a firm will simply sell its product to one segment of the population at a time, gradually decreasing the price to get consumers on lower segments and eventually reaching the competitive price[36].

It is unclear if the logic of durable goods apply to intellectual assets. There are some assets where it could apply such as e-books. This is because if someone gains the e-book they could still have value to the owner even ten years down the line and said owner could just re-read the same book. However there are some domains where intellectual assets may not be durable goods, for instance a movie would not fit into this since if a movie is valued for the memory it gives agents, and their memory is imperfect, this would in fact imply that after a few years, the demand would rise again.

The alternatives to intellectual property can also be separated into ex-ante or ex-post. A firm can seek funds before it undertakes innovation. Patronage was historically the main source of funding for music, architecture, books, etc, whether this be patronage for pleasure or for profit[37]. In the modern world, in addition to patronage, there is equity financing, debt financing or crowd-sourcing (examples are Kickstarter, Gogofundme, Patreon, etc). The advantage of patronage and crowd-sourcing is that they do not rely on a future stream of

[35]For details about the history see Plant (1934b)

[36]the Coase conjecture essentially states that the demand distribution would get truncated every period until it approached the marginal cost of production

[37]For an analysis of patronage in 19th century Austria see Carletti (2013)

revenues since the funding arrives directly from the consumer. Ex-post financing is often what the economic arguments target as infeasible, for after an innovation exists other agents can also use it. This usually applies for services or products that use the innovation in some way[38].

Ex-post financing is often what the pro-intellectual property economics arguments are target as unrealistic because once an innovation exists other agents will dissipate its value. This usually means that agents can monetize the intellectual property by the use of services or products which capitalize on it. The premise that this is not a feasible option relies on a number of assumptions: 1) That the discovery will spread fast enough to be quickly depreciated; 2) there is no reputational mechanism at work, firms cannot use the fact of their creation as a signal; and 3) that firms who developed the innovation did not also develop some other expertise that gives them an edge. [39]

0.4.1 Assumptions of intellectual property

There are two fundamental assumptions that underlie the theoretical justification for intellectual property.

The first of these is foresightedness and is perhaps the most commonly ignored in the economics literature. Since the argument for intellectual property is that after the innovation exists there will be some kind of market structure where the agent cannot extract rents, the agent will not undertake the investment. This fundamentally implies that innovation is directed, that is, that there is an agent who will invest based on his expected future payoff.

There are theoretical alternatives to innovation through investing in foresightedly profitable projects. Why might an agent take on a costly project that is not foreseeably profitable? One could drop the 'profitable' component and simply answer that that the agent is a hobbyist, that is, someone who enjoys the prospect of creation. Alternatively the agent will be seeking profits but without perfect foresight.

Fundamentally the investor has two kinds of unknowns. 1) The most common way of modelling in the literature is that investors know the outcome space but have uncertainty about which outcome will incur, this is usually modelled as a probability distribution. 2) The more unusual way of seeing investors is as not knowing the outcome space of a specific investment.

[38] this has a number of assumptions: perfect information, no reputational mechanism, no developed expertise, etc

[39] For some historical evidence of reputation playing a role in the development of new seeds in the 19th century see, Charnley (2013)

Arguably in this case, investors would focus less on the choice of projects and instead invest in their capabilities to deal with different situations.

These two kinds of unknowns give rise to two forces that determine what kind of investors survive in the long run. In the case where there is simply a probability distribution, agents with high deductive capabilities (those with the best knoweldge of the probability distribution) will survive in the long run, and they will be highly adaptive to market structure and government policy. If on the other hand the more important uncertainty is about the outcome space itself then the main force which would determine the long run equilibrium will be evolutionary selection. Agents who adopt certain positions/norms will simply survive better than those who do not adopt these norms. The norm may include all sorts of behaviors, such as tinkering and exploring, it may even imply false beliefs about how certain things function. It is easy to see how this is relevant in real markets. A father passing down a practice to his kin, without knowing why the practice works, is such an example.

If the market environment is fundamentally unknowable and irreducible, then norms or actions matter and belief functions do not. If there is a wide array of actions and firms choosing randomly among them, then the result depends on the kind of stochastic process associated with each action. If the environment is very competitive and firms which are not cutting edge don't survive, then only firms which pursue the actions with the highest mean and lowest variance will survive. It may be optimal that firms pursue strategies as a function of some personal characteristics(such as size) so that firms don't all take the same action even if they all have the same deterministic strategy. However if markets are not cutting edge then it is possible for firms which don't pursue the optimal strategy to survive in the long run. The set of possible long run competitors in a market with evolutionary selection is much larger than the set of firms under profit maximization. A fully deductive environment will result in only the highest strategy being pursued whilst an evolutionary environment implies that all firms with long run positive profits will survive, which is necessarily a larger set[40].

It is also important to look at competition between markets. If one market grows consistently slower than another one then the slower growth market eventually becomes trivially small. This kind of inter-market competition can imply that, in equilibrium, only markets which have a sufficiently high growth rate will be relevant, which in turn may imply evolutionary pressure towards research and development investments.

The view of economics as agents with heuristics has more traction than first appears if one looks at the frequency of occurrence of serendipitous discoveries. A large tome could be con-

[40]See: (Alchian, 1950)

cocted with a list of breakthroughs that is not directed but which are accidentally stumbled upon. Specifically for medicine, the list of drugs that were discovered accidentally is large: Periwinkle, Platinum, Aspirin, Thalidomide, Librium, Valium, Thorazine, etc. Similarly, the jet engine seems to have been developed without a clear sense of the developmental process and with researchers having a very limited background in physics. Even seemingly modern fields such as cybernetics (formalized by Wiener in 1948) was just an ex-post formalization of already existing systems. Perhaps, most noteworthy is how the processes of development in fields such as architecture caused developments in the structure of mathematics. Financial economics has its own examples: for instance the Black Scholes formula did not seem to have an impact on the prices of options (indeed the makers of the formula famously went bankrupt when trying to exploit their discovery). The general takeaway is that the ideal of "theory to practice" is often contradicted by evidence. In fact what seems to stand is that the practice is often what leads to the theory[41].

The idea that innovation occurs due to heuristics does not imply that the rate of innovation is policy invariant. Indeed, the competitive environment can matter a great deal since, for innovation to occur at the evolutionary stable state, it should increase the probability of survival of agents who innovate (nevertheless, the returns to imitation will be bounded and in equilibrium, innovation will occur)[42]. The consequences of viewing agents as heuristic creatures has widely been exaggerated, with much of micro-economic theory remaining intact, indeed the economic orthodoxy of patents would likely be the main casualty from such a paradigm shift [43].

The second fundamental assumption of intellectual property is that being first to market has a very limited effect on profits, "weak first mover advantage". This assumption can seemingly manifest in economic models in one of two ways. The common way is to assume that the structure of the market is such that once an innovation is created at some cost, F_1, the next person to enter the market can freely use it and pays less than the initial creator, $F_1 > F_2$ whilst having at least the same revenue as the initial firm, R. The specific description of this market structure is then $F_1 > R > F_2$. The argument against intellectual property then can take three forms: 1) the interval between the fixed costs is minimal (both firms entering does not affect whether the innovation is undertaken); 2) The revenue of the second firm is lower than its fixed cost (the second firm will not enter); and 3) the revenue of the first firm is not sufficient (innovation would not have taken place anyway).

[41]Medicine: Meyers (2007), Jet engine: Scranton (2006) and Planes: Meyer (2013),Cybernetics: Mindell (2002), Architecture: Unguru (1992)

[42]See Winter (1993) for a comparison of innovation with and without patents in an evolutionary model

[43]See Becker (1962)

Using this second assumption as their baseline, one of the first formalization of the "weak first mover advantage" assumption is attributed to Loury (1979). The general results are that: 1) as the number of firms increases, equilibrium investment decreases; 2) Increasing investment of a single firm decreases investment of all other firms by a smaller amount, entailing that the expected invention date is quicker with less firms; 3) Profits go to zero if technology for innovation is concave only in the limit (number of firms innovating goes to infinity); and 4) if technology is convex then profits may go to zero with a finite number of firms. However, this kind of model is fundamentally fragile to the cost structure of the firms. For instance, if innovation is not a fixed cost but a variable cost through time, Lee and Wilde (1980) show that the equilibrium investment is increasing with the number of firms. Additionally, if firms decide when to enter the market, this kind of framework can imply that it is optimal for firms to delay when they invest [44].

Fundamentally, if the assumptions that patents induce innovations are correct, patent length and patent breadth are the key policy tools. In an industry where products are substitutable and where the rate of new innovations is high, it is optimal for policy to focus on patent breadth, that is, to ensure that similar products cannot enter the market. If, on the other hand, innovations in the industry do not happen frequently, then it may be optimal to focus on patent length. Essentially, the cost of innovating must be repaid: this can be done either by having the profits realized early in high amount(breadth) or over time(length). [45]

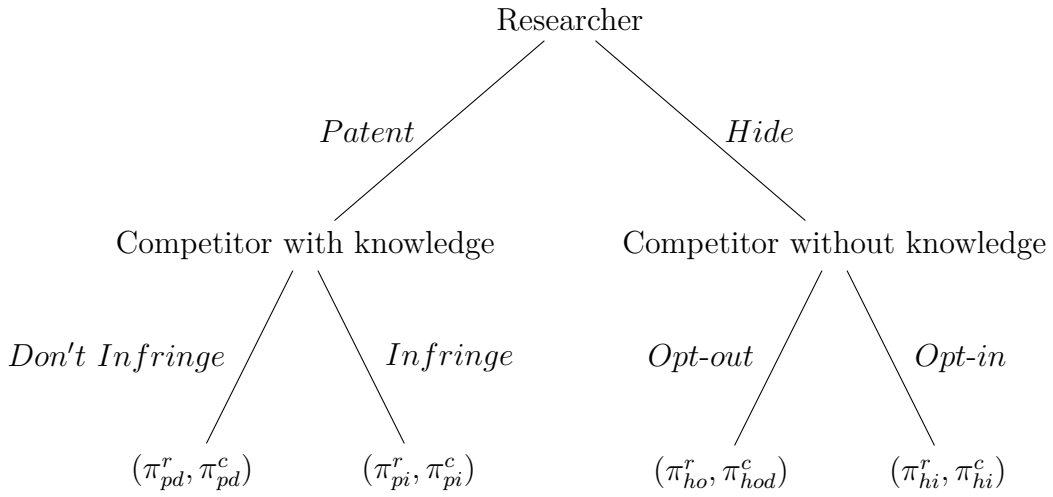
The truth of the two market structure assumptions does not entail that patents are in fact an optimal policy tool. Even if it is true that a patent will create innovations earlier, the same policy tool may delay later innovations. This is specifically an issue if the latest innovation is required to innovate to the next stage. That is, if the innovation created from the first stage is a required input for potential innovation in a second stage then follow-on innovations will be more delayed by a patent system[46]. In other words, the patent system may accelerate progress for the current stage of innovation but slow down follow-up inventions.

Will a firm that can patent always patent? The general economics view is that firms will not patent either due to the fixed cost of patenting or because they have to disclose information about their invention. The fixed cost of patenting (including the document drafting and the application itself) implies that smaller firms may have less of a reason to do so. The information disclosure is related to the broadness of the patent: if the patent is not broad, disclosing how an invention works can be sufficient to help competitors make a competitive product. If a firm chooses not to patent, then specific nuances of the patent system such as

[44] Weeds (2002). Uses investment under uncertainty framework of Dixit and Pindyck (1994)
[45] This is formalized by Takalo et al. (2001)
[46] This was first shown by Bessen and Maskin (2009) and generalized by Bryan and Lemus (2017)

"first to file" (European Patent office) or "first to invent" (United States patent and trademark office) can play an important role (for instance, the first to file rule can over-incentivize patenting over secrecy).

Models that formalize this choice take the form of a researcher choosing whether to hide or patent. If the competitor does not know about the innovation (because the researcher hid it), he simply has to choose whether to try and compete anyway or simply not enter at all. If, on the other hand, the competitor knows about the innovation, he can choose to compete anyway or infringe and have some probability of getting legal proceedings being initiated against him [47]. If there is significant overlap between innovations, the disclosure aspect of patenting may be the most important to avoid wasted effort from re-discoveries of existing technologies (Kultti et al., 2007).



The choice between patenting and secrecy can depend on the degree to which the innovation is radical. Firms may prefer to patent less valuable innovations and keep secret the more valuable ones[48] , for the simple reason that not disclosing a radical innovation offers more potential for a larger gap between the secret owner and competitors. However, if there are costs to renew the innovation and the degree of innovativeness can affect the probability of receiving the patent, the opposite may be true: high quality projects get patented and lower quality projects do not (Mose, 2011). The decision to patent also depends on the competitive gap between the potential patent owner and potential competitors: if the gap is already large, patenting is attractive; if the degree to which the innovation is radical depends on the firms investment, then weak patents are important but stronger patent protection

[47] for a survey of these models see, Hall et al. (2014), for a model that discusses first to file and first to invent see, Scotchmer and Green (1990)

[48] See Anton and Yao (2004)

does not necessarily increase innovation [49].

Perhaps another interesting question to ask is whether a company that can use its patent against someone will always do so. Tentatively, we may say that there may be a cost to enforcing patent rights; but even if that cost is zero, the firm may prefer not to initiate a legal action because the activity in question could be complementary. This kind of market dynamic, where firms willfully choose not to legally enforce their patents, may imply that the system is unnecessary[50].

0.4.2 Mechanism Design

Mechanism design is a natural way to study intellectual property since the system can be said to be designed by legislators from it's inception. The concept of a patent race, where firms invest to achieve a certain technological breakthrough and then only the winner will gain the patent, is indeed equivalent to a contest[51]. Mechanism design can also shed some light on when it may be optimal not to assign property rights at all. If agents have private information in a bilateral transaction (Coasian setting), the agent with the property right will have an incentive to overstate his cost or value because the other party will not have the option to go to court; whilst if it is unclear who will win in court, parties will have an incentive to tell the truth[52].

Part of the loss the patent system creates is an allocational efficiency problem. If the highest value user is changing in time, the system has no natural way to prevent the current owner from reaping most of the benefits from a transaction with a higher value user. A natural method for reducing this allocational efficiency is to use the Georgian scheme, and require firms to bid on their own patent every year (a sort of rent). This would establish a price reflecting the current owners value for the patent, and would enable the higher value user to purchase the asset.

However, the main problem with intellectual property is not the agent who owns the asset but the number of agents who own it. Above all, intellectual property is a monopoly, and as a monopoly it can induce deadweight loss. There are a number of tentative solutions to this: making all patents public (free for anyone to use) is one of them, but the methods are controversial. Suppose a system was instituted where all patents and future patents were

[49] See Kultti et al. (2006) for details

[50] For an example of firms not activating their patents in the development of short message services(SMS) see Corrocher (2013)

[51] Heidrun C. Hoppe and Baye (2003)

[52] Schmitz (2001)

bought over by the government at a fixed price. Whilst this would alleviate the deadweight loss problem, due to the variety of project values, two issues would arise.

The first issue is **under-rewarding**; projects which are worth more than the posted price would not be undertaken. This would not be a problem if the cost of inventing was below the bought price. The **under-rewarding** issue would entirely disappear if the consent of the patent owner was required, which transforms the payments into bids. However, for the projects which reject the government fixed bid, this would also entail that their dead-weight loss is still in place[53].

The second issue would be be that projects worth less than the bought value would be **over-rewarded**. This would naturally create incentives to maximize the number of patents instead of the value added of each invention. A method of alleviating the issue of over-reward would be to choose prices by creating a price system for each patent (allowing firms to bid for each patent[54]). Once the price of the patent is established, the government would then pay that price for the patent. The issue is then: why would a firm bid truthfully if it will not receive the patent? One could indeed imagine that a firm would have the incentive to overbid, and then split the surplus with the patent owner (in the language of game theory, the mechanism is not coalition proof). A partial solution to this problem would be that instead of the government buying out the patent deterministically, it would buy it out with some probability. Unfortunately, this does not alleviate the problem, for the reason that the firms could still make a positive coalition profit in expectation[55]; and if the probability of gaining the asset is low enough, then there would be very little incentive to participate in the mechanism.

Although generally alleviating the monopoly problem is too difficult, it is possible to play with the time dimension of patents to reduce it. To avoid the coalitional problems, one could devise a mechanism where a patent owner simply bids on its own patent and depending on the amount of the bid, the monopoly granted becomes longer. This mechanism would cause lower productivity sectors to have longer patents, and higher productivity sectors to have shorter duration patents[56].

[53] an interesting historical fact here is that in 18th century France in Lyon, Silk Factory innovation were compensated by the government not only per innovation but also by the dissemination of the idea Foray (2013)

[54] this method was first suggested by Kremer (1998) and was generalized by Weyl and Tirole (2012)

[55] Peters and Adamou (2018) show that even firms can exhibit risk avoiding behavior when they partially grow multiplicatively which implies the probabilistic mechanism can be effective

[56] This is because if an industry is innovative the value of the patent will quickly depreciate as more innovative products take over, this means that it would be worthwhile to increase the cost per unit of time for less productive sectors relative to more productive sectors so that the mechanism becomes more truth revealing. This mechanism was created by Scotchmer (1999) and built upon Cornelli and Schankerman

What should the relationship between patent length and the degree of innovation be? For any fixed period patent system, the patent duraction will incentivize projects which can earn sufficient profits within the period. However, the application for patenting may have agents over-applying because it is profitable to gain a patent ex-post on innovations which would not be profitable within the time period. If a patent is granted to these innovations, this would be a pure deadweight loss without an incentive effect. To clarify, the potential for a patent cannot be an incentive for projects which would be ex-ante loss making. But once a project exists, perhaps accidentally, an entrepreneur will still wish for a patent. This means that for the granting process to be effective and not incur accidental deadweight loss, it must use a criterion which has to do with how much profit can be attained within a constrained time period[57]. [58]

0.4.3 Scope and evidence

The spike in interest in intellectual property has occurred due to some attacks by economists arguing that such rights should be weakened. The main vanguard of this attack has been in the works of Dosi et al. (2006), Levine and Michele (2008) and Bessen et al. (2008). This attack has incited criticism (Scherer, 2009) along with responses (Boldrin and Levine, 2013). The arguments sometimes use specific historical cases to discuss counterfactuals. Perhaps the most radical claim being that James Watts' patent on the steam engine delayed the industrial revolution by decades (Levine and Michele, 2008) (Nuvolari, 2004), and similar claims have been made for the development of the plane and the car (Merges and Nelson, 1994). One of the most interesting studies on the topic is by Moser (2005), who studies historical exhibitions at the crystal palace and conclusively shows that relatively few of the inventions were patented (Moser, 2005)[59]. It seems difficult to evaluate intellectual property as a whole, as the evidence seems mixed[60]. Surveys with evidence on innovation seem to imply that it is an ineffective policy tool for the majority of industries in the United States, the exception generally being pharmaceuticals and chemicals [61]. The findings has been similar in Europe [62].

Take the premise that it is difficult to make sufficient profits from an innovation in some

(1999)

[57] O'Donoghue (1998)

[58] this class of models is presented with probabilistic enforcement by Chou and Haller (2007)

[59] A similar methodology has been applied to United States fairs, see Khan (2013)

[60] For evidence that the human genome patent reduced innovation see Williams (2013)

[61] (Mansfield, 1986) (Levin et al., 1987) replicated by (Cohen et al., 2000)

[62] See (Arundel and Kabla, 1998)

given market. Assume that some level of reward, say x, would balance the losses from the patent system and the incentives for innovation. Now suppose that through time, the market has expanded so that it is easier to gain large portions of profits in a quicker period of time (for instance, as is the case with globalization). If the cost of creating new innovations has remained the same after the market expansion, then the optimal patent system would lower the reward by, say, decreasing patent duration[63]. The intuitive implication is that as the world becomes more globalized, the requirement for patent protection is decreased because the potential payoffs of projects increase. An empirical measurement that could be relevant is the time from discovery to adoption. Ideally, as the adoption times decreases, the length of patent would also decrease [64].

While it is easy to imagine that policy makers optimize a social welfare function, in practice, the system itself relies on a bureaucracy. From an institutional point of view, balancing out incentives is crucial. In this context, if there is not sufficient incentive to reject bad projects, the issue of over-patenting emerges[65](this would naturally emerge as the costs of a welfare reducing patent would be diffuse while the benefits would be narrow). In the American system, court disagreements are discouraged thereby incentivizing institutions to over-grant patents in order to avoid appeals, leads to a sort of patent standards inflation(Masur, 2011).

A peculiar empirical fact with implications for patents is the practice of reverse settlements. Reverse settlements, which consist of extending patents by paying other firms not to use a technology, imply a few things about market structure. The simplest implication is that transaction costs must not be very high, so that it is possible for firms to strike Coasian bargains. Since these contracts are firm specific, the empirical implication is that smaller firms do not matter enough to change the profitability of such arrangements. The scope of reverse settlements is unknown, but if these contracts are possible before the creation of an innovation, it may imply that a large segment of the patent system is unnecessary, since firms can simply negotiate ex-ante with the limited number of firms who could use it. The ability to patent could then be interpreted as an increase in the bargaining power of the first firm (Green and Scotchmer, 1995). It is interesting to ponder if firms can foresee whether this bargain will occur: if firms do not have this capacity but undertake the investment anyway, it must be expected that patents over-reward.

An alternative explanation for reverse settlements is that patents are, in practice, probabilistic, and firms do not want to take the risk of a court failing to validate their patent.

[63]this kind of model is presented in Boldrin and Levine (2009)

[64]For evidence about the drop in adoption time see, Comin et al. (2006)

[65]see Caillaud and Duchêne (2012) for a model with pooling equilibria with good/bad projects and separating equilibria where only good projects are accepted

This probabilistic feature can have two effects: it can protect innovations which upon closer scrutiny, would not be protected. It can lead to firms to pursue secrecy strategies[66].

Perhaps the strangest empirical regularity is the practice of patent pooling. Firms jointly agree to enter a patent coalition and agree not to enforce their patent within that pool. This kind of structure inevitably leads to a standard for entering the patent pool, where entry will be granted only if one has a sufficiently important patent to the coalition. What is interesting is that this may create incentives for achieving this standard. However once in the pool, there may be little reason to keep innovating. Still, even without these dynamic notions, the presence of a patent pool may be welfare reducing if technologies are not complementary. [67]

This section on intellectual property has discussed various economic arguments justifying the legal practice. We have discussed when a firm may wish or may not wish to enforce it's patent, this is related to the first chapter, where it shown that when a firm has an intellectual asset, it may not wish to enforce its claim rights on all agents because value could emerge as a function of number of agents consuming. Perhaps most importantly, we have discussed the two fundamental assumptions that run through most of the economic literature. Foreseeability is relevant to chapter 3 in this thesis as it focuses on agents maximizing growth. This growth heuristic allows for agents to fit into evolutionary environments such as those described above. In other words, the growth heuristic is about agents surviving in the long run, not about deducing the expected value of pursuing a certain project.

[66] For details about probabilistic enforcement see, Lemley and Shapiro (2005), to see how reverse settlements can signal invalid patents, see Dolin (2011)

[67] see Lerner and Tirole (2004)

0.5 Contrbutions in this thesis

0.5.1 First chapter

The first chapter is a contribution to the economic modeling of pricing in an environment where a firm has a monopoly and agents value consuming as a group, a network good. The good in question is assumed to be an intellectual asset implying that the marginal cost to the firm of producing an additional unit is zero. The network good literature usually assumes that the intrinsic quality of the product increases due to market structure, for instance a library has more value as a function of the number of books it has available, if it is easier to find a book in a larget library, then that library may be able to increase its prices relative to other libraries. This is in constrast to our approach where the value of the network good depends on the size of the user base, that is the increase in value of the product is due to the number of consumers, not due to the internal characteristics of the good, this can be named as *extrinsic* value.

The concept of extrinsic value can be interpreted in numerous ways. Perhaps the simplest is psychological, the good itself is not what is important but the way the agent evaluates is value is what matters, perhaps a family heirloom. In our context, extrinsic value refers to *social* factors. For instance the product may be a signalling mechanism, for instance the product may signal that someone is a higher class. Perhaps more importantly the product may be a means to another social end, for instance there may be a book club and that meets weekly and discusses a specific book, this would increase the extrinsic value of the good. Alternatively, one could simply imagine a scenario where daily interactions revolve around the enjoyment of a certain good such as a television show or a football match.

We give a brief outline of the results. The main result is that increasing the cost of piracy has non-monotonic effect on profit and welfare. Whether the cost of piracy increases or decreases profits depends on the level of the network value, where if the network value is sufficiently large, firms will make more profits with piracy than without.

In an extension we endogenize the choice of product improvement. To test the effects of this endogenization we simulate the optimal choices of the firm, in these cases we also find that product improvement is decreasing in the level of piracy pursuit.

The discussion of of the paper revolves around what rights a firm should have for its product. The implication is that rights pertaining to intellectual property should be centralized, in the sense that if the firm has the right to exclude other firms, it should also have the right

to exclude users. The result goes against the way in which current court systems categorize piracy, mainly as a criminal offense. Categorizing piracy as a criminal offense implies that the firm does not have claim rights and can result in non pareto optimal equilibria.

0.5.2 Second chapter

The second chapter studies the choice of innovation in the presence of buyouts. The model can be interpreted as being within the incomplete contracts literature as firms cannot contract ex-ante. We present a two firm setting model with an entrant and incumbent where the entrant selects a technology. The choice of technologies is a sequential innovation and a radical one. We find that the ability to buyout affects the direction of innovation towards sequential innovations, that is, there exist cases where if no buyouts can occur, the radical innovation would have been pursued but the option to buyout creates a preference reversal. This effect only exists if the entrant has bargaining power. We show that this holds for both Bertrand and Cournot Competition. Finally we discuss the welfare implications of buyouts in this two technology paradigm and the link to the Coase theorem.

We give a brief numerical example here to illustrate:

Numerical example with two time periods

We have two time periods with the entrant selecting which innovation to pursue. The sequential technology will give the entrant the intermediate technology in period 1 and the advanced technology in period 2. The radical innovation will simply probabilistically give the advanced technology every period.

The profit of the entrant with the advanced technology is given by, 40. The profit of the incumbent with the advanced technology is given by 100. Additionally, let the profit of the incumbent when competing with an intermediate technology be 20, whilst the profit of the incumbent with the initial technology without competition is: $= 80$. Finally, we assume a Nash bargaining solution where firms have equal bargaining power, .5 and that the radical innovation has a 50% chance of succeeding.

We first do the case where there are no buyouts. If no buyouts do occur and the entrant chooses the sequential innovation then the entrant will earn, 40, which will only be realized in the second period. If the entrant chooses the radical innovation the payoff will be $.5(40 + 40) + (.5)^2(40) = 50$. Since $50 > 40$, if no buyout occurs the radical innovation will be chosen.

If buyouts do occur then the incentives change. The Nash surplus for the sequential in-

novation is, $NS_s = 100 + 80 - 20 - 40 = 120$. Therefore the payoff of the entrant after bargaining is $40 + \frac{1}{2}(120) = 100$. The radical innovation surplus is similarly $NS_r = .5(200) + (.5)^2 180 + (.5)^2 160 - (.5)^2 80 - (.5)^2 - 50 = 75$. Therefore the payoff after bargaining $50 + \frac{1}{2}(75) = 87.5$ so the entrant pursues the sequential innovation.

0.5.3 Third chapter

Whilst working on the second chapter this author noticed that firms were preferring earlier payments to later payment without an explicit discount rate. After some tinkering on the subject this led to the identification of the cause as coming from the multiplicative dynamic used. This dynamic was later removed from the second chapter and became its own project, the third chapter. Whilst the chapter focuses on agents discounting, the method used abstracts away from any subjectivity in the discount rate. Meaning that the discount rate could be interpreted as pertaining to the optimal long term use of objects in specific environments. This means that the theory can be directly applied for assets.

The problem question is about discounting. Specifically, in laboratory experiments agents discount the future in different ways. The two main kinds of empirical discounting reported are exponential and hyperbolic. Economists then try to conjure up reasons as to why the different sort of discounting may occur. There are essentially three approaches: psychological, agents ignore some information in some contexts; informational, agents have a specific information structure that causes the behavior; axiomatic, agents have a specific method of reasoning that implies this behavior.

These theories posit an initial behavior and explain the discounting behavior. The problem with this approach is that an explanation for the initial behavior is not given. Why do agents ignore some information (behavioral), why do agents have this specific information structure and why do agents have these axioms? If these theories are just different languages of describing the same behaviors then there is no issue, however it is quite clear that there will be circumstances where their predictions will not overlap.

The approach used in this paper appeals to consilience, that is, an axiom that is already used in another field, evolutionary theory. The axiom in question is growth, agents will simply maximize their growth, it is common sense why this axiom would naturally rise when evolutionary forces are at work. Simply, agents will maximize growth because agents who did not maximize growth would not be around in the long run. The growth axiom allows the explanation to be on physical environmental level instead of being an explanation about cognition.

Focusing on growth allows one to describe the environment in which the agent is in. Specifically we look at two kinds of dynamics, additive and multiplicative. These two are not exhaustive since in the real world some agents may evolve as mix of additive and multiplciative factors. However when the object under question most of the variation will be due to one factor or another.

We also focus on the choice environment by making explicit the assumption of whether future choices depend on current choices. There are two ways we characterize the time structure of the environment. The first is what we call a fixed structure, this means that choices are evaluated on some fixed time horizon. The second is what we call adaptive, this means that projects are evaluated on their own horizon, we illustrate below.

Suppose an agent would enjoy going to Spain for holidays but prefers to go to Cyprus. Now suppose the agent has two months holidays and has only the budget to go to one of the two. The agent can either go to Spain for the whole two months or go to Cyprus for one month. How should the agent evaluate these two choices?

If the agent considers choices by a fixed time horizon, it means that the agent will simply ask the question "When the two months of holidays are over, which of the two choices would have brought me more pleasure?". So in this scenario the agent will simply evaluate the total pleasure given by each choice, and choose the highest one. Implicitly the agent in this case the agent has to make an assumption about what happens with the free month under the Cyprus option.

The adaptive time frame implies the agent compares choices by looking within their own growth horizon. The question the agent will pose in this situation is "How much pleasure per unit of time will I gain?". This means that the Cyprus option will be considered within one month, the agent will not consider what happens in the free month remaining.

These are two ways of reasoning that can both be defended. Within the framework of pleasure it may not be an objective criterion, but within the framework of a firm deciding between projects the choice of which criterion to use does affect the maximum profit.

The general results of the paper are that we recover different kinds of discounting depending on the scenario. Specifically if an agent evaluates things in an additive and fixed time manner, the agent will not discount. On the other hand if the agent evaluates in a multiplicative and fixed time manner, the agent will discount exponentially. If the agent reasons in a additive and adaptive time frame, then the agent will discount hyperbolically. Finally, if the agent reasons in a multiplicative and adaptive time frame the agent will discount using some hybrid of hyperbolic and exponential discounting.

Chapter 1

Piracy and network value

1.1 Introduction

The music industry is often seen as the primary victim of piracy. It's long term reduction in profit has been attributed to the large number of pirates whose actions allegedly harm both artists and their representatives(Owsinski, 2015b). Still, more recent analysis estimates that as many copies of popular music are being pirated as are being accessed through legal means ((Owsinski, 2015a)).

The magnitude of the damage caused by piracy is ambiguous. Given the fact that some artists choose to give their work away for free, it may be inferred that piracy does not always harm the actors of the sector. Music is often uploaded for free on public platforms or even freely uploaded on piracy sites by their creators. The potential rise of the revenue of complementary goods such as concerts may compensate the direct loss on the sale of music. However, it remains unclear to which extent this entirely compensates for the loss in direct music sales. There also exist large movements against piracy. Many industry experts and artists argue that it is not possible to maintain a high level of innovation in the sector without protecting the market from pirates.

The main examples of digital goods which have a piracy option are music, movies or televised series. The main alternative to buying usually takes the form of torrents or online streaming. Buying on the other hand may take the form of renting or purchasing a copy of the product. For instance Spotify, the music platform, is essentially a streaming service which also allows users to download songs.

For the *cultural* goods that we consider, the users' utility depends mainly on two components:

intrinsic valuation and extrinsic valuation of the good. The intrinsic valuation of the good is the utility that the user derives when he consumes the good independently of the actions of other consumers. The extrinsic valuation is the utility that the user derives when he consumes the good which depends on others consumers' choices. With the second element, we intend to represent the fact that the value of a cultural good also depends on the possibility to exchange about it with members of your social network and the higher the possible exchanges are, the more consumption of the cultural is enjoyed.

There exist many types of goods which have large extrinsic portion to their value. Following our previous example, the value of a TV series to a consumer is not only the direct experience of watching it but also the socialization that follows it afterward. This is also true for sport performance. The FIFA world cup, for instance, attracts an extremely high number of viewers and followers. Seeing a high quality soccer match is not the only motive of the viewers. It is also a communal event. The viewer also wants to exchange comments, experiences, feelings with colleagues and friends during and after a match.

Software is another case where the extrinsic valuation may vary substantially. Statistical packages in general are software products which have the value which is directly provided by the firm and the value which leaks from other consumers and is generally dependent on the number of users because the number of packages and the versatility of the platforms depends on their users bases. This partly explains the rise of open source software, with Python, R, Ruby, etc. The active user bases produce packages which increase the value of these platforms. Proprietary software also gain value from a richer user base. For instance, STATA hosts events for licensees and there are many authors outside of the company that contribute.

We mainly consider digital goods with a marginal cost of production and distribution almost equal to zero. In these markets, consumers my decide to pirate the good, obtaining access to the good, or to a low quality version of the good, without paying it. The firm producing the good has several ways to react to piracy. It can increase the quality of the non pirated version of the good, adding bonuses and extra elements. It can modify its pricing policy. In some cases, it can also affect the cost piracy by increasing the protection of the good or by putting pressure on public authorities in order to increase the efforts spent on copyrights protection.

Product improvement or extra content is quite commonly employed in many industries. An example of this strategy in the entertainment industry is already seen through limited edition sets that include various extra content such as conceptual art or more information on the development process. In the case of video games, it is usually done by adding extra

functionalities such as the ability to pause and rewind games[1].

Often companies structure themselves in a way as to offer a free good of base value and giving an improved product to those who pay extra. This added content is (tautologically) most often coveted by the consumers who have a higher willingness to pay.

The situation can be framed as a choice between the relative level at which a firm will rely on the stick versus the carrot. Firms with copyright claims on their products have in their arsenal both a carrot and a stick. The carrot, in this case, is the ability to attract consumers by offering a high value product. In contrast, the stick is the ability to increase the cost of piracy.

A company that decreases the cost of pirating can expect two types of effects. The first is that the consumers who would have bought the product will instead pirate it and similarly the consumers who would have neither pirated nor bought it may decide to obtain it through piracy. The relative importance of these two effects would likely depend on the level of differentiation between the pirated product and bought product.

Now, the bought and pirated product, might differ in value naturally without extraordinary effort from the enterprise or government. For instance acquiring a product from a non-official digital source may entail some risk of downloading a virus or being hacked, this would be a natural level of a priori product degradation. Specific effects differentiating the socialization values between the two product can also be imagined. For instance a social stigma may cause the pirates to derive a lower proportion of value from socializing. Consumers may also derive additional socialization value from the bought product because it may be used as a signaling mechanism.

The main contribution of this paper is to show that because of the existence of this network effect for cultural good, the impact of piracy is ambiguous. We find that the effect of increasing the cost of piracy depends on network value. If the network value is low, then decreasing the cost of piracy is profit en-chancing, whilst if the network value is high, an increase in the cost of piracy may decrease profits. In other words, we find that if the network value is high, it may be profit and social surplus enhancing to decrease the cost of piracy. In the extension of the model we endogenize product improvement. The general result from this section is that the firm is less inclined to pursue product improvement if the network value is high than when the network value is low. Or in other words, product improvement starts out as complementary to network value but as the network value increases it becomes substitutable to network value. The paper is structured as follows. In the second section we

[1]To a certain extent, this is also equivalent to the rise of the *freemium* strategies in which firms offer a free good of base value and provide an improved version of the product to those who pay for it.

go through a brief literature review. In the third section we describe the setup, followed by a brief preliminary analysis. We then proceed to solve the cases where the cost of pirating is high and where there is no cost to piracy. The intermediate cost of piracy case is then considered followed by a brief extension to the model. Finally in section seven we briefly give some comments on welfare and policy implications.

1.2 Literature review

There exists a fairly rich literature on the pricing of network goods. The classic paper of networks in industrial organization by Katz and Shapiro (1986) where it is shown that under competitive paradigm, firms do not have an incentive to make their product compatible with other goods but they do have one to standardize. Fudenberg and Tirole (2000) show that under a network good paradigm, the incumbent may decide to keep low prices even without a direct competitor as long as the threat of entry exists.

There is also work that specifically studies the effect of the quality differential between the legal and illegal copy of a pirated product(Geng and Lee, 2013). The differential is said to be quite small when it comes to music, whilst in software this gap is larger. Most piracy models do in fact assume perfect substitution between the two goods when, dropping this assumption does induce different pricing and profit outcomes.

Similarly, Peitz and Waelbroeck (2006) use a sampling model to show that giving free samples to consumers may be profitable for firms. Chellappa and Shivendu (2005) show that much of the potential benefits from piracy can be extracted by the firm if it employs a sampling strategy to draw in consumers and then sell it to them. However in most of the domains this paper discusses, this is likely not a pertinent strategy, mainly because the ability to create samples for your product is very likely to be correlated with the ability to differentiate it.

There is also work showing that in the digital space, with perfect copying, stronger copyright may act as a coordinating device between firms so that they may collude(Jain, 2008). Indeed the work of Sunderajan (2004) shows that prices are reduced in the presence of piracy, a result that is duplicated in this paper.

Perhaps the closest model to our own is the model by Reavis Conner and Rumelt (1991) where they also have an intermediate option of piracy. However their model does not include a product improvement variable and does not give an explicit solution for a specific distribution. The approach is also quite different in that they assume a linear demand function, while the assumption here has to do with the distribution of agent valuations, which gives an

endogenous demand function. Additionally, the only difference between pirating and buying in their model is the cost of doing so, while our model includes a differential in valuation between the options. Another model of piracy which mimics our approach is by Machado et al. (2017), whom mimic our demand approach but focus more on the effects of piracy when open source competition is also a factor.

Finally, there is also work on the diffusion of innovation in more dynamic settings. Where piracy is said to boost innovation during the early stages of the product life-cycle process. Studying this setting yields a result that is also found on this paper, mainly that strengthening piracy controls is not necessarily an optimal strategy for digital markets(Gayer and Shy, 2003) (Moshe et al., 1995).

1.3 The model

We consider a monopolistic firm and a continuum of agents characterized by a parameter x distributed on the interval $[0, 1]$. Consumer i has a taste for the good, x_i. It is common knowledge that x is distributed according to F on $[0, 1]$ with associated density f. For the sake of simplicity we will assume that F is uniform on $[0, 1]$.

A consumer can choose between 3 options: not to use, buying the good or pirating the good. If a consumer either buys or pirates the good, we will say that the agent is a *user* of the good. If consumer i uses the good, whether she buys it or pirates it, she first derives a utility equal to her intrinsic valuation for the good x_i. She also derives a positive network effect that increases both with her taste for the good x_i and the number of other users of the good.

We will be assuming that buying the product gives the agent a higher network value. There are number of possible interpretations of this assumption. One could simply interpret that the legal version allows allows for easier socialization, either due to built in product feautures or simply physical considerations. It may also be that purchasing the good is associated with a signalling mechanism, where a buyer can signal to other agents that she bought the good. This interpretation may work either as the signal being positive, perhaps signalling that one is on the higher end of the wealth distribution, or as the lack of a signal showing something negative, for instance social stigma against piracy (a similar motivation is given in Reavis Conner and Rumelt (1991)). Finally the difference can be interpreted as having acess faster for a better version of the product. The speed of receival can affect the network value because it allows for more immediate socialization.

This difference is represented by the existence of two network parameters: α for the con-

sumers buying the good and β for the pirates. We assume that $\alpha > \beta$. The difference in the socialization parameters α(buying) and β(pirating) will play a key role in our results.

Though pirates do not pay the price for the product, they incur a subjective cost, r, which is exogenous to the firm. r can be interpreted as the expected cost of pirating due to local legislation. It can also be interpreted as normalized product degradation relative to the bought product, for example lower pixelation. Alternatively, r can be seen as the cost of resale, if the product is bought it may have a higher market value if sold back into the market.

Buying the good has an added value k, a 'bonus material' that is exogenously added to the product by the firm. It also has a price, p, which is also under control of the firm.

We introduce Q and G. Q is a function that takes the value 0 when agents with valuation t are not users and 1 if they are users. Similarly G is a function which takes the value 0 when consumers are not buying and 1 otherwise. Since Q includes both buyers and pirates and G only includes buyers, G is stochastically first order dominated by Q.

The fraction of consumers who are using the good is given by $\int_0^1 Q(t)f(t)dt$ and the fraction of consumers who are buying the good is given by $\int_0^1 G(t)f(t)dt$

For the sake of simplicity we assume that the network component of the utility is linear in all the parameters and consumers' utility is represented as follows.

$$U_i = \begin{cases} x_i(1 + \alpha(\int_0^1 Q(t)f(t)dt) + k) - p & if\ consumer\ i\ buys\ the\ good \\ x_i(1 + \beta(\int_0^1 Q(t)f(t)dt)) - r & if\ consumer\ i\ pirates\ the\ good \\ 0 & if\ consumer\ i\ does\ not\ buy\ the\ good \end{cases}$$

We assume the good is digital. The fact that the good is digital entails that the only cost to the firm is the fixed cost of production, FC. There is no marginal cost to having a larger user baser.

The firm has to choose the price, p with k and r being exogenous variables. The firms profit function is then given by:

$$\pi(p, r, k) = p \int_0^1 G(t)f(t)dt - FC \tag{1.3.1}$$

The timing of the model is that the firm sets its price and then agents simultanously choose. As such the concept of equilibrium used is a simple sub-game perfect nash equilibrium.

1.4 Preliminary Analysis

In this section we briefly show some results which naturally arise from our setup, these are independent of the distribution used.

Result 1. *If, at the equilibrium, a consumer with type $\tilde{x}$ buys the good with a strictly positive probability, then any consumer with a type $x > \tilde{x}$ buys the good with probability 1.*

Proof. If a consumer with type $\tilde{x}$ buys the good with a strictly positive probability, since consumers are rational, this means that :

$$\tilde{x}(1+\alpha(\int_0^1 Q(t)f(t)dt)+k)-p \geq \tilde{x}(1+\beta(\int_0^1 Q(t)f(t)dt))-r \tag{1.4.1}$$

and

$$\tilde{x}(1+\alpha(\int_0^1 Q(t)f(t)dt)+k)-p \geq 0 \tag{1.4.2}$$

If $\tilde{x}=1$, the proof is direct.

If $\tilde{x}<1$, since $\tilde{x}(1+\alpha(\int_0^1 Q(t)f(t)dt)+k)-p \geq 0$, $x(1+\alpha(\int_0^1 Q(t)f(t)dt)+k)-p>0$ for any $x>\tilde{x}$. This means that for any $x>\tilde{x}$, $Q(x)=1$ so that $\int_0^1 Q(t)f(t)dt>0$. Then, for any $x>\tilde{x}$

$$(x-\tilde{x})(\alpha-\beta)\int_0^1 Q(t)f(t)dt>0 \tag{1.4.3}$$

Therefore:

$$\tilde{x}(1+\alpha(\int_0^1 Q(t)f(t)dt)+k)-p+(x-\tilde{x})(\alpha-\beta)\int_0^1 Q(t)f(t)d > \tilde{x}(1+\beta(\int_0^1 Q(t)f(t)dt))-r \tag{1.4.4}$$

Hence:

$$x(1+\alpha(\int_0^1 Q(t)f(t)dt)+k)-p \geq x(1+\beta(\int_0^1 Q(t)f(t)dt))-r \tag{1.4.5}$$

So that for any $x>\tilde{x}$, a consumer strictly prefers buying rather pirating or non using the good.

□

Result 2. *If, at the equilibrium, a consumer with type $\tilde{x}$ does not use the good with a strictly positive probability, then any consumer with a type $x < \tilde{x}$ does not use the good with probability 1.*

Proof. If a consumer with type $\tilde{x}$ does not use the good with a strictly positive probability, since consumers are rational, this means that :

$$\tilde{x}(1 + \beta(\int_0^1 Q(t)f(t)dt)) - r \leq 0 \tag{1.4.6}$$

and

$$\tilde{x}(1 + \alpha(\int_0^1 Q(t)f(t)dt) + k) - p \leq 0 \tag{1.4.7}$$

Then, for any $x < \tilde{x}$:

$$x(1 + \beta(\int_0^1 Q(t)f(t)dt)) - r < 0 \tag{1.4.8}$$

and

$$x(1 + \alpha(\int_0^1 Q(t)f(t)dt) + k) - p < 0 \tag{1.4.9}$$

and the consumer strictly prefers not using the good rather pirating it or buying it.

□

Corollary 1. *If, at the equilibrium, a consumer with type $\tilde{x}$ pirates the good with a strictly positive probability, then all agents with a type $x > \tilde{x}$ are users of the good with a strictly positive probability and no consumer with a type $x < \tilde{x}$ buys the good with a strictly positive probability.*

Corollary 2. *For any value of the parameters of the game, any equilibrium is characterized by a pair $(\check{x}, \hat{x})$ with $0 \leq \check{x} \leq \hat{x} \leq 1$ such that If $x_i < \check{x}$, consumer i does not use the good, if $\check{x} < x_i < \hat{x}$, consumer i pirates the good and if $\hat{x} < x_i$, consumer i buys the good.*

Considering that the inequality $0 \leq \check{x} \leq \hat{x} \leq 1$ is not strict, any combinations of non-users, pirates and buyers remains possible depending on the value of the parameter of the model.

1.5 Solving the model

1.5.1 Equilibrium when r=0, no piracy cost

If $r = 0$, there is no reason not to use the good. Pirating is *free*. Consumers will only choose between pirating and buying.

Proposition 1. *If $r = 0$, the firm will choose $p = \frac{\alpha-\beta+k}{2}$, $\check{x} = 0$ and $\hat{x} = 1/2$ and $\pi = \frac{1}{4}(\alpha - \beta + k)$.*

Proof. Since all consumers uses the good, only $\hat{x}$ is relevant and it is defined by the following indifference condition, (since a consumer with type $\hat{x}$ must be indifferent between buying and pirating the good):

$$\begin{aligned} \hat{x}(1 + \alpha + k) &= \hat{x}(1 + \beta) \\ \Rightarrow \hat{x} &= \frac{p}{\alpha - \beta + k} \end{aligned}$$

Now, the profit function is

$$\tilde{\pi} = p\left(1 - \frac{p}{\alpha - \beta + k}\right)$$

The first order condition is given by:

$$\begin{aligned} \frac{\partial \tilde{\pi}}{\partial p} &= 1 - \frac{2p}{\alpha - \beta + k} \\ \Rightarrow p &= \frac{\alpha - \beta - k}{2} \end{aligned}$$

This gives $p^* = \frac{\alpha-\beta+k}{2}$ and $\pi = \frac{1}{4}(\alpha - \beta + k)$ if we put these values in the profit equation.

□

The results are reminiscent of standard monopoly tarification with $\alpha - \beta$ as a parameter for consumer's' taste for the good, plus the the product improvement which happens to be partly orthogonal. Quite intuitively, the price and the profit are increasing in $\alpha - \beta$ and

decreasing in c. But the quantity sold is independent of these parameters and it is always equal to 1/2. For any value of the parameters (as long as $r = 0$), the firm will choose p such that half of the consumers are buyers and half of the consumers are pirates. But let us remember that for the profit of the firm pirates are not equivalent to non users because a pirate as a user of the good is also a member of the network. since the network is larger with pirates, the value of the good increases for consumers with large x. We will show in the next session that we do not find identical results in an environment with only buyers and non-users (r sufficiently large so that no consumer pirates the good).

1.5.2 Equilibrium when r is high

Suppose that r is so high that piracy is never a good choice for a consumer. The minimum value of r in order to obtain such a result depends on the value of the other parameters of the model but for any value of these other parameters, there always exists a critical $\tilde{r}$, such that if $r > \tilde{r}$, no consumer wants to pirate the good. Although, we are back in a more standard Industrial Organization environment without any piracy, because of the network effect, the solution is not trivial and in order to simplify computations, we will assume that $c = 1$. The agent indifferent between buying and not using defined by:

$$\tilde{x} = \frac{\alpha + k + 1 - \sqrt{(\alpha + k + 1)^2 - 4\alpha p}}{2\alpha}$$

[2]

We can verify that $\hat{x}$ is decreasing in α and conversely, the proportion of users who are buying is increasing in α. We now give the expression for the price:

Proposition 2. *The optimal price of the firm is given by:*

$$\tilde{p} = \begin{cases} \frac{1+k}{2} & \textit{when: } \alpha = 0 \\ \frac{(\alpha+k+1)^2+2\alpha(k+1)-\omega}{9\alpha} & \textit{when: } \alpha \in]0, 1+k[\\ \frac{2(k+1)}{3} & \textit{when: } \alpha = 1+k \\ \frac{(\alpha+k+1)^2+2\alpha(k+1)+\omega}{9\alpha} & \textit{when: } \alpha \in]1+k, \infty[\\ \textit{where} \quad \omega = \sqrt{(1-\alpha+k)^2(\alpha^2+\alpha(k+1)+(k+1)^2)} & \end{cases}$$

[2] If $\alpha = 0$, then $\tilde{x} = \frac{p}{1+k}$

Proof. see appendix 1.8.2 □

The price is always increasing in the network value. However, the network value has an increasing effect on the price, when the network value is already large, then marginal increases will have a stronger effect on the price. The reason for this increasing effect of network value on price is the double effect of the network good. An increase in the network value, for the same price, increases both the valuation of users and their proportion. Which means that the firm can always find a p which is higher than before still have a higher proportion of users. This increasing effect eventually peaks at linearity[3], if α is large relative to $1+k$, the price can be approximated to:

$$p \approx \frac{2(1+\alpha+k)}{9} \tag{1.5.1}$$

Since a firm can always just ignore the network value and price according to the base value, the adaptive strategy implies that profit is increasing in network value. However if we use this approximated price, we can see that the share of agents buying will eventually be $\frac{2}{3}$

1.5.3 Does the firm prefer the high product degradation case or the no product degradation case?

Proposition 3. *There $\exists\overline{\alpha}$ such that if $\alpha > \overline{\alpha}$ the firm prefers $r = 0$ rather than $r \geq \tilde{r}$*

Proof. See appendix 1.8.3 □

In other words, the firm will prefer full piracy to no piracy if the network value is large. If the network value is low, $\overline{\alpha} < \alpha$, the firm is better off if only purchasers can use its product. Similarly, if the network value is high, $\overline{\alpha} < \alpha$, then the value added of pirates to the user base is large enough that profit is improved is higher with them.

We can also think of this from the inverse demand curve point of view. If there is no product degradation there is a simple linear inverse demand curve. On the other hand, the equilibrium with no pirates has a concave inverse demand function. More over, the concavity of the demand curve is increasing in α. So while the demand curve of the pirates is ever expanding towards the right, the concavity of the inverse demand curve ends up disabling the firm from being able to take advantage of the increasing network value. Which is why, at some point the α just stops adding value to the firm.

[3]The price is regularly varying with an index of $i = 1$, see appendix

Note that the interpretation of the above crucially depends on α increasing not independently but relative to β. If α is large then the slopes that need to be compared to have the equivalent result (eventually) are $\frac{\alpha-\beta}{4}$ and $\frac{2\alpha}{9}$. That is, the differential between the $\alpha-\beta$ must increase.

1.5.4 Equilibrium for intermediate values of the cost of piracy

We now begin to analyze the case where there are intermediate values of the cost of piracy. To simplify the analysis we now make an additional assumption, $\beta = 1$. This assumption implies that even if all agents are users, less than half the utility of pirating will come from the non-network value of the good.

There are now two indifference conditions to consider. There is the agent who is indifferent between pirating and buying, $\check{x}$, and the agent who is indifferent between buying and pirating, $\hat{x}$. Because of the monotonicity results in the preliminary analysis we also know that $\hat{x} > \check{x}$.

Proposition 4. *The proportion of agents who are users and consumers, respectively, are given by:*

$$1 - \check{x} = \sqrt{1-r} \tag{1.5.2}$$

$$1 - \hat{x} = 1 - \frac{p-r}{(\alpha-1)\sqrt{1-r}+k} \tag{1.5.3}$$

Proof. See the appendix, 1.8.4 □

Note that if r is 0, all the consumers are users. Which implies that all agents will be either pirates or buyers. From the user demand function we can see that a decrease in the cost of piracy necessarily increases the proportion of users. If $r = 0$ then all agents can be induced to prefer to pirate rather than not using and if $r > 1$, this is sufficient to ensure there are no pirates.

The exogenous interpretation of changes in r and α on demand has to do with timing. If the firm would first set it's price and product improvement and then either the state or some other exogenous force would shift the value of r and α we would see the described effect. In this interpretation α just increases the demand for buying. The effect of r is ambiguous.

Inside the profit of the firm only the buyer demand function directly enters. The profit of the firm is:

$$\pi = p\left(1 - \frac{p-r}{(\alpha-1)\sqrt{1-r}+k}\right)$$

Proposition 5. *The optimal price and profit with $r \in [0, \tilde{r}]$ are given by:*

$$p^*_{bpn} = \frac{k + (\alpha-1)\sqrt{1-r} + r}{2} \tag{1.5.4}$$

$$\pi^*_{bpn} = \frac{2\alpha\left(\sqrt{1-r}(k+r) + 3(r-1)\right) - \sqrt{1-r}(k+r) - r(r+3) + 3 - k(k+2) - 3\alpha^2(r-1)}{4(\alpha-1)\sqrt{1-r} - 4k} \tag{1.5.5}$$

Proof. Follows directly after setting the first order condition:

$$\frac{\partial \pi}{\partial p} = 1 - \frac{(2p-r)}{k + (\alpha-1)\sqrt{1-r}} = 0$$

We can also verify that the profit is strictly concave:

$$\frac{\partial^2 \pi}{\partial p^2} = -\frac{2}{k + (\alpha-1)\sqrt{1-r}} < 0$$

Recall that $\alpha \geq 1$ to see the concavity. □

The price can give us some interesting inights. We can notice that an increase in the cost of piracy, r can have both effects. If α is high it will decrease the optimal price, whilst if it is low it will increase the optimal price. Specifically we can see that if $\alpha > 1 + 2\sqrt{1-r}$, an increase in the cost of product degradation will decrease the price level.

Since have a general idea of the comparative statics of the product improvement, we can now make a final statement about the effect of the the cost of piracy.

Proposition 6. *There is always a network value, α where any increases in product degradation can only decrease profits.*

Proof. We can use the envelope theorem here (see 1.9.4)and only need to look at the direct

effect or r.

$$\frac{\partial \pi^*}{\partial r} = \frac{p^*}{(\alpha-1)\sqrt{1-r}+k}\left(1 - \frac{(\alpha-1)(p^*-r)}{2\sqrt{1-r}\left((\alpha-1)\sqrt{1-r}+k\right)}\right)$$

So the effect is negative if

$$\begin{aligned}
1 &< \frac{(\alpha-1)(p^*-r)}{2\sqrt{1-r}\left((\alpha-1)\sqrt{1-r}+k\right)} \\
2\sqrt{1-r}\left((\alpha-1)\sqrt{1-r}+k\right) &< (\alpha-1)(p^*-r) \\
2\sqrt{1-r}\left((\alpha-1)\sqrt{1-r}+k\right) &< (\alpha-1)\left(\frac{k+(\alpha-1)\sqrt{1-r}-r}{2}\right) \\
\sqrt{1-r}+\frac{k}{(\alpha-1)} &< \frac{k+(\alpha-1)\sqrt{1-r}-r}{4\sqrt{1-r}}
\end{aligned}$$

Notice that as α increases, the RHS diverges to $+\infty$ whilst the LHS converges to $\sqrt{1-r}$. □

This result is a generalization of the proposition in the previous section. It shows that even if the cost of piracy is not high or zero, it can be marginally true that decreasing product degradation increases profits.

Proposition 7. *If the buyer network value is larger than $\alpha > 1$, then further increases in the buyer network value increases the number of buyers.*

Proof.

$$\begin{aligned}
1-\hat{x} &= 1 - \frac{p^*-r}{(\alpha-1)\sqrt{1-r}+k} \\
&= 1 - \frac{k+(\alpha-1)\sqrt{1-r}-r}{2(\alpha-1)\sqrt{1-r}+2k} \\
\frac{\partial(1-\hat{x})}{\partial\alpha} &= -\frac{r\sqrt{1-r}}{2\left((\alpha-1)\sqrt{1-r}+k\right)^2}
\end{aligned}$$

Since $1-r>0$, this expression is always negative.

□

Note that the proportion of users, is independent of the value of α. In other words, increasing α decreases the number of pirates and increases the number of buyers even though the price is strictly increasing α.

1.6 Extension: Endogenous product improvement

The product improvement, k was assumed exogenous for the baseline model, we now endogenize it for a brief extension. The cost of k is given by a simple convex cost function, $c(k) = ck^2$ with $c \geq 0$.

How are the results affected when the firm can invest to improve its product? The statics relating to the cost of piracy(r) are not affected. The main result in this section is to show through a mix of analytic and point numerical estimations that product improvement(k) and network value(α) are complementary for low values of α and substitutable for high values of α.

We will be assuming that $c = 1$ after the cost-less piracy case. Extending the results to have endogenous product improvement does not always yield a closed form solution. The only case where there is a simple closed form solution is when piracy is cost-less. The most notable effect of endogenizing k in the cost-less case is that the optimal product improvement does not depend on α or β. It is simply given by $k = \frac{1}{8c}$. See appendix 1.9.4 for details.

There is no closed form solution for the corresponding case where $\tilde{r} < r$. We instead simulate the profit function as seen in figure 1.1. In this diagram both k and a are treated as endogenous variables. Notice that profit (π) is maximized around $k = 0.25$ even as α reaches infinity. Notice also that the product improvement is a strictly increasing function of α.

This implies that the product improvement pursued without pirates is strictly higher than the full piracy case. Indeed, they are only equal without network value, $\alpha = 0$.

Similarly we simulate the case where $0 < r < \tilde{r}$. Since we now have an additional dimension to represent, we use a 3D contour graph, see figure 1.2. In this case the k is the optimal k pursued. Notice that once again the equilibrium k is around $\frac{1}{8}$ for low values of product degradation r. However, in this scenario, the previous result is reversed, that is, higher piracy pursuit decreases the optimal product improvement. The intuition behind this counterintuitive result is that r and k are substitutes, the firm can use the higher r as an alternative to spending on k. In other words, the local effect of an increase in r within $0 < r < \tilde{r}$ is to decrease product improvement but once $r = \tilde{r}$, then there is a discontinous jump which

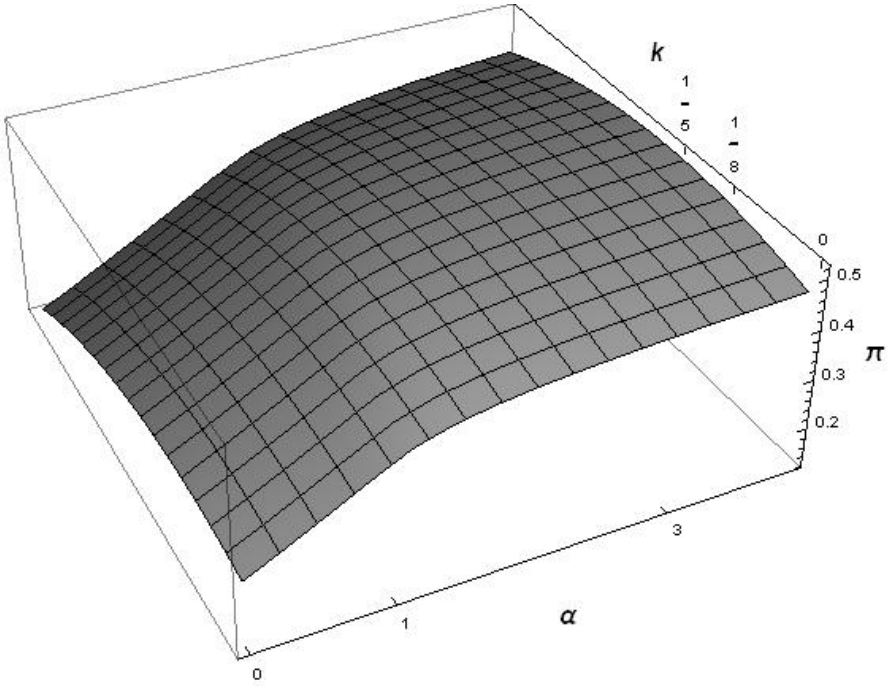


Figure 1.1: Simulation of profits with a high cost of piracy

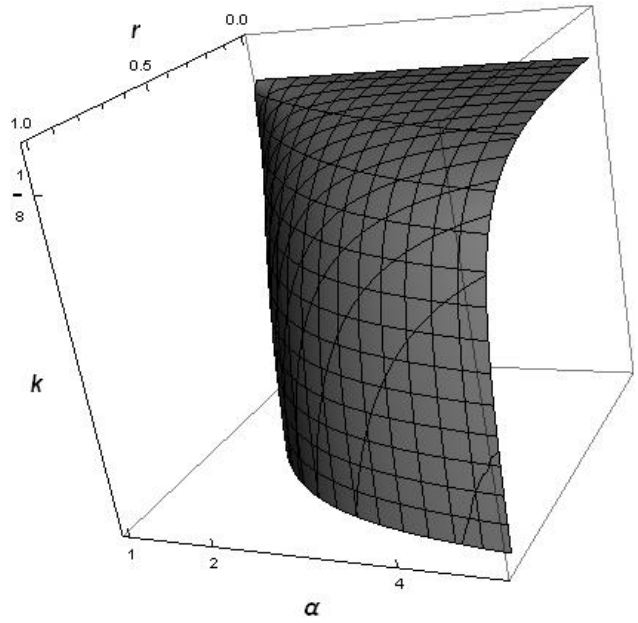


Figure 1.2: Intermediate cost of piracy case: Contour of the profit function maximized with repspect to the product improvement

depends on the network value which causes an increase in product improvement.

1.7 Discussion and Conclusion

The interpretation so far has been about network goods however a reputation interpretation could work equally well. If we interpret both α and β as reputational effects. Both α and β can be seen as signalling devices that can only be recognized by the portion of agents who are using. This interpretation may be less intuitive, however it more naturally explains why there may be a differential between the two network values.

The model has been assuming a kind of worse case scenario as to the effect of pirates. Though pirates can be used by the firm to increase its prices, there is no direct revenue mechanism for them. This could easily be imagined in a few cases, for instance, in the digital world a higher use base would allow for indirect profit opportunities. Take a social network as an example, a larger user base allows the network owner to sell advertising slots at a higher price, and the wider the reach the more profitable the enterprise. This kind of mechanism would only strengthen the profits of the firm in the pure piracy case whilist leaving the classic case unaffected.

Welfare in the model is always decreasing with respect to the cost of piracy. This is simply because the firm mostly does not produce the product since the welfare from an increase in user base is always positive. In the baseline model the product already exists, it only by endogenizing the product improvement that the firm can add value. This does not entail that product degradation should always be zero because profits must be higher than the fixed cost of creating the product. Instead, it should only be noted that the framework presented here actually implies the existence of pareto improving policies. That is, when the network value is high the firm prefers there to be a higher amount of piracy over no piracy at all.

Our model shows that even if the firm has total control of the level of piracy pursuit, it would not necessarily fully utilize this capability. Under the conditions discussed above, the firm often has incentive to not use deterrence to push consumers out of its pool of customers because reducing the number of consumers also decreases the value of the good for those who would buy. The intuition behind this carrot and stick approach is that the price has a single effect whilst piracy pursuit has a double effect. The price effect is less variable because it gives the firm the ability to be more precise in its targeting mechanism. Wile changing the price only works on the consumers who are marginally on the edge of the choice of buying or pirating, changing the degradation level affects all segments at once, resulting in more global network effects.

Increasing the product degradation affects both the proportion of users buying while also

decreasing the user base. The strength of these two effects depends on their valuation of the network value. The decrease in the user base corresponds to the product becoming less popular and the change in buying behavior represents either a consumer who no longer wishes to undertake the risk of pirating or a consumer who no longer wishes to buy the product because it has less value to her.

Though the more traditional way of seeing pirates is as a competitive force, in the model presented here pirates take the role of the path of least resistance. Indeed controlling the pirates to ensure that they are at the right level is a fine art and the if the instrument is too blunt, it may cause more harm than good. The conditions for piracy to be optimal are relatively mild, the bought network value need only reach a certain threshold for it to be worthwhile. Indeed though the model assumed that pursuing pirates is costless, in the real world, there are significant costs both from the point of view of the planner and the firm. The endogenization of these costs would only decrease the threshold α for which the firm would prefer piracy to exist.

One simple policy implication of the model is that pursuing pirates uniformly for all their activities may have welfare reducing effects. An optimal solution would be one where the pursuit of piracy is heterogeneous to each firm. Much like how firms only sue specific infringers and let others go, the same can be encouraged on the consumer side. From the legal point of view the implication is that piracy could be viewed as a civil or corporate issue and not a criminal one. For broadening the scope of tort law, see (Friedman, 1996).

Still, this approach is limited in that, whilst it is possible for a firm to identify other infringers and selectively sue them, it is likely that the number of pirates is too high to accurately identify. However, if we are to include the cost of identification, this same cost has to be applied on any mechanism that enforces piracy laws. A policy that focuses on enforcement against consumers is likely to have a much higher social cost than one which is enforced on firms in accordance with the actual number of consumers.

Put in another way, the problem with copyright enforcement is that it is unconditional. That is, unlike patents, where the patent right holder chooses to initiate an action against a patent infringer, presumably following a calculation which leads the patent right holder to the conclusion that such an action is worthwhile. The illegal act of piracy is unconditionally a criminal offense. In other words, regardless of whether the effects of the act aid or harm a copyright owner there is a consistent social cost imposed.

In practice, firms cannot control the probability of success when initiating an action. In industries where the pirates are many and are highly decentralized, it is quite plausible

that neither the government nor the firms can greatly increase their level of piracy pursuit without greatly invading personal data and privacy laws. Realistically, cases involving music or movies are more adapted and fit better into the above developed model.

It is also important to note that there are firm adaptation which can be used as alternatives to the law. These strategies may be less costly because they can focus on prevention rather than pursuit. In the digital space, these strategies are usually labeled, Digital Rights Management (DRM) strategies. Examples of DRM are idea's such as requiring users to use specific platforms to access the content, monitoring consumption, selling restricted usage copies etc. Such strategies may be complementary or substitutable with piracy enforcement. DRM strategies allow firms to select their own level of piracy pursuit. Piracy often has its own platforms which continously improve their own services. Indeed much of the historic incentive to pirate was due to the superior platform services of piracy software. The new wave of streaming services such as Netflix or Amazon prime represent competition to these piracy services.

A hidden assumption in the model presented above is that all users value socializing with all other users. In the real world, if there are segregation effects, this assumption may not be fully applicable. On the other hand, if we assume that segregation brings together more diverse valuations, such as perhaps a family unit, then this solidifies the conclusions of this model.

Generally, the assumption that utilities are independent is often too readily used. What is termed a "network good" in the industrial organization literature can be interpreted too narrowly. It is not simply digital social networks that have this property. In fact, it is possible to envision most goods as having both an intrinsic value and an extrinsic component. The relative ratio of intrinsic to extrinsic value will likely vary substantially between cultures, space and time. Additionally, the domain in which this is true is likely to extend much farther than what common intuition would entail. For instance, even perishable goods, such as food, are not necessarily exempt from this feedback process, for instance the consumption of cheese or wine is not something that is independent of the surrounding culture. Accordingly, much of what can be deemed "group identity" can be represented within a network good framework. A culture of baguette or chocolate eaters does not represent the same profit opportunities as groups without such culture.

One of the most common arguments against piracy is that it decreases the number of individuals who actually pay the real cost of the product. However, the implicit assumption in such a line of reasoning is that the number of purchasers and the value of the purchasing, are independent. Based on the model presented here, such a conclusion is drawn too hastily,

the effect of increasing the piracy cost may not have the desired effect. As pointed out in Reavis Conner and Rumelt (1991) this setup raises strange ethical questions. When we have possible parameter values that imply that higher pursuit of pirates may be worse for consumers and producers at public cost. With these factors in mind, it is unclear whether enforcing intellectual property rights passes even the most basic cost benefit analysis.

1.8 Appendix

1.8.1 Buyers and pirates

$$\hat{x} = \frac{p}{\alpha - \beta + k}$$

The upper condition is $\alpha - \beta + k \geq p$. Therefore the price is:

$$p = \frac{1}{2}(\alpha - \beta + k)$$

Note that these values imply that $\hat{x} = \frac{1}{2}$ regardless of the values of β or α. Profit is then:

$$\begin{aligned} \pi &= p(1 - \hat{x}) \\ &= \frac{1}{4}(\alpha - \beta + k) \end{aligned}$$

Social Surplus and welfare are:

$$\begin{aligned} \hat{S} &= \int_0^{\hat{x}} x(1+\beta)dx + \int_{\hat{x}}^1 (x(1+\alpha+k) - p)\, dx \\ &= \frac{1}{8}(\alpha + 2\beta + k + 3) \\ \hat{W} &= \frac{3}{8}(1 + \alpha + k) \end{aligned}$$

1.8.2 Proof of proposition 2

The demand function in the case of buyers only is given by:

$$1 - \frac{\alpha + k + 1 - \sqrt{(-\alpha - k - 1)^2 - 4\alpha p}}{2\alpha}$$
$$\frac{\alpha - k - 1 + \sqrt{(-\alpha - k - 1)^2 - 4\alpha p}}{2\alpha}$$

The profit function is then:

$$p * \frac{\alpha - k - 1 + \sqrt{(-\alpha - k - 1)^2 - 4\alpha p}}{2\alpha}$$

Derivative with respect to p :

$$\frac{\sqrt{(\alpha + k + 1)^2 - 4\alpha p} + \alpha - k - 1}{2\alpha} - \frac{p}{\sqrt{(\alpha + k + 1)^2 - 4\alpha p}} = 0$$
$$\frac{\sqrt{(\alpha + k + 1)^2 - 4\alpha p} + \alpha - k - 1}{2\alpha} = \frac{p}{\sqrt{(\alpha + k + 1)^2 - 4\alpha p}}$$
$$(\alpha + k + 1)^2 - 4\alpha p + (\alpha - k - 1)\sqrt{(\alpha + k + 1)^2 - 4\alpha p} = 2p\alpha$$
$$(\alpha + k + 1)^2 + (\alpha - k - 1)\sqrt{(\alpha + k + 1)^2 - 4\alpha p} = 6p\alpha$$
$$(\alpha - k - 1)\sqrt{(\alpha + k + 1)^2 - 4\alpha p} = 6p\alpha - (\alpha + k + 1)^2$$
$$(\alpha - k - 1)^2((\alpha + k + 1)^2 - 4\alpha p) = (6p\alpha - (\alpha + k + 1)^2)^2$$
$$0 = 9p^2\alpha - 2p(\alpha^2 + 4\alpha(1 + k) + (1 + k)^2) + (\alpha + k + 1)^2(1 + k)$$

This is gives two solutions for p:

$$\tilde{p} = \frac{(\alpha + k + 1)^2 + 2\alpha(k + 1) \pm \sqrt{(1 - \alpha + k)^2(\alpha^2 + \alpha(k + 1) + (k + 1)^2)}}{9\alpha}$$
$$= \frac{(\alpha + k + 1)^2}{9\alpha} + \frac{2(1 + k)}{9} + \frac{\pm\sqrt{(1 - \alpha + k)^2(\alpha^2 + \alpha(k + 1) + (k + 1)^2)}}{9\alpha}$$

If α is large relative to $1 + k$, the expression simplifies, we will represent the use of this assumption by $\approx$.

$$
\begin{aligned}
p &\approx \frac{\alpha}{9} + \frac{2(1+k)}{9} + \frac{\pm\sqrt{(\alpha^2 + \alpha(k+1))}}{9} \\
&= \frac{\alpha}{9} + \frac{2(1+k)}{9} + \frac{\pm\sqrt{(\alpha^2 + \alpha^2\frac{(k+1)}{\alpha})}}{9} \\
&= \frac{\alpha}{9} + \frac{2(1+k)}{9} + \frac{\pm\alpha\sqrt{\frac{k+1+\alpha}{\alpha}}}{9} \\
&= \frac{2\alpha}{9} + \frac{2(1+k)}{9}
\end{aligned}
$$

corollary:

$$\sqrt{(\alpha + k + 1)^2 - 4\alpha p}$$

Substitute in the approximated price:

$$
\begin{aligned}
&\approx \frac{1}{3}\sqrt{a^2 + 10a(k+1) + 9(k+1)^2} \\
&\approx \frac{1}{3}\sqrt{a^2 + 10\frac{a^2}{a}(k+1)} \\
&\approx \frac{a}{3}\sqrt{1 + \frac{10}{a}(k+1)} \\
&\approx \frac{a}{3}
\end{aligned}
$$

We can use this expression to represent the proportion of agents who will consume in equilibrium:

$$1 - \tilde{x} = 1 - \frac{\alpha + k + 1 - \frac{a}{3}}{2\alpha} \tag{1.8.1}$$

$$= \frac{4\alpha - 3(1+k)}{6\alpha} \approx \frac{2}{3} \tag{1.8.2}$$

1.8.3 Proof of proposition 3

We now look at the what the derivative of the profit function turns out to be. First note that in proposition 1.9.4 we deduce that the envelope theorem applies. We also use corollary

1.8.2 to simplify the square root and proposition to have the simplified price which assume that α is large.

$$\frac{\partial f}{\partial \alpha} = \frac{\frac{2(\alpha+k+1)-4p}{2\sqrt{(\alpha+k+1)^2-4\alpha p}}+1}{2\alpha} - \frac{\sqrt{(\alpha+k+1)^2-4\alpha p}+\alpha-k-1}{2\alpha^2}$$

$$= \frac{1}{2\alpha}\left(\frac{2(\alpha+k+1)-4p}{2\sqrt{(\alpha+k+1)^2-4\alpha p}}+1-\frac{\sqrt{(\alpha+k+1)^2-4\alpha p}+\alpha-k-1}{\alpha}\right)$$

$$= \frac{1}{2\alpha}\left(\frac{(\alpha+k+1)-2p}{\sqrt{(\alpha+k+1)^2-4\alpha p}}-\frac{\sqrt{(\alpha+k+1)^2-4\alpha p}-k-1}{\alpha}\right)$$

$$= \frac{1}{2\alpha}\left(\frac{\alpha(\alpha+k+1)-2\alpha p}{\alpha\sqrt{(\alpha+k+1)^2-4\alpha p}}-\frac{\sqrt{(\alpha+k+1)^2-4\alpha p}(\sqrt{(\alpha+k+1)^2-4\alpha p}-k-1)}{\alpha\sqrt{(\alpha+k+1)^2-4\alpha p}}\right)$$

$$= \frac{1}{2\alpha^2\sqrt{(\alpha+k+1)^2-4\alpha p}}\left(2\alpha p+(k+1)(\sqrt{(\alpha+k+1)^2-4\alpha p})-(\alpha+k+1)^2)\right)$$

We now assume that α is large relative to k+1 and substitute in the price and square root

$$= \frac{3}{2\alpha^3}\left(2\alpha p+(k+1)(\frac{a}{3}-(\alpha+k+1)^2)\right)$$

$$= \frac{3}{2\alpha^3}\left(2\alpha(\frac{2\alpha}{9}+\frac{2(1+k)}{9})+(k+1)(\frac{a}{3}-(\alpha+k+1)^2)\right)$$

$$= \frac{3}{2\alpha^3}\left(\frac{4\alpha^2}{9}+\frac{4\alpha(1+k)}{9}+(k+1)(\frac{a}{3}-(\alpha+k+1)^2)\right)$$

$$= \frac{3}{18\alpha^3}\left(4\alpha^2+4\alpha(1+k)+(k+1)(\frac{a}{3}-(\alpha+k+1)^2)\right)$$

Multiply by the price

$$p(a)\frac{\partial f}{\partial \alpha} = (\frac{2\alpha}{9}+\frac{2(1+k)}{9})\frac{3}{18\alpha^3}\left(4\alpha^2+4\alpha(1+k)+(k+1)(\frac{a}{3}-(\alpha+k+1)^2)\right)$$

$$= \frac{\alpha+1+k}{27\alpha^3}\left(4\alpha^2+4\alpha(1+k)+(k+1)(\frac{a}{3}-(\alpha+k+1)^2)\right)$$

$$= \frac{\alpha+1+k}{27\alpha^3}\left(4\alpha^2+(k+1)(\frac{13a}{3}-(\alpha+k+1)^2)\right)$$

If alpha is large relative to k+1

$$= \frac{\alpha}{27\alpha^3}\left(4\alpha^2 + (k+1)(\frac{13a}{3} - \alpha^2)\right)$$
$$= \frac{1}{27\alpha^2}\left(4\alpha^2 + (k+1)(\frac{13a}{3} - \alpha^2)\right)$$
$$= \frac{1}{27\alpha^2}\left(4\alpha^2 + (k+1)\frac{\alpha}{3}(13 - 3\alpha)\right)$$
$$= \frac{1}{71\alpha}\left(12\alpha + (k+1)(13 - 3\alpha)\right)$$

If the second term is negative, which occurs after α exceeds 5 we can say

$$= \frac{1}{71\alpha}\left(12\alpha + (k+1)(13 - 3\alpha)\right) < \frac{12}{71}$$

Recall that the slope of the profit function with pirates is $\frac{1}{4}$, therefore profit under piracy increases more than profit without piracy if α is relatively large

1.8.4 Proof of proposition 4

We first begin with the condition which renders an agents indifferent between pirating and not using to solve for the valuation x of the indifferent pirate.

$$x(1 + \beta(1 - F(x))) - r = 0$$
$$x(1 + \beta(1 - x)) = 0$$
$$x + x\beta - \beta x^2 - r = 0$$
$$x^2 - x\frac{(1+\beta)}{\beta} + \frac{r}{\beta} = 0$$
$$\check{x} = \frac{(1+\beta)}{2\beta} \pm \frac{\sqrt{\left(\frac{(1+\beta)}{\beta}\right)^2 - 4\frac{r}{\beta}}}{2}$$
$$\check{x} = \frac{(1+\beta)}{2\beta} \pm \frac{\sqrt{(1+\beta)^2 - 4r\beta}}{2\beta}$$

$$\check{x} = 1 \pm \sqrt{1 - r}$$

If $\check{x} > 1$ is verified this implies that no users will be pirating since all users prefer not to pirate. Therefore only the negative solution can be used. With the indifferent user, we need only take the difference $1 - \check{x}$ to know the proportion of agents who are users. This proportion is simply:

$$1 - \check{x} = \sqrt{1 - r}$$

We note that the proportion of agents who are users is entirely exogenous to the firm. What is endogenous to the firm is the proportion of agents who are buyers. The proportion of agents who are buyers, ceteris parabus, depends on the proportion who are pirates. The consumer who is indifferent between purchasing and pirating can similarly be found by equating the two options. Which gives the following value:

$$\begin{aligned} x(1 + \beta(1 - F(x))) - r &= x(1 + \alpha(1 - F(x))) - p \\ \Rightarrow \hat{x} &= \frac{p - r}{(\alpha - 1)\sqrt{1 - r} + k} \end{aligned}$$

Where the last step occurs because $1 - F(x) = 1 - \check{x}$. Recall that $\hat{x} \geq \check{x}$ from the preliminary section. The demand function for buying can then simply be written as:

$$\begin{aligned} D(\hat{x}) = 1 - \hat{x} &= 1 - \frac{(p - r)}{k + (\alpha - 1)\sqrt{1 - r}} \\ \frac{\partial D(\hat{x})}{\partial p} &= -\frac{1}{k + (\alpha - 1)\sqrt{1 - r}} \\ \frac{\partial D(\hat{x})}{\partial k} &= \frac{(p - r)}{(k + (\alpha - 1)\sqrt{1 - r})^2} \end{aligned}$$

1.9 Endogenous k

1.9.1 case where $r = 0$

Proposition 8. *If $r = 0$, the firm will choose $(p, k) = (\frac{1}{16c} + \frac{\alpha-\beta}{2}, \frac{1}{8c})$ $\check{x} = 0$ and $\hat{x} = 1/2$ and $\pi = \frac{1}{4}(\alpha - \beta) + \frac{1}{64c}$.*

Proof. As in the exogenous case we solve for the indifference condition:

$$\begin{aligned} \hat{x}(1 + \alpha + k) &= \hat{x}(1 + \beta) \\ \Rightarrow \hat{x} &= \frac{p}{\alpha - \beta + k} \end{aligned}$$

Now, the profit function is

$$\tilde{\pi} = p\left(1 - \frac{p}{\alpha - \beta + k}\right) - ck^2$$

The two first order conditions, relatives to p and k, for the profit maximization are:

$$\begin{aligned} \frac{\partial \tilde{\pi}}{\partial p} &= 1 - \frac{2p}{\alpha - \beta + k} \\ \frac{\partial \tilde{\pi}}{\partial k} &= \frac{p^2}{(\alpha - \beta + k)^2} - 2ck \end{aligned}$$

$$p = \frac{\alpha - \beta - k}{2}$$

and

$$2ck = \frac{p^2}{(\alpha - \beta + k)^2}$$

This gives $k^* = \frac{1}{8c}$, and $p^* = \frac{\alpha-\beta}{2} + \frac{1}{16c}$ and $\pi = \frac{1}{4}(\alpha - \beta) + \frac{1}{64c}$ if we put these values in the profit equation.

□

Observation 1. *The product improvement chosen by the firm does not depend on network values, β and α*

1.9.2 Intermediate r

$0 < r < \tilde{r}$

We need to also verify that the profit function satisfies the first and second order conditions with respect to the price level.

$$\begin{aligned}
\pi &= p\,(1-\hat{x}) - k^2 \\
&= p\left(1 - \frac{(p-r)}{(\alpha-1)\sqrt{1-r}+k}\right) - k^2 \\
\frac{\partial \pi}{\partial p} &= 1 - \frac{(2p-r)}{k+(\alpha-1)\sqrt{1-r}} \\
\frac{\partial^2 \pi}{\partial p^2} &= -\frac{2}{k+(\alpha-1)\sqrt{1-r}} \\
\frac{\partial \pi}{\partial k} &= \frac{p(p-r)}{((\alpha-1)\sqrt{1-r}+k)^2} - 2k \\
\frac{\partial^2 \pi}{\partial k^2} &= -\frac{2p(p-r)}{((\alpha-1)\sqrt{1-r}+k)^3} - 2
\end{aligned}$$

1.9.3 Some values for k

We now set $(\alpha-1)\sqrt{1-r} = w$ and proceed to solve for k using the second FOC.

$$\frac{\partial \pi}{\partial k} = \frac{p(p-r)}{(w+k)^2} - 2k$$

$$2k = \frac{p(p-r)}{(w+k)^2}$$

$$2k(w+k)^2 = p(p-r)$$

We now substitute the price into the this:

$$2k(w+k)^2 = \frac{k+w+r}{2}\left(\frac{k+w+r}{2} - r\right)$$

$$2k(w+k)^2 = \frac{k+w+r}{2}\left(\frac{k+w-r}{2}\right)$$

$$8k(w+k)^2 = (k+w+r)\,(k+w-r)$$

If we assume that $w = r$

$$\Rightarrow k = \frac{1}{16}\left(1 \pm \sqrt{32r+1}\right) - r$$

Only the positive solution yields positive values

$$= \frac{1}{16}\left(1 \pm \sqrt{32r+1}\right) - r$$

This function is maximized at $k = \frac{1}{8}$

We now take a limit case as α increases:

$$8k(w+k)^2 = (w+k)^2$$

$$k = \frac{1}{8}$$

Similarly if:

$$\alpha = 1 = \beta$$

$$\Rightarrow w = 0$$

$$\Rightarrow k = \frac{1}{8}$$

1.9.4 Notes

Why the envelope theorem applies

For notational simplicity we represent the demand function which is a function of α, p, k as $l(a, k(a), p(a))$.

$$\pi(a, p(a), k(a)) = p(a)l(a, k(a), p(a)) - k(a)^2$$

$$\frac{\partial \pi(a, p(a), k(a))}{\partial \alpha} = \frac{\partial p(a)}{\partial a}\left(l(a, k(a), p(a))\right) + p(a)\left(\frac{\partial l}{\partial a} + \frac{\partial l}{\partial k(a)}\frac{\partial k(a)}{\partial a} + \frac{\partial l}{\partial p(a)}\frac{\partial p(a)}{\partial a}\right) - 2k(a)\frac{\partial k(a)}{\partial a}$$

$$= \frac{\partial p(a)}{\partial a}\left(l(a, k(a), p(a)) + p(a)\frac{\partial l}{\partial p(a)}\right) + p(a)\frac{\partial l}{\partial a} + \frac{\partial k(a)}{\partial a}\left(p(a)\frac{\partial l}{\partial k(a)} - 2k(a)\right)$$

$$FOCS:$$

$$\frac{\partial \pi(a, p(a), k(a))}{\partial p(a)} = l(a, k(a), p(a)) + p(a)\frac{\partial l}{\partial p(a)} = 0$$

$$\frac{\partial \pi(a, p(a), k(a))}{\partial k(a)} = p(a)\frac{\partial l}{\partial k(a)} - 2k(a) = 0$$

applying the FOC's we can simplify to

$$\frac{\partial \pi(a, p(a), k(a))}{\partial \alpha} = p(a)\frac{\partial l}{\partial a}$$

Therefore we can deduce the effects of α by simply taking the direct derivative of the profit function. The argument is equivalent for r.

Regularly varying functions

Definition 1. *A regularly varyng function $L : (0, +\infty) \to (0, +\infty)$ has the following property:*

$$lim_{x\to\infty}\frac{L(tx)}{L(x)} = g(t) \tag{1.9.1}$$

It is known that in regular varying functions, $g(t)$ take the following form(Bojanic and Karamata, 1963):

$$g(t) = t^i \tag{1.9.2}$$

If $g(t) = t$ this implies that the function is eventually linear.

$$L(x) - L(ax) = L(x) - f(a)L(x) = L(x)(1 - f(a)) \tag{1.9.3}$$

$$L(y) - L(by) = L(y) - f(b)L(y) = L(y)(1 - f(b)) \tag{1.9.4}$$

$$\text{Linearity implies:} x - ax = y - yb \tag{1.9.5}$$

$$\rightarrow y = x\frac{(1-a)}{(1-b)} \tag{1.9.6}$$

$$\rightarrow L\left(x\frac{1-a}{1-b}\right)[1 - f(b)] \tag{1.9.7}$$

$$\rightarrow f\left(\frac{1-a}{1-b}\right)L(x)[1 - f(b)] \tag{1.9.8}$$

$$\text{L is linear in x if} \tag{1.9.9}$$

$$L(x)f\left(\frac{1-a}{1-b}\right)(1 - f(b)) = L(x)(1 - f(a)) \tag{1.9.10}$$

$$f\left(\frac{1-a}{1-b}\right) = \frac{1 - f(a)}{1 - f(b)} \tag{1.9.11}$$

$$\left(\frac{1-a}{1-b}\right)^i = \frac{1 - a^i}{1 - b^i} \tag{1.9.12}$$

Which is satisfied because $g(t) = t$ implies $i = 1$

Welfare

Welfare is just the sum of the social surplus and the profit.

$$W_{bn} = \int_0^{\tilde{x}} \left(x(1 + \alpha(1 - \tilde{x}) + \tilde{k}) - \tilde{p}\right) dx + \tilde{\pi}$$

$$W_{bp} = \int_0^{\hat{x}} x(1 + \beta)dx + \int_{\hat{x}}^1 \left(x(1 + \alpha + \hat{k}) - \hat{p}\right) dx + \hat{\pi}$$

$$W_{bpn} = \int_{\check{x}}^{\hat{x}} \left(x(1 + \beta(1 - \check{x})) - r\right) dx + \int_{\hat{x}}^1 \left(x(1 + \alpha(1 - \check{x}) + \check{k}) - \check{p}\right) dx + \check{\pi}$$

First and second order approximations of endogenous k

If we don't take the limit case then this is:

$$8k(w+k)^2 = (w+k)^2 - r^2$$

$$(w+k)^2(8k-1) = -r^2$$

If we take a first order approximation:

$$f(k) = (w+k)^2(8k-1) + r^2$$

$$\approx f(t) + f'(t)(k-t)$$

$$= (8t-1)(t+w)^2 + r^2 + (8(t+w)^2 + 2(8t-1)(t+w))(k-t)$$

$$\Rightarrow k = \frac{2t + w^2 + 2w - 15t^2 - 14tw - r^2}{8(t+w)^2}$$

If we approximate around $t = \frac{1}{8}$

$$k = \frac{(8w+1)^2 - 64r^2}{8(8w+1)^2}$$

If we approximate around $t = 0$

$$k = \frac{w(w+2) - r^2}{8w^2}$$

So k is decreasing with r

A second order approximation of the function:

$$f(k) = (w+k)^2(8k-1) + r^2$$

$$\approx f(t) + f'(t)(k-t) + \frac{f''(\alpha)}{2}(k-t)^2$$

$$= (8t-1)(t+w)^2 + r^2 + (8(t+w)^2 + 2(8t-1)(t+w))(k-t)$$

$$+ \frac{32(t+w) + 2(8t-1)}{2}(k-t)^2$$

$$= (8t-1)(t+w)^2 + r^2 + (8(t+w)^2 + 2(8t-1)(t+w))(k-t)$$

$$+ (16(t+w) + (8t-1))(k-t)^2$$

We solve for k around 0:

$$k = \frac{\pm\sqrt{(8w^2)^2 - 4(16w-1)\left(r^2 - w^2 - 2w\right)} - 8w^2}{2(16w-1)}$$
$$= \frac{\pm 2\sqrt{16w^4 - (16w-1)\left(r^2 - w^2 - 2w\right)} - 8w^2}{2(16w-1)}$$
$$= \frac{\pm\sqrt{16w^4 - (16w-1)\left(r^2 - w^2 - 2w\right)} - 4w^2}{16w-1}$$

only the positive solution gives a positive answer:

$$= \frac{\sqrt{16w^4 - (16w-1)\left(r^2 - w^2 - 2w\right)} - 4w^2}{16w-1}$$

Chapter 2

Cost-side innovation with project variance

2.1 Introduction

Models suggest that the possibility for a larger firm to purchase a smaller firm will incentivize smaller firms to innovate. Since this buyout is voluntary, it can only mean that the entrepreneur in question gains. It is difficult to argue against the proposition that increasing the reward of an activity encourages that activity. However this possibility, does not mean that the entrepreneur is more likely to invest in all projects. Quite the contrary, as the possibility increases the incentive to pursue some projects, it simultaneously decreases the incentive to pursue other projects. This paper explores the concept that entrepreneurs are incentivized to pursue projects which are correlated to existing firms activities. Empirical evidence for the link between innovation and the ability to buyout is tenuous because of the difficulty in controlling for causality. There is an empirical correlation which is that industries with higher measures of innovation tend to have a more buyouts (Haucap et al., 2016). However there is no clear causal mechanism to describe this empirical relationship, as it may also be inferred that as innovation slows down, industry consolidation occurs (Comanor and Scherer, 2013). The naive view of buyouts is simply that buyouts increase the potential payoff from innovation. We need only consider that an entrepreneur is considering the possible payoffs from his investment, ceteris paribus, a probability of being bought-out, can only increase the incentive to innovate. The naive view is correct in that an extra source of payoff can only increase the upside to the entrepreneur. Even if the entrepreneur is considering numerous projects, if at least one of the projects being considered has a higher

potential payoff, this can only increase the entrepreneurs incentive to innovate. This basic static logic does not necessarily generalize to long run dynamic situations. [1] Nevertheless the distortive effects of buyouts are not limited to being in the long run. The very option to buyout may incur immediate changes in the behavior of firms. The incentive boost from buyouts may change not only the *absolute* payoffs of projects but also their *relative* payoffs. To see why this may occur, we need only consider *who* is buying out the entrepreneur. An entrepreneur has some project, this project has some value in the marketplace that is known to all. If both the entrepreneur and the firm have the same discount factor without a dynamic framework or more restrictive assumptions it is unclear why a transaction would occur[2]. Why would a firm pay the entrepreneur more than the entrepreneurs project *isolated market value*? We explore two potential reasons why this may be the case. Consider first the case where the project is complementary with the buyers activity if bought by the buyer. The buyers willingness to pay for the project will then be what the project is worth individually and the complementary revenue that it brings in. This would shift the incentives of the entrepreneur towards projects that are complementary with existing technology. Now consider the case where the entrepreneur's technology is substitutable with the buyers technology. Here the buyers willingness to pay is more complex. If the buyer has the option of shutting the project down by buying it then the buyers excess willingness to pay does not depend on the projects value but depends on the buyers revenue loss if the project is not bought[3]. Either of these two cases have the same result, industry convergence. Even if a project would potentially be very profitable, if the damage it cause to an existing firm is significant enough, then there will be an incentive to buy it out and shut it down. The basic equivalence between complementarity and substitution will be shown in the discussion section below. The static effect can be extended to the dynamic case of the model with projects having different time structures. If the different projects of the entrepreneur will both end up with the same technology but with different patterns of arrival this can equally cause firms to pursue projects of different horizons due to the possibility of being bought out. Specifically we show that projects which pass through intermediate stages of efficiency are more threatening to incumbents than projects which can more directly reach their goal. Even a technology with a lower expected value can be preferred due to the chipping away of incumbent profits. The models presented can be interpreted in one of a few different ways. The most straightforward way is simply to say that a firm wishes to buyout another firm

[1] The dynamic effect is usually interpreted as a change in industry structure in such a way that less projects become profitable in the long run Bessen and Maskin (2009)

[2] barring of course the use of a utility function

[3] Assuming the projects market value is lower than the current activity of the buyer, if the project is worth more than the current activity then the buyer will not pay more than other buyers

and the regulatory authorities either allow this or forbid this transaction. A different way is simply to say that the entrepreneur cannot be bought out because the projects are not purchasable, perhaps they are not patented or the project is simply not visible to the buyer. The paper is structured as follows. In section 2.2 section we will make a few links with the literature, the model is general enough that it can be interpreted within a couple of strands. Section 2.3.1 will present the model and the results, finishing with a brief numerical example. In section 2.4 we apply the framework to Bertrand and Cournot competition. Finally in section 2.5 we discuss some of the results and in 2.4.4 we give a brief argument to show that complementarities and substitutabilities yield the same willingness to pay.

2.2 Link to the literature

The general trend for mergers and aqcuisition is that industry consolidation is occurring (White, 2002). Perhaps a rather radical example is the consolidation of the beer industry where from 1950 to 2000, the top four firms in the United States went from producing 22% to producing 95%. The empirical corrolations are numerous, for instance firms that are less innovative are often more likely to engage in buyouts. The work of Gerpott (1995) finds that for innovation to be well absorbed by the acquirer, the firms size must not differ excessively, or said otherwise, the closer the firms are in size, the more likely they are to merge successfully. Higgins and Rodriguez (2006) find that in the pharmaceutical industry, unproductive firms are more likely to engage in acquisition strategies. This is also supported by cross industry studies such as Zhao (2009). There is also empirical work showing that companies with larger patent portfolios and low research expenditure are more likely to acquire(Bena and Li, 2014). The basic theoretic framework used in this paper borrows from Cabral (2003), whose model involves two firms competing in $R\&D$. Each firm has the option of choosng a high variance strategy or a low variance strategy. The general result of the model is that when a firm is lagging behind, it prefers to a high variance strategy, and when it is ahead it chooses a low variance strategy. Our results borrow from this markov chain setup but pursues different questions. Instead of the incentive for projects occurring due to positioning in the market, it occurs due to the market structure itself. That is, the ability to merge distorts which projects are undertaken. Our subject matter is similar to the literature on firms innovating so that they can escape competition effects(Aghion et al. (2005),Aghion et al. (2001),Aghion et al. (1997)). Gilbert et al. (2016) shows these results only hold in duopolies and not oligopolies. Other work includes Phillips and Zhdanov (2013), where it is argued that large firms avoid engaging in $R\&D$ races. However these studies do not include different kinds of innovations,

only the relative pressure to innovate depending on market positioning. The paper can be linked to various strands of literature, we give a brief idea of how it relates. The idea can be framed as being an application of the Coase theorem to industry structure. The question of an incumbent being harmed by an entrant can be framed within the externality framework. When transaction costs are low enough, the Coase theorem allows a role for extortion: A may do some activity that he would have no interest in doing because B will be willing to pay for A to stop(Kuechle and Rios, 2012). It is also possible to interpret the results in a mechanism design framework. More specifically, auctions with allocative externalities. This literature is about preferences of ownership where agents have different utilities depending on who among the other agents owns the good, though no utility function is employed here, the results apply to other contexts for a survey of such models see Jehiel and Moldovanu (2005). Finally the model can be framed as being within an incomplete contract framework since there is a question of an inability for the firms to contract ex-ante. Though this is only one possible interpretation of the model, the usual reasons for uncontractibility apply(Hart and Moore, 1999).

2.3 Setup

2.3.1 The dynamic model

The model is made up of an incumbent and an entrant with asymmetric initial positions. The incumbent has three possible payoffs and the entrant has two. The incumbent and entrant start out with initial technologies described by the costs, c_i and c_e, respectively. We assume that the incumbent will initially earn the profit $\pi_i(c_i, c_e)$. The entrant earns no profit with the initial technology, $\pi_e(c_i, c_e) = 0$. We say that $c_e > c_i$ to denote the efficiency of production. The second kind of payoffs in the model are the payoffs once the entrant catches up to the incumbent. The cost associated with this level of development is c_1. This cost is between the other two, $c_i < c_1 < c_e$. The payoff of the incumbent when competing against this cost is lower than the payoff when competing with the less efficient entrant, $\pi_i(c_i, c_e) > \pi_i(c_i, c_{e1})$. If the incumbent were to acquire this intermediate technology, it would not use it since it is less efficient than c_i so the payoff would remain unchanged. If the entrant owns the intermediate technology, the associated payoff would be weakly higher than zero, $\pi_e(c_i, c_{e1}) \geq 0$. Finally the third kind of payoff is the payoff with the advanced technology, c_2. This technology is the most advanced technology available, $c_2 < c_i < c_1 < c_e$. Both firms would benefit from owning this technology and it represents their highest respective payoffs. The entrant in the

model must choose what kind of innovation to engage in. Both innovations have the same fixed cost, F. The innovation is the path to the technologies above which can be achieved. There are two options, the first option passes through both stages of efficiency, we call it the *sequential innovation*. The second jumps directly to the second stage of efficiency, we call it the *radical innovation*. Without loss of generality[4] we say that the sequential innovation has no risk associated with it and does not necessarily yield profits from the first stage of efficiency. The sequential option will achieve the intermediate technology, c_1 after t_1 periods and will achieve the advanced technology, c_2 after t_2 periods. The game ends immediately at time T. So if $T-1=t_2$ then there will only be one period where the advanced technology payoffs will be achieved. In the application to Bertrand and Cournot we will be assuming that $T-1=t_2=2$ and $t_1=1$ for simplicity. The radical innovation on the other hand will with some probability, q, give the entrant access to the advanced technology directly. So in each time period, with probability q, the firm will have cost c_2. If the technology fails to realize, which occurs with probability $(1-q)$ the entrant will remain at the initial technology c_e. If the radical innovation succeeds at any point, then the entrant will receive the advanced technology payoff until T. The difference to notice between these two methods is that the sequential innovation cannot reach the advanced payoff instantly, while the radical one can. These technologies can also be interpreted as high variance and low variance. Adoption cost of the innovations is assumed to be identical. The choice of the radical innovation and sequential technology will be represented by r and s, respectively. It is known that in most standard competitive frameworks, firms will want to merge because merger profits are higher than the sum of profits. So if bargaining is possible and credible through contracting there always exists a positive Nash surplus that can be shared. In our framework this takes the following form:

Assumption 1. *Sub-additive competitive profits*

$$\pi_i(c_{i2},c_e)\geq\pi_i(c_i,c_{e2})+\pi_e(c_i,c_{e2})$$

$$\pi_i(c_i,c_e)\geq\pi_i(c_i,c_{e1})+\pi_e(c_i,c_{e1})$$

We also make clear the assumptions about technology and competition we stated above. The incumbent payoff with the the advanced technology yields higher payoff than with the low technology, $\pi_i(c_{i2},c_e)>\pi_i(c_i,c_e)$. The default payoff of the incumbent yields higher payment than when the entrant has intermediate technology $\pi_i(c_i,c_e)\pi_i(c_i,c_{e1})$. Finally, the lowest payoff of the incumbent is when the entrant has the advanced technology $\pi_i(c_i,c_e)>\pi_i(c_i,c_{e2}$.

[4]All results also hold with a sequential innovation that has a weakly lower probability of going to the advanced technology in one go than the radical innovation

Assumption 2. *Profits increasing in technology and decreasing in competition*

$$\pi_i(c_{i2}, c_e) > \pi_i(c_i, c_e); \qquad \pi_i(c_i, c_e) > \pi_i(c_i, c_{e1}); \qquad \pi_i(c_i, c_e) > \pi_i(c_i, c_{e2})$$

Or simply:

$$\pi_i(c_{i2}, c_e) > \pi_i(c_i, c_e) > \pi_i(c_i, c_{e1}) > \pi_e(c_i, c_{e2})$$

We can re-arrange this assumption to get the following corollary:

Corollary 3. *Assumption 2 implies that* $\Rightarrow \pi_i(c_{i2}, c_e) + \pi_i(c_i, c_{e1}) > \pi_i(c_i, c_e) + \pi_i(c_i, c_{e2})$

We also make an additional assumption that is sufficient but not neccesary for the results[5]. We assume that what the incumbent can earn with the the advanced technology is not larger than the sum of all other possible profits the incumbent can earn in the three situations.

Assumption 3. *Bounded advanced technology profits*

$$\pi_i(c_{i2}, c_e) < \pi_i(c_i, c_e) + \pi_i(c_i, c_{e2}) + \pi_i(c_i, c_{e1})$$

We assume that firms do not have a time preference. Though there is no preference for present profits, the firms will still prefer the earlier payment because there is no risk associated with them, firms naturally will prefer one innovation over the other because of their time structure. We now specify the timing of the model. The first decision that occurs is the innovation decision of the entrant, the choice between sequential or radical innovation. If no buyouts are allowed, then the payoff streams occur. If there are buyouts, then after the choice of innovation occurs, negotiation occurs between the entrant and the incumbent for a buyout, after which the buyout payoff streams are realized. Notice that if it were possible for the buyout to materialize before the choice of innovation(ex-ante) this would result in the innovation with the highest expected payoff to happen. This could come about if the entrant was able to credibly signal to the incumbent that the incumbent is capable of undertaking the investment. An implication of an ex-ante buyout is that the incumbent pays a weakly higher amount for the innovation. It is easiest to see in the case where the entrant chooses an innovation which does not maximize market payoffs. This would be the blackmail case where the firm entrant can use the threat of substitutability to make the incumbent pay for the externalities imposed by the entrant. Note that if the radical innovation is bought, this does not guarantee the realization of the technology.

[5]The neccesary and sufficient condition is: $\pi_i(c_{i2}, c_e) \leq \pi_i(c_i, c_e) + (\pi_i(c_i, c_{e2}) + \pi_e(c_i, c_{e2}) + (\pi_i(c_i, c_{e1}) + \pi_e(c_i, c_{e1}))$

2.3.2 Sequential

The sequential innovation will give the intermediate technology after t_1 periods and the advanced technology after t_2 periods. Therefore the total profit of the entrant is:

$$\begin{aligned}\Pi_{es} &= \pi_e(c_i, c_{i1})(t_2 - t_1) + \pi_e(c_i, c_{e2})(T - t_2) - F \\ &= \pi_e(c_i, c_{i1}) + \pi_e(c_i, c_{e2})(T - t_2) - F\end{aligned}$$

Where we normalize $t_2 - t_1$ to 1. This normalization can done because only the relative length between $t_2 - t_1$ and $T - t_2$ matters. The payoff of the incumbent is similar except that there is an additional stream of payments for the first t_1 periods before the entrant is competitive enough to compete.

$$\Pi_{is} = \pi_i(c_i, c_e)t_1 + \pi_i(c_i, c_{i1}) + \pi_i(c_i, c_{e2})(T - t_2)$$

The merger profit of the incumbent will ignore the first t_1 periods because the intermediate technology is not an improvement on the initial technology. There is no direct value added to the incumbent payoff from variations in t_1. As we will see t_1 does matter for the effect it has on the bargaining disagreement payoff of the entrant. The merger profit is given by:

$$\Pi_s^m = \pi_i(c_i, c_e)t_2 + \pi_i(c_{i2}, c_e)(T - t_2)$$

From the two payoffs of the entrant we can deduce the willingness to pay for the sequential innovation:

$$WTP = \Pi_s^m - \Pi_{is} = (\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})) + (\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2}))(T - t_2)$$

We need only subtract the payoff of the entrant from the willingness to pay to have the bargaining surplus, that is the value added from merging:

$$S_s = \pi_i(c_i, c_e) - \pi_i(c_i, c_{i1}) - \pi_e(c_i, c_{i1}) + (\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2} - \pi_e(c_i, c_{e2}))(T - t_2)$$

The surplus will be split based on bargaining power $\omega \in [0, 1]$, where if $\omega = 1$, all the surplus is taken by the entrant and if $\omega = 0$, all surplus is taken by the incumbent. Note that there is no fixed cost in the surplus because it has already been incurred and therefore the entrant

cannot use it as a negotiation chip. The bargaining payoff of the entrant is therefore:

$$\begin{aligned}
B_{es}(\omega) &= \Pi_{es} + \omega S_s - F \\
&= \omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1}) + (1-\omega)\pi_e(c_i, c_{i1})) - F \\
&+ (\omega(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + (1-\omega)\pi_e(c_i, c_{e2}))(T - t_2) \\
B_{is}(\omega) &= \Pi_{is} + (1-\omega)S_s \\
&= \pi_i(c_i, c_e)t_1 \\
&+ \omega\pi_i(c_i, c_{i1}) + (1-\omega)(\pi_i(c_i, c_e) - \pi_e(c_i, c_{i1})) \\
&+ (\omega\pi_i(c_i, c_{e2}) + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})))(T - t_2)
\end{aligned}$$

Notice here is that the absolute value of t_1 plays a role because it represents how many periods the incumbent will be uninterrupted in pursuing the baseline profits. So the time when the intermediate innovation kicks in has no positive effect on the entrant but does increase the bargaining profit of the incumbent.

2.3.3 Radical

In the radical innovation, the probability that the entrant receives $T-1$ flows of advanced technology payoffs is given by q. Similarly the probability of receiving $T-2$ is $q(1-q)$, of $T-3$, $q(1-q)^2$, etc. Therefore the general payoff for the entrant is given by:

$$\begin{aligned}
\Pi_{er} &= \pi_e(c_i, c_{e2}) q \sum_{i=1}^{T-1} (1-q)^{i-1}(T-i) - F \\
&= \pi_e(c_i, c_{e2}) \left(T - \frac{1}{q}\left(1 - (1-q)^T\right)\right) - F \\
&= \pi_e(c_i, c_{e2}) \left(T - \rho\right) - F
\end{aligned}$$

Where we define $\rho \equiv \frac{1}{q}\left(1-(1-q)^T\right)$. Note that if $\lim_{q\to 1} \rho \to \rho = 1$ and $\lim_{q\to 0} \rho \to \rho = T$. So ρ can be interpreted as the expected number of periods the entrant will not receive a payment with the radical innovation. An assumption must be made on ρ for the simple reason that the radical innovation has a higher variance in payoffs, so if it also has a lower mean, it will never be pursued, so it must be that in expectation the number of higher payoffs received with the radical innovation is greater than the higher payoff projects with the sequential innovation, this is simply the following:

Assumption 4. *The expected number of payments of the high stream is higher with the radical innovation* ($t_2 > \rho$).

The incumbent payoff will take a similar form to the entrant but will receive non-zero payoffs regardless of the realization. Or to express this in another way, the expected number of payoffs wit competition is given by $T-1$. This means that we need only subtract the previous expected number of streams from $T-1$ to receive the expected number of streams with the initial technology. This is given below:

$$\begin{aligned}
\Pi_{ir} =& \pi_i(c_i, c_{e2}) \underbrace{q \sum_{i=1}^{T-1} (1-q)^{i-1}(T-i)}_{\text{Expected number of } \pi_i(c_i,c_{e2})} + \pi_i(c_i, c_e) \underbrace{(T-1-q\sum_{i=1}^{T-1}(1-q)^{T-1-i}i)}_{\text{Expected number of } \pi_i(c_i,c_e)} \\
=& \pi_i(c_i, c_{e2}) q \sum_{i=1}^{T-1}(1-q)^{T-1-i}i + \pi_i(c_i, c_e) \sum_{i=1}^{T-1}(T-1-q\sum_{i=1}^{T-1}(1-q)^{T-1-i}i) \\
=& \pi_i(c_i, c_{e2}) q \sum_{i=1}^{T-1}(1-q)^{i-1}(T-i) + \pi_i(c_i, c_e)(T-1-q\sum_{i=1}^{T-1}(1-q)^{T-1-i}i) \\
=& \pi_i(c_i, c_{e2}) \left(T - \frac{1}{q}\left(1-(1-q)^T\right)\right) + \pi_i(c_i, c_e)\frac{(1-q)}{q}\left(1-(1-q)^{T-1}\right) \\
=& \pi_i(c_i, c_{e2})\,(T-\rho) + \pi_i(c_i, c_e)\,(\rho - 1)
\end{aligned}$$

So with the buyout, the merger payoff of the incumbent is given by a similar expression, where the only difference is that the technology c_2 is owned by the incumbent instead of the entrant.

$$\begin{aligned}
\Pi_r^m &= \pi_i(c_{i2}, c_e) \left(T - \frac{1}{q}\left(1-(1-q)^T\right)\right) + \pi_i(c_i, c_e)\frac{(1-q)}{q}\left(1-(1-q)^{T-1}\right) \\
&= \pi_i(c_{i2}, c_e)\,(T-\rho) + \pi_i(c_i, c_e)(\rho - 1)
\end{aligned}$$

As before, the corresponding willingness to pay of the incumbent and Nash surplus to be shared are given by:

$$\begin{aligned}
WTP_r &= (\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2}))\,(T-\rho) \\
S_r &= (\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2}) - \pi_e(c_i, c_{e2}))\,(T-\rho)
\end{aligned}$$

The bargaining payoffs are then:

$$\begin{aligned}
B_{er}(\omega) &= \Pi_{er} + \omega S_r \\
&= \left(\omega\pi_i(c_{i2}, c_e) - \omega\pi_i(c_i, c_{e2}) + (1-\omega)\pi_e(c_i, c_{e2})\right)(T-\rho) \\
B_{ir}(\omega) &= \pi_i(c_i, c_{e2})\,(T-\rho) + \pi_i(c_i, c_e)(1-\rho) \\
&\quad + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2}) - \pi_e(c_i, c_{e2}))\,(T-\rho) \\
&= \pi_i(c_i, c_e)(\rho - 1) \\
&\quad + \left(\omega\pi_i(c_i, c_{e2}) + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2}))\right)(T-\rho)
\end{aligned}$$

The entrant will choose the radical innovation over the sequential innovation based purely on the expected values of the entrants payoffs.

Definition 2. *Let* $\Delta\Pi$ *denote the incentive to choose the radical innovation when there no buyouts. Specifically, if* $\Delta\Pi > 0$ *the radical innovation is pursued, and if* $\Delta\Pi < 0$ *the sequential innovation.*

Proposition 9. $\Delta\Pi$ *is continous and can take positive and negative values.*

Proof. The general form of the incentive is given by taking the difference between the expected values.

$$\begin{aligned}
\Pi_{er} &> \Pi_{es} \\
\pi_e(c_i, c_{e2})\,(T-\rho) - \pi_e(c_i, c_{i1}) - \pi_e(c_i, c_{e2})(T-t_2) &> 0 \\
\Delta\Pi \equiv \pi_e(c_i, c_{e2})\,(t_2-\rho) - \pi_e(c_i, c_{i1}) &> 0
\end{aligned}$$

We can notice that the domain of this function is in $\mathbb{R}$ to see that it is continous and well defined since $\rho < t_2$. We can also verify that it's derivative is well defined and positive:

$$\begin{aligned}
\frac{\partial\Delta\Pi}{\partial q} &= \pi_e(c_i, c_{e2})\left(\frac{1-(1-q)^T - Tq(1-q)^{T-1}}{q^2}\right) \\
&= \pi_e(c_i, c_{e2})\left(\frac{1-(1-q)^{T-1}((1-q)+Tq)}{q^2}\right)
\end{aligned}$$

The term $(1-q)^{T-1}((1-q)-Tq)$ is decreasing in q and at $q=0$ becomes 1.

We can see that $\Delta\Pi$ can take positive and negative values by taking the limit with respect to q:

$\lim_{q\to 1} \Delta\Pi = \pi_e(c_i, c_{e2})t_2 - \pi_e(c_i, c_{i1})$

This is positive due to the fact that: $t_2 > 1, \text{and}, \pi_e(c_i, c_{e2}) > \pi_e(c_i, c_{i1})$

$\lim_{q\to 0} \Delta\Pi = \pi_e(c_i, c_{e2})(t_2 - T) - \pi_e(c_i, c_{i1})$

To see that this is negative we need only recall that: $t_2 < T$. □

Intuitively, the higher the transition probability of the radical innovation, the more attractive it is to pursue. It can also similarly be seen that $\Delta\Pi$ is increasing linearly in t_2. We now present our first result:

Proposition 10. $\Delta\Pi$ *is decreasing and convex in* T.

Proof.

The first order condition with respect to T is:

$$\frac{\partial \Delta\Pi}{\partial T} = \frac{\pi_e(c_i, c_{e2})}{q}(1-q)^T \log[1-q]$$

The second condition with respect to T is:

$$\frac{\partial \Delta^2\Pi}{\partial T^2} = \frac{\pi_e(c_i, c_{e2})}{q}(1-q)^T \log[1-q]^2$$

Due to the fact that$(1-q) < 0, \text{the first is negative and second is positive}$

□

This first order effect is that as the time horizon under consideration increases, the incentive to pursue the radical technology is decreased. The second order effect is that the first order effect becomes marginally less important as the time horizon increases. This is because as the time horizon increases, the sequential project has a higher probability of having the extra payment.

Definition 3. *Let* ΔB *denote the incentive to choose the radical innovation when there are buyouts. Specifically, if* $\Delta B > 0$ *the radical innovation is pursued, and if* $\Delta B < 0$ *the sequential innovation.*

Proposition 11. ΔB *can take on positive and negative values and is continuous.*

Proof.

$$\begin{aligned}
B_{er}(\omega) - F > B_{es}(\omega) - F \\
\leftrightarrow \\
(\omega\pi_i(c_{i2}, c_e) - \omega\pi_i(c_i, c_{e2}) + (1-\omega)\pi_e(c_i, c_{e2}))\,(T-\rho) > \\
\omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})) + (1-\omega)\pi_e(c_i, c_{i1}) \\
+(\omega(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + (1-\omega)\pi_e(c_i, c_{e2}))(T - t_2) \\
\leftrightarrow \\
(\omega(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + (1-\omega)\pi_e(c_i, c_{e2}))\,(t_2 - \rho) \\
-\omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})) + (1-\omega)\pi_e(c_i, c_{i1}) \equiv \Delta B > 0
\end{aligned}$$

We can see that ΔB can take positive and negative values by taking the limit with respect to q:

$\lim_{q\to 1} \Delta B = (\omega(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + (1-\omega)\pi_e(c_i, c_{e2}))\,t_2 - \omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})) + (1-\omega)\pi_e(c_i, c_{i1})$

$= \omega(t_2(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + \pi_i(c_i, c_{i1}) - \pi_i(c_i, c_e)) + (1-\omega)(t_2\pi_e(c_i, c_{e2}) + \pi_e(c_i, c_{i1}))$

The first term is positive due to corollary 3 and $t_2 > 1$, and the second term is also positive.

$\lim_{q\to 0} \Delta B = (\omega(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + (1-\omega)\pi_e(c_i, c_{e2}))\,(t_2 - T)$

$\quad - \omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})) + (1-\omega)\pi_e(c_i, c_{i1})$

$= \omega((t_2 - T)(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + \pi_i(c_i, c_{i1}) - \pi_i(c_i, c_e)) + (1-\omega)((t_2 - T)\pi_e(c_i, c_{e2}) + \pi_e(c_i, c_{i1}))$

The ω term is negative due to the fact that $T - t_2 \geq 1$ and corollary 3

For the $1-\omega$ term we can see that it is negative due to $T - t_2 \geq 1$ and $\pi_e(c_i, c_{e2}) > \pi_e(c_i, c_{i1}))$

We can also see that the derivative of this function with respect to q is well defined and positive:

$$\frac{\partial \Delta B}{\partial q} = (\omega(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})) + (1-\omega)\pi_e(c_i, c_{e2}))\left(\frac{1 - (1-q)^{T-1}((1-q) + Tq)}{q^2}\right)$$

Where $\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})$ is positive by definition and the larger parenthesis is weakly positive by the same reasoning as in proposition 1. □

Proposition 12. *If the entrant has bargaining power ($\omega > 0$), a switch from the no-buyout to the buyout regimes, weakly increases the incentive for the entrant to pursue the sequential innovation ($\Delta B < \Delta\Pi$)*

Proof. We need that $\omega > 0$ since if $\omega = 0$, $\Delta B = \Delta\Pi$. We define: $\chi \equiv \Delta B - \Delta\Pi$. So if $\chi < 0$ this implies that the radical innovation is pursued less in the buyout regime and the

$\chi > 0$ implies the inverse. If we set $\Delta B \leq \Delta\Pi$.

$$\omega\left(\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2}) - \pi_e(c_i, c_{e2})\left(t_2 - \rho\right)\right.$$
$$\left. - \pi_i(c_i, c_e) - \pi_i(c_i, c_{e1}) - \pi_e(c_i, c_{e1})\right) = \chi \leq 0$$

Note that $\chi \leq 0$ is the necessary and sufficient condition for the sequential innovation to be over pursued relative to the radical innovation. We now show that, assumption 2 is also a sufficient condition. First note that the sign of χ is the sign of $\frac{\chi}{\omega}$. We can write $\frac{\chi}{\omega}$ in the following way:

$$\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_e) \leq \pi_e(c_i, c_{e2})\left(t_2 - \rho\right) + \pi_i(c_i, c_{e2}) + \pi_i(c_i, c_{e1}) + \pi_e(c_i, c_{e1})$$

We can note that by assumption 4, $(t_2 - \rho)$ is positive and since all the entrant profits are strictly positive. If we start from assumption 3

$$\pi_i(c_{i2}, c_e) \leq +\pi_i(c_i, c_e) + \pi_i(c_i, c_{e2}) + \pi_i(c_i, c_{e1})$$

We can see that the condition is verified.

□

As seen in the proposition, the bargaining power plays an important role for the distortion. At the extreme we have the following corollary.

Corollary 4. *If the entrant has no bargaining power, the buyout is not distortive.*

2.3.4 The willingness to lobby for buyouts

The buyout option will always benefit the entrant because by definition the entrant can always just ignore that option. However the buyout option does not always benefit the incumbent. It is worth considering how the willingness to pay of the incumbent for a policy change varies with parameters. Let $\Pi = max\{\Pi_{es}, \Pi_{er}\}$ and $B = max\{B_{es}, B_{er}\}$. If a change in the legality of buyouts does not affect the incentives of the entrant, $F > B > \Pi$ then there is no willingness to pay. The incumbent will have to balance out the risks of competition and blackmail with the possible synergies that could be present with the buyout.

Case 1: Buyouts affect decision to enter:

Suppose first that the possibility for a buyout affects the decision to enter. This corresponds to the fixed cost is larger than the non-merger choice but is smaller than the merger choice, $B > F > \Pi$.

Proposition 13. *If the buyout can effect the entry decision, there always exists a negotiating power,$\overline{\omega}$ where if $\omega > \overline{\omega}$, the incumbent will have a willingness to pay to prevent mergers, and if $\omega < \overline{\omega}$, the entrant will have a willingness to pay to legalize mergers.*

Proof. In this case the corresponding willingness to pay for policy change is given by:

$$
\begin{aligned}
&B_{is}(\omega) > (T-1)\pi_i(c_i, c_e) \\
&\qquad \leftrightarrow (\omega\pi_i(c_i, c_{i1}) + (1-\omega)(\pi_i(c_i, c_e) - \pi_e(c_i, c_{i1}))) \\
&\qquad + (\omega\pi_i(c_i, c_{e2}) + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})))(T - t_2) \\
&\qquad - \pi(c_i, c_e)(T - 1 - t_1) > 0 \\
&\text{If } \omega = 0 : \pi_i(c_i, c_e) - \pi_e(c_i, c_{i1}) + (\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2}))(T - t_2) - \pi(c_i, c_e)(T - 1 - t_1) > 0 \\
&\text{If } \omega = 1 : \pi_i(c_i, c_{i1}) + \pi_i(c_i, c_{e2})(T - t_2) - \pi(c_i, c_e)(T - 1 - t_1) > 0
\end{aligned}
$$

Notice that this expression can be negative. Depending on how large $\pi(c_i, c_e)$ is relative to the rest of the expression. This expression can be used to determine how much bargaining power the incumbent would need in order to have a zero willingness to pay.

It is also possible that the option to buyout affects is affects the option to enter but not the project undertaken. If the project that would have been undertaken was the sequential one then the willingness to pay would be the same as above. If on the other hand the entrant enters and pursues the radical innovation then the willingness to pay of the incumbent is:

$$
\begin{aligned}
&B_{ir}(\omega) > (T-1)\pi_i(c_i, c_e) \\
&\qquad \leftrightarrow (\omega\pi_i(c_i, c_{e2}) + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})) - \pi_i(c_i, c_e)) \left(T - \frac{1}{q}\left(1 - (1-q)^T\right)\right) > 0 \\
&\text{If } \omega = 0 : (\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2}) - \pi_i(c_i, c_e)) \left(T - \frac{1}{q}\left(1 - (1-q)^T\right)\right) \\
&\text{If } \omega = 1 : (\pi_i(c_i, c_{e2}) - \pi_i(c_i, c_e)) \left(T - \frac{1}{q}\left(1 - (1-q)^T\right)\right)
\end{aligned}
$$

The case where $\omega = 0$, where the incumbent has all the negotiating power, is strictly positive due to assumption 1. The case where all the negotiating power is given to the entrant, $\omega = 1$, results on the other hand is negative due to the competitive effect. □

Case 2: Buyouts only affect the preference

This leaves us with one case, the case where the firm would have entered anyway but the buyouts cause it to pursue the the sequential innovation. An intuitive scenario in which this case would occur is if the radical innovation is very effective, this would entail a large risk to the incumbent, which would create a high willingness to pay to avoid the potential losses. Due to proposition 12, we know that we only need to consider the case where the buyout shifts incentives to the sequential technology. In this case the willingness to pay would be given by the following expression:

Proposition 14. *If buyouts do not affect entry decision, the willingness to pay for buyouts by the incumbent is always positive.*

Proof.

$$
\begin{aligned}
& B_{is}(\omega) > \Pi_{ir} \\
& \leftrightarrow \pi_i(c_i, c_e)t_1 + \omega\pi_i(c_i, c_{i1}) + (1-\omega)(\pi_i(c_i, c_e) - \pi_e(c_i, c_{i1})) \\
& + (\omega\pi_i(c_i, c_{e2}) + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})))(T - t_2) \\
& > \pi_i(c_i, c_{e2})\left(T - 1 - \frac{1}{q}\left(1 - (1-q)^{T-1}\right)\right) + \pi_i(c_i, c_e)\frac{(1-q)}{q}\left(1 - (1-q)^{T-1}\right) \\
\leftrightarrow & \pi_i(c_i, c_e)(t_2 - \omega - \frac{1}{q}\left(1 - q - (1-q)^T\right)) + (\omega\pi_i(c_i, c_{i1}) - (1-\omega)\pi_e(c_i, c_{i1})) \\
& + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2}))(T - t_2) \\
& + \pi_i(c_i, c_{e2})\left(\omega(T - t_2) - T + \frac{1}{q}\left(1 - (1-q)^T\right)\right) > 0 \\
\text{If } \omega = 0 : & \pi_i(c_i, c_e)(t_2 - \frac{1}{q}\left(1 - q - (1-q)^T\right)) - \pi_e(c_i, c_{i1}) \\
& + (\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2}))(T - t_2) + \pi_i(c_i, c_{e2})\left(-T + \frac{1}{q}\left(1 - (1-q)^T\right)\right) > 0 \\
\text{If } \omega = 1 : & \pi_i(c_i, c_e)(t_2 - \frac{1}{q}\left(1 - (1-q)^T\right)) + \pi_i(c_i, c_{i1}) \\
& - \pi_i(c_i, c_{e2})\left(t_2 - \frac{1}{q}\left(1 - (1-q)^T\right)\right) > 0
\end{aligned}
$$

Some deduction is possible from the last term, with $\omega = 1$, when the the incumbent has none of the negotiating power. Since we know that the profits of the incumbent are higher when the entrant is less competitive, $\pi_i(c_i, c_e) > \pi_i(c_i, c_{e2})$, the willingness to pay is always positive. □

2.4 Applications:

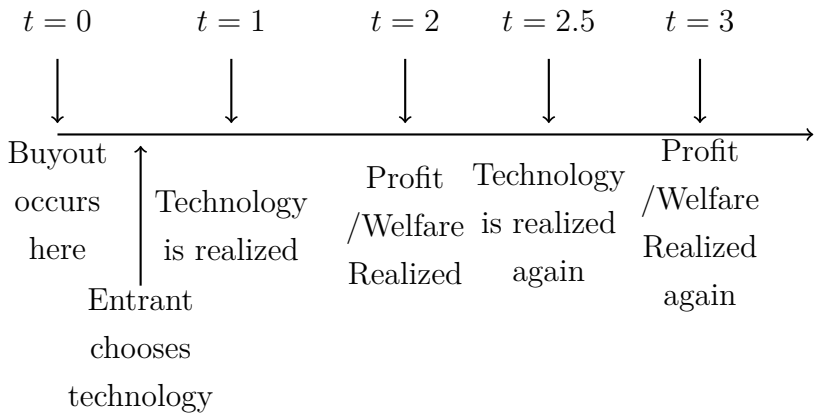
To clarify ideas and develop our intuition we now apply the general concept to Bertrand and Cournot competition. Without loss of generality we assume that both $\pi_i(c_i, c_e)$ and $\pi_i(c_{i2}, c_e)$ are monopoly profits. That is, the gap between the technologies is sufficiently large that the entrant sets the monopoly price. We also assume a linear demand function of the form $D(x) = 1 - x$ and linear cost structure. As such the monopoly payoffs are:

$$\pi_i(c_i, c_e) = \left(\frac{1 - c_i}{2}\right)^2; \quad \pi_i(c_{i2}, c_e) = \left(\frac{1 - c_{i2}}{2}\right)^2;$$

Note that the implicit assumption here is that $\frac{1+c_i}{2} < c_e$, due to the fact that the monopoly price must be lower than the cost of production of the competitor. Finally we assume that the game lasts two periods and that the sequential innovation gives the intermediate technology in period 1 and the advanced technology in period 2. That is, the entrant will earn one stream of low technology and one stream of high technology, $T - 1 = t_2 = 2$ and $t_1 = 1$.

2.4.1 Ex-ante buyout

As a baseline scenario we first briefly take a look at what occurs if the buyout is ex-ante. That is, the buyouts occurs before the entrant chooses his technology.



The analysis in this case is straightforward, we need only calculate the difference in profits in the case with the radical innovation and the sequential innovation.

Proposition 15. *If the buyouts are ex-ante, the decision criteria for the radical innovation to be chosen by the incumbent is:*

$$\frac{3-\sqrt{5}}{2} < q^*$$

Proof. We need only set

$$\Pi_r^m > \Pi_s^m$$
$$\pi_i(c_i, c_e)(1-q)(2-q) + \pi_i(c_{i2}, c_e)q(3-q) > \pi_i(c_i, c_e) + \pi_i(c_{i2}, c_e)$$

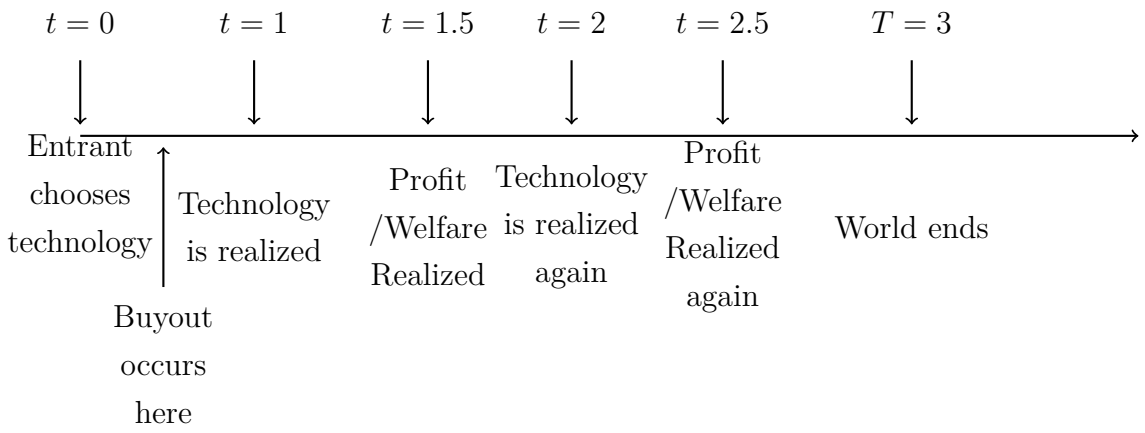
□

This is intuitive because, if the radical innovation has a high enough probability of being achieved, the incumbent will opt for it. Note that the ex-ante case is identical for both Cournot and Bertrand competition. If there is a reputational mechanism at work or perhaps a working relationship that already exists between the entrant and the incumbent then the ex-ante case becomes more plausible. Signaling mechanisms may also exist that enable the ex-ante buyout to occur. For instance if there is some way for the entrant to communicate why they can undertake a specific invention then this will also suffice.

2.4.2 Ex-post Bertrand

The ex-post case is the setup we used in the more general case. Apart from the incomplete contract justifications, it could occur because there are too many firms innovating and the incumbent cannot tell who has innovative capabilities before they invest. Bertrand competition reduces the distortion effect to a simpler form, this is because in Bertrand competition

only the highest technology firm makes profits. Additionally we know that the payoffs, when not the monopoly payoff, will be that the most advanced firm will price at the production cost of the second most advanced firm. Therefore $\pi_i(c_i, c_{e1}) = (1 - c_{e1})(c_{e1} - c_i)$ and $\pi_e(c_i, c_{e2}) = (1 - c_i)(c_i - c_{e2})$



If the buyout is not possible the bargaining payoffs are not available for either firm. Under such conditions the entrant will compare the direct market payoffs of the innovation.

Proposition 16. *If no buyouts can occur, the entrants preferences for the radical innovation are identical to the incumbent when the buyout is ex-ante.*

Proof.

$$\begin{aligned} \Pi_{er} &> \Pi_{es} \\ q\pi_e(c_i, c_{e2})(3 - q) &> \pi_e(c_i, c_{e2}) \\ \pi_e(c_i, c_{e2})(q(3 - q) - 1) &> 0 \end{aligned}$$

$$q > \frac{3 - \sqrt{5}}{2} = q^b$$

Notice that, $q^b = q^*$, therefore the preferences are identical. □

In other words, no distortion effect occurs if there are no buyouts. The result is not necessarily intuitive because the profits being compared are not of the same type. That is, the incumbents profits are monopoly profits whilst the entrants profits are competitive. Nevertheless since the absolute value of the gain does not play a role but only the relative gain does, this drives the result. We now proceed to compare the choice between the radical and sequential innovation when buyouts are allowed.

Proposition 17. *If buyouts are possible, then the q required for the radical innovation to be pursued will be higher than $q^b = q^*$.*

Proof. If buyouts are allowed, the radical innovation will be pursued if:

$$\begin{aligned} & B_{er}(\omega) > B_{es}(\omega) \\ \rightarrow\ & q\pi_e(c_i, c_{e2})(3-q)(1-\omega) + \omega q \pi_i(c_{i2}, c_e)(3-q) \\ & > \pi_e(c_i, c_{e2})(1-\omega) + \omega(\pi_i(c_i, c_e) + \pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e1})) \\ \leftrightarrow\ & \pi_e(c_i, c_{e2})(q(3-q)-1)(1-\omega) + \omega\pi_i(c_{i2}, c_e)(q(3-q)-1) - \omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{e1})) > 0 \end{aligned}$$

Note that the third term is negative because $\pi_i(c_i, c_e)$ is a monopoly profit whilst $\pi_i(c_{i2}, c_e)$ is a competitive profit. This implies that unlike before for the inequality to be satisfied, q must not only be large enough to make the expressions it interacts with positive but it must also be large enough to overcome the third term. □

Note that this is just a special case of proposition 12. But it serves to illustrate how the expression simplifies due to the Bertrand assumptions. Bertrand competition the preference shift is entirely due to the difference in profit of the incumbent between the default profit and the profit against an entrant with intermediate cost, in other words the externality, $\omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{e1}))$. The fact that the entrant can bargain on the externality is is the driving factor behind this result.

When does the buyout option help the incumbent?

We now return to the case where we consider the point of view of the incumbent. By looking at the preferences of the incumbent we can also derive a willingness to lobby. That is, if the incumbent loses from the ability to buyout because the competitive effect is larger than the potential technology boost. It is trivial to note that the incumbent does not have a willingness to lobby if the option to buyout does not change the preferences of the entrant. The incumbent will prefer to the buyouts to exist as a function of his own bargaining power, $1 - \omega$. We again look at the special case of the Bertrand competition with parameters $T - 1 = t_2 = t_1 + 1 = 2$. In what corresponds to case 2.3.4 in the general section, the conditions collapses to the following.

$$
\begin{aligned}
B_{is}(\omega) &> 2\pi_i(c_i, c_e) \\
&\leftrightarrow -\omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})) + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})) > 0 \\
1-\omega &> \frac{\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})}{\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1}) + \pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})} \\
B_{ir}(\omega) &> 2\pi_i(c_i, c_e) \\
&\leftrightarrow ((1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})) - \pi_i(c_i, c_e))q(3-q) > 0 \\
1-\omega &> \frac{\pi_i(c_i, c_e)}{\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e2})}
\end{aligned}
$$

The first result is the outcome if the buyout incentivizes the entrant to innovate with the sequential technology when the entrant would have otherwise not innovated at all. The bargaining power must be greater than the ratio of the profit loss from the externality,$\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1})$ to the profit loss from the externality and the difference in profit from what the incumbent can achieve with the best technology and what the entrant can achieve with the best technology,$\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})$.

Similarly with the radical innovation it must just be that the bargaining power of the incumbent is greater than the ratio of the default profit to the difference in profit from what the incumbent can achieve with the best technology and what the entrant can achieve with the best technology. Notice that this means that in the radical case, the willingness to lobby for buyouts is independent of the efficiency of the technology, q. This is because q is equally harmful as it is helpful, a high q increases the probability of achieving the high result but it also increases the negotiating power of the entrant by an equal amount.

Finally we have the case where the entrant would have entered anyway but will instead pursue the sequential innovation. Since the entrant would have entered anyway but the only thing that has changed is the choice of innovation this is the difference in payoff between bargaining for the sequential technology and competing with the radical technology.

$$
\begin{aligned}
&B_{is}(\omega) > \Pi_{ir} \\
\leftrightarrow &\pi_i(c_i, c_e)(q(3-q) - \omega) + \omega\pi_i(c_i, c_{i1}) + (1-\omega)(\pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})) > 0 \\
\leftrightarrow &(1-\omega) > \frac{\pi_i(c_i, c_e)(1 - q(3-q)) - \pi_i(c_i, c_{i1})}{\pi_i(c_i, c_e) - \pi_i(c_i, c_{i1}) + \pi_i(c_{i2}, c_e) - \pi_e(c_i, c_{e2})}
\end{aligned}
$$

Notice here that q does enter into the equation. The higher q is the higher is the willingness to lobby and the lower the required barganing power for the incumbent to wish to lobby.

2.4.3 Ex-Post Cournot

We will not detail the calculations the Cournot, our main purpose for this section is to show that the distortion of buyouts in Cournot is lower than in Bertrand. The payoffs of the entrant and incumbent in Cournot, with the same demand function as before, $D(q) = 1 - q$ are given by:

$$\pi_e(c_i, c_{e1}) = \left(\frac{1 - 2c_{e1} + c_i}{3}\right)^2; \pi_e(c_i, c_{e2}) = \left(\frac{1 - 2c_{e2} + c_i}{3}\right)^2;$$
$$\pi_i(c_i, c_{e1}) = \left(\frac{1 + c_{i1} - 2c_i}{3}\right)^2; \pi_i(c_i, c_{e2}) = \left(\frac{1 + c_{i2} - 2c_i}{3}\right)^2$$

If $c_i - c_{e1} = c_{e2} - c_i$ this would would imply the simplification that $\pi_e(c_i, c_{e1}) = \pi_i(c_i, c_{i2})$ and $\pi_e(c_i, c_{e2}) = \pi_i(c_i, c_{i1})$, this entailment would also hold in the case of Bertrand competition, $\pi^c_{e2} = \pi^c_{i2}$ and $\pi^c_{e1} = \pi^c_{i1}$. As before we assume the initial profit is a monopoly profit. The first result is that the sequential innovation is pursued more often in than in Bertrand.

Proposition 18. *Without buyouts, if the radical innovation is preferred in Cournot competition, it is also preferred in Bertrand.*

Proof.

$$\begin{aligned} \Pi_{er} &> \Pi_{es} \\ q\pi_e(c_i, c_{e2})(3 - q) &> \pi_e(c_i, c_{e1}) + \pi_e(c_i, c_{e2}) \\ q &> \frac{3}{2} - \frac{\sqrt{5\pi_e(c_i, c_{e2}) - 4\pi_e(c_i, c_{e1})}}{2\sqrt{\pi_e(c_i, c_{e2})}} = q^c \end{aligned}$$

We need only see that $q^c > q^b$. To do so we can notice that $\frac{\partial q^c}{\partial \pi_e(c_i, c_{e1})}$ is positive and that if $\pi_e(c_i, c_{e1}) = 0$, we are left with q^b □

The intuition behind this result is due to the lower advantage of being the market leader. In Cournot the entrant earns a profit even with the intermediate technology which makes the

lag time between the intermediate stage and the advanced stage less important. This means the entrant requires a higher q to be convinced to pursue the radical innovation.

Proposition 19. *If the payoff of the incumbent when the entrant has the advanced technology is the same in both Bertrand and Cournot competition, the cutoff point for the radical innovation to be pursued with buyout is lower than in Bertrand.*

Proof.

$$\begin{aligned}\Pi_{er} + \omega NS_r > \Pi_{es} + \omega NS_s \\ (q(3-q)-1)((1-\omega)\pi_e(c_i, c_{e2}) + \omega((\pi_i(c_{i2}, c_e) - \pi_i(c_i, c_{e1})))) \\ +(1-\omega)\pi_e(c_i, c_{e1}) - \omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{e2})) > 0\end{aligned}$$

Note here that this expression is identical to the expression in proposition 17 except for the extra term, $\pi_e(c_i, c_{e1})(1-\omega)$ which is always positive. Note however that this not entail the cuttoff point is always higher than in Bertrand, it depends on if gap between $\omega(\pi_i(c_i, c_e) - \pi_i(c_i, c_{e2}))$ is higher than the same gap in Bertrand competition. Note that the monopoly profit,$\pi_i(c_i, c_e)$ is identical in both cases. Therefore if the profit of the incumbent when the entrant has the advanced technology is identical in both Cournot and Bertrand, Cournot has a lower q for it to be pursued due to the extra positive term, $(1-\omega)\pi_e(c_i, c_{e1})$ □

The buyout case has two relevant effects which are in friction. On one hand the same effect as the non-buyout case, namely that being behind is not as important to the entrant. However there is now a second effect which is that the incumbent is less harmed by being overtaken, so there is a lower willingness to buyout. Note that since $q^b < q^c$ without buyout, and $q^b > q^c$ when there is a buyout, this means the difference between the buyout and the non-buyout case is smaller in Cournot than in Bertrand. Which means that the distortion effect is lower overall for the case of Cournot.

2.4.4 Static case: Reduced form version of the Coasian argument

We present the reduced form static version of the model because it illustrates that the specific complementarity or substitutability does not matter but instead it is only the relative value of purchasing that plays a role. We briefly present the taxonomy under the framework and discuss why each situation may occur.

The market profit potential of the innovation which the entrepreneur holds is given by π^e, this profit may be earned by whoever owns the entrepreneurial project. The profit

of the incumbent *if the innovation does not exist* is simply π^i. We denote the degree(or factor) of substitutability/complementarity **if not bought** by $\beta \in [0, \infty[$ and the degree of substitutability/complementarity **if bought** by $\alpha \in [0, \infty[$.

The payoff if not bought is $\beta\pi^i$. The payoff if bought is $\max\{\pi^i, \pi^e + \alpha\pi^i\}$. Therefore the willingness to pay for the product if $\max\{\pi^i, \pi^e + \alpha\pi^i\} = \pi^i$ is: $(1-\beta)\pi^i$ and if $\max\{\pi^i, \pi^e + \alpha\pi^i\} = \pi^e + \alpha\pi^i$ the willingness to pay is: $\pi^e + (\alpha - \beta)\pi^i$. The extra willingness to pay of the active firm is then simply: $(\alpha - \beta)\pi^i$. This form shows us that substitutability or complementarity do not matter for buyouts, instead it is only the *relative* effects of buyouts which affect the premium the incumbent is willing to pay. Note that π^i can then be seen as the *scale* parameter. Or to express it another way, let $\zeta = 1 + \frac{(\alpha-\beta)\pi^i}{\pi^e}$. If ζ is larger than 1, then the existing firm is willing to pay a premium and if the existing activity is of larger scale relative to the project, the incumbent is willing to pay a higher premium. We now briefly discuss the taxonomy of this framework.

Substitute if not bought and **Complementary** if bought implies: $\beta < 1$ and $\alpha > 1$. This case implies that the product will eat up the profits of the incumbent if allowed to compete with the current product but will expand profits if held together with the current activity.

Complementary if not bought and **Complementary** if bought implies: $\beta > 1$ and $\alpha > 1$. This is just the case where whether the innovation is bought or not, the firm will benefit from it.

Complementary if not bought and **Neutral** if bought implies: $\beta > 1$ and $\alpha = 1$. Why would the project not be complementary if bought? If consumers have a specific aversion to buying things from one firm.

Neutral if not bought and **Neutral** if bought implies: $\beta = 1$ and $\alpha = 1$. This is the case where the entrepreneur's project is uncorrelated to the incumbents current activity.

Neutral if not bought and **Complementary** if bought implies: $\beta = 1$ and $\alpha > 1$. If the technology is complementary this may result either because the production process becomes more efficient or because there is a bundling effect if both goods are sold together.

Neutral if not bought and **Substitute** if bought implies: $\beta = 1$ and $\alpha < 1$. This case would result in shutting down the project. It may be that if the firm markets some new product, the customers of this specific firm will flee to it.

Substitute if not bought and **Neutral** if bought implies: $\beta < 1$ and $\alpha = 1$. Why would a project not be substitable if bought? Perhaps there is a certain way of selling the product that would interact with the incumbents product market but if the incumbent owns it, they

can find a niche way to market it that allows it to be realized without eating away at their other products.

2.5 Discussion

The intuition behind result 17 is a consequence of the Coase theorem. The activity of the entrant can be interpreted to have an externality on the incumbent. Both the radical and sequential innovation have such an externality. However the sequential innovation has an externality with no associated direct benefit to the entrant beyond the ability to threaten the incumbent. In other words, if there was no bargaining, the entrant would be indifferent to increasing the damage done to the incumbent, it is a variable which does not enter into the decision criteria. However as soon as there is a buyout, the entrant now can negotiate on the negative externality being pushed on the incumbent. This incites the entrant to pursue the technology that has this externality relatively more than before.

The ability to blackmail has been studied in the context of the Coase theorem(H.Demsetz, 1972). In the rancher and farmer case study, the rancher has her cows graze whilst the farmer grows crops. If we suppose that the rancher does not have to compensate the farmer for the damage done, then the farmer will be willing to pay to stop the rancher. However in such a case, if the rancher has options which are not individually attractive but are very costly to the farmer, she can use these options to extort the farmer into giving a higher payment. It is important that the rancher be able to commit to applying this costly action, otherwise if the farmer can just say no to the offer, it is not credible that the rancher will act in a costly manner.

In fact this is very similar to our story here. When the entrant pursues the radical innovation there is an externality to the choice where it threatens to take away the profits of the incumbent, this is a productive action that can occur in either of the two periods. On the other hand the sequential innovation can be seen as an unproductive action followed by a productive action. This case is to be juxtaposed to a sequential innovation that would have no externality to the incumbent, this would reduce the payoff potential of the entrant if there is a buyout and be less distortionary.

It is interesting that there are different informational requirements in the two cases. In the case of no buyouts the informational pre-requisites on the entrant are simply to know the profit potential of the project, the cost of the project, and the probability of innovating. On the other hand the ability to buyout actually has a higher burden in terms of rationality on

the entrant, that is to compute the optimal decision one must know not only the potential payoffs of the project but also the revenue loss of the incumbent and the negotiating power. So while the model made abstractions from information asymmetries, it is quite clear that buyouts have a higher information burden, this could be captured merely by interpreting it as part of the cost.

We include the details of the welfare analysis in the appendix but give some cursory details here. In general since the buyout causes a monopoly this monopoly needs to be an improvement from what would occur otherwise, there are only two cases to consider. One alternative a less efficient monopoly, that is a monopoly with the high cost of production, this case occurs if without the buyout the entrant would not enter. Relative to this less efficient monopoly, the buyout can only improve the situation with a more efficient monopoly. The second alternative could be that the entrant would have pursued the innovation anyway, but the effect of the entrant on the market is less important and the more efficient monopoly gives a higher welfare. However this second case is not achievable in the modes of competition used here.

It is perhaps also interesting to ask which of the projects maximize welfare. The answer is a bit complex, the welfare maximizing choice with buyouts is the same as the profit maximizing choice without buyouts(see appendix). Since we have established that the choice of the firm in the ex-ante case pursue the radical innovation more often, this implies that whenever there is a preference reversal without affecting the decision entry, this reduces welfare. The reason this occurs is because the negative externality in the intermediate period is internalized when there is a buyout. This is consistent with coase as it implies that the efficient option is pursued either when the externality is ignored by entrant or when it is fully internalized due to the buyout.

The model predicts a number of things for industry structure. If the entrant is unknown to the incumbent until the the entrant starts to innovate, this immediately gives rise to distorting effects. This may occur if we have an industry where innovation occurs from many small entrants, the prediction is that the small entrant will over-pursue incremental innovations because it is the best way to make their project profitable. An example of such an industry is the relationship of biotechnological firms to the pharmaceutical industry. That is, numerous small entrants who threaten the incumbent who is already firmly established. On the other hand if an industry has endogenous mechanisms so that buyouts can occur before irreversible directional investments are undertaken, such as reputational mechanisms, then that industry will have have a higher tendency to pursue radical innovations.

The model presented is specifically about cost side innovations, the strength of the conclu-

sions depends on the ratio of production to development cost. A high production cost is about producing the marginal unit, if this is expensive then a proportional decrease in this cost will have greater effect on competitive pressure. A high development cost implies that the creation of the product has a sunk cost in the beginning which blocks entry, if this is low then industries may more easily enter and hence there will be more interactions of the sort described in this model. A high development cost is important for the buyouts described because such a cost, like all sunk costs, cannot be used for negotiating with the incumbent. Examples of industries with a high production to development cost are established industries where the good is generally larger, for instance cars, trains, airplanes, boats or metalworks are likely to have a high cost of production without there there being a high cost to development. An example of an industry where the model implies the effects will be weaker is an industry such as the information technology sector, this is because software exhibits very high development cost(programming) and a low cost to produce a unit of software.

A rather different method of envisioning a cost side innovation is management changes. Some entrant may have some cutting edge method of managing employees that can either reduce the cost of the firm gradually or it may have some scheme were a large structural change occurs and the costs are reduced quickly. The application of the model in this case is that incumbents will buyouts will lead to the entrant over-pursuing the slow employee cost reduction technique rather than the fast and risky one.

Another natural example of a cost side innovation is energy innovations. Perhaps rather intuitively, firms are more likely to buyout entrepreneurs who can cause damage even with an ineffective technology than firms with much better prospects for advancement but no intermediate stage. In energy this may look like firms buying out solar or wind technologies more than buying out nuclear plans.

2.6 Conclusion

The paper presents a preference reversal within two paradigms. We find that policy levers have ambiguous effects, enabling buyouts can have both a negative and positive effects on welfare and this is not necessarily a function of the willingness to pay. Instead it is a function of substitutability and complementarity. Empirically, the willingness to pay of incumbents for the entrants cannot be used as a proxy for reducing rent seeking since, the willingness to pay can stem equally from substitutability and complementarity.

We present a model which implies that mergers should not only be seen from a point of view

of competition vs efficiency but also from the point of view of innovative convergence. When firms can buy other firms this is in essence license for firms to play correlated strategies and there is a strong incentive for firms to choose industries which are already existing without reasons having to do with industry level characteristics. As such anti-trust policy should take into account the effects buyouts on industry convergence. Specifically, for patents, if the role of the patent is to exclude other firms, then the kind of patenting will not be affected, if on the other hand the role of patenting is to be bought over by larger firms, this will favor industry convergence. The basic reasoning implies that either patents requirement should more stringent when an innovation improves on a product with an existing industry or if possible, if entrants represent a larger threat to incumbents, buyouts rules should be more demanding. .

Finally, the model implies that there is a demand for lobbying. If we are in a paradigm where enabling buyouts create a preference reversal for the entrant and where this is not preferred by the incumbent, then this creates a willingness to pay from the incumbent which will disable buyouts.

Apendix 1: Welfare equations

There are four possible consumer surplus outcomes. The two monopoly outcomes, where the incumbent has the default or the highest technology, S_I and S_{I2}, respectively. Or the two competitive outcomes, where the incumbent must set a price when the entrant has an intermediate technology and when the entrant has the highest technology, S_{I1} and S_E respectively.

A reminder that the social surplus is found by computing: $\frac{1}{2}(1-p)(1-p)$. In the case of monopoly the price is simply the monopoly price in a Bertrand context. While if the outcome is competitive, the price is simply the competitors cost. The four possible social outcomes are given below:

$$S(c_i, c_e) = \frac{(1-c_i)^2}{8}; \quad S(c_{i2}, c_e) = \frac{(1-c_{i2})^2}{8}; \quad S(c_i, c_{e1}) = \frac{(1-c_{i1})^2}{2}; \quad S(c_i, c_{e2}) = \frac{(1-c_i)^2}{2}$$

Note that the consumer only prefers the buyouts if it incentives the entrant to pursue the projects. If the projects are already being pursued without the buyout then the consumer can only lose because whilst before there was some possibility of a competitive outcome, now there are only monopoly outcomes possible. From the welfare perspective the bargaining power only matters if it will change the choices of the firms. Otherwise bargaining power will be zero sum, therefore we need only look at the market profits and the social surplus of consumers to compute the welfare function. In the two cases where there is a monopoly this is either the monopoly with the default cost or the monopoly with the lower cost. We recall here that monopoly with the lower price is preferred over the monopoly with the default price for both consumers and the monopolist. These outcomes are given by:

$$\begin{aligned} w(c_i, c_e) &= \frac{(1-c_i)^2}{8} + \frac{(1-c_i)^2}{4} = \frac{3(1-c_i)^2}{8} \\ w(c_{i2}, c_e) &= \frac{3(1-c_{i2})^2}{8} \end{aligned}$$

Similarly the welfare payoffs of both consumers and the firms are given simply by the competitive profits and the consumer surplus. This represents a shift from firms to the consumers. From the consumer point of view it is preferred that the entrant be the market leader because

the price will necessarily be lower. However this does not neccesarily mean that the entrant will have less profits than the competitive case where the incumbent is ahead.

$$w(c_i, c_{e1}) = \frac{(1-c_{i1})^2}{2} + (1-c_{i1})(c_{i1}-c_i) = \frac{(1-c_{i1})}{2}\left((1-c_{i1}) + 2(c_{i1}-c_i)\right)$$
$$w(c_i, c_{e2}) = \frac{(1-c_i)}{2}\left((1-c_i) + 2(c_i - c_{i2})\right)$$

Something to note here is that while clearly if we compare the monopoly cases we have the relationship, $w(c_i, c_e) < w(c_{i2}, c_e)$, that is the monopoly outcome with the lower price is better for both consumers and the firms. However, no analogous relationship exists between $w(c_i, c_{e2})$ and $w(c_i, c_{e1})$. If the gap $c_i - c_{i2}$ and $c_{i1} - c_i$ are equal then we have the relationship, $w(c_i, c_{e2}) > w(c_i, c_{e1})$. This is for the same reason as for the monopolist outcome, the price is lower without the profits being lower, therefore a net gain for consumers.

Before proceeding to analyze the innovations effect on welfare, we note that the welfare without the innovation is simply:

$$w(c_i, c_e) + w(c_i, c_e) = \frac{3(1-c_i)^2}{4}$$

2.6.1 Sequential

In the sequential innovation case with no buyout, in the firs time period there will be the competitive outcome with the incumbent ahead and in the second time period the entrant will be ahead with another competitive outcome. Necessarily the price will decrease, therefore for the consumers there will be an increase in surplus in the second time period.

$$\begin{aligned}\overline{W}_S = w(c_i, c_{e1}) + w(c_i, c_{e2}) &= \frac{1}{2}\left((1-c_{i1})\left((1-c_{i1}) + 2(c_{i1}-c_i)\right) + (1-c_i)\left((1-c_i) + 2(c_i - c_{i2})\right)\right) \\ &= 1 - \frac{c_{i1}^2}{2} + c_{i1}c_i - \frac{c_i^2}{2} - c_i(1-c_{i2}) - c_{i2} \\ &= 1 - \frac{c_{i1}^2}{2} - \frac{c_i^2}{2} - c_i(1-c_{i2}-c_{i1}) - c_{i2}\end{aligned}$$

When the buyout occurs there is always a monopoly. So the consumers will simply have to deal with the default monopoly in the first period and with the lower cost monopoly in the second period.

$$W_S = w(c_i, c_e) + w(c_{i2}, c_e) = \frac{3}{8}\left((1-c_i)^2 + (1-c_{i2})^2\right) \tag{2.6.1}$$

2.6.2 Radical

Welfare when the radical innovation is pursued and there is no buyout is similarly given by:

$$\begin{aligned}\overline{W}_R &= q2w(c_i, c_{e2}) + (1-q)(w(c_i, c_e) + (1-q)w(c_i, c_e) + qw(c_i, c_{e2})) \\ &= qw(c_i, c_{e2})(3-q) + (1-q)w(c_i, c_e)(2-q) \\ &= \frac{1}{8}(1-c_i)\left(6 - c_i\left(7q^2 - 21q + 6\right) - (1-8c_{i2})q^2 - 3(8c_{i2}-1)q\right)\end{aligned}$$

If buyouts do occur and we are in the monopoly paradigm, the consumers are always in facin high prices but have a preference for the innovation to occur, the welfare when there are buyouts is given by the expression:

$$\begin{aligned}W_R &= qw(c_{i2}, c_e)(3-q) + (1-q)w_{m1}(2-q) \\ &= \frac{3}{8}\left((c_i-1)^2(2-q)(1-q) + (c_{i2}-1)^2(3-q)q\right)\end{aligned}$$

Appendix 2: Welfare results

Proposition 20. *The welfare maximizing choice in the case of buyouts is identical to the entrant's optimal choice in the case of no buyout and the incumbents ex-ante choice.*

2.6.3 Proof of proposition 20

Proof.

$$\begin{aligned} W_R &> W_S \\ qw(c_{i2}, c_e)(3-q) + (1-q)w(c_i, c_e)(2-q) &> w(c_i, c_e) + w(c_{i2}, c_e) \\ w(c_{i2}, c_e)(q(3-q)-1) + w(c_i, c_e)((1-q)(2-q)-1) &> 0 \\ w(c_{i2}, c_e)(3q - q^2 - 1) + w(c_i, c_e)(1 - 3q + q^2) &> 0 \\ w(c_{i2}, c_e)(3q - q^2 - 1) - w(c_i, c_e)(3q - q^2 - 1) &> 0 \end{aligned}$$

If the costs are the same, then radical will be preferred if:

$$q > \frac{3-\sqrt{5}}{2} \tag{2.6.2}$$

Which is identical to the cutoff point for the entrant to prefer the radical innovation.

□

In proposition 20 we look for the criterion under which welfare is maximized when **buyouts occur** and show that they are identical to the profit maximizing criterion of the entrant when **buyouts do not occur** and the ex-ante case. To state this another way, if absent a buyout, the entrant chooses the radical innovation, then, if there are buyouts, the welfare maximizing choice is also the radical innovation.

Proposition 21. *A necessary(but not sufficient) condition for the radical innovation to be welfare maximizing is that* $w(c_i, c_{e2}) + w(c_i, c_{e1}) - 2w(c_i, c_e) > 0$. *Similarly for it to be possible that sequential innovation is welfare maximizing it must be that:* $w(c_i, c_{e1}) - w(c_i, c_{e2}) > 0$

Proof.

$$\begin{aligned} \overline{W}_R &> \overline{W}_S \\ qw(c_i, c_{e2})(3-q) + (1-q)w(c_i, c_e)(2-q) &> w(c_i, c_{e1}) + w(c_i, c_{e2}) \\ w(c_i, c_{e2})(3q - q^2 - 1) + w(c_i, c_e)(2 - 3q + q^2) - w(c_i, c_{e1}) &> 0 \\ q^2(w(c_i, c_e) - w(c_i, c_{e2})) - 3q(w(c_i, c_e) - w(c_i, c_{e2})) - w(c_i, c_{e2}) + 2w(c_i, c_e) - w(c_i, c_{e1}) & \end{aligned}$$

$$\rightarrow q > \frac{3}{2} - \frac{\sqrt{w(c_i, c_e) - 5w(c_i, c_{e2}) + 4(w(c_i, c_{e1}))}}{2\sqrt{(w(c_i, c_e) - w(c_i, c_{e2}))}}$$

So the bound for the expression to be smaller than 1 is:

$$\frac{\sqrt{w(c_i, c_e) - 5w(c_i, c_{e2}) + 4(w(c_i, c_{e1}))}}{\sqrt{(w(c_i, c_e) - w(c_i, c_{e2}))}} > 1$$

$$w(c_i, c_e) - 5w(c_i, c_{e2}) + 4(w(c_i, c_{e1})) > (w(c_i, c_e) - w(c_i, c_{e2}))$$

$$w(c_i, c_{e1}) - w(c_i, c_{e2}) > 0$$

Similarly for the expression to be larger than 0 we must have:

$$\frac{\sqrt{w(c_i, c_e) - 5w(c_i, c_{e2}) + 4(w(c_i, c_{e1}))}}{2\sqrt{(w(c_i, c_e) - w(c_i, c_{e2}))}} < \frac{3}{2}$$

$$w(c_i, c_e) - 5w(c_i, c_{e2}) + 4(w(c_i, c_{e1})) > 9(w(c_i, c_e) - w(c_i, c_{e2}))$$

$$4(w(c_i, c_{e2}) + w(c_i, c_{e1})) - 8w(c_i, c_e) < 0$$

$$w(c_i, c_{e2}) + w(c_i, c_{e1}) - 2w(c_i, c_e) < 0$$

□

Chapter 3

Microfoundations of Discounting

With Alexander Adamou[1], Yonatan Berman[2] and Ole Peters[3]

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3.1 Introduction

3.1.1 Background

In economics and psychology, temporal discounting – or, simply, discounting – is a paradigm of how decision makers choose between rewards available at different times in the future. We write here of people and money payments, noting that discounting is also studied in other animals and for other reward types. The basic observation to be explained is this: for two payments of equal size, people prefer typically the earlier payment to the later one. In the discounting paradigm, the later payment is discounted relative to the earlier payment by multiplying it by a function of the time period between the payments, called the *delay*. This operation expresses the later payment as an equivalent payment at the earlier time, to be compared with the earlier payment actually on offer.

Why do people assign lower values to payments further in the future? One plausible answer is that a later payment is less likely to be received, because there is more time for something to

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go wrong with it. In other words, delay increases risk. Another is that, for equal payments, the later one corresponds to a lower growth rate which, if sustained over time, would result in being poorer. Modern treatments of discounting in economics tend to follow risk-based reasoning, while there is a more even split between risk and rate interpretations in psychology.

This paper studies the microfoundations of discounting using the rate interpretation in a riskless setting. In our model, a decision maker chooses between two known and different payments to be received at known and different times, such that the growth rate of her wealth is maximized. Our model assumes no behavioral bias and does not violate standard axioms of choice. It predicts a range of situation-dependent discount functions, including those documented in the discounting literature.

This literature abounds with models (Cohen et al., 2019). In some, theoretical considerations are used to construct the decision maker's maximand, from which the discount function is derived. Such models are "normative" in that they say what a decision maker should do if she wants to act optimally in the sense specified. In other models, the discount function is chosen to fit empirical data, with theoretical justification sought *post hoc* or not at all. These models are "descriptive" in that they predict what decision makers actually do, regardless of whether it is in any sense optimal.

The main normative model in economics is exponential discounting, in which the discount function decays exponentially with the delay (Samuelson, 1937). For money payments, this is derived straightforwardly: either by a no-arbitrage argument, assuming payments are guaranteed and the earlier payment can be invested during the delay to earn compound interest at a riskless rate; or by assuming the later payment has a constant hazard rate during the delay and the decision maker maximizes the expected payment (Kacelnik, 1997).

Exponential discounting is not descriptive. Experiments on human and non-human animals suggest that payments can be discounted more for shorter delays and less for longer delays than is well described by fitting the rate parameter of an exponential function. Furthermore, subjects exhibit a behavior known as preference reversal, where they switch from preferring the later to the earlier payment as time passes. Specifically, the switch happens as the time to the earlier payment – which we call the *horizon* – gets shorter, while the delay between the payments stays the same (Keren and Roelofsma, 1995, p. 288). Preference reversal is never predicted by standard exponential discounting (Green and Myerson, 1996, Fig 2). The primary evidence against the main normative model is summarized by Myerson and Green (1995) and references therein. In response to its falsification, descriptive models are proposed with discount functions better able to fit observations.

A widely-used descriptive model is hyperbolic discounting, where the discount function is a hyperbola in the delay. The function has one free parameter, known as the degree of discounting, which determines its steepness. Fitting this parameter to experimental data is more efficient than fitting an exponential function, both at group level and for individuals (Myerson and Green, 1995). Furthermore, preference reversal is compatible with this model (Green and Myerson, 1996, Fig. 2).

Kacelnik (1997) remarks on this divergence between normative and descriptive models, noting that the hyperbola "is not strongly explanatory because it did not emerge from an *a priori* analysis but purely from its power to describe data efficiently. In contrast, because of the strong appeal of the *a priori* argument favoring exponential discounting, several re-elaborations have been made to rescue the rationale that led to it."[4] Most such attempts to adapt the normative model introduce payment uncertainty, which we discuss in Sec. 3.1.3. Another approach, favored in behavioral economics, is to present non-exponential discounting as a cognitive bias – a deviation from optimal behavior – to be documented and quantified in mathematical functions that encode human psychology (Loewenstein and Prelec, 1992; Laibson, 1997).

3.1.2 Our model – growth rate maximization

Here we propose a model of temporal discounting compatible with hyperbolic discounting, in which neither payment risk nor behavioral bias are assumed. We study the basic temporal choice problem in which a decision maker must choose between two known, certain, and different payments to be made at known, certain, and different future times. In our model, she does so by comparing the growth rates of wealth associated with each option.

The temporal choice problem is underspecified. We specify it fully by introducing: the wealth dynamics, treating specifically additive and multiplicative cases; and the time frame of the decision, meaning the period over which it is appropriate to compute growth rates.

Depending on the specification, our model predicts four different forms of discounting: no discounting; exponential; hyperbolic; and a hybrid of exponential and hyperbolic. This is not an exhaustive list – other dynamics would produce other forms of discounting. Two of the discount functions nested in our model, hyperbolic and hybrid, are compatible with preference reversal.

[4] Said succinctly, a normative-only model provides rationale without fit, and a descriptive-only model provides fit without rationale. Ideally, models of discounting, as of other behavioral phenomena, would provide both and the distinction would be redundant.

The hybrid discount function depends not only on the delay and the horizon, but also on the decision maker's wealth and underlying growth rate. This produces a richer set of predicted behaviors than other specifications. One prediction is that decision makers can switch from preferring the earlier to the later payment as their wealth increases. In other words, richer people discount less steeply, consistent with empirical findings (Green et al., 1996; Epper et al., 2018). Another prediction is that, for small payments, discounting is close to hyperbolic for short delays and close to exponential for long delays.

The main contribution of this paper is the prediction of non-exponential discounting and preference reversal in a normative model that does not violate standard axioms of choice (von Neumann and Morgenstern, 1944). Our model assumes neither bias nor dynamic inconsistency (Cohen et al., 2019, p. 3) in the decision maker's behavior. At all times she prefers the option with the highest growth rate. In some specifications this translates into a reversal of preference between *payments* (not growth rates). In our perspective this reflects a change of circumstances and not of mind. The fundamental preference – for faster growth – never reverses.

Moreover, this paper marks a shift from psychological to circumstantial explanations of discounting. We predict that changes in the discount function arise from changes in wealth dynamics and time frame, which are properties not of the decision maker but of her circumstances. When these circumstances are included in the formalism, a single behavioral model – a single maximand – is capable of predicting a range of observed behaviors. Since psychological risk preferences, encoded in idiosyncratic utility functions, do not appear in our model, we sidestep recent controversy in the literature about the suitability of experiments involving money payments to test models of utility-of-consumption flows (Cohen et al., 2019). Such experiments are able to falsify our model and are planned.

The paper is organized as follows. Section 3.2 sets out the temporal choice problem and our decision model. In Sec. 3.3 we present different specifications of the problem and describe how a decision maker discounts future payments in each of them. We conclude in Sec. 3.4.

3.1.3 Related literature

This paper follows the tradition of adapting the normative discounting model to make it consistent with observations, *i.e.* to make it descriptive (Kacelnik, 1997). Our strategy is to postulate the growth rate of wealth as the decision maker's maximand. Computing this requires information about wealth dynamics and time frame, about which the basic temporal choice problem is silent.

Another strategy is to leave the maximand – usually an expected payment or utility gain – unchanged and introduce uncertainty in the amount or timing of the payment. The uncertainty is chosen so that the effective discount function takes a form known to fit the data. Adding risk can be viewed as another way of resolving the underspecification of the temporal choice problem. This approach is questionable because, whereas dynamics are unspecified in the problem, uncertainty is explicitly absent – a choice between certain payments at certain times. Payment risk is a sound microfoundation when the uncertainty absent from the problem formulation is important in reality. Such situations are plausible and likely widespread. For example, a predator declining a small but readily-caught prey to search for something more filling risks catching nothing at all. However, adding payment uncertainty to generate, say, hyperbolic discounting is not a general prescription. It fails when the real payment risk is negligible, as envisaged in the problem and presented in experiments, *e.g.* (Myerson and Green, 1995).

Furthermore, to recover the hyperbola as the discount function, specific forms of uncertainty or other adaptations are required. Green and Myerson (1996) point out that an expected payment model ('risk neutrality') with a hyperbolic hazard rate predicts hyperbolic discounting. They note also that the exponential function can be made consistent with observed behavior, including preference reversal, by allowing its rate parameter to vary with payment size. Sozou (1998) treats an expected payment model with an uncertain hazard rate, about which the decision maker learns through Bayesian updating. If the prior distribution of the hazard rate is exponential, then hyperbolic discounting is again obtained. Dasgupta and Maskin (2005) also assume risk neutrality but keep the hazard rate constant. To recover hyperbolic discounting they introduce the possibility of payment occurring before the anticipated time.

Such adaptations lead to a loss of generality. They make statements of the type: 'if there is uncertainty in a future payment, and if it takes this specific form, then the discount function is a hyperbola.' Such *ad hoc* assumptions reduce the generality of risk-based models further, since they are useful only when the real risk is of a particular nature.

The strategy we follow leaves payment certainty alone and changes the maximand. It is long established in biology and psychology that hyperbolic discounting is consistent with maximizing the rate of change of resources in a model of additive payments. This insight, traced back to the predation model of Holling (1959), is recognized in studies of human discounting (Myerson and Green, 1995; Kacelnik, 1997; Sozou, 1998). Kacelnik (1997, Fig. 2) offers a pictorial representation, similar to ours in Sec. 3.3, of how rate maximization predicts preference reversal. In the rate interpretation, the degree of discounting is no longer a free psychological parameter. Rather it is constrained to be the reciprocal of the horizon (Myerson

and Green, 1995). This is a testable prediction.

This paper extends this strand of the literature by setting the decision maker's maximand as the growth rate of resources under general dynamics. We generalize further by allowing the time frame of the decision – over which growth rates are computed – to be the period from the decision to either the chosen or the later payment. This captures circumstances akin to opportunity costs, specifically whether accepting the earlier payment frees the decision maker to pursue other payments.

Our work contributes to the growing field of ergodicity economics (Peters and Gell-Mann, 2016; Berman et al., 2017; Peters and Adamou, 2018) in which decision makers maximize the long-time growth rate of resources, rather than expectation values of psychologically-transformed resource flows (under, in prospect theory, psychologically-transformed probability measures). This study joins recent evidence of strong dependence on wealth dynamics of human decisions under uncertainty (Meder et al., 2019) and may be used to design similar experiments without uncertainty.

3.2 Theoretical Framework

3.2.1 Problem definition

We begin by formalizing a basic riskless temporal choice problem, where discounting is used to express a later payment as an equivalent payment at an earlier time. We define a *Riskless Intertemporal Payment Problem* (RIPP):

Definition 4. *Riskless Intertemporal Payment Problem.*

A Riskless Intertemporal Payment Problem (RIPP) is a comparison between two vectors, $a \equiv \{t_a, \Delta x_a\}, b \equiv \{t_b, \Delta x_b\}$. *A decision maker at time* t_0 *with wealth* $x(t_0)$ *must choose between two future cash payments, whose amounts and payment times are known with certainty. The two options are:*

a. an earlier payment of Δx_a *at time* $t_a > t_0$*; and*

b. a later payment of Δx_b *at time* $t_b > t_a$.

A criterion for choosing a or b is required. Here we explore what happens if that criterion is maximization of the growth rate of wealth, *i.e.* if a is chosen when it corresponds to a higher growth rate of the decision maker's wealth than b, and *vice versa*.

A growth rate is defined as the scale parameter of time in the growth function of wealth subject to dynamics. Dynamics can take different forms, each corresponding to a different form of growth rate. We treat explicitly multiplicative and additive dynamics (Peters and Gell-Mann, 2016), noting that more general dynamics can be treated similarly (Peters and Adamou, 2018).

Multiplicative dynamics

Ignoring, for the moment, payments Δx_a and Δx_b, a common assumption is that wealth grows exponentially in time at rate r. We label this dynamic as multiplicative. It corresponds to investing wealth in income-generating assets, where the income is proportional to the amount invested. Wealth grows as

$$x(t) = x(t_0)\, e^{r(t-t_0)}\,, \tag{3.2.1}$$

and the scale parameter of time in the exponential function is r. r resembles an interest rate or a rate of return on investment.

Additive dynamics

Another possibility is additive dynamics, where wealth grows linearly in time at a rate k. This resembles saved labor income or, more generally, situations where investment income is negligible and wealth changes by net flows that do not depend on wealth itself. In this case wealth grows as

$$x(t) = x(t_0) + k(t - t_0)\,, \tag{3.2.2}$$

and the scale parameter of time in the linear function is k.

The functional form of the growth rate differs between the dynamics. The growth rate between time t and $t + \Delta t$ can be extracted from the expression for the evolution of wealth over that period. Under multiplicative dynamics it is

$$r = \frac{\log x(t + \Delta t) - \log x(t)}{\Delta t}\,, \tag{3.2.3}$$

and under additive dynamics it is

$$k = \frac{x(t + \Delta t) - x(t)}{\Delta t}\,. \tag{3.2.4}$$

The matching of growth rate with dynamics is crucial. An additive growth rate applied to

wealth following a multiplicative process would vary with time, as would a multiplicative growth rate applied to additively-growing wealth. The correct growth rate extracts a stable parameter from the dynamics, allowing processes with the same type of dynamics to be compared.

Given the wealth dynamics, a RIPP implies two growth rates: g_a, associated with option a; and g_b, associated with option b. This permits a single function that characterizes the behavior of the agent. We say an agents behavior is *growth maximizing* if the following holds true:

Definition 5. *The Maximization of Growth.*

Given the wealth dynamics, a decision time t_0, an initial wealth $x(t_0)$, and payments $a \equiv (t_a, \Delta x_a)$ and $b \equiv (t_b, \Delta x_b)$, such that we have a RIPP:

1. $a \succ b$ *['a is preferred to b'] if and only if* $g_a > g_b$
2. $a \sim b$ *['indifference between a and b'] if and only if* $g_a = g_b$
3. $a \prec b$ *['b is preferred to a'] if and only if* $g_a < g_b$

In words, Definition 5 states that a decision maker prefers option a if her wealth grows faster under this choice than under option b, and *vice versa*. She is indifferent if the growth rates are equal. The growth function in Axiom 5 resembles a utility function and satisfies completeness and transitivity (see proof in Appendix .1). The function would also satisfy independence and continuity if we were in a non-deterministic setting.

3.2.2 Model setup

Figure 3.1 illustrates a RIPP, corresponding to the basic question that arises in temporal discounting, *e.g.* 'would you prefer to receive \$100 tomorrow or \$200 in a month's time?' Growth rates depend on time increments, not times themselves, so it is useful to define the two fundamental time increments in the problem: the period from the decision to the earlier payment, called the *horizon*,

$$H \equiv t_a - t_0 \,; \tag{3.2.5}$$

and the period between the payments, called the *delay*,

$$D \equiv t_b - t_a \,. \tag{3.2.6}$$

The discount function is a function of the delay. It is the multiplicative factor which, when applied to the later payment, renders the decision maker indifferent, *i.e.*

$$\delta(D) \equiv \left.\frac{\Delta x_a}{\Delta x_b}\right|_{a\sim b}. \tag{3.2.7}$$

Depending on the model specification, the discount function can also vary with other variables in the problem, such as the horizon, payment sizes, initial wealth, and underlying growth rate (r or k for the dynamics we study).

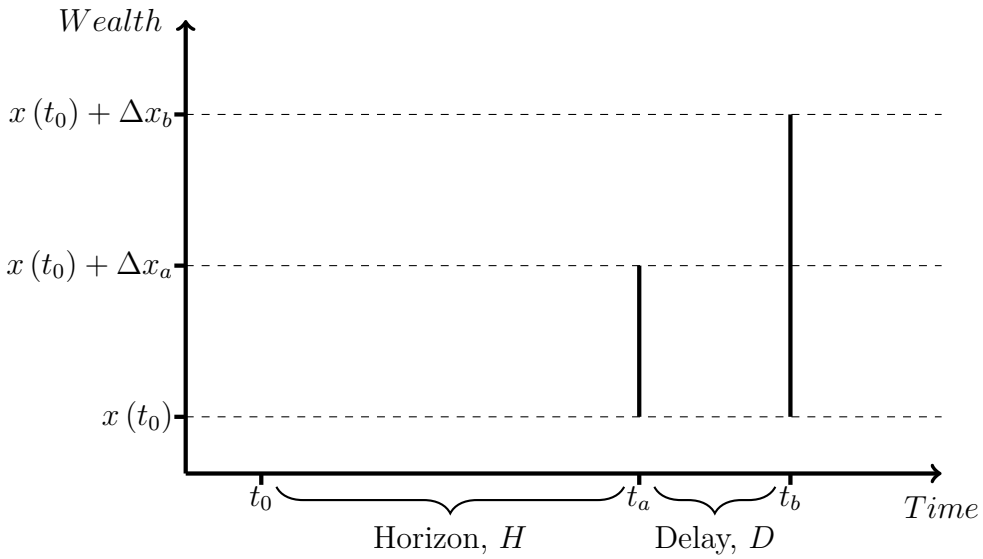

Figure 3.1: The basic setup of the model. A decision maker faces a choice at time t_0 between option a, which guarantees a payment of Δx_a at time t_a, and option b, which guarantees a payment of $\Delta x_b > \Delta x_a$ at time $t_b > t_a$. We define the time between the decision and the earlier payment as the *horizon*, $H \equiv t_a - t_0$; and the time between the two payments as the *delay*, $D \equiv t_b - t_a$.

Despite its apparent simplicity, solving the temporal choice problem requires additional assumptions. In our model, two assumptions are needed. The first concerns the dynamics under which the decision maker's wealth grows, as discussed in Sec. 3.2.1. This determines the appropriate form of the growth rate. The second assumption concerns what we call the *time frame* of the decision, specifically whether a decision maker accepting the earlier payment at t_a is free immediately to make her next decision, or whether she must wait until the later time t_b. This determines the appropriate time period for computing the growth rate under each option. Full specification allows the decision maker's maximand – the growth rate of her wealth – to be evaluated and her options compared. For concreteness we confine our attention to $\Delta x_b > \Delta x_a$, which is the most commonly considered dilemma.

We describe four different specifications of this basic setup. In each we calculate the growth rates of wealth, g_a and g_b, associated with options a and b. From this analysis we infer the discount function as

$$\delta(D) \equiv \left.\frac{\Delta x_a}{\Delta x_b}\right|_{g_a=g_b}, \tag{3.2.8}$$

i.e. the ratio of earlier to later payment under the constraint that the growth rates corresponding to each are equal. The four specifications predict four forms of discounting – no discounting, exponential, hyperbolic, and a hybrid of exponential and hyperbolic – and, in some cases, preference reversal.

3.3 Results

3.3.1 Specification

We begin by describing the four different model specifications. Each specifies two aspects necessary to quantify the growth rate of wealth: the time frame of the decision; and the dynamics under which wealth evolves.

Time frame

The time frame is a key aspect, often left unspecified or implicit in similar setups in the literature. To illustrate it, we consider the following scenarios:

1. Denise is a day laborer. Every morning she chooses a job to take on. Jobs pay different wages and take different amounts of time, although always less than a day. Denise is paid as soon as she completes the job and goes home. She cannot do more than one job each day.

2. Fiona is a freelancer. She works on projects ranging from a few days to many months and she can only work on one project at a time. As soon as she finishes a project, she gets paid and can move on to the next project.

In the first scenario, the important element to note is that no matter which choice is made, it does not affect the timing of future choices. Denise's next decision is always the next morning. The time frame is independent of the choice, so we say it is *fixed*.

In the second scenario, the time frame depends on the choice made. The timing of Fiona's next choice is determined by her current decision, *e.g.* choosing a shorter project frees her

sooner for the next opportunity. We call this the *adaptive* time frame.

In our model, we must choose the time period over which the growth rate of wealth is relevant to the decision maker. With a fixed time frame, this is always the period between the decision and the later payment, *i.e.* $t_b - t_0 = H + D$. In Denise's case, this is a working day. With an adaptive time frame, it is the period between the decision and the chosen payment, *i.e.* $t_a - t_0 = H$ for option a and $t_b - t_0 = H + D$ for option b. This specification is appropriate to Fiona's situation.

Dynamics

As described in Section 3.2, the wealth dynamics can also take different forms. We address two common cases: additive and multiplicative wealth dynamics. We note that under the multiplicative dynamics it is assumed that the payment itself is reinvested at the risk-free rate. For additive dynamics there is essentially no reinvestment of the payment. Income in this dynamic is independent of wealth.

We discuss the four specifications, as illustrated in Fig. 3.2. In each case we: compute the growth rates g_a and g_b associated with each option; compare them to determine the conditions under which each option is preferred; determine whether preference reversal is predicted; and, finally, elicit the form of temporal discounting equivalent to our decision model.

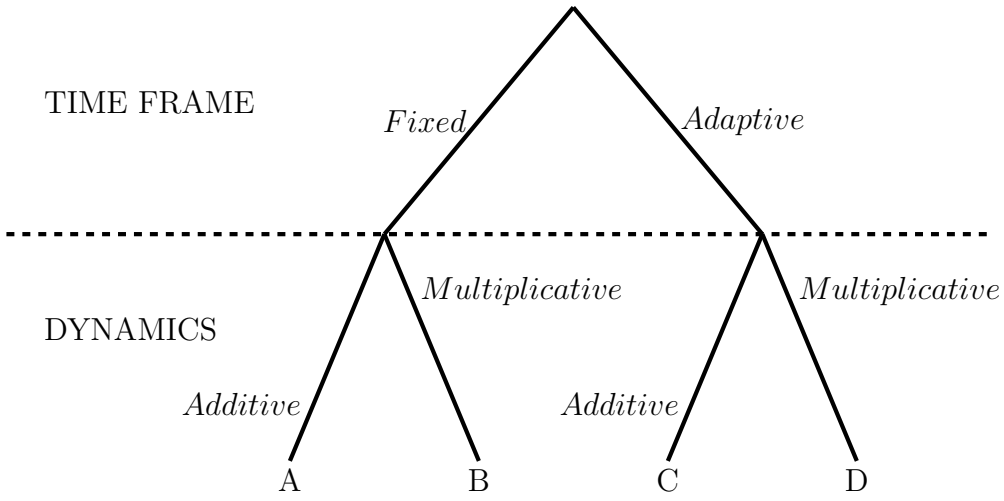

Figure 3.2: The four model specifications, determined by specifying a time frame and wealth dynamics. The labels A, B, C, and D, are used for the different cases.

3.3.2 Case A – Fixed time frame with additive dynamics

Specification: the period for computing the growth rate is that between the decision and the later payment, $t_b - t_0 = H + D$; and wealth dynamics are additive. Here, regardless of the initial wealth and in addition to the chosen payment, wealth grows at an additive rate k (corresponding, for example, to labor income).

We begin by writing down the final wealth under each of the two options, evaluated at t_b:

$$x_a(t_b) = x(t_0) + k(t_b - t_0) + \Delta x_a; \tag{3.3.1}$$

$$x_b(t_b) = x(t_0) + k(t_b - t_0) + \Delta x_b. \tag{3.3.2}$$

The growth rates are:

$$g_a = \frac{x_a(t_b) - x(t_0)}{t_b - t_0} = \frac{\Delta x_a}{H + D} + k; \tag{3.3.3}$$

$$g_b = \frac{x_b(t_b) - x(t_0)}{t_b - t_0} = \frac{\Delta x_b}{H + D} + k. \tag{3.3.4}$$

Since $\Delta x_b > \Delta x_a$, option b is always preferred to option a. This is a trivial case: under additive wealth dynamics and comparing growth rates over the same time period, only payment size matters. There is no discounting and the discount function δ is undefined, because the indifference condition is never satisfied.

3.3.3 Case B – Fixed time frame with multiplicative dynamics

Specification: the period for computing the growth rate is that between the decision and the later payment, $t_b - t_0 = H + D$; and wealth dynamics are multiplicative. This specification corresponds to the classical temporal discounting, where wealth compounds continuously at the risk-free rate, r, and payments are reinvested at this rate.

We note that in this case the earlier payment, Δx_a, if chosen, is treated as growing exponentially from its receipt at t_a to t_b. The wealths evolve from t_0 to t_b as follows:

$$x_a(t_b) = x(t_0)\, e^{r(t_b - t_0)} + \Delta x_a e^{r(t_b - t_a)}; \tag{3.3.5}$$

$$x_b(t_b) = x(t_0)\, e^{r(t_b - t_0)} + \Delta x_b. \tag{3.3.6}$$

The corresponding growth rates are:

$$g_a = \frac{\log x_a(t_b) - \log x(t_0)}{t_b - t_0} = \frac{1}{H+D} \log\left(1 + \frac{\Delta x_a e^{rD}}{x(t_0)\, e^{r(H+D)}}\right) + r\,; \tag{3.3.7}$$

$$g_b = \frac{\log x_b(t_b) - \log x(t_0)}{t_b - t_0} = \frac{1}{H+D} \log\left(1 + \frac{\Delta x_b}{x(t_0)\, e^{r(H+D)}}\right) + r\,. \tag{3.3.8}$$

The criterion $g_a > g_b$ is simple. Only the numerator in the second term of the logarithm is different, so only this must be compared. Thus, $g_a > g_b$ if

$$\Delta x_a e^{rD} > \Delta x_b\,. \tag{3.3.9}$$

We see that the decision criterion depends on a single time period, the delay, and on the underlying growth rate of wealth. The discount function is obtained by setting the growth rates to be equal, which happens when $\Delta x_a e^{rD} = \Delta x_b$. This yields

$$\delta(D; r) = \left.\frac{\Delta x_a}{\Delta x_b}\right|_{g_a = g_b} = e^{-rD}\,, \tag{3.3.10}$$

which is the classical exponential discounting result (Samuelson, 1937).[5] The interpretation is straightforward: if it is possible to reinvest the earlier payment such that it will exceed the later payment amount at the later payment time, then option a is preferred to option b (and *vice versa*).

3.3.4 Case C – Adaptive time frame with additive dynamics

Specification: the period for computing the growth rate is that between the decision and the chosen payment, either $t_a - t_0 = H$ or $t_b - t_0 = H + D$; and wealth dynamics are additive. Like in case A, regardless of the initial wealth and in addition to the chosen payment, wealth grows at an additive rate k. Unlike case A, options a and b are evaluated at t_a and t_b, respectively:

$$x_a(t_a) = x(t_0) + k(t_a - t_0) + \Delta x_a\,; \tag{3.3.11}$$

$$x_b(t_b) = x(t_0) + k(t_b - t_0) + \Delta x_b\,. \tag{3.3.12}$$

[5] Indeed, this result corresponds to the historical use of the term "rate of discount" to describe a risk-free interest rate in the money market, *e.g.* (Jevons, 1863).

The growth rates are:

$$g_a = \frac{x_a(t_a) - x(t_0)}{t_a - t_0} = \frac{\Delta x_a}{H} + k\,; \tag{3.3.13}$$

$$g_b = \frac{x_b(t_b) - x(t_0)}{t_b - t_0} = \frac{\Delta x_b}{H + D} + k\,. \tag{3.3.14}$$

It follows that the criterion $g_a > g_b$ is

$$\frac{\Delta x_a}{H} > \frac{\Delta x_b}{H + D}\,. \tag{3.3.15}$$

So, in this specification, the decision maker cares about the linear payment rate under each option.

Preference reversals are observed changes in decisions as time passes, *i.e.* as the horizon gets shorter. We can test whether they are predicted in our model by varying H while holding other variables constant. In the present specification, indifference occurs at the horizon for which Eq. (3.3.15) becomes an equality, *i.e.* at $H = H^{\mathrm{PR}}$ where

$$H^{\mathrm{PR}} \equiv \frac{D\Delta x_a}{\Delta x_b - \Delta x_a}\,. \tag{3.3.16}$$

Since $t_0 = t_a - H$, this can be expressed as a critical decision time at which the decision maker is indifferent:

$$t_0^{\mathrm{PR}} \equiv t_a - H^{\mathrm{PR}} = \frac{\Delta x_b t_a - \Delta x_a t_b}{\Delta x_b - \Delta x_a}\,. \tag{3.3.17}$$

For $H < H^{\mathrm{PR}}$ ($t_0 > t_0^{\mathrm{PR}}$), the payment rate under option a exceeds that under option b and the earlier payment is preferred. The converse is true for $H > H^{\mathrm{PR}}$ ($t_0 < t_0^{\mathrm{PR}}$). Fig. 3.3 illustrates how the dependence of payment rate on horizon leads to preference reversal under additive dynamics with an adaptive time frame, *c.f.* (Kacelnik, 1997, Fig. 2).

Finally, we compute the discount function under this specification. When $g_a = g_b$, we have

$$\delta(D; H) = \frac{\Delta x_a}{\Delta x_b} = \frac{H}{H + D} = \frac{1}{1 + D/H}\,. \tag{3.3.18}$$

Thus we recover the widely-used descriptive model of discounting in which the discount function, δ, is a hyperbola of the delay, D. We note that δ also depends on the horizon, H. Indeed, the degree of discounting parameter – usually treated as a psychological parameter – appears in our model as $1/H$, the reciprocal of the horizon (Myerson and Green, 1995). As the horizon gets shorter, $1/H$ becomes larger, δ gets smaller, and the later payment becomes

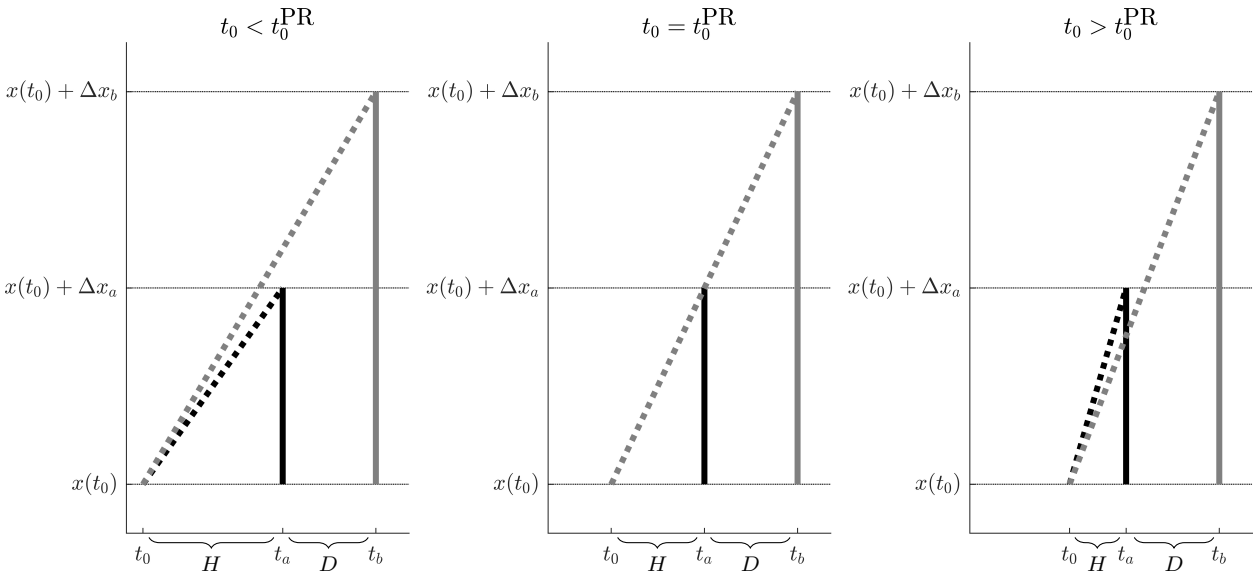

Figure 3.3: Preference reversal in case C. From left to right panel, t_0 increases and H decreases – *i.e.* both payments get closer – while all other parameters are held constant. Initially, option b is preferred, having the higher payment rate (slope of dashed line). At the critical time, $t_0 = t_0^{\mathrm{PR}}$, given by Eq. (3.3.17), both options imply the same payment rate. At later times, option a has the higher payment rate and is preferred.

less favorable. No knowledge of the decision maker's psychology is required in this setup, only the postulate that she prefers her wealth to grow faster rather than slower.

Finally, we note that k, the underlying growth rate of the decision maker's wealth, does not appear in the decision criterion. This is because wealth growth is not affected by exogenous cash flows under additive dynamics: the gain $k\Delta t$ over period Δt occurs regardless of other payments received. This contrasts with multiplicative dynamics, where early payments are subjected to the growth process through reinvestment.

3.3.5 Case D – Adaptive time frame with multiplicative dynamics

Specification: the period for computing the growth rate is that between the decision and the chosen payment, either $t_a - t_0 = H$ or $t_b - t_0 = H + D$; and wealth dynamics are multiplicative.

We follow the same steps as in the previous cases. Wealth evolves to:

$$x_a(t_a) = x(t_0)\, e^{r(t_a - t_0)} + \Delta x_a\,; \tag{3.3.19}$$

$$x_b(t_b) = x(t_0)\, e^{r(t_b - t_0)} + \Delta x_b\,. \tag{3.3.20}$$

The corresponding growth rates are:

$$g_a = \frac{\log x_a(t_a) - \log x(t_0)}{t_a - t_0} = \frac{1}{H}\log\left(1 + \frac{\Delta x_a}{x(t_0)\, e^{rH}}\right) + r\,; \tag{3.3.21}$$

$$g_b = \frac{\log x_b(t_b) - \log x(t_0)}{t_b - t_0} = \frac{1}{H+D}\log\left(1 + \frac{\Delta x_b}{x(t_0)\, e^{r(H+D)}}\right) + r\,. \tag{3.3.22}$$

Preference reversal

When the later payment is sufficiently large, $\Delta x_b > \Delta x_a e^{rD}$, preference reversal is predicted,[6] and a threshold horizon, H^{PR}, exists. For shorter horizons than H^{PR}, the earlier payment is preferred ($g_a > g_b$) and *vice versa*. The discount function and threshold horizon are not expressible in closed form for general parameter values. They become tractable in the limit of small payments, which we present below. If the later payment is too small, $\Delta x_b < \Delta x_a e^{rD}$, the earlier payment is always preferred in this specification.

Wealth effect

Our model predicts another type of preference reversal here, elicited by varying the initial wealth, $x(t_0)$, rather than the horizon. As $x(t_0) \to 0$, the earlier payment is preferred regardless of the size of the later payment. If the later payment is large enough, specifically if

$$\Delta x_b > \Delta x_a e^{rD}\left(\frac{H+D}{H}\right), \tag{3.3.23}$$

then it becomes preferable to the earlier payment as $x(t_0) \to \infty$. Thus, the decision maker switches from preferring earlier to later payments as her wealth increases. We label this the *wealth effect*. It is illustrated pictorially in Fig. 3.4, which shows the variation of growth rates, g_a and g_b, for each option as initial wealth, $x(t_0)$, increases from left to right. Other parameters are held constant.

Figure 3.5 shows the difference in growth rates, $g_a - g_b$, as a function of initial wealth, $x(t_0)$, for the same parameters as in Fig. 3.4. The earlier payment is preferred when this difference is positive, which happens for wealth below some threshold. For larger wealth, the growth rate difference is negative and the later payment is chosen.

We interpret this as follows. Assuming multiplicative dynamics and an adaptive time frame, it is growth-optimal for people of lower wealth to choose an earlier, smaller payment; and growth-optimal for wealthier individuals to hold out for the later, larger payment. This

[6] This can be shown by comparing the $H \to 0$ and $H \to \infty$ limits of g_a and g_b in Eq. (3.3.21) and Eq. (3.3.22).

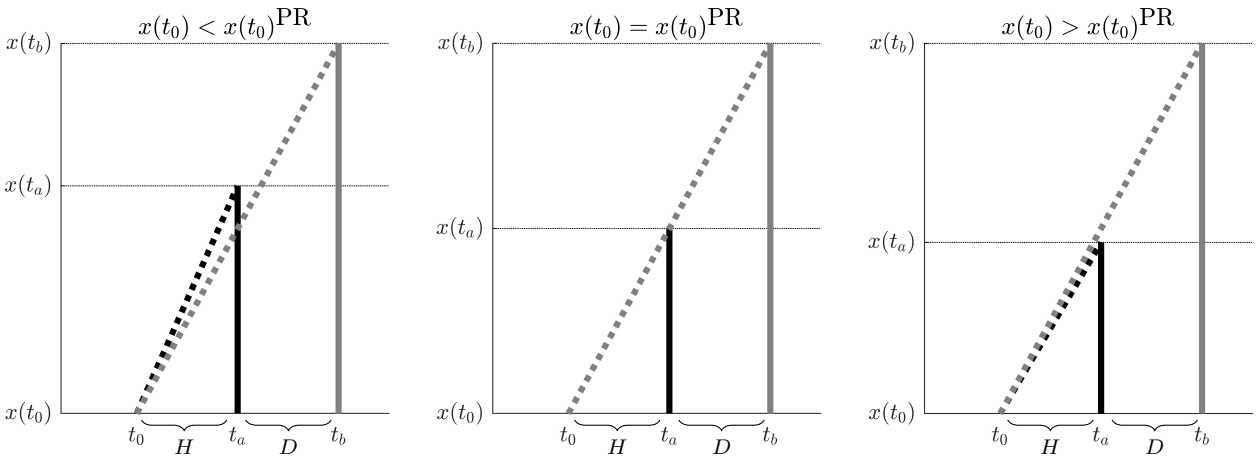

Figure 3.4: Wealth effect in case D, with logarithmic vertical scales. Initial wealth $x\,(t_0)$ increases from left panel to right panel (\$500, \$2277, \$5500) with all other parameters held fixed (t_0 = today, $t_a = 1$ year from today, $t_b = 2$ years from today, $\Delta x_a =$ \$1000, $\Delta x_b =$ \$2500, $r = 0.03$ per annum). At small wealth, option a is preferred, having the higher growth rate according to Eq. (3.3.21) and Eq. (3.3.22). At a larger wealth, $x\,(t_0)^{PR} \approx$ \$2277, both options imply equal growth, with preference reversal occurring as wealth increases further.

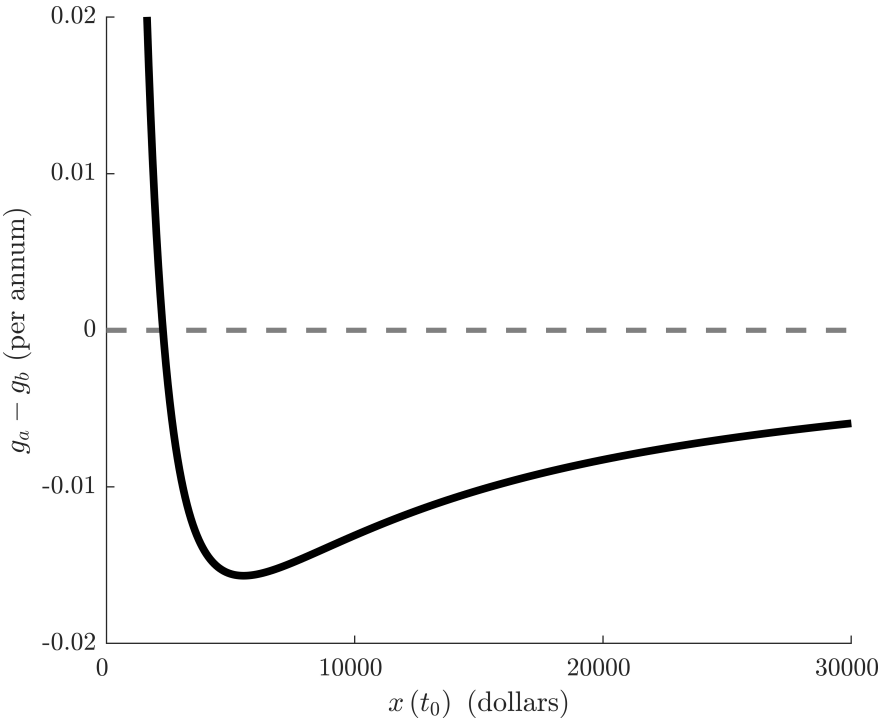

Figure 3.5: The difference in growth rates, $g_a - g_b$, as a function of initial wealth, $x\,(t_0)$, in case D. For small initial wealth the earlier, smaller payment is preferred, whereas for large initial wealth the later, larger payment is preferred. Parameters as used in Fig. 3.4.

is consistent with the findings of Epper et al. (2018), that "individuals with relatively low time discounting are consistently positioned higher in the wealth distribution." It is likely consistent with (Green et al., 1996), in which people with higher incomes were observed to discount less steeply.

We exemplify the wealth effect by presenting a calculation using the same parameters as in Fig. 3.4. Suppose a decision maker faces a choice between receiving \$1000 after one year (option a) or \$2500 after two years (option b), and that she has access to a risk-free interest rate of 0.03 per annum. If she has \$500 initially, she evaluates the growth rate corresponding to option a as

$$g_a = \frac{1}{1}\log\left(1+\frac{1000}{500e^{0.03\times 1}}\right)+0.03\approx 1.1 \text{ per annum}\,, \tag{3.3.24}$$

and to option b as

$$g_b = \frac{1}{2}\log\left(1+\frac{2500}{500e^{0.03\times 2}}\right)+0.03\approx 0.9 \text{ per annum}\,. \tag{3.3.25}$$

Thus, the decision maker would prefer the earlier, smaller payment, as $1.1 > 0.9$. If we assume that the decision maker had initially \$5500, *i.e.*, 11 times more than in the previous setting, a similar calculation yields $g_a \approx 0.19$ and $g_b \approx 0.21$, so the later, larger payment is preferred.

Discounting in the small payment limit

In many applications of discounting, it is plausible to assume that the payments are small relative to wealth:

$$\Delta x_a \ll x\left(t_0\right)e^{rH}\,; \tag{3.3.26}$$

$$\Delta x_b \ll x\left(t_0\right)e^{r(H+D)}\,. \tag{3.3.27}$$

We can express the threshold horizon and the discount function in closed form in this limit. Setting $g_a = g_b$ and using the first-order approximation $\log(1+\epsilon)\approx\epsilon$ for $\epsilon \ll 1$, we get

$$H^{\mathrm{PR}} \equiv \frac{D\Delta x_a e^{rD}}{\Delta x_b - \Delta x_a e^{rD}}\,, \tag{3.3.28}$$

and

$$\delta\left(D;H;r\right) = \left.\frac{\Delta x_a}{\Delta x_b}\right|_{g_a=g_b} \approx \frac{He^{rH}}{\left(H+D\right)e^{r(H+D)}} = \frac{e^{-rD}}{1+D/H}\,, \tag{3.3.29}$$

which is a product of hyperbolic and exponential discount functions. This hybrid case has interesting behavior in the long- and short-delay limits. As shown in Fig. 3.6, discounting is close to hyperbolic for short delays and to exponential for long delays. Thus, for the same dynamic and the same time frame, and assuming small payments relative to initial wealth, our growth definition predicts both hyperbolic and exponential discounting, depending on the delay.

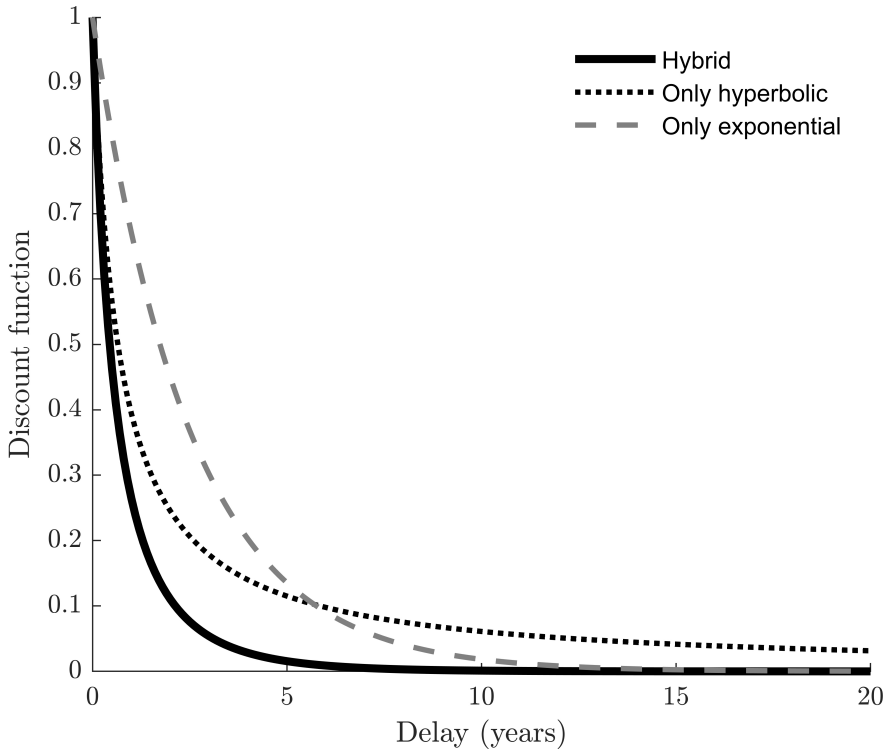

Figure 3.6: The discount function in the small payment limit in case D. The solid black curve is the hybrid $\delta = e^{-rD}/(1 + D/H)$, for $r = 0.4$ per annum and $H = 0.65$ years. This is close to the hyperbolic discount function $1/(1 + D/H)$ (black dotted) for short delays and to the exponential discount function e^{-rD} (grey dashed) for long delays.

3.4 Discussion

This paper explores temporal discounting under the postulate that decision makers maximize the growth rate of their wealth. We consider a basic temporal choice problem between two known, certain, and different payments at known, certain, and different future times. To compute growth rates, the problem must be further specified. We add information about wealth dynamics, treating additive and multiplicative cases, and the time frame of the decision, meaning the period over which growth is evaluated.

Preference reversal is an observed behavior in which decision makers switch from preferring later to earlier payments as time passes. It is incompatible with the classical normative model of exponential discounting. Our model generates four different forms of discounting, depending on the decision maker's circumstances – no discounting, exponential, hyperbolic, and a hybrid of exponential and hyperbolic. The hyperbolic and hybrid forms predict preference reversal without, as is commonly needed, assumptions of behavioral bias or payment risk.

The hybrid case – corresponding to multiplicative dynamics and an adaptive time frame – suggests another type of preference reversal, called the wealth effect. Here a decision maker switches from an earlier to a later payment as her wealth increases. In other words, richer people discount less steeply than poorer people, in line with empirical findings (Green et al., 1996; Epper et al., 2018).

Our main contribution is the prediction of non-exponential discounting and preference reversal in a model that does not violate standard axioms of choice (von Neumann and Morgenstern, 1944). Changes in the discount function arise only from changes in wealth dynamics and time frame. This marks a shift from psychological to circumstantial explanations of discounting. Our model assumes no dynamic inconsistency, in that the decision maker prefers at all times the option with the highest growth rate. If corroborated empirically, it would be both a normative and a descriptive model. Experimental tests are feasible because the model works directly with money payments rather than utility-of-consumption flows (Cohen et al., 2019).

The temporal choice problem we study is riskless. A planned extension of this work is to explore the consequences for our model of payment uncertainty. Uncertainty decreases the long-time growth rate associated with a payment (Peters and Gell-Mann, 2016). This would make a risky payment less desirable in our model, without reference to the risk preferences of the decision maker.

The dynamics discussed here do not cover the entire range of wealth dynamics. Although multiplicative and additive wealth dynamics are common and intuitive, other wealth dynamics are possible, which would lead to other forms of discounting in our model. Our decision criterion can be adapted to general dynamics using the growth rates described in (Peters and Adamou, 2018).

We end with a caveat. The temporal choice problem involves two *future* payments. In the small horizon limit, the growth rate corresponding to the earlier payment in the adaptive time frame diverges. This indicates a loss of model realism. We link this to the breakdown

of an implicit assumption: that the growth rates we compute are sustained over sufficiently long periods to be meaningful to the decision maker. They are, in effect, the growth rates of wealth achieved under repetition of the choice. We share the view of Kacelnik (1997, p. 60) that "the discounting process used for the one-off events seems to obey a law that evolved as an adaptation to cope with repetitive events." As the horizon shrinks, this imagined repetition occurs at a frequency so high that the choice problem no longer resembles a real situation.

.1 The Transitivity of Growth Rate Maximization

In this appendix we show that the maximization of growth, the growth definition in our model, satisfies transitivity for all four cases described in the paper. To prove transitivity we assume three vectors, $a \equiv (t_a, \Delta x_a)$, $b \equiv (t_b, \Delta x_b)$ and $c \equiv (t_c, \Delta x_c)$, where $t_a < t_b < t_c$. We also assume a decision time $t_0 < t_a$ and an initial wealth $x(t_0)$. The vectors $\{t_a, \Delta x_a\}$, $\{t_b, \Delta x_b\}$ and $\{t_c, \Delta x_c\}$ are thus RIPPs.

In each of the four cases we will show that if $a \prec b$ under the RIPP and $b \prec c$, then $a \prec c$. We will also show that if $a \sim b$ and $b \sim c$, then $a \sim c$.

Case A

In case A (see Sec. 3.3.2), we show that growth rate maximization is achieved by choosing the larger payment. Therefore, $a \prec b$ iff $\Delta x_a < \Delta x_b$ and $b \prec c$ iff $\Delta x_b < \Delta x_c$. It follows that $a \prec c$ because $\Delta x_a < \Delta x_c$. If $a \sim b$ and $b \sim c$ then $\Delta x_a = \Delta x_b$ and $\Delta x_b = \Delta x_c$, so $\Delta x_a = \Delta x_c$ and $a \sim c$.

Case B

In case B (see Sec. 3.3.3), we show that growth rate maximization is achieved by comparing the earlier payment to the later payment discounted by an exponential function, so

$$a \prec b \iff \Delta x_a < \Delta x_b e^{-r(t_b - t_a)}\,; \tag{.1.1}$$

$$b \prec c \iff \Delta x_b < \Delta x_c e^{-r(t_c - t_b)}\,. \tag{.1.2}$$

It follows that $\Delta x_b e^{-r(t_b - t_a)} < \Delta x_c e^{-r(t_c - t_b)} e^{-r(t_b - t_a)} = \Delta x_c e^{-r(t_c - t_a)}$, so

$$\Delta x_a < \Delta x_c e^{-r(t_c - t_a)} \Longrightarrow a \prec c\,. \tag{.1.3}$$

Similarly,

$$a \sim b \iff \Delta x_a = \Delta x_b e^{-r(t_b - t_a)}\,; \tag{.1.4}$$

$$b \sim c \iff \Delta x_b = \Delta x_c e^{-r(t_c - t_b)}\,. \tag{.1.5}$$

It follows that $\Delta x_b e^{-r(t_b - t_a)} = \Delta x_c e^{-r(t_c - t_a)}$, so

$$\Delta x_a = \Delta x_c e^{-r(t_c - t_a)} \Longrightarrow a \sim c\,. \tag{.1.6}$$

Case C

In case C (see Sec. 3.3.4) only the linear payment rate of each option matters to the decision maker, so

$$a \prec b \iff \frac{\Delta x_a}{t_a - t_0} < \frac{\Delta x_b}{t_b - t_0}\,; \tag{.1.7}$$

$$b \prec c \iff \frac{\Delta x_b}{t_b - t_0} < \frac{\Delta x_c}{t_c - t_0}\,. \tag{.1.8}$$

It follows that $\frac{\Delta x_a}{t_a - t_0} < \frac{\Delta x_c}{t_c - t_0}$, and $a \prec c$. Similarly,

$$a = b \iff \frac{\Delta x_a}{t_a - t_0} = \frac{\Delta x_b}{t_b - t_0}\,; \tag{.1.9}$$

$$b = c \iff \frac{\Delta x_b}{t_b - t_0} = \frac{\Delta x_c}{t_c - t_0}\,, \tag{.1.10}$$

so $\frac{\Delta x_a}{t_a - t_0} = \frac{\Delta x_c}{t_c - t_0}$, and $a \sim c$.

Case D

Like in case C, the time frame in case D (see Sec. 3.3.5) is adaptive. For this reason the growth rate associated with each payment depends only on the payment time and the decision time. In other words, under both comparisons between a and b, and the comparison between a and c, the growth rate associated with payment a, g_a, is the same. Similarly, g_b is the same in both the comparison between a and b, and the comparison between b and c. And g_c is the same in both the comparison between c b, and the comparison between c and a.

It follows that

$$a \prec b \iff g_a < g_b\,; \tag{.1.11}$$

$$b \prec c \iff g_b < g_c\,, \tag{.1.12}$$

so $g_a < g_c$, and $a \prec c$. Similarly,

$$a \sim b \iff g_a = g_b\,; \tag{.1.13}$$

$$b \sim c \iff g_b = g_c\,, \tag{.1.14}$$

so $g_a = g_c$, and $a \sim c$.

□

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RÉSUMÉ

Nous considérons des situations dans lesquelles des agents ont le choix entre plusieurs projets. Nous montrons comment les hypothèses sur la structure de marché influencent le type de projet qui est retenu par les agents. Cette thèse se constitue de trois parties: 1) nous déduisons des conditions pour lesquelles des firmes choisissent de laisser les agents piraté leur biens (non-rivaux), 2) nous analysons l'optimalité de la décision de fusionner deux firmes lorsque l'une d'entre elles détient le droit de propriété sur un projet innovant et peu risqué, enfin 3) nous montrons comment les caractéristiques d'un paiement (montant, fréquence) ainsi que l'environnement d'un agent (en termes de richesse) influencent les propriétés du taux d'escompte temporel de ce dernier.

MOTS CLÉS

théorie de la décision, escompte, ergodicité, organisation industrielle, contrats incomplets , innovation, monopole, biens de réseau

ABSTRACT

We consider situations where agents can choose between multiple projects. We show how specific market structure assumptions influence which choices agents pursue. The thesis has three parts 1) We deduce conditions under which firms will allow agents to pirate their non-rival products. 2) Analyze the decision for firms to merge when other firms can choose between projects of varying variances. 3) We show how the characteristics of a payment (amount, frequency) as well as the environment of agents (wealth, dynamics), influence the discount rates of agents.

KEYWORDS

decision theory, temporal discounting, ergodicity economics, industrial organization, incomplete contracts, innovation, monopoly, network goods