# Optimization of Retailers' Strategies in Price- and Carbon Emission- Sensitive Market 

Erfan Asgari

## To cite this version:

Erfan Asgari. Optimization of Retailers' Strategies in Price- and Carbon Emission- Sensitive Market. Modeling and Simulation. Université Grenoble Alpes [2020-..], 2021. English. NNT: 2021GRALI001 . tel-03228544

HAL Id: tel-03228544
https://theses.hal.science/tel-03228544
Submitted on 18 May 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## THÈSE

Pour obtenir le grade de

## DOCTEUR DE L'UNIVERSITÉ GRENOBLE ALPES

Spécialité : GI: Génie Industriel: conception et production
Arrêté ministériel : 25 mai 2016

Présentée par

## Erfan ASGARI

Thèse dirigée par Yannick FREIN, Professeur à Grenoble INP
et codirigée par Ramzi HAMMAMI,
Professeur à Rennes School of Business
préparée au sein du laboratoire G-SCOP
et de l'école doctorale I-MEP2
Optimisation des Stratégies des Détaillants sur Un Marché Sensible Au Prix et Aux Émissions de Carbone

Optimization of Retailers' Strategies in Price- and Carbon Emission- Sensitive Market

Thèse soutenue publiquement le 14/01/2021, devant le jury composé de :
M., Yacine REKIK

Professeur à EMLYON Business School, Rapporteur
Mme., Feng CHU
Professeur à Université d'Évry, Rapporteur
M., Ali SIADAT

Professeur à École Nationale Supérieure d'Arts et Métiers de Metz, Président
Mme., Maria DI MASCOLO
Directrice de Recherche CNRS à Grenoble INP, Examinatrice
Mme., Natacha GONDRAN
Professeur à IMT, Examinatrice
M., Yannick FREIN

Professeur à Grenoble INP, Directeur de thèse
M., Ramzi HAMMAMI

Professeur à Rennes School of Business, Co-Encadrant de thèse

## Acknowledgments

I would like to thank to my PhD advisors, Professor Yannick Frein and Dr. Ramzi Hammami, for supporting me during these past three years. Yannick is someone you will instantly love and never forget once you meet him. He is a warm and friendly advisor and one of the smartest people I know. I hope that I could be as lively, enthusiastic, and energetic as Yannick and to someday be able to command an audience as well as he can. Ramzi has been supportive and has given me the freedom to pursue various projects without objection. He has also provided insightful discussions about the managerial aspects of research. He is my primary resource for getting my questions answered and was instrumental in helping me crank out this thesis. I also have to thank the members of my PhD committee, Professors Ali Siadat, Maria Di Mascolo, Yacine Rekik and Feng Chu for their helpful career advice and suggestions in general.

A genuine thank you to our founder, Agence Nationale de la Recherche, support our work through project CONCLuDE.

I also thank my friends (too many to list here but you know who you are!) for providing support and friendship that I needed. I also thank my fellow labmates in G-SCOP for all the fun we have had in past three years.

I especially thank my mom, dad, brother, and sister. My hard-working parents have sacrificed their lives for my brother, sister and myself and provided unconditional love and care. I love them so much, and I would not have made it this far without them. My brother and sister have been my best friends all my life and I love them dearly and thank them for all their support. I know I always have my family to count on when times are rough.

> I dedicate this thesis to my family
> for their constant support and unconditional love
> I love you all dearly.

## Contents

List of Figures ..... vii
List of Figures ..... viii
List of Tables ..... ix
List of Tables ..... ix
Acronyms and Abbreviation ..... $\mathbf{x}$
Résumé en Français ..... 1
1 Introduction ..... 21
2 Literature review ..... 25
2.1 Customers' Environmental Awareness ..... 26
2.2 Product Differentiation In Supply Chain Models ..... 29
2.2.1 Optimal pricing strategies of substitutable products ..... 29
2.2.2 Optimal pricing and greenness strategies of substitutable products ..... 31
2.3 Competition Environment ..... 33
3 Benchmark Model ..... 39
3.1 Problem description ..... 39
3.2 Mathematical model and analytical solution ..... 43
3.3 Conclusion ..... 48
4 Product differentiation in retailing under price- and greenness-sensitive de- mand ..... 51
4.1 Problem description ..... 51
4.2 Mathematical models and analytical solutions ..... 55
4.2.1 Prices and stocks optimization ..... 55
4.2.2 New product optimization $\left(M_{2}\right)$ ..... 60
4.2.3 General products differentiation model $\left(M_{3}\right)$ ..... 63
4.3 Analysis and managerial insights ..... 67
4.3.1 Market Structures ..... 68
4.3.2 Insights ..... 69
4.4 Conclusion ..... 75
5 Optimization of Retailers' Strategies Under Dynamic Competition Environ- ment ..... 77
5.1 Problem description ..... 77
5.2 Mathematical models and analytical solutions ..... 80
5.2.1 Competition with full reaction $\left(M_{4}\right)$ ..... 80
5.2.2 Competition without reaction $\left(M_{5}\right)$. ..... 88
5.2.3 Competition with partial reaction $\left(M_{6}\right)$ ..... 90
5.3 Analysis and managerial insights ..... 94
5.3.1 Impact of competition on products' environmental performance ..... 94
5.3.2 Impact of transportation carbon emissions disparity ..... 99
5.3.3 Impact of refilling time on order size decisions ..... 101
5.3.4 Impact of competition on retailers' optimal profits ..... 102
5.4 Conclusion ..... 104
6 Complex Demands Functions ..... 105
$6.1 n$th root demand $\left(M_{7}\right)$ ..... 107
6.2 Capacitated $n$th root demand $\left(M_{8}\right)$ ..... 111
6.3 Capacitated linear demand ( $M_{9}$ ) ..... 113
6.4 Analysis and managerial insights ..... 114
6.5 Conclusion ..... 121
7 General conclusion ..... 123
7.1 Conclusion ..... 123
7.2 Future works and perspectives ..... 125
Bibliography ..... 127
A Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching ..... 135
A. 1 Introduction ..... 135
A. 2 Literature Review ..... 138
A. 3 Modeling framework ..... 140
A. 4 Delivery time quotation model with fixed stock and price ..... 144
A.4.1 Model solving ..... 145
A.4.2 Model analysis and insights ..... 146
A. 5 Delivery time quotation and stock level determination model with fixed price ..... 149
A.5.1 Model $\left(M_{l, S}\right)$ without stockout-based substitution $(\phi=0)$ : Ana- lytical resolution and insights ..... 150
A.5.2 Model ( $M_{l, S}$ ) with stockout-based substitution $(\phi>0)$ : Analysis and numerical insights ..... 152
A. 6 General model and effect of S-b-S ..... 156
A. 7 Conclusion and practical implications of our findings ..... 159
B Demand function's positivity in Lemma 3.2 ..... 161
C Objective function concavity in Proposition 4.6 ..... 163

D Demand function's Positivity in Lemma 5.2 165

## List of Figures

1 Cadre de la thèse ..... 4
2 Benchmark SC ..... 6
3 Modèle de référence du réseau de files d'attente ..... 6
4 Politique de réapprovisionnement des stocks utilisée pour estimer la prob- ..... 7
5 Différenciation des produits de la chaîne d'approvisionnement ..... 10
6 Le cadre général ..... 12
7 Competitive supply chains structure ..... 14
8 Supply chain ..... 17
$9 \quad$ Demands functions for a given price ..... 18
1.1 Framework ..... 24
3.1 Benchmark SC ..... 40
3.2 Benchmark queuing network ..... 40
3.3 Inventory refilling policy used to approximate the stock-out probability ..... 41
3.4 Optimal solutions' behavior in function of $\beta_{e}$ ..... 48
4.1 Collaboration supply chain ..... 52
4.2 General framework ..... 53
4.3 Inventory refilling policy used to approximate the stock-out probability ..... 54
4.4 Impact of $e_{1}$ on optimal order sizes ..... 72
4.5 Impact of market potential on optimal carbon emission level ..... 75
5.1 Competitive supply chains structure ..... 79
5.2 Illustration of the Nash equilibrium ..... 86
5.3 Optimal carbon emissions in function of market power ..... 98
5.4 Optimal prices in function of disparity in transportation emissions ..... 101
5.5 Effect of $\mu_{1}$ on retailers' optimal order sizes ..... 102
5.6 Existing retailer's profit under the different competition scenarios ..... 103
5.7 New retailer's profit under the different competition scenarios ..... 103
6.1 Supply chain ..... 105
6.2 Demands functions for a given price ..... 106
6.3 Possible demands functions ..... 109
6.4 Capacitated nth root demand for a given price ..... 112
6.5 Capacitated linear demand for a given price ..... 114
A. 1 Illustration of the studied hybrid system with a mix of DFS and drop- shipping ..... 136
A. 2 A hybrid delivery system with stock-out based substitution ..... 142
A. 3 Time differentiation strategy when the characteristics of the express product are fixed . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 146
A. 4 Profit as a function of DT for increasing $\pi_{2}(l)$. . . . . . . . . . . . . . . 155
A. 5 Impact of the stock-out based substitution on the stock level . . . . . . . 159

## List of Tables

2.1 Products differentiation comparison table ..... 32
2.2 Competition comparison table ..... 37
4.1 Parameters ..... 67
4.2 Gap between profits ..... 68
5.1 Impact of competition on the environmental performance ..... 96
5.2 Impact of market share on the environmental performance ..... 99
6.1 Parameters ..... 115
6.2 Gap between profits ..... 115
6.3 Gap between optimal carbon emissions improvements ..... 116
6.4 Gap between optimal prices ..... 116
6.5 Gap between optimal order sizes ..... 117
A. 1 Real examples of products delivered with a mix of DFS and drop-shipping ..... 136
A. 2 Optimal retailer's strategy under the setting of model $\left(M_{l, S}\right)$ with $\phi=0$. ..... 151
A. 3 Test cases ..... 154
A. 4 Comparison between the original model and the modified model ..... 157
A. 5 Effect of stock-out based substitution for the neutral market ..... 157
A. 6 Effect of stock-out based substitution for the SPD market ..... 157
A. 7 Effect of stock-out based substitution for the STD market ..... 158
A. 8 Effect of stock-out based substitution for the SPD market ..... 158

## Acronyms and Abbreviation

| Abbreviations | Explanation |
| :--- | :--- |
| ANR | Agence Nationale de la Recherche |
| CEA | Customers Environmental Awareness |
| GC | Green Credit |
| GDS | Greenness-Driven Switchovers |
| MILP | Mixed Integer Linear Programming |
| MS | Manufacturer Subsidy |
| MTO | Make-To-Order |
| MTS | Make-To-Stock |
| PDS | Price-Driven Switchovers |
| SC | Supply Chain |
| SS | Sales Subsidy |

# Résumé en Français 

## Chapitre 1: Introduction

La performance environnementale d'un produit mis sur le marché, souvent évaluée en fonction de la quantité d'émissions de carbone libérées pendant les phases de production et de transport, devient un critère d'achat important pour de nombreux clients (Palacios-Argüello et al., 2020; Hammami et al., 2018; Borin et al., 2013). Le rapport (AACE, 2015) sur l'analyse des émissions de carbone du produit révèle qu'environ $70 \%$ des émissions totales de carbone produites au cours du cycle de vie du produit sont liées au processus de production. Selon une enquête menée par Accenture en avril 2019 auprès de 6000 consommateurs dans 11 pays d'Amérique du Nord, d'Europe et d'Asie, $72 \%$ des consommateurs ont déclaré qu'ils achetaient actuellement plus de produits respectueux de l'environnement qu'il y a cinq ans, et $81 \%$ qu'ils prévoyaient d'en acheter davantage encore au cours des cinq prochaines années. En outre, plus de la moitié des consommateurs paieraient plus cher pour des produits respectueux de l'environnement (Accenture, 2019). Les enquêtes de Carbon Trust indiquent qu'environ $20 \%$ des clients préfèrent acheter des produits verts même s'ils sont plus chers que les produits ordinaires (Hong and Guo, 2019).

La plupart des études dans le domaine de la gestion de la chaîne d'approvisionnement partent du principe que la demande du client est connue (demande exogène). Cependant, nous savons que la demande est fortement influencée par les décisions internes (demande endogène). Le facteur le plus connu qui affecte le comportement d'achat du client est le prix. Mais il existe différents types de facteurs qui influent la demande d'un produit ou d'un service. Huang et al. (2013) ont fourni une étude intéressante sur les facteurs influençant la demande. Ils les ont répartis en six catégories : prix, rabais, délai, espace, qualité et publicité. Il est bien connu dans la littérature sur la logistique commerciale que l'un des éléments les plus importants du service à la clientèle, outre le prix, est le délai de livraison. Pour comprendre la logique, les idées et la méthodologie de résolution de modèles avec des demandes endogènes, nous avons commencé notre travail en considérant une demande endogène sensible au prix et au délai de livraison. Nous considérons un système hybride de vente de produits substituables, qui consiste en un système de fabrication à la commande (MTO) et un système de fabrication en stock (MTS) (voir l'annexe Appendix 1 pour plus de détails). Cependant, ce doctorat se concentre sur une demande endogène sensible au prix et à la qualité environnementale du produit, que l'on appellera greeness dans cette thèse.

La conscience environnementale modifie les décisions des consommateurs en matière d'achat de biens. Ils sont prêts à payer plus cher pour avoir des produits plus écologiques (Brécard, 2014). Par conséquent, la plus ou moins grande sensibilité des clients à l'environnement (CEA pour Customers' Environmental Awareness) va influencer directement la stratégie de l'entreprise à produire des produits plus ou moins verts et donc à investir dans des
technologies plus ou moins propres (Yalabik and Fairchild, 2011). Ce mouvement conduit aussi les fournisseurs à fournir des produits plus écologiques (Gu et al., 2015; Sheu and Chen, 2012; Ding et al., 2015; Cheng et al., 2017). Le consommateur est prêt à payer un prix plus élevé pour un produit vert, comme des voitures émettant moins de CO2 (Costa et al., 2019), mais le produit vert nécessite plus d'investissements dans la recherche et le développement ( $\mathrm{R} \& D$ ), ce qui générera des coûts plus élevés (Krass et al., 2013).

De nombreux détaillants ont bien saisi les préférences des clients pour des produits respectueux de l'environnement et leur acceptation à les payer plus chers et, par conséquent, ont adapté leurs stratégies d'achat pour offrir des alternatives plus écologiques. Ainsi, comme le soulignent Ramanathan et al. (2012), ces détaillants ont demandé à leurs fournisseurs de réduire les émissions de carbone dans les phases de production et de transport. Benjaafar et al. (2012) ont souligné que divers détaillants commencent à mettre des étiquettes d'empreinte carbone sur leurs produits. Deux grands détaillants, Tesco au Royaume-Uni et Casino en France, se sont déjà lancés dans des efforts d'étiquetage de ce type. De nombreux produits font l'objet d'un bilan carbone. Citons par exemple les smoothies aux fruits, les chaussures et la bière... Walmart, le plus grand détaillant au monde, a établi en octobre 2016 un plan de réduction des émissions et a invité ses fournisseurs à s'engager à réduire les émissions de gaz à effet de serre résultant de leurs activités et de leurs chaînes de valeur. L'objectif est de travailler avec les fournisseurs pour réduire les émissions de carbone de 1 gigatonne dans la production et l'utilisation des aliments et des produits à l'échelle mondiale entre 2015 et 2030. Bestseller, l'un des principaux détaillants de mode au Danemark, donne à ses clients la possibilité d'acheter une large gamme de produits respectueux de l'environnement. Le détaillant s'est engagé à améliorer en permanence l'empreinte écologique de ses produits, de ses activités et de sa chaîne d'approvisionnement. Ainsi, pour gérer la demande et augmenter la rentabilité, les détaillants vont se faire concurrence non seulement sur le prix mais aussi sur la performance environnementale (la greenness) de leurs produits (qui est ici évaluée en termes d'émissions de carbone).

Notre revue de la littérature, présentée dans le chapitre 2, a permis d'identifier de nouvelles perspectives pour le problème de l'optimisation des opérations d'un détaillant dans un contexte de demande sensible au prix et à la greeness. En particulier,

1. La plupart des études sont réalisées sur la base du système MTO (Make-To-Order). Elles ne sont donc pas adaptées au contexte de la vente au détail et de la gestion des stocks dans ce domaine. En effet, dans de nombreux cas, les détaillants passent commande à leurs fournisseurs et constituent un stock pour servir les clients immédiatement.
2. Dans un environnement compétitif, la plupart des articles ont considéré le jeu de Stackelberg dans lequel un joueur agit (prend des décisions) en premier, appelé leader, et le second, appelé suiveur, prend les décisions sur la base des décisions prises par le leader. Peu d'études ont considéré qu'il existe une compétition dynamique entre les joueurs qui peuvent prendre des décisions en même temps, et
donc des décisions mutuellement influencées.
3. Dans la littérature, les chercheurs considèrent la demande linéaire sans aucune contrainte, bien que nos collègues (Rennes School of Business et Ecole des Mines de St Etienne dans le projet ANR) aient constaté que la demande n'est pas linéaire et de plus ne n'augmentera pas au-delà d'un certain niveau de demande avec comme seul levier l'amélioration de la greeness.

Sur la base des observations ci-dessus, et après une revue de la littérature dans le chapitre 2, nous avons étudié les problèmes suivants, résumés Figure 1 .

- Nous développons tout d'abord dans le chapitre 3 un modèle de base : un seul détaillant offre un seul produit aux clients. La demande du produit est sensible au prix du produit et à son intensité carbone, dans un environnement stochastique pour la demande et la durée de livraison du fournisseur. Le détaillant conserve les produits en stock dans un entrepôt proche des clients pour les servir, et passe des commandes pour remplir le stock de produits auprès du fournisseur qui adopte une politique de fabrication MTS. Le stock est géré selon la politique standard $(\mathrm{q}, \mathrm{S})$ où q est le point de commande et S est la taille du lot de réapprovisionnement. Ce problème est considéré comme le modèle de référence pour les chapitres suivants.
- Différenciation des produits : un détaillant offre deux produits substituables à ses clients. L'objectif est de trouver les stratégies optimales, en termes de prix et de greeness, du détaillant, et ceci dans un environnement stochastique. La fonction de demande de chaque produit dépend non seulement de son prix et de son intensité carbone, mais aussi du prix et de l'intensité carbone de l'autre produit. Le problème est formulé dans un environnement stochastique. L'analyse est fournie dans le chapitre 4.
- Un jeu dynamique entre deux détaillants en concurrence est considéré dans un environnement sensible au prix et à la greeness, à nouveau dans un environnement stochastique, dans le chapitre 5 . Nous considérons la situation dans laquelle un détaillant agit sur le marché et où un nouveau venu arrive. Le nouveau venu offre un produit substituable. La part de marché potentielle n'est pas nécessairement égale pour les deux détaillants.
- Le modèle de référence est considéré avec différentes fonctions de demande (plus complexes) dans le chapitre 6. Nous obtenons les résultats avec les différentes fonctions de demande, puis les comparons pour analyser si la fonction de demande linéaire classique est satisfaisante ou non. Le chapitre 6 fournit plus de détails et de discussion.


Figure 1: Cadre de la thèse

## Chapitre 2: Revue de la Littérature

Notre étude est dans le champ des modèles analytiques de gestion des opérations dans une Supply Chain, avec des considérations environnementales et, en particulier, des études qui se concentrent sur la demande sensible au prix et à la greeness des produits. Nous avons distingué trois thèmes de la littérature sur la gestion d'une chaîne logistique verte. Dans un premier temps nous avons présenté les études qui considèrent la sensibilité des clients à la qualité environnementale (CEA pour Customers' Environmental Awareness), et donc le niveau des émissions comme variable de décision. Puis dans ce même contexte de marché dépendant du prix et de la qualité environnementale, nous avons présenté les études portant sur la différenciation des produits pour un acteur donné, et enfin les études concernant les stratégies dans un environnement concurrentiel entre acteurs.

## Chapitre 3: Modèle de Référence

Ce chapitre étudie le problème de la maximisation des profits d'un détaillant comme modèle initial et de référence pour le reste de la thèse. Nous comparerons en effet les résultats obtenus dans les autres chapitres aux résultats de ce chapitre. Ce premier cas nous permet également de présenter les principales idées utilisées pour résoudre analytiquement les problèmes futurs plus complexes. Nous considérons un détaillant qui vend
un produit, acheté à un fournisseur, sur un marché sensible aux prix et à la qualité environnementale du produit. Le détaillant décide du prix du produit, du niveau d'émission de carbone et de la taille du lot de commande des produits à son fournisseur afin de maximiser le profit total attendu. Les clients arrivent selon un processus de Poisson avec un taux d'arrivée moyen $\lambda$. La demande moyenne diminue linéairement en fonction du niveau des émissions de carbone et du prix. Les principales sources d'émission de carbone sont les activités de transport et de production. L'émission liée au transport par unité de produit, désignée par e, dépend de la distance parcourue par le produit depuis le site du fournisseur jusqu'à l'entrepôt du détaillant. Comme nous ne traitons pas de la sélection des fournisseurs, e n'est pas une variable de décision. Cependant, nous conservons e pour montrer l'effet des émissions de carbone dues au transport sur les décisions du détaillant. Sans perte de généralité, nous supposerons que les émissions liées au transport de l'entrepôt du détaillant vers les clients finaux peuvent être négligées, ce qui suppose implicitement que le détaillant est situé à proximité de la zone de demande. Pour les émissions liées à la production, nous considérons un contexte dans lequel le détaillant peut choisir le niveau d'émission de la production et demander au fournisseur de fabriquer le produit en conséquence. Ce contexte correspond aux exemples pratiques fournis en introduction où nous présentions des cas où de grands détaillants, tels que Walmart et Bestseller, demandent à leurs fournisseurs de fabriquer des produits plus écologiques. Nous appelons $x_{0}$ la quantité d'émissions de carbone en production par unité de produit fabriqué selon un processus de fabrication standard, et x la quantité d'émissions de production par unité de produit fabriqué selon le processus de fabrication effectivement mis en place. Offrir un produit plus vert (avec des émissions en production plus faibles) conduit à un coût de production et donc d'achat plus élevé pour le détaillant. Le coût d'achat unitaire d'un produit est donné par $c+b\left(x_{0}-x\right)^{2}$, où $c$ est le coût unitaire du produit standard, et $b$ est le facteur de coût de la réduction des émissions de production. Nous considérons une fonction de coût quadratique comme il est d'usage dans la littérature correspondante (par exemple, Liu et al., 2012, Ghosh and Shah, 2015).

## Description du problème

Nous considérons donc un détaillant qui commande un produit à son fournisseur et le vend à des clients qui tiennent compte du prix et de l'intensité carbone du produit dans leur décision d'achat. La Figure 2 montre la chaîne d'approvisionnement considérée.

L'arrivée du client suit une distribution de Poisson de taux ${ }^{\lambda}$. Ce taux est une fonction linéaire du prix et des émissions. A représente le potentiel du marché. Nous appelons respectivement $\alpha_{p}$ et $\beta_{e}$ la sensibilité du marché au prix et à l'intensité carbone. Le taux de demande est donné comme suit :

$$
\begin{equation*}
\lambda=A-\alpha_{p} p-\beta_{e}(x+e) \tag{1}
\end{equation*}
$$

Cette fonction de demande traduit le fait que le détaillant peut attirer $\alpha_{p}$ clients supplémentaires avec une baisse de prix d'une unité et $\beta_{e}$ clients supplémentaires avec une baisse de l'intensité carbone d'une unité.


Figure 2: Benchmark SC
$S$ est la taille de l'ordre de réapprovisionnement du détaillant. Le taux de service pour le réapprovisionnement du stock est réparti de manière exponentielle, avec un taux $\mu$. Le délai de réapprovisionnement ne dépend pas de la taille de la commande puisque le produit est supposé être toujours disponible sur le site du fournisseur, ce qui est une hypothèse courante (Zhu, 2015). Ainsi, le délai de réapprovisionnement correspond essentiellement aux activités de préparation et de transport. La Figure 3 montre le modèle réseau de files d'attente.


Figure 3: Modèle de référence du réseau de files d'attente

Le détaillant passe une commande pour remplir le stock auprès d'un fournisseur qui adopte une politique de fabrication MTS. Le stock est géré selon la politique standard $(q, S)$ où $q$ est le point de commande et $S$ est la taille du lot de réapprovisionnement. Pour simplifier l'analyse, nous supposons que le détaillant passe une commande de réapprovisionnement lorsqu'il n'y a plus de produit dans le stock principal $(q=0)$. Pendant la période de réapprovisionnement, la demande est alors satisfaite par un stock de sécurité. Nous supposons que le stock de sécurité est suffisamment important pour satisfaire toutes les demandes dans la grande majorité des cas et nous ignorons les rares cas où la demande ne peut être satisfaite. Le dimensionnement et la gestion du stock de sécurité n'entrent pas dans le cadre de cette étude. La politique de réapprovisionnement du stock utilisée pour calculer la probabilité de rupture de stock est illustrée dans la Figure 4.


Figure 4: Politique de réapprovisionnement des stocks utilisée pour estimer la probabilité de rupture de stock

Nous appelons $\psi$ la probabilité de rupture de stock principal du produit. Pour éviter des niveaux de stock irréalistes, nous imposons que la probabilité de satisfaire la demande à partir du stock principal, $(1-\psi)$, doit être supérieure à un niveau de service minimum prédéterminé. Nous appelons 1-r ce niveau de service minimum. Par conséquent, la contrainte de niveau de service pour le détaillant est donnée par $1-\psi \geq 1-r$, ce qui équivaut à $\psi \leq r$. Notez que $\psi$ représente la probabilité d'avoir zéro article dans le stock principal lorsqu'un client arrive et représente donc aussi la probabilité de servir un client à partir du stock de sécurité.

Étant donné que l'arrivée des clients et le temps de service de réapprovisionnement des stocks suivent des distributions exponentielles le nombre de pièces dans le stock principal est un processus markovien. Nous pouvons calculer la probabilité de rupture de stock, dans laquelle il n'y a aucun article dans l'entrepôt, en utilisant les propriétés du processus markovien continu. Après des calculs classiques on obtient :

Ainsi, la contrainte de niveau de service pour le détaillant est donnée par $\frac{\lambda}{\lambda+\mu S} \leq r$. Nous pouvons également calculer le niveau de stock moyen $\bar{S}$ et on obtient

Le niveau moyen du stock est complexe, mais en considérant l'hypothèse $1 \gg d f r a c \lambda \mu S$ (qui est une hypothèse logique puisqu'elle traduit simplement que le temps de réapprovisionnement moyen ( $\operatorname{frac} 1 \mu$ ) est très inférieur au temps moyen de consommation des $S$ pièces du stock ( $\frac{S}{\lambda}$ ), on peut utiliser l'approximation largement utilisée dans la littérature (par exemple, Cargal, 2003) consistant à considérer que la valeur moyenne du stock principal du détaillant est donnée par fracS2. Si $2 h$ indique le coût unitaire des stocks, le coût d'inventaire pour le détaillant sera donc de $h S$.

Les paramètres et les variables de décision sont présentés ci-après.

## Paramètres:

A : Potentiel de marché,
$c \quad:$ Coût fixe unitaire du produit,
$\mu \quad:$ Taux de remplissage moyen du produit,
$2 h \quad:$ Coût unitaire de détention des stocks pour le produit,
$x_{0} \quad:$ Émission de carbone par le processus de production standard,
$e \quad:$ L'émission de carbone par le transport du produit,
$b \quad:$ Facteur de coût de la réduction des émissions de carbone,
$r \quad$ : Stockage maximum garanti.

## Variables de décision indépendantes:

$x \quad:$ Émission de carbone par la production du produit,
$p \quad:$ Prix du détaillant
$S \quad:$ Taille de la commande.

## Variables de décision dépendantes:

$\lambda \quad:$ Taux de demande moyen pour le produit (donné par l'équation 1 ),
$\psi \quad:$ La probabilité d'une rupture de stock.

## Modèle mathématique et solution analytique

Dans ce modèle de référence, le détaillant est seul sur le marché. Le problème consiste à décider du prix $p$, du niveau d'émission de carbone $x$, et de la taille de la commande $S$ pour maximiser son profit. Un seul produit est proposé aux clients dans ce cas; rappelons que le taux de demande est donné par l'équation 1. Définissons $A^{\prime}=A-\beta_{e} e$, qui est une constante puisque $e$ est connu. Le modèle $\left(M_{0}\right)$ est donné ci-dessous.

$$
\begin{equation*}
\underset{x, p, S}{\operatorname{Maximize}} \pi=\left(p-\left(c-b\left(x_{0}-x\right)^{2}\right)\right) \lambda-h S \tag{2}
\end{equation*}
$$

Sous réserve des

$$
\begin{align*}
& \psi=\frac{\lambda}{\lambda+S \mu} \leq r  \tag{3}\\
& \lambda=A^{\prime}-\alpha_{p} p-\beta_{e} x  \tag{4}\\
& \lambda, p, S \geq 0,0 \leq x \leq x_{0} \tag{5}
\end{align*}
$$

L'objectif est de maximiser le profit total attendu donné dans l'équation 2. Ce bénéfice est égal au revenu (c'est-à-dire $p \boldsymbol{\lambda}$ ) - le coût d'approvisionnement (c'est-à-dire ( $c+b\left(x_{0}-\right.$ $\left.x)^{2}\right) \lambda$ ) - le coût d'inventaire (c'est-à-dire $h S$ ). La contrainte 3 correspond au taux de service par le stock principal souhaité. Elle garantit que la probabilité de rupture du stock principal ne dépasse pas un niveau prédéterminé de $r(1-r$ est le niveau de service minimum). Le taux de demande est donné dans l'équation 4. La contrainte 5 exprime que les variables de décision doivent être positives (et pour $x$ inférieur à $x_{0}$, niveau d'émission pour un processus de fabrication standard).

Le problème est formulé dans un environnement stochastique. Nous avons utilisé une approche analytique pour résoudre de façon exacte le problème et obtenir les expressions des solutions optimales. Le niveau optimal d'émission de carbone diminue linéairement en fonction de $\beta_{e}$ et augmente non linéairement en fonction de $\alpha_{p}$ et $b$. Le prix optimal est convexe en fonction de $\beta_{e}$; il diminue jusqu'à une valeur seuil, puis il augmente. La solution optimale est donné ci-après:
$x^{*}=\max \left\{0, x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right\}$,

$$
\begin{aligned}
& p^{*}= \begin{cases}\frac{A-\beta_{e} e+\alpha_{p}\left(c+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} & \text { for } x^{*}=0 \\
\frac{A-\beta_{e}\left(x_{0}+e\right)+\frac{3 \beta_{e}{ }^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} & \text { for } x^{*} \neq 0\end{cases} \\
& S^{*}=\left\{\begin{array}{ll}
\frac{(1-r)\left(A-\beta_{e} e-\alpha_{p}\left(c+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu}\right)\right)}{r \mu} & \text { for } x^{*}=0
\end{array},\right. \text { and } \\
& \frac{(1-r)\left(A-\beta_{e}\left(x_{0}+e\right)+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)}{r \mu} \\
& \pi^{*}= \begin{cases}\frac{\left(A-\beta_{e} e-\alpha_{p}\left(c+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu}\right)\right)^{2}}{4 \alpha_{p}} \quad \text { for } x^{*}=0\end{cases} \\
& \begin{array}{ll}
\frac{\left(A-\beta_{e}\left(x_{0}+e\right)+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)^{2}}{4 \alpha_{p}} & \text { for } x^{*} \neq 0
\end{array}
\end{aligned}
$$

## Chapitre 4: Différenciation des produits d'un détaillant dans le cadre d'une demande sensible au prix et à la qualité environnementale

Le chapitre précédent a abordé le problème de la maximisation des profits du détaillant, lorsque ce dernier offre un seul produit aux clients. Les résultats ont montré l'impact des facteurs de sensibilité aux émissions de carbone et au prix sur le profit optimal, et les variables de décision optimales (prix, niveau des émissions de carbone, taille des commandes). Dans ce chapitre, nous allons étudier les meilleures stratégies du détaillant lorsqu'il propose un deuxième produit substituable au produit initial, sur à nouveau un marché sensible au prix et à la qualité environnementale. Le nouveau produit est identique au premier produit en termes de performance, de fonction et d'utilisation, c'est-à-dire qu'il est substituable. Toutefois, ils peuvent différer en termes de prix et d'intensité carbone. Définissons le premier produit comme le produit "existant" et le second comme le "nouveau" produit. Les fournisseurs envoient les produits à l'entrepôt du détaillant, qui est proche des clients. Le détaillant conserve les produits pour servir les clients dès leur arrivée. La demande pour chaque produit dépend non seulement de son prix et de l'intensité de ses émissions de carbone, mais aussi du prix et de l'intensité des émissions de carbone de l'autre produit. Nous considérons un problème de maximisation du profit du détaillant, formulé dans un environnement stochastique. Les principales questions importantes de ce
chapitre sont les suivantes:

- Le détaillant a-t-il intérêt à proposer un nouveau produit substituable?
- Comment la différenciation des produits affecte-t-elle les stratégies optimales du détaillant?
- Les différentes structures de marché affectent-elles les stratégies optimales du détaillant?

La description du problème est présentée ci-après. Nous discutons des hypothèses du modèle et développons le cadre général. Nous considérons successivement différents ensembles de variables décisions et donc de modèles d'optimisation et les résolvons par des approches analytiques. Ensuite, nous en avons tirer des enseignements qualitatifs et quantitatifs.

## Description du problème

Comme nous l'avons indiqué ci-dessus, nous allons étudier le problème d'un détaillant proposant deux produits substituables. La Figure 5 montre le détaillant (ellipse rouge) et les fournisseurs des produits. Le détaillant passe des commandes auprès de deux fournisseurs différents. Chaque fournisseur prépare un type de produit.


Figure 5: Différenciation des produits de la chaîne d'approvisionnement

Nous considérons une demande linéaire en fonction du prix et de l'intensité en carbone avec substitution. Les clients arrivent selon un processus de Poisson avec un taux d'arrivée moyen $\lambda_{i}$ pour le produit $i(i=1,2$; produit existant et nouveau, respectivement). La demande moyenne de chaque produit diminue avec son niveau des émissions de carbone et son prix, mais augmente avec le niveau des émissions de carbone et le prix de l'autre produit. Le marché potentiel est indiqué par 2A (nous supposons qu'il est divisé en parts égales pour chaque produit). Les paramètres $\alpha_{p}$ et $\beta_{e}$ représentent la sensibilité
des clients au prix et à l'intensité des émissions de carbone (CEA), respectivement. $\alpha_{s}$ et $\beta_{s}$ représentent la sensibilité à respectivement la différence des prix et la différence entre les niveaux des émissions de carbone. Comme pour Liu et al. (2012) and Xiong et al. (2014), les fonctions de demande sont données comme suit.

$$
\begin{aligned}
& \lambda_{1}=A-\alpha_{p} p_{1}+\alpha_{s}\left(p_{2}-p_{1}\right)-\beta_{e}\left(x_{1}+e_{1}\right)+\beta_{s}\left(x_{2}+e_{2}-x_{1}-e_{1}\right) \\
& \lambda_{2}=A-\alpha_{p} p_{2}+\alpha_{s}\left(p_{1}-p_{2}\right)-\beta_{e}\left(x_{2}+e_{2}\right)+\beta_{s}\left(x_{1}+e_{1}-x_{2}-e_{2}\right)
\end{aligned}
$$

moyenne totale est sensible aux prix et aux intensités d'émissions de carbone mais pas aux différences de prix et d'émissions de carbone entre produits. Pour un produit i, la diminution du prix d'une unité attire $\alpha_{p}+\alpha_{s}$ plus de clients et la diminution d'une unité d'intensité de carbone attire $\beta_{e}+\beta_{s}$ plus de clients. Il est à noter que seule une partie de ces clients représente des nouvelles demandes du marché (pour le produit i, plus précisément $\alpha_{p}$ nouveaux clients pour une baisse unitaire du prix et $\beta_{e}$ nouveaux clients pour une baisse unitaire de l'intensité des émissions de carbone). Les autres clients attirés changent de produit ( $\alpha_{s}$ changements de clients pour une baisse unitaire du prix et $\beta_{s}$ changements de clients pour une baisse unitaire de l'intensité de carbone).

Les produits sont fournis par des fournisseurs différents. Nous considérons (sans perte de généralité) que le second fournisseur est plus proche du détaillant que le premier. Par conséquent, $e_{1} \geq e_{2}$. Puisque $e_{1}$ et $e_{2}$ sont des paramètres fixes, nous pouvons supposer, sans perte de généralité, que $e_{2}=0$ et, donc, $e_{1}$ peut être interprété comme la différence d'émissions du transport. En outre, nous considérons que l'intensité carbone du produit existant, $x_{1}$, est connue (c'est-à-dire un paramètre fixe) dans notre étude. Soient $A_{1}=$ $A-\beta\left(x_{1}+e_{1}\right), A_{2}=A+\beta_{s}\left(x_{1}+e_{1}\right), \beta=\beta_{e}+\beta_{s}, \alpha=\alpha_{p}+\alpha_{s}$. Notons que $\beta>\beta_{e}, \beta_{s}$ et $\alpha>\alpha_{p}, \alpha_{s}$. Après simplification, les demandes moyennes sont données comme suit.

$$
\begin{aligned}
& \lambda_{1}=A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2} \\
& \lambda_{2}=A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2}
\end{aligned}
$$

La politique de gestion des stocks du détaillant est similaire à celle du chapitre précédent. Il passe une commande pour remplir les stocks de produits auprès de son fournisseur qui adopte une politique de fabrication MTS. Pour le produit $i$, l'inventaire est géré selon la politique standard $\left(q_{i}, S_{i}\right)$ où $q_{i}$ est le point de commande, et $S_{i}$ est la taille du lot de réapprovisionnement. Le temps pour remplir le stock est distribué de manière exponentielle avec un taux moyen $\mu_{i}$ pour le produit i , qui ne dépend pas de la taille du lot. Rappelons que nous considérons le deuxième fournisseur comme étant plus proche du détaillant, donc, nous avons $\mu_{2} \geq \mu_{1}$. Le cadre général de la problématique de ce chapitre est présenté sur la figure 6 .

Nous avons formulé les problèmes dans différents contextes (en considérant différents ensembles de variables de décision) et les avons résolus par une approche analytique. Les solutions optimales sont fournies explicitement. Enfin, nous avons distingué différentes catégories de marché afin d'extraire des informations importantes de nos résultats. Ces


Figure 6: Le cadre général
marchés selon caractérisées selon les importances relatives de la sensibilité au prix ou aux émissions de carbone. Les résultats ont montré l'impact de ces caractéristiques du marché sur la stratégie du détaillant. Les résultats sont fournis dans différentes propositions pour couvrir tous les problèmes considérés dans cette étude. Un exemple numérique a également été présenté pour montrer la différence entre le profit du détaillant lorsqu'il offre un seul produit, et lorsqu'il offre deux produits substituables.

## Chapitre 5: Optimisation des stratégies des détaillants dans un environnement de concurrence dynamique

Le chapitre précédent se situait dans un contexte de différenciation des produits pour un détaillant donné. Dans ce chapitre, nous considérons deux détaillants concurrents qui offrent deux produits substituables sur un marché sensible à la qualité environnementale et au prix des produits. Chaque détaillant a son propre fournisseur. La demande et les délais de réapprovisionnement des stocks des détaillants sont aléatoires. La demande moyenne pour chaque produit diminue en fonction du niveau de ses émissions de carbone et de son prix, mais augmente en fonction du niveau des émissions de carbone et du prix de l'autre produit. Les émissions liées au transport dépendent essentiellement de la localisation du fournisseur. Les émissions liées à la production se produisent sur les sites des fournisseurs. Elles peuvent être réduites, mais cela entraîne un coût de fabrication plus élevé pour le fournisseur (Conrad, 2005), ce qui implique un coût d'achat plus élevé pour le détaillant. Dans le cas général, chaque détaillant décide du prix, de l'intensité des émissions de carbone et de la taille du lot de commande du produit qu'il propose afin de maximiser le profit attendu tout en satisfaisant à une contrainte de niveau de service.

Dans ce chapitre nous abordons les principales questions de recherche suivantes:

- Comment la concurrence affecte-t-elle les stratégies optimales des détaillants?
- Les différentes structures de marché ont-elles une incidence sur les meilleures stratégies des détaillants?


## Description du problème

Nous considérons deux détaillants qui vendent des produits substituables, distincts en termes de qualité environnementale (c'est-à-dire d'intensité des émissions de carbone) et de prix, sur un marché sensible à la greenness et au prix (voir la figure 6 ci-dessous). Les clients arrivent selon un processus de Poisson avec un taux d'arrivée $\lambda_{i}$ pour le détaillant $i$. Les émissions liées au transport par unité de produit, désignées par $e_{i}$ pour le détaillant $i$, dépendent de la distance parcourue par le produit entre le site du fournisseur et l'entrepôt du détaillant. Comme nous ne traitons pas de la sélection des fournisseurs, $e_{i}$ n'est pas une variable de décision. Sans perte de généralité, nous supposons que les émissions dues au transport des entrepôts des détaillants aux clients finaux peuvent être négligées, ce qui signifie implicitement que les détaillants sont situés à proximité de la zone de demande. En ce qui concerne les émissions liées à la production, nous considérons un contexte dans lequel le détaillant peut choisir le niveau d'émission de la production et, par conséquent, demander au fournisseur de fabriquer le produit en conséquence. Nous avons fourni au chapitre 1 quelques exemples qui illustrent la manière dont les principaux détaillants, tels que Walmart, demandent à leurs fournisseurs de fabriquer des produits plus écologiques. $x_{0}$ est la quantité d'émissions en production par unité d'un produit fabriqué par un processus standard. Proposer un produit plus écologique (avec des émissions de production plus faibles) implique un coût d'achat plus élevé pour le détaillant, car le fournisseur encourt un coût de fabrication plus élevé. Nous notons $x_{i}$ la quantité d'émissions de production par unité de produit $P_{i}$. L'intensité des émissions de carbone du produit $P_{i}$ est donc donnée par $x_{i}+e_{i}$. Le coût d'achat unitaire de $P_{i}$ est donné par $c+b\left(x_{0}-x\right)^{2}$, où $c_{i}$ est le coût unitaire du produit standard et $b$ est le facteur de coût pour la réduction des émissions de la production. A nouveau nous considérons une fonction de coût quadratique comme il est d'usage dans la littérature correspondante (par exemple, Liu et al., 2012, Ghosh and Shah, 2015).

La taille de l'ordre de réapprovisionnement du détaillant i est dénotée par $S_{i}$. Le temps du réapprovisionnement du stock est réparti de manière exponentielle avec un taux $\mu_{i}$ pour le détaillant $i$. Le délai de réapprovisionnement ne dépend pas de la taille de la commande puisque les produits sont supposés être toujours disponibles sur le site du fournisseur, ce qui est une hypothèse courante (Zhu, 2015). La politique de gestion de stocks des détaillants est similaire à celle des chapitres précédents. La probabilité de satisfaire la demande à partir du stock standard, dénotée par $1-\psi_{i}$ pour le détaillant $i$, doit être supérieure à $1-r$, pour les deux détaillants. Ainsi, la contrainte de niveau de service pour le détaillant i est donnée par $\left(1+\frac{\mu_{i} S_{i}}{\lambda_{i}}\right)^{-1} \leq r$ (voir le chapitre 3 , pour plus d'informations).

Comme chaque détaillant a son propre fournisseur, nous avons généralement une distance différente qui sépare chaque détaillant de son fournisseur. Sans perdre de vue la


Figure 7: Competitive supply chains structure
généralité, nous laissons le détaillant 1 représenter le détaillant qui a le fournisseur le plus éloigné. Nous avons donc $e_{1} \geq e_{2}$ et $\mu_{1} \leq \mu_{2}$.

Comme chaque détaillant a son propre fournisseur, nous avons généralement une distance différente qui sépare chaque détaillant de son fournisseur. Sans perdre en généralité, nous considérons que le détaillant 1 a le fournisseur le plus éloigné. Nous avons donc $e_{1} \geq e_{2}$ et $\mu_{1} \leq \mu_{2}$.

Notre modèle de demande est linéaire avec substitution. Nous rappelons que $\lambda_{i}$ fait référence au taux de la demande pour le détaillant i. Cette demande est très similaire à celle présentée au chapitre 4 (pour un produit sensible au prix et à la greeness de ce produit, mais aussi sensible à la différence de prix et de greeness entre les 2 produits).

Dans la pratique, les détaillants peuvent avoir des parts de marché différentes, par exemple lorsqu'un détaillant leader est établi et qu'un nouveau détaillant entre sur le marché. Dans ce cas, le premier a plus de parts de marché et attire donc plus de clients, même si le même produit est proposé par les deux détaillants. $\omega \in[0,1]$ représente la part de marché du détaillant 1 et, par conséquent, $(1-\omega)$ est la part de marché du détaillant 2 , lorsque les deux détaillants offrent le même produit (c'est-à-dire avec le même prix $p$ et la même intensité d'émission de carbone $x+e$ ). Les taux de demande sont donc donnés comme suit.

$$
\begin{gather*}
\lambda_{1}=\omega A-\omega \alpha_{p} p_{1}+\alpha_{s}\left(p_{2}-p_{1}\right)-\omega \beta_{e}\left(x_{1}+e_{1}\right)+\beta_{s}\left(\left(x_{2}+e_{2}\right)-\left(x_{1}+e_{1}\right)\right)  \tag{6}\\
\lambda_{2}=(1-\omega) A-(1-\omega) \alpha_{p} p_{2}+\alpha_{s}\left(p_{1}-p_{2}\right)-(1-\omega) \beta_{e}\left(x_{2}+e_{2}\right)+\beta_{s}\left(\left(x_{1}+e_{1}\right)-\left(x_{2}+e_{2}\right)\right) \tag{7}
\end{gather*}
$$

On a donc: $\lambda_{1}+\lambda_{2}=A-\omega \alpha_{p} p_{1}-(1-\omega) \alpha_{p} p_{2}-\omega \beta_{e}\left(x_{1}+e_{1}\right)-(1-\omega) \beta_{e}\left(x_{2}+e_{2}\right)$. Cela signifie que la demande moyenne totale est sensible aux prix et aux intensités des émissions de carbone.

Si le même produit est proposé par les deux détaillants, alors la demande moyenne est de $\omega\left(A-\alpha_{p} p-\beta_{e}(x+e)\right)$ pour le détaillant 1 et de $(1-\omega)\left(A-\alpha_{p} p-\beta_{e}(x+e)\right)$ pour le détaillant 2 , et la demande moyenne totale ne dépend pas de $\omega$. Le cas où les deux détaillants ont la même part de marché, qui est le cas typique étudié dans la littérature, correspond à $\omega=0.5$. Dans notre modèle, la part de marché $\omega$ représente la part de marché du détaillant 1 . Si $\omega>0.5$, alors le détaillant 1 a plus de parts de marché que le détaillant 2, et vice versa. Avec la prise en compte de cette notion de part de marché, nous généralisons la demande linéaire avec substitution qui est généralement adoptée dans la littérature correspondante.

Pour mieux comprendre notre fonction de demande, notons que le détaillant 1 peut attirer $\omega \alpha_{p}+\alpha_{s}$ clients avec une diminution d'une unité de son prix (respectivement, $(1-\omega) \alpha_{p}+\alpha_{s}$ clients pour le détaillant 2) et $\omega \beta_{e}+\beta_{s}$ clients avec une diminution d'une unité de son intensité d'émissions de carbone (respectivement, $(1-\omega) \beta_{e}+\beta_{s}$ our le détaillant 2). Seule une partie de ces clients représente une nouvelle demande créée sur le marché (pour le détaillant $1, \omega \alpha_{p}$ nouveaux clients pour une unité de baisse du prix et $\omega \beta_{e}$ nouveaux clients pour une unité de baisse de l'intensité des émissions de carbone), les autres clients attirés sont des clients qui changent de détaillant ( $\alpha_{s}$ changements de clients pour une unité de baisse de prix et $\beta_{s}$ changement de clients pour une unité de baisse de l'intensité des émissions de carbone).

Puisque $e_{1} \geq e_{2}$ et que les deux sont des paramètres fixes, nous pouvons considérer, sans perte de généralité, que $e_{2}=0$ et, donc, $e_{1}$ peut être interprété comme la différence des émissions de transport. Pour simplifier la notation, nous notons $A_{1}=\omega A-\theta_{1} e_{1}$, $A_{2}=(1-\omega) A+\beta_{s} e_{1}, \delta_{1}=\omega \alpha_{p}+\alpha_{s}, \delta_{2}=(1-\omega) \alpha_{p}+\alpha_{s}, \theta_{1}=\omega \beta_{e}+\beta_{s}$ et $\theta_{2}=$ $(1-\omega) \beta_{e}+\beta_{s}$. Les taux de demande sont finalement donnés comme suit.

$$
\begin{align*}
& \lambda_{1}=A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}  \tag{8}\\
& \lambda_{2}=A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1} \tag{9}
\end{align*}
$$

Sur la base du cadre décrit ci-dessus, nous étudions ensuite différents scénarios de concurrence et étudions leur impact sur le niveau de greeness et le prix des produits. Plus précisément nous considérons trois situations de concurrence, en partant du fait qu'il y a un détaillant établi et qu'un nouveau détaillant entre sur le marché. Plus précisément nous considérons que le détaillant établi réagit de 3 façons différentes : 1) le détaillant 1 ne réagit pas à l'apparition du nouveau détallant et donc au nouveau produit sur le marché ;2) il réagit partiellement en ajustant son prix $p_{1}$ et sa taille de commande $S_{1}$, mais sans modifier le produit et donc sans demander à son fournisseur de modification du niveau de greeness, $x_{1} ; 3$ ) il réagit en ajustant son prix $p_{1}$ et sa taille de commande $S_{1}$, mais aussi en demandant à son fournisseur de modifier le niveau de greeness, $x_{1}$.

Dans les scénarios de concurrence avec réaction (partielle ou complète), nous considérons un jeu non coopératif et déterminons la stratégie optimale de chaque détaillant par une recherche de l'équilibre de Nash.

Nous avons utilisé les résultats pour obtenir des considérations managériales, qui mettent en lumière l'impact de la concurrence sur les performances environnementales des produits. Par exemple nous avons constaté que lorsque le marché est davantage régi par la différence entre les qualités environnementales des produits (marché dit GDS pour Green Driven Switch), la concurrence conduit à proposer un produit plus vert. En revanche, lorsque le marché est davantage régi par la différenciation des prix (marché dit PDS pour Price Driven Switch), la concurrence conduit à proposer un produit moins vert. De plus, un détaillant qui acquiert un plus grand pouvoir de marché diminuera la greeness de son produit sur le marché du GDS et l'augmentera sur le marché du PDS.

Nous avons enfin étudié l'impact de la différence entre le niveau des émissions de carbone dans les transports sur la stratégie optimale des détaillants. Nous avons constaté que lorsque les deux détaillants ont la même part de marché, le détaillant qui s'appuie sur une chaîne d'approvisionnement plus courte offre un prix plus élevé si et seulement si la différence entre les émissions de carbone du transport est supérieure à une valeur seuil donnée.

## Chapitre 6: Fonctions de demandes complexes

Jusqu'à présent, nous avons considéré une demande linéaire, sensible au prix et aux émissions de carbone. À notre connaissance, la très grande majorité des études dans la littérature utilisent cette forme linéaire, avec laquelle nous avons formulé et résolu différents modèles. Dans ce chapitre, nous questionnons la pertinence de cette hypothèse de linéarité en nous intéressant à des fonctions de demande plus générale (en particulier non linéaire) concernant l'influence de la greeness. Pour cela, nous étudions le problème de base introduit dans le chapitre 3 sous différentes fonctions de demande. Nous étudions donc la stratégie optimale d'un détaillant qui offre un produit à ses clients sur un marché sensible au prix et aux émissions de carbone. Le détaillant conserve un stock, S , pour servir les clients. Le détaillant vend les produits au prix de détail, p. Nous considérons $x_{0}$ comme le niveau d'émission de référence, c'est-à-dire pour un processus de fabrication standard. $x_{0}$ peut donc être vu comme l'amélioration maximale possible des émissions de carbone. Le coût d'une amélioration x est donné par $b x^{2}$, où b est le facteur de coût, et $x$ est l'amélioration, c'est à dire la diminution du niveau des émissions. Rappelons que x dans les chapitres précédents représentait le niveau d'émission de carbone, alors que dans ce chapitre, pour faciliter le développement mathématique, x représente donc la diminution du niveau des émissions de carbone du produit. Le détaillant maximisera son profit en décidant du prix optimal, de l'amélioration des émissions de carbone et de la taille du lot de commande. La Figure 8 montre le problème décrit.

Comme nous l'avons dit précédemment, la fonction de demande linéaire est bien connue dans la littérature. Nos partenaires du projet ANR CONCLUDE à l'École des Mines de Saint-Étienne ont montré que la modélisation linéaire de la sensibilité de la demande aux émissions de carbone est insuffisante (Palacios-Argüello et al., 2020). Nous nous sommes donc intéressés à l'étude de nouvelles fonctions de demande pour voir en quoi les résultats diffèreraient de ceux obtenus avec des fonctions de demande linéaires tradi-


Figure 8: Supply chain
tionnelles, en procédant comme suit. Nous considérons que nous connaissons la demande du produit lorsque le détaillant offre le produit standard (c'est-à-dire sans amélioration). Nous supposons aussi connaître la forme de la courbe de la demande (par exemple, racine carrée de la diminution des émissions). Nous pouvons donc en déduire la courbe de pour x allant de 0 à $100 \%$ et en particulier le point à $100 \%$. Nous pouvons en déduire l'équation de la demande si on la suppose linéaire en rejoignant les deux points extrêmes ( 0 et $100 \%$ ) et on en déduit aussi le paramètre de sensibilité à x correspondant, $\beta_{e(L)}$. Par ailleurs dans la littérature, les chercheurs considèrent la demande linéaire sans aucune contrainte, c'est-à-dire considère que l'amélioration est linéaire de 0 à $100 \%$. Mais nous savons qu'en réalité, et cela a été aussi montré par nos partenaires du projet ANR, la demande n'augmente pas au-delà d'une valeur maximale. Cette valeur est obtenue pour un $x$ donné que nous appelons $x_{l}$. En d'autres termes, la demande augmente en fonction de $x$ jusqu'à $x_{l}$, et après cela, la demande est constante même si on continuait à diminuer les émissions. Ainsi, nous considérons une contrainte sur la demande que nous appelons cap. Nous considérons que l'augmentation maximale obtenue grâce à l'amélioration des émissions de carbone est de $\eta A$, pour un prix donné $p$. La Figure 9 montre alors les quatre fonctions de demande pour un prix donné que nous considérons : non linéaire, non linéaire avec cap, linéaire et linéaire avec cap.

Nous avons analysé le modèle de référence du chapitre 3, avec les trois nouvelles demandes : demande avec $x$ agissant selon une racine nième, avec ou sans cap, et demande linéaire avec cap. Les nouveaux problèmes ont été résolus par une approche analytique. Nous avons alors comparé les résultats en terme de profit et de solution optimale pour les 3 demandes linéaire avec ou sans cap, et demande non linéaire (racine nième) sans cap avec la demande que l'on considère de référence : non linéaire (racine nième) et cap. Les exemples numériques, faits pour une demande non linéaire de type racine carrée, montrent que lorsque le cap de la demande est faible (jusqu'à $20 \%$ environ), le modèle de linéaire avec cap fournit la meilleure approximation de la demande de référence, tandis que le modèle non linéaire fournit la meilleure approximation pour un cap plus élevé. Ce résultat est intéressant car le cas d'une augmentation limitée à $20 \%$ est très significatif dans de nombreuses applications et la résolution des modèles avec une demande type linéaire avec cap est une simple extension des résolutions présentées dans les chapitres


Figure 9: Demands functions for a given price
précédents.

## Chapitre 7: Conclusion

Les études sur les problèmes considérant un détaillant dans le cadre d'une demande sensible au prix et à la qualité environnementale des produits révèlent plusieurs lacunes dans ce domaine. Premièrement, la plupart des études considèrent un système de fabrication à la commande alors qu'il existe de nombreux cas dans la pratique qui suivent un système de fabrication pour stock et nécessitent donc de considérer les questions de politique de gestion de stocks, de taille des commandes, etc. Deuxièmement, dans un cadre de compétition, la plupart des études ont considéré un jeu de Stackelberg, où le leader décide d'abord et le suiveur agit ensuite, alors que dans la pratique, les différents aceteurs (par exemple, les entreprises, les détaillants, etc.) changent leurs stratégies en fonction de celles des autres concurrents. Troisièmement, la fonction de demande considérée est généralement linéairement décroissante en fonction du prix et de la qualité environnementale. Cependant, la qualité environnementale d'un produit peut avoir un effet non linéaire sur la demande. De plus, souvent au-delà d'un niveau donné d'amélioration de la qualité environnementale n'attire pas plus de clients. Ainsi, dans cette thèse, nous avons proposé différents problèmes et formulations pour combler les lacunes existantes dans la littérature. Nous avons commençé notre travail par l'étude d'un modèle de référence simple, qui comprend un fournisseur et un détaillant qui offre un produit aux clients sur un marché dépendant du prix et du niveau de la qualité environnementale. Le temps de remplissage de l'entrepôt du détaillant suit une distribution exponentielle. L'arrivée des clients suit un processus de Poisson. Ainsi, l'évolution du stock est un processus Markovien. Le détaillant décide du prix du produit, du niveau d'émission de carbone et
de la taille de la commande afin de maximiser son profit. Nous avons résolu le problème par une approche analytique. Ensuite, nous avons étendu le problème précédent en considérant un nouveau produit substituable. Dans ce cas, le problème inclut deux fournisseurs et un détaillant. Chaque fournisseur fournit au détaillant un produit qui permet au détaillant d'offrir deux produits substituables aux clients. La fonction de demande pour chaque produit est similaire au modèle de référence, sauf qu'il y a un effet de substitution entre produits. Cela signifie que la demande de chaque produit n'est pas seulement affectée par le prix et la qualité environnementale du produit, mais aussi par le prix et qualité environnementale de l'autre produit. Comme pour le problème précédent, ce problème est formulé dans un environnement stochastique et résolu, pour différents ensembles de variables de décision, par une approche analytique. Afin d'effectuer une analyse adéquate et de mettre en lumière les principaux éléments de gestion et de technique, nous distinguons le marché en fonction des paramètres de sensibilité. Le marché est catégorisé comme suit 1 . Marché PDS, où les changements sont davantage régis par la différenciation des prix que par la différenciation des qualités environnementales. 2. Marché GDS, où les changements sont davantage régis par la différenciation de la qualité environnementale que par la différenciation des prix. 3. Marché neutre, où les disparités de prix et de qualité environnemtale ont la même importance en ce qui concerne les changements de fournisseur. Les résultats montrent que les stratégies optimales des détaillants sont fortement impactées par ces caractéristiques du marché (PDS, GDS ou neutre). Par ailleurs les exemples numériques montrent que le détaillant est bien sûr gagnant lorsqu'il offre un produit nouveau et substituable, et nous avons quantifié ces gains. Nous avons poursuivi nos travaux sur les produits substituables dans un contexte de concurrence dynamique. Deux détaillants concurrents proposent deux produits substituables (un produit par détaillant) sur un marché sensible aux prix et à la qualité environnementale. Chaque détaillant a son fournisseur. Nous considérons la même politique de gestion de stock que pour les problèmes précédents. La fonction de demande pour chaque détaillant (produit) est similaire au problème précédent, sauf que nous avons introduit et appliqué le pouvoir du détaillant (sa part de marché) dans la fonction de demande. La demande et les délais de réapprovisionnement des entrepôts des détaillants sont aléatoires et suivent une distribution exponentielle. Les détaillants décident de leurs stratégies en fonction des stratégies des autres détaillants. Les stratégies optimales de chaque détaillant sont déterminées à l'équilibre de Nash. Nous avons étudié le modèle général dans lequel chaque détaillant décide du prix du produit, de la qualité environnementale et de la taille de la commande. Mais dans la pratique, il existe de nombreuses situations où un détaillant existant opère déjà sur le marché, et un nouveau détaillant entre sur le marché et propose un produit substituable. Nous avons donc proposé deux scénarios complémentaires: 1. La concurrence sans réaction, dans laquelle le détaillant existant ne réagit pas aux décisions du nouveau détaillant. 2. La concurrence avec réaction partielle, dans laquelle le détaillant existant met à jour son prix et la taille de sa commande mais ne change pas l'intensité des émissions de carbone car cela nécessite de nouveaux accords avec le fournisseur. Grâce aux résultats obtenus, nous avons étudié l'effet de la concurrence sur la performance environnementale des produits (par rapport au modèle de référence) ainsi que l'impact des
émissions de carbone du transport. Nous avons également étudié les effets du pouvoir des détaillants (part de marché) et des structures de marché (c.-à-d., PDS, GDS et marché neutre) sur la performance environnementale des produits. Les résultats ont montré que le détaillant qui a le plus de parts de marché diminuera la qualité environnementale de son produit sur le marché GDS et l'augmentera sur le marché PDS. Nous avons également constaté que lorsque les deux détaillants ont la même part de marché, le détaillant qui s'appuie sur une chaîne d'approvisionnement plus courte offre un prix plus élevé si et seulement si la différence des émissions de carbone liées au transport est supérieure à une valeur seuil donnée. Dans les chapitres 3 à 5 , nous avons considéré une fonction de demande linéairement décroissante en fonction du prix et du niveau d'émission de carbone. Dans le chapitre 6, nous avons considéré une fonction de demande linéairement décroissante en fonction du prix, mais qui augmente de façon non linéaire en fonction de l'amélioration des émissions de carbone. Nous avons également considéré une contrainte qui prend en compte que la demande ne peut pas augmenter au-delà d'un certain niveau. Les modèles avec les nouvelles fonctions ont été formulés et résolus, sur le problème de référence, étudié au chapitre 3 . Les exemples numériques montrent que lorsque la borne supérieure de la demande est relativement faible, le modèle linéaire avec borne fournit une très bonne approximation, ce qui est très intéressant car l'extension des modèles étudiés dans les chapitres 3 à 5 est relativement simple avec cette nouvelle forme de demande.

## Travaux futurs et perspectives

Notre étude peut être étendue de différentes manières. Comme nous l'avons mentionné dans l'introduction, notre travail a commencé par un système hybride en considérant une demande sensible au délai et au prix. Comme première extension, il serait intéressant d'étudier deux modes d'approvisionnement en compétition en considérant une demande sensible au délai, à la qualité environnementale et au prix. Une autre extension de notre travail serait de reprendre les modèles étudiés dans les chapitres 4 et 5 en considérant des fonctions de demande introduites dans le chapitre 6 . Le chapitre 6 introduit de nouvelles fonctions de demande et les compare à la demande linéaire. Ce travail peut être étendu en considérant d'autres formes de fonction de demande comme la fonction log-linéaire. Encore une fois, les problèmes de collaboration et de concurrence sont également intéressants à étudier avec d'autres fonctions de demande. Notre travail considère le CEA comme la seule source d'effet environnemental provenant des clients sur les stratégies des détaillants. Il serait également intéressant d'y ajouter la prise en compte de réglementations gouvernementales telles que la taxe carbone, le carbone cap, etc.

## Chapter 1

## Introduction

The environmental performance of a product, often assessed by the amount of carbon emissions released during the production and transportation phases, is becoming an essential purchase criterion for many customers (Palacios-Argüello et al., 2020; Hammami et al., 2018; Borin et al., 2013). Even it is highly dependent of products the relevant report on the product's carbon emission analysis reveals that approximately $70 \%$ of the total carbon emissions produced in the product's life cycle is related to the production process (AACE, 2015). According to a survey of 6,000 consumers, which is conducted by Accenture in April 2019, in 11 countries across North America, Europe, and Asia, 72\% of consumers said they are currently buying more environmentally-friendly products than they were buying five years ago. Also, $81 \%$ said they expect to buy more over the next five years. Moreover, more than half of consumers would pay more for environmentallyfriendly products (Accenture, 2019). Carbon Trust surveys indicate that approximately $20 \%$ of customers prefer to buy green products even if they are more expensive than regular products (Hong and Guo, 2019).

Most studies in supply chain management assume that customer's demand is known (exogenous demand). However, we know that demand is strongly impacted by internal decisions (endogenous demand). The most known factor that affects the customer's purchasing behavior is the price. However, there are different kinds of factors that affect the demand for a product or service. Huang et al. (2013) provided an extensive survey of papers where demand function depends on a factor. Those demand functions' models are in six categories: Price-Dependent, Rebate-Dependent, Lead Time-Dependent, Space-Dependent, Quality-Dependent, Advertising-Dependent Demand Model. It is well known in the business logistics literature that one of the most important customer service elements, in addition to price, is the delivery time. To understand the logic, the ideas, and solving methodology behind the endogenous demands, we started our work by considering an endogenous demand sensitive to price and lead time. We consider a hybrid system of selling substitutable products, which is consists of Make-To-Order (MTO) and Make-To-Stock (MTS) systems (see Appendix Afor more details). However, this Ph.D. focuses on endogenous demand sensitive to the product's price and greenness.

Motivated by empirical studies that show evidence of the relation between demand and environmental performance, modeling-based research has recently revised some classical problems in the operations management and Supply Chain (SC) literature while assuming the demand to be dependent on the environmental performance (see, e.g., Zhang et al., 2015, Hovelaque and Bironneau, 2015, and Liu et al., 2012). Environmental awareness changes consumer's decisions on purchasing goods. European Commission pointed out that $75 \%$ of Europeans choose green products. They are willing to pay more price to have
greener products (Brécard, 2014). Consequently, the Customers' Environmental Awareness (CEA) will directly influence the enterprise's strategy to produce green products and to invest and adopt cleaner technology (Yalabik and Fairchild, 2011). By recognizing these shifts in the marketplace, suppliers are considering to supply greener products (Gu et al., 2015; Sheu and Chen, 2012; Ding et al., 2015; Cheng et al., 2017). The consumer is willing to pay a premium price for a green product, such as lower CO2-emitting cars (Costa et al., 2019), but the green product needs more investment in research and development (R\&D), which will generate more cost (Krass et al., 2013). Therefore, the product's price as an important element of customer's purchasing decisions has been considered by many studies (e.g., Conrad, 2005; Sengupta, 2012).

Many retailers have developed a thorough understanding of customers' preferences for environmentally-friendly products and their willingness to pay more. Consequently, they have adapted their procurement strategies to offer greener alternatives to their customers. Thus, as highlighted by Ramanathan et al. (2012), these retailers asked their suppliers to reduce carbon emissions in production and transportation phases. Benjaafar et al. (2012) highlighted that various retailers are starting to attach carbon footprint labels to their products and position these products as greener alternatives. Two leading retailers, Tesco in the UK and Casino in France, have already embarked on aggressive labeling efforts. Recently, different products are becoming the object of carbon footprinting. Examples include fruit smoothies, shoes, and beer. Walmart, the world's biggest retail company, has set an emissions-reduction plan in October 2016 and invited the suppliers to join Walmart in committing to reduce greenhouse gas emissions resulting from their operations and value chains. The objective is to work with the suppliers to reduce carbon emissions by 1 Gigatonne in the production and use of food and products globally between 2015 and 2030 (Walmart, 2017). Bestseller (Bestseller, 2018), one of the leading fashion retailers in Denmark, allows its customers to buy a wide range of environmentallyfriendly products. The retailer (Bestseller) has committed to continuously improve the environmental footprint of its products, operations, and SC. Thus, to manage demand and increase profitability, retailers would have to compete not only on the price but also on their products' environmental performance (greenness) (which is here assessed in terms of carbon emissions).

Our review of the literature, presented in chapter 2, allowed to identify new perspectives for the problem of optimizing retail operations under a greenness- and price-sensitive demand context with endogenous demand. In particular,

1. Most studies are done based on MTO (Make-To-Order) system. As an exact term, they did not adapt to the retail context and inventory management in this area. However, in practice, we know that retailers make an order to their suppliers in many cases and make a stock to serve customers immediately. Therefore, retailers need to decide about order size and order point to optimize their expected profit.
2. In a competitive environment, most papers considered the Stackelberg game in which one player acts (makes decisions) first, called the leader. The second one follows and makes the decisions based on the leader's decisions, called the fol-
lower. Few studies considered a dynamic competition between players that they can make the decision simultaneously, and each player's decisions are affected by others' decisions.
3. In the literature, the researchers consider the linear demand without any constraint. However, our colleagues (Rennes School of Business and School of Mines of St Etienne in ANR project) found out that the demand is not linear and will not increase beyond a certain level of demand (Palacios-Argüello et al., 2020).

Based on the observations explained above, we propose the following extensions in this thesis:

- We first develop a basic model: a single retailer that offers one product to customers. The product's demand is sensitive to the product's price and carbon intensity. The retailer keeps the products as inventory in a warehouse near to customers to serve them. The retailer makes an order to fill the product's inventory from the supplier that adopts a MTS manufacturing policy. The inventory is managed according to the standard $(q, S)$ policy where $q$ is the reorder point, and $S$ is the replenishment lot size. As demand and inventory replenishment time are random, we develop the mathematical model in a stochastic environment (which is the case for the all models in this thesis). This problem is considered the benchmark model for the remaining chapters. More discussion and details are provided in chapter 3 .
- Product differentiation, where a retailer offers two substitutable products to customers, is considered. The goal is to find the retailer's optimal strategies. The demand function of each product depends on its price and carbon intensity and depends on other product's price and carbon intensity. The detailed analysis is provided in chapter 4
- A dynamic game between two retailers in a competition is considered in a price- and greenness- environment under a stochastic environment. We consider the situation where a retailer acts in the market, and a newcomer arrives. Newcomer offers a substitutable product. The market potential's share is not necessarily equal for both retailers. The detailed discussion is presented in chapter 5
- The benchmark model is considered with different (more complex) demands functions. We obtain and compare results thanks to different demand functions to analyze if the classical linear demand function is satisfying or not. More discussions and details are provided in chapter 6 .

Chapter 2 gives an overview of the related literature. The framework of the thesis is presented in the following.

## Chapter 2

- Literature review:
- How previous studies modelize Customer's Environmental Awaraness?
- What studies have been done in product differentiation with considering CEA?
- What studies have been done in competition with considering CEA?


## Chapter 3

- Benchmark model:
- Define a retailer's profit maximizing problem
- Deciding product's price, greenness, and order size


## Chapter 4

- Product's substitution
- Define a retailer's profit maximizing problem while offers two substitutable products
- Consider the problem with different settings
- Deciding products' prices, greenness, and order sizes


## Chapter 5

- Competition:
- Define two retailers' profit maximizing problems under dynamic game
- Consider the problem with different settings
- Deciding each product's price, greenness, and order size regarding to other player's decisions


## Chapter 6

Chapter 7

- Complex demand:
- Define a new and general demand in function of greenness
- Consider a limitation in demand's increasing in function of greenness
- Formulate the benchmark problem with new demands functions
- Compare different models and calculate the gap
- General conclusion
- Future work and perspective

Figure 1.1: Framework

## CHAPTER 2

## Literature review

Our study relates to the analytical research in operations and SC management with environmental considerations and, particularly, those studies that focus on price- and greenness- sensitive demand in SCs. Our work involves three streams of related literature on green supply chain management. The first section presents the studies that consider CEA (Customer Environmental Awareness) and, thus, carbon emission as a decision variable in a price- and greenness-dependant market. The second section of the literature review presents the studies that mainly worked on product differentiation. The third section reviews the literature concerning the best players' (retailers') strategies under a competitive environment.

The Growing number of customers who consider the environmental-friendly aspect of a product during their purchase inspires suppliers (e.g., manufacturers) to slowly begin to realize that they should use more advanced green production systems and green energies as an alternative to the traditional production system. The new alternative (green production system) decreases carbon emissions released into the environment during production. The green products release less carbon emission than the standard products, which refers to the friendliness of products to humans and the environment (Dangelico and Pontrandolfo, 2010). Carbon emission is becoming an essential factor that affects consumer purchase decisions. However, green products are often costlier to produce than the standard products produced traditionally, which makes green products are offered at higher prices than the standard products (Conrad, 2005). The stronger the consumers' environmental awareness is, the more the consumers are willing to pay a higher price for these green products (Chitra, 2007). Suchlike awareness and willingness may be considerably different among companies and industries, changeable during the time, and be different between consumer groups based on demographics, knowledge, values, attitudes, and behavior (Laroche et al., 2001; Carlson, 2005). Moon et al. gathered a survey of customers in prior "West" and "East" Berlin after Germany united. The survey provides the customers' willingness to spend a higher price for green foods produced by an environmental-friendly process (Moon et al., 2002). The results are presented for different groups of customers distinguished by geographic region and age. In the end, Moon et al. (2002), reported that a higher willingness to pay for green products would result in an instant effect that the higher price (or higher margin profit) will persuade more manufacturers to go after using environmentally-friendly production.

### 2.1 Customers' Environmental Awareness

Ghosh and Shah (2012) investigated the impact of greening decisions when one manufacturer and one retailer cooperate (i.e., centralized SC) or act individually (i.e., decentralized SC). Under the decentralized policy, the manufacturer moves first and decides the greenness level and the wholesale price. Improving greenness requires an investment that is modeled as a quadratic cost in greenness improvement. The retailer reacts by deciding the final price. Under the cooperative policy, the retailer and manufacturer first bargain on the greenness level; the manufacturer then decides the wholesale price, and the retailer reacts by deciding the final price. The study showed how channel structures influence greenness levels, prices, and profits.

Ma et al. (2013) considered a two-stage supply chain composed of one manufacturer and one retailer to investigate the effect of different contracts on the supply chain's profit and green improvement of product. There is one product to offer. The demand is a linear function of the retailer's price, product's quality level, and retailer's marketing efforts. A deterministic model is developed to adopt the contracts and analyze them. They consider two scenarios, centralized and decentralized supply chain. In a decentralized setting, three different contracts are presented. In the first contract, the retailer has to pay a fixed payment, which leads to a lower wholesale price. In the second contract, the quality improvement cost is shared between the retailer and the manufacturer. In the third contract, the quality improvement cost and marketing efforts cost are shared between the retailer and the manufacturer. The results show that the retailer will exert the highest marketing effort in the second contract if the retailer shares' quality cost is considerably high.

Nouira et al. (2014) presented the impact of considering environmental aspects and producing green products to industries. A linear demand function, which is increasing in the product's greenness, is considered. Each product has two attributes of greenness: the first one is related to the released carbon emission (which depends on the manufacturing process, etc.). The second one is related to raw materials. Two strategies are considered. In the first strategy, the manufacturer offers a single product to both customers (ordinary and green). The second strategy offers a particular type of product to each customer. A Mixed Integer Linear Programming (MILP) model is presented in order to maximize the profit. The model is solved by using real case study data. Results show that the second strategy leads the manufacturer to higher profit.

The studied problem in Ghosh and Shah (2015) is similar to Ghosh and Shah (2012). A two-stage supply chain with a manufacturer and a retailer under sensitive demand to the retailer price and green improvement is considered. The manufacturer produces one product to offer. However, in this study, "Cost Sharing Contract" is introduced. According to this contract, the retailer takes a share of green improvement costs. The results show that the green improvement, wholesale and retailer's price under this contract are higher than the decentralized policy. Under this contract (cost-sharing contract), the manufacturer's and retailer's profits also are higher than with the decentralized policy. Further, they discuss bargaining and take into the cost-sharing contract. The result shows that bargaining on the cost-sharing parameter provides a greater green improvement and greater
manufacturer's and retailer's profits.
Xu et al. (2016) considered a MTO closed-loop SC, composed of a manufacturer and a retailer, to investigate the effects of the CEA on the best decisions of the SC. In this SC, used products are not abandoned and returned to the life cycle by re-manufacturing. The product's demand increases linearly in the product's eco-friendly level and decreases the price's linear function. They formulated the problem in two ways to find out more about the supply chain system in economic, environmental, and social aspects: centralized and decentralized approaches. The manufacturer plays as the leader in the decentralized model. They also developed a revenue-sharing contract model to study more about collaboration (between manufacturer and retailer) in the decentralized model. The results showed that the centralized model provides better economic, environmental, and social performance than the decentralized model.

Li et al. (2016) investigated the pricing and greening strategies in a dual-channel SC. The manufacturer makes green products and sells them directly or through a retailer. The demand function is linearly decreasing in price and increasing in greenness level. The problem is formulated and solved in centralized and decentralized SCs. In the decentralized model, they propose coordination that provides the condition such that both the manufacturer and the retailer obtain win-win results. They also examine the effects of the greenness parameters (i.e., cost and sensitivity) on the manufacturer's and retailer's pricing decisions.

Liu and Yi (2017) brought the Big DATA (BDATA) concept into the green SC. The SC is composed of a manufacturer that produces green products and sells them through a retailer. The demand for products is sensitive to price and influenced by advertising. They presented four models: $1-\mathrm{M}$ model in which the manufacturer decides the product's wholesale price and greenness level and then the retailer decides the product's retail price; 2- R Model in which the retailer decides the product's retail price and the input level of BDATA and then the manufacturer decides the product's wholesale price and greenness level; 3- N Model in which is similar to R model in terms of decision variables, but, there is no leader and follower, and both manufacturer and retailer decide at the same time; and 4 - C Model that is the centralized setting, which the product's price is the only decision variable. They presented the close-form expression of optimal solutions and compared the profits of different models.

Hammami et al. (2018) studied the joint and alternative effect of the CEA and environmental regulations on the production policy, price, and greenness level of a product offered by a manufacturer. The authors considered linear and exponential demand functions that decrease in price and carbon intensity. Reducing the carbon intensity leads to increasing the production cost. The main results indicated that CEA is an efficient driver for better environmental performance, acting as a substitute for a carbon tax, unlike a carbon tax, leading to a lower price for customers. In the same research stream, many studies consider a SC that consists of one manufacturer and one retailer. These studies typically consider a linear demand function.

Hong and Guo (2019) considered a supply chain composed of one manufacturer and one retailer under different cooperation contracts (Price-only contract, Green-marketing
cost-sharing contract, Full channel coordination). The system follows a MTO production with a sensitive demand to the price and product's greenness. The demand function is increasing in the product's greenness and decreasing in price. A stochastic model is presented in order to maximize the profit (manufacturer and retailer). Manufacturer's decisions (wholesale price and greenness of product) and retailer's decisions (selling price and exerted effort for green marketing) are obtained by following the Stackelberg game. The retailer decides after the manufacturer's decisions. In conclusion, they declared that cooperation is needed to make the supply chain's environmental efficiency better. Especially in a high CEA market, the cooperative contracts are worth it.

Liu et al. (2020) considered a SC that is composed of one manufacturer and one retailer. The manufacturer invests in green innovation, produces a product, and sells it to the retailer. The retailer offers the product to customers in a price- and greenness- sensitive market. They consider the Stackelberg game between two players. The manufacturer acts as the leader and decides wholesale price and innovation investment, while the retailer, as the follower, decides the retail price. They also consider that the retailer acts as the leader, and the manufacturer acts as the follower. They attempt to investigate the effects of cooperative mechanisms (i.e., revenue sharing and cost-sharing contracts) on innovation investment and pricing strategies under different scenarios. The results show that when the retailer acts as the leader, the revenue sharing contract provides lower profit for both players (manufacturer and retailer) and lower investment in green innovation. They suggest that the cost-sharing contract provides a higher profit.

Huang et al. (2020) focused on government and bank loan effects on the green SC. The SC consists of a manufacturer who produces green products and sells them through a retailer. The government helps the manufacturer to produce greener products by offering the green credit (GC) that provides subsidies directly to the bank, manufacturer subsidy (MS) that provides subsidies directly to the manufacturer, and sales subsidies (SS) that provides subsidies directly to the retailer. The total subsidies of different modes are equal. The product's demand is decreasing in its price and increasing in greenness. They considered four scenarios: no-subsidy, green credit, manufacturer subsidy, and sales subsidy. The closed-form solutions are presented for all scenarios. The results showed that GC setting has better performance in terms of returns considering a green degree, market demand, social welfare, and environmental benefit when the bank loan is infinite.

The studies that have been done and presented in this section provide a comprehensive knowledge of customer environmental awareness subject, and guide us in the choices of the hypotheses we will make from chapter 3 .

### 2.2 Product Differentiation In Supply Chain Models

This section consists of two research areas: optimal pricing strategies of substitutable products, optimal pricing strategies under the green supply chains.

### 2.2.1 Optimal pricing strategies of substitutable products

The existence of substitutable products becomes an interesting area since the customers have more than one option in purchasing products. In pricing, customers switch between existing alternatives if they do not meet a preferred product (Shin et al., 2015). Therefore, the pricing strategies of substitutable products have become a popular research area.

Hsieh and Wu (2009) considered a SC, composed of two suppliers and one retailer, in a price-sensitive market. The suppliers produce two substitutable products and sell them through the common retailer. The problem consists of the retailer's pricing and ordering decisions in a stochastic environment. They considered different scenarios: revenue sharing, return policy, and combination of revenue sharing and return policy. The results reveal that in identical suppliers, the retailer's ordering and pricing decisions do not depend on demand or supply uncertainty. Also, they mentioned that whenever the retailer's cost per unit sold is smaller, he/she makes a higher order to suppliers.

Gürler and Yılmaz (2010) studied on a SC that includes one manufacturer and one retailer in a newsboy setting. The retailer makes an order for two substitutable products to the manufacturer. They assumed that each product's demand is independent of another. The customers switch between products (with a predetermined probability) when one of them is out of stock. The retailer has the right to return some or all of the unsold products to the manufacturer with some credit. The problem is formulated in a stochastic environment. They provided SC's total profit, manufacturer's profit, and retailer's profit.

Bish and Suwandechochai (2010) considered a SC that includes one supplier and one retailer. The supplier produces two substitutable products and sells them through a common retailer. The demand for each product is sensitive to its price (decreasing) and other product's price (increasing). They focused on the degree of products' substitution and the level of operational postponement. The problem is formulated in a stochastic environment that consists of deciding products' price and quantity. The results show the impacts of substitution and operational postponement on the optimal solution.

Zhao et al. (2014) considered a two-stage SC that consists of two manufacturers and one retailer to investigate the pricing strategies. The manufacturers sell the products through a common retailer. The products are substitutable. Each product's demand is sensitive to its and other product's prices. Different scenarios are considered to analyze the effects of the different SC's structures on the optimal pricing decisions. They are: 1Centralized pricing model, 2- Manufacturer Stackelberg pricing model, 3- Retailer Stackelberg pricing model, and 4 - Vertical Nash model. The closed-form expressions of all scenarios are provided by using an analytical approach. They derived some managerial insights.

Şen (2016) considered two suppliers that produce two substitutable products and sell them in a selling season. Each product's inventory is fixed; however, each product's demand is sensitive to the prices. At the beginning of the season, suppliers offer their products at a predetermined price and can change it once during the season. The goal is to know when is the best time for suppliers to reduce their products' prices to maximize the objective function (profit). The problem is formulated in a deterministic environment. The results showed that if a firm marks its price down after the season starts, it ensures that it runs out of inventory precisely when it ends.

Zhou et al. (2018) investigated the carbon tax policy's effect on a two-stage supply chain, which follows the MTO system. The model is deterministic, and the demand is a linear function of price and carbon emission's tax price. Demand is decreasing in price and carbon emission's tax price. The system is composed of three players. The government determines the carbon tax rate for the manufacturer. The manufacturer determines the wholesale price. The retailer determines the retail price. They declared that retail and wholesale prices are increasing in the carbon tax rate. The objective function is to maximize social welfare.

Luo et al. (2018) focused on customer values and different scenarios on the products' pricing strategy in a two-stage SC. They consider a manufacturer who produces two substitutable products and sells them through a retailer. The manufacturer and the retailer decide products' wholesale and retail prices, respectively. Each product's demand depends on the products' prices and customer acceptance. The goal is to maximize manufacture's and retailer's profit, and they formulate the problem in a deterministic environment. Different scenarios (i.e., manufacturer Stackelberg, retailer Stackelberg, and vertical Nash) are considered to investigate the effect of SC's structure on optimal solutions. They demonstrate that the vertical Nash structure creates a highly competitive environment between the manufacturer and the retailer, which retailer offers products at a lower price than other structures.

Ceryan (2019) investigated the pricing and replenishing strategies of two substitutable products: Regular and Seasonal. The regular products are refilled during horizon time, although the seasonal products are offer once. The products are different in the inventory management by the supplier. The supplier decides the regular product's price once (for all horizon times), while the seasonal product's price is determined at the beginning of the season. Each product's demand is sensitive to products' prices (i.e., decreases in its price and increases in other product's price). He formulated the problem through a multiperiod stochastic dynamic model. In addition to the initial problem, he considered two sub-models: Pricing with partial replenishment and Replenishment with Partial Pricing. The results provide a simple and effective heuristic policy.

Our study is different from the presented studies in the way that we not only consider price as a decision variable in product differentiation problem but also consider the carbon intensity of products as a decision variable. More details of the difference between our study and the presented studies are presented in Table 2.1.

### 2.2.2 Optimal pricing and greenness strategies of substitutable products

CEA's effect on the increasing demand for green products is another research area related to the current work. With the rise of CEA, it is expected to increase the pleasure of consuming a product for which an environment-friendly substitute is available (Conrad, 2005). With the increase of customer knowledge about environmental issues, the market purchase behavior is changed; people prefer to buy green products instead of usual ones but do not pay too much money (Datta et al., 2011). Green advertising is one of the best-known methods to increase CEA and change the people living style to a green life (Haytko and Matulich, 2008, Abd Rahim et al., 2012). A reasonable pricing strategy is critical for green products to acquire market share.

Liu et al. (2012) studied product differentiation in a SC that is composed of two manufacturers and one retailer. The manufacturers sell the products through a common retailer in a price and greenness sensitive market. A Stackelberg game is considered, such as manufacturers decide the product's price and greenness level as the game leader and then retailer follows and decides the retail price. The problem is formulated in a deterministic environment. They also considered other scenarios that we discuss in the next sub-section.

Zhang et al. (2015) considered a stochastic model to investigate the impact of CEA on the order quantity and channel coordination in a problem that the model is built on the multi-product newsvendor model. Demand is sensitive to price and product's environmental quality. The system is composed of one manufacturer that produces two substitutable products and one retailer. It is assumed that the products have one sale season. Products, which remain at the end of the season are sold at a lower price (first and second scenarios) or returned to the manufacturer with a return contract. The goal is to maximize the profit function in both centralized and decentralized policies. They showed that in decentralized policy, given that the manufacturer's profit is convex in CEA, the retailer's profit is increasing in CEA. Thus, the retailer makes more profit when offered products are better (greener). Also, they declared that the return contract is a win-win situation, even though the retailer's profit is less than the manufacturer's profit. In the end, the CEA's impact on traditional products depends on the differentiation between the greenness levels of two products.

Basiri and Heydari (2017) considered a two-stage supply chain, composed of a manufacturer and a retailer. The supply chain offers two substitutable products, which are traditional and green. The demand function is sensitive to price, green quality, and also sales efforts. The manufacturer decides the product's green quality, while, retailer decides retail price and sales efforts. Three scenarios are considered and compared: 1- Decentralized model, where both parties try to maximize their profits; 2- Integrated model, where one decision-maker exists; and 3-Collaborative model, which is supposed to offer a winwin situation. All of the scenarios' models are deterministic. An analytical approach solves the model. The numerical example results show that the manufacturer's profit in the collaboration setting is higher than in two other situations. However, for the retailer, the integrated setting's profit is higher than the profits in other settings.

Qin et al. (2017) considered a nonlinear stochastic model. The demand function is sensitive to price and carbon emission reduction. They studied a MTO supply chain system composed of one manufacturer and one retailer. This study's main point is to combine various demand forecasting and supply chain strategies (e.g., price and carbon emission reduction). In all scenarios, which are Without Information Sharing, Full Information Sharing, and Retailer-only Forecasting, the manufacturer is the leader that determines wholesale price and carbon emission reduction level, and the retailer is the follower which determines selling price. The results show that both the manufacturer and the retailer in the high uncertainty market tend to choose the third forecasting scenario (retailer-only Forecasting). In a low uncertainty market, the manufacturer chooses the second scenario, while the retailer chooses the first scenario.

To highlight the contributions of our work, Table 2.1 presents a summary of the existing literature. As Table 2.1 shows, there are few studies in this area that consider the order size as a decision variable. However, these studies consider endogenous demand sensitive only to the product's price, and then, the product's price and order size are the decision variables. The other studies that consider carbon emission as a decision variable do not consider the product's order size as a decision variable. Therefore, there is a gap, such as considering a stochastic system with these three variables (product's price, carbon emission, and order size). Our work that has been done, in Chapter 4, closes this gap of literature.

Table 2.1: Products differentiation comparison table

| Paper | Demand |  |  | Decision Variables |  |  |  | Model |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\stackrel{\rightharpoonup}{\dot{C}}$ |  | $\begin{aligned} & 0 \\ & \underset{\sim}{\square} \end{aligned}$ | $\stackrel{\rightharpoonup}{\underset{O}{8}}$ | O <br>  <br>  <br> 0 <br> 0 <br> 0. <br> 0. <br> 0 | $\begin{aligned} & \text { O} \\ & \stackrel{\rightharpoonup}{O} \\ & \underset{\sim}{\underset{\sim}{0}} \end{aligned}$ | $\begin{aligned} & \text { O} \\ & \stackrel{\rightharpoonup}{\top} \end{aligned}$ |  | $\begin{aligned} & \tilde{\omega} \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{n} \end{aligned}$ |
| Hsieh and Wu (2009) | $\bullet$ |  |  | - |  | $\bullet$ |  |  | $\bullet$ |
| Gürler and Yılmaz (2010) |  |  |  | $\bullet$ |  | $\bullet$ |  |  | $\bullet$ |
| Bish and Suwandechochai (2010) | $\bullet$ |  |  | $\bullet$ |  | $\bullet$ |  |  | $\bullet$ |
| Liu et al. (2012) | - | - |  | $\bullet$ | - |  |  | - |  |
| Zhao et al. (2014) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  |
| Zhang et al. (2015) | $\bullet$ |  |  | $\bullet$ |  | $\bullet$ |  |  | $\bullet$ |
| Şen (2016) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  |
| Basiri and Heydari (2017) | $\bullet$ |  | - | $\bullet$ |  |  | - | $\bullet$ |  |
| Qin et al. (2017) | $\bullet$ | - |  | $\bullet$ | - |  |  |  | $\bullet$ |
| Zhou et al. (2018) | $\bullet$ |  | - | $\bullet$ |  |  | - | $\bullet$ |  |
| Luo et al. (2018) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  |
| Ceryan (2019) | $\bullet$ |  |  |  | $\bullet$ |  | $\bullet$ |  | $\bullet$ |
| Our Study | $\bullet$ | - |  | - | - | - |  |  | $\bullet$ |

### 2.3 Competition Environment

Our study relates to the research on analytical models for operations and SC management with environmental considerations and, particularly, those studies that focus on greenness-based competition in SCs. There are valuable works on retailers' competition. Most of these studies, however, do not consider environmental aspects. In this section, we review the extant research in these areas and highlight our work's contributions.

Cachon (2001) considered a two-stage supply chain composed of a supplier and N retailers to study competitive and cooperative situations. The retailers hold the inventory in order to serve the customers, which has a cost. Also, there is a backorder cost if the stock is empty and a customer arrives. The supplier also has the inventory and backorder (situation that retailers face to empty stock) costs. The objective function is to find the optimal reorder points in order to minimize the cost. They formulated the problem in a stochastic demand by considering exogenous demand for each retailer. They use the Nash equilibrium to find the best solution. They declared that competition does not necessarily lead to supply chain inefficiency. In other settings, competition leads to costs that are substantially higher than optimal.

Dai et al. (2005) considered competitive firms to investigate pricing strategies. They considered a profit-maximizing problem in which the demand function is sensitive to the product's price. Each firm has its capacity (of satisfying demands), and so, the satisfied demand is equal to the minimum of the demand and the capacity. They solved the model under two situations according to the deterministic demand or stochastic demand. They did a sensitivity analysis of the optimal prices concerning cost and capacity parameters. They also declared that when the retailer's capacity is big enough, the optimal solution is independent of the capacities; otherwise, the optimal solutions depend on them.

McGuire and Staelin (2008) considered a price competition between two SCs. They considered different scenarios and models. However, the demand in all scenarios depends only on products' prices. Each manufacturer distributes its products through a single exclusive retailer, either a franchised outlet or a factory store. The problem is formulated in a deterministic environment. The results show that manufacturers prefer to follow a centralized system when competition's degree is low. However, the manufacturers prefer to follow a decentralized system when competition's degree is low. They mentioned that the products' substitutions have any effect on the Nash equilibrium. However, later in this work (Chapter 5), we show that the products' substitutions have significant effects on optimal solutions.

Xiao and Yang (2008) investigated the competition between two supply chains, which each of them is composed of one supplier and one retailer under demand uncertainty. They offer two substitutable products. The demand function is sensitive to the product's price and service level. The supplier leads and determines the wholesale price in each supply chain, while the retailer follows and determines the retail price and service level. They focused on the effects of the demand uncertainties, the service investment efficiencies, and the wholesale prices. They found that increasing one retailer's service investment efficiency leads to a lower optimal retail price and the other retailer's service level.

Wu et al. (2009) investigated the optimal strategies of two competing SCs with considering uncertain demand. They assume that the demand is uncertain with a high value, $h$, with probability $u$, and a low value, $l$, with probability $1-u$. They considered demand functions depend on three factors: the primary uncertain demand of the state, its retail price, and other product's retail price. The problem is formulated in a stochastic environment. They considered different games between SCs: 1- Manufacturer Stackelberg; 2- Vertical Integration; and 3- Bargaining over the wholesale price. They considered a single period ' and infinitely periods ' competition to compare the models. In infinity periods, a discount value is considered for each period. The products that are not sold have no value. They presented that vertical integration provides the unique Nash Equilibrium over single period, while other games may provide Nash Equilibrium over infinitely many periods.

Wu et al. (2012) investigated the competition effects on a two-stage supply chain composed of a supplier and two retailers. The supplier produces a product, and two retailers offer the same product to customers. They formulated the problem in a deterministic model with endogenous demand, sensitive to price. Six different scenarios that are the combination of horizontal, vertical, and Bertrand games are considered. The results showed that when retailers offer the same product, the vertical game has more influence on the scenarios' performances than the horizontal game.

Hafezalkotob (2015) studied the effects of government regulations on competing SCs. The government decides tariffs on products first, and the SCs follow. The government's goal is to maximize its profit or minimize pollution, or both. Each SC is composed of one manufacturer and one retailer that offers two substitutable products (regular and green) to customers (each offers one kind of product). The demand for each product decreases in its price and government tariffs and increases in other product's price and government tariffs. He considered six scenarios based on government tendencies and SC structures. The results show that: 1- in centralized or decentralized SCs, the environmental impact increases when the government's goal is to maximize its profit. 2- whatever the government's goal is, centralized SCs have a better environmental impact than decentralized SCs.

Baron et al. (2016) considered a competition between two two-stage SCs. Each SC is composed of one manufacturer and one retailer. They offer two substitutable products in a price-sensitive market. The problem is formulated in a deterministic environment. They focused on the effect of bargaining power within the SCs over wholesale prices. They considered three types of games: 1- Manufacturer Stackelberg; 2- Vertical Integration; and 3- Nash bargaining over the wholesale price. They showed that the first two games are a special case of the third game. They found out that just competition degree affects the optimal bargaining power, and thus, other parameters in the competing SCs have any effect on it. Also, the results revealed that when the SC is not alone in the market, vertical integration does not coordinate the SC, while manufacturer Stackelberg coordinates when the manufacturer has all the bargaining power.

Qi et al. (2017) considered a two-stage MTO supply chain composed of a supplier and two retailers to investigate the optimal pricing strategies under carbon cap policy.

The supplier produces a product and is the primary source of releasing carbon emission. The retailers sell the product to the customers. The retailers offer the same product to customers and compete to maximize their profits. The products' demand is sensitive to the retail price, decreasing in its retail price and increasing in other product's price. The model is deterministic, and a Stackelberg game is used between the supplier and retailers in the decentralized scenario, where the supplier is the leader and retailers are the followers. In the coordination scenario, they considered two settings: centralized and transfer payment mechanism. They analyzed and compared the performance of scenarios. The results showed that the best pricing decision for supply chain members could be achieved when wholesale prices (for both retailers) are the same, and retail prices are different from a transfer payment mechanism.

Jamali and Rasti-Barzoki (2018) considered two two-stage SC that produces two substitutable products and offer them to customers at a price- and greenness- sensitive market. They considered that one product is traditional, which they do not change its greenness level, and another one is green, in which the greenness level is considered as a decision variable. The problem is formulated in a deterministic environment for two scenarios; Centralized and Decentralized. They solved the models by an analytical approach and provided the closed-form expressions of optimal solutions. The results showed that the centralized setting provides more profit for the SC and higher greenness for the green product.

Zhou et al. (2018), which is presented earlier in the previous sub-section, considered a competition between $N$ retailers who offer substitutable products. As we mentioned, the SC includes government, manufacturer, and retailers. The problem consists of retailers' pricing decisions. In a scenario with multiple retailers, the government has to impose the carbon tax policy in order to decrease social welfare losses.

These studies by McGuire and Staelin (2008), Qi et al. (2017), and Zhou et al. (2018), however, do not consider a greenness-sensitive demand and do not investigate the greennessbased competition (in both studies, the greenness level is not a decision variable). Moreover, they assumed a deterministic setting which does not fit with most retail operations.

Giri et al. (2019) studied the effects of the government's policies on a two-stage SC that includes two manufacturers and one retailer. The manufacturers follow the MTO policy. The substitutable products, produced by manufacturers, are sold through a common retailer. The government's goal is to maximize its profit (not maximizing social welfare) by imposing carbon taxes, caps, and trade policies. They formulated the problem as a non-linear bi-level model such that the government is the leader of the Stackelberg game in a deterministic environment. The manufacturers horizontally follow Nash-equilibrium. Vertically, manufacturers are the leader in Stackelberg's leader-follower decisions and cooperating scenarios.

Sim et al. (2019) considered two two-stage SCs; each SC includes one manufacturer and one retailer. The products produced by manufacturers are sold through retailers. Each retailer offers only one product. The demand for each product depends on the products' quantity. The objective is to maximize social welfare. In their model, the manufacturers decide wholesale prices and abatement efforts concerning pollution emissions related to
manufacturing processes, whereas the retailers compete in quantities instead of prices. Vertical integration generally results in higher social welfare but more polluting emissions than vertical competition, whereas horizontal integration leads to lower welfare and lower emissions than horizontal competition.

Heydari et al. (2019) considered a three-stage supply chain that consists of a manufacturer, a distributor, and a retailer. The products are offered to customers either directly by the distributor (e-retailing) or by the retailer. The demand for each channel (distributor or retailer) is sensitive to the products' price and green level. The problem is modeled in different structures; open triad, the closed triad, and transitional triad that guarantees the manufacturer's profit. The results show that the closed triad provides more profits than the open triad to the supply chain.

Xu et al. (2020) considered different game models based on model structures that consist of two manufacturers and two retailers. The products produced by the two manufacturers are substitutable. In the model, the government subsidizes consumers who buy low carbon products but imposes a carbon tax on the manufacturer producing high carbon products. They found out that horizontal integration decreases competition. As a result, it causes higher prices and a lower value of social welfare.

Karray et al. (2020) investigated the pricing strategies of two retailers to answer the following question. "Can ignoring cross-category effect be a smart choice?". The retailers compete in the market to maximize their profit by offering two products. They categorized products' relations such as cross-category pricing effects are positive (complementary), negative (substitutable), or null (independent). The demand function is sensitive to the prices. Precisely, the product's demand is decreasing in both products' prices for the complementary category, while it is decreasing in its price and increasing in other product's price for substitutable products. A deterministic mathematical model is developed for different scenarios: 1- when both retailers choose to disregard cross-category effects; 2- when both retailers account for cross-category effects; and 3- when one retailer disregards cross-category effects while the other does not. The results revealed that higher complementary effects could invert prices' sensitivity to substitution levels within a category. The results also revealed that, in particular, ignoring the cross-category effect leads to lower prices when the two categories are substitutable or highly complementary and to higher prices otherwise.

The study that comes closest to our work is Liu et al. (2012). This study investigated the impact of CEA and competition intensity on the decision-makers' profits in different SC structures (one manufacturer and one retailer, two manufacturers and one retailer, and two manufacturers and two retailers). The manufacturer decides the greenness level and the wholesale price. The retailer decides its selling price. A Stackelberg game is used to model the manufacturer's problems as a first-mover and the retailer as a follower. The demand for each product linearly decreases in its price and increases in its greenness level. It also increases in the other product's price and decreases in the other product's greenness level. The main findings suggested that, as CEA increases, retailers and manufacturers with superior eco-friendly operations will benefit. In contrast, the inferior eco-friendly firm's profitability will tend to increase if the production competition level is low and de-
crease if the production competition level is high. Our study differs from that of Liu et al. (2012) in many aspects. First, we focus on the retailer's problem and consider that the retailer undertakes all the decisions (greenness level and price of the product and inventory decisions) while Liu et al. (2012) focused on the manufacturer's problem and assumed that the retailer's role is to price the product. Second, we consider the retailers' inventory decisions in a context of stochastic demand and stochastic inventory replenishment time. Liu et al. (2012), however, ignored the inventory and replenishment aspects, which does not fit with the retail context. Moreover, although there is a random parameter in their demand function, they worked with an average demand equivalent to a deterministic setting. Stockouts are, thus, not considered in their models. Third, concerning managerial implications, our main objective is to investigate the impact of the greenness- and price-based competition on the greenness of products, while Liu et al. (2012) analyzed the profits of the different SC actors.

To highlight our work's contribution, Table 2.2 presents a summary of the existing literature and our work that has been done to close the gap of literature in Chapter 5.

Table 2.2: Competition comparison table

| Paper | Demand |  |  | Decision Variables |  |  |  | Model |  | Product |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\stackrel{\rightharpoonup}{\underset{\sim}{8}}$ |  | $\begin{aligned} & \mathrm{O} \\ & \underset{\sim}{\mathrm{O}} \end{aligned}$ | $\stackrel{\rightharpoonup}{\vec{~}}$ |  | $\begin{aligned} & \text { O} \\ & \stackrel{\rightharpoonup}{0} \\ & \underset{\sim}{\underset{\sim}{\circ}} \end{aligned}$ | $\begin{aligned} & \stackrel{0}{\square} \\ & \stackrel{\oplus}{9} \end{aligned}$ |  |  | $\stackrel{0}{0}$ |  |
| Cachon (2001) |  |  |  |  |  | - | - |  | - | $\bullet$ |  |
| Dai et al. (2005) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ | $\bullet$ |  | $\bullet$ |
| McGuire and Staelin (2008) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  |  | $\bullet$ |
| Xiao and Yang (2008) | $\bullet$ |  | $\bullet$ | $\bullet$ |  |  |  | $\bullet$ | - |  | $\bullet$ |
| Wu et al. (2009) | $\bullet$ |  |  | $\bullet$ |  | - |  |  | $\bullet$ |  | $\bullet$ |
| Liu et al. (2012) | $\bullet$ | - |  | $\bullet$ | - |  |  | $\bullet$ |  |  | $\bullet$ |
| Wu et al. (2012) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  | $\bullet$ |  |
| Hafezalkotob (2015) | $\bullet$ |  | - | $\bullet$ |  |  | - | $\bullet$ |  |  | $\bullet$ |
| Baron et al. (2016) | $\bullet$ |  |  | $\bullet$ |  |  | - | $\bullet$ |  |  | $\bullet$ |
| Qi et al. (2017) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  | $\bullet$ |  |
| Zhou et al. (2018) | $\bullet$ |  | - | $\bullet$ |  |  | - | $\bullet$ |  |  | - |
| Jamali and Rasti-Barzoki (2018) | $\bullet$ | $\bullet$ |  | $\bullet$ | $\bullet$ |  |  | $\bullet$ |  |  | $\bullet$ |
| Giri et al. (2019) | $\bullet$ | $\bullet$ |  | $\bullet$ | $\bullet$ |  | - | $\bullet$ |  |  | $\bullet$ |
| Sim et al. (2019) |  |  | - | $\bullet$ | $\bullet$ | $\bullet$ |  | $\bullet$ |  |  | $\bullet$ |
| Heydari et al. (2019) | $\bullet$ | $\bullet$ |  | $\bullet$ | $\bullet$ |  |  | $\bullet$ | - | - |  |
| Xu et al. (2020) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  |  | $\bullet$ |
| Karray et al. (2020) | $\bullet$ |  |  | $\bullet$ |  |  |  | $\bullet$ |  |  | $\bullet$ |
| Our Study | $\bullet$ | - |  | $\bullet$ | - | $\bullet$ |  |  | $\bullet$ |  | $\bullet$ |

The results of the studies that had been done bring so many helpful references and great ideas for further studies in the green supply chain, but rarely combine consumer environmental awareness with dynamic competition in a stochastic environment. Approaching the green production process is important to construct supply chains with environmental benefits and high economic efficiency when we speak of green products. So, we are focusing on optimizing the carbon emission production process. Therefore, our works will fill the lack of green process consideration in the stochastic environment under SCs (players) competition.

In chapter 3, we present the benchmark model that is our initial model. We study an optimal retailer strategy that offers a product in a price- and greenness- sensitive market. The problem is formulated in a stochastic environment. Chapter 4 extends the benchmark model. In this chapter, we will study product differentiation. The retailer offers two substitutable products and decides price, greenness, and order size for each of them. In chapter 5, we study product differentiation in a competition context. We consider two retailers that each offers one product. We discuss the dynamic competition between them and their optimal strategies. Chapter 6 presents new demands functions (a general demand in the function of greenness, specifically non-linear demand). Finally, in chapter 7, we conclude the works that have been done in this thesis and provide future work directions.

## Benchmark Model

This chapter studies a retailer's profit-maximizing problem as an initial and benchmark model for the rest of the thesis. We will compare the results of the remaining chapters to this chapter's results. This first case also allows us to present the main ideas that we will use to solve analytically future problems. We consider one retailer that sells one product in a price- and greenness- sensitive market. The retailer decides the product's price, carbon emission level, and order size in order to maximize the total expected profit. Customers arrive according to a Poisson process with mean arrival rate of $\lambda$. The mean demand for the product is linearly decreasing in its carbon intensity and price. The demand function will be introduced and discussed later in this chapter. The retailer has his/her supplier. The main sources of carbon emission are transportation and production activities. The transportation emission per unit of product, denoted by $e$, depends on the distance traveled by the product from the supplier's site to the retailer's warehouse. As we do not deal with supplier selection, $e$ is not a decision variable. Without losing generality, we will assume that the retailer's warehouse's transportation emissions to end customers can be neglected, which implicitly assumes that the retailer is located close to the demand zone. However, we keep $e$ to show the transportation carbon emission's effect on the retailer's decisions. As for production emissions, we consider a context where the retailer can choose the production emission level and ask the supplier to manufacture the product accordingly. This context fits with the practical examples provided in Section 1 to illustrate how leading retailers, such as Walmart and Bestseller, are asking their suppliers to produce greener products. We let $x_{0}$ denote the amount of production emissions per unit of a standard product produced by a standard process. Offering a greener product (with lower production emissions) implies a higher purchasing cost for the retailer as it incurs a higher manufacturing cost. We let $x$ denote the amount of production emissions per unit of product. The carbon intensity is thus given by $x+e$. The unitary purchasing cost of a product is given by $c+b\left(x_{0}-x\right)^{2}$, where $c$ is the unitary cost of the standard product, and $b$ is the cost factor of the production's emission reduction. We consider a quadratic cost function as usual in the related literature (e.g., Liu et al., 2012, Ghosh and Shah, 2015).

### 3.1 Problem description

We consider a retailer who orders a product from his/her supplier and sells to customers who consider the product's price and carbon intensity in their purchase decision. Figure 3.1 shows the considered supply chain.


Figure 3.1: Benchmark SC

The customer's arrival follows a Poisson distribution with a rate of $\lambda$. Our demand model is linear in the function of price and greenness. $A$ denotes the market potential. We let $\alpha_{p}$ and $\beta_{e}$ respectively denote the market sensitivity to the price and the carbon intensity. The mean demand rate is given as follows.

$$
\begin{equation*}
\lambda=A-\alpha_{p} p-\beta_{e}(x+e) \tag{3.1}
\end{equation*}
$$

To better understand our demand function, be aware that the retailer can attract $\alpha_{p}$ more customers with one unit decrease in price and $\beta_{e}$ more customers with one unit decrease in carbon intensity.
$S$ denotes the replenishment order size of the retailer. The service rate to refill the stock is exponentially distributed, with a rate of $\mu$. The replenishment time does not depend on the order size since the product is assumed to be always available at the supplier's site, which is a common assumption (Zhu, 2015). Thus, the replenishment time corresponds basically to preparation and transportation activities. Figure 3.2 shows the Queueing network.


Figure 3.2: Benchmark queuing network

The retailer makes an order to fill inventory from a supplier that adopts a MTS manufacturing policy. The inventory is managed according to the standard $(q, S)$ policy where $q$ is the reorder point, and $S$ is the replenishment lot size. To simplify the analysis, we assume that the retailer places a replenishment order when there is no stock (product) in the warehouse. During the replenishment time, demand is satisfied with safety stock. We assume that the safety stock is large enough to satisfy all demands in the vast majority
of cases and ignore the rare cases in which demand cannot be satisfied. The sizing and management of the safety stock are out of the scope of this study.

We have the main stock that serves the customers in $(1-r) \%$ of the time. A safety stock serves the customer that faces an empty main stock. To simplify the analysis without losing the system's main trade-offs, we approximate the retailer's warehouse's stock-out probability with the stock-out probability obtained for $q=0$. This approximation is accurate when the value of $q$ considered in practice is relatively small, which is the case in many warehouses with storage capacity constraints. When the value of $q$ is relatively high, the impact of this approximation on the model's outcomes can be offset by increasing the minimum service level, $1-r$, to a higher value than that usually used in practice. The inventory refilling policy used to calculate the stock-out probability is illustrated in figure 3.3 .


Figure 3.3: Inventory refilling policy used to approximate the stock-out probability

As demand and inventory replenishment time are stochastic, there is a probability that the main stock is stock-out. We let $\psi$ denote the probability of stock-out of product. To avoid unrealistic inventory levels, we impose that the probability of satisfying demand from the main stock, denoted by $1-\psi$ for the retailer, must be greater than a predetermined minimum service level. We consider the minimum service level, $1-r$, for the retailer. Hence, the service level constraint for retailer is given by $1-\psi \geq 1-r$, which is equivalent to $\psi \leq r$. Notice that $\psi$ represents the probability of having zero items in the main stock when a customer arrives. Thus, it also refers to the probability of serving this customer from the safety stock.

Since customers' arrival and inventory refilling service time follow the Exponential distribution with mean rates $\lambda$ and $\mu$, respectively, the number of part in main stock is a Markovian process. In this case, we calculate the probability of stock-out, in which there is no item in the warehouse, using continuous Markov chain process properties. Regarding Figure 3.3, let us define $\psi_{i}$, which stands for the probability of system in state $i$ (i.e., having $i$ item in stock). For each state, the classical balance equations give:
for state 0 :

$$
\mu \psi_{0}=\lambda \psi_{1} \Leftrightarrow \psi_{1}=\frac{\mu}{\lambda} \psi_{0}
$$

for state 1 to S-1: $\quad \lambda \psi_{i}=\lambda \psi_{i+1} \Leftrightarrow \psi_{i}=\psi_{i+1}$
for state S: $\quad \mu \psi_{0}=\lambda \psi_{S} \Leftrightarrow \psi_{S}=\frac{\mu}{\lambda} \psi_{0}$
The summation of states' probability should be equal to 1 . Therefore:
$\sum_{i=0}^{S} \psi_{i}=1 \Leftrightarrow \psi_{0}+\psi_{1}+\cdots+\psi_{S}=1 \Leftrightarrow \psi_{0}+\frac{\mu}{\lambda} \psi_{0}+\frac{\mu}{\lambda} \psi_{0}+\cdots+\frac{\mu}{\lambda} \psi_{0}=1$
$\Leftrightarrow \psi_{0}+S \frac{\mu}{\lambda} \psi_{0}=1 \Leftrightarrow \psi_{0}\left(1+S \frac{\mu}{\lambda}\right)=1 \Leftrightarrow \psi_{0}=\left(1+S \frac{\mu}{\lambda}\right)^{-1}=\frac{\lambda^{\lambda}}{\lambda+\mu S}$
Hence, the service level constraint for retailer is given by $\frac{\lambda}{\lambda+S \mu} \leq r$. Also, we are going to calculate the expected inventory level.
$\bar{S}=\sum_{i=0}^{S} i \psi_{i}=\psi_{1}+2 \psi_{2}+3 \psi_{3}+\ldots+S \psi_{S}$
$\Leftrightarrow \bar{S}=\frac{\mu}{\lambda} \psi_{0}+2\left(\frac{\mu}{\lambda} \psi_{0}\right)+3\left(\frac{\mu}{\lambda} \psi_{0}\right)+\ldots+S\left(\frac{\mu}{\lambda} \psi_{0}\right)=\left(\frac{S(S+1)}{2}\right)\left(\frac{\mu}{\lambda} \psi_{0}\right)$
$\Leftrightarrow \bar{S}=\left(\frac{S+1}{2}\right)\left(\frac{\mu}{\frac{\lambda}{S}+\mu}\right)=\left(\frac{S+1}{2}\right)\left(\frac{1}{1+\frac{\lambda}{\mu S}}\right)$
The expected inventory level is a complex equation in function of $S$. However, by considering a simple assumption such that $S \gg \frac{\lambda}{\mu}$ (which is a logical assumption; otherwise, the system often faces empty stock), a close approximation that is widely used in the inventory literature (e.g., Cargal, 2003) can be considered. Thus, the expected value of the average inventory level for the retailer can be given by $\frac{S}{2}$. We let $2 h$ denote the unit inventory cost. Thus, the inventory cost for the retailer is $h S$.

The parameters and the decision variables are presented in the following.

## Parameters:

A : Market potential,
c : Unit fixed cost of the product,
$\mu \quad:$ Mean refilling rate of the product,
$2 h \quad:$ Unit inventory holding cost for product,
$x_{0} \quad:$ Standard production process carbon emission,
$e \quad:$ Product's transportation carbon emission,
$b \quad:$ Carbon emission reduction cost factor,
$r \quad:$ Maximum guaranteed stock-out.

## Independent Decision Variables:

$x$ : Production carbon emission of product,
$p \quad:$ Retail price
$S \quad$ : Order size.

## Dependent Decision Variables:

$\lambda \quad:$ Mean demand rate for product (given by Equation 3.1,
$\psi \quad:$ The probability of stock out of product.

### 3.2 Mathematical model and analytical solution

In this benchmark model, the retailer is alone in the market. The problem consists of deciding price $p$, carbon emission level $x$, and order size $S$ to maximize the retailer's expected profit. A single product is offered to customers in this case; let us recall that the mean demand rate is given by equation 3.1. Let us define $A^{\prime}=A-\beta_{e} e$ since $e$ is known. The model $\left(M_{0}\right)$ is given below.
$\underset{x, p, S}{\operatorname{Maximize}} \pi=\left(p-\left(c-b\left(x_{0}-x\right)^{2}\right)\right) \lambda-h S$
Subject to

$$
\begin{align*}
& \psi=\frac{\lambda}{\lambda+S \mu} \leq r  \tag{3.3}\\
& \lambda=A^{\prime}-\alpha_{p} p-\beta_{e} x  \tag{3.4}\\
& \lambda, p, S \geq 0,0 \leq x \leq x_{0} \tag{3.5}
\end{align*}
$$

The objective is to maximize the total expected profit given in equation 3.2, This profit is equal to the revenue (i.e., $p \lambda$ ) - the procurement cost (i.e., $\left.\left(c+b\left(x_{0}-x\right)^{2}\right) \lambda\right)$ - the inventory cost (i.e., $h S$ ). Constraint 3.3 is the service level constraint. It ensures that stock-out probability does not exceed a predetermined level of $r$ (i.e., $1-r$ is the minimum service level). The mean demand rate is given in equation 3.4. Constraint 3.5 presents decision variables and demand positivity. To solve this model, we first transform it into a single-variable model based on the results of Lemma 3.1 and 3.2 given below.

Lemma 3.1. For any given values of $p$ and $x$, service level constraint 3.3 is binding and the optimal order size is $S^{*}(p, x)=\frac{(1-r)\left(A^{\prime}-\alpha_{p} p-\beta_{e} x\right)}{r \mu}$.

Proof. The first derivative of profit function, $\pi$, with respect to order size, $S$, is negative (i.e. $\frac{\partial \pi}{\partial S}<0$ ). Considering that the second derivative of profit function with respect to $S$ is zero (i.e. $\frac{\partial^{2} \pi}{\partial S^{2}}=0$ ), then, the smallest $S$ is the optimal solution. Regarding to service level constraint (equation 3.3, we have $S \geq \frac{(1-r)\left(A^{\prime}-\alpha_{p} p-\beta_{e} x\right)}{r \mu}$. Therefore the optimal order size is $S^{*}(p, x)=\frac{(1-r)\left(A^{\prime}-\alpha_{p} p-\beta_{e} x\right)}{r \mu}$.

Thus, we replace $S^{*}$ with its expression given in Lemma 3.1 and obtain the following equivalent formulation of the model $\left(M_{0}\right)$ with two variables $p$ and $x$.
$\underset{x, p}{\operatorname{Maximize}} \pi=\left(p-\left(c-b\left(x_{0}-x\right)^{2}+\frac{h(1-r)}{r \mu}\right)\right) \lambda$
Subject to

$$
\begin{aligned}
& \lambda=A^{\prime}-\alpha_{p} p-\beta_{e} x \\
& \lambda, p \geq 0, x \leq x_{0}
\end{aligned}
$$

In the following Lemma, we determine the optimal price for any given carbon emission of $x$.

Lemma 3.2. For any given value of $x$, the optimal price is

$$
p^{*}(x)=\frac{\alpha_{p} b x^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x+A^{\prime}+\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} .
$$

Proof. The second derivative of $\pi$ with respect to $p$, which is presented in the following, is negative and, thus, the profit function is concave in $p$. Therefore, the root of the first derivative maximizes the objective function. Let us call this value $p^{* \max }$.
$\frac{\partial \pi}{\partial p}=-2 \alpha_{p} p+\alpha_{p} b x^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x+A^{\prime}+\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)$
$\frac{\partial^{2} \pi}{\partial p^{2}}=-2 \alpha_{p}<0$
$\frac{\partial \pi}{\partial p}=0 \leftrightarrow p^{* \max }=\frac{\alpha_{p} b x^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x+A^{\prime}+\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}$
If $p^{* \max }>0$ and $\lambda\left(p^{* \max }\right) \geq 0$, then it can be considered as the optimal price. The condition that $p^{* \max }$ is positive is presented in the following.
$p^{* \max }=\frac{\alpha_{p} b x^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x+A^{\prime}+\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}>0$
$\leftrightarrow \alpha_{p} b x^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x+A^{\prime}+\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)>0$
The discriminant of the above equation is equal to $\Delta_{1}=-4 \alpha_{p} b\left(A^{\prime}-\beta_{e} x_{0}-\frac{\beta_{e}^{2}}{4 \alpha_{p} b}+\right.$ $\left.\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)$. Let us notice that we consider assumptions such that the market
potential is sufficiently high to satisfy forthcoming conditions (in this thesis), which $A^{\prime}=$ $A-\beta_{e} e>\beta_{e} x_{0}+\frac{\beta_{e}^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$ is one of them. Thus, the $\Delta_{1}$ is negative and there is no root. Because $\alpha_{p} b$ (coefficient of $x^{2}$ ) is positive and $\Delta_{1}$ is negative, therefore the equation is always positive. Therefore the $p^{* \max }$ is always positive. The next condition that need to be satisfied is $\lambda\left(p^{* \max }\right) \geq 0$.
$\lambda \geq 0 \leftrightarrow \frac{-\alpha_{p} b x^{2}+\left(2 \alpha_{p} b x_{0}-\beta_{e}\right) x+A^{\prime}-\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} \geq 0$
$\leftrightarrow-\alpha_{p} b x^{2}+\left(2 \alpha_{p} b x_{0}-\beta_{e}\right) x+A^{\prime}-\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right) \geq 0$
The discriminant of the above equation is equal to $\Delta_{2}=4 \alpha_{p} b\left(A^{\prime}-\beta_{e} x_{0}-\frac{\beta_{e}^{2}}{4 \alpha_{p} b}+\alpha_{p}(c+\right.$ $\left.\left.\frac{h(1-r)}{r \mu}\right)\right) . \Delta_{2}=-\Delta_{1}$. Earlier we assume that $A^{\prime}>\beta_{e} x_{0}+\frac{\beta_{e}^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$, therefore, the $\Delta_{2}$ is positive and the above equation has two roots (called $R_{1}$ and $R_{2}$ ). The demand is positive between these two roots and negative outside them.
$R_{1}=\frac{-\left(2 \alpha_{p} b x_{0}-\beta_{e}\right)-\sqrt{\Delta_{2}}}{-2 \alpha_{p} b}=x_{0}+\frac{-\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}$
$R_{2}=\frac{-\left(2 \alpha_{p} b x_{0}-\beta_{e}\right)+\sqrt{\Delta_{2}}}{-2 \alpha_{p} b}=x_{0}-\frac{\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}$
We have $R_{1}>x_{0}$ and $R_{2}<0$ (see Appendix B for more details). Therefore, demand is positive in the feasible region $\left(\left[0, x_{0}\right]\right)$. Considering the condition that are mentioned before $p^{* \max }>0$ and $\lambda\left(p^{* \max }\right) \geq 0$, thus, we can consider $p^{* \max }$ as optimal price $\left(p^{*}\right)$.

We can now replace $p^{*}$ with its expression given in Lemma 3.2 and obtain the following equivalent formulation of the model $\left(M_{0}\right)$ with only one variable $x$.
$\underset{0 \leq x \leq x_{0}}{\operatorname{Maximize}} \pi=\frac{\left(-\alpha_{p} b x^{2}+\left(2 \alpha_{p} b x_{0}-\beta_{e}\right) x+A^{\prime}-\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)\right)^{2}}{4 \alpha_{p}}$
Subject to

$$
\begin{equation*}
\frac{-\alpha_{p} b x^{2}+\left(2 \alpha_{p} b x_{0}-\beta_{e}\right) x+A^{\prime}-\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)}{2} \geq 0 \tag{3.8}
\end{equation*}
$$

We finally solve this single-variable model and derive the optimal solution in Proposition 3.1 .

Proposition 3.1. The optimal solution of benchmark model $\left(M_{0}\right)$ is the following. $x^{*}=\max \left\{0, x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right\}$,
$p^{*}=\left\{\begin{array}{ll}\frac{A-\beta_{e} e+\alpha_{p}\left(c+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} & \text { for } x^{*}=0 \\ \frac{A-\beta_{e}\left(x_{0}+e\right)+\frac{3 \beta_{e}{ }^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} & \text { for } x^{*} \neq 0\end{array}\right.$,
$\left(\underline{(1-r)\left(A-\beta_{e} e-\alpha_{p}\left(c+b x_{0}^{2}+\frac{h(1-r)}{r \mu}\right)\right)}\right.$
$S^{*}= \begin{cases}\frac{r \mu}{(1-r)\left(A-\beta_{e}\left(x_{0}+e\right)+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)} & \text { for } x^{*}=0 \\ & \text {, and }\end{cases}$
$\frac{(1-r)\left(A-\beta_{e}\left(x_{0}+e\right)+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)}{r \mu}$ for $x^{*} \neq 0$
$\pi^{*}=\left\{\begin{array}{ll}\frac{\left(A-\beta_{e} e-\alpha_{p}\left(c+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu}\right)\right)^{2}}{4 \alpha_{p}} & \text { for } x^{*}=0 \\ \frac{\left(A-\beta_{e}\left(x_{0}+e\right)+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)^{2}}{4 \alpha_{p}} & \text { for } x^{*} \neq 0\end{array}\right.$.

Proof. Considering equation 3.8, the objective function (equation 3.7) can be presented as $\frac{\lambda^{2}}{\alpha_{p}}$. Since $\lambda \geq 0$, then maximizing $\lambda$ is equivalent to maximizing $\frac{\lambda^{2}}{\alpha_{p}}$ (profit function). The second derivative of demand with respect to $x$, which is presented in the following, is negative and the root of the first derivative, called $x^{* m a x}$, maximizes the demand (as well as objective function).
$\frac{\partial \lambda}{\partial x}=\frac{-2 \alpha_{p} b x+\left(2 \alpha_{p} b x_{0}-\beta_{e}\right)}{2}$
$\frac{\partial^{2} \lambda}{\partial x^{2}}=-\alpha_{p} b<0$
$\frac{\partial \lambda}{\partial x}=0 \leftrightarrow x^{* \max }=x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}$
It is clear that $x^{* \max }<x_{0}$. While the $x^{* \max }$ is in the feasible region, the optimal solution is equal to $x^{* \max }\left(x^{*}=x^{* \max }\right)$, otherwise, if $x^{* \max }<0$ then the optimal solution is equal to
zero. So $x^{*}=\max \left\{0, x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right\}$.
We obtain $p^{*}$ by substituting $x^{*}$ into 3.2. Then, we can obtain $S^{*}$ by substituting $x^{*}$ and $p^{*}$ into Lemma 3.1.

We deduce that the optimal carbon emission linearly decreases in CEA (i.e., $\beta_{e}$ ). The optimal price is convex functions in CEA. It first decreases until a threshold value of CEA $\left(\frac{\partial p^{*}}{\partial \beta_{e}}=0 \Leftrightarrow \beta_{e}=\frac{2}{3} \alpha_{p} b\left(x_{0}+e\right)\right)$, and then, increases. The calculations to obtain this threshold value are given in the following.
$\frac{\partial p^{*}}{\partial \beta_{e}}= \begin{cases}\frac{-e}{2 \alpha_{p}} & \text { for } x^{*}=0 \\ \frac{-\left(x_{0}+e\right)+\frac{6 \beta_{e}}{4 \alpha_{p} b}}{2 \alpha_{p}} & \text { for } x^{*} \neq 0\end{cases}$
$\frac{\partial^{2} p^{*}}{\partial \beta_{e}{ }^{2}}= \begin{cases}0 & \text { for } x^{*}=0 \\ \frac{3}{4 \alpha_{p}{ }^{2} b} & \text { for } x^{*} \neq 0\end{cases}$
In the case of $x^{*}=0$, the optimal price is linearly decreasing in CEA $\left(\frac{\partial p^{*}}{\partial \beta_{e}}<0\right.$ and $\frac{\partial^{2} p^{*}}{\partial \beta_{e}{ }^{2}}=0$ ). However in the case of $x^{*} \neq 0$, the optimal price is convex $\left(\frac{\partial^{2} p^{*}}{\partial \beta_{e}{ }^{2}}>0\right)$, and it is decreasing until $\beta_{e}=\frac{2}{3} \alpha_{p} b\left(x_{0}+e\right)$ and increasing after that. In addition, we deduce that the optimal price and order size are linearly decreasing in transportation carbon emission (i.e., $e$ ). It is an intuitive result. Increasing $e$ means that retailer offers dirtier product (i.e., higher carbon intensity). Increasing one unit of $e$ leads to increasing carbon intensity (dirtier product) that makes decreasing $\frac{\beta_{e}}{2 \alpha_{p}}$ unit of price and $\frac{\beta_{e}(1-r)}{r \mu}$ of order size.

Proposition 3.2. The optimal profit is convex with respect to $\beta_{e}$ and increasing $\beta_{e}$ leads profit to zero.

Proof. The first and second derivatives of $\pi^{*}$ with respect to $\beta_{e}$ are presented in the following.
$\frac{\partial \pi^{*}}{\partial \beta_{e}}=\frac{-\left(x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right)\left(A-\beta_{e}\left(x_{0}+e\right)+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)}{2 \alpha_{p}}$

The first derivative is negative that means the optimal profit is decreasing.
$\frac{\partial^{2} \pi^{*}}{\partial \beta_{e}{ }^{2}}=\frac{\left(x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right)^{2}+\frac{1}{2 \alpha_{p} b}\left(A-\beta_{e}\left(x_{0}+e\right)+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)}{2 \alpha_{p}}>0$

The second derivative is positive when $A>\beta_{e}\left(x_{0}+e\right)-\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$. We recall that this condition is already considered in Lemma 3.2. Therefore, optimal profit has a decreasing convex shape in $\beta_{e}$.

As we mentioned earlier, the optimal carbon emission is linearly decreasing in $\beta_{e}$ and leads to zero. Thus, the optimal demand also goes to $\lambda=A-\alpha_{p} p$. However, decreasing optimal carbon emission means increasing total production cost and consequently increasing the optimal price (after a threshold value that we mentioned earlier). Finally, increasing $\beta_{e}$ is decreasing product's demand and the margin profit that leads to decreasing profit. As an illustration, figure 3.4 shows the behavior of optimal solutions in $\beta_{e}$. To illustrate what it is explained, we consider the following numerical example: $A=1000$, $\alpha_{p}=8, c=20, x_{0}=100, \mu=30, b=0.01, h=5, r=0.05$. We vary $\beta_{e}$ from 2 to 10 .


Figure 3.4: Optimal solutions' behavior in function of $\beta_{e}$

### 3.3 Conclusion

This chapter contains a retailer's profit maximization problem in a price- and greennesssensitive market. The product's demand is a random variable that follows a Poisson dis-
tribution with a mean rate of $\lambda$. The mean rate is decreasing in the product's price and carbon intensity level. The retailer keeps the product to serve the customers immediately. The refilling time also is a random variable that follows Exponential distribution with a mean rate of $\mu$. The problem is formulated in a stochastic environment. We use an analytical approach to solve the problem and obtain the optimal solutions' expressions. The optimal carbon emission level is linearly decreasing in $\beta_{e}$ and non-linearly increasing in $\alpha_{p}$ and $b$. However, the optimal price is convex with respect to $\beta_{e}$, and it is decreasing until a threshold value, and after that, it is increasing.

## CHAPTER 4

# Product differentiation in retailing under price- and greenness-sensitive demand 

The previous chapter discussed the retailer's profit maximization problem, while the retailer offers one product to customers. The results showed the impacts of carbon emission and price sensitivity factors on optimal profit, price, carbon emission level, and order size. In this chapter, we are going to investigate the retailer's best strategies when he/she offers a new and substitutable product in a price- and greenness- sensitive market, in addition to the existing product. The new product is the same as the first product in terms of performance, function, and usage, i.e., substitutable. However, they can differ from pricing and carbon intensity. Let us define the first product as the "Existing" product and the second one as the "New" product. The suppliers send products to the retailer's warehouse, which is near to customers. The retailer keeps products to serve the customers as soon as one arrives. The demand for each product not only depends on its price and carbon emission intensity but also depends on the other product's price and carbon emission intensity. We consider a retailer-maximizing profit problem that is formulated in a stochastic environment. The main important questions of this chapter are:

- How much the retailer benefit from offering a new substitutable product?
- How product differentiation affects the retailer's best strategies?
- Do different market structures affect the retailer's best strategies?

The problem description is presented in the following. We discuss the model's assumptions and develop the general model framework. We consider different settings (sets of decisions) based on the general model and solve them by an analytical approach. After that, we derive analytical and numerical insights. Finally, the conclusion of this chapter is presented.

### 4.1 Problem description

As we briefly discussed at the beginning of this chapter, we are going to study the problem that a retailer offers two substitutable products. The retailer is already offering a product (called "Existing" product) to customers at a price- and carbon emission- sensitive market and plans to offer a new product (called "New" product). The new product

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
is substitutable to the existing product (i.e., similar in terms of usage, performance, function, etc.). However, they can be different from greenness and price points of view. The products' demands not only depend on their price and carbon emission but also depend on other product's price and carbon emission. Figure 4.1 shows the retailer (red ellipse) and the products' suppliers. The retailer makes orders from two different suppliers. Each supplier prepares one kind of product.


Figure 4.1: Collaboration supply chain

We consider a linear demand in the function of price and carbon intensity with substitution. Customers arrive according to a Poisson process with mean arrival rate $\lambda_{i}$ for product $i(i=1,2$; existing and new product, respectively). A unit of demand corresponds to one customer (order). Each product's mean demand decreases in its carbon intensity and price and increases in other products' carbon intensity and price. The market potential is denoted by $2 A$ (which we assume it is equally divided for each product). Parameters $\alpha_{p}$ and $\beta_{e}$ represent the customers' sensitivity to the price and the carbon emission intensity, respectively. Note that $\beta_{e}$ represents the CEA. We let $\alpha_{s}$ and $\beta_{s}$ represent the switchover's sensitivity toward price difference and carbon intensity difference, respectively. Similar to Liu et al. (2012) and Xiong et al. (2014), the demand functions given as follows.

$$
\begin{align*}
& \lambda_{1}=A-\alpha_{p} p_{1}+\alpha_{s}\left(p_{2}-p_{1}\right)-\beta_{e}\left(x_{1}+e_{1}\right)+\beta_{s}\left(x_{2}+e_{2}-x_{1}-e_{1}\right)  \tag{4.1}\\
& \lambda_{2}=A-\alpha_{p} p_{2}+\alpha_{s}\left(p_{1}-p_{2}\right)-\beta_{e}\left(x_{2}+e_{2}\right)+\beta_{s}\left(x_{1}+e_{1}-x_{2}-e_{2}\right) \tag{4.2}
\end{align*}
$$

Obviously, $\lambda_{1}+\lambda_{2}=2 A-\alpha_{p}\left(p_{1}+p_{2}\right)-\beta_{e}\left(x_{1}+e_{1}+x_{2}+e_{2}\right)$. This implies that total mean demand is sensitive to prices and carbon emission intensities; the switchovers do not affect the total demand.

Decreasing one unit of product $i$ 's price attracts $\alpha_{p}+\alpha_{s}$ more customers, and decreasing one unit of carbon intensity attracts $\beta_{e}+\beta_{s}$ more customers. It is noted that only a part of these customers represent the new created demand in the market (for product $i, \alpha_{p}$ new customers for one unit decrease in price and $\beta_{e}$ new customers for one unit decrease

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
in carbon emission intensity). The other attracted customers are switching from the other product ( $\alpha_{s}$ switching customers for a unit decrease in price and $\beta_{s}$ switching customers for a unit decrease in carbon intensity).

The products are supplied by different suppliers. We consider (without loss of generality) that second supplier is closer to the retailer than first supplier. Therefore, $e_{1} \geq e_{2}$. Since $e_{1}$ and $e_{2}$ are fixed parameters, we can assume, without loss of generality, that $e_{2}=0$ and, thus $e_{1}$ can be interpreted as the difference in transportation emissions. In addition, we consider that the existed product's carbon intensity, $x_{1}$, is known (i.e.; fixed parameter) in our study. We let $A_{1}=A-\beta\left(x_{1}+e_{1}\right), A_{2}=A+\beta_{s}\left(x_{1}+e_{1}\right), \beta=\beta_{e}+\beta_{s}, \alpha=\alpha_{p}+\alpha_{s}$. Note that $\beta>\beta_{e}, \beta_{s}$ and $\alpha>\alpha_{p}, \alpha_{s}$. After simplifications, the mean demands are given as follows.

$$
\begin{align*}
& \lambda_{1}=A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}  \tag{4.3}\\
& \lambda_{2}=A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2} \tag{4.4}
\end{align*}
$$

The retailer's inventories policy is similar to the previous chapter. $\mathrm{He} /$ She makes an order to fill products' inventory from their supplier that adopts an MTS manufacturing policy. For product $i$, the inventory is managed according to the standard $\left(q_{i}, S_{i}\right)$ policy where $q_{i}$ is the reorder point, and $S_{i}$ is the replenishment lot size. The service rate to refill the buffer is exponentially distributed with a mean rate of $\mu_{i}$ for product $i$, which does not depend on the lot size. Thus, the replenishment time corresponds basically to preparation and transportation activities. Recall that we consider the second supplier closer to the retailer. Thus, we have $\mu_{2} \geq \mu_{1}$. The general framework of this chapter's problem is presented in Figure 4.2.


Figure 4.2: General framework

As demands and inventories' replenishment time are stochastic, there is a probability of stock-out for each product. We let $\psi_{i}$ denote the probability of stock-out of product $i$. The probability of serving customers from the stock (i.e., $1-\psi_{i}$ ) must be higher than

## Chapter 4. Product differentiation in retailing under price- and greenness-sensitive

 demanda predetermined minimum service level. We consider the same service level, $1-r$, for both products. This assumption is motivated by the fact that we consider the same activity sector and the same product. To simplify the analysis without losing the system's main trade-offs, we approximate the retailer's warehouse's stock-out probability with the stockout probability obtained for $q_{i}=0$ (see Chapter 3 for more details). The inventory refilling policy used to calculate the stock-out probability is illustrated in Figure 4.3.


Figure 4.3: Inventory refilling policy used to approximate the stock-out probability

Like the previous chapter, we consider the inventory cost for product $i$ as $h S_{i}$, where $2 h$ is the unit inventory cost. Since customers' arrival and inventory refilling time follow the exponential distribution, the system follows the Markov chain process. Using continuous Markov chain process properties and the same approach and calculations as previous chapter, we obtain $\psi_{i}=\left(1+\frac{\mu_{i} S_{i}}{\lambda_{i}}\right)^{-1}$. The parameters and decision variables that we consider in this chapter are presented in the following.

## Parameters:

A : Each product's market potential,
$c_{1}, c_{2} \quad:$ Unit fixed cost of existing and new products, respectively,
$\mu_{1}, \mu_{2} \quad$ : Mean refilling rate of existing and new products, respectively,
$2 h \quad:$ Unit inventory holding cost for products,
$x_{0} \quad:$ Standard production process carbon emission,
$x_{1} \quad:$ Existing product's production process carbon emission,
$e_{1}, e_{2} \quad$ : Existing and new products transportation carbon emission of, respectively,
$b \quad:$ Carbon emission reduction cost factor,
$r \quad:$ Maximum guaranteed stock-out.

## Independent Decision Variables:

$x_{2} \quad$ : New product's production process carbon emission,
$p_{1}, p_{2} \quad:$ Retail price of existing and new products, respectively,
$S_{1}, S_{2} \quad$ : Order size of existing and new products, respectively.

## Dependent Decision Variables:

$\lambda_{1}, \lambda_{2} \quad$ : Mean demand rate for existing and new products (given by Equations 4.3 and 4.4), respectively,
$\psi_{1}, \psi_{2} \quad$ : The probability of stock out of existing and new products, respectively.

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand

We study different scenarios to investigate their impact on the greenness level and price of both products.

1. Retailer offers a new and substitutable product to the market and decides both products' prices, $p_{1}$ and $p_{2}$, and order sizes, $S_{1}$ and $S_{2}$. This scenario is the most common behavior of retailers in the real world.
2. The retailer offers a new and substitutable product to the market and decides the new product's carbon emission intensity, $x_{2}$, price, $p_{2}$, and both products' order sizes, $S_{1}$ and $S_{2}$. In this scenario, the retailer does not change the existed product price and carbon emission intensity but has the power of asking the second supplier (manufacturer) to provide him/her the product with $x_{2}$ carbon emission intensity.
3. Retailer offers a new and substitutable product to the market and decides new product's carbon emission intensity, $x_{2}$, both products' prices, $p_{1}$ and $p_{2}$, and order sizes, $S_{1}$ and $S_{2}$. This scenario is the general model of the retailer's discussed problem.

We compare the results of the above-described scenarios to investigate the impact of product differentiation.

### 4.2 Mathematical models and analytical solutions

In this technical section, we solve the problem under different settings (decision variables) and provide the closed-form expressions of the optimal solutions. Then, we analyze these optimal solutions to derive managerial insights.

### 4.2.1 Prices and stocks optimization

The retailer offers a new product to customers and decides the prices, $p_{1}$ and $p_{2}$, and the order sizes, $S_{1}$ and $S_{2}$, of products in order to maximize his/her expected profit. The retailer keeps the existing and new products' carbon intensity (i.e. $x_{1}+e_{1}$ and $x_{2}$, respectively). Here, the mean demand for existing (new) product, namely $\lambda_{1}\left(\lambda_{2}\right)$, depends not only on its price, $p_{1}\left(p_{2}\right)$, but also on the new (existing) product's price, $p_{2}\left(p_{1}\right)$. However, in this case $x_{1}$ and $x_{2}$ are known. Therefore, $\lambda_{1}$ is here given by $A_{1}^{\prime}-\alpha p_{1}+\alpha_{s} p_{2}$ and $\lambda_{2}$ is here given by $A_{2}^{\prime}-\alpha p_{2}+\alpha_{s} p_{1}$. As $x_{1}$ and $x_{2}$ are fixed, we let $A_{1}^{\prime}=A-\beta\left(x_{1}+\right.$ $\left.e_{1}\right)+\beta_{s} x_{2}$ and $A_{2}^{\prime}=A-\beta x_{2}+\beta_{s}\left(x_{1}+e_{1}\right)$.

In prices and stocks optimization, the retailer's profit optimization problem is denoted by $\left(M_{1}\right)$. We use the same methodology used for $\left(M_{0}\right)$ to solve the model $\left(M_{1}\right)$. However, the calculation steps are different since we do not have the same demand.

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$\underset{p_{1}, p_{2}, S_{1}, S_{2}}{\operatorname{Maximize}} \pi=\left(p_{1}-c_{1}\right) \lambda_{1}+\left(p_{2}-c_{2}\right) \lambda_{2}-h S_{1}-h S_{2}$
Subject to

$$
\begin{align*}
& \psi_{1}=\frac{\lambda_{1}}{\lambda_{1}+S_{1} \mu_{1}} \leq r  \tag{4.6}\\
& \psi_{2}=\frac{\lambda_{2}}{\lambda_{2}+S_{2} \mu_{2}} \leq r  \tag{4.7}\\
& \lambda_{1}=A_{1}^{\prime}-\alpha p_{1}+\alpha_{s} p_{2}  \tag{4.8}\\
& \lambda_{2}=A_{2}^{\prime}-\alpha p_{2}+\alpha_{s} p_{1}  \tag{4.9}\\
& p_{1}, p_{2}, S_{1}, S_{2}, \lambda_{1}, \lambda_{2} \geq 0
\end{align*}
$$

The retailer's objective function $(\pi)$ is to maximize his total profit (i.e., net profit of selling products - total inventory costs). Since the model is stochastic, the expected value of the stock is equal to $\sum_{i=0}^{S} i \psi_{i j}$ (where $\psi_{i j}$ represents the probability product $j ; j=$ 1,2 , represent the existing and the new product, respectively, of having $i ; i=1, \ldots, S$ items in stock when a random customer arrives). The expected value is too complicated. Therefore we consider a close approximation. The inventory cost is proportional to $S$, and we consider it as $h S_{j}$. Two service level constraints ensure that the probability of facing product j 's customer(s) to empty stock is less than or equal $r$. Other constraints are related to demands, prices, and order sizes positivity. To solve this model, we first transform it into a single-variable model based on the results of Lemma 4.1 and 4.2 given below.

Lemma 4.1. For any given values of $p_{1}$ and $p_{2}$, service levels constraints 4.6 and 4.7 are binding and the optimal stocks are $S_{1}^{*}=\frac{(1-r)\left(A_{1}^{\prime}-\alpha p_{1}+\alpha_{s} p_{2}\right)}{r \mu_{1}}$ and $S_{2}^{*}=\frac{(1-r)\left(A_{2}^{\prime}-\alpha p_{2}+\alpha_{s} p_{1}\right)}{r \mu_{2}}$.

Proof. Proof. Since the objective function is linearly decreasing in $S_{1}$, the smallest possible $S_{1}$ is the optimal stock. According to the service level constraint, $S_{1} \geq \frac{1-r}{r \mu_{1}} \lambda_{1}$, therefore, the optimal order size of the existing product is $S_{1}^{*}=\frac{(1-r)\left(A_{1}^{\prime}-\alpha p_{1}+\alpha_{s} p_{2}\right)}{r \mu_{1}}$, which implies that service level constraint is binding. In the same way, the optimal value of the new product's order size is equal to $S_{2}^{*}=\frac{(1-r)\left(A_{2}^{\prime}-\alpha p_{2}+\alpha_{s} p_{1}\right)}{r \mu_{2}}$.

Since we assume that $\lambda_{1}, \lambda_{2} \geq 0$, thus, $S_{1}^{*}, S_{2}^{*} \geq 0$. Thanks to Lemma 4.1, we can substitute $S_{1}^{*}$ and $S_{2}^{*}$ by their expressions, $\frac{(1-r)\left(A_{1}^{\prime}-\alpha p_{1}+\alpha_{s} p_{2}\right)}{r \mu_{1}}$ and $\frac{(1-r)\left(A_{2}^{\prime}-\alpha p_{2}+\alpha_{s} p_{1}\right)}{r \mu_{2}}$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
respectively, and obtain the following equivalent model with only two variables, $p_{1}$ and $p_{2}$.
$\underset{p_{1}, p_{2}}{\operatorname{Maximize}} \pi=\left(p_{1}-\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right) \lambda_{1}+\left(p_{2}-\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right) \lambda_{2}$
Subject to

$$
\begin{align*}
& \lambda_{1}=A_{1}^{\prime}-\alpha p_{1}+\alpha_{s} p_{2}  \tag{4.11}\\
& \lambda_{2}=A_{2}^{\prime}-\alpha p_{2}+\alpha_{s} p_{1}  \tag{4.12}\\
& \lambda_{1}, \lambda_{2}, p_{1}, p_{2} \geq 0
\end{align*}
$$

Lemma 4.2. For any given value of $p_{1}$, new product's optimal price is

$$
p_{2}^{*}=\frac{2 \alpha_{s} p_{1}+A_{2}^{\prime}+\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \alpha} .
$$

Proof. The second derivative of the objective function with respect to $p_{2}$, which is presented in the following, is negative and demonstrates that the objective function is concave with respect to $p_{2}$. Therefore the root of the first derivative (called $p_{2}^{* \max }$ ) maximizes the objective function.
$\frac{\partial \pi}{\partial p_{2}}=-2 \alpha p_{2}+2 \alpha_{s} p_{1}+A_{2}^{\prime}+\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)$
$\frac{\partial^{2} \pi}{\partial p_{2}{ }^{2}}=-2 \alpha<0$
$\frac{\partial \pi}{\partial p_{2}}=0 \leftrightarrow p_{2}^{* \max }=\frac{2 \alpha_{s} p_{1}+A_{2}^{\prime}+\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \alpha}$
If $p_{2}{ }^{* \max } \geq 0$, then it can be considered as the optimal price. The condition that $p_{2}{ }^{* \max }$ is positive is presented in the following.
$p_{2}^{* \max }=\frac{2 \alpha_{s} p_{1}+A_{2}^{\prime}+\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \alpha} \geq 0$
$2 \alpha_{s} p_{1}+A_{2}^{\prime}+\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right) \geq 0$
We know that $p_{1} \geq 0$. Then, $p_{2}{ }^{* \max } \geq 0$ while $A_{2}^{\prime} \geq-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(c_{1}+\right.$ $\left.\frac{h(1-r)}{r \mu_{1}}\right)$, which means that market potential is sufficiently high. The positivity of $p_{2}{ }^{* \max }$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
is a necessary condition but not sufficient. We have to also verify positivity of $\lambda_{1}$ and $\lambda_{2}$. By substituting $p_{2}{ }^{* \max }$ into $\lambda_{1}, \lambda_{2}$ we get following equivalent equations.
$\lambda_{1}=\frac{-2\left(\alpha^{2}-\alpha_{s}^{2}\right) p_{1}+2 \alpha A_{1}^{\prime}+\alpha_{s} A_{2}^{\prime}+\alpha \alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}^{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \alpha} \geq 0$
$\lambda_{2}=\frac{A_{2}^{\prime}-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2} \geq 0$
The existing product's demand, $\lambda_{1}$, is positive while $p_{1} \leq \frac{1}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(2 \alpha A_{1}^{\prime}+\alpha_{s} A_{2}^{\prime}+\right.$ $\left.\alpha \alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}^{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)$. The new product's demand, $\lambda_{2}$, is positive while we consider that $A_{2}^{\prime} \geq \alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)$. Again, these conditions are satisfying when we consider sufficiently high market potential. Under consideration of above conditions, $p_{2}{ }^{* \max }$ is considered as optimal solution.

Finally, we substitute $p_{2}^{*}$ with its expression, given in Lemma 4.2, and obtain the following equivalent single-variable model.
$\underset{p_{1} \geq 0}{\operatorname{Maximize}} \pi=\left(p_{1}-\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)$

$$
\begin{align*}
& \left(\frac{-2\left(\alpha^{2}-\alpha_{s}^{2}\right) p_{1}+2 \alpha A_{1}^{\prime}+\alpha_{s} A_{2}^{\prime}+\alpha \alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}^{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \alpha}\right) \\
& +\left(\frac{2 \alpha_{s} p_{1}+A_{2}^{\prime}-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \alpha}\right) \\
& \left(\frac{A_{2}^{\prime}-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2}\right) \tag{4.13}
\end{align*}
$$

Subject to

$$
\begin{equation*}
p_{1} \leq \frac{2 \alpha A_{1}^{\prime}+\alpha_{s} A_{2}^{\prime}+\alpha \alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}^{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)} \tag{4.14}
\end{equation*}
$$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand

Proposition 4.1. The optimal solutions for prices and stocks optimization model ( $M_{1}$ ) are properly presented in the following.

$$
\begin{aligned}
& p_{1}^{*}=\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha} A-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)+\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right), \\
& p_{2}^{*}=\frac{\alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha_{s}} A-\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha_{s}}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha_{s}} x_{2}\right)+\frac{1}{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right), \\
& S_{1}^{*}=\frac{1-r}{2 r \mu_{1}}\left(A-\beta\left(x_{1}+e_{1}\right)+\beta_{s} x_{2}-\alpha\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right), \text { and } \\
& S_{2}^{*}=\frac{1-r}{2 r \mu_{2}}\left(A-\beta x_{2}+\beta_{s}\left(x_{1}+e_{1}\right)-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right) .
\end{aligned}
$$

Proof. The second derivative, which is presented below, is negative with respect to $p_{1}$. Therefore, the root of the first derivative when it is equal to zero maximizes the objective function.
$\frac{\partial \pi}{\partial p_{1}}=-\frac{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}{\alpha} p_{1}+\left(\frac{\alpha+\alpha_{s}}{\alpha} A-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)+$
$\frac{\alpha^{2}-\alpha_{s}^{2}}{\alpha}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)$
$\frac{\partial^{2} \pi}{\partial p_{1}{ }^{2}}=-\frac{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}{\alpha}<0$
$\frac{\partial \pi}{\partial p_{1}}=0 \leftrightarrow p_{1}^{* \max }=\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha} A-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)+$ $\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)$
While $A \geq \frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha+\alpha_{s}}\left(x_{1}+e_{1}\right)-\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha+\alpha_{s}} x_{2}, p_{1}^{* \max }$ is positive. We also need to make sure that $p_{1}^{* \text { max }}$ satisfies constraint 4.14 . Thus, we have:
$p_{1}^{* \max } \leq \frac{2 \alpha A_{1}^{\prime}+\alpha_{s} A_{2}^{\prime}+\alpha \alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}^{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$
$\Leftrightarrow \frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha} A-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)+\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right) \leq$
$\frac{1}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(2 \alpha A_{1}^{\prime}+\alpha_{s} A_{2}^{\prime}+\alpha \alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}^{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)$
$A \geq \beta\left(x_{1}+e_{1}\right)-\alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)$
Under above condition, $p_{1}^{* m a x}=p_{1}^{*}$. By substituting $p_{1}^{*}$ into lemma 4.2 we get new product's optimal price, which is $p_{2}^{*}=\frac{\alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$\left(\frac{\alpha+\alpha_{s}}{\alpha_{s}} A-\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha_{s}}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha_{s}} x_{2}\right)+\frac{1}{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)$. Then by substituting $p_{1}^{*}$ and $p_{2}^{*}$ into lemma 4.1 we get $S_{1}^{*}$ and $S_{2}^{*}$, which are $S_{1}^{*}=\frac{1-r}{2 r \mu_{1}}\left(A-\beta\left(x_{1}+e_{1}\right)+\right.$ $\left.\beta_{s} x_{2}-\alpha\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)$ and $S_{2}^{*}=\frac{1-r}{2 r \mu_{2}}\left(A-\beta x_{2}+\beta_{s}\left(x_{1}+e_{1}\right)-\right.$ $\left.\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)$, respectively.

The prices and stocks optimization problem is solved by the analytical approach, and the closed-form expressions of optimal solutions are presented in Proposition 4.1. The optimal prices and order sizes have a linear relation with the inventory cost factor. Increasing one unit $h$ leads to increase $\frac{1-r}{2 r \mu_{1}}, \frac{1-r}{2 r \mu_{2}}, \frac{\alpha_{s}(1-r)^{2}}{2 r^{2} \mu_{1} \mu_{2}}$ unit of optimal existing and new product price, respectively. However, increasing one unit $h$ leads existing and new products' order sizes to increase $\frac{\alpha_{s}(1-r)^{2}}{2 r^{2} \mu_{1} \mu_{2}}$. We will use optimal solutions that are presented by Proposition 4.1 to pull managerial insights out of the shadow.

### 4.2.2 New product optimization $\left(M_{2}\right)$

Like previous case, the retailer offers a new product to customers. But in this case, the retailer keeps the existing product's price and carbon intensity (i.e., $p_{1}$ and $x_{1}$ ). Hence, we define $m_{1}$ as profit of selling one unit of existed product. This problem consists in deciding new product's price $p_{2}$, carbon emission intensity $x_{2}$, and both products' order sizes $S_{1}$ and $S_{2}$. Because in this case $x_{1}$ and $p_{1}$ are known, the demands functions (equation 4.1 and 4.2) can be simplified as follows. $\lambda_{1}$ is here given by $A_{1}^{\prime \prime}+\alpha_{s} p_{2}+\beta_{s} x_{2}$ and $\lambda_{2}$ is here given by $A_{2}^{\prime \prime}-\alpha p_{2}-\beta x_{2}$. We let $A_{1}^{\prime \prime}=A-\alpha p_{1}-\beta\left(x_{1}+e_{1}\right)$ and $A_{2}^{\prime \prime}=A+\alpha_{s} p_{1}+\beta_{s}\left(x_{1}+e_{1}\right)$.

In new product optimization, the retailer's profit optimization problem is denoted by $\left(M_{2}\right)$. We use the same methodology used for $\left(M_{1}\right)$ to solve model $\left(M_{2}\right)$. However, the calculation steps are different since we do not have the same decision variables.
$\underset{x_{2}, p_{2}, S_{1}, S_{2}}{\operatorname{Maximize}} \pi=m_{1} \lambda_{1}+\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}\right)\right) \lambda_{2}-h S_{1}-h S_{2}$
Subject to

$$
\begin{align*}
& \psi_{1}=\frac{\lambda_{1}}{\lambda_{1}+S_{1} \mu_{1}} \leq r  \tag{4.16}\\
& \psi_{2}=\frac{\lambda_{2}}{\lambda_{2}+S_{2} \mu_{2}} \leq r  \tag{4.17}\\
& \lambda_{1}=A_{1}^{\prime \prime}+\alpha_{s} p_{2}+\beta_{s} x_{2}  \tag{4.18}\\
& \lambda_{2}=A_{2}^{\prime \prime}-\alpha p_{2}-\beta x_{2}  \tag{4.19}\\
& p_{2}, S_{1}, S_{2}, \lambda_{1}, \lambda_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{align*}
$$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand

The mathematical model is similar to the prices and stocks optimization problem. Thus the explanation will be skipped. The rest of this sub-section aims to present theoretical proofs of optimal solutions.

Lemma 4.3. For any given values of $x_{2}$ and $p_{2}$, service levels constraints 4.6 and 4.7 are binding at optimality and the optimal stocks are $S_{1}^{*}=\frac{(1-r)\left(A_{1}^{\prime \prime}+\alpha_{s} p_{2}+\beta_{s} x_{2}\right)}{r \mu_{1}}$ and $S_{2}^{*}=\frac{(1-r)\left(A_{2}^{\prime \prime}-\alpha p_{2}-\beta x_{2}\right)}{r \mu_{2}}$.

The proof is similar to Lemma 4.1. Since we assume that $\lambda_{1}, \lambda_{2} \geq 0$, so, $S_{1}^{*}, S_{2}^{*} \geq 0$. Thanks to Lemma 4.3. we can substitute $S_{1}^{*}$ and $S_{2}^{*}$ by their expressions, $\frac{(1-r)\left(A_{1}^{\prime \prime}+\alpha_{s} p_{2}+\beta_{s} x_{2}\right)}{r \mu_{1}}$ and $\frac{(1-r)\left(A_{2}^{\prime \prime}-\alpha p_{2}-\beta x_{2}\right)}{r \mu_{2}}$, respectively, and obtain the following equivalent model with only two variables, $p_{1}$ and $p_{2}$.
$\underset{x_{2}, p_{2}}{\operatorname{Maximize}} \pi=\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right) \lambda_{1}+\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right) \lambda_{2}$
Subject to

$$
\begin{align*}
& \lambda_{1}=A_{1}^{\prime}-\alpha p_{1}+\alpha_{s} p_{2}  \tag{4.21}\\
& \lambda_{2}=A_{2}^{\prime}-\alpha p_{2}+\alpha_{s} p_{1}  \tag{4.22}\\
& \lambda_{1}, \lambda_{2}, p_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{align*}
$$

Lemma 4.4. For any given value of $x_{2}$, optimal price is

$$
p_{2}^{*}=\frac{\alpha b x_{2}^{2}-\left(\beta+2 \alpha b x_{0}\right) x_{2}+A_{2}^{\prime \prime}+\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha} .
$$

The proof approach is similar to Lemma 4.2. Finally, we substitute $p_{2}^{*}$ with its expression given in Lemma 4.4 and obtain the following equivalent single-variable model.

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$\underset{0 \leq x_{2} \leq x_{0}}{\operatorname{Maximize}} \pi=\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)\left(A_{1}^{\prime \prime}+\right.$
$\left.\frac{\alpha \alpha_{s} b x_{2}^{2}-\left(\alpha_{s} \beta+2 \alpha \alpha_{s} b x_{0}-2 \alpha \beta_{s}\right) x_{2}+\alpha_{s}^{2} m_{1}+\alpha_{s} A_{2}^{\prime \prime}+\alpha \alpha_{s}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha}\right)$
$+\left(\frac{\left(-\alpha b x_{2}^{2}-\left(\beta-2 \alpha b x_{0}\right) x_{2}+A_{2}^{\prime \prime}-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)^{2}-\left(\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)\right)^{2}}{4 \alpha}\right)$

Proposition 4.2. In the case of new product's optimization, if the root of following cubic equation is in the feasible region, it is the new product's optimal carbon emission.
$\alpha b^{2} x_{2}^{3}+\frac{3}{2} b\left(\beta-2 \alpha b x_{0}\right) x_{2}^{2}+\left(-b A_{2}^{\prime \prime}+3 \alpha b^{2} x_{0}^{2}-2 b \beta x_{0}+\alpha b\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right.$
$\left.+\alpha_{s} b\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{2 \alpha}\right) x_{2}+\left(\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)\left(\beta_{s}-\frac{\alpha_{p}\left(\beta+2 \alpha b x_{0}\right)}{2 \alpha}\right)-\frac{1}{2 \alpha}(\right.$
$\left.\left.\left(\beta-2 \alpha b x_{0}\right)\left(A_{2}^{\prime \prime}-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)\right)\right)=0$
Otherwise, in the case that there is no root in the feasible region, either 0 or $x_{0}$ (based on the profit in these two points) is the optimal solution.

Proof. The first and the second derivatives of objective function with respect to $x_{2}$ are presented in the following.
$\frac{\partial \pi}{\partial x_{2}}=\alpha b^{2} x_{2}^{3}+\frac{3}{2} b\left(\beta-2 \alpha b x_{0}\right) x_{2}^{2}+\left(-b A_{2}^{\prime \prime}+3 \alpha b^{2} x_{0}^{2}-2 b \beta x_{0}+\alpha b\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right.$
$\left.+\alpha_{s} b\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{2 \alpha}\right) x_{2}+\left(\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)\left(\beta_{s}-\frac{\alpha_{p}\left(\beta+2 \alpha b x_{0}\right)}{2 \alpha}\right)-\frac{1}{2 \alpha}(\right.$
$\left.\left.\left(\beta-2 \alpha b x_{0}\right)\left(A_{2}^{\prime \prime}-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)\right)\right)$
$\frac{\partial^{2} \pi}{\partial x_{2}^{2}}=3 \alpha b^{2} x_{2}^{2}+3 b\left(\beta-2 \alpha b x_{0}\right) x_{2}+\left(-b A_{2}^{\prime \prime}+3 \alpha b^{2} x_{0}^{2}-2 b \beta x_{0}+\alpha b\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right.$
$\left.+\alpha_{s} b\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{2 \alpha}\right)$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand

The second derivative is a quadratic function of $x_{2}$ that is negative between to roots and positive outside of them (because $x_{2}{ }^{2}$ coefficient is positive), only if the discriminant is positive; otherwise, the second derivative is always positive (i.e., convex). In the case that discriminant is positive, the objective function is concave between two roots and convex outside of them. Let's define $\Delta_{3}$ the discriminant, $\Delta_{3}=\left(3 b\left(\beta-2 \alpha b x_{0}\right)\right)^{2}-$ $12 \alpha b^{2}\left(-b A_{2}^{\prime \prime}+3 \alpha b^{2} x_{0}^{2}-2 b \beta x_{0}+\alpha b\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s} b\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{2 \alpha}\right)$.
As said above, if $\Delta_{3}<0$, then the second derivative is always positive. It means that the profit function is always convex. However, if $\Delta_{3}>0$, the second derivative is negative between the two roots and positive out of this range (two roots). Thus, the profit function is concave between two roots and convex out of them.

The condition that declares the $\Delta_{3}$ 's different situations are presented in the following.
$\Delta_{3}=12 \alpha b^{3}\left(A_{2}^{\prime \prime}-\beta x_{0}-\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{4 \alpha b}\right)$
Thus, while $A_{2}^{\prime \prime} \geq \beta x_{0}+\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)-\frac{\beta^{2}}{4 \alpha b}$, the $\Delta_{3} \geq 0$ and, thus, there are two roots (of the second derivative) that are:
$R_{3}=x_{0}+\frac{-3 \beta b+\sqrt{\Delta_{3}}}{6 \alpha b^{2}}$
$R_{4}=x_{0}-\frac{3 \beta b+\sqrt{\Delta_{3}}}{6 \alpha b^{2}}$
The objective function is concave between two roots, and out of them is convex. In addition, the first root is greater than $x_{0}$, and the second root is negative (see Appendix C for more details). According to the feasible region of $x_{2}\left(\left[0, x_{0}\right]\right)$, it can be said that the objective function is concave in the feasible region. Therefore, there is one optimal solution, at most, in the feasible region. Therefore, the optimal value of $x_{2}$ is the root of the first derivative $\left(\frac{\partial \pi}{\partial x_{2}}=0\right)$, only if there is a root in the feasible region. Suppose there is no root in the feasible region. In that case, the profit of the start point of the feasible region, 0 , and the profit of the last point of the feasible region, $x_{0}$, will be compared, and the point that has higher profit is the optimal solution.

### 4.2.3 General products differentiation model ( $M_{3}$ )

This case is the general scenario of retailer's maximizing profit problem. Like previous case, the retailer offers a new product to customers. In addition to prices and order sizes, the retailer also makes the decision of new product's carbon intensity, $x_{2}$. The existing product's carbon emission, $x_{1}$, remains without any changes. Equations 4.3 and 4.4 represent each product's demand. Therefore, $\lambda_{1}$ is here given by $A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}$ and $\lambda_{2}$ is here given by $A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2}$, which $A_{1}=A-\beta\left(x_{1}+e_{1}\right)$ and $A_{2}=$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$A+\beta_{s}\left(x_{1}+e_{1}\right)$. The mathematical model of discussed problem is presented in the following.
$\underset{x_{2}, p_{1}, p_{2}, S_{1}, S_{2}}{\operatorname{Maximize}} \pi=\left(p_{1}-c_{1}\right) \lambda_{1}+\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}\right)\right) \lambda_{2}-h S_{1}-h S_{2}$
Subject to

$$
\begin{align*}
& \psi_{1}=\frac{\lambda_{1}}{\lambda_{1}+S_{1} \mu_{1}} \leq r  \tag{4.25}\\
& \psi_{2}=\frac{\lambda_{2}}{\lambda_{2}+S_{2} \mu_{2}} \leq r  \tag{4.26}\\
& \lambda_{1}=A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}  \tag{4.27}\\
& \lambda_{2}=A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2}  \tag{4.28}\\
& p_{1}, p_{2}, S_{1}, S_{2}, \lambda_{1}, \lambda_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{align*}
$$

Lemma 4.5. For any given values of $x_{2}, p_{1}$, and $p_{2}$ service level constraints 4.25 and 4.26 are binding at optimality and the optimal stocks are

$$
S_{1}^{*}=\frac{(1-r)\left(A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}\right)}{r \mu_{1}} \text { and } S_{2}^{*}=\frac{(1-r)\left(A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2}\right)}{r \mu_{2}} \text {. }
$$

Proof is similar to that has been already given for Lemma 4.1. Since we assume that $\lambda_{1}, \lambda_{2} \geq 0$ and thanks to Lemma 4.5, $S_{1}^{*}, S_{2}^{*} \geq 0$ and we subsitute them by their expressions. By substituting $S_{1}^{*}$ and $S_{2}^{*}$, the model can be rewritten as presented in the following:
$\underset{x_{2}, p_{1}, p_{2}}{\operatorname{aximize}} \pi=\left(p_{1}-\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right) \lambda_{1}+\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right) \lambda_{2}$

Subject to

$$
\begin{aligned}
& \lambda_{1}=A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2} \\
& \lambda_{2}=A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2} \\
& p_{1}, p_{2}, \lambda_{1}, \lambda_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{aligned}
$$

Lemma 4.6. For any given value of $x_{2}$ and $p_{1}$, new product's optimal price is

$$
p_{2}^{*}=\frac{\alpha b x_{2}^{2}-\left(\beta+2 \alpha b x_{0}\right) x_{2}+A_{2}+2 \alpha_{s} p_{1}-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha} .
$$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand

The proof approach is similar to Lemma 4.2. Based on Lemma 4.6, we substitute $p_{2}^{*}$ by its expressions and obtain the following equivalent model with only two variables $p_{1}$ and $x_{2}$.
$\underset{x_{2}, p_{1}}{\operatorname{Maximize}} \pi=\left(p_{1}-\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right) \lambda_{1}+$
$\left(\frac{-\alpha b x_{2}^{2}+\left(-\beta+2 \alpha b x_{0}\right) x_{2}+A_{2}+2 \alpha_{s} p_{1}-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha}\right)$
$\left(\frac{-\alpha b x_{2}^{2}+\left(-\beta+2 \alpha b x_{0}\right) x_{2}+A_{2}+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha}\right)$

Subject to
$\lambda_{1}=A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}$
$\lambda_{2}=A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2}$
$p_{1}, \lambda_{1}, \lambda_{2} \geq 0,0 \leq x_{2} \leq x_{0}$

Lemma 4.7. For any given value of $x_{2}$, existed product's optimal price is

$$
p_{1}^{*}=\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(A_{1}+\frac{\alpha_{s}}{\alpha} A_{2}+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)+\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right) .
$$

The proof is similar to Proposition 4.1. Finally, we substitute $p_{1}^{*}$ with its expression given in Lemma 4.7 and obtain the following equivalent single-variable model.

$$
\begin{align*}
& \underset{0 \leq x_{2} \leq x_{0}}{\operatorname{Maximize}} \pi=\left(\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(A_{1}+\frac{\alpha_{s}}{\alpha} A_{2}+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)-\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right) \\
& \left(A_{1}-\alpha\left(\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(A_{1}+\frac{\alpha_{s}}{\alpha} A_{2}+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)-\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)+\right. \\
& \left.\alpha_{s}\left(\frac{\alpha b x_{2}^{2}-\left(\beta+2 \alpha b x_{0}\right) x_{2}+A_{2}+2 \alpha_{s} p_{1}-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha}\right)+\beta_{s} x_{2}\right) \\
& +\left(\frac{-\alpha b x_{2}^{2}+\left(-\beta+2 \alpha b x_{0}\right) x_{2}+A_{2}+2 \alpha_{s} p_{1}-\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha}\right) \tag{4.31}
\end{align*}
$$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$\left(\frac{-\alpha b x_{2}^{2}+\left(-\beta+2 \alpha b x_{0}\right) x_{2}+A_{2}+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \alpha}\right)$
Subject to
$\lambda_{1}=A_{1}-\alpha p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}$
$\lambda_{2}=A_{2}-\alpha p_{2}+\alpha_{s} p_{1}-\beta x_{2}$
$\lambda_{1}, \lambda_{2} \geq 0$

Proposition 4.3. In the case of general products differentiation, if the root of following cubic equation is in the feasible region, is the new product's optimal carbon

$$
\begin{aligned}
& \text { emission. } \\
& \alpha b^{2} x_{2}{ }^{3}-3 \alpha b^{2}\left(x_{0}-\frac{\beta}{2 \alpha b}\right) x_{2}^{2}+\left(\frac { \alpha \beta _ { s } - \beta \alpha _ { s } } { 2 ( \alpha ^ { 2 } - \alpha _ { s } ^ { 2 } ) } \left(\frac{4 \alpha \beta_{s}-3 \beta \alpha_{s}-2 \alpha \alpha_{s} b x_{0}}{2 \alpha}+b \alpha_{s}\left(x_{0}-\right.\right.\right. \\
& \left.\left.\left.\frac{\beta}{2 \alpha b}\right)\right)-b\left(A_{2}+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{1 \mu_{1}}\right)-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{1 \mu_{2}}\right)\right)+2 \alpha b^{2}\left(x_{0}-\frac{\beta}{2 \alpha b}\right)^{2}\right) x_{2} \\
& +\left(\frac { \alpha \beta _ { s } + \beta \alpha _ { s } } { 2 \alpha } \left(\frac{1}{2}\left(A_{1}-\frac{\alpha_{s}}{\alpha} A_{2}\right)-\alpha\left(c_{1}+\frac{h(1-r)}{1 \mu_{1}}\right)+\frac{\alpha_{s}}{2 \alpha}\left(A_{2}+\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{1 \mu_{2}}\right)\right.\right.\right. \\
& ))+\left(\frac{\alpha \beta_{s}+\beta \alpha_{s}}{2 \alpha}-\alpha_{s} b\left(x_{0}+\frac{\beta}{2 \alpha b}\right)\right)\left(\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(A_{1}+\frac{\alpha_{s}}{\alpha} A_{2}\right)-\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)+ \\
& \frac{1}{4 \alpha}\left(2 \alpha b\left(x_{0}-\frac{\beta}{2 \alpha b}\right)+2 \alpha_{s} \frac{\alpha \beta_{s}-\beta \alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\right)\left(A_{2}+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\alpha\left(c_{2}+b x_{0}^{2}+\right.\right. \\
& \left.\left.\frac{h(1-r)}{1 \mu_{2}}\right)\right)+\alpha b\left(x_{0}-\frac{\beta}{2 \alpha b}\right)\left(\frac{\left.A_{2}-\alpha\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{1 \mu_{2}}\right)\right)+\alpha b\left(x_{0}-\frac{\beta}{2 \alpha b}\right)}{2 \alpha}+\right. \\
& \left.\left.\frac{\alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(A_{1}+\frac{\alpha_{s}}{\alpha} A_{2}\right)\right)\right)=0
\end{aligned}
$$

Otherwise, in the case of having no root in the feasible region, either 0 or $x_{0}$ (based on the profit in these two points) is the optimal solution.

Proof is similar to Proposition 4.2.
In the following section, we are going to use the optimal values that we obtain to pull important managerial insights out from the shadow.

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand

### 4.3 Analysis and managerial insights

In this section, we present important insights of problems that are presented and solved earlier. First, we present a comprehensive numerical result to answer the first question of this study; how much the retailer benefit from offering a new substitutable product? We consider sets of parameters to do the numerical example. The parameters are presented in Table 4.1. The result is an average of 160 K different combinations.

Table 4.1: Parameters

| Parameter | Value | Parameter | Value |
| :---: | :---: | :---: | :---: |
| $A$ | $1200: 100: 2000$ | $x_{0}$ | 100 |
| $\alpha_{p}$ | $4: 1: 8$ | $e_{1}$ | $0: 0.2 x_{0}: 0.4 x_{0}$ |
| $\alpha_{s}$ | $3: 1: \alpha_{p}-1$ | $c_{1}$ | 10 |
| $\beta_{e}$ | $3: 1: 5$ | $c_{2}$ | $c_{1}: 0.2 c_{1}: 1.4 c_{1}$ |
| $\beta_{s}$ | $1: 1: \beta_{e}-1$ | $h$ | $1: 1: 3$ |
| $\mu_{1}$ | $1.5 \mu_{2}: \mu_{2}: 3.5 \mu_{2}$ | $b$ | $0.005: 0.005: 0.015$ |
| $\mu_{2}$ | 30 | $r$ | 0.05 |

Observation 4.1. On our large numerical example, the product differentiation provides $12.7 \%$ more profit on average than one product for retailer.

How much the retailer benefit from offering a new substitutable product? To answer the research question, we consider over 160k combinations of parameters' value that are presented in Table 4.1. The gap between the optimal profits when the retailer offers one product ( $\pi_{M_{0}}$ ) and when the retailer offers two substitutable products ( $\pi_{M_{2}}$ ) is calculated as $\frac{\left(\pi_{M_{2}}-\pi_{M_{0}}\right) * 100}{\pi_{M_{2}}}$. To be more precise, we confirm that in especial case when the retailer offers two identical products, total demand is the same as when he/she offers one product and so the profit is similar to the case we offer only one product (chapter 3). Table 4.2 shows the details concerning this gap. According to the results in table 4.2, the retailer gains 12.7 percent more profit on average when offers two substitutable product than offers one product. A more in-depth look into the results reveals that product differentiation, at the very least, provides the same profit as when retailer offers one product, which means that the retailer does not interest in offering a new substitutable product. It is intuitive, because the retailer offers two identical products in the worst case. The numerical results confirm that. The minimum value of the gap between the optimal profits is almost zero. However, the maximum value of the gap between the optimal profits is $46.93 \%$. So from a managerial perspective, when the gap between profits is sufficiently high enough, it is interesting to offer a new substitutable product. To completing the analysis, a T-student test is performed on the results. With regard to the confidence level $95 \%$, the mean gap

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
lies in [12.66 12.74], the confidence interval. In simple words, with a probability of $95 \%$, the retailer gains between 12.66 and 12.74 percent more profit when offers a new product to customers.

Table 4.2: Gap between profits

|  | Mean | Standard Deviation | 95\% Confidence Interval |
| :---: | :---: | :---: | :---: |
| $\pi_{M_{2}}$ V.s. $\pi_{M_{0}}$ | $12.7 \%$ | $9.04 \%$ | $[12.6612 .74] \%$ |

Comparing the optimal carbon emission level in benchmark and product differentiation models to highlight the substitutable effect is presented in the following results. The comparison is made in different market structures in order to answer the research questions.

### 4.3.1 Market Structures

To understand the following results, we distinguish the market based on price and greenness sensitivity factors (similar to Boyaci and Ray, 2003 and Boyaci and Ray, 2006).

In the case of $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$, a unit decrease in carbon intensity leads to a higher proportion of cannibalized customers (with respect to "new" customers) than the proportion resulting from a unit decrease in price. Thus, the switchovers are more governed by greenness differentiation than price differentiation, which will henceforth be referred to as a greenness-driven switchovers (GDS) market. To show that, let us first proof that GDS condition (i.e., $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ ) is also equivalent to $\frac{\alpha_{s}}{\alpha_{p}+\alpha_{s}}<\frac{\beta_{s}}{\beta_{e}+\beta_{s}}$.
$\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}} \Leftrightarrow \alpha_{s} \beta_{e}<\alpha_{p} \beta_{s} \Leftrightarrow \alpha_{s} \beta_{e}+\alpha_{s} \beta_{s}<\alpha_{p} \beta_{s}+\alpha_{s} \beta_{s} \Leftrightarrow \alpha_{s}\left(\beta_{e}+\beta_{s}\right)<\beta_{s}\left(\alpha_{p}+\alpha_{s}\right)$ $\Leftrightarrow \frac{\alpha_{s}}{\alpha_{p}+\alpha_{s}}<\frac{\beta_{s}}{\beta_{e}+\beta_{s}}$
Recall Equations 4.3 and 4.4 that are demand functions. If we decrease one unit of product 1's price, its demand increases of $\alpha$ units with $\alpha=\alpha_{p}+\alpha_{s}$, which $\alpha_{p}$ are the new customers and $\alpha_{s}$ are the customers who switch. Therefore, the fraction of customers who switch to the total customers is $\frac{\alpha_{s}}{\alpha_{p}+\alpha_{s}}$. In the same way for carbon emission, the fraction of customers who switch to the total customers (when we decrease one unit of carbon emission) is $\frac{\beta_{s}}{\beta_{e}+\beta_{s}}$. The fraction of "cannibalized" demand (with respect to the total demand generated) is higher for a carbon emission reduction rather than a price reduction. Therefore, in the GDS market, the switchover effect is stronger for carbon emission.

With the same analysis, we deduce in the case of $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ that the switchovers are more governed by price differentiation, which will henceforth be referred to as a pricedriven switchovers (PDS) market.

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand

Finally, when $\frac{\alpha_{s}}{\alpha_{p}}=\frac{\beta_{s}}{\beta_{e}}$, price and greenness disparities have the same importance with respect to switchovers, which is therefore referred to as a neutral market. The above market segregation builds an analysis framework that helps to understand the following findings.

Most of our results depend on the market type (PDS or GDS). We recall that a PDS market characterizes the case where switchovers are more governed by price differentiation than greenness differentiation, which does not necessarily mean that customers are less sensitive to greenness but may refer to situations where it is not possible for the customers to compare the greenness levels of products. For instance, for many products, there is no green labeling or the labeling does not allow customers to make reliable comparisons, such as when firms just highlight that the product is made from sustainable materials (e.g., Matt \& Nat handbags and wallets, some products offered by $\mathrm{H} \& \mathrm{M}$ and Zara) or with an environmentally-friendly process (e.g., Lobodis coffee). In these cases, the customers can compare the products only in terms of price, and switchovers will then be more governed by price differentiation (PDS market) even in the presence of environmentally-conscious customers. However, in a market characterized by a high customers' environmental awareness (e.g., agribusiness) and the possibility for customers to compare the greenness levels of products, switchovers can be more governed by greenness differentiation (GDS market). This motivated many agribusiness companies to focus on offering greener products while simplifying the greenness comparison for consumers. For instance, the British firm Innocent indicates the carbon footprint of some fruit smoothies on the packaging (Hammami et al., 2018). A GDS market may also correspond to products for which the greenness level implies a lower use cost (e.g., energy saving). The example of the environmentally-friendly bulb of the Philips-Carrefour SC in Europe, provided by Hong et al. (2019), is a good one. For this product, greenness is associated with energy labeling, which is a color-coded performance scale from G to A+++. The customers use this scale to compare green lamps with the conventional ones, and this comparison significantly influences their purchasing decisions (Hong et al., 2019).

### 4.3.2 Insights

Proposition 4.4. In prices and stocks optimization problem ( $M_{1}$ ), the greater the new product's carbon emission is, the lower the new product's price will be, and (i) the lower the existing product's price will be in the PDS market, (ii) the higher the existing product's price will be in the GDS market, and (iii) same price in the Neutral market (does not change).

Proof. Thank to proposition 4.1, the optimal prices are presented in the following.

$$
p_{1}^{*}=\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha} A-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)+\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)
$$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$p_{2}^{*}=\frac{\alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha_{s}} A-\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha_{s}}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha_{s}} x_{2}\right)+\frac{1}{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)$
The first derivatives of prices ( $p_{1}^{*}$ and $p_{2}^{*}$, respectievly) with respect to the new product's carbon emission, $x_{2}$, are presented in the following.
$\frac{\partial p_{1}^{*}}{\partial x_{2}}=\frac{\alpha \beta_{s}-\beta \alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$
$\frac{\partial p_{2}^{*}}{\partial x_{2}}=-\frac{\alpha \beta-\beta_{s} \alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$
Let us remember that $\alpha=\alpha_{p}+\alpha_{s}$ and $\beta=\beta_{e}+\beta_{s}$. Therefore, $\alpha \beta_{s}-\beta \alpha_{s}=\left(\alpha_{p}+\alpha_{s}\right) \beta_{s}-$ $\left(\beta_{e}+\beta_{s}\right) \alpha_{s}=\alpha_{p} \beta_{s}-\beta_{e} \alpha_{s}$. The existing product's price behavior is strongly related to the behavior of the customers (or market). In case that $\alpha \beta_{s}>\beta \alpha_{s}$ (equivalently, $\alpha_{p} \beta_{s}>\beta_{e} \alpha_{s}$ ) the existing product's price is increasing in $x_{2}$, while in case that $\alpha \beta_{s}<\beta \alpha_{s}$ (equivalently, $\alpha_{p} \beta_{s}<\beta_{e} \alpha_{s}$ ) the existing product's price is decreasing in $x_{2}$. In neutral market when $\alpha \beta_{s}=\beta \alpha_{s}$ (equivalently, $\alpha_{p} \beta_{s}=\beta_{e} \alpha_{s}$ ) the existing product's price is independent from $x_{2}$. However, the new product's price is always decreasing in $x_{2}$.

Intuitively, increasing the new product's carbon intensity (offer dirtier product) makes the retailer decrease its price. However, our findings show that the behavior of the existing product's price depends on market characteristics. In the neutral market, increasing the new product's carbon intensity has no effect on the existing product's price, while other markets either decrease or even increase its price. In the PDS market, the price switchover is stronger than the greenness switchover. Decreasing one unit of price attracts customers at a rate of $\alpha_{p}+\alpha_{s}$ for the existing product, which, $\alpha_{p}$ are "new" attracted customers and $\alpha_{s}$ are cannibalized from the new product. In this situation, the retailer gains more profit by decreasing the existing product's price. In the GDS market, the greenness switchover is stronger than the price. Unlike the previous case, the retailer is going to have more customers because of the cannibalization effect; therefore, the retailer increases the greener product's (new product's) price. In the neutral market, price and greenness are equally paid attention; the retailer's best strategy is to keep the existing product's price.

Proposition 4.5. In prices and stocks optimization problem ( $M_{1}$ ), increasing the gap between products' transportation carbon emission, $e_{1}$, leads to lower existing product's price and (i) lower the new product's price will be in PDS market and (ii) higher the new product's price will be in GDS market (iii) new product's price in the neutral market is independent of $e_{1}$.

Proof. The first derivatives of optimal prices ( $p_{1}^{*}$ and $p_{2}^{*}$, respectively), which are presented in proposition 4.1, with respect to the gap of the transportation carbon emission, $e_{1}$, are presented in the following.
$\frac{\partial p_{1}^{*}}{\partial e_{1}}=-\frac{\alpha \beta-\beta_{s} \alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$\frac{\partial p_{2}^{*}}{\partial e_{1}}=\frac{\alpha \beta_{s}-\beta \alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$
As we can see, the existing product's price is always decreasing in $e_{1}\left(\frac{\partial p_{1}^{*}}{\partial e_{1}}<0\right)$. However, the new product's price behavior is strongly related to the behavior of the customers (or market). In case that $\alpha \beta_{s}>\beta \alpha_{s}$ (equivalently, $\alpha_{p} \beta_{s}>\beta_{e} \alpha_{s}$ ) the new product's price is increasing in $e_{1}$, while in case that $\alpha \beta_{s}<\beta \alpha_{s}$ (equivalently, $\alpha_{p} \beta_{s}<\beta_{e} \alpha_{s}$ ) the new product's price is decreasing in $e_{1}$. In neutral market when $\alpha \beta_{s}=\beta \alpha_{s}$ (equivalently, $\alpha_{p} \beta_{s}=\beta_{e} \alpha_{s}$ ) the new product's price is independent from $e_{1}$.

The first part of proposition 4.3 is intuitive. Increasing $e_{1}$ means that the existing product's carbon intensity (i.e. $x_{1}+e_{1}$ ) increases. Therefore it is logical that the price decreases as dirtier as the existing product is to attract new customers. On the other hand, the new product's price behavior has a similar explanation as to the existing product's price in Proposition 4.3. As a result, we can say that in a product differentiation model, offering a dirtier product leads to a lower retail price for it. In contrast, the other product's retail price depends highly on the market. It can be higher in the GDS market or lower in the PDS market, or constant in the neutral market.

Proposition 4.6. In prices and stocks optimization problem ( $M_{1}$ ), increasing the gap between products' transportation carbon emission, $e_{1}$, decreases the existing product's order size and increases the new product's order size.

Proof. Thank to Proposition 4.1, the optimal order sizes are presented in the following.
$S_{1}^{*}=\frac{1-r}{2 r \mu_{1}}\left(A-\beta\left(x_{1}+e_{1}\right)+\beta_{s} x_{2}-\alpha\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)$
$S_{2}^{*}=\frac{1-r}{2 r \mu_{2}}\left(A-\beta x_{2}+\beta_{s}\left(x_{1}+e_{1}\right)-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)$
The first derivatives of stocks ( $S_{1}^{*}$ and $S_{2}^{*}$, respectievly) with respect to the gap of the transportation carbon emission, $e_{1}$, are presented in the following.
$\frac{\partial S_{1}^{*}}{\partial e_{1}}=-\frac{\beta(1-r)}{2 r \mu_{1}}$
$\frac{\partial S_{2}^{*}}{\partial e_{1}}=\frac{\beta_{s}(1-r)}{2 r \mu_{2}}$
As you can see, the existing product's stock is always decreasing in $e_{1}$. However, the new product's stock is increasing in $e_{1}$.

The result of Proposition 4.4 is quite intuitive. As we explained in the previous proposition, the retailer offers a dirtier product (existing product) to customers when $e_{1}$ increases. Despite that, the carbon intensity is not a decision variable in this model, but the demand is still sensitive. Thanks to Lemma 4.1, we know that the order sizes of products have a positive and direct relation with demands, which means the order size increases

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
when the demand increases. Thus, the retailer makes a higher order when there are more customers to respect the service level. Thus, as a result, there will be fewer customers for existing products and more customers for new products. Consequently, the existing product's order size decreases, and the new product's order size increases. Figure 4.4 illustrates what we discuss above.


Figure 4.4: Impact of $e_{1}$ on optimal order sizes

Proposition 4.7. In prices and stocks optimization problem $\left(M_{1}\right)$, the optimal profit is decreasing convex functions in transportation carbon emission, $e_{1}$.

Proof. The first and second derivatives of profit function are presented in the following.
$\frac{\partial \pi^{*}}{\partial e_{1}}=\frac{\partial\left(p_{1}^{*}-\left(c 1+\frac{h(1-r)}{r \mu 1}\right)\right)}{\partial e_{1}}\left(A_{1}^{\prime}-\alpha p_{1}^{*}+\alpha_{s} p_{2}^{*}\right)+\frac{\partial\left(A_{1}^{\prime}-\alpha p_{1}^{*}+\alpha_{s} p_{2}^{*}\right)}{\partial e_{1}}\left(p_{1}^{*}-\left(c 1+\frac{h(1-r)}{r \mu 1}\right)\right)$
$+\frac{\partial\left(p_{2}^{*}-\left(c 2+\frac{h(1-r)}{r \mu 2}\right)\right)}{\partial e_{1}}\left(A_{2}^{\prime}-\alpha p_{2}^{*}+\alpha_{s} p_{1}^{*}\right)+\frac{\partial\left(A_{2}^{\prime}-\alpha p_{2}^{*}+\alpha_{s} p_{1}^{*}\right)}{\partial e_{1}}\left(p_{2}^{*}-\left(c 2+\frac{h(1-r)}{r \mu 2}\right)\right)$
$=-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{A-\beta\left(x_{1}+e_{1}\right)+\beta_{s} x_{2}-\alpha\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\alpha_{s}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2}\right)$
$-\left(\frac{\beta}{2}\right)\left(\frac{\alpha}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha} A-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha}\left(x_{1}+e_{1}\right)+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha} x_{2}\right)-\frac{1}{2}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)$
$+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{A-\beta x_{2}+\beta_{s}\left(x_{1}+e_{1}\right)-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2}\right)$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$-\left(\frac{\beta_{s}}{2}\right)\left(\frac{\alpha_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(\frac{\alpha+\alpha_{s}}{\alpha_{s}} A+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha_{s}}\left(x_{1}+e_{1}\right)-\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha_{s}} x_{2}\right)-\frac{1}{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)$
$\frac{\partial^{2} \pi^{*}}{\partial e_{1}{ }^{2}}=\frac{\alpha \beta-\alpha_{s} \beta_{s}}{\alpha^{2}-\alpha_{s}^{2}}\left(\frac{\beta}{2}\right)+\frac{\alpha \beta_{s}-\beta \alpha_{s}}{\alpha^{2}-\alpha_{s}^{2}}\left(\frac{\beta_{s}}{2}\right)=\frac{\alpha \beta^{2}+\alpha \beta_{s}^{2}-2 \alpha_{s} \beta_{s} \beta}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$
$=\frac{\alpha_{p} \beta^{2}+\alpha_{s}\left(\beta_{e}+\beta_{s}\right)^{2}+\left(\alpha_{p}+\alpha_{s}\right) \beta_{s}^{2}-2 \alpha_{s} \beta_{s}\left(\beta_{e}+\beta_{s}\right)}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}=\frac{\alpha_{p}\left(\beta^{2}+\beta_{s}^{2}\right)+\alpha_{s} \beta_{e}^{2}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}$
It is proven that the second derivative of profit with respect to $e_{1}$ is positive and, then, the profit is convex in $e_{1}$.
$\frac{\partial \pi^{*}}{\partial e_{1}}=-\frac{\alpha_{p} \beta_{e}+2 \alpha_{s} \beta_{e}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)} A+\frac{\alpha_{p}\left(\beta^{2}+\beta_{s}^{2}\right)+\alpha_{s} \beta_{e}^{2}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(x_{1}+e_{1}\right)-\frac{\alpha_{s} \beta_{e}^{2}+2 \alpha_{p} \beta_{s}^{2}+2 \alpha_{p} \beta_{e} \beta_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)} x_{2}$ $+\left(\frac{\beta}{2}\right)\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\left(\frac{\beta_{s}}{2}\right)\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)$
The first derivative of optimal profit with respect to $e_{1}$ is negative considering that $A$ is sufficiently high to satisfy following condition:

$$
\begin{aligned}
& A \geq \frac{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}{\alpha_{p} \beta_{e}+2 \alpha_{s} \beta_{e}}\left(\frac{\alpha_{p}\left(\beta^{2}+\beta_{s}^{2}\right)+\alpha_{s} \beta_{e}^{2}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)}\left(x_{1}+e_{1}\right)-\frac{\alpha_{s} \beta_{e}^{2}+2 \alpha_{p} \beta_{s}^{2}+2 \alpha_{p} \beta_{e} \beta_{s}}{2\left(\alpha^{2}-\alpha_{s}^{2}\right)} x_{2}\right. \\
& \left.+\left(\frac{\beta}{2}\right)\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\left(\frac{\beta_{s}}{2}\right)\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)
\end{aligned}
$$

This result is intuitive. The market is sensitive to greenness, therefore, it is intuitive that the retailer loses profits when he/she offers a dirtier product. Increasing transportation carbon emission results in losing customers for the retailer at rate $\beta_{e}$, because total demand is $\lambda=\lambda_{1}+\lambda_{2}=2 A-\alpha_{p}\left(p_{1}+p_{2}\right)-\beta_{e}\left(x_{1}+x_{2}+e_{1}\right)$. This proposition demonstrates the importance of transportation carbon emission, which depends on the distance between the retailer and the supplier. The retailer's best strategy would be choosing the closer supplier if we assume that the other factors like production cost are the same.

Proposition 4.8. Increasing market potential to infinite leads to (i) increasing the new product's carbon intensity that approaches to $x_{1}$ (optimal decision in benchmark model) in the PDS market and (ii) decreasing the new product's carbon intensity that approaches to $x_{1}$ in GDS market. The new product's carbon intensity is constant in the neutral market.

Proof. The roots of cubic equation, which is presented in proposition 4.7, can be found using Cardano formula:
$r_{1}=\sqrt[3]{R+\sqrt{Q^{3}+R^{2}}}+\sqrt[3]{R-\sqrt{Q^{3}+R^{2}}}+T$

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
$r_{2}=-\frac{\sqrt[3]{R+\sqrt{Q^{3}+R^{2}}}+\sqrt[3]{R-\sqrt{Q^{3}+R^{2}}}}{2}+T+\frac{i \sqrt{3}}{2}\left(\sqrt[3]{R+\sqrt{Q^{3}+R^{2}}}-\sqrt[3]{R-\sqrt{Q^{3}+R^{2}}}\right)$
$r_{3}=-\frac{\sqrt[3]{R+\sqrt{Q^{3}+R^{2}}}+\sqrt[3]{R-\sqrt{Q^{3}+R^{2}}}}{2}+T-\frac{i \sqrt{3}}{2}\left(\sqrt[3]{R+\sqrt{Q^{3}+R^{2}}}-\sqrt[3]{R-\sqrt{Q^{3}+R^{2}}}\right)$
where
$T=x_{0}-\frac{\beta}{2 \alpha b}$
$Q=\frac{A_{2}^{\prime}-\beta x_{1}-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{4 \alpha b}}{-3 \alpha b}$
$R=\frac{\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)\left(\alpha \beta_{s}-\beta \alpha_{s}\right)}{-2 \alpha^{2} b^{2}}$
Without loss of generality, let us consider $r_{1}=\sqrt[3]{R+\sqrt{Q^{3}+R^{2}}}+\sqrt[3]{R-\sqrt{Q^{3}+R^{2}}}+T$ as the optimal solution. We can rewrite $r_{1}$ as $\sqrt[3]{Q\left(\frac{R}{Q}+\sqrt{Q+\frac{R^{2}}{Q}}\right)}+\sqrt[3]{Q\left(\frac{R}{Q}-\sqrt{Q+\frac{R^{2}}{Q}}\right)}+$ $T$ (we can do the same thing for $r_{2}$ and $r_{3}$ ). Increasing $A$ results in:
$\lim _{A \rightarrow \infty} \frac{R}{Q}=\lim _{A \rightarrow \infty} \frac{\frac{\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)\left(\alpha \beta_{s}-\beta \alpha_{s}\right)}{A_{2}^{\prime}-\beta x_{1}-\alpha\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{4 \alpha b}}}{-3 \alpha b}=0$
As much as $A$ increases, $\frac{R}{Q}$ goes to zero. In GDS market $\lim _{A \rightarrow \infty} r_{1}=T+\varepsilon$, which means that the optimal carbon emission converge towards $T$ (decreasing). On the other hand, in PDS market $\lim _{A \rightarrow \infty} r_{1}=T-\varepsilon$, which means that optimal carbon emission converge towards $T$ (increasing).

In a huge market that each product has an infinite demand, the difference between demands based on their price and carbon intensity is negligible. In other words, the differentiation between products has a small effect on the retailer's optimal profit. It means that even the retailer offers just one product instead of two substitutable products, he/she does not lose a considerable amount of profit. Consequently, the product differentiation problem becomes the benchmark problem where he/she offers one product. As a result, it is logical that we say as market potential increases, the more product differentiation

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
model becomes to benchmark, which means products become more similar to price and greenness point of view. Figure 4.5 illustrates the Proposition 4.8 .


Figure 4.5: Impact of market potential on optimal carbon emission level

Proposition 4.9. The new products that are offered in neutral market have, (i) higher price than new products in GDS market and (ii) lower price than new products in PDS market.

The first derivative of optimal price of new product $\left(p_{2}\right)$ is negative with respect to its carbon intensity ( $x_{2}$ ), consequently, $p_{2}$ has a negative relation with $x_{2}$. Thanks to proposition 4.8, we know that the new product is offered with higher carbon intensity in the GDS market (Let us call it $x_{2}^{* G D S}$ ) than the neutral market (Let us call it $x_{2}^{* N}$ ). In the same way, it is offered with higher carbon intensity in the neutral market than PDS (Let us call it $x_{2}^{* P D S}$ ). Briefly, $x_{2}^{* G D S}>x_{2}^{* N}>x_{2}^{* P D S}$. Intuitively, dirtier product (i.e., higher carbon intensity) has a lower price. Therefore, the retailer offers a new product with a higher price in the PDS market than the neutral market and a higher price in the neutral market than the GDS market.

### 4.4 Conclusion

In this chapter, a product differentiation problem is considered such that a retailer offers two substitutable products to customers in a price and carbon emission sensitive market. The retailer follows the MTS policy, keeps the products, and serves them as soon as a customer arrives. Since the products are substitutable, the products' demand not only

Chapter 4. Product differentiation in retailing under price- and greenness-sensitive demand
depends on their own price and carbon emission but also depends on other product's price and carbon emission. As a benchmark, we consider the retailer when it offers one product to customers. We formulate the problems under different settings (considering different sets of variables) and solve them through an analytical approach. The optimal solutions are provided by closed-form expressions. Finally, we distinguished different markets' categories to extract important insights into our results. The results showed the impact of market characteristics and substitution on the retailer's strategy. The insights are provided in different propositions to cover all the problems considered in this study. A numerical example is also done to show the difference between the retailer's profit when it offers one product, and when it offers two substitutable products. As future work, we would like to consider a more general demand function (e.g., non-linear concerning greenness) and compare the product differentiation scenario results.

# Optimization of Retailers' Strategies Under Dynamic Competition Environment 

The previous chapter discussed about products differentiation context. In this chapter, we consider two competing retailers that offer two substitutable products in a greennessand price-sensitive market. Each retailer has its own supplier. Demands and the replenishment lead times of retailers' stocks are random. The mean demand for each product decreases in its carbon emission intensity and price, and increases in other product's carbon emission intensity and price. Like previous chapters, the emissions of production and transportation activities are considered as the carbon emission intensity. The transportation emissions depend basically on the location of the supplier. The production emissions occur at suppliers' sites. They can be reduced, but this leads to a higher manufacturing cost for the supplier (Conrad, 2005), which implies a higher purchasing cost for the retailer. In the general case, each retailer decides the price, the carbon emission intensity, and the order size of the product that he/she is offering to maximize the expected profit while satisfying a service level constraint. This chapter closes the gap by studying the dynamic competition from a carbon emission perspective. We address these main research questions in this chapter:

- How greenness-driven competition affects the retailers' best strategies?
- Do different market structures affect retailers' best strategies?


### 5.1 Problem description

We consider two retailers that sell substitutable products, differentiated in terms of greenness level (i.e, carbon emission intensity) and price, in a greenness- and pricesensitive market (see Figure 5.1 below). Customers arrive according to a Poisson process with mean arrival rate $\lambda_{i}$ for the retailer $i$. The mean demand of each product is decreasing in its carbon emission intensity and price and increasing in other product's carbon emission intensity and price. The demand function will be introduced and discussed later in this section. Each retailer has its own supplier. The product sold by retailer $i$ is denoted by $P_{i}$. The main sources of carbon emissions are the transportation and production activities. The transportation emissions per unit of product, denoted by $e_{i}$ for retailer $i$, depend on the distance traveled by the product from the supplier's site to the retailer's warehouse.

## Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition

 EnvironmentAs we do not deal with supplier selection, $e_{i}$ is not a decision variable. Without loss of generality, we assume that the transportation emissions from the retailers' warehouses to end customers can be neglected, which implicitly means that the retailers are located close to the demand zone. As for production emissions, we consider a context where the retailer can choose the production emission level and, thus, ask for the supplier to manufacture the product accordingly. We provided chapter 1 some examples that illustrate how leading retailers, such as Walmart, are asking their suppliers to produce greener products. We let $x_{0}$ denote the amount of production emissions per unit of a standard product. Offering a greener product (with lower production emissions) implies a higher purchasing cost for the retailer as it incurs a higher manufacturing cost. We let $x_{i}$ denote the amount of production emissions per unit of product $P_{i}$. The carbon emission intensity of $P_{i}$ is thus given by $e_{i}+x_{i}$. The unitary purchasing cost of $P_{i}$ is given by $c_{i}+b\left(x_{0}-x_{i}\right)^{2}$, where $c_{i}$ is the unitary cost of the standard product and $b$ is the cost factor for production emissions reduction. We consider a quadratic cost function as usual in the related literature (e.g., Liu et al., 2012, Ghosh and Shah, 2015).

The replenishment order size of retailer $i$ is denoted by $S_{i}$. The service rate to refill the stock is exponentially distributed with mean rate $\mu_{i}$ for retailer $i$. The replenishment time does not depend on the order size since the products are assumed to be always available at the supplier's site, which is a common assumption (Zhu, 2015). Thus, the replenishment time corresponds basically to preparation and transportation activities. The retailers' inventory policy is similar to the previous chapter and, thus, we are going to skip explanation to avoid redundancy. As a reminder, the probability of satisfying demand from the standard stock, denoted by $1-\psi_{i}$ for retailer $i$, must be greater than $1-r$, for both retailers. Hence, the service level constraint for retailer $i$ is given by $\left(1+\frac{\mu_{i} S_{i}}{\lambda_{i}}\right)^{-1} \leq r$ (See Chapter 3 for more information).

Since each retailer has its own supplier, we typically have a different distance separating each retailer from its supplier. Without loss of generality, we let retailer 1 represents the retailer that has the farthest supplier. Therefore, we have $e_{1} \geq e_{2}$ and $\mu_{1} \leq \mu_{2}$.

Our demand model is linear with substitution. We recall that $\lambda_{i}$ refers to the mean demand rate for retailer $i$. The market potential is denoted by A . We let $\alpha_{p}$ and $\beta_{e}$ respectively denote the market sensitivity to the price and the carbon emission intensity. As for switchovers, we respectively denote by $\alpha_{s}$ and $\beta_{s}$, the sensitivity of switchover toward price difference and carbon emission intensity difference.

In practice, the retailers may have different market powers such as the case where there is an established leading retailer and a new retailer entering the market. In this case, the leading retailer has more market power and, thus, attracts more customers even when the same product is offered by both retailers. We let $\omega \in[0,1]$ denote the market share of retailer 1 and, thus, $(1-\omega)$ is the market share of retailer 2 , when both retailers offer the same product (i.e., with the same price $p$ and carbon emission intensity $x+e$ ). The mean demand rates are given as follows.

$$
\begin{equation*}
\lambda_{1}=\omega A-\omega \alpha_{p} p_{1}+\alpha_{s}\left(p_{2}-p_{1}\right)-\omega \beta_{e}\left(x_{1}+e_{1}\right)+\beta_{s}\left(\left(x_{2}+e_{2}\right)-\left(x_{1}+e_{1}\right)\right) \tag{5.1}
\end{equation*}
$$

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment


Figure 5.1: Competitive supply chains structure
$\lambda_{2}=(1-\omega) A-(1-\omega) \alpha_{p} p_{2}+\alpha_{s}\left(p_{1}-p_{2}\right)-(1-\omega) \beta_{e}\left(x_{2}+e_{2}\right)+\beta_{s}\left(\left(x_{1}+e_{1}\right)-\left(x_{2}+e_{2}\right)\right)$
Notice that $\lambda_{1}+\lambda_{2}=A-\omega \alpha_{p} p_{1}-(1-\omega) \alpha_{p} p_{2}-\omega \beta_{e}\left(x_{1}+e_{1}\right)-(1-\omega) \beta_{e}\left(x_{2}+\right.$ $e_{2}$ ). This means that the total mean demand is sensitive to prices and carbon emission intensities.

If the same product is offered by both retailers, then the mean demand is $\omega\left(A-\alpha_{p} p-\right.$ $\left.\beta_{e}(x+e)\right)$ for retailer 1 and $(1-\omega)\left(A-\alpha_{p} p-\beta_{e}(x+e)\right)$ for retailer 2 , and the total mean demand does not depend on $\omega$. The case where both retailers have the same market power, which is the typical case studied in the literature, corresponds to $\omega=0.5$. In our model, the market share $\omega$ represents the market power of retailer 1 . If $\omega>0.5$, then retailer 1 has more market power than retailer 2 , and vice versa. With the consideration of $\omega \in[0,1]$, we generalize the linear demand with substitution that is usually adopted in the related literature.

To better understand our demand function, notice that retailer 1 can attract $\omega \alpha_{p}+\alpha_{s}$ more customers with one unit decrease in price (respectively, $(1-\omega) \alpha_{p}+\alpha_{s}$ more customers for retailer 2) and $\omega \beta_{e}+\beta_{s}$ more customers with one unit decrease in carbon emission intensity (respectively, $(1-\omega) \beta_{e}+\beta_{s}$ for retailer 2 ). Only a part of these customers represent a new created demand in the market (for retailer $1, \omega \alpha_{p}$ new customers for one unit decrease in price and $\omega \beta_{e}$ new customers for one unit decrease in carbon emission intensity), and the other attracted customers are switching from the other retailer ( $\alpha_{s}$ switching customers for a unit decrease in price and $\beta_{s}$ switching customers for a unit decrease in carbon emission intensity).

Since $e_{1} \geq e_{2}$ and both of them are fixed parameters, we can consider, without loss of
generality, that $e_{2}=0$ and, thus, $e_{1}$ can be interpreted as the difference in transportation emissions. To simplify notation, we let $A_{1}=\omega A-\theta_{1} e_{1}, A_{2}=(1-\omega) A+\beta_{s} e_{1}, \delta_{1}=$ $\omega \alpha_{p}+\alpha_{s}, \delta_{2}=(1-\omega) \alpha_{p}+\alpha_{s}, \theta_{1}=\omega \beta_{e}+\beta_{s}$ and $\theta_{2}=(1-\omega) \beta_{e}+\beta_{s}$. The mean demand rates are finally given as follows.

$$
\begin{align*}
& \lambda_{1}=A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}  \tag{5.3}\\
& \lambda_{2}=A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1} \tag{5.4}
\end{align*}
$$

Based on the framework describe above, we then study different competition scenarios and investigate their impact on the greenness level and price of the products. In the competition with full reaction case, we consider that each retailer decides the greenness level of its product (i.e., $x_{i}$ ), the price $\left(p_{i}\right)$ and the order size $\left(S_{i}\right)$ to maximize its expected profit under the service level constraint while considering other retailer's decisions. We also consider two other particular settings that can be particularly relevant when there is an established retailer and a new retailer that enters the market. In fact, we consider that retailer 2 enters the market and decides $x_{2}, p_{2}$ and $S_{2}$ whereas, in the first particular setting, retailer 1 does not react and, in the second particular setting, retailer 1 reacts by adjusting its price $p_{1}$ and order size $S_{1}$, but without changing the greenness level, $x_{1}$, as this was agreed upon with its supplier. In the scenarios of competition with reaction (i.e., the general case and the second particular case), we consider a non-cooperative game and determine the optimal strategy of each retailer at the Nash equilibrium. In the following section, we formulate the different models and present the analytical solving approaches and the closed-from expressions of optimal solutions.

### 5.2 Mathematical models and analytical solutions

In this technical section, we solve the problem under different settings (decision variables) and provide the closed-form expressions of the optimal solutions. Then, we analyze these optimal solutions to derive managerial insights.

### 5.2.1 Competition with full reaction $\left(M_{4}\right)$

In this sub-section, we are looking forward to finding out that what first retailer's optimal strategies will be if he/she changes the product's carbon emission level also (i.e. considering $x_{1}$ as variable) with considering newcomer's decisions are and how newcomer reacts to the other retailer's new strategies. This chain of reaction between these two retailers, old one and newcomer, continues until there are no new strategies that improve both retailers' profit. The newcomer decides product's carbon emission level ( $x_{2}$ ), price $\left(p_{2}\right)$ and order size $\left(S_{2}\right)$. Meanwhile, the existed retailer decides product's carbon emission level $\left(x_{1}\right)$, price $\left(p_{1}\right)$ and order size $\left(S_{1}\right)$. We formulate the mathematical model of the problem as:

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Retailer 1's model
$\operatorname{Max}_{x_{1}, p_{1}, S_{1}} \pi_{1}=\left(p_{1}-\left(c_{1}+b\left(x_{0}-x_{1}\right)^{2}\right)\right) \lambda_{1}-h S_{1}$
Subject to
$\psi_{1}=\frac{\lambda_{1}}{\lambda_{1}+S_{1} \mu_{1}} \leq r$
$\lambda_{1}=A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}$
$p_{1}, S_{1}, \lambda_{1} \geq 0,0 \leq x_{1} \leq x_{0}$

Retailer 2's model
$\operatorname{Max}_{x_{2}, p_{2}, S_{2}} \pi_{2}=\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}\right)\right) \lambda_{2}-h S_{2}$
Subject to

$$
\begin{aligned}
& \psi_{2}=\frac{\lambda_{2}}{\lambda_{2}+S_{2} \mu_{2}} \leq r \\
& \lambda_{2}=A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1} \\
& p_{2}, S_{2}, \lambda_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{aligned}
$$

Each retailer objective is to maximize its profit function under other retailer's decisions. The profit function is the net profit of selling product, which is total revenue of selling product minus production and inventory costs. Each retailer has a service level constraint that ensures probability of having no stock is less than a predetermined amount. It is recalled that $A_{1}=\omega A-\theta_{1} e_{1}, A_{2}=(1-\omega) A+\beta_{s} e_{1}, \delta_{1}=\omega \alpha_{p}+\alpha_{s}, \delta_{2}=(1-\omega) \alpha_{p}+\alpha_{s}$, $\theta_{1}=\omega \beta_{e}+\beta_{s}$ and $\theta_{2}=(1-\omega) \beta_{e}+\beta_{s}$. It is also recalled that $\omega \in[0,1]$ represents the market power of retailer 1 and, thus, $1-\omega$ is the market power for retailer 2 .

To find the Nash equilibrium strategies, we first consider that retailer 2's decisions are known and determine retailer 1's best response, which means that we need to solve retailer 1's model. Then we consider that retailer 1's decisions are known and solve retailer 2's model to obtain retailer 2's best response. We finally determine the Nash equilibrium of the game.

We now determine the best strategy of retailer 1 given retailer 2's decisions. Thus, we need to solve model for a given values of $x_{2}, p_{2}$, and $S_{2}$. To solve this model, we first transform it into a single-variable model based on the results of Lemma 5.1 and 5.2 given below.

Lemma 5.1. For a given strategy of retailer 2 (i.e., given $x_{2}, p_{2}$, and $S_{2}$ ), the retailer l's optimal order size as a function of $x_{1}$ and $p_{1}$ is
$S_{1}^{*}=\frac{(1-r)\left(A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}\right)}{r \mu_{1}}$

Proof. Since the objective function is linearly decreasing in $S_{1}$, the smallest possible $S_{1}$ is the optimal stock. According to the service level constraint, $S_{1} \geq \frac{(1-r) \lambda_{1}}{r \mu_{1}}$, therefore, the optimal value is $S_{1}{ }^{*}=\frac{(1-r)}{r \mu_{1}}\left(A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}\right)$, which implies that service level constraint is binding.

We, thus, replace $S_{1}^{*}$ with its expression given in Lemma 5.1 and obtain the following equivalent formulation of model $\left(M_{4}\right)$ with two variables $p_{1}$ and $x_{1}$.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition
$\underset{p_{1}, x_{1}}{\operatorname{Maximize}} \pi=\left(p_{1}-\left(c_{1}-b\left(x_{0}-x_{1}\right)^{2}+\frac{h(1-r)}{r \mu_{1}}\right)\right) \lambda_{1}$
Subject to

$$
\begin{aligned}
& \lambda_{1}=A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2} \\
& \lambda_{1}, p_{1} \geq 0,0 \leq x_{1} \leq x_{0}
\end{aligned}
$$

In the following Lemma, we determine the optimal price.

Lemma 5.2. For a given strategy of retailer 2 (i.e., given $x_{2}, p_{2}$, and $S_{2}$ ), the retailer l's optimal price as a function of $x_{1}$ is

$$
p_{1}^{*}=\frac{\delta_{1} b x_{1}^{2}-\left(\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}
$$

Proof. The second derivative of objective function with respect to $p_{1}$, which is presented in the following, is negative and demonstrates that objective function is concave in $p_{1}$. Therefore the root of first derivative (called $p_{1}{ }^{* \max }$ ) maximize the objective function.
$\frac{\partial \pi}{\partial p_{1}}=-2 \delta_{1} p_{1}+\delta_{1} b x_{1}^{2}-\left(\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right)$
$\frac{\partial^{2} \pi}{\partial p_{1}{ }^{2}}=-2 \delta_{1}$
$\frac{\partial \pi}{\partial p_{1}}=0 \Leftrightarrow p_{1}{ }^{* \max }=\frac{\delta_{1} b x_{1}{ }^{2}-\left(\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$
If $p_{1}{ }^{* \max }>0$, then it can be considered as the optimal price. The condition that $p_{1}{ }^{* \max }$ is positive, is presented in the following.
$p_{1}{ }^{* \max }=\frac{\delta_{1} b x_{1}^{2}-\left(\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}} \geq 0$
$\Leftrightarrow \delta_{1} b x_{1}^{2}-\left(\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right) \geq 0$
The discriminant of above equation is $\Delta_{4}=-4 \delta_{1} b\left(A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\right.$ $\left.\theta_{1} x_{0}-\frac{\theta_{1}^{2}}{4 \delta_{1} b}\right)$. We know that $x_{2}, p_{2} \geq 0$, then, $\Delta_{4}$ is negative as long as we consider a con-

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment
straint such that $A_{1} \geq \theta_{1} x_{0}+\frac{\theta_{1}^{2}}{4 \delta_{1} b}-\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)$. Because $\delta_{1} b$ (coefficient of $x_{1}{ }^{2}$ ) is positive and $\Delta_{4}<0$, then, $p_{1}{ }^{* \max }$ is always positive.

The last constraint is related to the demand positivity. The $p_{1}{ }^{* m a x}$ is the optimal solution if $\lambda_{1}\left(p_{1}{ }^{* \max }\right) \geq 0$. The condition that $\lambda_{1} \geq 0$ is presented in the following.
$\lambda_{1}=\frac{-\delta_{1} b x_{1}^{2}+\left(-\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right)}{2} \geq 0$
$\Leftrightarrow-\delta_{1} b x_{1}^{2}+\left(-\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right) \geq 0$
The discriminant of the above equation is equal to $\Delta_{5}=4 \delta_{1} b\left(A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+\right.\right.$ $\left.\left.\frac{h(1-r)}{r \mu_{1}}\right)-\theta_{1} x_{0}+\frac{\theta_{1}^{2}}{4 \delta_{1} b}\right)$. Like earlier, We know that $x_{2}, p_{2} \geq 0$ and $\Delta_{5}$ is positive as long as we assume a constraint such that $A_{1} \geq \theta_{1} x_{0}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\frac{\theta_{1}^{2}}{4 \delta_{1} b}$. Therefore, the above equation has two roots (called $R_{5}$ and $R_{6}$ ), which is positive between these two roots and negative outside them. Therefore, the demand is positive between these roots.
$R_{5}=\frac{-\left(-\theta_{1}+2 \delta_{1} b x_{0}\right)-\sqrt{\Delta_{5}}}{-2 \delta_{1} b}=x_{0}+\frac{-\theta_{1}+\sqrt{\Delta_{5}}}{2 \delta_{1} b}$,
$R_{6}=\frac{-\left(-\theta_{1}+2 \delta_{1} b x_{0}\right)+\sqrt{\Delta_{5}}}{-2 \delta_{1} b}=x_{0}-\frac{\theta_{1}+\sqrt{\Delta_{5}}}{2 \delta_{1} b}$.
We have $R_{5}>x_{0}$ and $R_{6}<0$ (more details in Appendix D . Therefore, demand is positive in the feasible region $\left(\left[0, x_{0}\right]\right)$. As a result $p_{1}{ }^{* \max }=p_{1}{ }^{*}$.

Thanks to the result of the previous Lemma, we formulate model with only one variable $x_{1}$ as follows.
$\underset{0 \leq x_{1} \leq x_{0}}{\operatorname{Maximize}} \pi=$

$$
\left(-\delta_{1} b x_{1}^{2}+\left(-\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right)\right)^{2}
$$

$$
\begin{equation*}
4 \delta_{1} \tag{5.6}
\end{equation*}
$$

Subject to
$\lambda_{1}=\frac{-\delta_{1} b x_{1}^{2}+\left(-\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right)}{2}$

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Proposition 5.1. The optimal retailer I's carbon emission is $x_{1}^{*}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}$.
Proof. The objective function can be presented as $\frac{\lambda_{1}{ }^{2}}{\delta_{1}}$. Since $\lambda_{1} \geq 0$ (see Lemma 5.2, , maximizing $\lambda_{1}$ is equivalent to maximizing $\frac{\lambda_{1}{ }^{2}}{\delta_{1}}$ (objective function). The second derivative of demand with respect to the $x_{1}$, which is presented in the following, is negative and the root of the first derivative is the $x_{1}{ }^{* \max }$ that maximize the demand (equivalently objective function).

$$
\begin{aligned}
& \frac{\partial \lambda_{1}}{\partial x_{1}}=\frac{-2 \delta_{1} b x_{1}+\left(2 \delta_{1} b x_{0}-\theta_{1}\right)}{2} \\
& \frac{\partial^{2} \lambda_{1}}{\partial x_{1}{ }^{2}}=-\delta_{1} b \\
& \frac{\partial \lambda_{1}}{\partial x_{1}}=0 \Leftrightarrow x_{1}{ }^{* \text { max }}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}
\end{aligned}
$$

It is obvious that $x_{1}{ }^{* \max }<x_{0}$. While the $x_{1}{ }^{* \max }$ is in the feasible region, it is the optimal solution, otherwise, is equal to zero. Thus $x_{1}{ }^{*}=\max \left\{0, x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}\right\}$

We now assume that retailer 1's decisions (i.e., $x_{1}, p_{1}$ and $S_{1}$ ) are known and solve model for retailer 2. It is noted that models are symmetric. Therefore, we use the same approach to solve retailer 2's model.

Lemma 5.3. For a given strategy of retailer 1 (i.e., given $x_{1}, p_{1}$, and $S_{1}$ ), the retailer 2 's optimal order size as a function of $x_{2}$ and $p_{2}$ is
$S_{2}^{*}=\frac{(1-r)\left(A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1}\right)}{r \mu_{2}}$
The proof is similar to Lemma 5.1 . We, thus, replace $S_{2}^{*}$ with its expression given in Lemma 5.3 and obtain the following equivalent model with two variables $p_{2}$ and $x_{2}$.
$\underset{p_{2}, x_{2}}{\operatorname{Maximize}} \pi=\left(p_{2}-\left(c_{2}-b\left(x_{0}-x_{2}\right)^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right) \lambda_{2}$
Subject to

$$
\begin{aligned}
& \lambda_{2}=A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1} \\
& \lambda_{2}, p_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{aligned}
$$

In the following Lemma, we determine the optimal price.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Lemma 5.4. For a given strategy of retailer 1 (i.e., given $x_{1}, p_{1}$, and $S_{1}$ ), the retailer 2's optimal price as a function of $x_{2}$ is

$$
p_{2}^{*}=\frac{\delta_{2} b x_{2}^{2}-\left(\theta_{2}+2 \delta_{2} b x_{0}\right) x_{2}+A_{2}+\alpha_{s} p_{1}+\beta_{s} x_{1}+\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}
$$

The proof is similar to Lemma 5.2. Thanks to the result of the previous Lemma, we formulate model with only one variable $x_{2}$ as follows.
$\underset{0 \leq x_{2} \leq x_{0}}{\operatorname{Maximize}} \pi=$

$$
\left(-\delta_{2} b x_{2}^{2}+\left(-\theta_{2}+2 \delta_{2} b x_{0}\right) x_{2}+A_{2}+\alpha_{s} p_{1}+\beta_{s} x_{1}-\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)^{2}
$$

$$
\begin{equation*}
4 \delta_{2} \tag{5.9}
\end{equation*}
$$

Subject to
$\lambda_{1}=\frac{-\delta_{2} b x_{2}^{2}+\left(-\theta_{2}+2 \delta_{2} b x_{0}\right) x_{2}+A_{2}+\alpha_{s} p_{1}+\beta_{s} x_{1}-\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2}$

Proposition 5.2. The optimal retailer 2 's carbon emission is $x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}$.
The proof is similar to Proposition 5.1 .
Based on the analysis presented above, we can now derive the best response of each retailer to other retailer's decisions. Thanks to Proposition 5.1 and 5.2, we substitute $x_{1}$ and $x_{2}$ by their expression $x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}$ and $x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}$, respectively. Therefore, the optimal expression of $p_{1}$ given in Lemma 5.2 becomes:

$$
\begin{equation*}
p_{1}^{*}\left(p_{2}\right)=\frac{\alpha_{s}}{2 \delta_{1}} p_{2}+\frac{A_{1}+\left(\beta_{s}-\theta_{1}\right) x_{0}+\frac{3 \theta_{1}^{2}}{4 \delta_{1} b}-\frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}} \tag{5.11}
\end{equation*}
$$

Similarly, the optimal expression of $p_{2}$ given in Lemma 5.4 becomes:

$$
\begin{equation*}
p_{2}^{*}\left(p_{1}\right)=\frac{\alpha_{s}}{2 \delta_{2}} p_{1}+\frac{A_{2}+\left(\beta_{s}-\theta_{2}\right) x_{0}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}-\frac{\beta_{s} \theta_{1}}{2 \delta_{1} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}} \tag{5.12}
\end{equation*}
$$

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

The response of each player is, thus, linear in other player's decision. Consequently, the intersection of the two best response curves is the Nash equilibrium (Osborne et al., 2004), as illustrated in Figure 5.2. Based on the above analysis, we finally derive in the following


Figure 5.2: Illustration of the Nash equilibrium

Proposition the optimal strategy of each retailer at the Nash equilibrium.

Proposition 5.3. The optimal strategy of each retailer at the Nash equilibrium is the following.
For retailer 1: $x_{1}^{*}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}, p_{1}^{*}=\frac{2 \delta_{2}\left(2 \delta_{2} z_{1}+\alpha_{s} z_{2}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}$, and
$S_{1}^{*}=\frac{1-r}{r \mu_{1}}\left(A_{1}+\frac{2 \delta_{1}\left(-2 \delta_{1} \delta_{2}+\alpha_{s}^{2}\right) z_{1}+2 \alpha_{s} \delta_{1} \delta_{2} z_{2}}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}-\omega \beta_{e}\left(x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}\right)+\right.$
$\left.\frac{\beta_{s}}{2 b}\left(\frac{\theta_{1}}{\delta_{1}}-\frac{\theta_{2}}{\delta_{2}}\right)\right)$.
For retailer $2: x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}, p_{2}^{*}=\frac{2 \delta_{1}\left(2 \delta_{2} z_{2}+\alpha_{s} z_{1}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}$, and
$S_{2}^{*}=\frac{1-r}{r \mu_{2}}\left(A_{2}+\frac{2 \delta_{2}\left(-2 \delta_{1} \delta_{2}+\alpha_{s}^{2}\right) z_{2}+2 \alpha_{s} \delta_{1} \delta_{2} z_{1}}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}-(1-\omega) \beta_{e}\left(x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}\right)\right.$
$\left.+\frac{\beta_{s}}{2 b}\left(\frac{\theta_{2}}{\delta_{2}}-\frac{\theta_{1}}{\delta_{1}}\right)\right)$.
Where $z_{1}=\frac{A_{1}+\left(\beta_{s}-\theta_{1}\right) x_{0}+\frac{3 \theta_{1}{ }^{2}}{4 \delta_{1} b}-\frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$ and

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

$$
z_{2}=\frac{A_{2}+\left(\beta_{s}-\theta_{2}\right) x_{0}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}-\frac{\beta_{s} \theta_{1}}{2 \delta_{1} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}
$$

Proof. We know that the optimal prices $p_{1}^{*}$ and $p_{2}^{*}$ are obtained at the intersection of the two best response curves. If we take the optimal price $p_{1}^{*}$, we can determine its associated $p_{2}^{*}$ as a function of $p_{1}^{*}$ by using equation 5.12. The optimal price of retailer 1's product that is associated with this $p_{2}^{*}$ is, on the one hand, obtained by equation 5.11 and, on the other hand, equal to $p_{1}^{*}$ since we are at the intersection point.

Consequently, it comes that $p_{1}=\frac{\alpha_{s}}{2 \delta_{1}}\left(\frac{\alpha_{s}}{2 \delta_{2}} p_{1}+\frac{1}{2 \delta_{2}}\left(A_{2}+\left(\beta_{s}-\theta_{2}\right) x_{0}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}-\right.\right.$ $\left.\left.\frac{\beta_{s} \theta_{1}}{2 \delta_{1} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)\right)+\frac{1}{2 \delta_{1}}\left(A_{1}+\left(\beta_{s}-\theta_{1}\right) x_{0}+\frac{3 \theta_{1}{ }^{2}}{4 \delta_{1} b}-\frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right)$.
We then deduce by standard calculus that $p_{1}^{*}=\frac{2 \delta_{2}\left(2 \delta_{2} z_{1}+\alpha_{s} z_{2}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}{ }^{2}}$ and, consequently, $p_{2}^{*}=\frac{2 \delta_{1}\left(2 \delta_{2} z_{2}+\alpha_{s} z_{1}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}$. The optimal expressions of order sizes are given in Lemma 5.1 and 5.3 . The optimal order sizes are deduced from the formula $S_{1}^{*}=\frac{1-r}{r \mu_{1}}\left(A_{1}+\right.$ $\left.\frac{1}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}\left(2 \delta_{1}\left(-2 \delta_{1} \delta_{2}+\alpha_{s}^{2}\right) z_{1}+2 \alpha_{s} \delta_{1} \delta_{2} z_{2}\right)-\omega \beta_{e}\left(x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}\right)+\frac{\beta_{s}}{2 b}\left(\frac{\theta_{1}}{\delta_{1}}-\frac{\theta_{2}}{\delta_{2}}\right)\right)$ and $S_{2}^{*}=\frac{1-r}{r \mu_{2}}\left(A_{2}+\frac{1}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}\left(2 \delta_{2}\left(-2 \delta_{1} \delta_{2}+\alpha_{s}^{2}\right) z_{2}+2 \alpha_{s} \delta_{1} \delta_{2} z_{1}\right)-(1-\omega) \beta_{e}\left(x_{0}\right.\right.$ $\left.\left.-\frac{\theta_{2}}{2 \delta_{2} b}\right)+\frac{\beta_{s}}{2 b}\left(\frac{\theta_{2}}{\delta_{2}}-\frac{\theta_{1}}{\delta_{1}}\right)\right)$ by replacing $p_{1}, x_{1}, p_{2}$, and $x_{2}$ with their optimal expressions.

Until now, we have solved the competition with full reaction problem where both retailers undertake price, carbon emission, and inventory decisions. However in practice, there are many situations where an existing retailer (retailer 1) is already operating in the market, and a new retailer (retailer 2) enters the market and offers a substitutable product. In this case, two situations are relevant to study:

- Competition without reaction. The existing retailer does not react to the new retailer's decisions because, for instance, it has a much higher market power.
- Competition with partial reaction. The existing retailer just updates its price and order size but does not change the carbon emission intensity as this requires new deals with the supplier.

With respect to modeling and solving approaches, the case of competition without reaction is similar to the benchmark model and the case of competition with partial reaction

## Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition

 Environmentis a particular case of our general competition model. We present in what follows the optimal solutions in these two particular cases, but do not provide the details of modeling and solving approaches to avoid redundancy.

### 5.2.2 Competition without reaction ( $M_{5}$ )

In this case, retailer 1 is already selling its product in the market and retailer 2 enters the market and offers a substitutable product. Retailer 2 decides its product's greenness level, price, and order size ( $x_{2}, p_{2}$, and $S_{2}$, respectively) while retailer 1 changes nothing ( $S_{1}, p_{1}$, and $x_{1}$ are known). Hence, the problem consists in finding the optimal decisions for retailer 2 given that retailer 1 already operates in the market. Here, the mean demand for retailer 2's product, namely $\lambda_{2}$, depends not only on its carbon emission intensity and price ( $x_{2}$ and $p_{2}$ ) but also on the carbon emission intensity and price of retailer 1's product (see equations 5.3 and 5.4). Therefore, $\lambda_{2}$ is here given by $\omega A+\alpha_{s} p_{1}+\beta_{s}\left(x_{1}+e_{1}\right)-$ $\delta_{2} p_{2}-\theta_{2} x_{2}$. It is recalled that $\omega$ represents the market share of retailer 1 if both retailers offer products with the same characteristics. In the case of competition without reaction, $x_{1}, p_{1}$, and $S_{1}$ are fixed. We let $A_{2}^{\prime}=\omega A+\alpha_{s} p_{1}+\beta_{s}\left(x_{1}+e_{1}\right)$. Thus, the expression of $\lambda_{2}$ can be simplified as follows.

$$
\begin{equation*}
\lambda_{2}=A_{2}^{\prime}-\delta_{2} p_{2}-\theta_{2} x_{2} \tag{5.13}
\end{equation*}
$$

In the case of competition without reaction, the formulation of retailer 2's model, denoted by $\left(M_{5}\right)$, is similar to the benchmark model, although, the effectual market potential, the price sensitivity parameter, and the greenness sensitivity parameter are different. Consequently, we use the same methodology used for $\left(M_{0}\right)$ to solve model ( $M_{5}$ ). The calculation steps are similar, although, the demands are not the same (in aspect of parameters). The optimal solutions are provided in Proposition 5.4

Mathematical model:

$$
\begin{equation*}
\underset{x_{2}, p_{2}, S_{2}}{\operatorname{aximize}} \pi=\left(p_{2}-\left(c_{2}-b\left(x_{0}-x_{2}\right)^{2}\right)\right) \lambda_{2}-h S_{2} \tag{5.14}
\end{equation*}
$$

Subject to

$$
\begin{align*}
& \psi_{0}=\frac{\lambda_{2}}{\lambda_{2}+S_{2} \mu_{2}} \leq r  \tag{5.15}\\
& \lambda_{2}=A_{2}^{\prime}-\delta_{2} p_{2}-\theta_{2} x_{2}  \tag{5.16}\\
& \lambda_{2}, p_{2}, S_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{align*}
$$

Thanks to following Lemma 5.5 and 5.6, the model transforms into a single-variable model. The optimal solution is provided in Proposition 5.4.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Lemma 5.5. For a given values of $p_{2}$ and $x_{2}$, the optimal stock is
$S_{2}^{*}=\frac{(1-r)\left(A_{2}^{\prime}-\delta_{2} p_{2}-\theta_{2} x_{2}\right)}{r \mu_{2}}$
The proof is similar to Lemma 5.3 . Lemma 5.5 provides the optimal amount of order size that the retailer 2 needs to order in function of the product's price and carbon emission level. The service level (equation 5.13) is binding. Hence, we deduce that service level constraint will be relaxed after substituting $S_{2}^{*}$ by its expression into the model.
Since we assume that $\lambda_{2} \geq 0, S_{2}^{*}$ is positive too and can be substituted. By substituting $S_{2}^{*}$ in equation 5.12, the model can be rewritten as presented in the following:
$\underset{x_{2}, p_{2}, s_{2}}{\operatorname{Maximize}} \pi=\left(p_{2}-\left(c_{2}-b\left(x_{0}-x_{2}\right)^{2}-\frac{h(1-r)}{r \mu_{2}}\right)\right) \lambda_{2}$
Subject to

$$
\begin{aligned}
& \lambda_{2}=A_{2}^{\prime}-\delta_{2} p_{2}-\theta_{2} x_{2} \\
& \lambda_{2}, p_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{aligned}
$$

Lemma 5.6. For a given value of $x_{2}$, optimal price is

$$
p_{2}^{*}=\frac{\delta_{2} b x_{2}^{2}-\left(2 \delta_{2} b x_{0}+\theta_{2}\right) x_{2}+A_{2}^{\prime}+\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}
$$

The proof methodology is similar to Lemma 5.4. Lemma 5.6 provides the optimal price of retailer 2's product in function of carbon emission level. The optimal price is decreasing in carbon emission level $\left(\frac{d p_{2}^{*}}{d x_{2}}<0\right)$ that makes sense as we expected. As much as the carbon emission level of product decreases, retail price non-linearly increases. By substituting $p_{2}^{*}$ by its expression, the model transforms into single-variable model as presented in the following:
$\underset{0 \leq x_{2} \leq x_{0}}{\operatorname{aximize}} \pi=\frac{\left(-\delta_{2} b x_{2}^{2}+\left(2 \delta_{2} b x_{0}-\theta_{2}\right) x_{2}+A_{2}^{\prime}-\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)^{2}}{4 \delta_{2}}$

Subject to

$$
\lambda_{2}=\frac{-\delta_{2} b x_{2}^{2}+\left(2 \delta_{2} b x_{0}-\theta_{2}\right) x_{2}+A_{2}^{\prime}-\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2}
$$

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Proposition 5.4. The optimum amount of carbon emission, price and order size are

$$
\begin{aligned}
& \text { equal to } x_{2}{ }^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}, p_{2}^{*}=\frac{A_{2}^{\prime}-\theta_{2} x_{0}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}} \text {, and } \\
& S_{2}^{*}=\frac{(1-r)\left(A_{2}^{\prime}-\theta_{2} x_{0}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}-\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)}{2 r \mu_{2}} \text {, respectively. }
\end{aligned}
$$

The proof methodology is similar to Proposition 5.2 .
The optimal carbon emission level is affected by competition. The competition's effect and comparison are presented in the next section.

Like the optimal results of monopoly scenario, the optimal carbon emission level linearly decreases as customers' carbon emission sensitivity, $\theta_{2}$, increases. While optimal price and stock level non-linearly decrease until $\theta_{2}<\frac{2}{3} \delta_{2} b x_{0}$ and after that increase. When customers take more greenness level of products, when purchasing, into account, the retailer offers a greener product that leads to a higher price. In addition, optimal carbon emission level non-linearly decreases as carbon emission reduction cost factor, b , increases, while, optimal price and stock level non-linearly increase.

### 5.2.3 Competition with partial reaction $\left(M_{6}\right)$

In this section, retailer 2 offers a new substitutable product along with retailer 1's product to customers. Retailer 2's decisions are the carbon emission level $x_{2}$, the price $p_{2}$, and the order size $S_{2}$. Under the competition of retailer 2, retailer 1 can also update its retail price $p_{1}$ and order size $S_{1}$ strategy. Retailer 1 decides to keep his/her product's carbon emission level $x_{1}$. We formulate the problem as a non-cooperative game where there is no dominant retailer. Each retailer makes its decisions to maximize its expected profit, taking into account other retailer's decisions. We aim to determine the Nash equilibrium of this game, i.e., the set of optimal decisions such as no retailer (player, from a game theory perspective) can benefit by changing its own decisions while the other player keeps its decisions unchanged.

Since $x_{1}$ is known, we let $A_{1}^{\prime \prime}=\omega A-\theta_{1}\left(e_{1}+x_{1}\right)$ and $A_{2}^{\prime \prime}=(1-\omega) A+\beta_{s}\left(e_{1}+x_{1}\right)$ to simplify the notation. Thus, the mean demand for each retailer is given as follows.

$$
\begin{align*}
& \lambda_{1}=A_{1}^{\prime \prime}-\delta_{1} p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}  \tag{5.19}\\
& \lambda_{2}=A_{2}^{\prime \prime}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2} \tag{5.20}
\end{align*}
$$

The mathematical model for each retailer is given in the following.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

## Retailer 1's model

$\operatorname{Max}_{p_{1}, S_{1}} \pi_{1}=\left(p_{1}-c_{1}\right) \lambda_{1}-h S_{1}$
Subject to
$\psi_{1}=\frac{\lambda_{1}}{\lambda_{1}+S_{1} \mu_{1}} \leq r$
$\lambda_{1}=A_{1}^{\prime \prime}-\delta_{1} p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}$
$p_{1}, S_{1}, \lambda_{1} \geq 0$

Retailer 2's model
$\operatorname{Max}_{x_{2}, p_{2}, S_{2}} \pi_{2}=\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}\right)\right) \lambda_{2}-h S_{2}$
Subject to

$$
\begin{aligned}
& \psi_{2}=\frac{\lambda_{2}}{\lambda_{2}+S_{2} \mu_{2}} \leq r \\
& \lambda_{2}=A_{2}^{\prime \prime}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2} \\
& p_{2}, S_{2}, \lambda_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{aligned}
$$

We use same approach and steps since the problem is similar to general model $M_{4}$. To find the Nash equilibrium strategies, we first consider that retailer 2's decisions are known and solve retailer 1's problem. Second, we consider that retailer 1's decisions are known and solve retailer 2's problem. We finally deduce the Nash equilibrium.

Lemma 5.7. For a given strategy of retailer 2 (i.e., given $x_{2}, p_{2}$, and $S_{2}$ ), service constraint is binding and, thus, the optimal order size is $S_{1}^{*}=\frac{(1-r)\left(A_{1}^{\prime \prime}-\delta_{1} p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}\right)}{r \mu_{1}}$.

The proof is similar to Lemma 5.1. We substitute $S_{1}^{*}$ by its expression and transform model into a single-variable model in $p_{1}$ as the unique variable.
$\underset{p_{1}}{\operatorname{Maximize}} \pi_{1}=\left(p_{1}-\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)\right) \lambda_{1}$
Subject to

$$
\begin{align*}
& \lambda_{1}=A_{1}^{\prime \prime}-\delta_{1} p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2} \geq 0  \tag{5.22}\\
& p_{1} \geq 0
\end{align*}
$$

The retailer 1's optimal response to retailer 2's strategy is provided in the following.

Lemma 5.8. For a given strategy of retailer 2 (i.e., given $x_{2}, p_{2}$, and $S_{2}$ ), the optimal
retailer I's price is $p_{1}^{*}=\frac{A_{1}^{\prime \prime}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$.
The proof is similar to Lemma 5.2. Increasing one unit of second retailer product's price increases $\frac{\alpha_{s}}{2 \delta_{1}}$ unit of first retailer product's price.

So far, the first retailer's problem is solved. We now assume that retailer 1's decisions (i.e., $p_{1}$ and $S_{1}$ ) are known and solve second retailer's model.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Lemma 5.9. For a given values of $p_{1}, p_{2}$, and $x_{2}$, service constraint is binding at optimality and, thus, the optimal order size is $S_{2}^{*}=\frac{(1-r)\left(A_{2}^{\prime \prime}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}\right)}{r \mu_{2}}$.

The proof is similar to Lemma 5.3. We substitute $S_{2}^{*}$ by its expression and obtain an equivalent formulation of model ( $M_{2.2}$ ) with only two variables $p_{2}$ and $x_{2}$. The resulting model is given below.
$\underset{x_{2}, p_{2}}{\operatorname{Maximize}} \pi_{2}=\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right) \lambda_{2}$
Subject to

$$
\begin{align*}
& \lambda_{2}=A_{2}^{\prime \prime}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}  \tag{5.24}\\
& p_{2}, \lambda_{2} \geq 0,0 \leq x_{2} \leq x_{0}
\end{align*}
$$

The following Lemma provides the retailer 2's optimal price.

Lemma 5.10. For a given values of $p_{1}$ and $x_{2}$, the optimal price of retailer 2 's product

$$
\text { is } p_{2}^{*}=\frac{\delta_{2} b x_{2}^{2}-\left(2 \delta_{2} b x_{0}+\theta_{2}\right) x_{2}+A_{2}^{\prime \prime}+\alpha_{s} p_{1}+\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}} .
$$

The proof is similar to Lemma 5.4. The second retailer's product price is increasing in first retailer's product price and decreasing in its carbon emission. Increasing one unit of first retailer product's price increases $\frac{\alpha_{s}}{2 \delta_{2}}$ unit of second retailer product's price.

Thanks to the result of the previous Lemma, the mathematical model transforms into a single-variable model as function of $x_{2}$, which is presented in the following.
$\underset{0 \leq x_{2} \leq x_{0}}{\operatorname{Maximize}} \pi_{2}=\frac{\left(-\delta_{2} b x_{2}^{2}+\left(2 \delta_{2} b x_{0}-\theta_{2}\right) x_{2}+A_{2}^{\prime \prime}+\alpha_{s} p_{1}-\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)^{2}}{4 \delta_{2}}$

Subject to

$$
\lambda_{2}=\frac{-\delta_{2} b x_{2}^{2}+\left(2 \delta_{2} b x_{0}-\theta_{2}\right) x_{2}+A_{2}^{\prime \prime}+\alpha_{s} p_{1}-\delta_{2}\left(c_{2}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2}
$$

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Proposition 5.5. In case of competition with partial reaction, the optimal carbon emission level of retailer 2 's product is $x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}$. Consequently by substituting $x_{2}^{*}$, the optimal prices of retailer 1 and retailer 2 transform into
$p_{1}^{*}=\frac{A_{1}^{\prime \prime}-\frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\beta_{s} x_{0}+\alpha_{s} p_{2}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$ and
$p_{2}^{*}=\frac{A_{2}^{\prime \prime}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}-\theta_{2} x_{0}+\alpha_{s} p_{1}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}$ for a given strategy of $p_{1}$ and $p_{2}$,
respectively.

The proof is similar to Proposition 5.2
The optimal carbon emission level is decreasing in $\theta_{2}$. It means that either more sensitive and concern customers to greenness (i.e., higher $\beta_{e}$ and $\beta_{s}$ ) or higher power and market share (i.e., higher $1-\omega$ ) push second retailer to offer products with lower production's carbon emission.

Similar to general model $M_{4}$, the Nash equilibrium is the intersection of the retailers' optimal price response, the point that neither retailer 1 nor retailer 2 can not gain higher expected profit under other opponent's strategies. Thus, we derive in the following Proposition the optimal strategy of each retailer at the equilibrium.

Proposition 5.6. The optimum prices of retailer 1 and retailer 2 are
$p_{1}^{*}=\frac{2 \delta_{2}\left(2 \delta_{2} z_{1}^{\prime}+\alpha_{s} z_{2}^{\prime}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}$ and $p_{2}^{*}=\frac{2 \delta_{1}\left(2 \delta_{2} z_{2}^{\prime}+\alpha_{s} z_{1}^{\prime}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}$, respectively,
where $z_{1}^{\prime}=\frac{\omega A-\theta_{1}\left(e_{1}+x_{1}\right)-\frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\beta_{s} x_{0}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$ and
$z_{2}^{\prime}=\frac{(1-\omega) A+\beta_{s}\left(e_{1}+x_{1}\right)+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}-\theta_{2} x_{0}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}$.
The proof is similar to Proposition 5.3
In this technical section, different competition scenarios have been considered, formulated, and solved. The next section aims to bring different types of important managerial insights to the light.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

### 5.3 Analysis and managerial insights

In this section, we drive important insights of problems that are presented and solved earlier. We investigate in this section the effect of competition on products' environmental performance as well as the impact of transportation carbon emissions. First, we present the comparison of the optimal carbon emission level in benchmark and competitive scenarios to highlight the competition's effect. The comparison is done in different market structures (that are presented in Chapter 4), in order to answer the research questions. After that, we focus on the impact of competition on the second retailer's retail price. Finally, we drive some insights of important parameters' effect on benchmark and competitive scenarios.

### 5.3.1 Impact of competition on products' environmental performance

To understand the impact of competition on greenness, we compare the greenness level of retailer 2's product in competition scenarios to that obtained in the benchmark situation. We recall that the greenness level is measured in terms of carbon emission intensity and that the carbon emission intensity for retailer 2 is given by $x_{2}$. We focus on retailer 2's strategy because it is the retailer that undertakes all decisions (i.e., greenness, price and order size) in all studied situations, whereas retailer 1 does not decide the greenness level in the case of competition with partial reaction, and does not undertake any decision in the case of competition without reaction.

It is first noted that in all competition scenarios (regardless of whether and how retailer 1 reacts), retailer 2 offers a product with carbon emission $x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}=$ $x_{0}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{2\left((1-\omega) \alpha_{p}+\alpha_{s}\right) b}$. It is interesting to figure out that the carbon emission reduction (i.e., $x_{0}-x_{2}^{*}$ ) depends only on market characteristics and emission reduction cost, but does not depend on other retailer's strategy. The price and inventory decisions, however, depend on whether the other retailer reacts or not and the nature of decisions it makes. Hence, the retailer chooses its greenness strategy based on market characteristics and reacts to other retailer's decisions only by adjusting pricing and inventory policies.

We now compare the carbon emission obtained in the benchmark situation for retailer 2 (i.e.; $x_{2}^{*}=x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}$ ) to that resulting from the different competitions scenarios (i.e.; $\left.x_{2}^{*}=x_{0}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{2\left((1-\omega) \alpha_{p}+\alpha_{s}\right) b}\right)$. We obtain the following main result presented in Proposition 5.7

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

Proposition 5.7. When the switchovers are more governed by greenness differentiation (i.e., $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ ), the competition leads to offering a greener product.
When the switchovers are more governed by price differentiation (i.e., $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ ), the competition leads to offering a dirtier product.
In a neutral market (i.e., $\frac{\alpha_{s}}{\alpha_{p}}=\frac{\beta_{s}}{\beta_{e}}$ ), the competition does not affect the greenness of the product. The retailer offers the product with the same greenness level in both monopoly and competition situations.

Proof. Let us define $x_{2}^{* B}$ and $x_{2}^{* C}$ as optimal carbon emissions in benchmark and competition scenarios, respectively. The carbon emissions' difference in benchmark and competition scenario is $\Delta_{x}=x_{2}^{* B}-x_{2}^{* C}$.
$x_{2}^{* B}-x_{2}^{* C}=\left(x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right)-\left(x_{0}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{2\left((1-\omega) \alpha_{p}+\alpha_{s}\right) b}\right)=\frac{(1-\omega) \beta_{e}+\beta_{s}}{2\left((1-\omega) \alpha_{p}+\alpha_{s}\right) b}-\frac{\beta_{e}}{2 \alpha_{p} b}$
$\Leftrightarrow=\frac{\alpha_{p} \beta_{s}-\beta_{e} \alpha_{s}}{2 \alpha_{p}\left((1-\omega) \alpha_{p}+\alpha_{s}\right) b}$
$\Rightarrow \Delta_{x}= \begin{cases}>0 & , \text { if } \alpha_{p} \beta_{s}>\beta_{e} \alpha_{s} \\ =0 & \text {, if } \alpha_{p} \beta_{s}=\beta_{e} \alpha_{s} \\ <0 & \text {,if } \alpha_{p} \beta_{s}<\beta_{e} \alpha_{s}\end{cases}$
As we can see, the sign of $\Delta_{x}$ depends on market's structures.
Compared to the benchmark situation, one may expect that greenness- and price-based competition will lead to enhance the product's greenness. However, our results show that this holds only for some specific market conditions while, for other types of markets, competition either has no effect or even deteriorates the product's greenness. As we will see afterwards, the market characteristics influence many of our results, so they deserve deeper investigation.

We first recall that a unit decrease in carbon emission intensity of retailer 2's product (P2) generates customers at a rate of $(1-\omega) \beta_{e}+\beta_{s}$ for P 2 , out of which $(1-\omega) \beta_{e}$ are "new" attracted customers and $\beta_{s}$ are "cannibalized" customers, substituting P2 for P1. Similarly, a unit price decrease generates customers at a rate of $(1-\omega) \alpha_{p}+\alpha_{s}$ for P2, out of which $(1-\omega) \alpha_{p}$ are "new" attracted customers and $\alpha_{s}$ are cannibalized from P1.

As a quick reminder, we briefly mention present the market's structures. $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ implies GDS (governed by greenness differentiation) market, $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ implies PDS (governed by price differentiation) market, and $\frac{\alpha_{s}}{\alpha_{p}}=\frac{\beta_{s}}{\beta_{e}}$ implies neutral market.

## Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition

 EnvironmentBack to the findings of Proposition 5.7. The results indicate that whether switching of the customers is influenced more by the disparity between the prices or carbon emissions, governs how the competition influences the optimal greenness strategy and changes it with comparison to the monopoly case. The main difference between the competition case and the monopoly case is the structure of demand. In competition, the retailer's demand is sensitive not only to its own price and carbon emission level but also to the price and carbon emission differentiations with the other retailer as this determines the number of customers switching from one retailer to another. When the switchovers are more governed by greenness differentiation (i.e., in GDS market), there is a fierce competition on greenness and the retailer should therefore capitalize more on greenness performance, which leads to offering a greener product with comparison to the product offered in the monopoly case. In PDS market, in which the switchovers are more governed by price differentiation, the retailer should offer a more competitive price which requires to reducing the cost and, consequently, reducing the greenness performance. This leads to offering a dirtier product. Finally, in the neutral market, the switchovers are equally governed by price and carbon emission disparities. The impact of competition on the optimal carbon emission level is neutralized in this case, and the retailer offers the product with the same greenness level of the monopoly case. The results of Proposition 5.7 are summarized in the following table.

Table 5.1: Impact of competition on the environmental performance

| Market <br> structure | $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ | $\frac{\alpha_{s}}{\alpha_{p}}=\frac{\beta_{s}}{\beta_{e}}$ | $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ |
| :---: | :---: | :--- | :--- |
| Impact on the <br> environmental <br> performance | Competition im- <br> proves the environ- <br> mental performance | Competition does not <br> impact the environ- <br> mental performance | Competition deteri- <br> orates the environ- <br> mental performance |

The above discussion explains how competition impacts the greenness level with comparison to the monopoly case. In what follows, we focus on the impact of the market power on the greenness strategy of the retailer. The main result is given in Proposition 5.8

Proposition 5.8. In competition with full reaction, a retailer that gains more market power will decrease its product's greenness in GDS market and increase it in PDS market.

Proof. For retailer 2, we have $x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}=x_{0}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{2\left((1-\omega) \alpha_{p}+\alpha_{s}\right) b}$. Retailer 2 has more market power when $\omega$ decreases (i.e.; $1-\omega$ increases). We have $\frac{\partial x_{2}^{*}}{\partial \omega}=$ $\frac{\beta_{e} \alpha_{s}-\alpha_{p} \beta_{s}}{2 b\left((1-\omega) \alpha_{p}+\alpha_{s}\right)^{2}}$. Therefore, $\frac{\partial x_{2}^{*}}{\partial \omega}<0$ in GDS market (i.e., when $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ ) and

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment
$\frac{\partial x_{2}^{*}}{\partial \omega}>0$ in PDS market. Hence, an increase in market power for retailer 2 leads to an increase in $x_{2}^{*}$ in GDS market and a decrease in $x_{2}^{*}$ in PDS market, as stated in this Proposition.
As for retailer 1, we have $x_{1}^{*}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}=x_{0}-\frac{\omega \beta_{e}+\beta_{s}}{2\left(\omega \alpha_{p}+\alpha_{s}\right) b}$. Retailer 1 has more market power when $\omega$ increases. We have $\frac{\partial x_{1}^{*}}{\partial \omega}>0$ in GDS market and $\frac{\partial x_{1}^{*}}{\partial \omega}<0$ in PDS market.

Let us recall that $\omega$ represents the market power of retailer 1 and, thus $1-\omega$ is the market power of retailer 2 . One may expect that a retailer that sees an increase in its market power (i.e., for retailer $1, \omega$ increases and, for retailer 2 , $\omega$ decreases) will increase its price and, thus, will have more margin that justifies offering a greener product. However, our results indicate again that the retailer can have two different reactions according to market characteristics. A deeper analysis of the retailer's strategy under each market structure helps to shed light on the main trade-offs resulting from an increase in $\omega$, as we explain in the following. In GDS market, a retailer that loses market power needs to offer a greener product in order to attract more switching customers since switchovers are here governed by greenness. Offering a greener product increases the unit cost but the priority is here given to maintaining a profitable amount of demand. If we make this same analysis in the opposite way (increasing market power), we deduce that this retailer should offer a dirtier product when its market power increases under GDS market, as stated in Proposition 6. In PDS market, a retailer with decreasing market power needs to offer a cheaper product in order to attract more switching customers since switchovers are here governed by price. Offering a cheaper product requires to decreasing the unit cost which means reducing the greenness level. This analysis also means that this retailer should offer a greener product when its power increases under PDS market, wich explains the result of Proposition 5.8.

The results of Proposition 5.8 are illustrated in Figure 5.3. We see that an increase in the market power of retailer 1 (i.e., increase in $\omega$ ) leads an increase in $x_{1}^{*}$ (i.e., decrease in greenness) in GDS market and a decrease in $x_{1}^{*}$ (i.e., increase in greenness) in PDS market. For retailer 2, an increase in the market power (i.e., decrease in $\omega$ ) leads an increase in $x_{2}^{*}$ (i.e., decrease in greenness) in GDS market and a decrease in $x_{2}^{*}$ (i.e., increase in greenness) in PDS market.

We now compare the optimal carbon emissions $x_{1}^{*}$ and $x_{2}^{*}$ to determine which retailer buys the greener product (i.e., asks its supplier to provide the greener product). Notice that $x_{2}^{*}>x_{1}^{*}$ means that retailer 1 buys a greener product than the product bought by retailer 2. However, this does not necessarily mean that retailer 1 offers a greener product to its customers because we must add the transportation carbon emissions $e_{1}$ for P 1 while we have $e_{2}=0$ for P 2 . We consider here the general competition model, in which both retailers undertake greenness decisions. The result is given in Proposition 5.9.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment


Figure 5.3: Optimal carbon emissions in function of market power

Proposition 5.9. The retailer that has a smaller market power buys a greener product (than the product bought by the other retailer) under GDS market, and a dirtier product (than the product bought by the other retailer) under PDS market.

Proof. We have $x_{2}^{*}-x_{1}^{*}=\left(x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}\right)-\left(x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}\right)=\frac{1}{2 b}\left(\frac{(1-2 \omega)\left(\alpha_{p} \beta_{s}-\beta_{e} \alpha_{s}\right)}{\left(\omega \alpha_{p}+\alpha_{s}\right)\left((1-\omega) \alpha_{p}+\alpha_{s}\right)}\right)$.
We know that $\left(\omega \alpha_{p}+\alpha_{s}\right)\left((1-\omega) \alpha_{p}+\alpha_{s}\right)>0$. Hence, the sign of $x_{2}^{*}-x_{1}^{*}$ is given by the sign of $(1-2 \omega)\left(\alpha_{p} \beta_{s}-\beta_{e} \alpha_{s}\right)$. Notice that $\omega>0.5$ (respectively, $\omega<0.5$ ) means that retailer 1 has more power (respectively, less power). Thus, for $\omega<0.5$, we have $x_{2}^{*}>x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ (PDS market) and $x_{2}^{*}<x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ (GDS market). For $\omega>0.5$, we have $x_{2}^{*}>x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ (GDS market) and $x_{2}^{*}<x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ (PDS market). This demonstrates the results of this Proposition.

We provide a qualitative explanation of Proposition 5.9 in the following. If a given retailer (let us say retailer 2) has a smaller market power, then it needs to increase its market share. Thus, in GDS market, retailer 2 buys a greener product than the product bought by retailer 1 in order to convince more customers to substitute P2 for P1 since switchovers are, in this case, more governed by greenness. In PDS market, however, retailer 2 adopts an aggressive pricing strategy to attract more switching customers. Consequently, retailer 2 needs to decrease the cost and, thus, to reduce its greenness level which results in buying a dirtier product than the product bought by retailer 1. The result of Proposition 5.9 also

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment
means that the retailer that has a larger market power (let us say retailer 1) buys a dirtier product in GDS market and a greener product in PDS market. In the case where retailer 2 enters a market in which retailer 1 is already established, we can imagine that retailer 2 will start with a smaller market power. In this case, retailer 2 will buy a greener product than retailer 1's product under GDS market and a dirtier product under PDS market. This means that the new retailer will not always buy a greener product than the product bought by the existing retailer.

Table 5.2: Impact of market share on the environmental performance

| Market structure | $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ | $\frac{\alpha_{s}}{\alpha_{p}}=\frac{\beta_{s}}{\beta_{e}}$ | $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ |
| :---: | :---: | :---: | :---: |
| $\omega<0.5$ | $x_{1}^{*}>x_{2}^{*}$ | $x_{1}^{*}=x_{2}^{*}$ | $x_{1}^{*}<x_{2}^{*}$ |
| $\omega=0.5$ | $x_{1}^{*}=x_{2}^{*}$ | $x_{1}^{*}=x_{2}^{*}$ | $x_{1}^{*}=x_{2}^{*}$ |
| $\omega>0.5$ | $x_{1}^{*}<x_{2}^{*}$ | $x_{1}^{*}=x_{2}^{*}$ | $x_{1}^{*}>x_{2}^{*}$ |

### 5.3.2 Impact of transportation carbon emissions disparity

We now focus on the impact of transportation carbon emissions disparity on the optimal strategy of retailers. We recall that we considered, without loss of generality, that retailer 1 has the farthest supplier and, thus, generates more transportation carbon emissions per unit of product than retailer 2 . Since we considered, without loss of generality, that $e_{2}=0$, the transporation emissions per unit of retailer 1's product, namely $e_{1}$, also refers here to the transportation carbon emissions disparity. Moreover, given that retailer 1 relies on a longer SC (i.e., a more distant supplier), we have $\mu_{1} \leq \mu_{2}$. In this section, we assume that the retailer that relies on a longer SC has a smaller base cost. Therefore, we have $c_{1} \leq c_{2}$. This is the typical situation when, for instance, one retailer relies on a low-cost abroad supplier whereas the other retailer relies on a local but more expensive supplier. We consider here the general competition model. We have seen that the transportation emissions do not affect the optimal greenness levels of the products purchased from the suppliers (i.e., $x_{1}^{*}$ and $x_{2}^{*}$ ). However, they impact the pricing decisions. Hence, we focus on pricing and present the first result in Proposition 5.10.

Proposition 5.10. When both retailers have the same market power, the retailer that relies on a shorter SC offers a higher price if and only if the difference in transportation carbon emissions (i.e., $e_{1}$ ) is higher than a threshold value
$e_{0}=\frac{\left(\alpha_{p}+2 \alpha_{s}\right)\left(\frac{h(1-r)}{r}\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right)-\left(c_{2}-c_{1}\right)\right)}{\beta_{e}+4 \beta_{s}}$.
Proof. Recall Proposition 5.3 . We have $p_{2}^{*}>p_{1}^{*} \Leftrightarrow \frac{2 \delta_{1}\left(2 \delta_{2} z_{2}+\alpha_{s} z_{1}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}>\frac{2 \delta_{2}\left(2 \delta_{2} z_{1}+\alpha_{s} z_{2}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}$

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment
$\Leftrightarrow \frac{z_{2}}{z_{1}}>\frac{2 \delta_{1} \delta_{2}-\delta_{1} \alpha_{s}}{2 \delta_{1} \delta_{2}-\delta_{2} \alpha_{s}}$. When both retailers have the same market power, we have $\omega=0.5$ and $\delta_{1}=\delta_{2}$. Therefore, the above condition can be simplified as $\frac{z_{2}}{z_{1}}>1$, which is equivalent to $A_{2}-A_{1}+\delta_{2}\left(c_{2}-c_{1}+\frac{h(1-r)}{r}\left(\frac{1}{\mu_{2}}-\frac{1}{\mu_{1}}\right)\right)>0$. We know that $A_{2}-A_{1}=$ $\left(\omega \beta+2 \beta_{s}\right) e_{1}$. Hence, the above inequality is equivalent to $\left(0.5 \beta_{e}+2 \beta_{s}\right) e_{1}+\delta_{2}\left(c_{2}-\right.$ $\left.c_{1}+\frac{h(1-r)}{r}\left(\frac{1}{\mu_{2}}-\frac{1}{\mu_{1}}\right)\right)>0$, which is equivalent to
$e_{1}>e_{0}=\frac{\left(\alpha_{p}+2 \alpha_{s}\right)\left(\frac{h(1-r)}{r}\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right)-\left(c_{2}-c_{1}\right)\right)}{\beta_{e}+4 \beta_{s}}$.
One may expect that the retailer that has a shorter SC and bears a higher base cost (here, retailer 2) will always offer a higher price than the other retailer. This is true only when $e_{0} \leq 0$, which is verified when $\frac{h(1-r)}{r}\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right) \leq\left(c_{2}-c_{1}\right)$. In other words, the retailer that has a shorter SC will always offer a higher price when the difference in cost is too high (i.e., greater than $\left.\frac{h(1-r)}{r}\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right)\right)$. However, when the difference in cost is not that big (i.e., $c_{2}-c_{1} \leq \frac{h(1-r)}{r}\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right)$ ), $e_{0}$ is positive and, thus, we may have $e_{1} \leq e_{0}$. In this case, retailer 2 may offer a smaller price than retailer 1. Indeed, if the disparity in transportation carbon emissions is not that big (i.e., $e_{1}<e_{0}$ ), then retailer 2 offers a smaller price despite its higher base cost. The qualitative explanation is the following. A small disparity in transportation carbon emissions implies that retailer 2 should do more effort to offer a product that has an attractive carbon emission intensity level that enables to attract a profitable amount of greenness-sensitive customers. However, this increases the cost and reduces the net margin. Hence, it becomes more profitable for retailer 2 to capitalize on price-sensitive customers and, thus, offer a lower price than retailer 1's price.

We finally investigate the impact of the transportation carbon emissions disparity on the price differentiation between products. The result is given in Proposition 5.11.

Proposition 5.11. As long as the transportation carbon emissions gap (i.e., $e_{1}$ ) is smaller than $e_{0}$, the higher the transportation emission gap, the smaller the price differentiation between the products.
When the transportation carbon emission gap is greater than $e_{0}$, the higher the transportation emission gap, the higher the price differentiation between the products.

Proof is actually presented in the previous Proposition. It can be verified that $p_{1}^{*}$ is de-

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment
creasing in $e_{1}$, whereas $p_{2}^{*}$ is increasing in $e_{1}$. When $e_{1}<e_{0}, \mathrm{P} 2$ is offered at a lower price than P1 (see Proposition 5.10). A higher disparity in transportation emissions (i.e., an increase in $e_{1}$ ) will increase $p_{2}^{*}$ and decrease $p_{1}^{*}$, which closes the price gap. However, when $e_{1}>e_{0}, \mathrm{P} 2$ is offered at a higher price than P1 (Proposition 5.10), which implies that a higher disparity in transportation emissions will increase the price gap. This is illustrated in Figure 5.4 .


Figure 5.4: Optimal prices in function of disparity in transportation emissions

### 5.3.3 Impact of refilling time on order size decisions

We have obtained complex expressions of the optimal order size for each retailer. Theses expressions show that the inventory policy is impacted by all the input parameters of the problem (market characteristics, cost factors, supply characteristics, etc.). It is known in the inventory literature that the order size of a given retailer highly depends on its inventory refilling rate. In our context, we observe that the order size also depends on the refilling rate of the other retailer, as highlighted in the following proposition.

Proposition 5.12. An increase in the inventory refilling rate of one retailer not only leads to reducing the optimal order size of this retailer but also to reducing the optimal order size of the other retailer.

In Figure 5.5, we increase retailer 1's refilling rate $\left(\mu_{1}\right)$ and represent the optimal order size of each retailer. This provides an illustration of Proposition5.12. Note that we obtain the same behavior when we increase $\mu_{2}$.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment

An increase in the inventory refilling rate of a given retailer implies a shorter expected lead time between the moment when the retailer places the order and when it receives the products from the supplier. This means that the retailer can still satisfy the service level constraint (expressed in terms of maximum stock out probability level) while holding a lower stock. It is therefore expected that this retailer will reduce its order size as this allows to reducing the inventory cost. However, the inventory refilling rate of the other retailer does not change and, thus, it is not intuitive that this second retailer will also reduce its order size (even though this order size reduction is relatively small). This reaction is due to the price competition. Indeed, with an increase in its inventory refilling rate, the first retailer will decrease its price as its inventory cost goes down. This leads the other retailer to react and decrease its price (to limit the switchovers) but this price reduction is smaller than that of the first retailer. Hence, the second retailer will necessarily lose demand. Thus, it will need less inventory level which implies a smaller order size.


Figure 5.5: Effect of $\mu_{1}$ on retailers' optimal order sizes

### 5.3.4 Impact of competition on retailers' optimal profits

In this section, we aim to compare the optimal retailers' profits under the different competition scenarios. Given the complexity of the expressions of optimal profits, we base our analysis on numerical experiments. We conducted experiments to assess how the optimal profit of each retailer varies in function of the market power under the different competition scenarios. To obtain robust results, we considered different values of models parameters under the different market types (PDS, GDS, and neutral market). We tested 10 instances for each market type. In all cases, we obtained the same behavior for each retailer. This behavior is illustrated in Figures 5.6 and 5.7 for the existing retailer and the new retailer, respectively.

Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition Environment


Figure 5.6: Existing retailer's profit under the different competition scenarios


Figure 5.7: New retailer's profit under the different competition scenarios

When a new retailer enters the market, the existing retailer obtains the highest profit if it adopts a full reaction by adjusting its pricing, greenness and inventory policies (see Figure 5.6). We also observe that the partial reaction scenario is preferred to the scenario without reaction. All these observations are expected. As for the new retailer entering the market, we see in Figure 5.7 that this retailer makes the highest profit when the existing retailer does not react, which is also expected.

Less intuitive is that a full reaction of the existing retailer can be more beneficial for the new retailer than a partial reaction (see Figure 5.7). This result deserves a deeper

## Chapter 5. Optimization of Retailers' Strategies Under Dynamic Competition

 Environmentinvestigation. Under partial reaction, the existing retailer reacts by adjusting its price and order size. Thus, as the existing retailer does not change its product's greenness level, this retailer is obliged to adopt an aggressive pricing strategy to avoid a big loss of demand. This leads the new retailer to react by reducing its price, and finally leads to a significant price decrease for the new retailer at the equilibrium. This has a significant impact on new retailer's profit since its market potential is relatively low. However, under a full reaction scenario, the price reduction performed by the existing retailer is not that high, which incurs a smaller price reduction for the new retailer and leads to a relatively higher profit.

### 5.4 Conclusion

While considering a setting with two retailers that offer two substitutable products, this study investigated the effect of greenness- and price-based competition on the environmental performance of products. In the general case, each retailer decides the greenness of its product (carbon emission intensity), the price and the order size to maximize its expected profit under a service level constraint. We derived analytically the best response of each retailer to other retailer's decisions at the Nash equilibrium. We used the results to derive managerial insights that explain how competition affect the products' environmental performances. Some of our results are not intuitive.

We found that when switchovers are more governed by greenness differentiation (GDS market), the competition leads to offering a greener product. However, when switchovers are more governed by price differentiation (PDS market), the competition leads to offering a dirtier product. Moreover, a retailer that gains more market power will decrease its product's greenness in GDS market and increase it in PDS market. Our results also indicated that the retailer that has a smaller market power will buy a greener product (than the product bought by the other retailer) when switchovers are more governed by price differentiation, and will buy a dirtier product when switchovers are more governed by price differentiation.

We finally investigated the impact of transportation carbon emissions disparity on the optimal strategy of retailers. We found that when both retailers have the same market power, the retailer that relies on a shorter supply chain offers a higher price if and only if the difference in transportation carbon emissions is higher than a given threshold value.

## Chapter 6

## Complex Demands Functions

So far, we consider a linear demand that is sensitive to price and carbon emission. To the best of our knowledge, most studies in the literature used the same linear demand. We formulate different models and solved them. In this chapter, we are interested in considering a more general (especially non-linear) demand function concerning greenness. The objective of this chapter is to investigate the effects and differences of non-linear demand. For that, we would like to study the retailer's profit optimization problem under different demands functions for the benchmark model that was introduced in Chapter 3 , We study the optimal strategies of a retailer who offers a product to customers at a priceand carbon emission- sensitive market. The retailer keeps a stock, $S$, to serve customers. The inventory level decreases one unit as soon as a customer arrives. The retailer sells products at the retail price, $p$. We consider $x_{0}$ as the product's carbon emission reference. Therefore, $x_{0}$ is the maximum Carbon Emission Improvement (CEI) possible. CEI's cost is given by $b x^{2}$, where $b$ is the CEI's cost factor, and $x$ is the CEI. Let us remind that $x$ in previous chapters presents the carbon emission level. In this chapter for easier mathematical development, $x$ represents the product's carbon emission improvement (not the level of carbon emission). The retailer will maximize its profit by deciding the optimal price, carbon emission improvement, and order size. Figure 6.1 shows the described problem.


Figure 6.1: Supply chain

As we already said above, the linear demand function is well known in the literature. Our partners in project ANR CONCLuDE at École des Mines de Saint-Étienne found out that the linear sensitivity of demand to carbon emissions is insufficient (PalaciosArgüello et al., 2020). Therefore, we are interested in considering a new demand function (precisely, non-linear) to see how results differ from traditional linear demand functions. We consider that we know the product's demand when the retailer offers the standard product (i.e., without improvement). We also know the demand's shape of the curve
(e.g., square root). Therefore, we can deduce the last point (when we improve 100\%). Hence, we can deduce the linear demand from the two extreme points and calculate the corresponding greenness sensitivity parameter, $\beta_{e(L)}$.

In the literature, the researchers consider the linear demand without any constraint, but we know that in reality, the demand does not increase beyond a maximum value. This value is obtained for a given $x$ that we call $x_{l}$. In other words, the demand increases in CEI until $x_{l}$, and after that, the demand is constant even when CEI increases. Thus, we consider a constraint on the demand that we call cap. We consider that the maximum increasing obtained thanks to carbon emissions improvement is $\eta A$, for a given price $p$. Figure 6.2 shows four demands functions for a given price that we consider: non-linear, non-linear cap, linear, and linear cap.


Figure 6.2: Demands functions for a given price

We are going to analyze the benchmark model $\left(M_{0}\right)$ with three new demands: $n$th root demand $\left(M_{7}\right)$, Capacitated $n$th root demand $\left(M_{8}\right)$, and capacitated linear demand $\left(M_{9}\right)$.

## $6.1 n$th root demand $\left(M_{7}\right)$

In this problem, we consider a general demand function. However, when $n=2$, the demand function is the square root that we presented earlier (Figure 6.2). The demand function is linearly decreasing in retail price and non-linearly (for $n \geq 2$ ), increasing in carbon emission improvement. The demand function is presented in the following.

$$
\begin{equation*}
\lambda=A-\alpha_{p} p+\beta_{e(N L)} \sqrt[n]{x} \tag{6.1}
\end{equation*}
$$

Where $A$ is market potential, $\alpha_{p}$ and $\beta_{e(N L)}$ are price's- and CEI's- sensitivity parameters, respectively. $p$ and $x$ represent the product's retail price and carbon emission improvement, respectively, which are decision variables. Note that $x_{0}$ indicates the carbon emission reference (thus, maximum carbon emission improvement is equal to $x_{0} ; x \leq x_{0}$ ). The last decision variable of our model is order size, $S$, as we mentioned earlier.

The customer's arrival follows Poisson distributions with a rate of $\lambda$. The retailer's warehouse's refilling rate follows an exponential distribution with a mean rate of $\mu$. The mathematical model of the stochastic problem is presented in the following:

$$
\begin{equation*}
\underset{x, p, S}{\operatorname{Maximize}} \pi=\left(p-\left(c+b x^{2}\right)\right) \lambda-h S \tag{6.2}
\end{equation*}
$$

Subject to

$$
\begin{align*}
& \psi_{0}=\frac{\lambda}{\lambda+S \mu} \leq r  \tag{6.3}\\
& \lambda=A-\alpha_{p} p+\beta_{e(N L)} \sqrt[n]{x}  \tag{6.4}\\
& \lambda, p, S \geq 0,0 \leq x \leq x_{0} \tag{6.5}
\end{align*}
$$

The mathematical model and formulations are similar to the benchmark model that is presented in Chapter 3. In order to avoid redundancy, we skip the model explanation. The following of this section aims to present analytical proofs to obtain optimal solutions.

Lemma 6.1. For any given values of $p$ and $x$, service level constraint (6.3) is binding and the optimal amount of order size is $S^{*}=\frac{(1-r)}{r \mu}\left(A-\alpha_{p} p+\beta_{e(N L)} \sqrt[n]{x}\right)$.

Proof. Since the objective function is linearly decreasing in $S\left(\frac{\partial \pi_{M}}{\partial S}<0\right.$ and $\left.\frac{\partial^{2} \pi_{M}}{\partial S^{2}}=0\right)$, the smallest possible $S$ is the optimal stock. According to the constraint $6.3, S \geq \frac{(1-r) \lambda}{r \mu}$, therefore, the optimal value is $S^{*}=\frac{(1-r)}{r \mu}\left(A-\alpha_{p} p+\beta_{e(N L)} \sqrt[n]{x}\right)$, which implies that service level constraint (Equation 6.3) is binding.

Since we assume that $\lambda \geq 0$, then, $S^{*}$ is positive and can be substituted by its expression. The equivalent model with two variables, $p$ and $x$, is presented in the following.
$\underset{x, p}{\operatorname{Maximize}} \pi=\left(p-\left(c+b x^{2}+\frac{h(1-r)}{r \mu}\right)\right) \lambda$
Subject to

$$
\begin{aligned}
& \lambda=A-\alpha_{p} p+\beta_{e(N L)} \sqrt[n]{x} \\
& \lambda, p \geq 0,0 \leq x \leq x_{0}
\end{aligned}
$$

Lemma 6.2. For any given value of $x$, the optimal price is

$$
p^{*}=\frac{\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}
$$

Proof. The first and second derivatives of $\pi$ with respect to $p$ are presented in the following. The second derivative is negative that means objective function is concave in $p$. Therefore, the root of first derivative (called $p^{\max }$ ) maximizes the objective function.
$\frac{\partial \pi}{\partial p}=-2 \alpha_{p} p+\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$
$\frac{\partial^{2} \pi}{\partial p^{2}}=-2 \alpha_{p}<0$
$\frac{\partial \pi}{\partial p}=0 \Leftrightarrow p^{\max }=\frac{\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}$
To consider $p^{\max }$ as an optimal solution, we need to verify $p^{\max } \geq 0$ and $\lambda\left(p^{\max }\right) \geq 0$. Since $x>0$, this is obvious that $p^{\max } \geq 0$. Then we need to verify that $\lambda\left(p^{\max }\right)$ is greater than zero. To show demand positivity, we substitute $p^{\max }$ in the demand function, which is presented in the following:
$\lambda\left(p^{\max }\right) \geq 0 \Leftrightarrow \frac{-\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2} \geq 0$
$\Leftrightarrow-\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right) \geq 0$
The first and second derivatives of $\lambda\left(p^{\max }\right)$ with respect to $x$ are presented in the following.
$\frac{\partial \lambda\left(p^{\max }\right)}{\partial x}=\frac{1}{2}\left(-2 \alpha_{p} b x+\frac{\beta_{e(N L)}}{n} x x^{\frac{n-1}{2}}\right)$
$\frac{\partial^{2} \lambda\left(p^{\max }\right)}{\partial x^{2}}=\frac{1}{2}\left(-2 \alpha_{p} b-\frac{\beta_{e(N L)}(n-1)}{n^{2}} x \frac{2 n-1}{2}\right)<0$
The second derivative is negative, which means the demand is concave in $x$. The demand function is increasing from zero $\left(\lim _{x \rightarrow+\varepsilon} \frac{\partial \lambda}{\partial x}>0\right)$ to the root of the first derivative. If the first root is higher than $x_{0}$, it means that the demand function is positive in the feasible region. Otherwise, we have to make sure that the demand is positive when $x=x_{0}$. At first, we are going to consider that $\lambda\left(p^{\max }\right)$ is positive, when $x=0$, that lead us to $A \geq \alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$. Figure 6.3 shows the possible shapes of the demand.


Figure 6.3: Possible demands functions

As it has been described, there are two possibilities. The first possibility is that the root of the first derivative is higher than the upper bound, which is $x_{0}$. In this case, $\lambda\left(p^{\max }\right)$ is increasing in the feasible region $\left(\left[0, x_{0}\right]\right)$. Therefore if start point, $x=0$, is positive (we presented the condition earlier), then, the demand is always positive.

In the second possibility, the root of first derivative is in the feasible region, and the demand is decreasing after this root. Thus, also, we would need to make sure that demand is positive when $x=x_{0}$, which the condition is $A \geq \alpha_{p} b x_{0}^{2}-\beta_{e(N L)} \sqrt[n]{x_{0}}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$. However, it is not necessary because we show in the following (Proposition 6.1) that the root of the first derivative is also the point that maximizes the profit function. Therefore we do not interest in the region after the first derivative's root. As summary, while $A \geq$ $\max \left\{\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right), \alpha_{p} b x_{0}^{2}-\beta_{e(N L)} \sqrt[n]{x_{0}}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right\}$ the demand function
is positive in the feasible region. Therefore we can consider $p^{\max }$ as the optimal solution (i.e., $p^{*}=p^{\max }$ ).

By substituting $p^{*}$ in equation 6.6, the equivalent model with only one variable, $x$ is presented in the following:
$\underset{0 \leq x \leq x_{0}}{\operatorname{Mximize}} \pi_{M}=\frac{\left(-\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)^{2}}{4 \alpha_{p}}$
Subject to

$$
\begin{equation*}
\lambda=\frac{-\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2} \geq 0 \tag{6.8}
\end{equation*}
$$

Proposition 6.1. The optimal solutions for nth root demand model $\left(M_{7}\right)$ are properly presented in the following.

$$
x^{*}=\min \left\{\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}, x_{0}\right\}
$$

$p^{*}= \begin{cases}\frac{\frac{(2 n+1) \beta_{e(N L)}}{2 n}\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} & \text { for } x^{*} \neq x_{0}, \text { and } \\ \frac{\alpha_{p} b x_{0}^{2}+\beta_{e(N L)} \sqrt[n]{x_{0}}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} & \text { for } x^{*}=x_{0}\end{cases}$
$S^{*}=\left\{\begin{array}{ll}\frac{(1-r)\left(\frac{(2 n-1) \beta_{e(N L)}}{2 n}\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)}{2 r \mu} & \text { for } x^{*} \neq x_{0} . \\ \frac{(1-r)\left(-\alpha_{p} b x_{0}^{2}+\beta_{e(N L)} \sqrt[n]{x_{0}}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)}{2 r \mu} & \text { for } x^{*}=x_{0}\end{array}\right.$.
Proof. From Equation 6.8, the objective function (Equation 6.7) is equivalent to $\frac{\lambda^{2}}{\alpha_{p}}$.
Since $\lambda \geq 0$, then maximizing $\lambda$ is equivalent to maximizing $\frac{\lambda^{2}}{\alpha_{p}}$. We showed earlier
that the demand function is concave in $x$. Therefore, the root of the first derivative (called $x^{* m a x}$ ) maximizes the demand (equivalently, objective function).
$\frac{\partial \lambda}{\partial x}=\frac{1}{2}\left(-2 \alpha_{p} b x+\frac{\beta_{e(N L)}}{n} x^{-\frac{n-1}{n}}\right)=0 \Leftrightarrow-2 \alpha_{p} b x+\frac{\beta_{e(N L)}}{n} x^{-\frac{n-1}{n}}=0$
$\Leftrightarrow 2 \alpha_{p} b x=\frac{\beta_{e(N L)}}{n} x^{-\frac{n-1}{n}} \Leftrightarrow x . x^{\frac{n-1}{n}}=\frac{\beta_{e(N L)}}{2 \alpha_{p} b n} \Leftrightarrow x^{* \max }=\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}$
If $x^{* \max }$ is in the feasible region, then, it is the optimal solution. We know that $0 \leq x \leq x_{0}$. Therefore, $x^{*}=\min \left\{\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}, x_{0}\right\}$. By substituting $x^{*} \neq x_{0}$ into $p^{*}(x)$, the optimal price is $p^{*}\left(x^{*} \neq x_{0}\right)=\frac{\frac{(2 n+1) \beta_{e(N L)}}{2 n}\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p} \quad 1}$. In the same way, $S^{*}\left(x^{*} \neq x_{0}\right)=\frac{(1-r)\left(\frac{(2 n-1) \beta_{e(N L)}}{2 n}\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)}{2 r \mu}$.

### 6.2 Capacitated $n$th root demand $\left(M_{8}\right)$

In previous section, we have studied the problem with a non-linear demand function (precisely, $n$th root). In this section, we consider the same problem with considering one more constraint on demand function. As we mentioned in the beginning of this chapter, our colleagues found out that demand can not go beyond a certain value. Thus, we are introducing a new constraint that ensures demand function cannot go beyond a predetermined level. Therefore as already said previously, we define a new parameter, $\eta$ such that $0 \leq \eta \leq 1$. We consider that carbon emission improvement can at maximum attract $\eta A$ more customers. Thus for a given $p$, the demand non-linearly (for $n \geq 2$ ) increases in CEI up to $(1+\eta) A-\alpha_{p} p$, and after that, it is constant. Since improving carbon emission level after that point just increases the cost, it is logical that the optimal solution can not be after that. Therefore, we can introduce the constraint such that carbon emission improvement has a new upper bound, $x_{l}$, obtained from this inequality: $\beta_{e(N L)} \sqrt[n]{x} \leq \eta A$. Figure 6.4 shows the capacitated demand function under given price.


Figure 6.4: Capacitated nth root demand for a given price

The mathematical model of described problem is presented in the following:
$\underset{x, p, S}{\operatorname{Maximize}} \pi=\left(p-\left(c+b x^{2}\right)\right) \lambda-h S$
Subject to

$$
\begin{align*}
& \psi_{0}=\frac{\lambda}{\lambda+S \mu} \leq r  \tag{6.10}\\
& \beta_{e(N L)} \sqrt[n]{x} \leq \eta A  \tag{6.11}\\
& \lambda=A-\alpha_{p} p+\beta_{e(N L)} \sqrt[n]{x}  \tag{6.12}\\
& \lambda, p, S \geq 0,0 \leq x \leq x_{0} \tag{6.13}
\end{align*}
$$

The mathematical model is similar to the previous section. Therefore we skip their explanations. The optimal solutions are presented in the following.

Lemm 6.3. For any given value of $x$, the optimal order size and price are

$$
\begin{aligned}
& S^{*}=\frac{(1-r)}{r \mu}\left(A-\alpha_{p} p+\beta_{e(N L)} \sqrt[n]{x}\right) \text { and } p^{*}=\frac{\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}, \\
& \text { respectively. }
\end{aligned}
$$

The proof is similar to Lemma 6.1 and 6.2. By substituting optimal order size and price by their expressions, the equivalent model with only one variable, $x$ is presented in the following:
$\underset{0 \leq x \leq x_{0}}{\operatorname{Maximize}} \pi_{M}=\frac{\left(-\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)^{2}}{4 \alpha_{p}}$
Subject to

$$
\begin{align*}
& x \leq\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}  \tag{6.15}\\
& \lambda=\frac{-\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt[n]{x}+A-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2} \geq 0 \tag{6.16}
\end{align*}
$$

Proposition 6.2. The optimal carbon emission improvement for capacitated nth root

$$
\text { demand model }\left(M_{8}\right) \text { is } x^{*}=\min \left\{\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}},\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}, x_{0}\right\} .
$$

The proof is similar to the previous model except there is an extra constraint on $x$ that concerns maximum customers that can be attract when product's carbon emission level improves.

### 6.3 Capacitated linear demand ( $M_{9}$ )

In this section, we study the benchmark problem with capacitated traditional linear demand function. Like previous section, we consider that the retailer cannot keep getting more costumers than $\eta A$. So, after some point $\left(x_{l}\right)$, the demand is constant as carbon emission improvement is increasing. The figure 6.5 shows the demand function. The demand function is linearly increasing in carbon emission improvement until $x_{l}$ and is constant after that. The mathematical model of the stochastic problem is presented in the following:
$\underset{x, p, S}{\operatorname{Maximize}} \pi=\left(p-\left(c+b x^{2}\right)\right) \lambda-h S$
Subject to

$$
\begin{align*}
& \psi_{0}=\frac{\lambda}{\lambda+S \mu} \leq r  \tag{6.18}\\
& \beta_{e(L C)^{x} \leq \eta A}  \tag{6.19}\\
& \lambda=A-\alpha_{p} p+\beta_{e(L C)}  \tag{6.20}\\
& \lambda, p, S \geq 0,0 \leq x \leq x_{0} \tag{6.21}
\end{align*}
$$



Figure 6.5: Capacitated linear demand for a given price

We skip model explanation since it is similar to benchmark model. The optimal solution is presented in the following.

Proposition 6.3. The optimal carbon emission improvement for capacitated linear demand model $\left(M_{9}\right)$ is $x^{*}=\min \left\{\frac{\beta_{e(L C)}}{2 \alpha_{p} b}, \frac{\eta A}{\beta_{e(L C)}}, x_{0}\right\}$.

The proof is similar to the benchmark model that is presented in chapter 3 (see Proposition 3.1 for more details) except there is an extra constraint on $x$ (Equation 6.19) that concerns maximum customers that can be attract when product's carbon emission level improves.

### 6.4 Analysis and managerial insights

In this section, we provide some important insights. At first, we start to provide insights concerning to the optimal solutions that we obtained. Then, we compare the optimal profits of models with different demands functions.

Table 6.1 presents the parameters that are used to compare the gap between square root cap demand ( $n=2$ ), as the reference, and other demands' (square root, linear, and linear cap) profits. To present a comprehensive result, we vary the parameters to consider a large number of combinations.

Table 6.1: Parameters

| Parameter | Value | Parameter | Value |
| :---: | :---: | :---: | :---: |
| $A$ | $1200: 100: 2000$ | $c$ | $10: 5: 20$ |
| $\alpha_{p}$ | $4: 1: 8$ | $h$ | $1: 1: 3$ |
| $\beta_{e}$ | $3: 1: 6$ | $b$ | $0.005: 0.005: 0.015$ |
| $\mu$ | $30: 10: 50$ | $r$ | 0.05 |
| $x_{0}$ | 100 | $\eta$ | $0.1: 0.3$ |

Observation 6.1. The linear cap model is the closest approximation to the square root cap for small $\eta$, while increasing $\eta$ favors the square root demand model.

We consider a variation of parameters, which are presented in Table 6.1, that leads to 7.2 K different combinations. The gap between the optimal profit of the square root cap model $\left(\pi_{M_{8}}\right)$ and other models $\left(\pi_{M_{0,7,9}}\right)$ is calculated as $\frac{\left(\pi_{M_{8}}-\pi_{M_{0,7,9}}\right) * 100}{\pi_{M_{8}}}$.The results are presented in Table 6.2.

Table 6.2: Gap between profits

| Profit | Linear $\left(M_{0}\right)$ | Linear Cap $\left(M_{9}\right)$ | Square Root $\left(M_{7}\right)$ |
| :---: | :---: | :---: | :---: |
| $\eta=0.02$ | $-11.87 \%$ | $0.00 \%$ | $-25.36 \%$ |
| $\eta=0.04$ | $-7.41 \%$ | $0.00 \%$ | $-20.35 \%$ |
| $\eta=0.06$ | $-3.39 \%$ | $0.00 \%$ | $-15.84 \%$ |
| $\eta=0.08$ | $0.003 \%$ | $0.003 \%$ | $-12.00 \%$ |
| $\eta=0.10$ | $2.8 \%$ | $0.21 \%$ | $-8.88 \%$ |
| $\eta=0.12$ | $4.97 \%$ | $0.61 \%$ | $-6.44 \%$ |
| $\eta=0.14$ | $6.63 \%$ | $1.24 \%$ | $-4.56 \%$ |
| $\eta=0.16$ | $7.93 \%$ | $1.71 \%$ | $-3.35 \%$ |
| $\eta=0.18$ | $8.96 \%$ | $2.28 \%$ | $-2.41 \%$ |
| $\eta=0.19$ | $9.39 \%$ | $2.60 \%$ | $-2.03 \%$ |
| $\eta=0.20$ | $9.73 \%$ | $3.15 \%$ | $-1.64 \%$ |
| $\eta=0.30$ | $12.18 \%$ | $6.49 \%$ | $-0.28 \%$ |

As Table 6.2 shows, the linear cap demand is the closest approximation to the square root cap when $\eta$ is less than 0.18 , while, the square root demand provides better approximation to the square root cap when $\eta \geq 0.19$.

Table 6.3: Gap between optimal carbon emissions improvements

| $x^{*}$ | Linear $\left(M_{0}\right)$ | Linear Cap $\left(M_{9}\right)$ | Square Root $\left(M_{7}\right)$ |
| :---: | :---: | :---: | :---: |
| $\eta=0.02$ | $-117.11 \%$ | $0.00 \%$ | $-91.20 \%$ |
| $\eta=0.04$ | $-28.52 \%$ | $0.00 \%$ | $-22.05 \%$ |
| $\eta=0.06$ | $-12.12 \%$ | $0.00 \%$ | $-9.24 \%$ |
| $\eta=0.08$ | $-6.39 \%$ | $-0.01 \%$ | $-4.77 \%$ |
| $\eta=0.10$ | $-3.76 \%$ | $-0.04 \%$ | $-2.72 \%$ |
| $\eta=0.12$ | $-2.36 \%$ | $-0.07 \%$ | $-1.65 \%$ |
| $\eta=0.14$ | $-1.55 \%$ | $-0.10 \%$ | $-1.03 \%$ |
| $\eta=0.16$ | $-1.11 \%$ | $-0.12 \%$ | $-0.69 \%$ |
| $\eta=0.18$ | $-0.83 \%$ | $-0.14 \%$ | $-0.46 \%$ |
| $\eta=0.19$ | $-0.72 \%$ | $-0.15 \%$ | $-0.38 \%$ |
| $\eta=0.20$ | $-0.61 \%$ | $-0.16 \%$ | $-0.30 \%$ |
| $\eta=0.30$ | $-0.31 \%$ | $-0.25 \%$ | $-0.05 \%$ |

As Table 6.3 shows, the product obtained with a linear cap demand has the closest optimal carbon emission improvement approximation with respect to the product obtained with a square root cap when $\eta$ is less than 0.20 , while, the product obtained with a square root demand provides better approximation (closer carbon emission improvement) to the product obtained with a square root cap when $\eta \geq 0.20$.

Table 6.4: Gap between optimal prices

| $p^{*}$ | Linear $\left(M_{0}\right)$ | Linear Cap $\left(M_{9}\right)$ | Square Root $\left(M_{7}\right)$ |
| :---: | :---: | :---: | :---: |
| $\eta=0.02$ | $-17.87 \%$ | $0.00 \%$ | $-18.65 \%$ |
| $\eta=0.04$ | $-15.70 \%$ | $0.00 \%$ | $-16.47 \%$ |
| $\eta=0.06$ | $-13.52 \%$ | $0.00 \%$ | $-14.27 \%$ |
| $\eta=0.08$ | $-11.31 \%$ | $-0.08 \%$ | $-12.04 \%$ |
| $\eta=0.10$ | $-9.19 \%$ | $-0.27 \%$ | $-9.91 \%$ |
| $\eta=0.12$ | $-7.28 \%$ | $-0.47 \%$ | $-7.99 \%$ |
| $\eta=0.14$ | $-5.59 \%$ | $-0.55 \%$ | $-6.30 \%$ |
| $\eta=0.16$ | $-4.50 \%$ | $-0.71 \%$ | $-5.13 \%$ |
| $\eta=0.18$ | $-3.56 \%$ | $-0.92 \%$ | $-4.08 \%$ |
| $\eta=0.19$ | $-3.15 \%$ | $-1.02 \%$ | $-3.61 \%$ |
| $\eta=0.20$ | $-2.60 \%$ | $-1.16 \%$ | $-3.08 \%$ |
| $\eta=0.30$ | $-1.25 \%$ | $-2.40 \%$ | $-0.83 \%$ |

As Table 6.4 shows, the product obtained with a linear cap demand has the closest optimal price approximation with respect to the product obtained with a square root cap when $\eta$ is less than 0.20 , while, the product obtained with a square root demand provides better approximation (closer price) to the product obtained with a square root cap when $\eta \geq 0.20$.

Table 6.5: Gap between optimal order sizes

| $S^{*}$ | Linear $\left(M_{0}\right)$ | Linear Cap $\left(M_{9}\right)$ | Square Root $\left(M_{7}\right)$ |
| :---: | :---: | :---: | :---: |
| $\eta=0.02$ | $-5.61 \%$ | $0.00 \%$ | $-11.82 \%$ |
| $\eta=0.04$ | $-3.48 \%$ | $0.00 \%$ | $-9.56 \%$ |
| $\eta=0.06$ | $-1.53 \%$ | $0.00 \%$ | $-7.49 \%$ |
| $\eta=0.08$ | $0.15 \%$ | $0.01 \%$ | $-5.71 \%$ |
| $\eta=0.10$ | $1.53 \%$ | $0.11 \%$ | $-4.25 \%$ |
| $\eta=0.12$ | $2.61 \%$ | $0.31 \%$ | $-3.10 \%$ |
| $\eta=0.14$ | $3.45 \%$ | $0.63 \%$ | $-2.20 \%$ |
| $\eta=0.16$ | $4.12 \%$ | $0.87 \%$ | $-1.62 \%$ |
| $\eta=0.18$ | $4.65 \%$ | $1.16 \%$ | $-1.17 \%$ |
| $\eta=0.19$ | $4.87 \%$ | $1.33 \%$ | $-0.99 \%$ |
| $\eta=0.20$ | $5.04 \%$ | $1.61 \%$ | $-0.80 \%$ |
| $\eta=0.30$ | $6.33 \%$ | $3.34 \%$ | $-0.14 \%$ |

As Table 6.5 shows, the product obtained with a linear cap demand has the closest optimal order size approximation with respect to the product obtained with a square root cap when $\eta$ is less than 0.19 , while, the product obtained with a square root demand provides better approximation (closer order size) to the product obtained with a square root cap when $\eta \geq 0.19$.

The results show that the linear demand that is well known in the literature is never the best one. However, the linear cap is a good approximation when $\eta$ is small. The bright thing about linear cap is that we can easily adapt it to previous chapters (Chapter 4 and (5).

Proposition 6.4. In non-linear models ( $M_{7}$ and $M_{8}$ ), increasing n leads to zero carbon emission improvements and in this case $p^{*}=\frac{A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}$.

Proof. Thanks to Propositions 6.1 and 6.2 , we know that the optimal carbon emission improvements of non-linear and non-linear cap models are $\min \left\{\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}, x_{0}\right\}$ and $\min \left\{\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}},\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}, x_{0}\right\}$, respectively. It is clear that $n$ has no effect on $x_{0}$. However, the other elements, $\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}$ and $\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}$, depend on $n$. In the following, we show values of these elements when $n$ increases and goes to infinity.
$\lim _{n \rightarrow \infty}\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}=0$
$\lim _{n \rightarrow \infty}\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}= \begin{cases}0 & \text { for } \frac{\eta A}{\beta_{e(N L)}}<1 \\ 1 & \text { for } \frac{\eta A}{\beta_{e(N L)}}=1 \\ \infty & \text { for } \frac{\eta A}{\beta_{e(N L)}}>1\end{cases}$
The upper bound's value in non-linear cap, $\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}$, when $n$ goes to infinity has different values that depends on different situations. However, the other element's value, $\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}$, goes to zero. Therefore, increasing $n$ in non-linear model leads to decrease $x^{*}$ to zero. It is true for non-linear cap also (there is at least one element that goes to zero when $n$ increases to infinity). The optimal price when optimal CEI is equal to zero is $p^{*}=\frac{A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}$.

The $n$ is a introduction of customers' desire for carbon emission improvement. Increasing $n$ means the product's carbon emission improvement becomes less and less important to them. In other words, improving the carbon emission level attracts fewer and fewer new customers. So, improving one unit's carbon emission becomes less interesting from economical point of view. As a result, retailer's (or manufacturer's) motivation to offer greener products becomes fewer and fewer. Therefore for a given $\eta$, increasing $n$ leads to lower $\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}$, which is optimal solution without considering improvement constraint, and after some point, the constraint becomes non-binding at optimality.

Proposition 6.5. In non-linear cap model ( $M_{8}$ ), increasing $\eta$ from zero to
$\left(\frac{\beta_{e(N L)^{2 n}}^{2 n}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}}$
A not only leads to increasing optimal carbon emission improvement, but also leads to increasing profit.

Proof. Thanks to proposition 6.2. we know that the optimal solution is $x^{*}=\min \left\{\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}\right.$
$\left.,\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}, x_{0}\right\} \cdot\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}$ is the value that maximizes the profit function without considering any constraints. However, we know that Equation 6.15 does not allow to have the optimal CEI higher than $\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}$, which ensures demand does not go beyond a certain level.

$$
\begin{aligned}
& \left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}} \geq\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n} \Leftrightarrow \frac{\eta A}{\beta_{e(N L)}} \leq\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}} \\
& \Leftrightarrow \eta \leq \frac{\left(\frac{\beta_{e(N L)}}{2 n} \alpha_{p} b n\right.}{\alpha^{2 n}} \frac{1}{2 n-1} \\
& A
\end{aligned}
$$

As we presented, when $0 \leq \eta \leq \frac{\left(\frac{\beta_{e(N L)}{ }^{2 n}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}}}{A}$, the optimal solution is $\left(\frac{\eta A}{\beta_{e(N L)}}\right)^{n}$, which means Equation 6.15 is binding, and so, increasing $\eta$ leads to increase optimal carbon emission's improvement. As we mentioned earlier, $\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}$ is the value that maximizes the profit function when Equation 6.15 is non-binding in optimal solution. Recall that optimal profit is concave with respect to $x$ and $\left(\frac{\beta_{e(N L)}}{2 \alpha_{p} b n}\right)^{\frac{n}{2 n-1}}$ is the value that maximize the profit function. Therefore, as long as $\eta$ increases (between $0 \leq \eta \leq$ $\frac{\left(\frac{\beta_{e(N L)^{2 n}}^{2 \alpha_{p} b n}}{}\right)^{\frac{1}{2 n-1}}}{A}$, which means $x^{*}$ increases) the optimal profit increases.
 means Equation 6.15 is not binding, and $\eta$ 's changes do not affect the optimal solution (carbon emission improvement and profit).

Proposition 6.6. In case that $\eta \leq \frac{\left(\frac{\beta_{e(N L)}{ }^{2 n}}{2 \alpha_{p} b n}\right)^{\frac{1}{2 n-1}}}{A}$, the carbon emission improvement constraint is binding at optimality, and the optimal product's carbon emission improvement and price of the capacitated model is less than the non-capacitated model.

Proof. Thanks to Proposition 6.5 . we know that when $\eta \leq \frac{\left(\frac{\beta_{e(N L)^{2 n}}^{2 \alpha_{p} b n}}{}\right)^{\frac{1}{2 n-1}}}{A}$ Equation
6.15 is binding and as a result, the optimal carbon emission's improvement in binding situation (called $x^{* B}$ ) is less than the optimal carbon emission's improvement in the nonlinear model $\left(M_{7}\right)$, which called $x^{* N B}\left(x^{* B}<x^{* N B}\right)$. Also, the optimal price in the function of $x$, which has presented in Lemma 6.2 and 6.3, is strictly increasing in $x$. Indeed:
$\frac{d p}{d x}=2 \alpha_{p} b x+\frac{\beta_{e(N L)}}{n} x \frac{1-n}{n}>0$
Therefore $x^{* B}<x^{* N B} \Rightarrow p\left(x^{* B}\right)<p\left(x^{* N B}\right)$.

Proposition 6.7. In case such that optimal carbon emissions are less than $x_{0}$, the product obtained with a linear demand has a higher price than the product obtained with a square root demand if $\frac{\beta_{e(L)}}{\alpha_{p} b x_{0}}>\sqrt{\frac{125}{108}}$, otherwise, product obtained with a linear demand has equal or lower price than the product obtained with a square root demand.

Proof. The optimal CEIs are equal to $\frac{\beta_{e(L)}}{2 \alpha_{p} b}$ and $\left(\frac{\beta_{e(N L)}}{4 \alpha_{p} b}\right)^{\frac{2}{3}}$ for linear and square root demand (considering that both of them are lower than $x_{0}$ ), respectively. The optimal prices are presented in the following ( $p^{1}$ and $p^{2}$ indicate the linear and square root demands' prices, respectively).

$$
\begin{aligned}
& p^{1}=\frac{\alpha_{p} b x^{2}+\beta_{e(L)} x+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} \\
& =\frac{\alpha_{p} b\left(\frac{\beta_{e(L)}}{2 \alpha_{p} b}\right)^{2}+\beta_{e(L)}\left(\frac{\beta_{e(L)}}{2 \alpha_{p} b}\right)+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}=\frac{\frac{3 \beta_{e(L)}^{2}}{4 \alpha_{p} b}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} \\
& p^{2}=\frac{\alpha_{p} b x^{2}+\beta_{e(N L)} \sqrt{x}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} \\
& =\frac{\alpha_{p} b\left(\left(\frac{\beta_{e(L)}}{2 \alpha_{p} b}\right)^{\frac{2}{3}}\right)^{2}+\beta_{e(N L)} \sqrt[3]{\left(\frac{\beta_{e(L)}}{2 \alpha_{p} b}\right)^{2}}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}} \\
& =\frac{5 \sqrt[3]{\frac{\beta_{e(N L)}^{4}}{4^{4} \alpha_{p} b}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}}{2 \alpha_{p}}
\end{aligned}
$$

The condition that $p^{1}>p^{2}$ is provided in the following.
$p^{1}>p^{2} \Leftrightarrow \frac{\frac{3 \beta_{e(L)}^{2}}{4 \alpha_{p} b}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}>\frac{5 \sqrt[3]{\frac{\beta_{e(N L)}^{4}}{4^{4} \alpha_{p} b}}+A+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)}{2 \alpha_{p}}$
$\Leftrightarrow \frac{3 \beta_{e(L)}^{2}}{4 \alpha_{p} b}>5 \sqrt[3]{\frac{\beta_{e(N L)}^{4}}{4^{4} \alpha_{p} b}} \Leftrightarrow \frac{27 \beta_{e(L)}^{6}}{64 \alpha_{p}^{3} b^{3}}>\frac{125 \beta_{e(N L)}^{4}}{256 \alpha_{p} b} \Leftrightarrow \frac{\beta_{e(L)}^{6}}{\alpha_{p}^{2} b^{2} \beta_{e(N L)}^{4}}>\frac{125}{108}$
We know $\beta_{e(N L)}=\beta_{e(L)} \sqrt{x_{0}}$, therefore, $\frac{\beta_{e(L)}^{6}}{\alpha_{p}^{2} b^{2} \beta_{e(N L)}^{4}}=\frac{\beta_{e(L)}^{2}}{\alpha_{p}^{2} b^{2} x_{0}{ }^{2}}$. Thus, while $\frac{\beta_{e(N L)}}{\alpha_{p} b x_{0}}>$ $\sqrt{\frac{125}{108}}$ the product with linear demand function has higher price than the product with a square root demand function.

### 6.5 Conclusion

In this chapter, we consider more general and complex demand functions. The demand functions are linearly decreasing in price, while, non-linearly increasing in carbon emission improvement. We consider a similar problem as benchmark of chapter 3 and we formulated under new demand function. The new problem is solved by an analytical approach. We consider furthermore a constraint that allows demand to increase to a certain
level and we call it cap. The problems with new demands functions (i.e. linear cap and non-linear cap) are formulated and solved.

The numerical examples show that when demand's cap is small, the linear cap model provides the better approximation than others, whereas, the square root model provides the better approximation for the bigger cap.

The work that has been done in this chapter can extend in several ways. First, the new demand function can be used in product differentiation and competition models, which are presented in previous chapters. It should be interesting to see the effect of cap on optimal strategies of retailers, specially on greenness, along with retailers' power (i.e. market share). Another extension can be the government regulations. The government regulations such as carbon tax, carbon cap, and etc. have their own effects on greenness decisions. However, it is interesting to see the effects of demand's cap alongside the government regulations' effect.

## Chapter 7

## General conclusion

### 7.1 Conclusion

The studies on retailing problems under the consideration of a sensitive demand to the price and greenness reveal several gaps in this area. First, most of the studies considered a Make-To-Order system while there are many cases in practice that follows Make-ToStock system and deal with problems such as inventory policy, order size and etc. Second, most studies considered a Stackelberg game, where the leader decides first and the follower acts later, while in practice, there are many players (e.g., companies, retailers, etc.) that changes their strategies based on other competitors' strategies. Third, the considered demand function is generally linearly decreasing in price and greenness. However, greenness may have a non-linear effect on demand. Also, improving greenness level does not always attract more customers, beyond a given level.

Thus, in this study, we propose different problems and formulation to close the existed gaps in the literature. We start our work by a simple benchmark model, which includes one supplier and one retailer who offer one product to customers in a price- and greennessdependent market. The retailer's warehouse refilling time follows Exponential distribution. The customers' arrival follows Poisson process. Thus, the system follows Markov chain process. The retailer decides product's price, carbon emission level, and order size to maximize his/her profit. We solve the problem by an analytical approach.

Then, we extended previous problem by considering a new and substitutable product. In this case, the problem includes two suppliers and one retailer. Each supplier provides retailer with one kind of products that makes retailer to offer two substitutable products to customers. The demand function for each product is similar to the benchmark model except there are switchovers' effect in the demand function. It means that each product's demand not only is affected by its product's price and greenness, but also, affected by other product's price and greenness. Like previous problem, this problem is formulated in a stochastic environment under different settings (considering different sets of decision variables) and solved by an analytical approach. To do a proper analyze and bring most important managerial and technical insights to the light, we distinguish the market according to sensitivity parameters. The market is categorized as: 1. PDS market, the switchovers are more governed by price differentiation rather than greenness differentiation 2.GDS market, the switchovers are more governed by greenness differentiation rather than price differentiation 3. Neutral market, price and greenness disparities have the same importance with respect to switchovers. The results show that retailer's optimal strategies depend on the market characteristics (i.e. PDS, GDS, and Neutral markets).

The numerical examples show that retailer benefits when it offers a new and substitutable product.

We continued our work on substitutable products in dynamic competition context. Two competing retailers offer two substitutable products (one product per retailer) in a grenneess- and price-sensitive market. Each retailer has its supplier. We consider the similar inventory policy as previous problems. The demand function for each retailer (product) is similar to previous problem, except that we introduced and applied retailer's power (market share) into the demand function. Demand and the replenishment lead times of retailers' warehouse are random and follow Exponential distribution. The retailers (players) decide their strategies based on other retailer's (player's) strategies. It is a nondominated and non-cooperative game. Optimal strategies of each retailer are determined at the Nash equilibrium. We start with a general model which in each retailer decides product's price, greenness level, and order size. In practice there are many situations where an existing retailer is already operating in the market, and a new retailer enters the market and offers a substitutable product. Therefore we proposed two different scenarios: 1. Competition without reaction, in which the existing retailer does not react to the new retailer's decisions 2 . Competition with partial reaction, in which the existing retailer just updates its price and order size but does not change the carbon emission intensity as this requires new deals with the supplier. Both problems are solved by analytical approaches. Then, we investigated the effect of competition on products' environmental performance (compared to benchmark model) as well as the impact of transportation carbon emissions. We also investigated the effects of the retailers' power (market share) and the market structures (i.e., PDS, GDS, and Neutral market) on products' environmental performance (compare retailers' greenness level). The results showed that the retailer that gains more market power will decrease its product's greenness in GDS market and increase it in PDS market. We also found that when both retailers have the same market power, the retailer that relies on a shorter supply chain offers a higher price if and only if the difference in transportation carbon emissions is higher than a given threshold value.

So far, we considered a demand function that is linearly decreasing in price and carbon emission level. Finally, we consider the demand function that is linearly decreasing in price, while, non-linearly increasing in carbon emission improvement. We consider a similar problem as benchmark and formulated under new demand function. The new problem is solved by an analytical approach. Also, we considered a constraint that allows demand to increase only to a certain level (cap). The new demands functions (i.e. linear cap and non-linear cap) are formulated and solved. Finally, we highlight some technical insights to enrich this chapter. The results showed that increasing demand's cap not only leads to increasing carbon emission improvement, but also leads to increasing profit. The numerical examples show that when demand's cap is small, the linear cap model provides a better approximation than others, whereas, the square root model provides the better approximation for the bigger cap.

### 7.2 Future works and perspectives

Our study can be extended in different ways. As we mentioned in introduction, our work started by a hybrid system with considering a Lead time- and price- sensitive demand. As the first extension, it would be interesting to investigate two competitive supply chains systems while considering Lead time-, greenness-, and price sensitive demand. Thus in this case, there are two competitive retailers in the market, each of them offers one product. The demand of each product is decreasing in its price, carbon intensity, and lead time and increasing in other product's price, carbon intensity, and lead time. This work can be followed under different scenarios such as: Centralized supply chain and Decentralized supply chain (which two sub-scenarios; supplier as the leader, and retailer as the leader).

Another extension of our work would be to consider the works that have been done in chapters 4 and 5 with linear cap and non-linear demands functions. Thus, it is interesting to consider product differentiation and competition problems with new demands that are introduced in chapter 6.

Chapter 6 introduced new demands functions and compared them with linear demand. This work can be extended with considering different shapes of demand function like the log-linear. Different demands functions prepare the opportunities to develop our understanding of customers' behavior. Again both collaboration and competition problems are also interesting to investigate with other demands function.

Considering the government regulations such as carbon tax, carbon cap, and etc. would also be interesting. Our work considers the CEA as the only source of environmental effect that comes from customers on retailer's strategies. However, the government regulations can be considered as other sources (in addition to CEA) that affect the retailer's strategies (especially carbon intensity). This extension can be applied even with new demands functions that are introduced in chapter 6

## Bibliography

AACE
2015. China's carbon trading website. (n.d.). Analysis of Apple's Carbon Emissions(AACE). http://www.tanpaifang.com/jienenjianpai/2015/0306/ 42879.html.

Abd Rahim, M. H., R. Z. J. A. Zukni, F. Ahmad, and N. Lyndon
2012. Green advertising and environmentally responsible consumer behavior: The level of awareness and perception of malaysian youth. Asian social science, 8(5):46.

Accenture
2019. More than Half of Consumers Would Pay More for Sustainable Products Designed to Be Reused or Recycled, Accenture Survey Finds. https://newsroom.accenture.com/news/ more-than-half-of-consumers-would-pay-more-for-sustainable-products-designed-t htm.

Asgari, E., Y. Frein, and R. Hammami
2018. Lead time quotation under mto and mts delivery modes with endogenous demand. In 2018 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM), Pp. 22-26. IEEE.

Asgari, E., Y. Frein, and R. Hammami
2021a. On Retailers' Profit in A Price-and GreennessSensitive Market with Different Demand Functions. In Conférence Internationale Génie Industriel QUALITA, Grenoble, France.

Asgari, E., R. Hammami, and Y. Frein
2020a. Does Retailer Benefit from Offering a New substitutable Product? The Management of Retailer's Best Strategies under Price- and Greenness- Sensitive Market. Working paper.

Asgari, E., R. Hammami, and Y. Frein
2020b. Retailing Competition for Substitutable Products in Greenness- and PriceDependent Market. In The Eight International Conference on Information systems, Logistics and Supply chain (ILS), Pp. 236-243, Texas, USA.

Asgari, E., R. Hammami, Y. Frein, and I. Nouira
2021b. The effect of greenness-and price-based competition on a product's environmental performance. International Journal of Production Economics, 234:108062.

Baron, O., O. Berman, and D. Wu
2016. Bargaining within the supply chain and its implications in an industry. Decision Sciences, 47(2):193-218.

Basiri, Z. and J. Heydari
2017. A mathematical model for green supply chain coordination with substitutable products. Journal of cleaner production, 145:232-249.

Benjaafar, S., Y. Li, and M. Daskin
2012. Carbon footprint and the management of supply chains: Insights from simple models. IEEE transactions on automation science and engineering, 10(1):99-116.

Bestseller
2018. Taking action on climate change. https://about.bestseller.com/news/ taking-action-on-climate-change

Bish, E. K. and R. Suwandechochai
2010. Optimal capacity for substitutable products under operational postponement. European Journal of Operational Research, 207(2):775-783.

Borin, N., J. Lindsey-Mullikin, and R. Krishnan
2013. An analysis of consumer reactions to green strategies. Journal of Product \& Brand Management.

Boyaci, T. and S. Ray
2003. Product differentiation and capacity cost interaction in time and price sensitive markets. Manufacturing \& Service operations management, 5(1):18-36.

Boyaci, T. and S. Ray
2006. The impact of capacity costs on product differentiation in delivery time, delivery reliability, and price. Production and Operations Management, 15(2):179-197.

Brécard, D.
2014. Consumer confusion over the profusion of eco-labels: Lessons from a double differentiation model. Resource and energy economics, 37:64-84.

Cachon, G. P.
2001. Stock wars: inventory competition in a two-echelon supply chain with multiple retailers. Operations Research, 49(5):658-674.

Cargal, J. M.
2003. The eoq inventory formula. Mathematical Sciences, 1.

Carlson, D. K.
2005. Public priorities: environment vs. economic growth. Gallup Poll. http://www. gallup. com/poll/15820/Public-Priorities-Environment-vs-Economic-Growth. aspx.

Ceryan, O.
2019. Asymmetric pricing and replenishment controls for substitutable products. Decision Sciences, 50(5):1093-1119.

Cheng, J., B. Li, B. Gong, M. Cheng, and L. Xu
2017. The optimal power structure of environmental protection responsibilities transfer in remanufacturing supply chain. Journal of Cleaner Production, 153:558-569.

Chitra, K.
2007. In search of the green consumers: A perceptual study. Journal of Services Research, 7(1).

Conrad, K.
2005. Price competition and product differentiation when consumers care for the environment. Environmental and Resource Economics, 31(1):1-19.

Costa, E., D. Montemurro, and D. Giuliani
2019. Consumers' willingness to pay for green cars: a discrete choice analysis in italy. Environment, Development and Sustainability, 21(5):2425-2442.

Dai, Y., X. Chao, S.-C. Fang, and H. L. Nuttle
2005. Pricing in revenue management for multiple firms competing for customers. International Journal of Production Economics, 98(1):1-16.

Dangelico, R. M. and P. Pontrandolfo
2010. From green product definitions and classifications to the green option matrix. Journal of Cleaner Production, 18(16-17):1608-1628.

Datta, S. K. et al.
2011. Pro-environmental concern influencing green buying: A study on indian consumers. International Journal of Business and management, 6(6):124.

Ding, H., Q. Zhao, Z. An, J. Xu, and Q. Liu
2015. Pricing strategy of environmental sustainable supply chain with internalizing externalities. International Journal of Production Economics, 170:563-575.

Ghosh, D. and J. Shah
2012. A comparative analysis of greening policies across supply chain structures. International Journal of Production Economics, 135(2):568-583.

Ghosh, D. and J. Shah
2015. Supply chain analysis under green sensitive consumer demand and cost sharing contract. International Journal of Production Economics, 164:319-329.

Giri, R. N., S. K. Mondal, and M. Maiti
2019. Government intervention on a competing supply chain with two green manufacturers and a retailer. Computers \& Industrial Engineering, 128:104-121.

Gu, W., D. Chhajed, N. C. Petruzzi, and B. Yalabik
2015. Quality design and environmental implications of green consumerism in remanufacturing. International Journal of Production Economics, 162:55-69.

Gürler, Ü. and A. Yılmaz
2010. Inventory and coordination issues with two substitutable products. Applied Mathematical Modelling, 34(3):539-551.

Hafezalkotob, A.
2015. Competition of two green and regular supply chains under environmental protection and revenue seeking policies of government. Computers \& Industrial Engineering, 82:103-114.

Hammami, R., E. Asgari, and Y. Frein
2020. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching. Under revision.

Hammami, R., I. Nouira, and Y. Frein
2018. Effects of customers' environmental awareness and environmental regulations on the emission intensity and price of a product. Decision Sciences, 49(6):1116-1155.

Haytko, D. L. and E. Matulich
2008. Green advertising and environmentally responsible consumer behaviors: Linkages examined. Journal of Management and Marketing Research, 1:2.

Heydari, J., K. Govindan, and A. Aslani
2019. Pricing and greening decisions in a three-tier dual channel supply chain. International Journal of Production Economics, 217:185-196.

Hong, Z. and X. Guo
2019. Green product supply chain contracts considering environmental responsibilities. Omega, 83:155-166.

Hong, Z., H. Wang, and Y. Gong
2019. Green product design considering functional-product reference. International Journal of Production Economics, 210:155-168.

Hovelaque, V. and L. Bironneau
2015. The carbon-constrained eoq model with carbon emission dependent demand. International Journal of Production Economics, 164:285-291.

Hsieh, C.-C. and C.-H. Wu
2009. Coordinated decisions for substitutable products in a common retailer supply chain. European Journal of Operational Research, 196(1):273-288.

Huang, J., M. Leng, and M. Parlar
2013. Demand functions in decision modeling: A comprehensive survey and research directions. Decision Sciences, 44(3):557-609.

Huang, S., Z.-P. Fan, and N. Wang
2020. Green subsidy modes and pricing strategy in a capital-constrained supply chain. Transportation Research Part E: Logistics and Transportation Review, 136:101885.

Jamali, M.-B. and M. Rasti-Barzoki
2018. A game theoretic approach for green and non-green product pricing in chain-to-chain competitive sustainable and regular dual-channel supply chains. Journal of Cleaner Production, 170:1029-1043.

Karray, S., G. Martín-Herrán, and G. Zaccour
2020. Pricing of demand-related products: Can ignoring cross-category effect be a smart choice? International Journal of Production Economics, 223:107512.

Krass, D., T. Nedorezov, and A. Ovchinnikov
2013. Environmental taxes and the choice of green technology. Production and operations management, 22(5):1035-1055.

Laroche, M., J. Bergeron, and G. Barbaro-Forleo
2001. Targeting consumers who are willing to pay more for environmentally friendly products. Journal of consumer marketing, 18(6):503-520.

Li, B., M. Zhu, Y. Jiang, and Z. Li
2016. Pricing policies of a competitive dual-channel green supply chain. Journal of Cleaner Production, 112:2029-2042.

Liu, G., H. Yang, and R. Dai
2020. Which contract is more effective in improving product greenness under different power structures: Revenue sharing or cost sharing? Computers \& Industrial Engineering, P. 106701.

Liu, P. and S.-p. Yi
2017. Pricing policies of green supply chain considering targeted advertising and product green degree in the big data environment. Journal of Cleaner Production, 164:1614-1622.

Liu, Z. L., T. D. Anderson, and J. M. Cruz
2012. Consumer environmental awareness and competition in two-stage supply chains. European Journal of Operational Research, 218(3):602-613.

Luo, Z., X. Chen, and M. Kai
2018. The effect of customer value and power structure on retail supply chain product choice and pricing decisions. Omega, 77:115-126.

Ma, P., H. Wang, and J. Shang
2013. Contract design for two-stage supply chain coordination: Integrating manufacturer-quality and retailer-marketing efforts. International Journal of Production Economics, 146(2):745-755.

McGuire, T. W. and R. Staelin
2008. An industry equilibrium analysis of downstream vertical integration. Marketing Science, 27(1):115-130.

Moon, W., W. J. Florkowski, B. Brückner, and I. Schonhof
2002. Willingness to pay for environmental practices: implications for eco-labeling. Land Economics, 78(1):88-102.

Nouira, I., Y. Frein, and A. B. Hadj-Alouane
2014. Optimization of manufacturing systems under environmental considerations for a greenness-dependent demand. International Journal of Production Economics, 150:188-198.

Osborne, M. J. et al.
2004. An introduction to game theory, volume 3. Oxford university press New York.

Palacios-Argüello, L., N. Gondran, I. Nouira, M.-A. Girard, and J. Gonzalez-Feliu
2020. Which is the relationship between the product's environmental criteria and the product demand? evidence from the french food sector. Journal of Cleaner Production, 244:118588.

Qi, Q., J. Wang, and Q. Bai
2017. Pricing decision of a two-echelon supply chain with one supplier and two retailers under a carbon cap regulation. Journal of Cleaner Production, 151:286-302.

Qin, J., L. Ren, and L. Xia
2017. Carbon emission reduction and pricing strategies of supply chain under various demand forecasting scenarios. Asia-Pacific Journal of Operational Research, 34(01):1740005.

Ramanathan, R., U. Ramanathan, and H.-L. Hsiao
2012. The impact of e-commerce on taiwanese smes: Marketing and operations effects. International Journal of Production Economics, 140(2):934-943.

Şen, A.
2016. Competitive markdown timing for perishable and substitutable products. Omega, 64:24-41.

Sengupta, A.
2012. Investment in cleaner technology and signaling distortions in a market with green consumers. Journal of Environmental Economics and Management, 64(3):468-480.

Sheu, J.-B. and Y. J. Chen
2012. Impact of government financial intervention on competition among green supply chains. International Journal of Production Economics, 138(1):201-213.

Sim, J., F. El Ouardighi, and B. Kim
2019. Economic and environmental impacts of vertical and horizontal competition and integration. Naval Research Logistics (NRL), 66(2):133-153.

Walmart
2017. Walmart Launches Project Gigaton to Reduce Emissions in Company's Supply Chain. https://corporate.walmart.com/newsroom/2017/04/19/ walmart-launchesprojectgigaton-to-reduce-emissions-in-companys-supply-chain.

Wu, C.-H., C.-W. Chen, and C.-C. Hsieh
2012. Competitive pricing decisions in a two-echelon supply chain with horizontal and vertical competition. International Journal of Production Economics, 135(1):265-274.

Wu, D., O. Baron, and O. Berman
2009. Bargaining in competing supply chains with uncertainty. European Journal of Operational Research, 197(2):548-556.

Xiao, T. and D. Yang
2008. Price and service competition of supply chains with risk-averse retailers under demand uncertainty. International Journal of Production Economics, 114(1):187-200.

Xiong, Z., P. Zhang, and N. Guo
2014. Impact of carbon tax and consumers' environmental awareness on carbon emissions in supply chains. Systems Engineering-Theory \& Practice, 34(9):2245-2252.

Xu, C., C. Wang, and R. Huang
2020. Impacts of horizontal integration on social welfare under the interaction of carbon tax and green subsidies. International Journal of Production Economics, 222:107506.

Xu, Y., H. Xie, et al.
2016. Consumer environmental awareness and coordination in closed-loop supply chain. Open Journal of Business and Management, 4(03):427.

Yalabik, B. and R. J. Fairchild
2011. Customer, regulatory, and competitive pressure as drivers of environmental innovation. International Journal of Production Economics, 131(2):519-527.

Zhang, L., J. Wang, and J. You
2015. Consumer environmental awareness and channel coordination with two substitutable products. European Journal of Operational Research, 241(1):63-73.

Zhao, J., J. Wei, and Y. Li
2014. Pricing decisions for substitutable products in a two-echelon supply chain with firms different channel powers. International Journal of Production Economics, 153:243-252.

Zhou, Y., F. Hu, and Z. Zhou
2018. Pricing decisions and social welfare in a supply chain with multiple competing retailers and carbon tax policy. Journal of cleaner production, 190:752-777.

Zhu, S. X.
2015. Integration of capacity, pricing, and lead-time decisions in a decentralized supply chain. International Journal of Production Economics, 164:14-23.

# Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching 

## A. 1 Introduction

Many online retailers rely on a delivery mix that includes traditional delivery from stock (DFS) and drop-shipping to satisfy demand. With drop-shipping, a retailer simply forwards customer orders to manufacturers or wholesalers who fulfill those orders directly for a predetermined price to be paid by the retailer (Khouja and Stylianou 2009). Dropshipping has significant advantages for the retailer. There are savings in holding cost, but drop-shipping typically incurs longer delivery times (DTs), which may deter timesensitive customers (Khouja 2001, Khouja and Stylianou 2009, Rabinovich et al. 2008). DFS implies a shorter and more reliable DT, but incurs a holding cost that may increase with uncertainty. A mix of DFS and drop-shipping has proven to be an efficient distribution strategy, particularly when there is high uncertainty in demand and/or replenishment lead time. In such cases, drop-shipping provides retailers with the option to reserve stock internally for high-priority orders while fulfilling regular orders through drop-shipping (Ayanso et al. 2006).

With a mix of DFS and drop-shipping, a product is offered with two varieties that have different DTs and can generate different margins (Ayanso et al. 2006). Rabinovich et al. (2008) studied the handbag and luggage segment of online retailing (e-Bags.com) and provided empirical evidence suggesting that an internet retailer can command higher margins from customers when it is willing and able to promise a shorter DT. While shopping on Cdiscount.com (a French leading online retailer) over the last Christmas Holidays, one of the authors has been offered two options to buy a product. In the first option, the product is immediately delivered from Cdiscount store in France. In the second option, the product is delivered directly from the supplier in China. This second option is less expensive for the customer but leads to a much longer and less reliable DT. Two real examples are presented below.

Many other online retailers use this hybrid distribution mode to sell substitutable products, as highlighted by Li et al. (2019), Tian et al. (2018) and Hagiu and Wright (2015).

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
Table A.1: Real examples of products delivered with a mix of DFS and drop-shipping

|  |  |  |
| :--- | :--- | :--- |
| Option 1. <br> Delivery from <br> Cdiscount store | Delivery: $21-22$ November <br> Price: $179.99 €$ | Delivery: $14-15$ November <br> Price: $10.13 €$ |
| Option 2. <br> Delivery from <br> Chinese supplier | Delivery: $05-19$ December <br> Price: $169.99 €$ | Delivery: 13-17 December <br> Price: $7.99 €$ |

Examples include, but are not limited to, some globally recognized retailers such as Amazon, Home Depot, and NewEagle. For instance, Amazon offers substitutable goods that can be either drop-shipped from upstream suppliers or shipped directly from Amazon fulfillment centers (Ayanso et al. 2006, Rabinovich et al. 2008). Figure 2 shows a generic supply chain (SC) for a retailer that relies on the hybrid distribution mode presented in the above examples, where regular product refers to the drop-shipped product, and express product refers to the product delivered from stock. In compliance with the above examples, the express product has a shorter DT but a higher price than the regular product.


Figure A.1: Illustration of the studied hybrid system with a mix of DFS and drop-shipping

The customers observe the pair of quoted DTs and prices provided by the retailer for each product and make their purchase decisions. In case of stockout, some customers initially interested in the express product may switch to the regular product, whereas others may leave the system (lost sales). Many authors have provided practical examples to illustrate how customers can be served with drop-shipping in case of stockout (e.g.,

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
Khouja and Stylianou 2009, Ayanso et al. 2006). Stockout-based substitution (S-b-S) impacts the demand of both express and regular products. It is, therefore, important to consider S-b-S and study its effect on DTs, price, and inventory decisions. Overall, the rate of S-b-S increases when the retailer is the exclusive seller of the product and decreases if the product can be found elsewhere by customers.

This study develops a modeling framework that fits the practical situations described above in order to investigate the DT, price, and inventory decisions faced by a retailer that uses a mix of DFS and drop-shipping to serve a time- and price-sensitive market with two substitutable products that differ in terms of the guaranteed DT and price considering Sb -S. In its general form, the problem is deciding how to differentiate the products in terms of DT and price, and determining the stock level in retailer's warehouse while satisfying the service level constraint for each product. We examine this problem when the demand process is stochastic, inventory replenishment lead time is stochastic, and drop-shipping lead time is also stochastic. The products are substitutable, so the mean demand rate for each product decreases in price and quoted DT and increases in price and quoted DT for the competing product. In addition, as we consider S-b-S, the effective mean demand rate of each product depends also on the stockout probability, which leads to a non-linear demand function that is sensitive to DT, price, and stock level. Finally, we consider a timeand price-sensitive market and, consequently, the total initial demand (for both products) is not constant but depends on the quoted prices and DTs.

There are several trade-offs in the model. The express product can be sold with a higher margin but requires holding inventory, which implies an additional cost. If the express product is profitable enough, it may be interesting to hold more stock than the minimum level imposed by the service constraint as this reduces the stockout probability and potentially could satisfy more express customers. To increase the initial demand for the express product, the retailer can either lower the price or quote a longer DT for the regular product. However, an increase in the express demand may require holding more inventory to satisfy the service level constraint. S-b-S is expected to have a significant impact on such trade-offs. Additionally, with the presence of S-b-S, it may be interesting to hold less stock (to reduce the inventory cost) although this implies a loss of a certain amount of express demand since this loss will be partially transformed into additional demand for the regular product. However, this may lead to a longer waiting time in the drop-shipping channel and, consequently, a higher risk of violating the service level constraint. To provide insights into these complex trade-offs, we study different variants of the problem: (i) The retailer decides the DT differentiation where the stock and prices are fixed. (ii) The retailer decides the stock level and the DT differentiation where the prices are fixed. (iii) The retailer decides the DT and price differentiations as well as the stock level (general model). In the first setting, we determine the closed-form expression of the optimal solution. We show that it is not always optimal to adopt an extreme time differentiation strategy and that an intermediate strategy can be more valuable but should be adjusted as a function of the stock level. In the second setting, we determine the optimal solution when S-b-S is ignored and provide a near-optimal solution in the presence of S-b-S. In the case without S-b-S, there exists a threshold price below which it is optimal to

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
offer both products with minimum time differentiation and above which only the express product should be offered. Considering S-b-S, the profit function profile changes. We show that quoting the DT without considering the effect of S-b-S leads to considerable loss for the retailer. Finally, using the analytical results obtained in the first two settings, and based on an extensive numerical study, we transform the general model into a quasiequivalent single-variable model. We show that a higher rate of S-b-S should normally lead to greater time differentiation and more stock. However, this would not impact the price differentiation. As we shall show in the next section, this is the first paper that investigates time- and price-based differentiation along with inventory decisions for a retailer that relies on a hybrid distribution system (with a mix of DFS and drop-shipping) to satisfy the demand of a time- and price-sensitive market subject to S-b-S behavior. Section 2 gives an overview of the literature. In Section 3, we develop the general model framework. In Sections 4, 5 and 6 we solve different variants of the model and derive analytical and numerical insights. Finally, in Section 7, we conclude and discuss the practical implications of our findings.

## A. 2 Literature Review

This study relates to two streams of research: models on product differentiation in timesensitive markets and models on the design and/or management of hybrid distribution modes. In this section, we review the extant research in those areas and discuss the contributions of our work. Studies on time-based product differentiation are relatively scarce in the literature (Albana et al. 2018). Most studies focus on the problem of an MTO manufacturer that differentiates products by allocating a different production capacity to each product variety. Boyaci and Ray (2003) considered a firm selling two substitutable products to satisfy a linear price- and DT-sensitive demand. Demand is served from two separate facilities; one facility per type of product, and each facility is modeled as an M/M/1 queue. In its general form, the problem is choosing the price of each product and the DT of the express product where there is a cost associated with each capacity. The numerical illustrations show that the firm should reduce the time differentiation when the capacity cost for the regular product is ignored. When capacity costs are considered, the optimal strategy should be to guarantee a shorter DT for the express product. Boyaci and Ray (2006) extended the previous work by considering delivery reliability (i.e., the minimum service level) as a decision variable. The authors found that as long as the proportion of time-sensitive customers is approximately equal to the proportion of price-sensitive customers, the firm would typically offer the express product with a lower guarantee than the regular product. Zhao et al. (2012) compared the uniform quotation mode, when a firm offers a single DT and price, to the differentiated quotation mode, when a firm offers a menu of DT and prices. As with the previous works, the production system is modeled as an $\mathrm{M} / \mathrm{M} / 1$ queue, the demand is linear in price and DT, and there is a dedicated capacity for each product. The results indicated that when DT-sensitive customers value a product no more than price-sensitive customers, a firm should not use the differentiated quotation mode. Our research differs from the cited studies in three key aspects. First, we focus

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
on a different and more complex SC configuration since we consider a retailer that relies on a hybrid distribution channel with a mix of DFS and drop-shipping. Second, we study product differentiation along with inventory decisions while the above studies consider MTO manufacturing systems without inventory considerations, which does not fit with the retailing context. Third, in our work, switchovers are governed not only by price and DT disparities but also by stockout, which enables us to investigate the impact of S-b-S on the main trade-offs of the system and derive new managerial insights. Our study also contributes to the research literature on hybrid distribution systems. In this stream of research, there are valuable studies on dual-channel structure and procurement strategies. For instance, Hagiu and Wright (2015) investigated the decision of an intermediary to function as a marketplace or as a reseller. This decision is driven by the level of demandenhancing marketing activity, which is controlled by suppliers in the pure marketplace mode and the intermediary in the pure reseller mode. Which mode is preferred depends on whether independent suppliers or the intermediary have more important information relevant to the optimal tailoring of marketing activities for each specific product. Tian et al. (2018) considered an SC composed of two competing suppliers. The suppliers sell two substitutable products through a common online retailer that acts as a Stackelberg leader and decides whether to function as a pure reseller, a pure marketplace, or with a hybrid mode. The main findings indicated that the retailer's preferred mode depends on the level of order-fulfillment costs and on the level of competition among suppliers. Li et al. (2019) considered a similar framework to investigate the online retailer's choice between pure batch ordering (traditional shipping) and hybrid shipping where drop-shipping is used with one supplier and batch ordering is used with the other. The authors found that the retailer prefers simultaneous structure (when it approaches both suppliers simultaneously for a price quotation) over sequential structure in the traditional mode while, in the hybrid mode, the sequential structure is preferred. The cited works differ from our study in many ways. In particular, they do not consider time-based product differentiation and do not deal with operations related to inventory and lead times. More closely related to our study, another stream of research on hybrid distribution channels focuses on inventory control with the consideration of the drop-shipping option. Ayanso et al. (2006) considered an internet retailer relying on an ( $\mathrm{r}, \mathrm{q}$ ) inventory system and facing two customer classes with respectively short DT needs satisfied via air-shipment and regular needs satisfied via ground-shipment. The authors developed a Monte Carlo simulation model to identify the stock level below which low-priority orders are to be drop-shipped directly from the supplier rather than fulfilled from in-house stock, which is, in this case, exclusively reserved for high-priority orders. The authors demonstrated that differentiating customer orders in terms of their priority and reserving inventory for high-priority orders can provide retailers with greater profit opportunities. Our study builds on this finding and investigates further challenges, such as how to differentiate customer orders in terms of DT and price and how to adjust the stock level accordingly. Khouja and Stylianou (2009) revisited the ( $\mathrm{r}, \mathrm{q}$ ) inventory model by allowing a retailer to use the drop-shipping option in case of a shortage during replenishment lead time. Numerical experiments showed that the drop-shipping option is more valuable when the replenishment lead time is long,

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
the ordering cost relative to the holding cost is small, and the back order cost relative to the holding cost is also small. These two papers (Ayanso et al. 2006 and Khouja and Stylianou 2009) focused only on inventory decisions while respectively relying on simulation and numerical experiments and adopted a framework in which the demand, DT, and price of each product are exogenous. Our research adopts an analytical approach to study product differentiation with endogenous DT, price, inventory, and demand. Finally, an interesting study on DT quotation and pricing in dual-channel SCs has been proposed by Hua et al. (2010). The authors considered the problem of a manufacturer that sells a product through a retailer or directly to customers. The decision variables are the retail price, the direct channel price, and the quoted DT for the direct channel. Quoting a shorter DT requires a higher delivery cost. Similar to our model, demand is sensitive to price and DT. The authors studied both a centralized and decentralized setting (where the manufacturer is the leader, and the retailer reacts by choosing its selling price) and showed that DT strongly influences the manufacturer and the retailer's pricing strategies and profits. Our study differs from that of Hua et al. (2010)'s in three ways. First, we focus on the problem of a retailer that relies on a hybrid channel and, consequently, we also consider inventory decisions. Second, we consider a stochastic setting where demand, stock replenishment DT, and drop-shipping DT are random while Hua et al. (2010) assumed a deterministic context. Third, in our model, customer switching depends on the differentiation in terms of price and DT and on the inventory level (stockout). In summary, there are valuable studies on time-based differentiation for manufacturers, particularly in the MTO context where differentiation typically results from allocating different production capacity to each product variety (e.g., Zhao et al. 2012, Boyaci and Ray 2003, 2006). There is also a growing body of literature on dual-channel distribution with a focus on situations where the manufacturer (and not the retailer) introduces the direct channel (e.g., Hua et al. 2010), the retailer rethinks its inventory policy with drop-shipping (e.g., Ayanso et al. 2006, Khouja and Stylianou 2009), and the retailer is concerned with the choice of distribution channel structure (e.g., Hagiu and Wright 2015, Tian et al. 2018). However, the problem studied in this paper has not been thoroughly explored in the extant literature. Our findings can provide useful insights into product differentiation and inventory decisions for retailers relying on hybrid distribution with DFS and drop-shipping to serve substitutable products to time-and price-sensitive customers with S-b-S behavior.

## A. 3 Modeling framework

We consider a profit-maximizing retailer that uses a hybrid distribution with a mix of DFS and drop-shipping and, thus, offers two substitutable products differentiated in terms of DT and price to satisfy random time- and price-sensitive demand. With drop-shipping, the retailer forwards the customer order to the supplier (wholesaler or manufacturer). The order is then processed by the supplier and delivered to the customer directly. In this case, the product is sold as a regular product. With traditional DFS, the product is immediately delivered to the customer from the retailer's stock (if stock is available) and is sold as an express product. This SC structure has been illustrated in Figure 1.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

Our initial demand model (i.e., without incorporating S-b-S) is linear with substitution. Customers arrive according to a Poisson process with mean arrival rates $\lambda_{1}$ and $\lambda_{2}$ for regular and express products, respectively. One unit of demand corresponds to one customer (order). The mean demand of each product is decreasing in its DT and price and increasing in the other product's DT and price. The regular product is offered at a standard price $p_{1}$ and a guaranteed DT $l_{1}$. The express product is offered by the retailer at a higher price $p_{2}$ and a shorter DT $l_{2}$. The mean demand rates are given in equations (1) and (2) below. Notice that the total potential market size is $2 A$. Parameters $\alpha_{p}$ and $\beta_{l}$ are price-sensitivity and time-sensitivity, respectively, and $\alpha_{2}$ and $\beta_{2}$ represent the sensitivity of switchover toward price difference and DT difference, respectively.

$$
\begin{align*}
& \lambda_{1}=A-\alpha_{p} p_{1}+\alpha_{2}\left(p_{2}-p_{1}\right)-\beta_{l} l_{1}+\beta_{2}\left(l_{2}-l_{1}\right)  \tag{A.1}\\
& \lambda_{2}=A-\alpha_{p} p_{2}+\alpha_{2}\left(p_{1}-p_{2}\right)-\beta_{l} 2_{1}+\beta_{2}\left(l_{1}-l_{2}\right) \tag{A.2}
\end{align*}
$$

Thus, if the DT $l_{1}$ increases by one unit, ( $\beta_{2}+\beta_{l}$ ) units of demand (customers) will be lost from the drop-shipping channel of which $\beta_{2}$ units will transfer to the DFS channel, and $\beta_{l}$ units of the demand will be lost from the two channels. Note that the total initial demand, given by $\lambda_{1}+\lambda_{2}=2 A-\alpha_{p}\left(p_{1}+p_{2}\right)-\beta_{l}\left(l_{1}+l_{2}\right)$, decreases in price charged and DT quoted.

For the regular product, the supplier processes each single order forwarded by the retailer and ships it to the customer without passing through the retailer's stock. The drop-shipping channel is then modeled as an MTO M/M/1 queue where the service time is exponentially distributed with mean rate $\mu_{1}$. The $\mathrm{M} / \mathrm{M} / 1$ queue has been widely used in DT quotation literature since the pioneering paper of Palaka et al. (1998). The retailer targets a minimum service level $r_{1}$ for the regular product. This guarantees that the probability that a customer will be served within the quoted DT $l_{1}$ is greater than $r_{1}$ (i.e., $\operatorname{Pro}\left(w \leq l_{1}\right) \geq r_{1}$ where $w$ is the expected waiting time for a customer), which prevents an unreliable DT quote.

As the express product is immediately delivered from stock, our entire analysis, without loss of generality, considers that $l_{2}$ is normalized to be equal to zero. Thus, the DT of regular product represents also the DT difference between express and regular products. DT $l_{1}$ is a decision variable. It is, of course, affected by the drop-shipper service time (capacity), but the retailer controls $l_{1}$ with pricing and inventory decisions as these decisions impact the demand rate in the drop-shipping channel. Hua et al. (2010) argued that the decision on the DT for the direct channel is crucial for dual-channel management.

Each replenishment of retailer's inventory refills the stock to its target level $S$. The service time to refill the inventory is exponentially distributed with mean rate $\mu_{2}$. The service time does not depend on the replenishment lot size since the products are assumed to be always available at the supplier's site, which is a common assumption (Zhu, 2015). Thus, replenishment time corresponds to preparation and transportation activities. We let $\psi_{0}$ denote the probability of stockout in the retailer's warehouse. The probability of serving customers (i.e., $1-\psi_{0}$ ) must be greater than a predetermined minimum service level $r_{2}$. In case of stockout, the customer can either switch to the regular product (i.e., accept a longer DT $l_{1}$ but with a lower price $p_{1}$ ) or leave the system (lost sale). We

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
let $\phi \in[0,1]$ denote the $\mathrm{S}-\mathrm{b}-\mathrm{S}$ rate; it represents the percentage of customers initially interested in the express product but accepting the regular product in case of stockout. In particular, $\phi=1$ means that all customers switch to the regular product, whereas $\phi=0$ means that all customers leave the system. Note that a smaller $\phi$ can represent a more competitive market; the S-b-S typically increases when the retailer is the exclusive seller of the product and decreases if the product can be found elsewhere by customers.

We let $\overline{\lambda_{1}}$ and $\overline{\lambda_{2}}$, respectively, denote the effective mean demand rate of the regular and express products. $\overline{\lambda_{1}}$ is the sum of customers initially interested in the regular product (i.e., $\lambda_{1}$ ) and customers interested in the express product but deciding to switch to the regular product in case of stockout (i.e., $\phi \psi_{0} \lambda_{2}$ ). $\overline{\lambda_{2}}$ is the difference between the customers initially interested in the express product (i.e., $\lambda_{2}$ ) and the customers that are likely to find empty stock (i.e., $\psi_{0} \lambda_{2}$ ). This system is illustrated in Figure 3. To obtain


Figure A.2: A hybrid delivery system with stock-out based substitution
a tractable model while capturing the essential features and the key trade-offs, we make two approximations that we will discuss. The first approximation concerns the service constraint for the regular product. When S-b-S is considered (i.e., for $\phi>0$ ), $\overline{\lambda_{1}}$ is no longer exponentially distributed. In this case, we conducted extensive experiments to check whether it is still acceptable to use the service constraint formula of the M/M/1 queue (which assumes that demand is exponentially distributed). Thus, we simulated our hybrid system with Arena and ran it for the equivalent of 500,000 hours. We found that the probability of serving customers on time, obtained with simulation of the real system, is equal to 0.25495 , whereas this probability is equal to 0.25727 if $\overline{\lambda_{1}}$ is assumed to be exponentially distributed. Therefore, we deduced that the service constraint formula of the $\mathrm{M} / \mathrm{M} / 1$ queue provides an acceptable approximation. Note that this constraint is exact when $\phi=0$.

The second approximation relates to the stockout probability which is given by a very complex formula in the case of a general inventory refilling policy. To approximate the

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
stockout probability in our system, we use the stockout probability obtained in the case where the buffer is refilled to stock level $S$ when it is empty. This approximation is accurate when the reorder point considered in practice is relatively small, which is the case in many warehouses that have storage capacity constraints or in the presence of a second delivery channel as we consider in our model. When the reorder point is relatively high, the impact of this approximation on the model's outcomes can be offset by increasing the minimum service level $r_{2}$ to a higher value than that typically used in practice. Based on this assumption, we obtain $\psi_{0}=\left(1+\frac{\mu_{2} S}{\lambda_{2}}\right)^{-1}$.

Thus, the effective mean demand rates are given by the following equations where $\alpha_{1}=\alpha_{p}+\alpha_{2}$ and $\beta_{1}=\beta_{l}+\beta_{2}$ ( $l_{2}$ is normalized to be equal to zero without loss of generality).

$$
\begin{gather*}
\overline{\lambda_{1}}=\lambda_{1}+\phi \psi_{0} \lambda_{2}=\lambda_{1}+\phi \frac{\lambda_{2}^{2}}{\lambda_{2}+\mu_{2} S}=A-\alpha_{1} p_{1}+\alpha_{2} p_{2}-\beta_{1} l_{1}+\phi \frac{\left(A-\alpha_{1} p_{2}+\alpha_{2} p_{1}+\beta_{2} l_{1}\right)^{2}}{A-\alpha_{1} p_{2}+\alpha_{2} p_{1}+\beta_{2} l_{1}+\mu_{2} S} \\
\overline{\lambda_{2}}=\left(1-\psi_{0}\right) \lambda_{2}=\frac{\mu_{2} S \lambda_{2}}{\lambda_{2}+\mu_{2} S}=\frac{\mu_{2} S\left(A-\alpha_{1} p_{2}+\alpha_{2} p_{1}+\beta_{2} l_{1}\right)}{A-\alpha_{1} p_{2}+\alpha_{2} p_{1}+\beta_{2} l_{1}+\mu_{2} S} \tag{A.3}
\end{gather*}
$$

In the traditional shipping mode, retailers purchase products from suppliers for a wholesale price and then determine retail prices for consumers. We let $c_{2}$ denote the purchasing cost per unit of the product (shipping cost included), which incurs the retailer's unit margin $m_{2}=p_{2}-c_{2}$ (this does not account for the inventory holding cost). In contrast, in the drop-shipping mode, suppliers determine retail prices and share revenue with the retailer. Consequently, the retailer loses the flexibility of setting market prices (Khouja and Stylianou 2009, Tian et al. 2018). Hence, as in practice, we consider that the regular product has a given fixed price and generates a fixed unit margin $m_{1}=p_{1}-c_{1}$ for the retailer.

Hence, in its general form, the problem is determining (i) the DT differentiation by quoting the DT $l_{1}$, (ii) the price differentiation by setting the price $p_{2}$, and (iii) the stock level $S$ with the objective of maximizing the total expected profit under service level constraints for both regular and express products. The general model is given below.

$$
\begin{equation*}
\text { General Model }\left(M_{l_{1}, p_{2}, S}\right) \tag{A.5}
\end{equation*}
$$

$\underset{l_{1}, p_{2}, S}{\operatorname{Maximize}} \pi=\left(p_{1}-c_{1}\right) \overline{\lambda_{1}}+\left(p_{2}-c_{2}\right) \overline{\lambda_{2}}-h S$
Subject to

$$
\begin{align*}
& 1-e^{-\left(\mu_{1}-\overline{\lambda_{1}}\right) l_{1}} \geq r_{1}  \tag{A.6}\\
& \frac{\mu_{2} S}{\lambda_{2}+\mu_{2} S} \geq r_{2}  \tag{A.7}\\
& S, \lambda_{1}, \lambda_{2} \geq 0, l_{1}>0, p_{2}>p_{1}, \overline{\lambda_{1}}<\mu_{1}
\end{align*}
$$

Objective function (5) represents the total expected profit to be maximized (i.e., net revenue - inventory cost). Note that $\overline{\lambda_{1}}$ and $\overline{\lambda_{2}}$ are, respectively, given by equations (3) and

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
(4). The average inventory level is given by $\frac{S}{2}$ and the unit inventory cost is denoted by 2 h , which incurs the total average inventory cost hS . Constraint (6) expresses the service level constraint for the regular product, i.e., $\operatorname{Pr}\left(w \leq l_{1}\right) \geq r_{1}$. Using the M/M/1 queue properties (as discussed earlier), we obtain $\operatorname{Pr}\left(w \leq l_{1}\right)=1-e^{-\left(\mu_{1}-\overline{\lambda_{1}}\right) l_{1}}$ (see Hammami et al. 2020). The service level constraint for the express product, i.e., $1-\psi_{0} \geq r_{2}$, is given by constraint (7). We recall that $1-\psi_{0}=\frac{\mu_{2} S}{\lambda_{2}+\mu_{2} S}$. The other constraints specify the variable domains. Note that the stability condition of the $\mathrm{M} / \mathrm{M} / 1$ queue (i.e., $\overline{\lambda_{1}}<\mu_{1}$ ) is automatically satisfied when constraint (6) is satisfied, so we ignore the stability condition for the remainder of the paper.

For convenience, we let 1 denote the DT of the regular product (instead of $l_{1}$ ) and p denote the price of the express product (instead of $p_{2}$ ) in the rest of this manuscript. In the following sections, we solve and analyze different variants of this model. In Section 4 , we study the case where 1 is a decision variable while $S$ and $p$ are fixed. In Section 5, we study the case where 1 and $S$ are decision variables while $p$ is fixed. The general case, in which $1, S$, and $p$ are decision variables, is studied in Section 6.

## A. 4 Delivery time quotation model with fixed stock and price

In this section, we determine the optimal DT $l$ when $S$ and p are fixed. To simplify the notation, we let $\lambda_{1}=a_{1, p}-\beta_{1} l$ and $\beta_{2}=a_{2, p}+\beta_{2} l$, where $a_{1, p}=A-\alpha_{1} p_{1}+\alpha_{2} p$ and $a_{2, p}=A+\alpha_{2} p_{1}-\alpha_{1} p$. In addition, we let $z=\ln \left(\frac{1}{1-r_{1}}\right)$. The resulting model, denoted by $\left(M_{l}\right)$, is given below.

Maximize $\pi(l)=m_{1}\left(a_{1, p}-\beta_{1} l+\phi \frac{\left(a_{2, p}+\beta_{2} l\right)^{2}}{a_{2, p}+\beta_{2} l+\mu_{2} S}\right)+\frac{m_{2} \mu_{2} S}{a_{2, p}+\beta_{2} l+\mu_{2} S}\left(a_{2, p}+\beta_{2} l\right)-h S$

Subject to

$$
\begin{align*}
& \mu_{1}-a_{1, p}+\beta_{1} l-\phi \frac{\left(a_{2, p}+\beta_{2} l\right)^{2}}{a_{2, p}+\beta_{2} l+\mu_{2} S}-\frac{z}{l} \geq 0  \tag{A.9}\\
& \frac{\mu_{2} S}{a_{2, p}+\beta_{2} l+\mu_{2} S} \geq r_{2}  \tag{A.10}\\
& \max \left\{0, \frac{-a_{2, p}}{\beta_{2}}\right\}<l \leq \frac{a_{1, p}}{\beta_{1}}
\end{align*}
$$

Note that constraint (10) imposes lower and upper bounds on $l$ to guarantee positive values for $\lambda_{1}$ and $\lambda_{2}$ (also note that $\frac{-a_{2, p}}{\beta_{2}}$ is not necessarily negative). As highlighted by many authors (e.g., Ayanso et al. 2006, Rabinovich et al. 2008), the express product generates a higher margin than the regular product. Hence, we focus here on the case

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
of $m_{2} \geq m_{1}$. Since $\phi \leq 1$, we also have $m_{2} \geq \phi m_{1}$. We first solve model $\left(M_{l}\right)$ and then discuss the insights.

## A.4.1 Model solving

Notice that constraint (8) is equivalent to $f_{\phi(l)} \geq 0$ where $f_{\phi(l)}=\beta_{2}\left(\beta_{1}-\phi \beta_{2}\right) l^{3}+$ $\left(\beta_{1}\left(a_{2, p}+\mu_{2} S\right)-\beta_{2}\left(a_{1, p}-\mu_{1}+2 \phi a_{2, p}\right)\right) l^{2}-\left(\left(a_{1, p}-\mu_{1}\right)\left(a_{2, p}+\mu_{2} S\right)+\beta_{2} z+\phi a_{2, p}{ }^{2}\right) l-$ $z\left(a_{2, p}+\mu_{2} S\right)$. To solve model $\left(M_{l}\right)$, we first provide a simpler formulation of constraint (8). All proofs are given in the appendix.

Lemma A.1. For $\phi=0$, the service constraint for the regular product (constraint (8)) is equivalent to $l \geq l_{0}$, where $l_{0}=\frac{a_{1, p}-\mu_{1}+\sqrt{\left(a_{1, p}-\mu_{1}\right)^{2}+4 \beta_{1} z}}{2 \beta_{1}}$. For $\phi>0$, constraint ( 8 ) is equivalent to $l \geq l_{\phi}$, where $l_{\phi}$ is the unique root in $\left[l_{0}, \frac{a_{1, p}}{\beta_{1}}\right]$ of the cubic equation: $f_{\phi}(l)=0$.

Lemma 1 provides a new lower bound on 1. Furthermore, constraint (9) is equivalent to $l \leq \frac{\left(1-r_{2}\right) \mu_{2} S-r_{2} a_{2, p}}{r_{2} \beta_{2}}$. Hence, we deduce that the feasible domain for model $\left(M_{l}\right)$ is $\left[l_{\text {min }}, l_{\text {max }}\right]$, where $l_{\text {min }}=\max \left\{l_{\phi}, \frac{-a_{2, p}}{\beta_{2}}\right\}$ and $l_{\max }=\min \left\{\frac{a_{1, p}}{\beta_{1}}, \frac{\left(1-r_{2}\right) \mu_{2} S-r_{2} a_{2, p}}{r_{2} \beta_{2}}\right\}$. The optimal solution of model $\left(M_{l}\right)$ is given below.

Lemma A.2. The optimal DT of the regular product is:
$l^{*}= \begin{cases}l_{t h} & \text { if } l_{\text {min }} \leq l_{t h} \leq l_{\text {max }} \\ l_{\text {min }} & \text { if } l_{\text {th }}<l_{\text {min }} \\ l_{\text {max }} & l_{\text {th }}>l_{\text {max }}\end{cases}$
where $l_{\text {th }}=\frac{-a_{2, p}+\mu_{2} S\left(\sqrt{\frac{\beta_{2}\left(m_{2}-\phi m_{1}\right)}{m_{1}\left(\beta_{1}-\phi \beta_{2}\right)}}-1\right)}{\beta_{2}}, l_{\min }=\max \left\{l_{\phi}, \frac{-a_{2, p}}{\beta_{2}}\right\}$ and $l_{\max }=$ $\min \left\{\frac{a_{1, p}}{\beta_{1}}, \frac{\left(1-r_{2}\right) \mu_{2} S-r_{2} a_{2, p}}{r_{2} \beta_{2}}\right\}$. For $\phi=0$ (i.e., in the case without $S-b-S$ ), $l_{\phi}=$ $l_{0}=\frac{a_{1, p}-\mu_{1}+\sqrt{\left(a_{1, p}-\mu_{1}\right)^{2}+4 \beta_{1} z}}{2 \beta_{1}}$.
For $\phi>0, l_{\phi}$ is the unique solution in $\left[l_{0}, \frac{a_{1, p}}{\beta_{1}}\right]$ of the cubic equation $f_{\phi}(l)=0($ see Lemma 1).

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

## A.4.2 Model analysis and insights

The cases of $l^{*}=\frac{-a_{2, p}}{\beta_{2}}$ or $\frac{a_{1, p}}{\beta_{1}}$ correspond to extreme situations where only one product is offered to customers. These cases are not relevant to our study, so we exclude them from the analysis. Based on the result of Lemma 2, we deduce a set of managerial implications in the following series of propositions. In Proposition 1, we analyze the optimal time differentiation strategy.

Proposition A.1. If $\frac{m_{2}}{m_{1}} \leq \frac{\beta_{1}}{\beta_{2}}$, then the retailer should adopt a minimum time differentiation strategy (i.e., offering the regular product with the shortest feasible DT).
If $\frac{\beta_{1}}{\beta_{2}}<\frac{m_{2}}{m_{1}} \leq \frac{1}{r_{2}^{2}}\left(\frac{\beta_{1}}{\beta_{2}}-\phi\left(1-r_{2}^{2}\right)\right)$, then the optimal strategy depends on the stock level $S$. The retailer should adopt minimum time differentiation when $S$ is smaller than a given threshold value and should increase the time differentiation (without reaching the maximum).
If $\frac{m_{2}}{m_{1}}>\frac{1}{r_{2}^{2}}\left(\frac{\beta_{1}}{\beta_{2}}-\phi\left(1-r_{2}^{2}\right)\right)$, then the optimal strategy is maximum time differentiation (i.e., offering the regular product with the longest feasible $D T$ ).

The managerial guidelines provided in Proposition 1 are graphically illustrated in Figure 4.


Figure A.3: Time differentiation strategy when the characteristics of the express product are fixed

Note that $\frac{m_{2}}{m_{1}}$ represents the price differentiation between the express product and

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
the regular product. Furthermore, $\frac{\beta_{1}}{\beta_{2}}$ represents the ratio of customers lost from the dropshipping channel to customers transferring to the DFS channel due to a unitary increase in the DT of the regular product. A higher ratio $\frac{\beta_{1}}{\beta_{2}}$ means a higher number of lost customers relative to the number of customers switching to the express product when the retailer quotes a longer DT. This characterizes, for instance, a more competitive external market or more time-sensitive customers but who are not sensitive enough to switch to the express product and pay a higher price.

When $\frac{m_{2}}{m_{1}} \leq \frac{\beta_{1}}{\beta_{2}}$, quoting a longer DT results in loss of revenue caused by a decrease in regular demand that is greater than the additional revenue generated from the increase in the express demand. In this case, the retailer has an interest in quoting the shortest feasible DT given by $l^{*}=l_{\text {min }}=l_{\phi}$.

Then, when $\frac{\beta_{1}}{\beta_{2}}<\frac{m_{2}}{m_{1}} \leq \frac{1}{r_{2}^{2}}\left(\frac{\beta_{1}}{\beta_{2}}-\phi\left(1-r_{2}^{2}\right)\right)$, the trade-off between the regular demand and the express demand is affected by the stock level. Compared to the first case, the express product becomes more profitable. However, when S is relatively small, it is not in the retailer's interest to favor the express demand since the service level constraint for the express product cannot be satisfied. Thus, it is still optimal to adopt minimum time differentiation. When the stock level increases, it is possible to capitalize more on the express demand, which leads to greater time differentiation (i.e., $l^{*}=l_{t h}$ ). Nevertheless, the price differentiation is here not large enough to justify maximizing the express demand, so there is no need for maximum time differentiation. This situation is interesting as it shows that the optimal strategy is neither minimum nor maximum time differentiation but an intermediate strategy where $l^{*}=l_{t h}$, could be preferred.

Finally, when $\frac{m_{2}}{m_{1}}>\frac{1}{r_{2}^{2}}\left(\frac{\beta_{1}}{\beta_{2}}-\phi\left(1-r_{2}^{2}\right)\right)$, the express product becomes profitable. Thus, the loss of revenue for the regular product resulting from increasing the DT is offset by the gain in revenue from the express product. It is then optimal to maximize the express demand (the only limitation is the service constraint), which requires maximum time differentiation (i.e., $l^{*}=l_{\max }$ ).

In summary, the optimal strategy is minimum time differentiation when the price differentiation relative to the ratio $\frac{\beta_{1}}{\beta_{2}}$ is small, and the optimal strategy is maximum time differentiation when the price differentiation relative to the ratio $\frac{\beta_{1}}{\beta_{2}}$ is large. For intermediate levels of price differentiation, neither minimum nor maximum time differentiation is preferred, but the optimal strategy depends on the express product stock levels. We now investigate in Proposition 2 the effect of stock level on time differentiation.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

Proposition A.2. A higher stock level $S$ leads to the retailer offering the regular product with a shorter $D T$ when the retailer adopts a minimum time differentiation strategy and with a longer DT in all other cases.

We expect that holding more inventory leads to quoting a longer DT for the regular product to favor the express demand since more of this demand can be satisfied. However, in case of minimum time differentiation, we demonstrate that the optimal DT decreases in $S$ (see the proof of Proposition 2 in the appendix). The qualitative explanation is the following. An increase in $S$ reduces the number of switching customers in case of stockout, which implies lower regular demand and, consequently, leads to a non-binding service constraint for the regular product. Since we are analyzing the case where the shortest feasible DT is offered to customers, the model reacts by quoting a shorter DT until it reaches the binding situation once again. We now study in Proposition 3 the effect of S-b-S.

Proposition A.3. An increase in the $S$ - $b-S$ rate $\phi$ leads to the retailer offering the regular product with a longer $D T$ as long as the retailer does not adopt a maximum time differentiation strategy. In case of maximum time differentiation, the optimal DT does not depend on $\phi$.

Recall that a greater $\phi$ implies a higher percentage of express demand transformed into regular demand in the case of stockout. To understand the effect of S-b-S, three cases should be distinguished. In the case of minimum time differentiation, the service constraint for the regular product is binding. Consequently, when $\phi$ increases, the system must quote a longer DT to decrease the regular demand and satisfy the service constraint.

In the case of intermediate time differentiation (i.e., when $l^{*}=l_{t h}$ ), none of the service constraints are binding (neither for the regular product nor for the express product). This situation is less intuitive since the optimal strategy is composed of a trade-off between both types of demand and not by maximizing one of the demand types. In this case, the model reacts to an increase in $\phi$ by quoting a longer DT (as in the first case) as this enables offsetting the increase in regular demand and returning to a more profitable trade-off.

Finally, in case of maximum time differentiation, the express product is much more profitable than the regular product, but the system cannot increase the express demand any more since the service constraint for the express product is already binding. Thus, an increase in $\phi$ does not lead to any reaction. The analysis of this first model variant revealed the significant impact of $S$ on the optimal strategy and, consequently, the importance of optimizing $S$.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

## A. 5 Delivery time quotation and stock level determination model with fixed price

We now study a new variant of the model where $l$ and $S$ are decision variables, but $p$ is fixed. Recall that $\lambda_{1}=a_{1, p}-\beta_{1} l$ and $\beta_{2}=a_{2, p}+\beta_{2} l$. The resulting model, denoted by ( $M_{l, S}$ ), is given below.
$\underset{l, S}{\operatorname{Maximize}} \pi=m_{1}\left(a_{1, p}-\beta_{1} l+\phi \frac{\left(a_{2, p}+\beta_{2} l\right)^{2}}{a_{2, p}+\beta_{2} l+\mu_{2} S}\right)+\frac{m_{2} \mu_{2} S}{a_{2, p}+\beta_{2} l+\mu_{2} S}\left(a_{2, p}+\beta_{2} l\right)-h S$

Subject to

$$
\begin{align*}
& \mu_{1}-a_{1, p}+\beta_{1} l-\phi \frac{\left(a_{2, p}+\beta_{2} l\right)^{2}}{a_{2, p}+\beta_{2} l+\mu_{2} S}-\frac{z}{l} \geq 0  \tag{A.12}\\
& \frac{\mu_{2} S}{a_{2, p}+\beta_{2} l+\mu_{2} S} \geq r_{2}  \tag{A.13}\\
& l>0, S, \lambda_{1}, \lambda_{2} \geq 0
\end{align*}
$$

Note that $\pi(l, S)$ is concave in $S$ (for a given $l$ ) and reaches its maximum in $S_{t h}(l)=$ $\frac{\lambda_{2}}{\mu_{2}}\left(\sqrt{\frac{\mu_{2}\left(m_{2}-\phi m_{1}\right)}{h}}-1\right)$. To obtain a relevant problem, we must have $\frac{\mu_{2}\left(m_{2}-\phi m_{1}\right)}{h} \geq$ 1. Service constraints (11) and (12) are, respectively, equivalent to $S \geq S_{1}(l)=\frac{\lambda_{2}}{\mu_{2}}\left(\frac{\phi \lambda_{2}}{\mu_{1}-\lambda_{1}-\frac{z}{l}}-\right.$ 1) and $S \geq S_{2}(l)=\frac{\lambda_{2}}{\mu_{2}} \frac{r_{2}}{1-r_{2}}$. Hence, for a given feasible 1 , the optimal stock level is $S^{*}(l)=\max \left\{S_{t h}(l), S_{1}(l), S_{2}(l)\right\}$. To make more progress, we now need to distinguish two cases: $\phi=0$ and $\phi>0$. We first solve analytically the model for $\phi=0$ and derive managerial insights. Then, we analyze the case of $\phi>0$ and derive insights based on numerical experiments.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

## A.5.1 Model $\left(M_{l, S}\right)$ without stockout-based substitution $(\phi=0)$ : Analytical resolution and insights

To solve model ( $M_{l, S}$ ) for $\phi=0$, we first provide a simpler equivalent formulation in Lemma A. 3 .

Lemma A.3. For $\phi=0$, model $\left(M_{l, S}\right)$ is equivalent to the following single-variable model.
$\operatorname{Max} \pi(l)= \begin{cases}m_{1}\left(a_{1, p}-\beta_{1} l\right)+r_{2}\left(m_{2}-\frac{h}{\left(1-r_{2}\right) \mu_{2}}\right)\left(a_{2, p}+\beta_{2} l\right) & \text { if } m_{2} \leq \frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}} \\ m_{1}\left(a_{1, p}-\beta_{1} l\right)+\left(\sqrt{m_{2}}-\sqrt{\frac{h}{\mu_{2}}}\right)\left(a_{2, p}+\beta_{2} l\right) & m_{2}>\frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}}\end{cases}$ subject to $\max \left\{l_{0}, \frac{-a 2, p}{\beta_{2}}\right\} \leq l \leq \frac{a_{1, p}}{\beta_{1}}$, where $l_{0}=\frac{a_{1, p}-\mu_{1}+\sqrt{\left(a 1, p-\mu_{1}\right)^{2}+4 \beta_{1} z}}{2 \beta_{1}}$.

Note that $\pi(l)$ is linear in $l$. This result is not intuitive; it implies that the optimal DT is either the shortest feasible DT (i.e., $\max \left\{l_{0}, \frac{-a 2, p}{\beta_{2}}\right\}$ ) or the longest DT (i.e., $\frac{a_{1, p}}{\beta_{1}}$ ). When it is optimal to quote the shortest DT, we assume that $l_{0} \geq \frac{-a 2, p}{\beta_{2}}$ since, otherwise, the retailer offers only the regular product, which is an extreme case that we do not study. We provide the optimal solution in Lemma 4.

Lemma A.4. For $\phi=0$, the optimal solution of model $\left(M_{l, S}\right)$ is given as follows. case of $m_{2} \leq \frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}}$

- ifm $m_{2} \leq \frac{m_{1} \beta_{1}}{r_{2} \beta_{2}}+\frac{h}{\left(1-r_{2}\right) \mu_{2}}$, then $\left\{\begin{array}{l}l^{*}=l_{0}=\frac{a_{1, p}-\mu_{1}+\sqrt{\left(a_{1, p}-\mu_{1}\right)^{2}+4 \beta_{1} z}}{2 \beta_{1}} \\ S^{*}=\left(\frac{r_{2}}{1-r_{2}}\right) \frac{2 \beta_{1} a_{2, p}+\beta_{2}\left(a_{1, p}-\mu_{1}+\sqrt{\left(a_{1}, p-\mu_{1}\right)^{2}+4 \beta_{1} z}\right)}{2 \beta_{1} \mu_{2}}\end{array}\right.$
- if $m_{2}>\frac{m_{1} \beta_{1}}{r_{2} \beta_{2}}+\frac{h}{\left(1-r_{2}\right) \mu_{2}}$, then $\left\{\begin{array}{l}l^{*}=\frac{a_{1, p}}{\beta_{1}} \\ S^{*}=\left(\frac{r_{2}}{1-r_{2}}\right) \frac{\beta_{1} a_{2, p}+\beta_{2} a_{1, p}}{\beta_{1} \mu_{2}}\end{array}\right.$
case of $m_{2}>\frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}}$

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

- ifm $m_{2} \leq\left(\sqrt{\frac{m_{1} \beta_{1}}{\beta_{2}}}+\sqrt{\frac{h}{\mu_{2}}}\right)^{2}$, then $\left\{\begin{array}{l}l^{*}=l_{0}=\frac{a_{1, p}-\mu_{1}+\sqrt{\left(a_{1, p}-\mu_{1}\right)^{2}+4 \beta_{1} z}}{2 \beta_{1}} \\ S^{*}=\left(\sqrt{\frac{m_{2} \mu_{2}}{h}}-1\right) \frac{2 \beta_{1} a_{2, p}+\beta_{2}\left(a_{1, p}-\mu_{1}\right.}{2 \beta_{1} \mu_{2}} \sqrt{\left(a_{1, p}-\mu_{1}\right)}\end{array}\right.$
- if $m_{2}>\left(\sqrt{\frac{m_{1} \beta_{1}}{\beta_{2}}}+\sqrt{\frac{h}{\mu_{2}}}\right)^{2}$, then $\left\{\begin{array}{l}l^{*}=\frac{a_{1, p}}{\beta_{1}} \\ S^{*}=\left(\sqrt{\frac{m_{2} \mu_{2}}{h}}-1\right) \frac{\beta_{1} a_{2, p}+\beta_{2} a_{1, p}}{\beta_{1} \mu_{2}}\end{array}\right.$

Hence, according to Lemma 4, if $m_{2} \leq \frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}}$, then the minimum time differentiation is adopted when we also have $m_{2} \leq \frac{m_{1} \beta_{1}}{r_{2} \beta_{2}}+\frac{h}{\left(1-r_{2}\right) \mu_{2}}$, and only the express product is offered when $m_{2}>\frac{m_{1} \beta_{1}}{r_{2} \beta_{2}}+\frac{h}{\left(1-r_{2}\right) \mu_{2}}$. Furthermore, if $m_{2}>\frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}}$, then the minimum time differentiation is adopted when we also have $m_{2} \leq\left(\sqrt{\frac{m_{1} \beta_{1}}{\beta_{2}}}+\sqrt{\frac{h}{\mu_{2}}}\right)^{2}$, and only the express product is offered when $m_{2}>\left(\sqrt{\frac{m_{1} \beta_{1}}{\beta_{2}}}+\sqrt{\frac{h}{\mu_{2}}}\right)^{2}$. This is recapitulated in Table 1.

Table A.2: Optimal retailer's strategy under the setting of model $\left(M_{l, S}\right)$ with $\phi=0$

| $m_{2} \leq \frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}}$ | $m_{2}>\frac{h}{\left(1-r_{2}\right)^{2} \mu_{2}}$ |  |  |
| :--- | :--- | :--- | :--- |
| $m_{2} \leq \frac{m_{1} \beta_{1}}{r_{2} \beta_{2}}+$ | $m_{2}>\frac{m_{1} \beta_{1}}{r_{2} \beta_{2}}$ <br> $\frac{h}{\left(1-r_{2}\right) \mu_{2}}$ | $m_{2} \leq\left(\sqrt{\frac{m_{1} \beta_{1}}{\beta_{2}}}+\right.$ <br> $\left(1-r_{2}\right) \mu_{2}$ | $m_{2}>\left(\sqrt{\frac{m_{1} \beta_{1}}{\beta_{2}}}+\right.$ |
| Minimum time <br> differentiation | Only the express <br> product <br> is offered (DFS <br> channel) | Minimum time <br> differentiation | $\left.\frac{h}{\mu_{2}}\right)^{2}$ |
| Only the express <br> product <br> is offered (DFS <br> channel) |  |  |  |

Based on the result of Lemma 4, we derive insights into the optimal strategy of the retailer in the following two propositions. In Proposition 4, we investigate the effect of prices on time differentiation.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

$$
\begin{aligned}
& \text { Proposition A.4. There exists a threshold value defined by } \frac{\beta_{1} m_{1}}{r_{2} \beta_{2}}+\frac{h}{\left(1-r_{2}\right) \mu_{2}}+c_{2} \text {, } \\
& \text { if } \frac{\beta_{1} m_{1}}{\beta_{2}} \leq \frac{h}{\mu_{2}}\left(\frac{r_{2}}{1-r_{2}}\right)^{2} \text { and by }\left(\sqrt{\frac{\beta_{1} m_{1}}{\beta_{2}}}+\sqrt{\frac{h}{\mu_{2}}}\right)^{2} \text {, if } \frac{\beta_{1} m_{1}}{\beta_{2}}>\frac{h}{\mu_{2}}\left(\frac{r_{2}}{1-r_{2}}\right)^{2} \text {, such } \\
& \text { that: } \\
& \text { If the express product's price is smaller than this threshold value, then it is optimal to } \\
& \text { offer both products with a minimum time differentiation; otherwise, only the express } \\
& \text { product should be offered (i.e., the retailer must rely only on the DFS channel). }
\end{aligned}
$$

Observing Table 1, we expect that an increase in $m_{2}$ would have a non-monotonous effect on the time differentiation strategy. However, whatever the values of the parameters, we demonstrate that the intermediate intervals (second and third columns of Table 1) cannot be feasible simultaneously (see the proof of Proposition 4 in the appendix). Therefore, when $m_{2}$ increases, the model reacts only by moving from minimum time differentiation to giving up the drop-shipping channel. Thus, unlike model ( $M_{l}$ ) (in which the stock level was fixed), it is never optimal here to have an intermediate time differentiation strategy. The retailer should either offer both products with minimum time differentiation or only offer the express product. In Proposition 5, we investigate the case where both products are offered to customers.

Proposition A.5. If both products are offered to customers, then the higher the express product's price, the longer the DT of the regular product, and the higher the stock level.

From Proposition 4 we deduce that if both products are offered to customers, then the optimal strategy is minimum time differentiation. In this case, Proposition 5 states, as expected, that the optimal DT and the optimal stock are increasing in the express product's price as this favors express demand when the express product becomes more profitable.

Finally, note that the results discussed in this section are relative to the case without S-b-S and that the consideration of S-b-S will likely impact such trade-offs. This is the focus of the next section.

## A.5.2 Model $\left(M_{l, S}\right)$ with stockout-based substitution $(\phi>0)$ : Analysis and numerical insights

When $\phi>0$, it is not possible to solve model ( $M_{l, S}$ ) analytically. Hence, we first provide a simpler formulation of the model and then rely on this formulation to conduct our analysis.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

Lemma A.5. For $\phi>0$, Model $\left(M_{l, S}\right)$ is equivalent to the following single-variable model:
$\operatorname{Max\pi }(l)=$
$\left\{\begin{array}{l}\pi_{1}(l)=\left(m_{1}-\frac{m_{2}}{\phi}\right) \mu_{1}+m_{2} a_{2, p}+\frac{m_{2} a_{1, p}}{\phi}+\frac{h a_{2, p}}{\mu_{2}}-\left(m_{1}-\frac{m_{2}}{\phi}\right)^{z} \frac{z}{l}+ \\ \left(m_{2} \beta_{2}-\frac{m_{2} \beta_{1}}{\phi}+\frac{h \beta_{2}}{\mu_{2}}\right) l-\left(\frac{h \phi}{\mu_{2}}\right) \frac{\left(a_{2, p}+\beta_{2} l\right)^{2}}{\mu_{1}-a_{1, p}+\beta_{1} l-\frac{z}{l}} \text { if } l_{0} \leq l \leq l_{\theta} \\ \pi_{2}(l)=m_{1}\left(a_{1, p}-\beta_{1} l\right)+\left(m_{2}+\frac{h}{\mu_{2}}(1-\theta)-\frac{m_{2}-\phi m_{1}}{\theta}\right)\left(a_{2, p}+\beta_{2} l\right) \text { if } l_{\theta} \leq l \leq \frac{a_{1, p}}{\beta_{1}}\end{array}\right.$
where $\theta=\max \left\{\sqrt{\frac{\mu_{2}\left(m_{2}-\phi m_{1}\right)}{h}}, \frac{1}{1-r_{2}}\right\}$ and
$l_{\theta}=\frac{\left(\theta\left(a_{1, p}-\mu_{1}\right)+\phi a_{2, p}\right)+\sqrt{\left(\theta\left(a_{1, p}-\mu_{1}\right)+\phi a_{2, p}\right)^{2}+4 \theta\left(\theta \beta_{1}-\phi \beta_{2}\right) z}}{2\left(\theta \beta_{1}-\phi \beta_{2}\right)}$.

Observing Lemma 5 , note that when $\phi=0$, we have $l_{\theta}=l_{0}$, implying that $\pi_{1}(l)$ is no longer defined and that $\pi(l)=\pi_{2}(l)$ over $\left[l_{0}, \frac{a_{1, p}}{\beta_{1}}\right]$. Thus, we obtain the same linear profit function given in the previous section (Lemma 3). When $\phi>0$, we have two different behaviors of profit in function of 1 represented by the two functions $\pi_{1}(l)$ and $\pi_{2}(l)$ defined, respectively, over $\left[l_{0}, l_{\theta}\right]$, and $\left[l_{\theta}, \frac{a_{1, p}}{\beta_{1}}\right] . \pi_{1}(l)$ is not linear while $\pi_{2}(l)$ is linear. Therefore, the optimal DT is either the DT that maximizes $\pi_{1}(l)$ over $\left[l_{0}, l_{\theta}\right]$, or $l_{\theta}$ or $\frac{a_{1, p}}{\beta_{1}}$. Given the complexity of $\pi_{1}(l)$, it is not possible to obtain a closed-form expression of the optimal solution. Hence, the remainder of our analysis will be based on numerical experiments.

To conduct our experiments, we consider three market structures. The first market is characterized by $\frac{\beta_{2}}{\beta_{l}}>\frac{\alpha_{2}}{\alpha_{1}}$, i.e., the switchovers are more governed by time difference than price difference, which is referred to as a STD market. The second market is characterized by $\frac{\beta_{2}}{\beta_{l}}<\frac{\alpha_{2}}{\alpha_{1}}$, i.e., the switchovers are more governed by price difference than time difference, which is referred to as a SPD market. When $\frac{\beta_{2}}{\beta_{l}}=\frac{\alpha_{2}}{\alpha_{1}}$, the market is neutral (neither STD nor SPD). We consider the following parameters for each market structure. For the neutral market, $\alpha_{p}=50, \alpha_{2}=10, \beta_{l}=50$, and $\beta_{2}=10$. For the STD market, $\alpha_{p}=50, \alpha_{2}=10, \beta_{l}=25$, and $\beta_{2}=25$. For the SPD market, $\alpha_{p}=25, \alpha_{2}=25, \beta_{l}=50$, and $\beta_{2}=10$. For each type of market, we generated 168,000 instances by varying the values of the other parameters, as shown in Table 2.

For the neutral market, we obtained 56,705 feasible instances. In all of them, we have $\left(-\beta_{1} m_{1}+\beta_{2}\left(m_{2}+\frac{h}{\mu_{2}}(1-\theta)-\frac{m_{2}-\phi m_{1}}{\theta}\right)\right)<0$, implying that the linear function $\pi_{2}(l)$

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

Table A.3: Test cases

| Parameter | Value |
| :---: | :--- |
| $A$ | From 800 to 1,200 with a step of 100 |
| $\mu_{1}$ | Fixed to 150 |
| $\mu_{2}$ | From 200 to 400 with a step of 50 |
| $c_{1}$ | Fixed to 3 |
| $c_{2}$ | $c_{1}+0.5, c_{1}+1, c_{1}+1.5$, and $c_{1}+2$ |
| $h$ | From 0.5 to 2 with a step of 0.5 |
| $p_{1}$ | From 6 to 12 with a step of 1 |
| $p_{2}$ | From $m_{1}+c_{2}$ to 18 with a step of 1 |
| $\phi$ | From 0.2 to 1 with a step of 0.2 |
| $r_{1}=r_{2}$ | Fixed to 0.98 |

is decreasing in $l$. For the SPD market, we obtained 32,031 feasible instances, and $\pi_{2}(l)$ is also decreasing in all of them. Finally, for the STD market, $\pi_{2}(l)$ is decreasing in 24,904 instances and increasing in 39,875 instances. The analysis of the results leads to the following observations. We first highlight in observations 1 and 2 the implications in terms of DT quotation and then turn to the impact of stock-out in observation 3.

Observation A.1. If $\left(-\beta_{1} m_{1}+\beta_{2}\left(m_{2}+\frac{h}{\mu_{2}}(1-\theta)-\frac{m_{2}-\phi m_{1}}{\theta}\right)\right)>0$ (i.e., if $\pi_{2}(l)$ is increasing in $l$ ), then it is more profitable to offer only the express product (i.e., $l^{*}=\frac{a_{1, p}}{\beta_{1}}$ ) and to use the drop-shipping channel only in case of stock-out as a backup solution.

This result has been observed in all instances that have an increasing $\phi_{2}(l)(39,875$ instances in total). To illustrate, we consider the example of Figure 5 ( $A=1000, p_{1}=10$, $c_{1}=3, \mu_{1}=150, p_{2}=18, c_{2}=4.5, \mu_{2}=400, h=1, r_{1}=r_{2}=0.98$, and a STD market with $\phi=0.80$ ). We observe that the profit is increasing in 1 and, consequently, the optimal solution is offering the longest DT for the regular product. This means that the initial regular demand is equal to zero and that only the express product is to be offered. In this case, the drop-shipping channel is used only as a backup solution to serve the customers that choose to switch to the regular product in case of stock-out. The whole system can thus be viewed as a traditional shipping system with a combination of lost sales (customers who leave in case of stock-out) and back orders (customers who switch to the regular product in case of stock-out).

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching


Figure A.4: Profit as a function of DT for increasing $\pi_{2}(l)$

Observation A.2. If $\left(-\beta_{1} m_{1}+\beta_{2}\left(m_{2}+\frac{h}{\mu_{2}}(1-\theta)-\frac{m_{2}-\phi m_{1}}{\theta}\right)\right)<0$ (i.e., if $\pi_{2}(l)$ is decreasing in $l$ ) then it is optimal to offer both products. In this case, quoting the $D T l_{\theta}$ is a near-optimal solution.

For each generated feasible instance that has a decreasing $\pi_{2}(l)$ (113,640 instances in total), we calculated numerically the optimal DT and deduced the profit gap resulting from quoting $l_{\theta}$ (which is the intersection point between $\pi_{1}(l)$ and $\pi_{2}(l)$ ) instead of the real optimal DT. The gap is calculated as follows: $\left.\frac{\operatorname{Profit}(\text { optimalDT })-\operatorname{Profit}\left(l_{\theta}\right)}{\operatorname{Profit}(\text { optimalDT }}\right) \times 100$. For all types of markets, we obtained a minimal mean gap ( $0.007 \%$ for the neutral market, $0.0002 \%$ for the STD market, and $0.012 \%$ for the SPD market). This shows that $l_{\theta}$ is a near-optimal DT. To illustrate, we consider the following example: $A=1000, p_{1}=10$, $c_{1}=3, \mu_{1}=150, p_{2}=14, c_{2}=4.5, \mu_{2}=400, h=1, r_{1}=r_{2}=0.98$, with a neutral market structure and different values of $\phi$. The profiles of the profit functions are shown in Figure 6 (note that $\pi_{1}(l)$ and $\pi_{2}(l)$ are, respectively, represented by dashed and continuous lines over $\left[l_{0}, l_{\theta}\right]$ and $\left[l_{\theta}, \frac{a_{1, p}}{\beta_{1}}\right]$, and that $l_{\theta}$ is the intersection point). For $\phi=0.2, \pi_{1}(l)$ is increasing over $\left[l_{0}, l_{\theta}\right]$ and, consequently, $l^{*}=l_{\theta}$ (since $\pi_{2}(l)$ is decreasing). For the other values of $\phi, \pi_{1}(l)$ first increases and then slightly decreases before $l_{\theta}$. We observe that the curve is flat between the optimal DT and $l_{\theta}$, which means that $l_{\theta}$ is near-optimal.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

Observation A.3. $S$-b-S leads to higher time differentiation (i.e., offering the regular product with a longer $D T$ ), and ignoring $S$-b-S (by quoting the optimal DT obtained for $\phi=0$, namely $l_{0} \theta$ ) leads to considerable loss.

Since $l_{\theta}>l_{0}$, it is clear that the consideration of S-b-S leads to a retailer quoting a longer DT. In addition, similar to the results obtained from model ( $M_{l}$ ), we observe in Figure 6 that the higher the value of $\phi$, the longer the optimal DT for the regular product. Note that $\pi_{1}(l)$ increases rapidly at the neighborhood of $l_{0}$ (for all values of $\phi$ ). This means that ignoring S-b-S, and, consequently, quoting the optimal DT obtained for $\phi=0$ (i.e., $l_{0}$ ) can lead to substantial loss. We quantified this loss by calculating the gap $\frac{\operatorname{Profit}\left(l_{\theta}\right)-\operatorname{Profit}\left(l_{0}\right)}{\operatorname{Profit}\left(l_{\theta}\right)} \times 100$. We found a mean gap of $39.36 \%$ for the neutral market, $57.18 \%$ for the SPD market, and $45.28 \%$ for the STD market. This shows the motivation to consider the effect of S-b-S, as we do in our models.

## A. 6 General model and effect of S-b-S

In this section, we study the general model $\left(M_{l, S, p}\right)$ where $l, S$, and $p$ are decision variables and the $\mathrm{S}-\mathrm{b}-\mathrm{S}$ is considered. Given the complexity of this model, we first provide a simpler quasi-equivalent formulation using the results of the previous section. Then, we rely on the new formulation to conduct experiments and derive managerial implications.

To simplify the analysis, we focus on the more general case where both regular and express products are offered to customers. Therefore, we consider the values of $p$ such that $\left(-\beta_{1} m_{1}+\beta_{2}\left(p-c_{2}+\frac{h}{\mu_{2}}(1-\theta)-\frac{p-c_{2}-\phi m_{1}}{\theta}\right)\right)<0$ (see Observations 1 and 2). Under this condition, and for a given price $p$, we have shown in the previous section that the model can be written in the function of the single variable 1 and that $l=l_{\theta}$ is a near-optimal solution (for $\phi=0, l_{\theta}=l_{0}$ is the exact optimal solution). Hence, for a given price p , we replace 1 with $l_{\theta}$. Then, we show by standard (but long) calculus, that model $\left(M_{l, S, p}\right)$ can be written in the function of the single variable $p$. We denote this new model by $(\mathrm{M})$ and provide it below. Note that we add the constraint $p \geq p_{\text {min }}=$ $\max \left\{m_{1}+c_{2}, \frac{h}{\mu_{2}}+\phi m_{1}+c_{2}\right\}$ to guarantee model consistency as explained in the previous sections.

Before proceeding further, we must verify that model $(M)$ provides a good approximation of the original model ( $M_{l, S, p}$ ). We generate instances according to Table 2 (for $\phi=0$ and $\phi>0$ ) and calculate the gap between the optimal profit obtained with model ( $M$ ) and that of the original model ( $M_{l, S, p}$ ). To solve model ( $M_{l} l, S, p$ ), we used the algorithm SLSQP (sequential least squares programming) of the SciPy module. This is a recommended procedure for nonlinear problems with nonlinear constraints. For our modified model $(M)$, it can be easily solved to optimality with any computational software. The

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
gap between both models is calculated as $\frac{\left(\pi_{\text {originalmodel }}-\pi_{\text {modifiedmodel }}\right) \times 100}{\pi_{\text {orisinalmodel }}}$. Table 3 shows the results.

Table A.4: Comparison between the original model and the modified model

|  | $\phi=0$ |  |  |  | $\phi>0$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Test cases: 2800 |  |  |  | Test cases: 14000 |  |  |  |
| Market | Nb. Rel- <br> evant In- <br> stances | Mean <br> Gap | Standard <br> Devia- <br> tion | Confidenc <br> Interval | Nb. Rel- <br> evant In- <br> stances | Mean <br> Gap | Standard <br> Devia- <br> tion | Confidence <br> Interval |
| Neutral | 2449 | 0.0002 | 0.0003 | $(0.00022$, |  |  |  |  |
| 11955 | 0.0087 | 0.0294 | $(0.0082$, <br> $0.0092)$ |  |  |  |  |  |
| SPD | 2798 | 0.00014 | 0.0001 | $(0.00013$, <br> $0.00015)$ | 13917 | 0.0102 | 0.0494 | 0.0094, <br> $0.0110)$ |
| STD | 2154 | 0.95 | 3.0392 | $(0.8216$, <br> $1.0783)$ | 10491 | 0.8560 | 2.7181 | 0.8036, <br> $0.9076)$ |

The results confirm that the modified model $(M)$ is quasi-equivalent to the original model ( $M_{l, S, p}$ ), particularly for the neutral and SPD markets. Thus, the remainder of our analysis will be based on model $(M)$. Our main objective is to investigate the effect of S-b-S. We consider the basic example: $A=1000, p_{1}=8, c_{1}=3, \mu_{1}=150, c_{2}=4.5$, $\mu_{2}=400, h=1$, and $r_{1}=r_{2}=0.98$. We vary the value of $\phi$ and report the results in Tables 4, 5, and 6 for the neutral, the SPD, and the STD markets, respectively.

Table A.5: Effect of stock-out based substitution for the neutral market

| $\phi$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $p^{*}$ | 12.10 | 12.10 | 12.10 | 12.10 | 12.10 | 12.10 | 12.10 | 12.10 | 12.10 | 12.10 | 12.10 |
| $l^{*}$ | 8.19 | 8.20 | 8.22 | 8.23 | 8.25 | 8.26 | 8.28 | 8.29 | 8.31 | 8.32 | 8.34 |
| $S^{*}$ | 59 | 57 | 55 | 54 | 54 | 54 | 54 | 54 | 54 | 54 | 54 |

Table A.6: Effect of stock-out based substitution for the SPD market

| $\phi$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $p^{*}$ | 16.25 | 16.25 | 16.25 | 16.25 | 16.25 | 16.25 | 16.25 | 16.25 | 16.25 | 16.25 | 16.25 |
| $l^{*}$ | 14.28 | 14.29 | 14.30 | 14.32 | 14.33 | 14.35 | 14.37 | 14.38 | 14.40 | 14.42 | 14.45 |
| $S^{*}$ | 90 | 88 | 86 | 84 | 82 | 80 | 77 | 75 | 73 | 70 | 68 |

The analysis of the numerical results leads to the following main finding.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

Table A.7: Effect of stock-out based substitution for the STD market

| $\phi$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $p^{*}$ | 13.80 | 13.80 | 13.80 | 13.80 | 13.80 | 13.80 | 13.80 | 13.80 | 13.80 | 13.80 | 13.80 |
| $l^{*}$ | 10.18 | 10.19 | 10.20 | 10.22 | 10.24 | 10.27 | 10.29 | 10.31 | 10.33 | 10.35 | 10.37 |
| $S^{*}$ | 76 | 74 | 72 | 70 | 68 | 65 | 63 | 63 | 63 | 63 | 63 |

Observation A.4. In the general model, a higher rate of $S-b-S$ would normally lead to greater time differentiation and to holding less stock. However, it would not impact the price differentiation.

When $\phi$ increases, different trade-offs between price, DT, and stock govern the system. Nevertheless, we observe for all types of markets that an increase in the S-b-S rate does not affect the optimal price $p^{*}$. This result is not intuitive and implies that the price quoted for the express product should be the same whatever the extent of S-b-S. In particular, the optimal price is unchanged with or without S-b-S. To confirm this result, we extend our experiments to test, for each type of market, all the instances that are generated according to Table 2. For each instance, we calculate the difference between its optimal price (for $\phi>0$ ) and the optimal price obtained for $\phi=0$. The results, provided below in Table 7, confirm that the optimal price is not significantly sensitive to the S-b-S rate. The

Table A.8: Effect of stock-out based substitution for the SPD market

| Market | Mean price difference | Standard deviation | Confidence interval |
| :--- | :--- | :--- | :--- |
| Neutral | 0.0046 | 0.0066 | $(0.0044,0.0049)$ |
| SPD | 0.0080 | 0.0083 | $(0.0077,0.0083)$ |
| STD | 0.0769 | 0.2326 | $(0.0683,0.0856)$ |

numerical results presented in Tables 4, 5, and 6 also show that an increase in the S-b-S rate should lead to greater time differentiation and the holding of less stock. We illustrate in Figure 7 how the optimal stock varies as a function of $\phi$. When $\phi$ increases, it is preferable to capitalize on the express demand since the loss of regular demand can be offset by the additional number of customers switching from the express product to the regular product in case of stockout. Thus, as it is not profitable to reduce the price of the express product (we have just shown that this price does not change), the system reacts by quoting a longer DT for the regular product. In addition, when $\phi$ increases, the stockout becomes less penalizing for the retailer as there is a smaller proportion of customers leaving the system. Thus, the system favors reducing the inventory cost over satisfying more express demand. This explains why the stock decreases when $\phi$ increases. Nevertheless, beyond a threshold value, it is no longer possible to reduce the stock since the system must satisfy the service constraint. Therefore, the stock remains constant, as we observe in Figure 7.

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching


Figure A.5: Impact of the stock-out based substitution on the stock level

## A. 7 Conclusion and practical implications of our findings

We studied the problem of a retailer serving a time- and price-sensitive market with two substitutable products that differ in the guaranteed DT and price. The express product is delivered from stock, whereas the regular product is delivered directly from the supplier, which requires operating a hybrid distribution system with a mix of DFS and dropshipping. In case of stockout, some customers initially interested in the express product may switch to the regular product, which is referred to as S-b-S behavior.

When the characteristics of the express product (price and stock) are fixed and the DT of the regular product is a decision variable, we provided the closed-form expression of the optimal solution. We found that is not always optimal to adopt an extreme strategy (maximum or minimum time differentiation), and that an intermediate strategy could be preferred. An increase in the stock level leads to a retailer offering the regular product with a shorter DT when the retailer adopts a minimum time differentiation strategy and with a longer DT in all other cases. When the DT and the stock level are decision variables but the price is fixed, we distinguished two situations depending on whether S-b-S is considered. In cases without S-b-S, we solved the model to optimality and found that there exists a threshold value for the express product's price below which it is optimal to offer both products with minimum time differentiation and above which only the express product should be offered. The consideration of S-b-S renders the model much more complex. We characterized the situations where it is optimal to offer both products. In this case, we provided a near-optimal solution and showed that ignoring S-b-S by quoting

Appendix A. Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching
the optimal DT obtained without S-b-S leads to considerable loss. Finally, we studied the general model where DT, stock, and price are decision variables. We used the previous results to develop a quasi-equivalent formulation of the general model with the price as a unique variable. Based on numerical studies, we found that a higher rate of S-b-S would normally lead to greater time differentiation and less stock. However, it would not impact the price differentiation.

Practitioners might benefit from our findings in many ways. First, our study shows that similar to manufacturers, retailers can also use time-based product differentiation and segment pricing to enhance performance. In the retailing context, time-based differentiation can result from relying on a delivery mix including DFS and drop-shipping. Second, our results offer insights for retailers into how to differentiate products in terms of DT and price. Third, retailers can rely on our findings to rethink the inventory level for a given product when the drop-shipping option is also available. Finally, our study demonstrates that retailers must account for the effect of S-b-S, particularly in low competitive environments that are typically characterized by high S-b-S rate. Ignoring S-b-S leads to retailer undertaking operational decisions that are far from optimality, which implies a considerable loss.

This is the first study to investigate the time and price differentiation problem from the perspective of a hybrid distribution system and to include the effect of S-b-S. Our work can be extended to consider inventory capacity constraints in the retailer's warehouse as this may prevent the model from excessively increasing the time differentiation to favor the express demand. A future work can also investigate a more complex SC where a customer order may be delivered from the stock or from one of several potential suppliers. The retailer can quote a specific DT per supplier and, consequently, offer a menu of DTs to customers or quote a unique DT from all suppliers, which raises the issue of allocating orders to suppliers.

# Demand function's positivity in Lemma 3.2 

According to Lemma 3.2, there are two roots ( $R_{1}$ and $R_{2}$ ) that demand is positive between them and negative outside of them.
$R_{1}=\frac{-\left(2 \alpha_{p} b x_{0}-\beta_{e}\right)-\sqrt{\Delta_{2}}}{-2 \alpha_{p} b}=x_{0}+\frac{-\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}$
$R_{2}=\frac{-\left(2 \alpha_{p} b x_{0}-\beta_{e}\right)+\sqrt{\Delta_{2}}}{-2 \alpha_{p} b}=x_{0}-\frac{\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}$
where $\Delta_{2}=4 \alpha_{p} b\left(A-\beta_{e} x_{0}-\frac{\beta_{e}^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)$.
For the first root, we will prove that $x_{0}+\frac{-\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}>x_{0}$; for that let us prove:
$-\beta_{e}+\sqrt{\Delta_{2}}>0 \Leftrightarrow \sqrt{\Delta_{2}}>\beta_{e} \Leftrightarrow \Delta_{2}>\beta_{e}^{2}$
$\Leftrightarrow \Delta_{2}=4 \alpha_{p} b\left(A-\beta_{e} x_{0}-\frac{\beta_{e}^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)>\beta_{e}^{2}$
$\Leftrightarrow 4 \alpha_{p} b\left(A-\beta_{e} x_{0}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)>0$
While we assume that $A>\beta_{e} x_{0}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$, the above condition $\left(R_{1}>x_{0}\right)$ is true.
The same approach will be used for the second root. We will prove that $R_{2}<0$ and we have:

$$
\begin{aligned}
& x_{0}-\frac{\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}<0 \Leftrightarrow x_{0}<\frac{\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b} \Leftrightarrow \sqrt{\Delta_{2}}>2 \alpha_{p} b x_{0}-\beta_{e} \\
& \Leftrightarrow 4 \alpha_{p} b\left(A-\beta_{e} x_{0}-\frac{\beta_{e}^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)>\left(2 \alpha_{p} b x_{0}-\beta_{e}\right)^{2} \\
& \Leftrightarrow 4 \alpha_{p} b\left(A-\alpha_{p} \beta_{e} x_{0}^{2}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right)>0
\end{aligned}
$$

While $A>\alpha_{p} \beta_{e} x_{0}^{2}+\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)$, the above conclusion is true. Since we assume that $A$ is sufficiently high, it considers that the condition is authenticated. Thus, we have $R_{2}<0$.

We conclude that under a condition such that $A>\max \left\{\beta_{e} x_{0}-\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right), \alpha_{p} \beta_{e} x_{0}^{2}+\right.$ $\left.\alpha_{p}\left(c+\frac{h(1-r)}{r \mu}\right)\right\}$, we have $R_{1}>x_{0}$ and $R_{2}<0$, and thus, demand function is positive in feasible region $\left(\left[0, x_{0}\right]\right)$.

## Appendix C

## Objective function concavity in <br> Proposition 4.6

According to Proposition 4.6, there are two roots $\left(R_{3}\right.$ and $\left.R_{4}\right)$ that objective function is concave between them and convex outside of them.
$R_{3}=x_{0}+\frac{-3 \beta b+\sqrt{\Delta_{3}}}{6 \alpha b^{2}}$
$R_{4}=x_{0}-\frac{3 \beta b+\sqrt{\Delta_{3}}}{6 \alpha b^{2}}$
where $\Delta_{3}=12 \alpha b^{3}\left(A_{2}^{\prime \prime}-\beta x_{0}-\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{4 \alpha b}\right)$.
For the first root, we will prove that $x_{0}+\frac{-3 \beta b+\sqrt{\Delta_{3}}}{6 \alpha b^{2}}>x_{0}$; for that let us prove:

$$
\begin{aligned}
& -3 \beta b+\sqrt{\Delta_{3}}>0 \Leftrightarrow \sqrt{\Delta_{3}}>3 \beta b \Leftrightarrow \Delta_{3}>9 \beta^{2} b^{2} \\
& \Leftrightarrow \Delta_{3}=12 \alpha b^{3}\left(A_{2}^{\prime \prime}-\beta x_{0}-\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{4 \alpha b}\right)>9 \beta^{2} b^{2} \\
& \Leftrightarrow 12 \alpha b^{3}\left(A_{2}^{\prime \prime}-\beta x_{0}-\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)-\frac{\beta^{2}}{2 \alpha b}\right)>0
\end{aligned}
$$

While we assume that $A_{2}^{\prime \prime}>\beta x_{0}+\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{2 \alpha b}$, the above condition ( $R_{3}>x_{0}$ ) is true.
The same approach will be used for the second root. We will prove that $R_{4}<0$ and we have:

$$
\begin{aligned}
& x_{0}-\frac{3 \beta b+\sqrt{\Delta_{3}}}{6 \alpha b^{2}}<0 \Leftrightarrow x_{0}<\frac{3 \beta b+\sqrt{\Delta_{3}}}{6 \alpha b^{2}} \Leftrightarrow \sqrt{\Delta_{3}}>6 \alpha b^{2} x_{0}-3 \beta b \\
& \Leftrightarrow 12 \alpha b^{3}\left(A_{2}^{\prime \prime}-\beta x_{0}-\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{4 \alpha b}\right)>\left(6 \alpha b^{2} x_{0}-3 \beta b\right)^{2} \\
& \Leftrightarrow 12 \alpha b^{3}\left(A_{2}^{\prime \prime}-3 \alpha b x_{0}^{2}+2 \beta x_{0}-\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)-\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)-\frac{\beta^{2}}{2 \alpha b}\right)>0
\end{aligned}
$$

While $A_{2}^{\prime \prime}>3 \alpha b x_{0}^{2}-2 \beta x_{0}+\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{2 \alpha b}$, the above conclusion is true. Since we assume that $A$ is sufficiently high, it considers that the condition is authenticated. Thus, we have $R_{4}<0$.

We conclude that under a condition such that $A_{2}^{\prime \prime}>\max \left\{\beta x_{0}+\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\right.$ $\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+\frac{\beta^{2}}{2 \alpha b}, 3 \alpha b x_{0}^{2}-2 \beta x_{0}+\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)+\alpha_{s}\left(m_{1}-\frac{h(1-r)}{r \mu_{1}}\right)+$ $\left.\frac{\beta^{2}}{2 \alpha b}\right\}$, we have $R_{3}>x_{0}$ and $R_{4}<0$, and thus, objective function is concave in feasible region $\left(\left[0, x_{0}\right]\right)$.

# Demand function's Positivity in Lemma 5.2 

According to Lemma 5.2, there are two roots $\left(R_{5}\right.$ and $\left.R_{6}\right)$ that the demand function is positive between these two roots and negative out of them.
$R_{5}=x_{0}+\frac{-\theta_{1}+\sqrt{\Delta_{5}}}{2 \delta_{1} b}$,
$R_{6}=x_{0}-\frac{\theta_{1}+\sqrt{\Delta_{5}}}{2 \delta_{1} b}$,
where $\Delta_{5}=4 \delta_{1} b\left(A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\theta_{1} x_{0}+\frac{\theta_{1}^{2}}{4 \delta_{1} b}\right)$.
For the first root, we will prove that $x_{0}+\frac{-\theta_{1}+\sqrt{\Delta_{5}}}{2 \delta_{1} b}>x_{0}$; for that let us prove:

$$
\begin{aligned}
& -\theta_{1}+\sqrt{\Delta_{5}}>0 \Leftrightarrow \sqrt{\Delta_{5}}>\theta_{1} \Leftrightarrow \Delta_{5}>\theta_{1}^{2} \\
& \Leftrightarrow \Delta_{5}=4 \delta_{1} b\left(A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\theta_{1} x_{0}+\frac{\theta_{1}^{2}}{4 \delta_{1} b}\right)>\theta_{1}^{2} \\
& \Leftrightarrow 4 \delta_{1} b\left(A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\theta_{1} x_{0}\right)>0
\end{aligned}
$$

While we assume that $A_{1}>\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\theta_{1} x_{0}$, the above condition $\left(R_{5}>x_{0}\right)$ is true.

The same approach will be used for the second root. We will prove that $R_{6}<0$ and we have:

$$
\begin{aligned}
& x_{0}-\frac{\theta_{1}+\sqrt{\Delta_{5}}}{2 \delta_{1} b}<0 \Leftrightarrow x_{0}<\frac{\theta_{1}+\sqrt{\Delta_{5}}}{2 \delta_{1} b} \Leftrightarrow \sqrt{\Delta_{5}}>2 \delta_{1} b x_{0}-\theta_{1} \\
& \Leftrightarrow 4 \delta_{1} b\left(A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\theta_{1} x_{0}+\frac{\theta_{1}^{2}}{4 \delta_{1} b}\right)>\left(2 \delta_{1} b x_{0}-\theta_{1}\right)^{2} \\
& \Leftrightarrow 4 \delta_{1} b\left(A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)-\delta_{1} b x_{0}^{2}\right)>0
\end{aligned}
$$

While $A_{1}>\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\delta_{1} b x_{0}^{2}$, the above conclusion is true. Since we assume that $A$ is sufficiently high, it considers that the condition is authenticated. Thus, we have $R_{6}<0$.

We conclude that under a condition such that $A_{1}>\max \left\{\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)+\theta_{1} x_{0}, \delta_{1}\left(c_{1}+\right.\right.$ $\left.\left.\frac{h(1-r)}{r \mu_{1}}\right)+\delta_{1} b x_{0}^{2}\right\}$, we have $R_{5}>x_{0}$ and $R_{6}<0$, and thus, demand function is positive in feasible region ( $\left[0, x_{0}\right]$ ).

# Optimization of Retailers' Strategies in Price- and Carbon Emission- Sensitive Market 


#### Abstract

This work studies the retailer's profit maximization problem and investigates his/her optimal strategies in a price- and greenness- sensitive market. This work starts with a benchmark model where a retailer offers one kind of product to customers. The products are produced by a supplier and sent to the retailer. The retailer keeps the products in a warehouse near to the customers to serve them as soon as one arrives. The demand for the products is random and follows Poisson process. The customers' arrival mean rate is sensitive to retail price and carbon emissions level of the product. The refilling time of the retailer's warehouse is also random and follows Exponential distribution. The problem consists in deciding product's price, carbon emission level, and order size. We solve the problem by an analytical approach and provide the closed-form expressions of the optimal solutions. In the next step, the benchmark model is extended in the way that retailer offers two products to customers. The products, which are supplied by different suppliers, are the same in terms of performance, function, and etc., thus, they are substitutable. The demand function in this case is also affected by the substitution's effect. In another words, the demand for each product not only depends on its price and carbon emission level, but also, depends on the other product's price and carbon emission level. Like benchmark model, the retailer's profit maximization problem is formulated in an stochastic environment under different settings (decision variables) and are solved by an analytical approach. According to the results, the market is distinguished in three categories: 1-Greenness-Driven Switchovers (GDS) market, 2-Price-Driven Switchovers (PDS) market, and 3- Neutral market. These different types of market make it possible to structure and analyze the obtained results. A dynamic competition between two retailers, which each of them has its supplier, is considered. Retailers offer two substitutable products that each of them just offers one kind of product. There are two symmetric mathematical model that consist in deciding products' prices, carbon emission levels, and order sizes. Each retailer's decision affects the other retailer's decision. The general problem (price, greenness, and order size for each retailer as decision variables) is solved by an analytical approach and determined the Nash equilibrium. However, in practice, there are many situations where an existing retailer is already operating in the market, and a new retailer enters the market and offers a substitutable product. In this case, two situations are relevant to study are considered and solved: 1-Competition without reaction and 2-Competition with partial reaction. The close-form expressions of the optimal solutions are presented for all scenarios. This work ends its studies by introducing a non-linear demand function. In the literature, all studies consider a linear demand function (to the best of our knowledge). However, our partners in project ANR CONCLuDE found out that the linear function is not sufficient enough. Thus, a new non-linear demand function is considered with respect to carbon emission improvement. Furthermore, our partners' studies also reveal that product's demand can not go beyond a given level as carbon emission improvement increases. More precisely, they express that improving greenness leads to increase the demand to a certain amount of market potential and after that it is constant. The benchmark model is re-formulated with different demand functions (non-linear, non-linear cap, and linear cap) and solved by an analytical approach. Then, close-form expressions of optimal solutions are presented. A numerical example is conducted to compare profits with different demand functions. The non-linear cap is considered as reference and compare to other, the results reveal that when maximum attracted costumers possible is low (below than 20\%) the linear cap model performs better than others, but beyond that, the non-linear model performs better.


Keywords: supply chain management, retailers, endogenous demand, greenness, pricing, queuing theory, game theory

# Optimisation des Stratégies des Détaillants sur Un Marché Sensible Au Prix et Aux Émissions de Carbone 

Résumé - Ce travail étudie le problème de maximisation des profits d'un détaillant et examine ses stratégies optimales dans un marché sensible aux prix et à l'environnement (la greenness de produit). Ce travail commence par un modèle de référence où un détaillant propose un type de produit aux clients. Les produits sont fabriqués par un fournisseur et envoyés au détaillant. Le détaillant garde les produits dans un entrepôt à proximité des clients pour les servir dès leur arrivée. La demande pour les produits est aléatoire et suit un processus de Poisson. Le taux moyen d'arrivée des clients est sensible au prix de détail et au niveau des émissions de carbone du produit. Le temps de réapprovisionnement de l'entrepôt du détaillant est également aléatoire et suit la distribution exponentielle. Le problème consiste à décider du prix du produit, du niveau d'émission de carbone et de la taille de la commande. Nous résolvons le problème par une approche analytique et fournissons les expressions explicites des solutions optimales.
Dans l'étape suivante, le modèle de référence est étendu à un situation où le détaillant propose deux produits aux clients. Les produits, qui sont fournis par différents fournisseurs, sont les mêmes en termes de performances, de fonctions, etc., ils sont donc substituables. La fonction de demande dans ce cas est également affectée par l'effet de substitution. En d'autres termes, la demande pour chaque produit ne dépend pas seulement de son prix et de son niveau d'émission de carbone, mais aussi du prix de l'autre produit et de son niveau d'émission de carbone. Comme le modèle de référence, le problème de maximisation des profits du détaillant est formulé dans un environnement stochastique sous différents paramètres (variables de décision) et est résolu par une approche analytique. Les marchés se distinguent en trois catégories: 1-Greenness-Driven Switchovers (GDS) market, 2- Price-Driven Switchovers (PDS) market, et 3- Neutral market. Ces différentes types de marché permettent de structurer et analyser les résultats obtenu.
Une concurrence dynamique entre deux détaillants, dont chacun a son fournisseur, est ensuite envisagée. Les détaillants proposent deux produits substituables (chaque détaillant propose un type de produit). Le problème consiste à optimiser deux modèles mathématiques symétriques qui consistent à décider des prix des produits, des niveaux d'émission de carbone et des tailles de commande. La décision de chaque détaillant affecte la décision de l'autre détaillant. Le problème général (prix, greenness et taille de la commande pour chaque détaillant en tant que variables de décision) est résolu par une approche analytique en déterminant l'équilibre de Nash. Cependant, dans la pratique, il existe de nombreuses situations où un détaillant existant opère déjà sur le marché et un nouveau détaillant entre sur le marché et propose un produit substituable. Dans ce cas, deux situations pertinentes à étudier sont envisagées et résolues: 1Compétition sans réaction et 2 - Compétition avec réaction partielle. Les expressions des solutions optimales sont présentées pour tous les scénarios.
Ce travail se termine en introduisant une fonction de demande non linéaire. Dans la littérature, toutes les études considèrent une fonction de demande linéaire (au meilleur de nos connaissances). Cependant, nos partenaires du projet ANR CONCLuDE ont montré que la fonction linéaire n'est pas suffisante. Ainsi, une nouvelle fonction de demande non linéaire est considérée pour l'amélioration des émissions de carbone. De plus les études de nos partenaires révèlent également que la demande de produits ne peut pas dépasser un niveau donné à mesure que l'amélioration des émissions de carbone augmente. Plus précisément, l'amélioration de la greenness conduit à augmenter la demande jusqu'à un certain potentiel de marché et ensuite elle est constante. Le modèle de référence est reformulé avec trois différentes fonctions de demande (non linéaire bornée, non linéaire et linéaire bornée) et résolu par une approche analytique. Ensuite, des expressions des solutions optimales sont présentées. Un exemple numérique est effectué pour comparer les bénéfices avec différentes fonctions de demande. La demande non linéaire bornée est considérée comme une référence et est comparée aux autres, les résultats révèlent que lorsque la borne est relativement faible (moins de $20 \%$ pouvant être obtenue par l'amélioration des émissions), le modèle linéaire borné fonctionne mieux que les autres mais au delà, le modèle non linéaire fonctionne mieux.

Mots clés: gestion de la chaîne d'approvisionnement, détaillants, demande endogène, la greenness, tarification, théorie des files d'attente, théorie des jeux

