

Essays on the design of tax-benefit systems Antoine Ferey

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IP PARIS



Essais sur le barème et la structure des systèmes socio-fiscaux

Thèse de doctorat de l'Institut Polytechnique de Paris préparée à CREST, École polytechnique

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Thèse de doctorat

Institut Polytechnique de Paris

CREST

THÈSE

Pour l'obtention du grade de Docteure en Sciences Économiques de l'Institut Polytechnique de Paris

Présentée et soutenue publiquement le 26 Mars 2021 par:

Antoine FEREY

Essais sur le barème et la structure des systèmes socio-fiscaux

Sous la direction de

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Antoine FEREY

Essays on the design of tax-benefit systems

Under the supervision of

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Cette thèse de doctorat a été une aventure aussi bien humaine qu'intellectuelle, un voyage ponctué de rencontres, d'idées, de découvertes et d'apprentissage. Alors que le reste de ce manuscrit constitue un volumineux témoignage de l'aventure intellectuelle, je souhaite ici retracer l'aventure humaine, et remercier celles et ceux qui m'ont accompagné, soutenu et encouragé au cours de ces années. Cette thèse leur doit beaucoup.

Durant l'année 2015-2016, je suis étudiant en M2 à l'Ecole d'Economie de Paris. J'assiste au cours d'économie publique théorique de Pierre Boyer et Jean-Baptiste Michau, au cours d'économie publique appliqué de Thomas Piketty, et j'écris sous la direction d'Antoine Bozio un mémoire de master qui deviendra le premier chapître de cette thèse. Je me passionne à cette époque pour la théorie de la fiscalité optimale, et en particulier pour des papiers de recherche récents intégrant inattention, biais de perceptions et complexité du système sociofiscal. Je contacte alors Pierre Boyer, Philippe Choné, Etienne Lehmann et Jean-Baptiste Michau pour discuter d'un projet de thèse sur ces sujets. L'aventure commence, et elle se déroulera au CREST et à l'Ecole Polytechnique.

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L'environnement et la structure du laboratoire du CREST et de l'Ecole Polytechnique

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Résumé

Cette thèse de doctorat en économie publique étudie le barème et la structure des systèmes socio-fiscaux de manière positive et normative. Les principales contributions relèvent du champ de la théorie appliquée, et sont accompagnées de simulations numériques qui permettent de lier la théorie aux données.

Le chapitre 1 fournit une analyse descriptive du système socio-fiscal français et calcule les incitations financières au travail à l'aide du modèle de micro-simulation TAXIPP via le taux marginal effectif de prélèvements (TMEP) et le taux effectif de prélèvements à l'emploi (TEPE). Pour les célibataires sans enfant, le schéma des TMEP est en forme de tilde autour d'un TMEP moyen de 43%, tandis que le schéma des TEPE est en forme de bosse autour d'un TEPE moyen de 51%. La décomposition de ces mesures en termes des différents instruments de taxes et de transferts sous-jacents montre que les allocations logement jouent un rôle étonnamment important dans les incitations monétaires au travail. Dans l'ensemble, l'analyse souligne la complexité du système socio-fiscal français.

Le chapitre 2 est co-écrit avec Jeremy Boccanfuso et montre que l'inattention des agents à l'égard des impôts conduit les gouvernements à augmenter les taux d'imposition de façon inefficace. Ce *biais fiscal* résulte de l'incapacité des gouvernements à s'engager de façon crédible à implémenter une politique fiscale efficace et reflète le fait que l'inattention des agents réduit leurs réactions aux réformes fiscales, même si leurs perceptions s'ajustent à l'équilibre. À l'aide d'une formule exprimée en termes de statistiques suffisantes, nous estimons l'ampleur de ce biais pour l'économie américaine. L'inattention diminuant avec les revenus, ce biais fiscal est plus marqué sur les bas revenus et affecte donc la progressivité du système fiscal. Enfin, les gains de bien-être induits par d'éventuels biais de sous-estimations des taxes peuvent être dominés par les coûts de bien-être induits par ce biais fiscal, ce qui donne un nouvel éclairage sur les conséquences de la complexité des systèmes socio-fiscaux.

Le chapitre 3 analyse les politiques de redistribution optimale et d'assurance chômage optimale. Au niveau des ménages, les décisions d'offre de travail et de recherche d'emploi s'avèrent intrinsèquement liées, ce qui implique que les barèmes optimaux d'imposition et d'allocations chômage dépendent tous deux des élasticités de travail et de recherche d'emploi. De plus, l'allocation efficace des ressources requiert un équilibre entre taxes payées en emploi et prestations reçues au chômage, et montre l'existence d'externalités budgétaires. Une condition de Pareto-efficacité pour le système socio-fiscal, révèle qu'il ainsi est efficace d'implémenter des taux de remplacement qui diminuent en fonction du revenu d'activité. Il est donc efficace de redistribuer par le biais d'une assurance chômage progressive. Dans l'ensemble, ces résultats montrent que redistribution et assurance chômage interagissent, et que ces interactions sont importantes pour la conception des politiques publiques.

Le chapitre 4 est co-écrit avec Benjamin Lockwood et Dmitry Taubinsky. Il réexamine l'argument d'hétérogénéité des préférences comme motif d'imposition du capital. Pour des systèmes fiscaux continus et différentiables, nous montrons sous des hypothèses assez générales que l'allocation résultant du système fiscal optimal peut être obtenue avec des système fiscaux séparables, similaires à ceux utilisés en pratique. Nous caractérisons barèmes optimaux et conditions de Pareto-efficacité pour ces systèmes fiscaux séparables. Ces formules font apparaître une nouvelle statistique suffisante mesurant l'hétérogénéité des préférences et ouvre la voie à une estimation empirique de la fiscalité optimale du capital compte tenu de l'hétérogénéité des préférences empiriquement observable.

Discipline: Sciences économiques

Mots-clés: Système socio-fiscal; redistribution optimale; complexité; inattention; assurance chômage; fiscalité du capital

Summary

This PhD dissertation in public economics studies the design of tax-benefit systems from both positive and normative angles. The main contributions of this dissertation pertain to applied theory, and numerical simulations are used to link theory and data.

Chapter 1 provides a descriptive analysis of the French tax-benefit system, and computes agents' monetary incentives to work using TAXIPP micro-simulation model. I simulate effective marginal tax rates (EMTR) and effective participation tax rates (EPTR) which capture agents' incentives to, respectively, increase their earnings and become employed. Focusing on childless singles, the schedule of EMTR is tilde-shaped with a mean of 43%, and the schedule of EPTR is hump-shaped with a mean of 51%. Decomposing these measures into the different underlying tax and transfer instruments shows that housing benefits play a surprisingly large role in shaping monetary incentives to work, at both intensive and extensive margins. Overall, the analysis highlights the complexity of the French tax-benefit system.

Chapter 2 is joint with Jeremy Boccanfuso. It shows that agents' inattention to taxes leads governments to inefficiently increase tax rates. This *taxation bias* arises as a by-product of the government inability to credibly commit to an efficient tax policy and reflects the fact that agents' inattention reduces their responses to tax reforms, although their perceptions adjust in equilibrium. Deriving a sufficient statistics formula for the taxation bias, we estimate its magnitude for the US economy. Because inattention decreases with earnings, the taxation bias is more pronounced at low earnings levels and thus affects the progressivity of the tax system. Moreover, welfare gains from potential downward biases in tax perceptions may be dominated by the welfare cost of the taxation bias, which sheds a new light on the welfare effects of tax complexity.

Chapter 3 analyzes optimal redistribution and optimal unemployment insurance. At the individual level, work and job search decisions are both influenced by the tax-benefit system which implies that optimal tax and optimal benefit formulas both depend on work and search elasticities. Moreover, the efficient allocation of resources implies a balance between the taxes paid when employed and the benefits received when unemployed and highlights the existence of budget externalities between redistribution and insurance. Deriving a Pareto-efficiency condition for the tax-benefit system, I show that it is efficient to implement replacement rates that decrease with earnings. It is thus efficient to redistribute through progressive unemployment benefits even in the presence of an optimal nonlinear income tax. Overall, results highlight that redistribution and unemployment insurance interact in important ways, and that these interactions matter for the design of public policies.

Chapter 4 is joint with Benjamin Lockwood and Dmitry Taubinsky. It revisits the preference heterogeneity argument for capital taxation by providing a novel characterization of optimal nonlinear tax systems in terms of estimable sufficient statistics. Focusing on smooth tax systems, we derive surprisingly general conditions under which the allocation implemented by the optimal tax system can be implemented by the type of separable tax systems we observe in the real-world. We provide optimal tax formulas and Pareto-efficiency conditions for separable tax systems. They feature a new sufficient statistic allowing to measure preference heterogeneity directly from the data which paves the way for an empirical estimation of optimal capital taxes, given the extent of preference heterogeneity that can be observed.

Field: Economics

Key words: Tax-benefit systems; optimal redistribution; complexity; inattention; unemployment insurance; capital taxation

Introduction générale

L'économie publique est un champ de recherche qui étudie l'économie du secteur public. Elle analyse la manière dont les politiques publiques affectent l'économie et comment elles devraient être idéalement conçues pour atteindre leurs objectifs de façon efficace.

Cette thèse en économie publique étudie plus particulièrement le barème et la structure des systèmes socio-fiscaux qui sont une composante importante des politiques publiques. Elle analyse l'articulation des différents impôts et prestations en France (chapitre 1), l'impact de l'inattention des agents économiques à l'égard des impôts sur la politique fiscale (chapitre 2), les politiques optimales de redistribution et d'assurance chômage (chapitre 3), ainsi que les politiques optimales d'imposition des revenus du travail et du capital (chapitre 4).

La première partie de cette introduction générale est conçue comme une introduction à l'analyse des systèmes d'imposition et de prestations. Elle vise à fournir des connaissances de base sur le sujet en mettant l'accent sur les caractéristiques et les concepts utilisés dans la suite de la thèse. La deuxième partie fournit un résumé non technique des différents chapitres et explique comment ces travaux contribuent à notre compréhension des systèmes socio-fiscaux.

L'émergence de l'impôt. L'impôt semble au moins aussi ancien que l'écriture. En effet, le récit dominant concernant l'émergence de l'écriture en Mésopotamie vers 3200 avant J.-C. est qu'elle est apparue pour des motifs économiques (Schmandt-Besserat, 2010). Les signes de l'écriture tiraient leur forme et leur signification d'un système comptable préexistant de *jetons*. Les jetons étaient de petites formes modelées dans l'argile qui représentaient des unités particulières de marchandises spécifiques. Ils étaient utilisés pour répertorier les transactions, les stocks et les dettes, jusqu'à ce que l'administration du temple de la cité-état sumérienne d'Uruk commence à imprimer des jetons sur la surface de tablettes. Les signes ont ensuite été tracés à l'aide d'un stylet et, au lieu de répéter les signes pour ajuster les quantités comme on le faisait avec les jetons ("comptage concret"), l'écriture a conduit à l'introduction du "comptage abstrait" via des chiffres (Schmandt-Besserat & Erard, 2008).¹

En outre, l'écriture a permis le rassemblement, le traitement et la diffusion d'informations. Vers 2000 avant J.-C., ce qui est aujourd'hui appelé écriture cunéiforme avait évolué jusqu'à la représentation de syllabes et était utilisé pour des textes juridiques, historiques, religieux ou littéraires. Si l'utilisation du système comptable des jetons pour le versement d'impôts

¹Vers 3000 avant J.-C., des noms ont commencé à être ajoutés sur les tablettes, ce qui a marqué l'apparition des phonogrammes (signes retranscrivant un son). Vers 2800 avant J.-C., les noms ont ensuite été ajoutés aux tombes et sont devenus partie intégrante des rituels funéraires, rompant avec la fonction comptable de l'écriture.

n'est pas formellement avérée, les textes juridiques établissent clairement l'existence de taxes (miksu) sur les activités commerciales et agricoles dans le monde mésopotamien (Ellis, 1974). Les taxes les plus documentées sont une taxe de 10 % sur les bénéfices commerciaux imposée par le temple de Ningal à Ur, ou une taxe les marchandises en transit imposée par la cité-État de Mari. Nous connaissons également une taxe sur les terres cultivées qui semble avoir été liée à la surface de terre arable, ou à son rendement. Dans un cas, nous savons même que le produit de cet impôt a été utilisé pour financer des canaux et des systèmes d'irrigation.

La notion d'information est essentielle pour la capacité à collecter l'impôt. Outre l'écriture, Stasavage (2021) soutient que la possibilité de stocker les céréales a joué un rôle essentiel dans l'introduction d'impôts et plus largement la formation d'institutions étatiques. Contrairement aux racines et aux tubercules, les premières céréales comme le blé, l'orge et le millet pouvaient être stockées. De plus, étant récoltées de façon saisonnière, elles devaient être stockées afin d'être consommées à une date ultérieure. Par conséquent, les récoltes de céréales étaient à la fois observables et appropriables par les États, ce qui a conduit à l'apparition d'impôts (Mayshar, Moav, & Pascali, 2020). Le stockage permettait en outre de créer des excédents agricoles qui pouvaient être commercialisés, et encore une fois, taxés. Concomitamment, cela signifiait aussi que les céréales pouvaient être volées, créant un besoin de protection et légitimant peut-être l'imposition de taxes par une élite dirigeante capable de fournir et d'organiser cette protection. Ce modèle de société dans lequel l'État (ou l'élite dirigeante) impose des taxes et assure la fourniture d'une forme de protection et de biens publics devait dominer pour les siècles à venir.

L'émergence de l'État-providence. Cette thèse étudie les systèmes d'imposition et de prestations qui sont caractéristiques des États-providence ayant émergé en Europe occidentale et en Amérique du Nord au cours du XXème siècle. En effet, les dépenses publiques et les dépenses sociales restaient auparavant assez limitées, aussi bien en termes d'objectifs que de montants alloués. Au XIXème siècle, les dépenses publiques représentaient environ 10% du revenu national. Elles finançaient principalement les guerres et, dans une certaine mesure, l'éducation publique (Tanzi & Schuknecht, 2000). Les dépenses sociales étaient consacrées à la lutte contre l'extrême pauvreté et représentaient en moyenne moins de 1% du revenu national. Au début du XXème siècle, l'éducation de masse a déjà commencé à se développer et les dépenses sociales commencent à financer des formes d'indemnisation du chômage, de pensions de vieillesse, de dépenses de santé ou de subventions au logement. Le véritable essor se produit au lendemain de la Seconde Guerre mondiale, où les dépenses sociales décollent et atteignent en 1980 des valeurs comprises entre 10% et 30% du revenu national pour les pays de l'OCDE (Lindert, 2004). La transition vers un modèle de société dans lequel les États mettent en œuvre des programmes de redistribution et d'assurance sociale de grande ampleur financés par l'impôt est donc très récente et s'est produite en un peu moins d'un siècle.

Cette transition s'est accompagnée de "trois autres grandes transformations : une transition vers une démocratie plus complète, une transition démographique avec une réduction des naissances et un allongement de la vie, et une transition vers une période de croissance économique soutenue" (Lindert, 2004, page 20). Un certain nombre de théories ont été avancées pour articuler (certains de) ces éléments. D'abord, une littérature influente en économie politique soutient que la démocratisation et l'extension du droit de vote ont progressivement augmenté le poids politique des pauvres et donc la demande de redistribution (Acemoglu & Robinson, 2000). Scheve and Stasavage (2016) affirment en outre que ces demandes ont été jugées "justes" en raison du lourd tribut en vies humaines payé par la classe ouvrière durant la Première et la Seconde Guerre mondiale. Ensuite, le vieillissement de la population est associé à une augmentation des dépenses sociales, peut-être en raison de de besoins plus importants en matière d'assistance aux personnes âgées et de soins de santé ou de l'importance croissante des personnes âgées dans l'électorat (Lindert, 2004). Enfin, un vaste champ de recherche synthétisé par Besley and Persson (2013) analyse l'interdépendance entre fiscalité et développement. Elle suggère l'existence d'un cercle vertueux au sein duquel la croissance économique crée les moyens de financer dépenses et investissements publics qui stimulent en retour la croissance économique et donc une capacité de financement.

Il n'en demeure pas moins que la mise en place de programmes de redistribution et d'assurance sociale de grande ampleur nécessite des niveaux d'imposition élevés. Cela soulève la question suivante : "pourquoi les gouvernements modernes peuvent-ils prélever autant d'impôts ?" La réponse de Kleven, Kreiner, and Saez (2016) à cette question souligne que, là encore, l'information est essentielle pour la capacité à collecter des impôts. Ils montrent que l'augmentation des recettes fiscales au cours du XXe siècle est due à l'augmentation des "impôts modernes" comme les cotisations sociales, les impôts sur les salaires, les impôts sur le revenu ou les taxes sur la valeur ajoutée.² Ce qui rend ces impôts "modernes" est le recours à la collecte d'information par des tiers : le montant de l'impôt payé par une entité est déterminé par les informations transmises par une entité tierce. Par exemple, le paiement de cotisations sociales par un salarié se fait sur la base de son salaire déclaré par l'entreprise qui l'embauche. Rassemblant des enquêtes menées dans 100 pays et 140 ans de données historiques aux États-Unis, Jensen (2019) montre que la transition de long terme

 $^{^{2}}$ Dans cette décomposition, les "impôts traditionnels" sont les impôts fonciers, les droits de succession, les droits d'accise (forme d'impôt indirect perçu généralement sur la consommation), les taxes sur les ventes, les droits de douane.

du travail indépendant au travail salarié a permis aux États d'étendre considérablement le montant d'impôt sur le revenu collecté. Ce mécanisme de collecte d'information par des tiers semble donc essentiel pour limiter la fraude fiscale et permettre le financement des Étatsprovidence. Dans le même temps, le consentement à l'impôt a également augmenté avec les dépenses publiques et sociales (Delalande, 2011).

Des systèmes socio-fiscaux importants. Les États-providence modernes opèrent des systèmes socio-fiscaux de grande ampleur qui sont liés à la fois aux recettes et aux dépenses de l'État. Le tableau 1 présente les recettes fiscales moyennes des pays de l'OCDE en pourcentage du PIB par grandes catégories d'impôts. Schématiquement, les recettes fiscales se répartissent à parts égales entre les impôts sur le revenu, les bénéfices et les plus-values versés par les particuliers (8,2% du PIB) et les entreprises (3,3% du PIB), les cotisations sociales et les taxes sur les salaires versés par les salariés (3,4% du PIB), les entreprises (5,5% du PIB) et les travailleurs indépendants ou non salariés (0,9% du PIB), ainsi que les impôts sur les biens et services versés par les entreprises (10,9% du PIB), sont quelque peu marginaux et ne représentent en moyenne que 6,2% des recettes fiscales totales. Il est à noter que ces chiffres regroupent tous les niveaux de l'État (central, régional et local) et masquent une grande hétérogénéité entre les pays.

Type d'impôt	Recettes fiscales	
	(% du PIB)	(%)
Impôts sur les revenu, bénéfices et plus-values	11.5	33.9
Cotisations sociales et taxes sur les salaires	9.4	27.7
Impôts sur les biens et services	10.9	32.2
Autres impôts	2.1	6.2
Total	33.9	100

Table 1: Recettes fiscales, moyenne OCDE, 2018 (Global Revenue Statistics Database)

Le tableau 2 présente, pour les pays de l'OCDE, la répartition moyenne des dépenses publiques par fonction de l'État, exprimée en part des dépenses totales – qui s'avèrent être légèrement plus élevées que les recettes fiscales totales. Avec les dépenses de fonctionnement du service public et le financement des affaires économiques (transports et industries no-

³Cette décomposition est basée sur *l'incidence légale* des impôts qui reflète l'entité légalement responsable de la remise de l'impôt. Elle ne dit rien de *l'incidence économique* des impôts qui reflète l'entité qui supporte effectivement la charge de l'impôt. Par exemple, les cotisations sociales légalement versées par les employeurs peuvent être transférées aux salariés sous la forme de salaires plus faibles, et les taxes sur les biens et services légalement versées par les entreprises peuvent être transférées aux consommateurs sous la forme de prix plus élevés (voir par exemple Salanie, 2011).

tamment), les dépenses de santé, d'éducation et de protection sociale représentent la part la plus importante des dépenses publiques. Alors que la fourniture de biens publics, de soins de santé ou l'éducation peuvent être considérés comme des transferts en nature, l'analyse des systèmes d'imposition et de prestations se concentre généralement sur les transferts monétaires. Ces derniers apparaissent principalement sous la fonction *protection sociale* et représentent donc environ 35% des dépenses publiques. L'analyse considère donc que les 65% restants des dépenses publiques sont exogènes, au sens où ces dépenses n'interviennent pas dans l'analyse, si ce n'est qu'elles doivent être financées par les impôts collectés.

$\frac{Part (\%)}{12.6}$ 3.5
-
35
0.0
3.9
10.4
1.6
1.4
15.8
3.0
12.3
35.5
100

Table 2: Dépenses publiques par fonction, moyenne OCDE, 2018 (COFOG)

Protection sociale	Part $(\%)$
Maladie et invalidité	18.0
Vieillesse	51.7
Survivants	5.1
Famille et enfance	12.1
Chômage	5.2
Logement	1.8
Exclusion sociale	4.2
Autres	1.9
Total	100

Table 3: Dépenses de protection sociale par fonction, moyenne OCDE, 2018 (COFOG)

La répartition détaillée des différentes dépenses de protection sociale est fournie dans le tableau 3. Les dépenses vieillesse représentent la moitié des dépenses de protection sociale, ce qui reflète l'importance des systèmes de retraites publics dans les pays de l'OCDE.⁴ D'autres dépenses importantes de protection sociale sont liées à la maladie et à l'invalidité (18,0%), à la famille et à l'enfance (12,1%), au chômage (5,2%) ou à l'exclusion sociale (4,2%).

Des systèmes socio-fiscaux complexes. Ces chiffres et catégories agrégés sont une représentation largement simplifiée des systèmes d'imposition et de prestations qui sont en realité très complexes. Alors que quelques instruments fiscaux représentent généralement la plus grande partie des recettes fiscales, il existe une myriade d'instruments fiscaux de petite et très petite taille. Par exemple, une étude de la Cour des comptes réalisée en 2018 établit l'existence d'au moins 125 instruments fiscaux en France dont les recettes annuelles

 $^{^{4}}$ Ce résultat est en partie lié au fait que les variables COFOG de deuxième niveau utilisées pour construire le tableau 3 ne sont disponibles que pour les pays membres européens dans lesquels les systèmes publics de retraites sont prédominants.

sont inférieures à 150 millions d'euros (Cour des Comptes, 2018). En outre, les instruments fiscaux sont généralement truffés de déductions et d'exonérations qui contribuent encore à l'opacité du système d'imposition et de prestations. En France encore, la loi de finances 2019 fait état de 473 exonérations représentant 35% du total des recettes fiscales, les dix plus grandes exonérations représentant la moitié de ce montant et certaines exonérations n'étant utilisées que par quelques centaines de personnes (Assemblée Nationale, 2018).

Une partie de cette complexité est probablement inefficace et liée à des choix politiques. Souvent, les décideurs publics préfèrent en effet superposer de nouveaux dispositifs au lieu de réformer les dispositifs existants. Cela s'explique par le fait que ce type de réforme est généralement plus facile à mettre en oeuvre mais induit une complexité toujours croissante (Kawai, Lang, & Li, 2018; Bierbrauer, Boyer, & Peichl, 2021). Des exonérations fiscales peuvent également être accordées pour des raisons politiques. En particulier, l'octroi d'une exonération fiscale à des groupes d'intérêt (petits et influents) peut sembler économiquement indolore. Toutefois, à mesure que le nombre d'exonérations augmente, les coûts économiques peuvent devenir substantiels. Si substantiels qu'ils peuvent finir par handicaper l'économie au point qu'une remise à plat des exonérations bénéficie à tous, y compris les bénéficiaires de ces exonérations (Ilzetzki, 2018).

Cependant, une partie de cette complexité permet aussi de tenir compte de l'hétérogénéité des comportements (Kaplow, 1995) ou des caractéristiques (Kleven & Kopczuk, 2011). Elle permet ainsi aux gouvernements de mieux cibler les impôts et les prestations. En outre, la complexité des systèmes de prélèvements et de prestations est également inhérente à la planoplie d'objectifs, toujours plus nombreux et plus variés, poursuivis par les gouvernements ainsi que l'illustrent les tableaux 2 et 3. Différents objectifs conduisent souvent à la mise en œuvre de différents instruments de politiques publiques, et l'articulation de ces différents objectifs et instruments est au cœur de cette thèse de doctorat.

Redistribution et assurance sociale. En particulier, deux objectifs majeurs poursuivis par les États-providence modernes sont la redistribution et l'assurance sociale. La redistribution est définie comme le transfert de ressources entre individus différents, par opposition à l'assurance sociale définie comme le transfert de ressources entre individus similaires mais affectés différemment par la vie (Chetty & Finkelstein, 2013). Intuitivement, la redistribution est liée à l'hétérogénéité des caractéristiques tandis que l'assurance sociale est liée à la réalisation d'événements défavorables. Par conséquent, les politiques de redistribution comme l'impôt progressif sur le revenu ou les transferts sous condition de ressources sont généralement conditionnées au revenu et à la composition familiale, tandis que les politiques d'assurance sociale comme l'assurance maladie, la retraite ou l'assurance chômage sont généralement conditionnées à un événement spécifique, qu'il s'agisse de la maladie, la vieillesse ou le chômage.

Cette distinction entre redistribution et assurance sociale transparaît dans le vocabulaire utilisé. Le terme "impôts" est par exemple défini dans le système de comptabilité nationale (SCN) de l'OCDE comme "les paiements obligatoires sans contrepartie, en espèces ou en nature, effectués par des unités institutionnelles au profit des administrations publiques". Cette définition insiste sur le fait que les impôts ne sont associés à aucune contrepartie individuelle, par opposition aux "cotisations sociales" qui sont également obligatoires mais génèrent des droits individuels à des prestations futures. Les impôts financent donc biens publics et transferts redistributifs, ce qui signifie que la redistribution effective dépend aussi bien de la manière dont la charge fiscale est répartie entre les revenus que de la générosité des transferts. En revanche, les cotisations sociales ne financent que les programmes d'assurance sociale, et le montant des cotisations se reflète généralement dans le montant des prestations reçues. Ainsi, des cotisations à l'assurance chômage plus élevées en emploi donnent lieu à des allocations de chômage plus élevées au chômage, là où des taxes plus élevées n'induisent pas de contrepartie particulière. La redistribution et l'assurance sociale sont donc deux objectifs distincts implémentés en pratique par des instruments de politiques publiques différents.

Pourtant, cette distinction est quelque peu artificielle (Salanie, 2011). En se concentrant sur l'assurance chômage, cette thèse montre que cette distinction est en pratique difficile à opérer (chapitre 1) et que cela a des implications importantes pour le barème et la structure des systèmes d'imposition et de prestations (chapitre 3). Par exemple, les chômeurs peuvent recevoir à la fois des allocations de chômage et des transferts sous condition de ressources lorsque le montant des allocations de chômage est faible, ce qui brouille la frontière entre les deux. En outre, des montants plus élevés d'allocations de chômage réduisent les montants des transferts reçus sous condition de ressources, ce qui signifie que l'assurance sociale interagit avec la redistribution. Par ailleurs, les programmes d'assurance sociale comme l'assurance chômage peuvent être utilisés (et sont utilisés) comme des véhicules de redistribution en finançant les prestations des plus pauvres par les contributions des plus riches. Enfin, programmes de redistribution et programmes d'assurance sociale affectent ensemble le comportement des agents et les ressources du gouvernement, ajoutant par là une autre source d'interactions.

Arbitrages coûts-bénéfices. Les programmes de redistribution et d'assurance sociale induisent à la fois des bénéfices et des coûts. L'analyse coûts-bénéfices de ces programmes, et de la fiscalité en particulier, a une longue histoire en économie (Musgrave, 1985). Les bénéfices peuvent être liés à deux concepts importants : *l'équité* et *l'utilité marginale*. L'équité est liée à l'égale répartition des ressources dans la société, de sorte que le transfert d'un euro d'une personne riche vers une personne pauvre augmente l'équité. L'utilité marginale est la modification du bien-être d'un individu suite à une petite modification de ses ressources. En pratique, l'utilité marginale de la consommation diminue avec le revenu : une personne riche valorise moins qu'une personne pauvre un euro de consommation supplémentaire. Par conséquent, le transfert d'un euro d'une personne riche (ou en emploi) vers une personne pauvre (ou au chômage) augmente le bien-être dans la société. La mesure exacte des bénéfices des programmes de redistribution et d'assurance sociale dépend alors de la manière dont ces notions d'équité et d'utilité marginale sont paramétrées.⁵ Différentes paramétrisations reflètent différentes visions du monde et conduisent naturellement à des résultats quantitatifs différents. Mais, précisément parce qu'il s'agit de paramètres, on peut les faire varier et évaluer la robustesse des résultats en toute transparence.

Les coûts sont liés aux effets d'efficacité induits par les programmes de redistribution et d'assurance sociale. Intuitivement, ces programmes sont coûteux dans la mesure où ils doivent être financés par des taxes, et où le coût d'une taxe dépasse le montant des recettes perçues. La raison est que les agents ne se contentent pas de verser le montant de l'impôt, ils adaptent également leur comportement à la présence d'impôts : les impôts induisent des distorsions. Les réactions à la présence d'impôts varient en fonction de l'assiette fiscale visée par l'impôt. Au XVIIe siècle à Amsterdam, les taxes foncières étaient proportionnelles à la largeur des facades des maisons, ce qui a conduit à la construction de maisons hautes, profondes mais étroites. Au tournant du XXe siècle en Indonésie, le traitement fiscal défavorable des voitures a conduit à l'invention d'un nouveau type de cyclomoteur à trois roues avec un longs banc à l'arrière pouvant accueillir jusqu'à huit passagers (Gillitzer, Kleven, & Slemrod, 2017). De même, l'augmentation des taux d'imposition sur les revenus du travail tend à réduire ces revenus (Chetty, 2012; Saez, Slemrod, & Giertz, 2012), et l'augmentation des allocations de chômage tend à augmenter la durée du chômage (Schmieder & Von Wachter, 2016). Ces réponses sont mesurées en pratique par des paramètres d'élasticité et font l'objet d'une vaste littérature empirique, avec des désaccords sur l'ampleur de ces réponses et les mécanismes par lesquels elles opèrent.⁶

⁵Dans l'analyse traditionnelle des programmes d'assurance sociale, les motifs d'équité sont supprimés et le seul motif d'assurance sociale est la décroissance de l'utilité marginale de la consommation. Le chapitre 3 remet en question cette vision des choses, compte tenu des interactions existantes entre les problèmes d'assurance sociale et de redistribution.

⁶Si les réponses aux taxes sont dans ce contexte coûteuses pour la société, les taxes peuvent également réduire les mauvais comportements associées à des externalités négatives. Par exemple, une taxe sur le carburant réduit certes la consommation de carburant (distorsion) mais réduit également la pollution induite (externalité négative liée à la consommation). Ce type de taxes génère des réponses qui sont bénéfiques pour la société pigouviennes (Pigou, 1928).

Un champ de recherche plus récent souligne également que la manière dont le système socio-fiscal est effectivement mis en place peut influencer à la fois coûts et bénéfices (Slemrod & Gillitzer, 2013). Ce point est lié à la complexité des systèmes socio-fiscaux, qui crée des confusions et des biais considérables au sujet des impôts (Bernheim & Taubinsky, 2018; Stantcheva, 2020). Au niveau des bénéfices, différentes implémentations peuvent affecter par exemple la demande des transferts ou exemptions fiscales auxquelles les agents ont droit (Bhargava & Manoli, 2015; Aghion, Akcigit, Lequien, & Stantcheva, 2017; Finkelstein & Notowidigdo, 2019). Au niveau des coûts, cela peut affecter la manière dont les agents réagissent aux taxes et aux transferts et donc affecter les distorsions induites (Chetty, Looney, & Kroft, 2009; Taubinsky & Rees-Jones, 2018). Ces évolutions récentes soulèvent des points inédits sur une question centrale en économie publique : quel est le barème optimal et quelle est la structure optimale des systèmes d'imposition et de prestations ?

Barème optimal et structure optimale. S'appuyant sur la précédente analyse coûtsbénéfices, une littérature normative vise à caractériser la politique fiscale optimale, c'est à dire le barème optimal et la structure optimale des systèmes d'imposition et de prestations sociales. La première tentative d'apporter une réponse formalisée à cette question remonte à Ramsey (1927) et Pigou (1928) dont les travaux ont jeté les bases de la théorie moderne de la fiscalité optimale. Dans une contribution déterminante, Mirrlees (1971) formalise l'arbitrage entre équité et efficacité au cœur des politiques de redistribution en tant que problème d'asymétrie d'information. Le gouvernement souhaite effectuer de la redistribution entre des agents qui ont des capacités productives hétérogènes, mais il ne peut observer que leurs revenus du travail qui découlent à la fois de leurs capacités productives et de leurs efforts de travail. Par conséquent, la redistribution doit s'opérer par le biais d'impôts sur le revenu qui réduisent inévitablement les efforts de travail. Cette formulation élégante du problème lie la taille du gâteau (efficacité) à la distribution du gâteau (équité) dans un cadre théorique cohérent. Elle a ensuite été reliée aux données dans les travaux de Piketty (1997), Diamond (1998) et Saez (2001) grâce à l'utilisation de statistiques suffisantes, quantités estimables empiriquement capturant les effets des réformes fiscales sur l'équité et l'efficacité. La littérature sur l'assurance sociale optimale a suivi une évolution quelque peu parallèle. Le modèle fondateur d'assurance chômage optimal de Baily (1978) a été relié plus tard aux données par Chetty (2008). Cette thèse relie ces deux pans de la littérature en étudiant conjointement redistribution optimale et assurance chômage optimale (chapitre 3).

Un résultat influent sur la structure optimale des systèmes d'imposition et de prestations, dû à Atkinson and Stiglitz (1976), stipule que la redistribution doit être opérée par le biais d'impôts sur le revenu plutôt que par une imposition différenciée de catégories de consommation différentes. L'intuition est que les inégalités de consommation sont fondamentalement liées aux inégalité de revenus. Ainsi, il est plus efficace de redistribuer par le biais d'impôts sur le revenu que par le biais d'une taxe spécifique sur le champagne, même si le champagne est effectivement consommé par des gens riches. Ce résultat a eu une influence durable en économie publique. En effet, réinterprétant la consommation d'aujourd'hui et celle de demain comme deux biens différents, il fournit une justification à la non-imposition de l'épargne et du capital. Toutefois, cela n'est vrai que si les goûts des agents sont homogènes : une taxe spécifique sur le champagne est une politique redistributive efficace si les gens riches ont un goût relativement plus marqué pour le champagne (Saez, 2002b). Cette thèse réexamine la question de la structure optimale de la fiscalité en présence d'hétérogénéité des préférences (chapitre 4).

La politique fiscale analysée dans cette littérature théorique agrège généralement les instruments fiscaux existants, faisant ainsi l'hypothèse que c'est l'allocation finale des ressources générée par le système socio-fiscal qui importe. Lorsque les agents sont capables de relier parfaitement les instruments fiscaux existants à l'allocation finale des ressources, l'approche *mechanism design* centrée sur les allocations, proposée par Mirrlees (1971), est équivalente à l'approche *tax perturbation* centrée sur les instruments fiscaux proposée par Saez (2001) – voir par exemple Jacquet and Lehmann (2020) sur cette équivalence. Cependant, lorsque les agents sont inattentifs aux taxes et ont des perceptions erronées, cette équivalence n'est plus vérifiée (Rees-Jones & Taubinsky, 2018) et la politique fiscale optimale est sensiblement affectée (Farhi & Gabaix, 2020). Cette thèse montre que le gouvernement est aussi amené à ne plus choisir la politique fiscale optimale (chapitre 2).

Chapitre 1 – Allocations logement et incitations financières au travail : simulations pour la France

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Motivation. Le système des allocations logement vise à aider les ménages à faibles revenus à couvrir leurs dépenses de logement. En France, cela prend principalement la forme d'une aide financière versée aux locataires qui augmente avec le loyer (lien prestations-loyer) et diminue avec les revenus du ménage (condition de ressources). Des travaux antérieurs montrent que le lien entre aide et loyer fait que 50 à 80 % des allocations logement sont captées par les propriétaires via l'augmentation des loyers (Fack, 2006). Ces résultats ont donné lieu à plusieurs propositions de réforme visant à atténuer ce lien, transformant effectivement les allocations logement en un programme de transfert sous condition de ressources. Cela soulève des questions sur l'articulation des allocations logement avec les autres transferts sous condition de ressources afin de concevoir une politique de redistribution efficace qui préserve les incitations au travail et évite ainsi un phénomène de trappe à inactivité.

Les incitations monétaires au travail peuvent être décomposées entre les incita-Article. tions à travailler davantage lorsqu'on est en emploi (marge intensive) et les incitations à trouver un emploi lorsqu'on est sans emploi (marge extensive). Cet article caractérise les incitations financières au travail à ces deux marges à travers l'estimation de taux marginaux effectifs de prélèvements et de taux effectifs de prélèvement à l'emploi. Ils sont estimés à partir de l'enquête *Revenus fiscaux et sociaux* 2011 (ERFS, Insee) pour les célibataires sans enfants à l'aide du modèle de microsimulation TAXIPP, et sont décomposés par instruments fiscaux et prestations sociales. A la marge intensive, la modulation des allocations logement en fonction des conditions de ressources implique qu'une augmentation de 1 euro des revenus bruts du travail réduit les allocations de logement de 27 centimes en moyenne. Combiné à la réduction des autres transferts sous condition de ressources (30 centimes) et au paiement des cotisations sociales (21 centimes), cela se traduit par un taux marginal effectif de prélèvements proche de 80%. A la marge extensive, la réduction des allocations logement lors de l'accès à l'emploi agit comme une taxe à l'emploi. Du fait de la substituabilité des allocations chômage et des prestations sous conditions de ressources, l'importance de cette taxe varie selon que les individus recoivent ou non des allocations chômage en l'absence d'emploi.

Contribution. L'article contribue à la littérature sur les incitations financières au travail en France et souligne l'effet désincitatif induit par le fort conditionnement des allocations de logement aux ressources des ménages. Alors que (Sicsic, 2018) se concentre sur l'évolution historique d'estimateurs agrégés des taux marginaux effectifs de prélèvement et des taux effectifs de prélèvement à l'emploi pour différents types de ménages, cet article reporte des estimateurs individuels qui rendent compte de l'hétérogénéité des incitations au travail pour les célibataires sans enfant. Il examine en outre comment les incitations financières au travail sont affectées si (a) les individus perçoivent des allocations de chômage lorsqu'ils sont sans emploi, (b) les cotisations patronales sont en réalité payées par les salariés via des baisses de salaire plutôt que par les entreprises, (c) les cotisations d'assurance sociale sont traitées comme une épargne forcée associée à une consommation future plutôt que comme des taxes. L'analyse souligne la complexité du système français d'imposition et de prestations sociales et soulève des questions nouvelles sur l'articulation des transferts sous condition de ressources et des allocations de chômage.

Chapitre 2 – Inattention et biais fiscal (avec J. Boccanfuso)

Article ayant reçu le "ITAX PhD Student Award" lors de la conférence IIPF de 2019.

Motivation. Les systèmes d'imposition et de prestations sociales sont complexes. En conséquence, un nombre croissant d'études estiments que les agents économiques font preuve d'une inattention considérable et de perceptions erronées à l'égard des politiques fiscales (Bernheim & Taubinsky, 2018; Stantcheva, 2020). Cela n'est pas nécessairement mauvais. Par exemple, Rees-Jones and Taubinsky (2020) montrent que les ménages américains tendent à sous-estimer leurs taux d'imposition sur le revenu, ce qui réduit le coût d'efficacité de la fiscalité car les agents agissent comme si les impôts étaient moins élevés. À la lumière de ces travaux, une littérature normative analyse la politique fiscale optimale en présence d'inattention et d'erreurs de perceptions (Farhi & Gabaix, 2020). Cependant, étonnamment peu de travaux analysent la manière dont inattention et perceptions affectent les choix de politique fiscale des gouvernements, question que nous abordons dans cet article.

Article. Cet article montre que l'inattention des agents à l'égard des impôts interagit avec le choix de la politique fiscale et conduit les gouvernements à mettre en œuvre des taux d'imposition inefficacement élevés : l'inattention introduit un biais fiscal. Intuitivement, des agents inattentifs ignorent en partie les réformes fiscales introduites par les gouvernements, ce qui incite ces derniers à augmenter les taux d'imposition. Néanmoins, ces réformes fiscales induisent des ajustements d'équilibre dans les perceptions des agents, ce qui implique que ces hausses d'impôts s'avèrent trop importantes ex-post et révèle un problème d'engagement et de crédibilité dans le choix de la politique fiscale. Nous caractérisons ce biais fiscal en fonction de paramètres estimables empiriquement et, à l'aide de données, estimons son ampleur pour l'économie américaine. Étant donné que l'attention aux impôts augmente avec les revenus, le biais fiscal est plus prononcé sur les bas revenus et affecte la progressivité du système fiscal. Nous montrons en outre que les gains d'efficacité résultant d'une éventuelle sous-estimation des taux d'imposition peuvent être dominés par les coûts d'efficacité du biais fiscal lorsque l'attention est limitée. Dans l'ensemble, cet article jette un nouvel éclairage sur les conséquences de la présence d'inattention et d'erreurs de perceptions pour la politique fiscale.

Contribution. L'article contribue à l'émergente littérature d'économie publique comportementale en montrant que les gouvernements peuvent ne plus choisir la politique fiscale optimale en présence d'inattention. Il fournit ainsi un cadre théorique pour analyser les ajustements de politiques fiscales qui ont pu être observés après des changements dans la visibilité et lisibilité des impôts (Finkelstein, 2009; Cabral & Hoxby, 2012). Le fait que l'inattention induise un problème d'engagement et de crédibilité dans le choix de la politique fiscale établit un lien inattendu avec les travaux antérieurs sur l'incohérence des plans optimaux (Kydland & Prescott, 1977) et sur l'existence d'un biais d'inflation dans la politique monétaire (Barro & Gordon, 1983). Ce nouveau résultat soutient l'idée que l'inattention à l'égard des impôts peut entraîner des coûts substantiels et contribue à notre compréhension des systèmes complexes d'imposition et de prestations sociales.

Chapitre 3 – Incitations au travail ou à la recherche d'emploi? Politiques redistributives et assurance chômage

Motivation. Les États-providence modernes mettent en œuvre des programmes de redistribution et d'assurance sociale de grande ampleur, qui sont généralement étudiés séparément. Par exemple, la littérature sur la fiscalité optimale (Mirrlees, 1971; Saez, 2001) analyse un arbitrage entre équité et efficacité dans lequel les gouvernements valorisent la redistribution mais doivent préserver les incitations au travail. De même, la littérature sur l'assurance chômage optimale (Baily, 1978; Chetty, 2008) analyse un arbitrage dans lequel les gouvernements valorisent la fourniture d'une assurance aux chômeurs mais doivent préserver les incitations à la recherche d'emploi. Pourtant, politiques redistributives et assurance chômage sont difficiles à séparer en pratique, ce qui amène à se demander si ces problèmes interagissent et, dans l'affirmative, comment ces interactions affectent les politiques redistributives optimales et l'assurance chômage optimale.

Article. Cet article analyse politiques redistributives optimales et assurance chômage optimale. Premièrement, les deux problèmes interagissent parce que les décisions de travail et de recherche d'emploi des agents sont affectées à la fois par les taxes payés en emploi et par les allocations reçues au chômage. Il en résulte que les barèmes optimaux d'imposition et d'allocations chômage dépendent à la fois de l'élasticité du travail et de l'élasticité de la recherche d'emploi, ainsi que de l'objectif redistributif du gouvernement et du taux de chômage. Deuxièmement, l'allocation efficace des ressources implique un équilibre entre consommation des agents en emploi et consommation des agents au chômage. La condition d'efficacité induite montre qu'il est efficace de redistribuer à la fois par une fiscalité redistributive et par l'assurance chômage. Troisièmement, cette condition d'efficacité révèle également une externalité fiscale importante : le versement d'allocations chômage généreuses est non seulement coûteux pour l'assurance chômage, mais également coûteux pour la redistribution en raison des impôts non collectés si les agents restent plus longtemps au chômage. Il se trouve que cette externalité augmente fortement avec le revenu, ce qui nécessite un système d'assurance chômage avec des taux de remplacement décroissants. Une calibration sur données américaines montre que les taux de remplacement diminuent en pratique avec le revenu, mais suggère également l'existence de réformes bénéfiques à tous. Ces résultats montrent que politiques redistributives et assurance chômage interagissent de manière importante et que ces interactions sont importantes pour la conception des politiques publiques.

Contribution. L'article contribue à la littérature en économie publique en reliant deux champs de recherche importants et relativement déconnectés que sont la fiscalité optimale et l'assurance chômage optimale. Il contribue plus spécifiquement à la littérature sur la fiscalité optimale qui considère le chômage (voir Hungerbühler, Lehmann, Parmentier, & Van der Linden, 2006; Kroft, Kucko, Lehmann, & Schmieder, 2020) en introduisant des allocations de chômage qui dépendent des revenus en emploi, une caractéristique primordiale des systèmes d'assurance chômage qui joue un rôle important dans l'analyse. Alors que la littérature sur l'assurance chômage optimale étudie généralement des barèmes d'allocations chômage linéaires et des taux de remplacement constants, cet article applique les méthodes développées en fiscalité optimale pour caractériser les barèmes optimaux d'allocations chômage non linéaires et montre que les taux de remplacement optimaux diminuent avec le revenu. En outre, l'analyse souligne que lorsque les agents ont des revenus hétérogènes, implémenter une assurance chômage proportionnelle au revenu n'est possible (c'est-à-dire compatible en termes d'incitations) qu'en présence de conditions d'éligibilité telles que des durées minimales de cotisations à l'assurance chômage. Conceptuellement, l'article montre que les problèmes de redistribution optimale et d'assurance chômage optimale ne peuvent être séparés.

Chapitre 4 – Statistiques suffisantes et systèmes fiscaux non linéaires sous préférences hétérogènes (avec B. Lockwood et D. Taubinsky)

Motivation. Un résultat influent sur la structure optimale des systèmes d'imposition et de prestations, dû à Atkinson et Stiglitz (1976), stipule que la redistribution devrait s'opérer par le biais d'un impôt progressif sur le revenu plutôt que par une imposition différenciée des catégories de consommation. Ce résultat repose sur l'hypothèse que les préférences sont homogènes dans la population, et n'est plus vrai dès lors que les préférences sont corrélées avec la productivité, et donc les revenus du travail (Mirrlees, 1976). Une justification importante de l'imposition des revenus du capital, de l'épargne, des legs et de certains biens est donc qu'il est efficace de taxer les catégories de consommation préférées par ceux qui ont une productivité plus élevée. Pourtant, une grande partie de la littérature existante sur le sujet

se concentre sur des barèmes d'imposition linéaires, alors même que les taxes sur l'épargne ou sur les revenus du capital peuvent être non linéaires ou comporter des taux d'imposition linéaires dépendant des revenus du travail. En outre, les travaux existants sur l'hétérogénéité des préférences sont essentiellement qualitatifs ou limités à des hypothèses particulières sur les fonctions d'utilité des agents. Notre compréhension des systèmes fiscaux optimaux en présence d'hétérogénéité des préférences reste donc limitée.

Article. Cet article caractérise en termes de statistiques suffisantes les systèmes fiscaux non linéaires optimaux en présence de préférences hétérogènes. Nos résultats englobent les systèmes fiscaux non restreints permettant d'implémenter l'allocation optimale des ressources dans l'économie, ainsi que les systèmes fiscaux séparables qui combinent un impôt non linéaire sur les revenus du travail avec un impôt non linéaire sur les revenus du capital, ou avec un impôt linéaire sur les revenus du capital dépendant des revenus du travail. Sous certaines hypothèses étonnamment générales, nous montrons que ces systèmes fiscaux séparables permettent également d'implémenter l'allocation optimale. Dans l'ensemble des systèmes fiscaux, la statistique suffisante essentielle pour l'hétérogénéité des préférences est la différence entre la variation en coupe de la consommation en fonction du revenu et l'effet causal du revenu sur la consommation. Les formules caractérisant les systèmes fiscaux non linéaires optimaux que nous proposons correspondent à des généralisations empiriquement applicables du théorème d'Atkinson-Stiglitz, et prennent une forme familière dans la mesure où elles ressemblent à la caractérisation standard du barème non linéaire optimal d'imposition des revenus du travail (voir par exemple Saez, 2001).

Contribution. L'article contribue à notre compréhension des systèmes fiscaux non linéaires optimaux en présence d'hétérogénéité des préférences. Tout d'abord, il fournit des caractérisations empiriquement applicables de ces systèmes optimaux qui ne reposent pas sur l'estimation de paramètres de préférence structurels. La statistique suffisante mesurant l'hétérogénéité des préférences que nous proposons peut en effet être directement estimée à partir de données administratives ou de données d'enquêtes en utilisant des variations quasi-expérimentales comme des réformes fiscales ou des changements de salaires. Ensuite, nous déduisons les conditions sous lesquelles les systèmes fiscaux séparables, souvent utilisés en pratique, s'avèrent optimaux. Ce résultat sur la structure optimale des systèmes fiscaux montre qu'imposer une forme de séparabilité, condition a priori assez restrictive, peut se faire sans perte de généralité. Dans l'ensemble, l'analyse ouvre la voie à une évaluation empirique des systèmes fiscaux optimaux compte tenu du degré d'hétérogénéité des préférences mesuré dans les données.

General introduction

Public economics is a research field that studies the economics of the public sector. It analyzes how government policies affect the economy, and how they should be optimally designed to reach their goals efficiently.

This dissertation in public economics studies more specifically the design of tax-benefit systems which are a major component of government policies. It analyzes the articulation of different taxes and benefits in France (Chapter 1), the impact of agents' inattention to taxes on tax policy (Chapter 2), the optimal design of redistribution and unemployment insurance (Chapter 3), and the optimal design of earnings and capital taxes (Chapter 4).

This general introduction first introduces readers to the analysis of tax-benefit systems. It aims to provide basic knowledge on the topic with a focus on the features and concepts used in the remainder of the dissertation. The second part provides a non-technical summary of the different chapters and explains how they contribute to our understanding of the design of tax-benefit systems.

The emergence of taxes. Taxes are as old as writing. Indeed, the dominant narrative regarding the emergence of writing in Mesopotamia about 3200 BC is that writing appeared for economic motives (Schmandt-Besserat, 2010). Signs of writing derived their shape and meaning from a preexisting accounting system called *tokens*. Tokens were small shapes modeled in clay which represented particular units of specific commodities. They were used to keep track of transactions, stocks, and debts, until the Temple administration of the Sumerian city-state of Uruk started impressing tokens on the surface of tablets. Signs then became traced with a stylus, and, instead of repeating signs to adjust for quantities as was done with tokens ("concrete counting"), writing led to the introduction of "abstract counting" through numerals (Schmandt-Besserat & Erard, 2008).⁷

Beyond the ability to count quantities in an effective way, writing allowed to gather, process, and diffuse information. By 2000 BC, what became known as Cuneiform script had evolved to represent syllables and was used for legal, historical, religious, or literary texts. While we can only hypothesize that the accounting system of tokens was used for the remittance of taxes, legal texts clearly establish the existence of taxation (*miksu*) on commercial and agricultural activities in the Mesopotamian world (Ellis, 1974). In particular, we know of a 10 percent tax (*tithe*) imposed on trading profits by the Nigal Temple at Ur,

 $^{^7\}mathrm{About}$ 3000 BC, names started being added on the tablets which marked the appearance of phonograms (signs emulating the sound of speech). By 2800 BC, names were then added to tombs and became part of funerary rituals, breaking with the accounting function of writing.

or on goods in transit at the city-state of Mari. We also know of an impost on cultivated land that seems to have been tied to the land, or to its yield. In one instance, we even know that the proceeds of this impost was used to finance canals and irrigation systems.

Information is critical for the ability to collect taxes. In addition to writing, Stasavage (2021) argues that the storability of cereals was intrumental to the introduction of taxes and the formation of states. In contrast to roots and tubers, early cereals like wheat, barley and millet could be stored. Moreover, because they were harvested seasonally, they had to be stored for later consumption. As a result, cereals harvests were both observable and appropriable by states, which led to the emergence of taxes (Mayshar, Moav, & Pascali, 2020). Storability further enabled the creation of agricultural surpluses that could be traded, and again, taxed. Concomittantly, this also meant that cereals were appropriable by bandits, which created a demand for protection and perhaps legitimized the imposition of taxes by a ruling and protecting elite. This model of societies in which states (or the ruling elite) imposed taxes and provided some kind of protection and public goods would remain dominant for the centuries to come.

The emergence of modern welfare states. This dissertation studies the design of taxbenefit systems that are characteristic of modern welfare states which emerged in Western Europe and North America during the 20th century. Before that, both public and social spending remained fairly limited in scope and in size. In the 19th century, public spending was around 10% of national income. It mostly financed wars and to some extent public education (Tanzi & Schuknecht, 2000). Social spending was devoted to relief from extreme poverty and represented on average less than a percent of national income. By the beginning of the 20th century, mass education had started to develop and social spending started to finance forms of unemployment compensation, pension for old age, healthcare, or housing subsidies. The real boom came in the wake of World War II where social spending took off and reached by 1980 values between 10% and 30% of national income in OECD countries (Lindert, 2004). The transition to a model of societies in which states operate large-scale redistribution and social insurance programmes financed through taxes is thus very recent and occurred in a little less than a century.

This transition has been accompanied by "three other great social transformations: the transition to fuller democracy, the demographic transition toward fewer births and longer life, and the onset of sustained economic growth" (Lindert, 2004, page 20). A number of theories have been put forward to articulate (some of) these elements. First, an influential political economy literature argues that democratization and the extension of voting rights increased the political weight of the poor and thereby the demand for redistribution (Ace-

moglu & Robinson, 2000). Scheve and Stasavage (2016) further argue that these demands were deemed "fair" because of the heavy tribute in human lives paid by the working class during World War I and World War II. Second, population aging is associated with greater social spending, perhaps because of the growing importance of the elderly in the electorate or because of the greater needs for old age assistance and healthcare (Lindert, 2004). Third, a vast literature reviewed in Besley and Persson (2013) analyzes the interdependance between taxation and development. It suggests the existence of a virtuous circle in which economic growth creates the means to finance public expenditures and investments that further boost economic growth and tax collection capacities.

Still, operating large-scale redistribution and social insurance programmes requires high levels of taxation begging the question of "why can modern governments tax so much?". Kleven, Kreiner, and Saez (2016) answer to this question emphasizes that, again, information is critical for the ability to collect taxes. They show that the rise in tax revenue during the 20th century is driven by the rise of "modern taxes" like social contributions, payroll taxes, income taxes or value-added taxes.⁸ What makes these taxes "modern" is third-party reporting: the amount of tax (e.g. social contributions) paid by an entity (e.g. employed individual) is determined by the information transmitted by another entity (e.g. employing firm). Assembling surveys from 100 countries and 140 years of historical data within the US, Jensen (2019) shows that the long-run transition from self-employment to employee-employment has enabled states to considerably expand income tax collection. Information trails induced by third-party reporting thus appear to be key to limit tax evasion and finance modern welfare states. At the same time, the consent to taxes which is a profoundly contentious issue has also increased with public and social spending (Delalande, 2011).

Large-scale tax-benefit systems. Modern welfare states operate large-scale tax-benefit systems which relate to both revenue and expenditure sides of the government. Table 4 reports the average tax revenue in OECD countries as a share of GDP by type of taxes. Schematically, tax revenue is equally split between taxes on income, profits and capital gains remitted by individuals (8.2% of GDP) and firms (3.3% of GDP), social contributions and payroll taxes remitted by employees (3.4% of GDP), employers (5.5% of GDP), and self-employed or non-employed (0.9% of GDP), and taxes on goods and services remitted by firms (10.9% of GDP).⁹ Other taxes which include property taxes and custom duties (2.1%)

 $^{^{8}}$ In this decomposition, "traditional taxes" are property taxes, inheritance taxes, excise and sales taxes, custom duties.

⁹This decomposition is based on the *legal incidence* of taxes which reflects the entity legally responsible for remitting the tax. It says nothing about the *economic incidence* of taxes which reflects the entity who actually bears the burden of the tax. For instance, social contributions legally remitted by employers may

of GDP) are somewhat marginal and only represent 6.2% of total tax revenue on average. Note that these figures aggregate all levels of governments (central, state, local) and mask a large heterogeneity across countries.

Turna of tour	Tax revenue	Share of total
Type of tax	(% of GDP)	(%)
Taxes on income, profits and capital gains	11.5	33.9
Social security contributions and payroll	9.4	27.7
Taxes on goods and services	10.9	32.2
Other taxes	2.1	6.2
Total	33.9	100

Table 4: Tax revenue, OECD average, 2018 (Global Revenue Statistics Database)

Table 5 reports average government expenditures by functions in OECD countries as a share of total expenditures – which is in magnitude slightly higher than total tax revenue. Together with the financing of the government and the financing of economics affairs (e.g. transport, industries), spending on health, education and social protection represent the largest share of governments' expenditures. While public goods like healthcare and education can be seen as in-kind benefits, the analysis of tax-benefits systems usually focuses on monetary benefits which mostly appear under social protection and thus represent about 35% of government expenditures. The analysis thus takes the remaining 65% of government expenditures as exogenously given in the sense that they do not intervene in the analysis, except for the fact that they have to be financed by taxes.

Government expenditure	Share $(\%)$
General public services	12.6
Defence	3.5
Public order & safety	3.9
Economic affairs	10.4
Environment protection	1.6
Housing and amenities	1.4
Health	15.8
Recreation, culture, religion	3.0
Education	12.3
Social protection	35.5
Total	100

Table 5: Government expenditure by function, OECD average, 2018 (COFOG)

Social protection	Share (%)
Sickness and disability	18.0
Old age	51.7
Survivors	5.1
Family and children	12.1
Unemployment	5.2
Housing	1.8
Social exclusion	4.2
Others	1.9
Total	100

Table 6: Social protection expenditure by function, OECD average, 2018 (COFOG)

be shifted to workers in the form of lower wages, and taxes on goods and services legally remitted by firms may be shifted to consumers in the form of higher prices (see e.g. Salanie, 2011).

A detailed breakdown of the different social protection expenditures is provided in Table 6. Expenditures for old age represent half of social protection expenditures which reflects the importance of public pension systems in OECD countries.¹⁰ Other important social protection expenditures are related to sickness and disability (18.0%), family and children (12.1%), unemployment (5.2%) or social exclusion (4.2%).

Complexity of tax-benefit systems. These aggregate numbers and categories are a largely simplified representation of the actual complexity of tax-benefit systems. While few tax instruments usually make up for the largest share of tax revenue, there exists a myriad of small and very small tax instruments. For instance, a 2018 study of the French *Cour des Comptes* establishes the existence of at least 125 tax instruments in France with an annual revenue below 150 M \in (Cour des Comptes, 2018). Moreover, tax instruments are usually crippled with deductions and exemptions that further contribute to the opacity of the tax-benefit system. In France again, the 2019 budget law reported 473 exemptions accounting for 35% of total tax revenue, with the ten largest exemptions representing half of this amount and some exemptions only being used by few hundred individuals (Assemblée Nationale, 2018).

Part of this complexity is likely inefficient and driven by policymakers' decisions to overlay new policy instruments instead of reforming existing ones. This type of reform is usually more politically feasible but induces ever-increasing complexity (Kawai, Lang, & Li, 2018; Bierbrauer, Boyer, & Peichl, 2021). Tax exemptions may arise for similar political economy motives in that granting a tax exemption to (small and influential) interest groups may not appear economically costly. However, as the number of exemptions increases, economic costs may become so large that everyone may benefit from a tax reform, even the beneficiaries of these exemptions (Ilzetzki, 2018).

Yet, part of this complexity also allows to accomodate heterogeneity in behavior (Kaplow, 1995) or in characteristics (Kleven & Kopczuk, 2011). It thus enables governments to better target taxes and benefits. Moreover, the complexity of tax-benefit systems is also inherent to the wide-ranging policy objectives pursued by governments reflected in Table 5 and 6. Different policy objectives often lead to the implementation of different policy instruments, and the articulation of these different policy objectives and policy instruments are at the heart of this PhD dissertation.

 $^{^{10}}$ This is in part because second level COFOG variables used to construct Table 6 are only available for European member countries in which public pension systems are prevalent.

Redistribution and social insurance. In particular, two broad but distinct policy objectives pursued by modern welfare states are redistribution and social insurance. Redistribution is defined as the transfer of resources across individuals, in opposition to social insurance defined as the transfer of resources across states of the world (Chetty & Finkelstein, 2013). Intuitively, redistribution relates to heterogeneity in characteristics (e.g. income) whereas social insurance relates to the occurrence of adverse events (e.g. unemployment). As a result, redistributive policies like progressive income taxes or so-called *means-tested transfers* are typically conditional on income and family composition, while social insurance policies like health insurance, pension or unemployment insurance are typically conditional on an event, be it sickness, old age, or unemployment.

This distinction transpires in the terminology that we use. The term *taxes* is for instance defined in the OECD System of National Accounts (SNA) as "compulsory unrequited payments, in cash or in kind, made by institutional units to the general government". In particular, this means that taxes are not associated with any counterpart. This stands in contrast to *social contributions* which are also compulsory but open rights to future benefits. Taxes thus finance public goods and redistributive transfers meaning that actual redistribution depends both on how the tax burden is shared across income, and on the generosity of redistributive transfers. In contrast, social contributions only finance social insurance programmes, and the amount contributed is generally reflected in the amount of benefits received. For instance, higher unemployment contributions when employed give rise to higher unemployment benefits when unemployed. Providing redistribution and social insurance are thus two distinct objectives operated by different policy instruments.

Yet, this distinction is somewhat artificial (Salanie, 2011). Focusing on unemployment insurance, this dissertation shows that this distinction is in practice hard to operate (Chapter 1) and that this has important implications for the design of tax-benefit systems (Chapter 3). For instance, the unemployed may receive both unemployment benefits and meanstested transfers when the amount of unemployment benefits is low thereby blurring the frontier between the two. Moreover, higher amounts of unemployment benefits reduce, and thus crowd-out, amounts of means-tested transfers meaning that social insurance interacts with redistribution. In addition, social insurance programmes like unemployment insurance can be used (and are being used) as redistribution vehicles by financing the benefits of the poor through the contributions of the rich. Finally, both redistribution and social insurance programmes affect the behavior of agents and the resources of the government, thereby adding another layer of interactions. **Trade-offs: benefits and costs.** Redistribution and social insurance programmes induce both benefits and costs. The cost-benefit analysis of these programmes, and of taxation in particular, has a long history in economics reviewed in the work of Musgrave (1985). Benefits can be tied to two important concepts: *equity* and *marginal utility*. Equity relates to the equal distribution of resources in society, such that transferring one euro from the rich to the poor increases equity. Marginal utility is the change in an individual's well-being upon a small change in resources. In practice, the marginal utility of consumption is decreasing with income: richer people value one euro of consumption less than poorer ones. As a result, transferring one euro from the rich (or the employed) to the poor (or the unemployed) increases well-being in society. The exact measure of the benefits of redistribution and social insurance programmes then depends on how these notions of equity and marginal utility are parametrized.¹¹ Different parametrizations reflect different views of the world and naturally lead to different quantitative results. But, precisely because these are parameters, they can be varied and the robustness of the results can be transparently assessed.

Costs are related to the *efficiency* effects of redistribution and social insurance programmes. Intuitively, these programmes are costly in that they have to be financed through taxes, and that the burden of a tax exceeds the amount of revenue collected. The reason is that agents not only remit taxes, they also adapt their behavior to the presence of taxes: taxes induce *distortions*. Reponses to taxes vary depending on the tax base. In XVIIth century Amsterdam, property taxes were proportional to the street width of houses leading to the construction of tall, deep but narrow houses. In modern-day Indonesia, the preferential tax treatment of motorcycles led to the invention of a new type of motorcycle with three wheels and long benches at the back seating up to eight passengers (Gillitzer, Kleven, & Slemrod, 2017). Similarly, increases in labor income tax rates are shown to reduce labor income (Chetty, 2012; Saez, Slemrod, & Giertz, 2012), and increases in unemployment benefits are shown to increase the duration of unemployment (Schmieder & Von Wachter, 2016). These responses are measured through *elasticity parameters* and are the topic of a large empirical litterature, with some disagreements on both the magnitude and the mechanisms through which these responses operate.¹²

A recent strand of research also emphasizes that the way through which tax-benefits are actually implemented in practice may influence both benefits and costs (Slemrod &

¹¹In the traditional analysis of social insurance programmes, equity motives are assumed away and the only motive for social insurance is consumption smoothing in the occurrence of adverse events. Chapter 3 challenges this view given the existing interactions between social insurance and redistribution problems.

¹²While responses to taxes are in this context costly for society, taxes may also curb bad behavior that trigger negative externalities (e.g. pollution). This type of *Pigouvian taxes* generate responses that are beneficial to society (Pigou, 1928).

Gillitzer, 2013). This speaks in particular to the complexity of tax-benefits systems which creates substantial misunderstanding and confusion about taxes (Bernheim & Taubinsky, 2018; Stantcheva, 2020). On the benefit side, it may affect the take-up of transfers or tax exemptions agents are eligible to (Bhargava & Manoli, 2015; Aghion, Akcigit, Lequien, & Stantcheva, 2017; Finkelstein & Notowidigdo, 2019). On the cost side, it may affect how agents respond to taxes and transfers and thus the distortions that they induce (Chetty, Looney, & Kroft, 2009; Taubinsky & Rees-Jones, 2018). These recent development thus raise novel issues on an age-old question: what is the optimal design of tax-benefit systems?

Optimal design of tax-benefit systems. Building on the previous cost-benefit analysis, a normative literature aims at characterizing the optimal design of tax-benefit systems. The first formal attempt to tackle this question goes back to Ramsev (1927) and Pigou (1928) whose work laid the basis for the modern theory of optimal taxation. In a seminal contribution, Mirrlees (1971) formalized the equity-efficiency trade-off at the heart of redistributive policies as a problem of asymmetric information. The government wants to redistribute income across agents who are heterogeneous in their earnings abilities, but it can only observe agents' labor income that derives from both their abilities and work efforts. As a result, redistribution has to operate through income taxes which inevitably reduce work efforts. This elegant formulation of the problem ties the size of the pie (efficiency) to the distribution of the pie (equity) in a consistent theoretical framework. It was later connected to the data by Piketty (1997), Diamond (1998) and Saez (2001) through the use of empirically estimable sufficient statistics capturing equity and efficiency effects of tax reforms. The optimal social insurance litterature followed a somewhat parallel evolution. The seminal optimal unemployment insurance model of Baily (1978) was later connected to the data by Chetty (2008). This dissertation connects these two strands of the literature by jointly studying optimal redistributive taxation and optimal unemployment insurance (Chapter 3).

An influential result on the optimal design of tax-benefit systems due to Atkinson and Stiglitz (1976) states that redistribution should operate through progressive income taxes rather than through the differential taxation of consumption categories. The intuition is that inequality in consumption is due to inequality in income. Redistributing income is thus a more efficient way to redistribute than imposing differential consumption taxes that unnecessarily distort consumption choices: taxing income is better than taxing champagne at a higher rate than other alcoholic drinks. This result has had a long-lasting influence in public economics. Indeed, reinterpreting consumption today and consumption tomorrow as two different goods, it provides a basic rationale for savings and capital to go untaxed. However, this is only true if agents' tastes are the same: taxing champagne relatively more is an efficient policy if rich agents have relatively stronger preferences for champagne (Saez, 2002b). This dissertation revisits the issue of optimal tax design in the presence of preference heterogeneity (Chapter 4).

The tax policy considered in this theoretical literature usually aggregates existing tax instruments, assuming that the allocation of resources induced by the overall tax-benefit system is what matters. When agents are able to perfectly connect existing policy instruments to the actual allocation of resources, the *mechanism design* approach centered on allocations pioneered by Mirrlees (1971) is equivalent to the *tax perturbation* approach centered on tax instruments pioneered by Saez (2001) – see for instance Jacquet and Lehmann (2020). However, when agents are inattentive to policy and hold misperceptions, the equivalence breaks (Rees-Jones & Taubinsky, 2018) and optimal policy may be significantly affected (Farhi & Gabaix, 2020). This dissertation further shows that the government may no longer select the optimal policy (Chapter 2).

Chapter 1 – Housing benefits and monetary incentives to work: Simulations for France

Paper published in French and English in Economie & Statistique / Economics & Statistics.

Motivation. Housing benefits schemes aim at helping low-income households cover their housing expenditures. In France, this is primarily achieved through monetary transfers to tenants that are increasing with the rent (benefits-rent linkage) and decreasing with households' earnings (means-testing). Earlier work shows that the linkage with the rent causes 50 to 80% of housing benefits to be captured by homeowners through rents increases (Fack, 2006). These results have prompted several reform proposals aiming at alleviating this linkage, effectively turning the scheme into a means-tested transfer programme. This raises questions about the articulation of housing benefits with other means-tested transfers in order to design an effective redistributive policy that preserves work incentives and does not generate a poverty trap.

Paper. Monetary incentives to work can be decomposed between incentives to work more when in-work (intensive margin) and incentives to start working when out-of-work (extensive margin). This paper characterizes incentives to work at both margins through, respectively, effective marginal tax rates and effective participation tax rates. They are estimated using the 2011 *Enquête Revenus Fiscaux et Sociaux* (ERFS, Insee) for childless singles with the TAXIPP microsimulation model, and decomposed by tax and transfer instruments. Means-

testing implies that a 1-euro increase in gross labor earnings reduces housing benefits by 27 cents on average. Combined with reductions in other means-tested transfers (30 cents) and the payment of social contributions (21 cents) this translates into effective marginal tax rates close to 80%. Means-testing also induces a reduction in housing benefits upon taking a job which acts as a participation tax. Its magnitude depends on whether individuals receive unemployment benefits when out-of-work. Unemployment benefits increase overall participation tax rates by providing higher replacement earnings but decrease the participation tax linked to housing benefits by reducing the amounts of housing benefits received.

Contribution. The paper contributes to the literature on monetary incentives to work in France and underlines the adverse incentive effect of housing benefits means-testing. While (Sicsic, 2018) focuses on the historical evolution of aggregate estimates of effective marginal and participation tax rates for different household compositions, this paper provides individual estimates shedding light on heterogeneity of incentives to work for childless singles. It further investigates how incentives to work are affected by (a) whether individuals receive unemployment benefits when out-of-work, (b) whether employer contributions are actually paid by workers or firms, (c) whether social insurance contributions are treated as taxes or as forced savings that guarantee future consumption. The analysis highlights the complexity of the French tax-benefit system and raises novel questions about the articulation of means-tested transfers and unemployment benefits.

Chapter 2 – Inattention and the taxation bias (with J. Boccanfuso)

Paper awarded with the ITAX PhD Student Award at the 2019 IIPF Conference.

Motivation. Tax-benefit systems are complex. Accordingly, a growing body of evidence documents that agents exhibit substantial inattention to, and misperceptions about, tax policies (Bernheim & Taubinsky, 2018; Stantcheva, 2020). These need not entail negative effects. For instance, Rees-Jones and Taubinsky (2020) document that agents underestimate personal income tax rates in the US, and that this reduces the efficiency cost of taxation because agents act as if taxes were lower. In light of this evidence, a burgeoning normative literature analyzes the design of optimal tax policy accounting for these features (Farhi & Gabaix, 2020). Yet, surprisingly little work analyzes how these features affect the conduct of actual tax policy, a question we tackle in this paper.

Paper. This paper shows that agents' inattention to taxes interacts with policymaking and leads governments to implement inefficiently high tax rates: this is the *taxation bias*.

Intuitively, inattentive agents partially ignore tax reforms which induces governments to increase tax rates. Yet, these reforms induce equilibrium adjustments in agents' tax perceptions which imply that these tax increases are too large ex-post, reflecting a commitment problem in the choice of tax policy. Deriving a sufficient statistics formula for the taxation bias, we take it to the data and estimate the magnitude of the taxation bias for the US economy. Since inattention decreases with earnings, the taxation bias is more pronounced at low earnings levels and affects the progressivity of the tax system. Moreover, the efficiency gains from potential tax underestimation may be dominated by the efficiency costs of the taxation bias when attention is limited. Overall, the paper sheds a new light on the implications of tax inattention and misperceptions.

Contribution. The paper contributes to the expanding behavioral public finance litterature by showing that governments may no longer select the optimal tax policy in the presence of inattention to taxes. It thus provides a theoretical framework to analyze the policy adjustments that have been empirically observed after changes in the salience of taxes (Finkelstein, 2009; Cabral & Hoxby, 2012). The fact that inattention induces a commitment problem in the choice of tax policy builds an unexpected bridge to earlier works on the inconsistency of optimal plans (Kydland & Prescott, 1977) and on the existence of an inflation bias in monetary policy (Barro & Gordon, 1983). This novel finding supports the idea that inattention to taxes may entail substantial costs, and contributes to our understanding of complex tax-benefit systems.

Chapter 3 – Make work pay or make search pay? Redistributive taxation and unemployment insurance

Motivation. Modern welfare states operate large-scale redistribution and social insurance programmes, which are usually studied independently. For instance, the optimal taxation literature (Mirrlees, 1971; Saez, 2001) analyzes an equity-efficiency trade-off in which governments value redistribution but must preserve work incentives (*make work pay*). Similarly, the optimal unemployment insurance literature (Baily, 1978; Chetty, 2008) analyzes a tradeoff in which governments value insurance provision to the unemployed but must preserve job search incentives (*make search pay*). Yet, redistribution and unemployment insurance are hard to separate in practice, which begs the question of whether these problems interact and if so, how these interactions affect the optimal design of tax-benefit systems.

This paper analyzes the optimal design of redistributive taxation and unemploy-Paper. ment insurance. First, the two problems interact because agents' work and search decisions are affected by both taxes when employed and unemployment benefits when unemployed. Sufficient statistics formulas for optimal taxes and optimal benefits thus depend on both work and search elasticities, as well as on the redistributive tastes of the government and on unemployment. Second, the efficient allocation of resources implies a balance between agents' consumption when employed and when unemployed. The implied efficiency condition shows it is efficient to redistribute through both redistributive taxation and unemployment benefits. Third, this efficiency condition also reveals an important fiscal externality: providing generous unemployment benefits is not only costly to unemployment insurance but also costly to redistribution because of the foregone taxes people would pay if they were employed. This externality happens to be strongly increasing with income, calling for an unemployment insurance system with decreasing replacement rates. A calibration to the US economy shows actual net replacement rates decrease with earnings but suggests some scope for Pareto-improvements. These findings highlight that redistributive taxation and unemployment insurance interact in important ways and that these interactions matter for the design of public policies.

Contribution. The paper broadly contributes to the public economics literature by connecting two large and yet disconnected strands of research on optimal redistribution and optimal unemployment insurance. It more specifically contributes to the optimal income tax literature that considers unemployment (e.g. Hungerbühler, Lehmann, Parmentier, & Van der Linden, 2006; Kroft, Kucko, Lehmann, & Schmieder, 2020) by introducing unemployment benefits that depend on earnings when employed, a crucial feature of actual unemployment insurance systems which plays a prominent role in the analysis. While the optimal unemployment rates, this paper applies methods of the optimal taxation literature to characterize optimal nonlinear unemployment benefits and shows that replacement rates should be decreasing in income. Moreover, the analysis underlines that in settings with earnings heterogeneity, the provision of unemployment insurance is only possible (incentive compatible) in the presence of eligibility conditions such as minimal work durations. Conceptually, the paper shows that problems of optimal redistribution and optimal unemployment insurance cannot be separated.

Chapter 4 – Sufficient statistics for nonlinear tax systems with preference heterogeneity (with B. Lockwood and D. Taubinsky)

Motivation. An influential result on the optimal design of tax-benefit systems due to Atkinson and Stiglitz (1976) states that redistribution should operate through progressive income taxes rather than through differential taxation of consumption categories. This result relies on the assumption that tastes are homogeneous in the population, and does not generalize when tastes are correlated with earnings ability (Mirrlees, 1976). A prominent justification for the taxation of capital income, savings, bequests, and certain commodities is thus that it is efficient to tax consumption categories that are preferred by those with higher earnings ability. Yet, much of the existing literature on the topic focuses on linear taxation whereas actual taxes on savings, or on capital gains, can be nonlinear or feature earnings-dependent linear tax rates. Moreover, existing work on preference heterogeneity is mostly qualitative, or restricted to special assumptions about agents' utility functions. Our understanding of optimal tax systems in the presence of preference heterogeneity thus remains limited.

Paper. This paper provides sufficient statistics characterizations for optimal nonlinear tax systems in the presence of correlated preference heterogeneity. Our results encompass unrestricted tax systems that implement the optimal allocation of resources in the economy, as well as separable tax systems that combine a nonlinear earnings tax with a nonlinear capital income tax, or with an earnings-dependent linear capital income tax. Under some surprisingly general assumptions, we show that such separable tax systems also implement the optimal allocation. Across all tax systems, the key sufficient statistic for preference heterogeneity is the difference between the cross-sectional variation of consumption with income and the causal effect of income on consumption. Our formulas for optimal differential commodity taxation produce empirically-implementable generalizations of the Atkinson-Stiglitz theorem, and take a familiar form that resembles the sufficient statistics formula for the optimal nonlinear earnings tax (e.g. Saez, 2001).

Contribution. The paper contributes to our understanding of optimal tax systems in the presence of preference heterogeneity. First, it provides empirically implementable characterizations of optimal tax systems that do not rely on the estimation of structural preference parameters. The key sufficient statistic for preference heterogeneity can be directly estimated from administrative or survey data using quasi-experimental variations like tax reforms or wage changes. Second, we derive conditions under which the type of separable tax systems implemented in practice are optimal. This result on the optimal structure of tax systems shows that separability can be without loss of generality. Overall, the analysis paves the way for an empirical assessment of optimal tax systems given the amount of preference heterogeneity measured in the data.

Chapter 1 – Housing benefits and monetary incentives to work: Simulations for France

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Abstract. This paper characterizes the impact of housing benefits on monetary incentives to work in France both at the intensive and extensive margins. Effective marginal and participation tax rates are estimated using the 2011 enquête Revenus fiscaux et sociaux (ERFS, Insee) for employed childless singles with the TAXIPP microsimulation model and decomposed by tax and transfer instruments. Means-testing implies that a 1-euro increase in gross labor earnings reduces housing benefits by 27 cents on average. Combined with reductions in other means-tested transfers (30 cents) and the payment of social contributions (21 cents) this translates into effective marginal tax rates close to 80%. Means-testing also induces a reduction in housing benefits upon taking a job which acts as a participation tax. Its magnitude depends on whether individuals receive unemployment benefits when out-of-work. Unemployment benefits increase overall participation tax rates by providing higher replacement earnings but decrease the participation tax linked to housing benefits by reducing the amounts of housing benefits received.

Note. I wish to express my gratitude to Antoine Bozio for numerous and helpful discussions at the early stages of this work. I also wish to thank two anonymous referees, as well as Jérémy Boccanfuso, Pierre Boyer, Étienne Lehmann, Jean-Baptiste Michau, Thomas Piketty, Anasuya Raj, Michael Sicsic, Clémence Tricaud and members of CRED Taxation Seminar for their invaluable comments.

• ousing benefits schemes aim at helping low-income households cover their housing expenditures. In France, this is primarily achieved through monetary transfers to tenants that are increasing with the rent (benefits-rent linkage) and decreasing with households' earnings (means-testing). Seminal contributions (Laferrère & Le Blanc, 2004; Fack, 2005, 2006) show that the linkage with the rent causes 50 to 80% of housing benefits to be captured by homeowners through rents increase. This finding has prompted reform proposals, most recently by Trannov and Wasmer (2013), Bozio et al. (2015a) and Bargain et al. (2017), aiming at the alleviation of this linkage and the induced phenomenon of capture.

Another concern these reform proposals try to address is the potentially large disincentives to work associated with means-tested housing benefits. Indeed, as an increase in labor earnings translates into a decrease in benefits received, means-testing mechanically reduces the monetary gains from work. This may induce individuals to reduce their labor supply¹ and has thus important consequences for the design of means-tested transfer schemes (Saez, 2002; Brewer *et al.*, 2010). A poorly designed housing benefits scheme combining benefits capture by landlords and large disincentives to work for low-income tenants may be conducive to a poverty trap.

This paper aims at informing future reforms of the housing benefits scheme by providing a detailed analysis of monetary incentives to work in France. Following the labor supply literature (e.g. Heckman, 1993), the analysis distinguishes between incentives to increase work intensity when in-work (intensive margin) and incentives to join the workforce when out-of-work (extensive margin). Monetary incentives to work are accordingly measured by effective marginal and participation tax rates.

These measures are estimated at the individual level for a representative sample of employed childless singles aged 25 to 54 from the 2011 *enquête Revenus fiscaux et sociaux* (ERFS). Taxes and transfers are computed using the TAXIPP microsimulation model and include social contributions, the income tax and means-tested transfers. A decomposition of aggregate work incentives in terms of the underlying tax and transfer instruments clarifies the articulation of these instruments. Moreover, it allows to precisely characterize the disincentives

to work associated with housing benefits as well as their contribution to the aggregate.

The article begins with a brief review of related research and a discussion of the approach adopted here. The data, microsimulation tool and methodology are then carefully described along with the main features of the French tax and transfer system. The results derived in a baseline scenario show that housing benefits entail important disincentives to work. In particular, their joint withdrawal with other means-tested transfers imposes disincentives to work that are probably too large. These results are then shown to be qualitatively robust to alternative assumptions like treating unemployment and pension contributions as savings rather than taxes or assuming employer contributions are shifted to workers.

Analyzing Monetary Incentives to Work

In France, previous studies have analyzed work incentives either at the intensive margin (Bourguignon, 1998; Chanchole & Lalanne, 2012; Fourcot & Sicsic, 2017) or at the extensive margin (Legendre *et al.*, 2003; Gurgand & Margolis, 2008). Both margins are analyzed in Laroque and Salanié (1999), in Immervoll *et al.* (2007) who carry out a comparative analysis of monetary incentives to work in 15 EU countries and more recently in Sicsic (2018) who studies the evolution of monetary incentives to work in France over time by household composition.

At the intensive margin, past studies have focused on the redistribution operated by the overall tax and transfer system and on associated aggregate disincentives to work. Early results show that the distribution of effective marginal tax rates across earnings levels follows a U-shape pattern (e.g. Chanchole & Lalanne, 2012). In contrast, the present analysis shows that this distribution follows a tilde-shape which is consistent with more recent evidence (Fourcot & Sicsic, 2017) and can be explained by the move towards make-work-pay policies in France (Sicsic, 2018).

^{1.} Labor supply responses to monetary incentives to work are an important topic of the labor supply literature. Direct evidence is relatively scarce for France. Existing studies (Laroque & Salanié, 2002; Lehmann et al., 2013; Cabannes et al., 2014; Bargain et al., 2014) seem to suggest labor supply elasticities are around 0.05 at the intensive margin and between 0.15 and 0.35 at the extensive margin. These modest elasticities – in comparison to other estimates in the literature (see the reviews of Saez et al., 2012; Meghir & Phillips, 2010) – may nonetheless be attributed to adjustment frictions and the underlying elasticity parameters could well be larger (Chetty, 2012).

At the extensive margin, Legendre *et al.* (2003) and Gurgand and Margolis (2008) estimate the monetary gains to work of unemployed or inactive individuals using individual characteristics such as education or work experience to simulate plausible transitions to work. They conclude that agents have on average very little incentives to take a job (if any), calling for the move towards make-work-pay policies in France that has been recently observed.

Along with Fourcot and Sicsic (2017) and Sicsic (2018), this paper thus offers an update on monetary incentives to work in France after this important policy change. Beyond the valuable information brought by this exercise, the contribution of this paper to the literature is three-fold.

First, the decomposition of marginal and participation tax rates into the underlying tax and transfer instruments clarifies the role of the different instruments as well as their articulation.² In particular, alternating between simulation results for prototypical individuals and simulation results for the representative sample allows to directly connect the schedule of the instruments to monetary incentives to work.

Second, this study is the first to present results at the individual level which allows to better picture and understand heterogeneity in incentives to work. The important sources of heterogeneity between employed childless singles relate to housing statuses (as they determine potential eligibility to housing benefits) and the composition of their incomes (if individuals have other incomes beyond their wage earnings).

Third, to the best of my knowledge, this paper is the first to investigate how monetary incentives to work are affected by: (a) whether individuals receive unemployment benefits when out-of-work, (b) whether the incidence of employer contributions falls on employer or workers, and (c) whether contributory social contributions (unemployment and pension contributions) are treated as taxes or as savings. In practice, the right set of assumptions will likely be individual-specific and lie in between the polar cases analyzed here. Results can thus be interpreted as bounds for *true* effective marginal and participation tax rates.

The restriction to childless singles is admittedly the main limitation of this work given that the schedule of most taxes and transfers tends to vary with household composition. However, this restriction allows to connect the schedule of tax and transfer instruments to work incentives in a transparent way and to understand the heterogeneity in work incentives using graphical representations at the individual level. In addition, the analysis of Sicsic (2018) suggests that the results obtained for childless singles extend, at least qualitatively, to other demographic groups. Hence, one can be confident that the analysis presented here conveys useful information about work incentives in France.

Methodology

Monetary incentives to work are here characterized by the wedge between gross labor earnings and disposable income. The tax and transfer system corresponds to all fiscal instruments that operate between the two.³ First, the payment of social contributions legally divided between employer and employee contributions determine net labor earnings. Net earnings are then subject to income taxes (see details about contributions and income taxes Box 1). Finally, means-tested transfers and in particular housing benefits may be received if remaining income falls below the thresholds determining eligibility to the schemes.

The schedule of housing benefits consists in a fixed allowance at very low-income levels followed by a phasing-out region in which amounts received are decreasing with income. In that respect, it resembles the schedule of a minimum income support program. Housing benefits are nonetheless different in that they can only be claimed by tenants and that amounts received vary by geographical location to reflect local variations in rents (see details about housing benefits and other means-tested transfers in Box 2).

Simulation of Taxes and Transfers Using TAXIPP Microsimulation Tool

Taxes and transfers are here simulated at the individual level using TAXIPP microsimulation model. TAXIPP is the static microsimulation model of the *Institut des Politiques*

^{2.} A related decomposition also appears in Fourcot & Sicsic (2017), and Sicsic (2018).

Note that consumption taxes, local taxes and transfers in kind are here assumed away for the sake of simplicity.

Box 1 – Social Security Contributions and Income Taxes in France

Employer and Employee Contributions

Employer and employee contributions can be decomposed between contributions to contributory schemes (social insurance programs that open rights to future benefits) and contributions to non-contributory schemes (pure taxes). Following the classification of Landais *et al.* (2011) unemployment and pension contributions are treated as contributions to contributory schemes while health contributions, family contributions and all remaining contributions are treated as contributions to non-contributory schemes (see details in Online complement C1). Although standard, this decomposition between insurance and redistribution can be challenged as instruments may in practice respond to both motives.

Health contributions fall under the category of non-contributory components because they hold a substantial redistributive role (Rochet, 1996) but a small share of health contributions is also used to finance sick leaves which is a pure insurance scheme. Similarly, the French pension system responds primarily to an insurance motive but has also been shown to hold a moderate redistributive role (Dubois & Marino, 2015).

The schedule of employer and employee contributions can then be expressed as rates of contributions that apply to gross labor earnings, here defined as nominal posted earnings (*revenus bruts*). Statutory rates of contributions depend on several factors like the hourly wage rate, the status of the employee (executive/non-executive) or the size of the firm. Assuming individuals work in firms of 20 to 249 employees, do not qualify as executives and have an hourly wage rate below the 2011 Social Security Threshold (SST) of 22 euros per hour at which contributions are capped, rates of contributions can then be simply summarized (Table A).

Table A	
Statutory Rates of Contributions (0 to 1 SST Wage Bracket)

Statutory Rates of Contributions (of to 1.551 Wage Diacket)						
Contributions type	Employer rate (in %)	Employee rate (in %)				
Contributory schemes	20.0	13.0				
Unemployment scheme	4.4	2.4				
Pension scheme	15.6	10.6				
Non-contributory schemes	24.5	8.6				
Total	44.5	21.6				

Reading note: Employee rate of contribution to non-contributory schemes is 8.6% of gross labor earnings. Scope: On-executive workers with wage below SST and employed in medium-size firms (20 to 249 employees). Sources: Barèmes IPP, LégiSocial (2011 legislation).

Effective rates of employer contributions are nonetheless substantially lower than these statutory rates for low-wage workers because of reduction schemes aiming at reducing labor cost. The 2011 general reduction scheme (*réduction Fillon*) exonerates employers from remitting certain contributions for wage rates below 1.6 minimum wage rate (see details in Online complement C1). The effective rate of employer contributions falls down to approximately 18% at the minimum wage rate, 30% at 1.2 times the minimum wage rate and 38% at 1.4 times the minimum wage rate. Hence, effective rates of employer contributions are progressive and in practice equal to their statutory rates only for workers with wage rates higher than 1.6 times the minimum wage rate.

Income Tax

The income tax schedule in France is highly complex as it features several reduction and exemption mechanisms.

Assuming labor is the only source of income and ignoring non-standard reductions, a relatively simple formula may be derived for childless singles:

$$T_{IR} = \phi_{IR} \left(0.9 \left[y_{gross} - deductible contributions \right] \right) - D_{IR} \ge 0$$

Indeed, with only labor income, net taxable income is equal to gross labor earnings y_{gross} net of deductible contributions with a standard abatement of 10 percent. Additional earnings like financial income or unemployment benefits would increase net taxable income and thus the final amount of income tax paid. The main step in the computation of the income tax then lies in the application of the function ϕ_{ig} which is the known schedule of marginal tax rates by income tax brackets. In the 2011 legislation, there exists five income tax brackets described below (Table B).

Table B Income Tax Brackets ar	nd Associated Ma	rginal Tax Rates				
Bracket (in euros)	0 - 5,963	5,963 - 11,896	11,896 - 26,420	26,420 - 70,830	+70,830	
Marginal tax rate (in %)	0	5.5	14	30	41	
Reading note: Households with taxable income in the 5,963 - 11,896 bracket face a marginal tax rate of 5.5%. Sources: Barèmes IPP, LégiFiscal (2012 legislation on 2011 earnings).						

Box 1 (contd.)

This generally yields net income tax, i.e. what is effectively paid by the household. An important exception relates to households who benefit from the *décote* system which provides partial or full exemption to households with low income tax. For 2011, the deductible amount $D_{_{I\!R}}$ and the net income tax $T_{_{I\!R}}$ are given by

$$D_{IR} = \left(439 - \frac{1}{2}T_{IR}^{gross}\right) \mathbb{I}\left\{T_{IR}^{gross} \le 2^* \ 439\right\}$$

$$T_{IR} = max \left[T_{IR}^{gross} - D_{IR}; 0 \right]$$

In words, the scheme provides a total exemption for households with a gross income tax below 293 euros and a partial exemption for fiscal households with gross income tax between 293 and 878 euros. Consequently, the *décote* simultaneously reduces the income tax burden of low-income households and increases effective marginal tax rates above statutory ones. The global progressivity of the income tax schedule is thereby non-monotonic.

Publiques. It aims at simulating the entire French tax and benefit system and is composed of several modules simulating different parts of the legislation. Bozio *et al.* (2015b) offer a general presentation of the model with a description of the "social contributions" and "income tax" modules; a description of the "means-tested transfers" module can be found in Bozio *et al.* (2012).

As it is standard in the literature on monetary incentives to work, simulations abstract from

the problems of fraud and take-up. It is thus assumed that individuals who are eligible to a transfer scheme do receive the benefits they are eligible to, while non-eligible individuals do not. The perfect take-up assumption seems acceptable for housing benefits and the make-work pay policy of *prime pour l'emploi* (PPE) for which take-up rates are close to 100%, but may be problematic for the minimum income support scheme called *revenu de solidarité active* (RSA) as its take-up rate is somewhat lower (Lalanne, 2011). Furthermore, as entitlements

Box 2 - Housing benefits and other means-tested transfers

In the 2011 legislation, prime age childless singles are potentially eligible to the following means-tested transfer schemes: a minimum income support scheme named *Revenu de solidarité active* (RSA), an earned income tax credit called *prime pour l'emploi* (PPE) and housing benefits or *allocation logement* (AL).

The scheme of housing benefits in France is very complex and this description focuses on its main features with an emphasis on the relevant aspects for the analysis of work incentives. Childless singles eligibility to housing benefits is determined solely by housing status and earnings. Although the general scheme is divided into several sub-schemes specific to particular housing statuses the analysis focuses on the schedule for recipients who rent a home as they correspond to 85% of housing benefits recipients (*Minima sociaux et prestations sociales*, DREES 2015).

Renting a home thus determines *potential eligibility* to the scheme of housing benefits and individuals become *eligible* to the scheme if their earnings pass the means-test. This is the case if their entitlement to housing benefits, *AL*, turns out to be positive. Formally, *AL* is given by:

$$AL = \min[L; L_0] - T_p \max[y_h - y_0; 0] \ge 0$$

where L is the rent, L_q is a reference value that depends on geographical location to reflect prices of

the local housing market and on household composition. The benefit-rent linkage relates to this first term as a one-euro increase in a rent below L_0 is matched by a one-euro increase in benefits. However, 87% of rents are in practice higher than L_0 (ref) meaning that the amount received does not depend on the rent. Housing benefits are thus akin to a means-tested transfer scheme conditional on geographical location. Means-testing relates to the second term with T_p = 33.23% a parameter (see *Eléments de calcul des aides personnelles au logement*, Ministère du Logement, 2012) that governs the speed at which the amount of transfer decreases when net taxable income y_p increases above the reference income y_0 .

The schedule of other means-tested transfers can be summarized as follows: RSA guarantees a minimum monthly income, which is withdrawn at a 38% rate with net earnings. The earned income tax credit (PPE) kicks in at higher earnings, phases-in slowly at a 7.7% rate and is phased-out at a 19.9% rate. More details are provided in Online complement C2.

Finally, other transfer schemes are either targeted towards households with dependent children (*allocations familiales*) and typically not means-tested or targeted towards very specific categories like the handicapped (*allocation aux adultes handicapés*) or the elderly (*minimum vieillesse*).

to PPE strongly depend on amounts of RSA received, they are treated as a unified transfer scheme throughout the analysis⁴ although PPE is a negative income tax and thus means-tested using another earnings concept.

Some simplifying assumptions are also made regarding the timing of taxes and transfers. In particular, although housing benefits are means-tested against two-year-lagged income, they are here means-tested against current income.5 This assumption is suitable when earnings are relatively smooth over the years and, if there are sharp changes, it reflects the legislation which states that a contemporaneous evaluation of resources should in that case be made. Also, with annual data, it is not possible to follow individuals on a monthly basis. This implies that amounts of transfers are here simulated on the basis of average monthly earnings which may lead to averaging errors due to the non-linearity of the schemes – for instance, RSA is evaluated on rolling 3-months windows.

Another important source of non-linearity in the schedule of housing benefits is a peculiar rounding rule that imposes household income to be rounded up to the nearest hundred. To give a concrete example, a household with an income of 1,002 euros and another with an income of 1,098 euros will be imputed with the exact same value $y_h = 1100$ euros in the computation (Box 2). To focus on structural incentives to work and to ease the interpretation of the results, this rounding rule is here assumed away.

Since this paper focuses on monetary incentives to work, housing characteristics (housing status, rent, geographical location) are taken as given. Therefore, the incidence of a change in the amount of housing benefits stemming from a variation in labor earnings is supposed to be borne by the household and not by the landlord. This last assumption may seem contradictory with Fack (2005, 2006) who shows that housing benefits are captured by landlords through rents increases. However, this inflationary effect operates through market prices which are arguably orthogonal to the labor earnings of a specific individual. In other words, it is assumed that changes in housing benefits induced by changes in labor earnings will not be matched by a subsequent change in the rent and are thus effectively borne by households.

A Representative Sample of Employed Childless Singles

This paper uses ERFS (enquête Revenus Fiscaux et Sociaux, Insee) data which is a match between the Labor Force Survey and administrative income tax records. It provides all the variables required for the simulation of taxes and transfers, in particular detailed information on income (wage labor income, non-wage labor income, replacement income, capital and financial income) and hours worked on a yearly basis. ERFS data does not include rents but since 87% of rents are above the reference threshold of the housing benefits scheme (Bozio et al., 2015a), they do not affect the amount of housing benefits received in practice and are thus not necessary for this analysis. Another potential concern with the use of ERFS data to study the bottom of the income distribution is that very low-income households have been shown to be underrepresented in the survey (Lalanne, 2011). This typically poses a problem for recovering the budget devoted to means-tested transfers at an aggregate level, as underrepresentation leads to the underestimation of the number of recipients. However, it should not affect the analysis of monetary incentives to work at the individual level.

Simulations are based upon the 2011 wave of the ERFS – the latest version available when starting this project – and taxes and transfers are accordingly simulated using the 2011 tax code. Given the relative stability of the income distribution in France, the use of more recent data should not particularly affect the results. In contrast, the French tax code tends to be much more volatile and simulation results presented here accurately capture monetary incentives to work for the 2009-2015 period while recent reforms suggest they can be seen as illustrative for posterior years.⁶

^{4.} This convention is also in-line with the recent 2016 reform that introduced a unique make-work-pay transfer scheme named prime d'activité as a replacement for PPE and the make-work-pay component of RSA (see Appendix).

^{5.} The notion of earnings used for housing benefits means-testing corresponds to net taxable income in year N-2 except in a handful of cases described in the Appendix of Bozio et al. (2015a). In particular, job loss induces earnings means-testing in year N and grants individuals a 30% abatement on unemployment benefits. Similarly, although the income tax is in practice paid with a one-year lag, it is here assumed to be paid during the current year.

^{6.} On the transfer side, an important reform of means-tested transfers occurred in 2009 with the introduction of RSA schemes. Also, in 2016 the make-work-pay part of RSA called RSA activité and PPE were merged into a unified scheme called prime d'activité while maintaining the minimum income support part of RSA called RSA socle. On the tax side, reforms of the income tax schedule were implemented in 2012 (addition of a bracket at the top) and 2015 (deletion of the first bracket and changes in entry thresholds). In addition, employer social contributions were further reduced for low-wage workers in 2013 with the introduction of CICE (-4%) and its expansion in 2014 (-6%) and 2017 (-7%).

Sampling weights in ERFS data are defined at the household level and used throughout the analysis. The initial sample comprises 56,486 observations representative of the 28 million households living in France in 2011. The analysis focuses on a homogeneous demographic group: childless singles aged 25-54. This restriction simplifies the analysis and allows connecting the schedule of tax and transfer instruments to monetary incentives to work as well as understanding the sources of heterogeneity in work incentives that are unrelated to household composition. The sample is further restricted to employed individuals, defined using two conditions on labor earnings:

- Annual gross labor earnings exceed 1,365 euros (this corresponding to one-month full-time minimum wage earnings);

– Annual gross labor earnings times the replacement rate of 60% exceed the amount of gross unemployment benefits received.⁷

In other words, individuals are considered employed if they have a minimum amount of labor earnings during the year and, for those receiving unemployment benefits, if they spent at least more time employed than unemployed. With this definition, the employment rate among childless singles aged 25-54 in France is 81.1%.8 Finally, public sector employees (public sector variable) and self-employed workers (defined by non-wage labor income higher than wage labor income), two categories subject to specific social contributions schemes, are further excluded from the sample. In addition, compared to private sector employees, the self-employed have stronger work incentives and are less protected (no unemployment insurance and potentially large income variations) while public sector employees tend to have weaker

work incentives and to be more protected (job security for civil servants and public sector pay scales). The final sample of analysis comprises 3,745 observations representing the 2.2 million childless single households in France.

While the study of monetary incentives to work at the intensive margin (increasing work intensity when in-work) requires using a sample of employed individuals, the analysis of work incentives at the extensive margin (joining the workforce when out-of-work) involves making a choice: one can either use data on employed individuals and simulate their counterfactual situation if they were not employed as in Immervoll et al. (2007) or Sicsic (2018), or use data on individuals who are not employed and simulate their counterfactual situation if they were employed as in Gurgand and Margolis (2008). We follow the first route in order to characterize incentives to work at both margins for the same sample of individuals.

Descriptive statistics (Table 1) show that labor is the major source of earnings for all individuals in the sample.⁹ Nonetheless, some individuals do receive additional incomes which will turn out to be a main source of heterogeneity in monetary incentives to work. The other main source of heterogeneity relates to housing status determining potential eligibility to housing benefits. More than 80% of individuals are potentially eligible to housing benefits in the first income quartile and more than 70% in the second. The

Means	Sample	Q1	Q2	Q3	Q4
Gross labor earnings (euros / year)	28,173	11,846	21,252	27,776	51,842
Hours worked (hours / year)	1,855	1,516	1,877	1,902	2,130
Unemployment benefits (euros / year)	374	883	228	197	173
Financial income (euros / year)	1,298	413	682	857	3,219
Gender (% of men)	61.7	52.2	60.9	65.9	68.1
Age (in years)	38.9	38.7	38.5	38.9	39.7
Potential eligibility to housing benefits (%)	65.8	81.4	72.3	58.1	50.8

Table 1 Descriptive Statistics for the Whole Sample and by Quartiles of Labor Earnings

Reading note: On average, individuals in the first income quartile work 1,516 hours per year. Coverage: Sample of employed prime age childless singles.

Sources: Insee, *ERFS* 2011; author's computation.

^{7.} This is a proxy for the rules of unemployment insurance in France. Precise simulation of unemployment benefits would require information on past labor earnings which is not available in the data.

^{8.} The 2011 employment rate among all individuals aged 25-54 in France is 81.4% (Insee).

^{9.} The distribution of annual gross labor earnings in the sample of analysis is reported in Appendix.

schedule of housing benefits thus affects the work incentives of a large fraction of low-income individuals and is thereby important to take into account when analyzing incentives to work.

Definition and Estimation of Effective Marginal and Participation Tax Rates

Monetary incentives to work are captured in the wedge between labor income y and disposable income c. Given the relationship c = y - T(y), the characterization of incentives to work thus falls down to a characterization of taxes and transfers T(y). In order to reflect the dichotomy between labor supply decisions at the intensive margin and at the extensive margin (Heckman, 1993) this characterization is made through the estimation of marginal and participation tax rates.

This estimation requires precise definitions of income *y* and the components of the tax function T(y). As a benchmark, let's first consider a baseline scenario in which the real incidence of taxes coincides with their legal incidence. In that case, employer contributions are effectively paid by employers meaning that workers' labor earnings correspond to gross labor earnings (*salaires bruts*) and not to labor cost. The tax function then corresponds to employee contributions $T_W(y)$ and income taxes $T_{IR}(y)$ net of transfer benefits B(y):

$$T(y) = T_W(y) + T_{IR}(y) - B(y)$$

In this baseline scenario, let's also consider pension and unemployment contributions as taxes. This is the relevant assumption for agents who do not internalize the future expected benefits derived from pension and unemployment contributions in their labor supply decisions. It also provides what can be interpreted as an upper bound for disincentives to work.¹⁰

Incentives to work at the intensive margin are incentives to increase work intensity (e.g. hours worked) when employed. The standard measure associated with labor supply incentives at the intensive margin is the marginal tax rate defined as dT(y)/dy. Following a marginal increase in labor earnings, the marginal tax rate measures the fraction of additional earnings that will be paid in taxes. In other words, the marginal tax rate measures in labor earnings is taxed away.

Its empirical counterpart, the effective marginal tax rate (EMTR), is computed in TAXIPP using a 2% increase¹¹ in gross labor earnings y, simulating T(y) for the new earnings level and computing the effective differences $\Delta T(y)$ and Δy . To be consistent with the literature, this increase in labor earnings is associated to an increase in hours worked rather than to an increase in the hourly wage rate.¹² An exception is overtime hours that, following the legislation, are assumed to be paid at an hourly wage rate 25% higher than standard hours.¹³ Also, effective marginal tax rates are decomposed by tax and transfer instruments for the purpose of the analysis:

$$\operatorname{EMTR}(y) = \frac{\Delta T(y)}{\Delta y}$$
$$= \frac{\Delta T_{W}(y)}{\Delta y} + \frac{\Delta T_{IR}(y)}{\Delta y} + \frac{\Delta \left[-B(y)\right]}{\Delta y}$$

 $EMTR(y) = EMTR_{W}(y) + EMTR_{IR}(y) + EMTR_{B}(y)$

Incentives to work at the extensive margin are incentives to join the workforce when not employed. The standard measure associated with labor supply incentives at the extensive margin is the participation tax rate defined as [T(y) - T(0)]/y. Upon taking a job, individuals jump from zero labor earnings to labor earnings y > 0 and the effective participation tax rate measures the change in taxes net of transfers as a fraction of y. Importantly, this measure captures the resulting reduction in means-tested transfers which acts as a "participation tax" and reduces monetary incentives to participate in the workforce.

The computation of effective participation tax rates (EPTR) thus requires information on earnings and taxes net of transfers when employed (respectively y and T(y)) and transfers received when not employed T(0). Earnings y are taken from the data and taxes and transfers T(y) are simulated with TAXIPP microsimulation model. Last, T(0) is imputed as the amount of transfers received had individuals been out-of-work.

^{10.} These two assumptions and their impact on monetary incentives to work are analysed in the next Section.

Increases of earnings by 1% to 5% are common in the literature. Different values do not affect the results except at the entry and exit thresholds of tax and transfer schemes, where smaller increases in earnings tend to magnify the discontinuities associated to these thresholds (if any).
 This choice does not affect the results in the baseline scenario as it only impacts rates of employer contributions through the indexation of reduction schemes on hourly wage rates.

^{13. 25%} is the legal overtime rate unless a specific agreement is in place in the firm. Since this information is not in the data, this rate is applied to all individuals.

The imputation procedure for T(0) = -B(0) differs depending on whether individuals are assumed to receive unemployment benefits when out-of-work. Under the assumption that individuals do not receive unemployment benefits, transfers are simulated setting labor earnings to zero and holding all other individual characteristics constant. A similar imputation procedure is used by Laroque and Salanié (1999) and Sicsic (2018), who interpret their results as reflecting long-term incentives to join the workforce in the sense that individuals may receive unemployment benefits only for a limited period of time. Under the assumption that individuals receive unemployment benefits, the imputation is done in three steps: (1) assign gross unemployment benefits equal to 60% of observed annual gross labor earnings;14 (2) set labor earnings to zero; (3) simulate transfers. A similar procedure is used by Immervoll et al. (2007) and results can be interpreted as reflecting short-term incentives to join the workforce upon losing a job.

The computation of effective participation tax rates is then straightforward; for the purpose of this analysis they are decomposed by tax and transfer instruments:

EPTR
$$(y) = \frac{T(y) - T(0)}{y}$$

= $\frac{T_{W}(y)}{y} + \frac{T_{IR}(y)}{y} + \frac{-[B(y) - B(0)]}{y}$

 $EPTR(y) = EPTR_{W}(y) + EPTR_{IR}(y) + EPTR_{B}(y)$

Incentives to Work in the Baseline Scenario

This section characterizes monetary incentives to work in the baseline scenario with a focus on the role played by housing benefits. The budget set of childless singles is first depicted to get a sense of the importance of housing benefits in low-income workers' budget. Simulation results for effective marginal and participation tax rates are then presented, both for fictitious prototypical individuals (assumed to derive earnings only from labor) and for individuals from the representative sample. Alternating between simulation results for fictitious prototypical individuals and for individuals from the representative sample allows to directly connect the schedule of the instruments to monetary incentives to work and helps understand the heterogeneity in incentives to work.

The Importance of Housing Benefits in the Budget of Low-income Workers

The budget set of childless single workers (Figure I) reveals that housing benefits can be an important fraction of the budget of low-income workers. For instance, individuals working a half of a full-time job paid at the minimum wage rate earn 6,432 euros net per year, receive 3,548 euros in RSA and an additional 2,515 euros in housing benefits. Housing benefits thus account for 20% of total disposable income which is 12,495 euros per year. In contrast, an individual not eligible to housing benefits would only benefit from RSA and have a total disposable income of 9,980 euros.

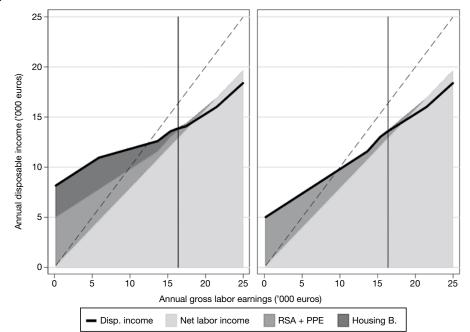
Housing benefits have thus two effects on incentives to work: first, means-testing induces disincentives to work for individuals eligible to the scheme. Second, housing benefits create substantial heterogeneity in incentives to work between individuals who are eligible to the scheme and those who are not.

Phasing-Out and Incentives at the Intensive Margin

At the intensive margin, housing benefits entail strong disincentives to work in the phasing-out region of the scheme. For a childless single who receives housing benefits, the phasing-out is such that a 1-euro increase in gross labor earnings reduces the amount of housing benefits by 27 cents on average (left panel of Figure II). Combined with the reduction in the amount of RSA-PPE received (30 cents) and the payment of employee contributions (21 cents), individuals thus face an extreme marginal tax rate of 78%. In other words, a 1-euro increase in labor earnings translates in a 22-cents increase in disposable income. In contrast, individuals not eligible to the scheme face a marginal tax rate of 51% in the same income region, meaning that a 1-euro increase in labor earnings yields a 49-cents increase in disposable income (right panel of Figure II).

^{14.} This is a proxy for the rules of unemployment insurance in France. Precise simulation of unemployment benefits requires detailed information on past labor earnings which is not in the data.

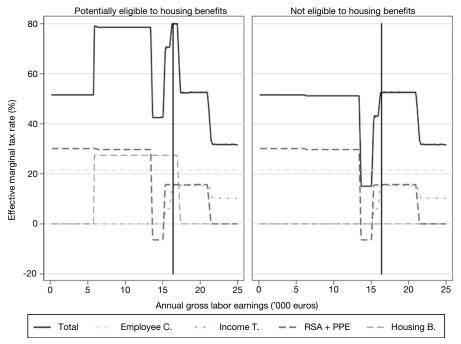
Figure I Budget Sets of Low-income Workers



Note: Individuals in the left and right panel only differ with respect to their eligibility to housing benefits (schedule of zone II). Baseline treats social insurance contributions as taxes and assumes employer contributions are paid by employers. The vertical line signals a full-time job paid at the minimum wage rate.

Scope: Simulations for fictitious childless singles assuming labor is the only source of earnings under the 2011 legislation. Sources: TAXIPP microsimulation model; author's computation.

Figure II Housing Benefits and Marginal Tax Rates



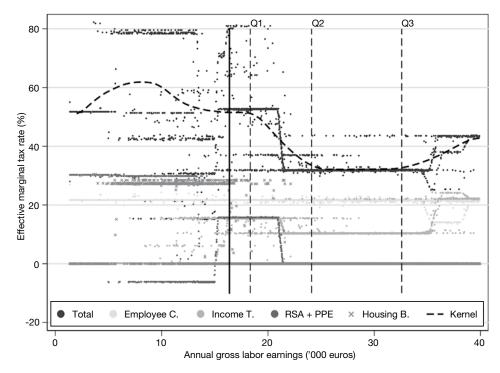
Note: Individuals in the left and right panel only differ with respect to their eligibility to housing benefits (schedule of zone II). Baseline treats social insurance contributions as taxes and assumes employer contributions are paid by employers. The vertical line signals a full-time job paid at the minimum wage rate.

Scope: Simulations for fictitious childless singles assuming labor is the only source of earnings under the 2011 legislation. Sources: TAXIPP microsimulation model; author's computation. Such extreme values for marginal tax rates are to be contrasted with the average estimated marginal tax rate of 43% for individuals in the representative sample. Simulation results on the representative sample show that such rates correspond to the top of the distribution of effective marginal tax rates across earnings levels (Figure III). Indeed, the distribution of local average marginal tax rates with earnings (dashed curve) follows a distinctive tilde-shape pattern with the top of the tilde located in the phasing-out region of housing benefits.¹⁵ This finding is consistent with those of Sicsic (2018) and reflects the recent policy move towards make-work-pay policies (RSA activité, PPE).16

The mechanisms behind this tilde-shape pattern are transparent from the decomposition. Marginal tax rates rise at the bottom of the earnings distribution due to the phasing-out of transfers. Marginal tax rates are then minimal around median earnings and increase with earnings afterwards as individuals fall into higher income tax brackets. Employee contributions have a uniform impact across the board, which reflects their flat schedule among the general working population.

Nonetheless, this tilde-shape pattern masks the important heterogeneity between individuals with similar labor earnings. Eligibility to the scheme of housing benefits is an important driver of heterogeneity together with differences in additional incomes (e.g. unemployment benefits, financial income). For instance, individuals with the lowest marginal tax rates in the first income quartile are those who are not eligible to housing benefits and not entitled to RSA after accounting for earnings other than labor.

Figure III Distribution of Effective Marginal Tax Rates



Note: Baseline treats social insurance contributions as taxes and assumes employer contributions are paid by employers. The dashed curve represents a kernel estimation of the local average EMTR (bandwidth of 4,000 euros). The vertical line signals a full-time job paid at the minimum wage rate and the dashed vertical lines indicate the quartiles of the earnings distribution.

Coverage: Employed prime age childless singles.

Sources: Insee, ERFS 2011; TAXIPP microsimulation model; author's computation.

^{15.} The three different marginal tax rates associated with the phasing-out of housing benefits corresponds to the geographical zoning into three zones and their specific schedules.

^{16.} Immervoll et al. (2007) also show that the distribution of marginal tax rates is tilde-shaped, although their study precedes the introduction of make-work-pay policies. However, their sample includes households with different demographic characteristics and they explain that the hump in their quasi-U-shape pattern is driven by high marginal tax rates imposed on secondary earners in couples.

Means-Testing and Incentives at the Extensive Margin

First assume individuals do not to receive unemployment benefits when out-of-work as in Laroque and Salanié (1999) and Sicsic (2018). This may be interpreted as a long-term perspective in the sense that it captures the incentives to work of long-term unemployed whose rights to unemployment insurance have expired. It also captures the work incentives of individuals who are not entitled to unemployment benefits (e.g. first entry on the labor market, job resignation).

Upon taking a job, housing benefits means-testing implies that the amount of housing benefits received decreases for individuals eligible to the scheme. The loss of housing benefits thus acts as a participation tax that can go up to 18% upon taking a full-time job paid at the minimum wage rate (left panel of Figure IV). The total participation tax then corresponds to 64% of labor earnings for individuals eligible to the scheme whereas it is 46% for non-eligible individuals (right panel of Figure IV). Moreover, eligibility to housing benefits generates a profile of participation tax rates that is increasing with earnings given the extreme marginal tax rates imposed in the phasing-out region of housing benefits.

Given the large fraction of individuals potentially eligible to the scheme of housing benefits, the distribution of participation tax rates estimated using the representative sample more closely resembles that of eligible individuals (Figure V). The average participation tax rate is 51% and local average participation tax rates are broadly increasing with earnings at low income levels and decreasing with earnings at higher income levels. The initial increase in participation tax rates reflects the increase in amounts of transfers lost through meanstesting upon taking a job. The subsequent decrease reflects the diminishing importance of this loss as labor earnings on-the-job grow.

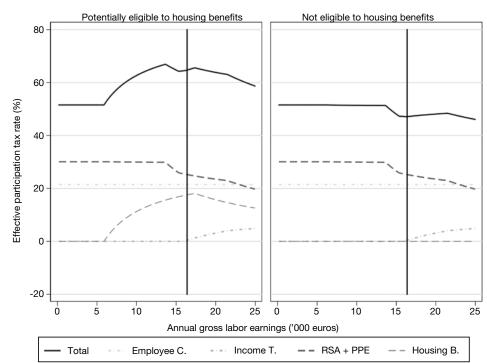


Figure IV Housing Benefits and Participation Tax Rates (No Unemployment Benefits)

Notes: Individuals in the left and right panel only differ with respect to their eligibility to housing benefits (schedule of zone II). Baseline treats social insurance contributions as taxes and assumes employer contributions are paid by employers. It is here assumed that individuals do not receive unemployment benefits when out-of-work. The vertical line signals a full-time job paid at the minimum wage rate. Scope: Simulations for fictitious childless singles assuming labor is the only source of earnings under the 2011 legislation. Sources: TAXIPP microsimulation model; author's computation.

These results are considerably impacted when unemployment benefits are included into the picture. Assuming individuals receive unemployment benefits when out-of-work as in Immervoll et al. (2007) may be interpreted as a short-term perspective in the sense that unemployment benefits entitlements are limited in time. Unemployment benefits have two effects on disposable income when not employed. First, disposable income increases as unemployment benefits are a new source of income. Second, as unemployment benefits enter means-testing, entitlements to meanstested transfers decrease: unemployment benefits and means-tested transfers are substitutes. Hence, beyond the overall increase in disposable income, the composition of disposable income when out-of-work drastically changes.

The impact of housing benefits on incentives to take up a job is thus strongly mitigated by the presence of unemployment benefits (Figure VI). Since unemployment benefits increase with labor earnings, higher labor earnings on-the-job imply higher unemployment benefits when not employed and by the substitution effect, lower entitlements to housing benefits. As a result, an increase in labor earnings decreases entitlements to housing benefits when in-work but also decreases entitlements to housing benefits when out-of-work. Hence, the participation tax associated to the loss of housing benefits is reduced and features an 8% plateau.

Furthermore, the presence of unemployment benefits overturns the impact of other means-tested transfers on incentives to join the workforce. Absent unemployment benefits, RSA and PPE scheme-specific participation tax is large and positive (around 30% at low earnings) as means-tested transfers decrease upon taking a job. With unemployment benefits, the RSA and PPE scheme-specific participation tax is still large but negative (around -27% at low earnings). Indeed, in-work

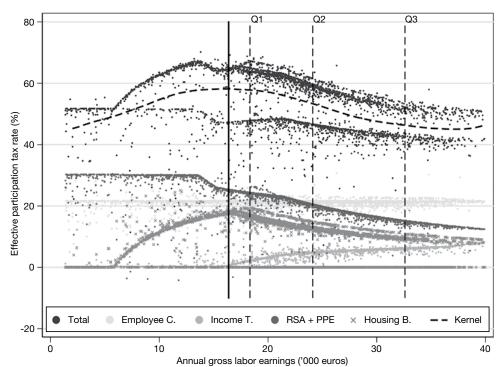


Figure V Distribution of Effective Participation Tax Tates (No Unemployment Benefits)

Notes: Baseline treats social insurance contributions as taxes and assumes employer contributions are paid by employers. It is here assumed that individuals do not receive unemployment benefits when out-of-work. The dashed curve represents a kernel estimation of the local average EPTR (bandwidth of 4,000 euros). The vertical line signals a full-time job paid at the minimum wage rate and the dashed vertical lines indicate the quartiles of the earnings distribution.

Coverage: Employed prime age childless singles. Sources: Insee, ERFS 2011; TAXIPP microsimulation model; author's computation.

transfers (*RSA activité* and *prime pour l'emploi*) are now larger than out-of-work transfers (*RSA socle*): make-work-pay schemes literally make work pay.

Looking at the distribution of participation tax rates (Figure VII), unemployment benefits increase the average participation tax rate in the sample to 77%. This increase in participation tax rates reflects the increase in the total amount of transfers received when not employed. Also, the loss of unemployment benefits upon taking a job becomes the main driver of participation tax rates.

The distribution of effective participation tax rates is now strongly increasing with earnings at low income levels and moderately increasing at higher income levels. This strong increase at the bottom is jointly driven by make-work-pay schemes and by the substitution effect between unemployment benefits and means-tested transfers. Indeed, as noted before, these two features imply that the amount of means-tested transfers received when in-work is higher than the amount received when out-of-work. This translates into negative participation tax rates attached to the RSA and PPE schemes. The strong increase in participation tax rates as earnings grow can thus be explained by the phasing-out of make-work-pay subsidies on-the-job. In contrast, moderately increasing participation tax rates at higher income levels are related to the increase of the income tax with earnings.

These findings are difficult to compare with previous findings in the literature, as Legendre *et al.* (2003) and Gurgand and Margolis (2008) do not report the distribution of participation tax rates with respect to earnings on-the-job. The only point of comparison is Immervoll *et al.* (2007), who obtain a distribution of participation tax rates that is increasing with earnings at low income levels and decreasing with earnings at higher income levels. They find an average participation tax rate close to 70%. However, in

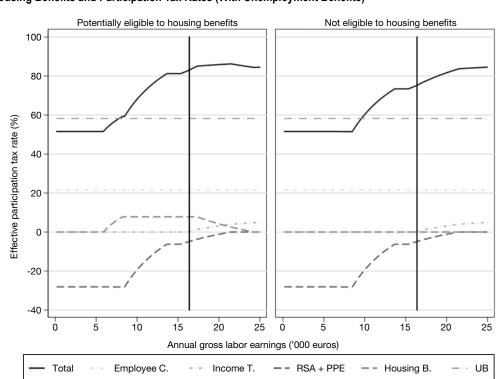


Figure VI Housing Benefits and Participation Tax Rates (With Unemployment Benefits)

Notes: Individuals in the left and right panel only differ with respect to their eligibility to housing benefits (schedule of zone II). Baseline treats social insurance contributions as taxes and assumes employer contributions are paid by employers. It is here assumed that individuals receive unemployment benefits when out-of-work. The vertical line signals a full-time job paid at the minimum wage rate. Scope: Simulations for fictitious childless singles assuming labor is the only source of earnings under the 2011 legislation. Sources: TAXIPP microsimulation model; author's computation.

addition to childless singles, their sample features couples and families with children whose tax treatments are different. Moreover, they randomly assign unemployment benefits to a part of their sample to reflect the fact that some but not all individuals receive unemployment benefits when out-of-work. Last, their study precedes the introduction of make-work-pay policies in France. Therefore, comparisons between the two sets of results involve too many differences to be truly informative.

Incentives to Work in Alternative Scenarios

The previous characterization of monetary incentives to work has been obtained under the assumption that employer contributions are effectively paid by employers and that contributions to social insurance programs (pension and unemployment contributions) are taxes although they primarily relate to an insurance motive and might thus be interpreted as savings rather than taxes.

Incidence of Employer Contributions on Workers

Assuming that the real incidence of taxes coincide with their legal incidence is a standard simplifying assumption commonly used as a benchmark (e.g. OECD data on labor wedges). Also, recent studies show that the legal incidence of taxes may distort their real incidence towards the legal taxpayer (Chetty *et al.*, 2009). However, there is in principle no reason for the real and the legal incidence of taxes to coincide. A standard result in economic theory due to Harberger (1964) states that the burden of a tax in a market is shared by both demand and supply sides in relative proportions that depend on the ratio of the respective elasticities. The more elastic one side of the market is, the more the

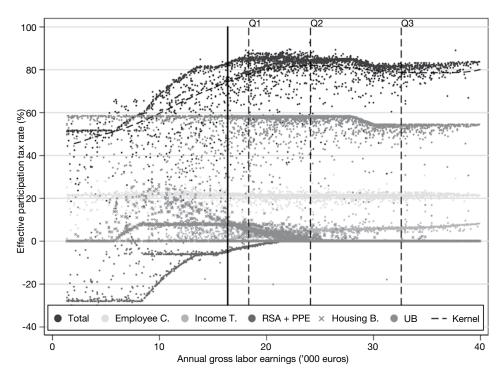


Figure VII Distribution of Effective Participation Tax Rates (With Unemployment Benefits)

Notes: Baseline treats social insurance contributions as taxes and assumes employer contributions are paid by employers. It is here assumed that individuals receive unemployment benefits when out-of-work. The dashed curve represents a kernel estimation of the local average EPTR (bandwidth of 4,000 euros). The vertical line signals a full-time job paid at the minimum wage rate and the dashed vertical lines indicate the quartiles of the earnings distribution.

Coverage: Employed prime age childless singles.

Sources: Insee, ERFS 2011; TAXIPP microsimulation model; author's computation.

burden of the tax is shifted to the other side of the market.

Empirical evidence on this question is mixed. In the short-run, Lehmann et al. (2013) show that wages are rigid and that an increase in employer contributions is borne by employers. Studying the medium-run effects of social security contributions reforms in France, Bozio et al. (2017) identify a partial shifting of employer contributions to workers.17 However, an important rigidity in the wage adjustment process in France is the existence of a relatively high minimum wage.¹⁸ This rigidity strongly suggests that, at least for wages close to the minimum wage, the real incidence of employer contributions should fall on employers. Hence, the baseline scenario seems relevant for the study of monetary incentives to work of low-wage individuals who are the major recipients of housing benefits. It is nonetheless interesting to understand how monetary incentives to work are affected when employer contributions are assumed shifted to workers. In this scenario, workers labor earnings y correspond to the labor cost, and taxes and transfers T(y) include employer contributions.

At the intensive margin, the average marginal tax rate increases to 57%, against 43% in the baseline. Moreover, assuming employer contributions are shifted to workers compresses the distribution of effective marginal tax rates towards a flat rate compared to the baseline (Figure VIII). Indeed, the progressivity of employer contributions stemming from the reduction schemes for low wage workers significantly increases marginal tax rates in the middle of the earnings distribution and at the top. As a result, the tilde-shape pattern of marginal tax rates is largely attenuated. The impact of housing benefits on incentives to work is qualitatively the same as in the baseline. The only

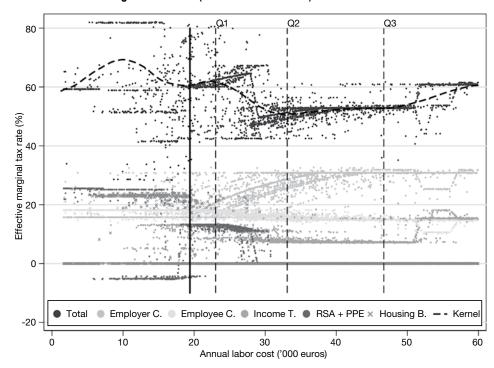


Figure VIII Distribution of Effective Marginal Tax Rates (Incidence on Workers)

Note: Social insurance contributions are treated as taxes while the incidence of employer contributions is assumed to fall on workers. The dashed curve represents a kernel estimation of the local average EMTR (bandwidth of 4,000 euros). The vertical line signals a full-time job paid at the minimum wage rate and the dashed vertical lines indicate the quartiles of the earnings distribution.

Coverage: Employed prime age childless singles.

Sources: Insee, ERFS 2011; TAXIPP microsimulation model; author's computation.

Bozio et al. (2017) provide micro-evidence for employed individuals. They argue that adjustments could also take place through other channels (e.g. job creation and destruction) that need to be further investigated.
 This rigidity seems well understood by policy makers as reductions in employer contributions tend to be concentrated around the minimum wage in order to obtain the largest effect on employment (Lehmann & L'Horty, 2014).

change is that housing benefits are phased-out at a 23% rate with respect to labor cost, against a 27% rate with respect to gross labor earnings.

At the extensive margin, assuming employer contributions are shifted to workers increases participation tax rates. Absent unemployment benefits, the average participation tax rates is 63%, against 51% in the baseline scenario. With unemployment benefits, the average participation tax rates climbs to 82%, against 77% in the baseline. However, this increase in participation tax rates does not strongly affect the pattern of participation tax rates (Figure not reported). The intuitive reason is that the incidence of employer contributions does not affect the amount of welfare benefits received when out-of-work but only taxes paid when in-work. Accordingly, housing benefits have once again the same qualitative impact on incentives to work while their magnitude is slightly reduced.

Social Insurance Contributions as Savings

Contributions to social insurance programs (pension and unemployment contributions), have so far been treated as taxes. However, these contributions are not pure taxes as they respond to an insurance motive: they aim at transferring resources from an individual currently employed to the same individual later in life, when either unemployed or retired. Hence, the tax hypothesis made in the baseline scenario corresponds to individuals who do not internalize future expected benefits in their labor supply decisions (e.g. myopic agents) or who anticipate that they will not benefit from unemployment insurance (e.g. no unemployment spell) or the pension system (e.g. early death).

In all generality, disincentives to work associated with social insurance contributions are equal to contributions costs net of future expected benefits (Disney et al., 2004). As future expected benefits have so far been assumed away (tax hypothesis), previous marginal and participation tax rates can be interpreted as upper bounds for their true values. Evaluating future expected benefits stemming from pension or unemployment contributions is beyond the scope of this work. Instead, it is assumed here that future expected benefits are exactly equal to contributions paid (savings hypothesis). This corresponds to the case in which social insurance programs are perfectly fair actuarially and operate no redistribution across individuals. In other words, pension and

unemployment contributions are akin to savings and perceived as such.¹⁹

While little evidence seems available on the redistribution operated by the French unemployment insurance system, a small literature characterizes the redistribution operated by the French pension system distinguishing between (1) redistribution within generations and (2) redistribution across generations. Paul-Delvaux (2015) shows that, within generations, the rate of return on general pension contributions is slightly decreasing with earnings. In other words, future expected benefits are relatively subsidized at low earnings levels and taxed at high earnings levels. Dubois and Marino (2015) characterize redistribution across generations and show that the rate of return on pension contributions is steadily decreasing across cohorts. This finding reflects the impact of global ageing on a pay-as-you-go pension system and tends to suggest that current workers are taxed to finance the pensions of retired individuals. Building on these contributions, redistribution within and across generations work in opposite directions for low-wage workers, meaning that marginal and participation tax rates derived under the savings hypothesis could be close to their true values. In contrast, they work in the same directions for high-wage workers, suggesting that marginal and participation tax rates derived under the savings hypothesis should rather be interpreted as lower bounds.

Under the savings hypothesis, assuming employer contributions are paid by employers, workers labor earnings *y* is gross labor earnings, while taxes and transfers T(y) no longer include worker pension and unemployment contributions.²⁰ At the intensive margin, treating pension and unemployment as savings decreases marginal tax rates by 13 percentage points across the board (Figure IX). The average marginal tax rate is then equal to 30%, against 43% in the baseline scenario. The impact of housing benefits on monetary incentives to work is not affected.

At the extensive margin, the impact of the savings hypothesis greatly depends on the treatment

^{19.} Using survey data, Dominitz et al. (2003) elicit Americans' expected returns on their pension contributions and show there exists substantial heterogeneity in perceptions. At the two extremes, some individuals do not expect the pension system to survive, while others tend to overestimate their future benefits.

^{20.} Assuming simultaneously that employer contributions are paid by workers and treating pension and unemployment contributions as savings generates hard-to-interpret results because reductions in employer contributions reduce the rates of contributions without reducing future benefits. Hence, future expected benefits are larger than contributions, which is not consistent with the savings hypothesis.

of unemployment benefits when out-of-work. When non-employed individuals do not receive unemployment benefits, effective participation tax rates decrease following the increase of disposable income on-the-job (Figure not reported). The average participation tax rate is then equal to 39%, against 51% in the baseline scenario. In contrast, when non-employed individuals receive unemployment benefits (short-term perspective), the savings hypothesis has a more drastic impact on participation tax rates. Indeed, if unemployment contributions are treated as savings, unemployment benefits are the depletion of past savings. Hence, unemployment benefits should not be treated as transfers. As a result, effective participation tax rates fall to unrealistically low values that contradict casual empiricism on monetary incentives to work in France (Figure X).

Such extremely low values do not only reflect the importance of unemployment benefits in disposable income when out-of-work, they once again highlight the important substitution effect between unemployment benefits and means-tested transfers. Indeed, effective participation tax rates are close to zero because means-tested transfers when out-of-work are substantially reduced in the presence of unemployment benefits. Hence, excluding unemployment benefits from means-tested transfers is misleading for the analysis of incentives to take up a job because unemployment benefits precisely replace means-tested transfers. In other words, the savings hypothesis seems of limited relevance for the analysis of incentives to work, at least in the kind of static framework considered here.

* *

This paper has analyzed monetary incentives to work in France and proposed a decomposition in terms of the underlying tax and transfer instruments. The decomposition reveals the interactions at play and allows to identify the impact of

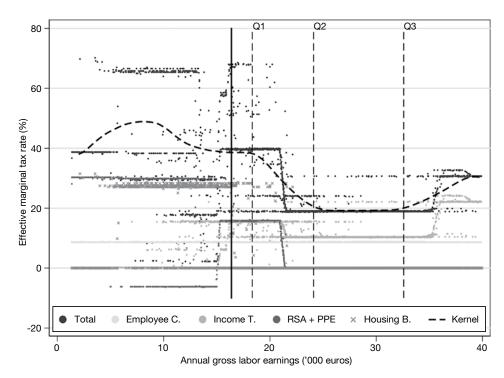


Figure IX Distribution of Effective Marginal Tax Rates (Savings Hypothesis)

Notes: The savings hypothesis treats social insurance contributions as savings while employer contributions are again assumed paid by employers. The dashed curve represents a kernel estimation of the local average EMTR (bandwidth of 4,000 euros). The vertical line signals a full-time job paid at the minimum wage rate and the dashed vertical lines indicate the quartiles of the earnings distribution. Coverage: Employed prime age childless singles.

Sources: Insee, ERFS 2011; TAXIPP microsimulation model; author's computation.

each instrument on incentives to work. Housing benefits entail substantial adverse effects on monetary incentives to work for individuals in the first income quartile.

At the intensive margin, a 1-euro increase in gross labor earnings reduces housing benefits by 27 cents on average in the phasing-out region of the scheme. The phasing-out of other means-tested transfers (30 cents) together with the payment of social contributions (21 cents) imply that in this region a 1 euro increase in gross labor earnings only translates into a 22 cents increase in disposable income. This corresponds to a marginal tax rate of nearly 80% and to the top of the tilde-shape distribution of marginal tax rates across earnings. In comparison, the average marginal tax rate is 43%.

At the extensive margin, monetary incentives to work greatly depend on whether individuals receive unemployment benefits when out-of-work. In the absence of unemployment benefits, the amount of housing benefits lost upon getting a job may represent up to 18 % of gross labor earnings on the job. Associated with the loss of other means-tested transfers (30%) and the payment of social contributions on the job (21%), transfers loss and tax payments may represent up to 70% of gross labor earnings. These top participation tax rates are attained upon taking a full-time job paid at the minimum wage rate and can be compared to the average participation tax rate of 51%. With unemployment benefits, the average participation tax rate shoots up to 77% as monetary gains to join the workforce decrease. However, because unemployment benefits and means-tested transfers act as partial substitutes, the amount of housing benefits received when out-of-work becomes rather small and the participation tax associated to the loss of housing benefits does not exceed 8%.

The identified substitutability of unemployment benefits (insurance) and means-tested transfers (redistribution) may bear substantial implications for the articulation of these

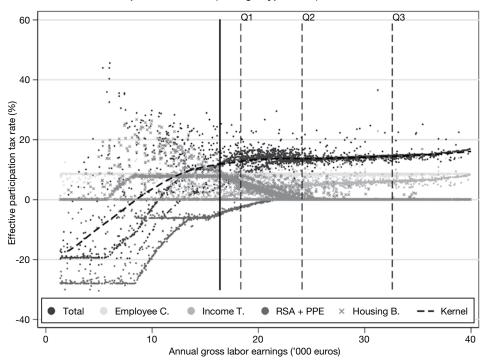


Figure X Distribution of Effective Participation Tax Rates (Savings Hypothesis)

Notes: The savings hypothesis treats social insurance contributions as savings while employer contributions are again assumed paid by employers. Individuals are here assumed to receive unemployment benefits when out-of-work, however unemployment benefits are now the depletion of past savings and thus not treated as a transfer scheme. The dashed curve represents a kernel estimation of the local average EPTR (bandwidth of 4,000 euros). The vertical line signals a full-time job paid at the minimum wage rate and the dashed vertical lines indicate the quartiles of the earnings distribution.

Coverage: Employed prime age childless singles.

Sources: Insee, ERFS 2011; TAXIPP microsimulation model; author's computation.

schemes. Surprisingly, although standard in modern welfare systems, this interaction between social insurance and redistribution instruments has received little attention in the normative literature.

These baseline results are derived under the assumptions that workers' social insurance contributions are treated as taxes and paid by workers while employer social contributions are paid by employers. Treating workers' pension and unemployment contributions as savings rather than taxes decreases marginal (-13 percentage points) and participation tax rates (-12 percentage points) across the board. In contrast, assuming employer contributions are being shifted to workers increases marginal and participation tax rates in a non-uniform way and compresses the tilde-shape pattern of marginal tax rates towards a flat rate because of the progressivity of employer contributions. The impact of housing benefits on monetary incentives to work is robust to these changes.

Last, housing benefits generate heterogeneity in incentives to work based on housing statuses

which determine potential eligibility to the scheme. While the tilde-shape distribution of local average marginal tax rates across earnings seems broadly consistent with policy recommendations of the optimal taxation literature (Saez, 2002), it seems likely that top marginal tax rates faced by individuals eligible to housing benefits are too high to be optimal.

Overall, housing benefits adverse effects on labor supply incentives are to be put into perspective with the phenomenon of capture identified in the literature. Since housing benefits are captured by home-owners through increases in rents (Laferrère & Le Blanc, 2004; Fack, 2005; 2006), low-income individuals may not effectively receive these benefits even when they effectively face reduced incentives to work. Housing benefits may thus contribute to generating a poverty trap. A structural reform of the scheme – for instance through its integration with other means-tested transfers as proposed by Bozio et al. (2015a) and Bargain et al. (2017) – could then be highly beneficial both for low-income individuals and for the French economy.

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Chapter 2 – Inattention and the taxation bias (with J. Boccanfuso)

Paper awarded with the ITAX PhD Student Award at the 2019 IIPF Conference.

Abstract. This paper studies how information frictions in agents' tax perceptions affect the design of actual tax policy. Developing a positive theory of tax policy, we show that agents' inattention interact with policymaking and induces the government to implement inefficiently high tax rates: this is the taxation bias. We quantify the magnitude of this policy distortion for the US economy. Overall, our findings suggest that existing information frictions – and thereby tax complexity – lead to undesirable, large and regressive tax increases.

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1 Introduction

A growing body of evidence documents substantial information frictions in agents' tax perceptions (Chetty, 2015; Bernheim & Taubinsky, 2018; Stantcheva, 2020). In particular, taxpayers tend to partially ignore non-salient taxes and transfers (Chetty, Looney, & Kroft, 2009; Miller, Mumford, et al., 2015; Taubinsky & Rees-Jones, 2018), to rely on linearizing heuristics (Liebman & Zeckhauser, 2004; Rees-Jones & Taubinsky, 2020) and to misunderstand some characteristics of income tax schedules (Saez, 2010; Aghion, Akcigit, Lequien, & Stantcheva, 2017). Taken together, these findings indicate that agents' tax perceptions are shaped by their attention to taxes and potential behavioral biases.

In light of this evidence, a burgeoning normative literature analyzes the design of optimal tax policy in the presence of information frictions in agents' tax perceptions.¹³ This literature characterizes optimal tax policies in terms of sufficient statistics that capture agents' earnings responses to tax changes and perception biases at the optimum. Doing so, it generally sidesteps the issue that agents' tax perceptions may adjust to changes in tax policy and remains agnostic about the mechanisms behind these adjustments.¹⁴ In their general treatment of optimal taxation with behavioral agents, Farhi and Gabaix (2015, p. 13) emphasize that "a difficulty confronting all behavioral policy approaches is a form of Lucas critique: how do the underlying biases change with policy?".

In practice, tax policy is likely influenced by the way agents' perceptions adjust to tax changes. Policymakers may for instance be tempted to increase taxes if agents are inattentive and only perceive a fraction of tax increase. In contrast to their normative counterparts, such positive policy implications remain surprisingly unexplored. This paper aims at filling this gap by studying how information frictions in agents' tax perceptions affect the design of actual tax policy.

We develop a positive theory of tax policy in a setting where agents' labor supply is determined by their tax perceptions. We show that the adjustment of agents' tax perceptions interacts with policymaking and generates a distortion in actual tax policy. Specifically, we show that inattention leads the government to implement inefficiently high tax rates: this is the taxation bias. The key insight is that inattention creates the illusion that earnings responses to tax reforms are lower than they actually are, thereby inducing a commitment

 $^{^{13}}$ For instance, Goldin (2015) shows that a government may implement non-salient taxes to reduce the deadweight loss of taxation. Gerritsen (2016) highlights that tax misperceptions introduce a new corrective motive for taxation and derives adjusted optimal tax formulas. Integrating both insights Allcott, Lockwood, and Taubinsky (2018) revisit the Atkinson and Stiglitz (1976) result when commodity taxes are not salient.

¹⁴This is at the essence of the sufficient statistics approach. See Chetty (2009) or Kleven (2020) for a general discussion and Reck (2016) for a discussion in a behavioral context.

problem in the choice of tax policy.

We then quantify the magnitude of this policy distortion through a simple sufficient statistics formula that we bring to the data. We further illustrate our theoretical results using numerical simulations which shed a new light on the implications of inattention and misperceptions. Overall, our findings suggest that existing information frictions lead to undesirable, large and regressive tax increases.

Our theoretical framework considers a population of heterogeneous and rationally inattentive agents who choose their earnings and consumption given their tax perceptions.¹⁵ We model agents' tax perceptions as resulting from a Bayesian learning model with a choice of information (Maćkowiak, Matějka, Wiederholt, et al., 2018; Gabaix, 2019). That is, taxpayers are endowed with a prior (or belief) about tax policy and can collect additional, but costly, information in the form of a signal. The precision of this signal is endogenous: the more attentive a taxpayer is, the more precise her signal and the more accurate her posterior (or perception). As a result, agents' perceived tax rate is in expectation given by a weighted average between their prior and the actual tax rate where the weight on the latter captures agents' attention to tax policy. Importantly, we allow the prior to be systematically biased to capture potential perception biases thereby building a bridge between behavioral models with ad-hoc misperceptions and standard rational inattention frameworks. This model thus captures the use of biased rule-of-thumbs as default while allowing taxpayers to improve their tax perceptions if they find optimal to do so (Morrison & Taubinsky, 2019).

Building on this general tax perception model, we develop a positive theory of tax policy that we formalize as a simultaneous game between rationally inattentive agents and a welfarist government.¹⁶ Agents endogenously choose their attention to taxes and the government sets tax policy to maximize social welfare taking attention choices into account. In equilibrium, (i) neither taxpayers nor the government has an incentive to deviate, and (ii) taxpayers' actions and perceptions are mutually consistent with the government's choice of tax policy. Our main result is that – irrespective of potential perception biases – inattention leads to the implementation of inefficiently high tax rates.

¹⁵The taxation bias follows from the presence of inattention, be it endogenous or exogenous. We nonetheless adopt a rational inattention model given the strong empirical support for the endogeneity of attention and this model in particular. For instance, Hoopes, Reck, and Slemrod (2015) find that rational inattention motives and shocks to tax salience drive taxpayers' online information search. Taubinsky and Rees-Jones (2018) show in a shopping experiment that tripling the tax rate nearly doubles agents' attention to taxes. Morrison and Taubinsky (2019) provide further compelling evidence that observed attention patterns are consistent with theoretical predictions of rational inattention models.

¹⁶None of our theoretical result hinges on a particular objective function for the government; it could as well be reflecting political economy forces or wider fairness concerns (Saez & Stantcheva, 2016).

Central to this result is a dichotomy between direct and indirect adjustments in perceptions upon changes in tax policy. Indeed, as agents' tax perceptions are determined by a combination of the actual tax rate and their prior, there are two margins through which perceptions may adjust: a *direct* margin capturing the attention agents devote to observing taxes and thus changes in tax policies, and an *indirect* margin capturing variations in the prior. For a given prior, inattentive agents only perceive a fraction of the change in tax policy which dampens their earnings responses. The government thus targets a higher tax rate than if agents were perfectly attentive. In equilibrium, agents' priors must however be consistent with the government's choice of tax policy. As a result, ex post earnings responses are larger than what anticipated ex ante. The government implements inefficiently high tax rates because it fails to internalize the indirect adjustment of the prior (arising as an equilibrium mechanism) in its choice of tax policy. In a nutshell, taxpayers' inattention to taxes creates the illusion that tax reforms induce lower efficiency costs than they actually do and ultimately prompts the government to misbehave from a normative perspective.

Fundamentally, this reflects a commitment problem. By implicitly restricting the set of tax policies to precommited policy rules, the aforementioned normative literature characterizes tax policy under commitment. This commitment tax policy is by definition the welfaremaximizing tax policy in the presence of information frictions in agents' tax perceptions. However, a side effect of information frictions is that actual policymakers cannot credibly commit to implement this optimal policy. Indeed, given agents' inattention, a discretionary government cannot resist the temptation to increase tax rates beyond their optimal levels, thereby introducing a taxation bias. We formally define the taxation bias as the difference between equilibrium tax rates under discretion and commitment and establish the existence of a positive taxation bias under a mild general requirement.

We then seek to illustrate the implications of this policy distortion and to quantify its magnitude. To do so, we parametrize our tax perception model with Gaussian distributions to provide further theoretical results and numerical simulations. They indicate that even small information frictions induce significant deviations in tax policy. Moreover, they allow to disentangle the implications of inattention to taxes from that of potential behavioral biases reflected in agents' priors. The optimal (or commitment) tax policy is mostly driven by equilibrium perception biases reflected in agents' posteriors. That is, the deviation in the optimal tax policy (Farhi & Gabaix, 2020) from a benchmark without information frictions (Saez, 2001) increases with the bias in the prior and decreases with attention. The actual (or discretionary) tax policy is similarly impacted by equilibrium perception biases but it also depends on taxpayers' attention through a second channel: the policy distortion induced

by inattention. The taxation bias, which measures the difference between the two, is thus primarily shaped by attention to taxes and relatively less by potential behavioral biases.

This result transpires in the simple sufficient statistics formula we derive for the taxation bias when income taxes are linear. Indeed, beyond the elasticity of earnings with respect to changes in the perceived marginal net-of-tax rate, it shows that the key sufficient statistic to estimate is the income-weighted average attention in the population. This statistic captures the fact that richer agents are more attentive to tax policy as documented by (Taubinsky & Rees-Jones, 2018) and as emerges in our endogenous attention model. While income taxes are nonlinear in the US economy, a linear tax model provides a reasonable first-order approximation (Piketty & Saez, 2013). Fitting a linear tax model to US tax data, we find a tax rate of 29.5 percentage points. Further relying on the existing empirical literature to calibrate our sufficient statistics, we estimate that the taxation bias is approximately equal to 3.7 percentage points. This means that the linearized US income tax rate is more than 12% higher than what would be optimal holding the government's objective constant: the taxation bias is large.¹⁷

We then show that our findings hold important and counterintuitive implications. Situations in which behavioral biases were previously thought to be welfare improving may actually turn out to be welfare decreasing. To illustrate this point, we carry out a welfare analysis in an economy where taxpayers' priors systematically underestimate tax rates (e.g. salience bias). As this downwards bias reduces the efficiency cost of taxation for any given tax rate, we unsurprisingly find that information frictions induce a welfare gain if the government was to implement the optimal tax policy. However, the optimal tax policy cannot be credibly implemented and the actual tax policy features an additional welfare loss due to the taxation bias. As inattention grows, the welfare loss from this policy distortion increases faster than the welfare gain from tax underestimation. Therefore, even if agents systematically underestimate tax rates, information frictions can be detrimental to welfare when agents are not sufficiently attentive to tax policy.

Last, we extend our analysis to nonlinear tax schedules. The government's incentive to increase the marginal tax rate at a given earnings level then depends on agents' attention at (or close to) this earnings level. As a consequence, the positive correlation between income and attention results in an income-specific taxation bias that is globally decreasing with income: the taxation bias is large at low income levels and virtually nonexistent at top income levels. The taxation bias thus attenuates the U-shape pattern of marginal tax rates (Saez,

¹⁷While the existence of a taxation bias does not depend on the objective of the policymaker, the magnitude of the taxation bias does. We use the social welfare weights that can be inferred from actual tax policy given agents' inattention as our baseline and provide a sensitivity analysis.

2001) and reduces the progressivity of actual income tax schedules.

This paper contributes to the behavioral public economics literature recently reviewed by Bernheim and Taubinsky (2018). It is the first to analyze the implications of information frictions in agents' tax perceptions for actual tax policy. We present a novel positive theory of tax policy that we link to the existing normative theory to show that inattention generates important policy distortions.^{18,19}

Central to this result is the dichotomy between direct and indirect perception adjustments induced by inattention and highlighted in recent empirical evidence. For instance, Sausgruber and Tyran (2005) and Fochmann and Weimann (2013) show that, with time and experience, taxpayers tend to internalize the impact of non-salient taxes they initially ignored.²⁰ Moreover, if taxpayers act upon their perceptions this dichotomy should also be reflected in earnings choices. Chetty (2012) documents a systematic difference between micro (capturing direct adjustments) and macro (capturing total adjustments) estimates of the elasticity of taxable income and rationalizes this difference by the existence of adjustment rigidities such as information frictions at the micro level.

By showing that information frictions induce a commitment problem leading to inefficient policy outcomes, this paper builds a perhaps unexpected bridge to an earlier literature on the inconsistency of policymaking (Kydland & Prescott, 1977). This justifies our use of the term *taxation bias* in analogy to the *inflation bias* (Barro & Gordon, 1983). A large body of evidence documents the existence of information frictions affecting consumers, firms or even professional forecasters (see e.g. Coibion and Gorodnichenko, 2015). Consequently, our analysis suggests that policy distortions may arise in a wide variety of settings in which the portable framework developed in this paper could be fruitfully applied.

Policymaking is, at least to some extent, discretionary. In the realm of taxation, discretion is usually discussed in the context of capital levies in which there is indisputable historical evidence of discretionary policies (e.g. Japan post WWII, Italy in 1992, Cyprus in 2013). While less salient for income taxes, discretionary behaviors are likely reflected in the obnoxious complexity of existing tax systems. The French constitutional court has

¹⁸While the term 'positive' sometimes refers to settings in which tax policy is determined as the outcome of a political economy process, it here refers to the discretionary nature of policymaking. See Matějka and Tabellini (2017) for the implications of (rational) inattention in a political economy process.

¹⁹This finding has potentially important implications for the inverse-optimum approach which aims at inferring the government's objective function from the shape of actual tax schedules (Bourguignon & Spadaro, 2012; Lockwood & Weinzierl, 2016; Jacobs, Jongen, & Zoutman, 2017).

 $^{^{20}}$ Indirect adjustments may also help explain why surveys of taxpayers' perceptions do not provide clear cut evidence of systematic tax rate underestimation (Fochmann, Kiesewetter, Blaufus, Hundsdoerfer, & Weimann, 2010; Gideon, 2017); a finding that is *prima facie* hard to reconcile with e.g. ironing behaviors or salience biases.

for instance repealed specific items of tax bills for their "excessive complexity" arguing they would not be understood by taxpayers (Conseil Constitutionnel, 2005, 2012). As a result, it should not come as a surprise that individuals strongly oppose tax complexity, even after acknowledging the potential advantages of differential tax treatments (Blesse, Buhlmann, & Doerrenberg, 2019). Indeed, our findings suggest that by inducing information frictions, tax complexity prompts the government to misbehave from a normative perspective.

The rest of the paper is organized as follows. To build up the intuition, we first characterize in Section 2 the taxation bias in a stylized representative agent model with exogenous attention. Section 3 microfounds the behavior of heterogeneous and rationally inattentive taxpayers. In Section 4, we formalize our positive theory of tax policy and establish the existence of a taxation bias. We then derive a simple sufficient statistics formula for the taxation bias that we take to the data and we illustrate our theoretical results with numerical simulations in Section 5. Section 6 turns to the welfare implications of information frictions in tax perceptions and Section 7 provides an extension to nonlinear tax schedules. The last section concludes. Unless stated in the text all proofs are relegated to the Appendix.

2 Taxation bias in a stylized model

Consider a canonical labor income taxation model where the government sets a linear tax rate τ to maximize tax revenue. Let $Y(1-\tau)$ be the aggregate earnings function. The tax revenue function $\tau Y(1-\tau)$ has an inverted U-shape and is nil when τ is equal to 0 or 100%. As is well-known (e.g. Piketty & Saez, 2013), the revenue maximizing tax rate follows an inverse elasticity rule and is equal to

$$\tau^r = \frac{1}{1+e} \tag{1}$$

where e is the elasticity of aggregate earnings with respect to the net-of-tax rate.

Assume now that because of information frictions taxpayers are unable to perfectly observe the tax rate. They must nonetheless form an estimate of the latter to decide how much to work. Call this estimate the perceived tax rate $\tilde{\tau}$ and suppose it is determined by a convex combination of a common prior $\hat{\tau}$ and the actual tax rate τ

$$\tilde{\tau} = \xi \tau + (1 - \xi)\hat{\tau},\tag{2}$$

where the weight $\xi \in [0,1]$ can be interpreted as a measure of taxpayers' attention to the

actual tax rate τ .²¹ Indeed, when $\xi = 1$, taxpayers perfectly observe changes in the tax rate whereas, when $\xi = 0$, they are completely inattentive to tax changes and fully anchor their perception on their prior. Since individual earnings choices depend on their perceived tax rate, aggregate earnings now write $Y(1 - \tilde{\tau})$. The tax revenue function becomes $\tau Y(1 - \tilde{\tau})$ which remains concave with respect to the actual tax rate τ given the tax perception model in equation (2).

How do information frictions interact with the design of actual tax policy? Consider a situation in which taxpayers expect the government to implement the optimal tax rate in the absence of information frictions, that is $\hat{\tau} = \tau^r$. Now, suppose the government sets $\tau = \tau^r$ and consider the effect of a policy deviation that consists in a small increase in the tax rate $d\tau$. This mechanically increases tax revenues by $M = Y(1 - \tilde{\tau})\Big|_{\tilde{\tau} = \tau^r} d\tau$ while it generates a behavioral response $dY = -\frac{\partial Y(1-\tilde{\tau})}{\partial 1-\tilde{\tau}}\Big|_{\tilde{\tau} = \tau^r} \xi d\tau$ as inattentive taxpayers only observe a fraction ξ of the increase in the tax rate $d\tau$. By definition of τ^r , the mechanical effect M outweighs the fiscal externality $FE = \tau^r dY$ induced by the behavioral response when agents are not fully attentive ($\xi < 1$). As a result, the government systematically deviates from tax policy τ^r and ends up choosing a higher tax rate.

Conceptually, an important consequence of inattention is to anchor taxpayers' perceptions on their prior. Because of this anchoring, the government has an incentive to implement policy deviations that taxpayers are going to partially ignore. This is a form of discretionary policy which arises as a side-effect of information frictions in tax perceptions. The government thus chooses its tax policy taking agents' priors and attention into account. Specifically, tax policy $\tau(\xi, \hat{\tau})$ is decreasing in taxpayers' prior $\hat{\tau}$ and attention parameter ξ

$$\tau(\xi, \hat{\tau}) = \begin{cases} \frac{1-(1-\xi)\hat{\tau}}{\xi(1+e)} & \text{if } \hat{\tau} \ge 1 - \frac{\xi}{1-\xi}e\\ 1 & \text{otherwise} \end{cases}$$
(3)

where the elasticity of aggregate earnings is defined with respect to the perceived net-of-tax rate $e \equiv \frac{1-\tilde{\tau}}{Y} \frac{\partial Y}{\partial(1-\tilde{\tau})} \geq 0$. The solution is interior whenever attention ξ or the prior $\hat{\tau}$ are high enough – otherwise the government finds it optimal to impose a 100% tax – and coincides with the inverse elasticity rule when agents are perfectly attentive ($\xi = 1$) and thus fully informed.

Figure 1 plots tax policy $\tau(\xi, \hat{\tau})$ as a function of agents' prior for different attention levels. It shows that small information frictions generate notable deviations in the government behavior. If agents' prior is that the government implements the inverse elasticity rule –

 $^{^{21}}$ Gabaix (2019) argues this is a unifying framework to modeling various behavioral biases and attention theories.

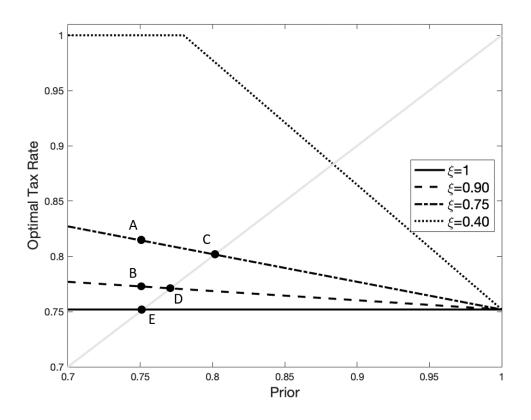


Figure 1: Optimal tax policy and equilibrium outcomes

NOTE: Optimal policy as a function of the prior $\hat{\tau}$ for different values of the attention parameter ξ . The elasticity of aggregate earnings with respect to the perceived net-of-tax rate is set to 0.33.

 $\tau^r = 75\%$ assuming e = 0.33 – the government chooses a tax rate of 82% (resp. 77%) when the attention parameter ξ is equal to 0.75 (resp. 0.90).²² This corresponds to point A (resp. B).

An equilibrium is as a situation in which (i) neither taxpayers nor the government has an incentive to deviate and (ii) taxpayers' actions and perceptions are mutually consistent with the government's choice of tax policy. We here focus on rational equilibria in which agents correctly anticipate the equilibrium tax policy $\hat{\tau} = \tau^{eq}$ and defer the introduction of biased equilibria to Section 4. Plugging this equilibrium condition in the government's choice of tax policy (3) we obtain that the equilibrium tax policy is

$$\tau^{\rm eq} = \frac{1}{1+\xi e} \tag{4}$$

 $^{^{22}}$ As an element of comparison, Gabaix (2019) states (p. 5) that "on average, the attention parameter estimated in the literature is 0.44, roughly halfway between no attention and full attention" while adding that "attention is higher when the incentives to pay attention are stronger" which should likely be the case when it comes to taxing 75% of one's income.

Graphically, the rational equilibrium is represented by the point where the 45-degree line $(\hat{\tau} = \tau^{eq})$ intersects the government policy function. Again, small information rigidities lead to large deviations in equilibrium. In a rational equilibrium, the tax rate is 80% (resp. 77%) when attention is such that 75% (resp. 90%) of a tax reform is directly internalized by taxpayers. This corresponds to point C (resp. D).

The government is unable to reach the top of the Laffer curve: the equilibrium tax rate is inefficiently high. We refer to this phenomenon as the *taxation bias* in analogy to the *inflation bias* (Barro & Gordon, 1983). While the government internalizes the direct impact of its choice on agents' perceptions (proportional to attention ξ), it does not internalize the equilibrium impact associated with the adjustment of the prior (proportional to inattention $1-\xi$). In Barro and Gordon's (1983) words, "the equality of policy expectations and realizations is a characteristic of equilibrium – not a prior constraint" (p. 591). Hence, inattention resurges the possibility that discretionary policies lead to inefficient outcomes (Kydland & Prescott, 1977).

To formalize this result, we characterize the optimal policy under commitment τ^* . This is the optimal policy of a government who can credibly commit to implement a tax level and thereby has to take into account the equilibrium effect of its choice of tax policy on perceptions. By definition, this is the optimal policy in the presence of information frictions and it coincides in this stylized rational equilibrium framework with the inverse elasticity rule $\tau^* = \tau^r$ (point E). However and as shown above, it cannot be an equilibrium policy under discretion in the presence of inattention.

Defining the taxation bias as the difference between the tax rates under discretion and commitment we have

$$\tau^{\rm eq} - \tau^{\star} = \frac{(1-\xi)e}{(1+\xi e)(1+e)} \ge 0$$
 (5)

Therefore, the taxation bias is strictly positive when agents are not fully attentive to taxes $(\xi < 1)$. Moreover, the (absolute) size of the taxation bias increases with agents inattention $1 - \xi$ and with the elasticity e as they intuitively both make policy deviations relatively more attractive.

To highlight the mechanisms that lead to a taxation bias, we have analyzed in this section a stylized representative agent model in which agents' behavior and attention are exogenously given and in which the government implements a linear tax policy to maximize tax revenue. In the remainder of the paper we broaden the scope of the analysis by studying the problem of a welfare-maximizing government facing a heterogeneous population of agents whose individual behavior is fully micro-founded and whose attention is endogenous. We also extend the equilibrium concept to allow for perception biases in equilibrium and examine how the taxation bias affects (the progressivity of) non-linear tax schedules. These extensions provide valuable insights on the magnitude and implications of the taxation bias in a policy relevant environment.

3 Agents' behavior, perceptions and attention

This section describes the behavior of taxpayers in the economy. Because of information frictions, taxpayers may not freely observe the tax rate implemented by the government. They rely on a Bayesian learning model with costly information acquisition to form their perceptions about the tax schedule in order to decide how much to earn and consume.

3.1 Primitives and assumptions

We consider a population of agents with heterogeneous productivities w which are private information and distributed from a well-defined probability distribution function $f_w(w)$. We assume taxpayers have a utility function U(c, y; w) where c is consumption and y earnings and where we impose U(.) to be continuously differentiable, increasing in consumption $(U_c > 0)$, decreasing in effort $(U_y < 0 \text{ and } U_w > 0)$ and such that the Spence-Mirrlees condition holds $(MRS_{yc}$ decreases with skill w). For simplicity, we consider a separable and quasi-linear utility U(c, y; w) = c - v(y; w) in the body of the paper and show in the Online Appendix how we can extend the analysis to more general utility functions.²³

Agents choose their consumption c and earnings y subject to an income tax T(y). Because of information frictions, we assume that taxpayers are unable to freely observe T(y) and instead rely on individual-specific perceived income tax schedules denoted $\tilde{T}(y)$.

Assumption 1 (linear representation). Individuals use a linear representation of the tax schedule $\tilde{T}(y) = \tilde{\tau}y - \tilde{R}$

To make their consumption and earnings choices, individuals rely on their perceptions of the tax liability at each earnings level. Assumption 1 imposes that taxpayers use a linear representation of the tax schedule. Hence, agents only need to form estimates of the marginal tax rate $\tilde{\tau}$ and the intercept \tilde{R} thereby reducing the dimensionality of the perceptions for-

²³Separability between earnings and consumption preferences combined with quasi-linearity guarantees the absence of income effects in labor supply decisions and considerably simplifies the analysis.

mation problem to two parameters.²⁴

In most of the paper, we consider that the actual tax schedule is also linear and denote by (τ_0, R_0) its slope and intercept. Consequently, we define (τ, R) as the associated random variables from the point of view of the agents. In Section 7 we extend the analysis to non-linear tax schedules.

3.2 Individual problem

Individuals jointly choose an allocation (c, y) and how much information to collect about the tax schedule. This one-step problem is equivalent to a two-step problem that we characterize. The first step identifies the optimal allocation choice given a perceived tax schedule $\tilde{T}(y)$ while the second step determines the optimal information acquisition taking into account how perceptions affect allocations.

Allocation choice. Agents choose consumption c and earnings y to maximize their utility subject to their perceived budget constraint which depends on their perceptions of the tax schedule. This problem writes

$$\max_{\substack{c,y \\ \text{s.t.}}} \int_{\tau} U(c,y;w) \ \tilde{q}(\tau) d\tau$$
(6)
s.t. $c \le R + (1-\tau)y$

where $\tilde{q}(\tau)$ is the perceived probability distribution of the marginal tax rate τ . With a separable and quasi-linear utility function, the first-order condition determining earnings writes

$$\frac{\partial v(y;w)}{\partial y} = 1 - \tilde{\tau} \tag{7}$$

with $\tilde{\tau} \equiv E_{\tilde{q}(\tau)}[\tau]$ the average perceived marginal tax rate. Consequently, the average perceived marginal tax rate $\tilde{\tau}$ is a sufficient statistics for labor supply and uniquely pins down optimal earnings $y^*(\tilde{\tau}; w)$. Hence, a direct implication of quasi-linear separable preferences is that tax liability, and in particular the perceived value of the demogrant \tilde{R} , is irrelevant for labor supply and only matters to determine agents' consumption levels.

 $^{^{24}}$ Beyond the fact that a linear approximation is usually a good approximation of existing tax schedules (Piketty & Saez, 2013), recent empirical evidence suggests that in practice taxpayers tend to use linear representations of tax schedules (Rees-Jones & Taubinsky, 2020).

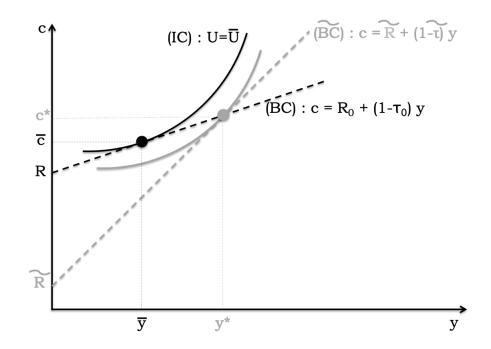


Figure 2: Allocation choice in a y-c diagram

NOTE: The figure displays an exemple of allocation choice when the agent underestimates the marginal tax rate $\tilde{\tau} < \tau_0$. The grey (resp. black) dot represents the allocation choice of an individual who misperceives (resp. correctly perceives) the tax schedule. The plain lines are the indifference curves and the dashed lines the budget contraints.

Assumption 2 (slack budget). Consumption adjusts such that agents exhausts their true budget i.e. $c^*(\tilde{\tau}; w) = R_0 + (1 - \tau_0)y^*(\tilde{\tau}; w)$

We assume consumption adjusts to ensure that the true budget constraint holds ex post.²⁵ The only parameter of interest for agents' allocation choice is thus the perceived marginal tax rate $\tilde{\tau}$.

Given this allocation choice, an agent's indirect utility is

$$V(\tilde{\tau}, \tau_0, R_0; w) = R_0 + (1 - \tau_0) y^*(\tilde{\tau}; w) - v \left(y^*(\tilde{\tau}; w); w \right)$$
(8)

Figure 2 summarizes the allocation choice in a y-c diagram. Perceptions of the tax schedule determine earnings (tangency condition with perceived budget line) while consumption adjusts to the true budget constraint (intersection with true budget line).

A natural observation from Figure 2 is that misperceptions induce utility misoptimization costs: the utility level \overline{U} associated with the choice under accurate perceptions (black

 $^{^{25}}$ This assumption is used throughout the behavioral tax literature as emphasized in Reck (2016) who discusses different budget adjustment rules in misperception models. See also Farhi and Gabaix (2020) for a related discussion.

dot) is higher than the utility level U^* associated with the choice under misperceptions (grey dot). However, when agents underestimate tax rates ($\tilde{\tau} < \tau_0$) misperceptions also induce efficiency gains: earnings y^* chosen under misperceptions are larger than earnings \bar{y} at the optimal allocation. As a result, tax underestimation may increase social welfare if efficiency gains dominate utility misoptimization costs.

Perceptions formation. Tax perceptions here follow from a Bayesian learning model with a choice of information (in Gabaix's (2019) terminology). They result from the combination of an exogenous and free prior (also referred to as a belief or an anchor) and an endogenous and costly information acquisition process. We choose this model for its wide use in economics, its well-understood micro-foundations and the fact that – as we show – it generates predictions that are consistent with the empirical evidence.

Let $\hat{q}(\tau)$ be the prior probability distribution about the tax rate and $\hat{\tau} \equiv E_{\hat{q}}[\tau]$ the expected tax rate derived from the prior. This probability distribution accounts for sources of structural and subjective uncertainty which may be related to policy primitives (e.g. hidden tastes for redistribution), economic fundamentals (e.g. shocks to the government expenditure requirements), institutions (e.g. inability to implementation a chosen policy), heuristic decision rules (e.g. ironing), etc.

In the following, we voluntarily remain agnostic about the origin of the prior and the sources of uncertainty it may capture for two reasons. First, the assumed ex ante uncertainty essentially represents a motive for taxpayers to learn in our setup and the main results of the paper will hold for a wide variety of well-defined smooth priors. Second, while the empirical literature clearly indicates that taxpayers tend to misperceive tax rates, there is yet no consensus on the exact rationale – or rationales – behind these misperceptions. Hence, we consider diverse situations ranging from priors that are correct on average to priors that are systematically biased due to cognitive or perception biases.

Information about the actual tax rate τ_0 takes the form of an unbiased Gaussian signal with precision $1/\sigma^2$. For a realization s of the signal, the posterior belief follows from Bayes law

$$\tilde{q}(\tau|s;\sigma) \propto \phi(s;\tau,\sigma^2)\hat{q}(\tau)$$
(9)

where $\phi(s; \tau, \sigma^2)$ is the Gaussian pdf with mean τ and variance σ^2 . Building on the rational inattention literature (Sims, 2003), the information content transmitted through the signal

is measured from the entropy reduction between the prior and the posterior

$$\mathcal{I}(\sigma) \equiv H(\hat{q}(\tau)) - E_{p(s)} \Big[H(\tilde{q}(\tau|s;\sigma)) \Big]$$
(10)

where $H(q(\tau)) \equiv -\int q(\tau) \log_2(q(\tau)) d\tau$ is the differential entropy (in bits) of the probability distribution $q(\tau)$ and $E_{p(s)}[.]$ the expectation taken over the marginal distribution of signals $p(s) \equiv \int \phi(s; \tau, \sigma) \hat{q}(\tau) d\tau$. Intuitively, $\mathcal{I}(\sigma)$ is a measure of the expected amount of information transmitted through the signal. To account for the energy and time devoted to acquiring and processing information, taxpayers suffer a utility cost κ per unit (bit) of processed information.²⁶ The attention strategy of a taxpayer with productivity w thus results from an arbitrage between improved private decisions thanks to more accurate information and the cost to acquire this information. More specifically, she chooses the signal's precision – or equivalently its standard error $\sigma^*(\hat{q}(\tau), \kappa, w)$ – to maximize her expected indirect utility

$$\max_{\sigma} \qquad \iint V\big(\tilde{\tau}(s,\sigma),\tau,R;w\big) \ \phi(s;\tau,\sigma) \ \hat{q}(\tau) \ ds d\tau - \kappa \mathcal{I}(\sigma) \tag{11}$$

where $\tilde{\tau}(s,\sigma) \equiv E_{\tilde{q}(\tau|s;\sigma)}[\tau]$ is the expected perceived marginal tax rate once the signal is observed and henceforth referred to as the perceived tax rate. Note that the decision to acquire information is here only based on the information contained in the prior distribution $\hat{q}(\tau)$ which ensures the internal consistency of this learning model.²⁷

In the following, we denote by $f_{\tilde{\tau}}(\tau | \tau_0, w)$ the posterior distribution of $\tilde{\tau}(s, \sigma^*)$ for a taxpayer with productivity w and signal s drawn from the Gaussian distribution with mean τ_0 and variance σ^* . This function summarizes the distribution of agent w perceptions in the economy. Moreover, for a given perceived tax rate $\tilde{\tau}$, agent w indirect utility net of information costs writes

$$\mathcal{V}(\tilde{\tau}, \tau_0, R_0; w, \kappa) = V(\tilde{\tau}, \tau_0, R_0; w) - \kappa \mathcal{I}(\sigma^*)$$
(12)

3.3 Tractable Gaussian model

The general Bayesian learning model presented above is generally intractable. We here focus on the Gaussian formulation in order to derive some predictions and implications of the model. As highlighted in the inattention literature (Maćkowiak & Wiederholt, 2015; Maćkowiak, Matějka, Wiederholt, et al., 2018), a closed form solution to problem (11) can be obtained under the following assumption.

²⁶Our results naturally extend to more general information cost functions.

²⁷In other words, agents "don't know what they don't know".

Assumption 3 (tractable Gaussian learning). Let the prior $\hat{q}(\tau)$ be the Gaussian distribution with mean $\hat{\tau}$ and variance $\hat{\sigma}^2$ and assume that agents use a quadratic approximation of their indirect utility to choose their attention strategies.

Under the assumption that the prior is Gaussian, the posterior will also be Gaussian and the information measure $\mathcal{I}(\sigma^*)$ takes a simple form.²⁸ Relying on a second-order approximation of indirect utility, the solution to this problem can then be derived.

In a tractable Gaussian learning model, the expected perceived marginal tax rate $\tilde{\tau}$ is given by

$$\tilde{\tau}(s,\sigma^{\star}) = \xi(\sigma^{\star})s + (1 - \xi(\sigma^{\star}))\hat{\tau}$$
(13)

where $\xi(\sigma^{\star}) \equiv \frac{\hat{\sigma}^2}{\hat{\sigma}^2 + \sigma^{\star 2}} \in [0, 1]$ is a measure of attention strategies.

Proof. See Appendix A.2

The perceived tax rate $\tilde{\tau}$ is given by a convex combination of the prior $\hat{\tau}$ and the realization of the signal s where the weight ξ is a measure of attention. Indeed, the lower the attention parameter ξ is, the more taxpayers rely on their prior $\hat{\tau}$ and the less attention they devote to acquiring information about the actual tax rate through the signal s. In other words, agents tend to choose to ignore their signal if they do not invest in information acquisition and the signal is hence relatively uninformative in comparison to the prior.

In a tractable Gaussian learning model, the optimal attention strategy ξ is given by

$$\xi = \max\left(0, 1 + \frac{\kappa}{\hat{\sigma}^2 \int \frac{\partial^2 y^{\star}}{\partial \tilde{\tau}^2}\Big|_{\tilde{\tau}=\tau} \phi(\tau; \hat{\tau}, \hat{\sigma}) d\tau}\right).$$
(14)

Proof. See Appendix A.2

Each taxpayer's attention strategy ξ is characterized by equation (14). Attention decreases with the information cost κ and increases with the uncertainty in the prior $\hat{\sigma}^2$. It also depends on the responsiveness of agents labor supply decisions to changes in perceived tax rates through $\frac{\partial^2 y^*}{\partial \tilde{\tau}^2}$. Indeed, an agent's responsiveness to changes in perceptions determines the value of information acquisition. As a result, attention increases with earnings

 \square

²⁸One may instead consider that the prior is a truncated Gaussian with support on [0, 1] in order to ensure that perceived tax rates $\tilde{\tau}$ always remain between zero and one. Doing so, the problem remains tractable but formulas become a lot less transparent. In practice, our simulations suggest that when the prior is sufficiently informative (low variance) and the tax rate not too extreme, the posterior support belongs to [0, 1] a.s. with a Gaussian prior.

ability w and with expected prior tax rates. Intuitively, agents who are more productive have a greater latitude in their earnings choice and will thus be more attentive – and hence responsive – to taxes. In a similar fashion, responsiveness to tax changes and thus attention increases when expected tax rates increase because it shifts the labor supply function to regions with a larger curvature.

Predictions and implications. Agents choose their attention strategies through the maximization of their expected indirect utility which is based on their prior. Attention is hence unaffected by an unanticipated change in the realized tax rate $d\tau_0$. As a result, an unanticipated change in the tax rate $d\tau_0$ induces a change in the distribution of the posterior $f_{\tilde{\tau}}(\tilde{\tau}|w)$ only through a change in the signal *s* which is now drawn from a novel distribution $\phi(s; \tau_0 + d\tau_0, \sigma^*)$. Therefore, taxpayers' perceived tax rate slowly adjusts to news and perceptions are anchored on the prior. Anchoring is a widely documented bias in the behavioral literature (**Gabaix2019**) which has two major implications in this context.

First, if agents' prior is biased ($\hat{\tau} \neq \tau_0$) and agents are not fully attentive ($\xi < 1$), the posterior and hence agents perceptions of the tax schedule will also be biased (almost surely). Indeed, taking expectations over signal realizations we have that

$$\tau_0 - E_{\phi(s)}[\tilde{\tau}(s,\sigma^*)] = \tau_0 - [\xi\tau_0 + (1-\xi)\hat{\tau}] = (1-\xi)(\tau_0 - \hat{\tau}).$$
(15)

Second, taxpayers labor supply responses to unanticipated changes in the tax rate are attenuated by anchoring. Indeed, taking the prior as given, responses to tax changes only transit through the variation of the signal which is weighted by attention ξ . Formally, this means $\frac{d\tilde{\tau}}{d\tau_0} = \xi$ such that

$$\frac{dy^{\star}}{d\tau_0} = \xi \frac{dy^{\star}}{d\tilde{\tau}} \tag{16}$$

Intuitively, $\frac{dy^{\star}}{d\tilde{\tau}}$ captures agents' preferences while ξ is a dampening factor that captures the fraction of the tax change that agents perceive. As a result the elasticity of labor supply with respect to unanticipated changes in the tax rate decreases in the presence of inattention.

The predictions derived from a Bayesian learning model with a choice of information thus seem consistent with the bulk of the empirical evidence on tax perceptions and behavioral responses to taxes. Most importantly, it can account for the presence of systematic perception biases and implies that elasticities will be lower when inattention is at play. In addition the model also generates dispersion in perceptions – through the noisiness of the signal – and features an increase in overall attention upon tax increases which hold potentially important welfare implications (Taubinsky & Rees-Jones, 2018).

4 Discretion, commitment and the taxation bias

This section introduces the problem of the government and formalizes our positive theory of tax policy. It characterizes tax policy under discretion and commitment and provides a formal definition of the equilibrium. A general result on the existence of a taxation bias concludes.

4.1 Government problem and discretion

We consider a welfarist government that maximizes a general social welfare function summing an increasing and weakly concave transformation G(.) of taxpayers' indirect utilities net of information costs. It chooses a target tax schedule (τ_g, R_g) , where τ_g is the marginal tax rate and R_g the demogrant, taking the distribution of skills $f_w(w)$ in the population as given.

Following Matějka and Tabellini (2017), we introduce implementation shocks ϑ as an underlying source of uncertainty in the model. The target tax rate is implemented up to a realization of this implementation shock such that the actual tax rate is $\tau_0 = \tau_g + \vartheta$ where ϑ is a white noise drawn from an exogenous distribution $f_{\Theta}(\vartheta)$ known to both taxpayers and the government. We assume the actual demogrant R_0 adjusts to the realization of the implementation shock ϑ to ensure that the government budget constraint is always binding. Conceptually, these implementation shocks are introduced to ensure that Bayesian taxpayers have an incentive to learn in equilibrium. They allow to formally close the model but have an otherwise negligible impact on the optimal tax policy. Hence, we sometimes use small shocks approximations in which case we explicitly disregard the small effects they may induce.

The government problem writes

$$\max_{\tau_g, R_g} \quad E_{\vartheta} \left[\iint G \Big(\mathcal{V}(\tilde{\tau}, \tau_0, R; \kappa, w) \Big) f_{\tilde{\tau}}(\tau | \tau_0; w) f_w(w) d\tau dw \right]$$
(17)

s.t.
$$\iint \tau_0 y^*(\tilde{\tau}; w) \ f_{\tilde{\tau}}(\tau | \tau_0; w) \ f_w(w) d\tau dw \ge R_0 + E$$
(18)

where E is an exogenous expenditure requirement, the expectation is taken over the implementation shock ϑ and $f_{\hat{\tau}}(\tau | \tau_0; w)$ is the posterior distribution of perceived rates for a taxpayer with productivity w given the actual tax rate $\tau_0 = \tau_g + \vartheta$. **Discretionary policy.** The government's optimal tax policy solves problem (17). When doing so, it takes the prior distribution $\hat{q}(\tau)$ as given. This is a form of Nash conjecture used to compute the best response of the government. While the problem is fundamentally simultaneous, it can be equivalently described by the following sequence of events which we here layout for the sake of clarity:

- 0. Agents are endowed with a common prior $\hat{q}(\tau)$ and the distribution of skills is $f_w(w)$.
- 1. The government sets the target tax policy (τ_g, R_g) to maximize (17).
- 2. The actual tax rate $\tau_0 = \tau_g + \vartheta$ is implemented up to an implementation shock drawn from a known distribution $f_{\Theta}(\vartheta)$ and the actual demogrant R_0 adjusts to the resource constraint.
- 3. Taxpayers choose their attention strategies using their common prior $\hat{q}(\tau)$, observe a Gaussian signal s about τ_0 which precision depends on their attention and decide how much to consume and earn.
- 4. The government levies taxes and redistributes through the demogrant.

The government understands that taxpayers will gather information and adjust their decisions in reaction to its choice of tax policy, it therefore plays "first" in the above-described sequence of events. However, it (i) treats the prior distribution $\hat{q}(\tau)$ and the skill distribution $f_w(w)$ as predetermined state variables and (ii) cannot directly influence agents' attention strategies since they are based on agents' predetermined prior. As a result, the government does not have a particular strategic advantage from playing "first" – thus reflecting the simultaneous nature of the problem. Importantly, the government is as rational and informed as in the standard Mirrlees (1971) model and the novelty relates to information frictions on the agents' side.

The tax policy of the government follows from Proposition 4.1, where first-order conditions have to hold in expectation of the realization of the implementation shock.

The discretionary tax policy (τ_g, R_g) is characterized by

$$(\tau_g): E_{\vartheta} \left[\int \left\{ \underbrace{\int \left[-\frac{G'(\mathcal{V})}{p} y^* + y^* \right] f_{\tilde{\tau}}(\tau | \tau_0; w) d\tau}_{\text{mechanical & welfare effects}} + \underbrace{\int \left[\frac{G(\mathcal{V})}{p} + \tau_0 y^* \right] \frac{df_{\tilde{\tau}}(\tau | \tau_0; w)}{d\tau_g} \Big|_{\hat{q}(.)} d\tau}_{\hat{q}(.)} d\tau \right\} f_w(w) dw \right] = 0$$
(19)
direct behavioral responses

$$(R_g): \quad E_{\vartheta} \bigg[\iint \bigg[\frac{G'(\mathcal{V})}{p} - 1 \bigg] f_{\tilde{\tau}}(\tau | \tau_0; w) f_w(w) d\tau dw \bigg] = 0$$

$$(20)$$

together with the resource constraint (18) and where p represents the social marginal cost of public funds.

Proof. See appendix A.3

The first order condition (19) captures the (expected) effects of a marginal increase in the target tax rate $d\tau_g$. The first line measures the impact of the reform on allocations when the distribution of perceptions remains fixed. It corresponds to the standard mechanical and welfare effects: a marginal increase in the tax rate mechanically increases tax revenue by $y^* d\tau_g$ additional dollars but reduces taxpayers' consumption and thus welfare by $\frac{G'(\mathcal{V})}{p}y^* d\tau_g$ dollars (**PikettySaez2013**).

The second line in condition (19) relates to the impact of the reform on the distribution of perceptions and thus captures behavioral responses to the reform. Indeed, behavioral responses transit through variations in the posterior distribution $f_{\tilde{\tau}}(\tau|\tau_0, w)$ of perceived tax rates $\tilde{\tau}$ which reflect changes in the actual tax rate τ_0 . A marginal increase in the tax rate increases, on average, the perceived tax rate by $d\tilde{\tau}$ and thus reduces tax revenue by $\tau_0 y^*(\tilde{\tau}) d\tilde{\tau}$. This is a reformulation of the standard behavioral effect. Moreover, because agents misoptimize, the envelope theorem no longer applies and a marginal deviation from taxpayers' perceived rate induces a welfare cost equal to $\frac{G(\mathcal{V}(\tilde{\tau}))}{p} d\tilde{\tau}$. This new welfare effect introduces a corrective motive for taxation in the presence of misperceptions common to optimal tax models with behavioral agents (Gerritsen, 2016; Farhi & Gabaix, 2020).

Condition (20) states that in the absence of income effects, social marginal welfare weights $g \equiv \frac{G'(\mathcal{V})}{p}$ average to 1 at the optimum: the government is indifferent between having an additional dollar or redistributing an additional dollar (Saez, 2001).

4.2 Equilibrium definition

An equilibrium is a set of target tax policy, denoted $(\tau_g^{\text{eq}}, R_g^{\text{eq}})$, and a set of attention, consumption and earnings decisions such that neither the government nor taxpayers have an incentive to deviate. Moreover, in equilibrium agents' prior $\hat{q}(\tau)$ must be mutually consistent with the government's target tax rate and with the uncertainty induced by the implementation shock.

As discussed in the introduction, there is a large body of evidence suggesting the existence of systematic perception biases. Therefore, we allow for a potential perception bias b in agents' common prior but remain agnostic on the origin of this potential bias. We henceforth call rational (resp. biased) an equilibrium in which agents correctly (resp. incorrectly) anticipate the target tax policy such that b = 0 (resp. $b \neq 0$).

Given the structure of the problem, the only free variables are the government's target tax rate τ_g , agents' attention strategies and the equilibrium distribution of the common prior $\hat{q}(\tau)$. Hence, for the sake of simplicity our formal definition of the discretionary equilibrium only involves these variables. Once they are set, all remaining variables may be mechanically deduced.

[equilibrium] Given the distribution of the implementation shock $f_{\Theta}(\vartheta)$, the equilibrium is a set of target tax rate τ_g^{eq} chosen by the government and attention strategies chosen by the agents such that

- (a) The target tax rate $\tau_g^{\text{eq}} \in [0, 1]$ solves the government's problem (17) given the common prior distribution $\hat{q}(\tau)$.
- (b) Attention strategies solve agents' problem (11) given the prior distribution $\hat{q}(\tau)$.
- (c) The common prior distribution $\hat{q}(\tau)$ is the pdf of $\tau_g^{\text{eq}} + b + \vartheta$.

Condition (a) and (b) guarantee that the government and the agents will not have an incentive to deviate while condition (c) ensures that agents' prior and actual tax policy are mutually consistent up to an arbitrary bias b. Indeed, condition (c) implies that the average prior is $\hat{\tau} = \tau_g^{\text{eq}} + b$ in equilibrium. Consequently, taxpayers correctly anticipate the government policy in the rational equilibrium (b = 0) and their attention strategies then reflect their willingness to observe the implementation shock ϑ – which is indeed the only information conveyed through the signals. Hence, implementation shocks are here essentially introduced to ensure that Bayesian taxpayers have an incentive to learn in equilibrium but do not otherwise play an economically meaningful role.

4.3 Commitment and the taxation bias

The discretionary equilibrium is socially suboptimal. To formalize this point, we characterize the welfare-maximizing feasible tax policy. It corresponds to the optimal policy that would be chosen by the government if it could credibly commit to a tax policy. We thus refer to it as the commitment tax policy.

Commitment policy. The commitment tax policy is the policy that would be chosen by a benevolent social planner who has the same information as the government but internalizes all equilibrium effects of tax policy. By implicitly restricting the set of tax policies to precommited policy rules, the normative literature (e.g. Farhi & Gabaix, 2020) characterizes this commitment tax policy which corresponds to the optimal tax policy in the presence of information frictions.

Formally, the commitment tax policy solves the government's problem (17) subject to the additional feasibility condition that agents' prior and actual tax policy realizations have to be mutually consistent in equilibrium (condition (c) in Definition 4.2). It is characterized by the following first order conditions.

The commitment tax policy $(\tau_q^{\star}, R_q^{\star})$ is characterized by

$$(\tau_{g}^{\star}) : E_{\vartheta} \left[\int \left\{ \underbrace{\int \left[-\frac{G'(\mathcal{V})}{p} y^{\star} + y^{\star} \right] f_{\tilde{\tau}}(\tau | \tau_{0}; w) d\tau}_{\text{mechanical \& welfare effects}} + \underbrace{\int \left[\frac{G(\mathcal{V})}{p} + \tau_{0} y^{\star} \right] \frac{df_{\tilde{\tau}}(\tau | \tau_{0}; w)}{d\tau_{g}} d\tau}_{\text{direct \& equilibrium behavioral responses}} \right\} f_{w}(w) dw \right] = 0$$
(21)

$$(R_g^{\star}) : E_{\vartheta} \left[\iint \left[\frac{G'(\mathcal{V})}{p} - 1 \right] f_{\tilde{\tau}}(\tau | \tau_0; w) f_w(w) d\tau dw \right] = 0$$

$$(22)$$

together with the resource constraint (18) and where p represents the social marginal cost of public funds. This is the policy implemented in a commitment equilibrium.²⁹

Proof. See Appendix A.3

As before, conditions have to hold in expectation because of the implementation shock

²⁹The commitment tax policy is not an equilibrium policy in the sense of Definition 4.2 because tax policy solves a different problem under commitment. Hence, notions of equilibrium under commitment implicitly refer to the equilibrium of a game in which tax policy would solve the commitment problem. The equilibrium tax policy is then simply equal to the commitment tax policy since all equilibrium adjustments are internalized in the choice of tax policy through the feasibility constraint.

 ϑ . The main difference between Propositions 4.1 and 4.3 is that the derivative $\frac{df_{\tilde{\tau}}(\tilde{\tau}|\tau_0;w)}{d\tau_g}$ in equation (21) now reflects changes in the signal received (direct adjustment) as well as changes in the prior (equilibrium adjustment). Hence, equilibrium adjustments are here internalized in the choice of tax policy.

Taxation bias. The discrepancy between the discretionary and commitment equilibria represents a taxation bias. It is a measure of the deviation from the welfare-maximizing feasible tax policy τ_q^{\star} .

[taxation bias] The taxation bias is the difference between the equilibrium tax rates under discretion τ_q^{eq} and commitment τ_q^{\star} .

The taxation bias arises as a consequence of the government's inability to internalize equilibrium adjustments in its choice of tax policy which induces a commitment problem. Proposition 4.3 relates the existence of a positive taxation bias to the associated aggregate equilibrium behavioral responses.

When both equilibria exist and are unique, there is a positive taxation bias if and only if

$$E_{\vartheta} \bigg[\iint \bigg(\frac{G(\mathcal{V})}{p} + (\tau_g^{\star} + \vartheta) y^{\star} \bigg) \bigg(\frac{df_{\tilde{\tau}}(\tau | \tau_g^{\star} + \vartheta; w)}{d\tau_g} - \frac{df_{\tilde{\tau}}(\tau | \tau_g^{\star} + \vartheta; w)}{d\tau_g} \Big|_{\hat{q}(.)} \bigg) f(w) d\tau dw \bigg] \le 0$$
(23)

Proof. τ_g^{\star} solves equation (21). Then, condition (23) implies that the left hand-side of equation (19) is ≥ 0 when evaluated at τ_g^{\star} . Hence, it directly follows from the existence and uniqueness of the discretionary equilibrium that $\tau_g^{\text{eq}} \geq \tau_g^{\star}$.

Equation (23) represents the expected change in welfare due to a marginal increase in the prior average. The term $G(\mathcal{V})/p$ stands for the welfare impact of the failure of the envelope condition. It is therefore of second order and can be overlooked when perception biases b are small. Therefore, the above condition holds whenever the expected change in aggregate tax revenue following a marginal increase in the prior average is negative (and of first order). In other words, when perception biases are small there is a positive taxation bias as long as agents tend to work less when they anticipate higher taxes – a mild condition. This shows that information frictions lead to upward distortions in actual tax policy: a discretionary government implements inefficiently high tax rates in equilibrium.

5 Gaussian illustration and sufficient statistics

This section presents an application to a setting with Gaussian implementation shocks. This allows us to derive simpler characterizations of the discretionary and commitment tax policies and to illustrate our findings with numerical simulations. We further provide a sufficient statistics formula for the taxation bias that we use to empirically assess its magnitude in the actual US economy.

5.1 Gaussian discretionary equilibrium

Let the implementation shocks be normally distributed, that is $f_{\Theta}(\vartheta)$ is the pdf of the Gaussian distribution $\mathcal{N}(0, \sigma_{\vartheta}^2)$. The common prior distribution $\hat{q}(\tau)$ is then also Gaussian in equilibrium to ensure that priors are consistent with actual tax policy realizations (condition (c) in Definition 4.2). Because the Gaussian family is self-conjugate with respect to a Gaussian likelihood, agent w posterior distribution $f_{\tilde{\tau}}(\tau|\tau_0, w)$ is Gaussian as well with (type-specific) mean $\mu = \xi \tau_0 + (1 - \xi)(\tau_g + b)$ in equilibrium. Introducing these equilibrium conditions into Proposition 4.1, we characterize the discretionary equilibrium tax policy.

4.1' Up to a first order approximation of the integrands in Proposition 4.1, the Gaussian discretionary equilibrium tax policy (τ_g^{eq}, R_g^{eq}) solves

$$E_{\vartheta} \left[\int \left\{ \underbrace{(1-g)y^{\star}}_{\text{mech. \& wel. effects}} + \underbrace{\left(g(1-\xi)(b-\vartheta) + \tau_0\right)\frac{dy^{\star}}{d\tilde{\tau}}\xi}_{\text{direct behavioral responses}} \right\} \Big|_{\tilde{\tau}=\mu} dF_w(w) \right] = 0$$
(24)

together with $E_{\vartheta} \left[\int g_{|\tilde{\tau}=\mu} dF(w) \right] = 1$, the government resource constraint (18) and where we have introduced social marginal welfare weights $g \equiv \frac{G'(\mathcal{V})}{p}$.

Proof. See Appendix A.4.

Equation (24) provides a simple expression of taxpayers' direct behavioral responses to tax changes and their impact for the discretionary equilibrium tax policy. Indeed, taxpayers adjust their earnings choice according to changes in their perceived tax rate. Behavioral responses $\frac{dy^*}{d\tilde{\tau}}$ are thus attenuated by the type-specific attention parameter ξ measuring the fraction of the change in taxes that agents observe. Moreover, the new welfare effect associated with the failure of the envelope theorem is directly proportional to the average size of the error in the posteriors $\mu - \tau_0 = (1 - \xi)(b - \vartheta)$ multiplied by social welfare weights g. It again captures the corrective motive for taxation in the presence of perception biases.

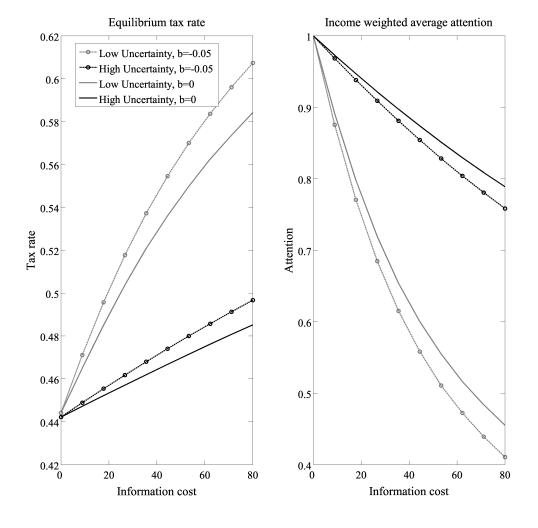


Figure 3: Gaussian discretionary equilibrium

NOTE: The left panel reports the equilibrium target tax rates for different values of the information cost κ expressed in \$ /bit /year. The right panel reports the average attention parameter ξ weighted by incomes. Low (resp. high) uncertainty corresponds to Gaussian implementation shocks with a standard deviation equal to 0.05 (resp. 0.1). b is the equilibrium perception bias in agents' prior. The government has a log social welfare function and its policy follows from Proposition 4.1. Taxpayers have an iso-elastic disutility to work $v(y,w) = (y/w)^{1+\epsilon}/(1+\epsilon)$ with $\epsilon = 1/0.33$. The distribution of skills $f_w(w)$ is calibrated using 2016 CPS data and a Pareto tail for high incomes. Figure 3 plots the tax rates (left panel) and income weighted average attention levels (right panel) in the Gaussian discretionary equilibria for different values of the information cost parameter κ . In these simulations, the distribution of skills is calibrated from the 2016 March CPS data and extended with a Pareto tail (k = 2) for incomes above \$200,000. We assume that the government has a log objective and agents have iso-elastic work disutility given by $v(y,w) = (y/w)^{1+\epsilon}/(1+\epsilon)$ where we set $\epsilon = 1/e$ with the structural elasticity parameter e = 0.33 (Chetty, 2012). A detailed presentation of the simulation procedure and the calibration strategy is available in the Online appendix.³⁰

These simulations highlight the importance of information rigidities for tax policy. Under discretion, the equilibrium tax rate (left panel) increases substantially when taxpayers are inattentive (right panel). As the information cost parameter κ increases, attention decreases and the equilibrium tax rate increases. For example, when the average attention level (weighted by incomes) is equal to 0.8, the tax rate at the rational equilibrium is 47.5% in comparison to a 44% tax rate without information frictions. Introducing a systematic downward bias of 5 percentage points in agents priors further increases the equilibrium tax rate, for instance by 1 to 2 percentage points when $\kappa =$ \$60/bit/year. The influence of the systematic perception bias b strengthens with taxpayers inattention because it is the equilibrium perception bias that matters for tax policy.

Finally, taxpayers are *ceteris paribus* less attentive when there is little prior uncertainty about the tax rate or, equivalently in equilibrium, when the variance of implementation shocks σ_{ϑ}^2 is small. In this case, the government has higher incentives to increase taxes and the discretionary equilibrium tax rate is higher. It should however be noted that the main effect of the parameter σ_{ϑ}^2 is to rescale the mapping between attention levels ξ and the information cost parameter κ . Indeed equilibrium attention strategies depend on the ratio $\kappa/\sigma_{\vartheta}^2$ as can be seen from equation (14). Therefore, once we consider pairs of $(\kappa, \sigma_{\vartheta}^2)$ that induce the same (income-weighted) average attention, tax rates in the low and high uncertainty equilibria are similar.³¹

5.2 Commitment and the taxation bias

With Gaussian implementation shocks, the characterization of the commitment tax policy can be simplified to

 $^{^{30}}$ Our simulations indicate that the loss in accuracy due to the approximation in Proposition 5.1 is very small (with our calibration). Comparing this tax rate to the one obtained directly from Proposition 4.1, the largest error is smaller than 1% in relative terms.

³¹For example, we find that when the (income-weighted) average attention level is 80%, the difference between the tax rates in the low and high uncertainty equilibria is only equal to 0.2 percentage points.

4.3' Up to a first order approximation of the integrands in Proposition 4.3, the Gaussian commitment (equilibrium) tax policy (τ_q^*, R_q^*) solves

$$E_{\vartheta} \bigg[\int \bigg\{ \underbrace{(1-g) y^{\star}}_{\text{mech. \& wel. effects}} + \underbrace{\left(g(1-\xi)(b-\vartheta) + \tau_0\right) \frac{dy^{\star}}{d\tilde{\tau}} \left(1 - \frac{d\xi}{d\tau_g}(b-\vartheta)\right)}_{\text{direct \& equilibrium behavioral responses}} \bigg\} \bigg|_{\tilde{\tau}=\mu} dF_w(w) \bigg] = 0 \qquad (25)$$

together with $E_{\vartheta} \left[\int g_{|\tilde{\tau}=\mu} dF(w) \right] = 1$, the government resource constraint (18) and where we have introduced social marginal welfare weights $g \equiv \frac{G'(\mathcal{V})}{p}$.

Proof. See Appendix A.4.

The direct and equilibrium responses to a change in taxes is now captured through the term $\frac{dy^*}{d\tilde{\tau}} \left(1 - \frac{d\xi}{d\tau_g}(b - \vartheta)\right)$. In the latter, the factor 1 stands for the fact that the government accounts for the equilibrium adjustment of priors when setting its policy. The supplemental term $\frac{d\xi}{d\tau_g}(b - \vartheta)$ captures the effect of changes in attention ξ following a marginal increase in the tax rate on equilibrium tax perceptions. This term vanishes (in expectation) when agents correctly anticipate the tax rate (b = 0) and prompts the government to decrease the tax rate when agents underestimate tax rates (b < 0). Indeed, increasing the tax rate increases attention and thereby reduces tax underestimation in equilibrium which is detrimental to efficiency.

In the left panel of Figure 4 we report the commitment equilibrium tax rates for different values of the information cost parameter κ . In the rational equilibrium (b = 0), the tax rate is only marginally higher in the presence of information frictions. Indeed, the policymaker finds it optimal to marginally increase taxes to prompt taxpayers to be more attentive to implementation shocks. In downward biased equilibria (b < 0), the government further increases the tax rate to exploit the efficiency gains from agents tax underestimation. Indeed, because taxpayers remain inattentive in equilibrium and the prior is downward biased, perceived tax rates are lower than the actual rate. Ultimately, this underestimation of tax rates reduces the efficiency costs of taxation allowing for tax increases. The leverage to increase tax rates is however limited here because the policymaker realizes that it also prompts increases in agents' prior and attention. This results in an increase of perceived tax rates which ultimately increases the efficiency costs of taxation. Consequently, commitment equilibrium tax rates are lower than discretionary equilibrium tax rates depicted in Figure 3.

As a consequence, and as depicted in the right panel of Figure 4, the taxation bias increases as the information cost parameter κ grows. Even small information frictions generate a significant taxation bias. Our simulations indicate that in a rational equilibrium, there is a taxation bias of 4 (resp. 3.5) percentage points in the presence of low (resp. high) uncer-

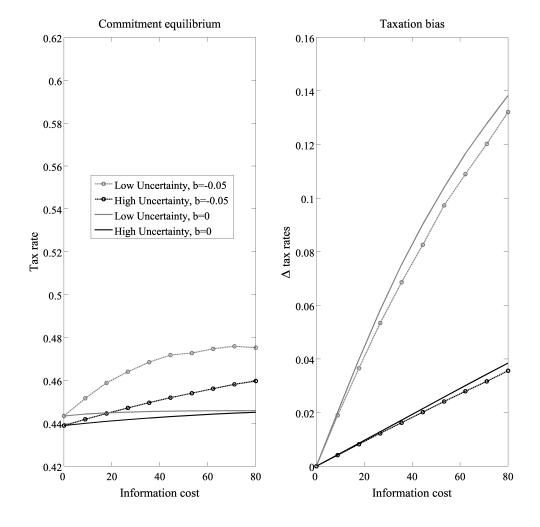


Figure 4: Taxation bias

NOTE: Difference between positive and normative tax rates for different values of the information cost κ expressed in annual \$ / bit. Low (resp. high) uncertainty corresponds to Gaussian implementation shocks with a standard deviation equal to 0.05 (resp. 0.1). b is the equilibrium perception bias in agents' prior. The government has a log social welfare function and its policy follows from Proposition 5.2 for the commitment equilibrium. Taxpayers' behavior relies on Assumptions 3.1-3.5 and an iso-elastic disutility to work $v(y,w) = (y/w)^{1+\epsilon}/(1+\epsilon)$ with $\epsilon = 1/0.33$. The distribution of skills $f_w(w)$ is calibrated using 2016 CPS data.

tainty when the income-weighted average attention parameter is 0.8. Moreover, the taxation bias is above 10 percentage points when the income-weighted average attention falls below 0.55.

The taxation bias can thus lead to significant upward distortions in actual tax rates when the income-weighted average attention turns out to be low. We now show theoretically that this is indeed a key sufficient statistic to empirically assess the magnitude of the taxation bias.

5.3 Sufficient statistics formulas and taxation bias in the US

We derive sufficient statistics formulas for the equilibrium tax policy under discretion and commitment that echo textbook optimal tax formulas and that we combine to obtain a sufficient statistics formula for the taxation bias. To obtain simple sufficient statistics formulas we further assume that preferences are iso-elastic such that the structural labor supply elasticity e - i.e. computed with respect to the perceived marginal net-of-tax rate – is constant and that implementation shocks and perception biases are small.

A sufficient statistics formula for the Gaussian discretionary equilibrium tax rate characterized in Proposition 5.1 is

$$\tau_g^{\rm eq} \simeq \frac{\overline{(1-g)y^{\star}}}{\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} \ e} - b \ \frac{\overline{g(1-\xi)y^{\star}\xi} \ e}{\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} \ e}$$
(26)

where all endogenous right hand side quantities are evaluated at τ_g^{eq} and we have introduced the mean operator $\bar{x} \equiv \int x(w) f(w) dw$.

Proof. See Appendix A.5.

The first term in equation (26) corresponds to the textbook optimal linear tax formula up to the presence of the income weighted average attention $\overline{y^*\xi}$. The second term corresponds to the corrective motive of taxation in the presence of perception biases.

A sufficient statistics formula for the Gaussian commitment equilibrium tax rate characterized in Proposition 5.2 is

$$\tau_g^{\star} \simeq \frac{\overline{(1-g)y^{\star}}}{\overline{(1-g)y^{\star}} + \overline{y^{\star}} e} - b \frac{\overline{g(1-\xi)y^{\star}} e}{\overline{(1-g)y^{\star}} + \overline{y^{\star}} e}$$
(27)

where all endogenous right hand side quantities are evaluated at τ_g^{\star} and we have introduced the mean operator $\bar{x} \equiv \int x(w) f(w) dw$.

Proof. See Appendix A.5.

The first term now exactly coincides with the textbook optimal linear tax formula while the second term again corresponds to the corrective motive of taxation in the presence of perception biases. It should however be noted that corrective terms in equations (26) and (27) are not identical. Their sign are however identical and both are proportional to the perception bias b.

Focusing on near-rational equilibria – that is equilibria with small perception biases $b \simeq 0$ such that corrective terms are second-order – we obtain a simple sufficient statistics formula for the taxation bias.

A sufficient statistics formula for the taxation bias in Gaussian near-rational equilibria is

$$\tau_g^{\rm eq} - \tau_g^\star \simeq \frac{\overline{(1-\xi)y^\star}}{(1-g)y^\star} \ e \ t^2 \tag{28}$$

where all endogenous right hand side quantities are evaluated at the actual tax rate t.

Proof. See Appendix A.5.

This simple formula for the taxation bias is reminiscent of the one provided Section 2 and is a generalization to a situation in which the government has tastes for redistribution and agents' attention is endogenous and hence type-specific.³² The income-weighted average attention $\overline{\xi y^{\star}}$ – or equivalently inattention $\overline{(1-\xi)y^{\star}}$ – thus becomes a key sufficient statistic for the taxation bias. *Ceteris paribus*, the taxation bias increases with the structural elasticity of labor supply *e*, with the square of the actual tax rate *t* and decreases with the government redistributive tastes.³³

In an attempt to gauge the empirical magnitude of the taxation bias in the actual US economy, we bring this sufficient statistics formula to the data. The meta-analysis of Gabaix (2019) combines existing measures of attention to sales taxes to trace out the evolution of average attention with the stakes. We find that income taxes in the US are well approximated by a linear tax schedule with a tax rate of t = 29.46% which would correspond to an average attention parameter of about 0.70. Focusing on the US personal income tax, Rees-Jones and Taubinsky (2020) estimate that agents' attention parameter to their marginal tax rate is equal to 0.81. Accordingly, we consider an average attention of 0.75 to taxes as our baseline.

$$\tau_g^{\text{eq}} - \tau_g^{\star} \simeq \frac{\text{cov}(\xi, y^{\star}) - \overline{(1-\xi)}\overline{y^{\star}}}{\text{cov}(g, y^{\star})} \ e \ t^2$$

 $^{^{32}}$ Equation (28) can also be expressed in terms of covariances as

³³Intuitively, the taxation bias *increases* with the government's redistributive tastes as it relatively increases the incentives to implement unanticipated tax increases. However, this first-order effect here transits through an increase in the actual tax rate t and we get the inverse relationship controlling for t.

Taxation bias		Average attention		
(percentage points)		0.65	0.75	0.85
Benthamite	0.27	5.21	3.66	2.17
redistribution	0.50	3.11	2.18	1.29
parameter	1.00	1.85	1.30	0.77

Table 7: Estimated taxation bias in the actual US economy

We are then able to compute the associated income-weighted average attention using our model of endogenous attention and the actual distribution of income.

Turning to other sufficient statistics, we take the structural elasticity parameter e = 0.33 estimated by Chetty (2012) and use an inverse optimum approach to deduce the US government's redistributive tastes from the actual tax policy.³⁴

In the actual US economy, we estimate that the taxation bias is roughly equal to 3.66 percentage points in our baseline calibration. This means that the US income tax rate is 12% higher than what would be optimal holding the government's redistributive objective constant. Table 7 provides a sensitivity analysis varying average attention and the government's redistributive objective. For the latter we use a Benthamite social welfare function for which we vary the value of the parameter that shapes the desire for redistribution. The value of 0.27 closely approximates the welfare weights we estimate using an inverse optimum approach and a value of 1 corresponds to a logarithmic social welfare function – which captures rather extreme redistributive tastes. For realistic redistributive tastes and attention parameters, the magnitude of the taxation bias in the actual US economy ranges from 1.29 to 5.21 percentage points and our baseline estimate of 3.66 lies in the middle of this range.

6 Welfare implications

This section analyzes the welfare implications of information frictions. It first decomposes the variation in aggregate social welfare between potential welfare gains that may be attained with information rigidities (commitment) and the welfare losses associated to actual policy distortions (discretion). It then quantifies the relative importance of the different

NOTE: Our estimation of the taxation bias (in percentage points) follows from the characterization in Proposition 5.3. A larger Benthamite parameter corresponds to a more redistributive objective. The value in bold corresponds to our baseline estimate for the 2016 US economy.

³⁴That is, we deduce $\overline{(1-g)y^{\star}}$ from equation (26) assuming $b \simeq 0$.

channels through which information rigidities ultimately affect welfare and redistribution at the individual level.

6.1 Information rigidities and aggregate welfare

Let $SW^{\text{eq}}(b,\kappa)$ be the social welfare from equation (17) evaluated at the discretionary equilibrium. The total welfare impact of information rigidities writes $\Delta SW^{\text{eq}}(b,\kappa) \equiv SW^{\text{eq}}(b,\kappa) - SW^{\text{eq}}(0,0)$.³⁵ It may be decomposed between the potential welfare gains from misperceptions and the welfare costs induced by the taxation bias as follows

$$\Delta SW^{\rm eq}(b,\kappa) = \underbrace{SW^{\rm eq}(b,\kappa) - SW^{\star}(b,\kappa)}_{\text{Taxation bias (<0)}} + \underbrace{SW^{\star}(b,\kappa) - SW^{\star}(0,0)}_{\text{Potential gains}}$$
(29)

where $SW^{\star}(b,\kappa)$ is the social welfare attained under commitment.

The welfare impact of the taxation bias is negative since the commitment tax policy is by definition the welfare-maximizing feasible policy. As a result, information rigidities are welfare improving if and only if this negative welfare impact is dominated by the welfare gains induced by misperceptions, that is $|SW^{eq}(b,\kappa) - SW^{\star}(b,\kappa)| \leq SW^{\star}(b,\kappa) - SW^{eq}(0,0)$.

This condition requires a downward bias in priors b < 0 such that agents underestimate tax rates. This can be easily seen when looking at the sufficient statistics formula for the commitment tax rate in equation (27). Indeed, when b = 0 the commitment tax rate is equal to the optimal tax rate without information up to a first order approximation. Therefore, there cannot be first order gains from information rigidities. However, equation (28) indicates that the taxation bias is nonetheless positive. Consequently, the total welfare impact is negative. The right panel of Figure 5 illustrates this result for Gaussian implementation shocks.³⁶

While a negative perception bias is necessary for information rigidities to be welfare improving, it is not a sufficient condition. Information rigidities should also not be too large

³⁵Note that $SW^{eq}(0,b) = SW^{eq}(0,0)$ since as soon as the information cost κ is nil, agents have perfect information and whether priors are biased is irrelevant.

³⁶Strangely enough, the potential gain is first decreasing and then increasing when b is small or nil. While the magnitude of the potential gain is small and thus negligible in comparison to the impact of the taxation bias, it deserves to be briefly explained. Consider the two extreme cases where $\kappa = 0$ and $\kappa \to \infty$. Hence, ξ is respectively equal to one or zero for each taxpayer. Everything else being equal, aggregate earnings are larger when $\kappa \to \infty$ as agents behave as if there were no implementation shocks when deciding how much to earn (individual earnings are a concave function of the perceived rate), while they fully adjust to these shocks when $\kappa = 0$. Consequently, the potential gain converges to a positive value as κ tends to infinity. However, when κ is small but strictly positive, some taxpayers noisily observe the implementation shocks so that the variance of their earnings choices increases. Ultimately, it lowers aggregate earnings and generates a negative and decreasing potential gain for small values of κ . Simulations indicate that the above described variations in aggregate earnings dominate other second order effects (e.g. misoptimization costs).

to ensure taxpayers are sufficiently attentive to tax policy. Indeed, as the information cost parameter κ grows, the welfare losses induced by the taxation bias increase more rapidly than the welfare gains from the negative perception bias. Indeed, the former is convex while the latter is concave.

The left panel of Figure 5 illustrates this mechanism with Gaussian implementation shocks. When the downward equilibrium perception bias is equal to 5 percentage points, the welfare gains induced by information rigidities dominate the welfare cost induced by the taxation bias as long as the information cost parameter κ is lower than \$25/bit/year. Above this threshold, inattention and the associated deviation from the commitment policy become too important such that information rigidities are welfare decreasing. Therefore, downwards biases in perceived marginal tax rates will be typically associated with a *decrease* – rather than an *increase* – in aggregate social welfare when agents are not sufficiently attentive to tax policy.

6.2 Redistributive impacts

We now turn to an analysis of the welfare implications of information rigidities at the individual level. Let $\Delta \mathcal{V} \equiv \mathcal{V}_{\kappa}(\tau_g^{\text{eq}}(\kappa), w) - \mathcal{V}_0(\tau_g^{\star}(0), w)$ be the variation in the expected utility of a taxpayer with skill w between the discretionary equilibrium when the information cost is κ and a counterfactual with perfect information. Using a quasi-linear and separable utility function allows us to decompose the variation in expected utility induced by information rigidities in the following way (see Appendix A.7 for precise definitions)

$$\Delta \mathcal{V} = \Delta^{\mathcal{V}} R + \Delta^{\mathcal{V}} \tau_g + \Delta^{\mathcal{V}} \text{info } \cot + \Delta^{\mathcal{V}} b + \Delta^{\mathcal{V}} \text{uncertainty}$$
(30)

that is, the welfare impact of information rigidities at the individual level arises from a variation in the demogrant R, the tax rate τ_g , the cost of information acquisition $\kappa \mathcal{I}(\sigma^*)$, the misoptimization costs induced by potential perception bias b, and the change in overall uncertainty. The increase in the demogrant has a positive effect on expected utility while all other terms are negative.

Figure 6 plots the above expected utility decomposition with Gaussian implementation shocks ($\sigma_{\vartheta} = 0.1$), no perception bias (b = 0) and an information cost κ of \$50/bit per year.

Information rigidities induce policy distortions that create losers and winners. Indeed, the redistributive impact of information rigidities is driven by the increase in the tax rate and thus in redistribution through the demogrant. This naturally benefits low skill workers at the extent of high skill workers.

Somewhat surprisingly, information costs represent a relatively small deadweight loss for

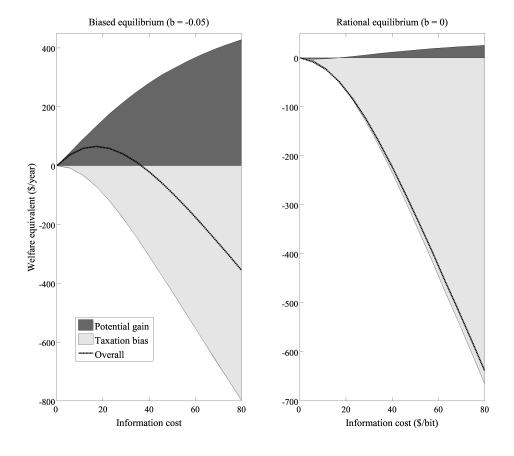


Figure 5: Welfare decomposition

NOTE: Welfare decomposition from equation (29) for different values of the information cost κ . The standard deviation for the Gaussian implementation shocks is equal to 0.05. b is the equilibrium perception bias in agents' prior. The government has a log social welfare function and its policy follows from Proposition 5.1. Taxpayers have an iso-elastic disutility to work $v(y,w) = (y/w)^{1+\epsilon}/(1+\epsilon)$ with $\epsilon = 1/0.33$ (Chetty2012). The distribution of skills $f_w(w)$ is calibrated using 2016 CPS data.

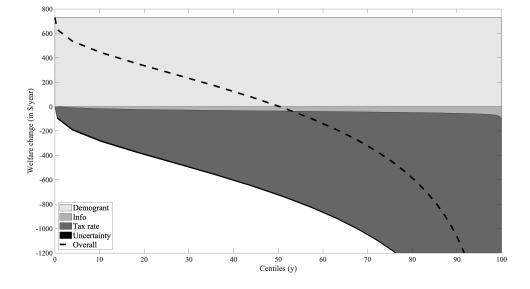


Figure 6: Variation in expected utility

NOTE: Expected utility decomposition from equation (30) by deciles of the productivity level w. The standard deviation for the Gaussian implementation shocks is equal to 0.1. There is no perception bias in agents' prior b = 0 and the information cost is $\kappa = 50$. The government has a log social welfare function and its policy follows from Proposition 5.1. Taxpayers' behavior relies on Assumptions 3.1-3.5 and an iso-elastic disutility to work $v(y,w) = (y/w)^{1+\epsilon}/(1+\epsilon)$ with $\epsilon = 1/0.33$ (Chetty2012). The distribution of skills $f_w(w)$ is calibrated using 2016 CPS data.

society in comparison to the large indirect impact of information frictions on tax policy and welfare. Moreover, it turns out that these information costs are higher for high skill workers because they have higher incentives to collect information and are thus more attentive. Extending our analysis to non-linear tax schedules, we show in the next section that this regressivity of attention has an impact on actual tax progressivity.

7 Tax progressivity and the taxation bias

In this section we extend the analysis to non-linear tax schedules. We find that the taxation bias becomes heterogeneous across income levels and ultimately reduces tax progressivity.

7.1 Introducing nonlinear tax schedules

We allow the government to use a nonlinear tax schedule T(y) but the setup introduced in Section 3 is otherwise unchanged. In particular, we maintain Assumption 1 that individuals use a linear representation of the tax schedule $\tilde{T}(y) = \tilde{\tau}y - \tilde{R}$ which now raises a new question: in the continuum of marginal tax rates $\{T'(y)\}_y$, what is the marginal tax rate $T'(y_w)$ agent w gathers information about?

Absent income effects, the perceived marginal tax rate $\tilde{\tau}_w$ remains a sufficient statistics for labor supply and uniquely pins down earnings $y^*(\tilde{\tau}_w; w)$.³⁷ Using this mapping, we define an agent w ex ante – before information acquisition – optimal earnings level $\hat{y}_w = y^*(\hat{\tau}_w; w)$ and make the following assumption.

Assumption 4 (prior reliance). Taxpayer w gathers information about the actual marginal tax rate $\tau_0(\hat{\tau}_w, w) \equiv T'_0(\hat{y}_w)$ at her ex-ante optimal earnings level \hat{y}_w .

Essentially, Assumption 4 guarantees the internal consistency of the perception formation process by ensuring agents have no additional information ex ante than that contained in their prior. Moreover, it gives a novel allocative role for the prior as taxpayers now linearize the tax schedule around the income level that they deem optimal ex ante.³⁸

³⁷While the perceived marginal tax rate $\tilde{\tau}$ was already type-specific in the previous sections – through type-specific attention choices –, we here introduce the subscript w to emphasize that it will in addition be type-specific through agents' type-specific priors (see Assumption 4).

³⁸To illustrate this new allocative role, consider the limit where the information cost κ goes to zero. Perceptions are then perfect $\tilde{\tau}_w = \tau_0(\hat{\tau}_w, w)$ and each agent chooses earnings $y_{w|\hat{\tau}_w} = y^*(\tau_0(\hat{\tau}_w, w); w)$. This is in contrast to the full information case in which earnings are the solution to a fixed-point problem characterized by $y_w = y^*(T'_0(y^*(.)); w)$. In a rational equilibrium (b = 0), both income concepts coincide. They will however differ in biased equilibria $b \neq 0$.

The presence of a nonlinear tax schedule does not fundamentally affect equilibrium concepts. Two refinements are nevertheless necessary. First, for the sake of simplicity we assume that implementation shocks ϑ uniformly affect marginal tax rates at all earnings levels y such that $T'_0(y) = T'_g(y) + \vartheta$. Second, the equilibrium condition (c) from Definition 4.2 – which characterizes the equilibrium adjustment of priors – now becomes

(c') The type-specific prior distribution $\hat{q}_w(\tau)$ is the pdf of $T'_a(\hat{y}_w) + b + \vartheta$.

That is, each taxpayer's prior is consistent with her marginal tax rate of interest up to an arbitrary perception bias b. Incidentally, the prior average $\hat{\tau}_w \equiv E_{\hat{q}_w(.)}[\tau]$ is thus necessarily type-specific in equilibrium when the government implements a nonlinear tax schedule. While natural in our context, this poses a potential challenge for the resolution of this nonlinear tax model.

We rely on a perturbation approach in order to derive the optimal tax schedule. Following Jacquet and Lehmann (2020), one needs three assumptions to solve for the optimal non-linear tax schedule using a tax perturbation approach. (i) The tax function $T_g(.)$ must be twice differentiable. (ii) The optimization program of each taxpayer must admit a unique global maximum. (iii) Agents' second-order conditions must hold strictly. While the first two conditions are generic requirements to ensure the global smoothness of the problem so that tax perturbations will not induce individuals to jump between different maxima, the third condition has less intuitive consequences.

In standard models, condition (iii) – combined with a single crossing assumption on individuals preferences – ensures the existence of an increasing mapping between earnings yand skills w. This is known as a monotonicity condition on allocations.³⁹ It is a requirement for the tax perturbation approach which disciplines the curvature of the tax function $T''_g(.)$. Here, allowing for type-specific priors and a perception bias b poses a potential threat to the existence of an increasing mapping between earnings y and skills w. In the Online Appendix, we show that under our assumptions the monotonicity condition is also expected to hold when $T''_g(.)$ is smooth enough. As a result, we solve for the optimal tax schedule assuming the monotonicity condition is verified and check ex post that it holds at the optimum.

³⁹It follows from agents incentive constraints in a mechanism design approach.

7.2 ABCD tax formula

We can now solve for the optimal nonlinear tax schedule. The government chooses a target nonlinear tax schedule $T_g(.)$ that consists in a continuum of marginal tax rates $\{T'_g(y)\}_y$ and a tax level indexed by the demogrant $T_g(0)$. It is implemented up to an implementation shock ϑ on marginal tax rates and the tax level adjusts such as to satisfy the government budget constraint ex post. The government problem writes

$$\max_{T'_g(.),T_g(0)} \quad E_{\vartheta} \left[\iint G \Big(\mathcal{V}(\tilde{\tau}_w, T_0(.); \kappa, w) \Big) \ f_{\tilde{\tau}}(\tau | \tau_0(\hat{\tau}_w, w); w) \ f_w(w) \ d\tau dw \right]$$
(31)

s.t.
$$\iint T_0(y^{\star}(\tilde{\tau}_w; w)) \ f_{\tilde{\tau}}(\tau | \tau_0(\hat{\tau}_w, w); w) \ f_w(w) \ d\tau dw \ge E$$
(32)

where E is an exogenous expenditure requirement, $f_{\tilde{\tau}}(\tau | \tau_0(\hat{\tau}, w); w)$ is the posterior distribution of agent w perceived tax rate and with the indirect utility function

$$\mathcal{V}(\tilde{\tau}_w, T_0(.); \kappa, w) = y^*(\tilde{\tau}_w; w) - T_0(y^*(\tilde{\tau}_w; w)) - v\left(y^*(\tilde{\tau}_w; w); w\right) - \kappa \mathcal{I}(\sigma^*)$$
(33)

We solve this problem using a perturbation approach. Namely, we consider the effect of a reform that consists in a small increase $\Delta \tau^r$ in marginal tax rates in a small bandwidth of earnings $[y^r - \Delta y, y^r]$ and characterize its impact on the objective function of the government. Following the tax perturbation literature, this reform may be apprehended through three mechanisms: a mechanical effect, a welfare effect and a behavioral effect. However, analyzing the impact of a reform in this setting with information frictions in tax perceptions calls for a careful identification of the agents affected by the reform.

The standard mechanical and welfare effects capture the change in taxes and welfare for individuals w whose earnings are higher than y^r given their perceived tax rates $\tilde{\tau}_w$. Following from the aforementioned monotonicity condition, it corresponds to all agents with a productivity $w \ge w^r$ where $y^*(\tilde{\tau}_{w^r}; w^r) \equiv y^r$. In contrast, the behavioral effect comes from taxpayers who are learning the marginal tax rate affected by the reform. That is, all agents whose ex ante optimal earnings level \hat{y} belong to $[y^r - \Delta y, y^r]$. Again using the monotonicity condition, we can equivalently identify these agents as those with a productivity $w \in [\hat{w}^r - \Delta \hat{w}, \hat{w}^r]$ where $y^*(\hat{\tau}_{\hat{w}^r}; \hat{w}^r) \equiv y^r$. Note that the two cut-offs w^r and \hat{w}^r differ almost surely.⁴⁰

⁴⁰The two cut-offs coincide only when $\hat{\tau}_w = \tilde{\tau}_w$. That is, when b = 0 and $\vartheta = 0$. Since we focus on Gaussian implementation shocks here, it is never the case (a.s.).

We then characterize the discretionary and commitment equilibrium tax schedules assuming Gaussian implementation shocks. As before, these conditions are easier to interpret after applying a small implementation shocks approximation which is what we report in Proposition 7.2, relegating general conditions to the Online Appendix.

[ABCD formula] Assuming small Gaussian implementation shocks, the equilibrium nonlinear tax schedule is to a first-order approximation characterized by

$$\frac{T'_{g}(y^{\star}(\mu_{\hat{w}^{r}};\hat{w}^{r})) + g(\hat{w}^{r})|_{\tilde{\tau}=\mu_{\hat{w}^{r}}} \left(\mu_{\hat{w}^{r}} - T'_{g}(y^{\star}(\mu_{\hat{w}^{r}};\hat{w}^{r}))\right)}{1 - \mu_{\hat{w}^{r}}} = \frac{1}{e^{\frac{d\mu_{\hat{w}^{r}}}{d\tau_{g}}}|_{\tilde{\tau}=\mu_{\hat{w}^{r}}}} \frac{1}{y^{\star}(\mu_{\hat{w}^{r}};\hat{w}^{r})} \frac{\frac{dy^{\star}(\hat{\tau}_{\hat{w}^{r}};\hat{w}^{r})}{dw}}{f_{w}(\hat{w}^{r})} \int_{w^{r}}^{\infty} \left(1 - g(w)|_{\tilde{\tau}=\mu_{w}}\right) f_{w}(w) \, dw \tag{34}$$

together with the transversality condition $\int g(w)_{|_{\bar{\tau}_w = \mu_w}} dF(w) = 1$ and the government budget constraint (32), where all endogenous quantities are evaluated at their equilibrium values.

Moreover the ex-post average perceived marginal tax rate is $\mu_w \equiv \xi \tau_0(\hat{\tau}_w, w) + [1 - \xi] \hat{\tau}_w$ such that $\frac{d\mu_w}{d\tau_g} = \xi$ under discretion and $\frac{d\mu_w}{d\tau_g} = 1 + \frac{d\xi}{d\tau_g} [\tau_0(\hat{\tau}_w, w) - \hat{\tau}_w]$ under commitment.

Proof. See Online Appendix.

Under commitment and absent perception biases (b = 0), the ABCD formula boils down to the ABC formula derived in Diamond (1998) and the standard interpretation prevails.⁴¹ The presence of perception biases $(b \neq 0)$ has several effects. First, it creates a disconnect between w^r and \hat{w}^r new to this non-linear setting. Second, it adds a welfare effect $(g(\hat{w}^r)(1-\xi)b$ in the numerator of the LHS) related to the failure of the envelope theorem when agents misoptimize. Third, it adds an efficiency term (fraction on the RHS with $\frac{d\mu_w}{d\tau_g} = 1 + \frac{d\xi}{d\tau_g} [\tau_0(\hat{\tau}_w, w) - \hat{\tau}_w]$ in the denominator) accounting for the variation in agents equilibrium misperception when their attention ξ changes in response to tax reforms.

As before, the emergence of a taxation bias comes from the discrepancy between the estimated impact of a reform under discretion and under commitment. Under discretion, the government fails to internalize the equilibrium impact of the reform on perceptions and accordingly considers that an increase $\Delta \tau^r$ in marginal tax rates only increases perceived marginal tax rates by $\xi \Delta \tau^r \leq \Delta \tau^r$. Increasing marginal tax rates is thus perceived as less costly in terms of efficiency than it really is. As a result, marginal tax rates are in a discretion equilibrium higher than in a commitment equilibrium. In other words, marginal tax rates are higher than they should be from a normative perspective: this is the taxation bias.

⁴¹The additional term on the RHS disappears since $\frac{d\mu_w}{d\tau_g} = 1$ while we have that $\hat{\tau}_w^{\text{eq}} \approx \tilde{\tau}_w^{\text{eq}}$ ensuring both $w^r \approx \hat{w}^r$ and $\tilde{\tau}_w^{\text{eq}} \approx T'_g(y^*(\mu_{\hat{w}^r}^{\text{eq}}; \hat{w}^r))$.

What is new to this non-linear setting is that the taxation bias affecting the marginal tax rate $T'_g(y^r)$ at a given level of earnings y^r is driven by the attention level of agents of type \hat{w}^r . Surprisingly, agents \hat{w}^r may not even be located at earnings y^r in the presence of perception biases. More importantly, if attention levels vary across the earnings distribution, the taxation bias will have an impact of the progressivity of the tax schedule.

7.3 Numerical illustration

To illustrate this property we represent in Figure 7 the target nonlinear tax schedules implemented under a discretion equilibrium (actual tax policy – dashed black line) and under commitment (optimal tax policy – full black line). Simulations are carried out absent systematic perception biases (b = 0) such that the optimal nonlinear tax schedule corresponds to the textbook optimal nonlinear tax schedule of **Saez2001**. We thus naturally retrieve the known U-shape pattern of marginal tax rates.

Because of the taxation bias, marginal tax rates are higher under discretion than under commitment. Strikingly, this difference in marginal tax rates is not constant across earnings levels. For instance, agents located at the first decile (resp. the median) of the earnings distribution face a marginal tax rate of 50% (resp. 44%) under commitment and a marginal tax rate of 63% (resp. 52%) under discretion. In contrast, the marginal tax rate faced by individuals in the top decile (resp. top percentile) increases by at most 4 (resp. 1) percentage points. This reflects the impact of the taxation bias on the progressivity of the tax schedule coming from the variation in attention ξ across earnings.

Attention levels represented in Figure 7 (grey lines) are indeed generally increasing in earnings. In our model, more productive agents have intuitively more latitude to choose the earnings level they see fit and attach thus a higher value to being informed about the tax schedule. As a result, attention globally increases with productivity and thus – through the monotonicity condition – with earnings.⁴² Note that this pattern is obtained assuming all individuals have the same cost of information κ . Therefore, assuming that more able workers are also more efficient at collecting information would only reinforce the striking result that, because it decreases with income, inattention to taxes induces regressive tax increases.

⁴²The pattern reverts at the very beginning of the earnings distribution because as the marginal tax rate approaches one, we approach the origin of the labor supply function where earnings become infinitely responsive to changes in the marginal tax rates. As a result, very low productive agents end up choosing very high attention levels.

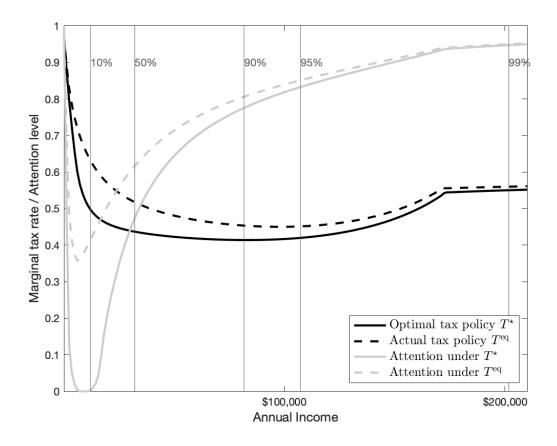


Figure 7: Non-linear tax schedules

NOTE: Target non-linear tax schedules (black curves) and attention levels (grey curves) under a discretion equilibrium (actual tax policy) and under commitment (optimal tax policy) for a value of the information cost $\kappa = 30$ \$/*bit/yr*. The government has a log social welfare function and follows the optimal policy from Proposition 7.2. Taxpayers' have an iso-elastic disutility to work $v(y,w) = (y/w)^{1+\epsilon}/(1+\epsilon)$ with $\epsilon = 1/0.33$ (Chetty2012). The distribution of skills $f_w(w)$ is calibrated using 2016 CPS data and extended with a Pareto-tail of parameter a = 2 (Saez2001).

Conclusion

We develop a positive theory of tax policy in the presence of information frictions and show that agents' inattention to taxes leads to a taxation bias. We find that this taxation bias is undesirable, large and regressive. Because it reflects a commitment problem, our results suggest that the welfare gains from using precommited tax rules could be large. Alternatively, the model identifies a key parameter to limit the government's deviations from the optimal tax policy: the information cost κ . Therefore, it would be interesting for future research to investigate the determinants behind information costs driving agents' inattention. Indeed, if the latter are related to the complexity of tax systems, monitoring and restricting tax complexity may be a simple and effective way to prevent the implementation of such inefficient and regressive tax increases.

While some of our results may be model-specific, our analysis sheds a new light on the welfare consequences of information frictions in agents' tax perceptions. It underlines that downward biases in tax perceptions are not necessarily welfare improving. They do lower the efficiency costs of taxation in existing tax systems, but existing tax systems without misperceptions are arguably not the right counterfactual to use for welfare analysis. Indeed, there may be other (equilibrium) effects at play – which here take the form of a taxation bias. We show that the welfare consequences of such effects may be dominant thereby delivering counterintuitive welfare implications. One should thus be very careful with the welfare implications drawn from the measurement of misperceptions. We believe that this general lesson applies outside of the realm of taxation.

A Appendix

A.1 Solution to the Toy Model with Imperfect Information

The government seeks to maximize tax revenue taking the prior as given. Its problem writes $\max_{\tau} \tau Y(1-\tilde{\tau})$ such that $\tilde{\tau} = \xi \tau + (1-\xi)\hat{\tau}, \{\tau, \hat{\tau}\} \in [0,1]^2$ and $\xi \in (0,1)$. The associated Lagrangian is $\mathscr{L}(\tau, \lambda) = \tau Y(1-\xi\tau-(1-\xi)\hat{\tau}) + \lambda(\tau-1)$. Following from the first order Kuhn and Tucker conditions, $\tau = 1$ if and only if $\hat{\tau} \leq 1 - \frac{\xi}{1-\xi}e$ and $\tau = \frac{1-(1-\xi)\hat{\tau}}{\xi(1+e)}$ otherwise. These conditions are also sufficient since the problem is convex under the assumption that $\tau Y(1-\tau)$ is concave.

At the rational equilibrium, the prior is correct $\hat{\tau} = \tau^*$. Guess that the rational equilibrium is interior. Hence, $\tau^* = \frac{1}{1+\xi e}$. Because e > 0, it implies that $\hat{\tau} > 1 - \frac{\xi}{1-\xi}e$ in equilibrium, thus confirming the guess. It is then straightforward to prove that $\tau^*Y(1-\tau^*) < \tau^rY(1-\tau^r)$ where $\tau^r \equiv \frac{1}{1+e}$ as $\tau^r = \arg \max_{\tau \in [0,1]} \tau Y(1-\tau)$. Moreover, the taxation bias $\tau^* - \tau^r = \frac{(1-\xi)e}{(1+\xi e)(1+e)}$ is strictly positive for all $\xi \in (0, 1)$.

A.2 Reformulation of the Tractable Gaussian Learning

The indirect utility of a taxpayer is given by equation (12). Performing a second order Taylor approximation of the latter around τ_0 gives

$$V_{\tau_0}^2(\tilde{\tau}, \tau_0, R_0; w) = V(\tau_0, \tau_0, R_0; w) + (\tilde{\tau} - \tau_0) \frac{\partial V}{\partial \tilde{\tau}} \Big|_{\tilde{\tau} = \tau_0} + \frac{(\tilde{\tau} - \tau_0)^2}{2} \frac{\partial^2 V}{\partial \tilde{\tau}^2} \Big|_{\tilde{\tau} = \tau_0}$$
(35)

where $\frac{\partial V}{\partial \tilde{\tau}}|_{\tilde{\tau}=\tau_0} = 0$ and $\frac{\partial^2 V}{\partial \tilde{\tau}^2}\Big|_{\tilde{\tau}=\tau_0} = \frac{\partial^2 y^*}{\partial \tilde{\tau}^2}$ from (7). Hence,

$$\iint V_{\tau}^{2}(\tilde{\tau},\tau,R;w)\phi(s;\tau,\sigma)\phi(\tau;\hat{\tau},\hat{\sigma})dsd\tau = \int \left[V(\tau,\tau,R;w) + \frac{\tilde{\sigma}^{2}}{2} \frac{\partial^{2}y^{\star}}{\partial\tilde{\tau}^{2}} \Big|_{\tilde{\tau}=\tau} \right] \phi(\tau;\hat{\tau},\hat{\sigma})d\tau$$

where $\tilde{\sigma}^2$ is the posterior variance and we are using the fact that with a Gaussian prior and a Gaussian signal, the posterior is also Gaussian. Accordingly, the expected information reduction writes

$$\mathcal{I}(\sigma) = \frac{1}{2} \Big(\log(2\pi e \hat{\sigma}^2) - \log(2\pi e \tilde{\sigma}^2) \Big) = \frac{1}{2} \log \frac{\hat{\sigma}^2}{\tilde{\sigma}^2}$$
(36)

where $\frac{1}{2}\log(2\pi e\sigma^2)$ is the differential entropy (in bits) of a Gaussian distribution with variance σ^2 . Therefore, in a Gaussian model, problem (11) becomes

$$\max_{\tilde{\sigma} \ge \hat{\sigma}} \quad \tilde{\sigma}^2 \int \frac{\partial^2 y^{\star}}{\partial \tilde{\tau}^2} \Big|_{\tilde{\tau} = \tau} \phi(\tau; \hat{\tau}, \hat{\sigma}) d\tau - \kappa \log \frac{\hat{\sigma}^2}{\tilde{\sigma}^2}$$
(37)

This problem has been extensively studied in the literature. For instance, a step-by-step derivation of the solution is provided in Maćkowiak, Matějka, Wiederholt, et al. (2018). It shows that the perceived tax rate is $\tilde{\tau} = \xi s + (1 - \xi)\hat{\tau}$ where $\xi \in [0, 1]$ is a measure of the attention level set optimally to

$$\xi = \max\left(0, 1 + \frac{\kappa}{\hat{\sigma}^2 \int \frac{\partial^2 y^{\star}}{\partial \tilde{\tau}^2} \Big|_{\tilde{\tau}=\tau} \phi(\tau; \hat{\tau}, \hat{\sigma}) d\tau}\right)$$
(38)

A.3 Proofs of Proposition 4.1 and 4.3

We here prove both propositions at the same time since the only difference between the two problems is in the nature of responses to tax changes that are taken into account. We thus solve the general problem where all agents' responses are taken into account (including equilibrium adjustments) to obtain Proposition 4.3 and from which Proposition 4.1 naturally follows.

The Lagrangian associated to problem (17) writes

$$\mathscr{L}(\tau_g, R, p) = E_{\vartheta} \bigg[\iint \bigg[G \Big(\mathcal{V}(\tilde{\tau}, \tau_g + \vartheta, R, \kappa; w) \Big) + p \Big((\tau_g + \vartheta) y^{\star}(\tilde{\tau}; w) - R_0 - E \Big) \bigg] f_{\tilde{\tau}}(\tilde{\tau} | \tau_g + \vartheta; w) f_w(w) d\tilde{\tau} dw \bigg]$$
(39)

The first-order condition associated with the choice of the marginal tax rate τ_g is

$$\frac{1}{p}\frac{d\mathscr{L}}{d\tau_g} = E_{\vartheta} \left[\int \left\{ \int \left[\frac{G'(\mathcal{V})}{p} \frac{d\mathcal{V}}{d\tau_g} + y^{\star} \right] f_{\tilde{\tau}}(\tilde{\tau} | \tau_g + \vartheta; w) d\tilde{\tau} + \int \left[\frac{G(\mathcal{V})}{p} + (\tau_g + \vartheta) y^{\star} - R_0 - E \right] \frac{df_{\tilde{\tau}}(\tilde{\tau} | \tau_g + \vartheta; w)}{d\tau_g} d\tilde{\tau} \right\} f_w(w) dw \right]$$
(40)

where $\frac{df_{\tilde{\tau}}(\tilde{\tau}|\tau_g+\vartheta;w)}{d\tau_g}$ is the change in the posterior distribution of perceived tax rate for type w and captures agents' responses to tax changes.

By definition $\int f_{\tilde{\tau}}(\tilde{\tau}|\tau_g + \vartheta; w) d\tilde{\tau} = 1$ thus $\int \frac{df_{\tilde{\tau}}(\tilde{\tau}|\tau_g + \vartheta; w)}{d\tau_g} d\tilde{\tau} = 0$. Moreover, the quasilinearity of utility implies that $\frac{d\mathcal{V}}{d\tau_g} = -y^*(\tilde{\tau}; w)$. Therefore, the optimality condition $\frac{1}{p} \frac{d\mathscr{L}}{d\tau_g} = 0$ writes

$$E_{\vartheta} \left[\int \left\{ \int \left[-\frac{G'(\mathcal{V})}{p} y^{\star} + y^{\star} \right] f_{\tilde{\tau}}(\tilde{\tau} | \tau_g + \vartheta; w) d\tilde{\tau} + \int \left[\frac{G(\mathcal{V})}{p} + (\tau_g + \vartheta) y^{\star} \right] \frac{df_{\tilde{\tau}}(\tilde{\tau} | \tau_g + \vartheta; w)}{d\tau_g} d\tilde{\tau} \right\} f_w(w) dw \right] = 0$$

$$(41)$$

This is equation (21) from Proposition 4.1 and characterizes the commitment tax rate. Equation (19) from Proposition 4.3 which characterizes the tax rate chosen by a discretionary government is obtained by when agents' responses to a change in the tax rate is computed holding agents' prior \hat{q} constant. That is $\frac{df_{\tilde{\tau}}(\tilde{\tau}|\tau_g+\vartheta;w)}{d\tau_g}\Big|_{\hat{q}(.)}$ replaces $\frac{df_{\tilde{\tau}}(\tilde{\tau}|\tau_g+\vartheta;w)}{d\tau_g}$ in equation (41).

The first-order condition associated with the choice of the demogrant R is

$$\frac{1}{p}\frac{d\mathscr{L}}{dR} = E_{\vartheta} \left[\iint \left[\frac{G'(\mathcal{V})}{p} \frac{d\mathcal{V}}{dR} - 1 \right] f_{\tilde{\tau}}(\tilde{\tau} | \tau_g + \vartheta; w) f_w(w) d\tilde{\tau} dw \right]$$
(42)

By quasi-linearity we have $\frac{d\mathcal{V}}{dR} = 1$. The optimality condition $\frac{1}{p}\frac{d\mathscr{L}}{dR} = 0$ thus writes

$$E_{\vartheta}\left[\iint \left[\frac{G'(\mathcal{V})}{p} - 1\right] f_{\tilde{\tau}}(\tilde{\tau}|\tau_g + \vartheta; w) f_w(w) d\tilde{\tau} dw\right] = 0$$
(43)

This is equation (20) from Proposition 4.1 and equation (22) from Proposition 4.3.

A.4 Optimal policies in tractable Gaussian case

Conditions (41) and (43) apply to any learning leading to a differentiable posterior distribution of perceptions $f_{\tilde{\tau}}(\tau|\tau_0; w)$ with positive support on [0, 1], where $\tau_0 = \tau_g + \vartheta$. Further insights may be gained by using a tractable Gaussian learning (Assumption 3). Indeed, in this case $f_{\tilde{\tau}}(\tau|\tau_0; w)$ is a Gaussian pdf $\phi(\tau; \mu, \sigma^2)$ with mean $\mu = \xi \tau_0 + (1 - \xi)\hat{\tau}$ and variance $\sigma^2 = \sigma^{*2}$. We can thus express agents' responses to tax reforms in terms of changes in the true tax rate τ_0 , changes in the prior mean $\hat{\tau}$ and induced changes in attention ξ that correspond to changes in the precision of the signal σ^* . To do so, we use a first-order approximation of the objective at the mean μ and exploit the following Lemma.

Let $\psi(x)$ be a differentiable real-valued function, $\psi_a(x) = \psi(a) + (x-a)\psi'(a)$ its first-order Taylor approximation evaluated at a and $\phi(x; \mu, \sigma^2)$ the pdf of the Gaussian distribution with mean μ and variance σ^2 . Then,

$$\int_{\mathbb{R}} \psi_{\mu}(x)\phi(x;\mu,\sigma^2)dx = \psi(\mu)$$
(44)

$$\int_{\mathbb{R}} \psi_{\mu}(x) \frac{\partial \phi(x; \mu, \sigma^2)}{\partial \mu} dx = \psi'(\mu)$$
(45)

$$\int_{\mathbb{R}} \psi_{\mu}(x) \frac{\partial \phi(x; \mu, \sigma^2)}{\partial \sigma} dx = 0$$
(46)

Proof. Equation (44) directly follows from $\int_{\mathbb{R}} (x-\mu)\phi(x;\mu,\sigma^2) = 0$ by definition of the mean. To prove equation (45), realize that $\int_{\mathbb{R}} \frac{\partial\phi(x;\mu,\sigma^2)}{\partial\mu} dx = 0$ and $\frac{\partial\phi(x;\mu,\sigma^2)}{\partial\mu} = \frac{x-\mu}{\sigma^2}\phi(x;\mu,\sigma^2)$ so that $\int_{\mathbb{R}} (\psi(\mu) + (x-\mu)\psi'(\mu))\frac{\partial\phi(x;\mu,\sigma^2)}{\partial\mu} dx = \frac{\psi'(\mu)}{\sigma^2} \int_{\mathbb{R}} (x-\mu)^2\phi(x;\mu,\sigma^2) = \psi'(\mu)$. Equation (46) follows from the fact that $\int_{\mathbb{R}} \frac{\partial\phi(x;\mu,\sigma^2)}{\partial\sigma} dx = 0$ such that the integral of a constant is nil and that $\frac{\partial\phi(x;\mu,\sigma^2)}{\partial\sigma}$ is symmetric such that the integral of x also nil by a symmetry argument. \Box

Rewriting equation (41) as

$$E_{\vartheta} \left[\int \left\{ \int \left[-\frac{G'(\mathcal{V})}{p} y^{\star} + y^{\star} \right] \phi(\tau; \mu, \sigma^2) d\tau + \int \left[\frac{G(\mathcal{V})}{p} + \tau_0 y^{\star} \right] \left(\frac{d\phi(\tau; \mu, \sigma^2)}{d\mu} \frac{d\mu}{d\tau_g} + \frac{d\phi(\tau; \mu, \sigma^2)}{d\sigma} \frac{d\sigma}{d\tau_g} \right) d\tau \right\} f_w(w) dw \right] = 0$$

$$(47)$$

allows us to apply Lemma A.4 to obtain with $\mu = \xi \tau_0 + (1 - \xi)\hat{\tau}$

$$E_{\vartheta} \left[\int \left\{ \left[-\frac{G'(\mathcal{V})}{p} y^{\star} + y^{\star} \right] \Big|_{\tilde{\tau}=\mu} + \left[\left(\frac{G'(\mathcal{V})}{p} (\tilde{\tau} - \tau_0) + \tau_0 \right) \frac{dy^{\star}}{d\tilde{\tau}} \frac{d\mu}{d\tau_g} \right] \Big|_{\tilde{\tau}=\mu} \right\} f_w(w) dw \right] = 0$$
(48)

since taking $\psi(\tau) = \left[\frac{G(\mathcal{V})}{p} + \tau_0 y^\star\right](\tau)$ implies $\psi'(\mu) = \left[\left(\frac{G'(\mathcal{V})}{p}(\tilde{\tau} - \tau_0) + \tau_0\right)\frac{dy^\star}{d\tilde{\tau}}\right](\mu)$ by the modified envelope condition. Recall that $\mu = \xi\tau_0 + (1-\xi)\hat{\tau}$. Now, in equilibrium we have by definition that $\hat{\tau} = \tau_g + b$ meaning $\mu = \tau_g + \xi\vartheta + (1-\xi)b$ and $\mu - \tau_0 = (1-\xi)(b-\vartheta)$. Hence, in equilibrium,

$$E_{\vartheta} \left[\int \left\{ \left[-\frac{G'(\mathcal{V})}{p} y^{\star} + y^{\star} \right] \Big|_{\tilde{\tau} = \tau_g + \xi \vartheta + (1-\xi)b} + \left[\left(\frac{G'(\mathcal{V})}{p} (1-\xi)(b-\vartheta) + \tau_g + \vartheta \right) \frac{dy^{\star}}{d\tilde{\tau}} \frac{d\mu}{d\tau_g} \right] \Big|_{\tilde{\tau} = \tau_g + \xi \vartheta + (1-\xi)b} \right\} f_w(w) dw \right] = 0$$

$$(49)$$

Last, we characterize taxpayers' average response to tax reforms $\frac{d\mu}{d\tau_g}$ as computed under

discretion and commitment. Under discretion, the policymaker takes agent's priors and thus attention strategies as given, hence $\frac{d\mu}{d\tau_g} = \xi \frac{d\tau_0}{d\tau_g} = \xi$ which yields equation (24). Under commitment, the policymaker internalizes the equilibrium condition that priors and thus attention strategies adjust to the tax policy such that $\frac{d\mu}{d\tau_g} = \xi \frac{d\tau_0}{d\tau_g} + (1-\xi) \frac{d\hat{\tau}}{d\tau_g} + \frac{d\xi}{d\tau_g} (\tau_0 - \hat{\tau}) = 1 + \frac{d\xi}{d\tau_g} (\vartheta - b)$ in equilibrium. This yields equation (25).

Transversality conditions follow from a direct application of Lemma A.4 to equation 43 with again $\mu = \tau_g + \xi \vartheta + (1 - \xi)b$:

$$E_{\vartheta}\left[\int \frac{G'(\mathcal{V})}{p}\Big|_{\tilde{\tau}=\mu}f(w)dw\right] = 1$$
(50)

A.5 Sufficient statistics formulas in tractable Gaussian case

Taking a small noise approximation, characterizations of equilibrium tax rates under discretion τ_g^{eq} and commitment τ_g^{\star} in this tractable Gaussian model write

$$\int \left[\left(1 - \frac{G'(\mathcal{V})}{p} \right) y^* + \left(\frac{G'(\mathcal{V})}{p} (1 - \xi) b + \tau_g^{\text{eq}} \right) \frac{dy^*}{d\tilde{\tau}} \xi \right] \Big|_{\tilde{\tau} = \tau_g^{\text{eq}} + (1 - \xi)b} f_w(w) dw = 0$$
$$\int \left[\left(1 - \frac{G'(\mathcal{V})}{p} \right) y^* + \left(\frac{G'(\mathcal{V})}{p} (1 - \xi) b + \tau_g^* \right) \frac{dy^*}{d\tilde{\tau}} \left(1 - \frac{d\xi}{d\tau_g} b \right) \right] \Big|_{\tilde{\tau} = \tau_g^* + (1 - \xi)b} f_w(w) dw = 0$$

Assuming preferences are iso-elastic, $U(c, y; w) = c - \frac{(y/w)^{1+\varepsilon}}{1+\varepsilon}$, the elasticity of earnings with respect to the perceived marginal net-of-tax rate e is constant

$$\forall \tilde{\tau}, w, \quad e \equiv \frac{1 - \tilde{\tau}}{y^{\star}} \frac{dy^{\star}}{d(1 - \tilde{\tau})} = \frac{1}{\varepsilon} \iff \frac{dy^{\star}}{d\tilde{\tau}} = -e \frac{y^{\star}}{1 - \tilde{\tau}} \tag{51}$$

Plugging in e we get

$$\int \left[\left(1 - \frac{G'(\mathcal{V})}{p}\right) y^{\star} - \left(\frac{G'(\mathcal{V})}{p}(1-\xi)b + \tau_g^{\text{eq}}\right) e^{\frac{y^{\star}}{1-\tilde{\tau}}\xi} \right] \Big|_{\tilde{\tau}=\tau_g^{\text{eq}}+(1-\xi)b} f_w(w)dw = 0$$
$$\int \left[\left(1 - \frac{G'(\mathcal{V})}{p}\right) y^{\star} - \left(\frac{G'(\mathcal{V})}{p}(1-\xi)b + \tau_g^{\star}\right) e^{\frac{y^{\star}}{1-\tilde{\tau}}\left(1-\frac{d\xi}{d\tau_g}b\right)} \right] \Big|_{\tilde{\tau}=\tau_g^{\star}+(1-\xi)b} f_w(w)dw = 0$$

To further simplify these formulas we now make a small perception bias approximation $b \ll 1$. This allows us to use the approximation $\frac{1}{1-\tau_g-(1-\xi)b} \approx \frac{1}{1-\tau_g}$ and to assume $\frac{d\xi}{d\tau_g}b \ll 1$

to simplify some terms⁴³. Defining social marginal welfare weights $g(w) \equiv \frac{G'(\mathcal{V})}{p}$ and the mean operator $\bar{x} = \int x(w) f(w) dw$ we get

$$\left\{ \overline{(1-g)y^{\star}} - \frac{\tau_g^{\rm eq}}{1-\tau_g^{\rm eq}} \overline{y^{\star}\xi} e - \frac{b}{1-\tau_g^{\rm eq}} \overline{g(1-\xi)y^{\star}\xi} e \right\} \Big|_{\tilde{\tau}=\tau_g^{\rm eq}+(1-\xi)b} = 0$$

$$\left\{ \overline{(1-g)y^{\star}} - \frac{\tau_g^{\star}}{1-\tau_g^{\star}} \overline{y^{\star}} e - \frac{b}{1-\tau_g^{\star}} \overline{g(1-\xi)y^{\star}} e \right\} \Big|_{\tilde{\tau}=\tau_g^{\star}+(1-\xi)b} = 0$$

which simplify to the compact sufficient statistics formulas

$$\tau_g^{\rm eq} = \frac{\overline{(1-g)y^{\star}}}{\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} \ e} - b \ \frac{\overline{g(1-\xi)y^{\star}\xi} \ e}{\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} \ e}$$
(52)

$$\tau_g^{\star} = \frac{\overline{(1-g)y^{\star}}}{\overline{(1-g)y^{\star}} + \overline{y^{\star}} e} - b \frac{\overline{g(1-\xi)y^{\star}} e}{\overline{(1-g)y^{\star}} + \overline{y^{\star}} e}$$
(53)

where all endogenous quantities on the right hand-side of the equations are evaluated at respectively $\tilde{\tau} = \tau_g^{\text{eq}} + (1 - \xi)b$ and $\tilde{\tau} = \tau_g^{\star} + (1 - \xi)b$. In other words formulas are expressed in terms of sufficient statistics evaluated at the optimum.

A.6 Taxation bias in tractable Gaussian case

A difficulty in comparing τ_g^{eq} and τ_g^{\star} is that some right-hand side quantities are endogenous to the tax rate and thus evaluated at different tax rates. To overcome this difficulty, we use a small taxation bias approximation $\tau_g^{\text{eq}} \approx \tau_g^{\star} = t$ such that quantities can be evaluated to a first-order approximation at the same tax rate. Furthermore, we assume that corrective motives associated to the presence of a perception bias b are evaluated at tax rate t such that we can finally directly compare

$$\tau_g^{\rm eq} = \frac{\overline{(1-g)y^{\star}}}{\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} \ e} - b \ \frac{\overline{g(1-\xi)y^{\star}\xi} \ e}{\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} \ e} \tag{54}$$

$$\tau_g^{\star} = \frac{(1-g)y^{\star}}{(1-g)y^{\star} + \overline{y^{\star}} e} - b \; \frac{\overline{g(1-\xi)y^{\star}} e}{(1-g)y^{\star} + \overline{y^{\star}} e} \tag{55}$$

The first terms on the right-hand side corresponds to the standard optimal tax formula (e.g. **PikettySaez2013**) whereas the second are corrective terms associated to the existence

⁴³In our simulations we do check that $\frac{d\xi}{d\tau_g}$ does not take large values (it takes values between 0.2 and 1 in equilibrium) as a way to confirm the validity of this approximation.

of a perception bias b. For small perception biases, these corrective terms are second-order and go in the same direction for both positive and normative tax rates. They are thus not driving the difference between the two and we disregard them to derive the following simple sufficient statistics formula for the taxation bias

$$\tau_g^{\text{eq}} - \tau_g^{\star} = \frac{\overline{(1-g)y^{\star}}}{\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} \ e} - \frac{\overline{(1-g)y^{\star}}}{\overline{(1-g)y^{\star}} + \overline{y^{\star}} \ e}$$
(56)

$$= \frac{\overline{(1-g)y^{\star}}\left(\overline{(1-g)y^{\star}} + \overline{y^{\star}} e\right) - \overline{(1-g)y^{\star}}\left(\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} e\right)}{\left(\overline{(1-g)y^{\star}} + \overline{y^{\star}\xi} e\right)\left(\overline{(1-g)y^{\star}} + \overline{y^{\star}} e\right)}$$
(57)

$$= \frac{e \tau_g^{\text{eq}} \tau_g^{\star}}{(1-g)y^{\star}} \left(\overline{y^{\star}} - \overline{y^{\star}\xi}\right)$$
(58)

$$\simeq \quad \overline{\frac{(1-\xi)y^{\star}}{(1-g)y^{\star}}} \ e \ t^2 \tag{59}$$

A.7 Utility decomposition

Let $\mathcal{V}_{\kappa}(\tau_g^{\star}(\kappa), w)$ be the expected utility of taxpayer w at the positive equilibrium when the information cost is κ and the optimal target tax rate of the government is $\tau_g^{\star}(\kappa)$. Then, with a separable utility,

$$\mathcal{V}_{0}(\tau_{g}^{\star}(0), w) = E_{\tau_{0}|\tau_{g}^{\star}(0)} \Big[R_{0} + (1 - \tau_{0}) y^{\star}(\tau_{0}; w) - v(y^{\star}(\tau_{0}; w); w) \Big]$$

$$\mathcal{V}_{\kappa}(\tau_{g}^{\star}(\kappa), w) = E_{\tau_{0}|\tau_{g}^{\star}(\kappa)} \Big[\int \Big(R_{0} + (1 - \tau_{0}) y^{\star}(\tau; w) - v(y^{\star}(\tau; w); w) \Big) f_{\tilde{\tau}}(\tau|\tau_{0}, w) d\tau \Big] - \kappa \mathcal{I}(\sigma^{\star}(\hat{q}(\tau), \kappa, w))$$

Using straightforward algebra,

$$\mathcal{V}_{\kappa}(\tau_g^{\star}(\kappa), w) - \mathcal{V}_0(\tau_g^{\star}(0), w) = \Delta^{\mathcal{V}}R + \Delta^{\mathcal{V}}\tau + \Delta^{\mathcal{V}}b + \Delta^{\mathcal{V}}\text{uncertainty} + \Delta^{\mathcal{V}}\text{info cost}$$
(60)

where

$$\Delta^{\mathcal{V}}R \equiv E_{\tau_0|\tau_q^{\star}(\kappa)}[R_0] - E_{\tau_0|\tau_q^{\star}(0)}[R_0]$$

is the change in the average demogrant,

$$\Delta^{\mathcal{V}}\tau \equiv E_{\tau_0|\tau_g^{\star}(\kappa)}[(1-\tau_0)y^{\star}(\tau_0;w) - v(y^{\star}(\tau_0;w);w)] - E_{\tau_0|\tau_g^{\star}(0)}[(1-\tau_0)y^{\star}(\tau_0;w) - v(y^{\star}(\tau_0;w);w)]$$

is the change in the expected utility due to the change in the tax target $\tau_g^\star,$

$$\Delta^{\mathcal{V}}b \equiv E_{\tau_0|\tau_g^{\star}(\kappa)} \Big[(1-\tau_0) \Big(y^{\star}(\tau_0+(1-\xi)b;w) - y^{\star}(\tau_0;w) \Big) \\ - \Big(v(y^{\star}(\tau_0+(1-\xi)b;w);w) - v(y^{\star}(\tau_0;w);w) \Big) \Big]$$

is the change in the expected utility due to the bias b,

$$\Delta^{\mathcal{V}} \text{uncertainty} \equiv E_{\tau_{0}|\tau_{g}^{\star}(\kappa)} \bigg[\int (1 - \tau_{0}) \big(y^{\star}(\tau; w) - v(y^{\star}(\tau; w); w \big) \\ \phi \big(\tau; \xi \tau_{0} + (1 - \xi) (\tau_{g}^{\star}(\kappa) + b), (\xi \sigma^{\star})^{2} \big) d\tau \\ - \big((1 - \tau_{0}) \big(y^{\star}(\tau_{0} + (1 - \xi)b; w) - v(y^{\star}(\tau_{0} + (1 - \xi)b; w); w) \big) \bigg]$$

is the change in the expected utility due to noisy information and $\Delta^{\mathcal{V}}$ info $\cos t = -\kappa \mathcal{I}(\sigma^{\star}(\hat{q}(\tau), \kappa, w)).$

Online Appendix (Not for publication)

This online appendix first provides detailed information on the numerical simulations. Second, it gives all proofs and derivations for the extension to nonlinear taxation. Third, we show how to incorporate income effects in the analysis.

B Numerical simulations

Simulations are implemented using Matlab and the algorithm may be summarized as follows. We first estimate a log-normal distribution of skills that we extend with a Pareto tail. This distribution of skills is then binned into a discrete approximation and taken as given for the rest of the exercise. Second, we find the optimal policy of the government using an iterative routine. Starting with a guess for the optimal policy, we compute the optimal attention strategies and allocations in equilibrium (i.e. when the priors are adjusted). We then compute a new optimal policy given taxpayers' choices and iterate until convergence to a fixed point solution.

This appendix provides details on these different steps. We first present the calibration strategy for the skill distribution. Second, we explain how to solve for the optimal attention strategies and allocations for a given tax schedule. Finally, we discuss how the government's problem is solved in the linear tax setting before turning to the nonlinear case.

B.1 Skill distribution

Simulations require an exogenous distribution of skills $f_w(.)$. We fit the adjusted gross incomes from the 2016 March CPS data to a log-normal distribution. The parameters of the log-normal are chosen to exactly match the mean and median of the observed distribution. Following Saez (2001), we extend the log-normal distribution with a Pareto tail (k = 2) for annual incomes above \$200,000. We then discretize the income distribution using evenly distributed bins over the [200; 200,000] interval and evenly distributed bins (in ln scale) over the [200,000; 4,000,000] interval. This allows us to approximate integrals with Riemann sums.

To translate this income distribution into a skill distribution, we invert agents' first-order conditions for labor supply. We first use OECD data on 2016 labor taxes in the US and fit a linear tax schedule $\{\tau_{obs}, R_{obs}\}$. Then, we impose a quasi-linear utility specification

 $u(c, y; w) = c - (y/w)^{1+\epsilon}/(1+\epsilon)$ with $e = 1/\epsilon = 0.33$ (Chetty, 2012). Assuming we are in a no bias equilibrium (i.e. rational expectation) such that agents' perceived tax rate coincide with the observed one τ_{obs} , this allows us to compute skills through $w = \left(y^{\epsilon}/(1-\tau_{obs})\right)^{\frac{1}{1+\epsilon}}$. We also use the estimated linear tax system $\{\tau_{obs}, R_{obs}\}$ together with the actual distribution of earnings to deduce an exogenous expenditure requirement E for the government budget constraint.

B.2 Taxpayers' behavior

Taxpayers' choices are presented in Section 3. For the simulations, we consider Gaussian implementation shocks. Under this assumption, the equilibrium prior distribution is Gaussian as well. Consequently, one may easily compute the attention parameter (ξ) , income (y) and consumption (c) for each taxpayer. Given an attention cost κ , a marginal tax rate τ – that potentially varies for each individual – and an uncertainty parameter σ_{ϑ} , the attention strategy in equilibrium follows from equation (14). Gaussian integrals are approximated using Gauss-Hermite quadratures. Using an agent's first-order condition (7) and budget constraint, we compute her income, consumption and utility for different signal realizations. These computations are made for each type of agent w. The demogrant R is computed from the government budget constraint.

B.3 Optimal linear tax

Unless stated otherwise, we assume throughout our numerical exercise that the social planner has a log objective $G(.) = \log(.)$.

In order to compute the optimal linear tax under discretion, we start with a guess $\tau_{g,0}$. Using this guess, we can deduce each taxpayer's attention strategy when the prior is adjusted to the guess $\hat{\tau}_0 = \tau_{g,0} + b$. We then consider this distribution of attention strategies as constant and use a Matlab optimization routine to find a new $\tau_{g,1}$ which maximizes social welfare for these attention strategies. We then update the prior $\hat{\tau}_1 = \tau_{g,1} + b$, recompute the attention strategies and re-optimize until convergence $|\hat{\tau}_i - \tau_{g,i+1}| \leq 1e^{-5}$. This method is intuitive and captures the essence of the discretionary policy: the government maximizes its objective taking attention strategies as fixed.

We also implement an alternative algorithm where instead of maximizing social welfare numerically we directly pick a new tax rate using the government FOCs in Proposition 4.1 under a small signals approximation. We find comparable equilibrium rates. Similarly, we compute the optimal policy under commitment using the FOCs in Proposition 4.3.

B.4 Optimal nonlinear tax

In order to compute the optimal nonlinear tax, we again use an iterative routine. We start with a guess – namely, a constant marginal rate – and iterate until convergence of the nonlinear tax schedule. We only present results for the unbiased equilibrium b = 0. We proceed in the same spirit as for the linear tax schedule:

- 1. Start with a guess for the nonlinear tax schedule
- 2. Compute the attention strategies $(\forall w)$ for a given adjusted prior $\hat{\tau}_w$
- 3. Compute allocations given attention strategies and tax schedule
- 4. Solve for the government FOCs at each w to deduce a new tax schedule
- 5. Repeat steps 1-4 until convergence.

To maintain the numerical stability of the algorithm we impose a slow adjustment of attention strategies ξ at each iteration. Indeed, marginal tax rates being sensitive to attention, one shall avoid large jumps in the attention parameter. The convergence criteria we use is the infinite norm for both marginal tax rates and attention strategies.

C Proofs for the extension to non-linear taxation

We here provide the proofs on the monotonicity condition and Proposition 7.2 (ABCD tax formula) of the main text.

C.1 Monotonicity

In this section, we demonstrate that the monotonicity condition is expected to hold for the quasi-linear and iso-elastic separable utility function that we consider in our simulations. For alternative specifications, we recommend to proceed using a guess-and-verify method. The latter is already implemented in our code and a warning is automatically displayed when the monotonicity does not hold ex post.

With a quasi-linear and iso-elastic separable utility function the first-order condition defining $y^{\star}(\tilde{\tau}_w; w)$ is

$$(FOC)_y : 1 - \tilde{\tau}_w - \frac{1}{w} \left(\frac{y^*}{w}\right)^\epsilon = 0 \tag{61}$$

Differentiating this equation with respect to w yields

$$\frac{\epsilon}{w^2} \left(\frac{y^*}{w}\right)^{\epsilon-1} \frac{dy^*(\tilde{\tau}_w; w)}{dw} = \frac{1+\epsilon}{w^2} \left(\frac{y^*}{w}\right)^{\epsilon} - \frac{d\tilde{\tau}_w}{dw}$$
(62)

Now – in expectation of the realization of the implementation shock ϑ – we also have $\tilde{\tau}_w = T'_g(y^*(\hat{\tau}_w; w)) + (1 - \xi)b$ which allows us to get

$$\frac{d\tilde{\tau}_w}{dw} = T_g''(y^*(\hat{\tau}_w; w)) \frac{dy^*(\hat{\tau}_w; w)}{dw} + \frac{d}{dw} \Big[(1-\xi)b \Big]$$
(63)

and we can show that

1. If agents correctly perceive marginal tax rates (b = 0), the equilibrium condition $\hat{\tau}_w = T'_g(y^*(\hat{\tau}_w; w)) + b$ becomes $\hat{\tau}_w = T'_g(y^*(\hat{\tau}_w; w)) = \tilde{\tau}$. We then have $\frac{dy^*(\hat{\tau}_w; w)}{dw} = \frac{dy^*(\tilde{\tau}_w; w)}{dw}$ such that plugging (63) with b = 0 into (62) the monotonicity condition boils down to

$$\frac{dy^{\star}}{dw} = \frac{\frac{1+\epsilon}{w^2} \left(\frac{y^{\star}}{w}\right)^{\epsilon}}{\frac{\epsilon}{w^2} \left(\frac{y^{\star}}{w}\right)^{\epsilon-1} + T_g''(y^{\star})} \ge 0 \iff -T_g''(y^{\star}) \le \frac{\epsilon}{w^2} \left(\frac{y^{\star}}{w}\right)^{\epsilon-1} \tag{64}$$

2. If agents exhibit a small perception bias $(b \approx 0)$ such that we have $\frac{dy^{\star}(\tilde{\tau}_w;w)}{dw} \approx \frac{dy^{\star}(\tilde{\tau}_w;w)}{dw}$ plugging (63) into (62) the monotonicity condition rewrites

$$\frac{dy^{\star}}{dw} = \frac{\frac{1+\epsilon}{w^2} \left(\frac{y^{\star}}{w}\right)^{\epsilon} - \frac{d}{dw} \left[(1-\xi)b \right]}{\frac{\epsilon}{w^2} \left(\frac{y^{\star}}{w}\right)^{\epsilon-1} + T_g''(y^{\star})} \ge 0 \iff \begin{cases} -T_g''(y^{\star}) \le \frac{\epsilon}{w^2} \left(\frac{y^{\star}}{w}\right)^{\epsilon-1} \\ \frac{d}{dw} \left[(1-\xi)b \right] \le \frac{1+\epsilon}{w^2} \left(\frac{y^{\star}}{w}\right)^{\epsilon} \end{cases}$$
(65)

where the equivalence comes from the fact that the other case in which we would have $-T_g''(y^*) \ge \frac{\epsilon}{w^2} \left(\frac{y^*}{w}\right)^{\epsilon-1}$ is infeasible.

Hence, the monotonicity condition will hold if the tax function $T_g(y)$ is sufficiently smooth such that its second derivative is bounded (in absolute value).

C.2 Proposition 7.2 (ABCD tax formula)

We proceed with a tax perturbation approach in order to characterize the nonlinear tax schedule chosen under discretion and under commitment. Consider a tax schedule $T_g(.)$ and a reform that consists in a small increase $\Delta \tau^r$ in marginal tax rates in a small bandwidth of earnings $[y^r - \Delta y^r, y^r]$ and let us compute its impact on the government's objective (written in Lagrangian form)

$$\mathscr{L} = E_{\vartheta} \bigg[\iint \left\{ G \Big(\mathcal{V}(\tilde{\tau}_w, T_0(.); \kappa, w) \Big) + p \Big(T_0(y^{\star}(\tilde{\tau}_w; w)) - E \Big) \right\} f_{\tilde{\tau}_w}(\tau | \tau_0; w) f_w(w) d\tau dw \bigg] (66)$$

where p is the multiplier associated to the government's budget constraint and is equal to the social marginal value of public funds at the optimum.

Impact of the reform For a given target tax schedule $T_g(.)$, the reform has

- a mechanical effect dM and a welfare effect dW that translate the lump-sum increase of $\Delta \tau^r \Delta y^r$ in the tax liabilities of agents $w \in [w^r, \infty[$ defined by $y^*(\tilde{\tau}_{w^r}; w^r) \equiv y^r$ where $E_s[\tilde{\tau}_w|\hat{\tau}_w] = \xi T'_0(y^*(\hat{\tau}_w; w)) + (1 - \xi)\hat{\tau}_w$ with $T'_0 = T'_g + \vartheta$
- a labor supply or behavioral effect dB that translates an increase $\Delta \tau^r$ in marginal tax rates that impacts the perceived marginal tax rates $\tilde{\tau}_w$ of agents $w \in [\hat{w}^r - \Delta \hat{w}^r, \hat{w}^r]$ defined by $y^*(\hat{\tau}_{\hat{w}^r}; \hat{w}^r) \equiv y^r$ and $y^*(\hat{\tau}_{\hat{w}^r}; \hat{w}^r - \Delta \hat{w}^r) \equiv y^r - \Delta y^r$

such that the total impact on the government's objective is

$$\frac{d\mathcal{L}}{p} = \frac{dM}{p} + \frac{dW}{p} + \frac{dB}{p} \tag{67}$$

with

$$\frac{dM}{p} + \frac{dW}{p} = \int_{w^r}^{\infty} E_{\vartheta} \left[\int \left\{ \Delta \tau^r \Delta y^r - \frac{G'(\mathcal{V}(w))}{p} \frac{\partial U}{\partial c} \Delta \tau^r \Delta y^r \right\} f_{\tilde{\tau}}(\tau | \tau_0; w) \, d\tau \right] f_w(w) \, dw$$
$$= \int_{w^r}^{\infty} E_{\vartheta} \left[\int \left(1 - g(w) \right) \Delta \tau^r \Delta y^r \, f_{\tilde{\tau}}(\tau | \tau_0; w) \, d\tau \right] f_w(w) \, dw \tag{68}$$

since we here have, holding $\tilde{\tau}_w$ constant,

$$d\mathcal{V} = \frac{d}{dc} \left\{ U\left(\underbrace{y^{\star}(\tilde{\tau}_w;w) - T_0(y^{\star}(\tilde{\tau}_w;w))}_{c}, y^{\star}(\tilde{\tau}_w;w);w\right) - \kappa \mathcal{I}(\sigma^{\star}) \right\} dc = -\frac{\partial U}{\partial c} dT_0 \quad (69)$$

and

$$\frac{dB}{p} = \int_{\hat{w}^r - \Delta \hat{w}^r}^{\hat{w}^r} E_{\vartheta} \left[\int \left\{ \frac{G(\mathcal{V}(w))}{p} + T_0(y^{\star}(\tilde{\tau};w)) \right\} \frac{df_{\tilde{\tau}}(\tau|\tau_0;w)}{d\tau_g} \Delta \tau^r \, d\tau \right] f_w(w) \, dw$$
$$\approx E_{\vartheta} \left[\int \left\{ \frac{G(\mathcal{V}(\hat{w}^r))}{p} + T_0(y^{\star}(\tilde{\tau};\hat{w}^r)) \right\} \frac{df_{\tilde{\tau}}(\tau|\tau_0;\hat{w}^r)}{d\tau_g} \Delta \tau^r \, d\tau \right] f_w(\hat{w}^r) \, \Delta \hat{w}^r \tag{70}$$

since we here have, holding $\tilde{\tau}_w$ constant,

$$d\mathcal{V} = \frac{d}{d\tau_g} \Big\{ U\Big(y^{\star}(\tilde{\tau}_w; w) - T_0(y^{\star}(\tilde{\tau}_w; w)), y^{\star}(\tilde{\tau}_w; w); w \Big) - \kappa \mathcal{I}(\sigma^{\star}) \Big\} d\tau_g = 0$$
(71)

Characterization of tax policy The optimality condition for the choice of tax policy $\frac{d\mathcal{L}}{p} = 0$ thus writes

$$E_{\vartheta} \left[\int \left\{ \frac{G(\mathcal{V}(\hat{w}^{r}))}{p} + T_{0}(y^{\star}(\tilde{\tau}_{\hat{w}^{r}};\hat{w}^{r})) \right\} \frac{df_{\tilde{\tau}_{\hat{w}^{r}}}(\tau|\tau_{0};\hat{w}^{r})}{d\tau_{g}} d\tau \right] \frac{f_{w}(\hat{w}^{r})}{\frac{dy^{\star}(\hat{\tau}_{\hat{w}^{r}};\hat{w}^{r})}{dw}} \\ + \int_{w^{r}}^{\infty} E_{\vartheta} \left[\int \left(1 - g(w) \right) f_{\tilde{\tau}_{w}}(\tau|\tau_{0};w) d\tau \right] f_{w}(w) dw = 0$$
(72)

where we have simplified through by $\Delta \tau^r \Delta y^r$ noting that

$$y^{\star}(\hat{\tau}; \hat{w}^r - \Delta \hat{w}^r) \equiv y^r - \Delta y^r \implies \Delta \hat{w}^r \frac{dy^{\star}(\hat{\tau}_{\hat{w}^r}; \hat{w}^r)}{dw} \approx \Delta y^r$$

Assuming we are in the tractable Gaussian case, the expost (after learning) distribution of the perceived marginal tax rate is Gaussian $f_{\tilde{\tau}_w}(\tau|\tau_0;w) \sim \mathcal{N}(\mu_w,\sigma^2)$ with mean $\mu_w = \xi\tau_0 + (1-\xi)\hat{\tau}_w$ and variance parameter $\sigma = \sigma^*$. Applying Lemma A.4 we can thus rewrite the optimality condition as

$$E_{\vartheta}\left[\left[\left\{\frac{G'(\mathcal{V}(\hat{w}^{r}))}{p}\left(\tilde{\tau}_{\hat{w}^{r}}-T_{0}'(y^{\star}(\tilde{\tau}_{\hat{w}^{r}};\hat{w}^{r}))\right) + T_{0}'(y^{\star}(\tilde{\tau}_{\hat{w}^{r}};\hat{w}^{r}))\right\}\frac{dy^{\star}}{d\tilde{\tau}}\frac{d\mu_{\hat{w}^{r}}}{d\tau_{g}}\right]\Big|_{\tilde{\tau}=\mu_{\hat{w}^{r}}}\frac{f_{w}(\hat{w}^{r})}{\frac{dy^{\star}(\hat{\tau}_{\hat{w}^{r}};\hat{w}^{r})}{dw}} + \int_{w^{r}}^{\infty}\left[1-g(w)\right]\Big|_{\tilde{\tau}=\mu_{w}}f_{w}(w)\ dw\right] = 0$$
(73)

where $\frac{d\mu_w}{d\tau_g} = \xi \frac{d\tau_0}{d\tau_g} = \xi$ under discretion since the government takes agents' priors as given whereas $\frac{d\mu_w}{d\tau_g} = \xi \frac{d\tau_0}{d\tau_g} + (1-\xi) \frac{d\hat{\tau}_w}{d\tau_g} + \frac{d\xi}{d\tau_g} (\tau_0 - \hat{\tau}_w)$ under commitment since the government internalizes that priors adjust to the choice of tax policy and are thus an endogenous object.

In addition, the Lagrange multiplier is – absent income effects – determined by the same transversality condition as before

$$E_{\vartheta} \left[\int_0^\infty \left[1 - g(w) \right] \Big|_{\tilde{\tau} = \mu_w} f_w(w) \ dw \right] = 0$$
(74)

which can be obtained in a perturbation approach by computing the impact of a uniform lump-sum increase in taxes.

ABCD formula To obtain our ABCD formula from equation (73), let us introduce e =

 $\frac{1-\tilde{\tau}_w}{y^*(\tilde{\tau}_w;w)}\frac{dy^*(\tilde{\tau}_w;w)}{d(1-\tilde{\tau}_w)} \text{ and assume that the shock } \vartheta \text{ is small to use } E_\vartheta[\psi(\vartheta)] \approx \psi(E_\vartheta[\vartheta]) \text{ regardless of function } \psi\text{'s curvature such that } E_\vartheta[\psi(\tau_0)] \approx \psi(\tau_g). \text{ This yields}$

$$\frac{T'_{g}(y^{\star}(\mu_{\hat{w}^{r}};\hat{w}^{r})) + g(\hat{w}^{r})|_{\tilde{\tau}=\mu_{\hat{w}^{r}}} \left(\mu_{\hat{w}^{r}} - T'_{g}(y^{\star}(\mu_{\hat{w}^{r}};\hat{w}^{r}))\right)}{1 - \mu_{\hat{w}^{r}}} = \frac{1}{e^{\frac{d\mu_{\hat{w}^{r}}}{d\tau_{g}}}|_{\tilde{\tau}=\mu_{\hat{w}^{r}}}} \frac{1}{y^{\star}(\mu_{\hat{w}^{r}};\hat{w}^{r})} \frac{\frac{dy^{\star}(\hat{\tau}_{\hat{w}^{r}};\hat{w}^{r})}{dw}}{f_{w}(\hat{w}^{r})} \int_{w^{r}}^{\infty} \left(1 - g(w)|_{\tilde{\tau}=\mu_{w}}\right) f_{w}(w) \, dw \tag{75}$$

where $\mu_w = \xi T'_g(y^*(\hat{\tau}_w; w)) + (1-\xi)\hat{\tau}_w$ and $\frac{d\mu_w}{d\tau_g} = \xi \frac{dT'_g(y^*(\hat{\tau}_w; w))}{d\tau_g} = \xi$ under discretion since the government takes agents' priors as given whereas $\frac{d\mu_w}{d\tau_g} = \xi + (1-\xi)\frac{d\hat{\tau}_w}{d\tau_g} + \frac{d\xi}{d\tau_g}(T'_g(y^*(\hat{\tau}_w; w)) - \hat{\tau}_w)$ under commitment since the government internalizes that priors adjust to the policy rule and are thus an endogenous object.

Note that with a quasi-linear and iso-elastic separable utility function we have $y^{\star}(\tilde{\tau}_w; w) = w^{1+\frac{1}{\epsilon}}(1-\tilde{\tau}_w)^{\frac{1}{\epsilon}}$ and $e = \frac{1}{\epsilon}$ such that

$$\frac{dy^{\star}(\tilde{\tau}_w;w)}{dw} = \left(1 + \frac{1}{\epsilon}\right) w^{\frac{1}{\epsilon}} (1 - \tilde{\tau}_w)^{\frac{1}{\epsilon}} = \frac{1 + e}{w} y^{\star}(\tilde{\tau}_w;w)$$
(76)

Assuming small (or no) perception biases such that $\tilde{\tau}_w \approx \hat{\tau}_w$ and $\frac{dy^{\star}(\tilde{\tau}_w;w)}{dw} \approx \frac{dy^{\star}(\hat{\tau}_w;w)}{dw}$ yields

$$\frac{T'_g(y^{\star}(\mu_{\hat{w}^r}^{\mathrm{eq}};\hat{w}^r)) + g(\hat{w}^r)(1-\xi)b}{1-\tilde{\tau}_{\hat{w}^r}^{\mathrm{eq}}} = \frac{1}{\frac{d\mu_{\hat{w}r}}{d\tau_g}} \frac{1+e}{e} \frac{1}{\hat{w}^r f_w(\hat{w}^r)} \int_{w^r}^{\infty} \left(1-g(w)\right) f_w(w) \ dw$$
(77)

D Income effects

In this final section of the Online Appendix, we illustrate how the (linear tax) model in the paper could be extended to account for income effects and accordingly characterize tax policy under discretion and commitment. We now have to account for the fact that the average posterior tax rate is no longer a sufficient statistics for taxpayers' earnings choices. This requires a mere reformulation of the initial problem without income effects: integration in the government's problem is now with respect to the signal distribution.

In order to introduce income effects, it will prove useful to slightly reformulate taxpayers' problem introduced in Section 3. To this end, consider that there is a continuum of individuals at each skill w of size f(w) and let $Y(\tau_0) \equiv \iint y^*(\cdot)\phi(s;\tau_0,\sigma^*)dsdF(w)$ be the aggregate earnings. Then, because the government budget constraint is binding at the optimum, the demogrant writes $R(\tau_0) = \tau_0 Y(\tau_0) - E$ as the overall population remains of size one. Further, and given that a taxpayer's budget constraint binds ex post, consumption adjusts such that $c_0 = R(\tau_0) + (1 - \tau_0)y$. Therefore, an agent's utility is $u(R(\tau_0) + (1 - \tau_0)y, y)$ for a realization τ_0 and earnings choice y.

Given the above reformulation, the only uncertainty arises from the randomness in the realized tax rate. An individual therefore chooses the signal precision σ and income y to maximize her expected utility

$$\sup_{\sigma,y|s} \iint u(R(\tau) + (1-\tau)y, y; w)\phi(s; \tau, \sigma)\hat{q}(\tau)dsd\tau - \kappa \mathcal{I}(\sigma)$$
(78)

where admissible earnings policies for this individual's choice may depend on the signal s. Now, guess that the optimal attention strategy σ^* depends only on w, $\hat{q}(.)$, and κ . As a consequence, the optimal earnings choice $y^*(s, w; \sigma^*, \hat{q}(.))$ now solves

$$\int [(1-\tau)u_c(R(\tau) + (1-\tau)y^*, y^*; w) + u_y(R(\tau) + (1-\tau)y^*, y^*; w)]f(\tau|s; \sigma^*, \hat{q}(.))d\tau = 0$$
(79)

where $f(\tau|s; \sigma^*, \hat{q}(.)) = \frac{\phi(s;\tau,\sigma^*)\hat{q}(\tau)}{\int \phi(s;\tau,\sigma^*)\hat{q}(\tau)d\tau}$ from Bayes rule. Assume that a solution to equation (79) exists. In turn, it implies that

$$\sigma^{\star}(w,\hat{q}(.),\kappa) = \arg\sup_{\sigma} \iint u(R(\tau) + (1-\tau)y^{\star}, y^{\star}; w)\phi(s;\tau,\sigma)\hat{q}(\tau)dsd\tau - \kappa\mathcal{I}(\sigma) \quad (80)$$

thus confirming the guess on σ^* (when it exists). We can now define agents' indirect utility function

$$\mathcal{V}(s,\tau_0;w,\kappa,\hat{q}(.)) \equiv u(R(\tau_0) + (1-\tau_0)y^\star, y^\star;w) - \kappa \mathcal{I}(\sigma^\star)$$
(81)

Turning to the government problem, it requires a mere variation from (17)

$$\max_{\tau_g} \quad E_{\vartheta} \left[\iint G \Big(\mathcal{V} \Big(s, \tau_0; w, \kappa, \hat{q}(.) \Big) \Big) \phi(s; \tau_0, \sigma^{\star}) f_w(w) d\tau dw \right]$$
(82)

Note that the inner integration is now with respect to the signal distribution $\phi(s; \tau_0, \sigma^*)$ and no longer with respect to the posterior distribution of perceived rates. This is because the perceived tax rate $\tilde{\tau}$ is no longer a sufficient statistics for earnings choices.

The first order condition for the target tax rate under discretion writes

$$E_{\vartheta} \left[\int \left\{ \int \left[G'(\mathcal{V}) \frac{d\mathcal{V}}{d\tau_g} \phi(s; \tau_0, \sigma^*) ds + \int G(\mathcal{V}) \frac{d\phi(s; \tau_0, \sigma^*)}{d\tau_g} ds \right\} f_w(w) dw \right] = 0$$
(83)

and the first order condition for the target tax rate under commitment writes

$$E_{\vartheta} \left[\int \left\{ \int \left[G'(\mathcal{V}) \left(\frac{d\mathcal{V}}{d\tau_g} + \frac{d\mathcal{V}}{d\hat{q}(.)} \frac{d\hat{q}(.)}{d\tau_g} \right) \phi(s;\tau_0,\sigma^{\star}) ds + \int G(\mathcal{V}) \frac{d\phi(s;\tau_0,\sigma^{\star})}{d\tau_g} ds \right\} f_w(w) dw \right] = 0$$
(84)

This characterizes tax policy under discretion and commitment in the presence of income effects. The key difference between the two equations is the fact that the commitment tax policy takes into account the adjustment in the prior $\frac{d\hat{q}(.)}{d\tau_g}$ whereas the discretion tax policy does not. This leads to a taxation bias.

Chapter 3 – Make work pay or make search pay? Redistributive taxation and unemployment insurance

Abstract. This paper analyzes the design of redistributive taxation and unemployment insurance. It develops a novel conceptual framework that nests the Mirrlees-Saez optimal income taxation model and the Baily-Chetty optimal unemployment insurance model. Heterogeneous agents choose whether to participate in the labor market, their earnings when employed as well as their job search efforts when unemployed. Using a perturbation approach, I provide sufficient statistics characterizations of (i) the optimal nonlinear tax and transfer schedule, (ii) the optimal nonlinear unemployment benefits schedule, (iii) a Pareto-efficiency condition for joint optimality. Using a mechanism design approach, I show that eligibility requirements to unemployment insurance are key to maintain incentive compatibility across agents while providing insurance. I find that it is efficient to provide redistribution through progressive unemployment benefits, even in the presence of an optimal nonlinear income tax. The policy implication is that net replacement rates should decrease across earnings levels. A calibration to the US economy shows actual net replacement rates decrease with earnings but suggests some scope for Pareto-improvements. Overall, these findings shed a new light on optimal taxation and optimal unemployment insurance by showing that the interactions between the two problems matter for the design of these policies.

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1 Introduction

The large-scale provision of redistribution and social insurance has become a defining feature of modern welfare states, and the design of such programmes a focal point in public and political discourse. Policymakers and academics alike tend to consider the design of these programmes as separate policy questions, echoing the separation between the public entities in charge of operating and managing them. Jointly analyzing the design of redistributive taxation and unemployment insurance, this paper studies the interactions between these two problems and shows they have important policy implications.

On the one hand, redistribution aims at reducing the inequalities induced by differences in pre-tax earnings across agents. Attributing these differences to heterogeneous earnings abilities and starting from the premise that generous redistributive policies do not *make work pay*, the optimal income taxation literature analyzes the design of redistribution as a trade-off between the social benefits from reductions in inequalities and the social costs from reductions in work incentives (Mirrlees, 1971; Saez, 2001, 2002a).

On the other hand, unemployment insurance aims at compensating the income loss agents incur upon being laid-off. Assuming insurance provision reduces job search efforts, the optimal unemployment insurance literature analyzes the design of insurance as a trade-off between the consumption smoothing benefits from insurance and the necessity to *make search pay* (Baily, 1978; Chetty, 2006a, 2008).

While parallels have been drawn between the two problems, suprisingly little work analyzes how they interact. First, redistribution and unemployment insurance are linked through the objective and the resource constraint of the government. Second, individual work and search decisions are partly shaped by the same factors (e.g. net earnings on the job), thereby connecting *make work pay* and *make search pay* concerns. Third, unemployment benefits depend on earnings on the job. An increase in redistribution towards agents at a given earnings level can thus transit either through a reduction in taxes or through an increase in unemployment benefits associated with that earnings level. Last but not least, low skill individuals earn less and face higher unemployment rates than high skill individuals which suggests that redistribution through unemployment benefits may be well targetted.

This paper develops a unifying conceptual framework that nests standard models of optimal income taxation and optimal unemployment insurance and takes these interactions into account. Using a perturbation approach, I provide sufficient statistics characterizations of (i) the optimal nonlinear tax and transfer schedule, (ii) the optimal nonlinear unemployment benefits schedule, (iii) a Pareto-efficiency condition for joint optimality. Using a mechanism design approach, I show that optimal allocations can be decentralized by these nonlinear tax-benefit schedules together with eligibility thresholds to unemployment insurance that mimicks actual eligiblity requirements. Finally, I bring the theory to US data and combine empirical estimates of sufficient statistics together with a structural model to empirically assess the importance of these interactions for actual policy making.

To begin with, I present a stylized version of the conceptual framework that builds on the extensive margin model of Saez (2002a). Agents differ in the type of job they may hold and decide wether to participate in the labor market. Non-participants are voluntarily not employed and receive a lump-sum transfer akin to social assistance. The novelty is that participating agents are exposed to unemployment risk. In this static model, this translates into agents spending a fraction of their time employed, and the remaining fraction of time unemployed.⁴⁴ When employed, agents get their labor earnings net of taxes and transfers. When unemployed, agents receive an amount of unemployment benefits indexed on their labor earnings and decide on their job search efforts. As in Chetty (2006a), search is costly but a higher search intensity maps into a higher fraction of time spent employed.

In this stylized setting, the optimal tax and transfer schedule depends on the government's redistributive tastes and agents' participation responses as in Saez (2002a), but also on unemployment risk and agents' job search responses. Unemployment risk calls for higher taxes because it decreases the efficiency cost of taxation. Intuitively, agents do not pay taxes while unemployed which reduces the net amount of revenue collected per participating agent and thus the necessity to make work pay. In contrast, search responses call for lower taxes when employed as a way to make search pay. If search responses are large, this gives a novel rationale to implement monetary transfers towards employed agents (e.g. an EITC policy). Interestingly, the net effect depends on the relative magnitude of participation and search elasticities which can be measured empirically.

Second, optimal unemployment benefits depend on unemployment risk, consumption smoothing benefits and job search responses as in standard optimal unemployment insurance models but also on the government's redistributive tastes and agents' participation responses. Redistributive tastes call for progressivity in the nonlinear schedule of unemployment benefits while participation responses generally call for higher unemployment benefits. Indeed, generous unemployment benefits boost incentives to participate in the labor market and thereby induce a positive *make work pay* externality. This is a pure efficiency motive for the provision of unemployment insurance. It generates a Pareto-lower bound for unemployment benefits defined by the revenue-maximizing level of unemployment insurance.

Third, I derive a Pareto-efficiency condition for joint optimality. Intuitively, it charac-

 $^{^{44}}$ I provide a dynamic micro-foundation for this static model by showing that it corresponds to the steady state representation of a dynamic model with a time-invariant tax-benefit system.

terizes the efficient allocation of consumption accross states of the world and follows from the fact that unemployment benefits depend on earnings on the job. In a pure insurance setting, this condition exactly corresponds to the Baily-Chetty formula. It pins down the optimal net replacement rate at each earnings level as a function of unemployment risk, the elasticity of job search and agents' coefficient of relative risk aversion. Assuming these elements are constant across earnings, as is usually assumed in the optimal unemployment insurance literature, net replacement rates are then constant across earnings.

However, with both redistribution and insurance, the optimal replacement rate at a given earnings level also depends on the net contributions agents make to the tax-benefit system. This additional term can receive a Pigouvian interpretation. Indeed, unemployment insurance imposes fiscal externalities on the financing of redistribution. For instance, keeping agents at the top of the income distribution unemployed has a large negative fiscal externality on the financing of redistribution. Taking these fiscal externalities into account, the Pareto-efficiency condition shows that net replacement rates should decrease with earnings. It highlights that it is efficient to redistribute not only through progressive taxes and transfers but also through progressive unemployment benefits.

Before looking at the quantitative implications of these interactions, I extend these results to a setting with an intensive margin of labor supply. Following Mirrlees (1971), I introduce heterogeneous earnings abilities to account for endogenous earnings decisions. When agents are free to decide on their earnings, I show in a mechanism design approach that the planner must introduce eligibility thresholds in order to provide unemployment insurance. These eligibility thresholds specify the minimum fraction of time agents must spend employed at a given earnings level in order to be eligible to the corresponding unemployment benefits when unemployed. They mimick eligibility requirements used in actual unemployment insurance systems such as minimal employment durations or maximal benefits durations.

Eligibility thresholds are not necessary to influence agents' behavior at their bliss point – I actually assume they don't – but to rule out non-local upward deviations to incentive compatibility. Such deviations consist in low ability individuals deciding to spend a very limited fraction of time employed at a high paying job, in order to enjoy the corresponding high unemployment benefits in the remaining fraction of time spent unemployed. Ruling out these types of deviations restores the planner's capacity to provide unemployment insurance. Moreover, by eliminating non-local binding incentive compatibility constraints, eligibility thresholds smooth out and concavify the problem. This paves the way for a characterization of the optimum through a perturbation approach since the smoothness of the problem is a prerequisite for the validity of this approach. Showing that nonlinear tax-benefit schedules together with eligibility thresholds to unemployment insurance decentralize opti-

mal allocations, I move on to characterize optimal policies in terms of sufficient statistics measuring agents' responses to tax-benefits reforms.⁴⁵

While the Pareto-efficiency condition is very similar to the one obtained before, optimal tax and optimal benefit schedules are now pinned down by ABC-type formulas (Diamond, 1998). Indeed, optimal marginal tax and benefit rates at a given earnings level depend on agents' earnings responses as well as participation and search responses *above* this earnings level. Although standard in the optimal tax literature, this type of formula is a completely novel characterization of optimal unemployment benefits.

When unemployment risk is concentrated at the bottom of the income distribution, the optimal tax formula calls for high marginal tax rates at low income levels. In contrast, large job search responses push for low marginal tax rates at the bottom of the income distribution as a way to *make search pay* at all incomes above. At the extreme, this may even call for negative marginal tax rates.

The optimal benefit formula suggests that large earnings reponses at the intensive margin call for large marginal benefit rates, and thus a steep schedule of unemployment benefits as a way to *make work pay*. Indeed, this is a way to provide incentives to choose higher earnings at the margin. In contrast, since optimal marginal benefit rates depend on the consumption smoothing benefits of individuals at earnings levels *above*, there is a counteracting force pushing for flat schedules of unemployment benefits, at least at high earnings. Since unemployment benefits are in practice capped at a maximum benefit level in almost all countries, I provide a test to determine whether increasing marginal benefit rates when they are equal to zero can be a desirable reform.

Bringing the theory to the data, I empirically assess the importance of these interactions for actual policy making in the US. Doing so requires a number of inputs. First, I use CPS data to compute the distribution of earnings as well as unemployment rates and participation rates across earnings levels. Second, I use earnings, participation and search elasticities estimated in the empirical literature combined with a calibrated structural model to be able to simulate counterfactuals, and in particular optimal policies. Last, I calibrate the tax-benefit schedules using the OECD microsimulation model TaxBEN.

So far, the tax and transfer schedule has been defined as the monetary flows from employed agents towards the government at different earnings levels, and conversely, the unemployment benefits schedule has been defined as the monetary flows from the government towards unemployed agents. Previous results thus characterize optimal tax-benefits sched-

⁴⁵This policy implementation is by no means unique but I argue it is the simplest and most natural. Formally, this tax-benefit system decentralizes third-best optimal allocations in a class of mechanisms that I call *threshold mechanisms*. I also characterize second-best optimal allocations but they cannot be implemented with smooth tax-benefit schedules and thus seem of little relevance for actual policy making.

ules in terms of net monetary flows in each state of the world. In practice, these monetary flows may be the sum of many different policy instruments. For instance in the US, the tax and transfer schedule corresponds to the income tax and social contributions net of EITC payments and social transfers (e.g. family benefits or SNAP), while the benefit schedule comprises unemployment benefits complemented by social transfers at low income levels and unemployment benefits net of the income tax liabilities (associated with high unemployment benefits) at high income levels.

The calibration shows actual replacement rates strongly decrease with earnings. It highlights a disconnect between standard models of unemployment insurance that seek to characterize *the* optimal replacement rate and actual policy. In contrast, optimal policies that account for the interactions between redistribution and unemployment show equally strong decreases in replacement rates across earnings. This framework thus captures an important aspect of real-world policies and thereby contributes to our understanding of the optimal design of redistributive taxation and unemployment insurance.

Related literature. This paper broadly contributes to the optimal income tax literature (Mirrlees, 1971; Diamond, 1980; Diamond, 1998; Saez, 2001, 2002a; Piketty & Saez, 2013) and to the optimal unemployment insurance literature (Baily, 1978; Chetty, 2006a, 2008; Chetty & Finkelstein, 2013; Schmieder & Von Wachter, 2016). It provides a unifying conceptual framework that nests standard models of optimal redistribution and optimal unemployment insurance that allows to study the interactions between these two problems and analyze their policy implications.

The key contribution of the paper to the optimal income tax literature is the introduction of a realistic unemployment insurance policy, in which the amount of unemployment benefits depends on earnings on the job. Indeed, a recent strand of the literature studies optimal redistribution in the presence of unemployment (see Hummel (2019), Costa, Maestri, and Santos (2019), Kroft, Kucko, Lehmann, and Schmieder (2020), as well as Boadway and Tremblay (2013) for a review of earlier work). However, these papers focus on the interactions between taxes and the equilibrium of the labor market in settings where all unemployed agents receive the same transfer regardless of their earnings on the job. Interestingly, Boadway and Cuff (2018) and Kroft, Kucko, Lehmann, and Schmieder (2020) derive analogous optimal tax formulas in settings with participation and search decisions that closely resemble the stylized model of this paper. They however do not provide similar optimal benefit formulas nor Pareto-efficiency conditions. To introduce unemployment benefits which depend on earnings on the job, the conceptual innovation of the paper is to consider a static setting derived as the steady state representation of a dynamic model.⁴⁶ This allows for the introduction of a realistic unemployment insurance policy in an otherwise standard static model of optimal income taxation.

By analyzing the design of unemployment insurance in a setting with heterogeneous earnings, the paper contributes to the optimal unemployment insurance literature in several dimensions. First, unlike most of the literature which optimizes a unique replacement rate, the paper characterizes the optimal nonlinear schedule of unemployment benefits and shows that optimal replacement rates should strongly decrease with earnings. This seems important to connect the theory to actual policy since net replacement rates starkly decrease with earnings in practice. Moreover, it speaks to the issue of "*optimal differentation*" of unemployment insurance identified by Spinnewijn (2020) as a key avenue for future research. Second, the paper relates to previous works on the desirability of eligibility requirements (e.g. minimal work durations, maximal benefits durations) to unemployment insurance. Existing justifications usually appeal to moral hazard arguments related to job search (Schmieder, Von Wachter, & Bender, 2012), or other opportunistic behaviors like quits (Hopenhayn & Nicolini, 2009) in representative agent settings. This paper provides a novel rationale for such eligibility requirements: they are necessary to maintain incentive compatibility across heterogeneous agents when providing insurance.

Last, this paper relates to a broader literature on the interactions between redistribution and social insurance. A classic result is that social insurance is an efficient way to provide redistribution when risk is negatively correlated with income (Rochet, 1991; Cremer & Pestieau, 1996; Boadway, Leite-Monteiro, Marchand, & Pestieau, 2006; Netzer & Scheuer, 2007). This result fundamentally relies on the premise that individuals are pooled within the same insurance contract (e.g. mandatory health insurance). By analyzing earnings-specific unemployment benefits I implicitly consider earnings-specific insurance contracts leaving little scope for this pooling logic. More recently, contributions in the new dynamic public finance literature (see Golosov, Tsyvinski, Werning, Diamond, and Judd, 2006; Kocherlakota, 2010) have also explored some of the interactions between redistribution and insurance. These papers generally focus on the redistribution and insurance motives for progressive income taxation in dynamic economies with stochastic shocks but do not introduce social insurance programs (e.g. Farhi and Werning, 2013; Golosov, Troshkin, and Tsyvinski, 2016; Findeisen and Sachs, 2017). Noteworthy exceptions are Golosov and Tsyvinski (2006) and Golosov, Shourideh, Troshkin, and Tsyvinski (2013a) who study the optimal design of dis-

 $^{^{46}\}mathrm{A}$ number of recent papers in public economics have successfully used steady state representations of dynamic models to study e.g. optimal capital taxation (Saez & Stantcheva, 2018), optimal unemployment insurance (Landais, Michaillat, & Saez, 2018b).

ability insurance and the optimal design of pensions. However, they do not specifically analyze the interactions between redistribution and social insurance and their implications for the design of these policies. This paper can be understood as providing a no separation result for optimal taxation and optimal unemployment insurance: separating these problems lead to Pareto-inefficient policies and creates a scope for Pareto-improving reforms.

The remainder of the paper is divided between the theory (Section 2) and the empirical application (Section 3). The theoretical part covers first a simple model with participation and job search decisions, and then extends the analysis to include earnings decisions. The empirical part presents a calibration to the US economy and tests the Pareto-efficiency of the US tax-benefit system.

2 Theory

This section covers the theory part of the paper. It begins with a presentation of a stylized model which is used to illustrate some of the main results in a simple environment. An extended model is then introduced and thoroughly analyzed using both mechanism design and perturbation approaches.

2.1 Stylized model with participation and job search decisions

2.1.1 Setting

Agents. Consider a continuum of heterogeneous agents indexed by $i \in [0, 1]$ with population density $f_i(i)$. This model is stylized in that agents cannot freely choose their jobs and earnings levels. That is, an agent *i* may only work in a single job, or occupation, characterized by a level of gross earnings z_i . There is thus a direct mapping between index *i* and potential earnings on the job z_i , where i < j implies $z_i < z_j$, and I henceforth index agents by their potential earnings on the job *z*. The population density $f_i(i)$ is then equivalently represented by a potential earnings density $f_z(z)$.

The model is static and agents make two sequential decisions. First, they decide whether they want to participate in the labor market. Non-participating agents become inactive: they will never be employed nor search for a job. In contrast, participating agents become active on the labor market: they may work in their occupation paying z if they are employed or search for a job paying z if they are unemployed. Such participation decisions are partly driven by fixed participation costs χ reflecting the potential monetary, time and psychological costs (or benefits) associated with being active on the labor market. These participation costs are heterogeneously distributed both within jobs and across jobs with density $f_{\chi|z}(\chi|z)$. This gives rise to job-specific participation decisions summarized by the function $h_z(z) \leq f_z(z)$ measuring the number of agents who are active on the labor market at potential earnings z.

Second, upon participation to the labor market, agents must decide on their job search efforts when unemployed. Indeed, agents are here exposed to unemployment risk on the labor market meaning that agents spend a fraction of time e employed and a fraction of time 1 - e unemployed. Job search efforts when unemployed are costly, but higher search intensity maps into a higher fraction of time spent employed. Job search efforts decisions are thus modelled in reduced-form as the choice of the time spent employed e(z) at job z given an increasing and convex search cost function $\psi(e; z)$. Saying agents are exposed to unemployment risk is here equivalent to having search costs $\psi(e; z)$ that become infinite as e approaches 1. Given agents' job search decisions, the number of agents employed at earnings z is equal to $e(z) h_z(z)$.

Assume that the government uses the following policy instruments: (1) a tax and transfer schedule $T_e(z)$ levied on the employed which specifies the amount of taxes paid (or transfers received) as a function of earnings on the job z, (2) an unemployment benefits schedule $B_u(z)$ which specifies the net amount of benefits received when unemployed as a function of earnings on the job z, (3) a social assistance program which provides a lump-sum transfer R_0 to non-participating agents who remain inactive.⁴⁷

Proceeding by backward induction, agents job search decisions upon participation solve

$$V(z) \equiv \max_{e} e \left[u \left(z - T_{e}(z) \right) - k(z) \right] + (1 - e) \left[u \left(B_{u}(z) \right) - \psi \left(e; z \right) \right]$$
(1)

where u(c) is a concave utility from consumption assumed stable across employment statuses, and k(z) is a disutility to work incurred when working in job z. This pins down the fraction of time e(z) spent employed at earnings z and defines indirect utility V(z).

Agents choose to participate in the labor market if and only if

$$V(z) - \chi \ge u(R_0) \iff \chi \le \tilde{\chi}(z) \equiv V(z) - u(R_0)$$
⁽²⁾

which defines a cut-off rule for participation. An agent with potential earnings on the job z decides to become active on the labor market if and only if her participation costs χ is lower than the job-specific participation cut-off $\tilde{\chi}(z) \equiv V(z) - u(R_0)$.

Implicit in this formulation of the problem is the hypothesis that agents are hand-tomouth and do not self-insure through savings against unemployment risk. Indeed, agents

⁴⁷I provide intuitions for the use of such policy instruments throughout the analysis of this stylized model. A more in-depth discussion on the optimality of these policy instruments and potential alternatives is included in the analysis of the general model.

here consume their net disposable income equal to $z - T_e(z)$ when employed and to $B_u(z)$ when unemployed. This hypothesis is adopted to reflect the fact that most jobseekers begin their unemployment spells without any liquidities, and I remain agnostic on the reasons behind this absence of self-insurance.⁴⁸ I discuss the introduction of savings and the provision of private insurance at the very end of the section.

A steady state approach. While the model presented thus far is static, there is a fundamental dynamic aspect to the provision of unemployment insurance: unemployment insurance depends in practice on the labor earnings realized in the past. I here reconcile these two views by showing that the simple static model described above can be micro-founded as the steady state representation of a dynamic model.

Consider a dynamic and infinite-horizon model where each time period (e.g. a week) is indexed by time t. To abstract from commitment problems on the side of the government, assume that the schedule of policy instruments $T_e(z)$, $B_u(z)$, R_0 is time-invariant and set at the origin of time. Moreover, assume that neither the government nor agents discount time.

At t = 0, agents must decide once and for all whether or not to participate in the labor market. Non-participating agents become inactive: they will never be employed nor search for a job in the future, and obtain the social assistance transfer R_0 every period. Participating agents become active on the labor market: they are then either employed or unemployed at time t.

If employed at t, an agent works in its dedicated occupation and receives gross earnings z net of the taxes and transfers $T_e(z)$. If unemployed at time t, an agent searches for a job in its dedicated occupation while receiving unemployment benefits $B_u(z)$ which depend on realized earnings z during the last employment spell. An exception are individuals joining the labor market: they initially start unemployed at t = 0 and must search for jobs while receiving the social assistance transfer R_0 .

Transitions between employment and unemployment are determined in the following way. When employed in a job paying earnings z, agents may become unemployed if their job gets destroyed which happens with an exogenous job-specific probability of separation s(z), assumed time-invariant. When unemployed, agents must search for jobs and search intensity determines the matching probability $m_t(z)$ that an agent finds a job in its dedicated occupation paying earnings z. Search decisions during the initial unemployment spell naturally differ from those made in subsequent unemployment spells. However, agents' problem in this infinite-horizon model converges to a stationary problem meaning that the matching probability converges to a time-invariant value m(z).

⁴⁸Potential explanations include missing markets, present bias or over-optimistic beliefs.

Lemma 1. The static model in which agents' participation and job search decisions follow from equations (1) and (2) is the steady state representation of the above dynamic model where the steady state probability of being employed is

$$e(z) = \frac{m(z)}{s(z) + m(z)}$$
(3)

Proof. For an agent working in a job paying earnings z, let $P_t(z) = \begin{pmatrix} e_t(z) \\ 1 - e_t(z) \end{pmatrix}$ be the probability vector measuring in period t the probability $e_t(z)$ to be employed, and the probability $1 - e_t(z)$ to be unemployed. Next period probability vector $P_{t+1}(z)$ is then asymptotically determined by $P_{t+1}(z) = M(z) \cdot P_t(z)$ where $M(z) = \begin{pmatrix} 1 - s(z) & m(z) \\ s(z) & 1 - m(z) \end{pmatrix}$ is a time-invariant transition matrix. This matrix has one eigenvalue equal to 1, and another equal to 1 - (s(z) + m(z)) which is lower than 1 in absolute value. As a result, the probability vector $P_t(z)$ converges to a steady state value P(z) characterized by $P(z) = M(z) \cdot P(z)$ or equivalently e(z) = (1 - s(z)) e(z) + m(z) (1 - e(z)). Rearranging yields equation (3).

By focusing on the steady state, the paper disregards transitional dynamics. This is key to obtain a simple and tractable model of redistribution and unemployment insurance as it eliminates issues of timing and history dependence. Indeed, agents spend in steady state a fraction of time e(z) employed at their occupation paying earnings z and a fraction of time 1 - e(z) unemployed. The government can thus perfectly separate unemployed individuals who spend some time employed at earnings z and to whom the government provides unemployment benefits $B_u(z)$ when unemployed, from inactive individuals who are never employed and who are only eligible to social assistance transfer R_0 .

Government. In this environment, the government aims to provide both redistribution and unemployment insurance. Assume it holds a welfarist objective with a social welfare function given by the sum of indirect utilities up to a concave transformation G(.). The government then sets its policy instruments $\{T_e(z), B_u(z), R_0\}$ to maximize

$$\int_{z} \left\{ \int_{\chi \leq \tilde{\chi}(z)} G\left(V\left(z\right) - \chi\right) f_{\chi|z}\left(\chi\right) d\chi + \int_{\chi \geq \tilde{\chi}(z)} G\left(u\left(R_{0}\right)\right) f_{\chi|z}\left(z\right) d\chi \right\} f_{z}\left(z\right) dz \tag{4}$$

where the first term relates to agents with potential earnings z who are active in the labor market and obtain indirect utility V(z) net of participation costs χ , and the second term to those who are inactive and only consume R_0 . Redistribution relates to the maximization of the weighted sum of utilities across heterogeneous agents, it is therefore driven both by the concavity of the government's G(.) function and by the concavity of agents utility from consumption u(.). In contrast, insurance provision relates to the maximization of utilities among homogeneous agents, it is therefore only driven by the latter.

Both redistribution and insurance are here achieved through the set of policy instruments $\{T_e(z), B_u(z), R_0\}$ which must respect the government's resource constraint

$$\int_{z} \left\{ \int_{\chi \leq \tilde{\chi}(z)} \left[e\left(z\right) T_{e}\left(z\right) - \left(1 - e\left(z\right)\right) B_{u}\left(z\right) \right] f_{\chi|z}\left(\chi\right) d\chi - \int_{\chi \geq \tilde{\chi}(z)} R_{0} f_{\chi|z}\left(\chi\right) d\chi \right\} f_{z}\left(z\right) d\omega \geq Exp$$
(5)

stating that the taxes levied on the employed $T_e(z)$ must finance the benefits provided to the unemployed $B_u(z)$, the transfer provided to those inactive R_0 , and an exogenous expenditure requirement Exp.

Although stylized, this model nests two workhorse frameworks in public economics. Assuming away unemployment risk and setting e(z) = 1, the model boils down to a continuous version of Saez (2002a) optimal taxation framework in an extensive margin model – in which case $\{T_e(z), R_0\}$ is the usual tax and transfer system. Similarly, assuming agents' participation decisions are exogeneous, this model corresponds to a simple version of Chetty (2008) optimal unemployment insurance model with the novelty that earnings z are here heterogeneous – in which case $\{T_e(z), B_u(z)\}$ is the usual unemployment insurance system for agents at earnings z with the additional constraint that insurance should be actuarially fair.

This stylized model is thus particularly well suited to study how the interactions between redistribution and unemployment insurance affect conventional wisdom regarding the optimal design of tax-benefits systems. I carry out this analysis expressing all results in terms of empirically estimable sufficient statistics that measure agents responses to tax-benefits reforms.

Sufficient statistics. Since agents make participation and job search decisions, there are two types of behavioral responses to tax-benefits reforms: participation responses and job search responses.

Participation responses are captured through participation semi-elasticities $\pi_e(z)$ and $\pi_u(z)$ (resp. $\pi_0(z)$) which measure the percentage increase (resp. decrease) in the number of agents $h_z(z)$ who are active on the labor market at potential earnings z upon a unit increase in disposable income when employed and unemployed (resp. inactive). Formally,

$$\pi_{e}(z) \equiv \frac{1}{h_{z}(z)} \frac{\partial h_{z}(z)}{\partial (z - T_{e}(z))} \qquad \pi_{u}(z) \equiv \frac{1}{h_{z}(z)} \frac{\partial h_{z}(z)}{\partial B_{u}(z)} \qquad \pi_{0}(z) \equiv -\frac{1}{h_{z}(z)} \frac{\partial h_{z}(z)}{\partial R_{0}} \quad (6)$$

where these semi-elasticity concepts are related through the following structural relationship

$$\frac{\pi_e(z)}{e(z)\,u'(z-T_e(z))} = \frac{\pi_u(z)}{(1-e(z))\,u'(B_u(z))} = \frac{\pi_0(z)}{u'(R_0)} \tag{7}$$

which follows from agents' participation decisions. It links participation semi-elasticities scaled by the corresponding marginal utility of consumption and time spent in each state.

Job search responses are captured through job search semi-elasticities $\mu_e(z)$ and $\mu_u(z)$ which measure the percentage increase in the time spent unemployed 1 - e(z) among agents who are active on the labor market at potential earnings z upon a unit increase in taxes when employed and benefits when unemployed. Formally,

$$\mu_e(z) \equiv \frac{1}{1 - e(z)} \frac{\partial \left(1 - e(z)\right)}{\partial T_e(z)} \qquad \mu_u(z) \equiv \frac{1}{1 - e(z)} \frac{\partial \left(1 - e(z)\right)}{\partial B_u(z)} \tag{8}$$

where these elasticity concepts are related through the following structural relationship

$$\frac{\mu_e(z)}{u'(z - T_e(z))} = \frac{\mu_u(z)}{u'(B_u(z))}$$
(9)

which follows from agents' job search decisions and links job search semi-elasticities scaled by the marginal utility of consumption in each state.

Although different in nature, another important type of sufficient statistics are social marginal welfare weights. Social marginal welfare weights $g_e(z)$, $g_u(z)$ and g_0 measure the social value of providing a unit increase in disposable income to respectively employed, unemployed and inactive agents. They are formally defined as

$$g_e(z) \equiv \frac{\overline{G'(V(z) - \chi)}}{\lambda} u'(z - T_e(z)) \qquad g_u(z) \equiv \frac{\overline{G'(V(z) - \chi)}}{\lambda} u'(B_u(z)) \qquad (10)$$

$$g_0 \equiv \frac{G'\left(u\left(R_0\right)\right)}{\lambda} u'\left(R_0\right) \tag{11}$$

and thus depend both on $\overline{G'(V(z)-\chi)} \equiv \int_{\chi \leq \tilde{\chi}(z)} G'(V(z)-\chi) f_{\chi|z}(\chi) d\chi$ and $G'(u(R_0))$ which capture the government's redistributive tastes and on agents' marginal utility of consumption in each state. The scaling factor λ measures the social marginal value of public funds. Intuitively, the government values giving $g_e(z)$ additional dollar to an agent employed in a job paying z as much as giving one additional dollar to every agents in the economy.

The government thus values transfering resources towards agents-states with higher social marginal welfare weights. As will become apparent, this logic applies here both to redistribution towards low income agents and to unemployment insurance provision towards the unemployed.

2.1.2 Optimal policies

This framework can be used to answer at least two types of questions regarding the design of policy instruments $\{T_e(z), B_u(z), R_0\}$. First, one can analyze the optimal schedule of one instrument, e.g. $T_e(z)$, given existing and potentially suboptimal schedules of other policy instruments, e.g. $B_u(z)$ and R_0 . Second, one can analyze joint optimality of policy instruments.

In the spirit of the first question, I sequentially derive the optimal tax and transfer schedule $T_e(z)$, and the optimal unemployment benefits schedule $B_u(z)$ taking in every instance other policies as given. Building on these results, I then move to the second type of question and provide a simple characterization for the Pareto-efficiency of the tax-benefits system $\{T_e(z), B_u(z)\}$.

Optimal tax and transfer schedule.

Proposition 1. Taking the unemployment benefits schedule $B_u(.)$ and the social assistance transfer R_0 as given, the optimal tax and transfer schedule $T_e(.)$ is characterized at each level of earnings z by

$$\left(T_{e}(z) + R_{0}\right) - (1 - e(z))\left(T_{e}(z) + B_{u}(z)\right)\frac{\pi_{e}(z) - \mu_{e}(z)}{\pi_{e}(z)} = e(z)\frac{1 - g_{e}(z)}{\pi_{e}(z)}$$
(12)

Proof. Consider a unit (e.g. a dollar) increase in the tax $T_e(z)$ levied on employed agents earnings z. This reform induces three different effects:

(1) There is a mechanical effect M on the mass $e(z) h_z(z)$ of employed agents at z. The reform raises one unit of tax revenue per agent which has a social benefit of 1 but also decreases the disposable income of each agent by this unit which has a social cost equal to $g_e(z)$. As a result, the mechanical effect changes the government objective by $M = e(z) h_z(z) (1 - g_e(z)).$

(2) There is a participation effect P. The unit tax increase reduces incentives to participate and induces $\pi_e(z) h_z(z)$ agents to leave the labor market. Each agent leaving the labor market reduces government's resources by $(T_e(z) + R_0) - (1 - e(z)) (T_e(z) + B_u(z))$. This *participation tax* reflects the fact that an agent who becomes inactive stops paying taxes $T_e(z)$ and starts receiving the transfer R_0 . However, an active agent is also unemployed during a fraction of time (1 - e(z)) in which it does not pay taxes $T_e(z)$ and receives $B_u(z)$, resources that are now saved by the government. The participation effect thus changes the government objective by $P = -\pi_e(z) h_z(z) [(T_e(z) + R_0) - (1 - e(z)) (T_e(z) + B_u(z))].$ (3) There is a job search effect S. The unit tax increase when employed reduces incentives to search for the mass $h_z(z)$ of active agents at earnings z. This induces a $\mu_e(z)(1 - e(z))$ increase in the time spent unemployed. This in turn reduces government's resources by the *employment tax* $T_e(z) + B_u(z)$ reflecting the fact that while unemployed agents do not pay taxes and receive benefits. Taking stock, the search effect changes the government objective by $S = -h_z(z) \mu_e(z)(1 - e(z))(T_e(z) + B_u(z))$.

The total effect of the reform is M + P + S, and the optimal schedule can by definition not be improved upon. Setting M + P + S = 0 and rearranging yields (12).

Assuming away unemployment risk (e(z) = 1), this boils down to Saez (2002a) optimal tax formula in an extensive margin model:

$$T_{e}(z) + R_{0} = \frac{1 - g_{e}(z)}{\pi_{e}(z)}.$$
(13)

The optimal tax then decreases with the redistributive motive towards agents working at z captured by $1 - g_e(z)$, and with participation responses captured by $\pi_e(z)$. Indeed, the government's ability to tax is bounded by the necessity to make work pay, that is boost participation to the labor market in order to collect revenue on a larger tax base.

Introducing unemployment risk into the picture has two effects. First, the redistributive motive on the right-hand side is now weighted by the time spent employed e(z) calling for higher taxes. Second, it introduces a new term on the left-hand side proportional to the time spent unemployed 1 - e(z) and to the employment tax $T_e(z) + B_u(z)$ whose net effect is ambiguous. On the one hand, unemployment risk on the labor market reduces the tax revenue collected upon participation which alleviates make work pay concern and calls for higher taxes. On the other, unemployment risk introduces the necessity to make search pay, that is to boost job search through lower taxes in order to increase the time spent employed and thus the tax base. The net effect depends on whether make work pay responses $\pi_e(z)$ dominate or whether make search pay responses $\mu_e(z)$ dominate at earnings z.

An important result of Saez (2002a) relates to the optimality of EITC types of policies. Indeed, he shows it is optimal to provide larger transfers to low-income individuals employed at z than to the inactive, $-T_e(z) \ge R_0$, if and only if social marginal welfare weights are sufficiently large, $g_e(z) \ge 1$. This result is here reinforced, meaning that such a policy is optimal for even lower social marginal welfare weights, either if (i) $B_u(z) \ge -T_e(z)$ and $\pi_e(z) \le \mu_e(z)$, or if (ii) $B_u(z) \le -T_e(z)$ and $\pi_e(z) \ge \mu_e(z)$.

Case (i) is the "normal" case in which transfers to the employed are large but not as large as unemployment benefits. The employment tax $T_e(z) + B_u(z)$ is then positive and the case for large transfers to the employed is reinforced if and only if make search pay responses dominate. Intuitively, an EITC policy becomes more desirable because the government saves resources by reducing unemployment through job search.

Case (ii) is the "special" case in which transfers to the employed are larger than unemployment benefits. The employment tax $T_e(z) + B_u(z)$ is then negative and the case for large transfers to the employed is reinforced if and only if make work pay responses dominate. Indeed, the government would then loose too much tax revenue if search responses were dominant – the government here paradoxically saves resources by *keeping* people unemployed.

Optimal unemployment benefits schedule.

Proposition 2. Taking the tax and transfer schedule $T_e(.)$ and the social assistance transfer R_0 as given, the optimal unemployment benefits schedule $B_u(.)$ is characterized at each level of reference earnings on the job z by

$$T_{e}(z) + B_{u}(z) = \frac{1}{\pi_{u}(z) + \mu_{u}(z)} \left[\left(g_{u}(z) - 1 \right) + \pi_{u}(z) \frac{T_{e}(z) + R_{0}}{1 - e(z)} \right]$$
(14)

Proof. Consider a unit (e.g. a dollar) increase in the benefits $B_u(z)$ provided to the unemployed with reference earnings on the job z. This reform induces three different effects.

(1) A mechanical effect $M = (1 - e(z)) h_z(z) (g_u(z) - 1)$. The mass of unemployed agents $(1 - e(z)) h_z(z)$ see their disposable income increase by one unit which has a social benefit of $g_u(z)$ and a resource cost of one unit which has a social cost of 1.

(2) A participation effect $P = \pi_u(z) h_z(z) [(T_e(z) + R_0) - (1 - e(z)) (T_e(z) + B_u(z))].$ Indeed, an additional unit of benefits when unemployed induces a mass $\pi_u(z) h_z(z)$ of agents to join the labor market, and each agent entering the labor market increases the government's resources by the participation tax $(T_e(z) + R_0) - (1 - e(z)) (T_e(z) + B_u(z)).$

(3) A job search effect $S = -h_z(z) \mu_u(z) (1 - e(z)) (T_e(z) + B_u(z))$ since the mass $h_z(z)$ of agents who are active on the labor market increase their time spent unemployed by $\mu_u(z) (1 - e(z))$ which reduces government's resources by the employment tax $T_e(z) + B_u(z)$.

The total effect of the reform is M + P + S, and the optimal schedule can by definition not be improved upon. Setting M + P + S = 0 and rearranging yields (14).

Assuming away participation responses $(\pi_u (z) = 0)$ and assuming the government has no redistributive tastes $(g_u (z) = u' (B_u (z)) / \lambda)$, this boils down to a formula that is reminiscent

of the Baily-Chetty logic:⁴⁹

$$T_{e}(z) + B_{u}(z) = \frac{1}{\mu_{u}(z)} \left[\frac{u'(B_{u}(z))}{\lambda} - 1 \right].$$
 (15)

Optimal unemployment benefits $B_u(z)$ increase with consumption smoothing benefits captured by $u'(B_u(z))$ and decrease with job search responses captured by $\mu_u(z)$ which reflect the necessity to make search pay.

There are however two novelties compared to the Baily-Chetty logic. First, optimal unemployment benefits decrease with the amount of taxes paid when employed $T_e(z)$ capturing *all* fiscal externalities that being unemployed imposes on the government's resources. Second, optimal unemployment benefits are characterized for every reference earnings z without any restriction on the shape of the benefits schedule.

Introducing participation responses and redistributive tastes generates additional effects. Redistributive tastes directly enter into the valuation of consumption smoothing benefits through $g_u(z)$ pushing towards higher benefits at low income levels and lower benefits at high income levels.⁵⁰ Participation responses captured by $\pi_u(z)$ generally push through the new right-hand side term for higher unemployment benefits as a way to boost participation. Yet, this make work pay aspect also comes with a moderating channel through the presence of $\pi_u(z)$ in the denominator. Indeed, unemployment benefits reduce the amount of the participation tax collected on new participants.

The fact that unemployment benefits actually make work pay defines a Pareto lower bound for unemployment benefits. Indeed, even if the government only cares about tax revenue and not about the well-being of agents in the economy, introducing unemployment benefits allows to attract people on the labor market and may therefore generates additional tax revenue. By this logic, any increase in unemployment benefits below this Pareto lower bound is a Pareto-improvement: it is more than self-financing and increases agents' utility.

Corollary 1. A Pareto lower bound for the unemployment benefits schedule $B_u(z)$ is $\underline{B_u}(z)$ defined by

$$\underline{B}_{u}(z) = \frac{1}{\pi_{u}(z) + \mu_{u}(z)} \left[-1 + \pi_{u}(z) \frac{T_{e}(z) + R_{0}}{1 - e(z)} \right] - T_{e}(z)$$
(16)

Proof. Setting $g_u(z) = 0$ in (14) yields the result.

This Pareto lower bound for unemployment benefits $\underline{B}_u(z)$ is driven by pure efficiency motives. The first efficiency motive is related to make work pay: increasing unemployment

⁴⁹Equation (21) below is the exact analogue to Baily-Chetty optimal formula.

⁵⁰I show in the mechanism part of the paper that providing generous unemployment benefits to low income individuals is an efficient and incentive-compatible way to achieve redistribution.

benefits is efficient whenever this increase triggers positive fiscal externalities from participation responses which are larger than the mechanical cost and the fiscal externalities induced by job search responses.

The second efficiency motive is unrelated as can be seen when setting $\pi_u(z) = 0$ in equation (16) to obtain:

$$\underline{B}_{\underline{u}}(z) = -\frac{1}{\mu_u(z)} - T_e(z).$$
(17)

If agents receive net transfers when employed, we have $T_e(z) < 0$ and thus $\underline{B}_u(z) > 0$. Hence, there is a strictly positive Pareto lower bound for the unemployment benefits schedule despite the fact that the government only cares about collecting tax revenue and no participation responses. This perhaps surprising result is best understood in the context of the general Pareto-efficiency characterization for tax-benefits system that I now turn to.

Joint optimality.

Proposition 3. The tax-benefit system $\{T_e(z), B_u(z)\}$ is Pareto-efficient if and only if at each earnings z

$$\frac{u'(B_u(z)) - u'(z - T_e(z))}{u'(z - T_e(z))} = \frac{\mu_u(z)}{e(z)} \left(T_e(z) + B_u(z)\right)$$
(18)

or equivalently, to a first-order approximation,

$$\frac{B_u(z)}{z - T_e(z)} \approx 1 - \frac{\mu_u(z) B_u(z)}{\gamma(z) e(z)^2} \left[1 + \frac{e(z) T_e(z) - (1 - e(z)) B_u(z)}{B_u(z)} \right]$$
(19)

with $\gamma(z) \equiv -\frac{u''(z-T_e(z))(z-T_e(z))}{u'(z-T_e(z))}$ the coefficient of relative risk aversion and $\frac{B_u(z)}{z-T_e(z)}$ the net replacement rate.

Proof. Consider a joint reform that consists in a unit increase in benefits $dB_u(z) = 1$ together with a small increase in taxes $dT_e(z) = \frac{(1-e(z))u'(B_u(z))}{e(z)u'(z-T_e(z))}$ such that the reform leaves the utility of agents participating at earnings z unchanged. Because agents' utility is unchanged, agents' participation decisions remain unchanged. Therefore, the reform only generates two effects:

(1) The mechanical effect captures the mechanical change in government's resources. A $dT_e(z)$ increase in taxes when employed and a unit increase in benefits when unemployed on the mass $h_z(z)$ of agents participating at earnings z generate a mechanical effect given by $M = [e(z) dT_e(z) - (1 - e(z))] h_z(z)$.

(2) The search effect captures the fiscal externality induced by job search responses. The tax and the benefits change both increase agent's time spent unemployed by respectively

 $(1 - e(z)) \mu_e(z) dT_e(z)$ and $(1 - e(z)) \mu_u(z)$. This induces a loss in revenue proportional to the employment tax $(T_e(z) + B_u(z))$ and the mass $h_z(z)$ of agents. The search effect is thus equal to $S = -(1 - e(z)) [\mu_e(z) dT_e(z) + \mu_u(z)] (T_e(z) + B_u(z)) h_z(z)$.

Replacing $dT_e(z)$ by its value and $\mu_e(z)$ by its expression as a function of $\mu_u(z)$, the two effects imply that the joint reform changes the government's resources by

$$M + S = (1 - e(z)) \left[\left(\frac{u'(B_u(z))}{u'(z - T_e(z))} - 1 \right) - \frac{\mu_u(z)}{e(z)} \left(T_e(z) + B_u(z) \right) \right] h_z(z)$$
(20)

If the term in bracket is positive (resp. negative), a joint increase (resp. decrease) in taxes and benefits generates a Pareto-improvement. The Pareto-efficiency characterization (18) follows. Doing a first-order Taylor expansion of the left-hand side, noting that $T_e(z) + B_u(z) = \frac{B_u(z)}{e(z)} + \frac{1}{e(z)} [e(z) T_e(z) - (1 - e(z)) B_u(z)]$ and rearranging yields the second expression. \Box

Pareto-efficiency condition (18) is a general characterization of joint optimality. It links the tax and transfer schedule $T_e(z)$ and the unemployment benefits schedule $B_u(z)$ to job search responses $\mu_u(z)$ scaled by the time spent employed e(z) and the difference in the marginal utility of consumption when employed and unemployed.

Intuitively, a Pareto-efficient tax-benefit system must allocate consumption efficiently between these two states of the world. This means that when policies are jointly optimal, there cannot be a reallocation of consumption between the two states which leaves agents' well-being unchanged while generating additional resources for the government.

An important benchmark is the pure insurance case in which tax proceeds are used to finance an actuarially fair unemployment insurance system. In that case, the taxes paid when employed only finance the benefits received when unemployed such that $e(z) T_e(z) = (1 - e(z)) B_u(z)$. The Pareto-efficiency characterization is then equivalent to Baily-Chetty optimal formula

$$\frac{u'(B_u(z)) - u'(z - T_e(z))}{u'(z - T_e(z))} = \frac{\mu_u(z)}{e(z)} \left(T_e(z) + B_u(z)\right)$$
(21)

where the left-hand side captures the social benefits related to consumption smoothing, and the right-hand side captures the social cost of unemployment benefits provision related to job search responses.⁵¹

⁵¹This is the exact equivalent of equation (20) in Chetty (2008). Beyond notational differences, Chetty (2008) uses a *total* elasticity concept $\varepsilon_{D,b} = \frac{b}{D} \frac{dD}{db}$ while this paper uses a *partial* semi-elasticity concept $\mu_u(z) = \frac{1}{1-e(z)} \frac{\partial(1-e(z))}{\partial B_u(z)}$. The two differs in that the total elasticity concept measures changes in the time spent unemployed upon a change in benefits accompanied by the corresponding change in taxes used to finance these benefits. Following Chetty (2008) and assuming $e(z) T_e(z) = (1-e(z)) B_u(z)$, the two are connected through $\frac{d(1-e(z))}{dB_u(z)} = \frac{1}{e(z)} \frac{\partial(1-e(z))}{\partial B_u(z)}$ which gives the equivalence.

Yet, in addition to unemployment insurance, government's resources are generally used to finance redistribution (and public goods). This means that low-income people are net receivers of government's resources, while high-income people are net contributors to government's resources. In that case, the unemployment benefits schedule defined by Baily-Chetty optimal formula is Pareto-inefficient: there exists a reallocation of consumption across states that generates a Pareto-improvement.

Corollary 2. When the tax-benefits system $\{T_e(z), B_u(z)\}$ is used to finance redistribution (and public goods) in addition to unemployment insurance, the Baily-Chetty optimal formula (21) defines a Pareto-inefficient tax-benefits system. Denote agents net contributions to the tax-benefits system $C(z) \equiv e(z) T_e(z) - (1 - e(z)) B_u(z)$:

- if $C(z) \leq 0$, there exists a Pareto-improving joint increase in $T_{e}(z)$ and $B_{u}(z)$.
- if $C(z) \ge 0$, there exists a Pareto-improving joint decrease in $T_e(z)$ and $B_u(z)$.

Proof. $C(z) \leq 0$ is equivalent to $T_e(z) + B_u(z) \leq \frac{B_u(z)}{e(z)}$. The Pareto-efficiency characterization (18) and Baily-Chetty optimal formula (21) then imply that

$$\frac{u'\left(B_{u}\left(z\right)\right)-u'\left(z-T_{e}\left(z\right)\right)}{u'\left(z-T_{e}(z)\right)}\Big|_{Pareto} \leq \frac{u'\left(B_{u}\left(z\right)\right)-u'\left(z-T_{e}\left(z\right)\right)}{u'\left(z-T_{e}(z)\right)}\Big|_{Baily-Chetty}$$

meaning that an economy in which the Baily-Chetty optimal formula is verified allocates consumption inefficiently across states: there exists a joint increase in unemployment benefits and in taxes which leads to a Pareto-improvement. If $e(z) T_e(z) \ge (1 - e(z)) B_u(z)$, the inequality is reversed and so is the direction of the reform.

Baily-Chetty optimal formula leads to unemployment benefits which are too low for low-income agents and too high for high-income agents. The intuition is that when net contributors (the rich) are employed they have a positive externality on the financing of redistribution, while net beneficiaries (the poor) have a negative externality. Because of these fiscal externalities, efficiency prescribes a more progressive schedule of unemployment benefits than the Baily-Chetty formula. This Pigouvian motive for progressivity in unemployment benefits is different from the incentive-compatibility motive highlighted in the mechanism design analysis next section.

2.2 Extended model with earnings decisions

2.2.1 Extended setting

Agents. In this extended setting, agents not only make participation and job search decisions but also earnings decisions. Following the optimal taxation literature (Mirrlees, 1971;

Saez, 2001), consider a continuum of agents with heterogeneous earnings ability ω with population density $f_{\omega}(\omega)$.

First, agents decide whether they want to participate in the labor market. As before, agents entering the labor market incur fixed participation costs (or benefits) χ which are heterogeneously distributed both within and across ability levels with density $f_{\chi|\omega}(\chi|\omega)$. Non-participating agents become inactive: they will never be employed nor search for a job. Participating agents become active on the labor market: they may work if they are employed or search for a job if they are unemployed.

Second, upon participation to the labor market, agents choose both the occupation in which they work when employed modelled here as a choice of earnings z, and their search efforts when unemployed.⁵² Agents are employed for a fraction of time e and unemployed for a fraction of time 1 - e.

When employed and working in a job paying z, an agent with earnings ability ω incurs a disutility to work equal to $k(z; \omega)$ reflecting the hours and efforts on the job. The disutility to work $k(z; \omega)$ is increasing and convex in earnings z as achieving higher earnings requires more work efforts, decreasing in ability ω as the effort requirement decreases with ability or skills. I also assume that the marginal disutility to work is decreasing in ability ω (Spence-Mirrlees condition when employed, $\frac{\partial k(y;\omega)}{\partial y \partial \omega} \leq 0$).

When unemployed, job search efforts are costly but higher search intensity maps into a higher fraction of time e spent on the job. Job search decisions s thus map in expectation into a fraction of time spent employed e(s) in a job paying earnings z given the search cost function $\psi(e(s), z; \omega)$. Search costs $\psi(e(s), z; \omega)$ are increasing and convex in the time spent employed e(s), increasing in earnings z to reflect that higher-paying jobs require higher search efforts, decreasing in ability ω to reflect that more able individuals may be more efficient or more succesful in their job search. I also assume that the marginal increase in the search cost associated with an increase in earnings z is decreasing in ability ω (Spence-Mirrlees condition when unemployed, $\frac{\partial^2 \psi(e(s), z; \omega)}{\partial z \partial \omega} \leq 0$).

Agent (ω, χ) expected utility upon participation writes

$$U(z, e(s), c_e, c_u; \omega, \chi) = e(s) [u(c_e) - k(z; \omega)] + (1 - e(s)) [u(c_u) - \psi(e(s), z; \omega)] - \chi$$
(22)

where c_e and c_u denote this agent's consumption levels when respectively employed and unemployed. Separability assumptions for the disutility to work $k(z;\omega)$ and the search effort cost $\psi(e, z; \omega)$ are adopted for tractability and may be relaxed. In contrast, the fact that these costs only depend on earnings ability ω considerably simplifies the problem by

 $^{^{52}}$ Thinking of this static setting as the steady state of a dynamic setting, it is natural to think of agents choosing an occupation before searching and working in that occupation.

imposing unidimensional heterogeneity on the labor market. Indeed, once fixed participation costs χ are sunk, earnings and job search decisions are only driven by ω .

Agent (ω, χ) utility if not participating in the labor market is $u(c_0)$ where c_0 denotes this agent's consumption level when inactive. Everywhere, the assumption that agents' utility from consumption is independent of agents' type is adopted for tractability and may be relaxed.

As before, this static model can be micro-founded as the steady state representation of a dynamic framework. For the redistribution problem, this means that agents characteristics are taken as exogenous and immutable. For the insurance problem, this means that the model disregards any issues pertaining to the timing of employment and unemployment spells. Also, savings are here ruled out by assumption and the government is the only insurance provider.

More importantly, the model is deterministic: I assume that search efforts s map into a unique realization of the time spent employed e(s) and disregard any uncertainty in the realization of employment and unemployment spells – this is akin to assuming that the law of large number holds at the individual level. In a mechanism design approach, this assumption has strong implications that I now turn to.

2.2.2 Mechanism design approach

Government. The government has two potential motives for policy intervention in this environment. First, it may want to redistribute across agents who differ in their innate characteristics. Second, it may want to insure people against unemployment risk.

In a mechanism design approach, the government is limited by asymetric information in its ability to intervene. Indeed, agents' characteristics (ω, χ) and job search efforts *s* are agents' private information and cannot be observed by the government who only observes labor market outcomes: time spent employed *e* and earnings *z*. Because the transfer of resources can only be conditioned on variables that are observable to the government, insurance and redistribution will be based on these proxies of search and work efforts. As a result, generous unemployment insurance will deter job search efforts thereby increasing the time spent unemployed (moral hazard problem). Similarly, generous redistribution will deter work efforts thereby reducing the earnings agents select (self-selection problem).

In this economy, an allocation is a quintuplet (z, e, c_e, c_u, c_0) . A direct mechanism consists in agents reporting their characteristics (ω, χ) and job search efforts s, and in the government assigning an allocation to each report. The problem of the government is to design the set of allocations $\mathcal{A} = \{z (\omega, \chi, s), e (\omega, \chi, s), c_e (\omega, \chi, s), c_u (\omega, \chi, s), c_0 (\omega, \chi, s)\}_{\omega,\chi,s}$ that maximizes its objective subject to a resource constraint and the information constraints inducing truthful reporting.

Asymetric information has some basic implications that considerably simplify the exposition of the problem. First, the government must provide the same consumption level c_0 to all non-participating agents because they are observationally equivalent (incentive compatibility). Second, the government has to provide the same labor market allocation $\{z (\omega, s), e (\omega, s), c_e (\omega, s), c_u (\omega, s)\}_{\omega,s}$ to all participating agents with ability ω and search effort s because they are also observationally equivalent (incentive compatibility). Third, since there is a unique dimension of heterogeneity ω on the labor market, all agents of ability ω make similar search decisions $s (\omega)$ such that labor market allocations can be written $\{z (\omega), e (\omega), c_e (\omega), c_u (\omega)\}_{\omega}$ with ability ω as the unique index. Last, participation decisions follow a cut-off rule: if an agent of ability ω with participation costs χ finds optimal to participate, then all agents with lower participation costs also find optimal to participate. I thus denote $\tilde{\chi} (\omega)$ the participation threshold of agents of ability ω .

Given these elements, the objective of a welfarist government with social welfare function given by the sum of indirect utilities up to a concave transformation G(.) writes

$$\int_{\omega} \left\{ \int_{\chi \leq \tilde{\chi}(\omega)} G\left(U\left(z\left(\omega\right), e\left(\omega\right), c_{e}\left(\omega\right), c_{u}\left(\omega\right); \omega, \chi\right) \right) f_{\chi|\omega}\left(\chi\right) d\chi + \int_{\chi \geq \tilde{\chi}(\omega)} G\left(u\left(c_{0}\right)\right) f_{\chi|\omega}\left(\chi\right) d\chi \right\} f_{\omega}\left(\omega\right) d\omega \quad (23)$$

and the resource constraint is

$$\int_{\omega} \left\{ \int_{\chi \leq \tilde{\chi}(\omega)} \left(e\left(\omega\right) \left[z\left(\omega\right) - c_{e}\left(\omega\right) \right] - \left(1 - e\left(\omega\right)\right) c_{u}\left(\omega\right) \right) f_{\chi|\omega}\left(\chi\right) \ d\chi - \int_{\chi \geq \tilde{\chi}(\omega)} c_{0} \ f_{\chi|\omega}\left(\chi\right) \ d\chi \right\} f_{\omega}\left(\omega\right) d\omega \geq Exp \quad (24)$$

where Exp is an exogenous expenditure requirement.

Incentive compatibility. A mechanism is incentive-compatible if agents make truthful reports and thereby pick the allocation designed for them. Basic implications of incentive compatibility outlined above imply that the set of incentive compatible allocations is here of the form $\mathcal{A} = \{A(\omega), c_0\}$, where $A(\omega) = \{z(\omega), e(\omega), c_e(\omega), c_u(\omega)\}$ is the allocation designed for agents of ability ω who are active on the labor market, and c_0 is the lump-sum transfer provided to non-participants.

Incentive compatibility imposes the set of incentive constraints

$$\forall \omega, \forall \omega', \quad V_m(\omega; \omega) \ge V_m(\omega'; \omega) \tag{25}$$

where $V(\omega'; \omega)$ is the gross utility level of an agent with ability ω choosing allocation $A(\omega')$ on the labor market, formally defined as

$$V(\omega';\omega) \equiv e(\omega') \left[u(c_e(\omega')) - k(z(\omega');\omega) \right] + (1 - e(\omega')) \left[u(c_u(\omega')) - \psi(e(\omega'),z;\omega) \right]$$
(26)

such that the *net* utility level of this agent on the labor market is $V(\omega'; \omega) - \chi$.

Individual rationality. A mechanism is individual-rational if agents participation decisions are utility maximizing. Imposing incentive compatibility, an agent of ability ω chooses allocation $A(\omega)$ on the labor market and thus obtain upon participation utility $V(\omega; \omega)$ net of participation costs χ . Therefore, individual rationality implies that an agent of type (ω, χ) decides to participate in the labor market if and only if

$$V(\omega;\omega) - \chi \ge u(c_0) \iff \chi \le \tilde{\chi}(\omega) \equiv V(\omega;\omega) - u(c_0)$$
(27)

Planner's problem. We can now write the planner's problem which is to design the set of allocations $\mathcal{A} = \{\{A(\omega)\}_{\omega}, c_0\}$ with $A(\omega) = \{z(\omega), e(\omega), c_e(\omega), c_u(\omega)\}$ solving

In what follows, I solve this general problem for three different mechanisms which highlight the interactions between redistribution and unemployment insurance.

In the optimal taxation literature, a known issue in solving this type of problem is how to deal with the set of incentive constraints. The usual way to handle incentive constraints is to note that the set of incentive constraints applying to an agent of ability ω is equivalent to $V(\omega; \omega) = \max_{\omega} V(\omega'; \omega)$. The set of incentive constraints can then be replaced by the first-order condition of this problem and by the second-order condition usually equivalent to a monotonicity condition on earnings. This monotonicity condition imposes that earnings $z(\omega)$ increase with ability ω . It is generally ignored in solving the planner's problem and checked ex-post.

In the first mechanism I consider, I show this procedure is valid and use it to characterize the optimum. In the two other mechanisms I consider, I am unable to find a set of necessary and sufficient conditions for $V(\omega; \omega) = \max_{\omega} V(\omega'; \omega)$. However, the first-order condition of this problem and two monotonicity conditions, one standard on earnings z and one novel on time spent employed e, turn out to be sufficient conditions. Keeping only the first-order necessary condition and ignoring these monotonicity conditions, I characterize the solution to the planner's problem and check ex-post that it is incentive-compatible. Given their technical nature, these results are left out from the body of the paper and relegated to the online appendix.

Optimal mechanism. The *optimal mechanism* is the mechanism in which the design of allocations is only restricted by information constraints, it thus correspond to a second-best world. As $e(\omega)$ belongs to $A(\omega)$, this implies that the planner may directly impose a specific value of time spent employed $e(\omega)$ to agents of ability ω . In practice, this would be achieved through e.g. very high taxes on those deviating from this specific value.

Given the assumption that the model is deterministic, search efforts s map into a unique realization of the time spent employed e. Hence, the government finds optimal to impose the value $e(\omega)$ that correspond to first-best search efforts: there is no moral hazard problem and the government provides full insurance to the unemployed.

Proposition 4. Given the assumption that the model is deterministic, there is no moral hazard problem and the second-best solution to (\mathcal{P}) includes first-best search efforts and full unemployment insurance:

$$\forall \omega, c_e(\omega) = c_u(\omega) = c(\omega).$$
(28)

Proof. Omitted.

This result crucially relies on: (i) the assumption that the model is deterministic, (ii) the fact that the planner may impose particular values of time spent employed e.

Relaxing (i) and introducing uncertainty in the model would break down the result as the government would no longer be able to dictate first-best search efforts s by imposing a specific value of time spent employed e. This could for instance be achieved through a parametrization of a probability distribution mapping search efforts s to potential values of time spent employed e. While interesting, I leave that to future research and choose to remain within the boundaries of this deterministic model.

In this deterministic model, the optimal mechanism is an interesting benchmark but it rests on (ii) which is at odds with how tax-benefits system operate in practice: tax agencies do not levy high taxes on individuals staying a bit too long employed or a bit too long unemployed. I thus relax (ii) and consider alternative mechanisms.

Constant mechanism. First, consider a mechanism in which the planner may not condition allocations on the fraction of time spent employed but only on the employment status. This is a *constant mechanism* in the sense that agents' allocations when employed and when unemployed are constant across all values of time spent employed. Formally, this corresponds to a third-best problem in which the planner designs allocations $\mathcal{A}_{CM} = \{\{A_{CM}(\omega)\}_{\omega}, c_0\}$ with $A_{CM}(\omega) = \{z(\omega), c_e(\omega), c_u(\omega)\}$.

Agents are now free to choose their search efforts which are endogenously determined through utility maximization. Upon choosing an allocation $A_{CM}(\omega')$, an agent of ability ω picks the job search effort $e_{CM}(\omega';\omega)$ that solves

$$\max_{e} e\left[u\left(c_{e}\left(\omega'\right)\right) - k\left(z\left(\omega'\right);\omega\right)\right] + (1-e)\left[u\left(c_{u}\left(\omega'\right)\right) - \psi\left(e,z\left(\omega'\right);\omega\right)\right].$$
(29)

This restores at allocation $A_{CM}(\omega')$ the trade-off at the heart of unemployment insurance: providing generous unemployment benefits $c_u(\omega')$ has consumption smoothing benefits but it negatively impacts search efforts and thus time spent employed $e_{CM}(\omega'; \omega)$.

However, assuming job search costs $\psi(e, z(\omega'); \omega)$ go to zero as agents' fraction of time spent employed *e* goes to zero, any agent may obtain the utility level $u(c_u(\omega'))$ by choosing to be employed an infinitesimal fraction of time (e.g. a day) at earnings $z(\omega')$ and unemployed the rest of the time consuming $c_u(\omega')$. Formally, for any ω

$$\lim_{e \to 0^+} e \left[u \left(c_e \left(\omega' \right) \right) - k \left(z \left(\omega' \right) ; \omega \right) \right] + (1 - e) \left[u \left(c_u \left(\omega' \right) \right) - \psi \left(e, z \left(\omega' \right) ; \omega \right) \right] = u \left(c_u \left(\omega' \right) \right)$$
(30)

Proposition 5. In the constant mechanism, the provision of unemployment insurance is bounded by the utility level of the agent who is the worst-off on the labor market:

$$\forall \omega', \ u\left(c_u\left(\omega'\right)\right) \le \underline{\mathcal{V}} \equiv \min_{\omega} V\left(\omega;\omega\right) \tag{31}$$

Proof. Any agent may obtain the utility level $u(c_u(\omega'))$, in particular the agent who is the worst-off on the labor market. Incentive compatibility for this agent implies (31).

The scope for unemployment insurance provision is thus extremely limited when the planner is completely unable to condition allocations on the time spent employed e.

This reflects a problem of "double deviation". Since agents choose both earnings z (through the reporting of their type ω) and time spent employed e, they can make a double deviation from $\{z(\omega), e_{CM}(\omega; \omega)\}$ towards $\{z(\omega'), 0^+\}$ where $z(\omega')$ is typically a higher

earnings level than $z(\omega)$ with potentially higher consumption when unemployed $c_u(\omega')$. As the planner is unable to condition allocations on the time spent employed e, agents then obtain this consumption level $c_u(\omega')$.

Intuitively, unemployment benefits become a form of social assistance transfer since agent (ω, χ) outside option is now max $\{u(\overline{c_u}) - \chi; u(c_0)\}$, with $\overline{c_u} \equiv \max_{\omega'} c_u(\omega')$. The reason is that in this case unemployment benefits are, just like social assistance transfers, not conditioned on a minimal employment duration or a maximal unemployment duration. Eligibility requirements of this kind are however ubiquitous in actual unemployment insurance systems which motivates the introduction of a third mechanism.

Threshold mechanism. In practice, the receipt of unemployment benefits is generally tied to (i) a minimal employment duration which determines eligibility to unemployment insurance upon lay-off, and to (ii) a maximal benefits duration after which agents are no longer eligible to unemployment insurance even if they are still unemployed.

In this static model, both types of eligibility requirements are somewhat equivalent.⁵³ They map into an eligibility threshold $\underline{e}(\omega')$ specifying the minimum fraction of time an individual has to be working at earnings $z(\omega')$ to obtain a consumption level $c_u(\omega')$ when unemployed, or equivalently the maximum fraction of time $1-\underline{e}(\omega')$ an individual can remain unemployed with a consumption level $c_u(\omega')$.

Formally, in this treshold mechanism the planner solves a third-best problem in which it designs allocations $\mathcal{A}_{TM} = \{\{A_{TM}(\omega)\}_{\omega}, c_0\}$ with $A_{TM}(\omega) = \{z(\omega), c_e(\omega), c_u(\omega), \underline{e}(\omega)\}$. An agent with ability ω at allocation $A_{TM}(\omega')$ picks the job search effort $e_{TM}(\omega'; \omega)$ that maximizes its utility given by

$$\begin{cases} e \left[u \left(c_{e} \left(\omega' \right) \right) - k \left(z \left(\omega' \right) ; \omega \right) \right] + (1 - e) \left[u \left(c_{u} \left(\omega' \right) \right) - \psi \left(e, z \left(\omega' \right) ; \omega \right) \right] - \chi & \text{if } e \geq \underline{e} \left(\omega' \right) \\ \alpha \left(e \right) \left\{ \underline{e} \left(\omega' \right) \left[u \left(c_{e} \left(\omega' \right) \right) - k \left(z \left(\omega' \right) ; \omega \right) \right] + (1 - \underline{e} \left(\omega' \right) \right) u \left(c_{u} \left(\omega' \right) \right) \right\} \\ + (1 - \alpha \left(e \right) \right) u \left(c_{0} \right) - (1 - e) \psi \left(e, z \left(\omega' \right) ; \omega \right) - \chi \end{cases} & \text{if } e \leq \underline{e} \left(\omega' \right) \end{cases}$$

$$(32)$$

with $\alpha(e) \equiv \frac{e}{\underline{e}(\omega')} \in [0, 1]$ the parameter that governs eligibility to unemployment insurance when $e \leq \underline{e}(\omega')$. That is, I assume agents who are employed a fraction of time $e \leq \underline{e}(\omega')$ can still be eligible to receive $c_u(\omega')$ during a subperiod of the time they spend unemployed, while they receive c_0 in the subperiod in which they are ineligible. Agents' utility level is thus continuous (but kinked) at $e = \underline{e}(\omega')$ and smoothly goes to $u(c_0)$ as the fraction of time

 $^{^{53}}$ When discussing extensions of the model, I provide a simple way to separate these two types of eligibility requirements in a static framework.

spent employed goes to zero.

This has two important implications. First, the presence of eligibility thresholds rules out profitable deviations from $\{z(\omega), e_{CM}(\omega; \omega)\}$ towards $\{z(\omega'), 0^+\}$ since agents obtain a utility level $u(c_0)$ when doing so. Now, if the eligibility threshold $\underline{e}(\omega')$ is very low, deviations towards $\{z(\omega'), \underline{e}(\omega')\}$ might still be profitable. Hence, the eligibility threshold $\underline{e}(\omega')$ must be sufficiently large to rule out such deviations and restore the planner's ability to provide unemployment insurance.

Second, whenever agents of ability ω pick job search efforts $e_{TM}(\omega; \omega) > \underline{e}(\omega)$ at the allocation $A_{TM}(\omega)$ designed for them, they face the same problem as in the constant mechanism and thus make identical choices $e_{TM}(\omega; \omega) = e_{CM}(\omega; \omega)$. In that case, the eligibility threshold $\underline{e}(\omega)$ does not directly affect their job search decisions, and thereby, does not enter the planner's problem: $\{z(\omega), c_e(\omega), c_u(\omega)\}$ are the only variables that matter. Hence, whenever the eligibility threshold $\underline{e}(\omega)$ is sufficiently low, the optimal allocation $A_{TM}(\omega)$ is independent of the exact value of the threshold $\underline{e}(\omega)$: the eligibility threshold only acts in the background as a deterrent for deviations of agents with different ability $\omega' \neq \omega$.

Assumption 1. Assume eligibility thresholds $\{\underline{e}(\omega)\}_{\omega}$ are exogenously given and that they are both (i) sufficiently large such that unemployment insurance provision is only restricted by local incentive constraints, (ii) sufficiently low such that they do not influence agents' job search decisions at the allocation designed for them.

I solve the problem in the threshold mechanism under Assumption 1. Part (i) guarantees the smoothness of the problem and thus the validity of the first-order approach in solving the mechanism design problem. Part (ii) guarantees that the exact values at which eligibility thresholds are set do not affect optimal allocations such that eligibility thresholds can in henceforth be ignored.⁵⁴ More specifically, I assume agents job search decisions are defined by the first-order condition derived from (29) and solve the planner's problem (\mathcal{P}) under this additional condition. I then check ex-post that there exists a set of eligibility thresholds $\{\underline{e}(\omega)\}_{\omega}$ verifying Assumption 1 – one can even characterize ex-post all sets of eligibility thresholds $\{\underline{e}(\omega)\}_{\omega}$ verifying Assumption 1.

Proposition 6. Optimal allocations $\{z(\omega), c_e(\omega), c_u(\omega)\}$ in the threshold mechanism are

⁵⁴Intuitively, thresholds concavify the problem at its extremities (e.g. rule out deviations towards $e = 0^+$) but do not affect the interior solution of the problem.

characterized at each ability level ω by the following first-order conditions

$$\eta(\omega) e_m(\omega; \omega) u'(c_e(\omega)) - \mu(\omega) \frac{\partial e_m(\omega; \omega)}{\partial c_e(\omega)} \frac{\partial FOC_{e_m(\omega'; \omega)}}{\partial \omega} \Big|_{\omega'=\omega} = \lambda \bigg[e_m(\omega; \omega) - \frac{\partial e_m(\omega; \omega)}{\partial c_e(\omega)} [z(\omega) - c_e(\omega) + c_u(\omega)] \bigg] \int_{\chi=\underline{\chi}}^{\mathcal{V}(\omega) - u(c_0)} f(\omega, \chi) d\chi$$
(33)

$$\eta(\omega)\left(1 - e_m(\omega;\omega)\right)u'(c_u(\omega)) - \mu(\omega)\frac{\partial e_m(\omega;\omega)}{\partial c_u(\omega)}\frac{\partial FOC_{e_m(\omega';\omega)}}{\partial \omega}\Big|_{\omega'=\omega}$$
$$= \lambda \left[\left(1 - e_m(\omega;\omega)\right) - \frac{\partial e_m(\omega;\omega)}{\partial c_u(\omega)}\left[z(\omega) - c_e(\omega) + c_u(\omega)\right]\right]\int_{\chi=\underline{\chi}}^{\mathcal{V}(\omega) - u(c_0)} f(\omega,\chi) d\chi \quad (34)$$

$$\eta(\omega) \left[e_m(\omega; \omega) k'_1(z(\omega); \omega) + (1 - e_m(\omega; \omega)) \psi'_2(e_m(\omega; \omega), z(\omega); \omega) \right] -\mu(\omega) \left[e_m(\omega; \omega) k''_{12}(z(\omega); \omega) + (1 - e_m(\omega; \omega)) \psi''_{23}(e_m(\omega; \omega), z(\omega); \omega) + \frac{\partial e_m(\omega; \omega)}{\partial z(\omega)} \frac{\partial FOC_{e_m(\omega'; \omega)}}{\partial \omega} \Big|_{\omega' = \omega} \right] = \lambda \left[e_m(\omega; \omega) - \frac{\partial e_m(\omega; \omega)}{\partial z(\omega)} \left[z(\omega) - c_e(\omega) + c_u(\omega) \right] \right] \int_{\chi = \underline{\chi}}^{\mathcal{V}(\omega) - u(c_0)} f(\omega, \chi) d\chi$$
(35)

with λ the multiplier on the resource constraint, $\eta(\omega)$ the multiplier on agents' indirect utility, $\mu(\omega)$ the multiplier on agents' incentive compatibility constraints, and $FOC_{e_m(\omega';\omega)}$ the first-order condition pinning down $e_m(\omega';\omega)$.

Proof. Omitted.

Although somewhat abstract, this characterization reveals a key insight from the mechanism design approach: optimal allocations $\{z(\omega), c_e(\omega), c_u(\omega)\}$ are affected both by distortions stemming from the redistribution problem, the unemployment insurance problem and the interactions between the two.

For instance, (33) pins down optimal consumption when employed $c_e(\omega)$ through a costbenefit trade-off. The first term on the left-hand side captures the social benefit of raising $c_e(\omega)$. It increases with the shadow value of utility $\eta(\omega)$, agents' marginal utility from consumption $u'(c_e(\omega))$, and with the time spent employed $e_m(\omega; \omega)$.

The right-hand side term captures the resource cost of raising $c_e(\omega)$. It increases with the shadow value of public funds λ and the time spent employed $e_m(\omega; \omega)$. It decreases with the positive fiscal externality induced by marginally higher search efforts $\frac{\partial e_m(\omega; \omega)}{\partial c_e(\omega)}$ which is proportional to the employment tax $z(\omega) - c_e(\omega) + c_u(\omega)$ and to the mass of employed individuals $\int_{\chi=\chi}^{\mathcal{V}(\omega)-u(c_0)} f(\omega, \chi) d\chi$.

The second term on the left-hand side captures the incentive compatibility cost of raising $c_e(\omega)$. It increases with $\mu(\omega)$ the shadow value of tightening this incentive constraint, $\frac{\partial e_m(\omega;\omega)}{\partial c_e(\omega)}$ the extent to which job search changes, and $\frac{\partial FOC_{em}(\omega';\omega)}{\partial \omega}\Big|_{\omega'=\omega}$ the extent to which individuals with marginally higher ability levels ω picker marginally higher time spent employed at allocation $A(\omega'=\omega)$.⁵⁵

Interestingly, this interaction term distorts $c_e(\omega)$ downwards because higher ability individuals pick higher time spent employed – they face both smaller disutility to work k(.) and smaller search costs $\psi(.)$ at the same allocation. This implies that increasing $c_e(\omega)$ tightens incentive compatibility constraints relatively more than increasing $c_u(\omega)$. As a result, providing generous consumption levels to the low-income when they are unemployed is an incentive-compatible way to provide redistribution.

Going beyond this abstract mechanism design characterization, I provide in what follows an equivalent characterization using a tax perturbation approach.⁵⁶ Indeed, under Assumption 1 the problem is smooth and agents are on their first-order conditions such that the perturbation approach is valid. Beyond the fact that it illuminates the economic intuitions behind the results, the perturbation approach allows to obtain a characterization expressed in terms of empirically estimable sufficient statistics which is key for empirical applications.

2.2.3 Perturbation approach

Policy implementation. In a perturbation approach, the optimal solution to the problem is pinned down through reforms of policy instruments. This approach thus fundamentally relies on the set of policy instruments under consideration. I here consider a set of policy instruments that allows to implement the optimal allocations under the threshold mechanism. This policy implementation is not unique and I discuss alternative policy implementations at the end of the section.

Definition 1. Let $\mathcal{T} = \{\{T_e(z), B_u(z), \underline{e}(z)\}_z, R_0\}$ be a tax-benefits system such that $T_e(z)$ is the tax and transfer schedule when employed, $B_u(z)$ is the unemployment benefits schedule conditional on being employed at z for a fraction of time $e \geq \underline{e}(z)$, and R_0 is the social assistance transfer given to non participating agents.

Maintaining the assumption that eligibility thresholds are not binding in agents' choices (Assumption 1), an agent of ability ω picks the earnings $z(\omega)$ and time spent employed $e(z(\omega); \omega)$ defined by the first-order conditions of this problem

$$\mathcal{V}(\omega) = \max_{z} \left\{ \max_{e \ge \underline{e}(z)} e \left[u \left(z - T_{e}(z) \right) - k \left(z; \omega \right) \right] + (1 - e) \left[u \left(B_{u}(z) \right) - \psi \left(e, z; \omega \right) \right] \right\}.$$
 (36)

⁵⁵The incentive compatibility cost here entirely operates through $e_m(\omega';\omega)$ because utility is separable, $U(c, z; \omega) = u(c) - k(z; \omega)$. With a general utility function, there would be an additional $U_{12}''(c, z; \omega)$ term. ⁵⁶See **jacquet2017optimal** on the link between mechanism design and perturbation approaches.

This pins down net utility upon participation as $\mathcal{V}(\omega) - \chi$ such that agents participation decisions obey the following cut-off rule:

$$\mathcal{V}(\omega) - \chi \ge u(R_0) \iff \chi \le \tilde{\chi}(\omega) \equiv \mathcal{V}(\omega) - \chi.$$
 (37)

Lemma 2. The tax-benefits system $\mathcal{T} = \{\{T_e(z), B_u(z), \underline{e}(z)\}_z, R_0\}$ may decentralize the optimal menu of allocations under the threshold mechanism.

Proof. Let $\mathcal{A}_{TM}^* = \{\{A_{TM}^*(\omega)\}_{\omega}, c_0^*\}$ with $A_{TM}^*(\omega) = \{z^*(\omega), c_e^*(\omega), c_u^*(\omega), \underline{e}^*(\omega)\}$ denote the optimal menu of allocations under the threshold mechanism. Define for all ω , $T_e(z^*(\omega)) \equiv z^*(\omega) - c_e^*(\omega), B_u(z^*(\omega)) \equiv c_u^*(\omega), \underline{e}(z^*(\omega)) = \underline{e}^*(\omega)$ and $R_0 \equiv c_0^*$ such that optimal allocations $A_{TM}^*(\omega)$ are by definition available in the decentralized economy.

Now, the question is whether agents make the same choices in both economics. The answer is positive because agents decisions solve the same problems in both economies: the inner problem of equation (36) pinning down $e(z;\omega)$ is exactly equivalent to (29) pinning down $e(\omega';\omega)$, and the outter problem of equation (36) pinning down earnings $z(\omega)$ is exactly equivalent to incentive compatibility i.e. $V(\omega;\omega) = \max_{\omega'} V(\omega';\omega)$. Participation decisions characterized by (37) and (27) are therefore also similar which completes the proof.

This result says that the tax-benefits system \mathcal{T} may allow to decentralize the optimal allocations characterized in the mechanism design approach. It does not say that the optimal tax-benefits system \mathcal{T}^* characterized in a perturbation approach decentralizes the optimal menu of allocations characterized in the mechanism design approach. This last statement is much more difficult to prove analytically and I check numerically that the two solutions coincide.

Sufficient statistics. All results are here expressed in terms of empirically estimable sufficient statistics measuring agents behavioral responses to tax-benefits reform. ⁵⁷

Just like in the stylized model, agents make participation and job search decisions measured through participation and job search semi-elasticities. Participation semi-elasticities $\pi_e(z)$, $\pi_u(z)$ and $\pi_0(z)$ are defined exactly as in (6) and linked through the structural equation (7). Job search semi-elasticities $\mu_e(z)$ and $\mu_u(z)$ are also defined exactly as in (8) and linked through the structural equation (9), where the choice of time spent employed $e(z; \omega)$ is made conditional on a given earnings level z and evaluated at $z = z(\omega)$.

⁵⁷The monotonicity condition on earnings $z(\omega)$ defines a unique and increasing mapping between earnings z and ability ω . Leveraging this mapping, I directly define sufficient statistics in terms of earnings z, and generally omit the dependency in ability ω .

However, agents now make earnings decisions on top of participation and job search decisions. There are thus additional sufficient statistics to consider when measuring agents responses to tax-benefits reforms: earnings substitution elasticities and income effects parameters as well as the cross-partial between job search and earnings which is specific to this setting.

Earnings substitution elasticities $\zeta_{T_e}^z(z)$ and $\zeta_{B_u}^z(z)$ capture substitution effects in earnings choices $z(\omega)$. They measure the percentage increase in earnings z upon a 1 percent increase in, respectively, the marginal keep rate $1 - T'_e(z)$ and the marginal benefits rate $B'_u(z)$. Formally,

$$\zeta_{T_e}^z(z) \equiv \frac{1 - T_e'(z)}{z} \frac{\partial z}{\partial \left(1 - T_e'(z)\right)} \qquad \zeta_{B_u}^z(z) \equiv \frac{B_u'(z)}{z} \frac{\partial z}{\partial B_u'(z)}.$$
(38)

where these elasticity concepts are related through the structural relationship

$$\frac{\zeta_{T_e}^z(z)}{(1 - T'_e(z)) e(z) u'(z - T_e(z))} = \frac{\zeta_{B_u}^z(z)}{B'_u(z) (1 - e(z)) u'(B_u(z))}$$
(39)

which follows from agents' earnings first-order condition. It links substitution elasticities scaled by, respectively, the marginal keep rate and the marginal benefit rate, and the corresponding time and marginal utility of consumption in each state.

Earnings income effects parameters $\eta_{T_e}^z(z)$ and $\eta_{B_u}^z(z)$ measure the change in earnings upon a unit increase in, respectively, the tax level $T_e(z)$ and the benefits level $B_u(z)$ scaled by the marginal keep rate $1 - T'_e(z)$ and the marginal benefits rate $B'_u(z)$. Formally,

$$\eta_{T_e}^z(z) \equiv (1 - T_e'(z)) \frac{\partial z}{\partial T_e(z)} \qquad \eta_{B_u}^z(z) \equiv -B_u'(z) \frac{\partial z}{\partial B_u(z)} \tag{40}$$

where these income effects parameters are related through the structural relationship

$$\frac{\eta_{T_e}^z(z)}{\left(1 - T_e'(z)\right)^2 e(z) \, u''(z - T_e(z))} = \frac{\eta_{B_u}^z(z)}{\left(B_u'(z)\right)^2 \left(1 - e(z)\right) \, u''(B_u(z))} \tag{41}$$

which follows from agents' earnings first-order condition. It links income effects parameters scaled by second derivatives of the utility from consumption because income effects intuitively have to do with risk aversion and the curvature of utility.

The cross-partial between job search and earnings $\xi_z^{1-e}(\omega)$ measures the change in the fraction of time spent unemployed $1 - e(z; \omega)$ upon a unit increase in earnings z, evaluated at $z = z(\omega)$:

$$\xi_z^{1-e}(\omega) \equiv \frac{\partial \left(1 - e\left(z;\omega\right)\right)}{\partial z}\Big|_{z=z(\omega)}.$$
(42)

It measures the causal effect of an increase in earnings on job search. While all previous sufficient statistics can be signed, and are here defined as positive quantities, the sign of this cross-partial is not pinned down by the theory. It depends on whether it is unemployment benefits $B_u(z)$ or job search costs $\psi(e, z; \omega)$ that rise faster upon an increase in earnings z.

With a slight abuse of notation I henceforth express this cross-partial as a function of earnings z rather than ability ω , and I comment on the numerical values of $\xi_z^{1-e}(z)$ induced by a realistic calibration of the model in the empirical part of the paper.

Last, and as the stylized model, social marginal welfare weights $g_e(z)$, $g_u(z)$ and g_0 defined in (10) measure the social value of providing a unit increase in disposable income to respectively employed, unemployed and inactive agents.

Optimal tax and transfer schedule.

Proposition 7. Taking the unemployment benefits schedule $B_u(.)$ and the social assistance transfer R_0 as given, the optimal tax and transfer schedule $T_e(.)$ is characterized at each level of earnings z^* by

$$\frac{e(z^{*}) T_{e}'(z^{*}) - (1 - e(z^{*})) B_{u}'(z^{*})}{1 - T_{e}'(z^{*})} - \frac{T_{e}(z^{*}) + B_{u}(z^{*})}{1 - T_{e}'(z^{*})} \xi_{z}^{1-e}(z^{*})$$

$$= \frac{1}{\zeta_{T_{e}}^{z}(z^{*})} \frac{1}{z^{*}h_{z}(z^{*})} \int_{z \ge z^{*}} \left\{ e(z) (1 - g_{e}(z)) + \frac{e(z) T_{e}'(z) - (1 - e(z)) B_{u}'(z)}{1 - T_{e}'(z)} \eta_{T_{e}}^{z}(z) - (T_{e}(z) + B_{u}(z)) \left((1 - e(z)) \mu_{e}(z) + \xi_{z}^{1-e}(z) \frac{\eta_{T_{e}}^{z}(z)}{1 - T_{e}'(z)} \right) - \left[(T_{e}(z) + R_{0}) - (1 - e(z)) (T_{e}(z) + B_{u}(z)) \right] \pi_{e}(z) \right\} h_{z}(z) dz$$

$$(43)$$

Proof. Consider a marginal increase $\delta \tau$ in the marginal tax rate $T'_e(z)$ that applies to agents in a small bandwidth of income $[z^*, z^* + \delta z]$ which translates in a lump-sum increase in taxes $\delta \tau \delta z$ on all agents employed at earnings levels $z \ge z^* + \delta z$. This reform induces four different effect:

(1) A mechanical effect δM capturing mechanical changes in the government budget and in agents' well-being from the lump-sum tax increase $\delta \tau \delta z$

$$\lim_{\delta z \to 0} \delta M = \delta \tau \delta z \, \int_{z \ge z^*} e(z) \left(1 - g_e(z)\right) h_z(z) \, dz$$

(2) A behavioral effect δB_z capturing earnings responses to the reform and that can be decomposed into substitution effects in response to the $\delta \tau$ increase in the marginal tax rate,

and into income effects in response to the $\delta \tau \delta z$ increase in the tax level

$$\lim_{\delta z \to 0} \delta B_z = -q(z^*) \frac{z^* \zeta_{T_e}^z(z^*)}{1 - T'_e(z^*)} \delta \tau h_z(z^*) \, \delta z + \int_{z \ge z^*} q(z) \frac{\eta_{T_e}^z(z)}{1 - T'_e(z)} \, \delta \tau \delta z \, h_z(z) \, dz$$

where $q(z) \equiv e(z) T'_e(z) - (1 - e(z)) B'_u(z)$ is the marginal tax-benefits rate capturing the net fiscal externality induced by a marginal change in earnings z.

(3) A behavioral effect δB_e capturing job search responses to the reform and that can be decomposed into direct effects in response to the $\delta \tau \delta z$ increase in the tax level, and into indirect effects induced by earnings changes

$$\lim_{\delta z \to 0} \delta B_e = \frac{T_e(z^*) + B_u(z^*)}{1 - T'_e(z^*)} z^* \xi_z^{1-e}(z^*) \zeta_{T_e}^z(z^*) \ \delta \tau \ h_z(z^*) \ \delta z$$
$$- \int_{z \ge z^*} \left(T_e(z) + B_u(z) \right) \left(\left(1 - e(z) \right) \mu_e(z) + \xi_z^{1-e}(z) \frac{\eta_{T_e}^z(z)}{1 - T'_e(z)} \right) \ \delta \tau \delta z \ h_z(z) \ dz$$

where $T_e(z) + B_u(z)$ is the employment tax capturing the net fiscal externality induced by changes in job search.

(4) A participation effect δP capturing negative participation responses to the reform due to the $\delta \tau \delta z$ increase in the tax level

$$\lim_{\delta z \to 0} \delta P = -\int_{z \ge z^*} \left\{ (T_e(z) + R_0) - (1 - e(z)) (T_e(z) + B_u(z)) \right\} \pi_e(z) \ \delta \tau \delta z \ h_z(z) \ dz$$

where $(T_e(z) + R_0) - (1 - e(z))(T_e(z) + B_u(z))$ is the participation tax capturing the net fiscal externality induced by changes in participation decisions.

The total effect of the reform is $\delta W = \delta M + \delta B_z + \delta B_e + \delta P$, and the optimal schedule can by definition not be improved upon. Setting $\delta W = 0$ and rearranging yields (43).

Formula (43) extends formula (12) derived in the stylized model to a setting with earnings choices. Assuming away unemployment risk ($\forall z, e(z) = 1, \xi_z^{1-e}(z) = 0$), it reduces to

$$\frac{T'_{e}(z^{*})}{1 - T'_{e}(z^{*})} = \frac{1}{\zeta^{z}_{T_{e}}(z^{*})} \frac{1}{z^{*}h_{z}(z^{*})} \int_{z \ge z^{*}} \left\{ (1 - g_{e}(z)) + \frac{T'_{e}(z)}{1 - T'_{e}(z)} \eta^{z}_{T_{e}}(z) - (T_{e}(z) + R_{0}) \pi_{e}(z) \right\} h_{z}(z) dz$$

which is Jacquet, Lehmann, and Van der Linden (2013) optimal tax formula in a model with both intensive and extensive margins of labor supply.

This is often referred to as an ABC formula. It says that the optimal marginal tax rate $T'_e(z^*)$ at earnings z^* depends on (A) the magnitude of the distortion induced by substitution effects at z^* through $\zeta^z_{T_e}(z^*)$, (B) the magnitude of the fiscal externality induced by this

distortion through $z^*h_z(z^*)$, (C) the magnitude of mechanical effects, income effects and participation effects affecting agents at earnings z above z^* .

Note that we obtain the same optimal formula for marginal tax rates at the top of the income distribution assuming that at top incomes (i) unemployment benefits are capped such that $B'_u(z) = 0$, (ii) unemployment risk is constant such that e(z) = e, (iii) job search responses can be neglected such that $\mu_e(z) = 0$ and $\xi_z^{1-e}(z) = 0$. For optimal tax rates at the top, the interaction between redistributive taxation and unemployment insurance is then limited to changes in marginal social welfare weights $g_e(z)$.

All of the forces highlighted in the ABC formula remain at play in formula (43) up to some adjustments, and new forces related to unemployment risk and job search emerge.

First, the optimal marginal tax rate at z^* is now pinned through the marginal taxbenefit rate $e(z^*)T'_e(z^*) - (1 - e(z^*))B'_u(z^*)$ because marginal changes in earnings do not only change the amount of taxes paid when employed but also the amount of unemployment benefits received when unemployed. Importantly, this implies that a higher marginal benefits rate $B'_u(z^*)$ or a higher unemployment risk $1 - e(z^*)$ at z^* push, ceteris paribus, for a higher marginal tax rate $T'_e(z^*)$. This provides an efficiency motive to increase marginal tax rates on low income individuals if they face high unemployment risk. Intuitively, if people are often unemployed, there is little distortion in increasing taxes when employed.

Second, and related, the appraisal of mechanical effects, income effects and participation effects now has to weight in the existence of unemployment risk. The integral term thus interestingly captures the gradient of unemployment risk at all earnings z above z^* . A negative gradient indicating lower unemployment risk at higher earnings pushes for a higher marginal tax rate $T'_e(z^*)$ through mechanical, income and participation effects but a lower marginal tax rate through direct job search effects.

Indeed, a third novel force relates to the impact of job search responses. Direct job search responses featured in the integral term through $\mu_e(z)$ push for lower marginal tax rates at earnings z^* in order to reduce the amount of taxes paid at earnings z, and thereby make search pay. In the limit case in which job search responses are very large around some level of earnings z, this make search pay channel may even call for negative marginal tax rates $T'_e(z^*)$ on all earnings z^* below the region of earnings z. This provides a potentially new rationale for EITC types of policies.

Fourth, indirect job search responses can push in either direction. If $\xi_z^{1-e}(z)$ is positive indicating that increases in earnings induce a larger fraction of time spent unemployed, they call for higher marginal tax rates through the fact that there are negative fiscal externalities induced by increases in earnings through substitution effects at z^* or income effects at earnings z above z^* . Otherwise, indirect job search responses induce positive fiscal externalities such that the effect is reversed.

Optimal unemployment benefits schedule.

Proposition 8. Taking the tax and transfer schedule $T_e(.)$ and the social assistance transfer R_0 as given, the optimal unemployment benefits schedule $B_u(.)$ is characterized at each level of earnings z^* by

$$\frac{(1-e(z^*))B'_{u}(z^*) - e(z^*)T'_{e}(z^*)}{B'_{u}(z^*)} + \frac{T_{e}(z^*) + B_{u}(z^*)}{B'_{u}(z^*)}\xi_{z}^{1-e}(z^*)$$

$$= \frac{1}{\zeta_{B_{u}}^{z}(z^*)}\frac{1}{z^*h_{z}(z^*)}\int_{z\geq z^*}\left\{ (1-e(z))(g_{u}(z)-1) - \frac{e(z)T'_{e}(z) - (1-e(z))B'_{u}(z)}{B'_{u}(z)}\eta_{B_{u}}^{z}(z) - (T_{e}(z) + B_{u}(z))\left((1-e(z))\mu_{u}(z) - \xi_{z}^{1-e}(z)\frac{\eta_{B_{u}}^{z}(z)}{B'_{u}(z)}\right) + \left[(T_{e}(z) + R_{0}) - (1-e(z))(T_{e}(z) + B_{u}(z))\right]\pi_{u}(z) \right\}h_{z}(z)dz$$
(44)
$$= \frac{1}{\zeta_{B_{u}}^{z}(z^*)}\frac{1}{z^*h_{z}(z^*)}\int_{z\geq z^*}\left\{ (1-e(z))(g_{u}(z)-1) - \frac{e(z)T'_{e}(z) - (1-e(z))B'_{u}(z)}{B'_{u}(z)}\right\}h_{z}(z)dz$$

Proof. Consider a marginal increase δb in the marginal benefits rate $B'_u(z)$ that applies to small bandwidth of earnings $[z^*, z^* + \delta z]$ which translates in a lump-sum increase in benefits $\delta b \delta z$ on all unemployed agents with reference earnings levels $z \ge z^* + \delta z$. This reform induces four different effects:

(1) A mechanical effect δM capturing mechanical changes in the government budget and in agents' well-being from the lump-sum benefits increase $\delta \tau \delta z$

$$\lim_{\delta z \to 0} \delta M = \delta b \delta z \, \int_{z \ge z^*} \left(1 - e\left(z\right) \right) \left(g_u\left(z\right) - 1 \right) h_z\left(z\right) dz$$

(2) A behavioral effect δB_z capturing earnings responses to the reform and that can be decomposed into substitution effects in response to the δb increase in the marginal benefits rate, and into income effects in response to the $\delta b \delta z$ increase in the benefits level

$$\lim_{\delta z \to 0} \delta B_{z} = q(z^{*}) \frac{z^{*} \zeta_{B_{u}}^{z}(z^{*})}{B_{u}'(z^{*})} \delta b h_{z}(z^{*}) \delta z - \int_{z \ge z^{*}} q(z) \frac{\eta_{B_{u}}^{z}(z)}{B_{u}'(z)} \delta b \delta z h_{z}(z) dz$$

where $q(z) \equiv e(z) T'_e(z) - (1 - e(z)) B'_u(z)$ is the marginal tax-benefits rate capturing the net fiscal externality induced by a marginal change in earnings z.

(3) A behavioral effect δB_e capturing job search responses to the reform and that can be decomposed into direct effects in response to the $\delta \tau \delta z$ increase in the tax level, and into indirect effects induced by earnings changes

$$\lim_{\delta z \to 0} \delta B_e = -\frac{T_e(z^*) + B_u(z^*)}{B'_u(z^*)} z^* \xi_z^{1-e}(z^*) \zeta_{B_u}^z(z^*) \ \delta b \ h_z(z^*) \ \delta z$$
$$- \int_{z \ge z^*} (T_e(z) + B_u(z)) \left((1 - e(z)) \ \mu_u(z) - \xi_z^{1-e}(z) \ \frac{\eta_{B_u}^z(z)}{B'_u(z)} \right) \ \delta b \delta z \ h_z(z) \ dz$$

where $T_e(z) + B_u(z)$ is the employment tax capturing the net fiscal externality induced by changes in job search.

(4) A participation effect δP capturing positive participation responses to the reform due to the $\delta b \delta z$ increase in the benefit level

$$\lim_{\delta z \to 0} \delta P = \int_{z \ge z^*} \left\{ (T_e(z) + R_0) - (1 - e(z)) (T_e(z) + B_u(z)) \right\} \pi_u(z) \ \delta b \delta z \ h_z(z) \ dz$$

where $(T_e(z) + R_0) - (1 - e(z))(T_e(z) + B_u(z))$ is the participation tax capturing the net fiscal externality induced by changes in participation decisions.

The total effect of the reform is $\delta W = \delta M + \delta B_z + \delta B_e + \delta P$, and the optimal schedule can by definition not be improved upon. Setting $\delta W = 0$ and rearranging yields (44).

Optimal unemployment benefits formula (44) extends formula (14) derived in the stylized model to a setting with earnings choices. It borrows both to Baily-Chetty and Mirrlees-Saez logics. Indeed, optimal unemployment benefits $B_u(z)$ do depend as in Baily-Chetty on consumption smoothing benefits through $(1 - e(z)) g_u(z)$ and job search responses through $\mu_u(z)$. But in a Mirrlees-Saez spirit, the optimal unemployment benefits schedule is pinned down at each earnings z^* through the marginal benefits rate $B'_u(z^*)$ expressed as a function of consumption smoothing benefits and job search responses at earnings z above z^* .

This novelty comes from the fact that the schedule of unemployment benefits here affects earnings decisions. For instance, a steeply increasing unemployment benefits schedule creates incentives to increase earnings at the margin in order to obtain high unemployment benefits when unemployed. This explains the presence of marginal benefits rates $B'_u(z)$ in the formula and why the Mirrlees-Saez logic kicks in a setting with earnings choices.

The optimal marginal benefits rate $B'_u(z^*)$ depends on (A) the magnitude of the distortion induced by substitution effects at z^* through $\zeta^z_{B_u}(z^*)$, (B) the magnitude of the fiscal externality induced by this distortion through $z^*h_z(z^*)$, (C) the magnitude of mechanical effects, income effects, search effects and and participation effects affecting agents at earnings z above z^* .

In particular, the government's redistributive tastes captured in $g_u(z)$ call for high marginal benefits rates at the bottom of the income distribution and low marginal benefits rates at the top, thus increasing the progressivity of unemployment benefits schedule $B_u(z)$. Moreover, large job search responses at earnings z above z^* call for lower marginal benefits rate $B'_u(z^*)$ as a way to reduce the amount of benefits $B_u(z)$ received at z, and thereby make search pay. In contrast, large participation responses at earnings z above z^* call for higher marginal benefits rate $B'_u(z^*)$ as a way to increase the amount benefits $B_u(z)$ received at z, and thereby make work pay.⁵⁸

An important question to get at the net effects is thus whether make search pay or make work pay responses are quantitatively more important. This is an empirical question that I try to answer in the empirical part of the paper.

Most countries implement a cap in unemployment benefits. That is unemployment benefits are constant above a certain income level meaning that the marginal benefit rate is zero. I here derive a sufficient condition for keeping the marginal benefit rate at zero, and actually calling for a negative marginal benefit rate – which may be hard to implement in practice.

Corollary 3. Assume unemployment benefits are capped $B_u(z) = \overline{B}$ at all earnings $z \ge \overline{z}$. It is then optimal to maintain the marginal benefit rate $B'_u(z)$ equal to zero, or even make it negative, if at each earnings $z^* \ge \overline{z}$

$$\left[e\left(z^{*}\right) T_{e}'\left(z^{*}\right) - \left(T_{e}\left(z^{*}\right) + \overline{B}\right) \xi_{z}^{1-e}\left(z^{*}\right) \right] z^{*} \frac{\zeta_{B_{u}}^{z}\left(z^{*}\right)}{B_{u}'\left(z^{*}\right)} h_{z}\left(z^{*}\right) + \int_{z \ge z^{*}} \left\{ \left(1 - e\left(z\right)\right) \left(g_{u}\left(z\right) - 1\right) - e\left(z\right) T_{e}'\left(z\right) \frac{\eta_{B_{u}}^{z}\left(z\right)}{B_{u}'\left(z\right)} - \left(T_{e}\left(z\right) + \overline{B}\right) \left(\left(1 - e\left(z\right)\right) \mu_{u}\left(z\right) - \xi_{z}^{1-e}\left(z\right) \frac{\eta_{B_{u}}^{z}\left(z\right)}{B_{u}'\left(z\right)} \right) + \left[\left(T_{e}\left(z\right) + R_{0}\right) - \left(1 - e\left(z\right)\right) \left(T_{e}\left(z\right) + \overline{B}\right) \right] \pi_{u}\left(z\right) \right\} h_{z}\left(z\right) dz \le 0$$
(45)

where $\frac{\zeta_{B_u}(z^*)}{B'_u(z^*)}$ and $\frac{\eta_{B_u}^z(z)}{B'_u(z)}$ capture agents responses to a marginal increase in the benefits rate and are thus perfectly well-defined when $B'_u(z) = 0$.

Proof. Computing the impact of a δb increase the marginal benefit rate in the income bandwidth $[z^*, z^* + \delta z]$ yields the result.

This gives an empirically testable condition to determine (i) whether implementing a cap in unemployment benefits is a sensible policy in the sense that positive marginal benefit rates are not optimal at high earnings levels, (ii) whether observed caps in unemployment benefits

 $^{^{58}}$ As in the stylized model, the fact that unemployment benefits make work pay defines a Pareto lower bound for the schedule of unemployment benefits, defined by the revenue maximizing schedule of unemployment benefits.

start at earnings levels that can be rationalized by the theory or whether there would be welfare gains from implementing caps at different earnings levels. I provide tentative answers to these questions in the empirical part of the paper.

Joint optimality.

Proposition 9. Assuming earnings elasticities $\zeta_{B_u}^z(z)$ and $\zeta_{B_u}^z(z)$ are locally constant, the tax-benefit system $\{T_e(.), B_u(.)\}$ is Pareto-efficient if and only if at each earnings z

$$\frac{u'(B_u(z)) - u'(z - T_e(z))}{u'(z - T_e(z))} = \frac{\mu_u(z)}{e(z)} \left(T_e(z) + B_u(z)\right) - \frac{Q(z)}{e(z)} \frac{u'(B_u(z))}{u'(z - T_e(z))} \left\{ \left[\frac{T_e''(z)}{1 - T_e'(z)} + \frac{B_u''(z)}{B_u'(z)} \right] \frac{\zeta_{T_e}^z(z)}{1 - T_e'(z)} z + \left[1 - \frac{B_u'(z)u''(B_u(z))u'(z - T_e(z))}{(1 - T_e'(z))u''(z - T_e(z))u'(B_u(z))} \right] \frac{\eta_{T_e}^z(z)}{1 - T_e'(z)} \right\}$$
(46)

where $Q(z) \equiv e(z) T'_e(z) - (1 - e(z)) B'_u(z) - (T_e(z) + B_u(z)) \xi_z^{1-e}(z)$ is the marginal taxbenefit rate inclusive of the cross-partial effect on the government budget.

Proof. Consider two simultaneous reforms. First, increase in a bandwidth $[z^*, z^* + \delta z]$ the marginal tax rate by $\delta \tau$, and simultaneously decrease in a bandwidth $[z^* + \varepsilon, z^* + \varepsilon + \delta z]$ the marginal tax rate by $\delta \tau$. Second, increase in a bandwidth $[z^*, z^* + \delta z]$ the marginal benefit rate by δb , and simultaneously decrease in a bandwidth $[z^* + \varepsilon, z^* + \varepsilon + \delta z]$ the marginal benefit rate by δb .

Letting δz the width of each reform go to zero, these two-bracket reforms generate in the bandwidth $[z^*, z^* + \varepsilon]$ a $\delta \tau \delta z$ lump-sum increase in taxes and a $\delta b \delta z$ lump-sum increase in benefits, with $\varepsilon \gg \delta z$. The former decreases agents' utility by $\delta \tau \delta z e(z) u'(z - T_e(z))$, while the latter increases agents' utility by $\delta b \delta z (1 - e(z)) u'(B_u(z))$.

Letting ε the width of the two-bracket reforms go to zero, set $\delta \tau = \delta b \frac{(1-e(z^*))u'(B_u(z^*))}{e(z^*)u'(z^*-T_e(z^*))}$ such that individuals in $[z^*, z^* + \varepsilon]$ are left exactly indifferent by the reform. By the envelope theorem, the utility level of agents in $[z^*, z^* + \delta z]$ and $[z^* + \varepsilon, z^* + \varepsilon + \delta z]$ are not affected by changes in marginal tax and benefit rates. Hence, these simultaneous two-bracket reforms leave all agents' utility and thus participation decisions unchanged. Computing how they impact the government budget leads to the Pareto-efficiency condition (46).

Formally, denote $\mathcal{F}_{T_e}(z^*, \delta z, \delta \tau)$ the impact of a prototypical reform of the tax schedule $T_e(z)$ that increases in a bandwidth $[z^*, z^* + \delta z]$ the marginal tax rate by $\delta \tau$. Similarly, denote $\mathcal{F}_{B_u}(z^*, \delta z, \delta b)$ the impact of a prototypical reform of the benefit schedule $B_u(z)$ that increases in a bandwidth $[z^*, z^* + \delta z]$ the marginal benefit rate by δb .

The impact on the government objective of these simultaneous two-bracket reforms is

$$\begin{split} &\lim_{\varepsilon \to 0} \left[\mathcal{F}_{T_e} \left(z^*, \delta z, \delta \tau \right) + \mathcal{F}_{T_e} \left(z^* + \varepsilon, \delta z, -\delta \tau \right) \right] + \left[\mathcal{F}_{B_u} \left(z^*, \delta z, \delta b \right) + \mathcal{F}_{B_u} \left(z^* + \varepsilon, \delta z, -\delta b \right) \right] \\ &= \lim_{\varepsilon \to 0} \left[\mathcal{F}_{T_e} \left(z^*, 1, 1 \right) - \mathcal{F}_{T_e} \left(z^* + \varepsilon, 1, 1 \right) \right] \delta z \delta \tau + \left[\mathcal{F}_{B_u} \left(z^*, 1, 1 \right) - \mathcal{F}_{B_u} \left(z^* + \varepsilon, 1, 1 \right) \right] \delta z \delta b \\ &= - \left[\frac{\partial \mathcal{F}_{T_e} \left(z^*, 1, 1 \right)}{\partial z^*} \delta \tau + \frac{\partial \mathcal{F}_{B_u} \left(z^*, 1, 1 \right)}{\partial z^*} \delta b \right] \varepsilon \delta z \end{split}$$

Computing these derivatives under the assumption that $\zeta_{T_e}^z(z^*)$ and $\zeta_{B_u}^z(z^*)$ are locally constant, plugging in $\delta \tau$, and exploiting the structural links between the elasticity concepts when employed and unemployed given in (9), (39), and (41), we get the following expression for the total impact of these simultaneous two-bracket reforms:

$$- h_{z}(z^{*}) \ \delta b \varepsilon \ \delta z \Biggl\{ (1 - e(z^{*})) \left[(T_{e}(z^{*}) + B_{u}(z)) \frac{\mu_{u}(z)}{e(z^{*})} - \left(\frac{u'(B_{u}(z^{*}))}{u'(z^{*} - T_{e}(z^{*}))} - 1 \right) \right] \\ - Q(z^{*}) \frac{(1 - e(z^{*}))}{e(z^{*})} \frac{u'(B_{u}(z^{*}))}{u'(z^{*} - T_{e}(z^{*}))} \left[1 - \frac{B'_{u}(z^{*}) u''(B_{u}(z^{*})) u'(z^{*} - T_{e}(z^{*}))}{(1 - T'_{e}(z^{*})) u''(z^{*} - T_{e}(z^{*})) u'(B_{u}(z^{*}))} \right] \frac{\eta^{z}_{T_{e}}(z^{*})}{1 - T'_{e}(z^{*})} \\ - Q(z^{*}) \frac{(1 - e(z^{*}))}{e(z^{*})} \frac{u'(B_{u}(z^{*}))}{u'(z^{*} - T_{e}(z^{*}))} \left[\frac{T''_{e}(z^{*})}{1 - T'_{e}(z^{*})} + \frac{B''_{u}(z^{*})}{B'_{u}(z^{*})} \right] \frac{\zeta^{z}_{T_{e}}(z^{*})}{1 - T'_{e}(z^{*})} z^{*} \Biggr\}.$$

When the tax-benefit system is Pareto-efficient, such reform cannot bring any additional tax revenue, meaning that this expression must be equal to zero. Rearranging yields (46). \Box

The Pareto-efficiency condition (46) extends the previous Pareto-efficiency characterization to an environment with endogenous earnings choices along the intensive margin. Earnings responses induce fiscal externalities proportional to the marginal tax-benefit rate Q(z)which are captured through an additional term. Absent earnings responses ($\zeta_{T_e}^z(z) = 0$, $\eta_{T_e}^z(z) = 0$), this additional term on the right-hand side is equal to zero, and we retrieve formula (18) derived in the stylized setting.

More specifically, earnings responses induce substitution and income effects. Income effects are measured through the income effect parameter $\eta_{T_e}^z(z^*)$. They stem from the fact that this Pareto-efficiency condition is obtained by simultaneously increasing the amount of taxes and benefits in a small bandwidth of income. While this simultaneous increase is designed to offset utility changes and thus leave agents' utility unchanged, it will typically not offset the fiscal externalities induced by income effects.

This simultaneous increase is here achieved through means of two-bracket reforms of taxes and benefits (see proof). When the tax-benefit system is linear, substitution effects in each of these brackets exactly cancel out. However, in the presence of curvature, a marginal change in earnings triggers a change in marginal tax or benefit rate and thus a further round of adjustment through substitution effects. Although such *circularity effects* are captured in the total substitution elasticity $\zeta_{T_e}^z(z)$ (see e.g. Jacquet, Lehmann, and Van der Linden (2013)), they do not cancel out within each bracket and thus entail substitution effects proportional to the substitution elasticity $\zeta_{T_e}^z(z)$ and to the curvature of tax-benefit schedules $T_e''(z)$ and $B_u''(z)$.

Overall, the empirical magnitude of the corrective terms on the right-hand side of (46) depends on the size of earnings responses and the different scaling factors. When these corrective terms are small, conclusions based on the simple characterization (18) remain largely valid.

3 Applications

3.1 Calibration to the US economy

The main steps of the calibration to the actual US economy involve pinning down (1) the distribution of earnings, (2) unemployment and participation rates, (3) existing tax-benefits schedules, (4) agents' work and search elasticities. I describe the calibration of each of these items before discussing what can and cannot be achieved relying exclusively on empirically estimable sufficient statistics. A presentation of the structural model that I use to simulate counterfactuals and optimal policies concludes the section.

3.1.1 Earnings distribution

The calibration of the earnings distribution is here more involved than in standard optimal taxation model. Indeed, annual taxable income includes both earned income when employed and unemployment benefits received when unemployed. As a result, the distribution of taxable income mixes earnings ability and the realization of unemployment risk in ways that may lead to important composition effects.

To circumvent this issue, I construct an earnings distribution using the variable *usual* weekly earnings available in Current Population Survey (CPS) data for all agents who participate in the labor force. This usual weekly earnings variable is defined as agents' earnings during weeks worked, it thus provides a proxy for earnings ability that is somewhat independent from the realization of unemployment risk.⁵⁹ I multiply this variable by 52 to obtain an

 $^{^{59}}$ A technical note on usual weekly earnings from the US Bureau of Labor Statistics (BLS) states: "The term "usual" is determined by each respondent's own understanding of the term. If the respondent asks for a definition of "usual," interviewers are instructed to define the term as more than half the weeks worked

annualized measure of the earnings agents would have earned if employed during the whole year.

Appending all monthly CPS files of 2019, I calibrate a log-normal distribution of annualized earnings when employed (Figure 8). The log-normal distribution broadly matches the shape of the earnings distribution. Moreover, it provides a smooth estimate of the density that eliminates any convergence issues that may arise with less smooth distributions (e.g. kernel density) when simulating optimal tax-benefit schedules.

CPS survey data is topcoded for weekly earnings above \$2,884.61 (annual earnings above \$150,000) which explains the mass at the extremity of the graph. Moreover, prior work has shown that top incomes are best represented by a Pareto distribution. I thus append to this log-normal distribution a Pareto-tail with parameter $\alpha = 2$ for earnings above \$200,000.

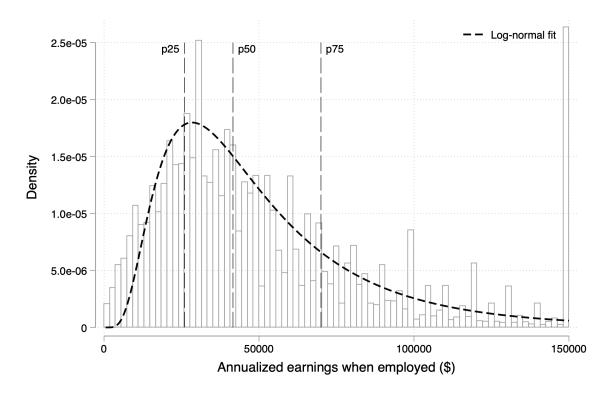


Figure 8: Distribution of earnings in the US (\$, 2019)

3.1.2 Unemployment and participation rates

The distribution of unemployment and participation rates across earnings is hard to construct since earnings cannot be observed for the unemployed and non-participants. I thus rely on measures of unemployment and participation rates by educational attainments which

during the past 4 or 5 months.".

is pre-compiled by the Bureau of Labor and Statistics (BLS) using Current Population Survey (CPS). Educational attainments are then mapped to earnings using 2019 median usual weekly earnings by educational attainment obtained from the same data source. This enables me to estimate distributions of unemployment and participation rates that are fully consistent with the above distribution of earnings.

For unemployment rates, I use annual statistics from 1992 to 2019. The data shows a significant downward gradient in unemployment rates across earnings (Figure 9). To obtain a smooth distribution of unemployment rates across earnings, I fit an exponential function to the data: $u(z) = a_u \exp(-b_u z)$. This functional form provides a good fit to the data while ensuring that the unemployment rate is always positive, and asymptotically goes to zero at top incomes. A drawback is that it may induce unrealistically high levels of unemployment at the very bottom of the income distribution leading to an upward bias in the average unemployment rate.⁶⁰

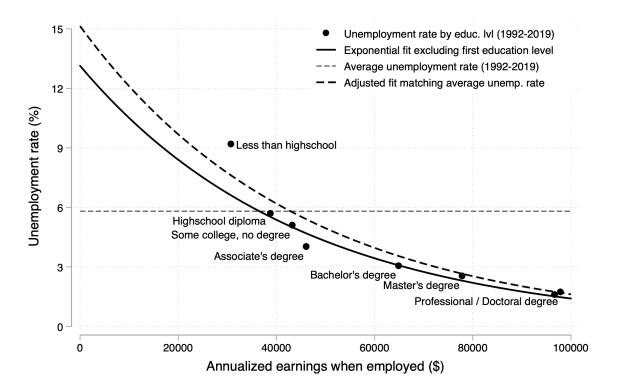


Figure 9: Unemployment rates across earnings in the US

Combining the fitted distribution of unemployment rates with the calibrated distribution of earnings yields an average unemployment rate of 5.04%. This micro estimate is closed to

⁶⁰This bias is particularly large when performing a nonlinear regression using all 8 education levels, and I thus choose to exclude the lowest education level and perform a nonlinear regression using 7 education levels only. Estimated parameters are equal to $a_u = 13.14$ and $b_u = 2.235 \times 10^{-5}$.

the unemployment rate estimated from macro data over the period which is equal to 5.81%. To resolve this discrepancy, I adjust the intercept a_u to match the latter. This preserves the gradient in unemployment rates measured at the micro level while providing a distribution that is consistent with macro data.

For participation rates, statistics seem unfortunately only available for the year 2016 and for all individuals aged 25 and above. The data shows a significant upward gradient in participation rates across earnings (Figure 10). As before, I fit an exponential function to the data to obtain a smooth distribution: $p(z) = 100 - a_p \exp(-b_p z)$.⁶¹

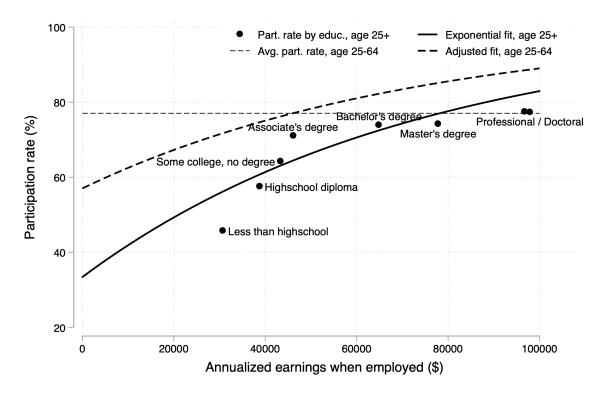


Figure 10: Participation rates across earnings in the US

Combining the fitted distribution of participation rates with the calibrated distribution of earnings yields an average participation rate of 67.35%. This largely reflects the fact that the data includes all individuals aged 25 and older – including those aged 65 and older whose participation rate is around 20%. To avoid capturing interactions between participation and retirement decisions of this older segment of the population, I adjust the intercept a_p to match the 2016 participation rate among individuals aged between 25 and 64 which is equal to 77.04% (OECD). This preserves the gradient in participation rates measured at the micro level while providing a distribution that is consistent with macro data.

⁶¹Excluding again the first education level, a nonlinear regression yields $a_u = 13.14$ and $b_u = 2.235 \times 10^{-5}$.

3.1.3 Taxes and benefits

The tax-benefit schedules $\{T_e(z), B_u(z)\}$ defined in the theoretical part of the paper do not directly correspond to actual tax or unemployment benefits schedules. The tax schedule $T_e(z)$ responds to the total amount of taxes net of transfers paid when employed at earnings z and the benefit schedule $B_u(z)$ to the total amount of benefits net of taxes and transfers when unemployed and previously employed at earnings z. The calibration of taxes and benefits aims at computing these schedules for existing tax-benefit systems.

For this computation, I rely on the OECD Tax-Benefit model (TaxBEN) which is freely accessible online. TaxBEN includes income taxes, social security contributions and the major cash (or near-cash) benefit programs, while it excludes wealth and capital income taxes, taxes on consumption and in-kind transfers. A particularity of the US economy is that taxes and benefits vary across states – this is particularly true of unemployment insurance programs which are operated at the state level. The solution adopted in TaxBEN, and thus in this paper, is to simulate the tax-benefit system applicable in the state of Michigan.⁶²

In practice, the amount of taxes and transfers can be determined either at the individual level (e.g. social security contributions) or at the household level (e.g. social assistance programs). In the latter case, household composition and the number of children is usually an important factor to take into account. To simplify things, and in particular to avoid having to incorporate interrelated labor supply decisions within households in the theoretical model, I simulate taxes and benefits for singles and more specifically childless singles.

Another simplifying assumption adopted in these simulations relates to the time-period used to compute taxes and benefits. While income tax payments are estimated every year based on annual income, transfer programs usually operate on an infra-annual basis. Following TaxBEN methodology, taxes and benefits are determined for a particular month and then multiplied by 12 to be reported on an annual basis.

Moreover, I assume that agents who become unemployed are (i) eligible to unemployment benefits and (ii) receive unemployment benefits during their entire unemployment spell. I thus abstract from issues related to eligibility, take-up and long-term unemployment.⁶³

Last, baseline TaxBEN simulations include employee social contributions, exclude employer social contributions and do not differentiate social contributions by types. This may be problematic for the US since employee social contributions typically fund old age, survivors,

⁶²The 2019 documentation of TaxBEN for the US is accessible at http://www.oecd.org/els/soc/ TaxBEN-United-States-2019.pdf. Simulations in TaxBEN include the personal income tax, social contributions, earned income tax credit (EITC), family benefits and tax credits (TANF, CTC, CCDF, CDCC), social assistance program (SNAP) and unemployment benefits, implemented at federal and state levels.

⁶³In the US, agents spend on average less than 20 weeks unemployed (the median is less than 10 weeks), while they are eligible to (the maximum amount of) unemployment benefits for a duration of 20 weeks.

and disability insurance whereas employer social contributions fund pensions and unemployment insurance. Results are however quantitatively robust to including social contributions funding unemployment insurance because they are small (federal tax of 0.6% on the first \$7,000 of earnings and state tax of 3.2% on the first \$9,000 of earnings) and qualitatively robust to alternative treatments of other social contributions.

The resulting tax and transfer schedule $T_e(z)$ can be directly obtained through simulations on TaxBEN web-calculator. They yield a schedule that is extremely close $(R^2 = 99\%)$ to a linear schedule with intercept $T_0^{US} = -\$4, 283$ and linear tax rate $\tau^{US} = 33.20\%$. Moreover, the amount of tax and transfer can be decomposed between the personal income tax, social contributions, in-work benefits and social assistance.

In contrast, the benefit schedule $B_u(z)$ cannot be directly obtained and I simulate it using the following two-step procedure. First, I simulate unemployment benefits using the legislation described in the TaxBEN documentation. Second, I apply to this amount of unemployment benefits a restricted tax and transfer schedule that only includes the personal income tax and social assistance components of the tax-benefit system. Indeed, unemployment benefits are subject to the personal income tax and agents may receive social assistance (SNAP) on top of unemployment benefits.

Results from this two-step procedure show that both unemployment benefits and net benefits $B_u(z)$ can be well approximated by a linear schedule at low earnings and by a constant after the earnings level at which unemployment benefits are capped (Figure 11). They also reveal that net benefits are higher than unemployment benefits at low incomes because of eligibility to social assistance, and lower than unemployment benefits at high incomes because of the personal income tax.

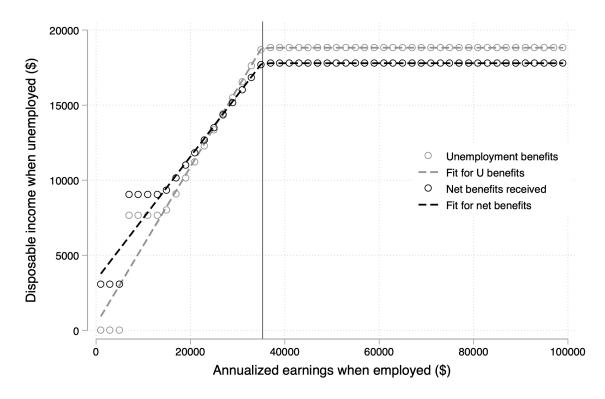


Figure 11: Benefits schedule $B_u(z)$ in the US (\$)

To check the accuracy of the two-step procedure used to simulate the benefit schedule $B_u(z)$, I compare the implied distribution of net replacement rates to values of net replacement rates for particular wage levels reported by the OECD (Figure 12). Net replacement rates are defined as the ratio of net benefits when unemployed to net income when employed, and the simulated distribution seems broadly consistent with OECD indicators. A striking feature of this graph is the heterogeneity of net replacement rates across earnings levels: above 70% at the federal minimum wage (\$15,080), around 40% at the average wage (\$57,055) and close to 25% at high earnings levels (e.g. \$100,000).

This contrasts with the policy debate on the design of unemployment insurance which usually centers on "the" optimal replacement rate. Indeed, a central policy parameter of unemployment insurance systems is the *gross replacement rate* defined as the ratio of unemployment benefits to gross earnings when employed. The gross replacement rate is constant below the cap which may explain the existing focus on "the" optimal replacement rate.

Yet, as shown in the optimal unemployment insurance literature, the theoretically important concept is the net replacement rate. Analyzing the design of unemployment insurance in a setting with earnings heterogeneity thus seems important in light of the existing heterogeneity of net replacement rates across earnings levels.

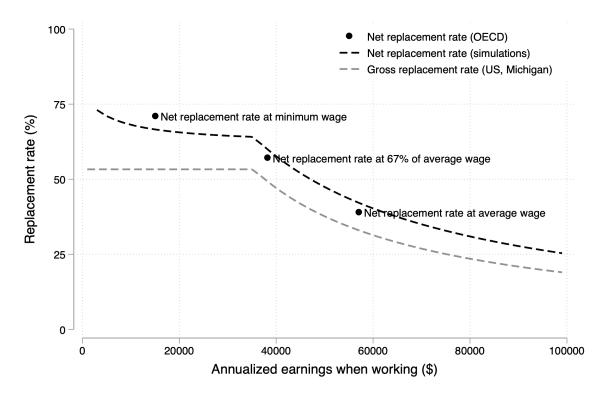


Figure 12: Benefits schedule $B_u(z)$ in the US (\$)

Finally, the amount of social assistance R_0 is calibrated as the transfer received by a childless single who is out of the labor force. In 2019, this individual would only be eligible to an annualized transfer of \$2,304 in the US tax-benefit system.

3.1.4 Work and search elasticities

The empirical literature has produced a large number of estimates of labor supply and search elasticities in the US.

Search elasticities. For the calibration of search elasticities, I rely on estimates of the elasticity of unemployment duration D – measured as the duration of unemployment benefits receipt – with respect to the amount of unemployment benefits B, that is $\mu^{elast} \equiv \frac{B}{D} \frac{\partial D}{\partial B}$.

Schmieder and Von Wachter (2016) provide a survey of the empirical literature and report existing quasi-experimental results (Table 2, column 5). All estimates lie in a range between 0 and 1, and there are two modes at 0.3 and 0.7. Using data from Missouri covering 2003 to 2013, Card, Johnston, Leung, Mas, and Pei (2015) exploit the kink induced by the cap in unemployment benefits for identification. They estimate an elasticity of 0.35 pre-recession and an elasticity of 0.78 in the midst and aftermath of the Great Recession. These two modes may thus reflect different labor market conditions and I use $\varepsilon_B^D = 0.5$ as a baseline average.

Theoretical results have nonetheless been expressed in terms of job search semi-elasticities $\mu_u(z) \equiv \frac{1}{B_u(z)} \frac{\partial (1-e(z))}{\partial B_u(z)}$ measuring the change in the fraction of time spent unemployed, or equivalently in the unemployment rate, upon changes in unemployment benefits. To translate the elasticity μ^{elast} measured empirically into this semi-elasticity concept $\mu_u(z)$ I follow Landais, Michaillat, and Saez (2018a) and write the unemployment rate as $1 - e \approx sD$ where s is the separation rate and D the average duration of unemployment (see footnote 5 of their paper). Taking a partial equilibrium view of the labor market and assuming separations are exogenous, this relationship implies

$$\mu^{elast} = B_u\left(z\right)\mu_u\left(z\right) \tag{47}$$

which allows to pin down $\mu_u(z)$ given the actual benefits schedule $B_u(z)$.

Work elasticities. Estimation of labor supply elasticities in the empirical literature can be divided between reduced-form papers exploiting tax reforms to directly identify elasticity parameters and structural papers that rely on the estimation of labor supply models.

Chetty, Guren, Manoli, and Weber (2013) provide a meta-analysis of extensive margin elasticities focusing on reduced-form estimates. They conclude that 0.2 is a reasonable value for participation elasticities, although they tend to be somewhat larger for certain subgroups of the population like the young, the old, and single mothers. For instance, Eissa, Kleven, and Kreiner (2008) suggest that the participation elasticity of single mothers is likely close to 0.7, while Kroft, Kucko, Lehmann, and Schmieder (2020) estimate a participation elasticity of 0.57 for single women.

Similar results are obtained using structural methods which allow to estimate elasticities by subgroups and by quintiles of earnings. Moreover, structural estimates tend to show that extensive margin responses of both single men and single women tend to be larger at lower earnings levels (e.g. Bargain, Orsini, and Peichl, 2014). I thus assume participation elasticities $\pi^{elast}(z)$ decrease with earnings z in line with prior optimal tax papers (Saez, 2002a; Immervoll, Kleven, Kreiner, & Saez, 2007; Kroft, Kucko, Lehmann, & Schmieder, 2020). More specifically, I set $\pi^{elast}(z)$ to 0.5 at the origin of the earnings distribution and assume it linearly decreases down to 0 at an earnings level of \$100,000 and above.

Participation elasticities reported in the empirical literature tend to measure changes in participation upon tax-induced changes in the difference between disposable income when employed $z-T_e(z)$ and when not participating R_0 , that is $\pi^{elast}(z) \equiv -\frac{z-T_e(z)-R_0}{h_z(z)} \frac{dh_z(z)}{d(T_e(z)+R_0)}$.⁶⁴ Yet, in the presence of risk aversion – the main motive for unemployment insurance provision – participation responses induced by changes in taxes $T_e(z)$ differ from those induced by changes in social assistance R_0 . Because many empirical studies rely on variations in taxes (or wages) when employed, I assume that $\pi^{elast}(z)$ measures the participation elasticity in response to changes in taxes $T_e(z)$ such that

$$\pi^{elast}(z) = (z - T_e(z) - R_0) \pi_e(z)$$
(48)

which allows to pin down the participation semi-elasticity $\pi_e(z) \equiv -\frac{1}{h_z(z)} \frac{dh_z(z)}{dT_e(z)}$ given actual values of $T_e(z)$ and R_0 .

For intensive margin responses, the variable of interest is the (compensated) elasticity of earnings which measures changes in earnings upon a change in the marginal net of tax rate, that is $\zeta_{T_e}^z(z) \equiv \frac{1-T'_e(z)}{z} \frac{\partial z}{\partial(1-T'_e(z))}$. The calibration is this time relatively straightforward since this is the elasticity concept used both in the empirical literature and in the theoretical model.

Elasticity estimates from quasi-experimental studies point to small elasticities of earnings around 0.1 (Saez, Slemrod, & Giertz, 2012). Chetty (2012) argues that many of these small estimates are likely driven by adjustment frictions. Doing a meta-analysis of the literature and accounting for the size of tax changes used to estimate elasticities, he concludes that a central value for intensive margin elasticities is 0.33 and I use this value as a baseline.

Risk aversion and income effects. The provision of unemployment insurance is mostly motivated by the consumption smoothing benefits it offers to the unemployed. These consumption smoothing benefits depend on the ratio of marginal utility of consumption when employed and when unemployed. As a result, the curvature of the utility of consumption is an important parameter for the design of unemployment insurance and it is usually measured through the coefficient of relative risk aversion γ .

There is a large body of work estimating coefficients of relative risk aversion in a broad range of settings. The review of Meyer and Meyer (2005) suggests that in the domain of wealth and consumption central estimates are between 1 and 2, while some estimates can be much higher. Incidentally, the curvature of the utility of consumption also determines income effects of labor supply in expected utility models. Given the relatively modest income effects

⁶⁴An interesting implication of the labor supply model presented in the theoretical part of the paper is that participation decisions also depend on the unemployment rate and the benefits received when unemployed. To the best of my knowledge, the impact of these two factors on participation elasticities has not been investigated in the empirical literature and may help explain some of the observed differences across countries.

measured in the existing labor supply literature, Chetty (2006b) shows that the coefficient of relative risk aversion must be below 2 and that the central value implied by estimates of income effects is 1.

In line with this evidence, most of the optimal unemployment insurance literature uses coefficients of relative risk aversion γ between 1 and 2 (Chetty, 2008; Landais, Michaillat, & Saez, 2018a), and I here adopt $\gamma = 1.5$ as a baseline value.

3.1.5 Simulations and sufficient statistics

Checking whether actual tax-benefit schedules are optimal or Pareto-efficient could in principle be done relying on empirically estimable sufficient statistics like the work and search elasticities. Yet, exclusively relying on a set of empirically estimated sufficient statistics can turn out to be infeasible if empirical estimates of certain statistics are not available. Furthermore, sufficient statistics may take different values in the actual economy and in counterfactual economies. Finally, exclusively relying on a set of empirically estimated sufficient statistics may lead to inconsistencies when doing numerical simulations.⁶⁵

In the present context, there does not seem to be any empirical estimates of the participation semi-elasticity upon changes in unemployment benefits $\pi_u(z)$ or the job search semi-elasticity upon changes in taxes $\mu_e(z)$. Moreover, and as highlighted by the previous discussion about the relationship between risk aversion and income effects, different sufficient statistics may simply not be consistent with one another – in the sense that they are inconsistent with the structure of the model. In addition, even if sufficient statistics are consistent under the actual tax-benefit schedule, they may no longer be consistent under the simulated optimal or Pareto-efficient tax-benefit schedules or take very different values.

While I try to rely as much as possible on estimable sufficient statistics to provide empirical results, I nonetheless specify a fully structural model of agents' behavior to overcome these challenges. This structural model is calibrated to match as closely as possible the available empirical evidence and is used only when needed. I thus combine sufficient statistics and structural methods as advocated by Chetty (2009).

I specify and calibrate the structural model as follows. First, the utility from consumption is $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ where γ directly corresponds to the coefficient of relative risk aversion. Second, the disutility to work is $k(z;\omega) = \frac{(z/\omega)^{1+\epsilon}}{1+\epsilon}$ where ϵ is set to match the earnings elasticity $\zeta_{T_e}^z(z)$ and ability ω is obtained by inverting the first-order condition for earnings z. The distribution of ability $f_{\omega}(\omega)$ is then calibrated to match the distribution of earnings $h_z(z)$.⁶⁶

⁶⁵See Chetty (2009) and Kleven (2020) for in-depths discussions of the sufficient statistics approach.

⁶⁶In the presence of unemployment risk (e(z) < 1) this specification does not give rise to a constant earnings elasticity $\zeta_{T_e}^z(z)$, and I calibrate ϵ such that the intensive margin elasticity at the average earnings

Third, the disutility to search is $\psi(e, z; \omega) = \frac{\psi_0}{\kappa} \frac{z^{\psi_z}}{\omega^{\psi_\omega}} \frac{1}{(1-e)^{\kappa+1}}$ where κ is set to match the search elasticity μ^{elast} and parameters $(\psi_0, \psi_z, \psi_\omega)$ are calibrated to match the observed distribution of unemployment rates 1 - e(z).

As a result, at earnings level z and time spent employed e, agents' utility when participating in the labor market writes

$$e\left[\frac{\left(z-T_{e}\left(z\right)\right)^{1-\gamma}}{1-\gamma}-\frac{k_{0}}{1+\varepsilon}\left(\frac{z}{\omega}\right)^{1+\varepsilon}\right]+\left(1-e\right)\left[\frac{\left(B_{u}\left(z\right)\right)^{1-\gamma}}{1-\gamma}-\frac{\psi_{0}}{\kappa}\frac{z^{\psi_{z}}}{\omega^{\psi_{\omega}}}\frac{1}{\left(1-e\right)^{\kappa+1}}\right]-\chi \quad (49)$$

and the last step is to calibrate the distribution of fixed costs χ . Following Jacquet, Lehmann, and Van der Linden (2013), I assume an ability-specific distribution of fixed-cost $f_{\chi|\omega}(\chi|\omega)$ given by

$$\int_{x \le \chi} f_{\chi|\omega}\left(x|\omega\right) dx = \frac{\exp\left(-\phi_1\left(\omega\right) + \phi_2\left(\omega\right)\chi\right)}{1 + \exp\left(-\phi_1\left(\omega\right) + \phi_2\left(\omega\right)\chi\right)} \tag{50}$$

where the parameters $\phi_1(\omega)$ and $\phi_2(\omega)$ are used to match the participation rate and participation elasticity at the earnings level $z(\omega)$ associated with ability level ω under the actual tax-benefit system.

3.2 Pareto-efficient tax-benefit schedules

3.2.1 Testing Pareto-efficiency

To determine whether the actual US tax-benefit system is Pareto-efficient, I test whether the Pareto-efficiency conditions derived in Proposition 3 and in Proposition 9 hold.

The former is derived in the context of the stylized setting and thus abstracts from earnings responses at the intensive margin. It states the tax-benefit schedule $\{T_e(z), B_u(z)\}$ is Pareto-efficient if at any earnings level z,

$$\frac{u'(B_u(z))}{u'(z - T_e(z))} - 1 = \frac{\mu^{elast}}{e(z)} \frac{T_e(z) + B_u(z)}{B_u(z)}$$
(51)

Consider a joint increase in taxes and benefits at earnings z that leaves agents indifferent. A dollar increase in unemployment benefits is then matched by a larger increase in taxes since the marginal utility of consumption is relatively higher when unemployed.⁶⁷ The left-hand side thus captures the *positive* mechanical fiscal effect from the reform. In contrast, the

level is equal to 0.33.

⁶⁷Formally, for a dollar increase in benefits $dB_u(z) = 1$, the corresponding increase in taxes is $dT_e(z) = \frac{1-e(z)}{e(z)} \frac{u'(B_u(z))}{u'(z-T_e(z))}$ which can be smaller or larger than 1. However, the fiscal impact of the tax increase is always larger because the increase in benefits is paid out during a fraction 1 - e(z) of time, while the tax increase is imposed during a fraction e(z) of time.

right-hand side measures the *negative* behavioral fiscal effect of the reform. Indeed, agents respond by marginally reducing their job search efforts when unemployed which reduces the government resources.

As a result, when the left-hand side is larger (resp. smaller) than the right-hand side, this means that a joint increase (resp. decrease) in taxes and benefits is Pareto-improving. Beyond the tax-benefits schedule, this condition only depends on the coefficient of relative risk aversion γ that pins down the marginal utility of consumption, the job search elasticity μ^{elast} estimated in the empirical literature, and the fraction of time spent employed e(z)which captures unemployment risk at earnings z.

In the baseline calibration with $\gamma = 1.5$ and $\mu^{elast} = 0.5$, the left-hand side and the right-hand side are not too far apart at low earnings levels i.e. below the cap in unemployment benefits. This suggests that the actual tax-benefit system is likely not too far from being efficient at the bottom of the income distribution. However, the left-hand side seems nonetheless larger than the right-hand side implying potential scope for a Pareto-improving *increase* in taxes and benefits. This conclusion is somewhat robust to decreasing (resp. increasing) the degree of risk aversion γ since this is more likely to be consistent with lower (resp. higher) search responses μ^{elast} given the structural relationship between the two.

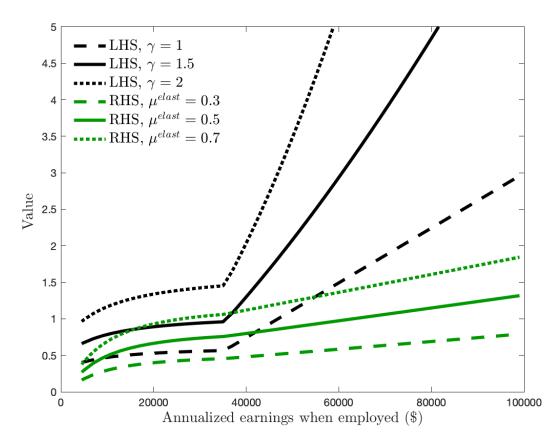


Figure 13: Pareto-efficiency of 2019 US tax-benefit schedule (no earnings responses)

In an attempt to quantify the scope for improvement, I calculate the revenue gains from moving to the Pareto-efficient system obtained by exhausting joint Pareto-improving increases in taxes and benefits at each earnings z. This is achieved through numerical simulations that rely on the calibrated structural model. Preliminary results suggest that going to the Pareto-efficient tax-benefit schedule induces a fiscal gain of 9\$ per year per adult affected by the reform i.e. below the cap. Hence, it seems that there is some scope for Pareto improvement under the baseline calibration.

So far, the discussion of Pareto-efficiency has been limited to agents at low earnings levels, that is below the cap in unemployment benefits. The main reason is that it seems important to account for earnings responses at higher earnings levels, that is above the cap. Indeed, the higher the earnings, the more likely are labor supply responses to be concentrated at the intensive margin.

Introducing an intensive margin of labor supply yields addition fiscal effects. Indeed, joint changes in taxes and benefits may now change not only job search efforts but also earnings choices. Assuming that the actual tax-benefit schedule is close to linear such that $T_e''(z) \approx$

0 and $B''_u(z) \approx 0$, the only fiscal externality induced by earnings responses stems from income effects. Further assuming that the utility from consumption exhibits constant relative risk aversion, the Pareto-efficiency condition adjusted for earnings responses (Proposition 9) writes

$$\frac{u'\left(B_{u}\left(z\right)\right)}{u'\left(z-T_{e}\left(z\right)\right)} - 1 = \frac{\mu^{elast}}{e\left(z\right)} \frac{T_{e}\left(z\right) + B_{u}\left(z\right)}{B_{u}\left(z\right)} - \frac{Q\left(z\right)}{e\left(z\right)} \frac{u'\left(B_{u}\left(z\right)\right)}{u'\left(z-T_{e}\left(z\right)\right)} \left[1 - \frac{B'_{u}\left(z\right)}{1 - T'_{e}\left(z\right)} \frac{z-T_{e}\left(z\right)}{B_{u}\left(z\right)}\right] \frac{\eta^{z}_{T_{e}}\left(z\right)}{1 - T'_{e}\left(z\right)}$$
(52)

where $Q(z) \equiv e(z) T'_e(z) - (1 - e(z)) B'_u(z) - (T_e(z) + B_u(z)) \xi_z^{1-e}(z)$ is the net marginal tax-benefit rate augmented with the marginal fiscal effect of cross responses.

The fiscal externality induced by income effects is captured by the second term on the right-hand side. Intuitively, at low earnings levels, changes in earnings affect both the amount of benefits received when unemployed and the amount of taxes paid when employed. The fiscal externality is then limited because these two effects go in opposite directions and seem empirically to almost cancel each other out. In contrast, at high earnings levels, the amount of benefits received when unemployed remains constant such that the variation in taxes paid when employed are no longer offset and become empirically significant.

Figure 14 illustrates this result. Below the cap of unemployment benefits, the impact of earnings responses on the right-hand side seems almost invisible: thin lines (no earnings responses) almost exactly overlap with thick lines (with earnings reponses). In contrast, the right-hand side jumps at the cap of unemployment benefits. The discrepancy between lefthand side and right-hand side at high earnings thus shrinks when taking earnings responses into account. This suggests that the actual tax-benefit system may be closer to Paretoefficiency than thought otherwise.

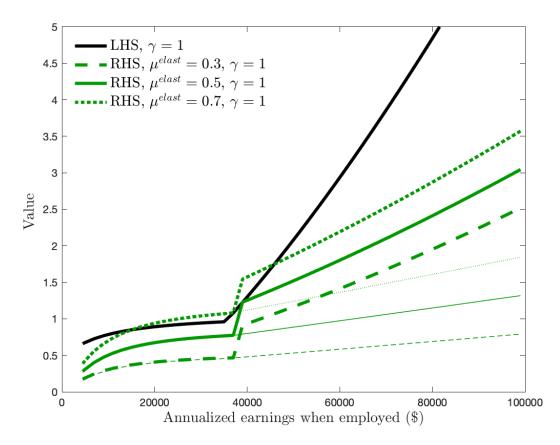


Figure 14: Pareto-efficiency of 2019 US tax-benefit schedule (with earnings responses)

However, reaching definitive conclusions about the Pareto-efficiency of the tax-benefit schedules at high income levels is hard in this setting without savings. While low earners tend to have little or no savings, this is typically not the case of high earners. They are thus able to smooth their income shocks which reduces the ratio of marginal consumptions across states. As a result, the present formula may overestimate the mechanical fiscal effect from the joint increase in taxes and benefits and thus lead to erroneous conclusions.

3.2.2 Comparison with Baily-Chetty formula

To illustrate the importance of the interactions between redistributive taxation and unemployment insurance, I contrast the Pareto-efficiency condition obtained in a setting with both redistribution and insurance to the standard Baily-Chetty formula from the optimal unemployment insurance literature. To do that, I use the formula

$$\frac{B_u(z)}{z - T_e(z)} \approx 1 - \frac{\mu^{elast}}{\gamma} \frac{1}{e(z)^2} \left[1 + \frac{e(z)T_e(z) - (1 - e(z))B_u(z)}{B_u(z)} \right]$$
(53)

which connects net replacement rates to sufficient statistics and calibrated parameters.

Recall that in a pure insurance setting, taxes only fund benefits such that $e(z) T_e(z) = (1 - e(z)) B_u(z)$. As a result, the Baily-Chetty formula writes

$$\frac{B_u(z)}{z - T_e(z)} \approx 1 - \frac{\mu^{elast}}{\gamma} \frac{1}{e(z)^2}$$
(54)

and the optimal unemployment insurance literature uses the average unemployment rate as a scaling factor to compute *the* optimal net replacement rate. In the baseline calibration with $\gamma = 1.5$, $\mu^{elast} = 0.5$ and an unemployment rate of 5.81%, this implies a *constant* net replacement rate equal to 62.43%.⁶⁸

Figure 15 depicts actual net replacement rates in the US (solid black line), the optimal net replacement rate implied by the Baily-Chetty formula (dashed grey line), and the optimal net replacement rates implied by the Pareto-efficiency condition with both redistribution and unemployment insurance (dashed green line).

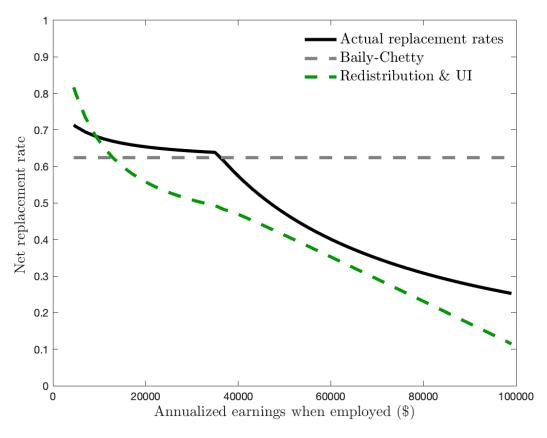


Figure 15: Actual and implied replacement rates in the US

⁶⁸Plugging in earnings-specific unemployment rates lead to *increasing* replacement rates.

The divergence between grey and green lines reveals that taking into account the interactions between redistribution and unemployment insurance leads to strikingly different policy recommendations. Intuitively, this is because the fiscal externality induced by keeping high earners unemployed is very large and thus critical for policymaking.

As a result, the policy recommendations from this model with redistribution and unemployment insurance seem a lot more in line with actual policies. Indeed, actual net replacement rates strongly decrease across earnings in a fashion that is very similar to optimal replacement rates. This suggests that this novel framework captures something important about the real-world and thereby contributes to our understanding of the optimal design of redistributive taxation and unemployment insurance.

Conclusion

This paper analyzes the design of redistribution and unemployment insurance in a general framework that nests two cornerstones of public economics: the Mirrlees-Saez optimal taxation model and the Baily-Chetty optimal unemployment insurance model. Building on the complementarity between mechanism design and perturbation approaches, I provide results on the right set of policy instruments to consider in this environment and characterize the optimal schedule of these instruments in terms of empirically estimable sufficient statistics. Overall, the analysis reveals that redistribution and unemployment insurance interact in many dimensions, and that these interactions have important policy implications.

Chapter 4 – Sufficient statistics for nonlinear tax systems with preference heterogeneity (with B. Lockwood and D. Taubinsky)

Abstract. A prominent justification for taxation of capital income, savings, bequests, and certain commodities is that taxes on consumption categories preferred by those with higher earnings ability are efficient even in the presence of nonlinear earnings taxation. This paper provides a method for characterizing optimal nonlinear tax systems in the presence of correlated preference heterogeneity using sufficient statistics that can be estimated from behavioral responses to tax reforms. Our results encompass tax systems that implement the optimal mechanism, as well as simpler tax systems such as those that involve a nonlinear earnings tax and a separable nonlinear capital income tax, or those that involve a nonlinear earnings tax and an earnings-dependent but otherwise linear capital income tax. All optimal tax systems can be expressed using a simple sufficient statistic for preference heterogeneity: the difference between the cross-sectional variation of consumption (or saving) with income, and the causal effect of income on consumption (or savings). Our formulas for optimal differential commodity taxes produce empirically-implementable generalizations of the Atkinson-Stiglitz theorem, and take a familiar form that resembles the formula for the optimal nonlinear earnings tax.

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1 Introduction

Most tax systems around the world involve taxes on capital income, estates, inheritances, and on some categories of consumption. The seminal theorem of Atkinson and Stiglitz (1976) suggests that this could be suboptimal: the theorem states that when preferences are homogeneous and weakly separable, an optimal tax system will involve an income tax only. However, as was understood in contemporaneous work by Mirrlees (1976),⁶⁹ and more recently explored by Saez (2002b), Diamond and Spinnewijn (2011), Gauthier and Henriet (2018) and others, the Atkinson-Stiglitz Theorem does not generalize when tastes for certain commodities (including consumption) are correlated with earnings ability. Yet while economists have made significant progress on empirically-implementable formulas for non-linear earnings taxes (e.g., Saez, 2001), the Atkinson-Stiglitz Theorem remains perhaps the sharpest practically implementable result about optimal commodity taxation, as existing insights about the limitations of this theorem are mostly qualitative,⁷⁰ or rely on structural models with strong assumptions about the functional form of utility.

In this paper, we fill this gap by developing sufficient statistics formulas for optimal commodity taxation in the presence of nonlinear taxation. Our formulas nest the Atkinson-Stiglitz Theorem and related results as special cases, and provide a characterization in terms of empirically-estimable elasticities much like those in the nonlinear income tax derivation of Saez (2001). Unlike much of the existing literature on commodity taxation, we closely consider the relationship between the optimal mechanism, which can be implemented with an arbitrarily complicated tax system, and several more common "simple" tax functions with restrictions on functional form, including those involving a nonlinear earnings tax and a separable tax on capital income (either linear or nonlinear), or those involving a nonlinear earnings tax.

Our model generalizes the model of Saez (2002b), where consumers with heterogeneous earning abilities and tastes choose labor supply and a consumption bundle that exhausts their after-tax income. For concreteness, we describe the consumption bundle as consisting of consumption and savings. The policymaker chooses a flexible nonlinear tax system that depends on both earnings and savings (non-dependence on consumption is without loss of generality) to maximize a (weighted) utilitarian aggregation of individuals' utilities. As in Mirrlees (1976), the policymaker does not observe earnings abilities or tastes for savings, and

 $^{^{69}\}mathrm{See}$ also Konishi (1995).

 $^{^{70}}$ Saez (2002b) answered the qualitative question of when a "small" commodity tax can increase welfare in the presence of preference heterogeneity, but left to future work the task of deriving an expression for the optimal tax, writing "It would of course be extremely useful to obtain optimal commodity tax formulas" in such a general framework.

thus is restricted to second-best policy tools that must tradeoff the policymakers redistributive goals against the distortionary effects on labor supply and savings. In the absence of any restrictions on the policymaker's choice of tax systems, our model matches the mechanism design framework of Golosov, Troshkin, Tsyvinski, and Weinzierl (2013b).

We establish the following four key results in this framework.

First, we show that under mild regularity assumptions, the optimal mechanism of Golosov, Troshkin, Tsyvinski, and Weinzierl (2013b) can be implemented by a smooth tax system. This allows us to then characterize the optimal mechanism not in terms of unobserved structural primitives, as in Golosov, Troshkin, Tsyvinski, and Weinzierl (2013b), but instead in terms of observable tax elasticities, as in Saez (2001).

Our second key result is then a characterization of optimal smooth tax systems. We show that optimal earnings tax rates take a form much like that in Saez (2001). Optimal savings tax rates depend on a key sufficient statistic for preference heterogeneity: the difference between the cross-sectional variation of savings s with earnings z, s'(z), and the causal effect of income windfalls on savings, which we denote $s'_{inc}(z)$. The residual, $s'_{pref}(z) :=$ $s'(z) - s'_{inc}(z)$ is the sufficient statistic for preference heterogeneity. We show how this statistic can be estimated from existing empirical data and from behavioral responses to policy reforms, avoiding the need for explicitly modeling the relationship between unobserved preferences and ability. The optimal savings tax rates take a form much like that of the optimal earnings tax rate, but with earnings z replaced by $s'_{pref}(z)$, and with the elasticity of taxable income replaced by the elasticity of savings with respect to the savings tax rate.⁷¹ This simple sufficient statistics formulation provides an immediate generalization of the Atkinson-Stiglitz Theorem, as it implies that the optimal savings tax rate is everywhere zero when $s'_{pref}(z) = 0$ for all earnings levels z.

In the second part of the paper, we explore what we call simple tax systems. We show that across a large number of countries, most taxes on various savings vehicles can be classified as one of three types: (i) a separable linear (SL) savings tax, as in Saez (2002b) (ii) a separable nonlinear (SN) savings tax, and (iii) a system with a linear earnings-dependent (LED) savings tax, which allows, e.g., for lower-income people to have their savings taxed at a lower rate, as is the case for Long Term Capital Gains in the U.S.

Our third contribution is to show that for all three of these simple tax systems, the optimal policy can expressed using a sufficient statistics formula like that of the optimal smooth tax system. In fact, the formulas for the savings tax rates in the SN and LED systems are identical to the formula derived for the optimal smooth tax system. The formula for the

⁷¹In the special case where $s'_{pref}(z) = s(z)$, our formula coincides with the formula of Saez and Stantcheva (2018).

optimal savings tax rate in an SL system is necessarily different, but still retains a similar form. These formulas are written in a form that also allows for interpretation as Pareto efficiency conditions, so that one can test whether an existing tax system is consistent (in the sense of Pareto efficiency) with the prevailing nonlinear income tax.

Our fourth contribution is to show that under a narrower—but still surprisingly general—set of assumptions, the SN and LED systems are capable of implementing the *optimal* mechanism. This suggests that the simple types of tax systems found frequently across the world allow for sufficient policy flexibility to achieve the welfare gains available even under much more complicated systems. And the methods we develop provide a practical and portable methodology for studying the optimal form of these tax systems using realistically available data.

Our paper contributes most directly to the literature studying optimal commodity and savings taxes in the presence of correlated preference heterogeneity. Saez (2002b) derives the conditions under which the optimal linear commodity tax is strictly positive or negative (thus departing from the benchmark case of the Atkinson-Stiglitz Theorem). More recently, Allcott, Lockwood, and Taubinsky (2019) derives a sufficient statistics formula for the optimal linear commodity tax in the presence of nonlinear income taxation and correlated preference heterogeneity. Relative to Allcott, Lockwood, and Taubinsky (2019), we characterize a much broader class of tax systems with varying degrees of simplicity, including some that can implement the allocation from the optimal mechanism. This paper also contributes to the literature characterizing optimal taxes on capital, inheritances, and estates in the presence of correlated preference heterogeneity. Golosov, Troshkin, Tsyvinski, and Weinzierl (2013b) derives conditions characterizing the optimal mechanism in a model like the one we study-these conditions on structural "wedges" are those that would be implemented by the sufficient statistics conditions we present in Section 3.3. In parallel work Gerritsen, Jacobs, Rusu, and Spiritus (2020) study necessary conditions for the optimal capital tax in the presence of a different type of correlated heterogeneity—rates of return on savings—which also generates an expression for optimal marginal capital tax rates akin to the wedges in Golosov, Troshkin, Tsyvinski, and Weinzierl (2013b) and the marginal tax rates in Section 3.3. Our paper is complementary, providing a strategy for implementing these formula from statistics that can be estimated in empirical data (Section 3.2), and conditions under which these formulas are not only necessary, but also sufficient to implement the optimal mechanism. Our paper also relates more broadly to recent works revisiting classical results that capital should go untaxed and showing that they fail to apply in practice (Saez & Stantcheva, 2018; Straub & Werning, 2020).

The rest of this paper proceeds as follows. Section 2 presents our model and assump-

tions; Section 3 shows that smooth tax systems can implement the optimal mechanism, and provides sufficient statistics for optimal smooth tax systems; Section 4 turns to simple tax systems, providing conditions under which they implement the optimal mechanism, and providing a sufficient statistics characterization of these systems; Section 5 concludes.

2 Model and assumptions

Agents. We consider a population of heterogeneous agents where θ denotes the vector of agents' characteristics. While θ may encapsulate several characteristics, we follow Saez (2002b) and Golosov, Troshkin, Tsyvinski, and Weinzierl (2013b) in assuming that all characteristics are perfectly correlated such that heterogeneity is one-dimensional and agents can be indexed along a single dimension. Without loss of generality, we thus assume that $\theta \in \mathbb{R}$, and to simplify some analysis we also make the technical assumption that types are supported on a compact set Θ , with a continuously differentiable distribution $F(\theta)$.

Agents live for two periods. In the first period, agents work and receive earnings z, consume c, and save s for consumption in the second period. In the second period, agents are passive and only consume the savings s they have accumulated.⁷² Agents' preferences over allocations (c, s, z) are represented by the general utility function $U(c, s, z; \theta)$ indexed by type θ . We impose the following assumption about the utility function for the duration of the paper.

Assumption 1. $U(c, s, z; \theta)$ is twice continuously differentiable, increasing and weakly concave in c and s, and decreasing and strictly concave in z. The first derivatives U'_c and U'_s are bounded.

For example, one frequently-used functional form (e.g. Saez, 2002b; Golosov, Troshkin, Tsyvinski, & Weinzierl, 2013b) involves additively separable utility, with heterogeneity in agents' productivity w and discount factor δ :

$$U(c, s, z; \theta) = u(c) + \delta(\theta)u(s) - k(z/w(\theta)), \qquad (1)$$

with u(.) the utility from consumption and k(z/w) the disutility to work. In the above example, we say that there is preference heterogeneity if $\delta(\theta)$, which determines agents' preferences for savings, varies with θ . More generally, we define preference heterogeneity as follows:

 $^{^{72}}$ More generally, these results extend to settings where s is some other commodity, or a bequest to a future generation as in the two-period model studied by Farhi and Werning (2010).

Definition 1. There exists preference heterogeneity if some agents prefer different consumptionsavings bundles conditional on having the same earnings level i.e.

$$\exists \theta_0, \forall (c, s, z), \left. \frac{\partial}{\partial \theta} \left(\frac{U'_s(c, s, z; \theta)}{U'_c(c, s, z; \theta)} \right) \right|_{\theta = \theta_0} \neq 0$$
(2)

where U'_s and U'_c denote partial derivative with respect to s and c.

Preference heterogeneity in consumption-savings bundles translates formally into different marginal rates of substitutions between consumption and savings across agents. For example, if agents with higher earnings have higher savings in part because they have a higher preference for savings, this implies that they have a higher marginal rate of substitution when evaluated at the same allocation.

Government. An agent's type θ is private information and cannot be observed by the government, which only observes the distribution of types $F(\theta)$. As a result, the government must design incentive-compatible allocations, or more pragmatically, design a tax system that only depends on the observable variables (c, s, z). Adopting as a normalization that consumption c is untaxed, such tax systems can be written as $\mathbb{R}^2 \to \mathbb{R}$ functions of the form $\mathcal{T}(s, z)$.

The government's objective is to maximize a weighted sum of agents' utility,

$$\int_{\theta} \alpha(\theta) U(c(\theta), s(\theta), z(\theta); \theta) dF(\theta), \qquad (3)$$

where $\alpha(\theta)$ are type-specific Pareto-weights that capture redistributive motives, and $(c(\theta), s(\theta), z(\theta))$ denotes the allocation assigned to type θ .

The government's resource constraint is

$$\mathcal{T}\left(s\left(\theta\right), z\left(\theta\right)\right) dF\left(\theta\right) \ge E \tag{4}$$

with $E \geq 0$ representing an exogenous expenditure requirement.⁷³ Without any restrictions on the form of the optimal tax system \mathcal{T} , the resulting optimal allocation $\mathcal{A} = \{(c^*(\theta), s^*(\theta), z^*(\theta))\}_{\theta}$ must solve the following mechanism design program:

$$\max \int_{\theta} \alpha\left(\theta\right) U\left(c\left(\theta\right), s\left(\theta\right), z\left(\theta\right); \theta\right) dF\left(\theta\right)$$
(5)

 $^{^{73}}$ This is the government's resource constraint in the first period. Since we assume agents are passive in the second period, it coincides with the intertemporal resource constraint if we assume a homogeneous interest rate with a gross rate of return normalized to 1.

subject to the resource constraint

$$\int_{\theta} \left[z\left(\theta\right) - s\left(\theta\right) - c\left(\theta\right) \right] dF\left(\theta\right) \ge E$$
(6)

and the incentive compatibility constraint

$$U(c(\theta), s(\theta), z(\theta); \theta) \ge U(c(\theta'), s(\theta'), z(\theta'); \theta) \ \forall \ \theta, \theta'$$
(7)

We refer to an allocation $\mathcal{A} = \{(c^*(\theta), s^*(\theta), z^*(\theta))\}_{\theta}$ that maximizes (5) subject to (6) and (7) as the optimal incentive-compatible allocation.

3 Optimal smooth tax systems

In this section, we provide two key results about *smooth tax systems*, by which we mean twice continuously differentiable functions $\mathcal{T} : \mathbb{R}^2 \to \mathbb{R}$. First, we show that the optimal incentive-compatible allocation is implementable by a smooth tax system under some basic regularity conditions. Second, we leverage our first result to derive a sufficient statistics characterization of optimal smooth tax systems.

This second result constitutes an important advance relative to previous characterizations of the optimal allocation in terms of structural parameters, such as Golosov, Troshkin, Tsyvinski, and Weinzierl (2013b), as it allows us to characterize optimal tax policy in terms of empirically-estimable elasticities. In particular, our formulas allow a characterization of optimal tax policy in a manner that does not require structural assumptions and direct measures of how savings preferences vary with earnings ability. Instead, we show that the relationship between the causal effect of earnings on savings, together with the cross-sectional covariation of savings with income, provide a sufficient statistic for the preference heterogeneity relevant for designing optimal taxation of savings, capital income, or bequests.

We derive these two results under the following assumption, which we maintain throughout the rest of our analysis.

Assumption 2. Under the optimal incentive-compatible allocation $\mathcal{A} = \{(c^*(\theta), s^*(\theta), z^*(\theta))\}_{\theta}$, c^* , s^* , and z^* are smooth functions of θ , and s^* and z^* are strictly increasing in θ . For any $\theta \neq \theta'$, type θ strictly prefers $(c^*(\theta), s^*(\theta), z^*(\theta))$ over $(c^*(\theta'), s^*(\theta'), z^*(\theta'))$.

3.1 Implementability with smooth tax systems

Definition 2. We say that an allocation $\mathcal{A} = \{(c(\theta), s(\theta), z(\theta))\}_{\theta}$ is implementable with a tax system \mathcal{T} if

- 1. \mathcal{T} satisfies type-specific feasibility: $c(\theta) + s(\theta) + \mathcal{T}(s(\theta), z(\theta)) = z(\theta)$ for all $\theta \in \Theta$, and
- 2. \mathcal{T} satisfies individual optimization: $(c(\theta), s(\theta), z(\theta))$ maximizes $U(c, s, z; \theta)$ for all $\theta \in$ Θ , subject to the constraint $c + s + \mathcal{T}(s, z) \leq z$.

Our first result shows that the optimal incentive-compatible allocation is implementable by some smooth tax system.

Proposition 1. Suppose that assumptions 1 and 2 hold. Then the optimal incentivecompatible allocation is implementable by a smooth tax system.

Proof. See Appendix A.1.

Although it is clear that the optimal incentive-compatible allocation $\{(c(\theta), s(\theta), z(\theta))\}_{\theta}$ can always be implemented by *some* two-dimensional tax system, by defining $\mathcal{T}(s(\theta), z(\theta)) =$ $z(\theta) - c(\theta) - s(\theta)$ for each θ and $\mathcal{T}(s, z) = \infty$ for all other combinations of s and z, such a tax system is not smooth. A smooth tax system effectively tightens the incentive compatibility constraints in Equation (7), as it must allow agents to independently adjust s and z locally, to points not chosen by any other type, and it is not obvious that these tightened constraints will continue to respect incentive compatibility. To see this, note that starting from any given allocation $\mathcal{A} = \{(c(\theta), s(\theta), z(\theta))\}_{\theta}\}_{\theta}$, a smooth tax system can implement it only by satisfing the first-order conditions

$$\mathcal{T}'_{s}\left(s\left(z\left(\theta\right)\right), z\left(\theta\right)\right) = U'_{s}(c(\theta), s(\theta), z(\theta); \theta) / U'_{c}(c(\theta), s(\theta), z(\theta); \theta) - 1$$
(8)

$$\mathcal{T}'_{z}\left(s\left(z\left(\theta\right)\right), z\left(\theta\right)\right) = U'_{z}(c(\theta), s(\theta), z(\theta); \theta) / U'_{c}(c(\theta), s(\theta), z(\theta); \theta) + 1$$
(9)

In the presence of preference heterogeneity, individuals' incentive to deviate from their assigned allocation $(c(\theta), s(\theta), z(\theta))$ are higher under a smooth tax system than under an optimal incentive-compatible mechanism. To see the intuition, suppose that higher types θ have a higher taste for savings. If they deviate downward to some other earnings level $z(\theta') < z(\theta)$, then under the optimal mechanism they will be forced to choose savings level $s(\theta')$. Under a smooth tax system, however, the deviating type θ will choose a higher savings level $s' > s(\theta')$ at earnings level $z(\theta')$, making the appeal of deviation higher.

Despite this, in Appendix A.1 we provide a constructive proof that there exists a smooth tax system that does ensure it is globally optimal for each agent to stick to their assigned bundle $(c(\theta), s(\theta), z(\theta))$. The idea behind the proof is to construct \mathcal{T} such that it satisfies type-specific feasibility and the first-order conditions above, but such that it is sufficiently convex in the savings choice to ensure that any type who chooses a level of earnings $z = z(\theta)$ must prefer to choose a level of savings that is "sufficiently close" to the level of savings $s(\theta)$. This ensures that the set of potential deviations available to a type θ under the smooth tax system is "sufficiently close" to the set of available deviations in the optimal mechanism.

3.2 Sufficient statistics for smooth tax systems

3.2.1 Definitions

We adopt the usage from Chetty (2009): sufficient statistics are high-level elasticities (or other "program-evaluation estimates") that can be estimated empirically. In our setting these are population statistics quantifying the relationship between savings and earnings, and measurable behavioral responses to tax reforms. To define these statistics, it is helpful to write agents' optimization problem under a tax system $\mathcal{T}(s, z)$ as

$$\max_{z} \left\{ \max_{c,s} U(c,s,z;\theta) \text{ s.t. } c \leq z - s - \mathcal{T}(s,z) \right\}$$
(10)

where the inner problem represents the optimal choices of consumption $c(z;\theta)$ and savings $s(z;\theta)$ for a given earnings level z, and the outer problem represents the optimal choice of earnings $z(\theta)$ taking into account endogenous consumption and savings choices.

Earnings responses to tax reforms are captured through $\zeta_z^c(\theta)$, the compensated elasticity of labor income with respect to the marginal labor income tax rate, and $\eta_z(\theta)$, the income effect parameter. Formally,

$$\zeta_{z}^{c}(\theta) \equiv -\frac{1 - \mathcal{T}_{z}'(\theta)}{z(\theta)} \frac{\partial z(\theta)}{\partial \mathcal{T}_{z}'(\theta)}$$
$$\eta_{z}(\theta) \equiv -(1 - \mathcal{T}_{z}'(\theta)) \frac{\partial z(\theta)}{\partial \mathcal{T}(\theta)}$$

where $\mathcal{T}(\theta) \equiv \mathcal{T}(s(z(\theta); \theta), z(\theta))$ is the tax liability, and $\mathcal{T}'_{z}(\theta) \equiv \frac{\partial \mathcal{T}(s(z(\theta); \theta), z(\theta))}{\partial z}$ is the marginal labor income tax rate of an agent of type θ . Since the earnings choice that pins down $z(\theta)$ takes into account endogenous consumption and savings choices, these elasticity concepts take into account the full sequence of adjustments due to changes in consumption and savings choices, as well as those due to any nonlinearities in the tax system.⁷⁴

Savings responses to tax reforms are captured through $\zeta_{s|z}^{c}(\theta)$, the compensated elasticity of savings with respect to the marginal savings tax rate, and $\eta_{s|z}(\theta)$, the income effect

 $^{^{74}}$ This corresponds to the type of circular adjustment process described in e.g. Jacquet and Lehmann (2020).

parameter:

$$\begin{split} \zeta_{s|z}^{c}\left(\theta\right) &\equiv -\frac{1 + \mathcal{T}_{s}^{\prime}\left(s\left(z;\theta\right),z\right)}{s\left(z;\theta\right)} \frac{\partial s\left(z;\theta\right)}{\partial \mathcal{T}_{s}^{\prime}\left(s\left(z;\theta\right),z\right)} \Big|_{z=z(\theta)} \\ \eta_{s|z}\left(\theta\right) &\equiv -\left(1 + \mathcal{T}_{s}^{\prime}\left(s\left(z;\theta\right),z\right)\right) \frac{\partial s\left(z;\theta\right)}{\partial \mathcal{T}\left(s\left(z;\theta\right),z\right)} \Big|_{z=z(\theta)} \end{split}$$

where $\mathcal{T}(s(z;\theta), z)$ is the tax liability, and $\mathcal{T}'_s(s(z;\theta), z)$ is the marginal savings tax rate of an agent of type θ who earns labor income z. These elasticity concepts are conditional on z: they measure responses of consumption and savings to tax reforms and nonlinearities in the tax system, holding labor income z fixed.

Correlated preference heterogeneity in consumption-savings choices is captured through $s'_{pref}(\theta)$, which measures the difference between the cross-sectional variation of savings along the earnings distribution and individuals' savings responses to changes in earnings. Formally,

$$s'_{pref}(\theta) \equiv \left(\underbrace{\frac{ds\left(z;\theta\right)}{dz}}_{s'(z)} - \underbrace{\frac{\partial s\left(z;\theta\right)}{\partial z}}_{s'_{inc}(z)}\right)\Big|_{z=z(\theta)}$$

where s'(z) measures cross-sectional changes in savings moving across agents' types, while $s'_{inc}(z)$ measures individual changes in savings for a given agent's type.

Finally, to encode the policymaker's redistributive objective, we define $\hat{g}(z)$ as the social marginal welfare weights augmented with income effects. These capture the social value of marginally increasing the disposable income of agents with earnings z. Formally,

$$\begin{split} \hat{g}(z) &:= \frac{\alpha\left(z\right)}{\lambda} U_{c}'\left(c\left(z\right), s\left(z\right), z; \theta\left(z\right)\right) + \mathcal{T}_{z}'\left(s(z), z\right) \frac{\eta_{z}(z)}{1 - \mathcal{T}_{z}'\left(s(z), z\right)} \\ &+ \mathcal{T}_{s}'\left(z, s(z)\right) \left(\frac{\eta_{s|z}(z)}{1 + \mathcal{T}_{s}'\left(s(z), z\right)} + s_{inc}'(z) \frac{\eta_{z}(z)}{1 - \mathcal{T}_{z}'\left(s(z), z\right)}\right). \end{split}$$

3.2.2 Measurement

Elasticities capturing earnings and savings responses to tax reforms are commonly estimated in the empirical literature, and we here discuss the measurement of the preference heterogeneity parameter $s'_{pref}(z)$. Since s'(z) is the cross-sectional variation of savings along the income distribution, it can be directly observed from survey or administrative data featuring both earnings and savings. The result below shows how $s'_{inc}(z)$ can be measured in the data.

Proposition 2. The sufficient statistic $s'_{inc}(z)$ can be measured as follows.

- If agents' preferences are weakly separable, $s'_{inc}(z) = \eta_{s|z}(z) \frac{1 \mathcal{T}'_{z}(s(z),z)}{1 + \mathcal{T}'_{z}(s(z),z)}$
- If agents wage rates w and hours h are observable, $s'_{inc}(z) = s(z) \frac{\xi_w^s(z)}{w(z) + h(z)\xi_w^h(z)}$

where $\xi_w^s(z)$ is the elasticity of savings with respect to the wage rate, and $\xi_w^h(z)$ is the elasticity of hours with respect to the wage rate.

Proof. See Appendix A.2.

This proposition shows how $s'_{inc}(z)$ can be related to empirically estimable elasticity concepts. If, as in example (1) above, agents' preferences are weakly separable between the utility of consumption and savings and the disutility of labor supply, then $s'_{inc}(z)$ is proportional to the causal income effect—it is for this reason that we use the subscript *inc*.

More generally, if agents' preferences are not weakly separable but wage rates w and hours h are observable, we can use the fact that earnings are given by the product of hours times the wage rate $z = h \cdot w$ and leverage wage changes to measure $s'_{inc}(z)$. In that case, it can be related to the elasticity of savings with respect to the wage rate and to the elasticity of hours with respect to the wage rate.

3.3 Sufficient statistics characterization of optimal tax systems

A key result for the sufficient statistics characterizations we provide is the following equivalence lemma for savings tax reforms. This result is a generalization of Lemma 1 from Saez (2002b) to a broader class of tax systems. It quantifies the change in earnings induced by a savings tax reform by characterizing the labor income tax reform inducing the exact same change in earnings.⁷⁵

Lemma 1. A small increase $d\tau_s$ in the marginal savings tax rate faced by agent θ at earnings z, induces the same earnings change as a small increase $s'_{inc}(z) d\tau_s$ in the marginal earnings tax rate.

Proof. See Appendix A.3.

Intuitively, an agent who reduces earnings by dz reduces savings by $s'_{inc}dz$. The Lemma shows that a $d\tau_s$ increase in the marginal savings tax generates a similar reduction in earnings as a $s'_{inc}d\tau_s$ increase in the earnings tax rate.

Lemma 1 relates the labor supply distortions induced by savings taxes to the labor supply distortions induced by income taxes. Specifically, it shows that increasing savings tax rates

⁷⁵We use the terms "income tax" and "earnings tax" interchangeably to refer to the tax on *labor* income z, as distinct from a tax on savings s, which in some settings can be interpreted as a tax on capital income—see the discussion of their relation in Section (4).

will be more distortionary when $s'_{inc}(z)$ is higher. We use this result to provide the following characterization of the optimal marginal income and savings tax rates:

Proposition 3. An optimal smooth tax system satisfies, at each bundle $(c^*(\theta), s^*(\theta), z^*(\theta))$ chosen by a type θ , the following marginal earnings tax rate condition

$$\frac{\mathcal{T}'_{z}\left(s^{*}(\theta), z^{*}(\theta)\right)}{1 - \mathcal{T}'_{z}\left(s^{*}(\theta), z^{*}(\theta)\right)} = \frac{1}{\zeta_{z}^{c}(z^{*}(\theta))} \frac{1}{z^{*}(\theta)h_{z}(z^{*}(\theta))} \int_{z^{*}}^{\bar{z}} (1 - \hat{g}(x)) \, dH_{z}(x) \\ - s'_{inc}(z^{*}(\theta)) \frac{\mathcal{T}'_{s}\left(s^{*}(\theta), z^{*}(\theta)\right)}{1 - \mathcal{T}'_{z}\left(s^{*}(\theta), z^{*}(\theta)\right)} \quad (11)$$

and the following marginal savings tax rate condition

$$\frac{\mathcal{T}_{s}'(s^{*}(\theta), z^{*}(\theta))}{1 + \mathcal{T}_{s}'(s^{*}(\theta), z^{*}(\theta))} = s_{pref}'(z^{*}(\theta)) \frac{1}{\zeta_{s|z}^{c}(z^{*}(\theta))} \frac{1}{s^{*}(\theta)h_{z}(z^{*}(\theta))} \int_{x \ge z^{*}(\theta)} (1 - \hat{g}(x)) \, dH_{z}(x) \quad (12)$$

Condition (11) on the optimal earnings tax rates constitutes a familiar condition analogous to Saez (2001), with one modification. Because there is a non-zero savings tax rate, the formula also accounts for the distortionary effects on savings caused by distortions to earnings levels.

Condition (12) presents a transparent generalization of the Atkinson-Stiglitz Theorem. When the sufficient statistic for preference heterogeneity, s'_{pref} , is equal to zero, the condition implies that the optimal savings tax rate must equal zero as well. When the statistic is $s'_{pref} > 0$, implying that higher earners have a higher taste for savings, the condition implies that the optimal savings tax rate must be positive. Interestingly, the optimal savings tax rates satisfy a formula that is remarkably similar to the standard ABC formula for optimal income tax rates. When $s'_{pref} > 0$, the magnitude of the optimal savings tax rate at point (s^*, z^*) is decreasing in the elasticity of savings with respect to the tax rate, increasing in the strength of redistributive motives, and decreasing in the relative density of individuals at point (s^*, z^*) .

We can combine conditions (11) and (12) to derive the following Pareto efficiency condition:

Corollary 1. Any Pareto efficient smooth tax system must satisfy

$$\frac{\mathcal{T}'_{s}(s^{*}(\theta), z^{*}(\theta))}{1 + \mathcal{T}'_{s}(s^{*}(\theta), z^{*}(\theta))} \zeta^{c}_{s|z}(z^{*}(\theta))s^{*}(\theta) \\
= \left[\underbrace{s'(z^{*}(\theta)) - s'_{inc}(z^{*}(\theta))}_{s'_{pref}(z^{*}(\theta))}\right] \frac{\mathcal{T}'_{z}(s^{*}(\theta), z^{*}(\theta)) + s'_{inc}(z^{*})\mathcal{T}'_{s}(s^{*}(\theta), z^{*}(\theta))}{1 - \mathcal{T}'_{z}(s^{*}(\theta), z^{*}(\theta))} \zeta^{c}_{z}(z^{*}(\theta))z^{*}(\theta) \quad (13)$$

Because the condition in (13) does not feature social marginal welfare weights, it is an efficiency condition that must hold for any tax system that is not Pareto dominated. An implication of Corollary 1 is that in the absence of any preference heterogeneity, positive savings tax rates are Pareto dominated, providing an extension of the Atkinson-Stiglitz Theorem for nonlinear tax systems. On the other hand, any Pareto efficient tax system must feature nonzero savings tax rates in the presence of preference heterogeneity.

4 Optimal simple savings tax systems

The class of all two-dimensional tax systems $\mathcal{T}(s, z)$ is very diverse, as it allows for tax liabilities to vary across every possible combination of earnings and savings. In practice, tax systems must be defined by policymakers, and implemented by institutions, who face constraints on the degree of complexity that can be accommodated. As a result, real tax functions impose strong restrictions on the set of functions considered. Although the details of these restrictions vary across institutions, most can be classified into a few common sets of functional forms. While these restrictions still allow for substantial flexibility (such as fully nonlinear income taxes) they are nevertheless far more restrictive than the fully general class of two-dimensional functions, and in this sense we consider them "simple." Two natural questions arise. First, are there circumstances under which such simple tax systems can achieve the outcomes that are feasible under the much larger unrestricted class of twodimensional tax functions? And second, is it possible to derive sufficient statistics formulas to characterize the optimal simple tax systems, akin to the general formulas presented in Proposition 3?

This section answers these questions. We first present a survey of savings tax policies across a large number of countries, and we identify three common categories of simple tax systems to which most belong: separable linear (SL) savings taxes, separable nonlinear (SN) savings taxes, and linear earnings-dependent (LED) savings taxes. Although it is clear that SL tax systems are generally unable to implement the optimum, due to their inability to satisfy the local savings first-order condition in Equation (8), the ability of SN and LED systems to implement the optimum is less obvious. In Section 4.2, we derive a strong result: under weak conditions on the utility function, a SN savings tax *can* implement the optimum (if any smooth tax system can at all). Under somewhat stronger conditions—which we make explicit—a LED savings tax can do the same. Finally, we derive sufficient statistics formulas that characterize the optimal SL, SN, and LED tax systems, as well as Pareto efficiency conditions, and we show that in each case the formulas can be written using the key statistic s'_{pref} to quantify preference heterogeneity.

4.1 A taxonomy of common simple savings tax systems

Many governments tax both labor income (earnings) and capital (savings) in some manner. These tax systems take a variety of forms, the details of which depend on the specifics of timing, the nature of the savings vehicle, and many other details. Nevertheless, with some simplifications, many of these tax policies can be interpreted as a $\mathbb{R}^2 \to \mathbb{R}$ function of earnings and savings, analogous to our generalized function $\mathcal{T}(s, z)$. Upon doing so, we observe that nearly all savings tax policies can be categorized as one of three simple types—separable linear (SL), separable nonlinear (SN), and linear income-dependent (LED)—characterized in Table 8.

Type of separable tax system	$\mathcal{T}\left(s,z ight)$	$\mathcal{T}_{z}^{\prime}\left(s,z ight)$	$\mathcal{T}_{s}^{\prime}\left(s,z ight)$
SL: separable linear	$\tau_{s}s + T_{z}\left(z\right)$	$T_{z}^{\prime}\left(z ight)$	τ_s
SN: separable nonlinear	$T_{s}\left(s\right) + T_{z}\left(z\right)$	$T_{z}^{\prime}\left(z ight)$	$T_{s}^{\prime}\left(s ight)$
LED: linear earnings-dependent	$\tau_{s}\left(z\right)s+T_{z}\left(z\right)$	$T_{z}^{\prime}\left(z\right)+\tau_{s}^{\prime}\left(z\right)s$	$ au_{s}\left(z ight)$

Table 8: Types of separable tax systems

To translate the highly detail-dependent nature of actual tax codes into something that can be interpreted as $\mathcal{T}(s, z)$, we impose a few important simplifications. First, we treat ordinary income as consisting primarily of labor income (earnings), written as z in our notation. Second, we separately consider taxes on five broad categories of savings vehicles: wealth, capital gains, real property, private pensions, and inheritances. (These categories may overlap—real property is a component of wealth, for example—but we use these groups to reflect the tax instruments that many governments use in practice.) Finally, in our model s represents the full amount of resources reserved for future consumption, and so we reinterpret these tax functions (if necessary) as taxes on that basis. Specifically, although taxes on wealth, inheritances, and property are generally written as a tax on the total asset value, taxes on capital gains and pensions are generally written as a tax on the flow of income generated by the underlying asset. As is well appreciated, if one abstracts from heterogeneous or uncertain rates of return (as we do) then one can translate between a tax on wealth and a tax on capital income, with linear taxes remaining linear under this translation.⁷⁶

It turns out that majority of savings tax policies (across countries and across savings vehicles) can be classified as having one of these simple structures. Indeed, each of the three tax systems in Table 8 can for instance be found in the US. Most property taxes (levied at the state and local level) take the form of separable linear taxes, with a flat tax rate (independent of one's labor earnings) applied to the assessed value of the total property. The estate tax takes the form of a separable nonlinear tax: the tax rate rises progressively with the value of the estate, but it does not vary with labor income of the donor or the recipient. Finally, taxes on long-term capital gains and on qualified dividends both take the form of linear earnings-dependent taxes: in 2020, for example, an individual with \$50,000 in labor earnings faced a long-term capital gains tax rate of 15%, whereas an individual earning \$500,000 faced a rate of 20%.

In Table 9, we categorize the tax policies on each class of savings vehicle for for 21 countries, most of which fit one of the three simple tax system types from Table 8. In cases where there is some ambiguity (such as the distinction between short-term and long-term capital gains in the United States) we provide additional detail in Appendix C.

⁷⁶For example, if the baseline amount of savings is s and the homogenous, deterministic gross rate of return is 1 + r, then a tax rate τ applied to total savings (1 + r)s is equivalent to a tax rate of $\tau(1 + 1/r)$ applied to capital income rs.

Country	Wealth	Capital Gains	Property	Pensions	Inheritance
Australia	_	Other	SL, SN	SL	_
Austria	_	Other	SL, SN	SN	—
Canada	_	Other	SL	SN	—
Denmark	—	SN	SL, SN	SL, SN	SN
France	_	Other	Other	SL, SN	SN
Germany	—	Other	SL	SN	SN
Ireland	_	SN	SL, SN	SN	SN
Israel	—	Other	Other	SN	—
Italy	SL, SN	SL	SL	SL	SL, SN
Japan	—	SL, SN	SN	SN	SN
Netherlands	SN	SL	SL, SN	SN	SN
New Zealand	—	Other	SN	SL, LED	—
Norway	SN	SL	SL	SN	—
Portugal	—	SL	Other	SN	SL
Singapore	—	Other	SN	SN	—
South Korea	—	SN	SN	SN	SN
Spain	SN	SN	SL, SN	SN	SN
Switzerland	SN	SN	SL, SN	SN	SN
Taiwan	—	SL, SN	SL, SN	SN	SN
United Kingdom	_	Other	SN	SN	SN
United States	_	LED	SL	SN	SN

Table 9: Tax systems applied to different savings vehicles, by country.

Given the ubiquity of these simple functional forms, we naturally wonder about their ability to achieve the policy outcomes available under more complicated tax systems, such as the fully general $\mathcal{T}(s, z)$ functions discussed in the previous section. We now turn to this question.

4.2 When simple savings tax systems can implement the optimum

We now derive sufficient conditions under which the optimal allocation can be implemented by a separable nonlinear (SN) tax system, and by a linear earnings-dependent (LED) tax system. Throughout this section, we assume that the optimal allocation satisfies the conditions in Proposition 1, implying that the allocation can be implemented by *some* smooth tax systems, so that the question of interest is whether SN or LED systems are among those that implement the optimum.

We proceed in three steps. First, we define SN and LED tax systems that satisfy typespecific feasibility and the individual first-order conditions, as in Section 3.1. Second, in Proposition 4 we present sufficient conditions under which these SN and LED tax systems also satisfy the local second-order conditions, implying that each type's assigned allocation is a local optimum under the SN or LED system. Third, in Proposition 5, we present sufficient conditions under which local optima are ensured to be global optima, implying that the SN and LED systems are indeed optimal.

We first define a potentially optimal SN system $\mathcal{T}(s, z) = T_s(s) + T_z(z)$, with the nonlinear functions T_s and T_z defined across all savings and earnings bundles in the optimal allocation as follows:

$$T_s(s^*(\theta)) = \int_{\vartheta=\theta_{min}}^{\theta} \left(U_s'(\vartheta) / U_c'(\vartheta) - 1 \right) s^{*'}(\vartheta) d\vartheta, \tag{14}$$

$$T_z(z^*(\theta)) = z^*(\theta_{min}) - s^*(\theta_{min}) - c^*(\theta_{min}) + \int_{\vartheta=\theta_{min}}^{\theta} \left(U_z'(\vartheta) / U_c'(\vartheta) + 1 \right) s^{*'}(\vartheta) d\vartheta$$
(15)

where θ_{min} denotes the lowest earning type, and the derivatives are evaluated at the bundle assigned in the optimal allocation (e.g., $U'_s(\vartheta) = U'_s(c^*(\vartheta), s^*(\vartheta), z^*(\vartheta); \vartheta)$). Note that under this tax system, the allocation satisfies by definition each type's first-order conditions for individual optimization in Equations (8) and (9), and we show that this system also satisfies type-specific feasibility (see Lemma 2 in Appendix).

We similarly define a potentially optimal LED system $\mathcal{T}(s, z) = \tau_s(z) \cdot s + T_z(z)$ as follows:

$$\tau_{s}(z^{*}(\theta)) = U'_{s}(\theta)/U'_{c}(\theta) - 1,$$

$$T_{z}(z^{*}(\theta)) = z^{*}(\theta_{min}) - s^{*}(\theta_{min}) - c^{*}(\theta_{min}) + \int_{\vartheta=\theta_{min}}^{\theta} (U'_{z}(\vartheta)/U'_{c}(\vartheta) + 1) s^{*'}(\vartheta) d\vartheta - s^{*}(z) \cdot (\tau_{s}(z) - \tau_{s}(z^{*}(\theta_{min})))$$

$$(17)$$

This system also satisfies type-specific feasibility and the local first-order conditions for individual optimization.

We can now derive sufficient conditions under which the above potentially optimal SN and LED tax systems satisfy the second-order conditions for individual optimization, implying that under these conditions assigned bundles are local optima. These conditions can be simply stated in terms of the marginal rates of substitution of savings and earnings vs. consumption:

$$\mathcal{S}(c, s, z; \theta) := \frac{U'_s(c, s, z; \theta)}{U'_c(c, s, z; \theta)}$$
(18)

and

$$\mathcal{Z}(c,s,z;\theta) := \frac{U'_z(c,s,z;\theta)}{U'_c(c,s,z;\theta)}$$
(19)

Note that by smoothness of the allocation and the utility function, these functions are

smooth in c, s, z, and θ . Using these definitions, sufficient conditions for local second-order conditions are given by the following proposition.

Proposition 4. Suppose that an allocation satisfies the conditions in Proposition 1. Then under the SN tax system defined by Equations (14) and (15), each agent's assigned choice of savings and earnings is a local optimum if the following conditions hold at each point in the allocation:

$$\mathcal{S}_c' \ge 0, \ \mathcal{S}_z' \ge 0, \ \mathcal{S}_\theta' \ge 0 \tag{20}$$

and

$$\mathcal{Z}_c' \le 0, \ \mathcal{Z}_s' \ge 0, \ \mathcal{Z}_\theta' \ge 0. \tag{21}$$

Under the LED tax system defined by Equations (16) and (17), each agent's assigned choice of savings and earnings is a local optimum if the utility function is additively separable in consumption, savings, and earnings, and additionally the following conditions hold at each point in the allocation:

$$\mathcal{S}'_{\theta} \ge 0, \ \mathcal{S}'_{\theta} < \mathcal{Z}'_{\theta}/s'(z), \ and \ \mathcal{S}'_{\theta} < (U'_s \mathcal{S}'_c - U'_c \mathcal{S}'_s) \ s'(\theta).$$
(22)

Proof. See Appendix B.

The sufficiency conditions (20) and (21) are quite weak; they are satisfied under many common utility functions used in calibrations of savings and income taxation models, including the simple example function in Equation (1). Conditions $S'_{\theta} \geq 0$ and $Z'_{\theta} \geq 0$ are single crossing conditions for savings and earnings, while other conditions intuitively relate to the concavity of preferences.

The sufficiency conditions for LED systems are more restrictive, and they place a constraint on the extent of local preference heterogeneity for savings, as compared with preference heterogeneity in earnings. In words, the preference for savings must not increase "too quickly" across types, or else the second-order condition for earnings may be violated. The intuition for this result can be seen from the definition of the potentially optimal LED system. If the marginal rate of substitution for saving, S, increases very quickly with income at some point in the allocation, then the savings tax rate $\tau_s(z)$ must rise very quickly with z at that point, by Equation (16). Since the savings tax rate $\tau_s(z)$ applies to total savings (including inframarginal savings), this increase in $\tau_s(z)$ must be offset by a sharp decrease in $T_z(z)$ at the same point in the distribution, by Equation (17). Yet a sufficiently steep decrease in $T_z(z)$ will cause the second-order condition for earnings choice—holding fixed savings choice—to be violated.

An implication of Proposition 4 is that under these sufficiency conditions, there is an equivalence between SN and LED tax systems, in that they can both implement the optimum. Hence, when we provide sufficient statistics characterization of these tax systems below, we are in fact providing a characterization of the same allocation. However, this equivalence relies on the use of different optimal nonlinear earnings tax schedules $T_z(z)$ in each case. Thus if there is an existing nonlinear income tax that cannot be jointly reformed with the savings tax, then the optimal SN and LED savings taxes (conditional on the suboptimal income tax) are generally not equivalent.

Having presented conditions under which the savings and earnings assignments for each type are local optima under SN and LED tax systems, we now present a set of regularity conditions ensuring that these individual local optima are also their global optima.

Proposition 5. Assume that $-\frac{U'_c(c,s,z;\theta)}{U'_z(c,s,z;\theta)}$ and $-\frac{U'_s(c,s,z;\theta)}{U'_z(c,s,z;\theta)}$, are strictly increasing in θ for all (c, s, z), and that $U''_{cs} \geq 0$, $U''_{zz} = 0$, $U''_{sz} = 0$. Suppose that $\mathcal{A} = \{(c^*(\theta), s^*(\theta), z^*(\theta))\}_{\theta}$ constitutes a set of local optima for types θ under a smooth tax system \mathcal{T} . Individuals' local optima correspond to their global optima when

- 1. \mathcal{T} is a SL system
- 2. \mathcal{T} is a SN system and $-\frac{T_{ss}'(s)}{1+T_s'(s)} < \frac{-U_{ss}''(c(s,\theta),s,z^*(\theta);\theta)}{U_s'(c(s,\theta),s,z^*(\theta);\theta)}$ for all s and θ , where $c(s,\theta) := z^*(\theta) s \mathcal{T}(s, z^*(\theta))$
- 3. \mathcal{T} is a LED system, $\frac{U'_s(c,s,z;\theta)}{U'_c(c,s,z;\theta)}$ is strictly increasing in θ , $\frac{-U''_{cc}(c(s,\theta),s,z^*(\theta);\theta)}{U'_c(c(s,\theta),s,z^*(\theta);\theta)} > \frac{-\tau'_s(z^*(\theta))}{(1+\tau_s(z^*(\theta)))\sigma_c(s,z^*(\theta))}$ for all $s < s^*(\theta)$ and θ , and $\frac{-U''_{ss}(c(s,\theta),s,z^*(\theta);\theta)}{U'_s(c(s,\theta),s,z^*(\theta);\theta)} > \frac{-\tau'_s(z^*(\theta))}{\sigma_c(s,z^*(\theta))}$ or all $s > s^*(\theta)$ and θ , where $\sigma_c(s,z) := 1 - \tau'_s(z)s - T'_z(z)$

Proof. See Appendix B.

In essence, global optimality is ensured under the following conditions. First, higher types θ derive higher gains from working and allocating those gains to consumption or savings—a generalized single-crossing condition. Second, additive separability of consumption and savings from labor. Third, the utility function U is sufficiently concave. For the case of SN systems, it must be sufficiently concave in savings. For the case of LED systems, the utility function must be sufficiently concave in both consumption and savings, as well as feature a monotonically increasing taste for savings in θ . Notably, these concavity conditions need only be checked when earnings are fixed at each type's assigned earnings level $z^*(\theta)$.

4.3 Sufficient statistics formulas for the optimal simple savings tax systems

We now characterize the optimal simple savings tax systems in terms of the empirically measurable sufficient statistics discussed in Section 3.2. These conditions can be stated in terms of the following proposition.

Proposition 6. For each type of simple tax system (SL, SN, LED), if an optimal such system exists and implements a smooth allocation wherein agents' private optima are unique, then the system satisfies the following conditions.

First, any optimal systems' marginal effective income tax rates must satisfy

$$\frac{\mathcal{T}_{z}'\left(s(z^{*}), z^{*}\right) + s_{inc}'(z^{*})\mathcal{T}_{s}'\left(s(z^{*}), z^{*}\right)}{1 - \mathcal{T}_{z}'\left(s(z^{*}), z^{*}\right)} = \frac{1}{\zeta_{z}^{c}(z^{*})} \frac{1}{z^{*}h_{z}(z^{*})} \int_{z^{*}}^{\bar{z}} (1 - \hat{g}(z)) \, dH_{z}(z) \tag{23}$$

Second, an optimal SN or LED system's effective marginal savings tax rates must satisfy

$$\frac{\mathcal{T}_{s}'\left(s(z^{*}), z^{*}\right)}{1 + \mathcal{T}_{s}'\left(s(z^{*}), z^{*}\right)}\zeta_{s|z}^{c}(z^{*})s(z^{*}) = \left[\underbrace{s'(z^{*}) - s'_{inc}(z^{*})}_{s'_{pref}(z^{*})}\right]\frac{\mathcal{T}_{z}'\left(s(z^{*}), z^{*}\right) + s'_{inc}(z^{*})\mathcal{T}_{s}'\left(s(z^{*}), z^{*}\right)}{1 - \mathcal{T}_{z}'\left(s(z^{*}), z^{*}\right)}\zeta_{z}^{c}(z^{*})z^{*},$$
(24)

whereas an optimal SL system's savings tax rate must satisfy

$$\frac{\tau_s}{1+\tau_s} \int_{z=\underline{z}}^{\overline{z}} \zeta_{s|z}^c(z) s(z) \, dH_z(z) = \int_{z=\underline{z}}^{\overline{z}} \left[\underbrace{s'(z) - s'_{inc}(z)}_{s'_{pref}(z)} \right] \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1-T'_z(z)} \zeta_z^c(z) \, z \, dH_z(z) \, . \tag{25}$$

Proof. See Appendix B.3.

Here we write the conditions (24) and (25) in terms of marginal tax rates, but without reference to marginal social welfare weights \hat{g} , so that they can be interpreted either as optimality conditions if (23) is satisfied, or as Pareto efficiency conditions regardless of whether (23) is satisfied. Intuitively, condition (23) determines optimal tax rates given the redistributive tastes of the government, while conditions (24) and (25) determine the optimal tax mix between earnings and savings given the amount of preference heterogeneity in consumption-savings decisions. As we show in the Appendix, the derivation of conditions (24) and (25) rely on joint reforms to the earnings tax and the savings tax schedules in the spirit of Konishi (1995), Laroque (2005) and Kaplow (2006).

The optimal earnings tax formula extends the standard ABC formula of Diamond (1998) and Saez (2001) to a setting with a consumption-savings choice and a savings tax. As in

the standard case, a reform of the earnings tax schedule induces changes in earnings. Yet, it here also induces changes in consumption-savings choices due to e.g. income effects. If the savings tax is zero, the latter do not trigger any fiscal effect, and the optimal tax formula exactly coincides with that derived in earlier works.

If savings are taxed and the marginal savings tax rate is positive at earnings z^* , an increase in the marginal earnings tax rate reduces both earnings tax revenue and savings tax revenue paid at this earnings level. It thus triggers a larger negative fiscal externality than in the standard case, and calls for lower marginal earnings tax rate. Moreover, if the marginal savings tax rate is also positive at earnings levels above, the increase in the tax liability further reduces savings tax revenue at higher earnings which similarly calls for lower marginal earnings tax rate. Hence, if the government relies on both an earnings tax and a savings tax for redistribution, it should rely less on the earnings tax than when the earnings tax is its sole instrument.

The Pareto-efficiency condition pins down the degree to which the government should tax savings relative to earnings at the margin. It is obtained by combining at earnings z^* an increase in the marginal savings tax rate by $d\tau_s$ with a decrease in the marginal earnings tax rate by $s'(z^*)d\tau_s$ tailored to offset all changes in tax liabilities. This still changes relative distortions at earnings z^* and thereby induces behavioral responses. The savings tax reform triggers a savings response proportional to $\zeta_{s|z}^c(z^*) d\tau_s$, and an earnings response proportional to $\zeta_z^c(z^*) (s'_{inc}(z^*) d\tau_s)$ by Lemma 1. The offsetting earnings tax reform only triggers an earnings response proportional to $\zeta_z^c(z^*) (s'(z^*) d\tau_s)$. The net earnings response thus depends on $s'_{pref}(z^*) = s'(z^*) - s'_{inc}(z^*)$, a sufficient statistic for local preference heterogeneity at z^{*} .⁷⁷

This highlights the role of preference heterogeneity in consumption-savings decisions for the taxation of capital. If higher earnings are the only reason why high earners save more than low earners $(s'_{pref}(z^*) = 0)$, Atkinson and Stiglitz (1976) result applies and Pareto-efficiency requires savings to go untaxed: it is inefficient to distort savings choices. In contrast, if high earners have stronger tastes for savings which contribute to their higher savings levels $(s'_{pref}(z^*) > 0)$, Atkinson and Stiglitz (1976) result no longer applies and Pareto-efficiency requires positive savings taxes: it is efficient to distort savings choices. In other words, if savings choices are informative about agents' productivity, then savings should be taxed and the size of the tax depends on the information that savings choices reveal.

While these formulas are presented jointly as a characterization of the optimum, they can be used on their own. The Pareto-efficiency condition (24) or (25) allows to test whether an

⁷⁷For a SL tax system, a change in the savings tax rate τ_s affects distortions at all earnings levels, and thus requires offsetting earnings tax reforms at all earnings levels. As a result, the Pareto-efficiency condition translates into the integral equation (25).

existing tax system is Pareto-efficient. Moreover, the optimal earnings tax formula (23) pins down the optimal earnings tax schedule for any given – potentially suboptimal – savings tax schedule.

We now present optimal savings tax formulas which characterize the optimal savings tax schedule for *any* given income tax schedule—including a potentially suboptimal one. This result may be useful in situations where it is not possible to reform both earnings and savings tax schedules.

Moreover, we have emphasized thus far the equivalence between optimal SN and optimal LED tax systems (Proposition 4) which transpires in the previous sufficient statistics characterization of the optimum (Proposition 6). However, conditional on a given earnings tax schedule, SN and LED tax systems are not equivalent.

Proposition 7. Given a (potentially suboptimal) earnings tax schedule $T_z(z)$, in an optimal SL tax system, τ_s satisfies

$$\tau_s \int_z \left\{ \frac{s(z)\zeta_{s|z}^c(z)}{1+\tau_s} + (s_{inc}'(z))^2 \frac{z\zeta_z^c(z)}{1-T_z'(z)} \right\} dH_z(z) = \int_z \left\{ [1-\hat{g}(z)] s(z) - s_{inc}'(z) T_z'(z) \frac{z\zeta_z^c(z)}{1-T_z'(z)} \right\} dH_z(z).$$
(26)

In an optimal SN system, the savings tax schedule $T_s(s)$ satisfies, at each level of savings $s(z^*)$,

$$T'_{s}(s(z^{*})) \left[\frac{s(z^{*})\zeta_{s|z}^{c}(z^{*})}{1+T'_{s}(s(z^{*}))} + (s'_{inc}(z^{*}))^{2} \frac{z^{*}\zeta_{z}^{c}(z^{*})}{1-T'_{z}(z^{*})} \right] h_{z}(z^{*})$$
$$= s'(z^{*}) \int_{z^{*}}^{\bar{z}} [1-\hat{g}(z)] dH_{z}(z) - s'_{inc}(z^{*})T'_{z}(z^{*}) \frac{z^{*}\zeta_{z}^{c}(z^{*})}{1-T'_{z}(z^{*})} h_{z}(z^{*}). \quad (27)$$

In an optimal LED system, the savings tax schedule $\tau_s(z)$ satisfies, at each level of earnings z^* ,

$$s_{inc}'(z^{*})\tau_{s}(z^{*})s(z^{*})\frac{\zeta_{z}^{c}(z^{*})z^{*}}{1-T_{z}'(z^{*})}h_{z}(z^{*}) + \int_{z^{*}}^{\bar{z}} \left\{\tau_{s}(z) \left[\frac{s(z)\zeta_{s|z}^{c}(z)}{1+\tau_{s}(z)} + (s_{inc}'(z))^{2}\frac{z\zeta_{z}^{c}(z)}{1-T_{z}'(z)}\right]\right\} dH_{z}(z)$$

$$= \int_{z^{*}}^{\bar{z}} \left\{ \left[1-\hat{g}(z)\right]s(z) - s_{inc}'(z)T_{z}'(z)\frac{z\zeta_{z}^{c}(z)}{1-T_{z}'(z)}\right\} dH_{z}(z) - T_{z}'(z^{*})s(z^{*})\frac{\zeta_{z}^{c}(z^{*})z^{*}}{1-T_{z}'(z^{*})}h_{z}(z^{*}).$$

$$(28)$$

Proof. See Appendix B.3.

These optimal savings tax formulas are all different, reflecting differences between the

savings tax instruments that we consider. Yet, they share common elements. Importantly, the preference heterogeneity term $s'_{pref}(z)$ no longer appears, yet $s'_{inc}(z)$ remains a key sufficient statistic for optimal savings tax schedules. The intuition is that outside of the full optimum, it may still be desirable to tax savings even though there might be no preference heterogeneity. As a result, optimality may clash with Pareto-efficiency when the earnings tax is suboptimal.

For a tax system with a SL tax system, Equation (26) characterizes the optimal linear savings tax rate τ_s . To understand this formula, consider a change in the savings tax rate by $d\tau_s$. This triggers at all earnings levels a direct savings response proportional to $\zeta_{s|z}^c(z) d\tau_s$ and an indirect savings response mediated by the change in earnings proportional to $s'_{inc}(z) \zeta_z^c(z) (s'_{inc}(z) d\tau_s)$ by Lemma 1. Moreover, it triggers mechanical effects proportional to the lump-sum change in tax liability $s(z) d\tau_s$ and earnings responses proportional to $\zeta_z^c(z) (s'_{inc}(z) d\tau_s)$ by the same result. Combining these different elements yields the result.

Similarly, for a tax system with a separable nonlinear savings tax (SN), equation (27) pins down the optimal nonlinear savings tax schedule $T'_s(s)$. The exact same forces as with a SL tax system are at play, except that we consider here a local reform: a $d\tau_s$ change in the marginal savings tax rate in a bandwidth of savings $[s(z^*), s(z^*) + ds]$. The only difference relates to mechanical effects affecting all agents above and proportional to the lump-sum change in tax liability $d\tau_s ds = d\tau_s s'(z^*) dz$. A larger $s'_{inc}(z)$ means that savings tax reforms impose higher distortions on earnings and thus generally calls for lower savings tax rate.

This intuition is particularly salient for a tax system with a linear earnings-dependent (LED). We here consider a $\delta \tau_s$ increase in the linear savings tax rate $\tau_s(z)$ that is phased-in over a bandwidth of earnings $[z^*, z^* + dz]$. As a result, the marginal savings tax rate is unchanged at z^* , while the marginal earnings tax rate increases by $s(z^*) d\tau_s$.⁷⁸ The savings response at earnings z^* thus entirely operates through the earnings response and is therefore proportional to $s'_{inc}(z)$. Other terms at play involve again the same forces as with tax system SL, and look actually very similar.

Tax systems SN and LED once again include tax system SL as a special case. However, they are no longer equivalent for a given earnings tax schedule $T_z(z)$. For a given marginal savings tax rate $T'_s(s(z^*)) = \tau_s(z^*)$, a LED tax system induces more distortions than a SN tax system at z^* if and only if $\tau'_s(z) \ge 0$. Yet, a LED tax system allows to tax all inframarginal units of savings at rate $\tau_s(z^*)$, while in a SN tax system the tax on infra-marginal units of savings depends on distortions imposed at lower earnings levels. Whether one is more advantageous than the other is thus a quantitative question.

⁷⁸Recall that for a LED tax system we have $\mathcal{T}'_{z}(s, z) = T'_{z}(z) + \tau'_{s}(z)s$.

5 Conclusion

This paper presents sufficient statistics formulas characterizing the optimal smooth tax system on earnings and savings (or other dimensions of consumption) in the presence of preference heterogeneity that is correlated with earnings ability. The formulas depend on a key sufficient statistic for preference heterogeneity, $s'_{pref}(z)$, which can be estimated from empirical data on cross sectional earnings and savings, and on observed behavioral responses to tax reforms. We show that such a smooth system can implement the optimal mechanism under weak conditions. Under only slightly stronger conditions, the optimal mechanism can also be implemented by two familiar types of "simple" separable tax systems which combine a nonlinear tax on earnings with a nonlinear tax on savings, or with a linear earningsdependent savings tax. We derive intuitive sufficient statistics formulas for these separable tax systems, as well as for linear separable savings taxes, and we present sufficient conditions under which these necessary conditions are also sufficient to characterize the optimal simple tax systems. Together, these results provide a practical and general method for quantifying optimal tax systems for saving, inheritances, and other commodities in the presence of correlated preference heterogeneity.

Appendix

A Proofs for Section 3

A.1 Proof of Proposition 1

Notation

Suppose $\{c^*(\theta), s^*(\theta), z^*(\theta)\}_{\theta}$ is an incentive-compatible allocation that is smooth in θ , in which $s^*(\theta)$ and $z^*(\theta)$ are strictly increasing in θ . Define

$$\mathcal{S}(c,s,z;\theta) := \frac{U'_s(c,s,z;\theta)}{U'_c(c,s,z;\theta)}$$
(29)

and

$$\mathcal{Z}(c,s,z;\theta) := \frac{U'_z(c,s,z;\theta)}{U'_c(c,s,z;\theta)}$$
(30)

to denote the marginal rates of subsitutition of savings and earnings (respectively) with respect to consumption.

Lemma for type-specific feasibility

Lemma 2. A smooth tax system \mathcal{T} satisfies type-specific feasibility if it satisfies the following conditions

1. $\mathcal{T}(s^*(\theta_{\min}), z^*(\theta_{\min})) = z^*(\theta_{\min}) - c^*(\theta_{\min}) - s^*(\theta_{\min})$ 2. $\mathcal{T}'_z(s^*(\theta), z^*(\theta)) = \mathcal{Z}(c^*(\theta), s^*(\theta), z^*(\theta); \theta) + 1$

3.
$$\mathcal{T}'_s(s^*(\theta), z^*(\theta)) = \mathcal{S}(c^*(\theta), s^*(\theta), z^*(\theta); \theta) - 1$$

Proof. Define $T^*_{\theta}(\theta) = z^*(\theta) - s^*(\theta) - c^*(\theta)$, and note that the lemma amounts to showing that $T^*_{\theta}(\theta) = \mathcal{T}(s^*(\theta), z^*(\theta))$ for all θ . To that end, note that the first-order condition for truthful reporting of θ under the optimal mechanism implies

$$U'_{c} \cdot (z'(\theta) - s'(\theta) - T^{*'}(\theta)) + U'_{s} \cdot s'(\theta) + U'_{z} \cdot z'(\theta) = 0,$$

$$T^{*\prime}(\theta) = \left(\frac{U'_s}{U'_c} - 1\right)s'(\theta) + \left(\frac{U'_z}{U'_c} + 1\right)z'(\theta)$$
$$= \mathcal{T}'_s(s^*(\theta))s^{*\prime}(\theta) + \mathcal{T}'_z(z^*(\theta))z^{*\prime}(\theta).$$

It thus follows that

$$\mathcal{T}(s^*(\theta), z^*(\theta)) - \mathcal{T}(s^*(\theta_{\min}), z^*(\theta_{\min})) = \int_{\vartheta=\theta_{\min}}^{\vartheta=\theta} \left(\mathcal{T}'_s(s^*(\vartheta))s^{*\prime}(\vartheta) + \mathcal{T}'_z(z^*(\vartheta))z^{*\prime}(\vartheta)\right) d\vartheta$$
$$= T^*_{\theta}(\theta) - T^*_{\theta}(\theta_{\min})$$

By construction, the smooth tax system \mathcal{T} therefore satisfies feasibility.

Lemma on second-order conditions

Lemma 3. Consider a smooth tax system \mathcal{T} satisfying the conditions in Lemma 2. Then

$$V_{ss}''(s(\theta), z(\theta); \theta) = \frac{U_z'}{s'(z)} \mathcal{S}_c' - \frac{U_c'}{s'(z)} \mathcal{S}_z' - \frac{U_c'}{s'(\theta)} \mathcal{S}_\theta' + \frac{U_c'}{s'(z)} \mathcal{T}_{sz}''$$
(31)

$$V_{zz}''(s(\theta), z(\theta); \theta) = U_s's'(z)\mathcal{Z}_c' - s'(z)U_c'\mathcal{Z}_s' - \frac{U_c'}{z^{*'}(\theta)}\mathcal{Z}_{\theta}' + U_c's'(z)\mathcal{T}_{sz}''$$
(32)

$$(V_{sz}'')^{2} - V_{ss}''V_{zz}'' = \left(\frac{U_{z}'}{s^{*'}(\theta)}\mathcal{S}_{c}' - \frac{U_{c}'}{s^{*'}(\theta)}\mathcal{S}_{z}'\right)U_{c}'\mathcal{Z}_{\theta}' + \frac{1}{s^{*'}(\theta)}V_{zz}''U_{c}'\mathcal{S}_{\theta}' + \mathcal{Z}_{\theta}'\frac{(U_{c}')^{2}}{s'(\theta)}\mathcal{T}_{sz}'', \quad (33)$$

Proof. Define the indirect utility function:

$$V(s, z; \theta) := U(z - s - \mathcal{T}(s, z), s, z; \theta).$$
(34)

Then the first-order conditions are

$$V'_{s}(s,z;\theta) = -(1 + \mathcal{T}'_{s}(s,z))U'_{c}(z - s - \mathcal{T}(s,z), s, z;\theta) + U'_{s}(z - s - \mathcal{T}(s,z), s, z;\theta)$$
$$V'_{z}(s,z;\theta) = (1 - \mathcal{T}'_{z}(s,z))U'_{c}(z - s - \mathcal{T}(s,z), s, z;\theta) + U'_{z}(z - s - \mathcal{T}(s,z), s, z;\theta)$$

The second derivatives are:

$$V_{ss}''(s,z;\theta) = -\mathcal{T}_{ss}''U_c' - (1+\mathcal{T}_s')\left(-(1+\mathcal{T}_s')U_{cc}'' + U_{cs}''\right) - (1+\mathcal{T}_s')U_{cs}'' + U_{ss}''$$
(35)

$$V_{zz}''(s,z;\theta) = -\mathcal{T}_{zz}''U_c' + (1-\mathcal{T}_z')\left((1-\mathcal{T}_z')U_{cc}'' + U_{cz}''\right) + (1-\mathcal{T}_z')U_{cz}'' + U_{zz}''$$
(36)

We compute \mathcal{T}''_{ss} by differentiating both sides of $\mathcal{T}'_{s}(s^*(\theta), z^*(\theta)) = \mathcal{S}(c^*(\theta), s^*(\theta), z^*(\theta); \theta) - 1$

with respect to θ :

$$\mathcal{T}_{ss}''s'(\theta) + \mathcal{T}_{sz}''z'(\theta) = \frac{d}{d\theta}\mathcal{S}(c(\theta), s(\theta), z(\theta); \theta)$$
(37)

$$= \mathcal{S}'_{c} \cdot c'(\theta) + \mathcal{S}'_{s} \cdot s'(\theta) + \mathcal{S}'_{z} \cdot z'(\theta) + \mathcal{S}'_{\theta}.$$
(38)

This can be rewritten as

$$\mathcal{T}_{ss}''(s) = \mathcal{S}_{c}' \cdot (1 - \mathcal{T}_{z}')/s'(z) - \mathcal{S}_{c}' \cdot (1 + \mathcal{T}_{s}') + \mathcal{S}_{s}' + \mathcal{S}_{z}'/s'(z) + \mathcal{S}_{\theta}' \cdot \frac{1}{s'(\theta)} - \mathcal{T}_{sz}''/s'(z).$$
(39)

Second, we can compute the following derivatives of \mathcal{S} by differentiating Equation (29):

$$S'_{c}(c^{*}(\theta), s^{*}(\theta), z^{*}(\theta); \theta) = \frac{U'_{c}U''_{sc} - U'_{s}U''_{cc}}{(U'_{c})^{2}}$$
$$= \frac{1}{U'_{c}} \left(-\frac{U'_{s}}{U'_{c}}U''_{cc} + U''_{sc} \right)$$
$$= \frac{1}{U'_{c}} \left(-(1 + T'_{s})U''_{cc} + U''_{sc} \right)$$
(40)

and similarly

$$S'_{s}(c^{*}(\theta), s^{*}(\theta), z^{*}(\theta); \theta) = \frac{U'_{c}U''_{ss} - U'_{s}U''_{cs}}{(U'_{c})^{2}}$$
$$= \frac{1}{U'_{c}} \left(-\frac{U'_{s}}{U'_{c}}U''_{cs} + U''_{ss} \right)$$
$$= \frac{1}{U'_{c}} \left(-(1 + T'_{s})U''_{cs} + U''_{ss} \right).$$
(41)

Substituting this and Equations (40) and (41) into (35), we have

$$V_{ss}''(s(\theta), z(\theta); \theta) = -U_c' \cdot \left(S_c' \cdot \frac{1 - \mathcal{T}_z'}{s'(z)} - S_c' \cdot (1 + \mathcal{T}_s') + S_s' + \frac{S_z'}{s'(z)} + \frac{S_{\theta}'}{s'(\theta)} - \frac{\mathcal{T}_{sz}''}{s'(z)} \right) - U_s' S_c' + U_c' S_s'$$

$$= -U_c' \cdot \left(\frac{1 - \mathcal{T}_z'}{s'(z)} S_c' + \frac{1}{s'(z)} S_z' + \frac{1}{s'(\theta)} S_{\theta}' - \frac{\mathcal{T}_{sz}''}{s'(z)} \right)$$

$$= \frac{U_z'}{s'(z)} S_c' - \frac{U_c'}{s'(z)} S_z' - \frac{U_c'}{s'(\theta)} S_{\theta}' + \frac{U_c'}{s'(z)} \mathcal{T}_{sz}''.$$
(42)

Similarly, we can derive

$$\mathcal{T}_{zz}''(z) = \mathcal{Z}_{c}' \cdot (1 - \mathcal{T}_{z}') - \mathcal{Z}_{c}' \cdot (1 + \mathcal{T}_{s}')s'(z) + \mathcal{Z}_{z}' + \mathcal{Z}_{s}' \cdot s'(z) + \mathcal{Z}_{\theta}' \cdot \frac{1}{z^{*'}(\theta)} - \mathcal{T}_{sz}''s'(z).$$
(43)

and thus

$$V_{zz}''(s(\theta), z(\theta); \theta) = U_s's'(z)\mathcal{Z}_c' - s'(z)U_c'\mathcal{Z}_s' - \frac{U_c'}{z^{*'}(\theta)}\mathcal{Z}_{\theta}' + U_c's'(z)\mathcal{T}_{sz}'',$$
(44)

Finally, we must check the second-order condition requirement $V_{ss}''V_{zz}'' > (V_{sz}'')^2$. This can found by noting that the first-order condition $V_s'(s^*(\theta), z^*(\theta); \theta)$ holds at every θ , by construction, and thus its total derivative with respect to θ is zero:

$$0 = \frac{d}{d\theta} V'_s(s^*(\theta), z^*(\theta); \theta)$$

$$= V''_{ss} s^{*'}(\theta) + V''_{sz} z^{*'}(\theta) + V''_{s\theta},$$
(45)

and by rearranging, this yields

$$-V_{sz}'' = V_{ss}''s'(z) + V_{s\theta}''/z^{*'}(\theta).$$
(46)

Similarly, by totally differentiating $V'_z(s^*(\theta), z^*(\theta); \theta) = 0$ and rearranging we find

$$-V_{sz}'' = V_{zz}''/s'(z) + V_{z\theta}''/s^{*'}(\theta).$$
(47)

We can thus write the square $(V''_{sz})^2$ as the product of the right-hand sides of Equations (46) and (47):

$$(V_{sz}'')^{2} = (V_{ss}''s'(z) + V_{s\theta}''/z^{*'}(\theta)) (V_{zz}''/s'(z) + V_{z\theta}''/s^{*'}(\theta))$$

= $V_{ss}''V_{zz}'' + \frac{1}{z^{*'}(\theta)}V_{ss}''V_{z\theta}'' + \frac{1}{s^{*'}(\theta)}V_{zz}''V_{s\theta}'' + \frac{1}{s^{*'}(\theta)z^{*'}(\theta)}V_{s\theta}''V_{z\theta}''$ (48)

We can derive

$$V_{s\theta}'' = \frac{\partial}{\partial \theta} V_s'(s, z; \theta)$$

= $-(1 + T_s') U_{c\theta}'' + U_{s\theta}''$
= $U_c' \cdot S_{\theta}',$ (49)

where the last step uses the expression for S'_{θ} derived from the quotient rule as in Equation (40), and similarly

$$V_{z\theta}^{\prime\prime} = U_c^{\prime} \cdot \mathcal{Z}_{\theta}^{\prime}. \tag{50}$$

Substituting these into Equation (48) and rearranging, we have

$$(V_{sz}'')^2 - V_{ss}''V_{zz}'' = \frac{1}{z^{*'}(\theta)}V_{ss}''U_c'\mathcal{Z}_{\theta}' + \frac{1}{s^{*'}(\theta)}V_{zz}''U_c'\mathcal{S}_{\theta}' + \frac{1}{s^{*'}(\theta)z^{*'}(\theta)}(U_c')^2\mathcal{S}_{\theta}'\mathcal{Z}_{\theta}'.$$
 (51)

By expanding $V_{ss}^{\prime\prime}$ from Equation (42) , we have

$$(V_{sz}'')^2 - V_{ss}''V_{zz}'' = \left(\frac{U_z'}{s^{*'}(\theta)}\mathcal{S}_c' - \frac{U_c'}{s^{*'}(\theta)}\mathcal{S}_z' + \frac{U_c'}{s^{*'}(\theta)}\mathcal{T}_{sz}''\right)U_c'\mathcal{Z}_{\theta}' + \frac{1}{s^{*'}(\theta)}V_{zz}''U_c'\mathcal{S}_{\theta}',$$

which gives the expression in the Lemma above.

Proof of the main result

Let $s^*(z)$ denote the savings level associated with earnings level z at the optimal incentivecompatible allocation. Let \bar{z} denote the maximal earnings level in the allocation, and assume for simplicity that the minimal earnings level is z = 0 (the proof for $\underline{z} > 0$ follows essentially the same lines). Let \underline{s} denote the minimal savings level in the optimal allocation. Let $s_{\mathcal{T}}(\theta)$ and $z_{\mathcal{T}}(\theta)$ denote the level of savings and earnings, respectively, that a type θ chooses given a smooth tax system \mathcal{T} .

Step 1: Defining the tax system. Consider now a smooth system $\mathcal{T}(s, z; k)$ defined as follows:

$$1. \ \mathcal{T}(s,z;k) = \begin{cases} T_s(s) + T_z(z) + k(s - s^*(z))^2 & \text{if } z \leq \bar{z} \\ \mathcal{T}(s,\bar{z};k) + k(z - \bar{z})^2 + T'_z(\bar{z})(z - \bar{z}) & \text{if } z > \bar{z}, s \leq s^*(\bar{z}) \\ \mathcal{T}(s^*(\bar{z}), z;k) + k(s - s^*(\bar{z}))^2 + T'_s(s^*(\bar{z}))(s - s^*(\bar{z})) & \text{if } z > \bar{z}, s > s^*(\bar{z}) \end{cases} \\ 2. \ T_s(s) = 0 \text{ for } s \leq \underline{s} \text{ and } T_z(z(\theta_{min})) = z^*(\theta_{min}) - c^*(\theta_{min}) - s^*(\theta_{min}) \\ 3. \ T'_z(z^*(\theta)) = \mathcal{Z}(c^*(\theta), s^*(\theta), z^*(\theta); \theta) + 1 \\ 4. \ T'_s(s^*(\theta)) = \mathcal{S}(c^*(\theta), s^*(\theta), z^*(\theta); \theta) - 1 \end{cases}$$

Step 2: Establishing first-order conditions and type-feasibility. Note that this implies that

$$\mathcal{T}'_{z}(s^{*}(\theta), z^{*}(\theta); k) = \mathcal{Z}(c^{*}(\theta), s^{*}(\theta), z^{*}(\theta); \theta) + 1$$
$$\mathcal{T}'_{s}(s^{*}(\theta), z^{*}(\theta); k) = \mathcal{S}(c^{*}(\theta), s^{*}(\theta), z^{*}(\theta); \theta) - 1$$

Therefore, \mathcal{T} satisfies type-specific feasibility by Lemma 2.

Define the indirect utility function:

$$V(s, z; \theta) := U(z - s - \mathcal{T}(s, z), s, z; \theta).$$
(52)

Then FOCs are

$$V'_{s}(s,z;\theta) = -(1 + \mathcal{T}'_{s}(s,z))U'_{c}(z - s - \mathcal{T}(s,z), s, z;\theta) + U'_{s}(z - s - \mathcal{T}(s,z), s, z;\theta)$$
$$V'_{z}(s,z;\theta) = (1 - \mathcal{T}'_{z}(s,z))U'_{c}(z - s - \mathcal{T}(s,z), s, z;\theta) + U'_{z}(z - s - \mathcal{T}(s,z), s, z;\theta)$$

and are satisfied at $(s^*(\theta), z^*(\theta))$ for each type θ by construction.

Step 3: Consequences of non-implementability for arbitrarily high values of k. Next, observe that since S is continuous, and the type space is compact, T'_s must be bounded. Moreover, U'_c and U'_s are bounded by assumption. Thus, V'_s is bounded. Thus, for each ε there exists a k large enough such that for all θ :

- $|s_{\mathcal{T}}(\theta) s^*(z)| < \varepsilon \text{ for } z_{\mathcal{T}}(\theta) \le \overline{z}$
- $|s_{\mathcal{T}}(\theta) s^*(\bar{z})| < \varepsilon \text{ for } z > \bar{z}$

•
$$z_{\mathcal{T}}(\theta) \leq \bar{z} + \varepsilon$$

We claim that there exists a k large enough such that $z_{\mathcal{T}}(\theta) = z^*(\theta)$ and $s_{\mathcal{T}}(\theta) = s^*(\theta)$. If not, then there exists an infinite sequence of types θ_k , choosing savings and earnings $s_{\mathcal{T}}^k(\theta)$ and $z_{\mathcal{T}}^k(\theta)$, and enjoying gains $\pi^k > 0$ from "deviating" to this allocation instead of staying at $(s^*(\theta), z^*(\theta))$. Since the type space Θ is compact, and since $s_{\mathcal{T}}^k, z_{\mathcal{T}}^k$ are bounded from above (by the three bullet points above), the Bolzano–Weierstrass theorem implies that there exists a convergent subsequence $(\theta^j, s^j(\theta), z^j(\theta)) \to (\hat{\theta}, \hat{s}, \hat{z})$, with $\hat{\theta} \in \Theta$. The third bullet point above implies that $\hat{z} \leq \bar{z}$, and the first two bullet points above imply that $\hat{s} = s^*(\hat{z})$. Now since $\pi^j > 0$ for all j, the payoffs $\hat{\pi}$ from choosing (\hat{s}, \hat{z}) over $(s^*(\hat{\theta}), z^*(\hat{\theta}))$ must be non-negative for type $\hat{\theta}$ by the continuity of U. But since $\hat{\pi} \geq 0$, we must have $\hat{s} = s^*(\hat{\theta})$ and $\hat{z} = z^*(\hat{\theta})$; otherwise this implies that the optimal allocation is not (strictly) incentive compatible.

In the last two steps of the proof, we first invoke Lemma 3 to show that $(s^*(\hat{\theta}), z^*(\hat{\theta}))$ is a local optimum for type $\hat{\theta}$ for sufficiently large k, and second we show that this is inconsistent with the the existence of the convergent subsequence defined above.

Step 4: SOCs satisfied for arbitrarily high k at a type's assigned allocation. Note that since $\mathcal{T}_{sz}'' = -2ks^{*'}(z)$, Lemma 3 implies that there exists a k^{\dagger} such that the SOCs are satisfied for type $\hat{\theta}$ for all $k \geq k^{\dagger}$. In fact, because for a fixed type $\hat{\theta}$, V_{ss}'', V_{zz}'' , and $(V_{sz}'')^2 - V_{ss}''V_{zz}''$ are strictly decreasing in k, there is an open set $N \in \mathbb{R}^2$ containing $(s^*(\hat{\theta}), z^*(\hat{\theta}))$ such that for all $k \geq k^{\dagger}$, $V(s, z; \hat{\theta})$ is strictly concave over $(s, z) \in N$ and attains it's highest value over N at $(s^*(\hat{\theta}), z^*(\hat{\theta}))$.

Step 5: Non-implementability is inconsistent with SOCs being satisfied for a type's assigned allocation. Consider now the convergent subsequence $(\theta^j, s^j(\theta), z^j(\theta)) \rightarrow (\hat{\theta}, s^*(\hat{\theta}), z^*(\hat{\theta}))$. By continuity, there exists a θ^j close enough to $\hat{\theta}$ such that the SOCs are satisfied for θ^j around the point $(s^*(\theta^j), z^*(\theta^j))$ for all $k \ge k^{\dagger}$. In fact, for any open set $N' \subsetneq N$ containing $(s^*(\hat{\theta}), z^*(\hat{\theta}))$, continuity implies that there exists a j^* such that for any $j \ge j^*$, $V(s, z; \theta^j)$ is strictly concave over $(s, z) \in N'$.

Now for j sufficiently large, $(s^*(\hat{\theta}), z^*(\hat{\theta})) \in N'$ and $(s^j(\theta), z^j(\theta)) \in N'$. This leads to a contradiction, because the FOCs derived in step 2, imply that $(s^*(\theta^j), z^*(\theta^j))$ is the optimum over N' for type θ^j .

A.2 Proof of Proposition 2

By definition, for an earnings level z, we have $s(z) \equiv s(z(\theta); \theta)$ where θ is the corresponding agent's type.

If agents' preferences are weakly separable, savings s only depend on earnings z through disposable income $y = z - s - \mathcal{T}(s, z)$, we thus have⁷⁹

$$s_{inc}'(z) \equiv \frac{\partial s(z;\theta)}{\partial z} = \frac{\partial s(z;\theta)}{\partial y} \frac{\partial y}{\partial z} = \frac{\eta_{s|z}(\theta)}{1 + \mathcal{T}_{s}'} (1 + \mathcal{T}_{z}')$$
(53)

If agents wage rates w and hours h are observable, we can use the fact that earnings z are given by $z = w \cdot h$ to infer s'_{inc} from changes in wages through

$$\frac{\partial s\left(\theta\right)}{\partial w\left(\theta\right)} = \frac{\partial s\left(w\left(\theta\right) \cdot h\left(\theta\right);\theta\right)}{\partial w\left(\theta\right)} = \frac{\partial s\left(z\left(\theta\right);\theta\right)}{\partial z\left(\theta\right)} \left(1 + \frac{\partial h\left(\theta\right)}{\partial w\left(\theta\right)}\right)$$
(54)

$$\iff \frac{\partial s\left(z\left(\theta\right);\theta\right)}{\partial z\left(\theta\right)} = \frac{\frac{\partial s\left(\theta\right)}{\partial w\left(\theta\right)}}{1 + \frac{\partial h\left(\theta\right)}{\partial w\left(\theta\right)}} = s\left(\theta\right) \frac{\frac{w\left(\theta\right)}{s\left(\theta\right)} \frac{\partial s\left(\theta\right)}{\partial w\left(\theta\right)}}{w\left(\theta\right) + h\left(\theta\right) \frac{w\left(\theta\right)}{h\left(\theta\right)} \frac{\partial h\left(\theta\right)}{\partial w\left(\theta\right)}} \tag{55}$$

$$\iff s_{inc}'(z) = s(z) \frac{\xi_w^s(z)}{w(z) + h(z) \xi_w^h(z)}$$
(56)

where $\xi_w^s(z) \equiv \frac{w(z)}{s(z)} \frac{\partial s(z)}{\partial w(z)}$ is individuals' elasticity of savings with respect to their wage rate,

⁷⁹Savings s may also depend on earnings z through the marginal savings tax rate $\mathcal{T}'_s(s, z)$, in which case we need to assume that the latter is locally constant with respect to z i.e. any nonlinearity is second-order.

and $\xi_w^h(z) \equiv \frac{w(z)}{h(z)} \frac{\partial h(z)}{\partial w(z)}$ is individuals' elasticity of hours with respect to their wage rate.

A.3 Proof of Lemma 1

Let

$$V\left(\mathcal{T}(.,z),z;\theta\right) = \max_{s} U\left(z-s-\mathcal{T}(s,z),s,z;\theta\right)$$

be agent θ indirect utility function at earnings z and (pre savings tax) disposable income y. Consider a savings tax reform $d\mathcal{T}_s(s)$ that consists in a small increase $d\tau_s$ in the marginal savings tax rate in a bandwidth of savings $[s^*, s^* + ds]$ associated with types $[\theta^*, \theta^* + d\theta]$, with $d\tau_s$ much smaller than ds:

$$d\mathcal{T}_{s}(s) = \begin{cases} 0 & if \ s \le s^{*} \\ d\tau_{s}(s-s^{*}) & if \ s \in [s^{*}, s^{*}+ds] \\ d\tau_{s}ds & if \ s \ge s^{*}+ds \end{cases}$$

The idea is to construct for each type θ a perturbation of the earnings tax $d\mathcal{T}_z^{\theta}(z)$ that induces the same earnings response as the initial perturbation $d\mathcal{T}_s(s)$. Suppose we define this perturbation for each type θ such that at all earnings z,

$$V(\mathcal{T}(.,z) + d\mathcal{T}_s(.), z; \theta) = V(\mathcal{T}(.,z) + d\mathcal{T}_z^{\theta}(.), z; \theta)$$

By construction, $dT_z^{\theta}(z)$ induces the same earnings response dz as the initial perturbation dT_s .

The Envelope Theorem implies that for $s(z; \theta) \in [s^*, s^* + ds]$

$$U_c' \cdot d\tau_s \left(s\left(z; \theta\right) - s^* \right) = U_c' \cdot d\mathcal{T}_z^{\theta}(z)$$

Thus, taking the limit with respect to $d\tau_s$ and then with respect to ds implies that a small increase $d\tau_s$ in the marginal savings tax rate faced by an agent $\theta \in [\theta^*, \theta^* + d\theta]$ induces the same earnings change dz as a small increase $s'_{inc}(z) d\tau_s$ in the marginal earnings tax rate.

A.4 Proof of Proposition 3

A.4.1 Earnings tax rates

Reform. For all types of smooth and separable tax systems $\mathcal{T}(s, z)$, we consider small reforms at earnings level $z^* = z(\theta^*)$ that consists in a small increase $d\tau_z$ of the marginal tax rate on earnings in a small bandwidth dz. Formally,

$$dT_z(z) = \begin{cases} 0 & \text{if } z \le z^* \\ d\tau_z(z - z^*) & \text{if } z \in [z^*, z^* + dz] \\ d\tau_z dz & \text{if } z \ge z^* + dz \end{cases}$$

We characterize the impact of this reform on the government objective function

$$\mathcal{L} = \int_{z} [\alpha(z) U(c(z), s(z), z; \theta(z)) + \lambda(\mathcal{T}(s, z) - E)] dH_{z}(z)$$

with λ the marginal value of public funds. Normalizing all effects by $1/\lambda$, the reform induces

• mechanical effects:

$$\int_{z^*}^{\bar{z}} \left(1 - \frac{\alpha(z)}{\lambda} U'_c(c(z), s(z), z; \theta(z)) \right) d\tau_z dz \, dH_z(z)$$

• behavioral effects from earnings changes:⁸⁰

$$-\mathcal{T}_{z}'(s(z^{*}), z^{*}) \frac{z^{*}}{1 - \mathcal{T}_{z}'(s(z^{*}), z^{*})} \zeta_{z}^{c}(z^{*}) d\tau_{z} dz h_{z}(z^{*}) -\int_{z^{*}}^{\bar{z}} \mathcal{T}_{z}'(s(z), z) \frac{\eta_{z}(z)}{1 - \mathcal{T}_{z}'(s(z), z)} d\tau_{z} dz dH_{z}(z)$$

• behavioral effects from savings changes:

$$-\mathcal{T}'_{s}(s(z^{*}), z^{*}) s'_{inc}(z^{*}) \left[\frac{z^{*}}{1 - \mathcal{T}'_{z}(s(z^{*}), z^{*})} \zeta^{c}_{z}(z^{*}) d\tau_{z} \right] dz h_{z}(z^{*}) -\int_{z^{*}}^{\bar{z}} \mathcal{T}'_{s}(s(z), z) \left[\frac{\eta_{s|z}(z)}{1 + \mathcal{T}'_{s}(s(z), z)} + s'_{inc}(z) \frac{\eta_{z}(z)}{1 - \mathcal{T}'_{z}(s(z), z)} \right] d\tau_{z} dz \, dH_{z}(z)$$

Summing over these different effects yields the total impact of the reform

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{dz} = \int_{z^*}^{\bar{z}} (1 - \hat{g}(z)) \ d\tau_z dH_z(z)
- \left(\mathcal{T}'_z(s(z^*), z^*) + s'_{inc}(z^*)\mathcal{T}'_s(s(z^*), z^*)\right) \frac{z^*}{1 - \mathcal{T}'_z(s(z^*), z^*)} \zeta_z^c(z^*) d\tau_z h_z(z^*) \quad (57)$$

⁸⁰Note that by definition of the elasticity concepts, savings and earnings changes are here given by

$$\begin{cases} dz = -\frac{z}{1-\mathcal{T}'_z}\zeta_z^c(z)\delta T'_z - \frac{\eta_z(z)}{1-\mathcal{T}'_z}\delta T_z\\ ds = -\frac{\eta_{s|z}(z)}{1+\mathcal{T}'_s}\delta T_z + s'_{inc}(z)dz \end{cases}$$

where $\hat{g}(z)$ represent social marginal welfare weights augmented with income effects, that is

$$\begin{split} \hat{g}(z) &= \frac{\alpha(z)}{\lambda} U_{c}'(c(z), s(z), z; \theta(z)) + \mathcal{T}_{z}'(s(z), z) \frac{\eta_{z}(z)}{1 - \mathcal{T}_{z}'(s(z), z)} \\ &+ \mathcal{T}_{s}'(s(z), z) \left(\frac{\eta_{s|z}(z)}{1 + \mathcal{T}_{s}'(s(z), z)} + s_{inc}'(z) \frac{\eta_{z}(z)}{1 - \mathcal{T}_{z}'(s(z), z)} \right) \end{split}$$

Optimality. A direct implication of this result is a sufficient statistics characterization of the optimal earnings tax schedule, given an existing and potentially suboptimal savings tax. Indeed, at the optimum the reform should have a zero impact on the government objective meaning that the optimal earnings tax schedule verifies at each earnings z^*

$$\frac{\mathcal{T}_{z}'\left(s\left(z^{*}\right),z^{*}\right)}{1-\mathcal{T}_{z}'\left(s\left(z^{*}\right),z^{*}\right)} = \frac{1}{\zeta_{z}^{c}(z^{*})}\frac{1}{z^{*}h_{z}(z^{*})}\int_{z^{*}}^{\bar{z}}(1-\hat{g}(z))\,dH_{z}(z) - s_{inc}'(z^{*})\frac{\mathcal{T}_{s}'\left(s(z^{*}),z^{*}\right)}{1-\mathcal{T}_{z}'\left(s\left(z^{*}\right),z^{*}\right)}$$

A.4.2 Savings tax rates

Reform. We consider a small reform of the savings tax at savings level $s^* = s(\theta^*)$ that consists in a small increase $d\tau_s$ of the marginal tax rate on savings in a small bandwidth δs . Formally,

$$\delta T_s(s) = \begin{cases} 0 & if \ s \le s^* \\ d\tau_s(s-s^*) & if \ s \in [s^*, s^* + ds] \\ d\tau_s ds & if \ s \ge s^* + ds \end{cases}$$

Assuming agents' preferences are smooth across the type distribution, there exists an increasing mapping between earnings z and savings s. Denote z^* the earnings level such that $s^* = s(z^*)$. We characterize the impact of this reform on the government objective function. Normalizing all effects by $1/\lambda$, the reform induces

• mechanical effects:

$$\int_{z \ge z^*} \left(1 - \frac{\alpha(z)}{\lambda} U'_c(c(z), s(z), z; \theta(z)) \right) \, d\tau_s ds \, dH_z(z)$$

• behavioral effects from earnings changes:⁸¹

$$-\mathcal{T}_{z}'(s^{*},z^{*})\left[\frac{z^{*}}{1-T_{z}'(z^{*})}\zeta_{z}^{c}(z^{*})\,s_{inc}'(z)d\tau_{s}\right]ds\,\frac{h_{z}(z^{*})}{s'(z^{*})}-\int_{z^{*}}^{\bar{z}}\mathcal{T}_{z}'(z)\,\frac{\eta_{z}(z)}{1-\mathcal{T}_{z}'(z)}d\tau_{s}ds\,dH_{z}(z)d\tau_{s}dt\,dH_{z}(z)$$

• behavioral effects from savings changes:

$$-\mathcal{T}_{s}'(s^{*},z^{*})\left[\frac{s(z^{*})}{1+T_{s}'(s^{*},z^{*})}\zeta_{s|z}^{c}(z^{*})d\tau_{s}+s_{inc}'(z^{*})\frac{z^{*}}{1-T_{z}'(s^{*},z^{*})}\zeta_{z}^{c}(z^{*})s_{inc}'(z^{*})d\tau_{s}\right]ds\frac{h_{z}(z^{*})}{s'(z^{*})}\\-\int_{z^{*}}^{\bar{z}}\mathcal{T}_{s}'(s(z))\left[\frac{\eta_{s|z}(z)}{1+T_{s}'(s^{*},z^{*})}+s_{inc}'(z)\frac{\eta_{z}(z)}{1-\mathcal{T}_{z}'(s^{*},z^{*})}\right]d\tau_{s}ds\,dH_{z}(z)$$

Summing over these different effects yields the total impact of the reform

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{ds} = s'(z^*)\int_{z^*}^{\bar{z}} (1-\hat{g}(z)) d\tau_s \, dH_z(z) - \left\{\mathcal{T}'_s(s(z^*))\frac{s(z^*)}{1+\mathcal{T}'_s(s^*,z^*)}\zeta^c_{s|z}(z^*) + \left[\mathcal{T}'_z(s^*,z^*) + s'_{inc}(z^*)\mathcal{T}'_s(s^*,z^*)\right]\frac{z^*}{1-\mathcal{T}'_z(s^*,z^*)}\zeta^c_z(z^*)s'_{inc}(z^*)\right\}d\tau_s \, h_z(z^*) \quad (59)$$

Optimality. A direct implication of this result is a sufficient statistics characterization of the optimal nonlinear savings tax schedule $T_s(s)$, given an existing and potentially sub-optimal nonlinear earnings tax $T_z(z)$. Indeed, at the optimum the reform should have a zero impact on the government objective, meaning that the optimal nonlinear savings tax schedule verifies at each savings $s^* = s(z^*)$

$$\frac{\mathcal{T}'_{s}(s^{*}, z^{*})}{1 + T'_{s}(s^{*}, z^{*})} \zeta^{c}_{s|z}(z^{*}) s(z^{*}) h_{z}(z^{*}) = s'(z^{*}) \int_{z^{*}}^{\bar{z}} (1 - \hat{g}(z)) dH_{z}(z)
- s'_{inc}(z^{*}) \frac{\mathcal{T}'_{z}(s^{*}, z^{*}) + s'_{inc}(z^{*}) \mathcal{T}'_{s}(s^{*}, z^{*})}{1 - \mathcal{T}'_{z}(s^{*}, z^{*})} \zeta^{c}_{z}(z^{*}) z^{*} h_{z}(z^{*})$$
(60)

Note that when $\mathcal{T}'_z(s^*, z^*)$ is optimal, this formula can be rewritten as

$$\frac{\mathcal{T}'_s(s^*, z^*)}{1 + \mathcal{T}'_s(s^*, z^*)} = s'_{pref}(z^*) \frac{1}{\zeta^c_{s|z}(z^*)} \frac{1}{h_z(z^*)s^*} \int_{z^*}^{\bar{z}} (1 - \hat{g}(z)) \, dH_z(z)$$

⁸¹Applying Lemma 1, earnings and savings changes are here given by

$$\begin{cases} dz = -\frac{z}{1-T'_z} \zeta_z^c(z) \delta T_z^{\theta'} - \frac{\eta_z(z)}{1-T'_z} \delta T_z^{\theta} \\ ds = -\frac{s(z)}{1+T'_s} \zeta_{s|z}^c(z) \delta T'_s - \frac{\eta_{s|z}(z)}{1+T'_s} \delta T_s + s'_{inc}(z) dz \end{cases}$$
(58)

where the equivalent reform T_z^{θ} is defined by equation (82), and the mass of individuals in the bandwidth is $\delta s \ h_s(s(z^*)) = \delta s \ \frac{h_z(z^*)}{s'(z^*)}$.

This optimal tax formula can also be obtained as a direct result through a joint reform of earnings and savings taxes at income level z^* . Starting from existing tax schedules, consider a *decrease* in marginal earnings tax rates by $d\tau_z$ in the bandwitdh $[z^*, z^* + dz]$ combined with an *increase* in marginal savings tax rates by $d\tau_s$ in the corresponding bandwidth $[s(z^*), s(z^* + dz)]$. Imposing $d\tau_z = s'_{inc}(z^*)d\tau_s$, this joint reform is such that the behavioral effects from earnings changes exactly cancel out (Lemma 1). Moreover, lump-sum changes induced by the two reforms are given by $d\tau_z dz = d\tau_s s'_{inc}(z^*)dz$ and by $d\tau_s ds = d\tau_s s'(z^*)dz$, meaning that the net lump-sum change is proportional to $s'_{pref}(z^*) = s'(z^*) - s'_{inc}(z^*)$. The total impact of the reform is thus

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{dz} = s_{pref}'(z^*)\int_{z^*}^{\bar{z}} (1-\hat{g}(z)) d\tau_s \, dH_z(z) - \mathcal{T}_s'(s^*, z^*) \, \frac{s(z^*)}{1 + \mathcal{T}_s'(s(z^*))} \zeta_{s|z}^c(z^*) d\tau_s \, h_z(z^*)$$

and characterizing the optimum as a situation in which this type of joint reform does not affect the government objective function yields the result.

Pareto-efficiency. We can combine formulas for optimal earnings and savings taxes to obtain a characterization of Pareto-efficiency as a corollary of the optimum. Indeed, leveraging the previous optimal tax formula written in terms of $s'_{pref}(z^*)$, and replacing the integral term by its value given from the optimal earnings tax formula yields

$$\frac{\mathcal{T}'_s(s^*, z^*)}{1 + T'_s(s^*, z^*)} \zeta^c_{s|z}(z^*) s^* = s'_{pref}(z^*) \frac{\mathcal{T}'_z(z^*) + s'_{inc}(z^*) \mathcal{T}'_s(s^*, z^*)}{1 - \mathcal{T}'_z(s^*, z^*)} \zeta^c_z(z^*) z^*$$

This Pareto-efficiency formula can also be obtained as a direct result through a joint reform of income and savings tax at income level z^* . Starting from existing tax schedules, consider an *increase* in marginal earnings tax rates by $d\tau_z$ in the bandwidth $[z^*, z^* + dz]$ and at the same time a *decrease* in marginal savings tax rates by $\delta\tau_s$ in the corresponding bandwidth $[s(z^*), s(z^* + dz)]$. Let construct these reforms such that it leave individuals above $z^* + dz$ unaffected by setting the two lump-sum changes equal, meaning $d\tau_z dz = d\tau_s ds$ i.e. $d\tau_z =$ $d\tau_s \frac{ds}{dz} = d\tau_s s'(z^*)$. Applying Lemma 1, the total impact of the reform on the government's tax revenue is

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{dz} = \mathcal{T}'_{s}\left(s^{*}, z^{*}\right) \frac{s^{*}}{1 + \mathcal{T}'_{s}\left(s^{*}, z^{*}\right)} \zeta^{c}_{s|z}(z^{*}) d\tau_{s} h_{z}(z^{*})
- \left(\mathcal{T}'_{z}(s^{*}, z^{*}) + s'_{inc}(z^{*})\mathcal{T}'_{s}(s^{*}, z^{*})\right) \frac{z^{*}}{1 - \mathcal{T}'_{z}(s^{*}, z^{*})} \zeta^{c}_{z}(z^{*}) d\tau_{s}\left(s'(z^{*}) - s'_{inc}(z^{*})\right) h_{z}(z^{*}) \quad (61)$$

while agents well-being remains unchanged. When the impact of this reform is non-zero, the type of joint reform we consider delivers a Pareto-improvement over the existing tax system. Characterizing a Pareto-efficient tax system as one that cannot be reformed in a Pareto-improving way yields the above sufficient statistics characterization.

B Proofs for Section 4

B.1 Proof of Proposition 4

The sufficiency conditions for the SN system follow directly from Lemma 3; the secondorder conditions are satisfied if Equations (31), (32), and (33) are negative under the SN tax system. The cross-partial derivative of the tax function \mathcal{T}_{sz}'' is zero, and thus it is easy to verify that the conditions (20) and (21) on \mathcal{S} and \mathcal{Z} in the proposition, combined with monotonicity (s'(z) > 0) and Assumption 1, jointly imply that each of these three equations is the sum of negative terms, implying the SOCs are satisfied.

To derive sufficient conditions for the LED tax system to be optimal, we again begin from Lemma 3. We consider the requirements $V_{ss}'' < 0$, $V_{zz}'' < 0$, and $V_{ss}''V_{zz}'' > (V_{sz}'')^2$ in turn. To show the first, under a LED tax system, $\mathcal{T}_{ss}'' = 0$, and therefore Equation (35) reduces to

$$\begin{aligned} V_{ss}''(s(\theta), z(\theta); \theta) &= -U_s' \mathcal{S}_c' + U_c' \mathcal{S}_s' \\ &= \frac{-U_c' U_s' U_{cs}'' + (U_s')^2 U_{cc}''}{(U_c')^2} + \frac{(U_c')^2 U_{ss}'' - U_c' U_s' U_{cs}''}{(U_c')^2} \end{aligned}$$

Therefore when utility is additively separable in c and s (implying $U''_{cs} = 0$), this expression is negative.

To show $V_{zz}'' < 0$, note that the potentially optimal LED system defined in Equations (16) and (17) satisfies

$$\mathcal{T}_{sz}''(s,z) = \tau_s'(z).$$

We can find an expression for $\tau'_s(z)$ at any point in the allocation in question by totally differentiating Equation (16) with respect to θ :

$$\begin{aligned} \tau'_s(z)z'(\theta) &= \frac{d}{d\theta} \mathcal{S}(c(\theta), s(\theta), z(\theta); \theta) \\ &= \mathcal{S}'_c \cdot (1 - \mathcal{T}'_z)z'(\theta) - \mathcal{S}'_c \cdot (1 + \mathcal{T}'_s)s'(\theta) + \mathcal{S}'_s s'(\theta) + \mathcal{S}'_z z'(\theta) + \mathcal{S}'_{\theta}, \end{aligned}$$

which can be rewritten as

$$\tau'_s(z) = \mathcal{S}'_c \cdot (1 - \mathcal{T}'_z) - \mathcal{S}'_c \cdot (1 + \mathcal{T}'_s)s'(z) + \mathcal{S}'_s s'(z) + \mathcal{S}'_z + \mathcal{S}'_{\theta}/z'(\theta).$$

Substituting this into the expression for V_{zz}'' in (44), we have

$$V_{zz}''(s(\theta), z(\theta); \theta) = U_s's'(z)\mathcal{Z}_c' - s'(z)U_c'\mathcal{Z}_s' - \frac{U_c'}{z'(\theta)}\mathcal{Z}_{\theta}' + U_c's'(z)\left[\mathcal{S}_c' \cdot (1 - \mathcal{T}_z') - \mathcal{S}_c' \cdot (1 + \mathcal{T}_s')s'(z) + \mathcal{S}_s's'(z) + \mathcal{S}_z' + \mathcal{S}_{\theta}'/z'(\theta)\right].$$
 (62)

Now employing the assumption that utility is separable in c, s, and z, (implying $U''_{cz} = U''_{cs} = 0$) we have

$$U'_{s}\mathcal{Z}'_{c} + U'_{c}\mathcal{S}'_{c}(1 - \mathcal{T}'_{z}) = U'_{s}\mathcal{Z}'_{c} - U'_{z}\mathcal{S}'_{c}$$

= $U'_{s}\frac{U'_{c}U''_{cz} - U'_{z}U''_{cc}}{(U'_{c})^{2}} - U'_{z}\frac{U'_{c}U''_{cs} - U'_{s}U''_{cc}}{(U'_{c})^{2}}$
= 0.

Substituting this result into Equation (62), and noting that $\mathcal{Z}'_s = \mathcal{S}'_z = 0$ by separability, yields

$$V_{zz}^{\prime\prime}(s(\theta), z(\theta); \theta) = s^{\prime}(z)^2 \left(U_c^{\prime} \mathcal{S}_s^{\prime} - U_s^{\prime} \mathcal{S}_c^{\prime} \right) + \frac{U_c^{\prime}}{z^{\prime}(\theta)} \left(s^{\prime}(z) \mathcal{S}_{\theta}^{\prime} - \mathcal{Z}_{\theta}^{\prime} \right)$$
(63)

Again employing separability, we have

$$U_{c}'\mathcal{S}_{s}' - U_{s}'\mathcal{S}_{c}' = U_{c}'\frac{U_{c}'U_{ss}'' - U_{s}'U_{cs}''}{\left(U_{c}'\right)^{2}} - U_{s}'\frac{U_{c}'U_{cs}'' - U_{s}'U_{cc}''}{\left(U_{c}'\right)^{2}} < 0,$$

implying that the first term on the right-hand side of Equation (63) is negative. Therefore condition (22) in the Proposition implies Equation (63) (and thus V''_{zz}) is negative.

Finally, to show $V_{ss}''V_{zz}'' > (V_{sz}'')^2$, we proceed from Equation (33) in Lemma 3, into which we substitute the expression for $\mathcal{T}_{sz}'' = \tau_s'(z)$, and we use the fact that $\mathcal{S}_z' = 0$ by separability:

$$(V_{sz}'')^2 - V_{ss}''V_{zz}'' = \left(\frac{U_z'}{s^{*'}(\theta)}\mathcal{S}_c' - \frac{U_c'}{s^{*'}(\theta)}\mathcal{S}_z'\right)U_c'\mathcal{Z}_{\theta}' + \frac{1}{s^{*'}(\theta)}V_{zz}''U_c'\mathcal{S}_{\theta}' + \mathcal{Z}_{\theta}'\frac{(U_c')^2}{s'(\theta)}\mathcal{T}_{sz}''$$
$$= -\mathcal{Z}_{\theta}'\frac{U_c'}{s'(\theta)z'(\theta)}\left[s'(\theta)\left(U_s'\mathcal{S}_c' - U_c'\mathcal{S}_s'\right) - \mathcal{S}_{\theta}'\right] + \frac{1}{s^{*'}(\theta)}V_{zz}''U_c'\mathcal{S}_{\theta}'.$$

We have already shown that $V''_{zz} < 0$. Thus the conditions $S'_{\theta} \ge 0$ and $S'_{\theta} < (U'_s S'_c - U'_c S'_s) s'(\theta)$ from (22) in the Proposition imply that both terms on the right-hand side are negative.

B.2 Proof of Proposition 5

We begin with a more general statement, and then derive Proposition 5 as a corollary. For a fixed type θ , let $c(z, \theta)$ and $s(z, \theta)$ be its preferred consumption and savings choices at earnings z, given the budget constraint induced by $\mathcal{T}(s, z)$

Proposition 8. Suppose that $\mathcal{A} = \{(c^*(\theta), s^*(\theta), z^*(\theta))\}_{\theta}$ constitutes a set of local optima for types θ under a smooth tax system \mathcal{T} . Individuals' local optima correspond to their global optima when

- 1. $z(\theta)$ is increasing
- 2. $-\frac{U'_c(c,s,z;\theta)}{U'_z(c,s,z;\theta)}$ and $-\frac{U'_s(c,s,z;\theta)}{U'_z(c,s,z;\theta)}$ are strictly increasing in θ for all (c,s,z)
- 3. For any two types θ and θ' , we cannot have both

$$\begin{split} &U_c'\left(c^*(\theta), s^*(\theta), z^*(\theta); \theta\right) \sigma_c\left(s^*(\theta), z^*(\theta)\right) + U_z'\left(c^*(\theta), s^*(\theta), z; \theta\right) \\ < &U_c'\left(c(\theta', z^*(\theta)), s(\theta', z^*(\theta)), z^*(\theta); \theta\right) \sigma_c\left(s(\theta', z^*(\theta)), z^*(\theta)\right) + U_z'\left(c(\theta', z^*(\theta)), s(\theta', z^*(\theta)), z^*(\theta); \theta\right) \end{split}$$

$$\begin{aligned} \text{and} \quad U_s'\left(c^*(\theta), s^*(\theta), z^*(\theta); \theta\right) \sigma_c\left(s^*(\theta), z^*(\theta)\right) + U_z'\left(c^*(\theta), s^*(\theta), z; \theta\right) \\ < U_s'\left(c(\theta', z^*(\theta)), s(\theta', z^*(\theta)), z^*(\theta); \theta\right) \sigma_s\left(s(\theta', z^*(\theta)), z^*(\theta)\right) + U_z'\left(c(\theta', z^*(\theta)), s(\theta', z^*(\theta)), z^*(\theta); \theta\right) \\ \text{where } \sigma_c\left(s, z\right) := 1 - \mathcal{T}_z'\left(s, z\right) \text{ and } \sigma_s\left(s, z\right) := \frac{1 - \mathcal{T}_z'(s, z)}{1 + \mathcal{T}_s'(s, z)}. \end{aligned}$$

In words, requirement 3 of the proposition is simply a requirement that if type θ preserves its earnings assigned earnings level $z^*(\theta)$, but chooses some other consumption level s(corresponding to a level that some other type θ' would choose if forced to choose earnings level $z^*(\theta)$), then at this alternative consumption bundle the type cannot have both higher marginal utility from increasing its savings through one more unit of work *and* increasing its consumption through one more unit of work. As shown for SL systems in Proposition 5, this amounts to a condition on the concavity of U with respect to consumption and savings. More generally, condition 3 in Proposition 8 will hold as long as U is sufficiently concave in consumption and savings when type θ chooses earnings level $z^*(\theta)$.

Proof. To prove agents' local optima are global optima, we want to show that for any given agent θ^* utility decreases when moving from allocation $(c^*(\theta^*), s^*(\theta^*), z^*(\theta^*))$ to allocation $(c(z, \theta^*), s(z, \theta^*), z)$.

The first step is to compute agent θ^* utility change. The envelope theorem applied to savings choices $s(z, \theta^*)$ implies

$$\frac{d}{dz}U(c(z,\theta^*)(z),s^*(z),z;\theta^*) = \frac{d}{dz}\left[U(z-s(z,\theta^*)-\mathcal{T}(s^*(z),z),s(z,\theta^*)),z;\theta^*)\right]$$

$$= U_c'(c(z,\theta^*)(z),s(z,\theta^*),z;\theta^*)\sigma_c(s(z,\theta^*),z) + U_z'(c(z,\theta^*)(z),s(z,\theta^*),z;\theta^*)$$
(64)
(65)

where $\sigma_c(s, z) = 1 - \mathcal{T}'_z(s, z)$. Note that, as established by Milgrom and Segal (2002), these equalities hold as long as U is differentiable in z (holding s and c fixed)—differentiability of $c(z, \theta^*)(z)$ or $s^*(z)$ is actually not required.

Similarly, the envelope theorem applied to consumption choices $c(z, \theta^*)(z)$ implies

$$\frac{d}{dz}U\left(c(z,\theta^*)(z),s(z,\theta^*),z;\theta^*\right) = U'_s\left(c(z,\theta^*)(z),s(z,\theta^*),z;\theta^*\right)\sigma_s\left(s(z,\theta^*),z\right) + U'_z\left(c(z,\theta^*)(z),s(z,\theta^*),z;\theta^*\right)$$
(66)

where $\sigma_s(s, z) = \frac{1 - \mathcal{T}'_s(s, z)}{1 + \mathcal{T}'_s(s, z)}$.

Therefore, agent's θ^* utility change when moving from allocation $(c(z, \theta^*), s^*, z^*)$ to allocation $(c(z, \theta^*)(z), s^*(z), z)$ is

$$U(c(z,\theta^{*})(z), s(z,\theta^{*}), z; \theta^{*}) - U(c(z,\theta^{*}), s(z,\theta^{*}), z^{*}; \theta^{*})$$

$$= \int_{x=z^{*}}^{x=z} \left[\min\left(U_{c}'(c(z,\theta^{*}), s(x,\theta^{*}), x; \theta^{*}) \sigma_{c}\left(s^{*}(x), x\right), U_{s}'(c(z,\theta^{*}), s(x,\theta^{*}), x; \theta^{*}\right) \sigma_{s}\left(s(x,\theta^{*}), x\right) + U_{z}'(c(x,\theta^{*}), s(x,\theta^{*})), x; \theta^{*} \right] dx \quad (67)$$

where the min operator is introduced without loss of generality given the fact that both terms are equal.

The second step is to show that under our assumptions, agent θ^* utility change (67) is negative. To do so, let θ_x be the type that chooses earnings x.

Suppose that $z > z^*$; the case $z < z^*$ follows identically. Now for any $x > z^*$, we have that $\theta_x > \theta^*$ by monotonicity. Then by the generalized single crossing condition (2), we have

$$-\frac{U_c'(c(x,\theta^*), s(x,\theta^*), x; \theta_x)}{U_z'(c(x,\theta^*), s(x,\theta^*), x; \theta_x)} \ge -\frac{U_c'(c(z,\theta^*), s(x,\theta^*), x; \theta^*)}{U_z'(c(z,\theta^*), s(x,\theta^*), x; \theta^*)}$$
(68)

as well as

$$-\frac{U_s'\left(c(x,\theta^*), s(x,\theta^*), x; \theta_x\right)}{U_z'\left(c(x,\theta^*), s(x,\theta^*), x; \theta_x\right)} \ge -\frac{U_s'\left(c(x,\theta^*), s(x,\theta^*), x; \theta^*\right)}{U_z'\left(c(x,\theta^*), s(x,\theta^*), x; \theta^*\right)}$$
(69)

This implies that

$$\min \left(U_{c}^{\prime}\left(c(x,\theta^{*}),s(x,\theta^{*}),x;\theta^{*}\right)\sigma_{c}\left(s(x,\theta^{*}),x\right),U_{s}^{\prime}\left(c(x,\theta^{*}),s(x,\theta^{*}),x;\theta^{*}\right)\sigma_{s}\left(s^{*}(x),x\right)\right)+U_{z}^{\prime}\left(c(x,\theta^{*}),s(x,\theta^{*}),s(x,\theta^{*})\right)$$

$$\leq \min \left(U_{c}^{\prime}\left(c(x,\theta^{*}),s(x,\theta^{*}),x;\theta_{x}\right)\sigma_{c}\left(s^{*}(x),x\right),U_{s}^{\prime}\left(c(x,\theta^{*}),s(x,\theta^{*}),x;\theta_{x}\right)\sigma_{s}\left(s^{*}(x),x\right)\right)+U_{z}^{\prime}\left(c(x,\theta^{*}),s(x,\theta^{*}),s(x,\theta^{*})\right)$$

$$\leq \max \left(U_{c}^{\prime}\left(c^{*}(\theta_{x}),s^{*}(\theta_{x}),x;\theta_{x}\right)\sigma_{c}\left(s^{*}(\theta_{x}),x\right),U_{s}^{\prime}\left(c^{*}(\theta_{x}),s^{*}(\theta_{x}),x;\theta_{x}\right)\sigma_{s}\left(s^{*}(\theta_{x}),x\right)\right)+U_{z}^{\prime}\left(c^{*}(\theta_{x}),s^{*}(\theta_{x}),x;\theta_{x}\right)$$

$$(70)$$

$$\leq \min \left(U_{c}^{\prime}\left(c^{*}(\theta_{x}),s^{*}(\theta_{x}),x;\theta_{x}\right)\sigma_{c}\left(s^{*}(\theta_{x}),x\right),U_{s}^{\prime}\left(c^{*}(\theta_{x}),s^{*}(\theta_{x}),x;\theta_{x}\right)\sigma_{s}\left(s^{*}(\theta_{x}),x\right)\right)+U_{z}^{\prime}\left(c^{*}(\theta_{x}),s^{*}(\theta_{x}),x;\theta_{x}\right)$$

$$(72)$$

where the last inequality follows from the fact that by assumption (*iii*), we either have⁸²

$$U_c'(c(x,\theta^*), s(x,\theta^*), x; \theta_x) \sigma_c(s^*(x), x) + U_z'(c(x,\theta^*), s(x,\theta^*), x; \theta_x)$$

$$\leq U_c'(c^*(\theta_x), s^*(\theta_x), x; \theta_x) \sigma_c(s^*(\theta_x), x) + U_z'(c^*(\theta_x), s^*(\theta_x), x; \theta_x)$$

or

$$U'_{s}(c(x,\theta^{*}), s(x,\theta^{*}), x; \theta^{*}) \sigma_{s}(s^{*}(x), x) + U'_{z}(c(x,\theta^{*}), s(x,\theta^{*}), x; \theta^{*})$$

$$\leq U'_{s}(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}) \sigma_{s}(s^{*}(\theta_{x}), x) + U'_{z}(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x})$$

Now, note that agent θ_x first-order conditions at its local optimum $(c^*(\theta_x), s^*(\theta_x), x)$ are

$$[1 - \mathcal{T}'_{z}(s^{*}(\theta_{x}), x)] U'_{c}(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}) + U'_{z}(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}) = 0$$
(73)

$$- [1 + \mathcal{T}'_{s}(s^{*}(\theta_{x}), x)] U'_{c}(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}) + U'_{s}(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}) = 0$$
(74)

which means

$$\max\left(U_{c}'\left(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}\right) \sigma_{c}\left(s^{*}(\theta_{x}), x\right), U_{s}'\left(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}\right) \sigma_{s}\left(s^{*}(\theta_{x}), x\right)\right) + U_{z}'\left(c^{*}(\theta_{x}), s^{*}(\theta_{x}), x; \theta_{x}\right) =$$

$$(75)$$

where the max operator is introduced without loss of generality given the fact that both terms are equal.

⁸²Not having $\{a > c \text{ and } b > c\}$ means having $\{a \le c \text{ or } b \le d\}$ which implies $\min(a, b) \le \max(c, d)$

Therefore,

$$\min\left(U_{c}'\left(c(x,\theta^{*}),s(x,\theta^{*}),x;\theta^{*}\right)\sigma_{c}\left(s^{*}(x),x\right),U_{s}'\left(c(x,\theta^{*}),s(x,\theta^{*}),x;\theta^{*}\right)\sigma_{s}\left(s^{*}(x),x\right)\right)+U_{z}'\left(c(x,\theta^{*}),s(x,\theta^{*}),x;\theta^{*}\right)\sigma_{s}(x,\theta^{*})$$
(76)

which shows that

$$U(c(z,\theta^*), s(z,\theta^*), z; \theta^*) - U(c^*(\theta^*), s^*(\theta^*), z^*(\theta^*); \theta^*) \le 0$$
(77)

The case with $z < z^*$ follows identically, proving Proposition 8.

Proof of Proposition 5

SL systems Since σ_c and σ_s are functions of z only, the result follows immediately from separability in labor and concavity of U.

SN systems If $s < s^*(\theta)$, then $c > c^*(\theta)$, and since σ_c is not a function of s, it follows that $U''_{cc} < 0$ and $U''_{cs} \ge 0$ imply that

$$U_c'\left(c^*(\theta), s^*(\theta), z^*(\theta); \theta\right) \sigma_c\left(s^*(\theta), z^*(\theta)\right) > U_c'\left(c, s, z^*(\theta); \theta\right) \sigma_c\left(s, z^*(\theta)\right)$$

Next, suppose that $s > s^*(\theta)$ and $c < c^*(\theta)$. The condition that $-\frac{T''_{ss}(s)}{1+T'_s(s)} < \frac{-U''_{ss}(c(s,\theta),s,z^*(\theta);\theta)}{U'_s(c(s,\theta),s,z^*(\theta);\theta)}$, together with $U''_{cs} > 0$, implies that $\frac{U'_s(c(s,\theta),s,z^*(\theta);\theta)}{1+T'_s(s)}$ is decreasing in s

$$V'_{s}(s, z^{*}(\theta); \theta) = -(1 + T'_{s}(s))U'_{c}(c, s, z^{*}(\theta); \theta) + U'_{s}(c, s, z^{*}(\theta); \theta) < 0$$

and thus that

$$\frac{U'_s(c, s, z^*(\theta); \theta)}{1 + T'_s(s)} < \frac{U'_s(c^*(\theta), s^*(\theta), z^*(\theta); \theta)}{1 + T'_s(s^*(\theta))}$$

LED systems First, consider a type θ' choosing $z > z^*(\theta')$. The condition that U'_s/U'_c is increasing in θ ensures that when a type θ chooses an earnings level $z = z^*(\theta) > z^*(\theta')$, her desired savings level is $s(z^*(\theta), \theta') < s^*(\theta)$. We show that in this case, the assumptions of the proposition imply that

$$U_c'(z-s-\tau_s(z)s-T(z),s,z^*(\theta);\theta)\,\sigma_c(s,z^*(\theta))$$

is increasing in s for $s < s^*(\theta)$, where $\sigma_c(s, z^*(\theta)) = 1 - T'_z(z) - \tau'_s(z)s$. This ensures that

$$U_{c}'(c(z^{*}(\theta), \theta'), s(z^{*}(\theta), \theta'), z^{*}(\theta); \theta) \sigma_{c}(s(z^{*}(\theta), \theta'), z^{*}(\theta))$$

$$< U_{c}'(c^{*}(\theta), s^{*}(\theta), z^{*}(\theta); \theta) \sigma_{c}(s^{*}(\theta), z^{*}(\theta))$$

To that end, we take the derivative with respect to s, which is

$$\frac{d}{ds}U'_{c} = -(1 + \tau_{s}(z))U''_{cc}\sigma_{c}(s, z^{*}(\theta)) + U''_{cs}\sigma_{c}(s, z^{*}(\theta)) - \tau'_{s}(z)U'_{c}$$

Since $U_{cs}'' \ge 0$, it is thus sufficient that

$$\begin{aligned} &-(1+\tau_{s}(z))U_{cc}''\sigma_{c}\left(s,z^{*}(\theta)\right)-\tau_{s}'(z)U_{c}'>0\\ \Leftrightarrow &\frac{-U_{cc}''(c(s,\theta),s,z^{*}(\theta);\theta)}{U_{c}'(c(s,\theta),s,z^{*}(\theta);\theta)}>\frac{-\tau_{s}'(z^{*}(\theta))}{(1+\tau_{s}(z))(1-\tau_{s}'(z^{*}(\theta))s-T_{z}'(z^{*}(\theta)))} \end{aligned}$$

Second consider a type θ' choosing $z = z^*(\theta) < z^*(\theta')$. The condition that U'_s/U'_c is increasing in θ ensures that when a type θ chooses an earnings level $z > z^*(\theta)$, her desired savings level is $s(z, \theta) > s^*(z)$. We show that in this case, the assumptions of the proposition imply that

$$U'_{s}(z-s-\tau_{s}(z)s-T(z),s,z^{*}(\theta);\theta)\sigma_{s}(s,z^{*}(\theta))$$

is decreasing in s for $s > s^*(z)$, where $\sigma_s(s, z^*(\theta)) = \frac{1 - T'_z(z) - \tau'_s(z)s}{1 + \tau_s(z)}$. This ensures that

$$U'_{s}(c(z^{*}(\theta), \theta'), s(z^{*}(\theta), \theta'), z^{*}(\theta); \theta) \sigma_{c}(s(z^{*}(\theta), \theta'), z^{*}(\theta))$$

$$< U'_{s}(c^{*}(\theta), s^{*}(\theta), z^{*}(\theta); \theta) \sigma_{c}(s^{*}(\theta), z^{*}(\theta))$$

To that end, we take the derivative with respect to s, which is

$$-(1+\tau_{s}(z))U_{sc}''\sigma_{s}\left(s,z^{*}(\theta)\right)+U_{ss}''\sigma_{s}\left(s,z^{*}(\theta)\right)-U_{s}'\frac{\tau_{s}'(z)}{1+\tau_{s}(z)}$$

Since $U_{sc}'' > 0$, it is sufficient to show that

$$U_{ss}'' \cdot (1 - T_z'(z) - \tau_s'(z)s) - U_s'\tau_s'(z) < 0$$

$$\Leftrightarrow \frac{-U_{ss}''(c(s,\theta), s, z^*(\theta); \theta)}{U_s'(c(s,\theta), s, z^*(\theta); \theta)} > \frac{-\tau_s'(z^*(\theta))}{1 - \tau_s'(z^*(\theta))s - T_z'(z^*(\theta))}.$$

B.3 Proof of Propositions 6 and 7

The characterization of the optimal income tax rates continues to hold.

A proof of the characterization of the SN as the proofs of Proposition 3, and thus does not need to be repeated. In particular, the derivation of equation (60) for the SN system is identical, which establishes Proposition 7 for the SN system. The proof of equation (61) is also identical for the SN system, which establishes Proposition 6 for the SN system. We thus provide proofs for the SL and LED savings tax rates here.

B.3.1 Earnings responses to the savings tax

We begin by establishing an analog of Lemma 1 for the SL and LED systems. The proof for the SN system is identical to the proof of Lemma 1.

Linear savings tax. A proof is available in Saez (2002b).

Linear earnings-dependent savings tax. Let

$$V\left(T_{s}(.|z), y, z; \theta\right) = \max_{s} U\left(y - s - T_{s}\left(s|z\right), s, z; \theta\right)$$

$$\tag{78}$$

be agent θ indirect utility function at earnings z and (pre savings tax) disposable income y. Consider a savings tax reform $dT_s(s|z)$ that consists in a small increase $d\tau_s$ of the savings linear tax rate phased-in over the earnings bandwidth $[z^*, z^* + dz]$ associated with types $[\theta^*, \theta^* + d\theta]$. Formally, this corresponds to the following reform $dT_s(s|z)$ of the savings tax function $T_s(s|z) = \tau_s(z) s$:

$$dT_s(s|z) = \begin{cases} 0 & \text{if } z \le z^* \\ d\tau_s (z - z^*) \, s & \text{if } z \in [z^*, z^* + dz] \\ d\tau_s dz \, s & \text{if } z \ge z^* + dz \end{cases}$$
(79)

The idea is to construct for each type θ a perturbation of the earnings tax $dT_z^{\theta}(z)$ that induces the same earnings response as the initial perturbation $dT_s(s)$. Suppose we define this perturbation for each type θ such that at all earnings z,

$$V(T_s(.) + dT_s(.), z - T_z(z), z; \theta) = V(T_s(.), z - T_z(z) - dT_z^{\theta}(z), z; \theta)$$
(80)

By construction, $dT_z^{\theta}(z)$ induces the same earnings response as the initial perturbation dT_s .

A first-order Taylor expansion of the previous equality around $V(T_s(.), z - T_z(z), z; \theta)$ yields

$$\mathbb{I}_{\{\theta^* \le \theta \le \theta^* + \mathrm{d}\theta\}} \left[\frac{\partial V}{\partial T'_s} d\tau_s \left(z - z^* \right) + \frac{\partial V}{\partial T_s} d\tau_s \left(z - z^* \right) s \left(z; \theta \right) \right] \\
+ \mathbb{I}_{\{\theta \ge \theta^* + \mathrm{d}\theta\}} \left[\frac{\partial V}{\partial T'_s} d\tau_s dz + \frac{\partial V}{\partial T_s} d\tau_s dz \, s \left(z; \theta \right) \right] = -\frac{\partial V}{\partial y} dT_z^{\theta}(z) \quad (81)$$

and recognizing that $\frac{\partial V}{\partial T'_s} = 0$ (envelope theorem) and that $\frac{\partial V}{\partial T_s} = -\frac{\partial V}{\partial y}$, we obtain by identification

$$\delta T_{z}^{\theta}(z) = \begin{cases} 0 & \text{if } z \leq z^{*} \\ d\tau_{s} \left(z - z^{*} \right) s \left(z; \theta \right) & \text{if } z \in [z^{*}, z^{*} + dz] \\ d\tau_{s} dz \, s \left(z; \theta \right) & \text{if } z \geq z^{*} + dz \end{cases}$$
(82)

This shows that a small increase $d\tau_s dz$ in the marginal savings tax rate faced by an agent $\theta \ge \theta^* + d\theta$ induces the same earnings change dz as a small increase $s'_{inc}(z) d\tau_s dz$ in the marginal earnings tax rate.

B.3.2 Linear savings tax

Reform. When the government uses a linear savings tax such that $\mathcal{T}(s, z) = \tau_s s + T_z(z)$, we consider a small reform of the linear savings tax rate τ_s that consists in a small increase $d\tau_s$. Formally, this corresponds to a reform $dT_s(s) = d\tau_s s$ of the savings tax function $T_s(s) = \tau_s s$.

We characterize the impact of this reform on the government objective function. Normalizing all effects by $1/\lambda$, the reform induces

• mechanical effects:

$$\int_{z} \left(1 - \frac{\alpha(z)}{\lambda} U'_{c}(c(z), s(z), z; \theta(z)) \right) d\tau_{s} s(z) dH_{z}(z)$$
(83)

• behavioral effects from earnings changes:

$$-\int_{z} T'_{z}(z) \left[\frac{z}{1 - T'_{z}(z)} \zeta_{z}^{c}(z) \, d\tau_{s} \, s'_{inc}(z) + \frac{\eta_{z}(z)}{1 - T'_{z}(z)} \, d\tau_{s} \, s(z) \right] \, dH_{z}(z) \tag{84}$$

• behavioral effects from savings changes:

$$-\int_{z} \tau_{s} \left[\frac{s(z)}{1+\tau_{s}} \zeta_{s|z}^{c}(z) d\tau_{s} + \frac{\eta_{s|z}(z)}{1+\tau_{s}} d\tau_{s} \, s\left(z;\theta(z)\right) \right] \, dH_{z}(z) \\ -\int_{z} s_{inc}'(z) \tau_{s} \left[\frac{z}{1-T_{z}'} \zeta_{z}^{c}(z) \, d\tau_{s} \, s_{inc}'(z) + \frac{\eta_{z}(z)}{1-T_{z}'} \, d\tau_{s} \, s\left(z;\theta(z)\right) \right] \, dH_{z}(z)$$
(85)

Summing over these different effects yields the total impact of the reform

$$\frac{d\mathcal{L}}{\lambda} = \int_{z} \left\{ \left(1 - \hat{g}(z)\right) s\left(z\right) - \frac{T'_{z}(z) + s'_{inc}(z)\tau_{s}}{1 - T'_{z}(z)} z\zeta_{z}^{c}(z) s'_{inc}(z) - \tau_{s} \frac{s(z)}{1 + \tau_{s}} \zeta_{s|z}^{c}(z) \right\} d\tau_{s} \, dH_{z}(z) \tag{86}$$

Optimality. A direct implication of this result is a sufficient statistics characterization of the optimal linear savings tax schedule τ_s , given an existing and potentially suboptimal nonlinear tax $T_z(z)$. Indeed, at the optimum the reform should have a zero impact on the government objective meaning that the optimal savings tax schedule verifies

$$\frac{\tau_s}{1+\tau_s} \int_z s(z)\zeta_{s|z}^c(z)dH_z(z) = \int_z \left\{ (1-\hat{g}(z)) \ s(z) - \frac{T'_z(z) + \tau_s \, s'_{inc}(z)}{1-T'_z(z)} z\zeta_z^c(z) \, s'_{inc}(z) \right\} dH_z(z) \tag{87}$$

Pareto-efficiency. To derive Pareto-efficiency, we combine savings tax and earnings tax reforms in a way that anihilates all lump-sum changes in tax liability in order to offset all utility changes.

We start with a small reform of the linear savings tax rate τ_s that consists in small increase $d\tau_s$. At the bottom of the earnings distribution $(z = \underline{z})$, the mechanical effect of the savings tax reform is an increase in tax liability by $s(\underline{z}) d\tau_s$. We thus adjust the earnings tax liability through a downward shift by $s(\underline{z}) d\tau_s$ at all earnings levels. This joint reform has the following impact on the government objective

$$\frac{d\mathcal{L}}{\lambda} = \int_{z=\underline{z}}^{\overline{z}} \left\{ [1 - \hat{g}(z)] \left[s\left(z\right) - s\left(\underline{z}\right) \right] - \frac{T'_{z}(z) + \tau_{s} \, s'_{inc}(z)}{1 - T'_{z}} z \zeta_{z}^{c}(z) \, s'_{inc}(z) - \tau_{s} \frac{s(z)}{1 + T'_{s}} \zeta_{s|z}^{c}(z) \right\} \, d\tau_{s} \, dH_{z}(z)$$

$$\tag{88}$$

meaning that the lump-sum change in tax liability is nil at earnings $z = \underline{z}$ but not at earnings above $z \ge \underline{z}$.

To cancel out lump-sum changes in tax liability at all earnings levels, we are going to construct a sequence of earnings tax reforms. We discretize the range of earnings $[\underline{z}, \overline{z}]$ into N bins and consider reforms in the small earnings bandwidths $dz = \frac{\Delta z}{N}$ where $\Delta z = \overline{z} - \underline{z}$. We proceed by induction to pin down a general formula for the tax reforms we consider:

• First, consider a decrease in the marginal earnings tax rate by $d\tau_z = s'(\underline{z}) d\tau_s$ over the bandwidth $[\underline{z}, \underline{z} + dz]$. In this bandwidth, this reform (i) cancels out lump-sum changes in tax liability to a first-order approximation since $[s(\underline{z} + dz) - s(\underline{z})] d\tau_s \approx s'(\underline{z}) dz d\tau_s$, and (ii) induces earnings responses through the change in marginal tax rates. Moreover, it decreases the lump-sum tax liability on all individuals with earnings $z \ge \underline{z} + dz$. The total impact of this sequence of reforms is then

$$\frac{d\mathcal{L}}{\lambda} = \int_{z=\underline{z}+dz}^{\overline{z}} \left\{ \begin{bmatrix} 1 - \hat{g}(z) \end{bmatrix} \begin{bmatrix} s(z) - s(\underline{z}) - s'(\underline{z}) dz \end{bmatrix} \right\} d\tau_s \, dH_z(z)
- \int_{z=\underline{z}}^{\overline{z}} \left\{ \frac{T'_z(z) + \tau_s \, s'_{inc}(z)}{1 - T'_z} z \zeta_z^c(z) \, s'_{inc}(z) - \tau_s \frac{s(z)}{1 + \tau_s} \zeta_{s|z}^c(z) \right\} d\tau_s \, dH_z(z)
+ \int_{z=\underline{z}}^{\underline{z}+dz} \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1 - T'_z(z)} z \zeta_z^c(z) \left(s'(\underline{z}) \, d\tau_s \right) \, dH_z(z) \quad (89)$$

• Second, consider a decrease in the marginal earnings tax rate by $d\tau_z = s'(\underline{z} + dz) d\tau_s$ over the bandwidth $[\underline{z} + dz, \underline{z} + 2dz]$, which again cancels out lump-sum changes in this bandwidth up to a first-order approximatince since $[s(\underline{z} + 2dz) - s(\underline{z}) - s'(\underline{z}) dz] \approx$ $s'(\underline{z} + dz) dz$. The total impact of this sequence of reforms is then

$$\frac{d\mathcal{L}}{\lambda} = \int_{z=\underline{z}+2dz}^{\overline{z}} \left\{ \begin{bmatrix} 1 - \hat{g}(z) \end{bmatrix} \begin{bmatrix} s(z) - s(\underline{z}) - s'(\underline{z}) dz - s'(\underline{z} + dz) dz \end{bmatrix} \right\} d\tau_s \, dH_z(z)
- \int_{z\geq\underline{z}} \left\{ \frac{T'_z(z) + \tau_s \, s'_{inc}(z)}{1 - T'_z} z \zeta_z^c(z) \, s'_{inc}(z) - \tau_s \frac{s(z)}{1 + \tau_s} \zeta_{s|z}^c(z) \right\} d\tau_s \, dH_z(z)
+ \int_{z=\underline{z}}^{\underline{z}+dz} \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1 - T'_z(z)} z \zeta_z^c(z) \left(s'(\underline{z}) \, d\tau_s \right) \, dH_z(z)
+ \int_{z=\underline{z}+dz}^{\underline{z}+2dz} \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1 - T'_z(z)} z \zeta_z^c(z) \left(s'(\underline{z} + dz) \, d\tau_s \right) \, dH_z(z)$$
(90)

• Iterating over to step k, in which we consider a decrease in the marginal earnings tax rate by $d\tau_z = s'\left(\underline{z} + (k-1)\frac{\Delta z}{N}\right) d\tau_s$ over the bandwidth $[\underline{z} + (k-1)\frac{\Delta z}{N}, \underline{z} + k\frac{\Delta z}{N}]$. The total impact of this sequence of reforms is then

$$\frac{d\mathcal{L}}{\lambda} = \int_{z=\underline{z}+k\frac{\Delta z}{N}}^{\overline{z}} \left\{ \left[1 - \hat{g}(z)\right] \left[s\left(z\right) - s\left(\underline{z}\right) - \frac{\Delta z}{N} \left[\sum_{p=0}^{k-1} s'\left(\underline{z} + p\frac{\Delta z}{N}\right)\right] \right] \right\} d\tau_s \, dH_z(z)
- \int_{z\geq\underline{z}} \left\{ \frac{T'_z(z) + \tau_s \, s'_{inc}(z)}{1 - T'_z} z \zeta_z^c(z) \, s'_{inc}(z) - \tau_s \frac{s(z)}{1 + \tau_s} \zeta_{s|z}^c(z) \right\} d\tau_s \, dH_z(z)
+ \sum_{p=0}^{k-1} \int_{z=\underline{z}+p\frac{\Delta z}{N}}^{\underline{z}+(p+1)\frac{\Delta z}{N}} \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1 - T'_z(z)} z \zeta_z^c(z) s'\left(\underline{z} + p\frac{\Delta z}{N}\right) d\tau_s \, dH_z(z) \quad (91)$$

• Pushing the iteration forward until k = N, the first integral disappears (integration over an empty set) such that the total impact of this sequence of reforms is given by

$$\frac{d\mathcal{L}}{\lambda} = -\int_{z\geq\underline{z}} \left\{ \frac{T'_{z}(z) + \tau_{s} \, s'_{inc}(z)}{1 - T'_{z}} z \zeta_{z}^{c}(z) \, s'_{inc}(z) - \tau_{s} \frac{s(z)}{1 + \tau_{s}} \zeta_{s|z}^{c}(z) \right\} d\tau_{s} \, dH_{z}(z) + \sum_{p=0}^{N-1} \int_{z=\underline{z}+p\frac{\Delta z}{N}}^{\underline{z}+(p+1)\frac{\Delta z}{N}} \frac{T'_{z}(z) + s'_{inc}(z)\tau_{s}}{1 - T'_{z}(z)} z \zeta_{z}^{c}(z) s'\left(\underline{z}+p\frac{\Delta z}{N}\right) d\tau_{s} \, dH_{z}(z)$$
(92)

Let's now compute the last term at the limit $N \to \infty$. We have

$$\sum_{p=0}^{N-1} \int_{z=\underline{z}+p\frac{\Delta z}{N}}^{\underline{z}+(p+1)\frac{\Delta z}{N}} \frac{T'_{z}(z) + s'_{inc}(z)\tau_{s}}{1 - T'_{z}(z)} z\zeta_{z}^{c}(z)s'\left(\underline{z}+p\frac{\Delta z}{N}\right) d\tau_{s} \, dH_{z}(z)$$
(93)

$$\approx \sum_{p=0}^{N-1} \frac{T_z'(\underline{z} + p\frac{\Delta z}{N}) + s_{inc}'(\underline{z} + p\frac{\Delta z}{N})\tau_s}{1 - T_z'\left(\underline{z} + p\frac{\Delta z}{N}\right)} \left(\underline{z} + p\frac{\Delta z}{N}\right) \zeta_z^c \left(\underline{z} + p\frac{\Delta z}{N}\right) s'\left(\underline{z} + p\frac{\Delta z}{N}\right) d\tau_s h_z \left(\underline{z} + p\frac{\Delta z}{N}\right) \frac{\Delta z}{N}$$

$$\tag{94}$$

$$\xrightarrow[N \to \infty]{} \int_{z=\underline{z}}^{\overline{z}} \frac{T'_{z}(z) + s'_{inc}(z)\tau_{s}}{1 - T'_{z}(z)} z\zeta_{z}^{c}(z) s'(z) d\tau_{s} h_{z}(z) dz$$

$$\tag{95}$$

where the last line follows from the Riemann definition of the integral in terms of Riemann sums. Hence, the total impact of this sequence of reforms is at the limit given by

$$\frac{d\mathcal{L}}{\lambda} = -\int_{z\geq\underline{z}} \left\{ \frac{T'_{z}(z) + \tau_{s} \, s'_{inc}(z)}{1 - T'_{z}} z \zeta_{z}^{c}(z) \, s'_{inc}(z) - \tau_{s} \frac{s(z)}{1 + \tau_{s}} \zeta_{s|z}^{c}(z) \right\} d\tau_{s} \, h_{z}(z) dz
+ \int_{z=\underline{z}}^{\overline{z}} \frac{T'_{z}(z) + s'_{inc}(z)\tau_{s}}{1 - T'_{z}(z)} z \zeta_{z}^{c}(z) \, s'(z) \, d\tau_{s} \, h_{z}(z) \, dz \quad (96)$$

By construction, the sequence of reforms we have constructed does not affect agents' utility, and only affects tax revenue through the expression above. When the impact of this reform is non-zero, the type of sequence of reforms we consider delivers a Pareto-improvement over the existing tax system. Characterizing a Pareto-efficient tax system as one that cannot be reformed in a Pareto-improving way yields the following sufficient statistics characterization:

$$\frac{\tau_s}{1+\tau_s} \int_{z=\underline{z}}^{\overline{z}} s(z) \zeta_{s|z}^c(z) h_z(z) dz = \int_{z=\underline{z}}^{\overline{z}} \left[s'(z) - s'_{inc}(z) \right] \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1 - T'_z(z)} z \zeta_z^c(z) h_z(z) dz \quad (97)$$

B.3.3 Linear earnings-dependent savings tax

Reform. When the government uses an linear earnings-dependent savings tax such that $\mathcal{T}(z,s) = T_z(z) + \tau_s(z) s$, we consider a small reform of the linear earnings-dependent

savings tax $\tau_s(z)$ that consists in a small increase $\delta \tau_s$ of the savings linear tax rate phased-in over the earnings bandwidth $[z^*, z^* + dz]$. This means that agents in this bandwidth face a linear tax rate equal to $\tau_s(z) + d\tau_s(z - z^*)$ and that all agents above face a linear tax rate equal to $\tau_s(z) + d\tau_s dz$. Formally, this corresponds to the following reform $dT_s(s|z)$ of the savings tax function $T_s(s|z) = \tau_s(z) s$:

$$dT_s(s|z) = \begin{cases} 0 & if \ z \le z^* \\ d\tau_s \left(z - z^*\right) s & if \ z \in [z^*, z^* + dz] \\ d\tau_s dz \ s & if \ z \ge z^* + dz \end{cases}$$
(98)

We characterize the impact of this reform on the government objective function. Normalizing all effects by $1/\lambda$, the reform induces

• mechanical effects:

$$\int_{z \ge z^*} \left(1 - \frac{\alpha(z)}{\lambda} U'_c(c(z), s(z), z; \theta(z)) \right) \, d\tau_s dz \, s(z) \, dH_z(z) \tag{99}$$

• behavioral effects from earnings changes:

$$-\mathcal{T}_{z}'(s(z^{*}), z^{*}) \left[\frac{z^{*}}{1 - \mathcal{T}_{z}'(s(z^{*}), z^{*})} \zeta_{z}^{c}(z^{*}) d\tau_{s}s(z^{*})\right] h_{z}(z^{*}) dz$$

$$-\int_{z^{*}}^{\bar{z}} \mathcal{T}_{z}'(s(z), z) \left[\frac{z}{1 - \mathcal{T}_{z}'(s(z), z)} \zeta_{z}^{c}(z) s_{inc}'(z) + \frac{\eta_{z}(z)}{1 - \mathcal{T}_{z}'(s(z), z)} s(z)\right] d\tau_{s} dz \, dH_{z}(z)$$
(100)

• behavioral effects from savings changes:

$$-\tau_{s}(z^{*})s_{inc}'(z^{*})\left[\frac{z^{*}}{1-\mathcal{T}_{z}'(s(z^{*}),z^{*})}\zeta_{z}^{c}(z^{*})\,d\tau_{s}\,s(z^{*})\right]h_{z}(z^{*})dz$$

$$-\int_{z^{*}}^{\bar{z}}\tau_{s}(z)\left[\frac{\zeta_{s|z}^{c}(z)+\eta_{s|z}(z)}{1+\tau_{s}(z)}\,s(z)+s_{inc}'(z)\left(\frac{z}{1-\mathcal{T}_{z}'(s(z),z)}\zeta_{z}^{c}(z)s_{inc}'(z)+\frac{\eta_{z}(z)}{1-\mathcal{T}_{z}'(s(z),z)}\,s(z)\right)\right]d\tau_{z}(z^{*})dz$$

Summing over these different effects yields the total impact of the reform

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{dz} = \int_{z^*}^{\bar{z}} \left\{ (1 - \hat{g}(z)) \, s\left(z\right) - \frac{\mathcal{T}'_z\left(s\left(z\right), z\right) + s'_{inc}(z)\tau_s(z)}{1 - \mathcal{T}'_z\left(s\left(z\right), z\right)} z\zeta_z^c(z) s'_{inc}(z) - \frac{\tau_s(z)}{1 + \tau_s(z)} s(z)\zeta_{s|z}^c(z) \right\} d\tau_s \, dH_z(z) - \frac{\mathcal{T}'_z\left(z^*, s\left(z^*\right)\right) + s'_{inc}(z^*)\tau_s(z^*)}{1 - \mathcal{T}'_z\left(s\left(z^*\right), z^*\right)} z^*\zeta_z^c(z^*) \, s(z^*) d\tau_s \, h_z(z^*) \quad (102)$$

Optimality. A direct implication of this result is a sufficient statistics characterization of the optimal linear earnings-dependent savings tax $\tau_s(z)$, given an existing and potentially suboptimal nonlinear earnings tax $T_z(z)$. Indeed, at the optimum the reform should have a zero impact on the government objective, meaning that the optimal linear earningsdependent savings tax schedule verifies at each earnings z^*

$$\frac{\mathcal{T}_{z}'\left(s\left(z^{*}\right), z^{*}\right) + s_{inc}'(z^{*})\tau_{s}(z^{*})}{1 - \mathcal{T}_{z}'\left(s\left(z^{*}\right), z^{*}\right)} z^{*}\zeta_{z}^{c}(z^{*})s(z^{*})h_{z}(z^{*})$$

$$= \int_{z^{*}}^{\bar{z}} \left\{ \left(1 - \hat{g}(z)\right)s\left(z\right) - \frac{\mathcal{T}_{z}'\left(s\left(z\right), z\right) + s_{inc}'(z)\tau_{s}(z)}{1 - \mathcal{T}_{z}'\left(s\left(z\right), z\right)} z\zeta_{z}^{c}(z)s_{inc}'(z) - \frac{\tau_{s}(z)}{1 + \tau_{s}(z)}s(z)\zeta_{s|z}^{c}(z) \right\} dH_{z}(z)$$
(103)

Pareto-efficiency. To derive Pareto-efficiency, we combine savings tax and earnings tax reforms in a way that anihilates all lump-sum changes in tax liability in order to offset all utility changes. To do so, consider the following sequence of reforms.

First, consider an *increase* $d\tau_s$ of the income-contingent savings tax $\tau_s(z)$ by phased-in over the earnings bandwidth $[z^*, z^* + dz]$, together with a *decrease* $d\tau_z$ of the marginal tax rate on earnings in the same bandwidth. Imposing $d\tau_z = s(z^*)d\tau_s$, this joint reform cancels out substitution effects at z^* and has the following impact on the government objective:

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{dz} = \int_{z^*}^{\bar{z}} \left\{ (1 - \hat{g}(z)) \, s\left(z\right) - \frac{\mathcal{T}'_z\left(s\left(z\right), z\right) + s'_{inc}(z)\tau_s(z)}{1 - \mathcal{T}'_z\left(s\left(z\right), z\right)} z\zeta_z^c(z)s'_{inc}(z) - \frac{\tau_s(z)}{1 + \tau_s(z)} s(z)\zeta_{s|z}^c(z) \right\} d\tau_s \, dH_z(z) - \int_{z^*}^{\bar{z}} (1 - \hat{g}(z)) \, s(z^*) d\tau_s \, dH_z(z) \quad (104)$$

Second, consider a *decrease* by $d\tau_s$ of the income-contingent savings tax $\tau_s(z)$ phased-in over the earnings bandwidth $[z^* + dz, z^* + 2dz]$, together with an *increase* $d\tau_z$ of the marginal tax rate on earnings in the same bandwidth. Imposing $d\tau_z = s(z^* + \delta z)d\tau_s$, this joint reform cancels out substitution effects at z^* and has the following impact on the government objective:

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{dz} = -\int_{z^*+dz}^{\bar{z}} \left\{ (1-\hat{g}(z))\,s\,(z) - \frac{\mathcal{T}'_z\,(s\,(z)\,,z) + s'_{inc}(z)\tau_s(z)}{1-\mathcal{T}'_z\,(s\,(z)\,,z)} z\zeta_z^c(z)s'_{inc}(z) - \frac{\tau_s(z)}{1+\tau_s(z)}s(z)\zeta_{s|z}^c(z) \right\} d\tau_s \, d\tau_s \,$$

The total impact of these two joint reforms on the objective function of the government is thus

$$\frac{d\mathcal{L}}{\lambda} = \int_{z^*}^{z^*+dz} \left\{ (1-\hat{g}(z)) \, s\left(z\right) - \frac{\mathcal{T}'_z\left(s\left(z\right), z\right) + s'_{inc}(z)\tau_s(z)}{1-\mathcal{T}'_z\left(s\left(z\right), z\right)} z\zeta_z^c(z) s'_{inc}(z) - \frac{\tau_s(z)}{1+\tau_s(z)} s(z)\zeta_{s|z}^c(z) \right\} \, d\tau_s dz \, dH \\
+ \left[s(z^*+dz) - s(z^*) \right] \int_{z^*}^{\bar{z}} (1-\hat{g}(z)) \, d\tau_s dz \, dH_z(z) - s(z^*) \int_{z^*}^{z^*+dz} (1-\hat{g}(z)) \, d\tau_s dz \, dH_z(z) \\$$
(106)

where at the limit $dz \to 0$, the term $(1 - \hat{g}(z)) s(z)$ in the integral on the first line excatly cancels out with the last integral term on the second line.

Third, we consider a *decrease* by $d\tau_z$ of the marginal tax rate on earnings in the bandwidth $[z^*, z^*+dz]$. Imposing $d\tau_z = [s(z^*+dz) - s(z^*)] d\tau_s$, this reform cancels out the other integral term on the second line and generates earnings responses at the margin. As a result, the total impact of this sequence of reforms is at the limit $dz \to 0$ given by

$$\frac{1}{\lambda}\frac{d\mathcal{L}}{dz} \rightarrow \left\{-s_{inc}'(z^{*})\frac{\mathcal{T}_{z}'(s(z^{*}), z^{*}) + s_{inc}'(z^{*})\tau_{s}(z^{*})}{1 - \mathcal{T}_{z}'(s(z^{*}), z^{*})}z^{*}\zeta_{z}^{c}(z^{*}) - \frac{\tau_{s}(z^{*})}{1 + \tau_{s}(z^{*})}s(z^{*})\zeta_{s|z}^{c}(z^{*})\right\}d\tau_{s}h_{z}(z^{*})dz + s'(z^{*})\frac{\mathcal{T}_{z}'(s(z^{*}), z^{*}) + s_{inc}'(z^{*})\tau_{s}(z^{*})}{1 - \mathcal{T}_{z}'(s(z^{*}), z^{*})}z^{*}\zeta_{z}^{c}(z^{*})d\tau_{s}h_{z}(z^{*})dz \quad (107)$$

where the $s'(z^*)$ in the last line comes from the fact that $[s(z^* + dz) - s(z^*)] d\tau_s \rightarrow s'(z^*) dz d\tau_s$. This expression is the impact of this sequence of reforms on the government's tax revenue, while agents well-being remains unchanged. When this impact is non-zero, the type of reforms we consider delivers a Pareto-improvement over the existing tax system. Characterizing a Pareto-efficient tax system as one that cannot be reformed in a Pareto-improving way yields the following sufficient statistics characterization:

$$\frac{\tau_s(z^*)}{1+\tau_s(z^*)}s(z^*)\zeta_{s|z}^c(z^*) = [s'(z^*) - s'_{inc}(z^*)]\frac{\mathcal{T}'_z(s(z^*), z^*) + s'_{inc}(z^*)\tau_s(z^*)}{1-\mathcal{T}'_z(s(z^*), z^*)}z^*\zeta_z^c(z^*) \quad (108)$$

C Details of tax systems by country

In Table 9, we consider five categories of savings subject to various taxation regimes in different countries: (i) wealth, (ii) capital gains, (iii) property, (iv) pensions, and (v) inheritance, which are typically defined in tax codes as follows. First, wealth, which is free from taxation in most advanced economies, is defined as the aggregate value of certain classes of assets, such as real estate, stocks, and bank deposits. Next, capital gains consist of realized gains from financial and real estate investments, and include interest and dividend payments. Third, property consists of real estate holdings, such as land, private residences, and commercial properties. Fourth, for our purposes, pensions are defined as private retirement savings in dedicated accounts, excluding government transfers to retired individuals, such as Social Security in the United States. Lastly, inheritances—also known as estates—are the collections of assets bequeathed by deceased individuals to living individuals, often relatives.

For each country, we label the tax system applied to each category of savings with the types described in Table 8 or "Other," which encompasses all other tax systems. An additional common simple tax structure is a "composite" tax, in which savings and labor income are not distinguished for the purposes of taxation. Composite taxes are often applied to classes of income for which it is unclear whether the income should be considered capital income or labor income. For example, in a majority of the countries in Table 9, rental income—which requires some active participation from the recipient of the income—is subject to composite taxation.

In the subsections below, we have included additional details about the tax system in each country in Table 9. Note that we characterize tax systems that feature a flat tax on savings above an exempt amount as having a separable nonlinear tax system. In addition, when benefits are withdrawn from pension accounts, they are often subject to the same progressive tax rates as labor income. We characterize these tax systems as separable nonlinear rather than composite since benefits are generally received after retirement from the labor force when the taxpayer's income is primarily composed of savings.

Australia

- Wealth: No wealth tax.
- **Capital gains:** Generally a composite tax applies. Gains from certain assets are exempt or discounted.
- Property: At the state level, land tax rates are progressive; primary residence land is

typically exempt. At the local level, generally flat taxes are assessed on property but the taxes can be nonlinear as well, depending on the locality.

- **Pensions:** A flat tax is assessed on capital gains made within the pension account. A component of pension benefits may be subject to taxation when withdrawn, in which case the lesser of a flax tax or the same progressive tax rates as apply to labor income is assessed.
- Inheritance: No inheritance tax.

Austria

- Wealth: No wealth tax.
- **Capital gains:** Generally a flat tax is assessed, with the rate depending on the type of asse; taxpayers with lower labor income can opt to apply their labor income tax rate instead. Gains from certain classes of assets are exempt.
- **Property:** Either flat or progressive tax rates are assessed on property, depending on its intended use. Rates vary by municipality.
- **Pensions:** Generally no tax on capital gains made within the pension account. Pension benefits are generally subject to the same progressive tax rates as labor income, with discounts applicable to certain types of withdrawals.
- Inheritance: No inheritance tax.

Canada

- Wealth: No wealth tax.
- **Capital gains:** For most capital gains, a discount is first applied to the gain and then the discounted gain is added to labor income and taxed progressively. For certain gains, such as interest income, no discount is applied. Lifetime exemptions up to a limit apply to gains from certain classes of assets.
- **Property:** Generally a flat tax is assessed on property, with rates varying by province and locality.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are generally subject to the same progressive tax rates as labor income, with exemptions applicable to certain types of withdrawals.

• Inheritance: No separate inheritance tax. A final year tax return is prepared for the deceased, including income for that year, that treats all assets as if they have just been sold and applies the relevant taxes (e.g., labor income and capital gains taxes) accordingly.

Denmark

- Wealth: No wealth tax.
- **Capital gains:** Progressive taxation with two tax brackets. Gains from certain classes of assets are exempt.
- **Property:** At the national level, property is subject to progressive taxation with two tax brackets. Pensioners under an income threshold can receive tax relief. Land taxes—assessed at the local level—are flat taxes, with rates varying by municipality.
- **Pensions:** A flat tax is assessed on capital gains made within the pension account. Pension benefits are generally subject to the same progressive tax rates as labor income (excluding a labor market surtax), a flat tax, or are exempt from taxation, depending on the type of pension.
- Inheritance: Generally a flat tax is assessed on the inheritance above an exemption, with a higher tax rate for more distant relatives. Transfers to spouses and charities are exempt. Inheritances above a certain value are subject to additional taxes.

France

- Wealth: No wealth tax.
- **Capital gains:** Different rates—progressive and flat—apply to gains from different classes of assets. Certain low-income individuals are either exempt from taxes or can opt to apply their labor income tax rate, depending on the type of asset. High-income individuals are subject to a surtax. Gains from certain assets are exempt or discounted.
- **Property:** Residence taxes are assessed on property users, while property taxes on developed and undeveloped properties are assessed on owners. Rates are set at the local level and apply to the estimated rental value of the property. Exemptions, reductions, and surcharges may apply depending on the taxpayer's reference income and household composition, certain events, and property characteristics. Surcharges may also apply

to higher-value properties. An additional property wealth tax applies at the national level; rates are progressive above an exemption.

- **Pensions:** Generally no tax on capital gains made within the pension account. Pension benefits beyond an exemption are generally subject to the same progressive tax rates as labor income. A flat tax is assessed on certain types of withdrawals, and special rules apply to certain types of accounts.
- Inheritance: Either a flat tax or progressive tax rates are assessed on the inheritance above an exemption, with rates and exemptions depending on the relation of the recipient to the deceased and their disability status. Transfers to spouses/civil partners are exempt. Certain shares are required to pass to the deceased's children.

Germany

- Wealth: No wealth tax.
- Capital gains: Generally a flat tax is assessed on gains above an exemption, but taxpayers with lower labor income can opt to apply their labor income tax rate instead. Gains from certain classes of assets are exempt or subject to special rules.
- **Property:** A flat tax is assessed on property, with rates depending on the class of property and subject to a multiplier, which varies by locality.
- **Pensions:** No tax on capital gains made within the pension account. A portion of pension benefits, which depends on the type of account, is subject to the same progressive tax rates as labor income.
- Inheritance: Progressive tax rates are assessed on the inheritance above an exemption, with tax rates and exemptions both depending on the relation of the recipient to the deceased. Pension entitlements are exempt.

Ireland

- Wealth: No wealth tax.
- **Capital gains:** A flat tax is assessed on gains above an exemption, with the rate depending on the type of asset. Certain classes of individuals, such as farmers and entrepreneurs, qualify for lower rates and additional exemptions.

- **Property:** Progressive tax rates are assessed on residential properties, with local authorities able to vary the rates to a certain extent. A flat tax is assessed on commercial properties, with rates varying by locality.
- **Pensions:** No tax on capital gains made within the pension account. Depending on the type of withdrawal, pension benefits are either subject to the same progressive tax rates as labor income or different progressive tax rates beyond an exemption. A surtax is assessed on high-value accounts.
- Inheritance: A flat tax is assessed on inheritances above an exemption. Exemptions are associated with the recipient and apply to the sum of all inheritances bequeathed to the recipient from certain classes of relatives.

Israel

- Wealth: No wealth tax.
- Capital gains: Generally a flat tax is assessed on real gains (i.e., the inflationary component of gains is exempt). High-income individuals are subject to a surtax.
- **Property:** Generally the tax increases in the area of the property, with amounts depending on property characteristics and varying by municipality. Tax relief may apply to certain taxpayers, such as new immigrants and low-income individuals, depending on the municipality.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are generally subject to the same progressive tax rates as labor income; certain taxpayers qualify for exemptions.
- Inheritance: No inheritance tax.

Italy

- Wealth: A flat tax is assessed on bank deposits and financial investments held abroad, with exemptions on bank deposits if the average annual account balance is below a certain threshold.
- **Capital gains:** Generally a flat tax is assessed on financial capital gains. For certain real estate capital gains, individuals can choose between separable or composite taxation, either applying a flat tax or their labor income tax rate.

- **Property:** Generally a flat tax is assessed on property, with rates depending on property characteristics and varying by municipality.
- **Pensions:** A flat tax is assessed on capital gains made within the pension account, with the rate depending on the type of asset. Pension benefits are also subject to flat taxes, with rates varying with the duration of the contribution period.
- Inheritance: A flat tax is assessed on inheritances, with higher rates for more distant relatives. Different amounts of the inheritance are exempt from taxation for certain close relatives.

Japan

- Wealth: No wealth tax.
- Capital gains: A flat tax is assessed on gains from certain classes of assets, such as securities and real estate, with the rate depending on the type of asset. Progressive tax rates, composite taxation, exemptions, and discounts apply to gains from different classes of assets.
- **Property:** A flat tax is assessed on property above an exemption, with a lower rate or reduction applicable to certain types of property.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are generally subject to progressive tax rates, with the rates depending on the type of withdrawal.
- Inheritance: Progressive tax rates are assessed on the inheritance above a general exemption and an exemption that depends on the relation of the recipient to the deceased and their disability status. A surtax applies to more distant relatives. Certain shares are required to pass to certain relatives.

Netherlands

- Wealth: A progressive, fictitious estimated return from net assets not intended for daily use is taxed at a flat rate depending on the amount above the exemption.
- **Capital gains:** Gains from a company in which an individual has a substantial stake are subject to a flat tax. Most other capital gains are not subject to taxation.

- **Property:** At the municipal level, a flat tax is assessed on property, with rates depending on property characteristics and varying by municipality. At the national level, progressive tax rates are assessed on the fictitious estimated rental values of primary residences, with substantial deductions applicable to the portion of the tax exceeding the mortgage interest deduction.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are generally subject to the same progressive tax rates as labor income, though certain accounts with taxed contributions allow tax-free withdrawals.
- Inheritance: Progressive tax rates are assessed on the inheritance above an exemption, with tax rates and exemptions depending on the relation of the recipient to the deceased and their disability status. Additional exemptions apply to certain classes of assets.

New Zealand

- Wealth: No wealth tax.
- **Capital gains:** Capital gains from financial assets are generally either subject to composite taxation or are exempt from taxation, depending on the type of gain. Special rules apply to certain classes of assets. Capital gains from real estate are generally subject to composite taxation. Depending on transaction characteristics, gains from the sale of commercial property may be subject to an additional tax, while gains from the sale of residential property may be exempt from taxation.
- **Property:** Generally a fixed fee plus a flat tax is assessed on property, with rates set at the municipal level. Low-income individuals qualify for rebates for owner-occupied residential property.
- **Pensions:** A flat tax is assessed on capital gains made within the pension account, with the rate depending on the type of account; for certain accounts, the rate depends on the taxpayer's labor income in prior years. Pension benefits are generally exempt from taxation.
- Inheritance: No inheritance tax.

Norway

- Wealth: A flat tax is assessed on wealth above an exemption, with the value of certain classes of assets, such as primary and secondary residences, discounted.
- Capital gains: A flat tax is assessed on gains from financial assets above the "risk-free" return (i.e., the counterfactual return on treasury bills of the same value). Gains from certain financial assets, such as dividends, are multiplied by a factor before the tax is assessed. A flat tax is assessed on real estate gains, with exemptions for certain types of property.
- **Property:** A flat tax is assessed on discounted property values, with rates varying by municipality and discounts varying by property type.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are generally subject to a lower tax rate than labor income, and taxpayers with smaller benefits qualify for larger tax deductions.
- Inheritance: No inheritance tax.

Portugal

- Wealth: No wealth tax.
- Capital gains: Generally a flat tax is assessed on gains from financial assets, but for certain types of gains, such as interest, low-income individuals can opt to apply their labor income tax rate. For real estate capital gains, a discount is first applied to the gain and then the discounted gain is added to labor income and taxed progressively. Certain classes of real estate are exempt.
- **Property:** Progressive tax rates are assessed on property, with exemptions for certain taxpayers. Rates and exemptions vary based on property characteristics, and an additional exemption applies to low-income individuals.
- **Pensions:** No tax on capital gains made within the pension account, except for dividends, which are generally subject to a flat tax. For different types of withdrawals above an exemption, capital gains are either subject to a flat tax or the same progressive tax rates as labor income when withdrawn. Depending on how contributions were initially taxed and the type of withdrawal, the non-capital gains component of benefits is exempt from taxation, or subject to a flat tax or the same progressive tax rates as labor income on the amount above an exemption.

• Inheritance: A flat tax is assessed on the inheritance, with a higher rate for real estate transfers. Transfers to spouses/civil partners, ascendants, and descendants are exempt (except for real estate transfers, which are subject to a low flat tax).

Singapore

- Wealth: No wealth tax.
- **Capital gains:** Most capital gains are not subject to taxation. Depending on transaction characteristics, composite taxation may apply.
- **Property:** Progressive tax rates are assessed on the estimated rental value of the property, with rates varying by property type and occupancy status.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are generally subject to the same progressive tax rates as labor income; benefits from contributions made before a certain year are exempt from taxation.
- Inheritance: No inheritance tax.

South Korea

- Wealth: No wealth tax.
- Capital gains: Various flat and progressive tax rates are assessed on gains above an exemption; rates and exemptions depend on the type of asset. Gains from certain classes of assets are entirely exempt. Dividends and interest are subject to flat taxation below a certain limit and composite taxation above that limit.
- **Property:** Progressive tax rates are assessed on property, with rates varying by property type.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits beyond a progressive exemption (i.e, greater portions are exempt at smaller benefit levels) are generally subject to the same progressive tax rates as labor income; the exempt amount may also depend on the type of withdrawal and taxpayer characteristics.
- Inheritance: Progressive tax rates are assessed on the inheritance above either a lump-sum or itemized deduction, which depends on the composition of the inheritance and relation of the recipient to the deceased. Transfers to spouses are exempt. The top tax rate increases for controlling shares in a company.

Spain

- Wealth: Progressive tax rates are assessed on net assets above an exemption, with an additional exemption for residences.
- **Capital gains:** Progressive tax rates are generally assessed on gains, with exemptions for elderly individuals under certain conditions and for certain real estate gains.
- **Property:** Generally a flat tax is assessed on property, with rates depending on the property type and varying by locality. Exemptions or discounts may apply depending on taxpayer and property characteristics, including taxpayer income.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are subject to the same progressive tax rates as labor income.
- Inheritance: Progressive tax rates are assessed on the inheritance above an exemption, with tax rates and exemptions depending on the relation of the recipient to the deceased and their disability status. Certain classes of assets, such as family businesses and art collections, are eligible for additional exemptions.

Switzerland

- Wealth: A flat tax is assessed on the net value of certain classes of assets and liabilities, with tax rates and exemptions varying by canton.
- **Capital gains:** Progressive tax rates are assessed on gains from real estate, with rates varying by canton. Most capital gains from financial assets are not subject to taxation. Dividends and interest are subject to composite taxation.
- **Property:** Generally a flat tax is imposed on property, with rates varying by canton; a minimum amount per property may apply. For owner-occupied properties not rented out, an estimated rental value is subject to composite taxation.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits are subject to either the same progressive tax rates as labor income or lower progressive tax rates, depending on the type of withdrawal.
- Inheritance: In most cantons, progressive tax rates are assessed on the inheritance and depend on the relation of the recipient to the deceased. Transfers to spouses and children are exempt in most cantons.

Taiwan

- Wealth: No wealth tax.
- Capital gains: Most capital gains from financial assets are subject to composite taxation; taxpayers can opt for a flat tax to be assessed on dividends, and certain gains are exempt from taxation. A flat tax is assessed on gains from real estate, with the rate depending on the type of asset, and an exemption for primary residences.
- **Property:** Flat or progressive tax rates are assessed on land, depending on its intended use. A flat tax is generally assessed on buildings, with rates depending on their intended use. Certain classes of land and buildings are exempt or subject to reduced rates.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits beyond an exemption—which depends on the duration of the contribution period—are subject to the same progressive tax rates as labor income.
- Inheritance: Progressive tax rates are assessed on the inheritance above an exemption, which depends on the relation of the recipient to the deceased, their disability status, and their age.

United Kingdom

- Wealth: No wealth tax.
- **Capital gains:** Either flat or progressive tax rates are assessed on gains, with rates depending on the taxpayer's labor income tax bracket; higher rates generally apply to taxpayers in higher labor income tax brackets. Exemptions for part or all of the gain apply to certain types of assets, such as dividends and primary residences.
- **Property:** Progressive tax rates are assessed on property, with rates varying by locality. Exemptions or discounts may apply to certain taxpayers depending on characteristics, such as age.
- **Pensions:** No tax on capital gains made within the pension account. Pension benefits beyond an exemption are subject to the same progressive tax rates as labor income. An additional flat tax may be imposed on accounts with a value exceeding a lifetime limit, with the tax rate depending on the type of withdrawal.
- Inheritance: A flat tax is assessed on the inheritance above an exemption, with larger exemptions for transfers to children. Transfers to spouses/civil partners, charities,

and amateur sports clubs are exempt. The tax rate is reduced if a certain share is transferred to charity.

United States

- Wealth: No wealth tax.
- Capital gains: Gains from "short-term" assets (held for less than a year) are subject to composite taxation. Gains from "long-term" assets are subject to a flat tax, with higher rates for higher-income individuals. Dividends are also subject to either composite taxation or flat taxes that increase with labor income, depending on their source.
- **Property:** Generally a flat tax is assessed on property, with rates varying by state, county, and municipality.
- **Pensions:** No tax on capital gains made within the pension account. Depending on the type of account, benefits are generally either exempt from taxation or subject to the same progressive tax rates as labor income.
- Inheritance: Progressive tax rates are assessed on the inheritance above an exemption. Transfers to spouses are generally exempt.

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DOCTORALE

ECOLE

Titre : Essais sur le barème et la structure des systèmes socio-fiscaux

Mots clés : redistribution optimale, système socio-fiscal, complexité, inattention, assurance chômage, fiscalité du capital

Résumé : Cette thèse de doctorat étudie le barème et la structure des systèmes socio-fiscaux. Le chapitre 1 propose une analyse descriptive du système sociofiscal français et calcule les incitations monétaires au travail à l'aide du modèle de micro-simulation TAXIPP. Le chapitre 2 montre que l'inattention des individus aux taxes conduit les gouvernements à augmenter de façon inefficace les taux d'imposition. Ce biais à l'imposition modifie la progressivité des taxes et éclaire d'un jour nouveau la complexité des systèmes socio-

fiscaux. Le chapitre 3 explore les interactions entre redistribution optimale et assurance chômage optimale. Il s'avère efficace, pour des motifs redistributifs, d'implémenter une assurance chômage avec des taux de remplacement qui décroissent avec le revenu. Le chapitre 4 revisite l'hétérogénéité des préférences comme justification à l'imposition du capital via une caractérisation du barème optimal en termes de statistiques suffisantes.

Title : Essays on the design of tax-benefit systems

Keywords : optimal redistribution, tax-benefit system, complexity, inattention, unemployment insurance, capital taxation

Abstract : This PhD dissertation studies the design of tax-benefit systems. Chapter 1 provides a descriptive analysis of the French tax-benefit system, and computes agents' monetary incentives to work using TAXIPP micro-simulation model. Chapter 2 argues that agents' inattention to taxes induces governments to inefficiently increase tax rates. This taxation bias affects the progressivity of tax systems and sheds a new light on the welfare effects of tax complexity. Chapter

3 explores the interactions between optimal redistribution and optimal unemployment insurance in a novel theoretical framework. It shows that it is efficient, for redistributive purposes, to implement an unemployment insurance with replacement rates that decrease with earnings. Chapter 4 revisits the argument of preference heterogeneity for capital taxation by providing a novel characterization of optimal taxes in terms of estimable sufficient statistics.

