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# Global Warming, Climate Policies and their Consequences on the Business Cycle: a general equilibrium Approach

Maxime Alexandre Bouter

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# THÈSE

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de l'Université de Pau et des Pays de l'Adour  
**Spécialité : Sciences Économiques**

## Global Warming, Climate Policies and their Consequences on the Business Cycle: a General Equilibrium Approach

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## Abbreviations

**AR5** fifth Assessment Report

**BAU** Business As Usual

**BTU** British Thermal Unit

**CDM** Clean development mechanism

**CSP** concentrating solar power

**DICE** dynamic integrated climate-economy

**DSGE** Dynamic Stochastic General Equilibrium

**EKC** environmental Kuznet curve

**eq.** equivalent

**ETS** Emissions trading system

**FOC** first order conditions

**GHG** greenhouse gas

**Gt** Gigaton

**GW** Giga Watt

**GWP** Global warming potential

**IPCC** Intergovernmental Panel on Climate Change

**kW** kilo Watt

**kWh** kilo Watt hour

**LCOE** Levelized cost of energy

**LHS** left-hand side



**LULCF** Land Use Land Use and Change and Forestry

**Mbd** Million barrels per day

**Mtoe** Million ton of oil equivalent

**MW** Mega Watt

**OECD** Organisation for economic co-operation and development

**ppm** parts per million

**RHS** right-hand side

**RICE** regional integrated climate-economy

**TWh** Terawatt-hour

## **Nomenclature**

$CO_2$  carbon dioxide

$CH_4$  methane

$N_2O$  nitrous oxide



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# Abstract:

Recognised by the United Nations as one of the eight Millennium Development Goals, the preservation of the environment and consequently of the climate, requires strong intervention by public authorities in order to limit the negative consequences of climate change. This thesis proposes to study some aspects of climate policies and the consequences they have on the economic cycle. We first address the issue of climate reversibility. One of the particularities of greenhouse gases is their particularly long lifetime. While on a geological scale, carbon sinks could in theory absorb a major part of anthropogenic emissions, on a human scale global warming appears irreversible without a transformation of the world economy towards a decarbonised model. In the absence of the discovery of a technology capable of reducing the carbon intensity of energy, public policy seems to be the solution most likely to meet the 2°C limit by the end of the 21<sup>st</sup> century. Secondly, we compare the consequences of introducing different climate policy instruments (carbon tax, allowances market, intensity target) in an economy that is subject to different exogenous shocks (productivity shock, shock on the price of fossil fuels). To this end, we use an RBC DSGE model to assess the economic consequences of these different measures. The implementation of one of these instruments can indeed have a contractionary effect, or on the contrary be inefficient from a climate point of view in periods of expansion. We therefore recommend the joint use of an allowance market, with the establishment of a floor price for carbon in order to achieve the emission reduction target. Finally, we look at the links between climate policy and monetary policy. We optimise the Taylor rule in the presence of either an allowance market or a carbon tax in order to make monetary

policy recommendations. We show that the Taylor rule coefficients that maximise consumer welfare are significantly different with the introduction of climate policy. These effects should therefore be taken into account in order to conduct the most effective environmental policy.

# Chapter 1

## A public policy, for a public good

### 1.1 Introduction

Global warming has been officially and internationally recognized as an issue in 1972 during the United Nations Conferences on the Human Environment which was held in Stockholm. It resulted in the creation of the United Nation Environment Program (UNEP). In December 1997 the Kyoto Protocol was adopted, and it forced developed countries to stabilize at least or to reduce their greenhouse gases (GHG) emissions. Implemented in 2005 for seven years, it could be seen as the first binding agreement whose objective was to reduce our environmental impact and to switch towards a mode of development more respectful of the current and future generations. Among the goals to aim at, the 2009 Copenhagen Summit has included "*limiting global temperature increase to 2 °C relative to the pre-industrial levels*". Nonetheless, it has been disappointing because no agreement has been found to include the United States and China, without which it seems impossible to find a solution to climate change (Colombier and Ribera, 2015). In 2015, 175 countries signed the Paris agreement which also recognizes the necessity to limit the temperature rise to an average of 2 °C. Nevertheless, current countries commitments to reduce GHG emissions do not allow to stay under this limit (Waisman et al., 2016).

To meet the target of either 2 or 1.5°C of global warming on average, developed

countries must drastically cut their GHG emissions. According to the fifth Assessment Report (AR5) of the Intergovernmental Panel on Climate Change (IPCC) carbon dioxide atmospheric concentration should not exceed 430 parts per million (ppm) to limit global warming to 1.5 °C and stay under 450 to 480 ppm for the 2 °C limit (Pachauri et al., 2014) . That corresponds to a cut in global emissions by 41 to 72% in 2050 compared with the 2010 level and of 78 to 118% at the end of the century for the latter scenario. The pathway that keeps global warming under 1.5 °C requires an annual emission flow below 35 Gigaton (Gt)  $CO_2$  equivalent (eq.) per year by 2030, which corresponds to a 45% decline of global anthropogenic emissions compared to the 2010 level (25% for the 2°C target) and the world should reach zero net emissions by 2070 (IPCC, 2018). According to the last Emissions Gap Report (UNEP, 2019) anthropogenic GHG emissions continue to rise in spite of scientific warning. Over the last decade GHG emissions have risen annually by 1.5 %, and emissions from fossil fuels consumption rose by 2% in 2018. Worse, there is currently no evidence of reaching a peak in the next years that would allow to reach the objective of either 1.5 or 2°C by the end of the century.

Almost 50 years after the Stockholm conference, almost 30 years after the signing of the Kyoto Protocol, the climate issue has still not been resolved. Introducing an effective climate policy is not easy. Climate and the negative externalities of emissions are global issues, and the consequences are unequally distributed geographically. Some countries will benefit from global warming while other countries will lose out. Some countries therefore have an interest in implementing climate policies, while others have no incentive to do so - especially fossil fuel producing countries (Sinn, 2012). Moreover, as climate is a global public good, it is subject to the free-rider behaviour. Some countries are shifting the effort to reduce emissions to other countries, thereby reducing the effectiveness of existing climate policies and introducing distortions that can lead to the relocation of polluting industries to these pollution havens. For these various reasons, since the Kyoto Protocol, no international agreement has really forced the various Parties to reduce their GHG emissions. The longer we wait, the greater the effort required to limit global warming to socially acceptable levels and the greater the risk that global warming will

become irreversible (Stern and Stern, 2007).

To be efficient, these climate policies must be properly designed, at the risk of being ineffective, or worse, counterproductive (Sinn, 2008, 2012). The economic crisis of 2008 has spilled over into the European carbon market, and due to a variety of mechanisms, the price of carbon has stagnated at an abnormally low level to allow the European Union to make the low-carbon investments necessary to achieve a sustainable reduction in GHG emissions (Laurent and Le Cacheux, 2009). In France, the continuous increase of the carbon tax has led to the protest movement of the Yellow Vests, as this tax has not adapted to international fluctuations in the price per barrel. Poorly designed, the consequences could be deplorable in the long term, if the population challenges this instrument. The current COVID health crisis may also have significant consequences for existing climate policy instruments if they have been poorly designed.

In the present thesis we contribute to the macroeconomics literature on climate policies. We first address the issue of climate irreversibility in Chapter 2. Chapters 3 and 4 compare the different climate policy instruments available to public authorities, when the economy is subject to different exogenous shocks, in general equilibrium models. The models of chapters 2 and 3 assumed a neoclassical analytical framework, where prices are perfectly flexible. The short-term consequences in an economy with nominal rigidities are considered in Chapter 4. The purpose of the current chapter is to describe the various concepts that will be of particular interest to us. Namely, the past causes of global warming, the future consequences, but also climate as a public good and possible solutions. Finally, we propose a review of the literature, and summarize our contribution to it.

## 1.2 The climate change issue

In this section, we briefly introduce the nature of GHG emissions, its implications on climate change, and the options currently available to manage it and diminish risks.

### 1.2.1 Greenhouse Gases and their Effect on Global Atmospheric Warming:

The earth's atmosphere is composed by several gases (78% of dinitrogen, 21% of dioxygen, and less than 1% of methane, water vapor, carbon dioxide, argon, helium), and one of its role is to regulate the temperature. Thanks to the presence of GHG<sup>1</sup> in the atmosphere (and obviously to a distance neither too near nor too far from the sun), the average temperature on the earth's surface is around 15°C which ensures conditions for life to exist, to compare with Venus 450 °C (due to the presence of an atmosphere) and Mars -50 °C (due to the lack of an atmosphere). The mechanism linking the presence of GHG and the earth temperature is the following: gases trap a part of the heat from sun rays, while another part is reflected and escaped into space.

In 1824, the French mathematician J. Fourier published "*On the Temperature of the Terrestrial Sphere and Interplanetary Space*" in which he described the greenhouse effect. In 1859, John Tyndall reproduces by experimentation the greenhouse gas effect in his laboratory, and identifies molecules responsible for it. In 1896, the Swedish chemist Svante August Arrhenius quantified the greenhouse effect and estimated that doubling the amount of GHG in the atmosphere would increase the temperature at the earth's surface by 5 °C on average. Thus, by increasing the quantity of GHG in the atmosphere, the greenhouse gas effect increases, more energy remains at the earth's surface and finally temperature mechanically rises, which could at last disturb life.

Since the Industrial Revolution, human activities have used fossil fuels (coal, oil, natural gas) that had for consequences to increase the concentration of  $CO_2$  and others GHG in the atmosphere. The GHG concentration present in the atmosphere depends on the difference between annual emissions flows, and the natural earth absorption capacities

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<sup>1</sup>GHG are gases that trap heat. The main GHG emitted from economic activities are carbon dioxide (or  $CO_2$ ), methane (or  $CH_4$ ) nitrous oxide (or  $N_2O$ ), and fluorinated gases (ex: hydrofluocarbons, perfluorocarbons, sulfur hexafluoride). They differ from each other by their lifespan - each of these gases remain in the atmosphere for different amounts of time from years to thousands of years, and by their warming potential - GHG affected more or less climate. Each GHG lifespan and warming impact are detailed in Chapter 2, in table 2.2, p.44.

(oceans, forests, and other sinks). When the former exceed absorptions capacity, GHG stock increases. According to Pachauri et al. (2014), in 2011 the  $CO_2$  stock in the atmosphere around was 390 parts-per millions or ppm, i.e. 40% higher than in 1750. Global temperature is increasing, and the last three decades have been the warmest over a period of 1400 years. On average, between 1880 and 2012, average temperature has risen between 0.65 and 1.06°C in the Northern hemisphere (Pachauri et al., 2014).

Consequences of climate change in the medium and long run highly depend on the future pathway of GHG concentration in the atmosphere. The IPCC considers four different scenarios, depending on the use of fossil fuels in the next decades. In the first case (RCP 2.6), climate change is limited thanks to a radical change in human behaviour corresponding to a decrease in the GHG emission flow by at least 41% in 2050. In the worst case, GHG stock will be multiplied by a factor 4 relatively to the current levels if we still continue to use coal and oil in the future, in that case it would yield an increase of at least 8°C of the earth's surface temperature. Two intermediates scenarios can lead to an increase of the temperature in between 3 and 4°C at the end of the 21<sup>st</sup> century.

### 1.2.2 Consequences and feedback effects of climate change

The effects of global warming are numerous, complex and uncertain (see Pachauri et al. (2014), Stern and Stern (2007) and Wagner and Weitzman (2015)). Forecasting the effects of climate change in medium and long run is not obvious because it depends both on a large number of parameters and also on exogenous stochastic phenomenons - volcanic eruptions, meteorite falls, variable solar activity. For example, in 1991, Mont Pinatubo erupted and emitted over 20 million tons of sulphur dioxide into the atmosphere, which had the effect of lowering the temperature on the earth's surface for a few months to pre-industrial levels (Wagner and Weitzman, 2015). However assuming no exogenous stochastic shocks, depending on the trend of GHG stock, it is possible to anticipate the main consequences of climate change.

The first visible effect of the rise in temperature is the polar ice melting, but also the permafrost, and Greenland ice sheet. Glaciers volume, from all around the world tends



to decrease, because of a diminishing number of snowfall days and ice melting. As a consequence, sea level is rising, but also affecting both pH and salinity, which eventually modify ecosystems affecting a large number of species. One purpose of ice is to reflect a part of sun rays. If a too large amount of ice disappear, one consequence is to increase the temperature because of the absence of the ice reflection capacity due to its white surface or albedo effect. In that sense, it is possible to reach a GHG stock threshold above which a feedback effect is self-sustaining. In other words, if the atmospheric GHG stock reaches this threshold, even if human activities stop using fossil fuels, rising in temperature will be self-sustained.

In Chapter 2 we look at this question of the irreversibility of climate. An exogenous reduction in GHG emissions is assumed and various scenarios (intervention versus no climate policy) are compared in a Real Business Cycle Integrated Assessment Model to see whether or not climate is reversible. We base our analysis on the Golosov et al. (2014) Real Business Cycle Integrated Assessment Model, in particular we use their climate module which reproduces the dynamics of the atmosphere and the carbon cycle. According to the results we obtain, climate change is irreversible and a climate policy must be put in place as soon as possible to achieve the IPCC objectives. In the absence of rapid intervention, adaptation to climate change would be the only option left, with potentially high costs.

The consequences are unevenly distributed among the different regions of the world (Change, 2014). Some, in cold or temperate climates, would be winners, while others would be more or less losers, but overall the negative consequences outweigh the positive ones. In the best case, a limited increase in temperature will affect essentially the Arctic Region, Africa, and the tropics. In this scenario, global ice surface would decrease by 20% (IPCC, 2014). In the worst case, a large part of arable land could become barren soils in Africa, South America, and in the south of the USA. Due to the sea level rise, archipelagos, coastal regions and low altitude areas could be flooded. Extremes events such as floods, droughts, hurricanes, storms would become more frequent and increase the costs for all societies (IPCC, 2014). Animal survival depends on their ability to move

to another territories in order to adapt to climate change. The faster climate change is, the higher the likelihood of species loss is. According to the United Nation Refugee Agency, in 2008 22.5 million people were displaced by climate, most of the concerned people are concentrated in the most vulnerable areas of the world, and the number of climate refugees will unfortunately increase in the next decades. In addition, climate change consequences on human life are various and would lead to an increase in poverty, conflicts and would affect health negatively .

### 1.3 The greenhouse gases externality and the climate as a public good

Climate change is inseparable from public goods and GHG emissions are negative externalities. A public good is characterized in economy by two properties: non-exclusion and non-rivalry. Samuelson (1954) defines a public good as having the property that "*one man's consumption does not reduce some other man's consumption*". The principle of externalities in economics is based on the market failure and miss-allocated properties rights. In the case of climate change, those who pollute do not take into account the negative consequences of their use of fossil resources.

#### 1.3.1 Greenhouse gases and externalities

The concept of externalities has been first introduced by Sidgwick (1901) and Pigou (1920). According to Pigou (1920) some times it exists a difference between the "*marginal social net product*" and the "*marginal private net product*" which he called the external effect<sup>2</sup>. Many economists tried after him to define the concepts of externalities. Among

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<sup>2</sup>Pigou (1920) defines the "*marginal net product of resources [or factor]*" as the "*result of the marginal increment of resources [or factor] employed there*". However, he distinguishes "*the marginal social net product*" from "*the marginal private net product*". He defines the former as "*the total net product of physical things or objective services due to the marginal increment of resources in any given use or place, no matter to whom any part of this product may accrue*". On the other hand, he defines the latter as "*the part of the total net product of physical things or objective services due to the marginal increment*"

others, we can successively cite Meade (1952), Scitovsky (1954), Buchanan and Stubblebine (1962), Mishan (1971), Baumol and Oates (1988). However we will retain the Baumol and Oates (1988) definition of externality :

*"Condition 1. An externality is present whenever some individual's (say A's) utility or production relationships include real (that is nonmonetary) variables, whose values are chosen by others (persons corporations governments) without particular attention to the effects on A's welfare. [...] It has also been suggested that for a relationship to qualify as an externality it must satisfy a second requirement: Condition 2. The decision maker, whose activity affects others' utility levels or enters their production functions, does not receive (pay) in compensation for this activity an amount equal in value to the resulting benefits (or costs) to others"* (Baumol and Oates, 1988, p. 17).

Following Bator (1958), Baumol and Oates (1988) make the distinction between "*public*" and "*private*" externalities. Indeed, many externalities affect not only one but several agents, and have the characteristics of a public good. For example, if a firm is responsible for polluting air, one person breathing the pollution will obviously not reduce the amount of pollution available to the rest of the population. In that case, the presence of the public externality results from a suboptimal system of price, because "*agents take into account only the direct effects upon themselves, not the effect on others, the decisions they make are likely not to be "efficient"*". *Air and water pollution are perhaps the most notable examples*" (Atkinson and Stiglitz, 2015, p. 7).

GHG emissions belong to the concept of negative externalities, but they also have specific characteristics that distinguish them from other externalities. Climate change is

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*of resources in any given use or place which accrues in the first instance to the person responsible for investing resources there".* When the marginal private net product exceeds the marginal social net product, costs are not supported by the right people. In the specific case of climate change, GHG emitters do not support the consequences of using fossil fuels. Climate change damage such as floods drought or diseases, will be supported by the entire society. It is the social cost, or the negative externality of GHG emissions. In order to achieve the social optimum again such that the "*the marginal social net product*" is equal to "*the marginal private net product*", Pigou's idea is to introduce a tax for the resource user responsible for emissions, which reflects the marginal social cost following the use of the resource.

global. When an agent emits GHG in one part of the world, the effect is not localized, it dissipates. It is also timeless, GHG have a relatively long lifespan<sup>3</sup>, so today's pollution will affect future generations. Finally, because of delayed and very long-term effects, they are also characterized by a high degree of uncertainty (Stern and Stern, 2007). Generally when a household or a firm uses fossil fuels, its price does not reflect the negative effects of GHG emissions. Marginal damages exceed marginal benefits because of a market failure, the price does not internalize the externality of fossil fuel consumption.

### 1.3.2 Possible solutions to climate change

A distinction has to be made between technological solutions and regulatory or policy solutions. Geoengineering is one possible technological solution. According to Wagner and Weitzman (2015) the 1991 eruption of the Mount Pinatubo volcano released 20 million tons of sulfur dioxide, a very small amount compared to anthropogenic GHG emissions since the beginning of the industrial era (585 billion tons). However, this small quantity of sulfur dioxide has reduced the global temperature by 0.5°C and has temporarily totally compensated for the rise in temperature due to the greenhouse effect. However, using this kind of geoengineering solution is not viable for at least two reasons. On the one hand, the long-term consequences linked to the emission of such particles are not known. According to Wagner and Weitzman (2015), these emissions of sulfur dioxide following the eruption may have been responsible for a rise of droughts and floods. On the other hand, the use of these particles does not solve the problem in the long term. Indeed, as long as fossil fuel consumption behaviour remains the same, GHG will continue to accumulate in the atmosphere. Consequently, the problem is postponed in time, and the consequences will be all the more important in the long term as the GHG stock in the atmosphere will be higher. Other technical solutions, such as renewable energy production, would make it possible to substitute the use of fossil fuels with non-polluting energy sources. As long as the production cost of these energies exceeds that of fossil

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<sup>3</sup>See table 2.2, p.44

fuels, substitution cannot take place<sup>4</sup>. Thus, in order to use relatively more non-polluting energies in the energy mix, the relative price of energies must be changed. In the absence of an increase in productivity, or a reduction in production costs, public authorities can use environmental policy instruments, which leads us to the second category of solutions that can limit global warming: policy regulations. Instruments available to policy-makers to avoid externalities of production or consumption may be divided into two groups. The first group contains prices instruments while the second uses quantity instruments. The next two sub-sections develop these two types of instruments: if well designed they are able to restore the social optimum through a better definition of property rights, or to reduce the cost of GHG abatement and consequently a greater use of low-carbon technologies.

### 1.3.2.1 Prices instruments

Pigou (1920) was the first to propose an instrument able to equalize marginal social cost to marginal social benefit of pollution (situation defined as the optimum). According to the polluter-payer principle, the implementation of a Pigouvian tax per unit of emissions, permits to equalize the marginal damage of pollution to the marginal benefit. Nevertheless, policy-makers need to exactly know the marginal damage function in order to implement the optimal tax rate: a lack of information on the real damage function would lead to a suboptimal situation. If the tax rate is lower than the optimum, polluters will emit more than the desired amount of GHG, and in contrast a higher tax will lead to the opposite result a suboptimal quantity of pollution. In order for the tax to be easily accepted by the population, some economists have argued in favor of recycling tax revenues by reducing another tax level such as capital income tax or labour taxes. The idea underlying is to attain a "*double dividend*": obviously the first goal is to increase

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<sup>4</sup>Another crucial element is the elasticity of substitution between polluting and non-polluting energies. In the presence of complementary energies, the use of renewable energies cannot be dissociated from fossil fuels. On the other hand, if they are substitutable, it is possible to replace the use of one energy source by another.

the environmental quality, the second one is to reduce the tax distortion thanks to the recycling revenue. This theory has been initially developed by Goulder (1995), Bovenberg (1999) and it has been challenged by Fullerton and Metcalf (1997). More recently, McKibbin and Wilcoxon (2013) have also discussed this subject.

The second price instrument is a subsidy to clean up production or consumption. Theoretically, a subsidy or a tax are equivalent in the short run, in their ability to reach the optimum. However if firms prefer the former instrument to the latter, they are not equivalent in the long run. Indeed the subsidy could permit to some projects to be profitable, while in the absence of this policy they certainly would not have existed. Therefore, at a micro level, it permits to reduce GHG emissions thanks to more efficient technology, nevertheless at a macro level, aggregate pollution can exceed initial level because of a larger number of emitter (see Spulber (1985)).

### 1.3.2.2 Quantity instruments

Instead of a price instrument, a policy-maker can use an instrument which have an effect on quantity, such as technical standards or quotas. Tietenberg and Lewis (2018) defined a standard as "*a legal limit on the amount of the pollutant an individual source is allowed to emit*" (Tietenberg and Lewis, 2018, p. 370). It consists in a ceiling or a bound not to be exceeded. This kind of instruments is not optimal from an economic point of view because it sets the same limit to all polluters. For some this standard would be too low while for others it would be too high. Moreover, it could also lead to a Jevons paradox<sup>5</sup>.

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<sup>5</sup>Think of a policy implementing a standard in order to increase the fuel efficiency of cars and therefore reducing oil consumption per kilometer. Apparently it seems equivalent to introduce a fuel consumption tax. However they are not the same. Indeed, on the one hand the fuel tax increases the marginal cost per kilometer driven so that drivers will tend to reduce their car utilisation or to substitute it to others means of transport (like bus, bicycle, train). On the other hand, a fuel-economy standard implies a reduction in the marginal cost of using cars. Because each kilometer driven costs less than before, car owners may use their cars more due to a more efficient engine. For this reason, there is no incentive to substitute their car utilization for others more environmental friendly means of transport. Eventually, a standard may have a positive, a negative or no effect. In the first case, the reduction of oil consumption

The last quantity instrument available to policy-makers is quotas market or cap-and-trade market. They are directly inspired by the analysis of Coase (1960). Often externalities result from the absence of market or from improperly designed property rights. A well defined property right must have the following three characteristics: 1. Exclusivity - benefits and costs are attributable to the owner ; 2. Transferability - a property right must be transferable (free of charge or not) ; 3. Enforceability - property right should be secured by law. Obviously, property right may not only be private, but also co-ownership or state-ownership (for example national parks). The global warming issue can be explained by a lack of property right because of an open-access to climate (nobody owns the atmosphere). This phenomenon has become famous with the "*Tragedy of commons*" developed by Lloyd (1833) and Hardin (2016). Coase's (1960) intuition to solve this market failure, was to argue that the policy-maker is not obliged to implement a Pigouvian tax, but on the contrary if government is able to establish adequate property rights, the market will by adjustment between supply and demand go towards equilibrium. Theoretically this solution seems to be easier to implement compared to a tax. As explained above, the policy-maker needs to know exactly the marginal damage function in order to introduce an efficient tax. In the case of a quota market, authorities first defined a target for emissions, and then distribute the quotas or Rights to Pollute to emitters. In a second step the latter may exchange these rights on a market. Those who have too many quotas will sell them, while those who have not enough will buy them on the market. If the number of quotas is well defined, a price will emerge and will be equal to the optimal price able to reduce the pollution to the desired level. However, if the number of marketable permits is initially over-evaluated the emission reduction will be lower than the optimal reduction in emissions. Because authorities do not know ex ante with certainty the marginal damage function, in order to reveal it ex post, it is necessary to adjust step by step the number of negotiable permits in a first phase.

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effect is larger than the increase of car utilization, in the second case the two effects are equivalent while in the last case the policy effect is lower than the income effect.

## 1.4 Energy and pollution in the business cycle:

Although macroeconomics has largely studied the role of energy in the economic cycle, little work has focused on the macroeconomic consequences of pollution from fossil fuel use, and by extension the consequences of environmental policies in the economic cycle. We contribute to this literature, which is still under-explored.

### 1.4.1 The role of energy in the business cycle:

The first models to include energy in DSGE models looked at the role of this production factor in the business cycle and in particular the consequences of exogenous oil price shocks on the economic cycle. Kim and Loungani (1992) are among the first to contribute to the RBC model literature in this regard. They integrate energy into the production function, and consider the price of energy as exogenous. The basic idea is to stress that the cycle cannot be explained exclusively by technology shocks as in Kydland and Prescott (1982) or Hansen (1985) models. Thus, they want to take into account the role played by the oil shocks of the 1970s and 1980s and their economic consequences on the United States, particularly their consequences in terms of unemployment. Rotemberg and Woodford (1996) start from the same empirical finding on the significant impact of energy prices on economic activity and on the negative relationship between higher intermediate costs and lower production levels. They examine energy price shocks by considering either a perfect competition or imperfect competition model. They show that the imperfectly competitive model is better able to reproduce the consequences of energy shocks in terms of lower output and lower wages. They find that nominal rigidities play a key role in the transmission of energy shocks to the rest of the economy. Finn (1991, 2000) also studied the role of energy shocks on economic activity. She introduces an endogenous capital utilization variable, and assumes that the energy shock will dissipate to the economy through a variation in capital utilization and consequently by capital services (Finn, 1991). Finn (2000) in a more recent article, she assumes a pure and perfect competition model, and still demonstrates that even in that case energy shocks have recessionary



effects. For this purpose, she assumes a variable capital utilisation rate. In addition, she also considers energy as essential to use capital and to produce goods. Capital without energy is unusable, so there is a complementary relationship between these two inputs. She thus shows that, even in a context of pure and perfect competition, and contrary to Rotemberg and Woodford (1996) conclusion, the negative consequences of energy shocks can be explained also in the absence of nominal rigidities. More recently Dhawan and Jeske (2008) investigated the issue of energy shocks in the presence of durable goods. In their DSGE model, they introduce two goods that they distinguish according to their lifetime: a consumable good and a durable good. They also introduce energy into the utility function of the consumer. According to their results, output volatility is lower in this context, following a shock on energy prices. The household has to make a trade-off between investment in capital, or consumption of durable goods. Because the shock will have a greater impact on the consumption of durable goods, the capital stock will fall, but relatively less than in the other DSGE models like Kim and Loungani (1992). The authors find that energy does not play a significant role in explaining the volatility of output.

These studies have so far considered the price of energy to be exogenous. However, recent empirical studies tend to show that the price of energy is not purely exogenous. It is determined both by the supply of fossil fuel producing countries and by global demand (see Kilian (2009), Kilian and Murphy (2012, 2014) and Baumeister and Peersman (2013a,b)). Thereafter, other studies made the effort to endogenize the price of oil. Among this literature are the work of Backus and Crucini (2000), who developed a three-country model. Bodenstein et al. (2011a,b), also developed an international trade model to analyse the issues of energy price shocks. Nakov and Pescatori (2010), and Bodenstein et al. (2012) looked at the issue of optimal monetary policy in response to energy-price shocks.

The importance of the role of oil as a factor of production and the consequences of oil prices shocks in the economic cycle no longer needs to be demonstrated. More broadly, we can extend this observation to various energies inputs like others fossil fuel, or non-

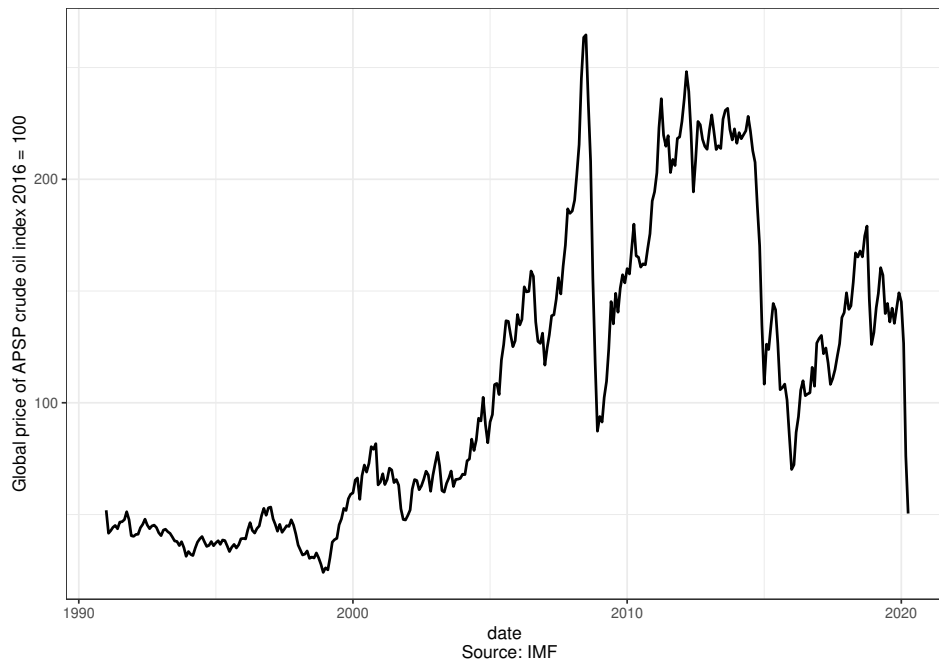


Figure 1.1 – Monthly crude oil price index 2016 = 100

polluting energies. As a result, not only oil price shocks may have consequences, but also renewable energies price shocks can have significant consequences for the rest of the economy. Obviously, these results would be influenced by the elasticity of substitution between energies inputs. For Kilian and Hicks (2013), the rise in the price of oil observed between 2000 and the economic and financial crisis of 2008 (see Figure 1.1) was due to the increase in demand for oil in emerging Asian economies, which experienced high rates of economic growth in the first decade of the 21<sup>st</sup> century. The decline in global demand at the time of the crisis resulted in a temporary drop in prices. The price is rising again despite the contraction of economic activity in developed countries, driven in particular by China's energy demand (Antonin et al., 2015). The decline in oil prices from 2014 onwards is partly due to lower demand from emerging and developing countries experiencing a slowdown in economic activity. Political instability in several oil-producing and exporting countries accounts for the other part of the decline in oil prices (Baumeister and Kilian,

2016). The exploitation of shale gas in the United States through hydraulic fracturing has reduced the demand for imported oil. As a result, the supply of available oil on world markets has increased, explaining the downward trend in oil prices (Kilian, 2017). On the other hand, as the break-even point for onshore oil production is lower in Saudi Arabia (see Figure 1.2), the latter has tried to exclude this new player from the market by increasing its production, thus further lowering the oil price. This strategy did not work since in 2015, the US Congress authorized oil exports, a first since the oil shocks of the 1970s, and the United States became the leading oil producer from 2017. Facing this failure, Saudi Arabia and the other members of OPEC decided to lower their production. The resulting increase in the world barrel price was reflected in France by the Yellow Vests movement, which saw the price of gasoline at the pump rise adding to the increase in the carbon tax. The global coronavirus health crisis, which has forced governments to lock-down more than a third of the world's population, has reduced the demand for oil. Saudi Arabia and Russia took advantage of this situation to try once again to exclude the United States from world markets. The imbalance between supply and demand was such that the West Texas intermediate crude oil price became negative on Monday April 20<sup>th</sup> 2020. These variations in the world price of oil, and more broadly in fossil fuel prices in general, play a determinant role in the demand for oil and consequently in GHG emissions. Including the possibility of exogenous shocks on polluting energies is therefore essential to understand the mechanisms of environmental policies in the business cycle. These exogenous variations in energy prices are taken into account in Chapters 3 and 4. Two DSGE models are developed in which energy is included as a factor of production and also as a source of GHG emissions. In Chapter 3, the DSGE model is based on the one developed by Fischer and Springborn (2011) (see next section), in which the price of energy is considered as an exogenous source of uncertainty. In the manner of Finn (1991) and Finn (2000), climate policies are evaluated by assuming a variable rate of capital utilization. Chapter 4 develops an NK DSGE model with nominal rigidities *à la* Calvo. The model uses the assumptions of Smets and Wouters (2003, 2007). The energy block is disaggregated in two parts and we assume on the one hand, polluting energies and,

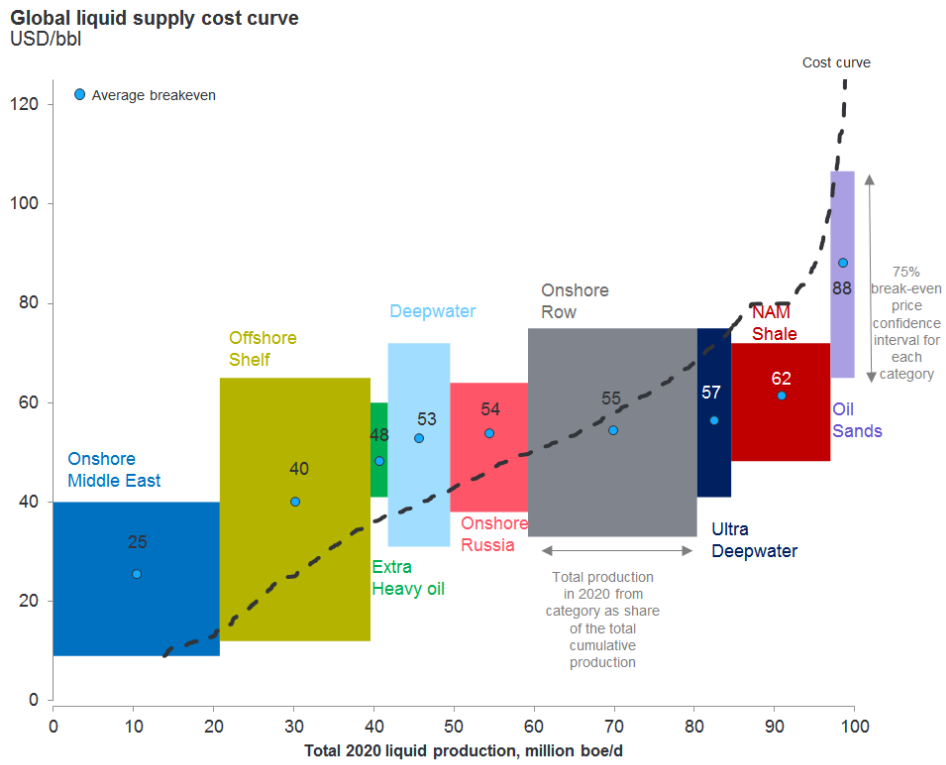


Figure 1.2 – Oil production cost curve - Source: Rystad Energy Research

on the other hand, non-polluting energies. It is assumed that the two sources of energy can be the cause of exogenous shocks. The literature on the business cycle and climate policies is developed in more detail, as is our contribution in the next section.

#### 1.4.2 Pollution and climate policy in the business cycle:

The evolution of the carbon price on the European cap-and-trade market shows the importance of studying the effectiveness of climate policies in the business cycle. Set up in 2005 to meet the binding GHG emission reduction targets of the Kyoto Protocol, after 15 years this market covers 12,000 production plants of the most polluting industries accounting for almost half of EU emissions. The EU ETS market has been established and extended over three successive phases: the first one or "*pilot phase*" (2005 - 2007) aimed to be on the one hand a learning phase, and on the other hand to determine correctly the volume of allowances for the next phases. The second phase from 2008 to 2012 was essentially the same as the first one. The third phase includes civil aviation, and allowances are no longer allocated entirely free of charge, but a fraction being auctioned. The third phase, which began in 2013 and runs until 2020, aims to achieve the emission reduction targets in accordance with the commitments of the Kyoto Protocol. During the first phase, allowance prices have been characterized by a high volatility from a range of €0.7 to 25. According to the market feature there was no restriction on banking or borrowing allowances. In fact they were issued annually, but the firms could use them whenever they wanted within the trading period. However, banking or borrowing allowances were not allowed between the first and second period. This is the reason why allowance price fell to approximately €0 well before the end of the trial period (Ellerman and Joskow, 2008). According to Ellerman and Joskow (2008) carbon price was higher than expected in 2005 and 2006 because of the weather: a cold winter during 2005 and a hot summer in 2006; and also due to a high price for oil and natural gas which made coal relatively more affordable, as coal emits more GHG than oil and natural gas, as a consequence the demand for allowances increased.

Carbon price forecasts was about €35 for 2012, but EUA carbon price fell below

€10 during the third phase. Among the various weaknesses, Laurent and Le Cacheux (2011) and de Perthuis and Trotignon (2014), identify the global financial crisis, which has been followed by an economic downturn and a decline in industrial activity. Indeed, industrial sectors such as steel and cement reduced their production, and GHG emissions decreased. Obviously, it had a strong effect on diminishing demand for allowances and therefore it led to a fall in carbon price. Because the number of quotas was stable, allowances prices fell below the price that had been forecasted to a new equilibrium in a context of a sluggish growth. At the end of the 2<sup>nd</sup> phase, another crisis hit the European economy known as the "*debt crisis*". It has also impacted the EU ETS market. Laurent and Le Cacheux (2011) also identify structural reasons such as the use of the Clean Development Mechanism allowed by the Kyoto protocol. Thanks to this mechanism, a firm that implement a emission-reduction project in developing countries gains access to an emission credit. Therefore it also reduces demand for emission permits.

From 2012 and until 2018, the third phase has been characterized by a stable and low carbon price. It is the result from the previous dysfunctional period, on the one hand the economic conjuncture (global financial crisis, and European sovereign debt crisis), and on the other hand by structural issues such as international credits due to the Clean Development Mechanism. Unlike the transition from the first to the second phase, where banking or borrowing allowances was not possible, the European Commission allowed it between the second and third period. Because of a low demand for allowances during the second phase, a surplus of allowances remain in the third period implying a low carbon price. Unfortunately, this situation could affect the functioning of the carbon market and in the long run make difficult or impossible to meet the GHG emissions reduction goal. To increase the carbon price, the European Commission decided for the forth phase to accelerate the annual rate of allowances decline to 1.74% to 2.2% and to implement the Market Stability Reserve, to limit carbon price volatility. Because this market is subject to exogenous shocks such as the 2008 financial crisis and the 2020 global COVID-19 pandemic, it is important to understand the consequences of the business cycle, and exogenous shocks on the effectiveness of climate policies.

The literature of macroeconomics business cycle addressing climate policies is recent. Among the first papers we can site Angelopoulos et al. (2010), Fischer and Springborn (2011), and Heutel (2012). The main extension of these environmental DSGE models has been to include pollution, but they differ from each other by the way they model it. Fischer and Springborn (2011) include energy as a factor of production and assume that pollution is proportional to the quantity of energy used in the production process. In Angelopoulos et al. (2010) and Heutel (2012), there is no energy input in the production function, and emissions are proportional to output. In these last two articles, environmental damages linked to pollution are explicitly modelled, whereas there is no explicit damage function in Fischer and Springborn (2011). They also differ in the research questions they address. Fischer and Springborn (2011) are interested in understanding how the economy will react using different environmental policies - a carbon tax, a cap-and-trade market or an intensity target - faced with a total factor productivity shock. Angelopoulos et al. (2010) also compare different environmental policies, but they assume in addition to a TFP shock, an exogenous variation of emissions per output ratio or "environmental uncertainty". They find that the cap-and-trade policy is the worst policy because the utility level will be the lowest in that case. But they are not able to identify which policy is better between a carbon tax a cap-and-trade market and a numerical target (an example of a numerical target is the Kyoto Protocol, which imposes a 25-40% reduction in emissions in 2020 compared to 1990 levels). Results will depend on which uncertainty dominate between the total factor productivity shock and the "environmental uncertainty". For Fischer and Springborn (2011), the intensity target is the best policy that can achieve the reduction in emissions at the lowest economic cost. Under a cap-and-trade regime, macroeconomic variables volatility will be lower, and carbon price volatility will increase, while under a carbon tax scenario, variables volatility will be higher because of the constant carbon price. Heutel (2012) is not interested in a comparison of environmental policies, but cares about how environmental policy variables (tax or an emission cap), may adapt to the cycle in order to maximize social welfare. For him, both of these instruments must vary in time and adapt to the business cycle.

The optimal tax must be procyclical, increasing during expansion and decreasing during recession. The same is true for the cap-and-trade market, if it is well designed it must react procyclically.

Annicchiarico and Di Dio (2015, 2017) are the first to develop a NK DSGE models to analyse the link between environmental policies and monetary policies. There is no energy in their model, they assume as in Heutel (2012) that emissions are proportional to output. In their first paper Annicchiarico and Di Dio (2015) focus on environmental policies comparison but with two other sources of uncertainty in addition to the common TFP shock: public consumption shock and monetary shock. Their results suggest that the cap-and-trade market will decrease the volatility of usual macroeconomic variables, while the carbon price will instead adapt to the shocks by a greater volatility, and this result is confirmed in the presence of nominal rigidities. They also find that with strong nominal rigidities, an intensity target will increase the volatility of macroeconomic variables. According to their results, the degree of price stickiness will play an important role for welfare analysis. In a pure competitive model, when prices adjust immediately, they found no difference between the tax and the intensity target, and only minor differences with the emission cap. When prices adjust quickly, welfare tend to be higher in the intensity or tax scenarios and lower in the cap scenario, whereas it is the contrary when prices adjust slowly. They also conduct a Ramsey analysis, and solve a Ramsey problem to derive the optimal environmental policy. In this model, the planner is able to control both the emission level and the environmental policy. Their results are consistent with the one obtained by Heutel (2012) and suggest that the carbon tax must be procyclical. Finally they consider different monetary policy regimes, and they conclude that the carbon tax response must be more aggressive when the monetary policy is less concerned by inflation. In their second paper Annicchiarico and Di Dio (2017) are more interested in the link between optimal monetary policy and optimal environmental policy. When the planner can control both monetary policy and environmental policy, it must fight against price distortion due nominal rigidities on the one hand and also fight against emission damages that affect negatively production. When the planner only controls



emissions and in the presence of an accommodative monetary policy, emissions must be countercyclical. When the planner only controls monetary policy, the inflation target rules only hold for the carbon tax case, if there is no environmental damage. Otherwise, in the presence of an emission cap or a carbon tax with environmental damage, prices must adapt in order to fight inflation or the negative consequences of emissions.

More recently, Argentiero et al. (2017) and Xiao et al. (2018) developed environmental DSGE models including energy in the production function. Xiao et al. (2018) assume energy as one factor of production, while Argentiero et al. (2017) disaggregate energy between polluting and non-polluting energies using a CES function. Argentiero et al. (2017) estimated a Bayesian DSGE models for China, the US and the EU 15 to understand how a carbon tax can result in incentives to use more renewable energies. Because their results suggest that the elasticity of substitution between the two energies inputs is higher in the EU, they find that a carbon tax generates a quicker reallocation of factors in the renewable energy sector. They also find that a carbon tax will diminish the use of fossil fuel, and consequently emissions in the case of a positive productivity shock in the fossil fuel sector. Xiao et al. (2018) developed a NK DSGE model following the approach of the Annicchiarico and Di Dio (2015) model and calibrate it for China. They also contribute to the literature by assuming different sources of uncertainty: technology shock, energy efficiency shock, government expenditures shocks, but also environmental tax rate shocks, emission cap shock or emission intensity shock. As expected, following a positive shock on the volume of allowances, the economy will react by increasing pollution because of the decrease in pollution price. The macro variables responses will be higher in the case of intensity target or in a carbon tax shock.

In Chapters 3 and 4, we contribute to this burgeoning literature by considering energy as a necessary production factor. Thus, in the third chapter, we make a comparison of the different climate policy instruments available to public authorities in a model with perfect price flexibility. We extend the model of Fischer and Springborn (2011) by considering, as in Finn (1991, 2000), that capital is not always fully utilized. The variable rate of capital utilization is then an important variable, since it allows the economy to

adapt to shocks more quickly. We also differ from Fischer and Springborn (2011) by considering in addition to the usual technology shocks, a new source of uncertainty: the price of energy. Our main results are as follows. In the case of a positive productivity shock, the volatility of the macro variables differs little or not at all from the base-line scenario when the government implements a tax or an intensity target. On the other hand, the volatility of the usual economic variables decreases in the case of an emission cap because the price of carbon rises sharply during a productivity shock to ensure the emission reduction target. On the other hand, in the case of an energy price shock, in the presence of a tax, the dynamic responses of the variables will follow the reference scenario (no policy). While the emission cap and intensity target will see the price of carbon decrease, which will further reduce the volatility of the model. The implications in terms of policy are important, since it suggests adapting the carbon tax in particular according to the volatility of the energy price to avoid the risk of Yellow Vests. Thus a flat tax would make it possible to reduce this risk. In the fourth chapter, we develop a DSGE model with nominal rigidities in the manner of Annicchiarico and Di Dio (2015). We add the energy factor like Xiao et al. (2018), but we differ from them by disaggregating the energy into two factors: the polluting energy source and the non-polluting energy using a CES. It allows us to consider these two sources of energies as complementary or substitutable. We further assume that emissions affect the utility of the consumer, not the level of production as it is usually the case in the literature. We compare two instruments of climate policy: a carbon tax and a cap-and-trade market. We consider 8 different exogenous shocks: TFP shock, shock in the polluting energy sector, in the non-polluting energy sector... According to our results, there is no significant difference between the two instruments in the event of a shock to the TFP. These results suggest the introduction of a floor price for carbon on the European allowance market, which would prevent the price from falling below a certain threshold, as was the case after the economic crisis of 2008. In the case of a shock in the fossil fuel sector, in the presence of a tax, emissions will increase because the marginal cost of pollution is constant. Consequently, emissions will reduce the consumer's utility. On the other hand, in the presence of an

emission cap, the price will adjust so as to limit emissions according to the cap. However, there is no significant difference between the instruments during a productivity shock in renewable production. When we optimize the monetary policy rule, we find that in the carbon tax case, the Central Bank must be more sensitive to inflation and less sensitive to the output gap compare to the usual coefficient of the Taylor rule. Whereas in the case of an emission cap it must be more responsive to both inflation and output gap.

## Chapter 2

# Irreversibility of emitting greenhouse gas and the cost of delaying climate policy

### 2.1 Introduction:

It has been ten years since the Copenhagen conference, ten years in which global demand for fossil energy has increased. This rise, due to the absence of an environmental policy instrument or to an excessively low carbon price, has resulted in a rise in emissions, which reduces the odds of limiting climate change.

The consequences of this increase in the GHG atmospheric stock are various. On the one hand, the current rise in emissions will result in even greater efforts if the objectives of the Paris agreement are to be met. On the other hand, this increase accentuates the feedback effects. The change in cloud formation, the concentration of water vapour in the atmosphere and the reflection effects of ice (albedo) modify the radiative forces and amplifying global warming (see IPCC (2014, 2018) and Stern and Stern (2007)). A higher stock of GHG in the atmosphere would also decrease the GHG absorption capacity of oceans and land. Thus, above a certain GHG threshold, climate change could be self-

sustaining, even if emissions were totally cut off. The possibility of such a threshold effect could then lead to a form of irreversibility of climate change. However, little work has been done on this notion of irreversibility. Economic theory is essentially based on the notion of reversibility of pollution. According to Grossman and Krueger (1995), pollution is a necessary stage, and it would suffice to continue to develop in order to return to the initial equilibrium where the environment is cleaned up.

We are interested in this question of irreversibility of climate change in absence of public intervention. To do so, we make the hypothesis, as in the Brock and Taylor (2010) model, that without public policy, the economy will be able to reduce its GHG emissions through an exogenous technological process per unit of fossil fuel energy consumed. We compare the long-term equilibrium - in terms of the stock of GHG in the atmosphere and their economic consequences - in the absence of intervention, to the results obtained with a public intervention. We show that climate dynamics is an irreversible phenomenon in the long term and consequently intervention is needed in the short term. We also show that the elasticity of substitution between the two sources of energy is a key parameter that also determines the long-term equilibrium. In order to limit global warming as much as possible, in addition to the fastest possible public intervention, a strong elasticity of substitution between polluting and non-polluting energies should be promoted. The aim of this chapter is not to reproduce IPCC projections or to calculate the optimal value of the carbon tax. The model is too simplistic to claim to make a recommendation on the optimal level of CO<sub>2</sub> in the atmosphere, or on the level of the tax to be put in place. All the more so as we do not know with certainty the true functional form of the damage function (see Pindyck (2017)). Instead, the aim is to focus on the question of climate reversibility, and the carbon cycle. The interest is therefore to make relative comparisons between the status quo and public intervention.

To explore this question of irreversibility, we use a real business cycle integrated assessment model (RBC IAM) initially developed by Golosov et al. (2014) which is based on Nordhaus's ones. Nordhaus (1977, 1994) acted as a precursors by including carbon cycle and the consequences of the GHG atmospheric stock on economic activity in its

DICE model. His RICE model (see Nordhaus and Yang (1996)) is a disaggregated version of its world-region model and includes 12 regions in its latest version. More recently, Hope (2006), and Waldhoff et al. (2011) developed respectively the PAGE and FUND models, but Nordhaus' model still remains the reference. Golosov et al. (2014) (GHKT hereafter) proposed a model based on Nordhaus' work. GHKT's and Nordhaus's work mainly differ in how they model the carbon cycle and the consequences of GHG damages. Unlike Nordhaus, GHKT admit that a significant proportion of emissions remain in the atmosphere for several thousand years. Moreover, according to them, the temperature response to the GHG atmospheric is immediate and not delayed over time. We use the same analytical framework as GHKT, including the same carbon cycle and the damage function of GHG emissions, but we differ from them in several ways. Their time reference is the decade and they assume a full depreciation of capital over this period (so that their capital depreciation rate equals 10% per year). On the contrary, we consider the year as a time unit, so we calibrate the capital depreciation rate to under 10% per year, more consistently with data. In addition we take into account the capital adjustment cost while GHKT consider this cost to be zero. Finally the structure of the energy sector differs from that of GHKT. They separate fossil energy into two regimes: a regime for exhaustible resources, and a regime for non-exhaustible resources such as coal. We only consider the non-depleting fossil fuels regime of coal regime. To see whether climate change is reversible (Grossman and Krueger (1995) model hypothesis), an exogenous variable, representing a form of technical progress, which reduces the carbon intensity of energy, is incorporated into the model.

Section 2 develops the concept of irreversibility, and its application in economics according to the 2<sup>nd</sup> principle of thermodynamics introduced by Georgescu-Roegen (1971). Section 3 summarises the different hypotheses of the model, section 4 presents the planner's problem in order to obtain the marginal cost of pollution. The decentralized competitive equilibrium is solved in section 5, the different parameters of the model are calibrated in section 6 and the results of the simulations are discussed in section 7 before concluding.

## 2.2 State of the art and contribution:

Much of the economic literature on growth and pollution is based on the Grossman and Krueger (1995) model. According to them, the relationship between demand for environmental quality and income would be an inverted U-shape, also known as the Environmental Kuznet Curve (EKC). It states that pollution diminishes as soon as an income threshold has been reached. There are three stages. In the early stage of development environmental quality is high, then as production increases (and therefore income), pollution increases, adversely affecting environmental quality. In a second step, pollution reaches a peak before falling while income is increasing. The other explanation of the EKC, given by Brock and Taylor (2010) would be related to technical progress. In time, a technological way would be found to reduce pollution. Here too, an inverted U-curve would be observed. It would then be necessary to degrade one's environment before cleaning it up. This theory is well suited for some pollutants that have the following characteristics: ideally, their lifespan should be short, and pollution remain locally concentrated. Indeed, if the pollution does not diffuse and if the pollutant can degrade quickly, then it is possible to reduce pollution over time. On the other hand, if pollution spreads, and/or if it lasts a long time, then public intervention becomes necessary to face the risk of inflicting irreversible damage.

The 1986 Chernobyl disaster is an example of irreversible environmental damage. The half-life of nuclear fuel ranges from ten to several tens of millions of years, so that if no technology is found to clean up the site, it will remain unusable for an infinite period of time (at the human life scale). The positive aspect is that nuclear pollution is (partly) geographically contained. Another example of irreversible damage is plastic waste found in the oceans. This time again the lifetime of plastic exceeds 100 years. It kills many species that ingest it before it degrades, and even when it decomposes as microplastics, marine wildlife ingests it and contaminates the entire food chain. The lack of technology to address this issue makes such pollution appear irreversible.

The same is true for global warming. Greenhouse gases have two characteristics that

make global warming irreversible if nothing is done to reduce our emissions in the short run. On the one hand, once emitted, GHG diffuse to the entire atmosphere. Thus, in the absence of a coordinated response, the efforts made by some may be less effective due to the free rider behaviour of other countries (see Stern and Stern (2007)). This well-known concept of public goods, is the consequence of market failure and misallocated property rights. The Trump administration's decision to leave in 2017 the Paris Agreement, while the US is the world's largest emitter of GHG in terms of  $CO_2$  eq. per capita illustrates this situation. On the other hand, several factors influence climate, whose laws of motion are very slow. In other words, once the initial equilibrium has been modified, it will be difficult to return to this equilibrium. The radiative forcing effect (or warming effect) is currently contained by the oceans, which act as a buffer and regulate temperature. However, once the ocean temperature have increased, due to this buffer effect, the time required to reverse the trend once emissions have sufficiently decreased is also very long. The phenomenon of global warming can also be self-sustaining once a certain threshold has been reached. For example, glaciers, sea ice and icecaps also acts as climate regulators. They do not only play the role of the planet's air conditioner. Indeed, ice's white surface reflects sunbeams and keeps our global climate mild. If ice melts, darker surfaces will absorb heat and increase temperature. For those different, albeit non-exhaustive reasons, in the absence of a technological solution that would allow us to absorb the stock of GHG currently present in the atmosphere, without a carbon price high enough to limit the use of fossil fuels climate change could have irreversible effects. Thus, even if income increases, we would not have the ability to avoid global warming as predicted by the EKC. In its report on green growth, the World Bank puts forward the idea, without developing it, of a possible irreversibility of the climate p.33: "*Climate change itself may be irreversible. This irreversibility is a clear incentive for early action, as the consequences of warming exceeding 2°C are highly uncertain and potentially severe[...]. The 2°C objective, for example is achievable only if significant emission reductions can be made before 2030*" (Bank, 2012)

In this chapter, we propose to investigate this concept of irreversibility of global



warming. Irreversibility is derived from the 2<sup>nd</sup> principle of thermodynamics (entropy law), and it has been introduced in economics by Georgescu-Roegen (1971). In his work, Georgescu criticizes the economic reasoning based on the classical laws of mechanics or Newtonian laws that are by nature reversible. He therefore recalls the origin of thermodynamics. This branch of physics, which appeared at the time of the industrial revolution, was intended to explain phenomena that the law of classical physics could no longer explain. Steam trains used the energy stored in the form of coal to convert it into mechanical work. This fuel allows the train locomotive to move forward. Once the fuel has been consumed, it decelerates before stopping. Hence, the mechanical movement is reversible. But another phenomenon has occurred, which is irreversible. Indeed, coal has not only been transformed into mechanical work, a share of energy has vanished in the form of heat, and ashes are the residue. This process is irreversible, it is unidirectional and irreversible. Just as birth always precedes death, life is unidirectional and irreversible.

This concept of irreversibility is a definition in the strict sense of the term. Irreversibility in the broad sense can be defined as a very low process of return to equilibrium on the scale of human life, as in the case of plastic or nuclear pollution. Thus, the dynamics governing climate change is slow. As a result, in the absence of technological innovation that can reduce emissions intensity, the damage caused is irreversible at least in the medium to long term on the scale of human life. Similarly if the implementation of a sufficiently high carbon price is delayed in the short term, consequence could also be non reversible in the distant future.

According to the Kaya's identity (see Kaya et al. (1997) and Waggoner and Ausubel (2002)) GHG emission are driven by four different factors: total population, GDP per capita, GDP energy intensity and carbon intensity of energy. It can be express as follows:

$$CO_{2,t} = POP_t \times \frac{GDP_t}{POP_t} \times \frac{ENE_t}{GDP_t} \times \frac{CO_{2,t}}{ENE_t}$$

Taking logs on both sides of previous equation and differentiating gives:

$$g_{CO_2} = g_{POP} + g_{GDP/POP} + g_{ENE/GDP} + g_{CO_2/ENE}$$

In other words, the growth in GHG emissions would be equal to the sum of the growth rates of the different factors. Figure 2.1 illustrates the evolution of global GHG emissions, as well as the evolution of the different factors. It can be seen that emissions are driven by population growth and rising per capita income, and are reduced by efficiency gains (decrease in the energy intensity of GDP by more than 15% in 2015 compared to the 1990 base year). The carbon intensity of energy, i.e. GHG emissions per unit of energy consumed, remains constant at the global level. In the rest of the chapter, we will focus on this energy carbon intensity. On the one hand, we will assume, as in the Brock and Taylor (2010) model, that an exogenous technological process will make it possible to decrease this carbon intensity over time. This is therefore a way of representing the famous ECK. This scenario will serve as a reference. The other way to reduce this carbon intensity is to use environmental policy instruments. Introducing a price on pollution is supposed to take into account the negative externalities linked to GHG emissions. In our model, we internalise the cost generated by the use of fossil resources and restore the social optimum. We therefore compare the reference scenario, in which the decrease in emissions is explained solely by technical progress, with different scenarios in which an optimal price for carbon is added.

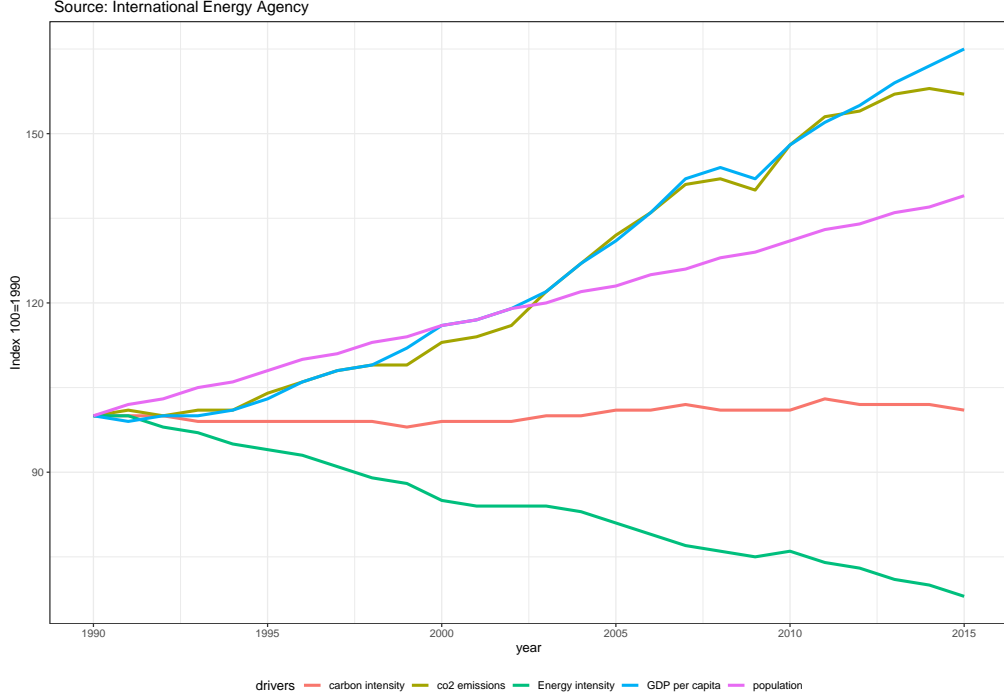


Figure 2.1 – Kaya's identity for the world - Source: IEA

### 2.3 General assumptions:

In this section, we describe the main assumptions of the model, on the one hand the construction of the environmental block and the carbon cycle, and on the other hand the economic block, energy use and environmental damage related to the use of fossil energy. GHG atmospheric stock evolves as follows:

$$S_t - \bar{S} = \sum_{s=0}^{t-T} (1 - d_s) \Phi \Omega_{t-s} e_{c,t-s} \quad (2.3.1)$$

$$1 - d_s = \psi_1 + (1 - \psi_1) \psi_2 (1 - \psi_3)^s, \quad d_s \in [0, 1] \quad (2.3.2)$$

Where  $S$  is the pre-industrial stock of GHG in the atmosphere,  $\Phi$  is the conversion factor that transforms one unit of fossil fuel  $e_{c,t}$  consumed (or polluting energy) into  $\Phi e_{c,t}$  units of GHG emitted in the atmosphere. According to GHKT, in each period the carbon cycle can be summarized by this mechanism. For each unit of emitted GHG, a share

$\psi_1 \in [0, 1]$  remains in the atmosphere forever. For the remaining share  $(1 - \psi_1)$ , one part  $(1 - \psi_2) \in [0, 1]$  is immediately absorbed, while the last part  $\psi_2$  slowly degrades at a rate  $\psi_3 \in [0, 1]$  in each period. Finally, we assume that exogenous technological progress  $\Omega_t \in [0, 1]$  will reduce the quantity of emissions per unit of burnt fossil fuel in the future. This is the central assumption in our model, it is a way to assume an EKC, its functional form is discussed more at length in the calibration section. This variable has the following properties, in  $t = 0$ ,  $\Omega_0 = 1$ , and its limits tends towards  $\epsilon$ , when  $t$  tends to infinity. The current stock of GHG depends on past uses of fossil fuels:

$$S_t = L(e_c^t)$$

$$\text{with } e^t = \{e_{c,-T}, e_{c,-T+1}, \dots, e_{c,t}\}$$

Another useful way to describe the evolution of the GHG atmospheric stock is to transform the previous sum of past emissions, into a recursive representation as the sum of two state variables  $S_t^1$  and  $S_t^2$ . In this representation, the accumulation of GHG can be broken down into two reservoirs whose dynamics are independent of each other, where  $S_t^1$  represents the first reservoir where the flow of emissions accumulates in the atmosphere forever, and  $S_t^2$ , on the other hand, represents the second reservoir in which the stock depreciates at a geometric rate  $\psi_3$ . This formulation can be represented by the three following equations:

$$\begin{cases} S_t = S_t^1 + S_t^2 \\ S_t^1 = \psi_1 \Omega_t \Phi e_{c,t} + S_{t-1}^1 \\ S_t^2 = (1 - \psi_1) \psi_2 \Omega_t \Phi e_{c,t} + (1 - \psi_3) S_{t-1}^2 \end{cases} \quad (2.3.3)$$

The infinitely lived representative household has the following logarithmic utility function:

$$U(C_t) = \log C_t \quad (2.3.4)$$

Output  $Y_t$  in the final goods sector is given by a Cobb-Douglas function and it requires three inputs capital  $K_{g,t}$ , labor  $L_{g,t}$  and energy  $E_{g,t}$ :

$$Y_t = \left(1 - D(S_t)\right) K_{g,t-1}^\alpha \left(A_g L_{g,t}\right)^{1-\alpha-\gamma} E_{g,t}^\gamma \quad (2.3.5)$$

$D(S_t(e_{c,t}))$  is the global damage equation of the atmospheric GHG stock. We follow GHKT and assume that:

$$D(S_t) = 1 - \exp\left(\kappa(\bar{S} - S_t)\right) \quad (2.3.6)$$

where  $\bar{S}$  is the pre-industrial atmospheric GHG concentration, and  $\kappa$  is a scale parameter to be calibrated.

It is assumed that energy production comes from several sources. We simplify reality by assuming that we can distinguish these two sources into non-polluting  $e_{r,t}$  and polluting sources  $e_{c,t}$ . Unlike the polluting source, the non-polluting source does not emit greenhouse gases. We assume that there is no Hotelling rent, in other words there is no rent linked to the exploitation of the exhaustible resource, we thus assume a coal regime. Indeed, coal is a finite, yet not a scarce resource, and the disposable quantity of coal is large enough to be used for several centuries, so that we do not consider it as a depletable resource. Energy is aggregated using a CES production function, where  $\omega$  is a share parameter, and  $\sigma = \frac{1}{1-\varphi}$  is the elasticity of substitution between polluting and non-polluting energies.

$$E_{g,t} = \left(\omega e_{r,t}^\varphi + (1 - \omega)e_{c,t}^\varphi\right)^{\frac{1}{\varphi}} \quad (2.3.7)$$

The aggregate accounting identity of the economy is:

$$C_t + I_t + \frac{\phi}{2} \left(\frac{I_t}{K_{g,t-1}} - \delta\right)^2 K_{g,t-1} + T_t = Y_t \quad (2.3.8)$$

It, includes investment adjustment costs:  $C(I_t, K_{g,t}) = \frac{\phi}{2} \left(\frac{I_t}{K_{g,t-1}} - \delta\right)^2 K_{g,t-1}$ , with  $\phi \geq 0$ , and where the cost function has the following properties: it is homogeneous of degree one, it is equal to zero in the steady state ( $C(\delta K_g, K_g) = 0$ ) and the marginal cost of investment increases with  $I_t$ , and decreases with the stock of capital  $K_{g,t}$ . This adjustment cost specification has been initially suggested by Hayashi (1982). Assuming first that  $\phi = 0$ , one obtains the usual capital accumulation equation. On the contrary, if  $\phi > 0$ , then outside the steady state ( $\frac{I_t}{K_{g,t}} \neq \delta$ ) increasing  $I_t$  makes capital depreciate faster. This functional form has been widely used in the literature (see Abel and Blanchard (1983), Blanchard et al. (1989), Obstfeld et al. (1996) or Jermann (1998) among

others). The law of motion for capital is given by:

$$K_{g,t} = I_t + (1 - \delta)K_{g,t-1} \quad (2.3.9)$$

We assume that coal production  $e_{c,t}$  and non-polluting energy production input only require labor, as in GHKT. In each sector, productivity depends on the parameter  $A_i$ ,  $i = \{r, c, g\}$  (each sector is defined by a letter:  $g$  for final good sector,  $r$  for non-polluting energy sector or renewable energy sector and  $c$  for polluting or coal sector),

$$\begin{cases} e_{c,t} = A_c L_{c,t} \\ e_{r,t} = A_r L_{r,t} \end{cases} \quad (2.3.10)$$

Finally, we assume that labor supply  $L_t$  is exogenous. Market clearing condition for labor is given by the following equation:

$$L_t = L_{g,t} + L_{c,t} + L_{r,t} \quad (2.3.11)$$

## 2.4 Planning problem

Now we describe the planner's problem. It is assumed that a planner maximizes the representative consumer's utility function (2.3.4) subject to equations (2.3.8), (2.3.5), (2.3.7), (2.3.10), (2.3.9), (2.3.11), (2.3.6) and (2.3.3). The goal is to find the expression of the optimal price of carbon in order to internalize the cost generated by GHG emissions.

The planning problem consists of the following maximization program.

$$\max_{\{C_t, Y_t, I_t, K_{g,t}, e_{r,t}, e_{c,t}, L_{g,t}, L_{c,t}, L_{r,t}, S_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad (2.4.1)$$

$$s.to : \begin{cases} Y_t = C_t + I_t + \frac{\phi}{2} \left( \frac{I_t}{K_{g,t-1}} - 1 \right)^2 K_{g,t-1} \\ Y_t = \left( 1 - D(S_t) \right) K_{g,t-1}^\alpha \left( A_g L_{g,t} \right)^{1-\alpha-\gamma} E_{g,t}^\gamma \\ E_{g,t} = \left( \omega e_{r,t}^\varphi + (1-\omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}} \\ e_{c,t} = A_c L_{c,t} \\ e_{r,t} = A_r L_{r,t} \\ K_{g,t+1} = I_t + (1-\delta) K_{g,t} \\ L_t = L_{g,t} + L_{c,t} + L_{r,t} \\ D(S_t) = 1 - \exp \left( \kappa (\bar{S} - S_t) \right) \\ S_t = \sum_{s=0}^{t+T} (1 - d_s) e_{c,t-s} \end{cases}$$

Where the planner's program is to choose the sequence  $\{C_t, Y_t, I_t, K_{g,t}, e_{r,t}, e_{c,t}, L_{g,t}, L_{c,t}, L_{r,t}, S_t\}_{t=0}^{\infty}$  that maximizes the intertemporal utility function. Let us define  $\lambda_t, \mu_t, \theta_t, \Upsilon_t, \zeta_t, \eta_t, \Xi_t$  and  $\nu_t$  as the Lagrange multipliers associated respectively with the overall resource constraint, the global production function, the aggregated demand for energy services, the production of non-polluting energy, the production of fossil fuel energy, the law of motion for capital, the market clearing condition for labor and with the law of motion for atmospheric GHG concentration. The associated Lagrangian to the planner's problem is:

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t & \left\{ U(C_t) + \lambda_t \left[ Y_t - C_t - I_t - \frac{\phi}{2} \left( \frac{I_t}{K_{g,t-1}} - 1 \right)^2 K_{g,t-1} \right] \right. \\ & + \mu_t \left[ \left( 1 - D(S_t) \right) K_{g,t-1}^\alpha \left( A_g L_{g,t} \right)^{1-\alpha-\gamma} E_{g,t}^\gamma - Y_t \right] + \theta_t \left[ \left( \omega e_{r,t}^\varphi + (1-\omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}} - E_{g,t} \right] \\ & + \Upsilon_t \left[ A_c L_{c,t} - e_{c,t} \right] + \zeta_t \left[ A_r L_{r,t} - e_{r,t} \right] + \eta_t \left[ L_t - L_{g,t} - L_{c,t} - L_{r,t} \right] \\ & \left. + \Xi_t \left[ -K_{g,t} + I_t + (1-\delta) K_{g,t-1} \right] + \nu_t \left[ S_t - \sum_{s=0}^{t+T} (1 - d_s) e_{c,t-s} \right] \right\} \quad (2.4.2) \end{aligned}$$

Usual first order conditions (FOC hereafter) are available in the technical appendix. We therefore focus on the most important FOC and their interpretations. Assuming

a positive installation cost for capital yields to the following FOC, when we take the derivative of the Lagrangian 2.4.2 with respect to gross investment  $I_t$ :

$$\lambda_t \left( 1 + \phi \left( \frac{I_t}{K_{g,t-1}} - \delta \right) \right) = \Xi_t \quad (I_t^{foc})$$

Recalling that  $\Xi_t$  is the shadow value of adding one unit of capital, it says that if the economy has already chosen the optimal capital stock  $K_{g,t}$ , adding one unit of capital, will reduce investment by one unit, and due to the adjustment cost function, it will reduce investment costs in the future (second term in brackets in LHS of the  $I_t^{foc}$ ).

We can use  $(C_t^{foc})$ ,  $(Y_t^{foc})$  and  $(I_t^{foc})$  to eliminate the Lagrange multipliers in order to express the Euler's equation:

$$U'(C_t) \left( 1 + \phi \left( \frac{I_t}{K_{g,t-1}} - \delta \right) \right) = \beta U'(C_{t+1}) \left[ \frac{\partial Y_{t+1}}{\partial K_{g,t}} + \left( 1 + \phi \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right) \right) (1 - \delta) - \frac{\phi}{2} \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right)^2 + \phi \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right) \frac{I_{t+1}}{K_{g,t}} \right] \quad (2.4.3)$$

The FOC obtained by taking the derivative of the Lagrangian w.r.t to the GHG atmospheric stock  $S_t$ , and fossil fuel energy  $e_{c,t}$  are:

$$-\mu_t \frac{dD(S_t)}{dS_t} K_{g,t-1} (A_g L_{g,t})^{1-\alpha-\gamma} E_{g,t}^\gamma + \nu_t = 0 \quad (S_t^{foc})$$

$$-\sum_{j=0}^{\infty} \beta^j \nu_{t+j} (1 - d_j) + \theta_t \left( \omega e_{r,t}^\varphi + (1 - \omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}-1} (1 - \omega) e_{c,t}^{\varphi-1} - \Upsilon_t = 0 \quad (e_{c,t}^{foc})$$

Where  $\Upsilon_t$  is the shadow value of coal energy production. Replacing  $(S_t^{foc})$ ,  $(C_t^{foc})$ ,  $(E_{g,t}^{foc})$ ,  $(G_t^{foc})$ ,  $(L_{c,t}^{foc})$  and  $(L_{g,t}^{foc})$  into  $(e_{c,t}^{foc})$  leads to:

$$\sum_{j=0}^{\infty} \beta^j \mu_{t+j} \frac{\partial D_{t+j}}{\partial S_{t+j}} \frac{dS_{t+j}}{de_{c,t}} K_{g,t}^\alpha L_{g,t}^{1-\alpha-\gamma} E_{g,t}^\gamma + \theta_t \left( \omega e_{r,t}^\varphi + (1 - \omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}-1} (1 - \omega) e_{c,t}^{\varphi-1} - \Upsilon_t = 0 \quad (2.4.4)$$

We can easily manipulate the latter equation to make it more understandable:

$$\begin{aligned} \theta_t \frac{\partial E_{g,t}}{\partial e_{c,t}} &= \Upsilon_t - \sum_{j=0}^{\infty} \beta^j \mu_{t+j} \frac{\partial Y_{t+j}}{\partial e_{c,t}} \\ U'(C_t) \frac{\partial Y_t}{\partial E_{g,t}} \frac{\partial E_{g,t}}{\partial e_{c,t}} &= \Upsilon_t - \sum_{j=0}^{\infty} \beta^j \mu_{t+j} \frac{\partial Y_{t+j}}{\partial e_{c,t}} \end{aligned} \quad (2.4.5)$$



As a classical result, in the optimum the marginal benefit of using one fossil fuel unit (Left Hand Side, LHS hereafter, of the previous equation) must be equal to its marginal costs. Marginal costs equal the sum of fossil fuel production cost, and global production loss because of the increase in the atmospheric GHG stock, as a consequence of burning one extra fossil fuel unit. Defining  $\Delta_t$  as the marginal damage cost of emission (2nd term in the RHS of the previous equation (2.4.5)), it is equal to the damage of one carbon unit emitted in the current period. We can rewrite the equation (2.4.5) as follows:

$$U'(C_t) \frac{\partial Y_t}{\partial E_{g,t}} \frac{\partial E_{g,t}}{\partial e_{c,t}} = \Upsilon_t + \Delta_t$$

$\Delta_t$  corresponds to the negative externality of using fossil fuel, and this term is the cornerstone of the present chapter. In the absence of public intervention, this negative effect is neglected so that the optimum is not reached. Using the logarithmic utility function (2.3.4), and the exponential function form (2.3.6) for the damage function, one obtains:

$$\frac{\Delta_t}{\lambda_t} = - \sum_{j=0}^{\infty} \beta^j \frac{C_t}{C_{t+j}} Y_{t+j} \frac{dS_{t+j}}{de_{c,t}} \quad (2.4.6)$$

GHG are characterized by both their lifetime in the atmosphere, and by their "Global Warming Potential" (see table 2.1). The GHG lifetime are relatively longer than one year, so that emitting one GHG unit will not only induce negative externality for the current period, but will also affect the future, depending on the carbon decay rate. We can now express future damages in terms of current GDP by dividing both sides by  $Y_t$  (in doing so, we assess damages for later periods relatively to the current one - see Carbon Cycle section for more details):

$$\frac{C_t \Delta_t}{Y_t} = - \sum_{j=0}^{\infty} \beta^j \frac{C_t}{Y_t} \frac{Y_{t+j}}{C_{t+j}} \frac{dS_{t+j}}{de_{c,t}} \quad (2.4.7)$$

Equation (2.4.7) is the optimal carbon tax rate, whose purpose is to internalize damages from fossil fuel GHG emissions, and consequently limit global warming. After some algebraic manipulation we finally obtain:

$$\frac{\Delta_t^s}{Y_t} = \sum_{j=0}^{\infty} \beta^j \frac{C_t}{Y_t} \frac{Y_{t+j}}{C_{t+j}} (1 - d_j) \kappa \Phi \Omega_{t+j} \quad (2.4.8)$$

This carbon tax depends on  $\beta$ ,  $\psi_3$  and  $\kappa$  parameters and the exogenous technological process that diminishes energy carbon intensity  $\Omega_t$ . Everything remaining equal, the higher discount rate, the lower the preference rate for the present time, the more future well-being is taken into account, the higher tax will be. The carbon tax depends negatively on the rate of carbon decay ( $\psi_3$ ). The higher this absorption rate, the lower the tax will be. Similarly, all other things being equal, the more the exogenous variable  $\Omega_t$  tends towards 0, the lower GHG intensity of consuming one unit of fossil fuel, the lower tax will be (environmental damage being low).

According to the  $(L_{c,t}^{foc}), (L_{r,t}^{foc}), (L_{g,t}^{foc})$ , marginal benefit of labor is equal across sectors:

$$\Upsilon_t A_c = \zeta_t A_r = (1 - \alpha - \gamma) \mu_t \left(1 - D(S_t)\right) K_{g,t}^\alpha \left(A_g L_{g,t}\right)^{1-\alpha-\gamma} L_{g,t}^{-1} E_{g,t}^\gamma = \eta_t$$

$$\Upsilon_t \frac{\partial e_{c,t}}{\partial L_{c,t}} = \zeta_t \frac{\partial e_{r,t}}{\partial L_{r,t}} = \mu_t \frac{\partial Y_t}{\partial L_{g,t}} = \eta_t$$

## 2.5 Decentralized competitive equilibrium:

### 2.5.1 Representative household:

We assume that the representative household owns capital  $K_{g,t}$ , makes investment decisions  $I_t$ , and supplies its labor force  $L_t$ . The representative household gets its income from the supply of its labor force  $L_t$  (because we assume a purely competitive market the household is a price-taker and consequently the wage  $w_t$  is given) and from renting capital  $K_{g,t}$  to the firm ( $r_t$ , which takes also as given). All its income is consumed or invested. We assume that government implements a tax on GHG emissions, and redistributes its revenue as a lump-sum subsidy  $T_t$ . The household gets utility only from consumption, so that leisure does not enter the utility function. Utility from future consumption is discounted according to  $\beta \in ]0, 1[$ . Given the sequence  $\{w_t, L_t, r_t\}_{t=0}^\infty$ , the representative household chooses the sequence  $\{C_t, I_t, K_{g,t}\}_{t=0}^\infty$  that maximizes its intertemporal utility function:

$$\max_{\{C_t, I_t, K_{g,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad (2.5.1)$$

subject to its budget constraint:

$$C_t + I_t + \frac{\phi}{2} \left( \frac{I_t}{K_{g,t-1}} - \delta \right)^2 K_{g,t-1} = r_t K_{g,t-1} + w_t L_t + T_t \quad (2.5.2)$$

Investment is given by the following equation:

$$K_{g,t} = I_t + (1 - \delta) K_{g,t-1} \quad (2.5.3)$$

By assuming the logarithmic utility function, and combining the FOC<sup>1</sup> for consumption and investment with the FOC for capital stock we obtain the Euler equation:

$$\begin{aligned} \frac{1}{C_t} \left( 1 + \phi \left( \frac{I_t}{K_{g,t-1}} - \delta \right) \right) &= \beta \frac{1}{C_{t+1}} \left[ r_{t+1} - \frac{\phi}{2} \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right)^2 \right. \\ &\quad \left. + \phi \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right) \frac{I_{t+1}}{K_{g,t}} + \left( 1 + \phi \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right) \right) (1 - \delta) \right] \end{aligned} \quad (2.5.4)$$

If the Euler equation (2.5.4) is satisfied, the household is indifferent between consuming today and consuming in the future. On the other hand, depending on the inequality that holds, household prefers either to save more in the current period and to postpone its consumption in the future, or on the contrary to increase its consumption level in the current period. This Euler equation is the same as the one obtained in the planning problem (equation (2.4.3)).

## 2.5.2 Firms:

We assume that all markets are competitive, so that firms are price takers in both final goods and intermediate goods sectors.

### 2.5.2.1 Final goods sector:

We first consider the firm in the final-good, it does not take into account the externalities linked to the use of polluting energy. In that case, in each period  $t$ , the representative firm chooses the sequence of inputs  $\{K_{g,t-1}, L_{g,t}, E_{g,t}\}$ , that maximizes its profit  $\Pi_{g,t}$  taking

<sup>1</sup>See Technical appendix for more details

the sequence  $\{r_t, w_t, p_{c,t}, p_{r,t}, \tau_{c,t}\}$  as given:

$$\begin{aligned} \max_{\{K_{g,t-1}, L_{g,t}, e_{c,t}, e_{r,t}\}} \quad & \Pi_{g,t} = Y_t - w_t L_{g,t} - r_t k_{g,t-1} - p_{c,t}(1 + \tau_{c,t})e_{c,t} - p_{r,t}e_{r,t} \\ \text{s.to} \quad & \begin{cases} Y_t = (1 - D(S_t)) K_{g,t-1}^\alpha (A_g L_{g,t})^{1-\alpha-\gamma} E_{g,t}^\gamma \\ E_{g,t} = (\omega e_{r,t}^\varphi + (1 - \omega)e_{c,t}^\varphi)^{\frac{1}{\varphi}} \end{cases} \end{aligned}$$

Demand for energy services from the final goods sector is given by the aggregated CES function  $E_{g,t}$ , and energy services are provided by intermediate goods firms. On these markets, the program is to choose the amounts of fossil fuel  $e_{c,t}$  and non-polluting energy  $e_{r,t}$  that minimize its costs  $p_{r,t}$  and  $(1 + \tau_{c,t})p_{c,t}$ . From the FOC we find the demand from the representative firm for each energy input:

$$\begin{cases} e_{r,t} = \left[ \frac{\gamma \omega Y_t}{p_{r,t}(\omega e_{r,t}^\varphi + (1 - \omega)e_{c,t}^\varphi)} \right]^{\frac{1}{1-\varphi}} \\ e_{c,t} = \left[ \frac{\gamma(1 - \omega)Y_t}{p_{c,t}(1 + \tau_{c,t})(\omega e_{r,t}^\varphi + (1 - \omega)e_{c,t}^\varphi)} \right]^{\frac{1}{1-\varphi}} \end{cases}$$

In the absence of public intervention (when  $\tau_c = 0$ ), the representative firm in the final goods sector does not take into account the damage caused by its use of fossil fuel. The demand for fossil fuel is greater than the optimum. Consequently, GHG emissions accumulate into the atmosphere, and increase long run damage.

When the firm is not myopic and takes into account the negative externalities of GHG emissions on the economy, it solves the following intertemporal maximisation program:

$$\begin{aligned} \max \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \exp \left( \kappa \left( \bar{S} - \sum_{s=0}^{t+T} (1 - d_s) e_{c,t-s} \right) \right) & K_{g,t-1}^\alpha (A_g L_{g,t})^{1-\alpha-\gamma} E_{g,t}^\gamma \\ & - w_t L_{g,t} - r_t k_{g,t-1} - p_{c,t}^* e_{c,t} - p_{r,t} e_{r,t} \end{aligned}$$

The FOC for the optimal use of fossil fuel is therefore:

$$\begin{aligned} \gamma (1 - D(S_t)) K_{g,t-1}^\alpha (A_g L_{g,t})^{1-\alpha-\gamma} (\omega e_{r,t}^\varphi + (1 - \omega)e_{c,t}^\varphi)^{\frac{\gamma}{\varphi}-1} (1 - \omega)e_{c,t}^{\varphi-1} \\ - \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} (1 - d_j) Y_{t+j} = p_{c,t}^* \end{aligned} \quad (2.5.5)$$

By moving the second term of the LHS to the RHS of the latter equation, we obtain the same result as in the planning problem (equation(2.4.7)). By setting the emissions tax at this level, public policy makes it possible to reach the social optimum.

### 2.5.2.2 Production of intermediate goods:

A typical firm in the energy sector uses only labor as input in its production function  $e_{i,t} = A_i L_{i,t}$   $i = c, r$ . Energy prices  $p_{c,t}$ ,  $p_{r,t}$  are given so that a representative firm in each of these sectors maximizes its profit:

$$\max_{\{L_{i,t}\}} \Pi_{i,t} = P_{i,t} e_{i,t} - w_t L_{i,t} \quad i = c, r$$

### 2.5.3 Closing the model:

Consolidating the household budget constraint with the necessary conditions on each market yields the aggregate accounting identity:

$$Y_t = C_t + I_t + \frac{\phi}{2} \left( \frac{I_t}{K_{g,t-1}} - \delta \right)^2 K_{g,t-1} \quad (2.5.6)$$

At each period, depending on the quantity of fossil fuel consumed, the atmospheric GHG stock evolves according to the carbon cycle, and generates damages on the economy.

$$\begin{cases} D(S_t) = 1 - \exp\left(\kappa(\bar{S} - S_t)\right) \\ S_t = S_t^1 + S_t^2 \\ S_t^1 = \psi_1 \Omega_t \Phi e_{c,t} + S_{t-1}^1 \\ S_t^2 = (1 - \psi_1) \psi_2 \Omega_t \Phi e_{c,t} + (1 - \psi_3) S_{t-1}^2 \end{cases}$$

## 2.6 Calibration:

In this section we attribute numerical values to the model parameters. The unit of time is the year, and we take 2010 as the initial period. For the Kyoto Protocol's first binding agreement in terms of emissions, the historical base year is 1990, but in the last IPCC report (IPCC, 2014) the different scenarios are calculated relatively to 2010 so that we use this year as a benchmark.

### 2.6.1 Carbon cycle:

Carbon dioxide  $CO_2$  is not the only GHG responsible for global warming. Methane, nitrous oxide among others are also involved in climate change. They differ from each other in their life time, and warming capacities (see Table 2.1). To be more accurate we should take into account these differences, however for simplicity reasons, and because  $CO_2$  remains the largest GHG in terms of global emissions (see Table 2.2), we aggregate GHG to focus only on  $CO_2$ <sup>2</sup> or  $CO_2$  equivalent.

Table 2.1 – Global warming potential (GWP)

Greenhouse gas	formula	lifetime (year)	$GWP_{100}$
Carbon dioxide	$CO_2$	$[1; 10^6]$	1
water vapor	$H_2O$		8
methane	$CH_4$	12.4	23
Nitrous oxide	$N_2O$	121	310
carbon tetrafluoride	$CF_4$	50,000	6,500
sulphur hexafluoride	$SF_6$		22,800

The  $GWP_{100}$  measures the global warming potential of one GHG molecule over a timescale of 100 years.  $CO_2$  is the reference, and for example one water vapor molecule's warming impact is 8 times that of one carbon dioxide molecule over a 100-year period.

Sources : *IPCC (2014)*, *Hannah Ritchie (2017)*

<sup>2</sup>hereafter, GHG will be used to refer to  $CO_2$  equivalent.

Table 2.2 – Global GHG emissions by gas (Gt-CO2 eq.)

year	Total GHG incl. LULCF	Total GHG excl. LULCF	C02 LULCF	excl.	$CH_4$	$N_2O$	F-gas
1990	33.82	29.96	21.24		5.94	2.52	0.26
1995	34.92	31.06	22.30		5.91	2.56	0.29
2000	36.86	33.26	24.15		6.11	2.58	0.43
2005	42.50	38.27	28.44		6.54	2.69	0.60
2010	45.77	42.78	32.31		6.96	2.84	0.67
2013	48.26	45.26	34.39		7.13	2.92	0.82

LULCF refers to Land Use, Land Use and Change and Forestry

Sources : *CAIT (2015)*

We now calibrate the different parameters of the equation characterizing the carbon cycle (equation (2.3.3)). Based on the 4<sup>th</sup> IPCC report, GHKT assume that 20% of GHG will remain forever in the atmosphere. However, in the 5<sup>th</sup> IPCC report (IPCC (2014); p.544-545), between 15% and 40% of emissions will remain in the atmosphere for more than 2000 years. Therefore,  $\psi_1$  must be included in this range, and we choose the mean value  $\psi_1 = 0.275$ , which is closer to the one given by Archer (2005) (0.25). The lifetime expectancy of one carbon unit varies a lot (see table 2.1), we also assume as in GHKT, that it is equal to 300 years. In other words, 50% will have a lifetime expectancy greater than 300 years, and 50% will have a lifetime expectancy lower than 300 years, so that  $(1 - \psi_3)^{300} = 0.5$ . Thus the long run carbon decay rate is  $\psi_3 = 0.0023$  GHG unit per year. Following GHKT based on the 4<sup>th</sup> IPCC report, it takes 30 years to eliminate 50% of the share of one carbon unit emitted in the atmosphere in the current period. It yields the value for  $\psi_2$ :

$$0.5 = 0.275 + 0.725\psi_2(1 - 0.0023)^{30}$$

$$\psi_2 = 0.3325$$

Each time one GHG unit is emitted in the atmosphere, 27.5% of it will remain forever in the atmosphere. This means that 72.5% will be absorbed: two thirds will be immediately removed from the atmosphere and the remaining third will be degraded at an annual rate of 0.23% per year.

GHKT used 2000 as a reference year and assume that the total GHG stock in the atmosphere was  $S_{2000} = 802$  Gt, which was distributed as follows: 684 Gt were stocked in the first tank  $S^1$ , and 118 Gt in the second tank  $S^2$ . According to the NOAA annual GHG index, in 2010, the GHG atmospheric concentration was 448 ppm  $CO_2$  eq., using the conversion factor  $1 \text{ ppm} \leftrightarrow 2.13 \text{ Gt C}$  (see Clark (1982)), the corresponding amount of GHG was 996 GT C. To find the corresponding initial GHG stock in each reservoir from 2000 to the 2010 year reference, we need to calibrate the conversion factor  $\Phi$ , and to assume a functional form for  $\Omega_t$  to match 996 GT C in 2010. In 2010, 2,879 of gas (Mtoe) were consumed, 3,636 Mtoe of coal and 4,085 Mt of oil were consumed, a total of 10.6 Gt of oil eq (see Dudley et al. (2015)).

The exogenous variable  $\Omega_t \in [0, 1]$  stands for the exogenous technology progress that reduces emission intensity per fossil fuel unit burned. It is the way we model the EKC supposed in the Grossman and Krueger (1995) model. According to Nordhaus (2010), we already have observed a reduction in emission intensity without implementing an environmental policy, he assumes that this decrease is the consequence of technological progress. This function is assumed to be decreasing over time, equals to 1 in  $t = 0$ , and tends towards 0 when  $t$  tends to infinity. We assume that  $\Omega_t$  follows a sigmoid-function as in Barrage (2014) and Krusell and Smith (2015), of parameters  $\Theta = \log\left(\frac{0.01}{0.99}\right)$ . Moreover, it is estimated that after 10 years ( $n_{99} = 10$ ), the emission intensity per unit of fossil fuel consumed will be reduced by 1%, and that it will take approximately 125 years to divide them by a factor 2. This means that after 298 years, emissions intensity will be reduced



by 99.9% (see Figure 2.2):

$$\left\{ \begin{array}{l} \Omega_t = \frac{1}{1 + \exp\left(\Theta\left(\frac{t-n_{50}}{n_{50}-n_{99}}\right)\right)} \quad 0 < t \leq 300 \\ \Omega_t = 0 \quad t > 300 \end{array} \right. \quad (2.6.1)$$

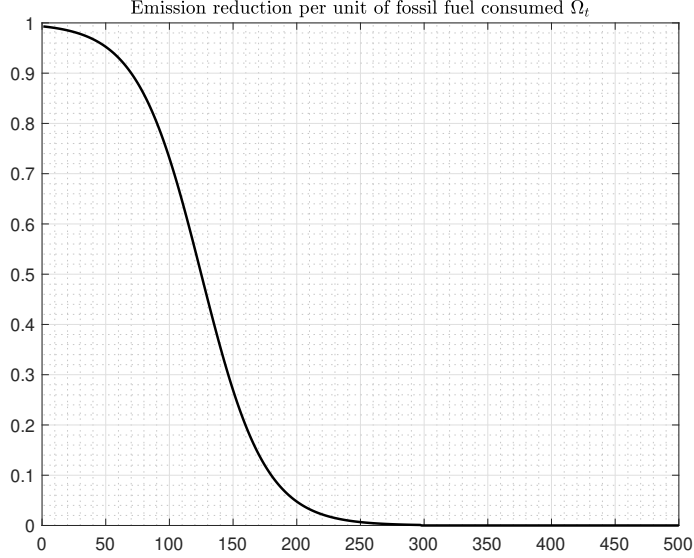


Figure 2.2 – Emissions intensity per fossil fuel unit

From the 2000 GHG stock in the atmosphere used in GHKT, the quantity of fossil fuel consumed, the function  $\Omega_t$  and using the conversion factor  $\Phi$  we can calculate the initial  $CO_2$  stock in 2010 by forward recursion:

$$\left\{ \begin{array}{l} S_{1,10} = S_{1,00} + \psi_1 \Phi \hat{e}_c \sum_{i=1}^{10} \Omega_i \\ S_{2,10} = (1 - \psi_3)^{10} S_{2,00} + (1 - \psi_1) \psi_2 \Phi \hat{e}_c \sum_{i=1}^{10} \Omega_i (1 - \psi_3)^{10-i} \end{array} \right. \quad (2.6.2)$$

By assuming that the quantity of fossil fuel consumed in 2000 was  $e_c = 10.6$  Gt oil eq., and a conversion factor of  $\Phi = 4$  and solving the system of equations (2.6.2), we finally find that in 2010, the total stock of GHG in the atmosphere was  $S_{10} = 993$  Gt  $CO_2$  eq., 788 Gt in the first reservoir  $S^1$ , and 206 Gt in the second reservoir  $S^2$ .

We calibrate the scale parameter  $\kappa = 5.3 \cdot 10^{-5}$  of the damage function, as in GHKT, and we assume that the pre-industrial GHG atmospheric stock was  $\bar{S} = 581$  Gt  $CO_2$  eq. To find the corresponding rise in temperature to the GHG atmospheric stock, we use the following equation as in GHKT:

$$T_t = \frac{3}{\log(2)} \log\left(\frac{S_t}{\bar{S}}\right) \quad (2.6.3)$$

Using the initial GHG stock in 2000  $S = 802$  of GHKT, the corresponding temperature increase was assumed to be 1.4 °C above pre-industrial levels. Assuming now an atmospheric stock of  $S = 993$  Gt  $CO_2$  eq. in 2010, we should see a temperature increase of 2.3 °C. Both of these values overestimate what we observe in reality. According to GHKT, Schwartz et al. (2010), and Stern and Stern (2007) (p.15), the warming observed is lower than the warming expected, because other pollutants in the atmosphere such as aerosols, cool down the temperature, and mask a share of warming effects of GHG.

### 2.6.2 Preferences and technology parameters:

We use the Penn World Table (Feenstra et al., 2015), to calibrate some parameters to reach steady state ratios. Out of 185 observations for the 2010 reference year, 47 countries are removed for which data are missing, resulting in a total of 135 countries. To calculate the  $\frac{K_g}{Y}$  steady state ratio, we aggregate the capital stock at 2011 constant price, and we divide it by the "*gross world product*", calculated as the sum of the GDP of each of the 135 countries. We find a capital to gross world product ratio of 3.83 in 2010. The global capital stock represents nearly 4 years of global production.

The average global depreciation rate is calculated in two steps. First, the depreciation rate of each country of the Penn World Table are used to calculate the share of capital that has depreciated, which gives us the global depreciated capital in 2010. Then, this share is divided by the global capital stock for the same year to calculate the average annual depreciation rate of capital:

$$\delta = \frac{\sum_j \delta_j K_{g,j}}{K_g} = 0.0441$$

We use each country's real internal rate of capital, and the above capital stock to production ratio to find  $\alpha = 0.325$ , and an aggregated steady state interest rate of  $r = 0.074$ . With an annual depreciation rate of capital  $\delta = 4.4\%$ , we find a gross investment to capital stock steady state ratio of  $\frac{I}{K_g} = 4.42\%$ , and an investment rate of  $\frac{I}{G} = 16.9\%$ . Consequently the income share consumed is equal to  $\frac{C}{G} = 83.1\%$ . We choose the discount factor  $\beta = 0.971$ , so that the rate of return of capital is  $r = 7.4\%$ .

The relative price  $\frac{P_n}{P_c}$  of non-polluting energy input in terms of polluting energy input is equal to the productivity ratio  $\frac{A_n}{A_p}$ . To calculate this ratio, we aggregate the amount of coal, gas and oil consumed into one polluting energy input and we calculate an aggregate price. The same technique is used to determine the amount of non-polluting energy (solar panel, wind, nuclear, hydro-energy, and biomass) consumed and its aggregated price. According to Dudley et al. (2015), in 2010 4,085 Mtoe of oil were produced, 2,879 Mtoe of gas and 3636 Mtoe of coal. The aggregated fossil fuel price  $p_c$  is calculated as a coal average price, natural gas average price and crude oil average price, weighted by the quantity of each fossil fuel consumed. According to the US Energy Information Administration, between 2000 and 2017, the annual WTI crude oil price was in average \$ 64.72 per barrel. The conversion factor of 1 barrel is 0.1364 metric ton, so on average one metric ton of oil cost \$ 474.46. For the same period, the Henry Hub Natural Gas Spot Price averaged \$ 5.2 per million British Thermal Unit (BTU). One million BTU is equal to 0.18 barrel of oil equivalent, so that the natural gas price averaged \$ 211.81 per metric ton. The global price of coal amounted to \$ 69.99 per metric ton. The aggregate price of fossil fuel is therefore:

$$P_c = \frac{211.81 \cdot 2879.7 + 69.99 \cdot 3635.6 + 474.46 \cdot 4085.42}{2879.7 + 3635.6 + 4085.42} = 264.39 \quad \$/toe$$

The aggregate price of non-polluting energy is calculated in the same way. In 2010, nearly 15% of primary energy consumed did not emit GHG: 2,767.3 TWh (626.2 Mtoe) was of nuclear<sup>3</sup> origin, 3,463.3 TWh (783.9 Mtoe) came from hydroelectricity, 31.4 TWh (7.1 Mtoe) from solar energy, 343.4 TWh (77.7 Mtoe) from wind energy and 367.6 TWh (83.

<sup>3</sup>Nuclear energy is by definition not a renewable energy, but since it does not emit GHG it is opposed to fossil fuels and consequently we include it into non-polluting energies in the sense of climate change.

Mtoe) from biomass (Dudley et al., 2015). The world price per MWh of nuclear energy is based on the costs of generating electricity calculated by the US Energy Information Administration <sup>4</sup>. The cost price is an average of operation, maintenance and fuel cost; its annual average between 2007 and 2017 amounted \$26.89 per MWh. According to IRENA<sup>5</sup>, the 2010-2017 average Levelised Cost<sup>6</sup> of hydropower is \$45 per MWh, \$230 per MWh for solar photovoltaic, \$ 140 per MWh for offshore wind, and \$ 70 per MWh for onshore wind. Using the conversion factor between MWh and ton of oil equivalent (1MWh  $\leftrightarrow$  0.085 toe), we find an aggregate price of  $P_r = 484.65 \text{ } \$/toe$ . Consequently, the we assume that  $\frac{e_r}{e_c} = 0.142$ , and we calibrate the productivity ratio as equal to:  $\frac{A_r}{A_c} = 0.544$

We simulate the model for different values of the elasticity of substitution between polluting and non-polluting energy. Pelli (2012) estimates the world elasticity of substitution between polluting and non-polluting inputs in the production of electricity to  $\sigma = 0.53$  using a sample of 20 countries, so that we calibrate  $\varphi = -0.88$ . The relative price of renewable in terms of fossil fuel is  $\frac{P_r}{P_c} = 1.84$ , using the relative energy demand  $\frac{e_r}{e_c} = 0.142$ , with the elasticity of substitution  $\sigma = 0.53$  we finally find  $\omega = 0.0442$ . Stern (2012), used 47 studies to conduct his meta-analysis on the elasticity of substitution of energies. The weighted mean of oil-electricity (1.06), coal-electricity (0.866) and gas-electricity (1.099) elasticities is greater than unity:  $\sigma = 1.004$ . GHKT used the same study to calibrate their elasticity of substitution but they only focused on oil and coal so that their elasticity of substitution is slightly less than unity (0.95). Finally, we consider two extreme cases where the elasticity of substitution is either equal to 2 or 5 (see table 2.3).

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<sup>4</sup>[https://www.eia.gov/electricity/annual/html/epa\\_08\\_04.html](https://www.eia.gov/electricity/annual/html/epa_08_04.html)

<sup>5</sup><http://resourceirena.irena.org/gateway/dashboard/?topic=3&subTopic=1065>

<sup>6</sup>The Levelised Cost of electricity is a measure of the overall costs necessary (investment and maintenance over time) to produce one unit of electricity over the lifetime of the system considered (25 years for wind power, solar PV, 30 years for hydropower and 20 years for biomass power).

Table 2.3 – Elasticity of substitution

$\sigma$	$\varphi$	$A_r/A_c$	$e_r/e_c$	$\omega$	ref.
0.53	-0.887	0.544	0.142	0.044	Pelli (2012)
1.004	$3.98 \cdot 10^{-3}$	0.544	0.142	0.208	Stern (2012)
2	0.5	0.544	0.142	0.408	
5	0.8	0.544	0.142	0.554	

According to Dudley et al. (2015), the world's primary energy consumption amounted to 12,110.765 Mtoe in 2010. This gives an energy intensity of the 2010 gross world product of  $\frac{E_g}{G} = 0.138$  kt oil eq. (ktoe) per million of 2011 US dollar constant, hence we calibrate  $\gamma = 0.0434$ , which is very close to the value of Golosov et al. (2014) ( $\gamma = 0.04$ ).

According to our sample of 135 countries from the Penn table, the corresponding population that of 5.9 billions in 2010. To determine the corresponding wage and labor demand across sectors we first assume that in 2010, the carbon tax was equal to zero, and consequently the household lump sum subsidy was also zero. Using the different parameter values for our calibration we find that in the final good sector, demand for labor is:  $L_g = 5.52$ . Labor demand for non-polluting energy is  $L_r = 0.0783$ , labor demand for the fossil fuel sector is  $L_c = 0.301$ , corresponding to a productivity in each sector respectively of  $A_r = 19.22$  and  $A_c = 35.22$ . Productivity in the final good sector is calibrated to  $A_g = 1.5 \cdot 10^4$ .

## 2.7 Simulations:

### 2.7.1 General methodology:

We use the Dynare software (Adjemian et al., 2011) to perform our simulations. This software, which is a set of MatLab routines, was developed to solve both stochastic and deterministic macroeconomic models. Dynare is able to solve deterministic non linear

dynamic models or perfect foresight models using a set of Newtonian algorithms (sparse matrices, sparse LU factorization, General Minimal Residual or GMRES, Stabilized Bi-conjugate Gradient or BICGSTAB, sparse Gaussian elimination or SPE and Juillard's algorithm). Usually, the way to solve this kind of model is to find the steady state value of all the state and control variables. Assuming that the model is in its equilibrium, one could find a way to determine the expression of the variables as functions of the deep parameters of the model as well as exogenous variables. A second approach to solve this kind of models is to solve paths of all the endogenous variables, depending on initial conditions and shocks (Fair and Taylor, 1983). The model can be reduced in the following form:

$$f(y_{t+1}, y_t, y_{t-1}, u_t) = 0 \quad t = 1, \dots, T$$

where  $y$  is a vector of endogenous variables,  $u$  is a vector of exogenous variables with initial condition  $y_0$  given. In each period  $t = 1, \dots, T$ , we need to solve the stacked system of nonlinear equations:

$$\left\{ \begin{array}{l} f(y_2, y_1, y_0, u_1) = 0 \\ f(y_3, y_2, y_1, u_2) = 0 \\ \vdots \\ f(y_{t+1}, y_t, y_{t-1}, u_t) = 0 \\ \vdots \\ f(y_{T+1}, y_T, y_{T-1}, u_T) = 0 \end{array} \right.$$

The compact stacked system can be written  $F(\mathbf{Y}) = 0$  with  $\mathbf{Y} = (y_1', y_2', \dots, y_T)'$ . The Newton method can be summarised as follows: first, we must provide an initial guess  $\mathbf{Y}^{(0)}$ , then iterate by updating solutions  $\mathbf{Y}^{(k+1)}$  after solving the system:

$$\left( \mathbf{Y}^{(k)} \right) + J_F \left( \mathbf{Y}^{(k)} \right) \left( \mathbf{Y}^{(k+1)} - \mathbf{Y}^{(k)} \right) = 0$$

where  $J_F(\mathbf{Y}^{(k)}) = \frac{\partial F}{\partial \mathbf{Y}}$  is the Jacobian matrix of  $F$ . Iterations are stopped if the terminal condition is satisfied:  $\|\mathbf{Y}^{(k+1)} - \mathbf{Y}^{(k)}\| < \epsilon$ , which is equivalent to  $\|F(\mathbf{Y}^{(k)})\| < \epsilon$ , where  $\epsilon$  is the convergence criterion.

In order to solve our system of non-linear equations, we must provide a boundary condition  $y_0$ . Usually this condition is the steady state value of the system. However this solution is irrelevant in our case. Indeed, by definition this implies that all variables are invariant, including the law of motion for the accumulation of GHG in the atmosphere. Yet, this assumption is incompatible with the current increase in GHG concentration in the atmosphere. Another solution would be to use the long-term equilibrium and by backward recursion calculate the trajectories from now that lead to this equilibrium. Once again, this solution is difficult to implement, since there is an infinite number of long-term steady states, that depend on the different possible fossil fuel energy consumption paths and so on emission trajectories. According to the model's assumptions, we know that even if the economy continues to use fossil fuels, emission intensity per unit of fossil energy consumed will tend towards 0, thanks to the exogenous variable  $\Omega_t$ . As a result, the GHG stock in the atmosphere will reach a maximum and then decrease over time towards its long-term equilibrium, and this equilibrium will define the equilibrium of the model. But the dynamics vary and the long-run equilibriums are determined from past GHG accumulations. In the second reservoir, the one in which  $CO_2$  is gradually eliminated, the very long term quantity of GHG in this reservoir will be equal to zero since it is assumed that the rate of carbon decay will be higher than the emissions rate, so that  $\lim_{t \rightarrow \infty} S_{2,t} = 0$ . In contrast, since GHG are not removed from the other reservoir  $S_1$ , the long term equilibrium of the overall stock of  $CO_2$  in the atmosphere will be determined by the amount of  $CO_2$  that has accumulated in this reservoir:  $\lim_{t \rightarrow \infty} S_t = \lim_{t \rightarrow \infty} (S_{1,t} + S_{2,t}) = \lim_{t \rightarrow \infty} S_{1,t}$ . Once we have this equilibrium, we would be able to determine environmental damages as well as the steady state of the entire model. There are as many possible trajectories as there are  $S_1$  limits.

Recalling the recursive representation of the carbon cycle (equation (2.3.3)):

$$\begin{cases} S_t^1 = \psi_1 \Omega_t \Phi e_{c,t} + S_{t-1}^1 \\ S_t^2 = (1 - \psi_1) \psi_2 \Omega_t \Phi e_{c,t} + (1 - \psi_3) S_{t-1}^2 \end{cases}$$

After 300 years,  $\Omega_t = 0$ , thus:

$$\Leftrightarrow \begin{cases} \psi_1 \Omega_t \Phi e_{c,t} = 0 \\ (1 - \psi_1) \psi_2 \Omega_t \Phi e_{c,t} = 0 \end{cases}$$

Consequently,

$$\begin{cases} \lim_{t \rightarrow +\infty} S_t^1 = S_0^1 + \sum_{j=1}^{300} \psi_1 \Omega_j \Phi e_{c,j} \\ \lim_{t \rightarrow +\infty} S_t^2 = 0 \\ \lim_{t \rightarrow +\infty} S_t = S_t^1 \end{cases}$$

The solution we have chosen for the initial guess is the following: it is assumed that all economic variables of the system are determined at their steady state, but not the carbon stock. Thus in the baseline scenario (the business as usual one) where externalities are not taken into account, the dynamics of the system will be induced by the carbon cycle. We thus simulated the model over a sufficiently large numbers of periods ( $t > 300$ ) to allow carbon dynamics to converge, but our analysis will focus on a shorter period. We choose  $T = 1000$ , the number of periods simulated. This strategy is reused for the different simulations environmental policy .

## 2.7.2 Results:

### 2.7.2.1 Environmental impacts:

In this section the 4 Business As Usual (BAU) scenarios (for the 4 different elasticities assumed see Table 2.3) are summarised in a single scenario. Since the productivity ratio of energies remains constant, the same results are obtained. The next section explores the case where the energy productivity ratio evolves in time, and then considers the 8 different scenarios. We thus consider one BAU scenario as the benchmark scenario (without an environmental policy), and 4 different scenarios with an optimal environmental policy for the different elasticities of substitution considered. Under the assumption of exogenous emission reduction, according to the results of the model, in all the different relevant scenarios we observe a decrease in emissions, yet in different proportions.



In the BAU scenario, the dynamics of GHG emission flow reduction is only due to technical progress  $\Omega_t$  (see Figure 2.3). The flow of emissions curve follows the sigmoid function defined above: emissions are supposed to reduce slowly in the next 50 years, then reduction in emission flows is supposed to accelerate and tends towards zero in the very long term. After 50 years, the model generates a reduction in emissions by 4% compared to 2010 levels and by 26% in a century thanks technological progress that will reduce emission intensity.

As we have already reported, as long as the emissions flow exceeds natural carbon sinks, GHG stock in the atmosphere increases. As shown in Figure 2.3, in the case of the BAU, the GHG stock increases over time although the decrease in the flow of emissions. The GHG stock reaches its maximum 170 years after 2010. Then in a second stage, as absorption capacity is greater than emissions flow, the GHG stock decreases and tends slowly towards its long-term equilibrium value. This accumulation of GHG results in a rise in temperature of 4 °C (see Figure 2.4) by the end of the century, and the damages caused by such a concentration would account for 8% of world GDP.

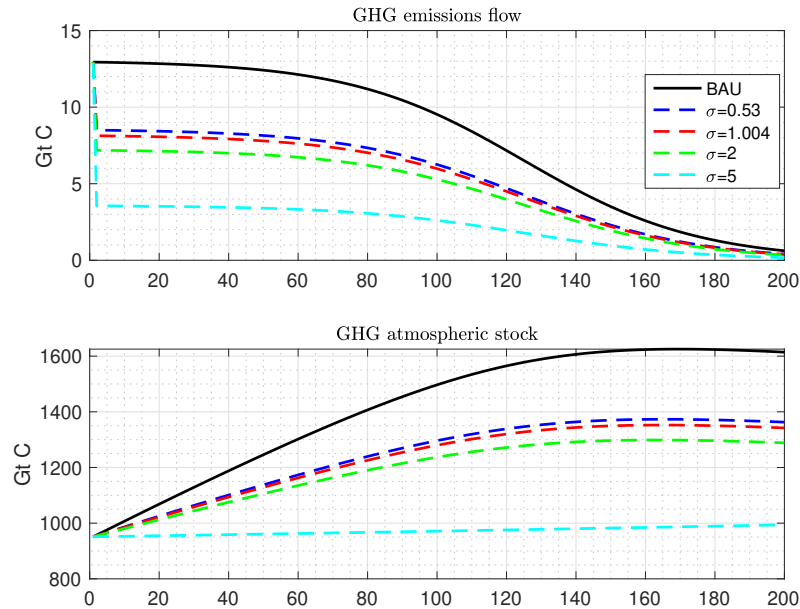


Figure 2.3 – GHG emissions flow and atmospheric stock.

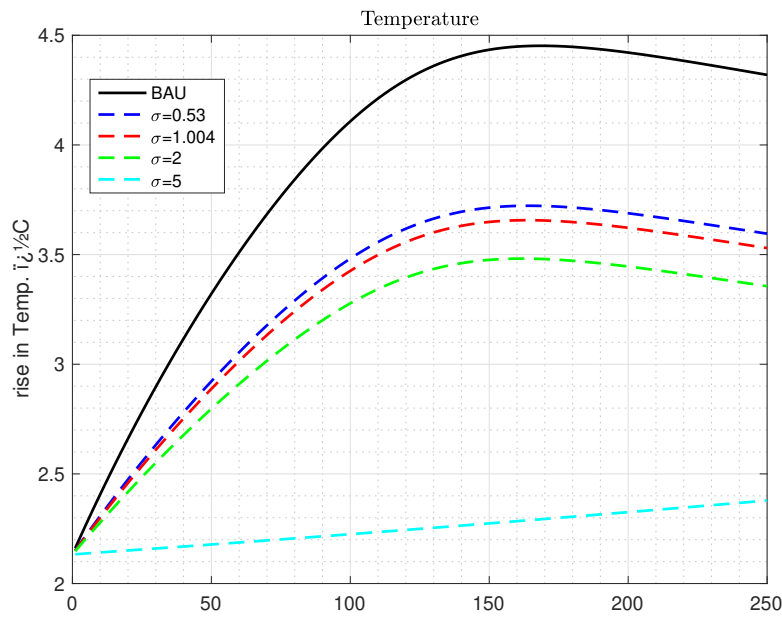


Figure 2.4 – Rise in temperature above the pre-industrial levels

When the economy internalizes the negative effects of its emissions by introducing a carbon tax (represented in the Figures by dashed lines), the reduction in emissions is explained, on the one hand, by technological progress as in the BAU, but also by the introduction of the tax. The tax explains the gap between the BAU and the different scenario in Figures 2.3 in the initial period. In the short run, the decreases in emissions is largely explained by the implementation of the tax. Differences between the four scenarios depend on the elasticity of substitution between energy inputs. In the case of the lowest elasticity ( $\sigma = 0.53$ ), fossil fuels and non-polluting energy are complementary inputs. However the introduction of the tax immediately reduces the annual flow of emissions by 34% with respect to the BAU. As expected, the higher the elasticity of substitution, the greater the effect of the tax will be (an immediate 37% reduction when  $\sigma = 1.004$  and by roughly 44% if  $\sigma = 2$ ). The economy is consequently constrained by this complementary effect, and reduction in emission flow is relatively low compared to the other scenarios. After 50 years, the model plans to reduce its emissions by at least 37% for  $\sigma = 0.53$  and by 40% for  $\sigma = 1.004$  (47% and 73% for  $\sigma = 2$  and  $\sigma = 5$

respectively) compared to 2010, thanks to both technological progress and to the carbon tax. At the end of the century relatively to the reference year, in the worst scenario the economy would be able to reduce by 47 % its emissions through an optimal carbon tax. As in the BAU, the model predicts that the GHG stock will peak in roughly 165 years, regardless of substitution elasticities. But, the atmospheric stock would be 184 Gt C lower in the case of complementary goods ( $\sigma = 0.53$ ) and 199 Gt C lower if they are substitutable ( $\sigma = 1.004$ ), 239 Gt C for  $\sigma = 2$  and 485 Gt C if  $\sigma = 5$ . Thus the damage would be respectively 6.6%, 6.4% , 6.0% and 3% of GDP by the end of the century, which would correspond to a rise in temperature of 3.4°C, 3.3°C, 3.1 °C and 2.2 °C respectively.

Finally, we can observe that due to the hypothesis of exogenous emission reduction we find the famous inverted U-shaped EKC, emissions flow reduce towards zero in the long run (see Figure 2.3 and 2.5). However, as expected, the stock of GHG is determined by the accumulation of emissions in the the first reservoir. Indeed, we observe that in the second reservoir, emissions diminishes towards zero, while they remain constant in the long-run in the first reservoir. Thus long-term environmental damage are determined by the stock in the latter reservoir. Emissions path flows depend on two criterion, the implementation or not of the carbon tax and the elasticity of substitution that will determine the emission flows if the climate policy is present. As a result, without public intervention or should it be delayed, it would be impossible to achieve certain long-term balances. In other words, if certain emission levels are reached, then certain temperature targets cannot be reached given the accumulation of GHG in the 1<sup>st</sup> reservoir. Consequently, global warming is irreversible if nothing is done in the short run.

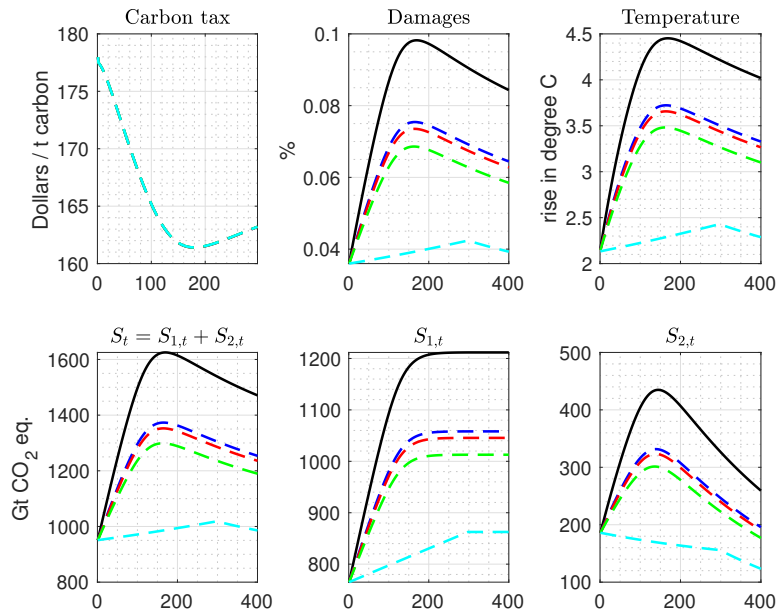


Figure 2.5 – GHG atmospheric stock and its consequences in terms of temperature and damages.

### 2.7.2.2 Economical impacts

In the absence of an environmental policy (BAU), the dynamics observed are the consequence of the variation in the stock of GHG in the atmosphere. The GHG increase results in higher environmental damage, which spreads throughout the economy owing to a decrease in the total productivity of factors. All other things being equal, production reaches its minimum when the GHG stock and damage are at their maximum.

The introduction of the carbon tax makes it possible to take into account the negative externalities generated by the use of fossil fuels. With an annual 3% per year discount rate, the amount of the optimal carbon tax obtained is about \$ 180 per ton of carbon emitted (see Figure 2.5). This result is very sensitive to the choice of the  $\beta$  parameter, that defines the time preference for the present in our economy. The lower this parameter, the greater the preference for the present. On the contrary, the more it tends towards 1, the more agents care about future generations. Thus, all other things being equal with a

$\beta = 0.96$ , the optimal carbon tax in the economy would be \$ 70 per ton of carbon emitted, whereas if  $\beta = 0.99$  the optimal tax would be \$ 345 per ton. However, by modifying this parameter, we also modify the saving behaviour of our representative household. In the first case, the investment rate is lower than what was actually observed (16%) with an interest rate of capital of  $r=8.6\%$ , and in the second case it is higher (26%) with an interest rate of 5%.

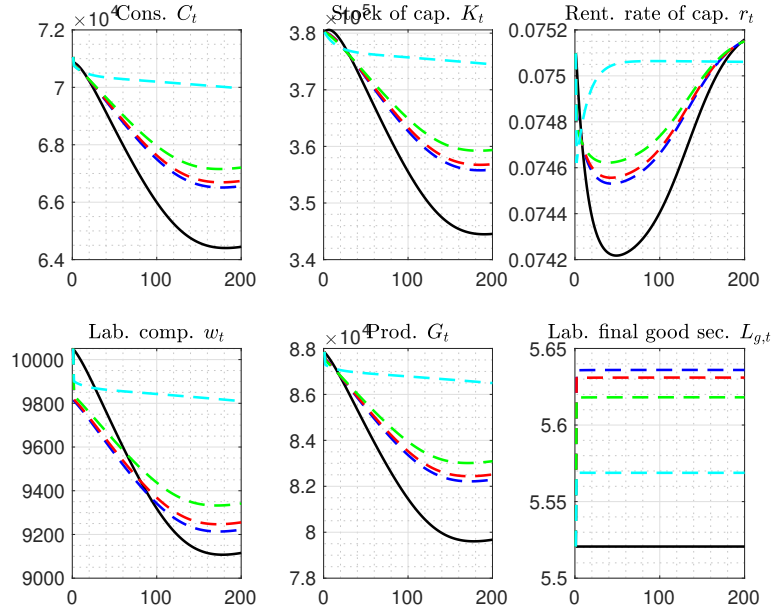


Figure 2.6 – Main economic variables of the stationary model:

By way of comparison, Nordhaus (2008) gets a carbon tax of \$ 35 per ton of carbon , while Stern and Stern (2007) obtain a tax of \$ 250 per ton and GHKT between \$25 and \$489 per ton of carbon (depending on various discount rate and if damages turn out catastrophic). Since they use different discount rates, the differences obtained can be explained in part by this factor. However, the assumptions in the model differ and also account for these differences. Indeed, for the sake of simplicity we have only considered a coal regime. Under this assumption, without Hotelling's rent, the price of the natural resource does not increase since it is not exhaustible. Whereas in models that assume resources as depletable (like oil or gas), the prices of these resources increase over

time, explaining why the optimal tax is relatively lower than in a pure coal regime. By introducing the tax, the economy sees its intermediate costs increase. This increase is reflected in a lower use of fossil fuels, but also in higher prices and therefore lower production relative to the BAU. But by emitting less GHG, environmental damage is relatively contained with a tax. After 15 to 19 years (depending on the elasticity of substitution  $\sigma$ ), the environmental cost in terms of GDP in the case of the BAU will exceed that of the tax.

The economic consequences of the tax will depend on the elasticity of substitution between energy factors. In case of complementarity between polluting and non-polluting energies ( $\sigma < 1$ ), the tax will reduce not only the use of fossil fuels, which will reduce the annual flow of emissions (see previous section), but also the use of non-polluting energy since they are complementary. The increase in carbon tax reduces demand for both energy inputs. As labor is the only factor used in energy production demand  $L_c$  and  $L_r$  falls, pulling down labor compensation. As this factor becomes relatively cheaper, demand for labor  $L_g$  in the final good sector tends to increase. In the short term, when environmental damage is limited, the dry loss due to the tax leads to a drop in demand. The demand for the final good is higher in the absence of the tax. But in the medium term, the effects will be reversed between the two scenarios. The cost in terms of environmental damage in the case of the BAU will exceed the dry loss of the tax. There is an intertemporal trade-off to be made: reducing emissions in the short term to limit climate change in the medium and long term requires a decrease in demand in the short term.

In the case where inputs are substitutable ( $\sigma > 1$ ), the decrease in fossil fuel use (that implies a decrease in emissions) will be partially offset by an increase in demand for non-polluting energy. Still, since productivity in this sector is lower than that of fossil fuels, the decline in labor demand in this sector ( $L_c$ ) is not fully offset by an increase in labor demand in the non-polluting energy sector ( $L_r$ ). This phenomenon of compensation in the various sectors following the tax implementation is positively related to the elasticity of substitution between energy inputs. The larger the elasticity of substitution, the more

the economy will use non-polluting energy, the greater the demand for labor in this sector. As in the previous case where energies were supposed to be complementary, the tax has a recessionary effect but this is offset by the use of the tax revenues in the form of subsidies which prevents consumption from collapsing. Nevertheless the negative effect of the tax is lessened by the ability of the economy to use non-polluting energy. In the case where energies are close substitute, the economy can reduce these GHG emissions in a larger proportion, the effect of the tax is then much more effective and thus limits the negative consequences related to environmental damage. With a larger production level, long-term consumption and utility are therefore higher than in the previous case.

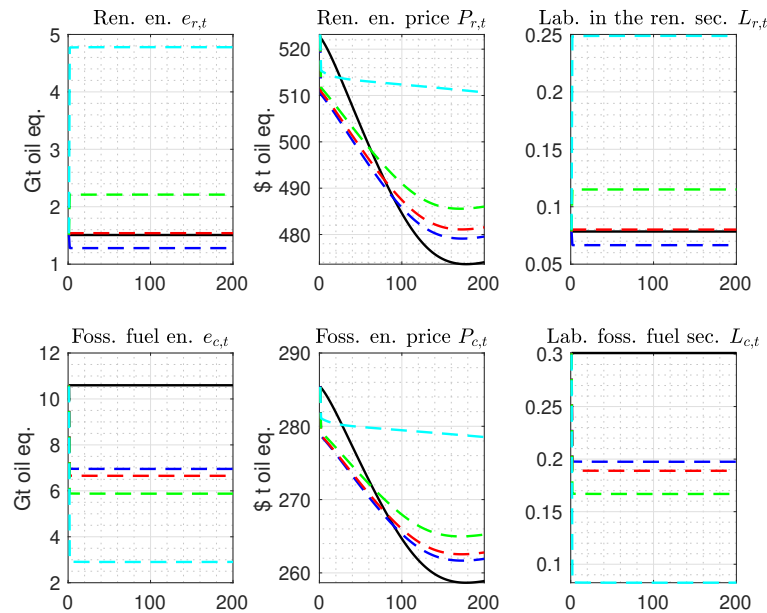


Figure 2.7 – Energy block:

### 2.7.3 Rise in renewable energy productivity:

We now consider the possibility of increased productivity in the non-polluting energy sector. Until now we have assumed that the productivity ratio ( $A_r/A_c$ ) remains constant over time. However over the past decade, there has been a decline in the global weighted-average levelized cost of electricity from non-polluting energies (bioenergy, geothermal,

hydropower, onshore and offshore projects, solar photovoltaic). The non-polluting energy sector is relatively young compared to the fossil fuel energy sector. It can be considered that the latter is now a mature industry. According to Kuuskraa et al. (2013), the main determinants of drilling costs is the well depth. Oil and gas firms began by exploiting first shallow fields. With the depletion of resources, they have to drill deeper and deeper both onshore and offshore (see Table 2.4). As a result, the cost of operating fossil fuels tends to increase, and the price per barrel must be high enough to reach the break-even point. In 2014, the onshore break-even point was \$ 44 per barrel of oil for US fields (conventional and shale gas fields) and higher than \$ 60 per barrel of oil for offshore projects (EIA, 2016).

Table 2.4 – US average depth of oil and natural gas wells drilled (feet per well)

year	1950	1960	1970	1980	1990	2000	2008
oil	3,893	3,889	4,385	3,801	4,483	4,593	5,094
natural gas	3,979	5,486	5860	5,275	5,114	4,923	6,500

Source: US EIA

On the other hand, the fact that the renewable energy industry is relatively young suggests that there may be declines in the LCOE. According to a study conducted by IRENA (2019), energy produced by new photovoltaic panels, and onshore wind turbines are expected to be relatively cheaper than the marginal operating cost of coal by power plants from 2020 and beyond. Between 2010 and 2018, on average the LCOE from onshore wind has been reduced by 35% and producing 1 kWh is worth \$ 0.056 (to be compared with the fossil fuel-fired power generation cost globally estimated to be in the range of \$0.046 and \$0.174<sup>7</sup>), thanks to a reduction in turbine prices due to a rise in technology (between 2010 and 2017, the rotor diameter increased by 76% in Ireland, by 42% in Brazil, 64% in Canada, 25% in France and 34% in the United States), economies of scale, regional

<sup>7</sup>All costs are expressed in constant 2018 US dollars



production etc. On the global weighted average, LCOE of solar photovoltaic was worth about \$ 0.085 in 2018, 77% lower than in 2010. This decrease is the result of a much lower raw material price (crystalline silicon module price fell by nearly 30% between 2017 and 2018). The total installed cost per kW was divided by a factor 4 between 2010 and 2018 (\$ 4,621 in 2010 and \$ 1,210 in 2018 global weighted average).

According to the same study, the LCOE from offshore wind power was worth \$0.127 per kWh in 2018, corresponding to a 20% reduction in costs between 2010 and 2018. The main drivers explaining that decrease in costs are innovations (larger rotor), economies of scales (installation and logistics). Larger turbines imply lower installation costs for a given power capacity. While the total installed cost remained constant because farms were installed in deeper ocean (\$ 4,572 in 2010 against \$4,353 in 2018). Setting up offshore wind turbines in deeper waters is more expensive, but the field covered is wider, wind is less volatile which increases the capacity factor. The rise in the capacity factor from 38% to 43% explained the decrease in LCOE for the same period.

Concentrating solar power (CSP) and bioenergy has also seen their LCOE fall over the past decade. A decrease of 46% per kWh for CSP and 15% for bioenergy compared to 2010. Energy production from CSP is in its early stages, global capacity was 5.5 GW, to be compared with 95 GW of new solar panel capacity installed the same year. China is developing several CSP projects, which should lower the LCOE in the near future. The total cost of installing a kW roughly amounted \$ 5,200 in 2018, against \$ 8,800 in 2010.

For geothermal power generation and hydropower, LCOE have increased between 2010 and 2018, by 40% for geothermal energies, and by 29% for hydropower. However, the LCOE for geothermal are very low \$ 0.07 per kWh, and \$ 0.047 per kWh for hydropower. The very small capacity (500 MW of new capacity in 2018) and specificities of each project (geography, country) explain this rise for geothermal energy. On the contrary, hydropower is a mature energy industry, and explains why new capacities increased its costs. Indeed, the best sites are already exploited, and developing this energy requires to overcome difficulties linked to geography like bigger rivers.

Based on these observations, it is assumed that productivity  $A_{r,t}$  in the non-polluting sector will increase. This increase will reflect on the one hand the reduction in investment cost per kW (or fixed cost), as a consequence of technological innovations or economies of scale. This will also represent the fact that, unlike the fossil fuel sector, the marginal cost of production is close to 0 for most of these energies. In the absence of public intervention, this rise in productivity may have opposite effects on GHG emissions, depending on the elasticity of substitution between clean and polluting inputs, as we will see in the next subsection. To take this scenario into account, we will assume that productivity evolves as:

$$A_{r,t} = X_t A_r$$

where  $X_t$  is a function with the following properties. The first derivative with respect to time is positive  $\dot{X}_t > 0$ , and the second derivative is negative  $\ddot{X}_t < 0$ . Initial condition supposes that  $X_0 = 1$ , while for the terminal condition it is assumed that  $\lim_{t \rightarrow \infty} X_t = C_1$ , where  $C_1$  is a positive constant. We assume that  $X_t$  takes the following function form:

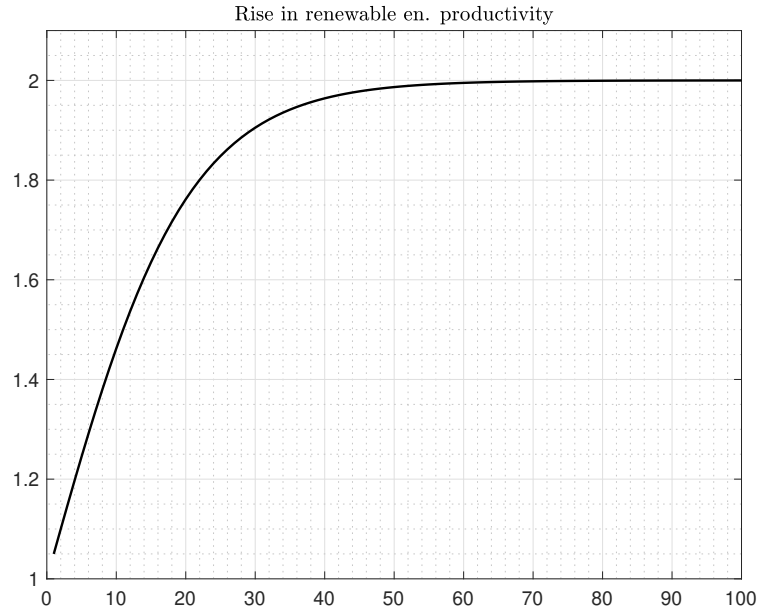


Figure 2.8 – Rise in clean energy productivity

$$\begin{cases} X_t = \frac{C_1}{1 + C_2 \exp(-C_3 \cdot t)} \\ C_1 > 0, \quad C_2 > 0, \quad C_3 > 0 \end{cases}$$

According to the market analysis of the International Energy Agency (IEA, 2019), between 2019 and 2049, clean energies capacities are supposed to expand by 50%. Hence, we calibrate the different constants of  $X_t$  so that we match this increase. Thus  $C_1 = 2$ ,  $C_2 = 1$ , and  $C_3 = 0.1$ . The value of  $C_3$  determines the rate of convergence towards the limit. The higher  $C_3$  is, the faster the limit is reached. With these different values, productivity in the non-polluting sector will increase by 98% in 50 years (see figure 2.8).

### 2.7.3.1 Environmental impacts:

The same model is solved, as before, but this time the increase in productivity is taken into account. Four benchmark scenarios are simulated, one for each of the elasticity of substitution between factors. Then we simulate 4 other scenarios, this time introducing the environmental tax on GHG.

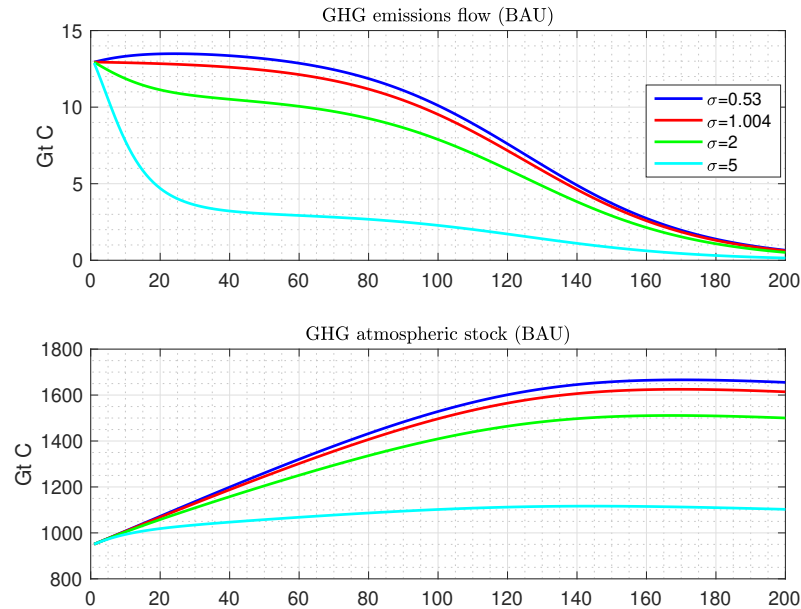


Figure 2.9 – BAU scenarios and consequences on emission flow and atmospheric stock

Figure 2.9 illustrates the changes in the flow of emissions and GHG atmospheric stock, due to the rise in productivity and the lack of environmental policy. With productivity increase in the non-polluting sector, we can notice that if the elasticity of substitution is greater than 1, then emission flow will be lower than in the previous scenario where the productivity ratio was constant. Consequently the GHG stock is relatively lower in the atmosphere. We can observe a considerable decrease in the extreme case considered, where the two energies are close substitute inputs ( $\sigma = 5$ ). With a productivity increase, the energy mix leaves more room to clean energy, thus a part of fossil fuels is abandoned, resulting in lower environmental damage and a relatively lower temperature increase. If, on the other hand, the two inputs are complementary (in our case we assumed  $\sigma = 0.53$ ), in the absence of a carbon tax, an increase in productivity in the clean energy sector can have negative consequences on the climate (increase in the flow of emissions and therefore increase in the GHG atmospheric stock relatively to the previous scenario).

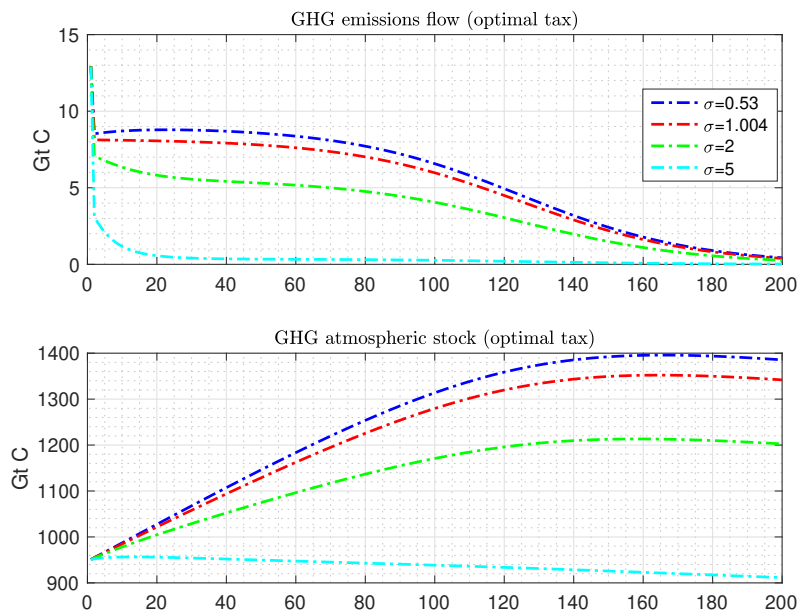


Figure 2.10 – GHG emission flow and GHG atmospheric stock with carbon tax

In all four cases, environmental policy makes it possible to limit the harmful effects of emissions (see figure 2.10). By using public policy to internalize the negative effect of

fossil fuels, annual emissions initially decrease by at least 34% in the case where energies are assumed to be complementary. The higher the elasticity of substitution, the less fossil fuels the economy uses, the more the climate is preserved in the long term. After 50 years, emissions would decrease by 34% when  $\sigma = 0.53$ , and by 59% when  $\sigma = 2$ . According to the simulations, the GHG stock reaches its maximum after 159 to 170 years (Figures 2.9 and 2.10) depending on the policy regime and the elasticity of substitution. In the extreme case where  $\sigma = 5$ , the maximum is reached 13 years after the tax. However the consequences are more significant in the absence of a policy. At the end of the century, the loss of global GDP is about 8.5%, 8.2% and 7.5% for elasticity of substitution or 0.53, 1.004 and 2 respectively for the BAU, while they are limited to 6.7%, 6.4% and 5.5% in case of public intervention. With a very high elasticity of substitution between factors ( $\sigma = 5$ ), annual emissions decrease sufficiently for the GHG stock to decrease monotonously. The absorption capacity of natural sinks is greater than the emission flow. In this case global warming is avoided.

### 2.7.3.2 Economic impacts:

As already explained, in the absence of environmental policy, if productivity increases in the non-polluting sector, the quantity of fossil fuels used will be more or less significant, depending on the elasticity of substitution between energy inputs. If energies are complementary ( $\sigma < 1$ ), increased productivity  $A_{r,t}$  increases demand for polluting energy (Figure 2.13), the annual emissions flow increases (Figure 2.9) and environmental damage increases (Figure 2.12). Environmental policy is in then essential to limit the consequences of global warming. In the absence of intervention, emissions will increase by almost 1% after 50 years and then decrease, thanks to technical progress, by almost 15% by the end of the century. With the tax, emissions will decrease by 34% after 50 years and by 44% in 2100.

In the event of weak substitutability between factors ( $\sigma = 1.004$ ), fossil energy use remains constant over time despite the increase in relative productivity  $A_{r,t}/A_c$ , if the tax is not introduced (Figure 2.13). The carbon tax thus permits to limit the harmful

consequences of the climate crisis. Once again, the introduction of the tax leads to an increase in intermediate costs and results in a deadweight loss. The objective of reducing emissions is achieved by a less intensive use of polluting energies, but the increase in intermediate costs will result in an increase in the price of the final good, a drop in demand and production in comparison with the BAU. However, the introduction of the tax limits environmental damage, which will preserve the economy in the medium and long term. Thus, between 12 to 18 years after the introduction of the tax, the deadweight loss is compensated since the environmental damage in the BAU generates higher losses. Since the mechanism is subsequently the same as the one where relative productivity remains constant, economic activity is supported by relatively lower environmental damage.

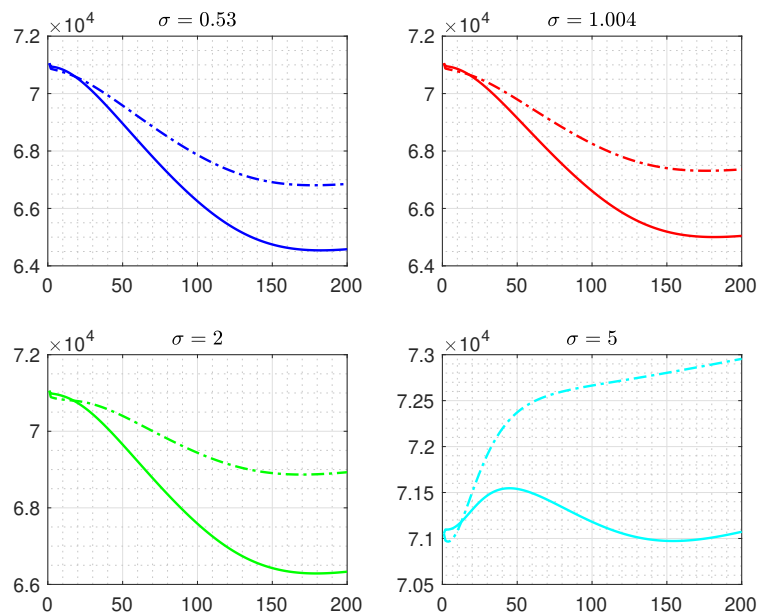


Figure 2.11 – Consumption

After 50 years, when  $\sigma = 2$ , thanks to a close substitutability between the two types of energy, the economy reduces the use of fossil fuels by nearly 21 % even in the absence of an environmental policy. However, the emission flow remains higher than the absorption capacity, so that the damage increases over time (even if it remains lower than in the two previous cases). The tax makes it possible to mitigate these environmental damages, and

to achieve an optimum by reducing the use of fossil fuels by almost 59% after 50 years. At the end of the century, emissions are reduced by 34% without tax and 65% with tax.

In the extreme case where  $\sigma = 5$ , the global economy reduces its use of fossil fuels by 75% after 50 years. In that case, the substitution is such that annual emission flow is lower than the absorption capacity of GHG natural sinks. Damage is decreasing, and since few GHG have accumulated in the first reservoir, the GHG atmospheric stock is decreasing monotonously even in the absence of an environmental policy. However, the latter would accelerate this mechanism.

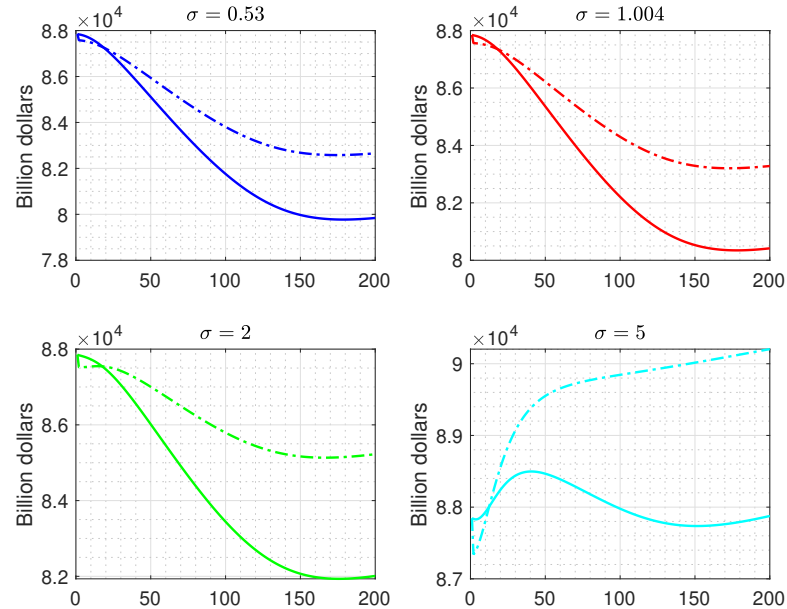


Figure 2.12 – Production

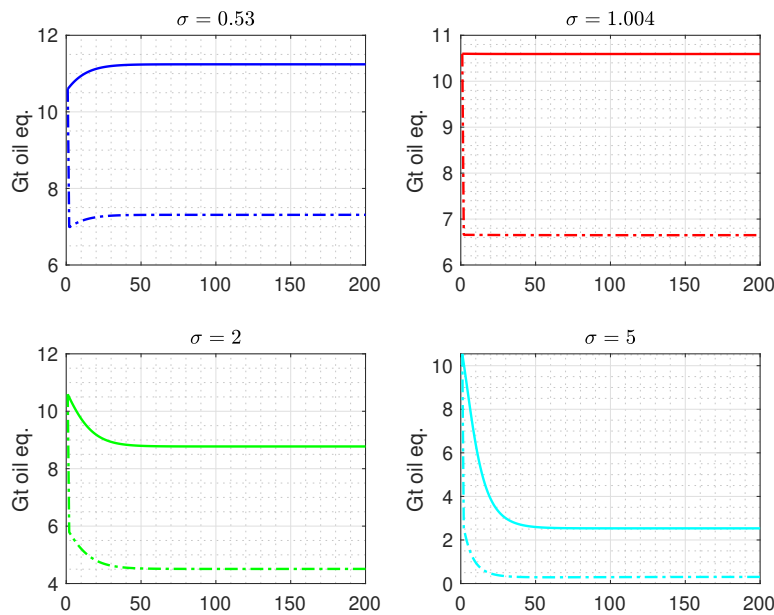


Figure 2.13 – Fossil fuel uses

Despite the weaknesses of the model (no scarcity rent for fossil fuels, no trend growth etc.), the results obtained by these simulations raise important elements that must be taken into account to effectively fight global warming effectively. In the most optimistic, yet unrealistic case where the elasticity of substitution is very high ( $\sigma = 5$ ), environmental policy does not appear to be essential. It would be possible to limit global warming only through technological progress, which would increase productivity in the non-polluting sector, and through increased energy efficiency, which would limit emission intensity. Nevertheless, when the elasticity of substitution is high ( $\sigma \gg 1$ ), it seems necessary to use environmental policy to contain the negative effect of GHG emissions. The lower the elasticity of substitution, the more important it is to internalise through a tax the negative consequences of GHG emissions in the short term. It is therefore relevant to question the notion of substitution between factors and what this implies.

A public policy recommendation would therefore be to invest in R & D in order to strongly increase the elasticity of substitution between energy sources. Most non-polluting energies are subject to climatic hazards (need for wind, and sunshine for exam-



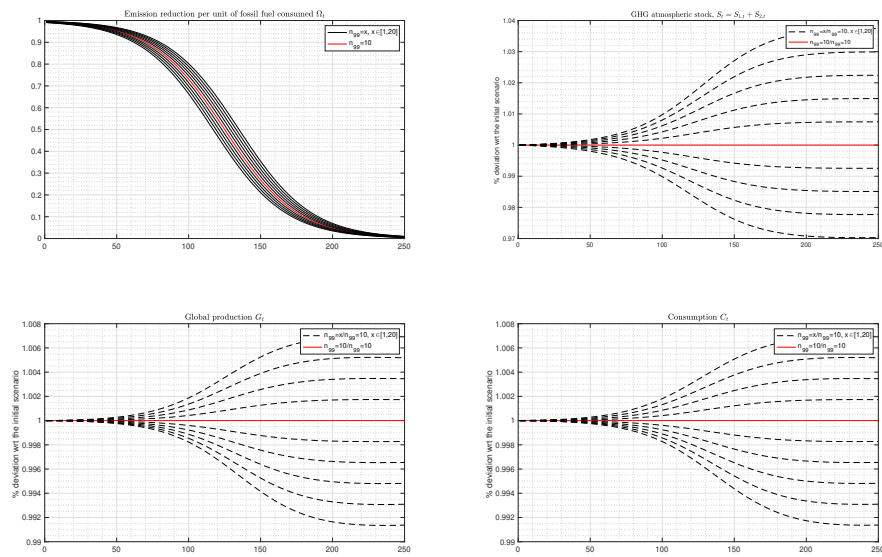
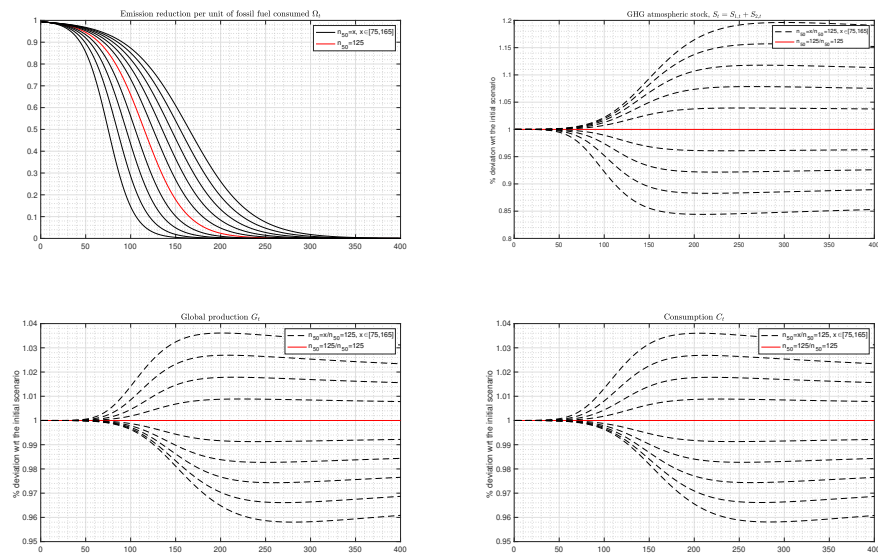
ple). However, the time when energy is produced does not necessarily coincide with the time when it is consumed. The substitution between non-polluting energies and fossil fuels raises the question of energy storage. In the absence of a storage solution, it is difficult to have a strong elasticity of substitution between energies. Increasing storage capacity as a result of clean energy production would increase this elasticity of substitution. Nevertheless, the production of clean energy faces other challenges. The development of non-polluting energies is constrained by physical limitations. For MacKay (2008), it would be necessary to cover Britain entirely with wind turbines to meet its energy demand. Using agricultural land to produce biofuels would only provide 12% of the energy required, without taking into account the consequences on quantities and prices of food commodities. Moreover, several non-polluting energies are in fact not recent, and will be confronted with physical limitations. We have already pointed out that hydropower has existed for centuries, and increasing the production capacity of this energy is technically challenging as it requires the building of new dams, as the most easily controllable rivers are already producing electricity. It is therefore difficult to imagine a reduction in the production cost of this energy. Wind energy is also quite old, and it will not be possible to increase the size of rotors or blades indefinitely.

#### 2.7.4 Robustness:

We now discuss how our results are affected by changes in the parameter of the sigmoid function (equation 2.6.1). Indeed to model an EKC, we have assumed that an exogenous technological progress will reduce emissions over time. The two parameters of interest are  $n_{99}$  and  $n_{50}$ , representing the number of year after the reference year when emissions will be exogenously reduced by 1% and 50% respectively. In the benchmark scenario, we have chosen  $n_{99} = 10$  and  $n_{50} = 125$  as in Barrage (2014) and Krusell and Smith (2015). One may argue that the choice of these parameters are uncertain and changing these values can affect significantly the results of our model previously obtained. We thus simulate the same model with a constant productivity ratio  $A_r/A_c$ , with an elasticity of substitution between the two energy inputs of  $\sigma = 1.004$ , by assuming first different

values for  $n_{99}$ , then focusing on the consequences of the choice of the second parameter by varying the values of  $n_{50}$  (and keeping  $n_{99} = 10$ ).

In the first series of simulations,  $n_{99}$  is chosen in the next 10-tuple:  $n_{99} = [2, 4, 6, 8, 10, 12, 14, 16, 18, 20]$ . In other words we run 10 different simulations, assuming for  $n_{99} = 2$  that emissions will be reduced by 1% after 2 years, for  $n_{99} = 4$  emissions will be reduced by 1% after 4 years etc. We also assume that the year in which emissions are divided by 2 will also change linearly. For example, if emissions are reduced by 1% after 2 years instead of 10, it is assumed that halving emissions will take 117 years instead of 125. Similarly, if it takes 4 years to reduce them by 1%, then it will take 119 years to reduce them by 50%, etc. So the associated 10-tuple for  $n_{50}$  is :  $n_{50} = [117, 119, 121, 123, 125, 127, 129, 131, 133, 135]$ . In the second series of simulations, it is assumed that it will take 10 years ( $n_{99} = 10$ ) to reduce GHG emissions by 1% per unit of polluting energy consumed. Different values are chosen for the  $n_{50}$  parameter, we estimate that in the best case it will take 75 years to halve emissions exogenously, and in the worst scenario 165 years. The values taken by  $n_{50}$  are therefore as follows:  $n_{50} = [75, 85, 95, 105, 115, 125, 135, 145, 155, 165]$ . Figures 2.14 and 2.15 plots the sigmoid function used for each simulation as well as the responses of the variables consumption, production and the total stock of GHG in the atmosphere. The black lines correspond to all the scenarios considered that are different from our benchmark scenario, while the red curve represents the initial sigmoid function used and calibrated in the previous sections. The black dashed lines for the three variables represent the ratio between the simulated variable considered value of the two parameters  $n_{99}$  and  $n_{50}$  for this robustness study over the simulated variable for the standard values of the benchmark scenario. By doing so, the benchmark scenario is normalized to 1, and for each of the 9 other scenarios considered, we observe the percentage deviation from the reference scenario.

Figure 2.14 – Robustness for different values of  $n_{99}$ Figure 2.15 – Robustness for different values of  $n_{50}$

We first describe the results obtained by the first series of simulations (Figure 2.14). By modifying the parameter  $n_{99}$ , the GHG stock will vary by more or less 1% after a century. The environmental damage linked to this variation in pollution emitted is in between -2% and +2% compared to the initial calibration. The repercussions on the economic block (production, consumption, investment or capital stock) are of the order of 2‰. The model becomes stable after two centuries, the GHG stock varies by more or less 4% and the deviations of the economic variables are less than 1% in absolute value. By modifying in a second step the parameter  $n_{50}$ , the response of the model is similar compared to the first series of simulations. Only the time needed to return to equilibrium and the level of deviation of the variables differ. As could be expected, the faster the economy halves the carbon intensity of polluting energy consumed, the lower the GHG stock is, relatively to the benchmark scenario (and vice versa). For the second series of simulations (Figure 2.15), when  $n_{50} = 75$ , after 100 years the GHG atmospheric stock is 7.5 % lower than in the initial scenario, and stabilises at a 15% lower level after two centuries. When on the contrary,  $n_{50} = 165$ , the GHG stock increases by 2.5% after 100 years, reaching a maximum of nearly 20 % more after 250 years before stabilising at this level. When emissions are reduced to their minimum ( $n_{50} = 75$ ) thanks to the exogenous technology world production increases by 4%, in the opposite case ( $n_{50} = 165$ ), world production decreases by 4%. The decrease or increase of other great economic variables (consumption, investment and stock of capital) are within more or less 4% with respect to the initial scenario.

Several important points emerge from this robustness study. First of all, the parameter  $n_{99}$  corresponding to the year when the economy will be able to reduce its GHG emissions by 1%, has relatively little influence on the GHG atmospheric stock, and therefore has little effect on the economy. This result is easily explained by the functional form chosen for the technological process, which exogenously reduces the carbon intensity of the polluting energy input used in the production of the final good. In fact, the sigmoid form can be broken down into three distinct phases. The first one is characterised by a fairly low slope, reflecting a relatively limited decrease in emissions. This phase illustrates

the fact that the probability of finding, in the near future, a technology capable of quickly reducing emissions is low. The second phase, where the slope is steeper, assumes that technical progress to limit emissions will occur more rapidly. Finally in the third phase, technical progress admits an asymptote in the long run, reflecting the fact that, beyond a certain threshold, reducing emissions becomes more and more difficult, in other words, it illustrates the fact that the marginal technological progress is decreasing. Thus, reducing emissions by 1% after 1, 10 or 20 years has relatively small influence on the whole model compared to the second parameter, which corresponds to the year when emissions will decrease by 50%. A substantial effect could therefore be expected when we assume that the economy will be able to reduce its emissions more or less rapidly over time by a factor of two. The results obtained are consistent for the GHG atmospheric stock, which varies significantly downwards if  $n_{50}$  is low, or upwards when  $n_{50}$  is high. Consequently, the variations in damage related to the level of GHG stock are also significant. They reach a maximum of 7.5% after 100 years when  $n_{50} = 75$ , and almost 12% when  $n_{50} = 165$  after more than 200 years. Nevertheless, it may be surprising that the consequences of such variations in terms of GHG stock and by extension environmental damage have a relatively small impact on the economic block. The small increase, or decrease, in all economic variables could be explained by the model's assumptions. We have indeed assumed that the GHG stock generates damage exclusively on the production supply. However, this assumption may be too simplistic. Indeed, we have overlooked the fact that the stock of GHG and thus global warming also has repercussions on the well-being of the representative household. To be more accurate, the variable  $S_t$  should have been included in the household utility function. Because climate change can be a source of illness, or if the representative household has an environmental awareness, then the increase in the GHG stock generates disutility. The household thus makes a trade-off between consumption and environmental quality. Either it continues to consume, which would generate more pollution, or it reduces its consumption in order to preserve the climate. Thus, by taking into account both the environmental damage into the production function and the disutility due to the GHG atmospheric stock, the model results

could better illustrate the negative consequences of climate change on the economy. In chapter 4, we integrate pollution into the household utility function in the manner of Heutel (2012) and Khan et al. (2019).

## 2.8 Conclusion:

Climate dynamics is a slow phenomenon with delayed consequences. Even if emissions flow are completely reduced to zero, the inertia is such that it would take several decades to find a new balance. To be fully effective, environmental policies must take into account the irreversibility of global warming. The irreversibility of climate change raises important questions, particularly about the efforts that current generations are prepared to make in the short term to preserve the climate in the long term, especially for future generations. While the problem of global warming has been highlighted by the scientific community since the 1970s, concluding that a strong policy response is needed, the lack of a drastic decision to reduce the use of fossil fuels shows that some countries are not ready to make the necessary efforts. The USA withdrawal from the Paris Agreement in 2017, the Yellow Vests movement in France in 2018, low share of emissions taxed worldwide, etc.

Policies to limit global warming are unpopular policies. The cost to current generations is substantial, while the positive effects will only be felt in the distant future. This chapter also shows that public intervention remains essential to internalize emissions, especially if productivity in the renewable energy sector increases and if energies are complementaries. Indeed, in the absence of a short-term solution to store non-polluting energy production, the use of fossil fuels remains essential to cope with the discontinuity of renewable energy production (lack of wind, sunshine, etc.). In the absence of a carbon tax, the effect could be counter-productive and the risk would be that emissions would increase rather than decrease (as demonstrated by the German experience following the closure of nuclear power plants after the Fukushima nuclear accident in 2011).

The simple model can be extended in many ways to refine the results obtained. First,

the model considers exclusively a coal regime. However, the results could differ taking into account Hotelling's rent related to the exploitation of exhaustible energy for gas and oil. Sinn (2008, 2012), through his concept of the green paradox, showed how the behaviour of the owners of these exhaustible resources could be changed as a result of environmental policies, leading to an increase in emissions in the short term. However, we have seen how short-term emissions can affect the long-term balance. The results obtained could then be modified by the reaction of polluting energy suppliers, whose Hotelling rent would be modified by the introduction of the tax.

Disaggregating the world economy into a multi-region economy in the Nordhaus way would make it possible to simulate several scenarios of cooperation or not in the fight against global warming. This would have the consequences of scenarios that consider free-riding behaviour at the national level in relation to global warming.

## Chapter 3

# Fossil fuel price shocks and climate policies in the business cycle

### 3.1 Introduction:

Reducing annual anthropogenic GHG emissions flow is one of the major challenges of the 21<sup>st</sup> century to limit global warming. The AR5 report and more recently the "*world scientists warning to humanity*" manifesto signed by more than 15,000 scientists highlight the urgency of the situation, as well as the need to change our production and consumption behaviour. The European Union (EU), which signed the Kyoto Protocol, has been committed to cut its emissions since the mid-1990s. In this sense, it adopted in 2008 the climate energy package whose objective was to achieve the "*3×20*" by 2020 (20% cut in GHG emissions, compared to its 1990 level, a rise of 20% of renewable in the EU energy mix, a 20% improvement in energy efficiency). This package has been revised in 2014 and set new targets for 2030: a 40% reduction in  $CO_2$  emissions, a 27% increase in renewable energies and a 27% increase in energy efficiency. To reduce its GHG emissions, the EU set up in 2005 the world's first carbon market where emissions rights are traded: the *European Union Emissions Trading Scheme* or EU ETS. The principle is first to set an emissions cap, then to distribute allowances to the 12,000 heavy energy-



using installations concerned and let them trade between those for which the demand for pollution exceed the number of allowances they have, and the others for which the demand for pollution is lower than the number of allowances distributed, so that the confrontation of supply and demand generates a carbon price. However, over the past decade, the price of carbon has remained at an abnormally low level, too low to hope to limit warming to  $2^{\circ}\text{C}$  by the end of the century. This low carbon price is explained on the one hand by the economic situation and recessions that have hit the EU over the past decade and on the other hand by structural reasons which have led to the reform of the carbon market in order to limit the volatility of the carbon price. Recent developments in the price of pollution rights show that the measure is starting to take effect. However, other instruments such as the carbon tax are subject to shocks, which can also have negative effects on the objective of containing climate change. Indeed, according to ECOFYS (2018), among the 57 existing environmental policy instruments worldwide, only 4 are at sufficiently high levels to limit our GHG emissions in line with the 2 or  $1.5^{\circ}\text{C}$  objective. The French carbon tax was one of these instruments at sufficiently high levels. However, the recent exogenous rise in the price of oil on the world markets and carbon taxes was reflected in a rise in the price of gasoline, which was one of the causes Yellow Vests movement. Following these demonstrations, the increase in the carbon tax was frozen in France, which could have harmful long-term effects for limiting global warming. Thus, in order to limit these dysfunctions, it is necessary to better understand the effect that can be caused not only by technological shocks, usually studied in the literature of DSGE models focusing on environmental policies, but also by the role of energy prices when the economy depends on the price of this exogenous factor.

The aim of this chapter is therefore to compare different environmental policies subject to various shocks. We use a Real Business Cycle (RBC) model based on the model developed by Fischer and Springborn (2011) to compare the economic consequences of several environmental policies whose objective is to reduce the anthropogenic impact on global climate subject to two exogenous shocks: a technology shock and an oil price shock. We consider four different scenarios: the first one corresponds to our benchmark

scenario without environmental policy, the second one reproduces the EU ETS carbon market and more generally a cap-and-trade market, the third one is the carbon tax case, and the last scenario considers the implementation of an intensity target. Our results are as follows. Like Annicchiarico and Di Dio (2015), for a technology shock, we find no significant difference over the business cycle between the BAU, the tax and the intensity target. However, setting an emissions cap reduces the positive effect of the shock by increasing the cost of carbon. In the case of a positive exogenous shock on the price of energy, the intensity target and the emissions cap will absorb the shock and limit the consequences on the business cycle, whereas the tax will have a significant pro-cyclical effect. Thus, if such an instrument is implemented, it should be accompanied by a floating tax to limit the rise in energy prices, which would contain both GHG emissions without penalising consumption or long-term climate policy.

The goal of section 2 is to present the research context, while section 3 shows both the state of the art and our contribution. The model is developed in section 4, we calibrate it in section 5, and we present our results in section 6 before concluding.

## 3.2 Research context:

### 3.2.1 The European carbon market subject to failures in the wake of the 2008 financial crisis:

Many studies have tried to calculate the optimal carbon price (see Quinet (2009) and Maibach (2008) for a literature review). According to the OECD, in 2008 the price per ton of  $CO_2$  was expected to be \$ 25 and would have increased by 2.4% per year (OECD, 2008) and a more recent OECD study evaluated that \$ 325 per ton of  $CO_2$  is needed in 2050 to limit global warming to 2°C (Marchal et al., 2011). The Stiglitz Stern commission estimated that the price should be between \$ 40 and \$ 80 per ton of  $CO_2$  in 2020 to be consistent with the 2°C target (Stern and Stiglitz, 2017). However between January 2013 (date of implementation of the 3<sup>rd</sup> phase of the EU ETS market) and June 2018, the average monthly carbon price was \$ 11 per ton, which is far from being enough to

limit global warming.



Figure 3.1 – EU ETS allowance price (€/ton)

For de Perthuis and Trotignon (2014), this very low carbon price can be explained by the different economic downturn (financial and debt crisis) and economic difficulties the EU has encountered during the last decade. The decrease in industrial activity has had negatively affected polluting rights demand and consequently the carbon price. The price volatility does not appear to be an issue if the variation in allowances price reflects the economic activity<sup>1</sup>. However sluggish economic growth is not the only factor that can explain the fact that the carbon price has remained at a very low price for six years. Indeed, structural reasons played a significant role (Laurent and Le Cacheux, 2011) and (de Perthuis and Trotignon, 2014): interactions between the EU ETS and other regulations such as national environmental policies diminish allowances demand

<sup>1</sup>However, this volatility can affect negatively investment in low carbon technology, for this reason several economists suggest to implement an allowance price floor (Goulder, 2013)

and reduce the carbon market efficiency. Similarly, according to Laurent and Le Cacheux (2009) the clean development mechanism (CDM) which has been first introduced in order to allow European firms to take into account projects they carried out in developing countries, has in fact introduced an additional weakness on the EU ETS market. It has reduced firstly the demand for allowances, and secondly this mechanism seems to be inefficient because between one and two thirds of the projects would not actually result in real emission reductions (see Wara and Victor (2008) for concrete examples). Lastly because it was possible to bank and transfer allowances that were not used from the 2<sup>nd</sup> to the 3<sup>rd</sup> phase, this rule played a role in the low carbon price observed during the current phase<sup>2</sup>. For these structural reasons, the carbon price was relatively too low to reduce GHG emissions in the long run.

Thus, to eliminate its dysfunctions and to strengthen its climate policy, the EU has undertaken a reform for the EU ETS market at the end of 3<sup>rd</sup> phase. On July 2015, the European Parliament adopted the proposal of the European Commission to implement the Market Stability Reserve (MSR) which has been started in January 2019. Its first objective is to diminish the number of allowances currently available. The second goal is to control carbon price volatility by creating a mechanism that can protect the system from an economic shock likely to unbalance the EU ETS, between supply and demand allowances. By doing so, legislators send a strong signal to investors and firms about the EU's commitment to meet its GHG emission reduction target. If the number of allowances available exceeds 833 Mega ton (Mt), then the authorities will remove 12% of them per year. On the contrary, they will reintroduce 100 Mt of allowances if their number is lower than 400 Mt. Between the two thresholds, there is no intervention. Moreover, the European Parliament decided for the 4<sup>th</sup> phase to accelerate the annual rate of decline of allowances number from 1.74% to 2.2% per year to reach the energy package targets. These policies should cause the carbon price to slowly increase during

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<sup>2</sup>This market failure should disappear with the suppression of the free allocation system. Indeed, in the auctioning system, firms must now buy every year the required quantity of allowances and no longer receive too many allowances and they are no longer supposed to bank them (de Perthuis and Trotignon, 2014).

the next decade.

### 3.2.2 Carbon tax, Yellow Vests and fossil fuel price changes:

Introduced in 2013 under François Hollande's presidency, the aim of the French carbon tax is to change consumption behavior of fossil fuel energies and thus achieve the objectives of reducing GHG emissions that France has set (reducing by 40% its GHG emission in 2030 compared to its 1990 level and by 75% by 2050). Before the Yellow Vests movements the target was to reach €100 per ton of  $CO_2$  by 2030 (for comparison the Swedish tax is €150 per ton in 2018). Little or no discussed at the time of its adoption in 2013, the French carbon tax went completely unnoticed at the time of its implementation in 2014 due to its low level (see Table 3.1) and relatively low barrel price.

Table 3.1 – Level of the French carbon tax, before the Yellow Vests movements

Year	2014	2015	2016	2017	2018	2019	2020	2030
Carbon tax (€/tCO <sub>2</sub> )	7	14.5	22	30.5	39	47.5	56	100

Source: French Ministry of Ecology

The decade 2010 is characterized by a transformation of the supply side of the global oil market. The exploitation of shale gas in the United States has increased oil supplied on world market. Saudi Arabia responded by pursuing a supply strategy to drive these new producers out of the market. Saudi Arabia's conventional oil resources are relatively easier to extract and consequently have a much lower break-even point than American shale gas. By supporting production, the global price barrel is kept at a low level, under the break-even point of this new player in order to maintain its oligopoly power thanks to this aggressive production policy. Between 2010 and 2016, oil production increased by 10% in OPEC countries and by 26% for the same period in Saudi Arabia (see Table 3.3). This strategy is proving ineffective, since in 2015, the US Congress decided to allow

oil exports that had been banned after the oil shocks of the 1970s (production was then exclusively reserved for the US market) and in 2017 the United States has become the leading oil producing country. As a result of the failure of Saudi Arabia's strategy and to a low barrel price, OPEC countries are emerging weakened, since their economies are mainly based on the rent provided by the exploitation of fossil fuels. Consequently they decided to reduce their production from 2017 onward (see Table 3.3). Trade tensions between the US and Iran, as well as the rise in global oil demand (1.4 Million barrels per day - Mbd) - two thirds of this increase comes from China (0.7 Mbd) and India (0.3 Mbd) while the remaining share comes from the rise of the US demand) drove oil prices up in 2018. In the same year, the French carbon tax increased by 27% compared to 2017 (see Table 3.1). Diesel which until recently benefited from lower taxation than regular gas, saw its taxes increase by 11% between 2017 and 2018, and by almost 40% between 2013 and 2018. While for regular gasoline, the tax increased by 5% between 2017 and 2018 and by 11% between 2013 and 2018 (see Table 3.2). As shown in Figure 3.2, the difference in gas price (including taxes) between diesel and gasoline decreased significantly in 2018. As a result of this exogenous increase on the global markets in the oil price per barrel, combined with the increase in the French carbon tax on fossil fuel products, the Yellow Vests movement began in France in November 2018. On December 5 2018, President Emmanuel Macron decided to freeze the initially planned increases in this tax. The interest of this research is therefore to better understand the consequences that an exogenous shock on fossil fuels price can have depending on the environmental policy in place.

Table 3.2 – Carbon tax per product:

Year	2013	2014	2015	2016	2017	2018	2019
Diesel (€/l)	0.43	0.43	0.47	0.5	0.53	0.59	0.59
E5 gas (€/l)	0.61	0.61	0.62	0.64	0.65	0.68	0.68
E10 gas (€/l)	0.61	0.61	0.62	0.62	0.63	0.66	0.66
Coal (€/Mwh)	1.19	2.29	4.75	7.21	9.99	14.62	14.62

Source: French Ministry of Ecology

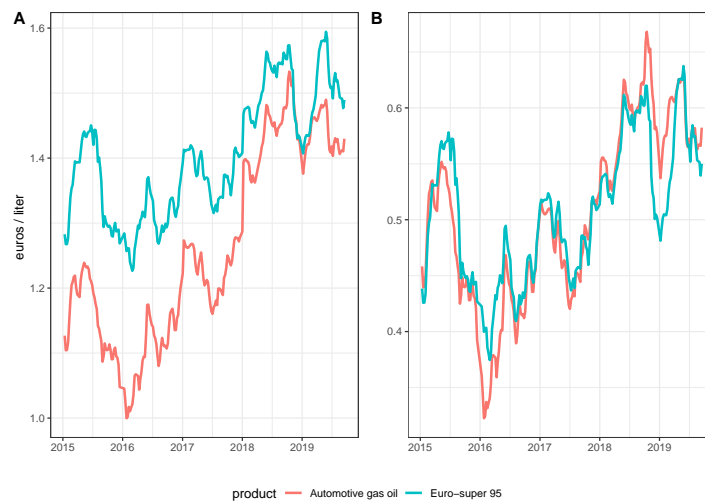


Figure 3.2 – French retail fuel price (€/l), panel A: with taxes, panel B : without tax.

Source: European Commission

Table 3.3 – OPEC’s oil production (thousand barrels per day)

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Algeria	1,250	1,240	1,210	1,159	1,118	1,113	1,105	1,047	1,042
Angola	1,786	1,667	1,738	1,600	1,639	1,769	1,718	1,634	1,505
Ecuador	475	490	499	526	553	543	530	515	519
IR Iran	3,706	3,628	2,977	2,673	2,773	2,836	3,515	3,818	3,553
Iraq	2,401	2,665	2,979	3,037	3,303	3,974	4,392	4,446	4,550
Kuwait	2,297	2,538	2,793	2,822	2,768	2,764	2,853	2,708	2,746
Libya	1,559	462	1,393	928	470	404	389	811	952
Nigeria	2,061	2,111	2,073	1,912	1,921	1,838	1,556	1,658	1,719
Qatar	791	794	753	732	n.a.	n.a	n.a	n.a	n.a.
Saudi Arabia	8,254	9,296	9,737	9,586	9,686	10,142	10,406	9,954	10,311
UAE	2,304	2,516	2,624	2,741	2,812	2,908	2,979	2,915	2,986
Venezuela	2,370	2,413	2,392	2,389	2,334	2,319	2,154	1,911	1,354
Total	29,255	29,821	31,168	30,231	30,028	31,240	32,209	32,013	31,865
OPEC									

Source: OPEC



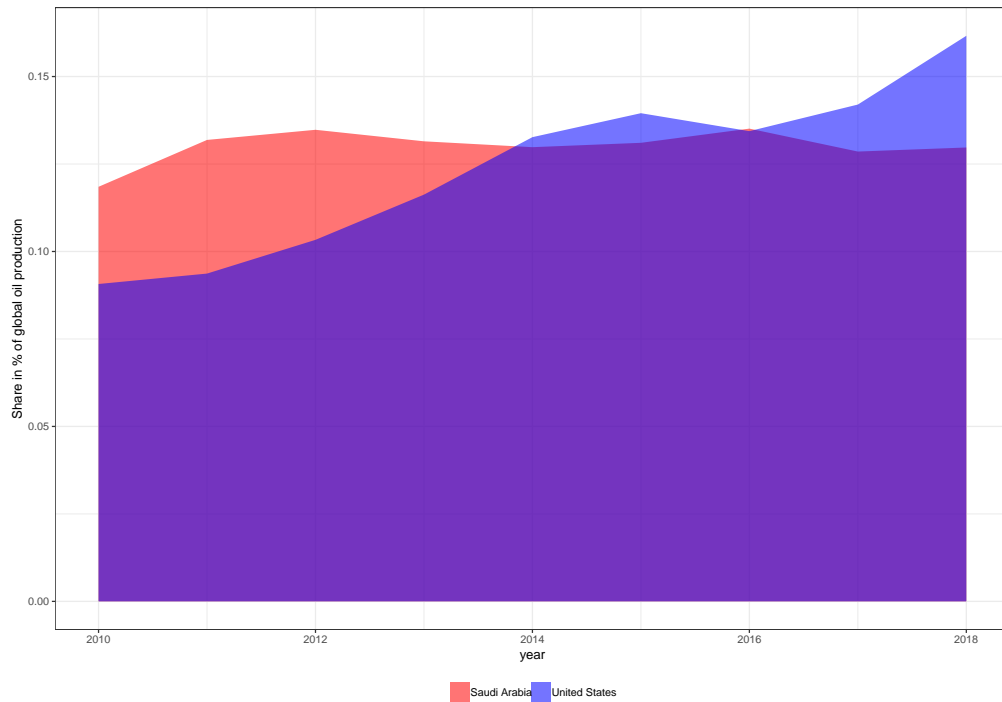


Figure 3.3 – Share of global oil production per country - Source: BP

### 3.3 State of the art and contribution to the literature:

Analysis of the role of environmental policies in the business cycle is recent. Angelopoulos et al. (2010), Fischer and Springborn (2011), and Heutel (2012) are the first to integrate these policies into a RBC model.

Angelopoulos et al. (2010) compare different environmental policies including the introduction of a carbon tax, an emission standard, or a "*Kyoto-like numerical rules*" in order to find the most effective instrument. They assume that pollution is the result of production: "*by product of output produced*", in other words they assume that there exists a relation between output and pollution, and they allow this pollution intensity of GDP to vary over time according to exogenous shocks. In their article they consider two sources of uncertainty, the first one is the classic exogenous productivity shock while the second is the environmental shock, or pollution technology shock. They compare for each of these

two shocks which instrument is best suited to reduce emissions and consequently which instrument has the greatest effect on welfare (in their model environmental quality enters the household utility function and the stock of pollution affects negatively its utility). They find that when volatility comes from productivity shock, introducing a tax is the best instrument, whereas when volatility comes from environmental technology shock, the introduction of a rule is more appropriate for pollution abatement. Finally, they find that in all cases the establishment of pollution permits is the least effective policy.

Fischer and Springborn (2011) also compare three different policies (a tax, an emission cap, and an output to pollution intensity target) but they differ from Angelopoulos et al. (2010) for several reasons. First, in their model it is the energy used as input in production which is source of pollution emissions. Secondly and contrary to Angelopoulos et al. (2010) who model the pollution stock as being the cause of pollution damage, Fischer and Springborn (2011) do not clearly explain the underlying relation between emissions and environmental damage, and lastly the only source of uncertainty comes from technological progress. They differ mainly from Angelopoulos et al. (2010) in the way they study the cost of each policy. Angelopoulos et al. (2010) evaluate the policy effects by comparing the gain in terms of welfare, on the contrary Fischer and Springborn (2011) compare the loss from the economic point of view. By doing so, they find that the intensity target achieves its goal in terms of reduction in pollution at the lowest economic cost.

Heutel (2012) is interested in the capacity of a tax or a cap to adapt to the cycle in order to maximize the welfare. The idea behind this proposal is that an environmental policy should adapt to the business cycle and consequently be stronger during economic expansion (when emissions tend to rise), and less restrictive during recession for the opposite reason (a lower growth means less emissions). For the two policies studied Heutel (2012) finds that it would be optimal to adapt them to the economic cycle: by increasing the tax rate and reducing the number of quotas during expansions periods and the opposite in the case of recession.

Annicchiarico and Di Dio (2015) are the first to develop a new keynesian model in this context by including in their model nominal price rigidities and monetary policy. They compare several environmental policies, and they care about the role play by these rigidities when we implement such a policy on the business cycle. They first find that a cap-and-trade system will diminish economic fluctuations, and will behave as an automatic stabilizer. As expected nominal price rigidities play an important role in terms of welfare variation as a response to environmental policy. The higher the price rigidities are, the higher the welfare loss is.

We contribute to the existing literature on environmental policy assessment in the context of a RBC model by analysing the consequences of energy price shock in the business cycle when the economy implements a climate policy. We use Fischer and Springborn (2011) model that includes energy as an input to conduct our research. We consider as in Kim and Loungani (1992) that one source of uncertainty comes from energy prices that are exogenously determined. Moreover and to our knowledge, it is the first article that takes into account the variation in the utilisation rate of capital in order to evaluate environmental policy. This assumption plays an important role in order to not over or under-estimate the impact of policy.

## 3.4 The model:

### 3.4.1 Households:

It is assumed that households are similar, they all adopt the same behaviour, so we can aggregate them such that there is one infinitely lived representative agent. We suppose the following CRRA utility function

$$U_t(c_t, l_t) = \frac{(c_t^\varphi (1 - l_t)^{1-\varphi})^{1-\sigma} - 1}{1 - \sigma} \quad (3.4.1)$$

$$\sigma > 1, \quad \varphi \in [0, 1]$$

where  $c_t$  is the final good consumption level in period  $t$ ,  $l_t$  is labor supply in period

$t$ , and where  $\varphi$  and  $\sigma$  are preferences parameter. We normalize to unity, household's time disposable in each period, such as  $1 - l_t$  corresponds to leisure time. Household maximises its intertemporal utility function subject to its budget constraint:

$$c_t + i_t = w_t l_t + r_t k_t u_t + T_t \quad (3.4.2)$$

The left hand side (LHS) of (3.4.2) corresponds to the household's demand, while right hand side (RHS) is household earnings from the labor force it supplies to the firm and as the owner of capital it also rents capital services to the firm.  $u_t$  is the rate of utilisation of capital (we suppose that the entire stock of capital is variable), therefore  $u_t k_t$  corresponds to the services derived from the utilisation of capital. The government redistributes tax revenues in the form of a lump-sum subsidy  $T_t$ . During each period, household divides a share of its income into capital investment, while the remaining goes to consumption of final goods. The law of motion for investment in capital is:

$$i_t = k_{t+1} - \left(1 - \delta(u_t)\right)k_t + \frac{B}{2} \left(\frac{k_{t+1} - k_t}{k_t}\right)^2 \quad (3.4.3)$$

$$\delta(u_t) = \frac{\omega_0}{\omega_1} u_t^{\omega_1}, \quad (3.4.4)$$

$$\delta(\cdot) \in ]0, 1[, \quad \omega_0 > 0, \quad \omega_1 > 1, \quad B > 0$$

where the last term in the RHS of (3.4.3) is the adjustment costs in capital investment,  $B$  is a positive parameter of the quadratic cost functions (to be calibrated). This assumption implies that increasing the capital stock is costly for the firm. Augmenting its investment could require for example formation of workers, such they are no longer working, so it could have a direct cost and an indirect cost by lowering labor productivity in the short run.

We assume that the depreciation rate of capital (3.4.4) and as the following properties:  $\frac{\partial \delta(u_t)}{\partial u_t} > 0$  and  $\frac{\partial^2 \delta(u_t)}{\partial u_t^2} > 0$ . Therefore the depreciation rate of capital is a convex and increasing function of  $u_t$ , as in Taubman and Wilkinson (1970), Greenwood et al. (1988) and Finn (2000). By doing so this formulation takes into account Keynes' definition

of usage cost of capital: in other words an increase in utilisation rate of capital will accelerate capital depreciation.

The household intertemporal program is to choose the sequence  $\{c_t, l_t, u_t, k_{t+1}\}$  that maximises its utility function 3.4.1 subject to its resource constraint 3.4.2 and the law of motion for capital accumulation (3.4.3).

$$\begin{aligned} & \max_{\{c_t, l_t, u_t, k_{t+1}\}} U_t(c_t, l_t) \\ & s.to \begin{cases} c_t + i_t = w_t l_t + r_t k_t u_t + T_t \\ i_t = k_{t+1} - (1 - \delta(u_t)) k_t + \frac{B}{2} \left( \frac{k_{t+1} - k_t}{k_t} \right)^2 \\ \delta(u_t) = \frac{\omega_0}{\omega_1} u_t^{\omega_1} \end{cases} \end{aligned}$$

The Lagrangian associated with this maximisation program is:

$$\begin{aligned} \mathcal{L} = \mathbb{E} \sum_{j=0}^{\infty} \beta^{t+j} & \left\{ \frac{\left( c_{t+j}^{\varphi} (1 - l_{t+j})^{\varphi} \right)^{1-\alpha} - 1}{1-\alpha} + \lambda_{1,t+j} \left( w_{t+j} l_{t+j} + r_{t+j} k_{t+j} u_{t+j} + T_{t+j} \right. \right. \\ & \left. \left. - c_{t+j} - k_{t+j+1} + (1 - \delta(u_{t+j})) k_{t+j} - \frac{B}{2} \left( \frac{k_{t+j+1} - k_{t+j}}{k_{t+j}} \right)^2 \right) \right\} \end{aligned}$$

where  $\lambda_{1,t}$  is the co-state variable associated with the budget constraint. FOC are detailed in the technical appendix.

### 3.4.2 Firms:

The final good is produced using three different inputs : labor, capital and energy. The production function of the final good is supposed to be a Cobb-Douglas form, where  $a_t$  is technological progress:

$$y_t = a_t (k_t u_t)^{\alpha} e_t^{\gamma} l_t^{1-\alpha-\gamma} \quad (3.4.5)$$

$e_t$  is the energy input used in production, and we assume that GHG emissions are proportional to its use, as in Fischer and Springborn (2011). It is assumed that each unit consumed of  $e_t$  generates  $\Xi$  units of GHG.

When the government decides to implement a climate policy, the firm faces another cost that modifies its optimizing program. Assuming that  $\bar{e}_t$  is exogenously decided by the public authorities, and represents one of the four policy at its disposal. In every period, the firm's objective is therefore to choose the sequence  $\{l_t, k_t u_t, e_t\}$  that maximizes its profits:

$$\max_{\{l_t, k_t u_t, e_t\}} a_t (k_t u_t)^\alpha e_t^\gamma l_t^{1-\alpha-\gamma} - w_t l_t - r_t k_t u_t - p_{e,t} e_t - p_t^C \bar{e}_t$$

Where  $p_t^C$  is the price of carbon that the company has to pay for each GHG unity emitted into the atmosphere. FOC are detailed in the technical appendix.

### 3.4.3 Climate policy and overall resource constraint:

We assume four different scenarios. In the first case nothing is done, the public authorities do not intervene, this is the benchmark, or Business As Usual scenario. In the other cases, they set up either a cap-and-trade market (or an emission cap), a carbon tax or an intensity target. When they decide to internalize GHG emissions externalities with a cap-and-trade market, in every period, they decide the quantity of emissions not to be exceeded, and to supply the corresponding quantity of allowances to the firms. Defining by  $q_t^e$ , the number of allowances supplied on the market in order to achieve the emission target, the emissions policy constraint is consequently:

$$\bar{e}_t \equiv q_t^e \geq \Xi e_t \quad (3.4.6)$$

According to the Kuhn and Tucker optimality conditions, if the emissions cap is not binding, i.e. if the firm emitted less than the upper bound  $q_t^e$ , then the carbon price will be zero. On the other hand, if the constraint binds, then the carbon price  $p_t^C$  will be positive. Next, we suppose that public authorities decide to implement a carbon tax. In that case, they decide exogenously the carbon price  $p_t^C = p_{e,t} \tau_t^C$  for each unit of GHG emitted. Finally, in the case of an intensity target, GHG emissions must not exceed a threshold  $A_t$  for each unit of GDP as in Fischer and Springborn (2011). The constraint is in this case:

$$\bar{e}_t \equiv A_t = \frac{\Xi e_t}{y_t} \quad (3.4.7)$$

We follow Annicchiarico and Di Dio (2015) and assume that, the shadow value  $p_t^C$  of emissions associated with the intensity constraint (3.4.7) represents carbon price. Indeed, in this case the government sells also permits to the firm, and  $p_t^C$  corresponds to the market price of emissions, that satisfies the exogenous constraint defined by the authorities. To close the model we need to find the overall resource constraint. We assume that the government implement the climate policy to contain GHG emissions flow, as described. It redistributes the tax revenues by a lump-sum subsidy to the household, so that government resource constraint is:

$$T_t = p_t^C e_t$$

Combining the government budget constraint with the household budget constraint and using optimality conditions of the firms, one obtains the overall resource constraint in the economy:

$$y_t = c_t + i_t + p_{e,t} e_t \tag{3.4.8}$$

### 3.4.4 Competitive equilibrium

Competitive equilibrium is obtained when the representative household and the representative firm optimize their program. In the case of a cap-and-trade market, it is determined by equations (3.4.3), (3.4.4), (3.4.5), (3.4.6), (3.4.8), and by the following set of equations<sup>3</sup>:

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<sup>3</sup>see appendix for more details.

$$\varphi w_t(1 - l_t) = (1 - \varphi)c_t \quad (3.4.9a)$$

$$r_t k_t = \delta'(u_t)k_t \quad (3.4.9b)$$

$$\begin{aligned} & \frac{(c_t^\varphi(1 - l_t)^{1-\varphi})^{1-\sigma}}{c_t} \left( 1 + B \left( \frac{k_{t+1} - k_t}{k_t} \right) \frac{1}{k_t} \right) \\ &= \beta \mathbb{E}_t \frac{(c_{t+1}^\varphi(1 - l_{t+1})^{1-\varphi})^{1-\sigma}}{c_{t+1}} \left( r_{t+1}u_{t+1} + 1 - \delta(u_{t+1}) + B \left( \frac{k_{t+2} - k_{t+1}}{k_{t+1}} \right) \frac{k_{t+2}}{(k_{t+1})^2} \right) \end{aligned} \quad (3.4.9c)$$

$$w_t = (1 - \alpha - \gamma) \frac{y_t}{l_t} \quad (3.4.9d)$$

$$r_t = \alpha \frac{y_t}{k_t u_t} \quad (3.4.9e)$$

$$p_{e,t} + \Xi p_t^C = \gamma \frac{y_t}{e_t} \quad (3.4.9f)$$

Combining FOC for consumption and labor supply gives (3.4.9a), it equates the marginal rate of substitution between leisure and consumption  $\frac{\partial U}{\partial l_t} \times \frac{1}{\frac{\partial U}{\partial c_t}}$  to the wage. Equation (3.4.9b) gives the optimal utilisation rate of capital. It equates the marginal benefit of increasing by 1% the utilisation rate of capital (LHS of equation (3.4.9b)) to its costs, that corresponds to an increase in capital depreciation due to a more intensive use of capital. Using FOC for consumption to eliminate the Lagrange multiplier  $\lambda_1$  into FOC for capital gives equation (3.4.9c). It is the well known Euler's equation and equates the marginal rate of substitution between consumption level in period  $t$  and consumption level in period  $t + 1$  to the relative price of today consumption. It states that the representative household is indifferent between consuming one unit today or postponing its consumption to the next period if this equation is satisfied. The terms in brackets corresponds to the gross return on capital ( $r_{t+1}u_{t+1}$ ) which is corrected from depreciation rate of capital. The last terms in the RHS of the Euler's equation is the reduction in capital adjustment cost in period  $t + 2$ , if capital as increased in period  $t + 1$ . Equations (3.4.9d) and (3.4.9e) equalise the marginal productivities of labor and capital to their respective costs. Equation (3.4.9f) equalises marginal productivity of oil to its cost as the sum of oil price plus carbon price. In the absence of climate policy, carbon price



$p_t^C = 0$ , and the use of fossil fuel is not high enough to limit GHG emissions in the aim of limiting global warming.

In the case of a carbon tax, equation (3.4.6) is not any more necessary in the optimizing program, and equation (3.4.9f) is replaced by  $p_{e,t}(1 + \tau_t^C \Xi) = \gamma \frac{y_t}{e_t}$ . Finally for the intensity target scenario, the constraint (3.4.6) is replaced by (3.4.7) and the optimality condition for the use of energy is modified:  $p_{e,t} + \Xi p_t^C \bar{e}_t = \gamma \frac{y_t}{e_t}$ .

### 3.5 Calibration:

We now calibrate the different parameters of the model using either 1<sup>st</sup> moment of observed variables (assuming that in the absence of stochastic shocks, the model is in its steady state), or past studies. The value for the rate of time preference  $\beta$ , for the utility function parameters  $\sigma$ , are the same as those used by Finn (2000). Labor supply ( $l$ ) is calculated as the average of weekly hours worked in the EU (Eurostat) divided by the total number of hours in a week (168 hours). Share of labor compensation into GDP ( $1 - \alpha - \gamma$ ) is measured as the annual average in the European Union for the period 1995-2017 from AMECO's database. It corresponds to the compensation of employees (wage and salaries plus employer's social contribution) divided by gross domestic product. Energy income share in the EU is calculated as follows: total primary energy in million tonne of oil equivalent (Mtoe) used in the EU is obtained thanks to Eurostat database. In a second step it is transformed into barrels by multiplying this quantity by the oil metric tonnes per barrel approximate conversion factor (BP, 2017). It is therefore multiplied by the WTI crude oil Brent price and expressed in Euros using the Euro-Dollar exchange rate<sup>4</sup> and we divide this value by the EU GDP (Eurostat) to finally obtain  $\gamma = 0.04$ . This value is consistent with the one used in Golosov et al. (2014) and Fischer and Springborn (2011). The utilization rate of capital comes from the Eurostat database, and it corresponds to the quarterly average of the current level of capacity utilization in the manufacturing industry. The Penn World Table is used to calculate

<sup>4</sup>The Euro-Dollar exchange rate can be found on: <https://fred.stlouisfed.org/series/CCUSMA02EZQ618N>

the capital depreciation rate in the EU 28 for the years 2010-2017. The total stock of capital that depreciates in the EU is calculated as follows. We sum the share of the capital stock that depreciates annually in each of the 28 EU countries, weighted by the annual depreciation rates in each of the 28 EU countries:  $\delta K = \sum_i \delta_i K_i$ . We divide this share of capital that has depreciated by the total stock of capital, it gives us the annual depreciation rate of capital. Finally we divide this result by 4 to obtain the quarterly depreciation rate. Assuming a capital interest rate of 2.7%, a capital depreciation rate of 1.25% and a capital utilization rate of 81% in steady state, equations (3.4.4) and (3.4.9b) can be used to determine  $\omega_0 = 0.033$  and  $\omega_1 = 1.80$ . We normalize to unity the conversion factor  $\Xi$  between energy use and GHG emissions. Finally, we have to calibrate the exogenous process of each climate policy. According to the Global Carbon Project, between 2005, our reference year when the EU ETS market was introduced, and 2017, GHG emissions from fossil fuel consumption have been reduced by 20% in the EU 28. Hence, Assuming a 20% reduction in GHG emissions, with an initial level of energy used at a stationary equilibrium of 0.026 in the absence of an environmental policy, the number of allowances offered on the market is therefore  $q^e = 0.021$ . In order to be comparable, the other environmental policy instruments are calibrated so that we can observe this 20% reduction in emissions. Thus, if we set the initial intensity target at  $A = 0.0313$ , this allows us to observe this 20% reduction in GHG emissions. The rate of the carbon tax that allows a 20% reduction in GHG emissions is  $\tau^C = 0.278$ . In the absence of an environmental policy, by calibrating the parameters of the model accordingly to the previously values (see table 3.4), in the steady state, the investment rate is 18%, the share of income consumed is 78% and the share allocated to energy is 4%. Finally, the capital stock represents roughly 15 times the quarterly GDP, which corresponds to 3.75 years of production. Assigned values, data sources, and calculation methods are summarized in table 3.4, and great steady state ratios in Table 3.5.

Table 3.4 – Variables and parameters calibration

Parameter or variable	Source	Period	method	value
$\beta$	Finn (2000)			0.99
$\sigma$	Finn (2000)			2
$1 - \alpha - \gamma$	AMECO	1995- 2017	average	0.63
$\gamma$	Eurostat, FRED, BP, EIA	1995:Q1- 2016:Q4	average	0.04
$\alpha$				0.33
$u$	Eurostat	1995:Q1- 2018:Q2	average	0.81
$\delta$	Penn World Table	2010- 2017	average	0.0125
$l$	Eurostat	2002- 2017	average of weekly worked	share hours 0.22
$\Xi$				1
$B$				1
$\varphi$				0.27

Table 3.5 – Steady state ratios

ratios	$\frac{c}{y}$	$\frac{i}{y}$	$\frac{k}{y}$	$\frac{e}{y}$
BAU	0.78	0.18	14.6	0.04
emissions cap	0.78	0.18	14.6	0.031
carbon tax				
intensity target	0.778	0.183	14.6	0.0313

We assume three different shocks in our model. Two of them are supposed to be stochastic while the last one is deterministic. The first stochastic shock considered is the common total factor productivity shock. Furthermore, we also assume that the economy is subject to a stochastic energy price shock. The EU is a net importer of oil, its production accounts for 1.6% of the global production, while it consumes 13.4% of the global production. We then assume that nowadays the EU is unable to influence oil price, but as a net importers it illustrates the vulnerability of this economy to an oil price shock. To close the model we assume that energy is exogenously supply, so that energy producers are supposed to control both the amount of energy supply and its price on the market. Consequently, the final goods firm takes the energy price as given, and it only controls the quantity of oil demanded. Stochastic shocks follow an exogenous  $AR(1)$  process:

$$p_{e,t} = \rho_{pe} p_{e,t-1} + \epsilon_{pe,t} \quad (3.5.1)$$

$$\epsilon_{pe,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_{pe}^2)$$

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \quad (3.5.2)$$

$$\epsilon_{z,t} \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_z^2)$$

We estimate the parameters of equation (3.5.1) using global price of Crude oil<sup>5</sup> from 1995:Q1 to 2017:Q2. To detrend the series we use a one-sided HP filter. The resulting estimation is  $\rho_{pe} = 0.725$  and  $\sigma_{pe} = 0.0536$ . The parameters of the Solow residual are estimated using the AMECO database for the capital stock and for labor used in production. The capital utilisation rate is taken from Eurostat, while the energy demand comes from the IEA database. Annual data are transformed into quarterly data using the Denton Cholette method. Finally, estimating the  $AR(1)$  process of equation (3.5.2) yields to the autocorrelation parameter  $\rho_z = 0.896$  and the standard deviation of the white noise is  $\sigma_z = 0.0353$ .

## 3.6 Simulations:

### 3.6.1 General Methodology:

We use Dynare software (Adjemian et al., 2011) to compute our stochastic model, in this subsection we briefly describe the underlying methodology used to solve and simulate DSGE models. We assume that the non-linear rational expectation model developed in this chapter can be represented in the following reduced form:

$$\mathbb{E}_t [f(y_{t+1}, y_t, y_{t-1}, \epsilon_t)] = 0$$

where  $y$  is an  $n_y \times 1$  vector of endogenous variables and  $\epsilon$  is an  $n_\epsilon \times 1$  vector of exogenous stochastic shocks, with the following properties  $\mathbb{E}_t(\epsilon_t) = 0$ ,  $\mathbb{E}_t(\epsilon_t \epsilon_t') = \Sigma_\epsilon$  and  $\mathbb{E}_t(\epsilon_t \epsilon_{t+j}) = 0, \forall j \neq 0$ . In period  $t$ , the only stochastic variables is  $y_{t+1}$ . To solve this problem, we use the perturbation approach (see Jin and Judd (2002)). It consists in linearizing the model by taking a first order Taylor expansion around the steady state, then in a second stage solving the linearized model using the generalized Schur decomposition, and assuming that the steady state is stable, the covariance matrix is solved using an algorithm for discrete time Lyapunov equations that allows us to find the impulse response function.

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<sup>5</sup>Nominal prices are available on fred website, and they are transformed into real prices: <https://fred.stlouisfed.org/series/POILWTIUSDQ>

We simulate the model over 400 periods, corresponding to 100 years simulated to calculate the long run mean and standard deviation of each variable, for each policy and we present the impulse response functions over 40 periods.

Table 3.6 – Variables mean and standard deviation

Policy	statistics	$y$	$c$	$k$	$l$	$i$	$e$	$u$
BAU	mean	0.693	0.538	10.498	0.225	0.127	0.028	0.80
	std	0.198	0.061	2.469	0.038	0.151	0.008	0.130
emissions cap	mean	0.686	0.533	10.436	0.226	0.125	0.021	0.80
	std	0.178	0.054	2.147	0.035	0.136	0	0.117
carbon tax	mean	0.684	0.531	10.385	0.225	0.125	0.022	0.80
	std	0.173	0.041	1.485	0.038	0.138	0.006	0.132
intensity target	mean	0.681	0.529	10.291	0.225	0.124	0.02	0.8
	std	0.193	0.06	2.429	0.038	0.147	0.006	0.129

### 3.6.2 Technology shock:

We first consider a total factor productivity shock. Figures 3.4 and 3.5 represent the impulse responses functions of the model variables following a technology shock for each of the four scenarios considered, depending on the environmental policy in place: no policy, emissions cap, carbon tax or intensity target. The first and second order moments of main variables are summarized in table 3.6.

By following a positive productivity shock, output increases. Both the supply and demand rises for the final good at the moment of the shock. To sustain production, in all scenarios demand for all production factors increases, except for the energy factor in the case of an emissions cap. Moreover, in order to meet this increase in supply, the firm

uses more intensively capital, which will consequently wear out more rapidly, resulting in greater capital depreciation. Following the technology shock, the rise in productivity is reflected in an increase in the remuneration of the factors of production - the latter being remunerated at their marginal productivity, except for the price of energy, which is regarded as exogenous - in other words, a rise in productivity in that economy is not capable of influencing world energy prices. The increase in household income not only satisfies the increase in demand for the final good, but also investment. As the latter exceeds the share that depreciates, the capital stock tends to increase. As the shock dissipates, productivity declines, as does the demand for factors of production. The decline in the remuneration of these factors is reflected in a decline in demand, which returns to its stationary equilibrium. Capital accumulation increases as long as investment exceeds depreciation. But investment slowly diminishes and the share of capital that depreciates becomes greater than the accumulation of newly installed capital, the capital stock declines and returns to its stationary equilibrium.

The implementation of a climate policy obviously affects the variables level in the model. According to Table 3.6, a negative effect is indeed observed in energy consumption level compared to the BAU model. It can also be seen that the other model variables are also affected in level. However, generally speaking, in the event of a productivity shock, there is no significant difference in terms of volatility between the BAU and the various environmental policies considered, with the exception of the cap-and-trade market. This result is consistent with the literature studying the consequences of climate policies on the business cycle, particularly the study by Annicchiarico and Di Dio (2015). This result can be explained in the case of a carbon tax by the constant cost of emitting GHG. In the case of an intensity target, the technology shock increases production, the economy can use more energy, thus equation (3.4.7) defining the constraint is satisfied. In the case of an intensity target, the price of carbon will not change. The firm's demand for the right to pollute remains constant, since the proportion of emissions per unit produced remains constant. In fact, the increase in productivity is reflected both in an increase in production and an increase in energy demand, so that the energy intensity

of GDP remains constant. However, since the economy produces more, and uses more energy in terms of level, it emits relatively more GHG during a positive productivity shock. In other words, emissions will tend to increase when the economy experiences a positive productivity shock. There is only a noticeable difference in the case of an emissions cap. This time (see Table 3.6), it can be seen that the standard deviation for energy is almost zero in this situation. This is the consequence of a constant supply of pollution allowances from the authorities. Indeed, with the increase in productivity, demand for energy rises. However, the economy is constrained by the emission target (equation (3.4.6)), which has to be binding. Adjustment can only be made by the price of pollution  $p_t^c$ , which is variable. As the demand for energy and consequently the demand for pollution goes up, for a constant supply of pollution rights, the price of allowances increases. However, energy accounts for a small share of total production relative to other factors of production. Although the rise in the price of carbon affects the whole economy, the differences remain insignificant compared to the other instruments studied.

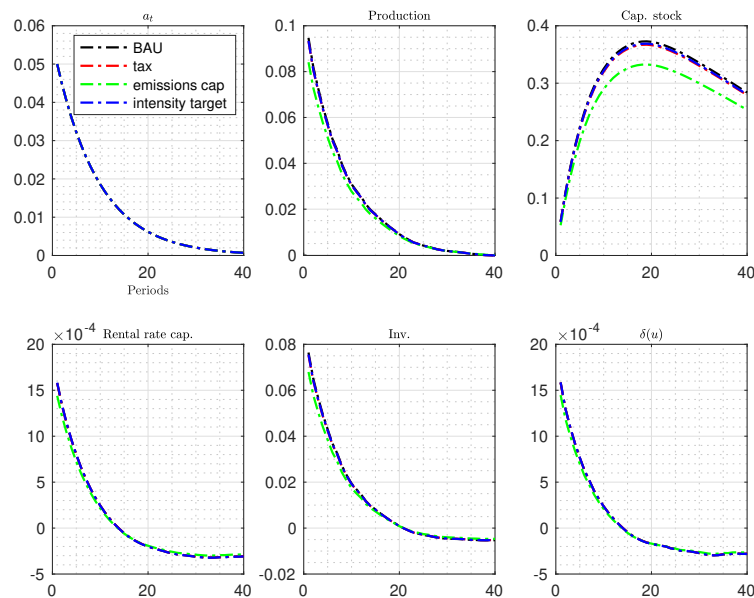


Figure 3.4 – Impulse response functions to a TFP shock (1/2)



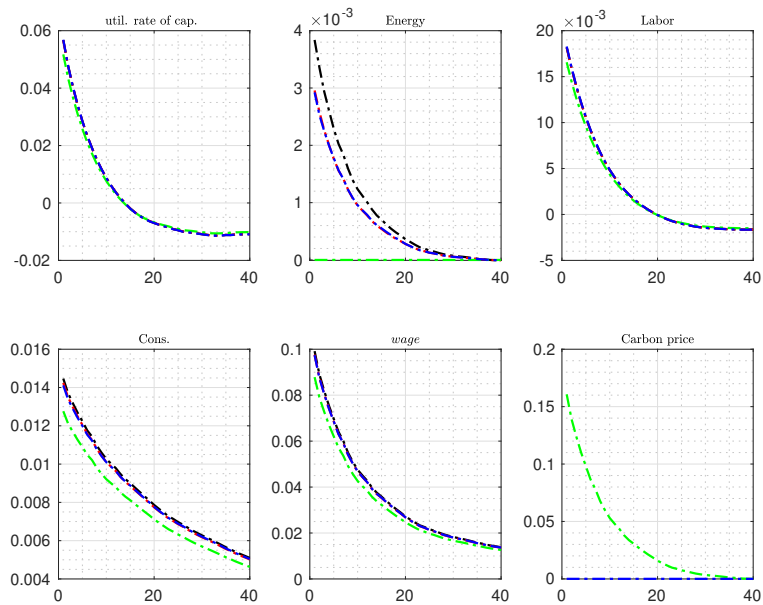


Figure 3.5 – Impulse response functions to a TFP shock (2/2)

### 3.6.3 Energy price shock:

According to Kilian (2009), we can distinguish three different oil shocks. An oil supply shock caused by the physical amount of oil disposable on the market<sup>6</sup>, and two demand shocks. The first demand shock is driven by the business cycle, in period of expansion the demand effect increase oil price - and the opposite during recession, while the second demand shock can be called "*Precautionary or speculative demand shock*". This last shock can be seen as the response to an anticipation of a price increase. Indeed oil consumers behave rationally and if they are expecting an increase in oil price, they would buy more oil in order to use it later, which eventually accelerate price inflation. In our case the economy takes the price as given so that, the second stochastic shock considered corresponds to Kilian's oil supply shock. Figures 3.16 and 3.17 represent impulses responses function to an energy price shock for the different environmental policies assumed.

<sup>6</sup>OPEC control 42% of the global oil production and 72% of the proved reserved.

Following a positive shock on the price of energy, two different reactions can be observed depending on the environmental policies implemented. On the one hand, the variables will react in the same way in the case of the BAU or a carbon tax. On the other hand, the variables will have a similar reaction in the case of an emission cap or an intensity target.

In the absence of an environmental policy, the exogenous rise in energy price results in a drop in demand for this factor of production. All other things being equal, the increase in this intermediate cost is reflected in a fall in demand for the final good, and thus a fall in consumption. Indeed, the price of the good reflects the marginal cost of production. Since the latter is now higher as a result of the rise in energy prices, the price of the final good also rises. Output production adapts to the fall in demand, which also decreases, leading at the same time to a fall in demand for the other inputs. The economy contracts at the time of the shock, and the firm remunerates its production factors at a relatively lower level compared to the stationary equilibrium. Physical capital is used less intensively, the depreciation rate of capital reacts pro-cyclically, and wears out less rapidly. However, the decrease in wear and tear on capital, linked to its less intensive use, does not compensate for the decrease in investment, but limits the share of capital that depreciates. The capital stock declines immediately and tends to decline further over time. Then, as the shock dissipates, the price of energy converges towards its initial value. The firm has a relatively greater recourse to all factors of production. As the price of the good declines, consumption rises again, supported by an increase in household income. Investment also tends to return to its initial level, but still does not compensate for the loss of capital. However, the capital stock is decreasing at a slower and slower pace, reaching its minimum 10 quarters after the shock before increasing again. In order to be able to satisfy its level of production, the firm will therefore use capital more intensively, which will increase its wear and tear, and this as long as the capital stock has not returned to its initial level.

In the case of a carbon tax, the impulse responses of the variables to an energy price shock are similar to the BAU scenario. A carbon tax corresponds to a constant marginal

cost of pollution abatement. As the price of energy rises, the firm must pay this cost for each unit of energy used in the production process. Since the price of pollution is constant, the variables respond in the same way as in the absence of climate policy. As the price of the final good reflects its marginal cost, the latter increases, and by reaction the demand for the final good decreases. The drop in household income, linked to a drop in demand for the factors of production, negatively influences the level of consumption of the good, etc. The return to equilibrium is also strictly identical to that described above in the BAU.

In the case of an emissions cap, the model reacts differently than described so far. Indeed, the impulse responses show no deviation from the steady state value of the system variables, following an energy price shock. When the government sets an emissions cap, the constraint (3.4.6) must be satisfied. Unlike the carbon tax where the cost of pollution abatement is constant, the carbon price is the variable that balances the supply of pollution with the demand for pollution. Thus, following an increase in the price of energy, there is a drop in the demand for the right to pollute, so that the price of pollution decreases and totally compensates for the increase in the price of energy. Since the total marginal price (exogenous price plus pollution price) remains constant, energy demand remains the same. The other variables of the model are then not affected by this shock.

The mechanism is essentially the same for the intensity target. In this case, the government issues permits not to achieve a certain level of emissions but to achieve a ratio of emissions per unit produced  $\frac{\Xi e}{y}$ . All other things being equal, as the price of energy rises, the demand for energy falls and at the same time, limits the demand for permits. The price of permits falls and, as with the emission cap, fully offsets the increase in the price of energy, so that the marginal cost of using this factor of production remains constant. Thus, energy demand remains the same as production. The constraint (3.4.7) for the intensity target imposed by climate policy is therefore satisfied.

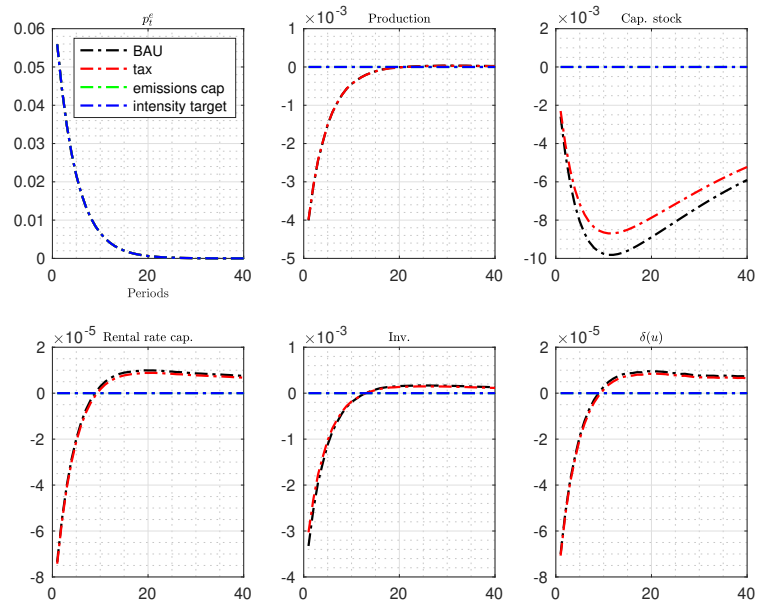


Figure 3.6 – IRF to an energy price shock (1/2)

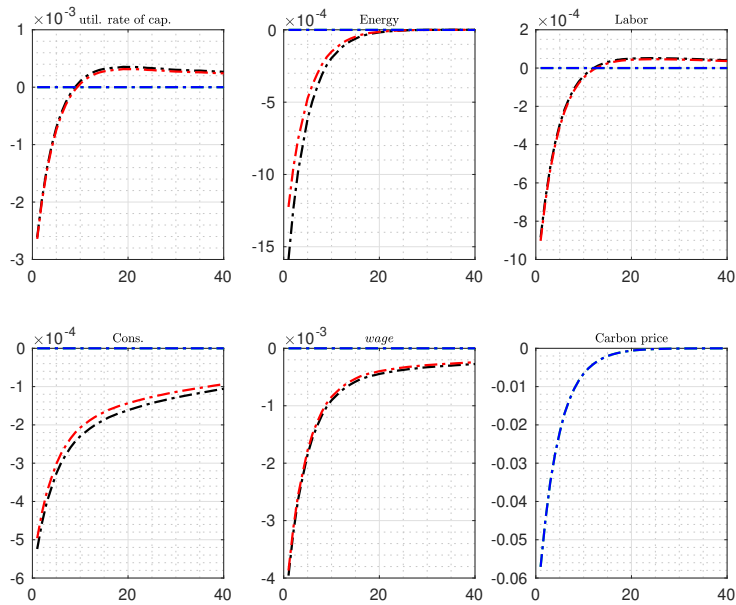


Figure 3.7 – IRF to an energy price shock (2/2)

### 3.6.4 Sensitivity analysis:

As Solow (1999) emphasizes, even if capital depreciation is considered constant in the literature, this assumption is "*empirically inaccurate*". Despite the fact that capital depreciation plays an important role, notably in Jorgenson's theory of investment and more particularly in the notion of user cost of capital that he developed (see Jorgenson (1963, 1967, 1971)), we can highlight that among the literature that seeks to assess the economic impact of environmental policies in a DSGE model (see Angelopoulos et al. (2010), Fischer and Springborn (2011), Heutel (2012), Angelopoulos et al. (2013) Golosov et al. (2014), Argentiero et al. (2017)), to our knowledge this article is the first to analyze the effects of environmental policy when the depreciation rate of capital is endogenous. This can be explained by the fact that several empirical studies have shown that geometric form is the function that best summarizes the depreciation of capital (see Hulten and Wykoff (1981) and more recently Pilat and Schreyer (2002), and Schreyer (2009)) and this geometric form implies a constant rate of depreciation. Consequently, if the depreciation rate of capital is constant, it will not be central in evaluating the economic impact of carbon pricing. However a recent study conducted by Albonico et al. (2014) using a Bayesian DSGE model leads to totally different results. On the contrary these authors show that the depreciation rate of capital in Canada and the US, is on the one hand not constant over time but volatile and responds with a *highly procyclical pattern*. Thus, if we assume as we previously did that the depreciation rate of capital is not constant but endogenous and responds to the cycle notably via the capital utilisation rate, then it is an important function in the model that need to be taken into account. Assuming that the depreciation rate of capital is constant when it is not, may lead to an over- or underestimation of the environmental policy impact. Indeed, in the analysis of the impulse responses of the variables following the two stochastic shocks, we were able to see that when the capital stock varies, as a result of an increase or decrease in investment, the utilization rate of capital moves pro-cyclically and is reflected in the rate of depreciation of capital, which also reacts pro-cyclically. To understand the role played by the utilization rate of capital in the model, we compare the previous results with a model where the

capital stock is fully utilized (assuming that  $u_t = u = 1$ ). For this purpose, equation (3.4.4) is replaced by a constant:

$$\delta(u_t) \equiv \delta = 0.0125 \quad \forall t \quad (3.6.1)$$

Hence, the law of motion for capital (3.4.3), and the Euler's equation (3.4.9c) are slightly modified while the marginal productivity of capital rate utilisation (3.4.9b) is removed from set of dynamic equations. The entire model with constant depreciation rate of capital is developed in the appendix.

Figures 3.7 to 3.19 illustrate for each of the four environmental policy scenarios the results with, on the one hand, where utilisation rate and capital depreciation are variables and, on the other hand, a model where they are considered as constant. The standard deviations that assume a full utilisation of the capital stock are summarised in Table 3.7. First, we note that the variables adopt the same response to shocks, depending on whether or not firms are allowed to use physical capital more or less intensively. On the other hand, it can be seen that, for the four scenarios considered, the magnitude of the reactions differs. In fact, it can be seen that when the firm uses the entire capital stock, the model reacts relatively less to the business cycles. Firms are unable to raise capital in response to periods of economic expansion or contraction. As well, the speed at which variables return to their stationary equilibrium is slower than when variable capital use is taken into account. Investment is a process that comes at a cost, there is an adjustment lag between the time when the decision to invest is made and the time when capital is productive. Adjustment to the cycle is therefore slower than in the case of variable capital use. Indeed, in this latter case, when capital is used more or less intensively, the firm has the capacity to mobilise immediately or, on the contrary, to reduce the proportion of capital used. The implementation of a climate policy will change the relative cost of energy compared to that of capital. By not taking account of this possible adaptation by the firm when substituting these two factors, because of the variation in relative cost, the reaction may be underestimated.

Table 3.7 – Variables mean and standard deviation when utilisation rate is constant

Policy	statistics	$y$	$c$	$k$	$l$	$i$	$e$
BAU	mean	0.776	0.601	11.478	0.228	0.144	0.031
	std	0.158	0.067	1.916	0.024	0.105	0.007
emissions cap	mean	0.757	0.587	11.184	0.228	0.140	0.021
	std	0.144	0.059	1.751	0.022	0.097	0
carbon tax	mean	0.768	0.595	11.399	0.228	0.143	0.006
	std	0.155	0.065	1.886	0.024	0.104	0.005
intensity target	mean	0.762	0.591	11.266	0.228	0.141	0.024
	std	0.154	0.061	1.815	0.024	0.105	0.005

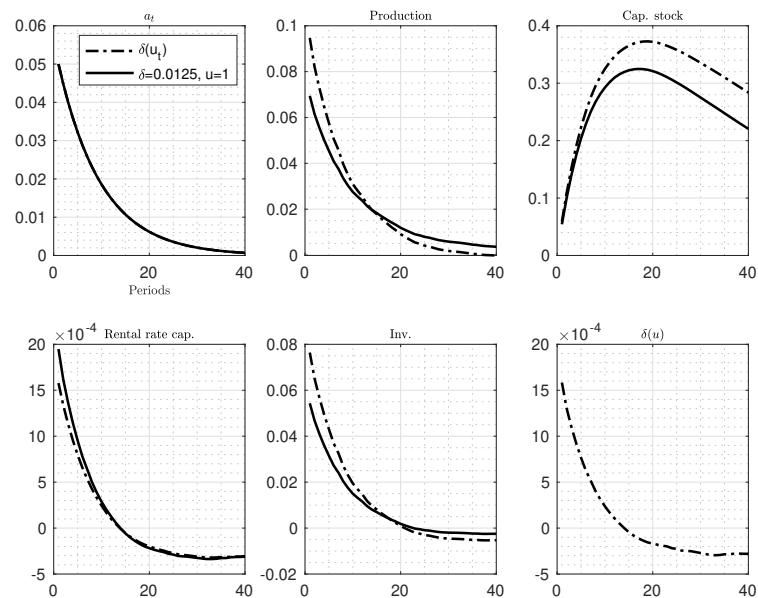


Figure 3.8 – IRF to a TFP shock in the BAU (1/2)

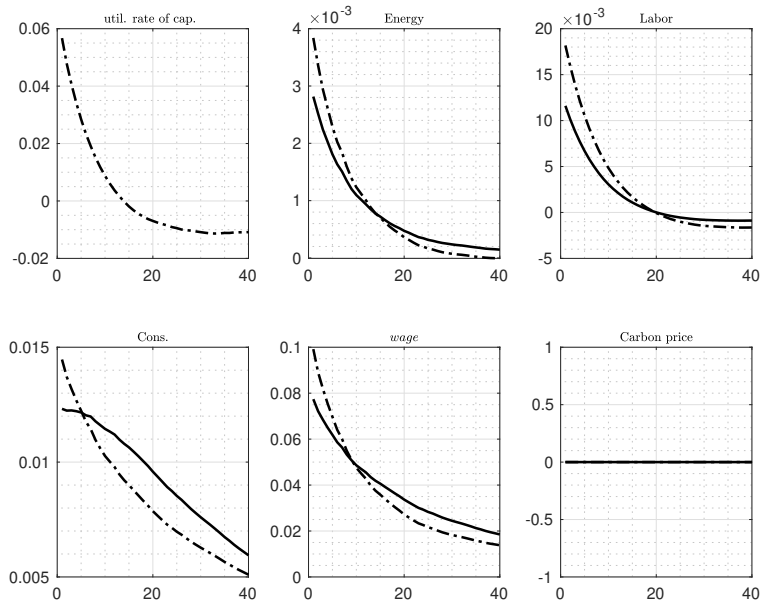


Figure 3.9 – IRF to a TFP shock in the BAU (2/2)

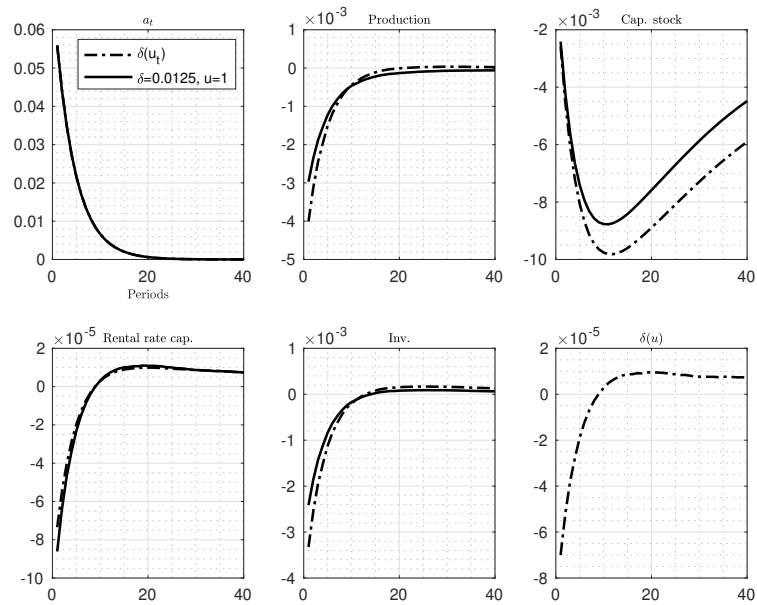


Figure 3.10 – IRF to an energy price shock in the BAU (1/2)



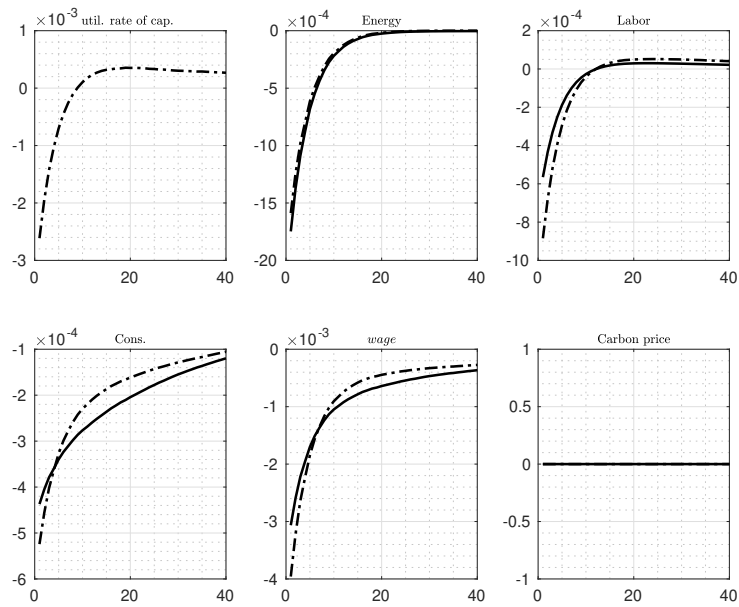


Figure 3.11 – IRF to an energy price shock in the BAU (2/2)

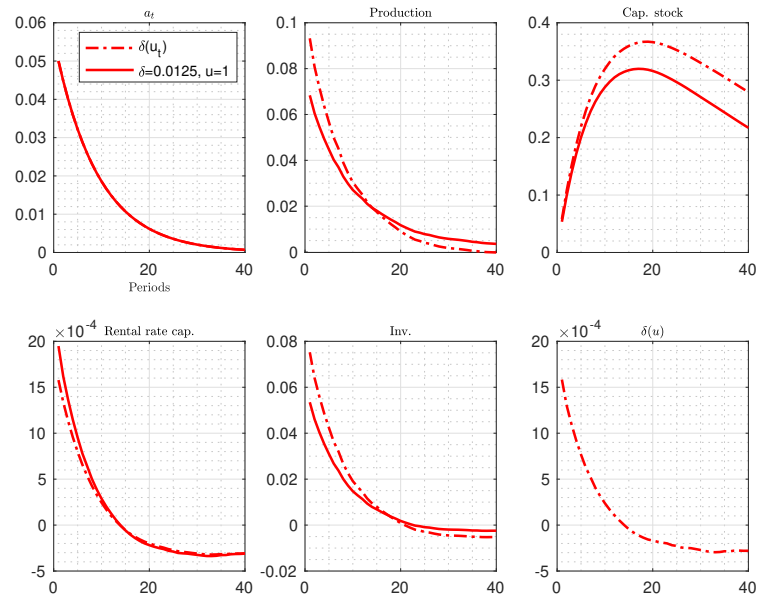


Figure 3.12 – IRF to a TFP shock in the carbon tax scenario (1/2)

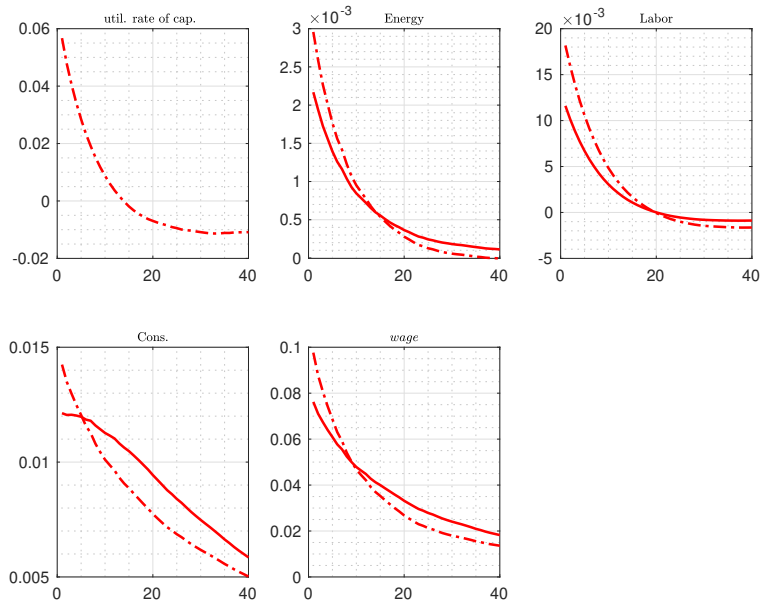


Figure 3.13 – IRF to a TFP shock in the carbon tax scenario (2/2)

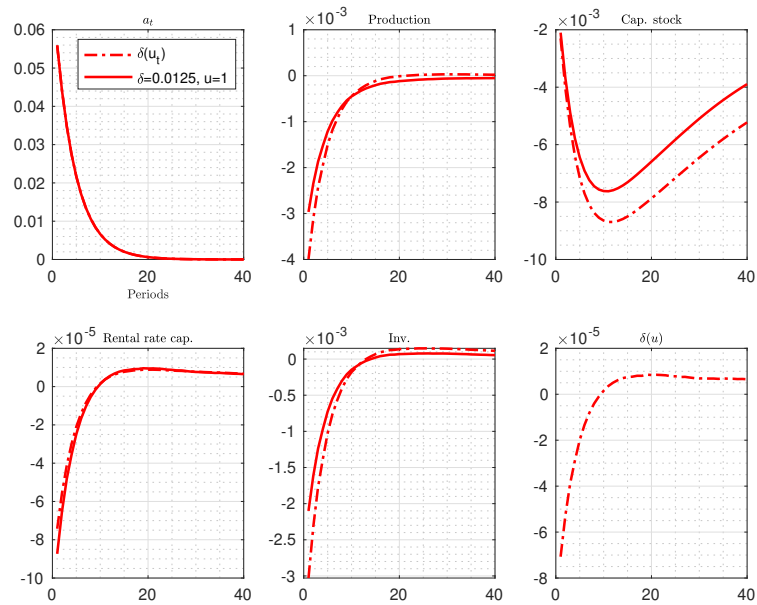


Figure 3.14 – IRF to an energy price shock in the carbon tax scenario (1/2)

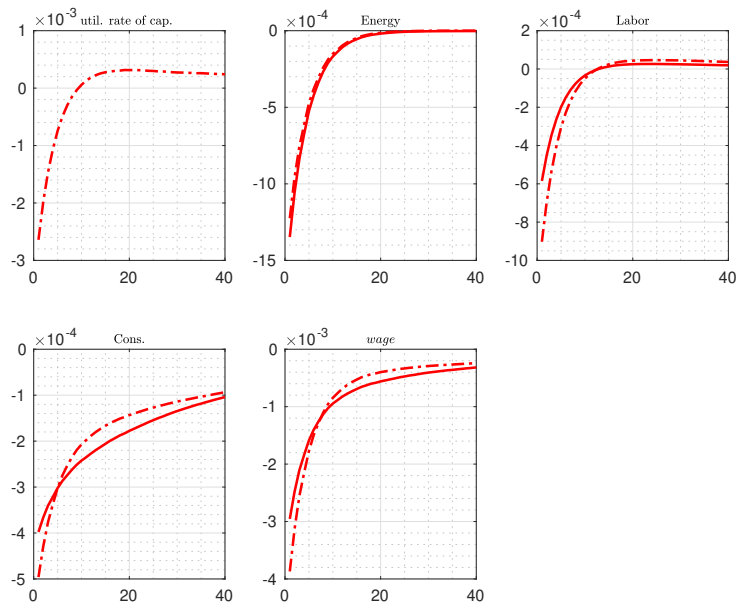


Figure 3.15 – IRF to an energy price shock in the carbon tax scenario (2/2)

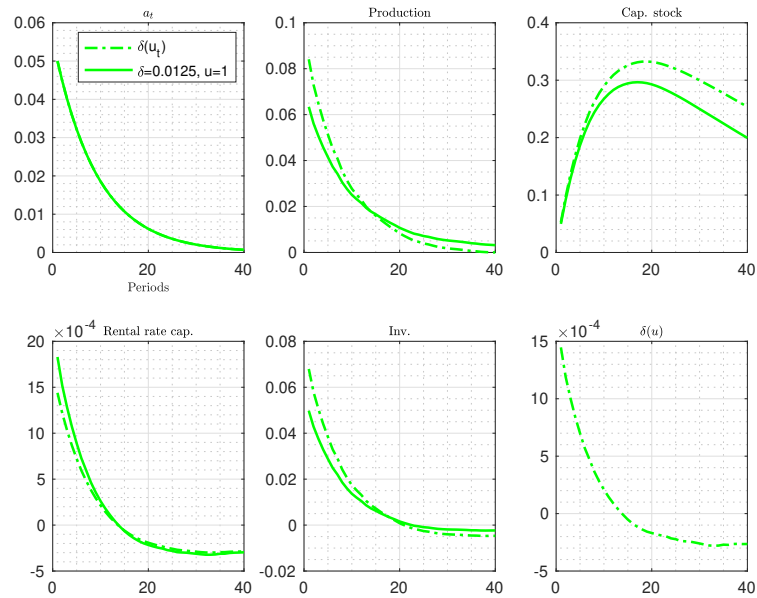


Figure 3.16 – The role of utilisation rate of capital(1/2)

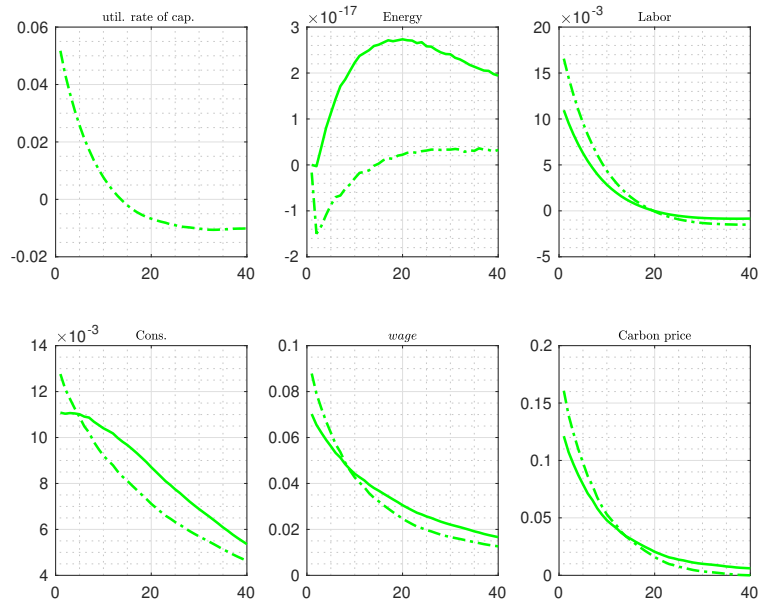


Figure 3.17 – The role of utilisation rate of capital (2/2)

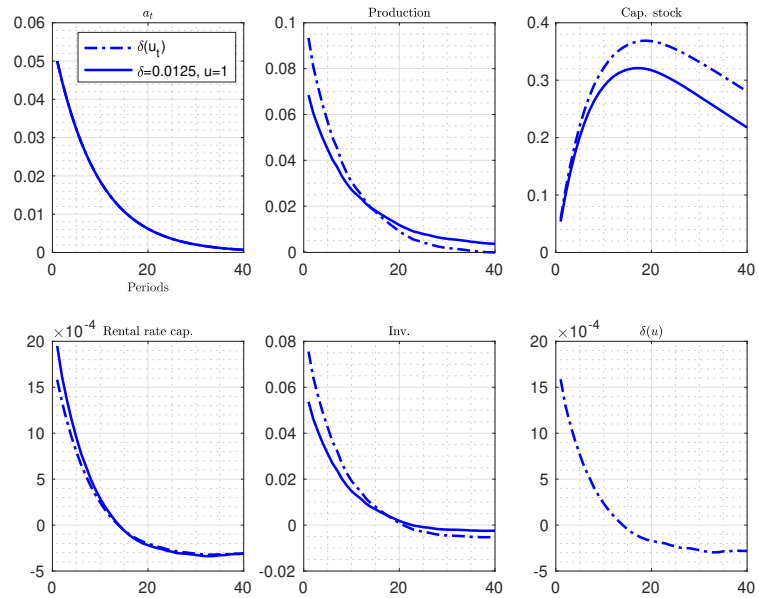


Figure 3.18 – IRF to a TFP shock shock in the intensity target scenario (1/2)

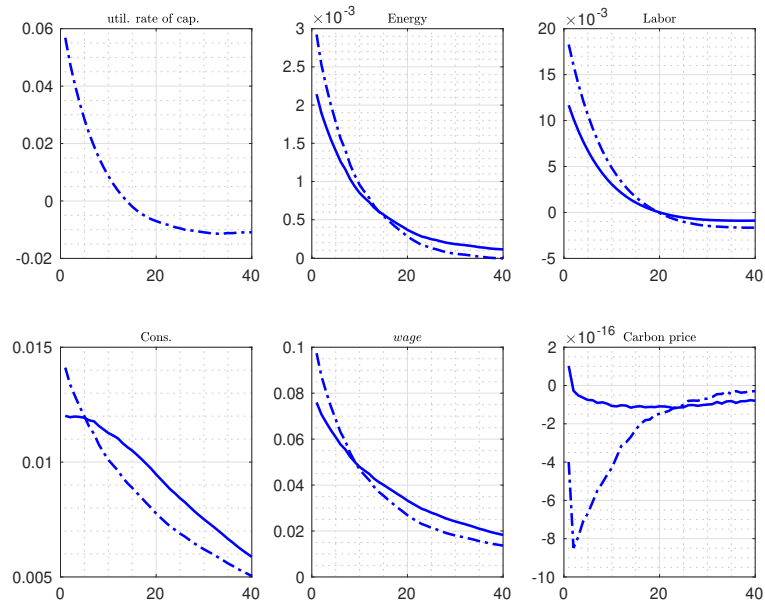


Figure 3.19 – IRF to a TFP shock in the intensity target scenario (2/2)

### 3.7 Conclusion:

In this chapter, we have used a DSGE model to compare the consequences of exogenous shocks on the business cycle when the economy implements a climate policy. For this purpose, we use the RBC model developed by Fischer and Springborn (2011), which includes energy as a factor of production. Thus, we were able to simulate not only the total factor productivity shock usually studied in the literature, but also a shock to energy prices. The Yellow Vests movement, which occurred in France following a rise in the price per barrel and an increase in the carbon tax, showed the importance of taking into account variations in the price of energy when the economy is energy dependent.

Our results show that the economy is significantly affected by a rise in energy prices. The introduction of a carbon tax raises the marginal cost of energy. This rigidity in the price of pollution will therefore accentuate the decrease in emissions in the short term at the time of the shock on the price of energy. However, with this increase in price of this input, demand will decrease and will tend to contract economic activity. Thus, there is a

risk that this type of instrument will be challenged, which may have a counter-productive effect on the level of emissions in the long term. On the contrary, the introduction of an instrument that reacts like a buffer to energy price shocks would make it possible to achieve the emission target set by the public authorities while limiting the negative effect on the rest of the economy. Indeed, the establishment of a pollution market or an intensity target allows the price of pollution to adjust in line with the price of energy. This price makes it possible, if the market is well constructed, to compensate upwards or downwards for the variation in the price of energy and sends firms the right price signal that takes into account both the marginal world price of energy plus the marginal cost of pollution, which represents the negative externality linked to the use of this resource. This result is therefore in favour of the implementation of a floating tax that would allow the carbon tax to rise over time in order to achieve the GHG emission reduction targets set by the IPCC. In addition, this would limit the negative consequences linked to the instability of energy prices set either by world demand or by the production of oil exporting countries. In addition, we develop the Fischer and Springborn (2011) model by integrating the capital utilization rate as a variable. We show that in the absence of the latter, the model tends to underestimate the volatility of the central variables.

In addition, the model can be extended in many ways. Indeed, it would be appropriate to integrate the environmental damage linked to GHG emissions in the manner of Heutel (2012). Moreover, it would be interesting to disaggregate the energy sector into several sources, with emitting energies on the one hand and non-polluting energies on the other. Finally, it would be interesting to introduce nominal rigidities into the model as in Annicchiarico and Di Dio (2015). Indeed, NK DSGE models that address environmental policy issues are still scarce. We try to contribute to this literature in the next chapter.



## Chapter 4

# The role of energy and greenhouse gas emissions for monetary policy design.

### 4.1 Introduction

The literature in macroeconomics that focuses on climate issues is still an under-explored field of research Schubert (2017), and this is especially true for the neo-Keynesian macroeconomics. Among the articles that have integrated environmental damage due to climate change, or studied the consequences of environmental policies on the economic cycle, are the work of Angelopoulos et al. (2013), Fischer and Springborn (2011), Heutel (2012) and more recently Annicchiarico and Di Dio (2015), Argentiero et al. (2017) and Xiao et al. (2018). Angelopoulos et al. (2013), Fischer and Springborn (2011), Heutel (2012) and Argentiero et al. (2017) use RBC models in their articles to analyse various climate policies. Annicchiarico and Di Dio (2015) are the first to develop a NK DSGE model with nominal rigidities à la Calvo. This model uses only two factors of production: capital and labor. Emissions is proportional to the quantity of final good produced. They compare different environmental policies instrument, and consider various shocks including



monetary policy shock. Xiao et al. (2018), use this model and estimate a Bayesian model for the Chinese economy, but incorporate a third factor of production, energy. Therefore, emissions depend on the quantity of energy used in production.

We contribute to the literature on environmental NK DSGE models in the following ways. We integrate as Xiao et al. (2018) energy into the model, but we disaggregate this factor into polluting and non-polluting energy using a CES function, which allows us to consider energies as complementary or substitutable. We use the introduce nominal rigidities in the model as in Annicchiarico and Di Dio (2015) and compare two environmental policy instruments: a carbon tax with an emission cap. Our results show that a total factor productivity shock does not generate significant differences depending on the instrument used, as suggested by Annicchiarico and Di Dio (2015) results. On the other hand, this result is similar to those obtained by Xiao et al. (2018). Only GHG emissions in the case of the tax will vary significantly. Thus, to be fully effective, the European carbon market could integrate a floor price, or a tax that would replace the allowance market if the price is too low. This would avoid a drop in the carbon price in the event of a major recession. On the other hand, responses of macroeconomic variables differ, depending on the instrument implemented in the event of a productivity shock in the polluting energy sector. Indeed, a positive productivity shock in this sector (such as fracking) will reduce the price of this energy source. In the case of a tax where the price of pollution remains constant, emissions will tend to increase relatively more than in the case of an emission cap. The combined use of these two instruments therefore appears to be the best solution to economic fluctuations. Finally, we contribute to the literature by optimizing the coefficients of the monetary policy rule used. In this sense, we show that when negative externalities related to GHG emissions are taken into account, it is optimal for the central bank to be more averse to inflation and output gap.

## 4.2 Carbon cycle. General assumptions

Since the industrial revolution, economic activity has generated a flow of GHG emissions by using fossil fuels in the production processes. Some of these emissions are naturally dissipated: either absorbed in the air through photosynthesis or diluted in oceans. When GHG is not assimilated by this degradation processes, it accumulates in the atmosphere. According to the AR5 (Pachauri et al., 2014), there exists a positive relationship between the rise in the stock of GHG that stays in the atmosphere and the damage caused by climate change and warmer temperatures (e.g., storms, sea level rise, coastal flooding, inland flooding, extreme heat etc.).<sup>1</sup>

The environmental damage associated with the increase in atmospheric GHG stock affects the production process. Negative consequences of GHG are usually modelled in such a way as to decrease total factors productivity. The DICE and RICE computable general equilibrium models developed by Nordhaus (1991, 1994, 2013) and Nordhaus and Yang (1996) integrate environmental damage into the production function. The Golosov et al. (2014) model derived from the DICE model also uses an exponential loss function that affects GDP. In the PAGE model (see Hope (2006)), damage is also accounted for as a loss as a percentage of GDP.

For the purposes of this chapter, we instead assume that negative externalities associated with emissions do not affect productivity but rather the household utility. Rising emissions generate disutility for the representative household. Incorporating the negative externalities of emissions into the household utility function allows us to optimize the coefficients of monetary policy rule. Moreover, the flow rather than the stock of GHG is considered responsible for these externalities. This approximation is made for a methodological reason. Resolution of DSGE models is based on Taylor's expansion, or perturbation method in the neighbourhood of steady state. The starting point is a deterministic stationary equilibrium that can be solved. This equilibrium is then perturbed by stochastic shocks, and in a second step, we observe the return of the variables to their

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<sup>1</sup>A systemic risk of a breakdown of infrastructure network can also be potentially connected to extreme weather. This could materialize in food insecurity, water stress or losses of ecosystem and biodiversity.

initial equilibrium as the shocks dissipate. This methodology assumes that all variables are stationary at equilibrium. In other words, in order to incorporate the stock instead of the flow, the stock would have to be invariant. However, this stock continues to grow as the annual flow of emissions exceeds the earth capacity to absorb GHG. Thus, assuming the stock to be invariant would be inconsistent with reality. Therefore, we consider the emissions flow and not the stock as a source of externalities. Thus, it is assumed that an increase in this flow will reduce household utility.

In the RBC model developed by Heutel (2012), and that of Annicchiarico and Di Dio (2015, 2017), there is no energy, but emissions are proportional to the level of production. In Fischer and Springborn (2011) and Xiao et al. (2018), production requires three factors of production : capital, labor and energy. But energy is an aggregate factor, so emissions are proportional to the use of this factor. Argentiero et al. (2017) also use three factors of production in their RBC model, including energy, which they disaggregate into polluting energy and renewable energy, in this case, emissions are proportional to the use of fossil energies and not to the aggregate energy as it is the case in Fischer and Springborn (2011) or Xiao et al. (2018). In our model, we assume as Argentiero et al. (2017) that emissions arise from the use of fossil fuels in the production process. Energy factor is similarly disaggregated into polluting energy and clean energy. Both energy inputs are aggregated using a CES production function. Emissions are then proportional to the quantity of polluting energy used in production. To the best of our knowledge, this is the first article to disaggregate energy use in a NK DSGE model. This approach allows for finer results, since the results will depend also on the elasticity of substitution between energy factors.

Without public intervention, fossil fuel users do not take into account negative externalities generated by the combustion of oil, gas or coal. Due to a lack of definition of property rights, costs of using these energies do not reflect real cost of pollution. In the absence of technological innovation that could increase energy efficiency and would result in a reduction in emissions per unit of fossil fuels consumed, the only way to reduce emissions is to implement a public policy instrument. Public authorities have different instruments at their disposal to internalize the cost of pollution: emission standard, a tax

or a cap-and-trade market. In our model we compare two instruments: a carbon tax and a cap-and-trade market. We assume that public policy decides in each period both the tax rate for emissions,  $\tau_t$ , and how to use tax revenues. In the second case, it is assumed that public authority set an emission cap not to be exceeded. They then offer a number of emission allowances on the market. The demand for allowances comes from firms that use fossil fuels in the production process, and let the market determine the pollution equilibrium price. Because we work at an aggregate level, we do not model allowances trade between firms. We focus on comparing a price instrument - a carbon tax - with an instrument acting on quantities, and variables responses for different exogenous shocks. In the latter case, the price of pollution will adapt to market fluctuations. This article expands the recent but dynamic literature between business cycle and climate policies. Fischer and Springborn (2011) compare a carbon tax, with an emission cap and an intensity target in a RBC model. The only source of exogenous variability comes from total factor productivity. Their results indicate that an intensity target instrument has least effect on economic growth compare to the two others instruments considered, and an emission cap leads to less volatility than a carbon tax. Thus, production and utility vary less in the case of an emissions cap. Heutel (2012) considers a tax and an emission cap, but is more interested in how these instruments should be adapted to maximise welfare. According to these results, both instruments should vary pro-cyclically. Annicchiarico and Di Dio (2015) will extend this literature by developing the first NK DSGE model in which they will consider various sources of uncertainty other than TFP, including public consumption and monetary policy shocks. Among other instruments they also compare a tax to an emission cap. They find that the cap reduces the volatility of the main macroeconomic variables of their model. Consequently, a climate policy instrument acts as an economic stabiliser. More recently Annicchiarico and Di Dio (2017) developed their model to analyse the optimal policy mix between environmental and monetary policy. They studied Ramsey's planner problem, where alternatively can control both environmental and monetary policy or monetary policy, and takes as given the environmental policy (carbon tax or cap-and-trade market), or in the last case it optimizes environ-

mental policy and takes as given monetary policy. When the planner can optimize both environmental and monetary policies, they find optimal to diminish the procyclicality of emissions (the planner is making a trade-off between the three imperfections present in the models: negative externalities of emissions, imperfect competitions and costly price adjustment). When the planner only has access to monetary policy, GHG emissions mitigation is not neutral for monetary policy, and depends on the climate policy instrument implemented. Under a carbon tax regime with no environmental damage, inflation targeting seems to be optimal while it is not under a cap-and-trade market or when damage occur with a carbon tax. In such cases, it appears to be optimal to allow changes in price in order "to address feedback of pollution on productivity". For Xiao et al. (2018), who add the energy factor in an NK DSGE into a model similar to the one develop by Annicchiarico and Di Dio (2015, 2017), they compare four environmental policy instruments to the BAU, which corresponds to a situation without policy. As expected, in the case of a total factor productivity shock, the model is more volatile in the absence of public intervention. The instruments will also act as stabilizers. They find that there is no significant difference between instruments of environmental policies. Only the volatilities of carbon prices or emissions will increase. Emissions are more volatile with a tax than with an emissions cap.

We contribute to this literature by comparing a carbon tax regime to an emission cap. We limit our analysis to these two instruments because they represent the major part of the instruments implemented to fight climate change ECOFYS (2018). The major contribution consists in the disaggregation of the energy sector in a NK DSGE model. This makes it possible to study various shocks, standard in recent macroeconomic literature, but we mainly focus on productivity shocks in the polluting energy sector, or in the clean energy sector. The model also allows us to assume that the two sources of energies are substitutable or complementary inputs. Our main findings are as follows. Our results are consistent with Xiao et al. (2018) who find that there are no significant differences between the two instruments used when a total factor productivity shock occurs in the economy. However, the tax generate more volatility in GHG emissions,

and an emissions cap generate more volatility in carbon price. Similarly, after a shock in the non-polluting energy sector, there is no significant difference between a tax and an emission cap. On the other hand, a tax generates more volatility than an emission cap in the case of a productivity shock in the polluting energy sector. Because polluting price remains constant, emissions will vary accordingly to the quantity of fossil fuel burnt, while in an emission cap regime, carbon price will adapt more or less to the situation and it lowers emissions flows. The elasticity of substitution between energies will play an essential role in variables responses following a productivity shock in the polluting energy sector in a carbon tax regime. There is indeed a significant difference if the energies are considered as complementary or substitutable. On the other hand, elasticity does not influence the response of variables in the case of a productivity shock in the non-polluting energy sector in the presence of a tax. In the presence of an instrument acting on quantities, the elasticity of substitution will essentially play a role on the carbon price. Depending on whether or not energy is substitutable, during a shock in the non-polluting energy sector, the price of allowances will either increase or decrease. But in the end, the rest of the economy is not affected by differences in elasticity.

Thus, based on these results, it seems appropriate to recommend the joint use of these two instruments in order to minimize GHG emissions as much as possible. Indeed, the 2008 financial crisis highlighted the shortcomings of using an allowance market alone. The European experience and its EU ETS market has shown that in the event of a crisis, the price of carbon tends to collapse. In line with IPCC recommendations, a carbon market alone seems insufficient to achieve the targets set. By remaining at a low level a carbon price does not send the right signal to investors. If the price is too low, it does not seem profitable to invest in low carbon technologies. So when the economy starts to recover, the economy starts using again more fossil fuels and emissions rise. The reform of the EU ETS market for the next period has put in place a mechanism to limit this phenomenon but does not totally prevent it Trotignon et al. (2015). The Californian carbon market, for its part, has a floor price below which carbon price cannot fall. In order to avoid a further sharp drop in the price of carbon in the event of an economic

recession, it seems wise to adopt the same system. Consequences are limited since we find no significant differences between both instruments in case of a TFP shock, as our results show.

### 4.3 The model

This section describes the way the behavioral equations of the model are obtained from the optimizing (rational) behavior of both households and firms.

#### 4.3.1 Households

There is a representative household in the economy whose welfare depends positively on consumption (shaped by habits from lagged consumption,  $0 < \chi < 1$ ) and negatively on labor supplied to firms.<sup>2</sup> Moreover, toxic (GHG) emissions reduce household welfare measured through a damage function  $d(G_t)$  which is increasing in the level of gas emissions  $G_t$ . At a given period  $t$ , this representative household seeks to maximize the intertemporal constant relative risk aversion (CRRA) utility function with constant discount factor  $\beta < 1$  at infinite time horizon:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left( (1 - d(G_{t+j})) \frac{(c_{t+j} - \chi c_{t-1+j}^A)^{1-\sigma}}{1 - \sigma} - \psi \frac{(n_{t+j}^s)^{1+\gamma}}{1 + \gamma} \right) \quad (4.3.1)$$

where  $c_t$  is current household-level consumption,  $c_{t-1}^A$  is lagged aggregate consumption,  $n_t^s$  is the current labor supply and  $G_t$  denotes the amount of economy-wide GHG emissions in period  $t$ . For the same current period  $t$ , the budget constraint in real terms is

$$\begin{aligned} c_t + i_t + a(z_t)k_{t-1} + \left( \exp\left(\varepsilon_t^b\right) (1 + r_t) \right)^{-1} b_{t+1} - b_t + m_t - (1 + \pi_t)^{-1} m_{t-1} + h(c_t, m_t) \\ = w_t n_t^s + r_t^k z_t k_{t-1} + div_t - tax_t \end{aligned} \quad (4.3.2)$$

where  $w_t$  is the real wage,  $r_t^k$  is real rental rate of capital,  $k_t$  is the stock of capital,  $z_t$  is the capital utilization rate and  $a(z_t)$  is its associated adjustment cost (with the

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<sup>2</sup>Consumption habits may be assumed with respect to lagged household-level consumption (internal habits) or lagged aggregate consumption (external habits).

steady-state properties  $z = 1$ ,  $a'(z) > 0$  and  $a''(z) = \sigma_a > 0$  as in Smets and Wouters, (2007),  $b_{t+1}$  is the amount of government bonds purchased in period  $t$  to be redeemed in period  $t + 1$  with a gross real return of  $\exp(\varepsilon_t^b)(1 + r_t)$  which can be decomposed in the contributions of the risk-free real interest rate  $r_t$  and the AR(1) risk-premium shock  $\varepsilon_t^b$ , the amount of real money holdings available for purchases in the current  $t$  period is  $m_t$ , there is a lump sum  $tax_t$  from the government,  $h(c_t, m_t)$  is a function that delivers the transaction costs depending positively on consumption and negatively on real money holdings, and  $div_t$  is the amount of real dividends collected from firm ownership.

Capital accumulation is costly as in Smets and Wouters (2007). Thus, the equation of motion for capital is,

$$k_t = (1 - \delta) k_{t-1} + \exp(\varepsilon_t^i) \left[ 1 - S\left(\frac{i_t}{i_{t-1}}\right) \right] i_t, \quad (4.3.3)$$

where  $\delta$  is the constant rate of capital depreciation rate,  $S(i_t/i_{t-1})$  is the investment adjustment cost function with the steady-state properties  $S(1) = S'(1) = 0$  and  $S''(1) = \varphi_k > 0$ , and  $\varepsilon_t^i$  is an stochastic AR(1) shock to the price of investment relative to consumption goods. Regarding the transactions technology, we follow Casares (2007) and the amount of income required to cover the transaction costs is determined by this function

$$h(c_t, m_t) = \begin{cases} 0 & \text{if } c_t = 0 \\ \phi_0 + c_t \phi_1 \left(\frac{c_t}{m_t}\right)^{\phi_2/(1-\phi_2)} & \text{if } c_t > 0 \end{cases}$$

where  $\phi_0$ , and  $\phi_1 > 0$ , and  $0 < \phi_2 < 1$ , so that the transaction costs increases with consumption, but falls with the stock of real money. The damage function that captures the gas emissions externality is quadratic as in Heutel (2012) and Khan et al. (2019)

$$d(G_t) = d_0 + d_1 G_t + d_2 G_t^2$$

The representative household chooses the sequence  $c_t$ ,  $n_t^s$ ,  $z_t$ ,  $k_t$ ,  $i_t$ ,  $b_t$ , and  $m_t$ . to maximize (4.3.1) subject to budget constraints and capital accumulation constraints such as (4.3.2) and (4.3.3), respectively for period  $t$  and analogous expected constraints for future periods.



### 4.3.2 Firms

The production sector is composed of both final-good firms (competitive packers) and intermediate-good firms that operate in monopolistic competition as in Dixit and Stiglitz (1977).

#### 4.3.2.1 Final-good firms

A continuum of intermediate goods is used in the production of the final good:

$$y_t = \left[ \int_0^1 y_t^{\frac{\theta-1}{\theta}}(i) di \right]^{\frac{\theta}{\theta-1}}$$

where  $y_t$  is the output of final good produced using  $y_t(i)$  quantity of intermediate goods of type  $i$ ,  $\theta > 1$  represents the elasticity of substitution between intermediates goods. The production of final goods is assumed to be under perfect competition. The representative firm chooses the quantity of intermediate goods  $y_t(i)$  that maximises its profit:

$$P_t y_t - \int_0^1 P_t(i) y_t(i) di$$

where  $P_t(i)$  is the nominal price of the intermediate good  $i$ , and  $P_t$  is the final good price.

#### 4.3.2.2 Intermediate-good firms

Intermediate goods are produced by monopolistically competitive firms using labor,  $n$ , capital,  $k$ , and energy,  $e$ , as inputs. The production technology of the representative  $i$  firm is given by:

$$y_t(i) = a_t k_t(i)^\alpha n_t(i)^{1-\alpha-v} e_t(i)^v \quad (4.3.4)$$

The intermediate goods firm determines the demand for inputs by solving the cost-minimization program

$$\text{Min } w_t n_t(i) + r_t^k k_t(i) + p_t^e e_t(i)$$

subject to technological constraint (4.3.4). We also assume that prices are set with nominal rigidities *à la* Calvo (1983). In each period a firm has the probability  $1 - \eta$  to adjust its price, and consequently has the remaining probability  $\eta$  to not adjust its

price. If the firm is not able to adjust its price optimally, the following indexation rule is applied

$$\frac{P_t(i)}{P_{t-1}(i)} = (1 + \pi_{t-1})^\kappa (1 + \pi + \nu_t)^{1-\kappa}$$

where  $\kappa \in [0, 1]$  is the indexation share parameter of lagged inflation  $\pi_{t-1}$ , and its complementary  $1 - \kappa$  is the indexation share for the steady-state rate of inflation  $\pi$  modified as determined by the  $\nu_t$  price-push shock generated from an exogenous AR(1) process. Between period  $t$  when firm has the market signal to set its price optimally and  $t + j$ , the price indexation factor is:

$$\Pi_{t,t+j}^P = \prod_{k=1}^j \left[ (1 + \pi_{t+k-1})^\kappa (1 + \pi + \nu_{t+k})^{1-\kappa} \right]$$

Intermediate-good firms face a Dixit-Stiglitz demand constraint which for the representative  $i$  firms reads as follows

$$y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} y_t \quad (4.3.5)$$

Since  $P_t(i)$  is the price chosen by firm  $i$  when it has the ability to choose its price optimally, it is set at the value that maximizes the discounted present value of real profits conditional to the lack of future optimal price setting:

$$\underset{P_t(i)}{\text{Max}} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j \left( \frac{P_t(i)\Pi_{t,t+j}^P}{P_{t+j}} y_{t+j}(i) - mc_{t+j} y_{t+j}(i) \right)$$

where  $mc_{t+j}$  is the real marginal cost in period  $t + j$ .

When the firm is able to set its price optimally in period  $t$ , using the demand constraint (4.3.5) in current and future periods gives this alternative way of writing the intertemporal profit function conditional to Calvo-type rigidity,

$$\underset{P_t(i)}{\text{Max}} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j \left( \left( \frac{P_t(i)\Pi_{t,t+j}^P}{P_{t+j}} \right)^{1-\theta} y_{t+j} - mc_{t+j} \left( \frac{P_t(i)\Pi_{t,t+j}^P}{P_{t+j}} \right)^{-\theta} y_{t+j} \right)$$

Defining  $\lambda_t(i)$  as the Lagrange multiplier associated with the technological constraint of the representative firm. It is the shadow price of increasing the production of intermediate output, or the marginal cost of inputs (capital, labor and energy) necessary to produce

one extra unit of output. From the solution of the minimizing cost optimisation program, it is easy to show that all the firms will have both same capital-labor ratio and energy-labor ratio. Consequently, real marginal cost  $mc_t$  is the same across firms, and it is equal to:<sup>3</sup>

$$mc_t = \frac{1}{\alpha^\alpha \nu^\nu (1 - \alpha - \nu)^{(1-\alpha-\nu)}} \frac{1}{a_t} (r_t^k)^\alpha (p_t^e)^\nu w_t^{(1-\alpha-\nu)} \quad (4.3.6)$$

Considering now the above price setting problem, if the intermediate good firm  $i$  has the opportunity to choose its price, then from the first order condition associated with this program the firm will impose the following price:

$$P_t(i) = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j mc_{t+j} \Pi_{t,t+j}^P \frac{-\theta}{P_{t+j}^\theta} y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j \Pi_{t,t+j}^P \frac{1-\theta}{P_{t+j}^{\theta-1}} y_{t+j}} \quad (4.3.7)$$

Since in the right hand side of the optimal price condition (4.3.7), no term depends on  $i$ , all firms able to set their price will do it at this level so we can remove the  $i$  index, and call this common optimal price  $P_t(i) = P_t^*$ . Bearing in mind that inflation rate between two successive periods is  $\pi_t = \frac{P_t}{P_{t-1}} - 1$ , the rate of inflation between  $t$  and  $t + j$  is therefore:  $\frac{P_{t+1}(i)}{P_t(i)} \frac{P_{t+2}(i)}{P_{t+1}(i)} \dots \frac{P_{t+j}}{P_{t+j-1}} = \frac{P_{t+j}}{P_{t-1}(i)} = \Pi_{k=1}^j (1 + \pi_{t+k})$ . Using this formulation into the optimal price gives us the optimal relative price:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j mc_{t+j} \Pi_{t,t+j}^P \frac{-\theta}{\Pi_{k=1}^j (1 + \pi_{t+k})^\theta} y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j \Pi_{t,t+j}^P \frac{1-\theta}{\Pi_{k=1}^j (1 + \pi_{t+k})^{\theta-1}} y_{t+j}}$$

In the extreme case when all the firms are able to adjust their price optimally in every period  $\eta = 0$ , we find the common result that firms will set their price to a fixed mark-up over its marginal cost:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} mc_t$$

The aggregated price index is an average between prices set by firms that have the capacity to do so  $(1 - \eta)$ , and other prices that could not be set optimally by the remaining firms and they are adjusted through the indexation rule. With the Dixit-Stiglitz aggregation scheme, it gives

$$P_t = \left[ (1 - \eta) P_t^*{}^{1-\theta} + \eta P_{t-1}^{1-\theta} \left[ (1 + \pi_{t-1})^\kappa (1 + \pi + \nu_t)^{1-\kappa} \right]^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (4.3.8)$$

<sup>3</sup>Proof available upon request.

### 4.3.2.3 Energy markets

For simplicity, we assume that the production of energy evolves in a competitive market and only require labour as input in its production (with diminishing marginal returns):

$$\begin{aligned} e_t^p &= a_t^p (n_t^p)^{\zeta^p} & \text{with } 0 < \zeta^p < 1 \\ e_t^{np} &= a_t^{np} (n_t^{np})^{\zeta^{np}} & \text{with } 0 < \zeta^{np} < 1 \end{aligned}$$

where  $a_t^p$  and  $a_t^{np}$  are productivity shocks for polluting and non-polluting energy, respectively. The firms producing polluting operate in perfect competition and determine the amount  $e_t^p$  by solving the maximisation program:

$$\begin{aligned} \text{Max} \quad & p_t^p e_t^p - w_t n_t^p \\ \text{s.t.} \quad & e_t^p = a_t^p (n_t^p)^{\zeta^p} \end{aligned}$$

Meanwhile, a similar program for firms producing non polluting energy:

$$\begin{aligned} \text{Max} \quad & p_t^{np} e_t^{np} - w_t n_t^{np} \\ \text{s.t.} \quad & e_t^{np} = a_t^{np} (n_t^{np})^{\zeta^{np}} \end{aligned}$$

**4.3.2.3.1 Energy demand in a carbon tax regime.** The composite energy input in the previous Cobb-Douglas production function  $e_t(i)$  uses both polluting  $e_t^p(i)$  and non polluting  $e_t^{np}(i)$  source of energy, and can be disaggregated into the following CES function:

$$e_t(i) = [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} \quad (4.3.9)$$

where  $\varphi \in (-\infty, 1)$ , the elasticity of substitution between polluting and non polluting energy demand is  $(1 - \varphi)^{-1}$ , and  $0 < \omega < 1$  is the share parameter for polluting energy.

The optimizing program for each energy input is:

$$\begin{aligned} \text{Max} \quad & p_t^e e_t(i) - p_t^p (1 + \tau_t \Xi) e_t^p(i) - p_t^{np} e_t^{np}(i) \\ \text{s.t.} \quad & e_t(i) = [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} \end{aligned}$$

The new notation includes  $p_t^p$  for the real price of polluting energy input,  $\tau_t$  as the unit tax on polluting energy, and  $p_t^{np}$  as the real price of non-polluting energy. From the first order conditions, we can obtain the following energy demand functions

$$\begin{aligned} e_t^p(i) &= \left( \frac{p_t^p(1 + \tau_t \Xi)}{\omega p_t^e} \right)^{\frac{1}{\varphi-1}} e_t(i) \\ e_t^{np}(i) &= \left( \frac{p_t^{np}}{(1 - \omega) p_t^e} \right)^{\frac{1}{\varphi-1}} e_t(i) \end{aligned}$$

where the CES energy price index is

$$p_t^e = \left[ \omega^{\frac{1}{1-\varphi}} [p_t^p(1 + \tau_t \Xi)]^{\frac{\varphi}{\varphi-1}} + (1 - \omega)^{\frac{1}{1-\varphi}} p_t^{np \frac{\varphi}{\varphi-1}} \right]^{\frac{\varphi-1}{\varphi}}$$

**4.3.2.3.2 Energy demand in an emission cap regime.** In this situation, the same CES function 4.3.9 for composite energy input applies. Only the optimizing program for the polluting energy input differs. The government set a cap of emissions not to be exceeded, and sell on the market allowances to polluting firms. For this reason, firms face another constraint that is suppose to be saturated. Let define  $\bar{G}_t$  the quantity of allowances exogenously set by the public authority, and  $P_{G,t}$  the marginal carbon price to pay for each unit of pollution emitted.

$$\begin{aligned} &Max \quad p_t^e e_t(i) - p_t^p e_t^p(i) - P_{G,t} \bar{G}_t - p_t^{np} e_t^{np}(i) \\ &s.t \quad \begin{cases} e_t(i) = [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} \\ \bar{G}_t = \Xi e_t^p(i) \end{cases} \end{aligned}$$

The first order conditions associated with the previous maximisation program leads to the following polluting energy demand:

$$e_t^p(i) = \left( \frac{p_t^p + P_{G,t} \Xi}{\omega p_t^e} \right)^{\frac{1}{\varphi-1}} e_t(i)$$

### 4.3.3 Public policies and environmental taxation

#### 4.3.3.1 Monetary policy

The central bank conducts the monetary policy accordingly to the following Taylor rule:

$$1 + R_t = ((1 + r)(1 + \pi)^{1 - \mu_\pi})^{1 - \mu_R} (1 + R_{t-1})^{\mu_R} (1 + \pi_t)^{(1 - \mu_R)\mu_\pi} (1 + \tilde{y}_t)^{(1 - \mu_R)\mu_y} \quad (4.3.10)$$

where  $R_t$  is nominal interest rate,  $\pi$  is the central bank's target inflation rate, and  $r$  the equilibrium real return and  $\tilde{y}_t$  is the output gap. The output gap measure the deviation of the current output to the potential or natural output:

$$\tilde{y}_t = \frac{y_t}{\bar{y}_t} - 1 \quad (4.3.11)$$

The natural output  $\bar{y}_t$  corresponds to the case where prices are flexible. The model with flexible prices is developed in the appendix. Nominal money grows at the rate  $g_M$ :

$$g_M = \frac{m_t}{m_{t-1}}(1 + \pi_t) - 1 \quad (4.3.12)$$

#### 4.3.3.2 Environmental and fiscal policies

In the absence of environmental policy, negative externalities of emissions are not taken into account by the firms. GHG emissions  $G_t$  are supposed to be proportional to the quantity of polluting input consumed in the production process:

$$G_t = \Xi \int_0^1 e_t^p(i) di$$

where  $\Xi$  is a parameter that convert quantity of fossil fuel consumed into GHG emissions.

The government exogenously chooses the level of taxation  $\tau_t$  to limit climate change. We assume that the atmospheric stock of GHG must not exceed a threshold, or equivalently that the economy must reduce its emissions flow in order to be sustainable. The environmental income tax is paid back to the household in the form of a lump sum subsidy. Therefore, the government budget constraint is:

$$g_t = tax_t + \tau_t p_t^p \Xi \int_0^1 e_t^p(i) di + \left( \exp(\varepsilon_t^b) (1 + r_t) \right)^{-1} b_{t+1} - b_t + m_t - (1 + \pi_t)^{-1} m_{t-1} \quad (4.3.13)$$

In the case of an emission cap, the government budget constraint is:

$$g_t = tax_t + P_{G,t} \Xi \int_0^1 e_t^p(i) di + \left( \exp \left( \varepsilon_t^b \right) (1 + r_t) \right)^{-1} b_{t+1} - b_t + m_t - (1 + \pi_t)^{-1} m_{t-1}$$

#### 4.3.4 Aggregation and market-clearing conditions

We assume that all the market clear:

$$n_t = \int_0^1 n_t(i) di$$

Labor supply is equal to labor demand:

$$n_t^s = n_t^p + n_t^{np} + n_t$$

Market clearing condition for capital:

$$k_t = \int_0^1 k_t(i) di$$

Market clearing condition for energy:

$$e_t = \int_0^1 e_t(i) di$$

Consolidating the household budget constraint with the government budget constraint and using the market clearing conditions for labor capital and energy delivers the overall resource constraint of the economy:

$$c_t + i_t + a(u_t)k_{t-1} + h(c_t, m_t) + g_t = PD_t y_t \quad (4.3.14)$$

Where  $PD_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di$ , is the price dispersion factor has defined by Schmitt-Grohé and Uribe (2005)

## 4.4 Calibration:

In this section we discuss the calibration of the different parameters of the model. Time period is the quarter. For the utility function, the relative risk aversion parameter is  $\sigma = 1.5$  which is a standard value in calibrated or estimated DSGE models (see Smets

and Wouters (2003, 2007) and Casares et al., (2019)). Empirical evidence (Altonji (1986), Pencavel (1986), Domeij and Floden (2006)) show that Frisch's labor supply elasticity must be low. We calibrate  $\gamma = 4$ , so that the labor elasticity with respect to wage turn out to be 0.25. We follow Smets and Wouters (2007), who estimated the habit formation parameter to 0.71 very close to the prior 0.7 using a Bayesian DSGE model. We consequently set  $\chi = 0.7$ . The inter-temporal discount factor for the household is set, as in Smets and Wouters (2007), to  $\beta = 0.9975$  so that the risk-free real interest rate is 2% in the steady state. The parameters of the transaction costs function are set as follow: the transaction cost parameter  $\phi_0 = 0.007$  implying a total transaction costs of about 1% of GDP in steady state, the scale parameter is  $\phi_1 = 0.05$  in order to reach a ratio of real money over consumption close to 1 (as considered a realistic average value for OECD countries), and the elasticity parameter is calibrated to have an nominal elasticity of nominal interest rest in money demand function as in the Baumol-Tobin model  $\phi_2 = 0.5$ . Finally we calibrate the parameters of the damage function  $d_0 = 0$ ,  $d_1 = 0.0278$  and  $d_2 = 0.3898$ , with a double criteria: i) the damage is null with zero emissions ( $d(G) = 0$  if  $G = 0$ ), the augmenting effect of pollution on consumption disutility is 5% in steady state, i.e  $1 - d(G) = 0.95$ .

According to Smets and Wouters (2003, 2007), they obtain a value of 6.77 for their adjustment cost parameter for investment in their Bayesian estimated model for the Euro area, while they obtain a value of 5.44 for their estimated Bayesian model on US data. The posteriors value are also close to the prior 4. Therefore, we use the average value for  $\varphi_k = 6.12$ . The depreciation rate of capital is supposed to be  $\delta = 0.025$ , corresponding to a 10% annual depreciation rate, which is a standard calibration in both RBC and NK literature. The value for the price mark-up parameter is  $\theta = 6$ , as in Casares (2010), Gali and Monacelli (2005) and Erceg et al. (2000).

The parameters of the Cobb-Douglas production function in the consumption good sector are calibrated as follow. Argentiero et al. (2017) estimate the energy share parameter at 0.06 in a Bayesian DSGE model for the EU-15 and 0.12 for the US. We use the mean value  $\nu = 0.09$  as a representative value for the OECD countries. We choose



$\alpha = 0.3$ , a standard value also assumed by Golosov et al. (2014), so that capital income share is 25%, labor income share is 55% and dividends income share is 20% in the steady state. Golosov et al. (2014) used Stern (2012) meta-study to calibrate the elasticity of substitution between energy inputs. In their article they used three different inputs (coal, oil and electricity) while we only distinguish polluting and non polluting energies. They calibrated the elasticity of substitution as an average between coal-oil, coal-electricity and oil-electricity, and finally set  $\varphi = -0.058^4$ . Using the same methodology as in Golosov et al. (2014), we find that our benchmark elasticity of substitution between clean and dirty energies is  $\varphi = 0.008$  assuming Stern (2012) estimations for elasticities of coal-electricity (0.866), oil-electricity (1.06) and gas-electricity (1.099). The share parameter for the CES energy function is set to  $\omega = 0.815$ , in order to match the average ratio of non-polluting energy over polluting energy in steady state suggested by Acemoglu and Rafey (2018)  $e^{np}/e^p = 0.23$ . The parameters of the two energy production functions are set  $\zeta^p = \zeta^{np} = 0.35$  so that employment share in the energy sector over total employment (4 to 5%) is consistent with US data (US Energy and Employment Report).

We use data from the BP statistical database (Dudley et al., 2018) to calculate energy demand. We aggregate energy inputs as follow. Between 2008 and 2018, for the OECD countries, the average annual quantity of oil consumed was 2111 million tons (million tons of oil equivalent or Mtoe henceforth), 1363 Mtoe of natural gas and 1004 Mtoe of coal. Corresponding to an average aggregate demand for fossil fuel of about  $e^p = 4478$  Mtoe. For the same period considered, the demand for non-polluting energy was in average  $e^{np} = 984$  Mtoe per year, which can be broken down as follows: 469 Mtoe of non-polluting energy consumed was of nuclear origin, 312 Mtoe from hydroelectric power plants, 32 Mtoe from solar energy, 102 Mtoe from wind and 70 Mtoe from biomass energy. According to the same database,  $CO_2$  emissions in OECD countries for the period 2008-2018 are on average  $12636 \text{ Mt } CO_2$  per year. The intensity of  $CO_2$  emissions per unit of fossil energy is therefore  $2.82 \text{ t } CO_2/toe$ . One ton of  $CO_2$  contains 0.27 ton of carbon.

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<sup>4</sup>If we define  $\epsilon$  as the elasticity of substitution between the polluting and non-polluting energy inputs, then  $\frac{\epsilon-1}{\epsilon} = \varphi$  and  $\epsilon = (1 - \varphi)^{-1}$ .

Thus the conversion factor in terms of carbon is  $\Xi = 0.76 \text{ tC/toe}$ . The demand for non-polluting energy thus represents 18% of the total energy demand, and the ratio of clean energy demand over polluting energy is  $\frac{e^p}{e^{np}} = 0.22$ .

The aggregate price of each energy input is calculated as a weighted average of the prices of each energy source:

$$p^p = \frac{\sum_i p^i e^i}{\sum_i e^i} \quad i = \{oil, gas, coal\}$$

According to the Crude oil Price time series for Europe from the U.S. Energy Information Administration database (US EIA), in average between 2008 and 2018 the price of oil per barrel was \$ 83 (2015 dollars constant). A barrel of oil corresponds to 0.1364 metric ton. Thus, the price of oil is \$ 610 per ton. We find an average price of \$ 99 per ton of coal, and an average price of \$ 9 per million of British Thermal Unit (Btu). One million of Btu is equivalent to 0.172 barrel of oil equivalent. So the average price of gas is \$ 54 per barrel of oil equivalent, and we finally find that the price of gas is  $p^p = 398\$/toe$ . The weighted average price of the polluting energy input is therefore \$326 per ton of oil equivalent. The carbon price on the EU Emissions Trading Scheme is used to calculate the carbon tax rate. Between 2019 and 2020, the average allowance price was  $\text{€}25/tCO_2$ , corresponding to  $\text{€}92/tC$  or  $\text{\$} 102/tC$  using the euro-dollar exchange rate for the same year (EUR 1  $\longleftrightarrow$  USD 1.11 according to the ECB euro reference exchange rate). Finally, by using the aggregate price  $p^p$ , the conversion factor  $\Xi$ , and the carbon price, we obtain a carbon tax rate of  $\tau = 0.24$ .

In their model, Smets and Wouters (2003) calibrate government expenditures to GDP ratio as 0.18. However, this value is well below the OECD data. Indeed the average is 0.41 for years 2007, 2009, 2014 and 2015. We thus calibrate  $\Phi_{gy} = 0.41$ . We assume, as it is standard in the literature that the Calvo probability is  $\eta = 0.75$ , so that the firm is able to set the optimal price once a year. The indexation share parameter of inflation is  $\kappa = 0.35$ . The value is calculated as the average value between Smets and Wouters (2003) Bayesian estimation for the Euro area (0.47) and their estimation for the US (0.24). We assume that the elasticity of the marginal cost of adjusting capital utilization rate with

respect to the capital utilisation rate is  $\sigma_a = 1.17$ , which is consistent with Smets and Wouters (2003, 2007) estimations.

We choose the same parameters for the monetary policy rule as in Taylor (1993), thus the coefficient for responses of the interest rate to inflation deviations is  $\mu_\pi = 1.5$ , the response of the interest to output gap  $\mu_y = 0.125$  (we divide the initial value by 4, because we consider quarter time period), and the interest rate smoothing  $\mu_R = 0.85$ .

Next, we calibrate the exogenous processes of the model. We assume that the autoregressive parameter of the Solow residual is  $\rho_a = 0.9$ , and its standard deviation is  $e_a = 0.007$ , a standard value in the literature. Autocorrelation parameters of the stochastic processes in both energies sectors are also supposed to be equal, we set  $\rho_{a^p} = \rho_{a^{np}} = 0.9$ , and their white noise have for standard deviation  $e_{a^p} = e^{a^{np}} = 0.014$ . Stochastic parameters of the three demand shocks - risk premium shock, investment cost shock, and government spending shock - are calibrated so that they explain around 50% of total output growth variance, a realistic value of business fluctuations. Consequently, we take  $\rho_b = \rho_i = \rho_g = 0.8$  and the standard deviations of white noises are respectively  $e_b = 0.0039$ ,  $e_i = 0.0024$ , and  $e_g = 0.0045$ . The price-push shock parameters are  $\rho_P = 0.6$  and  $e_P = 0.01$ , so that this shock explain 40% of inflation variance.

## 4.5 Simulations:

The following section uses the model to analyse the economy's response to various exogenous shocks considered under two environmental policy regimes. We analyse shocks under a carbon tax regime and under a cap-and-trade market (or emission cap). The model is computed using a perturbation approach after a first order approximation (or Taylor's expansion) around the steady state with Dynare software (Adjemian et al., 2011). The log-linearised model is developed in the technical appendix. We have considered seven exogenous shocks, among which five are standard in the NK DSGE literature (total factor productivity shock, investment cost shock, risk premium shock, inflation shock and government spending shock) plus two productivities shocks in each energy sectors. We

focus our analysis on three main shocks: total factor productivity shock and the two energies shocks. Results for the remaining shocks are available in the appendix.

#### 4.5.1 Simulations for various shocks:

##### 4.5.1.1 Dynamics under a technology shock:

First, the effects of an exogenous shock of total factor productivity  $a_t$  is considered. Variables responses to this shock are displayed in Figures 4.1 to 4.3. Following a positive productivity shock, production increases instantaneously, but to a lesser extent than in the model without rigidity. This results from firms ability to influence market price in presence of nominal rigidities. Thus, instead of seeing the price fall immediately, the price remains too high (sub-optimal), so that consumption increases, but relatively less than in the case of the flexible-price market. Thanks to this market power, and as a result of productivity shock, real marginal cost of the firm falls. It is able to be as productive using less production factors. As a result, the shock leads to a fall in labor demand, a fall in demand for capital, a fall in utilisation rate of capital and in energy demand. These in turn translate into lower real wages, lower returns on capital, and lower aggregated energy price.

In the absence of nominal rigidities, in order to support the increase in production, capital utilization rate (although it decreases relative to its stationary state, because it is more productive) decreases relatively less than in the stickiness price model. Increase in output also leads to an increase in demand for production factors, and consequently to an increase in real wage, return on physical capital and energy prices. However, rise in real wages is relatively greater than increases in return on capital and the price of energy. So there is a substitution effect between capital and labor and energy and labor. The demand for labor decreases, especially as a substitution effect takes place between working time and leisure time.

Inflation falls as total factor productivity rises, resulting in a lower marginal cost of production than at its steady-state equilibrium. Because of the output gap, and the fall in inflation, the central bank reacts by reducing its nominal interest rate. As a

result, the demand for money increases, which lowers transactions costs and encourages consumption of the final good. Moreover, a relatively low nominal interest rate reduces the incentive for households to save and, on the contrary, encourages investment so that the capital stock increases.

To meet rising demand, production increases, requiring more factors of production (capital, labor and energy). Rise in demand for inputs (which is accompanied by an increase in the rate of capital utilization) positively influences their respective remuneration, which increases marginal cost of production. Price rises, output gap narrows, for these reasons central bank implements a restrictive monetary policy. Rise in interest rates encourages savings and pushes households to postpone consumption. Investment becomes relatively more expensive, but capital stock continues to increase as long as depreciation of capital is lower than its accumulation.

The introduction of an instrument such as a carbon tax or an emission cap has no impact on the direction of variation and the amplitude of impulse responses functions. Only the variables of the energy block are affected. This results from an elasticity of substitution greater than unity. Productivity shock has a relatively greater effect when carbon price is constant (carbon tax), whereas in the case of a cap-and-trade market, if the quantity of allowances exogenously supplied by the authorities does not decrease, then the carbon price will tend to decrease sharply. As the cost of pollution decreases, the amount of fossil energy used remains constant. This result is also certainly influenced by the nature of the energy production function. Indeed, by using labor alone, this sector is able to adapt very quickly to shocks. Thus, following a productivity shock, in the presence of a carbon tax, the demand for labor in the polluting energy sector decreases, before increasing again very quickly as soon as the shock dissipates. It can thus be assumed that if capital is included as a factor of production, these results may be different.

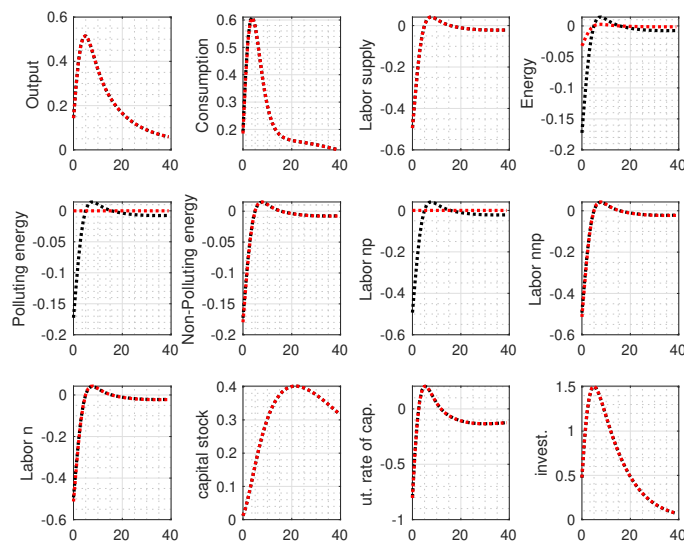


Figure 4.1 – Impulse responses functions (IRF) to a total factor productivity (TFP) shock (1/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

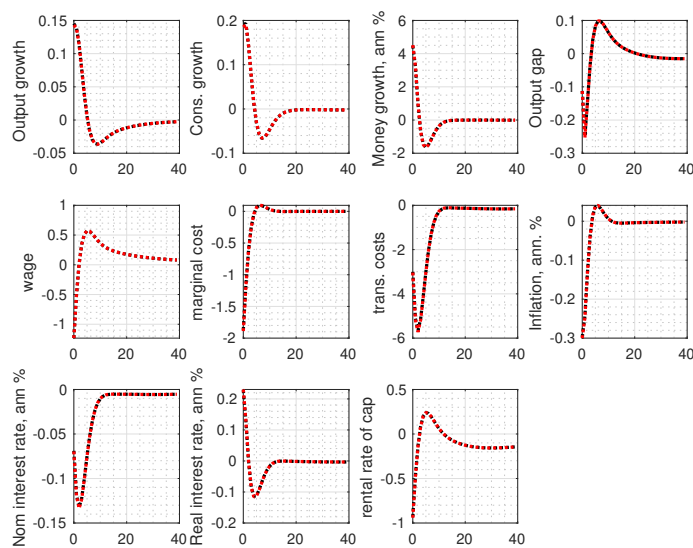


Figure 4.2 – IRF to a TFP shock (2/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

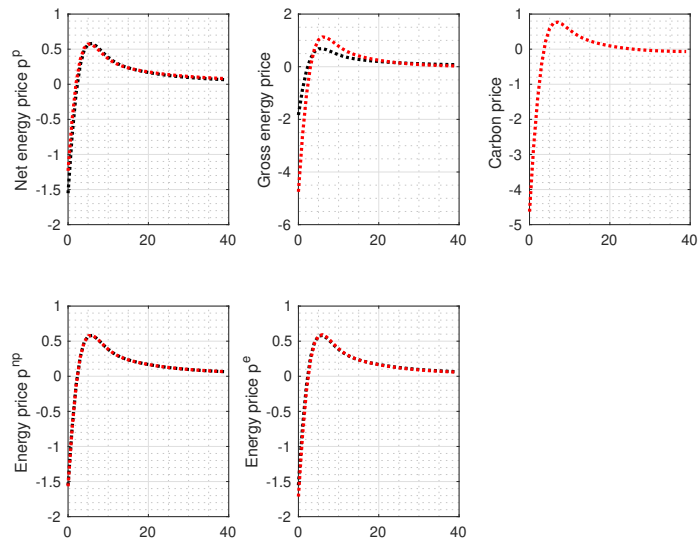


Figure 4.3 – IRF to a TFP shock (3/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

#### 4.5.1.2 Technology shock in the polluting sector:

In the case of a positive productivity shock in the polluting energy sector, the consequences will differ depending on the environmental policy instrument implemented. Figures 4.4 to 4.6 illustrate this case. With a carbon tax, the price to be paid for each ton of pollution emitted remains constant. Therefore, all other things being equal, the increase in productivity results in a decrease in gross price for fossil fuels. Relative price of energies changes in favour of polluting energies. Demand for polluting energy increases, and because of possible substitution between the two energy sources, demand for non-polluting energy tends to decrease at the time of the shock. Aggregate price of energy falls sharply, which reduces demand for other factors of production, capital and labor. Energy intensity of final good increases. As production costs fall, real marginal cost also falls, but since firms have market power, the price they set is above the optimal price relatively to the one in absence of nominal rigidities. This results in a negative output gap. Fall in prices and this output gap encourages the central bank to react by lowering its nominal interest rate. As the cost of investment is lower, investment rises,

which increases capital stock. With a lower nominal interest rate, demand for money increases, which reduces transaction costs. However, despite lower prices, lower transaction costs and lower interest rates, consumption diminishes under its steady state level. This is the result of increased polluting energy use. Indeed, for a carbon tax that remains constant, rise in productivity in this sector, increases the production of fossil energies, and at the same time generates an increase in emissions. This increase in emissions is explained because energy intensity is higher, consequently production of consumable good uses relatively more fossil fuels, as a result GHG emissions rise. However, emissions are responsible for externalities that affect negatively household's well-being and, more precisely, its consumption level. After the shock, polluting energy price rises again, demand for this factor decreases in favour of non-polluting energy. In order to produce, firms use relatively more capital and labor. As a result, GHG emissions fall. However, since transaction costs are also low, consumption rise again. Inflation remains below its steady state, the central bank continues its expansionary monetary policy, although it is slowly starting to raise its nominal interest rate. Then, in a second stage, the output gap turns positive, output is higher than in pure competitive situation. Rise in consumption and in real marginal cost of production generates inflation. The central bank restores its interest rate to equilibrium. The household substitutes consumption with savings and investment level also falls.

In the case of an allowance market, increase in productivity in polluting energy sector does not change the quantity of polluting energy used. As the supply of allowances remains constant, adjustment is made through prices. Increase in demand for allowances pushes carbon price upwards. However, this increase in price of carbon makes it possible to limit the use of fossil fuel resources. Moreover, rise in productivity reduces demand for labor in polluting energy sector, which tends to reduce wages. As a result of this fall in wages, price of non-polluting energies decreases and demand for this factor of production increases. This is the result of an elasticity of substitution greater than unity between energies inputs. Aggregate price of energy decreases, but relatively less than in the case of the carbon tax. Real marginal cost of production also falls, but to a



lesser extent compared to the tax. Due to the presence of nominal rigidities, output gap widens and central bank reacts here too to reduce these distortions. However, as the economy emits relatively less GHG (constant use of fossil fuels), consumption increases immediately, especially as transaction costs have decreased due to the increase in money supply. However, since real marginal cost of production falls relatively less than in the case of the carbon tax, increase in production is also relatively limited. When the productivity shock begins to unwind, demand for pollution rights falls, which lowers allowances price. The amount of polluting energy used remains constant, as does the flow of emissions. On the other hand, the use of non-polluting energy decreases, firms substitute aggregate energy by a more intensive use of capital and labor, the mechanism for returning to a stationary equilibrium is the same as in the case of the tax.

In the case of a shock in the productivity of polluting energies, introducing a tax has a greater effect on firms' margins because of a relatively greater fall in real marginal cost of production. In this case, production increases relatively more. Indeed, in the case of an allowance market, productivity gains are limited due to the rise in the price of carbon, but the absence of negative externalities linked to the constant use of fossil fuels and therefore emissions makes it possible to sustain demand.

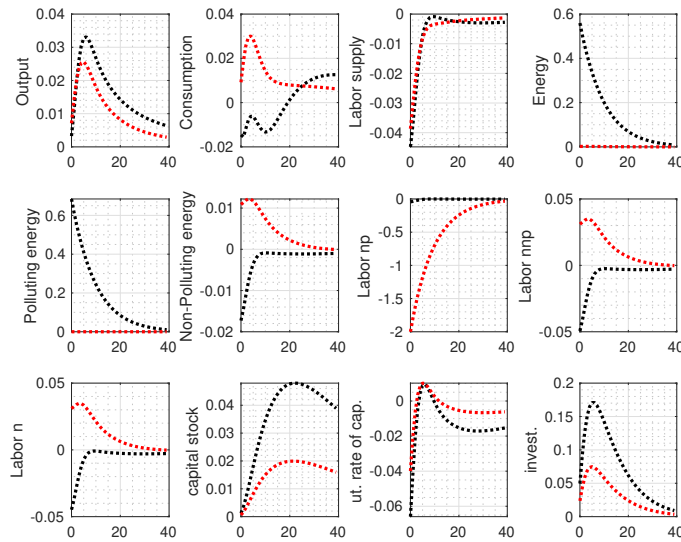


Figure 4.4 – IRF to a productivity shock in the polluting sector (1/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

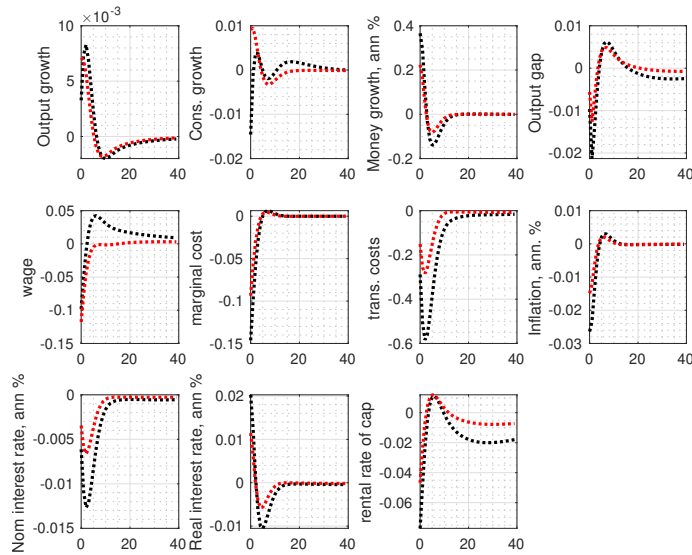


Figure 4.5 – IRF to a productivity shock in the polluting sector (2/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

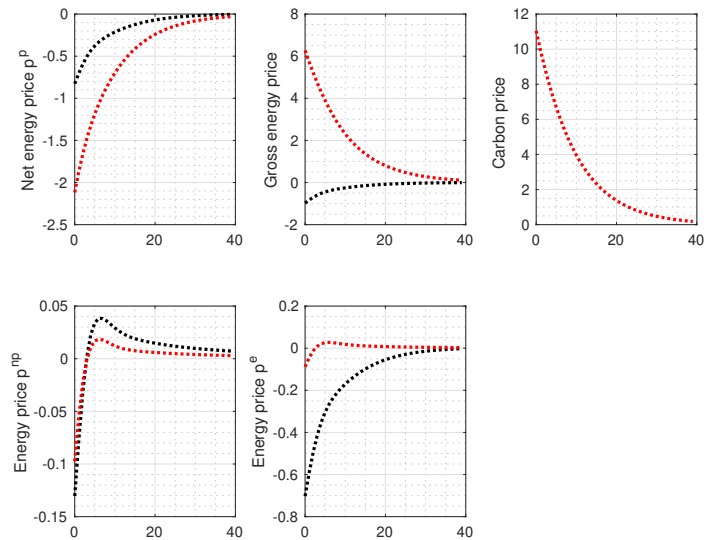


Figure 4.6 – IRF to a productivity shock in the polluting sector (3/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

#### 4.5.1.3 Technology shock in the non-polluting sector:

The use of a price, or quantity instrument has little, or no impact on the economy in the case of a technology shock in the non-polluting energy sector (see figures 4.7 to 4.9). Rising productivity immediately increases the use of non-polluting energy. In the presence of a carbon tax, demand for polluting energy decreases, due to changes in relative prices. In the case of an emission cap, if the supply of allowances remains constant, demand for pollution decreases, carbon price falls and demand for polluting energy remains constant. Aggregate energy demand increases at the expense of other factors of production. This increase in productivity is reflected in a fall in real marginal cost of production, the output gap widens and a fall in inflation is observed. The central bank then implements its expansionary monetary policy, which supports demand. This favours economic activity, inflation rises again and output gap becomes positive. The return to a stationary equilibrium is achieved by a further substitution of energy by a more intensive use of capital and labor. The central bank raises its nominal interest rate again, which favours savings, and households reduce their consumption.

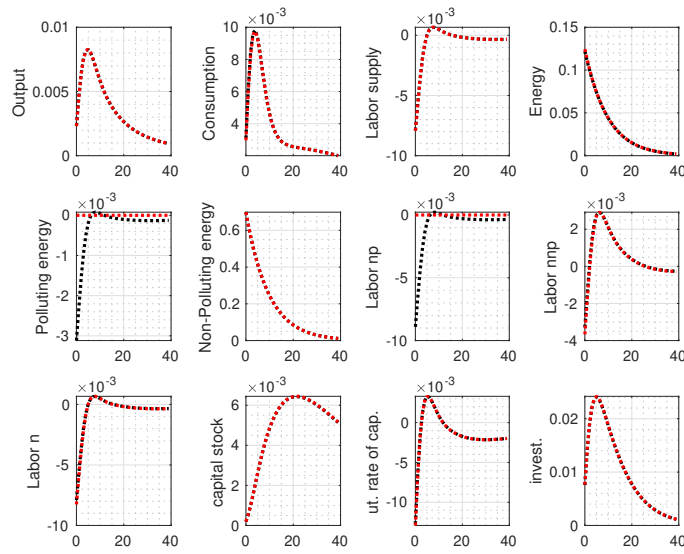


Figure 4.7 – IRF to a productivity shock in the non-polluting sector (1/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

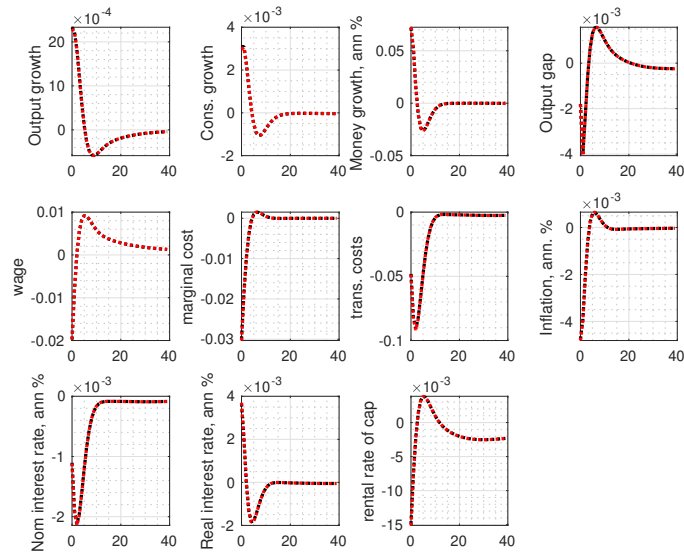


Figure 4.8 – IRF to a productivity shock in the non-polluting sector (2/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

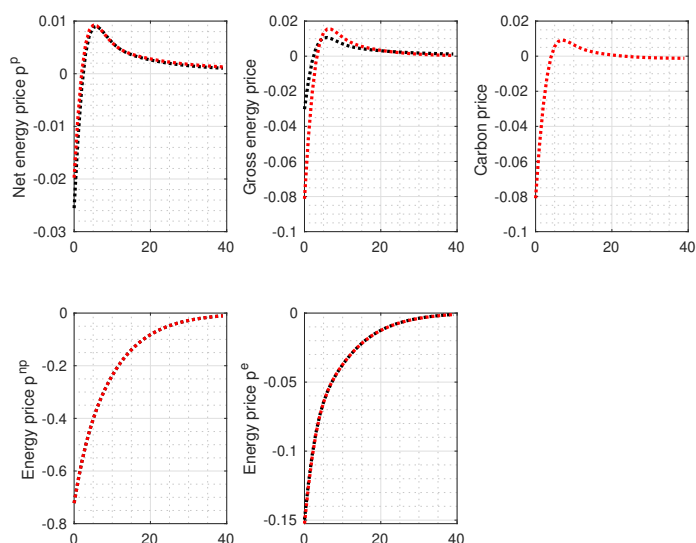


Figure 4.9 – IRF to a productivity shock in the non-polluting sector (3/3) - red dotted line: cap-and-trade market; black dotted line: carbon tax

#### 4.5.2 Robustness:

This section compares the results obtained previously using other values for the elasticity of substitution between the two energies. Indeed this parameter plays a determining role, and the results will differ if we consider that polluting and non-polluting energies are complementary inputs, close or strong substitute. However, according to empirical estimates, there is no consensus on the value of this elasticity. We therefore compare the results based on the elasticity estimated by Stern (2012), which will serve as a benchmark, with the results obtained using other values of  $\epsilon$ . Based on Acemoglu et al. (2012) model, Pelli (2012) estimates the elasticity of substitution for several regions and countries, and finds that polluting and non polluting energy inputs are complements. In average, according to its estimation the elasticity of substitution is equal to  $\epsilon = 0.51$ . Estimated over 10 years between 1996 and 2006, the elasticity of substitution for the EU is equal to  $\epsilon = 0.453$ . He aggregates oil products, natural gas and coal into the polluting input and hydroelectric, biomass, geothermal, solar energy wind and nuclear into non-polluting energies. Assuming an elasticity of  $\epsilon = 0.51$  implies that  $\varphi = -0.96$ . Argentiero et al.

(2017), estimated the elasticity of substitution between renewable and fossil energies at  $\epsilon = 1.22$  for the EU-15 using Bayesian methods, which gives us  $\varphi = 0.18$ . Papageorgiou et al. (2017) (PSS hereafter) used the World Input Output database to estimate the elasticity of substitution between clean and dirty inputs. They construct a panel data of OECD countries and conclude (like Argentiero et al. (2017)) that energy inputs are not complementary but substitute. At the aggregate level, they find that elasticity of substitution is  $\epsilon = 1.85$ , and consequently  $\varphi = 0.46$ . The elasticity of substitution between energy inputs is a key parameter of the model, we will use different value summarized in table 4.1.

Table 4.1 – Elasticity of substitution between polluting and non-polluting energies

Ref.	elasticity $\epsilon$	$\varphi = \frac{\epsilon-1}{\epsilon}$	$\omega$
Pelli (2012)	0.51	-0.96	0.951
Argentiero et al. (2017)	1.22	0.18	0.78
PSS (2012)	1.85	0.46	0.70

#### 4.5.2.1 Elasticity of substitution in the case of a carbon tax:

In the case of a TFP shock, the elasticity of substitution does not influence the response of the variables in the model. The shock hits the whole economy symmetrically. Increase in productivity leads to a decrease in energy demand because the firm is able to produce the same quantity of goods but with fewer production factors. The decrease in demand for polluting and non-polluting energy occurs in the same proportions.

In the case of a productivity shock in the polluting energies sector (see figures 4.10 to 4.12), it will have different consequences, depending on the elasticity of substitution between energies factors. However, despite these different reactions, the rest of the economy is not affected if energies are substitute. Let us first assume that energies are substitutable, a case similar to that described in the previous section. The shock will

have the same positive effect on the amount of fossil fuels used. Rise in GHG emissions will be the same, no matter the elasticity of substitution. On the other hand, the higher the elasticity of substitution, the lower the demand for non-polluting energy will be. The higher the elasticity of substitution the lower labor demand in non-polluting energy is. This division of labor in the two energy sectors does not modify the aggregate price of energy, regardless of the elasticity of substitution between factors. For these two reasons, despite differences in the elasticity of substitution, we do not observe significant differences in the response of other variables. This is due in particular to the parameter  $\nu$ , which measures the share of energy in production. A relatively weak parameter when compared to the shares of capital and labor in production. Let us now assume that the two energies are complementary. Rising productivity in the polluting energy sector leads to a joint increase in the demand for polluting and non-polluting energy. Unlike the previous case, it cannot use more fossil fuels without increasing the use of clean energy. The firm then prefers to pass on the increase in productivity by using relatively less labor in this sector and just as much non-polluting energy. As a result, polluting energies are used relatively less compared to the situation where energies inputs are substitute, and the economy emits less GHG. By containing its emissions, the negative externalities are relatively lower, which explains why consumption increases, contrary to the case described above, where energy was assumed to be substitutable.

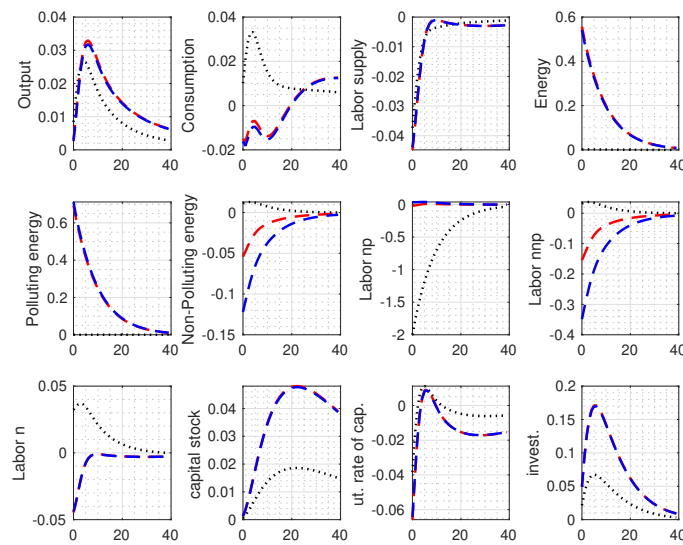


Figure 4.10 – IRF to a productivity shock in the polluting sector in a carbon tax regime  
 (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

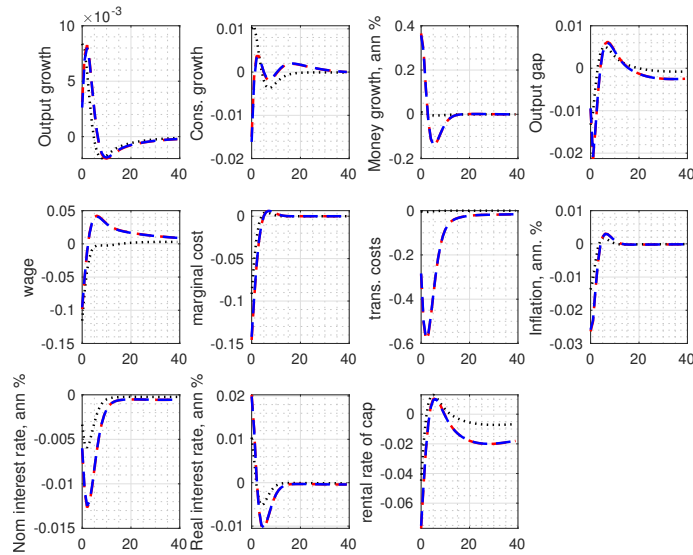


Figure 4.11 – IRF to a productivity shock in the polluting sector in a carbon tax regime  
 (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$



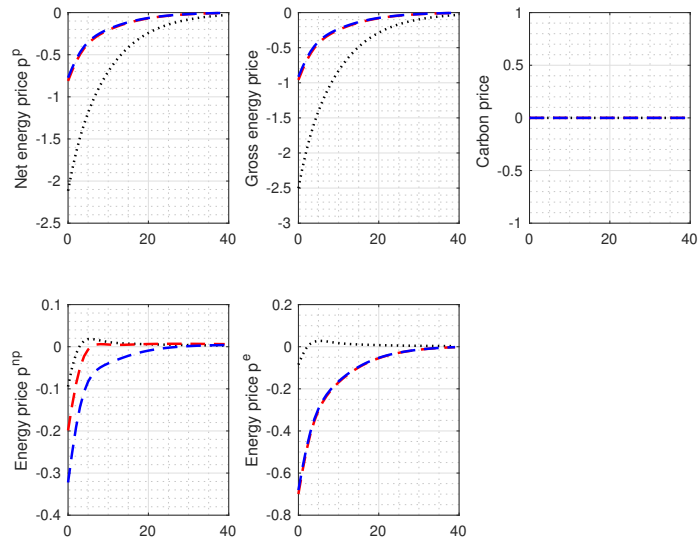


Figure 4.12 – IRF to a productivity shock in the polluting sector in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

Assume now a productivity shock in the non-polluting energy sector (figures 4.13 to 4.15). The elasticity of substitution between energy factors will more or less amplify the responses of variables in the energy market. When energies are substitutable, the higher the elasticity, the less the economy will use fossil resources, but the difference remains tiny. Consequently, volatility of emission is relatively low, and do not affect consumption level due to externalities. The transmission mechanism is the same as that described in the previous section. Finally, let us consider that the two sources of energy are complementary. The rise in productivity in the clean energy sector then leads to an increase in the demand for polluting energy. In the case of complementary energy factors, the firm prefers to pass on increase in productivity through a less intense use of labor input in the production of clean energy. Emissions remain constant because the use of fossil fuel remain the same as before the shock.

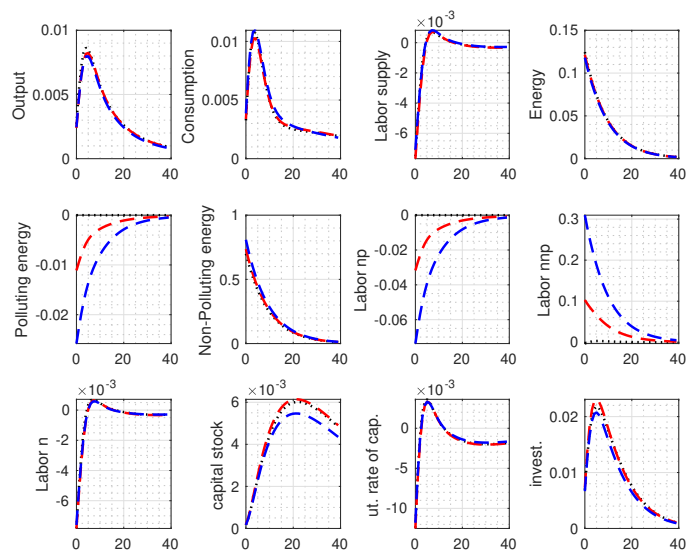


Figure 4.13 – IRF to a productivity shock in the non-polluting sector in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

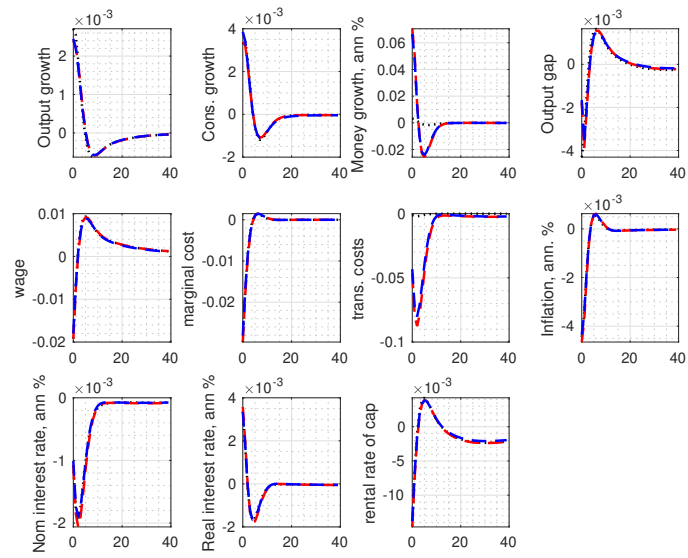


Figure 4.14 – IRF to a productivity shock in the non-polluting sector in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

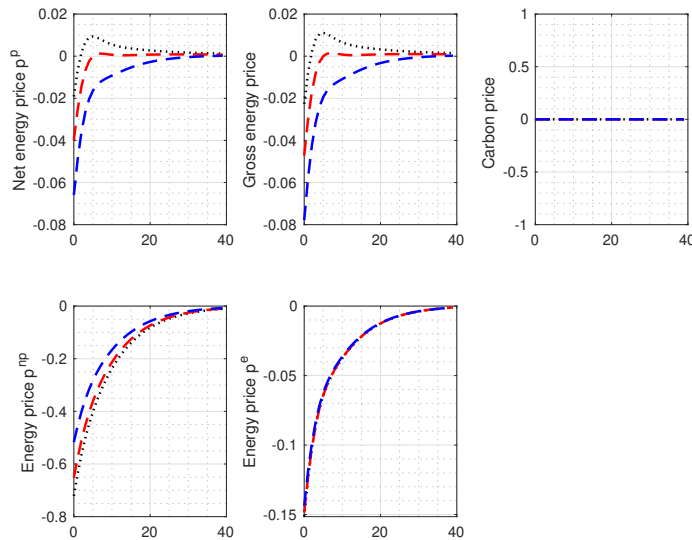


Figure 4.15 – IRF to a productivity shock in the non-polluting sector in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

#### 4.5.2.2 Elasticity of substitution in the case of a cap-and-trade market:

The case of a cap emission regime is examined in this subsection. As with the carbon tax, a TFP shock does not modify variables response for the different values considered of the elasticity of substitution. The same reasoning holds. The shock acts symmetrically, so energy demands change in the same proportions. The mechanism of return to equilibrium is therefore the same as the one described for the initial calibration.

Consider a positive productivity shock in polluting energy sector (figures 4.16 to 4.18). This increase in productivity is reflected in a drop in the price of polluting energy, consequently demand for this input is supposed to rise. However, in every period, the constraint for emissions must hold, allowances supply do not change for this reason price of carbon rises sharply so that demand for pollution remains constant. This drop in polluting energy market price does not compensate for the rise in the price for pollution. The relative price of energy changes in favour of non-polluting energy, it becomes profitable to use more clean energy in the energy mix. However, the variations are relatively

small despite differences in elasticity of substitution. Consequently, in the case of a cap emission regime, the elasticity of substitution does not play a decisive role in the event of a productivity shock in the polluting energy sector

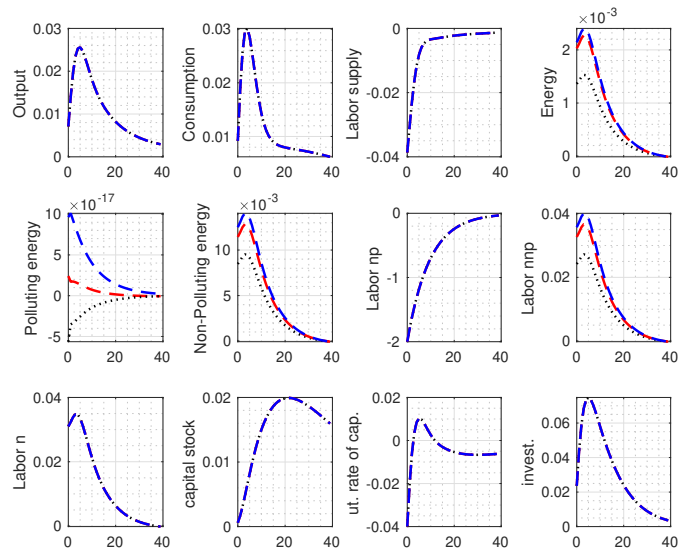


Figure 4.16 – IRF to a productivity shock in the polluting sector in an emission cap regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

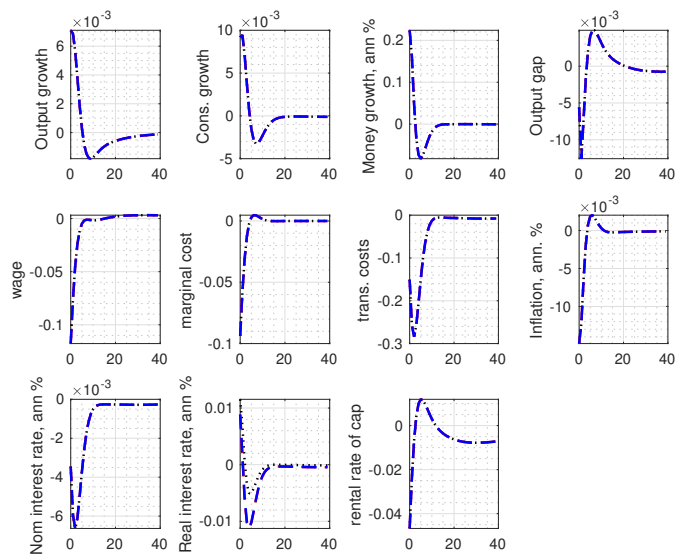


Figure 4.17 – IRF to a productivity shock in the polluting sector in an emission cap regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

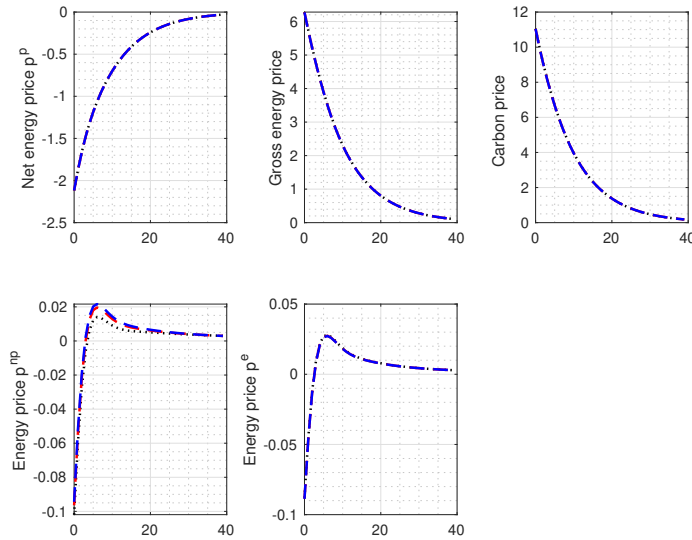


Figure 4.18 – IRF to a productivity shock in the polluting sector in an emission cap regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

The same applies in the case of a productivity shock in non-polluting energy sector (figures 4.19 to 4.21). However, it can be observed that the energy market evolves differently according to the elasticities. When the two sources of energy are complementary, an increase in productivity leads to an increase in demand for polluting energies. The economy is constrained because emissions cannot exceed the cap decided by public authorities, carbon price rise and demand for polluting input remains constant over time. If, on the contrary, energies are substitutable, increase in productivity leads to an increase in the use of non-polluting energies. The more substitutable energies are, the greater the demand for clean energy will be in the event of a productivity shock in this sector. Fossil fuels demand diminishes and leads to a decrease in allowances demand, and therefore a drop in the price of carbon. Since pollution is relatively cheaper, the amount of polluting energy used remains more or less constant, so the level of emissions does not change. However, in spite of these variations, we can see that the aggregate price remains in the same orders of magnitude, whether the energies are complementary or substitutable.

Consequently, because on the one hand emissions remain constant, and because aggregate energy price are identical despite differences in elasticity of substitution, the responses of the other variables to this shock remain the same.

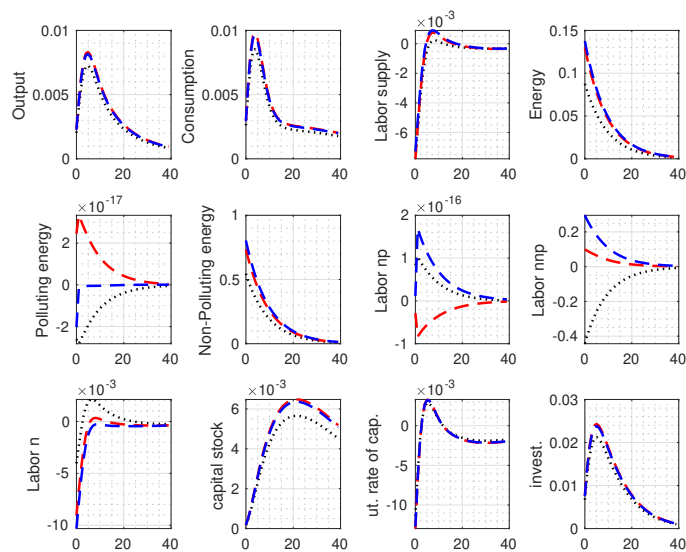


Figure 4.19 – IRF to a productivity shock in the non-polluting sector in an emission cap regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$



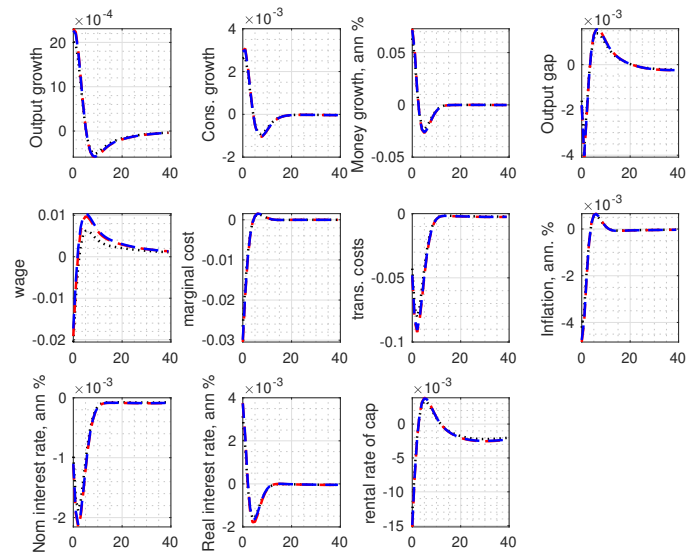


Figure 4.20 – IRF to a productivity shock in the non-polluting sector in an emission cap regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

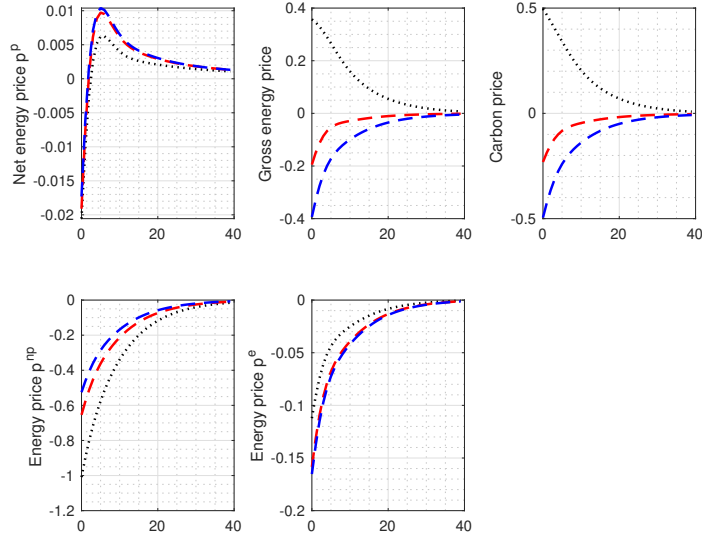


Figure 4.21 – IRF to a productivity shock in the non-polluting sector in an emission cap regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

## 4.6 Monetary policy analysis

In this section, we assume that the central bank behaves as a social planner that seeks to maximise household's utility subject to the set of equation that characterizes the competitive equilibrium of the economy. As described in the technical appendix, the quadratic approximation to the instantaneous household utility function is

$$\begin{aligned}
 U(\bar{c}_t, G_t) - V(n_t^s) \cong & \frac{(1 - d(G))}{1 - \sigma} \bar{c}^{1-\sigma} - \frac{\psi}{1 + \gamma} - \frac{(1 - d(G)) (\sigma - 1) \bar{c}^{1-\sigma}}{2(1 - \chi)^2} (\hat{c}_t - \chi \hat{c}_{t-1})^2 \\
 & + (1 - d(G)) \bar{c}^{1-\sigma} \frac{(\hat{c}_t - \chi \hat{c}_{t-1})}{1 - \chi} - \frac{\bar{c}^{1-\sigma}}{1 - \sigma} G \left( d_1 \left( \hat{G}_t + \frac{\hat{G}_t^2}{2} \right) + 2d_2 G \hat{G}_t \right) \\
 & - \frac{(d_1 + 2d_2 G)}{1 - \chi} \bar{c}^{1-\sigma} (\hat{c}_t - \chi \hat{c}_{t-1}) G \hat{G}_t - \psi \hat{n}_t^s - \frac{\psi(1 + \gamma)}{2} (\hat{n}_t^s)^2
 \end{aligned}$$

The unconditional expectation of the second-order approximation to  $U(U(\bar{c}_t, G_t) - V(n_t^s))$  is

$$\begin{aligned} \mathbb{E}[U(\bar{c}_t, G_t) - V(n_t^s)] &= \frac{(1 - d(G))}{1 - \sigma} \bar{c}^{1-\sigma} - \frac{\psi}{1 + \gamma} - \frac{(1 - d(G))(\sigma - 1)}{2} \text{var}(\hat{c}) \\ &\quad - \frac{\psi(1 + \gamma)}{2} \text{var}(\hat{n}^s) - \frac{\bar{c}^{1-\sigma} G d_1}{1 - \sigma} \frac{1}{2} \text{var}(\hat{G}) - (d_1 + 2d_2 G) \bar{c}^{1-\sigma} G \text{cov}(\hat{c}, \hat{G}) \end{aligned}$$

We can define  $\mathbb{W}_1$ ,  $\mathbb{W}_2$ ,  $\mathbb{W}_3$ ,  $\mathbb{W}_4$  and  $\mathbb{W}_5$  to rewrite the last expression into a more compact form:

$$\begin{aligned} \mathbb{W}_1 &\equiv \frac{(1 - d(G))}{1 - \sigma} \bar{c}^{1-\sigma} - \frac{\psi}{1 + \gamma} \\ \mathbb{W}_2 &\equiv -\frac{(1 - d(G))(\sigma - 1)}{2} \text{var}(\hat{c}) \\ \mathbb{W}_3 &\equiv -\frac{\psi(1 + \gamma)}{2} \text{var}(\hat{n}^s) \\ \mathbb{W}_4 &\equiv -\frac{\bar{c}^{1-\sigma} G d_1}{1 - \sigma} \frac{1}{2} \text{var}(\hat{G}) \\ \mathbb{W}_5 &\equiv -(d_1 + 2d_2 G) \bar{c}^{1-\sigma} G \text{cov}(\hat{c}, \hat{G}) \end{aligned}$$

so that:

$$\mathbb{E}[U(\bar{c}_t, G_t) - V(n_t^s)] = \mathbb{W}_1 + \mathbb{W}_2 + \mathbb{W}_3 + \mathbb{W}_4 + \mathbb{W}_5$$

#### 4.6.1 Optimized instrument rules

Household's welfare loss depends on consumption, labor supply, and greenhouse gas emissions variances, but also on the covariance between gas emissions and consumption. It is the consequence of variables deviations from their steady state, due to various exogenous shocks considered in our model. The central bank strategy is to choose coefficients values  $\mu_\pi$  and  $\mu_y$ , that minimizes the household's welfare loss for fixed values of smoothing coefficients  $\mu_R = 0.85$ . The optimized coefficients depend upon specific calibrations. The larger the value of  $\mu_\pi$ , the more aggressively the central bank reacts to inflation (and vice versa). Similarly, the greater the value of  $\mu_y$ , the more the central bank is concerned with stabilizing the output gap. To find the combination that maximizes household welfare,

we proceed in two steps. First, we replicate the model for different configurations of the coefficients on the monetary policy instrument. Thus, we assume that  $\mu_\pi(i)$ , can take its values between 1.1 and 4.5, and we increase its initial value by 0.01 at each iteration. Similarly, the second coefficient  $\mu_y(j)$  is assumed to take its values in the interval  $[0, 1]$ , and is iterated in steps of 0.01. Consequently, we replicate 34,000 times the model. Then, defining by  $\mathbb{W}$  the household welfare, we are looking for the combination of  $\mu_\pi(i)^*$ , and  $\mu_y(j)^*$ , that maximizes  $\mathbb{W}$ . More formally it can be expressed as follows:

$$\operatorname{argmax}_{\mu_\pi^*, \mu_y^*} \mathbb{W} \left( \hat{G}_t, \hat{n}_t^s, \hat{c}_t, \mu_\pi, \mu_y \right)$$

#### 4.6.1.1 Optimal monetary policy in the carbon tax case:

For different calibration, including the optimal value for monetary policy instrument considered, table 4.2 shows standard deviations of fluctuations of habit-adjusted consumption, employment, gas emissions, nominal interest rate and nominal money growth as well as the contribution to household welfare. Table 4.2 reports also results for the benchmark Taylor rule, when  $\mu_\pi = 1.5$ ,  $\mu_y = 0.125$ , and  $\mu_R = 0.85$ .

The values that maximize consumer welfare are  $\mu_\pi^* = 3.44$  and  $\mu_y^* = 0.08$ , which are different from the standard values in every respect. Compared to the initial calibration for the Taylor rule, according to the optimal values obtained that maximize household's welfare, the central bank must be more aggressive against inflation and less sensitive to output gap. Figure 4.22 shows welfare as a function of the two coefficients of the Taylor rule. Welfare is an inverted U-shape function of  $\mu_y$ , and reaches its maximum in  $\mu_y^* < 0.125$ . In other words, the central bank must give less consideration to output gap when authorities introduce a carbon tax. The existence of this output gap is the consequence of a lack of competition and to the markup. Although this is a sub-optimal situation that central bank must normally avoid, when it acts to reduce the gap between optimal output and real production, the flow of emissions will vary depending on the more or less accommodative monetary policy implemented. Indeed, the carbon tax is constant over time. In the presence of nominal rigidities, when a productivity shock occurs, firm uses fewer factors of production to produce as much. By fighting this production gap,

monetary policy will generate an increase in the use of fossil fuels and in GHG emissions. Consequently, since emissions generate disutility, central bank, by fighting output gap, also generates a loss of well-being for the household. In the same way, well-being is an increasing function of  $\mu_\pi < \mu_\pi^*$  and decreasing beyond this threshold. On the contrary, by placing greater emphasis on inflation control, the central bank will allow market imperfections to substitute. Indeed, by stabilising prices during a productivity shock, the central bank will not encourage companies to change their behaviour, so that this will limit the flow of emissions and, at the same time, the negative externalities of greenhouse gases. Thus, with such a monetary policy, in a carbon tax regime, it is desirable for the central bank to focus on price stability and not on the lack of competition.

Figure 4.23 shows the standard deviation for each of the variables on which household well-being depends, as a function of the parameters used for the monetary policy instrument. The standard deviation of consumption increases with  $\mu_\pi$  and  $\mu_y$ , for  $\mu_\pi > 1.3$ . Beyond this threshold ( $\mu_\pi > 1.3$ ), when  $\mu_\pi$  increases, the central bank attaches relatively more importance to inflation. All other things being equal, beyond this threshold, the more  $\mu_y$  increases, the more the central bank will take the output gap into account. However, nominal interest rate volatility will increase if central bank is more reactive to these variations. Consequently, money supply and consumption transaction costs will vary relatively more. Indeed, household observes a greater volatility of the nominal interest. It thus makes more trade-offs between consumption and savings. Thus, the higher  $\mu_\pi$  is, the more the household will adapt its consumption or savings levels.

Standard deviation of emissions depends positively on  $\mu_y$ . By tackling output gap more aggressively, the central bank will increase output volatility, and thus GHG emissions in order to be closer to the level of production in perfect competition. The standard deviation of emissions decreases with  $\mu_\pi < \mu_\pi'$ , and increases with  $\mu_\pi > \mu_\pi'$ , however its variation remains weak. For low values of  $\mu_\pi$ , the central bank has little control over inflation. The standard deviation of emissions is relatively high because they depend on consumption, which in turn depends on prices. If prices are subject to variations, emissions will tend to have a relatively high variance. Then this volatility will decrease as

the central bank takes inflation into account, reaching a minimum in  $\mu'_\pi$ , before increasing again. The reason for this increase is the central bank's desire to smooth inflation. Indeed, the more it fights inflation, the more the central bank's nominal interest rate will vary. As a result, consumption and saving vary accordingly, and will have indirect effect on production generating an increase in GHG emissions volatility.

Finally, the standard deviation of labor supply decreases with  $\mu_\pi$ , and increases with  $\mu_y$ . By limiting the output gap as much as possible, the central bank increases the variance of labor supply. When the central bank is not very reactive to inflation or to the output gap, labor supply adapts relatively more than when the central bank is averse to either inflation or the output gap, or both.

Table 4.2 and Figure 4.24 illustrate the contribution of each variable to household's welfare. Indeed, standard deviation analysis alone is not sufficient to explain the values that maximize well-being. Semi-elasticities must also be taken into account. The contributions of habit-adjusted consumption and labor supply to well-being vary in opposite ways with respect to their standard deviations. However, the functional form is identical for the three variables, only the direction of variation changes for  $\mathbb{W}_2$  and  $\mathbb{W}_3$  and  $\mathbb{W}_4$ . Thus,  $\mathbb{W}_2$  is a negative function of  $\mu_\pi$ , while  $\mathbb{W}_3$  is an increasing function of  $\mu_\pi$  and  $\mathbb{W}_4$  a inverted U shape function of  $\mu_\pi$ .

The orders of magnitude of standard deviations of consumption and GHG emissions are comparable (see Table 4.2 and Figure 4.23), while the standard deviation of labor supply is roughly three times smaller. In contrast, the contribution of habit-adjusted consumption is 15 times smaller than the contributions of labor supply to well-being while GHG emissions contribution to welfare is 7 times smaller than labor supply. This difference explains why, despite the increase in the value of  $\mu_\pi$ , which leads to a decrease in well-being because consumption volatility rises, this decrease is compensated by the increase in well-being linked to labor supply and well-being is maximal when  $\mu_\pi = 3.44$ . After this threshold, the increase in well-being linked to labor supply no longer compensates for the decrease in well-being linked to consumption and GHG emissions.

Table 4.2 – Optimized coefficients for monetary policy rules

$\mu_\pi$	1.5	1.1	3.44	4.5
$\mu_y$	0.125	0	0.08	1
$\sigma(\hat{c}_t)$	2.0787	2.0769	2.2612	2.4289
$\sigma(\hat{\hat{c}}_t)$	2.4381	2.4129	2.7017	2.9478
$\sigma(\hat{n}_t^s)$	0.7894	0.8149	0.7632	0.7560
$\sigma(\hat{G}_t)$	1.8358	1.8361	1.8358	1.8358
$\sigma(\hat{R}_t)$	0.3601	0.3513	0.4565	0.5707
$\sigma(\hat{g}_{Mt})$	14.9008	14.3607	16.9672	23.1929
$\mathbb{W}_1 \times 10^4$	-3.9297	-3.9297	-3.9297	-3.9297
$\mathbb{W}_2 \times 10^4$	-1.5513	-1.5194	-1.9049	-2.2677
$\mathbb{W}_3 \times 10^4$	-14.3470	-15.2902	-13.4103	-13.1587
$\mathbb{W}_4 \times 10^4$	-0.2278	-0.2279	-0.2279	-0.2279
$\mathbb{W}_5 \times 10^4$	0.0606	0.0589	0.0592	0.0568
$\mathbb{W} \times 10^{-3}$	-0.79981	-0.83633	-0.7765	-0.78108

Figure 4.22 – Monetary policy optimized coefficients and household welfare

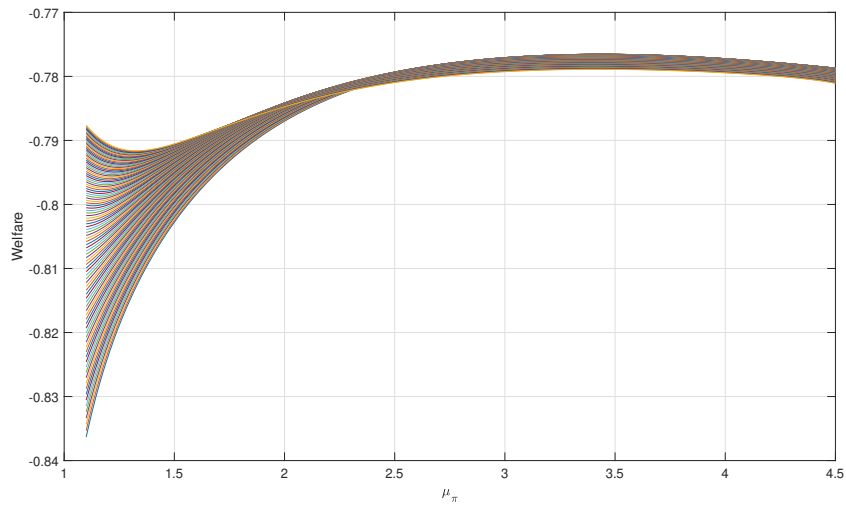


Figure 4.23 – Standard deviations for different monetary policies

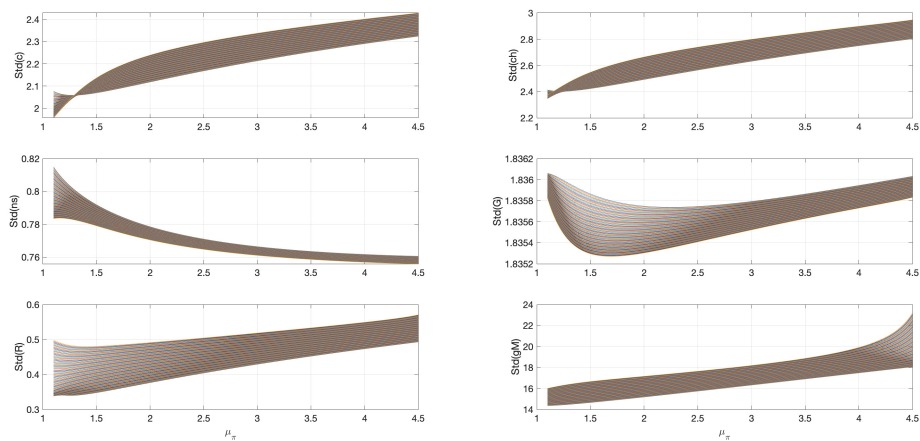
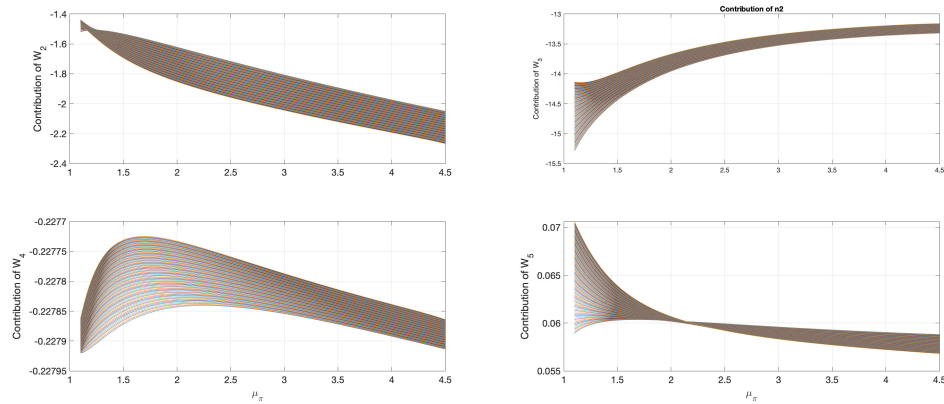




Figure 4.24 – Contributions of variables to household's welfare



#### 4.6.1.2 Optimal monetary policy in a cap-and-trade market:

We simulate the model 34,000 times in the same way, but this time replacing the carbon tax with an emissions cap. Table 4.3 summarizes the results of these simulations, including the values for each of the bounds considered (lower and upper bounds), as well as the results for the standard calibration of the Taylor rule, and finally the results for the optimized values of the monetary policy coefficients. Figure 4.25 illustrates the relationship between household welfare as a function of the monetary policy coefficients, Figure 4.26 details the standard deviations for the different variables of interest, and Figure 4.27 shows the contribution of  $W_2$  to  $W_5$  to household's welfare.

First, it can be noted that when coefficients of the monetary policy instrument are optimized, household welfare in the case of an emissions cap is higher than in the case of a carbon tax. We also find that the values  $\mu_\pi^*$  and  $\mu_y^*$  that maximize household welfare are higher than those optimized in the presence of a carbon tax, but also higher than the usual values of the Taylor rule. Thus, in this case, the central bank is more responsive to both the output gap and inflation. When the government sets the emissions cap and distributes allowances, price of pollution adjusts according to demand in order to ensure

a balance between supply and demand for pollution. Thus, by setting up an allowance market, standard deviation of emissions is zero (see table 4.3 figure 4.26 and 4.27). On the other hand, as the price of allowances adapts to the economic cycle and to various shocks, standard deviation of price of pollution increases (whereas it was zero in the case of the carbon tax). This significant difference in the effects of the two environmental policy instruments explains why such differences in the optimized coefficients of monetary policy are observed.

Indeed, in the case of the carbon tax, the weight that the central bank gives to the output gap when setting its nominal interest rate is close to zero ( $\mu_y^* = 0.08$ ). On the other hand, it will give much greater importance to the output gap when setting an emissions cap ( $\mu_y^* = 0.29$ ). With a carbon tax, as the price of pollution remains constant. The higher the value of  $\mu_y$ , the more the central bank is concerned by the output gap. It fights the lack of competitiveness at the source of this gap between optimal production and real production, by adapting its nominal interest rate and generates variations in the use of production factors, and in particular in demand for polluting energy. This variation in demand for fossil energy increases household disutility and promotes a loss of welfare. On the contrary, a cap-and-trade market will prevent an increase in the use of these energies. Demand for polluting energy will then remain constant and at the same time limit the negative consequences of GHG emissions. From then on, the central bank can take the output gap into account and use its instrument to compensate for the lack of competition by a more aggressive monetary policy.

By varying the price of pollution, with an emissions market, aggregate cost of energy is modified according to variation in demand for allowances. At the same time, this leads to variation in intermediate costs and affects the price level. This environmental policy instrument generates a greater source of price variability. Here again, central bank needs to be more reactive regarding inflation, in comparison with the carbon tax.

Except for the standard deviation for emissions, which in this case is zero (see Table 4.3 and Figures 4.26 and 4.27), other variables reactions is identical to that obtained with the carbon tax. The same analysis is therefore applied in case of an emission

cap. As we already noticed, household welfare is higher in this situation because of the absence of variance in GHG emissions. Moreover, welfare is driven by the contribution of labor supply, and the optimum is reached when  $\mu_\pi = 3.59$  and  $\mu_y = 0.29$ . Beyond this threshold, the gain from the contribution of labor supply does not compensate for the loss of welfare due to the increasing variance of consumption.

Thus, between the two instruments studied, an emission cap makes it possible to limit welfare losses, and the establishment of such markets should therefore be encouraged. It would be advisable to take into account negative externalities of issuance in the design of monetary policy. Finally, suppose that a new shock hits the cap-and-trade market, such as the EU ETS market, and significantly increases the abatement cost of pollution. One can imagine the implementation of a carbon tax or floor price that would replace the allowance market if carbon price falls below a certain threshold. This would limit the reduction in incentives to invest in low-carbon technologies.

Table 4.3 – Optimized coefficients for monetary policy rules

$\mu_\pi$	1.5	1.1	3.59	4.5
$\mu_y$	0.125	0	0.29	1
$\sigma(\hat{c}_t)$	2.0672	2.0652	2.2914	2.4238
$\sigma(\hat{\hat{c}}_t)$	2.4214	2.3962	2.7418	2.9432
$\sigma(\hat{n}_t^s)$	0.7922	0.8177	0.7641	0.7591
$\sigma(\hat{G}_t)$	0	0	0	0
$\sigma(\hat{R}_t)$	0.3617	0.3521	0.4827	0.5808
$\sigma(\hat{g}_{Mt})$	14.9262	14.3740	17.5770	25.6845
$\mathbb{W}_1 \times 10^4$	-3.6333	-3.6333	-3.6333	-3.6333
$\mathbb{W}_2 \times 10^4$	-1.3128	-1.2857	-1.6833	-1.9397
$\mathbb{W}_3 \times 10^4$	-14.4492	-15.3953	-13.4419	-13.2690
$\mathbb{W}_4 \times 10^4$	0	0	0	0
$\mathbb{W}_5 \times 10^4$	0	0	0	0
$\mathbb{W} \times 10^{-3}$	-0.77581	-0.81257	-0.75034	-0.75368

Figure 4.25 – Monetary policy optimized coefficients and household welfare

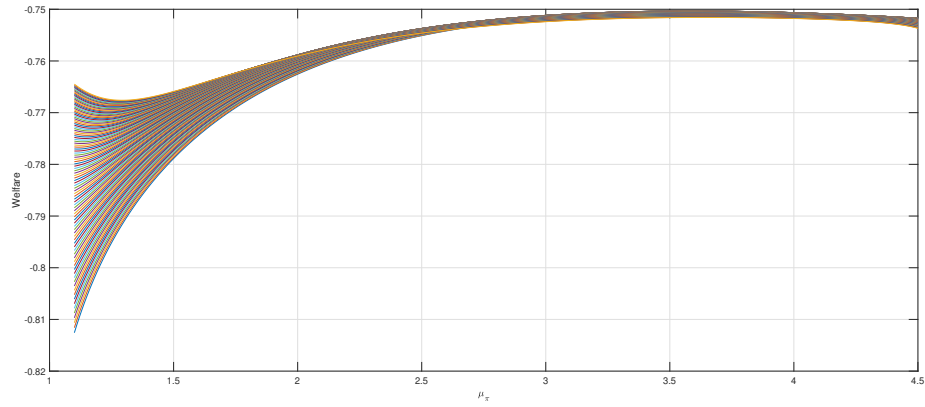


Figure 4.26 – Standard deviations for different monetary policies

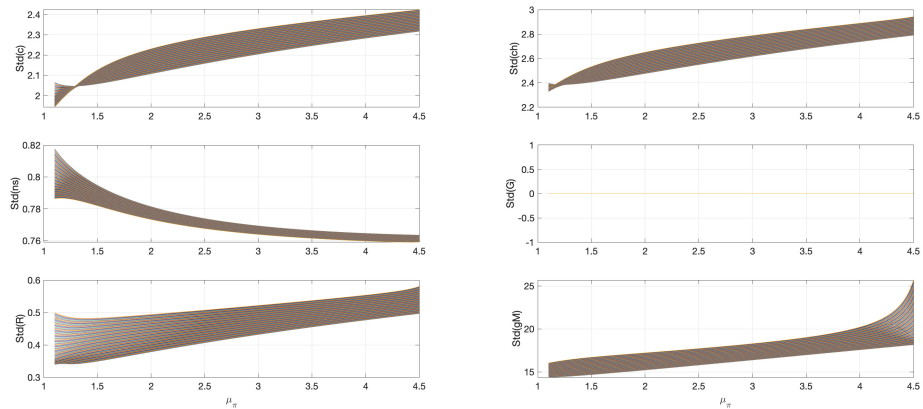
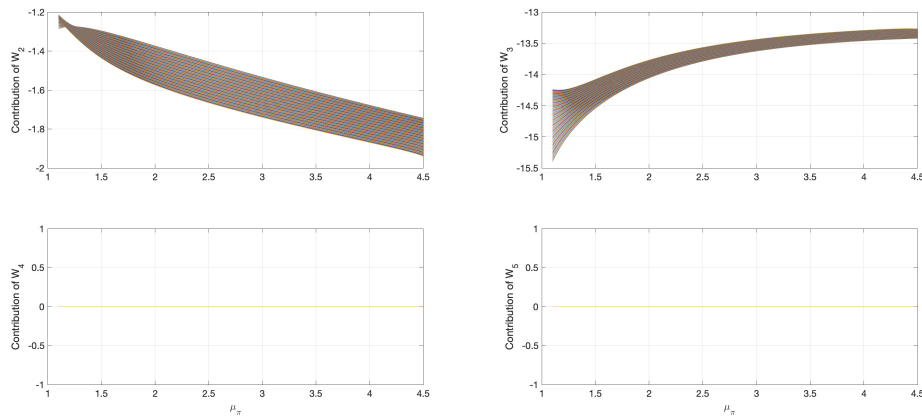


Figure 4.27 – Contributions of variables to household's welfare



## 4.7 Conclusion:

We have developed an NK DSGE model to compare two climate policy instruments: a carbon tax and an emissions cap. This work is among those that attempt to understand the interactions that different forms of environmental policies can have in the business cycle. It echoes the work of Annicchiarico and Di Dio (2015, 2017), who are the first to integrate nominal rigidities by assuming prices stickiness, and Xiao et al. (2018), the first NK DSGE model with an energy factor. We contribute to this literature, on the one hand by disaggregating the energy sector in two sectors. The energy mix is made up on the one hand of fossil energy - which emits GHG, while on the other hand the clean energy sector does not emit. We then use the model to understand how different shocks will affect the economy according to the two environmental policy instruments assumed.

We show that the differences are not significant between the two instruments considered after a productivity shock. The implementation of an emission cap allows the price of pollution to adjust according to demand in polluting energy. Thus, the joint use of the two instruments can be recommended, particularly for the European carbon market. The

establishment of a floor price for pollution below which pollution allowances could not be traded would avoid the situation encountered after the economic and financial crisis of 2008. On the other hand, in the event of a productivity shock in the polluting energy sector - such as new extracting method for non-conventional fossil fuels, the emission cap is preferable to the carbon tax, as it allows the price adjustment mechanism to reduce emissions. We obtain the same results for different values of the elasticity of substitution between energy factors. On the contrary, in the case of a carbon tax, when a productivity shock in the fossil fuel sector occurs, increase in emissions is linked to a more intensive use of this factor and therefore more negative externalities, due to a constant pollution price. In this case, the elasticity of substitution can more or less amplify this result. The increase in emissions will be all the more important as the factors are substitutable. In the case of a productivity shock in the renewable energy sector, the use of a tax or an emission cap does not change the response of the variables, either in their direction of variation or in the amplitude of the responses. Following such a shock, the elasticity of substitution will play a role in the level of emissions in presence of a carbon tax. Rise in productivity in this sector will result in a relatively greater drop of GHG emissions the higher the elasticity of substitution. While the variables of the energy market will be affected by changes in energy demand, responses of the other variables in the model are essentially the same. In the case of an emissions cap, the model responds in exactly the same way, no matter the elasticity of substitution considered. This can be explained by the variation in the carbon price that will adapt to pollution demand.

Finally, we seek to optimize the Taylor rule in order to make a recommendation for the monetary policy. Negative externalities related to the use of polluting energies will modify household behaviour. Using the initial values of the Taylor rule parameters, welfare losses are not optimized. Indeed, by fighting inflation or the output gap, the central bank influences the flow of emissions. Depending on the central bank's aversion to the output gap or to inflation, GHG emissions will vary more or less, which will affect the household's utility. We show that, in the presence of an emissions cap, although the volatility of emissions is reduced by the nature of the instrument, central bank must be

more aggressive towards inflation and output gap. Similarly, if the public authorities introduce a carbon tax, the use of this instrument will generate more or less volatility in emissions, as the price of pollution remains constant. In this case, the coefficients that make it possible to minimise welfare losses are even higher than in the case of an emissions cap.

This work contributes to expand the literature on business cycle and climate policies. Like all research work, it has its own limitations. On the one hand, we considered that only work was needed to produce polluting or non-polluting energy. It can be assumed that by integrating physical capital, the results obtained may thus differ. Adaptation to shocks may thus be more or less long. Furthermore, it can be assumed that negative externalities do not affect the demand for goods but rather labor supply. Another extension would be to incorporate another source of rigidities such as wage stickiness. Finally, the monetary policy analysis should take account of the current situation of very low nominal interest rates, and consequently analyse the special situation of the zero lower bound.





## Chapter 5

### General conclusion:

In this thesis, we analyzed the consequences of climate change and the different environmental policy instruments from a number of different perspectives. First, we looked at the reversibility of climate change. The model developed by Grossman and Krueger (1995) suggests that environmental degradation is a necessary evil that we have to go through. Applied to the climate, this means that we must observe a rise in the global average temperature before returning to an equilibrium where temperatures will be lower, and this without public intervention. To study this question of reversibility, we have assumed, in accordance with the theory, that an exogenous process would make it possible to reduce GHG emissions over time. We applied this assumption in a RBC model with a climate module capable of reproducing the dynamics of the atmosphere and the damage caused by climate change. We have thus shown that climate change is not reversible in the long term. Therefore, in order to achieve the objectives defined by the IPCC and included in the Paris agreement, climate policy instruments must be used to internalise the negative consequences of GHG emissions and also to reduce our emissions in the short term, which is a necessary condition for combating climate change in the long term. Indeed, beyond the uncertainty as to the discovery of a process capable of reducing GHG emissions, once they are released into the atmosphere or the oceans, the emissions persist and have a lasting effect on the climate. Thus, in the absence of rapid intervention, it would become impossible to limit the rise in temperature to 2 degrees,

and it would then be more relevant to adapt to this new situation rather than try to fight against it. Finally, by increasing the elasticity of substitution between energy factors we show that it is possible to achieve a relatively lower temperature increase than when the elasticity is low. A third line of thought in terms of policy implications would therefore be to favour this elasticity. There would be at least two tracks. On the one hand, favouring the storage of energy produced from renewable resources. Indeed, the production of renewable energy is subject to the hazards of the weather. Consequently, there can be imbalances between energy supply and demand. By supporting the storage of energy produced from renewable resources, this would limit these imbalances and therefore reduce the use of power plants using fossil fuels. Secondly, there is necessarily the question of the use of nuclear power in the energy mix. Indeed, nuclear energy, although it produces waste, is not polluting in the sense of the climate. Moreover, the production of energy from nuclear power is flexible and adaptable. However, it is also subject to external stochastic processes (sunshine, wind, swell, etc.) like renewable energy sources. Indeed, when temperatures are too high, nuclear power plants cannot produce energy because the discharge of water into rivers endangers the eco-systems because of too high water temperatures.

In the 3<sup>rd</sup> chapter, we compared different climate policy instruments in a closed economy model with energy as a factor of production. In this model we assume two sources of uncertainty, TFP and energy prices. We distinguished ourselves from the rest of the literature by assuming that capital was not fully mobilized in the production process. We show that the volatility of the model is roughly the same in the case of a productivity shock for the carbon tax and for the intensity target. For these two instruments during a productivity shock, emissions increase. First, with an emissions cap, the volatility of the economic variables will decrease because of the increased volatility of the carbon price, which adjusts to guarantee the emissions target. Nevertheless, during an energy price shock, intensity target and emissions cap are equivalent instruments, they will fully absorb the shock by lowering the carbon price. Whereas, in the presence of a tax, as the marginal cost of pollution remains constant, this instrument has a recessive effect. Thus,

in the case of exogenous energy price shocks, in order to limit the risk of behaviour that calls into question environmental policy, such as the movement of yellow jackets, it may be wise to introduce a floating tax that would adapt according to the price per barrel. Since the aim is to reduce emissions, if the price of energy rises exogenously, this increase will reduce demand and can replace the tax. The objective of reducing emissions is then met, and the cost of using energy remains constant over time.

In the fourth chapter, we analysed different climate policy instruments in the presence of nominal rigidities. Energy as a factor of production has been integrated and disaggregated into two sources: polluting and non-polluting energy sources. With a TFP shock, there is no significant difference between the two instruments. After a productivity shock in the fossil fuel sector, in the presence of a tax, because of the constant marginal cost of pollution, as the price of polluting energies falls, the economy sees these production costs fall, which will have an expansionary effect. As a result, emissions will tend to increase. Consumption will tend to decrease due to the disutility generated by the increase in emissions. Consequently, investment will replace consumption. In the presence of an emissions cap, the fall in the price of fossil energy will be offset by a rise in the price of carbon, which will limit GHG emissions. As intermediate production costs remain more or less the same, the effect in terms of production is limited. On the other hand, not using fossil fuels does not lead to disutility, and the household consumes relatively more. In terms of policy recommendations, the lack of difference between the two instruments during a productivity shock suggests that it would be wise to introduce floor prices in the allowance markets. Below this threshold, a carbon tax-type instrument would replace the allowance market. By doing so, this would send the message that the carbon price cannot fall below a certain level. The use of an instrument combining a carbon tax with an allowance market would avoid the phenomenon that was observed on the European carbon market after the 2008 financial crisis, which resulted in a price below €10 per tonne for almost 10 years. We do not currently know the consequences of the COVID-19 crisis on this market, the MSR may not be sufficient to limit the shock, and in this case the floor price would be more effective. Finally, we have also shown that climate policy

is not neutral for monetary policy.

This work deserves to be deepened and further developed. We have indeed reasoned exclusively in a closed economy. But it is important to understand what could happen in an open economy, when different countries adopt asymmetrical climate policies, in order to see how we can limit the possible behaviour of pollution havens, which could be done with a carbon tax at the borders. Moreover, by imagining a country importing and exporting fossil fuels, we could analyse the risk of the green paradox, a concept developed by Sinclair (1992) and later taken up by Sinn (2008, 2012). According to them, when a country exploits an exhaustible resource, if a climate policy is put in place, its Hotelling rent will be modified, and it will be led to over-exploit its resource in the short term, which would induce more pollution and reduce the effectiveness of the climate policy.

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# Appendix A

## Chapter 2 technical appendix:

### A.1 Appendix: rise in renewable energy productivity

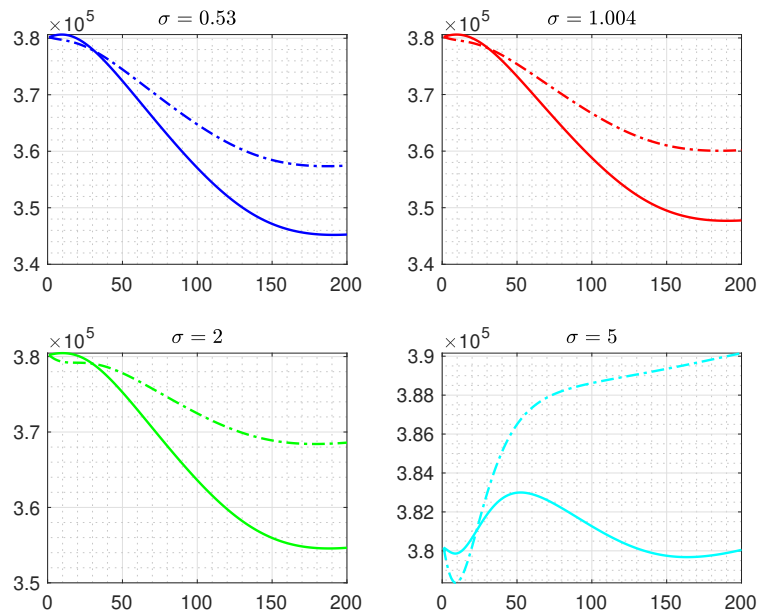


Figure A.1 – Stock of capital

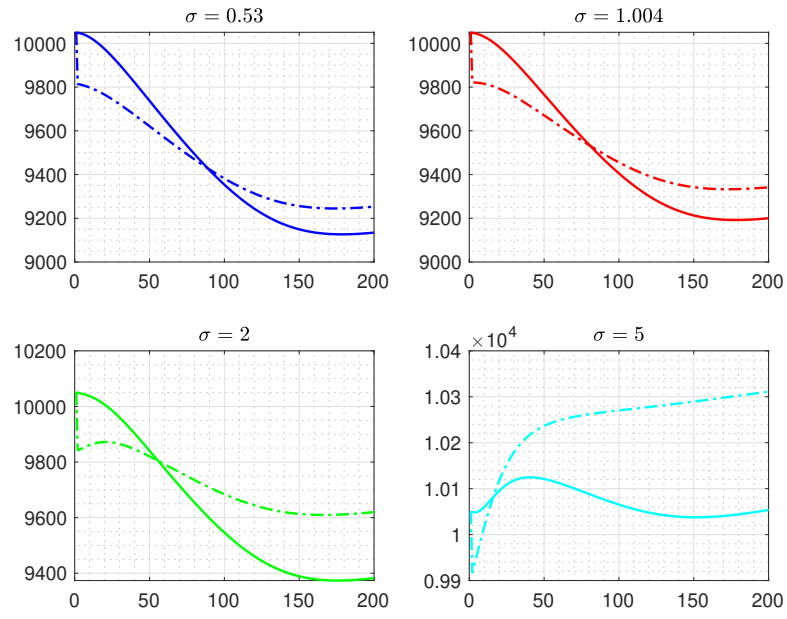


Figure A.2 – Stationarized model:

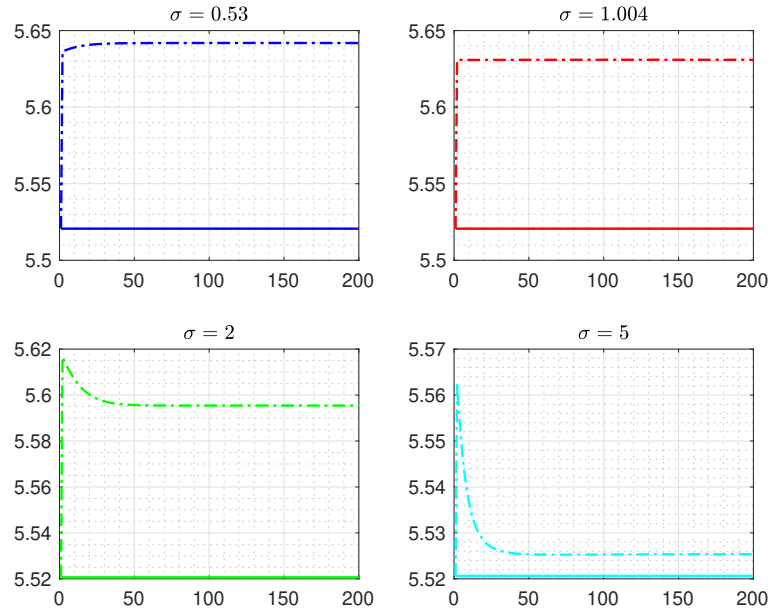


Figure A.3 – Labor in the global sector:

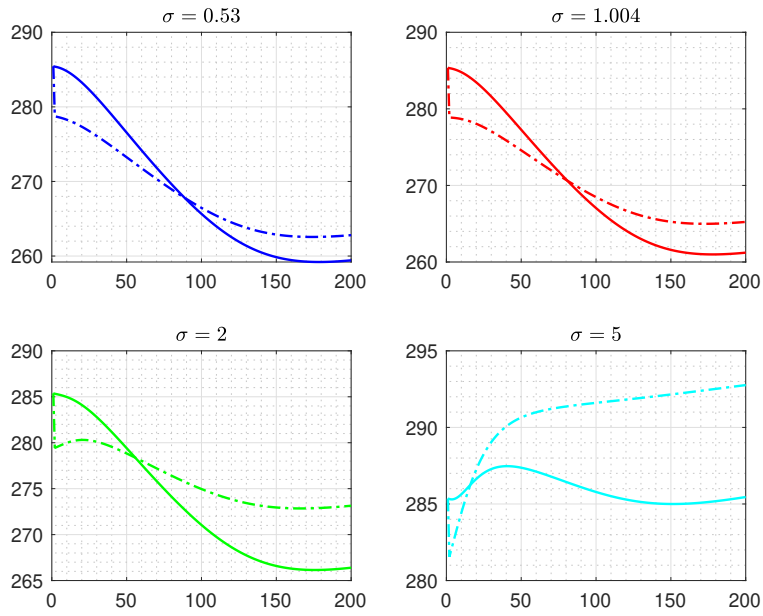


Figure A.4 – Fossil fuel price:

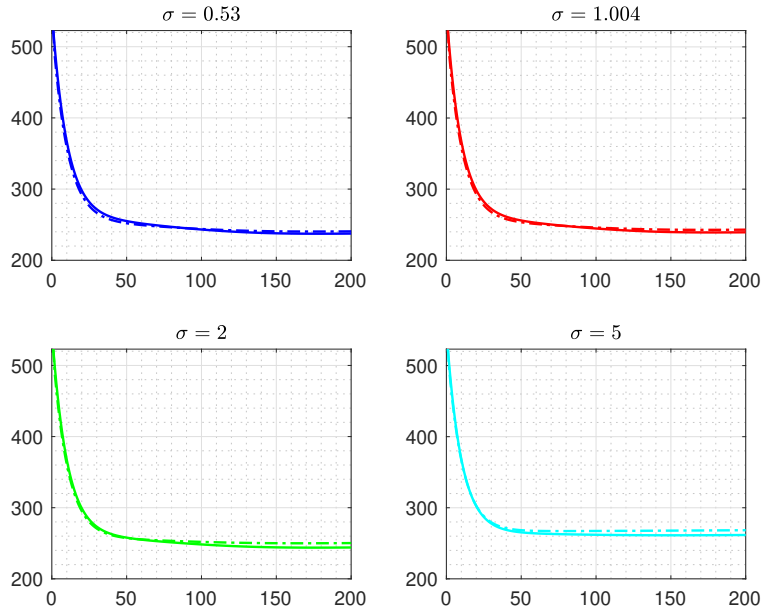


Figure A.5 – Green energy price:

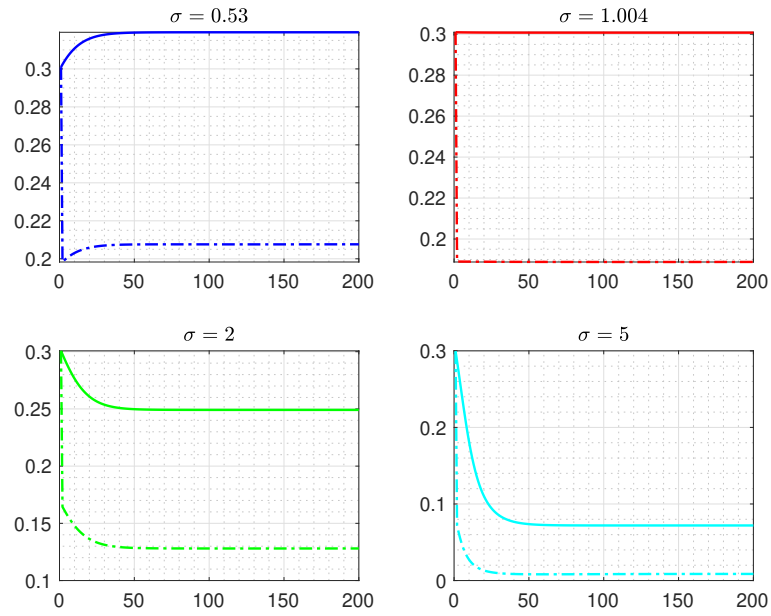


Figure A.6 – Labor in the fossil fuel sector:

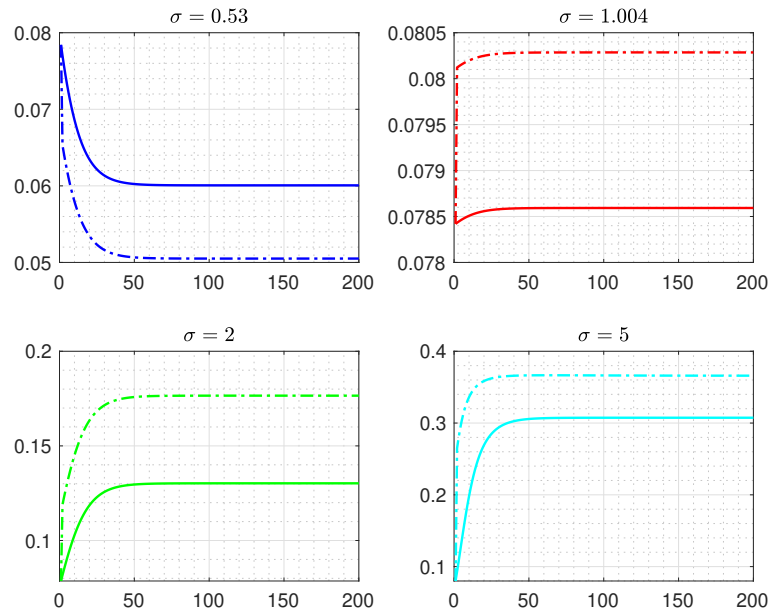


Figure A.7 – Labor in the ren. energy sector:

## A.2 Technical appendix

### A.2.1 Planning problem:

The associated Lagrangian to the planner's problem is:

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t & \left\{ U(C_t) + \lambda_t \left[ \tilde{Y}_t - C_t - I_t - \frac{\phi}{2} \left( \frac{K_{g,t} - K_{g,t-1}}{K_{g,t-1}} - 1 \right)^2 K_{g,t-1} \right] \right. \\ & + \mu_t \left[ \left( 1 - D(S_t) \right) K_{g,t-1}^\alpha \left( \rho^t A_g L_{g,t} \right)^{1-\alpha-\gamma} E_{g,t}^\gamma - \tilde{Y}_t \right] + \theta_t \left[ \left( \omega e_{r,t}^\varphi + (1-\omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}} - E_{g,t} \right] \\ & + \Upsilon_t \left[ A_{c,t} L_{c,t} - e_{c,t} \right] + \zeta_t \left[ A_{r,t} L_{r,t} - e_{r,t} \right] + \eta_t \left[ L_t - L_{g,t} - L_{c,t} - L_{r,t} \right] \\ & \left. + \Xi_t \left[ -K_{g,t} + I_t + (1-\delta) K_{g,t-1} \right] + \nu_t \left[ \sum_{s=0}^{t+T} (1-d_s) e_{c,t-s} - S_t \right] \right\} \end{aligned}$$

The first order conditions are:

$$\lambda_t = \mu_t \quad (\tilde{Y}_t^{foc})$$

$$U'(C_t) = \lambda_t \quad (C_t^{foc})$$

$$\lambda_t \left( 1 + \phi \left( \frac{I_t}{K_{g,t-1}} - \delta \right) \right) = \Xi_t \quad (I_t^{foc})$$

$$\gamma \mu_t \left( 1 - D(S_t(e_{c,t})) \right) K_{g,t-1}^\alpha \left( \rho^t A_g L_{g,t} \right)^{1-\alpha-\gamma} E_{g,t}^{\gamma-1} = \iff \mu_t \frac{\partial \tilde{Y}_t}{\partial E_{g,t}} = \theta_t \theta_t \quad (E_{g,t}^{foc})$$

$$\theta_t \left( \omega e_{r,t}^\varphi + (1-\omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}-1} \omega e_{g,t}^{\varphi-1} - \zeta_t = 0 \iff \theta_t \frac{\partial E_{g,t}}{\partial e_{r,t}} = \zeta_t \quad (e_{r,t}^{foc})$$

$$\Upsilon_t \rho^t A_c = \eta_t \iff \Upsilon_t \frac{\partial e_{c,t}}{\partial L_{c,t}} = \eta_t \quad (L_{c,t}^{foc})$$

$$\zeta_t \rho^t A_r = \eta_t \iff \zeta_t \frac{\partial e_{r,t}}{\partial L_{r,t}} = \eta_t \quad (L_{r,t}^{foc})$$

$$(1-\alpha-\gamma) \mu_t \left( 1 - D(S_t(e_{c,t})) \right) K_{g,t}^\alpha \left( \rho^t A_g L_{g,t} \right)^{1-\alpha-\gamma} L_{g,t}^{-1} E_{g,t}^\gamma = \eta_t \quad (L_{g,t}^{foc})$$

$$\iff \mu_t \frac{\partial Y_t}{\partial L_{g,t}} = \eta_t \quad (L_{g,t}^{foc})$$



$$\begin{aligned} \Xi_t = & \beta^t \lambda_{t+1} \left( -\frac{\phi}{2} \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right)^2 + \phi \left( \frac{I_{t+1}}{K_{g,t}} - \delta \right) \frac{I_{t+1}}{K_{g,t}} \right) \\ & + \beta^t \mu_{t+1} \alpha \left( 1 - D(S_{t+1}) \right) K_{g,t}^{\alpha-1} \left( \rho^{t+1} A_g L_{g,t+1} \right)^{1-\alpha-\gamma} E_{g,t+1}^\gamma + \beta^t \Xi_{t+1} (1 - \delta) \end{aligned} \quad (K_{g,t}^{foc})$$

$$-\mu_t \frac{dD(S_t)}{dS_t} K_{g,t-1} \left( \rho^t A_g L_{g,t} \right)^{1-\alpha-\gamma} E_{g,t}^\gamma - \nu_t = 0 \quad (S_{g,t}^{foc})$$

$$\sum_{j=0}^{\infty} \beta^j \nu_{t+j} (1 - d_j) + \theta_t \left( \omega e_{r,t}^\varphi + (1 - \omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}-1} (1 - \omega) e_{c,t}^{\varphi-1} + -\Upsilon_t = 0 \quad (e_{c,t}^{foc})$$

## A.2.2 Decentralized Competitive Equilibrium:

### A.2.2.1 Household program

The Lagrangian associated with the representative household program is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t) + \lambda_t \left( r_t K_{g,t-1} + w_t L_t + T_t - C_t + (1 - \delta) K_{g,t-1} - K_{g,t} - \frac{\phi}{2} \left( \frac{K_{g,t} - K_{g,t-1}}{K_{g,t-1}} \right)^2 K_{g,t-1} \right) \right\}$$

Taking the derivative with respect to  $C_t$ ,  $K_{g,t}$ , and  $\lambda_t$  yield the following FOC:

$$U'(C_t) = \lambda_t \quad (C_t^{foc})$$

$$\lambda_t \left( 1 + \phi \left( \frac{K_{g,t}}{K_{g,t-1}} - 1 \right) \right) = \beta \lambda_{t+1} \left( r_{t+1} + 1 - \delta - \frac{\phi}{2} \left( \frac{K_{g,t+1}}{K_{g,t}} - 1 \right)^2 + \phi \left( \frac{K_{g,t+1}}{K_{g,t}} - 1 \right) \frac{K_{g,t+1}}{K_{g,t}} \right) \quad (K_{g,t}^{foc})$$

$$r_t K_{g,t-1} + w_t L_t + T_t = C_t + K_{g,t} - (1 - \delta) K_{g,t-1} + \frac{\phi}{2} \left( \frac{I_t}{K_{g,t-1}} - \delta \right)^2 K_{g,t-1} \quad (\lambda_t^{foc})$$

We can easily eliminate the Lagrange multiplier  $\lambda_t$  in the second foc, assuming a logarithmic utility function:

$$\frac{1}{C_t} \left( 1 + \phi \left( \frac{K_{g,t}}{K_{g,t-1}} - 1 \right) \right) = \beta \frac{1}{C_{t+1}} \left( r_{t+1} + 1 - \delta - \frac{\phi}{2} \left( \frac{K_{g,t+1}}{K_{g,t}} - 1 \right)^2 + \phi \left( \frac{K_{g,t+1}}{K_{g,t}} - 1 \right) \frac{K_{g,t+1}}{K_{g,t}} \right) \quad (A.2.1)$$

**A.2.2.2 Firms maximisation program:**

The maximization program for the final good firm gives the usual FOC:

$$\alpha(1 - D(S_t))K_{g,t-1}^{\alpha-1}(\rho^t A_g L_{g,t})^{1-\alpha-\gamma} E_{g,t}^\gamma = r_t \quad (K_{g,t-1}^{foc})$$

$$(1 - \alpha - \gamma)(1 - D(S_t))K_{g,t-1}^\alpha (\rho^t A_g L_{g,t})^{1-\alpha-\gamma} L_{g,t}^{-1} E_{g,t}^\gamma = w_t \quad (L_{g,t}^{foc})$$

$$\gamma(1 - D(S_t))K_{g,t-1}^\alpha (\rho^t A_g L_{g,t})^{1-\alpha-\gamma} (\omega e_{r,t}^\varphi + (1 - \omega)e_{c,t}^\varphi)^{\frac{\gamma}{\varphi}-1} \omega e_{r,t}^{\varphi-1} = p_{r,t} \quad (e_{r,t}^{foc})$$

$$\gamma(1 - D(S_t))K_{g,t-1}^\alpha (\rho^t A_g L_{g,t})^{1-\alpha-\gamma} (\omega e_{r,t}^\varphi + (1 - \omega)e_{c,t}^\varphi)^{\frac{\gamma}{\varphi}-1} (1 - \omega)e_{c,t}^{\varphi-1} = p_{c,t}(1 + \tau_{c,t}) \quad (e_{c,t}^{foc})$$

The following system of equations  $Y_t, C_t, I_t, K_{g,t}, w_t, E_{g,t}, e_{c,t}, e_{r,t}$  summarizes the

model:

$$\left\{ \begin{array}{l}
 \frac{1}{C_t} \left( 1 + \phi \left( \frac{K_{g,t}}{K_{g,t-1}} - 1 \right) \right) = \beta \frac{1}{C_{t+1}} \left( r_{t+1} + 1 - \delta - \frac{\phi}{2} \left( \frac{K_{g,t+1}}{K_{g,t}} - 1 \right)^2 + \phi \left( \frac{K_{g,t+1}}{K_{g,t}} - 1 \right) \frac{K_{g,t+1}}{K_{g,t}} \right) \\
 K_{g,t} = I_t + (1 - \delta)K_{g,t-1} \\
 r_t K_{g,t-1} + w_t L_t + T_t = C_t + I_t + \frac{\phi}{2} \left( \frac{K_{g,t} - K_{g,t-1}}{\hat{K}_{g,t-1}} \right)^2 K_{g,t-1} \\
 Y_t = (1 - D(S_t)) (K_{g,t-1})^\alpha (A_g L_{g,t})^{1-\alpha-\gamma} E_{g,t}^\gamma \\
 r_t = \alpha \frac{Y_t}{K_{g,t-1}} \\
 w_t = (1 - \alpha - \gamma) \frac{G_t}{L_{g,t}} \\
 E_{g,t} = \left( \omega e_{r,t}^\varphi + (1 - \omega) e_{c,t}^\varphi \right)^{\frac{1}{\varphi}} \\
 e_{r,t} = \left[ \frac{\gamma \omega Y_t}{p_{r,t} (\omega e_{r,t}^\varphi + (1 - \omega) e_{c,t}^\varphi)} \right]^{\frac{1}{1-\varphi}} \\
 e_{c,t} = \left[ \frac{\gamma (1 - \omega) Y_t}{p_{c,t} (1 + \tau_{c,t}) (\omega e_{r,t}^\varphi + (1 - \omega) e_{c,t}^\varphi)} \right]^{\frac{1}{1-\varphi}} \\
 e_{c,t} = A_c L_{c,t} \\
 p_{c,t} A_c = w_t \\
 e_{r,t} = A_r L_{r,t} \\
 P_{r,t} A_r = w_t \\
 L_t = L_{g,t} + L_{c,t} + L_{r,t} \\
 S_t = S_{1,t} + S_{2,t} \\
 S_{1,t} = \psi_1 \Omega_t \Phi e_{c,t} + S_{1,t-1} \\
 S_{2,t} = (1 - \psi_1) \psi_2 \Omega_t \Phi e_{c,t} + (1 - \psi_3) S_{2,t-1}
 \end{array} \right. \tag{A.2.2}$$

### A.2.3 Steady state model:

There is a total of 20 equations for 20 endogenous variables (see the system (A.2.2)) :

$$\hat{C}_t, \hat{I}_t, \hat{K}_{g,t}, r_t, \hat{w}_t, L_{g,t}, \hat{S}_t, D_t, \hat{E}_{g,t}, \hat{Y}_t, P_{E,t}, \hat{e}_{r,t}, p_{r,t}, \hat{e}_{c,t}, p_{c,t}, L_{c,t}, L_{r,t}, L_t, \hat{S}_{1,t}, \hat{S}_{2,t}.$$

The steady state is characterized by the fact that all the variables are invariant:

$$x_{t-1} = x_t = x_{t+1} = x \quad (\text{A.2.3})$$

In the steady state, we can express the investment to GDP ratio, using the law of motion for capital:

$$\frac{I}{Y} = \delta \frac{K_g}{Y} \quad (\text{A.2.4})$$

From the Euler's equation we find the expression for the steady state interest rate as a function of the parameters:

$$r = \frac{1}{\beta} - 1 + \delta \quad (\text{A.2.5})$$

Using the condition leading to the optimal demand for capital with the previous result, we can express the capital stock to GDP steady state ratio:

$$\frac{K_g}{Y} = \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \quad (\text{A.2.6})$$

The accounting constraint of the economy combined with the investment, and the capital to GDP ratio gives the consumption rate:

$$\frac{C}{G} = 1 - \frac{\delta \alpha}{\frac{1}{\beta} - 1 + \delta} \quad (\text{A.2.7})$$

The relative price of fossil fuel energy over non polluting energy price is:

$$\frac{P_c}{P_r} = \frac{A_r}{A_c} \quad (\text{A.2.8})$$

The relative demand of non-polluting energy to polluting energy is:

$$\frac{e_r}{e_c} = \left( \frac{\omega(1 + \tau_c)A_r}{(1 - \omega)A_c} \right)^{\frac{1}{1-\varphi}} \quad (\text{A.2.9})$$

Using again the accounting identity, we can express the demand for labour in the final good sector:

$$L_g = \frac{\left(1 - \alpha - \frac{T}{Y}\right)L}{1 - \alpha - \gamma} \quad (\text{A.2.10})$$

Demand for non-polluting energy can be manipulated to be expressed as follow:

$$e_r = \frac{\gamma\omega Y}{p_r \left( \omega + (1-\omega) \left( \frac{e_c}{e_r} \right)^\varphi \right)} \quad (\text{A.2.11})$$

Replacing  $e_r$ ,  $p_r$  and the relative demand for energy by the expressions found above, demand for labor in the non-polluting energy sector:

$$L_r = \frac{\gamma\omega L}{(1-\alpha-tg)} \frac{1}{\left( \omega + (1-\omega) \left( \frac{e_c}{e_r} \right)^\varphi \right)} \quad (\text{A.2.12})$$

The demand for fossil fuel energy is symmetrical to that of polluting energies. We can therefore replicate the same steps as before to find the expression of labor demand in the second energy sector:

$$L_c = \frac{\gamma\omega L}{(1-\alpha-tg)(1+\tau_c)} \frac{1}{\left( \omega \left( \frac{e_r}{e_c} \right)^\varphi + (1-\omega) \right)} \quad (\text{A.2.13})$$

The production of energy in both sectors depends only on labor input and productivity, thus in the equilibrium, the amount of energy available in both sector is:

$$e_r = \frac{A_r \gamma \omega L}{(1-\alpha-tg)} \frac{1}{\left( \omega + (1-\omega) \left( \frac{e_c}{e_r} \right)^\varphi \right)} \quad (\text{A.2.14})$$

$$e_c = \frac{A_c \gamma \omega L}{(1-\alpha-tg)(1+\tau_c)} \frac{1}{\left( \omega \left( \frac{e_r}{e_c} \right)^\varphi + (1-\omega) \right)} \quad (\text{A.2.15})$$

We have assessed the GHG levels in each of the two atmospheric reservoirs  $S_1$  and  $S_2$  for 2010. we can therefore calculate the loss in terms of environmental damages caused by such a stock compared to the pre-industrial GHG stock:

$$D(S_{1,10} + S_{2,10}) = 1 - \exp\left(\kappa(\bar{S} - S_{1,10} - S_{2,10})\right) \quad (\text{A.2.16})$$

Now we can combine this last result with the expressions obtained above for steady state value of labor in the final good sector, as well as the equilibrium values for each of the two energy inputs and the capital-to-GDP ratio to calculate production:

$$Y = \left( 1 - D(S_{1,10} + S_{2,10}) \right)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}} L_g^{\frac{1-\alpha-\gamma}{1-\alpha}} E_g^{\frac{\gamma}{1-\alpha}} \quad (\text{A.2.17})$$

Using this expression for  $Y$ , we are now able to calculate all the steady state value of the remaining variables:

$$\left\{ \begin{array}{l} K_g = \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} Y \\ I = \frac{\delta \alpha}{\frac{1}{\beta} - 1 + \delta} Y \\ C = \left( 1 - \frac{\delta \alpha}{\frac{1}{\beta} - 1 + \delta} \right) Y \\ W = (1 - \alpha - \gamma) \frac{Y}{L} \\ p_c = (1 - \alpha - \gamma) \frac{Y}{A_c L} \\ p_r = (1 - \alpha - \gamma) \frac{Y}{A_r L} \end{array} \right.$$

#### A.2.4 Carbon Cycle:

- We assume as in Golosov et al. (2014), the atmospheric concentration of carbon evolves in each period :

$$S_t = \sum_{s=0}^{t+T} (1 - d_s) e_{2,t}, \quad d_s \in [0, 1] \quad (\text{A.2.18})$$

- We can rewrite this sum as:

$$S_t = (1 - d_0) e_{2,t} \sum_{s \geq 1}^{t+T} (1 - d_s) e_{2,t} \quad (\text{A.2.19})$$

$$\frac{dS_t}{de_{2,t}} = (1 - d_0) \quad (\text{A.2.20})$$

- For the next period, in  $t + 1$ :

$$S_{t+1} = (1 - d_1) e_{2,t} + \sum_{s=0, s \neq 1}^{t+1+T} (1 - d_s) e_{2,t+1} \quad (\text{A.2.21})$$

$$\frac{dS_{t+1}}{de_{2,t}} = (1 - d_1) \quad (\text{A.2.22})$$

- For the period  $t + 2$ :

$$S_{t+2} = (1 - d_2)e_{2,t} + \sum_{s=0, s \neq 2}^{t+2+T} (1 - d_s)e_{2,t+2} \quad (\text{A.2.23})$$

$$\frac{dS_{t+2}}{de_{2,t}} = (1 - d_2) \quad (\text{A.2.24})$$

- consequently:

$$\sum_{j=0}^{\infty} \frac{dS_{t+j}}{de_{2,t}} = \sum_{j=0}^{\infty} (1 - d_j) \quad (\text{A.2.25})$$

## Appendix B

### Chapter 3 technical appendix:

#### B.1 Appendix: model equilibrium

The langrangian associated with the consumer's program is the following:

$$\begin{aligned} \mathcal{L} = & \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left( c_t^\varphi (1-l_t)^\varphi \right)^{1-\alpha} - 1}{1-\alpha} \right. \\ & + \lambda_{1,t} \left( w_t l_t + r_t k_t u_t + P_{e,t} e_t - c_t \right. \\ & \left. \left. - k_{t+1} + \left( 1 - \delta(u_t) \right) k_t - \frac{B}{2} \left( \frac{k_{t+1} - k_t}{k_t} \right)^2 \right) \right\} \end{aligned} \quad (\text{B.1.1})$$

First order condition associated with consumption:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} = 0 & \iff \beta^t \left( \varphi \frac{\left( c_t^\varphi (1-l_t)^{1-\varphi} \right)^{1-\alpha}}{c_t} - \lambda_{1,t} \right) = 0 \\ & \iff \lambda_{1,t} = \varphi \frac{\left( c_t^\varphi (1-l_t)^{1-\varphi} \right)^{1-\alpha}}{c_t} \end{aligned} \quad (\text{B.1.2})$$

First order condition with respect to labour:

$$\frac{\partial \mathcal{L}}{\partial l_t} = 0 \iff \beta^t \left( - (1-\varphi) \frac{\left( c_t^\varphi (1-l_t)^{1-\varphi} \right)^{1-\alpha}}{l_t} + \lambda_{1,t} w_t \right) = 0 \quad (\text{B.1.3})$$



Replacing (B.1.2) in the last expression yields:

$$(1 - \varphi)c_t = \varphi w_t(1 - l_t) \quad (\text{B.1.4})$$

Euler's equation for capital:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial k_{t+1}} = 0 &\iff \\ \lambda_{1,t} \left( 1 + \frac{B}{k_t} \left( \frac{k_{t+1} - k_t}{k_t} \right) \right) & \\ = \beta \mathbb{E} \lambda_{1,t+1} \left( r_{t+1} + 1 - \delta(u_{t+1}) - B \left( \frac{k_{t+2} - k_{t+1}}{k_{t+1}} \right) \frac{k_{t+2}}{(k_{t+1})^2} \right) & \end{aligned} \quad (\text{B.1.5})$$

Replacing (B.1.2) into (B.1.5):

$$\begin{aligned} \varphi \frac{\left( c_t^\varphi (1 - l_t)^{1-\varphi} \right)^{1-\alpha}}{c_t} \left( 1 + \frac{B}{k_t} \left( \frac{k_{t+1} - k_t}{k_t} \right) \right) & \\ = \beta \mathbb{E}_t \varphi \frac{\left( c_{t+1}^\varphi (1 - l_{t+1})^{1-\varphi} \right)^{1-\alpha}}{c_{t+1}} \left( r_{t+1} + 1 - \delta(u_{t+1}) - B \left( \frac{k_{t+2} - k_{t+1}}{k_{t+1}} \right) \frac{k_{t+2}}{(k_{t+1})^2} \right) & \end{aligned}$$

FOC relatives to the utilisation rate of capital:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial u_t} = 0 &\iff \lambda_{1,t} \left( r_t k_t - \delta'(u_t) k_t \right) = 0 \\ \iff r_t &= \omega_0 u_t^{\omega_1 - 1} \end{aligned} \quad (\text{B.1.6})$$

Firm's first order conditions:

$$\min_{l_t, k_t, u_t, e_t} \left\{ w_t l_t + r_t k_t u_t + \left( P_{e,t} + \lambda_{2,t} \right) e_t \right\} \quad (\text{B.1.7})$$

$$s.t. \quad y_t = (z_t l_t)^\theta (k_t u_t)^\gamma e_t^{1-\theta-\gamma} \quad (\text{B.1.8})$$

$$\gamma \frac{y_t}{k_t u_t} = r_t \quad (\text{B.1.9})$$

$$\theta \frac{y_t}{l_t} = w_t \quad (\text{B.1.10})$$

$$(1 - \theta - \gamma) \frac{y_t}{e_t} = P_{e,t} + \lambda_{2,t} \quad (\text{B.1.11})$$

## B.2 Appendix: solving the steady state equilibrium

In the steady state, the economy is characterized by the following system of equations:

The utility function

$$U(c, l) = \frac{\left(c^\varphi(1-l)^{1-\varphi}\right)^{1-\alpha} - 1}{1-\alpha} \quad (\text{B.2.1})$$

The national income accounting identity:

$$c + i = wl + rku + P_e e \quad (\text{B.2.2})$$

The law of motion of capital:

$$i = \delta(u)k \quad (\text{B.2.3})$$

Depreciation rate of capital function:

$$\delta(u) = \frac{\omega_0}{\omega_1} u^{\omega_1} \quad (\text{B.2.4})$$

Production function:

$$y = (zl)^\theta (ku)^\gamma e^{1-\theta-\gamma} \quad (\text{B.2.5})$$

$$e = (1 - \sigma)E \quad (\text{B.2.6})$$

Opportunity cost of leisure:

$$(1-l) = \frac{1-\varphi}{\varphi} \frac{c}{w} \quad (\text{B.2.7})$$

Euler's equation for capital:

$$1 = \beta \left( ru + 1 - \delta(u) \right) \quad (\text{B.2.8})$$

$$r = \delta'(u) \quad (\text{B.2.9})$$

$$\gamma \frac{y}{ku} = r \quad (\text{B.2.10})$$

$$\theta \frac{y}{l} = w \quad (\text{B.2.11})$$

$$A = \iota e \quad (\text{B.2.12})$$

The remaining steady state value of variables are calculated as follow:

- Substituting B.2.10 into B.2.8, yields to the capital stock to gdp ratio:

$$\frac{k}{y} = \frac{\gamma}{\frac{1}{\beta} - 1 + \delta(u)} \quad (\text{B.2.13})$$

From table ??, we found the numerical application for the capital gdp ratio:  $\frac{k}{y} = 14.533$ .

- from the steady state law of motion for capital we can express the investment rate:

$$\begin{aligned} \frac{i}{y} &= \delta(u) \frac{k}{y} \\ \frac{i}{y} &= 0.1825 \end{aligned} \quad (\text{B.2.14})$$

- Rearranging (B.2.2) using the previous result gives the consumption to gdp ratio:

$$\begin{aligned} \frac{c}{y} &= 1 - \delta(u) \frac{k}{y} \\ \frac{c}{y} &= 0.8175 \end{aligned} \quad (\text{B.2.15})$$

- The steady state value for the GDP can therefore be calculated:

$$\begin{aligned} y &= l^\theta (14.533yu)^\gamma (0.04y)^{1-\theta-\gamma} \\ y &= 0.6541 \end{aligned} \quad (\text{B.2.16})$$

Steady state values for capital, consumption, investment and energy are:

$$\left\{ \begin{array}{l} k = 9.55 \\ i = 0.12 \\ c = 0.53 \\ e = 0.026 \\ E = 0.037 \end{array} \right. \quad (\text{B.2.17})$$

- we can calculate household's steady state wage (B.2.10), and return on capital (B.2.11):

$$w = 1.87 \quad (\text{B.2.18})$$

$$r = 0.027 \quad (\text{B.2.19})$$

- Equation (B.2.7) with (B.2.17) and (B.2.18) give the consumption leisure elasticity of substitution :

$$\begin{aligned} \varphi &= \frac{c}{c + w(1-l)} \\ \varphi &= 0.268 \end{aligned} \quad (\text{B.2.20})$$

- Next, we need to determine the values for the depreciation parameters  $\omega_0$  and  $\omega_1$ .

$$\begin{cases} r = \omega_0 u^{\omega_1 - 1} \\ \delta(u) = \frac{\omega_0}{\omega - 1} u^{\omega_1} \end{cases}$$

Resolving this system we obtain:  $\omega_0 = 0.033$ ,  $\omega_1 = 1.80$ .

First we can express the utilization rate of capital as a function of the capital to GDP ratio using B.2.9 and B.2.10:

$$u = \left( \frac{\gamma y}{\omega_0 k} \right)^{\frac{1}{\omega_1}} \quad (\text{B.2.21})$$

substituting this last result with B.2.4 and B.2.10 into B.2.8 yields to the capital to GDP ratio:

$$\frac{k}{y} = \frac{\gamma \left( 1 - \frac{1}{\omega_1} \right)}{\frac{1}{\beta} - 1} \quad (\text{B.2.22})$$

Dividing both side of B.2.3 by  $y$  and substituting ?? into its expression, gives the steady state investment to GDP ratio:

$$\frac{i}{y} = \frac{\gamma}{\omega_1} \quad (\text{B.2.23})$$

Consumption share of GDP is consequently:

$$\frac{c}{y} = 1 - \frac{\gamma}{\omega_1} \quad (\text{B.2.24})$$

We can combine B.2.24, B.2.7 and B.2.11 to obtain the steady state value of labour:

$$l = \frac{1}{\frac{1-\varphi}{\theta} \left( 1 - \frac{\gamma}{\omega_1} + 1 \right)} \quad (\text{B.2.25})$$

Substituting the steady state value B.2.22 into B.2.21 yields to:

$$u = \left( \frac{\frac{1}{\beta} - 1}{\omega_0 - \frac{\omega_0}{\omega_1}} \right)^{\frac{1}{\omega_1}} \quad (\text{B.2.26})$$

We can also use B.2.22 to find the steady state value of the interest rate:

$$r = \omega_0 \left( \frac{\frac{1}{\beta} - 1}{\omega_0 - \frac{\omega_0}{\omega_1}} \right)^{\frac{\omega_1 - 1}{\omega_1}} \quad (\text{B.2.27})$$

The steady state value for the GDP is:

$$y = \frac{\left[ \frac{\gamma}{\omega_0^{1/\omega_1}} \left( \frac{1/\beta - 1}{1 - 1/\omega_1} \right)^{1/\omega_1 - 1} \right]^{\frac{\gamma}{\theta}} (1 - \theta - \gamma)^{\frac{1 - \theta - \gamma}{\theta}}}{\frac{1 - \varphi}{\theta} (\omega_1 - \gamma) + 1} \quad (\text{B.2.28})$$

We can combine these expressions to find the remaining steady state value.

### B.3 Simulation results:

#### B.3.1 Business As Usual:

Table B.1 – Moments of simulated variables with variable depreciation rate of capital - BAU

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>c</i>	0.544127	0.059517	0.003542	0.271530	-0.032787
<i>i</i>	0.127850	0.149201	0.022261	0.524346	0.472166
<i>y</i>	0.700044	0.195606	0.038262	0.603954	0.487723
$\delta(u)$	0.012394	0.003382	0.000011	0.631858	0.489991
<i>u</i>	0.796177	0.125182	0.015670	0.219046	0.041386
<i>k</i>	10.745654	2.436030	5.934244	0.576926	0.355088
<i>l</i>	0.225310	0.037564	0.001411	-0.090128	0.018117
<i>E</i>	0.028121	0.008306	0.000069	0.673020	0.623880
<i>e</i>	0.028121	0.008306	0.000069	0.673020	0.623880
<i>w</i>	1.927760	0.267021	0.071300	0.449529	0.243837
<i>r</i>	0.027451	0.003518	0.000012	0.119056	0.006483
$\lambda_1$	0.712601	0.084436	0.007130	0.546404	-0.072445
<i>z</i>	0.003110	0.111071	0.012337	-0.024049	0.009994
<i>pe</i>	0.000578	0.091337	0.008342	0.032880	0.001178
<i>Pe</i>	1.004750	0.091717	0.008412	0.306026	0.136990

Table B.2 – Autocorrelation of simulated variables with variable depreciation of capital  
- BAU

<i>VARIABLE</i>	1	2	3	4	5
<i>c</i>	0.9721	0.9455	0.9202	0.8959	0.8733
<i>i</i>	0.8726	0.7588	0.6581	0.5681	0.4904
<i>y</i>	0.8873	0.7863	0.6966	0.6162	0.5464
$\delta$	0.8864	0.7846	0.6945	0.6141	0.5448
<i>u</i>	0.8908	0.7925	0.7051	0.6262	0.5577
<i>k</i>	0.9989	0.9958	0.9910	0.9848	0.9773
<i>l</i>	0.8750	0.7630	0.6635	0.5735	0.4954
<i>E</i>	0.8750	0.7644	0.6692	0.5853	0.5131
<i>e</i>	0.8750	0.7644	0.6692	0.5853	0.5131
<i>w</i>	0.9345	0.8749	0.8211	0.7718	0.7282
<i>r</i>	0.8915	0.7940	0.7071	0.6286	0.5602
$\lambda_1$	0.9915	0.9818	0.9710	0.9594	0.9471
<i>z</i>	0.8940	0.7987	0.7136	0.6365	0.5692
<i>pe</i>	0.7920	0.6282	0.4987	0.3962	0.3105
<i>Pe</i>	0.7915	0.6273	0.4977	0.3952	0.3095

Table B.3 – Variance decomposition simulating one shock at a time (in percent) - BAU

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	99.86	0.07	99.93

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Table B.3 – (continued)

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>i</i>	99.94	0.13	100.07
<i>y</i>	99.93	0.11	100.05
$\delta(u)$	99.90	0.15	100.05
<i>u</i>	99.90	0.14	100.04
<i>k</i>	99.77	0.06	99.84
<i>l</i>	99.90	0.14	100.04
<i>E</i>	88.69	10.97	99.65
<i>e</i>	88.69	10.97	99.65
<i>w</i>	99.90	0.09	99.98
<i>r</i>	99.90	0.13	100.03
$\lambda_1$	99.91	0.08	99.99
<i>z</i>	100.00	0.00	100.00
<i>pe</i>	0.00	100.00	100.00
<i>Pe</i>	0.00	100.00	100.00

Table B.4 – Moments of simulated variables with constant depreciation rate of capital-BAU

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>c</i>	0.602615	0.063885	0.004081	0.237013	-0.015986
<i>i</i>	0.144023	0.106208	0.011280	0.528772	0.501903

(Continued on next page)



Table B.4 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>y</i>	0.777792	0.157358	0.024762	0.550368	0.406220
<i>k</i>	11.522039	1.902966	3.621278	0.381806	0.146628
<i>l</i>	0.227630	0.023292	0.000542	0.004155	0.051260
<i>E</i>	0.031296	0.006994	0.000049	0.636242	0.592066
<i>e</i>	0.031296	0.006994	0.000049	0.636242	0.592066
<i>w</i>	2.136547	0.261360	0.068309	0.350552	0.112904
<i>r</i>	0.022529	0.003940	0.000016	0.388136	0.164785
$\lambda_1$	0.625720	0.076067	0.005786	0.510120	-0.072814
<i>z</i>	0.001751	0.110193	0.012143	0.001873	0.037060
<i>pe</i>	-0.001423	0.091627	0.008396	-0.041233	-0.020655
<i>Pe</i>	1.002776	0.091514	0.008375	0.230031	0.040232

Table B.5 – Autocorrelation of simulated variables with constant depreciation rate of capital - BAU

<i>VARIABLE</i>	1	2	3	4	5
<i>c</i>	0.9822	0.9640	0.9455	0.9269	0.9081
<i>i</i>	0.8705	0.7556	0.6533	0.5626	0.4827
<i>y</i>	0.9051	0.8202	0.7439	0.6755	0.6145
<i>k</i>	0.9985	0.9944	0.9881	0.9799	0.9700
<i>l</i>	0.8664	0.7478	0.6424	0.5493	0.4671

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Table B.5 – (continued)

<i>VARIABLE</i>	1	2	3	4	5
<i>E</i>	0.8814	0.7798	0.6933	0.6190	0.5550
<i>e</i>	0.8814	0.7798	0.6933	0.6190	0.5550
<i>w</i>	0.9580	0.9189	0.8823	0.8480	0.8160
<i>r</i>	0.8712	0.7573	0.6565	0.5679	0.4895
$\lambda_1$	0.9930	0.9842	0.9739	0.9623	0.9496
<i>z</i>	0.8904	0.7927	0.7052	0.6274	0.5583
<i>pe</i>	0.7898	0.6235	0.4958	0.3911	0.3096
<i>Pe</i>	0.7882	0.6213	0.4932	0.3885	0.3068

Table B.6 – Variance decomposition simulating one shock at a time with constant depreciation rate of capital (in percent)- BAU

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	99.99	0.07	100.06
<i>i</i>	99.86	0.13	99.99
<i>y</i>	99.91	0.11	100.01
<i>k</i>	99.99	0.07	100.05
<i>l</i>	99.92	0.15	100.06
<i>E</i>	80.93	18.43	99.36
<i>e</i>	80.93	18.43	99.36
<i>w</i>	99.97	0.08	100.04

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Table B.6 – (continued)

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>r</i>	99.96	0.14	100.09
$\lambda_1$	100.04	0.07	100.11
<i>z</i>	100.00	0.00	100.00
<i>pe</i>	0.00	100.00	100.00
<i>Pe</i>	0.00	100.00	100.00

Table B.7 – Matrix of covariance of exogenous shocks

<i>Variables</i>	$\epsilon_z$	$\epsilon_{pe}$
$\epsilon_z$	0.0025	0
$\epsilon_{pe}$	0	0.003126

**B.3.2 Models with a carbon tax:**

Table B.8 – Moments of simulated variables with variable depreciation rate of capital - carbon tax

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>c</i>	0.532345	0.060785	0.003695	0.251082	0.022765
<i>i</i>	0.124940	0.148793	0.022139	0.597965	0.600521
<i>y</i>	0.684763	0.195813	0.038343	0.661567	0.585668
$\delta$	0.012493	0.003494	0.000012	0.650616	0.469575
<i>u</i>	0.799478	0.128480	0.016507	0.256675	-0.009750
<i>k</i>	10.444833	2.487123	6.185782	0.600013	0.633494
<i>l</i>	0.225116	0.038090	0.001451	-0.033091	-0.009198
<i>E</i>	0.021507	0.006447	0.000042	0.728826	0.739742
<i>e</i>	0.021507	0.006447	0.000042	0.728826	0.739742
<i>w</i>	1.885941	0.270354	0.073091	0.454866	0.255033
<i>r</i>	0.027542	0.003605	0.000013	0.160904	-0.056127
$\lambda_1$	0.733428	0.091440	0.008361	0.633575	0.087415
<i>z</i>	-0.000324	0.113578	0.012900	0.014780	-0.017321
<i>pe</i>	0.000756	0.090350	0.008163	0.025690	-0.019833
<i>Pe</i>	0.557345	0.025216	0.000636	0.292320	0.077599

Table B.9 – Autocorrelation of simulated variables with variable depreciation rate of capital - carbon tax

<i>VARIABLE</i>	1	2	3	4	5
<i>c</i>	0.9742	0.9493	0.9255	0.9027	0.8805
<i>i</i>	0.8773	0.7664	0.6683	0.5809	0.5013
<i>y</i>	0.8921	0.7944	0.7076	0.6299	0.5589
$\delta$	0.8918	0.7942	0.7075	0.6294	0.5591
<i>u</i>	0.8951	0.8008	0.7160	0.6392	0.5699
<i>k</i>	0.9989	0.9960	0.9914	0.9854	0.9782
<i>l</i>	0.8782	0.7693	0.6712	0.5829	0.5032
<i>E</i>	0.8776	0.7695	0.6767	0.5956	0.5231
<i>e</i>	0.8776	0.7695	0.6767	0.5956	0.5231
<i>w</i>	0.9386	0.8821	0.8310	0.7843	0.7409
<i>r</i>	0.8957	0.8020	0.7176	0.6411	0.5720
$\lambda_1$	0.9921	0.9831	0.9730	0.9619	0.9501
<i>z</i>	0.8982	0.8068	0.7241	0.6493	0.5814
<i>pe</i>	0.7857	0.6178	0.4873	0.3835	0.3039
<i>Pe</i>	0.7852	0.6173	0.4868	0.3833	0.3038

Table B.10 – Variance decomposition simulating one shock at a time with variable depreciation rate of capital (in percent)- carbon tax

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	100.19	0.05	100.25
<i>i</i>	100.05	0.10	100.15
<i>y</i>	100.07	0.11	100.18
$\delta$	100.01	0.13	100.14
<i>u</i>	100.02	0.12	100.15
<i>k</i>	100.29	0.04	100.34
<i>l</i>	100.03	0.14	100.16
<i>E</i>	90.42	10.60	101.02
<i>e</i>	90.42	10.60	101.02
<i>w</i>	100.14	0.07	100.22
<i>r</i>	100.03	0.12	100.15
$\lambda_1$	100.11	0.06	100.17
<i>z</i>	100.00	0.00	100.00
<i>pe</i>	0.00	100.00	100.00
<i>Pe</i>	0.00	100.00	100.00

Table B.11 – Moments of simulated variables with constant depreciation rate of capital-carbon tax

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>c</i>	0.590590	0.064156	0.004116	0.262438	-0.084147
<i>i</i>	0.140749	0.105529	0.011136	0.526641	0.420694
<i>y</i>	0.761884	0.156918	0.024623	0.552783	0.326618
<i>k</i>	11.262566	1.909724	3.647045	0.405616	0.034330
<i>l</i>	0.227469	0.023609	0.000557	0.012305	-0.020145
<i>E</i>	0.023940	0.005437	0.000030	0.592086	0.390364
<i>e</i>	0.023940	0.005437	0.000030	0.592086	0.390364
<i>w</i>	2.093671	0.261911	0.068597	0.366375	0.049351
<i>r</i>	0.022581	0.004008	0.000016	0.395516	0.119627
<i>lambda_1</i>	0.642183	0.079732	0.006357	0.462565	-0.052073
<i>z</i>	-0.000694	0.111925	0.012527	0.013541	-0.045008
<i>pe</i>	0.000925	0.090644	0.008216	0.023194	-0.067576
<i>Pe</i>	0.557399	0.025299	0.000640	0.284486	0.034066

Table B.12 – Autocorrelation of simulated variables with constant depreciation rate of capital - carbon tax

<i>VARIABLE</i>	1	2	3	4	5
<i>c</i>	0.9829	0.9654	0.9474	0.9293	0.9110
<i>i</i>	0.8736	0.7619	0.6614	0.5728	0.4949

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Table B.12 – (continued)

<i>VARIABLE</i>	1	2	3	4	5
<i>y</i>	0.9080	0.8259	0.7512	0.6848	0.6256
<i>k</i>	0.9985	0.9945	0.9883	0.9802	0.9705
<i>l</i>	0.8696	0.7540	0.6503	0.5585	0.4783
<i>E</i>	0.8853	0.7871	0.7006	0.6274	0.5641
<i>e</i>	0.8853	0.7871	0.7006	0.6274	0.5641
<i>w</i>	0.9596	0.9221	0.8864	0.8532	0.8223
<i>r</i>	0.8743	0.7639	0.6652	0.5778	0.5010
$\lambda_1$	0.9932	0.9846	0.9744	0.9629	0.9503
<i>z</i>	0.8933	0.7983	0.7126	0.6362	0.5689
<i>pe</i>	0.7867	0.6179	0.4829	0.3785	0.2950
<i>Pe</i>	0.7863	0.6176	0.4828	0.3786	0.2953

Table B.13 – Variance decomposition simulating one shock at a time with constant depreciation rate of capital (in percent)- carbon tax

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	99.95	0.05	100.00
<i>i</i>	99.89	0.10	99.99
<i>y</i>	99.89	0.10	99.99
<i>k</i>	100.01	0.05	100.06
<i>l</i>	99.80	0.15	99.96

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Table B.13 – (continued)

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>E</i>	81.52	17.53	99.05
<i>e</i>	81.52	17.53	99.05
<i>w</i>	99.93	0.06	99.99
<i>r</i>	99.80	0.13	99.93
$\lambda_1$	99.73	0.05	99.78
<i>z</i>	100.00	0.00	100.00
<i>pe</i>	0.00	100.00	100.00
<i>Pe</i>	0.00	100.00	100.00

### B.3.3 Models with an emissions cap:

Table B.14 – Moments of simulated variables with variable depreciation rate of capital - cap-and-trade

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>c</i>	0.532228	0.054342	0.002953	0.277320	-0.000302
<i>i</i>	0.125107	0.134885	0.018194	0.578341	0.621917
<i>y</i>	0.684781	0.177205	0.031402	0.633716	0.562797
$\delta$	0.012436	0.003182	0.000010	0.636982	0.446052
<i>u</i>	0.799392	0.116770	0.013635	0.257538	0.038845
<i>k</i>	10.392754	2.192082	4.805221	0.560970	0.372317

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Table B.14 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>l</i>	0.225794	0.034498	0.001190	-0.009770	0.061767
<i>E</i>	0.020931	0.000000	0.000000	1.000321	-1.999140
<i>e</i>	0.020931	0.000000	0.000000	1.000321	-1.999140
<i>w</i>	1.885631	0.243053	0.059075	0.452897	0.234353
<i>r</i>	0.027552	0.003274	0.000011	0.165238	0.002577
$\lambda_1$	0.730832	0.081764	0.006685	0.488553	-0.115291
$\lambda_2$	0.304621	0.350249	0.122674	0.564099	0.444916
<i>z</i>	0.001298	0.113018	0.012773	0.023914	0.034525
<i>pe</i>	-0.000109	0.090751	0.008236	0.006814	-0.007128
<i>Pe</i>	1.308630	0.338642	0.114678	0.633716	0.562797

Table B.15 – Autocorrelation of simulated variables with variable depreciation rate of capital - cap-and-trade

<i>VARIABLE</i>	1	2	3	4	5
<i>c</i>	0.9732	0.9482	0.9240	0.9002	0.8774
<i>i</i>	0.8772	0.7708	0.6750	0.5869	0.5074
<i>y</i>	0.8915	0.7972	0.7121	0.6335	0.5624
$\delta$	0.8911	0.7973	0.7122	0.6340	0.5649
<i>u</i>	0.8943	0.8026	0.7194	0.6428	0.5748
<i>k</i>	0.9988	0.9957	0.9909	0.9845	0.9769

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Table B.15 – (continued)

<i>VARIABLE</i>	1	2	3	4	5
<i>l</i>	0.8774	0.7710	0.6751	0.5869	0.5085
<i>E</i>	4692.7983	4692.8886	4692.9841	4693.1048	4693.2288
<i>e</i>	4692.7983	4692.8886	4692.9841	4693.1048	4693.2288
<i>w</i>	0.9372	0.8817	0.8306	0.7827	0.7385
<i>r</i>	0.8948	0.8035	0.7207	0.6444	0.5766
$\lambda_1$	0.9918	0.9826	0.9721	0.9607	0.9485
$\lambda_2$	0.8850	0.7868	0.7000	0.6203	0.5487
<i>z</i>	0.8959	0.8054	0.7233	0.6477	0.5802
<i>pe</i>	0.7898	0.6233	0.4939	0.3882	0.3049
<i>Pe</i>	0.8915	0.7972	0.7121	0.6335	0.5624

Table B.16 – Variance decomposition simulating one shock at a time with variable depreciation rate of capital(in percent) - cap-and-trade

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	100.00	0.00	100.00
<i>i</i>	100.00	0.00	100.00
<i>y</i>	100.00	0.00	100.00
<i>delta</i>	100.00	0.00	100.00
<i>u</i>	100.00	0.00	100.00
<i>k</i>	100.00	0.00	100.00

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Table B.16 – (continued)

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>l</i>	100.00	0.00	100.00
<i>E</i>	0.02	0.00	0.02
<i>e</i>	0.02	0.00	0.02
<i>w</i>	100.00	0.00	100.00
<i>r</i>	100.00	0.00	100.00
$\lambda_1$	100.00	0.00	100.00
$\lambda_2$	93.48	6.74	100.23
<i>z</i>	100.00	0.00	100.00
<i>pe</i>	0.00	100.00	100.00
<i>Pe</i>	100.00	0.00	100.00

Table B.17 – Moments of simulated variables with constant depreciation rate of capital  
- cap-and-trade

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>c</i>	0.590988	0.060420	0.003651	0.258484	-0.021411
<i>i</i>	0.141437	0.099869	0.009974	0.493761	0.363974
<i>y</i>	0.762983	0.148226	0.021971	0.520869	0.307376
<i>k</i>	11.319509	1.815150	3.294770	0.408307	0.134868
<i>l</i>	0.227829	0.022333	0.000499	0.018312	-0.044953
<i>E</i>	0.020931	0.000000	0.000000	1.000236	-1.999370

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Table B.17 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>e</i>	0.020931	0.000000	0.000000	1.000236	-1.999370
<i>w</i>	2.095448	0.246869	0.060944	0.352914	0.089105
<i>r</i>	0.022488	0.003775	0.000014	0.384717	0.079706
$\lambda_1$	0.640862	0.074905	0.005611	0.466496	-0.068955
$\lambda_2$	0.453099	0.298917	0.089352	0.440079	0.281130
<i>z</i>	0.004056	0.113230	0.012821	0.012867	-0.079359
<i>pe</i>	0.000845	0.090900	0.008263	0.037062	0.030160
<i>Pe</i>	1.458075	0.283263	0.080238	0.520869	0.307376

Table B.18 – Autocorrelation of simulated variables with constant depreciation rate of capital - cap-and-trade

<i>VARIABLE</i>	1	2	3	4	5
<i>c</i>	0.9834	0.9662	0.9487	0.9312	0.9135
<i>i</i>	0.8765	0.7672	0.6700	0.5858	0.5119
<i>y</i>	0.9100	0.8296	0.7574	0.6940	0.6378
<i>k</i>	0.9985	0.9945	0.9884	0.9803	0.9706
<i>l</i>	0.8724	0.7587	0.6580	0.5709	0.4942
<i>E</i>	6324.1381	6324.4197	6324.5365	6324.8645	6325.1998
<i>e</i>	6324.1381	6324.4197	6324.5365	6324.8645	6325.1998
<i>w</i>	0.9605	0.9237	0.8891	0.8573	0.8276

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Table B.18 – (continued)

<i>VARIABLE</i>	1	2	3	4	5
<i>r</i>	0.8777	0.7695	0.6742	0.5915	0.5191
$\lambda_1$	0.9934	0.9850	0.9749	0.9636	0.9511
$\lambda_2$	0.8994	0.8123	0.7347	0.6676	0.6091
<i>z</i>	0.8963	0.8032	0.7205	0.6483	0.5843
<i>pe</i>	0.7885	0.6203	0.4839	0.3779	0.2967
<i>Pe</i>	0.9100	0.8296	0.7574	0.6940	0.6378

Table B.19 – Variance decomposition simulating one shock at a time with constant depreciation rate of capital(in percent) - cap-and-trade

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	100.00	0.00	100.00
<i>i</i>	100.00	0.00	100.00
<i>y</i>	100.00	0.00	100.00
<i>k</i>	100.00	0.00	100.00
<i>l</i>	100.00	0.00	100.00
<i>E</i>	0.01	0.00	0.01
<i>e</i>	0.01	0.00	0.01
<i>w</i>	100.00	0.00	100.00
<i>r</i>	100.00	0.00	100.00
$\lambda_1$	100.00	0.00	100.00

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Table B.19 – (continued)

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
$\lambda_2$	88.85	9.83	98.68
$z$	100.00	0.00	100.00
$pe$	0.00	100.00	100.00
$Pe$	100.00	0.00	100.00

### B.3.4 Models with an intensity target:

Table B.20 – Moments of simulated variables with variable depreciation rate of capital - intensity target

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
$c$	0.533912	0.060102	0.003612	0.204230	-0.007266
$i$	0.125382	0.149301	0.022291	0.587172	0.753241
$y$	0.686834	0.196029	0.038427	0.660797	0.783481
$\delta$	0.012442	0.003469	0.000012	0.643280	0.468587
$u$	0.797656	0.128136	0.016419	0.229853	0.035377
$k$	10.507683	2.440787	5.957444	0.484271	0.329751
$l$	0.225118	0.038210	0.001460	-0.072626	0.061514
$E$	0.021498	0.006136	0.000038	0.660797	0.783481
$e$	0.021498	0.006136	0.000038	0.660797	0.783481
$w$	1.891527	0.268761	0.072232	0.442521	0.373798

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Table B.20 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>r</i>	0.027491	0.003599	0.000013	0.129957	-0.000494
$\lambda_1$	0.730301	0.090520	0.008194	0.646906	0.008441
$\lambda_2$	0.272778	0.090556	0.008200	-0.252914	0.023325
<i>z</i>	0.001039	0.113450	0.012871	-0.013453	0.056380
<i>pe</i>	0.001098	0.090315	0.008157	-0.008942	-0.058853
<i>Pe</i>	1.277955	0.000000	0.000000	-1.029638	-1.919898

Table B.21 – Autocorrelation of simulated variables with variable depreciation rate of capital - intensity target

<i>VARIABLE</i>	1	2	3	4	5
<i>c</i>	0.9735	0.9484	0.9246	0.9011	0.8786
<i>i</i>	0.8775	0.7695	0.6746	0.5866	0.5078
<i>y</i>	0.8919	0.7962	0.7118	0.6333	0.5628
$\delta$	0.8910	0.7953	0.7120	0.6344	0.5656
<i>u</i>	0.8950	0.8022	0.7203	0.6438	0.5758
<i>k</i>	0.9989	0.9958	0.9910	0.9847	0.9771
<i>l</i>	0.8788	0.7719	0.6771	0.5887	0.5102
<i>E</i>	0.8919	0.7962	0.7118	0.6333	0.5628
<i>e</i>	0.8919	0.7962	0.7118	0.6333	0.5628
<i>w</i>	0.9376	0.8815	0.8310	0.7832	0.7395

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Table B.21 – (continued)

<i>VARIABLE</i>	1	2	3	4	5
<i>r</i>	0.8957	0.8034	0.7218	0.6455	0.5776
$\lambda_1$	0.9921	0.9831	0.9732	0.9622	0.9505
$\lambda_2$	0.7846	0.6166	0.4883	0.3899	0.3113
<i>z</i>	0.8977	0.8071	0.7265	0.6510	0.5838
<i>pe</i>	0.7854	0.6178	0.4892	0.3903	0.3116
<i>Pe</i>	50.3357	50.3230	50.3098	50.2958	50.2811

Table B.22 – Variance decomposition simulating one shock at a time with variable depreciation rate of capital (in percent) - intensity target

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	100.00	0.00	100.00
<i>i</i>	100.00	0.00	100.00
<i>y</i>	100.00	0.00	100.00
$\delta$	100.00	0.00	100.00
<i>u</i>	100.00	0.00	100.00
<i>k</i>	100.00	0.00	100.00
<i>l</i>	100.00	0.00	100.00
<i>E</i>	100.00	0.00	100.00
<i>e</i>	100.00	0.00	100.00
<i>w</i>	100.00	0.00	100.00

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Table B.22 – (continued)

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>r</i>	100.00	0.00	100.00
$\lambda_1$	100.00	0.00	100.00
$\lambda_2$	0.00	100.00	100.00
<i>z</i>	100.00	0.00	100.00
<i>pe</i>	0.00	100.00	100.00
<i>Pe</i>	1.98	0.00	1.99

Table B.23 – Moments of simulated variables with constant depreciation rate of capital  
- intensity target

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
<i>c</i>	0.592405	0.063615	0.004047	0.396332	0.564933
<i>i</i>	0.141585	0.106912	0.011430	0.602402	0.913766
<i>y</i>	0.764619	0.157928	0.024941	0.655066	0.999837
<i>k</i>	11.317836	1.905926	3.632553	0.653725	1.541997
<i>l</i>	0.227570	0.023792	0.000566	0.027694	0.119778
<i>E</i>	0.023933	0.004943	0.000024	0.655066	0.999837
<i>e</i>	0.023933	0.004943	0.000024	0.655066	0.999837
<i>w</i>	2.100353	0.260939	0.068089	0.492302	0.732611
<i>r</i>	0.022559	0.004021	0.000016	0.417819	0.260338
$\lambda_1$	0.639714	0.077184	0.005957	0.452197	-0.115672

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Table B.23 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>	<i>SKEWNESS</i>	<i>KURTOSIS</i>
$\lambda_2$	0.274464	0.090751	0.008236	-0.276210	0.076693
$z$	0.000887	0.112259	0.012602	0.033626	0.140423
$pe$	-0.000612	0.090587	0.008206	0.008845	-0.027316
$Pe$	1.277955	0.000000	0.000000	-1.239406	-1.304628

Table B.24 – Autocorrelation of simulated variables with constant depreciation rate of capital - intensity target

<i>VARIABLE</i>	1	2	3	4	5
$c$	0.9826	0.9647	0.9463	0.9277	0.9088
$i$	0.8770	0.7674	0.6698	0.5836	0.5065
$y$	0.9093	0.8277	0.7543	0.6888	0.6294
$k$	0.9985	0.9943	0.9879	0.9795	0.9693
$l$	0.8726	0.7587	0.6575	0.5681	0.4882
$E$	0.9093	0.8277	0.7543	0.6888	0.6294
$e$	0.9093	0.8277	0.7543	0.6888	0.6294
$w$	0.9594	0.9213	0.8854	0.8519	0.8202
$r$	0.8761	0.7650	0.6672	0.5812	0.5049
$\lambda_1$	0.9928	0.9836	0.9727	0.9603	0.9468
$\lambda_2$	0.7857	0.6195	0.4858	0.3807	0.3000
$z$	0.8946	0.7997	0.7150	0.6397	0.5719

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Table B.24 – (continued)

<i>VARIABLE</i>	1	2	3	4	5
<i>pe</i>	0.7863	0.6203	0.4867	0.3814	0.3004
<i>Pe</i>	5.6165	5.6080	5.5975	5.5855	5.5722

Table B.25 – Variance decomposition simulating one shock at a time with constant depreciation rate of capital - intensity target

	$\epsilon_z$	$\epsilon_{pe}$	<i>Tot.lin.contr.</i>
<i>c</i>	100.00	0.00	100.00
<i>i</i>	100.00	0.00	100.00
<i>y</i>	100.00	0.00	100.00
<i>k</i>	100.00	0.00	100.00
<i>l</i>	100.00	0.00	100.00
<i>E</i>	100.00	0.00	100.00
<i>e</i>	100.00	0.00	100.00
<i>w</i>	100.00	0.00	100.00
<i>r</i>	100.00	0.00	100.00
$\lambda_1$	100.00	0.00	100.00
$\lambda_2$	0.00	100.00	100.00
<i>z</i>	100.00	0.00	100.00
<i>pe</i>	0.00	100.00	100.00
<i>Pe</i>	17.78	0.00	17.78

## **B.4 Simulations:**

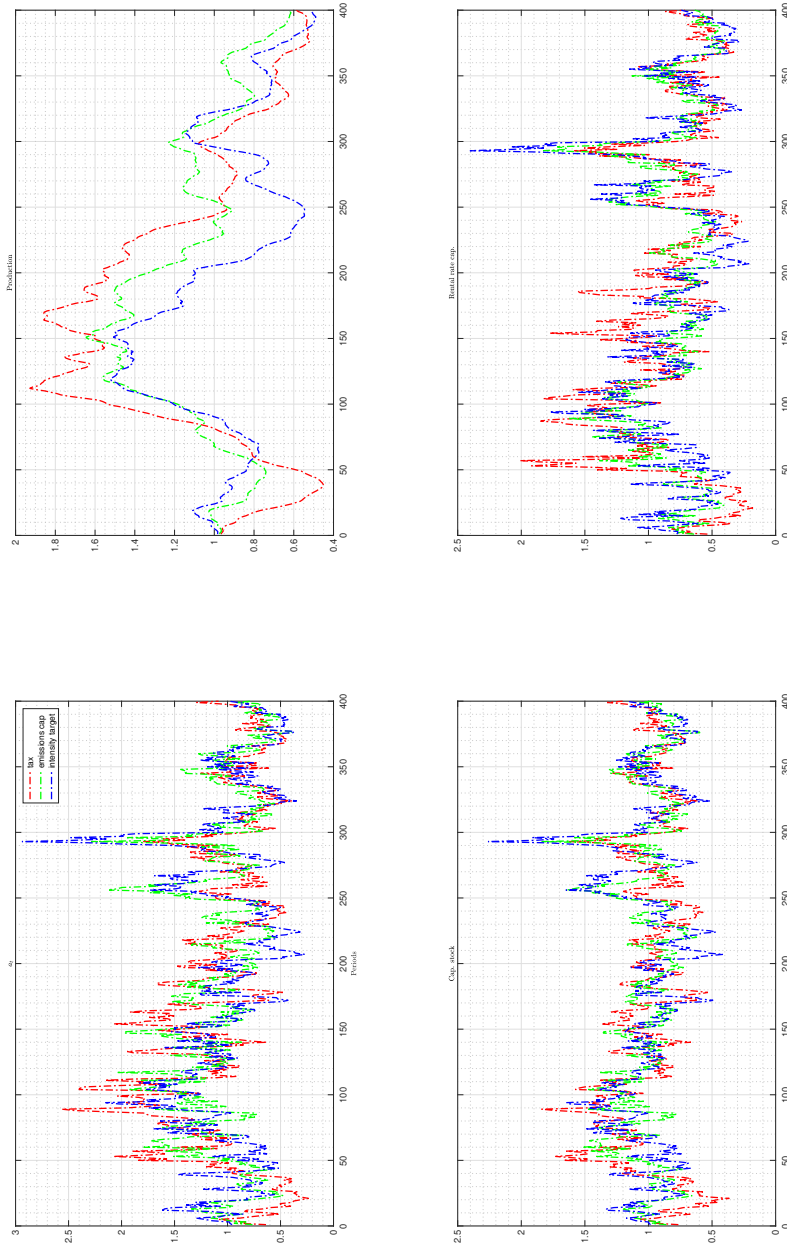


Figure B.1 – Variables simulated for a total factor productivity shock

## B.5 Sensitivity analysis

Table B.26 – steady state value for different discount factor

$\beta$	$\frac{k_{ss}}{y_{ss}}$	$\frac{i_{ss}}{y_{ss}}$	$\frac{c_{ss}}{y_{ss}}$	$l_{ss}$	$r_{ss}$	$y_{ss}$	$k'_{ss}$	$c_{ss}$	$i_{ss}$	$w_{ss}$	$e_{ss}$	$E_{ss}$
0.95	5.07	0.06	0.94	0.19	0.08	0.34	1.71	0.32	0.02	1.08	0.01	0.02
0.953	5.34	0.07	0.93	0.19	0.08	0.35	1.86	0.32	0.02	1.11	0.01	0.02
0.956	5.64	0.07	0.93	0.20	0.07	0.36	2.02	0.33	0.03	1.14	0.01	0.02
0.959	5.97	0.07	0.93	0.20	0.07	0.37	2.22	0.34	0.03	1.17	0.01	0.02
0.962	6.35	0.08	0.92	0.20	0.06	0.38	2.44	0.35	0.03	1.21	0.02	0.02
0.965	6.77	0.08	0.92	0.20	0.06	0.40	2.71	0.37	0.03	1.25	0.02	0.02
0.968	7.24	0.09	0.91	0.20	0.06	0.42	3.01	0.38	0.04	1.30	0.02	0.02
0.971	7.79	0.1	0.90	0.20	0.05	0.44	3.40	0.39	0.04	1.35	0.02	0.02
0.974	8.42	0.1	0.90	0.20	0.05	0.46	3.84	0.41	0.05	1.40	0.02	0.03
0.977	9.16	0.11	0.89	0.21	0.04	0.48	4.40	0.43	0.06	1.47	0.02	0.03
0.98	10.03	0.13	0.87	0.21	0.04	0.51	5.11	0.45	0.06	1.54	0.02	0.03
0.983	11.08	0.14	0.86	0.21	0.04	0.54	6.02	0.47	0.08	1.62	0.02	0.03
0.986	12.36	0.15	0.85	0.21	0.03	0.58	7.22	0.49	0.09	1.72	0.02	0.03
0.989	13.97	0.17	0.83	0.22	0.03	0.63	8.86	0.52	0.11	1.83	0.03	0.04
0.992	16.05	0.20	0.80	0.22	0.02	0.70	11.22	0.56	0.14	1.97	0.03	0.04
0.995	18.83	0.24	0.76	0.23	0.02	0.79	14.82	0.60	0.18	2.14	0.03	0.04
0.998	22.75	0.28	0.72	0.24	0.01	0.91	20.80	0.65	0.26	2.36	0.04	0.05

Table B.27 – steady state value for different depreciation rate of capital

$\delta$	$\frac{k_{ss}}{y_{ss}}$	$\frac{i_{ss}}{y_{ss}}$	$\frac{c_{ss}}{y_{ss}}$	$l_{ss}$	$r_{ss}$	$u_{ss}$	$y_{ss}$	$k_{ss}$	$c_{ss}$	$i_{ss}$	$w_{ss}$	$e_{ss}$	$E_{ss}$
0.01	16.42	0.16	0.84	0.22	0.02	0.81	0.68	11.22	0.57	0.11	1.99	0.03	0.04
0.011	15.64	0.17	0.83	0.22	0.02	0.81	0.67	10.50	0.56	0.12	1.94	0.03	0.04
0.012	14.93	0.18	0.82	0.22	0.03	0.81	0.66	9.85	0.54	0.12	1.90	0.03	0.04
0.013	14.29	0.19	0.81	0.22	0.03	0.81	0.65	9.27	0.53	0.12	1.85	0.03	0.04
0.014	13.69	0.19	0.81	0.22	0.03	0.81	0.64	8.74	0.52	0.12	1.81	0.03	0.04
0.015	13.15	0.20	0.80	0.22	0.03	0.81	0.63	8.26	0.50	0.12	1.77	0.03	0.04
0.016	12.64	0.20	0.80	0.22	0.03	0.81	0.62	7.82	0.49	0.13	1.73	0.02	0.04
0.017	12.18	0.20	0.79	0.23	0.03	0.81	0.61	7.42	0.48	0.13	1.70	0.02	0.03
0.018	11.74	0.21	0.79	0.23	0.03	0.81	0.60	7.05	0.47	0.13	1.67	0.02	0.03
0.019	11.34	0.22	0.78	0.23	0.03	0.81	0.59	6.71	0.46	0.13	1.64	0.02	0.03
0.02	10.96	0.22	0.78	0.23	0.04	0.81	0.58	6.40	0.46	0.13	1.61	0.02	0.03
0.021	10.61	0.22	0.78	0.23	0.04	0.81	0.58	6.11	0.45	0.13	1.58	0.02	0.03
0.022	10.28	0.23	0.77	0.23	0.04	0.81	0.57	5.84	0.44	0.13	1.56	0.02	0.03
0.023	9.97	0.23	0.77	0.23	0.04	0.81	0.56	5.59	0.43	0.13	1.53	0.02	0.03
0.024	9.68	0.23	0.77	0.23	0.04	0.81	0.55	5.36	0.43	0.13	1.51	0.02	0.03
0.025	9.40	0.24	0.76	0.23	0.04	0.81	0.55	5.14	0.42	0.13	1.49	0.02	0.03





# Appendix C

## Chapter 4 technical appendix:

### C.1 Decentralized Competitive Equilibrium:

#### C.1.0.1 Household optimisation program

$$\underset{c_t, n_t^s, k_t, b_{t+1}, m_t, i_t, z_t}{Max} \quad \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left( (1 - d(G_{t+j})) \frac{(c_{t+j} - \chi c_{t-1+j}^A)^{1-\sigma}}{1 - \sigma} - \psi \frac{(n_{t+j}^s)^{1+\gamma}}{1 + \gamma} \right)$$

subject to

$$\begin{aligned} c_{t+j} + i_{t+j} + a(z_{t+j})k_{t-1+j} + \left( \exp(\epsilon_{t+j}^b) (1 + r_{t+j}) \right)^{-1} b_{t+1+j} - b_{t+j} \\ + m_{t+j} - (1 + \pi_{t+j})^{-1} m_{t-1+j} + h(c_{t+j}, m_{t+j}) = \\ w_{t+j} n_{t+j}^s + r_{t+j}^k z_{t+j} k_{t-1+j} + d_{t+j} - tax_{t+j} \quad \text{for } j = 0, 1, 2, \dots \end{aligned}$$

and:

$$k_{t+j} = (1 - \delta) k_{t-1+j} + \exp(\epsilon_{t+j}^i) \left[ 1 - S \left( \frac{i_{t+j}}{i_{t-1+j}} \right) \right] i_{t+j}$$

$$\begin{aligned}
\mathcal{L}_t = & \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left( (1 - d(G_{t+j})) \frac{(c_{t+j} - \chi c_{t-1+j}^A)^{1-\sigma}}{1-\sigma} - \psi \frac{(n_{t+j}^s)^{1+\gamma}}{1+\gamma} \right) \\
& + \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} (w_{t+j} n_{t+j}^s + r_{t+j}^k z_{t+j} k_{t-1+j} + d_{t+j} - tax_{t+j} - c_{t+j} - i_{t+j} - a(z_{t+j}) k_{t-1+j} \\
& - \left( \exp(\epsilon_{t+j}^b) (1 + r_{t+j}) \right)^{-1} b_{t+1+j} + b_{t+j} - m_{t+j} + (1 + \pi_{t+j})^{-1} m_{t-1+j} - h(c_{t+j}, m_{t+j}) \\
& + \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} q_{t+j} \left( (1 - \delta) k_{t-1+j} + \exp(\epsilon_{t+j}^i) \left[ 1 - S \left( \frac{i_{t+j}}{i_{t-1+j}} \right) \right] i_{t+j} - k_{t+j} \right)
\end{aligned}$$

where  $\lambda_{t+j}$  is Lagrange multiplier associated with household's budget constraint in period  $t+j$  and  $\lambda_{t+j} q_{t+j}$  is Lagrange multiplier associated with capital accumulation equation.

The first order conditions for the choice variables are:

$$\begin{aligned}
(1 - d(G_t)) (c_{t+j} - \chi c_{t-1+j}^A)^{-\sigma} - \lambda_t (1 + h_{c_t}(c_t, m_t)) &= 0 & (c_t^{foc}) \\
-\psi (n_t^s)^\gamma + \lambda_t w_t &= 0 & (n_t^{foc}) \\
-\lambda_t q_t + \beta \mathbb{E}_t \lambda_{t+1} \left( r_{t+1}^k z_{t+1} - a(z_{t+1}) + q_{t+1} (1 - \delta) \right) &= 0 & (k_{t+1}^{foc}) \\
-\lambda_t \left( \exp(\epsilon_t^b) (1 + r_t) \right)^{-1} + \beta \mathbb{E}_t \lambda_{t+1} &= 0 & (b_{t+1}^{foc}) \\
\lambda_t (1 + h_m(c_t, m_t)) - \beta \mathbb{E}_t \lambda_{t+1} (1 + \pi_{t+1})^{-1} &= 0 & (m_t^{foc}) \\
-\lambda_t + \lambda_t q_t \exp(\epsilon_t^i) \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) - S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] + \\
\beta \mathbb{E}_t \lambda_{t+1} q_{t+1} \exp(\epsilon_{t+1}^i) S' \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2 &= 0 & (i_t^{foc}) \\
r_t^k = a'(z_t) & & (z_t^{foc})
\end{aligned}$$

It should be noticed the following notation regarding partial derivatives of the transaction costs function

$$\begin{aligned}
h_{c_t}(c_t, m_t) &\equiv \frac{\partial h(c_t, m_t)}{\partial c_t} \\
h_{m_t}(c_t, m_t) &\equiv \frac{\partial h(c_t, m_t)}{\partial m_t}
\end{aligned}$$

The consumption Euler equation obtained from combining consumption and bonds optimality condition,  $(c_t^{foc})$  and  $(b_{t+1}^{foc})$  respectively, is

$$\frac{(1-d(G_t))(c_t - \chi c_{t-1}^A)^{-\sigma}}{(1+h_{c_t}(c_t, m_t))(1+r_t)\exp(\epsilon_t^b)} = \beta \mathbb{E}_t \left( \frac{(1-d(G_{t+1}))(c_{t+1} - \chi c_t^A)^{-\sigma}}{(1+h_{c_{t+1}}(c_{t+1}, m_{t+1}))} \right)$$

which includes a real balance effect through the transactions-facilitating role of money (assuming  $\frac{\partial c_t(c_t, m_t)}{\partial m_t} < 0$ ).

Meanwhile, the labor supply curve is reached by inserting the consumption first order condition,  $(c_t^{foc})$ , into the work hours first order condition,  $(n_t^{sfoc})$ , to get

$$w_t = \frac{\psi(n_t^s)^\gamma (1+h_{c_t}(c_t, m_t))}{(c_t - \chi c_{t-1}^A)^{-\sigma} (1-d(G_{t+j}))}$$

and also observe the influence of real money holdings on the household supply of labor.

Capital accumulation is governed by the optimal portfolio choice that results from combining first order conditions of capital,  $(k_{t+1}^{foc})$ , and bonds,  $(b_{t+1}^{foc})$ . It yields

$$\exp(\epsilon_t^b) (1+r_t) = \frac{r_{t+1}^k z_{t+1} - a(z_{t+1}) + q_{t+1}(1-\delta)}{q_t}$$

that equalizes the expected returns on the financial asset (left hand side) and the physical asset (right hand side). Optimal choice of investment is given by:

$$1 = q_t \exp(\epsilon_t^i) \left[ 1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} \right] + \exp(\epsilon_t^b) (1+r_t) \mathbb{E}_t q_{t+1} \exp(\epsilon_{t+1}^i) S'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2$$

Finally, the money demand equation can be derived by inserting the bonds first order condition,  $(b_{t+1}^{foc})$ , into the real money optimality condition,  $(m_t^{foc})$ , and then use the Fisher relation,  $1+r_t = \frac{1+R_t}{\mathbb{E}_t(1+\pi_{t+1})}$  to introduce the nominal interest rate and obtain (after some easy algebra)

$$-h_m(c_t, m_t) = \frac{R_t}{1+R_t} = \frac{1 - \exp(\epsilon_t^b) (1+R_t)}{(1+R_t) \exp(\epsilon_t^b)}$$

which equates the marginal benefit of money (left side) to its marginal (opportunity) costs (right side).

### C.1.0.2 Firms optimisation program

**C.1.0.2.1 Final-good firms** The representative firm chooses the quantity of intermediate goods that maximises its profit:

$$\begin{aligned} \underset{y_t(i)}{\text{Max}} \quad & P_t y_t - \int_0^1 P_t(i) y_t(i) di \\ \text{s.to} \quad & y_t = \left[ \int_0^1 y_t^{\frac{\theta-1}{\theta}}(i) di \right]^{\frac{\theta}{\theta-1}} \end{aligned}$$

. From the FOC associated with the previous optimisation program, we find the demand for each intermediate good  $i$ :

$$y_t(i) = y_t \left( \frac{P_t(i)}{P_t} \right)^{-\theta}$$

**C.1.0.2.2 Intermediate-good firms** The intermediate-goods firm chooses the quantity of inputs that minimises its costs subject to its technological constraint solving the following program

$$\begin{aligned} \underset{n_t(i), k_t(i), e_t(i)}{\text{Min}} \quad & w_t n_t(i) + r_t^k k_t(i) + p_t^e e_t(i) \\ \text{s.to} \quad & y_t(i) = a_t k_t(i)^\alpha n_t(i)^{1-\alpha-v} e_t(i)^v \end{aligned}$$

The Lagrangian function is

$$\mathcal{L}_t = w_t n_t(i) + r_t^k k_t(i) + p_t^e e_t(i) + \lambda_t(i) \left( y_t(i) - a_t k_t(i)^\alpha n_t(i)^{1-\alpha-v} e_t(i)^v \right)$$

where  $\lambda_t(i)$  is the Lagrange multiplier associated with the constraint from the production technology of firm  $i$ . The optimality conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial n_t(i)} = 0 & \iff w_t = (1 - \alpha - v) \lambda_t(i) \frac{y_t(i)}{n_t(i)} & (n_t(i)^{foc}) \\ \frac{\partial \mathcal{L}_t}{\partial k_t(i)} = 0 & \iff r_t^k = \alpha \lambda_t(i) \frac{y_t(i)}{k_t(i)} & (k_t(i)^{foc}) \\ \frac{\partial \mathcal{L}_t}{\partial e_t(i)} = 0 & \iff p_t^e = v \lambda_t(i) \frac{y_t(i)}{e_t(i)} & (e_t(i)^{foc}) \\ \frac{\partial \mathcal{L}_t}{\partial \lambda_t(i)} = 0 & \iff y_t(i) = a_t k_t(i)^\alpha n_t(i)^{1-\alpha-v} e_t(i)^v & (\lambda_t(i)^{foc}) \end{aligned}$$

Dividing  $n_t(i)^{foc}$  by  $k_t(i)^{foc}$  gives us the capital to labor ratio

$$\frac{k_t(i)}{n_t(i)} = \frac{\alpha}{(1-\alpha-v)} \frac{w_t}{r_t^k}$$

Then, by replacing this result into  $n_t(i)^{foc}$ , we obtain

$$w_t = (1-\alpha-v)\lambda_t(i)a_t \left[ \frac{\alpha}{(1-\alpha-v)} \frac{w_t}{r_t^k} \right]^\alpha \left[ \frac{e_t(i)}{n_t(i)} \right]^v$$

Again, dividing  $n_t(i)^{foc}$  by  $e_t(i)^{foc}$  gives the energy to labor ratio

$$\frac{e_t(i)}{n_t(i)} = \frac{v}{1-\alpha-v} \frac{w_t}{p_t^e}$$

Finally, replacing this ratio into our previous result, we can express the Lagrange multiplier  $\lambda_t(i)$ , i.e. the real marginal cost, as:

$$\lambda_t(i) = \frac{1}{\alpha^\alpha v (1-\alpha-v)^{(1-\alpha-v)}} \frac{1}{a_t} (r_t^k)^\alpha (p_t^e)^v w_t^{(1-\alpha-v)}$$

#### *Optimal pricing and aggregate inflation dynamics*

The optimal choice of the intermediate-good price is

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j mc_{t+j} \Pi_{t,t+j}^P {}^{-\theta} \Pi_{k=1}^j (1+\pi_{t+k})^\theta y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j \Pi_{t,t+j}^P {}^{1-\theta} \Pi_{k=1}^j (1+\pi_{t+k})^{\theta-1} y_{t+j}}$$

where  $\Pi_{t,t+j}^P = \prod_{k=1}^j [(1+\pi_{t+k-1})^\kappa (1+\pi_{t+k})^{1-\kappa}]$  is the price indexation factor between periods  $t$  and  $t+j$ . Following Schmitt-Grohe and Uribe (2005), let us introduce the following auxiliary variables

$$A_t = y_t mc_t + \beta\eta \mathbb{E}_t \left[ A_{t+1} \left( \frac{(1+\pi_{t+1})}{(1+idx_{t+1})} \right)^{\theta/(1-\alpha)} \right]$$

and

$$B_t = mc_t + \beta\eta \mathbb{E}_t \left[ B_{t+1} \left( \frac{(1+\pi_{t+1})}{(1+idx_{t+1})} \right)^{\theta-1} \right]$$

which allows us to express the relative price function in the following way:

$$B_t \left( \frac{P_t^*}{P_t} \right) = \frac{\theta}{\theta-1} A_t$$

By using the inflation definition,  $1 + \pi_t = P_t/P_{t-1}$  as well as the price indexation factor in period  $t$ , leads to obtaining the following and equivalent expression from the Dixit-Stiglitz aggregate price level,  $P_t = \left[ (1 - \eta) (P_t^*)^{1-\theta} + \eta ((1 + id_x) P_{t-1})^{1-\theta} \right]^{1/(1-\theta)}$ , in terms of the relative optimal price

$$\left( \frac{P_t^*}{P_t} \right)^{\theta-1} = (1 - \eta) + \eta \left[ (1 + \pi_{t-1})^{\kappa_p} (1 + \pi + \nu_t)^{1-\kappa_p} \right]^{1-\theta} \left[ (1 + \pi_t) \left( \frac{P_t^*}{P_t} \right) \right]^{\theta-1}$$

These last four equations comprise the inflation dynamics block, which determine solution paths for  $\frac{P_t^*}{P_t}$ ,  $\pi_t$ ,  $A_t$  and  $B_t$ , for given values of  $y_t$  and  $mc_t$ .

### C.1.0.3 Energy services market

**C.1.0.3.1 Optimizing program in the case of a carbon tax:** The intermediate firm needs energy for the production of its good. It has an energy demand characterized by the composite energy  $e_t(i)$ . A part of the composite energy comes from fossil fuels  $e_t^p(i)$ , and the complementary part comes from non-polluting energies  $e_t^{np}(i)$ . Alternatively, the intermediary firm's program can be decomposed into the following cost minimization program. As previously, the intermediate-goods firm chooses the quantity of inputs  $k_t(i)$ ,  $n_t(i)$  but instead of choosing the quantity of composite energy  $e_t(i)$  it chooses directly  $e_t^p(i)$  and  $e_t^{np}(i)$  that minimises its costs subject to its technological constraints:

$$\begin{aligned} \text{Min} \quad & w_t n_t(i) + r_t^k k_t(i) + p_t^p (1 + \tau_t \Xi) e_t^p(i) + p_t^{np} e_t^{np}(i) \\ \text{s.to} \quad & \begin{cases} y_t(i) = a_t k_t(i)^\alpha n_t(i)^{1-\alpha-v} e_t(i)^v \\ e_t(i) = [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} \end{cases} \end{aligned}$$

By replacing the CES production function in the Cobb-Douglas function, the Lagrangian associated with this optimization program becomes (where  $\lambda_t(i)$  is the Lagrange multiplier associated with the  $i^{th}$  production function):

$$\begin{aligned} \mathcal{L}_t = & w_t n_t(i) + r_t^k k_t(i) + p_t^p (1 + \tau_t \Xi) e_t^p(i) + p_t^{np} e_t^{np}(i) \\ & + \lambda_t(i) \left( y_t(i) - a_t k_t(i)^\alpha n_t(i)^{1-\alpha-v} [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{v}{\varphi}} \right) \end{aligned}$$

The conditions of optimality for labor  $n_t(i)$  and for capital  $k_t(i)$  are exactly the same as the one obtained above and are consequently omitted. The intermediate-good firm chooses the quantity of polluting  $e_t^p(i)$  and non-polluting  $e_t^{np}(i)$  that minimizes its costs:

$$\frac{\partial \mathcal{L}_t}{\partial e_t^p(i)} = 0 \iff p_t^p(1 + \tau_t \Xi) = v \lambda_t(i) \frac{y_t(i)}{e_t(i)^\varphi} \omega e_t^p(i)^{\varphi-1} \quad (e_t^p(i)^{foc})$$

$$\frac{\partial \mathcal{L}_t}{\partial e_t^{np}(i)} = 0 \iff p_t^{np} = v \lambda_t(i) \frac{y_t(i)}{e_t(i)^\varphi} (1 - \omega) e_t^{np}(i)^{\varphi-1} \quad (e_t^{np}(i)^{foc})$$

The two demands function for each energy input ( $e_t^p(i)^{foc}$ ) and ( $e_t^{np}(i)^{foc}$ ) obtained by the minimization cost program are the same as the one obtained by the maximization program described in the article:

$$\begin{aligned} \text{Max} \quad & p_t^e e_t(i) - p_t^p(1 + \tau_t \Xi) e_t^p(i) - p_t^{np} e_t^{np}(i) \\ \text{s.t.} \quad & e_t(i) = [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} \end{aligned}$$

$$\mathcal{L}_t = p_t^e [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} - p_t^p(1 + \tau_t \Xi) e_{p,t}(i) - p_t^{np} e_t^{np}(i)$$

Optimality conditions obtained in the following case are:

$$\frac{\partial \mathcal{L}_t}{\partial e_t^p(i)} = 0 \iff p_t^e \frac{e_t(i)}{e_t(i)^\varphi} \omega e_t^p(i)^{\varphi-1} = p_t^p(1 + \tau_t \Xi) \quad (e_t^{p'}(i)^{foc})$$

$$\frac{\partial \mathcal{L}_t}{\partial e_t^{np}(i)} = 0 \iff p_t^e \frac{e_t(i)}{e_t(i)^\varphi} (1 - \omega) e_t^{np}(i)^{\varphi-1} = p_t^{np} \quad (e_t^{np'}(i)^{foc})$$

Now using the optimality condition for the composite energy demand ( $e_t(i)^{foc}$ ), the energy price index  $p_t^e$  can be eliminated in the expressions of ( $e_t^{p'}(i)^{foc}$ ) and ( $e_t^{np'}(i)^{foc}$ ). Finally, we find the same demand for each energy inputs so that ( $e_t^{p'}(i)^{foc} = e_t^p(i)^{foc}$ ) and ( $e_t^{np'}(i)^{foc} = e_t^{np}(i)^{foc}$ ).

$$\frac{\partial \mathcal{L}_t}{\partial e_t^p(i)} = 0 \iff v \lambda_t(i) \frac{y_t(i)}{e_t(i)^\varphi} \omega e_t^p(i)^{\varphi-1} = p_t^p(1 + \tau_t \Xi) \quad (e_t^p(i)^{foc})$$

$$\frac{\partial \mathcal{L}_t}{\partial e_t^{np}(i)} = 0 \iff v \lambda_t(i) \frac{y_t(i)}{e_t(i)^\varphi} (1 - \omega) e_t^{np}(i)^{\varphi-1} = p_t^{np} \quad (e_t^{np'}(i)^{foc})$$

We have just demonstrated that the intermediate-good firm's cost minimization program with the two energy inputs ( $e_t^p(i)$  and  $e_t^{np}(i)$ ) is equivalent to the two-step program of



cost minimization with composite energy input ( $e_t(i)$ ) (step 1), and then maximizing program for each energy input (step 2).

The producers of energy decide their labor demand by maximizing their profit in perfect competition. They have access to a linear technology based on employing labor to produce energy at constant productivity. Hence, the representative firm that produces polluting energy solves this program

$$\begin{aligned} \underset{n_t^p}{Max} \quad & p_t^p e_t^p - w_t n_t^p \\ \text{s.to} \quad & e_t^p = a_t^p (n_t^p)^{\varsigma^p} \end{aligned}$$

and the first order condition brings the following optimal demand for labor in the polluting energy sector

$$n_t^p = \left( \frac{p_t^p a_t^p \varsigma^p}{w_t} \right)^{\frac{1}{1-\varsigma^p}}$$

Meanwhile, the representative firm that produces non-polluting energy solves

$$\begin{aligned} \underset{n_t^{np}}{Max} \quad & p_t^{np} e_t^{np} - w_t n_t^{np} \\ \text{s.to} \quad & e_t^{np} = a_t^{np} (n_t^{np})^{\varsigma^{np}} \end{aligned}$$

which implies holding this optimality condition to determine the optimal labor demand in the non-polluting energy sector

$$n_t^{np} = \left( \frac{p_t^{np} a_t^{np} \varsigma^{np}}{w_t} \right)^{\frac{1}{1-\varsigma^{np}}}$$

**C.1.0.3.2 Optimizing program in the case of cap-and-trade market:** Next we consider the case of cap-and-trade market. In that case, the government decides exogenously the quantity of permits  $\bar{G}_t$  and sells it to the firm at market price  $P_{\bar{G},t}$ . The maximisation program of the firm is consequently: the maximization program described in the article:

$$\begin{aligned} \text{Max} \quad & p_t^e e_t(i) - p_t^p e_t^p(i) - P_{G,t} \bar{G}_t - p_t^{np} e_t^{np}(i) \\ \text{s.t} \quad & \begin{cases} e_t(i) = [\omega e_t^p(i)^\varphi + (1-\omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} \\ \bar{G}_t = \Xi e_t^p(i) \end{cases} \end{aligned}$$

$$\mathcal{L}_t = p_t^e [\omega e_t^p(i)^\varphi + (1 - \omega) e_t^{np}(i)^\varphi]^{\frac{1}{\varphi}} - p_t^p e_{p,t}(i) - P_{G,t} \Xi e_{p,t}(i) - p_t^{np} e_t^{np}(i)$$

Optimality conditions obtained in the following case are:

$$\frac{\partial \mathcal{L}_t}{\partial e_t^p(i)} = 0 \iff p_t^e \frac{e_t(i)}{e_t(i)^\varphi} \omega e_t^p(i)^{\varphi-1} = p_t^p + P_{G,t} \Xi \quad (e_t^{p'}(i)^{foc})$$

$$\frac{\partial \mathcal{L}_t}{\partial e_t^{np}(i)} = 0 \iff p_t^e \frac{e_t(i)}{e_t(i)^\varphi} (1 - \omega) e_t^{np}(i)^{\varphi-1} = p_t^{np} \quad (e_t^{np'}(i)^{foc})$$

Using again the same technique as in the case of the carbon tax by eliminating the energy price index  $p_t^e$  in the first order conditions for each energy inputs leads to the demand for each energy inputs:

$$\frac{\partial \mathcal{L}_t}{\partial e_t^p(i)} = 0 \iff v \lambda_t(i) \frac{y_t(i)}{e_t(i)^\varphi} \omega e_t^p(i)^{\varphi-1} = p_t^p + P_{G,t} \Xi \quad (e_t^{p'}(i)^{foc})$$

$$\frac{\partial \mathcal{L}_t}{\partial e_t^{np}(i)} = 0 \iff v \lambda_t(i) \frac{y_t(i)}{e_t(i)^\varphi} (1 - \omega) e_t^{np}(i)^{\varphi-1} = p_t^{np} \quad (e_t^{np'}(i)^{foc})$$

#### C.1.0.4 Overall resources constraint

**C.1.0.4.1 For the carbon tax:** To close the model we must now find the expression of the overall resource constraint. Each monopolistic firm  $i$  distributes dividends  $div_t(i)$  to its owner the representative household. The dividends paid are:

$$div_t(i) = \frac{P_t(i)}{P_t} y_t(i) - w_t n_t(i) - r_t^k k_{t-1}(i) - (1 + \tau_t \Xi) p_t^p e_t^p(i) - p_t^{np} e_t^{np}(i)$$

Aggregate dividends across the mass of firms allocated in the unit interval are

$$\begin{aligned} div_t = \int_0^1 div_t(i) di &= \int_0^1 \frac{P_t(i)}{P_t} y_t(i) di - w_t \int_0^1 n_t(i) di - r_t^k \int_0^1 k_{t-1}(i) di \\ &\quad - (1 + \tau_t \Xi) p_t^p \int_0^1 e_t^p(i) di - p_t^{np} \int_0^1 e_t^{np}(i) di \end{aligned}$$

where inserting the Dixit-Stiglitz demand constraint, (4.3.5), and the market clearing conditions, give

$$div_t = y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di - w_t (n_t^s - n_t^p - n_t^{np}) - r_t^k k_{t-1} - (1 + \tau_t \Xi) p_t^p e_t^p - p_t^{np} e_t^{np}$$

The zero profit condition in the energy sector implies both  $p_t^p e_t^p = w_t^p n_t^p$  and  $p_t^{np} e_t^{np} = w_t^{np} n_t^{np}$ , which can be substituted in the previous expression to obtain

$$div_t = y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di - w_t n_t^s - r_t^k k_{t-1} - \tau_t \Xi p_t^p e_t^p \quad (C.1.1)$$

Plugging (C.1.2) into the household budget constraint (4.3.2) yields

$$c_t + i_t + a(z_t)k_{t-1} + \left( \exp(\epsilon_t^b) (1 + r_t) \right)^{-1} b_{t+1} - b_t + m_t - (1 + \pi_t)^{-1} m_{t-1} + h(c_t, m_t) = y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di - \tau_t \Xi p_t^p e_t^p - tax_t$$

Finally, using the government budget constraint, (4.3.13), the aggregation for polluting energy  $e_t^p = \int_0^1 e_t^p(i) di$ , and the definition of the price dispersion indicator of Schmitt-Grohe and Uribe (2005),  $PD_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di$ , result in the overall resources constraint:

$$c_t + i_t + a(z_t)k_{t-1} + h(c_t, m_t) + g_t = PD_t y_t$$

**C.1.0.4.2 For the cap-and-trade market:** We use the same mechanism to find the overall resource constraint in the case of the cap-and-trade market:

$$div_t(i) = \frac{P_t(i)}{P_t} y_t(i) - w_t n_t(i) - r_t^k k_{t-1}(i) - (p_t^p + P_{G,t} \Xi) e_t^p(i) - p_t^{np} e_t^{np}(i)$$

Aggregate dividends across the mass of firms allocated in the unit interval are

$$div_t = \int_0^1 div_t(i) di = \int_0^1 \frac{P_t(i)}{P_t} y_t(i) di - w_t \int_0^1 n_t(i) di - r_t^k \int_0^1 k_{t-1}(i) di - (p_t^p + P_{G,t} \Xi) \int_0^1 e_t^p(i) di - p_t^{np} \int_0^1 e_t^{np}(i) di$$

where inserting the Dixit-Stiglitz demand constraint, (4.3.5), and the market clearing conditions, give

$$div_t = y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di - w_t (n_t^s - n_t^p - n_t^{np}) - r_t^k k_{t-1} - (p_t^p + P_{G,t} \Xi) e_t^p - p_t^{np} e_t^{np}$$

The zero profit condition in the energy sector implies both  $p_t^p e_t^p = w_t^p n_t^p$  and  $p_t^{np} e_t^{np} = w_t^{np} n_t^{np}$ , which can be substituted in the previous expression to obtain

$$div_t = y_t \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\theta} di - w_t n_t^s - r_t^k k_{t-1} - P_{G,t} \Xi e_t^p \quad (C.1.2)$$

Plugging (C.1.2) into the household budget constraint (4.3.2) yields

$$c_t + i_t + a(z_t)k_{t-1} + \left(\exp\left(\epsilon_t^b\right)(1+r_t)\right)^{-1} b_{t+1} - b_t + m_t - (1+\pi_t)^{-1} m_{t-1} + h(c_t, m_t) = y_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} di - P_{G,t} \Xi e_t^p - tax_t$$

In a cap-and-trade market, the government budget constraint is:

$$g_t = tax_t + P_{G,t} \Xi \int_0^1 e_t^p(i) di + \left(\exp\left(\epsilon_t^b\right)(1+r_t)\right)^{-1} b_{t+1} - b_t + m_t - (1+\pi_t)^{-1} m_{t-1}$$

Using this previous result with the aggregation for polluting energy  $e_t^p = \int_0^1 e_t^p(i) di$ , and the definition of the price dispersion indicator of Schmitt-Grohe and Uribe (2005),  $PD_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} di$ , result in the same overall resources constraint as in the carbon tax scenario:

$$c_t + i_t + a(z_t)k_{t-1} + h(c_t, m_t) + g_t = PD_t y_t$$

### C.1.0.5 The natural-rate block

We finally introduce the output gap as the fractional deviation of current output from the scenario where prices are flexible. Defining  $\tilde{y}_t$  as the output gap, it implies that:

$$\tilde{y}_t = \frac{y_t}{\bar{y}_t} - 1 \quad (\text{C.1.3})$$

where  $\bar{y}_t$  is the amount of output produced with full price flexibility.<sup>1</sup> In the absence of nominal rigidities, prices would be set optimally by all the firms. This corresponds to the extreme case where  $\eta$  is equal to 0. We will derive the natural-rate block as the general equilibrium conditions of our economy when price stickiness has been removed ( $\eta = 0$ ).

Symmetric behavior across firms makes prices and marginal costs be the same (and optimal) for all of them, which implies  $P_t^* = P_t$  and  $mc_t = \frac{\theta-1}{\theta}$ . The real wage  $\bar{w}_t$ , the real interest rate  $\bar{r}_t^k$  and the real price of energy are, respectively, fractions of marginal product of labor, capital, and energy (equations 1 through 3)

$$\bar{w}_t = \frac{\theta-1}{\theta} (1-\alpha-v) \frac{\bar{y}_t}{\bar{n}_t}$$

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<sup>1</sup>In terms of notation, any variable topped with a horizontal bar represents its value assuming flexible prices in the economy.

$$\bar{r}_t^k = \frac{\theta - 1}{\theta} \alpha \frac{\bar{y}_t}{\bar{k}_t}$$

$$\bar{p}_t^e = \frac{\theta - 1}{\theta} \nu \frac{\bar{y}_t}{\bar{c}_t}$$

In the absence of nominal rigidities, equations from the optimizing household program are the labor supply curve (equations 4 and 5)

$$\bar{w}_t = \frac{\psi(\bar{n}_t^s)^\gamma (1 + h_{\bar{c}_t}(\bar{c}_t, \bar{m}_t))}{(\bar{c}_t - \chi \bar{c}_{t-1}^A)^{-\sigma} (1 - d(\bar{G}_t))}$$

with

$$h_{\bar{c}_t}(\bar{c}_t, \bar{m}_t) = \frac{\phi_1}{1 - \phi_2} \left( \frac{\bar{c}_t}{\bar{m}_t} \right)^{\phi_2/(1-\phi_2)}$$

the consumption Euler equation (equation 6)

$$\frac{(1 - d(\bar{G}_t)) (\bar{c}_t - \chi \bar{c}_{t-1}^A)^{-\sigma}}{(1 + h_{\bar{c}_t}(\bar{c}_t, \bar{m}_t)) (1 + \bar{r}_t) \exp(\epsilon_t^b)} = \beta \mathbb{E}_t \left( \frac{(1 - d(\bar{G}_{t+1})) (\bar{c}_{t+1} - \chi \bar{c}_t^A)^{-\sigma}}{(1 + h_{\bar{c}_{t+1}}(\bar{c}_{t+1}, \bar{m}_{t+1}))} \right)$$

the arbitrage condition between capital and bonds (equations 7 to 8)

$$\exp(\epsilon_t^b) (1 + \bar{r}_t) = \frac{\bar{r}_{t+1}^k \bar{z}_{t+1} - a(\bar{z}_{t+1}) + \bar{q}_{t+1}}{\bar{q}_t}$$

with

$$\bar{r}_t^k = a'(\bar{z}_t)$$

the optimal condition of investment (equations 9 through 11)

$$1 = \bar{q}_t \exp(\epsilon_t^i) \left[ 1 - S\left(\frac{\bar{i}_t}{\bar{i}_{t-1}}\right) - S'\left(\frac{\bar{i}_t}{\bar{i}_{t-1}}\right) \frac{\bar{i}_t}{\bar{i}_{t-1}} \right] \\ + \exp(\epsilon_t^b) (1 + \bar{r}_t) \mathbb{E}_t \bar{q}_{t+1} \exp(\epsilon_{t+1}^i) S'\left(\frac{\bar{i}_{t+1}}{\bar{i}_t}\right) \left(\frac{\bar{i}_{t+1}}{\bar{i}_t}\right)^2$$

and law of motion for capital:

$$\bar{k}_t = (1 - \delta) \bar{k}_{t-1} + \exp(\epsilon_t^i) \left[ 1 - S\left(\frac{\bar{i}_t}{\bar{i}_{t-1}}\right) \right] \bar{i}_t$$

and the money demand equation (equations 12 and 13)

$$-h_{\bar{m}}(\bar{c}_t, \bar{m}_t) = \frac{\bar{R}_t}{1 + \bar{R}_t} = \frac{1 - \exp(\epsilon_t^b) (1 + \bar{R}_t) (1 + \bar{R}_t) \exp(\epsilon_t^b)}{1 + \bar{R}_t}$$

with

$$h_{\bar{m}}(\bar{c}_t, \bar{m}_t) = -\frac{\phi_1 \phi_2}{1 - \phi_2} \left( \frac{\bar{c}_t}{\bar{m}_t} \right)^{1/(1-\phi_2)}$$

Equations from the energy market are (equations 14 through 20)

$$\begin{aligned} \bar{e}_t &= [\omega(\bar{e}_t^p)^\varphi + (1 - \omega)(\bar{e}_t^{np})^\varphi]^{\frac{1}{\varphi}} \\ \bar{e}_t^p &= \left( \frac{(1 + \tau_t \Xi) \bar{p}_t^p}{\omega \bar{p}_t^e} \right)^{\frac{1}{\varphi-1}} \bar{e}_t \end{aligned}$$

In the case of the cap-and-trade market demand for polluting energy input is:

$$\begin{aligned} \bar{e}_t^p &= \left( \frac{1 \bar{p}_t^p + \bar{P}_{G,t}}{\omega \bar{p}_t^e} \right)^{\frac{1}{\varphi-1}} \bar{e}_t \\ \bar{e}_t^{np} &= \left( \frac{1}{(1 - \omega) \bar{p}_t^e} \right)^{\frac{1}{\varphi-1}} \bar{e}_t \\ \bar{n}_t^p &= \left( \frac{\bar{p}_t^p a_t^p \varsigma^p}{\bar{w}_t} \right)^{\frac{1}{1-\varsigma^p}} \\ \bar{e}_t^p &= a_t^p (\bar{n}_t^p)^{\varsigma^p} \\ \bar{n}_t^{np} &= \left( \frac{\bar{p}_t^{np} a_t^{np} \varsigma^{np}}{\bar{w}_t} \right)^{\frac{1}{1-\varsigma^{np}}} \\ \bar{e}_t^{np} &= a_t^{np} (\bar{n}_t^{np})^{\varsigma^{np}} \end{aligned}$$

The Cobb-Douglas production function is (equation 21)

$$\bar{y}_t = a_t \bar{k}_t^\alpha \bar{n}_t^{1-\alpha-v} \bar{e}_t^v$$

The market clearing condition for labor (equation 22)

$$\bar{n}_t^s = \bar{n}_t^p + \bar{n}_t^{np} + \bar{n}_t$$

The monetary policy rule consistent with flexible prices (zero output gap) is (equation 23)

$$1 + \bar{R}_t = ((1 + r)(1 + \pi)^{1-\mu_\pi})^{1-\mu_R} (1 + \bar{R}_{t-1})^{\mu_R} (1 + \bar{\pi}_t)^{(1-\mu_R)\mu_\pi}$$

and the Fisher relation used in the model is (equation 24)

$$1 + \bar{r}_t = \frac{1 + \bar{R}_t}{\mathbb{E}_t(1 + \bar{\pi}_{t+1})}$$

In the absence of nominal rigidities, price dispersion is null, consequently the overall resource constraint is (equations 25 and 26)

$$\bar{c}_t + \bar{i}_t + a(\bar{z}_t)\bar{k}_{t-1} + h(\bar{c}_t, \bar{m}_t) + g_t = \bar{y}_t$$

and total transaction costs (discarding null consumption)

$$h(\bar{c}_t, \bar{m}_t) = \phi_0 + \bar{c}_t \phi_1 \left( \frac{\bar{c}_t}{\bar{m}_t} \right)^{\phi_2/(1-\phi_2)}$$

Number of endogenous variables (27):  $\bar{y}_t, \bar{c}_t, \bar{w}_t, \bar{r}_t^k, \bar{p}_t^e, \bar{n}_t, \bar{n}_t^s, \bar{n}_t^p, \bar{n}_t^{np}, \bar{e}_t, \bar{k}_{t+1}, \bar{e}_t^p, \bar{e}_t^{np}, \bar{p}_t^p, \bar{p}_t^{np}, \bar{m}_t, \bar{R}, \bar{r}_t, \bar{\pi}_t, a(u_t), h(\bar{c}_t, \bar{m}_t), h_{\bar{m}}(\bar{c}_t, \bar{m}_t), h_{\bar{c}_t}(\bar{c}_t, \bar{m}_t), i_t, z_t, M_t$  and  $a'(u_t)$

### Set of dynamic equations

The following set of equations summarises the model. There is a total of 53 equations for 53 endogenous variables:

From the household optimizing program we have a total of eight equations that comprises, the labor supply curve:

$$w_t = \frac{\psi(n_t^s)^\gamma (1 + h_{c_t}(c_t, m_t))}{(c_t - \chi c_{t-1}^A)^{-\sigma} (1 - d(G_t))}$$

with the marginal transaction costs of consumption

$$h_{c_t}(c_t, m_t) = \frac{\phi_1}{1 - \phi_2} \left( \frac{c_t}{m_t} \right)^{\phi_2/(1-\phi_2)},$$

the consumption Euler equation (featuring a real balance effect through the marginal transaction costs)

$$\frac{(1 - d(G_t)) (c_t - \chi c_{t-1}^A)^{-\sigma}}{(1 + h_{c_t}(c_t, m_t)) (1 + r_t) \exp(\epsilon_t^b)} = \beta \mathbb{E}_t \left( \frac{(1 - d(G_{t+1})) (c_{t+1} - \chi c_t^A)^{-\sigma}}{(1 + h_{c_{t+1}}(c_{t+1}, m_{t+1}))} \right)$$

the arbitrage condition for the portfolio choice between bonds and capital

$$\exp(\epsilon_t^b) (1 + r_t) = \frac{r_{t+1}^k z_{t+1} - a(z_{t+1}) + q_{t+1}(1 - \delta)}{q_t}$$

and the marginal utilization cost of capital

$$r_t^k = a'(z_t)$$

the law of motion for capital:

$$k_t = (1 - \delta) k_{t-1} + \exp(\epsilon_t^i) \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right] i_t,$$

and the optimal investment choice:

$$1 = q_t \exp(\epsilon_t^i) \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) - S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] + \exp(\epsilon_t^b) (1 + r_t) \mathbb{E}_t q_{t+1} \exp(\epsilon_{t+1}^i) S' \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \quad (\text{C.1.4})$$



the money demand function

$$-h_m(c_t, m_t) = \frac{R_t}{1 + R_t} = \frac{1 - \exp(\epsilon_t^b)(1 + R_t)}{(1 + R_t) \exp(\epsilon_t^b)}$$

with the marginal transaction-facilitating return on money

$$h_m(c_t, m_t) = -\frac{\phi_1 \phi_2}{1 - \phi_2} \left( \frac{c_t}{m_t} \right)^{1/\phi_2}$$

From the inflation block, firm pricing behavior, sticky prices and the Dixit-Stiglitz aggregation scheme result in the following 4 equations (accumulated total is 12):

$$A_t = y_t m c_t + \beta \eta \mathbb{E}_t \left[ A_{t+1} \left( (1 + \pi_{t+1}) / \left( (1 + \pi_t)^\kappa (1 + \pi + \nu_{t+1})^{1-\kappa} \right) \right)^{\theta/(1-\alpha)} \right]$$

$$B_t = m c_t + \beta \eta \mathbb{E}_t \left[ B_{t+1} \left( (1 + \pi_{t+1}) / \left( (1 + \pi_t)^\kappa (1 + \pi + \nu_{t+1})^{1-\kappa} \right) \right)^{\theta-1} \right]$$

$$B_t \left( \frac{P_t^*}{P_t} \right) = \frac{\theta}{\theta - 1} A_t$$

$$\left( \frac{P_t^*}{P_t} \right)^{\theta-1} = (1 - \eta) + \eta \left[ (1 + \pi_{t-1})^{\kappa p} (1 + \pi + \nu_t)^{1-\kappa p} \right]^{1-\theta} \left[ (1 + \pi_t) \left( \frac{P_t^*}{P_t} \right) \right]^{\theta-1}$$

From the firm's optimizing program, we also have 4 equations for the demand for inputs (labor, capital, and energy) and the production function (accumulated total is 16):

$$\frac{w_t}{m c_t} = (1 - \alpha - v) \frac{y_t}{n_t}$$

$$\frac{r_t^k}{m c_t} = \alpha \frac{y_t}{k_t}$$

$$\frac{p_t^e}{m c_t} = v \frac{y_t}{e_t}$$

$$y_t = a_t k_t^\alpha n_t^{1-\alpha-v} e_t^v$$

From the energy market, we have 7 equations for the CES production function, the demand for polluting and non-polluting inputs,  $y$  and the production functions for polluting and non-polluting energies and demand for labor in both sectors (accumulated total is 23):

$$e_t = [\omega (e_t^p)^\varphi + (1 - \omega) (e_t^{np})^\varphi]^{\frac{1}{\varphi}}$$

$$e_t^p = \left( \frac{(1 + \tau_t \Xi) p_t^p}{\omega p_t^e} \right)^{\frac{1}{\varphi-1}} e_t$$

In the case of a cap-and-trade market, the previous equation is modified:

$$e_t^p = \left( \frac{1 p_t^p + P_{G,t} \Xi}{\omega p_t^e} \right)^{\frac{1}{\varphi-1}} e_t$$

Plus the constraint:

$$\begin{aligned} \bar{G}_t &= \Xi e_t^p \\ e_t^{np} &= \left( \frac{1 p_t^{np}}{(1 - \omega) p_t^e} \right)^{\frac{1}{\varphi-1}} e_t \\ n_t^p &= \left( \frac{p_t^p a_t^p s^p}{w_t} \right)^{\frac{1}{1-\varsigma^p}} \\ e_t^p &= a_t^p (n_t^p)^{\varsigma^p} \\ n_t^{np} &= \left( \frac{p_t^{np} a_t^{np} s^{np}}{w_t} \right)^{\frac{1}{1-\varsigma^{np}}} \\ e_t^{np} &= a_t^{np} (n_t^{np})^{\varsigma^{np}} \end{aligned}$$

The market clearing condition for labor

$$n_t^s = n_t^p + n_t^{np} + n_t$$

plus the overall resource constraint bring 2 more equations (accumulated total is 25):

$$PD_t y_t = c_t + i_t + a(u_t) k_{t-1} + h(c_t, m_t) + g_t$$

with exogenous  $g_t$ , this transaction costs technology

$$h(c_t, m_t) = \phi_0 + \phi_1 c_t \left( \frac{c_t}{m_t} \right)^{\phi_2 / (1 - \phi_2)},$$

and the price dispersion indicator

$$PD_t = (1 - \eta) \left( \frac{P_t^*}{P_t} \right)^{1-\theta} + \eta \left( (1 + \pi_{t-1})^{\kappa_p} (1 + \pi + \nu_t)^{1-\kappa_p} \right)^{1-\theta} PD_{t-1}$$

We assume a monetary block that includes 4 more equations (accumulated total is 29):

a Taylor (1993)-type monetary policy rule

$$1 + R_t = ((1 + r) (1 + \pi)^{1-\mu_\pi})^{1-\mu_R} (1 + R_{t-1})^{\mu_R} (1 + \pi_t)^{(1-\mu_R)\mu_\pi} (1 + \tilde{y}_t)^{(1-\mu_R)\mu_y}$$

the definition of the output gap

$$\tilde{y}_t = \frac{y_t}{\bar{y}_t} - 1,$$

the Fisher relationship

$$1 + r_t = \frac{1 + R_t}{1 + \mathbb{E}_t \pi_{t+1}},$$

and the relationship between nominal money growth,  $g_{M_t}$ , real money and inflation:

$$g_{M_t} = \frac{m_t}{m_{t-1}}(1 + \pi_t) - 1$$

The flexible-price (natural-rate) block comprises 24 more equations (accumulated total is 53)

$$\begin{aligned} \bar{w}_t &= \frac{\theta - 1}{\theta} (1 - \alpha - v) \frac{\bar{y}_t}{\bar{n}_t} \\ \bar{r}_t^k &= \frac{\theta - 1}{\theta} \alpha \frac{\bar{y}_t}{\bar{k}_t} \\ \bar{p}_t^e &= \frac{\theta - 1}{\theta} v \frac{\bar{y}_t}{\bar{e}_t} \\ \bar{w}_t &= \frac{\psi (\bar{n}_t^s)^\gamma (1 + h_{\bar{c}_t}(\bar{c}_t, \bar{m}_t))}{(\bar{c}_t - \chi \bar{c}_{t-1}^A)^{-\sigma} (1 - d(\bar{G}_t))} \\ h_{\bar{c}_t}(\bar{c}_t, \bar{m}_t) &= \frac{\phi_1}{1 - \phi_2} \left( \frac{\bar{c}_t}{\bar{m}_t} \right)^{\phi_2 / (1 - \phi_2)} \\ \frac{(1 - d(\bar{G}_t)) (\bar{c}_t - \chi \bar{c}_{t-1}^A)^{-\sigma}}{(1 + h_{\bar{c}_t}(\bar{c}_t, \bar{m}_t)) (1 + \bar{r}_t) \exp(\epsilon_t^b)} &= \beta \mathbb{E}_t \left( \frac{(1 - d(\bar{G}_t)) (\bar{c}_{t+1} - \chi \bar{c}_t^A)^{-\sigma}}{(1 + h_{\bar{c}_{t+1}}(\bar{c}_{t+1}, \bar{m}_{t+1}))} \right) \\ \exp(\epsilon_t^b) (1 + \bar{r}_t) &= \frac{\bar{r}_{t+1}^k \bar{z}_{t+1} - a(\bar{z}_{t+1}) + \bar{q}_{t+1}}{\bar{q}_t} \\ \bar{r}_t^k &= a'(\bar{z}_t) \end{aligned}$$

$$\begin{aligned} 1 &= \bar{q}_t \exp(\epsilon_t^i) \left[ 1 - S \left( \frac{\bar{i}_t}{\bar{i}_{t-1}} \right) - S' \left( \frac{\bar{i}_t}{\bar{i}_{t-1}} \right) \frac{\bar{i}_t}{\bar{i}_{t-1}} \right] + \\ &\quad \exp(\epsilon_t^b) + (1 + \bar{r}_t) \mathbb{E}_t \bar{q}_{t+1} \exp(\epsilon_{t+1}^i) S' \left( \frac{\bar{i}_{t+1}}{\bar{i}_t} \right) \left( \frac{\bar{i}_{t+1}}{\bar{i}_t} \right)^2 \\ \bar{k}_t &= (1 - \delta) \bar{k}_{t-1} + \exp(\epsilon_t^i) \left[ 1 - S \left( \frac{\bar{i}_t}{\bar{i}_{t-1}} \right) \right] \bar{i}_t \\ -h_{\bar{m}}(\bar{c}_t, \bar{m}_t) &= \frac{\bar{R}_t}{1 + \bar{R}_t} = \frac{1 - \exp(\epsilon_t^b) (1 + \bar{R}_t)}{(1 + \bar{R}_t) \exp(\epsilon_t^b)} \end{aligned}$$

$$h_{\bar{m}}(\bar{c}_t, \bar{m}_t) = -\frac{\phi_1 \phi_2}{1 - \phi_2} \left( \frac{\bar{c}_t}{\bar{m}_t} \right)^{1/(1-\phi_2)}$$

$$\bar{e}_t = [\omega(\bar{e}_t^p)^\varphi + (1 - \omega)(\bar{e}_t^{np})^\varphi]^{\frac{1}{\varphi}}$$

$$\bar{e}_t^p = \left( \frac{(1 + \tau_t \Xi) \bar{p}_t^p}{\omega \bar{p}_t^e} \right)^{\frac{1}{\varphi-1}} \bar{e}_t$$

In the case of a cap-an-trade market the previous equation is replaced by:

$$\bar{e}_t^p = \left( \frac{1 \bar{p}_t^p + \bar{P}_{G,t} \Xi}{\omega \bar{p}_t^e} \right)^{\frac{1}{\varphi-1}} \bar{e}_t$$

$$\bar{G}_t = \Xi \bar{e}_t^p$$

$$\bar{e}_t^{np} = \left( \frac{1 \bar{p}_t^{np}}{(1 - \omega) \bar{p}_t^e} \right)^{\frac{1}{\varphi-1}} \bar{e}_t$$

$$\bar{n}_t^p = \left( \frac{\bar{p}_t^p a_t^p \zeta^p}{\bar{w}_t} \right)^{\frac{1}{1-\zeta^p}}$$

$$\bar{e}_t^p = a_t^p (\bar{n}_t^p)^{\zeta^p}$$

$$\bar{n}_t^{np} = \left( \frac{\bar{p}_t^{np} a_t^{np} \zeta^{np}}{\bar{w}_t} \right)^{\frac{1}{1-\zeta^{np}}}$$

$$\bar{e}_t^{np} = a_t^{np} (\bar{n}_t^{np})^{\zeta^{np}}$$

$$\bar{y}_t = a_t \bar{k}_t^\alpha \bar{n}_t^{1-\alpha-v} \bar{e}_t^v$$

$$\bar{n}_t^s = \bar{n}_t^p + \bar{n}_t^{np} + \bar{n}_t$$

$$1 + \bar{R}_t = ((1 + r)(1 + \pi)^{1-\mu_\pi})^{1-\mu_R} (1 + \bar{R}_{t-1})^{\mu_R} (1 + \bar{\pi}_t)^{(1-\mu_R)\mu_\pi}$$

$$1 + \bar{r}_t = \frac{1 + \bar{R}_t}{\mathbb{E}_t(1 + \bar{\pi}_{t+1})}$$

$$\bar{c}_t + \bar{i}_t + a(\bar{z}_t) \bar{k}_{t-1} + h(\bar{c}_t, \bar{m}_t) + g_t = \bar{y}_t$$

$$h(\bar{c}_t, \bar{m}_t) = \phi_0 + \bar{c}_t \phi_1 \left( \frac{\bar{c}_t}{\bar{m}_t} \right)^{\phi_2/(1-\phi_2)}$$

## C.2 Log-Linearized model

Introduce hat variables as the log-deviations of the variables around its steady state, and dropping times subscript to denote its steady state value, thus for a variable  $X_t$  then:

$$\hat{X}_t = \frac{X_t}{X} - 1$$

For sake of easy, let define  $F_t = (c_t - \chi c_{t-1}^A)^{-\sigma}$  and  $J_t = 1 + h_{ct}(c_t, m_t)$  and replace them into the consumption Euler equation:

$$\frac{(1 - d(G_t)) (c_t - \chi c_{t-1}^A)^{-\sigma}}{(1 + h_{ct}(c_t, m_t)) (1 + r_t) \exp(\epsilon_t^b)} = \beta \mathbb{E}_t \left( \frac{(1 - d(G_{t+1})) (c_{t+1} - \chi c_t^A)^{-\sigma}}{(1 + h_{ct+1}(c_{t+1}, m_{t+1}))} \right)$$

Taking the logs both side of the previous equation yield to:

$$\begin{aligned} \log(1 - d(G_t)) + \log F_t - \log(1 + h_{ct}(c_t, m_t)) - \log(1 + r_t) - \epsilon_t^b = \\ \log \beta - \mathbb{E}_t (\log(1 - d(G_{t+1})) - \log F_{t+1} + \log(1 + h_{ct+1}(c_{t+1}, m_{t+1}))) \end{aligned} \quad (\text{C.2.1})$$

In the steady state, the previous equation becomes:

$$\begin{aligned} \log(1 - d(G)) + \log F - \log(1 + h_c(c, m)) - \log(1 + r) \\ = \log \beta + \log(1 - d(G)) + \log F - \log(1 + h_c(c, m)) \end{aligned}$$

Making the difference between the two last expression, and recalling that  $\log(1 + r_t) \simeq r_t$  and  $\log(1 + h_{c,t}(c_t, m_t)) \simeq h_{c,t}(c_t, m_t)$ :

$$\begin{aligned} \log \left( \frac{1 - d(G_t)}{1 - d(G)} \right) + \log \left( \frac{F_t}{F} \right) - (h_{ct}(c_t, m_t) - h_c(c, m)) - (r_t + \epsilon_t^b - r) \\ = \mathbb{E}_t \left( \log \left( \frac{1 - d(G_{t+1})}{1 - d(G)} \right) + \log \left( \frac{F_{t+1}}{F} \right) - (h_{ct+1}(c_{t+1}, m_{t+1}) - h_c(c, m)) \right) \\ \\ 1 - \widehat{d(G_t)} + \hat{F}_t - \hat{h}_{ct} h_c - (r_t + \epsilon_t^b - r) = \mathbb{E}_t \left( 1 - \widehat{d(G_{t+1})} - \hat{F}_{t+1} + \hat{h}_{ct+1} h_c \right) \\ - \frac{d(G)}{1 - d(G)} \widehat{d(G_t)} + \hat{F}_t - \hat{h}_{ct} h_c - (r_t + \epsilon_t^b - r) = -\mathbb{E}_t \left( \frac{d(G)}{1 - d(G)} \widehat{d(G_{t+1})} + \hat{F}_{t+1} - \hat{h}_{ct+1} h_c \right) \end{aligned} \quad (\text{C.2.2})$$

Where,  $d(G) = d_0 + d_1G + d_2G^2$ , and  $\widehat{d(G_t)} = \frac{d_1 + d_2}{d(G)} \hat{G}_t$

We now need to find the expression of  $\hat{F}_t$ :

$$F_t = (c_t - \chi c_{t-1}^A)^{-\sigma}$$

take logs on both side:

$$\log F_t = -\sigma \log (c_t - \chi c_{t-1}^A)$$

Take total differential in steady-state:

$$\frac{1}{F} dF_t = -\sigma \frac{1}{(1-\chi)} \left( \frac{dc_t}{c} - \chi \frac{dc_{t-1}}{c} \right)$$

$$\hat{F}_t \simeq -\sigma \frac{1}{(1-\chi)} (\hat{c}_t - \chi \hat{c}_{t-1}) \quad (\text{C.2.3})$$

Substitute (C.2.3) into (C.2.2)

$$\begin{aligned} -\frac{d(G)}{1-d(G)} \widehat{d(G_t)} - \sigma \frac{1}{(1-\chi)} (\hat{c}_t - \chi \hat{c}_{t-1}) - \hat{h}_{ct} h_c - (r_t + \epsilon_t^b - r) = \\ -\mathbb{E}_t \left( \frac{d(G)}{1-d(G)} \widehat{d(G_{t+1})} + \sigma \frac{1}{(1-\chi)} (\hat{c}_{t+1} - \chi \hat{c}_t) + \hat{h}_{ct+1} h_c \right) \end{aligned} \quad (\text{C.2.4})$$

Substitute this last result into the linearised Euler consumption equation (C.2.4), and after some easy algebra, one obtains:

$$\begin{aligned} \hat{c}_t = \frac{\chi}{1+\chi} \hat{c}_{t-1} + \frac{1}{1+\chi} \mathbb{E}_t \hat{c}_{t+1} - \frac{(1-\chi)h_c}{\sigma(1+\chi)} (\hat{h}_{ct} - \mathbb{E}_t \hat{h}_{ct+1}) \\ - \frac{(1-\chi)}{(1+\chi)\sigma} (r_t + \epsilon_t^b - r) - \frac{(1-\chi)}{(1+\chi)\sigma} \frac{d(G)}{(1-d(G))} (\widehat{d(G_t)} - \mathbb{E}_t \widehat{d(G_{t+1})}) \end{aligned} \quad (\text{C.2.5})$$

Taking the log on both sides of the relation between polluting energy  $e_t^p$  and GHG emissions  $G_t$ :

$$\log G_t = \log \Xi + \log e_t^p$$

Using the steady state condition  $\log G = \log \Xi + \log e^p$  gives the log linearised equation:

$$\hat{G}_t = \hat{e}_t^p \quad (\text{C.2.6})$$

Using the same technique for the marginal transaction cost of consumption leads to:

$$\hat{h}_{ct} = \frac{\phi_2}{1 - \phi_2} (\hat{c}_t - \hat{m}_t) \quad (\text{C.2.7})$$

Equalizing the money demand demand function with the marginal transaction-facilitating return on money, using the following approximation  $R_t/(1 + R_t) \simeq R_t$ . Then taking the logs on both sides and subtracting the steady state condition gives, a:

$$\log\left(\frac{R_t}{R}\right) = \frac{1}{1 - \phi_2} \left( \log\left(\frac{c_t}{c}\right) - \log\left(\frac{m_t}{m}\right) \right)$$

Replacing  $\hat{R}_t = (R_t - R)/R$ , demand for money is:

$$\hat{m}_t = \hat{c}_t - (1 - \phi_2) \frac{(R_t - R)}{R} \quad (\text{C.2.8})$$

According to Uhlig (1999), we can transform any variable  $X_t$  by this expression:  $X_t = X \exp(\hat{X}_t)$ . Let replace each variables in the transaction cost function  $h_t$ :

$$h \exp(\hat{h}_t) = \phi_0 + \phi_1 c^{\frac{1}{1-\phi_2}} m^{\frac{\phi_2}{\phi_2-1}} \exp\left(\frac{1}{1-\phi_2} \hat{c}_t + \frac{\phi_2}{\phi_2-1} \hat{m}_t\right)$$

Then using the following approximation  $\exp(a\hat{X}_t) \simeq 1 + a\hat{X}_t$ :

$$h (1 + \hat{h}_t) = \phi_0 + \phi_1 c^{\frac{1}{1-\phi_2}} m^{\frac{\phi_2}{\phi_2-1}} \left( 1 + \frac{1}{1-\phi_2} \hat{c}_t + \frac{\phi_2}{\phi_2-1} \hat{m}_t \right)$$

Using steady state condition  $h = \phi_0 + \phi_1 c^{\frac{1}{1-\phi_2}} m^{\frac{\phi_2}{\phi_2-1}}$  into the previous equation gives:

$$\hat{h}_t = \frac{h - \phi_0}{h} \frac{1}{1 - \phi_2} (\hat{c}_t - \phi_2 \hat{m}_t) \quad (\text{C.2.9})$$

Taking the logs on both sides of the Cobb-Douglas production function, and using the same technique as before, one gets:

$$\hat{y}_t = \epsilon_t + \alpha \hat{k}_t + (1 - \alpha - \nu) \hat{n}_t + \nu \hat{e}_t \quad (\text{C.2.10})$$

We follow Smets and Wouters (2007), and assume that capital supplied in period  $t$  is:

$$k_t^s = z_t k_{t-1}$$

In terms of deviation from steady state, the previous equation becomes:

$$\hat{k}_t^s = \hat{z}_t + \hat{k}_{t-1}$$

For the marginal utilization cost of capital. We use the following properties: in steady state  $z = 1$ , there's no adjustment cost for utilizing capital  $a(z) = a(1) = 0$ , and the marginal utilization cost of capital is supposed to be positive  $a'(z_t) > 0$ . Let define  $\sigma_a$  as the elasticity of marginal cost of adjusting capital utilization rate wrt the capital utilization rate:

$$\sigma_a \equiv \frac{\frac{\partial a'(z)}{a'(z)}}{\frac{\partial z}{\partial z}} = \frac{a''(z)}{a'(z)} \quad (\text{C.2.11})$$

taking a 1<sup>st</sup> Taylor order approximation of  $a'(z_t)$  yield:

$$a'(z_t) = a'(z) + \frac{a''(z_t)}{1!} (z_t - z)$$

$$a'(z_t) - a'(z) = a''(z_t) (z_t - z)$$

$$\frac{a'(z_t) - a'(z)}{a'(z)} = \sigma_a (z_t - z)$$

Thus the deviation of the marginal utilization cost of capital is:

$$\hat{a}'(z_t) = \sigma_a (z_t - z) \quad (\text{C.2.12})$$

Taking the log on both sides of the optimal condition for adjustment cost of capital:

$$\frac{\hat{r}_t^k}{z} = \sigma_a \hat{z}_t$$

$$\hat{z}_t = \frac{1}{\sigma_a} \hat{r}_t^k$$

Using (C.2.12) into the previous result:

$$\hat{r}_t^k = \hat{a}'(z_t)$$

Assuming the following properties for the adjusment cost of investment,  $S(1) = 0$ ,  $S'(1) = 0$ , and  $S''(1) = \phi_k$ . Using a 2<sup>nd</sup> order approximation for the FOC with respect to investment yields to the Smet and Wouters (2007) log-linearized equation for investment:

$$0 = \hat{q}_t + \epsilon_t^i + \left( 0 - 0 - S''(1)(\hat{i}_t - \hat{i}_{t-1}) \right) \\ + \beta \left( 0 \cdot \epsilon_{t+1}^i + S''(1)(\mathbb{E}_t \hat{i}_{t+1} - \hat{i}_t) + 0 \cdot \mathbb{E}_t \hat{q}_{t+1} - 0 \cdot (r_t - r) \right)$$



$$0 = \hat{q}_t + \hat{\epsilon}_t - \phi_k(1 + \beta)\hat{i}_t + \beta\phi_k\mathbb{E}_t\hat{i}_{t+1} - \phi_k\hat{i}_{t-1}$$

Rearranging the last equation:

$$\hat{i}_t = \frac{1}{1 + \beta}\hat{i}_{t-1} + \frac{\beta}{1 + \beta}\mathbb{E}_t\hat{i}_{t+1} + \frac{1}{(1 + \beta)\phi_k}(\hat{q}_t + \hat{\epsilon}_t^i)$$

In the Steady state the law of motion for capital is:

$$k = (1 - \delta)k + 1(1 - 0)i$$

Thus:

$$\begin{aligned} k_{t+1} - k &= (1 - \delta)(k_{t+1} - k + \exp(\epsilon_t^i) \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right)) i_t - (1 - 0)i \\ \hat{k}_t &= (1 - \delta)\hat{k}_{t-1} + \delta\epsilon_t^i + \delta i_t \end{aligned} \quad (\text{C.2.13})$$

In the steady state, the trade off condition between bonds and capital is:

$$(1 + r)q = (1 - \delta)q + zr^k - a(z)$$

Recalling that in the steady state  $q = 1$ ,  $z = 1$  and  $a(z) = 0$  then:

$$(1 + r) = r^k - \delta$$

Bonds return must be equal to the net return on physical asset. Consequently:

$$\begin{aligned} r_t + \epsilon_t^b - r + \hat{q}_t &= \frac{1 - \delta}{(1 + r)q} q \mathbb{E}_t q_{t+1} + \frac{zr^k}{(1 + r)q} \left( \mathbb{E}_t \hat{z}_{t+1} + \mathbb{E}_t \hat{r}_{t+1}^k \right) + \frac{a(z)}{(1 + r)q} \mathbb{E}_t \hat{a}(z_{t+1}) \\ \hat{q}_t &= \frac{1 - \delta}{1 + r^k - \delta} \mathbb{E}_t \hat{q}_{t+1} + \frac{r^k}{1 + r^k - \delta} \mathbb{E}_t \hat{r}_{t+1}^k - (r_t + \epsilon_t^b - r) \end{aligned}$$

Log-linearized equations for the demand inputs are easily obtained:

$$\hat{r}_t^k = \hat{y}_t - \hat{k}_t + \hat{m}c_t \quad (\text{C.2.14})$$

$$\hat{w}_t = \hat{y}_t - \hat{n}_t + \hat{m}c_t \quad (\text{C.2.15})$$

$$\hat{p}_t^e = \hat{y}_t - \hat{e}_t + \hat{m}c_t \quad (\text{C.2.16})$$

Using again  $F_t = F_t = (c_t - \chi c_{t-1}^A)^{-\sigma}$ , into the labor supply curve then taking the logs:

$$\hat{w}_t = \gamma \hat{n}_t^s + h_c \hat{h}_{ct} - \hat{F}_t - 1 - \widehat{d(G_t)}$$

Replacing  $1 - \widehat{d(G_t)}$  and  $\widehat{F}_t$  by their expression previously calculated:

$$\widehat{w}_t = \gamma \widehat{n}_t^s + h_c \widehat{h}_{ct} + \sigma \frac{1}{(1 - \chi)} (\widehat{c}_t - \chi \widehat{c}_{t-1}) + \frac{d(G)}{1 - d(G)} \widehat{d(G_t)} \quad (\text{C.2.17})$$

Using Uhlig's method for the CES production function:

$$e^\varphi \exp(\varphi \widehat{e}_t) = \omega (e^p)^\varphi \exp(\varphi \widehat{e}_t^p) + (1 - \omega) (\widehat{e}^{np})^\varphi \exp(\varphi \widehat{e}_t^{np})$$

$$e^\varphi (1 + \varphi \widehat{e}_t) = \omega (e^p)^\varphi (1 + \varphi \widehat{e}_t^p) + (1 - \omega) (\widehat{e}^{np})^\varphi (1 + \varphi \widehat{e}_t^{np})$$

Using the steady state condition  $e^\varphi = \omega (e^p)^\varphi + (1 - \omega) (\widehat{e}^{np})^\varphi$  to simplify the last expression:

$$\widehat{e}_t = \omega \left( \frac{e^p}{e} \right)^\varphi \widehat{e}_t^p + (1 - \omega) \left( \frac{\widehat{e}^{np}}{e} \right)^\varphi \widehat{e}_t^{np}$$

Log-linearized demand for each energy inputs are:

$$\widehat{e}_t^{np} = \frac{1}{\varphi - 1} (\widehat{p}_t^{np} - \widehat{p}_t^e) + \widehat{e}_t \quad (\text{C.2.18})$$

$$\widehat{e}_t^p = \frac{1}{\varphi - 1} (\widehat{p}_t^p - \widehat{p}_t^e + \Xi(\tau_t - \tau)) + \widehat{e}_t \quad (\text{C.2.19})$$

In the case of a cap-and-trade market, log-linearized energy demand for polluting input is given by:

$$\widehat{e}_t^p = \frac{1}{\varphi - 1} \left( \frac{\widehat{p}_t^p p^p}{p^p - p^q} + \frac{\widehat{p}_t^q p^q}{p^p - p^q} \right) + \widehat{e}_t \quad (\text{C.2.20})$$

Log-linearized demand for labor in both energy sectors are:

$$\widehat{n}_t^p = \frac{1}{1 - \varsigma^p} (\widehat{p}_t^p + (a_t^p - a^p) - \widehat{w}_t) \quad (\text{C.2.21})$$

$$\widehat{n}_t^{np} = \frac{1}{1 - \varsigma^{np}} (\widehat{p}_t^{np} + (a_t^{np} - a^{np}) - \widehat{w}_t) \quad (\text{C.2.22})$$

Log-linearized production functions for polluting and non-polluting energies:

$$\widehat{e}_t^p = (a_t^p - a^p) + \varsigma^p \widehat{n}_t^p \quad (\text{C.2.23})$$

$$\widehat{e}_t^{np} = (a_t^{np} - a^{np}) + \varsigma^{np} \widehat{n}_t^{np} \quad (\text{C.2.24})$$

The log-linearized version of the market clearing condition for labor is:

$$\hat{n}_t^s = \frac{n^p}{n^s} \hat{n}_t^p + \frac{n^{np}}{n^s} \hat{n}_t^{np} + \frac{n}{n^s} \hat{n}_t \quad (\text{C.2.25})$$

Assuming that price dispersion has no effect up to 1<sup>st</sup> order approximation. In the steady state, the overall resource constraint of the economy is:

$$y = c + i + a(z)k + h + g$$

Thus

$$\hat{y}_t = \frac{c}{y} \hat{c}_t + \frac{i}{y} \hat{i}_t + \frac{a(z)k}{y} \left( \widehat{a(z)} + \hat{k}_{t-1} \right) + \frac{h}{y} \hat{h}_t + \frac{g}{y} \hat{y}_t$$

In the steady state,  $a(u) = a(1) = 0$ , then:

$$\hat{y}_t = \frac{c}{y} \hat{c}_t + \frac{i}{y} \hat{i}_t + \frac{h}{y} \hat{h}_t + \frac{g}{y} \hat{y}_t \quad (\text{C.2.26})$$

Recalling equation (4.3.7):

$$P_t(i) = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j m c_{t+j} \Pi_{t,t+j}^P {}^{-\theta} P_{t+j}^\theta y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j \Pi_{t,t+j}^P {}^{1-\theta} P_{t+j}^{\theta-1} y_{t+j}}$$

In steady state:

$$P_t(i) = \frac{\theta}{\theta - 1} \frac{\frac{1}{1-\beta\eta} m c \cdot y}{\frac{1}{1-\beta\eta} y P^{-1}}$$

$$P_t(i) = \frac{\theta}{\theta - 1} P \cdot m c$$

From the Dixit-Stiglitz price aggregator:

$$P_t = \left[ (1 - \eta) P_t^{*1-\theta} + \eta P_{t-1}^{1-\theta} \left[ (1 + \pi_{t-1})^\kappa (1 + \pi + \nu_t)^{1-\kappa} \right]^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$P_t(s) = \left[ (1 + \pi_{t+1})^{-\kappa} (1 + \pi + \nu_t)^{1-\kappa} \right] P_{t-1}(s)$$

$$\pi_{t,t+1} = \pi_{t-1,t}^p$$

$$P_t(i) \cdot \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j \Pi_{t,t+j}^P {}^{1-\theta} P_{t+j}^{\theta-1} y_{t+j} = \frac{\theta}{\theta - 1} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\eta)^j m c_{t+j} \Pi_{t,t+j}^P {}^{-\theta} P_{t+j}^\theta y_{t+j}$$

$$\begin{aligned}
P_t(i) \left[ P_t^{\theta-1} y_t + \beta \eta P_{t+1}^{\theta-1} (\pi_{t,t+1}^p)^{1-\theta} y_{t+1} + \dots \right] \\
= \frac{\theta}{\theta-1} \left[ P_t^\theta y_t m c_t + \beta \eta P_{t+1}^\theta (\pi_{t,t+1})^{-\theta} y_{t+1} m c_{t+1} + \dots \right]
\end{aligned}$$

Multiplying both side by  $P_t$ :

$$\begin{aligned}
P_t(i) \left[ P_t^\theta y_t + \beta \eta P_{t+1}^\theta \left( \frac{P_t}{P_{t+1}} \right) (\pi_{t,t+1}^p)^{1-\theta} y_{t+1} + \dots \right] \\
= \frac{\theta}{\theta-1} \left[ P_t^{\theta+1} y_t m c_t + \beta \eta P_{t+1}^{\theta+1} (1 + \pi_{t+1})^{-1} (\pi_{t,t+1})^{-\theta} y_{t+1} m c_{t+1} + \dots \right]
\end{aligned}$$

$$\begin{aligned}
P_t(i) P_t^\theta \left[ y_t + \beta \eta (1 + \pi_{t+1})^{\theta-1} \left( 1 + \pi_{t,t+1}^p \right)^{1-\theta} y_{t+1} + \dots \right] \\
= \frac{\theta}{\theta-1} P_t^{\theta+1} \left[ y_t m c_t + \beta \eta y_{t+1} m c_{t+1} \left( \frac{1 + \pi_{t+1}}{\pi_{t,t+1}} \right)^\theta + \dots \right] \\
\frac{P_t(i)}{P_t} X_t = \frac{\theta}{\theta-1} Z_t
\end{aligned}$$

Taking logs on both sides:

$$\hat{P}_t(i) - \hat{P}_t + \hat{X}_t = \hat{Z}_t$$

In steady state:

$$X = y + \beta \eta y + (\beta \eta)^2 y + \dots$$

$$X = \frac{1}{1 - \beta \eta} y$$

$$Z = y m c + \beta \eta y m c + (\beta \eta)^2 y m c + \dots$$

$$Z = \frac{1}{1 - \beta \eta} y m c$$

$$\hat{X}_t = \frac{y}{X} \hat{y}_t + \frac{\beta \eta y}{X} ((\theta - 1) \pi_{t+1} - (\theta - 1) \text{ind}_{t+1} + y_{t+1}) + \dots \quad (\text{C.2.27})$$

Where

$$\log(1 + \pi_{t+1}) = \pi_{t+1}$$

and

$$\begin{aligned}
\log[(1 + \pi_t)^\kappa (1 + \pi + \nu_{t+1})] &= \kappa \pi_t + (1 - \kappa)(\pi + \nu_{t+1}) = \text{ind}_{t+1} \\
\hat{Z}_t &= \frac{y m c}{Z} (\hat{y}_t + \widehat{m c}_t) + \frac{\beta \eta y m c}{Z} (\theta \pi_{t+1} - \theta \text{ind}_{t+1} + \hat{y}_{t+1} + \widehat{m c}_{t+1}) + \dots
\end{aligned}$$

$$\frac{y}{X} = \frac{y}{(1 - \beta\eta)^{-1}y} = 1 - \beta\eta$$

and:

$$\frac{ymc}{Z} = \frac{ymc}{(1 - \beta y)^{-1}ymc} = 1 - \beta\eta$$

Thus:

$$\hat{P}_t(i) - \hat{P}_t + \hat{X}_t = \hat{Z}_t$$

$$\hat{P}_t(i) - \hat{P}_t = (1 - \beta\eta) [\widehat{mc}_t + \beta\eta\widehat{mc}_{t+1} + \dots] + \beta\eta [\pi_{t+1} - ind_{t+1} + \beta\eta(\pi_{t+2} - ind_{t+2}) + \dots]$$

$$\hat{P}_t(i) - \hat{P}_t = (1 - \beta\eta) \sum_{j=0}^{\infty} (\beta\eta)^j \widehat{mc}_{t+j} + \sum_{j=0}^{\infty} (\beta\eta)^j (\pi_{t+j} - ind_{t+j}) \quad (1^*)$$

We now have to log-linearized the Dixit-Stiglitz price aggregator:

$$P_t^{1-\theta} = (1 - \eta)P_t(i)^{(1-\theta)} + \eta \left( P_{t-1} (1 + \pi_{t-1})^\kappa (1 + \pi + \nu_t)^{1-\kappa} \right)^{(1-\theta)}$$

$$(1 - \theta)\hat{P}_t = (1\eta)(1 - \theta)\hat{P}_t(i) + \eta(1 - \theta) \left( \hat{P}_{t-1} + ind_t \right)$$

$$\hat{P}_t = (1 - \eta)\hat{P}_t(i) + \eta \left( \hat{P}_{t-1} + ind_t \right)$$

We want to find the expression for  $\hat{P}_t(i) - \hat{P}_t$ :

$$\hat{P}_t = (1 - \eta)\hat{P}_t(i) + \eta \left( \hat{P}_{t-1} + \hat{P}_t - \hat{P}_t + ind_t \right)$$

$$(1 - \eta)\hat{P}_t = (1 - \eta)\hat{P}_t(i) + \eta(-\pi_t + ind_t)$$

$$\hat{P}_t(i) - \hat{P}_t = \frac{\eta}{1 - \eta} (\pi_t - ind_t) \quad (2^*)$$

(1\*) = (2\*):

$$\frac{\eta}{1 - \eta} (\pi_t - ind_t) = \hat{P}_t(i) - \hat{P}_t = (1 - \beta\eta) \sum_{j=0}^{\infty} (\beta\eta)^j \widehat{mc}_{t+j} + \sum_{j=0}^{\infty} (\beta\eta)^j (\pi_{t+j} - ind_{t+j})$$

$$\pi_t = \frac{(1 - \beta\eta)(1 - \eta)}{\eta} \sum_{j=0}^{\infty} (\beta\eta)^j mc_{t+j} + \frac{1 - \eta}{\eta} \sum_{j=1}^{\infty} (\beta\eta)^j (\pi_{t+j} - ind_{t+j}) + ind_t$$

$$\pi_t - \beta\eta\mathbb{E}_t\pi_{t+1} = \frac{(1 - \beta)(1 - \eta)}{y} \widehat{mc}_t + \frac{(1 - \eta)}{\eta} \beta\eta (\pi_{t+1} - ind_{t+1}) + ind_t - \beta\eta\mathbb{E}_t ind_{t+1}$$

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \frac{(1 - \beta\eta)(1 - \eta)}{\eta} \widehat{mc}_t - \beta\mathbb{E}_t (\kappa\pi_t + (1 - \kappa)\nu_{t+1}) + \kappa\pi_{t-1} + (1 - \kappa)\nu_t$$

$$(1 + \beta\kappa)\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \frac{(1 - \beta\eta)(1 - \eta)}{\eta}\widehat{m}c_t + (1 - \kappa)\nu_t - \beta(1 - \kappa)\mathbb{E}_t\nu_{t+1}$$

Which gives the New-Keynesian Phillips curve:

$$\pi_t - \pi = \frac{\beta}{1 + \beta\kappa}\mathbb{E}_t(\pi_{t+1} - \pi) + \frac{\kappa}{1 + \beta\kappa}(\pi_{t-1} - \pi) + \frac{(1 - \beta\eta)(1 - \eta)}{\eta(1 + \beta\kappa)}\widehat{m}c_t + \frac{(1 - \kappa)(1 - \rho_\nu)}{1 + \beta\kappa}\nu_t \quad (\text{C.2.28})$$

Where  $\nu_t$  follows an AR(1) process  $\nu_{t+1} = \rho_\nu\nu_t + \epsilon'_t$ . Taking the logs on both side of monetary policy rule, and using the steady state condition gives:

$$R_t - R = \mu_R(R_{t-1} - R) + (1 - \mu_R)\mu_\pi(\pi_t - \pi) + (1 - \mu_R)\mu_\pi\tilde{y}_t + u_t \quad (\text{C.2.29})$$

Finally, log-linearizing the Fisher relation :

$$r_t - r = R_t - R - \mathbb{E}_t\pi_{t+1} + \pi \quad (\text{C.2.30})$$

## C.3 Optimal monetary policy:

### C.3.1 Second-order approximation of instantaneous utility

Recalling that the second order (quadratic) approximation to  $f(x, y)$  about  $(a, b)$  is:

$$\begin{aligned} f(x, y) \equiv & f(a, b) + \frac{\partial f}{\partial x}(x - a) + \frac{\partial f}{\partial y}(y - b) \\ & + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x - a)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}(y - b)^2 + \frac{\partial^2 f}{\partial x\partial y}(x - a)(y - b) \end{aligned}$$

Second-order approximation to the utility function:

$$(1 - d(G_t))\frac{(c_t - \chi c_{t-1})^{1-\sigma}}{1 - \sigma} - \psi\frac{(n_t^s)^{1+\gamma}}{1 + \gamma}$$

where the damage function raising consumption disutility is

$$d(G_t) = d_0 + d_1G_t + d_2G_t^2$$

Let us define habit-adjusted consumption

$$\bar{c}_t = c_t - \chi c_{t-1}$$

and insert it into the utility function:

$$(1 - d(G_t)) \frac{\bar{c}_t^{1-\sigma}}{1-\sigma} - \psi \frac{(n_t^s)^{1+\gamma}}{1+\gamma}$$

Defining two transitory variables  $U(\bar{c}_t, G_t)$  and  $V(n_t^s)$  for each of the two separable terms of the utility function:

$$U(\bar{c}_t, G_t) = (1 - d(G_t)) \frac{\bar{c}_t^{1-\sigma}}{1-\sigma}$$

$$V(n_t^s) = \psi \frac{(n_t^s)^{1+\gamma}}{1+\gamma}$$

The second-order approximation of  $U(\bar{c}_t, G_t)$  is

$$U(\bar{c}_t, G_t) \cong U(\bar{c}) + U_{\bar{c},G}(\bar{c}_t - \bar{c}) + \frac{U_{\bar{c}\bar{c}}}{2}(\bar{c}_t - \bar{c})^2 + U_G(\hat{G}_t - G) + \frac{U_{GG}}{2}(\hat{G}_t - G)^2 + U_{\bar{c},G}(\hat{c}_t - \bar{c})(\hat{G}_t - G) \quad (\text{C.3.1})$$

First and second partial derivatives of  $U(G_t, \bar{c}_t)$  wrt to  $\bar{c}$  are:

$$U_{\bar{c}} \equiv \frac{\partial U}{\partial \bar{c}} = (1 - d(G)) \bar{c}^{-\sigma}$$

$$U_{\bar{c}\bar{c}} \equiv \frac{\partial U_{\bar{c}}}{\partial \bar{c}} = -\sigma (1 - d(G)) \bar{c}^{-\sigma-1}$$

First and second partial derivatives of  $U(G_t, \bar{c}_t)$  wrt to  $G$  are:

$$U_G \equiv \frac{\partial U}{\partial G} = -(d_1 + 2d_2G) \frac{\bar{c}^{1-\sigma}}{1-\sigma}$$

$$U_{GG} \equiv \frac{\partial U_G}{\partial G} = -2d_2 \frac{\bar{c}^{1-\sigma}}{1-\sigma}$$

And the cross derivative of  $U(G_t, \bar{c}_t)$  wrt to  $\bar{c}$  and  $G$  is:

$$U_{\bar{c}G} \equiv \frac{\partial U_{\bar{c}}}{\partial G} = -(d_1 + 2d_2G) \bar{c}^{-\sigma}$$

Substituting the results for the partial derivatives into the second-order approximation yields:

$$U(\bar{c}_t, G_t) \cong (1 - d(G)) \frac{\bar{c}^{1-\sigma}}{1-\sigma} + (1 - d(G)) \bar{c}^{-\sigma} (\bar{c}_t - \bar{c}) - \frac{\sigma (1 - d(G)) \bar{c}^{-\sigma-1}}{2} (\bar{c}_t - \bar{c})^2 - (d_1 + 2d_2G) \frac{\bar{c}^{1-\sigma}}{1-\sigma} (G_t - G) - d_2 \frac{\bar{c}^{1-\sigma}}{1-\sigma} (G_t - G)^2 - (d_1 + 2d_2G) \bar{c}^{-\sigma} (\bar{c}_t - \bar{c})(G_t - G)$$

Second-order Taylor expansion for any variable  $z_t$  around its steady state:

$$\frac{z_t}{z} = 1 + \hat{z}_t + \frac{1}{2} \hat{z}_t^2 + O(\|\xi\|^3)$$

Where  $\hat{z}_t = \log\left(\frac{z_t}{z}\right)$ . Thus:

$$z_t - z \simeq z \hat{z}_t + \frac{z}{2} \hat{z}_t^2$$

and

$$(z_t - z)^2 \simeq z^2 \hat{z}_t^2$$

Using these properties for the second-order approximation of  $U(\bar{c}_t)$  yields:

$$\begin{aligned} U(\bar{c}_t, G_t) \cong & (1 - d(G)) \frac{\bar{c}^{1-\sigma}}{1-\sigma} + (1 - d(G)) \bar{c}^{-\sigma} \left( \bar{c} \hat{c}_t + \frac{\bar{c}}{2} \hat{c}_t^2 \right) - \frac{\sigma(1 - d(G)) \bar{c}^{-\sigma-1}}{2} \bar{c}^2 \hat{c}_t^2 \\ & - (d_1 + 2d_2 G) \frac{\bar{c}^{1-\sigma}}{1-\sigma} \left( G \hat{G}_t - \frac{G}{2} \hat{G}_t^2 \right) - d_2 \frac{\bar{c}^{1-\sigma}}{1-\sigma} G^2 \hat{G}_t^2 \\ & - (d_1 + 2d_2 G) \bar{c}^{-\sigma} \left( \bar{c} \hat{c}_t + \frac{\bar{c}}{2} \hat{c}_t^2 \right) \left( G \hat{G}_t - \frac{G}{2} \hat{G}_t^2 \right) \end{aligned}$$

Putting terms together and dropping those of third order or higher, it is obtained:

$$\begin{aligned} U(\bar{c}_t, G_t) \cong & U(\bar{c}, G) + (1 - d(G)) \bar{c}^{1-\sigma} \hat{c}_t - \frac{(1 - d(G))(\sigma - 1)}{2} \bar{c}^{1-\sigma} \hat{c}_t^2 \\ & - \frac{\bar{c}^{1-\sigma}}{1-\sigma} G \left[ d_1 \left( \hat{G}_t + \frac{1}{2} \hat{G}_t^2 \right) + 2d_2 G \hat{G}_t \right] - (d_1 + 2d_2 G) \bar{c}^{1-\sigma} \hat{c}_t \hat{G}_t \end{aligned}$$

The second-order approximation of  $V(n_t)$  is

$$V(n_t^s) \cong V(n^s) + V_n (n_t^s - n^s) + \frac{V_{nn}}{2} (n_t^s - n^s)^2$$

where first and second derivatives with respect to the labor supply  $n^s$  are

$$V_n \equiv \frac{\partial V}{\partial n^s} = \psi (n^s)^\gamma$$

and

$$V_{nn} = \frac{\partial V_n}{\partial n^s} = \psi \gamma (n^s)^{\gamma-1}$$

In steady state, labor supply is normalized at 1.0 ( $n^s = 1$ ), consequently:

$$V_n = d(G) \psi$$



$$V_{nn} = d(G)\psi\gamma$$

Thus, the second-order approximation to  $V(G_t, n_t^s)$  becomes

$$V(n_t^s) \cong \frac{\psi}{1+\gamma} + \psi(n_t^s - n^s) + \frac{\psi\gamma}{2}(n_t^s - n^s)^2$$

where, using  $(n_t^s - n^s)^2 = (n^s)^2(\hat{n}_t^s)^2 = (\hat{n}_t^s)^2$ ,  $n_t^s - n^s = n^s\hat{n}_t^s + \frac{n^s}{2}(\hat{n}_t^s)^2$  and recalling  $n^s = 1$ , we have,

$$V(n_t^s) \cong \frac{\psi}{1+\gamma} + \psi\left(\hat{n}_t^s + \frac{1+\gamma}{2}(\hat{n}_t^s)^2\right)$$

Therefore, the second-order approximation to the instantaneous utility function becomes

$$\begin{aligned} U(\bar{c}_t, G_t) - V(n_t^s) &\cong (1-d(G))\frac{\bar{c}^{1-\sigma}}{1-\sigma} + (1-d(G))\bar{c}^{1-\sigma}\hat{c}_t \\ &\quad - \frac{(1-d(G))(\sigma-1)}{2}\bar{c}^{1-\sigma}\hat{c}_t^2 - \frac{\bar{c}^{1-\sigma}}{1-\sigma}G\left[d_1\left(\hat{G}_t + \frac{1}{2}G_t^2\right) + 2d_2G\hat{G}_t\right] \\ &\quad - (d_1 + 2d_2G)\bar{c}^{1-\sigma}\hat{c}_t\hat{G}_t - \frac{\psi}{1+\gamma} - \psi\left(\hat{n}_t^s + \frac{1+\gamma}{2}(\hat{n}_t^s)^2\right) \end{aligned}$$

The unconditional expectation of the second-order approximation to  $U(\bar{c}_t) - V(G_t, n_t^s)$  is

$$\begin{aligned} \mathbb{E}[U(\bar{c}_t, G_t) - V(n_t^s)] &= (1-d(G))\frac{\bar{c}^{1-\sigma}}{1-\sigma} - \frac{\psi}{1+\gamma} - \frac{(1-d(G))(\sigma-1)\bar{c}^{1-\sigma}}{2}\text{var}(\hat{c}) \\ &\quad - \frac{\bar{c}^{1-\sigma}G}{2(1-\sigma)}G\text{var}(\hat{G}_t) - \frac{\psi(1+\gamma)}{2}\text{var}(\hat{n}^s) - \psi(d_1 + 2d_2G)\bar{c}^{1-\sigma}G\text{cov}(\hat{c}, \hat{G}) \end{aligned}$$

If we recall  $\hat{c}_t = \frac{1}{1-\chi}(\hat{c}_t - \chi\hat{c}_{t-1})$ , we can rewrite the approximation for  $U(\bar{c}_t) - V(G_t, n_t^s)$  in terms of current and lagged consumption<sup>2</sup>

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<sup>2</sup>It should be noticed that

$$\hat{c}_t^2 = \left(\frac{1}{1-\chi}(\hat{c}_t - \chi\hat{c}_{t-1})\right)^2$$

which implies

$$\hat{c}_t^2 = \frac{1}{(1-\chi)^2}\hat{c}_t^2 + \frac{\chi^2}{(1-\chi)^2}\hat{c}_{t-1}^2 - \frac{2\chi}{(1-\chi)^2}(\hat{c}_t\hat{c}_{t-1})$$

### C.3.2 Ramsey optimal monetary policy

The planner's optimizing program is to maximize the instantaneous household utility ((?)) derived in the previous sub-section subject to the set of equation that describes the competitive equilibrium. Formally, the central bank chooses the sequence:  $\{\hat{c}_{t+j}, \pi_{t+j} - \pi, R_{t+j} - R, \hat{n}_{t+j}^s, \hat{w}_{t+j}, \hat{k}_{t+j}, \hat{z}_{t+j}, \hat{m}_{t+j}, \hat{r}_{t+j}^k, \hat{e}_{t+j}, \hat{e}_{t+j}^p, \hat{e}_{t+j}^{np}, \hat{p}_{t+j}^e, \hat{p}_{t+j}^p, \hat{p}_{t+j}^{np}, \hat{n}_{t+j}^p, \hat{n}_{t+j}^{np}, \hat{n}_{t+j}, \hat{G}_{t+j}, \hat{q}_{t+j}\}_{j=0}^{\infty}$ , that maximizes:

$$\max \sum_{j=0}^{\infty} \mathbb{E}_t \beta^j U_{t+j} \left( \hat{c}_t, \hat{n}_t^s, \hat{G}_t \right)$$

subject to:

$$\begin{aligned} & - \left( 1 + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} \right) \hat{c}_{t+j} + \frac{\chi}{(1+\chi)} \hat{c}_{t-1+j} + \left( \frac{1}{1+\chi} + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} \right) \mathbb{E}_{t+j} \hat{c}_{t+1+j} \\ & + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} (\hat{m}_{t+j} - \mathbb{E}_{t+j} \hat{m}_{t+1+j}) - \frac{(1-\chi)}{\sigma(1+\chi)} ((R_{t+j} - R) - (\mathbb{E}_{t+j} \pi_{t+1+j} - \pi)) = 0 \\ & (\pi_{t+j} - \pi) - \frac{\kappa}{1+\beta\kappa} (\pi_{t-1+j} - \pi) - \frac{\beta}{1+\beta\kappa} (\mathbb{E}_{t+j} \pi_{t+1+j} - \pi) \\ & - \frac{(1-\beta\eta)(1-\eta)}{\eta(1+\beta\kappa)} \left( (1-\alpha-\nu) \hat{n}_{t+j} + \alpha \hat{r}_{t+j}^k + \nu \hat{p}_{t+j}^e \right) - \frac{(1-\kappa)(1-\beta\rho\nu)}{1+\beta\kappa} \varepsilon_{t+j}^\nu = 0 \\ & \hat{w}_{t+j} - \gamma \hat{n}_{t+j}^s - \left( \frac{\sigma}{(1-\chi)} + \frac{h_c\phi_2}{(1-\phi_2)} \right) \hat{c}_{t+j} + \frac{\sigma\chi}{(1-\chi)} \hat{c}_{t-1+j} + \frac{h_c\phi_2}{(1-\phi_2)} \hat{m}_{t+j} - \frac{d_1 G + 2d_2 G^2}{d(G)} \hat{G}_{t+j} = 0 \\ & \varepsilon_{t+j}^a + \alpha \hat{k}_{t+j} + \alpha \hat{z}_{t+j} + (1-\alpha-\nu) \hat{n}_{t+j} + \nu \hat{e}_{t+j} - \left( \frac{c}{y} + \frac{h-\phi_0}{y(1-\phi_2)} \right) \hat{c}_{t+j} \\ & - \frac{i}{y} \left( \frac{1}{\delta} \hat{k}_{t+j} - \frac{1-\delta}{\delta} \hat{k}_{t-1+j} - \varepsilon_{t+j}^i \right) - \frac{g}{y} \varepsilon_{t+j}^g - \frac{(h-\phi_0)\phi_2}{y(1-\phi_2)} \hat{m}_{t+j} = 0 \\ & \hat{q}_{t+j} - \frac{1-\delta}{1+r^k-\delta} \mathbb{E}_{t+j} \hat{q}_{t+1+j} - \frac{r^k}{1+r^k-\delta} r_{t+1+j}^k + (R_{t+j} - R) - \mathbb{E}_{t+j} (\pi_{t+1+j} - \pi) = 0 \\ & \hat{z}_{t+j} - \frac{1}{\sigma_a} \hat{r}_{t+j}^k = 0 \\ & \hat{w}_{t+j} - \hat{p}_{t+j}^e - \hat{e}_{t+j} + \hat{n}_{t+j} = 0 \\ & \hat{w}_{t+j} - \hat{r}_{t+j}^k - \hat{k}_{t-1+j} - \hat{z}_{t+j} + \hat{n}_{t+j} = 0 \\ & \hat{e}_{t+j} - \omega \left( \frac{e^p}{e} \right)^\varphi \hat{e}_{t+j}^p - (1-\omega) \left( \frac{e^{np}}{e} \right)^\varphi \hat{e}_{t+j}^{np} = 0 \end{aligned}$$

$$\begin{aligned}
\hat{e}_{t+j}^{np} - \frac{1}{\varphi-1} \left( \hat{p}_{t+j}^{np} + \hat{p}_{t+j}^e \right) - \hat{e}_{t+j} &= 0 \\
\hat{e}_{t+j}^p - \frac{1}{\varphi-1} \left( \hat{p}_{t+j}^p + \hat{p}_t^e - \Xi \varepsilon_{t+j}^T \right) - \hat{e}_{t+j} &= 0 \\
\hat{n}_{t+j}^p - \frac{1}{1-\zeta^p} \left( \hat{p}_{t+j}^p + \varepsilon_{t+j}^{ap} - \hat{w}_{t+j} \right) &= 0 \\
\hat{n}_{t+j}^{np} - \frac{1}{1-\zeta^{np}} \left( \hat{p}_{t+j}^{np} + \varepsilon_{t+j}^{anp} - \hat{w}_{t+j} \right) &= 0 \\
\hat{e}_{t+j}^p - \varepsilon_{t+j}^{ap} - \zeta^p \hat{n}_{t+j}^p &= 0 \\
\hat{e}_{t+j}^{np} - \varepsilon_{t+j}^{anp} - \zeta^{np} \hat{n}_{t+j}^{np} &= 0 \\
\hat{n}_{t+j}^s - \frac{n^p}{n^s} \hat{n}_{t+j}^p - \frac{n^{np}}{n^s} \hat{n}_{t+j}^{np} - \frac{n}{n^s} \hat{n}_{t+j} &= 0 \\
\hat{m}_{t+j} - \hat{c}_{t+j} + \frac{1-\phi_2}{R} (R_{t+j} - R) &= 0 \\
\hat{G}_{t+j} - \hat{e}_{t+j}^p &= 0
\end{aligned}$$

$$\begin{aligned}
\left( \frac{1}{\delta} \hat{k}_{t+j} - \frac{1-\delta}{\delta} \hat{k}_{t-1+j} - \varepsilon_{t+j}^i \right) - \frac{1}{1+\beta} \left( \frac{1}{\delta} \hat{k}_{t-1+j} - \frac{1-\delta}{\delta} \hat{k}_{t-2+j} - \varepsilon_{t-1+j}^i \right) \\
- \frac{\beta}{1+\beta} \left( \frac{1}{\delta} \mathbb{E}_{t+j} \hat{k}_{t+1+j} - \frac{1-\delta}{\delta} \hat{k}_{t+j} - \varepsilon_{t+1+j}^i \right) - \frac{1}{(1+\beta)\phi_k} \hat{q}_{t+j} - \varepsilon_{t+j}^i = 0
\end{aligned}$$

Defining  $\lambda_{i,t+j}$  with  $i = 1 \cdots 19$  the Lagrange multipliers respectively associated with the 19 constraints of the central bank optimizing program, the set of first order conditions comprises the 19 equations above and the following equations from the central-bank choice variables (we set  $j = 0$  in order to obtain expressions for period  $t$ ):

$$\begin{aligned}
\bar{c}^{1-\sigma} \left( \frac{1-\beta\chi}{1-\chi} \right) - \frac{(\sigma-1)\bar{c}^{1-\sigma}(1+\beta\chi^2)}{(1-\chi)^2} \hat{c}_t + \frac{\beta(\sigma-1)\bar{c}^{1-\sigma}\chi}{(1-\chi)^2} \mathbb{E}_t \hat{c}_{t+1} + \frac{(\sigma-1)\bar{c}^{1-\sigma}\chi}{(1-\chi)^2} \hat{c}_{t-1} \\
+ \beta^{-1} \left( \frac{1}{1+\chi} + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} \right) \lambda_{1,t-1} - \left( 1 + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} \right) \lambda_{1,t} + \frac{\beta\chi}{1+\chi} \mathbb{E}_t \lambda_{1,t+1} \\
- \left( \frac{\sigma}{1-\chi} + \frac{h_c\phi_2}{1-\phi_2} \right) \lambda_{3,t} + \frac{\beta\sigma\chi}{1-\chi} \mathbb{E}_t \lambda_{3,t+1} - \left( \frac{c}{y} + \frac{h-\phi_0}{y(1-\phi_2)} \right) \lambda_{4,t} - \lambda_{17,t} = 0 \quad (\hat{c}_t^{foc}) \\
\frac{(1-\chi)}{\beta\sigma(1+\chi)} \lambda_{1,t-1} + \lambda_{2,t} - \frac{1}{1+\beta\kappa} \lambda_{2,t-1} - \frac{\beta\kappa}{1+\beta\kappa} \mathbb{E}_t \lambda_{2,t+1} - \lambda_{5,t-1} = 0 \quad ((\pi_t - \pi)^{foc}) \\
- \frac{(1-\chi)}{\sigma(1+\chi)} \lambda_{1,t} + \lambda_{5,t} + \frac{(1-\phi_2)}{R} \lambda_{17,t} = 0 \quad ((R_t - R)^{foc}) \\
-d(G)\psi - d(G)\psi(1+\gamma)\hat{n}_t^s - \psi(d_1 + 2d_2G)G\hat{G}_t - \lambda_{3,t}\gamma + \lambda_{16,t} = 0 \quad ((\hat{n}_t^s)^{foc}) \\
- \frac{(1-\beta\eta)(1-\eta)(1-\alpha-\nu)}{\eta(1+\beta\kappa)} \lambda_{2,t} + \lambda_{3,t} + \lambda_{7,t} + \lambda_{8,t} + \frac{1}{1-\zeta^p} \lambda_{12,t} + \frac{1}{1-\zeta^{np}} \lambda_{13,t} = 0 \quad (\hat{w}_t^{foc})
\end{aligned}$$

$$\begin{aligned}
& -\frac{i}{y}\frac{1}{\delta}\lambda_{4,t} + \beta\left(\alpha + \frac{i}{y}\frac{(1-\delta)}{\delta}\right)\mathbb{E}_t\lambda_{4,t+1} - \beta\mathbb{E}_t\lambda_{8,t+1} - \frac{1}{\delta}\frac{1}{1+\beta}\lambda_{19,t-1} + \left(\frac{1}{\delta} + \frac{\beta(1-\delta)}{(1+\beta)\delta}\right)\lambda_{19,t} \\
& \quad - \beta\left(\frac{1-\delta}{\delta} + \frac{1}{(1+\beta)\delta}\right)\mathbb{E}_t\lambda_{19,t+1} + \frac{\beta^2(1-\delta)}{(1+\beta)\delta}\mathbb{E}_t\lambda_{19,t+2} = 0 \quad (\hat{k}_t^{foc}) \\
& \quad \lambda_{4,t}\alpha + \lambda_{6,t} - \lambda_{8,t} = 0 \quad (\hat{z}_t^{foc}) \\
& \quad \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)}(\lambda_{1,t} - \beta^{-1}\lambda_{1,t-1}) + \frac{h_c\phi_2}{1-\phi_2}\lambda_{3,t} + \frac{(h-\phi_0)\phi_2}{y(1-\phi_2)}\lambda_{4,t} + \lambda_{17,t} = 0 \quad (\hat{m}_t^{foc}) \\
& \quad -\frac{(1-\beta\eta)(1-\eta)\alpha}{\eta(1+\beta\kappa)}\lambda_{2,t} - \frac{r^k}{\beta(1+r^k-\delta)}\lambda_{5,t-1} - \frac{1}{\sigma_a}\lambda_{6,t} - \lambda_{8,t} = 0 \quad ((\hat{r}_t^k)^{foc}) \\
& \quad \nu\lambda_{4,t} - \lambda_{7,t} + \lambda_{9,t} - \lambda_{10,t} - \lambda_{11,t} = 0 \quad (\hat{e}_t^{foc}) \\
& \quad -(\omega\left(\frac{e^p}{e}\right)^\varphi)\lambda_{9,t} + \lambda_{10,t} + \lambda_{14,t} - \lambda_{18,t} = 0 \quad ((\hat{e}_t^p)^{foc}) \\
& \quad -((1-\omega)\left(\frac{e^{np}}{e}\right)^\varphi)\lambda_{9,t} + \lambda_{9,t} + \lambda_{11,t} - \lambda_{15,t} = 0 \quad ((\hat{e}_t^{np})^{foc}) \\
& \quad -\frac{(1-\beta\eta)(1-\eta)\nu}{\eta(1+\beta\kappa)}\lambda_{2,t} - \lambda_{7,t} + \frac{1}{\varphi-1}(\lambda_{10,t} + \lambda_{11,t}) = 0 \quad ((\hat{p}_t^e)^{foc}) \\
& \quad \quad -\frac{1}{\varphi-1}\lambda_{10,t} - \frac{1}{1-\zeta^p}\lambda_{12,t} = 0 \quad ((\hat{p}_t^p)^{foc}) \\
& \quad \quad -\frac{1}{\varphi-1}\lambda_{11,t} - \frac{1}{1-\zeta^{np}}\lambda_{13,t} = 0 \quad ((\hat{p}_t^{np})^{foc}) \\
& \quad \lambda_{4,t}(1-\alpha-\nu) + \lambda_{7,t} + \lambda_{8,t} - \frac{n}{n^s}\lambda_{16,t} = 0 \quad (\hat{n}_t^{foc}) \\
& \quad \lambda_{12,t} - \lambda_{14,t}\zeta^p - \frac{n^p}{n^s}\lambda_{16,t} = 0 \quad ((\hat{n}_t^p)^{foc}) \\
& \quad \lambda_{13,t} - \zeta^{np}\lambda_{15,t} - \frac{n^{np}}{n^s}\lambda_{16,t} = 0 \quad ((\hat{n}_t^{np})^{foc}) \\
& \quad -\frac{\psi(d_1+2d_2G)G}{1+\gamma} - \frac{\psi(d_1+4d_2G)G}{1+\gamma}\hat{G}_t - \psi(d_1+2d_2G)G\hat{n}_t^s - \frac{d_1G+2d_2G^2}{d(G)}\lambda_{3,t} + \lambda_{18,t} = 0 \quad ((\hat{G}_t)^{foc}) \\
& \quad \quad -\frac{(1-\delta)}{\beta(1+r^k-\delta)}\lambda_{5,t-1} + \lambda_{5,t} - \frac{1}{(1+\beta)\varphi_k}\lambda_{19,t} = 0 \quad (\hat{q}_t^{foc})
\end{aligned}$$

Next, we rewrite the first order conditions by subtracting their steady state version in order to have zero-mean expression and fluctuations with respect to steady state of Lagrange multipliers (e.g:  $\hat{\lambda}_{1,t} = \lambda_{1,t} - \lambda_1$ ). Consequently, we have:

$$\begin{aligned}
& -\frac{(\sigma-1)\bar{c}^{1-\sigma}(1+\beta\chi^2)}{(1-\chi)^2}\hat{c}_t + \frac{\beta(\sigma-1)\bar{c}^{1-\sigma}\chi}{(1-\chi)^2}\mathbb{E}_t\hat{c}_{t+1} + \frac{(\sigma-1)\bar{c}^{1-\sigma}\chi}{(1-\chi)^2}\hat{c}_{t-1} \\
& \quad + \beta^{-1}\left(\frac{1}{1+\chi} + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)}\right)\hat{\lambda}_{1,t-1} - \left(1 + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)}\right)\hat{\lambda}_{1,t} + \frac{\beta\chi}{1+\chi}\mathbb{E}_t\hat{\lambda}_{1,t+1} \\
& \quad - \left(\frac{\sigma}{1-\chi} + \frac{h_c\phi_2}{1-\phi_2}\right)\hat{\lambda}_{3,t} + \frac{\beta\sigma\chi}{1-\chi}\mathbb{E}_t\hat{\lambda}_{3,t+1} - \left(\frac{c}{y} + \frac{h-\phi_0}{y(1-\phi_2)}\right)\hat{\lambda}_{4,t} - \hat{\lambda}_{17,t} = 0 \quad (\hat{c}_t^{foc})
\end{aligned}$$

$$\begin{aligned}
& \frac{(1-\chi)}{\beta\sigma(1+\chi)}\hat{\lambda}_{1,t-1} + \hat{\lambda}_{2,t} - \frac{1}{1+\beta\kappa}\hat{\lambda}_{2,t-1} - \frac{\beta\kappa}{1+\beta\kappa}\mathbb{E}_t\hat{\lambda}_{2,t+1} - \hat{\lambda}_{5,t-1} = 0 & ((\pi_t - \pi)^{foc}) \\
& -\frac{(1-\chi)}{\sigma(1+\chi)}\hat{\lambda}_{1,t} + \hat{\lambda}_{5,t} + \frac{(1-\phi_2)}{R}\hat{\lambda}_{17,t} = 0 & ((R_t - R)^{foc}) \\
& -d(G)\psi(1+\gamma)\hat{n}_t^s - \psi(d_1 + 2d_2G)G\hat{G}_t - \gamma\hat{\lambda}_{3,t} + \hat{\lambda}_{16,t} = 0 & ((\hat{n}_t^s)^{foc}) \\
& -\frac{(1-\beta\eta)(1-\eta)(1-\alpha-\nu)}{\eta(1+\beta\kappa)}\hat{\lambda}_{2,t} + \hat{\lambda}_{3,t} + \hat{\lambda}_{7,t} + \hat{\lambda}_{8,t} + \frac{1}{1-\zeta^p}\hat{\lambda}_{12,t} + \frac{1}{1-\zeta^{np}}\hat{\lambda}_{13,t} = 0 & (\hat{w}_t^{foc}) \\
& -\frac{i}{y}\frac{1}{\delta}\hat{\lambda}_{4,t} + \beta\left(\alpha + \frac{i}{y}\frac{(1-\delta)}{\delta}\right)\mathbb{E}_t\hat{\lambda}_{4,t+1} - \beta\mathbb{E}_t\hat{\lambda}_{8,t+1} - \frac{1}{\delta}\frac{1}{1+\beta}\hat{\lambda}_{19,t-1} + \left(\frac{1}{\delta} + \frac{\beta(1-\delta)}{(1+\beta)\delta}\right)\hat{\lambda}_{19,t} \\
& \quad - \beta\left(\frac{1-\delta}{\delta} + \frac{1}{(1+\beta)\delta}\right)\mathbb{E}_t\hat{\lambda}_{19,t+1} + \frac{\beta^2(1-\delta)}{(1+\beta)\delta}\mathbb{E}_t\hat{\lambda}_{19,t+2} = 0 & (\hat{k}_t^{foc}) \\
& \quad \alpha\hat{\lambda}_{4,t} + \hat{\lambda}_{6,t} - \hat{\lambda}_{8,t} = 0 & ((\hat{z}_t)^{foc}) \\
& \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)}\left(\hat{\lambda}_{1,t} - \beta^{-1}\hat{\lambda}_{1,t-1}\right) + \frac{h_c\phi_2}{1-\phi_2}\hat{\lambda}_{3,t} + \frac{(h-\phi_0)\phi_2}{y(1-\phi_2)}\hat{\lambda}_{4,t} + \hat{\lambda}_{17,t} = 0 & (\hat{n}_t^{foc}) \\
& -\frac{(1-\beta\eta)(1-\eta)\alpha}{\eta(1+\beta\kappa)}\hat{\lambda}_{2,t} - \frac{r^k}{\beta(1+r^k-\delta)}\hat{\lambda}_{5,t-1} - \frac{1}{\sigma_a}\hat{\lambda}_{6,t} - \hat{\lambda}_{8,t} = 0 & ((\hat{r}_t^k)^{foc}) \\
& \nu\hat{\lambda}_{4,t} - \hat{\lambda}_{7,t} + \hat{\lambda}_{9,t} - \hat{\lambda}_{10,t} - \hat{\lambda}_{11,t} = 0 & (\hat{e}_t^{foc}) \\
& -(\omega\left(\frac{e^p}{e}\right)^\varphi)\hat{\lambda}_{9,t} + \hat{\lambda}_{10,t} + \hat{\lambda}_{14,t} - \hat{\lambda}_{18,t} = 0 & ((\hat{e}_t^p)^{foc}) \\
& -((1-\omega)\left(\frac{e^{np}}{e}\right)^\varphi)\hat{\lambda}_{9,t} + \hat{\lambda}_{11,t} + \hat{\lambda}_{15,t} = 0 & ((\hat{e}_t^{np})^{foc}) \\
& -\frac{(1-\beta\eta)(1-\eta)\nu}{\eta(1+\beta\kappa)}\hat{\lambda}_{2,t} - \hat{\lambda}_{7,t} + \frac{1}{\varphi-1}\left(\hat{\lambda}_{10,t} + \hat{\lambda}_{11,t}\right) = 0 & ((\hat{p}_t^e)^{foc}) \\
& \quad -\frac{1}{\varphi-1}\hat{\lambda}_{10,t} - \frac{1}{1-\zeta^p}\hat{\lambda}_{12,t} = 0 & ((\hat{p}_t^p)^{foc}) \\
& \quad -\frac{1}{\varphi-1}\hat{\lambda}_{11,t} - \frac{1}{1-\zeta^{np}}\hat{\lambda}_{13,t} = 0 & ((\hat{p}_t^{np})^{foc}) \\
& (1-\alpha-\nu)\hat{\lambda}_{4,t} + \hat{\lambda}_{7,t} + \hat{\lambda}_{8,t} - \frac{n}{n^s}\hat{\lambda}_{16,t} = 0 & ((\hat{n}_t^s)^{foc}) \\
& \hat{\lambda}_{12,t} - \zeta^p\hat{\lambda}_{14,t} - \frac{n^p}{n^s}\hat{\lambda}_{16,t} = 0 & ((\hat{n}_t^p)^{foc}) \\
& \hat{\lambda}_{13,t} - \zeta^{np}\hat{\lambda}_{15,t} - \frac{n^{np}}{n^s}\hat{\lambda}_{16,t} = 0 & ((\hat{n}_t^{np})^{foc}) \\
& -\frac{\psi(d_1+4d_2G)G}{1+\gamma}\hat{G}_t - \psi(d_1 + 2d_2G)G\hat{n}_t^s - \frac{d_1G+2d_2G^2}{d(G)}\hat{\lambda}_{3,t} + \hat{\lambda}_{18,t} = 0 & ((\hat{G}_t)^{foc}) \\
& -\frac{(1-\delta)}{\beta(1+r^k-\delta)}\hat{\lambda}_{5,t-1} + \hat{\lambda}_{5,t} - \frac{1}{(1+\beta)\varphi_k}\hat{\lambda}_{19,t} = 0 & (\hat{q}_t^{foc})
\end{aligned}$$

The set of dynamic equations for the Ramsey type optimal monetary policy consists in the 20 equations so far. Additionally the 19 constraints turn binding within the solution of the optimizing program:

$$- \left( 1 + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} \right) \hat{c}_t + \frac{\chi}{(1+\chi)} \hat{c}_{t-1} + \left( \frac{1}{1+\chi} + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} \right) \hat{c}_{t+1} \\ + \frac{h_c(1-\chi)\phi_2}{\sigma(1+\chi)(1-\phi_2)} (\hat{m}_t - \hat{m}_{t+1}) - \frac{(1-\chi)}{\sigma(1+\chi)} ((R_t - R) - \mathbb{E}_t(\pi_{t+1} - \pi)) = 0$$

$$(\pi_t - \pi) - \frac{\kappa}{1+\beta\kappa} (\pi_{t-1} - \pi) - \frac{\beta}{1+\beta\kappa} (\mathbb{E}_t\pi_{t+1} - \pi) \\ - \frac{(1-\beta\eta)(1-\eta)}{\eta(1+\beta\kappa)} \left( (1-\alpha-\nu)\hat{n}_t + \alpha\hat{r}_t^k + \nu\hat{p}_t^e \right) - \frac{(1-\kappa)(1-\beta\rho\nu)}{1+\beta\kappa} \varepsilon_t^\nu = 0$$

$$\hat{w}_t - \gamma\hat{n}_t^s - \left( \frac{\sigma}{(1-\chi)} + \frac{h_c\phi_2}{(1-\phi_2)} \right) \hat{c}_t + \frac{\sigma\chi}{(1-\chi)} \hat{c}_{t-1} + \frac{h_c\phi_2}{(1-\phi_2)} \hat{m}_t - \frac{d_1G+2d_2G^2}{d(G)} \hat{G}_t = 0$$

$$\varepsilon_t^a + \alpha\hat{k}_t + \alpha\hat{z}_t + (1-\alpha-\nu)\hat{n}_t + \nu\hat{e}_t - \left( \frac{c}{y} + \frac{h-\phi_0}{y(1-\phi_2)} \right) \hat{c}_t \\ - \frac{i}{y} \left( \frac{1}{\delta} \hat{k}_t - \frac{1-\delta}{\delta} \hat{k}_{t-1} - \varepsilon_t^i \right) - \frac{g}{y} \varepsilon_t^g - \frac{(h-\phi_0)\phi_2}{y(1-\phi_2)} \hat{m}_t = 0$$

$$\hat{q}_t - \frac{1-\delta}{1+r^k-\delta} \mathbb{E}_t\hat{q}_{t+1} - \frac{r^k}{1+r^k-\delta} \mathbb{E}_t r_{t+1}^k + (R_t - R) - \mathbb{E}_t(\pi_{t+1} - \pi) = 0$$

$$\hat{z}_t - \frac{1}{\sigma_a} \hat{r}_t^k = 0$$

$$\hat{w}_t - \hat{p}_t^e - \hat{e}_t + \hat{n}_t = 0$$

$$\hat{w}_t - \hat{r}_t^k - \hat{k}_{t-1} - \hat{z}_t + \hat{n}_t = 0$$

$$\hat{e}_t - (\omega \left( \frac{e^p}{e} \right)^\varphi) \hat{e}_t^p - ((1-\omega) \left( \frac{e^{np}}{e} \right)^\varphi) \hat{e}_t^{np} = 0$$

$$\hat{e}_t^{np} - \frac{1}{\varphi-1} (\hat{p}_t^{np} + \hat{p}_t^e) - \hat{e}_t = 0$$

$$\hat{e}_t^p - \frac{1}{\varphi-1} \hat{p}_t^p + \hat{p}_t^e - \Xi \varepsilon_t^\tau - \hat{e}_t = 0$$

$$\hat{n}_t^p - \frac{1}{1-\varsigma^p} (\hat{p}_t^p + \varepsilon_t^{a^p} - \hat{w}_t) = 0$$

$$\hat{n}_t^{np} - \frac{1}{1-\varsigma^{np}} (\hat{p}_t^{np} + \varepsilon_t^{a^{np}} - \hat{w}_t) = 0$$

$$\hat{e}_t^p - \varepsilon_t^{a^p} - \varsigma^p \hat{n}_t^p = 0$$

$$\hat{e}_t^{np} - \varepsilon_t^{a^{np}} - \varsigma^{np} \hat{n}_t^{np} = 0$$

$$\hat{n}_t^s - \frac{n^p}{n^s} \hat{n}_t^p - \frac{n^{np}}{n^s} \hat{n}_t^{np} - \frac{n}{n^s} \hat{n}_t = 0$$

$$\hat{m}_t - \hat{c}_t + \frac{1-\phi_2}{R} (R_t - R) = 0$$

$$\hat{G}_t - \hat{c}_t^p = 0$$

$$\begin{aligned} & \left( \frac{1}{\delta} \hat{k}_t - \frac{1-\delta}{\delta} \hat{k}_{t-1} - \varepsilon_t^i \right) - \frac{1}{1+\beta} \left( \frac{1}{\delta} \hat{k}_{t-1} - \frac{1-\delta}{\delta} \hat{k}_{t-2} - \varepsilon_{t-1}^i \right) \\ & - \frac{\beta}{1+\beta} \left( \frac{1}{\delta} \hat{k}_{t+1} - \frac{1-\delta}{\delta} \hat{k}_t - \mathbb{E}_t \varepsilon_{t+1}^i \right) - \frac{1}{(1+\beta)\phi_k} \hat{q}_t - \varepsilon_t^i = 0 \end{aligned}$$

Plus 6 additional equations:

$$\hat{k}_t^s - \hat{k}_{t-1} - \hat{z}_t = 0$$

$$\hat{y}_t - \varepsilon_t^a + \alpha \hat{k}_t^s + (1 - \alpha - \nu) \hat{n}_t + \nu \hat{e}_t = 0$$

$$\hat{h}_t - \frac{h-\phi_0}{h} \left( \frac{1}{1-\phi_2} \hat{c}_t - \frac{\phi_2}{1-\phi_2} \hat{m}_t \right) = 0$$

$$\hat{i}_t - \frac{1}{\delta} \hat{k}_t + \frac{1-\delta}{\delta} \hat{k}_{t+1} + \varepsilon_t^i = 0$$

$$\hat{m}c_t - (1 - \alpha - \nu) \hat{w}_t - \alpha \hat{r}_t^k - \nu \hat{p}_t^e + \varepsilon_t^a = 0$$

$$(\mu_t - \mu) - (\hat{m}_t - \hat{m}_{t-1}) - (\pi_t - \pi) = 0$$

We finally obtain a set of 45 equations for 45 endogenous variables.

## C.4 Simulations

### C.4.1 IRF to stochastic exogenous shocks:

#### C.4.1.1 IRF to a risk premium shock:

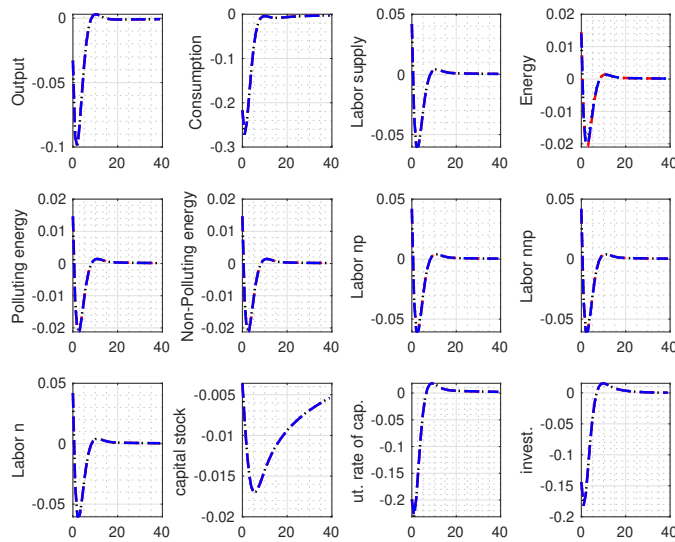


Figure C.1 – IRF to a risk premium shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$



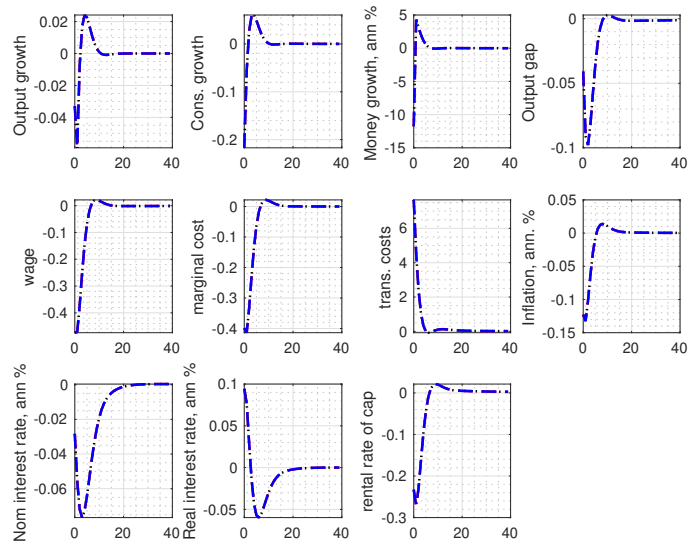


Figure C.2 – IRF to a risk premium shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

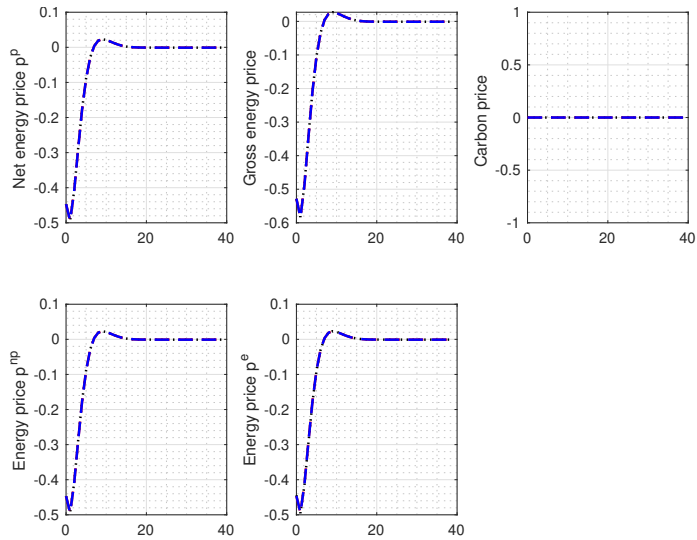


Figure C.3 – IRF to a risk premium price shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

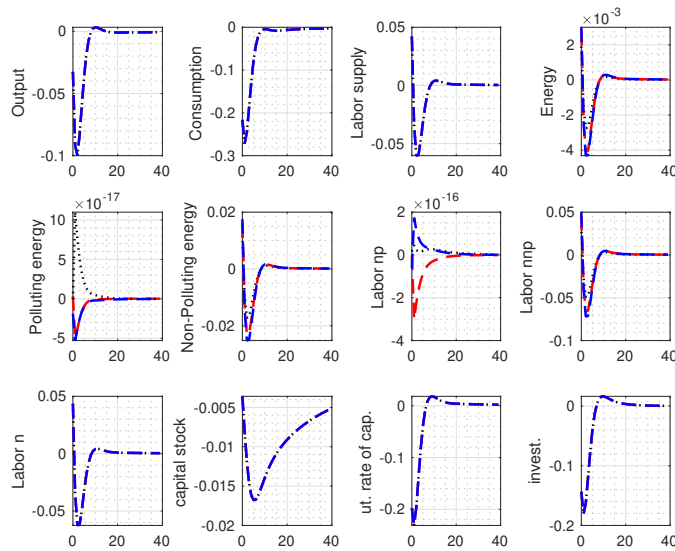


Figure C.4 – IRF to a risk premium shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

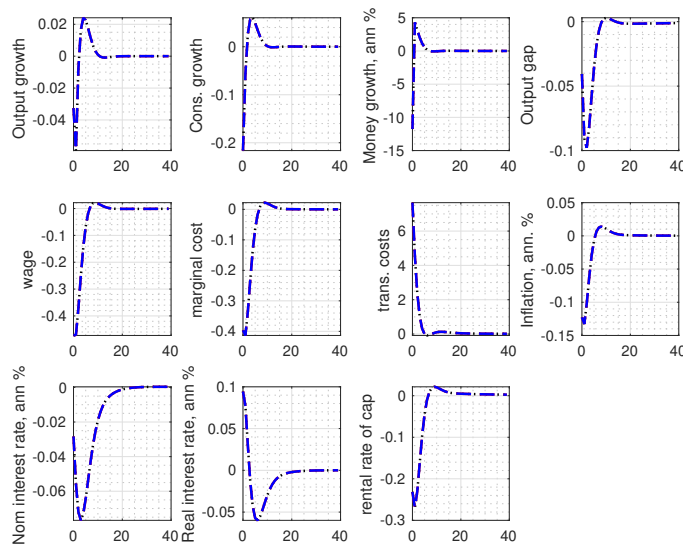


Figure C.5 – IRF to a risk premium shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

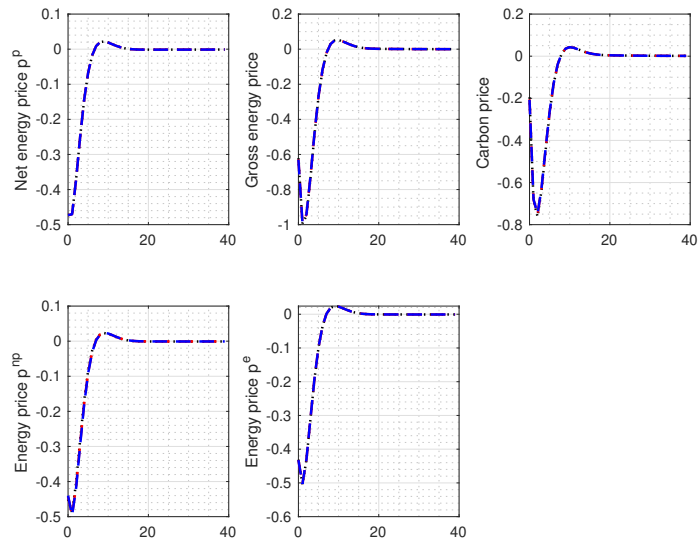


Figure C.6 – IRF to a risk premium shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

#### C.4.1.2 IRF to an investment shock:

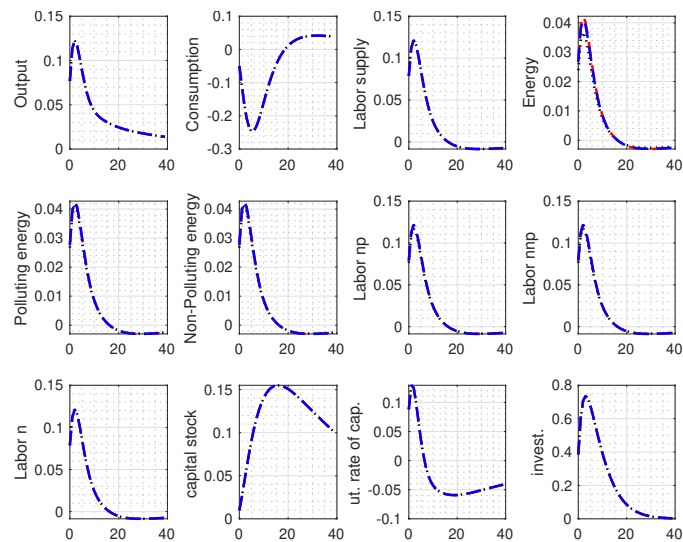


Figure C.7 – IRF to an investment shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

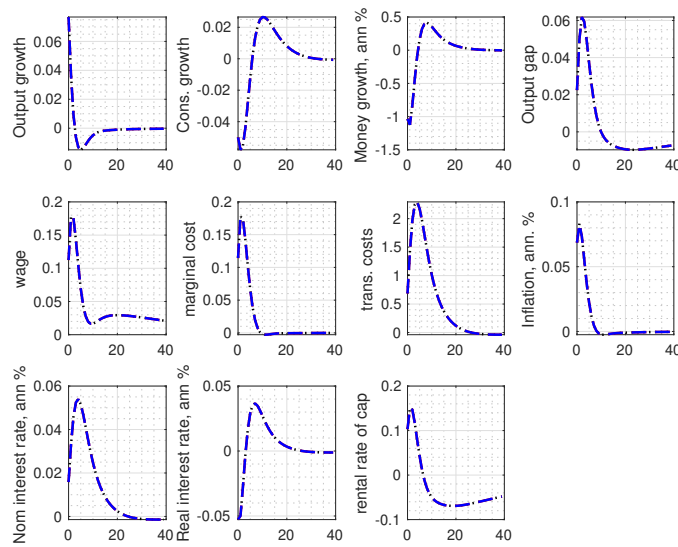


Figure C.8 – IRF to an investment shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

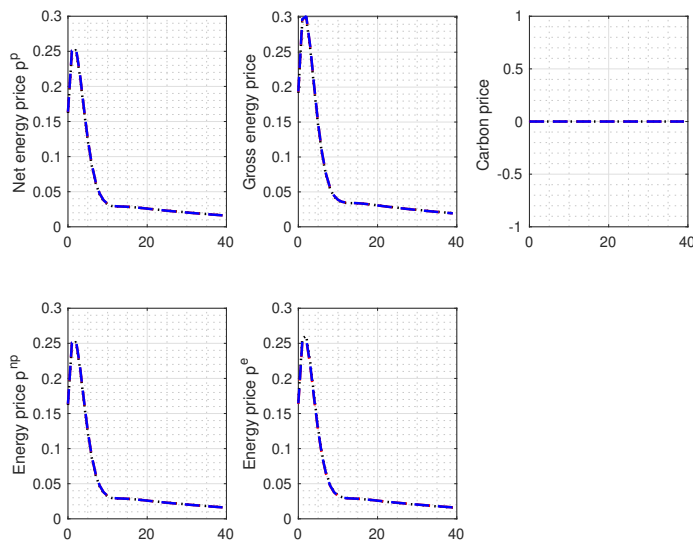


Figure C.9 – IRF to an investment shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

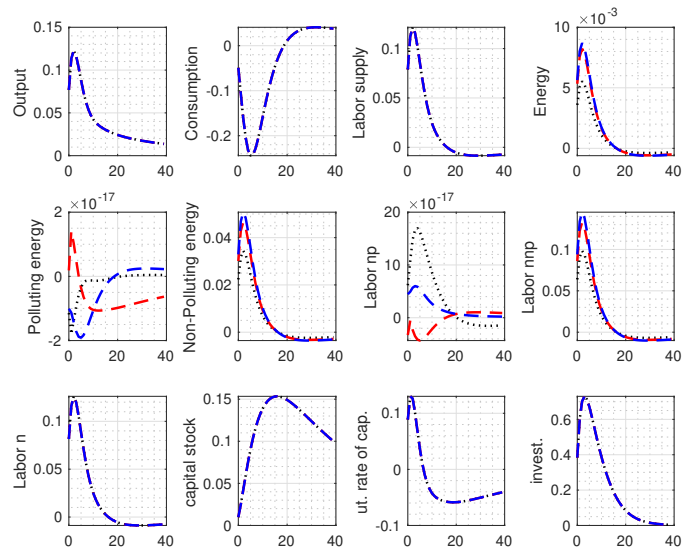


Figure C.10 – IRF to an investment shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

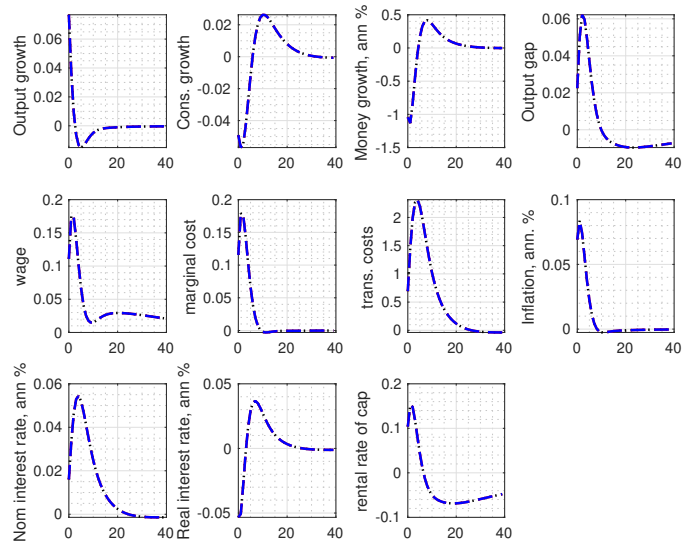


Figure C.11 – IRF to an investment shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

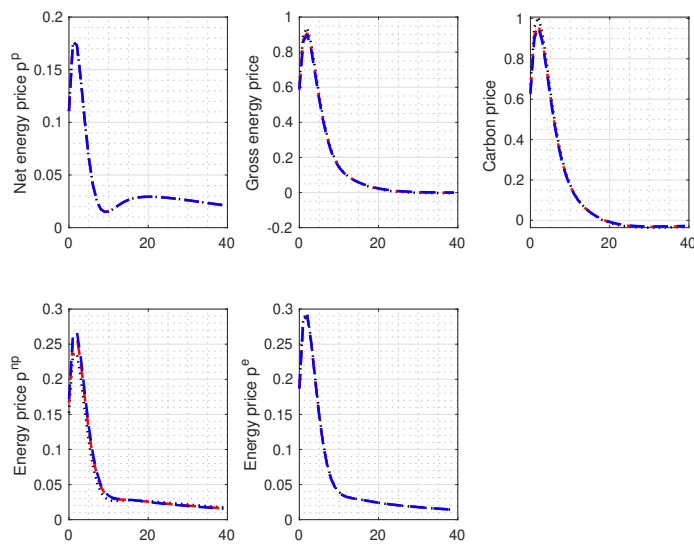


Figure C.12 – IRF to an investment shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

**C.4.1.3 IRF to an inflation shock:**

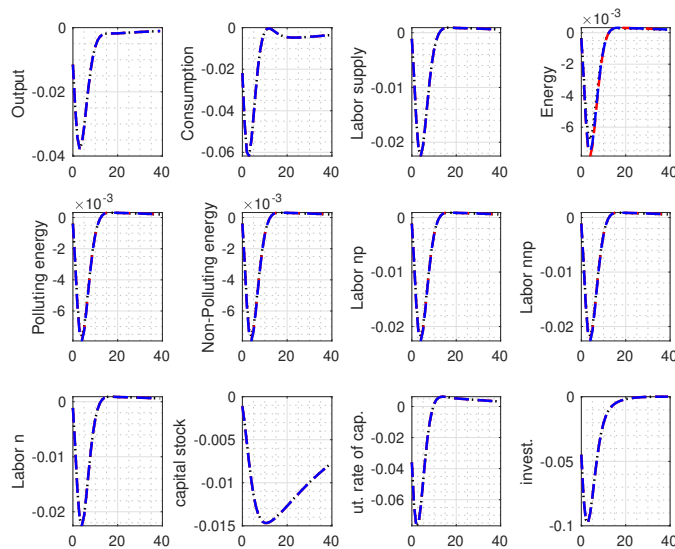


Figure C.13 – IRF to an inflation shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

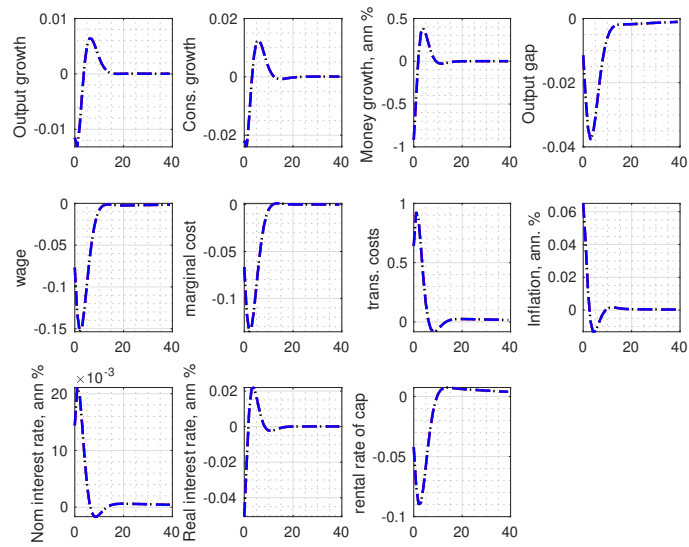


Figure C.14 – IRF to an inflation shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

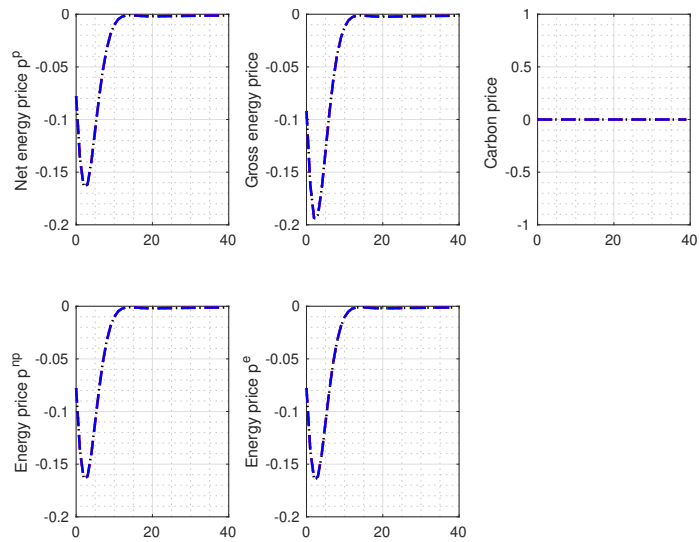


Figure C.15 – IRF to an inflation shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

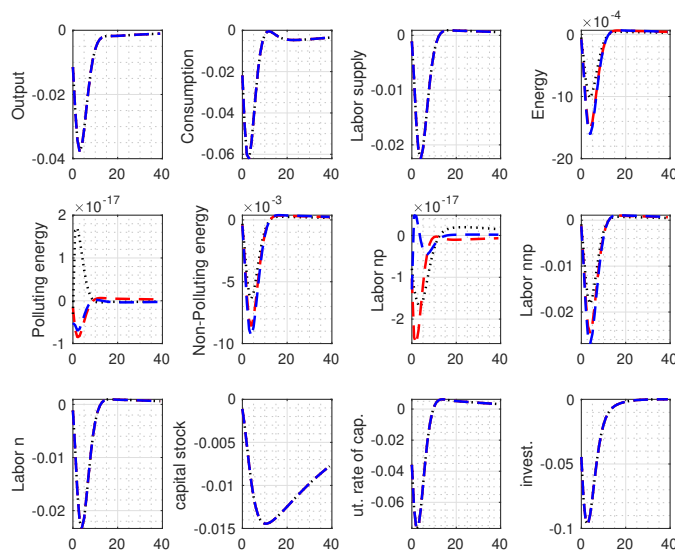


Figure C.16 – IRF to an inflation shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

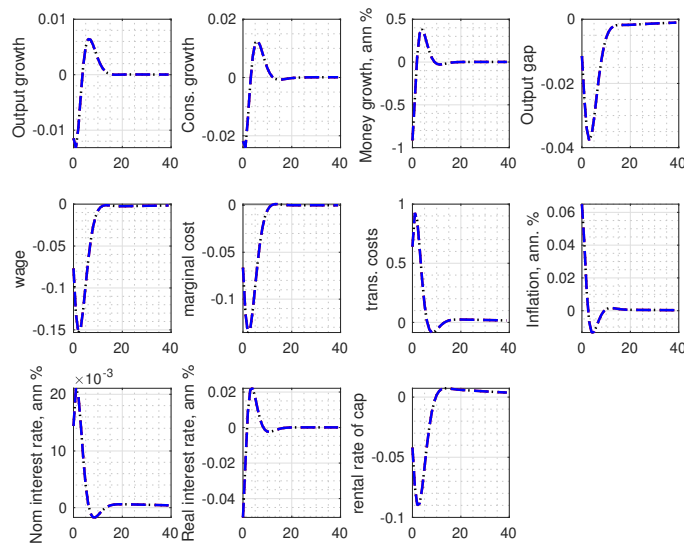


Figure C.17 – IRF to an inflation shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$



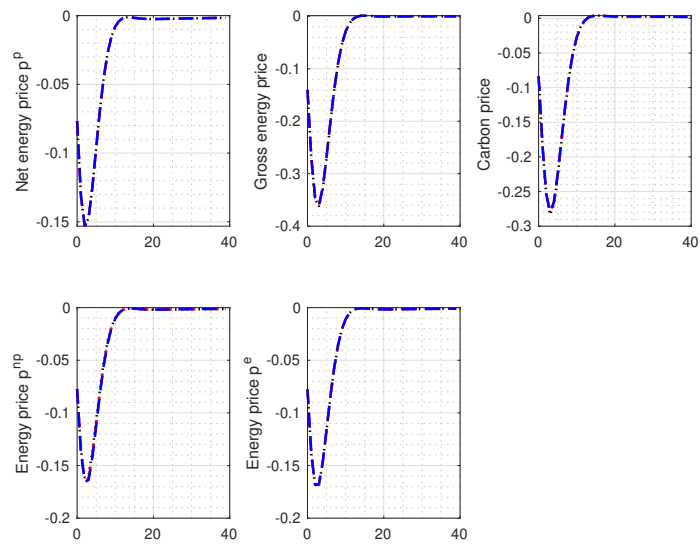


Figure C.18 – IRF to an inflation shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

#### C.4.1.4 IRF to a government spending shock:

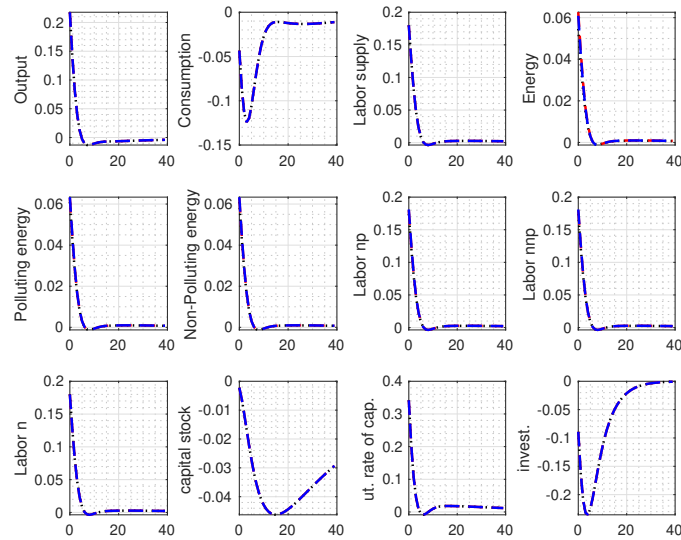


Figure C.19 – IRF to a government spending shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

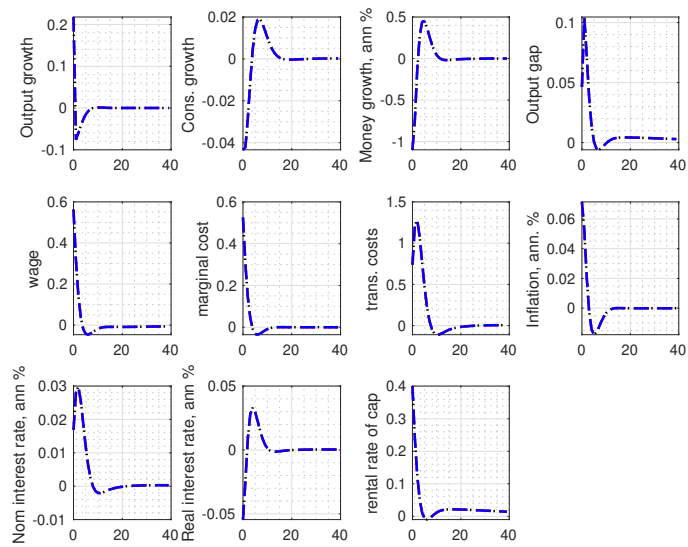


Figure C.20 – IRF to a government spending shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

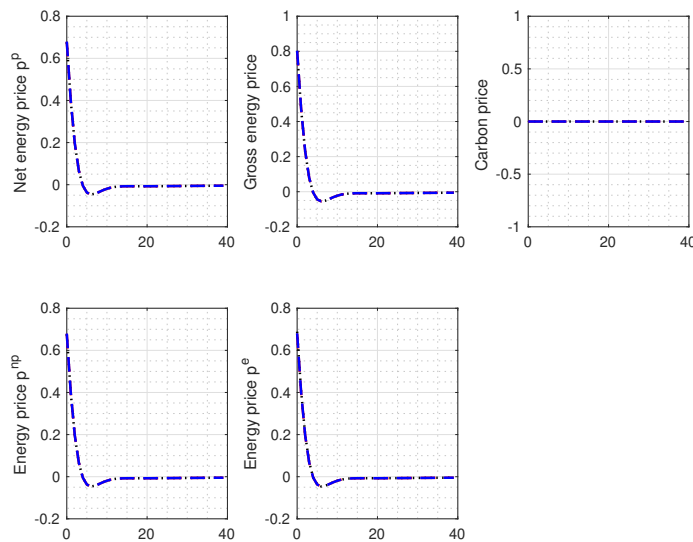


Figure C.21 – IRF to a government spending shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

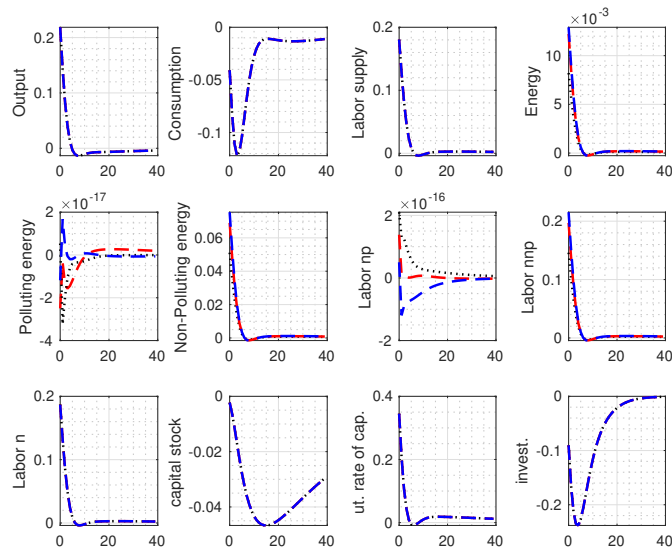


Figure C.22 – IRF to a government spending shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

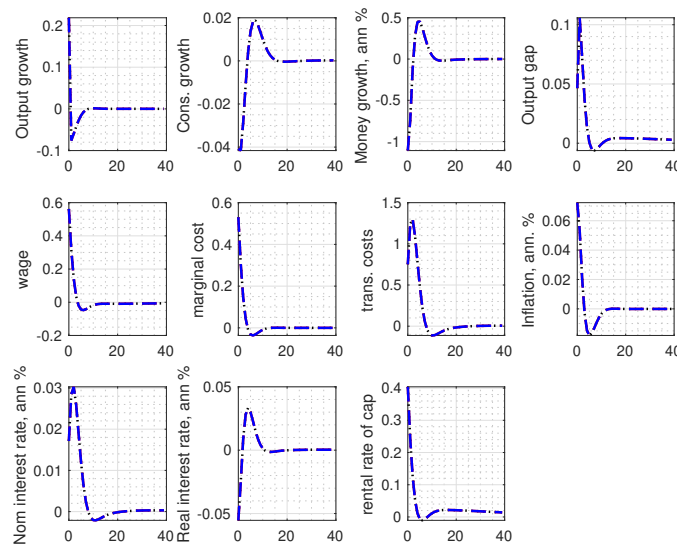


Figure C.23 – IRF to a government spending shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

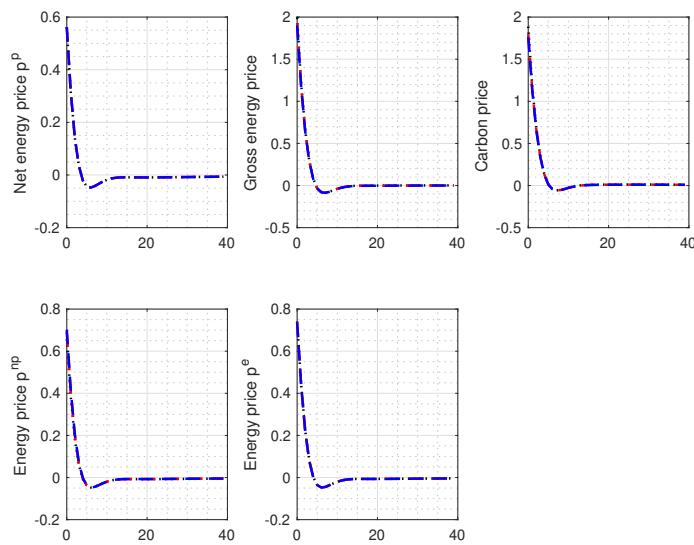


Figure C.24 – IRF to a government spending shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

C.4.1.5 IRF to a carbon tax shock:

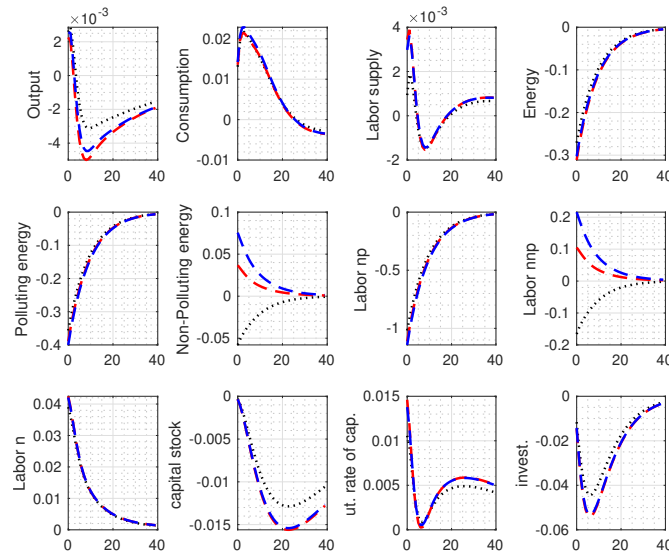


Figure C.25 – IRF to a carbon tax shock in a carbon tax regime (1/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

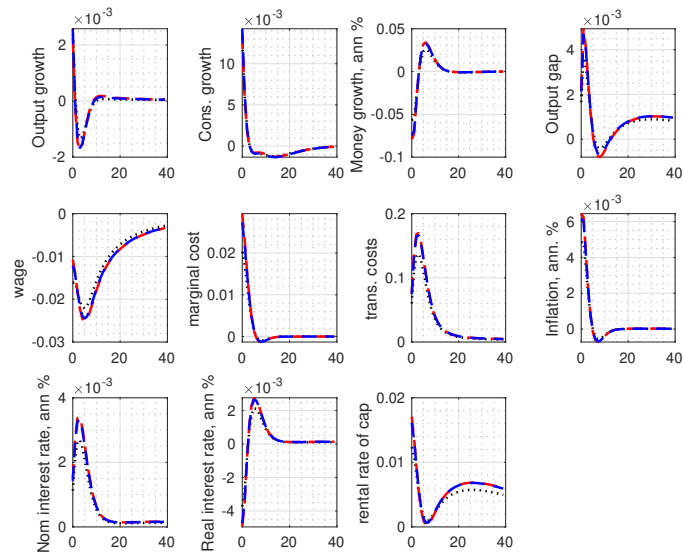


Figure C.26 – IRF to a carbon tax shock in a carbon tax regime (2/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

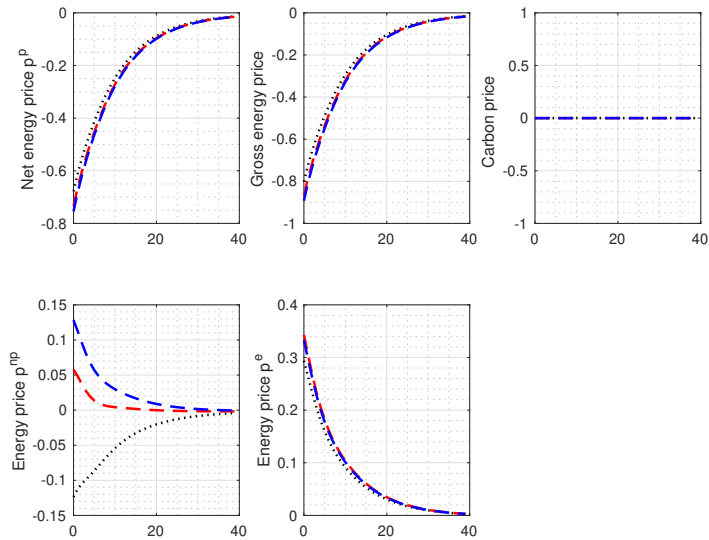


Figure C.27 – IRF to a carbon tax shock in a carbon tax regime (3/3) - black dotted line :  $\epsilon = 0.51$ ; red dashed line  $\epsilon = 1.22$ ; blue dashed line  $\epsilon = 1.85$

## C.4.2 Simulations results

## C.4.2.1 Models with a carbon tax:

Table C.1 – Theoretical moments for an elasticity of substitution of  $\epsilon = 0.51$  with a carbon tax

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1456
<i>dybar</i>	0.0000	0.4525	0.2048
<i>dc</i>	0.0000	0.4572	0.2090
<i>dcbar</i>	0.0000	1.2009	1.4423
<i>di</i>	0.0000	0.9541	0.9102
<i>dibar</i>	0.0000	0.9372	0.8784
<i>y</i>	0.0000	1.7372	3.0178
<i>ybar</i>	0.0000	1.6838	2.8353
<i>ygap</i>	0.0000	0.4859	0.2361
<i>c</i>	0.0000	2.0794	4.3239
<i>cbar</i>	0.0000	2.9584	8.7522
<i>i</i>	0.0000	5.4716	29.9386
<i>ibar</i>	0.0000	4.2505	18.0665
<i>R</i>	0.0000	0.3602	0.1298
<i>Rbar</i>	0.0000	0.8098	0.6557
<i>dM</i>	0.0000	14.9062	222.1941
<i>dMbar</i>	0.0000	57.8078	3341.7426
<i>r</i>	0.0000	0.4106	0.1686
<i>rbar</i>	0.0000	2.7497	7.5607

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Table C.1 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pi</i>	0.0000	0.5100	0.2601
<i>pibar</i>	0.0000	3.8178	14.5756
<i>mc</i>	0.0000	2.5382	6.4426
<i>nbar</i>	0.0000	0.8179	0.6689
<i>n</i>	0.0000	0.7961	0.6338
<i>np</i>	0.0000	2.4963	6.2315
<i>npbar</i>	0.0000	2.5012	6.2559
<i>nnp</i>	0.0000	1.5103	2.2809
<i>nnpbar</i>	0.0000	1.5200	2.3105
<i>m</i>	0.0000	26.7339	714.6991
<i>mbar</i>	0.0000	52.9325	2801.8532
<i>h</i>	0.0000	16.0896	258.8753
<i>hbar</i>	0.0000	33.6494	1132.2788
<i>ks</i>	0.0000	2.3267	5.4134
<i>ksbar</i>	0.0000	2.3199	5.3821
<i>k</i>	0.0000	2.7459	7.5402
<i>kbar</i>	0.0000	2.1252	4.5165
<i>z</i>	0.0000	1.5023	2.2569
<i>zbar</i>	0.0000	0.2903	0.0843
<i>q</i>	0.0000	1.8208	3.3151
<i>qbar</i>	0.0000	3.8270	14.6462
<i>rk</i>	0.0000	1.7577	3.0894
<i>rkbar</i>	0.0000	1.9977	3.9908

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Table C.1 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>w</i>	0.0000	2.4395	5.9509
<i>wbar</i>	0.0000	2.0587	4.2383
<i>pe</i>	0.0000	3.0861	9.5240
<i>pebar</i>	0.0000	2.1444	4.5986
<i>e</i>	0.0000	1.3289	1.7661
<i>ebar</i>	0.0000	1.3299	1.7687
<i>ep</i>	0.0000	1.7055	2.9086
<i>epbar</i>	0.0000	1.7063	2.9116
<i>enp</i>	0.0000	1.3250	1.7555
<i>enpbar</i>	0.0000	1.3266	1.7600
<i>pp</i>	0.0000	3.6653	13.4344
<i>ppbar</i>	0.0000	2.9604	8.7638
<i>pp2</i>	0.0000	3.7872	14.3428
<i>pnp</i>	0.0000	3.5890	12.8812
<i>pnpbar</i>	0.0000	2.9374	8.6285
<i>ns</i>	0.0000	0.7904	0.6247
<i>nsbar</i>	0.0000	0.8121	0.6595
<i>hc</i>	0.0000	25.0974	629.8807
<i>hcbar</i>	0.0000	51.0205	2603.0935
<i>dG</i>	0.0000	3.4502	11.9037
<i>dGbar</i>	0.0000	3.4519	11.9159



Table C.2 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 0.51$  with a carbon tax

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\tau}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.39	0.13	0.01	4.46	5.68	40.93	0.40	0.01
<i>dybar</i>	74.72	0.21	0.02	0.07	1.52	23.47	0.00	0.00
<i>dc</i>	57.94	0.13	0.01	29.77	8.21	2.95	0.88	0.11
<i>dcbar</i>	86.26	0.20	0.02	0.10	2.34	11.08	0.00	0.01
<i>di</i>	67.45	0.65	0.02	3.14	26.05	2.16	0.48	0.05
<i>dibar</i>	77.67	0.68	0.02	0.01	15.55	6.02	0.00	0.05
<i>y</i>	92.37	0.36	0.02	1.05	3.38	2.59	0.23	0.01
<i>ybar</i>	95.94	0.43	0.02	0.00	2.36	1.23	0.00	0.01
<i>ygap</i>	67.01	0.42	0.02	13.24	7.71	8.65	2.93	0.03
<i>c</i>	79.70	0.17	0.01	5.57	12.03	2.02	0.39	0.12
<i>cbar</i>	85.95	0.25	0.02	0.09	8.16	5.50	0.00	0.04
<i>i</i>	82.78	0.93	0.02	0.35	14.32	1.35	0.17	0.08
<i>ibar</i>	81.31	1.04	0.02	0.01	15.37	2.16	0.00	0.10
<i>R</i>	56.40	0.46	0.01	24.39	15.20	2.45	1.06	0.03
<i>Rbar</i>	74.10	0.42	0.02	10.92	4.82	9.70	0.00	0.02
<i>dM</i>	18.74	0.11	0.00	77.43	1.94	1.12	0.65	0.00
<i>dMbar</i>	86.35	0.38	0.02	0.55	0.60	12.09	0.00	0.01
<i>r</i>	61.45	0.46	0.01	22.07	10.25	3.94	1.79	0.03
<i>rbar</i>	85.68	0.33	0.02	1.18	0.33	12.46	0.00	0.00
<i>pi</i>	66.20	0.46	0.02	18.63	8.53	3.60	2.54	0.02
<i>pibar</i>	84.08	0.37	0.02	3.09	0.66	11.77	0.00	0.01

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Table C.2 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$mc$	82.43	0.44	0.02	8.03	1.74	6.10	1.23	0.01
$nbar$	77.83	0.65	0.02	0.03	7.37	12.89	0.00	1.21
$n$	73.87	0.60	0.01	2.18	12.48	9.27	0.37	1.22
$np$	7.01	2.41	0.48	0.21	1.18	0.88	0.03	87.80
$npbar$	7.76	2.41	0.49	0.00	0.73	1.29	0.00	87.32
$nnp$	19.15	25.96	42.20	0.56	3.23	2.40	0.09	6.39
$nnpbar$	21.02	25.65	41.60	0.01	1.99	3.48	0.00	6.25
$m$	53.35	0.36	0.01	29.05	14.05	2.24	0.93	0.02
$mbar$	82.49	0.45	0.02	0.74	5.65	10.63	0.00	0.02
$h$	49.29	0.49	0.01	33.38	13.56	2.22	1.00	0.04
$hbar$	82.62	0.48	0.02	0.80	5.08	10.98	0.00	0.02
$ks$	83.91	0.73	0.02	3.19	7.77	3.70	0.61	0.05
$ksbar$	84.48	1.16	0.02	0.00	12.68	1.53	0.00	0.12
$k$	86.83	1.01	0.02	0.07	10.95	0.96	0.08	0.09
$kbar$	85.25	1.13	0.02	0.00	12.07	1.42	0.00	0.11
$z$	75.18	0.66	0.02	7.13	7.70	8.11	1.16	0.06
$zbar$	86.97	0.84	0.02	0.01	9.74	2.34	0.00	0.09
$q$	52.15	0.42	0.01	17.26	26.74	2.53	0.87	0.03
$qbar$	80.41	0.42	0.02	0.03	9.30	9.80	0.00	0.01
$rk$	75.18	0.66	0.02	7.13	7.70	8.11	1.16	0.06
$rkbar$	86.97	0.84	0.02	0.01	9.74	2.34	0.00	0.09
$w$	76.44	0.42	0.02	11.38	2.40	7.54	1.71	0.09
$wbar$	96.77	0.62	0.02	0.00	1.80	0.55	0.00	0.24

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Table C.2 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$pe$	61.05	14.46	0.67	7.81	3.12	7.26	1.28	4.33
$pebar$	65.85	23.28	1.11	0.00	1.29	0.28	0.00	8.19
$e$	2.42	71.38	3.47	0.07	0.41	0.30	0.01	21.93
$ebar$	2.69	71.28	3.47	0.00	0.25	0.45	0.00	21.86
$ep$	1.84	74.39	0.13	0.05	0.31	0.23	0.01	23.04
$epbar$	2.04	74.31	0.13	0.00	0.19	0.34	0.00	22.98
$enp$	3.05	4.13	90.80	0.09	0.51	0.38	0.02	1.02
$enpbar$	3.38	4.13	90.61	0.00	0.32	0.56	0.00	1.01
$pp$	42.19	25.51	0.11	5.48	2.07	4.95	0.89	18.79
$ppbar$	35.52	33.62	0.23	0.00	0.67	0.12	0.00	29.85
$pp2$	39.51	23.89	0.10	5.14	1.94	4.64	0.83	23.94
$pnp$	44.00	2.28	38.96	5.72	2.16	5.16	0.93	0.79
$pnpbar$	36.07	4.82	56.92	0.00	0.68	0.12	0.00	1.38
$ns$	74.72	0.68	0.02	2.20	12.62	9.38	0.37	0.01
$nsbar$	78.72	0.73	0.02	0.03	7.45	13.04	0.00	0.01
$hc$	51.39	0.42	0.01	31.11	13.85	2.23	0.96	0.03
$hcbbar$	82.54	0.46	0.02	0.77	5.37	10.81	0.00	0.02
$dG$	1.84	74.39	0.13	0.05	0.31	0.23	0.01	23.04
$dGbar$	2.04	74.31	0.13	0.00	0.19	0.34	0.00	22.98

Table C.3 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 0.51$  with a carbon tax

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3984	0.2372	0.1023	0.0045	-0.0571
<i>dybar</i>	0.3124	-0.0142	-0.0619	-0.0636	-0.0584
<i>dc</i>	0.6630	0.3446	0.0924	-0.0770	-0.1705
<i>dcbar</i>	0.0567	-0.0806	-0.0863	-0.0730	-0.0592
<i>di</i>	0.7145	0.4608	0.2565	0.1053	0.0018
<i>dibar</i>	0.3751	0.1890	0.0982	0.0399	-0.0004
<i>y</i>	0.9759	0.9325	0.8778	0.8180	0.7581
<i>ybar</i>	0.9639	0.9052	0.8476	0.7944	0.7458
<i>ygap</i>	0.8284	0.5111	0.2526	0.0805	-0.0181
<i>c</i>	0.9758	0.9196	0.8468	0.7694	0.6958
<i>cbar</i>	0.9176	0.8259	0.7474	0.6832	0.6309
<i>i</i>	0.9848	0.9479	0.8969	0.8382	0.7763
<i>ibar</i>	0.9757	0.9331	0.8814	0.8249	0.7664
<i>R</i>	0.9459	0.8208	0.6696	0.5236	0.3989
<i>Rbar</i>	0.3513	0.2085	0.1578	0.1280	0.1062
<i>dM</i>	0.0215	-0.0547	-0.0894	-0.0936	-0.0813
<i>dMbar</i>	-0.3963	-0.0706	-0.0151	-0.0051	-0.0029
<i>r</i>	0.6804	0.3836	0.1703	0.0407	-0.0251
<i>rbar</i>	0.2037	0.0618	0.0316	0.0216	0.0161
<i>pi</i>	0.6995	0.3454	0.0904	-0.0521	-0.1101
<i>pibar</i>	-0.3752	-0.0653	-0.0127	-0.0035	-0.0016

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Table C.3 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6014	0.3082	0.1179	0.0100	-0.0405
<i>nbar</i>	0.3160	0.1855	0.1517	0.1371	0.1281
<i>n</i>	0.7143	0.4656	0.2731	0.1394	0.0562
<i>np</i>	0.8822	0.7772	0.6857	0.6072	0.5400
<i>npbar</i>	0.8416	0.7475	0.6712	0.6041	0.5440
<i>nnp</i>	0.8538	0.7246	0.6160	0.5281	0.4582
<i>nnpbar</i>	0.7460	0.6451	0.5764	0.5185	0.4674
<i>m</i>	0.8408	0.6690	0.5125	0.3847	0.2880
<i>mbar</i>	0.3251	0.1858	0.1419	0.1182	0.1014
<i>h</i>	0.8120	0.6199	0.4517	0.3192	0.2229
<i>hbar</i>	0.2668	0.1253	0.0871	0.0693	0.0574
<i>ks</i>	0.8898	0.7959	0.7246	0.6749	0.6423
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9710
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9777
<i>z</i>	0.7442	0.5388	0.3952	0.3074	0.2621
<i>zbar</i>	0.9633	0.9017	0.8389	0.7791	0.7230
<i>q</i>	0.8567	0.6838	0.5165	0.3744	0.2636
<i>qbar</i>	0.3174	0.1728	0.1241	0.0961	0.0753
<i>rk</i>	0.7442	0.5388	0.3952	0.3074	0.2621
<i>rkbar</i>	0.9633	0.9017	0.8389	0.7791	0.7230
<i>w</i>	0.7290	0.5042	0.3418	0.2372	0.1773
<i>wbar</i>	0.8831	0.8202	0.7713	0.7287	0.6907

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Table C.3 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.7252	0.4990	0.3342	0.2257	0.1608
<i>pebar</i>	0.9380	0.8688	0.8041	0.7453	0.6920
<i>e</i>	0.8956	0.8018	0.7182	0.6438	0.5777
<i>ebar</i>	0.8828	0.7916	0.7118	0.6404	0.5761
<i>ep</i>	0.8968	0.8041	0.7212	0.6472	0.5813
<i>epbar</i>	0.8873	0.7963	0.7162	0.6443	0.5797
<i>enp</i>	0.8933	0.7975	0.7124	0.6372	0.5709
<i>enpbar</i>	0.8763	0.7846	0.7054	0.6348	0.5714
<i>pp</i>	0.7797	0.5964	0.4582	0.3614	0.2968
<i>ppbar</i>	0.9190	0.8402	0.7682	0.7032	0.6446
<i>pp2</i>	0.7857	0.6067	0.4710	0.3750	0.3100
<i>pnp</i>	0.7811	0.5981	0.4600	0.3630	0.2984
<i>pnpbar</i>	0.9208	0.8432	0.7721	0.7079	0.6500
<i>ns</i>	0.7122	0.4618	0.2680	0.1337	0.0503
<i>nsbar</i>	0.3095	0.1785	0.1452	0.1312	0.1228
<i>hc</i>	0.8272	0.6457	0.4835	0.3533	0.2566
<i>hcbars</i>	0.2960	0.1555	0.1143	0.0935	0.0791
<i>dG</i>	0.8968	0.8041	0.7212	0.6472	0.5813
<i>dGbar</i>	0.8873	0.7963	0.7162	0.6443	0.5797

Table C.4 – Theoretical moments for an elasticity of substitution of  $\epsilon = 0.95$  with a carbon tax

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3955	0.1564
<i>dybar</i>	0.0000	1.4942	2.2326
<i>dc</i>	0.0000	0.5868	0.3444
<i>dcbar</i>	0.0000	3.6590	13.3881
<i>di</i>	0.0000	0.9975	0.9950
<i>dibar</i>	0.0000	1.0166	1.0335
<i>y</i>	0.0000	1.7902	3.2049
<i>ybar</i>	0.0000	1.8517	3.4286
<i>ygap</i>	0.0000	0.8772	0.7695
<i>c</i>	0.0000	2.4072	5.7945
<i>cbar</i>	0.0000	3.9105	15.2919
<i>i</i>	0.0000	5.4422	29.6174
<i>ibar</i>	0.0000	3.9715	15.7731
<i>R</i>	0.0000	0.5518	0.3044
<i>Rbar</i>	0.0000	3.3992	11.5543
<i>dM</i>	0.0000	20.4997	420.2387
<i>dMbar</i>	0.0000	388.1557	150664.8173
<i>r</i>	0.0000	0.5275	0.2782
<i>rbar</i>	0.0000	10.7109	114.7238
<i>pi</i>	0.0000	0.2934	0.0861
<i>pibar</i>	0.0000	8.5948	73.8699

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Table C.4 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	1.8250	3.3305
<i>nbar</i>	0.0000	1.6161	2.6119
<i>n</i>	0.0000	0.7621	0.5808
<i>np</i>	0.0000	2.6264	6.8982
<i>npbar</i>	0.0000	2.9852	8.9116
<i>nnp</i>	0.0000	0.7607	0.5787
<i>nnpbar</i>	0.0000	1.6151	2.6085
<i>m</i>	0.0000	38.3225	1468.6174
<i>mbar</i>	0.0000	227.6688	51833.0859
<i>h</i>	0.0000	23.5978	556.8556
<i>hbar</i>	0.0000	152.2848	23190.6696
<i>ks</i>	0.0000	2.1816	4.7596
<i>ksbar</i>	0.0000	2.2015	4.8468
<i>k</i>	0.0000	2.7412	7.5144
<i>kbar</i>	0.0000	2.0130	4.0520
<i>z</i>	0.0000	1.2766	1.6298
<i>zbar</i>	0.0000	0.3164	0.1001
<i>q</i>	0.0000	2.2045	4.8598
<i>qbar</i>	0.0000	7.3420	53.9044
<i>rk</i>	0.0000	1.4937	2.2310
<i>rkbar</i>	0.0000	2.1773	4.7408
<i>w</i>	0.0000	2.0492	4.1991
<i>wbar</i>	0.0000	2.1059	4.4349

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Table C.4 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	2.6940	7.2577
<i>pebar</i>	0.0000	2.2724	5.1640
<i>e</i>	0.0000	1.5351	2.3566
<i>ebar</i>	0.0000	1.6117	2.5975
<i>ep</i>	0.0000	1.8284	3.3430
<i>epbar</i>	0.0000	1.8928	3.5826
<i>enp</i>	0.0000	1.5994	2.5581
<i>enpbar</i>	0.0000	1.6753	2.8067
<i>pp</i>	0.0000	3.2365	10.4748
<i>ppbar</i>	0.0000	2.9049	8.4387
<i>pp2</i>	0.0000	3.2347	10.4635
<i>pnp</i>	0.0000	2.7813	7.7359
<i>pnpbar</i>	0.0000	2.4200	5.8562
<i>ns</i>	0.0000	0.7566	0.5725
<i>nsbar</i>	0.0000	1.6134	2.6029
<i>hc</i>	0.0000	36.3874	1324.0452
<i>hcbar</i>	0.0000	225.1723	50702.5851
<i>dG</i>	0.0000	3.6989	13.6817
<i>dGbar</i>	0.0000	3.8291	14.6622

Table C.5 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 0.95$  with a carbon tax

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\tau o}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	61.56	0.21	0.02	1.76	2.53	33.41	0.52	0.00
<i>dybar</i>	82.26	0.36	0.02	0.01	0.31	17.02	0.00	0.01
<i>dc</i>	77.61	0.08	0.02	9.97	5.95	4.57	1.77	0.04
<i>dcbar</i>	86.55	0.36	0.02	0.01	0.41	12.64	0.00	0.01
<i>di</i>	76.42	0.88	0.02	0.66	18.53	2.83	0.58	0.08
<i>dibar</i>	78.63	0.74	0.02	0.00	12.86	7.68	0.00	0.06
<i>y</i>	94.97	0.48	0.02	0.22	2.05	1.99	0.25	0.02
<i>ybar</i>	93.72	0.52	0.02	0.00	1.97	3.75	0.00	0.02
<i>ygap</i>	84.54	0.41	0.02	0.85	1.00	12.11	1.05	0.01
<i>c</i>	84.57	0.12	0.02	1.82	9.80	2.84	0.76	0.07
<i>cbar</i>	86.54	0.31	0.02	0.01	4.41	8.69	0.00	0.02
<i>i</i>	84.75	1.18	0.02	0.05	12.26	1.47	0.15	0.12
<i>ibar</i>	79.77	1.34	0.02	0.00	17.08	1.62	0.00	0.16
<i>R</i>	72.62	0.58	0.02	14.31	5.86	4.10	2.48	0.04
<i>Rbar</i>	85.66	0.40	0.02	0.79	0.45	12.67	0.00	0.01
<i>dM</i>	51.52	0.31	0.01	40.47	0.87	4.44	2.36	0.01
<i>dMbar</i>	86.47	0.39	0.02	0.02	0.23	12.86	0.00	0.01
<i>r</i>	72.50	0.56	0.02	13.84	4.88	5.92	2.25	0.03
<i>rbar</i>	86.35	0.38	0.02	0.09	0.28	12.87	0.00	0.01
<i>pi</i>	78.03	0.56	0.02	8.59	2.06	4.36	6.35	0.03
<i>pibar</i>	86.40	0.39	0.02	0.09	0.24	12.85	0.00	0.01

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Table C.5 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$mc$	86.30	0.49	0.02	3.04	0.25	6.89	2.99	0.02
$nbar$	84.08	0.51	0.02	0.01	2.06	12.96	0.00	0.35
$n$	76.41	0.81	0.02	0.87	10.29	9.65	0.51	1.45
$np$	6.43	0.10	0.00	0.07	0.87	0.81	0.04	91.67
$npbar$	24.64	0.18	0.01	0.00	0.60	3.80	0.00	70.77
$nnp$	76.68	0.34	1.16	0.87	10.32	9.69	0.51	0.42
$nnpbar$	84.19	0.37	0.28	0.01	2.06	12.98	0.00	0.11
$m$	74.53	0.53	0.02	12.08	6.27	4.12	2.43	0.03
$mbar$	86.31	0.40	0.02	0.02	0.48	12.76	0.00	0.01
$h$	73.09	0.65	0.02	13.74	5.62	4.21	2.63	0.05
$hbar$	86.33	0.40	0.02	0.02	0.43	12.78	0.00	0.01
$ks$	88.02	0.97	0.02	0.69	6.45	2.94	0.82	0.09
$ksbar$	83.32	1.48	0.02	0.00	13.81	1.19	0.00	0.18
$k$	87.89	1.27	0.02	0.00	9.62	0.99	0.06	0.13
$kbar$	84.13	1.44	0.02	0.00	13.18	1.05	0.00	0.17
$z$	80.76	0.91	0.02	1.90	6.81	7.52	1.99	0.09
$zbar$	86.52	0.90	0.02	0.00	8.12	4.33	0.00	0.10
$q$	68.09	0.64	0.02	3.73	22.52	3.78	1.18	0.05
$qbar$	84.83	0.43	0.02	0.01	2.68	12.02	0.00	0.01
$rk$	80.76	0.91	0.02	1.90	6.81	7.52	1.99	0.09
$rkbar$	86.52	0.90	0.02	0.00	8.12	4.33	0.00	0.10
$w$	86.13	0.57	0.02	3.19	0.80	6.02	3.06	0.21
$wbar$	95.89	0.78	0.02	0.00	1.70	1.30	0.00	0.29

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Table C.5 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$pe$	58.19	22.84	1.06	1.97	0.89	5.77	2.05	7.23
$pebar$	59.60	27.85	1.31	0.00	1.12	0.52	0.00	9.60
$e$	2.30	71.65	3.42	0.03	0.31	0.29	0.02	21.99
$ebar$	10.34	64.82	3.10	0.00	0.25	1.59	0.00	19.88
$ep$	1.63	74.75	0.00	0.02	0.22	0.21	0.01	23.17
$epbar$	7.51	69.58	0.00	0.00	0.18	1.16	0.00	21.56
$enp$	2.13	0.01	97.26	0.02	0.29	0.27	0.01	0.01
$enpbar$	9.58	0.04	88.65	0.00	0.23	1.48	0.00	0.01
$pp$	40.32	23.94	0.01	1.36	0.61	4.00	1.42	28.33
$ppbar$	36.47	26.54	0.02	0.00	0.69	0.32	0.00	35.97
$pp2$	40.36	23.97	0.01	1.37	0.61	4.00	1.42	28.25
$pnp$	54.59	0.40	34.93	1.85	0.83	5.41	1.93	0.05
$pnpbar$	52.55	0.60	45.30	0.00	0.99	0.46	0.00	0.11
$ns$	77.52	0.82	0.02	0.88	10.44	9.79	0.52	0.01
$nsbar$	84.37	0.51	0.02	0.01	2.07	13.01	0.00	0.01
$hc$	73.84	0.59	0.02	12.87	5.96	4.17	2.53	0.04
$hcbbar$	86.32	0.40	0.02	0.02	0.46	12.77	0.00	0.01
$dG$	1.63	74.75	0.00	0.02	0.22	0.21	0.01	23.17
$dGbar$	7.51	69.58	0.00	0.00	0.18	1.16	0.00	21.56

Table C.6 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 0.95$  with a carbon tax

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.4208	0.2638	0.0971	-0.0112	-0.0809
<i>dybar</i>	-0.6478	0.3472	-0.2035	0.1032	-0.0659
<i>dc</i>	0.6991	0.3231	0.0434	-0.1407	-0.2194
<i>dcbar</i>	-0.5994	0.2968	-0.1934	0.0843	-0.0651
<i>di</i>	0.6777	0.4003	0.1942	0.0553	-0.0282
<i>dibar</i>	-0.0908	0.3588	-0.0065	0.1107	-0.0165
<i>y</i>	0.9756	0.9307	0.8729	0.8103	0.7483
<i>ybar</i>	0.6744	0.7706	0.6408	0.6434	0.5788
<i>ygap</i>	-0.3618	0.3115	-0.1116	0.0983	-0.0281
<i>c</i>	0.9703	0.8990	0.8086	0.7155	0.6309
<i>cbar</i>	0.5622	0.6493	0.4765	0.4729	0.3955
<i>i</i>	0.9832	0.9436	0.8906	0.8311	0.7697
<i>ibar</i>	0.9672	0.9404	0.8901	0.8402	0.7830
<i>R</i>	0.9039	0.7250	0.5324	0.3719	0.2538
<i>Rbar</i>	-0.5152	0.3169	-0.1515	0.1031	-0.0423
<i>dM</i>	0.1816	-0.0274	-0.1461	-0.1567	-0.1335
<i>dMbar</i>	-0.7748	0.4294	-0.2387	0.1321	-0.0736
<i>r</i>	0.8209	0.6206	0.4047	0.2530	0.1459
<i>rbar</i>	-0.5451	0.3100	-0.1662	0.0966	-0.0503
<i>pi</i>	0.6146	0.2172	-0.0124	-0.0952	-0.0899
<i>pibar</i>	-0.7723	0.4282	-0.2379	0.1318	-0.0733

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Table C.6 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.5200	0.1896	0.0189	-0.0563	-0.0679
<i>nbar</i>	-0.4730	0.3332	-0.1226	0.1238	-0.0178
<i>n</i>	0.6763	0.3885	0.1951	0.0733	0.0150
<i>np</i>	0.8817	0.7755	0.6854	0.6086	0.5436
<i>npbar</i>	0.4989	0.6707	0.4801	0.5004	0.4125
<i>nnp</i>	0.6766	0.3890	0.1957	0.0739	0.0155
<i>nnpbar</i>	-0.4740	0.3328	-0.1233	0.1234	-0.0183
<i>m</i>	0.8546	0.6541	0.4615	0.3118	0.2084
<i>mbar</i>	-0.5183	0.3163	-0.1529	0.1026	-0.0429
<i>h</i>	0.8357	0.6192	0.4154	0.2618	0.1593
<i>hbar</i>	-0.5330	0.3125	-0.1606	0.0987	-0.0476
<i>ks</i>	0.9231	0.8552	0.8083	0.7760	0.7558
<i>ksbar</i>	0.9928	0.9913	0.9837	0.9768	0.9671
<i>k</i>	0.9991	0.9963	0.9917	0.9856	0.9779
<i>kbar</i>	0.9991	0.9964	0.9919	0.9859	0.9785
<i>z</i>	0.7861	0.6112	0.5046	0.4429	0.4174
<i>zbar</i>	0.6803	0.7700	0.6368	0.6319	0.5606
<i>q</i>	0.8428	0.6378	0.4515	0.3027	0.1992
<i>qbar</i>	-0.4521	0.3331	-0.1193	0.1181	-0.0246
<i>rk</i>	0.7861	0.6112	0.5046	0.4429	0.4174
<i>rkbar</i>	0.6803	0.7700	0.6368	0.6319	0.5606
<i>w</i>	0.8060	0.6326	0.5130	0.4330	0.3868
<i>wbar</i>	0.7369	0.7977	0.6888	0.6826	0.6267

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Table C.6 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.7762	0.5869	0.4593	0.3745	0.3247
<i>pebar</i>	0.9073	0.8401	0.7706	0.7129	0.6581
<i>e</i>	0.8955	0.8014	0.7179	0.6439	0.5783
<i>ebar</i>	0.7379	0.7536	0.6282	0.5928	0.5179
<i>ep</i>	0.8971	0.8044	0.7218	0.6481	0.5825
<i>epbar</i>	0.7831	0.7694	0.6563	0.6104	0.5381
<i>enp</i>	0.8944	0.7993	0.7153	0.6411	0.5756
<i>enpbar</i>	0.7452	0.7562	0.6329	0.5960	0.5218
<i>pp</i>	0.8151	0.6571	0.5444	0.4635	0.4090
<i>ppbar</i>	0.9050	0.8291	0.7556	0.6922	0.6335
<i>pp2</i>	0.8123	0.6522	0.5382	0.4568	0.4023
<i>pn<sub>p</sub></i>	0.7913	0.6130	0.4905	0.4069	0.3557
<i>pn<sub>p</sub>bar</i>	0.9096	0.8397	0.7705	0.7116	0.6566
<i>ns</i>	0.6732	0.3828	0.1880	0.0655	0.0073
<i>nsbar</i>	-0.4776	0.3316	-0.1255	0.1220	-0.0198
<i>hc</i>	0.8456	0.6374	0.4393	0.2875	0.1845
<i>hcbar</i>	-0.5258	0.3143	-0.1569	0.1006	-0.0453
<i>dG</i>	0.8971	0.8044	0.7218	0.6481	0.5825
<i>dGbar</i>	0.7831	0.7694	0.6563	0.6104	0.5381

Table C.7 – Theoretical moments for an elasticity of substitution of  $\epsilon = 1.008$  with a carbon tax

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1456
<i>dybar</i>	0.0000	0.4521	0.2044
<i>dc</i>	0.0000	0.4569	0.2088
<i>dcbar</i>	0.0000	1.1964	1.4315
<i>di</i>	0.0000	0.9548	0.9117
<i>dibar</i>	0.0000	0.9375	0.8789
<i>y</i>	0.0000	1.7379	3.0204
<i>ybar</i>	0.0000	1.6842	2.8366
<i>ygap</i>	0.0000	0.4855	0.2357
<i>c</i>	0.0000	2.0787	4.3208
<i>cbar</i>	0.0000	2.9557	8.7361
<i>i</i>	0.0000	5.4793	30.0232
<i>ibar</i>	0.0000	4.2574	18.1252
<i>R</i>	0.0000	0.3601	0.1296
<i>Rbar</i>	0.0000	0.8073	0.6518
<i>dM</i>	0.0000	14.9008	222.0324
<i>dMbar</i>	0.0000	57.4956	3305.7484
<i>r</i>	0.0000	0.4101	0.1682
<i>rbar</i>	0.0000	2.7309	7.4578
<i>pi</i>	0.0000	0.5094	0.2595
<i>pibar</i>	0.0000	3.7979	14.4238

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Table C.7 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	2.5327	6.4145
<i>nbar</i>	0.0000	0.8156	0.6652
<i>n</i>	0.0000	0.7948	0.6317
<i>np</i>	0.0000	2.6443	6.9925
<i>npbar</i>	0.0000	2.6490	7.0171
<i>nnp</i>	0.0000	0.7964	0.6342
<i>nnpbar</i>	0.0000	0.8172	0.6678
<i>m</i>	0.0000	26.7248	714.2159
<i>mbar</i>	0.0000	52.7601	2783.6270
<i>h</i>	0.0000	16.0854	258.7405
<i>hbar</i>	0.0000	33.5321	1124.4010
<i>ks</i>	0.0000	2.3272	5.4156
<i>ksbar</i>	0.0000	2.3230	5.3965
<i>k</i>	0.0000	2.7493	7.5589
<i>kbar</i>	0.0000	2.1279	4.5280
<i>z</i>	0.0000	1.5013	2.2539
<i>zbar</i>	0.0000	0.2905	0.0844
<i>q</i>	0.0000	1.8199	3.3119
<i>qbar</i>	0.0000	3.8150	14.5544
<i>rk</i>	0.0000	1.7565	3.0853
<i>rkbar</i>	0.0000	1.9996	3.9986
<i>w</i>	0.0000	2.4377	5.9422
<i>wbar</i>	0.0000	2.0586	4.2378

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Table C.7 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	3.1573	9.9686
<i>pebar</i>	0.0000	2.2694	5.1502
<i>e</i>	0.0000	1.5364	2.3604
<i>ebar</i>	0.0000	1.5375	2.3640
<i>ep</i>	0.0000	1.8358	3.3700
<i>epbar</i>	0.0000	1.8366	3.3732
<i>enp</i>	0.0000	1.6311	2.6603
<i>enpbar</i>	0.0000	1.6327	2.6656
<i>pp</i>	0.0000	3.6213	13.1140
<i>ppbar</i>	0.0000	2.9000	8.4102
<i>pp2</i>	0.0000	3.6293	13.1721
<i>pnp</i>	0.0000	3.1776	10.0974
<i>pnpbar</i>	0.0000	2.3762	5.6462
<i>ns</i>	0.0000	0.7894	0.6231
<i>nsbar</i>	0.0000	0.8101	0.6563
<i>hc</i>	0.0000	25.0898	629.4980
<i>hcbar</i>	0.0000	50.8485	2585.5716
<i>dG</i>	0.0000	3.7138	13.7922
<i>dGbar</i>	0.0000	3.7156	13.8054

Table C.8 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 1.008$  with a carbon tax

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\tau o}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.32	0.17	0.01	4.46	5.69	40.93	0.40	0.01
<i>dybar</i>	74.59	0.27	0.02	0.07	1.53	23.52	0.00	0.00
<i>dc</i>	57.90	0.14	0.01	29.81	8.20	2.94	0.88	0.11
<i>dcbar</i>	86.20	0.27	0.02	0.10	2.35	11.05	0.00	0.01
<i>di</i>	67.26	0.81	0.02	3.14	26.07	2.15	0.48	0.08
<i>dibar</i>	77.47	0.85	0.02	0.01	15.59	5.99	0.00	0.08
<i>y</i>	92.24	0.46	0.02	1.05	3.39	2.59	0.23	0.02
<i>ybar</i>	95.80	0.55	0.02	0.00	2.36	1.23	0.00	0.03
<i>ygap</i>	66.85	0.52	0.02	13.28	7.73	8.61	2.95	0.04
<i>c</i>	79.67	0.19	0.02	5.58	12.02	2.01	0.39	0.12
<i>cbar</i>	85.89	0.32	0.02	0.09	8.16	5.48	0.00	0.04
<i>i</i>	82.53	1.15	0.02	0.35	14.33	1.34	0.17	0.12
<i>ibar</i>	81.05	1.28	0.02	0.01	15.35	2.15	0.00	0.14
<i>R</i>	56.31	0.57	0.01	24.39	15.17	2.44	1.06	0.05
<i>Rbar</i>	73.93	0.53	0.02	10.99	4.84	9.67	0.00	0.03
<i>dM</i>	18.67	0.14	0.00	77.48	1.93	1.11	0.66	0.01
<i>dMbar</i>	86.25	0.49	0.02	0.55	0.61	12.07	0.00	0.02
<i>r</i>	61.33	0.57	0.02	22.09	10.23	3.92	1.80	0.04
<i>rbar</i>	85.58	0.43	0.02	1.20	0.33	12.43	0.00	0.01
<i>pi</i>	66.07	0.58	0.02	18.65	8.51	3.58	2.55	0.04
<i>pibar</i>	83.96	0.48	0.02	3.12	0.66	11.74	0.00	0.02

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Table C.8 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
<i>mc</i>	82.29	0.56	0.02	8.06	1.74	6.08	1.23	0.03
<i>nbar</i>	77.49	0.88	0.02	0.03	7.38	12.83	0.00	1.36
<i>n</i>	73.56	0.82	0.02	2.17	12.46	9.23	0.37	1.38
<i>np</i>	6.65	0.07	0.00	0.20	1.13	0.83	0.03	91.09
<i>npbar</i>	7.35	0.08	0.00	0.00	0.70	1.22	0.00	90.65
<i>nnp</i>	73.27	1.04	0.01	2.16	12.41	9.19	0.36	1.55
<i>nnpbar</i>	77.20	1.08	0.02	0.03	7.35	12.79	0.00	1.53
<i>m</i>	53.26	0.45	0.01	29.08	14.02	2.22	0.93	0.03
<i>mbar</i>	82.37	0.58	0.02	0.74	5.67	10.59	0.00	0.03
<i>h</i>	49.17	0.61	0.01	33.41	13.52	2.21	1.00	0.06
<i>hbar</i>	82.49	0.60	0.02	0.80	5.10	10.95	0.00	0.04
<i>ks</i>	83.72	0.91	0.02	3.19	7.78	3.68	0.61	0.08
<i>ksbar</i>	84.19	1.42	0.02	0.00	12.67	1.53	0.00	0.17
<i>k</i>	86.56	1.24	0.02	0.07	10.95	0.95	0.08	0.13
<i>kbar</i>	84.97	1.38	0.02	0.00	12.05	1.41	0.00	0.16
<i>z</i>	75.00	0.82	0.02	7.13	7.71	8.08	1.16	0.08
<i>zbar</i>	86.74	1.04	0.02	0.01	9.74	2.34	0.00	0.12
<i>q</i>	52.04	0.52	0.01	17.28	26.71	2.52	0.87	0.04
<i>qbar</i>	80.28	0.54	0.02	0.03	9.34	9.77	0.00	0.03
<i>rk</i>	75.00	0.82	0.02	7.13	7.71	8.08	1.16	0.08
<i>rkbar</i>	86.74	1.04	0.02	0.01	9.74	2.34	0.00	0.12
<i>w</i>	76.30	0.53	0.02	11.40	2.41	7.52	1.71	0.12
<i>wbar</i>	96.52	0.81	0.02	0.00	1.79	0.53	0.00	0.31

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Table C.8 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$pe$	57.08	18.37	0.83	7.41	2.85	6.72	1.21	5.53
$pebar$	59.82	27.88	1.31	0.00	1.15	0.22	0.00	9.63
$e$	2.41	71.44	3.41	0.07	0.41	0.30	0.01	21.95
$ebar$	2.67	71.34	3.41	0.00	0.25	0.44	0.00	21.88
$ep$	1.69	74.60	0.00	0.05	0.29	0.21	0.01	23.15
$epbar$	1.87	74.54	0.00	0.00	0.18	0.31	0.00	23.10
$enp$	2.14	0.03	97.08	0.06	0.36	0.27	0.01	0.05
$enpbar$	2.37	0.03	96.93	0.00	0.23	0.39	0.00	0.05
$pp$	43.39	20.40	0.01	5.64	2.17	5.11	0.92	22.37
$ppbar$	36.63	26.13	0.01	0.00	0.70	0.13	0.00	36.39
$pp2$	43.20	20.31	0.01	5.61	2.16	5.09	0.91	22.71
$pnp$	56.35	0.38	25.28	7.32	2.82	6.64	1.19	0.02
$pnpbar$	54.56	0.34	43.80	0.00	1.05	0.20	0.00	0.05
$ns$	74.57	0.83	0.02	2.20	12.63	9.36	0.37	0.01
$nsbar$	78.55	0.89	0.02	0.03	7.48	13.01	0.00	0.02
$hc$	51.28	0.52	0.01	31.14	13.81	2.22	0.97	0.04
$hcbars$	82.41	0.59	0.02	0.77	5.40	10.78	0.00	0.03
$dG$	1.69	74.60	0.00	0.05	0.29	0.21	0.01	23.15
$dGbar$	1.87	74.54	0.00	0.00	0.18	0.31	0.00	23.10

Table C.9 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 1.008$  with a carbon tax

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3985	0.2373	0.1026	0.0047	-0.0569
<i>dybar</i>	0.3132	-0.0132	-0.0617	-0.0637	-0.0585
<i>dc</i>	0.6626	0.3444	0.0924	-0.0768	-0.1701
<i>dcbar</i>	0.0590	-0.0803	-0.0866	-0.0734	-0.0595
<i>di</i>	0.7147	0.4613	0.2570	0.1058	0.0023
<i>dibar</i>	0.3772	0.1900	0.0986	0.0401	-0.0004
<i>y</i>	0.9759	0.9326	0.8779	0.8182	0.7583
<i>ybar</i>	0.9640	0.9054	0.8478	0.7946	0.7460
<i>ygap</i>	0.8291	0.5125	0.2542	0.0818	-0.0172
<i>c</i>	0.9758	0.9197	0.8469	0.7696	0.6960
<i>cbar</i>	0.9181	0.8265	0.7480	0.6838	0.6316
<i>i</i>	0.9848	0.9479	0.8970	0.8383	0.7764
<i>ibar</i>	0.9758	0.9332	0.8815	0.8249	0.7665
<i>R</i>	0.9460	0.8210	0.6701	0.5242	0.3996
<i>Rbar</i>	0.3542	0.2103	0.1589	0.1289	0.1069
<i>dM</i>	0.0211	-0.0547	-0.0893	-0.0934	-0.0812
<i>dMbar</i>	-0.3949	-0.0714	-0.0154	-0.0052	-0.0030
<i>r</i>	0.6805	0.3838	0.1707	0.0411	-0.0248
<i>rbar</i>	0.2065	0.0629	0.0321	0.0219	0.0163
<i>pi</i>	0.6997	0.3459	0.0910	-0.0515	-0.1096
<i>pibar</i>	-0.3737	-0.0660	-0.0130	-0.0035	-0.0016

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Table C.9 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6015	0.3086	0.1183	0.0104	-0.0401
<i>nbar</i>	0.3200	0.1883	0.1539	0.1390	0.1298
<i>n</i>	0.7149	0.4668	0.2745	0.1409	0.0577
<i>np</i>	0.8837	0.7799	0.6893	0.6112	0.5441
<i>npbar</i>	0.8459	0.7519	0.6752	0.6076	0.5471
<i>nnp</i>	0.7155	0.4679	0.2760	0.1426	0.0594
<i>nnpbar</i>	0.3220	0.1904	0.1559	0.1408	0.1314
<i>m</i>	0.8408	0.6692	0.5128	0.3851	0.2884
<i>mbar</i>	0.3280	0.1875	0.1430	0.1190	0.1021
<i>h</i>	0.8120	0.6202	0.4521	0.3198	0.2235
<i>hbar</i>	0.2697	0.1268	0.0879	0.0699	0.0579
<i>ks</i>	0.8904	0.7969	0.7259	0.6764	0.6437
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9710
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9777
<i>z</i>	0.7450	0.5403	0.3971	0.3095	0.2642
<i>zbar</i>	0.9635	0.9020	0.8393	0.7796	0.7235
<i>q</i>	0.8568	0.6840	0.5169	0.3748	0.2641
<i>qbar</i>	0.3202	0.1743	0.1250	0.0967	0.0758
<i>rk</i>	0.7450	0.5403	0.3971	0.3095	0.2642
<i>rkbar</i>	0.9635	0.9020	0.8393	0.7796	0.7235
<i>w</i>	0.7297	0.5054	0.3433	0.2388	0.1788
<i>wbar</i>	0.8841	0.8213	0.7722	0.7296	0.6915

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Table C.9 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.7360	0.5185	0.3591	0.2531	0.1884
<i>pebar</i>	0.9324	0.8610	0.7948	0.7347	0.6803
<i>e</i>	0.8959	0.8024	0.7189	0.6446	0.5785
<i>ebar</i>	0.8836	0.7923	0.7124	0.6409	0.5766
<i>ep</i>	0.8973	0.8051	0.7225	0.6487	0.5828
<i>epbar</i>	0.8889	0.7980	0.7177	0.6457	0.5810
<i>enp</i>	0.8951	0.8008	0.7168	0.6422	0.5761
<i>enpbar</i>	0.8831	0.7918	0.7122	0.6409	0.5769
<i>pp</i>	0.7764	0.5904	0.4506	0.3529	0.2882
<i>ppbar</i>	0.9203	0.8420	0.7704	0.7056	0.6472
<i>pp2</i>	0.7741	0.5862	0.4451	0.3467	0.2817
<i>pnp</i>	0.7448	0.5336	0.3781	0.2737	0.2092
<i>pnpbar</i>	0.9318	0.8600	0.7937	0.7335	0.6790
<i>ns</i>	0.7125	0.4624	0.2688	0.1344	0.0510
<i>nsbar</i>	0.3124	0.1802	0.1463	0.1322	0.1236
<i>hc</i>	0.8272	0.6459	0.4839	0.3537	0.2571
<i>hcbar</i>	0.2989	0.1571	0.1153	0.0943	0.0798
<i>dG</i>	0.8973	0.8051	0.7225	0.6487	0.5828
<i>dGbar</i>	0.8889	0.7980	0.7177	0.6457	0.5810



Table C.10 – Theoretical moments for an elasticity of substitution of  $\epsilon = 1.22$  with a carbon tax

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1456
<i>dybar</i>	0.0000	0.4521	0.2044
<i>dc</i>	0.0000	0.4569	0.2088
<i>dcbar</i>	0.0000	1.1967	1.4320
<i>di</i>	0.0000	0.9548	0.9117
<i>dibar</i>	0.0000	0.9375	0.8789
<i>y</i>	0.0000	1.7379	3.0203
<i>ybar</i>	0.0000	1.6842	2.8366
<i>ygap</i>	0.0000	0.4855	0.2357
<i>c</i>	0.0000	2.0788	4.3216
<i>cbar</i>	0.0000	2.9559	8.7373
<i>i</i>	0.0000	5.4794	30.0239
<i>ibar</i>	0.0000	4.2575	18.1260
<i>R</i>	0.0000	0.3601	0.1297
<i>Rbar</i>	0.0000	0.8074	0.6520
<i>dM</i>	0.0000	14.9011	222.0413
<i>dMbar</i>	0.0000	57.5103	3307.4306
<i>r</i>	0.0000	0.4101	0.1682
<i>rbar</i>	0.0000	2.7318	7.4626
<i>pi</i>	0.0000	0.5094	0.2595
<i>pibar</i>	0.0000	3.7988	14.4309

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Table C.10 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	2.5330	6.4160
<i>nbar</i>	0.0000	0.8157	0.6654
<i>n</i>	0.0000	0.7949	0.6318
<i>np</i>	0.0000	2.6739	7.1499
<i>npbar</i>	0.0000	2.6785	7.1742
<i>nnp</i>	0.0000	0.9139	0.8352
<i>nnpbar</i>	0.0000	0.9326	0.8698
<i>m</i>	0.0000	26.7256	714.2574
<i>mbar</i>	0.0000	52.7687	2784.5328
<i>h</i>	0.0000	16.0861	258.7628
<i>hbar</i>	0.0000	33.5380	1124.7971
<i>ks</i>	0.0000	2.3272	5.4161
<i>ksbar</i>	0.0000	2.3232	5.3970
<i>k</i>	0.0000	2.7494	7.5593
<i>kbar</i>	0.0000	2.1280	4.5284
<i>z</i>	0.0000	1.5014	2.2542
<i>zbar</i>	0.0000	0.2906	0.0844
<i>q</i>	0.0000	1.8199	3.3122
<i>qbar</i>	0.0000	3.8156	14.5588
<i>rk</i>	0.0000	1.7566	3.0858
<i>rkbar</i>	0.0000	1.9997	3.9988
<i>w</i>	0.0000	2.4378	5.9429
<i>wbar</i>	0.0000	2.0586	4.2380

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Table C.10 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	3.1547	9.9522
<i>pebar</i>	0.0000	2.2646	5.1283
<i>e</i>	0.0000	1.5285	2.3364
<i>ebar</i>	0.0000	1.5297	2.3399
<i>ep</i>	0.0000	1.8574	3.4501
<i>epbar</i>	0.0000	1.8583	3.4532
<i>enp</i>	0.0000	1.7202	2.9590
<i>enpbar</i>	0.0000	1.7218	2.9645
<i>pp</i>	0.0000	3.6157	13.0735
<i>ppbar</i>	0.0000	2.8944	8.3777
<i>pp2</i>	0.0000	3.6049	12.9955
<i>pnp</i>	0.0000	3.1076	9.6571
<i>pnpbar</i>	0.0000	2.2720	5.1618
<i>ns</i>	0.0000	0.7895	0.6232
<i>nsbar</i>	0.0000	0.8103	0.6565
<i>hc</i>	0.0000	25.0907	629.5418
<i>hcbar</i>	0.0000	50.8571	2586.4468
<i>dG</i>	0.0000	3.7576	14.1199
<i>dGbar</i>	0.0000	3.7593	14.1327

Table C.11 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 1.22$  with a carbon tax

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\tau o}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.33	0.16	0.01	4.46	5.69	40.93	0.40	0.01
<i>dybar</i>	74.60	0.27	0.02	0.07	1.53	23.51	0.00	0.00
<i>dc</i>	57.89	0.15	0.02	29.80	8.20	2.94	0.88	0.12
<i>dcbar</i>	86.20	0.26	0.02	0.10	2.35	11.05	0.00	0.01
<i>di</i>	67.26	0.81	0.02	3.14	26.07	2.15	0.48	0.08
<i>dibar</i>	77.47	0.85	0.02	0.01	15.59	5.99	0.00	0.08
<i>y</i>	92.24	0.45	0.02	1.05	3.39	2.59	0.23	0.02
<i>ybar</i>	95.80	0.55	0.02	0.00	2.36	1.23	0.00	0.03
<i>ygap</i>	66.85	0.52	0.02	13.28	7.73	8.61	2.95	0.04
<i>c</i>	79.66	0.20	0.02	5.57	12.02	2.01	0.39	0.12
<i>cbar</i>	85.89	0.32	0.02	0.09	8.16	5.48	0.00	0.05
<i>i</i>	82.52	1.15	0.02	0.35	14.33	1.34	0.17	0.12
<i>ibar</i>	81.05	1.28	0.02	0.01	15.35	2.15	0.00	0.15
<i>R</i>	56.31	0.57	0.01	24.39	15.17	2.44	1.06	0.05
<i>Rbar</i>	73.93	0.53	0.02	10.98	4.84	9.67	0.00	0.03
<i>dM</i>	18.67	0.14	0.00	77.48	1.93	1.11	0.66	0.01
<i>dMbar</i>	86.25	0.48	0.02	0.55	0.61	12.07	0.00	0.02
<i>r</i>	61.33	0.57	0.01	22.09	10.23	3.92	1.80	0.04
<i>rbar</i>	85.58	0.42	0.02	1.20	0.33	12.44	0.00	0.01
<i>pi</i>	66.08	0.58	0.02	18.65	8.51	3.58	2.55	0.04
<i>pibar</i>	83.96	0.48	0.02	3.12	0.66	11.74	0.00	0.02

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Table C.11 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$mc$	82.29	0.55	0.02	8.06	1.74	6.08	1.23	0.03
$nbar$	77.50	0.88	0.02	0.03	7.38	12.83	0.00	1.36
$n$	73.56	0.82	0.02	2.17	12.46	9.23	0.37	1.37
$np$	6.50	0.02	0.05	0.19	1.10	0.82	0.03	91.28
$npbar$	7.19	0.03	0.05	0.00	0.68	1.19	0.00	90.84
$nnp$	55.69	11.93	7.32	1.64	9.43	6.99	0.28	6.73
$nnpbar$	59.33	11.52	7.09	0.02	5.65	9.83	0.00	6.56
$m$	53.26	0.45	0.01	29.08	14.02	2.22	0.93	0.03
$mbar$	82.37	0.57	0.02	0.74	5.67	10.60	0.00	0.03
$h$	49.17	0.61	0.01	33.40	13.53	2.21	1.00	0.06
$hbar$	82.49	0.60	0.02	0.80	5.10	10.95	0.00	0.04
$ks$	83.72	0.91	0.02	3.19	7.78	3.68	0.61	0.08
$ksbar$	84.19	1.43	0.02	0.00	12.67	1.53	0.00	0.17
$k$	86.55	1.25	0.02	0.07	10.95	0.95	0.08	0.13
$kbar$	84.97	1.39	0.02	0.00	12.05	1.41	0.00	0.16
$z$	75.00	0.82	0.02	7.13	7.71	8.08	1.16	0.08
$zbar$	86.74	1.04	0.02	0.01	9.74	2.33	0.00	0.12
$q$	52.05	0.52	0.01	17.28	26.71	2.52	0.87	0.04
$qbar$	80.28	0.54	0.02	0.03	9.33	9.77	0.00	0.03
$rk$	75.00	0.82	0.02	7.13	7.71	8.08	1.16	0.08
$rkbar$	86.74	1.04	0.02	0.01	9.74	2.33	0.00	0.12
$w$	76.30	0.53	0.02	11.40	2.41	7.52	1.71	0.12
$wbar$	96.53	0.81	0.02	0.00	1.79	0.54	0.00	0.31

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Table C.11 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\tau o}$
$pe$	57.22	18.24	0.82	7.43	2.86	6.74	1.21	5.48
$pebar$	60.04	27.71	1.31	0.00	1.15	0.22	0.00	9.58
$e$	2.41	71.44	3.41	0.07	0.41	0.30	0.01	21.95
$ebar$	2.67	71.34	3.41	0.00	0.25	0.44	0.00	21.88
$ep$	1.65	74.62	0.01	0.05	0.28	0.21	0.01	23.17
$epbar$	1.83	74.56	0.01	0.00	0.17	0.30	0.00	23.12
$enp$	1.93	0.41	96.80	0.06	0.33	0.24	0.01	0.23
$enpbar$	2.13	0.41	96.66	0.00	0.20	0.35	0.00	0.24
$pp$	43.54	19.61	0.02	5.65	2.18	5.13	0.92	22.96
$ppbar$	36.77	25.02	0.00	0.00	0.70	0.13	0.00	37.36
$pp2$	43.80	19.73	0.02	5.69	2.19	5.16	0.93	22.50
$pnp$	58.94	0.74	21.44	7.65	2.94	6.94	1.25	0.09
$pnpbar$	59.68	0.14	38.76	0.00	1.14	0.22	0.00	0.05
$ns$	74.57	0.83	0.02	2.20	12.63	9.36	0.37	0.01
$nsbar$	78.55	0.90	0.02	0.03	7.48	13.01	0.00	0.02
$hc$	51.28	0.52	0.01	31.14	13.82	2.22	0.97	0.04
$hcbar$	82.42	0.59	0.02	0.77	5.39	10.78	0.00	0.03
$dG$	1.65	74.62	0.01	0.05	0.28	0.21	0.01	23.17
$dGbar$	1.83	74.56	0.01	0.00	0.17	0.30	0.00	23.12

Table C.12 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 1.22$  with a carbon tax

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3985	0.2373	0.1026	0.0047	-0.0569
<i>dybar</i>	0.3131	-0.0132	-0.0617	-0.0637	-0.0585
<i>dc</i>	0.6626	0.3443	0.0924	-0.0768	-0.1701
<i>dcbar</i>	0.0589	-0.0803	-0.0865	-0.0734	-0.0595
<i>di</i>	0.7147	0.4613	0.2570	0.1058	0.0023
<i>dibar</i>	0.3771	0.1900	0.0986	0.0401	-0.0004
<i>y</i>	0.9759	0.9326	0.8779	0.8182	0.7583
<i>ybar</i>	0.9640	0.9054	0.8477	0.7946	0.7460
<i>ygap</i>	0.8291	0.5125	0.2541	0.0817	-0.0172
<i>c</i>	0.9758	0.9197	0.8469	0.7696	0.6960
<i>cbar</i>	0.9181	0.8265	0.7480	0.6838	0.6315
<i>i</i>	0.9848	0.9479	0.8970	0.8383	0.7764
<i>ibar</i>	0.9758	0.9332	0.8815	0.8250	0.7665
<i>R</i>	0.9460	0.8210	0.6700	0.5242	0.3996
<i>Rbar</i>	0.3541	0.2102	0.1589	0.1288	0.1069
<i>dM</i>	0.0211	-0.0547	-0.0893	-0.0935	-0.0812
<i>dMbar</i>	-0.3950	-0.0713	-0.0154	-0.0052	-0.0030
<i>r</i>	0.6805	0.3838	0.1707	0.0411	-0.0248
<i>rbar</i>	0.2063	0.0629	0.0321	0.0219	0.0163
<i>pi</i>	0.6997	0.3459	0.0910	-0.0515	-0.1096
<i>pibar</i>	-0.3737	-0.0659	-0.0130	-0.0035	-0.0016

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Table C.12 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6015	0.3085	0.1183	0.0104	-0.0401
<i>nbar</i>	0.3199	0.1883	0.1539	0.1390	0.1298
<i>n</i>	0.7149	0.4667	0.2745	0.1409	0.0577
<i>np</i>	0.8840	0.7806	0.6903	0.6123	0.5452
<i>npbar</i>	0.8471	0.7532	0.6764	0.6087	0.5480
<i>nnp</i>	0.7577	0.5460	0.3793	0.2597	0.1805
<i>nnpbar</i>	0.4520	0.3300	0.2853	0.2575	0.2358
<i>m</i>	0.8408	0.6692	0.5128	0.3851	0.2884
<i>mbar</i>	0.3279	0.1875	0.1429	0.1190	0.1021
<i>h</i>	0.8120	0.6202	0.4521	0.3198	0.2235
<i>hbar</i>	0.2695	0.1267	0.0879	0.0699	0.0579
<i>ks</i>	0.8904	0.7968	0.7258	0.6763	0.6437
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9710
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9777
<i>z</i>	0.7450	0.5403	0.3971	0.3094	0.2641
<i>zbar</i>	0.9635	0.9020	0.8393	0.7796	0.7235
<i>q</i>	0.8568	0.6840	0.5169	0.3748	0.2641
<i>qbar</i>	0.3200	0.1742	0.1250	0.0967	0.0758
<i>rk</i>	0.7450	0.5403	0.3971	0.3094	0.2641
<i>rkbar</i>	0.9635	0.9020	0.8393	0.7796	0.7235
<i>w</i>	0.7297	0.5054	0.3432	0.2387	0.1787
<i>wbar</i>	0.8841	0.8212	0.7722	0.7296	0.6915

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Table C.12 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.7357	0.5177	0.3582	0.2520	0.1874
<i>pebar</i>	0.9326	0.8612	0.7951	0.7350	0.6807
<i>e</i>	0.8959	0.8024	0.7189	0.6446	0.5785
<i>ebar</i>	0.8835	0.7922	0.7124	0.6409	0.5766
<i>ep</i>	0.8974	0.8052	0.7227	0.6489	0.5830
<i>epbar</i>	0.8891	0.7983	0.7180	0.6459	0.5812
<i>enp</i>	0.8955	0.8016	0.7178	0.6434	0.5773
<i>enpbar</i>	0.8846	0.7935	0.7137	0.6424	0.5782
<i>pp</i>	0.7761	0.5899	0.4499	0.3522	0.2875
<i>ppbar</i>	0.9204	0.8422	0.7706	0.7059	0.6475
<i>pp2</i>	0.7726	0.5834	0.4415	0.3427	0.2778
<i>pnp</i>	0.7368	0.5195	0.3602	0.2543	0.1897
<i>pnpbar</i>	0.9345	0.8643	0.7991	0.7400	0.6865
<i>ns</i>	0.7125	0.4623	0.2688	0.1344	0.0510
<i>nsbar</i>	0.3123	0.1802	0.1464	0.1322	0.1236
<i>hc</i>	0.8272	0.6459	0.4839	0.3537	0.2571
<i>hcbar</i>	0.2988	0.1571	0.1153	0.0943	0.0797
<i>dG</i>	0.8974	0.8052	0.7227	0.6489	0.5830
<i>dGbar</i>	0.8891	0.7983	0.7180	0.6459	0.5812

Table C.13 – Theoretical moments for an elasticity of substitution of  $\epsilon = 1.85$  with a carbon tax

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1456
<i>dybar</i>	0.0000	0.4522	0.2045
<i>dc</i>	0.0000	0.4572	0.2090
<i>dcbar</i>	0.0000	1.1980	1.4352
<i>di</i>	0.0000	0.9547	0.9115
<i>dibar</i>	0.0000	0.9375	0.8790
<i>y</i>	0.0000	1.7377	3.0197
<i>ybar</i>	0.0000	1.6842	2.8365
<i>ygap</i>	0.0000	0.4856	0.2358
<i>c</i>	0.0000	2.0797	4.3250
<i>cbar</i>	0.0000	2.9570	8.7438
<i>i</i>	0.0000	5.4785	30.0139
<i>ibar</i>	0.0000	4.2568	18.1201
<i>R</i>	0.0000	0.3601	0.1297
<i>Rbar</i>	0.0000	0.8081	0.6530
<i>dM</i>	0.0000	14.9026	222.0864
<i>dMbar</i>	0.0000	57.5948	3317.1624
<i>r</i>	0.0000	0.4103	0.1683
<i>rbar</i>	0.0000	2.7369	7.4905
<i>pi</i>	0.0000	0.5096	0.2597
<i>pibar</i>	0.0000	3.8042	14.4719

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Table C.13 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	2.5345	6.4237
<i>nbar</i>	0.0000	0.8162	0.6662
<i>n</i>	0.0000	0.7951	0.6322
<i>np</i>	0.0000	2.7490	7.5568
<i>npbar</i>	0.0000	2.7533	7.5805
<i>nnp</i>	0.0000	1.4022	1.9663
<i>nnpbar</i>	0.0000	1.4157	2.0042
<i>m</i>	0.0000	26.7284	714.4094
<i>mbar</i>	0.0000	52.8157	2789.5034
<i>h</i>	0.0000	16.0881	258.8260
<i>hbar</i>	0.0000	33.5701	1126.9515
<i>ks</i>	0.0000	2.3274	5.4169
<i>ksbar</i>	0.0000	2.3230	5.3965
<i>k</i>	0.0000	2.7492	7.5578
<i>kbar</i>	0.0000	2.1279	4.5280
<i>z</i>	0.0000	1.5019	2.2556
<i>zbar</i>	0.0000	0.2905	0.0844
<i>q</i>	0.0000	1.8202	3.3132
<i>qbar</i>	0.0000	3.8188	14.5836
<i>rk</i>	0.0000	1.7572	3.0876
<i>rkbar</i>	0.0000	1.9996	3.9985
<i>w</i>	0.0000	2.4386	5.9467
<i>wbar</i>	0.0000	2.0587	4.2384

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Table C.13 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	3.1401	9.8603
<i>pebar</i>	0.0000	2.2388	5.0124
<i>e</i>	0.0000	1.4859	2.2078
<i>ebar</i>	0.0000	1.4869	2.2109
<i>ep</i>	0.0000	1.9076	3.6390
<i>epbar</i>	0.0000	1.9083	3.6418
<i>enp</i>	0.0000	1.9053	3.6301
<i>enpbar</i>	0.0000	1.9068	3.6360
<i>pp</i>	0.0000	3.6070	13.0103
<i>ppbar</i>	0.0000	2.8843	8.3191
<i>pp2</i>	0.0000	3.5527	12.6217
<i>pnp</i>	0.0000	3.0214	9.1287
<i>pnpbar</i>	0.0000	2.1302	4.5379
<i>ns</i>	0.0000	0.7899	0.6239
<i>nsbar</i>	0.0000	0.8109	0.6576
<i>hc</i>	0.0000	25.0935	629.6820
<i>hcbar</i>	0.0000	50.9041	2591.2297
<i>dG</i>	0.0000	3.8592	14.8932
<i>dGbar</i>	0.0000	3.8606	14.9045

Table C.14 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 1.85$  with a carbon tax

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\tau o}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.35	0.15	0.01	4.46	5.69	40.93	0.40	0.01
<i>dybar</i>	74.64	0.25	0.02	0.07	1.52	23.50	0.00	0.00
<i>dc</i>	57.89	0.16	0.02	29.77	8.20	2.94	0.88	0.13
<i>dcbar</i>	86.22	0.24	0.02	0.10	2.35	11.06	0.00	0.01
<i>di</i>	67.29	0.79	0.01	3.14	26.06	2.15	0.48	0.07
<i>dibar</i>	77.50	0.83	0.02	0.01	15.58	5.99	0.00	0.07
<i>y</i>	92.28	0.43	0.02	1.05	3.39	2.59	0.23	0.01
<i>ybar</i>	95.84	0.52	0.02	0.00	2.36	1.23	0.00	0.02
<i>ygap</i>	66.89	0.50	0.01	13.27	7.72	8.62	2.94	0.04
<i>c</i>	79.63	0.21	0.02	5.57	12.02	2.01	0.39	0.14
<i>cbar</i>	85.89	0.30	0.02	0.09	8.16	5.49	0.00	0.05
<i>i</i>	82.55	1.13	0.02	0.35	14.32	1.34	0.17	0.12
<i>ibar</i>	81.07	1.27	0.01	0.01	15.35	2.15	0.00	0.14
<i>R</i>	56.32	0.56	0.01	24.39	15.18	2.44	1.06	0.04
<i>Rbar</i>	73.97	0.50	0.02	10.96	4.83	9.68	0.00	0.03
<i>dM</i>	18.69	0.13	0.00	77.46	1.93	1.12	0.66	0.01
<i>dMbar</i>	86.28	0.46	0.02	0.55	0.61	12.07	0.00	0.01
<i>r</i>	61.35	0.56	0.01	22.08	10.23	3.93	1.80	0.04
<i>rbar</i>	85.61	0.40	0.02	1.19	0.33	12.44	0.00	0.01
<i>pi</i>	66.10	0.56	0.01	18.64	8.52	3.59	2.54	0.04
<i>pibar</i>	84.00	0.45	0.02	3.11	0.66	11.75	0.00	0.01

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Table C.14 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$mc$	82.32	0.53	0.02	8.05	1.74	6.08	1.23	0.02
$nbar$	77.52	0.87	0.02	0.03	7.37	12.84	0.00	1.36
$n$	73.58	0.81	0.02	2.17	12.45	9.23	0.37	1.37
$np$	6.23	0.21	0.33	0.18	1.05	0.78	0.03	91.17
$npbar$	6.90	0.22	0.33	0.00	0.66	1.14	0.00	90.76
$nnp$	23.96	29.43	26.42	0.71	4.05	3.01	0.12	12.30
$nnpbar$	26.09	28.92	26.01	0.01	2.48	4.32	0.00	12.16
$m$	53.28	0.43	0.01	29.07	14.03	2.23	0.93	0.03
$mbar$	82.40	0.55	0.02	0.74	5.66	10.60	0.00	0.03
$h$	49.19	0.60	0.01	33.39	13.53	2.21	1.00	0.06
$hbar$	82.52	0.58	0.02	0.80	5.09	10.96	0.00	0.04
$ks$	83.75	0.89	0.02	3.19	7.78	3.69	0.61	0.08
$ksbar$	84.21	1.41	0.01	0.00	12.66	1.53	0.00	0.17
$k$	86.58	1.23	0.02	0.07	10.94	0.95	0.08	0.13
$kbar$	84.99	1.38	0.02	0.00	12.05	1.41	0.00	0.16
$z$	75.03	0.80	0.02	7.13	7.71	8.08	1.16	0.08
$zbar$	86.76	1.03	0.02	0.01	9.73	2.33	0.00	0.12
$q$	52.07	0.51	0.01	17.27	26.71	2.52	0.87	0.04
$qbar$	80.32	0.51	0.02	0.03	9.32	9.78	0.00	0.02
$rk$	75.03	0.80	0.02	7.13	7.71	8.08	1.16	0.08
$rkbar$	86.76	1.03	0.02	0.01	9.73	2.33	0.00	0.12
$w$	76.33	0.50	0.02	11.39	2.41	7.52	1.71	0.11
$wbar$	96.56	0.78	0.02	0.00	1.79	0.54	0.00	0.30

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Table C.14 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$
$pe$	58.03	17.45	0.77	7.51	2.92	6.85	1.22	5.25
$pebar$	61.21	26.82	1.27	0.00	1.18	0.23	0.00	9.30
$e$	2.41	71.43	3.41	0.07	0.41	0.30	0.01	21.95
$ebar$	2.68	71.33	3.42	0.00	0.25	0.44	0.00	21.87
$ep$	1.59	74.61	0.08	0.05	0.27	0.20	0.01	23.19
$epbar$	1.76	74.56	0.08	0.00	0.17	0.29	0.00	23.14
$enp$	1.59	1.95	95.12	0.05	0.27	0.20	0.01	0.82
$enpbar$	1.76	1.95	95.00	0.00	0.17	0.29	0.00	0.82
$pp$	43.86	17.80	0.08	5.68	2.19	5.17	0.93	24.29
$ppbar$	36.99	22.59	0.07	0.00	0.71	0.14	0.00	39.51
$pp2$	45.21	18.35	0.08	5.86	2.26	5.33	0.95	21.96
$pnp$	62.51	2.87	14.00	8.10	3.13	7.37	1.32	0.70
$pnpbar$	67.81	2.59	27.04	0.00	1.30	0.25	0.00	1.01
$ns$	74.60	0.82	0.01	2.20	12.62	9.36	0.37	0.01
$nsbar$	78.57	0.88	0.02	0.03	7.47	13.01	0.00	0.02
$hc$	51.30	0.51	0.01	31.13	13.82	2.22	0.97	0.04
$hcbars$	82.44	0.56	0.02	0.77	5.39	10.79	0.00	0.03
$dG$	1.59	74.61	0.08	0.05	0.27	0.20	0.01	23.19
$dGbar$	1.76	74.56	0.08	0.00	0.17	0.29	0.00	23.14

Table C.15 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 1.85$  with a carbon tax

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3985	0.2372	0.1025	0.0046	-0.0569
<i>dybar</i>	0.3129	-0.0135	-0.0618	-0.0637	-0.0584
<i>dc</i>	0.6626	0.3443	0.0923	-0.0768	-0.1701
<i>dcbar</i>	0.0582	-0.0804	-0.0864	-0.0732	-0.0594
<i>di</i>	0.7147	0.4612	0.2569	0.1057	0.0022
<i>dibar</i>	0.3767	0.1898	0.0986	0.0401	-0.0004
<i>y</i>	0.9759	0.9326	0.8778	0.8181	0.7582
<i>ybar</i>	0.9640	0.9053	0.8477	0.7945	0.7459
<i>ygap</i>	0.8289	0.5121	0.2537	0.0814	-0.0174
<i>c</i>	0.9758	0.9196	0.8468	0.7695	0.6960
<i>cbar</i>	0.9179	0.8263	0.7479	0.6836	0.6314
<i>i</i>	0.9848	0.9479	0.8970	0.8383	0.7764
<i>ibar</i>	0.9757	0.9332	0.8815	0.8250	0.7665
<i>R</i>	0.9460	0.8210	0.6699	0.5240	0.3994
<i>Rbar</i>	0.3533	0.2098	0.1586	0.1286	0.1067
<i>dM</i>	0.0212	-0.0547	-0.0893	-0.0935	-0.0812
<i>dMbar</i>	-0.3953	-0.0711	-0.0153	-0.0052	-0.0030
<i>r</i>	0.6805	0.3837	0.1706	0.0410	-0.0249
<i>rbar</i>	0.2056	0.0626	0.0319	0.0218	0.0162
<i>pi</i>	0.6997	0.3458	0.0908	-0.0517	-0.1097
<i>pibar</i>	-0.3742	-0.0657	-0.0129	-0.0035	-0.0016

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Table C.15 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6015	0.3084	0.1182	0.0102	-0.0402
<i>nbar</i>	0.3193	0.1880	0.1538	0.1390	0.1297
<i>n</i>	0.7149	0.4667	0.2744	0.1408	0.0576
<i>np</i>	0.8848	0.7820	0.6920	0.6143	0.5474
<i>npbar</i>	0.8493	0.7557	0.6786	0.6107	0.5498
<i>nnp</i>	0.8374	0.6936	0.5746	0.4810	0.4095
<i>nnpbar</i>	0.7001	0.5960	0.5314	0.4789	0.4332
<i>m</i>	0.8408	0.6692	0.5127	0.3850	0.2883
<i>mbar</i>	0.3271	0.1870	0.1427	0.1188	0.1019
<i>h</i>	0.8120	0.6201	0.4521	0.3197	0.2234
<i>hbar</i>	0.2688	0.1264	0.0877	0.0698	0.0578
<i>ks</i>	0.8902	0.7966	0.7256	0.6760	0.6434
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9710
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9777
<i>z</i>	0.7448	0.5400	0.3967	0.3090	0.2637
<i>zbar</i>	0.9635	0.9020	0.8393	0.7796	0.7235
<i>q</i>	0.8568	0.6839	0.5168	0.3747	0.2640
<i>qbar</i>	0.3193	0.1738	0.1248	0.0966	0.0757
<i>rk</i>	0.7448	0.5400	0.3967	0.3090	0.2637
<i>rkbar</i>	0.9635	0.9020	0.8393	0.7796	0.7235
<i>w</i>	0.7295	0.5050	0.3428	0.2382	0.1783
<i>wbar</i>	0.8839	0.8211	0.7720	0.7294	0.6914

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Table C.15 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.7335	0.5138	0.3531	0.2465	0.1818
<i>pebar</i>	0.9337	0.8628	0.7969	0.7371	0.6829
<i>e</i>	0.8958	0.8022	0.7187	0.6444	0.5783
<i>ebar</i>	0.8833	0.7920	0.7122	0.6407	0.5764
<i>ep</i>	0.8974	0.8053	0.7228	0.6491	0.5832
<i>epbar</i>	0.8895	0.7986	0.7183	0.6463	0.5815
<i>enp</i>	0.8962	0.8029	0.7195	0.6453	0.5792
<i>enpbar</i>	0.8871	0.7962	0.7162	0.6446	0.5803
<i>pp</i>	0.7754	0.5887	0.4483	0.3505	0.2858
<i>ppbar</i>	0.9207	0.8426	0.7711	0.7065	0.6482
<i>pp2</i>	0.7690	0.5769	0.4332	0.3336	0.2687
<i>pnp</i>	0.7253	0.4991	0.3343	0.2260	0.1615
<i>pnpbar</i>	0.9387	0.8709	0.8076	0.7500	0.6979
<i>ns</i>	0.7124	0.4622	0.2686	0.1343	0.0509
<i>nsbar</i>	0.3116	0.1798	0.1462	0.1321	0.1235
<i>hc</i>	0.8272	0.6459	0.4838	0.3536	0.2570
<i>hcbare</i>	0.2980	0.1567	0.1151	0.0941	0.0796
<i>dG</i>	0.8974	0.8053	0.7228	0.6491	0.5832
<i>dGbar</i>	0.8895	0.7986	0.7183	0.6463	0.5815



**C.4.2.2 Models with an emission cap:**Table C.17 – Theoretical moments for an elasticity of substitution of  $\epsilon = 0.51$  with an emission cap

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1455
<i>dybar</i>	0.0000	0.4537	0.2059
<i>dc</i>	0.0000	0.4522	0.2045
<i>dcbar</i>	0.0000	1.2035	1.4483
<i>di</i>	0.0000	0.9531	0.9085
<i>dibar</i>	0.0000	0.9380	0.8799
<i>y</i>	0.0000	1.7352	3.0108
<i>ybar</i>	0.0000	1.6820	2.8291
<i>ygap</i>	0.0000	0.4890	0.2391
<i>c</i>	0.0000	2.0675	4.2745
<i>cbar</i>	0.0000	2.9550	8.7318
<i>i</i>	0.0000	5.4496	29.6984
<i>ibar</i>	0.0000	4.2276	17.8727
<i>R</i>	0.0000	0.3618	0.1309
<i>Rbar</i>	0.0000	0.8176	0.6685
<i>dM</i>	0.0000	14.9276	222.8333
<i>dMbar</i>	0.0000	58.6372	3438.3203
<i>r</i>	0.0000	0.4135	0.1710
<i>rbar</i>	0.0000	2.7958	7.8164
<i>pi</i>	0.0000	0.5130	0.2632

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Table C.17 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pibar</i>	0.0000	3.8715	14.9882
<i>mc</i>	0.0000	2.5556	6.5313
<i>nbar</i>	0.0000	0.8518	0.7256
<i>n</i>	0.0000	0.8268	0.6836
<i>np</i>	0.0000	4.5883	21.0526
<i>npbar</i>	0.0000	4.5883	21.0526
<i>nnp</i>	0.0000	1.1978	1.4347
<i>nnpbar</i>	0.0000	1.2078	1.4587
<i>m</i>	0.0000	26.8067	718.5978
<i>mbar</i>	0.0000	53.4801	2860.1168
<i>h</i>	0.0000	16.1491	260.7935
<i>hbar</i>	0.0000	34.0324	1158.2056
<i>ks</i>	0.0000	2.3265	5.4126
<i>ksbar</i>	0.0000	2.3098	5.3353
<i>k</i>	0.0000	2.7361	7.4860
<i>kbar</i>	0.0000	2.1162	4.4785
<i>z</i>	0.0000	1.5065	2.2696
<i>zbar</i>	0.0000	0.2894	0.0838
<i>q</i>	0.0000	1.8276	3.3402
<i>qbar</i>	0.0000	3.8663	14.9481
<i>rk</i>	0.0000	1.7626	3.1069
<i>rkbar</i>	0.0000	1.9918	3.9674
<i>w</i>	0.0000	2.4374	5.9411

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Table C.17 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>wbar</i>	0.0000	2.0705	4.2871
<i>pe</i>	0.0000	2.9391	8.6381
<i>pebar</i>	0.0000	1.7014	2.8946
<i>e</i>	0.0000	0.2046	0.0419
<i>ebar</i>	0.0000	0.2048	0.0420
<i>ep</i>	0.0000	0.0000	0.0000
<i>epbar</i>	0.0000	0.0000	0.0000
<i>enp</i>	0.0000	1.2728	1.6200
<i>enpbar</i>	0.0000	1.2743	1.6238
<i>pp</i>	0.0000	5.2915	27.9998
<i>ppbar</i>	0.0000	5.0654	25.6578
<i>pp2</i>	0.0000	6.3120	39.8419
<i>pnp</i>	0.0000	3.5097	12.3179
<i>pnpbar</i>	0.0000	2.8969	8.3922
<i>ns</i>	0.0000	0.7922	0.6276
<i>nsbar</i>	0.0000	0.8158	0.6655
<i>hc</i>	0.0000	25.1780	633.9308
<i>hcbar</i>	0.0000	51.5749	2659.9733
<i>dG</i>	0.0000	0.0000	0.0000
<i>dGbar</i>	0.0000	0.0000	0.0000
<i>pq</i>	0.0000	26.7526	715.7009
<i>pqbar</i>	0.0000	33.9331	1151.4520

Table C.18 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 0.51$  with an emission cap

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3982	0.2375	0.1026	0.0045	-0.0574
<i>dybar</i>	0.3096	-0.0155	-0.0621	-0.0635	-0.0581
<i>dc</i>	0.6670	0.3487	0.0949	-0.0763	-0.1712
<i>dcbar</i>	0.0540	-0.0810	-0.0859	-0.0725	-0.0587
<i>di</i>	0.7140	0.4595	0.2545	0.1029	-0.0006
<i>dibar</i>	0.3690	0.1846	0.0953	0.0381	-0.0014
<i>y</i>	0.9758	0.9324	0.8775	0.8177	0.7576
<i>ybar</i>	0.9636	0.9047	0.8469	0.7937	0.7450
<i>ygap</i>	0.8270	0.5084	0.2492	0.0768	-0.0219
<i>c</i>	0.9761	0.9203	0.8478	0.7707	0.6973
<i>cbar</i>	0.9171	0.8252	0.7467	0.6825	0.6304
<i>i</i>	0.9847	0.9476	0.8964	0.8374	0.7753
<i>ibar</i>	0.9754	0.9326	0.8807	0.8242	0.7657
<i>R</i>	0.9456	0.8199	0.6681	0.5215	0.3965
<i>Rbar</i>	0.3456	0.2042	0.1544	0.1253	0.1039
<i>dM</i>	0.0234	-0.0540	-0.0897	-0.0943	-0.0821
<i>dMbar</i>	-0.3981	-0.0697	-0.0147	-0.0050	-0.0029
<i>r</i>	0.6806	0.3832	0.1693	0.0392	-0.0269
<i>rbar</i>	0.2000	0.0599	0.0305	0.0209	0.0156
<i>pi</i>	0.6993	0.3446	0.0889	-0.0541	-0.1121
<i>pibar</i>	-0.3774	-0.0646	-0.0124	-0.0033	-0.0015

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Table C.18 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6021	0.3087	0.1177	0.0093	-0.0414
<i>nbar</i>	0.3160	0.1871	0.1536	0.1388	0.1294
<i>n</i>	0.7169	0.4695	0.2774	0.1436	0.0601
<i>np</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>npbar</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>nnp</i>	0.8458	0.7094	0.5957	0.5049	0.4342
<i>nnpbar</i>	0.7214	0.6195	0.5532	0.4981	0.4498
<i>m</i>	0.8412	0.6691	0.5121	0.3837	0.2866
<i>mbar</i>	0.3197	0.1818	0.1388	0.1156	0.0991
<i>h</i>	0.8125	0.6202	0.4512	0.3180	0.2211
<i>hbar</i>	0.2619	0.1219	0.0846	0.0673	0.0557
<i>ks</i>	0.8884	0.7930	0.7207	0.6702	0.6372
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9711
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9855	0.9778
<i>z</i>	0.7423	0.5351	0.3900	0.3013	0.2557
<i>zbar</i>	0.9630	0.9010	0.8379	0.7779	0.7216
<i>q</i>	0.8570	0.6837	0.5158	0.3730	0.2619
<i>qbar</i>	0.3123	0.1692	0.1215	0.0941	0.0738
<i>rk</i>	0.7423	0.5351	0.3900	0.3013	0.2557
<i>rkbar</i>	0.9630	0.9010	0.8379	0.7779	0.7216
<i>w</i>	0.7290	0.5039	0.3413	0.2365	0.1767
<i>wbar</i>	0.8770	0.8137	0.7651	0.7230	0.6854

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Table C.18 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.6835	0.4235	0.2366	0.1175	0.0510
<i>pebar</i>	0.9639	0.9051	0.8473	0.7942	0.7456
<i>e</i>	0.8947	0.8001	0.7157	0.6410	0.5748
<i>ebar</i>	0.8813	0.7900	0.7105	0.6395	0.5756
<i>enp</i>	0.8947	0.8001	0.7157	0.6410	0.5748
<i>enpbar</i>	0.8813	0.7900	0.7105	0.6395	0.5756
<i>pp</i>	0.8582	0.7363	0.6365	0.5565	0.4925
<i>ppbar</i>	0.8967	0.8108	0.7347	0.6665	0.6052
<i>pp2</i>	0.8706	0.7582	0.6640	0.5861	0.5217
<i>pnp</i>	0.7812	0.5983	0.4603	0.3636	0.2994
<i>pnpbar</i>	0.9174	0.8401	0.7701	0.7069	0.6499
<i>ns</i>	0.7130	0.4624	0.2682	0.1334	0.0498
<i>nsbar</i>	0.3057	0.1762	0.1437	0.1300	0.1217
<i>hc</i>	0.8277	0.6459	0.4830	0.3522	0.2550
<i>hcbar</i>	0.2908	0.1518	0.1116	0.0913	0.0772
<i>pq</i>	0.8850	0.7839	0.6955	0.6188	0.5523
<i>pqbar</i>	0.7654	0.5910	0.4632	0.3732	0.3115

Table C.19 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 0.51$  with an emission cap

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.11	0.12	0.01	4.46	5.70	41.21	0.40	0.00	0.00	0.00
<i>dybar</i>	74.60	0.18	0.01	0.07	1.51	23.62	0.00	0.00	0.00	0.00
<i>dc</i>	57.63	0.14	0.01	30.26	8.18	2.87	0.91	0.00	0.00	0.00
<i>dcbar</i>	86.27	0.21	0.02	0.10	2.31	11.09	0.00	0.00	0.00	0.00
<i>di</i>	68.18	0.17	0.01	3.13	25.81	2.22	0.48	0.00	0.00	0.00
<i>dibar</i>	78.33	0.19	0.02	0.01	15.31	6.14	0.00	0.00	0.00	0.00
<i>y</i>	92.48	0.23	0.02	1.05	3.37	2.62	0.23	0.00	0.00	0.00
<i>ybar</i>	96.16	0.24	0.02	0.00	2.34	1.24	0.00	0.00	0.00	0.00
<i>ygap</i>	67.48	0.17	0.01	13.06	7.67	8.72	2.89	0.00	0.00	0.00
<i>c</i>	79.87	0.20	0.02	5.65	11.88	1.98	0.40	0.00	0.00	0.00
<i>cbar</i>	86.11	0.21	0.02	0.09	8.06	5.51	0.00	0.00	0.00	0.00
<i>i</i>	83.68	0.21	0.02	0.35	14.20	1.38	0.16	0.00	0.00	0.00
<i>ibar</i>	82.27	0.20	0.02	0.01	15.29	2.21	0.00	0.00	0.00	0.00
<i>R</i>	56.75	0.14	0.01	24.29	15.27	2.49	1.04	0.00	0.00	0.00
<i>Rbar</i>	74.51	0.18	0.01	10.70	4.79	9.80	0.00	0.00	0.00	0.00
<i>dM</i>	18.98	0.05	0.00	77.22	1.96	1.14	0.65	0.00	0.00	0.00
<i>dMbar</i>	86.49	0.21	0.02	0.54	0.60	12.15	0.00	0.00	0.00	0.00
<i>r</i>	61.90	0.15	0.01	21.89	10.27	4.00	1.77	0.00	0.00	0.00
<i>rbar</i>	85.81	0.21	0.02	1.14	0.32	12.50	0.00	0.00	0.00	0.00
<i>pi</i>	66.61	0.16	0.01	18.49	8.56	3.66	2.50	0.00	0.00	0.00
<i>pibar</i>	84.30	0.21	0.02	3.00	0.65	11.83	0.00	0.00	0.00	0.00

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Table C.19 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$mc$	82.74	0.20	0.02	7.92	1.75	6.15	1.22	0.00	0.00	0.00
$nbar$	77.99	1.79	0.01	0.03	7.27	12.90	0.00	0.00	0.00	0.00
$n$	74.03	1.75	0.01	2.17	12.40	9.28	0.36	0.00	0.00	0.00
$np$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$npbar$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$nnp$	21.45	0.51	71.02	0.63	3.59	2.69	0.11	0.00	0.00	0.00
$nnpbar$	23.60	0.54	69.75	0.01	2.20	3.90	0.00	0.00	0.00	0.00
$m$	53.69	0.13	0.01	28.86	14.12	2.27	0.92	0.00	0.00	0.00
$mbar$	82.75	0.20	0.02	0.73	5.59	10.71	0.00	0.00	0.00	0.00
$h$	49.82	0.12	0.01	33.10	13.68	2.28	0.99	0.00	0.00	0.00
$hbar$	82.90	0.20	0.02	0.78	5.04	11.06	0.00	0.00	0.00	0.00
$ks$	84.50	0.21	0.02	3.19	7.70	3.77	0.61	0.00	0.00	0.00
$ksbar$	85.56	0.21	0.02	0.00	12.64	1.57	0.00	0.00	0.00	0.00
$k$	87.78	0.22	0.02	0.06	10.86	0.98	0.08	0.00	0.00	0.00
$kbar$	86.29	0.21	0.02	0.00	12.02	1.45	0.00	0.00	0.00	0.00
$z$	75.73	0.19	0.02	7.08	7.61	8.21	1.15	0.00	0.00	0.00
$zbar$	87.70	0.22	0.02	0.01	9.68	2.38	0.00	0.00	0.00	0.00
$q$	52.54	0.13	0.01	17.13	26.76	2.57	0.86	0.00	0.00	0.00
$qbar$	80.68	0.20	0.02	0.03	9.19	9.89	0.00	0.00	0.00	0.00
$rk$	75.73	0.19	0.02	7.08	7.61	8.21	1.15	0.00	0.00	0.00
$rkbar$	87.70	0.22	0.02	0.01	9.68	2.38	0.00	0.00	0.00	0.00
$w$	76.69	0.38	0.01	11.34	2.34	7.54	1.70	0.00	0.00	0.00
$wbar$	97.50	0.06	0.01	0.00	1.81	0.62	0.00	0.00	0.00	0.00

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Table C.19 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$pe$	75.13	0.19	0.49	8.94	4.37	9.39	1.50	0.00	0.00	0.00
$pebar$	95.38	0.21	1.12	0.00	2.22	1.06	0.00	0.00	0.00	0.00
$e$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$ebar$	2.60	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$ep$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$epbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$enp$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$enpbar$	2.60	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$pp$	16.27	78.86	0.00	2.41	0.50	1.60	0.36	0.00	0.00	0.00
$ppbar$	16.29	83.30	0.00	0.00	0.30	0.10	0.00	0.00	0.00	0.00
$pp2$	11.44	55.42	0.00	1.69	0.35	1.12	0.25	29.72	0.00	0.00
$pnp$	44.38	0.13	41.61	5.87	2.00	5.08	0.94	0.00	0.00	0.00
$pnpbar$	39.31	0.02	59.89	0.00	0.70	0.09	0.00	0.00	0.00	0.00
$ns$	74.89	0.59	0.02	2.20	12.54	9.39	0.37	0.00	0.00	0.00
$nsbar$	78.98	0.54	0.02	0.03	7.36	13.07	0.00	0.00	0.00	0.00
$hc$	51.81	0.13	0.01	30.88	13.94	2.28	0.95	0.00	0.00	0.00
$hcbar$	82.81	0.20	0.02	0.75	5.33	10.89	0.00	0.00	0.00	0.00
$dG$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$dGbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$pq$	5.25	92.76	0.22	0.25	0.69	0.78	0.05	0.00	0.00	0.00
$pqbar$	35.18	55.87	0.12	2.85	1.68	3.81	0.48	0.00	0.00	0.00

Table C.20 – Theoretical moments for an elasticity of substitution of  $\epsilon = 0.95$  with an emission cap

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1455
<i>dybar</i>	0.0000	0.4536	0.2058
<i>dc</i>	0.0000	0.4521	0.2044
<i>dcbar</i>	0.0000	1.2025	1.4459
<i>di</i>	0.0000	0.9531	0.9085
<i>dibar</i>	0.0000	0.9379	0.8797
<i>y</i>	0.0000	1.7351	3.0107
<i>ybar</i>	0.0000	1.6819	2.8287
<i>ygap</i>	0.0000	0.4889	0.2390
<i>c</i>	0.0000	2.0672	4.2735
<i>cbar</i>	0.0000	2.9542	8.7274
<i>i</i>	0.0000	5.4499	29.7014
<i>ibar</i>	0.0000	4.2279	17.8750
<i>R</i>	0.0000	0.3617	0.1308
<i>Rbar</i>	0.0000	0.8171	0.6676
<i>dM</i>	0.0000	14.9262	222.7919
<i>dMbar</i>	0.0000	58.5715	3430.6226
<i>r</i>	0.0000	0.4133	0.1708
<i>rbar</i>	0.0000	2.7919	7.7947
<i>pi</i>	0.0000	0.5128	0.2630
<i>piibar</i>	0.0000	3.8673	14.9557

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Table C.20 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	2.5543	6.5246
<i>nbar</i>	0.0000	0.8500	0.7226
<i>n</i>	0.0000	0.8252	0.6810
<i>np</i>	0.0000	4.5883	21.0526
<i>npbar</i>	0.0000	4.5883	21.0526
<i>nnp</i>	0.0000	0.8158	0.6656
<i>nnpbar</i>	0.0000	0.8400	0.7056
<i>m</i>	0.0000	26.8030	718.4000
<i>mbar</i>	0.0000	53.4416	2856.0054
<i>h</i>	0.0000	16.1468	260.7186
<i>hbar</i>	0.0000	34.0064	1156.4384
<i>ks</i>	0.0000	2.3262	5.4112
<i>ksbar</i>	0.0000	2.3098	5.3352
<i>k</i>	0.0000	2.7361	7.4860
<i>kbar</i>	0.0000	2.1162	4.4783
<i>z</i>	0.0000	1.5061	2.2684
<i>zbar</i>	0.0000	0.2894	0.0838
<i>q</i>	0.0000	1.8273	3.3390
<i>qbar</i>	0.0000	3.8636	14.9275
<i>rk</i>	0.0000	1.7622	3.1052
<i>rkbar</i>	0.0000	1.9918	3.9674
<i>w</i>	0.0000	2.4374	5.9409
<i>wbar</i>	0.0000	2.0694	4.2826

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Table C.20 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	2.9345	8.6114
<i>pebar</i>	0.0000	1.7151	2.9417
<i>e</i>	0.0000	0.2855	0.0815
<i>ebar</i>	0.0000	0.2858	0.0817
<i>ep</i>	0.0000	0.0000	0.0000
<i>epbar</i>	0.0000	0.0000	0.0000
<i>enp</i>	0.0000	1.6025	2.5680
<i>enpbar</i>	0.0000	1.6044	2.5742
<i>pp</i>	0.0000	5.2913	27.9978
<i>ppbar</i>	0.0000	5.0647	25.6510
<i>pp2</i>	0.0000	6.3119	39.8399
<i>pnp</i>	0.0000	3.2108	10.3091
<i>pnpbar</i>	0.0000	2.4162	5.8381
<i>ns</i>	0.0000	0.7922	0.6275
<i>nsbar</i>	0.0000	0.8156	0.6652
<i>hc</i>	0.0000	25.1744	633.7528
<i>hcbar</i>	0.0000	51.5367	2656.0319
<i>dG</i>	0.0000	0.0000	0.0000
<i>dGbar</i>	0.0000	0.0000	0.0000
<i>pq</i>	0.0000	26.6456	709.9905
<i>pqbar</i>	0.0000	33.7554	1139.4238

Table C.21 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 0.95$  with an emission cap

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3982	0.2375	0.1026	0.0045	-0.0573
<i>dybar</i>	0.3097	-0.0153	-0.0620	-0.0635	-0.0581
<i>dc</i>	0.6670	0.3487	0.0949	-0.0763	-0.1712
<i>dcbar</i>	0.0544	-0.0810	-0.0860	-0.0726	-0.0588
<i>di</i>	0.7140	0.4596	0.2546	0.1030	-0.0005
<i>dibar</i>	0.3694	0.1847	0.0954	0.0381	-0.0014
<i>y</i>	0.9758	0.9324	0.8775	0.8177	0.7576
<i>ybar</i>	0.9636	0.9047	0.8469	0.7937	0.7450
<i>ygap</i>	0.8272	0.5087	0.2495	0.0770	-0.0217
<i>c</i>	0.9761	0.9203	0.8478	0.7707	0.6974
<i>cbar</i>	0.9172	0.8253	0.7469	0.6827	0.6305
<i>i</i>	0.9847	0.9476	0.8964	0.8374	0.7753
<i>ibar</i>	0.9754	0.9326	0.8807	0.8242	0.7657
<i>R</i>	0.9457	0.8200	0.6682	0.5216	0.3966
<i>Rbar</i>	0.3461	0.2045	0.1546	0.1254	0.1040
<i>dM</i>	0.0233	-0.0540	-0.0896	-0.0943	-0.0820
<i>dMbar</i>	-0.3978	-0.0699	-0.0147	-0.0050	-0.0029
<i>r</i>	0.6806	0.3833	0.1694	0.0393	-0.0268
<i>rbar</i>	0.2006	0.0601	0.0306	0.0209	0.0156
<i>pi</i>	0.6994	0.3447	0.0890	-0.0539	-0.1120
<i>pibar</i>	-0.3771	-0.0647	-0.0124	-0.0034	-0.0015

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Table C.21 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6021	0.3087	0.1178	0.0094	-0.0413
<i>nbar</i>	0.3165	0.1874	0.1538	0.1390	0.1295
<i>n</i>	0.7170	0.4696	0.2775	0.1437	0.0602
<i>np</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>npbar</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>nnp</i>	0.7185	0.4726	0.2815	0.1482	0.0649
<i>nnpbar</i>	0.3215	0.1927	0.1587	0.1434	0.1335
<i>m</i>	0.8412	0.6691	0.5121	0.3837	0.2867
<i>mbar</i>	0.3203	0.1821	0.1390	0.1158	0.0993
<i>h</i>	0.8125	0.6202	0.4512	0.3180	0.2212
<i>hbar</i>	0.2624	0.1222	0.0847	0.0674	0.0558
<i>ks</i>	0.8885	0.7932	0.7209	0.6705	0.6374
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9711
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9778
<i>z</i>	0.7425	0.5353	0.3903	0.3016	0.2559
<i>zbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>q</i>	0.8570	0.6837	0.5158	0.3731	0.2619
<i>qbar</i>	0.3128	0.1695	0.1217	0.0942	0.0738
<i>rk</i>	0.7425	0.5353	0.3903	0.3016	0.2559
<i>rkbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>w</i>	0.7290	0.5040	0.3413	0.2366	0.1767
<i>wbar</i>	0.8774	0.8141	0.7655	0.7233	0.6857

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Table C.21 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.6850	0.4262	0.2401	0.1213	0.0550
<i>pebar</i>	0.9634	0.9043	0.8465	0.7932	0.7445
<i>e</i>	0.8947	0.8002	0.7159	0.6411	0.5750
<i>ebar</i>	0.8814	0.7902	0.7107	0.6396	0.5757
<i>enp</i>	0.8947	0.8002	0.7159	0.6411	0.5750
<i>enpbar</i>	0.8814	0.7902	0.7107	0.6396	0.5757
<i>pp</i>	0.8582	0.7363	0.6365	0.5565	0.4925
<i>ppbar</i>	0.8967	0.8109	0.7348	0.6666	0.6052
<i>pp2</i>	0.8706	0.7582	0.6640	0.5861	0.5216
<i>pnp</i>	0.7468	0.5369	0.3820	0.2778	0.2132
<i>pnpbar</i>	0.9299	0.8575	0.7908	0.7303	0.6756
<i>ns</i>	0.7130	0.4625	0.2684	0.1335	0.0499
<i>nsbar</i>	0.3062	0.1765	0.1438	0.1302	0.1218
<i>hc</i>	0.8277	0.6459	0.4831	0.3522	0.2551
<i>hcbar</i>	0.2914	0.1521	0.1117	0.0914	0.0773
<i>pq</i>	0.8860	0.7854	0.6973	0.6207	0.5541
<i>pqbar</i>	0.7661	0.5922	0.4648	0.3750	0.3134

Table C.22 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 0.95$  with an emission cap

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.10	0.12	0.01	4.46	5.70	41.22	0.40	0.00	0.00	0.00
<i>dybar</i>	74.58	0.18	0.02	0.07	1.51	23.64	0.00	0.00	0.00	0.00
<i>dc</i>	57.62	0.14	0.01	30.28	8.18	2.87	0.91	0.00	0.00	0.00
<i>dcbar</i>	86.27	0.21	0.02	0.10	2.31	11.08	0.00	0.00	0.00	0.00
<i>di</i>	68.17	0.17	0.02	3.13	25.82	2.22	0.48	0.00	0.00	0.00
<i>dibar</i>	78.32	0.19	0.02	0.01	15.32	6.14	0.00	0.00	0.00	0.00
<i>y</i>	92.47	0.23	0.02	1.05	3.37	2.63	0.23	0.00	0.00	0.00
<i>ybar</i>	96.16	0.24	0.02	0.00	2.34	1.24	0.00	0.00	0.00	0.00
<i>ygap</i>	67.46	0.17	0.02	13.07	7.68	8.72	2.89	0.00	0.00	0.00
<i>c</i>	79.87	0.20	0.02	5.66	11.88	1.98	0.40	0.00	0.00	0.00
<i>cbar</i>	86.11	0.21	0.02	0.09	8.07	5.51	0.00	0.00	0.00	0.00
<i>i</i>	83.67	0.21	0.02	0.35	14.21	1.38	0.16	0.00	0.00	0.00
<i>ibar</i>	82.26	0.20	0.02	0.01	15.30	2.21	0.00	0.00	0.00	0.00
<i>R</i>	56.75	0.14	0.01	24.29	15.26	2.49	1.04	0.00	0.00	0.00
<i>Rbar</i>	74.49	0.18	0.02	10.72	4.80	9.80	0.00	0.00	0.00	0.00
<i>dM</i>	18.97	0.05	0.00	77.23	1.96	1.14	0.65	0.00	0.00	0.00
<i>dMbar</i>	86.49	0.21	0.02	0.54	0.60	12.14	0.00	0.00	0.00	0.00
<i>r</i>	61.90	0.15	0.02	21.90	10.27	4.00	1.77	0.00	0.00	0.00
<i>rbar</i>	85.80	0.21	0.02	1.14	0.32	12.50	0.00	0.00	0.00	0.00
<i>pi</i>	66.60	0.16	0.02	18.50	8.56	3.65	2.51	0.00	0.00	0.00
<i>pibar</i>	84.29	0.21	0.02	3.00	0.65	11.83	0.00	0.00	0.00	0.00

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Table C.22 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$mc$	82.73	0.20	0.02	7.93	1.75	6.15	1.22	0.00	0.00	0.00
$nbar$	77.98	1.79	0.02	0.03	7.28	12.90	0.00	0.00	0.00	0.00
$n$	74.02	1.75	0.02	2.17	12.40	9.28	0.36	0.00	0.00	0.00
$np$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$npbar$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$nnp$	73.27	1.73	1.03	2.15	12.28	9.19	0.36	0.00	0.00	0.00
$nnpbar$	77.26	1.77	0.95	0.03	7.21	12.78	0.00	0.00	0.00	0.00
$m$	53.68	0.13	0.01	28.87	14.11	2.27	0.92	0.00	0.00	0.00
$mbar$	82.74	0.20	0.02	0.73	5.60	10.71	0.00	0.00	0.00	0.00
$h$	49.81	0.12	0.01	33.12	13.67	2.28	0.99	0.00	0.00	0.00
$hbar$	82.89	0.20	0.02	0.78	5.04	11.06	0.00	0.00	0.00	0.00
$ks$	84.49	0.21	0.02	3.19	7.70	3.77	0.61	0.00	0.00	0.00
$ksbar$	85.55	0.21	0.02	0.00	12.64	1.57	0.00	0.00	0.00	0.00
$k$	87.77	0.22	0.02	0.06	10.86	0.98	0.08	0.00	0.00	0.00
$kbar$	86.29	0.21	0.02	0.00	12.02	1.45	0.00	0.00	0.00	0.00
$z$	75.72	0.19	0.02	7.09	7.62	8.21	1.16	0.00	0.00	0.00
$zbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$q$	52.53	0.13	0.01	17.13	26.76	2.57	0.86	0.00	0.00	0.00
$qbar$	80.67	0.20	0.02	0.03	9.20	9.89	0.00	0.00	0.00	0.00
$rk$	75.72	0.19	0.02	7.09	7.62	8.21	1.16	0.00	0.00	0.00
$rkbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$w$	76.68	0.38	0.02	11.34	2.34	7.54	1.70	0.00	0.00	0.00
$wbar$	97.50	0.06	0.02	0.00	1.81	0.61	0.00	0.00	0.00	0.00

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Table C.22 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$pe$	74.80	0.19	0.94	8.94	4.31	9.32	1.50	0.00	0.00	0.00
$pebar$	94.40	0.19	2.25	0.00	2.17	0.99	0.00	0.00	0.00	0.00
$e$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$ebar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$ep$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$epbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$enp$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$enpbar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$pp$	16.27	78.86	0.00	2.41	0.50	1.60	0.36	0.00	0.00	0.00
$ppbar$	16.28	83.31	0.00	0.00	0.30	0.10	0.00	0.00	0.00	0.00
$pp2$	11.43	55.42	0.00	1.69	0.35	1.12	0.25	29.72	0.00	0.00
$pnp$	55.71	0.15	26.53	7.14	2.73	6.57	1.16	0.00	0.00	0.00
$pnpbar$	53.36	0.04	45.43	0.00	1.00	0.18	0.00	0.00	0.00	0.00
$ns$	74.89	0.59	0.02	2.20	12.55	9.39	0.37	0.00	0.00	0.00
$nsbar$	78.97	0.54	0.02	0.03	7.37	13.06	0.00	0.00	0.00	0.00
$hc$	51.80	0.13	0.01	30.89	13.93	2.28	0.95	0.00	0.00	0.00
$hcbar$	82.80	0.20	0.02	0.75	5.33	10.89	0.00	0.00	0.00	0.00
$dG$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$dGbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$pq$	5.00	93.30	0.00	0.25	0.66	0.74	0.05	0.00	0.00	0.00
$pqbar$	34.89	56.34	0.01	2.86	1.64	3.77	0.49	0.00	0.00	0.00

Table C.23 – Theoretical moments for an elasticity of substitution of  $\epsilon = 1.008$  with an emission cap

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1455
<i>dybar</i>	0.0000	0.4536	0.2058
<i>dc</i>	0.0000	0.4521	0.2044
<i>dcbar</i>	0.0000	1.2024	1.4458
<i>di</i>	0.0000	0.9531	0.9085
<i>dibar</i>	0.0000	0.9379	0.8797
<i>y</i>	0.0000	1.7351	3.0107
<i>ybar</i>	0.0000	1.6819	2.8287
<i>ygap</i>	0.0000	0.4888	0.2390
<i>c</i>	0.0000	2.0672	4.2734
<i>cbar</i>	0.0000	2.9542	8.7272
<i>i</i>	0.0000	5.4499	29.7015
<i>ibar</i>	0.0000	4.2279	17.8751
<i>R</i>	0.0000	0.3617	0.1308
<i>Rbar</i>	0.0000	0.8171	0.6676
<i>dM</i>	0.0000	14.9262	222.7904
<i>dMbar</i>	0.0000	58.5691	3430.3356
<i>r</i>	0.0000	0.4133	0.1708
<i>rbar</i>	0.0000	2.7918	7.7939
<i>pi</i>	0.0000	0.5128	0.2630
<i>pibar</i>	0.0000	3.8671	14.9545

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Table C.23 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	2.5543	6.5244
<i>nbar</i>	0.0000	0.8499	0.7223
<i>n</i>	0.0000	0.8251	0.6808
<i>np</i>	0.0000	4.5883	21.0526
<i>npbar</i>	0.0000	4.5883	21.0526
<i>nnp</i>	0.0000	0.8270	0.6839
<i>nnpbar</i>	0.0000	0.8518	0.7256
<i>m</i>	0.0000	26.8028	718.3927
<i>mbar</i>	0.0000	53.4402	2855.8522
<i>h</i>	0.0000	16.1467	260.7158
<i>hbar</i>	0.0000	34.0055	1156.3725
<i>ks</i>	0.0000	2.3262	5.4111
<i>ksbar</i>	0.0000	2.3098	5.3352
<i>k</i>	0.0000	2.7361	7.4860
<i>kbar</i>	0.0000	2.1162	4.4783
<i>z</i>	0.0000	1.5061	2.2684
<i>zbar</i>	0.0000	0.2894	0.0838
<i>q</i>	0.0000	1.8273	3.3390
<i>qbar</i>	0.0000	3.8635	14.9267
<i>rk</i>	0.0000	1.7621	3.1052
<i>rkbar</i>	0.0000	1.9918	3.9674
<i>w</i>	0.0000	2.4374	5.9411
<i>wbar</i>	0.0000	2.0694	4.2823

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Table C.23 – (continued)

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<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	2.9343	8.6102
<i>pebar</i>	0.0000	1.7162	2.9454
<i>e</i>	0.0000	0.2910	0.0847
<i>ebar</i>	0.0000	0.2914	0.0849
<i>ep</i>	0.0000	0.0000	0.0000
<i>epbar</i>	0.0000	0.0000	0.0000
<i>enp</i>	0.0000	1.6328	2.6659
<i>enpbar</i>	0.0000	1.6347	2.6723
<i>pp</i>	0.0000	5.2913	27.9978
<i>ppbar</i>	0.0000	5.0646	25.6506
<i>pp2</i>	0.0000	6.3119	39.8399
<i>pnp</i>	0.0000	3.1880	10.1637
<i>pnpbar</i>	0.0000	2.3756	5.6434
<i>ns</i>	0.0000	0.7922	0.6275
<i>nsbar</i>	0.0000	0.8156	0.6652
<i>hc</i>	0.0000	25.1743	633.7462
<i>hcbar</i>	0.0000	51.5353	2655.8850
<i>dG</i>	0.0000	0.0000	0.0000
<i>dGbar</i>	0.0000	0.0000	0.0000
<i>pq</i>	0.0000	26.6378	709.5702
<i>pqbar</i>	0.0000	33.7398	1138.3747

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Table C.24 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 1.008$  with an emission cap

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3982	0.2375	0.1026	0.0045	-0.0573
<i>dybar</i>	0.3097	-0.0153	-0.0620	-0.0635	-0.0581
<i>dc</i>	0.6670	0.3487	0.0949	-0.0763	-0.1712
<i>dcbar</i>	0.0545	-0.0810	-0.0860	-0.0726	-0.0588
<i>di</i>	0.7140	0.4596	0.2546	0.1030	-0.0005
<i>dibar</i>	0.3694	0.1847	0.0954	0.0381	-0.0014
<i>y</i>	0.9758	0.9324	0.8775	0.8177	0.7576
<i>ybar</i>	0.9636	0.9047	0.8469	0.7937	0.7450
<i>ygap</i>	0.8272	0.5087	0.2495	0.0770	-0.0217
<i>c</i>	0.9761	0.9203	0.8478	0.7707	0.6974
<i>cbar</i>	0.9172	0.8253	0.7469	0.6827	0.6305
<i>i</i>	0.9847	0.9476	0.8964	0.8374	0.7753
<i>ibar</i>	0.9754	0.9326	0.8807	0.8242	0.7657
<i>R</i>	0.9457	0.8200	0.6682	0.5216	0.3966
<i>Rbar</i>	0.3462	0.2045	0.1546	0.1254	0.1040
<i>dM</i>	0.0232	-0.0540	-0.0896	-0.0943	-0.0820
<i>dMbar</i>	-0.3978	-0.0699	-0.0147	-0.0050	-0.0029
<i>r</i>	0.6806	0.3833	0.1694	0.0393	-0.0268
<i>rbar</i>	0.2006	0.0601	0.0306	0.0209	0.0156
<i>pi</i>	0.6994	0.3447	0.0890	-0.0539	-0.1120
<i>pibar</i>	-0.3771	-0.0647	-0.0124	-0.0034	-0.0015

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Table C.24 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6021	0.3087	0.1178	0.0094	-0.0413
<i>nbar</i>	0.3165	0.1874	0.1538	0.1390	0.1295
<i>n</i>	0.7170	0.4696	0.2775	0.1437	0.0602
<i>np</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>npbar</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>nnp</i>	0.7170	0.4696	0.2775	0.1437	0.0602
<i>nnpbar</i>	0.3166	0.1874	0.1538	0.1390	0.1296
<i>m</i>	0.8412	0.6691	0.5121	0.3837	0.2867
<i>mbar</i>	0.3203	0.1821	0.1390	0.1158	0.0993
<i>h</i>	0.8125	0.6202	0.4512	0.3180	0.2212
<i>hbar</i>	0.2624	0.1222	0.0847	0.0674	0.0558
<i>ks</i>	0.8885	0.7932	0.7209	0.6705	0.6374
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9711
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9778
<i>z</i>	0.7425	0.5353	0.3903	0.3016	0.2560
<i>zbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>q</i>	0.8570	0.6837	0.5159	0.3731	0.2620
<i>qbar</i>	0.3128	0.1695	0.1217	0.0942	0.0738
<i>rk</i>	0.7425	0.5353	0.3903	0.3016	0.2560
<i>rkbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>w</i>	0.7290	0.5040	0.3413	0.2365	0.1767
<i>wbar</i>	0.8774	0.8141	0.7655	0.7233	0.6857

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Table C.24 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.6851	0.4264	0.2403	0.1216	0.0553
<i>pebar</i>	0.9633	0.9043	0.8464	0.7931	0.7444
<i>e</i>	0.8948	0.8002	0.7159	0.6411	0.5750
<i>ebar</i>	0.8814	0.7902	0.7107	0.6396	0.5757
<i>enp</i>	0.8948	0.8002	0.7159	0.6411	0.5750
<i>enpbar</i>	0.8814	0.7902	0.7107	0.6396	0.5757
<i>pp</i>	0.8582	0.7363	0.6365	0.5565	0.4925
<i>ppbar</i>	0.8968	0.8109	0.7348	0.6666	0.6052
<i>pp2</i>	0.8706	0.7582	0.6640	0.5861	0.5216
<i>pnp</i>	0.7435	0.5311	0.3746	0.2697	0.2050
<i>pnpbar</i>	0.9313	0.8594	0.7930	0.7329	0.6784
<i>ns</i>	0.7130	0.4625	0.2684	0.1335	0.0499
<i>nsbar</i>	0.3062	0.1765	0.1438	0.1302	0.1218
<i>hc</i>	0.8276	0.6459	0.4831	0.3522	0.2551
<i>hcbar</i>	0.2914	0.1521	0.1117	0.0914	0.0773
<i>pq</i>	0.8861	0.7855	0.6974	0.6208	0.5543
<i>pqbar</i>	0.7662	0.5924	0.4649	0.3752	0.3136

Table C.25 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 1.008$  with an emission cap

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{tao}$	$bigGbar$	$bigG$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.09	0.12	0.01	4.46	5.70	41.22	0.40	0.00	0.00	0.00
<i>dybar</i>	74.58	0.18	0.02	0.07	1.51	23.64	0.00	0.00	0.00	0.00
<i>dc</i>	57.62	0.14	0.01	30.28	8.18	2.87	0.91	0.00	0.00	0.00
<i>dcbar</i>	86.27	0.21	0.02	0.10	2.31	11.08	0.00	0.00	0.00	0.00
<i>di</i>	68.16	0.17	0.02	3.13	25.82	2.22	0.48	0.00	0.00	0.00
<i>dibar</i>	78.32	0.19	0.02	0.01	15.32	6.14	0.00	0.00	0.00	0.00
<i>y</i>	92.47	0.23	0.02	1.05	3.37	2.63	0.23	0.00	0.00	0.00
<i>ybar</i>	96.16	0.24	0.02	0.00	2.34	1.24	0.00	0.00	0.00	0.00
<i>ygap</i>	67.46	0.17	0.02	13.07	7.68	8.72	2.89	0.00	0.00	0.00
<i>c</i>	79.87	0.20	0.02	5.66	11.88	1.98	0.40	0.00	0.00	0.00
<i>cbar</i>	86.11	0.21	0.02	0.09	8.07	5.51	0.00	0.00	0.00	0.00
<i>i</i>	83.67	0.21	0.02	0.35	14.21	1.38	0.16	0.00	0.00	0.00
<i>ibar</i>	82.26	0.20	0.02	0.01	15.30	2.21	0.00	0.00	0.00	0.00
<i>R</i>	56.75	0.14	0.01	24.29	15.26	2.49	1.04	0.00	0.00	0.00
<i>Rbar</i>	74.49	0.18	0.02	10.72	4.80	9.80	0.00	0.00	0.00	0.00
<i>dM</i>	18.97	0.05	0.00	77.23	1.96	1.14	0.65	0.00	0.00	0.00
<i>dMbar</i>	86.49	0.21	0.02	0.54	0.60	12.14	0.00	0.00	0.00	0.00
<i>r</i>	61.90	0.15	0.02	21.90	10.27	4.00	1.77	0.00	0.00	0.00
<i>rbar</i>	85.80	0.21	0.02	1.14	0.32	12.50	0.00	0.00	0.00	0.00
<i>pi</i>	66.60	0.16	0.02	18.50	8.56	3.65	2.51	0.00	0.00	0.00
<i>pibar</i>	84.29	0.21	0.02	3.00	0.65	11.83	0.00	0.00	0.00	0.00

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Table C.25 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$mc$	82.73	0.20	0.02	7.93	1.75	6.15	1.22	0.00	0.00	0.00
$nbar$	77.98	1.79	0.02	0.03	7.28	12.90	0.00	0.00	0.00	0.00
$n$	74.01	1.75	0.02	2.17	12.40	9.28	0.36	0.00	0.00	0.00
$np$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$npbar$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$nnp$	74.02	1.75	0.01	2.17	12.40	9.28	0.36	0.00	0.00	0.00
$nnpbar$	77.98	1.79	0.02	0.03	7.28	12.90	0.00	0.00	0.00	0.00
$m$	53.68	0.13	0.01	28.87	14.11	2.27	0.92	0.00	0.00	0.00
$mbar$	82.74	0.20	0.02	0.73	5.60	10.71	0.00	0.00	0.00	0.00
$h$	49.81	0.12	0.01	33.12	13.67	2.28	0.99	0.00	0.00	0.00
$hbar$	82.89	0.20	0.02	0.78	5.04	11.06	0.00	0.00	0.00	0.00
$ks$	84.49	0.21	0.02	3.19	7.70	3.77	0.61	0.00	0.00	0.00
$ksbar$	85.55	0.21	0.02	0.00	12.64	1.57	0.00	0.00	0.00	0.00
$k$	87.77	0.22	0.02	0.06	10.86	0.98	0.08	0.00	0.00	0.00
$kbar$	86.29	0.21	0.02	0.00	12.02	1.45	0.00	0.00	0.00	0.00
$z$	75.72	0.19	0.02	7.09	7.62	8.21	1.16	0.00	0.00	0.00
$zbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$q$	52.53	0.13	0.01	17.13	26.76	2.57	0.86	0.00	0.00	0.00
$qbar$	80.67	0.20	0.02	0.03	9.20	9.89	0.00	0.00	0.00	0.00
$rk$	75.72	0.19	0.02	7.09	7.62	8.21	1.16	0.00	0.00	0.00
$rkbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$w$	76.68	0.38	0.02	11.34	2.34	7.54	1.70	0.00	0.00	0.00
$wbar$	97.50	0.06	0.03	0.00	1.81	0.61	0.00	0.00	0.00	0.00

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Table C.25 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$pe$	74.77	0.19	0.98	8.94	4.31	9.31	1.50	0.00	0.00	0.00
$pebar$	94.31	0.19	2.34	0.00	2.16	0.99	0.00	0.00	0.00	0.00
$e$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$ebar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$ep$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$epbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$enp$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$enpbar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$pp$	16.27	78.86	0.00	2.41	0.50	1.60	0.36	0.00	0.00	0.00
$ppbar$	16.28	83.31	0.00	0.00	0.30	0.10	0.00	0.00	0.00	0.00
$pp2$	11.43	55.42	0.00	1.69	0.35	1.12	0.25	29.72	0.00	0.00
$pnp$	56.77	0.16	25.12	7.26	2.80	6.71	1.18	0.00	0.00	0.00
$pnpbar$	54.92	0.04	43.82	0.00	1.03	0.19	0.00	0.00	0.00	0.00
$ns$	74.89	0.59	0.02	2.20	12.55	9.39	0.37	0.00	0.00	0.00
$nsbar$	78.97	0.54	0.02	0.03	7.37	13.06	0.00	0.00	0.00	0.00
$hc$	51.80	0.13	0.01	30.90	13.93	2.28	0.95	0.00	0.00	0.00
$hcbar$	82.80	0.20	0.02	0.75	5.33	10.89	0.00	0.00	0.00	0.00
$dG$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$dGbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$pq$	4.98	93.33	0.00	0.25	0.65	0.73	0.05	0.00	0.00	0.00
$pqbar$	34.86	56.38	0.01	2.86	1.64	3.77	0.49	0.00	0.00	0.00

Table C.26 – Theoretical moments for an elasticity of substitution of  $\epsilon = 1.22$  with an emission cap

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1455
<i>dybar</i>	0.0000	0.4536	0.2058
<i>dc</i>	0.0000	0.4521	0.2044
<i>dcbar</i>	0.0000	1.2024	1.4457
<i>di</i>	0.0000	0.9531	0.9085
<i>dibar</i>	0.0000	0.9379	0.8797
<i>y</i>	0.0000	1.7351	3.0107
<i>ybar</i>	0.0000	1.6819	2.8287
<i>ygap</i>	0.0000	0.4888	0.2390
<i>c</i>	0.0000	2.0672	4.2734
<i>cbar</i>	0.0000	2.9542	8.7271
<i>i</i>	0.0000	5.4499	29.7017
<i>ibar</i>	0.0000	4.2279	17.8752
<i>R</i>	0.0000	0.3617	0.1308
<i>Rbar</i>	0.0000	0.8170	0.6676
<i>dM</i>	0.0000	14.9261	222.7886
<i>dMbar</i>	0.0000	58.5663	3430.0141
<i>r</i>	0.0000	0.4133	0.1708
<i>rbar</i>	0.0000	2.7916	7.7930
<i>pi</i>	0.0000	0.5128	0.2630
<i>piibar</i>	0.0000	3.8669	14.9531

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Table C.26 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	2.5542	6.5241
<i>nbar</i>	0.0000	0.8496	0.7218
<i>n</i>	0.0000	0.8248	0.6803
<i>np</i>	0.0000	4.5883	21.0526
<i>npbar</i>	0.0000	4.5883	21.0526
<i>nnp</i>	0.0000	0.9028	0.8150
<i>nnpbar</i>	0.0000	0.9283	0.8617
<i>m</i>	0.0000	26.8027	718.3844
<i>mbar</i>	0.0000	53.4386	2855.6805
<i>h</i>	0.0000	16.1466	260.7127
<i>hbar</i>	0.0000	34.0044	1156.2987
<i>ks</i>	0.0000	2.3262	5.4110
<i>ksbar</i>	0.0000	2.3098	5.3352
<i>k</i>	0.0000	2.7361	7.4860
<i>kbar</i>	0.0000	2.1162	4.4783
<i>z</i>	0.0000	1.5061	2.2683
<i>zbar</i>	0.0000	0.2894	0.0838
<i>q</i>	0.0000	1.8273	3.3389
<i>qbar</i>	0.0000	3.8634	14.9259
<i>rk</i>	0.0000	1.7621	3.1051
<i>rkbar</i>	0.0000	1.9918	3.9674
<i>w</i>	0.0000	2.4376	5.9418
<i>wbar</i>	0.0000	2.0692	4.2815

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Table C.26 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	2.9339	8.6079
<i>pebar</i>	0.0000	1.7190	2.9548
<i>e</i>	0.0000	0.3044	0.0927
<i>ebar</i>	0.0000	0.3048	0.0929
<i>ep</i>	0.0000	0.0000	0.0000
<i>epbar</i>	0.0000	0.0000	0.0000
<i>enp</i>	0.0000	1.7176	2.9500
<i>enpbar</i>	0.0000	1.7196	2.9571
<i>pp</i>	0.0000	5.2913	27.9981
<i>ppbar</i>	0.0000	5.0645	25.6494
<i>pp2</i>	0.0000	6.3119	39.8403
<i>pnp</i>	0.0000	3.1290	9.7909
<i>pnpbar</i>	0.0000	2.2656	5.1331
<i>ns</i>	0.0000	0.7922	0.6276
<i>nsbar</i>	0.0000	0.8157	0.6653
<i>hc</i>	0.0000	25.1742	633.7388
<i>hcbar</i>	0.0000	51.5337	2655.7204
<i>dG</i>	0.0000	0.0000	0.0000
<i>dGbar</i>	0.0000	0.0000	0.0000
<i>pq</i>	0.0000	26.6190	708.5733
<i>pqbar</i>	0.0000	33.6995	1135.6543

Table C.27 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 1.22$  with an emission cap

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3982	0.2375	0.1026	0.0045	-0.0573
<i>dybar</i>	0.3097	-0.0153	-0.0620	-0.0635	-0.0581
<i>dc</i>	0.6670	0.3487	0.0949	-0.0763	-0.1712
<i>dcbar</i>	0.0545	-0.0810	-0.0860	-0.0726	-0.0588
<i>di</i>	0.7140	0.4596	0.2546	0.1030	-0.0005
<i>dibar</i>	0.3694	0.1847	0.0954	0.0381	-0.0014
<i>y</i>	0.9758	0.9324	0.8775	0.8177	0.7576
<i>ybar</i>	0.9636	0.9047	0.8469	0.7937	0.7450
<i>ygap</i>	0.8272	0.5087	0.2496	0.0770	-0.0217
<i>c</i>	0.9761	0.9203	0.8478	0.7707	0.6974
<i>cbar</i>	0.9172	0.8253	0.7469	0.6827	0.6305
<i>i</i>	0.9847	0.9476	0.8964	0.8374	0.7753
<i>ibar</i>	0.9754	0.9326	0.8807	0.8242	0.7657
<i>R</i>	0.9457	0.8200	0.6682	0.5216	0.3966
<i>Rbar</i>	0.3462	0.2045	0.1546	0.1254	0.1040
<i>dM</i>	0.0232	-0.0540	-0.0896	-0.0943	-0.0820
<i>dMbar</i>	-0.3978	-0.0699	-0.0147	-0.0050	-0.0029
<i>r</i>	0.6806	0.3833	0.1694	0.0393	-0.0268
<i>rbar</i>	0.2006	0.0601	0.0306	0.0209	0.0156
<i>pi</i>	0.6994	0.3447	0.0890	-0.0539	-0.1120
<i>pibar</i>	-0.3770	-0.0647	-0.0124	-0.0034	-0.0015

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Table C.27 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6021	0.3087	0.1178	0.0094	-0.0413
<i>nbar</i>	0.3166	0.1874	0.1538	0.1390	0.1296
<i>n</i>	0.7170	0.4696	0.2775	0.1437	0.0602
<i>np</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>npbar</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>nnp</i>	0.7308	0.4952	0.3114	0.1820	0.0997
<i>nnpbar</i>	0.3576	0.2311	0.1942	0.1752	0.1618
<i>m</i>	0.8412	0.6691	0.5121	0.3837	0.2867
<i>mbar</i>	0.3203	0.1821	0.1390	0.1158	0.0993
<i>h</i>	0.8125	0.6202	0.4512	0.3180	0.2212
<i>hbar</i>	0.2624	0.1222	0.0848	0.0674	0.0558
<i>ks</i>	0.8885	0.7932	0.7209	0.6705	0.6374
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9711
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9778
<i>z</i>	0.7425	0.5353	0.3903	0.3016	0.2560
<i>zbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>q</i>	0.8570	0.6837	0.5159	0.3731	0.2620
<i>qbar</i>	0.3129	0.1695	0.1217	0.0942	0.0738
<i>rk</i>	0.7425	0.5353	0.3903	0.3016	0.2560
<i>rkbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>w</i>	0.7290	0.5040	0.3413	0.2365	0.1766
<i>wbar</i>	0.8775	0.8142	0.7655	0.7234	0.6858

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Table C.27 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.6853	0.4268	0.2409	0.1223	0.0560
<i>pebar</i>	0.9632	0.9041	0.8462	0.7928	0.7442
<i>e</i>	0.8948	0.8002	0.7159	0.6412	0.5750
<i>ebar</i>	0.8815	0.7902	0.7107	0.6396	0.5757
<i>enp</i>	0.8948	0.8002	0.7159	0.6412	0.5750
<i>enpbar</i>	0.8815	0.7902	0.7107	0.6396	0.5757
<i>pp</i>	0.8582	0.7363	0.6364	0.5565	0.4925
<i>ppbar</i>	0.8968	0.8109	0.7348	0.6666	0.6052
<i>pp2</i>	0.8706	0.7582	0.6640	0.5861	0.5216
<i>pnp</i>	0.7345	0.5149	0.3539	0.2470	0.1822
<i>pnpbar</i>	0.9353	0.8650	0.7997	0.7404	0.6867
<i>ns</i>	0.7130	0.4625	0.2684	0.1335	0.0499
<i>nsbar</i>	0.3063	0.1765	0.1438	0.1302	0.1218
<i>hc</i>	0.8276	0.6459	0.4831	0.3522	0.2551
<i>hcbar</i>	0.2914	0.1521	0.1117	0.0914	0.0773
<i>pq</i>	0.8865	0.7860	0.6980	0.6214	0.5548
<i>pqbar</i>	0.7664	0.5928	0.4655	0.3758	0.3143

Table C.28 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 1.22$  with an emission cap

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.09	0.12	0.01	4.46	5.70	41.22	0.40	0.00	0.00	0.00
<i>dybar</i>	74.58	0.18	0.02	0.07	1.51	23.64	0.00	0.00	0.00	0.00
<i>dc</i>	57.62	0.14	0.01	30.28	8.18	2.87	0.91	0.00	0.00	0.00
<i>dcbar</i>	86.27	0.21	0.02	0.10	2.31	11.08	0.00	0.00	0.00	0.00
<i>di</i>	68.16	0.17	0.02	3.13	25.82	2.22	0.48	0.00	0.00	0.00
<i>dibar</i>	78.32	0.19	0.02	0.01	15.32	6.14	0.00	0.00	0.00	0.00
<i>y</i>	92.47	0.23	0.02	1.05	3.37	2.63	0.23	0.00	0.00	0.00
<i>ybar</i>	96.16	0.24	0.03	0.00	2.34	1.24	0.00	0.00	0.00	0.00
<i>ygap</i>	67.46	0.17	0.02	13.07	7.68	8.72	2.89	0.00	0.00	0.00
<i>c</i>	79.87	0.20	0.02	5.66	11.88	1.98	0.40	0.00	0.00	0.00
<i>cbar</i>	86.11	0.21	0.02	0.09	8.07	5.51	0.00	0.00	0.00	0.00
<i>i</i>	83.67	0.21	0.02	0.35	14.21	1.38	0.16	0.00	0.00	0.00
<i>ibar</i>	82.26	0.20	0.02	0.01	15.30	2.21	0.00	0.00	0.00	0.00
<i>R</i>	56.75	0.14	0.01	24.29	15.26	2.49	1.04	0.00	0.00	0.00
<i>Rbar</i>	74.49	0.18	0.02	10.72	4.80	9.80	0.00	0.00	0.00	0.00
<i>dM</i>	18.97	0.05	0.00	77.23	1.96	1.14	0.65	0.00	0.00	0.00
<i>dMbar</i>	86.48	0.21	0.02	0.54	0.60	12.14	0.00	0.00	0.00	0.00
<i>r</i>	61.89	0.15	0.02	21.90	10.27	4.00	1.77	0.00	0.00	0.00
<i>rbar</i>	85.80	0.21	0.02	1.14	0.32	12.50	0.00	0.00	0.00	0.00
<i>pi</i>	66.60	0.16	0.02	18.50	8.56	3.65	2.51	0.00	0.00	0.00
<i>pibar</i>	84.29	0.21	0.02	3.00	0.65	11.83	0.00	0.00	0.00	0.00

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Table C.28 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$mc$	82.73	0.20	0.02	7.93	1.75	6.15	1.22	0.00	0.00	0.00
$nbar$	77.98	1.79	0.02	0.03	7.28	12.90	0.00	0.00	0.00	0.00
$n$	74.01	1.75	0.02	2.17	12.40	9.28	0.36	0.00	0.00	0.00
$np$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$npbar$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$nnp$	68.73	1.62	7.15	2.02	11.52	8.62	0.34	0.00	0.00	0.00
$nnpbar$	72.66	1.67	6.83	0.03	6.78	12.02	0.00	0.00	0.00	0.00
$m$	53.68	0.13	0.01	28.87	14.11	2.27	0.92	0.00	0.00	0.00
$mbar$	82.74	0.20	0.02	0.73	5.60	10.71	0.00	0.00	0.00	0.00
$h$	49.81	0.12	0.01	33.12	13.67	2.28	0.99	0.00	0.00	0.00
$hbar$	82.89	0.20	0.02	0.78	5.04	11.06	0.00	0.00	0.00	0.00
$ks$	84.49	0.21	0.02	3.19	7.70	3.77	0.61	0.00	0.00	0.00
$ksbar$	85.55	0.21	0.02	0.00	12.64	1.57	0.00	0.00	0.00	0.00
$k$	87.77	0.22	0.02	0.06	10.86	0.98	0.08	0.00	0.00	0.00
$kbar$	86.29	0.21	0.02	0.00	12.02	1.45	0.00	0.00	0.00	0.00
$z$	75.72	0.19	0.02	7.09	7.62	8.21	1.16	0.00	0.00	0.00
$zbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$q$	52.53	0.13	0.01	17.13	26.76	2.57	0.86	0.00	0.00	0.00
$qbar$	80.67	0.20	0.02	0.03	9.20	9.89	0.00	0.00	0.00	0.00
$rk$	75.72	0.19	0.02	7.09	7.62	8.21	1.16	0.00	0.00	0.00
$rkbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$w$	76.67	0.38	0.02	11.34	2.34	7.54	1.70	0.00	0.00	0.00
$wbar$	97.49	0.06	0.03	0.00	1.81	0.61	0.00	0.00	0.00	0.00

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Table C.28 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$pe$	74.70	0.19	1.07	8.94	4.30	9.30	1.50	0.00	0.00	0.00
$pebar$	94.11	0.19	2.57	0.00	2.15	0.98	0.00	0.00	0.00	0.00
$e$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$ebar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$ep$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$epbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$enp$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$enpbar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$pp$	16.27	78.86	0.00	2.41	0.50	1.60	0.36	0.00	0.00	0.00
$ppbar$	16.27	83.32	0.00	0.00	0.30	0.10	0.00	0.00	0.00	0.00
$pp2$	11.44	55.42	0.00	1.69	0.35	1.12	0.25	29.72	0.00	0.00
$pnp$	59.70	0.16	21.22	7.57	3.00	7.11	1.24	0.00	0.00	0.00
$pnpbar$	59.53	0.05	39.04	0.00	1.14	0.24	0.00	0.00	0.00	0.00
$ns$	74.89	0.59	0.02	2.20	12.55	9.39	0.37	0.00	0.00	0.00
$nsbar$	78.97	0.54	0.02	0.03	7.37	13.06	0.00	0.00	0.00	0.00
$hc$	51.80	0.13	0.01	30.90	13.93	2.28	0.95	0.00	0.00	0.00
$hcbbar$	82.80	0.20	0.02	0.75	5.33	10.89	0.00	0.00	0.00	0.00
$dG$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$dGbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$pq$	4.90	93.41	0.02	0.25	0.64	0.72	0.05	0.00	0.00	0.00
$pqbar$	34.76	56.48	0.03	2.87	1.62	3.76	0.48	0.00	0.00	0.00

Table C.29 – Theoretical moments for an elasticity of substitution of  $\epsilon = 1.85$  with an emission cap

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>epsa</i>	0.0000	1.6059	2.5789
<i>dy</i>	0.0000	0.3815	0.1455
<i>dybar</i>	0.0000	0.4536	0.2058
<i>dc</i>	0.0000	0.4521	0.2044
<i>dcbar</i>	0.0000	1.2025	1.4461
<i>di</i>	0.0000	0.9531	0.9085
<i>dibar</i>	0.0000	0.9379	0.8797
<i>y</i>	0.0000	1.7351	3.0107
<i>ybar</i>	0.0000	1.6819	2.8288
<i>ygap</i>	0.0000	0.4889	0.2390
<i>c</i>	0.0000	2.0673	4.2736
<i>cbar</i>	0.0000	2.9543	8.7277
<i>i</i>	0.0000	5.4499	29.7012
<i>ibar</i>	0.0000	4.2279	17.8748
<i>R</i>	0.0000	0.3617	0.1308
<i>Rbar</i>	0.0000	0.8171	0.6677
<i>dM</i>	0.0000	14.9263	222.7952
<i>dMbar</i>	0.0000	58.5767	3431.2293
<i>r</i>	0.0000	0.4133	0.1708
<i>rbar</i>	0.0000	2.7922	7.7964
<i>pi</i>	0.0000	0.5128	0.2630
<i>pibar</i>	0.0000	3.8676	14.9583

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Table C.29 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>mc</i>	0.0000	2.5544	6.5251
<i>nbar</i>	0.0000	0.8491	0.7209
<i>n</i>	0.0000	0.8243	0.6795
<i>np</i>	0.0000	4.5883	21.0526
<i>npbar</i>	0.0000	4.5883	21.0526
<i>nnp</i>	0.0000	1.1755	1.3819
<i>nnpbar</i>	0.0000	1.1995	1.4388
<i>m</i>	0.0000	26.8033	718.4155
<i>mbar</i>	0.0000	53.4446	2856.3294
<i>h</i>	0.0000	16.1470	260.7245
<i>hbar</i>	0.0000	34.0085	1156.5776
<i>ks</i>	0.0000	2.3262	5.4113
<i>ksbar</i>	0.0000	2.3098	5.3352
<i>k</i>	0.0000	2.7361	7.4860
<i>kbar</i>	0.0000	2.1162	4.4783
<i>z</i>	0.0000	1.5062	2.2685
<i>zbar</i>	0.0000	0.2894	0.0838
<i>q</i>	0.0000	1.8273	3.3391
<i>qbar</i>	0.0000	3.8638	14.9291
<i>rk</i>	0.0000	1.7622	3.1054
<i>rkbar</i>	0.0000	1.9918	3.9674
<i>w</i>	0.0000	2.4380	5.9439
<i>wbar</i>	0.0000	2.0689	4.2804

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Table C.29 – (continued)

<i>VARIABLE</i>	<i>MEAN</i>	<i>STD.DEV.</i>	<i>VARIANCE</i>
<i>pe</i>	0.0000	2.9337	8.6068
<i>pebar</i>	0.0000	1.7227	2.9678
<i>e</i>	0.0000	0.3216	0.1034
<i>ebar</i>	0.0000	0.3220	0.1037
<i>ep</i>	0.0000	0.0000	0.0000
<i>epbar</i>	0.0000	0.0000	0.0000
<i>enp</i>	0.0000	1.8775	3.5250
<i>enpbar</i>	0.0000	1.8797	3.5334
<i>pp</i>	0.0000	5.2915	27.9996
<i>ppbar</i>	0.0000	5.0644	25.6477
<i>pp2</i>	0.0000	6.3120	39.8417
<i>pnp</i>	0.0000	3.0377	9.2275
<i>pnpbar</i>	0.0000	2.0764	4.3114
<i>ns</i>	0.0000	0.7924	0.6279
<i>nsbar</i>	0.0000	0.8159	0.6657
<i>hc</i>	0.0000	25.1747	633.7668
<i>hcbar</i>	0.0000	51.5397	2656.3425
<i>dG</i>	0.0000	0.0000	0.0000
<i>dGbar</i>	0.0000	0.0000	0.0000
<i>pq</i>	0.0000	26.5975	707.4290
<i>pqbar</i>	0.0000	33.6385	1131.5513

Table C.30 – Coefficients of autocorrelation for an elasticity of substitution of  $\epsilon = 1.85$  with an emission cap

<i>Order</i>	1	2	3	4	5
<i>epsa</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>dy</i>	0.3982	0.2375	0.1026	0.0045	-0.0573
<i>dybar</i>	0.3097	-0.0153	-0.0620	-0.0635	-0.0581
<i>dc</i>	0.6670	0.3487	0.0949	-0.0763	-0.1712
<i>dcbar</i>	0.0544	-0.0810	-0.0860	-0.0726	-0.0588
<i>di</i>	0.7140	0.4596	0.2546	0.1030	-0.0005
<i>dibar</i>	0.3694	0.1847	0.0954	0.0381	-0.0014
<i>y</i>	0.9758	0.9324	0.8775	0.8177	0.7576
<i>ybar</i>	0.9636	0.9047	0.8469	0.7937	0.7450
<i>ygap</i>	0.8272	0.5087	0.2495	0.0770	-0.0217
<i>c</i>	0.9761	0.9203	0.8478	0.7707	0.6974
<i>cbar</i>	0.9172	0.8253	0.7468	0.6827	0.6305
<i>i</i>	0.9847	0.9476	0.8964	0.8374	0.7753
<i>ibar</i>	0.9754	0.9326	0.8807	0.8242	0.7657
<i>R</i>	0.9457	0.8200	0.6681	0.5216	0.3966
<i>Rbar</i>	0.3461	0.2045	0.1546	0.1254	0.1040
<i>dM</i>	0.0233	-0.0540	-0.0896	-0.0943	-0.0821
<i>dMbar</i>	-0.3979	-0.0698	-0.0147	-0.0050	-0.0029
<i>r</i>	0.6806	0.3833	0.1694	0.0393	-0.0268
<i>rbar</i>	0.2005	0.0601	0.0306	0.0209	0.0156
<i>pi</i>	0.6994	0.3447	0.0890	-0.0539	-0.1120
<i>pibar</i>	-0.3771	-0.0647	-0.0124	-0.0034	-0.0015

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Table C.30 – (continued)

<i>Order</i>	1	2	3	4	5
<i>mc</i>	0.6021	0.3087	0.1178	0.0094	-0.0413
<i>nbar</i>	0.3165	0.1874	0.1538	0.1390	0.1296
<i>n</i>	0.7170	0.4696	0.2775	0.1437	0.0602
<i>np</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>npbar</i>	0.9000	0.8100	0.7290	0.6561	0.5905
<i>nnp</i>	0.7816	0.5895	0.4364	0.3238	0.2465
<i>nnpbar</i>	0.5131	0.3970	0.3474	0.3129	0.2845
<i>m</i>	0.8412	0.6691	0.5121	0.3837	0.2867
<i>mbar</i>	0.3202	0.1821	0.1390	0.1158	0.0992
<i>h</i>	0.8125	0.6202	0.4512	0.3180	0.2212
<i>hbar</i>	0.2623	0.1221	0.0847	0.0674	0.0558
<i>ks</i>	0.8885	0.7932	0.7208	0.6705	0.6374
<i>ksbar</i>	0.9983	0.9942	0.9881	0.9803	0.9711
<i>k</i>	0.9990	0.9962	0.9916	0.9854	0.9776
<i>kbar</i>	0.9990	0.9962	0.9916	0.9854	0.9778
<i>z</i>	0.7424	0.5353	0.3903	0.3016	0.2559
<i>zbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>q</i>	0.8570	0.6837	0.5158	0.3731	0.2619
<i>qbar</i>	0.3128	0.1694	0.1217	0.0942	0.0738
<i>rk</i>	0.7424	0.5353	0.3903	0.3016	0.2559
<i>rkbar</i>	0.9630	0.9010	0.8380	0.7780	0.7217
<i>w</i>	0.7289	0.5039	0.3411	0.2363	0.1765
<i>wbar</i>	0.8776	0.8143	0.7656	0.7235	0.6858

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Table C.30 – (continued)

<i>Order</i>	1	2	3	4	5
<i>pe</i>	0.6857	0.4275	0.2417	0.1232	0.0569
<i>pebar</i>	0.9630	0.9038	0.8459	0.7925	0.7438
<i>e</i>	0.8947	0.8002	0.7159	0.6411	0.5749
<i>ebar</i>	0.8814	0.7902	0.7106	0.6396	0.5757
<i>enp</i>	0.8947	0.8002	0.7159	0.6411	0.5749
<i>enpbar</i>	0.8814	0.7902	0.7106	0.6396	0.5757
<i>pp</i>	0.8581	0.7362	0.6364	0.5564	0.4924
<i>ppbar</i>	0.8968	0.8109	0.7348	0.6666	0.6052
<i>pp2</i>	0.8706	0.7581	0.6639	0.5861	0.5216
<i>pnp</i>	0.7182	0.4857	0.3165	0.2059	0.1407
<i>pnpbar</i>	0.9435	0.8765	0.8133	0.7557	0.7036
<i>ns</i>	0.7130	0.4625	0.2683	0.1335	0.0498
<i>nsbar</i>	0.3062	0.1764	0.1438	0.1301	0.1218
<i>hc</i>	0.8277	0.6459	0.4831	0.3522	0.2551
<i>hcbar</i>	0.2913	0.1521	0.1117	0.0914	0.0773
<i>pq</i>	0.8871	0.7868	0.6989	0.6223	0.5558
<i>pqbar</i>	0.7669	0.5936	0.4666	0.3770	0.3155

Table C.31 – Variance decomposition (in percent) for an elasticity of substitution of  $\epsilon = 1.85$  with an emission cap

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{tao}$	$bigGbar$	$bigG$
<i>epsa</i>	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>dy</i>	48.10	0.12	0.01	4.46	5.70	41.22	0.40	0.00	0.00	0.00
<i>dybar</i>	74.58	0.18	0.02	0.07	1.51	23.64	0.00	0.00	0.00	0.00
<i>dc</i>	57.62	0.14	0.01	30.28	8.18	2.87	0.91	0.00	0.00	0.00
<i>dcbar</i>	86.27	0.21	0.02	0.10	2.31	11.08	0.00	0.00	0.00	0.00
<i>di</i>	68.17	0.17	0.02	3.13	25.82	2.22	0.48	0.00	0.00	0.00
<i>dibar</i>	78.32	0.19	0.02	0.01	15.32	6.14	0.00	0.00	0.00	0.00
<i>y</i>	92.47	0.23	0.02	1.05	3.37	2.63	0.23	0.00	0.00	0.00
<i>ybar</i>	96.16	0.24	0.02	0.00	2.34	1.24	0.00	0.00	0.00	0.00
<i>ygap</i>	67.46	0.17	0.02	13.07	7.68	8.72	2.89	0.00	0.00	0.00
<i>c</i>	79.87	0.20	0.02	5.66	11.88	1.98	0.40	0.00	0.00	0.00
<i>cbar</i>	86.11	0.21	0.02	0.09	8.07	5.51	0.00	0.00	0.00	0.00
<i>i</i>	83.67	0.21	0.02	0.35	14.20	1.38	0.16	0.00	0.00	0.00
<i>ibar</i>	82.26	0.20	0.02	0.01	15.30	2.21	0.00	0.00	0.00	0.00
<i>R</i>	56.75	0.14	0.01	24.29	15.26	2.49	1.04	0.00	0.00	0.00
<i>Rbar</i>	74.49	0.18	0.02	10.72	4.79	9.80	0.00	0.00	0.00	0.00
<i>dM</i>	18.97	0.05	0.00	77.23	1.96	1.14	0.65	0.00	0.00	0.00
<i>dMbar</i>	86.49	0.21	0.02	0.54	0.60	12.14	0.00	0.00	0.00	0.00
<i>r</i>	61.90	0.15	0.02	21.90	10.27	4.00	1.77	0.00	0.00	0.00
<i>rbar</i>	85.80	0.21	0.02	1.14	0.32	12.50	0.00	0.00	0.00	0.00
<i>pi</i>	66.60	0.16	0.02	18.50	8.56	3.65	2.51	0.00	0.00	0.00
<i>pibar</i>	84.29	0.21	0.02	3.00	0.65	11.83	0.00	0.00	0.00	0.00

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Table C.31 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$mc$	82.73	0.20	0.02	7.93	1.75	6.15	1.22	0.00	0.00	0.00
$nbar$	77.97	1.79	0.03	0.03	7.28	12.90	0.00	0.00	0.00	0.00
$n$	74.00	1.75	0.03	2.17	12.40	9.28	0.36	0.00	0.00	0.00
$np$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$npbar$	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$nnp$	48.44	1.14	34.56	1.42	8.12	6.08	0.24	0.00	0.00	0.00
$nnpbar$	52.01	1.19	33.32	0.02	4.85	8.60	0.00	0.00	0.00	0.00
$m$	53.68	0.13	0.01	28.87	14.11	2.27	0.92	0.00	0.00	0.00
$mbar$	82.74	0.20	0.02	0.73	5.60	10.71	0.00	0.00	0.00	0.00
$h$	49.81	0.12	0.01	33.12	13.67	2.28	0.99	0.00	0.00	0.00
$hbar$	82.89	0.20	0.02	0.78	5.04	11.06	0.00	0.00	0.00	0.00
$ks$	84.49	0.21	0.02	3.19	7.70	3.77	0.61	0.00	0.00	0.00
$ksbar$	85.55	0.21	0.02	0.00	12.64	1.57	0.00	0.00	0.00	0.00
$k$	87.77	0.22	0.02	0.06	10.86	0.98	0.08	0.00	0.00	0.00
$kbar$	86.29	0.21	0.02	0.00	12.02	1.45	0.00	0.00	0.00	0.00
$z$	75.73	0.19	0.02	7.09	7.62	8.21	1.15	0.00	0.00	0.00
$zbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$q$	52.53	0.13	0.01	17.13	26.76	2.57	0.86	0.00	0.00	0.00
$qbar$	80.67	0.20	0.02	0.03	9.19	9.89	0.00	0.00	0.00	0.00
$rk$	75.73	0.19	0.02	7.09	7.62	8.21	1.15	0.00	0.00	0.00
$rkbar$	87.69	0.22	0.02	0.01	9.69	2.38	0.00	0.00	0.00	0.00
$w$	76.67	0.38	0.02	11.34	2.34	7.54	1.70	0.00	0.00	0.00
$wbar$	97.49	0.06	0.03	0.00	1.81	0.61	0.00	0.00	0.00	0.00

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Table C.31 – (continued)

	$e_a$	$e_{ap}$	$e_{anp}$	$e_b$	$e_i$	$e_g$	$e_P$	$e_{\text{tao}}$	$\text{bigGbar}$	$\text{bigG}$
$pe$	74.62	0.19	1.19	8.94	4.29	9.29	1.50	0.00	0.00	0.00
$pebar$	93.82	0.19	2.89	0.00	2.14	0.96	0.00	0.00	0.00	0.00
$e$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$ebar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$ep$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$epbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$enp$	2.33	0.05	96.86	0.07	0.39	0.29	0.01	0.00	0.00	0.00
$enpbar$	2.59	0.06	96.67	0.00	0.24	0.43	0.00	0.00	0.00	0.00
$pp$	16.28	78.85	0.00	2.41	0.50	1.60	0.36	0.00	0.00	0.00
$ppbar$	16.27	83.32	0.01	0.00	0.30	0.10	0.00	0.00	0.00	0.00
$pp2$	11.44	55.42	0.00	1.69	0.35	1.13	0.25	29.72	0.00	0.00
$pnp$	64.93	0.17	14.23	8.11	3.39	7.83	1.34	0.00	0.00	0.00
$pnpbar$	69.07	0.07	29.12	0.00	1.37	0.36	0.00	0.00	0.00	0.00
$ns$	74.89	0.59	0.02	2.20	12.55	9.39	0.37	0.00	0.00	0.00
$nsbar$	78.97	0.54	0.02	0.03	7.37	13.06	0.00	0.00	0.00	0.00
$hc$	51.80	0.13	0.01	30.89	13.93	2.28	0.95	0.00	0.00	0.00
$hcbbar$	82.80	0.20	0.02	0.75	5.33	10.89	0.00	0.00	0.00	0.00
$dG$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
$dGbar$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
$pq$	4.78	93.45	0.15	0.25	0.63	0.70	0.05	0.00	0.00	0.00
$pqbar$	34.57	56.62	0.12	2.87	1.60	3.73	0.48	0.00	0.00	0.00





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Résumé :

Reconnue par les Nations-Unies comme l'un des huit objectifs du millénaire, la préservation de l'environnement, et par conséquent du climat, nécessite une intervention forte des pouvoirs publics afin d'endiguer les conséquences négatives liées au dérèglement climatique. La présente thèse se propose d'étudier quelques aspects des politiques climatiques et des conséquences qu'elles engendrent sur le cycle économique. Nous nous intéresserons tout d'abord à la question de la réversibilité du climat. L'une des particularités des gaz à effets de serre est leur durée de vie particulièrement longue. Si à l'échelle géologique, les puits de carbone pourraient absorber une majeure partie des émissions d'origine anthropogénique, à l'échelle humaine le réchauffement climatique apparaît comme irréversible sans une transformation de l'économie mondiale vers un modèle décarboné. En l'absence de la découverte d'une technologie capable de réduire drastiquement l'intensité carbone de l'énergie, le recours aux politiques publiques est la solution la plus à même de respecter la limite des 2°C d'ici la fin du 21<sup>ème</sup> siècle. Dans un deuxième temps, nous comparerons les conséquences de l'instauration de différents instruments de politique climatique (taxe carbone, marché des quotas, cible d'intensité) dans une économie qui est sujette à différents chocs exogènes (choc de productivité, choc sur le prix des énergies fossiles). A cette fin, nous utilisons un modèle RBC DSGE pour évaluer les conséquences économiques de ces différentes mesures. La mise en place d'un de ces instruments peut en effet avoir un effet récessif, ou au contraire être inefficace du point de vue du climat en période d'expansion. Nous recommandons ainsi l'utilisation jointe d'un marché des quotas, avec la mise en place d'un prix plancher du carbone afin d'atteindre l'objectif de réduction des émissions. Finalement, nous nous intéressons aux liens entre politique climatique et politique monétaire. Dans ce but, nous optimisons la règle de Taylor en présence soit d'un marché des quotas, ou d'une taxe carbone afin de faire des recommandations de politique monétaire. Nous montrons que les coefficients de la règle de Taylor qui maximisent le bien-être du consommateur sont sensiblement différents avec l'instauration d'une politique climatique. Il conviendrait ainsi de tenir compte, de ces effets pour conduire une politique environnementale la plus efficace possible.

Mots clés : Changement climatique, macroéconomie environnementale, politique environnementale, cycle économique, dynamique macroéconomique, politique monétaire

Abstract :

Recognised by the United Nations as one of the eight Millennium Development Goals, the preservation of the environment and consequently of the climate, requires strong intervention by public authorities in order to limit the negative consequences of climate change. This thesis proposes to study some aspects of climate policies and the consequences they have on the economic cycle. We first address the issue of climate reversibility. One of the particularities of greenhouse gases is their particularly long lifetime. While on a geological scale, carbon sinks could in theory absorb a major part of anthropogenic emissions, on a human scale global warming appears irreversible without a transformation of the world economy towards a decarbonised model. In the absence of the discovery of a technology capable of reducing the carbon intensity of energy, public policy seems to be the solution most likely to meet the 2°C limit by the end of the 21st century. Secondly, we compare the consequences of introducing different climate policy instruments (carbon tax, allowances market, intensity target) in an economy that is subject to different exogenous shocks (productivity shock, shock on the price of fossil fuels). To this end, we use an RBC DSGE model to assess the economic consequences of these different measures. The implementation of one of these instruments can indeed have a contractionary effect, or on the contrary be inefficient from a climate point of view in periods of expansion. We therefore recommend the joint use of an allowance market, with the establishment of a floor price for carbon in order to achieve the emission reduction target. Finally, we look at the links between climate policy and monetary policy. We optimise the Taylor rule in the presence of either an allowance market or a carbon tax in order to make monetary policy recommendations. We show that the Taylor rule coefficients that maximise consumer welfare are significantly different with the introduction of climate policy. These effects should therefore be taken into account in order to conduct the most effective environmental policy.

Keywords: Climate change, Environmental Macroeconomics, Environmental Policy, Business Cycle, New Keynesian Model, Macroeconomic Dynamics, Monetary Policy.

Classification: E32, E50, H2, O44, Q43, Q50, Q52, Q58.