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Condition-based maintenance policies for multi-component systems considering stochastic dependences

Heping Li

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Heping LI

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Condition-based Maintenance Policies for Multi-component Systems Considering Stochastic Dependences

JURY

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Abstract

Nowadays, industrial systems contain numerous components so that they become more and more complex regarding the logical structures as well as the various dependences (economic, stochastic and structural dependences) between components. The dependences between components have an impact on the maintenance optimization as well as the reliability analysis. Condition-based maintenance which enables to manage maintenance activities based on information collected through monitoring has gained a lot of attention over recent years but stochastic dependences are rarely used in the decision making process. Therefore, this thesis is devoted to propose condition-based maintenance policies which take advantage of both economic and stochastic dependences for multi-component systems. In terms of economic dependence, the proposed maintenance policies are designed to be maximally effective in providing opportunities for maintenance grouping. A decision rule is established to permit the maintenance grouping with different inspection periods. Stochastic dependence due to a common degradation part is modelled by Lévy and Nested Lévy copulas. Condition-based maintenance policies with non-periodic inspection scheme are proposed to make use of stochastic dependence. Our studies show the necessity of taking account of both economic and stochastic dependences in the maintenance decisions. Numerical experiments confirm the advantages of our maintenance policies when compared with other existing policies in the literature.

Keywords:

- monte-Carlo method
- condition-based maintenance
- mathematical models
- dependence (statistic)
- decision making
- reliability (engineering)
- stochastic process

Résumé

De nos jours, les systèmes industriels sont de plus en plus complexes tant du point de vue de leur structure logique que des diverses dépendances (dépendances économique, stochastiques et structurelles) entre leurs composants qui peuvent influencer l'optimisation de la maintenance. La Maintenance conditionnelle qui permet de gérer les activités de maintenance en fonction de l'information de surveillance a fait l'objet de beaucoup d'attention au cours des dernières années, mais les dépendances stochastiques sont rarement utilisées dans le processus de prise de décision. Par conséquent, cette thèse a pour objectif de proposer des politiques de maintenance conditionnelle tenant compte des dépendances économiques et stochastiques pour les systèmes multi-composant. En termes de dépendance économique, les politiques proposées sont conçues pour permettre de favoriser les opportunités de grouper des actions de maintenance. Une règle de décision est établie qui permet le groupement de maintenances avec des périodes d'inspection différentes. La dépendance stochastique causée par une part de dégradation commune est modélisée par copules de Lévy. Des politiques de maintenance conditionnelle sont proposées pour profiter de la dépendance stochastique. Nos travaux montrent la nécessité de tenir compte des dépendances économiques et stochastiques pour la prise de décision de maintenance. Les résultats numériques confirment l'avantage de nos politiques par rapport à d'autres politiques existant dans la littérature.

Mots-clés :

- méthode Monte-Carlo
- maintenance conditionnelle
- modèles mathématiques
- dépendance (statistique)
- prise de décision
- fiabilité
- processus stochastique

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Chapter 1

General Introduction

Maintenance activities play an important role in industry as they can prevent sudden failures to guarantee the productivity and quality, to avoid severe damages and losses, to improve the reliability and safety of systems. Maintenance can be defined as the combination of all technical and associated administrative actions intended to retain an item or system in, or restore it to, a state in which it can perform its required function. Maintenance is considered an essential element to pursue economic competition between companies as it ensures the productivity of assets, the quality of product so as to accomplish mission in a short-time [1]. According to Robertson and Jones [2], maintenance budget varies from low in light manufacturing to high in equipment intensive industry with 2% to 90% of the total plant operating budget, the average being 20.8%. In order to reduce the maintenance cost, the maintenance optimization which decides when and how to repair components is widely studied since the early sixties [3, 4, 5, 6, 7, 8]. Various maintenance policies have been proposed in the literature to address the maintenance optimization problem which can be divided into two main categories: time-based maintenance policies and condition-based maintenance policies. Condition-based maintenance has gained a lot of attention over a few decades as it recommends maintenance actions based on the information collected through condition monitoring [9]. When compared to time-based maintenance, condition-based attempts to avoid unnecessary maintenance activities by carrying out maintenance actions only when there is evidence of severe degradation so that the maintenance cost is reduced. In recent years, with rapid developments of science and technology, industrial systems become more efficient but also more complex. Indeed, the increasing number of components in a system leads to not only the complicated system structure, but also various interactions between components which can be classified into three categories: economic dependence, stochastic dependence and structural dependence [10]. Interactions

between components increase the difficulty of maintenance optimization but provide opportunities at the same time. On one hand, economic dependence between components provide opportunities to group maintenance activities so that the maintenance cost is reduced. Many researches in the literature are devoted to propose grouping strategies for multi-component systems. Nevertheless, most of them proposed grouping strategies basing on the time-based maintenance. Grouping strategy for condition-based maintenance is less explored as well as the inspection strategies for multi-component systems. In fact, when the inspection cost is not negligible, it is more economic to apply an adaptive inspection period for each component in the system. However, the diversity of inspection periods of components increases the difficulty of maintenance grouping as the maintenance action can only be made at inspection. Therefore, the first objective of this these is to propose grouping strategy for condition-based maintenance as well as the inspection strategy. On the other hand, stochastic dependence which implies that the behavior of one component may influence other components in the system have a significant impact on the reliability analysis. Most literatures concerning the stochastic dependence model the stochastic dependence as failure interactions or degradation interactions whereas the stochastic dependence caused by common degradation part is less investigated. Moreover, few literatures take account stochastic dependence in their maintenance modeling but it is verified that stochastic dependence have an impact on the maintenance optimization. As a consequence, the second objective of this thesis is to model the stochastic dependence caused by common degradation part and then take full advantage of stochastic dependence for maintenance optimization.

Generally, the maintenance optimization can be divided into two parts: degradation modeling and maintenance modeling. The [chapter 2](#) and [chapter 3](#) are devoted to present and discuss the existing models concerning two parts respectively to give a framework for our study and clarify our problem. In [chapter 2](#) which review the degradation modeling for both single-unit and multi-component systems, we focus on the gradual degradation modeling with continuous state space modeled by stochastic processes and the stochastic dependence modeling. In [chapter 3](#), the existing maintenance policies in the literature are introduced and discussed which concern the effect of maintenance actions, maintenance policies for single-unit systems and maintenance policies for multi-component systems.

[chapter 4](#) and [chapter 5](#) are dedicated to model the gradual degradation with stochastic dependence and propose maintenance policies in order to take account of both economic and stochastic dependence. In [chapter 4](#), with respect to adaptive inspection periods for heterogeneous components in the system, grouping strategy for condition-based maintenance in order to take advantage of economic dependence is proposed. The stochastic dependence modeled by Lévy copulas is also investigated. The study of [chapter 4](#) can be seen as a first essay to consider stochastic and economic dependences simultaneously

which is rarely investigated in the literature with a two-component system. [chapter 5](#) is devoted to further investigate the stochastic dependence with asymmetric dependence and the stochastic dependence is fully explored by being integrated in the proposed inspection and replacement strategies. To end this thesis, conclusions and perspectives are drawn in [chapter 6](#).

Chapter 2

State of the art part I: Degradation modeling

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2.1 Introduction

In order to analyse the reliability of systems, predict the remain useful life or minimize the maintenance cost, the first step is to model the physical properties of components or systems such as how they degrade and when they fail. That is modeling the degradation behavior of a component or a system through a mathematical modeling. In this chapter, we review and analyse the existing degradation models for systems in the literature (either single-unit systems or multi-component systems).

The remainder of this chapter is organised as follows. Section 2.2 presents the degradation modeling for a single component which can be divided into two categories: lifetime distribution based models and gradual degradation based models. When considering failures of multi-component systems, stochastic dependences as well as the structure of the system should be take into account. Therefore section 2.3 describes the stochastic dependence modeling between components and section 2.4 is devoted to the structures of multi-component systems. The conclusions are drawn in section 2.5.

2.2 Individual degradation modeling

Before the component is put in service, it is important to measure its degradation speed of component and predict its failure date because a sudden failure of component may cause unpredictable losses. Before introducing the degradation modeling, some concepts are reviewed as follows if T stands for the lifetime.

Definition 2.1. Lifetime distribution function is the probability that a component fails up to and including t time units. If T is a continuous random variable, it can be expressed as:

$$F(t) = P(T \leq t) = \int_0^t f(x)dx, \quad (2.1)$$

where $f(x)$ is the probability density function of T and t is the length of the period of time (which is assumed to start from time zero).

Definition 2.2. Reliability is the probability that a component performs well during t time units as:

$$R(t) = P(T > t) = 1 - F(t) = \int_t^{\infty} f(x)dx, \quad (2.2)$$

Definition 2.3. The failure rate function is defined as:

$$\lambda(t) = f(t)/R(t), \quad (2.3)$$

provided that function $F(\cdot)$ is differentiable.

2.2.1 Lifetime distribution models

In lifetime distribution models, components have only two states (either functioning or failed) for the cases where the intermediate state of components is unobservable or hard to be measured. The exponential families and proportional hazard function are introduced where the former are widely used in the literature as they are able to derive explicit results and the latter is efficient in taking into account effects of covariables on the failure of component.

2.2.1.1 Exponential families

Exponential distribution, Weibull distribution and Gamma distribution are introduced in this section.

Exponential distribution

The component whose lifetime distribution follows an exponential distribution has following properties:

- Probability density function:

$$f(t; \lambda) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- Lifetime distribution function:

$$P(T \leq t) = F(t; \lambda) = \begin{cases} 1 - e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- Failure rate function:

$$\lambda(t) = f(t)/(1 - F(t)) = \lambda, t \geq 0 \quad (2.4)$$

- Mean and variance: $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$
- Remaining useful lifetime distribution:

$$P(T > y | T > x) = e^{-\lambda(y-x)}, y > x \quad (2.5)$$

The exponential distribution is a basic lifetime distribution whose failure rate is constant such that is an excellent model for the long flat "intrinsic failure" portion of the Bathtub

Curve. Furthermore, it works well for inter arrival times while the Poisson distribution describes the total number of events in a given period. Therefore, when the failures arrives which follows standard Poisson process, the exponential life distribution model will naturally apply. Nevertheless, the exponential distribution has only one parameter which can not fit all types of lifetime data such that distributions like Gamma distribution and Weibull distribution with two parameters (shape and scale) are more flexible in applications.

Gamma distribution

The component whose lifetime distribution follows a Gamma distribution has following properties:

- Probability density function:

$$f(t; \alpha, \beta) = \begin{cases} t^{\alpha-1} \frac{\beta^\alpha e^{-\beta t}}{\Gamma(\alpha)} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$ is the Gamma function.

- Lifetime distribution function:

$$P(T \leq t) = F(t; \alpha, \beta) = \begin{cases} \int_0^t x^{\alpha-1} \frac{\beta^\alpha e^{-\beta x}}{\Gamma(\alpha)} dx & t \geq 0 \\ 0 & t < 0 \end{cases}$$

when α is an integer, $F(t; \alpha, \beta)$ is called an Erlang distribution and its form can be obtained as below:

$$P(T \leq t) = F(t; \alpha, \beta) = \begin{cases} 1 - \sum_{k=1}^{\alpha-1} \frac{(\beta t)^k e^{-\beta t}}{k} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

whereas for an arbitrary α , $F(t; \alpha, \beta)$ has not closed form.

- Failure rate function:

$$\lambda(t) = f(t)/(1 - F(t)) = \left[\int_0^\infty \left(1 + \frac{x}{t}\right)^{\alpha-1} e^{-\beta x} dx \right]^{-1}, t \geq 0 \quad (2.6)$$

- Mean and variance: $\frac{\alpha}{\beta}$ and $\frac{\alpha}{\beta^2}$

Therefore, Gamma distribution generalizes the exponential model and is more flexible when compared to the exponential distribution in terms of the failure rate function. When $\alpha < 1$ the failure rate is decreasing with time; when $\alpha = 1$, the Gamma distribution reduces to the exponential distribution and the failure rate is constant; when $\alpha > 1$,

the failure rate is increasing. For any integer α , the sum of α independent and identically distributed exponential random variables with parameter β is Gamma distributed with parameters (α, β) . Moreover, the sum of independent and identically distributed Gamma random variables has a Gamma distribution so that it has a nice physical interpretation. The Gamma distribution has found a number of applications in different fields. For example, in [11], the Gamma distribution is used to assess and predict fatigue reliability of steel bridges based on long-term monitoring data. But the major disadvantage of the Gamma distribution is that the distribution function or survival function cannot be expressed in a closed form if the shape parameter is not an integer whereas one needs to obtain the distribution function, survival function or the failure rate by numerical integration. This makes Gamma distribution less popular compared to the Weibull distribution.

Weibull distribution

The lifetime (T) of a component follows the Weibull distribution with shape α and scale parameter β if its probability density function is given as follows:

$$f(t; \alpha, \beta) = \begin{cases} \alpha\beta(\alpha t)^{\beta-1}e^{-(\alpha t)^\beta} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Therefore the lifetime distribution function is:

$$P(T \leq t) = F(t; \alpha, \beta) = \begin{cases} 1 - e^{-(\alpha t)^\beta} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Then the failure rate function is obtained as follows:

$$\lambda(t) = f(t)/(1 - F(t)) = \alpha\beta(\alpha t)^{\beta-1}, t \geq 0 \quad (2.7)$$

and the remaining useful lifetime distribution is:

$$P(T > y|T > x) = R(y)/R(x) = \exp((\alpha x)^\beta - (\alpha y)^\beta), y > x \quad (2.8)$$

The Weibull distribution is the most used lifetime distribution in literature as it is powerful and flexible in modeling the ageing of various components according to [12] in many applications such as fatigue failure, vacuum tube failure and ball-bearing failure. Moreover, with Weibull distribution, most indicators are known in closed form such that it allows the mathematical developments. The impact of the shape parameter on the failure rate, lifetime, age-based maintenance policies and residual life is studied in [13]. For effect of scale parameter, when $\beta = 1$, the Weibull distribution reduces to

the exponential distribution where the failure rate is constant; when $\beta > 1$, the failure rate is increasing which means that the degradation speed is accelerated with increasing operating time of components; on the contrast, when $\beta < 1$, the failure rate is decreasing which means that the degradation speed is decelerated. A vast maintenance applications have been made based on the Weibull distribution. In [14], the bridge maintenance and replacement model is addressed with renewal theory and Weibull distribution is used to model the degradation process of the bridge deck in [15]. In [16], grouping maintenance strategies taking into account the economic dependence are proposed for multi-component systems.

2.2.1.2 Proportional hazards model

Components degrades along with usage time but it also can be affected by other covariables. In order to take covariables into consideration, the proportional hazards model is proposed which assumes the hazard function can be written as follows:

$$\lambda(t; z) = g(z)\lambda_0(t) \quad (2.9)$$

where z is a vector of explanatory variables which are considered to influence the lifetime parameters and $\lambda_0(t)$ is the baseline hazard function for a nominal set z_0 where all the explanatory variables take the value of zero. The proportional hazards model was first proposed by Cox in [17] to incorporate the effects of covariates such as the operating environment, the maintenance effect which might have impact on the failure rate. The advantage of proportional model is that, without making any specific assumptions about the form of the baseline hazard function, it is able to analyze experimental data, compute maximum likelihood estimates and use likelihood ratio tests to determine which explanatory variables are highly significant. The Weibull hazard function is mostly used as a baseline hazard function see [9] which is defined as follows:

$$\lambda_0(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \quad (2.10)$$

and a common useful form for $g(z)$ is log linear model: $g(z) = \exp(\alpha z)$ where α is a vector of parameters which defines the effects of the associated covariate.

The proportional hazards model is thereby powerful for modeling lifetime distribution to take not only the age but also covariates which may influence the lifetime distribution into consideration [18], for example, the state of components [19, 20], the effect of maintenance [21, 22].

2.2.2 Monotone gradual degradation models

Gradual degradation models are based on the measurement of intermediate states between perfect functioning and total failure. According to the number of states, the gradual degradation models can be further classified into discrete space degradation models which can be modeled by Markov or semi-Markov process [23, 24, 25, 26] and continuous space degradation models. In this document, we focus on the modeling of continuous space gradual degradation. In effect, continuous space gradual degradation models can be further divided into monotone degradation models and non-monotone degradation models. In this section, two monotone degradation models are introduced.

2.2.2.1 Gamma process

Definition 2.4. A stochastic process $(X_t)_{t \geq 0}$ is called Gamma process if it has the following properties:

- $X_0 = 0$
- $(X_t)_{t \geq 0}$ has independent increments
- For $t > 0$ and $h > 0$, $X_{t+h} - X_h$ follows a Gamma distribution with shape parameter $a(t+h) - a(h)$ and scale parameter β and the probability density function of $(X_{t+h} - X_h)$ is given by:

$$f_{a(t+h)-a(h),\beta}(x) = \frac{\beta^{a(t+h)-a(h)} x^{a(t+h)-a(h)-1} \exp(-\beta x)}{\Gamma(a(t+h) - a(h))}, \quad (2.11)$$

where $\Gamma(\cdot)$ is Gamma function and

$$\Gamma(a(t+h) - a(h)) = \int_0^\infty u^{a(t+h)-a(h)-1} e^{-u} du, \alpha > 0 \quad (2.12)$$

and $a(t)$ is be an increasing, right-continuous real-valued function of time t , with $a(0) \equiv 0$. When $a(t)$ is a linear function of time then the Gamma process is stationary otherwise it is non-stationary.

Therefore, some interesting properties of homogenous Gamma process are noted as follows:

- If component fails when the degradation modeled by a Gamma process with parameter $(a(t), \beta)$ exceeds a preset threshold L , then the reliability of component

at t is

$$P(T > t) = P(X_t < L) = \int_0^L f_{a(t),\beta}(u) du. \quad (2.13)$$

- If degradation level of component at s is known as x_s then

$$P(T > t | X_s = x_s) = P(X_t < L | X_s = x_s) = \int_0^{L-x_s} f_{a(t)-a(s),\beta}(u) du. \quad (2.14)$$

- $E(X_t) = \frac{a(t)}{\beta}$ and $var(X_t) = \frac{a(t)}{\beta^2} t$. For stationary Gamma process, the mean and variance are all linear functions which means that homogeneous Gamma process can model degradation with linear tendency over time.
- Gamma process is a pure jump Lévy process and the Lévy measure of stationary Gamma process is $\nu(dx) = \frac{\alpha e^{-\beta x}}{x} dx$.

Note that the Gamma process is time-homogeneous with independent and positive increments so it is sensible to use this process to describe a gradual deterioration especially the gradual damage monotonically accumulating over time in a sequence of tiny increments, such as wear, fatigue, corrosion, crack growth, erosion, consumption, creep, swell, degrading health index, etc [27]. Another advantage of Gamma process is that it has an explicit probability distribution function which permits both the mathematical developments and simulation. It has been extensively used to model single-unit systems as well as variants such as non-linear shape function and/or parameters depending on covariates (see [28, 29, 30, 31, 32, 33, 34, 35, 36, 37] for example).

2.2.2.2 Inverse Gaussian process

Definition 2.5. A stochastic process $(X_t)_{t \geq 0}$ is called Inverse Gaussian (IG) process if it has the following properties:

- $X_0 = 0$
- $(X_t)_{t \geq 0}$ has independent increments
- For $t > 0$ and $h > 0$, $X_{t+h} - X_h$ follows a IG distribution with shape parameter $\Lambda(t+h) - \Lambda(h)$ and scale parameter $\eta[\Lambda(t+h) - \Lambda(h)]^2$ and the probability density function of IG distribution with parameter (a, b) is given by:

$$f_{IG}(x) = \sqrt{\frac{b}{2\pi x^3}} \exp\left(-\frac{b(x-a)^2}{2a^2 x}\right), x > 0 \quad (2.15)$$

where $\Lambda(t)$ is a monotone increasing function. If $\Lambda(t) = \zeta t$, the IG process is a stationary process.

Therefore, some interesting properties of IG process are noted as follows:

- If component fails when the degradation modeled by an IG process with parameter $(\zeta t, \eta t^2)$ exceeds a preset threshold L , then the reliability of component at t is

$$P(T > t) = P(X_t < L) = \phi\left[\sqrt{\frac{\eta}{L}}\left(\frac{L}{\zeta} - t\right)\right] + \exp\frac{2\eta t}{\zeta}\phi\left[-\sqrt{\frac{\eta}{L}}\left(t + \frac{L}{\zeta}\right)\right]. \quad (2.16)$$

- If degradation level of component at s is known as x_s then

$$\begin{aligned} P(T > t|X_s = x_s) &= P(X_t < L|X_s = x_s) \\ &= \phi\left[\sqrt{\frac{\eta}{L-x_s}}\left(\frac{L-x_s}{\zeta} - t + s\right)\right] + \exp\frac{2\eta t}{\zeta}\phi\left[-\sqrt{\frac{\eta}{L-x_s}}\left(t - s + \frac{L-x_s}{\zeta}\right)\right]. \end{aligned} \quad (2.17)$$

- $E(X_t) = \zeta t$ and $var(X_t) = \frac{\zeta^3}{\eta}t$ if $\Lambda(t) = \zeta t$ such that degradation level of component increases linearly over time.

As a consequence, the IG process can be also used to model the monotone continuous gradual degradation of components like the Gamma process. In effect, the IG distribution is used as a lifetime distribution in [38] as it describes an increasing failure rate at the beginning and then a decreasing failure rate after a certain time. The IG process is used to fit the degradation data in [39] which has shown that Gamma process is not sufficient to well fit all types of data such that the IG process can be an alternative choice for monotone gradual modeling. As well random effects can be investigated with IG process. Considered as the first passage process of Brownian motion, two more random effects models are proposed in [40]. It has been proved in [40] that the IG process can be approximated as compound Poisson process. A general Bayesian method is proposed for degradation analysis with inverse Gaussian process models in [41]. A condition-based maintenance policy is proposed in [42] based on the inverse Gaussian degradation modeling and the remaining useful life estimation is investigated with respect to random effect in [43].

2.2.3 Non-monotone gradual degradation models

While the Gamma process is widely adopted in the literature to model continuous space degradation, however some cases cannot be explained by monotone degradation models due to internal mechanisms such as self-repair ability in short term or external impacts such as measurement errors. Therefore non-monotone degradation models should be proposed in order to depict the non-increasing increment in short term.

2.2.3.1 Drifted Brownian motion

The Brownian motion is also called Wiener process and it is defined as follows:

Definition 2.6. A stochastic process $(B_t)_{t \geq 0}$ is called standard Brownian motion if it has the following properties:

- $B_0 = 0$
- $(B_t)_{t \geq 0}$ has independent and stationary increments
- For $t > 0$ and $h > 0$, $B_{t+h} - B_h$ follows a Normal distribution with zero mean and variance t and the probability density function of Normal distribution with parameter $(0, t)$ is given by:

$$f(x|0, t) = \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{x^2}{2t}\right), x > 0. \quad (2.18)$$

Then a drifted Brownian motion is defined as:

$$X_t = \mu t + \sigma B_t, \mu > 0, \sigma > 0 \quad (2.19)$$

So that $X_{t+h} - X_h$ is normally distributed with mean μt and variance $\sigma^2 t$ for $t > 0$ and $h > 0$. Some interesting properties of drifted Brownian motion are noted as follows:

- If component fails once the degradation modeled by a drifted Brownian motion with parameter (μ, σ) exceeds a preset threshold L , then the reliability of component at t is

$$P(T > t) = P(X_t < L) = \phi\left(\frac{L - \mu t}{\sigma\sqrt{t}}\right) - \exp\left(\frac{2\mu L}{\sigma^2}\right) \phi\left(-\frac{L + \mu t}{\sigma\sqrt{t}}\right). \quad (2.20)$$

In effect, the first passage time of the drifted Brownian motion is distributed according to an Inverse Gaussian distribution with parameter $(\frac{L}{\mu}, \frac{L^2}{\sigma^2})$.

- If degradation level of component at s is known as x_s then the remaining lifetime distribution is

$$\begin{aligned} P(T > t | X_s = x_s) &= P(X_t < L | X_s = x_s) \\ &= \phi\left(\frac{L - x_s - \mu(t-s)}{\sigma\sqrt{t-s}}\right) - \exp\left(\frac{2\mu(L-x_s)}{\sigma^2}\right) \phi\left(-\frac{L - x_s + \mu(t-s)}{\sigma\sqrt{t-s}}\right). \end{aligned} \quad (2.21)$$

- $E(X_t) = \mu t$ and $var(X_t) = \sigma^2 t$ such that degradation level of component increases linearly over time in long term. However, the system modeled by Brownian motion

is not always degrading, but sometimes its degradation level is reduced without maintenance due to self-repair.

Brownian motion is widely used to model the non-monotone degrading system see [44, 45, 46] for example.

2.2.3.2 Variance Gamma process

The symmetric variance Gamma (VG) model was first introduced by Madan and Seneta in [47] to find an alternative to the Brownian motion in the modelling of option pricing. Different from the Brownian motion, the symmetric VG model is able to control the kurtosis of the distribution as well as the long-tailedness. Later, a 3-parameter model which permits to control both skewness and kurtosis is developed in [48]. The 2-parameter symmetric VG and the 3-parameter asymmetric VG option pricing models are empirically tested in [49]. They have shown that the VG models are better than the Black-Scholes approach in approximating the observed market prices. Three methods for sequential simulation and two bridge sampling methods of the VG process are proposed in [49].

The VG Process is a Lévy process with independent stationary increments. A 2-parameter (σ, β) symmetric VG process $(X_t)_{t \geq 0}$ can be obtained from the Brownian motion subjected to a random time change as follows:

$$X_t = W(G_t) \quad (2.22)$$

Where G_t is a Gamma process with parameter $(\frac{t}{\beta}, \frac{1}{\beta})$ and $W(t)$ is an independent Brownian motion of zero drift and variance $\sigma^2 G_t$. For $t > 0$ and $h > 0$, increment $X_{t+h} - X_h$ follows a Normal distribution with zero mean and variance following a Gamma distribution as follows:

$$f(x|\sigma^2, \beta) = \int_0^\infty \exp(-\frac{x^2}{2\sigma^2 v}) g(v|\beta) / (\sigma\sqrt{2\pi v}) dv, \quad (2.23)$$

with

$$g(v|\beta) = \frac{1}{\beta} v^{\frac{t}{\beta}-1} \exp(-\frac{1}{\beta}v) / \Gamma(\frac{t}{\beta}). \quad (2.24)$$

Such that

$$\begin{cases} E[X_t] = 0 \\ E[X_t - E[X_t]]^2 = \sigma^2 t \\ E[X_t - E[X_t]]^3 = 0 \\ E[X_t - E[X_t]]^4 = 3\sigma^4(t^2 + \beta t) \end{cases}$$

The kurtosis ($E[X_t - E[X_t]]^4 / \text{var}(X - t)$) therefore is $3(1 + \beta/t)$ which signifies that long-tailedness presents for short term data whereas long term data tend to be normally distributed.

A 3-parameter (θ, σ, β) VG process can be defined from Brownian motion with drift in Equation 2.22 such as:

$$X_t = \theta G_t + W(G_t) \quad (2.25)$$

Thus

$$\begin{cases} E[X_t] = \theta t \\ E[X_t - E[X_t]]^2 = \sigma^2 t + \theta^2 \beta t \\ E[X_t - E[X_t]]^3 = 2\theta^2 \beta^2 t + 3\sigma^2 \theta \beta t \\ E[X_t - E[X_t]]^4 = 3\sigma^2 \beta t + 12\sigma^2 \theta^2 \beta^2 t + 6\theta^4 \beta^3 t + 3\sigma^4 t^2 + 6\sigma^2 \theta^2 \beta t^2 + 3\theta^4 \beta^2 t^2 \end{cases}$$

The parameter θ is introduced to control the skewness. It is worth mentioning that the kurtosis and skewness of Gamma process are $3(1 + \frac{2}{\alpha t})$ and $\frac{2}{\sqrt{\alpha t}}$ such that the 3-parameter VG process is more flexible in the control of statistical characters.

Two simulation methods are introduced in this section to sample the increments of a VG process according to [50]:

Algorithm 1

- Set $X_0 = 0, i = 1, t_0 = 0$
- For given $\Delta t_i = t_i - t_{i-1}$, generate independent $G_i \sim \Gamma(\Delta t_i / \beta, \beta)$ and $W_i \sim N(0, 1)$
- Return $X_{t_i} = X_{t_{i-1}} + \theta G_i + \sigma \sqrt{G_i} W_i$.

Algorithm 2

- Set $X_0 = 0, i = 1, t_0 = 0$
- Set $u_p = \frac{1}{2} \sqrt{\theta^2 + 2\sigma^2 / \beta} + \frac{\theta}{2}, u_q = \frac{1}{2} \sqrt{\theta^2 + 2\sigma^2 / \beta} - \frac{\theta}{2}$
- For given $\Delta t_i = t_i - t_{i-1}$, generate independent $G_i^1 \sim \Gamma(\Delta t_i / \beta, \beta u_p)$ and $G_i^2 \sim \Gamma(\Delta t_i / \beta, \beta u_q)$
- Return $X_{t_i} = X_{t_{i-1}} + G_i^1 - G_i^2$.

Figure 2.1 illustrates a sampling path of degradation for a component when modeled by a Gamma process and by a VG process with the same mean and variance. The mean degradation of component per time unit is 9 and the variance is 45. Hence parameters of two process are: for the Gamma process, $\alpha = 9/5, \beta = 5$ with $t = 1$; for the VG process, $\theta = 9, \sigma^2 = 9, \beta = 4/9$ with $t = 1$. It can be seen that the VG process is

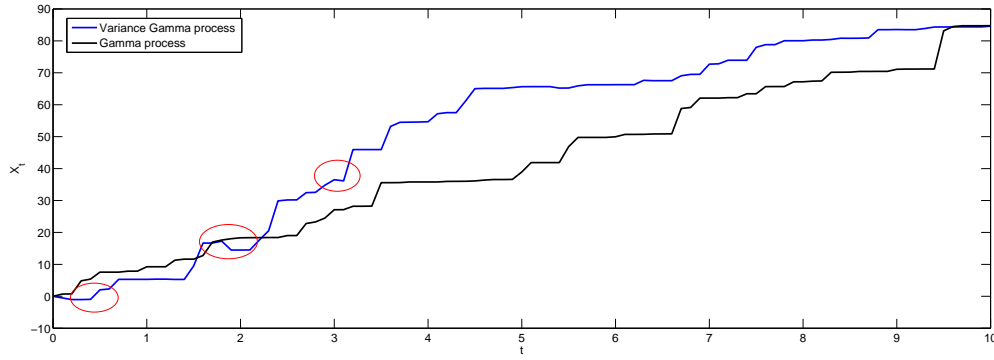


Figure 2.1: A sampling of degradation level of component which is model by the Gamma process and the Variance Gamma process respectively

not a monotone process. In fact, in long term the VG process has increasing trend while in short term the degradation level may decrease. Similar to Brownian motion, it can be used to model non-monotone degrading systems and has better properties than Brownian motion. However, it is much more complex such that it has not yet been used in reliability engineering.

2.2.3.3 Time-Dependent Ornstein-Uhlenbeck Process

Besides the drifted Brownian motion and the VG process, another non-monotone stochastic process can be used for degradation modeling is the time-dependent Ornstein-Uhlenbeck (OU) process. The general OU process is defined as follows which satisfies a stochastic differential equation:

$$dX_t = \theta(\mu - X_t)dt + \sigma dB_t \quad (2.26)$$

where θ , μ , σ are parameters and B_t is a standard Brownian motion. The parameter μ is the mean of the process, θ is the tendency of the process to return to the mean and σ is the degree of volatility around it caused by shocks. In fact, when μ is a constant, the solution of Equation 2.26 is:

$$X_t = \mu(1 - e^{-\theta t}) + x_0 e^{-\theta t} + e^{-\theta t} \int_0^t \sigma e^{\theta s} dB_s, \quad (2.27)$$

Such that:

$$E(X_t) = x_0 e^{-\theta t} + \mu(1 - e^{-\theta t}), \quad (2.28)$$

$$Var(X_t) = \frac{\sigma^2}{2\theta}(1 - e^{-2\theta t}), \quad (2.29)$$

$$cov(X_s, X_t) = \frac{\sigma^2}{2\theta} e^{-\theta(s+t)} (e^{2\theta \min(s,t)} - 1). \quad (2.30)$$

Equation 2.27 shows that no matter what the initial state is, the long-term value of process is a constant (μ) which is noted as the mean-reverting property of the OU process. The general OU process is widely used in the field of such as finance, biology [51, 52, 53] to describe systems which stabilize at their equilibrium point. However, in order to be adapted to the degradation modeling with a degrading trend, a time-dependent OU process is more suitable. By considering the time-dependent parameters in Equation 2.26, it can be given as:

$$dX_t = (a(t)X_t + b(t))dt + \sigma(t)dB_t \quad (2.31)$$

where $a(t), b(t), \sigma(t)$ are smooth enough functions and B_t is a standard Brownian motion. Several notations are introduced below to simplify the subsequent expressions:

$$\begin{aligned} \alpha(t, s) &= - \int_s^t a(u)du \\ \beta(t, s) &= - \int_s^t b(u)e^{\alpha(u,s)} du \\ \gamma(t, s) &= \int_s^t \frac{\sigma^2(u)}{2} e^{2\alpha(u,s)} du \end{aligned}$$

Therefore, the time-dependent OU process has following properties [54]:

- The solution of Equation 2.31 can be explicitly given as:

$$X_t = e^{-\alpha(t,0)}(X_0 - \beta(t,0) + \int_0^t \sigma(s)e^{\alpha(s,0)} dB_s). \quad (2.32)$$

- The mean of the process is given as:

$$E(X_t) = e^{-\alpha(t,0)}(E(X_0) - \beta(t,0)) \quad (2.33)$$

- The covariance and variance of the process are given as:

$$cov(X_t, X_s) = e^{-(\alpha(t,0)+\alpha(s,0))} (var(X_0) + \int_0^{\min(t,s)} \sigma^2(u)e^{2\alpha(u,0)} du) \quad (2.34)$$

$$var(X_t) = e^{-2\alpha(t,0)} (var(X_0) + \int_0^t \sigma^2(u)e^{2\alpha(u,0)} du) \quad (2.35)$$

As a consequence, the time-dependent OU process is able to model the degradation with continuous sample path, non-stationary increments. In effect, the time-dependent OU process is very flexible in controlling the statistical properties such as mean, variance and covariance with Equations 2.33 2.34 2.35. Moreover, the mean-reverting property of the time-dependent OU process is very attractive when compared to drifted Brownian

motion because it tends to return to the mean in long-term which provides a localizing mechanism to control uncertainties in stochastic models such that uncertainties can be adjusted. As a diffusion process, the OU process is investigated for the first passage time problem with fixed boundaries in [55, 56, 57] and random boundaries in [58, 59]. In [60], the residual useful lifetime is estimated when the degradation is modeled by the time-dependent OU process.

2.2.4 Competing-risk failures with damages and shocks

In practice, system performance gradually degrades due to such as wear, fatigue, erosion meanwhile the system may also suddenly fail due to such as hidden manufacturing defects, excessive loads, shocks. For these systems subjected to competing failure process, the failure happens when the degradation reaches a critical threshold or when the shock arrives, whichever happens first. In [61], three failure processes are considered where two degradation processes are modeled by so-called random-coefficient degradation path function and so-called randomized logistic degradation path function and the shock process is modeled by a compound Poisson process. Besides, these three processes are mutually independent. An average long-run maintenance cost rate function is derived based on the expressions for the degradation paths & cumulative shock damage, which are measurable. In [62], a single-unit system subjected to two failure mechanisms is considered. The failure of system happens when the cumulative damages due to degradation exceeds a fixed threshold or when the lethal shock arrives whose failure rate is related to both degradation level and the environment stress, whichever occurs first. The competing risk model is proposed to describe the reliability of the cylinder liners of a marine Diesel engine in [63]. Two dominant failure modes are presented: wear degradation described through a stochastic process and thermal cracking whose failure time is described by the Weibull distribution.

While most of previous literatures treated failure processes as independent, it is more appropriate to consider that they are dependent. In many practical situations, the dependence between them is of importance and should not be neglected. In [64, 65], the system fails when the degradation process reaches a critical threshold or when a shock occurs although the degradation process has not reached the threshold. The degradation is modeled by a Gamma process whereas shocks arrival times can be modeled by a nonhomogeneous Poisson process with stochastic increasing intensity which depends on the degradation level. When the degradation level exceeds a fixed level, the failure rate function due to shocks becomes $r_2(t)$ instead of $r_1(t)$ and $r_2(t) > r_1(t), \forall t > 0$. In [66], the system failure can be classified into K mutually exclusive failure modes where

each failure mode depends on both time t and the system degradation level. The degradation of system is modeled by a stochastic process which satisfies an Ito differential equation. In [67], a system subjected to competing risks due to two degradation processes and random shocks is considered. Each shock causes a sudden increment jump to both degradation processes simultaneously such that the two degradation processes are dependent. The system fails if the cumulative deterioration of any degradation process exceeds a certain critical failure threshold. In [19], the degradation process is described by a random-coefficient autoregressive model with time effect such that the system age, previous state observations, and the item-to-item variability of the degradation are jointly combined. The failure rate corresponding to the hard failure is characterized a proportional hazards function where both system age and the degradation level are involved. In [68], a s-dependent competing failures model is developed and both shock and wear indicators are involved. The system is subjected to M degradation processes and a shock process where the arrival of random shocks follows a homogeneous Poisson process. The shock could be fatal to cause the failure of system at time point t with probability $p(t)$ and be nonfatal with a probability $1 - p(t)$ however the nonfatal shock brings both a sudden increment jump and degradation rate acceleration to the degradation process. The system fails when one of the degradation exceeds a fixed threshold or a fatal shock arrives, whichever occurs first.

2.3 Stochastic dependence modeling

When dealing with the multi-component systems, it is more realistic to consider that components within system are dependent. According to Thomas [10], the dependences or interactions between components can be classified into three categories: economic dependence, stochastic dependence and structural dependence which are related to three aspects: maintenance cost, failure behavior and structure of system. In this chapter which deals with the failure behavior modeling, the stochastic dependence which means that the state or failures of components can influence the other components where the state can be referred to age, failure rate, failure or other measure is discussed. In addition to failure interactions which are mostly used, the construction of multi-dimensional stochastic processes with stochastic dependence is also discussed to be adaptive with the gradual degradation modeling.

2.3.1 Failure interactions models

Murthy and Ngugen in [69] define three types of failure interactions in a two-component system as follows:

- Type I: the failure of a component can induce the failure of the other component with probability p .
- Type II: the failure of component 1 can induce the failure of component 2 with probability p whereas the failure of component 2 acts as a shock to component 1 instead of immediate failure.
- Type III: the failure of a component can act as a shock to the other component and influences the failure rate of that one.

Nicolai and Dekker puts the types II and III failure interactions together with one definition that the failure of a component affects either the failure rate or causes a random amount of damage to the state of one or more of the remaining components [8]. In this work, the definition of [8] is adopted to review the existing results.

2.3.1.1 Type I failure interactions

The type I failure interactions was first studied in [69] for a two-unit system where the failure of component induces the failure of other component with a probability p and has no influence on the other component with probability $1 - p$. Murphy and Ngugen in [70] extended this model for multi-component systems where the failure of a component induces the failure of the whole system with probability p and does not induce other components to fail with probability $1 - p$ and proposed two maintenance policies. To deal with a two-unit system with both economic dependence and type I failure interactions, failure-based, aged-based as well as opportunistic maintenance policies to take advantage of economic dependence is proposed in [71]. Later, based on the same failure interaction model, block-replacement policies are proposed in [72]. Sharp and sheu in [73] derived the long-run expected cost for age and block replacement models for the multi-component systems with failure interactions. Liu et al in [74] extended the failure interactions between components in a more general way where the failure of component i can induce the failure of component j with probability p_{ij} and a warranty cost model is derived. All the above mentioned literatures modeled the failure of component with a life time distribution.

2.3.1.2 Type II failure interactions

The type II failure interactions was first discussed in [69] for a two-unit system where the failure of component does not induce the immediate failure of the other component but it affects the failure rate of the other component. In [75], the failure of component 1

is subject to a Poisson process and each failure of component 1 causes a random damage to component 2. Component 2 fails when its cumulative damage exceeds a threshold K . A two parameter maintenance policy is proposed where the system is preventively replaced either system ages to T or cumulative damage of component 2 exceeds K and the expected long-run maintenance cost is thereby derived. In [76], a two-component parallel system is studied and the failure of one component can modify the failure probability of the other component with probability p and does not interact with probability $1 - p$. In [77, 78], failures of component 1 occur according to a non-homogeneous Poisson process with increasing failure rate whereas the failure of component 2 occur according to a homogeneous Poisson process (HPP) with constant failure and increases the failure rate of component 1 by p percent.

2.3.2 Degradation interactions

While failure interactions models depict the dependence between components when a failure occurs, the state of the component might also influence degradation processes of other components in the system even when it does not fail. Such type of stochastic dependence can be defined as degradation interactions. Degradation interactions exist in many mechanical systems. For example, in wind turbines, the degradation of hydrodynamic bearings may increase the looseness of primary transmission shafts such that the vibration levels in the gearbox increase [79]. Several papers investigate degradation interactions for multi-component systems. Degradation rate interactions model is proposed in [79] and a system with n dependent components is considered. Components in the system continuously degrade but their states can be divided into discrete degradation states according to their degradation signals. When component i transitions to a more severe degradation state, it increases the degradation rate of the other components by an amount δ_{ij} ($\forall j \neq i$). Therefore, the overall degradation rate of component in the system consists of two parts: its inherent degradation rate and the increments of degradation rate due to the degradation state transitions of the other components. As a result, stochastic modeling of degradation signals of component is established by taking the degradation rate interactions into account. In [80], the degradation state of component is modeled by the general degradation path model where the degradation state of component at time nT $S(nT)$ can be expressed by $S(nT) = S(n(T-1)) + \Delta_n$ where Δ_n is the changes between two inspection intervals which is characterised by a probability distribution constructed with the historical data. The component interaction model is proposed by discussing the influence of degradation state of a component on the state or the changes of another component. For example, if the state of component 2 affect the state of component 1, then $S^1(nT) = S^1(n(T-1)) + \Delta_n^1 + g(S^2(nT))$ where S^i is the

state of component i ($i = 1, 2$) and g can be estimated by using regression technique. An industrial case study is provided and it shows that degradation interactions can affect maintenance of the system. This model is further developed in [81].

2.3.3 Multivariate models with common degradation parts

While failure and degradation interactions describe the interactions between components due to the influence of a failure or state of components, the degradation of components could depend on each other due to the common degradation part which can be referred to the impact of the operating environment. As Gamma process is widely used in gradual degradation modeling for single-unit systems to optimize various maintenance policies, a bivariate Gamma process is proposed in [82, 83] for two-unit systems and defined as follows:

$$\begin{aligned} X_t^1 &= (Y_t^1 + Y_t^3)\beta_1, \\ X_t^2 &= (Y_t^2 + Y_t^3)/\beta_2, \end{aligned} \quad (2.36)$$

where $(X_t^1)_{t \geq 0}$ and $(X_t^2)_{t \geq 0}$ describes the degradation of components 1 and 2 respectively. $(Y_t^i)_{t \geq 0}$ ($i = 1, 2, 3$) are mutually independent univariate Gamma processes with parameter $(\alpha_i, 1)$ such that the degradation of component i ($i = 1, 2$) is Gamma process with parameter $(\alpha_i + \alpha_3, \beta_i)$. In the bivariate Gamma process model, the degradation of components are correlated by a common degradation part which can be interpreted as the influence of environment. Therefore the joint probability density function of $(X_t^1, X_t^2)_{t \geq 0}$ is given as follows:

$$f(x_1, x_2) = \beta_1 \beta_2 \int_0^{\min(\beta_1 x_1, \beta_2 x_2)} f_{\alpha_1 t, 1}(\beta_1 x_1 - y_3) f_{\alpha_2 t, 1}(\beta_2 x_2 - y_3) f_{\alpha_3 t, 1}(y_3) dy_3, \quad (2.37)$$

where $f_{\alpha_1 t, 1}$ refers to the probability density function of Gamma distribution with parameters $(\alpha_1 t, 1)$. The Spearman's rank correlation coefficient is used in bivariate Gamma process to measure the dependence degree between components and it $\rho_{X_t^1 X_t^2}$ can be obtained as

$$\begin{aligned} \rho_{X_t^1 X_t^2} &= [E(X_t^1 X_t^2) - E(X_t^1)E(X_t^2)] / [\sqrt{\text{var}(X_t^1)} \sqrt{\text{var}(X_t^2)}] \\ &= \text{var}(Y_t^3) / [\sqrt{\text{var}(X_t^1)} \sqrt{\text{var}(X_t^2)} \beta_1 \beta_2] \\ &= \alpha_3 / [\sqrt{\alpha_1 + \alpha_3} \sqrt{\alpha_2 + \alpha_3}]. \end{aligned} \quad (2.38)$$

Equation 2.38 implies that the dependence between two components is decided by the common degradation process $(Y_t^i)_{t \geq 0}$ such that it has limitation in modeling the strong dependence degree.

The common degradation is also presented in [67] where the system is subjected to two dependent competing degradation processes and each degradation process is composed by a individual Gamma process and a common shock process. As the reliability of system is difficult to be derived analytically because of dependence, copulas are used to estimate the reliability under such of modeling.

2.3.4 Copulas and Lévy copulas

Copulas have been widely used in finance to deal with the multivariate distribution function model [84, 85, 86, 87]. On one hand, they can separate the dependence structure and marginal distributions from joint distribution. On the other hand, they permit different types of marginal processes (e.g. for a two-unit system, one component is Gamma process and the other one is inverse Gaussian process). In [88], ordinary copulas are chosen to describe the stochastic dependence between components with Gamma marginal distributions at time t . Three ordinary copula functions (Gumbel, Clayton, Normal) are investigated in [88] with different dependence degrees from independence to strong dependence for both series and parallel systems. It has been shown that the optimal results of series system are not sensitive to the stochastic dependence whereas those of parallel systems are influenced. The higher stochastic dependence exists, the higher expected cost rate is obtained and optimal inspection period is shorter because the parallel systems have higher probability to fail when components within system are more dependent on each other. Besides, it has been shown that type of ordinary copula function under the same Kendall's tau has no impact on optimal results of neither parallel nor series systems. In [68], the dependence structure between M degradation processes is modeled by a copula function instead of using multivariate distributions. A two-stage Maximum Likelihood Estimation is used to perform the statistical inference for copulas: firstly, the marginal reliability probability of each degradation process with the given parameters is calculated; secondly, the Maximum Likelihood Estimation is used to estimate the parameters of the joint copula reliability function with the underlying of the dependent relationship between multiple degradation processes. Similarly, the dependence between two degradation processes due to the additional damages caused by shocks in [67] make it difficult to calculate the system reliability such that the copula approach is applied to predict the system reliability. The modeling of s -dependence between two components with multi-state are proposed in [89]. The components in the systems have $M+1$ discrete state spaces and s -dependencies between the two components for all state subsets is considered. For example, the time spent by component 1 in state subset $\{0, 1, 2, \dots, N_1\}$ ($N_1 < M$) depends on the time spent by component 2 in state subset $\{0, 1, 2, \dots, N_2\}$ ($N_2 < M$). The computation of joint state probabilities is presented by

using copula functions for all combinations of state subsets of two components. It can be concluded from the above mentioned studies that copulas are very efficient and flexible to model the dependence structure for multi-component system so as to compute the reliability of system. Nevertheless, maintenance actions do not take into consideration in [87, 89] such making us think about the problem whether the dependence relationship remains the same after maintenance. Not clear instructions are given in [88] how the increments are generated by ordinary copulas. In fact, the increments generated As a consequence, the infinite divisible property of multi-dimensional Gamma process might not be kept which signifies that the dependence structure modeled by ordinary copulas depends on time. Furthermore, the results in [88] show that maintenance policies taking advantage of stochastic dependence should be proposed. To overcome this drawback, Cont & Tankov extend the ordinary copulas to Lévy copulas dedicated to Lévy processes in [90] as a time-invariant solution. Application of Lévy copulas can be seen in [91, 92, 93, 94, 95]. In effect, Lévy copulas model the dependence structure by coupling the jump paths of components thus they are time independent and are not influenced by maintenance actions.

2.4 Structure of system modeling

Since we consider the multi-component systems, the structure of system is briefly introduced in this section. The structure of system can be various types according to the configurations of components.

- Series system: the system is functioning if and only if all components in this system are functioning and the reliability of system is given as follows:
- Parallel system: the system is functioning if at least one component in this system is functioning and the reliability of system.
- k-out-of-n system: the system is functioning if at least k components in this system are functioning and the reliability of system. When $k = 1$, it becomes parallel system and when $k = n$, it becomes series system.
- Series-parallel system: the system is composed by several series subsystems in parallel.
- Parallel-series system: the system is composed by several parallel subsystems in series.
- Complex structure system: the system is composed by numbers of components and can not be classified into above mentioned types.

The parallel and series systems are the simplest multi-component system whereas they differ in the sensitivity with stochastic dependence. In fact, for parallel systems, the stronger stochastic dependence exists, the less reliability the systems have [88]. On the contrast, reliability of series systems is not sensitive to stochastic dependence because the system fails if one component fails. In [96, 97, 98, 99, 100], k-out-of-n systems are studied. Complex structure systems are less investigated, examples can be seen in [16, 101].

2.5 Summary and analysis

This chapter is devoted to presenting an overview of the current researches about the degradation modeling which is an important aspect for maintenance optimization. The three aspects relating to degradation modeling are briefly discussed and reviewed: individual degradation modeling, stochastic dependence modeling and structure of system modeling. For individual degradation modeling, various models have been proposed: the lifetime distribution models (Weibull distribution, proportional hazards model etc), the gradual degradation models with discrete state space (Markov process and semi-Markov process etc) and with continuous state space (Gamma process, IG process, drifted Brownian motion etc). Since the lifetime distribution can not express the real state of component, the gradual degradation models have more advantages for making maintenance decisions based on the state of component. Even though a vast results have been obtained for single-unit systems with more and more complex models taking account for multiple sources of degradation, covariates, non-stationary trend that may depend on covariates, the degradation modeling for multi-component systems is less explored and when studied, stochastic dependence between components is not often taken into account. Existing stochastic dependence models mainly focus on the failure interactions or degradation interactions whereas the inherent dependence between components due to common environment or the structure of system is rarely investigated. Therefore, one objective of this work is to propose a gradual degradation modeling for multi-component systems in which stochastic dependence is considered.

To address such a problem, several issues should be tackled such as how to couple several degradation processes while preserving their individual properties within a common structure or how to propose a dependence modelling framework flexible enough to be used with various system structures and number of components and allowing to represent not only various dependence degrees but also structures. Moreover an easy to implement framework is required for the evaluation of a cost criterion for performance optimization.

In term of stochastic dependence which is the main concern of degradation modeling

of multi-components, Lévy copulas appear to be very flexible in the dependence structure as well as the range of dependence degree when compared to dependence modeling through a common degradation part such as the multi-variate Gamma process obtained from a trivariate reduction. In particular, Lévy copulas are able to depict symmetric or non-symmetric dependence structures. Moreover Lévy copulas which are designed for the Lévy process family apply for a wide range of individual processes from a continuous path Brownian motion to pure jumps processes such as compound Poisson or Gamma processes as well as mixed models with continuous part and jump part. When compared to ordinary copulas, it models the dependence in the aspect of increment path instead of increment over a specific time which ensures the stable dependence structure and the individual properties of marginal processes such as the infinite divisible property for a Gamma process. Furthermore, Lévy copulas can separate the dependence structure and the marginal distributions. As a result, Lévy copulas can be a good choice to model the stochastic dependence for degradation processes of multi-component systems. Regarding the individual deterioration modeling of components, we have reviewed Gamma process is very popular among other Lévy processes to model the degradation of components because it has independent and stationary increments and increasing degradation path such that it is more suitable for our case where monotonically degrading system is considered. Besides, the inverse tail integral of Lévy measure of Gamma process can be numerically computed which permits the simulation of multi-dimensional process with Lévy copulas.

In the next chapter, we will review the maintenance modeling which is another aspect of maintenance optimization.

Chapter 3

State of the art part II: Maintenance modeling

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3.1 Introduction

In this chapter, the literature review on the maintenance modeling will be introduced. Given history data or degradation modeling, maintenance modeling aims to schedule maintenance activities so as to minimize the maintenance cost or maximize the availability or the reliability of system according to the context. In the earlier years, the system is maintained or replaced when it fails such a strategic is called the failure-based maintenance. However such maintenance strategies are very expensive since the replacement can not be planned and the failure may lead to catastrophic results due to the fact that failure time is random. In effect, the failure of system can result in significant economic losses (e.g. the delay of a mission), physical damage or threats to human life (e.g. the chemical plant, nuclear power plant). To overcome this drawback, preventive maintenance strategies are developed to prevent failures of system. Many preventive maintenance strategies have been developed over a few of decades. According to the number of components within the system, they can be classified as maintenance strategies for single-unit systems and maintenance strategies for multi-component systems [7]. Maintenance strategies for single-unit systems consider that the whole system can be synthesized as a unique component. In this way, the maintenance decision can be made based on a scalar indicator which can be seen as a characteristic of an abstract component (e.g. failure rate, degradation level, age). The proposed maintenance strategies for single-unit systems are more and more efficient by taking account of different kinds of competing degradation processes but is hard to extend these approaches to multi-component systems due to the existing interactions between components. Therefore, the adaptive maintenance strategies for multi-component systems should be developed. The remainder of this chapter is organized as follows. Before introducing maintenance strategies, section 3.2 discusses the impact of maintenance on the maintained component. Maintenance actions are called perfect, imperfect or minimal according to their efficiency. Section 3.3 presents and summarizes the maintenance strategies for single-unit systems and section 3.4 is devoted to maintenance models for multi-component systems. Some conclusions and perspectives are drawn in section 3.5.

3.2 Types of Maintenance actions

According to the state of component when it is maintained, the maintenance actions can be categorized into two major classes: corrective maintenance and preventive maintenance. Corrective maintenance is the maintenance that occurs when a component fails such that it brings the failed component back to operation while the preventive one is the maintenance that occurs when the component is still functioning but considered

to be degraded enough to need a maintenance. Preventive maintenance is designed to avoid or mitigate the sudden failure of component through planned operations so that it is often more economic than unplanned corrective maintenance. Therefore, appropriate maintenance activities should be scheduled to prevent the occurrence of failures of system and to improve the system reliability and/or the availability.

According to the efficiency of maintenance actions on the maintained component, maintenance actions can be also classified into three categories: perfect maintenance, minimal maintenance and imperfect maintenance [102]. A maintenance is perfect if component state after maintenance is as good as new (AGAN). For the ease of analytical results, most of literatures consider perfect maintenance in their models [29, 77, 103, 104]. A minimal maintenance means that the system is restored to a functioning state but its characteristics are the same as just before the maintenance that is named as bad as old (ABAO). Minimal maintenance is often applied to the corrective maintenance however it is less used for gradual degradation modeling. In [65], a single-unit system is subjected to two failure modes and the failure caused by external shocks is maintained with minimal repair. In [105], the component is subjected to two types of failures and type I failure is corrected by a minimal failure whereas type II failure is replaced with probability p and minimally maintained with probability $1 - p$. While perfect and minimal maintenance models describe two extreme states after maintenance actions, the imperfect maintenance which can bring a system to any condition between AGAN and ABAO seems more suitable for practical applications.

In order to measure the degree of improvement after maintenance actions, several models are proposed in literatures. The (p, q) rule is proposed to model imperfect maintenance by the combination of perfect maintenance and minimal repair where the state of component after maintenance is AGAN with probability p and is ABAO with probability $1 - p$, see [105, 106, 107, 108]. Even though the (p, q) rule is efficient for deriving optimal maintenance policies, it is more realistic to return the system in an intermediate state between AGAN and ABAO after the maintenance action especially for the gradually degrading components. The improvement factor method is introduced in [109] where the maintenance actions change the system time of the failure rate curve to some newer time but not all the way to zero (not new). The investigation of improvement factor method for imperfect maintenance can be found in [110, 111, 112] where the failure rate of system after maintenance is reduced proportionally to maintenance cost, the age of system or by a fixed reduction. Similar to the improvement factor model, the virtual age model is introduced in [113]. This model assumes that the system has a so-called virtual age V_n immediately after the $(n - 1)th$ maintenance operation and the failure time distribution after the $(n - 1)th$ maintenance operations depends on V_{n-1} . Between two maintenance actions the virtual age is incremented by a random amount X_n . When the nth imperfect maintenance action takes place, the virtual age V_n is reduced by an

amount proportional to virtual age before maintenance or by an amount proportional to the additional age X_n as following:

$$V_n = V_{n-1} + a_n X_n \quad (3.1)$$

$$V_n = a_n(V_{n-1} + X_n) \quad (3.2)$$

where a_n is the age reduction factor of n th maintenance action. Equation 3.1 indicates that the maintenance action can only reduce additional age whereas the imperfect maintenance modeled by Equation 3.2 reduces the cumulative virtual age. Researches have used the concept of virtual age model for imperfect maintenance, see [114, 115, 116]. While the improvement factor and the virtual age models restore the system to a newer state but do not modify failure rate function, the hazard rate model is introduced in [117] where the maintenance restore the system to AGAN but the failure rate $h_n(\cdot)$ after the n th maintenance action is modified as $h_n(t) = \beta_n h_{n-1}(t)$ where $\beta_n \geq 1$ is called adjustment factor. Therefore, the slope of the failure rate function increases with the number of imperfect maintenance operations. A hybrid hazard rate model is proposed in [118] where maintenance action not only reduces the effective age to a newer one, but also further changes the slope of the failure rate curve. The hybrid hazard rate model is further discussed in [119, 120]. The improvement factor model, virtual age model and hazard rate model are based on the failure rate function and Figure 3.1 gives an illustration of the impact of these models on the failure rate function when a maintenance takes place.

For a gradually deteriorating system, a cumulative damage shock model with imperfect periodic preventive maintenance actions is proposed in [121]. Each preventive maintenance reduces the degradation level by $(1 - b)\%$ where $b \in [0, 1]$ so that b can be seen as the improvement factor of imperfect maintenance. The cumulative degradation level after n th imperfect preventive maintenance is $X_{n+} = bX_{n-}$ and the related maintenance cost is unique and independent from b . In [122], the degradation level after preventive maintenance is returned to a fixed value x_r and the cost of preventive maintenance action depends on the degradation reduction. The randomly distributed improvement factor of imperfect maintenance is used in [123, 124, 125]. As imperfect maintenance also impacts the evolution of the system deterioration, the mean degradation of system after each imperfect maintenance action is modeled as an exponentially distributed increment in [125].

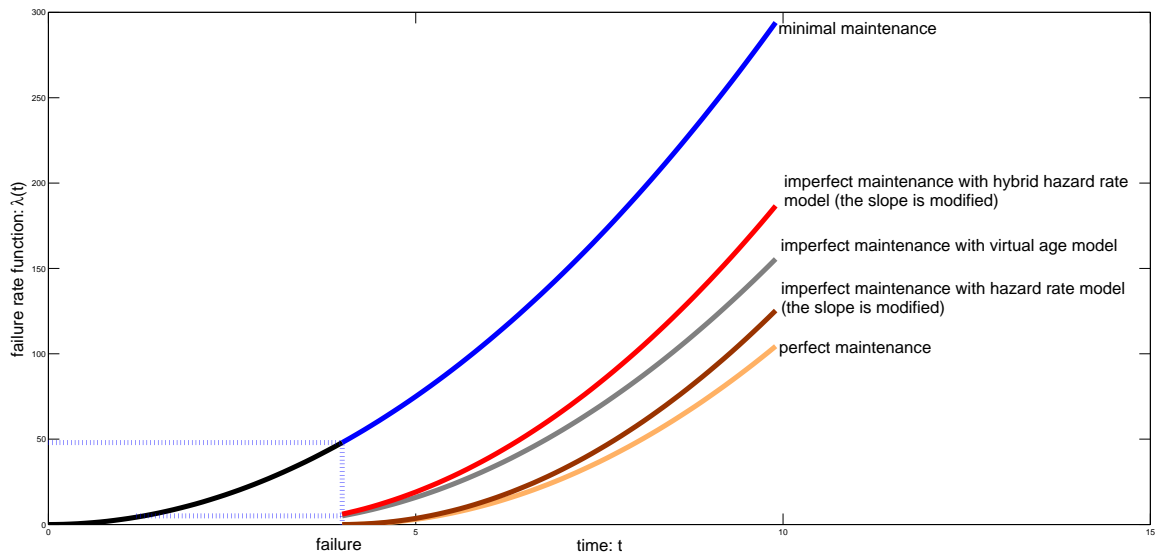


Figure 3.1: A sketch of the impact on the failure rate curve of minimal, imperfect and perfect maintenance

3.3 Maintenance modeling for single-unit systems

Several kinds of maintenance policies for single-unit systems are proposed in the literature and they can be classified into two major categories: time-based maintenance policies and condition-based maintenance policies. The difference between them is whether the degradation information of component is available or taken into consideration for decision-making. For time-based maintenance policies, the decision-making process is based on the operating time of component and its general failure time distribution without considering the real state of component. With the development of sensor techniques, condition-based maintenance which enables to manage maintenance activities based on information collected through system condition monitoring has gained a lot of attention over recent years.

3.3.1 Time-based maintenance policy

Age-based maintenance policy is the most used preventive maintenance policy. According to the age-based maintenance policy, the component is preventively maintained when its age arrives at T or correctively maintained when it fails, whichever occurs first, with the maintained component to be as good as new. In order to optimize the age-based maintenance policy, a mathematical form is proposed in [126] as follows:

$$C(T) = \frac{C_c F(T) + C_p (1 - F(T))}{\int_0^T (1 - F(u)) du} \quad (3.3)$$

where C_c is the unit corrective maintenance cost, C_p is the unit preventive maintenance cost, $F(\cdot)$ is the cumulative distribution function of the failure time. The optimal value of T is determined by minimizing the cost function $C(T)$ in equation 3.3. When preventive maintenance cost is equal to corrective maintenance cost, the age-based maintenance policy tends to be failure-based maintenance policy and T can be infinity. By considering minimal and imperfect maintenance, various extensions of the age-based maintenance policy are proposed. The age-based maintenance policy is extended in [127] by replacing the component to a new one at time T or when it has suffered N failures. If component fails before time T and the number of failures is less than N , the system is minimally repaired at the failure. In [128], the system suffers two types of failure where type I failure (minor failure) is corrected with minimal repair whereas type II failure (catastrophic failure) is corrected with replacement. An age-based maintenance policy is implemented such that the system is replaced when type II failure occurs or at the opportunity after age T , whichever occurs first and the optimal value of T is derived. In [129], the system undergoes imperfect maintenance either upon failure or after having reached a predetermined age T_i , whichever of them occurs first and it is replaced at the time of N^{th} maintenance such that the system is back to as good as new. The imperfect maintenance brings the age of the system back to zero but with a higher failure rate. Then a mathematical model is proposed for determining the optimal value of N and the sequence of preventive maintenance dates $(T_1, T_2, \dots, T_{N-1})$ to minimize the long-term average cost. Two age-based maintenance policies with minimal repair are proposed in [65] for a single-unit system whose failures are due to the competing causes of degradation and shocks. To summarize the age-based maintenance policy and its extensions, the maintenance decision is mainly made based on the age of component and the number of failures.

Similar to age-based maintenance policy, periodic preventive maintenance policy is proposed which maintains the component at fixed time kT ($k = 1, 2, \dots, n$) independently of the failure history of the component. When the component is found in a failed state at time kT , it is repaired; otherwise, a preventive maintenance is carried out. The periodic maintenance policy is suitable for the gradual degrading component where the failure is soft and can only be detected at certain times. The difference between age-based and periodic maintenance policies is that the failure is not corrected immediately in the latter one so that a cost of downtime is involved in the cost function $C(T)$ which is shown below:

$$C(T) = \frac{C_c F(T) + C_p(1 - F(T)) + C_d E[(T - d)\mathbb{1}_{\{T > d\}}]}{T} \quad (3.4)$$

where C_d is the cost of downtime per time unit, d is the failure time and $\mathbb{1}_{\{T > d\}} = 1$ for $T > d$ otherwise $\mathbb{1}_{\{T > d\}} = 0$. The maintenance in periodic maintenance policy

can be perfect, imperfect or minimal. For example, in [130], the system is maintained preventively at periodic times kT ($k \in \mathbb{N}$) where the preventive maintenance reduces the rate of degradation and the system is replaced by a new one at the N^{th} preventive maintenance. If the system fails between two preventive maintenance, it undergoes minimal repair. In [131], the component is repaired imperfectly at $(i - 1)^{\text{th}}$ failure ($i \in \mathbb{N}, i < k$) and after $k - 1$ failures, a periodic maintenance policy is applied. The periodic policy preventively maintains the system at times $T, 2T, 3T, \dots$ and brings system to as good as new with probability p and to as bad as old with probability $1 - p$ and an imperfect repair is performed when a failure occurs between preventive maintenances. Non periodic maintenance policy is also proposed so called sequential preventive maintenance policy because of the imperfect maintenance, see [117, 118, 132].

3.3.2 Condition-based maintenance policy

If the degradation information of component can be collected by monitoring, condition-based maintenance can be performed in order to provide flexible maintenance dates to avoid failures of component as well as lengthening the operating time when the component is still in good condition. A condition-based maintenance policy monitors components in the system and the inspections will reveal the information of components. For condition-based maintenance policy, the replacement strategy is: at each inspection, if the component is revealed to be failed, a corrective maintenance is carried out on the component; if no failure is detected but the degradation information (such as failure rate, wear or accumulative damage) of component is revealed to exceed a preventive maintenance threshold M , a preventive maintenance is carried out. Therefore condition-based maintenance is based on the collected information through condition monitoring process [9] in which two aspects are involved: inspection frequency and the collected information.

In practice, information can be gathered either continuously or at discrete times. When the system is continuously monitored, the information of component is available all the time so that the maintenance policy is more sensitive and react faster when a failure occurs. Nevertheless, the cost of continuous monitoring is very expensive because special inspection devices are required. In contrast, periodic inspection is less expensive but system may fail between two inspection times. As a consequence, condition-based maintenance policies with non-periodic inspections where the next inspection interval depends on the current state of component are often implemented [29, 133, 134]. Inspections are more and more frequent as the component deteriorates or ages.

Irrespective of whether the system is monitored continuously or through inspections,

two cases relating to the observability level may arise: completely observable component and partially observable component [9]. For a completely observable component, the state of component can be completely observed or identified such that the collected information can be used directly to make a maintenance decision. Several models are proposed to investigate condition-based maintenance policies for observable components. Among them, various degradation models as well as the types of maintenance are used. Examples which use different degradation models can be seen in [29, 103, 135, 136]. In [135], a condition-based maintenance policy is proposed based on a random coefficient growth model where the coefficients of the regression growth model are assumed to follow known distribution functions. Both the optimal critical level and inspection interval in condition based maintenance is determined based on this model in terms of a criterion of interest, which can be cost-based or reliability-based. In [29, 103], Gamma process is used to describe the degradation process and non-periodic inspection scheme as well as preventive maintenance threshold are optimized by using semi-regeneration technics. A Markov chain is used in [136] where the inspection period and preventive maintenance threshold are found to maximize the system availability. Although a lot of papers deal with the condition-based maintenance policy assume that the maintenance restore the component back to as good as new, condition-based maintenance policies with imperfect maintenance are also investigated, see [125, 137].

For partially observable components, the state of component cannot be fully observed or identified. For example, in [138], failure of component can be detected with a probability $1 - p$ or can not be detected with a probability p due to monitoring problems and the condition-based maintenance policy with control-limit rule is implemented where the interval of two inspections depends on the revealed state and the component is repaired if and only if it is known that the component is in condition i for every $i > \pi$. In [139], the system state is not observable, only the failure can be observed. Then the replacement problem is formulated as an optimal stopping problem with partial information and transformed to a problem with complete information by applying the projection theorem to a smooth semi-martingale process in the objective function. The dynamic equation is derived and analyzed in the piecewise deterministic Markov process framework. The system's degradation is represented by a proportional hazards model in [22] and the optimization of the optimal maintenance policy is formulated as a partially observed Markov decision process since the state of component is unknown. In [140], the problem is also formulated as a partially observed Markov decision process with heterogeneous parameters. The model is solved using a backward dynamic programming method and a dynamic strategy is produced.

3.4 Maintenance modeling for multi-component systems

Even though maintenance policies for single-unit systems have been widely studied, the results can not be directly extended for multi-component systems. Maintenance policies should be developed for taking into account the economic dependence for the multi-unit systems. Hence, in this section we discuss the extension of maintenance policies from single unit systems to multi-component systems by taking into account the economic dependence.

3.4.1 Economic dependence

Economic dependence applies if the maintenance cost depends on whether the components are maintained jointly or separately. The economic dependence can be positive or negative.

Positive economic dependence implies that maintenance cost can be saved if components are jointly maintained rather than separately, due to saving of so-called the set-up cost, ‘general economies of scale’ or downtime opportunity in [8]. The term economies of scale refers to the fact that grouping maintenance activities allows to save maintenance costs. Nicolai and Dekker distinguish two major forms of economies of scale that are set-up and ‘general economies of scale’. The concept of set-up cost relies on the fact that the cost of each maintenance action (either corrective or preventive) can consist of two parts: a set-up cost which is related to human activities and can be shared and an individual cost for each component incurred by materials consuming. In other words only one set-up cost is incurred when maintenance activities are grouped, regardless of the number of maintained components. The ‘general economies of scale’ cannot be modeled by a single set-up cost. While the cost saving due to set-up cost is linear to the number of maintained components, that due to ‘general economies of scale’ can be concave or convex. The downtime opportunity consists in turning the downtime of system into an opportunity to carry out preventive maintenance for non-failed components. The downtime opportunity is also different from the set-up cost because the cost saving depends on the maintenance duration.

In terms of negative economic dependence, it is less investigated in the literature. Negative economic dependence between components occurs when maintaining components simultaneously is more expensive than maintaining them individually. The cause of negative maintenance can be resources and manpower restrictions, safety requirement or production-loss, see [8]. Firstly, resources and manpower restrictions depict that the increasing number of maintenance may lead to the fact that the maintenance requirements are hardly satisfied or costly. For example, maintaining several components at the same

time requires to hire more labour and the storage of spare parts but the maintenance activities are less frequent. As a consequence, the balance between workload fluctuation and grouping maintenance should be found. Secondly, there are often restrictions on the use of equipment, when executing maintenance activities simultaneously. Moreover, if the breakdown of system may cause potential danger, it is better not to maintain component simultaneously. As an example, in a system with high safety requirement, redundancy is used to improve the reliability of system but joint maintenance of redundant parts causes the loss of its benefit. Production loss may also increase more than linearly with the number of components out of operation.

3.4.2 Maintenance models taking advantage of economic dependence

Considering the economic dependence, grouping maintenance strategies should be developed to provide more opportunities for costs saving by taking full advantage of positive economic dependence whereas the maintenance grouping should be avoided when negative economic dependence applies. Various optimal grouping maintenance models have been proposed in the literature, an overview of this problem can be seen in [7, 8, 141, 142, 143]. Among those proposed in the literature, the maintenance models considering economic dependence can be classified into several categories relating to different aspects [142]. Firstly, maintenance models can be identified as stationary or dynamic according to the planning characteristics. In stationary models, a long-term stable situation is assumed and static rules for maintenance are applied on an infinite horizon. The stationary models can be further classified into three categories: grouping of corrective maintenance, grouping of preventive maintenance and opportunistic maintenance regarding to types of maintenance action. In dynamic models, short-term information such as changes in the degradation parameters of components or unexpected opportunities of grouping can be taken into account. These models generate dynamic decisions that may change over the planning horizon. Therefore, maintenance models are reviewed according to the three categories of stationary models and then dynamic models in the following.

3.4.2.1 Grouping of corrective maintenance

In some types of system such as parallel systems or k-out-of-n systems, redundancy of components make it possible to postpone the corrective maintenance of a failed component to benefit from costs saving achieved by grouping the maintenance tasks when subsequent failure occur. In [144], an optimum age replacement policy is proposed to

minimize the maintenance cost and production loss due to failure of machine for N -unit systems. All the failed components are replaced only at $(kT)_{k \in \mathbb{N}}$. With the increasing number of failed machines, the maintenance cost per machine is reduced due to the shared set-up cost whereas the production loss per machine is increased. A nomograph was developed for machines with exponentially distributed failure time and the period T for repair as well as the total repair cost per cycle can be obtained easily from the nomograph. In [145], an optimal replacement policy is proposed which resets all failed machines at the moment when the number of failed machines reaches some prescribed number k . This model is similar to that in [144] but the optimal number k instead of the period T has to be found to minimize the cost criterion so that the replacement dates are not deterministic. A grouping corrective maintenance strategy which consider two types of components and leaves the non-critical components in failed state until the failure of system occurs is proposed in [146]. In [147], an optimal maintenance policy is proposed for k -out-of- n system based on a dynamic programming model and the optimal result turns out that the component should wait and be left in failure state until the sum of degradation level of other components exceeds a certain level.

3.4.2.2 Grouping of preventive maintenance

An advantage of preventive maintenance is that it can be planned so that necessary resources can be prepared which enable a large number of maintenance executed at the same time with lower cost. The existing grouping methods for preventive maintenance can be identified as direct grouping methods or indirect grouping methods [142]. In direct grouping methods, components are set into a fixed group and they are always preventively maintained with the other components in the same group at an appropriate moment. The groups of components as well as the optimal maintenance time for each group should be decided. The number of possibilities for splitting N components into groups is $2^N - 1$ (see [148]) such that the optimization of direct grouping is NP-hard. A direct grouping model is studied in [149] where the date of maintenance in a group is set to the shortest individual optimal replacement date of component in the group. In this way, the problem is solved in polynomial time and a dynamic programming algorithm is developed.

Since the optimization of direct grouping approaches is very complicated, indirect grouping methods are proposed. Two types of indirect grouping approaches exist in the literature: the standard indirect grouping method and the joint-overhaul problem. The standard indirect grouping is to maintain the component i at $(k_i T)_{k_i \in \mathbb{N}}$ such that T and $k_i \in \mathbb{N}$ should be optimized, see [150, 151, 152]. As the components heterogeneously degrade in the system, the standard grouping approach does not fix the components in

a group so that it is more flexible than direct grouping. The joint-overhaul problem is similar to the standard indirect grouping but it overhauls the system every T time units and the component i undergoes minor overhaul every $(\frac{T}{k_i})_{k_i \in \mathbb{N}}$ time units [153, 154, 155]. The overhaul of system brings back all the components to as good as new while the minor overhaul of component may be not.

3.4.2.3 Opportunistic maintenance

In the previously reviewed grouping maintenance strategies, the maintenance activities are jointly carried out in a group when a fixed number of failures is reached or after a fixed period. Nevertheless, this kind of grouping strategies does not consider the downtime opportunities such that an opportunistic maintenance policy should be proposed. Opportunistic maintenance applies when the failure of a component or the system downtime is turned into an opportunity to group corrective maintenance with preventive maintenance. Set-up cost as well as downtime cost can thus be saved. Since opportunistic maintenance policy groups maintenance by chance, it can either bring forward some preventive maintenance tasks at the failure time of a critical component or delay the corrective maintenance of non-critical components to the planned time of preventive maintenance of critical components. A component is said to be a critical component if its failure causes immediately the system break down whereas the failure of non-critical component does not influence the operation of system.

Numerous literatures use opportunistic maintenance model for optimizing multi-unit systems maintenance problem. The opportunistic maintenance models for a k -out-of- n : G system is studied in [156] and two (τ, T) opportunistic maintenance policies are proposed. The (τ, T) model corrects each failure occurring in time interval $[0, \tau]$ with minimal repair. Once m components fail in $[\tau, T)$, the corrective maintenance for failed components is immediately carried out and combined with the preventive maintenance of unfailed components so as the system is renewed. If less than m components fail in $[\tau, T)$, the whole system is preventively replaced at time T . Decision variables are τ and T whereas m is considered as a predetermined parameter. The imperfect maintenance is also considered in a way that when preventively maintained, the system is renewed (that is system returns to as good as new) with probability p and minimally repaired (that is as bad as old) with probability $1 - p$. In [157], the preventive maintenance of series multi-component systems leads to the downtime of system such that it provides the opportunity to maintain the other components to save the downtime cost and maintenance cost but a penalty cost incurs at the same time since the useful working time of components is reduced. A dynamic opportunistic strategy is proposed where a component is preventively maintained when its reliability reaches a threshold then the

possibility of preventive maintenance of the other components is studied based on the difference between cost saving (maintenance cost and downtime cost) and penalty cost. In [158], a series multi-component system which consists of N components is considered. The preventive maintenance of component i is only carried out at discrete times $(k_i\tau)_{k \in \mathbb{N}, i \in (1, 2, \dots, N)}$. At each failure of component, it should be decided whether other components can be opportunistically replaced with it or wait their scheduled replacements. A solution procedure based on Monte-Carlo simulations with informative search method is proposed. At the failure time t_i of component i , component j is preventively replaced with component i if the expected cost of taking opportunity to bring forward the preventive maintenance of component j is lower than that of leaving it until the next planned preventive maintenance (given that it may fail before). In [159], the real-time remaining useful life (RUL) of components is predicted with stochastic dependence based on stochastic filtering theory and historic condition monitoring data and then the optimal preventive maintenance date is analyzed based on the RUL prediction within a time window. The component is maintained at its optimal preventive maintenance date or as soon as it fails which turns out to be an opportunity to carry out preventive maintenance for other components. An opportunistic preventive maintenance strategy is proposed where the component j is preventively replaced when component i should be maintained with the condition that its individual optimal preventive maintenance date fails in the optimal opportunistic maintenance zone. Other opportunistic maintenance models can be seen [160, 161, 162, 163, 164, 165]. In general, opportunistic maintenance approach is to group the corrective maintenance with preventive maintenance based on the cost gain evaluation criteria.

3.4.2.4 Dynamic grouping

According to planning horizon, the dynamic grouping methods can be categorized into two types: methods with finite horizon and methods with rolling horizon [148, 166]. The methods with finite horizon apply for systems that are used during a given period and not afterwards so that the maintenance optimization is performed within this finite mission time. The methods with rolling horizon optimize the maintenance cost with tentative optimal long-term parameters at decision time but they can be updated when short-term information is available. Rolling-horizon models are proposed to take advantage of infinite horizon models to have more stable solutions compared to finite horizon models and to make use of short-term information as well.

Finite horizon models are less investigated in the literature because it is more difficult to discuss the optimal policies for these models. An overview of cost evaluation of maintenance policies for single-unit systems on a finite horizon can be found in [167]. In [168], a

finite horizon model is proposed for evaluating the expected cost associated with a periodic condition-based maintenance policy. A generalized renewal equation is formulated for the expected cost in a finite horizon. It is shown that the finite time cost curve is less smooth than the expected long-term cost curve. Since the analytical equations of the expected cost in a finite horizon for multi-component systems are more complex than single-unit systems, the simulation method is adopted in [88] to evaluate the expected cost and its probability distribution. In [169], the planning horizon is segmented into J discrete intervals and at the end of each interval, a decision should be made: either a preventive maintenance, a corrective maintenance or no maintenance action is carried out. When maintenance actions are carried out simultaneously, downtime cost can be saved. Therefore, the problem can be formulated as an integer linear programming problem and can be solved with heuristic methods.

The basic idea of dynamic grouping with rolling horizon is to group the maintenance activities following four steps:

- Individual optimization: the individual optimizations are first performed by considering each component as a mono-component system and minimizing the expected long-term cost on an infinite horizon. A penalty cost function is established in order to take account for changes caused by the shift (advance or delay) of the replacement date. On one hand the useful life of components is reduced if maintenance dates are advanced and on the other hand the probability of failure is increased if the maintenance dates are delayed.
- Tentative planning: from time t , the planning horizon PH is defined based on the individual optimization as $PH = [t, \max_{1 \leq i \leq n} \{t_i^*\}]$ where t_i^* is the optimal replacement date of component i after time t with considering short-term information. In the defined planning horizon, tentative preventive activities of components are displayed.
- Grouping optimization: on the planning horizon PH , groups of maintenance activities are determined at times $t_{G_i}^*$ by minimizing the total cost where G_i is a group i of maintenance activities. The cost saving of one group is twofold. On one hand, grouping maintenance activities of components jointly saves the set-up cost or reduces the downtime cost (if maintenance duration is non-negligible). On the other hand, a penalty cost is induced due to the shift of execution time of maintenance.
- Updating of the maintenance planning: when new short-term information is available, the maintenance planning is updated by repeating the steps of tentative planning and grouping optimization. In this way, the maintenance planning is dynamic and adaptive.

The main challenge of dynamic grouping strategy is the optimization of the group structure over the planning horizon. A fast approach to derive the grouping structure is proposed in [148] which reduced the computation complexity of set-partitioning algorithm in [170]. In [171], a dynamic grouping method for age-based replacement with rolling horizon is developed. The gradual degradation model is used and condition monitoring information is incorporated at decision time. This method is further developed in [172] on infinite horizon considering different levels and combinations of dependences between components. In [173], a dynamic grouping maintenance strategy based on failure rate distribution is proposed where the preventive maintenance duration is taken into consideration. In [16, 100], complex system structures are then considered. In [21], a grouping strategy based on proportional hazards model is proposed for a condition-based maintenance policy with periodic inspection. At each inspection, failed components are correctively replaced and unfailed components can be preventively replaced according to the following decision rule: if the hazard rate of a component exceeds the level-1 risk threshold d_1 , it is preventively replaced; moreover if a maintenance (either preventive or corrective) takes place, then any component whose hazard rate exceeds the level-2 risk threshold d_2 ($d_2 < d_1$) is preventively replaced. Therefore, the level-2 risk threshold is introduced to group the maintenance activities for condition-based maintenance. In [174], a multi-level predictive maintenance policy is proposed for multi-component systems with complex structure. The preventive maintenance activities are triggered only when the estimated system reliability before the next planned inspection is under a reliability threshold R_0 to be optimized. If this condition is met then the optimal group of components to be preventively maintained is obtained by minimizing its cost-based group improvement factor then the components in the optimal group is preventively maintained. Therefore the decision parameters of the proposed maintenance policy in [174] are (T, R_0) where T is inspection interval.

3.5 Summary and analysis

This chapter is devoted to presenting the existing maintenance models in the literature. Maintenance policies for single-unit and multi-component systems are discussed as well as the maintenance effect. Maintenance strategies for single unit systems are constantly being improved, combining several maintenance tasks and/or inspection scheme according to the degradation cause. For multi-component systems, economic dependence has been substantially addressed but stochastic dependence even when taken into account the degradation model is not often used in the decision making process. To deal with economic dependence, grouping strategy has to be proposed since it can reduce the maintenance cost. Nevertheless, the condition-based maintenance strategies which

make a decision based on the state of component are often more efficient than age-based maintenance strategies. In both reviewed papers using condition-based maintenance ([172] and [174]) the inspection schemes is not taken into account. Moreover, all the components in the system are inspected at the same period which might lead to high inspection cost because faster degrading components require high frequency inspection while the slower ones do not need. Therefore, the objective of this thesis regarding the maintenance modelling is twofold. Firstly, we aim to propose grouping strategies for condition-based maintenance with considering stochastic dependence. Secondly we aim to explore the use of stochastic dependence in the decision making process.

In the next two chapters, we aim to address the following problems:

- How to use the condition monitoring information to make a decision about the opportunities to group maintenance activities.
- How to benefit from cost saving due to grouping opportunities while preserving a maintenance scheme adapted to the individual behaviour of each component.
- How to profit the stochastic dependence between components to adapt the online decision.
- How to provide different inspection schemes for a number of components without increasing the complexity of optimization.
- Which indicator(s) should be used to make a decision regarding a multi-component system.

[chapter 4](#) is devoted to addressing the first three problems with a case study for a two-component system combining stochastic and economic dependences. The impact of each type of dependence is discussed in relation to individual behaviour of components. [chapter 5](#) increases the number of components and considers more complex dependence structures to deal with on the last three problems.

Chapter 4

Maintenance modeling of degrading systems with symmetric stochastic dependence between components

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4.1 Introduction

This chapter is written mainly based on [175]. The objective of this chapter is to propose condition-based maintenance decision rule evaluating grouping opportunities with individual inspection schemes to take advantage of both economic and stochastic dependence.

Positive economic dependence applies when maintaining two or more components simultaneously is cheaper than maintaining them separately is considered in this document. In such a context, the maintenance policies should be designed to maximize the opportunities for grouping maintenance activities. A decision rule based on the available information is proposed to decide at each inspection time how many components should be maintained. In this way the replacement of a component may be postponed or preempted when compared to the date that would be set if only individual information was considered.

Unlike those works that assume that the components are independent in most literatures, the stochastic dependence modeled by Lévy copulas is considered in our proposed degradation model. Since the multi-component systems may contain various components, it leads to various dependence between components. We first consider a rather simple case of symmetric dependence in this chapter and a two-component system in parallel is presented as an example. The asymmetric dependence will be considered in the next chapter.

The rest of this chapter is organized as follows. Section 4.2 is devoted to the description of individual degradation and stochastic dependence model. Section 3.3 describes the new condition-based maintenance policy and the implemented classic maintenance policies. In Section 3.4, numerical experiments are presented and sensitivity analysis is carried out. Finally, conclusions and perspectives are drawn in Section 5.

4.2 Stochastic degradation modeling

In the aim of dealing with multi-unit systems with stochastic dependence when components suffer gradual degradation, we propose to use Lévy copulas which are well suited to model stochastic dependences for Lévy processes. We first detail the individual degradation model of components in the system. Then we briefly introduce main properties of Lévy copulas. Simulation methods of multi-dimensional Gamma processes with Clayton Lévy copula are given to end this section.

4.2.1 Marginal Gamma process

In [chapter 2](#), we have reviewed some degradation models and justified that the Gamma process can be used to model the individual gradual degradation of components (noted as $(X_t)_{t \geq 0}$) in this work. Recall that $(X_t)_{t \geq 0}$ is a Gamma process if given shape parameter α and scale parameter β , it has the following properties:

- $X_0 = 0$
- $(X_t)_{t \geq 0}$ has independent and stationary increments
- For $t > 0$ and $h > 0$, $X_{t+h} - X_h$ follows a Gamma distribution with shape parameter αt and scale parameter β and the probability density function of $(X_{t+h} - X_h)$ is given by:

$$f_{\alpha t, \beta}(x) = \frac{\beta^{\alpha t} x^{\alpha t - 1} \exp(-\beta x)}{\Gamma(\alpha t)}, \quad (4.1)$$

where $\Gamma(\alpha t)$ is Gamma function and

$$\Gamma(\alpha) = \int_0^{\infty} u^{\alpha-1} e^{-u} du, \alpha > 0 \quad (4.2)$$

In order to simulate the degradation of a components modeled by the Gamma process, the Inverse Lévy Measure Method proposed in [\[176\]](#) is used for the purpose of being combined with Lévy copulas. Thus the degradation level of component at time t is defined as follows:

$$X_t = \sum_{n=1}^{\infty} U^{-1}\left(\frac{\Gamma_n}{T}\right) \mathbb{1}_{[0, t]}(v_n), \quad (4.3)$$

where U^{-1} is the inverse tail integral of Lévy measure of Gamma process and it is defined as:

$$U^{-1}(y) = \inf\{x > 0, U([x, \infty)) < y\} \quad (4.4)$$

$U([x, \infty))$ is the tail integral of Lévy measure of Gamma process. As the Lévy measure of Gamma process is $\nu(dx) = \frac{\alpha e^{-\beta x}}{x} dx$, then

$$\begin{aligned} U([x, \infty)) &= \int_x^{\infty} \nu(dx), \\ &= \int_x^{\infty} \frac{\alpha e^{-\beta x}}{x} dx, \\ &= \alpha \mathbf{E}_1(\beta x), \end{aligned} \quad (4.5)$$

where $\mathbf{E}_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$ is known as the exponential-integral function. Using Equations 4.5, Equation 4.3 is rewritten as:

$$X_t = \sum_{n=1}^{\infty} \mathbf{E}_1^{-1}\left(\frac{\Gamma_n}{\alpha T}\right) / \beta \mathbb{1}_{[0,t]}(v_n), \quad (4.6)$$

where $t \in [0, T]$, $\{\Gamma_n\}_{n \in \mathbb{N}}$ is a sequence of arrival times of a standard Poisson process and $\{v_n\}_{n \in \mathbb{N}}$ is a sequence of mutually independent and uniformly distributed random variables on $[0, T]$. Variables $\{\Gamma_n\}_{n \in \mathbb{N}}$ and $\{v_n\}_{n \in \mathbb{N}}$ are also mutually independent. Note that \mathbf{E}_1^{-1} can not be given explicitly (neither \mathbf{E}_1) thus an approximate evaluation should be developed. When the degree of freedom of χ^2 -tail probabilities tends to zero, its survival function tends to the exponential-integral function. Therefore, we adopt the inverse χ^2 -tail function in Matlab for simulation which is proposed in [177].

Since \mathbf{E}_1^{-1} is a decreasing function, for the feasibility of simulation, the series representation can be truncated. The arrival times of a standard Poisson process are generated until the arrival time Γ_n is greater than τ , because very large values of Γ_n relate to too small size of jump and have no significant contribution to the increment. An example of a sample path of a Gamma process using series representation is sketched in Figure 4.1. It can be seen that the sampling procedure is in three steps. Firstly in Figure 4.1.(a) the arrival times of a standard Poisson process $(\Gamma_n)_{n \in \mathbb{N}}$ (red circles on x-axis) are simulated which are used to calculate the jumps sizes (blue lines between circle and star) by applying the inverse tail integral of Levy measure of Gamma process. It should be noticed that the jump sizes are in decreasing order at this step and reduce quickly so only 17 points of Poisson process are depicted because the jump size becomes negligible after that. Figure 4.1.(b) shows that a jump time is allocated to each jump size using uniform random variables on $[0, 1]$. Finally, by combining the jump sizes obtained in Figure 4.1.(a) and jump times in Figure 4.1.(b), Figure 4.1.(c) illustrates the resulting increment path Gamma process.

In fact, other methods exist for simulating the Gamma process in a more simple but efficient way. Among those that are based on the series representations of Gamma process, the Bondesson's method is simpler as the involved functions are in closed form. The reason why we adopt the Inverse Lévy Measure Method in this work is that it is based on the Lévy measure where the Gamma process is totally determined by its Lévy measure. Therefore, this method can be easily extended to multi-dimension case when stochastic dependence is considered.

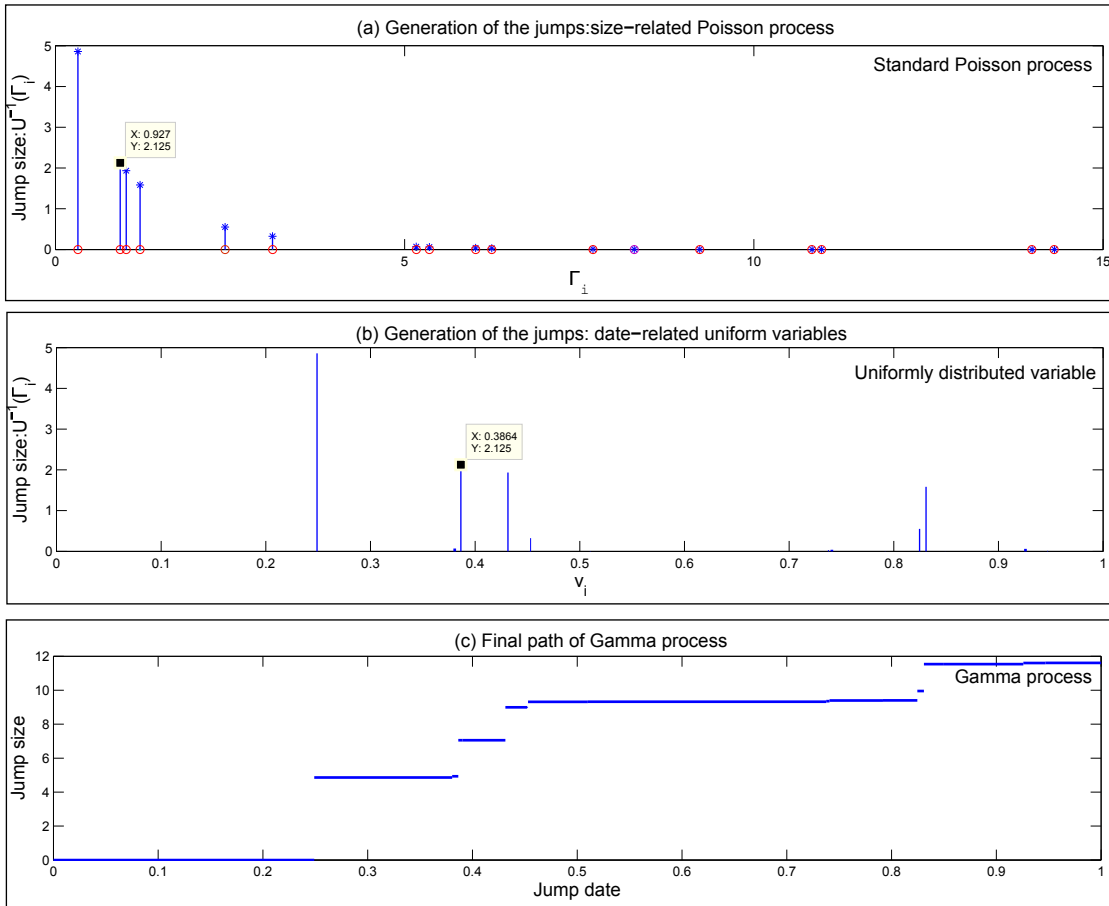


Figure 4.1: Example of series representations for gamma process on $[0,1]$ with $\alpha = 7/5$, $\beta = 1/5$

4.2.2 Symmetric Stochastic dependence modeling using Lévy copulas

When extended to multi-component systems, a multi-dimensional Gamma process which represents the state of system should be proposed and the stochastic dependence should be considered. As it has been reviewed in [chapter 2](#), while failure and degradation interaction models are focused on how a change in a component state impacts the other components, the inherent stochastic dependence due to a common cause of degradation is less investigated. Even though a bivariate Gamma process with common degradation part model (BGCD model) proposed by Mercier *et al.* in [82] can be used to describe such type of stochastic dependence, it is limited in some dependence degrees (e.g. the complete dependence can not be expressed). Moreover, BGCD model considers that the influence of a common cause is the same on all components whereas it may be different. Therefore, Lévy copulas which model the dependence between jumps is more suitable for multi-dimensional degradation processes case as well as its flexibility in a wide range of dependence structures and degrees. Since Lévy copulas are strongly related to ordinary copulas, we first introduce and review the main features of ordinary copulas.

We discuss a two-component system in this chapter, hence only two-dimensional copulas are presented but all results can be easily extended to d-dimensional copulas.

4.2.2.1 Dependence modeling with ordinary copulas

The word copula was first employed in a mathematical or statistical sense by Sklar in [178] to describe the functions that join together one-dimensional distribution functions to form multivariate distribution functions. In other words, copulas are the functions which contain the dependence information of multivariate distributions. In this way, multivariate distributions can be studied by their marginal distributions and the dependence. General introduction of copulas can be seen in [179]. In this section, the main properties of copulas are introduced as well as two copula functions.

Definition 4.1. A two-dimensional copula function C is a function defined from $[0, 1]^2$ to $[0, 1]$ who has the following properties:

1. C is grounded and 2-increasing.
2. For $u \in [0, 1]$ and $v \in [0, 1]$,

$$C(u, 1) = u, \quad (4.7)$$

and

$$C(1, v) = v. \quad (4.8)$$

Obviously, property 1 is a necessary and sufficient condition for the copula function to be a multivariate cumulative distribution function (cdf). Property 2 implies that the marginal distributions of C are both uniform. Hence copulas are joint cdfs on $[0, 1]^2$ with standard uniform marginal distributions.

Theorem 4.2. Sklar's Theorem. Let F be a joint distribution function with margins F_1, F_2 . Then there exists a copula C such that for all x_1, x_2 in R ,

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \quad (4.9)$$

If F_1 and F_2 are continuous, then C is unique; otherwise, C is uniquely determined on the product of range of F_1 and F_2 . Conversely, if C is a copula and F_1 and F_2 are distribution functions, then the function F defined by 4.9 is a joint distribution function with margins F_1 and F_2 .

This theorem is firstly proposed by Sklar in [178]. It shows that a multivariate distribution can be constructed from the pair of its marginal distributions and a given copula

function. All the dependence information is contained in the copula function. Consequently, it is important to choose an appropriate copula function for its underlying dependence.

Several copula functions have been proposed and widely studied such as Gaussian copula, student t copula and Archimedean copulas to name a few. In this work, we are interested in the Archimedean copulas which allow for a wide range of dependence and have the advantage of being known in closed form.

Definition 4.3. Let φ be a continuous, strictly decreasing function from $[0, 1]$ to $[0, \infty]$ such that $\varphi(1) = 0$, and let φ^{-1} be the inverse of φ . Let C be the function from $[0, 1]^2$ to $[0, 1]$ given by

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v)) \quad (4.10)$$

If φ is convex then any function C defined by Equation 4.10 is a copula function and it is said to be an Archimedean copula.

The function φ is called the generator of the Archimedean copula. The proof can be seen in [179] (Lemma 4.1.2.).

Among Archimedean copulas, two copulas are introduced and used in this work which are Gumbel copula and Clayton copula. The generator of Gumbel copulas is

$$\varphi(x) = (-\ln x)^\theta, \theta \in [1, \infty],$$

then

$$C(u, v) = \exp\{-[(-\ln u)^\theta + (-\ln v)^\theta]^{1/\theta}\}.$$

The generator of Clayton copulas is

$$\varphi(x) = x^{-\theta} - 1, \theta \in [-1, \infty] \setminus \{0\},$$

then

$$C(u, v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}.$$

In Figure 4.2, the scatterplots are shown with different values of θ . The figure shows that Clayton and Gumbel copulas have different dependence structure. In effect, Clayton copula has lower tail dependence where small values of variables are more correlated whereas Gumbel copula has upper tail dependence where dependence is stronger for large values of variables.

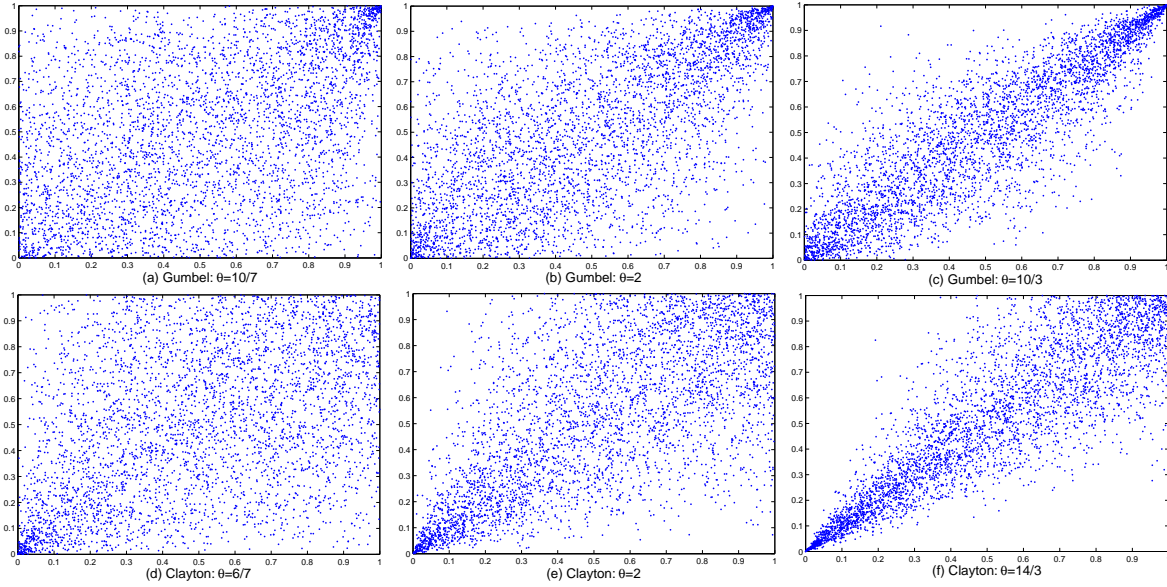


Figure 4.2: Scatterplots for Gumbel and Clayton copulas with different values of θ

4.2.2.2 Stochastic dependence modeling with Lévy copulas

In the previous section, we have reviewed the definition and main properties of ordinary copulas. For a two-dimensional Lévy process, the probability distribution at a specific time t can be characterized by the marginal distributions and an ordinary copula function C_t . However, since the marginal distributions depend on time, C_t may be also changed through time such that the infinitely divisible property is not guaranteed in this way. In order to model the dependence between Lévy processes, Lévy copulas are proposed in [92] to model the dependence structure in the field of Lévy measure instead of probability.

Definition 4.4. A two-dimensional tail integral of Lévy measure is a function U from $[0, \infty]^2$ to $[0, \infty]$ such that:

1. U is a d-increasing function.
2. U is equal to zero if one of its arguments is equal to ∞ .
3. U is finite everywhere except at zero and $U(0, 0) = \infty$.
4. $U(x_1, x_2) = v([x_1, \infty) \times [x_2, \infty))$, $x_1, x_2 \in (0, \infty)^2$ and v is a Lévy measure.
5. $U(x_1, 0) = U_1(x_1)$ and $U(0, x_2) = U_2(x_2)$. U_i is called the marginal tail integral of Lévy measure.

Therefore function U is similar to a survival function of a probability density function but the Lévy tail integral relates to the Lévy measure and it is not a bounded function.

Theorem 4.5. *Let U be a two-dimensional tail integral with margins U_1 and U_2 . There exists a positive Lévy copula \mathfrak{C} such that*

$$U(x_1, x_2) = \mathfrak{C}(U_1(x_1), U_2(x_2)) \quad (4.11)$$

If U_1 and U_2 are continuous, this Lévy copula is unique. Otherwise it is unique on $\text{Ran}U_1 \times \text{Ran}U_2$, the product of ranges of one-dimensional tail integrals. Conversely, if \mathfrak{C} is a positive Lévy copula and U_1, U_2 are one-dimensional tail integrals then Equation 4.11 defines a two-dimensional tail integral.

Notation C is reserved for ordinary copulas in the subsequent sections and \mathfrak{C} is reserved for Lévy copulas. The proof of this theorem can be seen in [92] and it is an extension of Sklar's theorem to the tail integrals of Lévy measure.

Theorem 4.6. *Let $(X_t)_{t \geq 0} = (X_t^1, X_t^2)_{t \geq 0}$ be a two-dimensional Lévy process with positive jumps having tail integral U and marginal tail integrals U_1 and U_2 . There exists a two-dimensional positive Lévy copula \mathfrak{C} which characterizes the dependence structure of $X_t = (X_t^1, X_t^2)$, that is, for all $x_1, x_2 \in [0, \infty)$,*

$$U(x_1, x_2) = \mathfrak{C}(U_1(x_1), U_2(x_2)) \quad (4.12)$$

If U_1 and U_2 are continuous, this Lévy copula is unique. Otherwise it is unique on $\text{Ran}U_1 \times \text{Ran}U_2$. Conversely, let $(X_t^1)_{t \geq 0}$ and $(X_t^2)_{t \geq 0}$ be two one-dimensional Lévy processes with positive jumps having tail integrals U_1 and U_2 and let \mathfrak{C} be a two-dimensional positive Lévy copula. Then there exists a two-dimensional Lévy process with Lévy copula \mathfrak{C} and marginal tail integrals U_1 and U_2 . Its tail integral is given by Equation 4.12.

This theorem states that the dependence between components of a multi-dimensional Lévy process can be modeled by using Lévy copulas to couple the marginal tail integral of Lévy measure. Thus the multivariate Lévy process models can be constructed by specifying separately jump dependence structure and one-dimensional margins.

Now come back to the series representations of Gamma process introduced in Section 4.2.1. When extended to a two-dimensional Gamma process $(X_t)_{t \geq 0} = (X_t^1, X_t^2)_{t \geq 0}$ with margin tail integrals U_1 and U_2 and Lévy copula \mathfrak{C} , the series representation is as follows:

$$\begin{aligned} X_t^1 &= \sum_{n=1}^{\infty} U_1^{-1}\left(\frac{\Gamma_n^1}{T}\right) \mathbb{1}_{[0,t]}(v_n), \\ X_t^2 &= \sum_{n=1}^{\infty} U_2^{-1}\left(\frac{\Gamma_n^2}{T}\right) \mathbb{1}_{[0,t]}(v_n). \end{aligned} \quad (4.13)$$

$(\Gamma_n^1)_{n \in \mathbb{N}}$ is a sequence of arrival times of a standard Poisson process. Conditionally to Γ_n^1 , Γ_n^2 has distribution function $\frac{\partial \mathfrak{C}(x,y)}{\partial x} |_{x=\Gamma_n^1}$. Therefore with a specific Lévy copula function, the two-dimensional Lévy process can be constructed. By analogy with Archimedean copulas introduced in Section 4.2.2.1 for the ordinary copulas, Archimedean Lévy copulas can be constructed. Since the domain and range of Lévy copula functions are different from that of ordinary copula functions, the generators should be defined to satisfy that: φ is a decreasing function defined on $[0, \infty)$, $\varphi(0) = \infty$ and $\varphi(\infty) = 0$.

In order to meet the above conditions, the generator of Gumbel Lévy copula can be given as

$$\varphi(x) = (\ln(x+1))^{-\theta}, \theta \in (0, \infty]$$

then

$$\mathfrak{C}(u, v) = \exp\{[(\ln(u+1))^{-\theta} + (\ln(v+1))^{-\theta}]^{-\frac{1}{\theta}}\} - 1.$$

Similarly, the generator of Clayton copula is

$$\varphi(x) = x^{-\theta}, \theta \in (0, \infty]$$

then

$$\mathfrak{C}(u, v) = (u^{-\theta} + v^{-\theta})^{-\frac{1}{\theta}}.$$

In the next section, we discuss simulation methods focusing on Clayton Lévy copula as an example for convenience purposes.

4.2.3 Simulating bivariate Gamma processes with Lévy copulas dependence

In order to simulate a bivariate Gamma process with Lévy copulas dependence, two simulation methods are introduced in this section. The first one is based on the conditional copula function and the second is based on the Marshall-Olkin algorithm.

4.2.3.1 Conditional distribution method

Simulating $(\Gamma_n^1, \Gamma_n^2)_{n \in \mathbb{N}}$ directly from Equation 4.13 needs to calculate the first derivative of copula function \mathfrak{C} with respect to its first variable. When the Lévy copula is in Clayton family with the form $\mathfrak{C}(u, v) = (u^{-\theta} + v^{-\theta})^{-\frac{1}{\theta}}$, then the conditional distribution function is:

$$\begin{aligned} \mathfrak{C}(v|u) &= \partial \mathfrak{C}(u, v) / \partial u \\ &= (1 + (u/v)^\theta)^{-(\theta+1)/\theta}. \end{aligned} \tag{4.14}$$

Since $\mathfrak{C}(u, v)$ is a two-dimensional positive Lévy copula, $\mathfrak{C}(v|u)$ exists for all values of $u \geq 0$ and is continuous for all $v \in [0, \infty]$ moreover $0 \leq \mathfrak{C}(v|u) \leq 1$. Thus with conditional method, the arrival times $(\Gamma_n^1)_{n \in \mathbb{N}}$ of a standard Poisson process are first generated and then Γ_n^2 is sampled from Γ_n^1 as:

$$\Gamma_n^2 = \mathfrak{C}^{-1}(\Gamma_n^2 | \Gamma_n^1) = \Gamma_n^1 (y^{-\theta/(1+\theta)} - 1)^{-1/\theta}, \quad (4.15)$$

where y is a random variable uniformly distributed on $[0, 1]$. Algorithm 1 is then developed.

Algorithm 1 Simulating two-dimensional Gamma process modeled by Clayton Lévy copula by the conditional distribution method

Input: $\tau, \theta, T, \alpha_1, \alpha_2, \beta_1, \beta_2$

$n \leftarrow 0, \Gamma_0^1 \leftarrow 0$

while $\Gamma_n^1 \leq \tau$ **do**

$n \leftarrow n + 1$

$w \leftarrow \exp(1)$

$\Gamma_n^1 \leftarrow \Gamma_{n-1}^1 + w$

$y \leftarrow \text{rand}(1)$

$\Gamma_n^2 \leftarrow \Gamma_n^1 (y^{-\theta/(1+\theta)} - 1)^{-1/\theta}$

$v_n \leftarrow \text{rand}(1)$

end while

$N \leftarrow n$

Output: $X_t^1 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^1}{\alpha_1 T}) / \beta_1 \mathbb{1}_{[0, t]}(v_n), X_t^2 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^2}{\alpha_2 T}) / \beta_2 \mathbb{1}_{[0, t]}(v_n)$

As the series representations are truncated for the reason of simulation time, the convergence of Algorithm 1 is slow regarding X_t^2 when the parameter θ is small related to a weak dependence. The reason is that generating enough significant jumps of component 2 from those of component 1 is more difficult when the dependence between components is weak. Indeed the stopping criterion of Algorithm 1 ($\Gamma_n^1 \leq \tau$) relates to the required precision. It ensures that all jumps generated for component 1 are greater than $U_1^{-1}(\tau)$. By properly setting τ , the mean degradation speed of component 1 can be controlled and maintained at the desired value. Conversely, no precision is ensured for component 2: when generating Γ_n^2 using Equation 4.15, some big and/or medium-size jumps can be missing for component 2. Thus, the performance of Algorithm 1 depends on the combination of (θ, T, τ) (see [180]). The more θ is low and the more τ/T should be set to a high value in order to control not only the mean degradation speed of component 1 but also that of component 2. Anyway, it is difficult to get convergence when $\theta < 1$ and in this case Algorithm 1 is not recommended to use.

4.2.3.2 Marshall-Olkin algorithm

In [181], another method is proposed to construct a random vector (u, v) with distribution function given by the Archimedean copula. According to Marshall-Olkin algorithm, a two-dimensional random vector (U_1, U_2) subjected to Archimedean copulas can be simulated in three steps:

1. First simulate a common variable V with distribution function G such that its Laplace function is φ^{-1} in Equation 4.10 (inverse of the generator).
2. Then simulate two individual independent uniformly distributed variables X_1 and X_2 .
3. Finally set U_i ($i = 1, 2$) equal to $\varphi(-\ln(X_i/V))$.

Similar algorithms have been derived for sampling Archimedean Lévy copulas (see [180]). In particular, Algorithm 2 below can be used to simulate a two-dimensional Gamma process with dependence modeled by Clayton Lévy copula.

Algorithm 2 A Marshall-Olkin type algorithm for sampling a two-dimensional Gamma with dependence modeled by Clayton Lévy copula

Input: $\tau, \theta, T, \alpha_1, \alpha_2, \beta_1, \beta_2$

$n \leftarrow 0, \omega_0^1 \leftarrow 0$

while $\omega_n^1 \leq \tau$ **do**

$n \leftarrow n + 1$

$w \leftarrow \exp(1)$

$\omega_n^1 \leftarrow \omega_{n-1}^1 + w$

$y \leftarrow \text{rand}(1)$

$z_n \leftarrow (\omega_n \Gamma(\frac{1}{\theta}) / \theta)^\theta$

$x_n \leftarrow \exp(1), y_n \leftarrow \exp(1)$

$v_n \leftarrow \text{rand}(1)$

$\Gamma_n^1 = (x_n/z_n)^{-\frac{1}{\theta}}, \Gamma_n^2 = (y_n/z_n)^{-\frac{1}{\theta}}$

end while

$N \leftarrow n$

Output: $X_t^1 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^1}{\alpha_1 T}) / \beta_1 \mathbb{1}_{[0,t]}(v_n), X_t^2 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^2}{\alpha_2 T}) / \beta_2 \mathbb{1}_{[0,t]}(v_n)$

As a result, components are processed symmetrically when simulated by Algorithm 2 so that the resulting precision is similar for both components. Algorithm 2 can be used even if the dependence degree between components is weak however practical use of the Marshall-Olkin type algorithms is limited to cases where the underlying function G can be deduced from its Laplace transform φ^{-1} (G is a probability distribution function or a more general mass function depending on whether we consider an ordinary copula or a Lévy copula). For the Clayton Lévy copula, function G is $G(x) = \frac{\theta x^{\frac{1}{\theta}}}{\Gamma(\frac{1}{\theta})}$. Some cases of Lévy copulas are detailed in [180] but for other Archimedean copula function, function

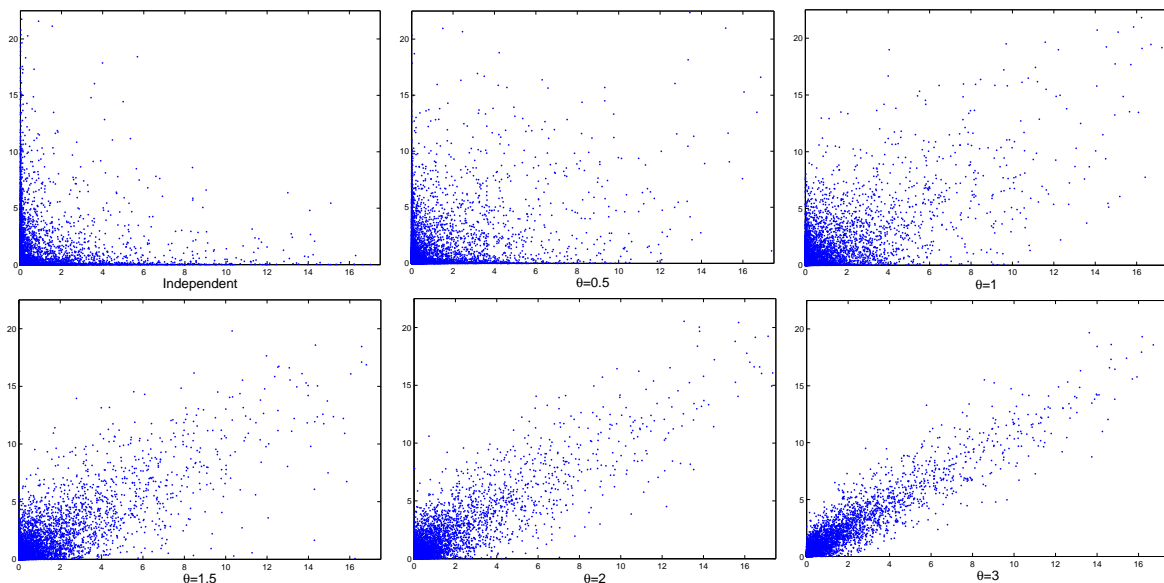


Figure 4.3: Scatterplots of the increments two-dimensional Gamma process on $[0, 0.1]$ modeled by Clayton Lévy copula with different values of θ and $\alpha_1 = 7/5$, $\alpha_2 = 9/5$, $\beta_1 = \beta_2 = 1/5$

G may be very difficult to derive.

Finally, Figure 4.3 is given which illustrates the dependence of the increments of a two-dimensional Gamma process on $[0, 0.1]$. It indicates that unlike Clayton ordinary copula, Clayton Lévy copula has stronger dependence for the big jumps which can better represent the reality. In fact, when a component suffers a very significant damage (e.g. a big jump occurs), the impact on the other components can be stronger as they try to reach a new balance working state. Figure 4.4 illustrates the degradation paths of components over time and it indicates that when the greater θ is, the stronger dependence exists between components such that the degradation paths are more similar.

4.3 Maintenance policies

In this section, we propose some maintenance policies for a two-component system when individual degradation of components is modeled by a Gamma process and dependence between components by a Clayton Lévy copula. Firstly classic maintenance policies such as periodic and condition-based maintenance policies are studied for this type of two-component system. Then a grouping strategy is developed in order to take advantage of the economic dependence between components.

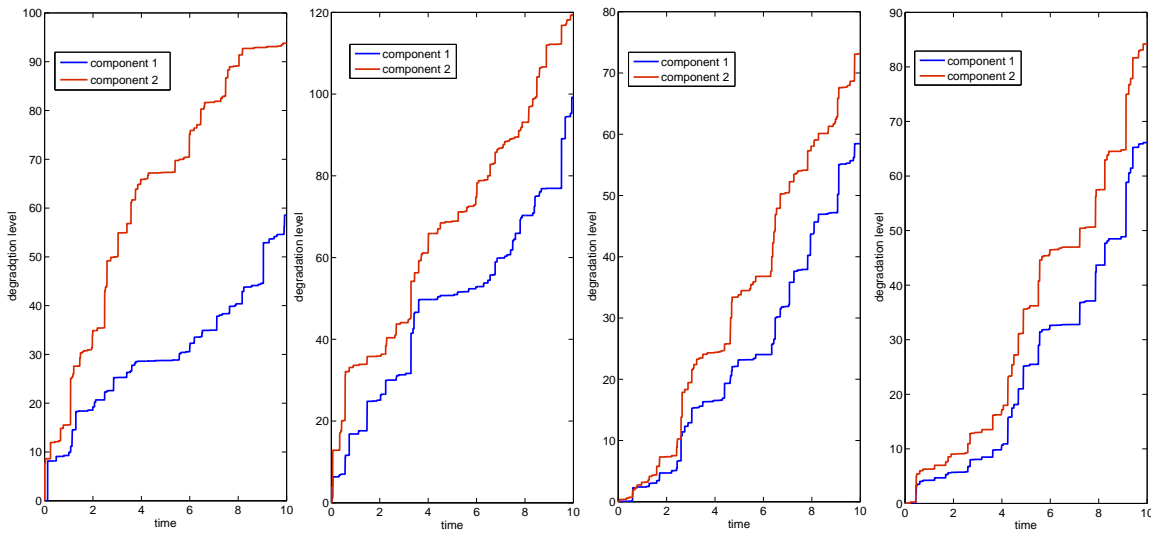


Figure 4.4: Evolution of degradation over time for components modeled by Gamma process with Clayton Lévy copula with different values of $\theta = 0.4, 0.8, 1.6, 6$ and $\alpha_1 = 7/5, \alpha_2 = 9/5, \beta_1 = \beta_2 = 1/5$

4.3.1 Main assumptions and cost model

For all considered maintenance policies, we make the following assumptions:

- Each component degrades gradually and when its degradation level exceeds a preset corrective threshold L , it is considered as failed and a corrective replacement is needed. When the component is replaced before failure, replacement is said to be a preventive replacement. Both replacements (preventive and corrective) are assumed to be perfect and instantaneous (the duration of maintenance is negligible).
- The degradation level of each component can be obtained only by inspection. Besides, if the component fails, it remains unavailable until its next inspection by then it is replaced. We assume that each inspection reveals the degradation level of component perfectly.
- The maintenance cost criterion depends on the following unitary costs:
 - A preventive maintenance cost ($c_p^i, i = 1, 2$) if a preventive maintenance is carried out.
 - A corrective maintenance cost ($c_c^i, i = 1, 2$) if a failure happens.
 - A penalty cost per time unit (c_u) if one component fails but the other one is still working.

- A downtime unitary cost per time unit (c_d) when the system is down.
- An inspection cost (c_{insp}) if an inspection operation is executed

To evaluate maintenance policies, we use the criterion of long-run expected cost of system per time unit which is defined as:

$$C_\infty = \lim_{t \rightarrow \infty} \frac{E[c(t)]}{t}.$$

The cumulative cost of system $c(t)$ depends on the number of preventive and corrective replacements ($N_P^i(t)$, $N_C^i(t)$), the number of inspections ($N_I^i(t)$), the individual cumulative downtime of components ($u_i(t)$), $i = 1, 2$ and the cumulative downtime of system ($D(t)$). Therefore, $c(t)$ can be written as:

$$c(t) = \sum_{i=1}^2 (c_p^i N_P^i(t) + c_c^i N_C^i(t) + c_{insp} N_I^i(t) + u_i c_u(t)) + c_d D(t). \quad (4.16)$$

To investigate the influence of economic dependence, we consider the cost of any replacement (both preventive and corrective) to consist of two parts: a set-up cost c_r which can be shared, mainly incurred by human activities and an individual cost for each component incurred by materials consuming. In this way, two individual replacements (either corrective or preventive) incur two c_r while simultaneous replacement of both components incurs a single c_r . Therefore a set-up cost c_r is saved as soon as two maintenance actions are carried out simultaneously. Obviously, taking account of set-up cost will affect the optimization of maintenance. When the set-up cost is significant, the maintenance policy tends to find an appropriate replacement date by delaying the preventive maintenance of one component or bringing forward the replacement of the other one. Therefore, the maintenance cost on $[0, t]$ is reduced by the number of grouped maintenance actions $N_g(t)$ multiplied by the set-up cost as in the following equation:

$$c(t) = \sum_{i=1}^2 (c_p^i N_P^i(t) + c_c^i N_C^i(t) + c_{insp} N_I^i(t) + u_i c_u(t)) + c_d D(t) - c_r N_g(t). \quad (4.17)$$

4.3.2 Classic maintenance policies

4.3.2.1 Periodic maintenance policy

For periodic maintenance policy, denoted as *Policy 4* in this work, component i ($i = 1, 2$) are replaced every TB_i time units in order to avoid failures. At time nTB_i ($n \in \mathbb{N}$), if component i is failed, a corrective maintenance cost is incurred otherwise a preventive maintenance cost is incurred. To minimize the global maintenance cost, the parameters

(TB_1, TB_2) have to be optimized. For single-component systems, periodic maintenance policy can be optimized using the renewal reward theorem if the component returns to be as good as new after maintenance. In this case, the average long-run maintenance cost per unit of time for single-component system with perfect maintenance, denoted by EC , is the expected maintenance cost incurred in a renewal cycle divided by the preventive replacement TB such as:

$$\begin{aligned}
EC &= \lim_{t \rightarrow \infty} E(c(t))/t = E(c(TB))/TB \\
&= \{P(X_{TB} > L)c_c + P(X_{TB} < L)c_p + E(TB - T_L \mathbf{1}_{\{TB > T_L\}}c_u)\}/TB \quad (4.18) \\
&= c_p + (c_c - c_p) \int_L^\infty f_{\alpha TB, \beta}(x)dx + \int_0^{TB} \int_L^\infty f_{\alpha s, \beta}(x)dx ds \}/TB,
\end{aligned}$$

where $f_{\alpha TB, \beta}(x) = x^{\alpha TB - 1} \exp(-\beta x) \beta^{\alpha TB} / \Gamma(\alpha TB)$ is the density function of Gamma distribution, T_L denotes the time when degradation level of component exceeds L and X_t stands for the degradation level of component at time t without maintenance actions (initial degradation level at $t = 0$ is assumed to be zero). For two-component systems, the renewal cycle is long and can be regarded as the moment when two components are replaced simultaneously that is the least common multiple of TB_1 and TB_2 noted as LCM . An example is given below when $TB_1 = 10$ and $TB_2 = 6$ then $LCM=30$:

$$\begin{aligned}
EC &= \lim_{t \rightarrow \infty} E(c(t))/t = E(c(LCM))/LCM \\
&= \sum_{i=1}^2 \frac{c_p + (c_c - c_p) \int_L^\infty f_{\alpha_i TB_i, \beta_i}(x)dx}{TB_i} + \frac{\int_0^{LCM} P(\tilde{X}_u^1 > L, \tilde{X}_u^2 > L)du c_d}{LCM} \quad (4.19) \\
&\quad + \frac{(\int_0^{LCM} P(\tilde{X}_u^1 > L, \tilde{X}_u^2 < L)du + \int_0^{LCM} P(\tilde{X}_u^1 < L, \tilde{X}_u^2 > L)du)c_u}{LCM}
\end{aligned}$$

When \tilde{X}_t^i is used to denote the degradation level of component i ($i = 1, 2$) with maintenance so that \tilde{X}_t^i returns to zero after maintenance action. Therefore the joint distribution of two-component are involved. When two components are independent with $TB_1 = 10$ and $TB_2 = 6$ then $LCM=30$, the expected downtime cost of system within a

renewal cycle of system can be calculated as follows:

$$\begin{aligned}
\mathbb{E}(LCM - u\mathbb{1}_{\{\tilde{X}_u^1 > L, X_u^2 > L\}})c_d &= \int_0^{30} P(\tilde{X}_u^1 > L, \tilde{X}_u^2 > L)du \\
&= \left\{ \int_0^6 P(X_u^1 > L)P(X_u^2 > L)du + \int_6^{10} P(X_u^1 > L)P(X_{u-6}^2 > L)du \right. \\
&+ \int_{10}^{12} P(X_{u-10}^1 > L)P(X_{u-6}^2 > L)du + \int_{12}^{18} P(X_{u-10}^1 > L)P(X_{u-12}^2 > L)du \quad (4.20) \\
&+ \int_{18}^{20} P(X_{u-10}^1 > L)P(X_{u-18}^2 > L)du + \int_{20}^{24} P(X_{u-20}^1 > L)P(X_{u-18}^2 > L)du \\
&\left. + \int_{24}^{30} P(X_{u-20}^1 > L)P(X_{u-24}^2 > L)du \right\}c_d
\end{aligned}$$

where $P(X_u^i > L) = \int_L^\infty f_{\alpha_i u, \beta_i}$ ($i = 1, 2$). Equation 4.20 shows that even for independent case, the calculation of maintenance cost is more complex for two-component systems than single-unit systems. Thus the Monte-Carlo simulation method is used in the following to optimize (TB_1, TB_2) for periodic maintenance policy (*Policy 4*).

4.3.2.2 Condition-based maintenance policy

With periodic replacement policy, component is replaced without identifying its real degradation level which may lead to unnecessary replacement when component is at a low degradation level. Some condition-based maintenance strategies are now proposed to replace each component according to its degradation level instead of its age. Health condition of components is supposed to be monitored through periodic inspections every ΔT time units. The degradation level is supposed to be revealed perfectly at the inspection moment. If the revealed degradation level exceeds the corrective threshold L , the component is correctively maintained; otherwise, in order to avoid failures, a preventive replacement is carried out as soon as the revealed degradation level exceeds a preventive threshold M . To this end, condition-based maintenance policy for single-unit systems, with parameters $(\Delta T, M)$ should be optimized. When ΔT is too small, expensive inspection costs are incurred whereas it can not perform well in avoiding failures; when M is small, expensive preventive maintenance cost are incurred.

To calculate the average long-run maintenance cost per time unit, the semi-regenerative theory can be used as after each inspection the component state is perfectly known. The single-unit system case is studied and developed in [29]. When extended to two-component systems, an inspection scheme with two individual periods combined with a decision rule based on two individual replacement thresholds (parameters $(\Delta T_1, \Delta T_2, M_1, M_2)$) is first considered. In order to investigate the influence of a joint parameter (either a

joint inspection period or a joint replacement threshold) for the components in multi-component systems, we propose following maintenance policies. In all cases parameters are optimized using Monte Carlo simulations.

Policy 1 Condition-based maintenance policy with two independent preventive thresholds (M_1, M_2) and inspection periods $(\Delta T_1, \Delta T_2)$.

Policy 2 Condition-based maintenance policy with two independent inspection periods $(\Delta T_1, \Delta T_2)$ and a joint preventive threshold M .

Policy 3 Condition-based maintenance policy with two independent preventive thresholds (M_1, M_2) and a joint inspection period ΔT .

Policy 2 is implemented to investigate the sensitivity to the preventive threshold and *Policy 3* is implemented to investigate the sensitivity to the inspection period. Indeed when the number of components in the multi-component systems increases, the optimization time will increase dramatically if the inspection scheme is implemented with individual parameters for all components.

All the maintenance policies proposed so far are a simple extension from the single-unit systems and they are not structurally designed for grouping maintenance actions. Two components can be maintained simultaneously in some cases, e.g., if the selected values of ΔT_1 and ΔT_2 allow some inspections of components 1 and 2 to take place at the same time and additionally if the degradation levels of both components have exceeded their own preventive threshold at such common inspection times. As the set-up cost increases, these policies favour the appropriate parameters values especially the parameters ΔT_i ($i = 1, 2$) to gain more opportunities for maintenance grouping. Even though some maintenance actions can take place simultaneously in classic maintenance policies, a grouping strategy should be developed to take full advantage of economic dependence.

4.3.3 A new condition-based maintenance policy with grouping opportunities

From a point of view on $[0, t]$, when two components are maintained closely, it would be beneficial to maintain them together as one set-up cost can be saved. Thus, in order to find the opportunities to group maintenance actions, a new decision rule is proposed. The maintenance policy proposed in this section is denoted as *Policy 0*.

As the state of component will be revealed by inspection, we only take the inspection moment as decision time. Suppose that at time $n\Delta T_1$ (date of n^{th} inspection of component 1), the state of component 1 exceeds the preventive threshold M_1 . Then,

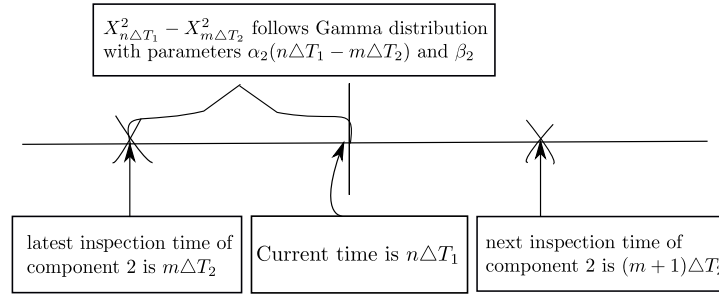


Figure 4.5: Sketch of inspection time

according to the condition-based maintenance decision rule, a preventive maintenance action is triggered. On one hand, if the degradation level of component 2 also exceeds the preventive threshold M_2 at this time, we could preempt the preventive maintenance of component 2. On the other hand, if the next inspection time $(m+1)\Delta T_2$ (date of $(m+1)^{th}$ inspection of component 2) is closed to $n\Delta T_1$, it is also feasible to postpone the preventive maintenance of component 1. Since the state of component 2 is unknown at time $n\Delta T_1$, we have to consider two cases as follows (see Figure 4.5):

1. Set $p_a = P(X_{n\Delta T_1}^2 > M_2 | X_{m\Delta T_2}^2 = x)$ be the probability that component 2 would need a preventive replacement at time $n\Delta T_1$ (the state of component 2 is updated at time $m\Delta T_2 < n\Delta T_1$ and denoted as x). If $p_a \rightarrow 1$, then component 2 will probably be replaced soon at time $(m+1)\Delta T_2$. In this case, termed preempting case, the replacement of component 2 is preempted in order to save one set-up cost as well as avoid the failure of component 2 before $(m+1)\Delta T_2$. However such an action may replace component 2 too early such that it leads to an increase of preventive maintenance cost. Parameter p_a only depends on the last known degradation level of component 2, it is computed as follows:

$$\begin{aligned} p_a &= P(X_{n\Delta T_1}^2 > M_2 | X_{m\Delta T_2}^2 = x) \\ &= P(X_{n\Delta T_1 - m\Delta T_2}^2 > M_2 - X_{m\Delta T_2}^2 | X_{m\Delta T_2}^2 = x), \end{aligned}$$

where $X_{n\Delta T_1 - m\Delta T_2}^2$ follows Gamma distribution with parameter $(n\Delta T_1 - m\Delta T_2)\alpha_2$ and β_2 . Therefore $p_a = \bar{F}_{(n\Delta T_1 - m\Delta T_2)\alpha_2, \beta_2}(M_2 - x)$, where

$$\bar{F}_{(n\Delta T_1 - m\Delta T_2)\alpha_2, \beta_2}(x) = \int_x^{+\infty} f_{(n\Delta T_1 - m\Delta T_2)\alpha_2, \beta_2}(u) du. \quad (4.21)$$

If the preventive maintenance of component 2 is preempted, the change of maintenance cost can be expressed as:

$$\Delta c(t) = c_r - c_{penalty} P(X_{n\Delta T_1}^2 < M_2), \quad (4.22)$$

where $c_{penalty}$ is the penalty cost which contains two parts: the gain from avoiding the failure of component 2 before time $(m+1)\Delta T_2$ and the loss due to shortening its lifetime. In order to minimize the expected maintenance cost, $\Delta c(t)$ should be positive. Under this condition,

$$\begin{aligned}\Delta c(t) &= c_r - c_{penalty}P(X_{n\Delta T_1}^2 < M_2) > 0, \\ p_a &= P(X_{n\Delta T_1}^2 > M_2) > \frac{c_{penalty} - c_r}{c_{penalty}}.\end{aligned}$$

Obviously $c_{penalty}$ is hard to be derived accurately but we can use a probability threshold PA to cope with it by implementing the test: $p_a > PA$. In other words, when the value of p_a is greater than a selected probability threshold PA ($0 \leq PA \leq 1$), we consider to preempt the replacement of component 2. Then PA is optimized using Monte Carlo simulations for different values of set-up cost.

2. Set $p_p = P(X_{(m+1)\Delta T_2}^1 > L | X_{n\Delta T_1}^1 = y)$ be the probability that component 1 will fail before the inspection time $(m+1)\Delta T_2$ (current state of component 1 is updated and denoted as y). If $p_p \rightarrow 0$, then component 1 will almost impossibly fail before time $(m+1)\Delta T_2$. In this case, termed postponing case, the replacement of component 1 can be postponed to time $(m+1)\Delta T_2$ in order to save one set-up cost as well as extend the working hours of component 1. However such an action may take the risk that component 1 fails before it is replaced whereas the cost of corrective replacement is more expensive than that of preventive replacement. Similarly to case 1, $p_p = \overline{F}_{((m+1)\Delta T_2 - n\Delta T_1)\alpha_1, \beta_1}(L - y)$ and

$$\begin{aligned}\Delta c(t) &= c_r - c'_{penalty}P(X_{(m+1)\Delta T_2}^1 > L) > 0, \\ p_p &= P(X_{(m+1)\Delta T_2}^1 > L) < \frac{c_r}{c'_{penalty}},\end{aligned}$$

where $c'_{penalty}$ represents the penalty cost due to postpone the preventive maintenance action of component 1. We can optimize a probability threshold PP ($0 \leq PP \leq 1$) with the test: $p_p < PP$ in the same way.

For general case,

$$p_a = \overline{F}_{(n\Delta T_i - m\Delta T_j)\alpha_j, \beta_j}(M_j - x). \quad (4.23)$$

$$p_p = \overline{F}_{((m+1)\Delta T_j - n\Delta T_i)\alpha_i, \beta_i}(L - y). \quad (4.24)$$

where $X_{m\Delta T_j}^j = x$, $X_{n\Delta T_i}^i = y$, current inspection time is $n\Delta T_i$ and $M_i < X_{n\Delta T_i}^i$.

Note that condition $p_a \rightarrow 1$ should be checked first because if p_a is too small, it would be useless to postpone the replacement of component 1 as component 2 will be rarely replaced at next inspection time. As a result, we give priority to the verification of

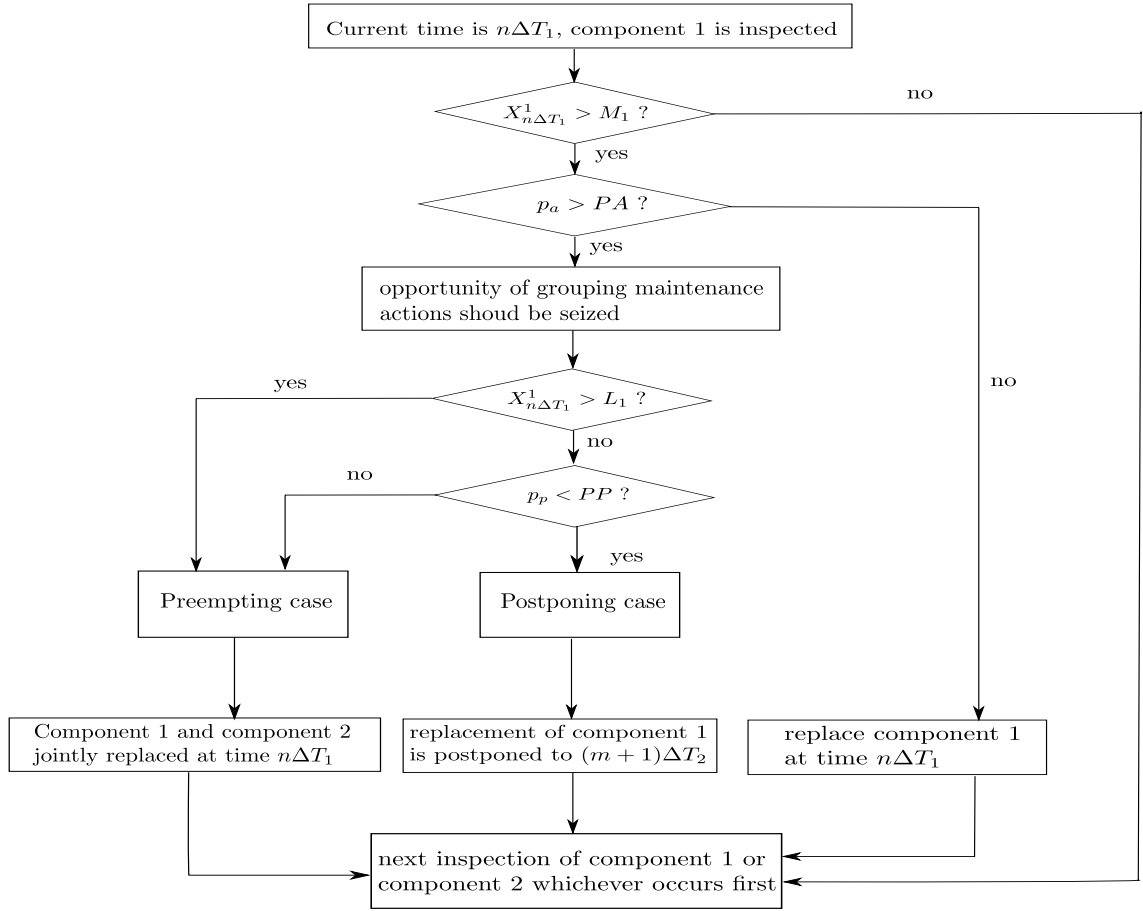


Figure 4.6: Sketch of decision rule for the example of component 1 is inspected at time $n\Delta T_1$

$p_a \rightarrow 1$ condition and define our decision rule in Figure 4.6. Pay attention that the replacement of component 1 can not be postponed if $X_{n\Delta T_1}^1 > L$.

Condition 1 is met if p_a is greater than PA whereas condition 2 is met when p_p is smaller than PP (see Figure 4.6). When PA is small, condition 1 will be often met which leads to more opportunities to group maintenance tasks. When PP is small, preempting the replacement of component 2 is more preferable than postponing that of component 1.

4.4 Numerical experiments

4.4.1 Description of the case studies

In this section, we investigate three cases of parallel systems related to the diversity of individual degradation behavior of components and to the global degradation speed of the multi-unit system. In terms of the degradation behavior of components, the “quasi-homogeneous” case corresponds to a system with two quite similar components whereas the “heterogeneous” case is related to a system with greater diversity between

Table 4.1: three cases of systems

		α	β	slow degradation	fast degradation
quasi-homogeneous	component 1	7/5	1/5	L=180	L=50
	component 2	9/5	1/5		
heterogeneous	component 1	5/7	1/7		
	component 2	9/5	1/5		

Table 4.2: unitary cost parameter

$c_p^i(i = 1, 2)$	$c_c^i(i = 1, 2)$	c_u	c_d
20	100	10	100

components. Fast and slow degrading sub-cases differ on the limit degradation level that should not be exceeded. The parameters of three cases are shown in Table 4.1 and Table 4.2 details the corresponding cost parameters. It is worth mentioning that the values of unitary costs are in arbitrary units, only the proportion of each type of maintenance cost is of interest for our study. In a general way, the corrective maintenance is much more expensive than the preventive one as the unpredicted failures often cause incalculable loss in practice while inspecting component costs much less than replacing it. Moreover, the values of dependence degree between components are involved in subsequent sections so as to investigate the influence of stochastic dependence.

4.4.2 Heterogeneous fast degrading system (*HTFD* system)

In *HTFD* system, component 2 degrades nearly twice as fast as component 1. To investigate the influence of economic dependence, the set-up cost varies from 0 to 5. The long-run maintenance cost is evaluated by Monte-Carlo simulation. In terms of optimization for preventive thresholds and inspection periods, a grid search method is used for *Policy* 1, 2, 3 and 4. It should be noted that *Policy* 0 is optimized by genetic algorithm considering that there are 6 parameters to manage so the grid search method will be time-consuming. A relatively strong dependence exists between components with $\theta = 2$ (*kendall's tau* ≈ 0.64). After implementing *Policy* 0, 1, 2, 3 and 4, the results are shown in Tables 4.3 4.4 where LC_i ($i = 1, 2$) refers to the mean lifecycle length of component i , PG_i refers to the proportion of grouped replacements among replacements of component i and $UNAV$ refers to the system's unavailability.

For this fast degrading system, components tend to fail early and the variance of failure dates is large. Consequently, in order to avoid failures *Policy* 4 has to replace components early (the values of $LC1/LC2$ with *Policy* 4 are smaller than those with other

Table 4.3: optimal parameters for *HTFD* system

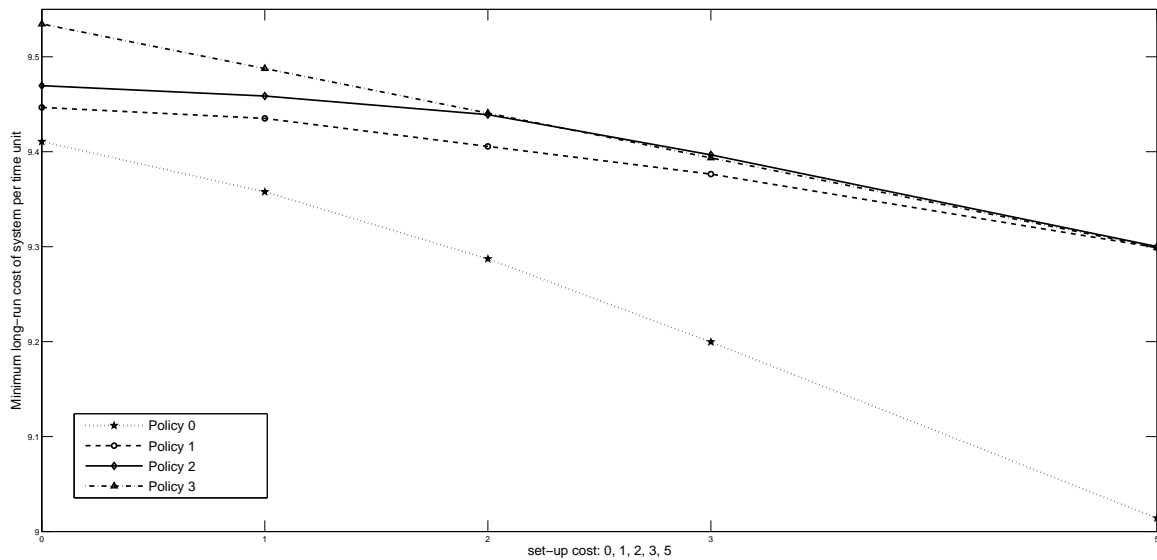
cost parameters	$C_p=20$	$C_c=100$	$C_{insp}=0.3$	$C_u=10$	$C_d=100$	$\theta=2$			
component	$\alpha_1=5/7$		$\beta_1=1/7$		$L=50$				
	$\alpha_2=9/5$		$\beta_2=1/5$						
set-up cost	policies	minimum cost	$LC1/LC2$	$\Delta T_1/TB_1$	$\Delta T_2/TB_2$	M_1	M_2	PA	PP
0	<i>Policy 0</i>	9.4107	7.50/4.29	1	0.55	32.2	33.6	0.13	0.01
	<i>Policy 1</i>	9.4467	7.58/4.30	1	0.55	31.8	33.6		
	<i>Policy 2</i>	9.4696	7.65/4.17	1	0.60	32.2	32.2		
	<i>Policy 3</i>	9.5346	7.65/4.20	0.75	0.75	32.8	31.8		
	<i>Policy 4</i>	12.7667	5.65/3.30	5.65	3.30				
1	<i>Policy 0</i>	9.3578	7.33/4.25	1	0.60	31.6	33.1	0.10	0.01
	<i>Policy 1</i>	9.4351	7.58/4.29	1	0.50	31.8	33.8		
	<i>Policy 2</i>	9.4587	7.65/4.17	1	0.60	32.2	32.2		
	<i>Policy 3</i>	9.4876	7.65/4.20	0.75	0.75	32.8	31.8		
	<i>Policy 4</i>	12.7305	5.50/3.30	5.50	3.30				
2	<i>Policy 0</i>	9.2871	7.34/4.27	1.2	0.55	32.3	33.5	0.07	0.01
	<i>Policy 1</i>	9.4056	7.38/4.27	1.1	0.55	30.6	33.4		
	<i>Policy 2</i>	9.4391	7.85/4.23	1	0.50	33.2	33.2		
	<i>Policy 3</i>	9.4407	7.65/4.20	0.75	0.75	32.8	31.8		
	<i>Policy 4</i>	12.6263	6.10/3.05	6.10	3.05				
3	<i>Policy 0</i>	9.1996	7.35/4.23	1	0.55	33.0	33.2	0.05	0.01
	<i>Policy 1</i>	9.3764	7.38/4.27	1.1	0.55	30.6	33.4		
	<i>Policy 2</i>	9.3967	7.45/4.20	0.75	0.75	31.8	31.8		
	<i>Policy 3</i>	9.3936	7.65/4.15	0.75	0.75	32.8	31.4		
	<i>Policy 4</i>	12.4624	6.10/3.05	6.10	3.05				
5	<i>Policy 0</i>	9.0141	6.92/4.27	1.35	0.55	33.1	33.6	0.03	0.01
	<i>Policy 1</i>	9.2991	7.65/4.15	0.75	0.75	32.8	31.4		
	<i>Policy 2</i>	9.3001	7.45/4.20	0.75	0.75	31.8	31.8		
	<i>Policy 3</i>	9.2991	7.65/4.15	0.75	0.75	32.8	31.4		
	<i>Policy 4</i>	12.1313	5.90/2.95	5.90	2.95				

policies). Thus, condition-based maintenance policies have better performance for fast degrading systems.

Figure 4.7 illustrates the minimum long-run maintenance cost of condition-based policies with different values of set-up cost. *Policy 1* is better than *Policy 2* and *3* because components degrade differently and a joint parameter (inspection period or preventive threshold) is not relevant. As shown in Figure 4.7, the curve of *Policy 0* is nonlinear which illustrates that *Policy 0* can take more and more advantage of economic dependence. In fact, by controlling the values of PA and PP , *Policy 0* provides more opportunities for maintenance grouping by preempting replacements of component 1 and offers flexible replacement dates as it can be seen from slightly lower values of LC_1 for *Policy 0* than for *Policy 1* even when the selected inspection periods are similar in both policies. That's why in Table 4.4, the value of PG_i ($i = 1, 2$) increases with set-up cost and it is much higher when *Policy 0* is applied rather than another condition-based maintenance policy. Moreover, when set-up cost is 0, *Policy 0* still performs better than *Policy 1*. For the reason of strong dependence, when a big jump occurs for component 2, component 1 would also be affected. In such cases, preempting component 1 can avoid some of its failures. Meanwhile, *Policy 0* provides flexible replacement date for

Table 4.4: Proportion of grouped maintenance actions and system unavailability of maintenance policies for HTFD system

	set-up cost	<i>Policy 0</i>	<i>Policy 1</i>	<i>Policy 2</i>	<i>Policy 3</i>	<i>Policy 4</i>
<i>PG</i> ₁	0	35.97%	2.06%	8.31%	35.88%	1.51%
	1	44.47%	20.35%	8.31%	35.88%	33.33%
	2	57.43%	21.52%	20.66%	35.88%	100%
	3	62.65%	21.52%	35.96%	36.13%	100%
	5	78.59%	36.13%	35.96%	36.13%	100%
<i>PG</i> ₂	0	21.69%	1.17%	4.52%	19.68%	0.88%
	1	25.78%	11.53%	4.52%	19.68%	20%
	2	33.38%	12.46%	11.11%	19.68%	50%
	3	36.02%	12.46%	20.26%	19.61%	50%
	5	48.45%	19.61%	20.26%	19.61%	50%
<i>UNAV</i>	0	0.37‰	0.21‰	0.37‰	0.45‰	2.63‰
	1	0.43‰	0.24‰	0.43‰	0.45‰	2.82‰
	2	0.43‰	0.24‰	0.43‰	0.45‰	3.50‰
	3	0.43‰	0.24‰	0.43‰	0.44‰	3.50‰
	5	0.43‰	0.44‰	0.43‰	0.44‰	3.00‰

Figure 4.7: Optimal values of C_∞ for *Policy 0*, *Policy 1*, *Policy 2*, *Policy 3* with $\theta = 2$ in HTFD system when set-up cost varies

both components without additional inspection cost. It should also be noted that even though the classic maintenance policies do not perform as well as Policy 0 in taking advantage of the economic dependence, they try to offer opportunities for maintenance grouping as many as possible.

As shown in Table 4.3, *Policy 1*, *Policy 2* and *Policy 3* inspect both components at the same time when the set-up cost is greater than 3. In this way, the opportunity of maintaining two components simultaneously increases.

Policy 1 succeeds to offer grouping opportunities while adapting parameters to the individual behavior of components in order to minimize the mean cost criterion of each component. When set-up cost is high, *Policy 1* favors maintenance opportunities by selecting a common inspection period of 0.75 and the resulting number of grouped replacements is nearly twice as high compared to the case $\Delta T_1 = 1$ and $\Delta T_2 = 0.5$. With a common inspection period for both components, *Policy 3* is designed to offer many opportunities for grouping maintenance as can be seen with rather high values of PG_i ($i = 1, 2$) and thus set-up cost savings are greater when compared to *Policy 1*. However, as *Policy 3* selects the mid-point between the individual inspection periods as the common period for both components, some failures can no longer be detected in time. The resulting increase of each individual maintenance cost of components can not be balanced by the set-up cost savings except when the unit set-up cost is greater than 5 (in this case *Policy 1* and *Policy 3* select the same parameters). When the maintenance threshold is constrained to be the same for both components, *Policy 2* fails to balance the objectives of grouping replacements and adapting the inspection periods to individual degradation speed of components. When the set-up cost is less than 2, *Policy 2* favors an individual inspection scheme whereas opportunities to group are favored when set-up cost is high.

Nevertheless, the maximal value of PG_i that *Policies 1, 2, 3* can provide is less than 50% due to the diversity of components.

4.4.3 Quasi-homogeneous cases

For quasi-homogeneous case, we study a slow degrading and highly correlated system (*QHSD*) and a fast degrading (*QHFD*) system. A strong stochastic dependence is considered in *QHSD* system with $\theta = 10$ (*kendall's tau* ≈ 0.94). Table 4.5 shows that *Policy 0* selects parameters for component 1 in order to be a block-replacement policy (with adaptive replacement time) due to the set-up cost. That is, when set-up cost is greater than 1, component 1 does not need to be monitored and it is replaced simultaneously with component 2. This is because both components degrade slowly and they are strongly correlated so that it merely needs to monitor the faster degrading component

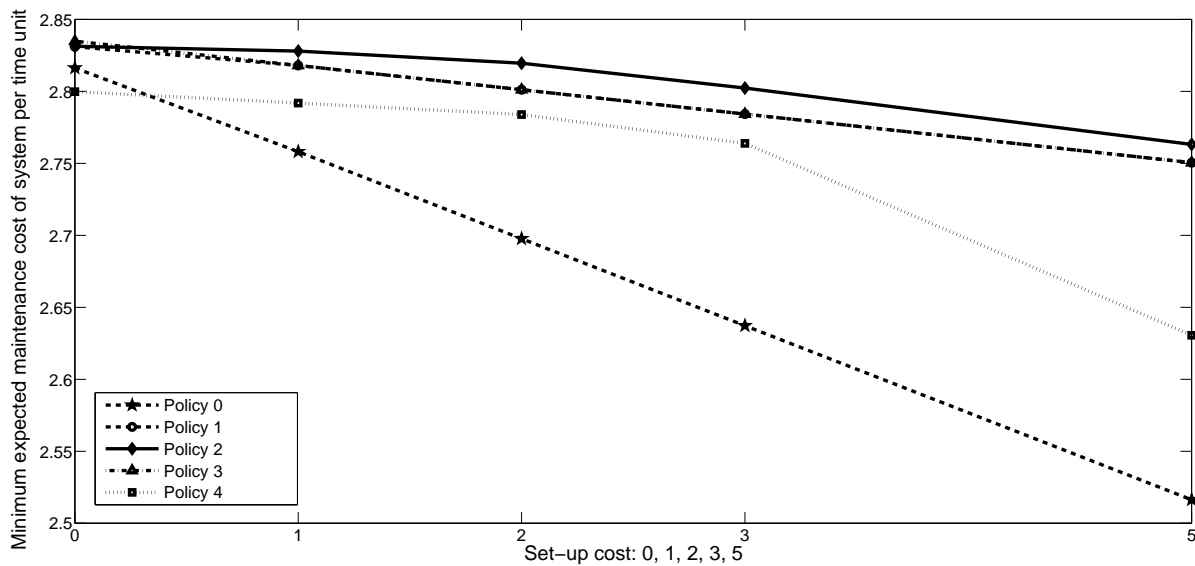


Figure 4.8: Optimal values of C_∞ for *Policy 0*, *Policy 1*, *Policy 2*, *Policy 3*, and *Policy 4* in *QHSD* system with $\theta = 10$ when set-up cost varies

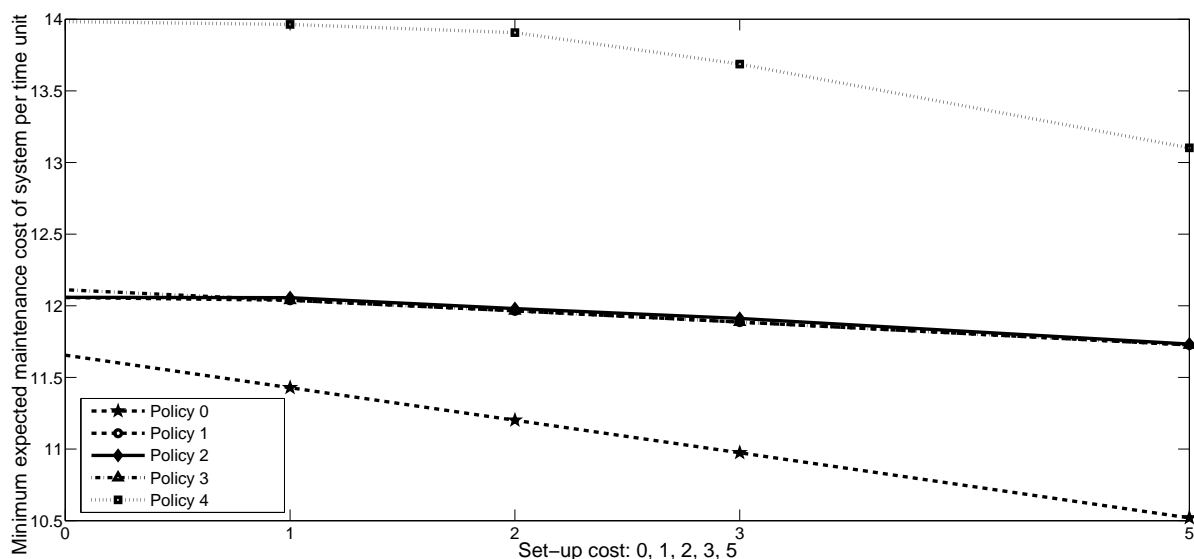


Figure 4.9: Optimal values of C_∞ for *Policy 0*, *Policy 1*, *Policy 2*, *Policy 3*, and *Policy 4* in *QHFD* system with $\theta = 10$ when set-up cost varies

and use its state as an indicator for the other component. Even though such an action may cause a certain increase of preventive replacements of component 1, it avoids some corrective replacements and saves inspection cost. As a consequence, only component 2 is inspected and replaced at optimal dates while component 1 is always replaced with it.

With the help of Figure 4.8, a global impression of performance of each policy is given. *Policy 4* is always better than other policies except *Policy 0* because both components

Table 4.5: Optimal parameters for *QHSD* system

cost parameters	$C_p=20$	$C_c=100$	$C_{insp}=1$	$C_u=10$	$C_d=100$	$\theta=10$			
component	$\alpha_1=7/5$		$\beta_1=1/5$		$L=180$				
	$\alpha_2=9/5$		$\beta_2=1/5$						
policies	set-up cost	minimum cost	$LC1/LC2$	$\Delta T_1/TB_1$	$\Delta T_2/TB_2$	M_1	M_2	PA	PP
<i>Policy 0</i>	0	2.8163	20.17/15.50	6	5	118	114	0.64	0.04
	1	2.7581	16.55/16.55	∞	4	0	128	0	0
	2	2.6977							
	3	2.6372							
	5	2.5164							
<i>Policy 1</i>	0	2.8311	19.93/15.88	6	4	116	122		
	1	2.8180	20.29/15.49	5	5	122	114		
	2	2.8012							
	3	2.7843							
	5	2.7505							
<i>Policy 2</i>	0	2.8314	20.29/15.88	5	4	122			
	1	2.8280	19.43/15.71	5	5	116			
	2	2.8196							
	3	2.8023							
	5	2.7632							
<i>Policy 3</i>	0	2.8349	20.29/15.49	5	5	122	114		
	1	2.8180							
	2	2.8012							
	3	2.7843							
	5	2.7505							
<i>Policy 4</i>	0	2.7998	18/14	18	14				
	1	2.7919	15/15	15	15				
	2	2.7840							
	3	2.7639							
	5	2.6306							

degrade slowly whereas the inspection cost is relatively high. In this case the periodic maintenance policy can well perform the preventive maintenance activities to avoid failures and the inspection cost can be saved. *Policy 1* and *Policy 3* recover the same optimal results when set-up cost increases because a stronger economic dependence provides more opportunities for maintenance grouping to reduce maintenance cost. However, the maintenance action is only possible at inspection times. Consequently, the two components must have a joint inspection period. *Policy 2* has worst results because of the joint preventive threshold. When compare *Policy 3* with *Policy 4* with a zero set-up cost, it shows that the inspection period has more influence than preventive threshold. When set-up cost is 0, *Policy 0* is better than other condition-based policies because it takes advantage of available information about components and the replacement dates of components become more flexible. When set-up cost is greater than 1, *Policy 0* turns out to be an adaptive block-replacement policy. Such a strategy reduces the inspection cost, saves the set-up cost and offers flexible dates for block replacement. For instance, when set-up cost is 5, *Policy 0* offers dates 4, 8, 12, 16, 20... for block replacement whereas *Policy 4* only offers dates 15, 30, 45... Therefore *Policy 0* performs better than *Policy 4*.

For *QHFD* system, we first evaluate the performance of maintenance policies with the same level of dependence degree ($\theta = 10$) to make sure that our proposed maintenance

policy still maintains the advantage. Figure 4.9 presents the minimum maintenance cost of system of each maintenance policy with different values of set-up cost. It reveals that the condition-based maintenance policies dominate the periodic maintenance policy for *QHFD* system. Among the condition-based maintenance policies, it remains the same dominance as for *QHSD* system. Nevertheless, *Policy 0* turns out again to be an adaptive block-replacement policy. We investigate the influence of stochastic dependence on *Policy 0* by testing weaker dependences (independent case, $\theta = 1$ and $\theta = 2$). The results are given in Figure 4.10 and Table 4.6.

Figure 4.10 reveals clearly that the stochastic dependence level has a significant impact on *Policy 0*. When the two components are strongly related ($\theta = 2$ and 10), *Policy 0* is an adaptive block-replacement policy which only inspects the faster degrading component and uses its degradation level as a criterion of block replacement. However when the two components are independent, failures of slower degrading component can not be avoided with such a strategy because the information about the faster degrading component has no relation with it. It is worth mentioning that when $\theta = 1$, the optimal parameters of *Policy 0* appear to be very different where $c_r = 2$ is the demarcation point. In fact, when $c_r = 2$ and $\theta = 1$, the difference between inspecting component 1 or not is: the gain of saving inspection cost is 0.5555, the loss of increasing preventive and corrective maintenance cost is 0.7178 and the gain of saving set-up cost is 0.2649. As a consequence, when set-up cost is greater than 2, the gain of maintenance grouping opportunities compensates the loss which is mainly caused by replacing component 1 too early.

Studying the independent case and that of $\theta = 1$, we can see that when stochastic dependence is weaker *Policy 0* accepts a greater *PA* and a smaller *PP* such that less grouped maintenance actions are allowed. The cases of $\theta = 2$ and 10 indicate that for strong dependence, the adaptive block-replacement policy has to replace components earlier in order to avoid failures where $\theta = 2$ has a smaller values of $LC1/LC2$ than $\theta = 10$. Compare the curves of $\theta = 2$ and $\theta = 10$, component 1 suffers more failures when two components are less dependent so that the expected maintenance cost of system is greater. On the contrary, with the same dependence degree and with the stronger economic dependence, the adaptive block replacement policy replaces components earlier. The curve of $\theta = 0$ in figure 4.10 shows that the economic dependence also plays a role in the optimization. When the set-up cost increases, *Policy 0* tends to find appropriate values of parameters in order to have more grouped maintenance actions to take advantage of economic dependence.

Table 4.6: optimal parameters of *Policy 0* for *QHFD* system with different degrees of dependence

cost parameters	$C_p=20$	$C_c=100$	$C_{insp}=1$	$C_u=10$	$C_d=100$				
component	$\alpha_1=7/5$		$\beta_1=1/5$		$L=50$				
	$\alpha_2=9/5$		$\beta_2=1/5$						
θ	set-up cost	minimum cost	$LC1/LC2$	ΔT_1	ΔT_2	M_1	M_2	PA	PP
$\theta \rightarrow 0$	0	12.0317	5.23/4.06	1.5	1.1	29.4	29.4	0.54	0
	1	11.9851	5.19/4.05	1.5	1.1	29.4	29.4	0.41	0
	2	11.9221	5.18/4.02	1.5	1.2	29.6	28.8	0.25	0.01
	3	11.8344	5.17/4.02	1.5	1.1	30.4	29.8	0.19	0
	5	11.6031	4.95/3.94	1.5	1.1	30.4	29.8	0.09	0
$\theta=1$	0	11.9669	5.28/4.05	1.5	1.0	29.9	29.6	0.26	0.02
	1	11.8781	5.20/4.08	1.5	1.0	29.7	29.7	0.15	0.03
	2	11.6712	4.26/4.26				31.8		
	3	11.4351		∞	0.9	0	31.4	0	0
	5	10.9607	4.22/4.22						
$\theta=2$	0	11.7812							
	1	11.5524	4.37/4.37				33.2		
	2	11.3236		∞	0.8	0		0	0
	3	11.0956	4.33/4.33				32.8		
	5	10.6334							
$\theta=10$	0	11.6422							
	1	11.4185	4.47/4.47				34		
	2	11.1948		∞	0.8	0		0	0
	3	10.9706							
	5	10.5186	4.42/4.42					33.6	

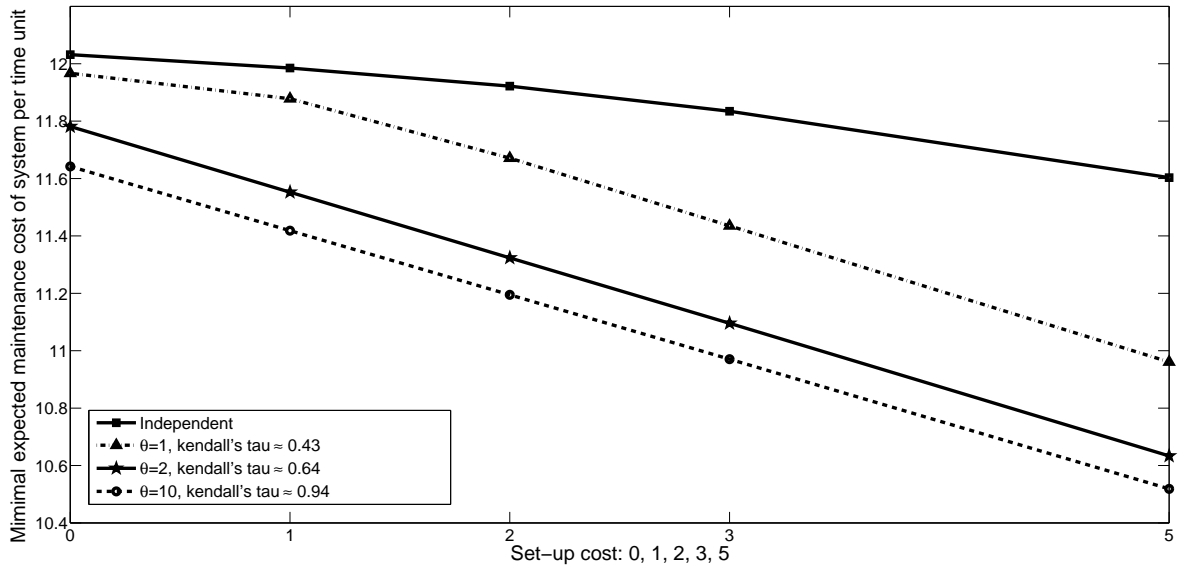


Figure 4.10: Optimal values of C_∞ for *Policy 0* in *QHSD* system with different degrees of dependence

4.4.4 Sensitivity analysis

In order to test the robustness of *Policy 0*, we consider two possible sources of error about the input parameters: the mean degradation speed (μ_1 and μ_2) of components and the corrective threshold L . The baseline case is *HTFD* system with mean degradation speed $\mu_1 = 5$, $\mu_2 = 9$ and corrective threshold $L = 50$. Errors of $e = \pm 10\%$ and

Table 4.7: Modified parameters of mean degradation speed

	L	θ	α_1	β_1	α_2	β_2
$\mu_{(-10\%)}$	50	2	4.5*4.5/35	4.5/35	8.1*8.1/45	8.1/45
$\mu_{(-5\%)}$			4.75*4.75/35	4.75/35	8.55*8.55/45	8.55/45
$\mu_{(0\%)}$			5/7	1/7	9/5	1/5
$\mu_{(5\%)}$			5.25*5.25/35	5.25/35	9.45*9.45/45	9.45/45
$\mu_{(10\%)}$			5.5*5.5/35	5.5/35	9.9*9.9/45	9.9/45

Table 4.8: Modified parameters of individual corrective threshold

	L	θ	α_1	β_1	α_2	β_2
$L_{(-10\%)}$	45	2	5/7	1/7	9/5	1/5
$L_{(-5\%)}$	47.5					
$L_{(0\%)}$	50					
$L_{(5\%)}$	52.5					
$L_{(10\%)}$	55					

$e = \pm 5\%$ about μ_i ($i = 1, 2$) and L are considered.

Let $C_{\mu(e)}$ be the minimal expected cost optimized using the real values of mean degradation speeds of components, $C_{L(e)}$ be the minimal expected cost optimized using the real parameters of the corrective threshold. $C_{OPT(e)}^\mu$ ($C_{OPT(e)}^L$) is the expected cost when *Policy 0* is implemented with optimized parameters for ($L = 50$, $\alpha_1 = 5/7$, $\beta_1 = 1/7$, $\alpha_2 = 9/5$, $\beta_2 = 1/5$) whereas the actual values of mean degradation speeds μ_i in real is $\mu_i(1 + e)$ (Table 4.7) (or $L(1 + e)$ (Table 4.8) respectively). The relative additional cost $V\mu$ (or VL respectively) incurred when implementing the maintenance policy with the optimized setting for a wrong value of mean degradation speed (corrective threshold respectively) instead of the actual value is defined as follows:

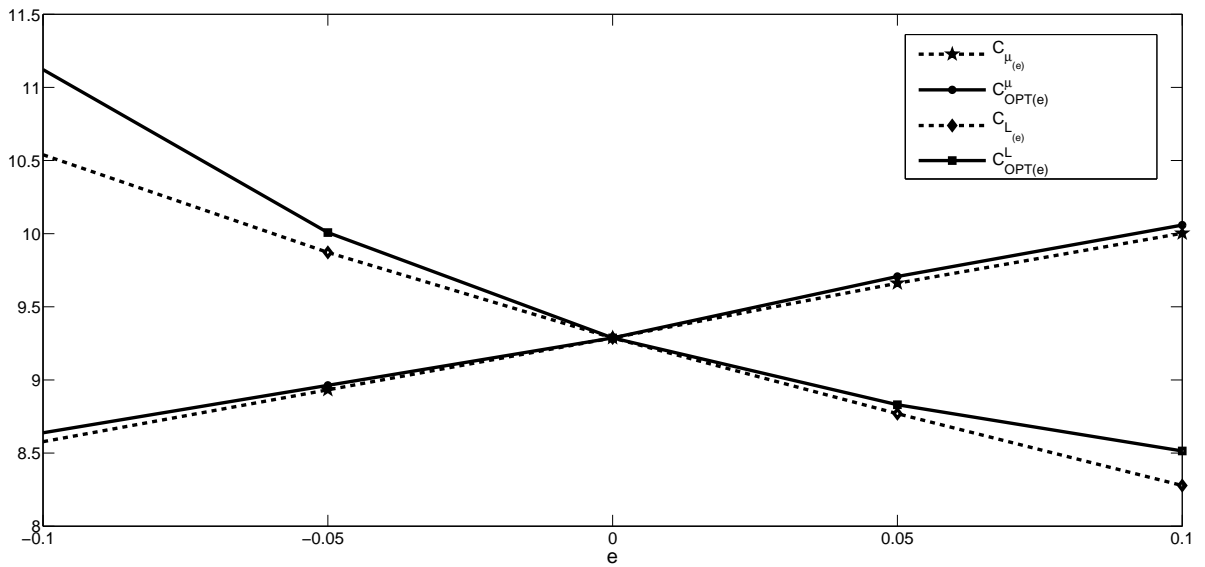
$$V\mu_{(e)} = \frac{C_{OPT(e)}^\mu - C_{\mu(e)}}{C_{\mu(e)}}, e = \pm 10\%, \pm 5\%, 0.$$

$$VL_{(e)} = \frac{C_{OPT(e)}^L - C_{L(e)}}{C_{L(e)}}, e = \pm 10\%, \pm 5\%, 0.$$

They measure the influence of parameters on the optimal solution and the results are given in Table 4.9 and Figure 4.11. As reflected, the mean degradation speed of components have a weak influence on *Policy 0*. When the mean degradation speeds of components increase, components will fail at earlier dates and have to be replaced more frequently. However the balance between the mean number of preventive and corrective replacements remains quite stable. In contrast, it changes a lot when the error is related to the corrective threshold. On one hand, when the corrective threshold decreases, the

Table 4.9: Relative variation for mean degradation speeds and corrective threshold

e	-10%	-5%	0%	5%	10%
$C_{\mu(e)}$	8,5781	8,9322	9,2871	9,6614	10,0030
$C_{OPT(e)}^{\mu}$	8,6389	8,9628	9,2871	9,7071	10,0587
$V\mu(e)$	0.7090%	0.3423%	0	0.4728%	0.5575%
$C_{L(e)}$	10,5395	9,8718	9,2871	8,7693	8,2779
$C_{OPT(e)}^L$	11,1214	10,0077	9,2871	8,8304	8,5145
$VL(e)$	5,5216%	1,3771%	0	0,6965%	2,8571%

Figure 4.11: Sensitivity analyse in *HTFD* system with two sources of error

component will fail in mean at earlier dates whereas the mean entrance time in preventive zone is unchanged. Consequently, the probability of failure of component between two inspections (that is $P(X_{(n+1)\Delta T} > L | X_{n\Delta T} < M)$) increases when L is actually smaller than the estimated value. It means that the proportion of corrective replacements is larger in this case. On the other hand, when the corrective threshold is in real greater than its estimated value, the proportion of preventive replacements increases due to too early replacement. As the preventive maintenance cost is lower than the corrective one, the impact on the global cost is less significant when L is underestimated than when it is overestimated. It is also worth mentioning that the decision rule is only related to the corrective threshold in the postponing case and the results of numerical experiments ($PP < 0.03$) show that postponing requirement is very rarely met. Therefore, the corrective threshold has little impact on the decision rule. In conclusion, the system parameter (corrective threshold L) has more influence than component parameter (mean degradation speed) on the condition-based maintenance policies.

4.5 Conclusions and perspectives

A new condition-based maintenance policy with grouping strategy is proposed in this chapter. As a result, we have shown that the proposed condition-based maintenance policy (*Policy 0*) can adapt to both quasi-homogeneous and heterogeneous cases, fast and slow degrading systems, and it outperforms classic maintenance policies. Based on an adaptive decision rule, *Policy 0* are able to take full advantage of the economic dependence by providing more opportunities as well as flexible replacement dates for maintenance grouping. The robustness of *Policy 0* is evaluated through sensitivity analysis by testing two sources of error.

The influences of the economic and stochastic dependences were investigated in this chapter. It revealed that both of the economic and stochastic dependences have a significant influence on the maintenance optimization. Therefore, for multi-component systems, appropriate stochastic dependence modelling and economic dependence should be taken into account in the design of maintenance policies.

Even though the proposed condition-based maintenance policy performed well experimentally, the classic Monte-Carlo simulation combined with optimization schemes is time-consuming, thus becoming impractical in view of the systems composed of numerous components. To extend the proposed strategy to such systems remains an open question. Furthermore, the stochastic dependence is only involved in the degradation modeling but it is not directly involved in the design of maintenance policies. Thus in the next chapter, we will consider systems with more than two components and more complex dependence structure. In the next chapter, we focus on the use of stochastic dependence,. In this aim, several maintenance policies seeking to make the best use of the available information about stochastic dependence modeling are proposed.

Chapter 5

Maintenance modeling of degrading system with hierarchical stochastic dependences

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5.1 Introduction

This chapter is written mainly based on [182, 183]. In [chapter 4](#), it has been proved through numerical experiments that stochastic dependence has a significant impact on the maintenance optimization. Therefore the objectives of this chapter is to develop maintenance policies which can take full advantage of the stochastic dependence. In terms of degradation modeling of system, a four-component system with hierarchical stochastic dependences is modeled by Gamma process with Nested Lévy copulas. In such a dependence modeling, system can be divided into subsystems according to different dependence degrees and components within subsystems are more dependent on each other than those between subsystems such that it can be applied in various cases of systems.

In terms of maintenance modeling, maintenance policies proposed in this chapter can be divided into two parts: maintenance policies with the parameters based on subsystems level which explore economic dependence and stochastic dependences both within subsystem and between subsystems and maintenance policies applied by the whole system which explore deeply the stochastic dependence. In the first part, we consider a type of systems with the assumption that components within subsystem contribute to the same tasks and have homogenous degradation behaviour such that the operations can be done at subsystem level instead of component level. In the first part, five maintenance policies are proposed to apply on such type of systems with respect to whether taking advantage of economic dependence or not, whether making use of within-subsystem stochastic dependences or not and whether profiting between-subsystem dependence or not. In the second part, we focus on developing maintenance strategies which can take full advantage of stochastic dependence when components in a subsystem can be treated separately. Two main maintenance policies are proposed in the second part and two classic maintenance policies are also implemented to evaluate the profit gained when stochastic dependences are used.

The rest of this chapter is organised as follows. [Section 5.2](#) is devoted to the hierarchical stochastic dependence modeling. [Section 5.3](#) describes maintenance policies. In [Section 5.4](#), numerical experiments are presented and sensitivity analysis is carried out. Finally, Conclusions and perspectives are drawn in [Section 5.5](#).

5.2 Asymmetric Stochastic dependence modeling with Nested Archimedean Lévy copulas

In [chapter 4](#), we dealt with the stochastic dependence of two-unit degrading systems with Archimedean Lévy copulas. In this chapter, the stochastic dependence is further investigated within a four-component system. When the system is composed of more than two components, the dependence between components is often asymmetric. According to the structure of system, the physical properties of components or the influence of working environment, the four-component system is supposed to be composed of two subsystems. Within subsystems, components share characteristics or tasks such that a symmetric and strong dependence exists between them. However dependence between subsystems is often observed at a lower degree which is related to a common environment and mission profiles. Therefore, components in the system are dependent on each other at different levels. In order to model hierarchical dependence, the Nested Lévy copulas model could be used.

5.2.1 Modeling of hierarchical stochastic dependences

Nested Clayton Lévy copulas are used with parameters $(\theta_0, \theta_1, \theta_2)$ to model such hierarchical dependencies, see [Equation 5.1](#).

$$C(u_1, u_2, u_3, u_4) = ((u_1^{-\theta_1} + u_2^{-\theta_1})^{-\frac{\theta_0}{\theta_1}} + (u_3^{-\theta_2} + u_4^{-\theta_2})^{-\frac{\theta_0}{\theta_2}})^{-\frac{1}{\theta_0}}. \quad (5.1)$$

Components within subsystem i are coupled by Clayton Lévy copulas which depicts symmetric dependence with parameter θ_i ($i = 1, 2$) and then both copulas are aggregated by a new Clayton Lévy copula with parameter θ_0 at a lower degree to model the dependence at system level. [Figure 5.1](#) depicts the dependence structure as a tree where C_{ij} refers to the copula function relating components i and j , $(i, j) \in \{(1, 2); (3, 4)\}$ and C^N refers to the copula function relating subsystems 1 and 2.

5.2.2 Simulating four-dimensional Gamma processes with Nested Lévy copulas dependence

To sample the increment path of four dependent related components, an adapted version of the Marshall-Olkin algorithm to the Clayton Nested Lévy copulas for Gamma process is used and it is detailed as follows:

It should be noticed that $S(\frac{\theta_0}{\theta_1}, 1, (\cos(\alpha\pi/2)x_n)^{\frac{\theta_1}{\theta_0}}, 0; 1)$ is a α -stable distribution and Υ

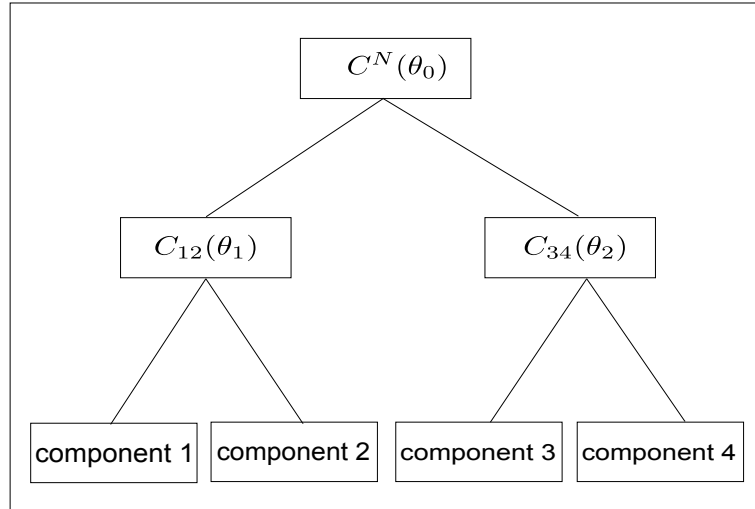


Figure 5.1: Sketch of dependence structure of system

Algorithm 3 Simulating four-dimensional Gamma process modeled by Nested Clayton Lévy copula by Marshall-olkin algorithm

Input: $\Upsilon, \theta_0, \theta_1, \theta_2, T, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4$

$n \leftarrow 0, \omega_0^1 \leftarrow 0$

while $\omega_n^1 \leq \Upsilon$ **do**

$n \leftarrow n + 1$

$w \leftarrow \exp(1)$

$\omega_n^1 \leftarrow \omega_{n-1}^1 + w$

$y \leftarrow \text{rand}(1)$

$z_n \leftarrow (\omega_n \Gamma(\frac{1}{\theta_0}) / \theta_0)^\theta$

$x_n \leftarrow S(\frac{\theta_0}{\theta_1}, 1, (\cos(\alpha\pi/2)z_n)^{\frac{\theta_1}{\theta_0}}, 0; 1), y_n \leftarrow S(\frac{\theta_0}{\theta_2}, 1, (\cos(\alpha\pi/2)z_n)^{\frac{\theta_2}{\theta_0}}, 0; 1)$

$a_n \leftarrow \exp(1), b_n \leftarrow \exp(1), c_n \leftarrow \exp(1), d_n \leftarrow \exp(1)$

$v_n \leftarrow \text{rand}(1)$

$\Gamma_n^1 = (a_n/x_n)^{-\frac{1}{\theta_1}}, \Gamma_n^2 = (b_n/x_n)^{-\frac{1}{\theta_1}}, \Gamma_n^3 = (c_n/y_n)^{-\frac{1}{\theta_2}}, \Gamma_n^4 = (d_n/y_n)^{-\frac{1}{\theta_2}},$

end while

$N \leftarrow n$

Output: $X_t^1 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^1}{\alpha_1 T}) / \beta_1 \mathbb{1}_{[0,t]}(v_n), X_t^2 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^2}{\alpha_2 T}) / \beta_2 \mathbb{1}_{[0,t]}(v_n),$

$X_t^3 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^3}{\alpha_3 T}) / \beta_3 \mathbb{1}_{[0,t]}(v_n), X_t^4 = \sum_{n=1}^N \mathbf{E}_1^{-1}(\frac{\Gamma_n^4}{\alpha_4 T}) / \beta_4 \mathbb{1}_{[0,t]}(v_n)$

is a parameter for truncating the series representation for reasonable simulation time and its value should be carefully chosen to make sure the convergence. For Laplace-Stieltjes transforms of the Nested Archimedean copulas and Nested Archimedean Lévy copulas generators, more details can be found in [180, 184]. Figure 5.2 illustrates the deterioration level of components with hierarchical dependence that is modeled by Nested Clayton Lévy copula. It shows that the stronger the stochastic dependence between components within subsystem is, the more often big damage arrives at both components. Moreover, the dependence within the system is asymmetric.

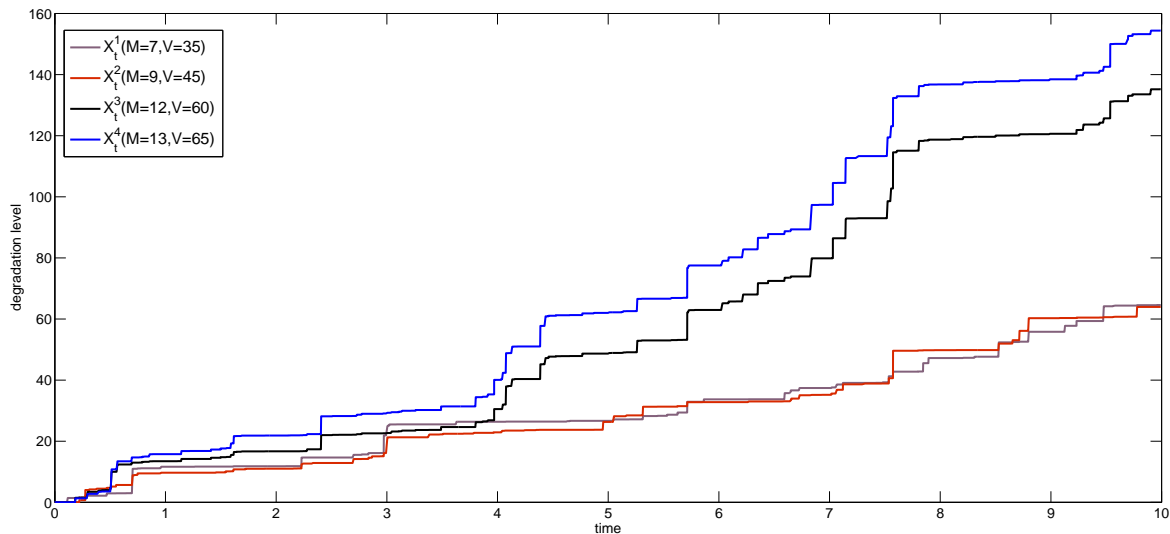


Figure 5.2: An illustration of deterioration paths of components when nested Clayton Lévy copula is used ($\theta_0 = 0.4$, $\theta_0 = 1.5$, $\theta_0 = 4$)

5.3 From Lévy copulas to ordinary copulas when dependence is considered at fixed time

In this section, we investigate the relationship between ordinary copulas and Lévy copulas at a fixed time. The dependence between components modeled by copulas and Lévy copulas is studied through Kendall's tau which shows the dependence degree as well as tail dependence which is regarded as dependence structure. At specific time, Gumbel ordinary copula is used to describe the dependence between components modeled by Clayton Lévy copula with adaptive parameter. Goodness-of-fit test is done which shows that Gumbel ordinary copula can fit the simulation data generated by Clayton Lévy copula.

5.3.1 Measure of dependence

In this section, we aim to study and measure the dependence in order to explore how the dependence between components can be expressed by ordinary and Lévy copulas. Two concepts are reviewed in this section where Kendall's tau can be considered to measure the dependence degree and tail dependence is used to investigate the dependence structure.

5.3.1.1 Kendall's tau

The concept of Kendall's tau was developed by Kendall in 1938 to measure the correlation between two random variables in [185] and it is defined as follows:

Definition 5.1. Let $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ be a random sample of n observations of random pair of (X, Y) . For any pair of observation (x_i, y_i) and (x_j, y_j) ($i \neq j$), it is said to be concordant if $\{x_i < x_j, y_i < y_j\}$ or $\{x_i > x_j, y_i > y_j\}$ and if $\{x_i > x_j, y_i < y_j\}$ or $\{x_i < x_j, y_i > y_j\}$, the pair is discordant. Let c denote the number of concordant pairs and d denote the number of discordant pairs. Therefore the Kendall's tau τ is defined as:

$$\tau = \frac{c - d}{c + d} \quad (5.2)$$

That means the population version of Kendall's tau is to measure the difference between the probability of concordance and discordance between two variables as follows:

$$\tau = P((x_1 - x_2)(y_1 - y_2) > 0) - P((x_1 - x_2)(y_1 - y_2) < 0). \quad (5.3)$$

Nelsen has proved that Kendall's tau can be calculated using the copula functions through following equation if C is the underlying copula of (X, Y) in [179] (Theorem 5.1.3.).

$$\begin{aligned} \tau &= 4 \iint_{[0,1]^2} C(u, v) dC(u, v) - 1 \\ &= 4E(C(u, v)) - 1. \end{aligned} \quad (5.4)$$

It can be proved that for Archimedean copulas with generator φ (see chapter 4.2.2.1), Kendall's tau can be given as follows:

$$\tau = 1 + 4 \int_0^1 \varphi(t)/\varphi'(t) dt. \quad (5.5)$$

The proof is seen in [179] (Corollary 5.1.4.). Therefore, the Kendall's tau of two variables where the dependence between them is modelled by Archimedean ordinary copulas can be easily achieved by Equation 5.5. For Clayton ordinary copula,

$$\begin{aligned} \tau &= 1 + 4 \int_0^1 (t^{-\theta} - 1)/(-\theta t^{-\theta-1}) dt \\ &= 1 + 4 \int_0^1 (t^{\theta+1} - t)/\theta \\ &= \theta/(\theta + 2), \end{aligned} \quad (5.6)$$

and for Gumbel ordinary copula,

$$\begin{aligned}
 \tau &= 1 + 4 \int_0^1 (-\ln t)^\theta / (-\theta(-\ln t)^{\theta-1}/t) dt \\
 &= 1 + 4 \int_0^1 t \ln t / \theta \\
 &= (\theta - 1) / \theta.
 \end{aligned} \tag{5.7}$$

The calculation of Kendall's tau for copulas displays the dependence degree between variables which is represented by the copula function and the value of parameter θ . Moreover, the estimation of Kendall's tau is very easy with a given sample such that it can be used for parametric estimation when the underlying copula function is known.

5.3.1.2 Tail dependence

Another important concept is the tail dependence which measures the dependence for extreme values. For a pair of random variables X_1 and X_2 , the upper tail dependence parameter is given by:

$$\begin{aligned}
 \lambda_U &= \lim_{t \rightarrow 1^-} P(X_1 > t \mid X_2 > t) \\
 &= \lim_{t \rightarrow 1^-} \frac{P(X_1 > t, X_2 > t)}{P(X_2 > t)} \\
 &= \lim_{t \rightarrow 1^-} \frac{1 - 2t + c(t, t)}{1 - t},
 \end{aligned} \tag{5.8}$$

and the lower tail dependence parameter is given by:

$$\begin{aligned}
 \lambda_L &= \lim_{t \rightarrow 0^+} P(X_1 < t \mid X_2 < t) \\
 &= \lim_{t \rightarrow 0^+} \frac{P(X_1 < t, X_2 < t)}{P(X_2 < t)} \\
 &= \lim_{t \rightarrow 0^+} \frac{C(t, t)}{t}.
 \end{aligned} \tag{5.9}$$

A copula function C is said to have lower (upper) tail dependence if $\lambda_L \neq 0$ ($\lambda_U \neq 0$). For Clayton ordinary copula, it has low tail dependence with

$$\begin{aligned}
 \lambda_L &= \lim_{t \rightarrow 0^+} (2t^{-\theta} - 1)^{-\frac{1}{\theta}} / t \\
 &= 2^{-\frac{1}{\theta}},
 \end{aligned} \tag{5.10}$$

and for Gumbel ordinary copula, it has upper tail dependence with

$$\begin{aligned}\lambda_U &= \lim_{t \rightarrow 1^-} (1 - 2t + \exp\{-[(-\ln t)^\theta + (-\ln t)^{\theta \frac{1}{\theta}}]\}) / (1 - t) \\ &= 2 - 2^\theta.\end{aligned}\tag{5.11}$$

However λ_U for Clayton ordinary copula and λ_L for Gumbel ordinary copula are 0 respectively. That's why in Figure 4.2, Clayton and Gumbel ordinary copulas have different dependence drawings even when their Kendall's tau is identical. For Lévy copulas, the jump tail dependence is investigated in [186]. It has been proved that the Clayton Lévy copula has tail dependence of large jumps and the jump tail dependence coefficient is $2^{-\frac{1}{\theta}}$.

The tail dependence is very important when dealing with the stochastic dependence modeling for maintenance optimization because the influence of large jump is more significant than small jump due to expensive corrective maintenance cost.

5.3.2 Clayton Lévy copula and Gumbel ordinary copula

Even though Lévy copulas are able to model a time-independent dependence structure and can be used to sample correlated degradation path, it can not lead to a direct statistical inference. The limit theorem of Lévy copulas in Equation 5.12 shows that when time tends to zero, there is an unique ordinary copulas C_t corresponding to the Lévy copula \mathfrak{C} (see [93]).

$$\mathfrak{C}(u_1, u_2) = \lim_{t \rightarrow 0} \frac{1}{t} C_t(tu_1, tu_2),\tag{5.12}$$

Our main idea is thus to use an ordinary copula as an approximation of the Lévy copula to derive information at fixed times about non observed components and only for the purpose of deciding if component needs an intervention. Since the relationship between the Lévy copulas and the ordinary copulas at specific time is rarely investigated in literature, we propose to fit an ordinary copula to the degradation levels observed at fixed times when the degradations of components are simulated by Nested Lévy copulas using Algorithm 3. When dealing with which copula function can fit better the data, the tail dependence should be considered because it measures the dependence for the extreme values. Ordinary copula which has the same tail dependence properties as Clayton Lévy copula should be chosen.

As we have reviewed in Section 5.3.1.2, due to the fact that the dependence modeled by Clayton Lévy copula is stronger when the jump size is great, the Gumbel ordinary copula with upper tail dependence can fit the data better than the Clayton ordinary copula. Thus stochastic dependence between two components modeled by Gumbel ordinary

copula at a fixed time t is defined as follows:

$$\begin{aligned} F(x_i, x_4) &= C_t^G(F_i(x_i), F_4(x_4)) = C_t^G(u_i, u_4) \\ &= \exp\{-[(-\ln u_i)^{\theta_G^{i4}} + (-\ln u_4)^{\theta_G^{i4}}]^{\frac{1}{\theta_G^{i4}}}\}, i = 1, 2, 3 \end{aligned} \quad (5.13)$$

The parameter of Gumbel ordinary copula function has to be estimated at each value of time. Figure 5.3 illustrates the Maximum Likelihood estimated values of parameter of each component when time varies. It verifies that ordinary copula is time-dependent and we can notice as well that θ_G^{34} is greater than θ_G^{14} and θ_G^{24} because the dependence degree within subsystems is greater than that between subsystems.

We propose a simple Kolmogorov–Smirnov test (see [187]) to choose between Clayton and Gumbel ordinary copulas because in our case, marginal distributions are well known. A goodness-of-fit test proposed in [188] with the method “ $S_n^{(B)}$ ” is also used as it has been proved that the best procedures overall are those based on “ $S_n^{(B)}$ ”. In fact, the Kolmogorov–Smirnov test with sample size $n = 10000$ between *component 1* and *component 4* rejects the hypothesis that the underlying distributions of two samples generated by the Clayton Lévy copula and Gumbel ordinary copula respectively differ with P-value 0.2000 and Kolmogorov–Smirnov statistic 0.0189. However, the Kolmogorov–Smirnov test for Clayton ordinary copula accepts that hypothesis with P-value 1.2395e-17 and Kolmogorov–Smirnov statistic 0.0700. The goodness-of-test for copulas with method “ $S_n^{(B)}$ ” between *component 1* and *component 4* shows that: when null hypothesis is Gumbel ordinary copula, P-value is 0.1673 which lead to accept the null hypothesis; however when it is Clayton ordinary copula, P-value is 0.0004995 such that the null hypothesis is rejected. As a conclusion, Gumbel ordinary copula can fit the data generated by Clayton Lévy copula. Thus, in the subsequent sections, Gumbel ordinary copula with parameter adaptive to time is used for for statistic inference instead of Clayton Lévy copula.

5.4 Maintenance policies

In this section, we propose some maintenance policies which can be divided into two parts. Firstly, some maintenance policies considering both economic and stochastic dependences but easy to be implemented in practice with decision parameter set at subsystem level. In the second part, we focus on taking advantage of stochastic dependences when no practical constraint holds hence maintenance policies taking into account stochastic dependences at all levels are proposed. Besides, some classic maintenance policies are also implemented for the purpose of comparison with the performance of our proposed maintenance policies.

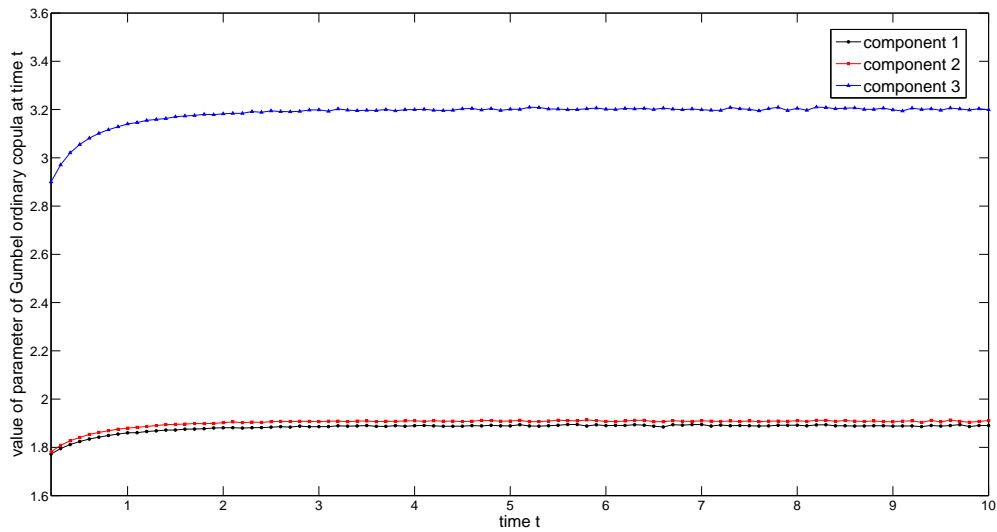


Figure 5.3: Estimating the values of parameter of Gumbel ordinary copula between component i ($i = 1, 2, 3$) and component 4 with $\theta_0 = 1$, $\theta_1 = 1.5$, $\theta_2 = 2$

For general assumption, the degradation level of components are revealed through inspections instantaneously and perfectly and an inspection cost c_{insp} is incurred. In order to avoid failures, components are preventively replaced with a preventive maintenance cost c_p when their degradation levels are revealed to have crossed over a preset threshold M . If the degradation level of component i ($i = 1, 2, 3, 4$) is revealed to have exceeded a corrective threshold L , it is considered failed and then a corrective maintenance with a cost c_c should be carried out. Both replacements (preventive and corrective) are assumed to be perfect and instantaneous (e.g. the duration of maintenance is negligible). The system is out of service as soon as a component fails and a downtime cost c_d is incurred per time unit.

5.4.1 Part I: Maintenance strategies based on subsystems dependence

It has been shown in [chapter 4](#) that the condition-based maintenance policy multi-component has lowest maintenance cost when each component is inspected and replaced according to individual inspection and replacement parameters. For example, for HTFD system in [chapter 4](#), 0.24% loss is observed for joint preventive threshold and 0.93% loss is observed for joint inspection period when compared to individual setting. However, the optimization of such a maintenance policy is time-consuming because eight parameters are involved. In this part, we consider a type of systems in which the degradation speeds of components within subsystem are closed and components are related. Therefore, the maintenance parameters can be set at subsystem level. In this way, if both components in a subsystem are monitored, they are inspected simultaneously (a joint inspection

period for both components in a subsystem) and the preventive maintenance threshold is the same for both components. Five maintenance policies are proposed below to take advantage of the economic dependence, the stochastic dependence within subsystem and the stochastic dependence between subsystems.

- Policy I1: Both components in subsystem i ($i = 1, 2$) are inspected simultaneously every T_i time units. Each component in subsystem i is replaced when its own degradation level is revealed to have exceeded the preventive threshold M_i .
- Policy I2: Both components in subsystem i ($i = 1, 2$) are inspected simultaneously every T_i time units. The whole subsystem i is replaced as soon as the degradation level of a component is revealed to have exceeded the preventive threshold M_i .
- Policy I3: A single component in subsystem i ($i = 1, 2$) is inspected every T_i time units in turns. The faster degrading component is to be inspected at first. The currently inspected component in subsystem i is replaced when its degradation level is revealed to have exceeded the preventive threshold M_i . Then the inspection scheme switches to the other component immediately to permit that components are replaced simultaneously. The inspection scheme on one component is started with the replacement of the other component and ended when it is replaced so that both components in a subsystem are alternately monitored.
- Policy I4: The faster degrading component in subsystem i ($i = 1, 2$) is inspected every T_i time units. The whole subsystem i is replaced when the degradation level of the inspected component in subsystem i is revealed to have exceeded the preventive threshold M_i .
- Policy I5: The faster degrading component among the whole system (that is *component 4* in subsystem 2) is inspected every T_2 time units. When the state of *component 4* in subsystem 2 is revealed to have exceeded the preventive threshold M_2 , the replacement of the whole subsystem 2 is carried out and the inspection scheme of subsystem 1 starts T_1 time units later. The component which degrades faster in subsystem 1 is inspected every T_1 time units until its degradation level is revealed to have exceeded the preventive threshold M_1 . Then an extra inspection of *component 4* in subsystem 2 is carried out instantaneously. If *component 4* is also revealed to have exceeded its preventive threshold M_2 then the whole system is replaced and the inspection scheme restarts. Otherwise only subsystem 1 is replaced. In all cases, as soon as subsystem 1 is replaced, its inspection scheme is stopped until subsystem 2 is replaced in turn so that inspections for subsystem 1 are always triggered by the replacement of subsystem 2.

To briefly summarize these five policies, policies I1 and I2 inspect both components within subsystem whereas policies I3, I4 and I5 inspect only one component within subsystem in order to take advantage of within-subsystem dependence. Policies I1 and I3 are independent replacements within subsystem but policies I2, I4 and I5 are block-replacements which can better profit economic dependence. Specially, policy I5 applies non-periodical inspection schemes for slower subsystem.

5.4.2 Part II: Maintenance policies making use of dependence at both subsystem and system levels

In previous part, in order to optimize the condition-based maintenance in a reasonable time as well as grouping maintenance activities, the number of maintenance parameters is reduced by setting at subsystem level. Nevertheless, it has to find appropriate parameters to balance both components within subsystem which lead to more inspections for slow degrading component hence high inspection cost whereas less inspections for fast degrading component hence high corrective maintenance cost. As a consequence, periodic inspection scheme is easy to manage but when applied to heterogeneous components, it fails to balance the objectives of grouping maintenances tasks to save set-up cost (individual maintenance parameters at component level) and adapting inspection periods to the specific degradation speeds of each component (joint maintenance parameters at subsystem level). As components are correlated, if degradation level of component is revealed within a time interval, it can be used to estimate the degradation information of other components. The stronger the dependence is, the more accurate the estimation. Moreover, the slow degrading component is not so sensitive to inspection frequency when compared to fast degrading component. Therefore, information about slow degrading components can be derived from revealed information of fast degrading components based on the stochastic dependence. To this end, we explore the stochastic dependence deeply and aim to propose strategies which can reduce unnecessary inspections and avoid failures by using stochastic dependences.

5.4.2.1 Deriving information about an unknown component from a known component

In Section 5.3, we have discussed how the increments generated by Lévy copulas can be presented by time-dependent ordinary copulas. Suppose that a pair of two random variables (X, Y) is coupled by Gumbel ordinary copula, that is the joint cumulative distribution function H can be written as a function of individual CDF F_X and F_Y as

following

$$H(x, y) = C(F_X(x), F_Y(y)) \quad (5.14)$$

where

$$C(u, v) = \exp\{-[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}}\}. \quad (5.15)$$

Then the conditional distribution copula is

$$\begin{aligned} C_{uv}(u \mid v) &= \int_0^u \frac{\partial^2 C(t, v)}{\partial t \partial v} dt \\ &= \frac{\partial C(u, v)}{\partial v} \\ &= \exp\{-[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}}\} [(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}-1} (-\ln v)^{\theta-1} \frac{1}{v}, \end{aligned} \quad (5.16)$$

and the density function is

$$\begin{aligned} c(u, v) &= \exp\{-[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}}\} [(-\ln u)^\theta + (-\ln v)^\theta]^{2(\frac{1}{\theta}-1)} (-\ln u)^{\theta-1} (-\ln v)^{\theta-1} \frac{1}{u} \frac{1}{v} \\ &\quad + \exp\{-[(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}}\} [(-\ln u)^\theta + (-\ln v)^\theta]^{\frac{1}{\theta}-2} (-\ln u)^{\theta-1} (-\ln v)^{\theta-1} \frac{1}{u} \frac{1}{v}. \end{aligned} \quad (5.17)$$

Since $E(X|Y)$ is a so-called minimum-mean-square-error estimation for X given the value of Y , it can be used to estimate the state of a non-inspected component if the state of another related component is known. The subsequent equation details the calculation of $E(X \mid Y = y)$ when X and Y are coupled by a copula function C with density function c in Equation 5.17.

$$E(X \mid Y = y) = \int_0^\infty xc(F_X(x), F_Y(y))f_X(x)dx. \quad (5.18)$$

With the estimation $E(X \mid Y = y)$, the mean-square error $MSE(y)$ is given as follows:

$$\begin{aligned} MSE(y) &= \int_0^\infty (x - E(X \mid Y = y))^2 f_X(x) dx \\ &= E(X^2) - 2E(X \mid Y = y)E(X) + (E(X \mid Y = y))^2. \end{aligned} \quad (5.19)$$

Therefore, the error of estimation $E(X \mid Y = y)$ for X depends not only on the marginal law of X but also on the dependence relationship between X and Y and the observed value of Y . Figure 5.4 illustrates the conditional expectation of X_i ($i = 1, 2, 3$) with all the possible values of Y (the values of Y are standardized to $[0, 1]$ by $F_Y(y)$) where X_i and Y are Gamma distributed, M_i and V_i is the mean and variance of x_i respectively

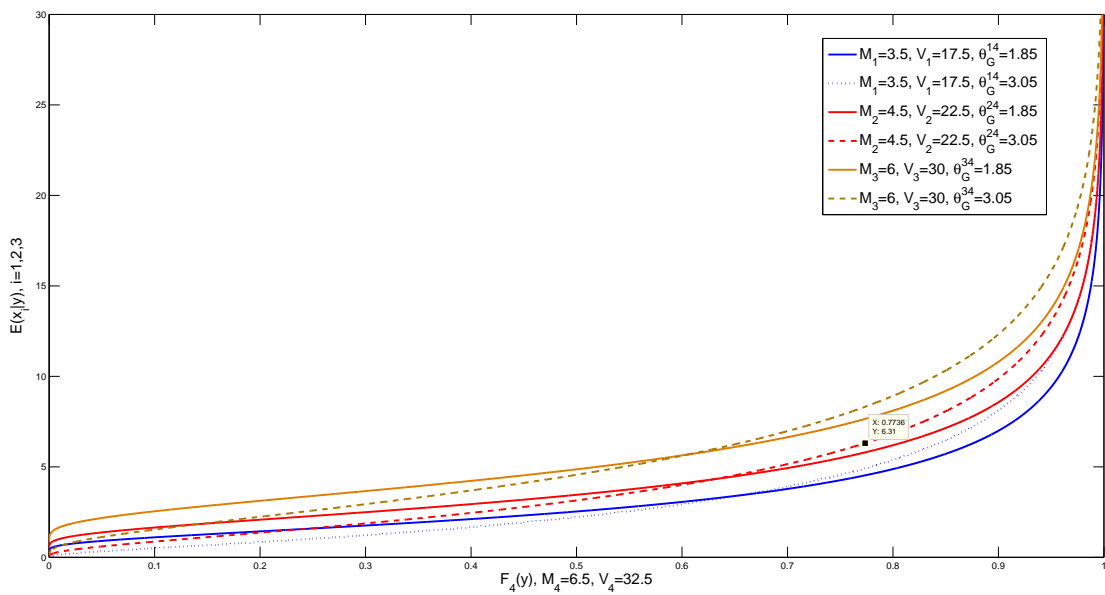


Figure 5.4: Conditional expectation of X knowing the observed value of Y served in function of $F_Y(y)$ with different mean and variances and dependence degrees

and θ_G^{i4} is parameter of Gumbel function between X_i and Y . Figure 5.4 shows that the greater θ_G^{i4} is, the more $E(X | Y = y)$ depends on the observed value of Y and the estimated value of X_i are more accurate. For example, compare two blue curves 5.4 which are solid and dotted respectively, the blue solid curve is more flat than the blue dotted curve hence it is less dependent on the observed value of Y . In fact, when X_1 and Y are independent, $E(X | Y = y) = E(X)$ is constant. As a result, if components are highly correlated, it is reasonable to inspect only some components and the degradation level of other components can be estimated e.g. by using conditional expectations. Nevertheless, the conditional expectation $E(X | Y = y)$ with ordinary Gumbel copula given by Equation 5.18 has not explicit form which leads to time-consuming numerical integral at each inspection time when it is implemented in maintenance policies. Thus the estimated value of the probability that components exceed the threshold is used as an alternative to make the maintenance decision. In this case, the conditional distribution function of a component given the observed state of another related component should

be derived as follows:

$$\begin{aligned}
H(x \mid y) &= \int_0^x \frac{h(t, y)}{f_Y(y)} dt \\
&= \int_0^x \frac{\partial^2 C}{\partial u \partial v}(F_X(t), F_Y(y)) f_X(t) dt \\
&= C_{uv}(F_X(x) \mid F_Y(y)) \\
&= \exp\{-[(-\ln F_X(x))^\theta + (-\ln F_Y(y))^\theta]^{\frac{1}{\theta}}\} [(-\ln F_X(x))^\theta \\
&\quad + (-\ln F_Y(y))^\theta]^{\frac{1}{\theta}-1} (-\ln F_Y(y))^{\theta-1} \frac{1}{F_Y(y)}
\end{aligned} \tag{5.20}$$

where $h(x, y) = \frac{\partial^2 H(x, y)}{\partial x \partial y}$. We give an example in Figure 5.5 of how to derive the probability that component i exceeds the preventive threshold at nT with the last known degradation level being $x_{m_i T}^i$ at $m_i T$ and the increment of a related component j on $[m_i T + T_1, m_i T + T_1 + W_i]$ being revealed as D_i with $m_i T + T_1$ and $m_i T + T_1 + W_i$ being two inspection moments of component j . Within an interval from last operation time $m_i T$ to current decision moment nT , the increment of component i can be divided into three parts: a random variable x_i following the Gamma distribution with parameters $(T_1 \alpha_i, \beta_i)$, a random variable y depending on both D_i and the marginal distribution function of component i during time W_i (conditional distribution given D_i is noted as $H_{W_i}(y \mid D_i)$) and a random variable x_2 following the Gamma distribution with parameters $(T_2 \alpha_i, \beta_i)$ (see in Figure 5.5(a)). Since the increments of Gamma process are independent and exchangeable, the increments of component i can also be divided in two parts: a random variable x following the Gamma distribution with parameters $((T_1 + T_2) \alpha_i, \beta_i)$ and a random variable y related to the revealed increments of component j (see in Figure 5.5(b)). Therefore,

$$\begin{aligned}
P(X_{nT}^i > M \mid X_{m_i T}^i = x_{m_i T}^i, D_i, W_i) \\
&= 1 - \iint_{x+y < M - x_{m_i T}^i} f_{\alpha_i(nT - m_i T - W_i), \beta_i}(x) h(y \mid D_i) dx dy \\
&= 1 - \int_{x < M - x_{m_i T}^i} f_{\alpha_i(nT - m_i T - W_i), \beta_i}(x) H(M - x_{m_i T}^i - x \mid D_i) dx
\end{aligned} \tag{5.21}$$

It is worthwhile to notify that when x is zero which mean that the observation of component j is complete for component i such that $P(X_{nT}^i > M \mid X_{m_i T}^i)$ can be directly computed without numerical integral. Therefore the computation of the probability that components exceed the threshold consumer less time than the conditional expectation especially when component j is always completely inspected at the decision time of component i . For subsequent sections, components are noted as component 1, 2, 3, 4

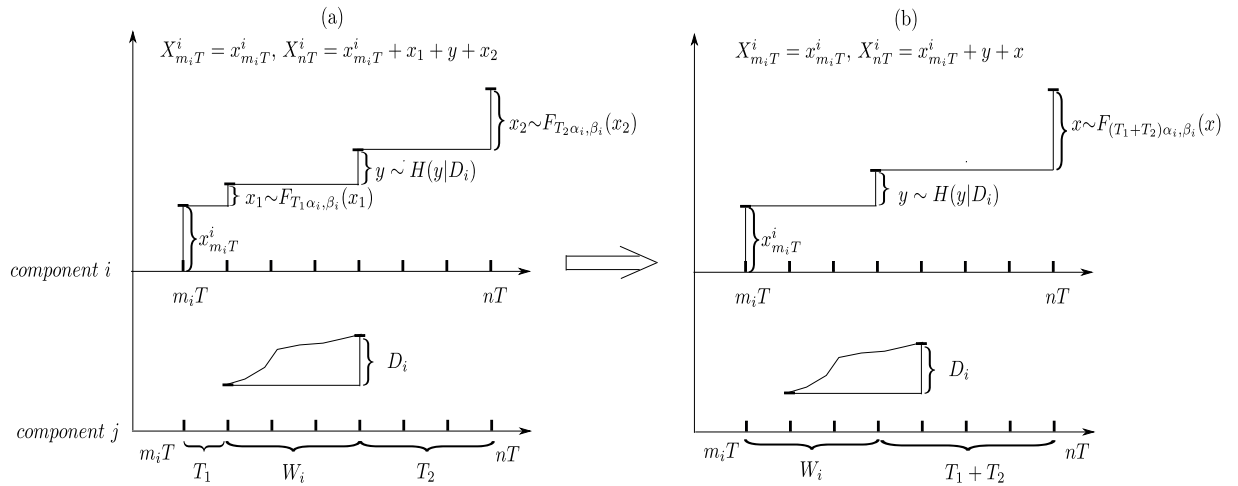


Figure 5.5: An example for calculation estimate the probability of component i exceeding preventive threshold M with the information of component j

with degradation speeds in increasing order so that *component 4* is the fastest degrading component.

5.4.2.2 Semi non-periodic strategy considering stochastic dependence (policy III)

On the basis of a four-component system, we propose a new condition-based maintenance policy denoted as policy III which is designed to inspect periodically the fastest degrading component and the remaining components are inspected only when needed. The main idea is that the fastest degrading component should be inspected more frequently and its revealed degradation information can be used to estimate the degradation level of the other components, thus the other components are inspected only when their degradation levels are estimated to exceed the preventive threshold. Policy III is detailed as follows:

- **Inspection strategy:** we suppose that the degradation level of component can only be known by inspection operations. *component 4* is inspected every T time units. Once the degradation level of *component 4* is observed at time nT ($n \in \mathbb{N}$), it is used to calculate the estimated probability PM_i that the degradation level of *component i* exceeds the preventive threshold (that is $PM_i \simeq P(X_{nT}^i > M | X_{nT}^4)$). *component i* ($i = 1, 2, 3$) is inspected when and only when the estimated probability PM_i is high enough while remaining below a direct replacement threshold : if $R2 \leq PM_i \leq R1$ then *component i* is inspected. Each inspection operation incurs a cost C_{insp} .

- Replacement strategy: *component* i ($i = 1, 2, 3, 4$) is replaced when inspected if its revealed degradation level exceeds a preventive threshold M and corrective replacement with cost C_c takes place if the preset corrective threshold L is reached otherwise preventive replacement incurs a cost C_p . Moreover, component i ($i = 1, 2, 3$) is replaced without inspection operation when the estimated PM_i exceeds the direct replacement threshold $R1$. If the component i fails, corrective maintenance incurs a cost C_c otherwise preventive maintenance incurs a cost C_p .

The degradation level of *component* 4 is revealed every T time units such that the estimated probability PM_i ($i = 1, 2, 3$) can be updated using the revealed increments of *component* 4 based on their dependence relationship. Having the following assumptions:

- nT is current decision time, *component* 4 is already inspected and the revealed increment of *component* 4 since time $(n - 1)T$ is D_{nT}^4 ,
- the last operation of component i ($i = 1, 2, 3$) took place at m_iT with $(n - 1)T < m_iT < nT$ and the last revealed degradation level is $x_{m_iT}^i$,

then

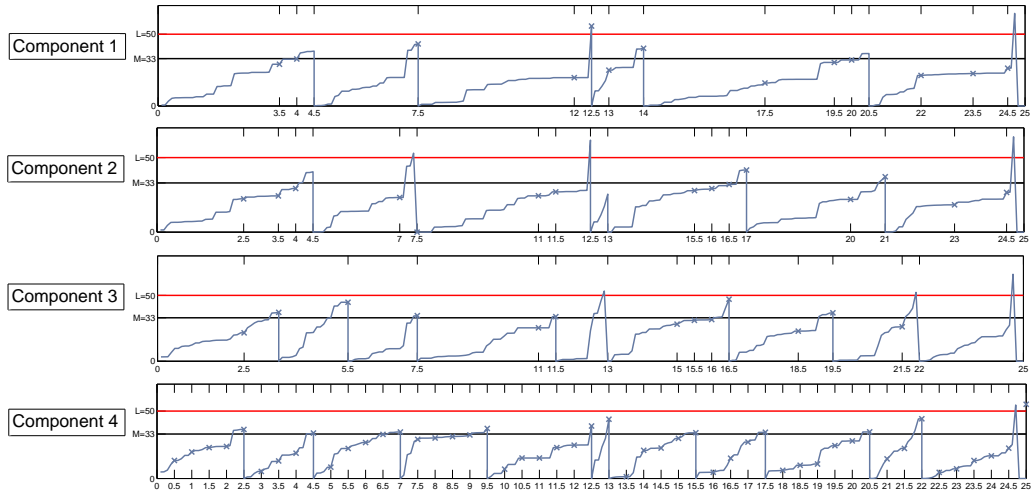
$$\begin{aligned} PM_i &= P(X_{nT}^i > M | X_{m_iT}^i = x_{m_iT}^i, \sum_{k=m_iT}^{k=nT} D_k^4) \\ &= 1 - \exp(-\wp_{\Delta t}^{i4}(x, y)^{\frac{1}{\theta_{i4}^G}}) \wp_{\Delta t}^{i4}(x, y)^{\frac{1}{\theta_{i4}^G} - 1} (-\ln \Re_{\Delta t}^4(x))^{\theta_{i4}^G - 1} \frac{1}{\Re_{\Delta t}^4(x)}, \end{aligned} \quad (5.22)$$

where

$$\begin{aligned} \Delta t &= (n - m_i)T, \\ x &= \sum_{k=m_iT}^{k=nT} D_k^4, \\ y &= M - x_{m_iT}^i, \\ \Re_{\Delta t}^4(x) &= F_{\Delta t \alpha_4, \beta_4}(x), \\ \wp_{\Delta t}^{i4}(x, y) &= (-\ln \Re_{\Delta t}^4(x))^{\theta_{i4}^G - 1} + [-\ln \Re_{\Delta t}^i(y)]^{\theta_{i4}^G - 1}, \end{aligned}$$

and $F_{(n-m_i)T \alpha_4, \beta_4}$ is Gamma distribution function with shape and scale parameters $(n - m_i)T \alpha_4$ and β_4 respectively. Equation 5.22 is obtained from Equation 5.20 with specific information.

As a result, the degradation increments of *component* 4 contribute to estimate PM_i of component i ($i = 1, 2, 3$) without inspection operations. The quality of predicted PM_i depends on the time elapsed since the last inspection of component i ($i = 1, 2, 3$) as well as the dependence degree between *component* 4 and component i . The inspection operations of slower degrading components are triggered only when the value of PM_i

Figure 5.6: An illustration of *policy III1*

exceeds an inspection threshold $R2$. When their degradation levels are high enough and exceed a direct replacement threshold $R2$, components are supposed to have deteriorated enough and they must be replaced. To make subsequent equations more succinct and readable, we name a new function $\mathfrak{S}_t^{ij}(x, y)$ as:

$$\mathfrak{S}_t^{ij}(x, y) = \exp(-\varphi_t^{ij}(x, y)^{\frac{1}{\theta_{ij}^G}}) \varphi_t^{ij}(x, y)^{\frac{1}{\theta_{ij}^G}-1} (-\ln \mathfrak{R}_t^j(x))^{\theta_{ij}^G-1} \frac{1}{\mathfrak{R}_t^j(x)}, \quad (5.23)$$

such that $PM_i = 1 - \mathfrak{S}_\Delta^{ij}t$. To have a better understanding of *policy III1*, we give an example in Figure 5.6 where the time of inspection is indicated by cross on the deterioration curve. It shows that by inspecting component 4 periodically and taking into account the observed increments, the inspection cost is saved. For example, on the time interval $[7.5, 11]$, *component 4* is revealed to deteriorate slowly so that *components 1, 2, 3* start to be inspected lately. On the contrary, on the time interval $[24.5, 25]$, a big jump is detected for *component 4* hence the other components are replaced directly without inspection.

5.4.2.3 Non-periodic strategy considering stochastic dependence (*policy I-I2*)

The purpose of *policy III1* is to avoid unnecessary inspection operations for the slower degrading components using stochastic dependences between the critical component in the system (the faster degrading one) and the others. Aiming to benefit from non-periodic inspection scheme for all components, we propose a new maintenance policy denoted as *policy II2*.

In *policy II2*, the inspection scheme of each component (even the fastest degrading

component) is based on the evaluation of probability PM_i of exceeding the preventive threshold every T time units so as to allow non-periodic inspections for all components. Moreover probability PM_i is calculated using the observed degradation data of any other components. Components are treated sequentially in the procedure, beginning with the fastest degrading one. The figure 5.10 illustrates the procedure of decision-making of *policy 3* in which some key elements are highlighted (marked in the flowchart from (1) to (7)) and explained in the following list if nT is the current decision time and m_iT is the last operation (either inspection or replacement) date of *component i*:

1. Initialisation “ $i = 4$ ”. When asking if inspections are needed, *component 4* is treated first because *component 4* degrades fastest so it should be inspected more frequently which leads to more available information. As soon as it is inspected, its revealed degradation level can be devoted to make decision for other components.
2. Instruction “find $j: j = \arg \max_k (INFOR_{ik3}), k = 1, \dots, 4$ and $k \neq i$ ” allows to identify the information that can be used for *component i*. Before the next inspection being carried out for *component i* ($i = 1, 2, 3, 4$), some other components may have been inspected hence the acquired information is available for calculating PM_i . To simplify the calculation, we only use the information observed on the component which corresponds to the longest effective activity interval denoted as $INFOR_{ik3}$. The matrix $INFOR$ stores the necessary information: $INFOR_{ij1}$ and $INFOR_{ij2}$ are the start and end time of observation interval of *component j* used for *component i* respectively; $INFOR_{ij3}$ is the length of effective activity interval of *component j* used for *component i* and $INFOR_{ij4}$ is the corresponding revealed increment of *component j* used for *component i* on the effective activity interval; $INFOR_{ij5}$ is the degradation level of *component j* observed at the last inspection.
3. Test “ $INFOR_{ij1} < m_iT$?” allows to know whether available information about *component j* can be used for *component i* or not. It should be noticed here that dependence between components is represented by dependent increments. If $m_iT > INFOR_{ij1}$, it signifies that even though the increment of *component j* is acquired, it is useless because the degradation level of *component j* is unknown at m_iT . W_i refers to the length of the selected interval to be used for *component i* where $W_i = 0$ represents no information is available for *component i*.
4. Estimation “ $PM_i = P(X_{nT}^i > M | X_{m_iT}^i = x_{m_iT}^i, m_iT, D_i, W_i, \alpha_i, \beta_i, \alpha_j, \beta_j)$ ”. If $W_i = 0$, then

$$\begin{aligned}
 PM_i &= P(X_{nT}^i > M | X_{m_iT}^i = x_{m_iT}^i, m_iT, D_i, W_i, \alpha_i, \beta_i, \alpha_j, \beta_j) \\
 &= 1 - F_{\alpha_i(nT-m_iT), \beta_i}(M - x_{m_iT}^i),
 \end{aligned} \tag{5.24}$$

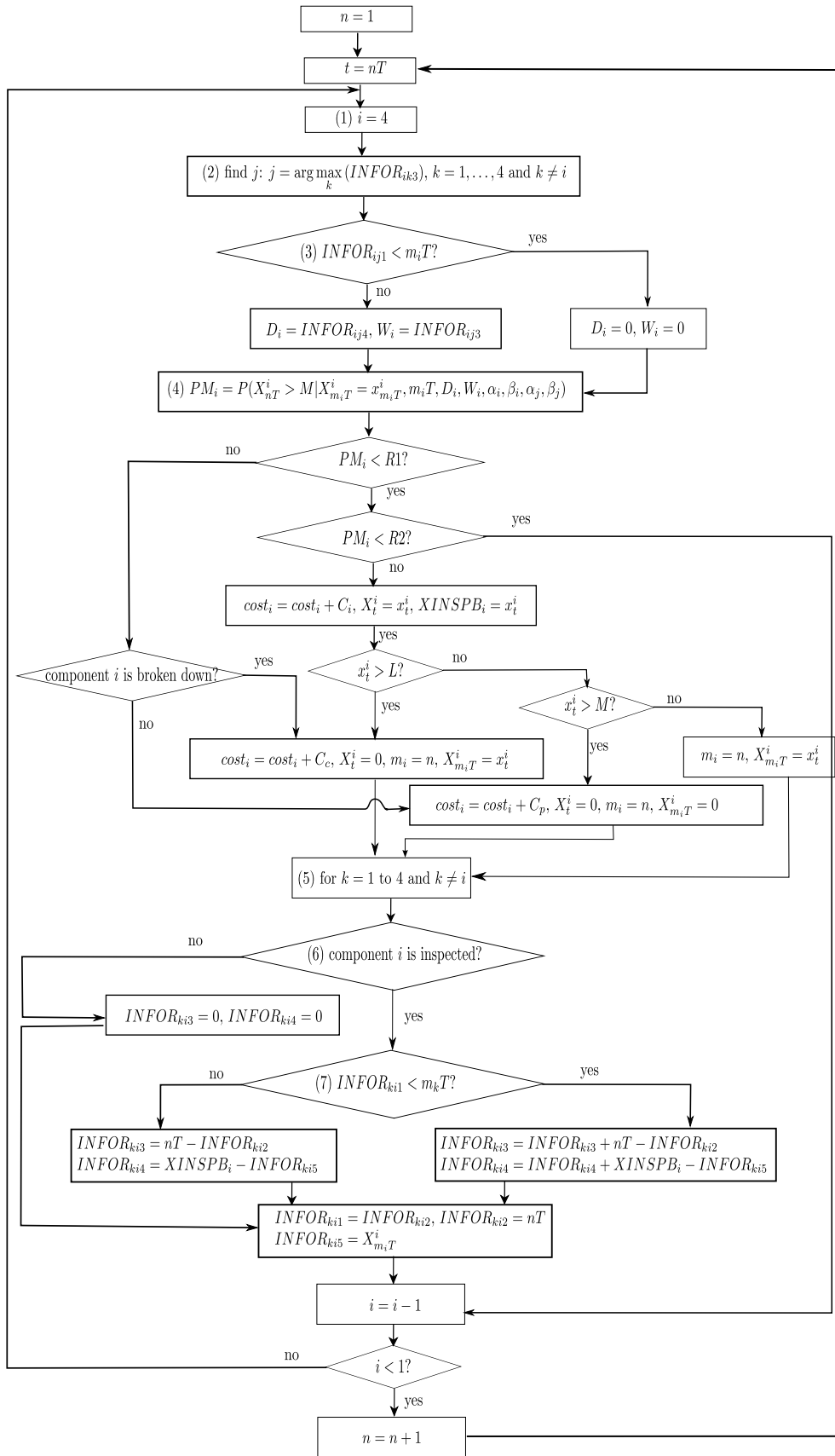


Figure 5.7: Flowchart of policy 3

and in this case, PM_i is completely decided by its marginal distribution; otherwise if *component j* with observed increment D_i is selected to be used for *component i*

$$\begin{aligned}
PM_i &= P(X_{nT}^i > M | X_{m_i T}^i = x_{m_i T}^i, m_i T, D_i, W_i, \alpha_i, \beta_i, \alpha_j, \beta_j) \\
&= 1 - \iint_{x+y < M - x_{m_i T}^i} f_{\alpha_i(nT - m_i T - W_i), \beta_i}(x) h(y | D_i) dx dy \\
&\simeq 1 - \int_{x < M - x_{m_i T}^i} f_{\alpha_i(nT - m_i T - W_i), \beta_i}(x) \mathfrak{S}_{W_i}^{ij}(D_i, M - x_{m_i T}^i - x).
\end{aligned} \tag{5.25}$$

where $h(y | D_i)$ is the conditional distribution of *component i* during time W_i knowing that the observed increment of *component j* during W_i is D_i . This distribution function depends on the dependence degree θ_{ij} between *components i* and *j* and individual degradation parameters of components. Equation 5.25 implies that the shorter W_i is, the calculation of PM_i depends more on its marginal distribution. Since stochastic dependences exist between components, the observed increments of one component certainly can serve for predicting increments of other components. That's why we choose the longest W_i in “(2) find j : $j = \arg \max_k (INFOR_{ik3})$, $k = 1, \dots, 4$ and $k \neq i$ ”.

5. Loop “ $k = 1, \dots, 4, k \neq i$ ”. After verifying $PM_i < R1$, *component i* is surely operating, therefore the new available information should be stocked for other components except itself.
6. Test “*component i* is inspected?” is dedicated to the updating of *component i*. If answer is yes, the degradation level of *component i* is revealed such that the information stock for *component j* can be updated; otherwise, *component i* is replaced without inspection hence not useful information can be updated.
7. Test “ $INFOR_{ki1} < m_k T$?” allows to determine which inspection of *component i* can bring new information used for which component. To update the information about *component i* available for other components, three cases are detailed in Figure 5.8 where $INFOR'$ is values of $INFOR$ after being updated using the revealed information of *component i* and $m_k T$ (*case j*) ($j = 1, 2, 3$) is the last operation (either inspection or replacement) time of *component k* in case j . Case 1 ($m_k T < INFOR_{ki1}$) means that since last operation (either inspection or replacement) time, *component i* have been inspected more than twice, such that the available information of *component i* for *component k* should be accumulated. Case 2 ($INFOR_{ki1} < m_k T < INFOR_{ki2}$) signifies that interval of information of *component i* is included in $[m_k T(\text{case2}), nT]$ such that this information can be updated for *component k*. In case 3 ($m_k T > INFOR_{ki2}$), the information of *component i* is no available for *component k* because its degradation level at time

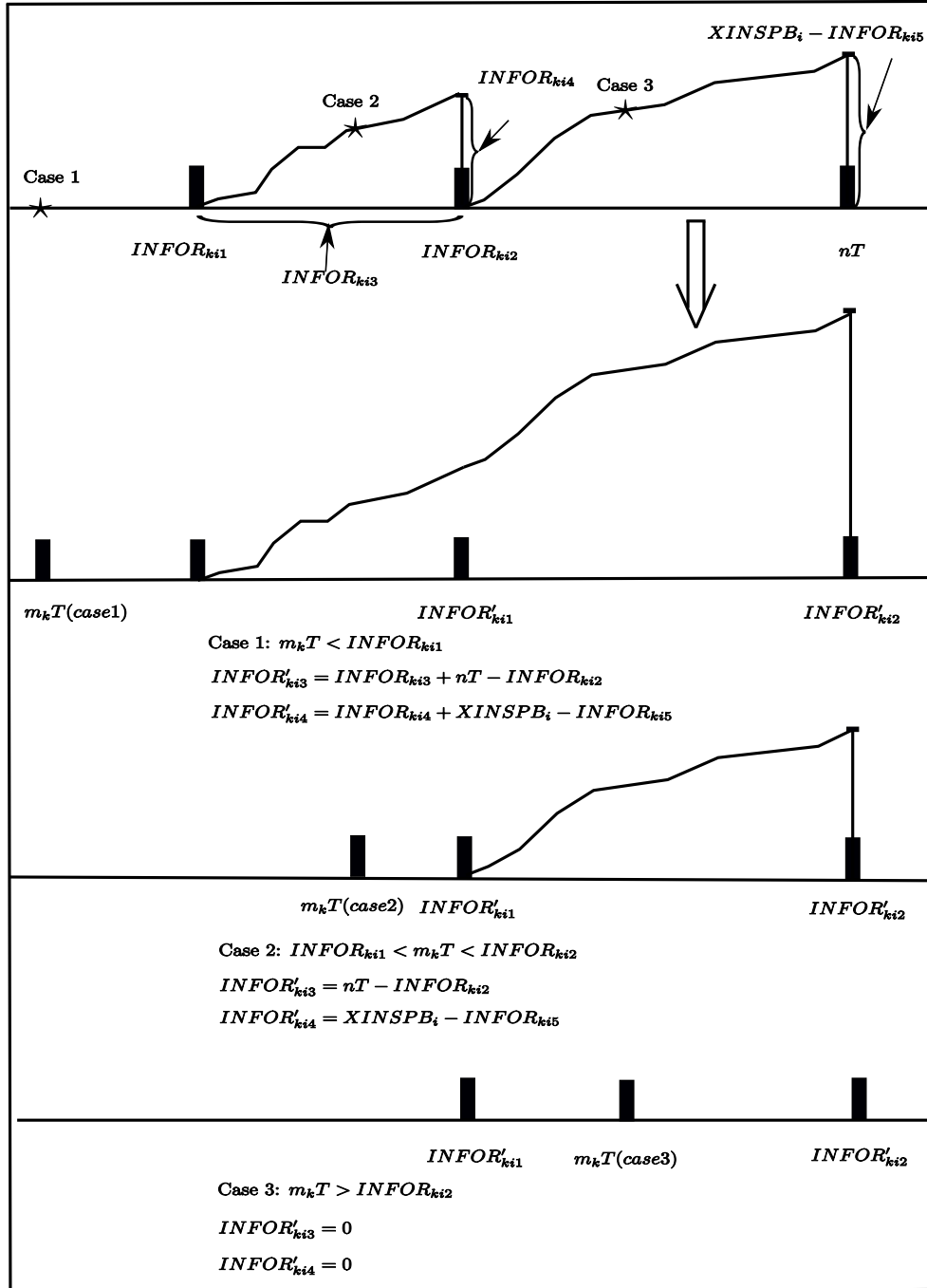


Figure 5.8: An interpretation of (7) in Figure 5.10

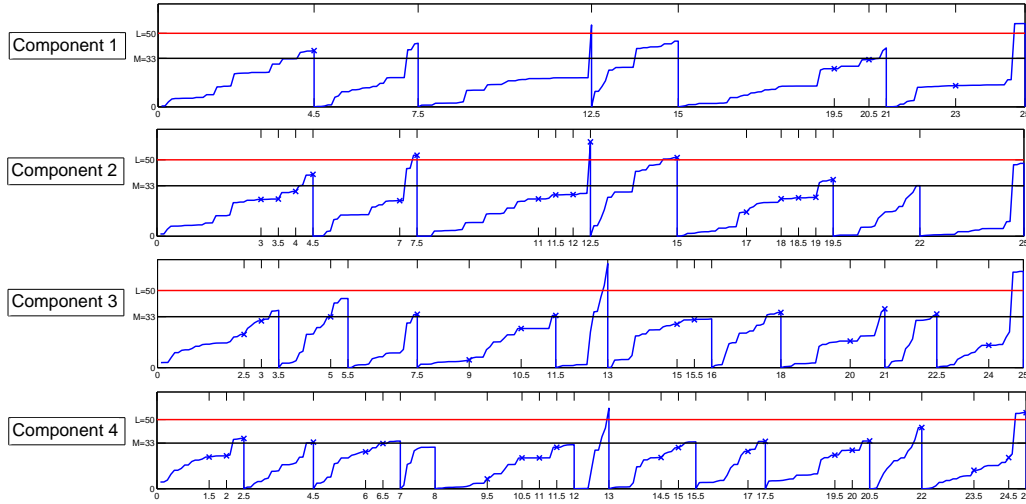


Figure 5.9: An illustration of policy II2

$m_k T(\text{case3})$ is unknown. In order to simplify the flowchart in Figure 5.10, case 3 is treated as case 2. In fact, even though the information of *component i* is updated for *component k* in case 3 in Figure 5.10, it is not used since the condition “(3) $INFOR_{k,j1} < m_k T$ ” is met then $D_k = 0, W_k = 0$.

An example is given in Figure 5.9 with the same degradation trajectory as that used in Figure 5.6. It shows that in policy II2, all components are non-periodically inspected and the next inspection time depends on not only the last revealed deterioration level and its deterioration parameters but also the revealed deterioration information of other components. For example, as *component 4* deteriorates the fastest and has the biggest variance so that it starts to be inspected first. Without taking into account the stochastic dependence, the first inspection of *component 4* always takes place 1.5 time units after it is maintained because at that time PM just exceeds the inspection threshold R2. However, in Figure 5.9, the first inspection of *component 4* takes place 2 time units after it is replaced at $t=2.5$ because *component 3* and *component 2* are revealed to deteriorate slowly during the time intervals $[2.5,3]$ and $[3,3.5]$ respectively whereas the *component 4* is directly replaced at $t=13$ only 1 time unit after the beginning of the lifecycle because a big jump has been detected for *component 2* on the time interval $[12,12.5]$.

5.4.2.4 Two strategies not considering stochastic dependence

In order to evaluate the profit gained by taking advantage of stochastic dependences, two maintenance policies which do not take into account stochastic dependences are proposed. Policy II3 is similar to policy II2 except that the calculation of PM_i relies

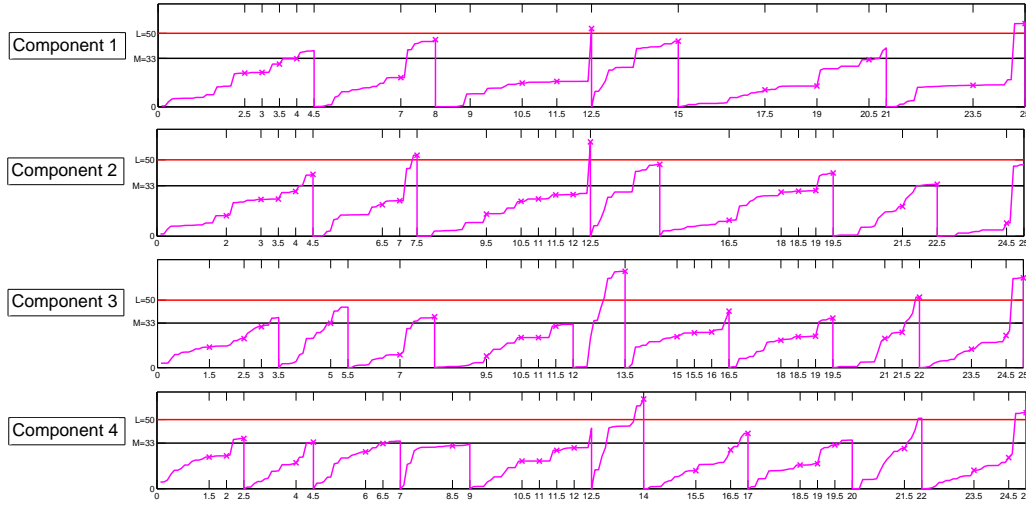
completely on the distribution of component i and do not take the revealed information of other components into consideration.

- Inspection strategy: at every T time units, the inspection operation of *component* i ($i = 1, 2, 3, 4$) is possible according to the value of PM_i where $PM_i = P(X_t^i > M | X_{t'}^i = x_{t'}^i) = 1 - F_{(t-t')\alpha_i, \beta_i}(M - x_{t'}^i)$ (t is current time, $x_{t'}^i$ and t' is last known degradation level and date respectively). If $R2 < PM_i < R1$, component i is inspected with a cost c_{insp} .
- Replacement strategy: at decision time, if $PM_i > R1$, component i ($i = 1, 2, 3, 4$) is replaced without inspection operation: when component is broken, a corrective maintenance incurs a cost c_c ; otherwise, a preventive maintenance incurs a cost C_p . *component* i ($i = 1, 2, 3, 4$) is replaced if the revealed degradation level exceeds a preventive threshold M : when the revealed degradation level exceeds a preset corrective threshold, a corrective maintenance incurs a cost c_c ; otherwise, a preventive maintenance incurs a cost c_p .

The replacement strategy of policy II3 is the same as that of policy III1 for *component* i ($i = 1, 2, 3$) and it applies on all components. It is noteworthy that both policies II2 and II3 are condition-based maintenance policies with non-periodic inspections where the inspection intervals depend on the condition of components but they are limited to the multiple of T . Furthermore, even though components are non-periodically inspected, the reliability of components is controlled to be at least over $1 - R1$.

An example is given in Figure 5.10 with the same degradation trajectory as that used in Figures 5.6 and 5.9. It shows that all components are non-periodically inspected and the next inspection time depends only on the last revealed deterioration level and individual deterioration parameters. The first inspection of *components* 1, 2, 3 and 4 always takes place 2.5, 2, 1.5, 1.5 time units respectively after they are replaced because at that times the value of PM of components just exceeds the inspection threshold $R2$. Once the first inspection takes place, the next ones can be decided according to revealed condition of component. For example, *component* 1 is inspected soon at $t=3$ when the deterioration level at $t=2.5$ is revealed to be close to the preventive threshold but the inspection takes place late at $t=19$ when the deterioration level at $t=17.5$ is revealed to be in low level.

It can be seen in Figures 5.6, 5.9, 5.10 that *policies* 1, 2, 3 have 2, 5, 3 groups of two maintenance operations carried out simultaneously, 4, 2, 3 groups of three maintenance operations and 1, 1, 0 groups of four maintenance operations on $[0,5]$ respectively that indicates a significant set-up cost saving of 13, 12, 9 respectively. As a result, the proposed condition-based maintenance policies can also take advantage of economic dependence.

Figure 5.10: An illustration of *policy II3*

A predictive maintenance policy with periodic inspections is proposed and denoted as *policy II4*.

- Inspection strategy: *component i* ($i = 1, 2, 3, 4$) is inspected every T time unit and its degradation level is revealed perfectly denoted as x_{nT}^i ($n = 1, 2, \dots, \infty$).
- Replacement strategy: at inspection time nT , if $x_{nT}^i > L$, a corrective maintenance is carried out for *component i*; if $x_{nT}^i < L$ but its conditional reliability at next inspection time given the observed current degradation level is less than a threshold R that is:

$$P(X_{(n+1)T}^i > L | X_{nT}^i = x_{nT}^i) = F_{T\alpha_i, \beta_i}(L - x_{nT}^i) < R, \quad (5.26)$$

then a preventive maintenance is carried out.

Such a strategy with periodic inspection scheme and predictive indicator is used for multi-component systems to develop grouping maintenance strategies taking account of economic dependence (see [100, 174]).

5.5 Numerical experiments

In this section, we evaluate the performance of the proposed maintenance policies using Monte-Carlo simulation. Table 5.1 and Table 5.2 state components and cost parameters. Components within subsystems degrade with similar speeds and subsystem 1 degrades in mean more slowly than subsystem 2. Cost values are arbitrarily fixed with the assumption that the corrective maintenance is much more expensive than the preventive

Table 5.1: Degrading parameters for three configurations of system

component i	1	2	3	4
mean	7	9	12	13
variance	35	45	60	65
α_i	7/5	9/5	12/5	13/5
β_i	1/5			

Table 5.2: Cost parameters

L	C_c	C_p	C_{insp}	C_d
50	100	20	1	10

one because the unpredicted failures often cause incalculable loss in practice and that inspecting component costs much less than replacing it.

5.5.1 Performance analysis of maintenance strategies based on subsystems dependence

5.5.1.1 Configurations of system regarding dependences

In order to investigate the influences of stochastic dependences within and between subsystems, we propose three cases of system. System 1 has moderate between-subsystems dependence and strong within-subsystems dependence. To investigate the influence of dependence within subsystems, system 2 has weaker within-subsystems dependence than system 1. Likewise, to study the effect of between-subsystems dependence, system 3 has independent subsystems. The dependence parameters of systems are defined in Table 5.3, where $\theta_0, \theta_1, \theta_2$ is the parameter of Clayton Lévy copula for the dependence between subsystems, the dependence within subsystem 1 and the dependence within subsystem 2 respectively. τ in Table 5.3 represents approximate value of Kendall's tau related to each given value of θ of Clayton Lévy copula because unlike ordinary copulas, the Kendall's tau of dependence modeled by Lévy copulas does not have closed form.

5.5.1.2 Comparison of maintenance policies for system 1 as the baseline case

Table 5.3: Dependence parameters for three configurations of system for maintenance policies of part I

parameter of clayton function	system 1	system 2	system 3
θ_0	1.0 ($\tau \simeq 0.43$)	1 ($\tau \simeq 0.43$)	0 ($\tau \simeq 0$)
θ_1	1.5 ($\tau \simeq 0.56$)	1.1 ($\tau \simeq 0.46$)	1.5 ($\tau \simeq 0.56$)
θ_2	2.0 ($\tau \simeq 0.65$)	1.2 ($\tau \simeq 0.59$)	2.0 ($\tau \simeq 0.65$)

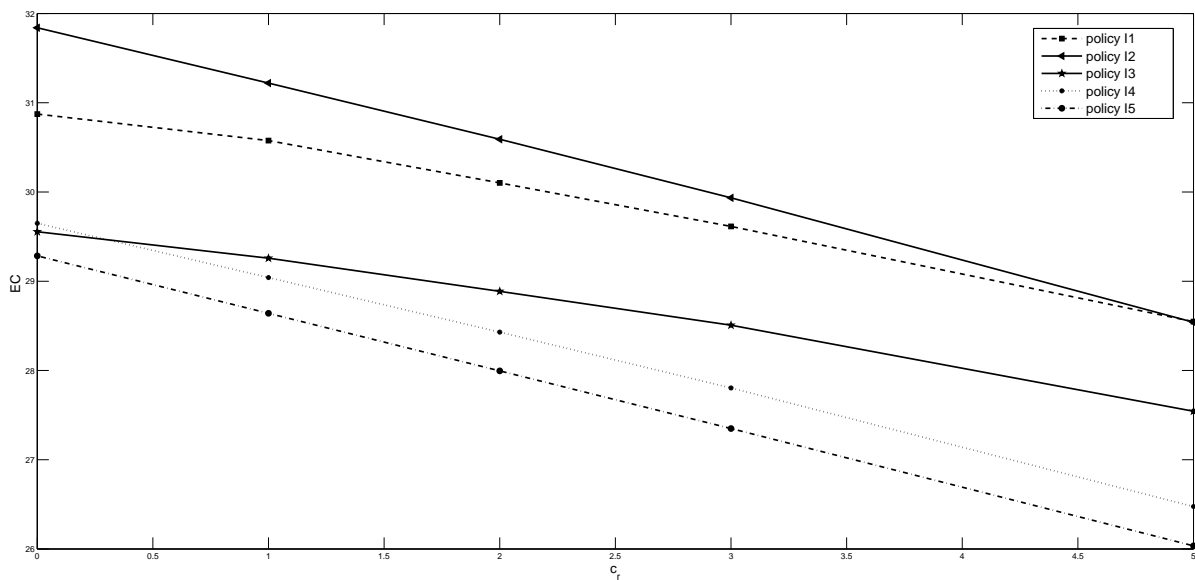
Figure 5.11: Optimal values of EC for policies I1, I2, I3, I4 and I5 in system 1 when set-up cost varies

Table 5.4: Optimal parameters and main characteristics of maintenance policies for system 1

	c_r	EC	T_1	T_2	M_1	M_2	$\bar{A}(\%)$	Pc_i (%)				LC_i				NG^p (%)			
								1	2	3	4	1	2	3	4	1	2	3	4
PI1	0	30.87	1.2	0.8	29.4	29.5	9.6	3.7	5.7	4.7	5.4	5.2	4.1	3.1	2.9	66.2	28.4	4.1	1.3
	1	30.58	0.9	0.9	30.9	27.7	7.46	3	4.4	4.3	5	5.2	4.2	3	2.8	52.7	30.9	12.5	3.9
	2	30.1	1	1	30.1	26.8	8.64	3	4.7	4.6	5.4	5.2	4.1	2.9	2.8	48.8	31.9	14.2	5
	3	29.61	1	1	29.5	26.7	8.23	2.7	4.2	4.6	5.4	5.1	4.1	2.9	2.8	48.6	31.8	14.4	5.2
	5	28.55	1.1	1.1	28.6	25.3	8.82	2.7	4.3	4.4	5.3	5	4	2.9	2.7	43.9	32.9	16.5	6.7
PI2	0	31.84	1.2	0.8	31.9	30.9	9.37	2	8.1	3.7	5.9	4.4	4.4	2.9	2.9	0	92.5	0	7.5
	1	31.22	1.2	0.8	31.1	30.9	8.76	1.8	7.2	3.7	5.9	4.3	4.3	2.9	2.9		92.5		7.5
	2	30.59	1.2	0.8	31.1	30.4	8.45	1.8	7.2	3.4	5.4	4.3	4.3	2.8	2.8		92.4		7.6
	3	29.93	1.6	0.8	27.9	30.4	10.19	1.9	7.4	3.4	5.4	4.1	4.1	2.8	2.8		84.3		15.7
	5	28.54	1	1	32	27.8	7.78	1.5	6.2	3.6	5.5	4.3	4.3	2.7	2.7		76.9		23.1
PI3	0	29.55	0.9	0.6	30.5	30.8	7.48	3	5.7	4.3	4.2	5.2	4.2	3.1	2.9	67.9	27.8	3.3	1
	1	29.26	1	0.5	29.9	31.9	7.21	3.1	5.9	4.1	4	5.2	4.2	3.2	2.9	65.2	27.6	5.7	1.6
	2	28.89	1.2	0.6	27.9	30.6	7.8	3	5.4	4.2	4.1	5	4	3.1	2.9	60.3	30.4	7	2.3
	3	28.51	0.8	0.8	30.6	28.5	7.78	2.6	5.2	4.7	4.7	5.2	4.2	3	2.8	52.8	31.6	11.6	4
	5	27.54	0.9	0.9	29.1	26.8	7.55	2.3	4.4	4.5	4.4	5	4	3	2.7	48.1	33	13.5	5.4
PI4	0	29.65	0.9	0.6	32.2	31.4	6.07	1.9	5.6	4.2	4.5	4.3	4.3	2.9	2.9	0	94.1	0	5.9
	1	29.04	0.9	0.6	32.2	31.3	6.02	1.9	5.6	4.1	4.5	4.3	4.3	2.9	2.9		94.1		5.9
	2	28.43	0.9	0.6	32.2	30.6	5.69	1.9	5.6	3.6	3.9	4.3	4.3	2.9	2.9		94		6
	3	27.8	0.7	0.7	33.7	29.9	5.45	1.8	5.1	4	4.6	4.4	4.4	2.8	2.8		83.5		16.5
	5	26.48	0.7	0.7	33.4	29.9	5.31	1.7	4.9	4	4.6	4.3	4.3	2.8	2.8		83.4		16.6
PI5	0	29.28	0.6	0.6	34.4	31.7	4.97	1.8	5	4.3	4.8	4.4	4.4	2.9	2.9	0	85.3	0	14.8
	1	28.64	0.6	0.6	34.4	31.3	4.76	1.7	4.9	4.1	4.5	4.4	4.4	2.9	2.9		85.2		14.8
	2	28	0.6	0.6	34.4	31.3	4.76	1.7	4.9	4.1	4.5	4.4	4.4	2.9	2.9		85.2		14.8
	3	27.35	0.6	0.6	34.4	30.6	4.43	1.7	4.9	3.6	3.9	4.4	4.4	2.9	2.9		85.1		14.9
	5	26.03	0.6	0.6	33.3	30.6	4.08	1.5	4	3.6	3.9	4.3	4.3	2.9	2.9		84.6		15.4

In this section, we compare the performance of the maintenance policies I1 to I5 when the stochastic dependences between subsystems are moderate. The optimal parameters and main characteristics of five proposed maintenance policies for system 1 are detailed in Table 5.4 where *PI1* stands for policy I1, \bar{A} is unavailability of system, Pc_i ($i = 1, 2, 3, 4$) is the proportion of corrective maintenance of component i , Lc_i is the mean lifecycle length of component i and NG^i is the proportion of maintenance activities that are performed on i components simultaneously. Figure 5.11 illustrates the performance of all maintenance policies when the set-up cost c_r varies from 0 to 5. Considering maintenance policies which inspect both of two components within subsystem, policy I2 is worse than policy I1 when c_r is small. In fact, policy I2 replaces the whole subsystem as soon as the degradation level of a component in the subsystem exceeds the preventive threshold, hence the service time of the other component is shortened. In fact, when set-up cost is 0, policy I2 reduces the lifetime of *component 1* from 5.2 to 4.4 and increases that of *component 2* from 4.1 to 4.4 when compared with policy I1. Neither of components within subsystem can achieve its optimum such that the maintenance cost augments. However policy I2 has much more grouped maintenance activities so that when c_r increases, it profits more economic dependence than policy I1 and the gap is narrowed. It is noteworthy that policy I2 begins to dominate policy I1 when c_r is greater than 5. Even though policy I1 tries to maximize the grouping opportunities by a joint inspection interval when set-up cost is 5, it is slightly worse than policy I2 as still 43.9% of the maintenance activities are operated individually. In practice, when applying the maintenance policies to some systems, the whole subsystem has to be replaced due to technical constraints. For example, some related bearing parts in a vehicle, such as timing belt and tension wheel are often replaced simultaneously and it is better to also replace water pump with them because it is not so easy to implement the maintenance operations on a vehicle. In cases where practical constraints lead to replace components in a subsystem at the same time, policy I2 has to be applied instead of policy I1 leading to around 3% of loss at most for small values of c_r when compared to policy I1.

Policies I3, I4 and I5 are designed to take advantage of the dependence between components and they inspect a single component within each subsystem. When compared to policy I1, policy I3 profit the stochastic dependence between components within subsystem such that the alternate inspection strategy begins to inspect components when component degrades to a certain level. Unnecessary inspection cost is saved when components is newly maintained in policy I3 hence the optimal inspection period is smaller than that of policy I1 (0.9 for subsystem 1 and 0.6 for subsystem 2 in policy I3 whereas in I1, that for subsystem 1 and 2 is 1.2 and 0.9 respectively). Meanwhile policy I3 allows each component within subsystem to be replaced according to its own degradation speed whereas policy I4 forces both components within a subsystem to be replaced

simultaneously. In fact when compared to policy I4, policy I3 reduces a lot the amount of preventive maintenances of *components* 1 and 3 but increases few of the amounts of their failures. Hence policy I3 is better than policy I4 when c_r is small. Nevertheless, policy I3 groups maintenance activities only when components happen to be inspected and they exceed the preventive threshold, so it performs the worst in taking advantage of economic dependence (48.1% of maintenance activities are performed on a single component when c_r is 5).

Like policy I3, policy I4 takes advantage of stochastic dependences between components within subsystem therefore it is better than policies I1 and I2 for all selected values of c_r . Besides, policy I4 replaces both components in a subsystem simultaneously, it has much more opportunities for maintenance grouping than policy I3. As it profits both economic and stochastic dependence, policy I4 is more profitable than policies I1, I2 and I3 when c_r is great.

Policy I5 profits further the stochastic dependences between subsystems such that it performs better than policies I3 and I4. Through the analysis of numerical experiments, we can see the advantage of policy I5 is twofold. On one hand, when two subsystems degrade dependently, triggering the inspections of the slower degrading subsystem (subsystem 1) according to the replacement time of the faster degrading subsystem (subsystem 2) can save some inspection costs without increasing the amount of failures of the slower degrading subsystem too much. On the other hand, when the slower degrading subsystem exceeds its preventive threshold between consecutive inspections, it reveals that it may have suffered some big jumps which can also be happened to faster degrading subsystem because of stochastic dependences at system level. Therefore adding an inspection of the faster degrading subsystem at these moments can avoid some of its failures. As a conclusion, maintenance policies which inspect only the faster degrading component per subsystem and replace the whole subsystem are preferable in multi-component systems in considering economic and stochastic dependence.

To briefly summarize the numerical results for system 1, it can be concluded that when components are dependent on each other, inspecting all components in the system is too expensive and unnecessary and adaptive block-replacement strategies can take advantage of economic dependence.

5.5.1.3 Comparison of maintenance policies for system 2 and system 3

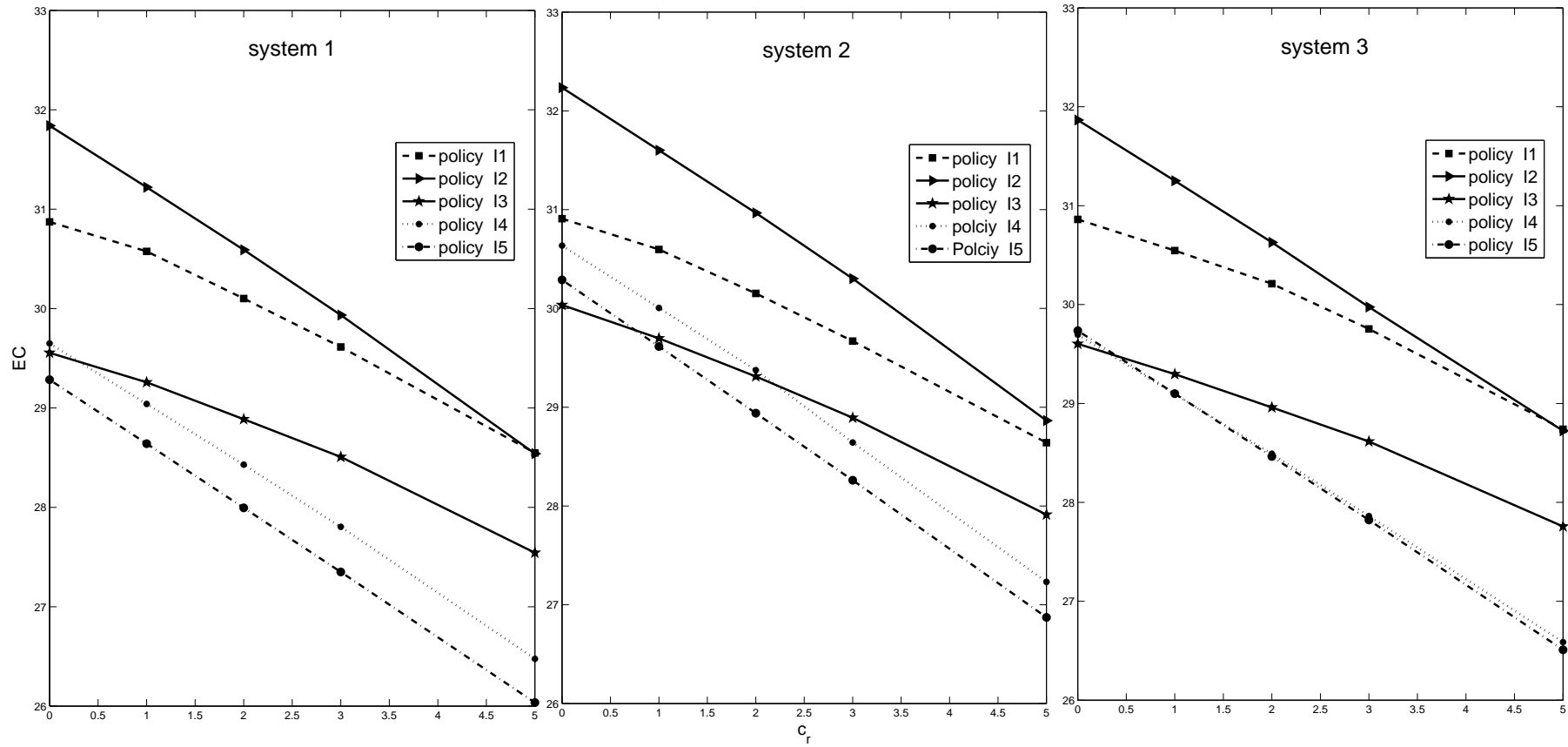


Figure 5.12: Optimal values of EC of policies I1, I2, I3, I4 and I5 for systems 1, 2, and 3

Table 5.5: Optimal parameters and main characteristics of maintenance policies for system 2

	c_r	EC	T_1	T_2	M_1	M_2	$\bar{A}(\%)$	Pc_i (%)				LC_i				NG^p (%)			
								1	2	3	4	1	2	3	4	1	2	3	4
PI1	0	30.91	1.2	0.7	29.4	30.1	8.9	3.6	5.7	4.1	4.8	5.2	4.1	3.1	2.9	75.8	23	0.9	0.3
	1	30.6	1.2	0.8	29.4	28.7	9.3	3.6	5.7	4.1	4.8	5.2	4.1	3	2.8	68	26.8	4	1.2
	2	30.15	1	1	30.3	26.7	8.9	3.2	4.8	4.5	5.4	5.2	4.1	2.9	2.7	50.6	31.3	13.6	4.5
	3	29.67	1	1	29.7	26.4	8.3	2.8	4.3	4.3	5.1	5.1	4.1	2.9	2.7	50.1	31.3	13.8	4.7
	5	28.64	1.1	1.1	29	25.3	9.5	2.9	4.6	4.5	5.4	5	4	2.9	2.7	46	32.2	15.8	5.9
PI2	0	32.23	1.2	0.8	31.8	30.9	9.6	2.1	7.7	3.6	5.5	4.3	4.3	2.8	2.8	0	92.4	0	7.6
	1	31.6	1.2	0.8	31.3	30.9	9.2	2	7.1	3.6	5.5	4.3	4.3	2.8	2.8		92.4		7.6
	2	30.97	1.2	0.8	31.3	30.9	9.2	2	7.1	3.6	5.5	4.3	4.3	2.8	2.8		92.4		7.6
	3	30.3	1	1	32.6	28.6	8.9	1.8	6.5	3.8	5.8	4.3	4.3	2.7	2.7		76.9		23.1
	5	28.86	1	1	32	28.2	8.2	1.6	5.9	3.6	5.4	4.2	4.2	2.7	2.7		76.5		23.5
PI3	0	30.03	1	0.6	29.4	29.8	9.5	3.2	6	4.9	3.8	5.1	4.1	3.1	2.8	67.7	29.4	2.2	0.7
	1	29.7	1	0.5	29	31.1	8.6	2.9	5.6	4.8	3.7	5.1	4.1	3.1	2.8	63.3	29	6	1.7
	2	29.31	1.2	0.6	28.1	29.5	9.8	3.4	6.1	4.6	3.6	5	4.1	3.1	2.8	59	31.4	7.1	2.4
	3	28.89	0.8	0.8	30.6	27.6	10.0	2.9	5.8	5.2	4.2	5.2	4.2	3	2.7	52	32	11.7	4.2
	5	27.91	0.9	0.9	29.1	26	9.5	2.6	5	5	4	5	4	2.9	2.6	47.6	33.3	13.6	5.5
PI4	0	30.64	0.9	0.6	32.5	29.8	8.7	3	6	5.3	3.4	4.3	4.3	2.8	2.8	0	94	0	6
	1	30.01	0.9	0.6	31.6	29.8	8.1	2.6	5	5.3	3.4	4.2	4.2	2.8	2.8		93.9		6.1
	2	29.38	0.9	0.6	31.6	29.8	8.1	2.6	5	5.3	3.4	4.2	4.2	2.8	2.8		93.9		6.1
	3	28.64	0.8	0.8	32	27.4	8.0	2.4	4.5	4.9	3.8	4.2	4.2	2.7	2.7		80.8		19.2
	5	27.23	0.8	0.8	31.7	27.4	7.8	2.3	4.2	4.9	3.8	4.2	4.2	2.7	2.7		80.7		19.3
PI5	0	30.29	0.7	0.7	33.2	29.4	8	2.7	4.8	5.9	4.2	4.3	4.3	2.8	2.8	0	82.9	0	17.1
	1	29.61	0.7	0.7	33.1	29.4	8.3	2.6	4.7	5.9	4.2	4.3	4.3	2.8	2.8		82.9		17.1
	2	28.94	0.7	0.7	33.1	29.4	8.3	2.6	4.7	5.9	4.2	4.3	4.3	2.8	2.8		82.9		17.1
	3	28.26	0.7	0.7	32.9	28.3	7.2	2.5	4.5	4.9	3.5	4.3	4.3	2.7	2.7		82.7		17.3
	5	26.87	0.7	0.7	32	28	6.5	2.2	3.8	4.6	3.3	4.2	4.2	2.7	2.7		82.2		17.8

Table 5.6: Optimal parameters and main characteristics of maintenance policies for system 3

	c_r	EC	T_1	T_2	M_1	M_2	$\bar{A}(\%)$	Pc_i (%)				LC_i				NG^u (%)			
								1	2	3	4	1	2	3	4	1	2	3	4
PI1	0	30.86	1.2	0.8	29.6	28.9	9.9	3.7	5.9	4.2	4.9	5.2	4.2	3	2.8	66.1	29.6	3.6	0.6
	1	30.55	1.2	0.8	29.1	28.8	9.4	3.4	5.4	4.1	4.8	5.1	4.1	3	2.8	65.9	29.8	3.7	0.6
	2	30.24	1	1	30.4	26.6	9.8	3.2	4.9	4.5	5.3	5.2	4.2	2.9	2.7	49.7	36.7	11.6	2.1
	3	29.75	1.1	1.1	29.9	25.5	11.2	3.4	5.3	4.6	5.5	5.2	4.1	2.9	2.7	45.2	38.3	13.8	2.8
	5	28.74	1.2	1.2	28.7	23.8	11.4	3.2	5.1	4.3	5.2	5.1	4.1	2.8	2.6	39.4	40.1	16.6	3.9
PI2	0	31.87	1.2	0.8	31.7	30.8	9.7	2	7.9	3.7	5.8	4.4	4.4	2.9	2.9	0	94.1	0	5.9
	1	31.25	1.2	0.8	30.7	30.3	8.6	1.7	6.7	3.3	5.3	4.2	4.2	2.8	2.8		94		6
	2	30.63	1.2	0.8	30.6	30.3	8.5	1.7	6.6	3.3	5.3	4.2	4.2	2.8	2.8		94		6
	3	29.98	1.4	0.7	29.3	31.9	9.1	1.8	7.1	3.4	5.5	4.2	4.2	2.9	2.9		89		11
	5	28.72	0.9	0.9	32.4	29	7.3	1.3	5.4	3.4	5.4	4.3	4.3	2.8	2.8		85.4		14.6
PI3	0	29.60	1	0.6	29.7	31	8.2	3	5.7	4.5	4.4	5.1	4.1	3.1	2.9	69.8	28.2	1.7	0.3
	1	29.39	1	0.5	29.7	31.8	7.4	3	5.7	4	3.9	5.1	4.1	3.1	2.9	66	28.9	4.4	0.7
	2	28.96	1.2	0.6	28.7	30.4	9	3.4	6.1	4	4	5.1	4.1	3.1	2.8	61.3	31.7	5.8	1.2
	3	28.62	0.8	0.8	30.9	27.9	8.2	2.7	5.5	4.2	4.1	5.2	4.2	3	2.7	54.3	35.4	8.6	1.7
	5	27.76	0.9	0.9	29.4	26.9	8.6	2.4	4.7	4.5	4.5	5	4.1	3	2.7	49.5	37.4	10.6	2.4
PI4	0	29.70	0.9	0.6	32.4	31.2	6.4	1.9	5.7	4	4.5	4.3	4.3	2.9	2.9	0	95.7	0	4.3
	1	29.09	0.9	0.6	32.4	31.2	6.4	1.9	5.7	4	4.5	4.3	4.3	2.9	2.9		95.7		4.3
	2	28.49	0.9	0.6	32.4	31	6.3	1.9	5.7	3.9	4.3	4.3	4.3	2.9	2.9		95.7		4.3
	3	27.86	0.7	0.7	33.8	30.1	6	1.8	5.2	4	4.6	4.4	4.4	2.9	2.9		89.3		10.7
	5	26.59	0.7	0.7	33.8	29.2	5.5	1.8	5.2	3.5	4	4.4	4.4	2.8	2.8		89.2		10.8
PI5	0	29.7	0.6	0.6	34.7	30.5	8.8	2.6	7.2	3.5	3.9	4.5	4.5	2.8	2.8	0	88.1	0	11.9
	1	29.10	0.6	0.6	34.7	30.5	8.8	2.6	7.2	3.5	3.9	4.5	4.5	2.8	2.8		88.1		11.9
	2	28.46	0.6	0.6	34.7	30.5	8.8	2.6	7.2	3.5	3.9	4.5	4.5	2.8	2.8		88.1		11.9
	3	27.82	0.7	0.7	33.7	30	10.1	2.6	7.4	4	4.6	4.4	4.4	2.8	2.8		86.1		13.9
	5	26.51	0.7	0.7	33	28.9	8.8	2.3	6.6	3.3	3.8	4.4	4.4	2.8	2.8		86		14

Table 5.7: The difference of optimal value of EC of five maintenance policies between system i ($i = 2, 3$) and system 1

	c_r	$PI1$	$PI2$	$PI3$	$PI4$	$PI5$
$\frac{EC_{s2}-EC_{s1}}{EC_{s1}}(\%)$	0	0.13	1.22	1.62	3.34	3.45
	1	0.07	1.22	1.5	3.34	3.39
	2	0.17	1.24	1.45	3.34	3.36
	3	0.2	1.24	1.33	3.02	3.33
	5	0.32	1.12	1.34	2.83	3.23
$\frac{EC_{s3}-EC_{s1}}{EC_{s1}}(\%)$	0	-0.03	0.09	0.17	0.17	1.57
	1	-0.1	0.1	0.14	0.17	1.61
	2	0.37	0.13	0.24	0.21	1.64
	3	0.47	0.17	0.39	0.22	1.72
	5	0.67	0.63	0.8	0.42	1.84

In order to investigate the influence of dependences within and between subsystems on maintenance policies, we compare the performance of policies I1, I2, I3, I4 and I5 for systems 2 and 3 with that for system 1. The results are shown in Figure 5.12 and Tables 5.5 and 5.6.

For system 2 where the within-subsystem dependence is weaker when compared to system 1, the performance of policies I4 and I5 reduce so much that policy I3 is the best when c_r is smaller than 1. In fact, when the dependence between components is weaker, the amount of information about the inspected component that can be used for the other related component is reduced. In such cases, policies I4 and I5 are less efficient to avoid the failures of non-monitored components than policy I3 which inspects both components in subsystems. Nevertheless policy I4 is still better than policy I1 even though the difference between them has been narrowed. Actually policies I2, I3, I4 and I5 profit the dependence between components. It is inevitable that their performances are reduced for system 2. However policies I4 and I5 always take better advantage of economic dependence than policy I1. The greater is c_r , the difference between them becomes larger. As a conclusion, when the dependence between components is tiny and c_r is small, it is better to apply policy I3 rather than policies I4 and I5.

In system 3 where two subsystems are independent, policies I1, I2, I3 and I4 are ranked in the same way when compared to system 1 except that policy I4 performs as well as policy I5. Policy I5 is better than policy I4 for systems 1 and 2 because it takes advantage of stochastic dependence between subsystems. When the subsystems are independent, there is no more advantage in applying policy I5.

5.5.1.4 Evaluation and relevance of maintenance policies

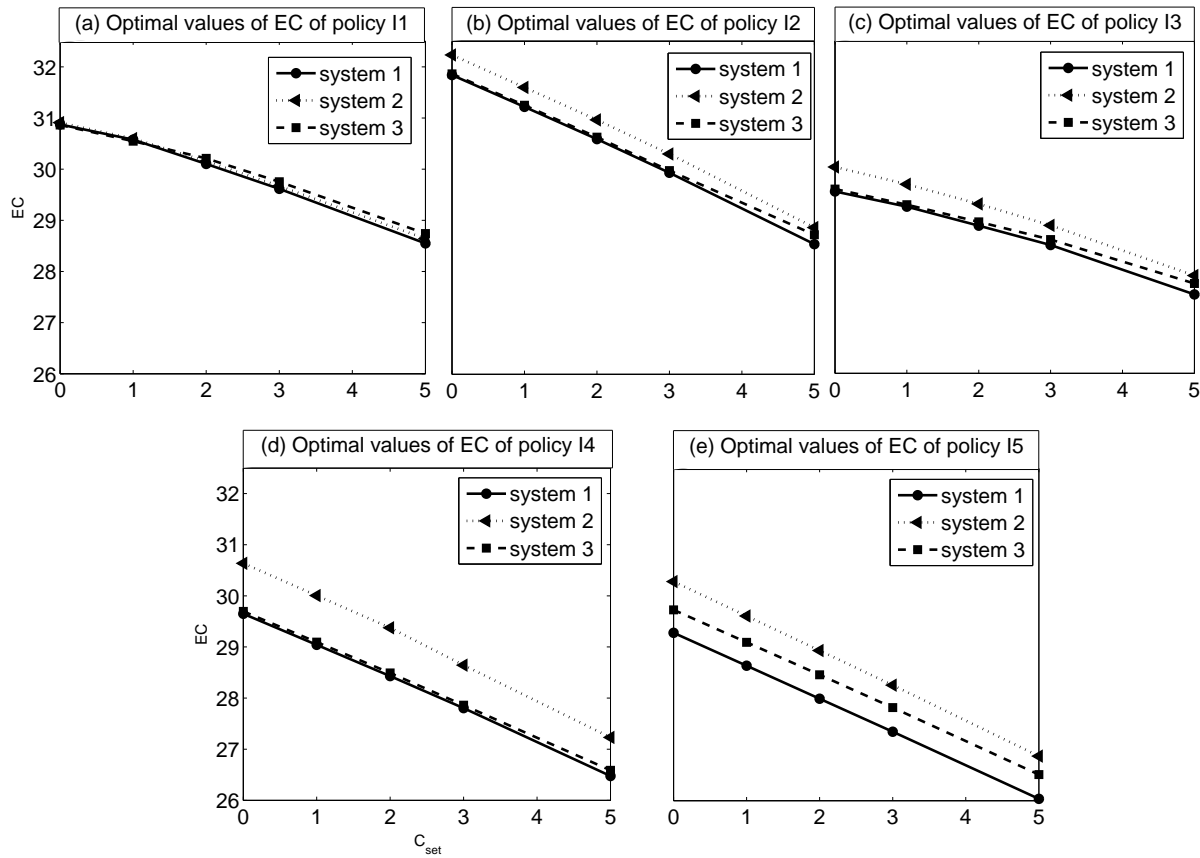


Figure 5.13: Optimal values of EC of policies I1, I2, I3, I4 and I5 when set-up cost varies from 0 to 5 for the three cases of system

In this section, we aim to summarize the performance of the five proposed maintenance policies for different systems and discuss their applications with Figure 5.13 and Table 5.7. EC_{si} ($i = 1, 2, 3$) in Table 5.7 represents the optimal value of EC of maintenance policy for system i . Figure 5.13 (a) to (e) illustrates the minimum EC of Policy I1 to I5 when the set-up cost varies from 0 to 5 for three systems. Figure 5.13 shows that policy I1 is almost insensitive to stochastic dependences within and between subsystems when c_r is small. When c_r is greater, the more stochastic dependences within and between subsystems, policy I1 has more opportunities to group maintenance activities such that $\frac{EC_{s2}-EC_{s1}}{EC_{s1}}$ is slightly increased when c_r is greater. However, the influence of stochastic dependences on policies I2, I4, I5 remains stable regardless the value of c_r . In fact, policies I2, I4 and I5 apply block-replacement within subsystems such that the influence of stochastic dependence is not sensitive to economic dependence. Figure 5.13(b), 5.13(c), 5.13(d) and 5.13(e) show that policies I2, I3, I4 and I5 are sensitive to stochastic dependences within subsystems. Moreover, policies I4 and I5 are more sensitive than policies I2 and I3. In policies I2 and I3, the slower degrading components within subsystem are inspected and they are replaced according to their revealed degradation level whereas in policies I4 and I5, they are not inspected and the decision have to be made based on the degradation levels of faster degrading components. As a consequence, when within-subsystem dependence is weaker, the maintenance decision based on the revealed degradation levels of faster degrading components is less reliable. For example, when the faster degrading component is found to exceed the preventive threshold soon after the block replacement of subsystem, the slower degrading component may be in relatively new state or when the faster degrading component is found to exceed the preventive threshold long after the block replacement of subsystem, the slower degrading component may have failed. In short, when the dependence between components is weak, the degradation levels of components are less dependent hence the maintenance decision of slower degrading components can not be made based on the information of faster degrading components.

Policies I1, I2, I3 and I4 do not take advantage of the stochastic dependence between subsystems such that the influence of between-subsystem dependence on them is much less significant than policy I5. Their performances depend slightly on the stochastic dependence between subsystems when c_r is great because stronger stochastic dependence between subsystem leads to more opportunities of maintenance grouping. Figure 5.13(e) reveals that Policy I5 is sensitive to stochastic dependences within and between subsystems. Only policy I5 considers and takes advantage of dependences at all levels.

As a conclusion, policies I4 and I5 basing the maintenance decision on a single component in a subsystem are fully exploited when the stochastic dependences are not so weak. However, such strategies are very sensitive to the stochastic dependence and they seem to request a right identification of the faster degrading component as only one

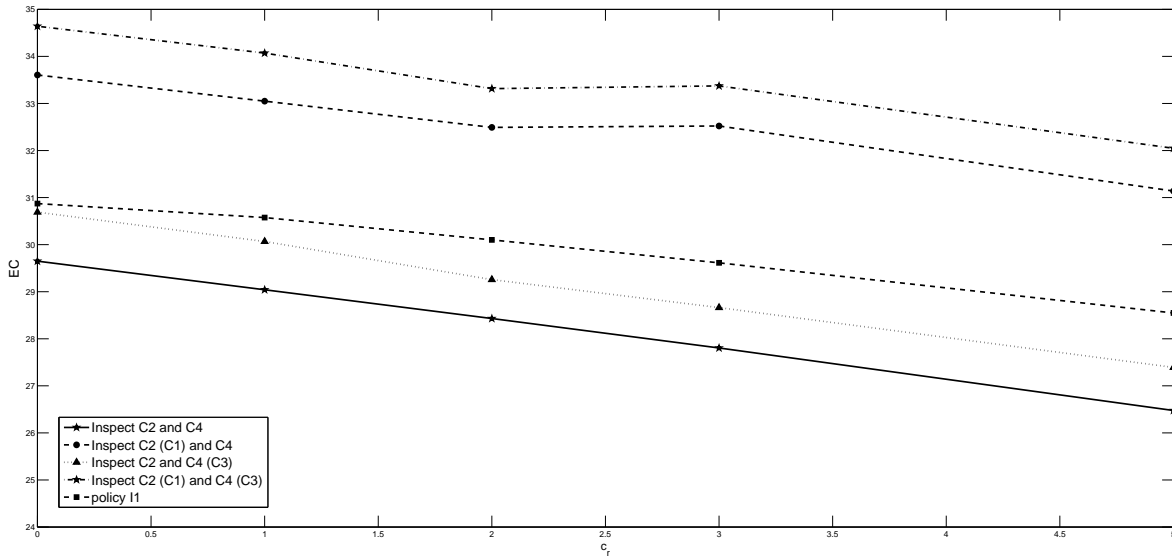


Figure 5.14: EC of policy I4 used with parameters of the faster degrading components when different components in subsystems are monitored

component is inspected. Otherwise, policy I1 is more preferable. Policy I2 is interesting when the whole subsystem has to be replaced at the same time and the system asks for a high reliability. In regard to Policy I3, it is less attractive unless c_r is small. In the next paragraph, we explore the impact of possible errors in the identification of the faster degrading components.

5.5.1.5 Sensitivity analysis

The performances of policies I4 and I5 depend not only on the degree of dependence between components but also on the identification of degradation speed of component. In practice, when two components degrade closely, it is hard to identify which is the faster one. Thus we test how policy I4 performs when errors are made in identifying the faster degrading component. The results which are applied the optimal parameters of policy I4 in system 1 are presented in Figure 5.14 and 5.15, where “Inspect C2(C1) and C4” represents that policy I4 inspects *component 1* instead of *component 2* in subsystem 1 but with parameters of *component 2* and replaces the whole subsystem 1 when degradation level of *component 1* exceeds preventive threshold. In doing so, the error is twofold: an estimation error about the parameters of the inspected component is cumulated to a mistaken strategy basing the maintenance decision on the wrong component. Three cases are considered depending on whether the error relates to subsystem 1 or subsystem 2 or both subsystems. Figure 5.14 shows that the amount of cost incurred by an error about one subsystem is much higher when the error relates to *component 2* rather than *component 4*. The error of subsystem 1 leads to a maximum of 17.61% loss

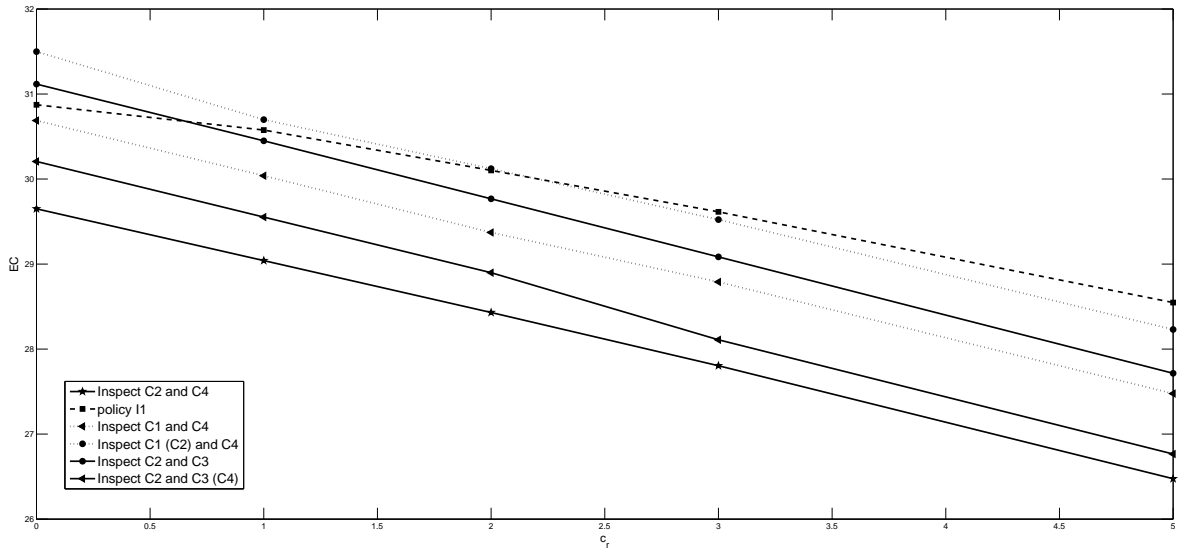


Figure 5.15: EC of policy I4 used with parameters of the lower degrading components when different components in subsystems are monitored

of maintenance cost whereas those of subsystem 2 leads to 3.53%. The reason is twofold. On one hand, *components* 1 and 2 have weaker dependence than *components* 3 and 4 so that *component* 1 is unable to be representative of *component* 2 as *component* 3 can be for *component* 4. On the other hand, the difference of mean degradation speed between *components* 1 and 2 is wider than that between *components* 3 and 4: around 20% in subsystem 1 while only 8% in subsystem 2. As *component* 2 degrades much faster and *component* 1 cannot represent it completely then basing the maintenance decision on *component* 1 with parameters of *component* 2 leads to replace *component* 2 much too late. Consequently, policy I1 with independent inspections of both components should be applied in this case. It is worth mentioning that in Figure 5.14, the curve of "Inspect C2 (C1) and C4" increases slightly between $c_r = 2$ and $c_r = 3$. Because when c_r is great, policy I4 has to find a joint inspection period for both subsystems in order to have more opportunities of maintenance grouping. When c_r passes from 2 to 3, the inspection period of subsystem 1 has been shortened to meet the inspection period of subsystem 2. When applied to *component* 1 instead of 2, this leads to more failures of *component* 2 and more downtime of system. The gain of grouping more maintenance activities cannot compensate the loss resulting from failures and downtime.

These results highlight the fact that when there is any doubt about which component degrades fast in a subsystem, policy I4 is suggested not to be used at least not be used without adapting the parameters of inspected component (inspection period and maintenance threshold). In fact as shown in Figure 5.15, if policy I4 is applied with optimal parameters of *component* 1 for subsystem 1, it performs better than policy I1 whatever the component monitored in subsystem 1 as soon as c_r is greater than 2.

Table 5.8: Dependence parameters for three configurations of system for maintenance policies of part II

parameter of clayton function	system 1	system 2	system 3
θ_0	1.0 ($\tau \simeq 0.43$)	1.5 ($\tau \simeq 0.56$)	0.8 ($\tau \simeq 0.36$)
θ_1	1.5 ($\tau \simeq 0.56$)	2.0 ($\tau \simeq 0.65$)	2.5 ($\tau \simeq 0.70$)
θ_2	2.0 ($\tau \simeq 0.65$)	1.6 ($\tau \simeq 0.58$)	2.0 ($\tau \simeq 0.65$)

Table 5.9: Optimal parameters of policy III

system	EC	T/M/R1/R2	$LC_1/LC_2/LC_3/LC_4$	$EC_1/EC_2/EC_3/EC_4$
1	28.45	0.5/33/0.52/0.12	5.36/4.22/3.20/2.98	4.64/5.98/7.82/9.97
2	28.28	0.5/33/0.54/0.16	5.37/4.22/3.21/2.98	4.52/5.83/7.92/9.97
3	28.62	0.5/33/0.54/0.12	5.37/4.22/3.20/2.98	4.70/6.07/7.82/9.97
system	UNAV	$Pc_1/Pc_2/Pc_3/Pc_4$	$\overline{NI}_1/\overline{NI}_2/\overline{NI}_3/\overline{NI}_4$	
1	4.54‰	3.06%/3.76%/4.42%/4.71%	2.40/2.21/1.47/5.96	
2	4.42‰	2.98%/3.60%/4.91%/4.71%	1.87/1.75/1.50/5.96	
3	5.07‰	3.31%/4.06%/4.46%/4.71%	2.62/2.41/1.48/5.96	

5.5.2 Performance analysis of maintenance policies making use of stochastic dependences

5.5.2.1 Configurations of system regarding the dependences

In order to investigate the influence of stochastic dependences within and between subsystems, we also propose three configurations of system for part II. Dependence parameters are detailed in Table 5.8. System 1 is the same as that in 5.5.1.1 as the baseline case. System 2 has strong within-subsystems and between-subsystems dependences. System 3 have weak between-subsystems dependence and strong within-subsystems dependence.

5.5.2.2 Performance analysis

The optimal parameters and main characteristics of policies III1, III2, III3 and III4 are detailed in Tables 5.9, 5.10, 5.11 and 5.12 where LC_i ($i = 1, 2, 3, 4$) refers to the life-cycle length of component i , EC_i refers to the expected long-run maintenance cost of component i , UNAV is the unavailability of system, Pc_i is the proportion of corrective maintenance and \overline{NI}_i is the mean number of inspections before replacement.

In policy III1, component 4 is inspected periodically whereas the other components are inspected according to the inspection strategy which is based on the value of PM_i .

Table 5.10: Optimal parameters of policy II2

system	EC	T/M/R1/R2	$LC_1/LC_2/LC_3/LC_4$	$EC_1/EC_2/EC_3/EC_4$
1	28.05	0.5/33/0.51/0.06	5.25/4.12/3.14/2.92	4.74/6.12/8.11/9.03
2	27.94	0.5/33/0.49/0.07	5.24/4.12/3.13/2.91	4.67/6.05/8.15/9.03
3	28.14	0.5/33/0.5/0.06	5.23/4.12/3.13/2.91	4.76/6.16/8.12/9.04
system	UNAV	$Pc_1/Pc_2/Pc_3/Pc_4$	$NI_1/NI_2/NI_3/NI_4$	
1	5.02% ₆₀	2.43%/3.14%/4.16%/4.76%	2.94/2.73/2.10/2.53	
2	5.20% ₆₀	2.44%/3.19%/4.26%/4.84%	2.54/2.37/2.09/2.38	
3	5.18% ₆₀	2.51%/3.16%/4.17%/4.73%	2.93/2.83/2.09/2.54	

Table 5.11: Optimal parameters of policy II3

system	EC	T/M/R1/R2	$LC_1/LC_2/LC_3/LC_4$	$EC_1/EC_2/EC_3/EC_4$
1	28.67	0.4/35/0.29/0.07	5.42/4.23/3.17/2.93	4.92/6.28/8.35/9.05
2	28.67	0.4/35/0.29/0.07	5.42/4.23/3.17/2.93	4.92/6.28/8.35/9.05
3	28.67	0.4/35/0.29/0.07	5.42/4.23/3.17/2.93	4.92/6.28/8.35/9.05
system	UNAV	$Pc_1/Pc_2/Pc_3/Pc_4$	$NI_1/NI_2/NI_3/NI_4$	
1	7.03% ₆₀	3.45%/4.21%/4.46%/4.88%	3.95/3.24/2.92/2.67	
2	6.87% ₆₀	3.45%/4.21%/4.46%/4.88%	3.95/3.24/2.92/2.67	
3	7.10% ₆₀	3.45%/4.21%/4.46%/4.88%	3.95/3.24/2.92/2.67	

Therefore, the performance of policy III1 mainly depends on the dependence degree between *component* 4 and *component* i ($i = 1, 2, 3$). The optimal values of $(T, M, R1, R2)$ for three systems indicate that when the stochastic dependence between *component* i and *component* 4 is weaker, policy III1 tends to use a higher $R1$ and a lower $R2$ to inspect components more frequently and avoid failures. System 3, with weak between-subsystems dependence, has the largest observed mean number of inspections before replacement for *components* 1 and 2. Despite this, we can see that the proportion of corrective maintenance of *components* 1 and 2 also augments in system 3 which means that the increasing number of inspections can not compensate the loss of validity of the revealed information of *component* 4. In other words, the stronger dependence exists, the better policy III1 performs in estimating the degradation level of non-inspected components by using the information of inspected component. By taking full advantage of stochastic dependence while ensuring that the risk of failure is below threshold $R1$ for each component, we can see that the system's unavailability is in a very low level (5.07% at most). It's also worth mentioning that when looking at the mean length of life cycle of four components, they are almost the same in the three systems. It signifies that the life cycle of components is mainly decided by their marginal degradation process.

When compare policy III1 to policy II4, the maintenance cost is saved by 8.76% of system 1, by 9.27 % of system 2 and by 8.21 % of system 3 and policy III1 saves not only the inspection cost but also the preventive and corrective maintenance cost. Even though

Table 5.12: Optimal parameters of *policy* II4

system	EC	T/R	$LC_1/LC_2/LC_3/LC_4$	$EC_1/EC_2/EC_3/EC_4$
1	31.18	0.9/0.08	5.82/4.29/2.99/2.69	5.46/6.80/9.01/9.82
2	31.17	0.9/0.08	5.82/4.29/2.99/2.69	5.46/6.80/9.01/9.82
3	31.18	0.9/0.08	5.82/4.29/2.99/2.69	5.46/6.80/9.01/9.82
system	UNAV	$Pc_1/Pc_2/Pc_3/Pc_4$	$NI_1/NI_2/NI_3/NI_4$	
1	8.71% ₆₀	6.64%/5.51%/4.52%/4.24%	6.47/4.77/3.32/2.99	
2	8.44% ₆₀	6.64%/5.51%/4.52%/4.24%	6.47/4.77/3.32/2.99	
3	8.85% ₆₀	6.64%/5.51%/4.52%/4.24%	6.47/4.77/3.32/2.99	

the mean observed number of inspection of policy II4 is twice times greater than that of policy III1, it performs worse in avoiding failures due to the fact that policy II4 has identical inter-inspection interval which can not be adapted to the condition of components. Component should be inspected more frequently when its degradation is in high level. As a conclusion, by making use of the stochastic dependence between components, policy III1 inspects the components in right time such that it not only saves the inspection cost but also avoids the failures.

To evaluate the performance of four proposed maintenance policies, non-periodic inspection schemes dominate the periodic inspection schemes. It can be seen in Tables 5.9 and 5.12 with EC_4 being 9.82 and 9.97 respectively, however EC_4 in Tables 5.10 and 5.11 is 9.03 and 9.05 respectively. In fact, periodic inspection schemes spends unnecessary cost to inspect components even when they are just maintained. Furthermore, they are very sensitive to the inter-inspection interval such that the interval can not be too short otherwise the inspection cost will be very expensive. Conversely, the non-periodic inspection polices are more flexible. It can be achieved that when component is just maintained, during a certain time, no inspection is implemented but consecutive and more frequent inspection is carried out when component degrades enough. Therefore, non-periodic inspection policies performs better to inspect component in right time and avoid failure for slower degrading components. That's why *components* 1 and 2 have higher value of Pc_1 and Pc_2 respectively in policy II4 than in policy II3.

Policies II3 and II4 do not take advantage of stochastic dependences between components so that all the four components have the same cost in three systems except that of system's unavailability. When components are more dependent on each other (the case of system 2), the system's unavailability is reduced (6.87% and 8.44% in policy II4 and policy II3 respectively) due to the fact that for series systems, once a component fails, it leads to the breakdown of system. Therefore, the series system's unavailability is lower when components are more dependent. Such a result shows that even if the stochastic dependence may not be so important to the economic part to minimize the

maintenance cost, it is very significant for analysing the reliability and availability of multi-component systems. The system structure and the stochastic dependence should be taken into consideration.

Policy II2 performs the best when compared to policies III1, II3 and II4 not only because it implements the non-periodic inspection strategy for all the four components but also because it takes advantage of the stochastic dependence at all levels. All the available information is fully utilized. Policy II3 can be regarded as special case of policy II2 when stochastic dependences are not considered. By taking advantage of the stochastic dependence, the maintenance cost is reducing through saving inspection cost as well as avoiding failures. For example, policy II2 saves 1.5 inspection cost unit in each life cycle and reduce the proportion of failures for *component 1* in system 2 by 1% when compared to policy II3. We analyse the maintenance cost saving of policy III1 and that of policy II2 at components level and Table 5.14 depicts the results. It can be concluded that policy III1 can take full advantage of the stochastic dependence at each potential inspection time by using the revealed degradation information of components. The effectiveness of prediction depends on the dependence degree between *component i* ($i = 1, 2, 3$) and *component 4*. For example, *component 3* saves more than *components 1* and *2* in systems 1 and 3 due to the stronger dependence. Nevertheless, in policy II2, all components are non-periodically inspected such that useful information level is lower at each potential inspection time. Therefore, *component i* ($i = 1, 2, 3$) saves less maintenance cost in policy II2 than in policy III1 even if *component 4* gains so much that policy 3 outperforms policy III1 at system level. But when analysing the profit of components, it can be found that in policy II2, *component 1* gains the most even better than *component 3*. It's because *component 1* degrades slowest, it can take advantage of information of the three other components before its replacement. Furthermore, *component 1* is more dependent on *component 2* than *component 4*. It is good to use the information of *component 2* for *component 1* as soon as it is possible.

Overall, the non-periodic inspection policies (policies III1, II2, II3) is better than periodic policy (policy II4). The stochastic dependence should be taken into consideration to minimize the maintenance cost as well as increase the system's availability. Even though policy II2 has the best performance, it is not easy to be implemented because the calculation of PM_i involves the integration and it is time-consuming. Thus policy III1 is more suitable for application.

5.5.2.3 Sensitivity analysis

In order to test the robustness of policy III1, we propose to investigate how policy III1 performs when errors are made in estimating degradation parameter of components. For

Table 5.13: Sensitivity analysis of policy III

$e\%$	-10%	-5%	0%	5%	10%
EC_e	26.67	27.55	28.45	29.50	30.56
$\frac{EC_e - EC_0}{EC_0}$	-6.26%	-3.16%	0	3.69%	7.42%

Table 5.14: maintenance cost saving of policies II1 and II2 when compared to policy II3

component	policy II1			policy II2		
	1	2	3	1	2	3
system 1	5.69%	4.78%	6.35%	3.66%	2.55%	2.87%
system 2	8.13%	7.17%	5.15%	5.05%	3.66%	2.40%
system 3	4.68%	3.34%	6.35%	3.25%	1.91%	2.75%

the baseline case of system 1, we consider the worst case that both the mean degradation speed and variance of non-inspected components are overestimated or underestimated by $e\%$ ($e = \pm 10, \pm 5$). The cost of policy II1 when implemented for system 1 with parameters in Table 5 whereas errors are made is shown in Table 5.13. When all non-inspected components increase their mean degradation speed by 10%, the whole system increase its mean degradation by 6.83% whereas the maintenance cost increases only 7.42%. Hence, policy II1 is robust. In fact, policy II1 is not as sensitive to the parameters (T, M) as the maintenance policy with periodic inspection. The change of degradation parameters impacts the inspection scheme but not so much since the inspection and replacement strategy of policy II1 is very flexible. In fact the times to inspect and/or replace *components* 1, 2 and 3 are decided not only on the basis of individual input parameters but also on the revealed degradation level of *component* 4.

5.6 Conclusions and perspectives

A four-component system with hierarchical dependences is proposed and investigated in this chapter. It is shown that the Nested Clayton Lévy copula enables modelling of time-independent structure as well as sampling. For maintenance decision making, five maintenance policies are firstly proposed in part I with the aim to investigate both economic and stochastic dependences and to be implemented easily in practice. Results show that policy I5 outperforms the other maintenance policies by considering all types of dependences, up to 11% of total cost compared to policy II1. Besides, even with an error in identifying faster degrading components, policies I4 and I5 can maintain the profits by adapting the parameters of slow degrading components.

Nevertheless, policy I4 and I5 are very sensitive to stochastic dependence as well as

the implemented values of parameters. Therefore, in part II, maintenance policies that fully exploit the stochastic dependence are proposed. It has been shown in this work that inference about the reliability at a fixed time of non-observed components can be made using an ordinary copula instead of the Lévy copula. The proposed maintenance strategy that uses these estimates derived from the monitoring of the critical component in the system to decide which other components should be inspected or replaced succeeds in reducing the global inspection cost while preventing failures. Therefore the maintenance cost is reduced with policy III1 when compared to classical maintenance strategies such that predictive maintenance policy with periodic inspection and condition-based maintenance policy with non-periodic inspection without considering stochastic dependence. An additional and more sophisticated maintenance strategy is proposed, allowing non-periodic inspections for all components in the system and taking advantage of stochastic dependence at all levels. Unlike the previous strategy, the monitoring information is gathered in certain time which provides more opportunities to group maintenance activities or inspection operations.

These results are a good foundation for future work that will consider both stochastic and economic dependences. As well, though we find an ordinary copula to fit the data generated by Lévy copula, further research on the relationship between ordinary copulas and Lévy copulas dependence structure when considered at a fixed time may lead to even better performances of policy III1.

Chapter 6

Conclusions and perspectives

This work is dedicated to the maintenance modelling for multi-unit system with dependent and gradually deteriorating components. More precisely, we are interesting in developing grouping strategies for condition-based maintenance to take advantage of economic dependence, modeling stochastic dependence between components caused by common degradation part and making use of stochastic dependence in maintenance decision process. To begin the work, a literature review which interests on the degradation modeling and maintenance modeling is presented first to clarify our problem and choose the adaptive models. In [chapter 2](#), by comparing with other models, the Gamma process and Lévy copulas are justified to model degradation of multi-component system with stochastic dependence caused by common degradation part. In [chapter 3](#), the necessity of developing grouping strategy for condition-based maintenance is highlighted in order to take advantage of positive economic dependence. Based on the chosen degradation model and the stochastic dependence model, the maintenance modeling is investigated according to the different aspects: the taking advantage of economic dependence when the condition-based maintenance with periodic inspection is applied, the taking advantage of stochastic dependence when the condition-based maintenance with non-periodic inspection is applied.

[chapter 4](#) works on the maintenance modelling for a two-component system with symmetric stochastic dependence. The degradation of system is modeled by two-dimensional Gamma process with Lévy copulas. Condition-based maintenance policies with periodic inspections and inspection period adaptive to the degradation speeds of components are proposed in order to take advantage of both economic and stochastic dependences. Additionally an adaptive decision rule is proposed to asses online opportunities of maintaining both components simultaneously with different inspection periods. The results show that the condition-based maintenance based on a static decision rule regarding

replacement dates is sensitive to the inspection period and the opportunities of maintenance grouping provided by a joint inspection period are very limited and the stochastic dependence can not be profited neither. On the contrast, the proposed adaptive maintenance decision rule provides flexible dates for profitable maintenance grouping. Furthermore, by investigating the different degrees of stochastic dependence between components, it shows that the proposed maintenance policy can make use of stochastic dependence where the maintenance decision of slower degrading component is made based on the revealed degradation of faster degrading component in the system.

[chapter 5](#) works on the maintenance modelling for a four-component system with asymmetric stochastic dependence. When extended to four components, the stochastic dependences between components are modeled by Nest Lévy copulas allowing non-symmetric dependence structure. Inspired by results obtained in [chapter 4](#) that the slower degrading component does not need to be inspected in some cases where the degradation speeds are closely related and stochastic dependence between components is strong, the work in [chapter 5](#) aims to propose maintenance policy which can use the stochastic dependence for more general cases and in a more efficient way. The decision making process of components is based on the total information of system (individual degradation information of components and the dependence relationship between them) with non-periodic inspection scheme so that the proposed maintenance policies are more robust to dependence degrees and inspection periods when compared to that proposed in [chapter 5](#). Moreover, maintenance policies set on subsystem level are also proposed with the objective of being easy to implement in practice as well as investigating the economic dependence, the stochastic dependences within subsystems and the stochastic dependences between components.

The maintenance models proposed in this thesis are aimed at investigating how to improve the decision making process by the use of dependences between components. Nevertheless further research should deal with expanding the scope of applications relating to the following aspects: possible failure modes for components in the system, complexity of the structure of system regarding the number of components, maintenance implementation, resource constraints consideration. For further research, we propose to investigate the maintenance models as follows:

- For degradation modeling which relates to individual degradation modeling and stochastic dependence modeling, the stochastic dependence caused by common degradation part is modeled in this thesis, however more complicated dependence relationship can be developed in future work. In effect, in a complex system, three types of stochastic dependence which are identified as failure interactions,

degradation interactions and common degradation part may exist simultaneously in reality. For condition-based maintenance, failure interactions or degradation interactions lead to an increasing frequency of inspections and preventive maintenance because the failure or the degradation of one component causes immediate failure or accelerates the degradation speed of other components so they have negative influence on the maintenance cost. On the contrast, the common degradation part can result in avoiding unnecessary inspection cost and less failures by taking advantage of stochastic dependence between components so that it is considered to have a positive impact on the maintenance optimization. Moreover, in this thesis, components are supposed to suffer only one failure mode but in reality the system can degrade due to several reasons, such as the wear and shock. Despite the dependence between components, the different sources of degradation can also depend on each other. As a consequence, it is necessary to take into account various stochastic dependences and competing failure modes even though it increases the difficulty of degradation modeling.

- Consequently new maintenance policies should be developed in order to adapt to more complex deterioration modeling. For example, when various types of stochastic dependences are considered in the multi-component system, the maintenance modeling becomes more complicated in order to reduce the effect of negative stochastic dependence and take advantage of positive stochastic dependence. Moreover, the complexity of maintenance modeling is also increased when considering complex logical system structure and economic dependence. The logical system structure is significant in reliability analysis and it impacts on the maintenance cost when downtime cost is great. Multi-level setup cost could be studied further in order to benefit economic dependence and provide opportunities of grouping maintenance activities while the cost saving is not a linear function of the number of components which undergo maintenance simultaneously.
- Some aspects related to maintenance should be considered such as imperfect maintenance, maintenance execution time, maintenance delay and the limited available maintenance resources. Besides, when condition-based maintenance policy is proposed, the inspection quality can also be discussed. For statistics aspects, parameter estimation methods such as for stochastic dependence, individual degradation behavior, maintenance effect need to be developed. In terms of optimization, some efficient and heuristic algorithms should be proposed when the number of maintenance decision parameters is large or when the exact methods are time-consuming.

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Politiques de maintenance conditionnelle pour des systèmes multi-composants avec dépendances stochastiques

De nos jours, les systèmes industriels sont de plus en plus complexes tant du point de vue de leur structure logique que des diverses dépendances (dépendances économique, stochastiques et structurelles) entre leurs composants qui peuvent influencer l'optimisation de la maintenance. La Maintenance conditionnelle qui permet de gérer les activités de maintenance en fonction de l'information de surveillance a fait l'objet de beaucoup d'attention au cours des dernières années, mais les dépendances stochastiques sont rarement utilisées dans le processus de prise de décision. Par conséquent, cette thèse a pour objectif de proposer des politiques de maintenance conditionnelle tenant compte des dépendances économiques et stochastiques pour les systèmes multi-composant.

En termes de dépendance économique, les politiques proposées sont conçues pour permettre de favoriser les opportunités de grouper des actions de maintenance. Une règle de décision est établie qui permet le groupement de maintenances avec des périodes d'inspection différentes. La dépendance stochastique causée par une part de dégradation commune est modélisée par copules de Lévy. Des politiques de maintenance conditionnelle sont proposées pour profiter de la dépendance stochastique. Nos travaux montrent la nécessité de tenir compte des dépendances économiques et stochastiques pour la prise de décision de maintenance. Les résultats numériques confirment l'avantage de nos politiques par rapport à d'autres politiques existant dans la littérature.

Mots clés : Monte-Carlo, méthode de - maintenance conditionnelle - modèles mathématiques - dépendance (statistique) - prise de décision - fiabilité (ingénierie) - processus stochastique.

Condition-based Maintenance Policies for Multi-component Systems Considering Stochastic Dependences

Nowadays, industrial systems contain numerous components so that they become more and more complex regarding the logical structures as well as the various dependences (economic, stochastic and structural dependences) between components. The dependences between components have an impact on the maintenance optimization as well as the reliability analysis. Condition-based maintenance which enables to manage maintenance activities based on information collected through monitoring has gained a lot of attention over recent years but stochastic dependences are rarely used in the decision making process. Therefore, this thesis is devoted to propose condition-based maintenance policies which take advantage of both economic and stochastic dependences for multi-component systems. In terms of economic dependence, the proposed maintenance policies are designed to be maximally effective in providing opportunities for maintenance grouping. A decision rule is established to permit the maintenance grouping with different inspection periods. Stochastic dependence due to a common degradation part is modelled by Lévy and Nested Lévy copulas. Condition-based maintenance policies with non-periodic inspection scheme are proposed to make use of stochastic dependence.

Our studies show the necessity of taking account of both economic and stochastic dependences in the maintenance decisions. Numerical experiments confirm the advantages of our maintenance policies when compared with other existing policies in the literature.

Keywords: Monte-Carlo method - condition-based maintenance - mathematical models - dependence (statistics) - decision making - reliability (engineering) - stochastic processes.

Thèse réalisée en partenariat entre :

