

### Application of fractional order system in the acoustics domain: Modeling and synthesis of a wind instrument Gaby Abou Haidar

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# THÈSE

PRÉSENTÉE À

## L'UNIVERSITÉ DE BORDEAUX

ÉCOLE DOCTORALE DES SCIENCES PHYSIQUES ET DE L'INGÉNIEUR

par

### **Gaby ABOU HAIDAR**

POUR OBTENIR LE GRADE DE

### DOCTEUR

SPÉCIALITÉ : AUTOMATIQUE

## Systèmes à Dérivées Non Entières dans le domaine de l'acoustique: étude des pertes viscothermiques dans un instrument à vent

Soutenance prévue le 15 Juillet 2021

Après avis de :

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| Xavier MOREAU,     | Professeur à l'Université de Bordeaux              | Directeur de thèse |

## Avant-Propos

Cette thèse s'inscrit dans le cadre d'une collaboration internationale entre Lebanese German University (LGU), American University of Science and Technology (AUST) et l'Université de Bordeaux (UBx), au sein de l'équipe CRONE du Groupe Automatique du laboratoire IMS, UMR 5218 du CNRS.

Cette collaboration a débuté en 2008 (figure 1) avec Clovis FRANCIS, aujourd'hui Professeur à l'Université Libanaise et membre du Conseil National de la Recherche Scientifique du Liban (CNRS-L). En 13 ans, cette collaboration internationale a donné lieu à 4 thèses soutenues, 1 thèse en cours (objet de ce mémoire), une prochaine thèse (septembre 2021), une HDR en cours de préparation, 13 publications dans des Revues Internationales avec Comité de Lecture (RICL), 33 communications dans des Conférences Internationales avec Comité de Lecture (CICL), 5 chapitres d'ouvrages, 2 ouvrages de synthèse, ...



Figure 1 – Chronogramme illustrant 13 ans de collaboration avec le Liban

Les travaux présentés dans ce mémoire de thèse concernent les Systèmes à Dérivées Non Entières (SDNE) dans le domaine de l'acoustique avec l'étude des pertes viscothermiques dans un instrument à vent.

Contrairement aux travaux menés, par exemple, à l'IRCAM (*Institut de Recherche et Coordination Acoustique/Musique*) ou dans des laboratoires d'acoustique, les travaux présentés dans ce mémoire de thèse n'abordent pas le sujet sous l'angle de l'acoustique musicale, mais sous l'angle de l'Automatique, c'est-à-dire l'étude de la dynamique des systèmes complexes. La flûte à bec ne constitue qu'un support dont le domaine d'étude défini dans ce mémoire est beaucoup plus restreint que celui envisageable avec un instrument à vent.

## Organisation et Contenu de la Thèse

Le mémoire de thèse comporte quatre chapitres dont la logique d'enchainement est illustrée à l'aide de la Figure 2.



Figure 2 – Illustration de la progression et de la logique d'enchainement des chapitres du mémoire de thèse

Ainsi, le *chapitre 1*, intitulé: « *Etude des pertes viscothermiques au sein du résonateur d'un instrument à vent* », est consacré, dans un premier temps, à la modélisation du résonateur seul en tant que tube acoustique (tous les trous permettant de faire varier la longueur apparente sont supposés fermés), modélisation qui aboutit à l'impédance générale Y(x,s,L) (débit/pression) du résonateur en fonction, notamment, de la longueur L et de la position x comprise entre 0 (entrée du résonateur) et L (sortie du résonateur). Ensuite, une approche système (au sens de l'Automatique) permet de décomposer l'impédance d'entrée  $Y_{in}(s,L) = Y(0,s,L)$  en sous-systèmes, facilitant ainsi l'analyse notamment lors du passage d'un système semi-infini à un système fini. De plus, l'introduction d'une extension de l'expression fractionnaire utilisée pour la prise en compte des pertes viscothermiques, où l'ordre m est habituellement égal à 0.5, permet de faire une première analyse de l'influence de l'ordre m, compris en 0 (système conservatif) et 1, sur la réponse fréquentielle de l'impédance du résonateur. Enfin, ce chapitre se termine par la présentation d'une méthode permettant de passer de la forme fractionnaire de l'impédance à ses formes rationnelles cascade et parallèle indispensables pour la simulation temporelle.

Le chapitre 2, intitulé: « Etude du couplage entre l'excitateur non linéaire et le résonateur d'un instrument à vent », est dédié, dans le cas habituel où l'ordre *m* est égal à 0.5, à l'étude du couplage entre l'excitateur non linéaire et le résonateur. Sur la base d'une synthèse bibliographique, un modèle non linéaire fréquemment utilisé dans la littérature est développé. Ensuite, après avoir insisté sur le mauvais conditionnement numérique d'un tel modèle, une solution est proposée permettant la mise en oeuvre d'un simulateur numérique programmé sous MatLab/Simulink. Pour un domaine d'étude défini par une pression constante à l'entrée du bec de flûte bornée par une valeur minimale de 400 Pa et une valeur maximale de 1000 Pa, une analyse détaillée des réponses temporelles simulées met en évidence la présence de trois phases durant lesquelles les variations

- de la pression à l'entrée du résonateur,
- de la vitesse acoustique
- et du déplacement latéral du jet d'air

restent petites autour de zéro. Ce constat permet le développement de manière légitime et réaliste de deux modèles linéarisés, l'un pour l'analyse de la phase de démarrage de la simulation, l'autre pour celle de la phase des auto-oscillations (régime périodique stationnaire), facilitant ainsi la compréhension des phénomènes mis en jeu.

Le chapitre 3, intitulé: « Conception et implémentation d'une bouche artificielle pour un instrument à vent », se focalise sur la démarche de conception et sur l'implémentation d'une bouche artificielle nécessaire pour contrôler la pression à l'entrée du bec de flûte. Durant la phase de conception, un premier simulateur a été développé sur la base d'une synthèse bibliographique concernant les bouches artificielles utilisées en acoustique musicale. Ce premier simulateur a permis de bien comprendre le fonctionnement d'un tel système, facilitant ainsi le choix et le dimensionnement des composants de la boucle de régulation (servovalve de débit, capteurs de débit et de pression, volume de la capacité pneumatique,...). A l'issue de la réalisation de la bouche artificielle, un travail de modélisation de ce dispositif expérimental a conduit à un deuxième simulateur. Ce dernier a fait l'objet d'un recalage à partir de comparaisons entre des résultats expérimentaux et de simulation. Ainsi, pour une flûte à bec bien réelle (en plastique dont la longueur du résonateur L = 30 cm, son rayon moyen r = 5 mm, l'ordre fractionnaire associé aux pertes viscothermiques m = 0.5, avec une fenêtre de l'excitateur dont les dimensions sont 1 cm x 0.4 cm, une hauteur du canal du bec h = 1 mm,...), le simulateur reproduit avec une bonne précision les conditions d'étude spécifiées au chapitre 2. Ce comportement bien réel (reproduit en simulation) est considéré au chapitre 4 comme le comportement nominal de référence.

Enfin, l'objectif du *chapitre 4*, intitulé: «*Analyse de l'influence de l'ordre fractionnaire sur le régime périodique stationnaire* », est d'étudier l'influence de l'ordre m compris entre 0 et 1, autour de sa valeur nominale  $m_0 = 0.5$ , sur le régime périodique stationnaire. Ce chapitre commence par présenter l'organisation du simulateur global développé sous MatLab/Simulink à partir des travaux présentés dans les trois premiers chapitres. Ensuite, une analyse dans le domaine fréquentiel de l'influence de l'ordre m est développée, d'abord dans le cas de l'impédance du résonateur établie au chapitre 1, puis dans le cas de la fonction de transfert en boucle ouverte définie au chapitre 2. Cette analyse se poursuit dans le domaine temporel avec les réponses issues du simulateur global en reprenant les scénarios des deux exemples du chapitre 2. Ainsi, à partir de l'extension du modèle fractionnaire proposée au chapitre 1, il est possible avec un seul paramètre de haut niveau, l'ordre m en l'occurrence, de faire varier facilement les pertes viscothermiques, alors que d'un point de vue expérimental, il faudrait fabriquer et tester un nombre important de résonateurs avec des dimensions, des rugosités et des matériaux différents.

Xavier MOREAU Directeur de thèse

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## Motivation

Music has always been the source of creativity and imagination for most thinkers and a relaxing tool for stress and depression. The greatest minds and thinkers like Albert Einstein, Mozart, and Frank Lloyd Wright all had something in common in that they were constantly exploring their imagination and creativity through music to make discoveries, innovations and many other different fields. Music and sound synthesis are two faces for the same coin. Sound synthesis started with the wish to generate any kind of music (or sound) using mathematical techniques, and by research improvement, it became possible to generate or transform any sound conceivable. The available sound synthesis techniques are capable of a perfect reproduction of sound but not actually a perfect generation of a sound; this is why model-based digital instruments should be available. The main advantage behind model-based synthesis is the more control over the physical variables (which influence the sound reproduction) it gives for the player.

This work aims to model and control a flute wind musical instrument for better numerical performance. To have a completely automated system, the mouth and the flute musical instrument parts will be modeled and implemented. So, based on the delivered air pressure by the artificial mouth, a certain frequency will be generated from the instrument.

To achieve the preset goal, the physical implementation of the artificial mouth is first established. Then, a study for every part of the flute will be conducted, starting by the resonator and then the exciter. This resonator is of a particular importance as it has been shown that the air flow within its boundaries is subject to visco-thermal losses that are being modeled by a fractional transfer function of order 0.5.

The last stage of this work will be to confront the already presented theoretical study with the experimental measurements and the implementation of a system that most closely resembles the real system. Note that this Ph.D. thesis will be the first block in a much larger project that will aim to reach the final objective already listed. Thus, the outcome of this work is not to have a complete autonomous system as a first step but to be able to model this complicated system and to generate sound while controlling the mouth pressure and/or flow.

# List of Abbreviations

| CRONE          | Command Robuste d 'Order Non Entier   |
|----------------|---------------------------------------|
| PID            | Proportional Integrator Derivative    |
| V              | Speed of light                        |
| р              | Accoustic Pressure                    |
| U              | Particle Velocity                     |
| k              | wave number                           |
| $J_m$          | Bessel Function                       |
| ζ              | Damping Factor                        |
| γ              | Normalized elastic material constant. |
| dB             | Decibel                               |
| W              | Angular Frequency                     |
| Wr,m           | Transitional Frequency                |
| h(t)           | Impulse response                      |
| $Q_{\nu}(t)$   | Air Flow                              |
| Q              | Resonance Factor                      |
| Е              | Visco-thermal losses                  |
| γ              | Specific Heat Ratio                   |
| S              | Cross Section                         |
| S              | Laplace variable                      |
| Γ(j <i>ω</i> ) | Complex Wave Number                   |
| tanh           | Tangent hyperbolic function           |
| L              | Length                                |
| r              | Radius                                |
| Y              | Acoustic Admittance                   |
| Z              | Acoustic Impedance                    |
| Н              | Transfer Function                     |
| αi             | Amplification Factor                  |
| $C_v^e$        | Convection speed                      |

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## General Overview

#### 1 – Introduction

The flute wind musical instrument family contains a large number of instruments types, which may seem from a first approach very different from each other, whether from the produced sounds perspectives, the geometries of the instrument, the used materials, or the contexts in which they are applied. This family of instruments includes indeed, in a non-exhaustive manner, the notched flutes such as Latin American *quena* or Japanese *shakuhachi*, the pan flutes, the globular flutes such as the ocarina, as well as the transverse flutes, the recorders and the mouth organ pipes. In addition, a general overview of all wind instruments will be presented in more details in the following section.

The diversity of these instruments is illustrated - among other criteria - by the relative influence of instrument and musician factor on the geometry of the instrument, and therefore on the sound production. For some instruments, such as the recorder or the mouth organ pipes, a large part of the geometry (jet formation channel, distance between the channel output and the bevel), is fixed by the instrument builder at the time of construction. However, in other flute types such as the *quena* or the *transverse*, the instrumentalist/musician has complete freedom to adapt these geometric parameters while playing music since the channel is formed by the musician's lips. This freedom gives the music player more autonomy, but makes it more difficult to control the instrument.

Despite this diversity, the instruments of the flute family share a mechanism of production of the common sound, involving the oscillation of an air jet around a bevel. A state of the art of the different wind musical instruments, the sound formation, the main components of the flute and the problem statement will be discussed in the following sections of this general overview.

#### 2 – Wind Musical Instrument

In this section, the human voice will be presented as it is the main source for the generation of sound. The voice is one of the most colorful interfaces for musical expression. It is certainly the most personal of all instruments (each voice is unique) – and the only one capable of presenting both words and musical sound simultaneously. Voices are classified by their tone-color, register and range. One can identify the following groups (FactMonster, 2001):

- soprano: highest female voice;
- mezzo-soprano: rich female voice mixing soprano and alto colors;

- contralto or "alto": lowest female voice;
- *countertenor*: a very high male voice;
- *tenor:* high male voice;
- *baritone:* rich male voice mixing tenor and bass colors;
- *bass:* very low male voice;

Hereafter, the different families of the wind musical instruments will be presented. For each family, the corresponding instruments will be shown along with their properties.

#### 2.1 – Woodwind Family

The woodwind family as in Figure 3 is composed of wooden instruments that the player must blow into to create a musical sound. Most members of the modern woodwind family are "reed" instruments (a piece of wooden reed attached to its mouthpiece that adds character to the sound). The complete woodwind family became a standard part of the orchestra by the early 1800's. Common woodwind instruments include (FactMonster, 2001) :

- the piccolo (very high) and various-sized flutes;
- the oboe and the "English" Horn (a tenor oboe);
- the bassoon and contrabassoon (very low bassoon);
- various-sized clarinets;
- various-sized saxophones;
- various-sized recorders (ancestors to the modern flute family);

The standard range categorization of this family is the *flute* (soprano), the *oboe* (alto), the *clarinet* (tenor) and the *bassoon* (bass).



Figure 3 – Common Woodwind Instruments

#### 2.2 – Brass Family

The brass family in Figure 4 is composed from powerful metallic instruments that must be blown into by the player to produce a musical sound. The tone-color (timbre) of most brass instruments can be altered by the use of various types of mutes which are inserted into the large end of the instrument. Until the invention of the valve, brass instruments could only produce a limited number of pitches, which lessened their usefulness to composers. Instrument builders experimented with various valves that would avail more pitch varieties to these instruments.

Many modern brass instruments have a system of three valves that can be combined in various combinations to produce different pitches. The modern "rotary" valve was invented in the early romantic period (c. 1830) – an invention that made the brass family more responsive and reliable. As a result, romantic composers made greater use of the brass family. The most common types of brass instruments include (FactMonster, 2001):

- various-sized trumpets (use valves to change pitch);
- various-sized trombones (use a slide instead of valves to change pitch);
- the "French Horn" (use valves to change pitch);
- the Tuba—a very low brass instrument (uses valves to change pitch);

The standard range categorization of this family is the *trumpet* (soprano), the *alto trombone* (alto), the *tenor trombone* that is equivalent to the *upper french horn* (tenor), the *bass trombone* that is equivalent to the *low french horn* (bass).



**Figure 4 – Common Brass Instruments** 

#### 2.3 – Flute Musical Instrument

The first published stage in the modeling and functioning of flute-like instruments probably came from two different works conducted by Mersenne and Bernoulli and they consisted of the analysis of the resonance frequencies of the bore. This approach is still followed today and is very useful for the understanding of several construction properties mainly from the resonator's position, diameter, height of the chimney and tone holes, shape corrections, etc... refer to Figure 5 (Mersenne, 1636) (Bernoulli, 1762).



**Figure 5 – Overall Flute System** 

The next stage in modeling consists of describing the source mechanisms. Helmholtz and Rayleigh were the first to propose such a description. Helmholtz described them as flow sources or a monopole while Rayleigh argued that since the sources act at an open end of the bore, they should be described as a dipole (Rayleigh, 1984).

In fact, close to the open end of the resonator there are large velocity fluctuations but the pressure fluctuations are small. Helmholtz rapidly changed his opinion, as he modified the comments on the second version of his book. Surprisingly, the same discussion reappeared this century in the sixties between Elder Coltman and Fletcher. Like in many of his works, Coltman's pragmatic, careful and clever approach unquestionably resolved the discussion (Coltman J., 1968) (Elder, 1973).

At the beginning of the sixties, the works by Powell on the edge-tone suggested a feedback loop model. The different components of the model were analyzed as independent blocks that were then connected together. The analysis of the loop gain fixed the phase and magnitude necessary for the loop to oscillate, and predicted the conditions and frequency for a linear model of oscillations. However, the actual amplitude of the oscillation can only be predicted by a non-linear model. His approach has inspired many other descriptions based on loop systems (Fletcher, 1979).

An important period in the development of flute models came with the work of Mc Intyre *et al.*, who were the first to propose a time simulation from a set of simplified equations derived from physical models. Their models were so neat and elegant that many researchers were persuaded that the secrets of the flute functioning had been found and captured in a physical model. Unfortunately, as it turned out, the model was better suited to the violin and clarinet but the flute doesn't even oscillate with the values published. After fine tuning the parameters and adding nonphysical elements, like a DC blocking filter and feeding the system with white noise, the model produces sound but is impossible to relate the quantities in the model with physical variables (McIntyre, 1983).

A more rigorous approach considering fluid mechanics and aero-acoustics has been followed by Howe for the flute and by Crighton and Elder for the edge-tones. In parallel, a group centered in Eindhoven, Netherlands adopted an intermediate path inspired by the work of different researchers in the field. Their approach lies between the rigorous formulation and the simplified description, producing good quality sound synthesis through models that use lumped elements with a sufficient amount of aero-acoustics (Elder, 1973) (Howe, 1975).

Modern flute musical instruments are usually depicted from old versions of this instrument with some additions on the shape, size, and other effects. Although the normal old versions of flutes are more likely preferred by musicians, modern flutes are being used numerically with the help of embedded systems and computers adding new features to the way of playing and to the quality of sound (Heller, 2017). However, what is no doubt with is that playing a typical old musical flute naturally will produce the perfect sound associated with feelings and that what numerical flutes are not able to do so far.

The flute, as shown in the Figure 6, is mainly composed from three main parts: the exciter which contains the mouth piece and the whistle (exciter), the resonator that contains the holes, and the horn which act as a reflector of the output waves.



Figure 6 – Traditional flute musical instrument

Figure 7 shows a block diagram representation of this musical instrument. The system is composed from the musician's ear that serves as a sensor to form the loop and to act accordingly on the flow delivered by its mouth towards the flute exciter. The radiation at the output of this system is generated at the flute horn level.



Figure 7 – Artificial loop based on the ear of the musician

Flutes are generally made from plastic, steel, or bamboo. Each type of material has a different effect on the quality of sound produced. This is due to the different types of materials used and to the visco-thermal effect of each material.

There are several types of flutes some are named according to the culture it is being used in (Chinese, Western, Indian, Irish, etc.), and others according to its shape and style (modern D, classical C, modern B, classical D, etc.). Every type has its own number of holes and their sizes and its own different shape and thus a different number of notes. For example, classical C flute has 6 holes and due to its shape, it can offer 8 notes. As for the classical D flute, it has the same number of holes (6 holes) but with only 6 notes because the D foot is smaller than the C foot; same number of holes, but different shape and size, give different number of notes. Classical flared flute has a larger number of holes (8 holes) and thus can offer more notes (23 notes) whereas baroque flute has 7 holes (less than classical flared flute) but yet can provide more notes due to its shape and size (32 notes to be precise). Figure 8 shows an illustration of these three types of flutes.



Figure 8 – Types of flutes: (a) Class D, (b) Classical Flared and (c) Baroque

#### **3 – Sound Formation in Flutes**

When the musician blows into the instrument, the overpressure inside his mouth creates a flow inside the channel, which can be a part of the instrument (refer to Figure 5 – case of the recorders), or constituted by the lips of the musician, like in the flute. At the output of the channel, the viscosity of the air prevents the flow from following the geometry of the boundaries: the flow takes off from the flute boundaries, and gives rise to a jet of air. Due to this unstable nature, the fewer disturbances to which the jet is subjected are amplified over time.

In the case of flutes, the jet created at the output of the channel is convicted inside the window excitement, until meeting the bevel. This jet-bevel interaction causes a hydrodynamic return on the jet, a mechanism which is notably at the origin of bevel sounds (Powell, 1961) (Coltman J., 1976). If this phenomenon is negligible once the steady state of the instrument is reached (Verge M., 1995), on the other hand, it plays a role in the initialization of the oscillation of the jet, by providing it with its first disturbances. Due to the natural instability of the air jet, disturbances are amplified naturally along this jet, from the exit of the channel to the bevel, and cause its oscillation on both sides of the bevel.

This oscillation causes a flow injection alternately inside and outside of the instrument, and is the aero-acoustic source of pressure which supplies energy to the resonator (Chaigne & Kergomard, 2008). The acoustic waves thus created in the resonator propagate and reflect (mainly) at the first open hole. The superposition of the outward and return waves created a standing wave system in the resonator, which disturbs the jet back to the channel output level, thus closing the self-oscillation loop. This initial disturbance jet, amplified and convicted from the channel output to the bevel, maintains the oscillation of the jet, and therefore the sound production mechanism.

Unlike percussion and struck or plucked string instruments, in which sound production is based on a phenomenon of free oscillations, the instruments of the flute family form with the reed instruments, strings rubbed, and brass instruments, the family of self-oscillating instruments. The self-oscillation mechanism, characterized by the ability of the instrument to produce an oscillation (in this case a wave acoustic) from a continuous source of energy - or at least varying slowly from acoustic variables - provided by the musician, is inseparable from non-linear nature of the instrument. A classic approach in musical acoustics consists of representing the instrument, as in Figure 9, by a nonlinear excitation system coupled to a passive linear resonant system (McIntyre, 1983) (Rayleigh, 1984).



Figure 9 – Block diagram of the mechanism of sound production in auto-oscillatory musical instruments

#### **3.1 – Acoustic Resonator of Wind Instruments**

The acoustic resonator in Figure 9 is a passive linear resonator that has been widely studied for several centuries. By its linear nature, it is also considered the easiest element to describe (at least taking into account a certain number of approximations). As in all wind instruments, the flute resonator is formed by the air column contained inside the body of the instrument. This is delimited by the hole in the instrument, that is to say the geometric shape of the resonator.

The geometrical characteristics of this air column, such as the conditions at the ends, the size and position of the different note holes, or the variations in the section of the bore, condition the propagation of acoustic waves inside the resonator, and therefore the resonator's response to the excitement provided.

As can be seen in Figure 5, which provides an overview of the main elements of a recorder, a large part of the instruments in the flute family have two open ends, which distinguish them from the most other wind instruments.

#### 3.2 – Flute Exciter

If the propagation phenomena inside the resonator still pose many questions, the characteristics of the exciter and its coupling with the resonator (for which it is the energy source) appears even more complex due to its non-linear nature.

The peculiarity of flutes lies precisely in the nature of the exciter where it involves the vibration of a solid body for all other wind instruments - reed blade for reed instruments, instrumentalist's lips for brass instruments - it is made up of flutes from the oscillation of a naturally unstable air jet around a bevel (refer to Figure 5).

In flutes, the self-oscillation mechanism is therefore based on a synchronization of the oscillation jet and acoustic waves: the same way as a person who wants to maintain the oscillations of a swing must restore energy to it during each period of oscillation (or a multiple of this period), the jet-bevel system must excite the resonator at the periodicity of the sound field.

#### 3.3 – Closed-loop system

Generic modeling of self-oscillating instruments by the looped system given in Figure 9 can then be very specific to the flute family instruments. The diagram represented in Figure 10 underlines the principal physical phenomena retained in the state of the art model of this family of instruments.



Figure 10 - Closed-Loop system of the self-oscillation mechanism of flute family instruments

#### 4 – Literature Review

The production of a note by a musical instrument results from a complex coupling between the instrument and the musician, where to obtain the desired sound requires the instrumentalist a constant adaptation of the parameters which are at his disposal. Experimentally, it is then difficult to ask an instrumentalist to set all these control parameters.

In fact, the system as shown in Figure 10 depends on two main entities, the musician and the flute instrument. As the ear and the musical experience of the musician plays a major role in controlling the air flow blowing in the exciter of the flute, the objective of this work will be to model all the parts of the system and then find some physical equivalence in order to be able to get the required frequency output precisely based on a closed loop control system.

Many researchers have been interested in the field of wind musical instruments (Mignot R., 2009) (Lefebvre, 2011). They have studied this field from many perspectives. Some tried to model wind instruments mathematically as will be demonstrated in the coming discussions, others tried to control the blowing part, and few went deeply in discussing the visco-thermal losses and their effects on the musical output.

The work of Mignot dealt with the physical modelling of cylindrical acoustic tubes for digital simulation in real-time. The main application is the sound synthesis of wind instruments, with a realistic model, a modular method and a low-cost digital implementation. The acoustic model of "Webster-Lokshin", used in this work, is a unidimensional model which takes into account the "curvature" of the profile and the visco-thermal losses at the wall. With this acoustic model, a framework for simulation which is compatible with the "Waveguides" approach was obtained: a tube is then represented by a system with delays and closed loops, involving several sub-systems without internal delay (Mignot R. , 2009). Figure 11 shows a representation of the flute using the tools that Mignot has developed.

Digital waveguide synthesis models are computational physical models for certain classes of musical instruments (string, winds, brasses, etc.) which are made up of delay lines, digital filters, and often nonlinear elements. Digital waveguide models typically share the following characteristics:

- Sampled acoustic traveling waves;
- Follow geometry and physical properties of a desired acoustic system;
- Efficient for nearly lossless distributed wave media (strings, tubes, rods, membranes, plates, vocal tract);

• Losses and dispersion are consolidated at sparse points along each waveguide (Smith, 2006).



Figure 11 – Example of the musical wind instrument array

The structure shown in Figure 11 disregards the effects of sections (representing the coupling between two adjacent blocks), slopes at each extremity of the section, curvature and loss, and isolates wave propagators through the tube. Thus, it does not pretend to replace them, but allows to extend these models to a higher degree of refinement, considering the curvature and visco-thermal losses of the tube. Finally, the number of sections is imposed by the sampling frequency.

Another work presented a number of methods for the computational analysis of woodwind instruments which is the Transmission-Matrix Method (TMM) for the calculation of the input impedance. The latter is an approach based on the Finite Element Method (FEM) and it is applied to the determination of the transmission-matrix parameters of woodwind instrument tone holes, from which new formulas are developed that extend the range of validity of current theories. This approach is applied as well to tone holes on a conical bore, and, as a result, the tone hole transmission matrix parameters developed on a cylindrical bore are equally valid for use on a conical bore. A boundary condition for the approximation of the boundary layer losses for use with the FEM was developed, and it enables the simulation of complete woodwind instruments. The comparison of the simulations of instruments with many open or closed tone holes with calculations using the TMM reveal discrepancies that are most likely attributable to internal or external tone hole interactions. The maximal error is found to be smaller than 10 cents (Musical intervals are often expressed in cents, a unit of pitch based upon the equal tempered octave such that one tempered semitone is equal to 100 cents) (Lefebvre, 2011). Figure 12 shows an example of the different musical notes with their corresponding frequencies and the active holes that produces them.

|                 | 1 | 2 | 3 | 4 5 | 6 |
|-----------------|---|---|---|-----|---|
| 0               | 0 | 0 | 0 | 0 0 | 0 |
| D4 [293.66 Hz]  | • | • | • | • • | • |
| E4 [329.63 Hz]  | • | • | • | • • | 0 |
| F‡4 [369.99 Hz] | • | • | • | • • | 0 |
| G4 [392.00 Hz]  | • | • | • | 0 0 | 0 |
| A4 [440.00 Hz]  | • | • | 0 | 0 0 | 0 |
| B4 [493.88 Hz]  | • | 0 | 0 | 0 0 | 0 |
| C#5 [554.37 Hz] | 0 | 0 | 0 | 0 0 | 0 |
|                 |   |   |   |     |   |

Figure 12 – Diagram of Keefe's flute

Moreover, preliminary studies on an artificial mouth adapted to the recorder have highlighted phenomena still unknown on this instrument, which raise many questions.

*First phenomenon:* for some fingerings, sounds are detected for a lower feed pressure than that usually used, sometimes less than the oscillation threshold of the desired note. These oscillations systems recall the "Aeolian sounds" observed on certain organ pipes or on the cross-section, which appear for very low values of the supply pressure. They can be heard at the organ during the extinction, and are sometimes used musically at the peak crossbred, their existence on the record was only highlighted recently, and their differences with the systems of "usual" oscillations remain rather poorly known.

**Second phenomenon:** it seems to exist, especially at the level of registry changes, a great diversity of behaviors according to the fingering used. We can observe for some fingerings a bifurcation towards a higher note (and, starting from this new note, a bifurcation towards the initial note when one feels less and less strong). If we reproduce the same experience on other fingerings, such as that of B flat, we observe a transition to a quasiperiodic system, corresponding audibly to a "rolling" note (Terrien S., 2011).

Added to that, Matignon classified the wind instrument into three main blocks which are the cup, pipe, bell, and radiation as shown in Figure 13. This design was represented in the form of blocks of delay lines which allowed each block to be modelled as a separate waveguide element, thus allowed to a more advanced sound synthesis technique (Matignon D., 2014).



Figure 13 - Trumpet-like instrument- Decomposition into elementary subsystems

Moreover, the musical instruments of the brass family are complex physical systems. They involve mechanical phenomena of deformable solids (muscles, tissues), fluid mechanics (jet, turbulence) including acoustic propagation. From a dynamic systems perspective, they correspond to self-oscillating non-linear and chaotic systems. In addition, their control is delicate and requires a long learning from the musician. For all these reasons, the modeling of these instruments, their analysis, simulation, inversion input / output determination of a command providing a target sound and as well as control are still active research topics, as is the case of an artificial mouth coupled to an acoustic system as shown in Figure 14. In order to carry out experiments that are calibrated and reproducible, the robotic design of an artificial mouth dedicated to brass instruments was initiated at IRCAM (Institut de Recherche et Coordination Acoustique/Musique). This work aims to propose models and simulations for the mouth system, (lips, instrument) while developing its robotic version, in order to consider the creation exploitation and control laws. An approach well adapted to the modeling of complex physical systems is based on the "Hamiltonian formulation with ports": it leads to well-balanced energy systems and naturally preserves the passivity. Thus, this work falls within the following three scientific fields: musical acoustics, robotics, and control (Lopes, 2016).



Figure 14 – Diagram of copper type instrument operation coupled to the musician (first resonance)

Current modeling of the flute family instruments allows a good prediction of their operation in transient and steady state. They allow to interpret a large part of the phenomena observed when the control parameters of the instrument vary slowly with respect to the time response of the instrument. The musician, however, constantly varies these parameters to articulate the musical discourse. During sudden spikes in particular, an instant variation in the supply pressure induce a gradual oscillation of the instrument. The birth of sound plays an important role in the perception of sound, the control of these sudden spikes is therefore an essential point in learning the concept of making flute instruments. The works presented focus on the study of sudden transients in flute instruments. The study is restricted to instruments whose jet geometry is fixed by the manufacturer.

At the end of this section, we have to mention the recent work presenting a comprehensive study of visco-thermal effects in quasi-2D artificial structures which are designed to exhibit double-negative behavior using a digital waveguide approach. The building units consist of structured cylinders made of a rigid material having air cavities penetrating deeply into the waveguide. The reported experimental characterization of these types of metamaterials is unable to demonstrate any features confirming double-negative effects. Extensive numerical simulations based on the boundary-element method were treated, which has been improved and adapted to tackle visco-thermal losses in these metamaterial structures. These structures are very efficient in absorbing the energy of acoustic waves traveling through them. For frequencies within the first passband, the absorbed energy reaches values as high as 80% of the amount of absorbance being directly proportional to the reciprocal of the group velocity in the band. The visco-thermal losses, which are enhanced due to the extremely low value of the group velocity inside this narrowband region, in addition to the possibility of reducing the relevance of losses by applying a scaling factor to the dimensions of the initial samples were treated. So, the study lead to the conclusion that final based metamaterial structures are not able to exhibit the predicted

double-negative behavior because of the strong dissipation associated with visco-thermal losses. A large amount of losses has also been reported in other rigid-based structures with embedded resonances, where the lack of a significant signal in the transmitted energy made them unfeasible for developing practical devices. These results may indicate that visco-thermal losses are relevant to any rigid-based double negative metamaterials. The rigid-based metamaterial structures could become interesting alternatives to conventional absorbers in particular situations, e.g., when treating low frequencies or when the excitation is narrow banded, such as damped and low-frequency resonances in room acoustics (Henríquez & Chocano, 2017).

### Chapter 1- Study of Viscothermal Losses Within the Resonator of a Wind Musical Instrument

#### 1.1 – Introduction

Historically, the first efficient and inexpensive simulations introduced for simple models (conservative plane waves) were based on signal processing tools: the so-called digital waveguide formalism (Tassart, 1999) (Helie T. , 2003) (Helie, Mignot, & Matignon, 2011) and more specifically, a factored form introduced by Kelly-Lochbaum (Matignon, Mignot, & Helie, 2007). The initial idea rested on the factoring of the alembertian of the equation of plane waves into two transport operators who each govern decoupled "round trip" progressive waves, from which we could derive a form in efficient delay system for the simulation (Mignot R., 2009) (Helie T., 2006).

Furthermore, 3D and 2D models with realistic boundary conditions are far too complex to be considered, especially in real-time sound synthesis. They can be effectively reduced to a 1D wave equation including a term that models the tube profile. It is about the *equation of the pavilions* also called model of Webster. A more elaborated version of this conservative model includes the effect of visco-thermal losses due to the boundary layers in the vicinity of the walls. This dissipative model, known as *de Webster-Lokshin 1D* (Lokshin & Rok, 1978) (Haddar, Hélie, & Matignon, 2003) (Haddar & Matignon, 2008), includes a term which involves a fractional derivation in time of order 3/2. This operator plays a crucial role from a perceptual point of view on sound realization (Vigué, Vergez, Lombard, & Cochelin, 2019).

The objective of this chapter is, first, to establish a *knowledge model* from partial differential equations which define the Webster-Lokshin model of an acoustic tube of constant radius r. Thus, a conventional resolution in the operational field leads to the analytical expression of the acoustic impedance and admittance of the tube as a function of the position x, its length L and its radius r. This working methodology will be similar to the one used for the modelling of the diffusive phenomenon in a semi-infinite homogeneous tube already proposed in previous works (Assaf, 2015). Then, a system vision is proposed aiming to causally decompose the whole model into sub-models in order to facilitate the analysis in the frequency domain. In addition, the introduction of uncertainty at the fractional order level makes it possible to study its influence on visco-thermal losses. Finally, for the time domain simulation in Chapters 2 and 4, two rational forms composed of an integrator and N second-order cells, one in cascade and the other in parallel, are introduced in as a *role model*. The parametric values of the cascaded rational form are then determined using the *Frequency Domain System Identification* (FDSI) module of the CRONE Toolbox, with the frequency

response of the knowledge model as target. As for the parametric values of the parallel form, they are obtained by the decomposition of the cascade form into simple elements.

#### 1.2 – Modelling

#### 1.2.1 – System Definition

Consider an acoustic tube of length *L* and constant radius *r* subjected to an acoustic flow (also called volume flow)  $Q_v(t)$  with x = 0 where  $x \in [0; L]$  (Figure 1.1).



Figure 1.1 – One-dimensional description of an acoustic tube of radius r = constant and of finite length L subjected to an acoustic flow  $Q_v(t)$  with x = 0

When an acoustic wave propagates in the air, this sets the particles of the fluid in motion which vibrate at a speed v(t) around their equilibrium position. The acoustic flow  $Q_v(t)$  then measures the flow [in  $m^3/s$ ] of this speed through a surface and present it as a scalar quantity (Blanc, 2009) (Ducasse E., 1990) (Ségoufin C., 2000) (Ducasse E., 2001) (Terrien., 2014).

The acoustic impedance  $Z_{ac}$  (also called *specific* acoustic impedance, because it is an intensive quantity) of a medium is defined in steady state by the ratio between the acoustic pressure [in Pa] and the speed [in m/s] of the associated particle. When the medium is air,  $Z_{ac}$  is equal to the product between the density of air,  $\rho_a$ , and the speed of sound in air,  $c_a$ , thus  $Z_{ac} = \rho_a c_a$ . These two parameters depend also on the air temperature  $T_a$ . For more illustration, Table 1.1 gives the values of the speed of the sound  $c_a$ , the density  $\rho_a$  and the characteristic acoustic impedance  $Z_{ac}$  as a function of the temperature  $T_a$  of the air.

Table 1.1 – Values of the speed of sound  $c_a$ , the density  $\rho_a$  and the characteristic acoustic impedance  $Z_{ac}$  as a function of the air temperature  $T_a$ 

| <i>T</i> <sub>a</sub> (°C)         | -10   | -5    | 0     | 5     | 10    | 15    | 20    | 25    | 30    |
|------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| <i>c</i> <sub><i>a</i></sub> (m/s) | 325.4 | 328.5 | 331.5 | 334.5 | 337.5 | 340.5 | 343.4 | 346.3 | 349.2 |
| $\rho_a (\text{kg/m}^3)$           | 1.341 | 1.316 | 1.293 | 1.269 | 1.247 | 1.225 | 1.204 | 1.184 | 1.164 |
| $Z_{ac}$ (Pa s/m)                  | 436.5 | 432.4 | 428.3 | 424.5 | 420.7 | 417   | 413.5 | 410   | 406.6 |

The model used in this work is that of Webster-Lokshin (Haddar, Hélie, & Matignon, 2003). It is a model with mono-spatial dependence which characterizes the linear propagation of acoustic waves in tubes with axial symmetry. This model takes also into account visco-

thermal losses at the wall boundaries with the assumption of wide tubes (Mignot R. , 2009). Thus, in an axisymmetric tube of constant section  $S = \pi r^2$ , the acoustic pressure P(x,t,L) and the acoustic flow  $Q_v(x,t,L)$  are governed by the *equation of the pavilions*, also called Webster-Lokshin, and Euler equation, leading to system (1.1):

$$\begin{cases} \frac{r}{c_{a}} \frac{\partial^{2}}{\partial t^{2}} P(x,t,L) + 2\varepsilon \frac{r}{c_{a}} \frac{\partial^{3/2}}{\partial t^{3/2}} P(x,t,L) - r \frac{\partial^{2}}{\partial x^{2}} P(x,t,L) = 0 , x \in [0;L], t > 0 \\ \\ \frac{\rho_{a}}{S} \frac{\partial}{\partial t} Q_{\nu}(x,t,L) + \frac{\partial}{\partial x} P(x,t,L) = 0 \end{cases}$$

$$(1.1)$$

where  $\varepsilon$  is a parameter associated with visco-thermal losses. More precisely,  $\varepsilon$  is given by the relation:

$$\varepsilon = \frac{K_0}{r}, \quad \text{with} \quad K_0 = \sqrt{l_v} + (\gamma - 1)\sqrt{l_h} \quad , \tag{1.2}$$

where  $l_{\nu}$  and  $l_h$  represent the characteristic lengths of viscous ( $l_{\nu} = 4 \ge 10^{-8}$  m) and thermal ( $l_h = 6 \ge 10^{-8}$  m) effects,  $\gamma$  being the ratio of specific heats.

The phenomenon of visco-thermal losses is a dissipative effect at the wall of the tube, which is due to the viscosity of the air and to the thermal conduction (Mignot R., 2009) (Boutin, Le Conte, Le Carrou, & Fabre, 2018). For the case of wind musical instruments resonators, the assumption of wide tubes is used. This hypothesis is expressed by the following relation:

$$r \gg \max[r_v = (l_v \lambda)^{0.5}; r_h = (l_h \lambda)^{0.5}],$$
 (1.3)

where  $\lambda = c_a/f$  represents the wavelength (in m) and *f* the frequency (in Hz).

Thus, for a speed of the sound  $c_a$  and a frequency  $f_{min}$  corresponding to the lower limit of the frequency domain of study of the model, it is possible to determine the minimum value of the radius  $r_{min}$  of the acoustic tube below which the model is not valid.

As an illustration where  $l_v = 4 \ge 10^{-8}$  m,  $l_h = 6 \ge 10^{-8}$  m and  $c_a = 346.3$  m/s (at a constant temperature  $T_a = 25^{\circ}$ C), Figure 1.2 presents the curves of  $r_v(f) = (l_v c_a/f)^{0.5}$  (in red) and  $r_h(f) = (l_h c_a/f)^{0.5}$  (in blue) with respect to the frequency f.

If we consider that the frequency domain of study of the Webster-Lokshin model is that of the frequencies audible by the human ear, frequencies ranging between 20 Hz (most serious frequency) and 20 000 Hz (most acute frequency), then  $f_{min} = 20$  Hz. For this value of  $f_{min}$ , the model is valid for acoustic tube radius greater than 1 mm (in next section, entitled **1.2.3 - Frequency Response Analysis**, r = 5 mm).



Figure 1.2 – Curves  $r_v(f) = (l_v c_a/f)^{0.5}$  (in red) and  $r_h(f) = (l_h c_a/f)^{0.5}$  (in blue) with respect to the frequency f

#### 1.2.2 Resolution in the Symbolic Domain

Under the assumption of zero initial conditions, the Laplace transformation applied to the system (1.1) leads to:

$$\begin{cases} \left[ \left( \left( \frac{s}{c_a} \right)^2 + 2\varepsilon \left( \frac{s}{c_a} \right)^{3/2} \right) - \frac{\partial^2}{\partial x^2} \right] r \,\overline{P}(x, s, L) = 0 \\ \frac{\rho_a}{S} s \,\overline{Q}_v(x, s, L) + \frac{\partial}{\partial x} \,\overline{P}(x, s, L) = 0 \end{cases}$$
(1.4)

with  $\overline{P}(x, s, L) = \text{TL}\{P(x, t, L)\}$  and  $\overline{Q}_{v}(x, s, L) = \text{TL}\{Q_{v}(x, t, L)\}$ , s being the Laplace variable and TL its transformation.

Solving the Webster-Lokshin equation (Haddar, Hélie, & Matignon, 2003) gives the solution  $\overline{P}(x,s,L)$  in the general form:

$$\overline{P}(x,s,L) = \frac{A(s)}{r} e^{x \Gamma(s)} + \frac{B(s)}{r} e^{-x \Gamma(s)} , \qquad (1.5)$$

where A(s) and B(s) are rational functions of s which depend on the boundary conditions, and  $\Gamma(j\omega) = j k(\omega), k(\omega)$  being a standard complex wave number.  $\Gamma(s)$  is given in the Laplace domain by (Mignot, Hélie, Matignon, & ., 2011):

$$\Gamma(s) = \sqrt{\left(\frac{s}{c_a}\right)^2 + 2\varepsilon \left(\frac{s}{c_a}\right)^{3/2}}, \text{ with } R_e(\Gamma(s)) \ge 0 \text{ if } \varepsilon \ge 0. (1.6)$$

The expression of the solutions of  $\overline{Q}_{v}(x,s,L)$  is deduced in two ways:

1- Using the Euler equation in the Laplace domain (second equation of the system (1.4)), that is:

$$\overline{Q}_{\nu}(x,s,L) = -\frac{S}{\rho_a} \frac{1}{s} \frac{\partial}{\partial x} \overline{P}(x,s,L), \qquad (1.7)$$

2- By introducing the general solution of  $\overline{P}(x, s, L)$  in relation (1.7), that is:

$$\overline{Q}_{\nu}(x,s,L) = -\frac{S}{\rho_a} \frac{1}{s} \frac{\partial}{\partial x} \left( \frac{A(s)}{r} e^{x \Gamma(s)} + \frac{B(s)}{r} e^{-x \Gamma(s)} \right).$$
(1.8)

Finally, the solution of  $\overline{Q}_{\nu}(x,s,L)$  is expressed in relation (1.9):

$$\overline{Q}_{\nu}(x,s,L) = -\left(\frac{S}{\rho_{a} r}\right) \frac{1}{s} \Gamma(s) A(s) e^{x \Gamma(s)} + \left(\frac{S}{\rho_{a} r}\right) \frac{1}{s} \Gamma(s) B(s) e^{-x \Gamma(s)}.$$
(1.9)

Taking into account the boundary conditions makes it possible to determine the two unknowns A(s) and B(s), and finally the impedance  $Z(x,s,L) = \overline{P}(x,s,L)/\overline{Q}_{v}(x,s,L)$  of the finite medium of length L.

As an example, consider a zero impedance at x = L, that is Z(L, s, L) = 0, which leads to  $\overline{P}(L, s, L) = 0$ . According to relation (1.5), we obtain:

$$\frac{A(s)}{r} e^{L \Gamma(s)} + \frac{B(s)}{r} e^{-L \Gamma(s)} = 0 \quad , \tag{1.10}$$

from which we deduce that

$$B(s) = -A(s)e^{2L \Gamma(s)} . (1.11)$$

Then, replacing B(s) by its expression (1.11) in the relation (1.5) of  $\overline{P}(x,s,L)$ :

$$\overline{P}(x,s,L) = \frac{A(s)}{r} e^{x \Gamma(s)} \left(1 - e^{2(L-x)\Gamma(s)}\right) . \qquad (1.12)$$

In the same way, by replacing B(s) by its expression (1.10) in the relation (1.8) of  $\overline{Q}_{v}(x,s,L)$ , we obtain:

$$\overline{Q}_{\nu}(x,s,L) = -\left(\frac{S}{\rho_{a} r}\right) \frac{1}{s} \Gamma(s) A(s) e^{x \Gamma(s)} \left(1 + e^{2(L-x) \Gamma(s)}\right).$$
(1.13)

Finally, the impedance  $Z(x,s,L) = \overline{P}(x,s,L)/\overline{Q}_{v}(x,s,L)$  is given by the relationship of equations (1.11) and (1.12), that is:

$$Z(x,s,L) = -\frac{\frac{A(s)}{r} e^{x \Gamma(s)} \left(1 - e^{2(L-x) \Gamma(s)}\right)}{\left(\frac{S}{\rho_a r}\right) \frac{1}{s} \Gamma(s) A(s) e^{x \Gamma(s)} \left(1 + e^{2(L-x) \Gamma(s)}\right)} , \qquad (1.14)$$

which gives after simplification:

$$Z(x,s,L) = -\frac{\rho_a}{S} \frac{s}{\Gamma(s)} \left( \frac{1 - e^{2(L-x)\Gamma(s)}}{1 + e^{2(L-x)\Gamma(s)}} \right) , \qquad (1.15)$$

knowing that  $tanh(y) = -(1 - e^{2y})/(1 + e^{2y})$ , tanh(.) being the tangent hyperbolic function,

$$Z(x,s,L) = \frac{\rho_a}{S} \frac{s}{\Gamma(s)} \tanh((L-x) \Gamma(s)) \quad . \tag{1.16}$$

From the perspective of a system approach, the function  $\Gamma(s)$  defined by the relation (1.5) is rewritten by putting the term  $s/c_a$  into factor, that is:

$$\Gamma(s) = \left(\frac{s}{c_a}\right) \sqrt{1 + 2\varepsilon \left(\frac{c_a}{s}\right)^{1/2}} \quad , \tag{1.17}$$

or again, in canonical form,

$$\Gamma(s) = \left(\frac{s}{c_a}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}} \quad \text{with} \quad \begin{cases} m = 0.5\\ \varepsilon = 2 \ m \frac{K_0}{r} \\ \omega_{r,m} = (2 \ \varepsilon)^{1/m} \ c_a \end{cases}$$
(1.18)

where  $\omega_{r,m}$  is a transitional frequency (in rad/s). Note that in the theoretical case where the system is conservative, that is to say  $\varepsilon = 0$ , the function  $\Gamma(s)$  (relation (1.17)) is reduced to  $\Gamma(s) = s/c_a$ . By replacing  $\Gamma(s)$  of relation (1.18) in relation (1.14), Z(x,s,L) can be expressed as follows:

$$Z(x,s,L) = \frac{\rho_a c_a}{S} \sqrt{\frac{\left(\frac{s}{\omega_{r,m}}\right)^m}{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}} \tanh\left((L-x)\left(\frac{s}{c_a}\right)\sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}}\right), \quad (1.19)$$

or again, by introducing the characteristic acoustic impedance  $Z_{ac} = \rho_a c_a$  and the transitional frequency  $\omega_{Lx} = c_a/(L-x)$  (in rad/s),

$$Z(x,s,L) = \frac{Z_{ac}}{S} \sqrt{\frac{\left(\frac{S}{\omega_{r,m}}\right)^m}{1 + \left(\frac{S}{\omega_{r,m}}\right)^m}} \quad \tanh\left(\frac{S}{\omega_{L,x}}\right) \sqrt{\frac{1 + \left(\frac{S}{\omega_{r,m}}\right)^m}{\left(\frac{S}{\omega_{r,m}}\right)^m}}\right) \quad . \tag{1.20}$$

Thus, from the analytical expression of the impedance Z(x,s,L) (1.20), knowing the flow  $\overline{Q}_{v}(x,s,L)$  at any point x of the acoustic tube of length L makes it possible to deduce the pressure  $\overline{P}(x,s,L)$  (Hélie, Gandolfi, & Hezard, 2014).

#### Remarks

At  $\mathbf{x} = \mathbf{L}$ ,  $1/\omega_{Lx} = 0$  hence, knowing that tanh(0) = 0, we verify that Z(L, s, L) = 0.

At x = 0, the input impedance  $Z_{in}(s,L) = Z(0,s,L)$  of the finite medium of length L is given by:

$$Z_{in}(s,L) = \frac{\rho_a}{s} \frac{s}{\Gamma(s)} \tanh(L \ \Gamma(s)) .$$
(1.21)

Always at x = 0, but for a semi-infinite medium  $(L \to \infty)$ , knowing that  $\lim_{y \to \infty} \tanh(y) = 1$  the input impedance  $Z_{in}(s, \infty) = Z(0, s, \infty)$  becomes:

$$Z_{in}(s,\infty) = \lim_{L \to \infty} Z(x=0,s,L) = \frac{\rho_a}{S} \frac{s}{\Gamma(s)} \quad .$$
(1.22)

Finally, in the theoretical case of a purely **conservative** system ( $\varepsilon = 0$ ) the acoustic impedance Z(x, s, L), noted in this case  $Z_0(x, s, L)$ , of a **finite** medium is reduced to

$$Z_0(x,s,L) = \frac{Z_{ac}}{S} \tanh\left(\frac{s}{\omega_{Lx}}\right) , \qquad (1.23)$$

and that of a semi-infinite medium, denoted  $Z_0(x,s,\infty)$ , at

$$Z_0(x,s,\infty) = \lim_{L \to \infty} Z(x,s,L) = \frac{Z_{ac}}{S} = cst \quad .$$
(1.24)
*Note that the conservative case, although purely theoretical, allows by comparison to better observe the effect of visco-thermal losses.* ■

To conclude this paragraph concerning the resolution in the symbolic domain, the study of asymptotic behaviors of Z(x,s,L), that is

$$\lim_{s \to 0} Z(x, s, L) = \frac{Z_{ac}}{S} \frac{1}{\omega_{r,m}^m} \frac{1}{\omega_{L,x}} \lim_{s \to 0} s^{m+1}$$
(1.25)

and

$$\lim_{s \to \infty} Z(x, s, L) \to \frac{Z_{ac}}{S} = cst \quad , \tag{1.26}$$

highlights that Z(x,s,L) tends towards a behavior of the following types:

- *fractional derivative* of order m+1, *i.e.* 1.5 with m = 0.5, when *s tends to zero*;

- *proportional*, whose gain value is fixed by *Z<sub>ac</sub>/S*, when *s tends to infinity*.

## 1.2.3 - Frequency Response Analysis

In stationary harmonic system, the frequency response  $Z(x,j\omega,L)$  is given by:

$$Z(x, j\omega, L) = \frac{Z_{ac}}{S} \sqrt{\frac{\left(\frac{j\omega}{\omega_{r,m}}\right)^m}{1 + \left(\frac{j\omega}{\omega_{r,m}}\right)^m}}} \tanh\left(\frac{j\omega}{\omega_{L,x}}\right) \sqrt{\frac{1 + \left(\frac{j\omega}{\omega_{r,m}}\right)^m}{\left(\frac{j\omega}{\omega_{r,m}}\right)^m}}\right), \quad (1.27)$$

where the transitional frequencies  $\omega_{r,m}$  and  $\omega_{L,x}$  have the following expressions:

$$\begin{cases}
\omega_{r,m} = \frac{c_a}{\left(\frac{r}{4mK_0}\right)^{1/m}} \\
\omega_{L,x} = \frac{c_a}{L-x}
\end{cases}$$
(1.28)

Thus, showing that  $\omega_{r,m}$  decreases when the radius *r* increases and that, on the contrary,  $\omega_{L,x}$  increases when the position *x* considered moves away from the origin and approaches the end of the acoustic tube, *L*.

For illustration, and in order to plot the frequency response  $Z(x,j\omega,L)$  in stationary harmonic system, let us consider an acoustic tube whose nominal dimensions are as follows:

- radius  $r = 5 \times 10^{-3}$  m (value perfectly in accordance with the domain of validity of the Webster-Lokshin model);

- length L = 0.3 m;
- temperature of 25°C;
- density  $\rho_a = 1.184 \text{ kg/m}^3$  and a speed of sound in the air  $c_a = 346.3 \text{ m/s}$  (refer to Table 1.1).

Note that these values of *r* and *L* representing the resonator of a recorder lead to the values of the transitional frequencies  $\omega_{r,m} = 4.92 \text{ rad/s} (0.78 \text{ Hz})$  and  $\omega_{L,x} = 1154 \text{ rad/s} (184 \text{ Hz})$  with x = 0, thus showing for such an instrument that  $\omega_{r,m} \ll \omega_{L,x}$ .

Figure 1.3 presents in x = 0 the Bode diagrams of  $Z(0,j\omega,L)$  on the range [20; 20,000] Hz of the audible frequencies by the human ear (Figure 1.3.a) and on the range [20; 4000] Hz of the frequencies attainable with a recorder (Figure 1.3.b).

Although an in-depth analysis of the frequency response of the model is developed in the following paragraph, the observation of these diagrams over the range of audible frequencies leads to a remark which concerns the existence of two very distinct behaviors:

- The first on the frequency range [20;  $\omega_{L,x}/2\pi = 184$ ] Hz with a derivative behavior of order 1, highlighting the absence of the fractional behavior on this frequency range which exists for frequencies lower than  $\omega_{r,m}/2\pi = 0.78$  Hz;
- The second on the frequency range  $[\omega_{L,x}/2\pi = 184; 20\ 000]$  Hz with an alternation of resonances and anti-resonances.



Figure 1.3 – Bode Diagrams of  $Z(0,j\omega,L)$  on the range [20 ; 20 000] Hz of the frequencies audible by the human ear (a) and on the range [20 ; 4 000] Hz of frequencies attainable with a recorder (b)

## 1.3 - System Approach

As a reminder, the resonator is only a part of our simulator which includes a non-linear exciter (Chapter 2), an artificial mouth (Chapter 3), and the simulation of the whole system (Chapter 4).

From a causal point of view, the input of the resonator at x = 0 is defined by the pressure at the output of the nonlinear exciter. This is the reason why the system approach developed in this paragraph considers the admittance  $Y(x,s,L) = Z^{-1}(x,s,L)$  and not the impedance Z(x,s,L). More specifically, it is the input admittance at x = 0, denoted  $Y_{in}(s,L) = Y(0,s,L)$ . Note that this consideration of the admittance  $Y_{in}(j\omega,L)$  leads to an integrative behavior for frequencies lower than  $\omega_{L,x}$  (derivative for Z(x,s,L)), thus respecting integral causality, a fundamental notion in a system approach.

In addition, the global admittance Y(x,s,L) is broken down into a cascade of local transfer functions of which all the parameters, as well as all the input and output variables will have a physical meaning. This decomposition then facilitates the frequency analysis of the Webster-Lokshin model, thus reaching a reduced model to be implemented in the simulator.

#### 1.3.1 Decomposition of Admittance Y (x, s, L) into Subsystems

The admittance  $Y(x,s,L) = Z^{-1}(x,s,L)$  of an acoustic tube of length *L* at a point *x* between 0 and *L* is therefore defined by the expression:

$$Y(x,s,L) = \frac{\overline{Q}_{\nu}(x,s,L)}{\overline{P}(x,s,L)} = \frac{S}{Z_{ac}} \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}} \frac{1}{\tanh\left(\frac{s}{\omega_{L,x}}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}}\right)}, \quad (1.29)$$

relation that can be expressed as follows,

$$Y(x,s,L) = H_0 I_m(s) T(x,s,L) , \qquad (1.30)$$

by taking

$$H_0 = \frac{S}{Z_{ac}} = \frac{S}{\rho_a \, c_a} \,\,, \tag{1.31}$$

$$I_{m}(s) = \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^{m}}{\left(\frac{s}{\omega_{r,m}}\right)^{m}}}, \qquad (1.32)$$

and

$$T(x,s,L) = \frac{1}{\tanh(F(x,s,L))} \quad , \tag{1.33}$$

where

$$F(x,s,L) = \left(\frac{s}{\omega_{L,x}}\right) I_m(s) \quad . \tag{1.34}$$

For the following, the concept of *acoustic admittance*  $Y(x,s,L) = \overline{Q}_v(x,s,L)/\overline{P}(x,s,L)$  is replaced by the concept of *transfer function* H(x,s,L) defined between the pressure source  $\overline{P}_{in}(s) = \overline{P}(x=0,s)$  at the input of the tube at x = 0 and the flow  $\overline{Q}_v(x,s,L)$  at any point x of the tube of length L (semi finite) and of constant radius r, that is:

$$H(x,s,L) = \frac{\overline{Q}_{v}(x,s,L)}{\overline{P}_{in}(s)} = H_{0} I_{m}(s) T(x,s,L).$$
(1.35)

At x = 0, for this finite medium, the admittance of input  $Y_{in}(s,L)$  has the expression:

$$Y_{in}(s,L) = H(0,s,L) = \frac{\overline{Q}_{v}(0,s,L)}{\overline{P}_{in}(s)} = H_{0} I_{m}(s) T(0,s,L).$$
(1.36)

At x = 0, but for a semi-infinite medium  $(L \to \infty)$ , the admittance of input  $Y_{in}(s, \infty)$  is reduced to:

$$Y_{in}(s,\infty) = H(0,s,\infty) = \frac{\overline{Q}_{v}(0,s,\infty)}{\overline{P}_{in}(s)} = H_0 I_m(s).$$
(1.37)

Figure 1.4 presents the block diagrams associated with this system approach where the different transfer functions are defined by:

$$\begin{cases} H_{0} = \frac{\overline{Q}_{in}(s)}{\overline{P}_{in}(s)} = \frac{S}{Z_{ac}} = cst \\ I_{m}(s) = \frac{\overline{Q}_{v}(0, s, \infty)}{\overline{Q}_{in}(s)} = \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^{m}}{\left(\frac{s}{\omega_{r,m}}\right)^{m}}} \\ T(x, s, L) = \frac{\overline{Q}_{v}(x, s, L)}{\overline{Q}_{v}(0, s, \infty)} = \frac{1}{\tanh\left(\left(\frac{s}{\omega_{L,x}}\right)I_{m}(s)\right)} \end{cases}$$
(1.38)

Note that the quantity  $H_0 \overline{P}_{in}(s)$  is homogeneous at a flow, noted  $\overline{Q}_{in}(s)$ , corresponding to the conversion of the pressure source applied at x=0 (Dirichlet condition) into an equivalent source of flow always applied at x=0 (Neumann condition) (Assaf, 2015).



Figure 1.4 – Block diagrams associated with the system approach: Whatever x is between 0 an L (a), at x=0 for the finite system L (b), at x=0 for a semi finite system (c)

#### 1.3.2 - Analysis in the Frequency Domain

In stationary harmonic system, the relation (1.35) becomes:

$$H(x, j\omega, L) = \frac{\overline{Q}_{v}(x, j\omega, L)}{\overline{P}_{in}(j\omega)} = H_{0} I_{m}(j\omega) T(x, j\omega, L) , \qquad (1.39)$$

with

$$I_{m}(j\omega) = \frac{\overline{Q}_{v}(0, j\omega, \infty)}{\overline{Q}_{in}(j\omega)} = \sqrt{\frac{1 + \left(j\frac{\omega}{\omega_{r,m}}\right)^{m}}{\left(j\frac{\omega}{\omega_{r,m}}\right)^{m}}}$$
(1.40)

and

$$T(x, j\omega, L) = \frac{\overline{Q}_{\nu}(x, j\omega, L)}{\overline{Q}_{\nu}(0, j\omega, \infty)} = \frac{1}{\tanh(F(x, j\omega, L))} , \qquad (1.41)$$

where

$$F(x, j\omega, L) = \left(\frac{j\omega}{\omega_{L,x}}\right) I_m(j\omega) \quad . \tag{1.42}$$

The remaining part of this paragraph is dedicated to a detailed analysis of the frequency responses  $I_m(j\omega)$ ,  $F(x,j\omega,L)$  and  $T(x,j\omega,L)$  of each subsystem, then of the frequency response  $H(x,j\omega,L)$  of the overall system.

## 1.3.2.1 - Analysis of I<sub>m</sub>(jω)

The analysis of  $I_m$  (j $\omega$ ) highlights two behaviors whose transition zone is fixed by the transitional frequency  $\omega_{r,m}$ , these behaviors are:

- for  $\omega \ll \omega_{r,m}$ , a fractional integrative behavior of order m/2 = 0.25,

$$\forall \, \omega << \omega_{r,m} \,, \quad I_m(j\omega) = \sqrt{\frac{1 + \left(j\frac{\omega}{\omega_{r,m}}\right)^m}{\left(j\frac{\omega}{\omega_{r,m}}\right)^m}} \approx \left(\frac{\omega_{r,m}}{j\omega}\right)^{m/2} \Rightarrow \begin{cases} |I_m(j\omega)| = \left(\frac{\omega_{r,m}}{\omega}\right)^{m/2} \\ \arg I_m(j\omega) = -m\frac{\pi}{4} \end{cases} ; (1.43)$$

- for  $\omega >> \omega_{r,m}$ , unit proportional behavior,

$$\forall \, \omega >> \omega_{r,m} \,, \quad I_m(j\omega) = \sqrt{\frac{1 + \left(j\frac{\omega}{\omega_{r,m}}\right)^m}{\left(j\frac{\omega}{\omega_{r,m}}\right)^m}} \approx 1 \quad \Rightarrow \quad \begin{cases} |I_m(j\omega)| = 1\\ \arg I_m(j\omega) = 0 \end{cases} . \quad (1.44)$$

As an illustration, let us take the acoustic tube whose nominal dimensions are fixed by a radius  $r = 5 \ge 10^{-3}$  m and a length L = 0.3 m at a temperature of 25 °C, with  $\rho_a = 1.184$  kg/m<sup>3</sup> and  $c_a = 346.3$  m/s. In this case, and as a reminder, the numerical value of the transitional frequency  $\omega_{r,m}$  (relation (1.28)) and 4.92 rad/s (0.784 Hz).

Figure 1.5 presents the Bode diagrams of the frequency response  $I_m(j\omega)$  over the range  $[10^{-4}; 10^4]$  Hz. The two behaviors appear clearly:

- for  $\omega \ll \omega_{r,m}$ , a gain diagram with a straight line with slope p = -m/2\*20 dB/dec = -5 dB/dec and a phase diagram with a horizontal line at  $-m/2*90^\circ = -22.5^\circ$ ;
- for  $\omega >> \omega_{r,m}$ , a gain diagram with a horizontal line at 0 dB and a phase diagram with a horizontal line at 0°.

Figure 1.6 presents the same frequency response of  $I_m(j\omega)$  but only on the range [20; 20,000] Hz of the frequencies audible by the human ear, and this with a gain diagram in linear-linear scale and a phase diagram with the frequency axis also in linear scale. The observation of this answer makes it possible to affirm, for this representative example of a recorder, that the unit proportional behavior is dominant, that is:

$$\forall \quad \omega \ge 2\pi \ 20 \text{ rad/s}, \quad I_m(j\omega) = \frac{\overline{Q}_v(0, j\omega, \infty)}{\overline{Q}_{in}(j\omega)} \approx 1 \quad .$$
 (1.45)

Thus, for the area of study considered in this work, area defined by the range [20; 20,000] Hz of the audible frequencies, the transfer  $I_m(s)$  can be reduced to the unit which leads to  $\overline{Q}_v(0,s,\infty) = \overline{Q}_{in}(s)$ , leading to a reduction in the block diagrams of Figure 1.4.

The direct consequence is that in the case of a semi-infinite medium at x = 0 (Figure 1.4.c), the fractional integrative behavior has no influence on the range of audible frequencies.



Figure 1.5 – Bode diagrams of  $I_m$  (j $\omega$ ) on the range [10<sup>-4</sup>; 10<sup>4</sup>] Hz



Figure 1.6 – Frequency response of  $I_m(j\omega)$  on the range [20; 20,000] Hz of the audible frequencies

## 1.3.2.2 - Analysis of F(0,jω,L)

Knowing that in the case of a recorder  $\omega_{r,m} \ll \omega_{L,x}$ , the analysis of  $F(0,j\omega,L)$  again highlights two behaviors whose transition zone is fixed by the transitional frequency  $\omega_{r,m}$ , that is:

- for  $\omega \ll \omega_{r,m}$ , a *fractional derivative* behavior of order (1 - m/2) = 0.75,

$$\forall \, \omega << \omega_{r,m} , \quad F(x, j\omega, L) = \left(\frac{j\omega}{\omega_{L,x}}\right) \sqrt{\frac{1 + \left(j\frac{\omega}{\omega_{r,m}}\right)^m}{\left(j\frac{\omega}{\omega_{r,m}}\right)^m}} \approx \left(\frac{\omega_{r,m}^{m/2}}{\omega_{L,x}}\right) (j\omega)^{(1-m/2)} , \quad (1.46)$$

hence the module and the argument are defined as follows:

$$\begin{cases} \left| F(x, j\omega, L) \right| = \left( \frac{\omega_{r,m}^{m/2}}{\omega_{L,x}} \right) \omega^{(1-m/2)} \\ \arg F(x, j\omega, L) = \left( 1 - \frac{m}{2} \right) \frac{\pi}{2} \end{cases}; \quad (1.47)$$

- for  $\omega_r \ll \omega$ , a *derivative* behavior of order 1,

$$\forall \omega_{r,m} \ll \omega, \quad F(x, j\omega, L) = \left(\frac{j\omega}{\omega_{L,x}}\right) \sqrt{\frac{1 + \left(j\frac{\omega}{\omega_{r,m}}\right)^m}{\left(j\frac{\omega}{\omega_{r,m}}\right)^m}} \approx \left(\frac{j\omega}{\omega_{L,x}}\right) \Rightarrow \begin{cases} |F(x, j\omega, L)| = \frac{\omega}{\omega_{L,x}} \\ \arg F(x, j\omega, L) = \frac{\pi}{2} \end{cases} . (1.48)$$

Figure 1.7 presents at x = 0 the Bode diagrams of the frequency response  $F(0,j\omega,L)$  on the range  $[10^{-4}; 10^4]$  Hz. The two behaviors appear clearly:

- for  $\omega \ll \omega_{r,m}$ , a gain diagram with a straight line with  $p_1 = (1-m/2)*20$  dB/dec = 15 dB/dec and a phase diagram with a horizontal line at  $(1-m/2)*90^\circ = 67.5^\circ$ ;
- for  $\omega >> \omega_{r,m}$ , a gain diagram with a straight line with slope  $p_2 = 20$  dB/dec of this a phase diagram with a horizontal straight line at 90 °.



Figure 1.7 – Bode diagrams of  $F(0, j\omega, L)$  on the range [10<sup>-4</sup>; 10<sup>4</sup>] Hz

## 1.3.2.3 - Analysis of T(0,jω,L)

The analysis of  $T(x,j\omega,L)$  highlights three behaviors whose transition zones are fixed by the transitional frequencies  $\omega_{r,m}$  and  $\omega_{Lx}$ :

- for  $\omega \ll \omega_{Lx}$ , an *integrative behavior with two different orders* according to the frequency range. Indeed,

$$\forall \, \omega \ll \omega_{L,x} , \quad T(x, j\omega, L) = \frac{1}{\tanh(F(x, j\omega, L))} \underset{\omega \ll \omega_{L,x}}{\approx} \frac{1}{F(x, j\omega, L)} , \qquad (1.49)$$

with, for  $\omega \ll \omega_{r,m}$ , an orderly *fractional integrative* behavior - (1 - m/2) = -0.75, that is

$$\forall \, \omega \ll \omega_{r,m}, \quad \frac{1}{F(x,j\omega,L)} \approx \left(\frac{\omega_{L,x}}{\omega_r^{m/2}}\right) \frac{1}{(j\omega)^{(1-m/2)}} \Rightarrow \begin{cases} \left|T(x,j\omega,L)\right| = \left(\frac{\omega_{L,x}}{\omega_r^{m/2}}\right) \frac{1}{\omega^{(1-m/2)}} \\ \arg T(x,j\omega,L) = \left(1-\frac{m}{2}\right) \frac{\pi}{2} \end{cases}$$
(1.50)

and, for  $\omega_{r,m} \ll \omega$ , a *derivative* behavior of order 1, that is

$$\forall \omega_{r,m} \ll \omega, \quad \frac{1}{F(x, j\omega, L)} \approx \left\{ \frac{|T(x, j\omega, L)|}{m} \Rightarrow \begin{cases} |T(x, j\omega, L)| = \frac{\omega_{L,x}}{\omega} \\ \arg T(x, j\omega, L) = -\frac{\pi}{2} \end{cases}; \quad (1.51)$$

(

- for  $\omega_{L,x} \ll \omega$ , a behavior composed of an alternation of *anti-resonances* and *resonances*, giving the expression of  $T(x,j\omega,L)$ ,

$$\forall \omega_{L,x} \ll \omega, \quad T(x, j\omega, L) = \frac{1}{\tanh\left(\left(\frac{j\omega}{\omega_{L,x}}\right)\sqrt{\frac{1+\left(j\frac{\omega}{\omega_{r,m}}\right)^m}{\left(j\frac{\omega}{\omega_{r,m}}\right)^m}}\right)}.$$
 (1.52)

Figure 1.8 shows the Bode diagrams of  $1/F(0,j\omega,L)$  (in red) and of  $T(0,j\omega,L)$  (in blue) over the range  $[10^{-4}; 10^4]$  Hz (Figure 1.8.a) and on the range [20; 4000] Hz of the audible and achievable frequencies with a recorder (Figure 1.8.b).

Below the first cutoff frequency  $[10^{-4}; \omega_{Lx}/2\pi = 184]$  Hz, the responses of  $1/F(0,j\omega,L)$  (in red) and  $T(0,j\omega,L)$  (in blue) overlap where:

- a *fractional integration behavior* of order -0.75 over the range  $[10^{-4}; \omega_r/2\pi = 0.784]$  Hz is observed;

- an *integrative behavior* of order 1 over the range  $[\omega_{r,m}/2\pi = 0.784; \omega_{Lx}/2\pi = 184]$  Hz is observed.

Beyond 184 Hz, the frequency response  $T(0,j\omega,L)$  (in blue) clearly presents an alternation of anti-resonances and resonances introduced by the hyperbolic tangent function.



Figure 1.8 – Bode diagrams of 1/F(0,jω,L) (in red) and of T(0,jω,L) (in blue) on the range [10<sup>-4</sup>; 10<sup>4</sup>] Hz
(a) and on the range [20; 4000] Hz of audible and achievable frequencies with a recorder (b)

### 1.3.2.4 - Analysis of H(x,jω,L)

The analysis of  $H(x,j\omega,L)$  highlights three behaviors whose transition zones are fixed by the transitional frequencies  $\omega_r$  and  $\omega_{Lx}$ :

- for  $\omega \ll \omega_r \ll \omega_{Lx}$ , an orderly *fractional integrative behavior* - (1 - m/2) = -0.75, that is

$$\forall \, \omega << \omega_{r,m} \,, \quad H(x, j\omega, L) \underset{\omega << \omega_{r,m}}{\approx} \left( \frac{\omega_{L,x}}{\omega_{r,m}^{m/2}} \right) \frac{H_0}{(j\omega)^{(1-m/2)}} \Rightarrow \begin{cases} |H(x, j\omega, L)| = \left( \frac{\omega_{L,x}}{\omega_{r,m}^{m/2}} \right) \frac{H_0}{\omega^{(1-m/2)}}; (1.53) \\ \arg H(x, j\omega, L) = \left( 1 - \frac{m}{2} \right) \frac{\pi}{2} \end{cases}$$

- for  $\omega_{r,m} \ll \omega \ll \omega_{Lx}$ , a *derivative behavior* of order 1, that is:

$$\forall \omega_{r,m} << \omega << \omega_{L,x}, \quad H(x, j\omega, L) \underset{\omega_{r,m} << \omega << \omega_{L,x}}{\approx} H_0\left(\frac{\omega_{L,x}}{j\omega}\right) \Rightarrow \begin{cases} \left|H(x, j\omega, L)\right| = H_0 \frac{\omega_{L,x}}{\omega} \\ \arg H(x, j\omega, L) = -\frac{\pi}{2} \end{cases}; (1.54)$$

- for  $\omega_{Lx} \ll \omega$ , a behavior composed of an alternation of *anti-resonances* and *resonances*, that is:

$$\forall \omega_{L,x} << \omega, \quad H(x, j\omega, L) \approx \frac{H_0}{\left(\frac{j\omega}{\omega_{L,x}}\right)} \left(\frac{1 + \left(j\frac{\omega}{\omega_{r,m}}\right)^m}{\left(j\frac{\omega}{\omega_{r,m}}\right)^m}\right). \quad (1.55)$$

Figure 1.9 presents at x = 0 ( $\omega_{Lx}/2\pi = 184$  Hz), in x = L/2 ( $\omega_{Lx}/2\pi = 368$  Hz) and at x = 3L/4 ( $\omega_{Lx}/2\pi = 735$  Hz) the Bode diagrams of  $H(0,j\omega,L)$  (in black), of  $H(L/2,j\omega,L)$  (in blue) and of  $H(3L/4,j\omega,L)$  (in red) on the range [20; 4000] Hz of the audible and achievable frequencies with a recorder.

Over the range [20;  $\omega_{Lx}/2\pi$ ] Hz, the three responses of  $H(x,j\omega,L)$  present an integration behavior of order 1. The fractional integration behavior of order -0.75 does not appear over this range as it is present within a much lower frequency (0.784 Hz). Beyond  $\omega_{Lx}$ , the three responses present a succession of alternation of anti-resonances and resonances introduced by the hyperbolic tangent (*tanh*) function. In the journal (Abou Haidar, Moreau, & Abi Zeid Daou, 2021), the authors show that the influence of the order *m* is essentially located:

- for *gain diagrams*, at the peaks of resonances and anti-resonances, quantifiable effects using quality factors for anti-resonances and for resonances illustrating well the phenomenon of dissipation associated with visco-thermal losses;
- for *phase diagrams*, at the crossing points at  $0^{\circ}$  with a local slope which is important as the order is small where the slope becomes infinite for m = 0 (purely conservative case).

Moreover, note that the farther the position x moves away from the origin, the higher the transitional frequency  $\omega_{Lx}$  pushes the anti-resonance and resonance frequencies towards the high frequencies.

In addition, the position x has no influence on the transitional frequency  $\omega_{r,m}$ , the fractional integrative behavior of order -0.75 still does not appear on this frequency range.



Figure 1.9 – Bode diagrams in  $x = \theta$  ( $\omega_{Lx}/2\pi = 184$  Hz), in x = L/2 ( $\omega_{Lx}/2\pi = 368$  Hz) and in x = 3L/4( $\omega_{Lx}/2\pi = 735$  Hz) of x = 3L/4 ( $\omega_{Lx}/2\pi = 735$  Hz) (in black), of  $H(L/2,j\omega,L)$  (in blue) and of  $H(3L/4,j\omega,L)$  (in red) in the range [20; 4000] Hz of audible and achievable frequencies

## 1.3.3 - Study of the Influence of the Fractional Order m

In the Webster-Lokshin model, the fractional order *m* has the value 0.5. The objective of this paragraph and the rest of this chapter is to analyze the influence of the order *m* on the behavior of the resonator by considering that *m* belongs to the interval [0; 1] with a nominal value  $m_0 = 0.5$ , a consideration which facilitates the introduction of the concept of parametric uncertainty (additive or multiplicative) at the fractional order level. Thus, by generalizing the expression of the parameter  $\varepsilon = K_0/r$  (relation (1.2)) associated with visco-thermal losses in the Webster-Lokshin model at  $\varepsilon = 2 m K_0/r$  (relation (1.18) which for m = 0.5 gives the same expression), the analytical link is naturally established between visco-thermal losses and fractional order.

Thus, the fractional order occurs only in the presence of visco-thermal losses. In the theoretical case of a purely *conservative* system, the parameter  $\varepsilon$  is zero which is equivalent

to m = 0, taking into account the relation (1.17). In this case, the expression of the acoustic transfer H(x, s, L), denoted as  $H_0(x, s, L)$ , of a *finite medium* is reduced to:

$$H_0(x,s,L) = \frac{H_0}{\tanh\left(\frac{s}{\omega_{L,x}}\right)} \quad . \tag{1.56}$$

Figure 1.10 shows the Bode diagrams at x = 0 of  $H(0,j\omega,L)$  for different values of the fractional order over the range [20; 4000] Hz of the audible and achievable frequencies with a recorder (Figure 1.10).

In order to magnify the different curves in Figure 1.9 for better observation, Figure 1.11 presents the reduced frequency responses  $H(0,j\omega,L)/H_0$  with the frequency axis on a linear scale over the range [20; 1000] Hz.

The observation of these frequency responses shows that the influence of the order m is essentially located:

- for *gain diagrams*, at the peaks of resonances and anti-resonances, quantifiable effects using quality factors  $Q_{zi}$  for anti-resonances and  $Q_{pi}$  for resonances illustrating well the phenomenon of dissipation associated with visco-thermal losses;
- for *phase diagrams*, at the crossing points at  $0^{\circ}$  with a local slope which is all the more important as the order is small, slope which becomes infinite for m = 0 (purely conservative case).



Figure 1.10 – Bode diagrams at x = 0 of  $H(0, j\omega, L)$  for different values of the fractional order *m* on the range [20; 4000] Hz of audible and achievable frequencies with a recorder



Figure 1.11 – Reduced frequency responses  $H(0, j\omega, L) / H_0$  with the frequency axis on a linear scale over the range [150; 750] Hz

## 1.4 - From the Simplified Fractional Model to its Rational Forms

For the area of study defined by the range [20; 20,000] Hz of the audible frequencies, the analysis presented in the previous paragraph shows that the frequency response  $I_m(j\omega)$ , is:

$$I_{m}(j\omega) = \sqrt{\frac{1 + \left(j\frac{\omega}{\omega_{r,m}}\right)^{m}}{\left(j\frac{\omega}{\omega_{r,m}}\right)^{m}}},$$
(1.57)

can be reduced to the unit (see paragraph 1.3.2.1). This is the reason why for this field of study the frequency response  $H(x, j\omega, L)$  defined, as a reminder, by

$$H(x, j\omega, L) = \frac{\overline{Q}_{\nu}(x, j\omega, L)}{\overline{P}_{in}(j\omega)} = H_0 I_m(j\omega) T(x, j\omega, L) , \qquad (1.58)$$

is simplified and noted  $\widetilde{H}(x, j\omega, L)$ , that is:

$$\widetilde{H}(x, j\omega, L) = H_0 T(x, j\omega, L) , \qquad (1.59)$$

with, always as a reminder,

$$T(x, j\omega, L) = \frac{\widetilde{H}(x, j\omega, L)}{H_0} = \frac{1}{\tanh\left(\left(\frac{j\omega}{\omega_{L,x}}\right)I_m(j\omega)\right)} = \coth\left(\left(\frac{j\omega}{\omega_{L,x}}\right)I_m(j\omega)\right). (1.60)$$

It is important to note that this simplification does not affect  $T(x,j\omega,L)$  because the absence of  $I_m(j\omega)$  in relation (1.59) corresponds to the conservative case where  $\varepsilon$  and m are zero (see paragraph 1.3.3 relation (1.55)). Figure 1.12 shows the block diagrams associated with this simplification in the field of study.



Figure 1.12 – Block diagrams associated with the simplified model: whatever x is between 0 and L (a), at x = 0 for the finite system of length L (b) and at x = 0 for a semi-infinite system (c)

In general, the time simulation of fractional models often requires the use of rational models (Assaf, 2015). Thus, the fractional form defined by the relation (1.58) can be put in a rational form of N cells in cascade, noted  $\tilde{H}_{N,c}(x,s,L)$ , that is:

$$\widetilde{H}_{N,c}(x,s,L) = \frac{A_0}{s} \prod_{i=1}^{N} \left( \frac{1+2\zeta_{zi} \frac{s}{\omega_{zi}} + \left(\frac{s}{\omega_{zi}}\right)^2}{1+2\zeta_{pi} \frac{s}{\omega_{pi}} + \left(\frac{s}{\omega_{pi}}\right)^2} \right), \qquad (1.61)$$

or in a rational form of N cells in parallel, noted  $\widetilde{H}_{N,p}(x,s,L)$ ,

$$\widetilde{H}_{N,p}(x,s,L) = \frac{A_0}{s} + \sum_{i=1}^{N} \left( \frac{A_i \ s + B_i}{1 + 2 \zeta_{pi} \frac{s}{\omega_{pi}} + \left(\frac{s}{\omega_{pi}}\right)^2} \right),$$
(1.62)

with  $A_0 = H_0 \omega_{L,x}$  where the  $\omega_{zi}$  and  $\zeta_{zi}$  represent the frequencies and the damping factors associated with the anti-resonances,  $\omega_{pi}$  and  $\zeta_{pi}$  the frequencies and the damping factors associated with the resonances while changing from the cascade form (1.60) to the parallel form (1.61) by decomposing into simple elements. Note that the parallel rational form facilitates the return to the time domain by inverse Laplace transform and that it is often associated with a decomposition in a modal space (Debut, 2004).

From a theoretical point of view,  $\omega_{zi}$  corresponds to the roots of the numerator of  $T(x, j \omega, L)$ , that is:

$$\cosh\left(\left(\frac{j\omega}{\omega_{L,x}}\right)\sqrt{\frac{1+\left(j\frac{\omega}{\omega_{r,m}}\right)^{m}}{\left(j\frac{\omega}{\omega_{r,m}}\right)^{m}}}\right) = 0 \quad , \tag{1.63}$$

and the  $\omega_{pi}$  correspond to the roots of the denominator of  $T(x, j\omega, L)$ , that is:

$$\sinh\left(\left(\frac{j\omega}{\omega_{L,x}}\right)\sqrt{\frac{1+\left(j\frac{\omega}{\omega_{r,m}}\right)^m}{\left(j\frac{\omega}{\omega_{r,m}}\right)^m}}\right) = 0 \quad . \tag{1.64}$$

From a practical point of view, finding these roots by analytical resolution is complex, if not impossible (Fabrice, 2009). On the other hand, the search by numerical resolution does not pose any particular problem. For example, it is possible to use the fact that the alternation of  $\omega_{zi}$  and  $\omega_{pi}$  appears clearly on the phase from when passing at 0°, from -90° to + 90° for  $\omega_{zi}$ , and from + 90° to -90° for  $\omega_{pi}$ .

In the context of the work of this thesis, the rational forms of N cells in cascade and in parallel are considered as behavior models whose numerical values of the parameters are obtained using an optimal approach aiming to minimize the difference between the target frequency response defined by the fractional form  $\tilde{H}(x,s,L)$  and the frequency response of the rational cascade form  $\tilde{H}_{N,c}(x,s,L)$ . This digital procedure is available in the *Frequency Domain System Identification* (FDSI) module of the CRONE Toolbox (Malti & Victor, 2015).

As an illustration, let us take the acoustic tube used as an example throughout this chapter:  $r = 5 \times 10^{-3}$  m, L = 0.3 m,  $\rho_a = 1.184$  kg/m<sup>3</sup>,  $c_a = 346.3$  m/s,  $\omega_{L,x} = 1,154$  rad/s at x = 0,  $H_0 = 19.15 \times 10^{-8}$  m<sup>3</sup>s<sup>-1</sup>Pa<sup>-1</sup> and  $A_0 = 22.11 \times 10^{-5}$  rad/s. Figure 1.13 shows two screenshots from the CRONE Toolbox before optimization (Figure 1.13.a) and after optimization (Figure 1.13.b) at x = 0 in the nominal case m = 0.5.

The procedure consists, in a first step, from generating the target frequency response  $\widetilde{H}(x, j\omega, L)$  of the fractional form which appears in red (Data) in Figure 1.13.a. Then, cells are added one after the other by clicking on the "*Add New Cell*" command in the "*Cells actions*" menu (in green at the top right), then by positioning the mouse cursor on the phase diagram at point considered where the cutting phase of  $\widetilde{H}(x, j\omega, L)$  axis 0°, and going from the lowest values (left) to the most important (right). Note that in this graphical interface, the term "cell" corresponds to a polynomial (numerator or denominator). Thus, with each addition in the "Transfer function" menu (in purple at the bottom) a column in blue for the numerator and in yellow for the denominator appears. The first line "*Cell Frequency*" gives the value in rad/s of  $\omega_{zi}$  or  $\omega_{pi}$ , the second "*Cell Order*" the highest order of the polynomial (here +2 for the numerator and -2 for the denominator), the third "Local Order" is equal to +1 (for the numerator) and -1 (for the denominator) insofar as these are explicit forms (Malti & Victor, 2015), and finally the fourth "*Damping Factor*" gives the value of  $\zeta_{zi}$  or  $\zeta_{pi}$ . All these values in the columns can be modified by clicking in the corresponding box. Thus, in the case of the resonator, all the values of  $\zeta_{zi}$  or  $\zeta_{pi}$  are initialized to 0.01.

This first stage of the procedure therefore makes it possible to fix the structure of the behavior model, as well as the initial values of its parameters. The second step is an optimization step launched using the "*Tools*" menu (in orange at the bottom right). For the example of illustration, the result appears in Figure 1.13.b in particular the optimal values of  $\omega_{zi}$ ,  $\omega_{pi}$ ,  $\zeta_{zi}$  and  $\zeta_{pi}$ .

Thus, the blue curve corresponds to the frequency response  $\widetilde{H}_{N,c}(j\omega,s,L)$  of the rational cascade form (gain and phase) obtained before optimization (Figure 1.13.a) and after optimization (Figure 1.13.b). As for the green lines, these are asymptotic lines.



#### (a)



Figure 1.13 – Screenshots from the CRONE Toolbox before optimization (a) and after optimization (b) at x = 0 in the nominal case m = 0.5

In summary, the cascade form defined by the relation (1.60) comprises N = 4 cells, that is 2N = 8 "*cells*" as defined in Figure 1.13, to which we must add the integrating cell  $A_0/s$ , either in total N + 1 = 5 cells, or else 2N + 1 = 9 "cells".

## Remark

In linear systems dynamics, in the general case of a polynomial of order 2 having 1 pair of conjugate complex roots, the damping factor  $\zeta$  and the resonance factor Q associated with this pair are linked by a relation of the form :

$$Q = \frac{1}{2\zeta \sqrt{1 - \zeta^2}}$$
 (1.65)

In instrument acoustics (Chaigne & Kergomard, 2013), and in particular in the specific case of resonators of wind instruments, the damping factors are very small compared to the unit. This is the reason why the relation (1.64) is reduced to:

$$\forall 0 < \zeta <<1, \ Q \approx \frac{1}{2\zeta} \iff \zeta \approx \frac{1}{2Q}.$$
 (1.66)

Note that in instrument acoustics, the term quality factor is used in place of the **resonance factor**. Thus, in many works, the taking into account of the visco-thermal losses is made directly using the parallel rational form defined by the relation (1.62) in which the  $\zeta_{pi}$  are replaced by the corresponding  $Q_{pi}$  (relation (1.66)) (Chaigne & Kergomard, 2013) without going through fractional models.

Table 1.2 summarizes the final numerical values of the parameters  $\omega_{zi}$ ,  $\zeta_{zi}$ ,  $Q_{zi}$ ,  $\omega_{pi}$ ,  $\zeta_{pi}$ and  $Q_{pi}$  of the N = 4 cells of the cascade form (relation (1.60)) to which we must not forget the cell number zero, namely the integrator  $A_0/s$ .

As for Table 1.3, it gives the numerical values of the parameters  $A_i$ ,  $B_i$ ,  $\omega_{pi}$ ,  $\zeta_{pi}$  and  $Q_{pi}$  of N = 4 cells of the parallel form (relation (1.61)) to which cell  $A_0/s$  is added.

| N | <i>w</i> <sub>zi</sub> (rad/s) | 5 zi                  | $Q_{zi}$ | $\boldsymbol{\omega}_{pi}$ (rad/s) | ζ <sub>pi</sub>       | $Q_{pi}$ |
|---|--------------------------------|-----------------------|----------|------------------------------------|-----------------------|----------|
| 1 | 1780                           | 18 10 <sup>-3</sup>   | 27.78    | 3580                               | 12.9 10 <sup>-3</sup> | 38.76    |
| 2 | 5380                           | $10.5 \ 10^{-3}$      | 47.62    | 7190                               | 9.2 10 <sup>-3</sup>  | 54.35    |
| 3 | 8990                           | 8.12 10 <sup>-3</sup> | 61.8     | 10800                              | 7.47 10 <sup>-3</sup> | 66.93    |
| 4 | 12600                          | 6.92 10 <sup>-3</sup> | 72.25    | 14500                              | 6.37 10 <sup>-3</sup> | 78.5     |

Table 1.2 – Final numerical values of the parameters  $\omega_{zi}$ ,  $\zeta_{zi}$ ,  $Q_{zi}$ ,  $\omega_{pi}$ ,  $\zeta_{pi}$  and  $Q_{pi}$  of the N = 4 cells of the cascade form

Table 1.3 – Numerical values of parameters  $A_{i}$ ,  $B_{i}$ ,  $\omega_{pi}$ ,  $\zeta_{pi}$  and  $Q_{pi}$  of N = 4 cells of the parallel form

| N | A <sub>i</sub>          | $\boldsymbol{B}_i$      | $\boldsymbol{\omega}_{pi}$ (rad/s) | $\zeta_{pi}$          | $Q_{pi}$ |
|---|-------------------------|-------------------------|------------------------------------|-----------------------|----------|
| 1 | 36.02 10 <sup>-12</sup> | 16.63 10 <sup>-10</sup> | 3580                               | 12.9 10 <sup>-3</sup> | 38.76    |
| 2 | 98.24 10 <sup>-13</sup> | 64.99 10 <sup>-11</sup> | 7190                               | 9.2 10 <sup>-3</sup>  | 54.35    |
| 3 | 53.38 10 <sup>-13</sup> | 43.07 10 <sup>-11</sup> | 10800                              | 7.47 10 <sup>-3</sup> | 66.93    |
| 4 | 58.98 10 <sup>-13</sup> | 54.47 10 <sup>-11</sup> | 14500                              | 6.37 10 <sup>-3</sup> | 78.5     |

Figure 1.14 presents the Bode diagrams of the response  $\widetilde{H}_{N,c}(j\omega,s,L)$  of the cascade form (in blue) and the response  $\widetilde{H}_{N,p}(j\omega,s,L)$  of the parallel form (in green) on the range [100: 2000] Hz where we observe the perfect superposition of the two curves.



Figure 1.14 – Bode diagrams of the response  $\widetilde{H}_{N,C}(j\omega, s, L)$  of the cascade form (in blue) and of the response  $\widetilde{H}_{N,P}(j\omega, s, L)$  of the parallel form (in green) over the range [100: 2000] Hz

Thus, the procedure presented in this paragraph, and illustrated in the nominal case m = 0.5, must be repeated for each value of *m* considered over the interval [0; 1].

## 1.5 – Conclusion

The structure and progression of this chapter are organized in a didactic way so that readers with no idea about visco-thermal losses in wind instruments can understand the dynamic behavior of an acoustic tube of constant radius. From the two partial differential equations which define the Webster-Lokshin model, a classical resolution in the operational domain leads to the analytical expression of the acoustic impedance and admittance of the function tube of position x, its length L and its radius r.

Moreover, a system vision is proposed aiming to causally decompose the global model into sub-models, thus facilitating analysis in the frequency domain. One of the conclusions of this frequency analysis is that the fractional model can be simplified over the range [20; 20,000] Hz of the audible frequencies. In Addition, the introduction of an uncertainty at the level of the fractional order (whose value considered as nominal is that of the initial Webster-Lokshin model, namely  $m_0 = 0.5$ ) allows to study the influence of the order *m* when this varies between 0 (conservative case) and 1. Although the fractional order behavior (fractional integrator) is only present for very low non-audible frequencies (less than 1 Hz), the influence of the fractional order *m* does appear at resonances and anti-resonances (in the audible frequencies), illustrating well the phenomenon of dissipation associated with viscothermal losses. The simulation in the time domain of fractional models requires the establishment of rational forms. Thus, two rational forms composed of an integrator and *N* second-order cells, one in cascade and the other in parallel, are introduced. The parameters of the cascade form are then determined using the *Frequency Domain System Identification* (FDSI) module of the CRONE Toolbox. As for the parameters of the parallel form, they are obtained by a decomposition into simple elements of the cascade form.

More generally in the fractional model, this study of visco-thermal losses within the resonator of a wind instrument leads to a finding similar to that already made in other fields. Indeed, the main interest of the fractional form resides in the parametric parsimony, that is to say the capacity which the integral-differential operator of non-integer order has to model the greatest number dynamic phenomena with a minimum of parameters. Thus, the study of parametric sensitivity, in particular in the frequency domain, is simpler.

In the case of visco-thermal losses treated in this chapter, the study of the sensitivity is reduced to the only parameter m. The same parametric sensitivity study in the frequency domain with one of the two rational forms presented here is much more complex because this form has 4N parameters (4 parameters for each of the N cells, *i.e.* 16 for the example of illustration presented corresponding at the only value m = 0.5). The transition to one of the rational forms is then carried out only for time simulation.

## Chapter 2- Study of the Nonlinear Exciter of a Wind Musical Instrument

## 2.1 – Introduction

The self-oscillation mechanism is characterized by the ability of the instrument to produce an acoustic wave from a stationary or quasi-stationary source of energy with respect to the acoustic variables. This mechanism is highly associated with the non-linear nature of the instrument. A classic approach in musical acoustics (Kergomard & Chaigne, 2013) represents the instrument (Figure 2.1) by a nonlinear excitation system coupled to a passive linear resonant system (Chapter 1).



Figure 2.1 – Schematic representation of the mechanism of sound production in self-oscillating musical instruments

Due to its non-linear nature, modeling the exciter and its coupling with the resonator that is the energy source is very complex. Indeed, the phenomenon results from the oscillation of a naturally unstable air jet around a bevel (Figure 2.2). Thus, the self-oscillation mechanism relies on a synchronization of the oscillation of the jet and the acoustic waves. It is therefore necessary that the jet-bevel system excites the resonator at the periodicity of the acoustic field.

In order to fully understand the phenomena involved and to be able to reproduce them by numerical simulation, this chapter proposes, first of all, a modeling by the proper diagram and parameters of the various essential elements of the system, then by recalling the mechanism of sound production. Then, from a bibliographic synthesis, a complete nonlinear model frequently used in the literature is developed. Moreover, after focusing on the poor digital conditioning of such a model, a solution is proposed in order to be able to develop a digital simulator programmed in MatLab/Simulink. A scenario is then defined for several values of the pressure at the input of the mouthpiece of the flute. This scenario is defined by respecting, not only the domain of validity of the nonlinear model, but also the values of pressure achievable experimentally by the artificial mouth presented in chapter 3. Thus, for this field of study, a detailed analysis of the time responses simulated is proposed. This analysis makes it possible, in particular, to observe that the variations of a certain number of physical quantities (pressure at the input of the resonator, acoustic speed, etc.) are small and around zero. On the basis of this observation, a linearization is developed leading to two linearized models, one for the analysis of the start-up phase of the simulation (*phase 1*), the other for that of the self-oscillations (*phase 3*). Finally, the last paragraph analyzes the conditions of self-oscillation of such a system.

## 2.2 – Modelling

## 2.2.1 – Schematic and Configuration of the Exciter

Figure 2.2 (which is a more detailed representation of the flute shown in Figure 1.3 in Chapter 1) shows a cross section of a recorder, as well as a schematic representation of its exciting mechanism (consisting of the interaction of an air jet with a bevel) where the notations that appear in the figure are defined by:

- 1: outlet of the channel; 2: bevel; 3: resonator (air column); 4: exciter;
- $R = (o; \vec{a}, \vec{a})$ : local reference associated with the jet;
- *h*: height of the spout channel;
- *w*: distance between the outlet of the channel and the tip of the bevel;
- $x_0$ : lateral offset of the tip of the bevel from the longitudinal  $\vec{j}$  axis of the channel;
- $P_m(t) = P_m^e + p_m(t)$ : pressure generated at the input of the mouthpiece,  $P_m^e$  is the stationary

component at the operating point and  $p_m(t)$  the variation around  $P_m^e$ .





## 2.2.2 - Sound Production Mechanism: Reminder

From a causal point of view, the generation of a pressure input  $P_m(t)$  at the mouthpiece of the flute (imposed by the musician or by an artificial mouth device as defined in chapter 3) gives rise to a flow in the spout channel (Figure 2.2). At the exit of the channel, the viscosity of the air prevents the flow from following the geometry of the walls (Terrien S. , 2015). The consequence is that the flow takes off from the walls and gives rise to an air jet whose speed at x = 0 is denoted  $U_j(x = 0, t)$ . This air jet is unstable in nature (Kergomard & Chaigne, 2013). Indeed, if in the vicinity of the origin at x = 0 the flow is well ordered, eddies appear when the flow moves away from the starting point. Due to this unstable nature, the slightest disturbance to the jet is amplified over time (Kergomard & Chaigne, 2013).

The jet created at the channel output flows inside the excitation window. It meets the bevel, the point of which is located at a distance w from the channel output and laterally offset by  $x_0$  with respect to the longitudinal axis  $\vec{j}$  of the flow. This jet-bevel interaction causes a hydrodynamic return on the jet. This phenomenon plays an essential role in the initiation of the oscillation of the jet by providing it with its first disturbances. Due to the natural instability of the air jet, disturbances naturally amplify along this jet, from the exit of the channel to the bevel, and cause it to oscillate on either side of the bevel. This oscillation causes a flow injection alternately inside and outside the instrument. It forms the *aero-acoustic source* of pressure which supplies energy to the resonator (Kergomard & Chaigne, 2013).

The acoustic waves thus created in the resonator (chapter 1) propagate and are reflected (mainly) at the level of the first open hole. The superposition of the outward wave and the return wave gives rise to a system of standing waves in the resonator, which disturbs the return jet at the channel output, thus closing the self-oscillation loop. This initial disturbance of the jet maintains its oscillation and therefore the sound production mechanism.

# **2.2.3 -** From a Phenomenon with Distributed Parameters to a Model with Localized Parameters

The models of sound production in flutes are based on the loop system in (Figure 2.3) and involve the modeling of each of the three main phenomena:

- the instability of the jet;
- aero-acoustic sources related to the jet-bevel interaction;
- the propagation of acoustic waves in the resonator.



Figure 2.3 – Functional diagram of the loop system describing the self-oscillation mechanism of instruments of the flute family [Fabre, 2000]

The separation of the three main phenomena in operation and the fact that their interactions are supposed to be localized create a characteristic and an important limitation of this representation. This approach may seem counter-intuitive, in the sense that the boundaries between these different elements remain relatively blurred. For example, where to locate the sources of flow resulting from the oscillation of the jet?

However, the phenomena involve different scales and therefore different hypotheses. Thus, for low frequencies, corresponding to the very first acoustic resonances, the area of development of the unstable air jet is small compared to the wavelengths of the acoustic waves. The jet can then be studied under the hypothesis of incompressibility, a hypothesis which is of course no longer compatible with the description of the propagation in the resonator. As discussed by Fabre and Hirschberg (Hirschberg, 2000), it is precisely because the different phenomena are described under different assumptions (which may at first appear paradoxical) that such a representation by blocks with localized interactions is possible. For each of the blocks highlighted in the functional diagram of (Figure 2.3), the physical phenomena in action, as well as their models retained are presented in the continuation of this paragraph. The complete model thus obtained is undoubtedly the most used one in the literature (Hirschberg, 2000).

## 2.2.3.1 - Receptivity: Initial Disturbance of the Jet

Once the self-oscillations have been established, the disturbance of the jet is brought about by the acoustic field present in the resonator. Due to the non-viscous fluid hypothesis adopted for the description of the jet, this phenomenon, called *receptivity*, is considered localized at the point of separation of the jet, *i.e.* at the channel output at x = 0 (Kergomard & Chaigne, 2013). Different receptivity models are proposed in the literature, in particular by Fabre and Hirschberg (Hirschberg, 2000) and by Blanc (F Blanc, 2014). In the rest of this chapter, the model used is the one proposed by de la Cuadra (Cuadra P. d., 2005) based on Schlieren visualizations of a jet subjected to a transverse acoustic field. One of the hypotheses which reduces the domain of validity of this model is that the pressure  $P_m(t) = P_m^e + p_m(t)$  in the musician's mouth or the artificial mouth (see Chapter 3) is assumed to be stationary (or quasi-stationary), an assumption which leads to a central jet speed  $U_j(x=0,t) = U_j^e + u_j(x=0,t)$  at the exit of the channel also stationary. This is the reason why in the rest of this chapter, we consider that:

$$\begin{cases} P_m(t) \approx P_m^e \\ U_j(x=0,t) \approx U_j^e \end{cases}, \tag{2.1}$$

where, according to Bernoulli's law in stationary system at the channel exit (Cuadra P. d., 2005),

$$U_j^e = \sqrt{\frac{2 P_m^e}{\rho}} \quad , \tag{2.2}$$

 $\rho$  being the density of air.

Thus, for this domain of validity, the initial disturbance of the jet at x = 0 at the exit of the channel is represented by a transverse displacement  $\eta_0(x = 0, t)$ , noted  $\eta_0(t)$ , linked to the acoustic speed  $v_{ac}(t)$  of the resonator always at x = 0 by a relation of the form:

$$\eta_0(t) = \frac{h}{U_j^e} v_{ac}(t) . \qquad (2.3)$$

## 2.2.3.2 - Jet Instability: Amplification and Convection of Disturbances

As noted above, the air jet that propagates between the channel output and the bevel is naturally unstable. This instability of the jet, known under the name of Kelvin-Helmholtz instability (Reid, 2002), results from the natural instability of shear layers animated at different speeds, that is to say of the interfaces between the jet and the external environment at rest. The natural instability of the jet results in amplification and convection of the slightest disturbance. Rayleigh (Rayleigh, 1984) first proposed a linear description of this phenomenon in the case of small perturbations of a semi-infinite jet in the form of an exponential amplification of the perturbations with the convection distance.

In the flute family instruments, the jet develops inside an excitation window limited at one end by the channel at x = 0 and at the other by the bevel with which it interacts at x = w. An experimental study by de la Cuadra (Cuadra P. d., 2005) showed that the exponential amplification of the initial disturbance  $\eta_0(t)$  with the convection distance x remained a reasonable approximation. In the case of the flute, at the level of the bevel at x = w, we

therefore obtain the transverse displacement  $\eta(w, t)$  of the disturbance whose expression is given by:

$$\eta(x = w, t) = \eta_0(t - \tau) \exp(\alpha_i w) , \qquad (2.4)$$

where, still according to de la Cuadra (Cuadra P. d., 2005), the estimate of the amplification factor  $\alpha_i$  as a function of the height *h* of the channel is of the order:

$$\alpha_i \approx \frac{0.4}{h} . \tag{2.5}$$

The delay  $\tau$  introduced in equation (2.4) is related to the convection time of the initial disturbance  $\eta_0(t)$  along the jet, from the exit of the channel at x = 0 to the wedge at x = w. Various experimental results (Cuadra, Vergez, & Fabre, 2007) (Nolle, 1998), as well as the theoretical work of Rayleigh (Rayleigh, 1984) have shown that in a steady state the convection speed  $C_v^e$  of transverse disturbances along a jet is linked to the difference in speed between the jet and the external environment is in general  $C_v^e \in \left[0.3 U_j^e; 0.5 U_j^e\right]$ . In this chapter, the mean value  $C_v^e = 0.4 U_j^e$  is adopted, hence the expression of the delay  $\tau$ :

$$\tau = \frac{w}{0.4 U_j^e} \ . \tag{2.6}$$

### 2.2.3.3 - Jet-Bevel Interaction: Aero-Acoustic Sources

In wind instruments, the oscillation of the jet around the bevel acts as an exciter, producing the acoustic energy transmitted to the resonator. While historically this source was first described by Helmholtz using a monopole, the dipole modeling, initiated by Rayleigh (Rayleigh, 1984), is now used. The jet-drive model, initially proposed by Coltman (Coltman J., 1976), then by Verge (Verge M., 1995) (Verge, Hirschberg, & Caussé, 1997) is based on this principle of representing the source term by a force. Figure 2.4 schematically illustrates the behavior of the jet.



Figure 2.4 – Schematic representation of the behavior of the jet, according to Fabre in [Chaigne, 2013]: (a) disturbance of the jet at the outlet of the channel by the acoustic field present in the resonator; (b) convection and amplification of the disturbance (c) oscillation of the jet around the bevel which gives rise to the aero-acoustic sources

Upon meeting the bevel (Figure 2.4.a), the jet subjected to a transverse displacement  $\eta(w,t)$  oscillates on either side of this bevel (Figure 2.4.b). The flow rate linked to the jet is then divided into two flow rates: the first  $Q_{in}(t)$  enters the resonator, while the second  $Q_{out}(t)$  leaves the resonator (Figure 2.4.c). These two flow sources in phase opposition are considered to be located at two points close to the bevel and separated by an equivalent distance  $\delta_d$ . From a calculation of a potential difference, Verge (Verge M., 1995) estimated:

$$\delta_d \approx \frac{4}{\pi} \sqrt{2 h w} . \tag{2.7}$$

Since this equivalent distance  $\delta_d$  is small compared to the acoustic wavelengths, the mass of air between the two flow injection points can be considered incompressible. The alternating sloshing of this air mass, induced by the oscillation of the jet, exerts a force on the acoustic field, which is modeled by a pressure difference, noted  $\Delta p_{src}(t)$ , whose expression is given by (Terrien S. , 2015):

$$\Delta p_{src}(t) = \left(\frac{\rho \,\delta_d \, b \, U_j^e}{w}\right) \, \frac{d}{dt} \left[ \tanh\left(\frac{\eta(w,t) - x_0}{b}\right) \right], \qquad (2.8)$$

where *b* represents the estimated half-thickness of the jet, according to Verge (M P Verge B. F., 1994) and Ségoufin (Ségoufin C. , 2000), by:

$$b = \frac{2}{5}h$$
, (2.9)

and where *tanh(.)* is the hyperbolic tangent function.

Moreover, the excitation window of the flute between the channel output at x = 0 and the bevel at x = w, from the point of view of the resonator, constitutes an open end, but also a constriction. In fact, its section is most of the time smaller than the section of the bore of the instrument. The acoustic speeds observed in this zone can therefore be high and most often cannot be neglected in relation to the speed of the jet. The presence of a sharp edge (the bevel), combined with the effects of viscosity, results in a separation of the flow at this point (Kergomard & Chaigne, 2013). This then causes detachments of vortices at the level of the bevel, as demonstrated experimentally by Fabre and Hirschberg (B Fabre, 1996).

Taking into account this phenomenon linked to energy loss in the instrument models appears to be decisive for correctly describing the mechanisms of saturation of the oscillation amplitude (B Fabre, 1996) (R Auvray, 2012). If different models have been proposed to account for this phenomenon (B Fabre, 1996), an approach often retained (mainly because of its simplicity (Kergomard & Chaigne, 2013)) consists in modeling it by an additional pressure difference  $\Delta p_{los}(t)$  between the two sides of the bevel, namely:

$$\Delta p_{los}(t) = -\frac{\rho}{2} \left( \frac{v_{ac}(t)}{\alpha_{vc}} \right)^2 \operatorname{sign} \left[ v_{ac}(t) \right], \qquad (2.10)$$

where  $\alpha_{vc}$  is a factor corresponding to an effect of *vena contracta* estimated at 0.6 in the case of a sharp arrest (Falkovich, 2011) and where sign represents the sign function.

Finally, the aero-acoustic pressure source  $\Delta p(t)$  at the bevel is given by (Terrien S., 2015):

$$\Delta p(t) = \Delta p_{src}(t) + \Delta p_{los}(t) . \qquad (2.11)$$

#### 2.2.3.4 – Complete Model

The complete model of the instrument retained within the framework of this study is finally composed of a system of three equations, each one being linked to a given element of the looped system shown in (Figure 2.5), namely:

$$\eta(w,t) = \frac{h \exp(\alpha_i w)}{U_j^e} v_{ac}(t-\tau)$$

$$\Delta p(t) = \left(\frac{\rho \,\delta_d \, b \, U_j^e}{w}\right) \frac{d}{dt} \left[ \tanh\left(\frac{\eta(w,t) - x_0}{b}\right) \right] - \frac{\rho}{2} \left(\frac{v_{ac}(t)}{\alpha_{vc}}\right)^2 \operatorname{sign}\left[v_{ac}(t)\right] ,(2.12)$$

$$v_{ac}(t) = \frac{1}{S} \operatorname{TL}^{-1}\left\{Y_{in}(s,L) \ \Delta P(s)\right\}$$
with  $\tau = \frac{w}{0.4 \, U_j^e}$ 

where S and L represent the section and the length of the resonator,  $Y_{in}(s,L)$  its input admittance (defined in Chapter 1) and  $\Delta P(s) = \text{TL}\{\Delta p(t)\}$ , TL designating the Laplace Transform.



Figure 2.5 – Functional diagram of the complete model

## 2.2.3.5 - Limits of the Model: Reminder of the Validated Domain

The capacity of the complete model to qualitatively reproduce many phenomena observed experimentally has been demonstrated in various works (Auvray, Fabre, & Lagrée, 2012) (S Terrien, 2012). However, like any model, this one is only valid in an area that must be taken into account when interpreting the results.

The linear description of the jet instability is theoretically valid only for small amplitudes of transverse deflection  $\eta(x,t)$ . When the value of  $\eta(x,t)$  becomes close to the thickness of the jet, the jet rolls up on itself giving rise to a series of vortices which can be modeled as a Von Karman alley (Kergomard & Chaigne, 2013). The model described by the system of equations (2.12) does not take this nonlinear phenomenon into account.

On the other hand, when the jet speed  $U_j^e$  becomes sufficiently high, a transition to a turbulent state can be observed. As a reminder (Terrien S., 2015), the transition zone from a laminar (linear) flow to a turbulent (nonlinear) flow is located for a Reynolds number

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between 2500 and 3000, knowing that the Reynolds number represents the ratio between inertial forces and viscous forces. According to Fabre (Kergomard & Chaigne, 2013), in the case of a turbulent flow at the outlet of the channel, the modeling should take into account various elements that are not described in this model.

Finally, the model is well suited (Terrien S., 2015) for:

- low values of the number of Strouhal  $\theta$  defined by:

$$\theta = \frac{w}{U_j^e} f_0 , \qquad (2.13)$$

where  $f_0$  represents the self-oscillation frequency in Hz of the waves within the resonator; - jets with a thin thickness compared to the length w;

- rather large hydrodynamic wavelengths.

## 2.3 – Numerical Simulation

## 2.3.1 – Digital Packaging Problem

The complete model, as defined by the system of equations (2.12), is poorly conditioned from a numerical point of view. Indeed, the presence of a derivation in the second equation of the system (2.12) reveals a causality problem. Moreover, assuming that the derivation can be programmed without posing a numerical problem, if the initial conditions associated with the variables  $\eta(w,t)$ ,  $v_{ac}(t)$  and  $\Delta p(t)$  are zero (which is the case physically as long as the pressure  $P_m(t)$  at the mouthpiece input is zero), then there is no generating process to initiate the start of a transient system which must then make it possible to reach a steady periodic system imposed by the self-oscillation conditions. Finally, the presence of the flow speed  $U_j(t)$  at the denominator of the first equation of the system (2.12) excludes any zero initial value of this speed at the start of the digital simulation, and therefore of the pressure  $P_m(t)$  at the entry of the mouthpiece of the flute, a situation however very physically realistic.

The solution adopted in this thesis concerning the derivation is the use of a frequency truncated differentiator whose expression D(s) in the symbolic domain is defined by:

$$D(s) = \frac{s}{\left(1 + \frac{s}{\omega_c}\right)^{n_h}},$$
(2.14)

 $\omega_c$  being the cutoff angular frequency (in rad/s) and  $n_h$  the order of the low-pass filter. The useful effect of the low-pass filter is to prevent D(s) from tending to infinity as *s* tends to infinity (i.e. the first instants taking into account the applicable time-frequency duality to this linear operator). The parasitic phase shift effect is as important as the order  $n_h$ . Faced with this dilemma, a good compromise is  $n_h = 2$  which leads to:

$$\lim_{s \to \infty} D(s) \to 0 .$$
 (2.15)

This value of  $n_h$  must be combined with a cut-off angular frequency  $\omega_c$  compared to the largest natural un-damped angular frequency  $\omega_{pi}$  of the rational form of the impedance  $Y_{in}(s,L)$  as defined in Chapter 1.

It is important to note that the choice of this operator makes it possible, not only to solve the numerical problem related to the derivation, but also to introduce a generator process allowing, in the presence of zero initial conditions, to initially trigger the start of a transient system then leading to a stationary periodic system (self-oscillation conditions). The following paragraph illustrates and then demonstrates this result.

#### 2.3.2 - Simulator Developed using MatLab / Simulink

The complete model in the presence of operator D(s) is programmed in MatLab / Simulink with the following parametric values:

| $\rho = 1.184 \text{ kg/m}^3$                    | density of air at 25°C.   |
|--|---|
| p 1.10 ( Kg/III )                                |   |
| $v = 15.6 \times 10^{-6} \text{ m}^2/\text{s}$ : | kinematic viscosity of air at 25°C;   |
| $h = 10^{-3}$ m:                                 | height of the spout channel;  |
| $w = 4.25 \times 10^{-3} \text{ m}$ :            | distance between the outlet of the nozzle channel and the tip of the            |
|  | bevel;  |
| $b = 0.4 \text{x} 10^{-3} \text{ m}$ :           | half-thickness of the jet;  |
| $\delta_d = 3.7 \mathrm{x} 10^{-3} \mathrm{m}$ : | equivalent distance between the 2 sources of flow $Q_{in}$ and $Q_{out}$ at the |
|  | level of the bevel  |
| $\alpha_{vc} = 0.6$ :                            | effect factor vena contracta;   |
| $x_0 = 0.1 \times 10^{-3} \text{ m}$ :           | lateral offset of the tip of the bevel from the longitudinal axis of the        |
|  | channel;  |
| L = 0.3  m:                                      | length of the resonator tube;   |
| $r = 5 \times 10^{-3}$ m:                        | radius of the resonator tube;   |
| $S = 7.854 \text{x} 10^{-5} \text{ m}^2$ :       | section of the resonator tube;  |
| $n_h = 2$ :                                      | order of the low-pass filter of the operator $D(s)$ ;                           |
| $\omega_c = 2\pi x 50\ 000\ rad/s$               | : cutoff angular frequency of the low-pass filter of the operator $D(s)$ .      |

As a reminder (see Chapter 1), Figure 2.6 shows the frequency response of the input admittance  $Y_{in}(s,L)$  of the resonator on the range [100; 2000] Hz and Table 2.1 gives the numerical values of the parameters  $A_i$ ,  $B_i$ ,  $\omega_{pi}$ ,  $\zeta_{pi}$  and  $Q_{pi}$  of its parallel rational form for the N = 4 first modes of the resonator.



Figure 2.6 – Frequency response of the input admittance *Yin(s, L)* of the resonator considered over the range [100; 2000] Hz.

Table 2.1 – Numerical values of the parameters  $A_i$ ,  $B_i$ ,  $a_{pi}$ ,  $\zeta_{pi}$  and  $Q_{pi}$  of the parallel rational form of the input admittance  $Y_{in}(s,L)$  for the N = 4 first modes of the resonator

| N | $A_i$                   | $\boldsymbol{B}_i$      | $\boldsymbol{\omega}_{pi}$ (rad/s) | $\zeta_{pi}$          | $Q_{pi}$ |
|---|-------------------------|-------------------------|------------------------------------|-----------------------|----------|
| 1 | 36.02 10 <sup>-12</sup> | 16.63 10 <sup>-10</sup> | 2π 570                             | 12.9 10 <sup>-3</sup> | 38.76    |
| 2 | 98.24 10 <sup>-13</sup> | 64.99 10 <sup>-11</sup> | 2π 1145                            | 9.2 10 <sup>-3</sup>  | 54.35    |
| 3 | 53.38 10 <sup>-13</sup> | 43.07 10 <sup>-11</sup> | 2π 1720                            | 7.47 10 <sup>-3</sup> | 66.93    |
| 4 | 58.98 10 <sup>-13</sup> | 54.47 10 <sup>-11</sup> | $2\pi  2309$                       | 6.37 10 <sup>-3</sup> | 78.5     |

## **Remark 2.1**

With a ratio  $\omega_c/\omega_{p4} = 21.6$  the cut-off angular frequency  $\omega_c = 2\pi \times 50,000$  rad/s of the low-pass filter of the operator D(s) is sufficiently large compared to the largest natural nondamped angular frequency  $\omega_{p4} = 2\pi \times 2,309$  rad/s of the rational form of the impedance  $Y_{in}(s,L)$ . Thus for times  $t >> \tau_c = 1/\omega_c = 3.18 \times 10^{-6}$  s, the two differentiation operators, D(s) and s have the same behavior.

## **Remark 2.2**

In the dynamics of linear systems, in the general case of a polynomial of order 2 having 1 pair of conjugate complex roots, the natural undamped angular frequency  $\omega_n$  and the resonance angular frequency  $\omega_r$  are linked by a relation of the form  $\omega_r = \omega_n \sqrt{1-2\zeta^2}$ where  $\zeta$  represents the associated damping factor. In acoustic instruments, and in particular in the particular case of wind instrument resonators, the damping factors  $\zeta_{pi}$  are very small compared to unity (see table 2.1). This is the reason why the natural un-damped angular frequencies  $\omega_{pi}$  and the resonance angular frequencies  $\omega_{ri}$  associated with each of the i modes of the resonator are almost equal, which is to say that the ratio  $\omega_{ri}/\omega_{ni} = \sqrt{1-2\zeta_{pi}^2}$  is close to unity. For more illustration, table 2.2 gives the numerical values of this ratio for the first 4 modes of the resonator considered in table 2.1.

| N | $\zeta_{pi}$          | ω <sub>ri</sub> /ω <sub>ni</sub> |
|---|-----------------------|----------------------------------|
| 1 | 12.9 10 <sup>-3</sup> | 0.9998                           |
| 2 | 9.2 10 <sup>-3</sup>  | 0.9999                           |
| 3 | 7.47 10 <sup>-3</sup> | 0.9999                           |
| 4 | 6.37 10 <sup>-3</sup> | 1                                |

Table 2.2 – Numerical values of the ratio for the first 4 modes of the resonator considered

Figure 2.7 shows the parametric configuration chosen for the Simulink solver.

| Simulation time  |  |
|--|--|
| Start time: 0.0  | Stop time: tmax  |
| Salvar options   |  |
|  |  |
| Type: Fixed-step                                       | Solver: ode3 (Bogacki-Shampine)  |
| Fixed-step size (fundamental sample time):             | 1e-6   |
|  |  |
| lasking and sample time options                        |  |
| Periodic sample time constraint:                       | Unconstrained •  |
| Tasking mode for periodic sample times:                | Auto   |
| Automatically handle rate transition for data transfer |  |
| Deterministic data transfer:                           | Whenever possible •  |
| ✓ Higher priority value indicates higher task priority |  |
|  | Simulation time         Start time:       0.0         Solver options         Type:       Fixed-step         Fixed-step size (fundamental sample time):         Tasking and sample time options         Periodic sample time constraint:         Tasking mode for periodic sample times:         Image: Automatically handle rate transition for data transfer         Deterministic data transfer:         Image: Higher priority value indicates higher task priority |

Figure 2.7 – Parametric configuration retained for the Simulink solver

The initial conditions of the various simulations carried out are:

$$\begin{cases}
P_m(0^+) = P_m^e \implies U_j(0^+) = U_j^e \\
v_{ac}(0^+) = 0 \\
\eta(w, 0^+) = 0 \\
\vdots \qquad \vdots 0 \implies \text{ thanks to the presence of } D(s) \\
\Delta p(0^+) = 0
\end{cases}$$
(2.16)

and this for 13 different values of the pressure  $P_m^e$  between 400 Pa and 1000 Pa, with a step of 50 Pa. For each of its simulations, three phases appear:

- phase 1: start-up with a convergent transient system;

- phase 2: divergent transient system;
- *phase 3:* stationary periodic system.

Among these 13 tests, two (corresponding to the limits of the membership interval of  $P_m^e$ ) are presented and analyzed by way of illustration:

- example 1:  $P_m^e = 400 \text{ Pa}$ ,  $U_j^e = 26 \text{ m/s}$  and  $\tau = 0.409 \text{ ms}$ ;

- example 2:  $P_m^e = 1000 \text{ Pa}$ ,  $U_i^e = 41 \text{ m/s}$  and  $\tau = 0.259 \text{ ms}$ .

For each of these two examples, three families of time responses are proposed for:

- the total duration of the simulation from  $t_0$  to  $t_{max}$ , with  $t_0 = 0$  s and  $t_{max} = 1$  s;
- phase 1 from  $t_0$  to  $t_1$ ;
- phase 3 from  $t_2$  to  $t_{max} = 1$  s.

Figure 2.8, for a pressure  $P_m^e = 400$  Pa (on the left) and  $P_m^e = 1000$  Pa (on the right), shows the time responses of the pressure  $\Delta p(t)$  at the input of the resonator ((a) and (b)), the acoustic velocity  $v_{ac}(t)$  ((c) and (d)) and the transverse displacement  $\eta(w,t)$  of the jet ((e) and (f)). Observing the results allows us to make the following remarks:

- the duration of *phase 1* depends on the value of the delay  $\tau$  and therefore on the value of  $P_m^e$ . Thus, the lower the delay  $\tau$  (that is, the greater the  $P_m^e$  pressure), the shorter *phase 1*. Focusing on  $P_m^e = 400$  Pa (left), the convergent transient system of *phase 1* is not visible for  $P_m^e = 1000$  Pa (right) because the divergent transient system of *phase 2* quickly overlaps;

- The shorter the duration of *phase 2* is the greater the pressure  $P_m^e$ , thus leading to the onset of the steady-state periodic system of *phase 3* more quickly.

Taking into account these remarks, Figure 2.9 presents, only for a pressure  $P_m^e = 400$  Pa, the time responses obtained during **phase 1** of the pressure  $\Delta p(t)$  at the input of the resonator (a), of the acoustic speed  $v_{ac}(t)$  (b) and the transverse displacement  $\eta(w,t)$  of the jet (c). A detailed analysis of this **phase 1** is proposed in paragraph **2.4** - Linearized model.



Figure 2.8 – Time responses obtained for a pressure = 400 Pa (on the left) and = 1000 Pa (on the right) concerning: - the pressure  $\Delta p(t)$  at the input of the resonator (a) and (b); - the acoustic speed  $v_{ac}(t)$  (c) and (d); - the transverse displacement  $\eta$  (w, t) of the jet (e) and (f)



Figure 2.9 – Time responses obtained in phase 1 for a pressure = 400 Pa concerning: - the pressure  $\Delta p(t)$  at the input of the resonator (a); - the acoustic speed  $v_{ac}(t)$  (b); - the transverse displacement  $\eta$  (w, t) of the jet (c)

Figure 2.10, for Example 1, and Figure 2.11, for Example 2, show the time responses obtained during phase 3:

- (a) the pressure  $P_m^e$  (in blue) and the pressure  $\Delta p(t)$  at the input of the resonator (in green);
- (b) speed  $U_i^e$  (in blue) and acoustic speed  $v_{ac}(t)$  (in green);
- (c) transverse displacements  $\eta_0(t)$  (in blue),  $\eta_0(t-\tau)$  (in green) and  $\eta(w,t)$  (in red);
- (d) pressures  $\Delta p(t)$  (in black),  $\Delta p_{src}(t)$  (in red) and  $\Delta p_{los}(t)$  (in blue).

The observation of these responses leads to the following remarks:

- Figures 2.10.a and 2.11.a: the pressure  $\Delta p(t)$  (in green) at the input of the resonator (relation (2.11)), which oscillates around 0, has a low amplitude compared to the pressure  $P_m^e$  (in blue);

- Figures 2.10.b and 2.11.b: at the level of the channel output at x = 0, same remark for the acoustic speed  $v_{ac}(t)$  (in green) which oscillates around 0 and has a low amplitude compared to flow velocity  $U_j^e$  (in blue);

- Figures 2.10.c and 2.11.c: the instability of the jet (which results in an amplification of its transverse displacement  $\eta(x,t)$  (relation (2.4)) appears clearly by comparing the amplitudes of  $\eta(x=w,t)$  (in red) and  $\eta(x=0,t-\tau) = \eta_0(t-\tau)$  (in green);

- Figures 2.10.d and 2.11.d: the pressure  $\Delta p_{los}(t)$  (in blue) linked to the loss of energy at the level of the bevel (relation (2.10)) remains low compared to the pressure  $\Delta p(t)$  (in black) at the input of the resonator (relation (2.11)).



Figure 2.10 – Time responses obtained for a pressure = 400 Pa: (a) pressure (in blue) and pressure  $\Delta p(t)$  at the input of the resonator (in green); (b) speed (in blue) and acoustic speed  $v_{ac}(t)$  (in green); (c)



transverse displacements  $\eta_{\theta}(t)$  (in blue),  $\eta_{\theta}(t-\tau)$  (in green) and  $\eta(w,t)$  (in red) (d) pressures  $\Delta p(t)$  (in black),  $\Delta p_{src}(t)$  (in red) and  $\Delta p_{los}(t)$  (in blue)

#### (d) pressures $\Delta p(t)$ (in black), $\Delta p_{src}(t)$ (in red) and $\Delta p_{los}(t)$ (in blue)

### 2.4 – Linearized Models

In order to facilitate the analysis of *phase 1* (start of the simulation) and *phase 3* (self-oscillation conditions), a linearization is developed leading to two linearized models, one for the analysis of the *phase 1*, the other for that of *phase 3*. Note that in *phase 3*, that is to say for time  $t >> \tau_c = 1/\omega_c = 3.18 \times 10^{-6} s$  (time constant of the low pass filter, see *Remark 2.1*) the two linearized models exhibit the same behavior.

Thus, the observation of the simulations presented in the previous paragraph allows us to affirm that the variations of  $\eta(w,t)$ ,  $v_{ac}(t)$  and  $\Delta p(t)$  are indeed around zero. This is the reason why the linearization of the system of equations (2.12) is done around this value.

More precisely, a first non-linearity appears in expression (2.8) of  $\Delta p_{src}(t)$  with the function tanh(.) and a second non-linearity appears in expression (2.10) of  $\Delta p_{los}(t)$  with the

acoustic speed  $v_{ac}(t)$  squared. The expansion limited to the first order of the function tanh(.) around  $\eta = 0$  leads to:

$$\tanh\left(\frac{\eta(w,t)-x_0}{b}\right)\Big|_{\eta=0} \approx \tanh\left(-\frac{x_0}{b}\right) + \left(\frac{1-\left(\tanh\left(-\frac{x_0}{b}\right)\right)^2}{b}\right)\eta(w,t), \quad (2.17)$$

form relation

$$\tanh\left(\frac{\eta(w,t)-x_0}{b}\right)\Big|_{\eta=0} \approx c_0 + c_1 \eta(w,t) , \qquad (2.18)$$

taking

$$\begin{cases} c_0 = \tanh\left(-\frac{x_0}{b}\right) \\ c_1 = \frac{1 - \left(\tanh\left(-\frac{x_0}{b}\right)\right)^2}{b} \end{cases}, \qquad (2.19)$$

 $c_0$  and  $c_1$  being constants.

As for the first-order limited expansion of  $[v_{ac}(t)]^2$  around 0, its expression is given by:

$$\left[ v_{ac}(t) \right]^{2} \Big|_{v_{ac}=0} \approx 0 + 2 \times 0 \times v_{ac}(t) \approx 0 \quad .$$
(2.20)

From these developments limited to the first order, two linear models are proposed:

- a consideration in the time domain of the derivation operator d/dt, called the *theoretical linearized model;* 

- the other considering in the operational field the derivative operator D(s), called the *programmed linearized model*.

#### 2.4.1 – Theoretical Linearized Model

The derivative with respect to time of relation (2.17) gives:

$$\frac{d}{dt} \left[ \tanh\left(\frac{\eta(w,t) - x_0}{b}\right) \right] \approx \left( \frac{1 - \left(\tanh\left(-\frac{x_0}{b}\right)\right)^2}{b} \right) \frac{d}{dt} \eta(w,t) .$$
(2.21)

Thus, the expression (2.8) of  $\Delta p_{src}(t)$  linearized around 0 is given by:

$$\Delta p_{src}(t) \approx \left(\frac{\rho \,\delta_d \, b \, U_j^e}{w}\right) \left(\frac{1 - \left(\tanh\left(-\frac{x_0}{b}\right)\right)^2}{b}\right) \frac{d}{dt} \eta(w, t) , \qquad (2.22)$$

or again, after simplification by b,

$$\Delta p_{src}(t) \approx K_1 \ \frac{d}{dt} \eta(w, t) , \qquad (2.23)$$

taking,

$$K_1 = \frac{\delta_d}{w} \rho \left( 1 - \left( \tanh\left(-\frac{x_0}{b}\right) \right)^2 \right) U_j^e , \qquad (2.24)$$

 $K_l$  being a constant.

Finally, the linearization of the system of equations (2.12) around 0 leads to:

$$\begin{cases} \eta(w,t) = K_0 \quad v_{ac}(t-\tau) \\ \Delta p(t) \approx K_1 \quad \frac{d}{dt} \eta(w,t) \\ v_{ac}(t) = \frac{1}{S} \quad \text{TL}^{-1} \{Y_{in}(s,L) \quad \Delta P(s)\} \end{cases}$$
(2.25)

where  $K_0 = h \exp(\alpha_i w) / U_j^e$  is a constant. The Laplace transform of the system (2.25), under the assumption of zero initial conditions, is then given by:

$$\begin{cases} \overline{\eta}(w,s) = K_0 e^{-\tau s} V_{ac}(s) \\ \Delta P(s) = K_1 s \overline{\eta}(w,s) \\ V_{ac}(s) = \frac{1}{S} Y_{in}(s,L) \Delta P(s) \end{cases}$$
(2.26)

where  $\overline{\eta}(w,s) = \mathrm{TL}\{\eta(w,t)\}.$ 

This *theoretical linearized model* is used in section 2.5 for the analysis of the autooscillation conditions (*phase 3*).

#### 2.4.2 Programmed Linearized Model

As specified in paragraph 2.3.1 - Numerical conditioning problem, the solution adopted to solve the causality problem lies in the use of a derivation operator D(s) as defined by the relation (2.14) with  $n_h = 2$  and  $\omega_c = 2\pi$  50 000 rad/s. Thus, the application in the symbolic domain of this derivation operator D(s) to relation (2.17) leads to an expression of the pressure  $\Delta P(s)$  of the form:

$$\Delta P(s) = K_1^* D(s) \left( \frac{c_0}{s} + c_1 \,\overline{\eta}(w, s) \right), \qquad (2.27)$$

where  $K_1^* = \frac{\rho \, \delta_d \, b}{w} U_j^e$  is a constant.

By distributing the operator D(s), we show that the pressure  $\Delta P(s)$  (relation (2.27)) is composed of the sum of two terms,  $\Delta P_0(s)$  and  $\Delta P_1(s)$ , that is:

$$\Delta P(s) = \Delta P_0(s) + \Delta P_1(s) , \qquad (2.28)$$

taking,

$$\begin{cases} \Delta P_0(s) = D(s) \frac{C_0}{s} \\ \Delta P_1(s) = C_1 D(s) \overline{\eta}(w, s) \end{cases}, \qquad (2.29)$$

with,

where it is

the station

$$\begin{cases} C_0 = c_0 K_1^* = \left(c_0 \frac{\rho \, \delta_d \, b}{w}\right) U_j^e \\ C_1 = c_1 \, K_1^* = \left(c_1 \frac{\rho \, \delta_d \, b}{w}\right) U_j^e \end{cases}, \qquad (2.30) \end{cases}$$
  
important to note that the two constants  $C_0$  and  $C_1$  are proportional to the value of ary speed  $U_j^e$  and therefore (taking into account the relation (2.2)) to the value of

the pressure  $P_m^e$ . Thus,  $\Delta p_0(t) = TL^{-1}\{\Delta P_0(s)\}$  represents the step response of the operator D(s) to a step of amplitude  $C_0$  (proportional to  $P_m^e$ ), or again, to the extent that

$$\frac{D(s)}{s} = \frac{1}{\left(1 + \frac{s}{\omega_c}\right)^{n_h}},$$
(2.31)

 $\Delta p_0(t)$  also represents the impulse response of the low-pass filter of order  $n_h = 2$  multiplied by the constant  $C_0$ , that is

$$\Delta p_0(t) = C_0 T L^{-1} \left\{ \frac{1}{\left(1 + \frac{s}{\omega_c}\right)^{n_h}} \right\} = C_0 \frac{\omega_c^{n_h}}{(n_h - 1)!} t^{(n_h - 1)} e^{-\omega_c t} .$$
(2.32)

For a pressure  $P_m^e = 400$  Pa and for the first moments of **phase 1**, Figure 2.12 presents the time responses of the pressure  $\Delta p_0(t)$  obtained with the relation (2.32) (curve 1 in green), the nonlinear model (curve 2 in red) and the linearized model (curve 3 in blue).

Thus, in the first moments of *phase 1* where the variations are of low amplitude, the responses of the linearized model (curve 3 in blue) are identical to those of the nonlinear model (curve 2 in red). These answers clearly illustrate the legitimacy of the analysis made in

the first moments using the linearized model which leads to the analytical expression (2.32) of the pressure signal (curve 1 in green).



Figure 2.12 – For a pressure equal to 400 Pa and for the first moments of phase 1, time responses of the pressure ∠p₀(t) obtained with:

relation (2.30) (curve 1 in green);
the nonlinear model (curve 2 in red);
the linearized model (curve 3 in blue)

Figure 2.13 presents the time responses obtained for a pressure  $P_m^e = 400$  Pa with the linearized model (in blue) and the nonlinear model (in red) during the first instants of **phase 1** (start-up) over a time interval [0; 5] x 10<sup>-5</sup> s (left) and [0; 5] x 10<sup>-4</sup> s (right), time responses of the pressure  $\Delta p(t)$  at the input of the resonator ((a) and (b)), of the acoustic speed  $v_{ac}(t)$  ((c) and (d)) and the transverse displacement  $\eta(w,t)$  ((c) and (d)) at the level of the bevel at x = w.

Observation of these responses allows us to make the following remarks:

- the pressure angular frequency  $\Delta p_0(t)$  as defined by relation (2.32) clearly appears from t = 0 s to  $t = 3x10^{-5}$  s with a maximum amplitude of 310 Pa (Figure 2.13.a). This pressure angular frequency at the input of the resonator is at the origin of the appearance of the response of the acoustic speed  $v_{ac}(t)$  (Figure 2.13.c) and of the transverse displacement  $\eta(w,t)$  of the jet in x = w (Figure 2.13.e);

- from  $t = 3x10^{-5}$  s to  $t = 4x10^{-4}$  s, the pressure  $\Delta p_0(t)$  is zero (the pressure angular frequency is over, Figure 2.13.b), while the transient system of the acoustic speed  $v_{ac}(t)$  (Figure 2.13.d) and the transverse displacement  $\eta(w,t)$  is not finished (Figure 2.13.f);

- from  $t = 4 \times 10^{-4}$  s (value of the order of the delay  $\tau = 0.4$  ms), under the effect of the looping of the system, a transient system associated with the pressure  $\Delta p_0(t)$  appears (Figure 2.13 .b) leading to an additional transient system for the acoustic speed  $v_{ac}(t)$  (Figure 2.13.d) and the

transverse displacement  $\eta(w,t)$  (Figure 2.13.f), transient system which is superimposed on that still in course resulting from the initial pressure angular frequency.



Figure 2.13- Time responses obtained for a pressure = 400 Pa with the linearized model (in blue) and the non-linear model (in red) during the first moments of phase 1 (start-up) over a time interval [0; 5]x10<sup>-5</sup> sec (left) and [0; 5]x10<sup>-4</sup> s (right): - pressure  $\Delta p(t)$  at the input of the resonator ((a) and (b));

- acoustic speed  $v_{ac}(t)$  ((c) and (d));
- transverse displacement  $\eta(w,t)$  ((c) and (d))

## 2.5 – Analysis of Auto-Oscillation Conditions

Figure 2.14 shows the variation in flow velocity  $U_i^e$  and the variation in frequency  $f_0$ 

(in Hz) of *phase 3* self-oscillations as a function of pressure  $P_m^e$ . For constant pressures  $P_m^e$  included:

- between 400 Pa and 550 Pa, the average frequency  $f_0$  of the self-oscillations is 569.6 Hz, lower value and close to the resonance frequency  $f_{rl}$  of the 1<sup>st</sup> mode of the resonator (Figure 2.6);

- between 600 Pa and 1000 Pa, the average frequency  $f_0$  of the self-oscillations is 1130 Hz, a value also lower and close to the resonance frequency  $f_{r2}$  of the 2<sup>nd</sup> mode of the resonance (Figure 2.6).



Figure 2.14 - Variations in the flow speed of the jet at the outlet of the channel (a) and the frequency  $f_0$  of the self-oscillations (b) as a function of the pressure

The remainder of this paragraph is devoted to the detailed analysis of the stationary periodic system present during *phase 3*.

#### 2.5.1 - Hydrodynamic Modes of the Jet and Acoustic Modes of the Resonator

The sound production mechanism is linked to the coupling between the jet and the acoustic resonator. Different *playing modes* exist according to the hydrodynamic k and acoustic *i* modes involved. Indeed, like the acoustic resonator (which has several resonance modes), the jet can oscillate on different *hydrodynamic modes*, shown in Figure 2.15.



between the outlet of the channel and the bevel;

for the second k = 2 (b) and third k = 3 (c) hydrodynamic modes, we observe respectively 3/2 and 5/2 wavelengths over this same distance.

The *normal* functioning of the instrument corresponds to the coupling of an acoustic mode i with the first hydrodynamic mode k = 1 of the jet (M P Verge B. F., 1997). In this case, we observe between the channel output and the bevel about 1/2 hydrodynamic wavelength. The coupling of this hydrodynamic mode with each of the acoustic modes of the resonator gives rise to the *registers* of the instrument. Thus, the  $i^{th}$  register corresponds to the coupling of this hydrodynamic mode k = 1 of the jet with the  $i^{th}$  acoustic mode of the resonator. The appearance of one register or another is linked to the choice of parameters controlled by the musician and set by the instrument maker.

In certain particular cases, the coupling of an acoustic mode with a hydrodynamic mode of order k greater than 1 is observed, and gives rise to a *wind system* (Kergomard & Chaigne, 2013) (Meissner, 2001) (Terrien, Vergez, & Fabre, 2013). For a wind system linked to the  $k^{th}$  hydrodynamic mode of the jet (with k > 1), we then observe (2k - 1) / 2 wavelength along the jet (Figure 2.10). For the  $k^{th}$  hydrodynamic mode, this condition on the wavelength is written:

$$w = \frac{2k-1}{2}\lambda \quad , \tag{2.33}$$

where  $\lambda$  is the wavelength whose expression is given by  $\lambda = C_v^e T_0$ , with  $T_0 = 1/f_0$  the period of the self-oscillations. Taking into account the expression for the delay  $\tau = w/C_v^e$  (relation 2.6), we obtain:

$$C_{\nu}^{e} \tau = \frac{2k-1}{2} C_{\nu}^{e} T_{0} , \qquad (2.34)$$

or, simplifying by  $C_v^e$ ,

$$\tau = \frac{2k-1}{2} T_0 \quad . \tag{2.35}$$

Thus, for wind systems (k > 1), the delay  $\tau$  is greater than the oscillation period  $T_0$ , while in **normal** operation (k = 1)  $\tau < T_0$ . In a musical context, the wind systems are obtained for very low values of the pressure  $P_m(t)$ .

#### 2.5.2 - Self-Oscillation Conditions

The analysis of the self-oscillation conditions of the loop system (Figure 2.5) sheds light on the mechanisms of appearance of these different types of systems (we can also refer to (Terrien, Vergez, & Fabre, 2013) for more information. details). Such a looped system can enter self-oscillation if the frequency response  $\beta(j\omega)$  of the open-loop transfer of the linearized system is equal to 1, *i.e.* if it satisfies the two self-oscillation conditions (Fletcher N. H., 1993):

- its modulus,  $\beta_u = |\beta(j\omega_u)|$ , is equal to 1,  $\omega_u$  being the angular frequency at unity gain,

- its phase,  $\phi_u = \arg[\beta(j\omega_u)]$ , is zero or equal to a relative whole multiple *n* of  $2\pi$ ,  $n \in \mathbb{Z}$ .

In steady harmonic system, the system of equations (2.26) becomes:

$$\begin{cases} \overline{\eta}(w, j\omega) = K_0 e^{-\tau j\omega} V_{ac}(j\omega) \\ \Delta P(j\omega) \approx K_1 j\omega \ \overline{\eta}(w, j\omega) \\ V_{ac}(j\omega) = \frac{1}{S} Y_{in}(j\omega, L) \ \Delta P(j\omega) \end{cases}$$
(2.36)

from which we easily deduce the expression of  $\beta(j\omega)$ , that is:

$$\beta(j\omega) = \beta_0 \ j\omega \ e^{-\tau j\omega} \ Y_{in}(j\omega, L) , \qquad (2.37)$$

with  $\beta_0$  a constant whose expression is given by:

$$\beta_0 = \frac{K_0 K_1}{S} = \frac{h \exp(\alpha_i w)}{S U_j^e} \frac{\delta_d}{w} \rho \left( 1 - \left( \tanh\left(-\frac{x_0}{b}\right) \right)^2 \right) U_j^e , \qquad (2.38)$$

or again, after simplification by  $U_j^e$ ,

$$\beta_0 = \frac{h \exp(\alpha_i w)}{S} \frac{\delta_d}{w} \rho \left( 1 - \left( \tanh\left(-\frac{x_0}{b}\right) \right)^2 \right) .$$
 (2.39)

The gain and the phase of  $\beta(j\omega)$  therefore ultimately have the following expression:

$$\begin{cases} \beta(\omega) = |\beta(j\omega)| = \beta_0 \ \omega |Y_{in}(j\omega,L)| \\ \phi(\omega) = \arg \beta(j\omega) = \frac{\pi}{2} - \tau \ \omega + \arg Y_{in}(j\omega,L) \end{cases}$$
(2.40)

It is to highlight that:

- the gain  $\beta(\omega)$  of the open loop does not depend on the flow speed  $U_j^e$  at the channel output and therefore on the pressure  $P_m^e$  at the mouthpiece input;

- the phase  $\phi(\omega)$  of the open loop depends on the speed of flow  $U_j^e$  through the delay  $\tau = \frac{w}{0.4 U_j^e}$ , the latter being as much greater as  $U_j^e$  is weak.

By illustration, Figure 2.16 shows the frequency responses of the gain  $\beta(\omega)$  and of the phase  $\phi(\omega)$  for Examples 1 (in black) and 2 (in green) defined in paragraph 2.3.



Figure 2.16 – Frequency responses of gain  $\beta(\omega)$  and phase  $\phi(\omega)$  for examples 1 (in black) and 2 (in green)

From a physical point of view, the value of *n* is related to the hydrodynamic mode of the jet. Indeed, from the relation concerning the phase  $\phi(\omega)$  of the open loop given by relation (2.40), we can verify posteriori that for a given steady harmonic system, the self-oscillation threshold is located at a frequency  $f_0$  lower than the resonant frequency. Thus, at  $\omega_0 = 2\pi f_0$ , it is possible to make the approximation. The self-oscillation condition on the phase of the open loop, namely  $\arg Y_{in}$  ( $j\omega_0, L$ )  $\approx \pi/2$  then reduces to:

$$\pi - \tau \ \omega_0 = n \ 2\pi \quad , \tag{2.41}$$

or again, knowing that  $\omega_0 = 2\pi/T_0$ ,

$$1 - 2\frac{\tau}{T_0} = 2n , \qquad (2.42)$$

finally:

$$\tau = \frac{1 - 2n}{2} T_0 \quad . \tag{2.43}$$

By comparing relations (2.35) and (2.43),

$$\frac{2k-1}{2}T_0 = \frac{1-2n}{2}T_0 , \qquad (2.44)$$

here comes

$$n = 1 - k$$
 . (2.45)

Thus, the value of *n* is directly related to the hydrodynamic mode *k* of the jet. If the instrument is in *normal* operation (n = 0) then k = 1. All cases where |n| > 1 correspond to the wind systems necessarily obtained for low values of  $P_m^e$  and  $U_j^e$ , therefore large values of  $\tau$ . Then, once the wind system can no longer be obtained, it possible to check the oscillation

conditions  $U_j^e$  for registers of higher orders (i.e. for resonance modes of larger order). We go from the first register, to the second, then to the third etc.

These findings correspond to experimental observations (Kergomard & Chaigne, 2013). In recorders, wind sounds have long been considered non-existent, due to the low value of the w/h ratio (Kergomard & Chaigne, 2013). The development of an artificial mouth regulated in pressure (see Chapter 3) makes it possible to demonstrate the appearance of these systems in recorders for particularly low pressure  $P_m^e$  values which are not used by the musician in normal music playing conditions.

For the two examples of illustrations presented in this chapter, the observation of (Figure 2.16) makes it possible to affirm that the self-oscillation conditions in the two cases are obtained for n = 0 (normal operation), that is to say  $\phi_u = 0$ , which targets frequencies that satisfy the condition:

$$\arg Y_{in}(j\omega,L) + \frac{\pi}{2} = \tau \ \omega \ . \tag{2.46}$$

Graphically, the solution frequencies of this equality are located at the intersections between the phase of the input admittance  $Y_{in}(j\omega,L)$  increased by  $\pi/2$  and the phase lines  $\tau \omega$  whose slopes  $\tau$  depend on the speed flow  $U_j^e$  and therefore pressure  $P_m^e$ . Figure 2.17 shows the frequency responses of  $\arg(Y_{in}(j\omega,L)+90^\circ)$  (in blue) and of  $\tau \omega$  for examples 1 ( $P_m^e = 400$  Pa,  $U_j^e = 26$  m/s, in black) and 2 ( $P_m^e = 1000$  Pa,  $U_j^e = 41$  m/s, in green) defined in paragraph 2.3.



Figure 2.17 – Frequency responses of  $argY_{in}$   $(j\omega, L) + 90^{\circ}$  (in blue) and of  $\tau^*\omega$  for example 1 ( $P_m = 400$  Pa,  $U_j = 26$  m/s, in black) and example 2 ( $P_m = 1000$  Pa,  $U_j = 41$  m / s, in green) defined in paragraph 2.3

Taking into account the very low values of the damping factors of the input admittance  $Y_{in}(j\omega,L)$  of the resonator (see Remark 2.2), the anti-resonance and resonance frequencies are easily identified in (Figure 2.17). They correspond to the intersection of the horizontal dotted line at 90° with the blue phase curve in the areas:

- positive slopes for the anti-resonance frequencies;

- negative slopes for the resonant frequencies.

Thus, still observing (Figure 2.17), it is interesting to note that the two intersections of the black line and the three intersections of the green line with the areas of positive slopes of the blue phase curve correspond to highly unstable operating points. (around anti-resonance frequencies) (Auvray R., 2010). Conversely, the two intersections of the black line and the three intersections of the green line with the areas of negative slopes of the blue phase curve correspond to operating points where ranges of playing frequencies (registers) are accessible (around resonant frequencies).

This phase analysis must be completed by an analysis of the gain  $\beta(\omega)$  of the open loop. If for a given playing frequency the latter is strictly greater than unity, then a perturbation  $\eta(w,t)$  at the operational frequency is amplified. When this disturbance becomes too great, the non-linear phenomena (saturation of the *tanh(.)* function) provide the conditions for self-oscillation.

### 2.6 - Conclusion

The different elements of the sound production mechanism recalled in this chapter make it possible to capture the main phenomena observed, in particular, in recorder-type instruments. From the complete nonlinear model frequently used in the literature, a solution to improve its digital conditioning is proposed, allowing its programming using MatLab / Simulink. For the defined field of study, a detailed analysis of the simulated time responses highlights the presence of three phases during which the variations of the pressure at the input of the resonator, of the acoustic speed and of the lateral displacement of the air jet remain small around zero. The assumption of small variations being thus perfectly legitimate and realistic, two linearized models, one for the analysis of *phase 1* (start of the simulation), the other for that of *phase 3* (self-oscillations) are used to further understand the phenomena involved.

However, one should be aware of the limitations of this model when interpreting the results and comparing them with experimental data. This is the subject of Chapters 3 and 4.

# Chapter 3- Design and Implementation of an Artificial Mouth for Wind Musical Instrument

## 3.1 – Introduction

Blowing machines, so-called artificial mouths, are used in musical acoustics when studying wind instruments, at least since 80 years (Ferrand & Vergez, 2008). As proposed in the literature review (Ferrand D., Vergez, Fabre, & Blanc, 2010) (Takanishi, 2010) (Maki-Patola, 2004) (Hamilton, 2019) (Paine, 2013) (R. Saar, 2014) (V. Chatziioannou, 2008), the objective of controlling the pressure inside the artificial mouth is considered in many applications:

- to ensure quasi-static variation in order to analyze experimental bifurcation diagrams;
- to reproduce typical signals (Heaviside step function, sinus, ramps...) to compare the dynamics of the real instrument to the one obtained through numerical simulations;
- to mimic the time evolutions recorded on real musicians in order to analyse the strategies discovered over the years of practice.

The objective of this chapter is the study of visco-thermal losses in a wind musical instrument from a hardware-in-the-loop simulation platform. The hardware part of the platform is made up of an automatic blowing machine connected to the mouthpiece of a wind instrument (Abou Haidar, Abi Zeid Daou, & Moreau, 2019).

The software part of the platform is composed of resonator numerical model of the wind instrument including visco-thermal losses based on fractional model and the Control System (CS) used for regulating the pressure inside the artificial mouth. The first part of the project was the Computer Aided Design (CAD) of the platform. Thus, a digital simulator was developed with MatLab/Simulink software based on wind instruments artificial mouths found in several publications (Ferrand & Vergez, 2008) (Ferrand D., Vergez, Fabre , & Blanc, 2010). The second part, presented in this chapter, is based on the realization of the test bench from the first part. The objective is to adjust the dynamic behavior of the numerical simulator to the real dynamic behaviour of test bench. The third and last part consists in modelling and analysing the visco-thermal losses present in the resonator of the wind instrument from fractional models (Andréa & Matignon , 1995).

### 3.2 – System Description

The test bench is presented in Figure 3.1. A servo-valve is connected to an air compressor through a pressure reducer. The maximum pressure available is around 6 bars, and the pressure reducer (with its manometer) is used to adjust the pressure  $P_1$  upstream the servo-valve. The servo-valve is connected at the entrance of the artificial mouth whose internal volume V = 343 cm<sup>3</sup> is the place where the air pressure  $P_m$  must be controlled. The artificial mouth blows into the mouthpiece of a recorder flute. A MatLab/Simulink/LabVIEW program is used in order to control the air pressure  $P_m$ . Added to that, a flow meter, a temperature and a pressure transducers are used in order to characterize the behavior of the different parts of this system.

#### Remark

For the rest of the chapter, the following notation is adopted for a variable X(t):

$$X(t) = X^{e} + x(t)$$
, (3.1)

where  $X^{e}$  is a constant value fixed by a given operating point and x(t) the fluctuation around  $X^{e}$ . Moreover,  $\tilde{X}(t)$  represents a measurement of X(t) and  $\hat{X}(t)$  an estimate of X(t).

Figure 3.2 presents the scheme of the experimental setup and Figure 3.3 the block diagram associated.





**Figure 3.1 – Test bench photos** 







Figure 3.3 – Block diagram of the experimental setup

### **3.3 – Specifications for the Control**

In the classical use of an artificial mouth, the pressure  $P_m(t) = P_m^e + p_m(t)$  upstream the mouthpiece of the recorder flute (inside the artificial mouth) is tuned by hand through the pressure reducer and its manometer. When the compressed air is produced by a compressor such as that presented in Figure 3.1, the pressure  $P_1(t) = P_1^e + p_1(t)$  upstream the servo-valve, fluctuates because the compressor tank being in need to be recharged once its pressure becomes below a certain level. This is the reason why it is difficult to manually control the pressure. Thus, the purpose of automatic pressure fluctuation  $p_1(t)$  considered as a disturbance, while satisfying robust tracking of the reference pressure  $P_{ref}(t)$  (Ferrand & Vergez, 2008). To recall, the final objective of this work project is the study of visco-thermal losses in a wind instrument based on fractional model. In order to facilitate the analysis of this complex problem, the reference pressure is chosen such as:

$$P_{ref}(t) = P_{ref}^{e} + p_{ref}(t) = P_{ref}^{e} + P_{ref}^{0} \cos(2\pi f_{0} t).$$
(3.2)

For all these reasons, the architecture of the control system presented in Figure 3.4 consists of a  $P_{ref}(t)$  reference generator, a  $U_{ff}(t)$  feedforward control and a  $U_{fb}(t)$  feedback control, the robust controller of which is designed with the CRONE methodology (Oustaloup A., 1995) (Oustaloup A., 1991) (P. Lanusse, 2013) (Lanusse P., 1994) (Lanusse P., 2010).



Figure 3.4 – Architecture of the Control System (CS)

## 3.4 – System Modelling and Validation

As presented in Figure 3.3, the experimental setup is divided into two parts:

- the first one consists of a voltage-current amplifier, a servo-valve connected to an air compressor via a pressure reducer. The inputs of this part are  $P_1(t)$  the pressure upstream the servo-valve and  $U_c(t) = U_{ff}(t) + U_{fb}(t)$  the control signal generated from MatLab/Simulink/ LabVIEW, whereas the output is  $Q_{sv}(t)$  the flow rate delivered by the servo-valve. This last device presents a nonlinear behavior;
- the second part is the artificial mouth and the mouthpiece of the recorder flute. The input of this part is  $Q_{sv}(t)$  the flow rate from the servo-valve, whereas the output is  $\widetilde{P}_m(t)$  the pressure measured within the artificial mouth,  $\widetilde{P}_m(t) = P_m(t) + n(t)$  where n(t) is the measurement noise.

#### 3.4.1 - Modelling and Validation of the Servo-Valve

The servo-valve is designed by Bürkert firm (ref. Bürkert 2871). Based on previous works (Ferrand & Vergez, 2008) (Ferrand D., Vergez, Fabre, & Blanc, 2010) (Abou Haidar., Abi Zeid Daou, & Moreau, 2018) (G. Abou Haidar, 2019), it has been shown that the servo-valve's behavior can be divided in two parts: a nonlinear static part and a linear dynamic part.

Figure 3.5 shows the block diagram of the servo-valve. In order to express the output of the nonlinear part, a static flow  $Q_{stat}$  is introduced.



Figure 3.5 – Block diagram of the servo-valve

The linear dynamic part between  $Q_{stat}$  and  $Q_{sv}(t)$  is represented by a second order transfer function  $H_{sv}(s)$  with a unit static gain (Ferrand & Vergez, 2008) (Ferrand D., Vergez, Fabre, & Blanc, 2010) (Abou Haidar., Abi Zeid Daou, & Moreau, 2018) (Abou Haidar, Abi Zeid Daou, & Moreau, 2019):

$$H_{sv}(s) = \frac{1}{1 + 2\zeta s / \omega_0 + (s / \omega_0)^2} , \qquad (3.3)$$

where  $\zeta = 0.3$  and  $\omega_0 = 2\pi 240$  rad/s.

As already mentioned, the output flow rate  $Q_{sv} = f(U_c, P_l)$  depends on the control voltage signal  $U_c$  (that may vary between 0 and 10V) and the pressure  $P_l$  upstream the servo- $(\max[P_l] = 6 \text{ bar})$ .

Figure 3.6 shows the static operating domain obtained from measurements for  $U_c \in [0; 10]$  V and  $P_1 \in [1; 6]$  bar.



Figure 3.6 – Static operating domain obtained from measurements

We can see a dead zone where  $Q_{sv} = 0$  whatever the value of  $U_c \in [0; U_{\min}(P_1)]$ , and a variation zone of  $Q_{sv}$  where the saturation,  $\max[Q_{sv}]$ , depends on  $\max[U_c] = 10$  V and  $\max[P_1] = 6$  bar. In the variation zone, for a given operating point O defined by  $O = (U_c^e; P_1^e)$ , the flow rate  $Q_{sv}(t, U_c, P_1)$  can be written as follow:

$$Q_{sv}(t, U_c, P_1) = Q_{sv}^e (U_c^e, P_1^e) + q_{sv}(t, U_c, P_1),$$
(3.4)

where

$$q_{sv}(t, U_c, P_1) = K_{qu}(U_c^e, P_1^e) u_c(t) + K_{qp}(U_c^e, P_1^e) p_1(t) , \qquad (3.5)$$

and

$$K_{qu}\left(U_{c}^{e},P_{1}^{e}\right) = \frac{\partial Q_{sv}}{\partial u_{c}}\bigg|_{P_{1}^{e}}, \quad K_{qp}\left(U_{c}^{e},P_{1}^{e}\right) = \frac{\partial Q_{sv}}{\partial p_{1}}\bigg|_{U_{c}^{e}}.$$
(3.6)

The estimated value  $\hat{K}_{qu}$  of the static gain  $K_{qu}$  is obtained from:

$$\hat{K}_{qu}(P_1) = \frac{\Delta \widetilde{Q}_{sv}}{\Delta U_c(P_1)} = \frac{\max[\widetilde{Q}_{sv}(U_{\max}, P_1)] - 0}{\max[U_c] - U_{\min}(P_1)}, \qquad (3.7)$$

with  $U_{max} = \max[U_c] = 10$  V.

Figure 3.7 shows the variation of  $\hat{K}_{qu}$  versus  $P_1 \in [1; 6]$  bar that can be considered as linear and modelled by:

$$\hat{K}_{qu}(P_1) = A P_1 + B, \qquad (3.8)$$

$$\lim_{l \to m^{-1} \cdot V^{-1}} \sum_{[bar]} P_1 + B,$$

$$\begin{cases} A = 0.99 \ 1.\text{mm}^{-1}.\text{V}^{-1}.\text{bar}^{-1} \\ B = 3.4 \ 1.\text{mm}^{-1}.\text{V}^{-1} \end{cases}$$
(3.9)



Figure 3.7 – Variation of  $\hat{K}_{qu}$  versus  $P_1 \in [1; 6]$  bar

The non-linear static part and the linear dynamic part of the servo-valve are integrated in a digital simulator programmed with MATLAB / Simulink.

#### 3.4.2 – Choosing an Operating Point

The limits of the servo-valve operating domain being estimated, the choice of an operating point  $O = (U_c^e; P_1^e)$  and the amplitude of the variations around this point is essential in order to avoid the risks of saturation of the flow rate  $Q_{sv}(t)$ .

After observing the pressure  $P_1(t)$  during numerous tests, the fluctuations  $p_1(t)$  are considered to be limited between -1 and + 1bar. This is the reason why we chose  $P_1^e = 5$  bar, and so  $P_1 \in [4; 6]$  bar. For this variation range of  $P_1$ , the variation range of  $U_c$  without saturation of  $Q_{sv}$  has as limits  $U_{\min}(P_{1\min} = 4bar) = 7.55$  V and  $U_{max} = 10$  V, so:

$$7.55 \text{ V} \le U_c(t) = U_{ff}(t) + U_{fb}(t) \le 10 \text{ V}, \qquad (3.10)$$

with

with

$$U_{ff}(t) = U_c^e + U_c^0 \cos(2\pi f_0). \tag{3.11}$$

By choosing  $U_c^e = 8.5 \text{ V}$  and  $U_c^0 = 0.5 \text{ V}$ , the variation range of the feed-forward control signal  $U_{ff}$  is given by:

$$\begin{cases} \max \left[ U_{ff}(t) \right] = U_{c}^{e} + U_{c}^{0} = 9 \text{ V} \le U_{\max} = 10 \text{ V} \\ \min \left[ U_{ff}(t) \right] = U_{c}^{e} - U_{c}^{0} = 8 \text{ V} \ge U_{\min} = 7.55 \text{ V} \end{cases}$$
(3.12)

and the variation range of the feedback control signal  $U_{fb}$  is given by:

$$\begin{cases} \max \left[ U_{fb}(t) \right] = U_{\max} - \max \left[ U_{ff}(t) \right] = +1 \text{ V} \\ \min \left[ U_{fb}(t) \right] = U_{\min} - \min \left[ U_{ff}(t) \right] = -0.45 \text{ V} \end{cases}$$
(3.13)

Figure 3.8 shows the chosen operating point  $O = (U_c^e = 8.5 \text{ V}; P_1^e = 5 \text{ bar})$  and the linear static operating domain defined by  $U_c \in [7.55; 10] \text{ V}$  and  $P_1 \in [4; 6] \text{ bar}$ .



Figure 3.8 – Operating point O and linear static operating domain

#### 3.4.3 – Modelling and Validation of the Artificial Mouth

The artificial mouth is implemented using a cubic box whose input is the flow rate  $Q_{sv}(t)$  coming from the servo-valve and the output is the pressure  $P_m(t)$  inside the box. The expression of the pressure  $P_m(t)$  is given by the state equation of perfect gases:

$$P_m(t) = \frac{r T}{V} M(t), \qquad (3.14)$$

where

- $r = 287 \text{ J.kg}^{-1}$ .K<sup>-1</sup>, the thermodynamic constant of air;
- $T = 293.5^{\circ}$ K, the temperature of air inside the box;
- M(t) is the mass of the air inside the box of volume V. This value depends on the variation of the flow between the input (represented by  $Q_{sv}(t)$ ) and the output (represented by  $Q_{mp}(t)$ ). As small variations are considered, the air density  $\rho$  is considered to be constant. Thus, the expression of M(t) is given by:

$$M(t) = \rho \int_{0}^{t} \left( Q_{sv}(\tau) - Q_{mp}(\tau) \right) d\tau + M(0), \qquad (3.15)$$

where the expression of  $Q_{mp}(t)$  is given by Bernoulli law (Ferrand D., Vergez, Fabre , & Blanc, 2010):

$$Q_{mp}(t) = \alpha \sqrt{\Delta P_{mp}(t)} , \qquad (3.16)$$

where  $\alpha$  is a coefficient estimated from measurements and  $\Delta P_{mp}(t) = P_m(t) - P_{atm}$  ( $P_{atm}$  is the relative atmospheric pressure which is zero by definition, so  $\Delta P_{mp}(t) = P_m(t)$ ). By introducing in relation (3.15) the pressure  $P_m(t)$  from relation (3.14), namely:

$$P_{m}(t) = \frac{\rho \ r \ T}{V} \int_{0}^{t} \left( Q_{sv}(\tau) - Q_{mp}(\tau) \right) d\tau + \frac{r \ T}{V} M(0), \qquad (3.17)$$

we obtain

$$P_{m}(t) = \frac{1}{C_{am}} \int_{0}^{t} (Q_{sv}(\tau) - Q_{mp}(\tau)) d\tau + P_{m}(0), \qquad (3.18)$$

where  $C_{am}$  is the pneumatic capacity associated with the volume V of the artificial mouth given by:

$$C_{am} = \frac{V}{\rho \, r \, T} \quad , \tag{3.19}$$

and  $P_m(0)$  the initial value of the pressure  $P_m(t)$  given by

$$P_m(0) = \frac{r T}{V} M(0) \quad . \tag{3.20}$$

It is important to note that in static  $Q_{mp}^e = Q_{sv}^e$ , and for a quasi-static variation  $Q_{mp}(t) \approx Q_{sv}(t)$ .

Figure 3.9 shows the plot of  $Q_{sv}(t)$  versus  $P_m(t)^{0.5}$  for a quasi-static variation. The linear fitting (in red) of the measurements (x) leads to an estimate  $\hat{\alpha}$  given by:

$$\hat{\alpha} = 1.1 \ \text{l.mn}^{-1}.\text{Pa}^{-0.5} = 18.33 \ 10^{-6} \ \text{m}^3.\text{s}^{-1}.\text{Pa}^{-0.5}.$$
 (3.21)



Figure 3.9 - Plot of  $Q_{sv}(t)$  vs  $sqrt(P_m(t))$  for a quasi-static variation

From relations (3.16), (3.17) and (3.20), the behavior of the artificial mouth is integrated in the digital simulator programmed with MATLAB / Simulink.

#### 3.4.4 - Linearized Model of the Plant

In order to design the CRONE controller in frequency domain, a linearized model of the plant is derived around the operating point O. Thus, the pressure  $P_m(t)$  and the flow rate  $Q_{mp}(t)$  can be expressed as follows:

$$\begin{cases} P_m(t) = P_m^e + p_m(t) \\ Q_{mp}(t) = Q_{mp}^e + q_{mp}(t) \end{cases},$$
(3.22)

with

$$Q_{mp}^{e} = \alpha \sqrt{P_{m}^{e}}$$
 and  $q_{mp}(t) = \frac{1}{R_{mp}} p_{m}(t)$ , (3.23)

$$\frac{1}{R_{mp}} = \frac{\partial Q_{mp}}{\partial P_m} \bigg|_{P_m = P_m^e} = \frac{\alpha}{2\sqrt{P_m^e}} .$$
(3.24)

Figure 3.10 shows the block diagram of the artificial mouth linearized model.



Figure 3.10 – Block diagram of the artificial mouth linearized model

The transfer function  $H_l(s)$  between  $\overline{P}_m(s) = LT\{p_m(t)\}\$  and  $\overline{Q}_{sv}(s) = LT\{q_{sv}(t)\}\$ , where LT represents Laplace Transform, is given by:

$$H_{1}(s) = \frac{\overline{P}_{m}(s)}{\overline{Q}_{sv}(s)} = \frac{R_{mp}}{1 + R_{mp} C_{am} s} = \frac{H_{0}}{1 + s / \omega_{1}},$$
(3.25)

where

$$H_0 = R_{mp}$$
,  $\tau_1 = R_{mp} C_{am}$  and  $\omega_1 = 1/\tau_1$ . (3.26)

Finally, the complete linearized model used for the design of the control law is represented by the transfer function  $G(s, P_l)$  between  $\overline{P}_m(s)$  and  $\overline{U}_c(s)$  as follow:

$$G(s, P_1) = \frac{G_0(P_1)}{\left(1 + 2\zeta \ s \ / \ \omega_0 + \left(s \ / \ \omega_0\right)^2\right) \left(1 + s \ / \ \omega_1\right)} , \qquad (3.27)$$

where

$$G_0(P_1) = K_{qu}(P_1) H_0.$$
(3.28)

For the design of the robust control law, the main parameter's values used are:

$$\begin{cases} P_{1} \in [4; 6] \text{ bar}; P_{m}^{e} = 750 \text{ Pa} \\ K_{qu} \in [12.27; 15.57] 10^{-5} \text{ m}^{3} \text{s}^{-1} \text{V}^{-1} \\ \hat{R}_{bec} = 2.65 10^{6} \text{ Pa s m}^{-3}; C_{am} = 3.38 10^{-9} \text{ Pa m}^{-3} \\ G_{0} \in [325; 412.5] \text{ Pa V}^{-1}; \omega_{1} = 2\pi 17.8 \text{ rad/s} \end{cases}$$

$$(3.29)$$

and three transfer functions are defined for the minimum (4 bar), the nominal (5 bar) and the maximum (6 bar) cases:

$$G_{min}(s, 4 \text{ bar}), G_{nom}(s, 5 \text{ bar}), G_{max}(s, 6 \text{ bar}).$$
 (3.30)

Figure 3.11 presents the Bode plots of  $G(j\omega)$  for three values of the pressure:  $P_1 = 4$  bar (in blue),  $P_1 = 5$  bar (in black) and  $P_1 = 6$  bar (in red).



Figure 3.11 – Bode plots of  $G(j\omega)$  for three values of the pressure:  $P_1 = 4$  bar (in blue),  $P_1 = 5$  bar (in black),  $P_1 = 6$  bar (in red)

## **3.5 – Controller Design**

The feedforward part is based on the nominal inverse static gain of the linearized model  $(G_{0 nom} = G_0(5 \text{ bar}))$  used for the design of the feedback control law, namely:

$$U_{ff}(t) = G_{0\ nom}^{-1} P_{ref}(t) \quad . \tag{3.31}$$

#### 3.5.1 – User Specifications

The user specifications of the control system defined from a preliminary work (Ferrand & Vergez, 2008) (Lanusse P., 1994) are the following:

- a phase margin  $M_{\emptyset} > 40^{\circ}$ ;
- an open-loop gain crossover frequency  $\omega_u \ge 2\pi \ 10 \text{ rad/s};$
- a steady-state error equal to 0;
- a variation range of  $U_{fb}$  given by  $-0.45 \text{ V} \le U_{fb} \le 1 \text{ V}$ .

#### 3.5.2 - CRONE Control-System Design (CSD) Methodology

The CRONE CSD methodology is a frequency-domain approach developed since the eighties (Oustaloup A., 1995) (Oustaloup A., 1991) (Lanusse P., 1994). It is based on the common unity-feedback configuration presented in Figure 4. Three CRONE CSD methods have been developed, each one of them denotes a generation of CRONE design. The general

form of the nominal open-loop transfer function  $\beta_{nom}(s)$  of the second generation CRONE control is defined by:

$$\beta_{nom}(s) = \beta_0 \left(\frac{1+s/\omega_l}{s/\omega_l}\right)^{n_l} \left(\frac{1+s/\omega_h}{1+s/\omega_l}\right)^n \left(1+s/\omega_h\right)^{-n_h} .$$
(3.32)

The first part of the above equation (3.32) represents the behavior at low frequencies with an integer order  $n_l$ , the second represents the behavior at middle frequencies with non-integer order *n* varying between 1 and 2 around  $\omega_u$ , and the last represents the behavior at high frequencies with an integer order  $n_h$ . As for the gain  $\beta_0$ , it is defined by (Lanusse P., 1994):

$$\beta_{0} = \left(\omega_{u} / \omega_{l}\right)^{n_{l}} \left(1 + \left(\omega_{u} / \omega_{l}\right)^{2}\right)^{(n-n_{l})/2} \left(1 + \left(\omega_{u} / \omega_{h}\right)^{2}\right)^{(n_{h}-n)/2}.$$
(3.33)

With  $M_{\emptyset} = 45^{\circ}$ ,  $\omega_u = 2\pi 10 \text{ rad/s}$ ,  $G_0 \in [325; 412.5] \text{ Pa/V}$  and in accordance with the methodology described in (Oustaloup A. , 1991), the parameter's values of the open-loop transfer function are:

$$\begin{cases} n_l = 2, n = 1.5, n_h = 2\\ \omega_l = 5.79 \text{ rad/s}, \omega_h = 20444 \text{ rad/s}, \beta_0 = 32.92 \text{ SI} \end{cases}$$
(3.34)

When the nominal open-loop transfer is determined, the fractional controller  $C_F(s)$  is defined by its frequency response:

$$C_F(j\omega) = \beta_{nom}(j\omega) / G_{nom}(j\omega) . \qquad (3.35)$$

The synthesis of the ideal frequency response  $C_F(j\omega)$  consists of identifying a rational frequency response  $C_R(j\omega)$  given by:

$$C_R(j\omega) = B(j\omega) / A(j\omega) , \qquad (3.36)$$

where  $B(j\omega)$  and  $A(j\omega)$  are polynomials of specified integer degrees  $n_B$  and  $n_A$ . All the frequency-domain system identification techniques can be used (Oustaloup A., 1991). Figure 3.12 presents the Bode plot of the controller  $C_R(j\omega)$ .



Figure 3.12 – Bode plot of  $C_R(j\omega)$ 

## **3.6 – System Performance**

#### 3.6.1 – Frequency Domain

Figure 3.13 presents the Bode plots (a) and the Nichols loci (b) of the open-loop transfer function  $\beta(j\omega)$  and Figure 3.14 the Bode plots of complementary sensitivity function  $T(j\omega)$  (a), of sensitivity function  $S(j\omega)$  (b), of control effort sensitivity function  $CS(j\omega)$  (c) and of plant input sensitivity function  $GS(j\omega)$  (d) obtained with the CRONE controller for the three cases (min, nom, max).

As one can observe, the phase margin  $M_{\emptyset}$  (Figure 3.13.b) and the resonant peaks  $Q_T$  of  $T(j\omega)$  (Figure 3.14.a) and  $Q_S$  of  $S(j\omega)$  (Figure 3.14.b) remain constant for all the cases thus showing the robustness of stability degree (Oustaloup A., 1991).



Figure 3.13 – Bode plots (a) and Nichols loci (b) of  $\beta(j\omega)$  obtained with the CRONE controller for the three cases: min (4 bar in blue), nom (5 bar in black) and max (6 bar in red)



Figure 3.14 – Bode plots of T(jω) (a), of S(jω) (b), of CS(jω) (c) and GS(jω) (d) obtained with the CRONE controller for the three cases: min (4 bar in blue), nom (5 bar in black) and max (6 bar in red)

#### **3.6.2 – Time Domain**

The reference pressure  $P_{ref}(t)$  is chosen such as:

for 
$$\begin{cases} t = 0 \text{ to } t_0, P_{ref}(t) = P_{ref}^e \\ t = t_0 \text{ to } t_1 = t_0 + T_0, P_{ref}(t) = P_{ref}^e + P_{ref}^0 \cos(2\pi t/T_0) , \\ t = t_1 \text{ to } t_2 = 2t_0 + T_0, P_{ref}(t) = P_{ref}^e \end{cases}$$
(3.37)

with  $t_0 = 0.1$  s,  $T_0 = 1$  s,  $P_{ref}^e = 750$  Pa and  $P_{ref}^0 = 185$  Pa.

It is important to note that the gain of the feedforward part is calculated only for the nominal case  $P_1 = 5$  bar and that it is not adjusted when  $P_1$  varies upstream the servo-valve.

Figure 3.15 presents time responses of  $P_{ref}(t)$  and  $P_m(t)$  (a) (b), of error signal  $\varepsilon(t) = P_{ref}(t) - P_m(t)$  (c) (d), and of control signal  $U_c(t)$  (e) (f) obtained without feedback (a) (c) (e) and with feedback (b) (d) (f) for the three cases (min, nom, max).

We observe that the robust feedback and feedforward control system ensures a good pressure tracking (b) (d), not only for the nominal case ( $P_1 = 5$  bar), but also for the minimal ( $P_1 = 4$  bar) and maximal ( $P_1 = 6$  bar) cases. Without the robust feedback control system (a) (c), pressure tracking is less effective. In all cases, the control signal  $U_c(t)$  (e) (f) remains within the variation range defined by  $U_{min}$  and  $U_{max}$ .



Figure 3.15 – Time responses of  $P_{ref}(t)$  and  $P_m(t)$  (a) (b), of error signal  $\varepsilon(t)$  (c) (d), and of control signal  $U_c(t)$  (e) (f) obtained without feedback (a) (c) (e) and with feedback (b) (d) (f) for the three cases: min (4 bar in blue), nom (5 bar in black) and max (6 bar in red)

## 3.7 – Conclusion

The first important work of this chapter was to mathematically describe, model, and simulate the linear and nonlinear parts of the servo valve and the artificial mouth and try to linearize their modeled equations. Moreover, understanding the influence of pressure  $P_1$  on the limits of the servo-valve operating range is essential in order to be then able to define a strategy for automatic pressure  $P_m$  control inside the artificial mouth. The second important step is the design of the control architecture for a robust control of the pressure  $P_m$ . The application of the CRONE system design methodology was able to achieve the target set with very good dynamic performance and respecting the linear operating range of the servo valve.

# Chapter 4- Analysis of the Influence of Fractional Order on the Stationary Periodic System

## 4.1 – Introduction

This chapter 4 uses the developments of chapters 1, 2 and 3 in order to analyze the influence of the fractional order m on the stationary periodic system. As a reminder, Figure 4.1 illustrates the progression and sequencing of the chapters of this thesis.



Figure 4.1 - Illustration of the progression and sequencing of the chapters of the thesis dissertation

Thus, from the extension of the fractional model proposed in chapter 2, it is possible with a single high-level parameter, the order m in this case, to easily vary in numerical simulation, the visco-thermal losses, while from an experimental point of view, it would be necessary to manufacture and test a large number of resonators with dimensions (length L and radius r), roughness (or surface condition) and different materials (wood, plastic, etc.).

This chapter therefore begins by recalling the organization of the global simulator developed under MatLab / Simulink from the work presented in the first three chapters. Then, an analysis in the frequency domain of the influence of the order m is developed, first in the case of the impedance of the resonator established in Chapter 2, then in the case of the defined open-loop transfer function in Chapter 3. This analysis continues in the time domain with the responses from the global simulator by using the scenarios of the two examples of Chapter 3. Finally, the main contributions of this chapter are recalled in the conclusion.
# 4.2 - Simulator of the Artificial Mouth - Recorder Set: Reminder

Figure 4.2 shows the block diagram of the global simulator used in this chapter. It consists of:

- the artificial mouth simulator (Chapter 3);
- the exciter simulator (Chapter 2).

The artificial mouth simulator has undergone a readjustment with respect to the test bench developed specifically for this study. The regulation loop makes it possible, in particular, to limit the sensitivity of the pressure  $P_m(t)$  within the pneumatic capacity to variations in the supply pressure  $P_1(t)$  of the servo valve. As part of the analysis of the influence of fractional order on the stationary periodic system, the reference pressure  $P_{ref}(t)$  is a constant equal to 400 Pa (example 1) and 1000 Pa (example 2).

The exciter simulator, on the other hand, is the one developed in Chapter 2. The fractional order is present in the acoustic tube model studied in Chapter 1 as a resonator.



Figure 4.2 - Functional diagram of the entire simulator

### 4.3 – Frequency domain analysis of the influence of fractional order

### 4.3.1 - Case of the Resonator Impedance

As a reminder (refer to Chapter 2) and for the field of study defined within the framework of this thesis work, the input admittance  $Y_{in}(s)$  of the acoustic tube (resonator) of length *L* and radius *r* has the expression:

$$Y_{in}(s) = \frac{S}{Z_{ac}} I_m(s) \frac{1}{\tanh(z(s))} , \qquad (4.1)$$

Figure 4.3 – Plot of 
$$\omega_{r,m}$$
 as a function of the order *m*



Still as a reminder, the numerical values retained for the study are 
$$r = 5 \times 10^{-3}$$
m,  $L = 0.3$ m, with a temperature of 25°C where  $\rho_a = 1.184$  kg/m<sup>3</sup> and  $c_a = 346.3$  m/s,  $K_0 = 2.98 \times 10^{-4}$ . In this case,  $\omega_L = 1.154$  rad/s (184 Hz). As for  $\omega_{r,m}$ , Figure 4.3 shows its plot as a function of the order *m* and Table 4.1 gives precisely the values corresponding to the 5 orders retained thereafter.

$$\omega_{r,m} = c_a \left(\frac{4 \ m \ K_0}{r}\right)^{1/m}$$
 and  $\omega_L = \frac{c_a}{L}$ . (4.4)

where 
$$Z_{ac} = \rho_a c_a$$
 represents the characteristic acoustic impedance, *S* the section of the acoustic tube, and where the transitional frequencies  $\omega_{r,m}$ , and  $\omega_L$  have the expression:

 $I_{m}(s) = \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^{m}}{\left(\frac{s}{\omega_{r,m}}\right)^{m}}}$ 

 $\omega_{r,m}$ 

ransitional frequencies 
$$\omega_{r,m}$$
, and  $\omega_L$  have the expression:  
 $(4 \ m \ K_0)^{1/m}$ 

$$\omega_{r,m} = c_a \left(\frac{4 \ m \ K_0}{r}\right)^{1/m} \quad \text{and} \quad \omega_L = \frac{c_a}{L}. \tag{4.4}$$

$$z(s) = \left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}} , \qquad (4.3)$$

(4.2)

with

and

Table 4.1 – Values of  $\omega_{r,m}$  as a function of the order *m* 

| т                |       | 0 | 0.25            | 0.5   | 0.75  | 1     |
|------------------|-------|---|-----------------|-------|-------|-------|
| Ø <sub>r,m</sub> | rad/s | 0 | $4.4 \ 10^{-3}$ | 4.92  | 34.88 | 82.55 |
|                  | Hz    | 0 | $7  10^{-4}$    | 0.783 | 5.55  | 13.14 |

In addition, the expression (4.1) of  $Y_{in}(s)$  can be put in a form  $Y_N(s)$  composed of an integrator of order 1 in cascade with N fractional cells of order 2, namely:

$$Y_{in}(s) = \lim_{N \to \infty} Y_N(s) \quad , \tag{4.5}$$

with

$$Y_N(s) = \frac{Y_0}{s} \prod_{i=1}^N \left( \frac{1 + 2\zeta_{zi} \left(\frac{s}{\omega_{zi}}\right)^{2-m} + \left(\frac{s}{\omega_{zi}}\right)^2}{1 + 2\zeta_{pi} \left(\frac{s}{\omega_{pi}}\right)^{2-m} + \left(\frac{s}{\omega_{pi}}\right)^2} \right), \qquad (4.6)$$

where

$$Y_0 = \frac{S}{\rho_a L} \tag{4.7}$$

$$\begin{cases} \omega_{zi} = c_{zi} \ \omega_L & ; \quad \zeta_{zi} \left( m \right) = \frac{2 \ K_0}{r} \left( \frac{L}{c_{zi}} \right)^m m \\ \omega_{pi} = c_{pi} \ \omega_L & ; \quad \zeta_{pi} \left( m \right) = \frac{2 \ K_0}{r} \left( \frac{L}{c_{pi}} \right)^m m \end{cases}$$
(4.8)

and

where  $c_{zi}$  and  $c_{pi}$  are constants resulting from the procedure allowing to pass from  $Y_{in}(s)$  to  $Y_N(s)$ . This procedure, presented in appendix A, is composed of 4 steps:

- Step 1: finite expansion of the function tanh(z) into a continuous fraction in z;
- Step 2: reduction of the continuous fraction to a ratio of two polynomials in z;
- Step 3: factorization of each polynomial in *z*;

- Step 4: rewriting  $Y_{in}(s)$  in the form  $Y_N(s)$ , namely an integrator of order 1 in cascade with a product of N fractional cells of order 2.

From a numerical point of view, there is no limitation on the choice of the number of cells N, the precision being the better the number N is high. Theoretically, N must be infinite. Thus, any finite value of N causes an edge effect on the last cell linked to the truncation. This is the reason why we must choose an expansion of order N + I to have good precision with N cells.

From an analytical point of view, the search for the roots of the polynomials obtained in step 2 (research necessary for factorization at the level of step 3) limits N to 2. Taking into account the edge effects, the analytical developments do not can only be used for the first cell which corresponds to the first mode of the resonator.

Figure 4.4 shows on the range [100; 2000] Hz for m = 0.5 the frequency responses of  $Y_{in}(s)$  (in black) and of  $Y_N(s)$  (in red) for N = 2. The integrative behavior of order 1 present at low frequencies is perfectly reproduced by the equation (4.6), as well as the behavior associated with anti-resonance and resonance of the first mode. Beyond that, the side effect linked to the truncation (N = 2) appears clearly.



Figure 4.4 – In the range [100; 2000] Hz and for m = 0.5, frequency responses of  $Y_{in}(s)$  (in black) and of  $Y_N$ (s) (in red) for N = 2

However, the analysis of expressions (4.6) and (4.8) shows that each cell of rank *i* is completely defined as soon as the two constants  $c_{zi}$  and  $c_{pi}$  are known. Based on this observation, a specific method is proposed in 3 steps.

Step 1 consists of determining the angular frequencies  $\omega_{zi}$  and  $\omega_{pi}$  solutions of the equation:

$$\arg Y_{in}(j\omega) = 0 \quad , \tag{4.9}$$

knowing that the phase curve alternates between -90 ° and + 90 ° (Figure 4.4), and that the  $\omega_{zi}$  correspond to solutions for which the slope of the phase curve of  $Y_{in}(j\omega)$  is positive (anti- resonance), i.e.:

$$\left. \frac{d}{d\omega} \arg Y_{in}(j\omega) \right|_{\omega = \omega_{zi}} > 0 \tag{4.10}$$

and the  $\omega_{pi}$  to the solutions for which the slope of the phase curve of  $Y_{in}(j\omega)$  is negative (resonance), namely:

$$\left. \frac{d}{d\omega} \arg Y_{in}(j\omega) \right|_{\omega = \omega_{pi}} < 0.$$
(4.11)

Given the complexity of the expression of  $Y_{in}(j\omega)$ , the search for the roots is done numerically at the scale of each cell of rank *i*.

Then, for a given angular frequency  $\omega_L$ , the constants  $c_{zi}$  and  $c_{pi}$  are calculated in step 2 in accordance with relations (4.8), namely:

and

$$\begin{cases} c_{zi} = \frac{\omega_{zi}}{\omega_L} \\ c_{pi} = \frac{\omega_{pi}}{\omega_L} \end{cases}$$
(4.12)

Finally, for a given order *m*, step 3 makes it possible to calculate the damping factors  $\zeta_{zi}$  and  $\zeta_{pi}$  of  $Y_N(s)$  in accordance with relations (4.8), namely:

$$\begin{cases} \zeta_{zi}(m) = \frac{2K_0}{r} \left(\frac{L}{c_{zi}}\right)^m m \\ \vdots \\ \zeta_{pi}(m) = \frac{2K_0}{r} \left(\frac{L}{c_{pi}}\right)^m m \end{cases}$$
(4.13)

Figure 4.5 shows on the range [100; 4000] Hz for m = 0.75 the frequency responses of  $Y_{in}(s)$  (in black) and of  $Y_N(s)$  (in red) for N = 4, minimum value to cover the field of study defined within the framework of this thesis work. Comparing the responses in Figure 4.4 allows us to appreciate the effectiveness of the proposed method. Indeed, the behavior associated with the anti-resonance and the resonance of the first three modes is perfectly reproduced. Beyond that, the side effect linked to the truncation (N = 4) appears clearly.



Figure 4.5 – In the range[100; 4000] Hz and for m = 0.75, frequency responses of  $Y_{in}(s)$  (in black) and of  $Y_N(s)$  (in red) for N = 4

Remark

The denominator Den(s) of  $Y_N(s)$  is defined by:

$$Den(s) = s \prod_{i=1}^{N} Den_i(s) \quad , \tag{4.14}$$

$$Den_i(s) = 1 + 2\zeta_{pi} \left(\frac{s}{\omega_{pi}}\right)^{2-m} + \left(\frac{s}{\omega_{pi}}\right)^2 \quad . \tag{4.15}$$

with

The roots of Den(s) are solution of Den(s) = 0. Apart from the obvious root s = 0, the search for the roots is done by solving

$$\prod_{i=1}^{N} Den_i(s) = 0 \quad , \tag{4.16}$$

which amounts to looking for the roots of each  $Den_i(s)$  polynomial.

The interested reader will find in the thesis (Ivanova, 2017) developments concerning, in particular, the search for the roots of a polynomial of the form of  $Den_i(s)$ .

Note that the ratio  $\omega_{pi}/\omega_{zi}$  only depends on  $c_{pi}$  and  $c_{zi}$  namely:

$$\begin{cases} \frac{\omega_{pi}}{\omega_{zi}} = \frac{c_{pi}}{c_{zi}} , \qquad (4.17) \end{cases}$$

and that the ratio  $\zeta_{pi}/\zeta_{zi}$  depends on  $c_{pi}$ ,  $c_{zi}$  and m, that is to say:

$$\frac{\zeta_{p_i}}{\zeta_{z_i}} = \left(\frac{c_{z_i}}{c_{p_i}}\right)^m . \tag{4.18}$$

Like the recursive factors  $\alpha$  and  $\eta$  used by Oustaloup (Oustaloup A., 1995) to establish the link between non-integer derivation and frequency recursion, ratios  $\alpha_i$  and  $\eta_i$  (whose values depend here on rank *i*) are introduced:

$$\begin{cases} \alpha_{i} = \frac{\omega_{pi}}{\omega_{zi}} = \frac{c_{pi}}{c_{zi}} > 1 \\ \\ \eta_{i} = \frac{\omega_{z_{i+1}}}{\omega_{p_{i}}} = \frac{c_{z_{i+1}}}{c_{p_{i}}} > 1 \end{cases}$$

$$(4.19)$$

Taking into account relations (4.18) and (4.19), the ratio of the damping factors  $\zeta_{zi}$  and  $\zeta_{pi}$  of the same cell of rank *i* is then written only as a function of  $\alpha_i$  and of the order *m*, that is to say:

$$\frac{\zeta_{p_i}}{\zeta_{z_i}} = \frac{1}{\alpha_i^m} < 1 \quad , \tag{4.20}$$

and the passage relations of the parameters of the cell of rank i and those of the cell of rank i + l, are given by

$$\begin{cases} \frac{\omega_{z_{i+1}}}{\omega_{z_i}} = \alpha_i \eta_i > 1 \quad ; \quad \frac{\zeta_{z_{i+1}}}{\zeta_{z_i}} = \frac{1}{\left(\alpha_i \eta_i\right)^m} < 1 \\ \frac{\omega_{p_{i+1}}}{\omega_{p_i}} = \alpha_i \eta_i > 1 \quad ; \quad \frac{\zeta_{p_{i+1}}}{\zeta_{p_i}} = \frac{1}{\left(\alpha_i \eta_i\right)^m} < 1 \end{cases}$$

$$(4.21)$$

Relations (4.21) show that:

- the resonance frequencies  $\omega_{pi}$  and anti-resonance  $\omega_{zi}$  are independent of the order *m* and increase with rank *i*;

- the damping factors depend on the order *m* and decrease with rank *i* for a fixed *m*.

Table 4.2 specifies the different parametric values which characterize the cells of ranks 1 to 4 within the framework of the work of this thesis.

| Rank i                | 1     | 2     | 3     | 4      |
|-----------------------|-------|-------|-------|--------|
| $\omega_{zi}$ (rad/s) | 1814  | 5412  | 9030  | 12660  |
| $\omega_{pi}$ (rad/s) | 3639  | 7218  | 10860 | 14460  |
| C <sub>zi</sub>       | 1.572 | 4.688 | 7.823 | 10.967 |
| C <sub>pi</sub>       | 3.153 | 6.253 | 9.408 | 12.527 |
| $\alpha_i$            | 2     | 1.334 | 1.203 | 1.142  |
| $\eta_i$              | 1.487 | 1.251 | 1.166 |        |
| $\alpha_i \eta_i$     | 2.983 | 1.668 | 1.402 |        |

Table 4.2 – Parametric values which characterize the cells of ranks 1 to 4 in the framework of the work of this thesis

Figure 4.6 presents the variation of the damping factors  $\zeta_{z1}$  and  $\zeta_{p1}$  of rank 1 (1<sup>st</sup> mode of the resonator) according to the order *m* ranging between 0 and 1. These plots are obtained from the analytical relations (4.13).

These plots show two zones:

- Zone 1, where the damping factors increase with the order *m*, with m = 0 (conservative case) up to a maximum value, that is to say: max  $[\zeta_{zl}] = 0.0265$  for m = 0.6 and max  $[\zeta_{pl}] = 0.0186$  for m = 0.43;

- Zone 2, where, on the contrary the damping factors decrease until a value of  $\zeta_{zl} = 0.0228$ and  $\zeta_{pl} = 0.0113$  for m = 1.



Figure 4.6 – Variation of the damping factors  $\zeta_{zl}$  and  $\zeta_{pl}$  of the first mode of the resonator as a function of the order *m* between 0 and 1

Moreover, the expressions (4.13) of  $\zeta_{z1}$  and  $\zeta_{p1}$  clearly show that for a given rank *i* and an order *m*, the damping factors are all the greater (and therefore the visco-thermal losses) as the length *L* of the acoustic tube is large and its radius *r* is small.

Figures 4.7 and 4.8 show the frequency responses of the input admittance  $Y_{in}(s)$  of the resonator for different values of the fractional order *m* between 0 and 1, with the frequency axis scaled:

- logarithmic over the range [100; 2000] Hz (Figure 4.7);
- linear over the range [100; 1400] Hz (Figure 4.8).



Figure 4.7 – Bode plots of the admittance  $Y_{in}(s)$  of the resonator for different values of the fractional order *m* over the range [100; 2000] Hz



Figure 4.8 – Frequency responses *Y<sub>in</sub>(jω)* of the resonator admittance for different values of the fractional order *m* with the linear scale frequency axis in the range [100; 1400] Hz

In addition to Figures 4.7 and 4.8, Figures 4.9 to 4.11 present the distributions:

- anti-resonance  $\omega_{zi}(o)$  and  $\omega_{pi}(x)$  resonance frequencies (Figure 4.9),
- damping factors  $\zeta_{zi}$  (Figure 4.10.a) and  $\zeta_{pi}$  (Figure 4.10.b),
- constants  $c_{zi}$  and  $c_{pi}$  (Figure 4.11.a), as well as factors  $\alpha_i$ ,  $\eta_i$  and  $\alpha_i^* \eta_i$  (Figure 4.10.b),

according to the first 4 modes admittance of the  $Y_{in}(s)$  of the resonator.

All its figures clearly illustrate the remarks already made concerning the different parameters of  $Y_N(s)$  as a function of rank *i*.



Figure 4.9 – Distribution of anti-resonance  $\omega_{zi}(o)$  and resonance  $\omega_{pi}(x)$  frequencies according to the first 4 modes of the admittance Yin(s) of the resonator



Figure 4.10 – Distribution of damping factors  $\zeta_{2i}$  (a) and  $\zeta_{0i}$  (b)according to the first 4 ranks of the  $Y_N(s)$  admittance of the resonator for 5 values of the order *m* (0; 0.25; 0.5; 0.75; 1)







Figure 4.11 – Distribution of the constants  $c_{zi}(o)$  and  $c_{pi}(x)$  (a);  $\alpha_i$  (o),  $\eta_i(x)$  and  $\alpha_i(o) * \eta_i(x)$  (b) according to the first 4 ranks of the admittance  $Y_N(s)$  of the resonator

### 4.3.2 - Case of the Open Loop Transfer Function

The frequency domain analysis of the influence of the fractional order *m* continues in this section with the open-loop transfer function,  $\beta(s)$ , established in Chapter 3.

As a reminder, its frequency response  $\beta(j\omega)$  in steady harmonic system resulting from the linearization carried out in Chapter 3 has the expression:

$$\beta(j\omega) = \beta_0 \ j\omega \ e^{-\tau j\omega} \ Y_{in}(j\omega) \ , \tag{4.22}$$

with  $\beta_0$  a constant whose expression is given by:

$$\beta_0 = \frac{h \exp(\alpha_i w)}{S} \frac{\delta_d}{w} \rho \left( 1 - \left( \tanh\left(-\frac{x_0}{b}\right) \right)^2 \right) , \qquad (4.23)$$

the gain and the phase of  $\beta(j\omega)$  expressed as:

$$\begin{cases} \beta(\omega) = |\beta(j\omega)| = \beta_0 \ \omega |Y_{in}(j\omega)| \\ \phi(\omega) = \arg \beta(j\omega) = \frac{\pi}{2} - \tau \ \omega + \arg Y_{in}(j\omega) \end{cases}.$$
(4.24)

Always as a reminder:

- the gain  $\beta(\omega)$  of the open loop does not depend on the flow speed  $U_j^e$  at the output of the channel and therefore on the pressure  $P_m = \text{cst}$  at the input of the mouthpiece;

- on the other hand, the phase  $\phi(\omega)$  of the open loop depends on the speed of flow  $U_j^e$  through the delay  $\tau = \frac{w}{0.4 U_j^e}$ , the latter being as much greater as  $U_j^e$  is weak.

In addition, for this linearized loop system within the scope of study of this thesis work, the two auto-oscillation conditions are:

and

$$\begin{cases} \beta_{u} = \left| \beta \left( j \omega_{u} \right) \right| = \beta_{0} \, \omega_{u} \left| Y_{in} \left( j \omega_{u} \right) \right| = 1 \\ \phi_{u} = \arg \beta \left( j \omega_{u} \right) = \frac{\pi}{2} - \tau \, \omega_{u} + \arg Y_{in} \left( j \omega_{u} \right) = 0 \end{cases}, \tag{4.25}$$

 $\omega_u$  being the angular frequency at unity gain.

In fact, as already specified in Chapter 3, the stationary periodic system observed in the time domain (see following paragraph), in particular in example 1 ( $P_m = 400$  Pa) and example 2 ( $P_m = 1000$  Pa), does not result from these self-oscillation conditions. This system is the result of an unstable behavior (in closed loop) which associated with the nonlinear phenomenon of saturation of the *tanh(.)* function leads to this steady periodic system.

These conditions of stationary periodic system (in closed loop) result in open loop by a gain  $\beta(\omega_0)$  strictly greater than the unit with the angular frequency  $\omega_0$  for which the phase  $\phi(\omega_0)$  is zero, that is to say:

If at  $\omega = \omega \omega$ ,

$$\begin{cases} \phi(\omega_0) = \arg \beta(j\omega_0) = \frac{\pi}{2} - \tau \,\omega_0 + \arg Y_{in}(j\omega_0) = 0 \\ , \qquad (4.26) \end{cases}$$

$$\beta(\omega_0) = \left| \beta(j\omega_0) \right| = \beta_0 \,\omega_0 \left| Y_{in}(j\omega_0) \right| > 1$$

then the system is unstable (closed loop).

For the five values of the order *m* (0; 0.25; 0.5; 0.75; 1), Figures 4.12 and 4.13 show the frequency responses  $\beta(j\omega)$  of the open-loop transfer function in the planes:

- from Bode (Figure 4.12);
- from Black-Nichols (Figure 4.13),

for

-  $P_m = 400$  Pa (Figures 4.12.a and 4.13.a);

-  $P_m = 1000$  Pa (Figure 4.12.b and 4.13.b).

Figure 4.14 is a zoom of Figure 4.13 in the Black-Nichols plane around the point  $(0^\circ, 0 \text{ dB})$ , or  $(0^\circ, 1)$ , thus allowing to better visualize the conditions (4.26).







for  $P_m = 400$  Pa (a) and for  $P_m = 1000$  Pa (b),

for 5 values of the order *m* (0; 0.25; 0.5; 0.75; 1)



*(b)* 

Figure 4.14 – Zoom of the frequency responses  $\beta(j\omega)$  in open loop in the Black-Nichols plane around the point (0°; 0 dB) for  $P_m = 400$  Pa (a) and for  $P_m = 1000$  Pa (b), for 5 values of the order *m* (0; 0.25; 0.5; 0.75; 1)

The observation of the frequency responses  $\beta(j\omega)$  of the open loop, in particular in the Black-Nichols plane with Figure 4.13, makes it possible to affirm that for the two values of the pressure  $P_m = 400$  Pa and 1000 Pa, and whatever the value of *m* between 0 and 1, at the angular frequency  $\omega_0$  for which the phase of the open loop is equal to 0° (horizontal line in blue on the phase diagrams of Figures 4.12.a and 4.12. b), the open loop gain value is greater than 1 (i.e. 0 dB).

As a reminder, the analysis of the stationary periodic system presented in Chapter 3 shows that for a constant pressure  $P_m$  at the entrance of the mouthpiece (that is to say at the level of the artificial mouth) belonging to a very precise interval, the angular frequency  $\omega_0$  (for which the phase  $\phi(\omega_0)$  is zero) is very close to the angular frequency  $\omega_{pi}$  of the mode *i* concerned. To illustrate graphically the search for the value of  $\omega_0$ , the condition  $\phi(\omega_0) = 0$  is rewritten in the form:

$$\phi(\omega_0) = 0 \implies \frac{\pi}{2} + \arg Y_{in}(j\omega_0) = \tau \omega_0 , \qquad (4.27)$$

the solution  $\omega_0$  resulting from the intersection of the plots associated with the expressions located to the right and to the left of the equal sign of relation (4.27).

Thus, Figure 4.15 presents the frequency responses of  $\pi/2 + \arg(Y_{in}(j\omega))$  for five values of the order *m*, as well as the line associated with the expression  $\tau \omega$  for:

- **example 1** where  $P_m = 400$  Pa (right in blue Figure 4.15.a) and where mode 1 is concerned ( $\omega_{p1} = 3639$  rad/s, is 579.5 Hz), the solution being  $\omega_0 = 3577$  rad/s (570 Hz);

- example 2 where  $P_m = 1000$  Pa (right in purple Figure 4.15.b) and where mode 2 is concerned ( $\omega_{p2} = 7.218$  rad/s, is 1.149 Hz), the solution being  $\omega_0 = 7.096.4$  rad/s (1.130 Hz).

Finally, Figure 4.16 shows for the two examples  $P_m = 400$  Pa (\_\_\_\_) and  $P_m = 1000$  Pa (\_\_\_\_), the variation of the open loop gain  $\beta(\omega_0)$  in dB scale (Figure 4.16.a) and in linear scale (Figure 4.16.b) as a function of the fractional order *m* between 0 and 1 on a linear scale.

Observation of these plots leads to two remarks:

- for a given order *m*, the value of the open loop gain  $\beta(\omega_0)$  greater than unity increases with the pressure  $P_m$ ;

- for a given pressure  $P_m$ , the curves of  $\beta(\omega_0)$  as a function of the order *m* show a different local minimum depending on the value of  $P_m$ .







Figure 4.16 – For  $P_m = 400$  Pa ( \_\_\_\_\_) and for  $P_m = 1000$  Pa ( \_\_\_\_\_), variation of the open loop gain  $\beta(\omega_0)$  in dB scale (a) and in linear scale (b) as a function of the order *m* 

# 4.4 - Time Domain Illustration of the Influence of Fractional Order

As a reminder (see Chapter 3), the time simulations carried out at  $P_m$  = cst reveal three phases:

- phase 1: start-up with a convergent transient system;

- phase 2: divergent transient system;

- phase 3: stationary periodic system.

The objective of this paragraph is to illustrate through time simulation the influence of order m on the periodic system of phase 3.

Thus, for this periodic system of phase 3, Figures 4.17 to 4.19 show for:

- the two pressures  $P_m = 400$  Pa (example 1) and  $P_m = 1000$  Pa (example 2)

- and the five values of the order *m* (0; 0.25; 0.5; 0.75; 1),

time responses:

- transverse displacement  $\eta(w, t)$  of the jet (Figure 4.17),

- the pressure  $\Delta p(t)$  at the input of the resonator (Figure 4.18),

- the acoustic speed  $v_{ac}(t)$  (Figure 4.19).

To help the reader to identify the variables  $\eta(w,t)$ ,  $\Delta p(t)$  and  $v_{ac}(t)$ , the functional diagram of the recorder is recalled at the beginning of each of these figures with the concerned variable in red.

Observation of all of these time responses leads to the following remarks.

For a given pressure  $P_m$  and for the five values of the order m (0; 0.25; 0.5; 0.75; 1), the amplitudes of the time responses are all maximum for m = 0 (conservative case) and minimum for an order m included between 0.5 and 0.75.

Although one cannot analytically establish a link between open-loop behavior and that observed in closed-loop due to the non-linear character, a trend emerges regarding the influence of the order m on the amplitudes of the time responses. Indeed, we observe in the time domain a similarity with the influence of the order m on the open loop gain  $\beta(\omega_0)$  (Figure 4.16). An analysis based on the method of the first harmonic or a decomposition in series of Volterra, for example, is a prospect which should make it possible to confirm this tendency.







Figure 4.17 – Time responses of the transverse displacement  $\eta(w, t)$  of the jet (a) for 5 values of the order m (0; 0.25; 0.5; 0.75; 1), for a pressure  $P_m = 400$  Pa (b) and  $P_m = 1000$  Pa (c)









Figure 4.18 – Time responses of the pressure  $\Delta p(t)$  at the input of the resonator (a) for 5 values of the order *m* (0; 0.25; 0.5; 0.75; 1), for a pressure  $P_m = 400$  Pa (b) and  $P_m = 1000$  Pa (c)

(a)











(c)

Figure 4.19 – Time responses of the acoustic velocity  $v_{ac}(t)$  (a) for 5 values of the order *m* (0; 0.25; 0.5; 0.75; 1), for a pressure  $P_m = 400$  Pa (b) and  $P_m = 1000$  Pa (c)

### 4.5 – Conclusion

One of the most important contributions of this chapter is related to the impedance of the resonator with the development of a method in several stages allowing to pass from the form  $Y_{in}(s)$  (relation (4.1)) from Chapter 2 to the form  $Y_N(s)$  (relation (4.5)).

Indeed, even if the form  $Y_{in}(s)$  remains the reference for the plot of the frequency responses, the form  $Y_N(s)$  makes it possible to reveal in an explicit way the parameters  $\omega_{zi}$ ,  $\zeta_{zi}$ ,  $\omega_{pi}$ ,  $\zeta_{pi}$  which characterize the resonator modes.

The step of determining the constants  $c_{zi}$  and  $c_{pi}$  (relations (4.12)) makes it possible to define each cell of rank *i* from the 4 parameters  $\omega_{z1}$ ,  $\zeta_{z1}$ ,  $\omega_{p1}$ ,  $\zeta_{p1}$  of the cell of rank 1 (obtained using an analytical development: Appendix A) and the 2 factors  $\alpha_i$  and  $\eta_i$  (relations (4.19)) of the cell of rank *i* (obtained numerically).

This method, allowing to switch from the form  $Y_{in}(s)$  to the form  $Y_N(s)$ , facilitates the analysis of the influence of the order *m*. Thus, it is demonstrated that the order *m*:

- has no influence on the angular frequencies  $\omega_{zi}$  and  $\omega_{pi}$ ,

- has an influence on the damping factors  $\zeta_{zi}$  and  $\zeta_{pi}$ .

More precisely, for a given rank i and for a variation of the order of m from 0 to 1, the damping factors are zero for m = 0 (conservative case), go through a maximum, then decrease. In addition, for a fixed order m, the damping factors decrease with rank i.

Note that the expressions (4.8) of  $\zeta_{zi}$  and  $\zeta_{pi}$  clearly show that for a given rank *i* and an order *m*, the damping factors are all the greater (and therefore the visco-thermal losses) as the length *L* of the acoustic tube is large and its radius *r* is small.

Finally, the analysis of the influence of the order *m* on the conditions of existence of the stationary periodic system observed in the time domain is an important contribution which completes the first analysis presented in Chapter 3. Indeed, these conditions are reflected in open loop by a gain  $\beta(\omega_0)$  strictly greater than the unit with the angular frequency  $\omega_0$  for which the phase  $\phi(\omega_0)$  (of the open loop) is null. Thus, for a given order *m*, the value of the open loop gain  $\beta(\omega_0)$  (greater than unity) increases with the pressure  $P_m$  at the mouthpiece input (i.e. in the artificial mouth) and for a given pressure  $P_m$ , the curves of  $\beta(\omega_0)$  as a function of the order *m* have a different local minimum depending on the value of  $P_m$ .

# General Conclusion and Perspective

The interpretation and discussion of the main results presented in this thesis are the subject of the general conclusion.

**Chapter 1** constitutes the first essential modeling step for the reader unfamiliar with wind instruments. Indeed, due to its structuring and its didactic progression, it allows to "soak up" the dynamic behavior of an acoustic tube of constant radius including the effect of visco-thermal losses due to the boundary layers in the vicinity of the walls. The **knowledge model** used in this work, called Webster-Lokshin 1D, is mono-spatial dependent. It characterizes the linear propagation of acoustic waves in axially symmetrical tubes taking into account visco-thermal losses. Thus, in an axisymmetric tube of constant section, the sound pressure and the sound flow rate are governed by the horn equation, known as Webster-Lokshin, and the Euler equation. This dissipative model includes a term that involves a fractional derivation of order m = 0.5. The classical resolution in the operational domain of the system of partial differential equations leads to the analytical expression of the acoustic impedance and admittance of the tube as a function of its length L, of its radius r and of the position x of the tube point considered.

One of the contributions of this chapter is the introduction of an extension of the fractional expression used to take into account visco-thermal losses where the order m is usually equal to 0.5. This extension makes it possible to consider an uncertainty of the order m, whose domain bounds are 0 (conservative system) and 1. Another contribution is the introduction of a system vision aiming to causally decompose the global model into sub models, to facilitate analysis in the frequency domain. One of the conclusions of this frequency analysis is that the fractional model can be simplified over the audible frequency range [20; 20,000]. Finally, for the simulation in the time domain, two rational forms composed of an integrator of order one and N cells of the second order, one in cascade and the other in parallel, are introduced as a *model of behavior*. The parametric values of the rational cascade form are determined using the *Frequency Domain System Identification* (FDSI) module of the CRONE Toolbox with the frequency response of the *knowledge model* as target. As for the parametric values of the parallel form, they are obtained by a decomposition into simple elements of the cascade form.

**Chapter 2** falls within the framework of the dynamics of complex systems. Indeed, the self-oscillation mechanism, characterized by the ability of the instrument to produce an acoustic wave from a stationary or quasi-stationary energy source with respect to the acoustic variables, is inseparable from the non-linear nature of the exciter and its coupling with the resonator. This phenomenon results from the oscillation of a naturally unstable air jet around a bevel. Thus, the self-oscillation mechanism relies on a synchronization of the oscillation of

the jet and the acoustic waves. The jet-bevel system must therefore excite the resonator at the periodicity of the acoustic field.

From a bibliographic synthesis, a complete nonlinear model frequently used in the literature is developed in this chapter. Faced with the poor digital conditioning of this model, a solution is proposed in order to be able to develop a digital simulator programmed under MatLab / Simulink. This solution constitutes a first contribution in this chapter. A scenario is then defined while respecting, not only the domain of validity of the nonlinear model, but also the values of pressure attainable experimentally by the artificial mouth presented in chapter 4. The detailed analysis of the simulated time responses makes it possible, in particular, to observe that the variations of a certain number of physical quantities (pressure at the input of the resonator, acoustic speed, etc.) are small and around zero. This observation allows us to legitimize the realism of the hypothesis of small variations. Thus, two linearized models, one for the analysis of *phase 1* (start of the simulation), the other for that of *phase 3* (self-oscillations) are developed and used to better understand the phenomena with the self-oscillation conditions.

Chapter 3 presents the model-based design approach of an artificial mouth, its realization and the experimental characterization of each of its components, in order to adjust the nonlinear models integrated in the digital simulator associated with this part. The use of an artificial mouth, in the context of comparative studies of the complex dynamic phenomena present in wind instruments, is necessary to ensure reproducibility of test conditions that are difficult to achieve with a human being. This three-step engineering approach is essential to fully understand the non-linear behavior of each component, define the limits of their operating range and establish the influence of different physical quantities on the pressure at the mouthpiece input. Note that this last point is essential to understand the need for a pressure regulation loop. Thus, the first step in this approach was the development of a first simulator on the basis of a bibliographic synthesis concerning artificial mouths used in musical acoustics. This first simulator made it possible to fully understand the operation of such a system, thus facilitating the choice and sizing of the components of the regulation loop (the servo valve, pressure and flow sensors, temperature sensors, the volume of the pneumatic capacity, etc.), as well as the summary of the pressure control (feedforward and feedback parts). The second step was the realization of the test bench with the assembly of the various parts and their tuning. Finally, the third step consisted of a second iteration, not only of modeling applied precisely to the experimental device produced, but also of synthesis of the pressure control. This step lead to a second digital simulator. The latter was the subject of a retiming from comparisons between experimental and simulation results. Thus, for a real recorder (made of plastic with a length of the resonator L = 30 cm, an average radius r = 5mm, a fractional order associated with visco-thermal losses m = 0.5, ..., the simulator reproduces with a good precision the study conditions specified in chapters 2 and 3. This very real behavior (reproduced in simulation) is considered in chapter 4 as the nominal reference behavior.

Finally, *Chapter 4* brings together the developments and results of the first three chapters to deepen the analysis of the influence of the fractional order m on the mechanism leading to the stationary periodic system.

One of the strong points of this chapter is the development of a method allowing to pass from the initial fractional form of the  $Y_{in}(s)$  impedance from chapter 1, where the modes of the resonator *appear implicitly* through the periodicity from the *tanh(.)* function, to another fractional form  $Y_N(s)$  where, on the contrary, the modes of the resonator *appear explicitly* through the product of N fractional cells of order 2. Indeed, even if the form  $Y_{in}(s)$  remains the reference for the plot of the frequency responses, the form  $Y_N(s)$  makes it possible to reveal in an explicit way the parameters  $\omega_{zi}$ ,  $\zeta_{zi}$ ,  $\omega_{pi}$ ,  $\zeta_{pi}$  which characterize the modes of the resonator. This method facilitates the analysis by showing that the order m has no influence on the angular frequencies  $\omega_{zi}$  and  $\omega_{pi}$ , but that it does have an influence on the damping factors  $\zeta_{zi}$  and  $\zeta_{pi}$ . For a given rank i and for a variation of the order of m from 0 to 1, the damping factors are zero for m = 0 (conservative case), go through a maximum, then decrease. In addition, for a fixed order *m*, the damping factors decrease with rank *i*. Finally, the analysis of the influence of the order m on the conditions of existence of the stationary periodic system observed in the time domain using the complete simulator (artificial mouth + nonlinear exciter + resonator) is an important contribution. which completes the first analysis made in chapter 2. Indeed, these conditions result in open loop by a gain  $\beta(\omega_0)$  strictly greater than the unit with the angular frequency  $\omega_0$  for which the phase  $\phi(\omega_0)$  (of the open loop) is zero. Thus, for a given order m, the value of the open loop gain  $\beta(\omega_0)$  (greater than unity) increases with the pressure  $P_m$  at the entrance to the mouthpiece (i.e. in the artificial mouth) and for a given pressure  $P_m$ , the curves of  $(\omega_0)$  as a function of the order m have a different local minimum depending on the value of  $P_m$ .

More generally in terms of fractional models, this study of the influence of order m on visco-thermal losses leads to a finding similar to that already made in other fields. Indeed, the main interest of the fractional form lies in the parametric parsimony, that is to say the capacity possessed by the integro-differential operator of non-integer order to model with a minimum of parameters the greatest number of dynamic phenomena.

The medium-term outlook falls directly within the continuity of the work in progress, that is to say within the framework of the dynamics of complex systems.

From a modeling point of view, a first perspective concerns the introduction of the extension of the fractional expression used to take account of visco-thermal losses in the more general framework of the pavilion equation as encountered in the literature of wind instruments. Indeed, in this case, the radius of the acoustic tube depends on the longitudinal position x of the point considered within the resonator. In addition, its apparent length may vary depending on the number of closed holes. Thus, taking into account a fractional order m

between 0 and 1, associated with variations in radius and apparent length, should make it possible to extend the domain of validity of the model developed in Chapter 1.

At the same time, the increase in the field of study defined in chapters 2 and 4 by considering a pressure  $P_m$  at the entrance to the mouthpiece, no longer constant, but variable (like that generated by a musician) constitutes a second perspective. To achieve this goal, it is necessary to solve the problem of causality due to the presence of a shunt in the model used to describe the interaction between the air jet and the bevel. The solution proposed in this thesis, namely the use of a frequency bounded derivation operator, only makes it possible to better condition the model numerically for a constant pressure  $P_m$  from the initial moment of the simulation.

*From a systems theory point of view*, an interesting perspective is the definition of a fractional resonator with one degree of freedom (1 dof). The first step will consist in defining its structure from the decomposition into simple elements of the product of the *N* fractional cells of order 2 present in the expression of the acoustic impedance  $Y_N(s)$  introduced in chapter 4. Then, the structure being perfectly defined, the study of the fundamental properties of this fractional resonator with 1 dof will constitute the second stage. The progression proposed within the framework of systems whose dynamics are described by differential equations of integer orders, with first the study of systems with 1 dof, then that of systems with *q* dof which in the modal space are in the form of *q* 1-DOF systems, will be generalized to SDNE (Non-Integer Derivative Systems which include the dynamic systems of integer orders).

Finally, *from an experimental point of view* at the level of the artificial mouth, the integration of components designed specifically for the medical sector, in particular those used in automatic respiratory assistance devices (servo valves, sensors, etc.), should make it possible to improve the performance of the test bench.

# From the initial form of the fractional impedance $Y_{in}(s)$ from chapter 1 to its fractional form $Y_N(s)$ from chapter 4

This appendix presents the details of the analytical developments allowing to pass from the initial form of the fractional impedance  $Y_{in}(s)$  of chapter 1 to its form  $Y_N(s)$  of chapter 4.

Thus, as a reminder and for the field of study defined within the framework of this thesis work, the input admittance  $Y_{in}(s)$  of the acoustic tube (resonator) of length *L* and radius *r* has the expression:

$$Y_{in}(s) = \frac{S}{Z_{ac}} I_m(s) \frac{1}{\tanh(z(s))} , \qquad (A.1)$$

$$I_m(s) = \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}} \qquad (A.2)$$

$$(A.2)$$

with

and

 $z(s) = \left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}} \quad . \tag{A.3}$ 

The expression (A.1) of  $Y_{in}(s)$  can be put in a form  $Y_N(s)$  composed of an integrator of order 1 in cascade with N fractional cells of order 2, namely:

$$Y_{in}(s) = \lim_{N \to \infty} Y_N(s) \quad , \tag{A.4}$$

$$Y_N(s) = \frac{Y_0}{s} \prod_{i=1}^N \left( \frac{1 + 2\zeta_{zi} \left(\frac{s}{\omega_{zi}}\right)^{2-m} + \left(\frac{s}{\omega_{zi}}\right)^2}{1 + 2\zeta_{pi} \left(\frac{s}{\omega_{pi}}\right)^{2-m} + \left(\frac{s}{\omega_{pi}}\right)^2} \right), \qquad (A.5)$$
$$Y_0 = \frac{S}{\rho_a L} \qquad (A.6)$$

with

where

and

$$\omega_{zi} = c_{zi} \omega_L \quad ; \quad \zeta_{zi}(m) = \frac{2 K_0}{r} \left(\frac{L}{c_{zi}}\right)^m m \qquad (A.7)$$
$$\omega_{pi} = c_{pi} \omega_L \quad ; \quad \zeta_{pi}(m) = \frac{2 K_0}{r} \left(\frac{L}{c_{pi}}\right)^m m$$

where  $c_{zi}$  and  $c_{pi}$  are constants resulting from the procedure allowing to pass from  $Y_{in}(s)$  to  $Y_N(s)$  This procedure consists of 4 steps:

- Step 1: finite expansion of the function tanh(z) into a continuous fraction in z;

- Step 2: reduction of the continuous fraction to a ratio of two polynomials in z;

- Step 3: factorization of each polynomial in z;

- Step 4: rewriting  $Y_{in}(s)$  in the form  $Y_N(s)$ , namely an integrator of order 1 in cascade with a product of N fractional cells of order 2.

### Step 1

The finite expansion of the function tanh(z) into a continuous fraction in z is given by (Wyman & Wyman, 1985)

$$tanh(z) = \frac{z}{1 + \frac{z^2}{3 + \frac{z^2}{5 + \frac{z^2}{7 + \frac{z^2}{9 + \dots}}}},$$
 (A.8)

or, by adopting a more condensed writing convention,

$$tanh(z) = \frac{z}{|1|} + \frac{z^2}{|3|} + \frac{z^2}{|5|} + \frac{z^2}{|7|} + \frac{z^2}{|9|} + \dots \quad .$$
(A.9)

### Step 2

The reduction of the continued fraction defined by relation (A.8) leads to an expression in the form of a product of the variable z and a ratio of two polynomials in z voluntarily limited to an order of 4/4 for the analytical developments, namely:

$$tanh(z) = z \left( \frac{num(z)}{den(z)} \right) ,$$
 (A.10)

$$\begin{cases} num(z) = n_0 + n_2 z^2 + n_4 z^4 \\ den(z) = d_0 + d_2 z^2 + d_4 z^4 \end{cases},$$
(A.11)

or 
$$\begin{cases} n_0 = 3 \times 5 \times 7 \times 9 = 945 \\ n_2 = 3 \times 5 + 7 \times 9 + 3 \times 9 = 105 \\ n_4 = 1 \end{cases}$$
 and 
$$\begin{cases} d_0 = 3 \times 5 \times 7 \times 9 = 945 \\ d_2 = 3 \times 5 + 7 \times 9 + 3 \times 9 + 5 \times 7 \times 9 = 420 \\ d_4 = 1 + 5 + 9 = 15 \end{cases}$$
 (A.12)

# Step 3

Factoring each polynomial in z requires finding their roots. To do this, we perform the change of variable by setting  $X = z^2$ , hence the system to be solved:

with

$$\begin{cases} num(z) = 0 \implies n_0 + n_2 X + n_4 X^2 = 0 \\ den(z) = 0 \implies d_0 + d_2 X + d_4 X^2 = 0 \end{cases}$$
(A.13)

Solving this system made up of two quadruple equations in X does not present any particular difficulty. Each polynomial in X has two distinct real roots, namely:

- for the numerator:

$$\begin{cases} X_{n1} = -95 \\ ; \\ X_{n2} = -9.94 \end{cases}$$
(A.14)

- for the denominator:

$$\begin{cases} X_{d1} = -25.533 \\ \\ X_{d2} = -2.467 \end{cases}$$
(A.15)

Finally, the expression (A.10) of tanh(z) is rewritten as:

$$tanh(z) = z \left( \frac{(z^2 + 9.94)(z^2 + 95)}{15(z^2 + 2.467)(z^2 + 25.533)} \right)$$
(A.16)

# Step 4

Finally, knowing that

$$z(s) = \left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}}, \qquad (A.17)$$

the introduction of the expression (A.17) of z(s) in the relation (A.16) of tanh(z) leads to:

$$tanh(z) = \left( \left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}} \right) \times \left( \frac{\left(\left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}}\right)^2 + 9.94} \right) \left( \left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}}\right)^2 + 95} \right) \right) \\ \frac{15 \left( \left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}\right)^2} + 2.467} \right) \left( \left(\frac{s}{\omega_L}\right) \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^m}{\left(\frac{s}{\omega_{r,m}}\right)^m}\right)^2} + 25.533} \right) \right) .$$
(A.18)

Taking into account the expression (A.1) of  $Y_{in}(s)$ , where  $I_m(s)$  is defined by:

$$I_{m}(s) = \sqrt{\frac{1 + \left(\frac{s}{\omega_{r,m}}\right)^{m}}{\left(\frac{s}{\omega_{r,m}}\right)^{m}}}, \qquad (A.19)$$

the introduction of the relation (A.18) in the expression (A.1) allows (after some manipulations and simplifications) the transition to the form  $Y_N(s)$  for N = 2, that is:

$$Y_{N=2}(s) = \frac{Y_0}{s} \prod_{i=1}^{2} \left( \frac{1 + 2\zeta_{zi} \left(\frac{s}{\omega_{zi}}\right)^{2-m} + \left(\frac{s}{\omega_{zi}}\right)^2}{1 + 2\zeta_{pi} \left(\frac{s}{\omega_{pi}}\right)^{2-m} + \left(\frac{s}{\omega_{pi}}\right)^2} \right),$$
(A.20)

with 
$$\begin{cases} c_{z1} = \sqrt{2.467} \\ c_{p1} = \sqrt{9.94} \end{cases}$$
 (A.21)

with the generalization of the relation (A.20) to any N is developed in chapter 4.

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