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Operating Theatre Planning and Scheduling under Material and Human Resource Constraints

Roberto Baretto

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Roberto Bargetto

Planification et programmation du bloc
opérateur sous contraintes de
ressources matérielles et humaines

Devant le jury composé de :

Della Croce, Federico, Prof.
Alfieri, Arianna, Prof.
Battaïa, Olga, Prof.
Demeulemeester, Erik, Prof.
Xie, Xiaolan, Prof.
Garaix, Thierry, HDR

Politecnico di Torino
Politecnico di Torino
KEDGE Business School
KU Leuven
Ecole des Mines de Saint-Etienne
Ecole des Mines de Saint-Etienne

Président
Rapporteure
Rapporteure
Examineur
Directeur de thèse
Co-encadrant

Spécialités doctorales
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 MECANIQUE ET INGENIERIE
 GENIE DES PROCEDES
 SCIENCES DE LA TERRE
 SCIENCES ET GENIE DE L'ENVIRONNEMENT

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 S. Drapier, professeur
 F. Gruy, Maître de recherche
 B. Guy, Directeur de recherche
 D. Graillot, Directeur de recherche

Spécialités doctorales
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 SCIENCES DES IMAGES ET DES FORMES
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 O. Boissier, Professeur
 JC. Pinoli, Professeur
 N. Absi, Maître de recherche
 Ph. Lalevé, Professeur

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YUGMA	Gallian	CR	Génie industriel	CMP

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Roberto BARGETTO

Operating Theatre Planning and Scheduling under Material and Human Resource Constraints

Speciality : Operations Research

Keywords : Operating Theatre, Operating Room Scheduling, Bed Management, Patient Admission, Emergency Scheduling, Human Resource Constraint, Nurse Scheduling, Discrete Event Simulation, Combinatorial Optimization, Stochastic Optimization

Abstract :

Operating rooms are bottleneck resources for the flow of surgical patients and represent the main costs hub in acute care hospitals. Operating room planning and scheduling methods need to meet requirements of economic and operational efficiency as well as those of surgery safety. This thesis focuses on operating room scheduling under material and human resource constraints.

Three decision levels are considered. Policies including bed management for the patients to surgery dates assignment and emergencies admission are evaluated by means of discrete event simulation and by considering resource shortages and wait times minimization objectives. The weekly scheduling of surgery is performed in two stages. A branch-and-price-and-cut algorithm is applied to schedule surgeries for a set of operating rooms under surgeon and nurse capacity constraints, surgery durations are deterministic. The second stage problem performs the nurse to surgery assignment considering objectives of nurse constant training, overtime minimization, visited operating rooms and by considering uncertain surgery duration. Finally, the insertion of emergency surgeries considering waiting time targets into given schedules of elective surgery is optimized by means of stochastic optimization techniques of queues systems. Developed problem solution methods are tested by means of numerical experiments and using real-world data from hospitals.

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Roberto BARGETTO

Planification et programmation du bloc opératoire sous contraintes de ressources matérielles et humaines

Spécialité: Recherche Opérationnelle

Mots clefs : Bloc opératoire, Programmation de interventions, Gestion des lits, Admission des patients, Insertion des urgences, Contraintes de ressource humaine, Affectation infirmiers, Simulation à événements discrets, Optimisation combinatoire, Optimisation stochastique

Résumé :

Le bloc opératoire représente une ressource critique de l'activité chirurgicale et constitue un goulot d'étranglement pour le flux des patients ainsi que le principal générateur de coût des établissements hospitaliers. Les outils d'aide à la planification de l'activité chirurgicale doivent être à la hauteur des enjeux en termes d'efficience économique, opérationnelle et de qualité des soins. Dans cette thèse, on aborde des problématiques connexes de planification du bloc opératoire en focalisant l'attention sur la programmation des procédures chirurgicales sous contraintes des ressources matérielles et humaines.

Trois niveaux de décisions sont traités. Les stratégies de planification des jours d'intervention sont optimisées en s'appuyant sur un modèle à simulation à événements discrets qui permet de jouer sur la projection des places disponibles en amont et en val du bloc, avec les critères d'améliorer les délais d'attente et l'insertion des urgences. La planification hebdomadaire se fait en deux étapes. Un algorithme de branch-and-price-and-cut planifie les chirurgies dans les salles sous des contraintes capacitaires de ressources infirmières et en considérant des durées opératoires déterministes. Une deuxième étape affecte le personnel infirmier avec des critères plus fins de rotation de l'activité et des dépassements horaires, en tenant compte de l'incertitude des durées opératoires. Enfin, l'insertion des urgences en cours de journée est étudiée par l'optimisation d'un système dynamique stochastique de file d'attente avec plusieurs délais d'urgence. Les méthodes sont validées par des expérimentations numériques utilisant des données hospitalières réelles.

Operating Theatre Planning and Scheduling under Material and Human Resource Constraints

Roberto Baretto

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Introduction en langue française

Les salles du bloc opératoire représentent les lieux physiques fondamentaux pour le déroulement de l'activité chirurgicale et constituent le goulot d'étranglement pour le flux des patients. En raison du haut niveau de standardisation, de technicité et professionnel du personnel que l'activité chirurgicale mobilise, le bloc opératoire représente aussi le principal centre de coût des établissements hospitaliers. D'ici l'exigence, de plus en plus grandissant, pour des outils de planification et programmation de l'activité chirurgicale à la hauteur des enjeux en termes d'efficacité économique, opérationnelle et de sécurité des patients. Dans cette thèse, on aborde des problématiques connexes de planification et programmation du bloc opératoire en focalisant l'attention sur la programmation des procédures chirurgicales sous contraintes des ressources matérielles, salles et lits des services, et humaines, chirurgiens et infirmiers du bloc opératoire.

Les différents problèmes traités se positionnent à différents niveaux de décision, du long-moyen terme de décision jusqu'à la décision en temps réel. Pour certaines problématiques, une attention spécifique est dédiée au flux des patients urgents.

Les méthodes prises en considération pour la résolution des problèmes traités couvrent une large ampleur des approches possibles. Ces approches incluent politiques simples évaluées par moyen de simulation à événement discrets et méthodes avancées d'optimisation combinatoire exacte et d'optimisation stochastique approchée. L'efficacité des méthodes développées et les gains pratiques apportés avec la résolution des problèmes considérés sont montrés par moyen de expérimentations numériques.

Dans le Chapitre 1, nous donnons une description du bloc opératoire, de la conception du lieu physique, du personnel impliqué dans l'activité chirurgicale et du parcours des patients chirurgicaux. La description du bloc opératoire est suivie d'une analyse concise de la littérature scientifique pour les problèmes de planification et programmation du bloc opératoire dans le domaine de la recherche opérationnelle.

Dans le Chapitre 2, une simulation à événements discrets permet d'évaluer des politiques de programmation des jours d'intervention chirurgical et d'admission des patients urgents en considérant comme critères d'évaluation les délais d'attente et d'insertion des urgences. Soit les politiques de programmation des chirurgies que d'admission des urgences incluent la gestion de lits des services en raison d'une projection des places disponibles en amont et en val du bloc opératoire. Les possibles combinaisons des politiques, programmation chirurgies et admission urgences, sont comparées par moyen des résultats numériques de la simulation.

La programmation hebdomadaire détaillé de la chirurgie se fait en deux étapes. Une méthode exact d'optimisation linéaire en nombres entiers résout le problème de la première étape, c'est à dire programmer, sous contraintes de ressource humaine, chirurgiens et les infirmiers du bloc opératoire, des chirurgies dans en ensemble des salles opératoires et sur un horizon de planification de plusieurs jours ; c'est décrit dans le Chapitre 3.

La solution de la première étape est complétée avec une affectation optimisée des infirmiers aux chirurgies considérant des objectifs opérationnels et de formation continue des infirmiers : dépassements horaires, nombre de salles opératoires visitées en cours de journée et rotation de l'activité ; ce sujet est traité dans le Chapitre 4. La formation continue avec rotation de l'activité est obtenu en privilégiant l'affectation des infirmiers pour l'exécution des chirurgies avec un nombre le plus large possible des chirurgiens. Le modèle d'affectation est présenté soit dans

sa version déterministe que dans sa version stochastique qui tient compte de l'incertitude des durées opératoires ; le modèle stochastique est résolu par moyen d'échantillonnage des scénarios.

L'efficacité de la programmation hebdomadaire des chirurgies dans les salles opératoires et de l'affectation des infirmiers par moyen des modèles proposés, et des méthodes d'optimisation développées pour les résoudre, est montrée avec les résultats des expérimentations numériques reportés dans le Chapitre 3 et dans le Chapitre 4.

Dans le Chapitre 5, on étudie des politiques pour la programmation en temps réel des urgences au bloc opératoire. Le problème de l'insertion en cours de journée des urgences dans un programme donnée de chirurgie planifiée est modélisé comme un problème d'optimisation stochastique d'un système dynamique de files d'attente. Les urgences sont caractérisées par différents délais d'urgence et ciblent de temps d'attente tolérés. L'optimisation prend comme objective la minimisation du coût total encouru pour les attentes excessives des urgences, les retards des chirurgies planifiées et les heures supplémentaires du personnel des salles opératoires. Les politiques sont comparées moyennant des expérimentation numériques.

Enfin, le Chapitre 7 résume le contenu de la thèse et réunit les perspectives de possibles recherches futures qui peuvent être développées ensuite des sujets traités dans la thèse ; le même texte en langue anglaise est dans le Chapitre 6.

Introduction

Operating rooms are the core physical components of the operating theatre and, if correctly dimensioned, the bottleneck for the flow of surgical patients. The high standardization of operating rooms points out and reflect the need for efficiency in the execution of surgical procedures and their planning and scheduling. The operating theatre, with its composing operating rooms, is the main costs hub of modern acute care hospitals and a unit that is both technology and labour intensive. The execution of surgeries and the high maintenance of equipment require high skills and technical knowledge of all the involved human resources, their costs account for a large part of the costs of running the operating theatre.

In reason of the high operational and maintenance costs, operating room scheduling has received relevant attention in the field of operations research and management science, but some gaps to close remain between the available literature and issues rising in practical contexts. We are persuaded that operating room scheduling that considers constraints of human resources as surgeons and nurses, and other relevant material resources as ward beds, needs to be studied in depth.

This thesis is motivated by the will to close or, at least, reduce research gaps by investigating the planning and scheduling of surgical cases considering not only the operating room capacity but also relevant resources other than operating rooms as human resources and ward beds. Special attention is devoted to a class of surgical patients, the emergency patients, that complicates the scheduling of surgery in the real-world as well as from a perspective of research. This thesis addresses the planning and scheduling of the operating theatre focusing on operating room scheduling under constraints of material and human resources, *i.e.*, ward beds, surgeons and surgical nurses.

The different problems addressed belong to different decision levels, from the middle-long planning to the real-time scheduling. The solution methods applied to problems cover a wide set of possible approaches, from simple scheduling policies evaluated by means of discrete event simulation to advanced algorithms of combinatorial optimization and approximated stochastic optimization. The effectiveness of developed solution methods, and the possible improvement for practical applications, are shown with extended numerical experiments.

In Chapter 1, we describe the operating theatre, its physical setting, the HRs involved in the surgery execution and the health-care pathway of surgical patients. A brief review of the literature about operating room planning and scheduling in the domain of operations research and management science is also reported in the chapter.

In Chapter 2, we describe a discrete event simulation model we developed for assessing a set of policies including bed management for the scheduling and the admission of elective and emergency patients. Admission and surgery days of elective patients are scheduled considering a forecast of the operating room capacity consumption and bed occupancy. The real-time admission of emergency patients is performed by assessing the current bed occupancy and the impact of the admission of emergencies on the future bed occupancy. The assessment of policies and their paring, scheduling and admission, is provided by means of numerical outcomes of simulation.

The weekly scheduling of surgeries is performed in two stages. An exact optimization algorithm, a branch-and-price-and-cut algorithm, we developed for solving an integrated operating

room planning and scheduling problem including resource constraints for surgeons and surgical nurses is described in Chapter 3. Surgeries selected from a waiting list and characterised by due dates are scheduled for a set of operating rooms under surgeon and nurse capacity constraints and over a planning horizon of several days. The resolution of this scheduling problem corresponds to the first stage of the weekly scheduling of surgeries.

In Chapter 4, we present the nurse to surgery assignment problem, a combinatorial optimization problem, in a new perspective by considering the constant training of nurses for non-technical skills (*i.e.*, interpersonal communication, coordination, judgement, etc.) as a problem objective. Nurses non-technical skills have crucial importance for the avoidance of errors inside the OR and the safety of surgery. The problem is presented in both its deterministic and stochastic version, the stochastic problem formulation considers the uncertainty of surgery durations and is solved by means of sample averaging of scenarios. The nurse to surgery assignment problem corresponds to the second stage of the weekly scheduling of surgeries.

In Chapter 5, we present a study of dynamic scheduling policies for the real-time insertion of emergency surgeries into operating room surgery schedules. Operating room schedules are of elective surgery and assumed to be given in the planning horizon. Emergency surgeries are characterized by different emergency levels and a waiting time target is associated to every surgery. The problem objective minimizes the total cost incurred by exceeding waiting time targets of emergency surgeries, delaying elective surgeries and incurring overtime for operating rooms staff. The problem is modelled as an event-based stochastic programming problem and both simple policies and advanced stochastic optimization methods are applied for the problem solution.

The thesis conclusion and possible perspectives for future researches beyond this thesis are reported in Chapter 6.

Chapter 1

The operating theatre

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Abstract

In this chapter, we describe the physical setting of the operating theatre, its functioning with respect to the flow of surgical patients, the human resources involved in the surgical activity and the health-care pathway of surgical patients. The typical layout of a modern operating theatre is outlined providing a description of the more important subunits. The professionals involved in the surgical activity are described according to their role and function at the operating theatre. The different types of surgical patients are introduced and their patterns of timing and resources consumption are outlined. This chapter provides, moreover, a brief review of the available operations research and management science literature for planning and scheduling problems of the operating theatre.

1.1 Introduction

Hospitals are the largest cost component of the health expenditure of most countries in the *Organisation for Economic Cooperation and Development* (OECD) and, from the early 80s of the 20th century, have faced consolidated trends of increasing pressure for patients admission [73]. Such increasing pressure has risen for many reasons (population ageing, emergent diseases, new in-hospital treatments, etc.) and countries have been challenged for improvements and adaptation efforts [99]. In developed countries, strategies for hospital efficiency improvement have focused on reduction of inappropriate admissions, more efficient patient care pathways, shorter patients length-of-stay and development of facilities for specific pathways of secondary

and tertiary care (*e.g.*, nursing homes, cancer management services, etc.), see [99] for a detailed discussion. Such strategies have transformed acute care hospitals into specialised facilities characterised by high rates of resource utilisation.

The *Operating Theatre* (OT), also known as *Operating Room* (OR) suite, is a hospital unit that is both labour and technology intensive and recognised as the main costs hub for modern hospitals, more than 40% of hospital expenses [9]. The execution of surgery, as well as the high maintenance of the OT equipment, requires high skills and technical knowledge of all the involved *Human Resources* (HRs), not for anything else, HRs account for most of the cost of running an OR [89, 36].

Resources that are frequently considered in strict relation with the OT, and that have been revealed of important interest for modern acute care hospital, are intensive care and ward beds. Surgical treatments require, in many cases, a hospital stay for the patient postoperative recovery and sometimes, a preoperative stay for the patient preparation, some type of surgical treatments require postoperative intensive care treatments, *e.g.*, cardiac surgery. According to [100], the bed capacity of acute care hospital has decreased significantly, all over the world, since 1990 pursuing rationalisation and efficiency improvement objectives. Modern hospitals, specialized for specific care pathways, have nowadays very narrow slack bed capacity and effective bed management policies have become of vital importance. The relation between the OT and *intensive care unit* (ICU) beds is even more important than for ward beds. The dimensioning of the ICU, *i.e.*, how many places/beds, is usually done on the basis of the need for surgical patients and non-surgical patients from the emergency department [12]; surgical patients are normally dominant over non-surgical patients. As the ICU represents a significant component of the hospital expenditures, around 15% [12], its slack capacity is targeted to be as small as possible.

In such context of increased pressure for hospital admissions, included those for surgical patients, it rises as certain the need for applying manufacturing industry systems of planning and scheduling to surgery and patients admission [86]. OT planning and scheduling is a widely investigated research area in the field of operations research and management science applied to health-care systems. One of the oldest review of papers about OT scheduling problems is [91]. The emergent, at that time, OT planning and scheduling problems have been identified by authors of preeminent interest for the management of health-care systems given the high hospital expenditures for running an OT and, at that time, the low utilization rate of OT resources. By then, research on the topic has been advanced and the OT technology intensity has increased tremendously, the OT management practice probably did not follow such advancements to an equivalent extent. An important review of the literature of *Operations Research and Management Science* (ORMS) applied to health-care systems, structured according to a taxonomic classification of problems, is in [77]. The large papers production of the ORMS research for health-care systems appears in [77] as an organic work and the central role of acute care hospitals, OTs and health-care professionals emerges as pieces of evidence.

In Section 1.2, we describe the physical setting of the OT, the HRs necessary for running an OT from the perspective of surgery and the different types of surgical patients. In Section 1.3, we describe the planning and scheduling of the OT from the perspective of the different decision levels (strategic, tactical and operational) and references to the relevant literature are provided. Section 1.4 introduces the research positioning of this thesis.

1.2 The operating theatre

In this section, we provide an overview of the physical setting of a standard modern OT and describe the HRs involved in the surgical process and the OT maintenance. The pathway of surgical patients, through and outside the OT, is also described.

1.2.1 Physical setting

As a technology-intensive, highly specialised and high safety requirements environment, guidelines for the OT design have been developed and investigated by research [65]. In general principals, an OT has to be compliant with requirements like the location within the hospital (lower floors), lightening (natural and artificial), safety and continuity of electricity, gases and water supply, ease of movements for patients and equipment, and criss-cross avoidance for paths that must stay apart. See [137] for further details.

In its standard configuration, An OT is composed of one or more ORs, a patients preparation atrium, usually called *Preoperative Holding Unit* (PHU) and a postoperative recovery atrium, usually called *Post Anaesthesia Care Unit* (PACU). Modern OTs are also equipped with several induction rooms to perform the patient induction of anaesthesia outside the OR concurrently to the use of the OR. Induction rooms allow reducing mean times of the OR turnover [123]. Frequently, some *Intensive Care* (IC) beds are present in the PACU as some patients require IC stay after surgery, by default (*e.g.*, open-heart cardiac surgery), or incidentally (*e.g.*, adverse surgery outcome), but the *Intensive Care Unit* (ICU) of a hospital is, in general, a unit separated from and outside the OT. Other important components of an OT are warehouses for sterile/non-sterile equipment, instruments and consumables. A strictly OT related hospital unit is the *Sterile Supply Unit* (SSU), sometimes *Central SSU* (CSSU), that is frequently adjacent to the OT to ease the OT supply of sterilized material over a clean path and the flow of used material to sterilize from the OT to the SSU over a dirty path. In Figure 1.1, we reported the layout of a modern OT, the figure is from [1].

ORs are core OT subunits where surgical procedures are executed and, together with warehouses of sterile material, are places, within the OT, that are subject to the tightest constraints for material and personnel flows and clean air supply. ORs are the more technology-intensive setting of the OT and the entire OT is designed around the ORs and the flow of patients, personnel and material from/to the ORs. In a correctly designed OT, the ORs are the physical bottleneck for the flow of surgical patients through the OT. With respect to the flow of patients, the PHU is in the OR upstream and the PACU, with the eventually installed IC subunit, are in the OR downstream. A logical scheme of the flow of surgical patients through the OT is reported in Figure 1.2 (the figure is from [62]).

Modern hospitals are equipped with large OTs that can count up to more than thirty ORs for serving several surgical specialities. One large OT shared by several surgical specialities allows exploiting scale economies for the OT subunits (PHU, PACU, SSU, OR, etc.) and resources (staffs included). Fully multifunctional ORs for executing surgeries of any specialty are available, but, despite this fact, some kind of specialization survives also in modern and large OTs. The set-up of multifunctional OR for switching from one specialty to another one may require the handling of fragile instruments and devices. It is so quite common that the OR set-up is done once for executing some type of surgeries requiring given devices (*e.g.*, extra-corporal circulation for cardiac surgeries or microscope for neurosurgeries) and not changed except that in the long term, the OR multi-functionality is so downgraded in practice.

The efforts for designing large OTs characterized by high standardization and multifunctional ORs point out and reflect the growing need for efficiency in the execution of surgical procedures and, indeed, their planning and scheduling.

1.2.2 Human resources

As we already argued in this section, the OT is the most labour intensive hospital unit. At the OT, several types of HR work and cooperate for running the OT and advance with the planned surgery. HRs directly involved in surgery are organised in surgical teams. A surgical team comprises of:

Surgeon She/he is responsible for performing the surgical procedure. She/he may be assisted

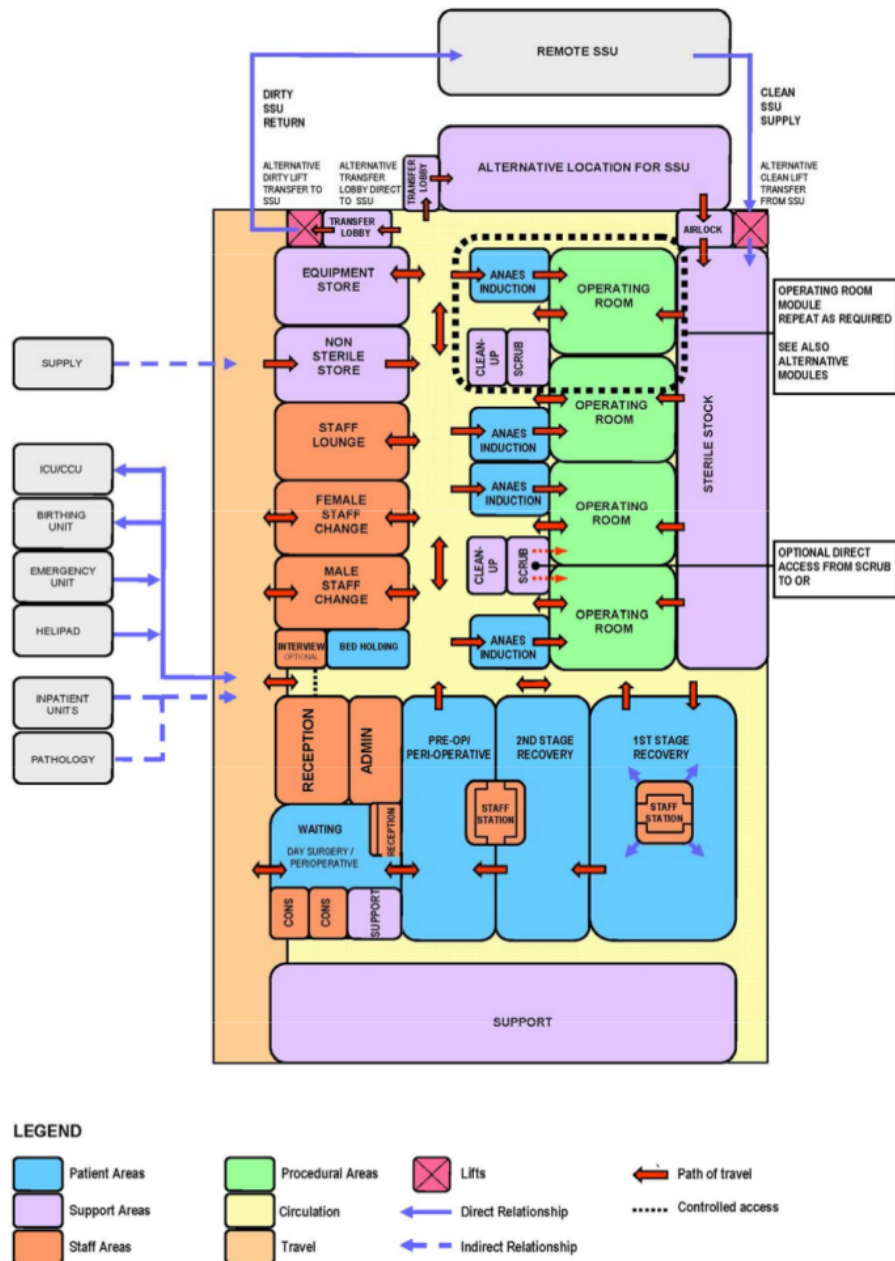


Figure 1.1: Single corridor operating theatre

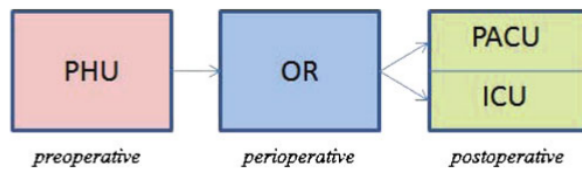


Figure 1.2: Flow of surgical patients

by a trainee surgeon.

Anaesthetist She/he is responsible for the patient anaesthesia induction and prophylactic drugs administration. She/he is responsible for patient vital functions monitoring during surgery.

Anaesthetist nurse She/he provides assistance to the anaesthetist.

Scrub nurse She/he provides assistance to the surgeon during the surgery execution. She/he is responsible for providing the surgical instruments and material to the surgeon with good awareness of the ongoing surgery and its steps. She/he has to be able to anticipate the next surgery step to promptly provide the right equipment.

Circulating nurse She/he is responsible for conducting equipment and materials from their storage to the scrub nurse and for the nurse tasks coordination. She/he is in middle between the scrub nurse and the OR/OT environment. More than one individuals can assist a surgery at the same time.

More details about surgical teams are in [137].

Among different countries, some differences hold for the role of nurses at the OT and the set of tasks for which a nurse has to be trained. Some terminology differences hold as well, *e.g.*, anaesthetist nurses are identified as *anaesthetic practitioner* in the United Kingdom, or simply *operating department practitioner* (a wider classification that includes some other surgical practitioners as the circulating nurse). In France, qualification for surgical practitioner is obtained through a bachelor of science degree according to two possible specialisation: *Infirmier(e) de Bloc Opératoire Diplômé(e) d'État* (IBODE, *i.e.*, Degree in OT nurse practice) and *Infirmier(e) Anesthésiste diplômé(e) d'État* (IADE, *i.e.*, Degree in anaesthesia nurse practice). IBODE are trained to provide surgery assistance as both scrub and circulating nurse and IADE are trained to provide assistance to anaesthetists. In many countries (France included), surgical practitioners as French IBODE are trained to perform autonomously also some important maintenance tasks of the OT equipment, the most important tasks are the surgical kits assembly and the sterilization operations of surgical materials and instruments at the SSU [124]. For information on the IADE profession in France, see [120]. An extensive and detailed description of the nurse activity in the CSSU is in [40].

1.2.3 Surgical patients

Surgical patients are patients to whom a surgical treatment have been prescribed by health-care professionals (typically physicians and surgeons) after clinical evaluation. The surgical care pathway can be complex, from the first consultation to the hospital admission for surgery, it may involve many steps of evaluation related to clinical exams and patient preparation. In general, for what concerns OT planning and scheduling, only a relatively small part of the surgical care pathway requires to be considered in detail, *i.e.*, the passage through the OT and eventually the passage through the immediately adjacent upstream and downstream resources (*e.g.*, the hospital admission department, the ward beds, ICU beds, etc.).

Common classification of surgical patients divides patients according to the history of the decision for surgical treatment. If the surgical treatment is chosen by the patient accordingly with the advice of consulted health-care professionals, the patient is said to be an elective patient, *i.e.*, a patient for which surgery is considered beneficial, but the patient health condition is not expected to change in the near future. Otherwise, the surgical treatment arises as the necessary action to take as soon as possible for the patient health, the patient is considered a non-elective patient. Non-elective patients are further divided into emergency patients and urgent patients, the first are patients for those the treatment is lifesaving, the second ones are patients not in a life-endangering condition. According to these patients classification, elective/non-elective, the surgical activity related to elective patients is called elective surgery and that one related to non-elective patients is called non-elective surgery. Non-elective surgery is further divided into emergency surgery and urgent surgery.

Another patients classification important for acute care hospitals is related to the type of patients hospital stay. Patients that do not need to occupy a hospital bed for their treatments

administration are outpatients. Patients that otherwise need to occupy a hospital bed for one or more days for their treatments administration are inpatients. Also surgical patients, elective and non-elective, are subject to the classification inpatients/outpatients, surgical outpatients are typically ambulatory surgery patients.

1.3 Operating theatre planning and scheduling

As we have already argued in the introduction of this chapter, the planning and scheduling of the OT has great attention in the research of ORMS applied to health-care systems. Literature reviews with different perspectives and problem classifications for this large research subject have been periodically published over the time. Relevant reviews are [62, 97, 115, 146], that provide wider view of the literature and [33, 131] that address more specific aspects of the subject.

Authors of [62] review the available OT management literature considering the hierarchical decision level of tackled problems (strategic, tactical, operational) for classifying papers. The survey addresses the planning and scheduling of the OT, but in practice focuses on ORs. ORs are generally the bottleneck resource of the OT and OR scheduling is thus the most studied subject for OT planning and scheduling. Despite the focus on ORs, according to [62], the OT staff and ward beds emerge as the most relevant resources after the ORs. In [97], the reviewed papers are classified according to the problem decision epoch and the time horizon. An interesting part of the review discusses the process of reengineering/redesign. Authors are persuaded that economic and project management aspects of the scheduling of surgery might be promising lines for future researches. In [115], a large review of the OR planning and scheduling literature is presented. Papers are classified according to a wide spectrum of characteristics (patient type, performance measure, decision, uncertainty, etc.). Authors try to provide an “anthology” of published papers with the aim to help researchers for future studies according to some suggested guidelines. The literature review provided in [146] about OR planning and scheduling, at the best of our knowledge, is the most recent one. In [146], the literature is reviewed considering different perspectives (the decision level, the patient characteristics, the considered uncertainty, the solution method, etc.). Authors conclude the review with two main observations: (1) there is still an open gap to close between the health-care management practice and published research about OR planning and scheduling and (2) some further efforts to cope with certain sources of uncertainty as the random consumption of resources due to emergency surgery are required.

In [33], authors classify the OR planning and scheduling literature according to the performance measure, the patient type, the uncertainty considered and the problem solution technique. The review is quite specific for operations research optimization techniques. Authors observe a discrepancy between the efforts made to study the scheduling of elective patients and the efforts made for the scheduling of non-elective patients, they suggest that further efforts may be necessary to close this gap. In [131], authors emphasise the necessary trade-offs between elective and emergency performance measures for surgery scheduling. More specifically, trade-offs between OT oriented (*e.g.*, staff overtime) and patient-oriented (*e.g.*, wait time) performance measures are discussed in reason of the type of scheduling policy and the scheduling method. The pros and cons of dedicated versus shared resources and the variability induced by the various sources of uncertainty (patients arrival, surgery duration, etc.) are discussed. Authors conclude that trade-offs between elective and emergency surgery are not yet well studied and that both dedicated and flexible capacity allocation solutions for the management of the OR capacity can be efficient strategies, but also that hybrid strategies are not yet well investigated.

In the remainder of this section, we report a brief summary of the OT planning and scheduling literature according to the different decision levels: strategic, tactical and offline/online operational. References to the relevant literature are also provided. The decision levels hierarchy is the common one for operations research and the same of [77] and [62]. The reader can

find a wider discussion of operations research techniques for solving planning and scheduling problems of surgical care services (and other health-care application domains) in [77]. A more specific literature review focusing on OT problems is in [62].

1.3.1 Strategic planning

The strategic planning of an OT can imply the resolution of several problems. According to [62], OT planning problems belonging to this decision level are essentially resource capacity planning and allocation problems. On the contrary in [77], resource capacity allocations are considered tactical level decisions and strategic level decisions include capacity dimensioning problems as well. For authors of [77], capacity dimensioning problems are those as facility layout planning, case mix and other planning decisions beyond OT planning and scheduling as regional coverage problems (*e.g.*, facility/hospital location problems) and service mix problems (*e.g.* hospital selection/assignment of surgical specialties). If the context is restrained to the OT planning and scheduling of a single facility/hospital, decisions that do not consider as given and unchangeable the OT physical setting maybe fall outside the subject.

Typical strategic decisions for an OT are: how many ORs have to be open daily [43, 19, 140], the OR time to schedule [42, 122, 134] and surgical case mix [79, 44]. Nevertheless, ORs are not the only OT resources considered by strategic decisions, as we have already argued previously, HRs have a relevant role at the OT and their strategic planning have received the attention of research over the time. HR long term OT scheduling mainly involves surgeons and surgical nurses (scrub and circulating), HR long term planning problems are essentially calendars generation [29] and staffing [139] problems. HR long term planning policies (*e.g.*, nurses outsourcing) can dramatically impact staffing costs [45] reducing them. OT nurse staffing problems are largely studied [30].

1.3.2 Tactical scheduling

At the tactical decision level, the scheduling of the OT usually involves the scheduling of resource capacity for the different patient groups. An important capacity allocation problem is related to the patient types elective and non-elective, a studied problem is the assignment of units of the available OR capacity (typically OR daily sessions) to one of the two patient types [107, 28, 147] in order to allocate an appropriate amount of OR capacity to every patient type. One of the most studied OT scheduling problem of tactical level is certainly the design of the *Master Surgery schedule* (MSS) [62], *i.e.*, assigning OR sessions to different surgical specialties (or groups of specialties) [129] as a cyclical calendar [130]. MSS problem extensions have been investigated to consider downstream resources [23, 56], different sources of uncertainty [83, 92], multiple objectives [24], or integrate other decisions [6]. If the assignment is not cyclical, the problem is identified with the more general term of *block scheduling* [43]. Not cyclical block scheduling improves flexibility, can decrease staffing costs and patient access time, but cyclical MSSs improve the downstream demand predictability [77]. A policy not very used in practice is the *open block* scheduling [62], *i.e.*, surgeries are scheduled for OR session without having computed a MSS or a block assignment [66]. In some cases, OT managers resort to the called “modified” block scheduling where only a part of the OR capacity is not assigned with block scheduling and some capacity is let open to cope with fluctuations of demand [62].

Tactical decision level problems for HR scheduling are shift scheduling problems, *i.e.*, the selection of shifts and the number of practitioners to be on duty for the selected shifts in order to cope with the surgery demand [50, 118].

One problem that does not belong to OT planning and scheduling, but that has a great impact on the OT operations, is the patient admission control [25] (surgical patients included indeed). Admission management is the set of scheduling and dispatching activities of the hospital admission control unit, this unit evaluates and coordinates the admission of any type of

patients requiring an in-hospital treatment [103]. An important, frequently dominant, part of the management of hospital admission is the management of bed capacity. Reviews of bed management problems are [67, 16]. The admission of surgical patients is strictly related to OT capacity planning and case-mix problems, their resolution is in practice a prescription for controlling the admission of surgical patients [20, 3].

1.3.3 Offline operational scheduling

One of the most studied operational problem for OT panning and scheduling is the surgical case scheduling problem [33, 97, 115], also called OR scheduling, *i.e.*, the problem of generating a complete and detailed proactive plan of surgery for a given time horizon (typically from one day up to two weeks) [33]. Such offline scheduling of surgeries has been frequently proposed as decomposed into two problems: *advanced scheduling* and *allocation scheduling* [33], the first one is the assignment of each surgery to a day and an OR and the second one is the sequencing of surgeries in each OR. The problem not decomposed is generally very difficult to solve, researchers have frequently addressed only one problem at a time, *i.e.*, only the advanced scheduling problem [63, 52] or the allocation scheduling problem [32, 127]. The problem not decomposed is usually called *integrated OR planning and scheduling* and has received attention with several papers, the more relevant papers are [93, 94, 34, 66].

Probably, the most studied HR scheduling problem in the health-care domain is nurse rostering, *i.e.*, the assignment of staff individuals to planned shifts [30], but in the specific case of surgical nurses, there is no paper [77]. The off-line scheduling of nurses for surgical cases is a subject that has received limited attention [88, 104, 141].

1.3.4 Online operational scheduling

The most relevant online operational scheduling problem of the OT is probably the online scheduling of emergency surgery. Emergency patients are routed from the ED to the OT to undergo surgery [76]. The OT has to quickly react to these events and schedule emergency surgeries according to the adopted policy [131], *i.e.*, to schedule the surgery in a dedicated OR [132, 142] or in an OR shared with the elective surgery, by cancelling or postponing elective surgery, and by balancing emergencies wait and elective plan disruption [15, 115].

Another important online operational problem of OT scheduling is the surgery rescheduling problem [146]. The need for surgery rescheduling can rise for many reasons included the arrival of emergencies [49, 2, 80]. There is also some literature considering as more relevant the adaptive/online scheduling of surgeries rather than the offline (proactive) scheduling, these papers focus on the online scheduling of elective surgery [145, 143].

To the best of our knowledge, there is no paper for the online scheduling/rescheduling of the OT staff and few papers have studied the nurse rescheduling outside the OT [14, 60].

1.4 Thesis positioning

In this thesis, we address a set of related OT planning and scheduling problems that belong to different decision levels. The focus is on OR scheduling under material and human resource constraints. Considered resources other than the ORs as material resources are ward beds and as human resources are surgeons and surgical nurses.

Through the modelling of problems we propose in this thesis and the optimization tools we develop for the solution of problems, we hope to be able to put forward the knowledge of the management of health-care systems, provide insight for health-care managers and possibly reduce the gap between research and real-world applications.

In Chapter 2, we assess by means of discrete event simulation a set of policies including bed management that we devise for the scheduling and admission of surgical elective and emergency

patients. Policies are based on bed occupancy thresholds and forecasts. Elective patients are scheduled for OR sessions considering the availability of OR capacity and ward beds, this is also known as the patient to date assignment problem, an operational problem that can consider even a large planning horizon, from few days up to some months. On the contrary, the emergency patient admission problem is a very short-term operational problem, some hours are the typical planning horizon and the admission of emergencies is essentially a real-time decision problem. Beyond the single-patient related decision making, the selection of policies for elective scheduling, emergency admission, and bed management, is an important tactical decision for the management of acute care hospitals. The management of the OR capacity also impact hospital performances, the scheduling and admission policies that we devise are also compared in relation to the applied OR capacity management policy, a strategic decision of OT management. We evaluate the application of an MSS and of the *Open Block* (OB) policy.

Chapter 2 provides also literature review for patient admission and bed management problems in Section 2.1. Recent works for the patient admission problem considering queues theory and discrete event simulation as problem solutions are reviewed as well as works resorting to other approaches like integer and stochastic programming. Occupancy thresholds seem to be promising and not burdensome solutions with the advantage of the ease of application in the real-world.

The first of the two stages of the weekly scheduling of surgery is addressed in Chapter 3. The first stage corresponds to an integrated operating room planning and scheduling problem that gathers together the most common constraints encountered in practice, *i.e.*, sequence, capacity and due date constraints, and constraints for human resources other than surgeons, *i.e.*, surgical nurses. Such surgery scheduling is a crucial offline operational decision problem of many acute care hospitals. The problem is difficult to solve and we devise for its solution a branch-and-price-and-cut algorithm based on the time-indexed formulation of the problem. The solution method relies on a label correcting algorithm we purposely develop for solving the column generation pricing problems that are proved to be strongly NP-Hard. The efficiency of the label correcting algorithm is ensured by dominance rules among labels and two algorithms for calculating the upper and lower bound of labels. A major contribution of the chapter is an effective cutting procedure, inspired by the Benders' decomposition and based on the duality theory for linear programming, that we develop for tightening the linear problem relaxation. The computational effectiveness of the solution method is demonstrated through numerical experiments. The solution method is tested with instances we generate and from the literature.

Chapter 3 provides a literature review of integrated operating room planning and scheduling problems and related solution methods in Section 3.1.

The second stage of the weekly scheduling of surgery is addressed in Chapter 4. The second stage problem considers as input a solution of the first stage problem, *i.e.*, a detailed schedule of surgery for every OR and day in the planning horizon, and completes the solution with the nurse to surgery assignment. We address the nurse to surgery assignment from a new perspective. The assignment problem is addressed by considering the constant training of nurses for non-technical skills (*i.e.*, interpersonal communication, coordination, judgement, etc.) as a problem objective. Nurses non-technical skills have crucial importance for error avoidance inside the OR and surgery safety. Other considered problem objectives are the minimization of the maximum nurse overtime and the maximum number of operating rooms visited by nurses on a day. The problem is presented in both its deterministic and stochastic version, the stochastic problem formulation considers the uncertainty of surgery durations and is solved by means of sample averaging of scenarios.

The nurse to surgery assignment problem is an offline operational decision that arises as necessary for the scheduling of real-world OTs. Despite the importance of this assignment problem for effective surgery schedules, the problem has not received great attention from researchers, as for many operational problems of surgical nurse scheduling. Chapter 4 in Section 4.1 includes

also a literature review of the subject and the related ones.

In Chapter 5, we study the problem of the real-time insertion of emergency surgeries into the schedule of elective surgery during its execution. A set of ORs is available for the execution of both the emergency and the elective surgery and the schedule of the elective surgery is assumed to be given in the problem planning horizon. Emergency surgeries are characterized by different emergency levels, waiting time targets and random arrival. An event-based stochastic programming model is proposed to minimize the total cost incurred by exceeding waiting time targets of emergency surgeries, delaying elective surgeries and incurring in ORs staff overtime. The optimality of the EDD (Early Due Date) first rule for prioritizing queued emergencies is proved. Simple heuristic policies and a stochastic optimization policy improvement approach derived from simple policies are applied for the problem solution and compared by means of numerical experiments.

As for the other chapters, also Chapter 5 provides a literature review of the addressed problem. The study discussed in Chapter 5 has lead to the published paper [15].

Chapter 2

Crossed evaluation of hospital bed management and surgical patient admission policies

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Abstract

In many hospitals, the bed capacity available for the hospitalization of patients is limited and the lack of bed management for scheduling and admitting patients can lead to bed shortages and disrupt the admission of inpatients. If bed shortages occur, elective inpatients need to be rescheduled for later admission dates and emergency inpatients diverged/rerouted to other hospitals. In this chapter, we devise a set of policies including bed management for scheduling elective patients and admitting emergency patients. Four key performance indicators are considered to evaluate policies: the percentage of refused emergencies, the percentage of overdue electives, the percentage of timed-out emergencies, and the percentage of elective rescheduling. Bed management is based on the evaluation of patient hospital stays and bed occupancy thresholds. Computation of forecasts for the patient length-of-stay and the actual/future bed occupancy is thus required for bed management. Two types of patient length-of-stay forecast are considered: the perfect information forecast and the expected value forecast. The long-term policy for managing the operating room capacity also affects the key performance indicators. We consider the application of a master surgery schedule as the default case and test the open block policy for assessing the benefit of flexible use of capacity. The policies devised for scheduling and admitting patients are tested through discrete event simulation of different simulation scenarios (combination of policies, flows of emergency patients, patients length-of-stay forecast, operating room capacity management policy, etc.). Numerical experiments reveal that elective scheduling and emergency admission policies including bed management with bed occupancy thresholds allow grasping the control of the key performance indicators.

2.1 Introduction and literature review

Since 1990 the hospital bed capacity has been decreased significantly all over the world in a heterogeneous manner because of changes in the population of patients (more elderly people) and the related needs, but also because of resources rationalization and/or budget constraints due to countrywide economic crisis (*e.g.*, former countries of the collapsed Union of Soviet Socialist Republics), see [100]. According to [100], strategies for an effective bed capacity reduction (not only in reason of costs cut) should include policies to reduce inappropriate admissions, pathways improvement for more efficient inpatient care, quicker patient discharges and the development of care facilities for specific care pathways (*e.g.*, nursing homes). This has been the path taken by most of the more advanced countries for rationalizing the available capacity, even if this has not necessarily reduced health-care costs. By pursuing these efficiency goals for the inpatients care, and to consequently reduce the available bed capacity, many modern hospitals have become more specialized for specific care pathways, reduced their role of general care facilities, and their slack bed capacity is nowadays very narrow. For these hospitals, an effective hospital admission system that provides patient admission management together with bed management is vital.

An early analysis of hospital admission systems and their organizational aspects that focus on the bed occupancy levels is in [103]; authors provide also a survey of the mathematical models and simulation approaches that were the most relevant at the time when the paper has been published. Since the time of [103], abundant literature about patient admission and bed management has been published to investigate both mathematical models, mainly queues theory and mathematical programming, and simulation approaches. The development of sophisticated mathematical models and/or simulations has been justified by the evidence of the poor results obtained by applying simple budget-driven strategies as target bed occupancy levels and clinical consolidation, see [59] and [109].

Admission management is intended to be the scheduling and dispatching activity of the hospital unit that evaluates and coordinates the admission of any type of patients requiring an in-hospital treatment with hospital resources (*e.g.*, the *Emergency Department* (ED), the

Operating Theatre (OT), the *Intensive Care Unit* (ICU), *Labour and Delivery units* (L&D), as well as the medical and the surgical wards). Bed management is part of admission management and plays a primary role in the admission of inpatients (inpatients are patients requiring a hospital stay over their health-care pathway, in contrast to outpatients that do not require any stay).

In [67], a review of the papers focusing on inpatient bed management is proposed. Authors assert that solutions based on system-wide approaches are preferable to studies focusing on only one or few hospital units, that system uncertainties need to be better addressed, and that simulation approaches are revealed to be better solutions to provide insights into uncertainties.

Another review of the literature for the bed management problem that focuses on operation research optimization methods (integer programming, goal programming) and simulation is in [16]. Authors of the review, as those of [67], conclude that bed management is a complex problem to address and that further research is required to provide tools and insight for hospital managers.

As for many other health-care topics, the distinction between elective and non-elective patients holds even for admission and bed management problems, and within inpatients. An inpatient can be either elective or non-elective, non-elective patients include urgent and emergency patients. From the medical point of view, the difference between an urgent and an emergency is related to the severity of the health patient conditions, an emergency is a life-endangering condition, an urgent patient is not. Concerning operations research and management science, the difference between urgent and emergency patients can be not relevant and non-elective patients are frequently indicated as emergency patients. Elective patients (or simply electives) are patients for which the service can be scheduled in advance, in contrast to emergency patients that cannot. By definition, emergency patients (or simply emergencies) arrive at the hospital for admission randomly and cannot be scheduled in advance. A literature review focusing on trade-offs for the scheduling of *Operating Room* (OR) capacity shared by elective and emergency patients is in [131]. A recent literature review about OR and surgical case scheduling that provides a patients classification (electives/emergencies, inpatients/outpatients, etc.) is in [146].

Bed management, *i.e.*, the management of the hospital bed capacity available, is usually treated in the literature as a scheduling problem that rises at two different decision epochs: (1) at the time of inpatient scheduling and (2) at the time of inpatient admission; see [77] for a taxonomy of decisions in health-care operations. Ward beds are commonly considered as resource constraint for inpatients scheduling problems and more bed management details are usually considered for inpatient admission (bed allocation) problems.

Papers focusing on the scheduling of elective inpatients usually resort to stochastic programming and rarely the random arrival of emergency patients is considered. In [135], a two-stage stochastic programming model that considers uncertain surgery durations and patients' *Length of Stay* (LOS) is proposed for scheduling the admission of elective inpatients; the operating theatre costs and the patient waiting time are minimized. The model includes chance constraints for the bed shortage probability and is solved by means of sample average approximation. The approach is tested through simulation and the optimized off-line patients scheduling is compared with some simple online policies of patient scheduling. According to their results, the advantage of the optimized off-line scheduling does not seem to be overall relevant. The random arrival of emergencies is not considered.

Also in [18], a two-stage stochastic programming model is presented for solving an admission scheduling problem for different patients types characterized by ward preference for bed allocation. The demand for the admission of patients is uncertain, first stage variables define the bed capacity for each type and time period and the problem bi-objective function targets a trade-off between resource utilization and costs of unmatched ward/patient-preference bed allocations; paper findings may provide insights for hospital managers.

In [98], a basic formulation of an admission scheduling stochastic problem considering bed

capacity and uncertain elective admissions, emergency patient arrivals and patients' LOS is presented. Three different model solution approaches, from the simple one up to the Monte Carlo evaluation, are evaluated and compared. The Monte Carlo method is revealed to provide the best performances in reasonable computation time and even for small-sized samples.

Queues theory received more attention with papers focusing on admission plans and admission thresholds. In [68], the optimality of two threshold policies for an inpatients admission model considering only the hospital bed occupancy census, and one policy considering also the length of the expedited elective call-in queue, is proved. The system is modelled as a Markov decision process. By means of simulation, authors evaluate the superiority of the devised policies with respect to the hospital current practice. As for performance indicators, the number of blocked emergencies, the number of cancelled electives, and the off-unit census are considered.

In [69], a mixed-integer linear programming model for computing the optimal plan of elective patients admission is presented. The model prescribes the number of patients of each given type that can be admitted each day of the planning horizon. Model parameters are computed resorting to queues theory and two hierarchical objectives are considered: the maximum number of elective admissions and the related minimum probability of blockage due to bed shortage. Rather than policies specific for elective scheduling, the model developed allows generating trade-off curves for blocking-vs-throughput in relation to case mix and volumes, by patient types, that can be used by decision-makers for hospital management.

In [21], a quantitative method for computing admissions quota of different types of inpatient is proposed, the method is based on a queuing model and a quadratic programming formulation of the quota computing problem. The study also covers the case of time-dependent patient arrivals, *i.e.*, ruled by a cyclical schedule for admission. As a practical insight, authors derive that a smoothed bed occupancy is achieved by evenly distributed patient arrivals, rather than reduced (or less variable) patients' LOS.

In [117], the admission process of elective inpatients and emergency patients from the ED is studied resorting to queues network modelling, the study focuses on the so-called boarding time faced by patients asking for general ward admission as inpatients, *i.e.*, the wait time in the ED. Authors investigate and show how policies ruling early discharge of inpatients through the day can reduce the emergency patients boarding time, smooth the ED congestion, and that more bed capacity not necessarily do that.

Case-mix solutions for planning the admission of inpatients have been also considered in the literature. In [20], a multidimensional inverse newsvendor problem is defined to compute the optimal portfolio of elective surgical procedures to compute daily according to the given bed and OR capacity, a random number of urgent surgeries is considered further. Authors make simplifying assumptions and the problem resolution does not provide policies, either for the admission of inpatients or bed management. The defined problem is close to the surgical case mix problem (see [74]), but decisions are intimately related to inpatient bed management and inpatient admission. The work provides useful insights and a basis for possible inpatients and bed management policies.

Some aspects of the admission of inpatients have received some attention, several papers have focused on details of inpatient admission as the allocation of patients to beds of wards different to that one of the related medical or surgical specialty (or group of specialties). Such allocation of patients to beds of wards not of the primary election is usually called non-primary allocation, or patient "misallocation". For this type of problems, characterized by dynamic and short-time horizon decisions, approximate dynamic programming methods and Markov decision processes have received remarkable attention.

In [41], an approximate dynamic programming algorithm is proposed for solving the Markov decision process of non-primary-ward bed allocation to inpatients, *i.e.*, how to decide if bed allocation that leads to primary ward patients overflow is suitable for the system performance. Authors resume the context of [117] and study such further aspect of bed allocation for inpatient

admission. Through numerical experiments, the authors show that the developed algorithm provides efficient control on the wards overflow compared to naive policies.

In [7], the problem of the in-hospital bed capacity redistribution to the wards is addressed by means of local search heuristic exploiting the Markov chain model of the hospital wards network system. The primary and secondary patients rejection, *i.e.*, blocking probability, is minimized. Authors show how their heuristic procedure provides good results in limited computation time and that the bed-to-ward reallocation, as well as a small bed capacity increase, allow a significant reduction of patient rejections.

In [10], an approximate dynamic programming algorithm is developed to solve a rolling-horizon and multi-resource (operating rooms and ward beds) inpatients scheduling problem, surgery and post-operative patient stay durations are uncertain, and patients to be scheduled are picked from waiting lists. Patient and resource-related objectives as patient lead time, OR overtime, and wards congestion, are optimized. Authors show that the devised algorithm outperforms the FIFO policy and is efficient for using the available resource capacity from several perspectives.

In [17], an approximate dynamic programming algorithm is devised to control the admission of elective patients by considering multiple resources for the care of patients (namely operating rooms and beds, even if the formulation is general). The hospital face also the random arrival of emergency patients that cannot be refused. The control process is formulated as a Markov decision process, the formulation includes uncertain evolving health of patients and care requirements. The devised algorithm is compared with two simpler algorithms. Computational experiments show the effectiveness of the devised algorithm for controlling the admission process, and that the newsvendor heuristic also provides good performances.

Other papers have addressed inpatient admission problems from a wider perspective. In [85], an algorithm for the admission of emergency inpatients is evaluated with different *Key Performance Indicators* (KPIs) and by means of a *Discrete Event Simulation* (DES). The bed capacity is the only resource considered for the admission of emergency inpatients, admission postponement or mismatched ward admission (admission to ward of different specialty patients) are minimized. According to the proposed algorithm, the admission of elective patients can be blocked in case of breaching the threshold for maximum overdue emergencies. Such an approach based on thresholds for patient types is revealed to be promising for controlling, and balancing, conflicting performance measures; it has tested with real-world data from a medium-sized public-funded hospital in Italy that is specialized in emergency care.

In [22], different bed allocation policies in the context of bed pooling are compared by means of a queuing model. Policies of bed earmarking and bed occupancy threshold for different patient types are compared by authors with cases of separated specialty wards and fully merged wards policies. The system performance measure is the probability of blocking due to bed shortage for the different types of patient. Authors conclude that bed occupancy threshold policies are effective for controlling the allocation of beds to different priority patients, such policies implies fully flexible wards and thus fit better for small- and medium-sized hospital. For big-sized hospital, fully flexible wards are usually not desirable given the wider set of patient types, ward staff skills do not necessarily fit with every patient type, in this case, due to the large scale advantages, bed earmarking policies provide sufficiently good results.

Inpatients admission problems have received much attention from the early times the topic has risen as an operation research/management topic [103], a recent literature review in [67]. Computing and enforcing thresholds for the bed occupancy has been revealed as an effective, or at least promising, approach to cope with some aspects of the inpatient admission problem as patient overflow in secondary wards, or the patient admission refusal because of a bed shortage. Two papers that we consider relevant for inpatients admission thresholds are [22, 68].

If inpatients are surgical patients, *i.e.*, patients requiring both an OR and a ward bed for their hospital care, the need to schedule/consider operating rooms and their management rises

as these are among the most expensive, and usually scarce, hospital resources.

In many hospitals, the access of the different surgical specialties to the OR capacity is ruled by *Master Surgery Schedule* (MSS), *i.e.*, a cyclical calendar that assigns surgical specialties to OR-sessions, according to this assignment, surgeries can be scheduled only for OR-sessions of the related surgical specialty. The MSS generation is usually a middle/long term decision that takes place every six or twelve months to adjust the OR capacity allocated to the different surgical specialties. An MSS mismatching the effective demand for surgery can lead to growing waiting lists for under-allocated specialties and OR capacity waste for over-allocated specialties. Note that growing surgery waiting lists reduce the ability to meet surgery due dates to a high extent.

As pointed out in [114], an effective way for reducing the elective patient time-to-surgery is to apply an *Open Block* (OB) policy for the management of the OR capacity. The OB policy is the opposite of the MSS policy, no surgical specialty is assigned to OR-session and surgeries can be scheduled for any open OR and day in the planning horizon, independently of the surgical specialty. More details on MSS and OB rule/policy for OR management are in [77, 62, 115].

In [77], a literature review of operation research problems is presented by following the taxonomy of decisions. Literature that focuses on OR scheduling at several decision levels is reviewed in [62] and papers on OR scheduling are reviewed and classified according to problem characteristics (patient type, decision epoch, performance measure, research method, etc.) in [115]. Studying inpatient scheduling and admission problems, OR capacity issues and scheduling rules/policies may have to be considered.

As usual for health-care problems, objectives and system performance measures are heterogeneous for inpatient scheduling and admission problems. The most common objectives/measures are: bed occupancy level, refused emergencies, blockage probability, cancelled admissions and overdue patients. The bed occupancy levelling/smoothing is considered in [68, 21, 20, 10], the patient waiting times in [135, 18, 117, 10], the amount of refused emergencies in [68], the probability of admission blockage because of bed shortage in [68, 69] and the amount of cancelled admission in [68].

Performance measures related to patient illness have received less attention for this kind of problems, but one of the most considered is the patient tardiness for service access. Such measure is based on patients due date, *i.e.*, the date given for each patient after which a further delay in the patient treatments administration (surgical or not) may endanger the patient health condition. Patient due dates are usually estimated at consultation time by the surgeon/physician that makes the patient consultation, a patient due date is commonly intended to be assigned on the basis of medical criteria exclusively. Patient due dates are considered in [136] for adjusting the OR capacity assignment to surgical specialties according to historical surgery data. In [114], patient due dates are considered as the driver for the dynamic scheduling of surgical patients. Authors evaluate by means of a DES simple patient scheduling policies and the early due date rule is shown to provide better performances with respect to the patient tardiness performance measure. In [15], waiting time targets based on patient due dates are used for evaluating the dynamic scheduling of emergency surgeries for a set of operating rooms.

In this chapter, we evaluate by means of DES some policies that we devise for the dynamic scheduling of elective patients and the admission of emergency patients. We consider the process of scheduling, admitting and serving only surgical patients (inpatients and outpatients), *i.e.*, only patients requiring an OR, and that may require a bed, for the hospital care. The OR capacity and the ward bed capacity are considered as constraints for scheduling elective patients and the sole bed capacity is considered as a constraint for the admission of emergency patients. We consider two possible cases for the management of the OR capacity: (1) an MSS is applied and (2) the OB policy is applied. The policies that we devise consider bed occupancy thresholds for controlling a set of *Key Performance Indicators* (KPIs).

The problem of the scheduling of elective patients and the admission of emergency patients, and the considered KPIs, are described in Section 2.2. The DES is described in Section 2.4 and

the data analysis of the surgery database from the Saint-Joseph Hospital in Paris (France) is reported in Section 2.5. The simulation scenarios that we test are described and the validation of simulation data are reported in Section 2.6. Section 2.7 presents the simulation results together with their discussion. Conclusions are finally reported in Section 2.8.

2.2 Problem setting, assumptions and key performance indicators

In this section, we describe the problem that we address, the assumptions that we make and the four KPIs that we consider for the evaluation of policies.

2.2.1 Problem setting

In this chapter, we focus on two scheduling decisions: (1) the middle-term dynamic scheduling of elective patients, frequently indicated as the patient-to-date assignment, and (2) the dynamic admission of emergency patients.

Both elective and emergency patients are characterized by due dates, we consider eight possible categories of due dates as proposed in [114], these categories are reported in Table 2.1. The patient type (elective or emergency) for the related category is reported in the first column, the due date category is in the second column, and the time interval for the due date computing (*i.e.*, the relative time from the consultation date/time within the patient should go under surgery) is reported in the third column.

Patient type	Category	Within
Emergency	1	1 hour
	2	6 hours
	3	12 hours
Elective	4	1 week
	5	2 weeks
	6	4 weeks
	7	8 weeks
	8	16 weeks

Table 2.1: Surgery due dates

The resources considered for scheduling elective patients are the ORs composing the OT and, for patients requiring hospitalization, the ward beds. For the admission or refusal of emergency patients, the sole ward beds are considered. The surgical activity of the OT is ruled by an MSS, elective patients are to be scheduled/served only for/in OR-sessions assigned by the MSS to the related surgical specialty and emergency patients are to be served in the first available OR with preference to OR-sessions assigned to the related specialty. Elective patients start to occupy a bed from on the scheduled admission date, if a bed is physically available. If there is no bed available on the scheduled day for admission, the elective patient has to be rescheduled. Once admitted to the hospital, elective patients are released to the OT at the scheduled dates. An emergency patient can be admitted to the hospital for surgery if a bed is (or becomes) available within the patient due date, otherwise, the patient must be refused and rerouted to another hospital, emergency patients are immediately moved to the OT after hospital admission.

Elective patients are divided into subtypes according to the type of hospital stay for recovering from surgery. An elective patient can belong to one of these subtypes: *Daily Care* (DC), *Day-0 Care* (DOC) and *Normal Care* (NC). No stay is foreseen for daily care patients, these patients require hospital resources as any other surgical patient (accordingly with their care pathway), but do not require a ward bed for recovering. A daily care patient is admitted and

discharged from the hospital on the same day of the surgery. Day-0 care patients are admitted the same day of the surgery and require a ward bed for at least one night to recover from surgery. Normal care patients are admitted the eve of the surgery and require a ward bed for at least one night to recover from surgery; these patients have a hospital stay of at least two nights of total duration. Except that for elective DC patients, each patient, elective or emergency, is characterized by a random hospital LOS that starts with the hospital admission and ends with the patient discharge; no stay is *a priori* foreseen for DC patients.

The hospital subsystem composed by the OT and its downstream resource of ward beds is shared by both elective and emergency patients. Scheduling the admission and the surgery of elective patients impacts the system capability to serve also emergency patients. The admission of emergency patients reduces the availability of resources for the scheduling of elective patients and may perturb the pathway of the already scheduled elective patients. The interlacing of these two flows of patients, elective and emergency, if not properly managed, can easily lead to a disrupted service, degraded Quality of Service (QoS), for both patient types.

We consider, as base-case, that an MSS slicing the available OR capacity among the different surgical specialties rules the scheduling of elective patients. The MSS is a cyclical calendar and limits, for every surgical specialty, the surgery time that can be scheduled for every OR-session. The MSS can incidentally create occupancy peaks for downstream resources, ward beds in this case, and significantly reduce the capability of emergency patient admission, if not purposely designed to avoid such side effects. OR-sessions assigned to surgical specialties with a prevalence of short surgeries and long patient LOSs produce peaks of elective patient admissions that can easily induce peaks of bed occupancy and reduce the number of possible emergency inpatient admissions.

Many strategies can be adopted to design an MSS that mitigate/compensate such that phenomena, a simple solution is to relax the MSS and allow the OB policy for the OT. The OB policy potentially produces OR-sessions with more heterogeneous patient mixes with respect to an MSS not purposely developed for such an objective. The OB policy can moreover reduce the time to surgery of elective patients. But, if the MSS cannot be redesigned to take into account scarce downstream resources of the OT, *i.e.*, ward beds in this case, the effective strategy is to devise policies for the scheduling of elective patients and the admission of emergency patients that directly control the bed occupancy.

In this chapter, we investigate some policies for the scheduling of elective patients and the admission of emergency patients that, combined together, target a QoS trade-off between elective and emergency patients. We also test the impact of the OB strategy as a benchmark of the MSS design.

2.2.2 Problem assumption

Let the assumptions that follow hold for the problem:

- A1. Elective surgeries are scheduled at the patient consultation time.
- A2. Every request for admission of an elective patient is scheduled, there is no unscheduled request of admission for elective patients.
- A3. The only limits to the scheduling of elective patients are the given ORs capacity and, if hospitalization is required, the availability of ward beds.
- A4. The only limit to the admission of both elective and emergency patients is the ward beds availability.
- A5. The MSS is given.
- A6. The OR capacity is assigned by the MSS to the different surgical specialties and includes an estimation of the random capacity consumption due to the execution of emergency surgeries.

- A7. Elective patients randomly arrive according to a non-homogeneous Poisson process at the hospital for clinical consultation and surgery scheduling. In practice, the consultation of an elective patient is scheduled as well and it is the consultation scheduler that actually faces the random arrival of elective patients, but, since this not impact the elective patient pathway that we study from the time of surgery scheduling to the time of the hospital patient discharge, we neglect consultation scheduling.
- A8. Emergency patients randomly arrive according to a non-homogeneous Poisson process at the hospital for admission and emergency surgery.
- A9. The LOS is equal to 0 always for DC elective patients, the minimum LOS is 2 nights for NC and 1 night for D0C.
- A10. The patient LOS distribution is known.
- A11. The patient surgery duration distribution is known.
- A12. The surgery duration and the LOS are mutually independent.

2.2.3 Key performance indicators

In order to evaluate the hospital QoS for the two types of patient, elective and emergency, we define four KPIs evaluated on the basis of weekly observations, the KPIs are:

1. The average percentage of refused emergency patients, *i.e.*, the percentage over the total known emergency patients that are not admitted to the hospital. The admission of an emergency patient can be refused according to a policy independently of the patient due date, or because the wait for admission of the patient exceeded the maximum allowed time, *i.e.*, exceeded the patient due date.
2. The average percentage of timed out emergency patients, *i.e.*, the percentage over the total number of emergency patients that are not admitted to the hospital and are refused because they reached the maximum wait time allowed, *i.e.*, the patient due date is exceeded. Such a percentage of timed out emergency patients can correspond to the total of the refused emergency if no policy is applied for deciding if an emergency patient has to be refused for admission before the patient due date is exceeded. Otherwise, if a policy is applied to early refuse the admission of emergency patients, such a percentage of timed out emergency patients corresponds to a fraction of the total of refused emergency patients.
3. The average percentage of elective rescheduling, *i.e.*, the percentage of scheduling events that correspond to the rescheduling of elective patients already scheduled at least once.
4. The average percentage of overdue electives, *i.e.*, the percentage over the total of admitted elective patients that are admitted to the hospital after their due date.

KPI 1 and KPI 2 are for the QoS as experienced by emergency patients, and KPI 3 and KPI 4 as experienced by elective patients.

2.3 Scheduling and admission policies

In this section, we first describe the five policies for the dynamic scheduling of elective patients, and the policy for the admission of elective patients, that we devised. We also describe the *Bed Occupancy Limit* (BOL), *i.e.*, the threshold to enforce for the bed occupancy of the different patient types. Then, we describe the five policies for the admission of emergency patients. Finally, we introduce the concepts of *Policy Setting* (PS) and *Bed Occupancy Limit Setting* (BOL Setting).

2.3.1 Elective patients scheduling

In this section, we describe the five policies that we devised for the scheduling of elective patients. One policy considers only the OR capacity free to be booked with surgeries as the criterion for scheduling elective patients, this is the base policy from which all the other policies are derived. The other four policies consider both the free OR capacity available for scheduling elective patients and the availability of beds for the admission of patients requiring a hospital stay.

Earliest OR Session (EORS). According to this policy, the surgery of an elective patient is scheduled for the earliest OR-session assigned to the related surgical specialty in the MSS and not yet fully booked with surgery. The earliest feasible date for the hospital admission of an elective patient is the day next to the consultation as the patient requires time to be prepared for the hospital admission. For D0C and DC patients, the earliest feasible date for surgery is thus at least one day next to the consultation date and, for NC patients, at least two days next to the consultation date, one day for preparation and one day of hospital stay before the surgery. This is the base policy for the scheduling of elective patients, it does not consider in any way the hospital bed capacity.

EORS with Free Bed on Admission Date (EORS-FBAD). This policy extends policy EORS and is based on a forecast of the future bed occupancy. We consider two possible forecasts for the bed occupancy: the *Expected Value* (EV) forecast and the *Perfect Information* (PI) forecast. For the EV LOS forecast, the LOS is foreseen to be equal to the expected value of the patient type/subtype LOS distribution if the patient is known but not admitted (*i.e.*, scheduled) and equal to the LOS conditional mean if the patient is an admitted patient (*i.e.*, a patient that has already started the hospital stay). The conditional mean is evaluated by means of the Monte Carlo simulation. For the PI LOS forecast, the foreseen patient LOS equals the effective LOS of the patient. With policy EORS-FBAD, a date for hospital admission selected with policy EORS is feasible for scheduling an elective patient if, according to the bed occupancy forecast, there is at least one free bed on the admission date. With this policy, the surgery of an elective patient is thus scheduled for the earliest date with an OR-session of the related surgical specialty with some OR capacity free to be booked and one or more beds foreseen to be free on the patient admission date.

EORS with Free Bed for Patient Stay (EORS-FBPS). As policy EORS-FBAD, this policy extends policy EORS and is based on a forecast of the future bed occupancy, EV forecast or PI forecast. With policy EORS-FBPS, a date for hospital admission selected with policy EORS is feasible for scheduling an elective patient if, according to the bed occupancy forecast, there is at least one free bed all along with the entire predicted patient LOS, from the admission date to the foreseen discharge date of the patient. With this policy, the surgery of an elective patient is thus scheduled for the earliest date with an OR-session of the related surgical specialty with some OR capacity free to be booked and one or more beds foreseen to be free for the entire predicted patient LOS.

EORS-FBAD with Bed Occupancy Upper Limit (EORS-FBAD-BOL). This policy extends policy EORS-FBAD by enforcing a BOL for scheduling patients. With policy EORS-FBAD-BOL, a date for hospital admission selected with policy EORS is feasible for scheduling an elective patient if, according to the bed occupancy forecast, there is at least one free bed on the admission date and the BOL is not exceeded if the patient is scheduled. With this policy, the surgery of an elective patient is thus scheduled for the earliest date with an OR-session of the related surgical specialty with some OR capacity free to be booked, one or more beds foreseen to be free, and the BOL is not exceeded by scheduling the patient.

EORS-FBPS with Bed Occupancy Upper Limit (EORS-FBPS-BOL). This policy extends policy EORS-FBPS by enforcing a BOL for scheduling patients. With policy EORS-FBPS-BOL, a date for hospital admission selected with policy EORS is feasible for scheduling an elective patient if, according to the bed occupancy forecast, there is at least one free bed all along the foreseen entire patient LOS, from the admission date to the foreseen discharge date, and without exceeding the BOL all over the foreseen LOS in the case the patient is scheduled. With this policy, the surgery of an elective patient is scheduled for the earliest date with an OR-session of the related surgical specialty with some OR capacity free to be booked, one or more beds foreseen to be free for the entire predicted patient LOS, and the BOL is not exceeded all over the foreseen entire patient LOS by scheduling the patient.

2.3.2 Elective patients admission

Elective outpatients are always admitted. Elective inpatients are always admitted to the hospital if there is some free bed capacity available. Every elective inpatient is admitted to the hospital if at least one bed is free at the patient admission time. The admission of an elective inpatient is refused and the admission and the surgery dates of the patient are rescheduled if there is no free bed at the scheduled patient admission time.

The outpatients and inpatients admission policy that we consider is the applied one in many real-world hospitals focusing on elective care pathways, as it is the hospital study case of this chapter. Hospital data are presented in Section 2.5.

2.3.3 Emergency patients admission

In this section, we describe the five policies that we devised for the admission of emergency patients. Four policies require knowing the current bed occupancy and a forecast of the bed occupancy over the future. As for policies for scheduling elective patients, EV and PI forecasts of the bed occupancy are used. Two policies consider BOL for emergency admission.

Undefined Wait (UW). An emergency patient is admitted to the hospital and promptly routed to the OT if there is a free bed at the patient arrival time. If there is no free bed at the patient arrival time, the patient waits indefinitely for a bed until its due date. If no bed becomes available within the time elapsing from the patient arrival time to the patient due date, the patient is refused and rerouted to another hospital.

Free Bed on Arrival Date (FBAD). An emergency patient is admitted to the hospital only if there is at least one free bed at the emergency arrival time or, if not already free, there is at least one bed foreseen to become free on the emergency arrival date, the emergency patient is otherwise refused. This policy requires a forecast of the LOS of patients to predict if some bed capacity is freed for the emergency arrival date. The patient LOS forecast can be EV or PI.

Free Bed for the Patient Stay (FBPS). This policy is based on a forecast of the current and future bed occupancy, EV forecast or PI forecast. The emergency patient is admitted if there is at least one bed free all along with the entire predicted patient LOS, from the patient arrival date to the predicted patient discharge date. At least one bed has to be already free at the emergency arrival time or a bed has to be foreseen freed for the patient arrival date, in both cases, a bed has to be free all along with the entire predicted patient LOS, the emergency patient is otherwise refused.

FBAD with Bed Occupancy Upper Limit (FBAD-BOL). This policy extends policy FBAD by enforcing a BOL for the admission of emergency patients. With this policy, an

emergency patient is admitted to the hospital only if, by admitting the patient, the BOL is not exceeded on the emergency admission date.

FBPS with Bed Occupancy Upper Limit (FBPS-BOL). This policy extends policy FBPS by enforcing a BOL for the admission of emergency patients. With this policy, an emergency patient is admitted to the hospital only if, by admitting the patient, the BOL is not exceeded all along with the foreseen patient LOS, *i.e.*, from the emergency patient admission date to the predicted patient discharge date.

2.3.4 Policy settings and BOL settings

Let a *Policies Setting* PS be defined as the pairing of an elective patient scheduling policy, one of those defined in Section 2.3.1, and an emergency patients admission policy, one of those defined in Section 2.3.3. In this chapter, we refer to a PS with the string given by the concatenation of the acronyms, separated by the string “-with-”, of the policy used for scheduling elective patients and the policy used for the admission of emergency patients. As an example, “EORS-with-UW” stands for the PS that pairs the elective scheduling policy EORS and the emergency admission policy UW. Since we defined five policies for the scheduling of elective patients and five for the admission of emergency patients, we have twenty-five possible PSs.

Let a BOL Setting, be defined as the couple of the BOLs selected for the two types of patient, elective and emergency. In this chapter, we limit to consider only BOLs for the two patient types (elective and emergency) that always sum to 100%. The two BOLs are expressed as the percentage of the total bed capacity in the order: elective patients BOL first and emergency patients BOL second separated by the minus character, *i.e.*, “-”. As an example, “80%-20%” stands for a BOL of the 80% of the bed capacity for electives and 20% as BOL for emergencies.

It worths pointing out that, given the two considered patient types (elective and emergency), enforcing the BOL for a patient type is implicitly as “to reserve” a percentage of the available bed capacity to the other patient type. By fixing the BOL to a given value, let this be for example 80% of the bed capacity for elective patients, is as to implicitly reserve the remaining 20% for emergency patients and *vice versa*, *i.e.*, fixing the BOL for emergency patients to 20% is as to implicitly reserve the 80% of the bed capacity to elective patients.

With a PS and a BOL-setting, the set of policies and parameters for the scheduling of elective patients and the admission of emergency patients are fully defined.

2.4 Discrete event simulation model

In this section, we describe the simulated pathways of elective patients, with the related subtypes, and of emergency patients. For both elective and emergency patients, we sketched the flow chart of the patient pathway.

The Discrete Event Simulation (DES) is implemented through the process modelling library of the commercial simulation software Anylogic and the policies of elective scheduling and emergency admission are implemented into a java code library linked to the simulation code.

The simulated elective pathway of patients is as follow: (1) the patient surgery is scheduled according to one of the scheduling policies defined in Section 2.3.1, (2) the patient waits until the planned date for hospital admission and shows up on the planned date for admission, (3) if no bed is required for serving the patient, the patient is admitted to the hospital, (4) if a bed is required for serving the patient and a bed is available, the patient is admitted, if no bed is available, the patient is rescheduled for a later date, (5) once admitted to the hospital, if no bed is required (DC patient), the patient is directly routed to the OT, if a bed is required, a bed is assigned to the patient and the patient follows the standard pathway of surgical patients: preoperative, surgery and postoperative steps. Note that, if the patient is D0C, the length of

the preoperative stay is 0 days and 1 day if the patient is NC. The flow chart of the simulated elective patient pathway of care is sketched in Figure 2.1. The flow chart is divided into two parts: the *Elective scheduling & rescheduling*, related to the consultation and scheduling (or eventual rescheduling) of elective patients, from the consultation time to the admission time, and the *Elective surgery service*, related to the surgery service of the hospital, from the admission to the discharge of elective patients.

The simulated emergency pathway of patients is as follow: (1) the admission of an emergency patient is evaluated according to one of the admission policies defined in Section 2.3.3, (2) if a bed is available for serving the emergency within the emergency time-out, the emergency patient is admitted to the hospital and routed to the OT, otherwise, the emergency is refused and the patient discharged, (3) once admitted to the hospital, a bed is assigned to the patient and the patient follows the pathway of surgery and postoperative step. In Figure 2.2, the flow chart of the simulated emergency patient pathway of care is sketched. The flow chart is divided into two parts: *Emergency admission management*, related to emergency admission management, from the emergency show up time to the emergency admission time (or refusal time), and *Emergency surgery service*, related to the emergency surgery service of the hospital, from the emergency patient admission to the patient discharge.

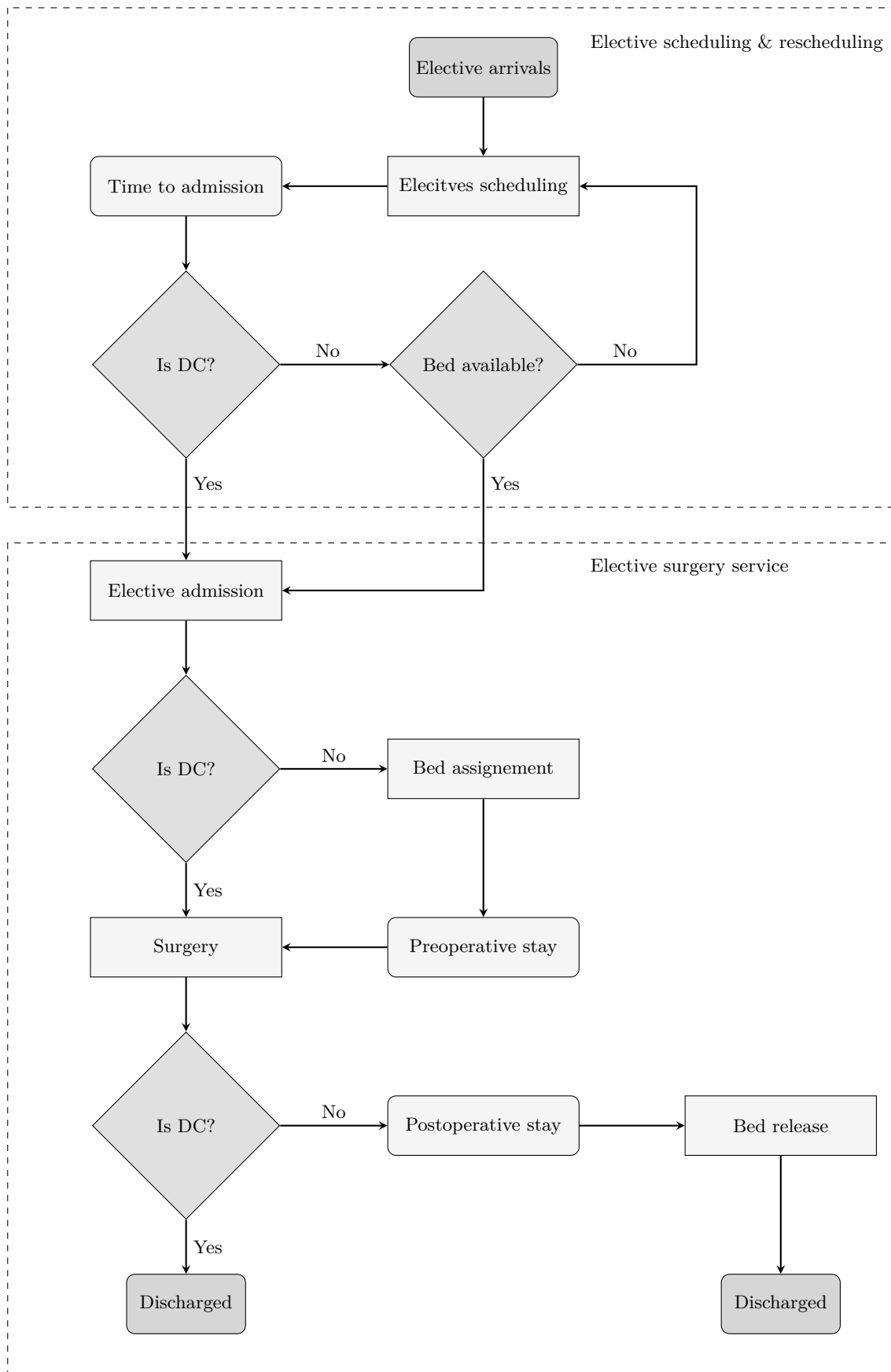


Figure 2.1: Elective patients flow chart

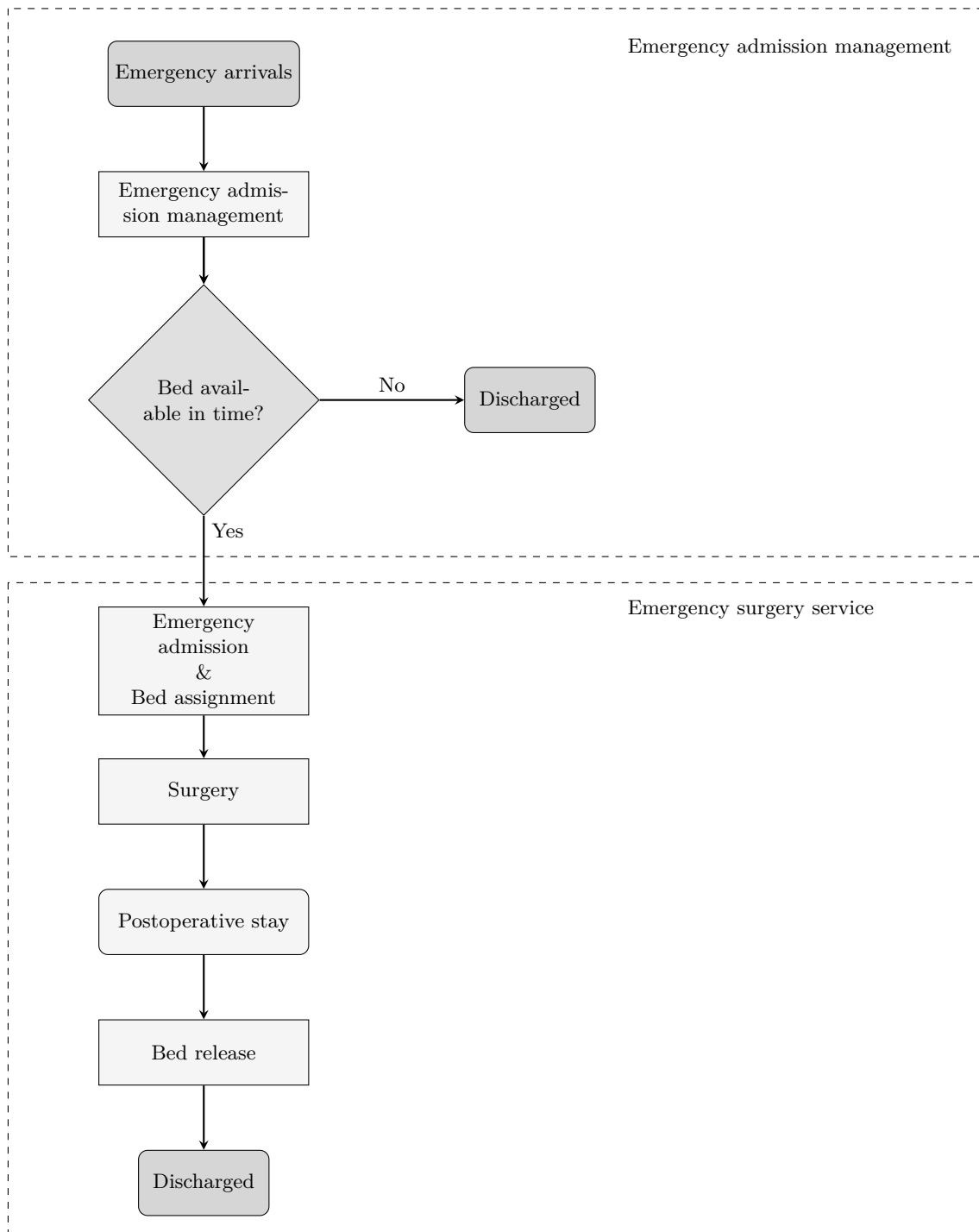


Figure 2.2: Emergency patients flow chart

2.5 Hospital data

In this section, we present an analysis of the data from the Saint-Joseph Hospital (Paris, France) that we used to feed our simulation model.

We obtained two databases from the Saint-Joseph Hospital. One database for the surgical activity of the OT recording 38237 surgeries performed in a period of 110 weeks, let this be the *OT Database* (OTDB), and one database tracking 9751 patient admissions in a period of 30 weeks, let this be the *Patient Admission Database* (PADB). Unfortunately, the two databases cannot be linked because surgery ids and patient ids in the two databases do not correspond.

In the two databases, no trends or relevant seasonality phenomena in terms of surgery volume, surgical cases count or patient admissions are revealed.

2.5.1 Patient flow

From the OTDB, we extracted the data reported in Table 2.2, the table reports, for each surgical specialty and patient type/subtype, the percentage of patients with respect to the total number of patients. The first column (*Label*) provides a string for each surgical specialty, the second column (*Specialty*) reports the name of the surgical specialty and the column (*Percentage*) reports the percentage of patients of the given specialty. The columns NC, D0C, DC and EM report the percentage of patients of the related patient type/subtype.

Label	Specialty	Percentage	NC	D0C	DC	EM
DG	Digestive	10.19	5.37	0.02	3.52	1.28
GO	Gynaecology/Obstetric	8.05	3.86	0.58	3.19	0.42
OP	Ophthalmology	7.51	0.57	0.10	6.77	0.07
OR	Orthopaedic	11.55	4.53	0.19	2.99	3.84
PL	Plastic	12.59	5.05	1.44	5.80	0.30
PR	Proctorial	19.63	5.91	0.43	12.47	0.82
ST	Stomatology/Otolaryngology	8.92	4.50	0.56	3.27	0.59
UR	Urology	12.28	7.36	1.42	2.51	0.99
VC	Vascular	9.28	4.71	0.02	3.65	0.90
	Total	100.00	41.86	4.76	44.17	9.21

Table 2.2: Flow of each patient type/subtype and surgical specialty

Table 2.2 shows that elective patients represent 90.79% of the patients flow, emergency patients the 9.21% and DC (daily care) patients represent a significant fraction of the elective patients, 44.17%. The patient types/subtypes mix is very heterogeneous through the different surgical specialty, for example, for specialty OP, 90% of the patients are DC, for specialty UR, more than 70% of the patients require hospitalization and the 30% of specialty OR are emergencies.

2.5.2 Length of Stay

In this section, we analyse the LOS for the different patient types/subtypes.

The average patient LOS is 2.6 days of hospitalization, it is slightly shorter for elective patients, 2.4 days, and significantly longer for emergency patients, 4.9 days. The maximum LOS in the PADB is 90 days. In Figure 2.3, we report the distribution of the LOS for elective patients and, in Figure 2.4, for emergency patients; values higher than 30 days are removed in the two figures.

The distribution for both the two types of patients, elective and emergency, seems to fit a bell-shaped asymmetric distribution that is skewed to the right (positively skewed), the most frequent value of patient LOS is one day for both the patient types, elective and emergency.

A box plot, for each surgical specialty, of the LOS of elective patients is reported in Figure 2.5 and of emergency patients in Figure 2.6. Outliers are removed from box plots. Note that there are no emergency patients for Ophthalmology.

DG, OR and VC are the surgical specialty with longer LOS for patients. LOS distributions for the different surgical specialties show some differences between elective and emergency patients. The third quartile value and the maximum value of the patient LOS are, on average, greater for emergency patients.

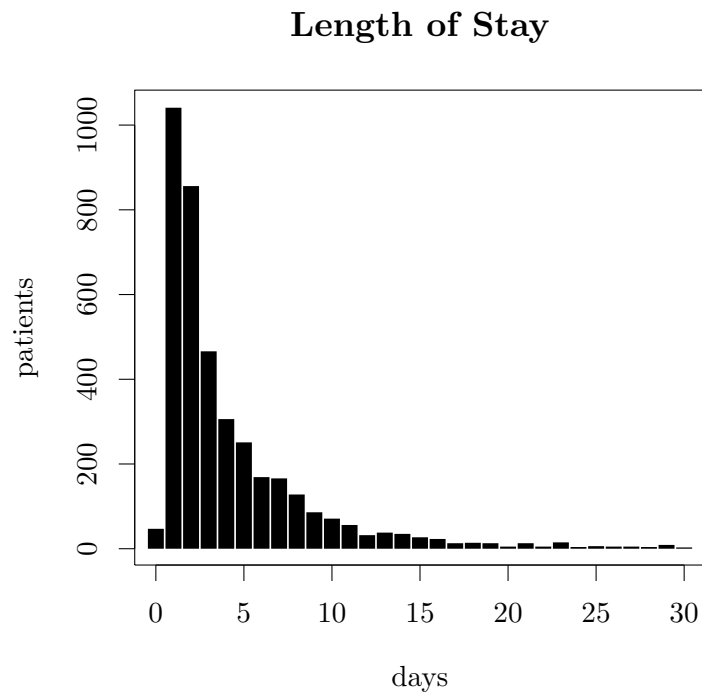


Figure 2.3: Elective patient LOS distribution

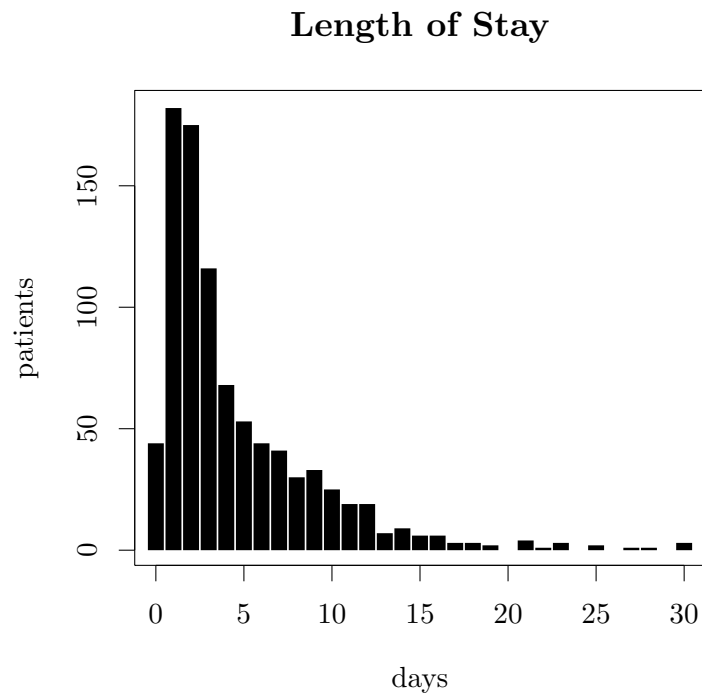


Figure 2.4: Emergency patient LOS distribution

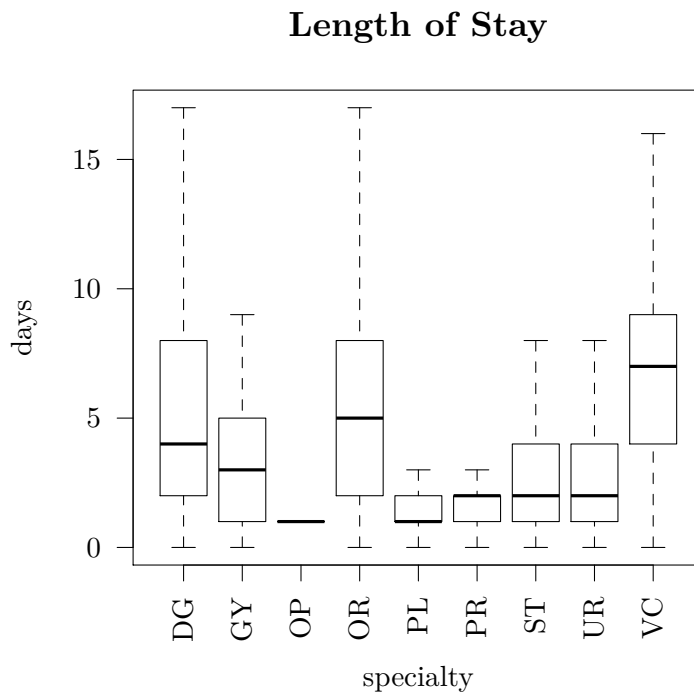


Figure 2.5: LOS distribution by specialty: elective patients

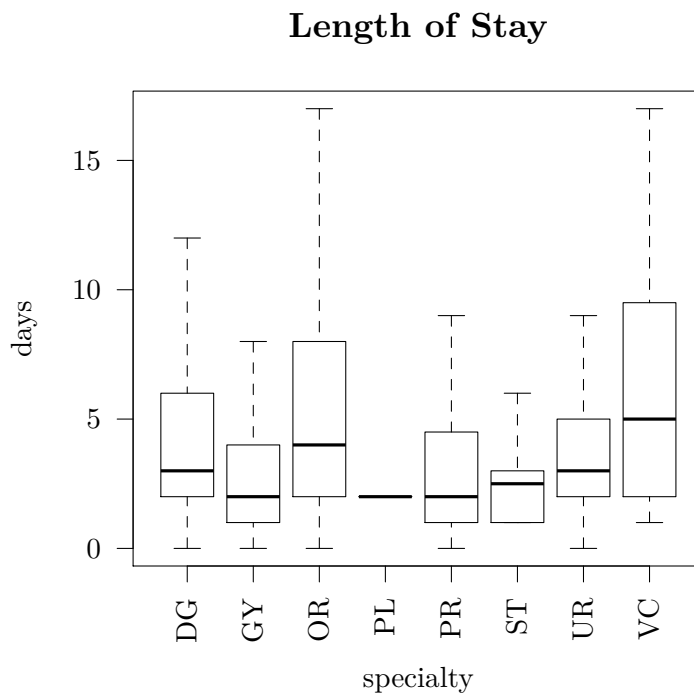


Figure 2.6: LOS distribution by specialty: emergency patients

2.5.3 Time to Surgery

In this section, we analyse, for the elective patients, the *time-to-surgery* (T2S), *i.e.*, the lag time between the date of surgery scheduling and the surgery date. Unfortunately, the only data

available in the OTDB for estimating the T2S is the date of the last scheduling event of each surgery because the date of the first consultation/scheduling has been overwritten every time a surgery has been rescheduled. These are the only data with which we can estimate the T2S distribution, but the real distribution, as well as its statistical parameters, might be different from those computed.

In Figure 2.7, the distribution of the time interval between the date of the last scheduling event and the surgery date is reported, values are in the range from 1 to 230 (in Figure 2.7, values greater than 150 are removed), the average value is 35.6 days and the standard deviation 28.8 days, the median and the mode are 29 days and 1 day respectively.

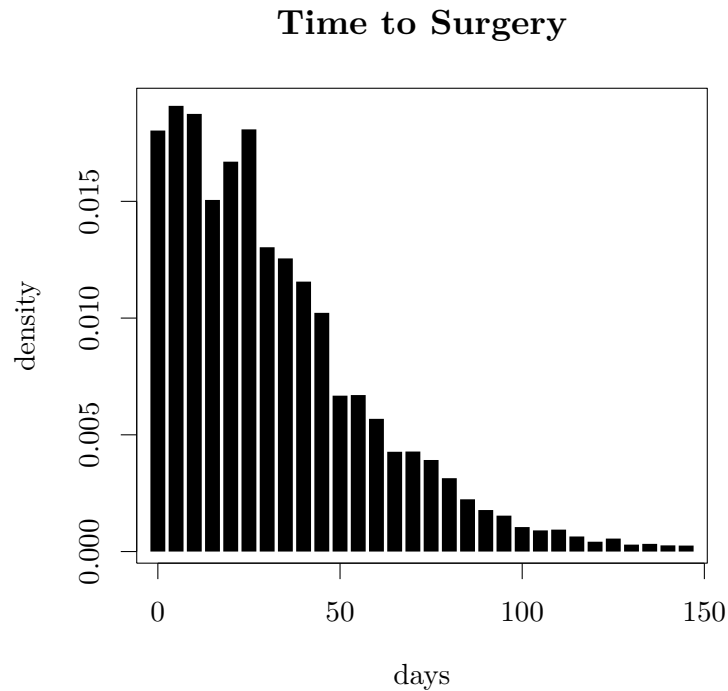


Figure 2.7: Distribution of the lag time between the last scheduling event date and the surgery date for elective patients

For the distribution in Figure 2.7, values of the first, second and third quartiles are 14 days, 29 days and 49 days respectively. The number of observations for an approximated T2S of 0 days shown in Figure 2.7 should correspond to rescheduled surgeries as elective surgeries are scheduled at least one day in advance.

In Figure 2.8, box plots of the (approximated) T2S distribution for the different surgical specialties are reported; outliers are removed. Except for Ophthalmology (OP) and Orthopaedic (OR) with a median T2S of 55 and 46 days respectively, the median value of the T2S is in the range from 20 and 30 days for all the other surgical specialties. The value of the first quartile is relatively greater for OP and OR, 31 days and 26 days respectively, than for the other specialties. This means that, for these two specialties, $\frac{3}{4}$ of the patients have a T2S that is, on average, longer than that of the other surgical specialties. The most unloaded/efficient surgical specialty is Urology (UR) with a value of 33 days for the third quartile of the T2S distribution, $\frac{3}{4}$ of the patients of this specialty face a T2S that is, on average, not longer than 33 days. DG, GY, PR and ST specialties show T2S performances comparable to that one of UR. PL and CV have median values for the T2S of 30 days and 29 days respectively (in the range from 20 and 30 days), but values of the third quartile are 52 days and 55 days respectively (quite longer T2S than other specialties if OP and OR are excluded).

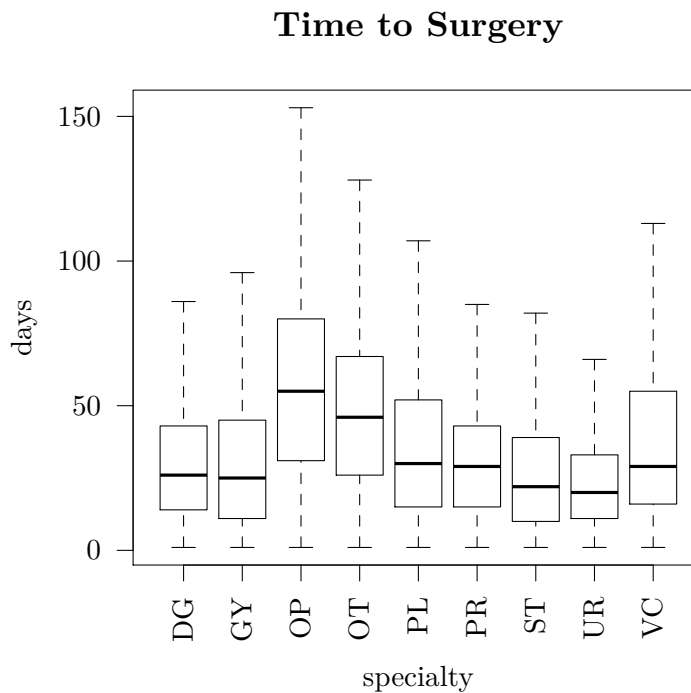


Figure 2.8: T2S distribution by specialty: elective patients

2.5.4 The Master Surgery Schedule and surgery data

At the HPPJ, the cyclical MSS that schedules the available OR capacity for the nine surgical specialties is applied on a basis of 6 or 12 months, *i.e.*, the MSS is eventually recomputed every 6 or 12 months accordingly with the contingent exigencies of surgical activity. The MSS adopted at the HPPJ spans a time period of 2 weeks, but, considering some exceptions that schedule different surgical specialties for the same OR-session, it technically spans a time period of 4 weeks. The given MSS schedules every OR with two OR-sessions per day (a.m. and p.m. session) and over 5 days (the workweek); the fixed duration of an OR-session is 4 hours. Rarely the a.m. and the p.m. OR-sessions of the same day and OR are assigned to two different surgical specialties. Let's thus talk simply about Full[-day] OR Sessions (FORS) and Half[-day] OR Sessions (HORS).

In Table 2.3, for the MSS that is used, we report the total number of FORS (column *FORS*), the total number of HORS (column *HORS*), the total amount of assigned hours over the full cycle (column *Total hrs*) and the number of used ORs (column *ORs*). Table 2.3 includes also the subtable (*Realized hrs per week*) that reports, for the realized hours of surgery of each specialty, the average (column *Ave.*), the standard deviation (column *Sd.*) and the maximum (*Max.*) calculated for the surgeries tracked in the OTDB.

Numbers reported in Table 2.3 show that the actual surgical activity is relatively congruent with the planned one by the MSS. As expected, surgical specialties with a higher fraction of emergency patients show a higher variability of the weekly surgical activity and a greater deviation from the amount planned by the MSS of the average and the maximum amount of hours realized weekly.

In Table 2.4, for each surgical specialty, we reported the average, the standard deviation and the maximum value of the surgery duration for each specialty, subtable (*Surgery duration*), and the average, the standard deviation and the maximum value of the duration of daily sessions of surgery, subtable (*Daily session duration*). For the daily sessions of surgery, we reported also,

Label	MSS					Realized hrs per week			
	FORS	HORS	Monthly total hrs	Weekly total hrs	ORs	Ave. ^a	Sd.	Max.	
DG	47	-	376	94(14%)	4	92 ± 8 (15%)	30	122	
GO	24	10	232	58(8%)	4	55 ± 3 (9%)	11	74	
OP	17	-	136	34(5%)	1	31 ± 1 (5%)	5	38	
OR	69	-	552	138(20%)	5	145 ± 12 (23%)	44	176	
PL	34	12	320	80(12%)	8	74 ± 2 (12%)	8	87	
PR	22	10	216	54(8%)	3	53 ± 1 (8%)	4	61	
ST	24	4	208	52(8%)	3	62 ± 2 (10%)	9	81	
UR	51	-	408	102(15%)	5	100 ± 8 (16%)	31	132	
VC	40	-	320	80(12%)	4	49 ± 4 (8%)	13	73	
Total	328	36	2768	692(100%)		634 ± 52 (100%)	191	735	

^a Average values with the half 95% CI after the “±” symbol. The value between parenthesis is given by ratio of the average for the specialty to the average for the all specialties reported in last row (*Total*), it is expressed as a percentage.

Table 2.3: MSS schedule data [hrs]

in the order by columns, the percentage of sessions with duration shorter than 480, 600 and 720 minutes.

Label	Surgery duration			Daily session duration					
	Ave.	Sd.	Max.	Ave.	Sd.	Max.	≤ 480	≤ 600	≤ 720
DG	157	88	909	424	171	778	53.50%	89.81%	98.73%
GO	98	66	573	236	145	619	92.26%	99.35%	100.00%
OP	43	16	143	417	94	563	77.55%	100.00%	100.00%
OR	140	81	685	424	155	732	56.10%	95.53%	98.78%
PL	135	78	700	370	181	670	63.64%	96.21%	100.00%
PR	44	15	139	313	159	541	85.59%	100.00%	100.00%
ST	139	109	823	348	187	823	72.03%	96.61%	98.31%
UR	140	98	775	418	194	814	54.65%	84.30%	97.67%
VC	132	77	734	378	161	656	69.77%	96.51%	100.00%

Table 2.4: Surgery and daily session durations [minutes]

Data reported in Table 2.4 suggest that the daily surgery sessions systematically overflow the regular daily work time of 8 hours prescribed with the MSS. We do not have further information on the OR staff work time like scheduled shifts and work contract details to assess quantitative or qualitative data of the effective overtime of human resources. We only know that emergency surgeries are executed by surgical staffs dedicated to emergency surgery and in ORs shared with the elective surgery (those scheduled with the MSS). Only one OR of the OT is purposely reserved to emergency obstetric surgery.

2.6 Simulation feeding, scenarios and validation

In this section, for first, we introduce the simulation scenarios and describe the simulation setting design of the numerical experiments, then, we present the simulation validation.

2.6.1 Simulation scenarios and setting design

We define a simulation scenario as the collection of simulation rules and parameters necessary to run the DES model that we described in Section 2.4, such a collection has a fixed number of ordered elements and each element represents a simulation scenario “factor”. A scenario of the DES model of this work has five factors, these are:

1. The OT scheduling policy, MSS or OB; this work focuses on the MSS case, but we evaluate also the OB policy as a benchmark given its characteristics and pros (see Section 2.2).
2. The PS that defines the elective scheduling policy and the emergency admission policy to simulate.
3. The patient LOS forecast type, EV or PI, used by the elective scheduling policy and/or the emergency admission policy for decision making.
4. The BOL-setting that defines the BOLs to enforce if required by the elective scheduling policy and/or emergency admission policy of the simulated PS.
5. The arrival-rate of each surgical specialty and patient type/subtype, given λ_{mj} the patients arrival-rate of surgical specialty m and type/subtype j , matrix $\Lambda := \{\lambda_{11}, \dots, \lambda_{MJ}\}$ is thus the matrix collecting all the arrival-rates.

For generating the simulation scenarios, we consider the whole set of possible values for some factors and only a representative subset of the possible values for the other factors. For factor *OT policy*, we consider both the policies MSS and OB, for *PS*, all the twenty-five possible PS that we defined, and both the PI and EV forecast for *LOS forecast*. For factor *BOL-setting*, we consider ten different BOL-settings for the BOL of emergency patients, from 4% to 40% of the bed capacity with a step of 4%, *i.e.*, from BOL-setting 96%-4% to 60%-40% (see Section 2.3.4 for more details on the BOL-settings). The arrival rate of every elective patient subtype is unvaried for every surgical specialty and in any scenarios. The arrival rate of emergency patients of every surgical specialty is multiplied for the same scalar value, ρ , to generate different matrices Λ of different simulation scenarios, ρ varies from 1.0 up to 4.0 with a step of 0.2.

In Table 2.5, we reported the cardinality of each one of the five factors of simulation scenarios. We reported also the product of the cardinalities that corresponds to the total number of simulation scenarios to generate for the exhaustive evaluation.

Factor	OT policy	PS	BOL-setting	LOS forecast	Arrival rates	Product ^a
Cardinality	2	25	10	2	16	16'000

^a The total number of simulation scenarios. This total number is indeed an upper bound on the number of scenarios to evaluate. The PS EORS-with-UW is based on elective scheduling and emergency admission policies using neither BOLs nor patient LOS forecasts, simulation results are thus not affected by a different BOL-setting or patient LOS forecast, *i.e.*, the minimum number of scenario to evaluate through simulation is slightly inferior.

Table 2.5: Simulation scenario dimension cardinalities

Numerical experiments are some series of simulation runs executed for different simulation scenarios for an *ex-post* comparison of the simulation outcomes of different simulations scenarios.

2.6.2 Patients generation

In this section, we describe how patients are generated in the DES and how a value is drawn for each patient attribute as a random variable realization.

Arrival rates Interarrival patients times are drawn from exponential distributions and the parameter (average interarrival time) for the distribution of each patient type/subtype and surgical specialty are extrapolated from the hospital databases.

Due dates Due dates assigned to patients are not recorded in the hospital database, we estimate the due date of each patient on the basis of two simple assumptions: (1) most of the patient are served on time, but neither very early nor at the last minute, and (2) the effort of the (real-world) surgery scheduler to plan surgeries as much as possible earlier is major if the patient due date is tight with respect to the scheduling date.

As already reported in Section 2.5.3, the scheduling date of patients recorded in the database is the date of the last scheduling event. Given t [days] the (approximated) T2S of a patient, we estimate the patient due date dd relative to the scheduling date according to:

$$dd = (a + b \cdot \alpha^t) \cdot t, \quad (2.1)$$

equation parameters a and b are integers, and α a decay coefficient in the range $(0, 1)$, we keep as reasonable values $a = 2$, $b = 1$ and $\alpha = 0.94$. For small values of t (patients served in a short delay), equation (2.1) provides relative due date values close to 3 times the patient T2S and, for greater values of t (patient served in a long time), values closer to 2 times the patient T2S.

On the basis of the estimation of the due date that we obtain with 2.1 for each patient in the database, we compute the due date distribution of the elective patients aggregating patient data by surgical specialty and patient type. Due dates in weeks are computed as $\lceil dd/7 \rceil$. Due date distributions are exploited to randomly assign due dates to each generated elective patients in the DES.

We do not have data tracked in the database to compute a due date distribution for emergency patients, we thus assign to each emergency patient generated in the DES a due date according to one of the three due date category of Table 2.1 with equal probability, *i.e.*, $1/3$, as in [114].

Lengths of Stay Values for the patient LOS are drawn from log-normal distributions and parameters (mean and standard deviation) of the distributions of each patient type/subtype and surgical specialty are extrapolated from the hospital databases. The minimum value for each patient LOS distribution is set according to the related patient type/subtype (2 days for NC, 1 day for D0C and 0 for other types/subtypes).

Surgery durations Values for the surgery duration of each patient are drawn from log-normal distributions and parameters (mean and standard deviation) of the distribution of each patient type/subtype and surgical specialty are extrapolated from the hospital databases.

2.6.3 Validation

The simulation validation is the important work step that ensures the simulation model credibility and the accuracy necessary to consider simulation outcomes exploitable for an evaluation of what would be the real-world outcome.

We assume that the process modelling library has passed a verification as it is provided with the commercial simulation software Anylogic. We verified the java code of the policies through dynamic testing (see [116]).

The simulation scenario corresponding to the hospital setting is reported in Table 2.6, let this be the *Hospital Setting Scenario* (HSS). In the HSS, the OT is ruled by the MSS, the PS for elective scheduling and emergency admission is EORS-with-UW and the patient arrival rates are those extrapolated from the hospital data. In the PS EORS-with-UW of the HSS, the BOL-setting and the patient LOS forecast do not matter as neither the elective scheduling policy nor the emergency admission policy require a BOL or a forecast of the patient LOS.

Factor	OT policy	PS	BOL-setting	LOS forecast	Arrival-rates
Values	MSS	EORS-with-UW	-	-	Hospital data

Table 2.6: Hospital setting scenario

The simulation is validated by comparing the simulation outcomes of the hospital setting scenario with the real hospital data. We consider of key importance for the simulation accuracy: (1) an arrival process of elective and emergency patients consistent with the real hospital data, (2) a realistic application of the MSS and (3) a distribution of the daily occupancy of the hospital bed capacity consistent with the real-world hospital data.

Over a time period of 110 weeks of surgical activity at OT, removing the first and the last week of the year that presents a very low surgical activity (Christmas period), the average OT throughput is 327.3 [patients/week] and 17'065 [patients/year], *i.e.*, a patient enters/exits the OT every 30.8 minutes. For 200 simulation runs considering the normal/tracked flow of emergency patients, the PS adopted by the hospital and for a simulated time of 1 year, the number of patients that undergo surgery is, on average, 17'053 and between minimum 16'624 and maximum 17'438 patients. We can assert that the aggregated flow of patients is consistent with the real-world hospital data.

In Table 2.7, we report the simulated surgery case-load of the OT and statistics of the weekly surgery time for a simulation run of one year (365 days, 52 weeks) of surgery and 17'109 operated patients. Subtable *Surgery case-load* is as Table 2.2, but for the simulated patient flow instead of the realized one. Subtable *Weekly surgery time* reports, by row, statistics for the total weekly surgery time of each surgical specialty: the average (column *Ave.*), the standard deviation (column *Sd.*) and the maximum (column *Max.*).

	Surgery case-load					Weekly surgery time [hours]		
		NC	D0C	DC	EM	Ave.	Sd.	Max.
DG	10.45%	5.31%	0.02%	3.72%	1.40%	90 ± 4 (15%)	16	129
GO	8.41%	3.90%	0.62%	3.38%	0.51%	48 ± 3 (8%)	11	69
OP	7.48%	0.58%	0.09%	6.74%	0.07%	17 ± 1 (3%)	3	25
OR	11.27%	4.52%	0.15%	2.95%	3.66%	88 ± 5 (15%)	17	122
PL	12.65%	5.04%	1.57%	5.69%	0.35%	93 ± 4 (15%)	16	116
PR	19.84%	6.15%	0.37%	12.54%	0.77%	47 ± 2 (8%)	6	60
ST	8.93%	4.23%	0.61%	3.44%	0.64%	66 ± 3 (11%)	10	79
UR	11.75%	7.12%	1.41%	2.34%	0.88%	91 ± 5 (15%)	18	125
VC	9.21%	4.72%	0.06%	3.49%	0.94%	65 ± 4 (11%)	15	89
Total	100.00%	41.58%	4.91%	44.29%	9.22%	604 ± 20 (100%)	73	673

Table 2.7: Simulated OT surgical case workload and surgery time.

By comparing values reported in subtable *Surgery case-load* of Table 2.7 with those of Table 2.2, we can argue that also proportions among the different surgical specialty and patient types/subtypes are consistent with the real-world hospital data and that the simulation has sufficient accuracy. By comparing subtable *Weekly surgery time* with Table 2.3, we reveal that also the simulated surgery time for the different surgical specialties is consistent with real-world hospital data and a realistic application of the MSS is validated. The discrepancy between the real weekly surgery time and the simulated one probably depends on the fact that the log-normal distribution does not provide the best fitting with the surgery duration distribution for every surgical specialty as, on the contrary, the real-world case workload and the simulated one are very close. The accuracy of the simulated surgery time is not of primary interest for the simulation purpose and not affects the KPIs we defined.

In Table 2.8, for each surgical specialty and the two patients type (elective and emergency),

we reported the average and maximum LOS for both the real hospital data (subtable *Realized*) and the simulation outcome (365 days, hospital PS and a normal emergency flow).

Specialty	Data	Electives			Emergencies		
		Ave. ^a	Sd.	Max.	Ave. ^a	Sd.	Max.
DG	Re.	4.4 ± 0.5	7.5	90.0	4.7 ± 0.7	4.1	25.0
	Sim.	4.4 ± 0.2	4.3	31.0	4.7 ± 0.2	1.9	11.0
GO	Re.	1.5 ± 0.2	2.4	19.0	3.5 ± 0.5	4.5	30.0
	Sim.	1.8 ± 0.1	2.0	13.0	3.2 ± 0.4	1.7	9.0
OP	Re.	0.1 ± 0.0	0.3	2.0	-	-	-
	Sim.	0.2 ± 0.0	0.6	2.0	-	-	-
OR	Re.	4.9 ± 0.4	7.8	88.0	5.7 ± 0.7	5.4	35.0
	Sim.	4.5 ± 0.2	4.3	28.0	5.8 ± 0.2	2.5	19.0
PL	Re.	1.1 ± 0.1	1.5	13.0	-	-	-
	Sim.	1.2 ± 0.1	1.3	13.0	-	-	-
PR	Re.	0.5 ± 0.1	1.2	13.0	3.1 ± 1.1	2.9	12.0
	Sim.	0.8 ± 0.0	1.3	16.0	3.0 ± 0.3	1.6	10.0
ST	Re.	1.6 ± 0.4	5.0	62.0	3.6 ± 1.8	4.6	23.0
	Sim.	3.5 ± 0.2	3.9	33.0	3.6 ± 0.4	2.3	16.0
UR	Re.	2.4 ± 0.2	3.5	37.0	4.1 ± 0.8	4.1	23.0
	Sim.	2.8 ± 0.1	2.5	26.0	4.4 ± 0.4	2.2	12.0
VC	Re.	5.0 ± 0.6	8.5	89.0	7.4 ± 1.6	8.2	56.0
	Sim.	5.3 ± 0.3	5.1	27.0	7.5 ± 0.5	3.1	21.0
All	Re.	2.4 ± 0.1	5.4	90.0	4.9 ± 0.3	5.3	56.0
	Sim.	2.5 ± 0.1	3.4	33.0	4.9 ± 0.1	2.7	21.0

^a The estimated 95% CI for the average patient LOS.

Table 2.8: Realized and simulated patient LOS statistics [days].

Values reported in Table 2.8 reveal that the simulated patient LOS is very close to the real-world data of the hospital, only the surgical specialty ST presents a simulated patient LOS that is, on average, longer than the that of the real world, but this seems to have no overall impact. In the real-world hospital data, the 95% CI of the average patient LOS is [2.3, 2.5] for elective patients, and [4.6, 5.2] for emergency patients. The simulated patient LOS is very close to the real-world data, *i.e.*, [2.4, 2.6] and [4.8, 5.0], for elective and emergency patients respectively. The average number of beds occupied daily in the simulation is 128, the 95% CI is [126 130], exactly the same value that we found by analysing the real-world data of the hospital. Simulation outcomes for the patient LOS have sufficient overall accuracy.

With the given simulation outcomes in terms of patient arrivals, MSS dependant surgery cases-load, surgery time, patient LOS and daily bed occupancy, we can assume the simulated daily bed occupancy fits that of the hospital in the real world and the simulation validated.

In Table 2.9, the control-table for the hospital setting and different flows of emergency patients, from 1.0 to 4.0 times the normal/tracked flow, is reported. By rows, for each KPI, the 95% CI of the KPI average value computed through 10 replications of the simulation is reported. Note that, with respect to the other control-tables reported in this chapter, this is a simplified version as different BOL-settings are not present because these are not considered in the hospital setting.

KPI	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
(%) Refused em.	0, 1	1, 2	2, 3	4, 5	9, 10	15, 17	18, 20	22, 24	25, 26	26, 27	28, 29
(%) Timedout em.	0, 1	1, 2	2, 3	4, 5	9, 10	15, 17	18, 20	22, 24	25, 26	26, 27	28, 29
(%) El. resched.	0, 0	0, 1	1, 1	1, 2	4, 6	9, 11	12, 13	18, 19	21, 23	25, 26	29, 30
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 3	4, 4	5, 6	8, 9

Table 2.9: Control-table of the hospital setting for the four KPIs and eleven emergency flows

2.7 Results

In this section, we report an analysis of the simulation experiments we run. For each one of the simulation scenarios described in Section 2.6.1, 10 runs, each one simulating 1 year of surgical activity and the scenario-related policies for the scheduling of elective patients and the admission of emergency patients, are executed for a total of 160'000 simulation runs.

For first, in Section 2.7.1, we present an overall and brief discussion of the simulation results. We present the main findings of this chapter in terms of PSs, PSs features and/or scenario factors that can be controlled/applied by the hospital manager and that provide the best performances in terms of the considered KPIs. In Section 2.7.1, we discuss two PSs that we consider as two possible solutions for practical application. Then, in Section 2.7.2, we describe the *benchmark scenario* and the underlying *benchmark PS*, *i.e.*, the simulation scenario and the PS characterized by some specific characteristics that make them suitable to be the benchmark for the other simulation scenarios and PSs.

A detailed analysis of the simulation outcomes in relation to the benchmark scenario and the benchmark PS is in Appendix A. A section is devoted to each one of the relevant scenario factors and PS features, *i.e.*, the patient LOS forecast PI vs EV, the type of bed availability FBPS and FBAD considered for scheduling elective patients and admitting emergency patients, and the enforcement or not of a BOL for elective and/or emergency patients. A section is devoted also to the analysis of the pros and cons of using the OB policy for the management of the OR capacity in place of the application of the MSS.

2.7.1 Overall discussion

In this section, we present a brief discussion of the simulation results and the main conclusions that are extrapolated from the analysis of the detailed simulation outcomes reported in Appendix A.

Such analysis of simulation outcomes reveals that enforcing (or not) the BOL for elective and/or emergency patients is the PS feature with the greater impact on all four KPIs that we consider, *i.e.*, the percentage of refused emergencies, the percentage of timed out emergencies, the percentage of elective rescheduling and the percentage of overdue electives. If the BOL is enforced for both the patient types, elective and emergency, for only one type of patients, or none of the two types, it can drastically change the observed KPI performance. Moreover, performances strictly depend on the selected BOL-setting if the BOL is enforced for at least one patient type. The BOL for elective patients is a needful PS feature to achieve the best all over the four KPIs. For emergency patients, the BOL may be unnecessary, but this strictly depends on the targeted performances. Performances in terms of percentage of refused emergencies and percentage of elective rescheduling remarkably depend also on the type of bed availability considered for the scheduling of elective patients and/or the admission of emergency patients (FBPS or FBAD), and the reliability of the patient LOS forecast (PI or EV) as well. The type FBPS of bed availability is always preferable to type FBAD and the patient LOS forecast PI always provides better performances than the EV forecast.

The percentage of refused emergencies can be controlled by enforcing the BOL for emergency

patients and selecting the BOL-setting that provides the desired performance; if the BOL is enforced also for elective patients, the performance is enhanced. The percentage of timed out emergencies can be controlled with the BOL for elective patients, but the type of bed availability and the patient LOS forecast play a crucial role. The FBPS bed availability is preferable to FBAD and the PI patient LOS forecast provides better performances than EV. The percentage of elective rescheduling can be controlled by enforcing the BOL for both the elective and emergency patients, or not enforcing either for elective or emergency patients; even in this case, both the type FBPS of bed availability and the PI patient LOS forecast have a relevant impact on the KPI, and are preferable to type FBAD and forecast EV respectively. Also, the percentage of overdue electives can be controlled by enforcing the BOL for both the elective and emergency patients, or not enforcing it at all, either for elective or emergency patients. The selected BOL-setting and the OT management policy play a crucial role, several BOL-setting can zero the number of overdue electives independently of the faced flow of emergencies, and the policy OB enhances performances with respect to policy MSS.

Notes for practitioners

Two PSs that fit with the summary of results of Section 2.7.1 are EORS-FBPS-BOL-with-FBAD-BOL and EORS-FBPS-BOL-with-FBAD. The policy for the scheduling of elective patients is the same for both the PSs, *i.e.*, the elective scheduling policy EORS-FBPS-BOL, such an elective scheduling policy is paired with the emergency admission policy FBAD or FBAD-BOL for obtaining the two PSs. For the elective scheduling policy we consider, as types of bed availability, the entire hospital patient stay (type FBPS), and, for the emergency admission policy, the sole admission day (type FBAD). Such a choice is because of the following considerations. Type FBPS of bed availability provides better performances for both elective and emergency patient types. So, FBPS is the considered type of bed availability for the scheduling of elective patients in the two PSs, the type of bed availability is relaxed to FBAD for the admission of emergency patients as the performances degradation is very limited and, for practical application, it may be considered inapplicable or unethical refusing the admission of an emergency patient because the bed is not available all along with the predicted patient hospital stay.

In PS EORS-FBPS-BOL-with-FBAD-BOL, the BOL is enforced for both the patient types (elective and emergency) and, in PS EORS-FBPS-BOL-with-FBAD, only for elective patients. PS EORS-FBPS-BOL-with-FBAD-BOL corresponds to a more conservative PS that enforces the BOL for both elective and emergency patients, with such PS, elective rescheduling is zeroed for every possible flow of emergencies, but the number of refused emergencies is higher for very large flows of emergencies. PS EORS-FBPS-BOL-with-FBAD enforces the BOL only for elective patients and corresponds to a greedier PS with respect to the admission of emergency patients, with such PS, elective rescheduling is not zeroed for every possible emergency flow, but the number of refused emergencies is more contained for very large flows of emergencies. The PS EORS-FBPS-BOL-with-FBAD-BOL is suitable for operational situations for which it is predicted an emergency flow that largely oversteps the number of emergency patients that the hospital is available to admit. The PS EORS-FBPS-BOL-with-FBAD is suitable for situations for which a too large emergency flow is unlikely.

Table 2.10 reports the control-table of PS EORS-FBPS-BOL-with-FBAD-BOL for the four KPIs; by rows, the 95% CI of the average value of the given KPI is reported for the given BOL-setting in the first column and, by columns, for the emergency flows in the table header. Table 2.11 is as Table 2.10, but for PS EORS-FBPS-BOL-with-FBAD.

Simulation outcomes in Table 2.11 and Table 2.10 are for the MSS policy of OR capacity management, as this is the OT policy currently used by the hospital, and for the PI patient LOS forecast; we estimate that human surgery schedulers (frequently the surgeons) are able to predict the LOS of patients with high reliability. The impact on the KPIs of a more flexible policy of OT management, *i.e.*, OB policy, and a less precise patient LOS forecast are investigated in

Section A.3 and Section A.1 respectively. For sake of completeness, the control-table of PS EORS-FBPS-BOL-with-FBAD-BOL and PS EORS-FBPS-BOL-with-FBAD for the OB policy are reported in Appendix A.4.

B.S.	Emergency flow											
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0	
	(%) Refused emergencies											
96%-4%	71, 72	76, 77	79, 80	82, 83	85, 86	88, 88	89, 89	90, 91	91, 91	92, 92	93, 93	
92%-8%	39, 41	50, 51	56, 57	61, 63	69, 70	73, 74	76, 77	79, 80	80, 81	83, 83	84, 85	
88%-12%	19, 21	29, 31	36, 38	45, 46	55, 56	62, 63	66, 67	70, 71	71, 72	74, 75	77, 78	
84%-16%	4, 5	11, 13	19, 21	27, 29	39, 41	49, 50	53, 54	58, 59	62, 62	66, 66	69, 69	
80%-20%	0, 1	3, 4	7, 9	14, 15	28, 29	37, 39	42, 44	49, 50	52, 53	58, 59	62, 63	
76%-24%	0, 0	0, 0	1, 2	4, 5	15, 16	26, 27	31, 32	39, 40	42, 44	49, 50	54, 55	
72%-28%	0, 0	0, 0	0, 0	1, 1	7, 8	17, 19	22, 24	30, 32	34, 36	41, 42	47, 48	
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 2	9, 10	13, 14	21, 23	25, 27	33, 34	39, 40	
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	3, 4	6, 7	15, 16	18, 20	26, 27	32, 33	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	3, 3	9, 10	12, 14	19, 20	26, 27	
	(%) Timeout emergencies											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
	(%) Elective rescheduling											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
	(%) Overdue electives											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
72%-28%	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	
68%-32%	4, 5	4, 4	5, 5	5, 6	4, 5	4, 5	5, 5	4, 5	4, 5	4, 5	4, 5	
64%-36%	8, 9	9, 10	8, 8	9, 10	8, 9	9, 10	9, 10	9, 10	9, 10	9, 9	8, 9	
60%-40%	12, 13	12, 13	11, 12	12, 13	12, 13	12, 13	12, 13	12, 13	12, 13	12, 13	12, 13	

In the table, the two values of the CI are reported separated by the a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table to fit into the page; the table remains meaningful.

Table 2.10: Control-table of PS EORS-FBPS-BOL-with-FBAD-BOL for the four KPIs (95% CI), MSS OT management policy and the PI patient LOS forecast

2.7.2 Benchmark simulation scenarios

In this section, we report the simulation outcomes for benchmark simulation scenarios. Benchmark scenarios are those related to PS EORS-FBPS-BOL-with-FBPS-BOL (let this be the benchmark PS), the MSS policy for the management of the OR capacity and the patient LOS forecast PI. The BOL-setting and the flow of emergency patients are let to vary.

We can define these scenarios as the benchmark scenarios because, with the PS EORS-FBPS-BOL-with-FBPS-BOL, the type of bed availability evaluated for both the scheduling of elective patients and the admission of emergency patients, *i.e.*, FBPS (first bed available all along with the patient stay), together with the PI patient LOS forecast, ensure that no elective patient is rescheduled and no emergency patient is refused because of the discrepancy between the predicted bed occupancy and the real (simulated) one; this is independent of the BOL-setting and the flow of emergency patients. So, there are no emergencies refused because of timeout, emergencies are refused at the admission request time only if there are no free beds for admitting new emergencies without overstepping the BOL defined by the BOL-setting, and there are no elective patients to be rescheduled because bed shortages. Every scheduled elective patient is

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
	(%) Refused emergencies										
96%-4%	1, 2	2, 3	3, 5	7, 8	13, 16	20, 23	24, 26	31, 34	34, 36	40, 42	44, 46
92%-8%	1, 2	2, 3	5, 6	8, 10	13, 15	20, 22	25, 27	33, 35	35, 37	39, 41	44, 46
88%-12%	1, 1	2, 3	4, 5	6, 8	13, 15	20, 23	24, 27	30, 32	34, 36	38, 40	44, 46
84%-16%	0, 1	1, 2	3, 4	6, 8	14, 16	19, 21	25, 27	31, 34	34, 36	40, 42	44, 46
80%-20%	0, 0	0, 1	2, 3	4, 6	12, 13	19, 21	24, 26	30, 32	33, 35	39, 41	44, 46
76%-24%	0, 0	0, 0	1, 1	2, 3	9, 11	19, 20	23, 24	29, 31	33, 34	38, 40	42, 44
72%-28%	0, 0	0, 0	0, 0	0, 1	6, 7	14, 16	18, 20	26, 27	30, 31	35, 37	41, 42
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 2	7, 8	11, 12	19, 21	23, 24	30, 31	36, 37
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	3, 4	5, 6	13, 15	17, 19	24, 26	30, 31
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 3	8, 9	11, 12	18, 20	24, 26
	(%) Timedout emergencies										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	(%) Elective rescheduling										
96%-4%	0, 0	0, 0	0, 1	1, 1	1, 2	2, 3	3, 3	4, 4	4, 5	6, 6	7, 8
92%-8%	0, 0	0, 0	0, 1	1, 1	1, 1	2, 3	3, 3	4, 5	5, 6	5, 6	7, 8
88%-12%	0, 0	0, 0	0, 0	0, 1	1, 2	2, 3	2, 3	3, 4	4, 5	5, 6	6, 7
84%-16%	0, 0	0, 0	0, 0	0, 1	1, 1	2, 2	3, 3	4, 5	4, 5	5, 6	6, 7
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 2	2, 3	3, 3	3, 4	4, 5	6, 7
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 2	2, 3	3, 4	4, 5
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 2	2, 3
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	(%) Overdue electives										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
72%-28%	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1
68%-32%	4, 5	5, 5	5, 6	4, 5	5, 6	5, 5	4, 5	4, 5	4, 5	4, 4	5, 5
64%-36%	8, 9	9, 10	9, 9	9, 9	9, 9	8, 9	9, 10	8, 9	8, 9	8, 9	9, 10
60%-40%	11, 12	13, 14	11, 13	13, 14	12, 13	11, 12	12, 13	11, 12	12, 13	12, 13	11, 12

In the table, the two values of the CI are reported separated by the a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table to fit into the page; the table remains meaningful.

Table 2.11: Control-table of PS EORS-FBPS-BOL-with-FBAD for the four KPIs (95% CI), MSS OT management policy and the PI patient LOS forecast

admitted to the hospital at the scheduled time as the booked bed is never used to serve other patients.

In Table 2.12, we report the control-table of PS EORS-FBPS-BOL-with-FBPS-BOL for the four KPIs given the PI patient LOS forecast and the MSS OT management policy. The 95% CI of the average value of each KPI is reported, by rows, for each BOL-setting and, by columns, for each flow of emergency patients. The control-table of PS EORS-FBPS-BOL-with-FBPS-BOL for the EV patient LOS forecast is in Section A.1, Table A.1, and for the OB OT management policy in Section A.3, Table A.13. The combined impact on each KPI of the OB policy, the EV forecast, and the PS feature, is investigated in Section A.3.

Table 2.12 shows that PS EORS-FBPS-BOL-with-FBPS-BOL and the PI forecast of patient LOS ensure that there are no emergencies timed out and no electives rescheduled; the percentage of timed out emergencies and elective rescheduling is always 0 independently of the BOL-setting and of the flow of emergencies. The percentage of refused emergencies and the percentage of overdue electives are effectively controlled by the BOL-setting. By selecting a BOL-setting, the

B.S.	Emergency flow											
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0	
	(%) Refused emergencies											
96%-4%	70, 71	75, 76	78, 79	81, 82	85, 85	87, 88	88, 89	90, 90	90, 91	91, 92	92, 92	
92%-8%	39, 42	49, 50	55, 57	61, 62	68, 69	73, 74	76, 76	79, 79	80, 80	82, 82	84, 84	
88%-12%	19, 21	30, 32	37, 39	44, 45	55, 56	61, 63	64, 65	69, 70	71, 72	74, 75	77, 77	
84%-16%	4, 5	11, 13	19, 21	27, 29	39, 41	48, 50	53, 54	59, 60	61, 62	65, 66	68, 69	
80%-20%	0, 1	2, 3	7, 9	14, 16	27, 29	37, 39	42, 43	49, 50	52, 53	57, 58	61, 62	
76%-24%	0, 0	0, 0	1, 1	4, 5	15, 17	26, 27	31, 33	39, 40	43, 44	48, 49	53, 54	
72%-28%	0, 0	0, 0	0, 0	1, 1	7, 9	16, 17	22, 23	30, 32	35, 36	41, 42	47, 48	
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 2	8, 10	12, 14	22, 23	26, 27	32, 34	39, 40	
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	3, 4	7, 8	14, 16	18, 19	25, 27	32, 34	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	3, 3	8, 10	12, 13	19, 20	26, 27	
	(%) Timedout emergencies											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
	(%) Elective rescheduling											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
	(%) Overdue electives											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
72%-28%	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	
68%-32%	5, 5	5, 5	5, 5	5, 5	5, 5	5, 5	5, 5	5, 5	5, 5	4, 4	5, 5	
64%-36%	8, 9	9, 9	8, 8	8, 9	9, 9	8, 9	8, 9	8, 9	9, 9	8, 9	8, 9	
60%-40%	11, 12	12, 13	12, 13	11, 12	11, 12	12, 13	12, 13	12, 13	12, 13	12, 13	12, 13	

In the table, the two values of the CI are reported separated by the a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table to fit into the page; the table remains meaningful.

Table 2.12: Control-table of PS EORS-FBPS-BOL-with-FBPS-BOL for the four KPIs (95% CI), MSS OT management policy and the PI patient LOS forecast

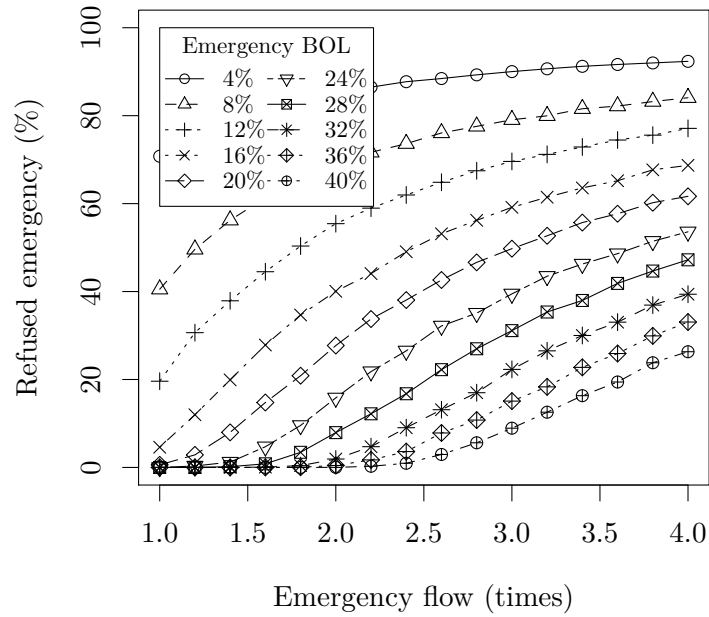
value of the KPIs can be lowered from high values to 0 or very low values. With BOL-setting 76%-24%, for the normal/tracked flow of emergencies (table value 1.0), the PS EORS-FBPS-BOL-with-FBPS-BOL with PI patient LOS forecast dominates the hospital PS as every KPI has a value of 0%.

In Figure 2.9, we report the graphical version of Table 2.12, the control-chart for KPIs percentage of refused emergencies and percentage of overdue electives.

Subfigure (a) of Figure 2.9 reveals that PS EORS-FBPS-BOL-with-FBPS-BOL enforcing the BOL for both the elective and the emergency patients is effective for controlling the percentage of refused emergencies for any flow of emergency patients by selecting the BOL-setting appropriated for the target value of refused emergencies. In subfigure (a), there is no overlapping of the curves. Even if not tested by simulation, it is reasonable to imagine that BOL-settings that enforce a BOL for emergency patients that is greater than the 40% can zero the refused emergencies also for flows of emergency patients that are greater than 2.5 times the normal/tracked flow of emergencies.

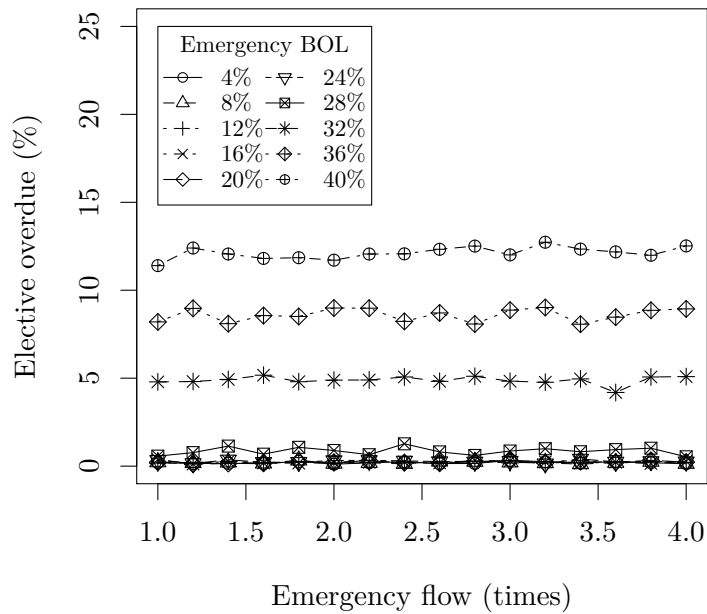
Subfigures (b) of Figure 2.9 reveals that by enforcing the BOL for both elective and emergency patients, *i.e.*, the given PS PS EORS-FBPS-BOL-with-FBPS-BOL, full control over the percentage of overdue electives is possible by selecting the correct BOL-setting. With BOL-settings providing a BOL for elective patients greater than 72%, the percentage of overdue electives is zero, or very close to zero, independently of the faced emergency flow.

EORS-FBPS-BOL with FBPS-BOL



(a) Control chart of PS EORS-FBPS-BOL-with-FBPS-BOL

EORS-FBPS-BOL with FBPS-BOL



(b) Control chart of PS EORS-FBPS-BOL-with-FBPS-BOL

Figure 2.9: Control-charts of PS EORS-FBPS-BOL-with-FBPS-BOL for the percentage of refused emergencies, subfigure (a), and the percentage of overdue electives, subfigure (b); the OT management policy is MSS and the patient LOS forecast is PI

For what concerns the percentage of overdue electives, we can make a further consideration about the bed capacity required for serving elective patients without disruption. Given a total bed capacity of 161 beds, for serving all the elective patients before their due date (*i.e.*, zero overdue elective patients), the BOL for elective patients has to be greater or equal than/to 70% of the bed capacity, *i.e.*, no less than 113 beds have to be available for serving only elective patients.

2.8 Conclusion

In this chapter, we studied some policies that include bed management for the scheduling of elective patients and the admission of emergency patients; the study is limited to surgical patients (elective or emergency). We considered four *Key Performance Indicators* (KPIs) for the policy evaluation: the percentage of refused emergencies, the percentage of timed out emergencies, the percentage of elective rescheduling and the percentage of overdue electives. The defined KPIs aim to measure the quality of service as experienced by emergency and elective patients, all four KPIs are related to the patient ease of access to the hospital surgical service. In the study, operating rooms and ward beds are considered the critical resources of the hospital for scheduling and admitting patients. Shortages of operating room and bed capacity can delay the access of elective patients and may impede the service of emergency patients. In case of resource shortage, elective patients may have to be rescheduled or served after their due date, and emergency patients may have to be rerouted to other hospitals.

For the policy evaluation, an elective scheduling policy and an emergency admission policy are paired to compose policy settings. A policy setting inherits from the composing policies bed management features, *i.e.*, the type of bed availability evaluated for the patient admission/scheduling and the bed occupancy limit enforced to avoid bed shortages. Two types of bed availability that have been considered are the availability of at least a bed on the patient admission date or at least a bed all along with the predicted patient hospital stay. Bed occupancy limits are defined as thresholds for the maximum bed occupancy allowed for the two patient types (elective or emergency). To compute the actual and/or future bed occupancy, a forecast of the patient length-of-stay is required. Two forecast types for the patient length-of-stay have been considered: the perfect information forecast and the expected value forecast.

Elective patient and emergency admission policies have been evaluated also with the applied operating room capacity management policy. As base-case, we considered that the scheduling of elective patients is constrained by a master surgery schedule (the common situation of many real-world hospitals). The open block policy has been also considered to assess the potential benefit of more flexible use of the operating room capacity.

The devised policies have been tested through a discrete event simulation. The flows of elective and emergency patients, as well as the processes of elective scheduling and emergency admission, are simulated. Different simulation scenarios of the flow of emergencies, operating room capacity management policy, and patient length-of-stay forecast, are evaluated to assess policy performances. Through an analysis of the simulation outcomes, we searched for the policy features and scenario factors that provide a good trade-off of the quality of service for emergency and elective patients.

The analysis of the simulation outcomes has revealed, as a general conclusion, that the devised policies including bed management allow grasping the control of the considered KPIs. The bed occupancy limit is revealed as the more relevant policy feature for controlling the KPIs. The patient length-of-stay forecast and the type of bed availability are revealed as less relevant than expected.

A narrow degradation of the KPIs, which arises only for large flows of emergencies, may be acceptable for using simpler policies. We observed that, for example, if the bed occupancy limit is enforced only for elective patients and emergency patients are admitted by evaluating

the availability of a bed only for the admission date (greedy/myopic availability type), the performances degradation is limited. Hospital managers can select the more fitting policy setting to apply according to targets (or thresholds) for the key performance indicators and the expected flow of emergencies.

Future research to improve the framework of policies that we devised may focus on bed management policies that consider dynamic/adaptive bed occupancy thresholds to fit the effective short-term demand of elective and emergency patient admissions, *i.e.*, more sophisticated policies for patient admission with improved flexibility. Otherwise, future research may focus on the introduction of waiting lists of elective patients and the periodic offline scheduling of this type of patients.

Chapter 3

A branch-and-price-and-cut algorithm for operating room scheduling under human resource constraints

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Abstract

In this paper, we address an integrated operating room planning and scheduling problem that includes, with fine detail, constraints commonly encountered in practice (*i.e.*, sequence, capacity and due date constraints) and for human resources other than surgeons, *i.e.*, nurses. A new model of the sequence-dependent operating room cleaning times that arise because of surgeries with different infection levels is considered. To solve this difficult integrated planning and scheduling problem, we devise a branch-and-price-and-cut algorithm based on the time-indexed formulation of the problem. The basic column generation scheme relies on a label-correcting algorithm that we purposely developed for solving the pricing problems that are modelled as single operating

room scheduling problems with time-dependent costs and sequence-dependent cleaning times. The pricing problems are strongly NP-Hard. The efficiency of the label-correcting algorithm is ensured by dominance rules among labels and by two algorithms for computing the upper and lower bound of labels. An effective cutting procedure, inspired by Benders' decomposition and based on duality theory for linear programming, is developed for tightening the linear relaxation of the problem. With instances from the literature and that we generated, we conduct a numerical study to demonstrate the computational effectiveness of the solution method.

3.1 Introduction and literature review

Surgery scheduling is usually treated as an operational problem with a planning horizon spanning one or several days. As pointed out in [33], surgery scheduling involves two main decisions: (i) the assignment of surgical cases to *Operating Room* (OR) sessions (*advanced scheduling*) and (ii) surgical case sequencing for ORs (*allocation scheduling*). Some papers focus only on advanced scheduling (e.g., [63, 52]) or allocation scheduling (e.g., [32, 127]) because, at the cost of suboptimal solutions, the two decisions can be taken sequentially in different decision epochs. Relevant papers dealing with both the advanced and allocation scheduling solved sequentially are [78, 53]. Some recent works focus on advanced and allocation scheduling as an integrated problem to overcome the suboptimality of sequential decisions. In these works, the problem is generally presented as *Integrated OR Planning and Scheduling* (IORPS); relevant papers are [96, 95, 34, 46]. A multi-operating theatre variant of the IORPS problem is proposed in [113].

In [96], a variant of the general IORPS problem considering surgery waiting lists with up to 1000 surgeries and 6 ORs is solved by combining an *Integer Programming* (IP) solver and an improvement heuristic. The problem spans a planning horizon of one week and includes sequence constraints and capacity constraints for ORs and surgeons, due date constraints for surgeries, and some practical constraints for the starting time of surgeries and the assignment of surgical specialties to ORs. A higher OR utilization rate is targeted by the problem objective, and the obtained solutions increase the OR utilization rate up to 40% with respect to hospital planning. The quality of solutions delivered by the IP solver is good, and the improvement heuristic further improves solutions by a few percentage points; however, the IP solver computation time is large in most cases (on the order of hours). While the results are promising, there is room for improvement in the solution approach. To solve the same problem proposed in [96], the same authors have developed in [95] a structured genetic algorithm able to quickly improve the best solution obtained in the previous work, but the quality of the improvement is in a limited range (few percentage points).

In [34], the same IORPS problem of [96, 95] is decomposed in advanced (master) and allocation (sub) problems. An IP solver-based algorithm relying on *Generalized Disjunctive Programming* (GDP) modelling of master and subproblems is applied for the problem solution. The *Mixed-Integer Linear Programming* (MILP) convex hull reformulation of the GDP model of both the advanced and allocation problem is given, and a hybrid continuous-time formulation with multiple time-grid (one per OR) and general precedence sequencing variables is adopted for the allocation problem. The authors promote the hybrid formulation (see [35]) as the best solution for the allocation problem. The obtained solutions quality is good, but the computation time is large (more than an hour).

A sophisticated branch-and-price-and-cut algorithm is proposed in [46] for a IORPS problem considering sequence constraints for infectious/noninfectious surgeries and capacity constraints for surgeons. The *Column Generation* (CG) algorithm is based on the time-indexed formulation of the OR scheduling problem proposed in [96, 95], the CG decomposition and the master problem formulation are inspired by [52], but subproblems are modelled by means of constraint programming and consider fine details of OR schedules. Dominance rules for columns and an infeasibility-detection algorithm are developed to speed up the column generation, and the

cutting procedure is based on lifted minimal cover inequality constraints (see [61] and [11]). Numerical results show the efficacy of the dominance rules and of the infeasibility-detection algorithm. The branch-and-price-and-cut algorithm outperforms the benchmark commercial solver, but the computation time is very large (on the order of hours) for tested medium-sized instances (6 ORs and up to 120 surgeries).

A *Logic-Based Benders' Decomposition* (LBBD) (see [75]) is applied in [113] to solve a multi-operating theatre IORPS problem integrated with the surgeon-to-surgery assignment. The available OR capacity is spread over some hospitals, and surgeries are assigned to surgeons according to patient's preferences. The decomposition master problem integrates advanced scheduling and surgeon assignment, and the subproblems are the OR allocation scheduling tasks. Numerical experiments reveal that Benders' decomposition methods are promising for IORPS problems and that computation times are not excessively large for medium-sized instances.

Nevertheless, few papers consider *Human Resources* (HR) other than surgeons. In [112], the authors address an IORPS problem with HR constraints, in which the modelling is inspired by resource-constrained project scheduling and a time unit of 15 minutes for the time discretization is adopted; the MILP formulation is effective only for very small-sized instances, and a heuristic algorithm is proposed for solving relatively large-sized instances (7 ORs and up to 80 surgeries). In [119], two IP-based heuristic methods are proposed to solve an IORPS problem involving HR constraints within restrained computation times at the expense of lower quality solutions; the time is discretized in very large units (30 minutes) to tackle large-sized instances with up to 11 ORs but no more than 90 surgeries.

The IORPS problem addressed in this paper includes constraints that are the most common in the literature for this kind of problem and provides an attempt for synthesizing state-of-the-art modelling of this kind of problem. Surgery due dates are hard constraints, as in [112, 96] and [46], as are readiness dates, as in [112]. The surgeon availability is given as in [112, 96] and [46], and the *Operating Theatre* (OT) upstream and downstream resources do not bind the scheduling of surgeries, as in [112, 96] and [46]. The scheduling of surgeries is instead constrained by the availability of specialized HRs other than surgeons, *i.e.*, nurses, as in [112] and [119]. As in [46], OR cleaning times depending on the sequence of surgery infection types are enforced and a very small unit for the time discretization is adopted. We assume that an OR cannot be shared by several specialties on the same day as in [96, 95], a practical constraint not well covered by the existing literature.

As argued in [46], time-indexed models, such as those of [96] and [112], are frequently used. Continuous time models involving big-M constraints, *e.g.*, [138], usually provide weaker *Linear Programming* (LP) relaxation and slow down the optimization time. The time-indexed formulation is also used in this paper.

From a practical point of view, failures to account for the most important features of the IORPS with HR constraints constitute the main obstacle to the implementation of surgery planning/scheduling algorithms in many hospitals. Such a problem is hard to solve, even in its simplified version without HR constraints. Simple scheduling algorithms usually fail to provide good (or even just feasible) solutions and lead to a reduced quality of service (*e.g.*, unmatched patient due dates) and direct or indirect economic losses (unused capacity, unscheduled surgeries, etc.). Weak (incomplete) scheduling (done manually, for example) frequently produces infeasible schedules because not all relevant details are considered and material or human resource clashes of surgeries can arise.

The remainder of this paper is organized as follows. In Section 3.2, the IORPS with HR constraints problem is described. The master problem formulation of the column generation decomposition is presented in Section 3.3 and that of one of the pricing problems in Section 3.6. The label-correcting algorithm we develop for solving the pricing problems is described in Section 3.6.2, and the cutting procedure we develop to improve the master problem LP relaxation is elaborated in Section 3.4. The B&P algorithm and the branching rules are described in

Section 3.5. The computational experiments through which we test the effectiveness of the developed algorithms are described in Section 3.7 with the report of the numerical results. The label-correcting algorithm, the Benders' cutting procedure and the evidence of their effectiveness, even for real-world-sized instances, represent the main contributions of this paper.

3.2 Problem statement

In this work, we address the problem of surgery scheduling for an OT composed of several ORs and a planning horizon of several days. The scheduling of ORs is constrained by the availability of two types of skilled HRs required for the execution of surgeries, *i.e.*, surgeons and nurses. The surgeon availability is characterized by a maximum daily time for surgery, whereas nurses are available according to a calendar. Every surgeon and every nurse belongs to a *Surgical Group* (SG), and an SG is a collection of surgical specialties characterized by surgical similarities (*e.g.*, head surgery, chest surgery, etc.). The surgeries to schedule are selected from a wait list. Each surgery is characterized by the surgical specialty, the duration, the infection type, the number of nurses required, the revenue, the release date and the due date. Surgeries with different infection types were first studied in [32], and then in [46], information on surgery infection types was required for correct scheduling of OR cleaning times. Revenue is a financial measure, but without changing the problem nature, it can be easily replaced by another measure such as a score calculated according to a set of criteria (*e.g.*, priority) or simply the surgery duration as in [46]. Surgeries with the due dates falling in the planning horizon are mandatory surgeries; these are surgeries that must be scheduled in the planning horizon, whereas other surgeries may be scheduled or not. The following assumptions hold throughout the paper.

1. The duration of each surgery is deterministic.
2. Every surgery is already assigned to a surgeon.
3. The maximum opening time of the ORs is eight hours and there is no OR or nurse overtime.
4. There is no fixed OR opening cost.
5. The ORs composing the OT are identical with respect to the surgery execution.
6. OT upstream and downstream resources do not constrain the OR scheduling.
7. Nurses can only assist in the execution of surgeries of their own SG.
8. Nurses are trained to assist the execution of surgeries as either scrub or circulating nurses.
9. An *Obligatory Cleaning Time* (OCT) of the OR may be required between two surgeries of a different infection type, where the OCT depends exclusively on the immediately preceding surgery. Moreover, we assume that:
 - (a) a surgery infection type is characterized by an infection level,
 - (b) infection types can be sorted according to the infection level,
 - (c) OCTs are required to switch from infection types of higher infection levels to those of lower infection levels and
 - (d) given any pair of infection types with infection levels f and f' such that $f > f'$ and $OCT(f, f')$ being the OCT required to switch from infection level f to f' , it holds that $OCT(f, f') \leq \sum_{g=f, \dots, f'-1} OCT(g, g+1)$. This is a special case of the triangular inequality, *i.e.*, it is more efficient (faster) to downgrade the OR infection level from f to f' directly with a unique OCT than by passing through the intermediate steps with a sequence of OCTs.

The duration of similar surgeries may vary considerably, but surgeons are usually able to accurately estimate the surgery duration for a specific patient; thus, given Assumption 2, Assumption 1 becomes reasonable. Assumption 2 is consistent with the practice of a wide range of hospitals, from university hospitals where each surgeon has her/his study cases to private hospitals where freelance surgeons perform surgeries on their patients. Eight hours is the regular open time for elective surgeries in many hospitals, and marginal costs of surgery, such as overtime or OR openings, are considered in some papers. However, we do not consider marginal costs, *i.e.*, Assumption 3 and Assumption 4, for the following reasons: (1) OR or nurse overtime is an undesirable event that may occur in the OT but should be avoided as much as possible; it thus seems unreasonable to schedule some overtime in advance as a regular practice, and (2) the marginal cost of allowing an OR to remain open is mainly given by its staffing. If the surgical staff is given (*i.e.*, the cost for staffing is already incurred), the remaining marginal cost is very low and negligible and given by surgical consumables. In regard to Assumption 5, only a few papers consider the opposite case of different OR equipment, and this does not provide a significant contribution. Assumption 6 is realistic for many hospitals. Assumption 7 is very realistic because nurses are usually assigned to an SG in the long term and are trained to assist only surgeries of their SG, whereas only veteran nurses have sufficient experience to assist, within a short time horizon (the same day or week), the execution of surgeries belonging to different SGs. Assumption 8 follows the common practice of many hospitals. Assumption 9 is as in [46] and is very realistic and necessary, but done in isolation as presented in [46], it is not sufficient for achieving full consistency with real-world functioning of ORs and OCT scheduling.

We extend Assumption 9 with Assumptions 9a, 9b, 9c and 9d; otherwise, by neglecting Assumptions 9a-9d, it leads to surgeries with “cleaning power”, which is a paradox. To better clarify, consider the schedule of three surgeries i_1 , i_2 and i_3 in the same OR with $OCT(i_1, i_2) = OCT(i_2, i_3) = 0$ but $OCT(i_1, i_3) > 0$ which does not meet Assumptions 9c. The sequence $i_1 \prec i_2 \prec i_3$ contains no OCT at all, and the insertion of i_2 between i_1 and i_3 has necessarily the paradoxical “cleaning power” on the OR.

The problem objective is the maximization of the total revenue of scheduled surgeries. A solution of the problem is obtained with the following decisions: (1) to assign SGs to ORs on a daily basis, (2) to select the surgeries to schedule, (3) to assign every selected surgery to an OR and (4) to assign a starting time to every selected surgery. These decisions are all correlated with respect to the problem objective, and an integrated decision is required to avoid suboptimal solutions. A feasible problem solution must respect the constraints that follow. Mandatory surgery must be scheduled only once in the planning horizon, and other surgeries may be scheduled at most once. A surgery can be scheduled for any day from on its release date to its due date, but not before/after its release/due date. The total daily OR opening time and the daily maximum surgery time of surgeons cannot be exceeded. The required number of nurses have to be available for the execution of each surgery. Surgeries cannot overlap in the same OR, and surgeries of the same surgeon cannot overlap. For every OR and day, only surgeries of the assigned SG can be scheduled in the OR. OR cleaning must be scheduled according to the OR sequence of surgeries and their infection type. No cleaning is required between two cases with no infection or the same type of infection. In Table 3.1, we report the notations for the problem we describe.

3.2.1 Mixed-integer linear programming model

The MILP of the problem is essentially the same as that of many other works on similar subjects, *e.g.*, [46, 96, 95], but with additional constraints for HRs other than surgeons (*i.e.*, nurses).

Decision variables:

x_{ijtk} , a binary variable that takes value 1 if the surgery i is scheduled for day j and time t in OR k .

Notation	
<i>Sets</i>	
A	Surgeons
B	Nurses
I	Surgeries
J	Days in the planning horizon
K	Operating rooms
SG	Surgical groups
T	Time slots in a day
F	OR cleaning times
<i>Subsets</i>	
B_m	Nurses of SG $m \in SG$
$B_{m,j}$	Nurses of SG $m \in SG$ available on day $j \in J$
I_{dd}	Mandatory surgeries
I_a	Surgeries of surgeon $a \in A$
I_m	Surgeries of SG $m \in SG$
I_j	Surgeries that can be scheduled for day $j \in J$
$I_{m,j}$	Surgeries of SG $m \in SG$ that can be scheduled for day $j \in J$
K_j	Operating rooms open on day $j \in J$
<i>Parameters</i>	
T_{aj}	Maximum surgery time of surgeon $a \in A$ for day $j \in J$
c_i	Revenue for surgery $i \in I$
rd_i	Release date of surgery $i \in I$
dd_i	Due date of surgery $i \in I$
p_i	Duration of surgery $i \in I$
$OCT(i, i')$	Duration of the OCT to execute after surgery i and before surgery i'
	The equivalent of $OCT(f_i, f_{i'})$
n_i	Required number of nurses to assist the execution of surgery $i \in I$
f_i	The infection level of surgery i

Table 3.1: Problem notation

y_{kmj} , a binary variable that takes value 1 if the OR k is assigned to SG m for day j .

The OR scheduling with HR constraints MILP reads:

$$\max \sum_{i \in I} \sum_{j \in J} \sum_{t \in T_j} \sum_{k \in K} c_i x_{ijtk} \quad (3.1)$$

$$\sum_{j \in J} \sum_{t \in T_j} \sum_{k \in K} x_{ijtk} = 1 \quad (i \in I_{dd}) \quad (3.2)$$

$$\sum_{j \in J} \sum_{t \in T_j} \sum_{k \in K} x_{ijtk} \leq 1 \quad (i \in I_{nc}) \quad (3.3)$$

$$\sum_{i \in I_k} \sum_{t'=t-p_i+1}^t x_{ijt'k} \leq 1 \quad (t \in T, j \in J, k \in K) \quad (3.4)$$

$$\sum_{i \in I_a} \sum_{t'=t-p_i+1}^t \sum_{k \in K} x_{ijt'k} \leq 1 \quad (t \in T, j \in J, a \in A) \quad (3.5)$$

$$\sum_{t'=t+p_i}^{t+p_i+OCT(i,i')-1} \bar{c}_{ii'} x_{ij't'k} \leq 1 - x_{ijtk} \quad ((i, i') \in I : i \neq i', t \in T, j \in J, k \in K) \quad (3.6)$$

$$\sum_{i \in I_a} \sum_{t \in T_j^a} p_i x_{ijtk} \leq T_{aj} \quad (j \in J, a \in A) \quad (3.7)$$

$$\sum_{t \in T_j} x_{ijtk} \leq y_{kmj} \quad (i \in I_m, j \in J, m \in SG, k \in K) \quad (3.8)$$

$$\sum_{m \in SG} y_{kmj} \leq |K_j| \quad (j \in J, k \in K) \quad (3.9)$$

$$\sum_{i \in I_m} \sum_{t'=t-p_i+1}^t n_i x_{ijt'k} \leq |B_{mj}| \quad (t \in T_j, j \in J, k \in K) \quad (3.10)$$

$$x_{ijtk} = 0 : j > dd_i \vee j < rd_i \quad (i \in I, t \in T, j \in J; k \in K) \quad (3.11)$$

$$x_{ijtk} \in \{0, 1\} \quad (i \in I, t \in T, j \in J; k \in K) \quad (3.12)$$

$$y_{kmj} \in \{0, 1\} \quad (k \in K, m \in SG, j \in J) \quad (3.13)$$

The objective function maximizes the total revenue of surgeries scheduled in the planning horizon. Constraints (3.2) enforce that mandatory surgeries are scheduled only once in the planning horizon. Constraints (3.3) enforce that not mandatory surgeries are scheduled at most once in the planning horizon. Constraints (3.4) enforce that surgeries do not overlap in the same OR. Constraints (3.5) enforce that surgeries of the same surgeon do not overlap in the schedule. Constraints (3.6) enforce that an OCT is scheduled between any pair of consecutive surgeries scheduled for the same OR and day in the planning horizon if an OCT is required in between. Constraints (3.7) enforce that the daily surgeon maximum surgery time is not exceeded in the schedule for every surgeon and day of the planning horizon. Constraints (3.8) enforce that only surgeries belonging to same SG can be scheduled for the same OR and day in the planning horizon. Constraints (3.9) enforce that only one SG can be assigned to an OR for a every day in the planning horizon. Constraints (3.10) enforce that surgeries scheduled do not require more nurses than those available; this is for every SG, day and time in the planning horizon. Constraints (3.11) enforce that any surgery is scheduled not before its release date and not after its due date. With (3.12) and (3.13) variable x_{ijtk} and y_m domains are defined.

3.3 Column generation master problem

In this section, we present the *Master Problem* (MP) of the column generation reformulation of the IORPS with HR constraints problem. In such an MP, a variable (*i.e.*, a column) stands for a schedule of one OR and one day in the planning horizon.

Let Γ^K be the set of all OR schedules and subset $\Gamma_j^K \subset \Gamma^K$ be the set of feasible schedules for day $j \in J$. Let r_γ^i be a binary value that takes value 1 if surgery i belongs to schedule $\gamma \in \Gamma^K$ and r_γ^{at} be a binary value that takes value 1 if surgeon a is occupied with surgery at time t in schedule $\gamma \in \Gamma^K$. Let also C_γ be the revenue of schedule $\gamma \in \Gamma^K$; this is the sum of the revenues of surgeries belonging to the schedule. Value p_γ^a is the total surgery time for surgeon a in schedule $\gamma \in \Gamma^K$, and value n_γ^{mt} is the number of nurses of SG m occupied with surgery at time t in schedule $\gamma \in \Gamma^K$. The decision variable ϑ_γ takes value 1 if the schedule $\gamma \in \Gamma_j^K$ is selected for day j (0, otherwise). The *OR Master Problem* (ORMP) formulation reads:

$$\max \sum_{\gamma \in \Gamma^K} C_\gamma \vartheta_\gamma \quad (3.14)$$

subject to

$$\sum_{j \in J} \sum_{\gamma \in \Gamma_j^K} r_\gamma^i \vartheta_\gamma = 1 \quad (i \in I_{dd}), \quad (3.15)$$

$$\sum_{j \in J} \sum_{\gamma \in \Gamma_j^K} r_\gamma^i \vartheta_\gamma \leq 1 \quad (i \in I \setminus I_{dd}), \quad (3.16)$$

$$\sum_{\gamma \in \Gamma_j^K} r_\gamma^{at} \vartheta_\gamma \leq 1 \quad (a \in A; j \in J; t \in T), \quad (3.17)$$

$$\sum_{\gamma \in \Gamma_j^K} p_\gamma^a \vartheta_\gamma \leq T_{aj} \quad (a \in A; j \in J), \quad (3.18)$$

$$\sum_{\gamma \in \Gamma_j^K} n_\gamma^{mt} \vartheta_\gamma \leq |B_{mj}| \quad (m \in SG; j \in J; t \in T), \quad (3.19)$$

$$\sum_{\gamma \in \Gamma_j^K} \vartheta_\gamma \leq |K_j| \quad (j \in J), \quad (3.20)$$

$$\vartheta_\gamma \in [0; 1] : \gamma \in \Gamma^K, \quad (3.21)$$

The objective function (3.14) maximizes the total surgery revenue for surgeries scheduled in the planning horizon. Constraints (3.15) and (3.16) enforce that mandatory surgeries are scheduled once in the planning horizon and nonmandatory surgeries are scheduled at most once in the planning horizon. Constraints (3.17) enforce that surgeries of the same surgeon do not overlap: a surgeon can execute only one surgery at a time. Constraints (3.18) enforce that the maximum daily time of surgery is not exceeded for every surgeon and day in the planning horizon. Constraints (3.19) enforce that, for every SG, day and time, the number of nurses occupied with surgery never exceeds the number of available nurses for the given SG and day. Constraints (3.20) enforce that at most $|K_j|$ OR schedules are selected for every day in the planning horizon. The domain of variables ϑ is defined by (3.21).

The LP relaxation of formulation (3.14)-(3.21) is solved by means of column generation techniques given the large, potentially huge, number of variables (feasible schedules) $\gamma \in \Gamma^K$. Column generation techniques tackle such huge numbers of variables considering a restricted number of variables, the subset $\tilde{\Gamma}^K \subset \Gamma^K$. The MP including only the restricted set of variables $\tilde{\Gamma}^K$ is the *Restricted Master Problem* (RMP). Let ORRMP be the OR restricted MP.

The master problem of the column generation scheme presented in this section is essentially the same as that of [46]; we added the resource constraints (3.19) for the nurse availabilities. In [46], the formulation of the master problem is solved by means of column generation (as in our work) and by resorting to a *Constraint Programming* (CP) model for solving subproblems. Since computational efficiency is crucial for generating new columns, we preferred to devise a label-correcting algorithm for solving subproblems, see Section 3.6.2, because we are persuaded that such an approach is likely faster than CP for generating new columns. It is also known that CP suffers from significant overhead times for the model solutions, as in *Integer Linear Programming* (ILP), and thus, it may not be the best for the generation of columns.

3.4 Benders' cutting procedure

In this section, we describe the Benders' cutting procedure that we developed for tightening the LP relaxation of model (3.14)-(3.21). This cutting procedure is inspired by the cutting phase of the three-phase algorithm proposed in [38] for the simultaneous assignment of locomotives and cars to passenger trains. In [38], the classic Benders' decomposition is combined with column generation to decompose and efficiently solve the problem. The same three-phase algorithm is successfully applied for the simultaneous scheduling of aircraft and crew as well; see [39, 101, 102]. Both the simultaneous assignment of locomotives and cars to passenger trains and the simultaneous scheduling of aircraft and crew present a formulation that uses a type of column variable for each resource. In both cases, once Benders' decomposition is applied, the master problem and the subproblem have column variables of/for, respectively, only one type/resource, and the applied Benders' decomposition is also a decomposition by resources.

The cutting procedure that we developed is inspired by Benders' decomposition and developed around the idea of a master resource and some slave resources. Each resource type has its own problem, and the solution of the master problem (that of the master resource) is enforced in the subproblems (those of slave resources) for generating cuts. ORs are the master resource, and surgeons and nurses are slave resources. Our approach differs from a classic Benders' decomposition because the OR solution that we enforce is partial. For generating cuts, we determine computationally efficient to enforce in subproblems the sole surgery selection given a complete schedule for the ORs; this is at the price of retaining resource constraints for surgeons and nurses in the master problem formulation. Through the generation of cuts and the convergence of the master problem and subproblems on a common selection of surgery and objective function value, the LP relaxation of the problem is tightened.

The ORMP of Section 3.3 is the master problem of the cutting procedure and the two Benders' subproblems are the *Surgeon Subproblem* (SSP) and the *Nurse Subproblem* (NSP). The SSP schedules surgeries for each surgeon and day in planning horizon. The NSP schedules surgeries for each nurse and day in planning horizon. At each iteration, for both the SSP and the NSP, the selection of surgeries is constrained by the current LP solution $\bar{\vartheta}_\gamma : \gamma \in \Gamma^K$ of the ORMP. Each time a Benders' subproblem is solved, a new cut can be added to the ORMP. The Benders' cutting procedure stops when the objective function of both the SSP and the NSP equal that of the ORMP, *i.e.*, the objective functions of the three problems have converged on the same value. In Figure 3.1, we present the flow diagram of the Benders' cutting procedure in relation to the basic column generation algorithm.

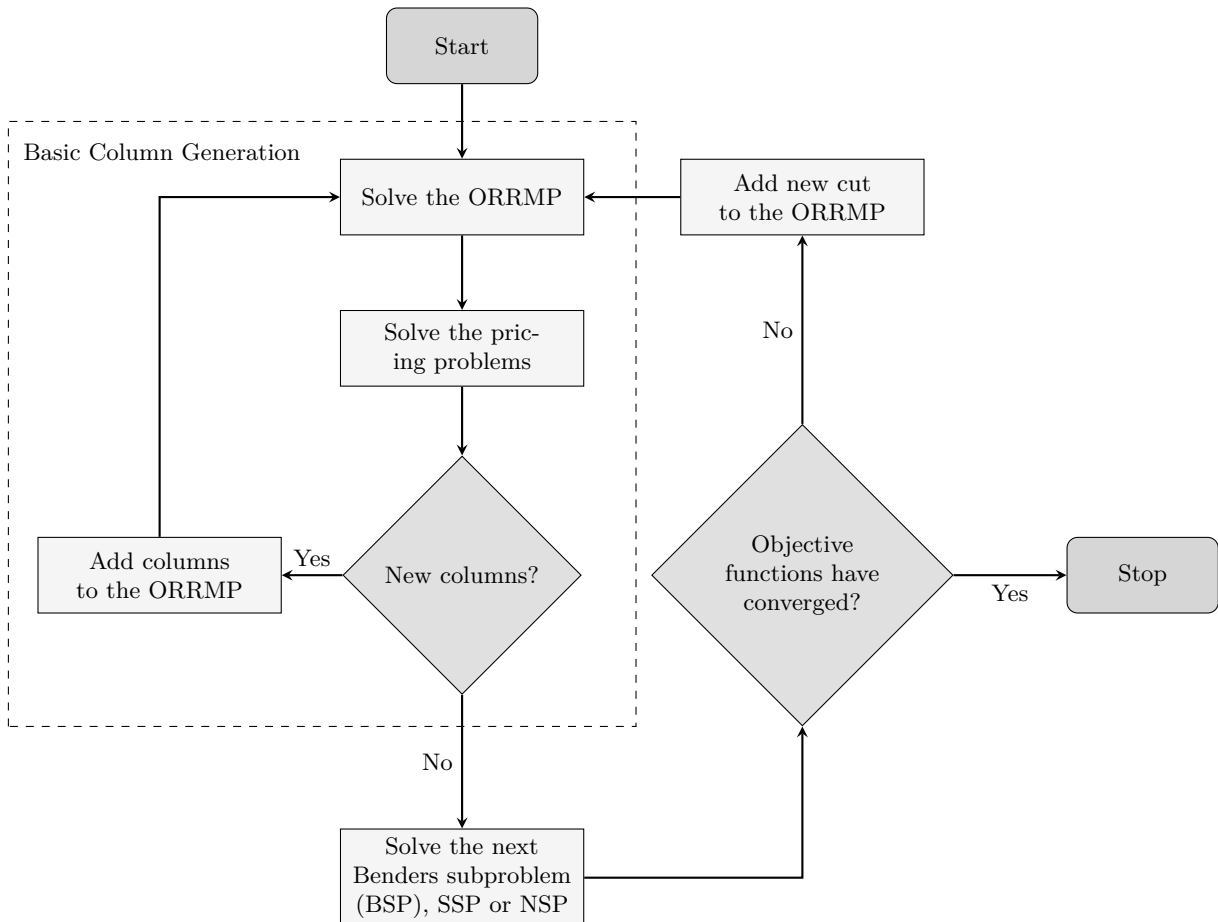


Figure 3.1: Column generation with Benders cutting procedure

In Section 3.4.1, we report the formulation of the *Surgeon Subproblem* (SSP) and, in Sec-

tion 3.4.2, that of the *Nurse Subproblem* (NSP). In Section 3.4.3, we report the restatement of the ORMP with included the formulation of the Benders' cuts generated through the SSP and NSP.

3.4.1 Surgeon subproblem

Let Γ_j^a be the subset of feasible schedules of surgeon $a \in A$ for day $j \in J$ and Γ^A be the set of all feasible schedules of all surgeons A . Let also C_γ be the total revenue of schedule $\gamma \in \Gamma^A$ defined as the sum of the revenues of surgeries belonging to the schedule. Decision variable ζ_γ defines whether the schedule $\gamma \in \Gamma^A$ is selected or not. Given nonnegative values $\bar{\vartheta}_\gamma$ such that $\gamma \in \Gamma^K$ satisfying constraints (3.15)-(3.20), the SSP reads:

$$\max \sum_{\gamma \in \Gamma^A} C_\gamma \zeta_\gamma \quad (3.22)$$

subject to

$$\sum_{j \in J} \sum_{\gamma \in \Gamma_j^a} r_\gamma^i \zeta_\gamma \leq \sum_{\gamma \in \Gamma^K} r_\gamma^i \bar{\vartheta}_\gamma \quad (a \in A, i \in I_a), \quad (3.23)$$

$$\sum_{\gamma \in \Gamma_j^a} \zeta_\gamma \leq 1 \quad (a \in A; j \in J), \quad (3.24)$$

$$\zeta_\gamma \geq 0 : \gamma \in \Gamma^A. \quad (3.25)$$

A solution of the SSP defines a schedule of surgery for every surgeon and day in the planning horizon. The SSP objective (3.22) maximizes the total revenue of the surgeries scheduled in the planning horizon. Constraints (3.23) bound the selection of each surgery in the range $[0; \sum_{\gamma \in \Gamma^K} r_\gamma^i \bar{\vartheta}_\gamma]$. Constraints (3.24) enforce that at most one schedule is selected for each day and each surgeon.

Note that to enforce that mandatory surgeries must be scheduled is not necessary for the SSP and constraints on mandatory surgeries are thus relaxed. To enforce the equality in Constraints (3.23) for mandatory surgeries is not necessary because the selection of surgeries in the SSP is the same as the ORMP when the two objective functions have converged on the same value with the Benders' cutting procedure. Given $\bar{\vartheta}_\gamma : \gamma \in \Gamma^K$ a solution of (3.15)-(3.20) such that the value of the SSP objective function equals that of the ORMP, it holds that

$$\sum_{j \in J} \sum_{\gamma \in \Gamma_j^a} r_\gamma^i \zeta_\gamma = \sum_{\gamma \in \Gamma^K} r_\gamma^i \bar{\vartheta}_\gamma \quad (a \in A, i \in I_a)$$

Let $\lambda^{(3.23)}$ and $\lambda^{(3.24)}$ be the dual variables for constraints (3.23) and (3.24), respectively. The *dual problem* of the SSP reads:

$$\min \sum_{i \in I} \left(\sum_{\gamma \in \Gamma^K} r_\gamma^i \bar{\vartheta}_\gamma \right) \lambda_i^{(3.23)} + \sum_{a,j} \lambda_{aj}^{(3.24)} \quad (3.26)$$

subject to

$$\sum_i r_\gamma^i \lambda_i^{(3.23)} + \lambda_{aj}^{(3.24)} \geq C_\gamma \quad (\gamma \in \Gamma_j^a, j \in J, a \in A), \quad (3.27)$$

$$\lambda_i^{(3.23)} \geq 0 : i \in I, \quad \lambda_{aj}^{(3.24)} \geq 0 \quad \forall a, j \quad (3.28)$$

The formulation of the Benders cut derived from the SSP dual problem is reported in Section 3.4.3.

3.4.2 Nurse subproblem

Let $\Gamma_j^{B_m}$ be the set of nurse schedules feasible for day $j \in J$ and surgical group $m \in SG$. Let Γ^B be the set of all nurse schedules. Decision variable $\varphi_\gamma : \gamma \in \Gamma_j^{B_m}$ defines the number of nurses of surgical group $m \in SG$ with schedule $\gamma \in \Gamma_j^{B_m}$ for day $j \in J$ in the problem solution. Let $C_\gamma : \gamma \in \Gamma^B$ be the schedule total revenue defined as: $\sum_{i \in \gamma} c_i/n_i$. For given nonnegative values $\bar{\vartheta}_\gamma : \gamma \in \Gamma^K$ satisfying Constraints (3.15)-(3.20), the NSP reads:

$$\max \sum_{\gamma \in \Gamma^B} C_\gamma \varphi_\gamma \quad (3.29)$$

subject to

$$\sum_{\gamma \in \Gamma_j^{B_m}} r_\gamma^i \varphi_\gamma \leq n_i \sum_{\gamma \in \Gamma_j^K} r_\gamma^i \bar{\vartheta}_\gamma \quad (i \in I_m, m \in SG), \quad (3.30)$$

$$\sum_{\gamma \in \Gamma_j^{B_m}} \varphi_\gamma \leq |B_{mj}| \quad (m \in SG; j \in J), \quad (3.31)$$

$$\varphi_\gamma \geq 0 : \gamma \in \Gamma^B. \quad (3.32)$$

A solution of the NSP defines a nurse schedule selection for every SG and day in the planning horizon. The NSP objective (3.29) maximizes the total revenue of the scheduled surgeries. Constraints (3.30) enforce that if a surgery is scheduled for the ORMP, the surgery can be scheduled at most for the required number of nurses of its SG. Constraints (3.31) enforce that the number of times that a schedule is selected cannot be greater than the number of available nurses of the related SG. Constraints on mandatory surgeries are relaxed also in the NSP, this is because when the Benders' cutting procedure stops, it holds that

$$\sum_{\gamma \in \Gamma_j^{B_m}} r_\gamma^i \varphi_\gamma = n_i \sum_{\gamma \in \Gamma_j^K} r_\gamma^i \bar{\vartheta}_\gamma \quad (m \in SG, i \in I_m).$$

Let $\lambda^{(3.30)}$ and $\lambda^{(3.31)}$ be the dual variables for Constraints (3.30) and (3.31), respectively. The *dual problem* of the NSP reads:

$$\min \sum_{i \in I_m} n_i \left(\sum_{\gamma \in \Gamma_j^K} r_\gamma^i \bar{\vartheta}_\gamma \right) \lambda_i^{(3.30)} + \sum_{m \in SG} \sum_{j \in J} |B_{mj}| \cdot \lambda_{mj}^{(3.31)} \quad (3.33)$$

subject to

$$\sum_{i \in I_m} r_\gamma^i \lambda_i^{(3.30)} + \lambda_{mj}^{(3.31)} \geq C_\gamma \quad (m \in SG, j \in J, \gamma \in \Gamma_j^{B_m}), \quad (3.34)$$

$$\lambda_i^{(3.30)} \geq 0 \quad \forall i, \quad \lambda_{mj}^{(3.31)} \geq 0 \quad \forall m, j \quad (3.35)$$

The formulation of the Benders' cut derived from the NSP dual problem is reported in Section 3.4.3.

3.4.3 ORMP restatement and cutting procedure

From dual problems (3.26)-(3.28) and (3.33)-(3.35), two types of Benders' cuts to add to the ORMP are derived.

Let Δ denote the polyhedron defined by Constraints (3.27) and (3.28) of the SSP; let also P_Δ and R_Δ be the set of extreme points and extreme rays of Δ , respectively. Note that, because of relaxed Constraints (3.23) on mandatory surgeries, the SSP is always feasible, and the set R_Δ of extreme rays is therefore empty. Let Ξ denote the polyhedron defined by Constraints (3.34)

and (3.35) of the NSP; let also P_{Ξ} and R_{Ξ} be the set of extreme points and extreme rays of the Ξ , respectively. The NSP is always feasible because of relaxed Constraints (3.30) on mandatory surgeries, and the set R_{Ξ} of extreme rays is therefore empty.

Let $z_0 \in \mathbb{R}^+$ be an auxiliary real variable to express the total revenue of scheduled surgeries. The ORMP, *i.e.*, formulation (3.14)-(3.21), can be restated for the Benders' cutting procedure as:

$$\max z_0 \quad (3.36)$$

subject to

$$(3.15), (3.16), (3.17), (3.18), (3.19), (3.20)$$

$$\sum_i \sum_{\gamma \in \Gamma^K} c_i r_{\gamma}^i \vartheta_{\gamma} - z_0 = 0 \quad (3.37)$$

$$\sum_i \sum_{\gamma \in \Gamma^K} \lambda_i^{(3.23)} r_{\gamma}^i \vartheta_{\gamma} - z_0 \geq - \sum_{a,j} \lambda_{aj}^{(3.24)} \quad ((\boldsymbol{\lambda}^{(3.23)}, \boldsymbol{\lambda}^{(3.24)}) \in P_{\Delta}), \quad (3.38)$$

$$\sum_i \sum_{\gamma \in \Gamma^K} n_i \lambda_i^{(3.30)} r_{\gamma}^i \vartheta_{\gamma} - z_0 \geq - \sum_{m,j} |B_{mj}| \cdot \lambda_{mj}^{(3.31)} \quad ((\boldsymbol{\lambda}^{(3.30)}, \boldsymbol{\lambda}^{(3.31)}) \in P_{\Xi}), \quad (3.39)$$

$$z_0 \geq 0, \quad \vartheta_{\gamma} \in [0; 1] : \gamma \in \Gamma^K. \quad (3.40)$$

Constraint (3.37) limits the z_0 variable to total revenue of scheduled surgeries expressed as a function of variables $\theta_{\gamma} : \gamma \in \Gamma^K$; this constraint allows to start the Benders' cutting procedure with an already good LP relaxation of the ORMP. Constraints (3.38) and (3.39) are the optimality Benders' cuts generated by solving the SSP and the NSP, respectively.

The optimality Benders cuts, Constraints (3.38) and (3.39), are not generated exhaustively. The Benders' cutting procedure generates only a subset of cuts sufficient to obtain an optimal solution. Feasibility Benders' cuts are never generated at all given the emptiness of R_{Δ} and R_{Ξ} ; we therefore omit the formulation of feasibility cuts. At each iteration of the algorithm, the relaxed ORMP is solved including the subset of cuts generated until the current iteration, *i.e.*, $P'_{\Delta} \subseteq P_{\Delta}$ and $P'_{\Xi} \subseteq P_{\Xi}$.

3.5 Branch and price

To obtain an optimal, or at least high-quality, integer solution of the ORMP, we developed a branch-and-price (B&P) procedure. Given $\bar{\vartheta}_{\gamma} \geq 0 : \gamma \in \Gamma^K$ the current LP solution of the ORMP, the B&P branching is based on the following rules:

1. branch on the total number of surgeries scheduled per SG in the planning horizon;
2. branch on whether a surgery is scheduled for a given day and time or not.

Branching rules 1 and 2 are applied hierarchically and according to the numerical order; rule 2 is applied if, with rule 1, the branching cannot be further applied.

To apply the branching rule 1, given $m' \in SG$, the surgical group with the most fractional number of surgeries scheduled in the current ORMP solution (*i.e.*, with the fractional part of the number closest to 0.5), one of the following two constraints has to be added to the ORMP:

$$\sum_{i \in I_{m'}} \sum_{\gamma \in \Gamma^K} r_{\gamma}^i \vartheta_{\gamma} \leq \lfloor \sum_{i \in I_{m'}} \sum_{\gamma \in \Gamma^K} r_{\gamma}^i \bar{\vartheta}_{\gamma} \rfloor, \quad (3.41)$$

$$\sum_{i \in I_{m'}} \sum_{\gamma \in \Gamma^K} r_{\gamma}^i \vartheta_{\gamma} \geq \lceil \sum_{i \in I_{m'}} \sum_{\gamma \in \Gamma^K} r_{\gamma}^i \bar{\vartheta}_{\gamma} \rceil. \quad (3.42)$$

Let Ω_1 be the set of all possible cuts of form (3.41) or (3.42) that can be generated.

To apply the branching rule 2, we branch on the most fractional scheduled time of a surgery in the current solution, *i.e.*, given r_γ^{ijt} a binary value that takes value 1 if surgery i is scheduled for day j and time t in schedule γ , one of the following two constraints has to be added to the ORMP:

$$\sum_{\gamma \in \Gamma^K} r_\gamma^{ijt} \vartheta_\gamma = \lfloor \sum_{\gamma \in \Gamma^K} r_\gamma^{ijt} \bar{\vartheta}_\gamma \rfloor, \quad (3.43)$$

$$\sum_{\gamma \in \Gamma^K} r_\gamma^{ijt} \vartheta_\gamma = \lceil \sum_{\gamma \in \Gamma^K} r_\gamma^{ijt} \bar{\vartheta}_\gamma \rceil. \quad (3.44)$$

Note that $0 \leq \sum_{\gamma \in \Gamma^K} r_\gamma^{ijt} \bar{\vartheta}_\gamma \leq 1$ for every $t \in T_j$, $j \in J$ and $i \in I$. Let Ψ be the set of all possible constraints of this type that can be generated.

Constraints in Ω_1 are effective for improving the column generation upper bound and detecting the problem infeasibility of a given B&P node, and constraints Ψ drive the search of an integer solution for the restricted ORMP. An integer solution for each node of the B&P is computed by enforcing integrality constraints to the column variables added to the ORMP and by solving the ORMP. The best integer solution of the ORMP found until the current iteration of the B&P provides a lower bound on the optimal solution of the problem and allows an early pruning of B&P nodes.

3.6 Column generation subproblem

The three problem formulations presented in Section 3.4 imply column generation for their solution. In this section, we describe the solution method that is applied for solving the pricing problems of the ORMP. The pricing problems of the SSP and the NSP are simplified versions of ORMP ones. So, we describe the solution method of the *OR Pricing Problem* (ORPP) and, when necessary, we point out what changes hold for solving the *Surgeon Pricing Problem* (SPP) or the *Nurse Pricing Problem* (NPP).

In Section 3.6.1, we define the ORPP as an extension of the *Rainbow Path* (RP) problem, see [82], that we call the *Maximum Revenue RP with Resource Constraints* (MRRPRC) problem, since searching for the OR schedule with maximum reduced cost is equivalent to searching for the MRRPRC over a graph. In Section 3.6.2, we describe the RP algorithm developed for solving the MRRPRC problem.

3.6.1 Description of the MRRPRC

Let $G = (V, E)$ be a *Directed Acyclic Graph* (DAG), where V is the set of nodes and E is the set of arcs. There is a node v_t for each possible starting time $t = 0, \dots, T$ of every surgery and three types of arcs: *surgery arcs*, *idle time arcs* and *OR-cleaning time arcs*. There is a surgery arc (i, v_t) of length p_i that reaches node v_{t+p_i} for every surgery $i \in I$ and feasible starting time t . There is an idle time arc from v_t to v_{t+1} for every time period $t \leq T - p_i$. There is an OR-cleaning time arc from v_t to $v_{t+OCT(f, f')}$ for every pair of infection levels (f, f') such that $OCT(f, f') > 0$ and time period $t \leq T - OCT(f, f')$. Every surgery arc that corresponds to the same surgery has the same colour $i \in I$. Idle time and OR-cleaning time arcs have no colour.

Let $d_{ai} \geq 0$ be the consumption of resource (surgeon) $a \in A$ along any arcs of colour i , it corresponds to the surgery duration (arc length) $d_{ai} = p_i$. For every resource $a \in A$, the capacity Q_a is defined.

Note that, in this MRRPRC problem, the graph is acyclic (any path is elementary by graph construction) and pairs of nodes are linked by many arcs instead of single arcs (as often encountered in column generation).

Let λ be the vector of dual costs of constraints from (3.15) to (3.19) of the master problem formulation plus the Benders cuts' constraints, (3.38) and (3.39), the branching constraints (3.43) or (3.44) and (3.43) or (3.44). For each element of λ , the constraint reference is in superscript

and the constraint indices are subscripts, *e.g.*, $\lambda_i^{(3.15,3.16)}$ are the dual costs of Constraint (3.15) or (3.16) depending on whether the surgery $i \in I$ is mandatory or not, respectively. Let x_{it} be a binary decision variable that takes value 1 if arc (i, v_t) is selected (0 otherwise). The reduced cost of a schedule $\gamma \in \Gamma^K$ is given by

$$\begin{aligned}
C_\gamma &= \sum_{i \in I_j} \sum_{t \in T_j} \lambda_i^{(3.15,3.16)} x_{it} - \sum_{i \in I_j} \sum_{t \in T_j} \sum_{t'=t}^{t+p_i-1} \lambda_{a_{ijt'}}^{(3.17)} x_{it} - \sum_{i \in I_j} \sum_{t \in T_j} \lambda_{a_{ij}}^{(3.18)} p_i x_{it} \\
&- \sum_{i \in I_j} \sum_{t \in T_j} \sum_{t'=t}^{t+p_i-1} \lambda_{m_{ijt'}}^{(3.19)} n_i x_{it} - \sum_{i \in I_j} \sum_{t \in T_j} \sum_{\nu \in P'_\Delta} \lambda_\nu^{(3.38)} \lambda_i^{(3.23)} x_{it} \\
&- \sum_{i \in I_j} \sum_{t \in T_j} \sum_{\nu \in P'_\Xi} \lambda_\nu^{(3.39)} n_i \lambda_i^{(3.30)} x_{it} - \sum_{\omega \in \Omega'_1} \lambda_\omega^{(3.41,3.42)} \sum_{i \in I_j} \sum_{t \in T_j} x_{it} \\
&- \sum_{(i,j,t) \in \Psi'} \lambda_{ijt}^{(3.43,3.44)} x_{it} - \lambda_j^{(3.20)}
\end{aligned} \tag{3.45}$$

that is separable according to starting times of surgeries; the part of the reduced cost depending on surgery $i \in I$ and time $t \in T_j$ is given by

$$\begin{aligned}
w_{it} &= c_i - \lambda_i^{(3.15,3.16)} - \sum_{t'=t}^{t+p_i-1} \lambda_{a_{ijt'}}^{(3.17)} - \lambda_{a_{ij}}^{(3.18)} p_i - \sum_{t'=t}^{t+p_i-1} \lambda_{m_{ijt'}}^{(3.19)} n_i \\
&- \sum_{\nu \in P'_\Delta} \lambda_\nu^{(3.38)} \lambda_i^{(3.23)} - \sum_{\nu \in P'_\Xi} \lambda_\nu^{(3.39)} n_i \lambda_i^{(3.30)} \\
&- \sum_{\omega \in \Omega'_1} \lambda_\omega^{(3.41,3.42)} - \sum_{(i',j,t) \in \Psi': i'=i} \lambda_{i'jt}^{(3.43,3.44)} x_{i't},
\end{aligned} \tag{3.46}$$

and the reduced cost can be written as

$$\sum_{i \in I_j} \sum_{t \in T_j} w_{it} x_{it} - \lambda_j^{(3.20)}. \tag{3.47}$$

Every surgery arc thus has revenue $w_{(i,v_t)} = w_{it}$. Idle time and OR-cleaning time arcs have revenue equal to zero.

The problem objective is to identify the RP from v_0 to v_T that satisfies resource constraints and maximize the revenue. The MRRPRC can be described with the following MILP:

$$\max \sum_{(i,v_t) \in E} w_{it} x_{it} - \sum_{m \in SG} -\lambda_j^{(3.20)} \tag{3.48}$$

subject to

$$\sum_{(i,v_t) \in E} x_{it} - \sum_{(i,v_t) \in E} x_{it} = 0 \quad (v_t \in V \setminus \{v_0, v_T\}), \tag{3.49}$$

$$\sum_{(i,v_0) \in E} x_{it} = 1, \tag{3.50}$$

$$\sum_{(i,v_T) \in E} x_{it} = 1, \tag{3.51}$$

$$s_{at} + d_{ai} \leq s_{ait+p_i} + M(1 - x_{it}), \tag{3.52}$$

$$s_{at} \leq Q_a, \tag{3.53}$$

$$\sum_{m \in SG} y_m \leq 1, \tag{3.54}$$

$$\sum_{i \in I} \sum_{(i, v_t) \in E} x_{it} \leq y_m \quad (m \in SG), \quad (3.55)$$

$$\sum_{(i, v_t) \in E} x_{it} \leq 1 \quad (i \in I), \quad (3.56)$$

$$\sum_{(i', v_{t'}) \in E_{t, (i, i')}^{oct}} x_{i't'} \leq 1 - x_{it} \quad ((i, i') \in I : i \neq i', v_t \in V), \quad (3.57)$$

$$x_{it} \in \{0; 1\} \quad \forall i \in I_j, t \in T_j. \quad (3.58)$$

Binary decision variable y_m takes value 1 if arcs of colours belonging to SG m can be selected (0 otherwise). Variable s_{at} measures the consumption of resource a reaching node v_t . The objective function (3.48) maximize the revenue, *i.e.*, search for the best paths. Constraints (3.49)-(3.53) are those for resources. Constraints (3.54) and (3.55) enforce that only colours of the selected SG can be selected. Constraints (3.56) enforce that paths have to be rainbow paths. Constraints (3.57) enforce infection level constraints on the sequence of surgery and OR cleaning arcs. With (3.58), the domain of variables x_{it} is defined.

The revenue w_{it} of the surgery arcs (i, v_t) have a simpler formulation in the SPP graph and the NPP graph, these are $w_{it} = c_i - \lambda_i^{(3.23)}$ and $w_{it} = c_i - \lambda_i^{(3.30)}$, respectively. The objective function becomes

$$\max \sum_{(i, v_t) \in E} w_{it} x_{it} - \lambda_{aj}^{(3.24)} \quad (3.59)$$

for the SPP and

$$\max \sum_{(i, v_t) \in E} w_{it} x_{it} - \lambda_{mj}^{(3.31)} \quad (3.60)$$

for the NPP.

In both the graph of the SPP and the NPP, there are no OR-cleaning time arcs and path colouring Constraints (3.54) and (3.55) have to be removed from the model. Even if there is no term that is a function of the time index t in the formulation of the surgery arc revenue w_{it} , the proposed graph modelling is effective as it allows considering constraints on surgery scheduling, *i.e.*, constraints in Ψ .

In the context of column generation, the MRRPRC optimization can be terminated as soon as some feasible paths are found, namely, paths that for the ORPP, satisfy

$$\sum_{(i, v_t) \in E} w_{it} x_{it} - \lambda_j^{(3.20)} > 0, \quad (3.61)$$

i.e., columns with a strictly positive reduced cost. As any feasible schedule can be represented as a path in the graph, the absence of such a rainbow path means that there exist no variables with a positive reduced cost with the given set of dual variables. A condition similar to (3.61) can be derived for the SPP and the MPP from objectives (3.59) and (3.60), respectively.

Proposition 1. The MRRPRC problem is strongly NP-Hard.

Proof. Proof. We prove the theorem by reducing the U problem to the MRRPRC problem. Problem U is a variant of the *single-machine scheduling problem with electricity costs*, and it is proven to be strongly NP-Hard by reduction of the 3-PARTITION problem; see [51]. Problem U is as follows. Jobs \mathcal{J} must be processed nonpreemptively at a uniform speed over a time horizon of distinct and contiguous time periods $\mathcal{P} = \{1, \dots, |\mathcal{K}|\}$. Let a_k and d_k be, respectively, the starting time and the duration of time period $k \in \mathcal{P}$. Each time period $k \in \mathcal{P}$ has an electricity price c_k per unit of energy. Each job $j \in \mathcal{J}$ has processing time p_j and a power demand q_j . It is assumed that processing times and the duration of time periods are given as integers. The relationship between processing time and power demand is arbitrary. We can

create an instance of the MRRPRC problem from any instance of the U problem as follows. Let $d_{jkt} = [t; t + p_j - 1] \cap [a_k, a_k + d_k - 1]$ be the processing time of job j that overlaps time period k if the job starts at time t . For each job $j \in \mathcal{J}$, a surgery i with execution time $p_i = p_j$ and revenue $w_{it} = \sum_{k \in \mathcal{P}} c_k q_j d_{jkt}$ is created. \square

3.6.2 Rainbow path algorithm

The idea of modelling the problem as the search of paths over a graph is inspired by [126] where the Dantzig-Wolfe decomposition combined with column generation is applied to compute the LP relaxation the time-indexed formulation of a single-machine scheduling problem. The LP relaxation is given by the optimal linear combination of generated pseudoschedules (columns). In [126], a pseudoschedule is a machine schedule in which a job can appear more than once. The approach of [126] has the advantage of polynomial time for computing pseudoschedules by means of Dijkstra's algorithm since the column generation subproblem is modelled as a shortest path search. As we are interested in searching in reasonable time problem integer solutions, and not only the LP relaxation, in this work, we extend the germinal idea of [126] to the search of true schedules as a feasible selection of true schedules is an integer problem feasible solution. For this purpose, we introduce coloured arcs in the problem graph at the price of upgraded problem complexity. The MRRPRC problem is NP-Hard.

The authors of [46] resort to a column generation algorithm with an NP-Hard pricing problem for generating true schedules. There, such an NP-Hard pricing problem is solved through CP to generate as many optimal columns as possible at the price of relatively high computation times. We are persuaded that generating true schedules is winning for fast detecting high-quality integer problem solutions and that dynamic programming, in reason of potential computational efficiency, is more promising than CP that offers the mere advantage of generating a large number of optimal columns. Beyond these considerations, some of the dominance and optimality rules that are developed for improving the label-correcting algorithm we present in this section are derived from those developed by [46] for reducing the number of optimal columns generated at each call to CP.

Problem (3.48)-(3.58) can be decomposed by building a graph for each SG, which allows removing colouring Constraints (3.54) and (3.55); consequently, the algorithm for the paths search is simpler. The search of the best path through the different SG graphs can be optimized by means of simple pruning strategies avoiding the full exploration of each SG graph; details are in Section 3.6.2. The description in this section is intended for solving the MRRPRC problem over a single SG graph.

Let a simplified notation be used for describing the algorithm: let set $I_{mj} \subseteq I$ be denoted (with an abuse of notation) with its membership set I . In this section, there is no need to differentiate. Surgeries provided as input of the RP algorithm are those that can be scheduled for day j and belong to SG m .

The algorithm that we developed to solve the MRRPRC is a label-correcting algorithm. Such an approach is an extension of the Ford-Bellman algorithm to take into account resource constraints and is quite common for vehicle routing problems with resource constraints, *e.g.*, papers [54] and [57] in the context of column generation. Labels depict paths and indicate the resource consumption of paths. Labels are generated throughout the algorithm and are sorted out as node attributes. Each node has its own set of labels that stands for the incoming paths that terminate with the node. Nodes in the graph are iteratively evaluated extending every incoming label toward every possible successor node. The algorithm terminates when no new labels are created. In Section 3.6.2, some dominance rules to limit the proliferation of labels are introduced. A detailed description of the algorithm is reported in Section 3.6.2.

The algorithm that we developed implements a *breadth-first search*, as do most labeling algorithms. Such a strategy has the benefit of a quick generation of a diversified set of paths searching the optimal one, but it can be longer than *depth-first search* in finding a path reaching

the destination node. To overcome this potential weakness of the breadth-first search, a heuristic algorithm for extending any new label (path) until the destination node v_T is developed; the heuristic algorithm is described in Section 3.6.2. The computation of an upper bound for the best RP is explained in Section 3.6.2. With upper and lower bounds, pruning strategies can be implemented to further limit the proliferation of labels.

Label definition and extension function

Let L be the label standing for a path from the origin node v_0 to node v_t . A label is defined by the following attributes: t_L the terminal node time index, W_L the path revenue, C_L the path colouration, S_{aL} the consumption of each resource $a \in A$ and f_L the label infection level. Note that the infection level is also a resource for labels. The label consumption of resource f is increased or decreased according to the label extension: any time a path is extended with a surgery arc or an OR cleaning arc, the infection level f_L of the label is updated accordingly, *i.e.*, lifted, lowered or kept unchanged.

For the label extension function, with an abuse of notation, let index i denote not only a colour $i \in I$ but also an OR cleaning $i \in F$ or an OR idling as well. A new label is created accordingly with the label extension function $Extend : L \rightarrow L'$

$$L' = Extend(L) = \begin{cases} W_{L'} = W_L + w_{it}, & \text{To maximize} \\ C_{L'} = C_L \cup i, & (i \notin C_L \wedge f_i \geq f_L) \vee i \notin I \\ t_{L'} = t_L + p_i, & - \\ S_{aL'} = S_{aL} + d_{ai}, & Q_a - S_{at} \geq d_{ai} \\ f_{L'} = f_i, & (i \in I \wedge f_i \geq f_L) \vee (i \in F : f' = f_L) \end{cases} \quad (3.62)$$

The label function, in the right-hand side, defines the attribute update operations of the label extension and the related constraints/conditions. In order, the constraints/conditions are: the revenue has to be maximized, the colour of the extending arc has to not already belong to the label or the extending arc has to be OR cleaning or OR idling, there is no constraint for the time-index update, the remaining capacity of resource $a \in A$ has to be sufficient and the arc infection level has to be the greater of or equal to that of the label if the extending arc is a surgery arc or, if it is OR cleaning, the infection level of the label to downgrade f_L has to equal that foreseen for the $OCT(f', f_i)$, *i.e.*, $f' = f_L$. Note that the label infection level remains unchanged if the extending arc is an OR-idling arc.

Dominance rules

The optimal solution of the MRRPRC problem can be found by considering for the label extension only nondominated labels (paths) and colours. In the following, we define some dominance rules to apply for limiting the proliferation of labels.

Proposition 2. Given labels L and L' reaching the same node v_t , label L' dominates label L if $W_{L'} \geq W_L$, $C_{L'} \subseteq C_L$ and $f_{L'} \leq f_L$.

Proof. *Proof.* Let L'' be a complete extension of L to node v_T . The extension of L' to node v_T through the same surgery arcs extending L is feasible since all colours in $C_{L''} \setminus C_L$ are reachable for L' , *i.e.*, $(C_{L''} \setminus C_L) \cap C_{L'} = \emptyset$, $f_{L'} \leq f_L$ and $S_{aL'} \leq S_{aL}$ for any $a \in A$. The revenue of the extended path from label L' equals $W_{L'} + W_{L''} - W_L$, and therefore, L' dominates L . \square

Let $w_{it}^{max} = \max\{w_{it} : t = t, \dots, T\}$ and $w_{it}^{min} = \min\{w_{it} : t = t, \dots, T\}$ be, respectively, the maximum and the minimum revenue among arcs of colour i from node v_t to node v_T .

Proposition 3. Given label L of node v_t and the two feasible extensions L' and L'' obtained extending L with an arc of colour i' and i'' , respectively, label L' dominates label L'' if

1. $w_{i't}^{min} > w_{i''t}^{max}$,
2. $p_{i'} + \widehat{OCT} \leq p_{i''}$ and
3. $d_{ai'} + d_{ai''} > Q_a - S_{atL} \forall a \in A : d_{ai'} > 0 \wedge d_{ai''} > 0$;

and colour i'' is thus not optimal for extending label L . Value \widehat{OCT} is an upper bound on the additional pre/post OR cleaning time required if an arc of colour i'' is substituted with an arc of colour i' given the infection level f_L . Possible values for $\widehat{OCT} > 0$ are reported in Table 3.2¹.

Proof. Proof. Let L^* be a complete extension of L'' to node v_T . According to condition 3, both colours i' and i'' cannot appear in the extension of L , and the extension of L' to node v_T through the same surgery arcs extending L'' is thus feasible since all colours in $C_{L^*} \setminus C_{L''}$ are reachable for L' , i.e., $(C_{L^*} \setminus C_{L''}) \cap C_{L'} = \emptyset$, and $S_{aL'} \leq S_{aL''}$ for any $a \in A$. By condition 2, the surgery arc of colour i'' can be replaced in label L^* with a surgery arc of colour i' , by condition 1, the label revenue is improved to at least $W_{L^*} + w_{i't}^{min} - w_{i''t}^{max}$, and therefore, L' dominates L'' . \square

Proposition 4. Given label L of node v_t , resource $a \in A$ and colour set $\bar{I}_{aL} \subseteq I_a$ such that $\bar{I}_{aL} \cap C_L = \emptyset$ and $d_{ai} < Q_a - S_{aL}$ for every $i \in \bar{I}_{aL}$, given colour $i \in \bar{I}_{aL}$ and set D_i of colours that dominate colour i with respect to conditions 1 and 2, colour i is not optimal for extending label L if it holds that

$$4. d_{ai} + \sum_{i' \in D_i \cap \bar{I}_{aL}} d_{ai'} > Q_a - S_{aL}.$$

Proof. Proof. Let L^* be a complete extension of L to node v_T that includes colour i . Because of condition 4, L^* can never include the entire colour set D_i , i.e., $D_i \setminus (C_{L^*} \cap D_i) \neq \emptyset$ always. Thus, by condition 2, the surgery arc of colour i can be replaced with an arc of a colour $i' \in D_i \setminus (C_{L^*} \cap D_i)$ and, by condition 1, the revenue of label L^* is improved to at least $W_{L^*} + w_{i't}^{min} - w_{i't}^{min}$. Colour i is therefore not optimal for extending label L of node v_t . \square

Proposition 5. Given label L and colour set $\bar{I}_L \subseteq I$ such that $\bar{I}_L \cap C_L = \emptyset$ and $d_{ai} < Q_a - S_{aL} \forall i \in \bar{I}_L, a \in A$, given colour $i \in \bar{I}_L$ and set D_i of colours that dominate colour i with respect to conditions 1 and 2, colour i is not optimal for extending label L if it holds that

$$5. p_i + \sum_{i' \in D_i \cap \bar{I}_L} p_{i'} > T - t.$$

Proof. Proof. Let L^* be a complete extension of L to node v_T that includes colour i . Because of condition 5, L^* can never include the entire colour set D_i , i.e., $D_i \setminus (C_{L^*} \cap D_i) \neq \emptyset$ always. Thus, by condition 2, the surgery arc of colour i can be replaced with an arc of a colour $i' \in D_i \setminus (C_{L^*} \cap D_i)$ and, by condition 1, the revenue of label L^* is improved to at least $W_{L^*} + w_{i't}^{min} - w_{i't}^{min}$. Colour i is therefore not optimal for extending label L of node v_t . \square

$f_{i'} \text{ vs. } f_L$	$f_{i'} \text{ vs. } f_{i''}$	$f_{i''} \text{ vs. } f_L$	\widehat{OCT}
$f_{i'} < f_L$	$f_{i'} < f_{i''}$	$f_{i''} < f_L$	$OCT(L, i') - OCT(L, i'')$
$f_{i'} < f_L$	$f_{i'} < f_{i''}$	$f_{i''} > f_L$	$OCT(L, i') - OCT(i'', L)$
$f_{i'} > f_L$	$f_{i'} > f_{i''}$	$f_{i''} < f_L$	$OCT(i', L) - OCT(L, i'')$
$f_{i'} > f_L$	$f_{i'} > f_{i''}$	$f_{i''} > f_L$	$OCT(i', L) - OCT(i'', L)$
$f_{i'} < f_L$	$f_{i'} < f_{i''}$	$f_{i''} = f_L$	$OCT(L, i')$
$f_{i'} > f_L$	$f_{i'} > f_{i''}$	$f_{i''} = f_L$	$OCT(i', L)$

Table 3.2: Value for OCT bound \widehat{OCT}

¹In the table, $OCT(L, i')$ and $OCT(L, i'')$ are the OR cleaning times required to add a surgery arc, respectively, of colour i' or i'' , given the label infection level f_L .

Heuristic RP and best RP lower bound computation

Given label L that stands for a path reaching node v_t , the lower bound $LB(L)$ of L is computed by means of a greedy procedure. The path is iteratively extended with feasible surgery arcs as long as some arcs are selectable or the destination node v_T is not reached. The label extension function is defined by (3.62). At each iteration, the most promising arc for the label extension is selected according to

$$\arg \max_{(i,v_t)} \frac{w_{it}}{p_i} \cdot \left(\frac{w_{it}}{w_{it}^{max}} \right)^2, \quad (3.63)$$

i.e., the marginal revenue of selecting a surgery arc of colour i for time t is pondered with the squared ratio of selecting a surgery arc of colour i for time t to the optimal time selection of a surgery arc of colour i . This rule for a greedy arc selection provides the best results among other possible rules we tested through numerical experiments.

If no surgery arc can be selected, the current label is extended with idle time arcs to reach v_T . Note that OR cleaning arcs are not considered in this heuristic.

Best RP upper bound computation

For the upper bound calculation of a label, we developed a method based on the well-known algorithm for the LP relaxation of the *Multidimensional 0-1 Knapsack Problem* (MKP), see [55].

Given a label L and its infection level f_L , the LP solution of the MKP is computed for every infection level from $f_{min} = \min\{f_i : i \notin C_L\}$ to f_L . For every infection level $f \in [f_{min}; f_L]$, the knapsack to fill has a dimension for each resource (surgeon) $a \in A$ with capacity $Q_a - S_{aL}$ and a dimension for the residual distance to reach the destination node v_T with capacity $d(v_t, v_T) = T - t - OCT(L, f)$. To fill the multidimensional knapsack, there is an item for each colour $i \notin C_L : f_i \geq f$ with volume equal to the arc length p_i and revenue equal to $w_{it'}^{max}$ with $t' = t + OCT(L, f)$ (the maximum arc revenue of arcs of colour i from $v_{t'}$ to v_T). Let thus $UB_f(L)$ be the LP solution for MKP computed for label L and infection level f . The upper bound for label L is finally $UB(L) = \max\{UB_f(L) : f \in [f_{min}; f_L]\}$.

Description of the algorithm

Let $G_m : m \in SG$ be the graph of SG m . Let also $LB_0(G_m)$ and $UB_0(G_m)$ be, respectively, the lower bound and the upper bound calculated over graph G_m given the empty label L_0 . Upper bound $LB_0(G_m)$ and lower bound $UB_0(G_m)$ are valid bounds for the best rainbow path that can be found over graph G_m . The upper bound $UB_0(I) = \max\{UB_0(G_m) : m \in SG\}$ is thus a valid upper bound on the best RP that can be found through the SG graphs created providing to the algorithm as input the set of surgeries I . In the following, we describe the algorithm for searching the best RP over SG graphs such that $UB_0(G_{m'}) \geq \max\{LB_0(G_m) : m \in SG\}$; other graphs are pruned.

Let Λ_t be the set of labels of node v_t (*i.e.*, the set of paths reaching and terminating with node v_t) and $\mathbf{\Lambda}$ be the set of all created labels. Let UB and LB be the global search upper and lower bound, respectively. Function *FAOE* is a set filtering function retaining as output the set of arcs that are feasible and optimal for extending the label provided as input. The arc feasibility is defined by constraints of function (3.62) and the optimality by Propositions 3, 4 and 5 defined in Section 3.6.2. Function *Extend* is the label extension function as defined with expression (3.62), and *LBproc* is the heuristic RP algorithm described in Section 3.6.2. Function *EFF* is a set filtering function retaining as output the set of nondominated labels passed as input.

The RP algorithm evaluates graph nodes according to their topological order, *i.e.*, from the source node v_0 to the sink node v_T . Each node v_t is evaluated extending every nondominated label that reaches the node with any feasible outgoing arc. Labels with an upper bound lower

than the best found RP that reaches the sink node v_T are pruned. The algorithm returns the set of nondominated RPs that start from node v_0 , reach node v_T , and cannot be further extended. Algorithm 1 provides the pseudocode of the RP algorithm.

Algorithm 1 Rainbow Path algorithm

```

1: procedure RP()
2:    $LB \leftarrow -\infty$ ;
3:    $UB \leftarrow +\infty$ ;
4:    $\Lambda_t = \{\emptyset\} \forall v_t \in V$ ;
5:    $\Lambda_0 = \{L_0\}$ ;
6:
7:   for  $v_t \in V : t = 0, \dots, T$  do
8:      $\Lambda_t \leftarrow$  set of  $v_t$  labels;
9:     for all  $L \in \Lambda_t$  do
10:      if  $UB(L) > LB$  then
11:         $E_L^* \leftarrow \mathbf{FAOE}(L, E_{v_t})$ ;
12:        for all  $(i, v_t) \in E_L^*$  do
13:           $L' \leftarrow \mathbf{Extend}((i, v_t), L)$ ;
14:           $L'' \leftarrow \mathbf{LBproc}(L')$ ;
15:           $LB \leftarrow \max\{LB; W_{L''}\}$ ;
16:           $\Lambda_{t'} \leftarrow \mathbf{EFF}(\Lambda_{t'} \cup L')$ ;
17:           $\Lambda_T \leftarrow \mathbf{EFF}(\Lambda_T \cup L'')$ ;
18:         $\Lambda_t \leftarrow \Lambda_t \setminus \{L\}$ ;
19:       $UB \leftarrow \max\{UB(L) : L \in \Lambda\}$ ;
20:
21:   return  $\{L \in \Lambda_T : W_L \geq LB\}$ ;

```

In the context of column generation, the RP algorithm can be terminated early, as soon as the size $|\Lambda_T|$ of rainbow paths reaches a given value. We found to explore all the SG graphs with a strictly positive upper bound $UB_0(G_m) > 0$ (those that may generate some columns with a positive reduced cost) more efficient; this is even the case if the upper bound $UB_0(G_m)$ is dominated by an already found lower bound $LB_0(G_{m'}) : m' \neq m$; the column generation is otherwise longer. In Algorithm 1, we omit implementation details about the column generation condition for termination and the search through the SG graphs to facilitate pseudocode clarity and readability as much as possible.

3.7 Computational results

In this section, we validate through a set of computational experiments the effectiveness of the algorithms that we developed, namely, the basic column generation, the column generation with Benders' cuts and the branch-and-price-and-cut. We test the algorithms with a set of instances that we generated based on data from the CHUSE (*Centre Hospitalier Universitaire de Saint-Étienne*, France). The instance generation procedure is described in Section 3.7.1. We also test the algorithms with the instances used in [46]. The CHUSE database tracks the surgical activity of several years.

Through the experiments, we first show the efficiency of the label-correcting algorithm used for solving the column generation pricing problems; the algorithm is described in Section 3.6.2. The effectiveness of the Benders' cutting procedure developed for tightening the LP relaxation of the problem computed by column generation is then evaluated, and the quality of the branch-and-price-and-cut algorithm for the search of integer problem solutions is compared with results of [46]. We also benchmarked our branch-and-price-and-cut with the results obtained with a commercial IP solver (CLPEX).

All the algorithms are coded in C++ and resort to the IBM ILOG CPLEX APIs (version

12.8) for LP and ILP routines. All the experiments are run with a processor *Intel Xeon E7-8890 v3* at 2.50 GHz.

3.7.1 Instances

In all the instances that we generated, the planning horizon is five days (a regular work week) and duration and times are expressed in time slots of 5 minutes. In the CHUSE database, the surgery duration, the surgical specialty and the number of nurses that assisted a surgery are known for every surgery. The CHUSE database records do not provide all the problem data; therefore, we complemented database records with randomly generated data. Even though not tracked in the CHUSE database, missing problem data are available information at the problem decision epoch.

For every surgery, the release date is drawn uniformly in the interval $[0; |J| - 1]$ and the due date is drawn uniformly in the interval $[ReleaseDate; 14]$. Two infection types are considered, infectious and noninfectious; the infectious type is randomly assigned to surgeries with probability $\frac{1}{2}$. A surgery can be normal or expensive. The average reimbursement is estimated, in France, at €3,375.00 for normal surgeries and at €7,830.00 for expensive surgeries. The expensive type is randomly assigned to surgeries with probability equal to a ratio we arbitrarily defined for every surgical specialty. The surgery reimbursement is drawn from a normal distribution with a mean equal to the reimbursement average value for the surgery type and standard deviation equal to $\frac{2}{3}$ of the mean (because the standard deviation is unknown); values lower than €500.00 and greater than €24,000.00 are discarded. Values of the ratio of expensive surgeries are reported in Table 3.3 for each surgical specialty, column *e.s. ratio*. In Table 3.3, we also reported the surgical group of the specialty (*SG*), the average surgery duration for the specialty (*ave.*), the percentage of surgeries with a duration lower than 60 minutes (< 60), lower than 90 minutes (< 90) and lower than 150 minutes (< 150), and the number of surgeries in the database for each surgical specialty.

SG	specialty	e.s. ratio	Duration			surgeries	
			ave.	< 60	< 90		< 150
OM	maxillofacial	65.00	101.4	5.47	46.42	88.83	5937
OM	otolaryngology	15.00	103.9	4.86	41.29	88.31	10372
NR	neurosurgery	85.00	134.8	0.72	15.76	65.13	9525
ORTR	bone	50.00	112.5	2.61	34.02	81.73	18901
CV	cardiovascular	75.00	133.8	0.87	20.85	64.74	7156
INF	pediatric	50.00	103.5	4.81	44.12	87.22	11326
DIG	alimentary canal	20.00	121.7	2.35	29.25	73.78	8185
GYURA	urology	20.00	97.6	6.16	50.58	91.11	6847
GYURA	gynecology	20.00	113.0	4.49	35.48	80.20	6973
GTPOT	oncological pneumology	40.00	83.9	8.99	62.23	99.32	1480
GTPOT	general and thoracic surgery	45.00	117.4	2.80	30.76	77.93	12692
AMB	daily	5.00	86.6	8.89	59.56	98.41	7275
OPT	ophthalmology	15.00	92.7	6.45	50.86	96.42	7292

Table 3.3: Expensive surgery ratio for specialties

Inputs for the instance generation are as follows: the desired number of surgeries, the desired number of surgeons, the selected SGs (then the surgical specialties), and the CHUSE database. It holds that the number of surgeons is always greater than the number of surgical specialties related to the selected SGs.

To generate an instance, a surgeon can be assigned to one and only one surgical specialty, one surgeon is assigned to every surgical specialty until all specialties have one surgeon assigned, and the remaining surgeons are then assigned to a specialty randomly selected. The number of days that a surgeon is available in the planning horizon is drawn from a triangular distribution with $min. = 1$, $max. = 5$ and $mode = 1$, the weekdays (from Monday to Friday) when the surgeon is

available are randomly selected, and the surgeon maximum daily time is drawn for every selected day from a triangular distribution with $min. = 4$, $max. = 8$ and $mode = 4$. Parameters of the two triangular distributions are estimated using the data available in the CHUSE database.

Surgeries are selected by iterating over surgeons. At each iteration, a given number of surgeries belonging to the surgeon specialty are randomly selected in the CHUSE database; this selection of surgeries is repeated until the desired number of surgeries belong to the instance. The number of surgeries to select at each iteration is calculated, for every surgical specialty, as 1 plus the rounding of the ratio between the average surgery duration multiplied by a scalar greater than 1 and the average surgery duration of the given specialty. This value is larger for surgical specialties with a prevalence of short surgeries and smaller for specialties with a prevalence of long surgeries; the scope of this value is to allow the generation of more balanced instances in terms of total surgery time of the different surgical specialties.

The number of nurses belonging to each SG m is computed as

$$\max \left\{ \frac{\sum_{i \in I_m} p_i n_i}{|J| - 1} \cdot T_m, \max\{n_i : i \in I_m\} \right\} \quad (3.64)$$

where T_m is the regular daily work time for nurses belonging to SG m . One day off, randomly selected in the planning horizon, is assigned to every nurse. For the other days, the cut off time of the regular work time is calculated as the OT opening time (conventionally 0) plus T_m . The value of T_m of 480 minutes is set for every $m \in SG$.

We generated three instance sets, denoted with capital letters S, M and H. For instances of set S, the selected surgeries belong to two SGs, OM and OPT , surgery durations are in the interval [40 minutes, 150 minutes] and the number of surgeries belonging to an instance is 60, 80 or 100; three surgeons and two equivalent ORs are available for surgery. For instances of set M, the selected surgeries belong to four SGs, OM , OPT , NR and INF , surgery durations are in the interval [40 minutes, 240 minutes] and the number of surgeries is from 60, 80 or 100; nine surgeons and six equivalent ORs are available for surgery. For instances of set H, surgeries belong to ten SGs (all considered SGs) and there are 160 surgeries with durations in the interval [40 minutes, 240 minutes]; twenty-four surgeons and twelve equivalent ORs are available for surgery. We generated five instances for each problem setting in S and M, and ten instances for set H.

The selection of SGs of sets S and M is driven by the purpose of designing a stress test for the developed algorithms, given that, as reported in Section 3.7.3, instances with only short surgeries are more difficult to solve. We verified with preliminary numerical experiments that other possible (equivalent) selections of SGs give similar results and lead to the same conclusions. For instances in S and M, we generated instances with an OR capacity lower than the expected total surgery time calculated as the product of the average surgery duration with the number of surgeries. The number of surgeons is chosen to obtain a tight total surgeon capacity with respect to the OR capacity. The 65 instances used in [46] instances are divided into three sets — A, B and C — reflecting different situations. For all these instance sets, there are no nurses, the revenue of each surgery is equal to the surgery duration and the OT management policy is open block, no SGs have to be assigned to ORs (there is virtually only one SG). In set A, surgeons are the more binding resources for scheduling the surgeries, and instances count from 40 up to 120 surgeries with only medium-long durations from 120 minutes up to 240 minutes. Instance set B differs from set A only with respect to the number of ORs open each day; in this set of instances, the OR capacity is also binding. In set C, both surgeons and ORs are binding resources, the number of surgeries is from 60 up to 100 and surgery durations are in the interval [40 minutes, 150 minutes]. For further details, see [46].

Instance benchmarking: a constructive heuristic algorithm

To test the effective hardness of evaluated instances, we developed a simple constructive heuristic algorithm. The heuristic algorithm keeps a priority queue of unscheduled surgeries, the surgery priority is assigned according to the early due date rule and, for equal due dates, surgeries with higher revenue have higher priority. Days in the planning horizon are evaluated in a rolling horizon fashion, and surgeries are scheduled greedily by processing the priority queue. Given the surgery ahead of the priority queue, the first starting time that fits the required capacity of the assigned surgeon, OR and required nurses is selected for scheduling the surgery, the concerned resource capacity is blocked from the surgery starting time to the surgery completion time and the surgery is removed from the queue. A surgery cannot be scheduled before its release date or after its due date.

3.7.2 Parameters

In the experiments we ran, we put a time limit of 3 hours for the total time of each run and a time limit of 2 minutes for the LB computing (solving the restricted master problem with integer variables). The target number of columns to add to the RMP at each iteration is defined by a function of the number of surgeries in the instance. This function is $\max\{4; 2^{7-0.05|I|}\}$ and, for the values 40, 60, 80, 100, 120 and 160 of $|I|$, produces the series $\{32, 16, 8, 4, 4, 4\}$. All the experiments are run as single-thread processes; there is no parallelism for both our code and calls to the commercial solver.

3.7.3 Results

In this section, we report numerical results for the experiments we ran for the 40 instances that we generated and the 65 instances by [46]. In all the tables we present, the results for instances of sets A, B and C are reported as average values. This is not only because the number of instances is very large but also because it provides a quick comparison with the results reported in [46], that are reported as average values.

Constructive heuristic algorithm

The solution quality of the simple heuristic algorithm that we developed to benchmark the hardness of the instances is by far inferior to that of the exact optimization algorithms we devised. The heuristic dramatically fails to find a feasible solution that schedules the whole set of mandatory surgeries for many instances; this happens for 80% of instances of set S and for 100% of instances of all the other sets, *i.e.*, sets M, H, A, B and C. For instances of set S, the average gap of the heuristic solution to the best known integer solution is 43%, and the minimum and the maximum are 25% and 58%, respectively. For instances of set M, the average gap is 22%, the minimum 9% and the maximum 32%. For instances of set H, the average gap is 31%, the minimum 20% and the maximum 44%. For instances of set A, B and C, respectively, the average gap is 12%, 10% and 3%, the maximum 20%, 19% and 13% and the minimum gap is 6% for set A and 0% for both sets B and C.

Basic CG

In Table 3.4, the basic CG algorithm is evaluated. We reported the LP relaxation computed by CG (*rev. LP*), the solution of the ORRMP with integer column variables (*rev. IP*) and the percentage gap of the IP solution with respect to the LP relaxation (*gap*). We also reported the time of the basic CG algorithm (*time LP*), the time spent solving the ORRMP (*time MP*), the time spent for the CG, *i.e.*, for solving the pricing problems, (*time CG*) and the time spent solving the ORRMP with integer column variables (*time IP*). We finally report the iteration

count of the basic CG algorithm (*iter. CG*) and the number of columns added to the ORRMP (*columns*). For instance sets A, B and C, in column *Rev. IP*, we reported between parenthesis the number of instances for which an integer solution is found by the algorithm; each row shows the average value for the given set.

Table 3.4 provides evidence that, for instances with only short surgeries (set S), most of the computation time is spent for the CG. This CG time decreases for instances with also longer surgeries, sets M and H. For set M, the time spent for the ORRMP solution and the CG is relatively balanced (the same order of magnitude). For set H, the time spent for the CG is significantly smaller than the time spent solving the ORRMP, in most of the cases smaller by one order of magnitude; this is because of the longer CG convergence due to the large number of surgeries and ORs in the instances of set H. The same relation between the time of the ORRMP solution and the CG time is evidenced through the results for sets A, B and C.

The average computation time for a call to CG can be calculated, for each instance, by dividing the *time CG* by *iter. CG*. As expected, the shorter the surgeries are, the longer this average time for a CG call is. The comparison between set S and M is emblematic. The average of this average time for a call to CG is 0.45 seconds for instances of set S and 0.07 seconds for instances of set M. The maximum average time for a call to CG is, respectively, 1.78 seconds for instances of set S, 0.19 seconds for instances of set M.

The gap of the IP solution with the LP relaxation of the basic CG algorithm seems not to be influenced by the instance characteristics of size and surgery durations for the instances that we generated: the gap is in the range of 0.0% and 18.7%. For sets of instances by [46] — A, B and C — the gap is in the range of 0.3% and 15.5%.

The number of columns generated is quite low for every set of instances. It is reasonably larger for set H, with many surgeries and many ORs available. It is also larger for instances of sets A, B and C, as expected, because of the open block strategy.

CG with Benders cuts

In Table 3.5, the CG with the Benders' cuts algorithm is evaluated. As in Table 3.4, we report values of the LP relaxation, of the integer solution and of the relative gap: (*rev. LP*), (*rev. IP*) and (*gap*), respectively. We report the total time of the CG-with-Benders'-cuts algorithm spent for computing the LP relaxation (*time LP*); this time includes the time spent solving the SSP (*time SSP*) and the NSP (*time NSP*). We finally report the total number of columns added to ORRMP (*columns*), the number of cuts generated by solving the SSP and the NSP, (*cuts SSP*) and (*cuts NSP*), respectively. As in Table 3.4, for instance sets A, B and C, in column *Rev. IP*, we report between parenthesis the number of instances for which an integer solution has been found by the algorithm.

The results in Table 3.5 show the efficacy of Benders' cuts in closing the IP optimality gap by improving the quality of the LP relaxation. For most of the small instances, sets S and M, the optimality of the IP solution is proven by applying the Benders' cutting procedure. For large instances (set H), the quality of the LP relaxation is significantly improved by Benders' cuts, but there are no IP solutions with proven optimality.

Although a remarkable improvement of the LP relaxation is attained, between 0% and 14% with an average of 3%, the Benders' cutting procedure requires a computation time that is not negligible. For instances of set S, the average time spent for solving the SSP and the NSP is, respectively, 16% and 13% of the total CG-with-Benders'-cuts time, 6% and 7% for instances of set M, respectively, and 2% (for both SSP and NSP) for instances of set H.

Branch-and-price-and-cut

Table 3.6 reports results for the IP commercial solver (CPLEX) and the branch-and-price-and-cut algorithm; the table is divided into two subtables, (*IP Solver*) and (*Branch-and-Price-and-*

Instance	rev. LP	rev. IP	gap (%)	time LP	time MP	time CG	time IP	iter. CG	columns
S60-1	117618	113604	3.41	19.2	4.5	14.4	0.1	95	752
S60-2	159934	149562	6.49	94.2	58.7	35.6	0.9	896	3141
S60-3	149045	145260	2.54	299.7	36.8	262.9	0.3	309	1817
S60-4	161093	161093	0.00	24.9	2.9	21.7	0.0	108	591
S60-5	111489	104192	6.55	2.1	0.5	1.3	0.0	31	214
S80-1	171743	170226	0.88	238.7	122.3	116.4	1.0	1355	3072
S80-2	145758	143487	1.56	140.5	19.4	121.1	0.1	306	1227
S80-3	136392	129849	4.80	35.9	13.6	22.4	0.2	318	956
S80-4	138825	121656	12.37	21.1	3.7	17.3	0.1	108	547
S80-5	160769	157079	2.30	1599.2	166.4	1432.7	1.3	803	2748
S100-1	148556	132610	10.73	270.3	21.8	248.5	0.1	727	1192
S100-2	154182	152521	1.08	502.0	24.8	477.2	0.1	493	960
S100-3	75948	73556	3.15	10.8	1.3	9.5	0.0	176	216
S100-4	168372	164986	2.01	897.0	130.6	766.4	0.2	1598	2183
S100-5	169078	160515	5.06	477.6	10.7	466.8	0.1	402	721
M60-1	179959	179489	0.26	5.4	2.1	3.3	0.1	91	413
M60-2	224376	213885	4.68	58.9	39.2	19.7	2.8	401	1561
M60-3	181254	180460	0.44	4.3	1.3	2.9	0.0	46	262
M60-4	220481	211937	3.88	37.5	21.7	15.7	0.5	316	1315
M60-5	227003	209209	7.84	9.4	3.6	5.8	0.0	131	462
M80-1	257464	235575	8.50	19.6	10.7	8.9	0.1	161	803
M80-2	206327	187989	8.89	112.8	42.2	70.6	1.1	771	1551
M80-3	236646	227641	3.81	144.7	67.4	77.3	1.0	1016	2001
M80-4	284881	280529	1.53	64.0	30.7	33.3	0.4	646	1334
M80-5	179000	145472	18.73	471.7	258.2	213.5	9.0	1126	3647
M100-1	296298	280485	5.34	481.4	266.3	215.0	1.5	1591	2957
M100-2	292928	277942	5.12	90.8	43.8	47.1	0.4	1056	1313
M100-3	263210	242692	7.80	151.9	84.3	67.6	0.6	946	1583
M100-4	216244	193964	10.30	212.0	130.3	81.7	0.7	1181	1848
M100-5	281826	268728	4.65	131.5	73.4	58.1	0.2	906	1375
H160-1	630674	593505	5.89	1186.2	1112.9	73.4	23.5	406	11286
H160-2	605497	558853	7.70	326.6	254.7	71.9	9.6	161	7749
H160-3	588978	542703	7.86	1391.6	991.7	399.9	94.3	2001	4867
H160-4	711602	667348	6.22	404.7	342.9	61.8	4.3	246	7597
H160-5	627127	577052	7.98	298.6	222.1	76.5	3.3	196	4343
H160-6	529833	496900	6.22	712.2	414.8	297.4	109.2	86	12255
H160-7	499528	459498	8.01	68.6	51.7	16.9	0.4	76	1514
H160-8	601766	574734	4.49	54.2	39.0	15.2	0.5	76	1133
H160-9	549814	537190	2.30	246.3	195.9	50.3	2.8	136	4554
H160-10	525786	506039	3.76	148.8	119.2	29.6	2.0	181	4405
Set									
A40	1431	1394(5)	2.59	3.4	1.8	1.2	0.3	47	641
A60	2020	1859(5)	7.96	24.9	18.3	6.6	37.3	143	1170
A80	2307	2104(5)	8.80	334.7	203.2	131.5	22.2	761	2605
A100	2401	2103(5)	12.41	658.5	372.8	285.7	68.8	1231	2820
A120	2423	2161(4)	11.03	521.3	261.5	259.8	18.2	948	2314
B40	1371	1355(5)	1.12	6.2	3.7	2.4	5.4	37	762
B60	1866	1824(5)	2.28	61.4	22.0	39.4	14.2	131	1274
B80	2205	2096(5)	4.92	183.6	105.6	78.0	56.6	344	1941
B100	2416	2176(5)	9.87	1532.8	673.3	859.6	108.6	1956	3788
B120	2480	2172(5)	12.40	1630.5	568.4	1062.1	108.1	1657	3526
C60	576	500(5)	13.19	39.1	16.8	22.4	2.8	74	1136
C80	792			35.0	17.8	17.2	0.2	102	780
C100	978			13.4	8.5	5.0	0.1	123	576

Table 3.4: Evaluation of the basic CG Algorithm

Cut). We reported, for both the IP solver and the B&P&Cut algorithm, the LP relaxation and the IP solution that result at the end of the IP optimization, (*rev. LP*) and (*rev. IP*), respectively. For the IP solver, we reported the percentage gap (*gap*) of the IP solution with

Instance	rev. LP	rev. IP	gap (%)	time LP	time SSP	time NSP	time IP	columns	cuts SSP	cuts NSP
S60-1	114168	114168	0.00	46.6	10.8	3.8	0.1	762	8	0
S60-2	149562	149562	0.00	238.0	2.5	6.0	1.7	4328	4	4
S60-3	147640	145260	1.61	377.2	26.7	15.2	0.4	1859	4	0
S60-4	161093	161093	0.00	27.6	1.3	1.3	0.0	591	0	0
S60-5	104192	104192	0.00	50.9	19.9	21.0	0.0	225	12	9
S80-1	170226	170226	0.00	473.4	23.2	18.2	2.1	3801	3	1
S80-2	143487	143487	0.00	372.4	11.6	8.0	0.4	1513	3	3
S80-3	129849	129849	0.00	240.2	35.9	166.4	0.2	957	2	0
S80-4	121656	121656	0.00	42.8	12.0	9.0	0.1	547	1	0
S80-5	158740	157079	1.05	1844.4	27.6	2.3	2.4	3306	10	3
S100-1	132610	132610	0.00	815.2	375.4	88.1	0.2	1228	7	1
S100-2	152521	152521	0.00	1381.8	246.4	218.6	0.3	1057	22	21
S100-3	73613	73556	0.08	44.4	12.5	4.5	0.1	262	7	0
S100-4	164986	164986	0.00	1712.2	68.1	54.4	0.4	2342	4	4
S100-5	160515	160515	0.00	597.9	116.6	11.7	0.1	723	1	1
M60-1	179489	179489	0.00	39.5	9.4	3.0	0.1	419	28	0
M60-2	215312	213885	0.66	191.5	2.6	3.8	13.3	1854	16	13
M60-3	180460	180460	0.00	8.9	1.2	1.6	0.0	271	2	0
M60-4	211937	211937	0.00	56.1	0.7	1.1	0.8	1813	2	1
M60-5	209209	209209	0.00	19.7	2.4	2.3	0.1	469	4	3
M80-1	241332	235830	2.28	52.3	9.8	4.3	0.1	854	8	2
M80-3	190041	187989	1.08	229.1	8.5	10.6	2.4	1944	6	5
M80-3	227641	227641	0.00	258.3	10.8	13.9	1.3	2117	14	5
M80-4	280529	280529	0.00	93.2	1.7	3.4	0.6	1632	4	5
M80-5	154405	145472	5.79	679.3	3.2	17.3	3.3	3957	3	5
M100-1	280485	280485	0.00	1214.9	8.1	39.1	2.9	3802	4	5
M100-2	277942	277942	0.00	162.5	8.2	29.5	0.6	1335	9	8
M100-3	243422	242692	0.30	208.8	3.7	10.0	0.9	1738	3	3
M100-4	193964	193964	0.00	200.3	3.9	5.4	0.6	1911	7	4
M100-5	268728	268728	0.00	192.1	3.3	6.1	0.4	1706	7	6
H160-1	605472	593505	1.98	3445.0	13.1	24.0	44.9	12653	15	13
H160-2	589814	560313	5.00	2617.7	17.5	32.3	36.3	11493	18	16
H160-3	549797	542703	1.29	2848.4	22.1	55.9	47.4	5031	29	28
H160-4	677788	669817	1.18	3151.8	16.8	33.5	55.4	11147	21	19
H160-5	605431	577052	4.69	1175.1	19.6	38.0	12.9	6725	12	14
H160-6	501733	494598	1.42	10635.6	27.5	28.1	110.1	12329	55	36
H160-7	483005	459498	4.87	637.9	32.8	28.9	1.0	2115	36	8
H160-8	586054	574734	1.93	121.4	10.8	7.2	0.6	1160	9	5
H160-9	539154	537190	0.36	10880.5	71.8	71.9	40.2	7894	105	29
H160-10	521027	506039	2.88	1678.3	14.5	14.0	36.6	7183	25	20
Set										
A40	1407	1394(5)	0.94	4.9	0.6	0.0	0.5	642	1	0
A60	1964	1876(5)	4.43	48.9	3.3	0.0	42.3	1321	4	0
A80	2198	2113(5)	3.82	660.9	18.4	0.0	53.6	3420	19	0
A100	2321	2180(5)	6.07	2632.5	33.5	0.0	109.1	5227	24	0
A120	2364	2201(4)	7.98	3377.2	75.5	0.0	67.5	5539	31	0
B40	1365	1355(5)	0.68	9.1	0.6	0.0	4.7	776	2	0
B60	1859	1824(5)	1.91	90.3	2.9	0.0	25.2	1323	4	0
B80	2143	2093(5)	2.31	574.1	21.7	0.0	76.5	2859	19	0
B100	2389	2222(5)	6.93	6101.5	58.3	0.0	117.5	6666	29	0
B120	2399	2223(5)	7.33	9937.2	112.5	0.0	117.9	8010	60	0
C60	576	500(5)	13.19	40.6	0.9	0.0	2.8	1136	0	0
C80	792			36.3	1.0	0.0	0.2	780	0	0
C100	978			16.2	1.0	0.0	0.1	576	0	0

Table 3.5: Evaluation of the Benders' cutting procedure

respect to the LP relaxation and the optimization time (*time IP*). For the B&P&Cut algorithm, we reported the percentage gap (*gap*), the optimization time at which the gap is reached (*time gap*) and the total optimization time (*total time*) that also includes the total time spent solving

the ORRMP with integer column variables (*time IP*). For instance of sets A, B and C, in the column *Rev. IP*, for both the *IP solver* and the *branch-and-price-and-cut*, we reported between parenthesis the number of instances for which an integer solution has been found by the algorithm.

Table 3.6 reveals that the branch-and-price-and-cut algorithm that we developed outperforms the IP commercial solver. Our algorithm is capable of finding the same or a better integer solution and stops the optimization because of the tight problem bounding provided by the computed LP relaxation. The IP commercial solver fails to identify an integer solution for large instances — set H, A120, B100 and B120 — and its performance decreases as the instance size (number of surgeries) increases, *i.e.*, very poor quality integer solutions and optimality gaps significantly larger than those found by the branch-and-price-and-cut. Computation times for the IP commercial solver are of the same order of magnitude of the branch-and-price-and-cut over instances of sets S and M. Computation times of the IP commercial solver grow significantly as the number of ORs available for the surgery scheduling increases.

Most of the improvement of the LP relaxation is given by the Benders' cuts added at the root node (see Table 3.5), but the branch-and-price-and-cut is even effective for further closing the IP optimality gap in both the directions, *i.e.*, tightening the LP relaxation and improving the IP solution quality. Such further gap closing for the LP relaxation is not greater than the 2.28% over all the sets of instances, and for the integer solution, it can be even 13%.

In Table 3.7, we report some further details of the branch-and-price-and-cut algorithm, but only average values for each set of instances. These are as follows: the number of columns added to the ORRMP (*columns*), the iteration count (*iter.*), the number of created nodes (*nodes*) and the number of closed nodes (*nodes closed*).

Assessing the value of the hybrid open block strategy for surgeons

It may be argued that the OT policy scheduling that does not consider an OR-to-surgeon assignment on a daily basis (at least) is not so common and unreasonably upgrades the problem complexity; note that we define the open block strategy as "hybrid" because, in our model, surgeons are constrained to use only ORs assigned to their SG. To step forward and better explain why we considered this OT scheduling policy, we remark that: (1) the policy is adopted by CHU hospital, (2) the policy is considered in [46] and (3) the numerical experiments we ran show that there is a significant gain in allowing more than one surgeon to use the same OR on the same day and for surgeons to be able to use more than one OR on the same day.

By enforcing OR-to-surgeon assignment constraints, we revealed the following. For instances of set S, one instance of fifteen becomes infeasible, for six instances the OR-to-surgeon assignment has no impact (the optimal solution value does not change) and the remaining eight instances lose on average 2.91% of the revenue, where the minimum and the maximum loss of revenue are 0.49% and 9.04%, respectively. For instances of set M, one instance of fifteen becomes infeasible, ten instances are not impacted and the remaining four instances lose on average 4.17%, where the minimum and the maximum loss of revenue are 1.52% and 6.21%, respectively. For instances of set H, three instances of ten become infeasible and for four instances the loss of revenue is on average the 1.25%, the minimum and the maximum losses are 0.34% and 2.39%, respectively. For instances of set A, there are nine instances of twenty-five that become infeasible and fifteen instances lose on average 3.38% of the revenue, where the minimum and the maximum losses of revenue are 0.07% and 8.13%, respectively. For instances of set B, fifteen instances over twenty-five become infeasible and ten instances lose on average the 17.74% of the revenue, where the minimum and the maximum losses of revenue are 10.08% and 24.95%, respectively. All the instances of set C become infeasible.

Note that for one instance of set A and three instances of set H, the comparison is not possible because for both models (with and without the OR-to-surgeon assignment), the integer optimization is not concluded within the given time-out and the UB resulting at the end of

Instance	IP Solver				Branch-and-Price-and-Cut					
	rev. LP	rev. IP	gap (%)	time IP	rev. LP	rev. IP	gap (%)	time gap	time total	time IP
S60-1	114168	114168	0.00	23.3	114168	114168	0.00	46.4	46.4	0.1
S60-2	149562	149562	0.00	57.8	149562	149562	0.00	240.3	240.3	1.7
S60-3	145260	145260	0.00	441.4	145260	145260	0.00	431.7	431.7	0.9
S60-4	161093	161093	0.00	27.2	161093	161093	0.00	27.8	27.8	0.0
S60-5	104192	104192	0.00	37.9	104192	104192	0.00	62.7	62.7	0.0
S80-1	170226	170226	0.00	68.9	170226	170226	0.00	471.1	471.1	2.1
S80-2	143490	143487	0.00	309.7	143487	143487	0.00	296.3	296.3	0.4
S80-3	129849	129849	0.00	11.4	129849	129849	0.00	240.6	240.6	0.2
S80-4	121656	121656	0.00	209.7	121656	121656	0.00	41.9	41.9	0.1
S80-5	158182	157079	0.70	10800.1	157079	157079	0.00	3842.0	3842.0	11.2
S100-1	132610	132610	0.00	200.9	132610	132610	0.00	660.4	660.4	0.2
S100-2	152521	152521	0.00	456.1	152521	152521	0.00	1383.3	1383.3	0.3
S100-3	73559	73556	0.00	62.3	73556	73556	0.00	69.0	69.0	0.2
S100-4	164986	164986	0.00	203.8	164986	164986	0.00	1709.4	1709.4	0.4
S100-5	160515	160515	0.00	175.8	160515	160515	0.00	597.2	597.2	0.1
M60-1	179489	179489	0.00	74.4	179489	179489	0.00	31.7	31.7	0.1
M60-2	213885	213885	0.00	550.6	213885	213885	0.00	215.9	215.9	15.5
M60-3	180460	180460	0.00	94.0	180460	180460	0.00	9.0	9.0	0.0
M60-4	211937	211937	0.00	225.7	211937	211937	0.00	72.3	72.3	1.1
M60-5	209209	209209	0.00	74.8	209209	209209	0.00	19.9	19.9	0.0
M80-1	235830	235830	0.00	137.2	235830	235830	0.00	61.0	61.0	0.3
M80-2	188002	187989	0.01	9536.3	187989	187989	0.00	209.5	209.5	3.2
M80-3	227641	227641	0.00	728.6	227641	227641	0.00	198.6	198.6	1.0
M80-4	280529	280529	0.00	133.7	280529	280529	0.00	121.7	121.7	0.7
M80-5	154405	154405	0.00	3657.1	154405	154405	0.00	878.9	878.9	17.3
M100-1	280485	280485	0.00	557.8	280485	280485	0.00	1259.3	1259.3	2.9
M100-2	277942	277942	0.00	1761.1	277942	277942	0.00	162.5	162.5	0.6
M100-3	242692	242692	0.00	666.9	242692	242692	0.00	234.7	234.7	2.5
M100-4	193964	193964	0.00	986.6	193964	193964	0.00	276.6	276.6	0.7
M100-5	268728	268728	0.00	453.9	268728	268728	0.00	242.7	242.7	0.5
H160-1					605433	593505	1.97	4037.1	10855.5	1155.8
H160-2					588932	560313	4.86	10748.2	12168.7	372.7
H160-3					548329	542703	1.03	4320.8	10815.7	3032.2
H160-4					677788	669817	1.18	3161.3	10822.4	1952.3
H160-5					605431	577052	4.69	1117.4	10831.5	1735.4
H160-6					501733	494598	1.42	10787.2	11093.4	235.7
H160-7					483005	459498	4.87	386.9	10813.6	509.9
H160-8					586054	583984	0.35	9160.9	10811.8	554.8
H160-9					539158	537190	0.37	10861.5	10861.5	40.5
H160-10					520908	509640	2.16	3569.8	10847.0	2385.8
A40	1403	1403(5)	0.00	692.8	1403	1403(5)	0.00	102.7	102.7	18.9
A60	1965	1960(5)	0.25	6559.3	1962	1944(5)	0.89	7859.6	10807.2	7740.2
A80	2241	1539(5)	31.34	12709.7	2196	2135(5)	2.73	5731.4	10823.3	5737.3
A100	2287	1339(1)	41.45	35356.5	2321	2196(5)	5.34	7181.0	10902.5	4937.0
A120					2364	2207(5)	6.61	7874.9	10997.1	2981.1
B40	1364	1361(5)	0.23	4682.8	1361	1360(5)	0.07	477.9	4698.4	762.7
B60	1849	1823(5)	1.37	8273.8	1854	1835(5)	1.00	5245.3	10809.3	4959.4
B80	2191	1376(5)	37.23	16244.4	2142	2121(5)	0.96	3877.9	10837.8	7233.4
B100					2387	2247(5)	5.83	8530.6	10925.6	2403.9
B120					2399	2242(5)	6.54	11129.5	11205.2	453.7
C60	576	576(5)	0.00	3190.6	576	574(5)	0.31	6592.1	9570.6	3371.4
C80	792	760(5)	4.02	11987.1	792	686(5)	13.34	4400.6	12217.4	2787.8
C100	978	360(5)	63.16	19969.2	977	733(4)	24.31	7585.7	10803.8	1333.4

Table 3.6: Evaluation of the Branch-and-Price-and-Cut Algorithm

the B&P&Cut optimization for the model with the OR-to-surgeon assignment is greater than the best integer solution found by the B&P&Cut for the model without the OR-to-surgeon assignment, so we cannot exclude that the two models can finally converge to the same optimal

Set	columns	cuts		iter.	nodes	nodes closed
		SSP	NSP			
S60	1553	6	3	1	1	1
S80	2114	5	2	2	2	1
S100	1122	9	5	1	1	1
M60	965	10	3	1	1	1
M80	2181	7	4	3	3	1
M100	2098	6	5	1	1	1
H160	11502	13	8	117	86	0
A40	739	1	0	68	131	1
A60	2189	11	0	637	1245	0
A80	3767	24	0	346	674	0
A100	5734	49	0	85	166	0
A120	5816	51	0	110	211	0
B40	9773	4	0	607	1045	2
B60	2588	42	0	799	1563	0
B80	3339	45	0	347	681	0
B100	7058	43	0	24	48	0
B120	8037	62	0	5	10	0
C60	4231	0	0	239	463	1
C80	4372	0	0	677	1353	0
C100	2957	0	0	1186	2367	0

Table 3.7: Evaluation of the Branch-and-Price Algorithm

integer solution.

Comparison with the literature

With Table 3.8, a comparison of the algorithms that we developed with those of [46] is proposed. We compare the average values of each instance set. Subtable *Column generation* focuses on the comparison of CG-with-Benders'-cuts with the enhanced CG with LCI cuts of [46]. For each set of instances, we reported the minimum upper bound (*Min. UB*) between the two upper bounds computed by the compared algorithms and the percentage gap of each one of the two upper bounds with the minimum; these gaps are (*gap CG-BC*) for the CG-with-Benders'-cuts and (*gap CG-LCI*) for the enhanced CG with LCI cuts. Subtables *BP Upper-Bound* and *BP Lower-Bound* focus on the comparison of the B&P&Cut that we developed with the B&P with LCI cuts of [46] (B&P&LCI). In subtable *BP Upper-Bound*, for each set of instances, we report the minimum UB (*Min. UB*) between the two upper bounds computed by the compared algorithms and the percentage gap of each upper bound with the minimum; the gaps are (*gap BP-BC*) for the B&P&Cut and (*gap BP-LCI*) for the B&P&LCI of [46]. Subtable *BP Lower-Bound* has the same structure of subtable *BP Upper-Bound*, but it is for a comparison of obtained integer solutions; we report the maximum lower bound (*Max. LB*) and the percentage gap of the lower bound of each algorithm with the maximum, where gaps are (*gap BP-BC*) for the B&P&Cut and (*BP-LCI*) for the B&P&LCI of [46].

Table 3.8 shows that our CG-with-Benders'-cuts provides a better LP relaxation than the CG with LCI of [46]; this is true for every instance of the sets A and B, but sets A100 and A120 make an exception (column *CG-BC* of subtable *Column generation*). For set C, our algorithm provides the same LP relaxation.

The algorithms that we developed are significantly faster than those of [46] (see the paper for details on computation times); this is especially the case for the comparison of the CG-with-Benders'-cuts with the CG with LCI. For small- and medium-sized instances of sets A and B, our algorithm is faster, and for large-sized instances (100 and 120 surgeries) of the same sets, computation times are comparable. Over sets C, our algorithm computes the LP relaxation of the problem in less than 1 minute, whereas the CG with LCI needs more than 1 hour.

The LP relaxation of the problem resulting at the end of the integer optimization of our

Instance	Column generation			BP Upper-Bound			BP Lower-Bound		
	Min. UB	gap (%) CG-BC	gap (%) CG-LCI	Min. UB	gap (%) BP-BC	gap (%) BP-LCI	Max. LB	gap (%) BP-BC	gap (%) BP-LCI
A40	1407	0.00	0.28	1403	0.00	0.00	1403	0.00	0.00
A60	1964	0.00	0.56	1962	0.00	0.46	1960	0.82	0.00
A80	2198	0.00	0.32	2196	0.00	0.32	2174	1.79	0.00
A100	2231	4.03	0.00	2297	1.04	0.00	2255	2.62	0.00
A120	2218	6.58	0.00	2298	2.87	0.00	2266	2.60	0.00
B40	1365	0.00	0.15	1361	0.00	0.00	1361	0.07	0.00
B60	1859	0.00	0.05	1853	0.05	0.00	1843	0.43	0.00
B80	2143	0.00	1.40	2142	0.00	0.89	2132	0.52	0.00
B100	2389	0.00	0.54	2387	0.00	0.59	2337	3.85	0.00
B120	2399	0.00	2.42	2399	0.00	2.42	2355	4.80	0.00
C60	576	0.00	0.00	576	0.00	0.00	574	0.00	0.52
C80	792	0.00	0.00	792	0.00	0.00	713	3.79	0.00
C100	978	0.00	0.00	977	0.00	0.10	860	14.77	0.00
Average		0.82	0.44		0.30	0.37		2.77	0.04

Table 3.8: Comparison with results from the literature

B&P&Cut is better in most of the cases. Even if our B&P&Cut significantly improves the LP relaxation of sets A100 and A120, sets A100 and A120 make an exception and the LP relaxation is of slightly lesser quality than that of the B&P&LCI.

Integer solutions delivered by our B&P&Cut are frequently of lesser quality and, in some cases, they are missing, but it is relevant to point out that, in [46], the B&P&LCI is executed with a warm start taking as input a feasible integer solution computed with a heuristic procedure, and we did not do that for executing our B&P&Cut.

3.8 Conclusion

In this work, we defined a IORPS problem including, with fine detail, constraints that are common in practice and in the literature for similar problems and constraints for HRs other than surgeons (*i.e.*, nurses). The defined problem has sequence-dependent OR cleaning times related to consecutive surgeries with different infection levels. The integrated planning and scheduling is important because operating theatres working without detailed schedules for the all relevant resources are frequently perturbed by unexpected resource unavailability. To solve this hard problem, we have devised a branch-and-price-and-cut algorithm relying on a label-correcting algorithm for solving the pricing problems and a Benders' cutting procedure for tightening the LP relaxation of the problem. The effectiveness of this solution method is demonstrated through a set of computational experiments. Our algorithm outperforms competing methods from the literature and the commercial solver (CPLEX). We hope that our devised Benders'-like cutting procedure can represent a starting point for future research of problems with several side resources to consider since the approach is sufficiently generic to be applied for an arbitrary number of resources. To this purpose, solving some technical difficulties is may be required.

Regarding the OR scheduling problem considered in this work, future research could advance in two possible directions: (*i*) to consider some source of uncertainty and (*ii*) to tackle problems that hierarchically follow the considered OR scheduling. Uncertain surgery durations and random resource consumptions due to add-on surgical cases that may show up after the OR scheduling can be considered as sources of uncertainty. For both cases, the optimization objective may be minimizing the average shortage of the available resource capacity or the chance of such shortages. Stochastic optimization methods are unfortunately able to solve only very small instances of IORPS problems, and an effort is required to tackle solution difficulties. Two problems that hierarchically follow the generation of OR schedules are the assignment of nurses to

surgical cases and the OR rescheduling problem to include add-on urgent surgeries that cannot wait until the next complete OR scheduling.

Chapter 4

Stochastic nurse to surgery assignment with multiple objectives

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Abstract

In this chapter, we present an operational problem for the scheduling of surgical nurses under a new perspective. We address the nurse to surgery assignment problem considering objectives related to the constant training of nurses for non-technical skills and nurse preferences. Non-technical skills (interpersonal communication, coordination, judgement, etc.) of nurses, and of surgical teams, are crucial to avoid the majority of errors and ensure the safety of surgery. Schedules that also satisfy nurse preferences limiting overtime and the number of operating rooms visited over a surgery day reduce stress situation and errors. We present both the deterministic and stochastic version of the problem. The problem that we present in this chapter requires as input a surgery schedule for the operating rooms. We assume the problem is solved sequentially to the problem presented in Chapter 3 for the generation of operating room schedules. Even though the two problems are to be solved sequentially, they belong to the same decision level and consider the same planning horizon. The two-stage stochastic modelling of the problem is solved by means of sample averaging of scenarios.

4.1 Introduction and literature review

In recent years, nurses have become scarce resources worldwide [26]. Many reasons have contributed to the nowadays nurse shortage. On one side, ageing populations have increased the demand for health-care services (and thus for nursing as well) and, during the same time, nursing professions have become less attractive because of salaries not always corresponding to the

demanding workload (*e.g.*, overwork is common practice [106]). On the other side, skills required for surgical nursing professions (and nursing in general) have increased over the past decades [64]. The variety of nurse tasks at the *Operating Theatre* (OT) is large [40] and the costs of *Human Resource* (HR), nurses included, are relevant components of expenditure for running an OT [89, 36]. Such elements have made the scheduling of nurses needed in many hospital units, included the OT.

Nurse scheduling problem problems at different decision levels have received attention over the time from the operations research scientific community. A review of the literature on nurse scheduling is in [30]. The common classification of problems according to the decision epoch — strategic, tactical and offline/online operational decision epoch — apply to nurse scheduling as well. Strategic problems of nurse scheduling are generally staffing problems [30], *i.e.*, staff budgeting, hiring/firing decisions and staffing [139, 13]. Tactical decisions may involve staffing as well, a staffing problem can be considered strategic or tactical according to the considered planning horizon [30]. Shift scheduling problems, *i.e.*, the selection of shifts and the number of practitioners to be on duty for the shifts [118], are tactical level problems of nurse scheduling. The probably most studied nurse scheduling problem is nurse rostering. The nurse rostering problem involves the assignment of nurses to the scheduled shifts considering nurse skills, preferences, constraints for the schedule of nurse shifts, etc. In [30], a review of the literature of nurse rostering is provided. Apparently, there is no research about nurse rostering for surgical nurses at the OT [77], no papers considering the nurse rostering of surgical nurses to schedule work shifts at the OT for assisting the execution of surgeries and/or performing maintenance activities. Very short-term rescheduling/adjustment of the nurse rosters is addressed in [14], but this is not a deeply investigated research subject.

Recent papers focusing on the nurse scheduling problem as a resource allocation problem are [31, 90]. In [31], a multi-objective problem for the short-term nurse scheduling, *i.e.*, the nurse to shift assignment, is solved by means of integer programming and variable neighbourhood search. The large set of objectives considered cover operational objectives and nurse satisfaction objectives, and constraints are those common for nurse rostering problems (weekends, stand-alone shift, free days, maximum/minimum consecutive shifts, forbidden successions of shift type, etc.). The addressed problem is a practical one rising in a Dutch hospital. Paper results suggest that combining exact and metaheuristics methods can be an interesting approach for future research. In [90], a multi-objective nurse scheduling problem is solved by means of an exact branch-and-price algorithm. The multi-objective formulation includes over/under-staffing and nurse dissatisfaction minimization. Common constraints for nurse rostering are included, *i.e.*, shift coverage, forbidden shift type successions, etc. The main findings are related to the solution method effectiveness. In [14], a short-term nurse rescheduling/adjustment problem is addressed; nurses have to be assigned to service units (OT included) and time periods (shifts). The need for nurse rescheduling rises to cope with daily shortages due to emergencies, sick leave, etc., or nursing demand fluctuations. The addressed nurse rescheduling problem covers a planning horizon of 24h in a rolling horizon fashion. Nurse preferences satisfaction and operational costs minimization are considered as problem objectives. The problem is solved by means of a branch-and-price algorithm.

Literature of nurse scheduling and staffing problems that focus on surgical nurses is very limited [77]. The staffing and the scheduling/rostering problems are addressed as an integrated problem for the specific case of surgical nurses in [13]; nurse training objectives of fairness are considered. The authors show that their optimization model can improve performance metrics and surgical demand coverage. A relevant problem of surgical nurse scheduling is the nurse to surgery assignment, but this problem has received limited attention; very few papers are available. In [88], the authors address the daily scheduling of nurses at the OT. Nurses have to be assigned to surgeries to form surgical teams. Operational efficiency metrics (shortages, overtime, idle time, room changes, etc.) are problem objectives and several constraints related to

nurses availability and nurse skills are considered in the problem. A column generation approach is proposed as the problem resolution method. The paper [88] essentially extends the problem of [104] considering the scheduling of nurse lunch-breaks and developing the column generation method for solving the problem.

Human factors as preferences, skills and training concern largely nurse planning and scheduling problems. Nurse preferences and nurse training at staffing and scheduling decision levels are considered in [13], at the best of our knowledge, it is the only paper addressing nurse preferences in the staffing problem and the scheduling of surgical nurses. Nurse preferences are otherwise criteria widely considered for nurse scheduling problems not concerning surgical activity [30]. Both preferences and skills are considered in [88], the preferences of nurses to work in a minimum number of ORs during the day and not too long uninterrupted surgery sessions are taken into account and compatibility constraints of nurse skills with the surgery specialty and procedure complexity. Less attention has been devoted to the training of nurses.

The OT is a high-reliability environment and, similarly to aviation, the majority of errors have non-technical causes [70]. For aviation crews, training programs that enhance non-technical skills (*i.e.*, interpersonal communication, judgement, teamwork, etc.), together with technical skills, are consolidated practice [71]. Surgical teams training programs similar to those for aviation crews, that can involve the simulation of crisis events, are so suggested as effective ways to reduce the occurrence of adverse events [5]. Even though the value of non-technical skills to acquire with specific training for surgical teams has been recognised, to enhance surgical team performances and surgery safety, the need for constant training for non-technical skills, especially with respect to team communication and teamwork, has been pointed out [125].

In this chapter, we address the problem of generating a surgery schedule for nurses, and for a planning horizon of several days (one week). The OT for the execution of surgical procedures is composed of several ORs. The surgery schedule that assigns surgeries to ORs and days in the planning horizon, and that defines the surgery sequence of each OR and day, is given. The given surgery schedule is detailed and generated considering resource constraints for nurses availability, but it does not provide the nurse to surgery assignment. In Chapter 3 of this thesis, the problem formulation and the solution method for generating the surgery schedule of the OT are described. In this chapter, we address the problem of generating a surgery schedule for nurses given the OT schedule, *i.e.*, to provide the nurse to surgery assignment. We present for first the problem in its deterministic version and then the formulation extended to the stochastic case.

The deterministic case considers two hierarchical objectives. As the primary objective is considered the minimization of the number of ORs visited by every nurse daily. Nurses prefer to visit the minimum number of ORs during a workday of surgical activity; this operational objective is considered also in [88]. Such preference rises in reason of the necessary preparation of the OR that is performed usually on the eve of the surgery day. The OR preparation consists of preparing/checking the surgical material (kits, devices, consumables, etc.) necessary for performing every surgical procedure in the OR. More ORs to visit involves more coordination for the ORs preparation with other nurses and surgeons that have to visit the same ORs. Moreover, as nurses are responsible to supervise the flow of patients and materials to/from the ORs and the other OT locations, the more the work environment of the day of surgery is physically dispersed (more ORs to supervise), the more is hard to keep everything under control and errors due to non-technical causes (communication, coordination, etc.) can happen. A secondary objective considered is the maximization of the number of surgeons assisted over the planning horizon for the execution of surgeries. This objective lets possible the constant training of nurses for non-technical skills. By assisting for surgery as much as possible different surgeons, nurses enhance and maintain the ability to correctly communicate and cooperate with every surgeon that they may have to assist. Nurses are assigned to *Surgical Groups* (SGs) in the long-term, an SG gather together different surgical specialties that are related from a surgical point of view. Nurses are

skilled to assist only surgeries of specialties of the SG to whom they belong and consequently they can have to assist only surgeons of that SG.

The stochastic version of the nurse to surgery assignment problem extends the deterministic version introducing the uncertainty of surgery durations. Given the uncertain duration of surgeries, a further objective, *i.e.*, the overtime minimization, that hierarchically precedes the two objectives already defined with the deterministic problem is introduced. The stochastic problem is modelled as a two-stage stochastic problem and solved by means of the sample averaging of scenarios.

The chapter is organised as reported in the following. In Section 4.2, the deterministic problem is presented with its modelling as a *Mixed-Integer Linear Program* (MILP) and, in Section 4.3, the extension of the problem to the stochastic case (two-stage modelling) is presented. In Section 4.4, the solution method is described and a computational study for the stochastic problem solution is presented in Section 4.5.

4.2 Deterministic nurse to surgery assignment problem

The problem we consider involves one decision, *i.e.*, the assignment of the required number of nurses to every surgery scheduled for the considered planning horizon. The following assumptions hold:

1. The durations of surgeries is deterministic.
2. A schedule of surgery is given for the planning horizon, *i.e.*, a day and an OR are already assigned to every surgery and a surgery sequence is given for each OR and day in the planning horizon.
3. Neither the assignment of surgeries to ORs nor the surgery sequence of the OT can be changed.
4. Every surgery has a surgeon already assigned and a surgery sequence is given for every surgeon and day in the planning horizon.
5. Every surgery requires for its execution the assistance of a given number of nurses that is the same for the entire surgery duration; from the surgery start to the surgery completion, nurses assigned to the surgery cannot accomplish other tasks unrelated to the surgery execution.
6. A global sequence for all surgeries, independent of the assigned ORs, is given for every day of the planning horizon.
7. Every nurse belongs to an SG, an SG gathers together surgical specialties characterized by surgical similarity (*e.g.*, head surgery, chest surgery, etc.).
8. Nurses are skilled in assisting the execution of surgeries only of their SG.
9. Nurses are trained for assisting the execution of surgeries as both scrub or circulating nurses.
10. Nurse regular work time is given for every nurse and day in the planning horizon.

We consider the optimality of a nurse to surgery assignment with respect to two practical objectives:

1. Minimize the maximum number of ORs visited by nurses on a day and
2. Maximize the minimum number of surgeons assisted by a nurse in the assignment solution.

These two objectives are hierarchical and the hierarchy follows the order with which the objectives are listed.

In Table 4.2, we reported the notation used in this chapter.

Notation	
<i>Sets</i>	
I	Nurses
J	Surgeries
D	Days in the planning horizon
R	Operating rooms
A	Surgeon
<i>Subsets</i>	
I_j	Nurses that can assist surgery j
I_a	Surgeries of surgeon a
J_d	Surgeries scheduled for day d
<i>Parameters</i>	
n_j	Number of nurses required for assisting surgery j
l_j	Duration of surgery j
F_{id}	Shift end time on day d for nurse i

Table 4.1: Problem notation

Decision variables of the *Deterministic Nurse to Surgery Assignment Problem* (DN2SAP) are:

x_{ij} a binary variable that takes value 1 if nurse i is assigned to surgery j ;

y_{ird} a binary variable that takes value 1 if nurse i assists at least one surgery in OR r on day d ;

z_{ia} a binary variable that takes value 1 if nurse i assists surgeon a at least once in the planning horizon.

The DN2SAP can be formulated as follows:

$$\min \max_{i,d} \left\{ \sum_{r \in R} y_{ird} \right\}, \quad (4.1)$$

$$\max \min_i \left\{ \sum_{a \in A} z_{ia} \right\}, \quad (4.2)$$

s.t.

$$\sum_{i \in I_j} x_{ij} = n_j \quad (j \in J), \quad (4.3)$$

$$x_{ij} \leq y_{ird} \quad (r \in R, d \in D, i \in I_j, j \in J_{rd}), \quad (4.4)$$

$$x_{ij} \leq z_{ia} \quad (a \in A, j \in J_a, i \in I_j), \quad (4.5)$$

$$C_j \leq F_{id} + M(1 - x_{ij}) \quad (d \in D, j \in J_d, i \in I_j), \quad (4.6)$$

$$C_j + l_k \leq C_k + M(2 - x_{ij} - x_{ik}) \quad (r \in R, d \in D, j, k \in J_d : j \prec k, i \in I_j \cap I_k), \quad (4.7)$$

$$C_j + l_k \leq C_k \quad (a \in A, d \in D, j, k \in J_{ad} : j \prec k), \quad (4.8)$$

$$C_j + l_k \leq C_k \quad (r \in R, d \in D, j, k \in J_{rd} : j \prec k), \quad (4.9)$$

$$l_j \leq C_j \quad (j \in J), \quad (4.10)$$

$$x_{ij} \in \{0, 1\}, y_{ird} \in \{0, 1\}, z_{ia} \in \{0, 1\}, C_j \in [0, +\infty). \quad (4.11)$$

Objective function (4.1) minimizes the maximum number of ORs visited by a nurse on a day. Objective function (4.2) maximizes the minimum number of surgeon assisted by a nurse. Constraints (4.3) enforce that the required number of nurses are assigned to each surgery. Constraints (4.4) link variables x_{ij} with variables y_{ird} , and constraints (4.5) with variables z_{ia} . Constraints (4.6) enforce the completion time of surgeries to not overstep the end time of nurse shifts. Constraints (4.7), (4.8), (4.9) and (4.10) are typical constraints of the time continuous models for parallel machine problems. With (4.11) the domain of integer variables x_{ij} , y_{ird} , z_{ij} and continuous variables C_j is defined.

4.3 Stochastic nurse scheduling problem

The DN2SAP described in Section 4.2 is herein extended to its stochastic version, the *Stochastic Nurse to Surgery Assignment Problem* (SN2SAP). The problem is modelled as a two-stage stochastic problem. All the assumptions listed for the DN2SAP hold also for the SN2SAP, except Assumption 1, in the SN2SAP the duration of surgeries is assumed to be uncertain.

As in the DN2SAP, the required number of nurses have to be assigned to each surgery in the SN2SAP. The two objectives of the DN2SAP, *i.e.*, minimize the number of visited ORs and maximize the number of assisted surgeons, are considered in the SN2SAP as well and a third additional objective is considered: minimize the expected maximum overtime for nurses.

First-stage decision variables of the SN2SAP are the same of the DN2SAP, *i.e.*, x_{ij} , y_{ird} and z_{ia} .

Let Ω be the set of all possible realizations, *i.e.*, scenarios, for the duration of every surgery belonging to the problem instance. We denote with $\omega \in \Omega$ an element of set Ω , *i.e.*, a scenario of durations of the surgeries in the problem instance. Second-stage decision variables of the SN2SAP are:

$C_j(\omega)$ the completion time of surgery j under scenario ω .

$O_{id}(\omega)$ the overtime of nurse i on day d under scenario ω .

The first-stage formulation reads:

$$Obj_1 := \min E[Q(x, \xi)], \quad (4.12)$$

$$Obj_2 := \min \max_{i,d} \left\{ \sum_{r \in R} y_{ird} \right\}, \quad (4.13)$$

$$Obj_3 := \max \min_i \left\{ \sum_{a \in A} z_{ia} \right\}, \quad (4.14)$$

s.t.

$$\sum_{i \in I_j} x_{ij} = n_j \quad (j \in J), \quad (4.15)$$

$$x_{ij} \leq y_{ird} \quad (r \in R, d \in D, j \in J_{rd}, i \in I_j), \quad (4.16)$$

$$x_{ij} \leq z_{ia} \quad (a \in A, j \in J_a, i \in I_j), \quad (4.17)$$

$$x_{ij} \in \{0, 1\}, y_{ird} \in \{0, 1\}, z_{ia} \in \{0, 1\}. \quad (4.18)$$

Objective function (4.12) minimizes the expected value of the second stage function $Q(x, \xi)$. Objective function (4.13) minimizes the maximum number of ORs visited by a nurses on a day. Objective function (4.14) maximizes the minimum number of surgeon assisted by a nurse. Constraints (4.15) enforce that the required number of nurses are assigned to each surgery. Constraints (4.16) link variables x_{ij} with variables y_{ird} and constraints (4.17) with variables z_{ia} . With (4.18) the domain of variables x_{ij} , y_{ird} and z_{ij} is defined.

Let ξ be the random data and $Q(x, \xi)$ the objective function of the second-stage problem for a given solution of the decision variables x of the first-stage problem. For $\xi(\omega) = l(\omega)$, which represents the realization of random data under scenario $\omega \in \Omega$, the second-stage problem reads:

$$Q(x, \xi(\omega)) = \min_i \max_{d \in D} \sum_{d \in D} O_{id}(\omega), \quad (4.19)$$

$$C_j(\omega) \leq F_{id} + O_{id}(\omega) + M(1 - x_{ij}) \quad (d \in D, j \in J_d, k \in K_j, i \in I_j), \quad (4.20)$$

$$C_j(\omega) + l_k(\omega) \leq C_k(\omega) + M(2 - x_{ij} - x_{ik}) \quad (r \in R, d \in D, j, k \in J_d : j \prec k, i \in I_j \cap I_k), \quad (4.21)$$

$$C_j(\omega) + l_k(\omega) \leq C_k(\omega) \quad (a \in A, d \in D, j, k \in J_{ad} : j \prec k), \quad (4.22)$$

$$C_j(\omega) + l_k(\omega) \leq C_k(\omega) \quad (r \in R, d \in D, j, k \in J_{rd} : j \prec k), \quad (4.23)$$

$$l_j(\omega) \leq C_j(\omega) \quad (j \in J), \quad (4.24)$$

$$C_j(\omega) \in [0, +\infty), O_{id}(\omega) \in [0, +\infty). \quad (4.25)$$

Objective function (4.19) minimize the maximum overtime for nurses, the overtime is computed all over the entire planning horizon. Constraints (4.20) link the completion time of surgeries with the daily overtime of each nurse. Constraints (4.21) enforce that surgeries assigned to the same nurse do not overlap (recall that the surgery sequence is given even for surgeries not assigned to the same OR or surgeon). Constraints (4.22) ensure that surgeries of the same surgeon do not overlap according to the given sequence. Constraints (4.23) ensure that surgeries scheduled in the same OR do not overlap according to the given sequence. Constraints (4.24) define the minimum feasible starting time for every surgery. With (4.25) the domain of variables $C_j(\omega)$ and $O_{id}(\omega)$ are defined.

4.4 Solution method

The three SN2AP objectives of the first-stage formulation (4.12)-(4.18) are assumed hierarchical, *i.e.*, $Obj_1 \succ Obj_2 \succ Obj_3$, and are solved sequentially. Objective Obj_1 is solved for first, the objective is reformulated as a constraint to the optimal value and added to the problem formulation. Objectives Obj_2 and Obj_3 are sequentially solved in the same fashion.

All three objectives of the SN2AP are solved by means of sample averaging of problem scenarios. This allows us to deal with a deterministic reformulation of the stochastic problem. Recalling Ω be the set of all problem scenarios for the duration of surgeries belonging to the problem instance and $\omega \in \Omega$ denote one problem scenario in Ω , let $\bar{\Omega} \subset \Omega$ be a subset of randomly selected problem scenarios.

With a standard reformulation of the objectives (4.12), (4.13) and (4.14) and considering $\bar{\Omega}$ in the place of Ω , the model (4.12)-(4.18) can be rewritten as a deterministic MILP that approximates the the two-stage formulation; the expected min-max daily overtime for nurses is approximated by means of sample averaging, *i.e.*, $\sum_{\omega \in \bar{\Omega}} \frac{1}{|\bar{\Omega}|} Q(x, \xi(\omega)) \approx E[Q(x, \xi)]$. See [81] for details about sample averaging of scenarios techniques. It follows the deterministic reformulation of the two-stage SN2AP.

Decision variables of the deterministic-equivalent MILP are the same of the SN2SAP, *i.e.*, x_{ij} , y_{ird} , z_{ia} , $C_j(\omega)$ and $O_{id}(\omega)$. The MILP read:

$$Obj_1 := \min E[Q(x, \xi)] : Q(x, \xi) = \max_{i, \omega} \sum_{d \in D} O_{id}(\omega), \quad (4.26)$$

$$Obj_2 := \min \max_{i, d} \left\{ \sum_{r \in R} y_{ird} \right\}, \quad (4.27)$$

$$Obj_3 := \max_i \min \left\{ \sum_{a \in A} z_{ia} \right\}, \quad (4.28)$$

s.t.

$$\sum_{i \in I_j} x_{ij} = n_j \quad (j \in J), \quad (4.29)$$

$$x_{ij} \leq y_{ird} \quad (r \in R, d \in D, j \in J_{rd}, i \in I_j), \quad (4.30)$$

$$x_{ij} \leq z_{ia} \quad (a \in A, j \in J_a, i \in I_j), \quad (4.31)$$

$$C_j(\omega) \leq F_{id} + O_{id}(\omega) + Mx_{ij} \quad (\omega \in \Omega, d \in D, j \in J_d, i \in I_j), \quad (4.32)$$

$$C_j(\omega) + l_k(\omega) \leq C_k(\omega) + M(2 - x_{ij} - x_{ik}) \\ (\omega \in \Omega, r \in R, d \in D, j, k \in J_d : j \prec k, i \in I_j \cap I_k), \quad (4.33)$$

$$C_j(\omega) + l_k(\omega) \leq C_k(\omega) \quad (\omega \in \Omega, a \in A, d \in D, j, k \in J_{ad} : j \prec k), \quad (4.34)$$

$$C_j(\omega) + l_k(\omega) \leq C_k(\omega) \quad (\omega \in \Omega, r \in R, d \in D, j, k \in J_{rd} : j \prec k), \quad (4.35)$$

$$l_j(\omega) \leq C_j(\omega) \quad (\omega \in \Omega, j \in J), \quad (4.36)$$

$$x_{ij} \in \{0, 1\}, y_{ird} \in \{0, 1\}, z_{ia} \in \{0, 1\}, \quad (4.37)$$

$$C_{jt}(\omega) \in [0, +\infty), O_{id}(\omega) \in [0, +\infty). \quad (4.38)$$

Objective (4.26) minimizes the maximum overtime among nurses. Objective (4.27) minimizes the maximum number of ORs visited by a nurse on a day. Objective (4.28) maximizes the minimum number of surgeons assisted by a nurse. Constraints (4.29) ensure that the required number of nurses are assigned to each surgery. Constraints (4.30) link variables x_{ij} with variables y_{ird} . Constraints (4.31) link variables x_{ij} with variables z_{ia} . Constraints (4.32) link the completion time of surgeries with variables $O_{id}(\omega)$ for the nurse overtime. Constraints (4.33) ensure that surgeries assigned to the same nurse do not overlap. Constraints (4.34) ensure that nurse of the same surgeon do not overlap. Constraints (4.35) ensure that nurse scheduled for the same OR do not overlap. Constraints (4.36) enforce surgery completion times to be greater equal to the surgery durations under every scenario. With (4.37) and (4.38) the domain of the problem variables are defined.

4.5 Computational study

In this section, we report the results of the numerical experiments we run. In Section 4.5.1, we describe the available surgery data exploited for generating the problem scenarios and the OT schedules that are instances of the SN2SAP. In Section 4.5.2, we describe how the problem scenarios are generated. Finally, numerical results are reported in Section 4.5.3.

4.5.1 Surgery schedules

A set of surgery schedules for the OT is available as input for the SN2SAP. These schedules are computed by means of the optimization method described in Chapter 3 of this thesis. As the optimization method includes resource constraints for nurses, a feasible solution for the assignment of nurses to surgeries always exists.

Schedules generated with the optimization method of Chapter 3 assign every surgery to an OR and a day in the planning horizon and sequences surgeries in every OR, surgeries assigned to the same OR belong to the same SG. The surgery schedule generation method considers a deterministic duration of surgeries.

4.5.2 Scenarios generation

For the sample averaging of scenarios, we generated 120 scenarios for each instance. For the generation of each scenario, we draw from a distribution a duration for each surgery. We assume a log-normal distribution for the duration of each surgery. The choice of the log-normal distribution is arbitrary. The duration of each surgery provided with the considered instance (schedule) is assumed to be the expected value parameter of the log-normal distribution of the surgery duration. The standard deviation parameter of the log-normal distribution is computed, for every surgery, through the given expected value and the *Coefficient of Variation* (CV) given for every surgical specialty, surgeries of the same surgical specialty share the same CV. The CV of each surgical specialty is reported in Table 4.2. Values of CV are computed from surgery data in the database of a French public-funded university hospital settled in Saint-Etienne (France), the *Centre Hospitalier Universitaire de Saint-Étienne* (CHUSE). The database stores surgery data from January 2010 to September 2017.

SG ^a	surgical specialty	CV ^b	surgeries ^c
OM	maxillofacial	0.38	5937
OM	otolaryngology	0.36	10372
NR	neurosurgery	0.33	9525
ORTR	bone	0.36	18901
CV	cardiovascular	0.37	7156
INF	pediatric	0.38	11326
DIG	alimentary canal	0.38	8185
GYURA	urology	0.38	6847
GYURA	gynecology	0.38	6973
GT POT	oncological pneumology	0.27	1480
GT POT	general and thoracic surgery	0.37	12692
AMB	daily	0.29	7275
OPT	ophthalmology	0.29	7292

^a Alphabetical code for identifying the surgical group SG.

^b Coefficient of Variation.

^c Surgeries count in the hospital database that is available.

Table 4.2: Surgery duration coefficients of variation by surgical group and specialty

In order to evaluate the quality of the nurse scheduling response to the realization of adverse or favourable scenarios, we consider five different probability settings for the drawn scenarios. To assign a probability to each scenario under a given setting, the generated scenarios are sorted out according to the percentage of surgeries with duration exceeding their expected value (this implies that the top scenario has the highest percentage of surgeries exceeding their expected durations) and scenario probabilities are then defined according to the rule of the given setting. The five probability settings we used are:

- I. **Average** — every scenario has the same probability $1/\bar{\Omega}$.
- II. **Worse than average** — The top 25% of the generated scenarios share the 50% of probability with uniform distribution and the remaining bottom 75% of the scenarios share the remaining 50% of probability.
- III. **Better than average** — The bottom 25% of the generated scenarios share the 50% of probability with uniform distribution and the remaining top 75% of the scenarios share the remaining 50% of probability.
- IV. **Severe** — The top 25% of the generated scenarios share the 75% of probability with

uniform distribution and the remaining bottom 75% of the scenarios share the remaining 25% of probability.

- V. **Lucky break** — The bottom 25% of the generated scenarios share the 75% of probability with uniform distribution and the remaining bottom 75% of the scenarios share the remaining 25% of probability.

As the names suggest, with respect to the maximum nurse overtime minimization objective, setting I. depicts the more balanced one, setting II. the moderately adverse one, setting III. the moderately favourable one, setting IV. the severely adverse one and setting IV. the very favourable one.

4.5.3 Numerical results

In table 4.3, we reported computation results for the ten instances selected from those used in Chapter 3 and the five scenarios probability settings that we defined in Section 4.5.2. We selected, for each instance set of type M, the worst and the best instance with respect to the maximum nurse overtime over the 120 generated scenarios. For instance set H, we selected four instances, the two worst instances and the two best instances with respect to the maximum nurse overtime. Every considered instance of type M counts 9 surgeons, 10 nurses, 4 SGs and 6 ORs. Every considered instance of type H counts 18 surgeons, 10 SGs and 12 ORs. Instances H160-W1 and H160-B1 count 27 nurses and instances H160-W2 and H160-B2 count 26 nurses.

Columns in Table 4.3 are (from the left to the right): the instance name, the scenario setting, the total time to solve the three objectives (in seconds), the optimal value for Obj_1 (in minutes), the half interval of confidence for Obj_1 over the 120 scenarios, the worst scenario value for objective Obj_1 and the distribution of objective Obj_1 values over the 120 scenarios (*i.e.*, the percentage of values equal to 0 minutes, less or equal than 60, less or equal than 120, less or equal than 180 and less or equal than 240); in the last two columns, we reported the optimal value of the objective Obj_2 and objective Obj_3 .

Analysing results in Table 4.3, we can observe that the computation times are limited for almost every considered instance of type M. The total computation time (*i.e.*, the time to solve the three objectives) becomes larger for instances of sets M100 and H160. For instance H160-W1 and scenario probability setting III, the computation time rises up to more than 20 minutes; such large computation time is the biggest one among all the instances considered in Chapter 3, computation times are otherwise of one order of magnitude smaller for any other considered instance and scenario probability setting. Considered that computation times greater than few minutes represent an exception for the considered instances, we can assert that the used solution method is effective, but there is anyway room for improvement in terms of computation time.

The choice of 120 scenarios for the evaluation of objective (4.12) (the average maximum overtime per nurse) seems reasonable, at least with respect to the objective value accuracy, as the uncertainty for the values is in the worst case ± 15 minutes considering a 95% CI. Throughout all the considered instances, the average maximum overtime for a nurse over the entire planning horizon is, in the worst case, a little bit greater than two hours (instance H160-W1 and scenario probability setting IV), which is quite reasonable, or acceptable at least. On the contrary, the maximum overtime throughout the entire set of scenarios, *i.e.*, the values of the worst-case scenario, can have a very large value, it can be also several hours. Anyway, the distributions of Obj_1 value across the instances suggest that very large values correspond to worst-case scenarios that have a low probability to happen. Looking at the distributions across the instances, by far more than the 50% of the cases the Obj_1 value is less than 2 hours (120 minutes). The considered uncertainty of surgery durations does not seem to provide poor quality schedules for nurses with respect to the nurse overtime, by solving the optimization problem that we formulated, good quality solutions can be delivered given the input instances.

Name	Sc. Setting	Tot. Time	Objective 1									Obj. 2	Obj. 3
			Obj. 1	CI95 ±	Max.	(%) ≤ 0	(%) ≤ 60	(%) ≤ 120	(%) ≤ 180	(%) ≤ 240			
M60-W	I	7.7	68.4	13.2	381	28.3	56.7	79.2	90.8	97.5	2	2	
M60-W	II	7.8	80.7	13.8	381	23.3	48.9	73.9	86.1	96.1	2	2	
M60-W	III	6.2	60.8	12.7	381	35.6	60.0	82.8	91.7	98.3	2	2	
M60-W	IV	6.9	92.9	14.1	381	18.3	41.1	68.6	81.4	94.7	2	2	
M60-W	V	7.1	53.3	12.0	381	42.8	63.3	86.4	92.5	99.2	2	2	
M60-B	I	6.1	43.0	8.7	252	32.5	72.5	90.0	99.2	99.2	1	2	
M60-B	II	5.8	47.2	8.5	252	27.2	69.4	88.9	99.4	99.4	1	2	
M60-B	III	6.1	39.1	8.9	252	39.4	75.0	92.2	98.3	98.3	1	2	
M60-B	IV	7.0	51.5	8.2	252	21.9	66.4	87.8	99.7	99.7	1	2	
M60-B	V	5.5	35.2	9.1	252	46.4	77.5	94.4	97.5	97.5	1	2	
M80-W	I	7.6	64.6	12.7	298	25.8	58.3	81.7	91.7	96.7	1	1	
M80-W	II	7.8	75.8	13.5	298	20.6	51.1	74.4	88.9	96.7	1	1	
M80-W	III	9.1	50.1	11.4	298	33.9	68.9	87.8	94.4	97.8	1	1	
M80-W	IV	8.3	87.0	13.9	298	15.3	43.9	67.2	86.1	96.7	1	1	
M80-W	V	9.3	35.6	9.1	298	41.9	79.4	93.9	97.2	98.9	1	1	
M80-B	I	15.8	23.6	6.2	143	45.0	86.7	96.7	100.0	100.0	3	1	
M80-B	II	16.6	28.8	6.7	143	34.4	83.3	94.4	100.0	100.0	3	1	
M80-B	III	14.5	19.4	6.0	143	55.6	88.9	96.7	100.0	100.0	3	1	
M80-B	IV	16.4	33.9	7.1	143	23.9	80.0	92.2	100.0	100.0	3	1	
M80-B	V	14.5	15.2	5.7	143	66.1	91.1	96.7	100.0	100.0	3	1	
M100-W	I	136.3	75.0	13.4	401	20.0	49.2	74.2	89.2	96.7	4	1	
M100-W	II	211.5	81.2	14.1	401	15.6	45.0	72.8	88.3	95.6	4	1	
M100-W	III	202.2	65.3	12.9	401	27.8	56.1	77.2	90.6	97.8	4	1	
M100-W	IV	149.6	87.3	14.7	401	11.1	40.8	71.4	87.5	94.4	4	1	
M100-W	V	173.3	55.6	12.1	401	35.6	63.1	80.3	91.9	98.9	4	1	
M100-B	I	94.0	61.4	13.1	331	31.7	60.0	80.0	92.5	96.7	3	1	
M100-B	II	69.7	66.3	13.1	331	27.8	55.6	78.9	92.8	96.7	3	1	
M100-B	III	122.3	53.9	13.0	331	38.9	66.7	81.1	91.7	97.8	3	1	
M100-B	IV	66.3	71.3	13.0	331	23.9	51.1	77.8	93.1	96.7	3	1	
M100-B	V	69.7	46.4	12.8	331	46.1	73.3	82.2	90.8	98.9	3	1	
H160-W1	I	397.8	108.9	14.8	388	4.2	31.7	65.0	83.3	90.0	3	1	
H160-W1	II	280.9	116.8	15.1	388	2.8	30.0	60.0	80.0	87.8	3	1	
H160-W1	III	1345.1	99.4	14.7	388	3.9	40.0	70.0	85.6	90.0	3	1	
H160-W1	IV	500.4	124.8	15.2	388	1.4	28.3	55.0	76.7	85.6	3	1	
H160-W1	V	500.8	89.8	14.5	388	3.6	48.3	75.0	87.8	90.0	3	1	
H160-W2	I	134.9	85.6	11.5	355	9.2	41.7	72.5	90.8	98.3	2	1	
H160-W2	II	151.9	92.4	11.2	355	6.1	36.7	69.4	90.6	97.8	2	1	
H160-W2	III	153.4	79.4	10.7	355	9.4	45.6	75.0	93.9	98.9	2	1	
H160-W2	IV	134.2	99.3	10.8	355	3.1	31.7	66.4	90.3	97.2	2	1	
H160-W2	V	142.4	73.2	9.8	355	9.7	49.4	77.5	96.9	99.4	2	1	
H160-B1	I	73.5	62.8	10.8	227	25.0	56.7	81.7	95.0	100.0	2	1	
H160-B1	II	81.5	70.3	11.5	227	22.2	51.1	77.8	92.2	100.0	2	1	
H160-B1	III	86.2	59.1	10.4	227	25.6	58.9	84.4	95.6	100.0	2	1	
H160-B1	IV	84.1	77.8	12.0	227	19.4	45.6	73.9	89.4	100.0	2	1	
H160-B1	V	79.0	55.5	10.0	227	26.1	61.1	87.2	96.1	100.0	2	1	
H160-B2	I	105.2	78.6	9.8	258	8.3	45.0	79.2	95.0	99.2	1	1	
H160-B2	II	109.0	83.2	10.0	258	6.7	41.7	75.0	94.4	99.4	1	1	
H160-B2	III	105.0	74.6	9.5	258	11.1	47.8	80.6	96.7	99.4	1	1	
H160-B2	IV	141.4	87.8	10.1	258	5.0	41.7	70.8	93.9	99.7	1	1	
H160-B2	V	107.9	70.6	9.1	258	13.9	50.6	81.9	98.3	99.7	1	1	

Table 4.3: Numerical results for the deterministic equivalent SN2AP formulation solution by means of sample averaging of scenarios

Result for the other two objectives, *i.e.*, objective (4.13) and objective (4.14), are maybe a little-bit less appealing. Among the ten considered instances, the maximum number of visited ORs for a nurse over one day, Obj_2 , is 1 OR for three instances, 2 ORs for three instances, 3 ORs for other three instances and, for one instance, the number of visited ORs for one day by

a nurse is 4 ORs. For Obj_3 , only two values are delivered by the solution of the problem over the ten instances, a nurse assists throughout the planning horizon of a week only one or two surgeons.

It worths pointing out that, for both objective (4.13) and objective (4.14), the values delivered by the problem solution for the considered instances represent worst cases. This given the min-max and max-min objective functions for the two objectives respectively. For Obj_2 , we can assert that for 6 instances over 10 instances (*i.e.*, 60% of the cases) a nurse visit, in the worst case, no more than 2 ORs during a day and for 9 instances over 10 no more than 3 ORs. Such worst-case values do not reveal the possibility of disruptive or harmful situations. For Obj_3 and instances of type M60, considering that there are, on average, 2.25 surgeon and 2.5 nurses for each SG (9 surgeons, 10 nurses and 4 SGs), to have a nurse assisting 2 surgeons (instances M60-W and M560-B) in the planning horizon means that nurses of every SG are able to assist every, or almost every, surgeon belonging to the same SG. For instances of type M80, M100, H160, the optimal value of Obj_3 is limited to 1 as in each instance there is at least one SG with scheduled surgeries of only one surgeon. In this case, 1 is the only possible value for Obj_3 .

Optimization results for the three problem objectives reveal as possible to find out quite good solutions to the nurse to surgery assignment problem. Such good quality of solutions allows assessing as a winning approach having addressed the surgery scheduling problem, in Chapter 3, and the nurse to surgery assignment problem, in this chapter, as two distinct problems solved sequentially, and not as one integrated problem solved in one stage. This is even if the uncertainty of surgery durations is considered only in the nurse to surgery assignment problem.

Even if numerical results reported in this chapter are not very extended, they foster for further development of the subject. In Section 4.6, together with the main conclusions, we highlight the possible future extensions of the work presented in this chapter.

4.6 Conclusion

In this chapter, we have presented two formulations of the nurse to surgery assignment problem, we presented the deterministic and the stochastic formulation of the problem. We have studied this problem from a new perspective, the problem has been studied considering the constant training of nurses for non-technical skills (interpersonal communication, coordination, judgement, etc.). The stochastic problem formulation presented in this chapter considers the uncertainty of surgery durations and three hierarchical objectives, *i.e.*, the overtime minimization, the minimization of ORs visited on a day by nurses and the maximization of the number of surgeons assisted by nurses in the planning horizon (this third objective is considered for the constant training of surgical nurses). The two-stage stochastic modelling of the problem is solved by means of sample averaging of scenarios.

Numerical results revealed that the used solution method is effective for solving the set of instances that we selected from Chapter 3. Computation times are shown to be limited, especially for small instances, but despite the good performances in terms of computation times, we observed that there is anyway room for improvements. We showed that the optimization problem that we proposed allows us to provide quite good quality solutions in terms of nurse overtime for the considered instances and despite the uncertainty of surgery durations. We showed also that the number of ORs visited by nurses on a day can be effectively optimized by solving the proposed problem, as well as the number of surgeons assisted by nurses over the planning horizon. Numerical results showed finally that having addressed the weekly scheduling of surgery, in Chapter 3, and the nurse to surgery assignment, in this chapter, as two distinct problems solved sequentially, and not as one integrated problem, is as a winning approach.

The study presented in this chapter can be extended in some directions. A more efficient method for solving the problem formulation can be developed. A typical stochastic programming (Benders' decomposition based) algorithm like the L-shaped method can be appropriated for

an enhanced problem solution. Instances with larger SGs can allow a wider range of possible values for the number of surgeons that can be assisted by nurses and make it possible to better appreciate the value of optimized solutions; the instances that we evaluated are characterized by small SGs with few surgeons. By introducing a partial rescheduling of surgeries, the objective related to the number of ORs visited on a day by nurses, and the objective related to the nurse overtime, can be improved. Nurses are very sensible to the schedule quality with respect to the number of ORs visited on a day and the overtime, any change in the surgery schedule improving one or both the objectives can have an important impact on the satisfaction of nurses. The problem formulation can be also extended to the case of more complex nurse shifts (shifts of different length, timing, etc.) to make the approach applicable in a wider set of practical settings.

Moreover, for a practical application of the problem solutions, running a second optimization to minimize the number of nurses experiencing the maximum overtime (computed with the first optimization), visiting the maximum number of operating rooms, and assisting the minimum number of surgeons, can provide problem solutions improved for practical use. In this way, the limitations of min-max/max-min objectives can be overcome.

Chapter 5

Dynamic insertion of emergency surgeries with different waiting time targets

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Abstract

This chapter addresses the problem of emergency surgery insertion into a given elective surgery schedule of an operating theatre composed of multiple operating rooms. Emergency surgeries with different emergency levels characterized by waiting time targets arrive according to a non-homogeneous Poisson process and can be inserted into any operating room. An event-based stochastic programming model is proposed to minimize the total cost incurred by exceeding waiting time targets of emergency surgeries, elective surgery delay and surgery team overtime. A perfect information-based lower bound is proposed and properties of the optimal policies proved. Simple heuristic policies and a stochastic optimization approach derived from the simple policies by policy improvement are proposed. Numerical experiments show that the stochastic optimization significantly outperforms the other evaluated methods and efficient emergency insertion significantly improves the system performance. Principal component analysis is performed to show how near-optimal policies differ from simple heuristic policies.

The chapter is motivated by enhancing the efficiency of operating theatres by sharing surgery capacity between elective and emergency surgeries. More specifically, we consider the problem

of inserting non-elective surgeries of different emergency levels in the execution of a given elective surgery schedule. A stochastic optimization approach is proposed to dynamically prioritize emergency and elective surgeries in order to best balance meeting emergency surgery requirement, perturbation of elective schedule and surgery team overtime. Numerical experiments based on data collected from Saint-Joseph Hospital in Paris show the significant benefit of efficient emergency insertion over the current hospital practice. Elective surgery schedule is shown to have the most important impact on the system performance but efficient emergency insertion always adds significant improvement.

5.1 Introduction

Many studies show that the *Operating Theatre* (OT) is the most expensive service of the hospital as it consumes a large number of expensive resources (surgeons, staff and equipment) [89, 130]. For private hospitals, *Operating Rooms* (ORs) are also the main source of income [144]. From a medical and organizational point of view, ORs are also critical resources. ORs have a sizeable impact on patients' safety and the workflow of other services and the health care system [110, 27]. The importance of ORs operation is also evidenced by the extensive literature on ORs planning and scheduling; see [33] for a review.

Sharing OR capacity between elective and emergency surgeries seems a natural way to improve the usage of the OR. Common emergency surgeries are those requiring a prompt surgical intervention to perform in an OR as a consequence of physical trauma, accident or rapid deterioration of health conditions. Hospitals usually use the so-called Emergency Severity Index to measure the emergency levels [37, 47]. The feasible delay for an emergency surgery varies from zero, the surgery has to be performed as soon as possible to avoid severe consequences, to several hours. Similar to the due dates in [114], we use instead the *Waiting Time Targets* (WTTs) to indicate the time that the hospital has to start an emergency surgery. The WTT varies from instantly up to 6h or merely within the current day in the study of [114].

Sharing OR capacity raises however significant challenges due to the random emergency arrival and the nature of emergency surgeries. Some authors show that unpredictable arrival of emergency surgeries make OR scheduling more complex [48, 72]. Most importantly, hospitals have limited time to respond to randomly arriving emergency demands. Two approaches may be investigated to alleviate the stress on OR management caused by emergency arrivals: robust schedules of elective patients and dynamic surgery scheduling. Our research focuses on the second approach.

This chapter considers the daily operation of an OT composed of multiple ORs shared between elective and emergency surgeries of different emergency levels characterized by different waiting time targets. We address particularly the problem of the insertion of randomly arriving emergency surgeries into a given elective surgery schedule. The goal is to find out the best balance between meeting the WTT requirement of emergency surgeries, the perturbation of elective schedule and the surgery team overtime. The problem is nontrivial. Inserting all emergencies instantly favours emergencies at the expense of excessive delay of elective surgeries and OR overtimes. Delaying all emergencies to the end of the day favours the execution of the elective surgery schedule at the risk of endangering the most urgent emergencies. How to dynamically prioritize emergency and elective surgeries taking into account different emergency levels is the main research question of this chapter.

More specifically, this chapter proposes a formal setting of the emergency insertion problem in which an elective schedule is given and elective surgeries can be delayed but cannot move to other ORs. Emergency surgeries arriving according to a non-homogeneous Poisson process with WTT known upon arrival can be inserted in any OR. The goal is to minimize the expected cost incurred by exceeding WTT of emergency surgeries (Quality of Care), delays of elective surgeries (Quality of Service) and overtime of surgery teams (Quality of Working Life). We then propose

an event-based stochastic programming model for determining the optimal emergency insertion policy. Based on this mathematical model, the closest waiting time target first is proved to be optimal for sorting emergencies and a tight perfect information-based lower bound taking into account this property is then proposed. Being the model intractable due to the hybrid state space, we propose a policy improvement procedure, a set of simple heuristic policies and a stochastic optimization approach built on policy improvement and simple policies. A numerical experiment based on data collected from a hospital is performed. The stochastic optimization approach is found to be by far the best policy and significantly improves a policy close to the current hospital practice. Further, whereas the elective schedule is found to have a higher impact on the system performance, the dynamic emergency insertion adds significant improvement.

To the best of our knowledge, this work is the first rigorous mathematical treatment of emergency surgery insertion in elective surgery schedules. The mathematical model and the stochastic optimization approaches proposed in this chapter are new. Interesting enough, with the efficient emergency insertion policy of the work, the elective schedule obtained by the BII ("*Break-In-Interval*") rule proposed by [127] is shown to be worse than the elective schedule obtained by the SEPT (Shortest Expected Processing Time first) rule.

The remainder of the chapter is organized as follows. In the following section, the literature review of OR management is given with a focus on considering emergency surgeries. Section 5.3 is dedicated to the formal setting of the emergency insertion problem, its mathematical modelling and the perfect information-based lower bound. The policies of dynamic scheduling are detailed in Section 5.4 and evaluated in Section 5.5. Section 5.6 is a conclusion.

5.2 Literature review

As in many operational systems, the planning decision process of the OT can be divided into three classes: strategical, tactical and operational. The reader may find more detailed surveys in the following references [115, 77, 62].

The planning of capacity is built at the strategical level. A cyclic master schedule is often used to assign each time slot of each OR to a specialty. Depending on the hospital the master schedule may be more or less flexible. The absence of a master schedule corresponds to the open scheduling strategy where assignments are dynamically decided according to the demand. In any case, the emergency demands are taken into account by means of slack times or some dedicated ORs [6, 133]. The master schedule has to face the seasonality of the activity and unpredictable fluctuations of elective and emergency demands. The medical staff timetabling is usually defined at the tactical level, whereas surgical cases are scheduled at the operational level under the rules and constraints from upper decision levels.

OR scheduling is one of the most studied health-care operation problems. The first models proposed were very close to the classical bin-packing problem, where surgeries are assigned to ORs (bins). Then, researchers added various extensions to the basic models. More recently published static and dynamic models include upstream and downstream resources (anaesthesia, wards and hospitalization beds) and robustness with respect to uncertainties on surgery durations and arrivals, like in [87, 105, 4, 8, 121, 43]. A wide variety of solution approaches have been investigated such as Markov decision process solution, linear programming, local search heuristic, etc. The computed surgery planning defines elective patient release times and staff working time.

Few tools implement specific strategies to optimize emergency surgeries insertions; see [111] for a review. Some authors use slack times to anticipate the insertion of emergency surgeries [84, 58]. In [127], the authors present a daily operation problem where surgeries are already assigned to ORs and only the sequencing of surgeries is considered. They call "*Break-In-Moment*" (BIM) the time when one surgery is completed and the next surgery starts in one OR. Thus, they define the "*Break-In-Interval*" (BII) as the time elapsing between two BIMs not interleaved by

another one regardless of the OR. The robustness criterion is to spread the BIMs over the day and then the maximum BII is minimized. Note that the practical motivation of our work is similar to that of [127].

Real-time decisions – starting time and surgery-to-OR assignment – can be optimized through dynamic decision models, like in [128, 108]. As in this work, the dynamic scheduling of surgery is treated in the papers [114, 143, 145]. A simulation model is proposed in [114] for evaluating the impact on the elective surgery schedule of emergency surgeries over a multiple-day horizon. Emergency levels are described by different due dates with the most urgent ones to be served as soon as possible and the less urgent ones that can be postponed to the end of the day. The WTTs of our work extend the emergency levels defined in [114]. In [145], the authors investigate the daily dynamic rescheduling of elective surgeries in an OT composed of several identical ORs. A dynamic stochastic programming approach is proposed to best balance surgeons waiting and OR idling/overtime. Note that emergency surgeries are not considered in that paper. In [143], the authors propose an adaptive dynamic surgery scheduling for a single OR and over the planning horizon of one day with random surgery times. Differing from the usual static or dynamic scheduling approaches, fixing in the schedule a set of not yet started surgeries, called “surgery committing”, is the main novelty of that paper. Our work differs from the papers on real-time surgery scheduling by taking into account different emergency levels of randomly arriving emergency surgeries. We propose a formal mathematical model and efficient dynamic emergency insertion policies.

5.3 Problem description

This section first provides a formal description of the dynamic emergency surgery insertion problem, then proposes the mathematical modelling of the problem and proves the optimality of the earliest due date first rule for emergency surgeries, finally proposes a perfect-information-based lower bound.

5.3.1 Problem setting

This chapter considers the daily operations of an OT composed of a set K of identical ORs. The OT serves two sets of surgeries: a given set R of elective surgeries also called regular surgeries and an unknown set E of randomly arriving non-elective surgeries also called emergency surgeries. Resources other than the ORs do not limit the surgical activity of the OT.

Each OR k is associated with an opening time a_k , a closing time b_k and unit overtime cost β_k for letting it open beyond the closing time.

The daily elective surgery plan is assumed given. An OR and an estimated surgery start time (also called surgery release time) are assigned to each elective surgery. The elective surgery plan can be described by the followings: (i) the release time r_i of surgery i and (ii) the ordered subset R_k of elective surgeries assigned to OR k such that $r_{(k,1)} < r_{(k,2)} < \dots < r_{(k,n)}$; (k, j) denotes the j -th elective surgery of OR k .

Emergency surgeries arrive randomly according to a non-homogeneous Poisson process of some given rate function $\gamma(t)$ for all time $t \geq 0$. The emergency level of an emergency surgery is described by a random WTT δ for the surgery start also called indifference interval; the WTT is the time after which letting a surgery be in wait becomes critical.

Each surgery i requires a random surgery time p_i also called processing time. Its probability distribution is assumed known. As a result, both elective and emergency surgeries are assumed to have random surgery durations. All random variables are assumed to be mutually independent.

Random surgery durations and random emergency arrival often result in perturbation of the elective surgery plan, tardy emergency insertion and OR overtime. The goal of this work is to determine the dynamic strategy for insertion of emergency surgeries in order to best balance

the fulfilment of waiting time targets of emergency surgeries, the respect of the elective surgery plan and the overtime usage.

Assumption A1 There exists a finite positive time $H \geq 0$ such that $\gamma(t) = 0, \forall t \geq H$.

Assumption A2 The surgery times p_i and the waiting time targets δ_i of emergency patients are mutually independent and are both i.i.d. (independent and identically distributed). Further, they have common tardiness cost α .

Assumption A3 The surgery-to-OR assignment and surgery sequencing of elective surgeries are fixed and cannot be changed.

Assumption A4 No elective surgery is deliberately delayed if its OR is free and no emergency surgery is assigned to it.

Assumption A5 No emergency surgery is inserted to an OR k after its closing time b_k and an OR is closed at or beyond its closing time b_k after the completion of the last surgery assigned to it. Further there exists at least one OR k such that $b_k = \infty$.

Even though not relevant from the application background, we extend the model by assigning to each elective surgery a WTT or indifference interval δ_i in addition to its tardiness cost α_i . This extension defines elective and emergency surgeries in a uniform way.

Remark 1 A1 is quite reasonable as late emergency surgeries are usually assigned to specific surgery teams on night duty in dedicated ORs. They are not relevant to the insertion in the elective surgery plan.

Remark 2 A2 is a restrictive assumption as breaching the WTT of a highly urgent patient might have more serious consequence and hence higher cost than breaching the WTT of a less urgent patient. A2 is however in line with our goal of best balancing between elective surgery waiting and WTT breaching of emergency surgeries. In this work, the emergency level is solely described by the WTT and the common tardiness cost allows us to better understand the insertion of emergency surgeries in elective surgery plan without the need to consider the tricky issue of the priority of emergency patients with different tardiness cost.

Remark 3 A3 is reasonable as surgical teams prefer to prepare in advance every elective surgery in the corresponding OR. The preparation of a surgery consists of withdrawing from the OT warehouse the specific material resources (*i.e.*, surgical devices and consumables), moving them into the OR and checking their completeness carefully. On the contrary, emergency surgeries are prepared just-in-time. The emergency condition justifies the risky task of preparing a surgery just-in-time. The A3 removal might bring additional improvement but implies that also elective surgeries are prepared just-in-time. A3 is also coherent with the study case hospital surroundings and consistent with the assumptions of [127].

Remark 4 Whereas it is reasonable to keep an OR free in anticipation of an upcoming elective surgery, doing so in anticipation of unknown future emergency surgery arrivals seems odd and A4 is quite reasonable.

Remark 5 The assumption of an OR without closing time ensures the feasibility of the problem. Such surgery teams can be considered as teams on night duty. An interesting extension beyond the scope of this work is to consider the OR closing as dynamic decisions.

Remark 6 As in the majority of the surgery scheduling literature, all surgeries are to be performed and surgery cancellation or postponement (to another day) is not allowed. The practical reasons of surgery cancellation/postponement go far beyond the OR usage. Our modelling approach can nevertheless be extended to take into account possible cancellation/postponement with the following rule: cancel/postpone with a penalty cost the elective surgeries that are not started before a deadline. However, we did not include cancellation/postponement in the experiments, since this issue is beyond the scope of our work focused on the insertion of emergency surgeries in the elective surgery plan.

5.3.2 Mathematical formulation

This subsection provides an event-based framework for the formal definition of the dynamic emergency surgery insertion problem.

Under the on-going assumptions, each surgery $i \in R \cup E$ is characterized by: a release time r_i equal to the planned release time if $i \in R$ or the random arrival time if $i \in E$, a random WTT δ_i known at the surgery release/arrival time, a due date $d_i = r_i + \delta_i$, a random surgery time p_i known only at the surgery completion, a starting time s_i , an OR o_i in which the surgery is performed, a completion time $c_i = s_i + p_i$, a tardiness $T_i = (s_i - d_i)^+$ where $(x)^+ = \max(0, x)$ and a unit tardiness cost α_i leading to tardiness cost $\alpha_i T_i$.

The overtime cost of each OR k depends on the completion time of the last surgery assigned to it, *i.e.*, $\max_{i \in R \cup E \wedge o_i = k} c_i$. No overtime cost is incurred if it is finished before the closing time b_k and an overtime cost $\beta_k (\max_{i \in R \cup E \wedge o_i = k} c_i - b_k)$ is incurred otherwise.

The Gantt chart 5.1 depicts the described surgery and OR variables.

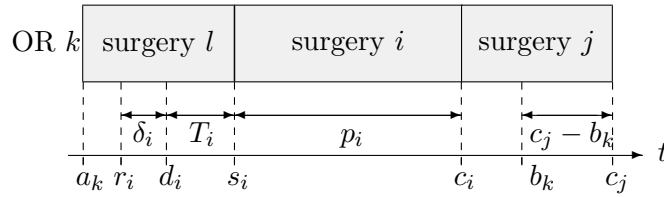


Figure 5.1: Surgery and OR variables.

The following events are considered: opening of an OR $k \in K$ at epoch a_k , release of an elective surgery $i \in R$ at epoch r_i , arrival of an emergency surgery $i \in E$ at epoch r_i , WTT breaching of an emergency surgery $i \in E$ at epoch d_i such that $s_i > d_i$ (the surgery is not yet started) and completion of a surgery $i \in R \cup E$ at epoch c_i . No event is associated with the WTT breaching of elective surgeries.

At the occurrence of an elective surgery release event, according to A4, the elective surgery starts if its OR is idle and no decision is needed.

The events: opening of an OR $k \in K$, releasing of an elective surgery $i \in R$ and completing of a surgery $i \in R \cup E$, and the related epochs, constitute a sufficient basis for defining starting time decisions of elective surgeries.

At all events other than the release of an elective surgery, a dynamic decision policy is needed to determine the optimal action. If no OR is available, then no decision is needed. If the emergency queue is empty, then start in each idle OR its earliest released elective surgery according to A4. If a surgery completion event occurs on an OR k at time $t \geq b_k$ and all its elective surgeries are completed, then close OR k . In all other cases, a decision is made based on the system state at time t denoted as $S(t)$.

State $S(t)$ at time t is defined by: (i) the emergency queue $E(t)$, (ii) the list $R(t)$ of remaining elective surgery, and (iii) the on-going surgery $i_k(t)$ of each OR k and its elapsed surgery time

$h_k(t)$ with $i_k(t) = \epsilon$ if the OR is idle.

Starting from state $S(t)$ and action u , a new state $S^{next}(S(t), u)$ is updated as follows:

- case $u = \text{no action}$, $S^{next}(S(t), u) = S(t)$;
- case $u = \text{assign an emergency surgery } i \text{ to OR } k$: set $i_k(t) \leftarrow i$, $h_k(t) \leftarrow 0$ and $E(t) \leftarrow E(t) - \{i\}$;
- case $u = \text{assign an elective surgery } i \text{ to OR } k$: set $i_k(t) \leftarrow i$, $h_k(t) \leftarrow 0$ and $R(t) \leftarrow R(t) - \{i\}$.

The goal of the dynamic insertion is to determine the action u that minimizes the total expected cost $Q(S(t), u, t)$ incurred at and beyond t by surgery tardiness and OR overtime under the assumption that the subsequent decisions are made by the optimal policy. More specifically,

$$Q(S(t), u, t) = E \left[\sum_{i \in R(t) \cup E^*(t)} \alpha_i (s_i - d_i)^+ + \sum_{k \in K} \beta_k \left(\max_{i \in R(t) \cup E^*(t) \wedge o_i = k} c_i - b_k \right)^+ : S(t) = S, u(t) = u \right] \quad (5.1)$$

where $E^*(t)$ is the complete set of emergency surgeries served at and beyond t .

Property 1. There exists an optimal policy such that all emergency surgeries are served in EDD order, *i.e.*, Earliest Due Date first.

Proof. Assume by contradiction that, at some time t and a state $S(t)$, the optimal action u is to assign an emergency surgery i to an OR k and there exists another emergency surgery j in $E(t)$ such that $d_j < d_i$. Define another feasible policy π identical to the optimal one but with service schedule of i and j switched. Since the surgery times of i and j are i.i.d. random variables, we also switch the surgery time of i and j . As a result, the two systems $Q(S(t), u, t)$ and policy π have exactly the same event times except the switched service order of i and j . Let $f^\pi(S(t), t)$ be the total expected cost at and beyond t by policy π . Since $d_j < d_i$ and $s_j > t$,

$$Q(S(t), u, t) - f^\pi(S(t), t) = E[\alpha(t - d_i)^+ + \alpha(s_j - d_j)^+ - \alpha(t - d_j)^+ - \alpha(s_j - d_i)^+] > 0 \quad (5.2)$$

which contradicts the optimality of action u and concludes the proof. \square

Let Ω be the set of all possible realizations, also called scenarios, of the number of emergency surgeries and variables: surgery time, surgery release/arrival time and surgery WTT. Then, let $E(\omega)$ be the set of emergency surgeries under scenario $\omega \in \Omega$, and $p_i(\omega)$, $r_i(\omega)$ and $d_i(\omega)$ be the surgery time, the release/arrival time and the due date of surgery $i \in R \cup E(\omega)$ under scenario $\omega \in \Omega$ respectively.

5.3.3 Perfect information bound

This subsection proposes a lower bound for the optimal total cost of the dynamic scheduling of emergency surgery.

Let $\theta(\omega)$ be the total cost resulting at the time of the latest surgery completion under scenario ω , *i.e.*, $\max_{i \in R \cup E(\omega)} c_i$. The lower bound cost is obtained by applying the perfect information

solution, *i.e.*, all pieces of uncertain information (number of emergency surgeries, surgery times and emergency surgery arrival times) are known at once at time 0. OR assignment of emergency surgeries and starting time of elective and emergency surgeries are determined independently for each scenario ω to minimize the total cost $\theta(\omega)$. This contradicts the progressive disclosure of uncertain information in our original model and hence provides a *Lower Bound* (LB) for the optimal total cost of the dynamic scheduling of emergency surgery.

More specifically, $LB = E_\omega[\theta(\omega)]$ where

$$\theta(\omega) = \min_{T_i(\omega), O_k(\omega)} \left\{ \sum_{i \in R \cup E(\omega)} \alpha_i \cdot T_i(\omega) + \sum_{k \in K} \beta_k \cdot O_k(\omega) \right\} \quad (5.3)$$

subject to

$$\sum_{k \in K} x_{ik}(\omega) = 1, \forall i \in R \cup E(\omega) \quad (5.4)$$

$$y_{ij}(\omega) + y_{ji}(\omega) \geq x_{ik}(\omega) + x_{jk}(\omega) - 1, \forall i, j \in R \cup E(\omega), k \in K \quad (5.5)$$

$$c_{ik}(\omega) \leq Mx_{ik}(\omega), \forall i \in R \cup E(\omega), k \in K \quad (5.6)$$

$$c_{ik}(\omega) \geq (a_k + p_i(\omega))x_{ik}(\omega), \forall i \in R \cup E(\omega), k \in K \quad (5.7)$$

$$c_{ik}(\omega) \geq (r_i(\omega) + p_i(\omega))x_{ik}(\omega), \forall i \in R \cup E(\omega), k \in K \quad (5.8)$$

$$\begin{aligned} c_{jk}(\omega) &\geq c_{ik}(\omega) + p_j(\omega) - M(1 - y_{ij}(\omega)) \\ &\quad - M(2 - x_{ik}(\omega) - x_{jk}(\omega)), \forall i, j \in R \cup E(\omega), k \in K \end{aligned} \quad (5.9)$$

$$O_k(\omega) \geq c_{ik}(\omega) - b_k, \forall i \in R \cup E(\omega), k \in K \quad (5.10)$$

$$T_i(\omega) \geq c_{ik}(\omega) - p_i(\omega) - d_i(\omega), \forall i \in R \cup E(\omega), k \in K \quad (5.11)$$

$$x_{ik}(\omega) = 1, \forall i \in R_k \quad (5.12)$$

$$y_{ij}(\omega) = 1, \forall k \in K, i, j \in R_k : i \text{ precedes } j \quad (5.13)$$

$$c_{ik}(\omega) \leq b_k + p_i(\omega), \forall i \in E(\omega), k \in K \quad (5.14)$$

$$(d_i(\omega) - d_j(\omega))y_{ij}(\omega) \leq Mz_{ij}(\omega), \forall i, j \in E(\omega) \quad (5.15)$$

$$\begin{aligned} c_{ik}(\omega) - p_i(\omega) &\leq r_j(\omega)z_{ij}(\omega) + M(1 - z_{ij}(\omega)), \\ &\quad \forall i, j \in E(\omega), k \in K \end{aligned} \quad (5.16)$$

$$\begin{aligned} c_{ik}(\omega) - p_i(\omega) + Mz_{ij}(\omega) &\geq r_j(\omega)(1 - z_{ij}(\omega)), \\ &\quad \forall i, j \in E(\omega), k \in K \end{aligned} \quad (5.17)$$

$$O_k(\omega), T_i(\omega) \geq 0, x_{ik}(\omega), y_{ij}(\omega), z_{ij}(\omega) \in \{0, 1\} \quad (5.18)$$

where $x_{ik}(\omega)$ is a binary variable equal to 1 if surgery i is assigned to OR k , $y_{ij}(\omega)$ is a binary variable equal to 1 if i precedes j , $z_{ij}(\omega)$ is a binary variable equal to 1 if i starts before the arrival of j , $c_{ik}(\omega)$ is the completion time of surgery i in OR k , $O_k(\omega)$ is the overtime of OR k , and M is a big number. The first part of the formulation from (5.4) to (5.11) is similar to the classical parallel machine scheduling mathematical programming model and the reader is referred to [145] for detailed explanation. Constraints (5.12)-(5.13) impose the fixed elective surgery plan. Constraint (5.14) forbids insertion of emergencies to an OR after its closing time b_k . Constraints (5.15)-(5.17) ensure the EDD order for queued emergency surgeries. The EDD rule must be applied only to emergency surgeries being queued concurrently; this concurrence condition is modelled by means of the variable $z_{ij}(\omega)$.

The EDD constraints (5.15)-(5.17) are very important for the tightness of the lower bound. A preliminary numerical experiment shows that the lower bound becomes very loose without these constraints. The gap observed between the lower bound obtained not-including EDD constraints and the best dynamic scheduling of surgery results is 15% farther (on average) than the lower bound obtained including EDD constraints.

5.4 Dynamic emergency insertion strategies

The exact resolution of the optimal dynamic insertion problem with continuous-time and hybrid state space with both discrete and continuous state variables is intractable. For this reason, we propose in this section a policy improvement procedure and several simple heuristic strategies.

5.4.1 A policy improvement procedure

From Section 5.3.2, the optimal total expected cost and the optimal control after the occurrence of an event at time t with state $S(t)$ are determined as follows:

$$V(S(t), t) = \min_{u \in A(S(t), t)} Q(S(t), u, t)$$

where $A(S(t), t)$ is the set of possible actions. $Q(S(t), u, t)$ is known as the Q-function and denotes the optimal total expected cost by starting with state $S(t)$ and action u under the assumption that the subsequent decisions are made by the optimal policy. To overcome the intractability of the Q-function, we resort to its approximation by a given policy π and define the following policy π' :

$$\pi'(S, t) = \operatorname{argmin}_{u \in A(S, t)} Q^\pi(S, u, t)$$

where $Q^\pi(S, u, t)$ denotes the total expected cost by starting with state S and action u under the assumption that the subsequent decisions are made by the policy π . This procedure is known in stochastic dynamic programming as policy improvement and the following result confirms the improvement of our problem with continuous-time and hybrid state space.

Property 2. $V^{\pi'}(S(t), t) \leq V^\pi(S(t), t)$ where $V^{\pi'}(S(t), t)$ and $V^\pi(S(t), t)$ are total expected cost under policies π' and π .

Proof. Modify the emergency arrival processes with emergency arrival cut off if L emergencies have arrived. It can be easily shown that the resulting cost functions $V^{\pi', L}(S(t), t)$ and $V^{\pi, L}(S(t), t)$ converge increasingly to $V^{\pi'}(S(t), t)$ and $V^\pi(S(t), t)$ as L increases. In the remaining proof, the index L is omitted for simplicity. Under the on-going assumption, there are at most $3L + 2|R| + |K|$ events for which a decision is needed. Let $V_n^{\pi'}(S(t), t)$ and $V_n^\pi(S(t), t)$ denote the cost functions after n events. We prove the property by induction on n . As no more decision is needed after $3L + 2|R| + |K|$ events, we have $V_{3L+2|R|+|K|}^{\pi'}(S(t), t) = V_{3L+2|R|+|K|}^\pi(S(t), t)$. Assume that the property holds for $V_{n+1}^{\pi'}(S(t), t)$ and $V_{n+1}^\pi(S(t), t)$ and we prove it for n . By definition,

$$\begin{aligned} V_{n+1}^{\pi'}(S(t), t) &= E[C_n(S, \pi'(S, t), t) + V_{n+1}^{\pi'}(S_{n+1}(t), t_{n+1})] \\ &\leq E[C_n(S, \pi(S, t), t) + V_{n+1}^\pi(S_{n+1}(t), t_{n+1})] \\ &= Q_n^\pi(S, \pi'(S, t), t) \\ &= \min_{u \in A(S, t)} Q_n^\pi(S, u, t) \\ &\leq Q_n^\pi(S, \pi(S, t), t) \\ &= V_{n+1}^\pi(S(t), t) \end{aligned}$$

with $C_n(S, \pi'(S, t), t)$ being the cost incurred before the occurrence of the next event, S_{n+1} and t_{n+1} the state and the time of the next event where the first inequality is from the induction assumption. The property is then shown by induction. \square

5.4.2 Heuristic strategies

This subsection proposes a stochastic optimization algorithm relied on the policy improvement procedure and introduces some simple emergency insertion rules that will be used to identify efficient implementation of the optimal dynamic emergency insertion strategies.

ASAP1 Emergencies are served in EDD order and *As Soon As Possible* whenever an OR is released. When two or more ORs are available, the emergency surgery is inserted into the OR that finishes for first all its remaining elective surgeries under the assumption of no emergency insertion and surgery times replaced by their mean. It is worth noticing that ASAP1 is close to the emergency insertion rule used in the hospital.

ASAP2 Similar to ASAP1 but with emergencies served in the *First Come First Served* (FCFS) order.

DDIP Similar to ASAP1 but with insertion of emergencies allowed only when their *Due Date Is Passed*.

MTC When an event occurs at time t and the emergency queue is not empty, this strategy determines the insertion of the emergency surgery with the earliest due date that minimizes the *Marginal Total Cost* (MTC). If the minimal marginal total cost is achieved by immediate insertion in an OR available at t , then the emergency is inserted, otherwise no action is taken at time t . The minimal MTC is checked for all possible insertions in any OR and at any location by replacing surgery times of ongoing surgeries by their conditional mean and all other surgery times by their mean. More specifically, let $\{[0], [1], \dots, [n]\}$ be the set of remaining surgeries of OR k with $[0]$ being the ongoing one and all others being the remaining elective surgeries of the OR. Consider the insertion of the emergency e of due date d_e after surgery $[i]$. Then the total cost $TC(k, i)$ of the OR after insertion becomes:

$$TC(k, i) = \min_{i \in \{0, \dots, n\}} \sum_{j=1}^n \alpha_{[j]} \left(c_{[j]} - E[p_{[j]}] - d_{[j]} \right)^+ \\ + \alpha_e (c_e - E[p_e] - d_e)^+ + \beta_k (c_{[n]} - b_k)^+$$

subject to

$$c_{[0]} = E[s_{[0]} + p_{[0]} : s_{[0]} + p_{[0]} \geq t] \\ c_{[j]} = \max(r_{[j]}, c_{[j-1]}) + E[p_{[j]}], \forall 0 < j \leq i \\ c_e = c_{[i]} + E[p_{[e]}] \\ c_{[i+1]} = \max(r_{[i+1]}, c_e) + E[p_{[i+1]}] \\ c_{[j]} = \max(r_{[j]}, c_{[j-1]}) + E[p_{[j]}], \forall j > i + 1$$

The mean conditional completion time $c_{[0]}$ of all ongoing surgeries is evaluated by Monte Carlo simulation. Our numerical experiments show that it is enough to check the earliest insertion of all OR plus the next insertion of ORs available at t . This is the MTC strategy implemented for numerical experiments.

SO This *Stochastic Optimization* strategy is a policy improvement of the MTC strategy that is proved numerically to be the best among the simple rules. More specifically, $\pi'(S, t) = \operatorname{argmin}_{u \in A(S, t)} Q^{MTC}(S, u, t)$. Further, the Q-function is evaluated by the sample average of

Table 5.1: Specialties Data

Specialty	case-mix	$E[p_j]$	$std_dev(p_j)$
Digestive Surg.	7.4%	161.2	90.5
Obstetrics	7.1%	98.8	73.8
Ophthalmology	9.2%	42.8	15.8
Orthopaedics	9.6%	146.5	86.7
Plastic Surg.	7.6%	135.4	77.9
Proctology	15.5%	43.1	14.6
Stomatology	5.2%	141.0	110.1
Urology	15.2%	97.4	100.8
Vascular Surg.	8.2%	125.3	71.4
Emergency Surg.	15.0%	109.7	68.0

a finite set of scenarios Ω^N , *i.e.*, $Q^{MTC}(S, u, t) \approx \frac{1}{|\Omega^N|} \sum_{\omega \in \Omega^N} Q^{MTC}(S, u, t; \omega)$. A set of 400 scenarios, used in all our numerical experiments, is found by preliminary experiments to be enough for a good trade-off between computational efficiency and solution quality. Further, for the sake of computational efficiency, we replace in MTC the mean conditional completion time $c_{[0]}$ of all ongoing surgeries by their actual completion times in the corresponding scenario $\omega \in \Omega^N$.

5.5 Numerical experiments

This section presents numerical results for (i) comparison of different heuristic policies, (ii) analysis of the impact of elective surgery plan and (iii) a principal component analysis to show how the near-optimal policy SO differs from best simple heuristic policy MTC.

5.5.1 Experimental setting

This subsection first presents the surgical activity data collected from a hospital that will serve as a basis for the test instance generation. We also discuss how different policies are evaluated.

Data collected from a hospital

The test instances of this work are based on real data from the Saint-Joseph Hospital (Paris, France) with 41,556 surgeries and a total surgery time of 71,120 hours in 2016. The hospital has 18 ORs. The regular opening time is 480 minutes from 8:00 to 16:00. Each weekday of an OR is split into OR blocks of either half-day of 4h or a day of 8h. Half of the OR blocks are blocks of 8h. There are nine surgery specialties managed according to a cyclic *Master Surgery Schedule* (MSS). The available OR-blocks are assigned to specialties and, within the same specialty, to surgeons.

The surgical activity collected data are reported in Table 5.1 in which column 2 is the case-mix of the specialty, column 3 the mean surgery time and column 4 the standard deviation. The surgery times are clearly specialty dependent and are assumed to be of log-normal distribution. Emergency surgeries, about 16% of the total surgery time, arrive according to a stationary Poisson process of rate of one emergency every 171 minutes. The hospital OT manager reserves a slack time for the emergency surgery demand and the OR blocks are reduced by 16% during the elective surgery planning.

Instances

For all test instances, there are 8 ORs. The waiting time targets are 60 minutes for all elective surgeries and uniformly distributed for emergency ones among three values: 0, 60 and 120. There are 9 types of elective surgeries corresponding to specialties of Table 5.1. Tardiness costs and overtime cost used in the experiments are given in Table 5.2.

To generate test instances, the following parameters are also considered: (i) elective surgery planning model, (ii) the OR blocks of each specialty, (iii) the surgery plan of each OR block, (iv) emergency arrival rate, (v) ORs without closing time.

The planning model for elective surgery can be either MSS which assigns OR blocks to specialties or *Open Schedule* (OS) without OR-block-to-specialty assignment. With equal probability, each OR has either a single block of 8h or 2 blocks of 4h. For each OR block, a specialty is randomly sampled according to the case-mix of Table 5.1. Emergency arrival rate is either $u=16\%$ of the OT activity or $2u = 32\%$. The length $BlockLength$ of each OR-block is reduced accordingly to $BlockLength^* = BlockLength \times (1 - u)$ or $BlockLength \times (1 - 2u)$. The OR with the smallest assigned elective surgery workload is selected to be on night duty, *i.e.*, with $b_k = \infty$.

The elective surgery plan depends on the model used. For each OR block, a new elective surgery n of the selected specialty for MSS and of a randomly generated specialty for OS is added as long as the following holds:

$$\sum_{j=1}^n E[p_j] \leq BlockLength^* + 0.5E[p_n] \quad (5.19)$$

The release times of all elective surgeries are determined by left-shifting and by using mean surgery time.

The surgery sequencing decision is needed for OS but unnecessary for MSS as all surgeries of each OR block are identical for MSS. Surgeries in different OR blocks of the OS model are sequenced according to one of the following priority rules:

BII Elective surgeries of different OR blocks are sequenced in order to minimize the maximal “break-in-interval” discussed in Section 5.2. That problem is not solved to optimality; the *Fixed Goal Values* greedy heuristic proposed in [127] has been implemented;

LEPT Elective surgeries in the same OR block are sequenced according to the *Longest Expected Processing Time first* rule;

SEPT Elective surgeries in the same OR block are sequenced according to the *Shortest Expected Processing Time first* rule.

24 MSS instances and 24 OS instances are generated with half instances for each emergency arrival rate ($u = 16\%$ and $2u = 32\%$).

Simulation setting

All five heuristic policies (SO, MTC, ASAP1, ASAP2, DDIP) are evaluated by simulation with 1000 replications and with common random variables for all policies. All policies and the simulation are coded in C++. All experiments are run on a machine equipped with a 3.5Ghz processor and 16GB of RAM.

In the following, we check the simulation accuracy and computation time on some preliminary test instances with OS elective schedule given by SEPT.

Table B.1 in Appendix B shows the simulation accuracy including mean total cost and 95% confidence half-width. The simulation accuracy seems good enough for a correct ranking of

Table 5.2: Cost Structures

Structure	elective tardiness	emergency tardiness	overtime
<i>Cost1</i>	0.33	0.33	0.33
<i>Cost2</i>	0.50	0.25	0.25
<i>Cost3</i>	0.25	0.25	0.50
<i>Cost4</i>	0.25	0.50	0.25

Table 5.3: Best vs. LB Deviation (%)

	u			$2u$		
	Avg.	Min.	Max.	Avg.	Min.	Max.
MSS	8.6	4.2	16.4	10.5	4.9	19.2
OS	12.4	8.8	19.8	16.7	13.0	22.0

different heuristic policies. A higher number of replications would lead to better simulation accuracy but requires significantly higher computation time, especially for the SO policy.

Table B.2 in Appendix B gives the computation time for decision making at each decision epoch. The simple heuristics (MTC, ASAP1, ASAP2, DDIP) require only a really short computation time. The most sophisticated SO policy takes at most 8 seconds with an average of less than 1 second. Such computation time is quite reasonable for health-care application.

5.5.2 Numerical results

For each instance, the cost $Cost^A$ of each policy A , the cost of the best heuristic policy $Cost^{Best}$ and the lower bound LB of the optimal cost are calculated. Then, the following indicators are determined

$$GAP^{Best, LB} := \frac{Cost^{Best} - LB}{LB} \quad (5.20)$$

$$GAP^{A, Best} := \frac{Cost^A - Cost^{Best}}{Cost^{Best}} \quad (5.21)$$

In order to evaluate the impact of the elective surgery schedule, the cost $Cost^{A, Y}$ of the coupling of policy A and elective schedule Y is calculated as well, the following indicator is then determined

$$Dev^{A, Y} := \frac{Cost^{A, Y}}{\min_{A', Y'} \{Cost^{A', Y'}\}} - 1 \quad (5.22)$$

$GAP^{Best, LB}$, about the tightness of the lower bound and the quality of the best policy, is given in Table 5.3. $GAP^{A, Best}$, about the percentage deviation of each heuristic policy A from the best policy, is given in Table 5.4 for MSS instances and in Table 5.5 for OS instances with cost structure *Cost1*, and in Appendix B for other cost structures. Each line gives the average, the minimal, the maximal and the number of best solutions reached over 12 instances. $Dev^{A, Y}$, on the impact of elective surgery schedule, concerns only OS instances and is given in Table 5.6 for cost structure *Cost1* and in the Appendix B for other cost structures. In OS instances tables, the first letter of the rule that sequences the initial elective surgery plan (S: SEPT, L: LEPT and B: BII) prefixes the name of the policy.

The main observations are as follows.

Best policy vs LB From Table 5.3, the deviation of the best policy from the lower bound is reasonably tight. As a result, it is meaningful to assess the performance of other heuristic policies with respect to the Best policy, *i.e.*, with respect to $GAP^{A, Best}$. Further, the perfect information bound integrating the property of EDD order of emergencies seems quite tight.

Table 5.4: MSS – Policy vs. Best Deviation (%)

<i>Cost1</i>	<i>u</i>				<i>2u</i>			
	Avg.	Min.	Max.	#1	Avg.	Min.	Max.	#1
SO	0.1	0.0	0.9	9	0.3	0.0	1.8	9
MTC	1.9	0.0	10.0	3	9.2	4.8	12.4	0
ASAP1	14.7	1.0	28.3	0	8.3	0.0	24.6	2
ASAP2	14.9	0.9	28.6	0	9.1	0.0	25.1	1
DDIP	18.1	8.0	31.1	0	31.3	25.1	34.8	0
<i>Cost2</i>	<i>u</i>				<i>2u</i>			
	Avg.	Min.	Max.	#1	Avg.	Min.	Max.	#1
SO	0.1	0.0	1.1	10	0.1	0.0	1.2	10
MTC	1.7	0.0	8.0	2	8.8	5.9	17.2	0
ASAP1	20.1	0.1	44.7	0	16.8	0.0	46.6	1
ASAP2	20.3	0.4	44.4	0	17.1	0.0	47.6	1
DDIP	18.5	6.3	33.9	0	31.6	21.8	44.2	0
<i>Cost3</i>	<i>u</i>				<i>2u</i>			
	Avg.	Min.	Max.	#1	Avg.	Min.	Max.	#1
SO	0.1	0.0	0.7	10	0.2	0.0	1.7	9
MTC	1.6	0.0	8.0	2	6.0	2.7	14.5	0
ASAP1	15.4	3.8	25.2	0	7.2	0.0	22.7	3
ASAP2	15.7	4.0	26.0	0	7.8	0.2	23.3	0
DDIP	18.2	8.5	30.3	0	27.5	20.5	31.7	0
<i>Cost4</i>	<i>u</i>				<i>2u</i>			
	Avg.	Min.	Max.	#1	Avg.	Min.	Max.	#1
SO	0.1	0.0	1.0	11	0.4	0.0	2.0	9
MTC	3.6	0.1	24.2	0	13.1	8.7	26.3	0
ASAP1	8.1	0.0	17.5	1	3.2	0.0	10.7	3
ASAP2	8.6	0.2	17.3	0	3.8	1.1	11.3	0
DDIP	20.1	11.8	25.7	0	32.5	29.4	34.4	0

Table 5.5: OS – Policy vs. Best Deviation (%)

<i>Cost1</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
B-MTC	4.2	1.6	8.2	0	13.9	1.0	22.6	0
B-ASAP1	8.1	6.0	9.5	0	6.4	3.4	10.6	0
B-ASAP2	8.4	6.1	9.9	0	6.8	3.9	11.0	0
B-DDIP	10.2	8.2	12.7	0	21.3	16.1	26.0	0
<i>Cost2</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
L-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
L-MTC	6.2	3.9	9.0	0	20.5	14.7	25.6	0
L-ASAP1	7.0	5.7	9.6	0	6.3	3.5	10.3	0
L-ASAP2	7.2	5.7	9.8	0	6.8	3.5	10.9	0
L-DDIP	9.9	8.6	11.8	0	21.4	17.2	24.7	0
<i>Cost3</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
S-SO	0.1	0.0	0.6	10	0.0	0.0	0.0	12
S-MTC	1.3	0.0	5.7	2	6.0	1.3	14.3	0
S-ASAP1	7.7	5.8	9.4	0	5.7	2.5	9.4	0
S-ASAP2	8.0	6.2	9.8	0	6.2	3.3	10.1	0
S-DDIP	10.4	8.5	12.2	0	21.5	17.1	26.0	0

Table 5.6: OS – The Impact of Proactive Schedule (%)

$Cost1$	u				$2u$			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	9.8	0.0	20.1	1	5.9	0.1	13.4	0
B-MTC	14.3	4.5	23.7	0	20.4	13.3	29.5	0
B-ASAP1	18.7	8.4	30.4	0	12.6	4.2	24.0	0
B-ASAP2	19.0	8.2	30.7	0	13.1	4.0	24.4	0
B-DDIP	21.0	9.9	32.2	0	28.4	16.2	38.8	0
L-SO	43.4	20.7	65.1	0	19.3	7.7	28.4	0
L-MTC	52.1	31.5	72.7	0	43.6	31.3	55.5	0
L-ASAP1	53.5	29.3	77.3	0	26.8	13.3	41.7	0
L-ASAP2	53.8	29.2	77.3	0	27.4	13.5	42.4	0
L-DDIP	57.7	33.2	80.8	0	44.7	28.6	58.2	0
S-SO	0.3	0.0	3.0	9	0.0	0.0	0.0	12
S-MTC	1.6	0.0	5.7	2	6.0	1.3	14.3	0
S-ASAP1	7.9	5.8	10.9	0	5.7	2.5	9.4	0
S-ASAP2	8.3	6.2	11.0	0	6.2	3.3	10.1	0
S-DDIP	10.7	8.5	13.9	0	21.5	17.1	26.0	0

Policy ranking The overall ranking is SO, MTC, ASAP1, ASAP2, DDIP. Results in tables 5.4-5.6 and B.3-B.6 show that SO is the best and DDIP the worst in the majority of instances. SO is close to the best when it is not the best. MTC is often ranked second. The poor performance of DDIP shows the importance of anticipating the waiting time targets of emergency surgeries. The superiority of SO can be explained as follows. Whereas MTC considers only the head of the emergency queue and neglects the entire emergency queue, the policy improvement makes it possible for SO to overcome the MTC blindness by taking into account all emergencies.

Benefit of efficient emergency insertion The gap between the best and worst policies is significant and often more than 20% (with the maximum gap of 47.6% and an average of 19.0%). With respect to ASAP1 which is similar to the hospital practice, the SO policy improves by 8.6% on average.

When hospital-like ASAP policies perform reasonably The performance of the ASAP policies improve when (i) the overtime cost is high ($Cost3$), (ii) the emergency tardiness cost is high ($Cost4$), and (iii) the emergency demand is high ($2u$). ASAP1 even becomes the best for some MSS instances fulfilling the above conditions. Under these scenarios, the capacity of closing ORs is quickly saturated. Then, the opportunity for a cleverer algorithm as SO to parallelize the work in the queue on multiple ORs is significantly reduced.

Impact of elective surgery schedule Whereas the elective schedule has a significantly higher impact on the performance than the emergency insertion policy, both contribute significantly to the overall performance of the system. Contrary to the observation of [127], the SEPT elective schedule is significantly better than the BII elective schedule. In all test instances, LEPT elective schedule is the worst.

How the near-optimal SO policy differs from simple MTC policy A *Principal Component Analysis* (PCA), given in Appendix B, is performed to understand how SO policy differs from MTC and how problem data and state information change the real-time decision. We summarize the key findings on the correlation of the emergency insertion decision with various

state information. First, a higher correlation with the time of the day for MTC than for SO is observed, implying the failure of MTC to insert appropriately late emergencies due to its short-sighted perspective. Second, a higher correlation with the night-duty OR insertion for MTC than for SO is observed, implying more ORs insertion exploited by SO and more the night-duty OR insertion by MTC. The correlation is surprisingly decreasing for SO but stable for MTC as the emergency demand increases from u to $2u$. Third, MTC has a higher sensitivity to the head-of-queue tardiness than SO, confirming again the myopic nature of MTC.

The PCA results sustain the conclusion about the SO capability to overcome the MTC short-sighted perspective over the future and the emergencies queue. SO appears able to foresee the decision impact over the performance and over future emergency arrivals. This result strengthens our conclusions about the quality of SO in recovering the weakness of MTC matching the purpose of a policy improvement algorithm.

5.6 Conclusion

This chapter addresses the dynamic scheduling problem of randomly arriving emergency surgeries in an operating theatre composed of several operating rooms shared between elective and emergency surgeries. We considered different waiting time targets to characterize different emergency levels of emergency surgeries. An event-based stochastic programming model is proposed to minimize the total cost incurred by exceeding waiting time targets of emergency surgeries, elective surgery delay and surgical team overtime. As the problem is hard to solve, we defined two simple *As Soon As Possible* emergency insertion policies (ASAP1 and ASAP2), we proved the optimality of the *Earliest Due Date* (EDD) first rule for queued emergencies and, on the basis of the EDD rule, we developed a simple heuristic policy (MTC) and a Stochastic Optimization (SO) policy improvement algorithm of the MTC policy. A perfect information lower bound for the cost of the dynamic scheduling of emergency surgery is provided as well.

A testbed of several instances that cover different specific initial surgery plan and emergency flow is used. Numerical results reveal that both MTC and SO overcome simple ASAP emergency insertion policies regardless of the initial schedule of elective surgeries and SO gives the best result in most of the cases. The MTC performance is reduced when the weight on emergency tardiness and the flow of emergencies increases. The obtained results also show that the initial schedule of elective surgeries has an important impact.

Future research can focus on the relaxation of the assumption that the initial schedule of elective surgeries is given and cannot be changed (Assumption A3). The impact on the system of a deep rescheduling has to be taken into account in this case. Another research direction can be to establish dynamically which operating rooms cope with night duty. Another one can be to extend the model proposed in this chapter to consider the cancellation and postponement to another day of elective surgeries. The challenge is to define how these decisions are made and under which conditions. Since the optimality of the EDD rule for queued emergencies relies on the unique distribution of their surgery times, a further research direction can be to consider different distribution functions for emergency surgeries involving this information in dynamic scheduling.

Chapter 6

Conclusion

In this thesis, we addressed a set of related problems for the *Operating Theatre* (OT) planning and scheduling focusing on the *Operating Room* (OR) scheduling under constraints of material and *human resource* (HR). ORs are the bottleneck for the surgical patients flow in a correctly designed OT, ward beds are the second cost hub after the OT in term of expenditures size for acute care hospital and HRs involved in the surgery execution account for most of the costs for running an OT. Resources other than ORs have been considered explicitly or as constraints, we have considered ward beds as material resources and surgeon and surgical nurses as HRs. Special attention has been devoted to a class of surgical patients, *i.e.*, the emergency patients. The OT has been introduced in Chapter 1, its physical setting, the HRs involved in surgery and the health-care pathway of surgical patients have been described.

Despite the relevant attention received in the field of operations research and management science, we revealed some gaps to close remaining between the available literature and issues rising in practical contexts. This thesis has been motivated by the will to close or, at least, reduce such gaps by investigating OR planning and scheduling considering, not only the OR capacity but also the other mentioned relevant resources. The different OT planning and scheduling problems addressed in this thesis belong to different decision levels, from the middle-long planning to the real-time scheduling. A short review of the operations research and management science literature about OT planning and scheduling has been provided according to the different decision levels in Chapter 1.

In Chapter 2, we studied a set of policies including bed management that we devised for the scheduling of elective patients and the admission of emergency patients. The study we proposed is limited to surgical patients and ORs and ward beds as hospital resources. Elective scheduling and emergency admission policies are paired to compose policy settings and evaluated by means of a discrete event simulation. Four *Key Performance Indicators* (KPIs) have been considered for the evaluation and comparison of policy settings: the percentage of refused emergencies, the percentage of timed out emergencies, the percentage of elective rescheduling and the percentage of overdue electives. The bed management policies we have considered relies on three components: the type of bed availability evaluated for the scheduling/admission of patients, this can be the single patient admission date or the entire predicted patient stay, the bed occupancy limit, *i.e.*, the threshold for the bed occupancy defined for limiting the scheduling/admission of patients, and the patients length-of-stay forecast used to compute the bed occupancy over the future, we considered a perfect information forecast and an expected value forecast as possible cases. As the used OR capacity management policy can impact largely on the scheduling of elective patients, we have considered two cases: (1) a master surgery schedule is applied and (2) elective surgeries are scheduled according to the open block policy.

Simulation outcomes have revealed that the devised policies including bed management can allow the hospital managers to control the considered KPIs and that the quality of the patients length-of-stay forecast, as well as the evaluated type of bed availability, are less relevant than

expected. Thresholds for the bed occupancy, *i.e.*, bed occupancy limits, have been revealed to be a major tool for controlling the flows of patients. Hospital managers can select the more appropriated policy setting to apply according to targets (or bounds) for the KPIs and the expected flow of emergencies.

Future researches extending the work presented in Chapter 2 may focus on bed management policies that consider dynamic/adaptive bed occupancy limits to fit better with the effective short-term demand of elective and emergency patient admissions, *i.e.*, to devise more advanced policies for the admission of patients with improved flexibility. Other researches may focus on the introduction of waiting lists for elective patients and the periodic off-line scheduling to optimize the resource utilization and assess the trade-off between patient-oriented KPIs and resource-oriented KPIs.

In Chapter 3, we have defined a problem for the scheduling of surgery that collects constraints identified as the most common in real-world practice (*i.e.*, sequence, capacity and due date constraints), and resource constraints for human resource, *i.e.*, surgeons and nurses. The problem includes also a new model for sequence-dependent OR cleaning times that arise because of surgeries with different infection levels. The interest for such problem including details of surgery sequences and HRs have been justified by the importance for OTs of detailed schedules to avoid unexpected resource unavailability. This difficult problem has been solved by devising a branch-and-price-and-cut algorithm based on the time-indexed problem formulation and relying on a label correcting algorithm for solving the pricing problems, which are proved to be strongly NP-Hard. The efficiency of the label correcting algorithm has been ensured by dominance rules among labels and label extensions upper and lower bound computation via *ad-hoc* algorithms. A Benders' cutting procedure for tightening the problem LP relaxation has been also developed. The cutting procedure has been inspired by the Benders' decomposition and is based on the theory of duality for linear programming. Such procedure is a major contribution of Chapter 3. A set of computational experiments revealed the effectiveness of such an exact algorithm for the problem solution. The algorithm outperformed competing methods from the literature and the commercial solver (CPLEX).

We hope that the Benders'-like cutting procedure we devised and described in Chapter 3 can be the basis of future researches for problems considering side resources. The developed approach is generic and, with maybe an effort to address the technical issues that can arise, can be applied for an arbitrary number of resources. The work presented in Chapter 3 can be extended in future researches advancing in two directions: (*i*) by considering some sources of uncertainty and (*ii*) by addressing problems that hierarchically follow the weekly scheduling of surgery. Main sources of uncertainty should include surgery durations and resources consumption due to add-on surgeries that may show up after the weekly scheduling of surgery. In both cases, the optimization objective may be minimizing the average shortage of resources or the chance of such a shortage. State-of-the-art stochastic optimization methods are nowadays unfortunately able to solve only very small instances of stochastic integrated operating room planning and scheduling problems, an effort is required to tackle solution difficulties.

In Chapter 4, the nurse to surgery assignment problem has been addressed, the problem has been presented in both its deterministic and stochastic formulation. This nurse scheduling problem has been studied in a new perspective, we studied the problem considering the constant training of nurses for non-technical skills (interpersonal communication, coordination, judgement, etc.). Nurses non-technical skills are of crucial importance for the avoidance of errors inside the OR and the safety of surgery. The stochastic problem formulation we have proposed considers uncertain surgery durations and three hierarchical objectives: the overtime minimization, the minimization of the number of ORs visited daily by nurses and the maximization of the number of surgeons assisted by nurses over the planning horizon. The minimization of the nurse overtime and the minimization of the number of visited ORs have been considered as nurse preference objectives, the maximization of the number of assisted surgeon has been considered for

the nurses non-technical constant training. The two-stage stochastic modelling of the problem has been solved by means of sample averaging of scenarios.

Numerical results of Chapter 4 revealed the effectiveness of the applied solution method for solving the problem: computation times are limited, especially for smaller instances. We can assert that the optimization problem that we proposed allows the computation of quite good quality solutions in terms of nurse overtime despite the uncertainty of surgery durations. Also, the number of ORs visited by nurses daily can be effectively optimized by solving the proposed problem, as well as the number of surgeons assisted by nurses over the planning horizon. Having addressed the weekly scheduling of surgery, in Chapter 3, and the nurse to surgery assignment, in Chapter 4, as two distinct problems solved sequentially and not as one integrated problem appears as a winning approach.

The study presented in Chapter 4 can be extended in the following directions. A more efficient method for solving the problem formulation can be developed. A stochastic programming algorithm (Benders' decomposition based) like the L-shaped method can be appropriated for enhancing the problem solution. The analysis of numerical results can benefit of instances with larger surgical groups since a wider range of possible values for the number of surgeons that can be assisted by nurses can allow to better evaluate the value of optimized solutions. A partial rescheduling of surgeries can allow an improved solution of objectives in terms of ORs visited daily by nurses and nurse overtime. As nurses are very sensible to the schedule quality with respect to the number of ORs visited daily and overtime, surgery schedules improving one or both these two objectives can have an important impact on the satisfaction of nurses. The problem formulation can be also extended to the case of more complex nurse shifts (shifts of different length, timing, etc.) to make the approach applicable in a wider set of practical settings. Moreover, for a practical application of the problem solutions, running a second optimization to minimize the number of nurses experiencing the maximum overtime, visiting the maximum number of operating rooms, and assisting the minimum number of surgeons, can provide problem solutions improved for practical use. In this way, the limitations of min-max/max-min objectives can be overcome.

In Chapter 5, we have addressed the problem of the dynamic scheduling of randomly arriving emergency surgeries in the OT. We have assumed that the OT is composed of several ORs shared between elective and emergency surgeries and that emergency surgeries are to be inserted into the given plan of elective surgery. Different emergency levels of emergency surgeries characterize the problem and a waiting time target is given for every surgery. An event-based stochastic programming model has been proposed to minimize the total cost incurred by exceeding waiting time targets of emergency surgeries, delaying elective surgeries and incurring surgical team overtime. The problem is hard to solve and we have devised two simple policies, a heuristic algorithm and a stochastic optimization algorithm for solving it. Solution methods have been compared against a perfect information lower bound for the cost of the dynamic scheduling of emergency surgery. The EDD (Earliest Due Date first) rule for queued emergencies exploited by insertion policies and algorithms has been proved to be optimal. Results of extended numerical experiments for testing the devised policies and algorithms have been reported in the chapter and a set of instances for different specific initial surgery plans and emergency flows have been used.

Numerical results reported in Chapter 5 have shown that both the heuristic algorithm and stochastic optimization algorithm overcome simple emergency insertion policies, this has been revealed regardless of the initial schedule of elective surgeries. The stochastic optimization algorithm has provided the best result in most cases. The obtained results revealed the initial schedule of elective surgeries to be of primary importance for the considered problem, a good initial plan of elective surgery allows to obtain the best performances. The SEPT (Shortest Expected Processing Time) first rule for sequencing elective surgeries in every OR for the initial schedule has been revealed to be the best rule.

Future research extending the study of Chapter 5 can focus on relaxing constraints of given plans of elective surgery, *i.e.*, to let possible a complete (or partial) rescheduling of the elective surgeries, the impact on the system of a deep rescheduling has to be taken into account in this case. The rescheduling of elective surgeries may consider also the cancellation and postponement to other dates of elective surgeries. The challenge will be to define how these decisions are made and which conditions are to be evaluated for decision making. As the optimality of the EDD rule for queued emergencies relies on the unique distribution of surgery times, future researches can consider different distribution functions for emergency surgery durations.

Chapter 7

Conclusion en langue française

Dans cette thèse, nous avons abordé des problématiques connexes de planification et programmation du bloc opératoire. L'attention a été focalisée sur des problèmes de programmation des chirurgies dans les salles opératoires sous contraintes des ressources matérielles, lits des services, et humaines, chirurgiens et infirmiers du bloc opératoire. Dans un bloc opératoire correctement conçu, les salles opératoires constituent le goulot d'étranglement pour le flux des patients accueillis et, avec le personnel mobilisé pour le déroulement de l'activité chirurgicale, représentent le principal centre de coût des établissements hospitaliers. En raison de ces éléments, une vaste littérature scientifique dans le domaine de la recherche opérationnelle a été produite pour des problèmes de planification et programmation de l'activité chirurgicales dans les salles opératoires. Bien que les efforts des recherches ont été productifs, des lacunes entre la littérature publiée et la pratique courante des hôpitaux sont visibles. Une analyse concise de la littérature scientifique du domaine de la recherche opérationnelle pour les problématiques de planification et programmation du bloc opératoire a été reportée dans le Chapitre 1 avec une description du bloc opératoire, de sa conception physique, du personnel impliqué dans l'activité chirurgicale et des possibles parcours des patients chirurgicaux.

Cette thèse a été motivée par l'ambition de remplir ces lacunes en focalisant l'attention sur les problématiques liées aux ressources autres que les salles opératoires. Une attention spécifique a été dédiée au flux des patients chirurgicaux urgents qui ne peuvent pas être planifiés à l'avance, mais qui posent des problématiques significatives de la gestion des flux des patients et des ressources du bloc opératoire.

Les politiques pour la programmation des chirurgies et l'admission des urgences décrites dans le Chapitre 2 incluent la gestion des lits des services et utilisent des seuils d'occupation des lits pour les décisions de programmation et admission des patients (exclusivement chirurgicaux). L'évaluation des politiques par moyen de simulation à événements discrets proposée a été basée sur quatre indicateurs de performance clés : le pourcentage des urgences dont l'admission est refusée, le pourcentage des urgences réorientées vers autres établissements cause d'attente dépassant un temps fixé, le pourcentage des chirurgies planifiées reprogrammées et le pourcentage des chirurgies programmées effectuées après date limite. Outre que les seuils d'occupation des lits, les politiques développées pour la gestion des lits sont caractérisées par : la prévision de la durée de séjours hospitaliers des patients (on a pris en considération les prévisions « information parfaite » et « durée espérée ») et le type de disponibilité du lit évaluée pour la programmation des chirurgies et l'admission des urgences (on a pris en considération la seule date prévue d'admission du patient, ou la durée totale prédite du séjour hospitalier du patient). Un facteur de gestion du bloc opératoire impactant sur les indicateurs de performance ciblés qui a été évalué avec les politiques développées est la politique de gestion de la capacité de la salle opératoire. On a pris en considération deux possibles politiques pour cette gestion de la capacité : application d'un calendrier cyclique pour l'affectation exclusive des spécialités chirurgicales aux créneaux de temps, politique connue en anglais comme « *Master Surgery Schedule* » (calendrier maître de la chirurgie),

ou gestion ouverte, « *Open Block* » en Anglais, de la capacité des salles opératoires ; dans cette dernière option, aucune affectation exclusive aux spécialités chirurgicales du temps d'ouverture des salles est appliquée.

Dans la simulation à événements discrets du Chapitre 2, les processus décisionnels pour la programmation des chirurgies et l'admission des urgences ont été modélisés en considérant les ressources salles opératoires et lits des services ; soit le flux de patients normaux que le flux de patients urgents sont simulés. Les résultats de simulation ont montré que les politiques développées incluant la gestion des lits sont efficaces pour maîtriser les indicateurs des performance ciblés. La qualité de la prédiction des durée des séjours des patients et le type de disponibilité du lit sont montré avoir un impact limité. Au contraire, les seuils d'occupation des lits ont été montrés comme des outils largement efficaces pour la gestion des flux des patients.

Recherches futures pour étendre l'étude présenté dans le Chapitre 2 peuvent focaliser des politiques de gestion des lits qui considèrent des schémas de décision adaptatives pour une gestion améliorée au fil du temps de la demande d'admission effective. Cette perspective implique le développement de politiques plus évoluées pour une majeure flexibilité d'admission en relation aux seuils d'occupation des lits. Autres recherches pourraient considérer une programmation des chirurgies cadencée et periodique dans le temps, au lieu que dynamique, pour une programmation qui puisse optimiser l'utilisation des ressources.

Le problème de programmation des salles opératoires que nous avons défini dans le Chapitre 3 collecte les contraintes que on a identifiées comme les plus communes dans les problèmes réels de programmation de chirurgies (contraintes d'ordonnancement, affectation, capacité, date limite) et inclut contraintes de capacité pour les ressources humaines mobilisées en chirurgie, les chirurgiens et les infirmiers du bloc opératoire. Le problème inclut aussi un nouveau modèle pour la programmation des nettoyages des salles qui sont dépendants de l'ordonnancement des chirurgies caractérisées par différents niveaux d'infection. L'intérêt pour ce problème de programmation incluant un ordonnancement détaillé des chirurgies et contraintes des ressources humaines naît en cause de l'importance de telle programmation détaillée pour éviter indisponibilités inattendues et conflits des ressources. Pour résoudre ce problème, nous avons développé une méthode exacte d'optimisation linéaire en nombres entiers. La méthode est basée sur la génération des colonnes et la qualité de la relaxation linéaire de la fonction objective est améliorée avec une méthode de génération des coupes type Benders. La résolution des sous-problèmes de la génération des colonnes, qui sont NP-difficile, est confiée à un algorithme de programmation dynamique de l'optimisation combinatoire dont l'efficacité a été amélioré avec règles de dominance. La méthode de génération de coupes est la contribution majeur du Chapitre 3. L'efficacité dans la résolution du problème et l'efficacité computationnelle de la méthode développée sont montrées moyennant des expérimentations numériques conduits en utilisant des instances soit de la littérature que générée avec des données réelles. Moyennant une comparaison avec résultats de la littérature et du solveur commercial CPLEX, la supériorité de la méthode développée a été montrée.

On souhaite que la méthode de génération des coupes développé dans le Chapitre 3 puisse être de base pour recherches futures traitant des problèmes qui considèrent contraintes de ressources. La méthode est générique et peut être étendu directement à un nombre arbitraire des ressources. Nous pensons que le travail présenté dans le Chapitre 3 pourra être ultérieurement développé en deux directions possible : (i) en considérant des aléas des paramètres du problème et (ii) en considérant des problématiques en val du problème traité. Les aléas d'intérêt majeur sont, à notre avis : des durée des chirurgies aléatoires et un consommation aléatoire des ressources due à la nécessité inattendue, suivant la planification, d'insérer autres chirurgies au programme calculé. Dans les deux cas, l'objectif de l'optimisation peut être la minimisation des dépassements horaires ou de la probabilité du manque de capacité des ressources. Un effort pour la conception d'une méthode de résolution efficace pour le problème incluant des aléas sera nécessaire, les méthode d'optimisation stochastique connus sont, au moment, capable des résoudre que des

instances de petite taille pour des problème de difficulté comparable à problème traité, mais incluant des aléas.

Dans le Chapitre 4, nous avons présenté le problème de l'affectation des infirmiers aux chirurgies. Ce problème prend comme entrée la solution du problème du Chapitre 3 ; les deux problèmes peuvent être vus comme deux étapes d'un seul problème de planification : générer un programme de chirurgie très détaillé incluant l'affectation des infirmiers aux chirurgies.

Le problème d'affectation des infirmiers du Chapitre 4 a été présenté dans sa version déterministe et stochastique. Ce problème d'affectation des infirmiers a été traité dans le chapitre en considérant une nouvelle perspective, la formulation inclut un objectif de rotation des infirmiers avec les différents chirurgiens au but de maintenir et perfectionner les habilités pas strictement techniques requises pour les infirmiers (communication interpersonnelle, coordination, jugement, etc.). Ces habilités pas-techniques sont d'importance crucial pour éviter une large partie des erreurs et accidentés que peuvent se produire en cours de chirurgie. Il est montré dans la littérature scientifique que la majorité des erreurs produits au cours des chirurgies est largement en cause de raisons qui ne sont pas techniques ; ces habilités pas-techniques des infirmiers, et de l'équipe chirurgical dans sa intégralité, contribuent significativement à la sécurité des patients dans les bloc opératoire. Le problème considère aussi des objectifs autre que rotation des infirmiers, le dépassements horaires des infirmiers et le nombre des salles opératoires visité au cours de la journée par infirmier dans le pire cas sont minimisés. La version stochastique du problème considère des durées des chirurgies aléatoires et a été résolue par moyen de la méthode à échantillonnage des scénarios.

Les résultats numériques du Chapitre 4 ont montré que la méthode de résolution appliquée à la formulation stochastique du problème est efficace pour les instances considérées. Les temps de calculs sont limités spécialement pour les instances les plus petite, parmi celles prises en considération. On peut affirmé que l'optimisation du problème donné permet de obtenir des solutions de une certaine qualité en terme de dépassements horaires, les dépassements sont contenus malgré les durées aléatoires des chirurgies. Le nombre des salles opératoires visitées dans le pire cas et le nombre des chirurgiens assistés en chirurgie sont effectivement optimisés aussi. De conséquence, la résolution du problème d'affectation des infirmiers aux chirurgies séparée de celle du problème de programmation des chirurgies, problème traité dans le Chapitre 3, semble une option valable.

Nous pensons que le travail présenté dans le Chapitre 4 puisse être entendu et développé dans ces directions : (i) développer une méthode de résolution plus efficiente du problème stochastique, les méthodes basées sur la décomposition de Benders peuvent être probablement les plus appropriées, (ii) étendre les expérimentations numériques en considérant des instances avec des caractéristiques différentes, par exemple avec des groupes des spécialités chirurgicales plus larges de celles déjà utilisées, ceci pour une évaluation qualitative plus large des solutions, (iii) permettre une reprogrammation partiel des chirurgies pour améliorer les différents objectifs et (iv) étendre la formulation du problème au cas d'horaires de travail des infirmiers plus structurés (différentes durées de horaires, heure de fin et début, etc.). En outre, on peut fournir des solutions améliorées, au but de leur application réelle, avec une deuxième optimisation du problème pour minimiser le nombre des infirmiers qui, dans l'évaluation des scénarios, expérimentent le nombre maximal des heures supplémentaires, visitent le nombre maximal des salles opératoires et assistent le nombre minimal des chirurgiens. Avec cette deuxième optimisation, il est possible de surmonter les limitations des objectifs min-max et max-min.

Dans le Chapitre 5, nous avons étudié des politiques pour l'insertion des urgences en cours de journée dans un programme de chirurgie donné. Nous avons fait l'hypothèse que un ensemble de salles opératoires équivalant du point de vue de la chirurgie est partagé entre la chirurgie programmée et les urgences et que plusieurs délais possible caractérisent les chirurgies. Le problème a été modélisé comme un problème d'optimisation stochastique d'un système dynamique des files d'attente en considérant comme objectif la minimisation du coût total des dépasse-

ments des délais des chirurgies et d'horaire des équipes chirurgicales. En raison de la difficulté de résolution du problème, nous avons conçu deux algorithmes simples d'insertion d'urgence, un algorithme heuristique et un algorithme d'optimisation stochastique approchée. Les algorithmes développés sont comparés entre eux et avec une borne d'information parfaite du coût total de l'insertion dynamique des urgences par moyen de expérimentations numériques. Dans le Chapitre 5 nous montrons aussi l'optimalité de la règle EDD (de l'Anglais *Earliest Due Date first*, délais plus proche en premier) pour l'ordonnancement de la file d'attente des urgences ; soit l'algorithme heuristique que ceci d'optimisation stochastique exploitent l'optimalité de la règle pour l'ordonnancement des urgences en attente d'insertion. Les instances utilisées dans les expérimentations numériques du chapitre permettent aussi une comparaison des programmes initiales de chirurgies générés en appliquant différentes règles d'ordonnancement des chirurgies dans les salles opératoires. Ordonnancer les chirurgies programmées de la plus courte à la plus longue est montré par les expérimentations numériques comme l'ordonnancement meilleur.

Des recherches futures en val du chapitre 5 peuvent considérer la relaxation des contraintes imposées par le programme initial de chirurgie. Au but d'une évaluation qualitative, considérer une reprogrammation complète, ou partielle, des chirurgies programmes peut être un sujet de recherche à développer. Cette reprogrammation pourrait inclure aussi l'annulation ou le décalage à autre dates des chirurgies. Dans ce cas, le défi comporte la définition des conditions de décision à évaluer en dynamique. Recherches futures peuvent aussi prendre en considération différent distributions des durées de chirurgie des urgences, dans ce cas, la règle EDD ne serait plus optimale car elle s'appuie sur l'hypothèse d'unicité de distribution des durées.

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Appendix A

Detailed analysis of the discrete event simulation outcomes

In this appendix, a systematic and detailed analysis of the simulation outcomes is presented. The impact on the four KPIs of the different PS features and scenario factors, in relation to the benchmark scenario and PS, is evaluated. Scenario factors and PS features considered as “dimensions” for the analysis are the patient LOS forecast, the type of bed availability considered for both elective and emergency patients, the enforcement or not of the BOL and the OR capacity management policy that is used.

In Section A.1, we discuss the impact on the four KPIs of the patient LOS forecast, PI or EV, in Section A.2, the impact of applying the BOL and, in Section A.3, the impact of applying the OB policy for the management of the OR capacity instead of applying the MSS.

A.1 PI vs. EV and BPS vs. BAD

In this section, we analyse the impact on the four KPIs of using the patient LOS forecast EV with respect to the PI forecast and how the two types of bed availability (FBPS and FBAD) considered for the scheduling of elective patients and the admission of emergency patients impact on the four KPIs. Simulation outcomes for the PSs are evaluated in relation to those of the benchmark PS.

Table A.1 is as Table 2.12 that reports outcomes for the benchmark PS and the patient LOS forecast PI, but for the patient LOS forecast EV. Table A.1 lets us make some considerations about the effect on the benchmark PS of the EV patient LOS forecast that introduces a discrepancy between the predicted patient LOS and the simulated (realized) one.

Table A.1 shows that a discrepancy between the predicted patient LOS and the simulated one has a very limited (even null) impact on the percentage of refused emergencies and the percentage of overdue electives. On the contrary, some emergencies timeout and the rescheduling of the elective patients arise. The percentage of timed out emergencies remains 0 for the normal/track flow of emergencies and it grows significantly for flows of emergencies greater than 2.0 times the normal/track flow. The percentage of elective rescheduling is limited even for flows of emergencies up to 4.0 times the normal/tracked flow.

With the simulation outcomes reported in Table A.1, we can assert that the benchmark PS EORS-FBPS-BOL-with-FBPS-BOL is quite robust with respect to the quality of the used patient LOS forecast. Even if there is a discrepancy between the predicted patient LOS and the simulated one, there is not a large perturbation of the KPIs.

B.S.	Emergency flow											
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0	
	(%) Refused emergencies											
96%-4%	71, 72	75, 76	78, 79	81, 82	85, 85	87, 88	88, 88	90, 90	90, 91	91, 92	92, 92	
92%-8%	40, 42	49, 50	55, 57	61, 62	68, 69	73, 74	75, 76	78, 79	80, 80	82, 83	84, 84	
88%-12%	19, 21	28, 30	37, 39	44, 46	54, 56	62, 63	64, 65	69, 70	71, 72	74, 75	77, 78	
84%-16%	4, 5	11, 12	19, 20	26, 28	39, 41	49, 50	52, 53	59, 60	61, 62	65, 66	69, 69	
80%-20%	1, 1	3, 4	7, 8	14, 16	26, 28	37, 39	43, 44	49, 50	52, 53	57, 58	62, 63	
76%-24%	0, 0	0, 1	2, 3	5, 6	16, 17	26, 28	31, 33	40, 41	43, 44	49, 50	54, 55	
72%-28%	0, 0	0, 0	1, 1	2, 3	9, 10	19, 21	24, 25	31, 33	36, 37	42, 43	48, 49	
68%-32%	0, 0	0, 0	0, 0	1, 1	5, 6	12, 13	16, 18	24, 25	28, 29	34, 36	40, 41	
64%-36%	0, 0	0, 0	0, 0	0, 0	2, 3	7, 8	10, 11	17, 18	21, 22	28, 30	34, 35	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 1	3, 3	5, 6	11, 13	14, 15	22, 23	28, 29	
	(%) Timedout emergencies											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
80%-20%	0, 0	0, 1	1, 1	1, 2	1, 2	1, 2	2, 2	1, 2	1, 1	1, 2	1, 2	
76%-24%	0, 0	0, 1	1, 2	2, 2	3, 4	4, 5	4, 4	4, 6	4, 5	5, 6	4, 5	
72%-28%	0, 0	0, 0	0, 1	2, 2	4, 5	7, 8	8, 9	8, 9	8, 10	9, 10	9, 10	
68%-32%	0, 0	0, 0	0, 0	1, 1	4, 5	8, 9	10, 11	11, 12	12, 14	12, 13	12, 13	
64%-36%	0, 0	0, 0	0, 0	0, 0	2, 2	5, 6	7, 8	10, 11	11, 12	13, 14	13, 14	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 3	4, 5	8, 9	9, 10	11, 12	13, 14	
	(%) Elective rescheduling											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	0, 1	
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 2	1, 2	2, 2	2, 3	2, 3	
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	2, 2	2, 2	3, 3	3, 3	3, 3	
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 2	2, 2	2, 3	3, 3	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	2, 2	2, 3	
	(%) Overdue electives											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	0, 1	1, 1	1, 1	
68%-32%	1, 1	2, 2	1, 1	1, 1	1, 2	2, 2	2, 3	2, 3	4, 4	3, 3	4, 4	
64%-36%	5, 6	5, 5	5, 6	5, 5	5, 5	6, 6	5, 6	6, 6	6, 7	7, 7	8, 8	
60%-40%	9, 10	9, 10	8, 9	9, 10	9, 10	10, 10	10, 11	10, 11	10, 11	10, 11	10, 11	

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.1: Control-table of PS EORS-FBPS-BOL-with-FBPS-BOL for the four KPIs (95% CI), MSS OT management policy and the EV patient LOS forecast

A.1.1 Percentage of refused emergencies

In this section, we discuss the impact of the patient LOS forecast and the type of bed availability on the percentage of refused emergencies.

In Table A.2, we report the 95% CI of the average weekly percentage of refused emergencies for PS EORS-FBAD-BOL-with-FBAD-BOL and both the patient LOS forecasts (PI and EV). Recall that, in PS EORS-FBAD-BOL-with-FBAD-BOL, with respect to PS EORS-FBPS-BOL-with-FBPS-BOL, for both the scheduling of elective patients and the admission of emergency patients, the type of bed availability is relaxed from FBPS to FBAD, for scheduling an elective patient or admitting an emergency patient, the availability of a bed is considered only for the admission day.

By comparing the simulation outcomes reported in Table A.2 for the percentage of refused

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBAD-BOL with FBAD-BOL, PI patient LOS forecast											
96%-4%	71, 72	75, 77	79, 80	82, 83	85, 86	88, 88	89, 89	90, 91	91, 91	92, 92	93, 93
92%-8%	40, 42	49, 51	56, 58	61, 63	68, 69	74, 75	76, 77	79, 80	80, 81	83, 83	85, 85
88%-12%	19, 21	29, 31	37, 39	44, 46	55, 56	62, 63	65, 66	70, 70	72, 72	74, 75	77, 78
84%-16%	4, 5	11, 12	19, 21	27, 28	39, 41	49, 50	52, 54	59, 60	61, 62	65, 66	69, 70
80%-20%	0, 1	2, 3	8, 9	14, 16	28, 29	37, 39	42, 43	50, 51	52, 53	57, 58	62, 63
76%-24%	0, 0	0, 0	1, 2	5, 6	15, 17	26, 27	31, 32	39, 40	43, 44	49, 50	53, 54
72%-28%	0, 0	0, 0	0, 0	1, 2	7, 8	17, 18	22, 24	31, 32	35, 36	42, 43	47, 48
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 3	9, 10	14, 15	22, 24	26, 28	34, 35	40, 41
64%-36%	0, 0	0, 0	0, 0	0, 0	1, 1	5, 6	8, 9	15, 17	19, 20	27, 28	33, 34
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	2, 3	3, 4	10, 11	13, 14	20, 22	27, 29
EORS-FBAD-BOL with FBAD-BOL, EV patient LOS forecast											
96%-4%	71, 73	76, 77	79, 80	82, 82	85, 86	88, 88	89, 89	90, 91	91, 91	92, 92	93, 93
92%-8%	40, 42	49, 50	56, 57	61, 62	68, 69	74, 75	76, 76	79, 80	80, 81	83, 83	84, 85
88%-12%	18, 20	29, 31	38, 40	45, 46	55, 56	62, 63	65, 66	69, 70	71, 72	75, 76	77, 78
84%-16%	4, 5	10, 12	19, 21	26, 28	39, 41	49, 50	52, 54	59, 60	61, 62	65, 66	69, 70
80%-20%	1, 1	3, 3	8, 9	15, 16	27, 28	38, 39	42, 43	50, 51	52, 54	58, 59	62, 63
76%-24%	0, 0	0, 1	2, 3	5, 6	16, 18	26, 28	31, 32	40, 41	43, 44	49, 50	54, 55
72%-28%	0, 0	0, 0	1, 1	2, 3	8, 10	19, 20	24, 25	32, 33	36, 37	42, 43	47, 48
68%-32%	0, 0	0, 0	0, 0	1, 2	5, 6	14, 15	16, 18	25, 26	28, 30	34, 36	41, 42
64%-36%	0, 0	0, 0	0, 0	0, 1	3, 4	9, 10	12, 13	19, 20	23, 24	29, 30	35, 36
60%-40%	0, 0	0, 0	0, 0	0, 0	1, 1	5, 6	7, 8	13, 15	17, 18	23, 24	29, 30

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.2: Control-table of PS EORS-FBAD-BOL with FBAD-BOL for the average percentage of refused emergencies (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

emergencies with those of Table 2.12 (outcomes for the benchmark PS and the patient LOS forecast PI), *i.e.*, comparing outcomes of PS EORS-FBPS-BOL-with-FBPS-BOL with those of PS EORS-FBAD-BOL-with-FBAD-BOL, we can argue that the impact on the KPI of considering the bed availability type FBAD instead of the type FBPS for both the scheduling of elective patients and the admission of emergency patients is very limited, as well as using the patient LOS forecast EV. We do not consider necessary to investigate what single element contributes the most.

A.1.2 Percentage of timedout emergencies

In this section, we discuss the impact on the percentage of timed out emergencies of the patient LOS forecast (EV or PI) and the type of bed availability (FBAD or FBPS).

In Table A.3, we report the 95% CI of the average weekly percentage of timed out emergencies for PS EORS-FBAD-BOL-with-FBAD-BOL and for both the patient LOS forecasts (PI and EV).

Table A.3 reveals that there are significant differences for the simulation outcomes of PS EORS-FBPS-BOL-with-FBPS-BOL compared to those of PS EORS-FBAD-BOL-with-FBAD-BOL, this is for both the patient LOS forecasts (PI and EV). The contribution of each elements is analysed with Table A.4.

From the outcomes in Table A.4, it emerges that the patient LOS forecast and the type of bed availability considered for scheduling elective patients have a major impact on the percentage of timed out emergencies than those considered for the admission of emergency patients. With PS EORS-FBAD-BOL-with-FBPS-BOL, elective patients are scheduled by evaluating the BOL only

B.S.	Emergency flow											
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0	
EORS-FBAD-BOL with FBAD-BOL, PI patient LOS forecast												
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	0, 0	0, 1	0, 0	0, 1	0, 1
72%-28%	0, 0	0, 0	0, 0	0, 0	1, 2	1, 2	3, 3	3, 4	2, 2	3, 3	3, 3	3, 4
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 2	4, 5	5, 6	7, 8	8, 9	8, 9	8, 9	8, 9
64%-36%	0, 0	0, 0	0, 0	0, 0	1, 1	3, 4	4, 5	7, 8	8, 9	9, 10	11, 12	11, 12
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 2	2, 3	5, 6	6, 7	8, 9	10, 11	10, 11
EORS-FBAD-BOL with FBAD-BOL, EV patient LOS forecast												
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84%-16%	0, 1	0, 0	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1
80%-20%	0, 0	0, 1	1, 1	1, 2	1, 2	1, 2	1, 2	2, 2	2, 2	1, 2	1, 2	1, 2
76%-24%	0, 0	0, 1	1, 1	2, 2	3, 4	4, 5	4, 5	5, 6	5, 6	5, 6	5, 6	4, 5
72%-28%	0, 0	0, 0	1, 1	2, 2	4, 5	7, 8	8, 9	9, 11	10, 11	10, 11	10, 11	9, 11
68%-32%	0, 0	0, 0	0, 0	1, 2	5, 6	10, 12	10, 12	13, 15	14, 15	15, 16	16, 17	16, 17
64%-36%	0, 0	0, 0	0, 0	0, 1	3, 4	8, 9	10, 11	14, 15	15, 16	16, 17	17, 18	17, 18
60%-40%	0, 0	0, 0	0, 0	0, 0	1, 1	4, 5	7, 8	11, 12	13, 15	16, 17	17, 19	17, 19

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.3: Control-table of PS EORS-FBAD-BOL with FBAD-BOL for the average percentage of timedout emergencies (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

for the patient admission date (availability type FBAD), for dates next to the patient admission date, the BOL can be overstepped systematically and the timeout of emergencies occur. Such loose enforcement of the BOL due to a myopic evaluation of the availability of beds provides an explanation for the timeout of emergencies, even in the case of the PI patient LOS forecast. Given the same patient LOS forecast, the type of bed availability used for scheduling elective patients has a greater impact on the KPI with respect to that used for admitting emergency patients.

The bed availability type considered for admitting emergency patients seems to have an impact on the KPI only in the case the predicted patient LOS introduces a discrepancy with the simulated patient LOS (*i.e.*, the forecast EV is used), the impact is relevant in this case.

We can conclude that a precise forecast of the patient LOS affecting the bed occupancy prediction is important to avoid as much as possible the timeout of emergencies and the type of bed availability used for scheduling elective patients is more important than that for admitting emergencies. FBPS is preferable to FBAD.

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBPS-BOL with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
EORS-FBPS-BOL with FBAD-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84%-16%	0, 0	0, 1	0, 1	1, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1
80%-20%	0, 0	0, 1	1, 1	1, 2	1, 2	1, 2	1, 2	1, 2	2, 2	1, 2	1, 2
76%-24%	0, 0	0, 0	1, 1	2, 2	4, 5	4, 5	4, 5	4, 5	4, 5	4, 5	4, 5
72%-28%	0, 0	0, 0	1, 1	1, 2	4, 5	7, 8	8, 10	9, 10	8, 10	9, 10	8, 9
68%-32%	0, 0	0, 0	0, 1	1, 1	4, 5	8, 9	10, 11	11, 12	12, 13	12, 13	12, 13
64%-36%	0, 0	0, 0	0, 0	0, 0	2, 2	5, 6	8, 9	10, 11	12, 13	12, 14	13, 15
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 3	4, 5	8, 9	9, 10	12, 13	12, 13
EORS-FBAD-BOL with FBPS-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	0, 0	0, 1	0, 1	0, 1	0, 1
72%-28%	0, 0	0, 0	0, 0	0, 0	1, 1	1, 2	2, 2	3, 4	2, 3	3, 4	3, 3
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 2	4, 5	5, 6	6, 7	7, 8	9, 10	8, 9
64%-36%	0, 0	0, 0	0, 0	0, 0	1, 1	3, 4	4, 5	7, 8	8, 9	10, 11	10, 11
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 3	5, 6	6, 7	8, 9	10, 11
EORS-FBAD-BOL with FBPS-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84%-16%	0, 0	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	1, 1	0, 1	0, 1
80%-20%	0, 0	0, 1	1, 1	1, 1	1, 2	1, 2	2, 2	1, 2	1, 2	1, 2	1, 2
76%-24%	0, 0	0, 0	1, 1	1, 2	3, 4	4, 5	4, 5	4, 5	4, 5	4, 5	4, 5
72%-28%	0, 0	0, 0	1, 1	2, 2	4, 5	6, 7	8, 10	8, 10	10, 11	9, 10	9, 10
68%-32%	0, 0	0, 0	0, 1	1, 2	4, 5	10, 11	11, 13	13, 14	14, 15	15, 16	15, 16
64%-36%	0, 0	0, 0	0, 0	0, 1	3, 4	8, 9	10, 11	14, 15	15, 16	17, 18	17, 18
60%-40%	0, 0	0, 0	0, 0	0, 0	1, 1	4, 5	7, 8	11, 12	13, 14	16, 17	17, 18

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.
Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.4: Control table of PS EORS-FBPS-BOL-with-FBAD-BOL and PS EORS-FBAD-BOL-with-FBPS-BOL for the average percentage of timedout emergencies (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

A.1.3 Percentage of elective rescheduling

In this section, we discuss the impact of the patient LOS forecasts and bed availability types on the percentage of elective rescheduling.

In Table A.5, we report the 95% CI of the average weekly percentage of elective rescheduling for PS EORS-FBAD-BOL-with-FBAD-BOL and both the patient LOS forecasts (PI and EV).

Table A.5 suggests that the type of bed availability for scheduling elective patients and admitting emergencies, together with the patient LOS forecast EV, may contribute relevantly to increase the KPI values. The contribution of each element, patient LOS forecast and bed availability type, is analysed with Table A.6.

According to values in Table A.6, the type of bed availability considered for scheduling elective patients contributes to increase the percentage of elective rescheduling more than that

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBAD-BOL with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	1, 1	1, 2
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 1	1, 1
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1
EORS-FBAD-BOL with FBAD-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	0, 1	0, 1
76%-24%	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 1	1, 2	1, 2	1, 2	1, 2
72%-28%	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2	2, 3	3, 3	3, 4	4, 5	4, 5
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 1	3, 4	3, 3	4, 5	6, 7	6, 7	8, 9
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 2	2, 3	4, 5	5, 6	7, 8	8, 9
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2	3, 4	4, 5	6, 7	7, 8

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.5: Control table of PS EORS-FBAD-BOL-with-FBAD-BOL for the average percentage of elective rescheduling (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

one considered for admitting emergency patients. Such a contribution is largely amplified by the use of the EV patient LOS forecast. Elective rescheduling increases relevantly for flows of emergencies greater than 2.0 times the normal/tracked flow.

We can conclude that a reliable forecast of the patient LOS is important to limit the rescheduling of elective patients and the type of bed availability FBPS is preferable to FBAD. The type of bed availability considered for the scheduling of elective patients is more relevant than that considered for the admission of emergencies.

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBPS-BOL with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
EORS-FBPS-BOL with FBAD-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	0, 1	0, 1
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 1	0, 1	1, 1	1, 1	1, 1	1, 1	1, 2
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 2	2, 2	2, 2	2, 3	2, 2
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	2, 2	2, 2	2, 3	3, 3	3, 4
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 2	2, 2	2, 3	3, 3
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	2, 2	2, 3
EORS-FBAD-BOL with FBPS-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	1, 1	1, 1
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 1	1, 1
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1
EORS-FBAD-BOL with FBPS-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	0, 0	0, 1	0, 1	0, 1
76%-24%	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 1	1, 1	1, 2	1, 2	1, 2
72%-28%	0, 0	0, 0	0, 0	0, 0	1, 1	1, 2	2, 2	3, 3	3, 4	4, 5	4, 5
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 1	3, 3	3, 4	5, 6	6, 7	7, 8	8, 9
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 2	3, 3	5, 5	6, 6	7, 8	8, 8
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2	4, 4	4, 5	5, 6	7, 8

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.6: Control table of PS EORS-FBPS-BOL-with-FBAD-BOL and PS EORS-FBAD-BOL-with-FBPS-BOL for the average percentage of elective rescheduling (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

A.1.4 Percentage of overdue electives

In this section, we discuss the impact of patient LOS forecasts and types of bed availability on the percentage of overdue electives.

In Table A.5, we report the 95% CI of the average weekly percentage of overdue electives for PS EORS-FBAD-BOL-with-FBAD-BOL and both the patient LOS forecasts (PI and EV).

Table A.7 shows that both the patient LOS forecast and the type of bed availability have a small impact on the percentage of overdue electives. It seems that the EV forecast may improve a little bit the simulation outcomes in terms of overdue electives, it may be that a lesser quality forecast of the patient LOS can advantage the scheduling of elective patients by allowing to schedule patients earlier. As differences with respect to the benchmark PS EORS-FBPS-BOL-with-FBPS-BOL of PS EORS-FBAD-BOL-with-FBAD-BOL in terms of overdue electives are narrow, we do not investigate further which element contributes the most to such a difference.

We can essentially conclude that performances in terms of overdue electives do not depend on the type of bed availability used for scheduling elective patients and/or admitting emergency patients. The scenario factor that seems to influence the most this KPI is the selected BOL-

B.S.	Emergency flow											
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0	
EORS-FBAD-BOL with FBAD-BOL, PI patient LOS forecast												
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
72%-28%	1, 1	0, 1	0, 1	1, 1	1, 1	1, 1	1, 1	1, 1	0, 1	1, 1	1, 1	1, 1
68%-32%	3, 4	4, 5	4, 4	3, 4	3, 3	4, 5	4, 4	5, 5	4, 5	4, 5	5, 5	5, 5
64%-36%	9, 10	8, 9	8, 9	8, 9	8, 9	9, 10	8, 8	8, 9	9, 9	8, 9	9, 10	9, 10
60%-40%	13, 14	14, 15	13, 14	13, 15	13, 14	13, 15	14, 15	13, 15	13, 15	14, 15	13, 15	13, 15
EORS-FBAD-BOL with FBAD-BOL, EV patient LOS forecast												
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
72%-28%	0, 0	0, 0	0, 1	0, 0	0, 0	0, 0	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1
68%-32%	1, 1	1, 1	1, 1	1, 1	1, 1	2, 2	1, 1	2, 3	3, 3	3, 4	4, 5	4, 5
64%-36%	5, 5	4, 4	4, 4	3, 4	4, 5	5, 5	5, 5	5, 6	6, 6	7, 8	7, 8	7, 8
60%-40%	9, 10	9, 10	8, 9	8, 9	9, 10	9, 10	10, 11	10, 11	10, 11	12, 13	11, 12	11, 12

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.7: Control table of PS EORS-FBAD-BOL-with-FBAD-BOL for the average percentage of overdue electives (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

setting.

A.2 With vs. Without BOL

In this section, we analyse the impact on the four KPIs of enforcing or not the BOL for elective patients and/or for emergency patients, and the control on the KPIs provided by the BOL-settings. A section is devoted to each KPI.

In Table A.8, we report the control-tables of the four KPIs for four PSs that consider only bed management and do not enforce any BOL, either for elective patients or emergency patients. For each PS, outcomes for the two types of patient LOS forecast (PI and EV) are reported. Table A.8 is reported as a reference for observing what happens if only bed management is applied. Control-tables of PSs EORS-FBPS-with-FBAD, EORS-FBAD-with-FBPS, EORS-with-FBAD and EORS-FBAD-with-UW are not reported as these PSs provides control-tables very similar to those reported in Table A.8.

PS EORS-FBPS-with-FBPS is like PS EORS-FBPS-BOL-with-FBPS-BOL, but without the BOL, either for electives or emergencies. Such a PS provides simulation outcomes that are close to those of PS EORS-FBPS-BOL-with-FBPS-BOL for BOL-settings between 80%-20% and 75%-25%, independently of the patient LOS forecast (PI or EV). Despite this fact, the percentage of refused emergencies is zeroed with PS EORS-FBPS-BOL-with-FBPS-BOL for several values of the flow of emergencies by selecting the right BOL-setting, PS EORS-FBPS-with-FBPS is thus dominated by PS EORS-FBPS-BOL-with-FBPS-BOL for this flows of emergencies.

If the type of bed availability for both the patients type is FBAD, *i.e.*, PS EORS-FBAD-with-FBAD, performance are very close to those of PS EORS-FBPS-with-FBPS, except that some elective rescheduling arises even in the case the patient LOS forecast is PI.

A.2.1 Percentage of refused emergencies

The analysis of the simulation outcomes reported in Section A.1 about the impact on the four KPIs of the patient LOS forecast and the type of bed availability considered for scheduling elective patients and admitting emergency patients reveals that both the elements have a very limited impact on the percentage of refused emergencies. In this section, we investigate the impact on the percentage of refused emergencies of enforcing the BOL for elective patients and/or for emergency patients.

Table A.9 reports the control-table of refused emergencies for PS EORS-FBPS-BOL-with-FBAD and PS EORS-FBPS-BOL-with-UW, control-tables are for both the patient LOS forecasts PI and EV. Table A.9 reports the 95% CI for the average percentage of emergencies refused weekly. The table allows us evaluating the impact on the KPI of removing the BOL for emergency patients, but preserving bed management for admitting emergency patients, PS EORS-FBPS-BOL-with-FBAD, and the impact of removing also bed management, PS EORS-FBPS-BOL-with-UW. We do not report the control-tables for PSs that do not consider ant BOL or bed management for scheduling elective patients as these PSs have performances in terms of refused emergencies very close to those of the benchmark PS EORS-FBPS-BOL-with-FBPS-BOL.

Figure A.1 reports the control-charts of the percentage of refused emergencies for PS EORS-FBPS-BOL-with-FBAD and PS EORS-FBPS-BOL-with-UW, and the patient LOS forecast PI; subfigure (a) subfigure and (b) respectively. The two control-charts are the graphical version of the related of control-tables.

With Figure A.1, we can visualize that, if the BOL is enforced only for elective patients, that is the case of both the PSs EORS-FBPS-BOL-with-FBAD and EORS-FBPS-BOL-with-UW, many BOL-settings provide very close performances for both the PSs, *i.e.*, curves of BOL-settings from 96%-4% to 72%-28% are almost completely overlapped. For BOL-settings with a tighter BOL for elective patients, from 72%-28% to 60%-40%, the curve is shifted to the right, the more the BOL is tighter for elective patients, the more the system is able to admit a higher percentage of emergency patients.

KPI	Emergency flow											
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0	
EORS-FBPS with UW, PI patient LOS forecast												
(%) Refused em.	0, 1	1, 1	2, 3	5, 6	9, 11	15, 17	19, 21	24, 25	26, 27	29, 30	31, 31	
(%) Timedout em.	0, 1	1, 1	2, 3	5, 6	9, 11	15, 17	19, 21	24, 25	26, 27	29, 30	31, 31	
(%) El. resched.	0, 0	0, 0	0, 0	1, 1	1, 2	3, 4	5, 5	8, 8	10, 10	13, 14	18, 18	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	2, 2	3, 3	4, 5	7, 8	
EORS-FBPS with UW, EV patient LOS forecast												
(%) Refused em.	0, 1	1, 2	2, 3	4, 5	8, 9	16, 17	19, 20	24, 25	25, 26	28, 29	30, 31	
(%) Timedout em.	0, 1	1, 2	2, 3	4, 5	8, 9	16, 17	19, 20	24, 25	25, 26	28, 29	30, 31	
(%) El. resched.	0, 0	0, 0	0, 1	1, 1	1, 2	4, 5	6, 7	10, 11	11, 12	16, 17	19, 20	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 2	3, 4	5, 6	8, 9	
EORS with FBPS, PI patient LOS forecast												
(%) Refused em.	1, 2	2, 3	4, 6	7, 9	13, 15	21, 24	23, 26	32, 34	35, 37	40, 42	45, 47	
(%) Timedout em.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
(%) El. resched.	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	1, 2	2, 2	2, 3	3, 3	4, 4	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
EORS with FBPS, EV patient LOS forecast												
(%) Refused em.	1, 2	2, 3	4, 5	7, 9	13, 15	21, 23	25, 27	30, 33	35, 38	39, 41	45, 47	
(%) Timedout em.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
(%) El. resched.	0, 0	0, 0	0, 1	0, 1	1, 1	2, 2	2, 2	2, 3	3, 3	3, 4	4, 5	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	
EORS-FBPS with FBPS, PI patient LOS forecast												
(%) Refused em.	1, 2	2, 4	4, 5	7, 9	13, 16	21, 24	25, 28	32, 34	34, 36	40, 42	45, 47	
(%) Timedout em.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
(%) El. resched.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
EORS-FBPS with FBPS, EV patient LOS forecast												
(%) Refused em.	1, 2	2, 3	4, 6	7, 9	12, 14	20, 23	23, 25	33, 35	33, 36	39, 41	45, 47	
(%) Timedout em.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
(%) El. resched.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	1, 1	1, 1	1, 1	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
EORS-FBAD with FBAD, PI patient LOS forecast												
(%) Refused em.	1, 2	2, 3	5, 7	7, 9	13, 15	20, 22	25, 27	32, 34	36, 38	39, 42	45, 47	
(%) Timedout em.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
(%) El. resched.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	1, 1	1, 1	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
EORS-FBAD with FBAD, EV patient LOS forecast												
(%) Refused em.	1, 2	2, 3	4, 5	6, 8	13, 15	19, 22	24, 27	31, 34	34, 36	39, 41	44, 46	
(%) Timedout em.	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	
(%) El. resched.	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	1, 1	1, 2	2, 2	2, 2	
(%) Overdue el.s	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.8: Control-tables for the four KPIs (95% CI) of some representative PSs that consider only bed management without enforcing the BOL; the OT management policy is MSS and both the PI and the EV patient LOS forecasts are reported

We can conclude that, for the percentage of refused emergencies, emergency admission policies that do not enforce the BOL for emergency patients are best effort policies for the admission of emergencies, and those that enforce the BOL are more conservative for the available bed capacity that is shared with elective patients. More conservative PSs, those like EORS-FBPS-BOL-with-FBPS-BOL or EORS-FBPS-BOL-with-FBAD-BOL that enforce the BOL for both elective and emergency patients, are suitable for operational situations such that an emergency flow is predicted to be largely greater than the number of emergency patients that the hospital is available to admit. The less conservative PSs, those like EORS-FBPS-BOL-with-FBPS,

B.S.	Emergency flow											
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0	
EORS-FBPS-BOL with FBAD, PI patient LOS forecast												
96%-4%	1, 2	2, 3	3, 5	7, 8	13, 16	20, 23	24, 26	31, 34	34, 36	40, 42	44, 46	
92%-8%	1, 2	2, 3	5, 6	8, 10	13, 15	20, 22	25, 27	33, 35	35, 37	39, 41	44, 46	
88%-12%	1, 1	2, 3	4, 5	6, 8	13, 15	20, 23	24, 27	30, 32	34, 36	38, 40	44, 46	
84%-16%	0, 1	1, 2	3, 4	6, 8	14, 16	19, 21	25, 27	31, 34	34, 36	40, 42	44, 46	
80%-20%	0, 0	0, 1	2, 3	4, 6	12, 13	19, 21	24, 26	30, 32	33, 35	39, 41	44, 46	
76%-24%	0, 0	0, 0	1, 1	2, 3	9, 11	19, 20	23, 24	29, 31	33, 34	38, 40	42, 44	
72%-28%	0, 0	0, 0	0, 0	0, 1	6, 7	14, 16	18, 20	26, 27	30, 31	35, 37	41, 42	
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 2	7, 8	11, 12	19, 21	23, 24	30, 31	36, 37	
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	3, 4	5, 6	13, 15	17, 19	24, 26	30, 31	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 3	8, 9	11, 12	18, 20	24, 26	
EORS-FBPS-BOL with FBAD, EV patient LOS forecast												
96%-4%	1, 2	2, 3	4, 6	6, 8	14, 16	20, 22	25, 27	31, 33	35, 37	40, 42	43, 45	
92%-8%	1, 2	2, 3	4, 5	7, 8	13, 16	21, 23	26, 28	31, 33	33, 35	40, 42	44, 46	
88%-12%	1, 2	2, 3	3, 5	6, 8	13, 16	20, 23	24, 26	32, 35	33, 35	39, 41	44, 46	
84%-16%	1, 1	1, 2	4, 5	6, 8	13, 15	21, 23	25, 27	31, 33	34, 36	40, 42	44, 46	
80%-20%	0, 1	1, 2	3, 4	5, 7	13, 16	20, 23	25, 27	30, 33	33, 35	39, 41	43, 45	
76%-24%	0, 0	1, 1	1, 2	4, 6	11, 13	18, 20	24, 26	31, 33	34, 35	37, 39	44, 45	
72%-28%	0, 0	0, 1	1, 2	2, 3	9, 11	19, 21	22, 24	28, 30	33, 35	38, 40	43, 44	
68%-32%	0, 0	0, 0	0, 1	1, 2	7, 8	15, 16	19, 21	26, 27	29, 31	35, 36	41, 42	
64%-36%	0, 0	0, 0	0, 0	0, 1	3, 4	8, 10	13, 15	20, 22	24, 26	30, 32	36, 38	
60%-40%	0, 0	0, 0	0, 0	0, 0	1, 1	4, 6	7, 8	14, 16	17, 19	24, 25	30, 32	
EORS-FBPS-BOL with UW, PI patient LOS forecast												
96%-4%	0, 1	1, 2	2, 3	4, 5	10, 12	16, 17	18, 19	23, 24	25, 26	27, 27	29, 30	
92%-8%	1, 1	1, 2	2, 3	4, 5	9, 11	16, 18	18, 20	23, 24	24, 25	27, 28	30, 30	
88%-12%	0, 1	1, 2	2, 3	4, 5	9, 11	15, 17	18, 19	23, 24	24, 25	27, 28	30, 31	
84%-16%	0, 0	1, 1	2, 2	4, 5	9, 11	17, 18	19, 21	23, 25	25, 26	28, 29	30, 31	
80%-20%	0, 0	0, 0	1, 1	3, 4	8, 9	16, 17	19, 20	23, 24	26, 26	28, 29	30, 31	
76%-24%	0, 0	0, 0	0, 0	1, 2	7, 8	15, 16	18, 19	24, 25	26, 27	29, 30	31, 32	
72%-28%	0, 0	0, 0	0, 0	0, 1	4, 5	11, 12	14, 16	21, 22	24, 25	28, 29	31, 32	
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 1	5, 6	8, 9	16, 18	19, 20	25, 26	29, 30	
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	2, 2	5, 5	10, 11	13, 15	20, 21	25, 26	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 2	6, 7	9, 10	15, 16	21, 22	
EORS-FBPS-BOL with UW, EV patient LOS forecast												
96%-4%	0, 1	1, 2	3, 4	3, 4	9, 11	16, 17	18, 20	22, 23	24, 25	27, 28	29, 30	
92%-8%	0, 1	1, 2	3, 4	4, 5	9, 11	15, 17	19, 20	23, 24	25, 25	27, 27	29, 30	
88%-12%	1, 1	1, 1	2, 3	4, 5	8, 10	16, 17	18, 19	24, 25	24, 26	27, 28	29, 30	
84%-16%	0, 1	1, 1	2, 3	4, 5	9, 10	16, 17	18, 20	23, 24	25, 26	28, 29	30, 31	
80%-20%	0, 0	1, 1	2, 2	4, 5	9, 10	14, 16	19, 20	24, 25	26, 27	29, 30	30, 31	
76%-24%	0, 0	0, 1	1, 2	3, 4	8, 9	15, 16	18, 20	24, 25	26, 27	28, 29	30, 31	
72%-28%	0, 0	0, 0	0, 1	1, 2	7, 8	13, 15	16, 17	22, 24	23, 24	27, 28	29, 30	
68%-32%	0, 0	0, 0	0, 0	1, 1	4, 5	10, 11	13, 14	19, 20	21, 22	25, 26	28, 29	
64%-36%	0, 0	0, 0	0, 0	0, 0	2, 2	6, 7	9, 10	14, 16	17, 18	22, 23	25, 26	
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 3	5, 6	10, 11	13, 14	18, 19	23, 24	

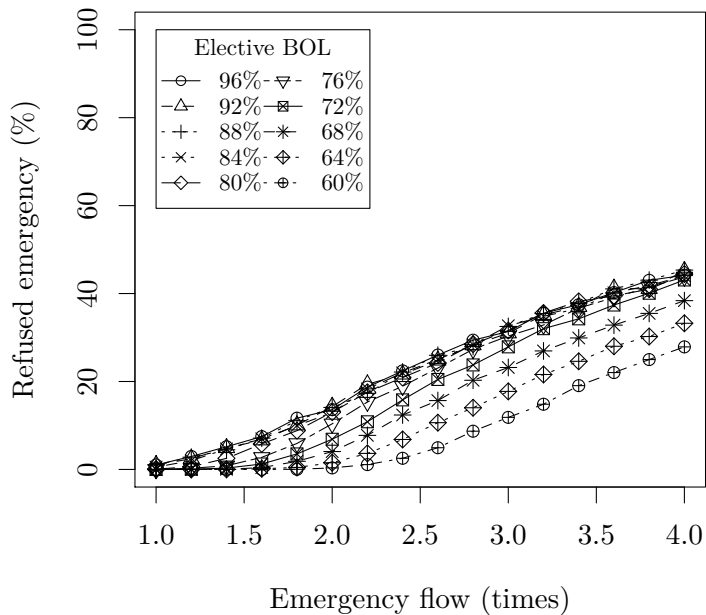
In the table, the two values of the CI are reported separated by the comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.9: Control table of PS EORS-FBPS-BOL-with-FBAD and PS EORS-FBAD-BOL-with-UW for the average percentage of refused emergencies (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

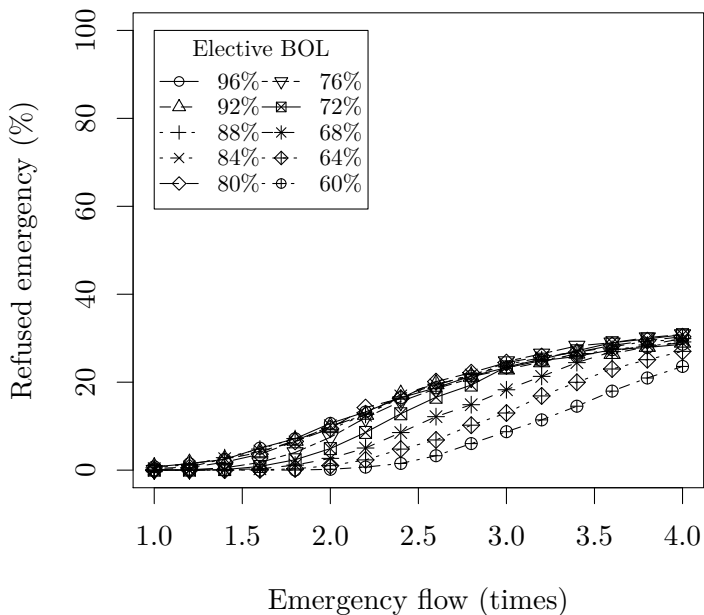
EORS-FBPS-BOL-with-FBAD or EORS-FBPS-BOL-with-UW, are suitable for situations for which a very large flow of emergencies is unlikely. PSs that enforce the BOL only for emergency patients are not of special interest as provide performances in terms of refused emergencies close to PSs that enforce the BOL for both the patient types, but the impact on the other KPIs of enforcing the BOL only for emergency patients is frequently bad.

EORS-FBAD-BOL with FBAD



(a) Control chart of PS EORS-FBPS-BOL with FBAD

EORS-FBAD-BOL with UW



(b) Control chart of PS EORS-FBPS-BOL with UW

Figure A.1: Percentage of refused emergencies, control chart for PS EORS-FBPS-BOL-with-FBAD, subfigure (a), and PS EORS-FBPS-BOL-with-UW, subfigure (b); MSS OT management policy and PI patient LOS forecast

A.2.2 Percentage of timedout emergencies

In this section, we investigate the impact on the percentage of timed out emergencies of enforcing (or not) the BOL for elective patients and/or for emergency patients.

As we have already argued in Section A.1, some emergencies are timed out because of the following reasons: (i.) there is no bed management for the admission of emergency patients, in this case, the percentage of timed out emergencies equals the percentage of refused emergencies (*i.e.*, every refused emergency is refused because of timeout), (ii.) elective patients are scheduled considering a myopic evaluation of bed availability, FBAD (only on the admission date) instead of FBPS (all along the patient stay), (iii.) the patient LOS forecast introduces an error between the predicted bed occupancy and the real/simulated one. In this section, we discuss the contribution provided by enforcing or not the BOL.

In Table A.10, the control-table for PS EORS-FBAD-BOL-with-FBAD, PS EORS-FBAD with FBAD-BOL and both the patient LOS forecasts (PI and EV) is reported. The two PSs EORS-FBAD-BOL-with-FBAD and PS EORS-FBAD with FBAD-BOL are sufficient to analyse the impact of the BOL on the KPI. Any other PS provides results comparable to one of the two mentioned PSs (or other PSs discussed in the previous sections, *i.e.*, Section 2.7.2 or Section A.1). PS EORS-FBAD-BOL-with-FBAD is as PS EORS-FBAD-BOL-with-FBAD-BOL already discussed in Section A.1, but without the BOL for the admission of emergency patients, and PS EORS-FBAD-with-FBAD-BOL is as PS EORS-FBAD-BOL-with-FBAD-BOL, but without the BOL for scheduling elective patients.

Simulation outcomes reported in Table A.10 reveal that removing the BOL and preserving bed management for emergency patients (PS EORS-FBAD-BOL-with-FBAD) is effective to zero the percentage of timed out emergencies; this is independent of the patient LOS forecast (PI or EV). On the contrary, removing the BOL for the scheduling of elective patients is quite disruptive, the percentage of timed out emergencies increases largely and is present even for a moderated flow of emergencies. Figure A.2 reports the control-chart of PS EORS-FBAD-with-FBAD-BOL for the percentage of timed out emergencies and PI patient LOS forecast.

Simulation outcomes for PS EORS-FBAD-with-FBAD-BOL reported in Table A.10 and visualized with the control-chart of Figure A.2 reveal that, the more the enforced BOL is loose for emergencies, the more the percentage of timed out emergencies increases. This happens because, the more the bed occupancy of emergency patients is let to expand, the more it is higher the probability that the bed occupancy of elective patients (that is not bounded by a BOL) oversteps the bed capacity implicitly reserved for elective patients by the BOL for emergency patients. Such phenomenon increases and becomes evident as much as the flow of emergencies increases.

We can conclude that enforcing the BOL for elective patients and removing it for emergency patients may be a solution to consider in order to limit as much as possible the number of timed out emergencies.

A.2.3 Percentage of elective rescheduling

In this section, we investigate the impact on the percentage of elective rescheduling of enforcing (or not) the BOL for elective patients and/or for emergency patients.

In Table A.11, we report the control-tables of PS EORS-FBAD-BOL-with-UW and PS EORS-with-FBAD-BOL, the first one is representative of PS enforcing the BOL only for elective patients and the second one of PSs enforcing the BOL only for emergency patients. For emergency patients, we consider the admission policy UW as representative for policies that not enforce the BOL, even if this policy does not consider at all bed management for the admission of emergency patients. With respect to elective rescheduling, PS EORS-FBAD-BOL-with-UW provides results comparable, but amplified, to EORS-FBAD-BOL-with-FBAD, a PS that pre-

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBAD-BOL with FBAD, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
EORS-FBAD-BOL with FBAD, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
EORS-FBAD with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84%-16%	0, 1	0, 1	1, 1	0, 1	1, 1	0, 1	1, 1	1, 1	0, 1	0, 1	0, 1
80%-20%	1, 1	1, 1	1, 2	2, 2	2, 3	2, 3	2, 2	2, 3	2, 2	2, 3	2, 3
76%-24%	0, 1	1, 2	2, 3	3, 4	5, 6	6, 7	6, 8	6, 8	6, 8	7, 8	5, 7
72%-28%	0, 1	1, 2	2, 3	4, 5	8, 10	10, 11	11, 13	12, 13	13, 15	13, 15	13, 14
68%-32%	0, 1	1, 2	3, 4	4, 5	8, 10	14, 15	17, 18	18, 19	19, 20	20, 21	20, 21
64%-36%	0, 1	1, 1	2, 3	4, 5	9, 11	15, 17	19, 20	22, 23	23, 24	25, 26	24, 25
60%-40%	0, 1	1, 2	3, 4	4, 5	9, 11	15, 17	19, 20	24, 25	25, 26	26, 27	27, 28
EORS-FBAD with FBAD-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84%-16%	0, 1	1, 1	0, 1	1, 1	1, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1
80%-20%	0, 1	1, 1	1, 2	2, 2	2, 3	2, 3	2, 3	2, 3	2, 3	2, 3	1, 2
76%-24%	0, 1	1, 2	2, 3	3, 4	5, 6	5, 6	6, 7	7, 9	6, 8	6, 8	5, 6
72%-28%	0, 1	1, 2	2, 3	3, 4	7, 9	9, 11	11, 13	12, 14	12, 14	12, 14	12, 14
68%-32%	0, 1	1, 2	2, 3	4, 5	9, 10	14, 16	16, 18	19, 21	20, 21	20, 21	20, 21
64%-36%	0, 1	1, 2	2, 3	4, 5	8, 10	15, 17	18, 19	22, 23	23, 24	23, 24	24, 25
60%-40%	0, 1	1, 2	2, 3	4, 5	9, 10	14, 15	18, 19	23, 24	25, 26	26, 27	27, 28

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.10: Control table of PS EORS-FBAD-BOL-with-FBAD and PS EORS-FBAD-BOL-with-FBAD-BOL for the average percentage of timedout emergencies (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

serve bed management. We preferred this in order to better visualize and describe the underlying phenomena. Similar considerations hold for the elective scheduling policy EORS and PS EORS-with-FBAD-BOL. For both the PSs, the control-table for patient LOS forecast PI, as well as for forecast EV, is reported.

In Figure A.3, the control-chart of the percentage of elective rescheduling for PS EORS-FBPS-BOL-with-UW is plotted in subfigure (a) and for PS EORS-with-FBAD-BOL in subfigure (b).

Subfigure (a) of Figure A.3 provides us with the basis for a counter-intuitive observation, *i.e.*, the higher is the BOL for elective patients, the greater can be the need for rescheduling elective patients. A simple explanation is related to the fact that, by enforcing the BOL for elective patients, the remaining bed capacity is implicitly “reserved” to emergency patients (see Section 2.3.4 for more details on this). If no BOL is enforced also for emergency patients, the bed occupancy of emergency patients can systematically overstep the bed capacity implicitly reserved and conflict with the scheduled bed occupancy of elective patients, in this case, the need for rescheduling arises. For example, if the BOL for elective patients is 80% of the hospital bed

EORS-FBAD with FBAD-BOL

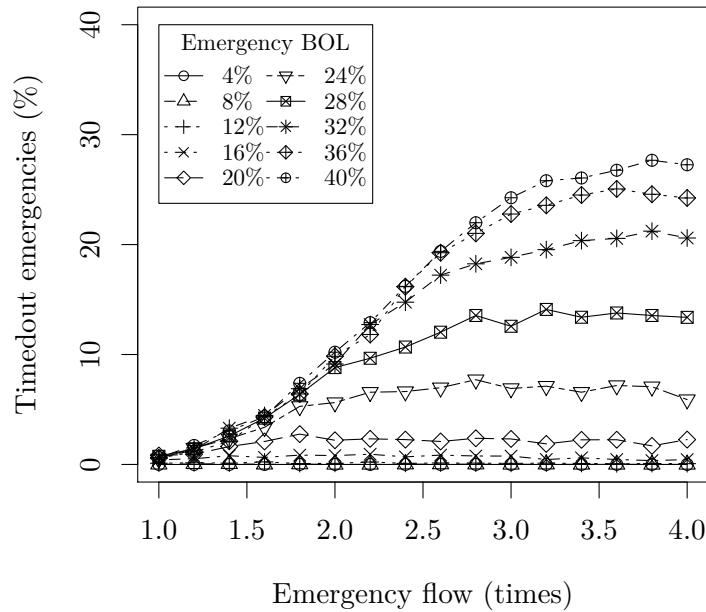


Figure A.2: Control-chart of PS EORS-FBAD-with-FBAD-BOL for the percentage of timedout emergencies, OT management policy is MSS and the patient LOS forecast is PI

capacity, it is not possible to schedule elective patients for a foreseen bed occupancy higher than 80%. The remaining 20% is foreseen to be available for the admission of emergency patients, but, without enforcing the BOL also for the emergency patients, emergency patients can be admitted to the hospital even if their bed occupancy oversteps 20%. The lower the value of the BOL is for elective patients, the bigger is the part of the bed capacity implicitly reserved to emergency patients, and the lower is thus the probability that the bed occupancy of emergency patients oversteps the implicitly reserved one.

Subfigure (b) of Figure A.3 shows that PSs that enforce the BOL only for the emergency patients does not provide a tight control for the rescheduling of elective patients. For such PSs, the bed occupancy of the elective patients is not limited by a BOL at the scheduling time and the bed occupancy of the scheduled electives can systematically overstep the bed capacity implicitly reserved with the BOL enforced for emergency patients, in this case, the need for rescheduling elective patients arises. For very small values of BOL for emergency patients, the bed occupancy of emergency patients is forced to be very small and the rescheduling of elective patients is negligible. If the BOL for emergency patients is set to a large value proportional to an emergency flow bigger than the usually served one, the extent of the required rescheduling of elective patients grows largely. In other words, the PS EORS-with-FBAD-BOL allows, as any PS not enforcing the BOL for elective patients, the scheduling of elective patients blind to the admission of emergency patients that can occur later than the scheduling of some elective patients, so, the need for rescheduling these elective patients may arise.

We can conclude that, with respect to the rescheduling of elective patients, the BOL for elective patients is an important PS feature for controlling the KPI, it may be sufficient in case of a moderate flow of emergency patients. The flow of emergencies that can be handled depends strictly on the applied BOL-setting, but also on the considered type of bed availability for both the patient types (see Section A.1). In face of a very large and unpredicted flow of emergencies the enforcement of the BOL for emergency patients, together with that for elective patients, can

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBAD-BOL with UW, PI patient LOS forecast											
96%-4%	0, 1	0, 1	1, 1	2, 3	5, 6	9, 10	12, 13	19, 20	21, 22	26, 27	29, 30
92%-8%	0, 0	0, 1	1, 1	1, 2	5, 6	10, 12	12, 14	18, 20	21, 23	26, 27	29, 30
88%-12%	0, 0	0, 1	1, 1	2, 3	4, 5	10, 11	12, 14	19, 20	21, 22	25, 26	29, 30
84%-16%	0, 0	0, 0	0, 1	1, 2	4, 5	10, 12	12, 14	19, 20	21, 22	26, 27	28, 29
80%-20%	0, 0	0, 0	0, 1	1, 1	3, 4	8, 10	12, 13	17, 19	20, 21	25, 26	28, 29
76%-24%	0, 0	0, 0	0, 0	0, 0	1, 2	5, 6	7, 9	15, 16	17, 19	24, 25	27, 28
72%-28%	0, 0	0, 0	0, 0	0, 0	1, 1	3, 3	5, 6	11, 12	14, 15	21, 23	26, 28
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 3	5, 6	8, 10	17, 19	23, 25
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	3, 3	5, 6	10, 12	16, 18
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2	5, 6	11, 13
EORS-FBAD-BOL with UW, EV patient LOS forecast											
96%-4%	0, 0	0, 1	1, 2	2, 2	4, 5	10, 11	13, 14	19, 20	21, 22	26, 27	29, 30
92%-8%	0, 0	0, 1	1, 1	2, 2	4, 5	10, 11	13, 15	18, 19	21, 22	25, 26	29, 30
88%-12%	0, 0	0, 1	1, 1	2, 3	4, 6	10, 12	12, 14	18, 19	21, 23	25, 26	29, 30
84%-16%	0, 0	0, 0	1, 1	2, 2	4, 6	9, 11	13, 14	19, 20	22, 23	25, 26	29, 30
80%-20%	0, 0	0, 0	0, 1	1, 1	3, 4	10, 11	12, 13	18, 19	20, 22	25, 26	28, 29
76%-24%	0, 0	0, 0	0, 0	0, 1	2, 3	7, 9	10, 11	18, 19	21, 22	24, 25	28, 29
72%-28%	0, 0	0, 0	0, 0	0, 0	1, 2	5, 6	8, 9	14, 15	19, 20	23, 24	27, 28
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 2	4, 4	6, 7	11, 12	14, 16	20, 22	26, 27
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 3	3, 4	7, 9	9, 11	15, 16	21, 22
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 2	2, 2	4, 5	6, 7	10, 12	16, 18
EORS with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84%-16%	0, 0	0, 0	0, 1	0, 1	0, 0	0, 1	1, 1	1, 1	0, 1	1, 1	0, 1
80%-20%	0, 0	0, 1	0, 1	1, 1	1, 2	1, 2	1, 2	1, 2	2, 2	2, 2	2, 2
76%-24%	0, 0	0, 1	1, 1	1, 2	2, 3	3, 4	3, 4	5, 6	5, 6	5, 6	7, 8
72%-28%	0, 0	0, 1	1, 2	1, 2	4, 5	6, 8	7, 8	8, 9	9, 10	10, 11	11, 12
68%-32%	0, 0	0, 1	1, 2	2, 3	5, 6	8, 10	10, 12	13, 15	15, 16	16, 17	19, 20
64%-36%	0, 0	1, 1	1, 1	2, 2	5, 6	9, 11	14, 15	18, 19	19, 20	21, 22	22, 24
60%-40%	0, 0	0, 1	1, 2	2, 2	5, 6	9, 10	13, 14	17, 19	20, 21	23, 24	25, 26
EORS with FBAD-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84%-16%	0, 0	0, 0	0, 0	0, 0	0, 1	0, 1	0, 1	0, 1	1, 1	0, 1	1, 1
80%-20%	0, 0	0, 1	0, 1	1, 1	1, 1	1, 2	1, 2	1, 2	2, 2	2, 3	2, 3
76%-24%	0, 0	0, 1	1, 1	1, 2	3, 4	3, 4	4, 5	4, 6	5, 6	5, 7	7, 8
72%-28%	0, 0	0, 1	1, 2	1, 2	4, 5	5, 6	6, 8	8, 10	9, 11	11, 13	13, 14
68%-32%	0, 0	0, 1	1, 1	1, 2	4, 5	8, 9	10, 12	14, 15	15, 17	17, 18	18, 19
64%-36%	0, 0	1, 1	1, 1	2, 2	5, 6	9, 11	12, 13	17, 18	18, 19	21, 22	22, 23
60%-40%	0, 0	1, 1	1, 1	2, 2	4, 5	10, 12	14, 15	19, 21	20, 21	23, 24	25, 26

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

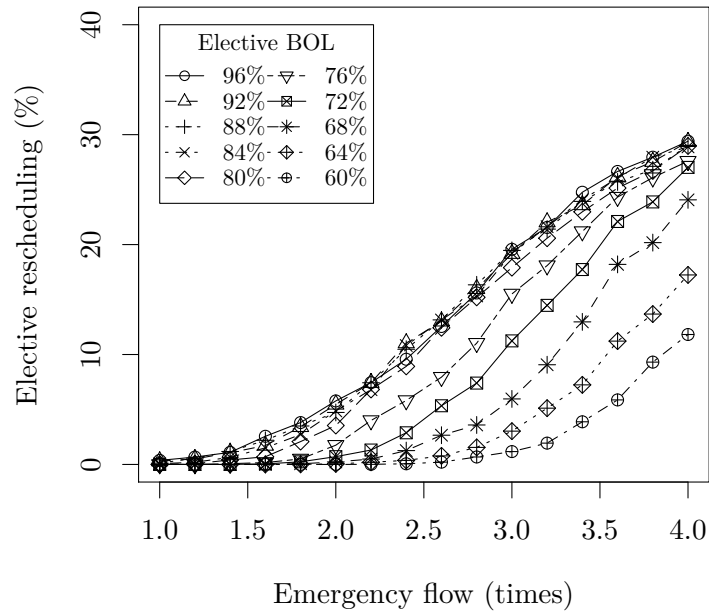
Table A.11: Control-table of PS EORS-FBAD-BOL-with-UW and PS EORS-with-FBAD-BOL for the average percentage of elective rescheduling (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

be a suitable solution.

A.2.4 Percentage of overdue electives

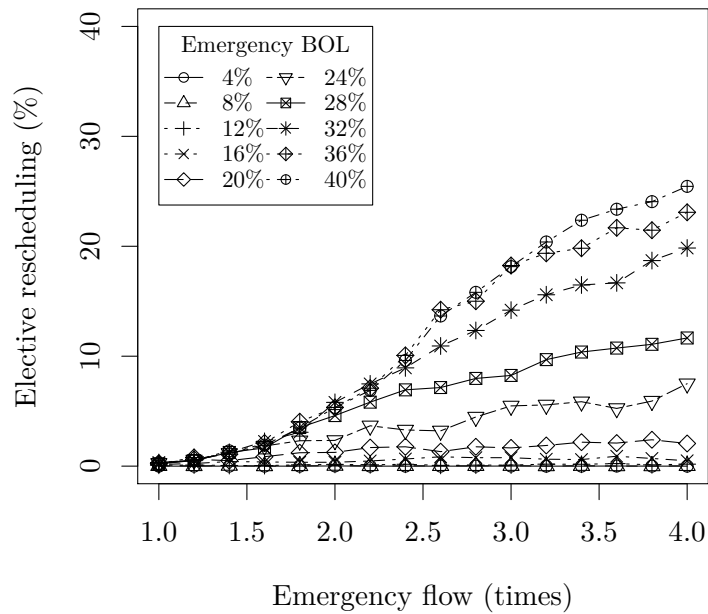
In this section, we investigate the impact on the percentage of overdue electives of enforcing (or not) the BOL for elective patients and/or for emergency patients.

EORS-FBAD-BOL with UW



(a) Control-chart of PS EORS-FBPS-BOL-with-UW

EORS with FBAD-BOL



(b) Control-chart of PS EORS-with-FBAD-BOL

Figure A.3: Control-chart of the percentage of elective rescheduling for PS EORS-FBPS-BOL-with-UW, subfigure (a), and PS EORS-with-FBAD-BOL, subfigure (b); OT management policy is MSS and the patient LOS forecast is PI

In Table A.12, as for the percentage of elective rescheduling in Table A.11, we report the control-tables of PSs EORS-FBAD-BOL-with-UW and EORS-with-FBAD-BOL, the first one is representative of PS enforcing the BOL only for elective patients and the second one of PSs enforcing the BOL only for emergency patients. For both the PSs, the control-table for the patient LOS forecasts PI and EV is reported. In this section, we consider three PSs to point out the value of enforcing the BOL for elective and/or emergency patients with respect to the percentage of overdue electives KPI, the PSs are EORS-FBPS-BOL-with-UW, EORS-FBPS-BOL-with-FBAD-BOL and EORS-with-FBAD-BOL. With PS EORS-FBPS-BOL-with-UW the BOL is enforced only for elective patients, with PS EORS-FBPS-BOL-with-FBAD-BOL, for both elective and emergency patients, and, with PS EORS-with-FBAD-BOL, only for emergency patients.

In Figure A.4, we report the control-chart of PS EORS-FBAD-BOL-with-UW, subfigure (a), and the control-chart of PS EORS-FBPS-BOL-with-FBAD-BOL, Subfigure (b). In both the subfigures, the percentage of overdue electives is plotted for ten BOL-settings, an emergency flow from 1.0 to 4.0 times the normal/tracked flow and the PI patient LOS forecast; the EV forecast provides very close results as pointed out in Table A.12.

Subfigure (a) of Figure 2.9 for PS EORS-FBAD-BOL-with-UW reveals that, if the BOL is enforced only for emergency patients, performances in terms of overdue electives are quite close to those of the benchmark PS EORS-FBPS-BOL-with-FBPS-BOL, this is evenly true for a flow of emergencies not greater than 2.5 times the normal/tracked flow of emergencies. For a flow of emergencies greater than 2.5, the absence of the BOL for emergency patients provides visible effects: the percentage of overdue electives grows significantly and independently of how much tighter is the BOL enforced for elective patients.

Subfigure (b) of Figure 2.9 for PS EORS-with-FBPS-BOL reveals that in the case of a PS enforcing the BOL only for emergency patients, *i.e.*, the scheduling of elective patients is not limited by a threshold for the bed occupancy, the percentage of overdue electives increases only for a flow of emergencies greater than 2.5 times the normal/tracked flow. Moreover, similar to that it happens for elective rescheduling, the more the BOL enforced for emergency patients is loose, the more the percentage of overdue electives increases. This happens because the BOL for emergency patients implicitly reserves the remaining bed capacity to elective patients, the scheduling of elective patients is thus not limited by a BOL and the bed occupancy of elective patients can systematically overstep the bed capacity implicitly reserved.

It worths pointing out that PS EORS-FBPS-BOL-with-FBAD not enforcing the BOL for the admission of emergency patients (only bed management) have performances in terms of overdue electives very close that of PS EORS-FBPS-BOL-with-FBAD-BOL and not with that of PS EORS-FBPS-BOL-with-UW. The same does not happen for the scheduling of elective patients, PS EORS-FBPS-with-FBAD-BOL provides performances comparable to those of PS EORS-with-FBAD-BOL in terms of overdue electives, which are by far performances not comparable to those of the benchmark PS. Resorting to bed management for the admission of emergency patients is enough to let the PS approach performances of the benchmark PS in terms of overdue electives.

We can conclude that, with respect to overdue electives, by enforcing the BOL only for elective patients can provide good performances, but the BOL-setting to apply have to be selected with attention, a tight BOL for elective patients can penalize patients of this type and generate a service for elective patients that is systematically late.

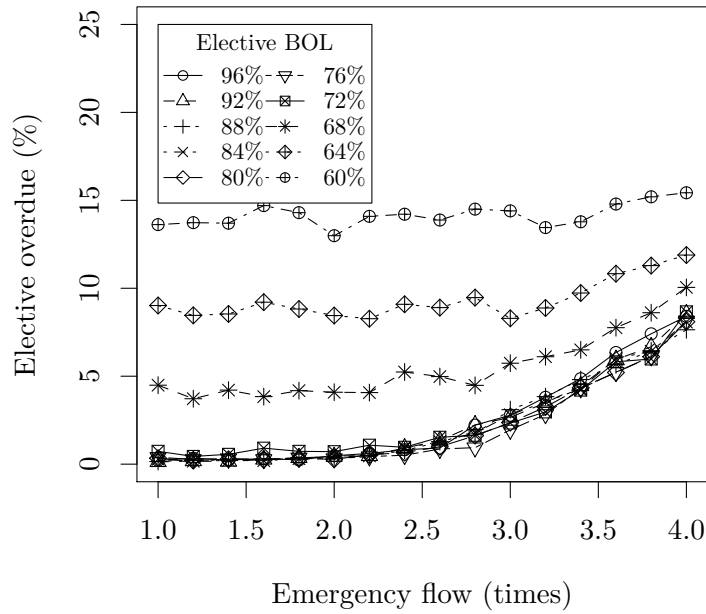
B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBAD-BOL with UW, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	3, 3	4, 4	6, 7	8, 9
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 3	3, 4	6, 6	8, 9
88%-12%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	3, 3	4, 4	5, 6	7, 8
84%-16%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 1	3, 3	3, 4	6, 6	8, 8
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 2	3, 3	5, 5	8, 8
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	2, 2	3, 3	5, 6	8, 9
72%-28%	1, 1	0, 0	0, 1	1, 1	1, 1	1, 1	1, 2	2, 2	3, 3	6, 6	8, 9
68%-32%	4, 5	4, 4	4, 4	4, 4	4, 4	5, 6	5, 5	5, 6	6, 6	7, 8	10, 11
64%-36%	9, 9	8, 9	8, 9	9, 10	8, 9	9, 10	9, 9	8, 9	8, 9	10, 11	11, 12
60%-40%	13, 14	13, 14	13, 14	14, 15	12, 14	14, 15	13, 14	14, 15	13, 14	14, 15	15, 16
EORS-FBAD-BOL with UW, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	3, 3	4, 4	6, 7	8, 8
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	3, 3	3, 3	5, 5	8, 9
88%-12%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 2	2, 3	3, 4	5, 6	8, 9
84%-16%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	3, 3	3, 4	5, 6	8, 8
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 3	3, 3	5, 5	7, 8
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 3	3, 4	5, 5	8, 9
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 3	4, 4	6, 7	8, 9
68%-32%	1, 1	1, 1	1, 1	1, 1	1, 1	2, 2	2, 2	5, 5	6, 6	7, 8	10, 11
64%-36%	3, 3	4, 4	4, 5	3, 3	4, 4	5, 6	6, 6	7, 8	8, 8	9, 10	12, 13
60%-40%	9, 9	9, 9	8, 8	8, 9	9, 10	10, 10	9, 10	10, 11	10, 11	12, 13	14, 15
EORS with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	0, 1	1, 1	1, 1	1, 1
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 2	2, 2	2, 2	3, 3
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	1, 2	2, 3	3, 3	4, 4	4, 5
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 2	3, 3	3, 4	4, 4	5, 6
EORS with FBAD-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 1	0, 0	1, 1	1, 1	1, 1	1, 1	1, 2
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	1, 2	2, 2	2, 3	2, 3
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 2	2, 3	3, 4	4, 5
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	3, 3	3, 4	5, 5	5, 6

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

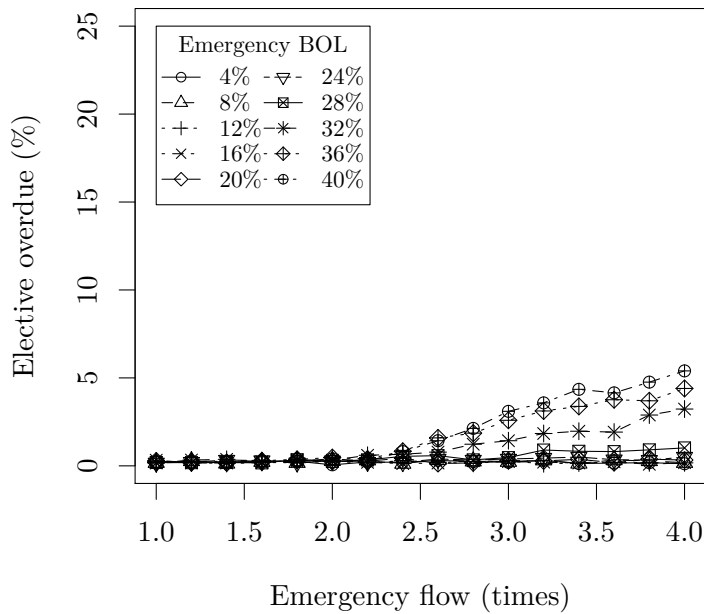
Table A.12: Control-table of PS EORS-FBAD-BOL-with-UW and PS EORS-with-FBAD-BOL for the average percentage of overdue electives (95% CI), MSS OT management policy and both the patient LOS forecasts PI and EV

EORS-FBAD-BOL with UW



(a) Control-chart of PS EORS-FBPS-BOL-with-UW

EORS with FBAD-BOL



(b) Control-chart of PS EORS-with-FBAD-BOL

Figure A.4: Control-chart of PS EORS-FBPS-BOL-with-UW, subfigure (a), and PS EORS-with-FBAD-BOL, subfigure (b), for the average percentage of overdue electives, OT management policy is MSS and the patient LOS forecast is PI

A.3 MSS vs. OB

In this section, we analyse the impact on each KPI of the OB policy for the management of the OR capacity. A section is devoted to each KPI. For each KPI, we select one or more PSs that show performances improved or degraded by the use of the OB policy.

As a reference, in Table A.13, we report the control-table of the benchmark PS EORS-FBPS-BOL-with-FBPS-BOL for the four KPIs, the OB policy and the PI patient LOS forecast (the EV forecast does not present significant differences). As for the other control-tables presented in this work, Table A.13 reports the 95% CI of the average KPIs value.

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
	(%) Refused emergencies										
96%-4%	70, 72	75, 76	79, 80	82, 82	85, 86	87, 88	88, 89	90, 90	90, 91	91, 92	92, 92
92%-8%	40, 42	48, 50	56, 57	61, 62	68, 69	73, 74	76, 76	79, 79	80, 80	82, 83	84, 84
88%-12%	18, 21	29, 32	37, 38	44, 45	55, 56	62, 63	65, 66	69, 70	71, 72	74, 75	77, 77
84%-16%	4, 5	11, 13	19, 21	27, 29	39, 41	49, 50	52, 54	58, 59	61, 62	65, 66	69, 69
80%-20%	0, 1	3, 4	8, 9	14, 16	27, 29	38, 39	42, 44	49, 51	52, 53	58, 58	62, 62
76%-24%	0, 0	0, 0	1, 2	4, 6	15, 17	25, 27	32, 33	39, 40	42, 44	49, 50	53, 54
72%-28%	0, 0	0, 0	0, 0	1, 1	7, 8	16, 17	22, 23	31, 33	34, 35	41, 42	47, 48
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 2	8, 9	12, 14	21, 23	26, 27	33, 35	39, 40
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	3, 4	6, 8	14, 16	18, 19	26, 27	33, 34
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	3, 3	9, 10	12, 14	19, 20	26, 27
	(%) Timedout emergencies										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	(%) Elective rescheduling										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	(%) Overdue electives										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
68%-32%	2, 3	2, 2	2, 3	2, 3	2, 2	3, 3	2, 2	2, 3	2, 3	2, 2	2, 3
64%-36%	4, 5	5, 5	5, 5	5, 5	5, 6	5, 5	5, 5	5, 6	5, 5	5, 5	5, 5
60%-40%	7, 7	7, 8	7, 8	7, 7	7, 8	7, 8	7, 8	7, 7	7, 8	7, 8	7, 8

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.13: Control-table of the benchmark PS EORS-FBPS-BOL-with-FBPS-BOL for the four KPIs (95% CI), OB OT management policy and both the patient LOS forecasts PI and EV

Table A.13 for the benchmark PS, if compared to Table 2.12 (control-table of the benchmark PS for the MSS OT management policy), puts in evidence that the OB policy has a very limited impact on the percentage of refused emergencies, performances are maybe slightly improved with the OB policy. As for the MSS OT management policy, even for OB policy, there is neither timed out emergencies nor rescheduling of elective patients. It worths observing that the percentage of overdue electives is evenly lowered if the OB policy is used. This means that, as expected, the OB policy eases the access of elective patients to the surgical service and reduces the time-to-surgery and the number of elective patients served after the due date.

A.3.1 Percentage of refused emergencies

The use of the OB policy for the management of the OR capacity has no significant effect on this KPI. For example, we report the control-table of the refused emergencies and PS EORS-FBPS-BOL-with-FBPS-BOL for the OB policy and both the patient LOS forecasts PI and EV in Table A.14; the control-table of the same PS for the MSS policy is in Table A.9 of Section A.2.1.

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
	EORS-FBPS-BOL with FBAD, PI patient LOS forecast										
96%-4%	1, 2	3, 4	4, 6	7, 9	13, 15	20, 23	24, 26	31, 33	33, 35	38, 40	43, 45
92%-8%	1, 2	3, 4	5, 6	7, 9	13, 15	19, 22	24, 27	31, 34	33, 36	38, 40	43, 45
88%-12%	1, 2	2, 3	4, 6	7, 8	13, 16	21, 23	23, 25	30, 32	34, 36	38, 40	43, 46
84%-16%	0, 0	2, 2	3, 5	6, 8	13, 15	19, 22	23, 26	31, 34	34, 36	39, 41	43, 45
80%-20%	0, 0	0, 1	2, 3	4, 6	12, 13	19, 21	24, 26	30, 32	33, 35	39, 41	43, 45
76%-24%	0, 0	0, 0	1, 1	2, 3	9, 11	19, 21	22, 24	29, 31	32, 34	38, 40	42, 44
72%-28%	0, 0	0, 0	0, 0	1, 1	6, 7	14, 16	19, 20	27, 28	30, 31	36, 37	41, 43
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 2	7, 8	12, 13	20, 21	23, 25	30, 32	36, 37
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	3, 3	6, 7	13, 15	17, 19	24, 26	31, 32
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 3	8, 9	11, 12	19, 20	25, 26
	EORS-FBPS-BOL with FBAD, EV patient LOS forecast										
96%-4%	2, 3	3, 4	4, 5	7, 8	14, 16	21, 23	24, 26	30, 33	33, 36	38, 40	43, 45
92%-8%	1, 2	3, 4	3, 5	7, 9	14, 16	20, 23	24, 27	30, 33	33, 36	38, 40	43, 45
88%-12%	1, 2	2, 3	5, 7	6, 8	13, 15	19, 22	24, 26	30, 33	34, 36	39, 41	43, 45
84%-16%	1, 1	2, 4	4, 5	6, 8	13, 15	20, 23	24, 26	31, 33	33, 35	39, 41	43, 45
80%-20%	0, 1	1, 2	3, 4	5, 7	12, 14	20, 22	23, 26	30, 32	33, 35	40, 42	43, 45
76%-24%	0, 0	0, 1	2, 2	4, 5	11, 13	20, 22	22, 24	29, 32	33, 35	39, 41	43, 45
72%-28%	0, 0	0, 0	1, 1	2, 3	7, 9	17, 19	22, 24	28, 30	32, 34	38, 40	43, 44
68%-32%	0, 0	0, 0	0, 0	1, 2	7, 8	14, 16	18, 20	26, 28	30, 31	35, 37	41, 42
64%-36%	0, 0	0, 0	0, 0	0, 0	2, 3	8, 10	12, 14	20, 21	23, 25	30, 32	35, 37
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 1	4, 5	7, 8	13, 15	17, 18	23, 25	30, 32

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.14: Control-table of PS EORS-FBPS-BOL-with-FBAD for the average percentage of refused emergencies (95% CI), OB OT management policy and both the patient LOS forecasts PI and EV

By comparing Table A.14 with Table A.9, it is evident that the OB policy does not provide significant differences, maybe a slightly inferior percentage of refused emergencies for a huge flow of emergencies.

A.3.2 Percentage of timedout emergencies

As for the MSS policy, the percentage of timed out emergencies increases, *i.e.*, it is not null, for few PSs, there is no PS (or patient LOS forecast) with timed out emergencies for the MSS policy, but not for the OB policy *or vice versa*. The only case that reveals a significant difference between the MSS policy and the OB policy is that of PSs enforcing the BOL for emergency patients but not for electives. In Table A.15, we report the control-table of PS EORS-FBAD-with-FBAD-BOL and the OB policy for the percentage of timed out emergencies, and for both the patient LOS forecasts PI and EV. The control-table of the same PS, but for the MSS OT management policy is in Table A.10.

The comparison of the simulation outcomes for the percentage of timed out emergencies of Table A.15 and Table A.10, respectively for the OB policy and the MSS policy, reveals that the OB policy may slightly degrade performances in terms of timed out emergencies for PS

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBPS-BOL with FBAD, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
88%-12%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
84%-16%	0, 1	0, 1	1, 2	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	0, 1	0, 1
80%-20%	0, 1	1, 2	2, 3	2, 3	2, 3	2, 3	2, 3	2, 3	2, 3	2, 3	2, 2
76%-24%	1, 1	1, 2	2, 3	3, 4	5, 7	6, 8	7, 8	7, 9	7, 9	7, 8	6, 7
72%-28%	1, 1	2, 3	2, 3	4, 5	8, 9	10, 11	11, 13	12, 14	13, 15	14, 16	14, 15
68%-32%	1, 1	1, 2	3, 4	4, 6	8, 10	13, 15	15, 17	19, 20	21, 23	22, 23	24, 25
64%-36%	1, 2	1, 2	2, 3	4, 5	9, 11	14, 16	19, 20	24, 25	26, 27	29, 30	30, 31
60%-40%	1, 1	1, 2	3, 4	4, 5	9, 11	16, 18	19, 21	25, 27	28, 29	30, 31	31, 32
EORS-FBPS-BOL with FBAD, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
88%-12%	0, 1	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
84%-16%	0, 1	0, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	1, 1	0, 1	0, 1
80%-20%	1, 1	1, 2	2, 2	2, 2	2, 3	3, 3	2, 3	2, 3	2, 3	2, 2	2, 2
76%-24%	1, 1	1, 2	2, 3	4, 5	5, 7	6, 7	6, 8	6, 8	6, 7	6, 8	5, 6
72%-28%	0, 1	1, 2	2, 3	4, 5	8, 9	9, 10	10, 12	11, 13	13, 15	13, 15	13, 15
68%-32%	1, 1	1, 2	2, 3	4, 5	8, 9	15, 16	15, 17	21, 22	22, 23	23, 25	22, 23
64%-36%	1, 1	1, 2	2, 3	4, 5	9, 11	15, 16	18, 20	23, 25	24, 26	28, 29	28, 30
60%-40%	1, 1	1, 2	3, 4	4, 5	9, 10	14, 16	18, 20	25, 26	27, 28	30, 31	30, 31

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.
 Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.15: Control-table of PS EORS-FBPS-BOL-with-FBAD for the average percentage of timedout emergencies (95% CI), OB OT management policy and both the patient LOS forecasts PI and EV

enforcing the BOL only for emergency patients, but this is limited to the case of a huge flow of emergencies and for BOL-settings that can be considered loose for emergency patients, *i.e.*, a BOL for emergency patients greater than 30%. Despite this fact, PSs not enforcing the BOL for elective patients, like EORS-FBAD-with-FBAD-BOL, have been revealed in the previous sections as not the most attractive, so, these partially degraded performances for the percentage of timed out emergencies that increases for these PSs, because of the OB policy, has minor importance.

A.3.3 Percentage of elective rescheduling

Even for the percentage of elective rescheduling, there are no cases of substantial changes of performances with the OB policy for OT management. If elective rescheduling is null for a PS in the case the MSS is applied, it is null also in the case the OB policy is applied. The need for rescheduling elective patients arises for many PSs, but it increases to relevant values only in few cases, these are: if neither a BOL nor bed management is enforced for emergency admission, but both are enforced for the scheduling of elective patients or *vice versa*, both the BOL and bed management are enforced for scheduling elective patients, but not for admitting emergency patients. In Table A.16, we report the control-table of PS EORS-FBAD-BOL-with-UW and PS EORS-with-FBAD-BOL for the percentage of elective rescheduling, the OB policy and both the patient LOS forecasts PI and EV. The control-table for the same two PS, but for the MSS policy is in Table A.11.

By comparing the simulation outcomes reported in Table A.16 with those in Table A.11, we can see that the rescheduling of elective patients increases more for the OB policy than for

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBAD-BOL with UW, PI patient LOS forecast											
96%-4%	1, 1	1, 2	2, 3	4, 5	9, 11	15, 17	18, 20	27, 28	29, 30	32, 33	33, 34
92%-8%	1, 1	1, 2	2, 3	4, 5	8, 10	16, 18	19, 21	27, 29	30, 31	32, 33	33, 34
88%-12%	0, 1	1, 2	2, 3	4, 5	8, 9	16, 18	19, 21	28, 29	29, 30	32, 33	33, 34
84%-16%	0, 0	0, 1	1, 2	3, 4	8, 10	15, 17	19, 21	26, 28	30, 31	32, 33	34, 34
80%-20%	0, 0	0, 0	0, 1	2, 3	6, 8	15, 17	19, 21	25, 27	29, 30	32, 33	34, 34
76%-24%	0, 0	0, 0	0, 0	0, 1	3, 4	11, 13	14, 16	23, 24	26, 28	31, 32	33, 34
72%-28%	0, 0	0, 0	0, 0	0, 0	1, 2	5, 6	8, 10	16, 18	20, 22	26, 28	32, 33
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 2	2, 3	7, 9	10, 12	18, 20	26, 28
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	3, 4	5, 7	12, 13	18, 20
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 2	2, 3	5, 7	11, 13
EORS-FBAD-BOL with UW, EV patient LOS forecast											
96%-4%	0, 1	1, 1	2, 3	3, 5	9, 11	15, 17	19, 21	26, 28	30, 31	32, 33	33, 34
92%-8%	0, 1	1, 2	2, 3	4, 5	9, 10	15, 17	19, 21	27, 28	30, 31	33, 33	33, 34
88%-12%	0, 1	1, 2	2, 3	4, 5	9, 11	14, 16	18, 20	25, 27	30, 31	32, 33	33, 34
84%-16%	0, 0	0, 1	2, 3	3, 4	8, 10	14, 16	17, 19	25, 27	30, 31	32, 33	34, 34
80%-20%	0, 0	0, 0	0, 1	2, 2	6, 8	15, 17	19, 21	27, 29	29, 30	33, 33	33, 34
76%-24%	0, 0	0, 0	0, 0	1, 1	4, 6	11, 12	15, 17	24, 26	27, 28	31, 32	33, 34
72%-28%	0, 0	0, 0	0, 0	0, 0	2, 3	6, 7	9, 10	17, 19	22, 24	28, 29	32, 33
68%-32%	0, 0	0, 0	0, 0	0, 0	1, 1	3, 4	5, 6	11, 13	14, 15	22, 24	27, 29
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 2	3, 4	6, 7	9, 10	15, 16	21, 23
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 2	2, 2	4, 5	5, 7	9, 10	15, 17
EORS with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
88%-12%	0, 0	0, 0	0, 0	0, 1	0, 1	0, 1	0, 0	0, 1	0, 1	0, 1	0, 1
84%-16%	0, 1	0, 1	1, 1	1, 1	1, 2	1, 2	1, 2	1, 2	2, 3	1, 2	1, 2
80%-20%	0, 1	1, 2	1, 2	2, 3	3, 4	3, 4	3, 4	3, 4	3, 5	3, 5	4, 6
76%-24%	1, 1	1, 1	2, 2	3, 4	4, 5	6, 8	7, 9	8, 10	10, 12	10, 12	12, 14
72%-28%	0, 1	1, 2	2, 3	3, 5	7, 9	10, 12	11, 13	14, 16	14, 17	17, 19	18, 20
68%-32%	1, 1	1, 2	2, 3	4, 6	9, 10	14, 16	16, 18	21, 23	21, 23	26, 27	30, 31
64%-36%	0, 1	1, 2	2, 3	3, 5	9, 11	15, 17	18, 20	25, 27	29, 30	31, 32	33, 33
60%-40%	1, 1	1, 2	2, 3	4, 5	8, 10	16, 18	20, 22	26, 28	28, 30	33, 33	34, 34
EORS with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
88%-12%	0, 0	0, 0	0, 0	0, 0	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1
84%-16%	0, 1	0, 1	1, 1	0, 1	1, 2	1, 1	1, 2	1, 2	1, 2	1, 2	2, 3
80%-20%	1, 1	1, 1	1, 2	2, 3	3, 4	3, 4	3, 4	3, 4	4, 6	4, 5	4, 6
76%-24%	0, 1	1, 2	2, 3	3, 4	4, 6	7, 8	7, 8	9, 11	9, 11	9, 11	11, 13
72%-28%	1, 1	1, 1	2, 3	4, 5	8, 9	9, 11	11, 13	15, 17	16, 18	15, 17	20, 22
68%-32%	0, 1	1, 2	2, 3	3, 4	9, 11	13, 15	18, 20	22, 23	24, 25	27, 29	30, 31
64%-36%	0, 1	1, 2	2, 3	3, 5	8, 10	15, 17	18, 20	26, 27	28, 29	31, 32	32, 33
60%-40%	0, 1	1, 2	2, 3	4, 5	8, 10	16, 18	20, 22	27, 29	29, 30	33, 33	33, 34

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.16: Control-table of PS EORS-FBAD-BOL-with-UW and PS EORS-with-FBAD-BOL for the average percentage of elective rescheduling (95% CI), OB OT management policy and both the patient LOS forecasts PI and EV

the MSS policy, the behaviour of the KPI is the same for both MSS and OB, and for both the PSs, but the increase of rescheduling is a little bit higher (few percentage points) for the OB policy. In Table A.16, we presented the case of two PSs that allows observing, at its most, the potential effect of changing the OT management policy from MSS to OB, even if these two PSs are among the less appealing with respect to their performances. Moreover, to apply

bed management for elective patients, *i.e.*, moving from PS EORS-with-FBAD-BOL to PS EORS-FBAD-with-FBAD-BOL, or for emergency patients, *i.e.*, moving from PS EORS-FBAD-BOL-with-UW to PS EORS-FBAD-BOL-with-FBAD, is enough to lower (almost to zero) the percentage of elective rescheduling.

A.3.4 Percentage of overdue electives

As we have already seen for the benchmark PS EORS-FBPS-BOL-with-FBPS-BOL in the introduction of this section (see Table A.13), the number of overdue electives can be lowered (or even zeroed) by using the OB management policy. This result is true for every PS and holds independently of the patient LOS forecast and the type of bed availability for scheduling elective patients and admitting emergency patients.

In this section, we extend the analysis of the impact on the percentage of overdue electives KPI to PS EORS-FBAD-BOL-with-UW and EORS-with-FBAD-BOL, those analysed in Section A.2 for evaluating the impact on the KPI of the BOL enforcement. For any other PS, when the OB policy is used, the percentage of overdue electives is null or similar to that of the PS covered in this section (or the benchmark PS, Table A.13). The two PSs analysed in this section represent extreme cases (large values).

In Table A.17, we report the control-tables of PS EORS-FBAD-BOL-with-UW and PS EORS-with-FBAD-BOL, OT management policy OB and both the patient LOS forecasts PI and EV. The control-tables of the same two PSs, but for the MSS policy, are in Table A.12.

Results in Table A.17, if compared to those of Table A.12, show that the percentage of overdue electives is zeroed for many values of the BOL-setting and the flow of emergencies by using the OB policy. If it is not zeroed, the percentage of overdue electives is at least reduced.

A.4 Supplementary tables

In this appendix, we report some control-table referenced in the main text. In Table A.18, the control-table of PS EORS-FBPS-BOL-with-FBAD-BOL for the 95% CI of the average value of the four KPIs is reported, in Table A.19, the control-table of PS EORS-FBPS-BOL-with-FBAD.

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
EORS-FBAD-BOL with UW, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 3	5, 6
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 2	6, 7
88%-12%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 2	5, 6
84%-16%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	3, 3	6, 7
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2	6, 7
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 2	5, 6
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	3, 3	6, 7
68%-32%	4, 4	3, 3	4, 4	3, 3	2, 3	3, 4	3, 4	5, 5	4, 5	6, 6	8, 9
64%-36%	8, 8	7, 8	7, 8	8, 9	7, 8	7, 8	7, 8	8, 9	8, 8	9, 10	10, 11
60%-40%	13, 14	13, 14	12, 14	13, 14	12, 14	14, 15	12, 13	13, 15	12, 13	13, 14	14, 15
EORS-FBAD-BOL with UW, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 3	6, 6
92%-8%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 2	6, 7
88%-12%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 2	6, 6
84%-16%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 3	6, 7
80%-20%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	3, 3	6, 7
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	3, 3	6, 7
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	3, 4	7, 8
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2	2, 3	5, 5	8, 8
64%-36%	3, 3	3, 3	3, 3	4, 4	3, 3	4, 5	5, 5	5, 6	6, 7	8, 9	11, 12
60%-40%	8, 9	8, 9	8, 9	8, 9	9, 10	8, 9	9, 10	10, 11	10, 11	11, 13	12, 13
EORS with FBAD-BOL, PI patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	2, 3	4, 4
EORS with FBAD-BOL, EV patient LOS forecast											
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 2	3, 4

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.
 Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.17: Control-table of PS EORS-FBAD-BOL-with-UW and PS EORS-with-FBAD-BOL for the average percentage of overdue electives (95% CI), OB OT management policy and both the patient LOS forecasts PI and EV

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
	(%) Refused emergencies										
96%-4%	71, 72	76, 77	79, 80	82, 82	86, 86	88, 88	89, 89	90, 91	91, 91	92, 92	93, 93
92%-8%	39, 42	50, 51	56, 57	61, 62	69, 70	73, 74	76, 77	79, 80	80, 81	83, 83	84, 85
88%-12%	18, 20	28, 30	38, 39	44, 46	55, 56	63, 64	65, 66	70, 70	71, 72	75, 75	77, 78
84%-16%	4, 5	11, 12	19, 21	27, 29	39, 40	48, 50	52, 53	58, 59	61, 62	66, 67	69, 70
80%-20%	0, 1	2, 3	7, 9	14, 16	28, 30	38, 39	42, 43	49, 50	53, 54	57, 58	62, 63
76%-24%	0, 0	0, 0	1, 2	5, 6	15, 16	26, 28	30, 32	39, 40	42, 44	49, 50	54, 55
72%-28%	0, 0	0, 0	0, 0	1, 2	7, 8	16, 18	22, 24	30, 32	34, 36	41, 42	47, 48
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 2	8, 10	13, 14	21, 22	25, 27	33, 34	39, 40
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 1	4, 4	6, 7	14, 16	18, 20	26, 28	33, 34
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 3	8, 10	12, 13	19, 21	26, 27
	(%) Timedout emergencies										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	(%) Elective rescheduling										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	(%) Overdue electives										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
68%-32%	2, 2	2, 2	2, 2	3, 3	2, 3	2, 2	2, 3	3, 3	2, 2	2, 3	2, 2
64%-36%	5, 5	5, 6	4, 5	6, 6	4, 5	5, 6	5, 6	5, 6	5, 6	5, 5	5, 6
60%-40%	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.18: Control-table of PS EORS-FBPS-BOL-with-FBAD-BOL for the four KPIs (95% CI), OB OT management policy and the PI patient LOS forecast

B.S.	Emergency flow										
	1.0	1.2	1.4	1.6	2.0	2.4	2.6	3.0	3.2	3.6	4.0
	(%) Refused emergencies										
96%-4%	1, 2	3, 4	4, 6	7, 9	13, 15	20, 23	24, 26	31, 33	33, 35	38, 40	43, 45
92%-8%	1, 2	3, 4	5, 6	7, 9	13, 15	19, 22	24, 27	31, 34	33, 36	38, 40	43, 45
88%-12%	1, 2	2, 3	4, 6	7, 8	13, 16	21, 23	23, 25	30, 32	34, 36	38, 40	43, 46
84%-16%	0, 0	2, 2	3, 5	6, 8	13, 15	19, 22	23, 26	31, 34	34, 36	39, 41	43, 45
80%-20%	0, 0	0, 1	2, 3	4, 6	12, 13	19, 21	24, 26	30, 32	33, 35	39, 41	43, 45
76%-24%	0, 0	0, 0	1, 1	2, 3	9, 11	19, 21	22, 24	29, 31	32, 34	38, 40	42, 44
72%-28%	0, 0	0, 0	0, 0	1, 1	6, 7	14, 16	19, 20	27, 28	30, 31	36, 37	41, 43
68%-32%	0, 0	0, 0	0, 0	0, 0	2, 2	7, 8	12, 13	20, 21	23, 25	30, 32	36, 37
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	3, 3	6, 7	13, 15	17, 19	24, 26	31, 32
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 3	8, 9	11, 12	19, 20	25, 26
	(%) Timedout emergencies										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
	(%) Elective rescheduling										
96%-4%	0, 0	0, 1	1, 2	2, 2	3, 4	5, 6	6, 8	8, 10	8, 9	9, 10	11, 12
92%-8%	0, 1	1, 1	1, 2	2, 3	3, 4	5, 6	7, 8	8, 9	8, 10	9, 10	11, 12
88%-12%	0, 0	0, 1	1, 1	2, 2	3, 4	5, 7	6, 7	8, 9	8, 10	9, 10	11, 12
84%-16%	0, 0	0, 0	0, 1	1, 2	3, 3	5, 6	6, 7	8, 9	8, 9	9, 10	10, 11
80%-20%	0, 0	0, 0	0, 0	0, 1	2, 2	4, 5	5, 6	6, 7	7, 8	8, 9	9, 10
76%-24%	0, 0	0, 0	0, 0	0, 0	0, 1	2, 2	2, 3	4, 4	5, 5	6, 6	7, 8
72%-28%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	2, 2	2, 2	3, 4	4, 5
68%-32%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	1, 1	1, 1	2, 2	3, 3
64%-36%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1	2, 2
60%-40%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 1	1, 1
	(%) Overdue electives										
96%-4%	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
68%-32%	2, 3	2, 3	2, 3	2, 2	3, 3	2, 2	2, 2	2, 2	2, 3	2, 2	3, 3
64%-36%	5, 6	5, 6	5, 6	5, 6	5, 5	5, 5	5, 5	5, 6	5, 5	5, 5	5, 6
60%-40%	7, 7	7, 8	7, 8	8, 8	8, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8

In the table, the two values of the CI are reported separated by a comma and, even if those values are percentages, the character “%” is omitted for space reasons.

Some values of the emergency flow are not reported always to let the table fit into the page; the table remains meaningful.

Table A.19: Control-table of PS EORS-FBPS-BOL-with-FBAD for the four KPIs (95% CI), OB OT management policy and the PI patient LOS forecast

Appendix B

Supplementary tables and principal component analysis

In this appendix, some further numerical results are reported in Section B.1 and results for the principal component analysis performed over the simulation outcomes are reported in Section B.2.

B.1 Tables

Table B.1 shows the simulation accuracy and Table B.2 gives the computation time for decision making at each decision epoch. Table B.3 shows the $GAP^{A,Best}$ average, minimum and maximum for each algorithm, over 24 OS instances and for costs structures $Cost2$, $Cost3$ and $Cost4$. Table B.4 shows the $Dev^{A,Y}$ average, minimum and maximum for each algorithm over 24 OS instances and for costs structures $Cost2$, $Cost3$ and $Cost4$. In both the tables, results for the three possible initial schedules SEPT, LEPT and BII are marked respectively with the letters S, L and B.

B.2 Principal Component Analysis (PCA)

In this section, we report detailed results of the PCA performed over a dataset collected running the simulation. The PCA dataset entry is:

- D1 replication,
- D2 event time t ,
- D3 total expected overtime cost at t ,
- D4 total expected electives delay cost at t ,
- D5 emergency queue head tardiness cost at t ,
- D6 total emergency queue tardiness cost at t ,
- D7 emergency insertion is on the night-duty OR,
- D8 decision (insert=1, not-insert=0)

The PCA analysis covers only methods SO and MTC. We keep such restriction since, for simple rule-based algorithms, the decision can be derived directly given the state of the system.

In Table B.5, for fields D2, D3, D4, D5, D6 and D7, the average, the minimum and the maximum correlation coefficient between the field and the decision D8 are shown for 12 MSS

Table B.1: Simulation Accuracy with 1000 replications

		MSS		OS	
		u	2u	u	2u
<i>Cost1</i>	SO	285 ± 18	321 ± 30	334 ± 26	430 ± 36
	MTC	314 ± 23	343 ± 36	339 ± 26	445 ± 40
	ASAP1	366 ± 25	366 ± 33	360 ± 27	450 ± 36
	ASAP2	367 ± 25	371 ± 33	360 ± 27	453 ± 36
	DDIP	353 ± 23	431 ± 38	370 ± 27	518 ± 41
	LB	250 ± 17	283 ± 29	304 ± 25	360 ± 34
<i>Cost2</i>	SO	215 ± 19	301 ± 28	408 ± 31	398 ± 31
	MTC	214 ± 19	325 ± 32	406 ± 31	406 ± 32
	ASAP1	221 ± 20	334 ± 30	452 ± 33	437 ± 32
	ASAP2	222 ± 20	340 ± 31	452 ± 33	438 ± 32
	DDIP	233 ± 20	378 ± 34	448 ± 33	481 ± 36
	LB	205 ± 19	275 ± 28	368 ± 29	342 ± 29
<i>Cost3</i>	SO	203 ± 17	263 ± 27	363 ± 24	431 ± 30
	MTC	203 ± 17	276 ± 30	362 ± 24	435 ± 32
	ASAP1	213 ± 17	259 ± 26	389 ± 25	465 ± 31
	ASAP2	214 ± 17	259 ± 26	390 ± 25	467 ± 31
	DDIP	228 ± 17	336 ± 33	390 ± 25	517 ± 35
	LB	193 ± 16	234 ± 26	323 ± 23	360 ± 28
<i>Cost4</i>	SO	208 ± 17	442 ± 48	294 ± 21	555 ± 52
	MTC	209 ± 18	481 ± 56	307 ± 24	652 ± 73
	ASAP1	205 ± 17	449 ± 48	305 ± 21	561 ± 53
	ASAP2	206 ± 17	450 ± 48	306 ± 21	568 ± 53
	DDIP	238 ± 19	587 ± 59	325 ± 23	697 ± 68
	LB	195 ± 17	413 ± 48	256 ± 19	478 ± 50

Table B.2: Algorithms Computational Time in milliseconds

algo	Avg.	St.Dev.	Min.	Max.
SO	911.9	1170.8	0.0	8241.7
MTC	0.4	1.0	0.0	43.8
ASAP1	0.1	1.0	0.0	44.1
ASAP2	0.1	1.0	0.0	42.8
DDIP	0.1	1.0	0.0	43.4

instances and for each considered costs structures. Table B.6 follows the same structure of Table B.5 and shows results for 12 OS instances and for each considered costs structures. For OS instance, no rule is applied for the elective surgeries release time. For each OR, the sequence of the elective surgeries is random.

A simulation run evaluates 1000 replications for each instance. The order of magnitude is 10 for the number of events for a simulation replication; so, the PCA analysis is performed on a large dataset giving a good level of accuracy.

In a global view, the variation of the costs structure does not have a great impact on the correlation coefficients of the fields and the emergency insertion decision.

The most interesting and useful result comes from looking at the correlation between the simulation time and the insertion decision. Such correlation is always stronger for MTC than for SO. This means that MTC is prone to insert emergencies later when SO inserts earlier.

Secondly, there is a strong correlation between the insertion decision and field D7. This is quite obvious since there is no overtime for the night-duty OR. A not obvious result is that such

correlation decreases for SO as the emergencies flow increases when it remains stable for MTC.

A quite strong correlation between the expected total overtime [D3](#) and the insertion decision is revealed, such correlation is stable as the costs structure and the emergencies flow vary. We can suppose that a certain amount of overtime is an intrinsic characteristic of evaluated instances.

The correlation of the emergency insertion decision with the emergency queue total tardiness [D6](#) and the queue head tardiness [D5](#) is significant; such correlations increase with the emergencies flow and mostly for OS instances, both for SO and MTC. MTC appears more sensible to the queue head tardiness, this is because it is prone to insert emergencies later.

The emergency insertion correlation with [D4](#) is weak throughout the experiments set, we can suppose that the 60 minutes target waiting time for the elective surgeries is a loose constraint on our instances.

Considering the dominance of SO over MTC (see Subsection [5.5.2](#)), we can argue that the PCA results sustain the conclusion about the SO capability to overcome the MTC shortsighted perspective over the future and the emergencies queue. SO appears able to foresee the decision impact over the performance estimating also the expectation of future emergency arrivals. This result strengthens our conclusions about the quality of SO in recovering the weakness of MTC matching the purpose of a policy improvement algorithm.

Table B.3: OS – Policy vs. Best Deviation (%)

<i>Cost2</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
B-MTC	1.4	0.1	3.7	0	7.7	1.1	12.9	0
B-ASAP1	12.9	10.2	15.5	0	15.7	9.5	25.9	0
B-ASAP2	13.0	10.4	15.7	0	16.0	10.0	25.9	0
B-DDIP	12.2	9.5	15.2	0	24.0	19.2	29.9	0
L-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
L-MTC	2.2	0.4	4.6	0	11.3	8.4	14.7	0
L-ASAP1	10.0	7.2	12.4	0	13.7	7.6	22.2	0
L-ASAP2	10.2	7.4	12.6	0	14.0	8.0	22.3	0
L-DDIP	11.2	8.9	13.1	0	23.1	16.9	29.3	0
S-SO	0.1	0.0	0.7	10	0.0	0.0	0.0	12
S-MTC	0.5	0.0	1.6	2	3.8	1.4	10.1	0
S-ASAP1	12.8	9.0	15.5	0	13.8	9.9	20.9	0
S-ASAP2	12.9	9.0	15.7	0	14.2	10.2	21.1	0
S-DDIP	11.9	9.5	15.1	0	23.3	19.7	27.3	0
<i>Cost3</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
B-MTC	2.7	0.8	5.3	0	12.3	0.3	25.0	0
B-ASAP1	8.0	5.8	9.6	0	6.7	3.4	10.0	0
B-ASAP2	8.1	5.9	9.7	0	7.0	3.9	10.7	0
B-DDIP	9.4	8.0	10.8	0	18.7	15.6	22.3	0
L-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
L-MTC	4.9	2.0	6.8	0	17.7	13.2	23.0	0
L-ASAP2	7.0	4.7	8.7	0	6.2	3.5	9.3	0
L-ASAP1	7.0	4.7	8.7	0	5.9	3.3	9.1	0
L-DDIP	9.2	7.5	10.5	0	18.6	15.9	21.8	0
S-SO	0.2	0.0	0.7	7	0.0	0.0	0.0	12
S-MTC	0.6	0.0	3.6	5	4.3	0.8	14.5	0
S-ASAP1	7.9	5.5	9.7	0	6.1	4.0	8.0	0
S-ASAP2	8.0	5.5	10.0	0	6.5	4.1	8.6	0
S-DDIP	9.2	7.4	10.9	0	19.0	16.4	21.3	0
<i>Cost4</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
B-MTC	10.9	7.4	18.1	0	27.7	10.7	41.5	0
B-ASAP1	4.9	2.3	6.4	0	2.6	0.7	7.1	0
B-ASAP2	5.2	2.6	6.9	0	3.4	1.9	7.9	0
B-DDIP	12.2	9.7	13.8	0	25.9	22.5	28.7	0
L-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
L-MTC	15.1	10.2	18.4	0	33.0	23.9	39.6	0
L-ASAP1	4.0	2.5	5.6	0	2.2	0.4	4.8	0
L-ASAP2	4.3	2.6	6.0	0	2.9	1.2	5.5	0
L-DDIP	10.8	8.9	13.5	0	24.6	21.3	26.9	0
S-SO	0.0	0.0	0.0	12	0.0	0.0	0.0	12
S-MTC	3.9	0.4	11.5	0	14.0	5.1	29.5	0
S-ASAP1	4.3	2.7	5.3	0	2.0	1.0	3.9	0
S-ASAP2	4.7	3.2	5.5	0	2.9	1.6	4.8	0
S-DDIP	12.5	10.4	14.0	0	26.4	24.5	29.5	0

Table B.4: OS – The Impact of Proactive Schedule (%)

<i>Cost2</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	13.1	0.0	32.8	1	9.6	0.0	21.7	3
B-MTC	14.7	1.6	34.2	0	17.9	6.5	31.5	0
B-ASAP1	27.8	12.0	49.9	0	26.8	11.9	48.7	0
B-ASAP2	28.0	12.3	50.1	0	27.2	12.3	48.8	0
B-DDIP	27.0	11.7	49.7	0	35.9	21.3	53.5	0
L-SO	61.8	31.7	94.7	0	33.7	15.5	51.9	0
L-MTC	65.3	33.9	98.0	0	48.7	30.9	64.9	0
L-ASAP1	78.2	46.1	115.0	0	52.1	29.4	74.2	0
L-ASAP2	78.4	46.3	115.2	0	52.5	29.5	74.5	0
L-DDIP	80.0	48.1	116.1	0	64.5	41.3	86.0	0
S-SO	0.3	0.0	2.8	9	0.4	0.0	3.0	9
S-MTC	0.7	0.0	3.0	2	4.2	1.5	10.1	0
S-ASAP1	13.0	9.0	15.5	0	14.3	9.9	20.9	0
S-ASAP2	13.2	9.0	15.7	0	14.7	10.2	21.1	0
S-DDIP	12.1	9.5	15.1	0	23.9	19.7	27.3	0
<i>Cost3</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	6.1	0.0	14.0	2	5.0	0.0	12.1	1
B-MTC	8.9	2.0	16.1	0	17.8	7.3	29.7	0
B-ASAP1	14.6	5.8	23.3	0	12.0	4.9	22.0	0
B-ASAP2	14.7	6.1	23.4	0	12.4	4.8	22.7	0
B-DDIP	16.0	8.0	24.4	0	24.6	15.6	32.7	0
L-SO	29.8	13.8	43.9	0	16.5	7.8	25.6	0
L-MTC	36.1	20.7	49.3	0	37.1	23.5	46.8	0
L-ASAP1	38.9	22.7	56.5	0	23.4	12.2	34.0	0
L-ASAP2	39.0	23.1	56.5	0	23.8	12.1	34.3	0
L-DDIP	41.7	25.6	57.9	0	38.2	25.0	49.9	0
S-SO	0.3	0.0	1.4	7	0.0	0.0	0.5	11
S-MTC	0.6	0.0	3.6	3	4.4	0.8	14.5	0
S-ASAP1	8.0	5.5	9.7	0	6.2	4.0	8.0	0
S-ASAP2	8.1	5.5	10.0	0	6.6	4.1	8.6	0
S-DDIP	9.3	7.4	10.9	0	19.0	16.4	21.3	0
<i>Cost4</i>	<i>u</i>				<i>2u</i>			
	Ave.	Min.	Max.	#1	Ave.	Min.	Max.	#1
B-SO	8.1	0.0	18.1	1	3.8	0.0	8.4	2
B-MTC	19.8	10.4	28.5	0	32.4	16.8	45.9	0
B-ASAP1	13.4	2.3	23.7	0	6.5	1.4	13.0	0
B-ASAP2	13.8	2.6	23.9	0	7.3	1.9	13.9	0
B-DDIP	21.2	12.2	32.2	0	30.5	26.5	35.3	0
L-SO	36.9	19.0	53.0	0	13.0	6.4	19.5	0
L-MTC	57.5	40.8	72.5	0	50.1	36.7	60.9	0
L-ASAP1	42.5	23.4	59.5	0	15.4	7.8	22.7	0
L-ASAP2	42.9	23.9	59.7	0	16.3	8.3	23.5	0
L-DDIP	51.7	35.1	69.2	0	40.7	33.4	46.8	0
S-SO	0.1	0.0	1.1	11	0.1	0.0	1.3	10
S-MTC	4.0	0.4	11.5	0	14.1	5.8	29.5	0
S-ASAP1	4.4	3.5	5.3	0	2.2	1.1	3.9	0
S-ASAP2	4.8	3.8	5.5	0	3.0	2.0	4.8	0
S-DDIP	12.6	10.4	14.8	0	26.6	24.5	29.5	0

Table B.5: MSS – PCA

			u			$2u$		
			Ave.	Min.	Max.	Ave.	Min.	Max.
<i>Cost1</i>	SO	D2	0.3	0.2	0.4	0.3	0.2	0.4
		D3	0.2	0.1	0.3	0.2	0.1	0.3
		D4	0.1	0.1	0.1	0.1	0.1	0.1
		D5	0.1	0.1	0.2	0.2	0.1	0.2
		D6	0.1	0.1	0.1	0.1	0.1	0.2
		D7	0.6	0.4	0.8	0.4	0.3	0.6
		D7	0.6	0.5	0.8	0.6	0.4	0.8
	MTC	D2	0.4	0.3	0.6	0.4	0.3	0.5
		D3	0.2	0.1	0.4	0.2	0.1	0.3
		D4	0.1	0.0	0.1	0.1	0.1	0.1
		D5	0.1	0.1	0.2	0.2	0.1	0.2
		D6	0.1	0.1	0.2	0.1	0.1	0.2
		D7	0.6	0.5	0.8	0.6	0.4	0.8
		D7	0.6	0.5	0.8	0.6	0.4	0.8
<i>Cost2</i>	SO	D2	0.4	0.1	0.5	0.3	0.2	0.5
		D3	0.2	0.1	0.4	0.2	0.1	0.3
		D4	0.1	0.0	0.1	0.1	0.1	0.1
		D5	0.1	0.1	0.2	0.2	0.1	0.3
		D6	0.1	0.1	0.2	0.1	0.1	0.2
		D7	0.6	0.4	0.7	0.5	0.3	0.6
		D7	0.6	0.5	0.8	0.6	0.4	0.8
	MTC	D2	0.5	0.2	0.6	0.4	0.3	0.6
		D3	0.2	0.1	0.5	0.2	0.1	0.3
		D4	0.1	0.0	0.1	0.1	0.1	0.1
		D5	0.1	0.1	0.2	0.2	0.1	0.3
		D6	0.1	0.1	0.2	0.1	0.1	0.2
		D7	0.6	0.5	0.8	0.6	0.4	0.8
		D7	0.6	0.5	0.8	0.6	0.4	0.8
<i>Cost3</i>	SO	D2	0.3	0.1	0.4	0.3	0.2	0.4
		D3	0.2	0.1	0.3	0.2	0.1	0.3
		D4	0.1	0.0	0.1	0.1	0.1	0.1
		D5	0.1	0.1	0.2	0.2	0.1	0.2
		D6	0.1	0.1	0.2	0.1	0.1	0.2
		D7	0.6	0.4	0.8	0.5	0.3	0.6
		D7	0.6	0.5	0.8	0.6	0.4	0.8
	MTC	D2	0.4	0.2	0.6	0.4	0.3	0.5
		D3	0.2	0.1	0.4	0.2	0.1	0.3
		D4	0.1	0.0	0.1	0.1	0.1	0.2
		D5	0.1	0.1	0.2	0.2	0.1	0.2
		D6	0.1	0.1	0.2	0.1	0.1	0.2
		D7	0.6	0.5	0.8	0.6	0.4	0.8
		D7	0.6	0.5	0.8	0.6	0.4	0.8
<i>Cost4</i>	SO	D2	0.3	0.2	0.4	0.3	0.2	0.4
		D3	0.2	0.1	0.2	0.2	0.1	0.2
		D4	0.1	0.1	0.1	0.1	0.1	0.2
		D5	0.1	0.1	0.2	0.2	0.1	0.2
		D6	0.1	0.1	0.1	0.1	0.1	0.2
		D7	0.5	0.4	0.7	0.4	0.3	0.5
		D7	0.6	0.5	0.8	0.6	0.4	0.8
	MTC	D2	0.4	0.2	0.6	0.4	0.3	0.5
		D3	0.2	0.1	0.4	0.2	0.1	0.3
		D4	0.1	0.1	0.1	0.1	0.1	0.1
		D5	0.1	0.1	0.2	0.2	0.1	0.2
		D6	0.1	0.1	0.1	0.1	0.1	0.2
		D7	0.6	0.5	0.8	0.5	0.4	0.7
		D7	0.6	0.5	0.8	0.5	0.4	0.7

Table B.6: OS – PCA

			u			$2u$		
			Ave.	Min.	Max.	Ave.	Min.	Max.
<i>Cost1</i>	SO	D2	0.3	0.3	0.4	0.3	0.3	0.3
		D3	0.3	0.2	0.3	0.2	0.2	0.2
		D4	0.0	0.0	0.1	0.0	0.0	0.1
		D5	0.2	0.1	0.2	0.2	0.2	0.2
		D6	0.1	0.1	0.2	0.2	0.1	0.2
		D7	0.6	0.5	0.6	0.5	0.4	0.5
		D7	0.6	0.6	0.7	0.6	0.6	0.7
	MTC	D2	0.5	0.4	0.6	0.5	0.5	0.6
		D3	0.4	0.3	0.4	0.3	0.3	0.4
		D4	0.1	0.0	0.1	0.1	0.1	0.1
		D5	0.2	0.1	0.2	0.2	0.2	0.3
		D6	0.1	0.1	0.2	0.2	0.1	0.2
		D7	0.6	0.6	0.7	0.6	0.6	0.7
		D7	0.6	0.6	0.7	0.6	0.6	0.7
<i>Cost2</i>	SO	D2	0.4	0.4	0.5	0.4	0.4	0.4
		D3	0.3	0.3	0.3	0.3	0.2	0.3
		D4	0.0	0.0	0.1	0.0	0.0	0.1
		D5	0.2	0.1	0.2	0.2	0.2	0.2
		D6	0.1	0.1	0.2	0.2	0.2	0.2
		D7	0.6	0.6	0.6	0.5	0.5	0.5
		D7	0.6	0.6	0.7	0.7	0.6	0.7
	MTC	D2	0.5	0.5	0.6	0.5	0.5	0.6
		D3	0.4	0.3	0.4	0.3	0.3	0.4
		D4	0.1	0.0	0.1	0.1	0.1	0.1
		D5	0.2	0.1	0.2	0.3	0.2	0.3
		D6	0.1	0.1	0.2	0.2	0.2	0.2
		D7	0.6	0.6	0.7	0.7	0.6	0.7
		D7	0.6	0.6	0.7	0.7	0.6	0.7
<i>Cost3</i>	SO	D2	0.4	0.3	0.4	0.3	0.3	0.3
		D3	0.3	0.2	0.3	0.2	0.2	0.3
		D4	0.0	0.1	0.1	0.0	0.0	0.1
		D5	0.2	0.1	0.2	0.2	0.2	0.2
		D6	0.1	0.1	0.2	0.2	0.1	0.2
		D7	0.6	0.6	0.6	0.5	0.4	0.5
		D7	0.6	0.6	0.7	0.6	0.6	0.7
	MTC	D2	0.5	0.4	0.6	0.5	0.4	0.6
		D3	0.4	0.3	0.4	0.3	0.3	0.4
		D4	0.1	0.0	0.1	0.1	0.1	0.1
		D5	0.2	0.1	0.2	0.2	0.2	0.3
		D6	0.1	0.1	0.2	0.2	0.1	0.2
		D7	0.6	0.6	0.7	0.6	0.6	0.7
		D7	0.6	0.6	0.7	0.6	0.6	0.7
<i>Cost4</i>	SO	D2	0.3	0.2	0.3	0.3	0.2	0.3
		D3	0.2	0.2	0.2	0.2	0.2	0.2
		D4	0.0	0.0	0.1	0.1	0.0	0.1
		D5	0.1	0.1	0.2	0.2	0.2	0.2
		D6	0.1	0.1	0.1	0.1	0.1	0.2
		D7	0.5	0.5	0.5	0.4	0.4	0.4
		D7	0.5	0.5	0.5	0.4	0.4	0.4
	MTC	D2	0.5	0.4	0.5	0.5	0.4	0.5
		D3	0.3	0.3	0.4	0.3	0.3	0.3
		D4	0.1	0.0	0.1	0.2	0.1	0.2
		D5	0.1	0.1	0.2	0.2	0.2	0.2
		D6	0.1	0.1	0.1	0.2	0.2	0.2
		D7	0.6	0.6	0.6	0.6	0.5	0.7
		D7	0.6	0.6	0.6	0.6	0.5	0.7