

## Centennial fluctuations of the geomagnetic intensity: from archeomagnetism to core processes

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# Centennial fluctuations of the geomagnetic intensity: from archeomagnetism to core processes

par

# Marie Troyano

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Thèse de doctorat de Sciences de la Terre et de l'environnement dirigée par **Alexandre Fournier** & **Yves Gallet** 

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#### Abstract

Direct measurements of the geomagnetic field being only available over the historical period (from 1590 to today), global reconstructions beyond that time therefore resort to indirect measurements provided by paleo- and archeomagnetism. In this respect, archeomagnetism can provide particularly well dated data. This thesis aims at analyzing the geomagnetic field intensity variations provided by archeomagnetism over multi-decadal to centennial timescales, from two different but complementary aspects.

A first study focuses on the acquisition of archeointensity data in central Asia and their consequences on the knowledge of regional and global geomagnetic field variations. In particular, global geomagnetic field models over the historical period based on direct measurements solely need additional constraints to overcome the absence of direct intensity measurements before  $\sim 1840$ . Two options have been proposed: either to linearly extrapolate backward the behavior of the axial dipole moment observed since 1840, as in the qufm1 model, or to rely on a global archeointensity dataset. In this study, a regional approach is used, based on new archeointensity data obtained from Bukhara for the historical period. This city is of particular interest owing to its outstanding, well-preserved historical center and the archives just as well preserved providing precise dating constraints on the buildings sampled for this study. The baked clay bricks fragments are analysed using the Triaxe experimental protocol. The obtained intensity variations curve shows a rapid decrease from  $1600 \text{ to} \sim 1750$  followed by an increase until the early 19th. This evolution is in good agreement with other Triaxe data acquired in western Europe and western Russia. These three Triaxe datasets are used to recalibrate the axial dipole moment from the *qufm1* model. The resulting evolution is non-linear, with a minimum amplitude during the second half of the 18th century. Although the results presented in this study need to be confirmed by further data acquisition worldwide, it nonetheless illustrates that archeointensity data can provide constraints on the geomagnetic intensity evolution over multi-decadal to centennial timescales at both regional and global scales.

The second study focuses on intensity variations inferred from archeomagnetic data, from a theoretical standpoint. Recently, extreme archeointensity events lasting only a few decades, termed geomagnetic spikes, have been proposed in the Near-East during the first millennium BC. They are associated with variations rates up to several  $\mu T/yr$ , while today's maximum is of order  $\sim 0.1 \,\mu\text{T/yr}$ . Magnetic flux expulsion at the core's surface has been proposed to explain such extreme events, but this process has not vet been studied in detail. In this study, a 2D kinematic model of magnetic flux expulsion is implemented, controlled by a single parameter: the magnetic Reynolds number Rm, the ratio of magnetic diffusion to advection times. This model allows for the monitoring of initially horizontal magnetic field lines, advected by a fixed flow pattern constituted by two counter-rotating eddies. As the magnetic field lines are distorted and folded by the flow, the magnetic flux is progressively expelled towards the domain's boundaries. If the boundary separates the conducting fluid from an insulating medium, the magnetic flux can diffuse through it. To follow the flux expulsion through the insulating boundary, the vertical component of the magnetic field is monitored during the system evolution. The characteristic rise time is found to scale as  $Rm^{0.15}$ , while the maximum instantaneous variation rate scales as  $Rm^{0.45}$ . These scaling laws are then extrapolated at the Earth's surface. The results show that geomagnetic spikes cannot be generated by flux expulsion. However, other intensity peaks of durations longer than one century and associated with much lower variation rates would be compatible with flux expulsion events.

**Keywords**: Geomagnetism, archeomagnetism, geomagnetic field intensity, axial dipole moment, rapid variations, core processes, modelling

#### Résumé

Les mesures directes du champ géomagnétique ne couvrant que la période historique (1590 à aujourd'hui), la reconstruction de son évolution au-delà de cette période repose sur des données indirectes, fournies par le paléo- et l'archéomagnétisme, ce dernier permettant d'acquérir des données particulièrement bien datées. Cette thèse a pour but d'analyser les variations d'archéointensité du champ sur des échelles multi-décennales à centennales, à travers deux aspects différents mais complémentaires.

Une première étude porte sur l'acquisition de données d'archéointensité en Asie centrale et leur interprétation en terme de variations d'intensité à une échelle régionale et globale. L'absence de données directes d'intensité avant  $\sim 1840$  nécessite une contrainte additionnelle pour construire des modèles globaux du champ magnétique à partir de données directes. Deux options sont proposées, la première extrapolant linéairement dans le passé les variations du dipôle axial observées en 1840, comme pour le modèle qufm1, la seconde s'appuyant sur un jeu global de données d'archéointensité. Dans cette étude, une approche régionale est proposée, basée sur de nouvelles données d'archéointensité obtenues à Boukhara. Cette ville est remarquable par la préservation de son centre historique, dont les bâtiments sont précisément datés par des archives documentaires. Les fragments de briques d'argiles cuites des bâtiments échantillonnés ont été analysés par le protocole expérimental du Triaxe. Les variations d'intensité obtenues montrent une décroissance rapide de 1600 à  $\sim$  1750 suivie d'une croissance modérée jusqu'au début du 19e siècle. Cette évolution est en accord avec d'autres données Triaxe acquises en Europe de l'Ouest et en Russie du Nord-Ouest. Toutes les données Triaxe sont donc utilisées pour recalibrer l'évolution du dipôle axial du modèle qufm1. La nouvelle évolution montre un minimum d'amplitude durant la seconde moitié du 18e siècle. Si les résultats présentés dans cette étude doivent être confirmés par de nouvelles données d'archéointensité, ils montrent néanmoins que ces dernières permettent de contraindre les variations multi-décennales à centennales du champ géomagnétique, à une échelle régionale et globale.

La seconde étude analyse les variations d'intensité archéomagnétique d'un point de vue théorique. Récemment, des pics extrêmes d'archéointensité, nommés spikes géomagnétiques, ont été proposés au Proche-Orient pendant le dernier millénaire av. NE. Ils sont caractérisés par une durée de quelques décennies et des taux de variations de plusieurs  $\mu$ T/an, le maximum actuel étant d'environ 0.1  $\mu$ T/an. L'expulsion de flux magnétique à la surface du novau a été proposée comme une origine possible. Cette étude propose l'analyse d'un modèle cinématique 2D d'expulsion de flux, contrôlée par un seul paramètre : le nombre de Reynolds magnétique Rm, rapport du temps magnétique de diffusion sur le temps d'advection. Ce modèle permet de suivre l'évolution les lignes de champ magnétique, initialement horizontales, alors qu'elles sont advectées par deux tourbillons de rotations opposées. Lorsque les lignes de champs sont déformées et pliées par l'écoulement, le flux magnétique est expulsé vers les bords du domaine. Si ce bord sépare le fluide d'un milieu isolant, le flux magnétique peut diffuser au travers. Pour étudier l'expulsion de flux au travers du bord isolant, la composante verticale du champ magnétique sur ce bord est suivie pendant l'évolution du système. Son temps caractéristique de croissance évolue comme  $Rm^{0.15}$  et le taux de variation instantanée maximum comme  $Rm^{0.45}$ . Ces lois d'échelles sont ensuite extrapolées à la surface de la Terre. Les résultats montrent que l'expulsion de flux ne peut pas générer des événements aussi extrêmes que les spikes. Cependant des événements présentant des durées supérieures au siècle et associés à des taux de variations nettement inférieurs pourraient être compatibles avec le processus d'expulsion de flux.

Mots clés: Géomagnetisme, archéomagnetisme, intensité du champ géomagnetique, dipôle axial, variations rapides, processus nucléaires, modélisation

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# Introduction

### 1.1 Historical overview

The study of the Earth's magnetic field "may well claim to be the oldest discipline in geophysics" [Stern, 2002]. Our knowledge of the geomagnetic field is the result of almost two millennia of research. In the following, a brief overview of key discoveries and advances in the study of the geomagnetic field in Europe are provided. For extensive reviews on the subject, see for example Merrill et al. [1998], Stern [2002], Courtillot and Le Mouël [2007], Kono [2015] and references therein, as well as the related chapters in Gubbins and Herrero-Bervera [2007].

The first recorded observation of the geomagnetic field is attributed to the ancient Chinese, who first notice the North-South alignment of magnets (lodestones) as early as the end of the first century AD, prefiguring the magnetic compass. The first description of magnetic compass and its use for navigation in Europe is attributed to the English abbot Alexander Neckam during the twelfth century AD. It is yet unclear how the magnetic compass arrives in Europe (or if it was invented independently) and starts to be used for navigation.

One of the first extensive study of the magnetic properties of lodestones is provided by Pierre de Maricourt (also known as Petrus Peregrinus), in his *Epistola de Magnete* (1269), often regarded as the first scientific study of magnetism (and even the first scientific study in the modern sense). Together with a thorough description of two magnetic compasses (see Fig. 1.1a), he introduces the concept of polarity (in Europe at least), dipolar magnets and magnetic meridians, which he infers parallel to the geographic meridians. It appears that, at that time, the Chinese are already aware that the direction of the geographic North is not perfectly aligned with the direction of the magnetic North and Yi-Xing, a Buddhist astronomer provides one of the first declination (D) measurement during the eighth century AD in China. The discovery of magnetic declination in Europe is unclear, but it seems that is has been known since the early fifteenth century. The discovery of magnetic inclination (I) is attributed to Georg Hartmann and Robert Norman by studying the dip angle between the magnetic needle of a compass and the horizontal plane.

William Gilbert is one of the first to propose an internal origin for the Earth's magnetic field, with his *Terella* model (Fig. 1.1b) described in his treatise *De Magnete* [Gilbert, 1600]. By studying the inclination variations at the surface of a spherical lodestone (the *Terella*), he hypothesizes that the Earth itself behaves like a giant magnet, implying the geomagnetic field is static. This idea is questioned in 1634 by Henry Gellibrand who discovers (or concludes) that the geomagnetic field is in fact varying based on analyses of magnetic declination records from London at different epochs. This discovery marks a turn in geomagnetism. It shows the major importance of constantly monitoring the Earth's magnetic field and awakes a strong interest in understanding its nature and sources.



FIGURE 1.1: a) Depiction of a floating compass described by Pierre de Maricourt in his *Epistola*.
b) *Terella* model of Gilbert. Source gallica.bnf.fr / Bibliothèque nationale de France.

With the end of the 17th century starts the era of scientific expeditions to study the geomagnetic field. From such an expedition in the Atlantic ocean, Edmund Halley proposes the first magnetic charts for declination in the Atlantic ocean (Fig. 1.2). This leads him to discover that some features of the magnetic field are drifting westward. Halley is also the first to propose a layered Earth model, with two layers, rotating at different velocities. This differential rotation would be responsible for the westward drift of magnetic structures observed at the Earth's surface.

During the 18th century, a lot of efforts are put into the geomagnetic field monitoring, through the invention and improvements of instruments, the collection and compilation of magnetic data and the creation of magnetic charts. A major event is the first measurement of the relative magnetic intensity, attributed to the French scientist Robert de Paul de Lamanon, during the de La Pérouse expedition (1785-1788), but the records were lost during the shipwreck of the expedition. The first global survey of magnetic intensity is attributed to Elisabeth Paul Edouard De Rossel during the D'Entrecasteaux expedition (1791-1794), sent in search of de La Pérouse expedition. The magnetic intensity is determined from the oscillation period of a magnetic needle displaced from its preferential position to return to it. This method is subsequently used by the German naturalist Alexander Von Humboldt during his expeditions in South America (1799-1804). These measurements evidence that the magnetic intensity is stronger towards the poles and weaker towards the equator.

The 19th century has seen two major advances in geomagnetism, both due to the German mathematician Carl Friedrich Gauss (named "Princeps mathematicorum" by his peers). His interest in geomagnetism is raised by Alexander Von Humboldt during a conference in Berlin. As a result, with the assistance of his collaborator Wilhelm Weber, Gauss designs the first instrument to measure the absolute magnetic intensity ['F', Gauss, 1833]. This discovery triggers the creation of the first global observatories network, the Göttingen Magnetic Union, in 1834. Its second major contribution is the first mathematical representation of the geomagnetic field achieved in 1839 [Gauss, 1839]. From geomagnetic data available from various observatories, he derives the spherical harmonic coefficients of the magnetic potential at the Earth's surface. His results show that the main field is mainly of internal origin.



Source gallica.bnf.fr / Bibliothèque nationale de France

FIGURE 1.2: Halley's chart of the declination in the Atlantic ocean. Source gallica.bnf.fr / Bibliothèque nationale de France.

Jonkers [2007] defines three period for the history of geomagnetism. The "proto-scientific stage (up to the 16th century)", the "early-modern stage (16th to early 19th century)", both described hereabove, and the "modern stage" which started with the first full vector measurement of the geomagnetic field by Gauss in the 1830s. He defines the modern discipline of geomagnetism as the study of the internal Earth's magnetic field and its secular variations, as well as the study of their origins as based on the geodynamo theory. The origin of the internal geomagnetic field has indeed puzzled the scientific community for a long time and was even described by Albert Einstein as one of the most important unsolved problem in physics. With the discovery of the Earth's core by Richard Oldham in 1906 [Oldham, 1906] and of its liquid exterior part in 1926 by Harold Jeffreys [Jeffreys, 1926], the idea of a self-sustained dynamo, first proposed by Larmor in 1919 is further explored. In particular, during the mid-20th, Elsasser [Elsasser, 1946, 1950] and Bullard [Bullard et al., 1950, Bullard and Gellman, 1954] give a new impulse to the geodynamo theory, with the first mathematical dynamo models based on magnetohydrodynamics.

In parallel another discipline linked with geomagnetism emerges during the middle of the 19th century. During the 1850s, Joseph Fournet [Fournet, 1849] and Achille Joseph Delesse [Delesse, 1849] notice that certain rocks are magnetized and this remanent magnetization is parallel to the Earth's magnetic field. Macedonio Melloni hypothetizes from experiments that the magnetization acquired by volcanic rocks is stable over time and is parallel to the Earth's magnetic field [Melloni, 1853]. Subsequently, the Italian scientist Giuseppe Folgheraiter provides the basis of archeomagnetism by studying the remanent magnetization of bricks and pottery. He shows that, knowing the position of firing of the bricks, it would be possible to recover the past field directions. From his results, he proposes the first paleosecular variation reconstruction and the first archeomagnetic dating [see Principe and Malfatti, 2020, and references therein]. This is "the birth of modern archaeomagnetic science" [Principe and Malfatti, 2020]. Based on these principles, Pierre David and Bernard Brunhes discover geomagnetic reversals by the analyses of paleomagnetic directions from natural clay baked by lava that flowed over it [David, 1904, Brunhes, 1906].

During the first part of the 20th, Émile Thellier pioneered the study of archeointensity, i.e. recovering the geomagnetic field intensity recorded by archeological artefacts. His experimental work in analysing and understanding the thermoremanent magnetization (TRM) acquisition triggers the interest of Louis Néel who subsequently theorizes the remanence acquisition for single-domain grains [Néel, 1949]. The experimental work of Émile Thellier and Odette Thellier, his wife, lead them to propose a method still used today for measuring paleointensities, the so-called Thellier and Thellier technique [Thellier and Thellier, 1959]. In addition, their dedicated work on ancient kilns allows Thellier to propose the first directional paleosecular variations curve for France, covering the last two millennia [Thellier, 1981].

As illustrated hereabove, between the 19th and 20th century, the study of geomagnetism gives birth to several scientific disciplines, ranging from rock magnetism and archeomagnetism, to dynamo theory. As highlighted in the Preface of the textbook from Merrill et al. [1998], unveiling the present and past field variations and their origins therefore requires an integrated approach.

#### 1.2 Sources of the geomagnetic field

The Earth's magnetic field results from the superposition of several contributions from internal and external origins, sketched in Figure 1.3a, together with their typical temporal



FIGURE 1.3: a) Sketch of the various sources of the Earth's magnetic field [Olsen et al., 2015]. b) Characteristic timescales of the Earth's magnetic field variations [from Turner et al., 2015] for the internal and external sources.

variations (1.3b).

By far, the largest contribution (more than 95% of the field observed at the Earth's surface) is generated by a self-sustained dynamo operating within the Earth's liquid outer core. At the Earth's surface, its amplitude varies from  $\sim 30000$  nT (at the Equator) to  $\sim 60000$  nT (at the poles). The main (or core) field time variations cover a broad range of timescales, from years to several million years [Constable and Johnson, 2005, see also Fig. 1.3b]. The second internal contribution is the crustal magnetic field, produced by the magnetization of lithospheric rocks. Its amplitude is on the order of a few hundred nT, with local anomalies reaching a few thousands nT [e.g. fig.1 in Lesur et al., 2016]. For main field modelling purpose, this contribution is often assumed static on the timescales of the secular variation (Fig.1.3b).

Turning to the external contributions, the first comes from the magnetosphere, enveloping the Earth up to 10 Earth's radii on the day side and up to several thousands Earth's radii on the night side. It is defined as the region below the boundary created by the interaction of the solar wind and the Earth's magnetic field. The magnetosphere's plasma is animated by electric currents inducing the magnetospheric field. These electric currents are strongly coupled with the electric currents in the ionosphere. This region defines an envelop between 80 km and a few thousands km above the Earth's surface. It corresponds to the high atmosphere ionized by ultraviolet rays from the Sun. These external fields are rather weak at the Earth's surface and only contributes to a few percent of the total measured field. However, they are strongly influenced by the Sun activity and can present large variations over short timescales (below a day) during magnetic storms. The typical variations range from seconds to a few years (see Fig.1.3b).

It should also be noted that the interactions of electric currents in the lithosphere, the mantle and the oceans with the time-varying external fields induce magnetic fields. These fields are however generally neglected as they are on the order of a few nT at the Earth's surface.



FIGURE 1.4: Spatial and temporal distribution of direct (a,d) and indirect records (b,c,e,f) of magnetic data (D, I, F) over the Holocene. From Korte et al. [2019].

### 1.3 Timescales of the main field variations

First and foremost, understanding the Earth's magnetic field starts by reconstructing its temporal variations, in the present and in the past. In the present, this is allowed by the continuous monitoring of the field by satellites and in magnetic observatories. The latter have been delivering full vector data since ~ 1850. Prior to this period, direct (or instrumental) observations can be recovered from archives, in particular from mariners minutes. The direct records present a good spatial coverage and cover the past ~ 400 years (see historical data, Fig. 1.4a,d) for directional data, but only starts in the ~ 1830s for absolute intensity measurements [Gauss, 1833].

Reconstructing the magnetic field variations further back in time requires analyses of indirect records of the magnetic field, provided either by archeological and volcanic material or by sediments. The former having experienced a heating fossilize an information on the geomagnetic field at the time of their cooling (see Section 2.1.2). As illustrated in Figure 1.4b,e, their spatial distribution is strongly biased towards the Northern hemisphere, onland, and they cover mostly the past three millennia. On the other hand, during the



FIGURE 1.5: Amplitude spectrum of the geomagnetic field. Figure from Constable [2007].

sedimentation process, magnetic grains can orientate within the Earth's magnetic field until a certain depth at which they are blocked by the sediments compaction, therefore fossilizing an information on the geomagnetic field. Although these records do not give an instantaneous representation of the geomagnetic field (but rather an averaged behavior over the duration of deposition), they provide the largest source of information on the past magnetic field from  $\sim 1000$  BC to  $\sim 10000$  BC. If the spatial distribution of the data is improved compared to volcanic and archeological material, there is still a strong bias towards the Northern hemisphere (Fig. 1.4c,f). Such indirect records are however scarcer in space and time (compared to direc measurements, Fig. 1.4), leading to lower resolution for the past field reconstructions.

Figure 1.5 shows the amplitude spectrum of the geomagnetic field and gives the associated internal and external processes contributing at various time scales. The main field contribution comprises periods ranging from one year or a few months to several million years. The amplitudes associated with timescales ranging from a few decades to a millennium, i.e. the so-called secular variation of the geomagnetic field, are not precisely known, for several reasons. First, the shortest timescales of the secular variation overlap the external contributions, making it difficult to isolate one contribution from the other [e.g. Constable, 2007]. Second, these ranges of timescales correspond to the lower limit of the temporal resolution of reconstruction based on historical data, but also to the upper limit of those based on paleomagnetic data, making these ranges of timescales difficult to accurately constrain. In addition, these variations probably result from different core processes, also covering a wide range of length and time scales, possibly overlapping each others, and further blurring the observed signal [e.g. Merrill et al., 1998, §4.2.8].

Over longer timescales, the best documented component of the field is the axial dipole, which is also the main component of the geomagnetic field. Its time variations cover a wide range of timescales, from a few years or less to several millions of years [Constable and Johnson, 2005]. Models of dipole moment power spectra obtained from paleomagnetic data and numerical dynamo simulations are shown in Fig. 1.6. The ultra-low (UF) and low (LF) frequency bands correspond to superchrons and chrons, i.e., long time intervals with no reversals of the magnetic field ( $\geq 10$  Myr for the former and  $10^5 - 10^7$  Myr for the latter). The transitional band (TF) corresponds to the paleomagnetic secular variations and the high frequency bands roughly corresponds to the convective turnover timescale in the outer core, which suggests that secular variations over these timescales are governed by convection in the outer core.

Therefore, both paleo-archeomagnetic data and direct records of the geomagnetic field are useful in assessing the geomagnetic secular variations over timescales of a few centuries or less. It remains that, owing to the scarcity of the presently available indirect data (Fig. 1.4), and their inherent uncertainties, global reconstructions of the paleomagnetic field cannot (or scarcely) reach the resolution needed to constraint secular variations over multidecadal and centennial timescales [e.g. Constable and Korte, 2015]. Nonetheless, over the past two decades, considerable efforts have been made by the archeomagnetic community in order to improve the quality, quantity and global coverage of archeomagnetic data. The dating constraints provided by archeology and/or archives and the generally stable thermoremanent magnetization carried by baked clay indeed makes archeological artefacts very suitable material for recovering the past geomagnetic field variations. The increasing amount of available indirect data, in particular in Africa [e.g. Osete et al., 2015, Tarduno et al., 2015, Kapper et al., 2017, 2020] and South-America [e.g. Hartmann et al., 2010, 2011, 2019, Poletti et al., 2016], is expected to improve the resolution and the accuracy of global paleomagnetic models.



FIGURE 1.6: Dipole moment spectra determined from paleomagnetic data (thick solid lines) and power law models for the spectrum obtained from numerical dynamo simulations obtained for various core advection times  $\tau_c$ . Various frequency bands with associated corner frequencies are identified (UF: Ultra-low, LF: low, TF: transitional, HF: high). From Olson et al. [2012].

### 1.4 Objectives and outlines of the manuscript

The studies carried in the present manuscript are part of these efforts to unravel the past geomagnetic field variations. They tackle two different and complementary aspects, one focusing on data acquisition and the other on a core process model.

A first study aims at constraining both regional and global geomagnetic field variations over the past four centuries. The reliability of global field reconstructions from direct measurements is restricted by the absence of intensity data between the end of the 16th and the middle of the 19th. To circumvent this issue, a solution is to rely on archeointensity data, perfectly adequate as over such recent periods, archeology and preserved archives can provide precise age constraints. The objective of this study is therefore twofold. First, it aims at further constraining regional intensity variations in Central Asia by the acquisition of new archeointensity data in Bukhara (Uzbekistan, central Asia), as well as at improving the global database. This city is especially attractive for archeomagnetic studies owing to its outstanding historical center, well preserved and dated thanks to a large amount of archives (just as well preserved). Second, these data can then be used to improve the global geomagnetic models based on direct measurements only and circumvent any approximation due to the absence of direct intensity measurement beyond 1830. Such an analysis also needs to discuss data quality.

The second study aims at analyzing extreme intensity events in terms of core processes. With the burgeoning number of archeomagnetic studies over the past two decades, the archeointensity field variations are incrementally improved, allowing to identify faster and/or sharper events in the geomagnetic field variations. However, the most extreme events recently proposed in the Near-East are puzzling as they are difficult to reconcile with our current understanding of core dynamics [e.g. Livermore et al., 2014]. A possible reason could be that the origin of such events lies beyond the current limits of this knowledge. These events therefore require to be further constrained by data. Here, a different approach is adopted, by proposing the analysis of a physical process possibly responsible for fast intensity variations observed from archeomagnetic data. More precisely, the process of magnetic flux expulsion has been proposed at the origin of extreme archeointensity spikes observed in the Near-East during the last millennium BC [Shaar et al., 2011], but also seems involved in the more recent, global variations of the geomagnetic field [e.g. Gubbins et al., 2006, Finlay, 2008. The aim of this study is therefore to propose a model for this process and to estimate its possible contributions to intensity variations of the Earth's magnetic field.

The outline of the manuscript is the following. In chapter 2, the various approaches allowing to study the geomagnetic field are described, with an account of their current limitations. A first part focuses on the acquisition of geomagnetic data, comprising direct and indirect measurements, with a particular focus on archeomagnetic data. The second section deals with the reconstructions of the past field variations, i.e., global models of the geomagnetic field. A third part briefly introduces the theoretical bases to understand the generation of the Earth's magnetic field and core dynamics. The results from all these approaches are integrated in a fourth part, summarizing the current knowledge on the geomagnetic field variations over the past four centuries. In chapter 3 is described the archeomagnetic analysis carried in Bukhara (Uzbekistan), allowing to reconstruct intensity variations at this location between the end of the 16th and the beginning of the 19th century. The results are then compared with archeomagnetic data available in Western Eurasia and with predictions from a global model built from direct measurements only. The implications in terms of global reconstructions of the geomagnetic field over this period, with a particular focus on the axial dipole variations are subsequently discussed. In chapter 4, a model of magnetic flux expulsion is described and analysed in details, with a thorough discussion on the implication for intensity variations at the Earth's surface and a comparison with extreme events observed from archeomagnetic data. Chapter 5 summarizes the main results and perspectives.

## Studying the Earth's magnetic field

### Résumé

Ce chapitre a pour but de présenter le contexte de cette thèse. Comme mentionné en introduction, l'étude du champ magnétique est un continuum de plusieurs disciplines, allant de l'archéomagnétisme à la magnétohydrodynamique. Ce chapitre présente donc une vue d'ensemble d'une partie de ces dernières, ainsi que les contraintes qu'elles peuvent apporter à notre compréhension des variations centennales du champ géomagnétique.

Dans une première section sont présentées les données disponibles pour reconstituer l'évolution du champ, comprenant des mesures directes sur la période historique, et des données indirectes sur des périodes plus anciennes, apportées par le paléo- et l'archéomagnétisme. Ces dernières étant au cœur du sujet de cette thèse, leur acquisition est traitée plus spécifiquement, suivi d'une discussion sur les bases regroupant ces données. Une deuxième section décrit comment les modèles globaux de champ magnétique sont construits à partir de ces données. L'attention est restreinte aux modèles qui couvrent uniquement ou en partie les quatre derniers siècles, dont la résolution spatio-temporelle est discutée. La troisième section se focalise sur l'origine du champ magnétique et ses variations. Le principe de la dynamo auto-entretenue et ses sources sont présentés ainsi que les équations de la magnéto-hydrodynamique sur lesquelles se basent les simulations numériques de la dynamo. La détermination de modèles de vitesse d'écoulement à la surface du noyau, à partir des données géomagnétiques et des connaissances théoriques et/ou apportées par l'analyse des simulations numériques est rapidement présentée. Enfin, une quatrième section intègre les connaissances apportées par ces différents domaines et présente ce qu'elles permettent de comprendre de l'évolution du champ sur les quelques derniers siècles.

#### 2.1 Measuring the Earth's magnetic field

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The magnetic field vector **B** is characterized at any point of the Earth's surface by its intensity F and directions (declination D, inclination I) (see Fig. 2.1). Considering a local Cartesian reference frame at the observation point, the vector field can be decomposed into three components (X, Y, Z), corresponding to the northward, eastward and downward components respectively, defined as

$$X = F \cos D \cos I = -B_{\theta},$$
  

$$Y = F \sin D \cos I = B_{\phi},$$
  

$$Z = F \sin I = -B_{r},$$
  
ith  $F = \sqrt{X^{2} + Y^{2} + Z^{2}}.$   
(2.1)

Measuring the magnetic field at the Earth's surface therefore amounts to determining either (F, D, I) or (X, Y, Z) (or equivalently  $(B_r, B_\theta, B_\phi)$  in spherical coordinates).



FIGURE 2.1: Components of the Earth's magnetic field. X, Y, Z are the northward, eastward and downward components of the magnetic field vector **B** respectively. F is the magnetic field intensity. H is the intensity of the magnetic field vector projected on the horizontal plane. Dis the declination of the magnetic field vector i.e. the angle between the geographic North and the magnetic North and I the inclination i.e. the angle between the magnetic field **B** and the horizontal plane.

#### 2.1.1 Direct measurements

Since the advent of compasses, the Earth's magnetic field is used for orientation, particularly at seas. However, the first measurements of the magnetic declination in Europe only date back to the early 16th century onland and late 16th century at seas, while first measurements of inclination date back to the second part of the 16th century [see for example Jonkers et al., 2003, Le Goff and Gallet, 2017]. The secular variation of the magnetic field is incidentally established in 1635 by Henry Gellibrand, from repeated measurement of declination in London. This discovery discloses the need for continuously monitoring the Earth's magnetic field. Since then, efforts had been put into compiling the existing data to study the Earth's field variations and morphology. Some compilations were assembled before that time but contained non-dated data, as there was apparently no need for precise dating. The term "direct measurements" therefore refers to instrumental measurements of the magnetic field components and the period over which they are available is usually referred to as the "historical period", the lower end of this interval being generally set at 1590 (see Fig. 1.4a,d).

#### a) Mariners observations and surveys

The largest amount of data available for the past four centuries originates from mariners observations (Fig. 1.4a,d). During the late Age of Discovery (16th and 17th), Europeans undertook large-scale explorations and colonisation overseas, particularly on the American continent and the Indies, hence providing a useful source for magnetic data, acquired during the voyages [Jonkers et al., 2003]. Despite the sastifying accuracy of such measurements [Jackson and Finlay, 2015], a number of bias and errors have to be accounted for. First, for a given measurement, the position of the observer has to be precisely known. While the determination of latitude is accurate since the end of the 15th, the determination of longitude remains quite problematic until the late 18th century, with the invention of the marine chronometer by John Harrison. Before that time, navigators used dead reckoning, which precision decreases with the duration of the journey. For modeling purpose, the positioning error arising from this method needs to be quantified. To this end, a method is proposed by Jackson et al. [2000], based on the assumption that the cumulated error on longitude follows a Brownian bridge model. Once data are corrected for this effect, the observational error has to be quantified and the crustal magnetic field needs to be corrected for. Devices for measuring the magnetic inclination were developed during the late 16th century [e.g. Jonkers et al., 2003, Courtillot and Le Mouël, 2007], but the complexity of such measurements at seas results in sparser marine observations of inclination.

With the 19th century starts the development of specific surveys and expeditions (marine, onland and aerial) dedicated to geomagnetic measurements, therefore supplying full vector measurements. One of the first of the kind has been led by Alexander Von Humboldt from 1799 to 1803, who measured both D, I and relative F [Courtillot and Le Mouël, 2007] (since the method for measuring absolute intensities only dates back to 1833, see below). Since then, the amount and quality of data have continuously increased, with the advent of magnetic observatories and later satellite era.

#### b) Magnetic observatories and repeat stations

The first magnetic observatories are established during the second part of the 17th century in Western Europe, where are carried out first systematic measurements of declination and later on inclination [e.g. Alexandrescu et al., 1996]. In 1833, Gauss' discovery of a method to measure absolute intensities [Gauss, 1833] triggers the establishment of the first magnetic observatories network, the Göttingen Magnetic Union, suggested by Alexander Von Humboldt and undertaken in 1836 [Courtillot and Le Mouël, 2007]. Nowadays, this task is fulfilled by the INTERMAGNET network (www.intermagnet.org) which continuously provides data from worldwide observatories, under the form of onesecond/one-minute/hourly/monthly or annual means. The INTERMAGNET observatories are required to meet certain criteria regarding data acquisition and processing. Other non-INTERMAGNET observatories provide one-minute or hourly means via the World Data Center (WDC) for Geomagnetism in Edinburgh (www.wdc.bgs.ac.uk). The entire network is constituted by ~ 190 observatories. Fig. 2.2 shows a map illustrating the distribution of the observatories. The spatial coverage is heavily biased towards Europe, as the first magnetic observatories were developed there. The observatories are located onland,


FIGURE 2.2: a) Distribution of the magnetic observatories listed by the WDC. b) Temporal distribution of the magnetic data provided by the observatories as annual means listed by the WDC.

leaving gaps in the oceans, particularly in the Pacific. Accordingly, the South hemisphere and the oceans are poorly covered.

In addition to the permanent observatories, repeat stations data improve the spatial coverage of ground-based observations. These measurements are regularly undertaken every five years at fixed locations onland and are currently used as a safety net in case satellite data are unavailable [Hulot et al., 2015].

# c) Satellite data

The advent of the satellite era marks a decisive turn in geomagnetism. The first satellite missions monitoring the Earth's magnetic field for the purpose of global field modeling are the POGO (Polar Orbital Geophysical Observatory) missions launched in the 60's and 70's, to measure the magnetic field intensity. The following missions then provide vector measurements of the magnetic field: Magsat (1979-1980); Ørsted (1999-2014) ; CHAMP (2000-2010) and SAC-C (2001-2004) [see Hulot et al., 2015, for a review]. More recently, the SWARM satellites constellation, launched in 2013, provides data with high precision (better than 1 nT, Olsen et al. [2013]).

Unlike magnetic observatories, satellite data allow for a global coverage of the Earth's magnetic field sampling (except for geographic poles), with the same instrumentation. However, the processing of these data raised a number of issues worth mentioning. First, as the satellite orbits are located within or above the ionosphere, the separation of the external field contribution from the internal one is not straightforward [e.g. Olsen et al., 2010]. As the magnetic field is never sampled twice at the same location at the same time, it is also difficult to distinguish between spatial and temporal variations. Finally, the satellite era covers the past  $\sim 60$  years, therefore sampling only the high frequency variations of the magnetic field [e.g Hulot et al., 2015]. The joint use of observatory data and satellite data can help to partially solve these issues.

# 2.1.2 Indirect measurements

Direct measurements provide information on the past geomagnetic field variations over the historical period (i.e. from ~ 1590 to today). However, there is paleomagnetic evidence that the geomagnetic field is at least ~ 3.5 Gyr old [e.g. Usui et al., 2009]. To understand the long-term variations of the Earth's magnetic field, we therefore need information from further back in time. Such information can be recovered from indirect records of the Earth's magnetic field, as provided by rocks and archeological artefacts, on timescales ranging from a few centuries to several billions of years ago. If these materials provide non-continuous records in space and time (particularly biased toward recent periods and Northern hemisphere, see Fig. 1.4b, e), they constitute the only window on the past geomagnetic field evolution. The magnetic minerals composing these recorders keep a "memory" of the field. To understand how we can reconstruct the field variations, it is therefore necessary to understand how the magnetic minerals record the field and how this information is extracted.

In this section, the basics of remanent magnetization acquisition and measurement are introduced, following the classical textbooks of Butler [1992], Dunlop and Özdemir [2001], Tauxe et al. [2018], to which the interested reader is referred to for more details and references.

#### a) Magnetic properties of ferromagnetic material

Magnetic remanence is a property carried by ferromagnetic material, i.e. material possessing their own magnetic field when no external field is applied. The magnetic properties of a material originate from the magnetic moments at the atomic level, resulting from electronic motions. Two sources contribute to this moment: the quantum state of theirs spins and their orbital angular momentum.

The magnetic minerals can be distinguished in two main groups: minerals possessing their own magnetic field in the absence of an external field (ferromagnetic minerals) and minerals possessing a magnetic field only when an external field is applied (dia- and paramagnetic minerals). In the case of a ferromagnetic material, the strong interactions of adjacent atomic moments (resulting from the interactions between unpaired spins of neighboring electrons from different atoms) produces an internal magnetic field. The magnetization resulting from the sum of these moments is called remanent magnetization, in contrast with induced magnetization.

The interaction between magnetic moments at the atomic level is quantified by the exchange energy. If the interatomic distance increase, for example under the effect of thermal expansion, the exchange energy weakens. At the Curie temperature  $(T_C)$ , the atomic moments are not coupled anymore. The material loses its ferromagnetic properties and becomes paramagnetic: the spins are randomly aligned and will align under the effect of an external field. On the other hand, if the interatomic distance decreases, the exchange energy increases. In a crystalline structure, electrons orbitals can partially overlap, resulting in a strong coupling of adjacent atomic moment. Therefore, an anisotropic crystal can lead to preferential spins alignment, and to preferential directions of magnetization acquisition (called easy axis). This dependency is referred to as magnetocrystalline anisotropy.

The crystalline structure and elements can lead to various spins alignment. If the spins alignment is parallel, the material is ferromagnetic (in a strict sense, see Fig. 2.3a), if the spins alignment is antiparallel resulting in a null magnetic moment, the material is antiferromagnetic (Fig. 2.3b). If the spin moments are antiparallel and of different magnitude such that they result in a net magnetization, the material is ferrimagnetic (Fig. 2.3c). Note that there is a fourth possible configuration, known as spin-canted antiferromagnetism, represented on Fig. 2.3d, arising when antiferromagnetic spins are not perfectly parallel.

Rocks or archeological artefacts consists of an assemblage of magnetic and non-magnetic mineral grains, each grain itself being an assemblage of crystals. If uniformely magnetized, the grains are single domain (SD) grains. However, depending on their size and shape,



FIGURE 2.3: Various configurations of spins alignements leading to a) ferromagnetism (stricto sensu), b) antiferromagnetism, c) ferrimagnetism, d) spin-canted antiferromagnetism. For a,c,d these alignments result in a net magnetization.

the grains can also be subdivided in several magnetic domains of uniform magnetization separated by domain walls, the latter defining the area where the spins orientation changes between two domains. Such grains are referred to as multi-domain (MD) grains.

The total energy of the grain is therefore the sum of the anisotropy energy (magnetocrystalline anisotropy, shape anisotropy introduced by the grain shape,...) and the exchange energy. In an external field, the total energy will be minimum when the magnetic moment is aligned with the external field. The anisotropy energy however acts as a barrier preventing the rotation of magnetic moments to align within the external field. Consequently, for a ferromagnetic material (in a large sense) to become a recorder of the Earth's magnetic field, the anisotropy barrier must be overcome in order for the magnetic moments to statistically align within the geomagnetic field.

#### b) Acquisition of remanent magnetization

In an external field, the direction in which the total magnetic energy of a moment is minimized is called easy axis of magnetization. To spin the magnetic moment, the applied energy has to overcome the anisotropy energy barrier, allowing the magnetic moment to align within the external field. In the case the applied energy is of thermal origin, it leads to the acquisition of a thermo-remanent magnetization (TRM).

In 1949, Néel formalised a theory of TRM acquisition for non-interacting single-domain grains [Néel, 1949]. The magnetic moment carried by an assemblage of single domain grains progressively loses its magnetization over time due to thermal agitation. The magnetization M evolves as

$$M(t) = M_0 \exp\left(-\frac{t}{\tau}\right),\tag{2.2}$$

with  $M_0$  the initial magnetization, t the time and  $\tau$  the relaxation time, defined as the time needed for M to decrease to  $M(\tau) = 1/eM_0$  (Fig. 2.4). The relaxation time is given by

$$\tau = \tau_0 \exp\left(\frac{KV}{kT}\right). \tag{2.3}$$

 $\frac{KV}{kT}$  is the ratio of anisotropy energy over thermal energy, with K an anisotropy constant, V the grains volume, k the Boltzmann constant and T the temperature.  $\tau_0$  is the attempt time, i.e. the atomic reorganisation time, on the order of  $10^{-9}$  s.

In an external magnetic field  $H_0$ , the magnetization of an assemblage of SD grains of initial magnetization  $M_0$  evolves as

$$M(t) = M_0 \exp\left(-\frac{t}{\tau}\right) + M_{\rm eq}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right],\tag{2.4}$$

with  $M_{\rm eq}$  the equilibrium magnetization

$$M_{\rm eq} = M_s \tanh\left(\frac{\mu_0 V M_s H_0}{kT}\right). \tag{2.5}$$



FIGURE 2.4: Left: Relaxation of the remanent magnetization with time, following Eq. 2.2. At the time  $\tau$ , the initial magnetization  $M_0$  have decreased from  $M_0$  to  $M_0/e$ . Right: Evolution of the relaxation time  $\tau$  with temperature, for single-domain magnetite. Above the blocking temperature  $T_B$ , the grains are superparamagnetic and below  $T_B$ , the grains are able to acquire a stable remanent magnetization. Figures from Butler [1992].

Here,  $M_s$  is the saturation magnetization (when all the spins are aligned) and  $\mu_0$  is the magnetic permeability of free space. The relaxation time evolved as

$$\tau = \frac{1}{C} \exp\left[\frac{KV}{kT} \left(1 - \frac{H_0}{H_c}\right)^2\right],\tag{2.6}$$

where  $H_c$  is the coercitive field (the field required to overcome the anisotropy barrier). After a time  $t \approx \tau$ , the magnetization is equivalent to the equilibrium magnetization  $(M(t) \approx M_{eq})$ .

For magnetic minerals to act as a memory of the Earth's magnetic field, this relaxation time must be on the order of the Earth's age. Eq. 2.6 shows that the relaxation time is highly sensitive to the grains volume and the temperature. Below a certain volume, the magnetic grains are too small to acquire a memory. Likewise, above a certain blocking temperature  $T_B$  the relaxation time is short (a few seconds). Above  $T_B$ , the grains are superparamagnetic (see, Fig. 2.4) and magnetic moments equilibrate quickly within an applied magnetic field. When cooled down below this temperature, the grains ( $M_{eq}$ ) are blocked by the anisotropy energy and the relaxation time is significantly increased. The grains hence acquire a TRM (Fig. 2.4). For weak fields such as the geomagnetic field

$$M_{\rm eq} \approx M_s \frac{\mu_0 V M_s H_0}{kT}$$

i.e., the TRM is directly proportional to the magnetic field intensity  $H_0$ . In addition,  $M_{eq}$  is demagnetized only when heated again at  $T_B$ , i.e. the unblocking temperature  $T_{UB} \approx T_B$ .

The ability of a magnetic mineral to acquire a stable remanent magnetization over timescales equivalent to the Earth's history is the keystone of paleomagnetism. This ability is highly dependent on the magnetic minerals properties carrying the natural remanent magnetization (NRM).

Hereabove is described the thermal acquisition of a NRM, but other processes allow to record the geomagnetic field. For example, during sediment deposition, the magnetic grains can orientate within the geomagnetic field. Below a certain depth, when the grains are compacted so that they cannot rotate, the magnetization is blocked in a so-called detrital remanent magnetization (DRM). An other example is chemical remanent magnetization (CRM) whereby ferromagnetic grains are formed by chemical alteration and acquire a remanence at the time of alteration.

Over the past few centuries, the recorders are mostly constituted by archeological and volcanic material (see Fig. 1.4a,b,d,e). These materials acquire a NRM by cooling below the Curie temperature, which is a TRM. For longer periods, the second largest source of data comes from DRM acquired by sediments (see Fig. 1.4c,f). As the subject of this thesis is focused on the past four centuries, only archeological and volcanic material will be considered in the following.

#### c) Measurement of a thermo-remanent magnetization

Reconstructing the Earth's magnetic field evolution requires records of directions and intensity of the ancient field, both of which can be estimated from intensity and directions of the recorder's NRM. Depending on the recorder's history, secondary magnetization components of viscous, chemical or thermal origin (Fig. 2.5) can be acquired. To recover the primary component of magnetization acquired during the object formation, it is therefore necessary to remove these components before determining the paleodirections and/or intensity. As illustrated in Fig. 2.5a, most often secondary components do not affect all the magnetic grains. In this example, the sample underwent a reheating up to a temperature  $T_2$  (Fig. 2.5a), lower than the blocking temperature, leading to the remagnetization of a certain proportions of the magnetic grains. In a grain assemblage, the less stable grains have a short relaxation time and will be affected by the ambiant magnetic field, say below  $T_1$  (Fig. 2.5a). Therefore the resultant NRM of the sample is the sum of the viscous remanent magnetization (VRM), the secondary partial TRM, and the primary TRM (Fig. 2.5b). In this case, the primary TRM can be identified and isolated. It is also possible to find samples which underwent a complete remagnetization, resulting from reheating above the Curie temperature or chemical alteration, for instance.

To recover the past directions, the recorder is sampled *in situ*, i.e. in the position it acquires its magnetization, as for instance lava flows or kilns (while the sample position is not needed to recover the past field intensity). The *in situ* sampling of archeological or volcanic structures is usually done either by core drilling or block sampling. In archeomagnetism, a common technique for block sampling is the "plaster cap technique": a part of the structure is isolated and molded in plaster (see Fig. 2.6a,b). This method allows the sampling of fragile or brittle structures that would be impossible to sample by core-drilling. The sample must be carefully oriented when collected for analyses. Generally, it is oriented with respect to the magnetic North and the Sun (geographic North), providing a precise orientation. For displaced material (found in a different position than during its firing), it is sometimes possible to determine the position of the artifact during its firing, for instance bricks, pottery or tiles, thereby allowing the determination of the ancient inclination [Thellier, 1938, Lanos, 1987].

In laboratory, the sample is usually step-wise demagnetized by thermal treatment or alternating field, and the three components of the magnetization are measured in the sample reference frame. Usually, such procedures are carried on small samples (of order  $\sim 10 \text{cm}^3$ ) due to instrumental constraints. The directions are then recovered from the *in situ* sample orientation. The protocol used at IPGP laboratory (formerly St-Maur laboratory) differs from the classical one described hereabove and is inherited from Thellier [described in Thellier, 1981]. In order to preserve the orientation precision obtained from the plaster cap technique, Émile Thellier and later Maxime Le Goff designed and constructed instruments

able to perform measurements directly on these large samples of typical size on the order of  $\sim dm^3$  [see Fig.2.6c,d,e; Thellier, 1967, Le Goff, 1975]. The method of Thellier [1981] relies on the assumption that the total NRM is the sum of a VRM and a TRM acquired during the last firing of the structure; i.e. there is no secondary magnetization (apart the VRM component). Therefore, archeological kilns, burried after their short use (as is the case for domestic kilns) are particularly suited for this method [Le Goff et al., 2020]. To estimate the effect of the VRM component, the sample is stored in laboratory in the same position it has on the field for several weeks. The remanent magnetization measured after this time is therefore the sum of the last TRM and the VRM. The sample is then stored in the opposite direction (compared to the first storage position) for several weeks (four weeks minimum). In the end, the measured magnetization should be the TRM minus the VRM. A magnetic viscosity index is defined by the ratio of these two measurements. If the index is lower than  $\sim 10\%$ , the effect of viscosity on TRM is assumed negligible and the TRM can be determined from simple vector subtraction. Otherwise, the sample is rejected. The absolute directions are then recovered from the sample orientation relative to the geographic North. Other classical protocols involve a step-wise demagnetization of the sample (by thermal treatment of alternating field) with measurements of the magnetization components in the sample reference frame at each step. In any case, several samples of the same thermal unit (a lava flow or an oven) are measured and the mean directions are statistically determined using a Fisher probability density function [Fisher, 1953].

On the other hand, the determination of a paleointensity is more complex. It relies on the assumption that the relationship between the applied field and the magnetization is linear for low field intensities, as is the case for the Earth (recall section 2.1.2, see also Fig. 2.7), such that

$$M_{NRM} = c_{anc} B_{anc}, \tag{2.7}$$

where  $M_{NRM}$  is the NRM,  $B_{anc}$  is the intensity of the ancient field and  $c_{anc}$  is a constant.

In principle, the measurement of a paleointensity requires to measure the NRM, and to replace it with a new thermo-remanent magnetization (TRM) acquired in a laboratory field of known intensity and directions. The new laboratory TRM is therefore

$$M_{lab} = c_{lab}B_{lab}.$$
 (2.8)

If  $c_{anc} = c_{lab}$ , then

$$B_{anc} = \frac{M_{NRM}}{M_{lab}} B_{lab}.$$
(2.9)

However, in practice the proportionality constant c depends on the material properties (for instance grain sizes, see Fig. 2.7) and the conditions of remanence acquisition (recall subsection 2.1.2). It is therefore prone to change if the experimental conditions are too far from the conditions of acquisition of the NRM or if the sample is altered during the laboratory treatment.

The most common method for paleointensity data acquisition was designed by Thellier and Thellier [1959], based on a step-wise demagnetization of the sample upon heating. Between two temperature steps, the sample is heated and cooled in a laboratory field to acquire a new partial thermo-remament magnetization (pTRM). Then the sample is heated and cooled again in a laboratory field of same intensity but opposite direction. Vector subtraction allows to recover the fraction of the remaining NRM at the end of the double heating step. The procedure is repeated until the NRM is fully demagnetized and replaced by a new TRM. The experiment is represented in Fig. 2.8a which shows at each temperature step the fraction of remaining NRM (black dots) and the fraction of gained



FIGURE 2.5: (a) Schematic of the grains proportions mobilized by different magnetization components.(b) Schematic of the various magnetization components. The sum of all components results in the total natural remanent magnetization (NRM). Figures from Turner et al. [2015].



FIGURE 2.6: a) Sampling of a medieval pottery workshop from Middle-Age (~ 1400) in Fiennes. b) Sampling of a medieval pottery workshop from a Middle-Age oven in Chamigny. c) The big sample spinner magnetometer [, or "Bellevue inductometer", Thellier, 1967]. d,e) Rotating inductometer for "big samples" [Le Goff, 1975]. Courtesy of Yves Gallet and Maxime Le Goff.



FIGURE 2.7: Predicted evolution of TRM normalised by sIRM (saturation isothermal remanent magnetization) as a function of the applied field, for single-domain grains of various sizes. The yellow box gives the range of the present field intensities. Figure from Tauxe and Yamazaki [2015].



FIGURE 2.8: (a) Example of step-wise demagnetization of the NRM and acquisition of a new TRM in a known laboratory field. (b) Example of an Arai plot representing the pTRM gained at each temperature steps versus the remaining NRM. Figures from Tauxe and Yamazaki [2015].

pTRM (white dots). The results are represented on an "Arai plot" [Nagata et al., 1963], giving the gained pTRM versus the remaining NRM (see Fig. 2.8b). The paleointensity is given by the slope of the Arai plot multiplied by the laboratory field intensity (recall Eq. 2.9).

Following Thellier and Thellier [1959], several alternative protocols had been proposed, categorized as double-heating methods. The family of double-heating methods relies on a number of assumptions on the pTRM behavior [Thellier and Thellier, 1959]. First, the reciprocity law states that the blocking and unblocking temperatures should be the same (if a pTRM is acquired by cooling from  $T_2$  to  $T_1$ , this pTRM is demagnetized by heating in zero field from  $T_1$  to  $T_2$ ). The independence law states that each pTRM acquired on a temperature interval should be independent from pTRM acquired on a different interval. The additivity law states that the final magnetization should be the sum of all the pTRM. For a paleointensity result to be reliable, these conditions must be tested and verified. To this end, various number of checks and corrections had been developed [for an extensive review on the Thellier-Thellier method and derivates as well as checks and corrections procedures, see Dunlop and Özdemir, 2001].

In particular, the large number of heating steps jeopardizes the thermal stability of the magnetic minerals, increasing the probability of alteration during the process. If there is alteration, the fundamental assumption of a unique proportionality constant between the field intensity and the magnetization is not verified. The "pTRM checks" (additional step to acquire a new pTRM at a certain temperature to compare with the pTRM acquired during the double-heating step) are supposed to check for the alteration during the heating process. Nowadays, this test is part of the standard procedure but most data before 1985 are not systematically provided with pTRM checks [Genevey et al., 2008]. Alternative methods had also been developed in an attempt to reduce possible magnetic alteration due to thermal treatment, for instance by using micro-wave [Walton et al., 1993] or alternating field [Shaw, 1974], or by decreasing the number of heating steps, for instance the multi-specimens method [Dekkers and Böhnel, 2006, Fabian and Leonhardt, 2010].

Another key parameter to control in paleointensity experiments is magnetic anisotropy. Depending on the crystalline structure, some directions of magnetization might be preferred during the remanence acquisition (easy axis). This is the case for example for archeological artefacts such as tiles or pottery shaped by the stretching of clays, hence resulting in a preferential orientation of the magnetic grains. For strong anisotropy effects, the anisotropy tensor can be determined from various measurements [thermo-remanent



FIGURE 2.9: a) Example of the effect of anisotropy correction on intensity determination from French ceramic fragments. From Genevey et al. [2008]. b) Example of the effect of anisotropy correction on the standard deviations distribution of obtained intensities. From Chauvin et al. [2000].

magnetization, anhysteretic remanent magnetization, isothermal magnetization or magnetic susceptibility, see Genevey et al., 2008, for details] in order to correct the effect on the TRM acquisition. However these tensors are not strictly equivalent and it has been experimentally shown that the anisotropy tensor of TRM gives the most accurate correction [Chauvin et al., 2000]. Effects of this correction are illustrated on Figure 2.9. Fig. 2.9a shows intensity values obtained for fragments of French ceramics with or without anisotropy correction. The correction tends to increase the intensity values from  $\sim 10$  to  $\sim 60\%$ . Note that the anisotropy correction does not always increase the intensity value, depending on the applied field directions with respect to the TRM. Figure 2.9b shows the histogram of the errors on the mean paleointensity for a set of French bricks and tiles fragments. The anisotropy correction has the effect of decreasing the errors (around  $\sim 5\%$ ) and narrowing the distribution. On the other hand, if the anisotropy is weak enough, applying the laboratory field parallel to the NRM directions reduces anisotropy effects on the new TRM acquisition and neglecting these effects becomes a reasonable approximation. Ideally, if the laboratory field is applied parallel to the ancient field, the anisotropy does not affect the TRM acquisition [Veitch et al., 1984].

Another important effect to take into account is the TRM dependency on the cooling rate (CR). The longer the cooling time, the larger the amount of magnetic grains reaching the equilibrium with the external field and the lower the blocking temperature Dodson and McClelland-Brown, 1980]. If the blocking temperature is lower, the remanent magnetization will be larger (lower thermal agitation and higher saturation magnetization for lower temperatures). However, in laboratory, the cooling rate is often faster (from  $\sim 30$  minutes to a few hours) than the original one (a day to several months for lavas), possibly leading to an overestimation of the intensity by more than 10% [Genevey and Gallet, 2002] up to 20% [Hervé et al., 2017, 2019] [see Genevey et al., 2008]. This cooling rate effect can be determined and corrected for, either by additional experiments on the samples to quantify it, or by an approximation of this effect ("educated guess"); the major challenge being to estimate the initial CR. Fig. 2.10a shows examples of overestimations of the intensity value for several fragments from Syrian bricks and ceramics and French ceramics as a function of a cooling time ratio (slow varying CR over fast constant CR of 30 minutes). The faster the experimental CR (compared to the original natural one), the larger the TRM overestimation. This figure also illustrates that these effects are variable from one sample to



FIGURE 2.10: Cooling rate dependency of TRM acquisitions. a) Exemple of the evolution of the TRM overestimations as a function of the cooling time for fragments of french and syrian ceramics.b) Proportions of fragments affected by various overestimates of TRM from groups of french ans syrian ceramics. From Genevey et al. [2008].

another. Fig. 2.10b) shows a histogram of intensity overestimations on these fragments, presenting a Gaussian distribution, centered around a value of  $\sim 7\%$ . Here, the CR effect is estimated between two TRM, one obtained from rapid cooling rate (30 minutes) and the other from slow cooling rate. If the former was chosen slower, the effect would therefore be lower as well as the overestimations.

Finally, for multi-domain grains, the domain walls can be displaced by the thermal treatment, leading to different blocking and unblocking temperature. In this case, the reciprocity law for double-heating method is not verified. Various tests had been developed to check anomalous behaviors resulting from MD grains during the thermal treatment [e.g. Riisager and Riisager, 2001].

Taking all these effects into account can prove challenging and time-consuming. In the present study, the method used for archeointensity measurements is the Triaxe protocol, designed by Le Goff and Gallet [2004] and derived from Thellier and Thellier [1959] [see Le Goff and Gallet, 2004, or the article presented in chapter 3 for a description], which allows to overcome these effects. This protocol allows to perform magnetization measurements automatically and continuously at high temperatures. In addition, the evaluated quantity for intensity determination has been experimentally proven independent of the cooling rate effect. The TRM dependency on anisotropy effects is also negligible, as the applied field  $H_{lab}$  is adjusted such that the new TRM is acquired parallel to the NRM. Finally, as the experimental conditions of TRM acquisition are as close as possible to the condition of acquisition of the NRM, the MD effects are negligible.

# d) Dating constraints

By their very nature, archeological and volcanic material provide non-continuous records in space and time of the Earth's magnetic field (recall 1.4b,e). The reconstruction of the past field variations therefore requires these records to be anchored in time. Dating techniques are divided in two main families. On the one hand, laboratory (or physico-chemical) methods provide absolute dating constraints, with for instance radiometry, chemistry or quantification of alteration caused by radiometry [Bahain and Antoine, 2012]. On the other



FIGURE 2.11: Age range covered by various dating techniques used in geology and archeology for the last million years. From Bahain and Antoine [2012].

hand, field-based approaches can provide relative and or/absolute age constraints, derived from archeological constraints and/or preserved archives. Over the period of interest, i.e. the historical period, the most common dating constraints are provided by thermoluminescence, radiocarbon, or historical/archeological constraints (archives, stratigraphy, typology, ...).

Thermoluminescence is a method based on the measurement of radioactivity stored in minerals, providing the age of the last firing of the material with uncertainties of order 7 - 10%. This degree of accuracy makes it used mostly to estimate a first chronological anchor when no other dating constraints are available [Evin et al., 2005]. The major interest compared to radiocarbon dating is the age interval covered by this method (see Fig. 2.11).

Radiocarbon dating is a method allowing to obtain the age of death of an organic material (if the latter happened over the past ~ 50000 yrs, see Fig. 2.11). The principle is to measure the amount of remaining <sup>14</sup>C, and from the law of radioactive decay, to deduce the age of death of the material. In archeology, this method can be applied on charcoal resulting from the last use of a kiln for instance. This method allows relatively accurate dating constraints (with typical errors of  $\pm 30 - 50$  yrs for the last millennium) providing that a number of conditions are fulfilled. For instance, a well-known error is the "oldwood" effect. This method indeed provides the age of death of the material. For charcoal found in oven for example, that would correspond to the age of death of the organism, i.e. when the tree or branch was cut down. If too much time passed between the cutting and the burning, the sample will provide a misleading result, older than the expected age. Therefore special care have to be taken to link the dated material and the archeological context, as for example the use of small branches or seeds for radiocarbon dating as these are not likely to have been reused or stored for a long time.

For recent periods, about 4% of data are dated from radiocarbon and 1% from thermoluminescence, while more than 50% of data are dated from archeological and historical constraints [combining directional and intensity data, Brown et al., 2015]. Archeological constraints are mostly based on stratigraphy and/or typology of discovered artefacts. In archeology, the principles of stratigraphy are the same than those used in geology (superposition, homogeneity, ...). They allow to establish a relative chronology between the layers of an archeological site, providing that these layers are sealed and unperturbed. If they contain objects than can be dated in an absolute sense ("diagnostic artifacts"), the relative chronology can then be anchored in time. For well-documented contexts, some objects possessing specific characteristics allow to assess their age (typology). It is worth noting that a key element here is the link between the objects used for archeomagnetic analyses and the dating constraints on the archeological context. A simple example is the analogous to the "old-wood" effect. In the framework of archeomagnetism, we are looking for dating constraints on the last firing of an object. If this object is found in a sealed archeological layer, how does the dating constraints relate to this object? It is important to ensure that the object is produced during the deposition of the layer and not after nor before as can be the case for rare pottery fragments. In this respect, very favorable contexts in archeomagnetism are manufacturing sites, as pottery production site for example.

Conversely, archeomagnetism can also be used as a dating tool. The comparison of direction and/or intensity data obtained for an undated archeological artefact with pale-osecular variation curves (PSVC) obtained in the same geographical area can give probable age intervals for the last firing of this object [Le Goff et al., 2002, Lanos, 2004]. The dating precision therefore relies on the precision of the reference curve, based itself on the precision of data used to construct it. It also depends on the variability of the PSVC: periods during which the PSVC present rapid fluctuations are more suitable to get precise dating constraints. For example, in France, the Middle-Age ( $\sim 800$  to  $\sim 1400$  AD) is particularly favorable for the use of archeomagnetic dating owing to the large directions variations of the field during this period (mostly in inclination, see Fig. 2.12) while for objects from the Roman Period, it is severely hindered by the tight loop in the variations.

To compute a reference curve from archeomagnetic data acquired at different locations, it is necessary to relocate the results at a single site, assuming that the Earth's magnetic field is an axial dipole. The relocation of data induces errors increasing with distance by a mean value of  $0.2^{\circ}/100$  km in directions and  $0.15 \ \mu T/100$  km [Casas and Incoronato, 2007]. Usually, reference curves take into account data available in a radius of ~ 1000 km. In regions where the amount of data is not sufficient to compute a regional reference curve, one can use global geomagnetic models to compute a variation curve at the location of the object to be dated. Due to the smoothness of such models (see subsections 2.2.3 and 2.2.4), the dating constraints obtained from such curves should be treated cautiously. Nonetheless, when no other constraints are available, this method can provide useful constraints for archeologists.

# 2.1.3 Paleo- and archeomagnetic databases

For global magnetic field modelling purposes, paleo-/archeomagnetic data are compiled in global databases, the aim being to standardize the input to feed to the model. Accounting for data complexity is therefore challenging owing to the large diversity of data. The amount of information entered in the database is up to its authors, who have to compromise between standardizing the data and capturing their complexity. The more information



FIGURE 2.12: Directional variations curves of the geomagnetic field in France for the last 2000 yrs obtained from archeomagnetic data. All directions were reduced to Paris. The mean curve is obtained from average over moving windows following Le Goff et al. [2002] with the corresponding 95% confidence ellipses in grey. The colored circles shows the 95% confidence interval of the archeointensity data. The blue curve gives variations obtained from direct measurements. From Le Goff et al. [2020].



FIGURE 2.13: Example of archeomagnetism dating of the burnt rampart of Beeston Castle (England). In this case, the obtained inclination and declination are compared separately to the corresponding reference curves. Their combined probability density functions give two dating interval, but only the younger one (520-425 BCE) is consistent with the archeological context. From Hervé and Lanos [2018].

kept, the larger the metadata content and the harder the global treatment of the data. On the contrary, if information are standardized, the data complexity is not entirely accounted for but the data processing is in turn made easier.

In this thesis, the main focus is on intensity data. As previously mentioned, their acquisition is complex and subject to a number of assumptions underlying the Néel theory and the experimental protocol (i.e., SD grains, pTRM laws,...) . The diversity of possible recorders (magnetic mineralogies, magnetization acquisition processes,...) results in a variety of methodologies for intensity determinations, tests and corrections, which adds to the complexity of data. A few examples are examined in the following.

For *in situ* structures such as archeological ovens, the definition of an archeological site is straightforward: the archeological site is the dated structure (the oven) and the mean directional or intensity values are determined from the average of N values obtained from the N fragments of the structure. For displaced objects such as potsherds, the definition of a site is variable depending on the context and on the authors. If several fragments are dated from the same age, the mean intensity value is derived from the average of intensities obtained at the fragment level. This could be the case for example when unambiguously sealed unearthed layer are identified during archeological excavations. The key assumption here is "sealed" in order to ensure the chronological homogeneity of the material within the layer. Note that, even if this chronological homogeneity is ensured, the field intensity obtained from mean of fragment values is averaged over the time during which the laver settled. In this case, the archeological site is the dated layer and the mean intensity is obtained from average at the fragment level [e.g. Gallet et al., 2015, Gómez-Paccard et al., 2016, Hervé et al., 2017. Alternatively, if the temporal homogeneity is not ensured for a group of fragments, each fragment must be considered as an individual site and the mean intensity for this site is obtained from the average of the intensities obtained from specimens of this individual fragment [e.g. Shaar et al., 2011, Cai et al., 2017]. In databases, these two cases should be unambiguously differentiated as the significance of the mean intensity value is different. In the studies carried in this PhD, for a dated archeological site, the mean intensity is derived from the average of values obtained at the fragments levels, themselves resulting from the average at the specimens (i.e., sub-fragment) level. This procedure allows to ascertain the internal self-consistency of the results.

Another complex case to examine is the necessity for checking alteration of the magnetic mineralogy during the thermal treatment. For double-heating methods, this is done from the pTRMs check. However, there is currently several ways to calculate the importance of alteration and to define the critical threshold for rejecting a measurement. In addition, this test can be applied on the whole collection or only on a part of it. When entering such information in a database, the authors have to choose the level of detail they want to implement: either simply give the information that alteration has been checked for [e.g., in GEOMAGIA, Brown et al., 2015, or to go into details and precise how the alteration is quantified and the threshold for rejection, and/or if the test has been applied for the whole collection [e.g., in ArcheoInt, Genevey et al., 2008]. Such details would undoubtedly increase the database complexity, but are necessary when one needs to apply strict selection criteria. Indeed, depending on the level of needed accuracy, data with pTRM checks defined at a certain threshold and performed only on a part of the collection might not be considered as equivalently reliable than data with pTRM checks with a different threshold performed on the whole collection. This example also highlights that not all data can be considered as equivalently reliable.

Another example is given by studies comparing several methods for intensity measurements. In some cases, the authors average the values from the various methods, or, they



FIGURE 2.14: Left: Intensity data obtained from archeological and volcanic material covering the past 2 millennia from GEOMAGIA [Brown et al., 2015], filtered using various selection criteria. Right: VADM computed from these data before (top panel) and after (bottom panel) data selection. From Poletti et al. [2018].

simply provide several intensity values for the same archeological site. When such studies are entered in a database, the authors must therefore choose either to enter them as different entries (i.e., each value obtained from one protocol is entered individually), which leads to the duplication of a value (resulting in an overweight of this value in global models); or to enter the mean intensity obtained from the average of several protocols. Therefore, constructing a database cannot be reduced to an objective treatment of paleo and archeomagnetic studies, but involves subjective choices.

These examples also illustrate that not all intensity data can be considered equivalent. Depending on the use of a global database, selection criteria can be applied to extract the information needed in an effort to keep only the most reliable data. These criteria might vary from author to author, but in general include tests on: experimental protocols, statistics (standard deviations and number of fragments and/or specimens), check for alteration, correction for anisotropy and cooling rate effects on TRM acquisition and/or test for MD grains effects (see Fig. 2.14). Depending on the purpose (global modelling, construction of regional variation curves, ...), the strictness of criteria is modulated to compromise between the amount of information needed and the reliability of data. The histogram in Fig. 2.14 shows the amount of intensity data obtained from archeological and volcanic material covering the past two millennia. The initial dataset comprises more than 2500 data. The histogram then shows the decrease of the amount of data after the application of each criterion set by the authors [Poletti et al., 2018]. The final dataset comprises less than 500 intensity data, i.e. only 16% of the total dataset meet these criteria. The effect of this set of selection criteria is illustrated on the right-hand side of Fig. 2.14. The top panel illustrate the temporal evolution of the virtual axial dipole moment computed from intensity data. Even though some patterns emerged from the data, the dispersion is fairly large and the confidence intervals of the box and whiskers plot are on the same order than the observed variations. Therefore, in an effort to extract significant trend in these variations, the authors apply a set of strict selection criteria. The results are shown on the bottom panel. The previously observed dispersion is reduced and the VADM variations curve shows less smoothed variations. Nonetheless, a closer examination of this figure shows that even though the dispersion previously observed is reduced, significant variations are still observed in each age intervals chosen for the box and whiskers plot. Only two reasons can explain such a dispersion: either it represents a true feature of the field, i.e., very fast and large variations of the field; or these data are not representative of the true field behavior. At this level, it is however impossible to discriminate one of these reasons. Therefore, refined analyses of the past field variations involves a trade-off between data selection to keep the most reliable data and still keep enough data to provide a meaningful reconstruction.

To conclude, this section illustrates how information on the past field variations can be recovered from direct and indirect measurements. In particular, it has been highlighted that intensity data obtained from archeological and volcanic material can provide very valuable information, knowledge that would be inaccessible otherwise. However, the acquisition of such data is intricate and currently subject to active research in order to increase the reliability and the amount of available data. Behind the necessary standardization of data compiled in database in order to construct global field models and understand the past field variations, conceals an inherent complexity that one should keep in mind when interpreting these data and/or global field models.

# 2.2 Modelling the Earth's magnetic field

Owing to the diversity and the scarcity of data (especially before the 19th century), the most appropriate method to study geomagnetic field variations is the analysis of global field models. Constructing such models first requires an appropriate set of basis functions to represent the field.

# 2.2.1 Spatial representation of the magnetic field

The Earth's magnetic field is a vector quantity characterized at any location by its intensity and direction. At the Earth's surface, it results from the superposition of several sources, divided into an internal and an external contribution. At the position  $(a, \theta, \phi)$  (with a = 6371.2 km, the mean radius of the Earth,  $\theta$  the colatitude and  $\phi$  the longitude) at a given time t, the magnetic induction **B** (hereafter the magnetic field) is expressed in spherical coordinates as

$$\mathbf{B}(a,\theta,\phi,t) = \mathbf{B}_i(a,\theta,\phi,t) + \mathbf{B}_e(a,\theta,\phi,t).$$
(2.10)

Assuming the observer is located outside the sources (as it is the case at the Earth's surface, above the internal sources and below the external ones), there is no electric current and the Ampère's law (see section 2.3.1, Eq. 2.25) gives

$$\nabla \times \mathbf{B} = 0. \tag{2.11}$$

The magnetic field  $\mathbf{B}$  thus derives from a scalar potential V such that

$$\mathbf{B} = -\nabla V, \tag{2.12}$$

and as there is no magnetic monopoles

$$\nabla \cdot \mathbf{B} = 0. \tag{2.13}$$

Therefore, the potential V satisfies the Laplace's equation

$$\nabla^2 V = 0. \tag{2.14}$$

A solution for the Laplace's equation in spherical coordinates is first proposed by Gauss [1839]. His formalism allows to separate the internal and external contributions of the field by the spherical harmonic analysis (SHA) of the potential such that

$$\mathbf{B}(a,\theta,\phi,t) = -\left[\nabla V_i(a,\theta,\phi,t) + \nabla V_e(a,\theta,\phi,t)\right].$$
(2.15)

For instance, the internal potential  $V_i$  is decomposed as

$$V_i(r,\theta,\phi,t) = a \sum_{\ell=1}^{\infty} \sum_{m=0}^{\ell} \left(\frac{a}{r}\right)^{\ell+1} \left[g_\ell^m(t)\cos(m\phi) + h_\ell^m(t)\sin(m\phi)\right] P_\ell^m(\cos\theta), \quad (2.16)$$

where  $\ell$  and m are the spherical harmonic degree and order respectively,  $g_l^m$  and  $h_l^m$  are the Gauss' coefficients, and  $P_l^m$  are the Schmidt quasi-normalised Legendre functions. Fig. 2.15 gives a picture of this decomposition up to degree 5.

The field components in spherical coordinates are therefore expressed as



FIGURE 2.15: Spherical harmonics as a function of the degree  $\ell$  and order m (arbitrary scale). The first column (m = 0) gives the zonal or axisymmetric modes and the main diagonal  $\ell = m$  gives the sectoral modes.

$$B_{r} = -\frac{\partial V}{\partial r} = \sum_{\ell=1}^{\infty} (\ell+1) \left(\frac{a}{r}\right)^{\ell+2} \sum_{m=0}^{\ell} \left[g_{\ell}^{m} \cos\left(m\phi\right) + h_{\ell}^{m} \sin\left(m\phi\right)\right] P_{\ell}^{m}(\cos\theta),$$
  

$$B_{\theta} = -\frac{1}{r} \frac{\partial V}{\partial \theta} = -\sum_{\ell=1}^{\infty} \left(\frac{a}{r}\right)^{\ell+2} \sum_{m=0}^{\ell} \left[g_{\ell}^{m} \cos\left(m\phi\right) + h_{\ell}^{m} \sin\left(m\phi\right)\right] \frac{\mathrm{d}P_{\ell}^{m}(\cos\theta)}{\mathrm{d}\theta},$$
  

$$B_{\phi} = -\frac{1}{r\sin\theta} \frac{\partial V}{\partial \phi} = -\sum_{\ell=1}^{\infty} \left(\frac{a}{r}\right)^{\ell+2} \sum_{m=0}^{\ell} \left[g_{\ell}^{m} \sin\left(m\phi\right) - h_{\ell}^{m} \cos\left(m\phi\right)\right] m P_{\ell}^{m}(\cos\theta).$$
  
(2.17)

In practice, these expansions are truncated at a certain degree L, depending on the data resolution. Using vector measurements of the magnetic field from several observatories, Gauss calculated these coefficients up to degree 4 by a least squares method. At that time, his results confirmed the predominance of an axial dipole field [Garland, 1979] and he showed that the main field sources are of internal origin.

With the advent of the satellite era, the quality and amount of data improved gradually, allowing to perform SHA to higher degrees. Using Magsat data, Langel and Estes [1982] analysed the internal geomagnetic field power spectrum [Lowes, 1974] up to degree 23 given by

$$R_{\ell} = (\ell+1) \sum_{m=0}^{\ell} \left[ (g_{\ell}^m)^2 + (h_{\ell}^m)^2 \right].$$
(2.18)

Figure 2.16 illustrates their results at the Earth's surface and at the core-mantle boundary (CMB). At the Earth's surface, most of the energy is contained in the lower degrees coefficients. Up to degree 13, the energy is dominated by the external core contribution (the geodynamo). This contribution falls off rapidly with decreasing length-scales. From degree 16 the energy is dominated by the crustal field with a nearly white contribution leading to a flat spectrum.

The SHA hence allows to separate internal and external contributions, relative to the altitude of data acquisition. At the Earth's surface, the internal contribution comprises the main field signal but also the crustal field. This signal must therefore be treated appropriately to recover only the core field signal. For satellite data, the acquisition is located within or above the ionosphere. Therefore, this signal must also be treated to isolate the main field signal.

# 2.2.2 Extracting the main geomagnetic field signal: data selection and pre-processing

#### a) Direct observations

The Earth's magnetic field resulting from the superposition of several sources, direct observations (described in 2.1.1) often require a pre-processing in order to filter out the crustal and the external fields signals to recover the main field variations.

The relative contribution of these sources in the signal depends on the location (i.e., altitude) of acquisition. Consequently, ground-based data and satellites data are treated differently.



FIGURE 2.16: Power spectrum of the geomagnetic field. Figure from Langel and Estes [1982].

**Ground-based observations** Geomagnetic observatory data are sensitive to the external magnetic field variations and must therefore be treated to separate internal and external field contributions. To this end, the external contribution can either be explicitly taken into account and modelled (comprehensive models solving for all sources of the field) or is simply filtered out. The external magnetic field indeed varies on shorter timescales than the internal contribution (less than a few years, recall Fig. 1.3). Its contribution is satisfactorily eliminated by taking annual means from observatory data [e.g. Gillet et al., 2010].

On the other hand, the crustal field resulting from the remanent magnetization of the lithosphere varies very slowly. On the timescales of centuries to millennia, it can be considered as steady. The order of magnitude is weak and on the same order than the observational errors for historical measurements. In this case, the signal is either neglected, or treated as an error using statistical models [e.g. Jackson et al., 2000]. For observatory data, as they are obtained at constant location, they include the crustal signal, which might introduce a bias when used with other types of data. To avoid this bias, a handful method is to take the first difference of the annual means [Bloxham and Jackson, 1992], therefore only accounting for the field variations due to the outer core signal.

**Satellite data** Satellites are either located within or above the ionosphere. The signal must therefore be treated to eliminate the external contributions from the internal ones. For models aiming at extracting only the main field signal, satellites data are filtered [Bloxham and Jackson, 1992]. Only data collected on the "night-side" are selected, during quiet magnetic time (with no strong activity of the external field). This procedure only partially removes the external field contribution. Therefore, most models currently try either to coestimate the external field contributions or parameterize the models to take them into account. For a comprehensive review on the subject, see for instance Finlay et al. [2017].

### b) Indirect observations

For indirect measurements of the magnetic field, the contributions of the external and the crustal fields are neglected. Depending on the spatial and temporal distributions of the data and on the period covered by the model, it is possible (mostly for recent periods) to filter the data with a set of selection criteria. However, owing to their global scarcity, most often, only known problematic records are eliminated. Some records presenting large dating uncertainties can also be removed [Campuzano et al., 2019]. In addition, most models include outliers rejection or reweighting schemes during in the procedure of data inversion.

### 2.2.3 Determination of time-dependent models of the geomagnetic field

### a) Time-dependency

The time-dependency of the magnetic field in global models is generally introduced by a temporal parameterization such that

$$g_{\ell}^{m}(t) = \sum_{i} {}^{i}g_{\ell}^{m}\Psi_{i}(t), \qquad (2.19)$$

with  $\Psi_i(t)$  a set of basis functions, most often B-splines. In this case, the time-resolution of the model will depend on the "knot points" of the splines allowed by the temporal resolution of data.

#### b) A non-linear inverse problem

Modeling the Earth's magnetic field amounts to determining the values of Gauss coefficients able to reproduce a finite set of observations. This is a classical inverse problem (the direct problem being the determination of the field components from a set of Gauss coefficients) described as

$$\mathbf{d} = f(\mathbf{m}) + \mathbf{e},$$

with  $\mathbf{d}$  the vector containing the observations and  $\mathbf{e}$  the corresponding errors,  $\mathbf{m}$  the vector containing the Gauss coefficients.

Historical measurements and paleo- and archeomagnetic data are mostly provided as scalar data (F, D, I) (and not vector data  $(B_r, B_\theta, B_\phi)$ ). In this case, the relation between these data and the Gauss coefficients is non-linear (Eqs. 2.1 and 2.17). Therefore, the inverse problem must be solved iteratively, by minimizing the least-squares difference between the data and model predictions. The cost function to minimize is given by

$$\mathcal{J}(\mathbf{m}) = [\mathbf{d} - H(\mathbf{m})]^T \mathbf{C}_{ee}^{-1} [\mathbf{d} - H(\mathbf{m})] + \mathbf{m}^T \mathbf{C}_{mm}^{-1} \mathbf{m}, \qquad (2.20)$$

with  $\mathbf{C}_{ee}$  the matrix containing the observational errors and  $\mathbf{C}_{mm}$  a matrix defining the model complexity. The last right-hand site term is added to regularize solutions of this ill-posed problem [see Gubbins, 2004].

### c) Non-uniqueness of the solution

This inverse problem therefore needs additional constraints. The choice of such a priori information varies from author to author. For archeomagnetic modelling, most models use spatial and temporal norms defining the field complexity. Most often, the temporal regularization penalizes the second derivative of the radial component of the field  $B_r$  at the CMB [Bloxham and Jackson, 1992]. The spatial norm minimizes the ohmic dissipation at the CMB [Gubbins, 1975]. Finally a damping parameter  $\lambda$  quantifies the trade-off between fitting the data and determining a reasonable model of minimum complexity. These regularizations indeed seek to obtain a smooth model in space and time in order that no unnecessary (and non-physical) complex behavior is introduced in the geomagnetic field reconstruction.

The prior information are not necessarily provided as norms. Taking advantage of the "Earth-likeness" of recent geodynamo simulations [Christensen et al., 2010, Aubert et al., 2013], Sanchez et al. [2016] propose to use a "dynamo norm" as a spatial prior, which simply amounts to using the time-average statistics of such simulations. Alternatively, Hellio and Gillet [2018] propose to use instead spatial but also temporal statistics of the field obtained from satellite, observatory and paleomagnetic data to avoid the use of such norms.

### d) Downward continuation of the field at the CMB

As mentioned in subsection 2.2.1, outside of sources, the magnetic field derives from a scalar potential V satisfying Laplace's equation. Therefore, assuming the mantle is insulating and neglecting the crustal field, the magnetic potential at the Earth's surface can be downward continued at the CMB. As expected from Eq. 2.17, the shorter wavelengths are more amplified during the process, by a factor  $(a/r)^{\ell+2}$ . Therefore, if the crustal and external field signals are not treated appropriately, the residual shortest wavelengths from these signals contaminate the results at the CMB, as is also the case for any type of noise in the data. With this procedure, it is possible to retrieve the radial component of the magnetic field  $B_r$  at the core-mantle boundary (CMB) from observations at the Earth's surface. Before analyzing global models of the geomagnetic field at the CMB, it is interesting to understand how observations at the Earth's surface sample the magnetic field at the CMB. Following Gubbins and Roberts [1983], the vertical component of the magnetic field at the Earth's surface Z at a given location  $\mathbf{r}_s$  sample the radial component of the field at the CMB  $B_r(r_c, \theta, \phi)$  as

$$Z(\mathbf{r}_s) = \int_S G_Z(\mathbf{r}_s | r_c, \theta, \phi) B_r(r_c, \theta, \phi) \mathrm{d}S, \qquad (2.21)$$

with  $G_Z(\mathbf{r}_s|r_c,\theta,\phi)$  the Green function corresponding to the observation Z at  $\mathbf{r}_s$ .

Subsequently, Johnson and Constable [1997] propose linearized Green functions (or data kernels) for non-linear function of  $B_z$ , i.e. declination and inclination. The latter are represented on Figure 2.17, together with the data kernel for an intensity observation. Comparing the results with data distribution at the Earth's surface (Fig. 1.4) shows that for the historical period, although there is some gaps in the data (Africa or in South Pacific), the available dataset allows for an extensive sampling of the core, which might not be the case for paleo- and archeomagnetic data. Figure 2.18 shows the sampling of the CMB achieved by the dataset used to construct the CALS10k.1b models covering the Holocene [the superseded version of CALS10k.2 described in the next section 2.2.4, Korte et al., 2011]. As expected from the data global coverage, the models are heavily biased towards the northern hemisphere and Europe in particular. In addition, the mapping of intensity does not allow to properly recover the magnetic field beneath South Atlantic, northern South-America and Africa, where most of the secular variations are concentrated. Therefore, special care is needed when interpreting the predicted field intensities in these regions.

# 2.2.4 Geomagnetic models covering the past four centuries

Table 2.1 summarizes the most recent models covering the past four centuries. Most of these models are constructed using the same general methodology described in the previous sections. The main differences are the data types and their pre-processing, the prior information injected in the model and the chosen truncature.

For the period of interest, gufm1 is regarded as a reference. This model is constructed from direct observations only: observatory and satellite data for recent periods, and mariners observations mostly for older ones [see Jonkers et al., 2003, for details on this compilation]. As the quality and coverage of direct observations is higher than those of indirect data, this model is provided with a higher spatial and temporal resolution (the main field is resolved up to degree 14). Therefore, models spanning longer durations are often anchored to gufm1 for the historical period (see last column of table 2.1).

Figure 2.19a displays the time-average power spectra (for the period 0 - 2000 A.D.) for some of these models, compared to the power spectrum of CHAOS-7 average between 1999 and 2020 [model built from satellite observations, Finlay et al., 2020]. Up to degree 4 - 5, a reasonable agreement is observed between the various models and CHAOS-7. Assuming the wavelength of the field at the Earth's surface  $\mathcal{L}$  is related to the spherical harmonic degree  $\ell$  by  $\mathcal{L} \approx \pi a/\ell$  [recall Figure 2.15, see also Thébault et al., 2010] means that these models are able to solve geomagnetic field features of characteristic lengthscales ~ 4000 km. Beyond degree 6, most paleomagnetic models show a rapidly decreasing power compared to CHAOS-7, apart from BIGMUDI4k.1 which displays a good agreement. This model is indeed obtained from the simultaneous inversion of both direct and indirect observations,



FIGURE 2.17: Data kernel for a a) declination, b) inclination, c) intensity observation at latitude  $0^{\circ}$  (1) and  $30^{\circ}$  (2). The star shows the location of the observation at the Earth's surface. Figures from Sanchez [2016].



FIGURE 2.18: Sampling of the CMB by the dataset incorporated in CALS10k.1b [Korte et al., 2011]. Figure from Constable and Korte [2015].



FIGURE 2.19: Time-averaged power spectra at the Earth's surface of the field (first column) and its secular variation (second column) over the period 0 - 2000 (first row) and 1590 - 1990 (second row) from various paleomagnetic models compared with those of CHAOS-7 average betweend 1999 and 2020 and time-averaged gufm1.



FIGURE 2.20: Resolution of the archeomagnetic field by archeomagnetic data from the AmK model [Sanchez et al., 2016]. From Sanchez et al. [2016].

covering the past four millennia. In addition, no spatial regularization is applied but only a temporal regularization on the secular variation. The spatial resolution is therefore improved compared to other models. For gufm1, this rapid drop-off (compared to CHAOS-7) starts around degree 8 - 9, reflecting the better spatial coverage of the available data over the past four centuries (recall Fig. 1.4a,d). The discontinuity observed between degree 5 and 6 in the COV-ARCH model has been attributed by the authors to the increase of the power stored in uncertainties.

These limits correspond to the maximum spatial resolution of the models. Figure 2.19b presents the same results but averaged over the historical period 1590 - 1990. The spatial resolution of the various paleomagnetic models is not improved, to the exception of CALS10k2, perfectly fitting the power spectrum of *gufm1* (but note that this model is constrained to fit *gufm1* over the historical period, see Table. 2.1). Using a dynamo norm, Sanchez et al. [2016] compute the resolution matrix of archeomagnetic data over the past three millennia (see Fig. 2.20), representing the ability of archeomagnetic data to resolve the archeomagnetic field. They highlight that archeomagnetic data cannot resolve the geomagnetic field above degree 5 on average, as foreseen by comparison of power spectra with recent field models.

The drop-offs in energy are mostly reflecting the data resolution (data coverage and uncertainties) and the influence of spatial regularizations used in the inverse problem [Korte and Constable, 2008]. Korte and Constable [2008] also studied the temporal resolution of depleted versions of the CALSxk models and they show that using B-splines with knotpoints of ~ 50 yrs leads to a temporal resolution of ~ 100 yrs, which is a minimum estimate owing to the large dating uncertainties of paleomagnetic data. For example, Nilsson et al. [2014] assessed a temporal resolution of 300 - 400 yrs for the pfm9k.1 model.

Comparison of the secular variation power spectrum of CHAOS-7 (time-averaged over 1999 - 2020) at the Earth's surface with those obtained for some paleomagnetic models (see Fig. 2.19c) shows that even the dipole component variations are not fully recovered (being either over or underestimated). This is the consequence of the data coverage and the

smoothing imposed by the strong temporal regularizations. Note that though the dipole is the dominant contribution in the spectrum of the geomagnetic field, this is not the case in the secular variation power spectrum, dominated by the quadrupole field. The axial dipole variations are however more complex to constrain and understand, as they cover a wide range of timescales (recall section 1 and Fig. 1.6).

The spatial and temporal resolutions of paleomagnetic models are essentially limited by the spatial and temporal coverage of data (recall Fig. 1.4b,c,e,f) and their uncertainties. However, the latter are difficult to assess. Usually, the standard error is taken as the experimental standard deviation, i.e., only the experimental uncertainty is considered in the total error budget [Suttie et al., 2011]. Based on the comparison of archeointensity data and gufm1 over the period 1840 - 1990 (predicted intensity values of which are considered as the "true" field intensities), they show that the archeointensities systematically present an uncertainty of  $\sim 6-7\%$  regardless the number N of samples considered to compute the mean intensity value. They underline that such a systematic bias does not necessarily come from the experimental protocol, but can originate from local magnetic anomalies or from the model used (in this case gufm1). On the other hand, Arneitz et al. [2017] take the analysis one step further by comparing directly direct and indirect measurements (instead of using a model itself based on data). They show that if archeomagnetic data present such a systematic bias, the effects are generally averaged, with an overall good agreement between direct and indirect records. This good agreement is found when using appropriate selection criteria with corresponding subsets of archeomagnetic data; e.g., archeointensity data corrected for anisotropy effects if the material is known to be strongly anisotropic (as pottery or tiles). Accurate estimations of uncertainties are critical to improve paleomagnetic reconstructions. They are usually accounted for by weighting the corresponding data. However, this approach is strongly limited by the large heterogeneity observed in errors assessment in databases (recall previous subsection), and in some cases not provided at all. The latter issue is generally handled by assigning underweighting uncertainties based on global statistics on the database [e.g. Licht et al., 2013]. Finally, databases are affected by a number of outliers that also need to be taken care of (either eliminated or underweighted).

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[2000]			crustal field errors	Outliers rejection			norms	$15 \text{ nT/yr} \ 15901840$
			(stochastic model) and					
			observational (est.)					
Constable et al.	-8000 - 1990	A, V, S	Fixed min estimate	Outliers rejection	10	BS $(40 \text{ yrs})$	Spatial and temporal	gufm1 constrained
[2016]			(A,V) & re-estimate				norms	
			(S)					
Constable et al. [2016]	-8000 - 1990	A, V, S	Fixed min estimate $(A, V)$ & re-estimate	Outliers rejection	10	BS $(40 \text{ yrs})$	Spatial and temporal norms	I
			from cross-validation (S)					
Licht et al. [2013]	-1200 - 2000	A, V	Quadrature combi-	Outliers reweight-	σī	BS $(40 \text{ yrs})$	Spatial and temporal	anchor to $gufm1$ in
			nation of fixed $\&$	ing scheme			norms	1990
			published errors to account for truncature					
Nilsson et al.	-7000 - 1900	A, V, S	Min estimate depending	Outliers rejection	10	BS $(50 \text{ yrs})$	Spatial and temporal	I
[2014]			on $N$ & overweighting of $A, V$				norms	
Pavón-Carrasco et al. [2014a]	-12000 - 1900	Α, V	Fixed min estimate	Outliers rejection	10	BS $(50 \text{ yrs})$	Spatial and temporal norms	gufm1 constrained
Campuzano et al.	-100 - 1900	Α, ν	Quality weighting dep.	Age constraints	10	BS $(25 \text{ yrs})$	Spatial and temporal	
[2019]			on methods, samples nbr	crit. & outliers rejection			norms	
Sanchez et al. [2016]	-1200 - 2000	Α, V	Fixed min estimate	Outliers rejection	τ <sup>υ</sup>	Single- epochs	Dynamo norm	I
Hellio and Gillet	-1200 - 2000	Α, ν	Published errors	Outliers rejection	10	Temporal	Statistics from sat.,	
[2018]						cross-	obs., paleomag. data	
						covariances functions (100 yrs)		
Gillet et al. [2015]	1840 - 2020	O, Sat, H	Estimated	Data selection $\&$	14	BS $(2 \text{ yrs})$	Statistics from recent	ı
				Outners rejection			satenne and observa- tory data	
Arneitz et al.	-2000 - 1900	A, V, H, O,	observational error, dat-	Data selection,	x	B-splines	minimization of secu-	Simultaneous
		Sat	ing uncertainties, posi- tion errors and crustal	outhers rejection, downweighing			lar variation energy	sion of direct and
			field errors	correlated records				indirect data
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[2013]       -1200 - 1900         [2014]       et al.       -7000 - 1900         [2014]       et al.       -7000 - 1900         [2014]       et al.       -11200 - 2000         Pavón-Carrasco       -12000 - 1900         [2019]       et al.       -1100 - 1900         [2016]       -100 - 1900         [2016]       -11200 - 2000         [2018]       et al.       -11200 - 2000         [2018]       Et al.       -1200 - 2000         [2018]       Et al.       -2000 - 2000         [2018]       Et al.       -2000 - 2000         [2018]       Et al.       -2000 - 2000         [2019]       Et al.       -2000 - 2000         [2019]       Et al.       -2000 - 2000	[2000]       Constable et al. $-8000 - 1990$ A, V, S         [2016]       A, V, S       A, V, S         [2014]       A, V, S       A, V         [2014]       A, V, S       A, V         [2019]       A, V       A, V         Sanchez       et al. $-1200 - 2000$ A, V         [2016]       A, V       A, V       A, V         [2018]       Gillet et al. 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Outlees rejection doservational (est.)         2016]       Constable et al.       -8000 - 1990       A.V.S       Fixed min estimate from cross-validation (S)       Outlees rejection (S)         2016]       Constable et al.       -8000 - 1990       A.V.S       Fixed min estimate from cross-validation (S)       Outlees rejection (S)         Licht et al.       12013       -1200 - 2000       A.V.S       Quadrature combination of fixed & ing scheme published errors to account for truncature for metionate et al.       Outlees rejection (S)         Pavón-Carrasco       -1200 - 1900       A.V.       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# 2.3 Origins of the Earth's magnetic field

Since the Earth's outer core cannot be directly observed, our current understanding of its dynamics results from the association of geomagnetic field observations at the Earth's surface and experimental and theoretical physics. Magnetohydrodynamics (MHD) gives the fundamental principles for magnetic field generation in a rotating conducting fluid. These equations can be solved numerically in a spherical geometry. The numerical cost and the uncertainties about some physical parameters in the Earth's outer core prevent such numerical dynamo simulations to reach the Earth's core regime, i.e., the operating conditions for the geodynamo. These conditions depend on the the physical, thermodynamic and transports properties of the outer core. If some of these properties are accurately known (depth of the outer core, rotation period, density, ...), there is still some uncertainties about others, in particular the thermodynamic and transports properties, estimated either experimentally [e.g. Ohta et al., 2016] or theoretically [ab initio calculations, e.g., Alfé et al., 2003, Pozzo et al., 2012. Nonetheless, the magnetic field resulting from such simulations present a number of common features with the geomagnetic field such as a predominant axial dipole component, polarity reversals, westward drift, etc... suggesting that, despite being far from the Earth's core parameters space, such simulations are very valuable tools to unveil Earth's core dynamics and the associated secular variation of the geomagnetic field.

On the other hand, theoretical physics also provide the basics to model flow motions at the core's surface from the variations of the geomagnetic field observed at the Earth's surface. Comparisons of core flow models with numerical simulations or experimental dynamos can bring additional constraints on core dynamics. Constructing such models requires a good knowledge of the magnetic field at the Earth's surface. The benefit of core flow models is therefore limited on recent periods covering the observatory (~ 1840-) and satellite (~ 1965-) eras.

#### 2.3.1 Foundation of electromagnetism

#### a) Maxwell's equation

The fundamental basis for electromagnetism is given by the following set of equations known as the Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{q}{\epsilon},\tag{2.22}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.23}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$
(2.24)

$$\nabla \times \mathbf{B} = \mu \mathbf{j} + \epsilon \mu \frac{\partial \mathbf{E}}{\partial t},\tag{2.25}$$

In the following, unless otherwise noted, the magnetic permeability  $\mu$  and the electric permittivity  $\varepsilon$  are those of the free space:

$$\mu \approx \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

and

$$\varepsilon \approx \varepsilon_0 = 8.8541910^{-12} \text{ F/m}.$$

Eq. 2.22 or the Maxwell-Gauss law relates the electric field  $\mathbf{E}$  with the electric charge density q and the electric permittivity. It states that the electric flux through a closed

surface is generated by the electric charges contained in this surface. In Eq. 2.23 or Maxwell-Thomson law, **B** refers to the magnetic induction, hereafter denoted magnetic field. It states that the magnetic flux through a closed surface is zero, i.e. there is no magnetic monopole. Eq. 2.24 gives the Maxwell-Faraday's law, which indicates that the electric field is induced by magnetic field variations. Finally, Eq. 2.25 or Maxwell-Ampère's law states that a magnetic field is induced by the electric current density **j** and the time variations of the electric field **E**.

In the framework of magnetohydrodynamics, the characteristic time for the system evolution is given by the characteristic flow velocity, negligible compared to the speed of light in vacuum c, with  $c = 1/\sqrt{\varepsilon\mu}$ . Consequently, the second term on the right-hand side (RHS) of Eq. 2.25 is neglected and the Maxwell-Ampère's law becomes

$$\nabla \times \mathbf{B} = \mu \mathbf{j}.\tag{2.26}$$

In a moving conducting fluid, the Ohm's law describes the matter response to an electric field. It relates the electric currents  $\mathbf{j}$  to the electric field  $\mathbf{E}$ 

$$\mathbf{j} = \sigma \left( \mathbf{E} + \mathbf{u} \times \mathbf{B} \right), \tag{2.27}$$

where  $\sigma$  is the electrical conductivity, **u** is the fluid velocity and the second term on the RHS corresponds to the electric field induced by the magnetic field.

#### b) The induction equation

Substituting Eq. 2.27 in Eq. 2.26, taking the curl and injecting in Eq. 2.24 leads to the induction equation which writes

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \Delta \mathbf{B}, \qquad (2.28)$$

where  $\eta = 1/(\mu\sigma)$  is the magnetic diffusivity. This equation relates the time variations of the magnetic field to two processes: advection and diffusion. More precisely, assuming an incompressible fluid and rewriting Eq. 2.28 yields

$$\partial_t \mathbf{B} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \eta \Delta \mathbf{B}. \tag{2.29}$$

In this equation, the induction term in 2.28 is decomposed in  $\mathbf{u} \cdot \nabla \mathbf{B}$ , corresponding to the advection of the field lines by the flow, and  $\mathbf{B} \cdot \nabla \mathbf{u}$ , corresponding to the stretching of the magnetic field by the flow.

Considering a typical field amplitude of B, the same typical lengthscale for advection and diffusion in the outer core  $\mathcal{L}$ , a typical velocity U and assuming that typical timescale of the system evolution is given by the advection time  $\tau_{adv} = \mathcal{L}/U$ , the dimensionless form of 2.28 writes

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Rm} \Delta \mathbf{B}, \qquad (2.30)$$

where

$$Rm = \frac{UL}{\eta} = \frac{\tau_{\rm dif}}{\tau_{\rm adv}} \tag{2.31}$$

is the magnetic Reynolds number and  $\tau_{\rm dif}~=~\mathcal{L}^2/\eta$  the diffusion timescale.

For the Earth's outer core, the root mean square (rms) velocity at the top of the core is estimated at  $U \sim 5 \times 10^{-4}$  m/s (see section 2.4.2) and the magnetic diffusivity  $\eta = 0.6 \text{ m}^2/\text{s}$  [Pozzo et al., 2012]. Assuming the characteristic lengthscale is the outer



FIGURE 2.21: Preliminary reference Earth model (PREM) as established by Dziewonski and Anderson [1981]. Figure from Olson [2015].

core depth  $\mathcal{L} \sim 3 \times 10^6$  m, the advection time  $\tau_{adv}$  is of order ~ 200 yrs. Assuming the field is not regenerated (i.e.  $\nabla \times (\mathbf{u} \times \mathbf{B}) = 0$ ), the magnetic field would simply diffuse away in a time  $\tau_{dif} \sim 5 \times 10^5$  yr. More precisely, accounting for the spherical geometry,  $\tau_{dif} = \mathcal{L}^2/(\pi^2 \eta) \sim 5 \times 10^4$  yr. The magnetic diffusion time is therefore much longer than the characteristic advection time, leading to high magnetic Reynolds number  $Rm \sim 2500$  in the outer core. The Earth's magnetic field being at least 3.5 Gyr old [e.g. Usui et al., 2009], induction must be powered by long-term energy sources to avoid the ohmic dissipation of the magnetic field.

# 2.3.2 Dynamics of the Earth's outer core

The main field of the Earth is generated within the Earth's liquid outer core by a selfsustained dynamo. A dynamo relies on the conversion of mechanic to electromagnetic energy. This mechanic energy is in fact supplied as kinetic energy by flow motions in the outer core, structured in columns by the Earth's rotation. Several processes are driving the convection in the Earth's core, although their relative contributions are still debated [e.g. Holme, 2015]. These processes will be briefly described in the following section, along with an overview of the current knowledge of core dynamics main features.

#### a) Core structure and energy sources for a self-sustained dynamo

The current knowledge of the Earth's structure is mainly brought by seismic data analysis. Figure 2.21 shows the velocity and density profiles of the Earth from PREM (Preliminary Reference Earth Model) obtained by Dziewonski and Anderson [1981]. The density profile shows a large density jump between the mantle and the core, at  $\sim 2900$  km below the Earth's surface. A second density jump is also visible between the liquid outer core and the solid inner core. From meteorites composition, it has been deduced that the high-density core is mainly composed of iron, or more precisely of a Fe-Ni alloy. The outer core's density as estimated from PREM is slightly lower than expected for iron at such pressures and temperatures. This density deficit is explained by the presence of lighter



FIGURE 2.22: Sketch of energy flows operating in the outer core. The kinetic energy is supplied by compositional and thermal convection, and by astronomical forcing (tidal, precession forcing), though the latter contribution is difficult to quantify [e.g. Jones, 2015]. The kinetic energy is converted into magnetic energy through the process of induction. The magnetic energy can be converted back into kinetic energy by the retroaction of Lorentz force on the fluid flow. The magnetic energy is converted to heat by ohmic dissipation. Figure from Deguen and Lasbleis [2020].

elements, of nature and proportions still debated. Prospective candidates are the oxygen, sulfur and silicon [Alfé et al., 2003, Badro et al., 2014], among others.

As mentioned hereabove, the high-conductivity of the outer core's iron [ $\sigma = 1.4 \times 10^6$  S/m, Pozzo et al., 2012] implies that an energy source is constantly supplying the magnetic field to compensate for ohmic dissipation. Fig. 2.22 shows a sketch representing the energy flows in the outer core. The kinetic energy supplying the magnetic field induction originates in two main sources: thermal and compositional convection and to a lesser extent astronomical forcing [although the contribution of the latter is unknown, e.g. Jones, 2015].

The inner core freezing provides a first energy source for convection. The heavier elements indeed crystallise first, leading to a release of lighter elements at the inner core boundary (ICB) producing a compositional buoyancy often thought to be the main driver for convection. The inner core is also the source of a second thermal buoyancy flux produced by the release of latent heat from its freezing. An additional thermal buoyancy flux originates in the heat loss from the core to the mantle. The precise relative contributions of these sources to the convection remain to be determined. For a detailed review on core's convection, see Jones [2015].

The fundamental ingredient to convert this kinetic energy into electromagnetic energy is the electromagnetic induction (Eq. 2.28, Fig. 2.22). An induced electric field is generated by motions of the conducting fluid in an initial embedding magnetic field. The induced



FIGURE 2.23: Sketch of the main features of the geodynamo. The conducting fluid motions in the outer core are organized in Taylor columns under the effect of rotation. A secondary motion in convection creates helicity in these columns. This specific flow motion induces a dipolar magnetic field. Figure from Sanchez [2016].

electric field variations generates in turns a magnetic field, to obtain a self-sustained dynamo. The magnetic energy is dissipated through ohmic diffusion into heat, preventing the magnetic energy to grow indefinitely, while the kinetic energy is dissipated through viscous effects, although the latter are supposed to be negligible in the outer core compared to ohmic diffusion [e.g. Roberts and King, 2013, Deguen and Lasbleis, 2020].

The main features of the geodynamo are sketched on Figure 2.23. The fluid flow is strongly compelled by the Earth's rotation through the Taylor-Proudman constraint [Proudman, 1916, Taylor, 1922]. When the Coriolis force dominates the force balance, the fluid motion is invariant along the rotation axis. In a spherical geometry, this constraint results in the formation of Taylor columns, aligned with the rotation axis. The convective motions also introduce secondary components in the flow (bold yellow arrows on Fig. 2.23, creating helicity in Taylor columns (thin orange arrows on Fig. 2.23).

Figure 2.24a illustrates the importance of columnar convection in magnetic field induction. Considering an initial poloidal field P, the magnetic field lines are twisted by the columnar vortice  $V_1$  (in yellow) converting the poloidal field into toroidal field. The helical flow between vortices  $V_1$  and  $V_2$  acts on the toroidal field, inducing a poloidal field P' which in turn enhances the initial poloidal field P. The columnar convection thus enhances the initial dipole field. The conversion mechanism from poloidal to toroidal magnetic field is


FIGURE 2.24: a) Mechanism for generating and enhancing the magnetic field. The red lines depict the poloidal (P) and toroidal (T) magnetic field lines. The axial vorticity is shown in yellow. At the core's surface, outward (resp. inward) directed magnetic field is given in red (resp. blue). Figure from Olson [2013]. b) Magnetic field lines from simulated geomagnetic field [Glatzmaier and Roberts, 1995]. c) Snapshot of the axial vorticity from a numerical dynamo simulation (red: positive, blue:negative) and d) the corresponding magnetic field lines. c,d) From Olson [2015].

the so-called  $\alpha$ -effect and can also produce poloidal field from toroidal field [e.g. Roberts and King, 2013]. The magnetic field pattern shown in Figure 2.24a is observed in outputs from numerical dynamo simulations, as illustrated in Figure 2.24b,d.

#### b) Equations of magnetohydrodynamics

The description of the core dynamics is obtained from the magnetohydrodynamics (MHD) equations, that is the Navier-Stokes and continuity equations coupled with the induction equation, together with a number of approximations and boundary conditions, derived in the following.

The geometry is the one pictured on Fig. 2.23: a spherical shell of outer radius  $r_0$  and inner radius  $r_i$ , filled of an electrically conducting fluid. The following equations are given in spherical coordinates  $(r, \theta, \phi)$ .

The mass conservation equation writes

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \qquad (2.32)$$

with  $\rho$  the fluid density and **u** the fluid velocity. Assuming the fluid is incompressible, the mass conversation equation becomes

$$\nabla \cdot \mathbf{u} = 0. \tag{2.33}$$

The momentum conservation equation for an incompressible newtonian fluid is given by

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \rho \nu \nabla^2 \mathbf{u} + \mathbf{f}_b$$
(2.34)

with p the pressure,  $\nu$  the kinematic viscosity and  $\mathbf{f}_b$  the volumetric body forces.

Now, supposing the shell is rotating with an angular velocity  $\Omega$ , the effects of rotation are more conveniently expressed in the rotating (non-inertial) reference frame. It allows to introduce two terms in Eq. 2.34 corresponding to the Coriolis  $(-2\Omega \times \mathbf{u})$  and the centrifugal accelerations  $(-\Omega \times \Omega \times \mathbf{r})$ . The latter is grouped in the scalar *P* together with the kinematic pressure, leading to

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P - 2\rho \mathbf{\Omega} \times \mathbf{u} + \rho \nu \nabla^2 \mathbf{u} + \mathbf{f}_b.$$
(2.35)

Finally, two body forces are comprised in the  $\mathbf{f}_b$  term: the Lorentz force and the buoyancy force.

The electromagnetic force of a magnetic field retroacting on the flow is called the Lorentz force and writes

$$\mathbf{f}_L = \mathbf{j} \times \mathbf{B} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}, \qquad (2.36)$$

As previously mentioned, the fluid is assumed incompressible, i.e.  $\rho$  is constant. However, the thermal convection is driven by thermally induced variations in density. In order to account for these variations, the fluid is assumed incompressible, of background density  $\rho_0$  except in the buoyancy term, induced by density perturbations  $\rho'$ . This approximation is known as the Boussinesq approximation [e.g. Braginsky and Roberts, 1995]. The buoyancy force induced by the density variations (arising from both thermal and compositional effects) is obtained from

$$\mathbf{f}_g = \rho' g \mathbf{r},\tag{2.37}$$

with g the gravity acceleration.

Braginsky and Roberts [1995] introduce a formalism to account for both thermal and compositional convection effects on buoyancy. They define a codensity function C as

$$C = \alpha \rho_0 T' + \Delta \rho \xi' \tag{2.38}$$

with T' the temperature perturbations,  $\alpha$  the thermal expansion coefficient,  $\xi'$  the light elements mass fraction and  $\Delta \rho$  the density variations between light and heavy elements. Assuming thermal and chemical diffusivity are of same order, the codensity satisfies

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \kappa \nabla^2 C + \varepsilon \tag{2.39}$$

with  $\varepsilon$  the sources (and sinks) of codensity and  $\kappa$  the thermal diffusivity.

To summarize, the complete system describing the geodynamo is given by

$$\begin{cases}
\nabla \cdot \mathbf{B} = 0 \\
\nabla \cdot \mathbf{u} = 0 \\
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho}\nabla P + \nu\nabla^{2}\mathbf{u} + Cg\mathbf{r} + \frac{1}{\mu\rho_{0}}(\nabla \times \mathbf{B}) \times \mathbf{B} \\
\frac{\partial t}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \eta\Delta \mathbf{B} \\
\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \kappa\nabla^{2}C + +\varepsilon
\end{cases}$$
(2.40)

together with the appropriate set of boundary conditions.

An appropriate boundary condition for the flow is to consider no-slip conditions ( $\mathbf{u} = 0$ ), often applied at both boundaries in numerical dynamo simulation. It is also possible to consider a free-slip boundary condition ( $u_r = 0$  and tangential components of the stress are zero) at the CMB to avoid the effect of a thick diffusive layer. In the Earth's outer core, the heat flux at the CMB is controlled by the lower mantle. The thermal boundary condition should therefore be expressed in terms of heat flux. In numerical dynamo simulations, this condition is either applied as a constant heat flux through the boundaries, or constant temperatures at the boundaries. At first order, the mantle can be regarded as insulating (though the lower mantle has a low conductivity). Finally the inner core is either treated as insulating, or as conducting (and of the same conductivity than the inner core in that case).

#### 2.3.3 Numerical dynamo simulations

A useful method to understand the force balance in the Earth's core dynamics and to obtain a convenient parameterization of the above system is the introduction of dimensionless parameters. In the following, time is scaled by the rotation period  $\Omega^{-1}$  (with  $\Omega = 7.292 \times 10^{-5}$  rad/s), the typical lengthscale is taken as  $D = r_o - r_i = 2260$  km (with  $r_o$  the radius of the outer core and  $r_i$  the radius of the inner core), and the magnetic field is scaled by  $(\rho\mu)^{1/2}\Omega D$ , with  $\rho = 1.09 \times 10^4$  km/m<sup>3</sup> [Olson, 2015] for the Earth's outer core. The system becomes

$$\nabla \cdot \mathbf{B} = 0$$
  

$$\nabla \cdot \mathbf{u} = 0$$
  

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + 2\mathbf{z} \times \mathbf{u} = -\nabla P + E\nabla^{2}\mathbf{u} + Ra^{*}C\frac{\mathbf{r}}{r_{o}} + (\nabla \times \mathbf{B}) \times \mathbf{B}$$
  

$$\partial_{t}\mathbf{B} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \frac{E}{Pm}\Delta \mathbf{B}$$
  

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \frac{E}{Pr}\nabla^{2}C + +\varepsilon$$
(2.41)

with  $\mathbf{z}$  a unit vector in the direction of the rotation axis.

The evolution of the system is controlled by four independent parameters.

- the Ekman number

$$E = \frac{\nu}{\Omega D^2}$$

represents the ratio of viscous to Coriolis force. For the Earth's outer core, the kinematic viscosity  $\nu$  is estimated at  $10^{-6\pm 2}$ m<sup>2</sup>/s [Olson, 2015], leading to  $E = \mathcal{O}(10^{-15})$ . The viscous forces are therefore negligible compared to the Coriolis force (as long as large

lengthscales are involved). However, such low values are not computationally reachable and currently, the lower reachable values are of order  $E \sim 10^{-7}$  [Schaeffer et al., 2017];

- the Prandtl number

$$Pr = \frac{\nu}{\kappa},$$

is the ratio of kinematic to thermal diffusivity, equivalent in the outer core such that  $Pr = \mathcal{O}(1)$ ;

- the magnetic Prandtl number

$$Pm = \frac{\nu}{\eta},$$

is the ratio of magnetic diffusion time to viscous diffusion time. For the Earth,  $Pm = \mathcal{O}(10^{-6})$ , suggesting that ohmic dissipation is much more important than viscous dissipation. However, due to the numerical constraint on E, Pm in numerical dynamos simulations is of order  $\sim \mathcal{O}(1)$ ;

- the mass anomaly flux Rayleigh number

$$Ra^* = \frac{g_0 F}{4\pi\rho\Omega^3 D^4}$$

with F the mass anomaly flux, measures the vigor of the thermo-chemical convection and is of order ~  $\mathcal{O}(10^{-12})$  for the Earth and reaches ~  $\mathcal{O}(10^{-9})$  in numerical dynamo simulations.

Accounting for the low viscosity of the fluid, the fluid motion is strongly constrained by the Earth's rotation. Retaining only the Coriolis and the pressure forces in the force balance leads to the Taylor-Proudman theorem, that is the fluid velocity is invariant along the rotation axis (recall Fig. 2.23). Such flows are called geostrophic. Small departures from geostrophy due to buoyancy or the Lorentz force result in quasi-geostrophic (QG) motions [e.g. Finlay et al., 2010].

Considering the low values of the Ekman and modified Rayleigh numbers, the dimensionless Navier-Stokes equation given in system 2.41 shows that the Earth's core dynamics is primarily governed by a balance between pressure, the Lorentz force (M for Magnetic), the buoyancy (A for Archimedean), and the Coriolis force (C) in a so-called MAC balance.

The core regime modelled by numerical dynamo simulations is evaluated by diagnostic dimensionless parameters. Taking U as the root mean square velocity in the shell and  $D = r_o - r_i$  as the characteristic lengthscale, the main diagnostic parameters are:

- the magnetic Reynolds number

$$Rm = \frac{UD}{\eta}$$

is the ratio of magnetic diffusion time to advection time (recall previous section). In other words, it measures the relative importance of magnetic induction to diffusion. In dynamo simulations, this parameter is of order  $\sim 10^2 - 10^3$ . For the Earth, it is estimated as  $\sim 10^3$ . As previously mentioned, the magnetic field must be constantly regenerated through magnetic induction. To maintain a self-sustained dynamo, the magnetic Reynolds number must exceed the critical value  $Rm_c \approx 50$  [Christensen and Wicht, 2015].

- the Reynolds number

$$Re = \frac{UD}{\nu}$$

is the ratio of advection to viscous dissipation. For the Earth, this ratio is estimated on the order of  $\sim 10^9$  as the viscous dissipation is supposed to be negligible, illustrating the turbulent regime operating in the outer core. In numerical dynamo simulations, this parameter is of order  $10^2 - 10^3$ , due to the overestimation of the viscous effects.

- the Rossby number

$$Ro = \frac{U}{\Omega D}$$

measures the ratio of inertia to Coriolis force, or in other words, the fluid vorticity to the planetary vorticity. This ratio is supposed to be rather low as inertia is negligible. For the Earth, it is found to be on the order of  $\sim 10^{-6}$  and for numerical dynamo simulations it ranges from  $\sim 10^{-2}$  to  $10^{-4}$ .

- the Elsasser number

$$\Lambda = \frac{B^2}{\rho \mu \eta \Omega}$$

is the ratio of Lorentz to Coriolis force. For the Earth, this ratio depends on estimates of the magnetic field strength inside the core. However, the toroidal magnetic field trapped into the outer core is difficult to estimate. The total rms field in the core is assumed to be on the order of 1-5 mT. With such values, the Elsasser number should be of order  $\sim 0.1$ . In numerical dynamo simulation, the Elsasser number is on the order of  $\mathcal{O}(10)$ .

The case for which  $\Lambda \geq 1$ , i.e., the Lorentz force is larger than the Coriolis force corresponds to strong field dynamo regime. In such a regime, the flow dynamics is mainly magnetostrophic (MAC balance force) and leads to a stable equilibrium of the magnetic field. Such a balance would favor a stable dipole moment as observed for the Earth [e.g. Deguen and Lasbleis, 2020]. However, it has been recently shown that numerical dynamo simulations reproducing a mainly dipolar magnetic field are controlled at leading order by a quasi-geostrophic balance between pressure and Coriolis force, and at first order by a MAC balance between the Lorentz force, buoyancy and ageostrophic Coriolis force, in a so-called QG-MAC balance [Schwaiger et al., 2019].

#### 2.3.4 Modeling the fluid velocities at the CMB

#### a) Frozen-flux approximation

In the outer core, the magnetic field variations are related to the flow as stated by the induction equation Eq. 2.28. As  $B_r$  is continuous through the CMB, time variations of  $B_r$  at the core's surface, are given by

$$\frac{\partial B_r}{\partial t} = -\nabla_H \cdot (\mathbf{u}_H B_r) + \frac{\eta}{r} \nabla^2 (r B_r), \qquad (2.42)$$

with  $\mathbf{u}_H$  the horizontal velocity and  $\nabla_H$  the horizontal divergence. Here, the core surface refers to the region just below the boundary layer, at the top of the free stream and the changes of  $B_r$  across this layer are neglected [as this layer is supposed fairly thin, e.g. Holme, 2015]. At the bottom of this layer, the free-slip ( $\mathbf{u} \cdot \mathbf{r} = 0$  and tangential components of the stress are zero) rather than the no-slip ( $\mathbf{u} = 0$ ) condition applies, allowing for tangential flows at this boundary.

If the radial component of the magnetic field and its time variation are known at the CMB, it should therefore allow to deduce the horizontal flow  $\mathbf{u}_H$  from the induction equation. However, the second term on the RHS of Eq. 2.42 involves radial derivatives of  $B_r$ , which cannot be determined from the knowledge of the field at the CMB solely. To

address this issue, magnetic diffusion is often neglected. By doing so, the magnetic field variations are simply linked to the flux advection, i.e. the magnetic field lines are frozen in the fluid. This is the so-called "frozen-flux" approximation [Roberts and Scott, 1965]. This approximation is motivated by the large value of the magnetic Reynolds number for the Earth ( $Rm \sim 2500$ , recall section 2.3.3) or in other words, the fact that the characteristic timescale for diffusion is far longer than the advection time. Therefore, looking at sufficiently short timescales, this approximation seems reasonable. The induction equation becomes

$$\frac{\partial B_r}{\partial t} = -\nabla_H \cdot (\mathbf{u}_H B_r). \tag{2.43}$$

#### b) Non-uniqueness

Eq. 2.43 however still contains two unknowns, i.e. the horizontal components of the fluid velocity. An additional constraint is therefore needed to overcome the non-uniqueness of this problem. Such constraints are usually based on dynamical assumptions on the flow [see Holme, 2015, for a review]. From a theoretical standpoint, the flow in rapidly rotating shell is dominated by the Coriolis force, constraining the flow to be invariant along the rotation axis through the Proudman-Taylor constraint. This is supported by observations of length of day (LOD) fluctuations. The rotation period of the Earth is varying under the influence of external torques and internal exchanges of angular momentum [e.g. Gross, 2015]. Assuming geostrophic core flow motion, Jault et al. [1988] show a good correlation between such flows and LOD decadal variations. Columnar convection is also supported by outputs from numerical simulations of the geodynamo [see for example Christensen and Wicht, 2015]. The flow is therefore often assumed geostrophic (balance between pressure and Coriolis forces, section 2.3.3) either at the Earth's core surface [Le Mouël, 1984, Bloxham and Jackson, 1991, tangentially geostrophic flow, e.g.] or within the outer core [e.g. quasi-geostrophic flows, Pais and Jault, 2008]. Other assumptions have been proposed such as steady flows or toroidal flows [see for example Holme, 2015].

Owing to the non-uniqueness of this problem, the models are also often regularized, in most cases by penalising the small-scale flows. As for geomagnetic field models, such norms aim at ensuring that no unnecessary small-scale features are introduced in the flow, and should therefore be seen as an additional assumption on the flow, imposed to be large scale [Holme, 2015].

While the various prior constraints incorporated in models can lead to significantly different flow patterns, some features have been found robust [Finlay et al., 2010, Holme, 2015]. Figure 2.25 compares the results obtained from different *a priori* assumptions on the flow (i.e., steady flows, toroidal flows or tangentially-geostrophic flows). All of them favor a lower activity in the Pacific hemisphere. They show a strong westward drift along the Equator in the Atlantic hemisphere. They also present a circulation in the Southern hemisphere, equatorward below the Southern Indian ocean, westward below Africa and poleward again beneath South America or beneath the South Pacific ocean.

In the case of the quasi-geostrophic flow assumption (QG), the flow is assumed to be mainly geostrophic, but effects of the Lorentz and buoyancy forces lead to departures from this state, resulting in axial flow component [e.g. Finlay et al., 2010]. In fact, this assumption requires the flow to be symmetric with respect to the equator plane. This additional constraint thus allows to infer the flow in the equatorial plane rather than at the core's surface only. With this constraint, Pais and Jault [2008] infer the existence of a large eccentric equatorial jet producing an anti-cyclonic flow closer to the core's surface in the Atlantic hemisphere (see Fig. 2.26a,c). Another prior constraint has been proposed by Aubert [2013], Fournier et al. [2011] using outputs from numerical dynamo simulations. The prior flow constraints are imposed from statistical properties of geodynamo simulations. The results, presented on Figure 2.26 are in agreement with core flow models obtained from QG flow assumption [e.g. Pais and Jault, 2008], recovering the planetary scale eccentric gyre, visible on Figure 2.26a,c. The gyre and its counterparts in the Southern hemisphere [Amit and Olson, 2006], are found persistent over the observatory era [Aubert, 2014, Pais et al., 2014]. For earlier period, the amount and quality of data presently hinder the determination of velocity flows at the core's surface.



FIGURE 2.25: Core flow derived from gufm1 in 1980 using the frozen-flux approximation with various additional constraints: a) steady b) toroidal and c) tangentially geostrophic flows. Figure from Holme [2015].



FIGURE 2.26: Core flow in 2001 obtained from inverse geodynamo modelling [Aubert et al., 2013]
from gufm-sat-Q3 [Finlay et al., 2012]. a) Surface core flow (arrows, arbitrary scale) and its toroidal component. b) Cylindrical radial velocity in the equatorial plane (top) and associated isosurfaces (levels given on the colour bar). c) Azimuthal velocity in the equatorial (top) and meridional (bottom) planes. Grey arrows gives the general flow circulation. Figure from Aubert [2013].

## 2.4 Geomagnetic field variations over the past four centuries

Over the past four centuries, the Earth's magnetic field has significantly evolved both at the Earth's surface and at the core-mantle boundary. Due to the largest amount of both direct and indirect measurements, global models covering this period (i.e., the historical period) can potentially achieve higher spatial (up to degree 8 - 9, recall section 2.2.4) and temporal resolution. Then, the joint analyses of geomagnetic field and flow models and comparison with numerical dynamo simulations can help unveiling the Earth's magnetic field variations and their origins. In this section are reviewed the main features of the secular variations over the past few centuries together with discussions on the possible processes at the origin of such variations.

#### 2.4.1 Morphology of present field at the Earth's surface and at the CMB

The best picture of the magnetic field that can be obtained is obviously provided by the present field and its variations captured by satellites and observatory data. The models constructed from such data thus often serve as benchmarks to analyse the spatial and temporal resolution from historical and archeomagnetic models. In addition, the *a priori* information incorporated in the latter are often derived from the behavior of the present field, described in the following.

Figure 2.27a illustrates a map of  $B_r$  at the Earth's surface obtained from CHAOS-7 [Finlay et al., 2020] in 2015 up to degree 13. At present, the field is mainly dipolar, with a south dip pole located in Siberia and a north dip pole located in Antarctic, south of Australia. The magnetic equator (thick black line on Figure 2.27a) is distorted in the Atlantic hemisphere (-90°E to 90°E) under the influence of the South Atlantic Anomaly (SAA), a region presenting abnormally low field intensities (~ 25  $\mu$ T). Another interesting feature of the recent field is observed by studying its variations. Figure 2.27b shows a map of the secular variation of the vertical component of the field  $\dot{B}_r$  in 2015. Most of the variations occurred in the Atlantic hemisphere while the Pacific hemisphere presents rather low activity.

Before comparing the geomagnetic field as observed at the Earth's surface with its downward continuation at the CMB, it is worth recalling that the geomagnetic field at the CMB can be recovered only up to degree 13, higher degrees being contaminated by the crustal magnetic field. The structures analysed from maps at the CMB are actually smaller. Figure 2.28 shows a snapshot from a numerical dynamo simulation truncated at various spherical harmonic degree. The radial magnetic field truncated at degree 13 share several common features with  $B_r$  obtained at the CMB from geomagnetic models (see below). Nonetheless, when the truncation degree is increased, the field morphology is significantly different, presenting much smaller structures. Therefore, special care must be taken, in particular when analyzing structures and processes in terms of characteristic lengthscales, as the observed magnetic field at the CMB probably arise from smaller and more complex structures [e.g. Christensen and Wicht, 2015].

Figure 2.29 shows a map of the radial component  $B_r$  in 2015 (a) and its secular variation (b) at the core's surface up to degree 13 from the model CHAOS-7 [Finlay et al., 2020]. At the CMB, closer to the source, the axial dipole component is less prominent than at the Earth's surface (Fig. 2.27 and Fig. 2.16). Shorter wavelengths structures emerge, in particular areas of more or less concentrated magnetic flux hereafter referred to as flux patches. In the northern hemisphere, two strong normal flux patches (same polarity than the overall hemisphere) are identified beneath Siberia and Arctic Canada, with counterparts in the southern hemisphere below Antarctica. These lobes are observed over the



FIGURE 2.27: Maps of radial component of the internal magnetic field  $B_r$  (a) and its secular variations  $\dot{B}_r$  (b) at the Earth's surface as given by CHAOS-7 [Finlay et al., 2020] in 2015 up to degree 13. The black contour gives the magnetic equator. The components have been computed using ChaosMagPy [Kloss, 2020].



FIGURE 2.28: Snapshot of the radial magnetic field at the CMB from a model output at various spherical harmonic truncation level. Figure from Aubert [2013].

historical period, as illustrated in Figure 2.31 showing maps of  $B_r$  at the CMB from gufm1 over the past four centuries [e.g. Hulot et al., 2002]; but also on the average field over millennial time scales [e.g. Korte and Holme, 2010, Constable and Korte, 2015]. Gubbins and Bloxham [1987] interpret these flux lobes as the signature of the columnar convection in the outer core. This interpretation is supported by theoretical and numerical results described in the previous section, that is in rapidly rotating system such as the Earth, the flow should be organised in columns parallel to the rotation axis. Such constraint would produce a long-term stable dipole, which is also consistent with the observations (recall Fig. 1.6). An other interesting feature is the presence of reversed flux patches (RFP), i.e. areas where the magnetic flux is of opposite sign compared to the hemisphere where they are located. In particular, several of them are observed beneath South Atlantic and are responsible for the South Atlantic anomaly observed at the Earth's surface [e.g. Terra-Nova et al., 2017].

Turning to the secular variation observed in 2015 (Fig. 2.29b), most of the variations are observed in the Atlantic hemisphere, as expected from the observations at the Earth's surface, in particular along the Equator. In addition, strong patches of SV are observed beneath Siberia, illustrating the North magnetic pole westward acceleration [e.g. Finlay et al., 2016b].

Analyses of the field evolution over the historical period shows that the geomagnetic field has significantly changed over the past four centuries [e.g. Jackson and Finlay, 2015]. Figure 2.30 compares the field intensity F in 1590 (a) and in 1990 (b) as provided by gufm1 (up do degree 14). The first noticeable difference is a global intensity decrease, observed from the decrease in intensity and spatial extent of the high latitudes patches. In addition, the low intensity anomaly located South of Africa in 1590 seems to have drifted westward to South America and has intensified. Figure 2.31 shows the evolution of the radial component of the field  $B_r$  at the CMB over the past four centuries from gufm1. The spatial resolution is significantly improved with time, as witnessed by the emergence of smaller scale features, with increasing quality and amount of data.



FIGURE 2.29: Maps of a)  $B_r$  and b) its secular variation at the core-mantle boundary (CMB) in 2015 from CHAOS-7 model [Finlay et al., 2020] (up to degree 13).



FIGURE 2.30: Maps of the field intensity F at the Earth's surface from gufm1 [Jackson et al., 2000] in 1590 (a) and 1990 (b).

Although gufm1 is a reference model to infer the geomagnetic field variations over the past four centuries, some inherent limits arise due to the scarcity of directional data, in particular before ~ 1750, added to the fact that no intensity data are available before ~ 1840. Several authors pointed to discrepancies between archeomagnetic data and gumf1 predictions between ~ 1600 and ~ 1750 [e.g. Genevey et al., 2009, Hartmann et al., 2011, Osete et al., 2015, Le Goff and Gallet, 2017, see also section 3.5]. Therefore, improving our knowledge of the geomagnetic field and its variations over the past centuries requires the joint use of direct and indirect data.

#### 2.4.2 The westward drift

Shortly after the discovery of secular variations of the Earth's magnetic field, Edmund Halley evidenced the westward drift of declination by comparing records from Europe, Africa and South Atlantic. Bullard et al. [1950] calculated the global average drift rate between 1907 and 1945 and found a rate of  $\sim 0.180 \pm 0.015^{\circ}/\mathrm{yr}$  for the non-dipole part of the field. This drift is better observed by looking at the evolution of the radial component of the field  $B_r$  at the CMB. Such a description is first provided by Bloxham and Gubbins [1985] from single-epoch models running from 1715 to 1980. This analysis is subsequently improved by Bloxham et al. [1989], with models starting in 1695. They find that the westward drift affect most structures from mid to low latitudes and is concentrated in the Atlantic hemisphere (in both Northern and Southern hemisphere). In order to provide a better description of this motion, Finlay and Jackson [2003] propose to remove the timeaveraged axisymmetric component of the magnetic field and filter out frequencies larger than 400 years. The results, analysed from time-longitude plots, show that the westward drift is most prominent in the Atlantic hemisphere, at low latitudes, over the past four centuries and is characterized by a rms velocity of  $\sim 17$  km/yr, or equivalently  $\sim 0.27^{\circ}/yr$ , close to the one found at the global scale by Bullard et al. [1950]. This observation strongly supports the planetary-scale gyre inferred from core flow models.

The persistent character of this equatorial westward drift observed from direct records of the field over longer timescales cannot be resolved by paleomagnetic reconstructions of the field, due to the data precision, the lack of data in the Southern hemisphere and the models regularizations [Dumberry and Finlay, 2007]. However, such models seem to favor the occurrence of west- and/or eastward drifts at mid- to high latitudes for the last 3 to 4,000 yrs [Dumberry and Finlay, 2007, Hellio and Gillet, 2018, Campuzano et al., 2019, Nilsson et al., 2020] with rates varying from ~ 0.1 to  $0.25^{\circ}$ /yr, upper bound similar to the drift rate observed over the historical period. This high-latitude drift is also observed on shorter timescales from recent field models spanning the satellite era [Finlay et al., 2016b].

Nonetheless, with the increasing amount of data and the recent efforts to collect data from the Southern hemisphere, Campuzano et al. [2019] propose a new paleomagnetic reconstruction for the past two millennia, from the analysis of which they suggest that the South Atlantic Anomaly results from the southwestward drift of a reverse flux patch beneath South India between  $\sim 1000$  A.D. and today. This would imply that the currently observed westward equatorial drift is persistent over the last millennium at least.

#### 2.4.3 Hemispherical asymmetries of the geomagnetic field

Albeit the current geomagnetic field is mainly dipolar, the largest non-dipolar contribution is the South Atlantic Anomaly, i.e. a region of abnormally low field intensities, with a minimum currently located in Southern Brazil [Thébault et al., 2015]. Figure 2.30 shows the magnetic field intensity F in 1590 (a) and 1990 (b). We can see that the anomaly



FIGURE 2.31: (continued next page)



FIGURE 2.31: Maps of the radial component of the field  $B_r$  at the Earth's core surface up to degree 13 from gufm1 from 1590 to 1990.

has intensified and drifted westward for at least the past four centuries. However, the long-term persistence of this feature is still under debate. While there is paleomagnetic evidence for similar geomagnetic anomalies in South Africa [Tarduno et al., 2015] and South Brazil [Trindade et al., 2018, Hartmann et al., 2019] during the last millennia [and even longer, e.g. Engbers et al., 2020], global paleomagnetic reconstructions analyses are mitigated. This is essentially due to the fact that the current global database contains only a small fraction of data from the Southern hemisphere. Consequently, tracking the evolution of the SAA over the past millennia is challenging. Nonetheless, from the analysis of a new weighted paleomagnetic model, Campuzano et al. [2019] suggest the emergence of the South-Atlantic anomaly around  $\sim 1000$  A.D. in South-Africa, which would have then drifted southwestward and intensified at various rates until today.

This result evidences a second hemispherical asymmetry, more visible by analyses of the secular variation of the field, and still visible today. It indeed appears that most of the field variations are located in the Atlantic hemisphere, while the Pacific hemisphere presents rather low activity [e.g. Jackson and Finlay, 2015]. Again, the lower resolution of long-term models, especially due to the low amount of data covering the Pacific hemisphere makes it difficult to ascertain a long-term persistence of this feature [e.g. Constable and Korte, 2015]. Nonetheless, from the analyses of CALS10k.2, Constable et al. [2016] find higher activity in the Atlantic hemisphere compared to the Pacific hemisphere.

At the CMB, Bloxham and Gubbins [1985] and Gubbins [1987] notice the emergence of an inverse flux patch around ~ 1715 beneath South Africa, which subsequently drift westward (situated beneath Patagonia in 1990, Fig. 2.31e). The formation of another RFP is visible in 1790 (Fig. 2.31c) and seems to also drift westward to merge with the first patch (2.31d). During the 20th century, several other RFP emerge in this same region below South Atlantic. The emergence and amplification of these RFP are at the origin of the intensifying South Atlantic anomaly observed at the Earth's surface [Tarduno et al., 2015, Terra-Nova et al., 2017]. As previously mentioned, the persistence of such anomalously low field intensity in the South Atlantic is still under debate, though it seems a persistent feature for at least the past four centuries as illustrated by the time averaged field from gufm1 [Hulot et al., 2002]. Such a persistence is suggestive of a lower mantle control on the emergence of reversed flux patches beneath South Atlantic [e.g. Tarduno et al., 2015, Terra-Nova et al., 2016].

Based on analysis of the archeomagnetic field, Terra-Nova et al. [2015] hypothesize that RFP are associated with spherical harmonic of at least order 4, though it should be noted that this roughly corresponds to the resolution of such models. By analysing the historical field, Metman et al. [2018] find that typical structures of RFP are of spherical harmonic order  $\ell \sim 4 - 5$ , corresponding to wavelengths of ~ 4000 to ~ 5000 km at the CMB. Understanding the evolution of these reversed flux patches is crucial as they have been linked to the current decay of the geomagnetic dipole observed at the Earth's surface [Gubbins, 1987, Gubbins et al., 2006, Hulot et al., 2002, Olson and Amit, 2006, Terra-Nova et al., 2015, Finlay et al., 2016a, Metman et al., 2018].

#### 2.4.4 The axial dipole decay

#### a) Observations at the Earth's surface

At the global scale, the most noticeable feature of the secular variation over the past four centuries is the axial dipole decay. This decay has been first inferred by Barraclough [1974], who presented snapshots of spherical harmonic models (up do degree 4) of the geomagnetic field between 1610 and 1900. As there is no direct absolute intensity measurements before



FIGURE 2.32: Axial dipole evolution (a) since 1840 obtained from *gufm1* (black), COV-OBSx1 (blue) and CHAOS-7 (orange) and its variations (b).

1830s [Gauss, 1833], additional constraints are needed to reconstruct the field evolution before 1840. To this end, Barraclough [1974] proposed a backward linear extrapolation of the decrease observed from the axial dipole component  $g_1^0$  (between 1829 and 1970, see Fig. 2.32). He therefore imposed a linear decay of 15.46 nT/yr, stating that "the linear extrapolation of the values of the coefficient  $g_1^0$ , which was used to overcome the shortage of early intensity data, leaves something to be desired but it has the merits of being simple to use and of being not too implausible". Jackson et al. [2000], following Barraclough [1974], impose a decay rate of 15 nT/yr between 1590 and 1840 to construct gufm1.

Ever since, various attempts had been made to use global paleomagnetic datasets to constrain the axial dipole decay between 1590 and 1840 (the archeomagnetic data are used to recalibrate the axial dipole evolution as provided by gufm1, see section 3.1). All these studies propose linear decay of the axial dipole moment ranging from ~ 0 to ~ 15 nT/yr. Analyses of the available global paleo- and archeointensity dataset shows that while the axial dipole moment variations are characterized at first order by a global decrease [Genevey et al., 2008, Usoskin et al., 2016, Poletti et al., 2018], it seems that the decay is not exactly linear (Fig. 2.14, right panel). However, the large scatter observed in global datasets, even after the application of very strict selection criteria (recall section 2.1.3), prevents a further detailed analysis of the axial dipole moment evolution.

This PhD therefore explores an alternative approach proposed by Genevey et al. [2009]. Instead of relying on global datasets, these authors propose to use a restricted but selfconsistent regional archeointensity dataset to recalibrate the axial dipole moment as provided by *gufm1*. The obtained evolution is non-linear, with a minimum of the axial dipole amplitude during the second part of the 18th century. If this approach allows to circumvent the dispersion of global archeomagnetic data, additional well-constrained data are needed to confirm the results. This approach is further explored in chapter 3, from the analysis of well-dated (dating uncertainties lower than ~ 25 years) baked clay brick fragments sampled in Bukhara (central Asia).

#### b) Analysis in terms of core processes

Following Olson and Amit [2006] and Finlay et al. [2016a] (and references therein), the dipole moment variations in the Earth's outer core are described by

$$\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t} = \frac{3}{2\mu_0} \int_V \frac{\partial \mathbf{B}}{\partial t} \mathrm{d}V. \tag{2.44}$$

Inserting the induction equation Eq. 2.28 and taking the cylindrical axial component  $\mathbf{z}$ , the variations of the axial dipole moment over the core's surface are given by

$$\frac{\mathrm{d}m_z}{\mathrm{d}t} = -\frac{3}{2\mu_0} \int_S \left[ \underbrace{u_\theta \sin \theta B_r}_{\text{meridional advection}} + \underbrace{\frac{\eta}{r_c} \frac{\partial (rB_\theta)}{\partial r} \sin \theta}_{\substack{\mathrm{radial diffusion}}} - \underbrace{\frac{\eta}{r_c} \frac{\partial B_r}{\partial \theta} \sin \theta}_{\substack{\mathrm{meridional diffusion}}} \right] \mathrm{d}S. \tag{2.45}$$

This equation illustrates that the axial dipole moment variations are controlled by the meridional advection of the radial magnetic field, and the radial and meridional diffusion of the magnetic field. The contribution of meridional advection and diffusion can be calculated from  $B_r$  at the CMB and Olson and Amit [2006] find that the contribution of meridional diffusion is negligible. It will therefore be ignored in the following.

At first order, the axial dipole has been decaying for at least the past four centuries, at a mean rate of ~ 15 nT/yr over the past two centuries (Fig. 2.32b). At this rate, it would take ~ 2000 yrs for the axial dipole to vanish. As the free Ohmic dissipation time is far longer (~  $5 \times 10^4$  yrs), it seems that the leading contribution to the decay is meridional advection.

This is evidenced by Olson and Amit [2006] and Finlay et al. [2016a], by mapping the first term of RHS in 2.45. The flow is either estimated based TG and helical flows [Olson and Amit, 2006] or QG flows [Finlay et al., 2016a]. In addition Finlay et al. [2016a] also perform forward and inverse geodynamo modelling to coestimate the effect of diffusion on the axial dipole decay. They both find that most of the axial dipole decay is accounted for by advection, one of the key ingredient being the eccentric gyres. The meridional flows of the Eastern limbs transport normal flux patches equatorward beneath Central Asia and South Indian ocean [Olson and Amit, 2006, , see also Fig. 2.31]. These flux patches that emerge at mid to low latitudes in the Atlantic hemisphere. The RFP are then advected polewards by the meridional flows of gyres western limbs [Gubbins, 1987].

When evaluating the contribution of the Southern and Northern hemisphere in the dipole decay, Olson and Amit [2006], Gubbins et al. [2006] and Finlay et al. [2016a] also find that the Southern hemisphere is the main contributor, the Northern hemisphere presenting a steady contribution of normal and inverse magnetic flux to the dipole evolution, and these contributions cancelling each other out. The dipole decay therefore seems the result of unbalanced sources and sinks of the dipole moment.

On the other hand, although the radial diffusion contribution seems fairly low [around or less than ~ 20%, Finlay et al., 2016a] and constant over time, it still plays a key role in the axial dipole decay through the emergence and growth of reverse flux patches at the origin of the hemispherical asymmetry (i.e., departure from balanced sources and sinks). In fact, by studying the evolution of RFP, Metman et al. [2018] show that their evolution account for ~ 2/3 of the axial dipole decay during the 20th century, comprising both the expansion and poleward migration of the RFP. This is consistent with the results of Olson et al. [2009], who found that the origin of dipole decay in numerical dynamo simulations lies in the emergence of RFP.

Bloxham [1986] links the emergence of RFP with a mechanism of magnetic flux expulsion at the CMB (see Chapter 4). The toroidal magnetic field lines confined within the core would be advected close to the CMB by a flow upwelling and would subsequently diffuse through it. Liu and Olson [2009] propose such a mechanism applied to poloidal field, to explain the axial dipole decay. They analyse kinematic MHD model in 3-D spherical geometry with a fixed 2-D flow pattern represented on Figure 2.33 (top). The magnetic field lines are distorted and advected close to the CMB by a flow upwelling. The magnetic field subsequently diffuses through the CMB to form reversed magnetic flux patches at the surface of the Earth's core. This mechanism first increases the axial dipole (see 2.33) by the effect of flux concentration close to the CMB. When diffusion takes over, it leads to the subsequent decay of the axial dipole component. They infer that the axial dipole decay observed over the past two centuries could be attributed to this process. Tarduno et al. [2015] also link the emergence of RFP to the process of magnetic flux expulsion through the CMB. To explain the preferential occurrence of RFP beneath South Atlantic, they propose that flux expulsion is constrained by the topography of the CMB below South Atlantic, associated with the African large low shear velocity province (LLSVP) in the lower mantle. They speculate that a small protrusion of the CMB would deflect the underlying flow and favor the concentration and subsequent diffusion of magnetic flux at the border of the protrusion. They propose a small-scale deflected flow that would lead to fairly low Rm, allowing diffusion to act on much smaller timescales than infer at the global scale of the Earth's outer core. Understanding the contribution of magnetic diffusion to the axial dipole decay therefore requires to further analyse the process of magnetic flux expulsion (which is the subject of Chapter 4) and the mechanism of growth and decay of RFP.

#### 2.4.5 Local intensity variations

The most prominent local intensity variation observed at the Earth's surface over the historical period is the South Atlantic anomaly, involved in the axial dipole decay [e.g. Finlay et al., 2016a]. In this region, the geomagnetic field is characterized by low intensities and this anomaly is intensifying since at least the past two centuries, with an almost constant linear rate of  $\sim -0.03 \ \mu T/yr$  [Pavón-Carrasco and De Santis, 2016]. Despite the interest aroused by this anomaly, its evolution and dynamical origin are still poorly understood. Terra-Nova et al. [2017] analyse the relation between the South Atlantic anomaly at the Earth's surface and the underlying reversed magnetic flux patches at the CMB. They show that the low field intensity at the Earth's surface results from the joint evolution of several reverse flux patches beneath South Atlantic and the axial field dipolarity. They suggest that the westward drift of the SAA observed at the Earth's surface is mostly linked with advection of magnetic flux patches at the CMB and their intensity variations, while its intensification is the result of radial diffusion.

Another interesting feature of local intensity variations over the historical period is the intensity peak observed in Western Europe in ~ 1600AD [Fig. 2.34, Genevey et al., 2009, 2013, 2019]. This intensity peak is associated with maximum variation rates of order ~ 0.06 - 0.07  $\mu$ T/yr. Interestingly, it also seems that this intensity peak is part of a series of intensity peaks observed in Western Europe [Genevey et al., 2013, 2016, 2019, see also Gómez-Paccard et al., 2016] during the past ~ 1500 years. These peaks show a pseudoperiod of ~ 250 yrs [Genevey et al., 2016] and maximum variation rates of ~ 0.1  $\mu$ T/yr, the same order than the maximum variation rates observed in the current field [Finlay et al., 2020]. Stronger variation rates have nonetheless been inferred in Western Europe



FIGURE 2.33: Sketches for the mechanism of mixing flow (or flux expulsion, top) and influence on the axial dipole decay (bottom), for a magnetic Reynolds number Rm = 260. In the sketches, the black lines on the LHS of the sphere shows the streamlines and arrows on the RHS give the magnetic field lines of the poloidal field. The magnetic field azimutal current density is shown on the right of the sphere. The left panel gives the corresponding axial dipole decay. The right panel shows the total rate of change of the axial dipole (dots), the contribution of meridional advection (asterisks), and radial diffusion (triangles). Figure from Liu and Olson [2009].



FIGURE 2.34: Intensity variations in Western Europe during the last millennium. The archeointensity data are selected in a 700 km radius from Beaune in France [see Genevey et al., 2019, for the description of the selection criteria]. The mean curve is computed from the algorithm AH-RJMCMC from [Livermore et al., 2018]. The \* indicates data arbitrarily corrected for cooling rate effect by a 5% decrease of the intensity. Figure from Genevey et al. [2019].

during the last millennium BC (~ 700 BC), reaching values on the order of ~ 0.25  $\mu$ T/yr, associated with an intensity peak lasting ~ 400 yrs [Hervé et al., 2017].

The fastest intensity variations inferred so far for the geomagnetic field are associated with the so-called geomagnetic "spikes" in the Near-East during the first half of the first millennim BC [see Fig. 2.35, Ben-Yosef et al., 2009, Shaar et al., 2011, 2016, Ben-Yosef et al., 2017]. These spikes are defined as extreme intensity peaks lasting a few decades, associated with variation rates recently reassessed at ~ 0.75 - 1.5  $\mu$ T/yr [compared to previous suggestions of ~ 4 - 5  $\mu$ T/yr Livermore et al., 2014]. If some studies suggest the occurence of such spikes at the same period in Turkey [Ertepinar et al., 2012] and Georgia [Shaar et al., 2013], they do not seem to be identified in paleosecular variations curves in the Balkans [Tema and Kondopoulou, 2011, Kovacheva et al., 2014], as well as in Central and Western Europe, and their spatial extent remains unclear.

These extreme events challenge our current understanding of the Earth's core dynamics. By analyzing the intensity variations produced by optimized core flows, Livermore et al. [2014] infer that the geomagnetic spikes are not compatible with our current understanding of the core's surface flows. They show that, assuming purely toroidal flows at the top of the Earth's core, the optimized core flows are large scales, but nonetheless produce intensity change at the Earth's surface up to 0.62  $\mu$ T/yr (see Fig. 2.36). By analysing spherical harmonics models based on indirect observations, Korte and Constable [2018] show that such extreme values can be recovered by increasing the dipole moment intensity and variability. They suggest that such features originate from growth and decay of intense flux patches at the CMB. These results are in agreement with the conclusions of Davies and Constable [2018] who analyse outputs from numerical dynamo simulations and find that the most extreme events lead to intensity variations of ~ 0.75  $\mu$ T/yr associated with rapid intensification of magnetic flux patches at the core's surface. If these studies have mostly



FIGURE 2.35: Intensity variations in the Near East (Levant) during the last two millennia BC. Intensities are expressed in terms of virtual axial dipole moment (VADM). Figure from Shaar et al. [2016].

focused on the origin of the extreme intensity variation rates inferred for the geomagnetic spikes, the temporal dimension of such events have been so far put aside.

The inferred origin for these extreme intensity events, i.e., growth and decay of magnetic flux patches would involve a diffusion process at the CMB. Magnetic flux expulsion of toroidal field [Bloxham, 1986] has been suggested as a possible explanation. Unravelling the contribution of growth and decay of normal and reverse flux patches at the CMB, at a global (the axial dipole) or local (regional intensity peaks) scales therefore requires a detailed analysis of this process, carried in chapter 4. The aim of this study is to constrain the intensity variations produced by flux expulsion events at the CMB in order to determine if the latter can be at the origin of geomagnetic spikes. To this end, a 2D kinematic model of magnetic flux expulsion is implemented, following Bloxham [1986]. A systematic analysis of the result is performed, allowing for the determination of scaling laws depending on the key control parameter: the magnetic Reynolds number Rm. The extrapolation of these scaling laws at the Earth's surface therefore allows the comparison with both variation rates and durations associated with extreme intensity events observed at the Earth's surface. As the model does not account for the Lorentz force, it is expected to provide upper bounds characterizing the magnetic flux expulsion process.



FIGURE 2.36: Intensity variations at the Earth's surface at the global (left) and regional (center) scales, induced by a purely toroidal flow at the top of the Earth's core (right), optimized at the site Timna-30 at the Earth's surface, marked by a yellow star (Israel). Figure from Livermore et al. [2014].

# Chapter 3

# Analyzing the geomagnetic axial dipole field moment over the historical period from new archeointensity results at Bukhara (Uzbekistan, Central Asia)

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## Résumé

Depuis le milieu du 19e siècle, les mesures directes de l'intensité et de la direction du champ géomagnétique permettent de déterminer avec précision ses variations spatio-temporelles. Avant cette époque, entre  $\sim 1600$  et 1840, ne sont disponibles que des données directes de directions. La construction de modèles globaux du champ sur cette période nécessite donc soit un traitement spécifique du dipôle axial, soit l'utilisation de données archéomagnétiques d'intensité. Dans cette étude, une approche régionale est proposée, basée sur la reconstruction des variations d'archéointensité en Asie centrale, obtenues par l'analyse de fragments de briques cuites échantillonnées à Boukhara (Ouzbékistan), datées entre la fin du 16e et le début du 19e siècle. Cette ville est particulièrement intéressante d'un point de vue archéomagnétique de par ses bâtiments remarquablement bien conservés et datés précisément grâce à des archives documentaires. Une série de données d'archéointensité est obtenue par le protocole expérimental du Triaxe, qui montre une décroissance de l'intensité entre  $\sim 1600$  et  $\sim 1750$ , ainsi que des valeurs d'intensités plus faibles que celles prédites par les modèles globaux du champ géomagnétique. Ces nouvelles données sont cohérentes avec d'autres données Triaxe acquises précédemment en Europe de l'Ouest et en Russie de l'Ouest, quand ces dernières sont transférées à Boukhara en utilisant la géométrie du champ proposée par le modèle qufm1. Ces données sont donc utilisées pour recalibrer l'évolution du dipôle axial de ce modèle. La nouvelle évolution ainsi obtenue est non-linéaire, avec un minimum relatif de l'intensité du dipole axial à la fin du 18e siècle. A l'heure actuelle, cette évolution ne peut être confirmée ou réfutée de façon satisfaisante par d'autres jeux de données présentement disponibles pour l'Eurasie de l'Ouest (ainsi qu'à une échelle plus globale), principalement du fait de la dispersion significative de ces données. Cette interprétation repose sur la précision de la géométrie du champ de qufm1, qui semble moins fiable avant ~ 1750. Néanmoins, le minimum proposé au cours du 18 siècle semble être une véritable caractéristique de l'évolution du dipôle axial.

## Abstract

Since the mid-19th century, direct measurements of both intensity and direction of the Earth's magnetic field have been available, allowing an accurate determination of its spatiotemporal variations. Prior to this time, between  $\sim 1600$  and 1840, only direct directional measurements are available. Therefore, the construction of global field models over this period requires either a specific treatment of the axial dipole field component or the use of archeomagnetic intensity data. In this study, we use a regional approach based on the construction of an archeointensity variation curve in Central Asia. We analyze baked clay brick fragments sampled in Bukhara (Uzbekistan), dated between the end of the 16th century and the beginning of the 19th century. This city is of particular interest for archeomagnetism due to the well-preserved old buildings accurately dated by documentary archives. A series of archeointensity results is obtained using the Triaxe experimental protocol, which shows a decreasing trend in intensity from  $\sim 1600$  to  $\sim 1750$ , with intensities during the 18th century lower than expected from global geomagnetic field models. These new data appear consistent with other Triaxe data previously obtained in western Europe and western Russia, when transferred to Bukhara using the field geometry of the qufm1 model. Together, these data are used to recalibrate the axial dipole moment evolution provided by this model. The resulting evolution appears non-linear, with a clear relative minimum in the magnitude of the axial dipole during the late 18th century. We illustrate the fact that at present this evolution can neither be satisfactorily confirmed nor refuted by other datasets available in western Eurasia (as well as at a wider spatial scale), mainly due to the significant dispersion of the data. Our interpretation relies on the accuracy of the field geometry of the qufm1 model, which appears less reliable prior to ~ 1750. Nevertheless, the minimum proposed in the 18th century seems to be a true feature of axial dipole behavior.

## 3.1 Introduction

Variations of Earth's magnetic dipole cover a wide range of timescales from a year or less to tens of millions of years. Three different frequency bands are evidenced by analyses of the dipole power spectrum from paleo- and geomagnetic data and simulations [Constable and Johnson, 2005, Ziegler et al., 2011, Olson et al., 2012, Panovska et al., 2013, Bouligand et al., 2016, Lesur et al., 2018: an ultra-low to low frequency band (UF), a transitional frequency band (TF), and a high frequency band (HF). The UF band comprises chrons and superchrons and is associated with the thermal evolution of the outer core. The TF band covers paleo-/archeomagnetic secular variations and is associated with geodynamo processes. Finally, the HF band contains the shortest periodicities of the axial dipole's variations (as observed from satellite data). These bands are separated by two cut-off frequencies  $T_s$  (between UF and TF) and  $T_f$  (between TF and HF), estimated by Hellio and Gillet [2018] from recent field statistics as  $T_s = 100$  kyr and  $T_f = 60$  yr, for the purpose of constructing the COV-ARCH model (more on global models below). The axial dipole's power spectrum from numerical dynamo simulations corroborates these results Olson et al., 2012, Bouligand et al., 2016, although the estimated characteristic timescale  $T_f$  is longer  $(T_f \sim 10^2 - 10^3 \text{ yrs})$ , which is probably associated with the convective timescale in the outer core of order 150 yr. While secular variations recovered from global archeomagnetic models are representative of the low-frequency TF band, regional variation curves spanning the last few millennia based on high-quality archeomagnetic data could be associated with the high-frequency band, on time scales on the order of the convective turnover time [e.g. Genevey et al., 2016, 2019].

Studying past field variations requires the construction of time-dependent global field models from the compilation of direct (or instrumental) and/or indirect geomagnetic field measurements. One of the most widely used models is the qufm1 model, which covers the past 400 years [Jackson et al., 2000] from 1590 to 1990, and which was constructed from a large set of direct geomagnetic measurements obtained in land-based observatories and by mariners during their voyages across the seas [e.g. Jonkers et al., 2003], as well as from satellite data for the most recent period. However, our ability to instrumentally measure geomagnetic field intensities only dates back to the 1830s [Gauss, 1833]. To overcome this lack of intensity data, Jackson et al. [2000], following Barraclough [1974], impose a linear decay rate of 15 nT/yr to the axial dipole component between 1590 and 1840, i.e. a rate corresponding to a crude extrapolation back in time of the behavior observed since  $\sim 1840$ . Since it is essential for the construction of the qufm1 model, and in general for our knowledge of geomagnetic field behavior during the historical period, this crude extrapolation has been tested against paleo- archeointensity data (i.e. indirect measurements) provided by the study of the thermoremanent magnetization carried by archeological artifacts and volcanic deposits [e.g. Gubbins et al., 2006, Finlay, 2008, Genevey et al., 2009, Hartmann et al., 2011, Suttie et al., 2011, Poletti et al., 2018]. Hulot et al. [1997] indeed establish that the geomagnetic field can be recovered from directional data alone, up to a constant multiplier (the uniqueness of the sought-after solution being guaranteed by the existence of two, and only two, poles at Earth's surface). The multiplicative constant is in practice provided by independent intensity measurements, each Gauss coefficient entering the mathematical description of the field being renormalized to account for the intensity measured at the specific location of interest.

Gubbins et al. [2006] follow this line of reasoning and this is the first study to use the set of indirect intensity data available between 1590 and 1840 to recalibrate the axial dipole component provided by gufm1 by the ratio of measured to predicted intensities at intensity determination sites. Due to scattered data, they assume that a linear fit is indeed the most

reasonable solution prior to 1840, but estimate that the axial dipole component between 1590 and 1840 had a rate of decrease of  $2.28 \pm 2.72 \text{ nT/yr}$ , which is significantly lower than that proposed by Barraclough [1974] and used by Jackson et al. [2000] (15.46 nT/yr and 15 nT/yr respectively).

Next, Finlay [2008] combines both direct and indirect geomagnetic measurements to calculate a new geomagnetic field model between 1590 and 1840, without imposing a linear decrease in the axial dipole during this period (but with an artificial overweighting of the indirect records). He shows that this approach does not provide better results than those favoring no change in axial dipole during the 17th and 18th centuries.

Suttie et al. [2011] propose a radically different approach based on the statistical analysis of errors in the paleo- archeointensity data. In particular, the dataset available between 1840 and 1990 is used to estimate reasonable errors in the data, which are best assigned as fractions (~ 15%) of the field intensity values expected from gufm1. When applied to data prior to 1840, and again assuming a linear evolutionary trend in axial dipole over this period, they find a rate of decay (~ 11.9 nT/yr) close to what Barraclough [1974] found. They further show that if data errors are assigned as fractions of measured intensities, the decay rate is similar to that proposed by Gubbins et al. [2006] and Finlay [2008] (i.e., with either a slight change or no change at all in the axial dipole component over the 17th and 18th centuries). However, this observation is the result of a bias toward lower field values, as their uncertainties are lower when given as a proportion of measured intensities.

For the different methods above, dispersion of paleo-archeomagnetic data is such that it prevents overcoming the assumption of a linear evolution of the axial dipole component over the historical period. In addition, Suttie et al. [2011] demonstrate that the use of quality criteria on the dataset does not significantly change the conclusions. More recently, Poletti et al. [2018] also use a selected global dataset with strict criteria covering the historical period (1590 – 2009). After converting intensity data into corresponding axial dipole moments and performing linear regression computations for datasets covering various time intervals, they reach a conclusion favoring a linear decreasing trend of the axial dipole over the historical period of ~ 12.5 nT/yr, thus close to that advocated by Barraclough [1974] and Suttie et al. [2011].

Given the dispersion observed in the global compilation of intensity data regardless of the selection criteria considered, Genevey et al. [2009] explore a different approach using a single consistent regional intensity dataset to recalibrate the  $q_1^0$  coefficient of qufm1. The principle remains the same as above [Hulot et al., 1997], which assumes that the geometry of the geomagnetic field as provided by qufm1 is correct. While it potentially avoids the problem of global data scatter, and the almost insoluble issue of selecting only the most reliable data, it does raise the pending issue of which dataset is sufficiently reliable to be used to recalibrate the Gauss coefficients (an evaluation that will surely vary from one author to another). Genevey et al. [2009] use the set of accurate and precisely dated archeointensity results obtained in western Europe (700 km around Paris, France). Instead of a linear decrease of the axial dipole magnitude over the historical period, they find a significant decrease between  $\sim 1590$  and the second half of the 18th century, with a minimum magnitude during this period, followed by a moderate increase from  $\sim 1800$ to  $\sim 1840$  and then, the well-established linear decrease up to the present. As a follow-up to this first study, Hartmann et al. [2010, 2011] analyse precisely dated architectural brick fragments from southern and northern Brazil. Despite a significant non-dipole field effect between these two regions associated with the South Atlantic Anomaly (SAA), the results obtained appear to support the evolution in dipole field moment proposed by Genevey et al. [2009]. As a new development, the present study carried out in Central Asia (Bukhara, Uzbekistan) focusing on the 1590 to 1850 period aims to further constrain the accuracy of the non-linear dipole moment evolution deduced from the western European dataset.



FIGURE 3.1: General location map of Uzbekistan and Bukhara

# 3.2 Historical context and sampling

Situated on the Silk Road, Bukhara (39.8°N, 64.5°E, Fig. 3.1) has long been an important place for trade, Islamic education and religion in Central Asia, as evidenced by the many madrasas (or religious schools) and mosques still standing in the city's historic center. These old buildings were built throughout the medieval period from fired clay bricks. Their history, and more generally that of the city itself, is well known through abundant written testimonies preserved in the state archives of Uzbekistan.

For the period covered by our study, the sampled buildings were erected during three successive dynasties that ruled Bukhara from the mid-16th century to the beginning of the 20th century: the Shaybanid dynasty during the 16th century, the Djanid dynasty from the 17th to the mid-18th century, and the Manghit dynasty from the mid-18th century to the early 20th century. The Shaybanid dynasty, which claimed to be descended from Genghis Khan, conquered Bukhara from the Timurids in the early 16th century and founded the khanate of Bukhara. Their domination for  $\sim 100$  years was interrupted by the Djanid dynasty (which also claimed to be descended from Genghis Khan), which then established its rule over Bukhara for about a century and a half. The Shah of Iran (Nader Shah) conquered the khanate around the mid-18th century, but the collapse of his empire a few years later led to the establishment of the Manghit dynasty. This dynasty was of Uzbek origin and ruled the Emirate of Bukhara until 1920 when Soviet Red Army troops invaded the city.

Throughout the above period, the city of Bukhara was divided into several small social

units called *guzars*, whose history is well documented in the archives. Each guzar was led by a chief (*aqsaqal*) nominated by the elders and had its own mosque around which the community was structured [Khalid, 1991]. Sukhareva [1976] [see Khalid, 1991] has done considerable work in compiling the oral and written testimonies on the guzars, providing extremely valuable information on the dating of even the smallest madrasas and mosques built in Bukhara over the past millennium.

Our archeomagnetic sampling was focused on several major and some minor buildings in and around Bukhara (Fig. 3.2, Table 3.1). Among the most important are three madrasas (with a group of fragments collected for each of them): two were built during the reign of Abdullah Khan (1583-1598), one of the most famous rulers of the Shaybanid dynasty. One was built for his own glory (Madrasa Abdullah Khan; BK03, Fig. 3.2a), and the second was to glorify his mother (Madrasa Modari Khan, built around 1561; BK04, Fig. 3.2b). The third madrasa was erected in  $\sim 1651 - 1652$  by the Khan Abd al-Aziz of the Djanid dynasty (Madrasa Abd al-Aziz Khan; BK01). We also carried out a sampling in the Chor Bakr necropolis, built near Bukhara at the time of the Shaybanid dynasty, and at the location of older tombs dating from the 10th century. There, two groups of fragments associated with the tomb (or *khazira*) of Khwādja Saad, son of Khwādja Islām Juybārī, leader of the Sufi order (i.e., a mystical order of Islam), erected just before his death at the end of the 16th century, have been collected (Khwādja Saad tomb's wall and floor, BK05 and BK06 respectively; Fig. 3.2d; Table 3.1). The sampling also included the Ark citadel. This ancient fortress, which was last destroyed in 1920 and rebuilt several times during the history of the city, was inhabited by the rulers. It comprised several buildings surrounded by an imposing wall, among which we sampled the kanaqah, which is a dwelling place for dervishes (adherents of Sufi orders) dating from the mid-18th century (BK14, Fig. 3.2f). In addition, we sampled fragment groups from three minor mosques (Mosque Dostum Chor Oghasi, BK09; Mosque Magoki Kurpa, BK12; and Mosque Kemuhtagaron, BK13) and three madrasas from smaller neighborhoods (i.e. the guzars; Madrasa Kunjak, BK07; Madrasa Rakhmanqul, BK08; Madrasa Rashid-al-Din, BK11, Table 3.1). It should also be mentioned that special care was taken to avoid restored wall segments and/or recycled bricks, which would result in an inaccurate dating.

In total, our archeomagnetic study is based on 13 groups of architectural brick fragments. The three above-mentioned dynasties are equally sampled: five buildings belong to the Shaybanid dynasty, four buildings to the Djanid dynasty and four buildings to the Manghit dynasty. For each of the two fragment groups BK08 and BK11, the samples were collected in different rooms of the same building; in this case, each sub-subset has been identified but all fragments are considered to come from the same ensemble (e.g. BK08A or BK08B; see supplementary Table A.1). Particular care was taken with the available dating constraints and we selected those buildings that have age uncertainties of less than  $\pm 25$  years, but in most cases these are less than 15 years. For some of these buildings, the construction is very well constrained by archives due to the social prominence of the people they were built for. In general, minor mosques and madrasas are first mentioned after construction in written documents, in particular for their inauguration or when a donor subsidized its use.

Sampling was carried out using an electrical driller with a water can. From 10 to 18 cores, 2.5 cm in diameter and from 5 to 10 cm in length, were drilled per group (Fig. 3.3). A total number of 160 cores were analyzed in the present study.



FIGURE 3.2: Example of buildings sampled in Bukhara: a) courtyard of Madrasa Abdullah Khan (BK03, 1578 – 1590), b) BK04: façade of Madrasa Modari Khan (1556 – 1567), c) BK08: façade of the Madrasa Rakhmanqul (1790 – 1795), d) BK05: tomb of Khwādja Saad in Chor Bakr (1589 – 1615), e) BK13: Mosque Kemuhtagaron (1700 – 1750), and f) BK14: kānaqāh inside the Ark citadel (1758 – 1785)



FIGURE 3.3: Examples of archeomagnetic sampling carried out in Bukhara: a) sampling of the Mosque Magoki Kurpa (BK12, 1631 – 1637), b) sampling of the Madrasa Rakhmanqul (BK08, 1790 – 1795), c) sampled wall in Madrasa Modari (BK04, 1556 – 1567), and d) cores sampled in the Madrasa Kunjak (BK07, 1700 – 1722).

## 3.3 Methods

### 3.3.1 Archeointensity determinations

All experiments were conducted in the paleomagnetic laboratory of the Institut de Physique du Globe de Paris (IPGP). The archeointensity determinations are based on the experimental protocol developed for the Triaxe magnetometer. This unique magnetometer allows continuous magnetization measurements (every  $\sim 5^{\circ}$  C) of a small individual specimen (less than 1 cm<sup>3</sup>) at high temperatures and under controlled field conditions, both in intensity and direction [Le Goff and Gallet, 2004].

The Triaxe procedure consists of five measurement series automatically performed between a low temperature  $T_1$ , usually 150°C, and a high temperature,  $T_2$ , at which most of the magnetization carried by the specimen is erased:

- Step 1: After rapid heating from room temperature to  $T_1$ , the specimen is heated in a zero field from  $T_1$  to  $T_2$  to demagnetize its natural remanent magnetization (NRM). The corresponding magnetization measurements give the  $M_1$  series;
- Steps 2 and 3: The specimen is cooled from  $T_2$  to  $T_1$  (step 2) and next heated from  $T_1$  to  $T_2$  (step 3), both steps in a zero field, to allow characterization of the thermal variability of the NRM fraction that remains blocked at  $T_2$ . The magnetization measurements give the  $M_2$  and  $M_3$  magnetization series, respectively;
- Step 4: The specimen is cooled from  $T_2$  to  $T_1$  in a laboratory field, the intensity of which is chosen close to the expected ancient field intensity, and its direction is automatically adjusted so that the direction of the newly acquired laboratory thermoremanent magnetization (TRM<sub>lab</sub>) is parallel to the direction of the original TRM (i.e., NRM). This step therefore leads to the acquisition of a new TRM with unblocking temperatures between  $T_2$  and  $T_1$  (magnetization series  $M_4$ );
- Step 5: The specimen is then heated again between  $T_1$  and  $T_2$  to demagnetize the TRM<sub>lab</sub> (magnetization series  $M_5$ ).

The procedure ends with rapid cooling of the specimen to room temperature.

Intensity determinations are based on the ratio between the NRM and  $\text{TRM}_{\text{lab}}$  fractions unblocked between T<sub>1</sub> and a temperature T<sub>i</sub>, varying from T<sub>1</sub> to T<sub>2</sub>. At any T<sub>i</sub>, these fractions are respectively determined by:

$$\Delta_1'(T_i) = (M_1(T_1) - M_1(T_i)) - (M_3(T_1) - M_3(T_i));$$
(3.1)

$$\Delta_5'(T_i) = (M_5(T_1) - M_5(T_i)) - (M_3(T_1) - M_3(T_i)).$$
(3.2)

And the intensity value at  $T_i$  is given by:

$$R'(T_i) = H_{\text{lab}} \frac{\Delta'_1(T_i)}{\Delta'_5(T_i)}.$$
(3.3)

An intensity value is obtained for each specimen from the average of the  $R'(T_i)$  data derived from all temperatures  $T_i$  between  $T_1$  and  $T_2$  [see more details and discussion in Le Goff and Gallet, 2004]. Since intensity values should only be determined on the primary and single-vector magnetization acquired during the manufacture of the archeological artifacts, if a secondary magnetization is observed above  $T_1$  up to  $T_1'$  (but below  $T_2$ ) from the analysis of the NRM demagnetization data, then the reference temperature  $T_1$  can be increased to  $T_1'$ .

Compared to more conventional paleo- archeointensity methods that rely on stepwise demagnetizations and magnetization measurements carried out at room temperature, the Triaxe procedure has several advantages, including the fact that the TRM<sub>lab</sub> is acquired under thermal and field conditions relatively similar to those that led to the NRM acquisition. This helps to mitigate possible spurious effects that would result from the presence of multi-domain grains. The fact that the direction of the TRM<sub>lab</sub> is parallel to that of the original TRM eliminates the need for anisotropy correction on TRM acquisition [Le Goff and Gallet, 2004]. In addition, experiments have shown that the use of R'(T<sub>i</sub>) data allows to overcome the effect of the cooling rate on TRM acquisition [Le Goff and Gallet, 2004, Genevey et al., 2009, Hartmann et al., 2010, 2011, see also a more general discussion on TRM anisotropy and cooling rate effects in Genevey et al., 2008].

The intensity data are then examined according to a set of quality criteria, which have remained the same as in previous studies in which archeointensity data obtained using the Triaxe procedure are reported [e.g. Genevey et al., 2013, 2016, 2019, Gallet et al., 2014, 2015, 2020]. At the specimen level, the  $R'(T_i)$  data must involve at least 50% of the NRM still blocked at  $T_1$  (or  $T_1$ ) and the relative variations of  $R'(T_i)$  between  $T_1$  (or  $T_1$ ) and  $T_2$  must not exceed 10%. A mean intensity value is determined at the fragment level when successful results are obtained from at least two specimens (note that in our study, we increase this number to three different specimens). In addition, the fragment-mean value is retained only if its standard deviation does not exceed 5% of the corresponding mean-intensity value. Finally, a mean intensity value is calculated at the level of a group of fragments when results are available from a minimum of three different fragments meeting the above criteria. The error is given as the standard deviation computed from the set of the retained intensity values at the fragment level. These criteria applied successively at the specimen, fragment, and fragment-group levels have proven to be extremely effective in constraining the quality and consistency of the intensity values obtained using the Triaxe method [e.g. Gallet and Le Goff, 2006, Genevey et al., 2009, Hartmann et al., 2010, 2011, Hervé et al., 2017.

#### 3.3.2 Magnetic mineralogy characterizations

In addition to the archeointensity experiments, we also performed different analyses on the retained fragments to identify the magnetic minerals present in the samples and to further assess whether this magnetic mineralogy alters during heating. Analyses include, for all retained fragments, low-field susceptibility vs. temperature measurements (using a KLY3 kappabridge from Agico coupled with a CS3 furnace) and for at least two fragments from each retained group, the acquisition (using a Vibrating Sample Magnetometer Model 3900) of isothermal remanent magnetization (IRM) and hysteresis loop measurements as well as the thermal demagnetization of three-axis IRM acquired (using a MMPM10 pulse magnetizer) in orthogonal fields of 1.5, 0.6 and 0.2 T [Lowrie, 1990]. Additionally, for a selection of representative samples, complementary low-temperature magnetization measurements are carried out using a magnetic property measurements system (MPMS XL-5 EverCool). The latter experiments include the following remanent magnetization measurements: 1) temperature cycling of an IRM acquired at room temperature (RT-SIRM) in a 2.5 T field down to 10 K and return to room temperature in a zero field (less than  $\pm$  500 nT), and 2) the thermal demagnetization from 10 K to 300 K of an IRM acquired at 10 K in a 2.5 T field following a zero-field cooling (ZFC) and 2.5 T-field cooling (FC) pre-treatments from 300 K to 10 K. Both the RT-SIRM and ZFC-FC measurements were duplicated in a second series of experiments where the initial IRMs acquired in 2.5 T are demagnetized in a 300 mT using the MPMS's superconducting magnet in a field oscillation mode, a method introduced and validated in Lagroix and Guyodo [2017]. The objective of the second series of experiments is to remove the contribution from low coercivity minerals to the total magnetization.


FIGURE 3.4: a) Normalized IRM acquisition obtained up to 1.5 T for 24 representative fragments of the selected groups, b-e) four examples of thermal demagnetization of 3-axis IRM acquired in orthogonal fields of 1.5 T (blue dots), 0.6 T (orange triangles), 0.2 T (green squares).

## 3.4 Archeointensity results

### 3.4.1 Magnetic mineralogy

IRM acquisition curves are reported in Fig. 3.4a. They show that saturation of the magnetization is often not completely achieved at 1T, but a clear inflexion in the magnetization curves is observed at  $\sim 0.1$  T. The thermal demagnetization of three-axis IRM further indicates that the magnetization is mostly carried by low-coercivity minerals with unblocking temperatures below 600°C, which is consistent with the presence of (titano)magnetite (Fig. 3.4b-e). Fig. 3.4b-e also shows the presence of high-coercivity minerals whose unblocking temperatures do not exceed  $\sim 550^{\circ}$ C, being sometimes as low as  $\sim 200^{\circ}$ C or with an inflexion around this temperature (Fig 3.4b-d). Fine grained hematite, lowering its unblocking temperature [e.g. Özdemir and Dunlop, 2014], and/or epsilon iron oxide, a magnetic phase often observed in archeological artifacts [e.g. Genevey et al., 2016, López-Sánchez et al., 2017, Kostadinova-Avramova et al., 2019] are mineral phases compatible with the above observations. A duality of low and high coercivity minerals is observed in a few hysteresis loops displaying slight constrictions (Fig. 3.5a). However, most of the hysteresis loops are not wasp-waisted, always exhibiting a monotonic decrease in loop opening with increasing field (Fig. 3.5b-c).

Low-field susceptibility versus temperature (heating and cooling) curves yield two main

observations (Fig. 3.5d-i). First, heating and cooling curves are reversible or very nearly, which attests to the stability upon heating of the magnetic mineralogy in the temperature range used for intensity determinations. Second, most susceptibility curves show a clear inflexion around 300°C, arising from a range of susceptibility evolutions from rapid rates of change (Figs. 3.5g,i) to slower monotonic ones (Figs. 3.5d,e,f,h), in addition to a higher temperature inflexion above 500°C. At this stage, we could propose that the inflexions indicate the presence of two families of (titano)magnetite differing by their grain size, their titanium content and/or their oxidation state (see below).

Low-temperature magnetization measurements bring additional insight into the magnetic mineralogy. Compared to the classic RT-SIRM and ZFC-FC experiments (left panels in Fig. 3.6 and 3.7 respectively), their 300 mT demagnetized counterparts highlight the temperature dependent behaviour of the high coervicity minerals (right panels in Fig. 3.6 and 3.7 respectively). Comparing the two provides information on the relative contribution of low or high coercivity minerals to the total remanence [Lagroix and Guyodo, 2017]. The lack of a Verwey transition in ZFC-FC data (Fig. 3.7) and primarily reversible RT-SIRM curves (Fig. 3.6a and c) or temperature suppressed Verwey transition (Fig. 3.6b) are compatible with titanomagnetite [Kakol et al., 1994, Moskowitz et al., 1998, Muxworthy and McClelland, 2000]. Hematite is unambiguously identified in the demagnetized RT-SIRM data (right panels of Fig. 3.6a and c) from the observed Morin transition, which displays a remanence loss and partial recovery over a wide temperature range ( $\sim 235$  K to 170 K) compatible with fine grained (0.1 to 1  $\mu$ m) hematite [Özdemir et al., 2008, Özdemir and Dunlop, 2014]. Another noteworthy observation is the kink seen at  $\sim 70$  K in both ZFC and FC curves which is also compatible with Ti-rich [50 to 60 % Ti; see Moskowitz et al., 1998] titanomagnetite and the persistence of the kink after 300 mT demagnetization (except for BK04-10) finds an explanation in the significant increase in coercivity at low temperature of Ti-rich titanomagnetite [see for example fig. 15c in Almeida et al., 2014]. The 70 K kink could alternatively be related to the epsilon iron oxide phase [López-Sánchez et al., 2016, 2017]. Lastly, behaviour suggestive of nanogoethite [Guyodo et al., 2003], which would be of weathering origin, is occasionally observed (right panels of Fig. 3.6b and Fig. 3.7a).

#### 3.4.2 New archeointensity data

We analyzed a total of 160 fragments (532 specimens) from 13 different archeological (historical) contexts. Most often, the magnetization of the samples is comprised between  $\sim 30$ and  $\sim 140 \times 10^{-8} \text{Am}^2$  (with a maximum of  $\sim 500 \times 10^{-8} \text{Am}^2$ ) and 42 of them are too weak to be measured with the Triaxe magnetometer ( $< 30 \times 10^{-8} \text{Am}^2$ ), which has a measurement sensitivity on the order of  $\sim 10^{-8}$  Am<sup>2</sup> [Le Goff and Gallet, 2004]. Among the 118 remaining fragments, 70 fragments are rejected due to non-linear or complex behavior compared to the nominal behavior described in Le Goff and Gallet [2004] [see also Genevey et al., 2009, Hartmann et al., 2010, 2011] and because of scattered magnetization measurements. In addition, 12 fragments are rejected because satisfactory results are obtained from only one specimen for each of them (whereas a minimum number of three specimens is required). Finally, 36 fragments from nine groups of fragments (112 specimens) provide archeointensity results that meet our set of selection criteria, while four groups are rejected (i.e. BK01 [1642 - 1652], BK02 [1735 - 1759], BK03 [1578 - 1590], BK09 [1580 - 1586]). This corresponds to a low success rate of 31% compared with the 118 fragments actually analyzed with the Triaxe. Details of the successful data are presented in Table A.1 in the supplementary material.

Three representative thermal demagnetization diagrams are shown in Fig. 3.8 (left panels), together with the corresponding  $R'(T_i)$  data (right panels). In general, the specimens



FIGURE 3.5: Examples (a-c) are hysteresis loops obtained for selected fragments where a) is an example of slightly constricted behavior, b-c) are examples of common behavior with narrow but open loops with a squared shape. d-i) are normalized low-field susceptibility vs. temperature curves obtained for some of the selected fragments up to  $\sim 500^{\circ}$ C. The orange curves (resp. blue) show the behavior during the heating (resp. cooling) step.



FIGURE 3.6: Representative examples of RT-SIRM cycles of a 2.5 T field IRM (left panels) and a 2.5 T IRM partially demagnetized (right panels) with a 300 mT field generated by the superconducting magnet operating in oscillation mode. The blue (resp. orange) dots correspond to the cooling (resp. heating) step. The results are normalized to  $M_n$  (corresponding to the initial RT-SIRM at 300 K).



FIGURE 3.7: ZFC-FC warming curves for the same fragments as in Fig. 3.6 of 2.5 T IRMs acquired at 10 K (left panels) and 2.5 T IRMs partially demagnetized at 10 K (right panels) with a 300 mT field generated by the superconducting magnet operating in oscillation mode. The blue (resp. orange) dots correspond to the ZFC (resp. FC) step. The results are normalized to  $M_n$ (corresponding to the initial FC at 10 K).

are fully demagnetized at relatively low temperatures, below  $450 - 500^{\circ}$ C. A single magnetization component is essentially isolated, even though a small secondary component probably of viscous origin is identified in most cases at low temperatures (below  $150^{\circ}$ C) but also in some cases having slightly higher temperatures (around  $200^{\circ}$ C).

Of the nine groups of fragments, the data from six groups are shown in Fig. 3.9 (with one panel each). In this figure, each curve represents the  $R'(T_i)$  data obtained for a specimen. It is also worth recalling that for each group of fragments, the  $R'(T_i)$  data are first averaged at the specimen level (over the temperature range between 140°C-260°C and 385°C-525°C), then at the fragment (brick) level, and finally all the fragment-mean values are averaged at the level of each group of fragments. Six groups of fragments are defined by data obtained from three different fragments (with a total of nine specimens) and the maximum number of fragments is seven (group BK11 with 21 specimens; Table 3.1). Fig. 3.9 also illustrates the overall consistency of the data obtained for each group of fragments, resulting in small standard deviations. They range from 0.9  $\mu$ T (BK08) to 2.0  $\mu$ T (BK11), or between 1.7% (BK06) and 4.5% (BK11) of the mean intensity values.

The new archeointensity data obtained at Bukhara cover a time interval of ~ 250 years, between the mid-16th century and ~ 1800 (Table 3.1; Fig. 3.10). A significant decrease in intensity values by ~ 14  $\mu$ T is observed from ~ 1560 to ~ 1725, leading to an average rate of change of ~ -0.1  $\mu$ T/yr. The second half of the 18th century is then marked by an increase of ~ 4  $\mu$ T until the early 19th century, leading to a variation rate of ~ 0.05  $\mu$ T/yr. This rate of change is fairly comparable to that of the present-day field in Bukhara (~ 0.08  $\mu$ T/yr).



FIGURE 3.8: Left panels: thermal demagnetization data obtained for three different specimens. Open (close) symbols refer to the inclinations (declinations). Right panels: corresponding  $R'(T_i)$  datasets obtained from the same specimens [see in Le Goff and Gallet, 2004]



FIGURE 3.9: New archeointensity results obtained at the specimen level for six groups of fragments (one panel each). Each curve shows the  $R'(T_i)$  data obtained for one specimen over the temperature range used for intensity determination (from  $T_1$  or  $T_1$ ' to  $T_2$ ).

TABLE 3.1: Mean archeointensity data obtained from nine groups of fragments collected at Bukhara. The historical context is indicated in the first column. The archeomagnetic reference of the groups of fragments is given in the second column. The dating of the context/group is provided in the third column. The number N of successful fragments (n specimens) used to compute the intensity value for each group is specified in the fourth column. The last column contains the corresponding mean archeointensity values.

Archeological site	Label	Age (yr AD)	N frag. (n spec.)	$F_{\rm mean} \pm \sigma F ~(\mu T)$
Madrasa Modari Khan	BK04	1556 - 1567	5(15)	$53.6 \pm 1.6$
Chor Bakr - Khwādja Saad tomb's wall	BK05	1589 - 1615	3(9)	$52.4 \pm 1.0$
Chor Bakr - Khwādja Saad tomb's ground	BK06	1558 - 1589	3(10)	$54.4\pm0.9$
Madrasa Kunjak	BK07	1700 - 1722	3(9)	$44.9 \pm 1.0$
Madrasa Rakhmanqul	BK08	1790 - 1795	3(9)	$42.3\pm0.9$
Madrasa Rashid- al-Din	BK11	1775 - 1825	7(22)	$44.2\pm2.0$
Mosque Magoki Kurpa	BK12	1631 - 1637	3(9)	$49.7 \pm 1.5$
Mosque Kemuh- tagaron	BK13	1700 - 1750	6(20)	$40.5\pm1.4$
Ark - $\underline{k}\overline{a}naq\overline{a}h$	BK14	1758 - 1785	3(9)	$44.3\pm1.5$

## 3.5 Discussion

## 3.5.1 Comparison of the new archeointensity data with model predictions at Bukhara

The new archeointensity data obtained in Bukhara are compared in Fig. 3.10 with the variations in intensities predicted by several geomagnetic field reconstructions (see Section 3.1). For the historical period, this is the gufm1 model [Jackson et al., 2000], and the models of Gubbins et al. [2006], Finlay [2008], Suttie et al. [2011] derived from gufm1 and calibrated for the 1590 - 1840 time interval from a global compilation of archeointensity measurements. Over this time interval, they predict the same pattern of variation but with various amplitudes corresponding to the different rates of decay imposed on the axial dipole component (recall the related commentary in the introduction). The comparison is also extended to geomagnetic models covering longer time intervals (between 3000 and 14,000 years): A FM [Licht et al., 2013], pfm9k.1 [Nilsson et al., 2014], SHA.DIF.14k [Pavón-Carrasco et al., 2014a], CALS10k.2 and ARCH10k.1b [Constable et al., 2016], COV-ARCH [Hellio and Gillet, 2018] and BIGMUDI4k.1 [Arneitz et al., 2019] constructed using global archeomagnetic datasets. A number of these models are constrained by gufm1 over the historical period (i.e. CALS10k.2, ARCH10k.1b, SHA.DIF.14k) and the corresponding predictions fall within the range of *gufm1*-recalibrated models. The pfm9k.1 model predicts a similar evolution, although the predicted dipole moment is higher compared to the other models for this period. The authors interpret this overestimation as resulting from the introduction of directional sedimentary data [Nilsson et al., 2014]. As this study is focused on the field variations over the historical period, the models constrained by qufm1 (CALS10k.2, ARCH10k.1b, SHA.DIF.14k) and those integrating sedimentary data (pfm9k.1) are not represented. A FM predicts an intensity evolution in Bukhara very close to the prediction from qufm1, with higher values than the observed intensities during the 18th and early 19th century. On the other hand, BIGMUDI4k.1 predicts a different intensity evolution, with a quasi-constant intensity during the 17th century and a wellmarked intensity peak during the 18th century. Unlike the other models, BIGMUDI4k.1 is built from the simultaneous inversion of both direct and indirect data. The authors note a significant decrease in the dipole energy associated with an increase of the non-dipole energy around ~ 1600. According to Arneitz et al. [2019], this is mainly due to the large increase in the amount of data at the onset of the historical geomagnetic era, rather than a true geomagnetic feature. The subsequent increase in dipole energy is therefore probably artificial, as is the resulting intensity peak. The modeled field behavior during this period should therefore be considered with caution. The COV-ARCH model, integrating only archeomagnetic data, predicts an evolution close to those of Gubbins et al. [2006] and Finlay [2008]. This evolution shows a minimum intensity at the end of the 18th century, slightly later than the minimum intensity observed from our data. Interestingly, in this model, the high-frequency range of the axial dipole variations is constrained by a timescale on the order of the convective turnover time (recall Introduction).

However, regardless of the model, none of the expected intensity evolutions reach the low intensity values observed in the 18th century from the new Bukhara archeointensity data, with a minimum overestimate of  $\sim 5-6 \mu$ T. Prior to this period, the Bukhara data show a rapid decrease in intensities between 1550 and the early 18th century with a rate quite similar to that of *gufm1* [see also the model of Suttie et al., 2011]. On the other hand, the new data require an increase in intensities during the first half of the 19th century, which is either absent or much more limited in model predictions.



FIGURE 3.10: Archeointensity data obtained in Bukhara (red dots). These data are compared with intensity values predicted from different global field models (continuous lines, errors are given as two standard deviations by shaded areas; see legend in the figure).

## 3.5.2 Dispersion of archeointensity results in regional datasets

Comparison with other archeointensity results previously obtained in the Bukhara area, as well as elsewhere in western Eurasia, raises a critical problem related to the dispersion that generally characterizes the regional datasets. Here we distinguish four geographical areas within a 700 km-radius around the cities of Bukhara (Uzbekistan), Moscow (western Russia), Tbilisi (Georgia), and Thessaloniki (Balkans). In each zone, the data are reduced to the latitude of the corresponding city. Most of these data are rather old and were described in the ArcheoInt compilation [Genevey et al., 2008]. In our study, they are selected using at first the same minimalist criteria as in Genevey et al. [2008] (referred to as G2008 below). A location map of the selected data is given in supplementary material (Fig. A.1). These criteria were originally proposed to allow the discussion of old data acquired without all the quality criteria now considered necessary for any new study, whereas applying modern criteria would eliminate most (if not all) of them. They do not consider the intensity methods directly but instead rely on 1) the error (most often a standard deviation) on the average intensity, which must be known and less than or equal to 15%; 2) the number of intensity determinations (Nint) used to derive an intensity mean. Nint is required to be greater than or equal to three when no pTRM-check was implemented or when this test does not apply. Otherwise, Nint must be greater than or equal to two. For objects recognized to be strongly anisotropic (such as pottery or tiles), Nint is required to be greater than or equal to three if anisotropy effects on TRM acquisition were not taken into account. In a second step, we consider stricter criteria requiring pTRM-check (when this test does apply) and intensity average derived from at least three independent fragments. In both cases, we select data whose age uncertainties are less or equal to  $\pm 50$ vears because we are interested in fairly rapid variations over a short time interval of  $\sim 300$ years. Further note that practically none of the available data have been corrected for the cooling rate effect on TRM acquisition.

Intensive work by Russian archeomagnetists in the 1970s and 1980s (S. Burlatskaya, I. Nachasova and K. Burakov) resulted in three regional datasets, in Uzbekistan, around

Moscow (Moscow, Gor'kiy, and Vologda) and in Georgia. These data share common features. They were acquired from analyzed baked bricks. The number of intensity determinations corresponds to the number of independent bricks studied. The method used is the original Thellier and Thellier [1959] protocol with the use of pTRM-check along the measurements cycle. This key element was, however, not specified in the articles (mainly published in Russian journals) but given by S. Burlatskaya, I. Nachasova and K. Burakov to A. Genevey in a personal communication (2004). A single dataset was obtained both using the Thellier and Thellier [1959] protocol and an original method developed by Burakov and Nachasova [1978], so-called thermal curves, derived from Wilson [1961]'s method (the TRM anisotropy being also taken into account). Both datasets are considered in our paper (identified by two different symbols), although we should point out that Nachasova and Burakov [1996] argue that the thermal-curves dataset is more reliable.

The results from Uzbekistan, more precisely obtained in the cities of Bukhara [Burlatskaya et al., 1977, 1986b], Samarkand [Burlatskaya et al., 1969, 1986b], and Khiva [Burakov and Nachasova, 1978] are of particular interest as they allow a direct comparison with the new archeointensity data reported in the present study (Fig. 3.11a). For Bukhara, the sampled sites are unfortunately not indicated in the original Russian publication [nor in the compilation of Burlatskaya et al., 1986b], but their estimation of the age of the buildings does not correlate with our new data, and the buildings sampled in the present study are therefore probably different. Despite some scatter, a fairly satisfactory agreement could be found for all results dating from ~ 1700 to ~ 1850, with values often lower than the intensities expected in Bukhara from the models. However, this satisfaction must be tempered by the fact that the older data (before ~ 1700) appear systematically weaker than our own intensity values. The discrepancy would be even larger if a cooling rate correction [for instance 5% as suggested by Genevey et al., 2008] was applied to the data. This ambiguous information is not improved by considering stricter selection criteria as most of the Uzbek data meet those criteria (see Fig. A.2 in supplementary data).

A fairly large scatter is observed for the data around Moscow, whether these data are selected using the G2008 set of criteria or stricter criteria (remember that only the number of fragments per site, Nint  $\geq 2$  or  $\geq 3$ , is the difference between the two selections, Fig. 3.11b and Fig. A.2b). This dispersion questions the reliability of at least part of these data, as previously discussed in Salnaia et al. [2017a,b]. The data seem consistent with a decreasing trend in intensities over the historical period. Nonetheless, as the cooling rate effect was not evaluated in the old Russian studies [Nachasova, 1972, Burlatskaya et al., 1986b], these data could also agree with lower than predicted values during the 18th century. Comparing the scant data obtained in Georgia with the model predictions leads to another contrast (see Fig. 3.11c). While fairly consistent agreement is observed for the data up to ~ 1700, the results are more scattered from ~ 1700 to ~ 1850, and in general the results are less consistent with the expected intensity values. With an arbitrary correction of the cooling rate effect of 5%, some of the latter results would be too low, in particular those dating from ~ 1800 to ~ 1850.

Finally, the Balkan area incorporates results from Greece and Bulgaria [Aitken et al., 1989a, Spassov et al., 2010, Kovacheva et al., 2009, 2014]. Contrary to the Russian datasets, whether pTRM checks are implemented or not is critical for distinguishing between the two selected datasets based on the G2008 versus the stricter set of criteria. (Fig. 3.11d and Fig. A.2d). With the loose selection (Fig. 3.11d), the data available between  $\sim 1550$  and  $\sim 1700$  appear relatively scattered, with the Bulgarian data in particular generally higher than the values expected from the models. A limited decrease to account for the cooling rate effect would improve the agreement. However, when stricter criteria are applied, all



FIGURE 3.11: Archeointensity results obtained in a 700-km radius from a) Bukhara, b) Moscow (Russia), c) Tbilisi (Georgia), d) Thessaloniki (Greece), reduced at the latitude of the corresponding location. The data are filtered using the G2008 set of criteria. Each panel also shows the predicted intensity evolution from various geomagnetic models at the corresponding location (continuous lines, errors are given as two standard deviations by shaded areas; see legend and text for details).

but one data point from this time interval are eliminated (Fig. A.2d). Still considering this selection, five data points remain for the 18th century (including two obtained by Spassov et al. [2010] from the same volcanic event) and only one for the 19th century. We may note that two Bulgarian results from the second half of the 18th century do not seem to indicate lower values than those predicted by the models. In contrast, this would be the case for the only result dating from the early 19th century. At this stage, it is therefore difficult to draw a firm conclusion from these rare data as well as from the entire Balkan dataset.

Overall, Fig. 3.11 shows that the data available in each of the four areas discussed above are too scattered to show any consistent pattern of intensity variations, at least at the century scale. As pointed out by many authors, determining a set of selection criteria that allows for significant reduction in the regional data scatter is a challenge. In the present case, increasing the strictness of the criteria does not alter our conclusion (at most, it leads to the rejection of most data in the Balkans), as this was also previously observed and discussed for western Europe [Genevey et al., 2009, 2013, 2019].

Our purpose is not to analyze and discuss in detail all datasets currently available worldwide [see for instance Poletti et al., 2018]. As observed in western Eurasia, data at the regional spatial scale are generally either too scant or too scattered to draw a clear evolution of intensities, which could lead to a meaningful comparison with the intensity values derived from the geomagnetic field models [see for instance Tema et al., 2017, Goguitchaichvili et al., 2018, Kapper et al., 2020, for Hawaii, Mesoamerica and West Africa, respectively]. Due to the general dispersion of the archeointensity data at the regional scale, a linear evolution of the dipole moment as constrained over the entire historical period, notably between 1600 and 1800, by a recalibration of  $g_1^0$  with these data, is a simple and reasonable approximation [e.g. Gubbins et al., 2006]. However, this does not demonstrate that the axial dipole moment evolution is actually linear; the scatter and the small amount of data leave room for more complex, possibly hidden variations in axial dipole moment.

## 3.5.3 A non-linear evolution of the axial dipole moment over the historical period

One might consider two options for explaining the dispersion of the data, either the frequent presence of biased results masking the "true" regional field intensity evolution or an inherent limitation in archeointensity determinations. In other words, due to their lack of resolution (and/or underestimation of their uncertainties), archeointensity data could not reliably detect and describe century-scale intensity variations. It is worth pointing out that this (dull) option is in clear contradiction with the convincing detection in western Europe of century-scale intensity variations over the past ~ 1500 years [Genevey et al., 2009, 2013, 2016, 2019]. For the historical period, western Europe benefits from a fairly dense archeointensity dataset showing a smooth evolution, with reduced dispersion (see description of the data in the mentioned studies). This leads Genevey et al. [2009] to explore a different approach for the recalibration of gufm1 Gauss coefficients by using a limited but consistent regional dataset.

Fig. 3.12a shows a direct comparison between the new data from Bukhara and the western European results recently upgraded and summarized in Genevey et al. [2019]. Most of these results share the same (Triaxe) experimental methodology and obey the same set of selection criteria. This comparison takes into account the geomagnetic field geometry given by the gufm1 model. Following Gubbins et al. [2006] and Genevey et al. [2009], a



FIGURE 3.12: a) New intensity evolution in Bukhara predicted by gufm1 recalibrated with the Triaxe data from western Europe (blue dots, Genevey et al. [2009, 2013, 2019]) and Russia (grey dots, [Salnaia et al., 2017a,b]), with the mean intensity variations curve and its 95% credible interval (in blue). This curve is computed using the AH-RJMCMC algorithm from Livermore et al. [2018] using the following input parameters:  $\sigma_{move} = 30$  yrs,  $\sigma_{change} = 5$  yrs,  $\sigma_{birth} = 5$  yrs,  $K_{max} = 150$ . The intensity priors are set to a minimum of 35  $\mu$ T and a maximum of 60  $\mu$ T, with a chain length of 100 million samples [see Livermore et al., 2018, for details on the parameters]. To stabilize the prediction for the younger period, the prediction is tied to the intensity value predicted by gufm1 in 1860 (47.5  $\mu$ T). b) Evolution of the axial dipole component  $g_1^0$  over the past four centuries. Dots gives the recalibration of  $g_1^0$  from gufm1 by the new archeointensity data from Bukhara (red dots), western Europe and the Russian datasets (blue and grey dots resp.), with the median variations curve and its 95% credible interval computed using the same parameters described above (except for the intensity priors set to  $-38 \ \mu$ T and  $-26 \ \mu$ T). The continuous lines give  $g_1^0$  as provided by gufm1 and various derived models (see text for details and Table A.2 for values). For BIGMUDI4k.1 and COV-ARCH, errors are given as two standard deviations by shaded areas

ratio of measured to predicted intensity is determined for each data point of Genevey et al. [2009, 2013, 2019]. It is then used to recalibrate all Gauss coefficients from *gufm1*, allowing the computation of a new field intensity prediction at Bukhara. This procedure applied to all the western European Triaxe data, to which we add three Triaxe data obtained in Russia by Salnaia et al. [2017a,b], allows for the determination of a consistent dataset (Fig. 3.12a). The results show a clear intensity decrease between  $\sim 1600$  and the first half of the 18th century, followed by an increase up to 1850. This intensity pattern is further evidenced by the computation of a mean intensity variation curve and its credible interval using the transdimensional Bayesian method recently developed by Livermore et al. [2018]. Based on this consistency, we also use the same dataset to recalibrate the axial dipole component  $(g_1^0)$  given by gufm1 (Fig. 3.12b). While the  $g_1^0$  values (provided in Table A.2) are rather compatible with those of the models during most of the 17th century, significant differences are then observed with all models, with smaller recalibrated values, throughout the entire 18th century and the early 19th century. On the other hand, our study also shows that there is currently no dense regional archeointensity dataset in western Eurasia, as elsewhere considering also the dispersion of the data, that could clearly contradict this  $g_1^0$  evolution. As previously suggested by Genevey et al. [2009] and now based on a larger collection of results, Fig. 3.12b strongly militates for a non linear evolution of the axial dipole field moment over the historical period, with a distinct minimum of  $|g_1^0(t)| \approx 29400$  nT during the 18th century. The average rate of decrease of  $|g_1^0(t)|$  during the 17th and the late 18th reaches  $\sim -26 \text{ nT/yr}$ , while the increase during the first half of the 19th reaches a rate of  $\sim 34 \text{ nT/yr}$ . These two variation rates are higher in amplitude than the one observed over the past 150 years [ $\sim -15 \text{ nT/yr}$ ; e.g. Barraclough, 1974, Jackson et al., 2000].

The regional approach used above is based on the reliability and accuracy of the geomagnetic field geometry of the gufm1 model [Jackson et al., 2000]. The implication on the axial dipole field moment's evolution between 1590 and 1850, as mainly constrained by the available Triaxe archeointensity data, therefore depends on this reliability. However, several studies have highlighted discrepancies between archeomagnetic directional data and the directions predicted by the gufm1 model [see for instance Tanguy et al., 2011, for the western Indian Ocean]. In France, Le Goff and Gallet [2017] have also shown that while satisfactory consistency is observed after ~ 1675, the gufm1 predictions differ significantly from most direct directional measurements prior to this date.

The relatively low reliability of the qufm1 model during the 17th century and part of the 18th century should not be surprising, given that very few, if any direct inclination data are available before 1700 - 1750, and more generally, given the poor spatial and temporal coverage of historical directional measurements between 1590 and  $\sim 1700$  [e.g. fig. 1a,e Jackson et al., 2000, in particular in Central Asia [see fig. 8 to 14 in Jonkers et al., 2003]. This calls for caution when interpreting recalibrated variations of axial dipole moments as shown in Fig. 3.12b. However, at the scale of western Europe to Central Asia), the satisfactory modelling of the non-dipole effects in qufm1 is evidenced by the good consistency, upon recalibration, of the Triaxe archeointensity datasets shown in Fig. 3.12a. In addition, it should be noted that this consistency is poorer when the data from western Europe and Russia are transferred to the latitude of Bukhara using a purely axial dipole field approximation (Fig. A.3). Nevertheless, the clear non-linear dipole moment evolution deduced using the *qufm1* model in a region across which significant nondipole field effects are not expected to occur over the historical period [e.g. Pavón-Carrasco et al., 2014b] does not demonstrate its truly dipole origin. For this, we need a large set of reliable and geographically distributed archeointensity data from around the world.

Away from western Eurasia, Hartmann et al. [2010, 2011] obtained in southern and

northern Brazil coherent historical archeointensity data using the Triaxe protocol (so far this is the only Triaxe data obtained outside western Eurasia) thus sharing the same criteria as before. As pointed out by Hartmann et al. [2011], these results also show the gufm1 model's lack of reliability for the period before ~ 1750. Note that based on an archeointensity result obtained in Ethiopia dated ~ 1615, Osete et al. [2015] also arrive at the same conclusion. On the other hand, using the same recalibration method as previously used by Genevey et al. [2009], the Brazilian data are consistent with a minimum of the axial dipole magnitude  $|g_1^0(t)|$  around the late 18th century, as shown by the western Eurasian Triaxe data (Fig. 3.12). This feature could thus represent a true dipole feature contradicting a linear evolution of the  $g_1^0$  term over the entire historical period. At this stage, however, we recognize that its global (dipole) nature has yet to be confirmed by the acquisition of new high quality archeointensity data.

## 3.6 Conclusions

The acquisition of nine new archeointensity data from Bukhara, Uzbekistan using the Triaxe experimental protocol allows for reconstruction of the geomagnetic field intensity variations in Central Asia from the mid-16th to the beginning of the 19th century. The evolution derived from the new data is marked by a rapid decrease of the intensities by  $\sim 14 \,\mu\text{T}$  from  $\sim 1560$  to  $\sim 1725$  followed by an intensity minimum during the late 18th century and then by an increase from the mid-18th to the beginning of the 19th century. Using the field geometry provided by the gufm1 model, we show that these results are consistent with other Triaxe data previously obtained in western Europe and in northwestern-central Russia.

When these data are used to recalibrate the axial dipole coefficient given by the gufm1 model, the resulting evolution appears non-linear over the historical period, with a clear minimum in magnitude of ~ 29400 nT during the 18th century. This trend contrasts with the linearity assumed by most global models so far. The validity of the global, dipolar nature of this analysis is contingent upon the reliability and accuracy of the field geometry provided by gufm1, both of which are well established from 1750 onward. The trend we find for  $g_1^0(t)$  can neither be satisfactorily confirmed nor refuted by the other regional datasets available in western Eurasia due to their dispersion. The sole data confirming the low of  $|g_1^0(t)|$  during the second half of the 18th century are Triaxe data from Brazil.

The conclusions we can draw from this study are twofold: first, it shows again that the acquisition and analysis of archeomagnetic data can provide useful information on the temporal behavior of the geomagnetic dipole on those time scales close to the convective turnover time (around the junction between the TF and HF frequency bands discussed in the introduction); second, it stresses that that information could be particularly useful to better constrain the geomagnetic secular variation during the historical period prior to the observatory era.

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## Chapter 4

# Imprint of magnetic flux expulsion at the core-mantle boundary on geomagnetic field intensity variations

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## Résumé

Au cours de la dernière décennie, des variations rapides ou extrêmes de l'intensité du champ géomagnétique, associées à des taux de variations supérieurs au maximum observé dans le champ actuel, ont été proposés à partir de données archéomagnétiques acquises au Proche-Orient et en Europe occidentale. Les événements les plus extrêmes, nommés spikes géomagnétiques, sont définis comme des pics d'intensité se déroulant sur une période très courte (quelques décennies), caractérisés par des taux de variation élevés, jusqu'à plusieurs  $\mu T/an$ . L'expulsion de flux magnétique à la surface du noyau de la Terre a été suggérée comme une explication possible de ces spikes mais n'a pas encore été étudiée en détail. Dans cette étude, nous développons un modèle cinématique bidimensionnel d'expulsion de flux magnétique dont le paramètre clé de contrôle est le nombre de Reynolds magnétique  $R_m$ , le rapport entre le temps de diffusion magnétique et le temps d'advection. Ce modèle permet de suivre l'évolution des lignes de champ magnétique qui sont déformées et repliées par un écoulement fixé. Deux processus régissent l'évolution magnétique du système. Le premier est l'expulsion du flux magnétique de lignes de courant fermées: le flux se concentrant progressivement près des limites du domaine, cela entraîne une augmentation de l'énergie magnétique du système. Si la limite supérieure du domaine sépare le fluide conducteur d'un milieu isolant, un second processus a lieu, celui de la diffusion à travers cette interface, que nous pouvons quantifier en suivant l'évolution de la composante verticale de l'induction magnétique le long de cette limite. C'est la conjonction de ces deux processus qui définit notre modèle d'expulsion du flux magnétique à travers la limite noyau-manteau. Nous analysons plusieurs configurations présentant des écoulements et des conditions aux limites magnétiques variables. Nous nous concentrons d'abord sur l'expulsion de flux d'un seul tourbillon. Cette configuration spécifique ayant été largement étudiée, la cohérence de nos résultats avec des solutions analytiques ainsi qu'avec des résultats numériques publiés précédemment valide notre méthode. Nous nous intéressons ensuite à une configuration caractérisée par deux tourbillons de sens opposés produisant un écoulement ascensionnel au centre du domaine, et une condition au limite supérieure isolante. Nous constatons que le temps de croissance caractéristique et le taux de variation instantané maximum de la composante verticale du champ magnétique expulsée à l'échelle du domaine sont respectivement de  $\sim R_m^{0,15}$  et  $\sim R_m^{0,45}$ . L'extrapolation de ces lois d'échelle à la Terre est comparée à divers pics d'archéointensité proposés au Proche-Orient et en Europe occidentale. D'après nos résultats, il est peu probable que l'expulsion de flux magnétique produise des spikes géomagnétiques, alors que des pics d'intensité de plus longue durée (un siècle et plus) et présentant des taux de variation plus faibles semblent être compatibles avec ce processus.

## Abstract

During the last decade, rapid or extreme geomagnetic field intensity variations associated with rates greater than the maximum currently observed have been inferred from archeomagnetic data in the Near-East and in Western Europe. The most extreme events, termed geomagnetic spikes, are defined as intensity peaks occurring over a short time (a few decades), and are characterized by high variation rates, up to several  $\mu T/yr$ . Magnetic flux expulsion from the Earth's outer core has been suggested as one possible explanation for these peaks but has not yet been examined in detail. In this study, we develop a twodimensional kinematic model for magnetic flux expulsion whose key control parameter is the magnetic Reynolds number  $R_m$ , the ratio of magnetic diffusion time to advection time. This model enables the tracking of magnetic field lines which are distorted and folded by a fixed flow pattern. Two processes govern the magnetic evolution of the system. The first one is the expulsion of magnetic flux from closed streamlines, whereby flux gradually concentrates near the boundaries of the domain, which leads to an increase of the magnetic energy of the system. If the upper boundary separates the conducting fluid from an insulating medium, the second process takes place, that of diffusion through this interface, which we can quantify by monitoring the evolution of the vertical component of magnetic induction along this boundary. It is the conjunction of these two processes that defines our model of magnetic flux expulsion through the core-mantle boundary. We analyse several configurations with varying flow patterns and magnetic boundary conditions. We first focus on flux expulsion from a single eddy. Since this specific configuration has been widely studied, we use it to benchmark successfully our implementation against analytic solutions and previously published numerical results. We next turn our attention to a configuration which involves two counter-rotating eddies producing an upwelling at the center of the domain, and comprises an upper boundary with an insulating medium. We find that the characteristic rise time and maximum instantaneous variation rate of the vertical component of the magnetic field that escapes the domain scale like  $\sim R_m^{0.15}$  and  $\sim R_m^{0.45}$ , respectively. Extrapolation of these scaling laws to the Earth's régime is compared with various purported archeointensity highs reported in the Near-East and in Western Europe. According to our numerical experiments magnetic flux expulsion is unlikely to produce geomagnetic spikes, while intensity peaks of longer duration (one century and more) and smaller variation rates appear to be compatible with this process.

Studying the variations of the Earth's magnetic field provides invaluable information on the dynamics in the Earth's outer core through the magnetohydrodynamic equations. These variations combine two processes: advection and diffusion. Considering that the typical timescale for diffusion is much longer than that for advection, diffusion is often neglected, especially when dealing with short-term geomagnetic variations. Such a simplification is referred to as the frozen-flux approximation [Roberts and Scott, 1965]. However, the validity of this approach is sometimes questioned [see Jackson and Finlay, 2015, for a review]. First, the diffusion term is negligible compared to the advection term only when considering the same large length scale for both processes ( $\sim 1000 \text{ km}$ ), which does not hold for peculiar flow and/or field configurations, such as that occurring when an upwelling drives expulsion of toroidal magnetic field. From the observational standpoint, constraints on the processes underlying secular variation come from the study of maps of the radial component of the magnetic field at the core-mantle boundary (CMB) inferred from data at the Earth's surface using inverse modelling [Jackson and Finlay, 2015]. These models have sometimes been interpreted as suggesting local failures of the frozen-flux approximation, for example related to the growth or decay in intensity of magnetic flux patches at the CMB, or the emergence of newly formed reverse flux patches [see Finlay et al., 2010, for a review]. The outputs of numerical dynamo simulations also point to a significant contribution of radial diffusion to the secular variation at a regional scale [Amit and Christensen, 2008]. This regional field-diffusion in specific locations is often interpreted as being due to flux expulsion events: the toroidal magnetic field confined within the outer core is advected by an upwelling of the flow, and concentrated at the CMB through which it then diffuses [Christensen and Olson, 2003, Christensen and Wicht, 2015]. Flux expulsion thereby contributes to the emergence of reverse flux patches or the weakening/strenghtening of existing flux patches.

Flux expulsion is often invoked to explain episodes of notable field intensity change in specific places such as the growth of the South Atlantic anomaly and associated dipole decay since 1850 [Gubbins, 1987, Gubbins et al., 2006], the acceleration of the drift of the north magnetic pole since the 1990s [Chulliat et al., 2010], or extreme geomagnetic field intensity variation rates as detected in archeomagnetic datasets [Shaar et al., 2011]. In the latter case, flux expulsion has been tentatively proposed as an explanation for two episodes of extreme field intensity variations during the first half of the first millennium BC in the Near-East [see for example Ben-Yosef et al., 2009, Shaar et al., 2011, 2016]. These features, named geomagnetic spikes, involve extreme variation rates up to several  $\mu T/yr$  and occur over only a few decades, a very short time interval for such large changes. Livermore et al. [2014] emphasize that such large rates are difficult to reconcile with our current understanding of core dynamics. An optimized core surface flow, given the estimated available energy, could generate a pointwise rate-of-change of geomagnetic intensity on the order of or lower than 1  $\mu$ T/yr, while the maximum observed in present-day variations is  $\sim 0.1 \mu T/yr$  [Olsen et al., 2014]. Davies and Constable [2018] study outputs of numerical dynamo simulations and find spike-like features, whose intensity variation rates reach 0.75  $\mu$ T/yr, corresponding to the lower-end of the values recently reassessed by Ben-Yosef et al. [2017] (i.e. between 0.75 and 1.5  $\mu T/yr$ ). Their study suggests that these events are linked to the intensification and migration of intense magnetic flux patches at the CMB. They also underline that current numerical dynamo simulations are not able to establish the occurrence of faster intensity variations at Earth-like parameters. Korte and Constable [2018] draw a similar conclusion from the analysis of series of global field models. The origin of such extreme intensity variations thus remains elusive; in particular, the imprint at Earth's surface of magnetic flux expulsion in the core remains to be studied in detail.

The process of flux expulsion from closed streamlines (or more generally, of the expulsion of a scalar from closed streamlines) has been widely studied. The simplest case to analyse is the kinematic one, which involves only the induction equation. The canonical configuration is that of a prescribed circular flow which interacts with an initially horizontal magnetic field in a closed domain surrounded with a perfect conductor. Weiss [1966] and Charbonneau [2013] performed the theoretical and numerical analysis of this setup, in a two-dimensional Cartesian geometry. In the situation in which advection dominates diffusion, the initially horizontal magnetic field lines are stretched and folded by the flow; this leads to an increase of the magnetic energy. Through this interaction, field lines are progressively expelled from the eddy by diffusion, and they concentrate near the edges of the domain, which leads ultimately to a global decrease of the total magnetic energy. This standard textbook configuration is characterized by a series of scaling laws relating for instance the timing and amplitude of the energy growth to the relative importance of transport by the flow to diffusion (quantified by the magnetic Reynolds number to be introduced below). In the case of the Earth, the setup departs from the textbook, as the mantle is to first order an insulator, which allows poloidal magnetic field lines to go through the CMB. Using a two-dimensional (2D) Cartesian model of a conducting fluid sandwiched between a lower perfectly conducting medium (the inner core in a first approximation) and an upper insulating medium (the mantle), and an imposed pair of counter-rotating eddies mimicking an upwelling, Bloxham [1986] demonstrated the plausibility of this process in the Earth's core, without attempting to derive scaling laws accounting for the characteristic time scale and magnitude of magnetic flux expulsion. Perhaps surprisingly, flux expulsion has not been systematically studied further in the decades since.

In this study we investigate whether flux expulsion events at the CMB could be a viable mechanism to explain rapid or extreme field intensity variations as revealed by archeomagnetic data recently obtained in the Near East and in Western Europe. We aim to derive scaling laws that provide detailed constraints on episodes of flux expulsion, with a particular focus on their duration. We will conduct this investigation in two dimensions, varying the geometry and the prescribed flows. First, we will follow Weiss [1966] and Charbonneau [2013] and consider only one eddy, the purpose being mostly to validate our numerical approach against previously published results. Then, a model similar to that of Bloxham [1986] involving two eddies is analysed. The results are compared with the maximum intensity variation rates as derived from different archeointensity datasets of different periods obtained in the Near East and in Western Europe.

This paper is organized as follows: the numerical method is introduced in section 4.2. The results are presented in section 4.3, with a detailed analysis of the theoretical and numerical scaling laws for the expulsion of magnetic flux from closed streamlines. A comparison with results from previous studies is conducted in order to validate our numerical method. Then, magnetic flux expulsion through an insulating boundary is studied for different configurations in order to underline the effects of different flow geometry. Scaling laws are derived for those different configurations. In section 4.4, these scalings laws are tentatively applied to the Earth and comparisons are made with various extreme archeointensity events inferred in the Near-East and in Western Europe. A conclusion follows in section 4.5. An appendix provides the interested reader with the details of the numerical implementation and its verification against analytical solutions.

## 4.2 Model and method

We are interested in the kinematic interaction of fluid flow and magnetic field in a closed domain  $\mathcal{D}$ , surrounded by an exterior domain  $\check{\mathcal{D}}$ . Two physical processes are at work. The first one is strictly-speaking the flux expulsion process, that is the expulsion of magnetic flux from closed streamlines. This expulsion occurs when advection dominates over diffusion, namely when the frozen-flux approximation holds. This is a standard phenomenon in magnetohydrodynamics whose description can be found in classical textbooks [see e.g. Roberts, 1967, §2.4], and whose numerical investigation was pioneered by Weiss [1966], who assumed that the outer region  $\check{\mathcal{D}}$  was filled by a perfect conductor. Rhines and Young [1983] subsequently proposed an analytical solution to the expulsion of a passive scalar from closed streamlines which is suitable for a variety of fluid-flow configurations.

When magnetic flux gets concentrated towards the edges of the domain, it can radially diffuse through the boundary, if the medium across the boundary is either finitely conducting or insulating. Magnetic field lines separate from fluid flow and the field diffuses in  $\check{\mathcal{D}}$ . This second process is of geophysical interest, as it can lead to a local increase (or decrease) of magnetic flux at the boundary. In the geophysical literature, flux expulsion often refers to the joint effect of flux concentration towards the boundary (in this case the CMB) and diffusion through it. It is the combination of these two processes which we seek to investigate.

Let (x, y, z) denote Cartesian coordinates. As illustrated in Fig. 4.1, we consider a twodimensional Cartesian domain  $\mathcal{D}$  of width  $L_x$  and height  $L_z$ , setting the top of the domain at z = 0. We focus on three configurations (see Fig. 4.1). Configuration 1 (Fig. 4.1a) is akin to one of the setups chosen by Weiss [1966]; it consists of a square  $\mathcal{D}(L_x = L_z)$ , filled by one vortex, between two perfect conductors. The system is periodic in the x-direction. The initial magnetic field is uniform and parallel to the x-axis. In this setup, only the flux expulsion from closed streamlines occurs, there is no subsequent diffusion through the boundary. We choose this setup, and a slightly modified version with a different streamfunction, previously studied by Charbonneau [2013] in order to confirm results previously published and to validate our numerical implementation. The stream-functions are shown in Fig. 4.1a and b respectively for Weiss [1966] and Charbonneau [2013], see the next section for details. For Configuration 2 (Fig. 4.1b), we introduce a single modification: the top boundary is assumed to be insulating. Consequently, diffusion through that boundary occurs after the initial expulsion phase, and we can isolate this effect in order to see if it has a global effect on the system, compared with Configuration 1 (for both stream-functions). Finally, Configuration 3 (Fig. 4.1c) retains the same boundaries (insulating top and perfectly conducting bottom), in a different geometry and with a different flow pattern. The domain is now rectangular ( $L_x = 2L_z$  throughout) and it contains two counter-rotating eddies, which induce an upwelling in the center of the domain. The initial magnetic field is again uniform and parallel to the x-axis. From the geophysical standpoint, this is the analog of the toroidal field in the vicinity of the CMB. This last configuration is the one designed and previously studied by Bloxham [1986].

Regardless of the configuration retained, the evolution of the system is governed by the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[ \mathbf{u} \times \mathbf{B} - \eta \times \nabla \times \mathbf{B} \right],\tag{4.1}$$

where **B** is the magnetic flux induction (hereafter the magnetic field), **u** the fluid velocity and  $\eta$  the magnetic diffusivity. In our two-dimensional framework, this equation can be conveniently recast in terms of the magnetic vector potential **A**, such that **B**(x, z, t) =



FIGURE 4.1: Sketch of the three configurations considered in this study. a) Configuration 1: square cell with perfectly conducting top and bottom boundaries, containing one anticlockwise vortex. b) Configuration 2: square cell with an insulating top boundary and a perfectly conducting bottom boundary, containing one anticlockwise vortex. Note that the represented stream-functions are different (see text for details). c) Configuration 3: rectangle domain with insulating top boundary and perfectly conducting bottom boundary, containing two counter-rotating vortices. Blue and red arrows represent magnetic field lines and field flow, respectively. Solid and dashed black lines denote perfectly conducting and insulating boundaries, respectively.

 $\nabla \times \mathbf{A}(x, z, t)$  and  $\mathbf{A} = A(x, z, t) \, \hat{\mathbf{y}}$ . If we further impose the Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$ , we obtain a scalar equation for A,

$$\frac{\partial A}{\partial t} = -\mathbf{u} \cdot \nabla A + \eta \nabla^2 A, \tag{4.2}$$

which has to be supplemented by initial and boundary conditions. The initial vector potential is  $A(x, z, t = 0) = -B_0 z$ , whereby  $B_0$  defines the amplitude of the initial field, which is parallel to the x-axis (recall the blue arrows in Fig. 4.1). Boundary conditions depend on the configuration chosen (see above). The flow **u** is steady and expressed in terms of a stream-function  $\Psi(x, z)$ , such that  $\mathbf{u} = \nabla \times \Psi(x, z)\hat{\mathbf{y}}$ . We choose the advection time  $L_z/U$  as the characteristic time scale, where U is the characteristic fluid velocity. Variables are non-dimensionalized according to

$$t^* = t \frac{U}{L_z}, \quad z^* = \frac{z}{L_z}, \quad A^* = \frac{1}{B_0 L_z} A_z$$

in which starred variables are dimensionless. This allows us to rewrite Eq. (4.2) in the following non-dimensional form

$$\left[\frac{\partial}{\partial t^*} - \frac{1}{R_m} \left(\frac{\partial^2}{\partial x^{*2}} + \frac{\partial^2}{\partial z^{*2}}\right)\right] A^* = \frac{\partial \Psi^*}{\partial z^*} \frac{\partial A^*}{\partial x^*} - \frac{\partial \Psi^*}{\partial x^*} \frac{\partial A^*}{\partial z^*},\tag{4.3}$$

with

$$R_m = \frac{UL_z}{\eta} \tag{4.4}$$

the magnetic Reynolds number. The magnetic Reynolds number is the ratio of the characteristic timescales of diffusion and advection

$$\tau_{\rm d} = \frac{L_z^2}{\eta}$$

and

$$\tau_{\rm adv} = \frac{L_z}{U},$$

respectively;  $R_m$  is the sole control parameter for the kinematic problem of interest here.

In what follows, the stars are omitted and, unless otherwise noted, time is expressed in units of advection time.

In order to approximate the solution of Eq. (4.3) numerically, a pseudo-spectral method is applied using a Fourier expansion along the x-direction together with a second-order accurate finite difference scheme in the z-direction. The resulting semi-discrete problem is advanced in time by means of an implicit-explicit scheme, along the lines described by Bloxham [1986]. A Fourier expansion in x is chosen because, for those configurations which possess an insulating upper boundary, the connection with an exterior potential solution is conveniently expressed using such a global basis [again, see Bloxham, 1986]. The reader is referred to Appendix B.1 for a complete description of the implementation of the method and a thorough analysis of its convergence properties.

## 4.3 Results

For each configuration described above, we vary systematically  $R_m$ , and integrate Eq. 4.2 until a steady state is reached. Snapshots of A will document the expulsion of flux by the flow towards the edges of the domain  $\mathcal{D}$ ; in addition, we will monitor the evolution of the z-component of the magnetic field at the upper boundary,  $B_{z0}(x,t) = B_z(x,z=0,t)$ , if that boundary is insulating (Configurations 2 and 3). In order to derive scaling laws describing flux expulsion, we shall follow the time evolution of the total magnetic energy in the domain,  $\mathcal{E}_B$ ,

$$\mathcal{E}_B(t) = \iint_{\mathcal{D}} \frac{\mathbf{B}^2(t)}{2} \,\mathrm{d}\mathcal{D},\tag{4.5}$$

and for the insulating upper boundary configurations also the evolution of the maximum of  $B_{z0}(t) = B(x, z = 0, t)$ ,

$$B_{z0}^{\max}(t) = \max_{x} |B_{z0}(x,t)|.$$
(4.6)

To characterize the evolution of these two quantities, we track their maximum amplitude,

$$\mathcal{E}_B^{\max} = \max_t \mathcal{E}_B(t), \quad \text{and} \quad (B_{z0}^{\max})^{\max} = \max_t B_{z0}^{\max}(t), \tag{4.7}$$

and the time  $\tau_{\text{max}}$  to reach the latter. The three different configurations each involve at least one perfectly conducting boundary (the lower one). As the magnetic field in a perfectly conducting medium is static, the magnetic field line on this boundary is kept fixed on it (see Appendix A for details). This condition prevents the magnetic energy from decaying, thereby enabling the establishment of a non-trivial steady state. We also characterize the steady state, defined as the moment at which the first order derivative in time of the monitored quantities remains under a threshold value close to zero. The criterion is set as follows:

$$\left|\frac{\partial \mathcal{E}_B}{\partial t}\right| < C_1, \quad \text{and} \quad \left|\frac{\partial (B_{z0}^{\max})^{\max}}{\partial t}\right| < C_2 \quad \text{for} \quad t \ge \tau_{\text{ss}}, \tag{4.8}$$

with  $\tau_{\rm ss}$  the time at which the steady state is reached, and  $C_1$  and  $C_2$  two constants. Once  $\tau_{\rm ss}$  is identified, the corresponding amplitude for each of the quantities is determined. The threshold value  $C_1$  is fixed at  $C_1 = 5 \times 10^{-3}$ .  $C_2$  is fixed depending on the order of magnitude reached by  $(B_{z0}^{\rm max})^{\rm max}$ . We also determine the instantaneous rate of change of  $B_{z0}^{\rm max}$  at an instant  $t_n$  defined as

$$R(t_n) = \frac{\mathrm{d}B_{z0}^{\max}}{\mathrm{d}t} \approx \frac{B_{z0}^{\max}(t_{n+1}) - B_{z0}^{\max}(t_n)}{\Delta t},\tag{4.9}$$

with  $t_{n+1} - t_n = \Delta t = 0.1 \tau_{adv}$ ; we track its maximum

$$R^{\max} = \max_{t} R(t). \tag{4.10}$$

In the following section, we describe the results for the 3 chosen configurations.  $\mathcal{E}_B^{\max}$ ,  $\mathcal{E}_B^{ss}$  and  $\tau_{ss}$  are mainly used to compare our findings with those obtained in previous studies in order to validate our numerical implementation. Those previous studies do not investigate the behaviour of  $\tau_{\max}$ . Here, the quantities  $(B_{z0}^{\max})^{\max}$ ,  $\tau_{\max}$  and  $R^{\max}$  are used to analyze the process of flux expulsion through an insulating boundary and to discuss this process with regards to extreme archeointensity events.

## 4.3.1 Configuration 1: flux expulsion from one eddy with a perfectly conducting upper boundary

This configuration presents a square domain  $\mathcal{D}$  of unit length L and contains a single counterclockwise eddy. The top and bottom boundaries are perfectly conducting. The initial magnetic field is horizontal. The magnetic field lines are "attached" to the left and right boundaries such that A[x = (0, L), z, t] = A[x = (0, L), z, t = 0]. Two different streamfunctions  $\Psi$  are considered, following Weiss [1966, fig. 2a], see Fig. 4.1a and Charbonneau [2013, chap. 2, fig. 2.8], see Fig. 4.1b

$$\Psi_1(x,z) = \frac{1}{\pi} \left(1 - 4x^2\right)^4 \sin(\pi z), \tag{4.11}$$

$$\Psi_2(x,z) = \frac{1}{4\pi} \left[ 1 - \cos\left(2\pi x\right) \right] \left[ 1 - \cos\left(2\pi z\right) \right], \tag{4.12}$$

respectively. Figs. 4.2a, 4.2b, and 4.2c illustrate the evolution of the scalar potential A after 0.5, 2, and 11 advection times respectively, for the stream-function  $\Psi_1$  and for a magnetic Reynolds number  $R_m = 1000$ . On each panel, isolines of A correspond to magnetic field lines, and the color scale represents the local strength of the magnetic field,  $F = \sqrt{B_x^2 + B_z^2}$ . Likewise, Figure 4.3 displays the same quantities when the chosen stream-function is  $\Psi_2$ , all other parameters being the same. Figs. 4.2a and 4.2b show that the initially horizontal field lines are wound by the eddy. This leads to the formation of magnetic stripes of alternating polarity whose width decreases with time, driving enhanced magnetic diffusion. After eleven advection times, the flux is expelled from the eye of the eddy and concentrated near the edges (Fig. 4.2c). This evolution is similar to the evolution obtained by Weiss [1966] for the same  $R_m$ . The initial shear of the field lines leads to an increase of  $\mathcal{E}_B$ , as shown in Fig. 4.2e (to be compared with Weiss [1966, fig. 5]). This first stage is dominated by the advection of the magnetic field. When magnetic diffusion becomes significant, the energy reaches a maximum  $\mathcal{E}_B^{\max}$  and starts to decrease until a steady state is obtained. The comparison of the energy evolution determined in our study and in Weiss [1966] study (for the same range of  $R_m$ ) shows the same behavior with a slight difference in energy amplitude (see below).

We follow the analysis by Weiss [1966] to derive theoretical scaling laws describing the behavior of  $\mathcal{E}_B$ . During the first phase of evolution, the velocity field builds the component of the magnetic field parallel to flow motions. This corresponds to an advective term of magnitude  $UB_0/L$  in the induction equation (4.1). In the meantime, as the magnetic field lines are wrapped, the stripes so formed become closer and closer to one another, leading to the decrease of the actual characteristic lengthscale of magnetic field variations,  $\ell$ , and thus inducing the gradual expulsion of the magnetic field towards the edges of the eddy (Fig. 4.2b) by reconnection of field lines. This expulsion is the product of magnetic diffusion in the fluid which becomes significant before the characteristic diffusion time is reached due to the decreasing lengthscale. The energy then reaches a maximum  $\mathcal{E}_B^{\max}$  (see the squares in Fig. 4.2) found by equating the advective term previously discussed with the diffusive term in the induction equation (4.1) based on the characteristic lengthscale of magnetic variations

$$UB_0/L \sim \eta B/\ell^2.$$

Replacing  $\ell$  using the conservation of magnetic flux  $B\ell \sim B_0 L$ , with B the current magnetic field strength and  $B_0$  its initial value, it follows that, at  $t = \tau_{\text{max}}$  (squares in Fig. 4.2),  $B \sim R_m^{1/3} B_0$  and hence  $\mathcal{E}_B^{\text{max}} \sim R_m^{2/3} B_0^2$ . As diffusion gradually balances advection, the energy starts to decrease, and at time  $t = \tau_{\text{ss}}$  (see the circles in Fig. 4.2), a steady state is reached when all the flux has been expelled from the cell and the magnetic energy becomes constant. This final state is represented in Fig. 4.2c. At that point, all the remaining flux



FIGURE 4.2: Time evolution of the magnetic field for Configuration 1 if the prescribed streamfunction is  $\Psi_1$  (Eq. 4.12). The black lines are iso-contours of the vector potential A (therefore magnetic field lines), shown every  $0.1B_0L_z$ . The colorscale represents the current magnetic field strength normalized by its initial value. The second row represents the magnetic field in the final stage (steady state) for a magnetic Reynolds number  $R_m = 1000$  (c) and for  $R_m = 40$  (d). e) shows the temporal evolution of the magnetic energy normalized by its initial value for each computed  $R_m$ . The squares (resp. circles) denote the time when the maximum (resp. steady-state) stage of the evolution is reached. See text for details.



FIGURE 4.3: Time evolution of the magnetic field for Configuration 1 if the prescribed streamfunction is  $\Psi_2$  (Eq. 4.11). The black lines are iso-contours of the vector potential A, shown every  $0.1B_0L_z$ . The colorscale represents the current magnetic field strength normalized by its initial value. The second row represents the magnetic field in the final stage (steady state) for a magnetic Reynolds number  $R_m = 1000$  (c) and for  $R_m = 50$  (d). e) shows the temporal evolution of the magnetic energy normalized by its initial value for each computed  $R_m$  for the stream-function  $\Psi_2$ (solid lines) compared with  $\Psi_1$  (dashed lines). The squares (resp. circles) denote the time when

the maximum (resp. steady-state) stage of the evolution is reached. See text for details.

is concentrated at the edges of the cell, inside a boundary layer of width  $\ell_{\rm ss}.$  The local diffusion time

 $\tau_{ld} \sim \ell_{\rm ss}^2/\eta$ 

decreases until it reaches  $\tau_{ld} \sim \tau_{adv}$ , leading to  $\ell_{ss} \sim R_m^{-1/2}L$ . All the magnetic energy is concentrated inside these boundary layers (Fig. 4.2c). Conservation of flux implies that in these boundary layers  $B_{bl} \sim R_m^{1/2}B_0$ . The total magnetic energy in the domain finally scales as

$$\mathcal{E}_B^{\rm ss} \sim R_m^{1/2} B_0^2.$$

Rhines and Young [1983] show that in general, the expulsion of a passive scalar presents two stages: a rapid stage of expulsion during which the scalar quantity is rapidly mixed along streamlines by advection and a slow stage during which this quantity is homogenized by diffusion. In the case we are interested in, the mixing along the streamlines leads directly to the homogenization of the field due to the initial condition (uniform magnetic field) and the geometry of the flow. As underlined by Moffatt and Kamkar [1983], the effect of diffusion is cumulative which prevents the determination of a simple scaling by the evaluation of this term at time t but requires an integration of the diffusion term from 0 to t. Their analysis leads to  $\tau_{\rm ss} \sim R_m^{1/3} \tau_{\rm adv}$ . It also prevents one from finding a scaling law for  $\tau_{\rm max}$ , that is the time at which diffusion starts to equilibrate advection but is still not predominant. However, this quantity should present a low dependency to  $R_m$  as  $\mathcal{E}_B^{\rm max}$  is reached quickly, after a few turnover times, even for large  $R_m$  (Fig. 4.2e).

As stressed by Rhines and Young [1983], the actual expulsion of the flux from the inside of the eddy towards its edges is a mechanism of "shear-augmented dispersion". During the stage of increasing magnetic energy, the field lines are stretched and wrapped by the flow. It leads to the formation of stripes of alternating polarity. If the magnetic Reynolds number is large ( $\geq 50$ ), the advection time is small compared to the diffusion time and magnetic field lines are further wrapped before the equilibrium between advection and diffusion occurs. When two field lines of opposite polarity are separated by a distance smaller than the diffusion scale, diffusion leads to the destructive folding of the field [Charbonneau, 2013]. On the contrary, when the magnetic Reynolds number is low, advection is not strong enough to sufficiently fold magnetic field lines. In this case, the balance between advection and diffusion is reached before the destruction of the field (Fig. 4.2d) and the decrease of energy is consequently less marked (see Fig. 4.2e, obtained for  $R_m = 40$ ).

Figure 4.4a illustrates the scaling laws (summarized in Table 4.1) that can be extracted from our suite of simulations. For comparison with the work of Weiss [1966],  $R_m$  lies between 40 and 10<sup>3</sup> (including larger values of  $R_m$  does not alter our findings significantly).

The maximum value of the energy  $\mathcal{E}_B^{\text{max}}$  is found to scale as  $R_m^{0.645\pm0.004}$ ; Weiss [1966] numerically finds  $R_m^{0.59}$  over the same range of  $R_m$ . Our results are close to the theoretical scaling law,  $R_m^{2/3}$ . In addition, the value of the energy of the steady state  $\mathcal{E}_B^{\text{ss}}$  scales as  $R_m^{0.504\pm0.005}$ . Weiss [1966] gives  $R_m^{0.42}$ , while the theoretical expectation is  $R_m^{1/2}$ , again close to our numerically determined value.

With regard to the time taken to reach this steady value, we find that it scales as  $R_m^{0.254\pm0.026}$ , while the theoretical scaling law is  $R_m^{1/3}$ , which is markedly different. Interestingly, if we use  $\Psi_2$ , the same characteristic time is found to scale as  $R_m^{0.412\pm0.001}$  (see Fig. 4.4b). This quantity appears to be highly dependent on the prescribed stream-function (this will be discussed in the following). Here the theoretical exponent lies in between the two numerical values. To summarize, our results are close to the results obtained by Weiss [1966]. As Weiss [1966] does not provide the details of his computational method, the small



FIGURE 4.4: Dependency of the monitored quantities on the magnetic Reynolds number  $R_m$  in Configuration 1, when the imposed stream-function is  $\Psi_1$  (a, see Eq. 4.12) and  $\Psi_2$  (b, see Eq. 4.11). Blue squares (circles) represent the maximum (steady-state) magnetic energy  $\mathcal{E}_B^{\max}$  ( $\mathcal{E}_B^{ss}$ ); the time to reach steady-state energy  $\tau_{ss}$  is shown with orange circles. The *y*-axis on the left features the scale for magnetic energy  $\mathcal{E}_B$ , while the *y*-axis on the right that for the time  $\tau$ . The straight lines illustrate the corresponding scaling laws found by least-squares fitting, whose form is given in the label at the top,  $\beta$  denoting the exponent found in each case. Slopes of 1/3, 1/2, and 2/3 indicated for reference.

discrepancies noted between his results and ours most probably arise from a difference in the numerical approach. Note that to further ensure the accuracy of our model, the results of additional tests are provided in Appendix B.1. We conclude this part by stating that the agreement between the theoretical expectations, previously published results and our numerical findings convinces us of the validity of our numerical implementation.

The stream-function  $\Psi_2$  gives qualitatively the same results as previously described (Figs. 4.3 a, b, c, d). The evolution of the field and of the magnetic energy are similar and correspond to the results presented by Charbonneau [2013, chap.2, fig. 2.9]. However, the scaling laws that can be numerically extracted are somewhat different from the ones obtained with  $\Psi_1$ . Fig. 4.3e shows the evolution of the magnetic energy as a function of time and it is interesting to note that the amplitude of the resulting energy for the same  $R_m$  is lower for  $\Psi_2$  (see Fig. 4.3e, where the dashed lines show for comparison the evolution that is obtained for  $\Psi_1$ ). The time to reach the maximum amplitude is equivalent for a given  $R_m$ . As a general rule, the steady state is reached later and the corresponding amplitude of the energy is lower for  $\Psi_2$ . For example, for  $R_m = 1000$ , using  $\Psi_1$ , a steady state is already reached at t = 6.5 whereas for  $\Psi_2$ ,  $R_m = 1000$ , it is reached later than t = 8.

The scaling laws obtained for the evolution of the magnetic energy with  $\Psi_2$  are shown in Fig. 4.4b. Note that these laws are computed for  $200 \leq R_m \leq 2 \times 10^5$  this time, and this will be the case in the remainder of this paper (in particular for the computation of the scaling laws), where we are interested in the large  $R_m$  limit. The maximum value of the energy  $\mathcal{E}_B^{\text{max}}$  is found to scale as  $R_m^{0.644\pm0.004}$ , similar to the scale previously determined. The energy at the steady state  $\mathcal{E}_B^{\text{ss}}$  scales as  $R_m^{0.334\pm0.0001}$ , much lower than previously  $(R_m^{0.504})$  and lower than the anticipated value of  $R_m^{1/2}$ . Likewise the time taken to reach the steady state is found to scale as  $R_m^{0.412\pm0.001}$ , as mentioned above. These differences can be ascribed to the choice of the stream-function.  $\Psi_1$  falls off rapidly at the upper and lower boundaries. This leads to a greater shear of the magnetic field lines close to these boundaries which in turn enhances the energy produced by the shear. Conversely,  $\Psi_2$ produces a stronger shear close to the lateral boundaries compared to  $\Psi_1$ . The magnetic stripes are thus subject to a greater folding, subsequently leading to an increase of the relaxation time and a decrease of the total magnetic energy. As the efficiency of advection increases with the magnetic Reynolds number, the shear of the magnetic field lines becomes larger with higher  $R_m$ , thereby enhancing this relaxation. This illustrates the fact that the flow pattern (in particular the occurrence of strong flow gradients) has a strong impact on the timing and intensity of the flux expulsion mechanism.

## 4.3.2 Configuration 2: flux expulsion from one eddy with an insulating upper boundary

Of importance (in particular when considering geophysical/astrophysical implications) is also the nature of the magnetic boundary condition that is imposed at the top of the domain. Our goal in Configuration 2 is to quantify this effect, using the previous configuration as a reference. In Configuration 2 (recall section 4.2), the vertical component of the field can also diffuse through the upper boundary. Fig. 4.5 shows the results for the same cases as considered in the previous section (Figs. 4.2 and 4.3), using  $\Psi_1$  as our imposed stream-function. In the different panels of Fig. 4.5, each blue curve shows the vertical component of the magnetic field at the upper insulating boundary,  $B_{z0}$ , as a function of x. For  $R_m = 1000$  (Fig. 4.5a,b,c), the results are comparable with those obtained in the previous configuration (compare with Fig 4.2). During the first (mainly advective) phase, magnetic field lines are twisted and folded (Fig. 4.5a). Then magnetic field lines start to reconnect to each other, leading to a progressive expulsion of the flux from the inside of the eddy toward its edges, which in turn leads to the increase of the amplitude of  $B_{z0}$  (Fig. 4.5b). The steady state is represented in Fig. 4.5c,d for two values of  $R_m$ . For  $R_m = 1000$  (Fig. 4.5c), the flux is entirely expelled from the eye of the eddy at steady state. For  $R_m = 50$  (Fig. 4.5d), advection is not strong enough for reconnection to occur and the expulsion of the magnetic field lines from the center of the eddy to be completed. Consequently, the amplitude of  $B_{z0}$  reached at steady state is lower in this low- $R_m$  case.

The same cases (same  $R_m$  and same times for snapshots) are presented in Fig. 4.6 for  $\Psi_2$ . The general behavior is similar. Note that the overall amplitude of the magnetic field intensity is lower in this case. The concentration of magnetic flux close to the upper boundary lead to an increase of the amplitude of the z-component of the field in z = 0. The maximum amplitude reached at the top and bottom is lower in this case, leading to lower values of  $B_{z0}$ . Another difference due to the geometry of the flow is that more flux is concentrated close to the lateral boundaries.

Fig. 4.7a shows the evolution of the energy for  $\Psi_1$  (dashed lines) and for  $\Psi_2$  (solid lines) for different  $R_m$ . Again, the amplitude of the energy is higher with  $\Psi_1$ . The same behavior as described in the previous section (Figs. 4.2 and 4.3) is observed. The first stage involves an increase of the energy corresponding to the advection phase. Then diffusion starts to be significant and the energy reaches a maximum and decreases, as magnetic field lines reconnect to each other, until all the flux is expelled from the eye, and steady state is reached. The scaling laws (Fig. 4.7b and c, summarized in Table 4.1) obtained to characterize the energy at its maximum and at steady state are close to those obtained for the previous configuration.  $\mathcal{E}_B^{\max}$  is found to scale as  $R_m^{0.641\pm0.001}$  and as  $R_m^{0.645\pm0.004}$  for  $\Psi_1$ and  $\Psi_2$ , respectively (exponents equal to 0.645 and 0.644 resp. for Configuration 1). In the first configuration, energy amplitudes are slightly higher; the scaling law shows that there is no difference regarding the trend of the evolution.  $\mathcal{E}_B^{\text{ss}}$  is found to scale as  $R_m^{0.533\pm0.003}$  and



FIGURE 4.5: Time evolution of the magnetic field for Configuration 2 if the prescribed streamfunction is  $\Psi_1$  (Eq. 4.12). The black lines are iso-contours of the vector potential A, shown every  $0.1B_0L_z$ . The colorscale represents the current magnetic field strength normalized by its initial value. Above each panel the blue curve shows the variation of the z-component of the magnetic field along the top boundary,  $B_{z0}$ , at the corresponding time. The second row represents the magnetic field in the final stage (steady state) for a high magnetic Reynolds number  $R_m$  (c) and for a low  $R_m$  (d).



FIGURE 4.6: Time evolution of the magnetic field for Configuration 2 if the prescribed streamfunction is  $\Psi_2$  (Eq. 4.11). The black lines are iso-contours of the vector potential A, shown every  $0.1B_0L_z$ . The colorscale represents the current magnetic field strength normalized by its initial value. Above each panel, the blue curve shows the variation of the z-component of the magnetic field along the top boundary,  $B_{z0}$ , at the corresponding time. The second row represents the magnetic field in the final stage (steady state) for a magnetic Reynolds number  $R_m = 1000$  (c) and for  $R_m = 50$  (d).



FIGURE 4.7: Evolution of the total magnetic energy for Configuration 2. (a): evolution of the magnetic energy with time for a range of magnetic Reynolds number  $R_m$  in the case of the stream-function  $\Psi_1$  (dashed lines) and  $\Psi_2$  (solid lines). (b) and (c): evolution of the maximum of the magnetic energy  $\mathcal{E}_B^{\max}$ , its amplitude at the steady state  $\mathcal{E}_B^{ss}$ , and their corresponding characteristic times,  $\tau_{\max}$  and  $\tau_{ss}$  respectively, as a function of  $R_m$  (ranging from 200 to 20000), for  $\Psi_1$  and  $\Psi_2$ , respectively. The straight lines illustrate the corresponding scaling laws found by least-squares fitting, whose form is given in the label at the top,  $\beta$  denoting the exponent found in each case.

 $R_m^{0.335\pm0.0002}$  for  $\Psi_1$  and  $\Psi_2$  respectively, similar to Configuration 1 in both cases (0.504 and 0.334 resp.). The time to reach the steady state scales also as in Configuration 1 for  $\Psi_2$ : the exponent is  $0.418 \pm 0.001$  (against 0.412). It is rather different for  $\Psi_1$ :  $0.578 \pm 0.030$  (against 0.254). These results show that the effect of the insulating boundary on the energy variations is limited. However, it has a noticeable effect on the characterization of the steady state. The diffusion of the flux through the insulating boundary takes longer than the expulsion of the flux from the center of the eddy which leads to longer relaxation time at higher  $R_m$ .

The same systematic analysis is applied to the evolution of  $B_{z0}^{\max}$  (Fig. 4.8). Fig. 4.8a shows the evolution with time of  $B_{z0}^{\max}$  for  $\Psi_1$  (dashed lines) and for  $\Psi_2$  (solid lines). It is striking that for  $\Psi_1$ , the amplitude of  $B_{z0}^{\max}$  is significantly higher than that obtained with  $\Psi_2$ . For the latter,  $B_{z0}^{\max}$  grows rather slowly until reaching a maximum, then it decreases to the steady state value (see the solid curves in figure 4.8). In contrast, for  $\Psi_1$ ,  $B_{z0}^{\max}$  grows faster (over a longer time than for  $\Psi_2$ , several tens of advection times for the
largest values of  $R_m$ ), until it plateaus at a maximum value  $(B_{z0}^{\max})^{\max}$  coinciding with the steady state. The resulting scaling laws (see Table 4.1) are represented in Fig. 4.8b,c for  $\Psi_1$ and  $\Psi_2$ , respectively.  $(B_{z0}^{\text{max}})^{\text{max}}$  is found to scale as  $R_m^{0.201\pm0.009}$  (for  $\Psi_2$ ). The threshold value  $C_2$  used to define the steady state (recall Eq.4.8) is chosen as  $C_2 = 5.10^{-4}$  for  $\Psi_1$ and  $C_2 = 5.10^{-5}$  for  $\Psi_2$ . At steady state,  $(B_{z0}^{\text{max}})^{\text{ss}}$  scales as  $R_m^{0.435\pm0.007}$  and  $R_m^{0.237\pm0.003}$ for  $\Psi_1$  and  $\Psi_2$ , respectively. The time to reach the steady state is found to scale as  $R_m^{0.569\pm0.007}$  and  $R_m^{0.425\pm0.012}$  for  $\Psi_1$  and  $\Psi_2$ , respectively. The different results for the different stream-functions are again a consequence of the behavior of the different flows at the boundaries. The larger flow gradients at the top and bottom boundaries lead to a greater energy concentration for  $\Psi_1$ , particularly at the top: this enhances the amplitude of  $B_{z0}^{\max}$ . Conversely, the slightly stronger flow gradients on the lateral boundaries for  $\Psi_2$  generate a stronger destructive folding of magnetic fields lines on those boundaries, and consequently to a decrease of  $B_{z0}^{\max}$ . We do not observe this behaviour if  $\Psi_1$  is prescribed. These results show that  $\Psi_1$  is more efficient for the expulsion of flux from the eddy and through the insulating upper boundary. We ascribe its efficiency to its geometrical properties. As described in the previous configuration, strong flow gradients enhance the magnetic energy at the boundary. Here, gradients in both z and x are important: the former allows the flux to concentrate in the vicinity of the boundary, while the latter concentrates the flux near downwellings, thereby enabling an amplification of  $B_{z0}$ .

In this configuration, the evolution of  $(B_{z0}^{\max})^{\max}$  is primarily controlled by the lateral boundary conditions. The magnetic field lines being fixed on lateral boundaries, they are driven downward by the downwelling on the left boundary which increases the z-component of the field in x = 0.  $(B_{z0}^{\max})^{\max}$  is thus located at x = 0, where the magnetic field lines are fixed.

### 4.3.3 Configuration 3: flux expulsion from two eddies with an insulating upper boundary

The third configuration retains the same top and bottom boundary conditions for the vector potential as those for Configuration 2, with an upper insulating boundary and a perfectly conducting bottom boundary. As shown in Fig. 4.1c), the domain  $\mathcal{D}$  is rectangular with a 2 : 1 aspect ratio ( $L_z = 1$ ,  $L_x = 2$ ), and the fluid is set in motion by two eddies producing a convective upwelling at the center of the domain. This flow is defined by the stream-function

$$\Psi(x,z) = \frac{1}{\pi} \sin(\pi x) \sin(\pi z).$$
(4.13)

This stream-function has the same behavior as  $\Psi_1$  along the z-direction and should therefore maximize concentration of flux near the top boundary. The initial magnetic field is again horizontal (see the blue arrows in Fig. 4.1c). A noticeable difference with regard to Configurations 1 and 2 is that the magnetic field lines are no longer tied to the lateral sides, as was the case in the one-vortex canonical configuration, in order to reproduce exactly the configuration used by Bloxham [1986]. This choice is further motivated by the fact that it is not relevant to fix the field lines at the lateral boundaries as they are supposed to reflect the behaviour of the toroidal magnetic field in the Earth's outer core.

Results are presented in Fig. 4.9 for  $R_m = 1000$  at t = 1.5 (Fig. 4.9.a), t = 3.5 (Fig. 4.9.b), and t = 30 (Fig. 4.9.c). As previously seen in the single-eddy configurations for high  $R_m$  cases, the magnetic field lines initially inside the eddies are at first wound and concentrated near the edges of the domain (Fig. 4.9a, bottom panel). Reconnection of field lines occurs when two stripes of opposite polarity are close enough (Fig. 4.9b,



FIGURE 4.8: Evolution of the maximum of the z-component of the magnetic field at the top insulating boundary,  $B_{z0}^{\max}$  (Eq. 4.6), for Configuration 2. (a) shows the evolution of  $B_{z0}^{\max}$  with time for a range of magnetic Reynolds number  $R_m$  for the stream-function  $\Psi_1$  (dashed lines) and  $\Psi_2$  (solid lines). (b) and (c) show the evolution of the monitored quantities as a function of  $R_m$ together with the associated scales for  $\Psi_1$  and  $\Psi_2$  respectively. The straight lines illustrate the corresponding scaling laws found by least-squares fitting, whose form is given in the label at the top,  $\beta$  denoting the exponent found in each case.



FIGURE 4.9: Time evolution of the magnetic field for Configuration 3 if the prescribed streamfunction is  $\Psi$  (Eq. 4.13), for a magnetic Reynolds number  $R_m = 1000$ . The black lines are iso-contours of the vector potential A, shown every  $0.1B_0L_z$ . The colorscale represents the current magnetic field strength normalized by its initial value. Above each panel the blue curve shows the variation of the z-component of the magnetic field along the top boundary,  $B_{z0}$ .



FIGURE 4.10: a) Evolution of the maximum of the z-component of the field at z = 0,  $B_{z0}^{\max}$  (Eq. 4.6), for a range of magnetic Reynolds number  $R_m$ . b) Shows the evolution of the monitored quantities as a function of  $R_m$  together with the associated scales. The straight lines illustrate the corresponding scaling laws found by least-squares fitting, whose form is given in the label at the top,  $\beta$  denoting the exponent found in each case.

bottom panel); this leads to the progressive expulsion of magnetic flux from the interior of each eddy to its edges. The evolution of  $B_{z0}$  (top panels of Figs. 4.9a, 4.9b, 4.9c) is however different from that observed in Configuration 2: it is now antisymmetric with respect to the center of the upwelling, with two intensity peaks of opposite signs and same amplitude, located above the downwellings. As the flux is further concentrated near the top left and right corners of the domain, the amplitudes of the peaks of  $B_{z0}$  increases with time (Fig. 4.9b), until it finally reaches a steady state (Fig. 4.9c).

The behaviour of the peak amplitude,  $B_{z0}^{\max}$ , normalized by the initial magnetic field strength  $B_0$ , is documented as a function of time for different magnetic Reynolds numbers in Fig. 4.10, left panel. Note that the integration times necessary to reach a steady state at large  $R_m$  are substantially longer (several tens of  $\tau_{adv}$ ) than those considered by Bloxham [1986], whose range of  $R_m$  was restricted between 10 and 200. When compared with Configuration 2, the same increase is seen at first, over a few  $\tau_{adv}$ , following the initial shear-augmented diffusion of flux. At any given  $R_m$ , the maximum amplitude that is reached is however larger than the one obtained for Configuration 2 (Fig. 4.8a). The evolution of  $B_{z0}^{\max}$  is not controlled by the lateral boundary conditions (as in Configuration 2), but by the flow gradients in x concentrating the flux near downwellings. The magnetic field lines move freely along the lateral boundaries, so the flow drags them downward and no vertical component is created as the magnetic field lines are not distorted. Once the maximum (of  $B_{z0}^{\max}$ ) is obtained at time  $t = \tau_{\max}$ , relaxation to steady state occurs. This relaxation can be oscillatory in nature, and its amplitude increases with  $R_m$ . We interpret these oscillations as originating from the arrival near the top of the domain of the magnetic field stripes of alternating polarity created by the distortion of magnetic field lines and their subsequent diffusion. Such oscillations do not occur in the case of Configuration 2 precisely because the evolution of  $B_{z0}^{\max}$  is in this case governed by field lines being tied to the lateral boundaries. In addition, note that these oscillations can exist by virtue of the two-dimensional geometry. Flux can not escape in the third direction of space. In three dimensions, for instance in the case of a helical upwelling flow, the situation may be different.

Our systematic survey of  $R_m$  values allows us to derive scaling laws describing the

TABLE 4.1: Summary of the scaling laws determined in this study, compared with the theoretical expectations.  $\mathcal{E}_B^{\max}$  and  $\mathcal{E}_B^{ss}$  are the maximum energy and the energy at the steady state respectively, with  $\tau_{\max}(\mathcal{E}_B^{\max})$  and  $\tau_{ss}$  their corresponding characteristic time scales.  $(B_{z0}^{\max})^{\max}$ is the maximal amplitude of the z-component of the field at the insulating top boundary, with  $\tau_{\max}((B_{z0}^{\max})^{\max})$  its associated time scale. For Configuration 2,  $\Psi_1$ , the scalings indicated for the latter quantities are the scalings at the steady state, which are the same only in this case (see text for details).

		$\mathcal{E}_B^{\max}$	$\tau_{\max}(\mathcal{E}_B^{\max})$	$\mathcal{E}_B^{\mathrm{ss}}$	$\tau_{\rm ss}$	$(B_{z0}^{\rm max})^{\rm max}$	$\tau_{\max}((B_{z0}^{\max})^{\max})$	$R^{\max}$
Theoretical scalings		$R_m^{2/3} B_0^2$	-	$R_m^{1/2}B_0^2$	$R_m^{1/3}\tau_{\rm adv}$	-	-	-
Conf. 1 $-$	$\Psi_1$	$R_m^{0.645} B_0^2$	$R_m^{0.231}\tau_{\rm adv}$	$R_m^{0.504} B_0^2$	$R_m^{0.254}\tau_{\rm adv}$	-	-	-
	$\Psi_2$	$R_m^{0.644}B_0^2$	$R_m^{0.313}\tau_{\rm adv}$	$R_m^{0.334}B_0^2$	$R_m^{0.412} \tau_{\rm adv}$	-	-	-
Conf. 2 —	$\Psi_1$	$R_m^{0.641} B_0^2$	$R_m^{0.283}\tau_{\rm adv}$	$R_m^{0.533}B_0^2$	$R_m^{0.578}\tau_{\rm adv}$	$R_m^{0.435}B_0$	$R_m^{0.569}\tau_{\rm adv}$	-
	$\Psi_2$	$R_m^{0.645} B_0^2$	$R_m^{0.313}\tau_{\rm adv}$	$R_m^{0.335} B_0^2$	$R_m^{0.418}\tau_{\rm adv}$	$R_m^{0.201}B_0$	$R_m^{0.477}  au_{ m adv}$	-
Conf. 3	Ψ	-	_	-	-	$R_m^{0.495}B_0$	$R_m^{0.152}  au_{ m adv}$	$R_m^{0.447} \frac{B_0}{\tau_{\rm adv}}$

evolution of  $B_{z0}^{\text{max}}$  (see Table 4.1). To define the steady state, we used a threshold value  $C_2 = 5 \times 10^{-3}$  (recall Eq. 4.8). As shown in Fig. 4.10, right panel, we find that

$$\tau_{\rm max} = 1.002 \pm 0.012 R_m^{0.152 \pm 0.004}$$
, and  $(B_{z0}^{\rm max})^{\rm max}/B_0 = 0.267 \pm 0.019 R_m^{0.495 \pm 0.006}$  (4.14)

provide an adequate least-squares fit (in log-log space) to the data. Restoring dimensions, these scaling laws become

$$\tau_{\max} = 1.002 \frac{L}{U} \left(\frac{UL}{\eta}\right)^{0.152}, \quad (B_{z0}^{\max})^{\max} = 0.267 B_0 \left(\frac{UL}{\eta}\right)^{0.495}. \tag{4.15}$$

To reiterate, the pseudo-convective flow prescribed here causes a more intense flux concentration near the top left and right corners of the domain, regardless of the value of  $R_m$ .

The increase of  $B_{z0}^{\max}$  between the initial state at t = 0 and the maximum at  $t = \tau_{\max}$ can be averaged, under the assumption of a linear increase. The ratio of  $(B_{z0}^{\max})^{\max}$  to  $\tau_{\max}$ therefore gives an estimate of this average variation rate and its dependency on  $R_m$ . We find that this average variation rate scales as  $\sim R_m^{0.343} B_0 U/L$ . However, as shown by the evolution of  $B_{z0}^{\max}$  (recall Fig. 10, left panel), the instantaneous rate of change R (Eq. 4.9) can be faster than this average at some instants. We determine the scale for the maximum instantaneous rate of change (defined by Eq. 4.10) by a systematic survey of this quantity for the same range of  $R_m$ . We find a stronger dependency on  $R_m$  (with an exponent equal to 0.447) which yields the following dimensional law

$$R^{\max} = 0.300 B_0 \frac{U}{L} \left(\frac{UL}{\eta}\right)^{0.447}.$$
 (4.16)

#### 4.4 Discussion

In order to provide plausible estimates for magnetic field variations that could be produced by flux expulsion from the Earth's core, we base the discussion on results obtained from Configuration 3. Although simplified, this configuration appears to be the most efficient to generate flux expulsion through the upper boundary. In addition, the specific geometry and fluid flow pattern imposed by Bloxham [1986] produces a pair of peaks obtained at the top insulating boundary (taking the periodicity of the solution into account, recall Figure 4.9) which is reminiscent of pairs of flux patches of opposite polarity seen at the top of the core (see for example Jackson and Finlay [2015]).

Configuration 3 is however based on a number of assumptions, the consequences of which need to be discussed. First, we consider a two-dimensional domain. As mentioned in the previous section, the same evolution for  $(B_{z0}^{\max})^{\max}(t)$  is not expected in 3D and the oscillations observed before the steady-state is reached may disappear. In their study, Galloway et al. [1978] also argued that in 3D, in the large  $R_m$  limit, the flow concentrates the flux in a rope, amplifying the field inside as a function of  $\sim R_m B_0$ , instead of  $\sim R_m^{1/2} B_0$ obtained in the 2D, sheet-like configuration. A fully 3D geometry therefore increases  $(B_{ro}^{\max})^{\max}$ , since it increases the concentration of magnetic flux. The work by Galloway et al. [1978] focuses on the solar dynamo, which is not dramatically influenced by the background rotation of the Sun. On Earth, however, rotational effects are of foremost importance, and the Coriolis force imparts an invariance of the flow in the direction of Earth's background rotation, making the flow two-dimensional in practice. Our 2D model could in this respect represent the situation occurring in the equatorial plane (or any plane parallel to the equatorial plane), where two counter-rotating eddies may contribute to the expulsion of magnetic flux. Second, we consider two perfectly symmetric counter-rotating eddies. If this symmetry were broken, the z-component of the magnetic field at the upper insulating boundary would no longer be symmetrical, as the shear of the magnetic field lines by the two eddies would no longer be the same. The stronger eddy would give rise to a stronger flux expelled through the boundary.  $(B_{z0}^{\max})^{\max}$  would still depend on the streamfunction, in particular on the gradient that controls the shear of the magnetic field lines. Third, we consider an initial magnetic field that is horizontal and uniform. Were this initial field not uniform, in the high  $R_m$  limit, magnetic field lines would still be rapidly swept aside and concentrated at the boundaries. We therefore do not expect a significant change in the scaling laws from a change in the initial magnetic configuration (provided that the average magnetic field strength remains the same). Fourth, the steady flow considered here is extremely efficient in concentrating and shearing magnetic field lines in the vicinity of the upper boundary. In the case of a non-steady flow, this optimal geometry would not be sustained over the time span it takes to generate the expulsion of flux. This would lower the efficiency of the process. We thus expect  $(B_{z0}^{\max})^{\max}$  to be either lower and/or achieved over a longer duration  $\tau_{\rm max}$ . Finally, it is also important to stress that this 2D model is kinematic and does not account for the Lorentz force: the above rates overestimate the rate of flux expulsion. In a dynamical context, the Lorentz force is likely to exert a feedback on the fluid flow, in particular in the case of a strong toroidal field. Magnetic tension decreases the flow speed, which is detrimental to both the amount of flux advected towards the boundary and the magnitude of the field gradients. In spite of these intrinsic limitations, we think that Configuration 3 gives relevant information to understand whether flux expulsion events can generate geomagnetic spikes as documented in the recent literature.

The extrapolation of our deduced scaling laws (Eq. 4.15) to the Earth requires constraints on the fluid velocity and on the magnetic diffusivity at the CMB, as well as on the intensity of the toroidal magnetic field close to the CMB and the typical lengthscale of the convective process. Reasonable estimates are available at least for the two first parameters. The root mean square (rms) velocity of the fluid at the CMB is ~ 15 km/yr [Jones, 2015], a value derived from the study of the magnetic flux patches advection at the CMB. The magnetic diffusivity is also usually assumed to take a value close to  $\eta \sim 0.6 \text{ m}^2 \text{s}^{-1}$  [Pozzo et al., 2012]. Below we discuss the impact of other, more speculative, choices for these parameters.

Constraining the two remaining parameters is more challenging. The degree 13 of spherical harmonic decomposition is the smallest wavelength of the contribution of the core field that is observable at the Earth's surface, the crustal magnetic field hiding its smaller wavelength contribution. Truncation at degree 13 is a common practice used for core field models, for instance those contributing to the IGRF [Thébault et al., 2015]. The corresponding minimum distance L at the CMB is then ~ 800 km. Adopting U = 15 km/yr,  $\eta = 0.6 \text{ m}^2 \text{s}^{-1}$ , L = 800 km leads to a local  $R_m \sim 630$ . Application of our scaling laws characterizing flux expulsion then gives a duration of  $\tau_{\text{max}} \sim 140 \text{ yr}$  associated with a total duration of  $\tau_{\text{tot}} \sim 280 \text{ yr}$  accounting for an increase and a decrease of the intensity (to obtain a peak, cf. Table 4.2). Based on these estimates, 140 yr is therefore the shortest timescale that may be detected at the Earth's surface due to flux expulsion at the CMB. On the other hand, one can instead take as the relevant lengthscale the thickness of the outer core, i.e.  $L \sim 2 \times 10^3 \text{ km}$ , which corresponds to spherical harmonic degree  $\sim 5$  at the Earth's surface. This leads to a local  $R_m \sim 1590$  and a duration  $\tau_{\text{max}} \sim 410 \text{ yr}$ , leading to a total duration  $\tau_{\text{tot}} \gtrsim 820 \text{ yr}$ .

Turning to the toroidal field confined within the Earth's core, it is clear that determining its intensity is far from trivial. Hori et al. [2015] suggest a lower bound of 3 mT from the analysis of the secular variation in terms of Rossby waves in numerical dynamo simulations but stressed that this value could be much higher (up to approx. 10 mT). The results reported in this study for Configuration 3 suggest that the maximal proportion of  $B_z$ expelled evolves as  $\sim R_m^{1/2}$ . Taking 3 mT as the toroidal field strength in the vicinity of the CMB, the minimal and maximal lengths L given above lead to an intensity of the expelled field of between 20 mT (L = 800 km) and 31 mT ( $L = 2 \times 10^3$  km). This corresponds to average intensity variation rates at the CMB of 75  $\mu$ T/yr to 137  $\mu$ T/yr. As for the maximum instantaneous rate  $R^{\text{max}}$ , it reaches 302  $\mu$ T/yr for L = 800 km and 182  $\mu$ T/yr for L = 2000 km, twice as much as the average rates. Assuming an attenuation of the field through the mantle on the order of  $\sim (r_c/r_a)^{\ell+2}$ , with  $\ell$  the degree of the spherical harmonic decomposition,  $r_c$  the radius of the core and  $r_a$  the radius of the Earth, allows us to roughly estimate the intensity variation rates that would be observed at the Earth's surface. Such attenuation would lead to average intensity variation rates at Earth's surface of ~ 0.016  $\mu T/yr$  (L = 800 km) to ~ 1.09  $\mu T/yr$  (L = 2000 km, cf. Table 2). Turning to the maximum instantaneous rate of change (Eq. 4.10), using the same parameters than previously, the extrapolation leads to values ranging from 0.035 to 2.64  $\mu$ T/yr.

Having established geophysically plausible ranges for the rate of flux expulsion according to our simple model, it is now of interest to compare these with field intensity variations reported for geomagnetic spikes. It should first be mentioned that the intensity variation rates that have been associated with the latter, as well as their duration, are only approximated from an experimental point of view. On the basis of the available archeomagnetic data, it is not yet possible to know precisely whether the values that have been proposed characterize the spikes as a whole (with an estimate of the average variation rate and of the total duration of the rising or falling part of the intensities), or only part of the spikes. For this reason, in our discussion, we have considered both the average intensity variation rate

TABLE 4.2: Extrapolation of the scaling laws given by Eqs. 15-16 at the CMB and at the Earth's
surface for various choices of characteristic scales. The last two columns give variation rates at
the CMB and at the Earth's surface respectively. These rates are comprised between the average
variation rate (lower value) and the maximum instantaneous variation rate (higher value).

$L \ (\mathrm{km})$	$\ell$	$U~({ m km/yr})$	$R_m$	$\tau_{\rm max}$ (yr)	$(B_{z0}^{\max})^{\max}$ ( $\mu T$ )	$ m CMB~(\mu T/yr)$	Surf. $(\mu T/yr)$
800	13	15	630	140	$20 \times 10^3$	137 - 302	0.016 - 0.035
2000	5	15	1590	410	$31 \times 10^3$	75 - 182	1.09 - 2.64
175	62	15	140	25	$9  imes 10^3$	372 - 700	$0.6-1.1\times10^{-14}\approx0$
175	62	28	260	15	$13  imes 10^3$	860 - 1726	$1.3-2.7\times10^{-14}\approx0$

and the maximum of the instantaneous rate of change over  $\tau_{\rm max}$ . Concerning the duration of geomagnetic spikes, it has been argued that their total duration may not exceed ~ 50 yr, i.e.  $\tau_{\rm max} \sim 25$  yr [Ben-Yosef et al., 2009, Shaar et al., 2011]. Assuming the same characteristic velocity and magnetic diffusivity as used above, the observation of such a short lasting event would require a characteristic lengthscale of ~ 175 km at the CMB and thus a local magnetic Reynolds number  $R_m \sim 140$ . These values would lead to a maximum amplitude of 9 mT and an average variation rate at the CMB of ~ 372  $\mu$ T/yr. The maximum instantaneous variation rate would reach ~ 700  $\mu$ T/yr in this case. However, and regardless of the variation rate we consider, such an event would not be detected at the Earth's surface, because of the geometric attenuation through the mantle, as it would correspond to degree  $\ell \sim 62$  (the signal would be practically invisible at the Earth's surface).

From the scaling laws reported in Eqs. (4.15-4.16), a more general analysis can be performed to determine if geomagnetic field intensity peaks suggested by recent archeomagnetic datasets could be reasonably attributed to the signature at the Earth's surface of flux expulsion events at the CMB. As we aim to give optimistic estimates of the intensity rate of change produced by flux expulsion, we will therefore only rely on the results obtained from the scaling law of  $R^{\max}$ , associated with the duration  $\tau_{\max}$ , corresponding to the time needed to reach the maximum of  $B_{z0}^{\max}$  (i.e. half the total duration of an intensity peak). Figure 11 shows color scale plots of the extrapolated maximum instantaneous variation rates at the Earth's surface as a function of the characteristic velocity and the characteristic wavelength of the process, for two different magnitudes of the toroidal field (3 and 10 mT, taken as plausible lower and upper values, see Hori et al. [2015]) and exploring two possible values of the magnetic diffusivity (0.6 and  $2 \text{ m}^2 \text{s}^{-1}$ ). The durations are represented by white dashed lines ( $\tau_{\rm max}$  being half of the total duration). The solid lines show different values for variation rates, given in  $\mu T/yr$ . From this figure, it is observed that the dashed line corresponding to  $\tau_{\rm max} \leq 50$  yr ( $\tau_{\rm tot} \leq 100$  yr), i.e. roughly two or three times the total duration initially suggested for geomagnetic spikes Ben-Yosef et al., 2009], never cross the line corresponding to a variation rate of 4  $\mu$ T/yr. Shaar et al. [2016] and Ben-Yosef et al. [2017] suggest a less extreme geomagnetic spike during the 8<sup>th</sup> century BC that would be associated with variation rates of  $\sim 0.75$ -1.5  $\mu T/yr$ . Considering the lowest intensity variation rate of 0.75  $\mu$ T/yr and a total duration of 100 yr, flux expulsion could account for geomagnetic spikes only if the intensity of the toroidal magnetic field was increased to 10 mT and if the lengthscale of the flow pattern was on the order of 1000 km. Such values would however correspond to a flow speed greater than 50  $\rm km/yr$  a rather extreme value according to inferences based on the present geomagnetic field [Hulot et al., 2002, Finlay and Amit, 2011, Holme, 2015]. For a total duration of 200 yr ( $\tau_{\rm max} = 100$ yr), a characteristic flow speed of 35 km/yr would be needed. For the same duration, a variation rate as high as 4  $\mu$ T/yr would require a characteristic flow speed of 50 km/yr. Adopting a higher value of the magnetic diffusivity  $(2 \text{ m}^2 \text{s}^{-1}, \text{Fig. 11c, d})$  does not significantly change these results. To summarize, Figure 11 clearly shows that according to our



FIGURE 4.11: Instantaneous rate of change of the intensity of the field generated by flux expulsion as a function of the characteristic velocity of the fluid in the Earth's core and the characteristic scale of the process in the core, given for different initial intensity and magnetic diffusivity.  $U_{\rm rms}$ is the root mean square velocity at the top of the core. The solid lines represent some constant rates of change. The dashed lines give different values of  $\tau_{\rm max}$  corresponding to half of the total duration of the expected peak of intensity induced by flux expulsion at the CMB.

analyses magnetic flux expulsion can conceivably produce extreme variation rates only for large-scale events (> 1000 km at the CMB) and for a minimum total duration longer than a century.

Although less extreme, other periods characterized by strong intensity variation rates are proposed both in Western Europe and the Near East. In the Near East, Yutsis-Akimova et al. [2018a,b] report during the 6<sup>th</sup> millennium BC two intensity peaks of about one century or less (see also Kovacheva et al. [2014]), associated with variation rates bounded between  $\sim 0.1$  and  $\sim 0.25 \ \mu T/yr$ . Considering a total duration of 100 yr, a variation rate of  $0.1\mu T/yr$ , and the commonly accepted value of  $0.6 \text{ m}^2 \text{s}^{-1}$  for the magnetic diffusivity, such feature would be reachable for a flow speed greater than 40 km/yr, i.e. more than twice its current rms value ( $\sim 15$  km/yr). For the third millennium BC, Gallet et al. [2020] also report two intensity peaks in Mesopotamia associated with durations of  $\sim 200$ yrs and variation rates of  $\sim 0.10 - 0.20 \ \mu T/yr$ . These events would be compatible with flux expulsion over large spatial scale, with a reasonable flow speed of 20-30 km/yr. In Western Europe, Hervé et al. [2017] report a field intensity peak centered around 600 BC. This peak would have lasted about 400 yr and is associated with variation rate of  $\sim 0.25 \ \mu T/yr$ during its ascending branch (7<sup>th</sup> century BC), about twice the maximum value known in the modern field [Livermore et al., 2014]. In this case, according to our experiments, such variations could be compatible with magnetic flux expulsion over large spatial scale ( $\sim 1000$  km corresponding to degree 10), associated with a flow speed of 15 km/yr, the rms flow speed at the CMB. Genevey et al. [2013, 2016] also show the existence in Western Europe of a series of geomagnetic field intensity peaks over the past  $\sim 1500$  yr. The total duration of these peaks is  $\sim 200$  yr and they are associated with intensity variation rates from ~ 0.05  $\mu$ T/yr, to a maximum of ~ 0.15  $\mu$ T/yr, as for instance during the 10<sup>th</sup> century AD. For such durations and variation rates, explaining these events by flux expulsion process appears also possible with a quite reasonable flow speed not exceeding 20-30 km/year and a lengthscale for the eddies that would approximately correspond to degree 13. This range of lengthscales for magnetic flux expulsion at the CMB would result in an extended large-scale signature at the Earth's surface, of about 1500 - 2000 km for  $\ell \sim 10 - 13.$ 

Our analyses therefore lead to different interpretations of the geomagnetic intensity peaks reported in the Near East and in Western Europe. In the Near-East, the peaks during the 6<sup>th</sup> millennium BC are obtained for unrealistically high characteristic flow speeds (for the considered lengthscales and durations) while for the intensity peaks of the third millennium BC and those observed in Western Europe, they could reasonably be attributed to a flux expulsion process. It is worth recalling that the kinematic configuration considered in this study constitutes a particular limit (leading to high variation rates), for which the feedback of the Lorentz force on the velocity field is not considered, and the intensity variation rates estimated at the CMB are roughly extrapolated to the Earth's surface. On another hand, analysis of outputs of numerical dynamo simulations would be of great interest to determine whether flux expulsion events in a 3D setting with self-consistent dynamics follow the scaling laws proposed here, or whether they are enhanced or attenuated by other processes.

#### 4.5 Conclusion

The analysis of simple two-dimensional kinematic models of magnetic flux expulsion allows the determination of scaling laws governing the two physical processes involved in flux expulsion, namely the expulsion of magnetic flux from closed streamlines and subsequent expulsion through an insulating boundary, depending on the magnetic Reynolds number  $R_m$ .

From the different cases studied here, Configuration 3 gives the better indication of the conditions prevailing in the outer core. For this configuration, we consider a rectangular domain containing two counter-rotating eddies with an upper insulating boundary. The maximum vertical component of the field that diffuses through the upper boundary  $B_{z0}^{\max}$  presents an intensity peak, characterized by a maximum amplitude reached over a time  $\tau_{\max}$  which scales as  $\sim R_m^{0.152}$ . The maximum instantaneous rate of change scales as  $\sim R_m^{0.447}$ .

The extrapolation of the above scaling laws to the Earth (using reasonable estimates for the material properties and the flow magnitude at the CMB) shows that extreme intensity variations rates such as proposed for geomagnetic spikes are difficult to obtain solely by magnetic flux expulsion. According to our experiments, extremely fast variations over a duration shorter than a century do not appear to be compatible with flux expulsion events. Durations and variation rates of more moderate intensity peaks detected in the Near East and Western Europe during the Holocene are more compatible with our results.

An extension of this study to the dynamic case, considering the Lorentz force, would help to better characterize the expected signature of flux expulsion in the outer core. Dynamical variations rates are however expected to be lower than the kinematic ones we have investigated, since the feedback of the Lorentz force on the flow should lower the efficiency of expulsion (from the eddies and thereby through the insulating boundary). Recent studies of inverse geodynamo modelling covering the historical period during which direct measurements of the magnetic field are available (1840-2010) identify magnetic flux expulsion at low latitudes [Aubert, 2014], arising from columnar vortices. To strengthen this conclusion, it would be interesting to look in details at outputs from numerical dynamos simulations presenting more realistic turbulent flows [Schaeffer et al., 2017] to seek magnetic flux expulsion events and study the underlying flow. It would also allow an assessment of whether the scaling laws determined in this study hold in the three-dimensional, fully dynamical case.

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### **Conclusions and perspectives**

The recovery of the past geomagnetic field variations is of particular importance in geomagnetism, with a number of applications as unveiling core dynamics and in archeology with the improvement of archeomagnetic dating. This thesis is part of the efforts being carried by the archeomagnetic community in that direction, addressing two aspects. The first, presented in chapter 3, focuses on archeointensity data acquisition in Central Asia to better constrain regional intensity variations. These data allow to illustrate how a wellconstrained archeointensity dataset can improve global reconstructions of the geomagnetic field. The second study, presented in chapter 4, focuses on the intensity variations inferred from archeomagnetic data from a theoretical standpoint. The process of magnetic flux expulsion at the CMB, proposed as a possible origin for extreme archeointensity events, is analysed in detail. The main results of both studies are summarized in the following.

# 5.1 Constraining the axial dipole moment variations from archeomagnetic data

In the archeomagnetic analysis presented in chapter 3, nine new archeointensity data are acquired in Bukhara, dated between the mid-16th century and the early 19th century. This new dataset is acquired using the Triaxe protocol, which allows to overcome a number of effects usually affecting archeointensity data acquisition, such as anisotropy or cooling rate effects on TRM acquisition. It shows that the intensity evolution in Bukhara undergoes a rapid decrease of ~ 14  $\mu$ T from ~ 1560 to ~ 1725, ending with an intensity minimum during the late 18th century, followed by a moderate increase during the early 19th. These results are compared then with archeointensity data available around Bukhara and in western Eurasia. Four areas are defined, in a 700-km radius of the cities of Bukhara, Tbilisi (Georgia), Moscow (Russia) and Thessaloniki (Balkans). In each case however, the dispersion observed at the regional level prevents deciphering the intensity evolution.

Interestingly, this evolution is in good agreement with archeointensity data from western Europe, accquired from the same Triaxe protocol and selection criteria. Based on this agreement, the Triaxe dataset (Bukhara data, western Europe data from Genevey et al. 2009, 2013, 2019 and northwestern Russia data from Salnaia et al. 2017a,b) is used to recalibrate the axial dipole moment  $g_1^0$  from gufm1, assuming that the geometry of the latter is reliable.

The obtained evolution is non linear, with a minimum of  $|g_1^0|$  of ~ 29400 nT during the second half of the 18th century. This result contradicts the linear axial dipole moment evolution proposed in previous studies following the same approach (i.e., the recalibration of  $g_1^0$  from gufm1 from archeointensity data) but using global archeomagnetic datasets [Gubbins et al., 2006, Finlay, 2008, Suttie et al., 2011, Poletti et al., 2018]. It should nonetheless be mentioned that, owing to the scatter observed in these datasets, a linear evolution is the most parsimonious interpretation. Even if, to first order, the axial dipole moment is decaying, it might also present shorter variations hidden by the data dispersion at the global scale.

The approach proposed in this study relies on the geometry of the magnetic field provided by gufm1. Yet, several studies have shown a lower reliability of the predicted directions before ~ 1750 by comparison with archeomagnetic data [e.g. Tanguy et al., 2011, Le Goff and Gallet, 2017], for instance. This is not surprising given the lower amount of data available between 1590 and ~ 1750, in particular, the limited number or absence of inclination data (recall Fig. 1.4d).

The axial dipole moment evolution proposed here remains to be confirmed by the acquisition of well-constrained archeointensity data globally distributed. Nevertheless, the archeointensity data obtained in this study and the good consistency observed from the Triaxe dataset show that archeomagnetic data can provide constraints on the geomagnetic intensity evolution over centennial to multi-decadal timescales at a regional and global scale.

#### 5.2 Intensity variations produced by magnetic flux expulsion at the CMB

In chapter 4, a two-dimensional kinematic model of magnetic flux expulsion is developed following Bloxham [1986]. From the various cases presented in this study, the most efficient configuration to generate flux expulsion consists of two counter-rotating eddies and an upper insulating boundary. The only control parameter of the model is the magnetic Reynolds number Rm (the ratio of diffusion to advection times).

A systematic analysis is carried, monitoring the evolution of initially horizontal magnetic field lines inside the domain, advected by a fixed flow pattern. Two physical processes are identified during the evolution of the system. First, the process of magnetic flux expulsion from closed streamlines. The magnetic field lines are sheared and folded by the flow, leading to the concentration of magnetic flux close to the boundary. If this boundary separates the conducting fluid from an insulating medium, a second physical process is observed, that of diffusion of the magnetic flux through the insulating boundary. The evolution of the maximum vertical component of the magnetic field  $(B_{z0}^{\max})^{\max}$  at the top insulating boundary is monitored. As the flux is progressively expelled from the eddies and concentrated close to the upper boundary,  $(B_{z0}^{\max})^{\max}$  increases, until reaching an intensity peak, followed by a decrease until a steady state is reached. The maximum instantaneous variation rate is found to scale as ~  $R_m^{0.447}$ , while the time to reach the intensity peak of  $(B_{z0}^{\max})^{\max}$  scales as ~  $R_m^{0.152}$ .

These scaling laws are then extrapolated to the Earth and the results are compared with rapid or extreme intensity events inferred from archeomagnetic data analyses. It is found that, considering reasonable estimates for Earth's parameters, flux expulsion cannot generate intensity peaks shorter than a century associated with variation rates of  $4 - 5 \ \mu T/yr$ , as first proposed for geomagnetic spikes [Ben-Yosef et al., 2009, Shaar et al., 2011], or even the lower range of values of  $0.75 - 1.5 \ \mu T/yr$  proposed more recently [Shaar et al., 2011]. Nonetheless, other archeointensity events characterized by longer durations and lower variation rates, closer to the maximum observed in the modern field [~ 0.1  $\mu T/yr$ , e.g. Finlay et al., 2020] appear compatible with flux expulsion events. This comprises for instance two other intensity peaks inferred in the Near-East during the third millenium BC, of ~ 200-years durations and associated with a variation rate of

~  $0.1 - 0.2 \ \mu T/yr$  [Gallet et al., 2020]. Other compatible events are found in Western Europe, as the 400-yr intensity peak inferred during the first millennium BC, associated with a maximum variation rate of ~  $0.25 \ \mu T/yr$  [Hervé et al., 2017], or the series of peaks proposed during the past ~ 1500 years, of duration 250 years, with variations rates of ~  $0.05 - 0.15 \ \mu T/yr$  [Genevey et al., 2013, 2016].

#### 5.3 Perspectives

The studies presented in this manuscript raise a number of interesting perspectives, some of which are described in the following.

The archeomagnetic analysis carried in Bukhara and the comparison with other regional archeointensity datasets underline once again the crucial importance of acquiring new well-constrained archemagnetic data. This perspective is pursued in a second ongoing archeointensity study. In line with the recent efforts to improve the geographic distribution of the archeointensity data especially in Africa and South-America, we had the opportunity of carrying a sampling of archeological material in French Guiana, covering the historical period. As a consequence of the colonial history of this region, the sugar cane manufacturing was implanted in French Guiana during the 17th century. The subsequent burgeoning number of factories and pottery production sites makes it a particularly favourable context for archeomagnetic sampling. The analysis of these samples is expected to improve the regional intensity variations in northwestern South-America. It would then allow to extend the constraints on the South Atlantic Anomaly effect to this region, over the historical period.

In the present study, the approach proposed to constrain the evolution of the axial dipole moment over the historical period relies on the geometry of the gufm1 model. However, the lack of direct directions measurements (especially inclination data) before ~ 1750 hinders accurate reconstructions of the axial dipole moment evolution as well as regional intensity variations between ~ 1590 and ~ 1750. It therefore calls for an improvement of the geometry proposed by gufm1 over this period. A possible option could be to integrate both direct and indirect measurements in the construction of a global field model. A first step in this direction has been taken by Arneitz et al. [2019] who proposed the joint inversion of both types of data. However, they show that the very large disparity in terms of amount of available data generates a number of model artifacts difficult to avoid. Alternatively, one could also resorts on a limited number of well-constrained archeomagnetic data to perform a joint inversion of both datasets over the historical period.

From a core processes standpoint, the study presented here presents a number of limitations, the effects of which have been discussed in section 4.4. In particular, the twodimensional flux expulsion model presented in this study operates in a kinematic regime, the deduced scaling laws are therefore expected to provide optimistic estimates of the resulting intensity variations. In a dynamical regime, it is expected that the Lorentz force would exert a feedback on the flow, lowering the efficiency of flux expulsion. To better understand how the process is operating in the outer core, the model will be extended to the dynamical regime to include the feedback of the Lorentz force. This will allow to ascertain the effect of the latter on the signature of flux expulsion at the core's surface. It would then be interesting to use these results to monitor flux expulsion events in numerical dynamo simulations and analyse the underlying flow. It would allow, for instance, to assess the effects of 3D geometry or asymmetric flow patterns (recall section 4.4) on signature of flux expulsion at the core's surface. Alternatively, the same kind of analysis could be applied to inverse geodynamo modelling. The identified flux expulsion events could be analysed in terms of inverted flow and field and subsequently compared with the geomagnetic model used for the inversion. Such analyses would provide additional constraints on the spatial and temporal intensity variations induced by flux expulsion, at both regional and global scales.

Indeed, this process also seems to be involved in the axial dipole decay of the geomagnetic field [e.g. Gubbins et al., 2006], through the growth of reverse flux patches at the CMB (section 2.4.4). An interesting application could therefore be to track these events in numerical dynamo simulations and analyse the effect of magnetic flux expulsion on the axial dipole moment evolution, with a particular focus on reversing dynamo simulations, as the latter present dramatic dipole decay during reversals. Such analyses would thus bring additional insights on the effects of diffusion on fluctuations of the axial dipole moment.

### Appendix A

# Supplementary material Analyzing the geomagnetic axial dipole field moment over the historical period from new archeointensity results at Bukhara (Uzbekistan, Central Asia)

TABLE A.1: Detailed archeointensity results obtained at the specimen level.  $T_1 - T_2$  is the temperature range over which the  $R'(T_i)$  data are averaged.  $H_{\text{lab}}$  gives the laboratory field intensity applied during the Triaxe protocol. K (resp. S) gives the NRM proportion (resp.  $R'(T_i)$  data slope) over the given temperature range. F is the intensity determined at the specimen level and F mean at the fragment level.

Site	Age (yr AD)	Fragment	Specimen	$T_1$ - $T_2$ (°C)	$H_{lab}(\mu T)$	K (%)	S (%)	$F(\mu T)$	F mean $(\mu T)$
BK04	1556 - 1567	BK04-02	BK04-02b	225-490	50	89	5	55.1	$54 \pm 1.2$
			BK04-02c	255 - 470	50	81	4	54.1	
			BK04-02d	260 - 455	50	76	5	52.8	
		BK04-03	BK04-03a	180 - 450	50	78	2	50.5	$51.8 \pm 1.3$
			BK04-03c	180-470	50	87	4	53.0	
			BK04-03d	175 - 470	50	83	5	51.9	
		BK04-08	BK04-08a	170-440	50	73	-3	53.5	$54 \pm 0.7$
			BK04-08b	175 - 480	50	86	6	53.8	
			BK04-08c	205 - 480	50	84	6	54.8	
		BK04-09	BK04-09a	175 - 440	50	71	-4	54.5	$55.9 \pm 1.3$
			BK04-09b	175 - 480	50	84	-1	56.8	
			BK04-09c	190-480	50	85	1	56.5	
		BK04-10	BK04-10a	175 - 440	50	88	2	52.0	$52.5\pm0.4$
			BK04-10b	175 - 440	50	88	-4	52.6	
			BK04-10c	190-440	50	87	4	52.8	
BK05	1589 - 1615	BK05-02	BK05-02a	175 - 440	50	78	-5	53.9	$53.5\pm0.5$
			BK05-02b	175 - 460	50	82	2	53.5	
			BK05-02c	175 - 460	50	84	0	53.0	
		BK05-03	BK05-03a	210-440	50	78	5	52.0	$51.6\pm0.3$
			BK05-03b	210-460	50	82	5	51.5	
			BK05-03d	210-440	50	76	1	51.4	
		BK05-05	BK05-05b	175 - 440	50	76	0	52.9	$52.1 \pm 1$
			BK05-05c	175 - 440	50	79	1	52.4	
			BK05-05d	175 - 450	50	71	3	50.9	
BK06	1558 - 1589	BK06-01	BK06-01a	180 - 450	50	81	2	53.6	$53.4\pm0.9$
			BK06-01d	180 - 450	50	81	1	52.4	
			BK06-01e	180 - 450	50	82	1	54.2	

		BK06-02	BK06-02a	180-450	50	81	2	54.9	$54.7 \pm 0.5$
			BK06-02d	180-435	50	77	0	54.1	
			BK06-02e	175-445	50	78	-2	55.1	
		BK06-06	BK06-06a	180-450	50	64	-3	56 1	$55.2 \pm 0.7$
		DII00 00	BK06-06b	195-490	50	80	2	55.4	00.2 ± 0.1
			BK06-06c	180-490	50	79	3	54.1	
			BK06-06d	210-480	50	75	6	54.6	
BK07	1700 - 1722	BK07_02	BK07-02a	175-450	45	77	3	14.0	$44 \pm 0.5$
DIX07	1100 1122	DI(07-02	BK07-02a BK07-02c	180 455	45	78	0	44.0	$44 \pm 0.0$
			BK07-020	100-450	45	70	0	44.0	
		DK07 02	DK07-020	190-450	45	74	6	45.5	$45.0 \pm 0.2$
		DK07-03	DK07-03a DK07-02h	190-400	45	74 75	5	45.7	$45.9 \pm 0.2$
			DK07-030	100-400	40	70	57	40.9	
		DV07 04	DK07-03C	180-400	40	() 61	1	40.1	11 2 1 0 1
		BK07-04	BK07-04a DV07-04-	185-450	45	01	1	44.9	$44.8 \pm 0.1$
			DK07-04C	100-490	40	70	2	44.9	
DIZOO	1700 1705	DV09A 09	BK07-040	205-490	40	13	-0	44.7	41 0 1 0 7
BK08	1790 - 1795	BK08A-02	BK08A-02a	175-440	40	07 C7	-4	41.5	$41.8 \pm 0.7$
			BK08A-02C	150-440	40	67	ა ე	42.6	
		DIZOOA OA	BK08A-02d	225-445	40	05	-3	41.4	41 7
		BK08A-04	BK08A-04a	175-440	40	87	1	41.6	$41.7 \pm 0.8$
			BK08A-04b	175-385	40	81	5	42.5	
		DIMOND ON	BK08A-04d	205-445	40	79	3	41.0	
		BK08B-02	BK08B-02a	180-455	45	93	1	42.5	$43.3 \pm 1.1$
			BK08B-02b	140-400	45	93	-1	44.6	
			BK08B-02c	175-400	45	91	-4	42.9	
BK11	1775 - 1825	BK11A-03	BK11A-03a	180 - 455	40	88	-3	43.7	$43.6 \pm 0.1$
			BK11A-03b	180 - 455	40	89	-1	43.6	
			BK11A-03c	180 - 455	40	89	-2	43.5	
		BK11A-04	BK11A-04a	180 - 450	40	92	1	43.3	$43.2 \pm 0.1$
			BK11A-04b	180 - 455	40	93	7	43.1	
			BK11A-04c	180 - 455	40	93	5	43.2	
		BK11A-05	BK11A-05a	180 - 455	40	61	7	46.5	$47.4 \pm 1$
			BK11A-05b	180 - 495	40	76	7	46.6	
			BK11A-05c	180-525	40	89	-1	48.6	
			BK11A-05d	140 - 455	40	66	7	47.9	
		BK11A-06	BK11A-06b	180 - 475	40	65	2	46.9	$46.5\pm0.8$
			BK11A-06c	180-505	40	75	2	47.1	
			BK11A-06d	175 - 505	40	81	4	45.6	
		BK11B-02	BK11B-02a	210-450	40	86	2	43.0	$42.6 \pm 1.1$
			BK11B-02c	215 - 450	40	85	2	41.3	
			BK11B-02d	230 - 450	40	84	4	43.4	
		BK11B-03	BK11B-03a	210-450	40	91	3	42.0	$42.3\pm0.3$
			BK11B-03b	180 - 450	40	81	2	42.6	
			BK11B-03e	175 - 450	40	74	3	42.2	
		BK11C-01	BK11C-01a	180 - 455	40	76	1	43.5	$43.6\pm0.2$
			BK11C-01b	180-470	40	81	4	43.6	
			BK11C-01c	180 - 475	40	82	4	43.8	
BK12	1631 - 1637	BK12-02	BK12-02a	180 - 450	50	82	-1	50.0	$49.4\pm0.6$
			BK12-02b	175 - 445	50	82	1	48.9	
			BK12-02c	180 - 450	50	81	3	49.2	
		BK12-04	BK12-04a	180 - 450	50	77	-1	48.2	$48.3\pm0.2$
			BK12-04b	180-490	50	88	6	48.1	
			BK12-04c	180-490	50	90	3	48.5	
		BK12-06	BK12-06a	220-450	50	70	3	51.2	$51.3 \pm 0.5$
			BK12-06b	230-470	50	78	3	51.8	
			BK12-06c	210-470	50	79	4	50.8	
BK13	1700 - 1750	BK13-01	BK13-01a	205-440	45	73	4	40.9	$41.8 \pm 0.8$

		BK13-01b	185-440	45	75	0	42.4	
		BK13-01c	175-440	45	75	0	42.0	
	BK13-02	BK13-02a	205-440	45	79	3	39.2	$39.5\pm0.4$
		BK13-02b	205-440	45	78	3	39.0	
		BK13-02c	200-440	45	80	3	39.9	
		BK13-02d	175 - 450	40	81	2	39.7	
	BK13-03	BK13-03a	175 - 440	45	72	7	40.3	$40.7\pm0.3$
		BK13-03b	245 - 460	45	75	8	40.9	
		BK13-03c	175 - 440	45	70	-4	40.8	
	BK13-05	BK13-05a	175-40	45	75	5	38.4	$39.5\pm1$
		BK13-05b	175 - 460	45	83	4	39.8	
		BK13-05d	215 - 450	40	72	1	40.3	
	BK13-06	BK13-06a	225 - 440	45	68	-5	42.7	$42.3\pm0.9$
		BK13-06b	175 - 460	45	80	-3	42.9	
		BK13-06c	175 - 465	45	80	4	41.3	
	BK13-07	BK13-07a	175 - 440	45	74	-3	39.1	$38.9\pm0.3$
		BK13-07b	175 - 460	45	80	0	38.9	
		BK13-07c	175 - 460	45	82	1	39.1	
		BK13-07d	175 - 445	40	75	2	38.4	
BK14 1758 - 1785	5 BK14-02	BK14-02a	175 - 440	40	79	0	43.9	$43.2\pm1.2$
		BK14-02b	245 - 440	40	71	3	41.8	
		BK14-02c	245 - 460	40	80	4	43.9	
	BK14-03	BK14-03a	170-440	40	79	1	45.7	$46\pm0.6$
		BK14-03b	205 - 440	40	80	6	46.7	
		BK14-03c	140-440	40	82	4	45.5	
	BK14-04	BK14-04a	175 - 440	40	71	-1	43.5	$43.7\pm0.9$
		BK14-04b	175 - 460	40	78	5	43.0	
		BK14-04c	175 - 440	40	72	-3	44.7	

TABLE A.2: Mean curve values and 95% credible interval (upper and lower limits) for  $g_1^0$  variations obtained from the recalibration of *gufm1* coefficients. The values are obtained with the AH-RJMCMC algorithm from Livermore et al. [2018] using the following input parameters:  $\sigma_{\text{move}} = 30 \text{ yrs}, \sigma_{\text{change}} = 5 \text{ yrs}, \sigma_{\text{birth}} = 5 \text{ yrs}, K_{\text{max}} = 150$ . The intensity priors are set to a minimum of  $-36 \ \mu\text{T}$  and a maximum of  $-26 \ \mu\text{T}$ .

Age (yr)	$g_1^0$ median ( $\mu$ T)	lower limit $(\mu T)$	upper limit $(\mu T)$
1542.0	-35.5	-37.7	-26.7
1542.6	-35.5	-37.6	-27.0
1543.3	-35.4	-37.6	-27.3
1543.9	-35.4	-37.6	-27.4
1544.6	-35.4	-37.5	-27.6
1545.2	-35.4	-37.5	-27.7
1545.9	-35.4	-37.5	-27.8
1546.5	-35.4	-37.4	-27.9
1547.1	-35.4	-37.4	-28.0
1547.8	-35.4	-37.4	-28.1
1548.4	-35.4	-37.3	-28.2
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1781.3	-29.4	-30.3	-28.5
1781.9	-29.4	-30.3	-28.5
1782.6	-29.4	-30.3	-28.5
1783.2	-29.4	-30.3	-28.5
1783.9	-29.4	-30.3	-28.5
1784.5	-29.4	-30.3	-28.6
1785.2	-29.4	-30.3	-28.6
1785.8	-29.4	-30.3	-28.6
1786.4	-29.4	-30.3	-28.6
1787 1	-29.4	-30.3	-28.6
1787.7	-29.4	-30.3	-28.6
1788.4	-29.5	-30.3	-28.7
1789.0	-29.5	-30.3	-28.7
1789.7	-29.5	-30.2	-28.7
1790.3	-29.5	-30.2	-28.7
1791.0	-29.5	-30.2	-28.7
1791.6	-29.5	-30.2	-20.1
1702.2	-29.6	-30.2	-20.0
1792.2	-29.6	-30.2	-20.0
1703 5	-29.6	-30.2	-20.0
1794.2	-29.6	-30.2	-28.8
1794.8	-29.6	-30.2	-28.8
1795.5	-29.6	-30.2	-28.8
1796.1	-29.6	-30.3	-28.9
1796.7	-29.6	-30.3	-28.9
1790.1 1797.4	-29.6	-30.3	-28.9
1798.0	-29.0	-30.3	-20.9
1798.7	-29.7	-30.3	-28.9
1700 3	-29.7	-30.3	-20.9
1800.0	-29.7	-30.3	-28.9
1800.6	-29.1	-30.3	-28.9
1801.2	-29.8	-30.3	-28.9
1801.2	-29.8	-30.3	-28.9
1802.5	-29.8	-30.3	-28.9
1802.0	-29.8	-30.3	-20.9
1803.8	-29.8	-30.4	-20.9
1804.5	-29.8	-30.4	-20.9
1804.5	-29.0	-30.4	-20.9
1805.1	-29.9	-30.4	-20.9
1806 4	-29.9 -29.9	-30.4	-20.9 -28 Q
1807.0	-20.0	-30.4	-20.9 -98 0
1807.7	-20.0	-30.5	-20.9 -98 0
1808.3	-29.9	-30.5	-20.9 -28 9
1809.0	-29.9	-30.5	-20.9 -28 9
1809.6	-30.0	-30.5	-20.9 -28 9
1810.3	-30.0	-30.5	-20.9 -28 9
1810.9	-30.0	-30.6	-28.9
TO TO 10	00.0	00.0	40.0

1811.5	-30.0	-30.6	-28.9
1812.2	-30.0	-30.6	-28.9
1812.8	-30.0	-30.6	-28.9
1813.5	-30.0	-30.6	-28.9
1814.1	-30.1	-30.7	-28.9
1814.8	-30.1	-30.7	-28.9
1815.4	-30.1	-30.7	-28.9
1816.0	-30.2	-30.7	-28.9
1816.7	-30.2	-30.7	-28.9
1817.3	-30.2	-30.8	-28.9
1818.0	-30.2	-30.8	-29.0
1818.6	-30.2	-30.8	-29.0
1810.3	-30.2	-30.8	-29.0
1810.0	-50.2	-30.0	-25.0
1019.9	-30.2	-30.9	-29.0
1020.0	-30.3	-30.9	-29.0
1021.2	-30.3	-50.9	-29.0
1821.8	-30.3	-30.9	-29.0
1822.5	-30.4	-30.9	-29.0
1823.1	-30.4	-31.0	-29.0
1823.8	-30.4	-31.0	-29.0
1824.4	-30.4	-31.0	-29.1
1825.0	-30.4	-31.0	-29.1
1825.7	-30.5	-31.0	-29.1
1826.3	-30.5	-31.1	-29.1
1827.0	-30.5	-31.1	-29.1
1827.6	-30.5	-31.1	-29.1
1828.3	-30.5	-31.1	-29.2
1828.9	-30.6	-31.2	-29.2
1829.5	-30.6	-31.2	-29.2
1830.2	-30.6	-31.2	-29.2
1830.8	-30.6	-31.2	-29.2
1831.5	-30.7	-31.2	-29.3
1832.1	-30.7	-31.3	-29.3
1832.8	-30.8	-31.3	-29.3
1833.4	-30.8	-31.3	-29.3
1834.1	-30.8	-31.3	-29.4
1834.7	-30.8	-31.4	-29.4
1835.3	-30.9	-31.4	-29.4
1836.0	-30.9	-31.4	-29.4
1836.6	-31.0	-31.4	-29.5
1837 3	-31.0	-31.4	-29.5
1837.0	-31.0	-31.5	-29.5
1838.6	31.0	31.5	20.6
1830.2	-51.0	-51.5	-29.0
1039.2	-01.1	-31.5	-29.0
1039.0	-01.1 21.1	-31.3 21.6	-29.0
1040.0	-31.1	-51.0 21.0	-29.7
1841.1	-31.1	-31.0	-29.7
1841.8	-31.2	-31.0	-29.7
1842.4	-31.2	-31.0	-29.8
1843.1	-51.2	-51.7	-29.8
1843.7	-31.2	-31.7	-29.8
1844.3	-31.3	-31.7	-29.9
1845.0	-31.3	-31.8	-29.9
1845.6	-31.4	-31.8	-29.9
1846.3	-31.4	-31.8	-29.9
1846.9	-31.4	-31.8	-30.0
1847.6	-31.4	-31.9	-30.0

1848.2	-31.5	-31.9	-30.0
1848.8	-31.5	-31.9	-30.0
1849.5	-31.6	-32.0	-30.1
1850.1	-31.6	-32.0	-30.1
1850.8	-31.6	-32.0	-30.1
1851.4	-31.6	-32.1	-30.1
1852.1	-31.7	-32.1	-30.1
1852.7	-31.7	-32.1	-30.2
1853.4	-31.7	-32.2	-30.2
1854.0	-31.8	-32.2	-30.2
1854.6	-31.8	-32.2	-30.2
1855.3	-31.8	-32.2	-30.3
1855.9	-31.8	-32.2	-30.3
1856.6	-31.9	-32.2	-30.4
1857.2	-31.9	-32.2	-30.5
1857.9	-32.0	-32.3	-30.6
1858.5	-32.0	-32.3	-30.9
1859.1	-32.0	-32.3	-31.4
1859.8	-32.1	-32.3	-31.8
1860.4	-32.2	-32.6	-31.9
1861.1	-32.2	-33.2	-31.9
1861.7	-32.2	-33.8	-31.8
1862.4	-32.2	-34.5	-31.6
1863.0	-32.3	-35.4	-31.1



FIGURE A.1: Location map of the four datasets selected using the G2008 (or loose) selection criteria. The colored symbols show the location of the selected data (same symbols as in Fig. 3.11). The black circles gives the 700-km radius area around Bukhara, Tbilisi, Moscow, Thessaloniki (from East to West).



FIGURE A.2: Archeointensity results obtained in a 700-km radius from a) Bukhara, b) Moscow (Russia), c) Tbilisi (Georgia), d) Thessaloniki (Greece), reduced at the latitude of the corresponding location. The data are filtered using the set of strict selection criteria. Each panel also shows the predicted intensity evolution from various geomagnetic models at the corresponding location (continuous lines, errors are given as two standard deviations by shaded areas; see legend and text for details).



FIGURE A.3: Comparison of a) the recalibrated prediction of gufm1 using the western European and Russian datasets and b) the same datasets simply reduced at the latitude of Bukhara, assuming an axial dipole field.

### Appendix B

# Supplementary material Imprint of magnetic flux expulsion at the core-mantle boundary on geomagnetic field intensity variations

#### **B.1** Numerical approximation

#### B.1.1 Numerical scheme and boundary conditions

To numerically approximate Eq. 4.3, we use a pseudo-spectral method involving a Fourier expansion along the x-direction together with a second-order accurate finite-difference scheme in the z-direction. The Fourier expansion depends on the lateral boundary conditions one intends to impose.

In configurations 1 and 2, magnetic field lines are anchored in the lateral boundaries. We decompose the vector potential A according to

$$A(x, z, t) = A_0(z) + A_v(x, z, t),$$
(B.1)

where  $A_0(z) = A(x, z, t = 0)$  is the initial vector potential (which is a function of z only), and the time-dependent  $A_v$  vanishes on the lateral boundaries, at x = 0 and  $x = L_x$ . To satisfy exactly this vanishing requirement,  $A_v$  is expanded in x using sine functions. This amounts to rewriting Eq. (B.1) as

$$A(x, z, t) \approx A_0(z) + \frac{1}{2M} \sum_{k=0}^{M-1} \hat{A}_{vk}(z, t) \sin\left(\pi k \frac{x}{L_x}\right),$$
(B.2)

where  $x \in [0, L_x[$  is discretely sampled at M - 1 equally spaced internal points

$$x_k = \frac{kL_x}{M}, \quad k = 1, \dots, M - 1,$$
 (B.3)

and

$$\hat{A}_{vk}(z,t) \equiv \sum_{m=1}^{M-1} A_v(x_m, z, t) \sin\left(\pi k \frac{m}{M}\right).$$
(B.4)

In configuration 3, magnetic field lines move freely along the lateral boundaries. The vector potential can then be approximated using a generic Fourier expansion, such that

$$A(x,z,t) \approx \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} \hat{A}_k(z,t) \exp\left(-2i\pi k \frac{x}{L_x}\right),\tag{B.5}$$

where  $x \in [0, L_x[$  is sampled at M equally spaced points

$$x_k = \frac{kL_x}{M}, \quad k = 0, 1, \dots, M - 1,$$
 (B.6)

and each Fourier mode  $\hat{A}_k$  reads

$$\hat{A}_{k}(z,t) \equiv \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} A(x_{m}, z, t) \exp\left(2i\pi k \frac{m}{M}\right) \equiv \mathcal{F}_{k}[A(x, z, t)], \quad k = 0, 1, \dots, M-1.$$
(B.7)

Regardless of the configuration, note that the  $\hat{A}_{vk}$  and  $\hat{A}_k$  undergo the same treatment. We will use the latter notation in the following to discuss in a common manner the treatment of configurations 1, 2, and 3.

If  $\hat{\mathcal{J}}_k(z,t)$  denotes  $\mathcal{F}_k[\mathcal{J}(x,z,t)]$ , the Fourier expansion of Eq. (4.3) yields

$$\frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} \left\{ \frac{\partial}{\partial t} - \frac{1}{R_m} \left[ \frac{\partial^2}{\partial z^2} - k_p^2 \right] \right\} \hat{A}_k(z,t) \exp\left(-2i\pi k \frac{x}{L_x}\right) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} \hat{\mathcal{J}}_k(z,t) \exp\left(-2i\pi k \frac{x}{L_x}\right).$$

with  $k_p = 2\pi k/L_x$ . The orthogonality of the Fourier basis then allows solving

$$\left[\frac{\partial}{\partial t} - \frac{1}{R_m} \left(\frac{\partial^2}{\partial z^2} - k_p^2\right)\right] \hat{A}_k(z,t) = \hat{\mathcal{J}}_k(z,t), \tag{B.8}$$

for each mode k.

Time  $t \in [0, T]$  is sampled at discrete times  $t_n = n\Delta t$  and the vertical direction  $z \in [0, L_z]$  is sampled at a set of discrete  $z_j = jL_z/(N-1) = j\Delta z$ . This prompts us to define  $\hat{A}_k(j\Delta z, n\Delta t) \equiv \hat{A}_{k,j}^n$  and  $\hat{\mathcal{J}}_k(j\Delta z, n\Delta t) \equiv \hat{\mathcal{J}}_{k,j}^n$ . The centered finite difference scheme in z yields

$$\frac{\hat{A}_{k,j}^{n+1} - \hat{A}_{k,j}^{n}}{\Delta t} - \frac{1}{R_m} \left[ \frac{\hat{A}_{k,j+1}^{n+1} - 2\hat{A}_{k,j}^{n+1} + \hat{A}_{k,j-1}^{n+1}}{2\Delta z^2} + \frac{\hat{A}_{k,j+1}^n - 2\hat{A}_{k,j}^n + \hat{A}_{k,j-1}^n}{2\Delta z^2} - \frac{k_p^2}{2} \left( \hat{A}_{k,j}^{n+1} + \hat{A}_{k,j}^n \right) \right] = \hat{\mathcal{J}}_{k,j}^n$$
(B.9)

for the interior points  $z_j$ , j = 1, ..., N-2. This equation is modified near the boundaries in order to accommodate boundary conditions.

The lower boundary (j = N - 1) is supposed to be perfectly conducting for all three configurations. As the field in a perfectly conducting medium is static, the vector potential for all non-zero modes is forced to zero at the lower boundary, and the k = 0 mode remains equal to its initial value

$$\hat{A}_{k,N-1}^{n} = 0 \ \forall \ k \neq 0, \ \forall n, \tag{B.10}$$

$$\hat{A}_{0,N-1}^{n} = \hat{A}_{0,N-1}^{0} \forall n.$$
(B.11)

In configuration 1, the top is also perfectly conducting. The same condition is then applied

$$\hat{A}_{k,0}^n = 0 \ \forall \ k \neq 0, \ \forall n, \tag{B.12}$$

$$\hat{A}^{n}_{0,0} = \hat{A}^{0}_{0,0} \forall n.$$
(B.13)

If  $\mathbf{A}_{k}^{n}$  denotes the column vector of unknowns  $\left[\hat{A}_{k,0}^{n}, \hat{A}_{k,1}^{n}, \dots, \hat{A}_{k,N-2}^{n}, \hat{A}_{k,N-1}^{n}\right]^{T}$  (in which T implies transposition), the linear system to solve at each time step for configuration 1 reads

$$\mathbf{M}_{k}^{(1)}\mathbf{A}_{k}^{n+1} = \mathbf{N}_{k}^{(1)}\mathbf{A}_{k}^{n} + \mathbf{J}_{k}^{n} = \mathbf{F}_{k}^{n},$$
(B.14)

where

$$\mathbf{M}_{k}^{(1)} = \begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \gamma_{k} & \rho & \ddots & \ddots & \vdots \\ 0 & \rho & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \gamma_{k} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \rho & 0 \\ \vdots & & \ddots & \rho & \gamma_{k} & 0 \\ 0 & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix}, \forall k,$$
(B.15)

with  $\gamma_k = \frac{1}{\Delta t} + \frac{1}{R_m} \left( \frac{1}{\Delta z^2} + \frac{k_p^2}{2} \right)$  and  $\rho = -\frac{1}{R_m} \frac{1}{2\Delta z^2}$ ;

$$\mathbf{N}_{k}^{(1)} = \begin{bmatrix} b_{k} & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \chi_{k} & -\rho & \ddots & \ddots & \vdots \\ 0 & -\rho & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \chi_{k} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -\rho & 0 \\ \vdots & & \ddots & -\rho & \chi_{k} & 0 \\ 0 & \cdots & \cdots & 0 & 0 & b_{k} \end{bmatrix}, \forall k,$$
(B.16)

with  $\chi_k = \frac{1}{\Delta t} - \frac{1}{R_m} \left( \frac{1}{\Delta z^2} + \frac{k_p^2}{2} \right)$ ;  $b_0 = 1$  and  $b_k = 0$  for  $k \neq 0$ , leading to

$$\mathbf{F}_{0}^{n} = \begin{bmatrix} \hat{A}_{0,0}^{0}, \chi_{0}\hat{A}_{0,1}^{n} - \rho\hat{A}_{0,2}^{n} + \hat{\mathcal{J}}_{0,1}^{n} + \frac{A_{0,0}^{0}}{R_{m}\Delta z^{2}}, \dots, -\rho\hat{A}_{0,j-1}^{n} + \chi_{0}\hat{A}_{0,j}^{n} - \rho\hat{A}_{0,j+1}^{n} + \hat{\mathcal{J}}_{0,j}^{n}, \\ \dots, -\rho\hat{A}_{0,N-3}^{n} + \chi_{0}\hat{A}_{0,N-2}^{n} + \hat{\mathcal{J}}_{0,N-2}^{n} + \frac{\hat{A}_{0,N-1}^{0}}{R_{m}\Delta z^{2}}, \hat{A}_{0,N-1}^{0} \end{bmatrix}^{T}$$
(B.17)

for the k = 0 mode and

$$\mathbf{F}_{k}^{n} = \begin{bmatrix} 0, \chi_{k} \hat{A}_{k,1}^{n} - \rho \hat{A}_{k,2}^{n} + \hat{\mathcal{J}}_{k,1}^{n}, \dots, -\rho \hat{A}_{k,j-1}^{n} + \chi_{k} \hat{A}_{k,j}^{n} - \rho \hat{A}_{k,j+1}^{n} + \hat{\mathcal{J}}_{k,j}^{n}, \dots, \\ -\rho \hat{A}_{k,N-3}^{n} + \chi_{k} \hat{A}_{k,N-2}^{n} + \hat{\mathcal{J}}_{k,N-2}^{n}, 0 \end{bmatrix}^{T}$$
(B.18)

for each  $k \neq 0$  mode.

In configurations 2 and 3, the top boundary is insulating. By using the fact that the magnetic field in the z < 0 region is a potential field, Bloxham [1986] showed that the corresponding boundary condition reads

$$k_p \hat{A}_k(z,t) = \frac{\partial \hat{A}_k(z,t)}{\partial z}, \quad \forall k \neq 0.$$
 (B.19)

Using a centered finite difference scheme, this becomes

$$\hat{A}_{k,-1}^n = \hat{A}_{k,1}^n - 2k_p \Delta z \hat{A}_{k,0}^n.$$
(B.20)

using a ghost node located at  $z_{-1} = -\Delta z$ . For k = 0, we impose

$$\hat{A}^n_{0,0} = 0. (B.21)$$
Adopting the same notations than for Configuration 1 (Eq. B.14), the components of the linear system to solve for Configuration 2/3 become

$$\mathbf{M}_{k}^{(2/3)} = \begin{bmatrix} \xi_{k} & 2\rho & 0 & \cdots & \cdots & 0 \\ \rho & \gamma_{k} & \rho & \ddots & \ddots & \vdots \\ 0 & \rho & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \gamma_{k} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \rho & 0 \\ \vdots & & \ddots & \rho & \gamma_{k} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix}, k \neq 0;$$
(B.22)

with  $\xi_k = \frac{1}{\Delta t} + \frac{1}{R_m} \left( \frac{1 + k_p \Delta z}{\Delta z^2} + \frac{k_p^2}{2} \right);$ 

$$\mathbf{M}_{0}^{(2/3)} = \begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \gamma_{0} & \rho & \ddots & \ddots & \vdots \\ 0 & \rho & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \gamma_{0} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \rho & 0 \\ \vdots & & \ddots & \rho & \gamma_{0} & 0 \\ 0 & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix};$$
(B.23)

$$\mathbf{N}_{k}^{(2/3)} = \begin{bmatrix} \zeta_{k} & -2\rho & 0 & \cdots & \cdots & 0 \\ -\rho & \chi_{k} & -\rho & \ddots & \ddots & \vdots \\ 0 & -\rho & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \chi_{k} & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & -\rho & 0 \\ \vdots & & & \ddots & -\rho & \chi_{k} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix}, \ k \neq 0;$$
(B.24)

with  $\zeta_k = \frac{1}{\Delta t} - \frac{1}{R_m} \left( \frac{1 + k_p \Delta z}{\Delta z^2} + \frac{k_p^2}{2} \right)$ 

$$\mathbf{N}_{0}^{(2/3)} = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \chi_{0} & -\rho & \ddots & \ddots & \vdots \\ 0 & -\rho & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \chi_{0} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -\rho & 0 \\ \vdots & & \ddots & -\rho & \chi_{0} & 0 \\ 0 & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix};$$
(B.25)

leading to

$$\mathbf{F}_{k}^{n} = \left[\zeta_{k}\hat{A}_{k,0}^{n} - 2\rho\hat{A}_{k,1}^{n}, -\rho\hat{A}_{k,0}^{n} + \chi_{k}\hat{A}_{k,1}^{n} - \rho\hat{A}_{k,2}^{n} + \hat{\mathcal{J}}_{k,1}^{n}, \dots, -\rho\hat{A}_{k,j-1}^{n} + \chi_{k}\hat{A}_{k,j}^{n} - \rho\hat{A}_{k,j+1}^{n} + \hat{\mathcal{J}}_{k,j}^{n}, \dots, -\rho\hat{A}_{k,N+2N-3}^{n} + \chi_{k}\hat{A}_{k,N-2}^{n} + \hat{\mathcal{J}}_{k,N-2}^{n}, 0\right]^{T}$$
(B.26)

for each  $k \neq 0$  mode and

$$\mathbf{F}_{0}^{n} = \left[0, \chi_{0}\hat{A}_{0,1}^{n} - \rho\hat{A}_{0,2}^{n} + \hat{\mathcal{J}}_{0,1}^{n}, \dots, -\rho\hat{A}_{0,j-1}^{n} + \chi_{0}\hat{A}_{0,j}^{n} - \rho\hat{A}_{0,j+1}^{n} + \hat{\mathcal{J}}_{0,j}^{n}, \dots, -\rho\hat{A}_{0,N-3}^{n} + \chi_{0}\hat{A}_{0,N-2}^{n} + \hat{\mathcal{J}}_{0,N-2}^{n} + \frac{\hat{A}_{0,N-1}^{0}}{R_{m}\Delta z^{2}}, \hat{A}_{0,N-1}^{0}\right]^{T}$$
(B.27)

for the k = 0 mode, respectively.

This pseudo-spectral method has been implemented using python and its linear algebra and Fourier transform libraries.

#### B.1.2 Validation tests

## a) Diffusive limit: theoretical solution inside the domain

Drew [1993] tested his numerical results for the diffusion equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla^2 \mathbf{B},\tag{B.28}$$

in the case of a perfectly conducting boundary at the bottom and an insulating boundary at the top. For a purely horizontal initial field  $\mathbf{B} = B(z, t)\hat{x}$ , this equation becomes scalar

$$\frac{\partial B(z,t)}{\partial t} = \frac{\partial^2 B(z,t)}{\partial z^2},\tag{B.29}$$

subject to the boundary conditions,

$$\frac{\partial B(z,t)}{\partial z} = 0 \text{ at } z = 1, \forall t, \tag{B.30}$$

$$B(z,t) = 0 \text{ at } z = 0, \forall t.$$
 (B.31)

Assuming an exponential decay  $e^{-pt}$  of the field gives the following diffusive modes

$$B_p(z,t) = B_0 \cos\left[\sqrt{p}(1-z)\right] e^{-pt}, \forall p$$
 (B.32)

with  $B_0$  a constant and  $\sqrt{p} = \frac{(2q+1)\pi}{2}, q = 0, 1, 2, ...$  the decay rate. The fundamental (q = 0) mode is

$$B_{p0}(z,t) = B_0 \cos\left[\frac{\pi}{2}(1-z)\right] e^{-\frac{\pi^2}{4}t}.$$
(B.33)

The problem is solved using the same set of parameters than those used by Drew [1993], with  $L_z = L = 1$  and  $L_x = 2L = 2$ . Using our method, the magnetic field is expressed in terms of the vector potential in the Fourier domain. Consequently, the condition

$$\hat{A}_p(z=1,t) = 0 \quad \forall t, \forall p \tag{B.34}$$

is imposed at the bottom. At the top, an insulating boundary is specified for all modes as follows

$$\frac{\partial \hat{A}_p(z=0,t)}{\partial z} = 0 \quad \forall t, \forall p.$$
(B.35)

The initial field is set to  $B_p(z,t=0) = \cos\left[\sqrt{p}(1-z)\right]$  and in terms of vector potential

$$A_p(z,t=0) = \frac{1}{\sqrt{p}} \sin\left[\sqrt{p}(1-z)\right] \quad \forall p.$$
 (B.36)

Calculations are carried out using a time step size  $\Delta t = 1.10^{-4}$  and N = 200 points in the z direction. Figure B.1 shows the evolution of the vector potential at the center of the domain  $(x_c = 1, z_c = 0.5)$ , for the analytical (straight lines) and the numerical (symbols) solutions as a function of time (expressed in advection time units). The slopes for the different modes give the corresponding decay coefficient p.



FIGURE B.1: Decay of the vector potential in the center of the layer as a function of time (expressed in advection time units) for the first three diffusive modes computed for N = 200 and  $\Delta t = 1.10^{-4}$ .

TABLE B.1: Theoretical and numerical values of the decay coefficient of the vector potential for different modes q (for q = 2, as the numerical solution reaches the numerical accuracy after t = 0.6, see Fig. B.1, the slope is computed for the previous timesteps, from t = 0 to t = 0.6).

	$\sqrt{p}$ theo	$\sqrt{p}$ num
q = 0	1.57079633	$1.57078058 \pm 2.26 \times 10^{-15}$
q = 1	4.71238898	$4.71204979 \pm 9.69 \times 10^{-8}$
q=2	7.85398163	$7.93229119 \pm 1.76 \times 10^{-1}$

For the fundamental mode q = 0, the numerical solution gives a decay coefficient very close to the theoretical expectation ( $\sqrt{p} = 1.5708$ ). The accuracy of the numerical computation is also checked for modes corresponding to q = 1 and q = 2 (cf. table B.1). Again, the numerical solution is very close to the analytical solution, although not as close as for the q = 0 mode.

We define the average cumulative error e

$$e = \frac{1}{T} \int_0^T \left\{ \frac{1}{\mathcal{D}} \iint_{\mathcal{D}} [A_{\text{ref}}(z,t) - A_{\text{num}}(z,t)]^2 \mathrm{d}\mathcal{D} \right\}^{\frac{1}{2}} \mathrm{d}t,$$
(B.37)

with  $A_{\text{ref}}(z, t)$  and  $A_{\text{num}}(z, t)$  the analytical and numerical solutions, respectively;  $\mathcal{D}$  is the domain and T the integration time. This error is computed for different sets of  $\Delta t$  and  $\Delta z$ . Results are shown in Figure B.2.

As expected, with decreasing  $\Delta t$  and  $\Delta z$ , the error tends to zero, in an algebraic fashion. In either case, the slope is close to 2, as expected for the Crank-Nicholson scheme in time and the centered finite difference scheme we chose in space.

## b) Diffusive limit: analytical solution in the case of an insulating upper boundary condition

In order to assess the numerical treatment of the insulating boundary condition (see Equations B.20 and B.21), an analytical solution for the diffusion equation is determined for a domain  $\mathcal{D}$  of aspect ratio 2 : 1, with a insulating top boundary and a perfectly conducting bottom boundary. The Fourier expansion of the diffusion equation for the vector potential is

$$\left[\frac{\partial}{\partial t} - \left(\frac{\partial^2}{\partial z^2} - k_p^2\right)\right] \hat{A}_k(z,t) = 0, \tag{B.38}$$



FIGURE B.2: Convergence analysis for the fundamental mode q = 0. Left: error as a function of the gridsize  $\Delta z$ . Right: error as a function of the timestep  $\Delta t$ . The slope of the straight line is the order of the numerical scheme in the z-direction (left) and in time (right).

with  $R_m = 1$  and  $k_p = \pi k/L_x \ \forall k$ . This equation can be solved for each mode k.

As this is a simple diffusion problem, we apply a root finding procedure. To satisfy the boundary conditions, we have to solve

$$\tan\left(\mu L\right) = \frac{-\mu}{k_p},\tag{B.39}$$

with  $\mu = \sqrt{-(\lambda + k_p^2)}$  and  $\lambda + k_p^2 < 0$ , with  $\lambda < 0$  a decay time coefficient. Let  $\lambda_m$  denote the *m*-th eigenvalue for m = 1, 2, 3, ...

The solution of Eq.(B.38) is given by

$$\hat{A}_k(z,t) = \sum_m E_m e^{\lambda_m t} \left[ \frac{\mu_m}{k_p} \cos\left(z\mu_m\right) + \sin\left(z\mu_m\right) \right],\tag{B.40}$$

with  $\mu_m = \sqrt{-(\lambda_m + k_p^2)}$ . At t = 0, Eq. B.40 becomes

$$\hat{A}_k(z,t=0) = \sum_m E_m \left[\frac{\mu_m}{k_p}\cos\left(z\mu_m\right) + \sin\left(z\mu_m\right)\right] = \sum_m E_m G_m.$$

Multiplying with  $G_{m'}$  and integrating over z gives:

$$\int_0^1 \hat{A}_k(z,t=0)G_{m'} dz = \int_0^1 \sum_m E_m G_m G_{m'} dz.$$

The orthogonality of the eigenfunctions leads to  $G_m G_{m'} = 0$  if  $m \neq m'$  and therefore

$$\int_0^1 \hat{A}_k(z,t=0)G_m \mathrm{d}z = E_m \int_0^1 G_m^2 \mathrm{d}z,$$

which leads to

$$E_m = \frac{\int_0^1 \hat{A}_k(z, t=0)G_m \mathrm{d}z}{\int_0^1 G_m^2 \mathrm{d}z}$$



FIGURE B.3: Decay time for the 0 mode computed for a model with N = 200 and  $\Delta t = 1.10^{-3}$ .  $A(x_c, z_c, t)$  is the value of the vector potential at the center of the layer, normalised by its initial value  $A(x_c, z_c, t = 0)$ .

Let us define the initial field as

$$\hat{A}_k(z,t=0) = \frac{\mu_m}{k_p} \cos\left(z\mu_m\right) + \sin\left(z\mu_m\right)$$

and focus on the m = 1 case. Figure B.3 shows the evolution of the numerical and theoretical solutions at the center of the domain ( $x_c = 1, z_c = 0.5$ ), precisely for m = 1, taking a time step size  $\Delta t = 10^{-3}$  and 200 points in the z direction. Both exhibit a decrease of the amplitude, following the decay rate  $-\lambda_1$ . The numerical value of  $\lambda_1$ ,  $\lambda_{num} = -15.992412$  is close to the theoretical expectation  $\lambda_{\text{theo}} = -15.991856$ .

The components of the magnetic field are

$$B_x(x,z,t) = -\frac{\partial A(x,z,t)}{\partial z}, \tag{B.41}$$

$$B_z(x, z, t) = \frac{\partial A(x, z, t)}{\partial x}.$$
(B.42)

In the Fourier domain the latter expands as

$$\hat{B}_{zk}(z,t) = -ik_p \hat{A}_k(z,t).$$
 (B.43)

Using Eq.(B.40), this yields

$$\hat{B}_{zk}(z,t) = -ik_p \sum_m E_m e^{\lambda_m t} \left[ \frac{\mu_m}{k_p} \cos\left(z\mu_m\right) + \sin\left(z\mu_m\right) \right].$$
(B.44)

To assess the imposition of the insulating top boundary condition, an average cumulative error is computed using a diagnostic similar to Eq. (B.37), restricted to the upper boundary for the *z*-component of the field

$$e = \frac{1}{T} \int_0^T \left\{ \frac{1}{L_x} \int_{x=0}^{x=L_x} [B_{z,\text{ref}}(x, z=0, t) - B_{z,\text{num}}(x, z=0, t)]^2 \mathrm{d}x \right\}^{\frac{1}{2}} \mathrm{d}t,$$
(B.45)

As previously, Figure B.4 shows that the numerical scheme is as expected of order 2 in both space and time.



FIGURE B.4: Convergence analysis for  $B_z$  in z = 0. Left: error as a function of the gridsize  $\Delta z$ , right: error as a function of the time-step  $\Delta t$ . The  $\alpha$  value gives the computed slope for each case.

### c) Advective limit

In order to test the numerical approximation of the advection term in the induction equation (Eq. 4.2) and to check that the time evolution is well-controlled in our modelling, we study the evolution of the vector potential in a domain  $\mathcal{D}$  of aspect ratio 2 : 1, periodic along the *x*-direction, with perfectly conducting upper and lower boundaries. A high magnetic Reynolds number  $R_m = 10^5$  is chosen to mimic a diffusionless setting. The initial magnetic field is a Gaussian bell of the form

$$A(x, z, t = 0) = \exp\left[-\frac{(x - x_0)^2 + (z - z_0)^2}{2\sigma^2}\right],$$
(B.46)

and the imposed velocity field is

$$u_x = -\frac{\partial \Psi}{\partial z} = -U_0 \hat{x}$$
  

$$u_z = \frac{\partial \Psi}{\partial x} = 0,$$
(B.47)

which amounts to a translation in the x direction.

The initial state is represented in Fig. B.5. The magnetic field takes the form of a bell whose size is controlled by the amplitude of  $\sigma$  in the Gaussian function. In order to meet the boundary conditions, we require  $\sigma \ll L_z$ . Here we choose to impose  $\sigma = 0.1L_z$ , with  $x_0 = L$  and  $z_0 = 0.5L$ . Taking the periodicity of the problem along the *x*-direction into account, with the chosen velocity field, the bell is expected to get back to its initial position after one advection time  $\tau_{adv}$ . Fig. B.5 depicts an evolution in line with what is expected in this configuration. To assess the order of the scheme, we define the error as

$$e = \max(|A - A_{\rm ref}|) \tag{B.48}$$

with A = A(x, z, t = 1) and  $A_{\text{ref}} = A(x, z, t = 0)$ . The error is computed for different  $\Delta z$  and  $\Delta t$ . Results are given in Fig. B.6. As expected for the advection term, the scheme is of order 1 in time. For very small  $\Delta t$  the error increases as  $\Delta t$  decreases. The imposed  $R_m$  is finite (though very small) which means that, at a certain point, diffusion eventually occurs, preventing the error to decrease with smaller  $\Delta t$  as expected.



FIGURE B.5: Evolution of a Gaussian-shaped vector potential. The upper, middle and lower panels shows vector potential at the initial time, after one half of an advection time, and one advection time, respectively. The color scale represents the intensity of the magnetic field.



FIGURE B.6: Convergence analysis for the advective component of the induction equation. The error is represented as a function of the time-step  $\Delta t$ . Each straight line of slope  $\alpha$  is obtained for each  $\Delta z$  by least-squares fitting.

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