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Family, marriage markets and inequality : a matching approach

Simon Weber

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Doctorat en économie

Family, marriage markets and inequality: a matching approach

Simon WEBER

Thesis supervised by Pr. Alfred GALICHON

defended in Paris on September 14th 2017

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ABSTRACT

“I know of only highly impressionistic evidence on the effects of the sex ratio, or for that matter any other variable, on the division of output between mates. This division usually has not been assumed to be responsive to market forces, so that no effort has been put into collecting relevant evidence.”

Gary Becker, 1973.

This dissertation is entitled “Family, marriage markets and inequality: a matching approach”. It deals with couple formation on the marriage market, and adopts a perspective that focuses on inequality both at the between- and within-household levels. The starting point of this work is to recognize that the marriage market has a deep influence on these inequalities. Therefore, it is crucial to model it properly, and by doing so, gain insights on the broad question of inequalities in family economics. My thesis contains three distinct chapters on this topic.

Chapter 1 deals with the role of marital preferences on growing income inequalities between households. Indeed, “who marries with whom” is going to determine, in part, the distribution of income. To measure the contribution of marital preferences to inequality, I use (in collaboration with Edoardo Ciscato) and estimate a structural matching model with transferable utility (TU). Most of the literature on marriage patterns and inequality is non-structural and focuses on education as the sole dimension on which partners match. Therefore, it is not possible to estimate marital preferences and a fortiori disentangle marital preferences from structural changes in the population supplies (for example, the rise in the number of women with a college degree). To overcome these difficulties, I propose to use a multidimensional and continuous TU model. I use mating patterns in the United States from 1962 to 2015 and measure the impact of changes in marital preferences on between-household inequality. I consider several dimensions on which partners match (age, education, wage, race). I find that, after controlling for other observables, assortative mating on education has become stronger. Moreover, if mating patterns had not changed since 1971, the 2015 Gini coefficient between households would be lower by 6%.

Chapter 2 puts emphasis on resource sharing within couples. I adopt the collective view of the household decision process, in which the division of bargaining power shapes household decisions regarding private consumption, but also public consumption and investment decisions in the human capital of children. Collective models are inseparable from marriage markets. Indeed, the division of bargaining power within the household is endogenous to the determination of equilibrium on the marriage market, and therefore depends on the position of men and women on the market (e.g. their relative scarcity or how attractive the outside option of singlehood is). Thus, I propose to bring together these two literature into a unified framework. To do so, I could not rely on TU models: although they are very convenient in several aspects (link with linear programming), they suffer from several drawbacks (in collective models with TU, “power” does not matter in explaining public good consumption). Hence, the second chapter is dedicated to a new class of matching models with Imperfectly Transferable Utility (ITU). This work is joint with Alfred Galichon and Scott Kominers. We show that our model embeds both the fully- and non-transferable utility models, and is suitable to integrate collective models. We allow agents to have heterogeneous tastes and we show existence and uniqueness of equilibrium. We provide two algorithms to compute the equilibrium. We also provide guiding steps for estimation by maximum likelihood as well as an empirical illustration.

Chapter 3 explores further the integration of collective models into the ITU matching framework. Here, I make both technical and empirical contributions. I characterize classes of collective models that can be embedded in the ITU setting. The application of the Galichon-Kominers-Weber framework relies on an essential analytic device, labeled distance-to-frontier function, which serves as a basis for estimation. In many cases, however, (in fact, as soon as public goods or time constraints are introduced) the distance function cannot be computed in close form. I provide a technique to compute the distance function at a minimal cost, and provide simultaneously the gradient of the distance function with respect to any model parameter. I provide a full application of my results using collective model that features private consumption, leisure and a public good produced from time inputs.

Contents

Introduction	9
1. The scope of family economics	9
2. The marriage market	10
3. Marriage, intra-household bargaining and inequality	11
4. Contributions to the literature	14
Bibliography	17
Chapter 1. Marriage and Inequality: The role of marital preferences in growing income inequality	21
1. Introduction.	22
2. Previous Findings.	25
3. Theoretical framework.	31
4. Data.	36
5. Trends in Matching Patterns.	43
6. Counterfactual Analysis.	51
7. Conclusions and Perspectives.	59
Bibliography	61
Appendix	65
1.A. Neutrality of Optimal Matching	65
1.B. Improvements to the Estimation Procedure.	66
1.C. Robustness Checks.	66
1.D. Additional Figures.	66
Chapter 2. Intra-couple bargaining and matching: the ITU framework	75

1. Introduction	76
2. Prelude: From TU matching to ITU matching	79
3. Framework	82
4. Aggregate equilibrium: motivation and definition	91
5. Aggregate Equilibrium: Existence, Uniqueness, Computation	96
6. The ITU-logit model	100
7. Conclusion	106
Bibliography	109
Appendix	113
2.A. Proofs of the results in the main text	113
2.B. Other models of interest	123
2.C. Remarks on equilibrium vs optimality	125
2.D. Identification	126
2.E. Relating individual and aggregate equilibria	128
2.F. Illustrative example	129
Chapter 3. Collective Models and the Marriage Market	135
1. Introduction	136
2. Bargaining Sets	139
3. A computation method	145
4. Estimation	152
5. Illustration	155
6. Conclusion	164
Bibliography	165
Appendix	167
3.A. Log-likelihood estimation	167
3.B. Preliminary aggregate results	168
Conclusion	173

Introduction

1. The scope of family economics

Gary Becker introduced to economics a series of topics that traditionally belonged to other fields of social sciences. In several key contributions, he built the theoretical framework to analyze a wide range of social issues, including discrimination, criminality, or social interactions¹ to name a few examples. But most prominently, Becker applied this “economic way of looking at life”² to family. In his 1981 book, *A Treatise on the Family*, Becker discussed marriage patterns, fertility, human capital formation, intergenerational mobility and even polygamy.

These topics are increasingly important in the economic literature, because they relate to fundamental questions on earning dynamics, labour force participation, child development and skill formation, inequalities and ultimately to economic development and growth. For example, building on Becker’s insight that human capital is in part the product of investment within the family, a recent literature has explored the technology of skill formation. Childhood is no longer a one-step period, but a process with several critical moments for skill accumulation (Cunha et al., 2006; Heckman and Cunha, 2007). Both cognitive and non cognitive skills are shown to play a crucial part in the determination of schooling and earnings later in life, but also determine many other outcomes including health, criminal behavior, marriage and fertility (Heckman, Stixrud, and Urzua, 2006). Interestingly, these new approaches to family economics have shed light on new types of policy interventions (García et al., 2016) targeting the family with the aim of fostering human capital accumulation and development.

¹see , e.g., Becker (1971), Becker (1968) and Becker (1974)

²Becker (1993)

The fact that families play a key role in most societies suggest to look at the process of family formation in the first place, instead of taking it as given. To do so, economists have introduced a new object, the “marriage market”. Talking about marriage markets might be shocking for the non-initiated, but to put it in Becker’s word (Becker, 1991, p. 81), “the phrase marriage market is used metaphorically and signifies that the mating of human populations is highly systematic and structured”. Therefore, we may study how the marriage market is structured, and how this structure is influencing individual and household decisions even after the decision to marry has been taken.

2. The marriage market

Although marriage rates have been declining in most developed countries in the past few decades³ marriage remains a dominant institution. In the United States, in 2016, 61.4% of individuals aged 30-34 have already been married⁴.

The marriage market structures the interaction between two distinct populations, men on one side and women on the other side. Each individual have heterogeneous preferences over the set of potential partners on the other side of the market, and a subset of these men and women may decide to marry. This is broadly representative of how families are formed in modern societies⁵. In the matching literature terminology, this corresponds to a classical bipartite, bilateral one-to-one matching market. This framework has been largely studied by the so-called matching literature, whether transfers between partners are allowed or on the contrary completely prohibited. Interestingly, when utility is assumed to be perfectly transferable across partners, the study of the equilibrium assignment on such market is tightly connected to Optimal Transportation Theory (Villani, 2003), and therefore traces back to the work of the French mathematician Gaspard Monge. Later on, the field has

³In the United States, from 10.6 marriages per 1000 population in 1970 to 6.9 per 1000 in 2014. In France, for the same period, the numbers are 7.8 and 3.5, respectively.

⁴Census Data. In 1970, the number was 92.2%. Stevenson and Wolfers (2007) documents that in most developed countries, about 60% of adults are married, and when this proportion is lower, other forms of substitutable living arrangements (such as cohabitation) makes up for the difference

⁵In this introduction, I will use the words “marriage” and “couple formation” interchangeably. I ignore polygyny, as monogamous marriages are the norm in developed countries, see Gould, Moav, and Simhon (2008), or Henrich, Boyd, and Richerson (2012) for an anthropological explanation. Same-sex couples are ignored here. In 2010, the US census report that about 1% of couple households are same-sex. For an application with same-sex couples, see Ciscato, Galichon, and Goussé (2015).

largely benefited from the contributions of Kantorovich and the introduction of linear programming techniques, which ultimately were used in an economical context by Koopmans and Beckmann (1957) and Shapley and Shubik (1971). Subsequently, Becker (1973) used the same theoretical apparatus to study the marriage market, including marriage patterns and how the marriage surplus is allocated within couples. Under the assumption of non-transferable utility, the marriage market has also received much attention in game theory. In that case, each person ranks their potential partners by order of preference, and in the simplest version of this “marriage game”, the stable outcome can be found by using a “deferred-acceptance” algorithm (Gale and Shapley, 1962), which somehow mimic the traditional proposal-engagement-marriage sequence. Although the absence of transfers across partners may look restrictive, the approach has been revived recently in Pollak (2016).

Thus, marriage markets have been substantially covered from a theoretical point of view. In this thesis, I argue that one of the reason these contributions are important is because studying the marriage market can shed light on some important topics such as marriage patterns and intra-household decision-making. And *in fine*, it is the relationship between the marriage market and inequality that comes into question.

3. Marriage, intra-household bargaining and inequality

3.1. “Who marries whom?”: Rationale and Methods. In appearance, the question of “*who marries whom?*”, or *how* people marry, may seem less fundamental than understanding *why* people do marry. Indeed, there are even evolutionary justification for the dominance of marriage, as it promotes survival by helping women receiving resources for their children and men solving the uncertain paternity problem⁶. In economics, marriage provides substantial gains to the partners involved in the relationship, whether these gains come from specialization, the sharing of public goods or simply spending time together or raising children.

This doesn’t mean, however, that we should elude the question of “*who marries whom?*”. Going back to Becker’s quote, it is quite remarkable that “[...] the mating of human populations is highly systematic and structured” in modern societies. Consequently, we may learn

⁶see, e.g. Daly and Wilson (2000), and Buss (1989) for some empirical support

important lessons from studying the matching process that takes place on the marriage market. And in fact, this has been central to family research in demography, sociology and economics. A key motivation is that examining intimate decisions such as whom to marry may be informative about broader societal issues. As an example, consider the matter of Black-White equality in the United States: while segregation laws and interracial marriage bans have been repealed and earnings differential are closing, there are still important barriers to interracial marriage (Kalmijn, 1998). Interestingly, such barriers are not as strong for other ethnic group (between White and Asians for example, see Fryer (2007)).

The benefits of looking at marriage patterns are better understood when considering education. The rationale for studying assortativeness in education is that education plays a key role in determining a variety of outcomes and decisions related to health, income, happiness, fertility and human capital investment in children. Therefore, if the mating of likes (in terms of education) is highly systematic, we may expect strong between-couple inequalities along these outcomes, as well as between-individual inequalities if education levels complement each other in producing positive outcomes. In addition, Fernandez et al. (2005) have explored the idea that there may be a reinforcing relationship between assortative mating and inequality.

Positive assortative mating in education is a well established fact (see Browning, Chiappori, and Weiss (2014) for a review). Therefore, most of the literature has moved on to measure trends in assortativeness over the last few decades. While this literature has not reached a consensus yet (Gihleb and Lang, 2016), it suggests that assortativeness has increased over time (see Mare (1991), Schwartz and Mare (2005), Chiappori, Salanié, and Weiss (2016), and Ciscato and Weber (2017)). However, it is difficult to determine from this literature whether observed assortativeness reflects preferences or is shaped by the environment (e.g. individuals from different education groups may meet partners from other groups at different rates, see Jaffe and Weber (2017)), although there is some supporting evidence (Hitsch, Hortasu, and Ariely, 2010; Bruze, 2011) that tastes play a key role.

Since earnings and education are key determinants of overall inequality, it is not surprising that economists have put more emphasis on sorting on both of these dimensions.

As a matter of fact, in 2011, the OECD (2011) reported that assortativeness contributed to 11% of the rise in household earnings inequality on average (over OECD countries) between the mid 1980s and mid 2000s, on par with the contribution of changes in household structure. This effect is not negligible, and in addition, hides the fact that there are many other dimensions to inequality that may be in part determined by marriage patterns.

3.2. “Who wears the trousers?” Decision making in the family. Let’s suppose a moment that a couple has been formed on the marriage market. What happens next? How do we model the decision process that takes place within this new entity? Is there some notion of balance of power that determines the kind of decisions that are taken? Are these decisions Pareto-efficient?

The traditional “unitary” model of the household ignores these questions altogether. In this context, family is a single decision unit. Decisions follow from the maximization of a utility function that takes into account individuals preferences using weights that are fixed and therefore do not respond to prices, income, distribution factors⁷ or any change in the structure of the marriage market. This modelling assumption is not entirely satisfying as it is rather limited and has found itself at odds with empirical findings (Fortin and Lacroix, 1997; Lundberg, Pollak, and Wales, 1997; Vermeulen, 2005).

The unitary view of the household has been progressively replaced by cooperative and noncooperative theories. We shall leave the noncooperative models aside (Lundberg and Pollak, 1993; Konrad and Lommerud, 1995), although some evidence suggests that couples may not always behave in a cooperative and efficient way. In contrast, the collective models (Chiappori, 1988; Bourguignon and Chiappori, 1992) rely on the assumption that households take Pareto-efficient decisions. The approach, which is not limited to the case with two decision-makers (Dauphin et al., 2011), introduces the notion of bargaining power. Consequently, collective models are important in at least two aspects. First, they convey the idea that power matters: the distribution of bargaining power determines labor market participation, fertility, the structure of household expenditures as well as investment decisions (such as human capital transmission to children). Therefore, it is crucial to understand

⁷Distribution factors are variables that would affect the decision process, but not preferences or budget constraints. Examples include sex ratios, wage ratios, etc.

how policy interventions may shift the balance of power and ultimately affect those decisions. Second, collective models introduce a new channel of between-individual inequality. It may well be the case that the sharing of resources within couples attenuate, or on the contrary exacerbate, the overall level of inequality (Lise and Seitz, 2011).

As we shall see, there is a natural connection between collective models and matching models of the marriage market. It is in the very nature of collective models to allow transfers between partners; whether these transfers are more favourable to men or women reflects the balance of bargaining power. Such transfers may not take the form of money (or any other numeraire good) but of adjustments in the structure of private and public consumption. In any case, the intuition, that can be found in Becker (1973), is that the intra-household allocation of resources responds to changes in the marriage market (Chiappori, Fortin, and Lacroix, 2002).

4. Contributions to the literature

In this work, I deal with the question of inequality which, as shown above, pertains to both the matching literature (marriage and between-household inequalities) and collective models (intra-household allocation of resources). I believe that this thesis' main contribution is to develop and bring tools from the matching literature to family economics in order to gain new insights on these important questions. In particular, I provide a unified empirical framework for matching and collective models.

The thesis contains three chapters.

Chapter 1 deals with marital preferences and between-household inequalities. This new channel is becoming an increasingly popular topic among economists, and several research suggest that the evolution of marriage patterns contributed positively to the overall level of between-household income inequality. However, most of this literature does not make use of structural methods that allow to construct proper counterfactual experiments. With Edoardo Ciscato, I propose to estimate marital preferences using a multidimensional and continuous matching model with transferable utility. We apply our setting to US data from 1962 to 2015. We show that assortativeness in education has risen in this period,

while assortativeness in age has dramatically declined. We also show that unobservables play a much larger role today than they used to fifty years ago. We use our model to conduct a counterfactual experiment: what would be the marriage patterns today if marital preferences had not changed since 1971? And how would this affect the distribution of income across households? This is the first paper to answer this question with the use of a matching framework. We show that, had marital preferences not changed since 1971, inequality, as measured by the Gini coefficient, would decline by 6%. Overall, we estimate that 20% of the rise in inequality since 1971 is due to change in marital preferences.

Chapter 2 begins to explore the connection between collective models and the marriage market. However, the literature on matching models is mostly split in two extreme cases: on one hand, non transferable utility (NTU) models (Gale and Shapley, 1962) prohibit entirely transfers between partners (and therefore do not fit in with collective models), while on the other hand, transferable utility (TU) models (Shapley and Shubik, 1971) assume that utility is perfectly transferable. The latter is a reasonable modelling assumption, but it has its shortcomings: in a collective model context, assuming TU puts restriction on the class of utility functions that can be used to represent preferences, and has implications that are at odds with the empirical literature. Thus, it is necessary to construct a matching model with imperfectly transferable utility (ITU) that can be easily brought to the data. With Alfred Galichon and Scott Kominers, we introduce an empirical framework for models of matching with imperfectly transferable utility and unobserved heterogeneity in tastes. The setting is general enough to encompass models with fully- and non-transferable utility as well as collective models. From a theoretical point of view, we contribute to the literature by showing existence and uniqueness of the equilibrium under minimal assumptions and providing algorithms to compute the equilibrium. On the empirical side, the model can easily be brought to the data, and the model of interest estimated by maximum likelihood. A simple illustration is provided.

In **Chapter 3**, I choose collective models as my starting point. I point out that unlike the classical unitary model, they emphasize that bargaining, and therefore bargaining power, play an important role in family decisions. Furthermore, it is implicit in this literature that bargaining power and marriage markets are somehow related. Thus, I argue that there

could be significant benefits from bringing together collective models and matching models of the marriage market into a unified framework. To do so, I make use of the models and techniques developed in the second chapter, and explore further the connection between the matching and the collective view of the household. I make theoretical, computational and empirical contributions. In particular, I characterize the class of collective models that can be embedded in a matching framework with imperfectly transferable utility (ITU) as developed in Chapter 2. I also point out that the estimation of these models depends crucially on the computation of the so-called *distance-to-frontier functions*. However, aside from some simple cases, in most models (e.g. with public goods and time use), computations are uneasy and must rely on numerical methods. I provide a general method that works for any model and also return the derivative of the *distance-to-frontier functions* at no additional computational cost. I illustrate my results with a fully-fledged application on PSID data, featuring private consumption, leisure and a public good produced from time inputs.

Bibliography

- [1] Gary Becker. *The Economics of Discrimination*. Tech. rep. University of Chicago Press, 1971.
- [2] Gary S Becker. “A Theory of Marriage: Part I”. In: *Journal of Political Economy* 81.4 (1973), pp. 813–846. ISSN: 0226740854.
- [3] Gary S Becker. “A Theory of Social Interactions”. In: *Journal of Political Economy* 82.6 (1974), pp. 1063–93.
- [4] Gary S. Becker. “Crime and Punishment: An Economic Approach”. In: *Journal of Political Economy* 76.2 (1968), pp. 169–217.
- [5] Gary S Becker. “Nobel Lecture: The Economic Way of Looking at Behavior”. In: *Journal of Political Economy* 101.3 (1993), pp. 385–409.
- [6] Francois Bourguignon and Pierre-André Chiappori. “Collective models of household behavior : An introduction”. In: *European Economic Review* (1992), pp. 355–364.
- [7] Martin Browning, Pierre-André Chiappori, and Y. Weiss. *Economics of the Family*. Ed. by Cambridge Surveys of Economic Literature. Cambridge Surveys of Economic Literature, 2014, pp. –.
- [8] Gustaf Bruze. “Marriage Choices of Movie Stars: Does Spouse’s Education Matter?”. In: *Journal of Human Capital* 5.1 (Mar. 2011), pp. 1–28. ISSN: 1932-8575.
- [9] David M Buss. “Sex differences in human mate preferences: Evolutionary hypotheses tested in 37 cultures”. In: *Behavioral and brain sciences* 12.01 (1989), pp. 1–14.
- [10] Pierre-André Pierre-André Chiappori, Bernard Fortin, and Guy Lacroix. “Marriage Market, Divorce Legislation, and Household Labor Supply”. In: *Journal of Political Economy* 110.1 (2002), pp. 37–72. ISSN: 00223808.
- [11] Pierre-André Chiappori. “Rational household labor supply”. In: *Econometrica: Journal of the Econometric Society* (1988), pp. 63–90.

- [12] Pierre-André Chiappori, Bernard Salanié, and Yoram Weiss. “Partner Choice and the Marital College Premium: Analyzing Marital Patterns Over Several Decades”. In: *Working Paper* (2016), pp. –.
- [13] Edoardo Ciscato, Alfred Galichon, and Marion Goussé. “Like Attract Like? A Structural Comparison of Homogamy Across Same-Sex and Different-Sex Households”. In: *Working Paper* (2015), pp. –.
- [14] Edoardo Ciscato and Simon Weber. “The Role of Evolving Marital Preferences in Growing Income Inequality”. In: *Working Paper* (2017).
- [15] Flavio Cunha et al. “Interpreting the evidence on life cycle skill formation”. In: *Handbook of the Economics of Education* 1 (2006), pp. 697–812.
- [16] Martin Daly and Margo I Wilson. “The evolutionary psychology of marriage and divorce”. In: *The ties that bind: Perspectives on marriage and cohabitation* (2000), pp. 91–110.
- [17] Anyck Dauphin et al. “Are Children Decision Makers within the Household?” In: *Economic Journal* 121.553 (2011), pp. 871–903.
- [18] R. Fernandez et al. “Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality”. In: *Quarterly Journal of Economics* 120.1 (Feb. 2005), pp. 273–344.
- [19] Bernard Fortin and Guy Lacroix. “A Test of the Unitary and Collective Models of Household Labour Supply”. In: *Economic Journal* 107.443 (1997), pp. 933–55.
- [20] Roland G Fryer. “Guess Who’s Been Coming to Dinner? Trends in Interracial Marriage over the 20th Century”. In: *Journal of Economic Perspectives* 21.2 (May 2007), pp. 71–90.
- [21] D Gale and L S Shapley. “College Admissions and the Stability of Marriage”. In: *The American Mathematical Monthly* 69.1 (Jan. 1962), pp. 9–15.
- [22] Jorge Luis García et al. *The life-cycle benefits of an influential early childhood program*. Tech. rep. National Bureau of Economic Research, 2016.
- [23] Rania Gihleb and Kevin Lang. *Educational Homogamy and Assortative Mating Have Not Increased*. Working Paper 22927. National Bureau of Economic Research, Dec. 2016.

- [24] Eric D Gould, Omer Moav, and Avi Simhon. “The mystery of monogamy”. In: *The American Economic Review* 98.1 (2008), pp. 333–357.
- [25] James Heckman and Flavio Cunha. “The Technology of Skill Formation”. In: *American Economic Review* 97.2 (2007), pp. 31–47.
- [26] James J. J. Heckman, Jora Stixrud, and Sergio Urzua. “The Effects of Cognitive and Noncognitive Abilities on Labor Market Outcomes and Social Behavior”. In: *Journal of Labor Economics* 24.3 (2006), pp. 411–482. ISSN: 0734306X.
- [27] Joseph Henrich, Robert Boyd, and Peter J Richerson. “The puzzle of monogamous marriage”. In: *Phil. Trans. R. Soc. B* 367.1589 (2012), pp. 657–669.
- [28] Gunter J. Hitsch, Ali Hortasu, and Dan Ariely. “Matching and Sorting in Online Dating”. In: *American Economic Review* 100.1 (2010), pp. 130–63.
- [29] Sonia Jaffe and Simon Weber. “The Effect of Meeting Rates on Matching Outcomes”. In: *Working Paper* (2017).
- [30] M Kalmijn. “Intermarriage and homogamy: causes, patterns, trends.” In: *Annual review of sociology* 24 (Jan. 1998), pp. 395–421.
- [31] Kai A Konrad and Kjell Erik Lommerud. “Family Policy with Non-cooperative Families”. In: *Scandinavian Journal of Economics* 97.4 (1995), pp. 581–601.
- [32] T.C. Koopmans and M.J. Beckmann. “Assignment Problems and the Location of Economic Activities”. In: *Econometrica* 25.1 (1957), pp. 53–76.
- [33] Jeremy Lise and Shannon Seitz. “Consumption Inequality and Intra-household Allocations”. In: *Review of Economic Studies* 78.1 (2011), pp. 328–355.
- [34] Shelly Lundberg and Robert A Pollak. “Separate Spheres Bargaining and the Marriage Market”. In: *Journal of Political Economy* 101.6 (1993), pp. 988–1010.
- [35] Shelly J. Lundberg, Robert A. Pollak, and Terence J. Wales. “Do Husbands and Wives Pool Their Resources? Evidence from the United Kingdom Child Benefit”. In: *Journal of Human Resources* 32.3 (1997), pp. 463–480.
- [36] Robert D Mare. “Five Decades of Educational Assortative Mating”. In: *American Sociological Review* 56.1 (1991), pp. 15–32.
- [37] OECD. “Divided We Stand”. In: *Report* (2011).
- [38] Pollak. “Marriage Market Equilibrium”. In: *Working Paper* (2016).

- [39] Christine R Schwartz and Robert D Mare. “Trends in Educational Assortative Marriage from 1940 to 2003”. In: *Demography* 42.4 (Nov. 2005), pp. 621–646.
- [40] Lloyd S Shapley and Martin Shubik. “The Assignment Game I: The Core”. In: *International Journal of Game Theory* 1.1 (1971), pp. 111–130.
- [41] Betsey Stevenson and Justin Wolfers. “Marriage and Divorce: Changes and their Driving Forces”. In: *Journal of Economic Perspectives* 21.2 (2007), pp. 27–52. ISSN: 08953309.
- [42] Frederic Vermeulen. “And the winner is... An empirical evaluation of unitary and collective labour supply models”. In: *Empirical Economics* 30.3 (2005), pp. 711–734.
- [43] C Villani. “Topics in Optimal Transportation, vol. 58 of Graduate Studies in Mathematics series”. In: *American Mathematical Society* (2003).

Chapter 1

Marriage and Inequality: The role of marital preferences in growing income inequality

Abstract

In this paper, we describe mating patterns in the United States from 1962 to 2015 and measure the impact of changes in marital preferences on between-household income inequality. We rely on the recent literature on the econometrics of matching models to estimate complementarity parameters of the household production function. Our structural approach allows to measure sorting on multiple dimensions and to effectively disentangle changes in marital preferences and in demographics, addressing concerns that affect results from existing literature. We answer the following questions: *has assortativeness increased over time? Along which dimensions? To which extent the shifts in marital preferences can explain inequality trends?* We find that, after controlling for other observables, assortative mating on education has become stronger. Moreover, if mating patterns had not changed since 1971, the 2015 Gini coefficient between households would be lower by 6%. We conclude that about 20% of the increase in between-household inequality is due to changes in marital preferences.¹

¹This chapter is based on “The Role of Evolving Marital Preferences in Growing Income Inequality”, written by Edoardo Ciscato and Simon Weber. I am indebted to Edoardo for letting me using this paper as Chapter 1.

1. Introduction.

The study of mating patterns, and especially assortativeness, traces back to the seminal work of Becker (1973), Becker (1974), and Becker (1991). Becker’s earliest model of a competitive marriage market aims to rationalize both household specialization and the homogamy observed in the data with respect to several non-labor market traits (e.g. education, ethnicity, religion). Becker points at the structure of the household production function to explain marriage patterns: complementarity between inputs leads to optimal positive assortative mating, whereas substitutability to negative assortative mating.

In light of such observations on Becker’s work, studying marriage patterns is primarily insightful because it reveals much about intra-household dynamics. Differences in mating dynamics over time and space may be the result of transformations in the institution of the family, labor market conditions, available household-production technology, gender roles, etc. For instance, one could wonder whether Becker’s observation that we should expect a negative association between spouses’ wage rates due to household specialization still applies to modern families despite the improvements in home technology and closing gender wage gap². Changes in the cultural and legal framework also matter for the evolution of marital preferences, due to their influence on marriage flows and on the allocation of resources across and within the couple.

In recent years, marital sorting has become the object of increasing attention because of its relationship with growing inequalities between households. Researchers have focused on the relationship between marriage patterns, between-household income inequality and long-run economic outcomes (e.g. Burtless, 1999; Fernández, Guner, and Knowles, 2005; Greenwood et al., 2014). The compelling research question is whether stronger assortativeness with respect to some crucial dimensions - notably, education - is associated with higher inequality.

The aim of this paper is to build a connection between changes in the structure of marital gains and the increasing income inequality observed in the United States. We address the

²The survey of Stevenson and Wolfers (2007) keeps track of the changes that the institution of the family has gone through in recent decades and presents several significant research questions that need to be answered.

following questions: has assortative mating increased over time? And, if yes, along which dimensions? What is the impact of shifts in marital preferences on household income inequality? The framework we adopt follows Choo and Siow (2006) and Galichon and Salanié (2015)’s observation that joint marital surplus can be identified with data on matches in a static, competitive matching framework. Following this observation, we employ the recent estimation technique proposed by Dupuy and Galichon (2014) and estimate the degree of complementarity and substitutability between the spouses’ traits. Such estimates stand as our measures of the strength of marital sorting. This structural approach allows us to contribute to the literature on sorting and inequality by overcoming some limitations affecting studies based on standard measures of assortativeness, such as correlation coefficients, homogamy rates, frequency tables, and so on. Disentangling changes in marital preferences and demographics is crucial because of important changes in the marginal distributions of people’s traits in the United States during the last decades (e.g., ageing of the population, overall increase in schooling attainment, closing of the gender wage gap and reversal of the gender gap in higher education). In addition, our analysis is not limited to educational assortativeness: the multidimensional matching model of Dupuy and Galichon (2014) provides tools to study complementarity on education, as well as interactions between other socio-economic traits. Following this new approach, we rediscuss the findings of several key papers in the marriage literature, such as Fernández, Guner, and Knowles (2005), Schwartz and Mare (2005) and the recent Greenwood et al. (2014), Eika, Mogstad, and Zafar (2014) and Greenwood et al. (2016).

As anticipated, the theoretical framework employed in this paper is drawn from the work by Dupuy and Galichon (2014), which is grounded on Transferable Utility models and logit formalism, and extends the seminal matching model by Choo and Siow (2006) to the multidimensional and continuous case. Agents are fully informed about potential partners’ characteristics, but the econometrician only observes traits $x \in X$ and $y \in Y$, respectively for men and women, where X and Y are continuous and multidimensional. The empirical strategy relies on a bilinear parametrization for the systematic marriage surplus function, i.e. $\Phi(x, y) = x' Ay$. It follows that we can measure the degree of complementarity or substitutability by estimating the *marital preference parameters*, i.e. the elements of the

affinity matrix A , since $\partial\Phi/\partial x_j\partial y_k = A_{jk}$. These will be our measures of assortativeness. In addition, after estimating A , we recover the optimal probability distribution of matches $\pi^A(x, y)$, which is in other words the joint frequency table of partners' types at equilibrium. The latter depends both on the structure of preferences given by A and the marginal distributions of observable types $f(x)$ and $g(y)$: operating on the parameters A , we can compute the predicted distribution of couples' traits under counterfactual preferences. For instance, we can artificially increase the value of one parameter of A , say the strength of assortative mating on education, and check how the distribution of partners' types $\pi^{A'}(x, y)$ changes at the new (counterfactual) marriage market equilibrium.

In practice, we estimate marital preference parameters for United States over the period 1962-2015 with Current Population Survey data (Flood et al., 2015) to keep track of sorting dynamics through the analysis of preferences. We consider the following observable variables: age, education, hourly wage, hours worked and ethnic background. We subsequently use the marriage patterns predicted by the model - the optimal matching function $\pi^A(x, y)$ - to construct counterfactual household income distributions. To do so, we substitute the actual preferences measured for a given wave with counterfactual preferences measured for a different wave. This means that we provide a prediction of how people would have sorted into married couples in a given year if their marital preferences had been equal to those of another cohort (e.g. to their parents' or grandparents'). In this way, we study the contribution of changes in marital preferences to the observed marriage patterns and to the evolution of inequality in several illustrative examples.

To the best of our knowledge, this is the first attempt to analyze marital preferences in the United States by means of structural estimation techniques³ in a multidimensional matching framework. We hereby provide a complete analysis of assortativeness along multiple observable socio-economic traits, track changes in sorting patterns over time and assess to which extent they can explain between-household income inequality rise in the last decades.

³Two closely related works are those by Chiappori, Salanié, and Weiss (2015) and Greenwood et al. (2016). The results of the former on educational assortativeness are discussed in Section 5. Greenwood et al. (2016) embed an educational choice and a household model in a matching framework and estimates the household production function parameters. It is thus more similar in spirit to ours, although the theoretical framework is much different and more explicit about household behaviour. We also discuss their results in Section 5.

The paper is organized as follows. Section 2 provides a brief literature review, while Section 3 introduces the theoretical framework. In Section 4, we describe CPS data and our sample selection criteria. Hence, we present and discuss our results: in Section 5, the trends of marital preferences, while in Section 6 the counterfactual analysis of inequality. Section 7 concludes.

2. Previous Findings.

2.1. Evolution of Mating Patterns. A crucial question that the literature has tried to answer in different ways is whether assortativeness has increased over time. The demographic and sociological literature often makes use of log-linear models to explain mating patterns and measure assortativeness. Log-linear models for contingency tables help to “specify how the size of a cell count depends on the levels of the categorical variables for that cell” (Agresti, 2013, Chapter 9). Several papers relying on this methodology focus on assortativeness on education: the contingency table of size $I \times I$ tells the frequency of couples by partners’ education ij , with $i, j \in \{1, \dots, I\}$ being the individual schooling level. If matching were random, the following regression would exhibit a good fit

$$\log \mu_{ij} = \lambda + \lambda_i^M + \lambda_j^W$$

where μ_{ij} is the frequency of a couple with education ij , λ_i^M is the vector of men’s educational level effects and λ_j^W is the vector of women’s educational level effects. Under random matching marginal distributions are sufficient to explain the entries of the contingency tables. Nonetheless, if matching is not random, then one needs to include other regressors to explain the couples’ joint distribution. “Homogamy models” contain an additional regressor measuring the impact of educational homogamy on the log-joint frequency $\log \mu_{ij}$ (e.g. Johnson, 1980; Kalmijn, 1991b; Schwartz and Mare, 2005). “Crossing models”, instead, contain additional regressors measuring the impact of crossing an educational barrier (e.g. a college-graduate marrying a dropout, see Mare, 1991; Smits, Ultee, and Lammers, 1998; Schwartz and Mare, 2005). Log-linear models can be rewritten as multinomial choice models (see Agresti, 2013; Schwartz and Graf, 2009), which in turn are surprisingly close in spirit

to the matching model class of Choo and Siow (2006). In the equivalent multinomial logit model, the categorical response variable would be the wife’s (or the husband’s) education to represent the choice of the husband (or the wife’s) conditional on his (her) schooling level. However, a basic choice model of this kind does not take into account that men and women actually seek a partner in a competitive environment: the choice of one agent affects the pool of partners available for other agents. As a consequence, it is not possible to interpret the coefficients as the “true” preference parameters. In the structural framework proposed by Choo and Siow (2006) and Galichon and Salanié (2015), it is instead possible to estimate the parameters of the model so that the matching market is indeed at equilibrium. In these equilibrium models, every agent’s choice is constrained by the choices of other “competitors” and the market must clear, i.e. the sum of singles and married must be equal to the total number of individuals by type and sex.

Several studies apply log-linear models or closely related ones to study changes in educational assortativeness in marriage patterns in the United States. Most agree that educational assortative mating strengthened in the second part of 20th century (Mare, 1991; Kalmijn, 1991a; Kalmijn, 1991b; Qian and Preston, 1993) and the first decade of the 21st (Schwartz and Mare, 2005), although some other studies argue that educational homogamy stayed constant or declined: for instance, Fu and Heaton (2008) observe a decline between 1980 and 2000, while Liu and Lu (2006) maintain that the intensity of educational homogamy increased from 1960 to 1980 but then started decreasing. Interestingly, most papers also agree that one of the strongest trend is the increase in the frequency of marriages between highly educated individuals. Several papers use log-linear models to explore other matching dimensions, sometimes in multidimensional frameworks, although the number of variables stay low (2 or 3 typically) because of methodological limitations). Johnson (1980) and Kalmijn (1991a) analyze religion, Schoen and Wooldredge (1989) and Fu and Heaton (2008) ethnicity, Qian and Preston (1993) age, Kalmijn (1991b) and Blackwell (1998) parents’ education, Stevens and Schoen (1988) language spoken. Some empirical findings on assortativeness in the United States are particularly interesting since they can be compared with ours. Qian and Preston (1993) find that homogamy with respect to age increased (from

1972 to 1987); Fu and Heaton (2008) find that racial homogamy decreased (from 1980 to 2000).

In the economic literature, some analyses of mating patterns rely on simple descriptive statistics: for instance, Fryer (2007) uses the probabilities of crossing racial barriers to describe the patterns of racial intermarriage in the United States and explore the possible drivers behind the trends. Other researchers assess the strength of educational assortativeness through the comparison with counterfactual distributions. The simplest indicators of this kind are “homogamy rates” which are the ratios between the actual frequency of a couples’ joint education and the counterfactual frequency computed under random matching. Contingency tables to compare actual and counterfactual joint distributions are similar (if not identical) to homogamy rates (e.g. Greenwood et al., 2014). Another possibility is to compare the actual distribution to the counterfactual under perfect positive assortative mating (e.g. Liu and Lu, 2006). While generally insightful, homogamy rates and similar measures are not suitable for comparisons across different populations and even across different categories within the same population. The *size* of the homogamy rate is hardly comparable when the marginals become smaller. Hence, it is hard to set the comparison between homogamy for PhD graduates, who represent a small share of the population, and high school diplomas, who represent a wide share. In consequence, researchers opt for aggregate measures of assortativeness that take into account the different size that each category has in the population (e.g. Greenwood et al., 2014; Eika, Mogstad, and Zafar, 2014). Using such measures based on homogamy rates, Eika, Mogstad, and Zafar (2014) conclude that marital sorting in the United States on education has slightly increased over the period 1980-2007. The findings of Greenwood et al. (2014) are similar: relying on several measures, some of which based on homogamy rates, they find that assortativeness on education has increased in the period 1960-2005.

2.2. Assortativeness and Inequality. Another crucial question is whether changes in mating patterns can partly explain the trend of income inequality between households. Many authors are concerned with the possibility that more assortativeness on socio-economic characteristics - particularly on education - can lead to higher household

income inequality. Since education is a primary dimension of assortativeness, and since highly educated individuals typically have higher income, more educational homogamy implies that high-income individuals will marry with each other more and more frequently. Nevertheless, it is not straightforward to disentangle the effect of changes in marital preferences from the shifts in the marginal distributions. This is particularly relevant because of the closing of the educational gap between men and women in the last decades and women's increased participation to the labor force.

The landmark contributions by Fernández and Rogerson (2001) and Fernández, Guner, and Knowles (2005) make an attempt to model the trends of household inequality in order to shed some light on the role played by sorting, fertility and children's education. Fernández and Rogerson (2001) set a model in which individuals are either skilled or unskilled and marry more or less frequently with partners of the same educational level according to an exogenous parameter accounting for the degree of homogamy on the marriage market. Since the children of highly educated families will be more likely to go to college, mating patterns are crucial in order to explain the steady state level of inequality. Fernández, Guner, and Knowles (2005) introduce a simple two-round matching model in order to endogenize the strength of sorting on education. They find that, at steady state, a higher degree of sorting - measured as the correlation between partners' income - is associated with higher income inequality⁴. Both papers argue that educational assortativeness exacerbates inequality in the long run, in disagreement with Kremer (1997), who states that sorting has a negligible impact on steady state inequality. Although the structural approach of these models is extremely insightful to understand through which channels mating patterns may influence inequality in the long run, we believe that their conclusions might - to some extent - depend on their specific measure of educational assortativeness. In particular, Fernández, Guner, and Knowles (2005) show that the Pearson correlation coefficient between partners' education correctly measures the degree of assortativeness. However, this conclusion can be reached only under the restrictive assumptions necessary for their two-round matching model. Indeed, in most alternative matching models, a change in the correlation may well be

⁴Both Fernández and Rogerson (2001) and Fernández, Guner, and Knowles (2005) use the skill premium as a measure of inequality.

due to a change in *marital preferences* as well as to a *shift in the marginals*. Hence, since a higher correlation rate does not necessarily imply more assortativeness, we propose to relate alternative measures of assortativeness to income inequality in order to check whether their conclusions are robust.

As previously mentioned, Greenwood et al. (2016) set up a model of educational choice, marriage and the household, and estimate its steady-state. With respect to the papers mentioned above, the focus is now more on household technology and changes in the wage distribution rather than intergenerational transmission. After comparing the estimated parameters and equilibrium outcomes for the US economy between 1960 and 2005, the authors run a number of counterfactual experiments that help to understand what forces contributed to raise inequality. In particular, they assess that the change in wage structure alone explains 39% of the rise of inequality. They subsequently stress that changes in marriage patterns account for 18.6% of the increase, which grow to 35.6% when allowing households to adjust their labor supply. In the present paper, we also disentangle changes in the wage distribution from transformations to the structure of marital gains, while we also control for changes in the marginal distribution of other observables (e.g. race and ethnicity). On the other hand, Greenwood et al. (2016) make explicit assumptions on household behavior and their model insightfully predicts how households adjust their labor supply. In this way, they separately assess the effects of changes in home technology and in taste for educational homogamy on income inequality. We compare our empirical findings to theirs in Section 5 and find encouraging similarities despite the differences between the two approaches.

Beside the above-mentioned papers, most research focuses on the empirics in the hope of assessing the impact of changes in marital preferences on income inequality in the United States correctly. Measuring the strength of educational assortativeness is not straightforward and several approaches have been tried out. The work by Burtless (1999) is an early example of counterfactual analysis of inequality. In order to assess the degree of inequality that we would observe in 1996 if matching patterns did not change since 1979, Burtless shuffles the observed married couples in 1996 and reassigns spouses as follows: if the man whose income had rank r married a woman with rank s in 1979, the man with rank r in

1996 is assigned to the woman with rank s from the same year. Cancian and Reed (1998) and Western, Bloome, and Percheski (2008) suggest using decomposition methods on the changes in the variance of household income. The methodology consists in dividing the household population into groups according to certain characteristics (e.g. age, education, children) and then studying the trends of income variance within and between groups.

Schwartz (2010) focuses on marital preferences and is thus more closely related to our analysis. She uses the log-linear models explained in Section 2.1 to build counterfactual distributions of partners' income⁵. The author concludes that inequality would have been lower without the shifts in income assortativeness⁶.

The works of Greenwood et al. (2014) and Eika, Mogstad, and Zafar (2014) also aim to assess the impact of changes in educational assortativeness on inequality. Using contingency tables, Greenwood et al. (2014) show that, under random matching, the counterfactual Gini coefficient in 2005 for United States is much lower than the actual (about 18% less). In addition, using standardized contingency tables with several variables (e.g. children, participation in the labor force), they assess that, had sorting patterns been constant since 1960, the 2005 Gini coefficient would have been much lower (always about 18% less). Eika, Mogstad, and Zafar (2014) conduct a similar analysis to study the trends of household income inequality⁷ in the United States between 1980 and 2007. They employ a methodology which consists in building counterfactuals by combining the partners' joint distribution of schooling attainments from a given year to the conditional distribution of income given the educational level from another year. They conclude that, had returns to schooling not changed since 1980, 2007 household income inequality would have been much lower (about 23% less). In addition, the authors also remark that, without the overall increase in schooling attainments at individual level, 2007 inequality would be even higher. Finally,

⁵The methodology consists in finding a log-linear model with good fit to explain a contingency table with the distribution of income by percentile (plus one category containing zero-income observations), one can compute predicted frequencies after removing certain regressors to reproduce counterfactual situations.

⁶Schwartz (2010) uses the ratios between the median income of the top 20% households (high class) over the median income of the middle 60% (medium class) or the median income of the top 20% (low class) as measures of inequality.

⁷Note that an important difference is that Greenwood et al. (2014) include one-person households, i.e. singles and divorced, when computing the Gini coefficient. Similarly to us, Eika, Mogstad, and Zafar (2014), instead, exclude them from the sample. Hence, the conclusions must be interpreted with caution.

they assess that, had 1980 marital preferences been the same as in 2007, we would have not observed any relevant difference in household income inequality: in this regard, their findings differ from those of Greenwood et al. (2014), although the time lapse considered is different.

3. Theoretical framework.

Dupuy and Galichon (2014, hereafter DG) extend the setting of Choo and Siow (2006) and Galichon and Salanié (2015) to the multidimensional and continuous case. Here, we closely follow the methodology of DG. Recently, Ciscato, Galichon, and Goussé (2015) have made a first attempt to conduct a cross-market analysis on differences in matching patterns across heterosexual and homosexual marriage markets in California. Here we briefly recall the theoretical framework and the estimation technique⁸.

3.1. Matching model. In this frictionless Transferable Utility framework, men and women are characterized by a vector of characteristics $x \in \mathcal{X}$ for men, and $y \in \mathcal{Y}$ for women. Note that, with a large set of continuous variables, every individual is virtually unique in his (her) *observable type* given by x (y). A *matching* is a probability distribution that tells the odds of a couple with observable types x and y to be matched. When a man x and a woman y match, they receive *systematic utility shares* U and V respectively, which both depend on the combination of observable types (x, y) only. In addition, a man of type x experiences a random *sympathy shock* ε^k that is individual-specific to the potential partner k of type y^k . Hence, the two components being additive, the man’s payoff from a match with a woman k of type y is given by $U(x, y^k) + \frac{\sigma}{2}\varepsilon^k$, where the scalar σ measures the relevance of the unobservable component. Women’s payoff can be written in an analogous way.

When the sympathy shock is of Gumbel type, the setting is completely analogous to Choo and Siow (2006). However, Dupuy and Galichon suggest assuming that each man chooses his partner within a set of infinite but countable “acquaintances”, each with characteristics (y^k, ε^k) over the space $\mathcal{Y} \times \mathbb{R}$: such set is the enumeration of a Poisson process

⁸For a more detailed exposition, see the original paper DG and Ciscato, Galichon, and Goussé (2015) for an extension to the unipartite case.

with intensity $dy \times e^{-\varepsilon} d\varepsilon$, which leads us to a continuous logit framework. Note that, under this assumption, the shock ε^k is independent from the observables. Every man solves the following problem

$$\max_k \left\{ U(x, y^k) + \frac{\sigma}{2} \varepsilon^k \right\}$$

and so do women with due changes in notation.

Dupuy and Galichon show that it is possible to recover the optimal matching $\pi(x, y)$ among those that satisfy the market scarcity constraints, as well as the equilibrium quantities $U(x, y)$ and $V(x, y)$, and that the equivalence $\Phi(x, y) \equiv U(x, y) + V(x, y)$, which defines the *systematic surplus*, holds. Provided two functions $a(x)$ and $b(y)$ so that $\pi(x, y)$ is *feasible* - the sum of married individuals of a given type does not exceed their initial number - the equilibrium is thus fully characterized by:

- (1) the optimal matching function $\pi(x, y)$, which tells the probability of matching (equivalently, the relative frequency at equilibrium) for a couple with observables (x, y) :

$$(3.1) \quad \pi(x, y) = \exp \left(\frac{\Phi(x, y) - a(x) - b(y)}{\sigma} \right).$$

- (2) the shares of systematic surplus at equilibrium for each couple with observables (x, y) :

$$(3.2) \quad U(x, y) = \frac{\Phi(x, y) + a(x) - b(y)}{2}$$

$$(3.3) \quad V(x, y) = \frac{\Phi(x, y) + b(y) - a(x)}{2}$$

so that $U(x, y) + V(x, y)$ gives the total systematic surplus at equilibrium, i.e. $\Phi(x, y)$.

3.2. Specification. In this paper, we consider the following parametrization of the systematic surplus, introduced by Ciscato, Galichon, and Goussé (2015):

$$\Phi(x, y) = x' Ay = \sum_{i,j \in \{1, \dots, O\}} x_i a_{ij} y_j + \sum_{i \in \{O+1, \dots, U\}} \lambda_i \mathbb{1}[x_i = y_i].$$

where the first O variables contained in the vectors of observables x and y are *ordered* and the last U are *unordered*. Examples of ordered variables are age, education and wage, whereas ethnicity and working sector are unordered.

Our specification implies that the matrix of parameters A - called *affinity matrix* - looks as follows:

$$A = \begin{bmatrix} \tilde{A} & 0 \\ 0 & \Lambda \end{bmatrix}.$$

The $O \times O$ entries of the submatrix \tilde{A} determine whether the (ordered) variables are complementary or substitutes, as well as the intensity of the affinity (or repulsion) between the two inputs. The elements of the diagonal submatrix Λ tell us whether homogamy with respect to one of the unordered variables results in an increase rather than in a decrease of the systematic surplus. All the other elements of the matrix are constrained to zero.

3.3. Estimation. To compute equilibrium quantities, we solve for $a(x)$ and $b(y)$ enforcing the market scarcity constraints through an Iterative Projection Fitting Procedure for given parameters A and σ . Hence, note that, according to the crucial result of Shapley and Shubik (1971), the equilibrium matching of a decentralised matching market is also the one that maximises social gain. We define the function $\mathcal{W}(A, \sigma)$ as follows:

$$\mathcal{W}(A, \sigma) \equiv \max_{\pi \in \mathcal{M}} \{E_{\pi}[x' Ay] - \sigma E_{\pi}[\log \pi(x, y)]\}$$

where \mathcal{M} is the set of feasible matchings and where expected values with subscript π are taken with respect to the optimal matching probabilities.

DG set the following convex optimisation problem in order to estimate the matrix $B = A/\sigma$:

$$\min_B \mathcal{W}(B, 1) - E_{\hat{\pi}}[x'By]$$

where the expected value with subscript $\hat{\pi}$ is taken with respect to the relative frequencies observed in the data. The First Order Conditions of the problem imply that we are matching the co-moments of men's and women's characteristics predicted by the model with the corresponding empirical co-moments observed in the data. In practice, we are computing B so that the following holds

$$E_{\pi}[X_i Y_j] = E_{\hat{\pi}}[X_i Y_j]$$

for each couple (i, j) of ordered characteristics. Similarly, B must be such that the following holds

$$E_{\pi}[\mathbb{1}[X_i = Y_i]] = E_{\hat{\pi}}[\mathbb{1}[X_i = Y_i]]$$

for each unordered characteristic i .

3.4. Identification with Multiple Markets. One drawback of the original model of DG is that only $B = A/\sigma$ is identified, i.e. A is identified up to a scalar. This is mainly irrelevant to study assortativeness on a single market, since comparing different entries of the matrix B is equivalent to comparing the elements of A . Nonetheless, Cisco, Galichon, and Goussé (2015) stress that it is not possible to compare the affinity matrices of different markets without a further restriction on A ⁹. Briefly, if the entries of B^t computed in year t are globally larger than those in B^s computed in year s , we do not know if this is due to an increase of the parameters of the affinity matrix rather than to a decrease of the scalar σ unless we impose a further assumption.

Denote A^t the affinity matrix in year t . In order to compare marriage markets over time, we assume that the Frobenius norm of the submatrix \tilde{A}^t is equal to one for every t ,

⁹Alternatively, one could put an additional restriction on the parameters σ , for instance $\sigma = 1$ on each market. Cisco, Galichon, and Goussé (2015) propose to normalise the social gain $\mathcal{W}(A, \sigma)$ so that both A and σ can vary across markets. However, this assumption makes welfare comparison impossible.

i.e. $\|\tilde{A}^t\| = 1 \forall t$. This implies that $\frac{\tilde{B}^t}{\|\tilde{B}^t\|} = \tilde{A}^t$, which in turn implies that $\sigma^t = \frac{1}{\|\tilde{B}^t\|}$. This means that we interpret large global changes in the submatrix \tilde{A} as due to a shift in the relative relevance of unobservables in mating.

Although we need to introduce this further restriction to proceed with cross-market analysis, note that the optimal matching function $\pi(x, y)$ only depends on $B = A/\sigma$. Hence, it stays unchanged under different identification assumptions. This makes the results of our counterfactual analysis of inequality in Section 6 robust with respect to different restrictions on the parameters A and σ . We provide a formal proof for this statement in Appendix 1.A.

3.5. Counterfactual Methodology. An interesting, but still unused, feature of Dupuy and Galichon’s model is the possibility to compute counterfactual equilibrium matching by operating on the matrix of preference parameters A . The idea is to infer the marital preferences (A^t, σ^t) from cross-sectional data on couples (X^t, Y^t) for a given year t and then compute the equilibrium matching $P(s, s; t) \equiv \pi(x^s, y^s; A^t, \sigma^t)$ for population data (X^s, Y^s) under the same marital preferences. In this way, by comparing the counterfactual $P(s, s; t)$ to the actual $P(s, s; s)$, we can tell *how people would match if preferences stayed unchanged between period s and t .*

Using $P(s, s; t)$ together with data (X^s, Y^s) , we can compute the counterfactual distribution of couples’ characteristics. For instance, we can compute the distribution of household income, as well as various measures of inequality, such as the Gini coefficient. In this way, we can tell to which extent the distribution of household characteristics has changed because of shifts in marital preferences.

Moreover, it is also possible to create a counterfactual match between subpopulations from different cross-sections. In fact, we can predict the matching $P(s, t; s)$ originating from a fictional situation in which men from year s met women from year t , with the preference parameters s . In this way, it is possible to assess how changes in the marginals influenced the match in order to address specific questions. Although we do not employ this last type of experiment, we recommend it for future research.

While this counterfactual analysis unveils the hidden potential of the model of DG, it also shows an important limitation concerning its empirical application to the marriage

market. In absence of a more explicit household model that explains how agents determine their labor supply and advance in their working career, we are forced to consider wage rates and working hours as exogenous characteristics. The counterfactual analysis does not take into account that spouses adjust their labor supply and take on different working careers according to the partners' characteristics and household decision-making process.

4. Data.

The paper uses CPS data from 1962 to 2015 (March Supplement) from the Integrated Public Use Microdata Series (Flood et al., 2015). CPS data provide a detailed representation of the married male and female populations in the United States over time. Hence, they provide us with reliable “photographs” of the marriage market equilibrium we aim to study. In reality, people are likely to meet and marry in small, local marriage markets: identifying the (fluid) geographic boundaries of such markets seems an extremely complicated task. Because of the limited sample size of CPS yearly database¹⁰, we do not account for heterogeneity in sorting patterns across smaller geographical units (such as states or counties) and we present aggregate trends at the United States level.

In this section, we describe the construction of the main variables of interest and the selection of the samples. We also present summary statistics on our population of couples before turning to estimation.

4.1. Construction of Variables. Our empirical analysis makes use of five key variables: age, education, wage, hours of work and race. In a few cases, such as age, the construction of the variable is straightforward as we take the raw data without further adjustments. In the following, we explain how we deal with other variables.

- *Educational attainment* is available for all years, but with various levels of detail. IPUMS provides a 12-level education variable, to which we refer as the “continuous education variable”. However, this variable is not entirely consistent across years

¹⁰In addition, in CPS waves before 1976, there is no state variable at household level. Only broad geographical areas are reported.

(the coding changed after 1992). To overcome this difficulty (and provide summary statistics on broader education groups), we constructed two other education variables, one with 5 levels and one with 4 levels¹¹. Robustness of the results is checked for each of these specifications.

- As concerns *hours of work per week*, the most consistent variable across waves is “hours worked last week”, as the usual hours of work are not available prior to 1976. However, we check the robustness of our main results obtained with the first definition by implementing checks with the latter, as well as with a combination of the two.
- We define *labor income* as the sum of salary, self-employment income and farming income. These components are top-coded. To deal with this issue, we use the income top-codes files provided by the Census Bureau, which are matched with our observations. These files cover the period 1976-2010. To deal with top-coded observations prior to 1976, we follow Schmitt (2003) and replace top-codes with the expected income above the top-code value. This can be done by assuming that the top of the income distribution is well-described by a Pareto distribution. We test the robustness of the results to these various procedures. As top-coded values account for only a (very) small fraction of the sample, the results are not affected.
- We compute *hourly wages* using labor income, hours of work per week and weeks of work per year¹². We constructed it as follows:

$$(4.1) \quad \text{wage} = \frac{\text{labor income}}{\text{hours} \times \text{weeks}}$$

However, the wage variable may feature abnormally low or abnormally high values.

We follow Schmitt (2003) advice to trim the data, dropping values below 1\$ or

¹¹The 5 levels variable is constructed as follows: (1) below high school degree, (2) high school degree, (3) some college, (4) college degree and (5) 5+ years of college. With 4 levels only, we distinguish: (1) below high school degree, (2) high school degree, (3) college degree and (4) 5+ years of college.

¹²The number of weeks working in the past year is usually available as a continuous variable. However, it is sometimes only available as a grouped variable, which we use to proxy the number of weeks worked in the past year.

above 100\$ (in 2002 dollars), while keeping observations with a zero wage. All income and wage variables are converted to 1999 dollars.

- There is no consistent *race/ethnicity* variable across years. In the early waves of the CPS data, individuals are only classified as White, Black or Other. After 1971, it becomes possible to separately identify Hispanics and, after 1988, Asians¹³. Across the years, the race variable became more detailed, allowing individuals to declare a mixed ethnic background. However, when comparing preferences across waves, we need to use a consistent specification of the variable. We mainly use three different specifications: (1) Black or White, available since 1962 and considering Hispanics as White after 1971; (2) Black, White, Hispanic and Other, available since 1971 and reallocating Asians into the residual category Others; (3) Black, White, Hispanic and Asian, available since 1988.
- IPUMS provides a consistent and detailed *occupation* variable, available after 1968 and based on the 1990 Census classification scheme. We use this information to construct a broad occupation sector variable with 11 categories, as it has been done in recent waves of CPS data¹⁴.

In most of our specifications, we use five variables, namely age, education, hourly wage, hours of work and race. We test the robustness of our results to the inclusion of other variables (such as occupation) or to alternative coding of the variables.

4.2. Default Sampling Procedure. For every cross-section (i.e. every wave of the survey), we consider the current matches as those resulting from the stable equilibrium of the marriage market. In our empirical analysis of the marriage market equilibrium, we need to decide what matches to include in the sample, which results in several practical issues. First of all, we recall that our analysis of the marriage market equilibrium does not include singles, i.e. never married, separated, divorced and widowed individuals. In addition, we do

¹³Comparing summary statistics before and after 1971 suggests that most Hispanics declared themselves as White, whereas the category Others mostly contain Asians before 1988.

¹⁴These categories are: (1) Management, business, and financial occupations, (2) Professional and related occupations, (3) Service occupations, (4) Sales and related occupations, (5) Office and administrative support occupations, (6) Farming, fishing, and forestry occupations, (7) Construction and extraction occupations, (8) Installation, maintenance, and repair occupations, (9) Production occupations, (10) Transportation and material moving occupations, (11) Armed Forces.

not consider unmarried couples: cohabitation out of wedlock can be a “trial period” before marriage but also an alternative to it, which makes it hard to distinguish the two cases in the data. Couples where spouses live in different households are also excluded from the sample. Finally, same-sex couples are excluded. On the other hand, we do not make any difference between individuals that married once and those who married more than once.

Most importantly, we select couples where at least one of the partners is aged between 23 and 35¹⁵. The bracket roughly corresponds to the core of prime adulthood and aims to exclude individuals still at school¹⁶. Although in reality the matches we observe took place at different points in time, we assume that, for each cross-section, individuals aged between 23 and 35 compete on the same marriage market. In this case, marriage markets are not rigidly defined by age brackets: particularly, the age difference between the partners and the age of first marriage may vary greatly. However, our empirical analysis relies on the assumption that sorting dynamics are relatively homogeneous for the age bracket 23-35 for each wave.

On this delicate point, we differ substantially from Chiappori, Salanié, and Weiss (2015). In the latter, the authors use 2010 Census data to construct the population vectors cohort by cohort. Their method relies on the assumption that each cohort is a separated marriage market¹⁷. Nonetheless, we aim to estimate the intensity of assortativeness on age and document its trend over time. The selection criterion proposed by Chiappori, Salanié, and Weiss (2015), instead, assumes an extremely rigid sorting pattern with respect to age.

One of the main concerns affecting our age restriction is the self-selection due to divorce. Separation and divorce allow us to observe only the prevailing unions at a given point in time and this may lead to some problems in the interpretation of the results. For example, assume that the cohort born in 1950 has been largely affected by the change in divorce laws in the 1970s and that the divorce rate is particularly high for this cohort. Divorce may

¹⁵Similar simple selection criteria by age are common in the literature. See Schwartz and Mare (2005) (where the wife must be between 18 and 40) or Schwartz and Graf (2009) (where both partners must be between 20 and 34).

¹⁶We also exclude students aged more than 23 by combining data on school attendance and reasons for not participating to the labor market

¹⁷More precisely, each cohort t of boys matches with the cohort $t + 1$ of girls. However, the problem is analogous.

primarily destroy non-assortative matches. Hence, the marriage patterns observed in 2010 for this cohort might result from a selection process through divorce, instead of being the result of the specific tastes at the moment of the match. In order to overcome this potential bias, it could be advised to work with a subsample of newlyweds (as also suggested by Schwartz and Mare, 2005). Unfortunately, in our case data on marital history are not always available and focusing on newlyweds would imply a drastic reduction of the sample size.

Finally, note that the estimation algorithm works best with samples with order of magnitude equal to 3. For some waves, the sample of observations respecting our selection criteria is greater than 10,000. In Appendix 1.B, we propose a methodology to ensure that the sample is highly representative of the sorting patterns when we must reduce its size.

4.3. Baseline Sample. The changes in the availability of data and potential problems arising from the construction of the variables motivate the use of alternative samples. In spite of this, we choose two baseline specifications described in Table 1 that we use to present our main findings. The first one employs all waves from 1962 to 2015, while the second one contains a more detailed race/ethnicity variable but only uses waves from 1971 to 2015. We introduce a second baseline sample since we cannot compare the racial homogamy parameter across waves consistently if racial/ethnic categories change over the years and since we are primarily concerned with potential biases due to the misspecification of ethnic traits and the exclusion of minorities from the sample. Several other robustness checks and additional results are summarised in the Appendix 1.C.

4.4. Summary Statistics. The population that we consider in this application has gone through major changes in the past fifty years. Many of these transformations directly concern the family and its structure and are also documented in our sample. The rise in educational achievement is depicted in panel (a) and (b) of Figure 1. Only a relatively smaller fraction of individuals now belongs to low education categories (below high school or high school degree), while an increasing share of the population falls into higher education categories (some college, college degree or above). Note that this trend is especially striking

TABLE 1. Baseline Samples

Model	Baseline A	Baseline B
Period	1962 – 2015	1971 – 2015
Age	At least one spouse aged between [23, 35] <i>(Figure 3)</i>	At least one spouse aged between [23, 35]
Education	“Continuous” <i>(Figure 4)</i>	“Continuous”
Wage	Trimmed <i>(Figure 5)</i>	Trimmed
Hours	“Hours worked last week” <i>(Figure 6)</i>	“Hours worked last week”
Occupation	Not Included	Not Included
Race	White (incl. Hisp.), Black <i>(Figure 7)</i>	White, Black, Hisp., Others (incl. Asian) <i>(Figure 7)</i>

The table describes the criteria used to select our two baseline samples starting from the main CPS database. We mainly use baseline samples A and B in our empirical analysis throughout the paper.

for women, who now appear to be more educated than men, while the reverse was true in the 1960s¹⁸.

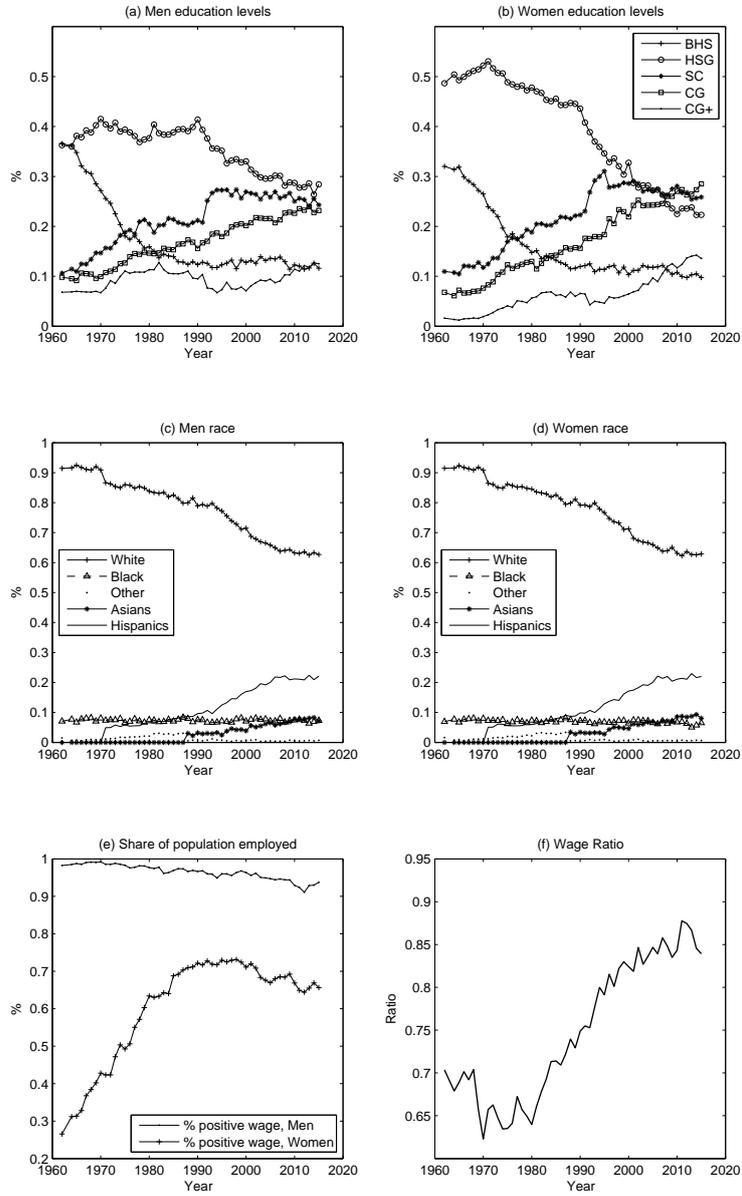
In panel (c) and (d), we describe the racial composition of our sample. We can separately identify the four major racial groups (White, Black, Hispanic and Asian) after 1988. From the graph, it seems that Hispanics used to declare themselves as White prior to 1971, while Asians composed the majority of the “Others” category. The share of Black in the samples is relatively constant, while Hispanics and Asians account for an increasing share of the population at the expense of the White category.

One major change in families in the past fifty years is the increased participation of women on the labor market. This is represented in panel (e) of Figure 1. Our measure of employment for our sample is the share of people with a strictly positive wage¹⁹: the graph shows a dramatic increase for women, although the rate stabilised after 1990. Finally panel (f) depicts the wage ratio for women relative to men (conditionally on having a strictly

¹⁸The graph also shows the discrepancy in the education variable in 1992, as a large share of the population previously categorised as having a high school degree now appears in the “Some College” category.

¹⁹The share of employed people may appear extremely high in some cases (for men at the beginning of the period for example), but this may be due to our sample selection criteria based on age and marital status. In addition, we consider a person as employed as long as we are able to compute a wage, that is, as long as she worked in the past year.

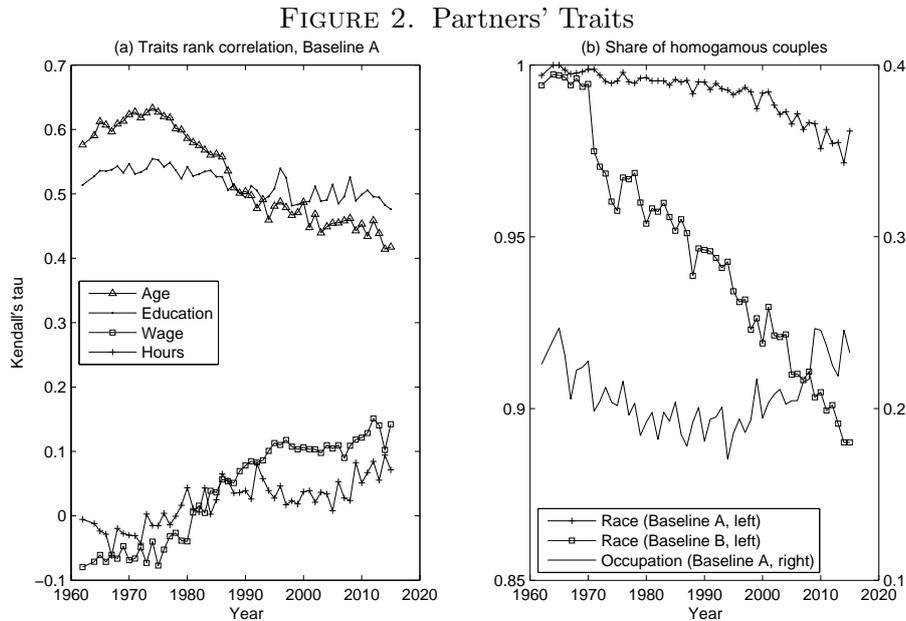
FIGURE 1. Summary Statistics



Married couples from CPS data 1962-2015. For a couple to be in our sample, at least one partner must be aged between 23-35. Couples where one partner is still at school are also excluded. Discontinuity around 1992 for schooling trends is due to a change in the variable specification made by the US Census Bureau. Discontinuities in the race trends are also due to the addition of new categories in the set of possible answers.

positive wage). This increase has been identified as one of the main factors of change for

families (see Becker, 1973; Becker, 1991, on specialization within households and human capital investment of women).



Samples used: baseline A. Baseline sample B is used for the trend of interracial marriage (right panel, square marker).

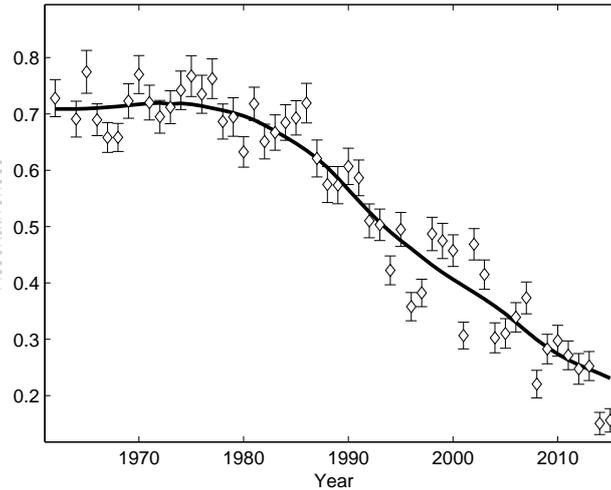
When we look at the joint characteristics of the spouses (Figure 2), we notice a strong positive correlation between the partners' age and education, which is a first hint that these traits are complements. While correlation by age decreases over time, the trend of correlation by education is instead unclear. On the other hand, we observe an increasing trend for the correlations by hours worked and hourly wage. Interestingly, these co-moments are first weakly negative and then weakly positive. Finally, the share of interracial marriages has increased over time, whereas the share of couples where the spouses are both employed in the same sector does not exhibit any clear trend.

5. Trends in Matching Patterns.

In this section, we describe trends for the diagonal elements of the affinity matrix estimated using the Baseline A Sample described in Section 4.3 over the period 1962-2015.

Estimation follows the steps explained in Section 3. Estimates of the A_{ij} entries are obtained for every year and shown in the graphs below. We display the point estimates, as well as the confidence intervals. Data are standardized so that the covariance matrices $E_{\hat{\pi}}[x'x]$ and $E_{\hat{\pi}}[y'y]$ have diagonal entries equal to one for a reference year²⁰: this allows us to compare different estimates of A within and across years. We also use local constant regression smoothing (LOWESS) to ease the interpretation of the results. Finally, we present several robustness checks in order to understand whether our baseline findings suffer from variable misspecification, sample selection or endogeneity problems: we provide a list of the checks in Appendix 1.C.

FIGURE 3. Assortativeness in age



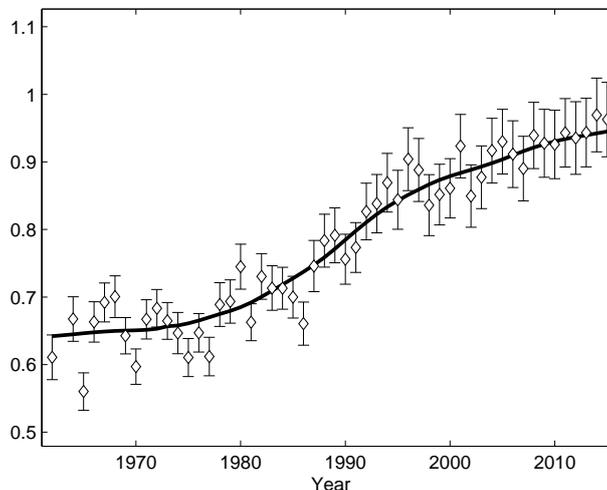
Sample used: baseline A. The figure displays the estimated trend of the diagonal element of the marital preference parameter matrix A capturing the interaction between husband's and wife's age. We observe a decrease in age complementarity.

5.1. Age. Our results show that spouses' ages are strongly complementary. However, Figure 3 also shows an unambiguous decrease in age assortativeness. This may appear as in contrast with previous results by Atkinson and Glass (1985) and Qian and Preston (1993), who claim that in the United States homogamy by age increased up to 1987. Nonetheless, this trend could be explained by a progressive passage from a traditional form of marriage

²⁰In practice, we first compute $(diag(E_{\hat{\pi}}[x'_{1991}x_{1991}]))^{-1/2}$ for men's population in 1991, and then use it as a scaling factor for every cross-section. Same for women's population.

- where the woman is slightly younger than the man - to a variety of different unions. For instance, Atkinson and Glass (1985) notice that spouses with similar socio-economic background tend to be of the same age more and more frequently. Moreover, couples where the husband is younger or where the difference in partners' ages is high are more and more socially acceptable. What we find is, in fact, that the *strength of sorting* decreased, which means that several age combinations now coexist at equilibrium.

FIGURE 4. Assortativeness in education

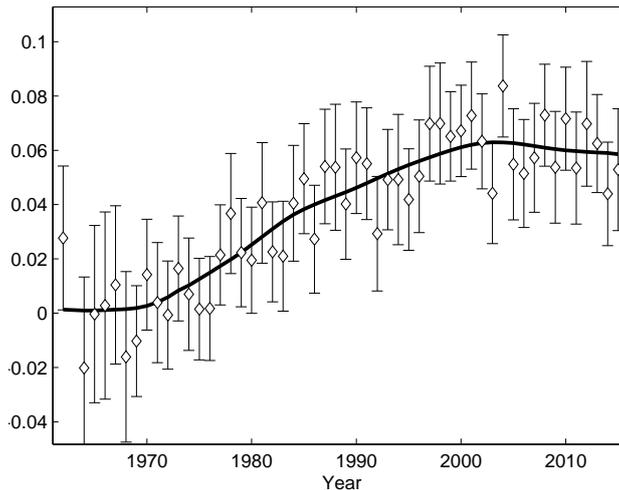


Sample used: baseline A. The figure displays the estimated trend of the diagonal element of the marital preference parameter matrix A capturing the interaction between husband's and wife's schooling level. We observe an increase in education complementarity.

5.2. Education. Figure 4 represents the trend of assortativeness in education between 1962 and 2015. We find a general increase in assortativeness in education. This is in line with Chiappori, Salanié, and Weiss (2015), who document such a rise in assortativeness for cohorts born between 1943 and 1972. This also coincides with the results of Greenwood et al. (2014) and with most of the findings in the literature (see Section 2). Nonetheless, as explained throughout the paper, we argue that our estimates only capture marital preferences and are cleansed from any demographic effect. In addition, since we are in a multivariate setting, we can “control” for other observables and also conclude that assortativeness in education is comparable in strength to age, whereas it is much higher than wage or hours of work (see Figures 5 and 6). As concerns possible misspecification of the schooling level

variable, we find that our results are robust to different measures of educational attainment, described in Section 4.1 and listed in Appendix 1.C (checks 2 and 3).

FIGURE 5. Assortativeness in wage



Sample used: baseline A. The figure displays the estimated trend of the diagonal element of the marital preference parameter matrix A capturing the interaction between husband's and wife's wage. We observe a possible rising of a relatively weak wage complementarity which was not observed in the early waves.

5.3. Wage. The estimates for wage assortativeness are presented in Figure 5. In the earliest waves, the estimates of the affinity matrix parameter for wages are not significantly different from zero. However, assortativeness in hourly wage rates has steadily increased up to the 2000s and is significantly positive in every wave since the mid-1980s. In the last 15 years, it seems that the estimate stabilised around a value of 0.05, which is still considerably smaller than the values taken by the corresponding estimates for age and education. The trend for the wage estimate is parallel to the closing of the wage gap and may suggest that men developed a stronger incentive to look for a spouse among high earners.

In spite of this (weak) positive assortativeness, Becker (1973) suggested that the spouses' wages should be substitutes because of household specialization, while main non-labor-market traits are expected to be complements. Unfortunately, our result is not a good test for Becker's predictions: since many women (as shown in panel (e), Figure 1) are not part of the labor force - especially in the earlier waves - we are not able to observe their wage potential. In other words, we are not able to determine the *shadow price* of time spent

away from the labor market to which Becker refers to in his analysis of the household. As a result, the estimates we present do not capture marital preferences because of this endogeneity issue affecting the observed hourly wage rates (see also Ciscato, Galichon, and Goussé, 2015).

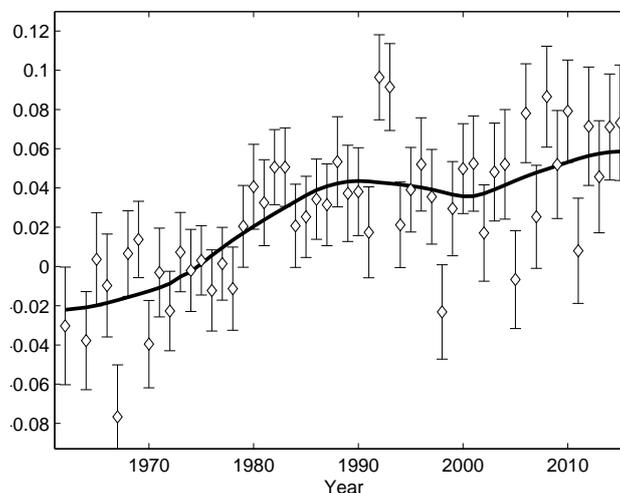
To understand to which extent our main findings are affected by endogenous workforce participation choices, we run two parallel estimations with different subsamples (see Appendix 1.C, checks 9 and 10). First, we only estimate the affinity matrix for a subsample of couples where both spouses have a positive wage. With respect to our baseline results (Figure 5), we find evidence of positive assortative mating on wages since the earliest waves and the strength of assortativeness is now constantly larger (Figure 17). Second, when we only consider the subsample of childless couples, the estimates for wages' complementarity are even higher (Figure 18). These checks seem to suggest that, for households where household specialization is expected to be less pronounced, sorting on spouses' wages is indeed stronger. Nonetheless, as both fertility and labor force participation are the outcomes of endogenous choices, none of the two subsamples can be considered as representative of the population preferences. Further research is needed in this direction.

Finally, we construct a measure of potential income that allows us to deal with non-participation (see Appendix 1.C, check 11). Following Bertrand, Kamenica, and Pan (2015), we divide our sample in demographic groups based on age, education and geographical area²¹. Hence, we assign a positive wage rate for individuals that are out of the labor force by randomly drawing among observations from the same demographic group but with positive income. We estimate the trends of marital preferences and report the result in Figure 19. The main important finding is that we observe positive assortative mating on wages since the earliest waves and increases in strength over time. We discuss the potential implications on inequality trends in Section 3.5 in light of the results of our decomposition exercise.

5.4. Hours Worked. Trends in mating preferences for hours of work are represented in Figure 6. Similarly to the case of wages, the estimates for the earliest waves are

²¹We build different geographical variables using CPS information on region, state and metropolitan area. Note that location refers to the household address.

FIGURE 6. Assortativeness in Hours of work

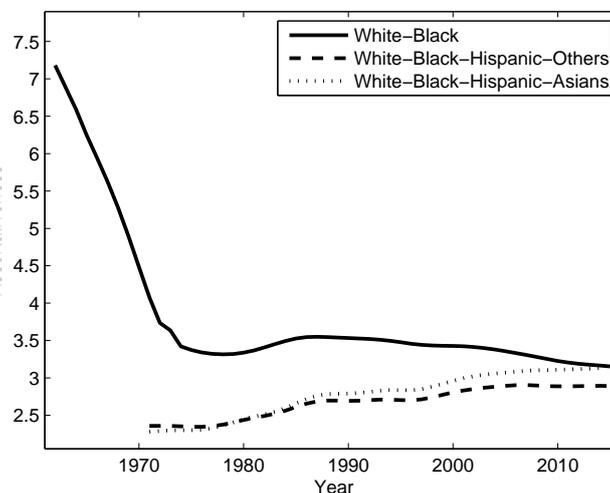


Sample used: baseline A. The figure displays the estimated trend of the diagonal element of the marital preference parameter matrix A capturing the interaction between husband's and wife's hours worked. We observe a possible rising of a relatively weak complementarity in hours worked which was not observed in the early waves.

not significantly different from zero and are even negative for some waves, whereas we observe an irregular increase starting from the 1980s. The increasing trend seems consistent with the shift from production complementarities as the main source of marriage gains to consumption-based complementarities (Stevenson and Wolfers, 2007): while in traditional families one spouse - typically the wife - focused on housework and the other on the labor market, now partners may benefit from similar time schedules.

Once again, what the estimate for hours worked captures cannot be interpreted in terms of preferences at the moment of the match, since spouses most likely adjust their labor supply after the marriage. Checks 9 and 10, described in Section 5.3, lead to the following results: for couples where both partners are employed, we observe positive assortative mating on time schedules for any wave (Figure 17), while for childless couples the positive sorting is even stronger (Figure 18). In both cases, the strength of complementarity increases over time, similarly to the baseline trend. Although these estimates are biased because the samples are not representative of the population, it seems that couples where household specialization is weaker indeed display more homogeneous working time schedules and leisure time spent together.

FIGURE 7. Assortativeness in Race, by number of race included



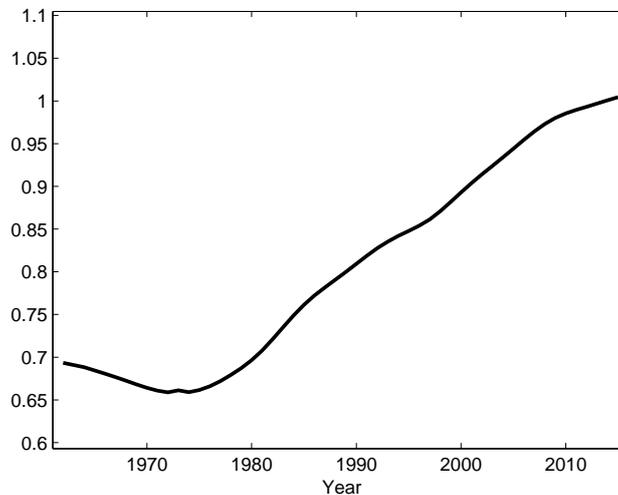
Sample used: baseline A, baseline B and check 1 (see Appendix 1.C). The figure displays the estimated trend of the homogamy preference parameter for race contained in the matrix A . We observe an increase in the preference for racial homogamy.

5.5. Race. Figure 7 reports our estimates of the racial homogamy parameter for baseline sample A and baseline sample B, described in Section 4.3. We observe a sharp decline in the taste for homogamy when considering the race specification Black-White: the most significant decrease took place during the 1960s, when the last anti-miscegenation laws were ruled unconstitutional, whereas we observe a steady but only slight decrease from the 1970s. Interestingly, when switching to the specification Black-White-Hispanic-Others (baseline sample B), the trend is instead slightly increasing over the period 1971-2015. Finally, we run a robustness check with the specification Black-White-Hispanic-Asian (Appendix 1.C, check 1), available since 1988: this time, the trend does not change much from the one obtained with the baseline sample B. In general, however, the results depend on how many groups are considered, that is, on the level of detail of the classification scheme. Studies on racial homogamy are facing the same issue, as the number of racial groups may vary depending on the availability of the data, or on how individuals are allowed to report their race. We can conclude that, although the data show a growing number of interracial marriages, the latter became less desirable since the 1970s when considering a detailed level of

ethnic fragmentation, i.e. when Hispanics and other smaller minorities (especially Asians) are considered as well-distinguished ethnic groups.

Our findings are not directly comparable with those by Fryer (2007), who uses a specification White-Black-Asian for his race variable and concludes that preference for homogeneity decreased throughout the last century. However, the main difference with our work is that we disentangle demographic effects - i.e. the changes in the ethnic composition of the male and female populations - from preferences in a multidimensional setting, which is crucial as the American population has become more and more multi-ethnic over the last decades. We can instead set a direct comparison with the work of Fu and Heaton (2008), who use the specification White-Black-Hispanic-Asian to describe ethnic groups. Our results are in contradiction with theirs, as they find that taste for racial homogeneity decreased over the period 1980-2000.

FIGURE 8. Sigma



Sample used: baseline A. The figure displays the estimated trend of the parameter σ capturing the relevance of idiosyncratic preference shocks in our matching model. We observe an increase of the relevance of unobservables in matching.

5.6. Unobservables. We recall that the scalar σ measures the relevance of the unobservable random preference shock: the higher σ , the more matching appears as random to

the observers. Figure 8 displays the values of σ obtained under our identification assumption given in Section 3.4 and with baseline sample A. The clear increasing trend suggests that socio-economic observables matter less today than they did fifty years ago.

The role played by the parameter σ in our theoretical framework suggests that there are two forces offsetting each other. On the one hand, we report that taste for racial homogamy and positive assortativeness in education have increased in strength. On the other hand, the relevance of the socio-economic observables that we take into account has decreased.

6. Counterfactual Analysis.

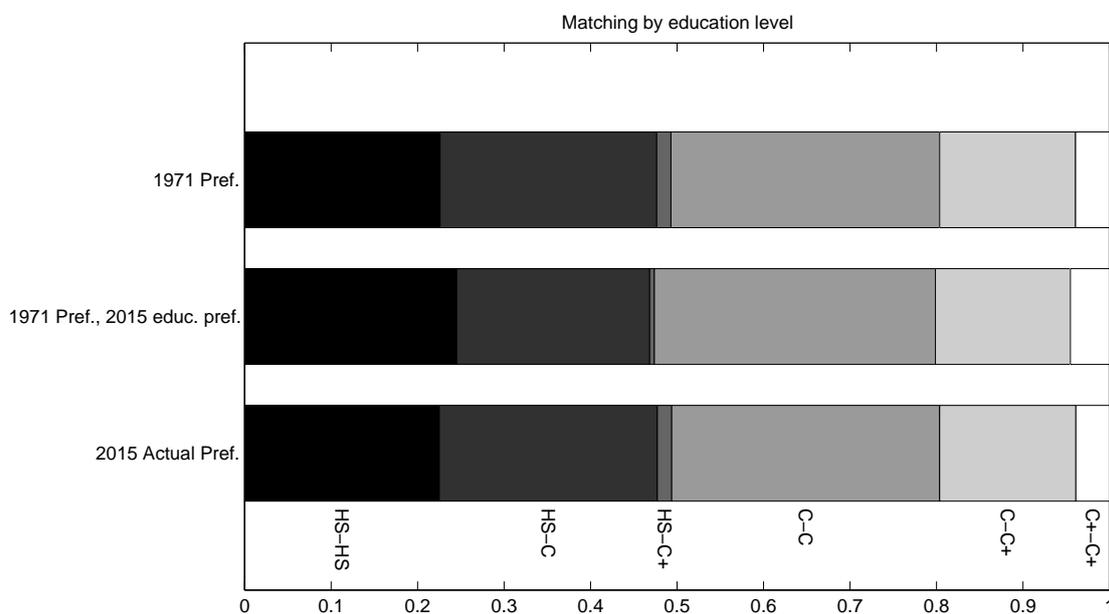
One key motivation behind the analysis of marital preferences is to understand their contribution to the changes in mating patterns and between-household inequality. To conduct our counterfactual analysis, we used CPS data for the years 1971 and 2015. We use the baseline sample B, which includes four racial groups (White, Black, Hispanic, Others). We try to answer two questions: (a) what would be the marriage patterns, for example the joint distribution of education, if individuals married as in 1971? (b) how inequality would change if individuals had the same marital preferences as in 1971?

As explained in Section 3.5, once estimated A^{1971} and with (X^{2015}, Y^{2015}) at hand, we can compute the counterfactual optimal matching $P(2015, 2015; 1971)$ in order to compare it with the marriage market equilibrium predicted by the model with the actual 2015 preferences²². In this section, we report the results of our counterfactual analysis for two variables, age and education. We subsequently proceed with the analysis of inequality trends and a decomposition exercise to understand which parameters are associated with the inequality rise.

6.1. Counterfactual Joint Distribution of Education at Equilibrium. To ease the representation of the results, we gather individuals in three educational types: high school and below (HS), some college (C), and college degree and above (C+). We first compute the counterfactual market equilibrium $P(2015, 2015; 1971)$: the upper distribution

²²Alternatively, we could compare the counterfactual matching $P(2015, 2015; 1971)$ with the actual frequencies observed in the data, rather than those predicted by the model. However, since the model does an excellent job in reproducing the actual frequencies, the two exercises are equivalent.

FIGURE 9. Assortativeness in education, counterfactual distribution

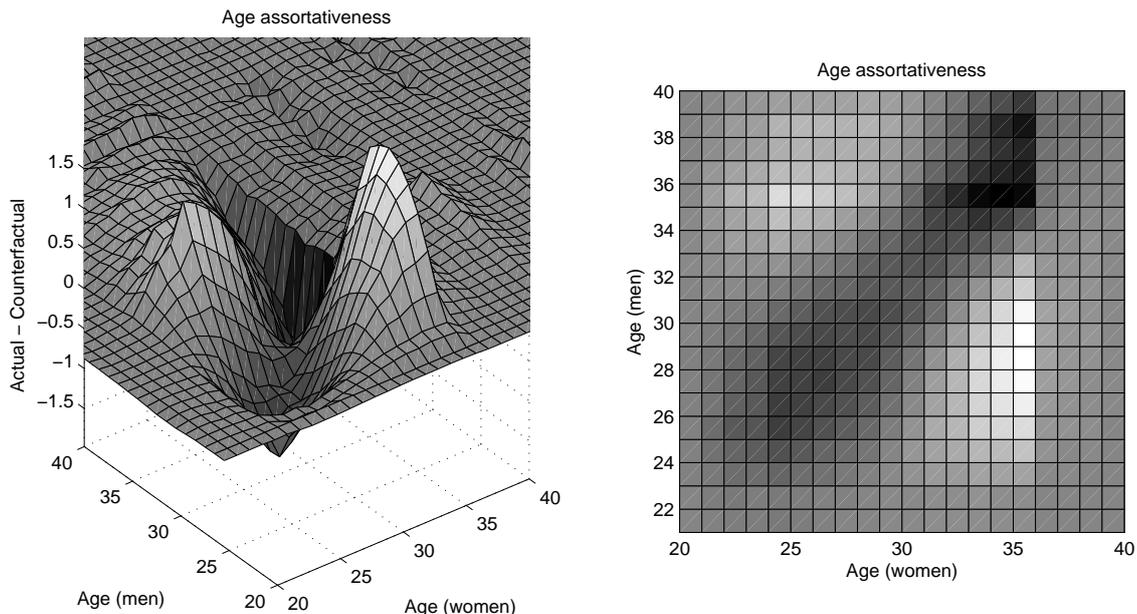


Sample used: baseline B. The marginal distributions of characteristics (X^{2015}, Y^{2015}) are taken from 2015 data for the three figures. In the first line, we show the counterfactual joint distribution of partners' educational levels obtained using 1971 marital preferences. In the second line, we show the counterfactual distribution obtained using 1971 marital preferences but allowing the schooling complementarity parameter to rise to its 2015 value as in Figure 4. In the third line, we show the actual distribution obtained with 2015 marital preferences.

in Figure 9 displays the relative frequencies of each of the six possible types of match that would result from matching if the preferences of the 2015 population were the same as in 1971. The relative frequencies reported in the second line are the result of a different counterfactual experiment. We now fix all the parameters to their 1971 values except for the one capturing the interaction between partners' education, which is allowed to take its actual 2015 value. In this way, we isolate the effect of the change in this single parameter on the marriage market outcome. We note that the increase in complementarity shrinks the shares of couples crossing educational barriers. On the other hand, endogamous marriages are more frequent. In the last line, we report the joint aggregate equilibrium distribution of educational types predicted by the model for year 2015. The changes observed in the second line turn out to be mostly offset by shifts in other parameters, which leads us to conclude that the evolution of marital preferences had little impact on the joint distribution

of partners' schooling levels. One main reason is likely to be the increase in the parameter σ , which decreases the relevance of socio-economic observables on the marriage market.

FIGURE 10. Assortativeness in age, counterfactual distribution



Sample used: baseline B. The figures depict the differences in joint frequencies of partners' ages between the actual distribution obtained with 2015 preferences and the counterfactual one obtained with 1971 preferences ($P(2015, 2015; 2015) - P(2015, 2015; 1971)$). We show such frequencies in a three-dimensional space and in the corresponding "elevation" map. The darker the block, the more couples of the corresponding age in the counterfactual outcome outnumber their peers in the actual. The lighter the block, the more couples of the corresponding age in the actual outcome outnumber their peers in the counterfactual. Remember that the sample may include individuals of any age, although we require that at least one partner is aged between 23 and 35 for the couple to be in the sample.

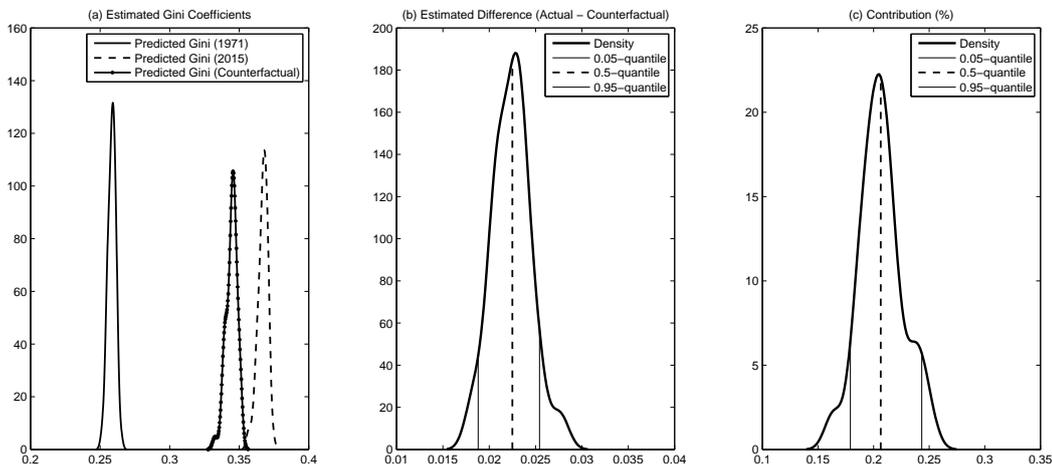
6.2. Counterfactual Joint Distribution of Age at Equilibrium. We repeat a similar experiment with age, as illustrated in Figure 10, where we computed the joint distribution of spouses' age with both actual and counterfactual marital preferences, then we looked at the difference between the two. Remember that, as discussed in Section 5.1, in 1971 there used to be a relatively stronger sorting on age than in 2015. From Figure 10, we note that, under counterfactual 1971 preferences, we would observe far more couples where the husband is around 2 or 3 years older than the wife (the darkest cells are mostly right above the diagonal) and slightly more couples with partners of the same age (the

cells on the diagonal are dark). These two types of couples, and especially those with a slightly older husband, can be considered as the most “traditional”. However, the change in preferences made them less frequent in favor of other types of marriages. Indeed, under actual 2015 preferences, we observe far more couples where the distance between spouses’ ages is greater (the white cells are far from the diagonal). In particular, there are many more couples where the wife is more than 5 years older than the husband.

6.3. Counterfactual Analysis of Inequality. The purpose of the previous sections was to show how the change of the affinity matrix directly translates into a different marriage market outcome. We can now compute household income distributions and then Gini coefficients. For each potential couple, we compute the total labor income of the household, while the optimal matching matrix $P(s, s; t)$ tells us the corresponding frequency of this type of couple at equilibrium. For any two years s and t , we use individual traits distribution from year s and marital preference parameters from year t and compute the Gini coefficient using the optimal matching matrix $P(s, s; t)$ - i.e. the counterfactual frequency table of the couples’ type - and the vectors of spouses’ incomes x^s and y^s . We denote $\mathcal{G}(s, s; t)$ the predicted Gini coefficient computed with male and female population vectors from year s and with marital preferences (A^t, σ^t) from year t .

We aim to study the evolution of inequality from 1971 to 2015. In particular, we ask the following question: *what inequality patterns would we observe if the 2015 population had the same marital preferences as in 1971?* To answer, we first need to compute the Gini coefficient predicted by the model with the actual 2015 preferences, $\mathcal{G}(2015, 2015)$. Similarly, we obtain the same coefficient for the reference year 1971. Hence, we fix marital preferences to their 1971 levels but predict marriage patterns for the 2015 population. Hence, we compute $\mathcal{G}(2015, 1971)$ using the counterfactual labor income distribution. Figure 11 presents the results from this experiment. The confidence intervals are obtained from 50 bootstrap replications. The effect of matching patterns on inequality is given by the difference $\mathcal{G}(2015, 2015) - \mathcal{G}(2015, 1971)$ and is shown in panel (b). Similarly to Eika, Mogstad, and Zafar (2014), we observe a clear increase in income inequality among married households over the last 45 years, from 25.84 points in 1971 to 36.70 in 2015. However,

FIGURE 11. Counterfactual analysis, Gini coefficients



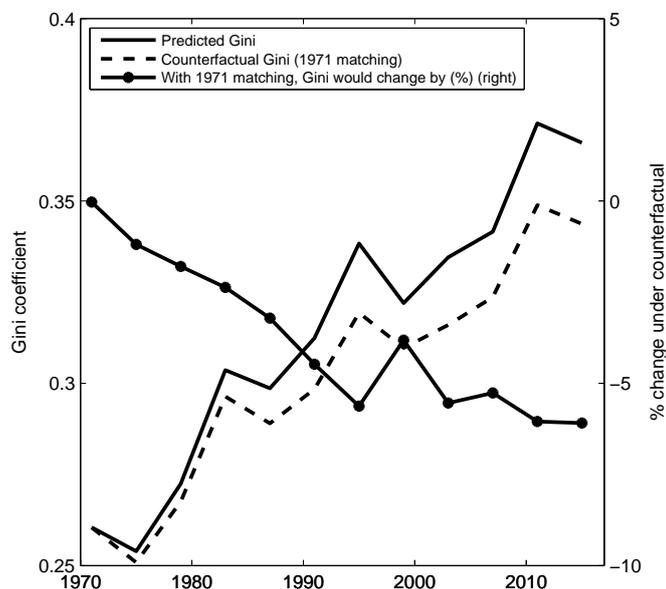
Sample used: baseline B. Graph (a) shows our estimates for the actual between-household Gini coefficient for 1971 and 2015, as well as a counterfactual measure obtained with 2015 data and 1971 marital preferences. Graph (b) shows the difference between the actual 2015 Gini coefficient and the counterfactual. Graph (c) shows the difference relative to the actual 2015 Gini coefficient. Confidence intervals are obtained through 50 bootstrap replications.

were marital preferences constant since 1971, the current Gini would be lower by 2.24 points (about 6% less). Our experiment indicates that 20.68% of the rise in inequality in household labor market income between 1971 and 2015 can be attributed to changes in preferences on the marriage market (Figure 11, panel (c)).

To deepen our analysis, we replicate the same exercise every four years starting from our reference year 1971 and always fixing marital preference parameters to their 1971 levels. The results are displayed in Figure 12. The blue lines represent the predicted Gini coefficients with actual preferences (solid) and with counterfactual preferences (dashed), while the red line depicts how the Gini coefficient would change (in percentage) if individuals had the same tastes as in 1971. While inequality is steadily increasing since 1971, the blue lines slowly diverge from each other, which means that the rise of household income inequality has been exacerbated by the shifts in marital preferences.

6.4. Decomposition. Finally, we decompose the share of the increase of the Gini coefficient that we attribute to shifts in marital preference parameters (Figure 13). On the right of the vertical axis, we find the main forces that contributed to the rise of household

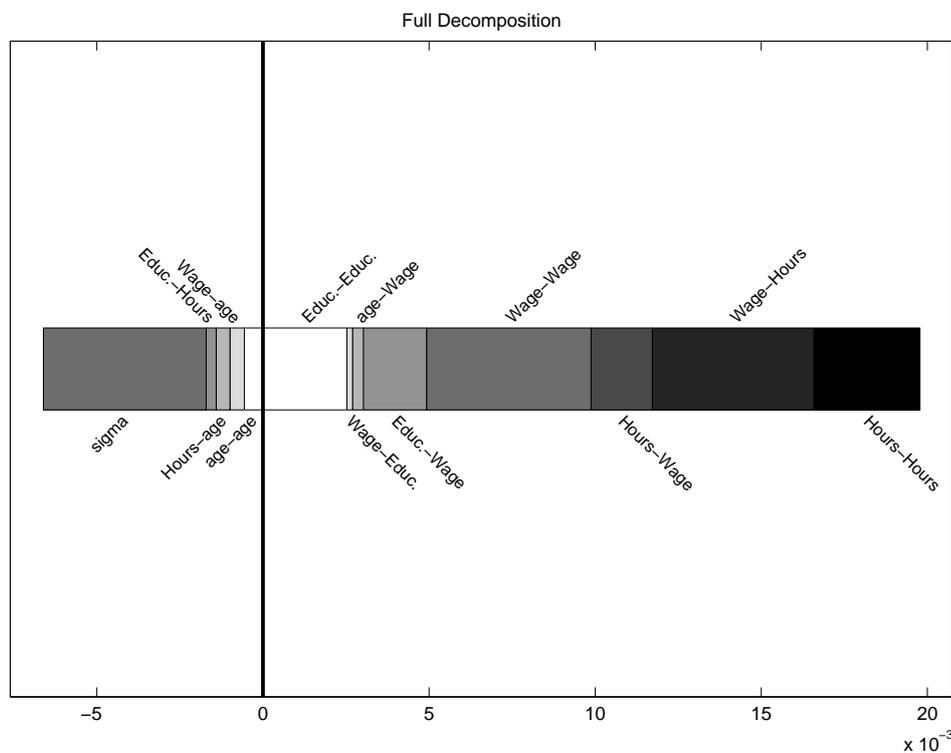
FIGURE 12. Counterfactual analysis, Gini coefficients since 1971



Sample used: baseline B. The figure shows the estimated actual trend of the between-household Gini coefficient and a counterfactual trend obtained by fixing marital preferences to their 1971 values.

inequality. Not surprisingly, increased complementarity in partners' education is one of them, albeit not the strongest. In fact, despite the modest size of their increases (see Figures 5 and 6), the changes that have concerned sorting on wage rates and working hours seem to be the main drivers for inequality rise. However, even small variations in the parameters may result in large fluctuations of macroeconomic outcomes if the marginal distributions change. Since the wage structures, the wage gender gap and women's participation to the workforce have radically changed (see panels (e) and (f) in Figure 1), the interaction of such transformations with marital preference evolution has amplified inequality growth. In addition, a large share of the change in Gini coefficient is due to shifts in cross-interactions, i.e. of those parameters that do not lie on the diagonal of the affinity matrix. In particular, the interactions between wage rates and hours and the one between husband's education and wife's wage matter. Their trends can be found in Appendix 1.D, Figure 14, and once again our estimates suggest that such interactions are relatively weak and have not changed

FIGURE 13. Decomposition of Gini coefficient shift 1971-2015 due to marital preferences



Sample used: baseline B. In the labels, the first trait is the husband's and the second is the wife's, e.g. Wage-Educ refers to the interaction between the husband's wage rate and the wife's education. On the right of the vertical axis, there can be found the parameters that contributed to raise inequality; on the left of the vertical axis, those that pushed in the opposite direction, leading to a decrease.

much over time²³. Finally, looking at the left of the vertical axis, we find that the increase of the parameter σ has hampered inequality by reducing the relevance of socio-economic observables in matching. The second main counterforce is the decrease in assortative mating on age.

6.5. Discussion. Our findings lie somewhere in between those of Greenwood et al. (2014) and those of Eika, Mogstad, and Zafar (2014). The latter found no empirical evidence showing that shifts in sorting patterns actually had any impact on inequality and point at changes in labor market participation and returns to education as the main causes explaining

²³The only trend that we find is a slight - and barely significant - decrease of the negative interaction between men's wage and women's working hours. This parameter may partly capture the wife's wealth effect on labor supply.

household income inequality trends. However, our counterfactual analysis does show that changes in marital preferences must be regarded as an important driver for the recent rise in inequality. On the other hand, we conclude that such changes only account for 20% of the total increase in the Gini coefficient, a significantly lower share than the one suggested by Greenwood et al. (2014), which is close to 100%²⁴. As anticipated in Section 2, our estimate of the Gini coefficient increase (about 20%) that we associate with changes in marital preferences is close to the 18.6% estimated by Greenwood et al. (2016), the only paper which addresses the same issue by means of a structural analysis. However, the results of the two papers are not fully comparable: in particular, we do not model singlehood, so our measure of inequality does not consider one-person households.

Our analysis may also serve as an indirect test to Fernández, Guner, and Knowles (2005)’s prediction that stronger educational assortativeness is linked with higher household income inequality in the long term, although our marriage market equilibrium lacks any characterisation as long-run steady state. From our findings, it seems that both the strength of positive assortative mating on education (Figure 4) and the Gini coefficient (Figure 12) increased over time, which is in line with the predictions of Fernández, Guner, and Knowles (2005) model. Nonetheless, our setting is multidimensional, which lets us wonder whether changes in other sorting dimensions had a similar or even greater impact on inequality: for instance, while assortativeness in education increased, age became less relevant and household specialization seems to be weaker. Most importantly, while we find no reason to reject Fernández, Guner, and Knowles (2005)’s theoretical predictions, our matching model only describes a series of static equilibria and says nothing about fertility and traits transmission: further research is needed to shed more light on the long-term dynamics of marriage and inequality.

Finally, the decomposition shows that marital preferences mainly boosted income inequality through changes in labor market traits’ interaction. As discussed in Section 5.3, we

²⁴Greenwood et al. (2014) use 1960 and 2005 as years of reference. However, after trying out different combinations of reference years, we have never found that changes in sorting patterns have such a sizable impact as claimed by their paper. In Appendix 1.C, check 7, we use 1965 as a reference year and still find that the 2015 Gini coefficient would be about 6% lower if preferences did not change, although now marital preferences account for 33% of the rise in inequality between 1965 and 2015. This is due to the fact that marital preferences already changed much between 1965 and 1971 (e.g. see the race/ethnicity trend).

are particularly concerned about the interpretation and the potential bias in our estimates of marital preference trends when it comes to labor market variables. A broader analysis would explicitly model household behaviour in order to predict spouses' labor supply decisions, similarly to what Greenwood et al. (2016) do. Nevertheless, our robustness checks with childless couples (Figure 18), two-earner households (Figure 17) and potential income (Figure 19) suggest that our estimates of the trends of wage and working hours interactions used for our counterfactual analysis do not capture sorting on the marriage market only but also labor supply preferences and home productivity. We show that our estimates are likely to be downward biased. On the other hand, the same robustness checks confirm that the direction and the size of the trend are not affected by the bias. All our estimates suggest that the strength of sorting on wage rates and working hours has increased over time and the size of such increase does not change relevantly across different estimations.

7. Conclusions and Perspectives.

Our analysis calls into question and updates previous results: it aims to provide the most recent and complete picture of mating patterns in the United States relying on a structural approach that is new to this literature. The framework introduced by DG not only allows us to disentangle preferences and demographics effectively, but also to work in a multidimensional and continuous setting. This flexible specification presents an advantage with respect to previous works in that it allows to analyze different dimensions of sorting at once, in order to understand to which extent marital preferences matter to explain inequality rise and which dimension actually has contributed the most to such increase. On the other hand, we limit ourselves to document the changes in sorting patterns and household dynamics without attempting to explain the drivers behind such transformations. Our work is thus complementary to those by Fernández, Guner, and Knowles (2005) and Greenwood et al. (2016). The first proposes a dynamic and intergenerational approach that allows to improve the understanding of the mutual influence of marital preferences and the changes in the structure of the population, through fertility and educational choices. The second analyzes changes in household specialization patterns by explicitly modeling home technology and endogenizing labor supply decisions.

Throughout our paper, we provide a detailed picture of the evolution of marital preferences in the United States over the period 1962-2015. In line with the majority of previous works, we find that, even after including several other personal traits, positive assortative mating on education has become stronger and stronger over time. At the same time, positive assortative mating on age has decreased and household specialization has become weaker. We also find that, overall, the relevance of socio-economic observable traits has decreased on the marriage market. Finally, preference for racial homogamy seems to have increased since the 1970s, although this conclusion is only reached when considering Whites, Blacks, Hispanics and Others/Asians as different racial categories.

In the second part, we run counterfactual experiments to assess the impact of the shifts in marital preferences on between-household income inequality. We find that, had preferences not changed since 1971, the Gini coefficient would have been 6% lower: this implies that about 20% of the rise of income inequality over the period 1971-2015 is due to changes in sorting patterns. Our results only partially confirm those of Greenwood et al. (2014), while they are closer to those of Greenwood et al. (2016). While we find that shifts in marital preferences do matter, we show that they only account for a significant but limited share of the inequality rise. Finally, when decomposing the contribution of marital preferences to the increase of the Gini coefficient, we find that changes in interactions among labor market traits can explain a large share of it. Since the 1980s, couples exhibit a weak but significant complementarity in wage rates and hours worked: this, jointly with important changes in the wage distribution, has crucially contributed to the rise of income inequality. The increased complementarity of spouses' education is also an important factor, although the decreased relevance of socio-economic observables and decreased complementarity of spouses' age are sufficient to offset its effect.

Bibliography

- [1] Alan Agresti. *Categorical Data Analysis, 3rd Edition*. John Wiley and Sons, 2013.
- [2] Maxine P Atkinson and Becky L Glass. “Marital Age Heterogamy and Homogamy, 1900 to 1980”. In: *Journal of Marriage and the Family* 47.3 (1985), pp. 685–691.
- [3] Gary S. Becker. “A Theory of Marriage: Part I”. In: *Journal of Political Economy* 81.4 (1973), pp. 813–846.
- [4] Gary S. Becker. “A Theory of Marriage: Part II”. In: *Journal of Political Economy* 82.2 (1974), S11–S26.
- [5] Gary S. Becker. *A Treatise On The Family*. Harvard University Press, 1991.
- [6] Marianne Bertrand, Emir Kamenica, and Jessica Pan. “Gender Identity and Relative Income within Households”. In: *The Quarterly Journal of Economics* 130.2 (2015), pp. 571–614.
- [7] Debra L Blackwell. “Marital Homogamy in the United States: the Influence of Individual and Paternal Education”. In: *Social Science Research* 27.2 (1998), pp. 159–188.
- [8] Gary Burtless. “Effects of Growing Wage Disparities and Changing Family Composition on the US Income Distribution”. In: *European Economic Review* 43.4 (1999), pp. 853–865.
- [9] Maria Cancian and Deborah Reed. “Assessing the Effects of Wives’ Earnings on Family Income Inequality”. In: *Review of Economics and Statistics* 80.1 (1998), pp. 73–79.
- [10] Pierre-André Chiappori, Bernard Salanié, and Yoram Weiss. *Partner Choice and the Marital College Premium: Analyzing Marital Patterns Over Several Decades*. Tech. rep. CEPR Discussion Paper, 2015.

- [11] Eugene Choo and Aloysius Siow. “Estimating a marriage matching model with spillover effects”. In: *Demography* 43.3 (2006), pp. 463–490.
- [12] Edoardo Ciscato, Alfred Galichon, and Marion Goussé. “Like Attract Like? A Structural Comparison of Homogamy Across Same-Sex and Different-Sex Households”. In: (2015).
- [13] Arnaud Dupuy and Alfred Galichon. “Personality Traits and the Marriage Market”. In: *Journal of Political Economy* 122.6 (2014), pp. 1271–1319.
- [14] Lasse Eika, Magne Mogstad, and Basit Zafar. *Educational Assortative Mating and Household Income Inequality*. Tech. rep. NBER Working Paper, 2014.
- [15] Raquel Fernández, Nezih Guner, and John Knowles. “Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality”. In: *Quarterly Journal of Economics* 120.1 (2005), pp. 273–344.
- [16] Raquel Fernández and Richard Rogerson. “Sorting and Long-Run Inequality”. In: *Quarterly Journal of Economics* 116.4 (2001), pp. 1305–1341.
- [17] Sarah Flood et al. *Integrated Public Use Microdata Series, Current Population Survey: Version 4.0*. Tech. rep. Minneapolis: University of Minnesota, 2015.
- [18] Roland G Fryer. “Guess Who’s Been Coming to Dinner? Trends in Interracial Marriage over the 20th Century”. In: *The Journal of Economic Perspectives* 21.2 (2007), pp. 71–90.
- [19] Xuanning Fu and Tim B Heaton. “Racial and Educational Homogamy: 1980 to 2000”. In: *Sociological Perspectives* 51.4 (2008), pp. 735–758.
- [20] Alfred Galichon and Bernard Salanié. *Cupid’s Invisible Hand: Social Surplus and Identification in Matching Models*. Tech. rep. SSRN Working Paper Series, 2015.
- [21] Jeremy Greenwood et al. “Marry Your Like: Assortative Mating and Income Inequality”. In: *American Economic Review* 104.5 (2014), pp. 348–353.
- [22] Jeremy Greenwood et al. “Technology and the changing family: A unified model of marriage, divorce, educational attainment, and married female labor-force participation”. In: *American Economic Journal: Macroeconomics* 8.1 (2016), pp. 1–41.
- [23] Robert A Johnson. “Religious Assortative Mating in the United States”. In: *Academic Press* (1980).

- [24] Matthijs Kalmijn. “Shifting Boundaries: Trends in Religious and Educational Homogamy”. In: *American Sociological Review* 56.6 (1991), pp. 786–800.
- [25] Matthijs Kalmijn. “Status Homogamy in the United States”. In: *American journal of Sociology* 97.2 (1991), pp. 496–523.
- [26] Michael Kremer. “How Much Does Sorting Increase Inequality?*”. In: *The Quarterly journal of economics* 112.1 (1997), pp. 115–139.
- [27] Haoming Liu and Jingfeng Lu. “Measuring the Degree of Assortative Mating”. In: *Economics Letters* 92.3 (2006), pp. 317–322.
- [28] Robert D Mare. “Five Decades of Educational Assortative Mating”. In: *American Sociological Review* 56.1 (1991), pp. 15–32.
- [29] Zhenchao Qian and Samuel H Preston. “Changes in American Marriage, 1972 to 1987: Availability and Forces of Attraction by Age and Education”. In: *American Sociological Review* 58.4 (1993), pp. 482–495.
- [30] John Schmitt. “Creating a Consistent Hourly Wage Series from the Current Population Survey’s Outgoing Rotation Group, 1979-2002”. In: *CEPR Working Paper* (2003), pp. –.
- [31] Robert Schoen and John Wooldredge. “Marriage Choices in North Carolina and Virginia, 1969-71 and 1979-81”. In: *Journal of Marriage and the Family* 51.2 (1989), pp. 465–481.
- [32] Christine R Schwartz. “Earnings Inequality and the Changing Association Between Spouses’ Earnings”. In: *American Journal of Sociology* 115.5 (2010), p. 1524.
- [33] Christine R Schwartz and Nikki L Graf. “Assortative Matching Among Same-Sex and Different-Sex Couples in the United States, 1990–2000”. In: *Demographic Research* 21.28 (2009), p. 843.
- [34] Christine R Schwartz and Robert D Mare. “Trends in Educational Assortative Marriage from 1940 to 2003”. In: *Demography* 42.4 (2005), pp. 621–646.
- [35] Lloyd S Shapley and Martin Shubik. “The Assignment Game I: The Core”. In: *International Journal of Game Theory* 1.1 (1971), pp. 111–130.

- [36] Jeroen Smits, Wout Ultee, and Jan Lammers. “Educational Homogamy in 65 countries: An Explanation of Differences in Openness Using Country-level Explanatory Variables”. In: *American Sociological Review* 63.2 (1998), pp. 264–285.
- [37] Gillian Stevens and Robert Schoen. “Linguistic Intermarriage in the United States”. In: *Journal of Marriage and the Family* 50.1 (1988), pp. 267–279.
- [38] Betsey Stevenson and Justin Wolfers. “Marriage and Divorce: Changes and Their Driving Forces”. In: *The Journal of Economic Perspectives* 21.2 (2007), pp. 27–52.
- [39] Bruce Western, Deirdre Bloome, and Christine Percheski. “Inequality Among American Families with Children, 1975 to 2005”. In: *American Sociological Review* 73.6 (2008), pp. 903–920.

Appendix

1.A. Neutrality of Optimal Matching

According to DG, the equilibrium matching is described by the function 3.1. Take the log of $\pi(x, y)$ so that

$$\log \pi(x, y) = x' \frac{A}{\sigma} y - \frac{a(x)}{\sigma} - \frac{b(y)}{\sigma}.$$

The first component is $x'By$: without the identification assumption with multiple markets described in Section 3.4, we are still able to identify $B = A/\sigma$ unequivocally. In fact, under any assumption to disentangle A from σ , a sample (X, Y) yields a unique estimate \hat{B} .

As concerns the second and third components $a(x)/\sigma$ and $b(x)/\sigma$, define $\tilde{a}(x) = \exp(a(x)/\sigma)$ and $\tilde{b}(x) = \exp(b(x)/\sigma)$. We can rewrite 3.1 as

$$\pi(x, y) = K(x, y; B) \tilde{a}(x) \tilde{b}(y)$$

and plug it into the accounting constraints:

$$\begin{aligned} f(x) &= \tilde{a}(x) \int_{\mathcal{Y}} K(x, y; B) \tilde{b}(y) dy \\ f(y) &= \tilde{b}(y) \int_{\mathcal{X}} K(x, y; B) \tilde{a}(x) dx. \end{aligned}$$

DG suggest solving this system by means of an IPFP algorithm. Notice that, we can conclude that, for a given set of parameters B , there is a unique solution given by vectors \tilde{a}^* and \tilde{b}^* , and thus a unique solution π^* .

1.B. Improvements to the Estimation Procedure.

Depending on the year, our samples may contain many individuals. However, for computational reasons, estimation can only be performed on a subset of the population. Doing so, we do not make full use of the data to compute the empirical variance-covariance matrix. If the subsamples's size is too small, this may even introduce some bias in the estimates. Since the estimation strategy relies on matching the theoretical co-moments to the empirical counterparts, we pick a random subsample whose co-moments of interest are close to those of the full sample. Hence, we use the following procedure to select the subsamples:

PROCEDURE 1. Let N be the number of couples in the population

Step 0. Compute the empirical variance-covariance $\hat{V} \equiv E[XY]$ with the full sample

Step 1. Draw a subsample of size $n < N$ and compute the empirical variance covariance matrix \hat{V}_n

Step 2. Check if $|\hat{V} - V_n| < \epsilon \times \hat{V}$ for a given level of precision ϵ .

Step 3. If Step 2 is satisfied, use V_n and the corresponding subsample to estimate the affinity matrix. Otherwise repeat Step 1-3.

1.C. Robustness Checks.

1.D. Additional Figures.

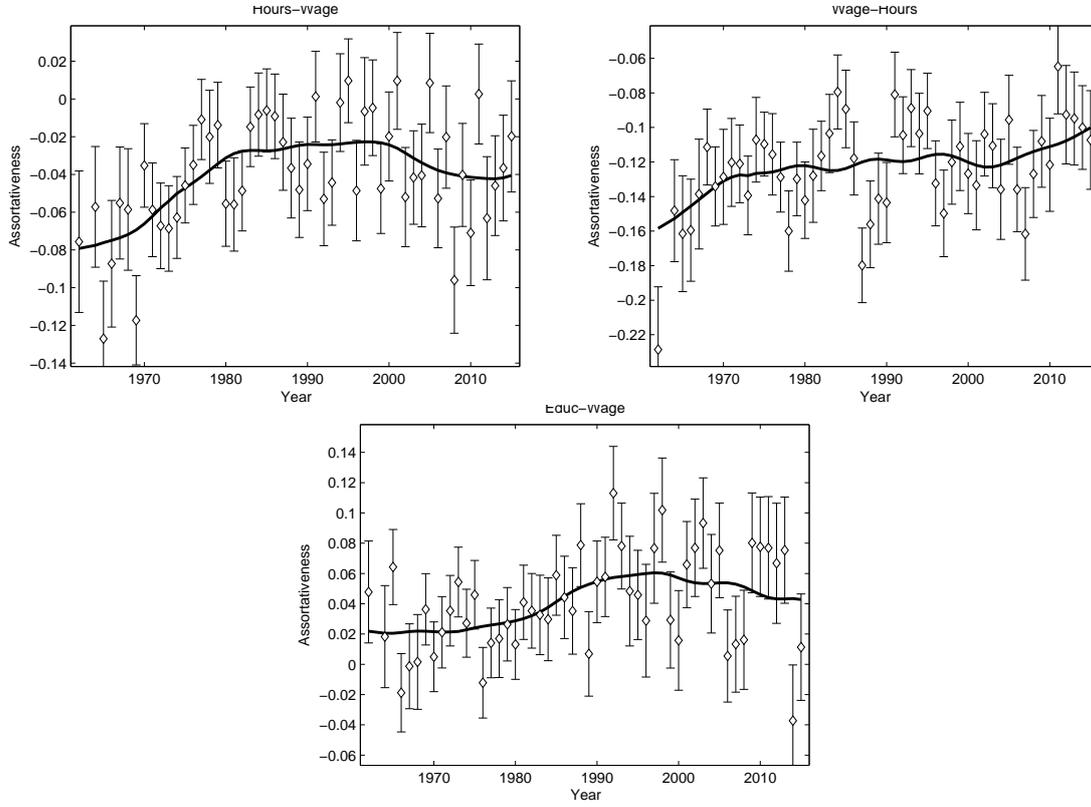
TABLE 2. Robustness Checks

Check #	Period	Age	Education	Wage	Hours	Race	Occ.	Comments
Baseline A	1962-	[23-35] ²⁵	Continuous	Topcodes; Trimming	Yes	White (incl. Hisp.) and Black	No	Fig. 3,4,5,6, 7
Baseline B	1971-	[23-35]	Continuous	Topcodes; Trimming	Yes	White, Black, Hisp. and Others	No	Fig. 7
1	1988-	[23-35]	Continuous	Topcodes; Trimming	Yes	White, Black, Hisp., Asians	No	Fig. 7
2	1962-	[23-35]	4 levels	Topcodes; Trimming	Yes	White (incl. Hisp.) and Black	No	–
3	1962-	[23-35]	5 levels	Topcodes; Trimming	Yes	White (incl. Hisp.) and Black	No	–
4	1995-	[23-35]	Continuous	Topcodes; Trimming	Yes	White (incl. Hisp.) and Black	No	Cohabiting couples. Fig. 15
5	1962-	[23-35]	Continuous	Topcodes; Trimming	Yes	1	No	Only Whites. Fig. 16
6	1962-	[23-35]	Continuous	Topcodes; Trimming	Yes	1	No	Only Blacks. Fig. 16
7	1965-	[25-35]	Continuous	Topcodes; Trimming	Yes	White (incl. Hisp.) and Black	No	Contribution to rise in inequality: 33%. Decrease in current inequality with 1965 matching : 6%
8	1971-	[23-35]	Continuous	Topcodes; Trimming	Yes	White, Black, Hisp. and Others	Yes	–
9	1962-	[23-35]	Continuous	Topcodes; Trimming; Positive	Yes	White (incl. Hisp.) and Black	No	Fig. 17
10	1962-	[23-35]	Continuous	Topcodes; Trimming;	Yes	White (incl. Hisp.) and Black	No	Childless couples. Fig. 18
11	1971-	[25-35]	Continuous	Topcodes; Trimming; Potential Income	No	White (incl. Hisp.) and Black	No	(see Bertrand, Kamenica, and Pan, 2015). Fig. 19

Note : Bold indicates main changes compared to the baseline.

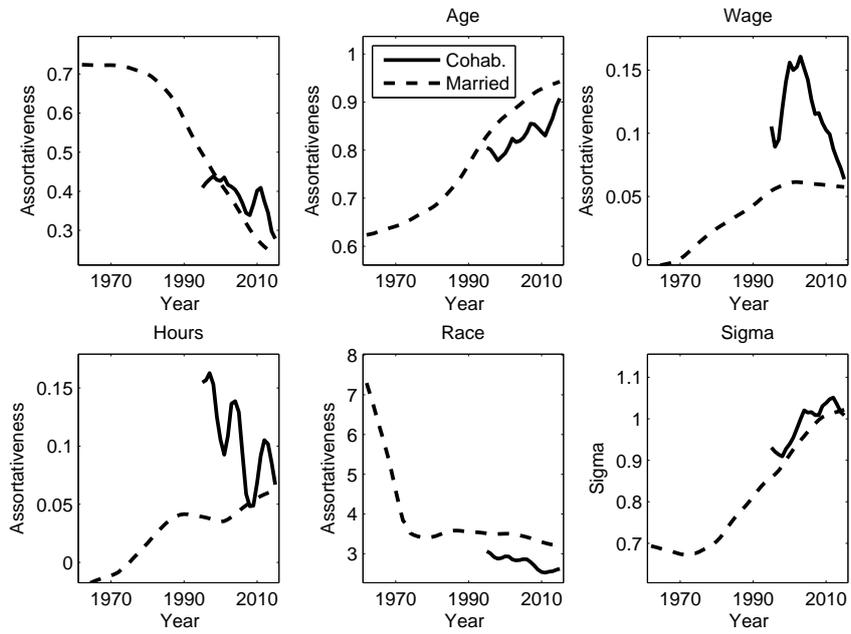
The table describes the criteria used to select 11 different samples starting from the main CPS database. These samples are used to run robustness checks throughout the paper.

FIGURE 14. Relevant cross-interactions



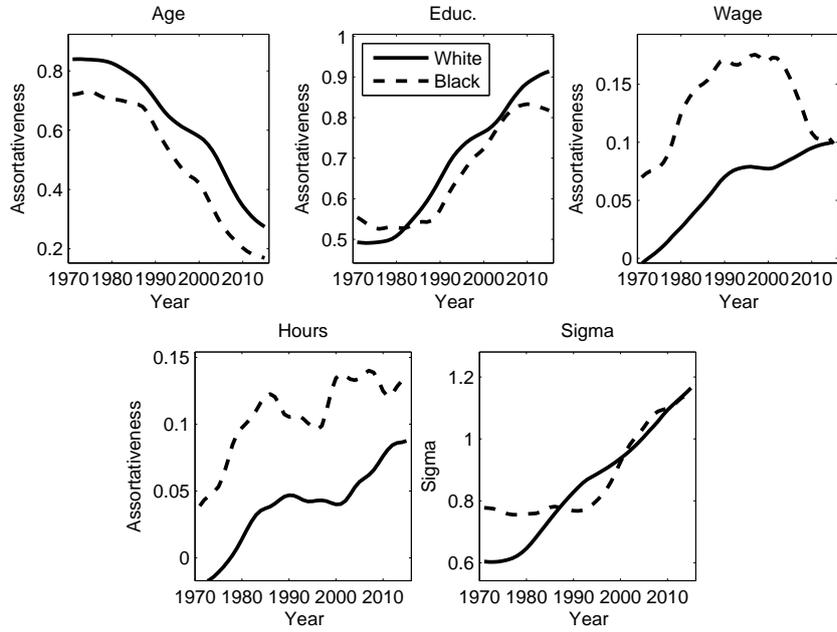
Sample used: baseline A. The figures display the estimated trends of the off-diagonal elements of the marital preference parameter matrix A that have some relevance in our decomposition exercise in Section 6.4. In the labels, the first trait is the husband's and the second is the wife's, e.g. Wage-Educ refers to the interaction between the husband's wage rate and the wife's education.

FIGURE 15. Cohabiting and married couples



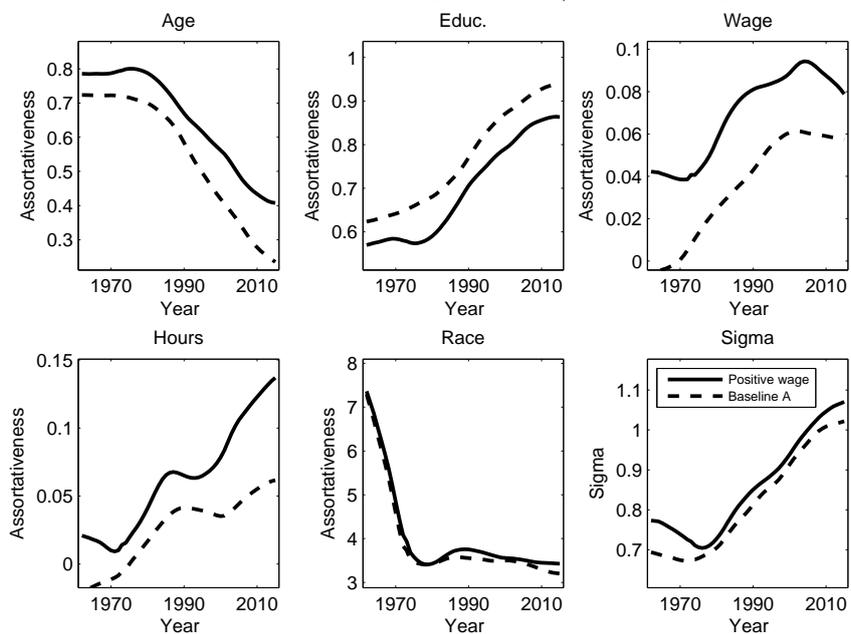
Sample used: check 4 (see Appendix 1.C). The figures compare our baseline results on estimated trends of the diagonal elements of the marital preference parameter matrix A with those obtained using a subsample of cohabitating couples. Data on cohabitating couples are only available since 1995 and the size of the sample is considerably smaller.

FIGURE 16. White and Black couples



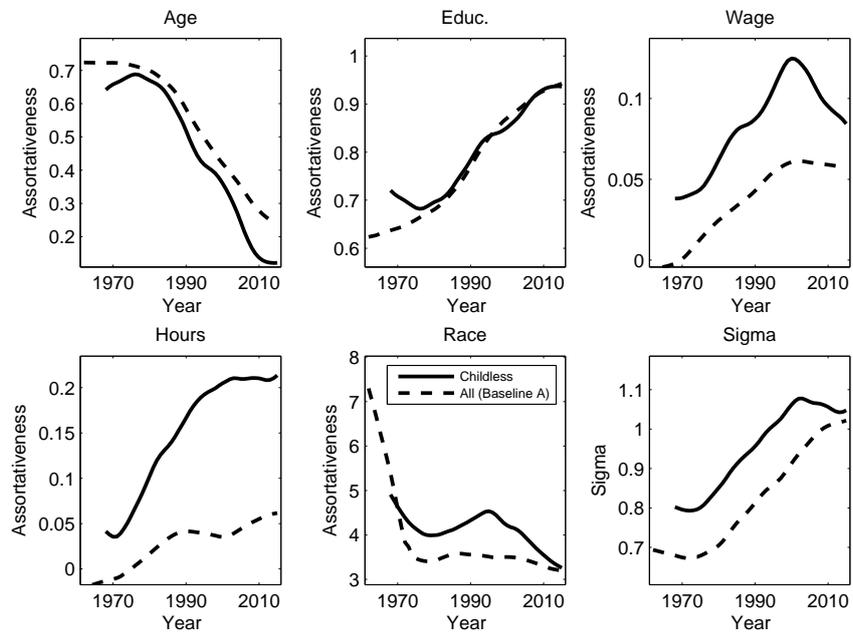
Sample used: checks 5 and 6 (see Appendix 1.C). The figures compare the estimated trends of the diagonal elements of the marital preference parameter matrix A obtained with a subsample of White couples with those obtained using a subsample of Black couples.

FIGURE 17. All couples and working couples (where both partners work)



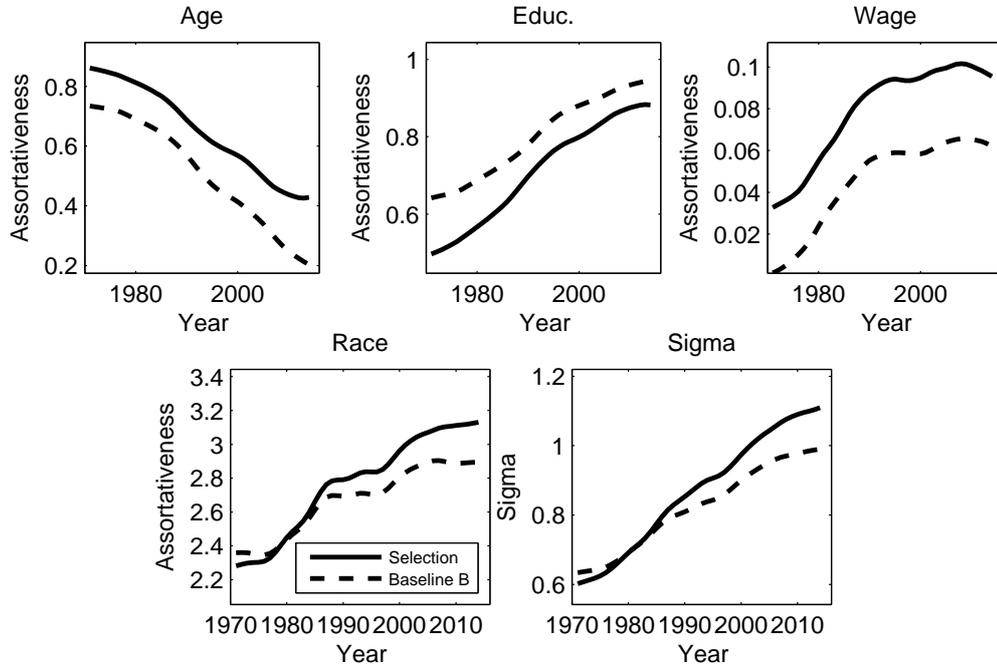
Sample used: check 9 (see Appendix 1.C). The figures compare our baseline results on estimated trends of the diagonal elements of the marital preference parameter matrix A with those obtained using a subsample of couples where both spouses work.

FIGURE 18. All couples and childless couples



Sample used: check 10 (see Appendix 1.C). The figures compare our baseline results on estimated trends of the diagonal elements of the marital preference parameter matrix A with those obtained using a subsample of childless couples.

FIGURE 19. Sorting on potential income



Sample used: check 11 (see Appendix 1.C). The figures compare our baseline results on estimated trends of the diagonal elements of the marital preference parameter matrix A with those obtained using a measure of potential income following Bertrand, Kamenica, and Pan (2015). In order to do so, we assigned a positive wage to non-working individuals by randomly drawing among a subset of observations that shared similar characteristics (education, age, geographic location, ethnicity).

Chapter 2

Intra-couple bargaining and matching: the ITU framework

Abstract

We introduce an empirical framework for models of matching with imperfectly transferable utility and unobserved heterogeneity in tastes. Our framework allows us to characterize matching equilibrium in a flexible way that includes as special cases the classical fully- and non-transferable utility models, collective models, and settings with taxes on transfers, deadweight losses, or risk aversion. We allow for the introduction of a general class of additive unobserved heterogeneity on agents' preferences. We show existence and uniqueness of an equilibrium under minimal assumptions. We then provide two algorithms to compute the equilibrium in our model. The first algorithm operates under any structure of heterogeneity in preferences; the second is more efficient, but applies only in the case in which random utilities are logit. We show that the log-likelihood of the model has a simple expression and we compute its derivatives. As an empirical illustration, we build a model of marriage with preferences over the partner type and private consumption, which we estimate on a British dataset.¹

¹This chapter is based on “Costly Concessions: An Empirical Framework for Matching with Imperfectly Transferable Utility”, written by Alfred Galichon, Scott Kominers and Simon Weber. I am indebted to Alfred and Scott for letting me use this paper as Chapter 2.

1. Introduction

The field of family economics has two principal approaches to modeling and understanding marriage patterns: *Matching models* emphasize market-level forces and take heterogeneous tastes over partners as primitives; *collective models*, by contrast, focus on the impact of intra-household bargaining. However, these two approaches have not been combined yet, because collective models necessarily include nonlinearities of a form absent from classical matching frameworks. In this paper, we develop an Imperfectly Transferable Utility (ITU) matching model with random utility that allows us to unify marriage matching with unobserved heterogeneity in tastes and the collective framework. Our setting moreover allows for the introduction of a general class of additive unobserved heterogeneity on agents' preferences, under which existence and uniqueness of an equilibrium is obtained. These models can be computed efficiently and easily estimated.

Naturally, matching models have been extensively used to model the marriage market, in which men and women with heterogeneous tastes may form pairs; this approach, pioneered in economics by Becker (1973) and Shapley and Shubik (1972), focuses mainly on matching patterns and the sharing of the surplus in a Transferable Utility (TU) setting. While appealing from a theoretical point of view, TU matching models have a significant limitation: TU models rely on the assumption there is a way to transfer utility between partners in an additive manner. Consequently, a man and a woman who match and generate a joint surplus Φ may decide on splitting this surplus between the utility of the man u and the utility of the woman v in any way such that $u + v \leq \Phi$. In this case, the bargaining frontier in the space of the partners' utilities is simply a straight line of slope -1 . However, the assumption that the bargaining frontier has this particular shape may be inappropriate—one can think of many cases in which there are nonlinearities that partially impede the transfer of utility between matched partners. Such nonlinearities arise naturally in marriage markets, where the transfers between partners might take any form (e.g. cash, favor exchanges, and change in time use or consumption patterns), and the utility cost of a concession to one partner may not exactly equal the benefit to the other. An extreme case is the Nontransferable Utility (NTU) framework (Gale and Shapley, 1962), in which there is no possibility of compensating

transfer between partners. However, although NTU matching seems well-suited to settings like school choice, where transfers are often explicitly ruled out, it is also not, in general, the most realistic assumption.

The collective model approach of Chiappori (1992), which focuses on intra-household bargaining over a potentially complex feasible utility set, cannot generally be expressed in terms of TU matching models, because imperfect substitutability in the bargaining process creates nonlinearities.² Indeed, in TU models, households are modelled as representative consumers, so that intra-household allocation of power does not affect the household’s aggregate behavior, and public good consumption in particular. In spite of their complementarity, the matching and collective approaches to modeling marriage have not yet been combined into a single empirical framework. Choo and Siow (2006) observed this issue explicitly, stating that “[their] model of marriage matching should be integrated with models of intra-household allocations”—an integration which, in over ten years since Choo and Siow (2006) were writing, has not been achieved prior to our work.

In our Imperfectly Transferable Utility framework, partners participate in a one-to-one bilateral matching market, but utility transfers within matches are not necessarily additive. This allows us to embed TU, NTU, and collective approaches to the marriage market. Our framework also makes sense for modeling labor markets—because of taxation, an employer must pay more in wages than its employees actually receive (Dupuy et al. (2017)). In contrast with prior ITU matching models, our setting allows for a compact characterization of equilibrium as a set of nonlinear equations, as well as efficient computational approaches and comparative statics in closed form. We prove existence and uniqueness of the equilibrium outcome in our ITU model with general heterogeneity in tastes. In the case that the heterogeneity is logit, we show how maximum likelihood estimation of our model can be performed in a very straightforward manner, which we illustrate by estimating a simple collective model of matching in a market with marital preferences and private consumption.

²There are exceptions—see, e.g., the model described in Bowning et al. (2014), pp. 83 and 118, in which one private good is assumed to provide the same marginal utility to both members of the household, and thus can be used to transfer utility additively.

Relation to the literature. The theory of ITU matching has (at least implicitly) been studied by a number of authors: Crawford and Knoer (1981), Kelso and Crawford (1982), and Hatfield and Milgrom (2005) found conditions and algorithms that can be used to find competitive equilibrium outcomes in ITU matching markets; they also analyzed the structure of the sets of equilibria. Kaneko (1982), Quinzii (1984), Alkan (1989), Alkan and Gale (1990), Gale (1984), and Demange and Gale (1985) provide results on the existence of equilibria and studied properties of the core. Pycia (2012) considers a general many-to-one matching setting with imperfectly transferable utility and characterizes the sharing rules that lead to pairwise alignment of preferences and existence of equilibria. Dupuy et al. (2017) study the problem of matching with linear taxes and provide comparative statics results. Legros and Newman (2007) find conditions under which positive assortativeness arises in ITU models; they apply these findings to problems of matching under uncertainty with risk aversion. Recently, Chiappori and Reny (2016) considered a similar model with risk sharing. Chade and Eeckhout (2014) extended the work of Legros and Newman (2007) to the case that agents have different risky endowments. Nöldeke and Samuelson (2015) connect ITU matching with abstract notions of convexity. Chiappori (2012) provides an illustrative example of how collective models naturally embed into ITU matching models.

However, the literature on the structural estimation of matching models has so far been restricted to the TU and NTU cases only. In the wake of the seminal work by Choo and Siow (2006), many papers have exploited heterogeneity in preferences for identification in the TU case (see Fox (2010), Chiappori, Oreffice and Quintana-Domeque (2012), Galichon and Salanié (2014), Chiappori, Salanié, and Weiss (2017), and Dupuy and Galichon (2015)). Choo and Seitz (2013) present one of the first attempts to reconcile the matching and the collective approaches, albeit still in the TU case. Other research in the collective model literature have endogenized the sharing rule, but mostly in a TU framework (see Chapters 8 and 9 in the textbook by Browning et al. (2014) for a review, and references therein, e.g. Chiappori et al. (2009) and Iyigun and Walsh (2007)). Cherchye et al. (2017) derive Afriat-style inequalities that result from ITU stability in collective models. Similar strategies have been successfully applied in the NTU case (see Dagsvik (2000), Menzel (2015), Hitsch,

Hortaçsu, and Ariely (2010), and Agarwal (2015)). To the best of our knowledge, our work is the first to provide an empirical framework for general ITU models with random utility.

Organization of the paper. The remainder of the paper is organized as follows. Section 2 provides an introduction to the ITU framework building off the classic TU case. Section 3 formally describes the model we consider, introduces important technical machinery used throughout, and provides a number of examples. Section 4 introduces heterogeneity in tastes, defines the notion of aggregate equilibrium, and relates it with the classical notion of individual stability. Then, Section 5 determines the equations characterizing the aggregate equilibrium, shows existence and uniqueness results, and provides an algorithm to find equilibria in our framework. Section 6 deals with the important special case of logit heterogeneity, providing a more efficient algorithm for find equilibria in that case, and discussing maximum likelihood estimation. Section 7 concludes. All proofs are presented in appendix 2.A. The appendix also contains an illustrative example, and some additional results.

2. Prelude: From TU matching to ITU matching

We start with a brief overview of the structure of our model, which we hope will be particularly useful for readers who have already some degree of familiarity with TU matching models. To guide intuition, we start with the classical TU model, and show how it extends to the more general ITU model. Although less popular than the more restrictive TU and NTU models, ITU models have been studied in various forms in the literature (see, e.g., Crawford and Knoer (1981), Kelso and Crawford (1982), Alkan (1989), Chapter 9 of Roth and Sotomayor (1990), and Hatfield and Milgrom (2005)). However, unlike in prior work, our presentation introduces ITU matching in a form that at the same time is general enough to embed both the TU and the NTU models, while still being amenable to the introduction of additive unobserved heterogeneity in preferences.

2.1. The TU matching model. We first recall the basics of the Transferable Utility model. In this model, it is assumed that there are finite sets \mathcal{I} and \mathcal{J} of men and women.

If a man $i \in \mathcal{I}$ and a woman $j \in \mathcal{J}$ decide to match, they respectively enjoy utilities α_{ij} and γ_{ij} ; the vectors α and γ are primitives of the model.

If man i and woman j match, they also may agree on a transfer w_{ij} (determined at equilibrium) from the woman to the man (positive or negative), so that their utilities after transfer are respectively $\alpha_{ij} + w_{ij}$ and $\gamma_{ij} - w_{ij}$. If i and j choose to remain unmatched, they enjoy reservation utilities \mathcal{U}_{i0} and \mathcal{V}_{0j} , which are exogenous.

Let μ_{ij} encode the “matching” (also determined at equilibrium), which is equal to 1 if i and j are matched, and 0 otherwise. Hence, a matching should satisfy the *feasibility conditions*

$$(\mathbf{F}) \quad \begin{cases} \mu_{ij} \in \{0, 1\} \\ \sum_{j \in \mathcal{J}} \mu_{ij} \leq 1 \\ \sum_{i \in \mathcal{I}} \mu_{ij} \leq 1, \end{cases}$$

Let u_i and v_j be the indirect payoffs of man i and woman j , respectively. These quantities are determined at equilibrium, and we have $u_i = \max_{j \in \mathcal{J}} \{\alpha_{ij} + w_{ij}, \mathcal{U}_{i0}\}$ and $v_j = \max_{i \in \mathcal{I}} \{\gamma_{ij} - w_{ij}, \mathcal{V}_{0j}\}$, which implies in particular that for any i and j , the inequalities $u_i \geq \alpha_{ij} + w_{ij}$ and $v_j \geq \gamma_{ij} - w_{ij}$ jointly hold, implying that $u_i + v_j \geq \alpha_{ij} + \gamma_{ij}$ should hold for every $i \in \mathcal{I}$ and $j \in \mathcal{J}$. Likewise, $u_i \geq \mathcal{U}_{i0}$ and $v_j \geq \mathcal{V}_{0j}$ should hold for all i and j . Thus, the equilibrium payoffs should satisfy the *stability conditions*

$$(\mathbf{S}) \quad \begin{cases} u_i + v_j \geq \alpha_{ij} + \gamma_{ij} \\ u_i \geq \mathcal{U}_{i0} \\ v_j \geq \mathcal{V}_{0j}. \end{cases}$$

Finally, we relate the equilibrium matching μ and the equilibrium payoffs (u, v) . If $\mu_{ij} > 0$, then $\mu_{ij} = 1$ and i and j are matched, so the first line of **(S)** should hold as an equality. On the contrary, if $\sum_j \mu_{ij} < 1$, then $\sum_j \mu_{ij} = 0$, so i is unmatched and $u_i = \mathcal{U}_{i0}$. Similar conditions hold for j . To summarize, the equilibrium quantities are related by the

following set of *complementary slackness* conditions:

$$(\mathbf{CS}) \quad \begin{cases} \mu_{ij} > 0 \implies u_i + v_j = \alpha_{ij} + \gamma_{ij} \\ \sum \mu_{ij} < 1 \implies u_i = \mathcal{U}_{i0} \\ \sum \mu_{ij} < 1 \implies v_j = \mathcal{V}_{0j} \end{cases} .$$

Following the classical definition, (μ, u, v) is an *equilibrium outcome* in the TU matching model if the feasibility conditions **(F)**, stability conditions **(S)**, and complementary slackness conditions **(CS)** are met. The characterization of the solutions to that problem in terms of linear programming is well known (see, e.g., Chapter 8 of Roth and Sotomayor). The equilibrium outcomes (μ, u, v) are such that μ maximizes the utilitarian social welfare $\sum_{ij} \mu_{ij} (\alpha_{ij} + \gamma_{ij} - \mathcal{U}_{i0} - \mathcal{V}_{0j})$ with respect to $\mu \geq 0$ subject to $\sum_j \mu_{ij} \leq 1$ and $\sum_i \mu_{ij} \leq 1$, which is the primal problem; and the payoffs (u, v) are the solution of the corresponding dual problem, hence they minimize $\sum_i u_i + \sum_j v_j$ subject to $u_i + v_j \geq \alpha_{ij} + \gamma_{ij}$, and $u_i \geq \mathcal{U}_{i0}$, $v_j \geq \mathcal{V}_{0j}$. However, this interpretation in terms of optimality is very specific to the present TU case, as discussed in appendix 2.C.

2.2. The ITU matching model. The ITU matching model is a natural generalization of the TU model. If man $i \in \mathcal{I}$ and woman $j \in \mathcal{J}$ agree to match with transfer w_{ij} , their utilities after transfer are respectively $\mathcal{U}_{ij}(w_{ij})$ and $\mathcal{V}_{ij}(w_{ij})$, where $\mathcal{U}_{ij}(\cdot)$ is a continuous and nondecreasing function and $\mathcal{V}_{ij}(\cdot)$ is a continuous and nonincreasing function. (Note that in the specialization to the TU case, $\mathcal{U}_{ij}(w_{ij}) = \alpha_{ij} + w_{ij}$ and $\mathcal{V}_{ij}(w_{ij}) = \gamma_{ij} - w_{ij}$). If i or j opt to remain unmatched, they enjoy respective payoffs $\mathcal{U}_{i0} \in \mathbb{R}$ and $\mathcal{V}_{0j} \in \mathbb{R}$, which are exogenous reservation utilities. As before, the matching μ has term μ_{ij} equal to 1 if i and j are matched, 0 otherwise; clearly, the set of conditions **(F)** defining feasible matchings is unchanged.

In equilibrium, the indirect payoffs are now given by $u_i = \max_{j \in \mathcal{J}} \{\mathcal{U}_{ij}(w_{ij}), \mathcal{U}_{i0}\}$ and $v_j = \max_{i \in \mathcal{I}} \{\mathcal{V}_{ij}(w_{ij}), \mathcal{V}_{0j}\}$, which implies in particular that for any i and j , the inequalities $u_i \geq \mathcal{U}_{ij}(w_{ij})$ and $v_j \geq \mathcal{V}_{ij}(w_{ij})$ jointly hold. However, in contrast to the TU case, adding up the utility inequalities does not cancel out the w_{ij} term. As a way out of this problem, we introduce in section 3 a function $D_{ij}(u, v)$, called *distance-to-frontier function*, which is

non-decreasing in u and v and has $D_{ij}(\mathcal{U}_{ij}(w), \mathcal{V}_{ij}(w)) = 0$ for all w . Then $u_i \geq \mathcal{U}_{ij}(w_{ij})$ and $v_j \geq \mathcal{V}_{ij}(w_{ij})$ jointly imply that $D_{ij}(u_i, v_j) \geq D_{ij}(\mathcal{U}_{ij}(w), \mathcal{V}_{ij}(w)) = 0$. Hence the equilibrium payoffs in an ITU model must satisfy the *nonlinear stability conditions*

$$(\mathbf{S}') \quad \begin{cases} D_{ij}(u_i, v_j) \geq 0 \\ u_i \geq \mathcal{U}_{i0} \\ v_j \geq \mathcal{V}_{0j}, \end{cases}$$

and the *nonlinear complementary slackness conditions*

$$(\mathbf{CS}') \quad \begin{cases} \mu_{ij} > 0 \implies D_{ij}(u_i, v_j) = 0 \\ \sum \mu_{ij} < 1 \implies u_i = \mathcal{U}_{i0} \\ \sum \mu_{ij} < 1 \implies v_j = \mathcal{V}_{0j}. \end{cases}$$

A triple (μ, u, v) is an equilibrium outcome in the matching model with Imperfectly Transferable Utility whenever conditions (\mathbf{F}) , (\mathbf{S}') and (\mathbf{CS}') are met.

3. Framework

We now give a complete description of the framework introduced in the previous section. We consider a population of men indexed by $i \in \mathcal{I}$ and women indexed by $j \in \mathcal{J}$ who may decide either to remain single or to form heterosexual pairs. It will be assumed that if i and j match, then they bargain over utility outcomes (u_i, v_j) lying within a *feasible set* \mathcal{F}_{ij} , the structure of which is described in section 3.1. If i and j decide to remain single, then they receive their respective reservation utilities \mathcal{U}_{i0} and \mathcal{V}_{0j} .

An *outcome* (formally defined in section 3.2) is comprised of (i) a *matching* $\mu_{ij} \in \{0, 1\}$, which is a binary variable equal to 1 if and only if i and j are matched; and (ii) the *payoffs* u_i and v_j , which are in \mathcal{F}_{ij} if i and j are matched, and are equal to the reservation utilities when i and j are unmatched.

Our concept of equilibrium, which we formalize in definition 4 of section 3.2, is based on *pairwise stability*: an outcome (μ, u, v) is an equilibrium outcome if there is no blocking coalition, i.e., if all the payoffs are above reservation value, and if there is no pair (i, j) of individuals who would be able to reach a feasible pair of utilities dominating u_i and v_j .

We give a number of examples of our model in section 3.3, including the classic TU and NTU models, as well as several intermediate cases of interest.

3.1. The feasible bargaining sets. If man $i \in \mathcal{I}$ and a woman $j \in \mathcal{J}$ are matched, then they bargain over a set of feasible utilities $(u_i, v_j) \in \mathcal{F}_{ij}$. We begin by describing the pairwise bargaining sets \mathcal{F}_{ij} ; then, we provide two different—but equivalent—useful descriptions. First, we represent the feasible sets “implicitly,” by describing the bargaining frontier as the set of zeros of a function, $\{(u_i, v_j) \in \mathbb{R}^2 : D_{\mathcal{F}_{ij}}(u_i, v_j) = 0\}$. Next, we represent the feasible sets “explicitly,” by their frontiers as the range of a map: $\{(\mathcal{U}_{\mathcal{F}_{ij}}(w_{ij}), \mathcal{V}_{\mathcal{F}_{ij}}(w_{ij})) : w_{ij} \in \mathbb{R}\}$.

3.1.1. *Assumptions on the feasible sets.* The following natural assumptions on the geometry of the sets \mathcal{F}_{ij} is employed extensively throughout the paper.

DEFINITION 1. The set \mathcal{F}_{ij} is a *proper bargaining set* if the three following conditions are met:

- (i) \mathcal{F}_{ij} is closed and nonempty.
- (ii) \mathcal{F}_{ij} is *lower comprehensive*: if $u' \leq u$, $v' \leq v$, and $(u, v) \in \mathcal{F}_{ij}$, then $(u', v') \in \mathcal{F}_{ij}$.
- (iii) \mathcal{F}_{ij} is *bounded above*: Assume $u_n \rightarrow +\infty$ and v_n bounded below then for N large enough $(u_n, v_n) \notin \mathcal{F}$ for $n \geq N$; similarly for u_n bounded below and $v_n \rightarrow +\infty$.

Some comments on the preceding requirements are useful at this stage. The closedness of \mathcal{F}_{ij} is classically needed for efficient allocations to exist. The fact that \mathcal{F}_{ij} is lower comprehensive is equivalent to free disposal; in particular, it rules out the case in which \mathcal{F}_{ij} has finite cardinality. The scarcity property rules out the possibility that both partners can obtain arbitrarily large payoffs. The fact that \mathcal{F}_{ij} is nonempty, combined with condition (ii), implies that if both partners’ demands are low enough, they can always be fulfilled. Finally, it is worth pointing out that these are the only restrictions that we shall impose on the bargaining sets; in particular, we do not require them to be convex sets.

3.1.2. *Implicit representation of the bargaining frontier.* We provide a first representation of the set \mathcal{F}_{ij} as the lower level set of a function D_{ij} , which we have called “distance-to-frontier function” because $D_{\mathcal{F}_{ij}}(u, v)$ measures the signed distance (up to a factor $\sqrt{2}$) of

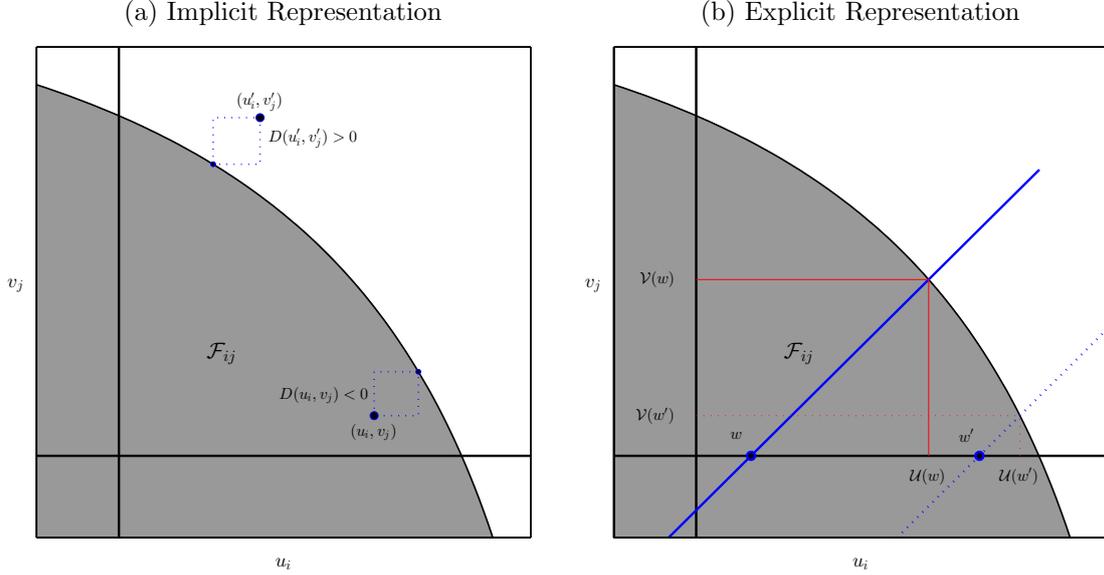


FIGURE 1. Implicit and explicit representations of the bargaining set \mathcal{F}_{ij} .

(u, v) from the bargaining frontier of \mathcal{F}_{ij} , when running along the diagonal. (See figure 1a.) $D_{\mathcal{F}_{ij}}(u, v)$ is positive if (u, v) is outside of the feasible set, and negative if (u, v) is in the interior of the feasible set; its value is 0 at the frontier. Formally:

DEFINITION 2. The distance-to-frontier function $D_{\mathcal{F}_{ij}} : \mathbb{R}^2 \rightarrow \mathbb{R}$ of a proper bargaining set \mathcal{F}_{ij} is defined by

$$(3.1) \quad D_{\mathcal{F}_{ij}}(u, v) = \min \{z \in \mathbb{R} : (u - z, v - z) \in \mathcal{F}_{ij}\}.$$

The function $D_{\mathcal{F}_{ij}}$ defined by (3.1) exists: indeed, the set $\{z \in \mathbb{R} : (u - z, v - z) \in \mathcal{F}_{ij}\}$ is closed because \mathcal{F}_{ij} is closed, bounded above because \mathcal{F}_{ij} is bounded above, and nonempty by condition (i) in definition 1; hence the minimum in (3.1) exists. By the definition of $D_{\mathcal{F}_{ij}}$, we have $\mathcal{F}_{ij} = \{(u, v) \in \mathbb{R}^2 : D_{\mathcal{F}_{ij}}(u, v) \leq 0\}$, and $D_{\mathcal{F}_{ij}}(u, v) = 0$ if and only if (u, v) lies on the frontier of \mathcal{F}_{ij} . The quantity $D_{\mathcal{F}_{ij}}(u, v)$ is interpreted as the distance (positive or negative) between (u, v) and the frontier of \mathcal{F}_{ij} along the diagonal. In particular, $D_{\mathcal{F}_{ij}}(a + u, a + v) = a + D_{\mathcal{F}_{ij}}(u, v)$ for any real a, u and v . By the same token, if $D_{\mathcal{F}_{ij}}$ is differentiable at (u, v) , then $\partial_u D_{\mathcal{F}_{ij}} + \partial_v D_{\mathcal{F}_{ij}} = 1$. The following lemma summarizes important properties of $D_{\mathcal{F}_{ij}}$.

LEMMA 1. Let \mathcal{F}_{ij} be a proper bargaining set. Then:

(i) $\mathcal{F}_{ij} = \{(u, v) \in \mathbb{R}^2 : D_{\mathcal{F}_{ij}}(u, v) \leq 0\}$.

(ii) For every $(u, v) \in \mathbb{R}^2$, $D_{\mathcal{F}_{ij}}(u, v) \in (-\infty, +\infty)$.

(iii) $D_{\mathcal{F}_{ij}}$ is \gg -isotone, meaning that $(u, v) \leq (u', v')$ implies $D_{\mathcal{F}_{ij}}(u, v) \leq D_{\mathcal{F}_{ij}}(u', v')$; and $u < u'$ and $v < v'$ jointly imply $D_{\mathcal{F}_{ij}}(u, v) < D_{\mathcal{F}_{ij}}(u', v')$.

(iv) $D_{\mathcal{F}_{ij}}$ is continuous.

(v) $D_{\mathcal{F}_{ij}}(a + u, a + v) = a + D_{\mathcal{F}_{ij}}(u, v)$.

3.1.3. *Explicit representation of the bargaining frontier.* We now give an explicit parametrization of the bargaining frontier, which will be useful in particular in section 5.

Given two utilities (u, v) such that $D_{\mathcal{F}_{ij}}(u, v) = 0$, let us introduce the *wedge* w to be the difference $w = u - v$.

DEFINITION 3. Define $\mathcal{U}_{\mathcal{F}_{ij}}(w)$ and $\mathcal{V}_{\mathcal{F}_{ij}}(w)$ as the values of u and v such that

$$(3.2) \quad D_{\mathcal{F}_{ij}}(u, v) = 0 \text{ and } w = u - v.$$

See figure 1b. Definition 3 (and the existence of the functions $\mathcal{U}_{\mathcal{F}_{ij}}$ and $\mathcal{V}_{\mathcal{F}_{ij}}$) is motivated by the following result.

LEMMA 2. Let \mathcal{F}_{ij} be a proper bargaining set. There are two 1-Lipschitz functions $\mathcal{U}_{\mathcal{F}_{ij}}$ and $\mathcal{V}_{\mathcal{F}_{ij}}$ defined on a nonempty open interval $(\underline{w}_{ij}, \bar{w}_{ij})$ such that $\mathcal{U}_{\mathcal{F}_{ij}}$ is nondecreasing and $\mathcal{V}_{\mathcal{F}_{ij}}$ is nonincreasing, and such that the set of (u, v) such that $D_{\mathcal{F}_{ij}}(u, v) = 0$ is given by $\{(\mathcal{U}_{\mathcal{F}_{ij}}(w), \mathcal{V}_{\mathcal{F}_{ij}}(w)) : w \in (\underline{w}_{ij}, \bar{w}_{ij})\}$. Furthermore, $\mathcal{U}_{\mathcal{F}_{ij}}(w)$ and $\mathcal{V}_{\mathcal{F}_{ij}}(w)$ are the unique values of u and v solving (3.2), and they are given by

$$(3.3) \quad \mathcal{U}_{\mathcal{F}_{ij}}(w) = -D_{\mathcal{F}_{ij}}(0, -w), \text{ and } \mathcal{V}_{\mathcal{F}_{ij}}(w) = -D_{\mathcal{F}_{ij}}(w, 0).$$

Whenever $\mathcal{U}_{\mathcal{F}_{ij}}$ and $\mathcal{V}_{\mathcal{F}_{ij}}$ are differentiable, it is easy to see that $\mathcal{U}'_{\mathcal{F}_{ij}}(w) = \partial_v D_{\mathcal{F}_{ij}}(0, -w)$ and $\mathcal{V}'_{\mathcal{F}_{ij}}(w) = -\partial_u D_{\mathcal{F}_{ij}}(w, 0)$. Further, as $\mathcal{U}_{\mathcal{F}_{ij}}(w)$ is increasing and 1-Lipschitz, \bar{w}_{ij} is finite if and only if the maximal utility u obtainable by the man for some feasible $(u, v) \in \mathcal{F}_{ij}$

is finite. Similarly, w_{ij} is finite if and only if the maximal utility v obtainable by the woman for some feasible $(u, v) \in \mathcal{F}_{ij}$ is finite.

3.2. Basic model. Having established the structure of the feasible bargains among matched couples, we describe the matching process. Men and women may form (heterosexual) pairs or decide to remain unmatched. If i (resp. j) decides to remain unmatched, he (resp. she) gets reservation utility \mathcal{U}_{i0} (resp. \mathcal{V}_{0j}). If i and j decide to match with each other, they bargain over a set \mathcal{F}_{ij} of feasible payoffs (u, v) , where \mathcal{F}_{ij} is a proper bargaining set, whose associated distance-to-frontier function is denoted $D_{ij} := D_{\mathcal{F}_{ij}}$ and whose functions $\mathcal{U}_{\mathcal{F}_{ij}}$ and $\mathcal{V}_{\mathcal{F}_{ij}}$ are respectively denoted \mathcal{U}_{ij} and \mathcal{V}_{ij} . We denote by u_i (resp. v_j) the equilibrium outcome utility of man i (resp. woman j). At equilibrium, we must have $u_i \geq \mathcal{U}_{i0}$ and $v_j \geq \mathcal{V}_{0j}$ as it is always possible to leave an arrangement which yields less than the reservation utility. Similarly, at equilibrium, $D_{ij}(u_i, v_j) \geq 0$ must hold for every i and j ; indeed, if this were not the case, there would be a pair (i, j) such that (u_i, v_j) is in the strict interior of the feasible set \mathcal{F}_{ij} , so that there would exist payoffs $u' \geq u_i$ and $v' \geq v_j$ (with at least one strict inequality) and $(u', v') \in \mathcal{F}_{ij}$, which would imply that i and j can be better off by matching together. Let μ_{ij} be an indicator variable which is equal to 1 if i and j are matched, and 0 otherwise. If $\mu_{ij} = 1$, we require that (u_i, v_j) be feasible, that is $D_{ij}(u_i, v_j) \leq 0$, hence equality should hold.

Combining the conditions just described, we are ready to define equilibrium in our ITU matching model. We call this equilibrium “individual” to distinguish it from the concept of “aggregate” equilibrium we introduce in section 4.

DEFINITION 4 (Individual Equilibrium). The triple $(\mu_{ij}, u_i, v_j)_{i \in \mathcal{I}, j \in \mathcal{J}}$ is an *individual equilibrium outcome* if the following three conditions are met:

- (i) $\mu_{ij} \in \{0, 1\}$, $\sum_j \mu_{ij} \leq 1$ and $\sum_i \mu_{ij} \leq 1$;
- (ii) for all i and j , $D_{ij}(u_i, v_j) \geq 0$, with equality if $\mu_{ij} = 1$;
- (iii) $u_i \geq \mathcal{U}_{i0}$ and $v_j \geq \mathcal{V}_{0j}$, with equality respectively if $\sum_j \mu_{ij} = 0$, and if $\sum_i \mu_{ij} = 0$.

The vector $(\mu_{ij})_{i \in \mathcal{I}, j \in \mathcal{J}}$ is an *individual equilibrium matching* if and only if there exists a pair of vectors $(u_i, v_j)_{i \in \mathcal{I}, j \in \mathcal{J}}$ such that (μ, u, v) is an individual equilibrium outcome.

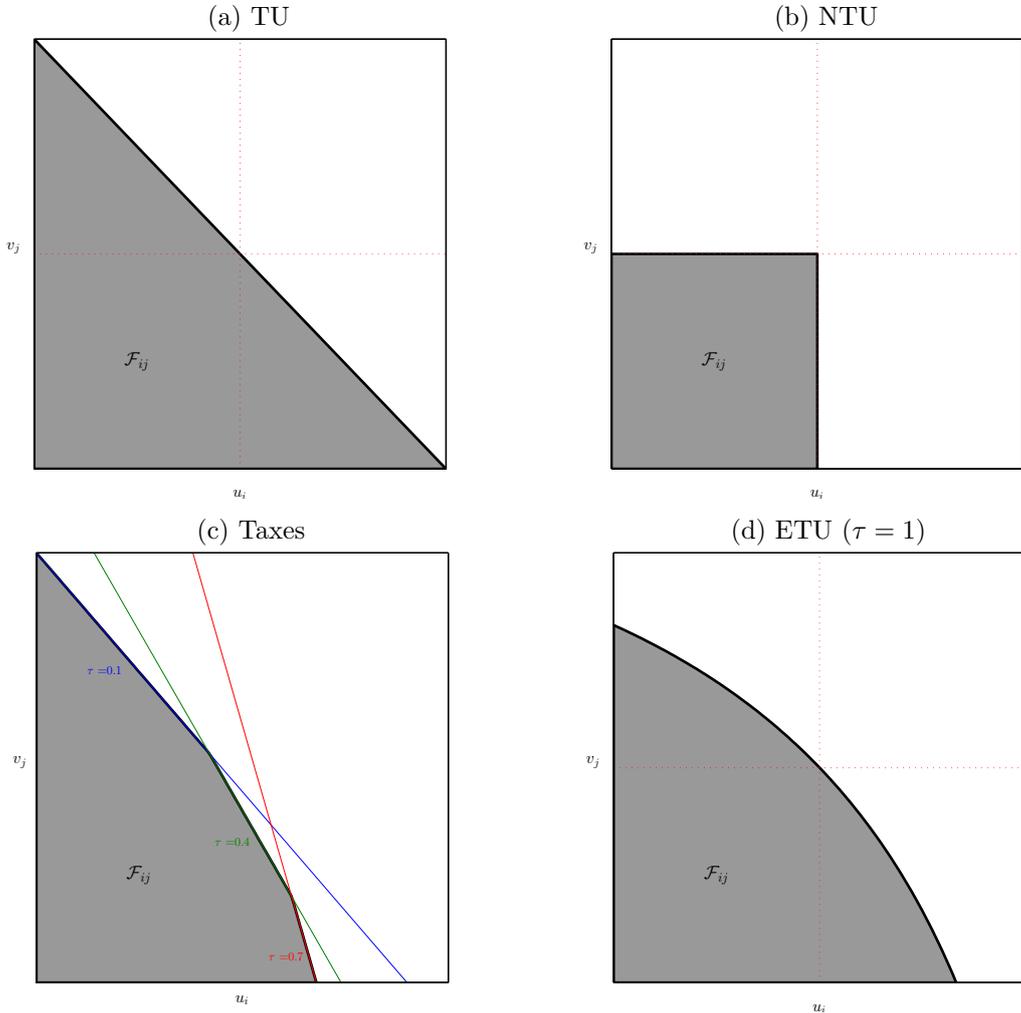


FIGURE 2. Examples of bargaining sets.

As we detail in the next section, our setting embeds the TU and the NTU matching models, as well as many other matching frameworks.

3.3. Example Specifications. Now, we provide examples of specifications of frontiers \mathcal{F} (or equivalently, distance-to-frontier functions D) that illustrate a number of applications encompassed by our framework. See also appendix 2.B.

3.3.1. *Matching with Transferable Utility (TU).* The classical TU matching model has been widely used in economics—it is the cornerstone of Becker’s marriage model, which has found applications in labor markets, marriage markets, and housing markets (Shapley and

Shubik, 1971; Becker, 1973). To recover the TU model in our framework, we take

$$(3.4) \quad \mathcal{F}_{ij} = \{(u, v) \in \mathbb{R}^2 : u + v \leq \Phi_{ij}\},$$

that is, for some (potential) surplus matrix Φ , the partners can additively share the quantity Φ_{ij} , which is interpreted as a *joint surplus* (see figure 2a). The Pareto efficient payoffs will be such that $u + v = \Phi_{ij}$. In this setting, utility is perfectly transferable: if one partner gives up one unit of utility, the other partner fully appropriates it. (In the specification of section 2.1, we have $\Phi_{ij} = \alpha_{ij} + \gamma_{ij}$.)

It is easily verified that in the TU case,

$$(3.5) \quad D_{ij}(u, v) = \frac{u + v - \Phi_{ij}}{2},$$

and as a result, $\mathcal{U}_{ij}(w) = (\Phi_{ij} + w)/2$, and $\mathcal{V}_{ij}(w) = (\Phi_{ij} - w)/2$.

3.3.2. Matching with Non-Transferable Utility (NTU). Equally important is the NTU matching model,³ which has frequently been used to model school choice markets and centralized job assignment. In this case, utility is not transferable at all, and the maximum utility that each partner can obtain is fixed and does not depend on what the other partner gets. Like the TU model, we can embed the NTU model in our ITU framework: in this case,

$$(3.6) \quad \mathcal{F}_{ij} = \{(u, v) \in \mathbb{R}^2 : u \leq \alpha_{ij}, v \leq \gamma_{ij}\},$$

which means that the only efficient pair of payoffs has $u = \alpha_{ij}$ and $v = \gamma_{ij}$ (see figure 2b). It is easily checked that

$$(3.7) \quad D_{ij}(u, v) = \max\{u - \alpha_{ij}, v - \gamma_{ij}\},$$

and we have $\mathcal{U}_{ij}(w) = \min\{\alpha_{ij}, w + \gamma_{ij}\}$ and $\mathcal{V}_{ij}(w) = \min\{\alpha_{ij} - w, \gamma_{ij}\}$.

Our notion of equilibrium matching (implied by definition 4) when the feasible set is given by (3.6) can be showed to coincide with NTU stability in the sense of Gale and Shapley.

³This model is closely connected to the model of Gale and Shapley (1962); but we depart from that classical setting in allowing for free disposal. However, as we argue at the end of the paragraph, both models yield the same set of stable matchings.

Indeed, it is shown elsewhere (see theorem 6 of Galichon and Hsieh (2017) that (i) given a matching μ which is stable in the sense of Gale and Shapley, then defining the outcome payoffs of i and j by $u_i = \sum_j \mu_{ij} \alpha_{ij}$, and $v_j = \sum_j \mu_{ij} \gamma_{ij}$, it follows that (μ_{ij}, u_i, v_j) is an equilibrium payoff in the sense of our definition 4; and that (ii) conversely, if (μ_{ij}, u_i, v_j) equilibrium payoff in the sense of definition 4, then (μ_{ij}) is a stable matching in the sense of Gale and Shapley.

3.3.3. Matching with a convex tax schedule. Our framework embeds matching with non-linear taxes, and our formulas take a convenient form when the tax schedule is convex (see Dupuy et al. (2017)). Assume i is an employee, receiving gross wage w_{ij} from employer j . Assume that the utility of the employee is equal to α_{ij} plus net (after-tax) wage, while the profit of the firm equals γ_{ij} minus gross wage.

Assume the tax thresholds are given by t^1, t^2, \dots, t^K , where no tax is due below t^1 , so that the marginal rate before that threshold is $\tau^0 = 0$. Assume income between thresholds t^k and t^{k+1} is taxed at rate τ^k , and income above t^K is taxed at rate τ^K . It is assumed that the tax rates are increasing, which means that the tax schedule is convex.

Let α_{ij}^k be the utility of the worker with a gross wage t^k . One has $\alpha_{ij}^0 = \alpha_{ij}$, and $\alpha_{ij}^{k+1} = \alpha_{ij}^k + (1 - \tau^k) (t^{k+1} - t^k)$, and, more generally, the utility of a worker with gross wage w_{ij} is given by $u_i = \min \left\{ \alpha_{ij}^k + (1 - \tau^k) w_{ij}, k = 0, \dots, K \right\}$, while the profit of the firm is given by $v_j = \gamma_{ij} - w_{ij}$. The feasible set is given by

$$\mathcal{F}_{ij} = \left\{ (u, v) \in \mathbb{R}^2 : \forall k \in \{0, \dots, K\}, u_i \leq \alpha_{ij}^k + (1 - \tau^k) (\gamma_{ij} - v_j) \right\},$$

and it is straightforward to verify that the distance-to-frontier function is given by

$$(3.8) \quad D_{ij}(u, v) = \max_{k \in \{0, \dots, K\}} \left\{ \frac{u - \alpha_{ij}^k + (1 - \tau^k) (v - \gamma_{ij})}{2 - \tau^k} \right\}.$$

See figure 2c.

3.3.4. Collective Models. Finally, we consider a situation in which a man i and a woman j have respective utilities $\mathbf{u}_i(c_i, l_i, g)$ and $\mathbf{v}_j(c_j, l_j, g)$ which depend on private consumptions c_i and c_j , private leisure l_i and l_j , and a public good g . The wages of man i and of woman j are respectively denoted w_i and w_j , and the price of the public good is denoted p . The

budget constraint of the household is therefore $c_i + c_j + w_i l_i + w_j l_j + pg = B_{ij}$, where the household budget is $B_{ij} = (w_i + w_j) T$, and T is the total amount of time available to each partner. The “collective” approach initiated by Chiappori (1992) assumes that the outcome (u, v) lies on the Pareto frontier of the feasible set \mathcal{F}_{ij} of achievable utilities, where the feasible set is given by

$$\mathcal{F}_{ij} = \left\{ \begin{array}{l} (u, v) \in \mathbb{R}^2 : u \leq \mathbf{u}_{ij}(c_i, l_i, g), v \leq \mathbf{v}_{ij}(c_j, l_j, g), \\ c_i + c_j + w_i l_i + w_j l_j + pg = B_{ij}, \\ c_i, c_j \geq 0, g \geq 0, \text{ and } 0 \leq l_i, l_j \leq T. \end{array} \right\}.$$

In this case, one can easily verify that the distance-to-frontier function is given by

$$\begin{aligned} D_{ij}(u, v) &= \min_{\substack{c_i, c_j, g \geq 0 \\ 0 \leq l_i, l_j \leq T}} \max(u - \mathbf{u}_{ij}(c_i, l_i, g), v - \mathbf{v}_{ij}(c_j, l_j, g)) \\ \text{s.t.} \quad & c_i + c_j + w_i l_i + w_j l_j + pg = B_{ij} \end{aligned}$$

In the very particular case when there is no utility for leisure and no public good⁴, and we assume log-utility of consumption, $\mathbf{u}_{ij}(c_i) = \alpha_{ij} + \tau_{ij} \log c_i$ and $\mathbf{v}_{ij}(c_j) = \gamma_{ij} + \tau_{ij} \log c_j$, it is possible to obtain D_{ij} in closed form:

$$(3.9) \quad D_{ij}(u, v) = \tau_{ij} \log \left(\frac{\exp\left(\frac{u - \alpha_{ij}}{\tau_{ij}}\right) + \exp\left(\frac{v - \gamma_{ij}}{\tau_{ij}}\right)}{B_{ij}} \right),$$

and we have

$$\mathcal{U}_{ij}(w) = -\tau_{ij} \log \left(\frac{e^{\frac{-\alpha_{ij}}{\tau_{ij}}} + e^{\frac{-w - \gamma_{ij}}{\tau_{ij}}}}{B_{ij}} \right) \text{ and } \mathcal{V}_{ij}(w) = -\tau_{ij} \log \left(\frac{e^{\frac{w - \alpha_{ij}}{\tau_{ij}}} + e^{\frac{-\gamma_{ij}}{\tau_{ij}}}}{B_{ij}} \right).$$

Without loss of generality (and up to a redefinition of the terms α and γ), one may assume that the total household budget is $B_{ij} = 2$. We call the resulting model an *Exponentially Transferable Utility* (ETU) model. A particular case of the ETU model can be found in Legros and Newman (2007, p. 1086). The terms α and γ play the role of “premuneration values,” as defined in Liu et al. (2014) and Mailath et al. (2013). The corresponding feasible set is displayed in figure 2d for $\tau_{ij} = 1$.

⁴In appendix 2.B.3, we provide close-forms expression for the model with a public good.

Note that the ETU model imposes that the total household budget is 2, namely $B_{ij} = 2$. In this case, we recover the NTU model (3.7) as $\tau_{ij} \rightarrow 0$, and the TU model (3.5) as $\tau_{ij} \rightarrow +\infty$. Hence, the ETU model interpolates between the nontransferable and fully transferable utility models. Here, the parameter τ_{ij} , which captures the elasticity of substitution between marital well-being and consumption, equivalently parameterizes the *degree of transferability*.

4. Aggregate equilibrium: motivation and definition

In this section we add structure to our previous model by assuming that agents can be grouped into a finite number of types, which are observable to the econometrician and vary according to an unobserved taste parameter. Section 4.1 precisely describes this setting. The individual, or “microscopic” equilibrium defined in section 3 above has a “macroscopic” analog: the *aggregate equilibrium*, which describes the equilibrium matching patterns and systematic payoffs across observable types; we define this concept in section 4.3.

4.1. Unobserved heterogeneity. We assume that individuals may be gathered in groups of agents of similar observable characteristics, or types, but heterogeneous tastes. We let \mathcal{X} and \mathcal{Y} be the sets of *types* of men and women, respectively; we assume that \mathcal{X} and \mathcal{Y} are finite. Let $x_i \in \mathcal{X}$ (resp. $y_j \in \mathcal{Y}$) be the type of individual man i (resp. woman j). We let n_x be the mass of men of type x , and let m_y be the mass of women of type y . In the sequel, we denote by $\mathcal{X}_0 \equiv \mathcal{X} \cup \{0\}$ the set of marital options available to women (either type of male partner or singlehood, denoted 0); analogously, $\mathcal{Y}_0 \equiv \mathcal{Y} \cup \{0\}$ denotes the set of marital options available to men (either type of female partner or singlehood, again denoted 0). For a man $x \in \mathcal{X}$ and a woman $y \in \mathcal{Y}$, let \mathcal{F}_{xy} be a proper bargaining set in the sense of definition 1. Let $D_{xy}(\cdot, \cdot)$ be the associated distance-to-frontier function, and recall from paragraph 3.1.3 that one can deduce an explicit representation of the feasible utilities by defining $\mathcal{U}_{xy}(w) = -D_{xy}(0, -w)$, and $\mathcal{V}_{xy}(w) = -D_{xy}(w, 0)$, so that $D_{xy}(u, v) \leq 0$ if and only if there exists a $w \in \mathbb{R}$ such that $u \leq \mathcal{U}_{xy}(w)$ and $v \leq \mathcal{V}_{xy}(w)$.

Consider a market in which men and women either decide to match or to remain single. Let u_i and v_j be the equilibrium utilities that man i and woman j obtain respectively on this

market. Our first assumption requires that these utilities arise as the sum of the outcome of a bargaining process plus an idiosyncratic term.

ASSUMPTION 1. *Assume that if i and j are matched, then there exists an endogenous $w_{ij} \in \mathbb{R}$ such that*

$$u_i \leq \mathcal{U}_{x_i y_j}(w_{ij}) + \varepsilon_{iy_j} \text{ and } v_j \leq \mathcal{V}_{x_i y_j}(w_{ij}) + \eta_{x_i j},$$

while if they remain single, then $u_i = \varepsilon_{i0}$ and $v_j = \eta_{0j}$, where the “idiosyncratic” parts of their utilities are the entries of exogenous random vectors $(\varepsilon_{iy})_{y \in \mathcal{Y}_0}$ and $(\eta_{xj})_{x \in \mathcal{X}_0}$ which are i.i.d. draws from distributions \mathbf{P}_x and \mathbf{Q}_y , respectively.

Assumption 1 immediately implies the following:

LEMMA 3. *If i and j are matched, there exists $(U_i, V_j) \in \mathcal{F}_{x_i y_j}$ such that $u_i = U_i + \varepsilon_{iy_j}$ and $v_j = V_j + \eta_{x_i j}$.*

In the case of TU models (see example 3.3.1 above), the restriction implied by lemma 3 simply states that the joint surplus Φ_{ij} can be decomposed in the form $\Phi_{ij} = \Phi_{x_i y_j} + \varepsilon_{iy_j} + \eta_{x_i j}$. This is the “additive separability” assumption in Choo and Siow (2006), who were the first to realize its analytical convenience; it has played a central role in the subsequent literature,⁵ see in particular Chiappori, Salanié, and Weiss (2017). Note that, while the transfers U_i and V_j are allowed to vary in an idiosyncratic manner within observable types, it will be a fundamental property of the equilibrium (stated in theorem 6 below in appendix 2.E) that U_i is the same for all men i of type x matched with a woman of type y , while V_j is the same for all the women j of type y matched with a man of type x .

We now introduce a technical restriction on the bargaining sets \mathcal{F}_{xy} .

ASSUMPTION 2. *The sets \mathcal{F}_{xy} are such that for each man type $x \in \mathcal{X}$, either all the \bar{w}_{xy} , $y \in \mathcal{Y}$ are finite, or all the \bar{w}_{xy} , $y \in \mathcal{Y}$ coincide with $+\infty$ (where \bar{w}_{xy} and \underline{w}_{xy} are as defined in section 3.1.3). For each woman type $y \in \mathcal{Y}$, either all the \underline{w}_{xy} , $x \in \mathcal{X}$ are finite, or all the \underline{w}_{xy} , $x \in \mathcal{X}$ coincide with $-\infty$.*

⁵In contrast, Dagsvik (2000) and Menzel (2015) assume that the heterogeneity in tastes is of the form ε_{ij} and η_{ij} , where the utility shocks are i.i.d. across partners, and hence is individual-specific.

This assumption expresses that given any agent (man or woman), the maximum utility that this agent can obtain with any partner is either always finite, or always infinite; this is needed to ensure existence of an equilibrium, and it is satisfied in all the examples we have.

We finally impose assumptions on \mathbf{P}_x and \mathbf{Q}_y , the distributions of the idiosyncratic terms $(\varepsilon_{iy})_{y \in \mathcal{Y}_0}$ and $(\eta_{xj})_{x \in \mathcal{X}_0}$, which are i.i.d. random vectors respectively valued in $\mathbb{R}^{\mathcal{Y}_0}$ and $\mathbb{R}^{\mathcal{X}_0}$.

ASSUMPTION 3. \mathbf{P}_x and \mathbf{Q}_y have non-vanishing densities on $\mathbb{R}^{\mathcal{Y}_0}$ and $\mathbb{R}^{\mathcal{X}_0}$.

There are two components to assumption 3: the requirement that \mathbf{P}_x and \mathbf{Q}_y have full support, and the requirement that they are absolutely continuous. The full-support requirement implies that given any pair of types x and y , there are individuals of these types with arbitrarily large valuations for each other; this implies that at equilibrium, any matching between observable pairs of types will be observed. The absolute continuity requirement ensures that with probability 1 the men and the women's choice problems have a unique solution.

Transposing definition 4 to the framework with parameterized heterogeneity, we see that (μ_{ij}, u_i, v_j) is an individual equilibrium outcome when:

- (i) $\mu_{ij} \in \{0, 1\}$, $\sum_j \mu_{ij} \leq 1$ and $\sum_i \mu_{ij} \leq 1$;
- (ii) for all i and j , $D_{x_i y_j}(u_i - \varepsilon_{iy_j}, v_j - \eta_{x_i j}) \geq 0$, with equality if $\mu_{ij} = 1$;
- (iii) $u_i \geq \varepsilon_{i0}$ and $v_j \geq \eta_{0j}$ with equality if respectively $\sum_j \mu_{ij} = 0$ and $\sum_i \mu_{ij} = 0$.

4.2. Informal preview of the next steps. To provide some intuition on the definition of aggregate equilibrium to follow, we summarize the next steps. We start with an equivalent condition to point (ii) in the definition of an individual equilibrium above (definition 4): for any pair of types $x \in \mathcal{X}$ and $y \in \mathcal{Y}$,

$$\min_{\substack{i: x_i=x \\ j: y_j=y}} D_{xy}(u_i - \varepsilon_{iy}, v_j - \eta_{xj}) \geq 0,$$

with equality if there is a matching between a man of type x and a woman of type y . Thus, defining $U_{xy} = \min_{i: x_i=x} \{u_i - \varepsilon_{iy}\}$ and $V_{xy} = \min_{j: y_j=y} \{v_j - \eta_{xj}\}$ yields $D_{xy}(U_{xy}, V_{xy}) \geq 0$.

0. We show that under weak conditions, this is actually an equality, hence:

$$(4.1) \quad D_{xy}(U_{xy}, V_{xy}) = 0.$$

Further, one sees from the definition of U_{xy} and V_{xy} that $u_i \geq \max_{y \in \mathcal{Y}} \{U_{xy} + \varepsilon_{iy}, \varepsilon_{i0}\}$ and $v_j \geq \max_{x \in \mathcal{X}} \{V_{xy} + \eta_{xj}, \eta_{0j}\}$. Again under rather weak conditions (stated in appendix 2.E), this actually holds as an equality, so that $u_i = \max_{y \in \mathcal{Y}} \{U_{xy} + \varepsilon_{iy}, \varepsilon_{i0}\}$ and $v_j = \max_{x \in \mathcal{X}} \{V_{xy} + \eta_{xj}, \eta_{0j}\}$. Hence, agents face discrete choice problems when choosing the type of their partner. At equilibrium, the mass of men of type x choosing type y women should coincide with the mass of women of type y choosing men of type x . Thus, we need to relate this common quantity μ_{xy} to the vector of systematic utilities (U_{xy}) and (V_{xy}). This is done in the next paragraph using results from the literature on Conditional Choice Probability (CCP) inversion (see Berry, 1994), which allows us to state a definition of aggregate equilibrium.

4.3. Aggregate Equilibrium. An *aggregate matching* (or just a *matching*, when no confusion is possible), is specified by a vector $(\mu_{xy})_{x \in \mathcal{X}, y \in \mathcal{Y}}$ measuring the mass of matches between men of type x and women of type y . Let \mathcal{M} be the set of matchings, that is, the set of $\mu_{xy} \geq 0$ such that $\sum_{y \in \mathcal{Y}} \mu_{xy} \leq n_x$ and $\sum_{x \in \mathcal{X}} \mu_{xy} \leq m_y$. For later purposes, we shall need to consider the strict interior of \mathcal{M} , denoted \mathcal{M}^0 , i.e. the set of $\mu_{xy} > 0$ such that $\sum_{y \in \mathcal{Y}} \mu_{xy} < n_x$ and $\sum_{x \in \mathcal{X}} \mu_{xy} < m_y$. The elements of \mathcal{M}^0 are called *interior matchings*.

We look for an individual equilibrium (μ_{ij}, u_i, v_j) with the property that there exist two vectors (U_{xy}) and (V_{xy}) such that if i is matched with j , then $u_i = U_{x_i y_j} + \varepsilon_{i y_j}$, and $v_j = V_{x_i y_j} + \eta_{x_i j}$.⁶

Under such an equilibrium, each agent is faced with a choice between the observable types of his or her potential partners, and man i and woman j solve respectively the following discrete choice problems

$$u_i = \max_{y \in \mathcal{Y}} \{U_{x_i y} + \varepsilon_{i y}, \varepsilon_{i 0}\} \quad \text{and} \quad v_j = \max_{x \in \mathcal{X}} \{V_{x y_j} + \eta_{x j}, \eta_{0 j}\}.$$

⁶While this may look like a restriction, we show in appendix 2.E that: (i) there always exists an individual equilibrium of this form, and (ii) under a very mild additional assumption on the feasible sets (namely, assumption 2' in appendix 2.E), *any* individual equilibrium is of this form.

This yields an important extension of Choo and Siow's (2006) original insight that the matching problem with heterogeneity in tastes is equivalent to a pair of discrete choice problems on both sides of the market. This allows us to relate the vector of utilities (U_{xy}) and (V_{xy}) to the equilibrium matching μ such that μ_{xy} is the mass of men of type x and women of type y mutually preferring each other. In order to establish this relation, we make use of the convex analytic apparatus of Galichon and Salanié (2015). We define the total indirect surplus of men and women by respectively

$$(4.2) \quad G(U) = \sum_{x \in \mathcal{X}} n_x \mathbb{E} \left[\max_{y \in \mathcal{Y}} \{U_{xy} + \varepsilon_{iy}, \varepsilon_{i0}\} \right] \quad \text{and} \quad H(V) = \sum_{y \in \mathcal{Y}} m_y \mathbb{E} \left[\max_{x \in \mathcal{X}} \{V_{xy} + \eta_{xj}, \eta_{0j}\} \right].$$

By the Daly-Zachary-Williams theorem, the mass of men of type x demanding a partner of type y is a quantity $\mu_{xy} = \partial G(U) / \partial U_{xy}$, which we denote in vector notation by $\mu \equiv \nabla G(U)$. Similarly, the mass of women of type y demanding a partner of type x is given by ν_{xy} , where $\nu \equiv \nabla H(V)$. At equilibrium, the mass of men of type x demanding women of type y should coincide with the mass of women of type y demanding men of type x , thus $\mu_{xy} = \nu_{xy}$ should hold for any pair, so reexpresses as $\nabla G(U) = \nabla H(V)$. Of course, U and V are related by the feasibility equation $D_{xy}(U_{xy}, V_{xy}) = 0$ for each $x \in \mathcal{X}$ and $y \in \mathcal{Y}$. This leads to the following definition.

DEFINITION 5 (Aggregate Equilibrium). The triple $(\mu_{xy}, U_{xy}, V_{xy})_{x \in \mathcal{X}, y \in \mathcal{Y}}$ is an *aggregate equilibrium outcome* if the following three conditions are met:

- (i) μ is an interior matching, i.e. $\mu \in \mathcal{M}^0$;
- (ii) (U, V) is feasible, i.e.

$$(4.3) \quad D_{xy}(U_{xy}, V_{xy}) = 0, \quad \forall x \in \mathcal{X}, y \in \mathcal{Y};$$

- (iii) μ, U , and V are related by the market clearing condition

$$(4.4) \quad \mu = \nabla G(U) = \nabla H(V).$$

The vector $(\mu_{xy})_{x \in \mathcal{X}, y \in \mathcal{Y}}$ is an *aggregate equilibrium matching* if and only if there exists a pair of vectors $(U_{xy}, V_{xy})_{x \in \mathcal{X}, y \in \mathcal{Y}}$ such that (μ, U, V) is an aggregate equilibrium outcome.

We discuss the equivalence of individual and aggregate equilibrium in theorem 6 of appendix 2.E.

4.4. Aggregate matching equation. Before ending this section, we rewrite the system of equations in definition 5 as a simpler system of equations which involves the matching vector μ only. To do this, we need to invert $\mu = \nabla G(U)$ and $\mu = \nabla H(V)$ in order to express U and V as a function of μ . For this purpose, we introduce the Legendre-Fenchel transform (a.k.a. convex conjugate) of G and H :

$$(4.5) \quad G^*(\mu) = \sup_U \left\{ \sum_{xy} \mu_{xy} U_{xy} - G(U) \right\} \quad \text{and} \quad H^*(\nu) = \sup_V \left\{ \sum_{xy} \nu_{xy} V_{xy} - H(V) \right\}.$$

It is a well-known fact from convex analysis (cf. Rockafellar 1970) that, under smoothness assumptions that hold here given assumption 3,

$$\mu = \nabla G(U) \iff U = \nabla G^*(\mu) \quad \text{and} \quad \nu = \nabla H(V) \iff V = \nabla H^*(\nu),$$

so we may substitute out U and V as an expression of μ in the system of equations in definition 5, so that equilibrium is characterized by a set of $|\mathcal{X}| \times |\mathcal{Y}|$ equations expressed only in terms of μ .

PROPOSITION 1. *Matching $\mu \in \mathcal{M}^0$ is an aggregate equilibrium matching if and only if*

$$(4.6) \quad D_{xy} \left(\frac{\partial G^*(\mu)}{\partial \mu_{xy}}, \frac{\partial H^*(\mu)}{\partial \mu_{xy}} \right) = 0 \text{ for all } x \in \mathcal{X}, y \in \mathcal{Y}.$$

Although the reformulation in proposition 1 is not used to obtain existence and uniqueness of an equilibrium in section 5, it is extremely useful in section 6 when the particular case of logit heterogeneity is considered; in that case, equation (4.6) can be inverted easily.

5. Aggregate Equilibrium: Existence, Uniqueness, Computation

In this section, we study aggregate equilibria by reformulating the ITU matching market in terms of a demand system. The couple types xy will be treated as goods; men as producers, and women as consumers. Each man of type x chooses to produce one of the goods of type xy , where $y \in \mathcal{Y}_0$; similarly, each woman of type y chooses to consume one of

the goods of type xy , where $x \in \mathcal{X}_0$. The wedges $W_{xy} = U_{xy} - V_{xy}$ are interpreted as prices, and $\partial G(\mathcal{U}(W)) / \partial U_{xy}$ is interpreted as the supply of the xy good, and $\partial H(\mathcal{V}(W)) / \partial V_{xy}$ is interpreted as the demand for that good if the price vector is W . An increase in W_{xy} raises the supply of the xy good and decreases the demand for it. We can define the excess demand function as

$$(5.1) \quad Z(W) := \nabla H(\mathcal{V}(W)) - \nabla G(\mathcal{U}(W)),$$

so that $Z_{xy}(W)$ is the mass of women of type y willing to match with men of type x minus the mass of men of type x willing to match with women of type y , if the vector of market wedges is W . At equilibrium, the market wedges are such that $Z(W) = 0$. In section 5.1, we show that our demand system satisfies the gross substitutability property of Kelso and Crawford (1982); this observation is the basis of our existence and uniqueness proofs in section 5.2. The machinery developed in this section is also useful to obtain results on identification which is the focus of appendix 2.D.

5.1. Reformulation as a demand system. Thanks to the explicit representation of the feasible sets, we obtain an alternative description of our matching model as a demand system, in the spirit of Azevedo and Leshno's (2016) approach to NTU models without unobserved heterogeneity. As we recall, $D_{xy}(U_{xy}, V_{xy}) = 0$ is equivalent to the existence of W_{xy} such that $U = \mathcal{U}(W)$ and $V = \mathcal{V}(W)$, where the xy -entries of $\mathcal{U}(W)$ and $\mathcal{V}(W)$ are $\mathcal{U}_{xy}(W_{xy})$ and $\mathcal{V}_{xy}(W_{xy})$, as introduced in definition 3.

PROPOSITION 2. *Outcome (μ, U, V) is an aggregate equilibrium outcome if and only if $\mu = \nabla G(U) = \nabla H(V)$, and there exists a vector (W_{xy}) such that $U = \mathcal{U}(W)$, $V = \mathcal{V}(W)$, and*

$$(5.2) \quad Z(W) = 0.$$

As we recall, $Z(\cdot)$ is to be interpreted as an excess demand function, and (W_{xy}) as a vector of market prices: if W_{xy} increases and all the other entries of W remain constant, the systematic utility V_{xy} of women in the xy category decreases and the utility U_{xy} of men in that category increases, hence Z_{xy} , the excess demand for category xy , decreases. It is

possible to express that in this demand interpretation various categories of goods xy are gross substitutes, in the following sense:

PROPOSITION 3 (Gross Substitutes). (a) If W_{xy} increases and all other entries of W remain constant, then:

(a.1) $Z_{xy}(W)$ decreases,

(a.2) $Z_{x'y'}(W)$ increases if either $x = x'$ or $y = y'$ (but both equalities do not hold),

(a.3) $Z_{x'y'}(W)$ remains constant if $x \neq x'$ and $y \neq y'$.

(b) for any $x \in \mathcal{X}$ and $y \in \mathcal{Y}$, the sum $\sum_{x' \in \mathcal{X}, y' \in \mathcal{Y}} Z_{x'y'}(W)$ is a decreasing function of W_{xy} .

The result implies that the excess demand function Z satisfies the *gross substitutability* condition. Point (a.1) means that when W_{xy} increases, one moves along the Pareto frontier of the feasible set \mathcal{F}_{xy} towards a direction which is more favorable to the men (U_{xy} increases, V_{xy} decreases), and thus there is *ceteris paribus* less demand from women and more from men for the category xy , and excess demand Z_{xy} decreases. Point (a.2) expresses that when the price of some category, say W_{xy} increases, and all the other entries of W remain constant, then the prospects of women in the category xy deteriorates, thus some of these women will switch to category $x'y$, and hence the excess demand $Z_{x'y}$ for category $x'y$ increases. Point (a.3) simply means that an agent (man or woman) does not respond to the price change of a category which does not involve his or her type. Finally, point (b) expresses that when the price of category xy increases, then singlehood becomes weakly less attractive for all men, and strongly less so for men of category x ; while singlehood becomes more attractive for women, which explains that the sum of $Z_{x'y'}$ over all categories, decreases.

5.2. Existence, uniqueness, and computation. We now state and prove a theorem that ensures the existence and uniqueness of an equilibrium using the characterization of aggregate equilibrium as a demand system introduced in proposition 2. We show that there is a unique vector of prices (W_{xy}) at which the value of excess demand is 0. This is stated in the following result:

THEOREM 1 (Existence and uniqueness of a price equilibrium). *Under assumptions 1, 2, and 3, there exists a unique vector W such that*

$$(5.3) \quad Z(W) = 0.$$

5.2.1. *Existence and computation.* The proof of equilibrium existence is constructive, and W is obtained as the outcome of the following algorithm. It is shown in the proof of theorem 1 that one can find an initial vector of prices (W_{xy}^0) for which excess demand is negative, that is $Z(W^0) \leq 0$. This suggests that prices (W_{xy}^0) are too high. The iteration consists of lowering these prices such that at each step, the excess demand at current price $Z(W^t)$ remains negative. More precisely, we set W_{xy}^t , the price of category xy at time t , to be such that $Z(W_{xy}^t, W_{-xy}^{t-1}) = 0$, where $(W_{xy}^t, W_{-xy}^{t-1})$ denotes the price vector which coincides with W^{t-1} on all entries except on the xy entry and which sets price W_{xy}^t to the xy entry. In other words, the prices of each category are updated in order to cancel the corresponding excess demand, holding the prices of other categories constant. Formally, it is possible to define a map $\mathcal{W} : \mathbb{R}^{\mathcal{X} \times \mathcal{Y}} \rightarrow \mathbb{R}^{\mathcal{X} \times \mathcal{Y}}$ such that $W' = \mathcal{W}(W)$ if and only if for all $xy \in \mathcal{X} \times \mathcal{Y}$

$$Z_{xy}(W'_{xy}, W_{-xy}) = 0,$$

and the procedure simply consists in setting $W^t = \mathcal{W}(W^{t-1})$. By proposition 3, point (a.1), it follows that $W_{xy}^t \leq W_{xy}^{t-1}$ for each xy . Because of the gross substitutability property (proposition 3, point (a.2)), $Z(W_{xy}^t, W_{-xy}^t) \leq Z(W_{xy}^t, W_{-xy}^{t-1}) = 0$, so that excess demand is still negative at step t . Finally, it is possible to show that W_{xy}^t remains bounded by below; thus, it converges monotonically. The limit is therefore a fixed point of \mathcal{W} , hence a zero of Z . This leads to the following algorithm:

ALGORITHM 1.

Step 0 | Start with $w^0 = \bar{W}$.

Step t | For each $x \in \mathcal{X}, y \in \mathcal{Y}$, define $W^{t+1} = \mathcal{W}(W^t)$.

The algorithm terminates when $\sup_{xy \in \mathcal{X} \times \mathcal{Y}} |W_{xy}^{t+1} - W_{xy}^t| < \epsilon$.

5.2.2. *Uniqueness.* The proof of uniqueness is based on a result of Berry, Gandhi, and Haile (2013), that implies that Z is inverse isotone. Hence, if there are two vectors W and

\tilde{W} such that $Z(W) = Z(\tilde{W}) = 0$, it would follow that $W \leq \tilde{W}$ and $\tilde{W} \leq W$ altogether, hence $W = \tilde{W}$. Note that the uniqueness of an equilibrium follows crucially from the full support assumption (assumption 3).

Combining theorem 1 and proposition 2, it follows that there exists a unique equilibrium outcome (μ, U, V) , where μ , U , and V are related to W by $U_{xy} = \mathcal{U}_{xy}(W_{xy})$, $V_{xy} = \mathcal{V}_{xy}(W_{xy})$, and $\mu = \nabla G(U) = \nabla H(V)$.

COROLLARY 1 (Existence and uniqueness of an equilibrium outcome). *Under assumptions 1, 2, and 3, there exists a unique equilibrium outcome (μ, U, V) , and μ , U , and V are related to the solution W to system (5.3) by $U_{xy} = \mathcal{U}_{xy}(W_{xy})$, $V_{xy} = \mathcal{V}_{xy}(W_{xy})$, and $\mu = \nabla G(U) = \nabla H(V)$.*

6. The ITU-logit model

In this section, and for the rest of the paper, we consider the model of matching with Imperfectly Transferable Utility and *logit* heterogeneity. We therefore replace assumption 3 by the stronger:

ASSUMPTION 3'. \mathbf{P}_x and \mathbf{Q}_y are the distributions of *i.i.d.* Gumbel (standard type I extreme value) random variables.

Of course, assumption 3' is a specialization of assumption 3, as the Gumbel distribution has a positive density on the real line. As we show in this section, the logit assumption carries strong implications. We show in section 6.1 that under assumption 3', the equilibrium matching equations (4.6) can be drastically simplified, and an algorithm more efficient than algorithm 1 can be used to solve them. Next, in section 6.2, we provide a number of illustrative applications of the logit assumption in the various example instances introduced in section 3.3. Finally, we will show in section 6.3 that maximum likelihood estimation is particularly straightforward in the logit context.

6.1. Equilibrium and computation, logit case. With logit random utilities, it is well-known (McFadden, 1974) that the systematic part of the utility U_{xy} can be identified

by the log of the ratio of the odds of choosing alternative y , relative to choosing the default option, and a similar formula applies to V_{xy} , hence $U_{xy} = \log(\mu_{xy}/\mu_{x0})$ and $V_{xy} = \log(\mu_{xy}/\mu_{0y})$, where $\mu_{x0} = n_x - \sum_{y \in \mathcal{Y}} \mu_{xy}$, and $\mu_{0y} = m_y - \sum_{x \in \mathcal{X}} \mu_{xy}$. Hence, the feasibility equation $D_{xy}(U_{xy}, V_{xy}) = 0$ in expression (4.6) becomes $D_{xy}(\log \mu_{xy} - \log \mu_{x0}, \log \mu_{xy} - \log \mu_{0y}) = 0$, which, given the translation invariance property (v) of lemma 1, yields

$$\log \mu_{xy} = -D_{xy}(-\log \mu_{x0}, -\log \mu_{0y}),$$

which explicitly defines μ_{xy} as a function of μ_{x0} and μ_{0y} :

$$(6.1) \quad \mu_{xy} = M_{xy}(\mu_{x0}, \mu_{0y}), \text{ where } M_{xy}(\mu_{x0}, \mu_{0y}) = \exp(-D_{xy}(-\log \mu_{x0}, -\log \mu_{0y})).$$

REMARK 6.1. By construction, the aggregate matching functions derived from our framework are homogeneous of degree 1 in the number of singles. That is, if μ_{xy}^* , μ_{x0}^* and μ_{0y}^* is the equilibrium matching for given population supplies, $\lambda \mu_{xy}^*$, $\lambda \mu_{x0}^*$ and $\lambda \mu_{0y}^*$ is the equilibrium matching when the population supplies are multiplied by λ . It is well known that the Choo and Siow model satisfies this constant return to scale property, while other models, such as those explored by Mourifié and Siow (2015) and Menzel (2015), do not.

The expression of μ_{xy} as a function of μ_{x0} and μ_{0y} , combined with the requirement that $\mu \in \mathcal{M}^0$, provides a set of equations that fully characterize the aggregate matching equilibrium, as argued in the following result:

THEOREM 2. *Under assumptions 1, 2, and 3', the equilibrium outcome (μ, U, V) in the ITU-logit model is given by*

$$\mu_{xy} = M_{xy}(\mu_{x0}, \mu_{0y}), \quad U_{xy} = \log \frac{\mu_{xy}}{\mu_{x0}}, \quad V_{xy} = \log \frac{\mu_{xy}}{\mu_{0y}},$$

where the pair of vectors $(\mu_{x0})_{x \in \mathcal{X}}$ and $(\mu_{0y})_{y \in \mathcal{Y}}$ is the unique solution to the system of equations

$$(6.2) \quad \begin{cases} \sum_y M_{xy}(\mu_{x0}, \mu_{0y}) + \mu_{x0} = n_x \\ \sum_x M_{xy}(\mu_{x0}, \mu_{0y}) + \mu_{0y} = m_y. \end{cases}$$

Theorem 2 implies that computing aggregate equilibria in the logit case is equivalent to solving the system of nonlinear equations (6.2)—a system of $|\mathcal{X}| + |\mathcal{Y}|$ equations in the same number of unknowns. It turns out that a simple iterative procedure provides an efficient means of solving (6.2). The basic idea is each equation in the first set of equations is an equation in the full set of (μ_{0y}) , but in the single unknown μ_{x0} . Hence, these can be inverted to obtain the values of (μ_{x0}) from the values of (μ_{0y}) . A similar logic applies to the second set of equations, where the values of (μ_{0y}) can be obtained from the values of (μ_{x0}) . The proposed algorithm operates by iterating the expression of (μ_{x0}) from (μ_{0y}) and vice-versa. Provided the initial choice of (μ_{0y}) is high enough, the procedure converges isototonically, as argued in the theorem below.

ALGORITHM 2.

- | | |
|---------------|---|
| Step 0 | Fix the initial value of μ_{0y} at $\mu_{0y}^0 = m_y$. |
| Step $2t + 1$ | Keep the values μ_{0y}^{2t} fixed. For each $x \in \mathcal{X}$, solve for the value, μ_{x0}^{2t+1} , of μ_{x0} such that equality $\sum_{y \in \mathcal{Y}} M_{xy}(\mu_{x0}, \mu_{0y}^{2t}) + \mu_{x0} = n_x$ holds. |
| Step $2t + 2$ | Keep the values μ_{x0}^{2t+1} fixed. For each $y \in \mathcal{Y}$, solve for which is the value, μ_{0y}^{2t+2} , of μ_{0y} such that equality $\sum_{x \in \mathcal{X}} M_{xy}(\mu_{x0}^{2t+1}, \mu_{0y}) + \mu_{0y} = m_y$ holds. |

The algorithm terminates when $\sup_y |\mu_{0y}^{2t+2} - \mu_{0y}^{2t}| < \epsilon$.

THEOREM 3. *Under assumptions 1, 2, and 3', algorithm 2 converges toward the solution (6.2), in such a way that (μ_{0y}^t) is nonincreasing with t , and (μ_{x0}^t) is nondecreasing with t .*

6.2. Example specifications, logit case.

6.2.1. *TU-logit specification.* In the logit case of the TU specification introduced in paragraph 3.3.1, the matching function becomes

$$(6.3) \quad M_{xy}(\mu_{x0}, \mu_{0y}) = \mu_{x0}^{1/2} \mu_{0y}^{1/2} \exp \frac{\Phi_{xy}}{2},$$

which is Choo and Siow's (2006) formula.

6.2.2. *NTU-logit specification.* In the logit case of the NTU specification introduced in paragraph 3.3.2, the matching function becomes

$$(6.4) \quad M_{xy}(\mu_{x0}, \mu_{0y}) = \min(\mu_{x0} e^{\alpha_{xy}}, \mu_{0y} e^{\gamma_{xy}}).$$

When $\mu_{x0}e^{\alpha_{xy}} \leq \mu_{0y}e^{\gamma_{xy}}$, $\mu_{xy} = \mu_{x0}e^{\alpha_{xy}}$ is constrained by the choice problem of men; we say that, relative to pair xy , men are on the *short side (of the market)* and women are on the *long side (of the market)*, and visa versa. Galichon and Hsieh (2017) study this model in detail. In particular, they show that existence and computation of equilibria in a more general version of this model can alternatively be provided via an aggregate version of the Gale–Shapley (1962) algorithm.⁷

6.2.3. *Convex tax schedule and logit specification.* Recall from paragraph 3.3.3 and expression (3.8) that when there is a convex tax schedule, the distance-to-frontier function is expressed as a maximum of linear terms, more specifically

$$D_{xy}(u_x, v_y) = \max_{k \in \{0, \dots, K\}} \left\{ \lambda_{xy}^k (u - \alpha_{xy}^k) + \zeta_{xy}^k (v - \gamma_{xy}) \right\}$$

where $\lambda_{xy}^k = 1 / (2 - \tau_{xy}^k)$, and $\zeta_{xy}^k = (1 - \tau_{xy}^k) / (2 - \tau_{xy}^k)$, and α_{xy}^k is obtained recursively by $\alpha_{xy}^0 = \alpha_{xy}$, and $\alpha_{xy}^{k+1} = \alpha_{xy}^k + (1 - \tau_{xy}^k) (t_{xy}^{k+1} - t_{xy}^k)$. In this case, the corresponding matching function obtains as

$$(6.5) \quad M_{xy}(\mu_{x0}, \mu_{0y}) = \min_{k \in \{0, \dots, K\}} \mu_{x0}^{\lambda_{xy}^k} \mu_{0y}^{\zeta_{xy}^k} e^{\lambda_{xy}^k \alpha_{xy}^k + \zeta_{xy}^k \gamma_{xy}}.$$

Of course, the NTU-logit case of the previous paragraph is a particular case of this expression of $K = 1$, $\lambda_{xy}^0 = 0$, $\zeta_{xy}^0 = 1$ and $\lambda_{xy}^1 = 1$, $\zeta_{xy}^1 = 0$.

6.2.4. *Collective model and logit specification.* In the case of the collective model introduced in paragraph 3.3.4 with a logit heterogeneity, the feasibility frontier takes the form

$$\exp\left(\frac{U_{xy} - \alpha_{xy}}{\tau_{xy}}\right) + \exp\left(\frac{V_{xy} - \gamma_{xy}}{\tau_{xy}}\right) = B_{xy},$$

which yields the following expression for the matching function:

$$(6.6) \quad M_{xy}(\mu_{x0}, \mu_{0y}) = \left(\frac{e^{-\alpha_{xy}/\tau_{xy}} \mu_{x0}^{-1/\tau_{xy}} + e^{-\gamma_{xy}/\tau_{xy}} \mu_{0y}^{-1/\tau_{xy}}}{B_{xy}} \right)^{-\tau_{xy}}.$$

⁷Note that Dagsvik (2000) and Menzel (2015) obtain $\mu_{xy} = \mu_{x0}\mu_{0y}e^{\alpha_{xy} + \gamma_{xy}}$, in contrast with our formula (6.4). The reason for this difference is that Dagsvik (2000) and Menzel (2015) assume that the stochastic matching affinities are given by $\alpha_{ij} = \alpha_{xy} + \varepsilon_{ij}$ and $\gamma_{ij} = \gamma_{xy} + \eta_{ij}$, where the ε_{ij} and η_{ij} terms are i.i.d. type I extreme value distributions. In contrast, in our setting, $\alpha_{ij} = \alpha_{xy} + \varepsilon_{ij_j}$ and $\gamma_{ij} = \gamma_{xy} + \eta_{x_{ij}}$.

As expected, when $B_{xy} = 2$ and $\tau_{xy} \rightarrow 0$, formula (6.6) converges to the NTU-logit formula, (6.4). Likewise, when $B_{xy} = 2$ and $\tau_{xy} \rightarrow +\infty$, (6.6) converges to the TU-logit formula, (6.3). But when $\tau_{xy} = 1$, then (up to multiplicative constants) μ_{xy} becomes the harmonic mean between μ_{x0} and μ_{0y} . We thus recover a classical matching function form—the “Harmonic Marriage Matching Function” that has been used by demographers for decades, see, e.g., Schoen (1981). To our knowledge, our framework gives the first behavioral/microfounded justification of the harmonic marriage matching function—see Siow (2008, p. 5).

6.3. Maximum likelihood estimation, logit case. In this paragraph, we assume that (D_{xy}) belongs to a parametric family (D_{xy}^θ) and we estimate $\theta \in \mathbb{R}^d$ by maximum likelihood. In this case, the matching function is then $M_{xy}^\theta(\mu_{x0}, \mu_{0y}) = \exp(-D_{xy}^\theta(-\log \mu_{x0}, -\log \mu_{0y}))$. We shall assume sufficient smoothness on the parametrization.

ASSUMPTION 4. *For each $xy \in \mathcal{XY}$, the map $(\theta, u, v) \mapsto (D_{xy}^\theta(u_x, v_y))_{xy}$ is twice continuously differentiable from $\mathbb{R}^d \times \mathbb{R}^{|\mathcal{X}|} \times \mathbb{R}^{|\mathcal{Y}|}$ to $\mathbb{R}^{|\mathcal{X}| \times |\mathcal{Y}|}$.*

The sample is made of N i.i.d. draws of household types $xy \in \mathcal{XY} \cup \mathcal{X}_0 \cup \mathcal{Y}_0$. Let $\zeta = (n, m)$ be the distribution vector of men and women’s types, respectively. Given the model’s primitives θ and ζ , let $(\mu_{x0}^{\theta, \zeta}, \mu_{0y}^{\theta, \zeta})$ be the solution to the system of equations (6.2), and let $\mu_{xy}^{\theta, \zeta} = M_{xy}^\theta(\mu_{x0}^{\theta, \zeta}, \mu_{0y}^{\theta, \zeta})$ be the equilibrium matching patterns. We define the predicted frequency $\Pi_{xy}(\theta, \zeta)$ of a household of type xy by

$$(6.7) \quad \Pi_{xy}(\theta, \zeta) = \frac{\mu_{xy}^{\theta, \zeta}}{N^h(\theta, \zeta)}, \text{ where } N^h(\theta, \zeta) := \sum_{xy \in \mathcal{XY} \cup \mathcal{X}_0 \cup \mathcal{Y}_0} \mu_{xy}^{\theta, \zeta}$$

and $N^h(\theta, \zeta)$ is the total mass of households predicted if the masses of individual men and women are given by ζ .

We let $\hat{\mu}_{xy}$ be the number of households of type xy in the sample, $\hat{N}^h = \sum_{xy \in \mathcal{XY} \cup \mathcal{X}_0 \cup \mathcal{Y}_0} \hat{\mu}_{xy}$ the number of households in the sample, and $\hat{\pi}_{xy} = \hat{\mu}_{xy} / \hat{N}^h$ be the empirical frequency of household xy in the sample. The log-likelihood of observation $\hat{\pi}$ is

$$l(\hat{\pi} | \theta, \zeta) = \sum_{xy \in \mathcal{XY} \cup \mathcal{X}_0 \cup \mathcal{Y}_0} \hat{\pi}_{xy} \log \Pi_{xy}(\theta, \zeta).$$

Consider \mathcal{I} the Fisher information matrix of $l(\pi|\theta, \zeta)$ with respect to its parameter vector (θ, ζ) , which is written blockwise as

$$\mathcal{I} = \begin{pmatrix} \mathcal{I}_{11} & \mathcal{I}_{12} \\ \mathcal{I}'_{12} & \mathcal{I}_{22} \end{pmatrix},$$

where \mathcal{I}_{11} is the Hessian of $-l(\hat{\pi}|\theta, \zeta)$ with respect to θ , \mathcal{I}_{12} is the matrix of cross-derivatives of $-l(\hat{\pi}|\theta, \zeta)$ with respect to the entries of θ and ζ , and \mathcal{I}_{22} is the Hessian of $-l(\hat{\pi}|\theta, \zeta)$ with respect to ζ .

In principle, (θ, ζ) could be estimated jointly by Maximum Likelihood; however, the dimensionality of ζ , the vector of the types distribution in the population is potentially large, so this could pose computational difficulties. Further, ζ is a nuisance parameter as the focus of the estimation procedure is on the estimation of θ ; thus we choose to use the consistent estimator of ζ provided by the distribution of types in the sample. Letting A be the matrix acting on $\mu = (\mu_{xy}, \mu_{x0}, \mu_{0y})$ such that $(A\mu) = \binom{n}{m}$ where $n_x = \sum_{y \in \mathcal{Y}_0} \mu_{xy}$ and $m_y = \sum_{x \in \mathcal{X}_0} \mu_{xy}$, we get that a consistent estimator of ζ is $A\hat{\pi}$. Thus, we shall define $\hat{\theta}$ to be the maximum likelihood estimator of θ conditional on the distribution of types being estimated by $A\hat{\pi}$, that is $\hat{\theta}$ is the value of θ that maximizes $l(\hat{\pi}|\theta, A\hat{\pi})$.

THEOREM 4. (i) *The log-likelihood is expressed using*

$$(6.8) \quad -l(\hat{\pi}|\theta, A\hat{\pi}) = \sum_{xy \in \mathcal{X} \times \mathcal{Y}} \hat{\pi}_{xy} D_{xy}^\theta(u_x^\theta, v_y^\theta) + \sum_{x \in \mathcal{X}} \hat{\pi}_{x0} u_x^\theta + \sum_{y \in \mathcal{Y}} \hat{\pi}_{0y} v_y^\theta + \log N^h(\theta, \zeta),$$

where the quantities $u_x^\theta = -\log \mu_{x0}^\theta$ and $v_y^\theta = -\log \mu_{0y}^\theta$ form the unique pair vectors (u, v) solution to

$$(6.9) \quad \begin{cases} e^{-u_x} + \sum_{y \in \mathcal{Y}} e^{-D_{xy}^\theta(u_x, v_y)} = \hat{n}_x \\ e^{-v_y} + \sum_{x \in \mathcal{X}} e^{-D_{xy}^\theta(u_x, v_y)} = \hat{n}_y. \end{cases}$$

(ii) *Letting $\hat{\theta}$ be the Maximum Likelihood Estimator, $N^{1/2}(\hat{\theta} - \theta) \Rightarrow \mathcal{N}(0, V_\theta)$ as the sample size $\hat{N}^h \rightarrow +\infty$, where the variance-covariance matrix V_θ can be consistently estimated by*

$$(6.10) \quad \hat{V}_\theta = (\mathcal{I}_{11})^{-1} (\mathcal{I}_{12}) A V_\pi A' \mathcal{I}'_{12} (\mathcal{I}_{11})^{-1}, \text{ with } V_\pi = \text{diag}(\pi) - \pi\pi'.$$

Expression (6.8) in Part (i) of the result has an interesting interpretation. For a matched pair (x, y) , $D_{xy}(u_x, v_y)$ is the signed distance to the bargaining frontier, which will be typically negative (i.e. (u_x, v_y) is an interior point in general). For a single individual man or woman of type x or y , u_x or v_y is also the signed distance to the bargaining frontier, which is 0. Hence, the value of minus the likelihood is the sum of two terms: (i) a first term

$$\sum_{xy \in \mathcal{X} \times \mathcal{Y}} \hat{\pi}_{xy} D_{xy}^{\theta} \left(u_x^{\theta}, v_y^{\theta} \right) + \sum_{x \in \mathcal{X}} \hat{\pi}_{x0} u_x^{\theta} + \sum_{y \in \mathcal{Y}} \hat{\pi}_{0y} v_y^{\theta}$$

which is the average distance of $u_x^{\theta} = -\log \mu_{x0}^{\theta}$ and $v_y^{\theta} = -\log \mu_{0y}^{\theta}$ to the bargaining frontier, and (ii) a second term which is $\log N^h(\theta, \zeta)$, the logarithm of the predicted number of households.

Let us comment on the intuitive explanation for the second term, i.e. the logarithm of the number of predicted households in the expression of the opposite of the likelihood. The maximization of the second term pulls the estimation towards overestimating the number of matched household. On the contrary, in the first term, single households are given the same weight as matched households in the objective function, thus the maximization of the first term tends to underestimate the number of matched households. The trade-off between the two effects is expressed by the $\log N^h(\theta, \zeta)$ term. In appendix 2.F, we provide an illustrative empirical example of this strategy.

7. Conclusion

The present contribution brings together a number of approaches. In terms of the techniques used, it builds on concepts from game theory, general equilibrium, and econometrics. In terms of models allowed, it embeds models with and without transferable utility. It also provides an integrated approach for both matching models and collective models. Lastly, we note that our work can be used in conjunction with reduced-form methodologies, as it allows to compute the equilibrium outcome's response to a shock in the matching primitives (e.g., a demographic shock) and to regress the former on the latter.

Beyond the class of problems investigated in the present paper, the methods developed here, based on fixed point theorems for isotone functions, may be more broadly applicable.

In particular, they may be a useful tool for the investigation of matching problems with peer effects put forward by Mourifié and Siow (2014). They may also prove useful for studying certain commodity flow problems in trading networks, and may also extend to one-to-many matching problems. We leave this last extension for further work.

Bibliography

- [1] Agarwal, N. (2015): “An Empirical Model of the Medical Match,” *American Economic Review* 105 (7), pp. 1939–1978.
- [2] Alkan, A. (1989): “Existence and computation of matching equilibria,” *European Journal of Political Economy*, 5(2-3), pp. 285–296.
- [3] Alkan, A., and Gale, D. (1990): “The core of the matching game,” *Games and Economic Behavior*, 2(3), pp. 203–212.
- [4] Azevedo, E., and Leshno, J. (2016): “A Supply and Demand Framework for Two-Sided Matching Markets,” *Journal of Political Economy*, 124: (5), pp. 1235–1268.
- [5] Becker, G. S. (1973): “A Theory of Marriage: Part I,” *Journal of Political Economy*, 81(4), pp. 813–46.
- [6] Berry, S. T. (1994): “Estimating discrete-choice models of product differentiation”. *The RAND Journal of Economics* 25(2), pp. 242-262.
- [7] Berry, S., Gandhi, A., and Haile, P. (2013). “Connected substitutes and invertibility of demand”. *Econometrica* 81 (5), pp. 2087–2111.
- [8] Browning, M., P.-A. Chiappori, and Y. Weiss (2014): *Economics of the Family*. Cambridge University Press.
- [9] Chade, H., and J. Eeckhout (2014): “Stochastic Sorting,” working paper.
- [10] Cherchye, L., T. Demuynck, B. De Rock and F. Vermeulen (2017): “Household consumption when the marriage is stable,” *American Economic Review*, 107(6), pp. 1507-1534.
- [11] Chiappori, P.-A. (1992): “Rational Household Labour Supply,” *Econometrica* 56 (1), pp. 63–89.
- [12] Chiappori, P.-A. (2012): “Modèles d’appariement en économie: quelques avancées récentes,” *Revue économique* 63 , pp. 433–448.

- [13] Chiappori, P.-A., M. Iyigun, and Y. Weiss (2009): “Investment in Schooling and the Marriage Market,” *American Economic Review*, 99(5), pp. 1689–1713.
- [14] Chiappori, P.-A., Oreffice, S., and C. Quintana-Domeque (2012): “Fatter Attraction: Anthropometric and Socioeconomic Matching on the Marriage Market.” *Journal of Political Economy*, 120(4), pp. 659–695.
- [15] Chiappori, P.-A. and Reny, P. J (2016): “Matching to share risk”. *Theoretical Economics*, 11, pp. 227-251.
- [16] Chiappori, P.-A., B. Salanié, and Weiss, Y. (2017): “Partner Choice, Investments in Children, and the Marital College Premium,” *American Economic Review*, forthcoming.
- [17] Choo, E. and Seitz, S. (2013): “The Collective Matching Model: Identification, Estimation and Testing,” chapter in *Advances in Econometrics* 32, Emerald Publishing Group.
- [18] Choo, E., and A. Siow (2006): “Who Marries Whom and Why,” *Journal of Political Economy*, 114(1), pp. 175–201.
- [19] Crawford, V. P., & Knoer, E. M. (1981): “Job Matching with Heterogeneous Firms and Workers,” *Econometrica*, 49(2), pp. 437–50.
- [20] Dagsvik, J. K. (2000): “Aggregation in Matching Markets,” *International Economic Review*, 41(1), pp. 27–57.
- [21] Demange, G., & Gale, D. (1985): “The Strategy Structure of Two-Sided Matching Markets,” *Econometrica*, 53(4), pp. 873–888.
- [22] Dupuy, A., and A. Galichon (2014): “Personality traits and the marriage market,” *Journal of Political Economy*, 122 (6), pp. 1271-1319.
- [23] Dupuy, A., Galichon, A., Jaffe, S., and Scott D. Kominers (2017): “On the Incidence of Taxation in Matching Markets,” working paper.
- [24] Fox, J. (2010): “Identification in Matching Games,” *Quantitative Economics* 1, pp. 203–254.
- [25] Ekeland, I., J. Heckman, and L. Nesheim (2004): “Identification and estimation of hedonic models,” *Journal of Political Economy*, 112, 60–109.
- [26] Gale, D. (1984): “Equilibrium in a discrete exchange economy with money,” *International Journal of Game Theory*, 13(1), pp. 61–64.

- [27] Gale, D., & Shapley, L. S. (1962): “College Admissions and the Stability of Marriage,” *The American Mathematical Monthly*, 69(1), pp. 9–15.
- [28] Galichon, A. and Hsieh, Y.-W. (2017): “A Theory of Decentralized Matching Markets Without Transfers, with an Application to Surge Pricing,” working paper, SSRN #2887732.
- [29] Galichon, A., and B. Salanié (2015): “Cupid’s Invisible Hand: Social Surplus and Identification in Matching Models,” working paper, available at SSRN: <http://dx.doi.org/10.2139/ssrn.1804623>.
- [30] Galichon, A. K. O’hara, and S. Weber (2015–2017): “TraME: Transportation Methods for Econometrics”. R package available at <https://github.com/TraME-Project/>.
- [31] Hatfield, J. W., and Milgrom, P. R. (2005): “Matching with Contracts,” *American Economic Review*, 95(4), pp. 913–935.
- [32] Heckman, J., R. Matzkin, and L. Nesheim (2010): “Nonparametric identification and estimation of nonadditive hedonic models,” *Econometrica*, 78(5), 1569–1591.
- [33] Hitsch, G. J., Hortasu, A., & Ariely, D. (2010): “Matching and Sorting in Online Dating,” *American Economic Review*, 100(1), pp. 130–63.
- [34] Iyigun, M. and R. P. Walsh (2007): “Building the family nest: Premarital investments, marriage markets, and spousal allocations,” *Review of Economic Studies*, 74, pp. 507–535.
- [35] Kaneko, M. (1982): “The central assignment game and the assignment markets,” *Journal of Mathematical Economics*, 10(2-3), pp. 205–232.
- [36] Legros, P., and Newman, A. F. (2007): “Beauty Is a Beast, Frog Is a Prince: Assortative Matching with Nontransferabilities,” *Econometrica*, 75(4), pp. 1073–1102.
- [37] Liu, Q., Mailath, G., Postlewaite, A., and Samuelson, L. (2014): “Stable Matching with Incomplete Information.” *Econometrica* 82 (2), pp. 541–587.
- [38] Mailath, G., Postlewaite, A., and Samuelson, L. (2013): “Pricing and Investments in Matching Markets.” *Theoretical Economics* 8 (2), pp. 535–590.
- [39] McFadden, D. (1974): “Conditional Logit Analysis of Qualitative Choice Behavior”. In *Frontiers in Econometrics*, edited by Zarembka, P. New York: Academic Press.

- [40] Menzel, K. (2015): “Large Matching Markets as Two-Sided Demand Systems,” *Econometrica*, 83(3), pp. 897-941.
- [41] Moré, J. (1972), “Nonlinear Generalizations of Matrix Diagonal Dominance with Applications to Gauss-Seidel Iterations.” *SIAM Journal on Numerical Analysis* 9 (2), 1972.
- [42] Mourifié, I., and Siow, A. (2014): “Cohabitation vs Marriage: Marriage matching with peer effects,” working paper, University of Toronto.
- [43] Nöldeke, G., & Samuelson, L. (2015): “The Implementation Duality,” *Cowles Foundation Discussion Paper*.
- [44] Office for National Statistics, Department for Environment, Food and Rural Affairs (2015): “Living Costs and Food Survey, 2013”. UK Data Service. Available from: <http://doi.org/10.5255/UKDA-SN-7702-1>
- [45] Quinzii, M. (1984): “Core and competitive equilibria with indivisibilities,” *International Journal of Game Theory*, 13(1), pp. 41–60.
- [46] Rheinboldt, W. (1970). “On M-Functions and their Application to Nonlinear Gauss-Seidel Iterations and to Network Flows.” *Journal of Mathematical Analysis and Applications* 32, pp. 274–307.
- [47] Rheinboldt, W. (1998). *Methods for Solving Systems of Nonlinear Equations*. Second Edition, CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM.
- [48] Rockafellar, R.T. (1970): *Convex Analysis*. Princeton University Press.
- [49] Roth, A. and M. Sotomayor (1990): *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Cambridge University Press.
- [50] Schoen, R. (1981). “The Harmonic Mean as the Basis of a Realistic Two-Sex Marriage Model.” *Demography* 18 (2), pp. 201–216.
- [51] Shapley, L., and M. Shubik (1972): “The Assignment Game I: The Core,” *International Journal of Game Theory*, 1, pp. 111–130.
- [52] Siow, A. (2008). “How does the marriage market clear? An empirical framework.” *Canadian Journal of Economics* 41 (4), pp. 1121–1155.
- [53] van der Vaart, A. (2000). *Asymptotic Statistics*. Cambridge University Press.

Appendix

This appendix contains supplementary material that could not be included in the main text. Appendix 2.A contains the proofs of the mathematical results in the main text. Appendix 2.B provides examples of other models of interest. Appendix 2.C offers remarks on equilibrium versus optimality in the matching models considered in the main paper. Appendix 2.D provides formal results on identification. Appendix 2.E provides results relating aggregate and individual equilibria. Finally, appendix 2.F contains an illustrative example based on the Living Costs and Food Survey (ONS, 2015).

2.A. Proofs of the results in the main text

2.A.1. Proof of lemma 1.

PROOF. (i) directly follows from the definition of D . (ii) is straightforward given requirements (i)–(iii) of definition 1. Let us show (iii). Assume $(u, v) \leq (u', v')$. Then by requirement (ii) of definition 1, for any $z \in \mathbb{R}$, $(u' - z, v' - z) \in \mathcal{F}_{ij}$ implies $(u - z, v - z) \in \mathcal{F}_{ij}$. Thus $D_{\mathcal{F}}(u, v) \leq D_{\mathcal{F}}(u', v')$, which is the first part of the claim.

Now assume $u < u'$ and $v < v'$ and $D_{\mathcal{F}}(u, v) = D_{\mathcal{F}}(u', v')$. Then $u - D_{\mathcal{F}}(u, v) < u' - D_{\mathcal{F}}(u', v')$ and $v - D_{\mathcal{F}}(u, v) < v' - D_{\mathcal{F}}(u', v')$. But this implies that there exists $\epsilon > 0$ such that $u - D_{\mathcal{F}}(u, v) + \epsilon \leq u' - D_{\mathcal{F}}(u', v')$ and $v - D_{\mathcal{F}}(u, v) + \epsilon \leq v' - D_{\mathcal{F}}(u', v')$; however, as $(u' - D_{\mathcal{F}}(u', v'), v' - D_{\mathcal{F}}(u', v')) \in \mathcal{F}_{ij}$, this implies, still by requirement (ii) of definition 1, that $(u - D_{\mathcal{F}}(u, v) + \epsilon, v - D_{\mathcal{F}}(u, v) + \epsilon) \in \mathcal{F}_{ij}$, a contradiction. Thus $D_{\mathcal{F}}(u, v) < D_{\mathcal{F}}(u', v')$, which completes the proof that $D_{\mathcal{F}}$ is \gg -isotone.

To show point (iv) ($D_{\mathcal{F}}$ is continuous), consider (u, v) and (u', v') , and assume that $v - u \geq v' - u'$. Then $u - D(u, v) \leq u' - D(u', v')$; indeed, assume by contradiction that

$u - D(u, v) > u' - D_{\mathcal{F}}(u', v')$, then by summation $v - D(u, v) > v' - D_{\mathcal{F}}(u', v')$. By the same argument as above, this leads to a contradiction; hence, $u - D(u, v) \leq u' - D_{\mathcal{F}}(u', v')$. It is easy to check that $v - u \leq v' - u'$ implies $u - D(u, v) \geq u' - D_{\mathcal{F}}(u', v')$. Hence, in general

$$\min(v' - v, u' - u) \leq D_{\mathcal{F}}(u', v') - D_{\mathcal{F}}(u, v) \leq \max(u' - u, v' - v)$$

which shows continuity of $D_{\mathcal{F}}$.

(v) One has $D_{\mathcal{F}}(u + a, v + a) = \min\{z \in \mathbb{R} : (u + a - z, v + a - z) \in \mathcal{F}\}$ by the very definition of $D_{\mathcal{F}}$, which immediately shows that $D_{\mathcal{F}}(u + a, v + a) = a + D_{\mathcal{F}}(u, v)$. ■

2.A.2. Proof of lemma 2.

PROOF. The proof is divided in several parts.

First part: let us show that the set of wedges w that can be expressed as $w = u - v$ for u and v such that $D_{\mathcal{F}}(u, v) = 0$ is an open interval. Consider u, u', v and v' such that $D(u, v) = 0$ and $D(u', v') = 0$, and $(u, v) \neq (u', v')$. Let $w = u - v$ and $w' = u' - v'$. Assume w.l.o.g $u' > u$, then one has necessarily $v \geq v'$, hence $u' - v' > u - v$. In this case, let $u_t = t(u' - u) + u$ and $v_t = t(v' - v) + v$. Let $\tilde{u}_t = u_t - D(u_t, v_t)$ and $\tilde{v}_t = v_t - D(u_t, v_t)$, so that $D(\tilde{u}_t, \tilde{v}_t) = 0$. One has $\tilde{u}_t - \tilde{v}_t = u_t - v_t = t(u' - v') + (1 - t)(u - v) = tw' + (1 - t)w$, which shows that the set of wedges is an interval, denoted I . Let us now show that this interval is open. Call \underline{w} the infimum of the interval, and assume it is finite. Then there is a sequence (u_n, v_n) such that u_n is decreasing, v_n is increasing, $D(u_n, v_n) = 0$ and $u_n - v_n \rightarrow \underline{w}$. Then by the scarcity of \mathcal{F} , u_n and v_n need to remain bounded, hence they converge in \mathcal{F} . Let (u^*, v^*) be their limit; one has $D(u^*, v^*) = 0$ and $u^* - v^* = \underline{w}$. For any $u' < u^*$, one has $D(u', v^*) \leq 0$; hence, by scarcity of \mathcal{F} , there is some $v' \geq v^*$ such that $D(u', v') = 0$. $u' < u^*$ and $v' \geq v^*$, thus $u' - v' < u^* - v^* = \underline{w}$, a contradiction. Thus $\underline{w} \in I$. A symmetric argument shows that if the supremum of I is finite, then it belongs in I . Thus, I is an open interval.

Second part: let us show that \mathcal{U} and \mathcal{V} are well defined on I . For $w \in I$, there exists by definition (u, v) such that $D(u, v) = 0$ and $u - v = w$. The argument at the beginning of part (i) implies that (u, v) is unique. Hence \mathcal{U} and \mathcal{V} are well defined.

Third part: let us show that \mathcal{U} is increasing and \mathcal{V} is decreasing. Suppose $w < w'$ and $\mathcal{U}(w) \geq \mathcal{U}(w')$. Then $w - \mathcal{U}(w) < w' - \mathcal{U}(w')$, hence $\mathcal{V}(w) > \mathcal{V}(w')$, a contradiction. Thus $\mathcal{U}(w) < \mathcal{U}(w')$, which shows that \mathcal{U} is increasing. By a similar logic, $\mathcal{V}(w) > \mathcal{V}(w')$, and \mathcal{V} is decreasing.

Fourth part: let us show that \mathcal{U} is 1-Lipschitz. Take $\epsilon > 0$ and assume by contradiction $u' > u + \epsilon$ where $u = \mathcal{U}(w)$ and $u' = \mathcal{U}(w + \epsilon)$. Then $D(u, u - w) = 0$ with and $D(u', u' - w - \epsilon) = 0$. Then because $u' > u$ and $u' - \epsilon > u$, it follows that $D(u', u' - w - \epsilon) > D(u, u - w) = 0$, a contradiction. Hence, $0 \leq \mathcal{U}(w + \epsilon) - \mathcal{U}(w) \leq \epsilon$, and thus \mathcal{U} is 1-Lipschitz. A similar argument for \mathcal{V} completes the proof.

Fifth part: let us show that expression (3.3) holds. By applying point (v) of lemma 1 twice, once with $a = -u$ and once $a = -v$, it follows respectively that $D_{\mathcal{F}}(0, v - u) = D_{\mathcal{F}}(u, v) - u$ and that $D_{\mathcal{F}}(u - v, 0) = D_{\mathcal{F}}(u, v) - v$. Hence, if (u, v, w) are solutions to (3.2), it follows that $u = -D_{\mathcal{F}}(0, -w)$, and thus $v = -D_{\mathcal{F}}(w, 0)$. Hence (3.3) holds. ■

2.A.3. Proof of lemma 3.

PROOF. Set $U_i = u_i - \varepsilon_{iy_j}$ and $V_j = v_j - \eta_{x_{ij}}$. Then by assumption 1, there exists $w_{ij} \in \mathbb{R}$ such that $U_i \leq \mathcal{U}_{x_{iy_j}}(w_{ij})$ and $V_j \leq \mathcal{V}_{x_{iy_j}}(w_{ij})$; by definition, $(\mathcal{U}_{x_{iy_j}}(w_{ij}), \mathcal{V}_{x_{iy_j}}(w_{ij})) \in \mathcal{F}_{x_{iy_j}}$, and because the latter set is a proper bargaining set, it follows from definition 1, part (ii) that $(U_i, V_j) \in \mathcal{F}_{x_{iy_j}}$. ■

2.A.4. Proof of proposition 1.

PROOF. Assume μ is an aggregate equilibrium matching. Then, by definition, there exists a pair of vectors U and V such that (μ, U, V) is an aggregate equilibrium outcome. Thus $D_{xy}(U_{xy}, V_{xy}) = 0$ for every $x \in \mathcal{X}$, $y \in \mathcal{Y}$, and $\mu_{xy} = \partial G(U) / \partial U_{xy}$ and $\mu_{xy} = \partial H(V) / \partial V_{xy}$, which inverts into

$$(2.A.1) \quad U_{xy} = \partial G^*(\mu) / \partial \mu_{xy} \text{ and } V_{xy} = \partial H^*(\mu) / \partial \mu_{xy},$$

and thus by substitution,

$$(2.A.2) \quad D_{xy} (\partial G^* (\mu) / \partial \mu_{xy}, \partial H^* (\mu) / \partial \mu_{xy}) = 0$$

holds for every $x \in \mathcal{X}$, $y \in \mathcal{Y}$. Conversely, assume (2.A.2) holds. Then, defining U and V by (2.A.1), one sees that (μ, U, V) is an aggregate equilibrium outcome. ■

2.A.5. Proof of proposition 2.

PROOF. Assume (μ, U, V) is an aggregate equilibrium outcome. Then $D_{xy} (U_{xy}, V_{xy}) = 0$ for every $x \in \mathcal{X}$, $y \in \mathcal{Y}$, and

$$(2.A.3) \quad \mu_{xy} = \partial G (U) / \partial U_{xy} = \partial H (V) / \partial V_{xy}$$

thus, by lemma 2, there exists a vector (W_{xy}) such that for every $x \in \mathcal{X}$, $y \in \mathcal{Y}$,

$$(2.A.4) \quad U_{xy} = \mathcal{U}_{xy} (W_{xy}) \quad \text{and} \quad V_{xy} = \mathcal{V}_{xy} (W_{xy}),$$

where \mathcal{U}_{xy} and \mathcal{V}_{xy} are defined in (3.3). Thus it follows that $Z(W) = 0$. Conversely, assume that $Z(W) = 0$. Then letting U and V as in (2.A.4), and μ such that $\mu_{xy} = \partial G (U) / \partial U_{xy}$, it follows that (μ, U, V) is an aggregate equilibrium outcome. ■

2.A.6. Proof of proposition 3.

PROOF. Recall that

$$Z_{x'y'} (W) = \frac{\partial H}{\partial V_{x'y'}} (\mathcal{V} (W)) - \frac{\partial G}{\partial U_{x'y'}} (\mathcal{U} (W))$$

and, because of assumption 3, $\partial G / \partial U_{x'y'} (U)$ is increasing in $U_{x'y'}$, decreasing in U_{xy} if either of the conditions $x = x'$ or $y = y'$ holds (but not both), a similar condition holds for H , and $W_{xy} \rightarrow \mathcal{V}_{xy} (W_{xy})$ is nonincreasing, while $W_{xy} \rightarrow \mathcal{U}_{xy} (W_{xy})$ is nondecreasing. At the same time, $\mathcal{U}_{xy} (W_{xy}) - \mathcal{V}_{xy} (W_{xy}) = W_{xy}$, so \mathcal{U}_{xy} and \mathcal{V}_{xy} cannot be stationary at the same point W_{xy} .

Proof of (a.1): One has $Z_{x'y'} (W) = \partial H / \partial V_{x'y'} (\mathcal{V} (W)) - \partial G / \partial U_{x'y'} (\mathcal{U} (W))$, thus the map $W_{x'y'} \rightarrow Z_{x'y'} (W)$ is nonincreasing. At the same time, as $\partial G / \partial U_{x'y'} (U)$ is increasing

in $U_{x'y'}$ and $\partial H/\partial V_{x'y'}(V)$ is increasing in $V_{x'y'}$ and as \mathcal{U}_{xy} and \mathcal{V}_{xy} cannot be stationary at the same point W_{xy} , it follows that $W_{x'y'} \rightarrow Z_{x'y'}(W)$ is decreasing.

Proof of (a.2): The proof is based on the same logic as above.

Proof of (a.3): When $x \neq x'$ and $y \neq y'$, then the quantity $\partial H/\partial V_{x'y'}(\mathcal{V}(W))$ does not depend on W_{xy} and nor does $\partial G/\partial U_{x'y'}(\mathcal{U}(W))$. Thus $Z_{x'y'}(W)$ does not depend on W_{xy} .

Proof of (b): One has

$$\sum_{x' \in \mathcal{X}, y' \in \mathcal{Y}} Z_{x'y'}(W) = \sum_{y'} m_{y'} - \sum_{x'} n_{x'} + \sum_{x'} \mu_{x'0}(\mathcal{U}(W)) - \sum_{y'} \mu_{0y'}(\mathcal{V}(W))$$

where $\mu_{x'0}(U)$ is defined as $n_{x'} - \sum_{y' \in \mathcal{Y}} \partial G(U)/\partial U_{x'y'}$, and $\mu_{0y'}(V)$ is defined as $m_{y'} - \sum_{x' \in \mathcal{X}} \partial H(V)/\partial V_{x'y'}$. But it is easy to check that $\mu_{x'0}(U) = n_{x'} \Pr(\varepsilon_{i0} > \max_{y' \in \mathcal{Y}} \{U_{x'y'} + \varepsilon_{iy'}\})$, thus $\mu_{x'0}(U)$ is decreasing with respect to all the entries of vector $U_{x'y'}$, $y' \in \mathcal{Y}$. A similar logic applies to show that $\mu_{0y'}(V)$ is decreasing with respect to all the entries of vector $V_{x'y'}$, $x' \in \mathcal{X}$. Hence, $\sum_{x' \in \mathcal{X}, y' \in \mathcal{Y}} Z_{x'y'}(W)$ is decreasing with respect to any entry of the vector W .

Remark: Conditions (a.1)–(a.3) express that $-Z$ is a Z-function, while conditions (a) and (b) together imply that $-Z$ is a M-function. ■

2.A.7. Proof of theorem 1. The existence part of theorem 1 is constructive, and consists in showing that algorithm 1 converges to a solution of equations (4.6); this convergence in turns follows from two claims, which are rather classical but included here for completeness. The uniqueness part relies on the fact that, by a result of Berry et al. (2013), the Gross Substitute property established in proposition 3 implies that the excess demand function Z is inverse antitone, thus injective.

We show that:

CLAIM 1. *There exist two vectors w^l and w^u such that $w^l \leq w^u$ and*

$$Z(w^u) \leq 0 \leq Z(w^l).$$

PROOF. By assumption 2 (iii), for each $x \in \mathcal{X}$, either all the men's payoffs U_{xy} are bounded above or they all converge to $+\infty$. Let \mathcal{X}_1 be the set of $x \in \mathcal{X}$ such that for each

$y \in \mathcal{Y}$, $\mathcal{U}_{xy}(w_{xy})$ all converge to $+\infty$ as $w_{xy} \rightarrow \bar{w}_{xy}$. For $x \in \mathcal{X}_1$, let $p_y = n_x(1 - 1/k) / |\mathcal{Y}|$, and let $U_{xy}^k = \partial G^* / \partial \mu_{xy}(p)$. It is easy to see that $U_{xy}^k \rightarrow +\infty$, thus $V_{xy}^k \rightarrow -\infty$. Hence there exists w_{xy} such that $\mathcal{U}_{xy}(w_{xy}) = U_{xy}^k$ and $V_{xy}^k = \mathcal{V}_{xy}(w^k)$. Now for $x \notin \mathcal{X}_1$, then for each $y \in \mathcal{Y}$, \bar{w}_{xy} is finite, and $\mathcal{U}_{xy}(w_{xy})$ all converge to a finite value $\bar{U}_{xy} \in \mathbb{R}$. Then, let $U_{xy}^k = \mathcal{U}_{xy}(\bar{w}_{xy} - 1/k)$ and $V_{xy}^k = \mathcal{V}_{xy}(\bar{w}_{xy} - 1/k)$, so that $V_{xy}^k \rightarrow -\infty$ and $U_{xy}^k \rightarrow \bar{U}_{xy} \in \mathbb{R}$. We have thus constructed vectors w^k such that $w_{xy}^k \rightarrow \bar{w}_{xy}$ for all x and y , and $\mathcal{V}_{xy}(w^k) \rightarrow -\infty$, while $\mathcal{U}_{xy}(w^k)$ converges to a vector of positive numbers. Thus, for k large enough, setting $w^u = w^k$ implies $Z(w^u) \leq 0$. A similar logic implies that there exists w^l such that $Z(w^l) \geq 0$. ■

CLAIM 2. Z is inverse antitone: if $Z(w) \leq Z(w')$ for some two vectors w and w' , then $w \geq w'$.

PROOF. We show that $-Z$ satisfies the assumptions in Berry et al. (2013), theorem 1, see also related results in Moré (1972), theorem 3.3. We verify the three assumptions in Berry et al. (2013). Assumption 1 in that paper is met because Z is defined on the Cartesian product of the intervals (w_{xy}, \bar{w}_{xy}) . Next, by part (a.2) of proposition 3 above, $-Z_{xy}(w)$ is weakly decreasing in $w_{x'y'}$ for $x'y' \neq xy$, and letting

$$Z_0(w) = \sum_{y'} m_{y'} - \sum_{x'} n_{x'} - \sum_{\substack{x \in \mathcal{X} \\ y \in \mathcal{Y}}} Z_{xy}(w),$$

it follows from part (b) of proposition 3 above that $-Z_0$ is strictly decreasing in all the w_{xy} . Thus assumption 2 and 3 in Berry et al. (2013) are also satisfied, hence $-Z$ is inverse isotone, Z is inverse antitone. ■

With these preparations, a proof of theorem 1 can be provided.

PROOF OF THEOREM 1. We prove existence first, then uniqueness.

Proof of existence: It is easy to see that Z is continuous, and by the results of proposition 3, it is strictly diagonally antitone, and off-diagonally isotone. Existence follows from theorem 3.1 in Rheinboldt (1970) jointly with proposition 3 and Claim 1. The proof there is based on a constructive argument based on nonlinear Gauss-Seidel iterations, as discussed in section 5.2.1.

Proof of uniqueness: As noted in Berry et al. (2013), uniqueness follows from Claim 2 as in corollary 1. Indeed, assume $Z(w) = Z(w')$. Then, by Claim 2, both inequalities $w \geq w'$ and $w' \geq w$ hold, and thus $w = w'$. ■

2.A.8. Proof of corollary 1.

PROOF. This corollary directly follows from a combination of proposition 2 and theorem 1. ■

2.A.9. Proof of theorem 5.

PROOF. (i) and (ii) follow as a direct consequence of equations (2.D.1), while (iii) follows from taking the pairwise difference of equations (2.D.2) and using the fact that $\mathcal{U}_{xy}(w_{xy}) - \mathcal{V}_{xy}(w_{xy}) = w_{xy}$. ■

2.A.10. Proof of theorem 2.

PROOF. By combining theorem 1 with proposition 1, it follows that equation (4.6), namely

$$D_{xy} \left(\frac{\partial G^*}{\partial \mu_{xy}}(\mu), \frac{\partial H^*}{\partial \mu_{xy}}(\mu) \right) = 0$$

has a unique solution. But when assumption 3 is strengthened into assumption 3', then

$$\partial G^* / \partial \mu_{xy}(\mu) = \log(\mu_{xy} / \mu_{x0}) \quad \text{and} \quad \partial H^* / \partial \mu_{xy}(\mu) = \log(\mu_{xy} / \mu_{0y})$$

where $\mu_{x0} = n_x - \sum_{y \in \mathcal{Y}} \mu_{xy}$ and $\mu_{0y} = m_y - \sum_{x \in \mathcal{X}} \mu_{xy}$. Hence equation (4.6) rewrites as

$$(2.A.5) \quad \begin{cases} D_{xy}(\log \mu_{xy} - \log \mu_{x0}, \log \mu_{xy} - \log \mu_{0y}) = 0 \\ \mu_{x0} + \sum_{y \in \mathcal{Y}} \mu_{xy} = n_x \\ \mu_{0y} + \sum_{x \in \mathcal{X}} \mu_{xy} = m_y \end{cases}$$

but $D_{xy}(\log \mu_{xy} - \log \mu_{x0}, \log \mu_{xy} - \log \mu_{0y}) = \log \mu_{xy} + D_{xy}(-\log \mu_{x0}, -\log \mu_{0y})$, thus system (2.A.5) rewrites as system (6.2). Conversely, assume (μ_{x0}, μ_{0y}) satisfy the system (6.2). Then, letting $\mu_{xy} = M_{xy}(\mu_{x0}, \mu_{0y})$, $U_{xy} = \log(\mu_{xy} / \mu_{x0})$ and $V_{xy} = \log(\mu_{xy} / \mu_{0y})$, one has \mathcal{M}^{int} , $D_{xy}(U_{xy}, V_{xy}) = 0$ and $U_{xy} = \log(\mu_{xy} / \mu_{x0})$ and $V_{xy} = \log(\mu_{xy} / \mu_{0y})$, thus (μ, U, V) is an aggregate equilibrium outcome. ■

2.A.11. Proof of theorem 3.

PROOF. The proof of theorem 2 is based on the following set of properties of $M_{xy}(\mu_{x0}, \mu_{0y}) = \exp(-D_{xy}(-\log \mu_{x0}, -\log \mu_{0y}))$, which are direct consequences of definition 1 and of lemma 1. For every pair $x \in \mathcal{X}$, $y \in \mathcal{Y}$:

(i) Map $M_{xy} : (a, b) \mapsto M_{xy}(a, b)$ is continuous.

(ii) Map $M_{xy} : (a, b) \mapsto M_{xy}(a, b)$ is weakly isotone, i.e. if $a \leq a'$ and $b \leq b'$, then $M_{xy}(a, b) \leq M_{xy}(a', b')$.

(iii) For each $a > 0$, $\lim_{b \rightarrow 0^+} M_{xy}(a, b) = 0$, and for each $b > 0$, $\lim_{a \rightarrow 0^+} M_{xy}(a, b) = 0$.

Given these properties, the existence of a solution (μ_{x0}, μ_{0y}) is essentially an application of Tarski's fixed point theorem; we provide an explicit proof for concreteness. We show that the construction of μ_{x0}^{2t+1} and μ_{0y}^{2t+2} at each step is well defined. Consider step $2t + 1$. For each $x \in \mathcal{X}$, the equation to solve is

$$\sum_{y \in \mathcal{Y}} M_{xy}(\mu_{x0}, \mu_{0y}) + \mu_{x0} = n_x$$

but the right-hand side is a continuous and increasing function of μ_{x0} , tends to 0 when $\mu_{x0} \rightarrow 0$ and tends to $+\infty$ when $\mu_{x0} \rightarrow +\infty$. Hence μ_{x0}^{2t+1} is well defined and belongs in $(0, +\infty)$. Denoting

$$\mu_{x0}^{2t+1} = \mathfrak{F}_x(\mu_{0.}^{2t}),$$

we see that \mathfrak{F} is antitone, meaning that $\mu_{0y}^{2t} \leq \tilde{\mu}_{0y}^{2t}$ for all $y \in \mathcal{Y}$ implies $\mathfrak{F}_x(\tilde{\mu}_{0.}^{2t}) \leq \mathfrak{F}_x(\mu_{0.}^{2t})$ for all $x \in \mathcal{X}$. By the same token, at step $2t + 2$, μ_{0y}^{2t+2} is well defined in $(0, +\infty)$, and we can denote

$$\mu_{0y}^{2t+2} = \mathfrak{G}_y(\mu_{0.}^{2t+1})$$

where, similarly, \mathfrak{G} is antitone. Thus, $\mu_{0.}^{2t+2} = \mathfrak{G} \circ \mathfrak{F}(\mu_{0.}^{2t})$, where $\mathfrak{G} \circ \mathfrak{F}$ is isotone. But $\mu_{0y}^2 \leq m_y = \mu_{0y}^0$ implies that $\mu_{0.}^{2t+2} \leq \mathfrak{G} \circ \mathfrak{F}(\mu_{0.}^2)$. Hence $(\mu_{0.}^{2t+2})_{t \in \mathbb{N}}$ is a decreasing sequence, bounded from below by 0. As a result $\mu_{0.}^{2t+2}$ converges. Letting $\bar{\mu}_{0.}$ be its limit, and letting $\bar{\mu}_{0x} = \mathfrak{F}(\bar{\mu}_{0.})$, it is not hard to see that $(\bar{\mu}_{0x}, \bar{\mu}_{0y})$ is a solution to (6.2). ■

2.A.12. Proof of theorem 4.

PROOF. Proof of part (i): One has $l(\hat{\pi}|\theta, \zeta) = \sum_{xy \in \mathcal{X}\mathcal{Y} \cup \mathcal{X}_0 \cup \mathcal{Y}_0} \hat{\pi}_{xy} \log \Pi_{xy}(\theta, \zeta)$, hence

$$l(\hat{\pi}|\theta, \zeta) = \sum_{xy \in \mathcal{X}\mathcal{Y} \cup \mathcal{X}_0 \cup \mathcal{Y}_0} \hat{\pi}_{xy} \log \mu_{xy}^{\theta, \zeta} - \log N^h(\theta, \zeta).$$

But as $\log \mu_{xy}^{\theta, \zeta} = -D_{xy}^\theta(u_x^\theta, v_y^\theta)$, we get

$$-l(\hat{\pi}|\theta, A\hat{\pi}) = \sum_{xy \in \mathcal{X}\mathcal{Y}} \hat{\pi}_{xy} D_{xy}^\theta(u_x^\theta, v_y^\theta) + \sum_{x \in \mathcal{X}} \hat{\pi}_{x0} u_x^\theta + \sum_{y \in \mathcal{Y}} \hat{\pi}_{0y} v_y^\theta + \log N^h(\theta, \zeta).$$

Proof of part (ii): This asymptotic result is classical in nonparametric estimation in the presence of a nuisance parameter; see e.g. van der Vaart (2000), Chapter 25. θ is such that $\nabla_\theta l(\pi|\theta, A\pi) = 0$, and $\hat{\theta}$ is such that $\nabla_\theta l(\hat{\pi}|\hat{\theta}, A\hat{\pi}) = 0$. As a result, $\nabla_\theta l(\hat{\pi}|\hat{\theta}, A\hat{\pi}) - \nabla_\theta l(\pi|\theta, A\pi) = 0$, and

$$(D_{\theta\pi} l)(\hat{\pi} - \pi) + (D_{\theta\zeta} l) A(\hat{\pi} - \pi) + (D_{\theta\theta} l)(\hat{\theta} - \theta) = o(n^{-1/2})$$

but by first order conditions, the first term is equal to zero, and the equation becomes

$$\hat{\theta} - \theta = \mathcal{I}_{11}^{-1} \mathcal{I}_{12} A(\hat{\pi} - \pi) + o(n^{-1/2})$$

and as the convergence in distribution $n^{1/2}(\hat{\pi} - \pi) \Rightarrow V_\pi$ holds as $n \rightarrow \infty$, expression (6.10) follows. ■

2.A.13. Proof of theorem 6.

PROOF. Proof of part (i): Let (μ, U, V) be an aggregate equilibrium matching, and let u_i and v_j as in (2.E.1). By definition of these quantities, one has $u_i - \varepsilon_{iy} \geq U_{xy}$ and $v_j - \eta_{xj} \geq V_{xy}$, thus $D_{xy}(u_i - \varepsilon_{iy}, v_j - \eta_{xj}) \geq D_{xy}(U_{xy}, V_{xy}) = 0$. Further, $u_i \geq \varepsilon_{i0}$ and $v_j \geq \eta_{0j}$, hence the stability condition holds. Let us show that one can construct μ_{ij} so that (μ_{ij}, u_i, v_j) is feasible. For $x \in \mathcal{X}$ and $y \in \mathcal{Y}$, let \mathcal{I}_{xy} be the set of $i \in \mathcal{I}$ such that $x_i = x$ and $y = \arg \max_{y \in \mathcal{Y}_0} \{U_{xy} + \varepsilon_{iy}, \varepsilon_{i0}\}$. Similarly, let \mathcal{J}_{xy} be the set of $j \in \mathcal{J}$ such that $y_j = y$ and $x = \arg \max_{x \in \mathcal{X}_0} \{V_{xy} + \eta_{xj}, \eta_{0j}\}$. The mass of \mathcal{I}_{xy} is $\partial G(U) / \partial U_{xy}$ and the mass of \mathcal{J}_{xy} is $\partial H(V) / \partial V_{xy}$. The equilibrium condition $\mu = \nabla G(U) = \nabla H(V)$ implies therefore that the mass of \mathcal{I}_{xy} and the mass of \mathcal{J}_{xy} coincide. One can therefore take any assignment of men in \mathcal{I}_{xy} to women in \mathcal{J}_{xy} . Let μ_{ij} be the resulting individual assignment.

If $\mu_{ij} > 0$, then $i \in \mathcal{I}_{x_i y_j}$ and $j \in \mathcal{J}_{x_i y_j}$, therefore $u_i = U_{xy} + \varepsilon_{iy}$ and $v_j = V_{xy} + \eta_{xj}$, thus $D_{xy}(u_i - \varepsilon_{iy}, v_j - \eta_{yj}) = D_{xy}(U_{xy}, V_{xy}) = 0$. Assume i is unassigned under (μ_{ij}) ; then for all $y \in \mathcal{Y}$, $u_i > U_{xy} + \varepsilon_{iy}$, and thus $u_i = \varepsilon_{i0}$. Similarly, if j is unassigned under (μ_{ij}) , then $v_j = \eta_{0j}$. Hence, (μ_{ij}, u_i, v_j) is an individual equilibrium.

Proof of part (ii): Now assume (μ_{ij}, u_i, v_j) is an individual equilibrium. Then for all i and j , the stability condition

$$D_{x_i y_j}(u_i - \varepsilon_{iy}, v_j - \eta_{jx}) \geq 0,$$

holds, and holds with equality if $\mu_{ij} > 0$. Hence, for all pairs x and y , we have the inequality

$$\min_{i:x_i=x} \min_{j:y_j=y} \{D_{x_i y_j}(u_i - \varepsilon_{iy}, v_j - \eta_{jx})\} \geq 0,$$

with equality if $\mu_{xy} > 0$, that is, if there is at least one marriage between a man of type x and a woman of type y . Taking U and V as (2.E.2), and making use of the strict monotonicity of D_{xy} in both its arguments, matching $\mu \in \mathcal{M}$ is an equilibrium matching if inequality $D_{xy}(U_{xy}, V_{xy}) \geq 0$ holds for any x and y , with equality if $\mu_{xy} > 0$. By definition of U and V , one has

$$u_i \geq \max_y \{U_{xy} + \varepsilon_{iy}, \varepsilon_{i0}\} \quad \text{and} \quad v_j \geq \max_{x \in \mathcal{X}} \{V_{xyj} + \eta_{xj}, \eta_{0j}\}.$$

Assume one of these inequalities holds strict, for instance $u_i > \max_y \{U_{xy} + \varepsilon_{iy}, \varepsilon_{i0}\}$. Then $u_i - \varepsilon_{iy} > U_{xy}$. Because D was assumed strictly increasing, this implies that for all j

$$D_{xy}(u_i - \varepsilon_{iy}, v_j - \eta_{yj}) > D_{xy}(U_{xy}, v_j - \eta_{yj}) \geq D_{xy}(U_{xy}, V_{xy}) \geq 0$$

thus for all j , $\mu_{ij} = 0$. Therefore i is single, but $u_i > \varepsilon_{i0}$ yields a contradiction. Now assumption 3 implies $\mu_{xy} > 0$ for all x and y , thus $D_{xy}(U_{xy}, V_{xy}) = 0$. ■

2.A.14. Proof of corollary 2.

PROOF. Let (μ_{ij}, u_i, v_j) be an individual outcome. By part (ii) of theorem 6, the aggregate outcome (μ, U, V) is such that

$$U_{xy} = \min_{i:x_i=x} \{u_i - \varepsilon_{iy}\} \quad \text{and} \quad V_{xy} = \min_{j:y_j=y} \{v_j - \eta_{jx}\},$$

hence $U_{xy} \geq u_i - \varepsilon_{iy}$ and $V_{xy} \geq v_j - \eta_{jx}$, but $D_{xy}(U_{xy}, V_{xy}) = 0$ and $D_{xy}(u_i - \varepsilon_{iy}, v_j - \eta_{jx}) = 0$, thus, by assumption 2', $U_{xy} = u_i - \varepsilon_{iy}$ and $V_{xy} = v_j - \eta_{jx}$. Hence $u_i = U_{xy} + \varepsilon_{iy}$ and $v_j = V_{xy} + \eta_{jx}$. ■

2.B. Other models of interest

2.B.1. Matching with a Linear Tax Schedule. Our framework can also model a labor market with linear tax: Assume the nominal wage W_{ij} is taxed at rate $1 - R_{ij}$ on the employee's side (income tax) and at rate $1 + C_{ij}$ on the firm's side (social contributions). The tax rates are allowed to depend on both employer and employee characteristics. Assume that if employee i and employer j match and decide on a wage W_{ij} , they respectively have (post-transfer) utilities $u_i = \alpha_{ij} + R_{ij}W_{ij}$ and $v_j = \gamma_{ij} - C_{ij}W_{ij}$, where α_{ij} is job j 's amenity to worker i , and γ_{ij} is the productivity of worker i on job j . This specification is called the *Linearly Transferable Utility (LTU)* model, and the feasible set is given by

$$\mathcal{F}_{ij} = \{(u, v) \in \mathbb{R}^2 : \lambda_{ij}u + \zeta_{ij}v \leq \Phi_{ij}\},$$

where $\lambda_{ij} = 1/R_{ij} > 0$, and $\zeta_{ij} = 1/C_{ij} > 0$, and $\Phi_{ij} = \lambda_{ij}\alpha_{ij} + \zeta_{ij}\gamma_{ij}$. Note that the TU case is recovered when $\lambda_{ij} = 1$ and $\zeta_{ij} = 1$. A simple calculation yields

$$(2.B.1) \quad D_{ij}(u, v) = \frac{\lambda_{ij}u + \zeta_{ij}v - \Phi_{ij}}{\lambda_{ij} + \zeta_{ij}},$$

and we have

$$\mathcal{U}_{ij}(w) = \frac{\Phi_{ij} + \zeta_{ij}w}{\lambda_{ij} + \zeta_{ij}} \text{ and } \mathcal{V}(w) = \frac{\Phi_{ij} - \lambda_{ij}w}{\lambda_{ij} + \zeta_{ij}}.$$

The LTU model (2.B.1) is studied in depth in Dupuy et al. (2017), who carry welfare analysis. In the case when the heterogeneity is logit, the matching function becomes

$$(2.B.2) \quad M_{xy}(\mu_{x0}, \mu_{0y}) = e^{(\lambda_{xy}\alpha_{xy} + \zeta_{xy}\gamma_{xy})/(\lambda_{xy} + \zeta_{xy})} \mu_{x0}^{\lambda_{xy}/(\lambda_{xy} + \zeta_{xy})} \mu_{0y}^{\zeta_{xy}/(\lambda_{xy} + \zeta_{xy})}.$$

In particular, when $\lambda_{xy} = 1$ and $\zeta_{xy} = 1$, one recovers the Choo and Siow (2006) matching function.

2.B.2. Matching with Uncertainty. Now, we consider a model of matching with risksharing; such a model is considered by Legros and Newman (2007), Chade and Eeckhout

(2014), and Chiappori and Reny (2016) who all focus on characterizing (positive or negative) assortativeness. Assume $i \in \mathcal{I}$ are the men and $j \in \mathcal{J}$ are the women. Assume that the joint endowment of the household ij is $\tilde{b}_{ij} \in \mathbb{R}^d$, which is stochastic. Let $\tilde{c}_i \in \mathbb{R}^d$ be the contingent consumption of the man, and $\tilde{c}_j \in \mathbb{R}^d$ be the contingent consumption of the woman. It is assumed that i and j are expected utility maximizers with respective (concave) utilities U_i and V_j . Hence, they enjoy respective utilities $\mathbb{E}[U_i(\tilde{c}_i)]$ and $\mathbb{E}[V_j(\tilde{c}_j)]$. Letting λ and $(1 - \lambda)$ be the respective Pareto weights associated to i and j 's consumption, the contingent consumptions conditional on household budget b are given by the program

$$S_{ij}(\lambda; b) = \max_{c_i, c_j \geq 0} \{ \lambda U_i(c_i) + (1 - \lambda) V_j(c_j) : c_i + c_j \leq b \}.$$

The set of feasible utilities for household ij is

$$\mathcal{F}_{ij} = \left\{ (u, v) \in \mathbb{R}^2 : \max_{\lambda \in [0, 1]} \left\{ \lambda u + (1 - \lambda) v - \mathbb{E} \left[S_{ij}(\lambda; \tilde{b}_{ij}) \right] \right\} \leq 0 \right\},$$

from which it is immediate that the corresponding distance-to-frontier is

$$D_{ij}(u, v) = \max_{\lambda \in [0, 1]} \left\{ \lambda u + (1 - \lambda) v - \mathbb{E} \left[S_{ij}(\lambda; \tilde{b}_{ij}) \right] \right\}.$$

2.B.3. Matching with a collective model and a public good. The model of section 3.3.4 can be extended to incorporate a public good, which is of particular interest in models of household bargaining. Assume utilities are given by

$$\mathbf{u}_{ij}(c_i, g) = \alpha_{ij}(g) + \tau_{ij} \log c_i \text{ and } \mathbf{v}_{ij}(c_j) = \gamma_{ij}(g) + \tau_{ij} \log c_j,$$

where the budget constraint is $c_i + c_j + pg = B_{ij}$. Then the distance-to-frontier function is given by

$$(2.B.3) \quad D_{ij}(u, v) = \min_{0 \leq g \leq B_{ij}/p} \tau_{ij} \log \left(\frac{\exp\left(\frac{u - \alpha_{ij}(g)}{\tau_{ij}}\right) + \exp\left(\frac{v - \gamma_{ij}(g)}{\tau_{ij}}\right)}{B_{ij} - pg} \right).$$

Indeed, $D_{ij}(u, v) \leq 0$ if and only if there is $0 \leq g \leq B_{ij}/p$ and $c_i = \exp\left(\frac{u - \alpha_{ij}(g)}{\tau_{ij}}\right)$, $c_j = \exp\left(\frac{v - \gamma_{ij}(g)}{\tau_{ij}}\right)$ with $c_i + c_j + pg = B_{ij}$ and $u = \alpha_{ij}(g) + \tau_{ij} \log c_i$ and $v = \gamma_{ij}(g) + \tau_{ij} \log c_j$.

In particular, when the preferences are Cobb-Douglas, we have

$$\mathbf{u}_{ij}(c_i, g) = \alpha_{ij} + \sigma_{ij} \log g + \tau_{ij} \log c_i \text{ and } \mathbf{v}_{ij}(c_j) = \gamma_{ij} + \sigma_{ij} \log g + \tau_{ij} \log c_j,$$

and a straightforward calculation shows that expression (2.B.3) simplifies to

$$(2.B.4) \quad D_{ij}(u, v) = \tau_{ij} \log \left(\frac{\exp \left(\frac{u - \alpha_{ij}}{\tau_{ij}} - \frac{\sigma_{ij}}{\tau_{ij}} \log \frac{\sigma_{ij} B_{ij}}{p(\tau_{ij} + \sigma_{ij})} \right) + \exp \left(\frac{v - \gamma_{ij}}{\tau_{ij}} - \frac{\sigma_{ij}}{\tau_{ij}} \log \frac{\sigma_{ij} B_{ij}}{p(\tau_{ij} + \sigma_{ij})} \right)}{B_{ij} \tau_{ij} / (\tau_{ij} + \sigma_{ij})} \right).$$

2.C. Remarks on equilibrium vs optimality

In this appendix, we comment on the contrast between equilibrium and optimality, which manifests itself in our model. It is a well known fact (which is studied in Monge-Kantorovich theory) that in the TU case, equilibrium and optimality coincide. This coincidence is not preserved under the more general ITU framework. The following remarks clarify this.

REMARK 2.C.1 (Equilibrium vs. Optimality). As argued in example 3.3.1 above, the TU matching model (also sometimes called the *optimal assignment model*), is recovered in the case $D_{ij}(u, v) = u + v - \Phi_{ij}$ for some joint surplus matrix Φ_{ij} , shared additively between partners. It is well known in this case that the equilibrium conditions are the complementary slackness conditions for optimality in a linear programming problem, so in this case, equilibrium and optimality coincide. However, outside of this case, our conditions are *not* the first-order conditions associated to an optimization problem, and equilibrium does not have an interpretation as the maximizer of some welfare function.

REMARK 2.C.2 (Abstract convexity). When D_{ij} is strictly increasing in each of its arguments (or equivalently, when the upper frontier of \mathcal{F}_{ij} is strictly downward sloping), one may define

$$\mathbb{U}_{ij}(v) = \max \{u : D_{ij}(u, v) \leq 0\} \text{ and } \mathbb{V}_{ij}(u) = \max \{v : D_{ij}(u, v) \leq 0\}$$

and it can be verified that \mathbb{U}_{ij} and \mathbb{V}_{ij} are continuous, strictly decreasing, and inverses of each other. In this case, if u and v are equilibrium payoff vectors, then

$$v_j = \max_{i \in \mathcal{I}} \{\mathbb{V}_{ij}(u_i), \mathcal{V}_{0j}\} \text{ and } u_i = \max_{j \in \mathcal{J}} \{\mathbb{U}_{ij}(v_j), \mathcal{U}_{i0}\}.$$

In particular, in the TU case studied in example 3.3.1 above, $\mathbb{U}_{ij}(v) = \Phi_{ij} - v$ and $\mathbb{V}_{ij}(u) = \Phi_{ij} - u$. The maps \mathbb{U}_{ij} and \mathbb{V}_{ij} are called *Galois connections*, and are investigated directly by Nöldeke and Samuelson (2015); they also appear implicitly in Legros and Newman (2007). Our setting is more general than Galois connections, as the NTU case studied in example 3.3.2 above cannot be described using Galois connections.

REMARK 2.C.3. In the TU setting, $D_{xy}(u, v) = (u + v - \Phi_{xy})/2$; thus, the matching equation (4.6) can be rewritten as $\nabla G^*(\mu) + \nabla H^*(\mu) = \Phi_{xy}$. In this case, Galichon and Salanié (2015) have shown the existence and uniqueness of a solution to (4.6) by showing that this equation coincides with the first-order conditions associated to the utilitarian welfare maximization problem, namely

$$\max_{\mu} \left\{ \sum_{xy} \mu_{xy} \Phi_{xy} - \mathcal{E}(\mu) \right\},$$

where $\Phi = \alpha + \gamma$ is the systematic part of the joint affinity, and $\mathcal{E} := G^* + H^*$ is an entropy penalization that trades-off against the maximization of the observable part of the joint affinity. However, just as in the context of Remark 2.C.1, besides the particular case of Transferable Utility, (4.6) cannot be interpreted in general as the first-order conditions of an optimization problem because the function defined by the left hand-side of (4.6) does not have a symmetric Jacobian. Hence, the methods developed in the present paper, which are based on gross substitutability, are very different than those of Galichon and Salanié (2015), which rely on convex optimization.

2.D. Identification

In this appendix, we discuss identification issues. Assume that the parameter to be identified is $\theta = (\alpha_{xy}, \gamma_{xy}, \tau_{xy})$, and the feasible sets are parameterized by θ so that the distance-to-frontier function is given by

$$D^{\theta}(u, v) = \tau_{xy} d_{xy} \left(\frac{u - \alpha_{xy}}{\tau_{xy}}, \frac{v - \gamma_{xy}}{\tau_{xy}} \right),$$

where $d_{xy}(a, b)$ is a known function, for instance, in the ETU model of paragraph 3.3.4, $d_{xy}(a, b) = \log \left(\frac{\exp(a) + \exp(b)}{2} \right)$. The interpretation of the parameters is straightforward;

α_{xy} represents the men's preferences; γ_{xy} represents the women's preferences, and τ_{xy} represents the curvature of the efficient bargaining frontier. Letting $\mathcal{U}_{xy}(w) = -d_{xy}(0, -w)$ and $\mathcal{V}_{xy}(w) = -d_{xy}(w, 0)$, one has that the feasibility condition $D^\theta(U_{xy}, V_{xy}) = 0$ holds if and only if $U_{xy} = \alpha_{xy} + \tau_{xy}\mathcal{U}_{xy}(w_{xy})$ and $V_{xy} = \gamma_{xy} + \tau_{xy}\mathcal{V}_{xy}(w_{xy})$, which, coupled with the optimality conditions $\nabla G^*(\mu) = U$ and $\nabla H^*(\mu) = V$, yields

$$(2.D.1) \quad \begin{cases} \nabla G^*(\mu) = \alpha_{xy} + \tau_{xy}\mathcal{U}_{xy}(w_{xy}) \\ \nabla H^*(\mu) = \gamma_{xy} + \tau_{xy}\mathcal{V}_{xy}(w_{xy}) \end{cases},$$

for some $w_{xy} = \frac{(U_{xy} - \alpha_{xy}) - (V_{xy} - \gamma_{xy})}{\tau_{xy}}$, which is equal to the (algebraic) quantity received by x minus the quantity received by y . Note that system (2.D.1) exhausts the equilibrium conditions of the model.

THEOREM 5. (i) *Assume the matching patterns (μ_{xy}) and the transfers (w_{xy}) are observed, and $\tau = (\tau_{xy})$ is known. Then α and γ are point-identified by*

$$(2.D.2) \quad \alpha_{xy} = \frac{\partial G^*(\mu)}{\partial \mu_{xy}} - \tau_{xy}\mathcal{U}_{xy}(w_{xy}) \quad \text{and} \quad \gamma_{xy} = \frac{\partial H^*(\mu)}{\partial \mu_{xy}} - \tau_{xy}\mathcal{V}_{xy}(w_{xy}).$$

(ii) *Assume only the matching patterns (μ_{xy}) are observed, and $\tau = (\tau_{xy})$ is known. Then α and γ are set-identified by*

$$(2.D.3) \quad (\alpha_{xy}, \gamma_{xy}) \in \left\{ \left(\frac{\partial G^*(\mu)}{\partial \mu_{xy}} - \tau_{xy}\mathcal{U}_{xy}(w_{xy}), \frac{\partial H^*(\mu)}{\partial \mu_{xy}} - \tau_{xy}\mathcal{V}_{xy}(w_{xy}) \right) : w_{xy} \in \mathbb{R} \right\}.$$

(iii) *Assume there are K markets where the matching patterns (μ_{xy}^k) and the transfers (w_{xy}^k) are observed. Then τ is identified by the fixed-effect regression*

$$(2.D.4) \quad \frac{\partial G^*(\mu^k)}{\partial \mu_{xy}} - \frac{\partial H^*(\mu^k)}{\partial \mu_{xy}} = (\alpha_{xy} - \gamma_{xy}) + \tau_{xy}w_{xy}^k,$$

and α and γ are in turn identified by (2.D.2).

Let $U_{xy}^k = \partial G^*(\mu^k) / \partial \mu_{xy}$ and $V_{xy}^k = \partial H^*(\mu^k) / \partial \mu_{xy}$. We have $\Delta U_{xy}^k - \Delta V_{xy}^k = \tau_{xy}\Delta w_{xy}^k$, where $\Delta z^k = z^k - \bar{z}$ and $\bar{z} = K^{-1} \sum_{k'} z_{k'}$. Hence

$$\hat{\tau}_{xy} = \frac{\sum_{k=1}^K (\Delta U_{xy}^k - \Delta V_{xy}^k) \Delta w_{xy}^k}{\sum_{k=1}^K (\Delta w_{xy}^k)^2},$$

and

$$\begin{cases} \alpha_{xy} = \bar{U}_{xy} - \hat{\tau}_{xy} K^{-1} \sum_{k=1}^K \mathcal{U}_{xy}(w_{xy}^k) \\ \gamma_{xy} = \bar{V}_{xy} - \hat{\tau}_{xy} K^{-1} \sum_{k=1}^K \mathcal{V}_{xy}(w_{xy}^k) \end{cases}.$$

Theorem 5 clarifies what can be identified depending on how much is observed. If only the matching patterns μ are observed, then part (i) of the theorem expresses that if the matching patterns and the transfers are observed, then the matching preferences α and γ on both sides of the market can be identified conditional on the knowledge of the curvature of the bargaining frontier. Part (ii) expresses that it is not possible to identify both α and γ simultaneously without observing the transfers. In order to identify simultaneously α , γ , and τ , then observations on multiple markets are needed, as shown in part (iii) of the theorem.

2.E. Relating individual and aggregate equilibria

In this appendix, we establish a precise connection between individual equilibria (defined in section 3), and aggregate equilibria (defined in section 4). It will be useful at some point to introduce a slightly stronger assumption than assumption 2, to handle the case when the frontiers of the bargaining sets are strictly downward sloping. This leads us to formulate:

ASSUMPTION 2'. The sets \mathcal{F}_{ij} satisfy assumption 2, and in addition, D_{ij} is strictly increasing in both its arguments for all i and j .

Note that in the NTU case, the frontier of the feasible set is not strictly downward-sloping, and therefore assumption 2' is not satisfied, while it is satisfied for all the other examples in section 3.3.

The following result relates the individual and aggregate equilibria.

THEOREM 6. *(i) Under assumptions 1, 2, and 3, let (μ, U, V) be an aggregate equilibrium outcome. Then, defining*

$$(2.E.1) \quad u_i = \max_{y \in \mathcal{Y}} \{U_{x_i y} + \varepsilon_{iy}, \varepsilon_{i0}\} \quad \text{and} \quad v_j = \max_{x \in \mathcal{X}} \{V_{xy_j} + \eta_{xj}, \eta_{0j}\},$$

there is an individual matching μ_{ij} such that (μ_{ij}, u_i, v_j) is an individual equilibrium outcome, which is such that $\mu_{ij} > 0$ implies $u_i = U_{x_i y} + \varepsilon_{iy}$ and $v_j = V_{xy_j} + \eta_{xj}$.

(ii) Under assumptions 1, 2', and 3, let (μ_{ij}, u_i, v_j) be an individual equilibrium outcome.

Then, defining

$$(2.E.2) \quad U_{xy} = \min_{i:x_i=x} \{u_i - \varepsilon_{iy}\} \quad \text{and} \quad V_{xy} = \min_{j:y_j=y} \{v_j - \eta_{jx}\},$$

and $\mu_{xy} = \sum_{ij \in \mathcal{I} \times \mathcal{J}} \mu_{ij} \mathbb{1}\{x_i = x\} \mathbb{1}\{y_j = y\}$, it follows that (μ, U, V) is an aggregate equilibrium outcome.

Note that deducing an aggregate equilibrium based on an individual equilibrium (part ii) requires a slightly stronger assumption than deducing an individual equilibrium based on an aggregate equilibrium (part i). The NTU case (not covered under assumption 2') thus deserves further investigations, which are carried in Galichon and Hsieh (2017).

Part (ii) of theorem 6 implies that agents keep their entire utility shocks at equilibrium, even when they could transfer them fully or partially.

COROLLARY 2. *Under assumptions 1, 2', and 3, consider a pair of matched individuals i and j of types x and y respectively. Then the equilibrium payoffs of i and j are respectively given by $u_i = U_{xy} + \varepsilon_{iy}$ and $v_j = V_{xy} + \eta_{jx}$, where U and V are aggregate equilibrium payoffs. Therefore, individuals keep their idiosyncratic utility shocks at equilibrium.*

This finding, which carries strong testable implications, was known in the TU case (see Chiappori, Salanié, and Weiss (2017)). Our theorem clarifies the deep mechanism that drives this result: the crucial assumption is that the distance-to-frontier function D_{ij} should only depend on i and j through the observable types x_i and y_j , and that some transfers are possible.

2.F. Illustrative example

2.F.1. A simple marriage model. We estimate the model introduced in paragraph 3.3.4—a model with marital complementarities and private consumption—using household consumption data. Following the setup of section 6, we assume logit heterogeneity. In the spirit of paragraph 3.3.4, the systematic utilities of a man of type x and a woman of type y paired together are specified as $\alpha_{xy} + \tau \log c_{xy}^m$ and $\gamma_{xy} + \tau \log c_{xy}^w$, respectively, where c_{xy}^m

and c_{xy}^w are respectively the private consumption levels of the man and the woman. Private consumption should satisfy the budget constraint $c_{xy}^m + c_{xy}^w = I_x + I_y$, where I_x and I_y are the income of men of type x and women of type y , respectively. It follows from theorem 6 that at equilibrium, c_{xy}^m and c_{xy}^w only depend on the man and the woman's observable types. In addition, we specify the utility of men and women of remaining single as $\alpha_{x0} + \tau \log I_x$ and $\gamma_{0y} + \tau \log I_y$.

The systematic parts of the matching surpluses, relative to singlehood, for a married pair x, y are given by

$$U_{xy} = \alpha_{xy} - \alpha_{x0} + \tau \log \left(\frac{c_{xy}^m}{I_x} \right) \text{ and } V_{xy} = \gamma_{xy} - \gamma_{0y} + \tau \log \left(\frac{c_{xy}^w}{I_y} \right).$$

Without loss of generality, we assume in the sequel that $\alpha_{x0} = 0$ and $\gamma_{0y} = 0$. The budget constraint $c_{xy}^m + c_{xy}^w = I_x + I_y$ implies an expression for the feasible set \mathcal{F} and the distance-to-frontier function D_{xy} . A calculation similar to the one in section 3 shows that

$$D_{xy}(U_{xy}, V_{xy}) = \tau \log \left(\rho_{xy} \exp \left(\frac{U_{xy} - \alpha_{xy}}{\tau} \right) + (1 - \rho_{xy}) \exp \left(\frac{V_{xy} - \gamma_{xy}}{\tau} \right) \right),$$

where $\rho_{xy} = \frac{I_x}{I_x + I_y}$ denotes the man's share of contribution to total income of the household.

We estimate the model on a marriage market based on data from the British Living Costs and Food survey (ONS, 2015), which contains the relevant information on incomes, demographics (such as age and education), and a proxy for private consumption. The moderate size of the sample we retain is well suited for the purpose of illustration; however, estimation scales up well in the ITU-logit framework, as we discuss in section 2.F.4 below.

2.F.2. The data. To estimate our model, we use the British Living Costs and Food Survey data set (which replaced the Family Expenditure Survey in 2008) for the year 2013 (see ONS, 2015). The data allows us to construct a toy marriage market that includes raw estimates of personal expenditures. We focus on married heterosexual pairs, in which case we gather information on both partners, as well as singles (never married, divorced, separated or widowed)⁸ who are heads of their households. We only keep couples in which both

⁸Ideally, it would be preferable to focus on first-time married couples and never-married singles, but such detailed information on marital history is usually missing in expenditures data sets.

members have positive income, and singles with positive income. Additionally, we restrict our attention to households of size 1 for singles and size 2 for couples (hence excluding households with children or relatives and non-relatives), as we focus our attention on the sharing of resources between the married partners. Another advantage of such restriction is that we exclude from the analysis a major public good, namely, investment in children and their education. Finally, we select households in which the head is between 25 and 40 years old, and drop singles or couples with missing information.

The total income of a matched pair is the sum of the partners' personal incomes. Ideally, our application would combine income data with data on private consumption. Of course, private consumption variables are rarely available, and researchers must instead use a proxy of personal expenditures. The data offers a variable called "Total Personal Expenditures"; this is an imperfect measure of consumption, however, it excludes major public goods such as rent, heating or car purchases, while aggregating individual-level expenditures on food, household equipment, leisure goods and services, and clothing. For singles, the variable is set equal to total personal income. For couples, personal expenditure is taken by breaking down the total income proportionally according to each partner's share of personal expenditures. This ensures that the sum of personal expenditures across partners coincides with the couple's total income.

TABLE 1. Summary statistics, full sample

	Married				Single			
	Male		Female		Male		Female	
	mean	sd	mean	sd	mean	sd	mean	sd
Age	32.37	4.60	30.33	4.94	33.43	4.61	32.58	4.35
White	0.92	0.27	0.90	0.30	0.87	0.34	0.89	0.31
Black	0.01	0.11	0.02	0.14	0.04	0.20	0.05	0.21
Education	19.84	2.99	20.14	3.08	18.80	2.97	19.17	2.65
Personal Income	638.05	325.88	485.36	264.95	544.27	330.86	478.32	275.12
Share Expenditures	0.47	0.23	0.53	0.23
Observations	161		161		76		66	

Summary statistics are displayed in Table (1). Our sample is mostly composed of White individuals. Married men appear to be older than married women (with an average age difference of two years, a fairly standard fact in marriage markets), but somewhat less

educated. The data displays large variations in personal income, and shows that women account for a slightly larger share of personal expenditures than men. However, this may be a consequence of measurement error on private consumption, as the latter is only imperfectly observed. Finally, due to our restrictions on couples and singles selection (children and age requirements), our sample is rather small but is well suited for our illustration exercise.

2.F.3. Estimation. Estimation follows the steps described in section 6. We assume that the weight of each man and woman in our sample is uniform, so that $n_x = m_y = 1$ for all $x, y \in \mathcal{X} \times \mathcal{Y}$. We do not worry here about the fact that the types are sampled from a continuous distribution; if we did, our model would have to be amended to the continuous logit framework used by Dupuy and Galichon (2014) and Menzel (2015), but the estimation would be identical. The likelihood function is similar to expression (6.8), but we augment it by making use of the fact that our model predicts private consumption, as described in section 2.F.4 below.

We use a simple parametrization of couples' pre-transfer utilities:

$$\begin{aligned}\alpha_{xy} &= \alpha_1|\text{educ}_x - \text{educ}_y| + \alpha_2|\text{age}_x - \text{age}_y| \\ \gamma_{xy} &= \gamma_1|\text{educ}_x - \text{educ}_y| + \gamma_2|\text{age}_x - \text{age}_y|,\end{aligned}$$

where educ_x and educ_y are the (standardized) ages at which the members of the couple left the schooling system—a proxy for years of education (we also standardized the age variables, age_x and age_y).

TABLE 2. Estimates

Parameters	α_1	α_2	γ_1	γ_2	s_ϵ	τ
Estimates	-1.26	-1.90	-1.80	-2.04	301.75	3.26
CI	[-1.84, -0.76]	[-2.42, -1.39]	[-2.35, -1.23]	[-2.78, -1.50]	[256.22, 353.61]	[1.79, 7.57]

Note: These estimates are obtained using the TraME package (Galichon, O'hara and Weber (2017)) and the NLOPT optimization routine. Parameters α_1 and γ_1 measure education assortativeness, α_2 and γ_2 measure age assortativeness, and τ captures the curvature of the bargaining frontier. The standard deviation of our measurement error is estimated as s_ϵ . Confidence intervals at the 5% level are computed using 200 bootstrap estimates.

The results of our maximum likelihood estimation are presented in Table 2. They are robust to random selection of starting points (multistart). As an additional robustness

check, we estimated the model for a range of fixed values of τ (from low, NTU-limit values to high, TU-limit values). The value of the log-likelihood is decreasing for small and large values of τ . Finally, we computed the hessian of the log-likelihood function at the optimal value of the parameters and checked its invertibility.

Table 2 calls for several comments. We provide bootstrapped confidence intervals for our parameter estimates. We used 200 replications and report percentiles intervals at the 5% level. The coefficients corresponding to education and age assortativeness are in line with the prior literature on marriage—they indicate that utility decreases as distance between the education level or age of the partners increases. Hence, our results unsurprisingly suggest positive assortative mating in education and age. We also obtain an estimate of the curvature of the efficient bargaining frontier that is suggestive an intermediate case between NTU and TU. Although our estimates are too imprecise to reject either TU or NTU, our illustration—focusing on a relatively simple model of marriage with education assortativeness and consumption, and making use of crude expenditure data—suggests the potential of the approach.

2.F.4. Computational Details. We modify the likelihood in expression (6.8) to account for the fact that the model predicts private consumption, which is imperfectly observed in the data. For man i , we have

$$c_{x_i y_j}^m = I_{x_i} \exp(u_{x_i} - D_{x_i y_j}(u_{x_i}, v_{y_j}) - \alpha_{x_i y_j})^{1/\tau}$$

with the notation $u_x = -\log \mu_{x0}$ and $v_y = -\log \mu_{0y}$. We assume further that we measure private expenditures with some measurement error, that is we observe men's private consumptions as $\hat{c}_i^m = c_{x_i, y_j}^m + \epsilon_{ij}$, where ϵ is a Gaussian measurement error with variance s_ϵ^2 , and independently distributed across the (x, y) pairs. Letting θ be a parameterizations of (α, γ, τ) , the log-likelihood (up to constants) is:

(2.F.1)

$$\log \mathcal{L}(\theta, s_\epsilon) = - \sum_{(i,j) \in \mathcal{C}} D^\theta(u_{x_i}^\theta, v_{y_j}^\theta) - \sum_{i \in \mathcal{S}_M} u_{x_i}^\theta - \sum_{j \in \mathcal{S}_F} v_{y_j}^\theta - \sum_{(i,j) \in \mathcal{C}} \frac{(\hat{c}_i^m - c_{x_i, y_j}^m)^2}{2s_\epsilon^2} - |\mathcal{C}| \log s_\epsilon,$$

where \mathcal{C} denotes the set of matched pairs (i, j) observed in the data, \mathcal{S}_M and \mathcal{S}_F respectively denote the set of single men and the set of single women observed in the data, and where u_x^θ and v_y^θ satisfy equilibrium equations (6.2).

We maximize the likelihood using the NLOPT package and the BFGS algorithm, with bound constraints on τ and on s_ϵ (these parameters are restricted to be positive). We test the robustness of the results against different starting points for the parameter values. We compute analytically the gradient to improve performance. At each step of the estimation process, the following computations are performed:

- (i) The parameters (θ, s_ϵ) are updated using the gradient computed in the previous step.
- (ii) The updated values of (θ, s_ϵ) are deduced.
- (iii) The equilibrium quantities u and v are computed using algorithm 2, and the predicted consumption levels are constructed.
- (iv) The log-likelihood is updated.

This procedure is part of a R package named TraME (Transportation Methods for Econometrics; Galichon, O'hara and Weber (2017)). It simplifies the computation and the estimation of a wide range of discrete choice and matching problems, as it relies on a flexible formulation of these models in terms of transferability or heterogeneity structure. Under TraME, user-defined models can be solved using core equilibrium algorithms (mainly via Linear Programming, Convex Optimization, Jacobi iterations, Deferred Acceptance, or Iterative Fitting of which algorithm 2 is an instance of) and estimated by maximum likelihood. he

From the estimation steps mentioned above, step (iii) is the most time-consuming. However, the IPFP algorithm is quite fast, especially given that we have a small sample. It takes about 3 seconds to obtain the equilibrium quantities u and v in a market of this size. With 300 men and women, computation time raises to 3.5 seconds, and to 9.5 seconds with 500 men and women. Overall, the max-likelihood estimation procedure converges in 45 minutes.

Chapter 3

Collective Models and the Marriage Market

Abstract

This paper contributes to closing the gap between collective models and matching models of the marriage market. It builds on previous work by Galichon, Kominers, and Weber (2016) and clarify the connection between these two literatures. In particular, I characterize the class of collective models that can be embedded in a matching framework with Imperfectly Transferable Utility (ITU). Estimation of these models depends crucially on the computation of the so-called “distance function”, which, in most cases, has no analytic form. I provide a fast computation method that works in any case. Finally, I illustrate the model with PSID data.¹

¹This chapter is based on my paper “Collective Models and the marriage market”. I am particularly grateful to Edoardo Ciscato, Arnaud Dupuy, Alfred Galichon, James Heckman, Sonia Jaffe, Jack Mountjoy, Juan Pantano and Paul Vertier, and seminar participants at the CEHD and University of Chicago for valuable comments.

1. Introduction

Collective models (Chiappori, 1988) are a cornerstone of modern family economics research. Unlike the classical unitary model, they emphasize that bargaining, and therefore bargaining power, is likely to play an important part in family decisions. Implicitly, this literature has recognized that bargaining power and marriage markets are somehow related. This is illustrated by the use of sex ratios in many empirical papers. Intuitively, a marriage market in which women are more scarce should shift the balance of power in their favor. Such idea is already found in Becker (1973), where the market structure governs the division of marital surplus. This suggests that there could be significant benefits from bringing together collective models and matching models of the marriage market into a unified framework.

In this paper, I focus on the connection between within-household bargaining and the marriage market, and propose a way to fill the gap between collective and matching models. I do so by integrating collective models into the imperfectly transferable utility (ITU) matching framework developed by Galichon, Kominers, and Weber (2016). As argued above, this approach has a number of advantages. First, Pareto weights (or, alternatively, sharing rules) are endogenous to the determination of the marriage market equilibrium, and can be recovered when the model is estimated. In addition, I am not restricted to use collective models featuring transferable utility (TU). Although this is synonymous of losing the computational tractability of TU models, this also means that we will avoid some of the undesirable properties of TU collective models. For example, from now on, power allocation will matter for determining public good expenditures. Finally, because the collective model is embedded into a structural matching framework, it is relatively easy to conduct counterfactual analysis. As an example, we could simulate the impact of increasing the sex ratio (or the wages of women) on public good consumption.

More formally, the paper contributes to the literature in several ways. First, I clarify the connection between collective models and the ITU framework. In particular, I give sufficient conditions under which the usual utility possibility set is a proper bargaining set as defined in Galichon, Kominers, and Weber (2016), and under which the equilibrium

utilities correspond to a strictly Pareto efficient allocation. Second, I provide a method to compute the distance function (an important analytical device introduced in the ITU framework that serves as a basis for estimation) in a simple way and for any collective model. Third, I propose to illustrate the framework with a collective model with private consumption, leisure and a public good using data from the PSID. I allow preferences to vary by education level. The model features imperfectly transferable utility, which, as we shall see, produces a richer set of results and counterfactuals than a transferable utility model. Finally, using a series of counterfactual experiments, I investigate how changes in the market structure (e.g, sex ratios or distribution of wages) affect the consumption of public good in the household and the sharing rule.

An overview of the literature. The starting point of this paper are collective models. Since the seminal work of Chiappori (1988), collective models have become a standard toolbox of the economics of the family. By explicitly modelling the bargaining process that takes place within the household, this literature asks a new array of questions. For example, how is bargaining power distributed within the household? And more importantly, how does the distribution of bargaining power affect household decisions? This is particularly relevant if one considers that the main source of marriage gains and public consumption in the household is the well being of the (eventual) children and their outcomes. In this case, if partners have heterogenous tastes over public consumption, a shift in bargaining power may influence the well being of children. Many efforts have been deployed to estimate bargaining power, or at least by how much bargaining power shifts when factors such as wage or sex ratios change.

The idea that intra-household bargaining and marriage market equilibrium are related is far from new. Becker (1973) already stated that “theory does not take the division of output between mates as given, but rather derives it from the nature of the marriage market equilibrium”. In a TU setting, and with a simple supply-demand framework, he showed how surplus sharing within couples depends on population supplies. And indeed, the subsequent collective model literature that started with Chiappori (1988) and Chiappori (1992) has used the sex ratio as a main example of distribution factors (variables that affect bargaining power but not preferences nor budget constraints). Despite these early

contributions, attempts to reconcile collective models and models of the marriage market are very recent. The mechanics are illustrated in Chiappori (2012), and Browning, Chiappori, and Weiss (2014) contains interesting examples.

Full applications can be found in Choo and Seitz (2013) and Chiappori, Costa Dias, and Meghir (2015), although in the TU case. The use of a TU setting might be explained by the fact that TU models are computationally easy to handle. Therefore, this paper contributes to the literature from a technical point of view by showing how very general ITU models can be solved. While TU has its own advantages, it has obvious drawbacks, the main issue being that public good consumption does not depend on bargaining power, only on prices, income or technology. This conclusion is at odd with the empirical collective model literature (see Lundberg, Pollak, and Wales (1997) and Attanasio and Lechene (2002)). There have been very few attempts to integrate collective models in the general ITU setting. A notable exception is Cherchye et al. (2014) that uses a revealed preference approach. The most complete application of the Galichon, Kominers, and Weber (2016) framework can be found in Gayle and Shephard (2016), although the paper focuses on optimal taxation questions.

Organization of the paper. Section 2 introduce bargaining sets starting from the collective models. In section 3, I introduce a computation method to solve general ITU models. Section 4 provide a reminder on estimation methods for ITU models. In section 5, I provide an illustration with a collective model featuring private and public consumption as well as labour supplies. Finally, section 6 concludes.

Notations. I consider a bipartite, bilateral one-to-one marriage market. It is populated by men and women, indexed by $i \in \mathcal{I}$ and $j \in \mathcal{J}$, respectively, who may form heterosexual pairs². At some point in this paper, I will assume that men and women can be gathered in groups of similar observable characteristics, called *types*. The sets of types are denoted \mathcal{X} for men and \mathcal{Y} for women, while $\mathcal{X}_0 = \mathcal{X} \cup \{0\}$ and $\mathcal{Y}_0 = \mathcal{Y} \cup \{0\}$ introduce singlehood as an option. I let n_x be the mass of men of type x , and m_y be the mass of women of type y . The mass of matches between men of type x and women of type y is the vector $(\mu_{xy})_{x \in \mathcal{X}, y \in \mathcal{Y}}$,

²I choose to ignore same-sex couples, as they account for only 1% of couple households in the US in 2010. See Ciscato, Galichon, and Goussé (2015) for an analysis with same sex couples

while the vectors $(\mu_{x0})_{x \in \mathcal{X}}$ and $(\mu_{0y})_{y \in \mathcal{Y}}$ denote the mass of single men of type x and single women of type y , respectively.

2. Bargaining Sets

In this section, I clarify the connection between the ITU framework as introduced in Galichon, Kominers, and Weber (2016) (hereafter GKW) and models of household bargaining.

2.1. Prerequisites. The starting point in GKW is that two potential partners who meet, say man i and woman j , bargain over utility outcomes (u, v) that lie in a bargaining set \mathcal{F}_{ij} . In addition, it is assumed that this set meet a certain number of conditions, in which case it is called a *proper bargaining set* (see definition 1 in GKW).

In this paper, I do not take the proper bargaining sets as given. I am more explicit about how it is constructed from individual preferences over a certain number of goods. Therefore, in this general case, there is not always money or a numeraire good that will allow utility to be transferred at a constant exchange rate between partners. I assume that man i and woman j derive utility from the consumption of private goods (the vectors q_i and q_j , respectively) and public goods (the vector Q). When man i and woman j meet, they choose an allocation $\omega = (q_i, q_j, Q) \in R_+^L$ and I denote Ω_{ij} the set of feasible allocations. For a given allocation $(q_i, q_j, Q) \in \Omega_{ij}$, man i and woman j receive utility $U_i(q_i, Q)$ and $V_j(q_j, Q)$, respectively. Therefore, preferences are egotistic. The model can be extended to accommodate for the presence of other household members (children, for example). However, I assume that there are only two decision makers in the household, namely the husband and the wife (collective models can have multiple decision makers, see e.g. Dauphin et al. (2011)).

With this framework in mind, I will assume that the bargaining set for the pair (i, j) is the utility possibility set as defined below

DEFINITION 1. The *utility possibility set* \mathcal{G}_{ij} is the set

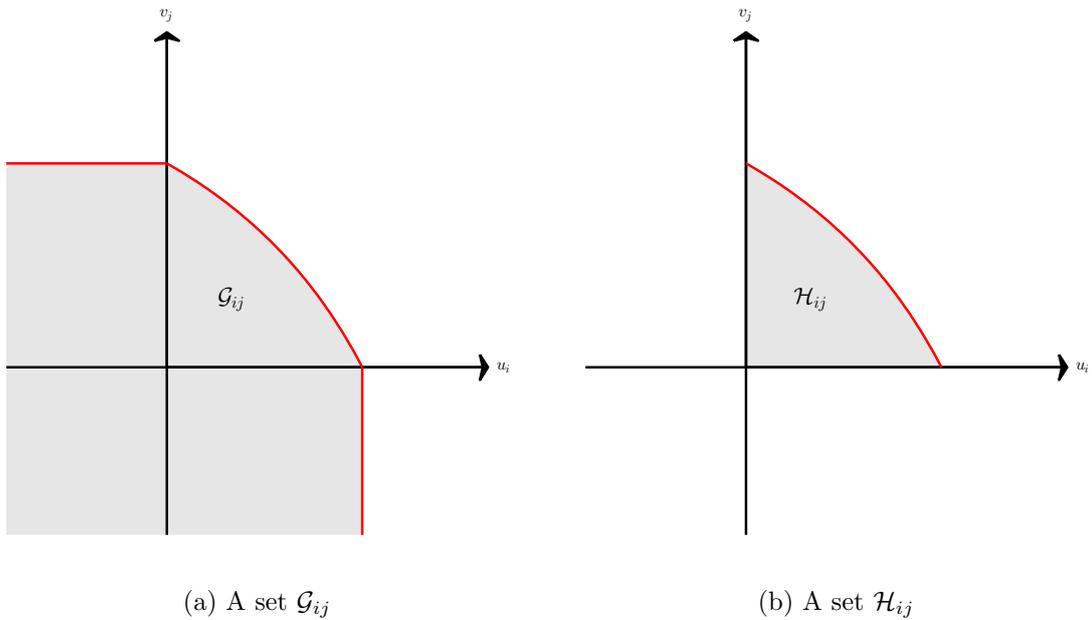
$$\mathcal{G}_{ij} = \{(u_i, v_j) : \exists(q_i, q_j, Q) \in \Omega_{ij} : u_i \leq U(q_i, Q), v_j \leq V(q_j, Q)\}$$

and the *strict utility possibility set* \mathcal{H}_{ij} is the set

$$\mathcal{H}_{ij} = \{(u_i, v_j) : \exists(q_i, q_j, Q) \in \Omega_{ij} : u_i = U(q_i, Q), v_j = V(q_j, Q)\}$$

In each case, I say that \mathcal{G}_{ij} and \mathcal{H}_{ij} are the utility possibility sets associated with (Ω_{ij}, U_i, V_j) . These sets are illustrated as shaded areas in figure 1.

FIGURE 1. Bargaining Sets



In the remainder of this paper, we are going to choose either \mathcal{G}_{ij} or \mathcal{H}_{ij} as our bargaining set. As we can see, it is easier to work with \mathcal{G}_{ij} because of the free-disposal underlying assumption. And indeed, we are going to show that under very mild conditions, \mathcal{G}_{ij} is a proper bargaining set. On the other hand, working with \mathcal{H}_{ij} as our bargaining set is more restrictive, but we will be able to say more about the boundary points of \mathcal{H}_{ij} . Before turning to the results, I shall recall the definition of a proper bargaining set from GKW.

DEFINITION 2 (Galichon, Kominers and Weber, 2016). A bargaining set \mathcal{F}_{ij} is a *proper bargaining set* if the three following conditions are met: (i) \mathcal{F}_{ij} is closed and nonempty (ii) \mathcal{F}_{ij} is *lower comprehensive*: if $(u, v) \in \mathcal{F}_{ij}$, then $(u', v') \in \mathcal{F}_{ij}$ provided $u' \leq u$ and $v' \leq v$. (iii) \mathcal{F}_{ij} is *bounded above*: Assume $u_n \rightarrow +\infty$ and v_n bounded below then for N large enough (u_n, v_n) does not belong in \mathcal{F} for $n \geq N$; similarly for u_n bounded below and $v_n \rightarrow +\infty$

2.2. Bargaining over \mathcal{G}_{ij} . We shall start by assuming that man i and woman j bargain over the *utility possibility set* \mathcal{G}_{ij} . I make the following assumption on U , V and Ω . For notational simplicity, I drop the indexes i , j and ij .

ASSUMPTION 1 (Feasible Allocations).

- (i) Ω is compact
- (ii) Ω is convex

ASSUMPTION 2 (Utility Functions).

- (i) U and V are upper semi continuous and bounded above on Ω
- (ii) U and V are strictly increasing
- (iii) U and V are strictly quasi-concave

Assumptions 1 and 2 are fairly standard in microeconomic theory. With the classic linear inequality budget and time constraints, the set Ω is indeed convex and compact. The assumption is still holding when some goods are home-produced and the production function is assumed to be concave. As we shall see, upper semi continuity is enough to prove closedness of the bargaining set. I also make the assumption that agents consume only “goods” (as opposed to “bads”) so that the utility is increasing in consumption. Finally, we can choose any cardinalization of the utility functions; here I simply impose a strictly concave cardinal representation.

PROPOSITION 1. *Under assumptions 1-(i) and 2-(i), the utility possibility set \mathcal{G}_{ij} associated with (Ω_{ij}, U_i, V_j) is a proper bargaining set*

PROOF. The fact that \mathcal{G} is lower comprehensive follows directly from its definition. Let us show that \mathcal{G} is closed. Consider a convergent sequence $\{w_n\}_{n \in \mathbb{N}}$ in \mathcal{G} such that $w_n = (u_n, v_n) \in \mathcal{G}, \forall n \in \mathbb{N}$. For each $w_n, \exists \omega_n \in \Omega : u_n \leq U(q_{i,n}, Q_n), v_n \leq V(q_{j,n}, Q_n)$. Since Ω is compact, there exist a convergent subsequence $\{\omega_{f(n)}\}_{n \in \mathbb{N}}$ in Ω (Bolzano–Weierstrass theorem). From $\{\omega_{f(n)}\}_{n \in \mathbb{N}}$, construct the subsequence $\{w_{f(n)}\}_{n \in \mathbb{N}}$. For any w_k in that subsequence, $\exists \omega_k \in \Omega : u_k \leq U(q_{i,k}, Q_k), v_k \leq V(q_{j,k}, Q_k)$. Since $u_k \rightarrow u, v_k \rightarrow v$ and $\omega_k \rightarrow \omega = (q_i, q_j, Q) \in \Omega$, we see by continuity of U and V that $u \leq U(q_i, Q)$ and $v \leq V(q_j, Q)$. Therefore $(u, v) \in \mathcal{G}$. Point (iii) in the definition of a proper bargaining set follows immediately from the fact that U and V are bounded above on Ω . ■

We have just shown under mild conditions, we can actually choose a *utility possibility set* as our bargaining set in the ITU framework, since this set is proper. We also know that in the ITU setting, equilibrium utilities belong to the boundary of the bargaining set. What can we say about these points in terms of Pareto efficiency?

PROPOSITION 2. *Whenever the set \mathcal{G} is a proper bargaining set, a boundary point of \mathcal{G} is weakly Pareto efficient.*

PROOF. The proof follows immediately from the fact that if \mathcal{G} is a proper bargaining set, it is lower comprehensive and closed. Indeed, take any (u, v) on the boundary of \mathcal{G} . If (u, v) is not weakly Pareto efficient, $\exists (u', v') \in \mathcal{G}$ such that $u' > u$ and $v' > v$, hence (u, v) is an interior point (consider the open ball of center (u, v) and radius ϵB_ϵ where $\epsilon < \min(u' - u, v' - v)$). ■

2.3. Bargaining over \mathcal{H}_{ij} . We have shown that it is possible to assume that partners bargain over the set \mathcal{G}_{ij} , because under mild conditions, it is a proper bargaining set. However, we cannot say much about the properties of the boundary points of this set. Therefore, we may want to assume that partners bargain over the *strict utility possibility set* \mathcal{H}_{ij} instead.

If we want to connect the ITU to collective models, we need to show two things: (i) if \mathcal{H} is a proper bargaining set, then a boundary point is Pareto efficient, and (ii) \mathcal{H} is indeed a proper bargaining set.

First, let's assume that the set \mathcal{H} is a proper bargaining set, and consider the following assumption.

ASSUMPTION 3 (Transferability). *If $(q_i, q_j, Q) \in \Omega$ and $q_{i,k} > 0$ for some private good k , then for any q'_i such that $q'_{i,-k} = q_{i,-k}$, $q'_{i,k} < q_{i,k}$ and $(q'_i, q_j, Q) \in \Omega$, there is a q'_j and some private good l such that $q'_{j,-l} = q_{j,-l}$, $q'_{j,l} > q_{j,l}$ and $(q'_i, q'_j, Q) \in \Omega$.*

Assumption 3 states that for any feasible allocation in which the man (or the woman) consumes a strictly positive amount of some private good, we can always find a feasible allocation that is identical except for the fact that we slightly decreased the consumption of the man for that private good and slightly increased the consumption of the woman for some private good. All in all, assumption 3 states that there is always some ways (even if imperfect) of transferring utility from one partner to the other. We can now get the following result

PROPOSITION 3. *Whenever the strict utility possibility set \mathcal{H} is a proper bargaining set, and under assumption 1, 2 and 3, a boundary point (u, v) of \mathcal{H} is strongly Pareto efficient.*

PROOF. Choose any (u, v) on the boundary of \mathcal{H} attained with the allocation $\omega = (q_i, q_j, Q)$ and suppose it is not Pareto efficient. Then, there is a point $(u', v') \in \mathcal{H}$ attained for some allocation $\omega' = (q'_i, q'_j, Q')$ such that $u' \geq u$ and $v' \geq v$, with at least one strict inequality. Without loss of generality, say $u' > u$. Note that $v' > v$ yields to a contradiction since (u, v) would be an interior point (see the proof of Proposition 2). Hence, $u' > u$ and $v' = v$. To reach u' , we need to change the amount of goods consumed by the man, so that either $q'_i \neq q_i$ or $Q' \neq Q$, or both.

If $Q' \neq Q$, then we can show that (u, v) is an interior point, a contradiction. Indeed, the allocation $(tq_i + (1-t)q'_i, tq_j + (1-t)q'_j, tQ + (1-t)Q') \in \Omega$ for $t \in (0, 1)$ by convexity of Ω , and gives both partners utility (u'', v'') with $u'' > u$ and $v'' > v$.

If $Q' = Q$, then $q'_i \neq q_i$ and there is at least one private good k for which $q'_{i,k} > q_{i,k} \geq 0$ since U is strictly increasing. By continuity of U and V , and assumption 3, we can always slightly decrease $q'_{i,k}$ and slightly increase $q_{j,l}$ for some private good l in a feasible way, and

reach a point (u'', v'') such that $u' > u'' > u$ and $v'' > v$. Hence, (u, v) is an interior point, a contradiction. ■

Using the *strict utility possibility set* \mathcal{H} as our bargaining set is attractive because under the conditions mentioned above, the ITU framework imposes that partners choose a Pareto efficient outcome. Hence, the remaining piece of the puzzle is showing that we can indeed use \mathcal{H} , that is, showing that it is a proper bargaining set.

ASSUMPTION 4 (Vital goods). *There exists two private goods, $q_{i,1}$ and $q_{j,1}$ such that*

$$(i) \lim_{q_{i,1} \rightarrow 0^+} U(q_i, Q) = -\infty \text{ and } \lim_{q_{j,1} \rightarrow 0^+} V(q_j, Q) = -\infty$$

(ii) *for any $(q_i, q_j, Q) \in \mathcal{G}$, $((q'_{i,1}, q_{i,-1}), (q'_{j,1}, q_{j,-1}), Q) \in \mathcal{G}$ whenever $q'_{i,1} \leq q_{i,1}$ and $q'_{j,1} \leq q_{j,1}$*

Assumption 4 states that both men and women consume a vital private good, in the sense that if the quantity consumed tends to zero, their utility tends to minus infinity. It also states that if the household decreases the quantity of these goods, he can buy at least the same amount of the other goods. In effect, this assumption plays the same role than the free-disposal restriction: if the couple agrees on an utility allocation (u, v) , it is always possible to reach an allocation $(u', v') : u' \leq u, v' \leq v$ by decreasing the amount of vital goods consumed. In practice, one can think of food or water filling this role³. We now reach our final result

PROPOSITION 4. *Under assumption 1, 2 and 4, the strict utility possibility set \mathcal{H}_{ij} associated with (Ω_{ij}, U_i, V_j) is a proper bargaining set*

PROOF. Closedness and boundedness are proven exactly as in Proposition 1. We need to show that \mathcal{H} is lower comprehensive. Take $(u, v) \in \mathcal{H}$, with $u = U(q_i, Q)$ and $v = V(q_j, Q)$ for some allocation $(q_i, q_j, Q) \in \Omega$ and consider a point (u', v') such that $u' \leq u, v' \leq v$. Without loss of generality, take $u' < u$. By assumption 2 and 4, we can find $q'_{i,1}$ such that $((q'_{i,1}, q_{i,-1}), q_j, Q) \in \Omega$, $U((q'_{i,1}, q_{i,-1}), Q) = u'$ and $V(q_j, Q) = v$.

If $v' = v$, then we stop.

³Although food preparation may be a public good, the consumption of food is private

If $v' < v$, there is a $q'_{j,1}$ such that $((q'_{i,1}, q_{i,-1}), (q'_{j,1}, q_{j,-1}), Q) \in \Omega$, $U((q'_{i,1}, q_{i,-1}), Q) = u'$ and $V((q'_{j,1}, q_{j,-1}), Q) = v'$. Therefore, $(u', v') \in \mathcal{H}$. ■

We can now combine all the results in the following proposition:

PROPOSITION 5. *Under assumption 1, 2, 3 and 4, the strict utility possibility set \mathcal{H} is a proper bargaining set and any point on the boundary of \mathcal{H} is strongly Pareto efficient.*

The proof of this proposition follows from putting together the previous results. Therefore, we have shown in the section that there is a whole class of collective models that can be embedded in the ITU framework from GKW. However, as we shall see, estimating these models requires more advanced computing techniques.

3. A computation method

In this section, I will assume that the econometrician choose a convenient parameterization of the utility functions introduced in the previous section, and I will denote θ the vector of preference parameters that we want to estimate. Given a value of θ , we can construct the bargaining sets \mathcal{G}_{ij} and \mathcal{H}_{ij} for any pair (i, j) , as shown in Section 2. GKW show that estimating θ is possible using a variety of techniques (including maximum likelihood estimation). However, they require the ability to compute distance functions.

3.1. Distance Functions. The *distance function* D_{ij} for the pair (i, j) is simply an analytical tool that describes whether a utility allocation (u_i, v_j) is a boundary point of the bargaining set or not. As shown in GKW, the distance function is not only useful to characterize the equilibrium in a simpler way, but also turns out to be crucial for estimation purposes. For a given pair (i, j) and bargaining set \mathcal{F}_{ij} , the distance function is defined as

$$D_{ij}(u, v) = \min \{z \in \mathbb{R} : (u - z, v - z) \in \mathcal{F}_{ij}\}.$$

Note that, indeed, $D_{ij}(u, v) = 0$ whenever the pair of utilities (u, v) belongs to the boundary of \mathcal{F}_{ij} , $D_{ij}(u, v) \leq 0$ if the pair (u, v) is feasible, and $D_{ij}(u, v) > 0$ otherwise. This is illustrated in figure 3 for a feasible point (u_i, v_j) . GKW contains many examples in which the distance function has a simple analytical expression. However, we might expect this

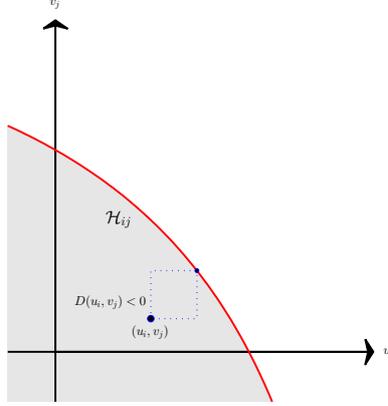


FIGURE 3. The distance function for a feasible point (u_i, v_j)

not the be the case anymore in models that include public goods and multiple inequality constraints.

EXAMPLE 1. Consider a case in which utility depends on the consumption of a private good q and a public good Q . All prices are set to one. The systematic utilities are as follows

$$\mathcal{U}_{xy}(q_i, Q) = \log q_i + A \log Q$$

$$\mathcal{V}_{xy}(q_j, Q) = \log q_j + B \log Q$$

where I assume $A \neq B$. The budget constraint is $q_i + q_j + Q \leq \Phi_i j$. According to Section 2, the bargaining set \mathcal{H}_{ij} is a proper bargaining set. To compute the distance function, we make use of the FOC in the corresponding collective model. We get the standard Bowen-Lindhal-Samuelson condition

$$Aq_i + Bq_j = Q$$

Say we want to compute the distance at point (u, v) , that is, we want to find z such that $(u - z, v - z)$ belongs to the frontier. It is easily seen that the BLS condition defines Q as an implicit function of $(u - z, v - z)$ with

$$A \exp(u - z - A \log Q(u - z, v - z)) + B \exp(v - z - B \log Q(u - z, v - z)) = Q(u - z, v - z)$$

Unless $A = B$, there is no close form formula for the optimal Q and therefore no close form for the distance function, which, in this case is

$$\min_{z \in \mathbb{R}} \{z : \exp(u - z - A \log Q(u - z, v - z)) + \exp(v - z - B \log Q(u - z, v - z)) + Q(u - z, v - z) \leq \Phi_{ij}\}$$

This example brings two remarks to mind. First, there is little computational gains in writing down the collective model and making use of the FOC. Indeed, we still need to minimize over z and for each value of z we must solve for the optimal Q using numerical methods. Second, this example is still relatively simple because we only need to make use of one constraint (the budget constraint) to compute the distance function. We do not need worry about non negativity constraints since we work with logs, and there are time constraints or corner solutions here. As we shall see, there are no easy way of solving richer models.

3.2. The method. In the following, I introduce a method to compute distance functions when the proper bargaining set is either \mathcal{G}_{ij} or \mathcal{H}_{ij} constructed from some utility functions U_i^θ and V_j^θ . It relies on the definition of the distance function but has interesting implications. While I keep things general, I will assume that the set of feasible allocations Ω_{ij} is fully characterized by R feasibility constraints

$$(3.1) \quad h_r(q_i, q_i, Q) \leq 0, \quad r \in \{1, \dots, R\}$$

where $\{h_r\}_{r \in \{1, \dots, R\}}$ are convex functions.

PROBLEM 1. *Given a proper bargaining set \mathcal{F}_{ij} , the distance from a point (u, v) to the frontier can be computed in the following way:*

(i) Suppose that \mathcal{G}_{ij} is a proper bargaining set associated with some $(U_i^\theta, V_j^\theta, \Omega_{ij})$. Then we can solve

$$(3.2) \quad \min_{z, q_i, q_j, Q} z$$

$$s.t \ u - z \leq \mathcal{U}_i^\theta(q_i, Q)$$

$$(3.3) \quad v - z \leq \mathcal{V}_j^\theta(q_j, Q)$$

$$h_r(q_i, q_i, Q) \leq 0, \ r \in \{1, \dots, R\}$$

when $D_{ij}^\theta(u, v) = z^*$, solution to the above program.

(ii) Similarly, suppose that \mathcal{H}_{ij} is a proper bargaining set associated with some $(U_i^\theta, V_j^\theta, \Omega_{ij})$. Then we can solve

$$(3.4) \quad \min_{z, q_i, q_j, Q} z$$

$$s.t \ u - z = \mathcal{U}_i^\theta(q_i, Q)$$

$$(3.5) \quad v - z = \mathcal{V}_j^\theta(q_j, Q)$$

$$h_r(q_i, q_i, Q) \leq 0, \ r \in \{1, \dots, R\}$$

when $D_{ij}^\theta(u, v) = z^*$, solution to the above program.

In practice, it is very fast to solve for such problems, and given the solution z^* to this problem, we have $D_{xy}(u, v) = z^*$. To ease computations, note that it is straightforward to derive the analytic gradient of the objective function, and that, in general, the analytic expression of the jacobian of the system of constraints can be obtained.

REMARK 3.1. The above technique is fairly general, but of course, it can be combined with the approach laid out in example 1 to reduce the number of variables over which the minimization is performed. For instance, we saw in example 1 that Q can be immediately deduced from q_i and q_j making use of the Bowen-Lindhal-Samuelson condition. Therefore, we can avoid minimizing over Q in Problem 1 above.

3.3. Gradient of the distance function. When the number of types of men and women is large, it is computationally costly to obtain the derivative of the distance function

for each pair (i, j) using standard numerical methods. Fortunately, the following results shows how to obtain the derivative of the distance function D_{ij}^θ with respect to u , v or θ .

THEOREM 1. *Introduce the Lagrange multipliers associated with the constraints in Problem 1 as λ_1, λ_2 , and $\{\xi_r\}_{r \in \{1, \dots, R\}}$. The gradient with respect to u , v and θ of the distance function D_{ij} can be obtained directly when solving Problem 1. Indeed, we have*

$$\begin{aligned}\partial_u D_{ij}^\theta(u, v) &= \lambda_1 \\ \partial_v D_{ij}^\theta(u, v) &= \lambda_2 \\ \partial_\theta D_{ij}^\theta(u, v) &= -\lambda_1 \partial_\theta U_i^\theta(q_j, Q) - \lambda_2 \partial_\theta V_j^\theta(q_j, Q)\end{aligned}$$

PROOF. The proof follows from the Envelop Theorem, since

$$D_{ij}^\theta(u, v) = \min_{z, q_i, q_j, Q} z \text{ s.t. (3.3) or (3.5)}$$

■

REMARK 3.2 (Gradient of the decision variables). Solving Problem 1 also returns optimal values for q_i , q_j , and Q . Can we compute the derivative of these values with respect to u , v or θ ? While there is no general way of proceeding, it is usually possible. We can combine the first order conditions of our minimization problem and use the Implicit Function Theorem to recover the derivatives. I will illustrate this point with an example. Suppose that the utility functions are given by

$$\begin{aligned}U_i &= a \log q_i + A \log Q \\ V_j &= b \log q_j + B \log Q\end{aligned}$$

and we are interested in computing the derivative of the optimal q_i , q_j , and Q with respect to a . Suppose that the only constraint is $q_i + q_j + Q \leq \Phi_{ij}$ where Φ_{ij} is the income of pair (i, j) . We can see that in this model, the utility possibility set \mathcal{H}_{ij} is a proper bargaining.

It is easy to see that a solution to Problem 1-(ii) satisfies

$$\begin{aligned} c_i + c_j + Q &= \Phi_{ij} \\ u - D_{ij}^\theta(u, v) &= a \log q_i + A \log Q \\ \frac{A}{a} q_i + \frac{B}{b} q_j &= Q \end{aligned}$$

where the first equality comes from the budget constraint (which is binding), the second is an equality constraint in Problem 1-(ii) and the third is obtained by combining the first order conditions. Therefore we have

$$\begin{pmatrix} \frac{a}{q_i} - \frac{A}{Q} & -\frac{A}{Q} \\ \frac{A}{a} + 1 & \frac{B}{b} + 1 \end{pmatrix} \begin{pmatrix} \partial_a q_i \\ \partial_a q_j \end{pmatrix} = \begin{pmatrix} -\partial_a D_{ij}^\theta(u, v) - \log q_i \\ \frac{A}{a^2} q_i \end{pmatrix}$$

which we can rewrite $M \times \begin{pmatrix} \partial_a q_i \\ \partial_a q_j \end{pmatrix} = d$, so that

$$\begin{pmatrix} \partial_a q_i \\ \partial_a q_j \end{pmatrix} = M^{-1} \times d$$

and ∂Q_a follows easily.

3.4. Connection with collective models. The computation methods introduced in the previous subsection highlights an interesting connection with collective models. It can be shown that the Lagrange multipliers associated with the constraint involving the utility functions of the partners can be interpreted as Pareto weights. This relationship is proven in the following theorem:

THEOREM 2. *Suppose that the bargaining set \mathcal{H}_{ij} , as introduced in Definition 1, is a proper bargaining set associated with some $(U_i^\theta, V_j^\theta, \Omega_{ij})$. Suppose that Ω_{ij} is characterized by a set of feasibility constraints $h_r(q_i, q_j, Q) \leq 0$, $r \in \{1, \dots, R\}$, where $\{h_r\}$ are convex functions. Suppose that assumption 1, 2 and 3 holds. Then*

(i) *the allocation q_i^*, q_j^*, Q^* solution to Problem 1-(ii) is Pareto efficient.*

(ii) *in addition, the Pareto weights for the man and the woman are, respectively, the λ_1^* and λ_2^* associated to q_i^*, q_j^*, Q^* . λ_1^* and λ_2^* also satisfy $\lambda_1^* + \lambda_2^* = 1$*

PROOF. To prove (i), we need to show that the point $(u - z^*, v - z^*)$ (where z^* solves Problem 1-(ii)) is a boundary point of \mathcal{H}_{ij} , and therefore it is Pareto efficient by Proposition 3. If $(u - z^*, v - z^*)$ is not a boundary point of \mathcal{H}_{ij} , then there is an open ball of center $(u - z^*, v - z^*)$ and radius ϵ completely contained in \mathcal{H}_{ij} . Therefore, the point $(u - z^* + \frac{\epsilon}{2}, v - z^* + \frac{\epsilon}{2})$ is also in \mathcal{H}_{ij} , but then z^* cannot be solution to Problem 1-(ii)).

To prove (ii), we write the Lagrangian associated to Problem 1-(ii)), except that we maximize $-z$ instead of minimizing z :

$$\mathcal{L}(z, q_i, q_j, Q, \lambda_1, \lambda_2, \xi_r) = -z - \lambda_1(u - z - U_i^\theta(q_i, Q)) - \lambda_2(u - z - V_j^\theta(q_j, Q)) - \sum_r^R \xi_r h_r(q_i, q_j, Q)$$

For a solution point (z^*, q_i^*, q_j^*, Q^*) , I introduce the associated nonnegative multipliers $\lambda_1^*, \lambda_2^*, \xi_r^*$. Consider now the following Lagrangian:

$$\tilde{\mathcal{L}}(q_i, q_j, Q, \xi_r) = \lambda_1^* U_i^\theta(q_i, Q) + \lambda_2^* V_j^\theta(q_j, Q) - \sum_r^R \xi_r h_r(q_i, q_j, Q)$$

and note that (i) $\tilde{\mathcal{L}}$ is a concave function of (q_i, q_j, Q) , (ii) taking $\xi_r = \xi_r^*$ and

$(q_i, q_j, Q) = (q_i^*, q_j^*, Q^*)$, we have $\xi_r^* \geq 0$ and

$$\frac{\partial \tilde{\mathcal{L}}(q_i^*, q_j^*, Q^*, \xi_r^*)}{\partial q_i} = 0, \quad \frac{\partial \tilde{\mathcal{L}}(q_i^*, q_j^*, Q^*, \xi_r^*)}{\partial q_j} = 0, \quad \text{and} \quad \frac{\partial \tilde{\mathcal{L}}(q_i^*, q_j^*, Q^*, \xi_r^*)}{\partial Q} = 0$$

Therefore, (q_i^*, q_j^*, Q^*) is solution to

$$\begin{aligned} & \max_{q_i, q_j, Q} \lambda_1^* U_i^\theta(q_i, Q) + \lambda_2^* V_j^\theta(q_j, Q) \\ & \text{s.t. } h_r(q_i, q_j, Q) \leq 0, \quad r \in \{1, \dots, R\} \end{aligned}$$

Hence, (q_i^*, q_j^*, Q^*) is a Pareto efficient allocation that maximizes a social welfare function with Pareto weights λ_1^* and λ_2^* . The fact that $\lambda_1^* + \lambda_2^* = 1$ follows directly from the first order conditions of Problem 1-(ii), since $\partial \mathcal{L} / \partial z = 0 \iff -1 + \lambda_1 + \lambda_2 = 0$. ■

The proof of theorem 2 makes use of the expression of the Lagrangian of the problem of computing the distance function. The main idea is to fix the irrelevant variables and recover the expression of the Lagrangian of a collective model problem. This result is important in

theory as well as in practice, since it states that we can compute the Pareto weights from the Lagrange multipliers (which will be provided by any solver).

4. Estimation

4.1. A reminder. In this section, I summarize how to estimate a household bargaining models using the ITU-logit framework as introduced in section 6 of GWK. I assume that individuals can now be gathered in groups of similar observable characteristics, or types (notational details can be found at the end of the introduction). Following GWK, the main assumption is that the utility received by man i and woman j is the sum of two components: (i) a systematic part, denoted U_i and V_j , respectively, that lie within the bargaining set \mathcal{F}_{ij} , and (ii) an idiosyncratic part, denoted ϵ_{iy_j} and $\epsilon_{x_i j}$, respectively. In addition, it is assumed that the bargaining set only depends on the observable types, that is, $\mathcal{F}_{ij} = \mathcal{F}_{x_i y_j}$.

One contribution of GWK is to show that we can actually focus on equilibrium where the systematic utilities, that I will denote $U_{x_i y_j}$ and $V_{x_i y_j}$, depend on the observable types of the partners. They show that doing so is in fact merely a restriction, as there always exist an individual equilibrium of this form. The advantage of focusing on such equilibria is that we can reduce the matching problem to a series of discrete choice problem, where each individual choose the observable type of his potential partner. In addition, if we choose carefully the distribution of the idiosyncratic component, we can greatly simplify the characterisation of the equilibrium.

In the ITU-logit framework that we follow here, it is assumed that the idiosyncratic shocks are i.i.d draws from an extreme value type I distribution. In this case, it is well known that the systematic utilities can be obtained from the matching patterns using the log odds ratio formula. Namely:

$$U_{xy} = \log \frac{\mu_{xy}}{\mu_{x0}} \text{ and } V_{xy} = \log \frac{\mu_{xy}}{\mu_{0y}}$$

In this setting, characterizing equilibrium is very simple. Indeed, in this case the triple $(\mu_{xy}, U_{xy}, V_{xy})$ is an aggregate equilibrium if and only if the systematic utilities given by

$U_{xy} = \log \frac{\mu_{xy}}{\mu_{x0}}$ and $V_{xy} = \log \frac{\mu_{xy}}{\mu_{0y}}$ satisfy

$$(4.1) \quad D_{xy}(\log \frac{\mu_{xy}}{\mu_{x0}}, \log \frac{\mu_{xy}}{\mu_{0y}}) = 0$$

and

$$(4.2) \quad \begin{aligned} \mu_{x0} + \sum_y \mu_{xy} &= n_x \\ \mu_{0y} + \sum_x \mu_{xy} &= m_y \end{aligned}$$

4.2. Parameterization. A key difference with GKW is that I will use parameterizations of the systematic utilities U_{xy} and V_{xy} that are very common in the collective model literature. Specifically, I will choose a parametric household bargaining model that satisfy the assumptions in section 2.

When a man of type x meets with a woman of type y , they choose vectors of private consumption, denoted q_x and q_y , as well as a vector of public consumption Q . Naturally, the set of feasible allocations (q_x, q_y, Q) must belong to some feasible set Ω_{xy} that satisfy assumption 1. For a given allocation (q_x, q_y, Q) , the partners receive the following amounts of utility

$$\mathcal{U}_{xy}^\theta(q_x, Q) \text{ and } \mathcal{V}_{xy}^\theta(q_y, Q)$$

where θ is a vector of preference parameters.

I shall also specify the outside options (singlehood) for a man of type x and a woman of type y . I assume that they have access to a subset of the goods they can consume while married. They choose vectors of private goods, denoted q_x^s and q_y^s that must belong to some feasible sets Ω_{x0} and Ω_{0y} , respectively. They receive utility

$$\mathcal{U}_{x0}^\theta(q_x^s) \text{ and } \mathcal{V}_{0y}^\theta(q_y^s)$$

Note that in general, it will be very straightforward to solve for the optimal consumption choices q_x^{s*} and q_y^{s*} of man x and woman y when they are single. Given θ and their chosen

allocation (q_x, q_y, Q) when married, we can simply compute the systematic utilities as

$$(4.3) \quad U_{xy} = \mathcal{U}_{xy}(q_x, Q) - \mathcal{U}_{x0}^\theta(q_x^{s*}) \text{ and}$$

$$(4.4) \quad V_{xy} = \mathcal{V}_{xy}(q_y, Q) - \mathcal{V}_{0y}^\theta(q_y^{s*})$$

4.3. Estimation method. To estimate a ITU-logit model, we make use of the simple characterization of the equilibrium in that particular case. Typically, we will look for the value of θ that maximizes some objective function under the constraints given by equations (4.1) together with the system of scarcity constraints (4.2). The objective function can be a log-likelihood function as in GKW or moment-based as in Gayle and Shephard (2016).

To clarify the process of estimation, let us introduce $u_x = -\log \mu_{x0}$ and $v_y = -\log \mu_{0y}$. Given \mathcal{H}_{xy} , u_x , v_y and θ , and from the definition of the distance function, it follows that $\mu_{xy} = \exp(-D_{xy}^\theta(u_x, v_y))$. Therefore, equilibrium is fully characterized by the system of equations

$$(4.5) \quad \begin{aligned} \exp(-u_x) + \sum_y \exp(-D_{xy}^\theta(u_x, v_y)) &= n_x \\ \exp(-v_y) + \sum_x \exp(-D_{xy}^\theta(u_x, v_y)) &= m_y \end{aligned}$$

Let us introduce the objective function, denoted F . At this point, there are two ways to proceed to estimate θ . A first solution, which is employed in GKW, is to maximize F with respect to θ , but in that case, one must solve for the (u_x^*, v_y^*) solution to (4.5) for each value of the parameters θ . Given θ , u_x^* , and v_y^* , one can then compute the value of the objective function. These steps are repeated until F is maximized. A second approach is to maximize F with respect to (θ, u_x, v_y) simultaneously, under the constraint (4.5). This is the so-called MPEC approach from Su and Judd (2012). In the following, I will rely on the second approach. Although there are efficient algorithms to solve for the equilibrium (u_x, v_y) in system (4.5), the computation cost is much higher when there are no analytical formula for the distance function.

Note that when solving Problem 1-(ii) for a particular (x, y) pair, we will obtain the predicted mass of marriage for this pair of types (that would be $\exp(-z^*)$), as well as

predicted consumption of public and private goods. Therefore, it is easy to construct the objective function based on the observed and predicted marriage patterns and household decisions.

5. Illustration

In this section, I explore the possibilities offered by the ITU-collective model approach. In particular, I consider a collective model in which the public good is produced within the household: household members combine goods and time to produce it. I use the PSID to construct a representative marriage market of the United States. I build on an earlier note of Weber (2016) in which I introduced a pure simulation exercise. Here, I choose a very parsimonious model that I bring to real data. Therefore, preference parameters estimates should be taken very cautiously. However, the main question remains whether a parsimonious collective model would fit the data well and allow for interesting counterfactual experiments. In particular, I build on the earlier collective model literature which estimate how the sharing rule (a monetary measure of power) varies with distribution factors such as the sex ratio. For example, in a classic contribution, Chiappori, Fortin, and Lacroix (2002) show that a one percentage point increase in the sex ratio (defined as the share of men in the total population) yields to an annual transfer from men to women equal to \$2000.

5.1. Model. I shall now be more specific regarding the parameterization of the systematic utilities. First, I assume that singles derive utility from private consumption and leisure, denoted c and ℓ , respectively. I normalize the price of private consumption to 1. All agents can spend time on the labor market, in which case they earn a hourly wage denoted w . The total time endowment is T . Consequently, a single man of type $x \in \mathcal{X}$ faces the following maximization program

$$\begin{aligned} \max_{c_x^s, \ell_x^s} & a_{e(x)} \log c_x^s + \alpha_{e(x)} \log \ell_x^s \\ \text{s.t.} & c_x^s + \ell_x^s w_x \leq T w_x \\ & \ell_x^s \leq T \end{aligned}$$

The indices $e(x)$ indicates that I allow preference parameters to vary with types. More precisely, preferences parameters may depend on a subset of the observable characteristics used to form types. In this particular model, preference parameters vary with three levels of education (see below for more details). In similar fashion, a single woman of type $y \in \mathcal{Y}$ solves

$$\begin{aligned} \max_{c_y^s, \ell_y^s} & b_{e(y)} \log c_y^s + \beta_{e(y)} \log \ell_y^s \\ \text{s.t.} & c_y^s + \ell_y^s w_y \leq T w_y \\ & \ell_y^s \leq T \end{aligned}$$

It is straightforward to find the optimal $(c_x^{s^*}, \ell_x^{s^*})$ and $(c_y^{s^*}, \ell_y^{s^*})$ from which we can compute the singlehood reservation utilities $\mathcal{U}_{x0}^\theta(c_x^{s^*}, \ell_x^{s^*})$ and $\mathcal{V}_{0y}^\theta(c_y^{s^*}, \ell_y^{s^*})$.

If two individuals choose to marry instead, I assume that they have access to a public good Q (e.g. children, heating, etc). Therefore, preferences do not change with marriage, but partners have access to a new source of utility. The public good Q that is produced from time inputs by both partners. Therefore, married partners receive

$$(5.1) \quad \mathcal{U}_{xy}(c_x, \ell_x, Q) = a_{e(x)} \log c_x + \alpha_{e(x)} \log \ell_x + A_{e(x)} \log Q$$

$$(5.2) \quad \mathcal{V}_{xy}(c_y, \ell_y, Q) = b_{e(y)} \log c_y + \beta_{e(y)} \log \ell_y + B_{e(y)} \log Q$$

where

$$Q = [\eta h_x^\sigma + (1 - \eta) h_y^\sigma]^{1/\sigma}$$

where the η and $1 - \eta$ are factor shares and $1/(1 - \sigma)$ is the elasticity of substitution⁴. Time spent on housework by the man and the woman are denoted h_x and h_y , respectively. I assume that σ and η are the same for all types of households. In some versions of this model, I allowed η to depend on the type of the woman. The budget and time constraints

⁴Recall that as $\sigma \rightarrow 1$, we recover a linear production function, while $\sigma \rightarrow 0$ corresponds to a Cobb-Douglas production function

are

$$(5.3) \quad c_x + c_y + (\ell_x + h_x)w_x + (\ell_y + h_y)w_y \leq T(w_x + w_y)$$

$$(5.4) \quad \ell_x + h_x \leq T$$

$$(5.5) \quad \ell_y + h_y \leq T$$

For simplicity, I assume that the preference parameters for private consumption and leisure are the same for singles and married individuals of the same type. The parameters to be estimated are the $(a_{e(x)}, \alpha_{e(x)}, A_{e(x)}, b_{e(y)}, \beta_{e(y)}, B_{e(y)}, \sigma, \eta)$, denoted θ . Given a value of θ and a choice of $(c_x, \ell_x, h_x, c_y, \ell_y, h_y)$ for when man x and woman y are married, we can construct the systematic utilities simply by taking the difference $\mathcal{U}_{xy}(c_x, \ell_x, Q) - \mathcal{U}_{x0}(c_x^{s*}, \ell_x^{s*})$ and $\mathcal{V}_{xy}(c_y, \ell_y, Q) - \mathcal{V}_{0y}(c_y^{s*}, \ell_y^{s*})$.

5.2. Data. The data used for estimating the model is PSID for year 1989. Because the dataset is relatively small, I assume that I can draw a sample that is representative of a static marriage market. To construct the vector of *mass* of marriages (μ_{xy}) , I use all singles aged between 30 and 60 years old, and I select couples in which at least one partner satisfy this age requirement. By taking 30 as a lower bound, I avoid taking into account individuals that are still in school. The PSID is a rich dataset: I use information on annual labour supply and labour income to construct hourly wages. When wages are not observed, I predict them using a Heckman selection model. I observe the number of years of education which allows me to categorize men and women into three types: “High School and below” (HS or E1), “Some College” (C or E2) and “College degree and plus” (C+ or E3). In addition, I observe how much time married partners spend on housework. Naturally, this variable is a very imperfect measure of time spent producing household goods, since it is self reported and it is hard to know what respondents take into account when providing an answer. We know however, that it includes time spend cooking, cleaning and doing work for the house. Descriptive statistics are displayed in tables 1 and 2. As expected, women spend on average less time on the labour market, and much more time on housework.

TABLE 1. Descriptive Statistics, Men (left=married, right=single)

	Mean.	SD.	Min.	Max.	N.		Mean.	SD.	Min.	Max.	N.
Age	41.17	8.05	30.00	60.00	2506.00	Age	38.67	7.89	30.00	60.00	486.00
Educ. Level 1	0.51	0.50	0.00	1.00	2506.00	Educ. Level 1	0.57	0.50	0.00	1.00	486.00
Educ. Level 2	0.22	0.41	0.00	1.00	2506.00	Educ. Level 2	0.22	0.42	0.00	1.00	486.00
Educ. Level 3	0.27	0.44	0.00	1.00	2506.00	Educ. Level 3	0.21	0.41	0.00	1.00	486.00
Wage	13.68	7.40	0.93	44.47	2506.00	Wage	10.47	6.25	0.93	39.00	486.00
Hours worked	41.16	15.53	0.00	112.31	2506.00	Hours worked	34.27	18.47	0.00	112.31	486.00
Housework	7.41	7.77	0.00	84.00	2506.00	Housework	8.50	7.91	0.00	56.00	486.00

Finally, I fix the time endowment to 112 hours per week and assume that leisure is equal to the total time endowment minus the time spent on the labour market and on housework.

TABLE 2. Descriptive Statistics, Women (left=married, right=single)

	Mean.	SD.	Min.	Max.	N.		Mean.	SD.	Min.	Max.	N.
Age	38.65	7.56	28.00	58.00	2506.00	Age	39.84	8.89	28.00	58.00	1050.00
Educ. Level 1	0.56	0.50	0.00	1.00	2506.00	Educ. Level 1	0.63	0.48	0.00	1.00	1050.00
Educ. Level 2	0.22	0.42	0.00	1.00	2506.00	Educ. Level 2	0.22	0.41	0.00	1.00	1050.00
Educ. Level 3	0.22	0.42	0.00	1.00	2506.00	Educ. Level 3	0.15	0.36	0.00	1.00	1050.00
Wage	9.28	5.28	0.87	40.82	2506.00	Wage	8.83	4.88	0.91	44.17	1050.00
Hours worked	24.31	17.43	0.00	98.46	2506.00	Hours worked	26.74	18.42	0.00	112.00	1050.00
Housework	23.00	14.34	0.00	84.00	2506.00	Housework	14.43	11.32	0.00	84.00	1050.00

5.3. Estimation. To estimate the model, I follow the steps described in section 4. I assume that there is a mass 1 of each type of man and woman in the sample, that is $n_x = m_y = 1$ for all $x, y \in \mathcal{X} \times \mathcal{Y}$. Preference parameters are allowed to vary with education, as explained above. The objective function used to estimate θ is the log-likelihood constructed from the predicted frequency that each pair will form on the marriage market, as well as the predicted labor supply and housework time. Appendix 3.A contains details about the log-likelihood. Finally, I estimate the model on a subsample of 250 households. However, promising results show that the model could be estimated with 1000 households on larger computing clusters.

5.4. Results.

5.4.1. *Preference Parameters and model fit.* To estimate the model, I used the observed marriage patterns, labour supplies of married couples and singles, and time spent on housework by married couples. To provide a better fit of the marriage patterns, the model includes dummies for each type in the specification of the systematic utilities as well as a dummy when the absolute difference in education level between the man and the woman

is equal to one, and one dummy for when the absolute difference in education level is two. Economically speaking, these components capture costs (or gains) of singlehood for each type, as well as preferences for education assortativeness.

Estimates are displayed in Table 3. Women appear to have a stronger preference for leisure compared to men, which is driven by the fact that women work less than men in the data. In addition, women seem to care more about public consumption, with only slight differences across types. The estimate for σ is not reported here, because estimation actually selects the Cobb-Douglas production function for the public good as the one giving the best fit. The estimate for η is about 1/5. The magnitude of this parameter is not surprising, since in a collective model with a home-produced nonmarketable good, inputs choice is driven by cost minimization. Therefore, the ratio $\eta/(1-\eta)$ is equal to the ratio of wages (w_x/w_y) times the ratio of housework (h_x/h_y). From the raw data, we can deduce that η should be around 0.30. With aggregated data (see appendix 3.B), this indeed what we observe, thus the difference here might be due to sampling error.

TABLE 3. Parameter Estimates

	Men			Women			
	Private C.	Leisure	Public C.	Private C.	Leisure	Public C.	Technology
	a	α	A	b	β	B	η
HS	0.33	0.64	0.03	0.23	0.60	0.18	0.20
C	0.34	0.61	0.04	0.26	0.57	0.17	0.20
C+	0.37	0.58	0.05	0.27	0.56	0.17	0.20

Note: types are displayed in rows (HS = high school or below, C = some college, C+ = college degree and above). In the estimation, I do not impose the preference parameters for consumption, leisure and public good to sum to 1. In the table above, the reported values have been divided by the sum of the parameters.

The model fit is represented in Figure 4. For the sake of clarity, I only display predicted and observed moments for certain types of households. Namely, I use the broad education types of man and woman to construct 15 types of households (9 types of married couples and 6 types of singles), and I use the predicted μ to compute estimated mean for each of these “cells”. The observed average marriage patterns⁵, labour supplies and housework time are represented on the x-axis, and their predicted counterparts are displayed on the y-axis.

⁵I rescale marriage patterns by the total predicted and observed number of households. Therefore, the numbers in the figure represents the share of households of a given type

The solid red line is a 45 degree line that would indicate perfect fit. Our basic model fits the data reasonably well, but the limited size of the sample introduces some noise in the predictions. Moreover, it seems to underestimate housework time of men.

5.4.2. *Conditional sharing rules.* The model predicts Pareto weights for each household. However, I use an alternative monetary measure of power, the conditional sharing rule. In the collective model literature, sharing rules are in general preferred since their interpretation is easier: they simply express how resources (a certain amount of money) are divided between partners. For a particular pair (x, y) , denote \hat{c}_x , \hat{c}_y , $\hat{\ell}_x$ and $\hat{\ell}_y$ the predicted private consumption and leisure. The conditional sharing rule (from the point of view of women) is computed as follow

$$S_{xy} = \frac{\hat{c}_y + w_y \hat{\ell}_y}{\hat{c}_x + w_x \hat{\ell}_x + \hat{c}_y + w_y \hat{\ell}_y}$$

The distribution of the estimated conditional sharing rules are showed in table 5 for various education levels of the partners. Men's education is displayed in rows and women's education is represented in columns. The estimates are well behaved in the sense that the distributions are shifting to the right whenever the education of women increases and shifting to the left whenever the education of men increases. This is consistent with the fact that outside options improve with education (since higher educated individuals are likely to have higher wages). Overall, it seems that women are less favoured than men, a prediction that can be explained by the more attractive outside options of men (since they have a higher wage), and the population imbalance (the sex ratio favours men).

5.4.3. *Counterfactual experiment.* The key benefit of the collective model-ITU approach is to be able to conduct counterfactual experiments. For example, how would the distribution of bargaining power vary following a change in the market structure? As an example, consider an increase in the sex ratio, defined as the share of men in the total population. As men become more abundant, theory predicts that they will transfer resources to their partner in the form of higher private consumption, more leisure hours or higher public good consumption if women have indeed a strong preference for these goods. This is, indeed, what we observe in our illustration. I simulate the effect of a 1 percentage point increase

FIGURE 4. Model Fit

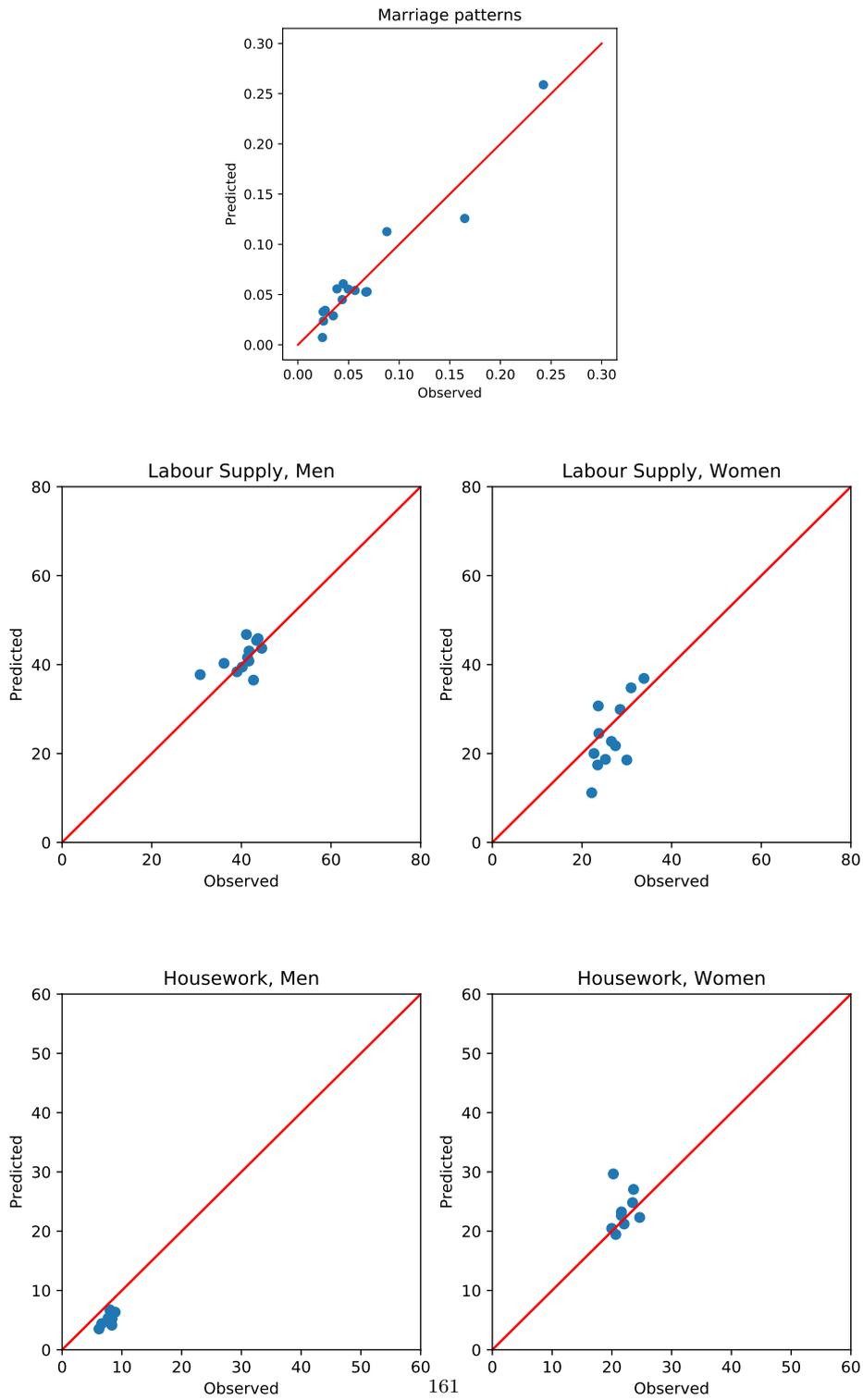
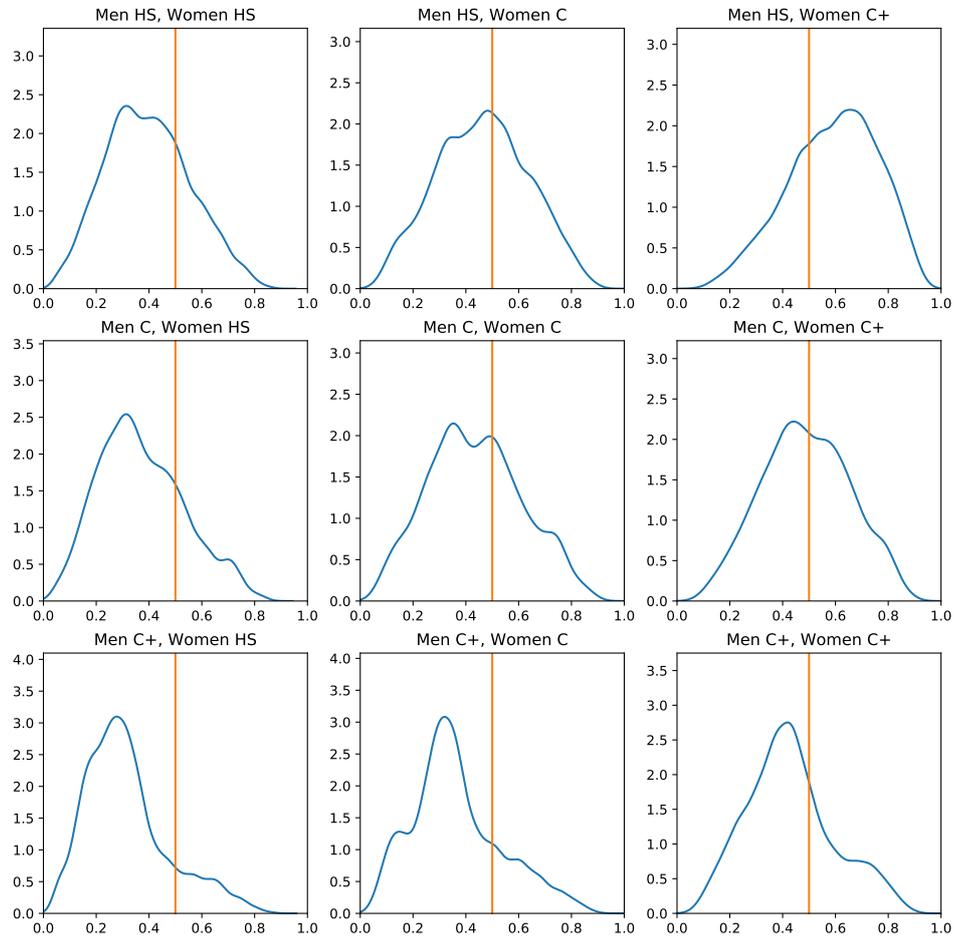


FIGURE 5. Conditional Sharing Rules



in the sex ratio. To compute the “transfers” that occur between men and women, I compute the changes in the conditional sharing rules (from women’s point of view) between the baseline case and the counterfactual. Estimated (annualized) transfers are showed in table 4. Again, for the sake of clarity, I only report transfers averaged by education of men and women. They range approximatively from \$150 to \$700 per year. Note that these are much lower figures than what Chiappori, Fortin, and Lacroix (2002) found (which was about \$2000 per year). To obtain a figure similar to theirs, I would need to consider a 4 percentage point increase of the sex ratio.

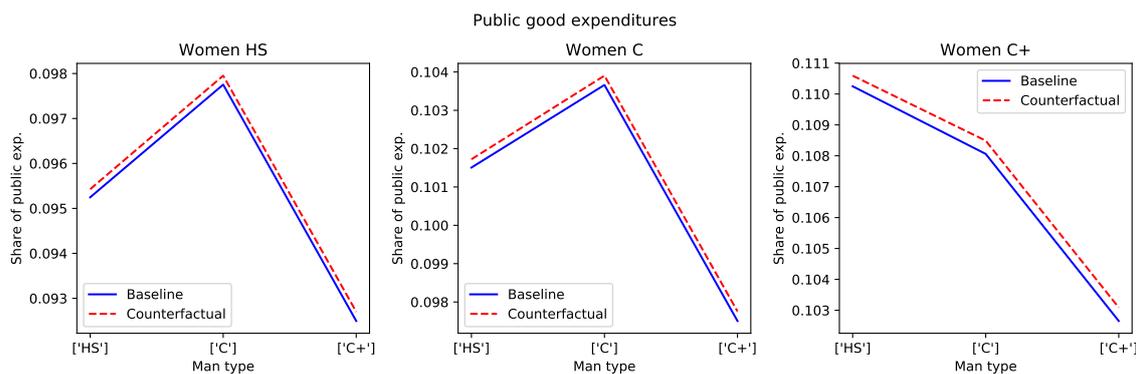
TABLE 4. Estimating transfers from husbands to wives

	HS	C	C+
HS	128.18	243.40	551.51
C	132.13	256.05	597.19
C+	144.58	275.94	683.64

Note: the table displays the transfers in private expenditures (private good plus leisure) from husbands to wives following a 1 percentage point increase in the sex ratio (share of men in total population). Men are displayed in rows and women in columns.

Using the same counterfactual experiment, I look at how public consumption change with sex ratio. As expected, and unlike in the TU case, public good expenditures are affected by the change in the sex ratio. To measure public good consumption in monetary terms, I compute the total amount spent on housework, where the housework times of the man and the woman are priced at their respective wage. The results are displayed in figure 6 (again, aggregated by education level): the blue line indicates the predicted share of public goods expenditures in total income for each broad type of household (the education of men is displayed on the x-axis and each subplot corresponds to a different education level of women). Not surprisingly, the share of public good expenditures is small, around 10% of total income.

FIGURE 6. Public Good expenditures



Given the preference of women for the public good, figure 6 shows that public good expenditures increases under the counterfactual, regardless of the education of men or women. These results illustrates the benefit of the collective model-ITU approach, as we obtain richer comparative statics than under the classical TU case.

6. Conclusion

The present contribution brings together two important parts of the family economics literature: the collective models, that describe the bargaining process that takes place within the household, and matching models, that have been used to model the marriage market. There have been a few attempts to achieve this goal using matching models with perfectly transferable utility. While this approach has produced impressive results, it is not completely satisfying. In particular, “power” does not matter in determining the public good consumption of married individuals. Therefore, I have proposed in this paper to use the empirical ITU matching framework developed previously in Galichon, Kominers, and Weber (2016) to close the gap between collective models and matching.

I have shown that under fairly standard assumptions, many collective models can be embedded in the ITU framework. I also propose a route for estimating these models. Since it is crucial to compute “distance functions”, which in general have no close forms, I discuss ways to avoid this pitfall. In particular, I propose a computation method that works for any collective model at a reasonable computational cost. In addition, the method underlines the connection that exists between the ITU framework and collective models. I provide an illustration using PSID data and a non-trivial model that includes leisure, private consumption and a public good that is produced from time inputs. I investigate whether a parsimonious model fit the data well, and show the strength of the approach with counterfactual experiments.

The ultimate purpose of this paper is to provide grounds for future, more complete, applications. For example, we could estimate a model with the production of a public good such as child quality and a greater number of types. This is part of my research agenda.

Bibliography

- [1] Orazio Attanasio and Valérie Lechene. “Tests of Income Pooling in Household Decisions”. In: *Review of Economic Dynamics* 5.4 (2002), pp. 720–748.
- [2] Gary S Becker. “A Theory of Marriage: Part I”. In: *Journal of Political Economy* 81.4 (1973), pp. 813–846. ISSN: 0226740854.
- [3] Martin Browning, Pierre-André Chiappori, and Y. Weiss. *Economics of the Family*. Ed. by Cambridge Surveys of Economic Literature. Cambridge Surveys of Economic Literature, 2014, pp. –.
- [4] Laurens Cherchye et al. “Household Consumption When the Marriage Is Stable”. In: *IZA Discussion Paper* 8070 (2014), pp. 0–25.
- [5] P.A. Chiappori. “Modèles d’appariement en économie”. In: *Revue Economique* 63.3 (2012), pp. 437–437. ISSN: 9782724632538.
- [6] Pierre-André Pierre-André Chiappori, Bernard Fortin, and Guy Lacroix. “Marriage Market, Divorce Legislation, and Household Labor Supply”. In: *Journal of Political Economy* 110.1 (2002), pp. 37–72. ISSN: 00223808.
- [7] Pierre-André Chiappori. “Collective Labor Supply and Welfare”. In: *Journal of Political Economy* 100.3 (1992), pp. 437–437. ISSN: 00223808.
- [8] Pierre-André Chiappori. “Rational household labor supply”. In: *Econometrica: Journal of the Econometric Society* (1988), pp. 63–90.
- [9] Pierre-André Chiappori, Monica Costa Dias, and Costas Meghir. “The Marriage Market, Labor Supply and Education Choice”. In: *NBER Working Paper* (Apr. 2015), pp. –.
- [10] Eugene Choo and Shannon Seitz. “The Collective Marriage Matching Model: Identification, Estimation, and Testing”. In: *Advances in Econometrics*. Vol. 31. Emerald Group Publishing Limited, Dec. 2013, pp. 291–336–.

- [11] Edoardo Ciscato, Alfred Galichon, and Marion Goussé. “Like Attract Like? A Structural Comparison of Homogamy Across Same-Sex and Different-Sex Households”. In: *Working Paper* (2015), pp. –.
- [12] Anyck Dauphin et al. “Are Children Decision Makers within the Household?” In: *Economic Journal* 121.553 (2011), pp. 871–903.
- [13] Alfred Galichon, Scott Duke Kominers, and Simon Weber. “Costly Concessions: An empirical framework for matching with imperfectly transferable utility”. In: *Working Paper* (2016).
- [14] George-Levi Gayle and Andrew Shephard. “Optimal Taxation, Marriage, Home Production, and Family Labor Supply”. In: *Working Paper* (2016).
- [15] Shelly J. Lundberg, Robert A. Pollak, and Terence J. Wales. “Do Husbands and Wives Pool Their Resources? Evidence from the United Kingdom Child Benefit”. In: *Journal of Human Resources* 32.3 (1997), pp. 463–480.
- [16] Che-Lin Su and Kenneth L. Judd. “Constrained Optimization Approaches to Estimation of Structural Models”. In: *Econometrica* 80.5 (2012), pp. 2213–2230. ISSN: 1468-0262.
- [17] Simon Weber. “Notes on Integrating Collective Models and the Marriage Market”. In: *Working Paper* (2016).

Appendix

3.A. Log-likelihood estimation

The construction of the log-likelihood follows the same step as in GKW. Our model predicts leisure and housework time for any (i, j) pair of individuals. I use this information and the assumption that that housework and leisure are observed in the data with some Gaussian measurement error to construct the likelihood function. Since I assumed a uniform distribution of types, we may as well reintroduce the notation (i, j) instead of types. We can form the log-likelihood as follows:

$$\begin{aligned}
\log \mathcal{L}(\theta, u_i^\theta, v_j^\theta) &= - \sum_{(i,j) \in \mathcal{C}} \left[D^\theta(u_i^\theta, v_j^\theta) + \frac{1}{2} \left(\frac{\ell_i - \hat{\ell}_i}{\hat{s}_1} \right)^2 + \frac{1}{2} \left(\frac{\ell_j - \hat{\ell}_j}{\hat{s}_2} \right)^2 \right] \\
&\quad - \sum_{(i,j) \in \mathcal{C}} \left[\frac{1}{2} \left(\frac{h_i - \hat{h}_i}{\hat{s}_3} \right)^2 + \frac{1}{2} \left(\frac{h_j - \hat{h}_j}{\hat{s}_4} \right)^2 \right] \\
&\quad - \sum_{i \in \mathcal{S}_M} \left[u_i^\theta + \frac{1}{2} \left(\frac{\ell_i - \hat{\ell}_i}{\hat{s}_1} \right)^2 \right] \\
&\quad - \sum_{j \in \mathcal{S}_F} \left[v_j^\theta + \frac{1}{2} \left(\frac{\ell_j - \hat{\ell}_j}{\hat{s}_2} \right)^2 \right] \\
&\quad - |\mathcal{I}| \log \hat{s}_1 - |\mathcal{J}| \log \hat{s}_2 - |\mathcal{C}| \log \hat{s}_3 - |\mathcal{C}| \log \hat{s}_4 \\
&\quad - \hat{N} \log(N)
\end{aligned}$$

In the above expression, ℓ_i, ℓ_j, h_i and h_j denote the predicted leisure and housework time of men and women, respectively. The observed counterparts are denoted $\hat{\ell}_i, \hat{\ell}_j, \hat{h}_i$ and \hat{h}_j . \hat{N} and N are the observed and predicted number of households. The set of matched pairs (i, j) observed in the is denoted \mathcal{C} , and \mathcal{S}_M and \mathcal{S}_F respectively denote the set of single men

and the set of single women observed in the data. The variances of the Gaussian noise on the observed leisure and housework time of men and women are taken from the data and denoted $(\hat{s}_1, \hat{s}_2, \hat{s}_3, \hat{s}_4)$.

3.B. Preliminary aggregate results

3.B.1. Estimation method. In this appendix, I report preliminary results obtained using aggregate types. In particular, I do not assume that there is a mass 1 of each man and women. Instead, I aggregate men and women into three education types as described in the main text. I compute the average wage for each type of men and women. The model predicts the mass of marriage of each household type xy , with $x \in \mathcal{X}_0$ and $y \in \mathcal{Y}_0$, as well as the predicted leisure and housework. In the data, I observe the mass of marriages and the average leisure and housework of each household type. To estimate the parameters θ , I use the same maximum likelihood estimator technique as before. The main advantage of the aggregate approach is that in our case there are only 3 types of men and 3 types of women. Therefore, the model need only to be solved for the 9 possible household types. In practice, estimation is greatly simplified, but as we shall see, the results remain very insightful.

3.B.2. Results. First, the model does a good job at fitting the observed marriage patterns, labour supplies of married couples and singles, and time spent on housework by married couples. This is illustrated in figure 7.

The parameter estimates are displayed in Table 5. They are mostly comparable to the estimates reported in the main text. In particular, the estimate for η is compatible with the cost minimization result from the collective model literature.

The sharing rule estimates reported in table 6 give a more concise view of the sharing of resources within couples. They range from 0.25 to 0.54 and are computed from the point of view of women. The estimates are well behaved in the sense that they are increasing in the education of women and decreasing in the education of men which is consistent with the fact that outside options improve with education (since higher types have higher wages). Overall, it seems that women are less favoured than men, a prediction that can be

FIGURE 7. Model Fit

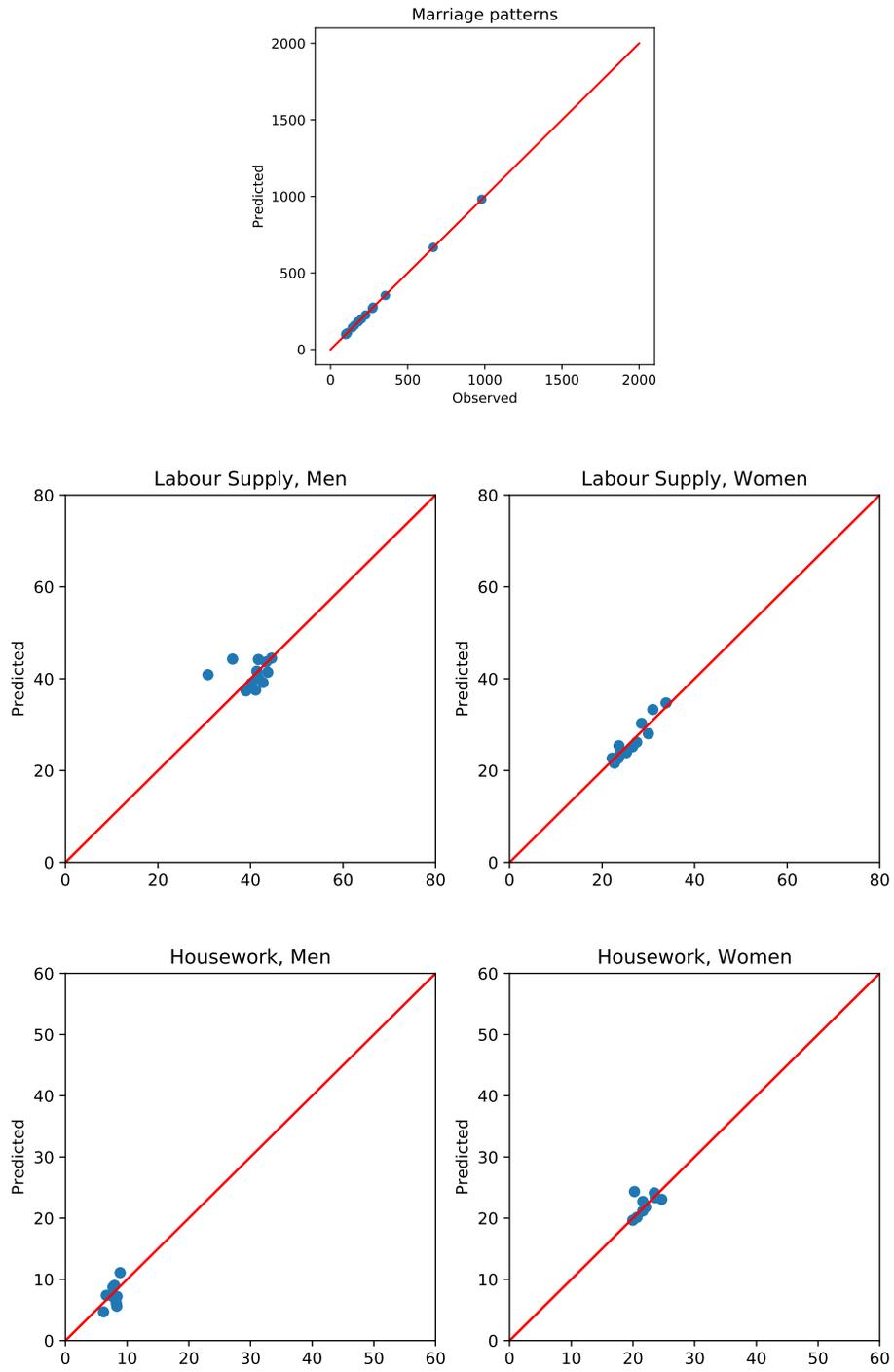


TABLE 5. Parameter Estimates

	Men			Women			
	Private C.	Leisure	Public C.	Private C.	Leisure	Public C.	Technology
	a	α	A	b	β	B	η
HS	0.34	0.59	0.08	0.18	0.62	0.19	0.31
C	0.37	0.56	0.07	0.24	0.57	0.19	0.31
C+	0.37	0.57	0.06	0.25	0.56	0.19	0.31

Note: types are displayed in rows (HS = high school or below, C = some college, C+ = college degree and above). In the estimation, I do not impose the preference parameters for consumption, leisure and public good to sum to 1. In the table above, the reported values have been divided by the sum of the parameters.

explained by the more attractive outside options of men (since they have a higher wage), and the population imbalance (the sex ratio favours men).

TABLE 6. Conditional Sharing Rules

	HS	C	C+
HS	0.37	0.45	0.54
C	0.30	0.38	0.46
C+	0.25	0.33	0.42

Note: the table displays the sharing rule from the woman point of view. Men displayed in rows and women in columns. Each entry is equal to the private expenditures of the woman (equal to her private good consumption plus her leisure consumption priced at the wage rate), divided by the private expenditures of the woman plus the private expenditures of the man. Public goods are not taken into account.

Finally, I perform a similar counterfactual experiment than the one presented in the main text. I simulate the effect of a 1 percentage point increase in the sex ratio (which in our case is equivalent to a 4% increase in the total number of men). The estimated (annualized) transfers are showed in table 7: they range roughly from \$200 to \$500 per year.

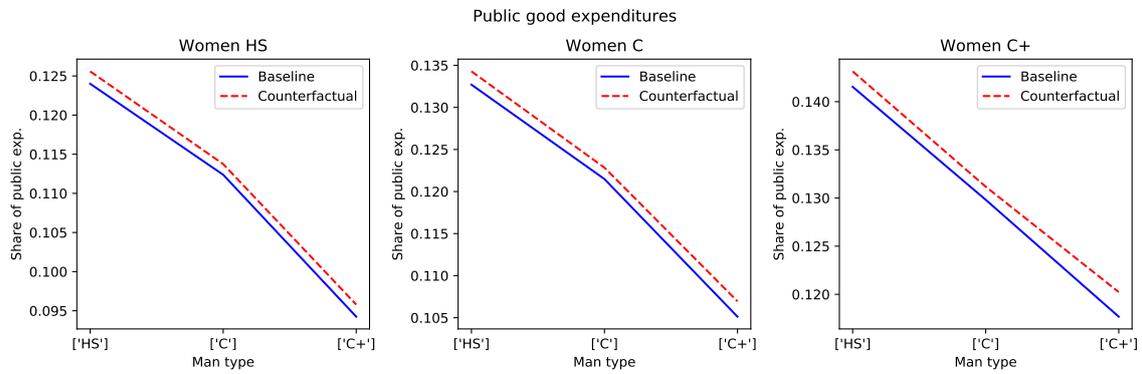
Finally, figure 8 investigates the effect of a change in the sex ratio on public good expenditures. I simulate the impact of a 5 percentage point increase in the sex ratio, because in that case, the transfers are roughly comparable to what has been obtained by Chiappori, Fortin, and Lacroix (2002). As expected, the model predicts that public good expenditures would increase under the counterfactual.

TABLE 7. Estimating transfers from husbands to wives

	HS	C	C+
HS	223.88	252.66	319.95
C	218.12	242.09	294.82
C+	249.13	325.11	539.57

Note: the table displays the transfers in private expenditures (private good plus leisure) from husbands to wives following a 1 percentage point increase in the sex ratio (share of men in total population). Men are displayed in rows and women in columns.

FIGURE 8. Public Good expenditures



Conclusion

The marriage market plays a central part in couple formation and intra-household bargaining. This intuition is the starting point of my research. It led me to investigate how the marriage market relates to inequalities, both at the individual and household levels. For example, the questions I had in mind were (i) to which extent do marital preference contribute to between-household income inequality? or (ii) how are resources shared within couples and how does it respond to factors such as changes in the market structure? Ultimately, it led me to adopt and develop new methods in the matching literature in order to model the marriage market appropriately, and doing so, gain new insights on these important empirical questions.

In chapter 1, I focused mainly on between-household inequalities. Unlike prior work, I used structural methods and I estimated a continuous and multidimensional matching model with transferable utility (TU) that allowed me to disentangle marital preferences from structural changes in the population supplies. Using US data, I showed that assortative mating on education has become stronger and that sorting on unobservables plays a larger role today. My research showed that if mating patterns had not changed since 1971, the 2015 Gini coefficient between households would be lower by 6%.

In chapters 2 and 3, I put emphasis on the intra-household allocation of resources. The idea is that this division *is* assumed to be responsive to market forces, as it is endogenous to the determination of equilibrium on the marriage market. I developed a matching model with imperfectly transferable utility that embeds both the fully- and non-transferable utility models, and is suitable to integrate collective models. Chapter 3 refines and applies the model.

My hope is that this work shows that many other important questions can be answered by using the methods introduced here. I will conclude with an example borrowed

from Heckman and Cunha (2007). Children skills are the product of parental investments. These skills are formed in the early stages of childhood, but have enduring effects in later life. Therefore, it is critical to understand how these skills are formed, and in particular how investment decisions, in a collective setting, may respond to dramatic changes in the marriage market structure (e.g. women's access to higher education) or a variety of policy interventions (e.g. transfers to women). I believe the models developed in this thesis can be used to that end. This is left for future research.