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Zineddine Alla

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Institut d'études politiques de Paris
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Optimal policies in International Macroeconomics

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Abstract

Optimal policies in International Macroeconomics

by Zineddine ALLA

Olivier Blanchard noted in its seminal contribution "Rethinking Macroeconomic Policy" (Blanchard and Paolo, 2010), that "the great moderation lulled macroeconomists and policymakers alike in the belief that we knew how to conduct macroeconomic policy". It was indeed attractive for macroeconomists to attribute (at least partially) the decline in the volatility of business cycle fluctuations starting in the mid-1980s, to the development and subsequent implementation of the inflation-targeting framework by independent central banks (see Bernanke (2004)). In standard New Keynesian Models (e.g. Gali and Monacelli (2005)), conventional monetary policy is sufficient to perfectly stabilize the economy, since the two objectives of stable inflation and zero output gap coincide (the 'divine coincidence'). However, the 2008 global financial crisis and the subsequent euro area sovereign debt crisis successively forced macroeconomists to reassess this conceptual framework.

The global financial crisis indeed compelled the central banks around the world to rethink their monetary and financial stability frameworks, both in terms of objectives and policy tools. The crisis highlighted frictions other than nominal rigidities, in particular those that originate in financial intermediation. Concerns about both financial-stability and the risk of deflation have thus led central banks to use a variety of policy instruments, from macro-prudential tools to balance sheet operations, including credit policy, quantitative easing, and foreign exchange intervention (the latter especially in emerging markets). As a result, old questions about the appropriate objectives of monetary policy and the instruments that should be in the central bank's toolkit, have re-emerged.

The Eurozone sovereign debt crisis then brought back the role of fiscal policy as a stabilization tool to the forefront of the economic agenda. If fiscal policy had been seen as an essential policy tool following the Great Depression, the success of the inflation-targeting framework established monetary policy as the most appropriate stabilization tool to respond to macroeconomic disturbances in the presence of nominal rigidities. Doubts about the real effects of fiscal policy (mostly related to Ricardian arguments), and the fear that fiscal policy would be mostly driven by short-term political motivation – inducing permanent deficits and a steady increase in the debt levels – restricted the scope of fiscal policy to a public goods provider that should be implemented as smoothly as possible.

The crisis in the euro area then shed light on the necessity for member states to proceed to internal/external adjustments to restore their competitiveness facing idiosyncratic shocks. Indeed,

countries that belong to a currency union and who decide to peg their bilateral exchange rates, are deprived of the possibility of an exchange rate adjustment following asymmetric shocks. Such a stabilization margin would then be particularly helpful to rebalance their terms of trade and their current account. The use of fiscal tools to replicate the effects of a flexible exchange rate and an independent monetary policy have then become a central political issue since 2010.

Blanchard also concluded that "In many ways, the general policy framework should remain the same. The ultimate goals should be to achieve a stable output gap and stable inflation. But the crisis has made clear that policymakers have to watch many targets, including the composition of output, the behavior of asset prices, and the leverage of different agents. It has also made clear that they have potentially many more instruments at their disposal than they used before the crisis. The challenge is to learn how to use these instruments in the best way." The following essays are a modest contribution to the huge efforts undertaken by macroeconomists following the crisis to meet this challenge, i.e. to develop some insights about the optimal use of unconventional policy tools.

To do so, this thesis is twofold. Each part intends to explore from a theoretical perspective a fundamental macroeconomic situation that called for the use of unconventional policy instruments in the recent years. The first part, "Optimal Unconventional Policy in An Open Economy" analyzes the optimal use of unconventional policy instruments by the central bank in an open economy framework. Assuming that the presence of financial frictions changes the way monetary policy affects the economy, or that the occurrence of exogenous shocks breaks the "divine coincidence", this part describes how a central bank should combine an unconventional policy instrument and conventional monetary policy to favor macroeconomic stabilization. A microfounded example of this general approach is then provided, and describes the optimal use of sterilized intervention in an open economy, assuming the portfolio balance channel is effective. The second part, "Optimal Fiscal Policy in a Currency Union", takes the standpoint of the government of a country located in a currency union (typically the euro area). Such a country being deprived of monetary policy autonomy, this part considers the opportunity of using fiscal policy as a stabilization tool, and describes the optimal use of fiscal devaluations following idiosyncratic exogenous shocks.

This first chapter analyzes the use of an unconventional policy instrument in New Keynesian setups where 'the divine coincidence' breaks down. It intends to provide a unified framework in which to cast New Keynesian Models with additional instruments (on top of monetary policy), and derives general results that are applicable to a wide variety of models. My work highlights the role of the unconventional instrument, discusses its coordination with standard interest rate policy, and presents theoretical results on equilibrium determinacy, the inflationary bias, the stabilization bias and the optimal central banker's preferences when two policy instruments are available. I show that the use of an unconventional instrument can help reduce the zone of equilibrium indeterminacy and the volatility of the economy following a real exogenous shock. However, in some circumstances, committing not to use the secondary instrument may be welfare improving (a result akin to Rogoff (1985a)). I also characterize the preferences of the optimal central banker, whom I find should be both aggressive against inflation, and interventionist when using the unconventional policy instrument. As long as price setting depends on expectations about the future, there are gains from establishing credibility by using any instrument that affects these expectations.

The second chapter proposes a microfounded example of the general approach described in Chapter 1, by modelling sterilized intervention in an open economy New Keynesian Model, assuming the

portfolio balance channel is effective. The paper assumes that on top of conventional monetary operations (via the policy rate, or for those central banks that have not yet moved to interest rate targeting, domestic credit operations), the central bank implements sterilized foreign exchange intervention, which consists in purchasing or selling foreign exchange, but offsetting any effect this has on the money supply and the interest rate with open market operations. The paper then highlights the role of foreign exchange intervention as a tool for exchange rate stability, inflation stability, and in general macroeconomic stability following exogenous risk premium shocks. I then show how the use of foreign exchange interventions can help reduce the zone of policy indeterminacy and the occurrence of speculative attacks in the presence of a simple financial accelerator. Indeed, although equilibrium is unique in the classical New Keynesian framework, multiple equilibria occur for a sufficiently strong financial accelerator (akin to third generation models of currency crises, see e.g. Aghion et al. (2000)). I determine under which conditions such indeterminacy occur, and highlight how the use of foreign exchange interventions reinforces the central bank's credibility.

The third chapter considers the situation of a country that is located in a currency union, and has consequently lost monetary policy autonomy. Indeed, the well-known Mundell's trilemma stresses that a country can not simultaneously have free capital flows, a fixed exchange rate and an independent monetary policy. Advanced economies, notably in the Eurozone, favored the first two features, consequently losing monetary policy autonomy. Such a margin is yet extremely useful. Currency union's members face the impossibility to engineer countercyclical monetary/exchange rate policies to promote macroeconomic stabilization. I then develop a standard small open economy model with a fixed exchange rate to study optimal fiscal devaluations in a currency union. The only fiscal tools available for the government are value-added and payroll taxes, subject to a revenue-neutrality constraint. I study both optimal time-varying and one-time fiscal devaluations, following a variety of macroeconomic shocks. The analysis provides closed-form results about optimal fiscal devaluations following exogenous shocks, and shows that optimal fiscal devaluations' sign is determined by the nature of the macroeconomic shock (demand/supply). It also highlights the importance of fiscal pass-through to consumer prices to design optimal fiscal devaluations, and describe analytically a policy trade-off between two channels respectively based on a consumption and a competitiveness stimulus. Finally, it shows that contrary to the main criticisms against fiscal devaluations, optimal fiscal devaluations (both time-varying and one-time) are of the same order of magnitude than the occurring macroeconomic shocks, and substantially efficient from a welfare point of view.

There are, of course, a number of related an important questions related to the optimal use of unconventional policy instruments which are being left over, and which leave room for future research. A natural on-going extension of my thesis is to turn to the definition of new policy objectives following the global financial crisis, and to cast these targets in an optimal policy framework. More precisely, the most important ongoing work intends to introduce a financial stability and systemic risk measure as an objective in the canonical optimal policy framework, and to analyze the optimal use of macroprudential tools in this framework. Other theoretical work is also in progress.

Résumé

Politiques optimales en macroéconomie internationale

par Zineddine ALLA

Peu après le déclenchement de la crise, Olivier Blanchard souligne dans son article fondateur "Re-thinking Macroeconomic Policy" (Blanchard and Paolo, 2010), que "la grande modération a bercé les macroéconomistes et les décideurs politiques, en les convainquant qu'ils savaient conduire la politique macroéconomique." Il était en effet très tentant pour les macroéconomistes d'attribuer (au moins partiellement) le déclin de la volatilité des cycles économiques observé depuis la moitié des années 80 au développement d'un cadre conceptuel fondé sur une cible d'inflation, et à sa mise en oeuvre par des banques centrales indépendantes (cf. Bernanke (2004)). Dans un cadre Néo-Keynésien standard (cf. Gali and Monacelli (2005)), la politique monétaire conventionnelle est en effet suffisante pour stabiliser parfaitement l'économie, puisque les deux objectifs de stabilisation de l'inflation et de l'écart de production coïncident (la "divine coïncidence"). Cependant, la crise financière mondiale qui a débuté en 2008, et la crise des dettes souveraines en zone euro qui l'a suivie, ont successivement forcé les macroéconomistes à repenser leur cadre conceptuel.

D'une part, la crise financière mondiale a obligé les banques centrales à repenser leur cadre de politique monétaire, tant en matière d'objectifs poursuivis que d'instruments. Cette crise a notamment mis en évidence des frictions autres que les rigidités nominales, en particulier celles provenant de l'intermédiation financière. Les inquiétudes liées à la fois aux enjeux de stabilité financière et aux risques de déflation ont ainsi conduit les banques centrales à élargir leur panel d'instruments, qu'il s'agisse d'outils macro-prudentiels ou d'opérations de bilans (politique de crédit, assouplissement quantitatif ou interventions de change – ces dernières concernant notamment les pays émergents). De fait, les questions supposées résolues et portant sur la définition des objectifs poursuivis par la politique monétaire, ainsi que sur les instruments qui devraient figurer dans la "boîte à outils" du banquier central, sont repassées au premier plan de la réflexion économique.

D'autre part, la crise des dettes souveraines en zone euro a placé le rôle stabilisateur de la politique budgétaire au coeur des débats économiques. Si la politique budgétaire était perçue comme un outil essentiel de politique économique au lendemain de la Grande Dépression, le succès (supposé) des politiques monétaires fondées sur une cible d'inflation a établi la politique monétaire comme l'outil de stabilisation le plus approprié pour répondre aux chocs macroéconomiques en présence de rigidités nominales. Les doutes portant sur les effets réels de la politique budgétaire (provenant essentiellement d'arguments reposant sur des formes d'équivalence ricardienne), ainsi que la crainte que la politique budgétaire soit essentiellement mue par des considérations politiques de court terme – conduisant à des déficits permanents et une augmentation continue du niveau de dette – ont

restreint le champ de la politique budgétaire à un pourvoyeur de biens publics dont l'implémentation doit être lissée autant que possible.

La crise en zone euro a cependant souligné l'importance pour les Etats membres d'une union monétaire d'être en mesure de procéder à des ajustements (internes et externes) pour rétablir leur compétitivité à la suite de chocs idiosyncratiques. En effet, les pays appartenant à une zone monétaire unifiée, et qui ont donc décidé de fixer leurs taux de change bilatéraux, sont privés de la possibilité d'ajuster leur taux de change en réponse à des chocs asymétriques. Une telle marge de stabilisation est cependant particulièrement utile pour rééquilibrer (notamment) les termes de l'échange et le compte courant. La mise en oeuvre d'outils budgétaires pour répliquer l'utilisation d'un taux de change flexible et d'une politique monétaire indépendante est ainsi devenue une question politique centrale depuis 2010.

Blanchard conclut ainsi que "Par bien des aspects, le cadre général de politique économique devrait rester le même. Les objectifs finaux demeurent la stabilité de l'inflation et de l'écart de production. Cependant, la crise a clairement mis en évidence que les décideurs politiques devaient surveiller de nombreux indicateurs, notamment la composition de la production, le comportement des prix des actifs, les effets de levier des différents agents. Elle a également souligné que ces mêmes décideurs politiques avaient beaucoup plus d'instruments à leur disposition que ceux utilisés avant la crise. Le défi consiste aujourd'hui à utiliser ces instruments de manière optimale." Les essais présentés ci-après sont une modeste contribution aux efforts colossaux déployés par les macroéconomistes à travers le monde pour faire face à ce défi: renforcer la compréhension de l'utilisation optimale des outils de politique économique non conventionnels.

A cette fin, cette thèse est construite en deux parties. Chaque partie vise à explorer au plan théorique un "contexte macroéconomique-type" au sein duquel des outils de politique économique non conventionnels ont été employés ces dernières années. La première partie, intitulée "Politique Non Conventionnelle Optimale en Economie Ouverte", analyse l'utilisation optimale d'instruments de politique économique non conventionnels par une banque centrale en économie ouverte. En présence de frictions financières qui modifient la manière dont la politique monétaire affecte l'économie, ou en présence de chocs exogènes qui mettent en défaut la "divine coïncidence", cette partie décrit comment un banquier central devrait combiner un instrument de politique monétaire non conventionnelle et la politique monétaire conventionnelle à des fins de stabilisation macroéconomique. Un exemple microfondé de cette approche générale est ensuite proposé, et décrit l'utilisation optimale des interventions de change en économie ouverte, en faisant l'hypothèse que le canal d'équilibrage des portefeuilles est effectif. La seconde partie, "Politique Budgétaire Optimale en Union Monétaire", adopte le point de vue du gouvernement d'un pays situé en union monétaire (typiquement la zone euro). Un tel pays ne disposant d'une politique monétaire autonome (au plan national), cette partie étudie la possibilité pour un tel pays d'utiliser la politique budgétaire comme un outil de stabilisation, et décrit l'utilisation optimale des dévaluations fiscales en réponse à des chocs exogènes idiosyncratiques.

Le premier chapitre analyse l'utilisation d'un instrument de politique économique non conventionnelle dans un cadre néokeynesien au sein duquel la "divine coïncidence" ne s'applique pas. Ce chapitre introduit un cadre unifié s'appliquant aux modèles néokeynesiens incluant des outils de politique économique non conventionnelle, et énumère plusieurs résultats généraux. Mon travail met en évidence le rôle de cet instrument non conventionnel, discute les enjeux de coordination entre cet instrument et la politique usuelle fondée sur les taux d'intérêt directs. Je présente ainsi des résultats théoriques portant sur la détermination des équilibres, les biais d'inflation et

de stabilisation, ainsi que sur les préférences optimales d'un banquier central disposant de deux instruments. Je montre ainsi que l'utilisation d'un instrument de politique économique non conventionnelle peut aider à réduire la zone d'indétermination des équilibres ainsi que la volatilité de l'économie en réponse à un choc réel exogène. Cependant, ce travail souligne que dans certaines circonstances, il peut être préférable de la part du banquier central de s'engager à ne pas utiliser d'instrument non conventionnel (un résultat dans l'esprit de celui de Rogoff (1985a)). Je caractérise enfin les préférences du "banquier central optimal", qui doit être à la fois conservateur vis-à-vis de l'inflation et interventionniste en utilisant l'instrument de politique monétaire non conventionnelle. Si la formation des prix dépend des perspectives d'utilisation d'un instrument non conventionnel, le banquier central peut renforcer la crédibilité de son action en utilisant cet instrument de manière volontariste.

Le second chapitre propose un exemple microfondé de l'approche générale introduite ci-dessus. Ce chapitre décrit ainsi l'utilisation optimale des réserves de changes dans un modèle néokeynesien de petite économie ouverte, en supposant que le canal d'équilibrage des portefeuilles est effectif. En sus de la politique monétaire conventionnelle (implémentée au moyen de taux directeurs, ou pour les banques centrales n'ayant pas adopté un tel pilotage, au moyen d'opérations de crédit domestique), la banque centrale utilise des interventions stérilisées sur le marché des changes, qui consistent à acheter ou à vendre des devises étrangères, puis à annuler l'impact de ces opérations sur la masse monétaire domestique au moyen d'opérations de marché. Ce chapitre met ainsi en évidence l'utilité des interventions sur le marché des changes en vue de favoriser la stabilité du taux de change et de l'inflation, et plus généralement la stabilité de l'ensemble des grandeurs macroéconomiques en réponse à des chocs exogènes sur le premium des bons domestiques. Je montre ainsi comment l'utilisation des réserves de change peut aider à réduire la zone d'indétermination des équilibres économiques et la possibilité d'attaques spéculatives en présence d'un accélérateur financier simplifié. En effet, si l'unicité de l'équilibre est assurée dans le cadre néokeynesien classique, je montre que la présence d'équilibres multiples est possible en présence d'un accélérateur financier suffisamment important (de manière semblable aux modèles de troisième génération de crise de balance des paiements, cf. Aghion et al. (2000)). Je détermine ainsi les conditions dans lesquelles une telle indétermination est possible, et met en évidence la manière dont l'utilisation de réserves de change peut renforcer la crédibilité de la banque centrale.

Le troisième chapitre analyse enfin le cas d'un pays situé dans une union monétaire, et qui par conséquent ne dispose pas d'une politique monétaire indépendante. En effet, le fameux trilemme de Mundell souligne que deux pays ne peuvent opter à la fois mutuellement pour une libre circulation des capitaux, un taux de change fixe et une politique monétaire indépendante. Les économies avancées, notamment en zone euro, ont fait le choix des deux premières caractéristiques, et par conséquent confié la mise en oeuvre de la politique monétaire à une banque centrale. Cependant, une politique monétaire et de change indépendante s'avère extrêmement utile pour promouvoir des politiques de stabilisation macroéconomique contracycliques. Je considère ainsi un modèle standard de petite économie ouverte en change fixe, afin d'étudier les dévaluations fiscales optimales en union monétaire. Les seuls outils budgétaires disponibles sont la taxe sur la valeur ajoutée (TVA) et les cotisations patronales, soumises à une contrainte de neutralité budgétaire. J'étudie à la fois les dévaluations fiscales optimales continues et ponctuelles, en réponse à un panel de chocs macroéconomiques. Cette analyse propose des résultats sous forme de formules closes, et montre que le signe des dévaluations fiscales optimales (dévaluation ou réévaluation fiscale) dépend de la nature du choc macroéconomique (offre/demande). Je souligne également que le degré de transmission de la politique fiscale sur les prix détermine de manière cruciale la forme des dévaluations fiscales

optimales, et décris ainsi un arbitrage entre deux canaux de transmission respectivement basé sur une stimulation de la consommation ou de la compétitivité. Enfin, je montre que contrairement à la plupart des critiques adressées à l'encontre des dévaluations fiscales, les dévaluations fiscales optimales (ponctuelles et continues) sont de l'ordre du choc macroéconomique auxquelles elles répondent, et sont substantiellement efficaces à des fins de stabilisation macroéconomique.

Bien entendu, de nombreuses questions essentielles en matière d'utilisation optimale des outils de politique monétaire non conventionnelle demeurent en suspens, laissant la voie ouverte à des travaux futurs. Une extension naturelle, et en cours, des travaux ici présentés est la définition de nouveaux objectifs de politique économique à l'issue de la crise financière, et leur intégration dans un cadre de politique optimale. Plus précisément, un travail en cours vise à introduire une mesure de stabilité financière dans un modèle de politique optimale canonique, et à analyser l'utilisation d'outils macroprudentiels dans ce cadre. D'autres travaux théoriques sont également en cours.

Contents

Acknowledgements	i
Abstract	iii
Résumé	ix
I Optimal Unconventional Policy in An Open Economy	2
1 Optimal Use Of Unconventional Policy Instruments	3
1.1 Abstract	3
1.2 Introduction	3
1.3 Analytical framework	5
1.3.1 The extended New Keynesian framework	5
1.3.2 The rationale for unconventional policy instruments	7
1.3.3 An example	7
1.4 The need for unconventional policy instruments	8
1.4.1 Equilibrium determinacy	8
1.4.2 Optimal stabilization policy following real shocks	11
1.5 Central banker's preferences	12
1.5.1 The stabilization bias	12
1.5.2 The stabilization bias when optimal preferences trigger multiple equilibria	15
1.5.3 The inflationary bias	16
1.6 Concluding Remarks	17
1.7 Appendix	19
1.7.1 Proof of Proposition 3	19
1.7.2 Proof of Proposition 4	22
2 Foreign Exchange Interventions in the New Keynesian Model	26
2.1 Abstract	26
2.2 Introduction	26
2.3 Related Literature	28
2.4 A small open economy	31
2.4.1 Households	31
2.4.2 Firms	32
2.4.3 The central bank, sterilized interventions and the government	34
2.4.4 Exchange rates and the terms of trade	36
2.4.5 Equilibrium Conditions	36

2.4.6	The log-linearized framework	38
2.4.7	Loss function	40
2.5	Optimal stabilization policy following risk premium shocks	42
2.5.1	Optimal use of FX intervention following risk premium shocks	42
2.5.2	Simulations	45
2.6	FX Intervention and speculative attacks	47
2.6.1	Optimal policy	47
2.6.2	Equilibrium Determinacy	48
2.7	Conclusion	50
2.8	Appendix	52
2.8.1	First-order conditions for Calvo price setting	52
2.8.2	Derivation of the intertemporal budget constraint	52
2.8.3	Derivation of the loss function	53
2.8.4	Proof of Proposition 6	54

II Optimal Fiscal Policy in a Currency Union 55

3	Optimal Fiscal Policy in a Currency Union: Fiscal Devaluation or Fiscal Reevaluation?	56
3.1	Abstract	56
3.2	Introduction	56
3.3	A small open economy	59
3.3.1	Households	59
3.3.2	Firms	62
3.3.3	Government	64
3.3.4	Equilibrium conditions with symmetric rest of the world	65
3.3.5	Steady State Taxes	67
3.3.6	Shocks	68
3.4	The log-linearized framework	68
3.4.1	The pass-through impact: two competing channels	69
3.4.2	The loss function and the role of trade openness	71
3.4.3	The Planning problem: comparison with the flexible exchange rate case	72
3.5	Optimal fiscal devaluations	74
3.5.1	Rigid prices	74
3.5.2	Flexible prices and rigid wages	76
3.5.3	Optimal fiscal devaluations under staggered price and wage settings	78
3.6	Conclusion	82
3.7	Appendix	83
3.7.1	First-order conditions for Calvo price and wage settings	83
3.7.2	Proof of Proposition 8	83
3.7.3	Proof of Proposition 9	85
3.7.4	Summarizing the economy	85
3.7.5	Derivation of the loss function	87
3.7.6	Proof of Proposition 10	88
3.7.7	Proof of Proposition 11	90

Part I

Optimal Unconventional Policy in An Open Economy

Chapter 1

Optimal Use Of Unconventional Policy Instruments

1.1 Abstract

This paper analyzes the use of an unconventional policy instrument in New Keynesian setups where 'the divine coincidence' breaks down. The objective of the paper is to highlight the role of this instrument, discuss its coordination with standard interest rate policy, and present theoretical results on equilibrium determinacy, the inflationary bias, the stabilization bias and the optimal central banker's preferences when these two policy instruments are available. We show that the use of an unconventional instrument can help reduce the zone of equilibrium indeterminacy and the volatility of the economy. However, in some circumstances, committing not to use the secondary instrument may be welfare improving (a result akin to Rogoff (1985a)). We also characterize the preferences of the optimal central banker, whom we find should be both aggressive against inflation, and interventionist when using the unconventional policy instrument. As long as price setting depends on expectations about the future, there are gains from establishing credibility by using any instrument that affects these expectations.

1.2 Introduction

Since the 2008 global financial crisis, central banks around the world have been forced to rethink their monetary and financial stability frameworks. Concerns about both financial-stability and the risk of deflation have led central banks to use a variety of policy instruments, from macro-prudential tools to balance sheet operations, including credit policy, quantitative easing, and foreign exchange intervention (the latter especially in emerging markets). As a result, old questions about the appropriate objectives of monetary policy, the desirability of targeting asset prices or other financial stability measures, and the instruments that should be in the central bank's toolkit, have re-emerged. These questions had seemed settled by the success that inflation-targeting central banks enjoyed during the so-called "Great Moderation". For instance, in his volume *Interest and Prices*, Woodford (2003) argues that central banks should *only* target the inflation of the basket of goods whose prices are updated the least frequently, because volatility in these prices would distort most relative prices.¹ On the other hand, the crisis highlighted frictions other than nominal rigidities, in particular those that originate in financial intermediation. Acknowledging the need to

¹For this reason, asset prices, which adjust at high frequency and thus reflect the market view of relative prices, should not be part of the inflation measure that guides monetary policy decisions.

include financial frictions in the standard framework, the literature has investigated the benefits of other policy regimes, starting from flexibilizing (e.g., Woodford (2012)) to more radical rethinking (Giavazzi and Giovannini (2010)) of inflation targeting.

In the standard model, however, the policy interest rate is sufficient to achieve economic stability because the inflation target and output at its first best level coincide: this is the famous *divine coincidence* (Blanchard and Gali (2007)). Optimal monetary policy then consists in indexing the real interest rate on the natural rate of interest.² But when additional elements are added to the model, this ‘divine coincidence’ often breaks down and the conduct of monetary policy becomes more challenging. These elements could be reduced-form, exogenous, cost-push shocks, as commonly included in New Keynesian Phillips Curves. Central banks then face a trade-off between reducing output volatility and inflation volatility (Taylor (1979), Clarida et al. (1999)). Or there could be frictions beyond the nominal rigidities already included in the New Keynesian Model. In models with real wage rigidities, stabilizing inflation and the output gap is not optimal (Blanchard and Gali (2007)). In models where interest rates affect marginal costs, standard policy rules may lead to indeterminacy (Surico (2008)) and monetary policy is inefficient (the output gap and inflation both fluctuate following productivity or demand shocks, see Ravenna and Walsh (2006)). Or there could be limits to the efficacy of standard interest rate policy. For instance, because of the zero-lower bound (Eggertsson and Woodford (2003)); because of risk premia in international capital markets (Farhi and Werning (2013)); or because of disruptions in the process of financial intermediation (Curdia and Woodford (2010)).

In each of these circumstances, it is natural to ask how a secondary, unconventional, policy instrument can alleviate the challenges faced by policymakers. Different instruments have been discussed, depending on the source of the friction: capital controls can lean against volatile capital flows when there are shocks to risk premia (Farhi and Werning (2013)); fiscal policy can support monetary policy if it is constrained (Correia et al. (2013)); quantitative easing can help reduce credit spreads that hamper financial intermediation (Curdia and Woodford (2011)); macroprudential policy can help resolve financial instability or aggregate demand externalities (e.g., De Paoli and Paustian (2013), Farhi and Werning (2013)). This literature has also touched upon the capacity of monetary policy alone to do the job (Woodford (2012)), and the need for coordination of the different policy instruments (Svensson (2014)).

In many of these papers, despite the diversity of circumstances considered, the formal models often boil down to an extended New Keynesian Model, where the linearized expected Investment Saving (IS) curve and Phillips curve are affected by the ‘friction’, by the new instrument, and where (the quadratic approximation of) the welfare function includes directly the unconventional policy instrument (typically penalizing its use). That is the general problem we study. Our objective is to provide a unified framework to draw general results on the use of additional policy instruments. We show that additional policy instruments can be useful in ruling out equilibrium indeterminacy and in reducing welfare losses after exogenous shocks or in the presence of a distorted steady state, although under some circumstances, committing *not* to use the unconventional instrument may be welfare improving. We also establish that the inflationary bias and the stabilization bias are mitigated if the central bank aggressively uses the secondary instrument. Finally, we characterize the optimal preferences for the central bank governor in cases when societal preferences would result in indeterminacy.

²The interest rate that would prevail at the flexible allocation.

Our contribution is to present a unifying framework in which to cast New Keynesian Models with multiple instruments and derive general results that are applicable to a wide variety of models. Section 1.3 presents the analytical framework, which is a general linear New Keynesian model, and discusses how it relates to different strands of the literature. Section 1.4 analyses equilibrium determinacy, and characterizes the optimal stabilization policy following exogenous real shocks. Section 1.5 discusses the optimal preferences (over inflation and over the use of the second instrument) of the central bank to mitigate the inflationary bias and the stabilization bias, given the weights in the social welfare function. Section 1.6 concludes by discussing some of the policy implications of our analysis.

1.3 Analytical framework

1.3.1 The extended New Keynesian framework

We want to analyze the optimal use of an unconventional policy instrument, denoted θ , in a general framework that comprises a Phillips curve, an expected IS curve and a quadratic loss function. Following Kydland and Prescott (1977), we know that as long as the dynamic system features expected terms, optimal policy under commitment is not time-consistent. We then consider a purely discretionary framework in which expected values of expected variables are taken as given, and analyze the ways in which a central bank can reinforce its credibility, we consider a purely .

Our approach is general enough to encompass various candidates for the unconventional instrument θ : public spending as in Gali and Monacelli (2008); fiscal policy (Alla et al. (2016)); capital controls (Farhi and Werning (2013)); foreign exchange intervention (Alla et al. (2015)), quantitative easing (Curdia and Woodford (2011)) or macroprudential policy (e.g. De Paoli and Paustian (2013), Farhi and Werning (2013)).

The dynamic equations

A fairly general model is one in which the New Keynesian Phillips Curve (NKPC) and the IS curve take the following forms:

$$\pi_{H,t} = \Phi(\pi_{H,t+1}^e, y_t, y_{t+1}^e, i_t, \theta_t, \theta_{t+1}^e, u_t) \quad , \quad y_t = \Psi(y_{t+1}^e, \pi_{H,t}, \pi_{H,t+1}^e, i_t, \theta_t, \theta_{t+1}^e, v_t)$$

where π_H is domestic inflation, y is the output gap, i is the policy interest rate, θ is the unconventional policy instrument, u and v are exogenous shocks, and Φ and Ψ are linear functions.

Note that this formulation is more general than the standard New Keynesian Model. In particular, in the standard model, there is no additional instrument ($\partial\Phi/\partial\theta_t = \partial\Phi/\partial\theta_{t+1}^e = \partial\Psi/\partial\theta_t = \partial\Psi/\partial\theta_{t+1}^e = 0$) and the interest rate does not enter the NKPC ($\partial\Phi/\partial i_t = 0$), and some of the coefficients in Φ and Ψ are constrained. Model modifications that change these coefficients are not minor as they can affect essential results in the monetary policy literature (equilibrium determinacy for instance). Substituting for the interest rate³ in the Phillips Curve,⁴ we can summarize the model's dynamics by:

$$\pi_{H,t} = k_\pi \pi_{H,t+1}^e + k_y y_t + k_{y^e} y_{t+1}^e + k_\theta \theta_t + k_{\theta^e} \theta_{t+1}^e + u_t \tag{1.1}$$

³This substitution is only possible if the interest rate enters the Phillips curve. If not, the NKPC directly takes the form of equation (1.1).

⁴We decide to keep the Phillips Curve since it is the relevant dynamic equation in the standard New Keynesian model, in which the interest rate allows to control output in the IS curve. However, this is without loss of generality.

It is important to note that our results would apply to any optimal control problem, not just models where the state variables are output and inflation. The only important ingredient is that the unconventional instrument and its expected value affect the variable the central bank wants to stabilize.

The objective function and the intertemporal constraint

The objective is to minimize the welfare loss function:

$$\sum_{t=0}^{\infty} \beta^t [\alpha_{\pi} \pi_{H,t}^2 + y_t^2 + \alpha_{\theta} \theta_t^2]$$

where α_{π} and α_{θ} are the weights on inflation and on the unconventional instrument. Since we are in a discretionary framework, the central banker problem boils down to minimizing the current term of the above expression:

$$\alpha_{\pi} \pi_{H,t}^2 + y_t^2 + \alpha_{\theta} \theta_t^2 \tag{1.2}$$

If the unconventional policy instrument has budgetary implications (for the Treasury, the central bank, or the country as a whole), one may need to take into account an intertemporal budget constraint of the form:⁵

$$\sum_{t=0}^{\infty} \beta^t \theta_t = 0 \tag{1.3}$$

In many models, this constraint can be derived endogenously; see for instance Farhi and Werning (2013) or Alla et al. (2015). Although our analysis will take into account the government's (or central bank's) inability to commit to specific future policies (on asset purchases, the deficit, etc), we assume they can commit not to default. Unless the government can commit not to default, the intertemporal budget constraint means that the second instrument could not be used⁶. We define Γ as the Lagrange multiplier associated to the intertemporal budget constraint (1.3). Γ can easily be set to 0 if this constraint is not relevant to the specific problem under study.

Limitations

Our framework does not encompass several complications that could be relevant in specific contexts. First, we do not consider an explicit stochastic setup. However, since we are working with linearized conditions, as most studies, introducing stochastic exogenous shocks would not change the results. Second, and more importantly, non-linear dynamics are not discussed. This may be relevant for financial stability problems, characterized by abrupt transitions and regime-switching (Woodford (2012)). However, it is likely that the gist of our results, which rely on very general assumptions, would hold under more complex dynamics.

⁵A No-Ponzi condition would imply that the discounted value of the instrument is smaller than a given value. This value is normalized to zero since a non-zero target would imply a steady-state deviation of $\bar{\Theta}$ in the non-linearized budgetary equation (θ is the log-deviation of the secondary instrument). For the same reason, the constraint is an equality.

⁶In a purely discretionary framework, the government would promise at each period to reimburse the current period deficit with future revenues: $\theta_t = -\sum_{s=t}^{\infty} \beta^{s-t} \theta_s$. However, this promise, renewed at each period, is not credible since it omits past deficits. The only solution consistent with rational expectations is then $\theta_t = 0$: $\theta_t = -\sum_{s=t+1}^{\infty} \beta^{s-t} \theta_s = -\beta \sum_{s=t+1}^{\infty} \beta^{s-(t+1)} \theta_s = 0$. Such an intertemporal constraint thus requires a commitment not to default.

1.3.2 The rationale for unconventional policy instruments

Breaking the divine coincidence

Our objective is to assess theoretically the relevance of additional policy instruments. We thus consider two cases where monetary policy can not perfectly stabilize the economy. The first case, which we refer to as *cost-push shocks*, represents any model element that leads to additive factors u_t in the Phillips curve. These exogenous shocks have appeared in recent models, for instance: risks to financial intermediation in the version of the model of Curdia and Woodford (2009) where leverage is exogenous (see Woodford (2012)); risks premia in models of capital flows (Farhi and Werning (2013)). Because they break the ‘divine coincidence’, they are important for much of our analysis. Exogenous shocks that affect the welfare criterion have implications that are similar to cost-push shocks and thus can also be analyzed in our framework.

We also introduce *financial frictions* in a reduced form, as any model component that implies that the (domestic or foreign) interest rate enters the Phillips curve. The cost channel of monetary policy is operative when firms’ marginal costs are affected directly by the interest rate, and is well documented in empirical studies (Tillman (2008) shows that the cost channel adds significantly to the explanation of inflation dynamics). Such a financial friction can lead to monetary policy indeterminacy, see Surico (2008), and reduces the efficacy of monetary policy as a stabilization tool (Ravenna and Walsh (2006)).

The forward-looking determinants of inflation

The presence of such a financial friction is however not required for our analysis. What matters is the presence of the expected terms y_{t+1}^e and θ_{t+1}^e in the Phillips Curve.⁷ There is no empirical consensus on the role of expectations in the new Keynesian Phillips Curve (Mavroeidis et al. (2014)), but this has not prevented the macroeconomic literature from restricting itself to future inflation as the sole forward-looking determinant of inflation. This assumption can lead to strong policy prescriptions. For instance, the argument in favor of a conservative central banker. Quoting Clarida et al. (1999):

If price-setting depends on expectations of future economic conditions, then a central bank that can credibly commit to a rule faces an improved short-run trade-off between inflation and output. The solution under [a rule]⁸ in this case perfectly resembles the solution that would obtain for a central bank with discretion that assigned to inflation a higher cost than the true social cost.

The proposition stating that the central banker should be more conservative than the social preferences relies on the implicit assumption that inflation is the only variable whose expectation affect current inflation. We investigate in this paper how other instruments, whose expected values could also affect current inflation, should be used by central banks.

1.3.3 An example

At this stage it may help intuition to provide an example of such an extended New Keynesian Model, in the context of an open economy where the firms’ marginal costs depend on the current

⁷One case where this is found is when the interest rate enters the Phillips Curve by substituting it using the IS curve, but this is not the only situation where this could happen.

⁸Clarida et al. (1999) use the word “solution under commitment”, but this is meant as a synonym for “solution under a rule”.

interest rate, as in Ravenna and Walsh (2006). Capital controls, modeled along the lines of Farhi and Werning (2013), are available to the central bank as an unconventional instrument. The policy problem at any date t is to minimize the quadratic loss function:

$$\min_{\{i_t, \pi_{H,t}, y_t, \theta_t\}} \alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \theta_t^2 \quad (1.4)$$

subject to the Phillips Curve and the IS curve

$$\begin{aligned} \pi_{H,t} &= \beta \pi_{H,t+1}^e + \kappa_y y_t + \kappa_\theta^\pi \theta_t + \kappa_{\theta^e}^\pi \theta_{t+1}^e + \kappa_f i_t + u_t \\ y_t &= y_{t+1}^e - (i_t - \pi_{H,t+1}^e - \rho) + \kappa_\theta^y \theta_t + \kappa_{\theta^e}^y \theta_{t+1}^e \end{aligned}$$

and to the intertemporal budget constraint:

$$\sum_{t=0}^{\infty} \beta^t \theta_t = 0$$

where θ_t represents the wedge, due to capital controls, between the foreign and domestic interest rates (and thus between the foreign and domestic intertemporal marginal rates of substitution) —see Farhi and Werning (2013) for details. Substituting for the interest rate in the Phillips curve by using the IS curve leads to the dynamic equation:

$$\pi_{H,t} = (\beta + \kappa_f) \pi_{H,t+1}^e + (\kappa_y - \kappa_f) y_t + \kappa_f y_{t+1}^e + (\kappa_\theta^\pi + \kappa_f \kappa_\theta^y) \theta_t + (\kappa_{\theta^e}^\pi + \kappa_f \kappa_{\theta^e}^y) \theta_{t+1}^e + u_t$$

The financial friction introduces the expected terms y_{t+1}^e and θ_{t+1}^e in the dynamic behavior of inflation. We will show how this affects several of the main results in the monetary policy literature. Note, in addition, that the financial friction also increases the forward coefficient on inflation, because any increase in expected inflation lowers the real interest rate, and thus requires a hike in interest rate to stabilize output. This increase in the interest rate affects firms' costs and thus contemporaneous inflation.

Finally, the intertemporal budget constraint represents a No-Ponzi condition on the country's net foreign assets position. Capital controls, by imposing a wedge between domestic and foreign interest rates, distort the path of domestic consumption and thus the trade balance. The present value of this distortion must be zero.

1.4 The need for unconventional policy instruments

1.4.1 Equilibrium determinacy

We first analyze under which conditions equilibrium determinacy is guaranteed under discretionary policy. To do so, we solve the maximization problem and substitute optimal policies in equation (1.1) to assess the dynamics of π_H . The first-order conditions for y_t and θ_t are respectively:

$$y_t = -\alpha_\pi k_y \pi_{H,t} \quad (1.5)$$

$$\theta_t = -\frac{\alpha_\pi}{\alpha_\theta} k_\theta \pi_{H,t} - \frac{\Gamma}{\alpha_\theta} \quad (1.6)$$

Domestic inflation thus obeys the following law of motion:

$$\pi_{H,t} = \frac{k_\pi - \alpha_\pi \left(k_y k_y^e + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \pi_{H,t+1} - \frac{\left(\frac{k_\theta + k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \Gamma \quad (1.7)$$

Equation 1.5 shows that the optimal policy is to choose a positive level of inflation together with a negative output gap (or a negative level of inflation with a positive output gap) —otherwise, if the output gap and the inflation were positive, the central bank could reduce both by increasing the interest rate. In other words, the central bank “leans against the wind”, engineering a contraction if inflation is excessive. Similarly, for a given Lagrange multiplier, the unconventional instrument is used to moderate inflation. However, as the budget constraint becomes tighter, the use of the secondary instrument is restrained ($|\theta|$ falls). We also use equation (1.7) to determine the conditions for equilibrium determinacy.

Proposition 1. Equilibrium Determinacy under Discretionary Policy

Equilibrium determinacy is ensured when the Blanchard-Kahn condition is satisfied, i.e. when⁹

$$\alpha_\pi > \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\alpha_\theta}} \tag{1.8}$$

Proof: The proof simply consists in applying the Blanchard-Kahn condition to equation (1.7), i.e. verifying that:

$$\left| \frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \right| < 1$$

□

What are the conditions under which the model leads to indeterminacy? In the standard New Keynesian Model, $k_{y^e} = k_\theta = k_\theta^e = 0$ and $k_\pi = \beta < 1$. This implies that the denominator of the right hand side of (1.8) is positive, that its numerator is negative, and, since $\alpha_\pi > 0$, the Blanchard-Kahn condition is satisfied.¹⁰ Equilibrium determinacy is then guaranteed. In the general model, there are parametrizations for which the Blanchard-Kahn condition could be violated. An important situation where this could happen is when the financial friction is non-negligible ($k_f > 1 - \beta$ in equation (1.4)), and more generally when current inflation is strongly determined by expected inflation. In this case, the numerator in (1.8) becomes positive.

To understand the role of the second instrument in ensuring determinacy, it is useful to first understand the determinacy condition when the second instrument is not used. This is found by adding a constraint $\theta_t = 0$ to the minimization problem (1.2). Alternatively, this is obtained by assuming that the cost of using the secondary instrument is infinite, i.e. $\alpha_\theta \rightarrow +\infty$ (and, as a result, $\theta \rightarrow 0$). The determinacy condition is then:

$$\alpha_\pi > \frac{k_\pi - 1}{k_y(k_y + k_{y^e})}$$

⁹And $\alpha_\pi < \frac{k_\pi + 1}{k_y(k_{y^e} - k_y) + \frac{k_\theta}{\alpha_\theta}(k_\theta^e - k_\theta)}$ if $k_y(k_{y^e} - k_y) + \frac{k_\theta}{\alpha_\theta}(k_\theta^e - k_\theta) > 0$.

If the expected impact of output and the second instrument on inflation is larger than the current one (a case that would occur under some parametrization), optimal policy may result in expected inflation being stabilized more efficiently than current inflation, leading to indeterminacy. Even though such a situation appears counterintuitive (in particular inflation would change sign at each date), it can be avoided by ensuring that the weight on inflation α_π is not too high: the expected impact is then offset by the indexation of current inflation on expected inflation, ensuring determinacy. We omit this condition in the rest of this section.

¹⁰Moreover, the second condition, detailed in the previous footnote, does not apply since $k_{y^e} = k_{\theta^e} = 0$

Denoting by X_y the recession engineered by the central bank when inflation is 1 percent (i.e. $X_y = \alpha_\pi k_y > 0$, from equation 1.5), the first condition for equilibrium determinacy is :

$$(k_y + k_{y^e})X_y > k_\pi - 1$$

Intuitively, determinacy requires that the optimal decisions by the central bank are to engineer recessions such that the total impact on today's inflation $1 + (k_y + k_{y^e})X_y$ is stronger than the dynamic impact of expected inflation on today's inflation, k_π : this ensures that current inflation is low, ruling out multiple equilibria.

However, with a financial friction, the decision to increase the interest rate also affects the marginal cost in the Phillips curve (k_π increases; this is the cost channel of monetary policy). The recession must thus be deeper, or the sensitivity of inflation to the output gap higher, to ensure marginal costs are sufficiently reduced. If the weight of inflation in the loss function is too low, the recession engineered by the central bank may be insufficient to offset the impact of the financial friction on inflation. Current inflation may then be too high, and resulting in multiple equilibria.

We now reintroduce the second instrument, and define $X_\theta = \frac{\alpha_\pi}{\alpha_\theta} k_\theta > 0$ as the marginal increase in the optimal use of the unconventional instrument for a decrease in the level of inflation. The determinacy condition becomes:¹¹

$$(k_y + k_{y^e})X_y + (k_\theta + k_\theta^e)X_\theta > k_\pi - 1 \quad (1.9)$$

The rationale is as before. The optimal use of the new instrument (and its use in period $t + 1$) can mitigate current inflation, the more so if the effect of the instrument on today's inflation is high (i.e., k_θ and k_θ^e are high) and if the central bank uses this instrument aggressively (if X_θ is high). Figure 1.1 shows the zone of indeterminacy provided by conditions (1.8) and (1.9). When the use of the unconventional policy instrument comes at no cost ($\alpha_\theta = 0$, see left-hand chart), or when the the new instrument has a strong effect on inflation (X_θ is high, see right-hand chart) the risk of indeterminacy is eliminated, even if the central bank is not willing to engineer recessions. The downward sloping frontier in the right-hand chart clarifies the trade-off: for given impacts of the interest rate and conventional policy instruments, the central bank must either be willing to engineer large recessions or to be activist with the second instrument.

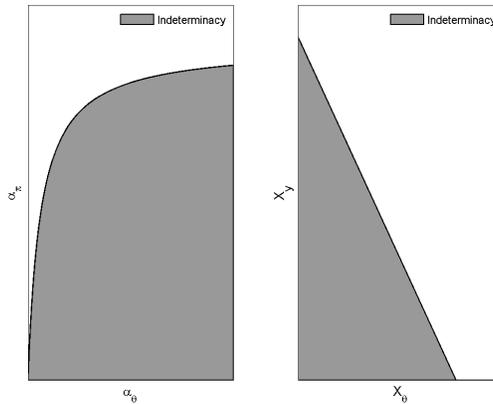


Figure 1.1: Optimal policy determinacy condition

¹¹A second condition is $\alpha_\pi < \frac{k_\pi + 1}{k_y(k_{y^e} - k_y) + \frac{k_\theta}{\alpha_\theta}(k_\theta^e - k_\theta)}$ if $k_y(k_{y^e} - k_y) + \frac{k_\theta}{\alpha_\theta}(k_\theta^e - k_\theta) > 0$.

1.4.2 Optimal stabilization policy following real shocks

In this section, we analyze the complementarity of policy tools by focusing on optimal stabilization policy after exogenous shocks. We assume that the model parameters are such that equilibrium determinacy is guaranteed. We thus focus on how the unconventional instrument is used in presence of a cost-push shock. Our objective is to find theoretical results, which is why we consider cost-push shocks that enter the Phillips Curve linearly; this allows us to obtain closed-form solutions. These cost-push shocks, which are common in the literature, can also capture financial disruption, as in e.g., Curdia and Woodford (2010). However, our results would stand for more general exogenous shocks that distort the Phillips Curve or the loss function.

Proposition 2. Optimal policy following cost-push shocks

Following a cost-push shock with autoregressive process $u_t = \rho_u^t u_0$, the optimal paths of inflation, output, and of the unconventional instruments are¹² :

$$\pi_{H,t} = \frac{1}{D(\rho_u)} u_0 \rho_u^t - \frac{k_\theta + k_\theta^e}{\alpha_\theta D(1)} \Gamma \quad , \quad y_t = -X_y \pi_{H,t} \quad , \quad \theta_t = -X_\theta \left[\rho_u^t - \frac{1-\beta}{1-\beta\rho} \right] u_0 \quad (1.10)$$

where¹³

$$D(\rho_u) = 1 - \rho_u k_\pi + \alpha_\pi \left[k_y (k_y + \rho_u k_{y^e}) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\alpha_\theta} \right]$$

Proof: The proof consists of iterating forward equation (1.7):

$$\pi_{H,t} = \sum_{i=0}^{\infty} \left(\frac{k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right)}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \right)^i \frac{1}{1 + \alpha_\pi \left(k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right)} \left(u_0 \rho_u^{t+i} - \frac{k_\theta + k_\theta^e}{\alpha_\theta} \Gamma \right)$$

We solve for the Lagrangian multiplier Γ by using the intertemporal constraint and the first-order condition $\theta_t = -X_\theta \pi_{H,t} + \frac{\Gamma}{\alpha_\theta}$, yielding:

$$\Gamma = \frac{\alpha_\theta (1-\beta)}{1-\rho\beta} \frac{X_\theta}{D(\rho_u) \left[1 - X_\theta \frac{k_\theta + k_\theta^e}{D(1)} \right]} u_0$$

□

Using the unconventional instrument enables the central bank to stabilize inflation and output more efficiently. The impact of the unconventional instrument is captured by the term $\frac{\alpha_\pi}{\alpha_\theta} k_\theta (k_\theta + \rho_u k_\theta^e)$ in $D(\rho_u)$. This formula is intuitive: the stabilization power of the second instrument is increasing in its current impact on inflation (coming from both current and expected actions), and is decreasing in the cost of using it.

As long as this term is positive, the impact of the cost-push shock on the economy is minimized thanks to the availability of the unconventional policy instrument.¹⁴ However, if the impact of the

¹³Note that $D(\rho_u) > 0 \Leftrightarrow \alpha_\pi > \frac{\rho_u k_\pi - 1}{k_y (k_y + \rho_u k_{y^e}) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\alpha_\theta}} \Leftrightarrow 1 + \alpha_\pi \left[k_y^2 + \frac{k_\theta^2}{\alpha_\theta} \right] > \rho_u \left[k_\pi - \alpha_\pi \left(k_y k_{y^e} + \frac{k_\theta k_\theta^e}{\alpha_\theta} \right) \right]$,

which is always true since the last inequality is verified for $\rho_u = 1$ in the Blanchard-Kahn condition (1.8) (and if the last bracket is negative, the result is trivial).

¹⁴This is always the case, for instance, if the future unconventional instrument does not enter the Phillips curve and the IS curve (in which case $k_\theta^e = 0$).

expected use of the instrument more than offsets the impact of its current use ($k_\theta(k_\theta + \rho_u k_\theta^e) < 0$), then it is preferable to commit not to use the secondary instrument. In that case, the availability of the secondary instrument makes the economy more volatile, and a commitment not to use that instrument may be welfare improving. This result is akin to that of Rogoff (1985a), who argued that international monetary policy coordination could affect inflation expectations and worsen the trade-off faced by central banks¹⁵.

The use of the unconventional instrument is however constrained by the intertemporal budget constraint (equation (1.3)). A tighter budget constraint (a higher Γ in absolute value in equation (1.10)) reduces the ability of policymakers to stabilize the economy.

1.5 Central banker's preferences

We analyzed above optimal policy assuming the central banker's and the social preferences coincide. However, the central bank's inability to commit to future policies restricts the space of feasible allocations, reduces its ability to stabilize the economy, and worsens social welfare. Kydland and Prescott (1977) and Barro and Gordon (1983) first showed how discretionary policy could lead to inefficient levels of inflation when the central bank targets a positive output gap (the *inflationary bias*). If the central bank cannot commit to future policies, it should thus target inflation more aggressively and tolerate a larger output gap in the current period in order to reduce inflation expectations, thus improving the trade-off characterized by the forward-looking Phillips Curve (Rogoff (1985b)). Clarida et al. (1999) extended this result by showing that even when the output objectives are realistic and the steady-state is efficient, the central bank could improve its short-run trade-offs by assigning to inflation a higher cost than the true social cost (the *stabilization bias*).

We investigate in this section which central banker's preferences (with respect to the weight on inflation and the unconventional policy instrument in the loss function) minimize welfare losses due to the stabilization bias and to the inflationary bias. Although alternative design strategies for central banks have been proposed (in particular in Walsh (1995) and in Svensson (1997)), we focus on preference weights for simplicity. We first explore the stabilization bias, and then present similar results for the inflationary bias (that may be seen as a particular case featuring a permanent shock).

1.5.1 The stabilization bias

If the weight that the central banker assigns to inflation is $\tilde{\alpha}_\pi$ and the weight on the unconventional instrument is $\tilde{\alpha}_\theta$, the central banker's objective is (using Proposition 2):¹⁶

¹⁵Rogoff's result may seem counter-intuitive. Since the central bank under coordination could always choose the same policies as it would under the Nash equilibrium, it would seem by revealed preferences that it could never be worse-off under the cooperative equilibrium than under the Nash. Likewise, here since the central bank could always choose not to use the second instrument, it would appear that its availability could never make the central bank worse off. In both examples, the revealed preferences argument breaks down because of the presence a forward-looking private sector, and the inability of the central bank to commit to future policies.

¹⁶In this section, we make the assumption that when the intertemporal constraint apply, its impact on optimal policy for inflation and output is small compared to the time-varying components of policy, i.e.: $\left| \frac{1}{D(\rho_u)} u_0 \right| \gg$

$\left| \frac{k_\theta + k_\theta^e}{\alpha_\theta D(1)} \Gamma \right| \Leftrightarrow 1 \gg \left| \frac{(k_\theta + k_\theta^e) X_\theta}{1 - k_\pi + \tilde{\alpha}_\pi \left[k_y (k_y + k_y^e) + \frac{k_\theta (k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta} \right]} \right| \frac{1 - \beta}{1 - \rho \beta}$. This assumption is valid for shocks that are transitory (where ρ is small enough). It is also easy to justify in a microfounded framework (Alla et al. (2015)).

$$W(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta) = \frac{\tilde{\alpha}_\pi + \tilde{\alpha}_\pi^2 k_y^2 + \alpha_\theta \frac{\tilde{\alpha}_\theta^2 k_\theta^2 \beta (1-\rho_u)^2}{(1-\beta\rho_u)^2}}{\tilde{D}(\rho_u, \tilde{\alpha}_\pi, \tilde{\alpha}_\theta)^2} \frac{u_0^2}{1 - \beta\rho_u^2}$$

where

$$\tilde{D}(\rho_u, \tilde{\alpha}_\pi, \tilde{\alpha}_\theta) = 1 - \rho_u k_\pi + \tilde{\alpha}_\pi \left[k_y(k_y + \rho_u k_y^e) + \frac{k_\theta(k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right]$$

The central banker who should be appointed is the the one whose preferences are:

$$\left\{ \tilde{\alpha}_\pi^{opt}, \tilde{\alpha}_\theta^{opt} \right\} = \operatorname{argmin} W(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta) \quad (1.11)$$

under the constraint that his preferences lead to equilibrium determinacy, i.e.

$$\tilde{\alpha}_\pi^{opt} > \frac{k_\pi - 1}{k_y(k_y + k_y^e) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{opt}}}$$

Proposition 3 present the solution assuming that social preferences remain in the area where equilibrium determinacy is guaranteed. Proposition 4, in the next section, next presents the solution for the ‘dual’ problem of minimizing the social cost function when the initial social preferences would be in an area of indeterminacy.

Proposition 3. A conservative and interventionist central banker

If the social preferences are such that equilibrium determinacy is guaranteed:

- (i) The central banker’s optimal preferences can *not* induce equilibrium indeterminacy ;
- (ii) When the shock is not highly persistent $\left(\rho_u < \frac{1}{k_\pi}\right)$, the central banker’s preferences that minimize welfare losses are:

$$\tilde{\alpha}_\pi = \frac{1 + \rho_u \frac{k_y^e}{k_y}}{1 - \rho_u k_\pi} \alpha_\pi \quad \tilde{\alpha}_\theta = \frac{1 + \rho_u \frac{k_y^e}{k_y} \beta (1 - \rho_u)^2}{1 + \rho_u \frac{k_\theta^e}{k_\theta} (1 - \beta\rho_u)^2} \alpha_\theta \quad ;$$

- (iii) If the shock is persistent enough $\left(\rho_u > \frac{1}{k_\pi}\right)$, the optimal preferences are:

$$\tilde{\alpha}_\pi = +\infty \quad \tilde{\alpha}_\theta = \frac{1 + \rho_u \frac{k_y^e}{k_y} \beta (1 - \rho_u)^2}{1 + \rho_u \frac{k_\theta^e}{k_\theta} (1 - \beta\rho_u)^2} \alpha_\theta$$

- (iv) When the intertemporal budget constraint (1.3) for the unconventional instrument θ is not applicable, the optimal weight for this instrument is:

$$\tilde{\alpha}_\theta = \frac{1 + \rho_u \frac{k_y^e}{k_y}}{1 + \rho_u \frac{k_\theta^e}{k_\theta}} \alpha_\theta$$

Proof: See section 1.7.1 in the Appendix.

□

Corollary 1. Optimal preferences

Using Proposition 3 it is possible to show how the optimal central bank's preferences deviate from social preferences:

- (i) $\tilde{\alpha}_\pi \geq \alpha_\pi$
- (ii) $\tilde{\alpha}_\pi$ is increasing in the persistence of the shocks and in the effect of future output ¹⁷ on current inflation k_y^e ;
- (iii) $\tilde{\alpha}_\theta < \alpha_\theta$ if $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$;
- (iv) $\tilde{\alpha}_\theta$ is decreasing in the persistence of the shock if $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$.

Proposition 3 first shows that the optimal central banker always improves credibility and economic stability in the following sense: if the social preferences are such that equilibrium determinacy is guaranteed, then determinacy is also guaranteed under optimal preferences. In addition, determinacy may be obtained under the optimal preferences even when the social preferences are in the indeterminacy area. In other words, when an unconventional instrument is available, the optimal central banker uses it to improve its short-run trade-off and in doing so, she reduces the possibility of indeterminacy.

Proposition 3 and its corollary also show that the weight given to inflation by the optimal central banker is higher than social preferences ($\tilde{\alpha}_\pi \geq \alpha_\pi$). The advantage of appointing a “conservative central banker” even when the target for the output gap is 0 was first explained in Clarida et al. (1999); because inflation depends on future output gaps, the central bank has always an incentive to promise strong future actions against inflation before reneging on its promises. As the private sector anticipates this, with rational expectations inflation is higher under discretionary policy than if the central bank could commit. A Rogoff conservative central banker can mitigate this bias. This result is valid in our more general framework.¹⁸ In addition, the more persistent the shock, or the stronger the effect of future output on inflation, the more averse to inflation the central banker should be (if the shocks are one-off, i.e. $\rho_u = 0$, then $\tilde{\alpha}_\pi = \alpha_\pi$ because expected inflation is always 0 and thus is unaffected by the commitment technology). The objective is indeed to tackle anticipations of inflation, and inflation expectations create inflation today (and the more so the higher k_π , for instance in presence of a financial friction). For very persistent shocks, when inflation is strongly influenced by expected inflation, the minimization problem (1.11) does not have an interior solution, and the optimal central banker is Mervyn King (1997)'s “inflation nutter”, as he cannot accept any deviation of inflation from his target. Finally, the optimal weight on inflation does not depend on the presence of the second instrument: indeed, the central banker's weight on inflation does not depend on the cost of using this instrument (α_θ) or on its impact in the IS curve or Phillips Curve.

Proposition 3 also determines what the optimal preferences for the unconventional instrument should be. The central bank should use the secondary instrument more actively than if it were following social preferences ($\tilde{\alpha}_\theta < \alpha_\theta$) if $\frac{k_y^e}{k_y} < \frac{k_\theta^e}{k_\theta}$. This condition is one where the effect of future unconventional policy on inflation (relative to current policy) is larger than the effect of future

¹⁷Which is equal to the financial friction coefficient for the example presented in subsection 1.3.3.

¹⁸Our results for $\tilde{\alpha}_\pi$ are the same as those in Clarida et al. (1999) when $k_y^e = 0$.

conventional policy (relative to current policy).¹⁹ This would be the case in the model of capital controls presented in section 1.3.3, for instance. Using the unconventional instrument aggressively enables the central banker to tackle expectations of high inflation, thus improving the short-run trade-off he faces. The optimal central banker should then not only be conservative, but also more interventionist with instruments whose future use affects current economic conditions more than its current use. Note also that the extent of deviation from social preferences for the use of θ (i.e. the ratio $\tilde{\alpha}_\theta/\alpha_\theta$) appears to be independent of the cost of inflation α_π .

Finally, findings (ii) and (iv) in Proposition 3 show that even when the shocks are one-off, the optimal weight for θ can be different from that of the social preferences because of the budget constraint (as mentioned earlier, the optimal weight for inflation $\tilde{\alpha}_\pi$ is equal to α_π when facing one-off shocks because there is no stabilization bias: expected inflation is always 0 independently from the policymaker's credibility). The difference between the solutions for $\tilde{\alpha}_\pi$ and for $\tilde{\alpha}_\theta$ comes from the intertemporal budget constraint $\sum_{t=0}^{\infty} \beta^t \theta_t$. Because of this constraint, even after one-off shocks, $\sum_{t=1}^{\infty} \beta^t \theta_t \neq 0$ since $\theta_0 \neq 0$. Since the unconventional instrument is used in the future even for one-off shocks, how it is used is important for today's inflation and thus it is possible to improve the inflation-output trade-off by choosing a central banker with preferences that differ from social preferences.

1.5.2 The stabilization bias when optimal preferences trigger multiple equilibria

The previous results were found assuming that under optimal preferences, equilibrium determinacy is guaranteed. But if this not the case, who should be appointed as central banker? Assuming that the social costs of indeterminacy are large enough that it needs to be ruled out altogether, the problem can be formalized as follows:

$$\left\{ \tilde{\alpha}_\pi^{copt}, \tilde{\alpha}_\theta^{copt} \right\} = \operatorname{argmin} W(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta) \quad (1.12)$$

subject to:

$$\tilde{\alpha}_\pi^{copt} > \frac{k_\pi - 1}{k_y(k_y + k_{ye}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{copt}}}$$

and knowing that:

$$\tilde{\alpha}_\pi^{opt} \leq \frac{k_\pi - 1}{k_y(k_y + k_{ye}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{opt}}}$$

Proposition 4. Optimal preferences in situations of equilibrium indeterminacy

If the optimal preferences described in Proposition 3 are indeterminate, then the optimal constrained choice $\left\{ \alpha_\pi^{copt}, \alpha_\theta^{copt} \right\}$:

- (i) is located on the determinacy frontier ;
- (ii) features a higher weight on inflation $\alpha_\pi^{copt} > \alpha_\pi^{opt}$;
- (iii) features a lower weight on the unconventional instrument $\alpha_\theta^{copt} < \alpha_\theta^{opt}$ if, and only if, $\frac{k_\theta^e}{k_\theta} > \frac{k_y^e}{k_y}$.

¹⁹Output is the reference since its weight is normalized to 1 in the objective function.

Proof: See section 1.7.2 in the Appendix. □

Proposition 4 shows that the optimal preferences are located on the determinacy frontier, to be as close as possible to social preferences. In addition, the optimal, constrained, choice always reinforces the central bank credibility, in the sense that it features a higher inflation weight, and a lower weight on the use unconventional instrument weight if and only if the effect of the future use of the instrument on today's inflation is strong enough. The intuition is similar to that of Proposition (3). If the central banker has a tool whose future use matters a lot, he should be more interventionist with the tool, even though the constraint on determinacy forces him to adopt "second-best" preferences.

1.5.3 The inflationary bias

Finally, we undertake a similar analysis to solve for the optimal central banker's preferences if the social welfare objective function targets a level of output \bar{y} that is higher than its steady state value (i.e in presence of the traditional inflationary bias). The social welfare loss is :

$$\min_{\{\pi_{H,t}, y_t, \theta_t\}} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\alpha_{\pi} \pi_{H,t}^2 + (y_t - \bar{y})^2 + \alpha_{\theta} \theta_t^2 \right]$$

subject to

$$\pi_{H,t} = k_{\pi} \pi_{H,t+1}^e + k_y y_t + k_{y^e} y_{t+1}^e + k_{\theta} \theta_t + k_{\theta^e} \theta_{t+1}^e$$

And the intertemporal budget constraint is:

$$\sum_{t=0}^{\infty} \beta^t \theta_t = 0$$

Proposition 5. The inflationary bias

Assume that the optimal preferences are determinate. We distinguish models in which there is an intertemporal budget constraints from models where this constraint does not apply.

(i) Assume that the tool is not constrained by the intertemporal budget constraint (the constraint (1.3) does not apply),

a. If current inflation depends on expected inflation ($k_{\pi} < 1$), the central banker's preferences that minimize welfare losses are:

$$\tilde{\alpha}_{\pi} = \frac{1 + \frac{k_y^e}{k_y}}{1 - k_{\pi}} \alpha_{\pi} \quad \tilde{\alpha}_{\theta} = \frac{1 + \frac{k_y^e}{k_y}}{1 + \frac{k_{\theta^e}}{k_{\theta}}} \alpha_{\theta} \quad ;$$

b. If current inflation strongly depends on expected inflation ($k_{\pi} > 1$), the central banker's preferences become:

$$\tilde{\alpha}_{\pi} = +\infty \quad \tilde{\alpha}_{\theta} = \frac{1 + \frac{k_y^e}{k_y}}{1 + \frac{k_{\theta^e}}{k_{\theta}}} \alpha_{\theta}$$

(ii) Assume that the tool is constrained intertemporally:

a. it is optimal not to use it, i.e. $\theta_t = 0$ (its weight is then irrelevant).

Proof: Similar to the proof of Proposition 3.

□

Since the problem is formally similar to that of the stabilization bias, the results and intuitions developed for Proposition 3 carry over. Item (ii) in Proposition 5 shows that the effect of the budget constraint on policy (captured by the term $(1 - \beta)/(1 - \beta\rho_u)$ in equation (1.10) is crucial. If the budget constraint is applicable to the problem at hand, when the shock is permanent ($\rho_u = 1$), the optimal use of the unconventional instrument would be constant, which is only possible if $\theta_t = 0$ given that $\sum_{t=0}^{\infty} \beta^t \theta_t = 0$. Intuitively, the intertemporal budget constraint on the second instrument, if applicable, means that any use today must be paid back by the opposite use in the future; hence, there is no purpose in using the instrument in the face of a permanent shock.

1.6 Concluding Remarks

According to the Tinbergen principle, a policymaker needs as many (independent) instruments as (independent) objectives in order to reach the desired equilibrium. In the New Keynesian Model, when the divine coincide occurs, the two objectives of zero inflation and zero output gap coincide, and one instrument (conventional monetary policy) is sufficient to stabilize the economy perfectly. In practice, policymaking is almost always more challenging than this result would suggest, because the divine coincide does not hold well; this situation is often captured in theoretical models by the presence of cost-push shocks. The optimal policy response when interest rate policy is the sole instrument available is then to maintain a positive output gap as long as inflation stays below target.

The depth of the global crisis, however, forced central banks to investigate the use of new instruments, either because interest rate policy was constrained (by the zero-lower bound, by fixed currency arrangements) or because new objectives arose (for financial stability, for asset prices, for the balance of payments or for exchange rates). The instruments used, which were chosen in order to attain as effectively as possible these new objectives, have included central bank balance sheet operations, sterilized forex intervention, macroprudential policy, fiscal devaluations, etc. The theoretical literature followed suit in justifying the use of such instruments in microfounded models.

However, this literature has not yet provided a consensus on when and how to use these instruments and how to coordinate their use with interest rate policy. A case in point is that of the central bank of Sweden, which split over the decision to use interest rate policy to reduce risks to financial stability (Svensson (2011), Svensson (2014)). Disagreements are fed by the lack of evidence on the effect of different instruments on the target variables and by the lack of models that address appropriately these issues.

The objective of this paper has been to contribute to this literature by addressing the issue of instruments and objectives in a relatively general but tractable framework of discretionary policy, and to investigate how some of the key results in the monetary policy literature (determinacy, inflationary bias, discretionary bias, conservative central banker) extend when additional instruments are available. We found that additional instruments are useful to ensure equilibrium determinacy and reduce economic volatility in presence of cost-push shocks, although under some specific parametrizations it is possible that committing not to use the unconventional instrument is efficient.

We also investigated whether the intuition of Rogoff (1985b)'s conservative central banker extends in the context of a model with several instruments. We found that if the future use of the unconventional instrument has relatively more importance for inflation than the future output gap, then the optimal central banker is more interventionist with the instrument than what social preferences would imply. In addition, we investigated how a conservative central banker could reduce the risk of equilibrium indeterminacy. Since the policy implications for this kind of theoretical results depend on the coefficients that parameterize the effects of current and future instruments on current inflation, we think it will be important for the empirical literature to delve further into the appropriate shape of the Phillips Curve and in particular the role of unconventional instruments in affecting economic activity and inflation.

1.7 Appendix

1.7.1 Proof of Proposition 3

Planning problem

The central banker has to solve the following problem to determine his optimal preferences:

$$W(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta) = \frac{\alpha_\pi + k_y^2 \tilde{\alpha}_\pi^2 + \alpha_\theta k_\theta^2 \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2} \frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}}{\left[1 - \rho_u k_\pi + \tilde{\alpha}_\pi \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right] \right]^2} \frac{u_0^2}{1 - \beta\rho_u^2}$$

subject to

$$\tilde{\alpha}_\pi > \frac{k_\pi - 1}{k_y (k_y + k_y^e) + \frac{k_\theta (k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta}}$$

We assume that the constraint is satisfied for the social preferences. We verify *ex post* that the constraint is also satisfied for the optimal preferences. We denote:

$$\tilde{W}(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}) = \frac{\alpha_\pi + k_y^2 \tilde{\alpha}_\pi^2 + \alpha_\theta k_\theta^2 \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2} \frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}}{\left[1 - \rho_u k_\pi + \tilde{\alpha}_\pi \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right] \right]^2} \frac{u_0^2}{1 - \beta\rho_u^2} = \frac{N}{D^2} \frac{u_0^2}{1 - \beta\rho_u^2}$$

where²⁰

$$N = \alpha_\pi + k_y^2 \tilde{\alpha}_\pi^2 + \alpha_\theta k_\theta^2 \frac{\beta(1-\rho_u)^2}{(1-\beta\rho_u)^2} \frac{\tilde{\alpha}_\pi^2}{\tilde{\alpha}_\theta^2}, \quad D = 1 - \rho_u k_\pi + \tilde{\alpha}_\pi \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right]$$

Optimal preferences

We then compute the partial derivatives:

$$\begin{aligned} \frac{\partial \tilde{W}(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta})}{\partial \tilde{\alpha}_\pi} &= 2 \frac{\left[k_y^2 + \frac{\alpha_\theta k_\theta^2 \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2 \tilde{\alpha}_\theta^2} \right] \tilde{\alpha}_\pi D - \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right] N}{D^3} \frac{u_0^2}{1 - \beta\rho_u^2} \\ &= 2 \frac{\left[k_y^2 + \frac{\alpha_\theta k_\theta^2 \beta(1-\rho_u)^2}{(1-\beta\rho_u)^2 \tilde{\alpha}_\theta^2} \right] [1 - \rho_u k_\pi] \tilde{\alpha}_\pi - \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right] \alpha_\pi}{D^3} \frac{u_0^2}{1 - \beta\rho_u^2} \end{aligned}$$

We need to consider two cases:

- if $\rho_u k_\pi < 1$, there is an interior point where the partial derivative $\frac{\partial \tilde{W}(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta})}{\partial \tilde{\alpha}_\pi}$ is equal to zero.
- if $\rho_u k_\pi > 1$, this derivative is negative for any value of $\{\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\}$, the optimal solution is then $\tilde{\alpha}_\pi = +\infty$. The welfare loss converges to a finite value since it is bounded from below by zero.

²⁰Since determinacy is ensured, $D > 0$.

The second partial derivative can be expressed as follows:

$$\begin{aligned} \frac{\partial \tilde{W} \left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta} \right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} &= 2 \frac{\tilde{\alpha}_\pi k_\theta \left[\frac{\alpha_\theta k_\theta \beta (1 - \rho_u)^2}{(1 - \beta \rho_u)^2} \frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} D - (k_\theta + \rho_u k_\theta^e) N \right]}{D^3} \frac{u_0^2}{1 - \beta \rho_u^2} \\ &= \frac{\tilde{\alpha}_\pi k_\theta \left[\frac{\alpha_\theta k_\theta \beta (1 - \rho_u)^2}{(1 - \beta \rho_u)^2} \frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} [1 - \rho_u k_\pi + k_y (k_y + \rho_u k_y^e) \tilde{\alpha}_\pi] - (k_\theta + \rho_u k_\theta^e) (\alpha_\pi + k_y^2 \tilde{\alpha}_\pi^2) \right]}{D^3} \frac{u_0^2}{1 - \beta \rho_u^2} \end{aligned}$$

If $1 > \rho_u k_\pi$, this second derivative necessarily admits an interior cancellation point. Let us first consider this case.

In this situation, each partial derivative cancels and changes signs in one point (for a given value of the other parameter). There is thus only one interior point in which the two derivatives cancel simultaneously. Since the also change signs in this point (from being negative to positive), this is the global minimum.

Using the partial derivatives formulations with N and D , we see that this interior point verifies:

$$\begin{aligned} \left[k_y^2 + \frac{\alpha_\theta k_\theta^2 \beta (1 - \rho_u)^2}{(1 - \beta \rho_u)^2 \tilde{\alpha}_\theta^2} \right] \tilde{\alpha}_\pi D &= \left[k_y (k_y + \rho_u k_y^e) + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{\tilde{\alpha}_\theta} \right] N \\ \frac{\alpha_\theta k_\theta^2 \beta (1 - \rho_u)^2}{(1 - \beta \rho_u)^2} \frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} D &= (k_\theta + \rho_u k_\theta^e) N \end{aligned}$$

By dividing the two equations, we find that:

$$\tilde{\alpha}_\theta^{opt} = \frac{(k_y + \rho_u k_y^e) k_\theta \beta (1 - \rho_u)^2}{k_y (k_\theta + \rho_u k_\theta^e) (1 - \beta \rho_u)^2} \alpha_\theta = \frac{1 + \rho_u \frac{k_y^e}{k_y} \beta (1 - \rho_u)^2}{1 + \rho_u \frac{k_\theta^e}{k_\theta} (1 - \beta \rho_u)^2} \alpha_\theta;$$

We then substitute for $\tilde{\alpha}_\theta^{opt}$ in any of the above equations, and find that the optimal choice for inflation is:

$$\tilde{\alpha}_\pi^{opt} = \frac{1 + \rho_u \frac{k_y^e}{k_y}}{1 - \rho_u k_\pi} \alpha_\pi;$$

If $1 < \rho_u k_\pi$, we saw that the optimal choice for the inflation coefficient is $\tilde{\alpha}_\pi^{opt} = +\infty$. Using the second equality for the partial derivative²¹ $\frac{\partial \tilde{W}(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta})}{\partial \frac{1}{\tilde{\alpha}_\theta}}$, we find that the optimal choice for $\tilde{\alpha}_\theta^{opt}$ verifies²²:

$$\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi) = \frac{\frac{\alpha_\theta k_\theta \beta (1 - \rho_u)^2}{(1 - \beta \rho_u)^2} \tilde{\alpha}_\pi [1 - \rho_u k_\pi + k_y (k_y + \rho_u k_y^e) \tilde{\alpha}_\pi]}{(k_\theta + \rho_u k_\theta^e) (\alpha_\pi + k_y^2 \tilde{\alpha}_\pi^2)} \xrightarrow{\tilde{\alpha}_\pi \rightarrow +\infty} \frac{1 + \rho_u \frac{k_y^e}{k_y} \beta (1 - \rho_u)^2}{1 + \rho_u \frac{k_\theta^e}{k_\theta} (1 - \beta \rho_u)^2} \alpha_\theta$$

²¹Since it cancels out only once when $\tilde{\alpha}_\pi$ is large enough

²²Formally, for any couple $\{\tilde{\alpha}_\pi, \tilde{\alpha}_\theta\}$ we have:

$$\tilde{W} \left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta} \right) > \tilde{W} \left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)} \right)$$

The coefficient for the unconventional tool is then unchanged, and the optimal choice is:

$$\tilde{\alpha}_\pi^{opt} = +\infty \quad , \quad \tilde{\alpha}_\theta^{opt} = \frac{1 + \rho_u \frac{k_y^e}{k_y} \beta (1 - \rho_u)^2}{1 + \rho_u \frac{k_\theta^e}{k_\theta} (1 - \beta \rho_u)^2} \alpha_\theta;$$

Optimal preferences determinacy

Let us finally prove that if the determinacy constraint is satisfied for the social preferences $\{\alpha_\pi, \alpha_\theta\}$, it is then satisfied for the optimal preferences chosen by the central banker.

Given that the frontier is concave (see Figure 1.1 and equation (1.8)), and since $\alpha_\pi^{opt} \geq \alpha_\pi$, we see that if $\alpha_\theta^{opt} \leq \alpha_\theta$, then the optimal preferences are also determined.

We then consider the case when the unconventional instrument is less forward-looking than output (i.e. $\frac{k_y^e}{k_y} > \frac{k_\theta^e}{k_\theta}$, potentially inducing an optimal cost that is higher than the social one. The slope of the optimal deviation is then:

$$S = \frac{\alpha_\pi^{opt} - \alpha_\pi}{\alpha_\theta^{opt} - \alpha_\theta} = \frac{\frac{k_y^e}{k_y} + k_\pi}{\left(\frac{k_y^e}{k_y} - \frac{k_\theta^e}{k_\theta}\right) (1 - \rho_u k_\pi)} \frac{\alpha_\pi}{\alpha_\theta} \geq \frac{\alpha_\pi}{\alpha_\theta}$$

We want to compare this slope to the frontier derivative for $\alpha_\theta = \tilde{\alpha}_\theta$. Since the frontier is strictly concave, if S is greater than its derivative, the optimal preferences are in the determinacy area. Figure 1.2 illustrated the proof.

The frontier can be parametrized as follows:

$$\tilde{\alpha}_\pi^{fr}(\tilde{\alpha}_\theta) = \frac{a\tilde{\alpha}_\theta}{1 + b\tilde{\alpha}_\theta}$$

where $a = \frac{k_\pi - 1}{k_\theta(k_\theta + k_\theta^e)}$ and $b = \frac{k_y(k_y + k_y^e)}{k_\theta(k_\theta + k_\theta^e)}$. Its derivative for $\tilde{\alpha}_\theta = \alpha_\theta$ is then equal to $D = \frac{a}{(1 + b\tilde{\alpha}_\theta)^2}$. Since the social preferences are located above the determinacy frontier, we have:

$$\alpha_\pi \geq \frac{a\alpha_\theta}{1 + b\alpha_\theta}$$

We finally get that:

$$S \geq \frac{\alpha_\pi}{\alpha_\theta} \geq \frac{a}{1 + b\tilde{\alpha}_\theta} \geq \frac{a}{(1 + b\tilde{\alpha}_\theta)^2} = D$$

Then, denoting $g(\tilde{\alpha}_\pi) = \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)$, we have:

$$g'(\tilde{\alpha}_\pi) = \frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \tilde{\alpha}_\pi} + \frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} \frac{\partial \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}}{\partial \tilde{\alpha}_\pi} = \frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \tilde{\alpha}_\pi}$$

by definition of $\frac{\partial \tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta^{opt}(\tilde{\alpha}_\pi)}\right)}{\partial \frac{1}{\tilde{\alpha}_\theta}}$. We then see that $g(\tilde{\alpha}_\pi) > \lim_{\tilde{\alpha}_\pi \rightarrow \infty} g(\tilde{\alpha}_\pi)$, e.g.:

$$\tilde{W}\left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta}\right) > \tilde{W}\left(+\infty, \frac{1}{\tilde{\alpha}_\theta(+\infty)}\right).$$

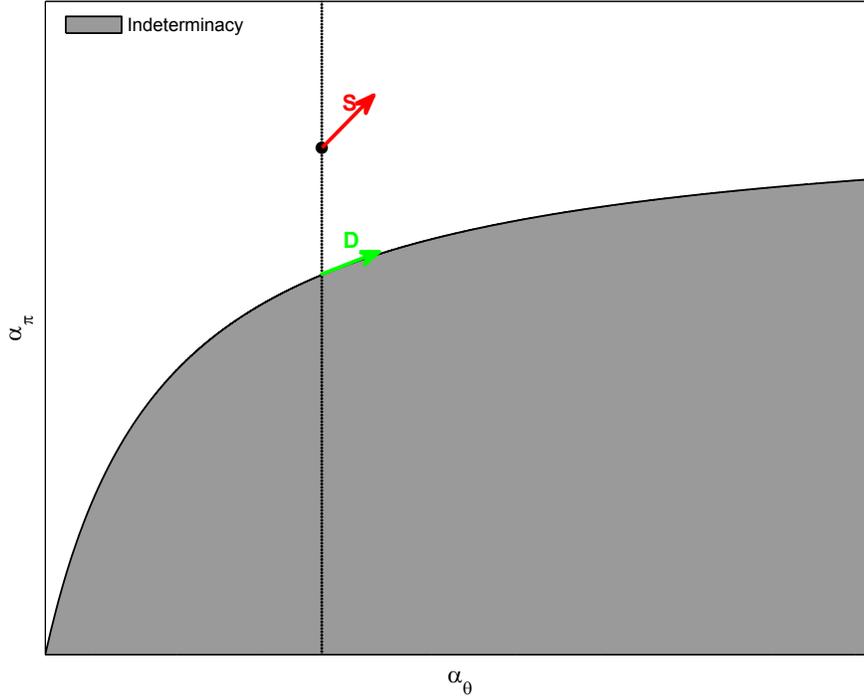


Figure 1.2: Optimal preferences determinacy

This proves that if the social preferences are determinate, the optimal ones are too. In this sense, the optimal central banker preferences strengthen its credibility.

1.7.2 Proof of Proposition 4

We consider that the optimal choice, as defined in section 1.7.1, leads to indeterminacy, e.g.:

$$\tilde{\alpha}_\pi^{opt} \leq \frac{k_\pi - 1}{k_y(k_y + k_{y^e}) + \frac{k_\theta(k_\theta + k_\theta^e)}{\tilde{\alpha}_\theta^{opt}}} \quad (1.13)$$

The determinacy constraint (1.8) assumes that the inflation weight should be strictly above the frontier. However, we show below that the solution to the problem that includes the border is unique, and located on the border.

It is then easy to see that the solution to the strict inequality problem will be in the neighbourhood of the above point (there would be no solution *per se*, but a sequence converging to this point). We will then consider that the solution to the problem (1.12) is located on the border.

Solution location

Let us first prove that the solution to the constrained problem is located on the determinacy frontier. To that end, we reformulate the partial derivatives:

$$\frac{\partial \tilde{W} \left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta} \right)}{\partial \tilde{\alpha}_\pi} = 2 \frac{k_y^2}{1 - \rho_u k_\pi} \frac{\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt} + \frac{k_\theta (k_\theta + \rho_u k_\theta^e)}{k_y (k_y + \rho_u k_y^e)} \left[\frac{\tilde{\alpha}_\theta^{opt}}{\tilde{\alpha}_\theta} \tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt} \right]}{D^3} \frac{u_0^2}{1 - \beta \rho_u^2}$$

We then see that if $\tilde{\alpha}_\pi > \tilde{\alpha}_\pi^{opt}$ and $\tilde{\alpha}_\pi > \frac{\tilde{\alpha}_\theta}{\tilde{\alpha}_\theta^{opt}} \tilde{\alpha}_\pi^{opt}$, the welfare loss is strictly increasing with $\tilde{\alpha}_\pi$. Similarly,

$$\frac{\partial \tilde{W} \left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta} \right)}{\partial \frac{1}{\tilde{\alpha}_\theta}} = 2 \frac{\tilde{\alpha}_\pi k_\theta k_y (k_\theta + \rho_u k_\theta^e)}{k_y + \rho_u k_y^e} \frac{\left(\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} \tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt} + k_y^2 \tilde{\alpha}_\pi^2 \tilde{\alpha}_\pi^{opt} \left[\frac{\tilde{\alpha}_\theta^{opt}}{\tilde{\alpha}_\theta} - 1 \right] \right)}{D^3} \frac{u_0^2}{1 - \beta \rho_u^2}$$

Since $\frac{\partial W(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta)}{\partial \tilde{\alpha}_\theta} = -\tilde{\alpha}_\theta^2 \frac{\partial \tilde{W} \left(\tilde{\alpha}_\pi, \frac{1}{\tilde{\alpha}_\theta} \right)}{\partial \frac{1}{\tilde{\alpha}_\theta}}$, the welfare loss is strictly decreasing (resp. increasing) with $\tilde{\alpha}_\theta$ when $\tilde{\alpha}_\pi > \frac{\tilde{\alpha}_\pi^{opt}}{\tilde{\alpha}_\theta^{opt}} \tilde{\alpha}_\theta$ (resp. $\tilde{\alpha}_\pi < \frac{\tilde{\alpha}_\pi^{opt}}{\tilde{\alpha}_\theta^{opt}} \tilde{\alpha}_\theta$) and $\tilde{\alpha}_\theta < \tilde{\alpha}_\theta^{opt}$ (resp. $\tilde{\alpha}_\theta > \tilde{\alpha}_\theta^{opt}$).

To get some intuition, let us represent graphically the above dynamics. The red arrows represent the gradient of $W(\tilde{\alpha}_\pi, \tilde{\alpha}_\theta)$ along its partial derivatives.

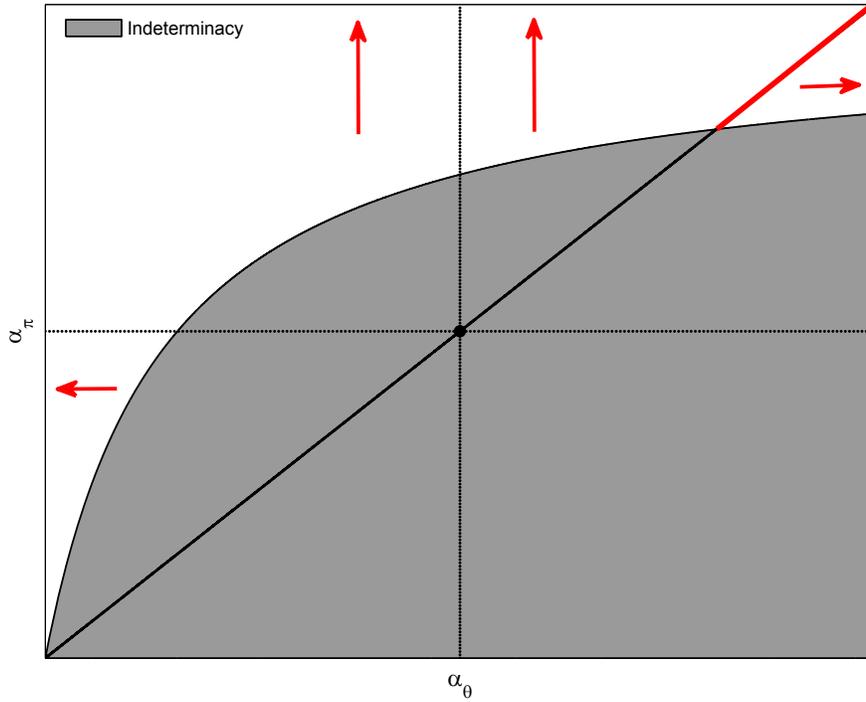


Figure 1.3: Welfare loss variations in the determinacy area

We then see that if the optimal preferences are located below the curve²³, starting from any point located above the frontier, it is optimal to move along a direction that brings you back to the frontier or to the red part of the line passing through the origin and the optimal point.

Along this line, denoting its slope $S^{opt} = \frac{\tilde{\alpha}_\pi^{opt}}{\tilde{\alpha}_\theta^{opt}}$ and the ratio $R = \frac{k_\theta(k_\theta + \rho_u k_\theta^c)}{k_y(k_y + \rho_u k_y^c)}$, the welfare loss can be expressed as follows:

$$W(\tilde{\alpha}_\pi) = \frac{\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^2}{\left[\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\tilde{\alpha}_\pi\right]^2}$$

This function derivative is:

$$W'(\tilde{\alpha}_\pi) = \frac{\left(\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt}\right)\left(\tilde{\alpha}_\pi - \tilde{\alpha}_\pi^{opt}\right)}{\left[\frac{\alpha_\pi}{k_y^2} + RS^{opt}\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}\tilde{\alpha}_\pi\right]^3}$$

We see that the welfare loss is strictly increasing along this ray for $\tilde{\alpha}_\pi > \tilde{\alpha}_\pi^{opt}$. It is then optimal to get back to the frontier on the red part of the line too.

We then proved that the solution to the problem featuring a lower or equal sign is located on the determinacy border.

Solution determination

Since the solution of the constrained problem is located on the determinacy frontier, using the frontier parametrization introduced in Appendix 1.7.1, the optimal parameters are linked by the following relation:

$$\frac{\tilde{\alpha}_\pi}{\tilde{\alpha}_\theta} = a - b\tilde{\alpha}_\pi \quad (1.14)$$

Using the above notations, the welfare loss can then be expressed as follows:

$$W(\tilde{\alpha}_\pi) = \frac{\frac{\alpha_\pi}{k_y^2} + R\tilde{\alpha}_\theta^{opt}(a - b\tilde{\alpha}_\pi)^2 + \tilde{\alpha}_\pi^2}{\left[\frac{\alpha_\pi}{k_y^2} + \tilde{\alpha}_\pi^{opt}[\tilde{\alpha}_\pi + R(a - b\tilde{\alpha}_\pi)]\right]^2} = \frac{\frac{\alpha_\pi}{k_y^2} + R\tilde{\alpha}_\theta^{opt}a^2 - 2abR\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi + \left(1 + Rb^2\tilde{\alpha}_\theta^{opt}\right)\tilde{\alpha}_\pi^2}{\left[\frac{\alpha_\pi}{k_y^2} + aR\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}(1 - bR)\tilde{\alpha}_\pi\right]^2}$$

Its derivative is then equal to:

$$W'(\tilde{\alpha}_\pi) = \frac{\left[\frac{\alpha_\pi}{k_y^2}\left(1 + Rb^2\tilde{\alpha}_\theta^{opt}\right) + aR\tilde{\alpha}_\pi^{opt}\left(1 + b\tilde{\alpha}_\theta^{opt}\right)\right]\tilde{\alpha}_\pi - \left[\frac{\alpha_\pi}{k_y^2}\left[\tilde{\alpha}_\pi^{opt} + bR\left(a\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt}\right)\right] + a^2R\tilde{\alpha}_\theta^{opt}\tilde{\alpha}_\pi^{opt}\right]}{\left[\frac{\alpha_\pi}{k_y^2} + aR\tilde{\alpha}_\pi^{opt} + \tilde{\alpha}_\pi^{opt}(1 - bR)\tilde{\alpha}_\pi\right]^3}$$

²³Since the constraint frontier is concave and the optimal point is located below the frontier, the line passing through the origin and the optimal point cuts the frontier once for $\tilde{\alpha}_\theta > \tilde{\alpha}_\theta^{opt}$.

The cancellation point, that corresponds to the constrained optimal, is then unique and defined by:

$$\tilde{\alpha}_\pi^{copt} = \frac{\frac{\alpha_\pi}{k_y^2} \left[\tilde{\alpha}_\pi^{opt} + bR \left(a\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt} \right) \right] + a^2 R \tilde{\alpha}_\theta^{opt} \tilde{\alpha}_\pi^{opt}}{\frac{\alpha_\pi}{k_y^2} \left(1 + Rb^2 \tilde{\alpha}_\theta^{opt} \right) + aR \tilde{\alpha}_\pi^{opt} \left(1 + b\tilde{\alpha}_\theta^{opt} \right)}$$

We want to compare this constrained optimal to the unconstrained optimal choice. After some algebra, we get:

$$\tilde{\alpha}_\pi^{copt} - \tilde{\alpha}_\pi^{opt} = \frac{R \left(b \frac{\alpha_\pi}{k_y^2} + a\tilde{\alpha}_\pi^{opt} \right) \left[a\tilde{\alpha}_\theta^{opt} - \left(1 + b\tilde{\alpha}_\theta^{opt} \right) \tilde{\alpha}_\pi^{opt} \right]}{\frac{\alpha_\pi}{k_y^2} \left(1 + Rb^2 \tilde{\alpha}_\theta^{opt} \right) + aR \tilde{\alpha}_\pi^{opt} \left(1 + b\tilde{\alpha}_\theta^{opt} \right)}$$

Since the optimal preferences are indeterminate, following equation (1.13), we have:

$$a\tilde{\alpha}_\theta^{opt} > \left(1 + b\tilde{\alpha}_\theta^{opt} \right) \tilde{\alpha}_\pi^{opt}$$

The optimal constrained inflation choice is then always above the optimal unconstrained point.

We now want to determine the location of the constrained optimum for the unconventional instrument. Using the frontier equation (1.14) and the above formula for $\tilde{\alpha}_\pi^{sb}$, we get after some algebra²⁴:

$$\tilde{\alpha}_\theta^{copt} = \frac{\frac{\alpha_\pi}{k_y^2} \left[\tilde{\alpha}_\pi^{opt} + bR \left(a\tilde{\alpha}_\theta^{opt} - \tilde{\alpha}_\pi^{opt} \right) \right] + a^2 R \tilde{\alpha}_\theta^{opt} \tilde{\alpha}_\pi^{opt}}{\frac{\alpha_\pi}{k_y^2} \left[a + b(bR - 1)\tilde{\alpha}_\pi^{opt} \right] + a^2 R \tilde{\alpha}_\pi^{opt}}$$

We finally compute the difference between the constrained optimal and the unconstrained optimal for the unconventional instrument. We get:

$$\tilde{\alpha}_\theta^{copt} - \tilde{\alpha}_\theta^{opt} = \frac{\frac{\alpha_\pi}{k_y^2} (bR - 1) \left[a\tilde{\alpha}_\theta^{opt} - \left(1 + b\tilde{\alpha}_\theta^{opt} \right) \tilde{\alpha}_\pi^{opt} \right]}{\frac{\alpha_\pi}{k_y^2} \left[a + b(bR - 1)\tilde{\alpha}_\pi^{opt} \right] + a^2 R \tilde{\alpha}_\pi^{opt}}$$

$bR = \frac{1 + \rho_u \frac{k_y^e}{k_y}}{1 + \rho_u \frac{k_\theta^e}{k_\theta}}$ is simply the ratio of output and unconventional instrument forward-looking impacts.

When it is smaller (resp. larger) than one, e.g. the unconventional instrument is more (resp. less) forward-looking than output, the constrained optimum uses more (resp. less) aggressively this instrument.

²⁴Since $a > b\tilde{\alpha}_\pi^{opt}$, the denominator is always strictly positive.

Chapter 2

Foreign Exchange Interventions in the New Keynesian Model

2.1 Abstract

We develop an open economy New Keynesian Model to assess the role of foreign exchange intervention in the presence of a financial accelerator mechanism. We first explore optimal discretionary policy following shocks to risk premia in international capital markets and explain how foreign exchange intervention can help reduce the volatility of the economy and welfare losses associated with such shocks. We also show how foreign exchange intervention can help reduce the zone of equilibrium indeterminacy, which we interpret as the zone where speculative attacks are possible. Indeed, although equilibrium is unique in the New Keynesian framework, multiple equilibria can occur for a sufficiently strong financial accelerator (this result is akin to that of third generation models of currency crises, see e.g. Aghion et al. (2000)). We determine the conditions under which indeterminacy occur and highlight how the use of foreign exchange intervention reinforces the central bank's credibility and limits such risk.

2.2 Introduction

Capital account liberalization and rising capital mobility over the last 30 years has made exchange rate management increasingly difficult. The Mundellian trilemma —the theoretical impossibility of fixing the exchange rate and maintaining monetary autonomy when the capital account is open— has indeed become more acute as countries removed restrictions to capital mobility, but a large number of central banks around the world have maintained an objective to achieve exchange rate stability.¹ And even countries with flexible exchange rates have found that financing conditions tended to be dictated by global conditions (especially US monetary policy and risk aversion), leading Rey (2015) to refer to the open economy *dilemma* (open capital account vs. independent monetary policy) as opposed to the Mundellian *trilemma*.

Although capital controls have been discussed as a possible answer and have attracted the interest of researchers and policy institutions (e.g., Korinek (2011), Ostry et al. (2011), IMF (2012)), the two main instruments commonly used by small economies' central banks to influence credit conditions remain conventional monetary operations (the policy rate, or for those central banks

¹Around 30 percent of countries can be considered, under either the *de jure* or the *de facto* criteria to manage their exchange rates.

that have not yet moved to interest rate targeting, domestic credit operations) and balance sheet operations, mostly in the form of sterilized foreign exchange (FX) intervention, which consists in purchasing or selling foreign exchange, but offsetting any resulting effect on the money supply and the interest rate via open market operations.

The purpose of this paper is to examine the role of FX intervention as a tool for exchange rate stability, inflation stability, and in general macroeconomic stability when conventional monetary policy is insufficient and FX intervention is effective. In standard New Keynesian Models (e.g., Gali and Monacelli (2005)), the policy interest rate suffices because the two objectives of stable inflation and zero output gap coincide (the ‘divine coincidence’). But in the presence of additional frictions, the divine coincidence breaks down, thus providing a rationale for the use of additional tools such as foreign exchange intervention. Although macroprudential policy and unconventional monetary policy instruments are also being considered, foreign exchange intervention is a natural instrument to use for small open economies. Indeed, central banks in such countries have shown a fear of floating (Reinhart and Calvo (2002)) and used sterilized intervention even as they moved towards inflation targeting.

Although such traditional fear of floating has been associated with emerging markets’ central banks lacking credibility and worries about excessive depreciation or excessive appreciation, it is also pertinent to advanced countries. Particularly in the aftermath of the global financial crisis, several advanced economy central banks have, at least on occasion, considered that the exchange rate could be used as a policy instrument or that movements in the value of their currency were unrelated to fundamentals and needed to be stabilized. The recent cases of the Czech Republic and of Switzerland are instructive as they provide examples of FX intervention being used to help stabilize inflation in the presence of real shocks (in the Czech Republic) and of FX intervention being used to limit the risk of self-fulfilling exchange rate movements (in Switzerland).

In September 2012, the Board of the Czech National Bank signaled that it was considering using the exchange rate as an additional monetary instrument, as the central bank faced a combination of low growth and negative core inflation, capital outflows and weak credit, thus highlighting the limits of conventional monetary policy, especially as the policy interest rate was near zero (IMF (2013)). The Czech National Bank announced a target exchange rate in November 2013 and based its exchange rate target on a specific calibration of the effect of the exchange rate on aggregate demand and inflation. Clear communication was crucial in making FX intervention effective and the exchange rate soon hit the target rate, contributing to a boost in inflation and aggregate demand (Alichi et al. (2015)).

In Switzerland, as the Euro crisis reduced investors’ appetite for Euro-denominated assets, financial flows to Switzerland threatened to appreciate the Swiss Franc above the ceiling announced in 2011 by the Swiss National Bank. The central bank backed the announcement with aggressive intervention, leading reserves of the Swiss National Bank to swell above US\$ 500 billion in 2013, more than twice the amount of reserves held in 2011 (IMF (2014)). Although the policy was initially effective in mitigating capital inflows and stabilizing the exchange rate, a renewed wave of capital inflows at the end of 2014 (driven by poor Euro area prospects and the ECB quantitative easing program) made the situation untenable: not only was reserve accumulation becoming very costly, but there were also fears that markets were anticipating appreciation and that such expectations would fuel either a spiral of appreciation or ballooning reserves at the central bank. In short, one policy instrument was not enough to contain speculative flows and the Swiss National Bank decided

to cut its policy rate from -0.25 to -0.75 percent in January 2015.²

The objective of this paper is to analyze, in a New Keynesian Model, the conditions under which FX intervention can help conventional monetary policy reduce economic volatility in the presence of real shocks or when there is a risk of self-fulfilling currency movements. The paper is organized as follows. Section 2.3 discusses the relevant literature. Section 2.4 presents the open economy model, in particular the mechanics of FX intervention, as well as a log-linearized version of the model. Section 2.5 describes optimal stabilization policy following shocks to the risk premium in international capital markets, and explains how the use of foreign exchange intervention can help reduce the volatility of the economy. Section 2.6 shows that FX intervention can reduce the risk of equilibrium indeterminacy. Section 2.7 provides some concluding remarks.

2.3 Related Literature

Recent literature on the use and accumulation of central bank reserves has been influenced by the dramatic rise in reserve holdings by emerging market central banks, with median holdings rising from some 3 percent of GDP in 1990 to 20 percent of GDP by 2010.³ While some of the reserve accumulation may have been a byproduct of mercantilist policies (e.g., Durdu et al. (2009)), precautionary motives may have also been at play (Ghosh et al. (2017)).

For sterilization intervention to work, it is necessary that the uncovered interest parity be violated, which may happen through several channels: (i) *the portfolio balance channel*, whereby sterilized intervention modifies the stock of debt held by markets and thus the risk premium (e.g., Dominguez and Frankel (2005)); (ii) *the signaling channel*, in which central bank “puts its money where its mouth is” is one way to communicate to markets the central bank’s views on the FX market (e.g. Ghosh (1992))⁴; and (iii) *the market microstructure channel*, where market participants’ trading decisions, especially when technical analysis is used, can be affected by the immediate impact of central bank purchases (e.g., Evans and Lyons (2002)). The effectiveness of these different channels will depend on country characteristics. For instance, for countries with smaller public debt markets, the portfolio balance channel would be more likely to be effective.

There is a large empirical literature on the effectiveness of intervention, and it is beyond the scope of this paper to review it here.⁵ Macroeconomic models have usually assumed that the portfolio balance channel is operative, following on the tradition of Kouri (1976) and Branson and Henderson (1985), and to some extent because modeling the other channels (credibility channel and market micro-structure channel) would involve departing significantly from the standard macro framework.

²Of course, similar stories could be told of central banks that attempted to limit depreciation using FX intervention and eventually had to raise rates. In fact, most central banks have followed such strategies in the major currency crises — e.g. Mexico in 1994; Thailand in 1997; Brazil in 1998; Russia in 1998; etc — although with limited success.

³See for instance Jeanne (2007), Barnichon (2009), and Jeanne and Ranciere (2011) who interpret reserves as precautionary savings, which would be used by a country in the event of a sudden stop. In these models, the economies are ‘real economies’ without a monetary sector.

⁴A central bank buying foreign currency would make losses if its domestic currency subsequently appreciates.

⁵Surveys include Sarno and Taylor (2001) and Neely (2005). Disyatat and Galati (2007) and Menkhoff (2013) are more recent surveys that focus on emerging markets. The recent literature that uses intraday data has been in general supportive of the effect of intervention on the exchange rate, at least in the short-term (Melvin et al (2009), Dominguez (2006)), although the significance of intervention is likely to depend on the communication policy of the central bank. For instance, Dominguez et al. (2013) find that irregular, discretionary, interventions had no impact, whereas regular sales of reserves did.

Most of the literature has simply assumed some form of financial frictions that make imperfect asset substitutability hold. This is how Schmitt-Grohe and Uribe (2003) close a linearized open economy model, for instance. Blanchard et al. (2005) also assumes that the risk premium is as a function of the share of different currencies in the US investor portfolio, and Benes et al. (2011) include sterilized intervention in a new Keynesian model assuming the portfolio balance channel is working. This portfolio balance channel is likely to be mostly relevant for emerging markets, where the domestic bond markets are small enough that risk premia can be affected by intervention, and where the signaling channel is weaker because central banks have less credibility (e.g., Domaç and Mendoza (2002)).⁶

Critiques of this approach, in particular Backus and Kehoe (1989), have noted that sterilized intervention would have no effect on the risk premium if the intervention has no fiscal implications beyond the currency composition of debt, even taking into account imperfect asset substitutability. This finding had a strong influence on the literature, contributing to the smaller role of FX intervention in open economy macro models (Blanchard et al. (2005)). However, there are several theoretical reasons why FX intervention could nonetheless matter. Kumhof (2010) noted that the Backus and Kehoe (1989)'s result relies on the assumption that any monetary and fiscal policy is available, whereas in practice fiscal policy tends to either be exogenous or to follow rules. Gabaix and Maggiori (2015) explicitly modeled imperfect international financial intermediation and showed how FX intervention that changes the balance sheet of financial intermediaries is effective. Cavallino (2016) integrates this model in a New Keynesian open economy framework to discuss the role of FX intervention in stabilizing shocks. Since our paper's contribution is on the dynamic implication of FX intervention for monetary theory, rather than on its finance micro-foundation, we stick to the simpler macroeconomic tradition of Branson and Henderson (1985) and Blanchard et al. (2005), and do not further analyze the foundation of the portfolio balance channel.

A risk premium is not enough to ensure FX intervention is needed or effective. As in Gabaix and Maggiori (2015) and Cavallino (2016), we also assume a financial friction to violate the divine coincidence and the UIP. To break the divine coincidence, we assume that exogenous shocks to foreigners' preference (as in Farhi and Werning (2014)) change the cost of borrowing of the Home country. These shocks drive a wedge in the Backus-Smith condition, i.e., a wedge between the consumption plans of Home households and Foreign households (e.g., a higher risk premium lowers consumption at Home) and such volatility reduces welfare. FX intervention can mitigate this wedge in our model if it affects the discount rate of Home households and Foreign households differently. To this end, we assume that Home households cannot invest large amounts abroad (this breaks their ability to run carry trades) and that the risk premium due to the portfolio balance channel is perceived differently by foreign investors and domestic investors. For foreign investors, changes in the risk premium originating from FX intervention have no effect on the discount rate since these sophisticated investors, who are used to take into account the risk of debt restructuring, internalize that higher risk premium only compensate, in an actuarially fair sense, for default losses—i.e., foreign investors expect to be bailed in. For Home households, on the contrary, the risk premium due to FX intervention changes the discount rate since, for political economy reasons, domestic bondholders are always bailed out (and they expect to be bailed out; to satisfy the budget constraint, the government will need to raise taxes, but we assume this is done via lump-sum taxes and that the link with the bailout is therefore not internalized by the domestic bondholders). Thus, FX

⁶Although some of the empirical literature findings are consistent with this view (Disyatat and Galati (2007)), Dominguez and Frankel (2005) find that the portfolio channel was significant even for the US and Germany.

intervention also creates a wedge in the Backus-Smith condition. Sterilized intervention is however not a perfect instrument to offset shocks to the foreign demand for bonds because the use of FX intervention is limited by a budget constraint.

Our paper focuses on the role of FX intervention under discretionary policy. In the standard open economy New Keynesian Model, ‘conventional’ monetary policy (which we will also call ‘interest rate policy’) is sufficient to guarantee the stability of an economy because: (i) the two objectives of stable inflation and zero output gap coincide ; and (ii) because equilibrium uniqueness is ensured under optimal policy. We discuss the role of central banks reserves in cases where either (i) or (ii) is not guaranteed.

The first such case is the occurrence of exogenous shocks that affect the economy’s dynamics and that cannot be perfectly offset with conventional monetary policy, for instance by choosing a nominal interest rate such that the real interest rate is equal to the natural rate of interest.⁷ These shocks are often represented as additive factors in the Phillips curve. We show in this paper that risk premium shocks in international capital markets, which affect the interest rate faced by domestic households but not the interest rate faced by foreign investors, distort domestic consumption and cannot be offset by stabilizing fully the output gap with interest rate policy. Foreign exchange intervention is then useful as a second instrument, as it allows the central bank to target a level of domestic consumption and thus to stabilize macroeconomic dynamics.

The second case in which FX intervention is useful is due specifically to the financial friction. In the standard open economy New Keynesian Model, equilibrium determinacy is guaranteed even under discretionary policy (Gali (2008)). But in open economy models that incorporate financial frictions (for instance, the effect of the exchange rate on balance sheets and borrowing costs), multiple equilibria are possible (see Aghion et al. (2000) and Aghion et al. (2004), who follow the tradition of third-generation models of speculative attacks, as in Obstfeld (1986) and Krugman (1999)). More recently, Mendoza (2010) showed that collateral constraints could explain economic dynamics after sudden stops in a calibrated business cycle model. Surico (2008) also showed that when the cost channel of monetary policy —a form of financial friction— is present, monetary policy in the absence of a commitment technology may not guarantee determinacy. Accordingly, in this paper, we also investigate the importance of FX intervention in reducing this risk of indeterminacy.

Our paper is closest to that of Farhi and Werning (2014) and of Cavallino (2016), although both papers focus on optimal policy under commitment whereas we focus on optimal discretionary policy. Farhi and Werning (2014) develop a New Keynesian open economy model where the Home country is affected by risk premium shocks but whose central bank can use capital controls as well as monetary policy to stabilize output and inflation. Cavallino (2016) develops a similar model, but integrates the financial intermediary of Gabaix and Maggiori (2015) to model the effect of FX intervention. A main difference with our paper is that we focus on discretionary policy; this also allows us to find closed-form solutions which are more easily interpretable.

Although our paper is specifically interested in FX intervention, other instruments may be available for central banks who seek to influence economic activity and the exchange rate. These include quantitative easing, macro-prudential policy, capital controls. This paper is thus related to the

⁷The natural rate of interest rate is the rate that would prevail for in the equilibrium with flexible prices.

literature on second instruments, for instance Farhi and Werning (2012), which discusses the role of capital controls in models with shocks to international risk premia, Curdia and Woodford (2011), which analyzes the role of quantitative easing in models with frictions in financial intermediation, Woodford (2012), which discusses the role of macroprudential policy in a similar setting, and Alla et al. (2016), which gives general results on the use of second instruments in New Keynesian models.

2.4 A small open economy

We extend a standard open economy New Keynesian model, adding a financial friction and FX sterilized intervention, which is effective because of a portfolio balance effect. We begin with the optimization problems of the representative household and the representative firm before discussing central bank policy.

2.4.1 Households

There is a continuum of countries indexed by $i \in [0, 1]$ but we focus on a single country, called Home, and denoted by the subscript H . In each country, the representative household's utility function is:

$$U(C_t, N_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right], \quad (2.1)$$

where N_t is the quantity of labor supplied and C_t is aggregate consumption at time t defined as:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

η denotes the elasticity of substitution between domestic and foreign goods; α is the openness coefficient that parameterizes the share in aggregate consumption of domestic and foreign consumption $C_{H,t}$ and $C_{F,t}$, respectively defined by:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}.$$

where $j \in [0, 1]$ is the index of individual good variety produced domestically, ε is the elasticity of substitution between domestic goods, γ is the elasticity of substitution between foreign goods, and $C_{i,t}$ is the consumption basket imported from country i , aggregated across products indexed by k : $C_{i,t} = \left(\int_0^1 C_{i,t}(k)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}.$

The openness parameter α is a measure of home bias: when $\alpha \rightarrow 0$, the share of foreign goods in domestic consumption reaches 0 and the economy can be considered closed. Conversely, when $\alpha \rightarrow 1$, the economy is fully open and since the country's size is infinitesimally small, the share of domestic goods in Home consumption is 0. The corresponding Consumer Price Index (CPI), domestic Producer Price Index (PPI) and Imported Price Index (IPI) are :

$$P_t = \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}, \quad P_{F,t} = \left[\int_0^1 P_{i,t}^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}.$$

Domestic households can borrow and lend using risk-free domestic assets. However, we assume that domestic households can only use domestic bonds, contrary to foreign investors who can buy

either domestic or foreign bonds (for more details, see Section 2.4.3). The portfolio of the domestic representative household is then made of D_{t-1}^h units of domestic bonds and S_{t-1}^b units of sterilization bonds sold by the central bank in period $t-1$, both of which pay the home interest rate i_{t-1} .

Thus, the (domestic) representative household's problem is to maximize (2.1) subject to the sequence of budget constraints, for each period t :

$$\begin{aligned} \left[\int_0^1 P_{H,t}^i(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(k) C_{i,t}(k) dk di \right] + D_t^h + S_t^b + T_t^{fin} \\ \leq (1 + i_{t-1}) \left(D_{t-1}^h + S_{t-1}^b \right) + W_t N_t + \Pi_t + T_t^{firm} + T_t^{gov} \end{aligned} \quad (2.2)$$

where all the variables, which we will describe now, are expressed in domestic currency. W_t , Π_t , T_t^{firm} and T_t^{gov} are respectively the nominal wage, profits and lump-sum transfers from the firms and the government. Π_t , T_t^{firm} and T_t^{gov} are rebated to the representative household. T_t^{fin} is a lump sum tax imposed to households to finance the domestic investors' bail-out when the government defaults (see Section 2.4.3 for details about the effect of FX intervention).

2.4.2 Firms

A typical firm j in the domestic economy produces a differentiated good (also indexed by j) under monopolistic competition with a linear technology using labor as unique input:

$$Y_t(j) = A_H N_t(j)$$

where A_H is (domestic) labor productivity (which is assumed to be constant in this paper).

A simplified financial accelerator for domestic firms is included in the model: at the beginning of each period, firms must borrow a fixed fraction $\mu \in [0, 1]$ of the nominal value of output $P_{H,t}(j)Y_t(j)$ they intend to produce. This cost captures working capital requirements at the beginning of the period, used to pay for labor costs or cover other liquidity requirements.

These short-term loans require repayment at the end of each period, after production takes place. Firms thus deduct from their profits the interest costs of working capital requirements, which are charged at the current interest rate i_t by the domestic investor (and thus are paid as a lump-sum transfer to the representative household). The financial friction has no direct impact on the country's aggregate wealth since this interest cost is charged domestically, but it is worth noting that since the UIP holds (see below), the cost charged to firms is affected by expected exchange rate movements and by the risk premium. In particular, an expected depreciation of the exchange rate leads to a higher domestic interest rate and higher marginal costs. Our model thus follows the spirit of Obstfeld (1986), Krugman (2000), Aghion et al. (2000), Aghion et al. (2004) and other third-generation speculative attack models in linking equilibrium indeterminacy to the feedback between the exchange rate and the financial situation of firms in the presence of a financial friction.

The price-setting behaviour of firms follows Gali and Monacelli (2005) in assuming Calvo pricing and that the Law of One Price holds. At each date $t \geq 0$, a randomly selected fraction $1 - \delta$ of firms is able to reset prices. Firm j chooses its price $P_{H,t}(j)$ by maximizing profits, i.e. by solving

the maximization problem:

$$\max_{P_{H,t}(j)} \sum_{k=0}^{\infty} \delta^k \left(\prod_{h=1}^k \frac{1}{1+i_{t+h}} \right) [P_{H,t}(j)Y_{t+k|t}(j) - P_{H,t}MC_t Y_{t+k|t}(j) - \mu P_{H,t}(j)Y_{t+k|t}(j)i_{t+k}]$$

where:

- $Y_{t+k|t}(j) \equiv \left(\frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k}$ is the demand for firm's j good at date $t+k$ if prices were reset for the last time at date t ;
- $MC_t \equiv \frac{1+\tau}{A_H} \frac{W_t}{P_{H,t}}$ is the real marginal cost deflated by home PPI⁸ ;
- $\mu P_{H,t}(j)Y_{t+k|t}(j)i_{t+k}$ represents the working capital cost bore by firms.

The solution to this problem is presented in Appendix 2.8.1.

Foreign investors

Foreign investors can invest in Home bonds, and changes in their demand for these bonds are an important source of shocks. Indeed, most discussions about the use of FX intervention in developing countries are related to the desire to smooth the impact of sudden capital inflows and outflows that result from fluctuations in foreign investors' perception of risk. To contribute to this discussion, we assume, as in Farhi and Werning (2014), that the Home country is hit by an exogenous risk-premium shock Ξ_t that captures changes in foreign investors' appetite for a particular country's bonds.⁹ It is possible to model the sources of this shock in more detail, as is done in Gabaix and Maggiori (2015) by assuming imperfect international financial intermediation and modeling the behavior of financial intermediaries, but we abstract from these issues since they inessential to our argument.

The shock Ξ_t is defined such that the gross return *perceived* by foreign investors in domestic currency $\frac{1+i_t}{\Xi_t}$ differs from the gross interest rate paid by the central bank, $1+i_t$ (see also UIP in equation (2.4) below). This shock allows us to analyze either the impact of capital flow surges (negative risk premium shock Ξ_t) or of sudden stops (positive risk premium shock Ξ_t). For a given exchange rate and foreign interest rate, a positive risk premium shock is thus akin to an increase in the domestic interest rate, but one affecting only domestic households. A positive risk premium shock can also be interpreted as a negative shock to domestic consumption: since the interest rate faced by domestic households suddenly increases, domestic consumption falls (this also leads to a trade surplus). Because this shock lowers domestic consumption, it reduces the required real wage (since labor supply is stimulated when consumption is low), even when there is no output gap. Thus, this shock breaks the divine coincidence, which is why it calls for a second policy instrument that would complement monetary policy. We now explain why FX intervention is particularly suitable as the second instrument.

⁸We allow for a constant labor tax τ to make the steady state efficient.

⁹ Ξ_t could also be thought of as capturing time-varying and country-specific borrowing constraints, the risk premium shock being simply the multiplier on the borrowing constraint.

2.4.3 The central bank, sterilized interventions and the government

The central bank

In each period, the central bank chooses the interest rate and performs sterilized foreign exchange intervention, offsetting any increase in reserves ($R_{i,t}$ denotes the Home central bank's holdings of currency i , valued in local currency using the exchange rate $E_{i,t}$) by issuing sterilization bonds S_t^b , so as to keep the money supply constant and the policy interest rate unchanged:

$$\int_0^1 E_{i,t} R_{i,t} di - \int_0^1 E_{i,t} (1 + i_{t-1}^i) \Xi_{t-1} R_{i,t-1} di = S_t^b - (1 + i_{t-1}) S_{t-1}^b \quad (2.3)$$

This intervention implies a change in the stock of public debt held by the private sector, and thus an increase in the risk premium Ψ_t on domestic bonds because of a portfolio balance channel effect. More precisely, we assume that the risk-premium is a function of the level of reserves, and thus of the level of (general) government debt held by the private sector:

$$\Psi_t = \left(\frac{\int_0^1 E_{i,t} R_{i,t} di}{\bar{R}} \right)^\varphi$$

where φ is the elasticity of the risk premium to reserves and \bar{R} is the equilibrium level of reserves for which there is no risk premium. This formulation is akin to Schmitt-Grohe and Uribe (2003)'s model of the risk premium (see also Blanchard et al. (2005) and Benes et al. (2011)). In Gabaix and Maggiori (2015), the effectiveness of FX intervention (captured by our elasticity φ) would depend on the importance of the friction affecting the international financial intermediaries, and on the relation of the size of intervention to the size of the intermediaries' balance sheet (which would be proxied by our parameter \bar{R}).

As will be shown in Section 2.4.5, an exogenous shock to foreigners' preference for Home bonds drives a wedge in the Backus-Smith condition, i.e., a wedge between the consumption plans of the Home households and of Foreign household, and such volatility reduces welfare. FX intervention can mitigate this wedge if it affects the discount rate of Home households and Foreign households differently. To achieve this, we assume that Home households cannot borrow from or invest abroad¹⁰ (this breaks their ability to run carry trades) and that FX intervention is perceived differently by foreign investors and domestic investors, as we explain now.

Foreign investors' view of FX intervention

We assume that foreign investors, who are sophisticated agents used to the risks of debt restructuring, internalize that the risk premium Ψ_t compensates exactly, in an actuarially fair sense, for a partial default they will suffer. Thus, for foreign investors from country i (e.g. in the US), the rate of return on Home bonds, taking into account the risk premium but also the default rate, should be equal to the rate of return in the US. As a result, since we also have to take into account the shock to preferences Ξ_t , for a foreign investors living in country i , the UIP is:

¹⁰In reality, households in small open economies do invest part of their net wealth abroad, but they do not run carry trade, i.e., borrow in local currency at a high interest rate to invest abroad at a low rate, even though the local exchange rate may be depreciating faster than what the UIP would imply. The reasons for limited carry trade from emerging economies may be due to liquidity constraints, short-termism (since future capital gains are offset by today's losses in interest income), or regulatory constraints on FX position. Such regulatory constraints certainly also prevent households to borrow significant amounts in foreign currency to invest at home.

$$\frac{1 + i_t}{\Psi_t \Xi_t} = (1 + i_t) \frac{E_{i,t+1}}{E_{i,t}} \quad (2.4)$$

Domestic households' view of FX intervention

On the other hand, we assume that domestic households correctly anticipate they will be bailed-out following the domestic government's partial default. Thus, domestic households do perceive a higher rate of return on domestic bonds, and thus face a higher interest rate in their intertemporal consumption Euler equation, by the value of the risk premium.

A lump sum tax T_t^{fin} is imposed to households to finance the bail-out of domestic bond holders. Since our model does not feature uncertainty, this lump-sum tax is paid at each period, and is equal to the risk premium paid on the total volume of domestic bonds hold by domestic households:

$$T_t^{fin} = \frac{\Psi_t - 1}{\Psi_t} (D_t^h + S_t^b)$$

Although the risk-premium affects the Euler equation, the lump sum tax to finance it does not. Thus, foreign exchange interventions affect the interest rate faced by foreign investors, but not the one faced by domestic households. Foreign exchange interventions then introduce a wedge between the domestic households' and the foreign households' consumption plans. As a result, FX intervention will distort domestic consumption, but without changing foreign demand and exports.

Why is FX intervention useful?

The "endogenous" risk premium Ψ_t allows the central bank to have influence on the UIP and thus on the exchange rate, for a given policy interest rate and a given exogenous shock Ξ_t .¹¹ A positive exogenous risk premium shock signals a lower appetite from foreign investors for domestic bonds. Foreign investors then have to be compensated when buying domestic bonds. The Home central bank has three possibilities:

- (i) accept a large instantaneous currency depreciation (and a subsequent appreciation);
- (ii) increase the interest rate;
- (iii) sell foreign exchange reserves to lower the endogenous risk premium Ψ_t (for simplicity, we assume that only the Home central bank uses FX intervention;¹² thus, if the central bank reduces its reserves to buy domestic debt, the currency appreciates).

As explained earlier, the exogenous risk premium shock is a shock that affects domestic consumption, and as a result it affects both the IS curve and the Phillips curve (from the labor supply side, wages are a function of consumption). The interest rate, however, is mostly an instrument to manage demand (i.e. the IS curve). Note also that the more open the economy, the more distinct are domestic consumption and domestic output, and thus the more penalizing it is to accept exchange rate movements or to use monetary policy to respond to a shock to domestic consumption. FX intervention has the advantage of targeting domestic consumption directly and is thus a better

¹¹Reserves may be insufficient to offset large expected depreciations but we do not discuss this possibility here.

¹²This assumption will be natural in our context of a small open economy taking the rest of the world as given.

instrument to offset shocks to the risk premium.

Absent FX intervention, domestic consumption would suffer because of a high risk premium, and because optimal policy would suggest a combination of monetary policy tightening and currency depreciation (see section 2.5.2 for a numerical example) that would affect the whole of domestic output and lead to greater economic volatility. By lowering its reserves, the central bank can moderate the impact on domestic consumption of capital outflows and of the risk premium shock. Such an intervention reduces the required exchange rate depreciation and limits the increase in interest rates, thus implying lower macroeconomic volatility.

The government

The central government's revenues are the tax receipts ($\tau W_t N_t$). In addition, the government sells bonds to domestic investors (the representative household, who holds D_t^h bonds) and to foreign investors (who hold D_t^f bonds). At each period, and in line with the portfolio balance channel, the government defaults partially on bonds held by domestic households thus receiving as debt relief $T_t^{fin} = \frac{\Psi_t - 1}{\Psi_t} (D_t^h + S_t^b)$. Finally, the government rebates to households the labor tax income through a lumps-sum transfer: $T_t^{gov} = \tau W_t N_t$. The government budget constraint is then:

$$\frac{1 + i_{t-1}}{\Psi_{t-1}} D_t^f + (1 + i_{t-1}) (D_t^h + S_t^b) + T_t^{gov} \leq D_{t+1}^f + D_{t+1}^h + \tau W_t N_t + T_t^{fin} \quad (2.5)$$

2.4.4 Exchange rates and the terms of trade

Since the Law of One Price holds, we have:

$$P_{F,t} = E_t P_t^*$$

where $P_t^* = \left[\int_0^1 P_{i,t}^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$ is the world price index, and $P_{i,t}^i$ is the country i 's domestic PPI in its own currency. The above equation then defines the effective nominal exchange rate E_t .

Finally, the terms of trade S_t and the real exchange rate Q_t of Home are defined as:

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \frac{E_t P_t^*}{P_{H,t}} \quad Q_t = \frac{P_{F,t}}{P_t} = \frac{E_t P_t^*}{P_t}.$$

2.4.5 Equilibrium Conditions

We now consider the situation in which Home takes the rest of the world as given and is the only country who implements foreign exchange interventions. The rest of the world is exogenous and all foreign countries are identical. Foreign variables are denoted with stars.

The supply side equations include:

- the optimal labor-leisure decision by households for a given CPI and nominal wage:

$$C_t^\sigma N_t^\phi = \frac{W_t}{P_t}$$

- the Calvo price setting optimal conditions which are complex in the non-linearized setup and presented in the Appendix 2.8.1

The demand side equations are:

- the Euler equation for domestic households

$$1 + i_t = \beta^{-1} \left(\frac{C_{t+1}}{C_t} \right)^\sigma \frac{P_{t+1}}{P_t},$$

We write current consumption as a function of foreign consumption C_t^* and the relative Pareto weight of Home in world consumption Θ_t :

$$C_t = \Theta_t C_t^* Q_t^{1/\sigma}. \quad (2.6)$$

This equation will be called the Backus-Smith condition, although it is an abuse of language since it is the definition of Θ_t in this model.

- the no-arbitrage condition (for a foreign household) between home and foreign bonds gives the law of motion of Θ_t ¹³ :

$$\left(\frac{\Theta_{t+1}}{\Theta_t} \right)^\sigma = \frac{1 + i_t}{1 + i_t^*} \frac{E_t}{E_{t+1}} = \Psi_t \Xi_t \quad (2.7)$$

where the risk premia Ψ_t and Ξ_t have already been introduced. The logarithm of Θ_t appears to follow an autoregressive process. Θ_t can thus be interpreted as a cumulative risk premium. Together with equation (2.6), equation (2.7) shows how exogenous shocks to foreign investors' preference for domestic bonds and FX intervention affect domestic consumption, since they introduce a wedge between the domestic and the foreign paths of consumption. Since monetary policy has symmetric effects on domestic and foreign consumption, it is not the ideal instrument to offset such shocks. On the contrary, FX intervention also moves domestic consumption, and is thus better suited as a second policy instrument in response to such exogenous risk premium shocks.

In addition, the market clearing conditions are:

- for the goods market:

$$Y_t = (1 - \alpha) \left(\frac{Q_t}{S_t} \right)^{-\eta} C_t + \alpha S_t^\gamma C_t^*; \quad (2.8)$$

- for the labor market:

$$N_t = \frac{Y_t}{A_H} \Delta_t \quad (2.9)$$

where Δ_t is the index of price dispersion defined by

$$\Delta_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj \quad ;$$

and where the relation between the terms of trade and the real exchange rate is

$$Q_t = \left[(1 - \alpha) S_t^{\eta-1} + \alpha \right]^{\frac{1}{\eta-1}} ; \quad (2.10)$$

¹³Using the definition of Θ_t , we have $\left(\frac{\Theta_{t+1}}{\Theta_t} \right)^\sigma = \left(\frac{C_{t+1}}{C_t} \right)^\sigma \frac{P_{t+1}}{P_t} \left(\frac{C_t^*}{C_{t+1}^*} \right)^\sigma \frac{E_t P_t^*}{E_{t+1} P_{t+1}^*}$. The Euler equations for domestic and foreign households give: $1 + i_t = \beta^{-1} \left(\frac{C_{t+1}}{C_t} \right)^\sigma \frac{P_{t+1}}{P_t}$ and $1 + i_t^* = \beta^{-1} \left(\frac{C_{t+1}^*}{C_t^*} \right)^\sigma \frac{P_{t+1}^*}{P_t^*}$. Dividing the domestic Euler equation by the foreign Euler equation, and using the first equation above, we find the law of motion of Θ_t , i.e. equation (2.7).

The country's budget constraint at date t , which is derived by adding the budget constraints of the households (equation (2.2)), of the central bank (equation (2.3)) and of the government (equation (2.5)), is:¹⁴

$$NFA_t = -\frac{1}{P_{F,t}C_t^{*\sigma}}NX_t + \beta\Xi_t^{-1}NFA_{t+1} \quad (2.11)$$

where the country net foreign asset position¹⁵ and trade balance are respectively:

$$NFA_t = \frac{(1+i_{t-1}^*)\Xi_{t-1}E_t}{P_{F,t}C_t^{*\sigma}} \left[R_t^* - \frac{D_t^f}{E_{t-1}} \right], \quad NX_t = P_{H,t}Y_t - P_tC_t$$

Finally, we get the intertemporal budget constraint by taking $NFA_0 = 0$, solving forward equation (2.11), and imposing the transversality condition:

$$\sum_{t=0}^{\infty} \beta^t \left(\prod_{s=0}^{t-1} \Xi_s \right)^{-1} C_t^{*\sigma} \left[\frac{Y_t}{S_t} - \frac{C_t}{Q_t} \right] \geq 0 \quad (2.12)$$

2.4.6 The log-linearized framework

Following Gali and Monacelli (2005) and Farhi and Werning (2014), our analytical results are derived for the log-linearized model using the Cole-Obstfeld parametrization, where $\sigma = \eta = \gamma = 1$ (Cole and Obstfeld (1991)). A lower case variable denotes the log-deviation from the deterministic steady-state of the variable.¹⁶

To disentangle the effects of the two risk premia, we decompose the Pareto weight Θ_t , i.e., the wedge between domestic and foreign consumption, as follows:

$$\theta_t = \hat{\theta}_t + \bar{\theta}_t \quad \text{where}$$

- $\bar{\theta}_t$ is the distortion to consumption resulting from the exogenous risk premium shock ξ_t ;
- $\hat{\theta}_t$ the the distortion to consumption resulting from the FX intervention's risk premium ψ_t .

Demand side The log-linearization of the goods market clearing condition gives

$$y_t = (1 - \alpha)[-(q_t - s_t) + c_t] + \alpha s_t = (1 - \alpha)\theta_t + s_t$$

since $c_t^* = 0$ and thus $q_t = c_t - \theta_t$ (from the Backus-Smith condition). Thus θ_t has an effect on aggregate demand (and thus on Home prices and the terms of trade; this is the *demand channel* of risk premia—see below for the supply channel), but it has a smaller effect on y_t than on c_t if q_t and s_t are small, i.e. in the short run where prices are sticky (since $c_t = \theta_t + q_t$). Overall, the demand side can be represented by the dynamic Investment Saving (IS) equation

$$y_t = y_{t+1}^e - (i_t - \pi_{H,t+1}^e - \rho) + \alpha(\psi_t + \xi_t) \quad (2.13)$$

$$= y_{t+1}^e - (i_t^* - \pi_{H,t+1}^e - \rho) + e_t - e_{t+1}^e - (1 - \alpha)(\psi_t + \xi_t) \quad (2.14)$$

¹⁴Since firms' borrowing is domestic, interest payments are collected by domestic investors and thus they do not affect the country's budget constraint at date t .

¹⁵We normalize the net foreign asset position by the foreign consumption and take the foreign price at home $P_{F,t}$ as the numeraire.

¹⁶Note that the model and the loss function are presented differently from that in Farhi and Werning (2014), who present variables in deviation from the natural allocation, whereas we express them in deviation from the deterministic steady state.

Equation (2.13) shows that for a given exchange rate and given expectations, higher risk premia, which increase the domestic interest rate, hurt consumption and thus output (the effect is proportional to $1 - \alpha$, the weight of domestic consumption in output). In addition, the UIP is:

$$e_t = e_{t+1}^e - (i_t - i_t^*) + \psi_t + \xi_t \quad (2.15)$$

Given exchange rate expectations and monetary policy decisions, a positive risk premium, either due to a preference shock or to an increase in the level of reserves, triggers a depreciation of the local currency.

Supply side

The supply side consists of the New-Keynesian Phillips Curve:

$$\pi_{H,t} = \beta \pi_{H,t+1}^e + \kappa_y y_t + \kappa_\theta (\hat{\theta}_t + \bar{\theta}_t) + \kappa_f (i_t - \rho) \quad (2.16)$$

$$\text{where } \kappa_y = \lambda(1 + \phi), \quad \kappa_\theta = \lambda\alpha, \quad \kappa_f = \lambda\mu$$

λ captures price flexibility and is increasing in the share of firms that can reset their prices.¹⁷

The new term $\kappa_\theta (\hat{\theta}_t + \bar{\theta}_t)$ represents the effect of the risk premium on domestic firms' marginal costs (the *supply channel* of risk premia). Following a negative risk premium shock, which leads to a positive shock to domestic consumption ($\bar{\theta}_t > 0$), real wages have to be increased in order to maintain labor supply. This is why firms' marginal costs and inflation increase. But FX intervention can “lean against the wind”: the central bank can increase the risk premium (by accumulating reserves and selling sterilization bonds), thus increasing the endogenous risk premium and depressing consumption ($\hat{\theta}_t < 0$).

As highlighted in Section 2.4.3, the exogenous risk premium targets specifically domestic consumption and not the whole of domestic output (exports are not affected), and this is what breaks the divine coincidence. This effect is clearly visible in equation (2.16). It is also worth noting that the effect of the exogenous risk premium on PPI inflation is increasing in the economy's openness ($\kappa_\theta = \lambda\alpha$) because the more open the economy, the larger the difference between output and consumption (which is the part of output that is independently affected by the risk premium).

Finally, the financial accelerator appears in the Phillips curve (through the term $\kappa_f (i_t - \rho)$) in a way similar to that of the cost channel of monetary policy (Surico (2008)). Since the UIP holds, the financial accelerator is also $\kappa_f (i_t - \rho) = \kappa_f [e_{t+1}^e - e_t + i_t^* - \rho + \theta_{t+1}^e - \theta_t]$. Exchange rate expectations affect firms' marginal costs and can thus have real effects. In Section 2.6, we explore the impact of the financial accelerator on equilibrium determinacy under optimal discretionary policy and discuss the ability of FX intervention to reduce the risk of indeterminacy.

The intertemporal budget constraint

The intertemporal budget constraint is :

$$\sum_{t=0}^{\infty} \beta^t \hat{\theta}_t = 0. \quad (2.17)$$

¹⁷ $\lambda = \frac{(1-\delta)(1-\beta\delta)}{\delta}$. As a result, $\lambda \rightarrow \infty$ when prices are fully flexible

The intertemporal budget constraint requires that the discounted value of the distortions to domestic consumption be zero. When the central bank sells reserves, the risk-premium decreases and domestic consumption increases, thus worsening the trade balance ($nx_t = -\alpha(\bar{\theta}_t + \hat{\theta}_t)$). A larger trade deficit has to be compensated by larger surpluses in the future, which is what this intertemporal budget constraint captures, and this is why it is impossible to use FX intervention to affect the path of consumption permanently.

2.4.7 Loss function

The welfare costs due to deviations from the steady state can be expressed using the following welfare loss function (proved in Appendix 2.8.3, using a second-order approximation):

$$\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\alpha_{\pi} \pi_{H,t}^2 + y_t^2 + \alpha_{\theta} \left[\hat{\theta}_t + \alpha_{\psi} \bar{\theta}_t \right]^2 \right]$$

where

$$\alpha_{\pi} = \frac{\varepsilon_P}{\lambda_P(1+\phi)}, \quad \alpha_{\theta} = \frac{\alpha}{1+\phi} \left(\frac{2-\alpha}{1-\alpha} + 1 - \alpha \right), \quad \alpha_{\psi} = \frac{1-\alpha}{\frac{2-\alpha}{1-\alpha} + 1 - \alpha}$$

The first two terms are familiar—they are identical to those obtained in Gali and Monacelli (2005). The third term captures the direct distortions induced by FX intervention on domestic consumption. Indeed, FX intervention targets specifically domestic consumption, thus it generates additional welfare costs, on top of those related to the distortion of output and the dispersion of relative prices. Sterilized intervention can reallocate domestic demand intertemporally (as do capital controls in Farhi and Werning (2012)). But such reallocation comes at a cost for welfare, expressed as follows:

$$\alpha_{\theta} \left[\hat{\theta}_t + \alpha_{\psi} \bar{\theta}_t \right]^2 = \alpha_{\theta} \underbrace{\hat{\theta}_t^2}_{\text{cost of external imbalance}} + 2\alpha_{\theta}\alpha_{\psi} \underbrace{\hat{\theta}_t \bar{\theta}_t}_{\text{costs of distorting consumption}} + \text{exogenous terms}$$

The welfare costs of FX intervention are thus due to:

- (i) *the cost of external imbalances:*

Selling reserves decreases the endogenous risk premium, and therefore increases domestic consumption without changing foreign demand or Home exports. This worsens the trade balance. Such distortion to the trade balance imposes a welfare cost because the intertemporal budget constraint has to hold (the second order approximation of the budget constraint is what is capture by the term $\alpha_{\theta}\hat{\theta}_t^2$); FX intervention can smooth domestic consumption but cannot increase it permanently. We also find that the more open the economy, the more costly it is to distort the trade balance. Indeed, the welfare cost is proportional to $\alpha_{\theta} = \frac{\alpha}{1+\phi} \left(\frac{2-\alpha}{1-\alpha} + 1 - \alpha \right)$, which is an increasing function of α .

- (ii) *the costs of distorting consumption:*

A positive shock to the exogenous risk premium increases the interest rate faced by domestic consumers and decreases domestic consumption. Even keeping output constant, the falls in domestic consumption also lead to a fall in domestic prices (because of Home bias in

consumption), which decreases the terms of trade.¹⁸ FX intervention has symmetric effects. This means that when FX intervention is used to offset the exogenous risk premium, it is able to stabilize consumption (which has welfare benefits in itself) as well as the terms of trade, which is also worthwhile given the intertemporal budget constraint. The welfare loss of distorting consumption is proportional to $\alpha\theta\alpha_\psi = \frac{\alpha(1-\alpha)}{1+\phi}$. When the economy is closed ($\alpha \rightarrow 0$), output and domestic consumption coincide, and there is no cost due to deviations in domestic consumption that is not already captured by the cost of a non-zero output gap. When the economy is fully open ($\alpha \rightarrow 1$), Home consumption is entirely made of imported goods, and there is no cost in distorting it as this does not affect the terms of trade or consumption.¹⁹

Finally, we note that contrary to the other elements in the welfare loss function (inflation, output gap, and terms of trade), the cost of external imbalance exists even when the country is fully open. Gali and Monacelli (2005) indeed showed that the welfare losses resulting from distortions in output and inflation go to zero when $\alpha \rightarrow 1$. However, since the cost of external imbalances affect the path of domestic consumption, this cost exists even when the economy is fully open. Because the welfare cost of a non-zero output gap distortion was normalized to 1, the welfare cost of external imbalances goes to infinity when $\alpha \rightarrow 1$.

Planning problem

We assume that the authorities are unable to commit to specific future policies (on the interest rate, asset purchases, the deficit, etc).²⁰ Working in a discretionary setup, as opposed to the commitment framework analyzed in Farhi and Werning (2014) and in Cavallino (2016), also allows us to derive simple and transparent closed-form formulas. The complete planning problem at date t is thus composed of the objective function; the Phillips curve; the IS equation; the UIP condition; the dynamic relation between the consumption wedge and FX intervention; the intertemporal budget constraint of the country;²¹ and the initial condition for the output gap:

¹⁸This effect exists as long as the economy is not fully open, i.e. as long as the share α of domestic goods consumed by domestic households is not zero.

¹⁹More precisely, as shown by Farhi and Werning (2014), optimal policy with a fully open economy would consist of maximizing the monopoly profits of exporters, but given that under the Cole-Obstfeld parameterization, exporters face a demand with elasticity of 1, the monopoly problem is degenerate. As a result, output should converge to 0 when $\alpha \rightarrow 1$, and distorting consumption does not help with the terms of trade, which is why FX intervention is useless.

²⁰However, we also assume that the government can commit *not* to default. Indeed, unless the government can commit not to default, the intertemporal budget constraint implies that sterilized interventions could never be used. In a purely discretionary framework, the government would promise at each period to reimburse the current period deficit with future revenues: $\theta_t = -\sum_{s=t}^{\infty} \beta^{s-t}\theta_s$. However, this promise, renewed at each period, is not credible since it omits past deficits. The only solution consistent with rational expectations is then $\theta_t = 0$: $\theta_t = -\sum_{s=t+1}^{\infty} \beta^{s-t}\theta_s = -\beta\sum_{s=t+1}^{\infty} \beta^{s-(t+1)}\theta_s = 0$. Such an intertemporal constraint thus requires a commitment not to default.

²¹The quadratic approximation of the budget constraint included in the welfare objective does not ensure that this constraint is satisfied; the presence of the intertemporal constraint in the objective function is necessary (due to the microfoundations) but not sufficient to ensure that the constraint is satisfied.

$$\begin{aligned}
\min_{\pi_{H,t}, y_t, i_t, \psi_t, \hat{\theta}_t, e_t} \quad & \alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \left[\hat{\theta}_t + \alpha_\psi \bar{\theta}_t \right]^2 \quad \text{s.t.} & (2.18) \\
\pi_{H,t} = & \beta \pi_{H,t+1}^e + \kappa_y y_t + \kappa_\theta \hat{\theta}_t + \kappa_\theta \bar{\theta}_t + \kappa_f (i_t - \rho) \\
y_t = & y_{t+1}^e - (i_t - \pi_{H,t+1}^e - \rho) + \alpha (\psi_t + \xi_t) \\
e_t = & e_{t+1}^e - (i_t - i_t^*) + \psi_t + \xi_t \\
\hat{\theta}_{t+1}^e = & \psi_t + \hat{\theta}_t \\
\sum_{t=0}^{\infty} \beta^t \hat{\theta}_t = & 0 \\
y_0 = & (1 - \alpha) (\hat{\theta}_0 + \bar{\theta}_0) + e_0
\end{aligned}$$

2.5 Optimal stabilization policy following risk premium shocks

In this section, we describe optimal monetary and FX intervention policies following an exogenous shock. We focus on exogenous risk premium shocks in international capital markets, as introduced in Section 2.4.1. These shocks capture the changes in foreign investors' sentiment that have historically led to volatile episodes of capital inflows (negative risk premium shocks) or capital outflows (positive risk premium shocks). Such "push factors" can be considered to be exogenous to the small open economy. We show how FX intervention allows the central bank to reduce economic volatility in the face of these external shocks.

2.5.1 Optimal use of FX intervention following risk premium shocks

A positive risk premium shock stands for a reduction in foreign investors' appetite for domestic bonds. This shock affects both domestic allocation and welfare (the latter through domestic terms of trade stabilization effect as was explained in Section 2.4.6). Proposition 6 provides a transparent analysis based on closed-form formula that details how FX intervention is optimally used following a risk premium shock. For the sake of clarity, the analytical results are presented in Proposition 6 with the financial accelerator turned off ($\mu = 0$) although the qualitative and quantitative results are similar when the financial accelerator is turned on (see footnote 21 and Section 2.5.2; on the contrary, our results on multiple equilibria in Section 2.6 depend crucially on the size of the financial accelerator).

Proposition 6. Optimal policy following risk premium shocks

Following an exogenous risk premium shock with autoregressive process $\xi_t = \xi_0 \rho^t$, the optimal paths for inflation, output, the endogenous consumption wedge and the endogenous risk premium are:²²

$$\begin{aligned}
\pi_{H,t} &= \frac{\kappa_\theta(1-\alpha_\psi)\xi_0}{1-\rho} \left(\frac{1-\beta}{1-\rho\beta} \frac{1}{D(\alpha_\theta, 1)} - \frac{1}{D(\alpha_\theta, \rho)} \rho^t \right) + \frac{\kappa_\theta}{\alpha_\theta D(\alpha_\theta, 1)} \Gamma \\
y_t &= -\alpha_\pi \kappa_y \pi_{H,t} \\
\hat{\theta}_t &= \frac{\xi_0}{1-\rho} \left[1 - (1-\alpha_\psi) \frac{D(+\infty, \rho)}{D(\alpha_\theta, \rho)} \right] \left(\rho^t - \frac{1-\beta}{1-\rho\xi\beta} \right) \\
\psi_t &= \xi_0 \left[(1-\alpha_\psi) \frac{D(+\infty, \rho)}{D(\alpha_\theta, \rho)} - 1 \right] \rho^t \\
e_{t+1} - e_t &= i_t - i_t^* - (\psi_t + \xi_t) \quad , \text{ with } e_0 = y_0 - (1-\alpha) \left(\hat{\theta}_0 + \bar{\theta}_0 \right)
\end{aligned} \tag{2.19}$$

where²³

$$D(\alpha_\theta, \rho\xi) = 1 - \rho\xi\beta + \alpha_\pi \left(\kappa_y^2 + \frac{\kappa_\theta^2}{\alpha_\theta} \right)$$

Proof: See Appendix 2.8.4 □

A positive risk premium shock decreases domestic consumption (and thus θ_t) in the short run since it increases the interest rate faced by domestic households.²⁴ Such a shock would lower inflation even if output were perfectly stabilized and FX intervention not used because of Home bias in consumption. Optimal monetary policy without FX intervention would consist in increasing the interest rate and letting the currency depreciate instantly, which would lead to a positive output gap.²⁵ These imbalances then recede as the shock vanishes: the output gap is positive, but starts decreasing; inflation is negative, but starts increasing; and the currency, which depreciated on impact, is appreciating.

Following a positive risk premium shock, FX intervention can lean against the wind: selling reserves reduces the endogenous risk premium, lowering the interest rate faced by domestic households. FX intervention limits the magnitude of the interest rate hike and exchange rate devaluation required after the shock, thus stabilizing output and inflation as well as supporting domestic consumption. However, as explained in Section 2.4.7, FX intervention generates an external imbalance that needs

²²When the financial accelerator is active (i.e. $\mu \neq 0$), the results become: $\pi_{H,t} = \frac{(1-\alpha_\psi)\xi_0}{1-\rho\xi} [K(1) - K(\rho\xi)\rho_\xi^t] + \frac{\kappa_\theta}{\alpha_\theta D(1)} \Gamma$; $y_t = -X_y \pi_{H,t}$ $\hat{\theta}_t = \frac{\xi_0}{1-\rho\xi} \left[(1-\alpha_\psi) \frac{D^y(\rho\xi)}{D(\rho\xi)} - 1 \right] \left(\frac{1-\beta}{1-\rho\xi\beta} - \rho_\xi^t \right)$; $\psi_t = \xi_0 \left[(1-\alpha_\psi) \frac{D^y(\rho\xi)}{D(\rho\xi)} - 1 \right] \rho_\xi^t$ where $\Gamma = \frac{(1-\alpha_\psi)\xi_0}{(1-\rho\xi) \left(1 - \frac{\alpha_\pi \kappa_\theta (\kappa_\theta - \alpha_\pi \kappa_f)}{\alpha_\theta D(1)} \right)}$ $\left(\frac{\kappa_\theta}{D(1)} - \frac{\kappa_\theta - \alpha(1-\rho\xi)\kappa_f}{D(\rho\xi)} \frac{1-\beta}{1-\rho\xi\beta} - \alpha_\theta \frac{\beta(1-\rho\xi)}{1-\beta\rho\xi} \right)$; $D(\rho\xi) = 1 + \alpha_\pi \left[(\kappa_y - \kappa_f)^2 + \frac{(\kappa_\theta - \alpha_\pi \kappa_f)^2}{\alpha_\theta} \right] - \rho\xi \left\{ \beta + \kappa_f \left[1 - \alpha_\pi \left(\kappa_y - \kappa_f + \frac{\alpha(\kappa_\theta - \alpha_\pi \kappa_f)}{\alpha_\theta} \right) \right] \right\}$ and $D^y(\rho\xi) = 1 + \alpha_\pi [(\kappa_y - \kappa_f)^2] - \rho\xi \left\{ \beta + \kappa_f [1 - \alpha_\pi (\kappa_y - \kappa_f)] \right\}$; $K(\rho\xi) = \frac{\kappa_\theta - \alpha(1-\rho\xi)\kappa_f}{D(\rho\xi)}$

²³And $\Gamma = \frac{(1-\alpha_\psi)\xi_0}{(1-\rho)} \frac{D(\alpha_\theta, 1)}{D(+\infty, 1)} \frac{1-\beta}{1-\rho\beta} \alpha_\pi \kappa_\theta^2 \left(\frac{1}{D(\alpha_\theta, 1)} - \frac{1}{D(\alpha_\theta, \rho)} \right)$

²⁴This effect is visible through the domestic consumption distortion that results from the exogenous risk premium shock $\bar{\theta}_t = -\frac{\xi_0}{1-\rho} \left(\rho^t - \frac{1-\beta}{1-\rho\beta} \right)$.

²⁵Adding a constraint that $\hat{\theta}_t = 0$ to the optimization problem of the central bank would make the problem collapse to that of Gali and Monacelli (2008), with the central bank facing standard cost-push shocks.

to be offset in the future. As a result, optimal FX intervention has to trade off between the stabilization objective and the cost of external imbalances.

Our closed-form formula enables us to understand how optimal foreign exchange intervention is chosen along this trade-off. The optimal choice for the (endogenous) risk-premium ψ_t is decomposed as follows:

$$\psi_t = - \left[\underbrace{\alpha_\psi}_{\text{terms of trade effect}} + \underbrace{(1 - \alpha_\psi) \left(1 - \frac{D(+\infty, \rho)}{D(\alpha_\theta, \rho)} \right)}_{\text{trade-off between stabilizing consumption and generating an external imbalance}} \right] \xi_t \quad (2.20)$$

(i) *the terms of trade effect:*

As explained in Section 2.4.6, FX interventions can stabilize the terms of trade. The first component in equation (2.20) represents this effect, i.e. the endogenous risk premium is chosen so as to offset the shock to the exogenous risk premium;

(ii) *the trade-off between stabilizing consumption and generating an external imbalance:*

- Stabilizing consumption would call for FX intervention to completely offset the exogenous risk premium, i.e. $\psi_t = -\xi_t$. This would occur if $D(\alpha_\theta, \bullet) = +\infty$, i.e. if FX intervention were perfectly efficient.²⁶
- But FX intervention also induces external imbalances that require future repayments, which is why ψ_t cannot fully offset ξ_t .

This trade-off has to weigh the benefits of stabilizing consumption and the terms of trade vs. the cost of external imbalances. The trade-off is captured by the ratio $\frac{D(+\infty, \rho)}{D(\alpha_\theta, \rho)}$. This ratio, which is always smaller than 1, is a measure of the efficiency of FX intervention because it is the ratio of $1/D(\alpha_\theta, \rho)$, which parametrizes the response of inflation and output to the shock, and $1/D(+\infty, \rho)$, which would parametrize the response of inflation and output to the shock if FX intervention was impossible (e.g. if the cost of external imbalances was prohibitive).

²⁶In the case where $D(\alpha_\theta, \bullet) = +\infty$, Proposition 6 shows that output and inflation are 0.

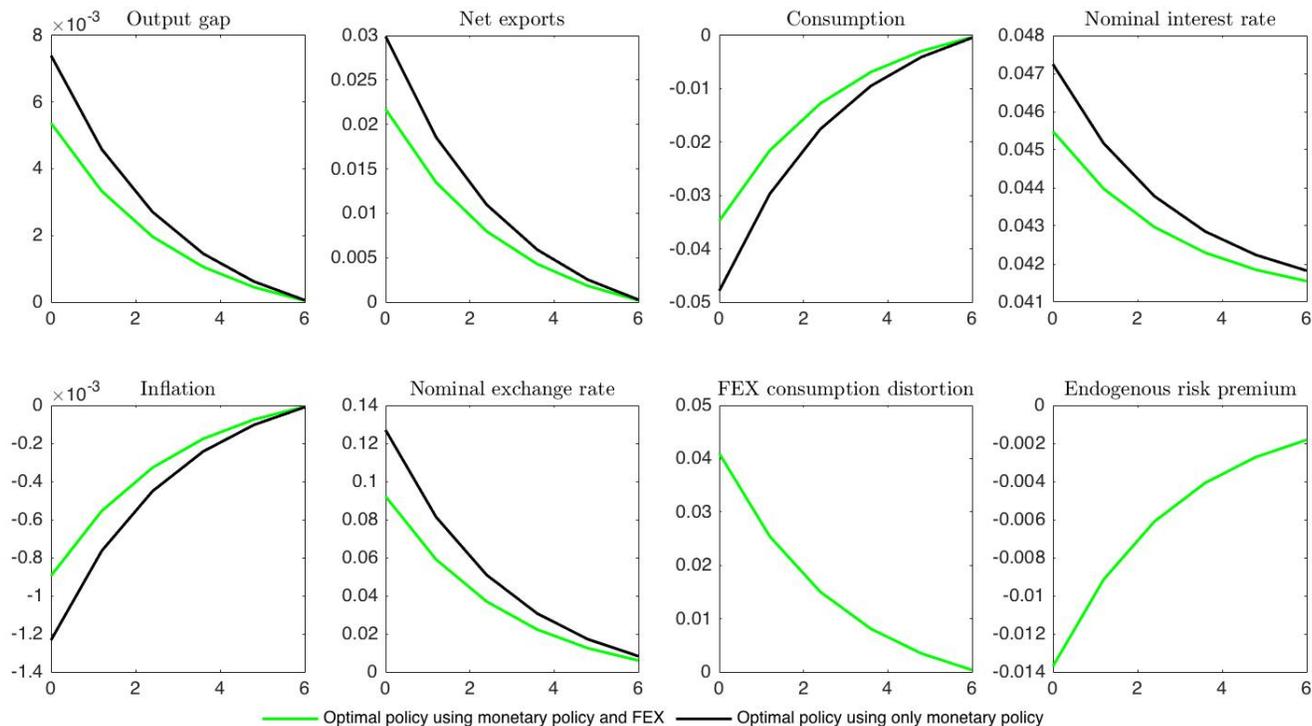


Figure 2.1: The effects of a risk-premium shock under optimal discretionary policy

2.5.2 Simulations

We calibrate our numerical exercise using the same parameters as those in Farhi and Werning (2014), who run a similar exercise to analyze the efficiency of capital controls:

$$\phi = 3, \quad \beta = 0.96, \quad \epsilon = 6, \quad \delta = 1 - 0.75^4, \quad \alpha = 0.2,$$

and the financial accelerator parameter is set to $\mu = 0.2$, following Mendoza (2010). As in Farhi and Werning (2014), the economy is hit with a 5% risk premium shock whose half-life is 2 years. We compare in Figure 2.1 the allocation in which the central bank can only use conventional monetary policy (black line) to the allocation in which the central bank can use both conventional monetary policy and FX intervention (green line).

When only monetary policy is allowed, a risk premium shock provokes a large depreciation of the exchange rate and forces the central bank to increase the interest rate to compensate foreign investors (see equation (2.4)). This policy reaction leads to a large trade surplus (since the currency has depreciated) and a large drop in domestic consumption, with relatively small net effects on domestic output and on (PPI) inflation. We see that the variables that are not directly included in the objective function (the exchange rate, the trade balance and domestic consumption) behave as shock absorbers.

As analyzed in Section 2.5.1, optimal foreign exchange interventions lean against the wind by decreasing the interest rate faced by domestic households and thus supporting domestic consumption. The central bank sells foreign currency reserves, reducing the endogenous risk premium by

about 1.4% initially. As a result, the fall in domestic consumption is mitigated (on impact, 3.5% vs. 4.8% when FX intervention was not allowed), the trade surplus is smaller (2.2% vs 3% on impact) and the depreciation of the exchange rate is also substantially smaller (9.2% vs. 12.7% at impact). This confirms that FX intervention proves useful as a stabilization tool, especially following risk-premium shocks whose impact on domestic consumption, the trade balance and the nominal exchange rate can be efficiently smoothed.

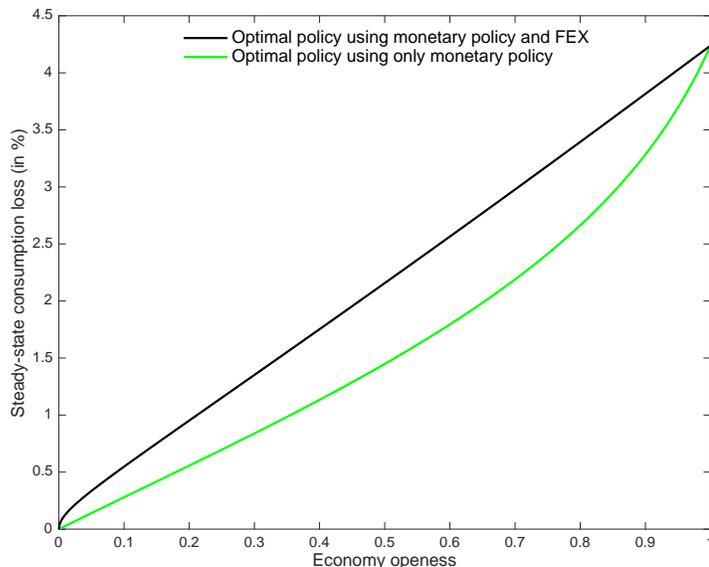


Figure 2.2: Welfare losses following an exogenous risk premium shock (5% shock, two years half-life)

We also quantify the welfare gains allowed by FX intervention by computing the welfare loss due to the exogenous risk premium shock, and comparing this welfare loss in the case where FX intervention is used with the case where it is not allowed. The welfare losses are expressed as a compensatory, per-period, percentage decline in permanent consumption (see Figure 2.2). As expected, welfare losses following the exogenous risk premium shock are initially increasing with the economy's openness. Indeed, the risk premium shock is similar to a shock to domestic consumption. The more open the economy, the larger the difference between consumption and output, and thus the less efficient is monetary policy after risk premium shocks. For reasonable value of openness, the steady-state consumption gains allowed by FX intervention impact are sizable (around 0.4% of permanent consumption for $\alpha = 0.2$; see LHS panel of Figure 2.3), and the relative impact is important too (40% of the welfare losses are canceled for $\alpha = 0.2$; see RHS panel of Figure 2.3).

However, the relative cost of FX intervention due to external imbalances is also increasing in the economy's openness. Indeed, the benefits of stabilizing output when the economy is fully open are null, and therefore the relative cost of external imbalances is infinite (see Section 2.4.7). From Proposition 6, we know that FX intervention is not useful ($\hat{\theta} \rightarrow 0$) when $\alpha \rightarrow 1$ since:

$$\alpha_{\psi} \xrightarrow{\alpha \rightarrow 1} 0 \quad \text{and} \quad \frac{D(+\infty, \rho)}{D(\alpha_{\theta}, \rho)} \xrightarrow{\alpha \rightarrow 1} 1$$

As a result, when $\alpha \rightarrow 1$, the welfare losses due to risk premium shocks are identical in the two cases.

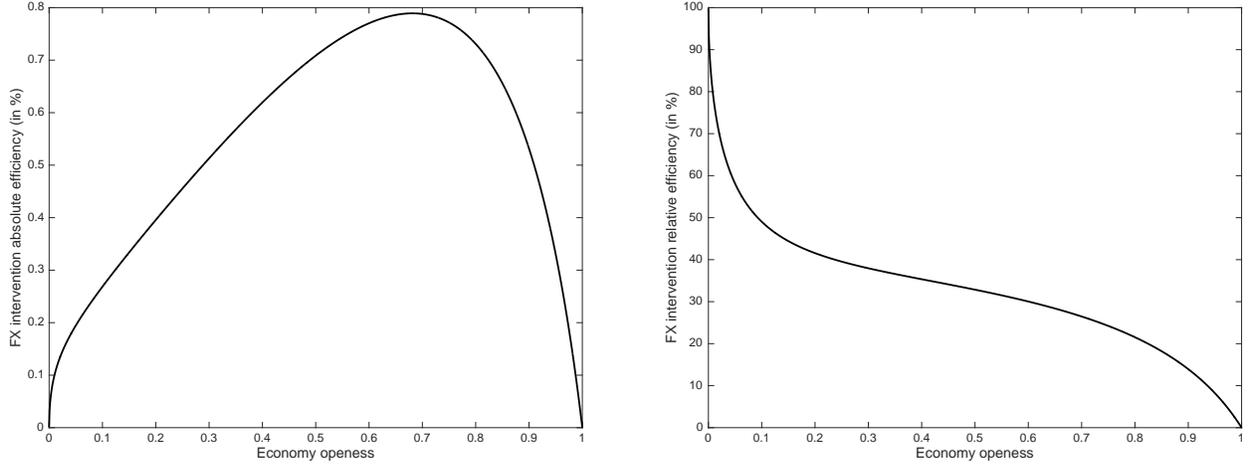


Figure 2.3: Welfare gains from using FX intervention following an exogenous risk premium shock

2.6 FX Intervention and speculative attacks

We now discuss the risks of multiple equilibria in the presence of a financial friction. We only consider speculative shocks (deviations in expected values), having in mind in particular self-fulfilling currency movements, when analyzing equilibrium uniqueness. Substituting for the interest rate in the Phillips Curve using the IS curve, the central bank's problem (2.18) becomes:

$$\begin{aligned} \min_{\pi_{H,t}, y_t, \theta_t} \quad & \alpha_\pi \pi_{H,t}^2 + y_t^2 + \alpha_\theta \hat{\theta}_t^2 \quad \text{s.t.} \\ \pi_{H,t} = & (\beta + \kappa_f) \pi_{H,t+1}^e + (\kappa_y - \kappa_f) y_t + \kappa_f y_{t+1}^e + (\kappa_\theta - \alpha \kappa_f) \hat{\theta}_t + \alpha \kappa_f \hat{\theta}_{t+1}^e \\ \sum_{t=0}^{\infty} \beta^t \hat{\theta}_t = & 0 \end{aligned} \quad (2.21)$$

2.6.1 Optimal policy

The extended Phillips Curve in equation (2.21) shows how the financial friction affects the dynamics of inflation (and thus of the exchange rate). Compared to the standard New Keynesian Phillips Curve, future inflation has a bigger weight as a determinant of current inflation (since $\kappa_f > 0$), whereas the coefficient on current output is smaller (also by κ_f). The first-order conditions on y_t and θ_t are:²⁷

$$\alpha_\pi (\kappa_y - \kappa_f) \pi_{H,t} + y_t = 0 \quad (2.22)$$

$$[\kappa_\theta - \alpha \kappa_f] \pi_{H,t} + \alpha_\theta \theta_t + \Gamma = 0 \quad (2.23)$$

²⁷Since there are no exogenous risk premium shock in this section, the total consumption distortion θ_t coincide with the consumption distortion resulting from foreign exchange interventions $\hat{\theta}_t$. We then simply refer to the total consumption distortion θ_t as the policy tool in this section.

where, Γ is, as before, the Lagrange multiplier for the intertemporal budget constraint. Domestic inflation thus obeys the following law of motion:

$$\pi_{H,t} = \frac{\beta + \kappa_f - \alpha_\pi \kappa_f \left(\kappa_y - \kappa_f + \frac{\alpha(\kappa_\theta - \alpha\kappa_f)}{\alpha_\theta} \right)}{1 + \alpha_\pi (\kappa_y - \kappa_f)^2 + \frac{\alpha_\pi (\kappa_\theta - \alpha\kappa_f)^2}{\alpha_\theta}} \pi_{H,t+1}^e - \frac{\frac{\kappa_\theta}{\alpha_\theta}}{1 + \alpha_\pi (\kappa_y - \kappa_f)^2 + \frac{\alpha_\pi (\kappa_\theta - \alpha\kappa_f)^2}{\alpha_\theta}} \Gamma \quad (2.24)$$

Equation (2.22) shows that optimal policy is to ‘choose’ a negative output gap when inflation is positive (or a positive output gap when inflation is negative) —otherwise, if the output gap and inflation were both positive, the central bank could reduce both by increasing the interest rate. In other words, the central bank “leans against the wind”, engineering a contraction if inflation is excessive.

Similarly, for a given Lagrange multiplier, equation (2.23) shows that optimal policy is to choose a negative consumption gap when inflation is positive —otherwise, if the consumption gap and inflation were both positive, the central bank could reduce both by increasing the level of reserves and the risk premium (an increase in the risk premium decreases current consumption, and thus aggregate demand, for a given level of future consumption; see equations (2.6) and (2.7)). The central bank again “leans against the wind”, increasing the level of reserves and the risk premium when inflation is positive. Conventional and unconventional policies steer inflation and output in the same direction. However, looking now at the exchange rate, the impact of an increase in the interest rate is partially offset by an increase in the endogenous risk ($e_{t+1}^e - e_t = i_t - i_t^* - \psi_t$). Sterilized intervention thus allows the central bank to limit the effect of its conventional monetary policy on the exchange rate.

2.6.2 Equilibrium Determinacy

In the standard New Keynesian Model, optimal policy ensures equilibrium uniqueness. We show here that this result does not hold in the presence of the financial accelerator and discuss how FX intervention can help.

Proposition 7. Equilibrium Determinacy under Discretionary Policy

Equilibrium determinacy is ensured when the Blanchard-Kahn condition is satisfied, i.e. when: ²⁸

$$\alpha_\pi > \frac{\beta + \kappa_f - 1}{\kappa_y (\kappa_y - \kappa_f) + \frac{\kappa_\theta (\kappa_\theta - \alpha\kappa_f)}{\alpha_\theta}} \quad (2.25)$$

Proof: The proof consists in applying the Blanchard-Kahn condition to equation (2.24). □

If the coefficient of the financial accelerator is larger than $1 - \beta$, equilibrium under discretionary policy can be indeterminate, a possibility absent in the standard open economy New Keynesian Model (see Galí (2008)). It is instructive to analyze the role of foreign exchange intervention in ensuring determinacy. To this aim, we first consider the case in which FX intervention is not possible.

²⁸If $(\kappa_y - \kappa_f)(2\kappa_f - \kappa_y) + \frac{(\kappa_\theta - \alpha\kappa_f)(2\alpha\kappa_f - \kappa_\theta)}{\alpha_\theta} > 0$, which implies $\mu > \frac{1}{2}$, the condition has an upper bound: $\frac{\beta + \kappa_f - 1}{\kappa_y (\kappa_y - \kappa_f) + \frac{\kappa_\theta (\kappa_\theta - \alpha\kappa_f)}{\alpha_\theta}} < \alpha_\pi < \frac{\beta + \kappa_f + 1}{(\kappa_y - \kappa_f)(2\kappa_f - \kappa_y) + \frac{(\kappa_\theta - \alpha\kappa_f)(2\alpha\kappa_f - \kappa_\theta)}{\alpha_\theta}}$

Equilibrium determinacy without FX intervention

If FX intervention is not available, the condition for determinacy is:

$$\alpha_\pi > \frac{\beta + \kappa_f - 1}{\kappa_y(\kappa_y - \kappa_f)} \quad (2.26)$$

Denoting by X_y the size of the output loss engineered by the central bank when inflation is 1 percent (i.e. $X_y = \alpha_\pi(\kappa_y - \kappa_f)$, obtained from equation (2.22)), the condition for equilibrium determinacy is rewritten as:

$$1 + \left(\underbrace{\kappa_y - \kappa_f}_{\text{impact of current output on current inflation}} + \underbrace{\kappa_f}_{\text{impact of expected output on current inflation}} \right) X_y > \beta + \kappa_f$$

or equivalently:

$$\kappa_y X_y > \beta + \kappa_f - 1$$

Intuitively, determinacy requires that, when inflation is positive, the central bank decides to engineer an economic slowdown such that the total impact on today's inflation is stronger than the impact of expected inflation on today's inflation $\beta + \kappa_f$ in the Phillips curve (see problem (2.21)). This ensures that current inflation is lower than expected inflation, ruling out multiple equilibria.

In absence of the financial accelerator (i.e. $\kappa_f = 0$), the equilibrium is always unique, because $\beta - 1 < 0$. The central bank's optimal policy would be to hike interest rates and engineer a recession if confronted with speculative increases in inflation. The negative output gap would lower inflation below what is expected by firms, thus ruling out self-fulfilling inflation and currency movements.²⁹

However, with a financial friction, the decision to increase the interest rate would also increase firms' marginal costs. The recession must thus be deeper, or the sensitivity of inflation to the output gap higher, to ensure marginal costs are sufficiently reduced. If the weight of inflation in the loss function is too low, the slowdown engineered by the central bank may be insufficient to offset the impact of the financial friction on inflation. Current inflation ends up being too high, thus justifying self-fulfilling, multiple, equilibria.

Equilibrium determinacy with FX intervention

We now reintroduce FX intervention, and using equation (2.23), we define $X_\theta = \frac{\alpha_\pi}{\alpha_\theta}(\kappa_\theta - \alpha\kappa_f) > 0$ as the marginal increase in the consumption wedge for a decrease in the rate of inflation. Reacting to an expected appreciation of the currency, the central bank can accumulate reserves to increase the risk premium. For a given increase in the interest rate, an increase in the risk premium would depreciate the currency. The Blanchard-Kahn condition becomes:

$$\kappa_y X_y + \kappa_\theta X_\theta > \beta + \kappa_f - 1 \quad (2.27)$$

The rationale is as before. The optimal use of reserves (and its use in period $t + 1$) can mitigate current inflation, the more so if the effect of the instrument on current and future inflation is high (i.e., κ_θ is high) and if the central bank uses this instrument aggressively (if X_θ is high).

²⁹More precisely, inflation is always lower than future inflation in the absence of a financial accelerator; thus, the only non-explosive path for inflation (and the exchange rate) is the one with zero inflation. This is why when the Blanchard-Kahn condition is always satisfied in this case.

Figure 2.4 shows the zone of indeterminacy provided by conditions (2.25) and (2.27). When using reserves comes at no cost ($\alpha_\theta = 0$, see left-hand chart), or when reserves have a strong effect on inflation (X_θ is high, see right-hand chart), the risk of indeterminacy is eliminated, even if the central bank is not willing to hike interest rates and engineer recessions. The downward sloping frontier in the right-hand chart depicts the trade-off: for a given impact of the interest rate, the central bank must either be willing to engineer large recessions (X_y is large) or to be activist with its FX reserves (X_θ large).

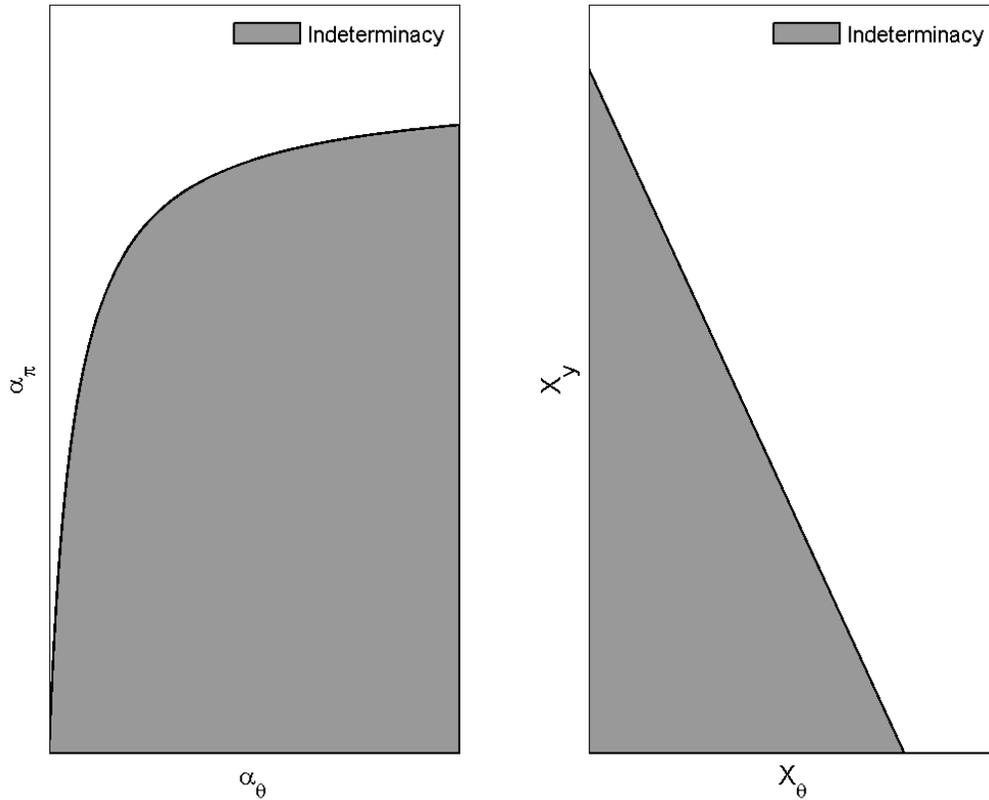


Figure 2.4: Optimal policy determinacy condition

2.7 Conclusion

Although empirical research has been sometimes skeptical of the capacity of central banks to influence the exchange rate, central banks often make use of their reserves with the hope that FX intervention can be used as a second instrument of monetary policy. In particular, it is likely that FX intervention does have an effect in open economies with small debt markets or small FX markets. Notwithstanding the aforementioned limitations of FX intervention, it is natural to investigate how these intervention should interact with interest rate policy, and in particular (ii) whether they help stabilize the economy following exogenous shocks to international capital flows (ii) whether they limit the risk of self-fulfilling currency movements. This paper provides answers to these questions in the context of an open economy New Keynesian Model.

To justify the use of a second policy instrument, the New Keynesian Model is extended to add ‘frictions’ to the model, since, from the Tinbergen principle, two instruments would be needed only if there are at least two frictions. To the nominal rigidities that underpin the role for monetary policy in the New Keynesian Model, we add a financial friction and shocks to the uncovered interest parity.

Such an exogenous risk premium shock affects domestic consumption and breaks the divine coincidence. We show that central bank reserves help stabilize domestic allocation following this shock and FX intervention is particularly efficient at stabilizing domestic consumption, the exchange rate and the trade balance. We also show that FX interventions are efficient from a welfare point of view, and can mitigate the welfare losses due to the exogenous risk premium shock.

We also find that self-fulfilling currency movements are possible in the New Keynesian model with financial frictions, along the lines of speculative attack models à la Aghion et al. (2000). We show that in that situation, the central bank’s ability to use FX intervention reduces the range of parameters for which multiple equilibria coexist. If FX intervention is very effective, or if central banks are keen to intervene in the FX market, the central bank can rule out multiple equilibria even if it is not perceived to be sufficiently active with conventional monetary policy.

2.8 Appendix

2.8.1 First-order conditions for Calvo price setting

In the non-linearized model, the Calvo Price setting first-order conditions can be expressed as follow:

$$\frac{1 - \delta(\Pi_{H,t})^{\varepsilon-1}}{1 - \delta} = \left(\frac{F_t}{K_t}\right)^{\varepsilon-1},$$

where

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \frac{1 + \tau}{A_H} Y_t N_t^\phi + \delta \beta \Pi_{H,t+1}^\varepsilon K_{t+1}, \quad F_t = Y_t C_t^{-\sigma} S_t^{-1} Q_t + \delta \beta \Pi_{H,t+1}^{\varepsilon-1} F_{t+1},$$

with PPI inflation denoted $\Pi_{H,t+1} = \frac{P_{H,t+1}}{P_{H,t}}$, and price dispersion Δ_t following the law of motion:

$$\Delta_t = h(\Delta_{t-1}, \Pi_{H,t}), \quad h(\Delta, \Pi) = \delta \Delta \Pi^\varepsilon + (1 - \delta) \left(\frac{1 - \delta \Pi^{\varepsilon-1}}{1 - \delta}\right)^{\varepsilon/(\varepsilon-1)}.$$

2.8.2 Derivation of the intertemporal budget constraint

Assuming that foreign countries are symmetric, Home's budget constraint at date t , which is derived by adding the budget constraints of:

- the households:

$$P_t C_t + D_{t+1}^h + S_{t+1}^b + T_t^{fin} \leq (1 + i_{t-1}) (D_t^h + S_t^b) + W_t N_t + \Pi_t + T_t^{firm} + T_t^{gov}$$

- the central bank:

$$E_t R_{t+1}^* - E_t (1 + i_{t-1}^*) \Xi_{t-1} R_t^* = S_{t+1}^b - (1 + i_{t-1}) S_t^b$$

- the government:

$$\frac{1 + i_{t-1}}{\Psi_{t-1}} D_t^f + (1 + i_{t-1}) (D_t^h + S_t^b) + T_t^{gov} \leq D_{t+1}^f + D_{t+1}^h + \tau W_t N_t + T_t^{fin}$$

The consolidated budget constraint is then:

$$E_t R_{t+1}^* - E_t (1 + i_{t-1}^*) \Xi_{t-1} R_t^* + \frac{1 + i_{t-1}}{\Psi_{t-1}} D_t^f \leq D_{t+1}^f + P_{H,t} Y_t - P_t C_t$$

We define the country net foreign asset position³⁰ and the trade balance as follows:

$$NFA_t = \frac{(1 + i_{t-1}^*) \Xi_{t-1} E_t}{P_{F,t} C_t^{*\sigma}} \left[R_t^* - \frac{D_t^f}{E_{t-1}} \right], \quad NX_t = P_{H,t} Y_t - P_t C_t$$

Introducing the net foreign asset position and the trade balance in the consolidated budget constraint, we have:

$$\frac{P_{F,t+1} C_{t+1}^{*\sigma}}{(1 + i_t^*) \Xi_t} NFA_{t+1} - P_{F,t} C_t^{*\sigma} NFA_t = NX_t$$

³⁰Normalizing by foreign consumption and taking the foreign price at home $P_{F,t}$ as the numeraire.

The Euler equation for foreign households is:

$$1 + i_t^* = \beta^{-1} \left(\frac{C_{t+1}^*}{C_t^*} \right)^\sigma \frac{P_{F,t+1}}{P_{F,t}}$$

Using the two above equations, we then have:

$$NFA_t = -\frac{1}{P_{F,t}C_t^*}NX_t + \frac{\beta}{\Xi_t}NFA_{t+1}$$

2.8.3 Derivation of the loss function

We first have the exact relationship:

$$c_t = (1 - \alpha)s_t + \theta_t$$

And the following second-order approximation of the goods market clearing condition $Y_t = S_t C^*[(1 - \alpha)\Theta_t + \alpha]$:

$$y_t = s_t + (1 - \alpha)\theta_t + \frac{1}{2}\alpha(1 - \alpha)\theta_t^2$$

We use this result to derive:

$$c_t = (1 - \alpha)y_t + \alpha(2 - \alpha)\theta_t - \frac{1}{2}\alpha(1 - \alpha)^2\theta_t^2$$

By the labor market clearing condition, we have up to second-order approximation:

$$n_t = y_t + \log\Delta_t^P + \frac{1}{2}y_t^2$$

By Woodford (2003), we have:

$$\sum_{t=0}^{\infty} \beta^t \log\Delta_t^P = \frac{\varepsilon_P}{2\lambda_P} \int_0^{\infty} e^{-\rho t} \pi_{H,t}^2$$

Finally, using $\bar{N}^{1+\phi} = (1 - \alpha)$ and integrating over time, we have the following expression for the objective function:

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - \bar{U}}{CU_c} \right) = \\ & - \frac{(1-\alpha)(1+\phi)}{2} \int_0^{\infty} e^{-\rho t} \left\{ \alpha\pi_{H,t}^2 + y_t^2 - \frac{2\alpha(2-\alpha)}{(1-\alpha)(1+\phi)}\theta_t + \alpha\frac{1-\alpha}{1+\phi}\theta_t^2 \right\} \end{aligned}$$

We now use a second order approximation of the budget constraint to replace the linear term θ_t in the expression above. We find:

$$- \sum_{t=0}^{\infty} \beta^t \theta_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\bar{\theta}_t^2 + (\bar{\theta}_0 - \bar{\theta}_t)\theta_t \right] + t.i.p.$$

The sum $\sum_{t=0}^{\infty} \beta^t \theta_t$ only has second order terms in θ_t , so we can get rid of $\bar{\theta}_0 \sum_{t=0}^{\infty} \beta^t \theta_t$ when developing up to the second order the welfare loss.

We then get the following loss function (up to additive terms independent of policy and multiplicative constants):

$$\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\alpha_{\pi} \pi_{H,t}^2 + y_t^2 + \alpha_{\theta} \left[\hat{\theta}_t + \alpha_{\psi} \bar{\theta}_t \right]^2 \right]$$

where

$$\alpha_{\pi} = \frac{\varepsilon_P}{\lambda_P(1+\phi)}, \quad \alpha_{\theta} = \frac{\alpha}{1+\phi} \left(\frac{2-\alpha}{1-\alpha} + 1 - \alpha \right), \quad \alpha_{\psi} = \frac{1-\alpha}{\frac{2-\alpha}{1-\alpha} + 1 - \alpha}$$

2.8.4 Proof of Proposition 6

For analytical convenience, we express the consumption distortion θ_t in deviation from the steady state. The problem boils down to:

$$\begin{aligned} \min_{\pi_{H,t}, y_t, \hat{\theta}_t, \psi_t, \theta_t, e_t} \quad & \alpha_{\pi} \pi_{H,t}^2 + y_t^2 + \alpha_{\theta} \left[\theta_t - (1 - \alpha_{\psi}) \bar{\theta}_t \right]^2 \quad \text{s.t.} \\ \pi_{H,t} = & \beta \pi_{H,t+1}^e + \kappa_y y_t + \kappa_{\theta} \theta_t \\ \sum_{t=0}^{\infty} & \beta^t \theta_t = 0 \end{aligned}$$

Proof. The first-order conditions on the two instruments are:

$$\alpha_{\pi} \kappa_y \pi_{H,t} + y_t = 0 \quad \alpha_{\pi} \kappa_{\theta} \pi_{H,t} + \alpha_{\theta} \left[\theta_t - (1 - \alpha_{\psi}) \bar{\theta}_t \right] - \Gamma = 0$$

Since $\bar{\theta}_t = \frac{\xi_0}{1-\rho} \left(\frac{1-\beta}{1-\rho\beta} - \rho^t \right)$ for exponentially decreasing risk premium shocks: $\xi_t = \xi_0 \rho^t$, substituting the first-order conditions in the Phillips Curve and iterating, we get:

$$\begin{aligned} \pi_{H,t} = & \frac{\kappa_{\theta}(1-\alpha_{\psi})\xi_0}{1-\rho} \left(\frac{1-\beta}{1-\rho\beta} \frac{1}{D(1)} - \frac{1}{D(\rho)} \rho^t \right) + \frac{\kappa_{\theta}}{\alpha_{\theta} D(1)} \Gamma \\ \text{with } \Gamma = & \frac{(1-\alpha_{\psi})\xi_0}{(1-\rho)} \frac{D(1)}{D^y(1)} \frac{1-\beta}{1-\rho\beta} \alpha_{\pi} \kappa_{\theta}^2 \left(\frac{1}{D(1)} - \frac{1}{D(\rho)} \right) \\ y_t = & -\alpha_{\pi} \kappa_y \pi_{H,t} \quad \text{and} \quad \theta_t = \frac{(1-\alpha_{\psi})\xi_0}{1-\rho} \frac{D^y(\rho)}{D(\rho)} \left(\frac{1-\beta}{1-\rho\beta} - \rho^t \right) \end{aligned}$$

Since $\bar{\theta}_t = \frac{\xi_0}{1-\rho} \left(\frac{1-\beta}{1-\rho\beta} - \rho^t \right)$, we find that the foreign exchange intervention impact on consumption can be described as follows:

$$\hat{\theta}_t = \frac{\xi_0}{1-\rho} \left[(1-\alpha_{\psi}) \frac{D^y(\rho)}{D(\rho)} - 1 \right] \left(\frac{1-\beta}{1-\rho\beta} - \rho^t \right)$$

□

Part II

Optimal Fiscal Policy in a Currency Union

Chapter 3

Optimal Fiscal Policy in a Currency Union: Fiscal Devaluation or Fiscal Reevaluation?

3.1 Abstract

We consider a standard small open economy model with a fixed exchange rate to study optimal fiscal policy in a currency union. The only fiscal tools available for the government are value-added and payroll taxes, subject to a revenue-neutrality constraint. We study both optimal time-varying and one-time fiscal policy, following a variety of macroeconomic shocks. First, we show that the optimal fiscal policy is determined by a policy trade-off between two channels respectively based on a consumption and a competitiveness stimulus, whose respective strength depends on the tax pass-through to producer prices. These two channels respectively call for a VAT/labor tax decrease in the optimal time-varying response. However, only the competitiveness channel matters for one-time fiscal devaluations: negative demand shock should be followed by a permanent labor tax decrease. Second, we show that the sign of the optimal time-varying fiscal policy (and thus the prevailing channel) depends on the nature of the macroeconomic shock (demand/supply). Following a negative demand shock and under plausible tax pass-through assumptions, it is optimal to decrease the VAT in the short term to support domestic consumption, thus replicating a fiscal reevaluation. Finally, and contrary to the main criticisms against fiscal devaluations, we find that optimal fiscal devaluations (both time-varying and one-time) are of the same order of magnitude than the occurring macroeconomic shocks, and substantially efficient from a welfare point of view to absorb the impact of the macroeconomic shock.

3.2 Introduction

Monetary policy is commonly seen as the most appropriate stabilization tool to respond to macroeconomic disturbances in the presence of nominal rigidities. However, countries belonging to a currency union, who decide to peg their bilateral exchange rates, are deprived of the possibility of such an adjustment following asymmetric shocks. Indeed, the well-known Mundell's trilemma states that a country can not simultaneously have free capital flows, a fixed exchange rate and an independent monetary policy. Advanced economies, notably in the Eurozone, favored the first two features, consequently losing monetary policy autonomy. Such a margin is yet extremely useful. Currency union's members face the impossibility to engineer countercyclical monetary/exchange

rate policies to promote macroeconomic stabilization. The current crisis in the Eurozone shed light on the necessity for the member states to proceed to internal/external adjustments to restore their competitiveness facing idiosyncratic shocks.

The use of fiscal tools to replicate the effects of a flexible exchange rate and an independent monetary policy have thus become a crucial political issue, notably in the euro area, these last years. Germany has for instance implemented a "so-called" fiscal devaluation in 2007¹, thus before the beginning of the financial crisis. Some years later and in the course of the eurozone sovereign debt crisis, France has engineered several labor tax decreases since 2012, notably the *Crédit Impôt Compétitivité Emploi* by 2013, to stimulate its domestic economy. Eurozone governments have thus generally been inclined to decrease the payroll tax these last years, sometimes compensating the subsequent fiscal loss with a consumption tax increase. However, this policy has also been strongly contested by "austerity opponents", who argue that such a fiscal scheme would depress consumption and have a recessive impact. The appropriate use of fiscal policy depending on the economic circumstances has then become one of the most controversial policy issues these last years, which calls for a reassessment of the role of fiscal policy in the light of the recent economic and financial crises.

Indeed, even though the idea of using fiscal policy to replicate exchange rate movements has been extensively discussed in the economic literature², as reviewed in details in Farhi et al. (2013), the usual approach consists in analyzing how exchange rate devaluations can be replicated with fiscal instruments: the optimal fiscal policy can then be explicitly derived using canonical monetary/exchange rate policy results. However, less efforts have been devoted to directly assessing the optimal fiscal policy, under realistic policy schemes (we detail below what we mean by this), and depending on the macroeconomic circumstances. The optimal fiscal policy to implement following a given macroeconomic shock is then still an open question.

A first branch of the literature on fiscal devaluations then assumed *ex ante* the fiscal policy to implement, namely a simultaneous VAT increase and a social security contribution decrease, and tried to assess the ability of this fiscal scheme to reduce external imbalances, similarly to an exchange rate devaluation. The results are quite mitigated, but rather conclude to a positive and moderate impact of fiscal devaluations on external imbalances. Feldstein and Krugman (1990), and more recently De Mooij and Keen (2012), have emphasized the idea that an increase in consumption taxes accompanied by a balanced-budget cut in labor taxes tends to have no long-run effect on trade patterns if changes in domestic goods and production factors prices undo the effects of the tax changes. Lipinska and Von Thadden (2009), following the same rationale, finds that fiscal devaluations long-run effectiveness is subject to a number of caveats. More recently, Engler and al. (2013) emphasize that fiscal devaluations have a small positive impact to correct structural imbalances. This branch of the literature describes the VAT increase/social security contribution decrease as the macroeconomic shock engineered to correct structural imbalances. Naturally in this situation, large fiscal devaluations are required (if they are implemented in response to external imbalances for instance), their long-run efficiency being small since prices and wages finally adjust. An other branch of the literature adopts a more analytical approach, and explores under which conditions fiscal policy and exchange rate policy are equivalent. In this context, a seminal contribution was proposed by Farhi et al. (2013), who show that with a rich enough set of distortionary taxes

¹The VAT rate was raised by 3 points, meanwhile the employer social security contributions decreased by 1.3 points, see Bernoth and al. (2014)

²Keynes started this tradition by arguing in 1931 that a uniform ad valorem tariff on all imports coupled with a uniform subsidy on all exports would have the same impact than an exchange rate devaluation.

and under varying degrees of price rigidity and asset market assumptions, the flexible exchange rate allocation can be achieved.

Our approach differs in two crucial ways from Farhi et al. (2013).

First, we restrict the policy toolbox to the tools usually considered in the political debate and actually used by governments these last years, namely the payroll and value-added taxes. These taxes have been intensively used by Eurozone countries since 2008, and have been proposed as potential candidates in policy circles, e.g. IMF (2011). Conversely, Farhi et al. (2013) first-best policy assumes that the VAT, the payroll tax, the labor income tax and a consumption subsidy are simultaneously available. However, it seems that less instruments are actually available for policy makers: for instance, there is no obvious existing candidate to implement a consumption subsidy which is proportional to the consumption value for each household. Fiscal devaluations incomplete implementations, i.e. fiscal policy which is only based on the VAT and the payroll tax as usually considered in the literature, are then keen to have different implications, as shown in our paper. Second, and more importantly from a theoretical point of view, we depart from Farhi et al. (2013) by allowing for arbitrary fiscal policy pass-through. Since our analysis takes place in a currency union, the exchange rate pass-through is indeed not a central issue: the bilateral nominal exchange rates being fixed between the currency union members. However, all previous works made implicit, but crucial assumptions about fiscal policy pass-through to producer prices. Lipinska and Von Thadden (2009), Franco (2011) and Engler and al. (2013) assume that the VAT short-run pass-through is complete, whereas the labor-tax one is zero. This assumption is rather consistent with the few existing empirical evidence (see Carbonnier (2007) and subsection 3.3.2). However, Farhi et al. (2013) results, which are valid both under Producer Currency Pricing (PCP) and Local Currency Pricing (LCP), require restrictive and rather implausible assumptions on fiscal pass-through³. We then decide to focus on the PCP case⁴, but allow for arbitrary fiscal pass-through to producer prices. We show that these pass-through implicit assumptions nest a policy trade-off, between competitiveness and consumption stimulus, that is at the heart of the "austerity debate".

Our contribution then intends to revisit the existing literature on fiscal devaluations in light of the recent policy experience. We then want to assess how fiscal devaluations can help a country member of a currency union to regain a stabilization margin and ease its adjustment path following exogenous shocks. We describe optimal fiscal devaluations following exogenous shocks in a standard New Keynesian open economy model with a fixed exchange rate. The only fiscal tools we use are value-added and payroll taxes, subject to a revenue-neutrality constraint⁵.

We then characterize the sign, size and efficiency of optimal fiscal devaluations in a linearized optimal policy framework. This setup is close to the one introduced by Gali and Monacelli (2005), and extended in Farhi and Werning (2012). Microfounding fiscal policy in it, we derive endogenous costs and dynamics, that enable us to derive optimal fiscal devaluations depending on the exogenous macroeconomic shock.

First, we show that the optimal fiscal policy is determined by a policy trade-off between two channels respectively based on a consumption and a competitiveness stimulus, whose respective strength depends on the tax pass-through to producer prices. These two channels respectively call for a VAT/labor tax decrease in the optimal time-varying response. However, only the competitiveness channel matters for one-time fiscal devaluations: negative demand shock should be followed by a permanent labor tax decrease.

³Assuming that the VAT and the payroll tax pass-through are equal.

⁴Which is without loss of generality for our analytical results derived under perfectly rigid or flexible prices.

⁵This question would then still be relevant in a closed economy located in a currency union.

Second, we show the sign of the optimal time-varying fiscal policy (and thus the prevailing channel) depends on the nature of the macroeconomic shock (demand/supply). Following a negative demand shock and under plausible tax pass-through assumptions, it is optimal to decrease the VAT in the short term to support domestic consumption, thus replicating a fiscal reevaluation. However, we show that only the competitiveness channel matters for one-time fiscal devaluations as long as fiscal revenues are rebated to households: negative demand shock should be followed by a permanent labor tax decrease.

Finally, and contrary to the main criticisms against fiscal devaluations and contrary to the traditional views fuelled by previous numerical works, we find that optimal fiscal devaluations (both time-varying and one-time) are of the same order of magnitude than the occurring macroeconomic shocks, and substantially efficient from a welfare point of view to offset the impact of the macroeconomic shock.

The paper is structured as follows. Section 2 introduces our small open economy model. Section 3 introduces the log-linearized framework and highlights the channels through which fiscal devaluations affect the economy. Section 4 characterizes optimal fiscal devaluations starting from polar cases to grasp some intuition, to a more realistic framework with staggered price and wage settings. It also provides numerical illustrations and quantify fiscal devaluations efficiency. Section 5 concludes.

3.3 A small open economy

We expand the model of Galí and Monacelli (2005) to explore optimal fiscal devaluations as stabilization policies for countries located in a currency union, and consequently deprived of monetary and exchange rate autonomy. We then normalize the nominal exchange rate between each pair of countries to one. Since the currency union as a whole can implement monetary policy and has a free exchange rate with the rest of the world, we are only interested in intra-union macroeconomic dynamics and idiosyncratic shocks.

As in Farhi and Werning (2012), we focus on one-time unanticipated shocks at date $t = 0$.

3.3.1 Households

There is a continuum measure one of countries $i \in [0, 1]$. We focus on a single country, called Home, which can be thought of as a particular value $H \in [0, 1]$. Only Home implements active fiscal policy in this paper.

Household utility

In each country, there is a continuum of households denoted by $h \in [0, 1]$. A typical household seeks to maximize:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t(h)^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\phi}}{1+\phi} \right] \quad (3.1)$$

where $N_t(h)$ is the quantity of labor supplied and $C_t(h)$ is household h aggregate consumption, with

$$C_t(h) = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^i(h)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^i(h)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

α denotes the openness parameter which defines the share of domestic consumption $C_{H,t}$:

$$C_{H,t}(h) = \left(\int_0^1 C_{H,t}(j, h)^{\frac{\epsilon_P - 1}{\epsilon_P}} dj \right)^{\frac{\epsilon_P}{\epsilon_P - 1}}$$

where $j \in [0, 1]$ is an individual good variety produced at Home.

This openness parameter can be interpreted as a measure of home bias. Since the VAT affects domestic consumption and thus the share of output that is domestically consumed, this openness parameter shapes the optimal fiscal response size and sign. Indeed, when $\alpha \rightarrow 0$, the share of foreign goods in domestic consumption vanishes and the economy can be seen as closed with almost no trading with the rest of the world. In this situation, using the VAT to stabilize domestic consumption will be an efficient way to affect domestic output. Conversely, when $\alpha \rightarrow 1$, the economy becomes fully open and the share of domestic goods in Home consumption becomes null. It is then inefficient to try to stabilize output through domestic consumption distortion.

Similarly, $C_{F,t}(h)$ is the consumption index of imported goods, defined by

$$C_{F,t}(h) = \left(\int_0^1 \Lambda_{i,t}^{\frac{1}{\gamma}} C_t^i(h)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

where $C_{i,t}(h)$ is an index of imported consumption by domestic household h from country i :

$$C_{i,t}(h) = \left(\int_0^1 C_{i,t}(j, h)^{\frac{\epsilon_P - 1}{\epsilon_P}} dj \right)^{\frac{\epsilon_P}{\epsilon_P - 1}}$$

As in Farhi and Werning (2012), we introduce a shifter $\Lambda_{i,t}$ which captures the taste for imports from country i . Changes in $\Lambda_{H,t}$ can then be seen as export-demand shocks for Home. This parameter's distribution is common to all countries and normalized so that $\int_0^1 \Lambda_{i,t} = 1$.

Household budget constraint

The typical domestic household h optimization problem consists in maximizing its utility (3.1) subject to the following sequence of budget constraints for each date t :

$$\begin{aligned} & \int_0^1 P_{H,t}(j) C_{H,t}(j, h) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j, h) dj di + D_{H,t+1}(h) + \int_0^1 D_{i,t+1}(h) di \\ & \leq (1 + i_{t-1}) D_{H,t}(h) + \int_0^1 (1 + i_{t-1}^i) \Psi_{t-1} D_{i,t}(h) di + \frac{W_t(h) N_t(h)}{1 + T^s} + \Pi_t + T_t \end{aligned}$$

where $P_{H,t}(j)$ is the price of the domestic variety j , $P_{i,t}(j)$ is the price of variety j imported from country i , $W_t(h)$ is the household h nominal wage, Π_t is nominal profits by domestic firms and T_t the government lump-sum transfer. The two latter are rebated equally between households (thus the absence of the index h).

Domestic consumption prices are inclusive of the VAT: the VAT and the payroll tax pass-through to consumer prices are indeed crucial to design optimal fiscal policy in the presence of price stickiness. We then assume that firms set prices including the VAT, and we allow for an arbitrary pass-through rate to consumer prices. The VAT and the payroll tax are then paid by firms, and do not appear explicitly in the consumer budget constraint. \bar{T}^s denotes the social contributions paid by households on top of wages: this tax is kept constant and will not be part of the fiscal adjustment policy. Proposition 8 details why we introduce this tax.

Finally, households can borrow and lend using riskfree assets provided by each country. The portfolio of an household h is then made of $D_{H,t}(h)$ units of domestic bonds and $D_{i,t}(h)$ units of bonds of country i . They respectively pay the home interest rate i_t and the interest rate in country i i_t^i . We introduce a domestic risk-premium shock Ψ_{t-1} as a wedge between foreign and domestic investors as in Farhi and Werning (2013). We detail the economic intuition for this shock in Subsection 3.3.6.

Price indexes, terms of trade and real exchange rate

Since we use Dixit-Stiglitz aggregators, the corresponding Consumer Price Index (CPI) and Producer Price Index (PPI) at Home are respectively:

$$P_t = \left[(1 - \alpha)(P_{H,t})^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\varepsilon_P} dj \right]^{\frac{1}{1-\varepsilon_P}}.$$

$P_{F,t}$ is the price index for imported goods:

$$P_{F,t} = \left[\int_0^1 \Lambda_{i,t} P_{i,t}^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$$

where $P_{i,t} = \left[\int_0^1 P_{i,t}(j)^{1-\varepsilon_P} dj \right]^{\frac{1}{1-\varepsilon_P}}$ is country's i PPI.

We denote with a star prices excluding the VAT⁶. Since the VAT is rebated to exporting firms, the export price is given by:

$$P_{H,t}^* = (1 - T_t^v) P_{H,t}.$$

Similarly, the price index for imported goods excluding the VAT corresponds to the world price index and is defined by:

$$P_t^* = (1 - T_t^v) P_{F,t}.$$

The terms of trade - the ratio of the import price index to the export price index adjusted for the VAT - is then:

$$S_t = \frac{P_{F,t}}{P_{H,t}}$$

Finally, the consumer-price real exchange rate - the ratio of foreign consumption price index to domestic consumption price index - is:

$$Q_t = \frac{P_t^*}{P_t}.$$

Wage setting

Wage setting is engineered by households. A typical household provides $N_t(h)$ units of labor with $h \in [0, 1]$. Each household is then specialized in the supply of a different type of labor, also indexed by h . Thus, it has some monopoly power in the labor market, and posts the nominal wage at which it accepts to supply specialized labor services to firms that demand them.

Aggregate labor is then defined as the Dixit-Stiglitz aggregator of these different varieties:

$$N_t = \left(\int_0^1 N_t(h)^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dh \right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}$$

⁶Following the law of one price, they also correspond to foreign prices since only Home uses fiscal policy and the exchange rates are normalized to 1.

with the corresponding wage index denoted by:

$$W_t^i = \left(\int_0^1 W_t(h)^{1-\varepsilon_W} dh \right)^{\frac{1}{1-\varepsilon_W}}.$$

3.3.2 Firms

There is a continuum $j \in [0, 1]$ of domestic firms producing monopolistically differentiated goods with a linear technology:

$$Y_t(j) = A_{H,t} N_t(j)$$

where $A_{H,t}$ represents domestic total factor productivity.

Price setting

As in Galí and Monacelli (2005), we assume that the law of one price holds at all times: the price of a given variety in different countries is identical once expressed in the same currency and adjusted for taxes (this assumption is also known as Producer Currency Pricing). This assumption is not restrictive when prices are perfectly flexible or perfectly rigid. Moreover, deriving optimal fiscal policy in a currency union, the key issue is not the exchange rate pass-through but the fiscal ones. We detail this point in the next paragraph.

The VAT is collected on firms: firm j perceives $(1 - T_t^v)P_{H,t}(j)$ on each unit sold. Moreover, on top of wages, firms pay the proportionnal labor tax T_t^n . The real marginal cost deflated by the Home PPI is then:

$$MC_t = \frac{1 + T_t^n}{A_{H,t}} \frac{W_t}{P_{H,t}}$$

When prices are perfectly flexible, firms simply maximize their current profit taking as given the PPI, the marginal cost and current aggregate output:

$$\max_{P_{H,t}(j)} \left[(1 - T_t^v) P_{H,t}(j) Y_{t|t}(j) - P_{H,t} MC_t Y_{t|t}(j) \right]$$

where the demand for firm's j good sold at price $P_{H,t}(j)$ is $Y_{t|t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon_P} Y_{t+k}$. As usual in this monopolistic setting with perfectly flexible prices, firms charge a constant mark-up over their nominal marginal cost:

$$(1 - T_t^v) P_{H,t} = P_{H,t}^* = \frac{\varepsilon_P}{\varepsilon_P - 1} \frac{(1 + T_t^n) W_t}{A_{H,t}}$$

However, in the New Keynesian literature, the need for policy intervention comes from nominal rigidities. We then introduce a sticky price framework.

Fiscal policy pass-through

When designing optimal fiscal policy, tax pass-through assumptions are crucial. One contribution of this paper is to show that the optimal fiscal policy is determined by these pass-through. We then consider a generalized Calvo pricing framework in which firms choose prices inclusive of the VAT, and pass on consumers a given fraction of the VAT and the payroll tax changes.

At each date $t \geq 0$, a randomly selected fraction $1 - \theta_P$ of firms are able to reset their prices. We assume that firms which can not reset their prices mechanically index them to changes in the VAT

and payroll tax, with arbitrary (and constant) index rates. The price dynamics at the firm level then satisfies:

$$P_{H,t}(j) = \begin{cases} \bar{P}_{H,t}, & \text{if adjusts, with probability } 1 - \theta_P \\ \left(\frac{1 - T_t^v}{1 - T_{t-1}^v}\right)^{-\zeta^v} \left(\frac{1 + T_t^n}{1 + T_{t-1}^n}\right)^{\zeta^n} P_{H,t-1}(j) & \text{if does not adjust, with probability } \theta_P \end{cases}$$

The intuition underlying this generalized Calvo pricing is that fiscal policy changes are more easily detected by firms than macroeconomic shocks (see below for a quick empirical review). An analogy could be that fiscal changes are less costly to collect since they are communicated by the government. Moreover, this specification is quite intuitive. If prices were flexible, tax pass-through would indeed be complete:

$$P_{H,t} = \left(\frac{1 - T_t^v}{1 - T_{t-1}^v}\right)^{-1} \left(\frac{1 + T_t^n}{1 + T_{t-1}^n}\right) \frac{W_t}{W_{t-1}} \frac{A_{H,t-1}}{A_{H,t}} P_{H,t-1}$$

This generalized Calvo pricing thus allows firms to erase some of the price distortion introduced by fiscal policy. Allowing for differentiated pass-through for fiscal policy changes and for macroeconomic shocks (nominal wage and productivity), this framework captures the specific impact of fiscal policy on firms behavior, justified by the few existing empirical evidence (see next paragraph). With extreme values, $\zeta^v = 1$ (resp. $\zeta^v = 0$), we find back the situation when prices exclusive (resp. inclusive) of the VAT are sticky.

Previous works make an implicit assumption on these parameters' values. A usual specification (see Franco (2011) and Engler and al. (2013)) assumes that the VAT pass-through is substantially larger than the payroll tax one, with $\{\zeta^v, \zeta^n\} = \{1, 0\}$. On the opposite, Farhi et al. (2013) assumes $\zeta^v = \zeta^n$, and most of the results provided in this paper do not hold when the pass-through values differ. We then allow for arbitrary values of these two parameters, and describe how they affect the fiscal policy impact.

Thus, when firms can reset their prices, they do so by choosing their price $\bar{P}_{H,t}(j)$ in the following maximization problem:

$$\max_{\bar{P}_{H,t}(j)} \sum_{s=t}^{\infty} \theta_P^{s-t} \left(\prod_{h=1}^{s-t} \frac{1}{1 + i_{t+h}} \right) \left[(1 - T_t^v) \hat{P}_{H,s}(j) Y_{s|t}(j) - P_{H,t} MC_s Y_{s|t}(j) \right]$$

with

$$\hat{P}_{H,t}(j) = \left(\frac{1 - T_s^v}{1 - T_t^v}\right)^{-\zeta^v} \left(\frac{1 + T_s^n}{1 + T_t^n}\right)^{\zeta^n} \bar{P}_{H,t}(j)$$

and where the demand for firm's j good at date s if prices were reset for the last time at date t is:

$$Y_{s|t}(j) = \left(\frac{\hat{P}_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_s.$$

The relevant inflation index to measure the price stickiness is then:

$$\Pi_{H,t+1} = \frac{P_{H,t+1}}{P_{H,t}} \left(\frac{1 + T_t^n}{1 + T_{t+1}^n}\right)^{\zeta^n} \left(\frac{1 - T_t^v}{1 - T_{t+1}^v}\right)^{-\zeta^v}$$

In our optimal policy framework (including an objective function), this generalized Calvo pricing works as the usual one, using the above inflation index.

Empirical evidence

As explained in Farhi et al. (2013), evidence on responses of domestic prices to VAT and payroll tax changes are very weak.

First, there is no clear evidence for the payroll tax pass-through⁷. As moving the labor tax affects the supply side of the economy (firms' profits), it is usually described as a medium/long-run policy that gradually affects the real economy due to the price stickiness. Previous works about fiscal devaluations all assumed a zero instantaneous pass-through $\zeta^n = 0$ for the labor tax. We also choose this value in our numerical calibration in subsection 3.5.3.

However, there are some evidence about the VAT impact. Carbonnier (2007) analyzes two steep decreases of the VAT for specific markets⁸, and finds that the instantaneous pass-through was 57 % in the car sales market and 77 % in the household repair service market. VAT pass-through to consumer prices, though differentiated between markets, is expected to be large, and significantly greater than the labor tax pass-through. We then retain $\zeta^v = 0.7$ as the calibrated value in subsection 3.5.3.

Finally, one should remember that we only use these calibrated values to provide numerical illustrations and quantify the efficiency of fiscal devaluations in Subsection 3.5.3, but allow for arbitrary tax pass-through in most of our analysis.

3.3.3 Government

The active policy tools considered in this paper have already been introduced: the VAT and the labor tax, both paid by firms but subject to asymmetric pass-through in the presence of price stickiness.

The budget-neutrality constraint

Our fiscal policy relies on a budget-neutral shift between these taxes. The budget-neutrality constraint is imposed as follows: the additional fiscal revenue following a tax increase must be used to decrease the amount levied by the other tax.

$$(T_t^v - \bar{T}^v) P_t C_t = (\bar{T}^n - T_t^n) W_t N_t$$

Fiscal devaluations can thus not be used to deliver proper fiscal stimulus. Without this constraint, it would be optimal to use the two taxes countercyclically. We then add this constraint to stand for fiscal restrictions Eurozone countries face following the financial crisis.

An other formulation of the budget-neutrality constraint is that the State fiscal revenue, rebated to households through the lump-sum transfer, must be constant once adjusted for the *ex post* aggregate outcomes:

$$T_t = (1 + T_t^v) P_t C_t - P_{H,t} Y_t + \left(T_t^n + \frac{\bar{T}^s}{1 + \bar{T}^s} \right) W_t N_t = (1 + \bar{T}^v) P_t C_t - P_{H,t} Y_t + \left(\bar{T}^n + \frac{\bar{T}^s}{1 + \bar{T}^s} \right) W_t N_t$$

Such a fiscal policy only has indirect effects, that should be positive since such a reform is supposed to stimulate the economic activity.

A final interpretation is that the tax deviations multiplied by the tax bases are constant up to first order. This interpretation will be clear in the log-linearized framework.

⁷The *ex post* analysis of the impact of the *Crédit Impôt Compétitivité Emploi* implemented in France by 2013 should provide empirical assessments of payroll tax pass-through.

⁸From 33.3 % to 18.6 % in the car sales market, from 20.6 % to 5.5 % in the household repair service market.

Fiscal policy schemes

Since we are interested in assessing the efficiency of implementable schemes, we consider two types of fiscal responses:

- time-varying fiscal devaluations in which taxes are reset at each date;
- one-time fiscal devaluations in which taxes are changed once for all at date $t = 0$.

Another contribution of this paper is to show that contrary to the results provided by Lipinska and Von Thadden (2009) or Engler and al. (2013), one-time fiscal devaluations, if carefully designed, can be efficient and substantially welfare improving. Adopting an explicit welfare criterion, and designing optimal fiscal devaluations according to it enables us to measure fiscal devaluations opportunity beyond its impact on output or trade balance.

A second-best environment

In the absence of monetary policy, our one-dimensional policy tool will be called, following the existing literature, a fiscal devaluation. Indeed, following certain types of shocks⁹, it is optimal to increase the VAT and to decrease the payroll tax. This tax change would then increase the imports price and decrease the exports price, similarly to an exchange rate devaluation.

However, this denomination must be regarded cautiously. First, because in some circumstances (following a negative demand shock for instance), the optimal fiscal policy recommended by our framework will rather consist in a fiscal reevaluation (based on a VAT decrease and a labor tax increase). Second, because the fiscal policy implemented in our paper do not replicate the allocations attained under a flexible exchange rate¹⁰. Indeed, the one-dimensional fiscal policy tool we consider is not sufficient to reproduce the flexible exchange rate allocation:

- the budget-neutrality constraint induces side effects from our one-dimensional fiscal tool, involving policy trade-offs¹¹ ;
- the VAT distorts the trade balance, impacts the country's intertemporal budget constraint and then induces a welfare cost.

Thus, this denomination rather refers to the stabilization purpose of our proposed fiscal policy. Farhi et al. (2013) analyzes in details how the flexible exchange rate allocation can be replicated, and naturally finds that the set of fiscal tools should be larger (including at least consumption and payroll subsidy on top of our tools) to offset the two obstacles identified above.

Our fiscal policy will then not be able to reproduce the flexible exchange rate allocation. However, we will see that optimal fiscal devaluations allow a currency union member to offset a substantial part of the macroeconomic shock that can affect this country.

3.3.4 Equilibrium conditions with symmetric rest of the world

We now summarize the equilibrium conditions. Only Home can implement active fiscal policy. The rest of the world is then exogenous and all foreign countries are identical¹². Foreign variables are

⁹We will characterize precisely under which conditions in Sec. 3.5.

¹⁰Farhi et al. (2013) describe in details the conditions under which such an equivalence holds.

¹¹Fiscal devaluations distort simultaneously aggregate consumption, price and wage settings, impacting macroeconomic dynamics in multiple ways. We explore these channels in Subsection 3.4.1.

¹²It was necessary to introduce the model with a continuum of countries, in order to define the elasticity of substitution between domestic and foreign goods. This parameter indeed plays an important role in shaping the channels through which fiscal policy affects the real economy (see Subsection 3.4.1)

denoted with stars. As in Farhi and Werning (2012), we group these equations into two blocks, which we refer to as the demand and the supply block.

The demand block

This block is independent of the nature of price and wage settings. It is made of:

- the Euler equation for each household h

$$1 + i_t = \beta^{-1} \left(\frac{C_{t+1}(h)}{C_t(h)} \right)^\sigma \frac{P_{t+1}}{P_t}$$

Given that initial conditions are symmetric across households, we then see that in each period, each household chooses the same consumption. We then omit the index h for consumption variables by now and express the current consumption as follows

$$C_t = \Theta_t C_t^* Q_t^{1/\sigma}$$

This equation will be called inappropriately the Backus-Smith condition since it is a definition of Θ_t , which is the relative Pareto weight of home in world consumption;

- the arbitrage condition between home and foreign bonds that gives the law of motion of Θ_t

$$\left(\frac{\Theta_{t+1}}{\Theta_t} \right)^\sigma = \frac{1 + i_t}{1 + i_t^*} = \Psi_t$$

where the domestic risk-premium shocks Ψ_t are defined as deviations from the Uncovered Interest Parity (UIP). Θ_t may then be interpreted as cumulative risk-premium along times;

- the relation between the terms of trade and the real exchange rate

$$Q_t = (1 - T_t^v) \left[(1 - \alpha) S_t^{\eta-1} + \alpha \right]^{\frac{1}{\eta-1}}$$

- the goods market clearing condition

$$Y_t = (1 - \alpha) \left[\frac{Q_t}{(1 - T_t^v) S_t} \right]^{-\eta} C_t + \alpha \Lambda_{H,t} S_t^\gamma C_t^*$$

- the labor market clearing condition

$$N_t = \frac{Y_t}{A_{H,t}} \Delta_t^P \Delta_t^W$$

where Δ_t^P and Δ_t^W are respectively indexes of price and wage dispersion defined by

$$\Delta_t^P = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon_P} dj, \quad \Delta_t^W = \int_0^1 \left(\frac{W_t(h)}{W_t} \right)^{-\varepsilon_W} dh$$

- the intertemporal country budget constraint in present-value form¹³, which is obtained by substituting for profits and lump-sum taxes in the representative household budget constraint, solving forward and imposing a no-Ponzi condition

$$0 = - \sum_{t=0}^{\infty} \beta^t \frac{C_t^{*-\sigma}}{\prod_{s=0}^{t-1} \Psi_s} \left[\frac{Y_t}{S_t} - (1 - T_t^v) \frac{C_t}{Q_t} \right] \quad ;$$

This constraint only applies to open economies, and will be crucial to assess the VAT impact on the intertemporal trade balance.

- and the fiscal devaluations budget-neutrality constraint

$$(T_t^v - \bar{T}^v) P_t C_t = (\bar{T}^n - T_t^n) W_t N_t$$

The supply block

This block varies with the nature of price and wage settings. With Calvo price or wage setting, the non-linearized equations are quite involved and presented in Appendix 3.7.1.

3.3.5 Steady State Taxes

Following Farhi and Werning (2012), we derive the optimal condition for steady state taxes set uncooperatively by each country in a symmetric steady state without any rigidity:

Proposition 8. Steady State Taxes

Suppose we are in a symmetric steady-state without macroeconomic fluctuation and rigidity. Then, the VAT, the labor tax and the social contribution tax are pinned down in the following way:

$$\frac{(1 + \bar{T}^s)(1 + \bar{T}^n)}{1 - \bar{T}^v} = \frac{1}{\mu_P \mu_W} \frac{(1 - \alpha)(\eta - 1) + \gamma}{(1 - \alpha)\eta + \gamma - 1}$$

$$\text{where } \mu_P = \frac{\varepsilon_P}{\varepsilon_P - 1} \text{ and } \mu_W = \frac{\varepsilon_W}{\varepsilon_W - 1}$$

and fiscal devaluations are Pareto inefficient.

We find back that at the deterministic steady-state, only one tax is necessary to achieve the trade-off between offsetting firms and households' monopoly power at home and exercising it abroad (see Gali (2008) that introduces a payroll subsidy). Since the steady state is set uncooperatively, the distortions are imposed to foreign consumers, justifying the term $\frac{(1-\alpha)(\eta-1)+\gamma}{(1-\alpha)\eta+\gamma-1}$ (larger than 1 and increasing in the trade openness) on the right hand side of the above formula.

As usual in the New Keynesian framework, this steady state relation does not fit well the taxes implemented in most countries. The choice of steady state taxes value is then sensitive. However, we are mostly interested in taxes deviation from this steady state rather than in tax levels. Since our fiscal policy relies on the VAT and the payroll tax, we decide to calibrate these steady state taxes at their observed value. The social contribution tax is then pinned down by the above equality and could stand in a stylized way for welfare state subsidies. These steady state taxes at their observed value. The social contribution tax is then pinned down by the above equality and could stand in a stylized way for welfare state subsidies.

¹³Since Ricardian equivalence holds here.

3.3.6 Shocks

In the spirit of Farhi and Werning (2012), we want to characterize optimal fiscal devaluations implemented in response to macroeconomic shocks. The economy is initially at the deterministic steady state and faces an unanticipated shock at date $t = 0$. We consider both supply and demand shocks.

Our supply shock is a productivity shock $\{A_{H,t}\}_{t \geq 0}$ that impacts firms' efficiency. A productivity decrease is followed by a price increase and a real wage decrease in the absence of nominal rigidity. We consider two types of idiosyncratic demand shocks¹⁴: a risk-premium shock on the domestic interest rate $\{\Psi_t\}_{t \geq 0}$, and an export-demand shock $\{\Lambda_{H,t}\}_{t \geq 0}$. The risk-premium shock on the domestic interest rate is a stylized description for sudden stops. Without being microfounded here, it stands for the risk of investing in the domestic country when foreign borrowers estimate it differently from domestic lenders. We use this shock to explore the impact of the required bonds yields increase in the euro area periphery, which notably led to the debt crisis by 2010 in the euro area. The export-demand shock indexes the taste for the imports from Home to foreign countries. This shock allows for heterogeneous export performances which are not related to price differentials (lower range level or differentiation for instance).

These shocks ; either because they are commonly used to explain heterogeneous economic situations in the Euro area (productivity, export-demand), or because they are part of the story of the recent debt crisis (risk-premium) ; seem compelling to deliver insights about the ability for advanced economies located in currency areas to regain some stabilization margin through fiscal policy adjustments.

A crucial insight from our analysis is that optimal fiscal devaluations are determined by the nature of the shock: similar macroeconomic paths or identical initial conditions (price/wage rigidity, public debt) may call for opposite fiscal policy. Contrary to most of the previous analyses (see Engler and al. (2013) for instance), we do not consider fiscal policy as the exogenous macroeconomic shock without which no adjustment would occur. Conversely, we describe fiscal devaluations as the policy tool that is useful to ease the adjustment path following a specific macroeconomic shock. The usefulness of active fiscal policy is then measured with a microfounded welfare criteria. Our structural analysis then allows us to get insights about both fiscal devaluations opportunity and efficiency.

3.4 The log-linearized framework

In this Section, we introduce the log-linearized framework and highlight the channels through which fiscal devaluations affect the economy¹⁵. Given our log-linearized framework, optimal fiscal devaluations are linear in the considered macroeconomic shocks.

We describe in Appendix 3.7.4 the log-linearized framework. Except for interest, inflation and tax rates, lower-case variables x_t^i refer to log-deviations from the deterministic steady state under flexible prices and wages. The natural allocation is defined as the allocation obtained without any rigidity or fiscal intervention (corresponding variables are denoted with double bars). In the staggered price and wage framework, variables are expressed in gaps from the natural allocation (corresponding variables being denoted with hats).

¹⁴We could also consider aggregate demand shocks, such as shocks to foreign consumption or inflation. However, for ease of reading, we only consider idiosyncratic shocks.

¹⁵It was easier to introduce the model in a discrete time version to describe households and firms behavior. For analytical reasons, we now turn to a continuous time version of the model.

3.4.1 The pass-through impact: two competing channels

The fiscal policy scheme we consider consists in shifting the fiscal burden between the payroll tax and the VAT. The common story behind fiscal devaluations is that it should result in decreasing the export prices and increasing the import ones. Besides readjusting the trade balance, fiscal devaluations should also have a positive effect on domestic output: the payroll tax decrease should enable domestic firms to lower their export prices and offset the VAT increase on domestic prices, leading to a demand increase both from foreign and domestic consumers (that face more expensive imports).

Thus, a crucial issue regarding fiscal devaluations impact is the tax changes pass-through to producer and consumer prices. When prices are perfectly flexible, the pass-through are complete and the above reasoning is valid. However, when the VAT and the payroll tax pass-through to consumer prices are asymmetric¹⁶, this may not be true anymore. Indeed, if the VAT pass-through is higher than the labor tax one, the rationale can be reversed and fiscal reevaluation favored: in order to prevent output from plummeting following a negative demand shock¹⁷, it may be preferable to decrease the VAT to favor domestic consumption.

We then see that tax pass-through nest a policy trade-off between competitiveness and consumption stimulus. To highlight the channels at stake, we assume in this Subsection that domestic prices are only moved by fiscal policy¹⁸. We refer to this extreme situation as "the rigid price case", that will be useful to understand the short and medium run dynamics. Indeed, if firms adjust their prices along time, in the short-run, prices only respond to fiscal changes.

Transitory fiscal policy

We first consider the transitory component of the optimal time-varying policy: we then describe the VAT deviation following a shock to the terms of trade derivative.

Fiscal policy impact on the output gap

In this situation, the following equation relates the output gap and the fiscal policy:

$$\dot{y}_t + \left(\begin{array}{ccc} \underbrace{\frac{1-\alpha}{\sigma}}_{\text{VAT impact on domestic consumption}} & - & \underbrace{\frac{1-\zeta^v}{\hat{\sigma}}}_{\text{VAT impact on the terms of trade}} & - & \underbrace{\frac{\kappa\zeta^n}{\hat{\sigma}}}_{\text{Payroll tax impact on the terms of trade}} \end{array} \right) \dot{\tau}_t^v = -\dot{s}_t$$

where $\hat{\sigma} = \frac{\sigma}{1+\alpha\omega}$ and $\omega = \sigma\gamma - 1 + (1-\alpha)(\sigma\eta - 1)$

We first see that the size of the VAT deviation required to offset the output gap derivative is proportional to the intertemporal elasticity of substitution σ , e.g. the household desire to smooth consumption.

The VAT impact on output is then twofold:

- it impacts the CPI and affects the domestic consumption through the Backus-Smith condition: a VAT increase decreases domestic consumption. This impact decreases with the economy openness. Indeed, when the economy becomes more open, the share of domestic consumption in domestic output decreases, reducing this stabilization margin ;

¹⁶Which is very likely from an empirical point of view, see Subsection 3.3.2.

¹⁷Under flexible prices, such a shock would involve an instantaneous price decrease to curb the output fall.

¹⁸ $\theta_P = 1$ in the generalized Calvo pricing introduced in Subsection 3.3.2.

- it impacts the terms of trade, thus distorting output and the trade balance: a VAT increase lowers imports consumption if domestic firms pass on consumer a smaller share of the VAT change than foreign firms¹⁹. This impact also increases with the elasticity of substitution between domestic and foreign goods η and between foreign goods γ . Indeed, as these parameters increase, domestic and foreign consumption become more sensitive to price differentials, reinforcing the terms of trade channel.

Thus, when the VAT pass-through increases or the trade openness decreases, the impact on domestic consumption increases whereas the trade effect decreases: a VAT increase is then less attractive following a terms of trade worsening.

The payroll tax effect on output is straightforward: it impacts the Home PPI and thus the terms of trade. Similarly to the VAT impact on the terms of trade, this effect increases with the payroll tax pass-through to the CPI.

The two competing channels

We then define two channels whose respective strength will drive the transitory component of the optimal time-varying policy:

$$\dot{\hat{y}}_t + \left(\underbrace{\frac{1-\alpha}{\sigma}}_{\text{The consumption channel}} - \underbrace{\frac{1-\zeta^v + \kappa\zeta^n}{\hat{\sigma}}}_{\text{The competitiveness channel}} \right) \dot{\hat{\tau}}_t^v = -\dot{\hat{s}}_t$$

The short-term effect of fiscal policy, that is crucial for transitory shocks, thus strongly depends on the pass-through assumptions in the presence of price stickiness.

We rigorously show (using the microfounded objective function) how these channels define the optimal fiscal policy in Section 3.5.

Permanent fiscal policy

We now turn to the opposite situation in which the fiscal policy response is constant over time.

The consumption channel (ir)relevancy

Let us first understand why the consumption channel is inactive in this situation. Since the VAT is constant over time, consumption is also constant if the terms of trade do not move²⁰: consumption then only changes once for all at date $t = 0$. Following the Backus-Smith condition, this change depends on the weight deviation $\hat{\theta}_0$ and on the VAT change. The intertemporal budget constraint gives:

$$\hat{\theta}_0 = \rho \int_0^\infty e^{-\rho t} \left(\frac{\tau^v}{\sigma} + K \hat{y}_t \right) dt = \frac{\tau^v}{\sigma} + K \rho \int_0^\infty e^{-\rho t} \hat{y}_t dt$$

The initial weight deviation then offsets the discounted VAT deviation, which is equal to the permanent VAT change here. This equality simply states that moving the VAT induces a trade imbalance that is offset by an initial consumption deviation. However, this equality holds if, and only if, the VAT moves once for all at date $t = 0$. Indeed, future VAT changes, even when the

¹⁹Their pass-through is normalized to 1 due to the PCP assumption.

²⁰See the Backus-Smith condition and the relation between the real exchange rate and the terms of trade in Subsection 3.3.4.

shock has vanished, affect the initial weight adjustment and turn the consumption channel on (see Subsection 3.5.3).

The consumption deviation is then up to first order:

$$\hat{c}_t = \frac{1-\alpha}{\sigma} \hat{s}_t + \hat{\theta}_0 - \frac{1}{\sigma} \tau^v = \frac{1-\alpha}{\sigma} \hat{s}_t - \rho K \int_0^\infty e^{-\rho t} \hat{y}_t dt$$

Proposition 9. Consumption channel irrelevancy

Assume that prices are perfectly rigid and that the economy is not closed. In this case, only the competitiveness channel matters for one-time fiscal devaluations. Fiscal policy impact on output can then be expressed as follows:

$$\left[1 - \tilde{K}\right] \hat{y}_t - \underbrace{\frac{1 - \zeta^v + \kappa \zeta^n}{\hat{\sigma}}}_{\text{The competitiveness channel}} \tau^v = - \left[1 - \tilde{K}\right] \bar{s}_t - \tilde{K} \int_0^\infty e^{-\rho s} \bar{s}_s ds$$

$$\text{where } \tilde{K} = \frac{(1-\alpha)(\frac{\omega}{\sigma}-1)\hat{\sigma}}{1+(1-\alpha)(\frac{\omega}{\sigma}-1)\hat{\sigma}} \leq 1$$

A terms of trade deterioration should then be followed by a permanent VAT increase (and labor tax decrease). It is interesting to note that in the spirit of Farhi and Werning (2012), the consumption channel irrelevancy is "discontinuous" in the closed economy limit. Indeed, when $\alpha \rightarrow 0$, even though the trade surplus goes to zero, the intertemporal budget constraint adjusts the Pareto weight to offset the trade imbalances, muting the consumption channel. However, when the economy is closed²¹, the intertemporal budget constraint does not apply and consumption falls once for all following a VAT increase.

We now introduce the objective function necessary to define our optimal policy problem.

3.4.2 The loss function and the role of trade openness

From now on, we focus on the Cole-Obstfeld case that imposes $\sigma = \eta = \gamma = 1$ and serves as a benchmark in the optimal policy literature (see Gali and Monacelli (2005) or Farhi and Werning (2012)). Since we are first interested in understanding the tax pass-through impact on optimal fiscal policy, we restrict ourself to this calibration to derive easily the loss-function around the deterministic steady-state. Deriving the optimal policy outside of the Cole-Obstfeld case is possible (forthcoming paper), but the results can not be expressed with transparent closed formula (the loss function derivation being long and involved).

For $\sigma = \eta = \gamma = 1$, we get the following second-order approximation for the welfare loss function²² that is valid to obtain a first-order approximation of optimal fiscal devaluations:

$$\frac{1}{2} \int_0^\infty e^{-\rho t} \left[\alpha_\pi \pi_{H,t}^2 + \alpha_{\pi_W} \pi_t^W + \left[\hat{y}_t + \frac{\alpha}{1+\phi} (\lambda_{h,t} - \theta_t) \right]^2 + \alpha_\tau \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1 - \alpha_\psi) \theta_0 \right]^2 \right] dt$$

where

²¹But still located in a currency union, i.e. without an independent monetary policy. The risk-sharing condition then applies as introduced above.

²²Up to additive and multiplicative terms independent of policies, see Appendix 3.7.5.

$$\alpha_\pi = \frac{\varepsilon_P}{\lambda_P(1+\phi)}, \quad \alpha_{\pi_W} = \frac{\varepsilon_W(1+\varepsilon_W\phi)}{\lambda_W(1+\phi)}, \quad \alpha_\tau = \frac{\alpha}{1+\phi} \left(\frac{2-\alpha}{1-\alpha} + 1 - \alpha \right), \quad \alpha_\psi = \frac{1-\alpha}{\frac{2-\alpha}{1-\alpha} + 1 - \alpha}$$

In the absence of export-demand and risk-premium shocks, the first three terms are familiar in the New Keynesian framework with price and wage rigidities, and identical to those obtained in Gali and Monacelli (2005). We then see that even in the absence of nominal rigidities, the first best would not consist in perfectly stabilizing output gap and inflation, leaving room for terms of trade manipulation. This desire to manipulate intertemporally the terms of trade in response to transitory shocks is explained in Costinot and al. (2011), and the optimal policy with capital controls is detailed in Farhi and Werning (2012). Although we could derive the optimal fiscal policy with flexible prices and wages, we prefer focusing on assessing optimal policies under realistic assumptions on nominal rigidities. Moreover, from a quantitative point of view, the per-period permanent consumption gain obtained from this manipulation is particularly small compared to the losses induced by the nominal rigidities (less than a percent). Thus, this terms of trade manipulation motive will not be relevant in the presence of nominal rigidities.

The fourth term captures the direct distortions induced by the VAT on the trade balance. Decreasing the VAT induces a consumption increase for both domestic and foreign goods, leading to an import increase while leaving exports unchanged: it would then imply a trade deficit. Since this deficit must be compensated in the future by trade surpluses, this intertemporal constraint enters the loss function with a quadratic cost indexed by α_τ . Without export-demand and risk-premium shocks (that affect the trade balance), this cost simply becomes $\alpha_\tau (\tau_t^v - \hat{\theta}_0)^2$, penalizing VAT deviations from its discounted average.

This VAT deviation cost is increasing with the openness parameter α . Indeed, in a closed economy limit, distorting the trade balance would incur no welfare loss; and since consumption and output coincide, a VAT change directly targets the output gap. However, as the economy opens, the welfare losses associated to the trade balance distortion increase, and consumption impacts less and less output. The opportunity of using the VAT as a stabilization tool then decreases with the economy openness, which is reflected by an increasing welfare cost.

3.4.3 The Planning problem: comparison with the flexible exchange rate case

The planner solves the following optimal control problem, which is made of:

- An objective function:

$$\min_{\{\pi_{H,t}, \pi_t^W, \hat{y}_t, \tau_t^v, \tau_t^n\}} \frac{1}{2} \int_0^\infty e^{-\rho t} \left[\alpha_\pi \pi_{H,t}^2 + \alpha_{\pi_W} \pi_t^{W2} + \left[\hat{y}_t + \frac{\alpha}{1+\phi} (\lambda_{h,t} - \theta_t) \right]^2 + \alpha_\tau \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1 - \alpha_\psi) \theta_0 \right]^2 \right] dt$$

- A dynamic system that comprises:

- The New Keynesian Phillips Curve:

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \lambda_P \alpha \hat{y}_t - \lambda_P \hat{\omega}_t + \lambda_P [\kappa - (1 + \alpha(1 - \alpha))] \tau_t^v + \lambda_P \alpha (1 - \alpha) \hat{\sigma} \hat{\theta}_0$$

- The New Keynesian Wage Phillips Curve:

$$\dot{\pi}_t^W = \rho \pi_t^W - \lambda_W [(1 - \alpha) + \phi] \hat{y}_t + \lambda_W \hat{\omega}_t + \lambda_W \alpha (2 - \alpha) (\tau_t^v - \hat{\theta}_0)$$

- The dynamic saving equation:

$$\dot{\hat{y}}_t = i_t^* + \psi_t - \pi_{H,t} - \bar{r}_t - (\zeta^v - \alpha - \kappa \zeta^n) \tau_t^v$$

– The law of motion of the real wage:

$$\dot{\hat{\omega}} = \pi_t^W - (1 - \alpha)\pi_{H,t} - [(1 - \alpha)(\zeta^v - \kappa\zeta^n) - \alpha]\dot{\tau}_t^v - \alpha\pi_t^* - \dot{\bar{\omega}}_t$$

– The balanced fiscal devaluation constraint:

$$0 = \kappa\tau_t^v + \tau_t^n$$

– The intertemporal budget constraint:

$$\hat{\theta}_0 = \rho \int_0^\infty e^{-\rho t} \tau_t^v dt$$

– The initial conditions on price and wage stickinesses:

$$\hat{y}_0 + (\zeta^v - \alpha - \kappa\zeta^n)\tau_0^v - (1 - \alpha)\hat{\theta}_0 = -\bar{s}_0 \quad \hat{\omega}_0 + [(1 - \alpha)(\zeta^v - \kappa\zeta^n) - \alpha]\tau_0^v = -\bar{\omega}_0$$

Comparison with the flexible exchange rate case

Since our fiscal policy is commonly called a fiscal devaluation, it is essential to understand the links between this policy and the flexible exchange rate situation.

With a flexible exchange rate and an independent monetary policy²³, the central bank interest rate i_t^* must be replaced by the domestic interest rate $i_t^* + \dot{e}_t$. Home can then drive domestic output and intertemporal consumption decisions by changing the interest rate, and monetary policy can be used as a stabilization tool.

Moreover, the initial conditions on price and wage stickinesses become:

$$\hat{y}_0 = e_0 - \bar{s}_0, \quad \hat{\omega}_0 = -\bar{\omega}_0 - \alpha e_0, \quad \text{where } e_0 \text{ refers to the nominal exchange rate.}$$

In the presence of nominal rigidities, a flexible exchange rate allows domestic prices and wages to jump. For instance, a terms of trade appreciation ($\bar{s}_0 < 0$) can be offset by an exchange rate depreciation ($e_0 > 0$) which results in stabilizing output.

As it is well known, in the presence of one nominal rigidity and when the natural allocation is the first best, a flexible exchange rate and an independent monetary policy allow to achieve perfect economic stabilization. When both prices and wages are sticky, the real wage is also rigid, the *divine coincidence* disappears and there is a trade-off between stabilizing inflation and the output gap (see Blanchard and Gali (2005)).

Our fiscal scheme shares some features with the flexible exchange rate situation. As the dynamic saving equation shows, the VAT allows to reallocate consumption intertemporally, and can then be used as a stabilization tool similar to monetary policy. However, as explained in Subsection 3.4.2, in our microfounded framework, the VAT is a distortive tool which is costly from a welfare point of view (contrary to monetary policy in the usual New Keynesian framework). Finally, fiscal policy also allows initial adjustments for the real wage and the terms of trade, as would a flexible exchange rate do.

We now formalize the intuitions developed in this section by deriving optimal fiscal devaluations depending on the nominal rigidities assumptions.

²³Since the Uncovered Interest parity holds, the two are equivalent here.

3.5 Optimal fiscal devaluations

In this Section, we derive insights about the features that make fiscal devaluations desirable. We then consider polar cases to get analytical results that clearly evidence the optimal policy rationale, depending on the nominal rigidities and fiscal pass-through.

3.5.1 Rigid prices

Since our policy tends to distort firms' pricing behavior through fiscal incentives, price stickiness is naturally going to play a decisive role in shaping the optimal fiscal response.

We first consider the "rigid price case" introduced in Subsection 3.4.1. Following our general Calvo pricing formulation, firms are allowed to pass a fixed share of the VAT and the payroll tax change at each period. The following proposition is valid for any wage dynamics, as long as there is no wage dispersion or if the wage dispersion cost verifies $\alpha_{\pi_W} = 0$.²⁴ For instance, it is then valid when wages are perfectly flexible or perfectly rigid.

Proposition 10. Rigid prices

Assume prices are perfectly rigid and wage inflation is not costly. The optimal time-varying fiscal devaluation in response to any change in the path $\{a_{h,t}, \lambda_{h,t}, \psi_t\}_{t \geq 0}$ is:

$$\begin{aligned} \tau_t^v = & - \frac{\zeta^v - \alpha - \kappa \zeta^n}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left[a_t - \rho \int_0^\infty e^{-\rho s} a_s ds \right] + \alpha \frac{\zeta^v - \alpha - \kappa \zeta^n - \frac{1-\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left(\lambda_t - \rho \int_0^\infty e^{-\rho s} \lambda_s ds \right) \\ & + (1 - \alpha) \frac{\zeta^v - \alpha - \kappa \zeta^n + \frac{\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left(\int_0^t \psi_s ds - \int_0^\infty \psi_s ds \right) + \frac{\rho}{1 - \zeta^v + \kappa \zeta^n} \int_0^\infty e^{-\rho s} (a_s - \lambda_{h,s}) ds \end{aligned}$$

which can be decomposed into:

- A mean-reverting-component:

$$\begin{aligned} \tau_t^{v,mr} = & - \frac{\zeta^v - \alpha - \kappa \zeta^n}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left[a_t - \rho \int_0^\infty e^{-\rho s} a_s ds \right] + \alpha \frac{\zeta^v - \alpha - \kappa \zeta^n - \frac{1-\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left(\lambda_t - \rho \int_0^\infty e^{-\rho s} \lambda_s ds \right) \\ & + (1 - \alpha) \frac{\zeta^v - \alpha - \kappa \zeta^n + \frac{\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left(\int_0^t \psi_s ds - \int_0^\infty \psi_s ds \right) \end{aligned}$$

- A permanent component:

$$\tau^{v,p} = \frac{\rho}{1 - \zeta^v + \kappa \zeta^n} \int_0^\infty e^{-\rho s} (a_s - \lambda_{h,s}) ds$$

The mean-reverting component

The rationale for fiscal devaluations is here guided by the nominal price rigidity. Since prices do not respond to demand shocks, all the adjustment goes through quantities, involving output overadjustment. Since prices also do not respond to supply shocks, it implies a too high real wage following negative productivity shocks. Thus, the fiscal action tends to reduce the output gap adjustment:

$$\dot{y}_t + (\zeta^v - \alpha - \kappa \zeta^n) \dot{\tau}_t^v = -\dot{\bar{s}}_t = -\dot{a}_t + \frac{\alpha \phi}{1 + \phi} \dot{\lambda}_t + \frac{1 + (1 - \alpha) \phi}{1 + \phi} \psi_t$$

²⁴Wage inflation is then not costly from a welfare point of view.

Positive productivity, negative export-demand and risk-premium shocks appreciate the terms of trade at the natural allocation. They should then be followed by a producer price decrease if prices were flexible.

Two channels compete to engineer this PPI decrease and curb the output fall:

- the consumption channel, that calls for a VAT cut to decrease the CPI and stimulate the domestic consumption;
- the competitiveness channel, that calls for a labor tax cut to make the domestic products cheaper and a VAT increase to make the imports more expensive

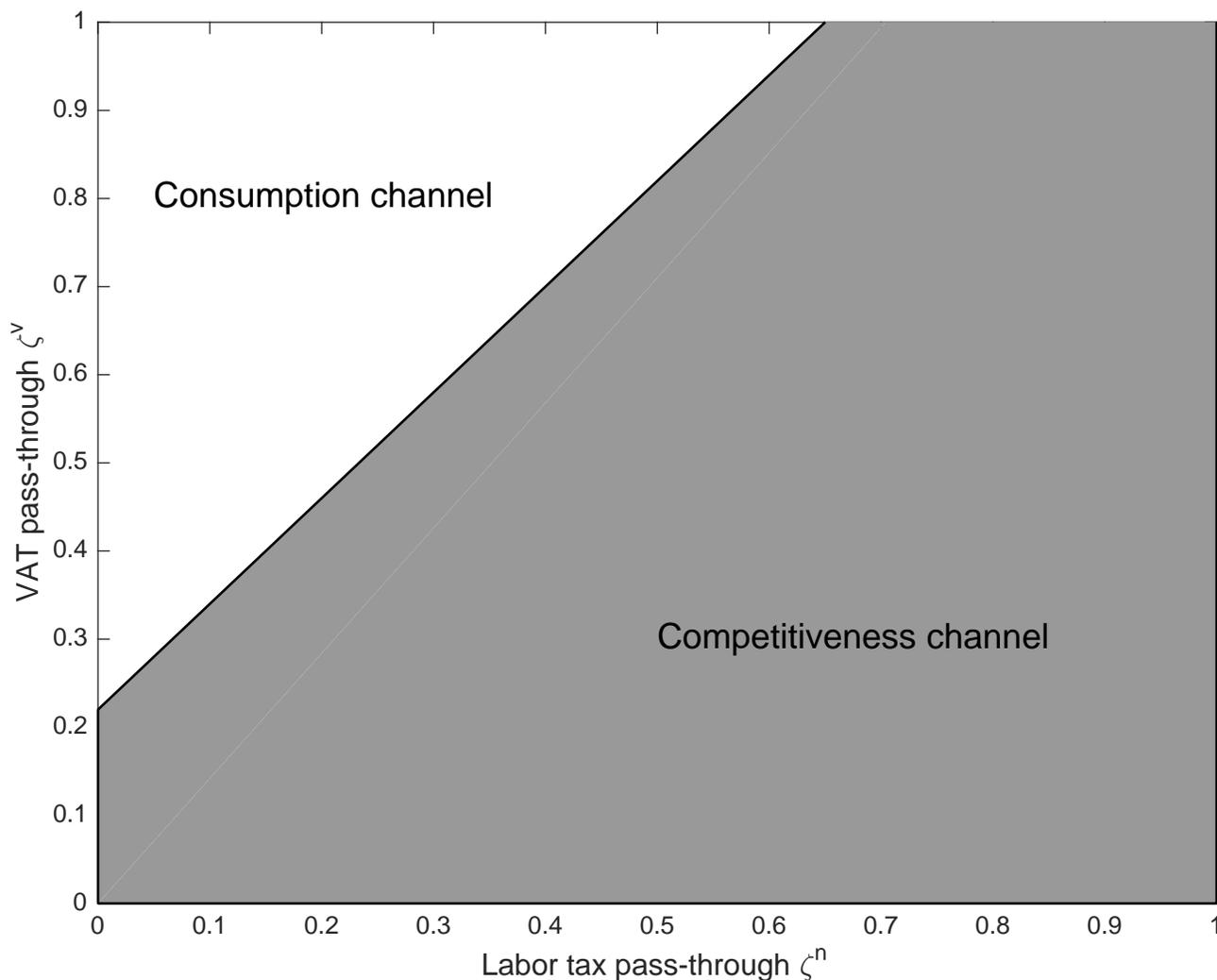


Figure 3.1: Optimal policy determinacy condition: the two competing channels

Our baseline assumption, following the few existing empirical evidence (see Subsection 3.3.2), is that the VAT pass-through to producer prices is notably larger than the payroll tax one. This assumption is implicit in Engler and al. (2013), Franco (2011) and in Lipinska and Von Thadden (2009). In that case, the consumption channel should be favored: the VAT decrease smooths the output overadjustment by supporting domestic consumption. Following a negative demand shock,

the VAT decrease then replicates a fiscal reevaluation in the short term. It is finally crucial to note that this result is valid for any wage dynamics, as long as wage inflation is not costly or if there is no wage dispersion: it will then be true in the short-run when both prices and wages are sticky (in this situation, wages are rigid in the short term and there is no wage dispersion).

Optimal one-time fiscal devaluations

Corollary 1 (Rigid prices) *The permanent component of the optimal time-varying fiscal devaluation and the optimal one-time fiscal devaluation coincide. They are given by:*

$$\tau^{n,p} = \frac{\rho\kappa}{1 - \zeta^v + \kappa\zeta^n} \int_0^\infty e^{-\rho s} \bar{s}_s ds = \frac{\rho\kappa}{1 - \zeta^v + \kappa\zeta^n} \int_0^\infty e^{-\rho s} (-a_s + \lambda_{h,s}) ds$$

It is first interesting to see that the two policy responses we consider, the time-varying and the permanent fiscal devaluation, are complementary. The optimal time-varying response simply adds a transitory component (see above) to the optimal one-time fiscal devaluation.

As explained in Subsection 3.4.1, the consumption channel is irrelevant for one-time fiscal devaluations. We then express the optimal policy in terms of the labor tax. We find back that a terms of trade appreciation (positive supply shock, negative demand shock) should be followed by a labor tax decrease to replicate a PPI decrease. One-time optimal fiscal devaluations then target the deviations induced by the terms of trade discounted value. It is then intuitive to see that they are not used to respond to purely transitory shocks such as risk-premia.

Finally, since this permanent policy induces no distortionary cost²⁵ here, the tax deviation is potentially unbounded and simply tries to adjust the PPI at its optimal average level: the stronger the competitiveness channel, the lower the required tax deviation.

3.5.2 Flexible prices and rigid wages

We now turn to the complementary situation in which prices are perfectly flexible and wages are completely fixed.

Proposition 11. Flexible prices and rigid wages

Assume that prices are perfectly flexible and wages are perfectly rigid. The optimal fiscal devaluation in response to any change in the path $\{a_{h,t}, \lambda_{h,t}, \psi_t\}_{t \geq 0}$ is:

$$\begin{aligned} \tau_t^v = & -\alpha \frac{\kappa - (1-\alpha) + \frac{(1-\alpha)\phi}{1+\phi}}{\alpha_\tau + [\kappa - (1-\alpha)]^2} \left(\lambda_t - \rho \int_0^\infty e^{-\rho s} \lambda_s ds \right) - (1-\alpha) \frac{\kappa - (1-\alpha) - \frac{\alpha}{1+\phi}}{\alpha_\tau + [\kappa - (1-\alpha)]^2} \left(\int_0^t \psi_s ds - \int_0^\infty \psi_s ds \right) \\ & - \frac{\rho}{\kappa} \int_0^\infty e^{-\rho s} \lambda_{h,s} ds \end{aligned}$$

which can be decomposed into:

- A mean-reverting-component:

$$\tau_t^{v,mr} = -\alpha \frac{\kappa - (1-\alpha) + \frac{(1-\alpha)\phi}{1+\phi}}{\alpha_\tau + [\kappa - (1-\alpha)]^2} \left(\lambda_t - \rho \int_0^\infty e^{-\rho s} \lambda_s ds \right) - (1-\alpha) \frac{\kappa - (1-\alpha) - \frac{\alpha}{1+\phi}}{\alpha_\tau + [\kappa - (1-\alpha)]^2} \left(\int_0^t \psi_s ds - \int_0^\infty \psi_s ds \right)$$

- A permanent component:

$$\tau^{v,p} = -\frac{\rho}{\kappa} \int_0^\infty e^{-\rho s} \lambda_{h,s} ds$$

²⁵There is no VAT distortion cost since the VAT moves once for all: $\tau_t^v = \hat{\theta}_0$ for any t in the welfare loss (Subsection 3.4.3). Given our assumption in this section, there are also no price or wage dispersion costs.

The mean-reverting component

Given that firms can freely adjust their price, it is intuitive to find that fiscal devaluations prove useless against supply shocks. However, against demand shocks, fiscal devaluations are in general Pareto improving. Indeed, since households can not post the nominal wage at which they accept to supply labor, firms simply adjust for productivity and tax changes, omitting the demand factors. In this setting, the terms of trade are then simply given by:

$$s_t = p_{F,t} - p_{H,t} = a_t - \tau_t^n.$$

Households are not on their optimal labor-supply curve, and domestic prices do not respond to demand shocks. Compared to the natural allocation, domestic output is then distorted as follows:

$$\dot{y}_t + (1 - \alpha)\dot{\tau}_t^v + \dot{\tau}_t^n = \dot{y}_t + \left(1 - \frac{1 - \alpha}{\kappa}\right) \dot{\tau}_t^n = \frac{\alpha\phi}{1 + \phi} \dot{\lambda}_t + \frac{1 + (1 - \alpha)\phi}{1 + \phi} \dot{\psi}_t$$

Since firms can freely adjust their prices, productivity shocks do not distort the real wage and consequently involve no fiscal intervention in this setup. However, the price rigidity following demand shocks implies an overadjustment in quantities. Once again, the two channels compete. Since prices fully adjust, which is similar (in terms of response to fiscal changes) to a situation with complete pass-through, the competitiveness channel prevails over the consumption one. Following negative demand shocks, optimal fiscal devaluations consist in lowering the labor tax to decrease producer prices and reduce the output fall.

The degree of price flexibility (following fiscal changes and macroeconomic shocks) and the nature of the shock are then the two main determinants of optimal fiscal devaluations. They are crucial to decide which channel - consumption or competitiveness - should be favored. Identical macroeconomic shocks can imply opposite fiscal policy depending on these parameters: a terms of trade deterioration and a trade deficit call for a VAT decrease if prices are not flexible enough.

This insight is crucial, and goes against the common view about the desirable adjustment to external imbalances in currency unions. If in the short term prices are expected to be rigid, negative demand shocks should be followed by a VAT decrease to limit their recessive impact. If prices become flexible enough to make the competitiveness channel attractive, then the pattern is reversed and the VAT change should be positive. Such a scenario will occur after a certain period of time with sticky prices that sluggishly adjust.

This point should finally be related to the situation in the Euro periphery following the debt crises. Assuming that these economies suffered from demand shocks (likely risk-premium or export-demand shocks), increasing the VAT would have been particularly harmful in the short and medium run: it would then have made the adjustment longer and more costly from a welfare point of view.

Optimal permanent fiscal devaluations

Corollary 1 (Flexible prices and rigid wages) *The permanent component of the optimal time-varying fiscal devaluation and the optimal one-time fiscal devaluation coincide. They are given by:*

$$\tau^{n,p} = \rho \int_0^\infty e^{-\rho s} (\bar{s}_s - a_s) ds = \rho \int_0^\infty e^{-\rho s} \lambda_{h,s} ds$$

This formula is similar to the one derived in the rigid prices case, with complete pass-through and no response to productivity shock. Its extreme simplicity and implementability makes this fiscal policy particularly appealing.

3.5.3 Optimal fiscal devaluations under staggered price and wage settings

Nominal Stickinesses and fiscal policy interactions

We finally detail the most realistic case, in which both prices and wages are sticky, to derive quantitative insights about fiscal devaluations. The optimal policy problem has been introduced in Subsection 3.4.3. Its dimensionality makes the derivation of intuitive closed formula impossible.

However, we can understand the optimal fiscal paths using the intuitions developed in the polar cases. It is thus interesting to understand how nominal rigidities and fiscal policy interact. As highlighted in Subsection 3.5.1, in the short term, when both prices and wages are rigid, the response is driven by the consumption channel. Then, as prices become more flexible, the remaining output gap is more efficiently reduced by taking advantage of the competitiveness channel.

	Optimal time-varying FD		Optimal one-time FD
	Short-run: consumption channel	Medium-run: competitiveness channel	
Negative demand shock	VAT decrease	Labor tax decrease	Labor tax decrease
Negative supply shock	VAT decrease	No response	No response

Table 3.1: Optimal fiscal devaluations' sign

It is important to understand why one-time fiscal devaluations are not used against supply shocks. We saw in Subsections 3.5.1 and 3.5.2 that the size of optimal one-time fiscal devaluations is inversely proportional to the competitiveness channel. However, with staggered prices and wages, large one-time fiscal devaluations induce costly price/wage inflation and substantial welfare losses. Since the competitiveness channel is expected to be small until prices become more flexible, the required tax deviation to stabilize output would be large and costly from a welfare point of view.

This explains why one-time fiscal devaluations will not prove efficient against supply shocks: when prices become flexible and this policy becomes effective, no action is required since firms have already adjusted their prices following a supply shock. However, against demand shocks, this policy fully takes advantage of the competitiveness channel.

It also explains why one-time fiscal devaluation efficiency decreases with wages flexibility. If wages are more flexible than prices, the time prices adjust and the competitiveness channel becomes effective, wages have already adjusted and reducing the output gap does not reduce the wage dispersion cost.

Calibration

We explore numerically optimal fiscal devaluations under price and wage stickinesses. We follow Farhi et al. (2013) calibration for Spain, except for the trade openness (see Bussiere and al. (2013)), effective tax rates (see Borselli and al. (2012)) and the VAT pass-through (see Carbonnier (2007)).

	Parameter	Value
Rate of time preference	ρ	0.02
Labor disutility	ϕ	2
Elasticity of substitution between individual goods varieties	ε_P	4
Calvo rate for price adjustments	θ_P	0.75 ⁴
Elasticity of labor demand for a household variety	ε_W	4
Calvo rate for wage adjustments	θ_W	0.85 ⁴
Economy openness	α	0.25
Steady state VAT	\bar{T}^v	12.1%
Steady state labor tax	\bar{T}^n	30%
VAT instantaneous pass-through	ζ^v	0.7
Labor tax instantaneous pass-through	ζ^n	0

Table 3.2: Calibration values

Fiscal devaluation efficiency

We compute the optimal fiscal policy efficiency with a calculation a la Lucas. We assess the share of the shock that is offset by computing the following ratio:

$$\text{Fiscal policy efficiency} = \frac{\mathbb{C}_{\text{no fiscal policy}} - \mathbb{C}_{\text{fiscal policy}}}{\mathbb{C}_{\text{no fiscal policy}} - \mathbb{C}_{\text{independent monetary policy}}} \quad (3.2)$$

where \mathbb{C} refers to the permanent per-period decline in consumption in the steady state required to match the welfare loss following the exogenous shock. The "maximum VAT deviation line" indicates the value along the VAT deviation path that is the largest in absolute terms.

	Productivity	Risk-premium	Export-demand
Half-life	3 years	1 year	$+\infty$
Optimal time-varying FD	53 %	37 %	85 %
Maximum VAT deviation (1 % shock)	0.8 %	-1 %	0.7 %

Table 3.3: Optimal time-varying fiscal devaluation efficiency and size

We find that optimal time-varying fiscal devaluations are substantially efficient from a welfare point of view, since they can erase a substantial part of the welfare losses (compared to the flexible exchange rate allocation). This result, that is based on the implementation of an optimal fiscal response, goes against the results of Lipinska and Von Thadden (2009) and Engler and al. (2013), who find that fiscal devaluations are quite inefficient under in a similar environment, but without using a normative approach.

Moreover, we find that the optimal fiscal devaluations size is moderate. This point was visible in the closed formula derived in the polar cases, in which optimal fiscal policy is of the same order of magnitude than the exogenous shock. Table 3 shows that this is still true in a more realistic framework with Calvo price and wage setting.

Finally, we see that the fiscal devaluations efficiency is increasing with the shock persistency, which highlights the importance of the competitiveness channel and long-run dynamics in the welfare

assessment. As is well known, in standard business cycle models, short-term fluctuations are not very costly from a welfare point of view. However, this may result in underestimating temporary recessions welfare cost, and thus the consumption channel opportunity in the short-term.

	Productivity	Risk-premium	Export-demand
Half-life	3 years	1 year	$+\infty$
Optimal permanent FD	1.5 %	18 %	82 %
VAT deviation (1 % shock)	-0.03 %	0.2 %	0.7 %
t_{lim}	69 years	46 years	29 years

Table 3.4: Optimal one-time fiscal devaluation efficiency and size

We see that one-time fiscal devaluations, taking advantage of the competitiveness channel, are efficient following demand-shocks that are sufficiently persistent.

However, it is important to notice that their efficiency heavily depends on the consumption channel irresponsiveness. This result is no longer valid if the VAT is set back to its steady state value at a given date. Indeed, in such a case, households anticipate that the VAT increase is temporary and consumption falls during this period²⁶. We then compute the earliest date t_{lim} at which the VAT can be set back to its steady state value²⁷ for the fiscal policy to be welfare-neutral (zero fiscal policy efficiency in 3.2). Unsurprisingly, since the discount factor ρ is small, this date is very large. One-time fiscal devaluations efficiency then heavily rest on fiscal authorities credibility, that must commit to move the VAT once for all following a given shock.

Counterfactual experiments

In this Subsection, we illustrate how fiscal devaluations are optimally used to respond to exogenous shocks. Following Farhi et al. (2013), we provide a simple counterfactual experiment by calibrating our model on the recent experience of Spain.

Spain 2008: 1.2 % risk-premium shock (3.5 years half-life)

We first explore the situation of Spain in 2008, assuming that the country was hit by an unexpected shock in the cost of borrowing. Our analysis follows exactly the one of Farhi et al. (2013) to compare our results.

We then calibrate the risk-premium shock to match the 4% decline in GDP in Spain between 2008-2009. This corresponds to a 1.2 % risk-premium shock (1.13 % in Farhi et al. (2013)). We similarly consider a 3.5 years half-life for the shock. The calibration has already been introduced. Since we do not have any empirical evidence for the fiscal pass-through in Spain, we keep the values available for France.

²⁶The Pareto weight $\hat{\theta}_0$ increases by less that the VAT deviation: $\hat{\theta}_0 = (1 - e^{-\rho t_{lim}}) \tau^v$, where t_{lim} is the date at which the VAT is set back to its steady state value.

²⁷Going from the optimal one-time fiscal devaluations value to the steady-state one.

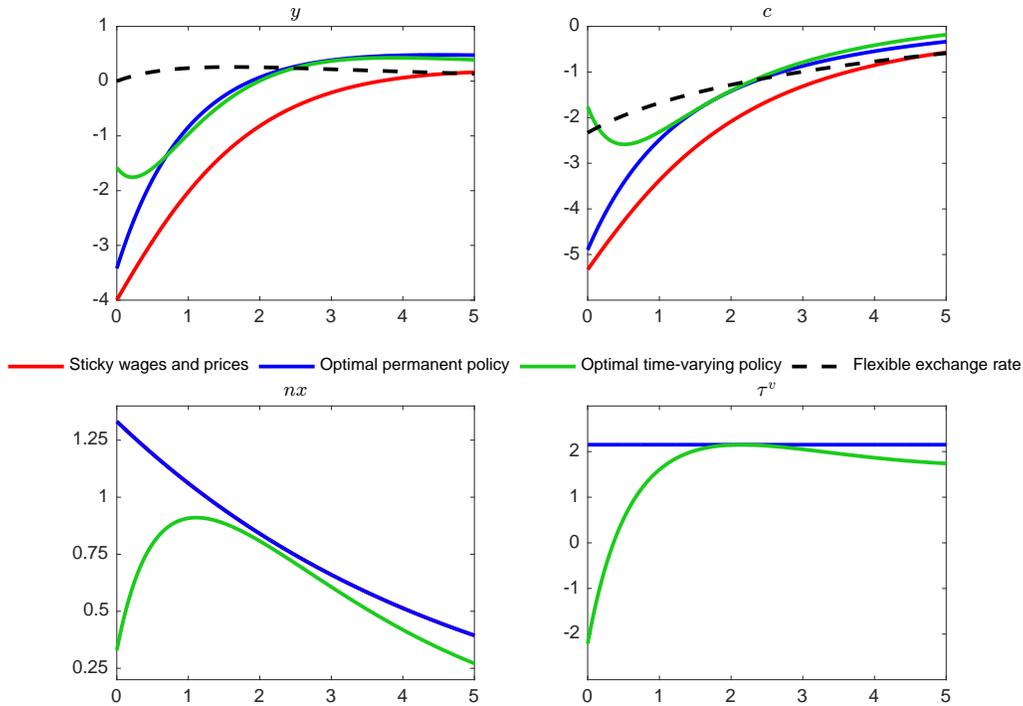


Figure 3.2: Optimal fiscal devaluations following a risk-premium shock

Despite its extreme simplicity, our model matches quite well the data: consumption falls by 5.2 % (4.9 % in the data, 3.8 % in Farhi et al. (2013)) and the trade balance as a ratio of GDP improves by 5.3 % (4.1 % in the data, 3.6 % in Farhi et al. (2013)).

At the flexible exchange rate allocation, consumption instantly decreases due to a higher interest rate. However, the terms of trade deterioration is followed by an exchange rate depreciation that stabilize output and reduce the consumption fall, implying a large trade surplus. As the shock vanishes, consumption and output go back to their steady state level and a constant share of the accumulated surplus is consumed at each period in the long run.

When prices are sticky and can not adjust in the short run, the consumption fall is twice larger since it is not curbed by a price decrease. Output then dramatically falls and the price decrease is gradually and weakly engineered through costly deflation. The output gap is then very large.

The optimal time-varying policy consists in decreasing the VAT in the short-run when the competitiveness channel is low, to reduce the consumption drop by decreasing the prices paid by consumers. Output decrease is then twice smaller than in the absence of fiscal intervention. As prices flexibility increases, the competitiveness channel prevails and the price decrease is engineered through a labor tax decrease (VAT increase): the two fiscal schemes allocations then coincide.

The optimal one-time policy, which only works through the competitiveness channel, then consists in increasing the VAT once for all by 2.1 %. It allows to finance a 2.9 % labor tax decrease that increasingly reduces the output gap.

Both policies allow substantial welfare gains by adjusting prices, thus lowering the output gap and the price and wage adjustment costs²⁸

²⁸60 % efficiency with a calculation a la Lucas for the time-varying scheme, 53 % with the one-time fiscal devaluations.

3.6 Conclusion

This paper describes optimal fiscal devaluations based on the VAT and labor tax for countries located in a currency union. Our microfounded and normative approach allows us to derive three main results.

First, optimal fiscal devaluations' sign are determined by the nature of the macroeconomic shock (demand/supply). Following a negative demand shock, it is optimal to decrease the VAT in the short term, thus engineering a fiscal reevaluation that aims at supporting domestic consumption: a trade deficit and a negative output gap do not require a VAT increase in general.

Second, fiscal pass-through to consumer prices also define optimal fiscal devaluations. Two channels are highlighted, relying respectively on a consumption and a competitiveness stimulus. We provide intuitive closed-form formula for these channels, and show that the consumption channel prevails in the optimal time-varying policy until prices have adjusted sufficiently. We also show that only the competitiveness channel matters for one-time fiscal devaluations as long as VAT revenues are rebated to households. However, this last result heavily rests on fiscal authorities credibility, that must commit to move the VAT once for all following a given shock.

Third, and contrary to a common view fuelled by previous empirical and numerical works, our optimal policy approach highlights that optimal fiscal devaluations (both time-varying and one-time) are of the same order of magnitude than the occurring macroeconomic shocks, and quite efficient to regain monetary power.

One limit of our analysis is the Cole-Obstfeld calibration that notably constrains the value of the risk-aversion. However, we showed in Subsection 3.4.1 how the constrained parameters affect the trade-off between output stabilization and fiscal policy distortion. A forthcoming paper will describe optimal fiscal devaluations outside of the Cole-Obstfeld case in a quantitative way, and show that the results derived here about fiscal devaluations sign, size and efficiency are valid for any calibration. Finally, our analysis did not consider fiscal devaluations spillovers if any country of the currency area can implement active fiscal policy. We will also consider these coordination issues in the above-mentioned forthcoming paper.

3.7 Appendix

3.7.1 First-order conditions for Calvo price and wage settings

In the non-linearized model, the Calvo price and wage setting first-order conditions can be expressed as follows:

Calvo price setting

$$\frac{1 - \theta_P(\Pi_{H,t})^{\varepsilon_P-1}}{1 - \theta_P} = \left(\frac{F_t}{K_t}\right)^{\varepsilon_P-1},$$

where

$$K_t = \frac{\varepsilon_P}{\varepsilon_P - 1} \frac{(1 + T_s^n)W_t C_t^{-\sigma}}{P_t} \left(\frac{Y_t}{A_{H,t}}\right) + \theta_P \beta \Pi_{H,t+1}^{\varepsilon_P} K_{t+1}, \quad F_t = Y_t C_t^{-\sigma} S_t^{-1} Q_t + \theta_P \beta \Pi_{H,t+1}^{\varepsilon_P-1} F_{t+1},$$

denoting the PPI inflation adjusted for mechanical indexation

$$\Pi_{H,t+1} = \frac{P_{H,t+1}}{P_{H,t}} \left(\frac{1 + T_t^n}{1 + T_{t+1}^n}\right)^{\zeta^n} \left(\frac{1 - T_t^v}{1 - T_{t+1}^v}\right)^{-\zeta^v}$$

. The price dispersion index Δ_t^P follows the law of motion:

$$\Delta_t^P = h(\Delta_{t-1}^P, \Pi_{H,t}), \quad h(\Delta, \Pi) = \theta_P \Delta \Pi^{\varepsilon_P} + (1 - \theta_P) \left(\frac{1 - \theta_P \Pi^{\varepsilon_P-1}}{1 - \theta_P}\right)^{\varepsilon_P/(\varepsilon_P-1)}.$$

Calvo wage setting

$$\frac{1 - \theta_W(\Pi_t^W)^{\varepsilon_W-1}}{1 - \theta_W} = \left(\frac{F_t^W}{K_t^W}\right)^{\frac{\varepsilon_W-1}{1+\varepsilon_W\phi}},$$

where

$$K_t^W = \frac{\varepsilon_W}{\varepsilon_W - 1} N_t^{1+\phi} + \theta_W \beta (\Pi_{t+1}^W)^{\varepsilon_W(1+\phi)} K_{t+1}^W, \quad F_t^W = \frac{C_t^{-\sigma} N_t W_t}{P_t(1 + T_s)} + \theta_W \beta (\Pi_{t+1}^W)^{\varepsilon_W-1} F_{t+1}^W,$$

denoting wage inflation $\Pi_{t+1}^W = \frac{W_{t+1}}{W_t}$. The wage dispersion index Δ_t^W follows the law of motion:

$$\Delta_t^W = h(\Delta_{t-1}^W, \Pi_t^W), \quad h(\Delta, \Pi) = \theta_W \Delta \Pi^{\varepsilon_W} + (1 - \theta_W) \left(\frac{1 - \theta_W \Pi^{\varepsilon_W-1}}{1 - \theta_W}\right)^{\frac{\varepsilon_W}{\varepsilon_W-1}}.$$

3.7.2 Proof of Proposition 8

This proof follows the lines of Farhi and Werning (2012).

We first consider a relaxed problem in which taxes are allowed to vary over time. We assume that the world is at a symmetric deterministic steady state. Home takes the rest of the world as given and uses the labor tax, the VAT and the social contribution tax paid by households to maximize its welfare.

Home then has to solve the following problem:

$$\max_{\{C_t, Y_t, N_t, Q_t, S_t, T_t^v, T_t^n, T_t^s\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right],$$

subject to:

$$\begin{aligned}
C_t &= \Theta C^* Q_t^{1/\sigma} \\
Y_t &= C^* \left[(1 - \alpha)(1 - T_t^v)^\eta Q_t^{\frac{1}{\sigma} - \eta} S_t^\eta \Theta + \alpha S_t^\gamma \right] \\
Q_t &= (1 - T_t^v) \left[(1 - \alpha) S_t^{\eta-1} + \alpha \right]^{\frac{1}{\eta-1}} ; \\
N_t &= \frac{Y_t}{A_{H,t}} \\
0 &= - \sum_{t=0}^{\infty} \beta^t C^{*\sigma} \left(\frac{Y_t}{S_t} - (1 - T_t^v) \frac{C_t}{Q_t} \right) ; \\
\Theta^{-\sigma} C^{*\sigma} S_t^{-1} &= \mu_P \mu_W \frac{(1 + T_t^s)(1 + T_t^n)}{A_{H,t}} N_t^\phi
\end{aligned}$$

Assuming that the problem is sufficiently convex, stationarity implies that $\{C_t, Y_t, N_t, Q_t, S_t\}$ are constant. This implies that T_t^v and $(1 + T_t^s)(1 + T_t^n)$ are constant. Using the fact that the country's budget constraint must be balanced, we express Θ as a function of S and Q :

$$\Theta = \frac{Q^{\eta - \frac{1}{\sigma}} S^{\gamma-1}}{(1 - T^v)^\eta}$$

We then substitute out into the utility function to rewrite the planning problem:

$$\max_S \frac{[(1 - \alpha)S^{\eta-1} + \alpha]^{\frac{\eta(1-\sigma)}{\eta-1}} S^{(\gamma-1)(1-\sigma)} C^{*1-\sigma}}{1 - \sigma} - \frac{1}{1 + \phi} \left(\frac{C^*}{A_H} \right)^{1+\phi} [(1 - \alpha)S^{\eta-1} + \alpha]^{1+\phi} S^{\gamma(1+\phi)}$$

The first-condition on S then gives:

$$\begin{aligned}
0 &= \left[\frac{(1 - \alpha)\eta S^{\eta-1}}{S [(1 - \alpha)S^{\eta-1} + \alpha]} + \frac{\gamma - 1}{S} \right] [(1 - \alpha)S^{\eta-1} + \alpha]^{\frac{\eta(1-\sigma)}{\eta-1}} S^{(\gamma-1)(1-\sigma)} C^{*1-\sigma} \\
&\quad - \left[\frac{(1 - \alpha)(\eta - 1)S^{\eta-1}}{S [(1 - \alpha)S^{\eta-1} + \alpha]} + \frac{\gamma}{S} \right] \left(\frac{C^*}{A_H} \right)^{1+\phi} [(1 - \alpha)S^{\eta-1} + \alpha]^{1+\phi} S^{\gamma(1+\phi)}
\end{aligned}$$

Since we are looking for a symmetric equilibrium, we now impose $S = 1$, that gives:

$$\frac{A_H^{1+\phi}}{C^{*\phi+\sigma}} = \frac{(1 - \alpha)(\eta - 1) + \gamma}{(1 - \alpha)\eta + \gamma - 1}$$

We then use the labor-leisure condition to derive:

$$\frac{A_H^{1+\phi}}{C^{*\phi+\sigma}} = \mu_P \mu_W \frac{(1 + T_t^s)(1 + T_t^n)}{1 - T_v}$$

Though only the above product is fixed, we impose all taxes to be constant, which leads to:

$$\frac{(1 + \bar{T}^s)(1 + \bar{T}^n)}{1 - \bar{T}_v} = \frac{1}{\mu_P \mu_W} \frac{(1 - \alpha)(\eta - 1) + \gamma}{(1 - \alpha)\eta + \gamma - 1}$$

3.7.3 Proof of Proposition 9

Assuming we are in the rigid case, the following equation relates the output gap and the fiscal policy:

$$\hat{y}_t + \left(\frac{1-\alpha}{\sigma} - \frac{1-\zeta^v + \kappa\zeta^n}{\hat{\sigma}} \right) \tau_t^v - (1-\alpha)\hat{\theta}_0 = -\bar{s}_t$$

We then assume than the VAT is constant over time. Since the economy is not closed, the intertemporal budget constraint applies, and can be expressed as follows (see Subsection 3.7.4):

$$\hat{\theta}_0 = \frac{\tau^v}{\sigma} + K\rho \int_0^\infty e^{-\rho t} \hat{y}_t dt$$

Plugging the second equation into the first one and integrating over time, we get:

$$[1 - (1-\alpha)K] \rho \int_0^\infty e^{-\rho t} \hat{y}_t dt = -\rho \int_0^\infty e^{-\rho t} \bar{s}_t dt + \frac{1-\zeta^v + \kappa\zeta^n}{\hat{\sigma}} \tau^v$$

We then find the following relation between the output gap, the fiscal policy and the terms of trade:

$$\left[1 - \tilde{K}\right] \hat{y}_t - \frac{1-\zeta^v + \kappa\zeta^n}{\hat{\sigma}} \tau^v = - \left[1 - \tilde{K}\right] \bar{s}_t - \tilde{K} \int_0^\infty e^{-\rho s} \bar{s}_s ds \quad \text{where} \quad \tilde{K} = \frac{(1-\alpha)\left(\frac{\omega}{\sigma} - 1\right)\hat{\sigma}}{1 + (1-\alpha)\left(\frac{\omega}{\sigma} - 1\right)\hat{\sigma}} \leq 1$$

3.7.4 Summarizing the economy

The natural allocation

We express endogenous variables at the allocation obtained without rigidity and fiscal intervention as linear combinations of the unanticipated macroeconomic shocks occurring at $t = 0$.

- Domestic output and terms of trade can be respectively expressed as follows

$$\bar{y}_t = \frac{\hat{\sigma}^{-1}(1+\phi)}{1+\phi\hat{\sigma}^{-1}} a_{h,t} + \frac{\alpha}{1+\phi\hat{\sigma}^{-1}} \lambda_{h,t} - \frac{\alpha\omega}{1+\phi\hat{\sigma}^{-1}} \theta_t,$$

where $\hat{\sigma} = \sigma/(1-\alpha + \alpha\omega)$

$$\bar{s}_t = \frac{1+\phi}{1+\phi\hat{\sigma}^{-1}} a_{h,t} - \frac{\alpha\phi}{1+\phi\hat{\sigma}^{-1}} \lambda_{h,t} - \frac{\sigma + (1-\alpha)\phi}{1+\phi\hat{\sigma}^{-1}} \theta_t.$$

- The country intertemporal budget constraint is

$$\int_0^\infty e^{-\rho t} \theta_t dt = (1-K) \int_0^\infty e^{-\rho t} \lambda_{h,t} dt + K \int_0^\infty e^{-\rho t} y_t dt,$$

where $\rho = -\ln(\beta)$, $K = \frac{(\frac{\omega}{\sigma}-1)\hat{\sigma}}{1+(1-\alpha)(\frac{\omega}{\sigma}-1)\hat{\sigma}}$ and $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$

In the Cole-Obstfeld case, the trade balance and the intertemporal budget constraint will be driven by exogenous shocks even in the presence of nominal rigidities. Only fiscal interventions will be able to distort these paths.

- The natural interest rate, defined as the real interest rate prevailing at the natural allocation, can be related to the international one using Home and foreign Euler equations plus the perfect risk sharing condition (assuming no aggregate shock):

$$\bar{r}_t = \rho + \sigma\psi_t + \dot{\bar{s}}_t$$

The natural interest rate is then equal to the world one adjusted by the risk-premium and by the terms of trade derivative. It can then finally be expressed as a linear combination of the exogenous macroeconomic shocks:

$$\bar{r}_t - \rho = \frac{1 + \phi}{1 + \phi\hat{\sigma}^{-1}}\dot{a}_t - \frac{\alpha\phi}{1 + \phi\hat{\sigma}^{-1}}\dot{\lambda}_t + \frac{\alpha\omega\phi}{1 + \phi\hat{\sigma}^{-1}}\psi_t$$

Gap equations

We now derive the first-order log-linearized equilibrium conditions as gaps from the natural allocation, under the Calvo price and wage settings framework. We denote these variables with hats, $\hat{x}_t^i = x_t^i - \bar{x}_t^i$. These equations are the log-linearized versions of the one presented in Subsection 3.3.4.

As explained in Subsection 3.3.2, in the presence of price stickiness, we allow for VAT and payroll tax asymmetric pass-through. We showed in Appendix 3.7.1 that in this setup, the relevant inflation index to follow is the price inflation exclusive of the mechanical adjustment on tax changes:

$$\tilde{\pi}_{H,t} = \pi_{H,t} - \zeta^v \dot{\tau}_t^v - \zeta^n \dot{\tau}_t^n$$

Since we only track this index for domestic inflation, we still denote it $\pi_{H,t}$. Foreign inflation π_t^* is defined exclusive of the domestic VAT.

The demand block can then be summarized by the three following equations:

- the gap Dynamic Saving (DS) equation

$$\dot{\hat{y}}_t = \frac{1}{\hat{\sigma}} \left(\rho + \dot{\theta}_t - \pi_{H,t} - \bar{r}_t \right) + \frac{\alpha\omega(1 - \zeta^v) - \zeta^v(1 - \alpha)}{\sigma} \dot{\tau}_t^{i,v} - \zeta^n \frac{1 - \alpha + \alpha\omega}{\sigma} \dot{\tau}_t^{i,n} \quad ;$$

- the intertemporal budget constraint

$$\hat{\theta}_0 = \rho \int_0^\infty e^{-\rho t} \left(\frac{1}{\sigma} \tau_t^v + K \hat{y}_t \right) dt \quad ;$$

- the balanced fiscal devaluation constraint

$$0 = \kappa \tau_t^v + \tau_t^n \quad \text{where } \kappa = \frac{\varepsilon_P}{\varepsilon_P - 1}.$$

To make our calculations simpler, the tax deviations are defined as follows:

$$1 - T_t^v = (1 - \bar{T}^v) e^{-\tau_t^v}, \quad \text{and} \quad 1 + T_t^n = (1 + \bar{T}^n) e^{\tau_t^n}$$

The VAT deviation will then always be smaller than the labor tax one for our calibration, which is consistent with a larger VAT basis. The constraint can also be formulated by calibrating this parameter on the tax bases for each country.

The supply block also consists in three equations

- the law of motion of the real wage²⁹

$$\dot{\hat{\omega}} = \pi_t^W - (1 - \alpha)\pi_{H,t} - [(1 - \alpha)(\zeta^v - \kappa\zeta^n) - \alpha]\hat{\tau}_t^v - \alpha\pi_t^* - \frac{\dot{\bar{\omega}}_t}{\bar{\omega}_t} \quad ;$$

- the New Keynesian Philips Curve in gaps

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \lambda_P\alpha\hat{\sigma}\hat{y}_t - \lambda_P\hat{\omega}_t - \lambda_P \left[\frac{(1 - \alpha^2 + \alpha\omega)\hat{\sigma}}{\sigma} \right] \tau_t^v + \lambda_P\alpha(1 - \alpha)\hat{\sigma}\hat{\theta}_0 - \lambda_P\tau_t^n$$

$$\lambda_P = \rho_P(\rho + \rho_P) \quad \text{and} \quad \rho_P = -\ln(\theta_P)$$

- the wage dynamic equation in gaps

$$\dot{\pi}_t^W = \rho\pi_t^W - \lambda_W((1 - \alpha)\hat{\sigma} + \phi)\hat{y}_t + \lambda_W\hat{\omega}_t + \lambda_W\alpha(1 - \alpha + \omega)\hat{\sigma} \left(\frac{\tau_t^v}{\sigma} - \hat{\theta}_0 \right).$$

$$\lambda_W = \rho_W \frac{(\rho + \rho_W)}{1 + \epsilon_W\phi} \quad \text{and} \quad \rho_W = -\ln(\theta_W)$$

Finally, we have the following initial conditions stating that the terms of trade and the real wage can not jump at $t = 0$ in the presence of nominal stickiness:

$$\hat{y}_0 - \frac{\alpha\omega(1 - \zeta^v) - \zeta^v(1 - \alpha)}{\sigma}\tau_0^v - (1 - \alpha)\hat{\theta}_0 + \zeta^n \frac{1 - \alpha + \alpha\omega}{\sigma}\tau_0^n = -\frac{\bar{s}_0}{\hat{\sigma}},$$

$$\hat{\omega}_0 + [\zeta^v(1 - \alpha) + \alpha]\tau_0^v + (1 - \alpha)\zeta^n\tau_0^n = -\bar{\omega}_0 = \alpha\bar{s}_0 - a_0$$

3.7.5 Derivation of the loss function

We now focus on the Cole-Obstfeld case. We first have the exact relationship:

$$c_t = (1 - \alpha)s_t + \theta_t - \tau_t^v$$

And the following second-order approximaton of the goods market clearing condition $Y_t = S_t C^*[(1 - \alpha)\Theta_t + \alpha\Lambda_t]$:

$$y_t = s_t + (1 - \alpha)(\theta_t - \tau_t^v) + \alpha\lambda_{h,t} + \frac{1}{2}\alpha(1 - \alpha)\lambda_{h,t}^2 + \frac{1}{2}\alpha(1 - \alpha)(\theta_t - \tau_t^v)^2 - \alpha(1 - \alpha)(\theta_t - \tau_t^v)\lambda_{h,t}$$

Using the two previous equations, we have in gaps:

$$\hat{c}_t = (1 - \alpha)\hat{y}_t - \alpha(2 - \alpha)\tau_t^v - \frac{1}{2}[\alpha\tau_t^v(\tau_t^v - 2\theta_t) + 2\alpha\tau_t^v\lambda_{h,t}]$$

We can now use this result to derive:

$$\log C_t = \bar{c}_t + \hat{c}_t = \bar{c}_t + (1 - \alpha)\hat{y}_t - \alpha(2 - \alpha)\tau_t^v - \frac{1}{2}[\alpha\tau_t^v(\tau_t^v - 2\theta_t) + 2\alpha\tau_t^v\lambda_{h,t}]$$

²⁹Defined as the nominal wage divided by the CPI.

Household h delivers $N_t(h)$ units of labor of type h . Defining aggregate employment as $N_t = \int_0^1 N_t(h)dh$, and following the same steps than Galí (2008) we have up to a second order approximation:

$$\int_0^1 \hat{n}_t(h)dh + \frac{1+\phi}{2} \int_0^1 \hat{n}_t(h)^2 dh = \hat{n}_t + \frac{1+\phi}{2} \hat{n}_t^2 + \varepsilon_W \phi \log \Delta_t^W$$

By the labor market clearing condition, we have up to second-order approximation:

$$\hat{n}_t = \hat{y}_t + \log \Delta_t^W + \log \Delta_t^P + \frac{1}{2} \hat{y}_t^2$$

By Woodford (2003), we have:

$$\int_0^\infty e^{-\rho t} \log \Delta_t^P dt = \frac{\varepsilon_P}{2\lambda_P} \int_0^\infty e^{-\rho t} \pi_{H,t}^2 dt, \quad \int_0^\infty e^{-\rho t} \log \Delta_t^W dt = \frac{\varepsilon_W}{2\lambda_W(1+\varepsilon_W\phi)} \int_0^\infty e^{-\rho t} \pi_t^{W^2} dt.$$

Finally, using $\bar{N}_t^{1+\phi} = (1-\alpha)[1+\alpha(\lambda_t - \theta_t)]$ and integrating over households and time, we have the following expression for the objective function:

$$\begin{aligned} & \int_0^\infty e^{-\rho t} \int_0^1 \left(\frac{U_t(h) - \bar{U}_t(h)}{CU_c} \right) dh dt = \\ & - \frac{(1-\alpha)(1+\phi)}{2} \int_0^\infty e^{-\rho t} \left\{ \alpha_\pi \pi_{H,t}^2 + \alpha_{\pi_W} \pi_t^{W^2} + \hat{y}_t^2 + \frac{2\alpha}{1+\phi} \hat{y}_t [\lambda_{h,t} - \theta_t] \right. \\ & \left. + \frac{2\alpha(2-\alpha)}{(1-\alpha)(1+\phi)} \tau_t^v + \frac{1-\alpha}{1+\phi} [\alpha \tau_t^v (\tau_t^v - 2\theta_t) + 2\alpha \tau_t^v \lambda_{h,t}] \right\} dt \end{aligned}$$

We now use a second order approximation of the budget constraint to replace the linear term τ_t^v in the expression above. We find:

$$\int_0^\infty e^{-\rho t} \tau_t^v = \int_0^\infty e^{-\rho t} \left[\frac{1}{2} \tau_t^v (\tau_t^v - 2\theta_t) - (\theta_0 - \theta_t) \tau_t^v \right] dt + t.i.p.$$

so we get the following loss function (up to additive and multiplicative constants):

$$\frac{1}{2} \int_0^\infty e^{-\rho t} \left[\alpha_\pi \pi_{H,t}^2 + \alpha_{\pi_W} (\pi_t^W)^2 + \left[\hat{y}_t + \frac{\alpha}{1+\phi} (\lambda_{h,t} - \theta_t) \right]^2 + \alpha_\tau [\tau_t^v + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1-\alpha_\psi)\theta_0]^2 \right] dt$$

where

$$\alpha_\pi = \frac{\varepsilon_P}{\lambda_P(1+\phi)}, \quad \alpha_{\pi_W} = \frac{\varepsilon_W(1+\varepsilon_W\phi)}{\lambda_W(1+\phi)}, \quad \alpha_\tau = \frac{\alpha}{1+\phi} \left(\frac{2-\alpha}{1-\alpha} + 1 - \alpha \right), \quad \alpha_\psi = \frac{1-\alpha}{\frac{2-\alpha}{1-\alpha} + 1 - \alpha}$$

3.7.6 Proof of Proposition 10

Assuming prices are fixed and there is no wage dispersion (or the wage dispersion cost is zero), the planning problem becomes:

$$\min_{\{\hat{y}_t, \tau_t^v\}} \frac{1}{2} \int_0^\infty e^{-\rho t} \left\{ \left[\hat{y}_t + \frac{\alpha}{1+\phi} (\lambda_{h,t} - \theta_t) \right]^2 + \alpha_\tau \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1-\alpha_\psi)\theta_0 \right]^2 \right\} dt$$

subject to

$$\dot{\hat{y}}_t = \rho + \psi_t - \bar{r}_t - [\zeta^v - \alpha - \kappa \zeta^n] \tau_t^v,$$

$$\hat{\theta}_0 = \rho \int_0^\infty e^{-\rho t} \tau_t^v dt.$$

$$\hat{y}_0 + [\zeta^v - \alpha - \kappa \zeta^n] \tau_0^v - (1 - \alpha) \hat{\theta}_0 = -\bar{s}_0$$

Let Γ be the multiplier on the budget constraint, $\mu_{\tau,t}$ and $\mu_{y,t}$ be the co-states. We have the following first-order conditions:

$$-\dot{\mu}_{y,t} + \rho \mu_{y,t} = \hat{y}_t + \frac{\alpha}{1 + \phi} (\lambda_{h,t} - \theta_t)$$

$$-\dot{\mu}_{\tau,t} + \rho \mu_{\tau,t} = \alpha_\tau \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1 - \alpha_\psi) \theta_0 \right] - \Gamma$$

$$0 = -[\zeta^v - \alpha - \kappa \zeta^n] \mu_{y,t} + \mu_{\tau,t}$$

$$0 = \int_0^\infty e^{-\rho t} \left(\alpha_\tau \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1 - \alpha_\psi) \theta_0 \right] - \Gamma \right) dt$$

Using the first three equations, we have:

$$\hat{y}_t + \frac{\alpha}{1 + \phi} (\lambda_{h,t} - \theta_t) - \frac{\alpha_\tau}{[\zeta^v - \alpha - \kappa \zeta^n]} \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1 - \alpha_\psi) \theta_0 \right] = -\frac{1}{[\zeta^v - \alpha - \kappa \zeta^n]} \Gamma \quad (3.3)$$

Differentiating and using the DS equation, we get:

$$\dot{\tau}_t^v = -\frac{\zeta^v - \alpha - \kappa \zeta^n}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \dot{a}_t + \alpha \frac{\zeta^v - \alpha - \kappa \zeta^n - \frac{1-\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \dot{\lambda}_t + (1 - \alpha) \frac{\zeta^v - \alpha - \kappa \zeta^n + \frac{\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \dot{\psi}_t$$

Since $\int_0^\infty e^{-\rho t} (\lambda_{h,t} - \theta_t) dt = 0$ ³⁰, the fourth first-order condition and the intertemporal budget constraint give:

$$\Gamma = -\alpha_\tau (1 - \alpha_\psi) \theta_0$$

We then decompose the fiscal response to highlight its transitory and permanent components:

$$\tau_t^v = g_t - I + \hat{\theta}_0 \quad \text{where}$$

$$g_t = -\frac{\zeta^v - \alpha - \kappa \zeta^n}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} a_t + \alpha \frac{\zeta^v - \alpha - \kappa \zeta^n - \frac{1-\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \lambda_t + (1 - \alpha) \frac{\zeta^v - \alpha - \kappa \zeta^n + \frac{\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left(\theta_0 + \int_0^t \psi_s ds \right)$$

$$\text{and } I = -\rho \frac{\zeta^v - \alpha - \kappa \zeta^n}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \int_0^\infty e^{-\rho s} a_s ds + \rho \alpha \frac{\zeta^v - \alpha - \kappa \zeta^n - \frac{1-\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \int_0^\infty e^{-\rho s} \lambda_s ds$$

$$+ (1 - \alpha) \frac{\zeta^v - \alpha - \kappa \zeta^n + \frac{\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa \zeta^n)^2 + \alpha_\tau} \left(\theta_0 + \int_0^\infty e^{-\rho s} \psi_s ds \right)$$

By construction, $I = \rho \int_0^\infty e^{-\rho t} g_t dt$, which justifies the above decomposition of the fiscal response. We then use the equation 3.3 and the initial condition to find $\hat{\theta}_0$:

$$-\bar{s}_0 - [\zeta^v - \alpha - \kappa \zeta^n] \tau_0^v + (1 - \alpha) \hat{\theta}_0 + \frac{\alpha}{1 + \phi} (\lambda_0 - \theta_0) - \frac{\alpha_\tau}{[\zeta^v - \alpha - \kappa \zeta^n]} [g_0 + I + \alpha_\psi (\lambda_0 - \theta_0)] = 0$$

³⁰By definition of the initial level of consumption θ_0 at the natural allocation, see the intertemporal budget constraint in Appendix 3.7.4.

We get:

$$(1 - \zeta^v + \kappa\zeta^n)\hat{\theta}_0 = \frac{(\zeta^v - \alpha - \kappa\zeta^n)^2 + \alpha\tau}{\zeta^v - \alpha - \kappa\zeta^n} I$$

The final expression for the fiscal reponse is then:

$$\begin{aligned} \tau_t^v = & -\frac{\zeta^v - \alpha - \kappa\zeta^n}{(\zeta^v - \alpha - \kappa\zeta^n)^2 + \alpha\tau} \left[a_t - \rho \int_0^\infty e^{-\rho s} a_s ds \right] + \alpha \frac{\zeta^v - \alpha - \kappa\zeta^n - \frac{1-\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa\zeta^n)^2 + \alpha\tau} \left(\lambda_t - \rho \int_0^\infty e^{-\rho s} \lambda_s ds \right) \\ & + (1 - \alpha) \frac{\zeta^v - \alpha - \kappa\zeta^n + \frac{\alpha}{1+\phi}}{(\zeta^v - \alpha - \kappa\zeta^n)^2 + \alpha\tau} \left(\int_0^t \psi_s ds - \int_0^\infty \psi_s ds \right) + \frac{\rho}{1 - \zeta^v + \kappa\zeta^n} \int_0^\infty e^{-\rho s} (a_s - \lambda_{h,s}) ds \end{aligned}$$

Optimal one-time fiscal devaluations

We now look for the optimal constant path, characterized by $\hat{\theta}_0 = \tau_t^v \quad \forall t$. We see that the VAT only enters the initial condition, thus enabling to control \hat{y}_0 . Using the dynamic saving equation that is now only made of exogenous terms, the optimal choice on \hat{y}_0 is:

$$\hat{y}_0 + \rho \int_0^\infty e^{-\rho s} [\rho + \psi_s - \bar{r}_s] ds = 0$$

Plugging that into the initial condition, we find:

$$[1 - \zeta^v + \kappa\zeta^n]\tau^{v,p} = \bar{s}_0 - \rho \int_0^\infty e^{-\rho s} [\rho + \psi_s - \bar{r}_s] ds = \rho \int_0^\infty e^{-\rho s} \bar{s}_s ds$$

We finally get:

$$\tau^{v,p} = \frac{\rho}{1 - \zeta^v + \kappa\zeta^n} \int_0^\infty e^{-\rho s} (a_s - \lambda_{h,s}) ds$$

3.7.7 Proof of Proposition 11

We proceed as in the above Subsection to find the optimal fiscal path when prices are flexible and wages are rigid. The planning problem is:

$$\min_{\{\hat{y}_t, \tau_t^v\}} \frac{1}{2} \int_0^\infty e^{-\rho t} \left\{ \left[\hat{y}_t + \frac{\alpha}{1+\phi} (\lambda_{h,t} - \theta_t) \right]^2 + \alpha\tau \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1 - \alpha_\psi)\theta_0 \right]^2 \right\} dt$$

subject to

$$\dot{\hat{y}}_t = \rho + \psi_t + [\kappa - (1 - \alpha)]\tau_t^v + \dot{a}_{H,t} - \bar{r}_t,$$

$$\hat{\theta}_0 = \int_0^\infty e^{-\rho t} \tau_t^v dt.$$

$$\hat{y}_0 - [\kappa - (1 - \alpha)]\tau_0^v - (1 - \alpha)\hat{\theta}_0 = a_0 - \bar{s}_0.$$

We have the following first-order conditions:

$$-\dot{\mu}_{y,t} + \rho\mu_{y,t} = \hat{y}_t + \frac{\alpha}{1+\phi} (\lambda_{h,t} - \theta_t)$$

$$-\dot{\mu}_{\tau,t} + \rho\mu_{\tau,t} = \alpha\tau \left[\tau_t^v - \hat{\theta}_0 + \alpha_\psi (\lambda_{h,t} - \theta_t) - (1 - \alpha_\psi)\theta_0 \right] - \Gamma$$

$$0 = [\kappa - (1 - \alpha)]\mu_{y,t} + \mu_{\tau,t}$$

$$0 = \int_0^{\infty} e^{-\rho t} \left(\alpha_{\tau} \left[\tau_t^v - \hat{\theta}_0 + \alpha_{\psi}(\lambda_{h,t} - \theta_t) - (1 - \alpha_{\psi})\theta_0 \right] - \Gamma \right) dt$$

We then have:

$$\hat{y}_t + \frac{\alpha}{1 + \phi}(\lambda_{h,t} - \theta_t) + \frac{\alpha_{\tau}}{\kappa - (1 - \alpha)} \left[\tau_t^v - \hat{\theta}_0 + \alpha_{\psi}(\lambda_{h,t} - \theta_t) - (1 - \alpha_{\psi})\theta_0 \right] = \frac{1}{\kappa - (1 - \alpha)}\Gamma$$

Differentiating and using the DS equation, we get:

$$\dot{\tau}_t^v = -\alpha \frac{\kappa - (1 - \alpha) + \frac{(1 - \alpha)\phi}{1 + \phi}}{\alpha_{\tau} + [\kappa - (1 - \alpha)]^2} \dot{\lambda}_t - (1 - \alpha) \frac{\kappa - (1 - \alpha) - \frac{\alpha}{1 + \phi}}{\alpha_{\tau} + [\kappa - (1 - \alpha)]^2} \dot{\psi}_t$$

We similarly determine $\hat{\theta}_0$ and find the optimal fiscal path:

$$\begin{aligned} \tau_t^v = & -\alpha \frac{\kappa - (1 - \alpha) + \frac{(1 - \alpha)\phi}{1 + \phi}}{\alpha_{\tau} + [\kappa - (1 - \alpha)]^2} \left(\lambda_t - \rho \int_0^{\infty} e^{-\rho s} \lambda_s ds \right) - (1 - \alpha) \frac{\kappa - (1 - \alpha) - \frac{\alpha}{1 + \phi}}{\alpha_{\tau} + [\kappa - (1 - \alpha)]^2} \left(\int_0^t \psi_s ds - \int_0^{\infty} \psi_s ds \right) \\ & - \frac{\rho}{\kappa} \int_0^{\infty} e^{-\rho s} \lambda_{h,s} ds \end{aligned}$$

Permanent response

We follow the same steps than in Subsection 3.7.6 to derive the optimal one-time fiscal devaluation when prices are flexible and wages are rigid:

$$\tau_0^n = \rho \int_0^{\infty} e^{-\rho s} \lambda_{h,s} ds$$

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