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Jeremy Rastouil

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THÈSE PRÉSENTÉE

POUR OBTENIR LE GRADE DE

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**THREE ESSAYS ON LABOR MARKET FRICTIONS
UNDER FIRM ENTRY AND FINANCIAL BUSINESS CYCLES**

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Soutenue le Lundi 25 Novembre 2019

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"L'esprit intuitif est un don sacré et l'esprit rationnel est un serviteur fidèle. Nous avons créé une société qui honore le serviteur et a oublié le don. "

A. Einstein

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Résumé en Français

Durant la Grande récession, les interactions entre fluctuations du prix de l'immobilier, du travail et de la création nette de firme ont mis en évidence l'existence de relations étroites entre ces différents marchés aux Etats-Unis. En dépit de ces évidences, la grande majorité des modèles DSGE (Dynamic Stochastic General Equilibrium) traite de manière séparée chacun de ces marchés. Ainsi, il existe des incertitudes concernant les canaux de transmissions entre ces différents marchés. Cette thèse, qui est composée de trois chapitres, incorpore ces différents marchés afin d'en rendre compte. Le choix des modèles DSGE comme référence de modélisation est multiple. Tout d'abord, ces derniers ont connu de grandes avancées tant au niveau théorique qu'empirique depuis leur début avec le modèle de [Kydland and Prescott \(1982\)](#), connu sous le nom de modèle de cycles d'affaires (Real Business Cycle). Ce modèle est constitué de ménages et de firmes qui répondent de manière optimale aux innovations technologiques dont l'économie est affectée. De cette manière les modèles de cycles d'affaire ont pu répliquer les variances des principaux indicateurs des Etats-Unis.

Par la suite, des rigidités nominales dites à la [Calvo \(1983\)](#) ou à la [Rotemberg and Woodford \(1999\)](#) ont été introduites dans ces modèles afin de rendre compte des dynamiques de l'inflation. Ainsi, grâce à ces rigidités, ces modèles ont pu accorder une place à la politique monétaire à travers la règle de Taylor ([Taylor \(1993\)](#)). Ces modèles, par la suite ont pu être utilisés pour étudier les conséquences des politiques monétaires sur l'économie. Cependant, un certain nombre de critiques furent adressées aux modèles DSGE quant à leur capacité à répliquer des données car ils reposaient sur des calibrations et non sur des estimations de paramètres. En réponse à ces critiques, les travaux de [Christiano](#)

[et al. \(2005\)](#) et [Smets and Wouters \(2003\)](#) ont permis d'estimer les modèles DSGE en recourant à de l'économétrie bayésienne. Ainsi, ces modèles sont utilisés par les institutions politiques pour deux raisons. La première est leur capacité à produire une description complète des dynamiques stochastiques et déterministes par lesquels les agrégats macroéconomiques évoluent et ainsi permettent de comparer les données simulées avec les données empiriques. La seconde est leur utilité pour l'implémentation de différentes réformes (fiscales, budgétaires, financières) afin d'analyser les conséquences du court jusqu'au long terme pour l'économie dans ce cadre ([Cette et al. \(2016\)](#), [Duval and Furceri \(2018\)](#)).

Ainsi, dans cette thèse nous utilisons ces deux aspects des modèles DSGE pour rendre compte des interactions entre le marché du travail, de la création de firme et du marché de l'immobilier à la fois pour de quantifier les liens entre eux et aussi pour apprécier l'impact de réforme structurelle sur le marché du travail et de potentielles politiques macroprudentielles. Concernant la modélisation du marché du travail, cette thèse utilise le modèle d'appariement à la [Diamond \(1982\)](#), [Mortensen \(1982\)](#) et [Pissarides \(1985\)](#) dont l'idée centrale repose sur l'imperfection de l'appariement entre les chercheurs d'emplois et les firmes. Ce processus imparfait rend compte de l'aspect couteux en termes de temps et de ressources, à la fois pour les entreprises à la recherche d'un employé mais aussi du côté de ceux qui recherchent un travail. De cette façon, ce type de modèle permet de rendre compte des différents mouvements sur le marché du travail. Ce cadre a naturellement été intégré dans les modèles DSGE afin de rendre compte des dynamiques de la marge extensive du travail ([Gertler et al. \(2008\)](#)). Intégré à des modèles DSGE estimé à l'aide de l'économétrie bayésienne, ce cadre a montré sa capacité à reproduire les faits stylisés du marché du travail comme la courbe de Beveridge, la forte volatilité du taux de chômage et sa forte autocorrélation. En se basant sur ces deux aspects i.e. les modèles DSGE et le modèle d'appariement cette thèse se décompose en trois chapitres.

Le premier reprend la structure des modèles DSGE à la [Smets and Wouters \(2003\)](#) avec un modèle d'appariement en y intégrant une structure de marché endogène à la [Bilbiie et al. \(2012\)](#). Cette dernière structure permet de rendre endogène la création de firmes. Ce faisant, nous avons pu étudier les interactions entre le marché du travail et celui des biens. Le premier constat est que même dans le cas où la création de firmes est contracyclique,

le niveau d'emploi lui est toujours procyclique. Le second est l'importance jouée par la chute de la création nette d'entreprise sur le chômage durant la Grande récession. Ainsi, ce second résultat rejoint l'analyse de [Jaimovich and Floetotto \(2008\)](#) où 20% de la volatilité de l'emploi sont dû à la création/destruction de firmes. Le troisième résultat de ce chapitre concerne le taux de marge des entreprises, i.e. le ratio entre le prix de vente et le coût marginal. En intégrant une structure d'appariement sur le marché du travail plutôt qu'une structure walrasienne se concentrant sur la marge intensive ([Bilbiie et al. \(2012\)](#); [Lewis and Stevens \(2015\)](#)), nous obtenons un taux de marge contracyclique même en l'absence d'entrée de firmes. Ce résultat est dû au caractère acyclique du coût marginal qui ne dépend pas seulement du coût du travail à savoir le salaire mais aussi de la valeur présente et future de l'embauche.

Le second chapitre se concentre sur le lien entre emploi et prêt immobilier des ménages. Nous sommes partis de deux études empiriques. La première provient du Survey of consumer finance (SCF) où en moyenne de 1990 à 2015 environ 20% des emprunteurs sont au chômage contre environ 55% des employés. La seconde provient des observations faites par le SCE housing survey où les saisies immobilières sont à 17.1% dues aux variations de la valeur immobilière et 36% à la perte de l'emploi. Ces deux études rentrent en conflit avec la contrainte de collatéral traditionnellement utilisé dans les modèles DSGE qui spécifient que le montant du prêt est égal au LTV (Loan-to-Value ratio) multiplié par le prix anticipé du bien immobilier et le bien en question ([Kiyotaki and Moore \(1997\)](#)). Pour intégrer les composantes du marché du travail sur le niveau des prêts, nous avons considéré la contrainte de collatéral de [Iacoviello \(2015\)](#) avec une composante exogène représentant la part des ménages remboursant leurs prêts précédemment contractés. Nous avons endogénéisé cette composante en considérant que ceux qui remboursaient leurs prêts sont ceux qui ont gardé leur emploi et que ceux qui en contractent font partie de ceux qui en ont trouvé un. En termes de résultat statistiques, nous montrons que la contrainte de collatéral ajustée avec les modèles d'appariements donne les meilleures performances statistiques tant en termes de prévision que de reproduction des données utilisées. dans cette logique, nous avons pu étudier l'impact à long terme de réforme libérale sur le marché du travail et de politique macroprudentielle restrictive, la première réforme étant

de faire diminuer l'assurance chômage et d'observer ses conséquences sur l'ensemble de l'économie. De manière traditionnelle, dans les modèles DSGE (Arpaia et al. (2007), Cacciatore and Fiori (2016)), nous avons obtenus une augmentation du niveau du PIB ainsi que d'une réduction du taux de chômage. Cependant, une réforme libérale sur le marché du travail a aussi pour conséquence une forte augmentation du niveau des prêts des ménages et du prix de l'immobilier lorsque l'on utilise cette contrainte de collatéral augmentée. Ce résultat est dû à la présence à l'équilibre de la probabilité de trouver un emploi dans le montant d'endettement. A l'inverse, une politique macroprudentielle restrictive qui diminue le LTV ratio, permet de limiter les effets potentiellement néfastes d'une réforme libérale sur le marché du travail.

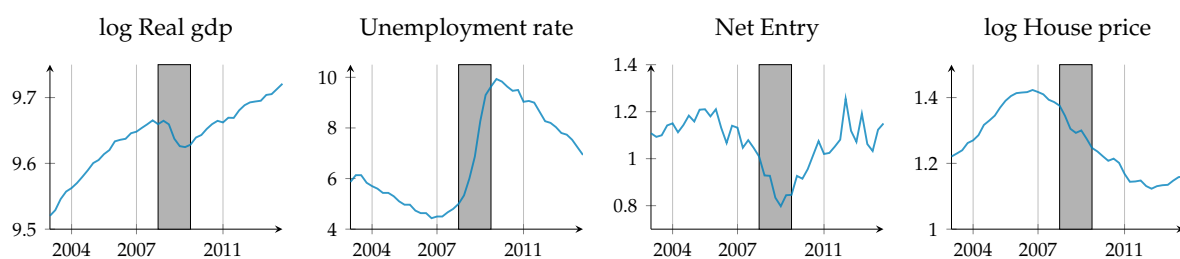
Le troisième chapitre intègre les éléments du précédent en se concentrant sur l'aspect financier des entreprises. Le premier travail effectué dans ce chapitre a été d'identifier quelle contrainte de collatéral était la plus adéquate pour rendre compte du lien entre marché de l'emploi et cycles financiers. Ainsi, nous avons comparé trois types de contraintes de collatéral, une n'incluant que du capital, une seconde avec capital et masse salariale et la dernière incluant en supplément les biens immobiliers commerciaux. Statistiquement, c'est la contrainte qui inclue ces trois composantes qui permet de mieux rendre compte des données utilisées. , nous avons pu de cette manière, étudier le rôle joué par les fluctuations du prix de l'immobilier et du crédit sur le marché de l'emploi. Ainsi, ces deux aspects expliquent une grande partie de la montée du chômage durant la Grande récession. Finalement, nous avons conduit des exercices de politique structurelle comme dans le chapitre précédent avec comme nouveauté des effets à long terme sur le niveau du PIB et du chômage pour les politiques macroprudentielles. On peut observer qu'une réforme libérale sur le marché du travail conduit aux mêmes résultats que ceux du chapitre précédent avec une augmentation substantielle du niveau des prêts des entreprises. Une politique macroprudentielle restrictive visant les ménages endettés a des effets vertueux à long terme sur l'emploi et le PIB, tout en limitant l'endettement des ménages et diminuant le prix de l'immobilier. Nous avons, enfin, conduit deux politiques macroprudentielles restrictives sur les entreprises avec d'un côté en diminuant le LTV ratio sur le capital et de l'autre sur l'immobilier. La première politique conduit à des effets plus négatifs sur

le long terme que la seconde mais avec une diminution moins marquée sur le marché de l'immobilier.

General Introduction

As a result of imbalances within the financial system, the Great Recession induced an unprecedented rise in unemployment rate combined with a large drop in firm entry and housing price. This movement between housing, labor and entry, as described as described by the Fig.1, highlights the existence of narrow propagation channels between these markets. Despite this evidence, most of the state-of-art DSGE model ways consider frictions on the labor market, on firm entry, or in the financial system in isolation without taking their interactions seriously. As a consequence, many uncertainties remain regarding the propagation channels across these three markets.

FIGURE 1: Housing, labor and firm entry during the Great recession



Notes: The shaded area displays the Great Recession period. Net entry is defined by the difference between entry and exit (from the BLS), United-States.

Therefore, the aim of this thesis is to shed a light on labor market interactions with endogenous market structures and financial business cycles, by building on the recent theoretical and empirical of Dynamics Stochastic General Equilibrium (DSGE hereafter) models.

This general introduction briefly sketches the main building blocks of this thesis. Section 1 illustrates the theoretical background⁷ with a recent historical perspective of the

canonical Diamond-Mortensen-Pissarides (DMP hereafter) framework and useful theoretical and econometric features of DSGE models. Section 2 provides a discussion on the three possible missing propagation channels. Section 3 provides a summary of the main contributions of the thesis to the existing literature. Section 4 describes the structure and the results of the thesis, organised in 3 chapters.

1 Theoretical background: search and matching frictions and DSGE models

Search-matching models have become the workhorse modelling framework in labor economics. The current search-matching models of the labor market have their theoretical foundations in an early study of Phelps (1970) who showed search theory can be useful in analysing both the natural rate of unemployment and the trade-off between inflation and unemployment. The following contributions of Diamond (1982), Mortensen (1982) and Pissarides (1985) lead to the development of an equilibrium model that produce more accurate predictions at a business cycle frequency of labor market variables than the standard neoclassical framework (Pissarides (2000)). Instead of solely focusing on the real wage cost of labor and its intensive margin, the DMP model centers on the dynamic labor market flows and the matching of firms and workers. The centerpiece of these models is the imperfect matching process between firms and workers. These models acknowledge that finding a worker for a job, or for a worker to find an open job vacancy, is a time-consuming and costly activity that is typically characterised by search frictions.¹ The DMP search and matching model has established itself as the standard theory of equilibrium unemployment.

Naturally, this labor market framework was quickly integrated in general equilibrium models (e.g. Cheron and Langot (2004); Gertler et al. (2008)). This class of models, referred to as DSGE models, surrounds a large variety of frameworks from the simple Real Business Cycle (RBC hereafter) model of Kydland and Prescott (1982), to the new

¹Petrongolo and Pissarides (2001) provide an extensive discussion on the matching process of unemployed workers and vacancies and on the microfoundations of the matching function.

classical growth model of [King et al. \(1988\)](#) or to the New Keynesian model of [Smets and Wouters \(2003\)](#). All these models are solved under the common assumption that agents solve intertemporal maximisation problems under rational expectations. This is a necessary assumption to fix the anomalies of the old Keynesian macroeconomic theory.²

Prior to the colonisation of DSGE in many economic fields, the building of what has now become a benchmark framework for policy discussions can be traced back to the RBC model of [Kydland and Prescott \(1982\)](#). Their model describes a frictionless economy populated by households and firms, who respond optimally to productivity shocks and are able to replicate key business cycles second moment for the US economy. To introduce nominal frictions, [Calvo \(1983\)](#) and [Rotemberg \(1982\)](#) provide two alternative micro-foundations of price stickiness and thus feature inflation dynamics in the RBC model. Following this contribution, the so-called Taylor rule ([Taylor \(1993\)](#)) was included to explicitly include the monetary policy as an autonomous agent fighting inflation. The resulting framework is successful for policy analysis.

However, some concerns have been raised about DSGE modeling regarding its empirical relevance since policy experiments were based on calibration rather than on the estimation of structural parameters. [Christiano et al. \(2005\)](#); [Smets and Wouters \(2003,0\)](#) solved this limitation by extending Bayesian econometrics to the estimation of DSGE mode. However, these three papers had a focus on the intensive margin of labor and did not consider frictions on the labor market as relevant features of the cycles. In contrast, [Gertler et al. \(2008\)](#) filled the gap by providing an estimated DSGE model focusing on the extensive margin of labor.

DSGE models are very popular in the policy-making institution. This approach provides a complete quantitative description of the joint stochastic processes by which a set of aggregate variables evolves, and provides a direct comparison of the simulated series with the relevant observed data. As an example, with this framework, we can simulate the

²Before DSGE models, policy-makers used a dynamic version of IS-LM. These models did a good job in fitting and explaining Western economies until the stagflation period of the 1970s. Nonetheless, during this period, large oil-price shocks raised both inflation and unemployment. Thus, the standard approach becomes irrelevant to explain higher levels of these two macroeconomic features and it became clear that models would require both expectations and microfoundations to address inflation and unemployment problems.

Beveridge Curve (i.e. the ratio between unemployment and vacancies) without having vacancies as an observable variable.

As a consequence, DSGE models are quickly emerging as a useful tool for quantitative policy analysis and for the conduct of monetary policy. However, the policy scope of this tool is not limited to monetary policy, as DSGE are also useful for fiscal and macroprudential policies, as well as structural reforms. For market reforms, DSGE models are an important complement to empirical models (Cette et al. (2017), Duval et al. (2018)). In our thesis, DSGE models will be employed to examine the implications of labor market reforms and macroprudential policies.

Nonetheless, as for DSGE models, the canonical DMP framework fails to reproduce the surge of the unemployment rate during the great recession (2007-2009) and then several criticisms have been made due to its lack of connection with other markets. The major critic towards DSGE models was that they ignore the role of financial factors in shaping macroeconomic fluctuations. Then, the financial crisis led to a renewed interest in the analysis made by Kiyotaki and Moore (1997) where durable assets serve for production and also as collateral for loans. This dual role amplifies the movement of asset prices and leaks to other sectors, in particular in the labor market.³ Thus, ignoring financial business cycles or firm entry in a DSGE model, as mentioned before, made them unable to reproduce the co-movement between unemployment rate, firm and housing price during the Great Recession.

Consequently, in this thesis, we consider the canonical DMP framework as an already relevant benchmark model, thus this thesis will provide many efforts in adding three important macroeconomic features for the labor market that will interact with the DMP structure.

³The financial crisis also led to a renewed interest in the analysis provided by Bernanke et al. (1999) linking financial distress to the financial accelerator.

2 The missing propagation channels

In this section, we describe the missing propagation channels that can contribute to account for the co-movement between housing, labor and product markets.

Endogenous market structure. According to [Jaimovich and Floetotto \(2008\)](#), approximately 20% of job creation (destruction) is due to the opening (closing) of the establishment. Based on this empirical observation, including an endogenous market structure (allowing for firm entry in the analysis of unemployment fluctuations) could be one of the missing ingredient in current DSGE models. [Bilbiie et al. \(2012\)](#) provide a framework that tackles the role of firm entry in shaping macroeconomic fluctuations through labor costs of a Walrasian labor market.⁴ In their framework, entry takes place subject to sunk product development costs; which are paid by investors in the expectation of future profits. As an example, an expansionary shock (i.e. one that raises output) leads to profit opportunities above entry costs, then new firms and products enter. In this setup, the markup, i.e. the price over the marginal of production of firms, depends on the number of entrants. Then, considering endogenous entry leads to an additional source of price fluctuations, other than the traditional price stickiness. As emphasised by [Lewis and Poilly \(2012\)](#), countercyclical fluctuations in the markup play a key role in the monetary transmission mechanism, driving inflation and hence aggregate demand.

Combining the firm entry mechanism into the DMP model offers two appealing features. The first feature concerns the role of firm entry on job creation. As more varieties of goods are created, this translates into more employment both for installed and existing firms. This creates a new propagation mechanism from firm creation to the labor market. The second feature concerns the markup, as firms must hire new workers to operate, frictions in the labor market reshape price dynamics by affecting the marginal cost. In addition, the creation of a new firm requires that new entrants have the same workforce as incumbents. Thus, a tightening in the labor market immediately transfers into a barrier to

⁴By Walrasian, we mean a perfectly competitive labor market based on the fluctuations of hours worked. The role of the extensive margin of the labor market and the sluggishness in the matching process are ignored into a Walrasian framework.

entry. These interplays between the labor market and firm dynamics create a yet powerful propagation channel. The latter is one of the missing ingredients that could explain the long-lasting recession of the US economy.

Concerning the existing research nesting the DMP framework into the entry mechanism, the literature is twofold. On the one hand, [Colciago and Rossi \(2011\)](#) merge these two markets in order to partially solve the so-called [Shimer \(2005\)](#) puzzle. Following a technology shock, their model induces a rise in the number of entrants. Then, the effect on employment is more important since it is equal to the number of incumbents times their labor force. On the other hand, some authors focus on improving the regulation of both labor and product market to enhance the structural allocation of an economy (e.g. [Cacciatore and Fiori \(2016\)](#)). The main result is that improving the regulation of one market induces positive effects for the other. Deregulating the labor market encourages incumbents to hire new workers at a lower cost, which in turn attracts more entrants as a result from a lower barrier to entry. Reducing the barrier to entry naturally leads to more goods and as a consequence, firms need to hire workers for their productions.

Job seekers and the level of mortgage debt. The second missing ingredient we investigate concerns the trivial role of labor market frictions in determining the level of mortgage debt of an economy. Financial intermediaries typically consider employment as an important criterion in their decision to grant a mortgage, which *de facto* excludes jobseekers from the financial market.⁵ In addition, the 2019 SCE housing survey suggests that job loss and income reduction account for 36% and 48% of foreclosure decisions. Given this evidence, employment fluctuations clearly shape the creation of new mortgages in the US. In this thesis, we propose a new channel to account for this evidence. This new channel statistically improves the empirical relevance of the DMP model in replicating the large co-movement between unemployment and mortgage debt at a business cycle frequency.

⁵The share of borrowers among employed is on average 50% and for unemployed 22% based on the Survey of Consumer Finance from 1990 to 2017.

Financial business cycles. The financial crisis of 2007 triggered an urgent need for a new policy framework that could address the key role of financial factors in shaping macroeconomics fluctuations. In their direction, many articles document the interaction between unemployment and financial markets through financial frictions based either on a collateral requirement *à la* [Kiyotaki and Moore \(1997\)](#) or on the financial accelerator *à la* [Bernanke et al. \(1999\)](#) for firms. Following this way of new financial frictions models, many articles have been devoted to link financial disruptions to unemployment peaks. As an example [Garin \(2015\)](#), [Zhang \(2018\)](#), [Christiano et al. \(2011\)](#) find an amplification mechanism from the financial market which partially drives the surge in unemployment. In contrast, [Liu et al. \(2016\)](#) focus on collateral requirements based on real commercial estate for firms in order to examine the relationship between financial and labor market. They find that the decline in housing prices during the Great Recession explains a fraction of the peak in unemployment rate. Given this evidence, there is a large consensus about the co-movement between housing worth and consumption cycle as documented by [Mian and Sufi \(2014\)](#).

However, about the papers previously cited, none of them consider both collateral requirements on real commercial estate and productive capital in a non-walrasian labor market. This thesis thus proposes to tackle the joint dynamics of unemployment, housing prices and industrial loans into a unified framework.

3 Key Contributions of the Thesis

The main contributions can be listed as follows:

- *An empirical quantification of the interaction between labor market frictions and endogenous entry.* Unlike the literature that combines entry and the DMP framework through calibration exercises, we propose to put the model to the data using Bayesian econometrics. The latter allows documenting the effect of entry on the mark-up through a data consistent model.

- *A new perspective on the cyclical nature of the markup* under labor market frictions. From an empirical standpoint, there is no well-established consensus in this literature on the cyclical nature of markups (e.g. [Bils \(1987\)](#); [Nekarda and Ramey \(2013\)](#); [Rotemberg and Woodford \(1999\)](#)).⁶ From a theoretical standpoint, either through a calibration exercise ([Bilbiie et al. \(2012\)](#)) or an estimated DSGE model ([Lewis and Stevens \(2015\)](#)), markup is found to be countercyclical. Given the absence of consensus on the cyclical nature of markup. Our contribution is thus to examine this cyclical nature, using a rich set of real frictions that are traditionally used in estimated DSGE model, combined with an imperfect labor market.
- *A new way of modelling collateral constraints for borrowers*. To account for the role of employment as a determinant of mortgage creation, we modify the traditional collateral constraint *à la* [Kiyotaki and Moore \(1997\)](#) to endogenise the Loan-To-Value (LTV hereafter) ratio through labor market frictions. To do so, we enrich the collateral constraint by limiting mortgage granting solely to households in employment. Given the presence of inflows and outflows in employment, the collateral constraint originally depends on employment flows balance.
- *An analysis of labor market reforms under financial frictions*. Following the standard practice in labor market reforms literature, we examine the effects induced by a decrease in unemployment insurance. As a key contribution with respect to the current literature, we find important leakages from labor market reform to the housing market. In particular, a structural reform in the labor market induces more borrowing in the economy as more households are in employment, which mechanically ease the borrowing constraint.
- *An analysis of macroprudential policy for entrepreneurs with spillovers on the labor market*. To offset imbalances in the industrial loan market, macroprudential policy can play a role by capping the amount of debt that firms can borrow. Assuming that the macroprudential authority decides the ratio of debt-to-assets that a firm

⁶[Bils \(1987\)](#); [Rotemberg and Woodford \(1999\)](#) finds countercyclical markups while [Nekarda and Ramey \(2013\)](#) observe the opposite.

can take, we examine how such a policy can be useful in mitigating financial imbalances. We consider two different policy tools that differ in the type of assets that is considered in the risk ratio of firms. One ratio considers physical capital as the asset, while the other considers the real commercial estate of firms. By doing so, we can quantify the long-run implications from the credit conditions for firms and the transition dynamics between the two equilibria. We find that tightening these ratios conduces to negative effect in the long run for the level of output and employment. However, tightening the LTV ratio for real commercial estate conduces to a less negative effect than the physical capital.

- *An analysis of macroprudential policy for household borrowers with spillover on the labor market.* We examine the spillover effect of a reform on the LTV ratio for borrowing agent (households) on the labor market. At the opposite of the result obtained for entrepreneurs, we find a positive long-run effect to tighten this LTV ratio due to the *microfoundation* of our collateral constraint.
- *The joint existence of two collateral constrained agents (i.e. entrepreneurs and households) under unemployment fluctuations.* While most of financial business cycle models have focused on the intensive margin of labor, we extend the scope of the analysis by examining the role of the extensive marginal of labor on industrial loans. More particularly, the incorporation of three types of constrained agents *à la Iacoviello (2005)* captures the so-called housing wealth effect, characterised by a joint rise in housing price and consumption.
- *A collateral constraint for entrepreneurs based on both real commercial estate and capital as financial assets.* This type of collateral constraint is not a new device in the literature, but it is only present with a Walrasian labor market. The integration of such a collateral constraint to study unemployment fluctuations highlights the key role of housing price fluctuations for collateral requirements on real commercial estate and also the effect of the investment fluctuation for the requirement on capital.

- *An endogenous bargaining power for workers.* The presence of the wage bill in advance constraint for firms originally makes the bargaining power of workers endogenous. This mechanism enlarges the number of factors that determines the bargaining power of households. In the current state of the literature, this bargaining power typically results from nominal rigidities on wages (e.g. [Abbritti and Fahr \(2011\)](#)). In this thesis, we find that the wage bill in advance is an important determinant of the bargaining power of firms, as tighter credit conditions encourage firms to further negotiate wages.

4 Structure and results of the Thesis

This thesis comprises three essays presented in three separate chapters where each of them highlights the main results obtained in this Thesis.

Chapter I, entitled “*Unemployment, entry and markup dynamics*”, introduces a DSGE framework that jointly addresses both search and matching friction on the labor market and an endogenous market structure, referred to as the firm entry mechanism. Unlike previous papers (e.g. [Cacciatore and Fiori \(2016\)](#); [Colciago and Rossi \(2011\)](#)) limiting the analysis a calibration exercise, we estimate the model using US data by adding a set of rigidities à la [Smets and Wouters \(2007\)](#) and [Lewis and Stevens \(2015\)](#).

We get three main results from this chapter. First, we confirm the finding of [Jaimovich and Floetotto \(2008\)](#), where entry plays a key role in generating unemployment fluctuations. Second, we find that one of the components of the markup, i.e. the marginal cost, becomes acyclical compared to a walrasian labor market. Third, we find a less important role of the entry mechanism generating a countercyclical markup and then a reduced *competition effect*.

Chapter II, entitled “*The role of labor market frictions on mortgage debt dynamics*”, considers an augmented collateral constraint where inflows and outflows of employment play a role in the determination of the level of mortgage debt. This chapter examines this new modelling device compared to the original collateral constraint à la [Kiyotaki and Moore \(1997\)](#) and an exogenous augmented collateral constraint à la [Iacoviello \(2015\)](#).

We get three main results. First, our collateral constraint outperforms the two others in terms of empirical fit. Second, we examine the consequence of deregulation on the labor market and find the existence of a strong amplification mechanism on the housing debt driven by unemployment fluctuations. Third, a macroprudential policy that aims at tightening the LTV ratio for borrowers can compensate the strong spillover effect implied by the first reform.

Chapter III, entitled “*Unemployment dynamics with financial business cycles*”, extends the analysis of the previous chapter by considering financially constrained firms, namely entrepreneurs, in addition to constraint households. In this setup, we include a large set of financial frictions limiting the entrepreneurs’ capacity to borrow from financial intermediaries through alternative LTV ratios on capital, real commercial estate and wage bills paid in advance.

We get five main results. First, we find that an entrepreneur collateral constraint that includes these three features outperforms those who do not take these into account. Second, the surge in the unemployment rate during the Great Recession is partly driven by the fall in the housing price through the combination of two effects: i) the housing wealth ii) the financial accelerator. Third, we examine the potential consequence of the deregulation of the labor market for both types of borrowers. For households, we have the same amplification channel as the previous chapter, while for entrepreneurs we have a substantial rise in their debt level. Fourth, a macroprudential policy characterised by a tightening of LTV ratio for households now has long-run positive effects on output and unemployment rate. Finally, we find that reducing LTV ratios for entrepreneurs has a negative long-run effect on output and unemployment rate. However, the LTV ratio on real commercial estate has a less negative effect than the LTV on capital for the long-run.

Chapter 1

Unemployment, entry and markup dynamics

Introduction

Firm entry cycles account for a large fraction of job creation (destruction) in the U.S. economy due to the birth (death) of firms. As pointed by [Jaimovich and Floetotto \(2008\)](#), the average fraction of quarterly job gain (losses) that can be explained by the opening (closing) of establishments is about 20%. Despite this evidence, the current state-of-art macroeconomic models with only firm entry are estimated on the intensive margin of labor, thus leaving the empirical nexus between labor frictions and firm entry yet undocumented. The main topic of this chapter is thus to close this gap by formulating and estimating a quantitative macroeconomic model that incorporates both labor and goods market frictions. The model is estimated on US time series from 1992Q3 to 2016Q4. The objective of the chapter is to explore the interaction of good and labor market frictions, and assess quantitatively the importance of firm entry in shaping the aggregate fluctuations of (un)employment for the US economy.

In the literature, two important papers related to the entry-unemployment nexus are close to our approach. First, [Colciago and Rossi \(2011\)](#) provide an important contribution based

on a friction on firm entry limiting the entry to startups that have reached the labor size of incumbents. Using the entry mechanism as well as search and labor market frictions, they find a strong amplification channel of technology shocks on labor market variables which partially solves the [Shimer \(2005\)](#) puzzle. We complement their analysis by having a time-varying markup determined endogenously by the number of producers operating in the market. The fraction of the labor force employed by incumbents is challenged by the number of firms willing to enter the market. According to the terminology of [Etro \(2007\)](#) and [Lewis and Stevens \(2015\)](#), we refer to the effect of entry on markup, and by extension on economic activity, as the *competition effect* in the rest of the chapter. The second paper is [Cacciatore et al. \(2016\)](#). They also validate the existence of the so-called *competition effect*. However, they focus on the impact of market reforms on labor and product and their welfare implications.

In these two main papers ([Colciago and Rossi \(2011\)](#) and [Cacciatore et al. \(2016\)](#)), the analysis is limited to a calibration exercise and validated with a comparison of simulated versus observed moments. Unlike these papers, we tackle the empirical dimension of firm entry and matching frictions by estimating the model to the data.

In this chapter, we formulate an original model with entry and labor market frictions, and take the model to the data through Bayesian techniques. By doing so, we formally evaluate the quantitative implication of entry to the overall model performance, and answer two questions: (i) *How important is the role of endogenous market structure to the fluctuations in unemployment ?* (ii) *Do search and matching frictions in the labor market generate countercyclical markup ?*

Our first question relates to the dynamics of unemployment in the traditional medium-scale DSGE models to study business cycle fluctuations. In this branch of the literature, unemployment is introduced using the setup of [Mortensen and Pissarides \(1994\)](#) (e.g. [Gertler et al. \(2008\)](#)). In these models, a positive demand shock raises the labor demand, firms post vacancies and, through the matching process, slowly hire new employees. In the case of a TFP shock, firms are more productive, which reduces their marginal cost but raises the marginal gain of hiring. Through the matching process, firms engage in hiring

new workers. However, the presence of endogenous firm creation significantly shakes this conventional propagation mechanism. Under firm entry, unemployment is proportional to the number of firms operating in the market, which reshapes the overall dynamics following demand and supply shocks. Since operating firms are the sum of incumbents and startup, their combination affects employment dynamics. As in [Colciago and Rossi \(2011\)](#) and [Cacciatore and Fiori \(2016\)](#), entry costs depend on two ingredients: the incumbent workforce size and the probability for firms to find a worker. The modification of these two components affect the entry cost, resulting in an amplification mechanism on entry throughout labor market fluctuations. In the case of an expansionary shock boosting entry, the number of producers rises but the size of the incumbent workforce can potentially drop due to the *competition effect*. If the expansionary shock crowds out entry, the number of producers naturally drops, but the *competition effect* can lead to an expansion of the incumbent labor force.

The second question is related to the cyclical propriety of markup. For central banks, fluctuations in the markup play a key role in the monetary transmission mechanism which describes how interest rate changes affect the rest of the economy (e.g. [Lewis and Poilly \(2012\)](#)). For example, a countercyclical markup drives inflation and hence aggregate demand. However, from an empirical point of view, markups of price over marginal costs are hard to measure and thus remain unobservable. Given this dark corner, important uncertainties remain on the cyclicity of markups. For example, [Bils \(1987\)](#) and [Rotemberg and Woodford \(1999\)](#) find evidence of countercyclical markups, while [Nekarda and Ramey \(2013\)](#) obtain the opposite result on the correlation link. Since there is no well-established consensus on the cyclicity of markups in the data or even conditional on shock, the estimated model can thus be employed to contribute to the measure of markup, and investigate its cyclical link.¹ Thus, the choice of search and matching friction in the vein of [Mortensen and Pissarides \(1994\)](#) introduces a wedge on the marginal cost that depends on both hiring costs and the future value of employment. This wedge is likely to reverse the markup cyclicity, with respect to a standard intensive margin labor model (e.g. [Lewis and Stevens \(2015\)](#) or [Bilbiie et al. \(2012\)](#)).

¹The simulation of the DSGE model allows for a smoothed simulate of unobserved variables.

Therefore, the main findings of the chapter read as follows. First, we find that entry shocks play a key role in generating unemployment fluctuations via a variance decomposition analysis and the time path of unemployment conditional to entry shock. Second, under matching frictions, markup becomes countercyclical and characterised by a less important *competition effect* than in previous studies. Finally, after conducting some robustness analysis with alternative hypothesis, we find that our model is preferred to other specifications.

1 The model

In this section, we present the model which combines the entry mechanism proposed by [Bilbiie et al. \(2012\)](#) and search and matching frictions in the labor market in the spirit of [Mortensen and Pissarides \(1994\)](#). We include a set of real and nominal frictions as in [Christiano et al. \(2005\)](#) and [Smets and Wouters \(2007\)](#), such as habit formation, investment adjustment costs and variable capital utilisation. These features are necessary in order to replicate the joint dynamics of investment, consumption and output as observed in the data. We consider an economy populated by households, firms (which could be producers or new entrants), and national authorities with a central bank and a government. We first described translog preferences of the representative household, as in [Feenstra \(2003\)](#), and its optimal decisions regarding investment and consumption. Second, we introduce the production sector and the accumulation of firm dynamics by separating new entrants and incumbents. The latter minimise its cost and fix its selling price subject to nominal rigidities *à la* [Rotemberg \(1982\)](#), while new entrants post vacancies in order to reach the size of the incumbent labor force. Third, we present the wage setting resulting from Nash bargaining between incumbents and its workforce. Finally, in two different sections, we report the role of national authorities and the general equilibrium conditions of the economy.

1.1 Household

Each household is designed as a large extended family containing a continuum of members along a unit interval. The number of family members currently employed is defined by L_t . Following [Andolfatto \(1996\)](#) and [Merz \(1995\)](#), the family provides perfect consumption insurance for its members such that there is no *ex-post* heterogeneity in consumption across households.

1.1.1 Translog preferences and product turnover

Preferences The final consumption baskets, denoted Y_t^C , is defined over a continuum \tilde{N} where only a subset of goods are available at time t : $N_t \in \tilde{N}$.² We follow [Feenstra \(2003\)](#) in assuming that the final consumption basket Y_t^C , which includes the consumption bundle C_t , takes a translog preference form.³ Translog preferences are characterised by defining the unit expenditure function (i.e. the price index P_t^C) associated with the preference aggregator. Let $P_{f,t}$ be the nominal price for the variety $f \in [0, N_t]$, then the unit expenditure function on the basket good Y_t^C is given by:

$$\ln P_t^C = \frac{\tilde{N} - N_t}{2\sigma\varepsilon_t^P \tilde{N} N_t} + \frac{1}{N_t} \int_{f \in N_t} \ln P_{f,t} df + \frac{\sigma\varepsilon_t^P}{2N_t} \int_{f \in N_t} \int_{f' \in N_t} \ln P_{f,t} (\ln P_{f',t} - \ln P_{f,t}) df df' , \quad (1.1)$$

where $\sigma > 0$ denotes the price elasticity of the spending share to an individual good and ε_t^P an exogenous price markup shock. The assumption of translog preferences originally provides endogenous markup by product.⁴ Translog preferences give an elasticity of demand θ_t which varies with the number of goods: $\theta_t = 1 + \sigma\varepsilon_t^P N_t$ and the following optimal

² \tilde{N} represents the maximum of goods that the economy can reach.

³Since aggregate demand Y_t^C also includes others sources than household consumption C_t , this assumption ensures that the consumption price index is also the price index for the aggregate demand.

⁴There are various ways to introduce endogenous markups, with demand-side complementarities as in this article or supply-driven competition effects working through changes in the market structure. As in [Bilbiie et al. \(2012\)](#) and [Lewis and Stevens \(2015\)](#), we choose the latter device for its ability to reproduce key features of business cycles. Moreover, a supply-driven mechanism needs to replace monopolistic competition by another type, such as Cournot ([Lewis and Poilly \(2012\)](#)) and complicates the comparison with the benchmark specification with CES preferences.

demand addressed for one good,

$$N_t P_{f,t} y_{f,t} = P_t^C Y_t^C, \quad (1.2)$$

where $y_{f,t}$ is the output of the f^{th} producer. Note that we use the terms 'goods', 'firms' and 'producers' interchangeably, assuming that each firm produces exactly one differentiated variety. According to [Bilbiie et al. \(2012\)](#), each unit is interpreted as a production line that could be part of a multi-product firm whose boundary is left undetermined. Household preferences thus make demand more elastic when the number of producers rises.

Product Turnover In each period, all (entering and existing) firms are affected by an exogenous exit shock $\delta^N \in [0, 1]$. Then, the law of motion of firms in the economy reads as:

$$N_t = (1 - \delta^N) (N_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E), \quad (1.3)$$

where AC_{t-1}^E denotes the endogenous failure rate of startups. Following [Lewis and Stevens \(2015\)](#), we assume that not all startups are successful in entering the good market. As a result, a fraction AC_{t-1}^E of new firms does not succeed in establishing their business. Formally, $AC_{t-1}^E = \frac{\varphi^E}{2} \left(\varepsilon_{t-1}^E \frac{N_{t-1}^E}{N_{t-2}^E} - 1 \right)^2$ with $\varphi^E \geq 0$ the degree of rigidity and ε_t^E an exogenous shock. Thus, the failure rate of startups increases with the growth rate of competitors. In the same vein as investment costs, this function captures the dynamic patterns of entry over business cycles and can be interpreted as a procyclical congestion cost.

1.1.2 Budget and intertemporal decisions

The representative household maximises its expected intertemporal utility function adjusted by the discount factor $\beta \in [0, 1]$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon_t^B \frac{(C_t - h^C C_{t-1})^{1-\sigma^C}}{1 - \sigma^C} \right\}, \quad (1.4)$$

where ε_t^B is an exogenous preference shock, $h^C \in [0, 1]$ denotes external habits and $\sigma^C > 0$ is the risk-aversion parameter for consumption.

Households' budget constraint in real terms reads as:

$$\begin{aligned} \frac{W_t}{P_t^C} L_t + (1 - L_t) b + X_t^K + (1 - \delta^N) (d_t + e_t) (x_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E) \\ = C_t + T_t + \frac{B_t}{P_t^C} - R_{t-1} \frac{B_{t-1}}{P_t^C} + N_t^E \phi_t^E + e_t x_t \end{aligned} \quad (1.5)$$

The income of the representative household is made up of labor income with $\frac{W_t}{P_t^C} L_t$ for employed members and $(1 - L_t) b$ for unemployed members, return on riskless bonds $R_{t-1} \frac{B_{t-1}}{P_t^C}$ with R_{t-1} the nominal interest rate, net return on owning capital stock X_t^K and return on share holdings x_t and on successful startups $(1 - AC_{t-1}^E) N_t^E$.

The expenditure side includes consumption C_t , taxes T_t , bond purchases B_t , investment on startups $N_t^E \phi_t^E$ and share purchases $e_t x_t$ (where e_t stands for the market price of a share x_t). The goal of the representative household is to maximise its utility (Eq.1.4) subject to its budget constraint (Eq.1.5), to choose consumption, saving (in the form of bonds), share holding, investment in startups, capital stock and utilisation rate. We therefore describe the decisions of households in three separate groups for clarity purposes.

Consumption and saving decisions The optimal consumption choice reads as follows:

$$\lambda_t^C = \varepsilon_t^B (C_t - h^C C_{t-1})^{-\sigma^C}, \quad (1.6)$$

where λ_t^C is the marginal utility of consumption. The First Order Condition (FOC hereafter) with respect to B_t , gives the Euler equation for bonds:

$$\frac{\lambda_t^C}{\beta \mathbb{E}_t \{ \lambda_{t+1}^C \}} = \frac{R_t}{\mathbb{E}_t \{ \pi_{t+1}^C \}}, \quad (1.7)$$

where $\pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$ is the welfare-based expected inflation rate. This condition determines the optimal path of consumption.

Share holdings and start-up financing Successful startups $(1 - AC_{t-1}^E) N_{t-1}^E$ and shares purchased in the previous period x_{t-1} pay dividends d_t and are worth e_t conditional

only on the exogenous survival probability $1 - \delta^N$.⁵ Each period, the representative household needs to spend its income on financing start-ups at a cost ϕ_t^E , as described later, and purchase new shares at the market price e_t .⁶

Then, optimal share purchasing leads to the Euler condition on shares:

$$e_t = (1 - \delta^N) \mathbb{E}_t \{ \beta_{t,t+1} (d_{t+1} + e_{t+1}) \}, \quad (1.8)$$

with $\beta_{t,t+1} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^C}{\lambda_t^C} \right\}$ the stochastic discount factor. Put in a recursive fashion, Eq.1.8 becomes $e_t = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta_t (1 - \delta^N)^t d_t$, where the current value of shares is equal to the discounted sum of expected dividends as in the standard corporate finance theory. The FOC with respect to new firms N_t^E gives the following free entry condition:

$$\phi_t^E = e_t \left(1 - \frac{\partial AC_t^E N_t^E}{\partial N_t^E} \right) - \mathbb{E}_t \left\{ \beta_{t,t+1} e_{t+1} \frac{\partial AC_{t+1}^E}{\partial N_t^E} N_{t+1}^E \right\}. \quad (1.9)$$

Assuming that the probability of success for new businesses is equal to one, such that $\varphi^E = 0$, the free entry condition is the same as in [Bilbiie et al. \(2007\)](#) ($\phi_t^E = e_t$). In this case, entry thus occurs until the firm value reaches the entry cost ϕ_t^E . Note that the adjustment cost (including exogenous shock ε_t^E) drives a wedge between the equity value (e_t) and the cost of establishing new firms (ϕ_t^E).⁷

Optimal decisions for capital The representative household owns the total stock of capital of the economy K_t and chooses the capital utilisation rate ν_t . The effective unit of capital K_t^ν is given by:

$$K_t^\nu = \nu_t K_t. \quad (1.10)$$

Adjusting the utilisation rate is costly for the representative household. It has to pay $\psi(\nu_t)$ per unit of physical capital if it wants to adjust the utilisation rate. Following [Christiano et al. \(2005\)](#), we assume that $\nu = 1$, $\psi(\nu) = 0$ and $\psi'(\nu) = r^K$.⁸ Formally, we have

⁵Formally, this corresponds to $(1 - \delta^N) (d_t + e_t) (x_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E)$ in the income side of the budget constraint (Eq.1.5)

⁶The cost ϕ_t^E is described in the subsection 1.2.2.

⁷This functional form of startup establishment captures financial frictions or institutional changes, making entry more desirable than other opportunities of investment.

⁸In the rest of the paper, variables without their time index refer to their steady-state value. Here, for example ν corresponds to the steady-state value of the utilisation rate.

$\psi(\nu_t) = \frac{1-\Psi}{\Psi} r^K \left(e^{\frac{\Psi}{1-\Psi}(\nu_t-1)} - 1 \right)$ with $\Psi \in [0, 1]$ the utilisation elasticity.⁹ Additionally to this cost, the investment in physical capital is costly, i.e. the representative household faces an adjustment cost AC_t^I on investment, such that $AC_t^I = \frac{\varphi^I}{2} \left(\varepsilon_t^I \frac{I_t}{I_{t-1}} - 1 \right)^2$, where $\varphi^I \geq 0$ is the degree of rigidity and ε_t^I is an exogenous shock to the efficiency of investment. The capital stock in the economy thus evolves according to:

$$K_t = (1 - \delta^K) K_{t-1} + (1 - AC_t^I) I_t. \quad (1.11)$$

According to these costs, effective capital is rented to firms at price r_t^K in a perfectly competitive capital market. The net return on owning the capital stock X_t^K in the budget constraint (Eq.1.5) reads as follows:

$$X_t^K = r_t^K \nu_t K_t - \psi(\nu_t) K_t - I_t, \quad (1.12)$$

where $r_t^K \nu_t K_t$ represents the household's earnings from supplying capital services to firms.

The First Order Conditions (FOC hereafter) with respect to physical capital reads as:

$$q_t^K = \mathbb{E}_t \left\{ \beta_{t,t+1} \left(r_{t+1}^K \nu_{t+1} - \psi(\nu_{t+1}) + q_{t+1}^K (1 - \delta^K) \right) \right\}, \quad (1.13)$$

where q_t^K denotes the shadow value of capital (in units of consumption).¹⁰ This shadow value is defined by the following FOC for investment I_t :

$$1 = q_t^K \frac{\partial(1 - AC_t^I)I_t}{\partial I_t} - \mathbb{E}_t \left\{ q_{t+1}^k \beta_{t,t+1} \frac{\partial AC_{t+1}^I I_{t+1}}{\partial I_t} \right\}. \quad (1.14)$$

Then, for $\varphi^I \neq 0$, the real shadow value of capital is time varying, and the exogenous shock ε_t^I with fluctuations in investment drives the value of capital q_t^K .

⁹For $\psi \rightarrow 1$, the cost of changing the utilisation rate is very high and the utilisation rate does not vary. For $\psi \rightarrow 0$, the marginal cost of changing the capital utilisation rate is constant and as a result, the rental rate of capital does not vary.

¹⁰ q_t^K also corresponds to the Lagrange multiplier associated with the capital law of motion (Eq.1.11) normalised by the marginal utility of consumption λ_t^C .

Finally, the optimal utilisation rate for capital is determined by:

$$\psi'(\nu_t) = r_t^K. \quad (1.15)$$

Under this specification, the utilisation rate is increasing with the cost of purchasing a new unit of physical capital.

1.2 Production

There is a continuum of monopolistically competitive firms, each producing a differentiated variety $f \in [0, N_t]$, and a continuum of new entrants $e \in [0, N_t^E]$. Prior to entry, new firms pay a sunk entry cost $\phi_{e,t}^E$, to be specified later on. As in [Cacciatore \(2014\)](#), all firms that enter the market start producing at the next period.

1.2.1 Incumbent

To introduce nominal rigidities, we assume that incumbents face a two-stage problem. First, they minimise their production cost subject to labor and capital law of motions. Second, they fix their selling prices subject to [Rotemberg \(1982\)](#) rigidities to introduce a sticky selling price as a time-varying markup over the marginal cost.

Marginal cost determination For incumbents, production requires both labor and capital. For labor, a representative firm is subject to matching frictions for hiring workers. To create a new job, a producer posts a vacancy and bears a cost denoted by f^V . The probability of finding a worker depends on a constant return to scale matching technology, which converts aggregate unemployed workers U_t and aggregate vacancies V_t into aggregate new jobs $M_t = m(V_t)^\zeta (U_t)^{1-\zeta}$ with $m > 0$ the degree of efficiency and $\zeta \in [0, 1]$ the elasticity of matches with respect to vacancies. Thus, the probability of filling a vacancy is given by: $q_t = \frac{M_t}{V_t}$ and the probability for an unemployed worker to find a job is: $f_t = \frac{M_t}{U_t}$. From the perspective of an individual firm, employment $l_{f,t}$ is the balance between inflows of new workers, represented by $q_t \nu_{f,t}$, and the outflow of workers due to

exogenous separation $\delta^L \in [0, 1]$. Formally, we have:

$$l_{f,t} = (1 - \delta^L) l_{f,t-1} + q_t v_{f,t}. \quad (1.16)$$

According to our timing assumption, the number of unemployed workers looking for a job is given by the difference between unity (the total population of workers) and the number of employed workers at the end of the period $t - 1$: $U_t = 1 - (1 - \delta^L) (1 - \delta^N) L_{t-1}$.¹¹

Thus, the total output of firm f is given by:

$$y_{f,t} = \varepsilon_t^Z (l_{f,t})^\alpha (k_{f,t}^\nu)^{1-\alpha}, \quad (1.17)$$

where $\alpha \in [0, 1]$ is the part of labor used in production and ε_t^Z is an exogenous productivity shock. Firm decides on the optimal amount of labor and vacancies subject to Eq.1.16 for the labor market, and also decide on the optimal amount of capital given the renting rate r_t^K . Then, the maximisation problem is given by:

$$\max_{l_{f,t}, v_{f,t}, k_{f,t}^V} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 - \delta^N)^t \left\{ \frac{P_{f,t}}{P_t} y_{f,t} - (1 + AC_{f,t}^W) \frac{W_t}{P_t} l_{f,t} - f^V v_{f,t} - r_t^K k_{f,t}^V \right\},$$

with $AC_{f,t}^W$ a quadratic adjustment cost for nominal wage *à la* Rotemberg (1982) as described later.¹²

The optimal employment per firm is given by:

$$\mu_{f,t}^L = \alpha \frac{mc_{f,t} y_{f,t}}{l_{f,t}} - (1 + AC_{f,t}^W) \frac{W_t}{P_t} + (1 - \delta^N) (1 - \delta^L) \mathbb{E}_t \{ \beta_{t,t+1} \mu_{f,t+1}^L \}, \quad (1.18)$$

with $\beta_{t,t+1}$ the stochastic discount factor of households (which are assumed to own firms) and $mc_{f,t}$ the real marginal cost associated with production. The marginal profit from hiring a new worker has three main determinants. The first determinant is the marginal product of employment, the second is the wage bill paid by firms to workers and the last one is the expected continuation value of the job at the next period accounting for the

¹¹Gertler et al. (2008) use a similar timing.

¹²Since the wage is determined through Nash bargaining, the firm does not determine the real wage on its own, we describe this property in the subsection 1.3.

probability of both separations (from the labor market δ^L and from the product market δ^N). Optimising with respect to vacancies, then the value of hiring a new worker $\mu_{f,t}^L$ emerges:¹³

$$\mu_{f,t}^L = \frac{f^V}{q_t}. \quad (1.19)$$

This value depends negatively on the probability that the vacancy is filled. Intuitively, the firm positively values hiring when it is difficult to find employees. Finally, the FOC with respect to effective capital reads as:

$$r_t^K = (1 - \alpha) \frac{mc_{f,t} y_{f,t}}{k_{f,t}^\nu}. \quad (1.20)$$

Since these first order conditions determine the marginal cost of production, we now turn to the second step, namely applying a time-varying markup on the marginal cost subject to nominal rigidities.

Price setting In the second step, the representative firms operate monopolistically and set prices according to [Rotemberg \(1982\)](#) technology. The quadratic adjustment cost is given by: $AC_{f,t}^P = \frac{\kappa^P}{2} \left(\frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right)^2 \frac{P_{f,t}}{P_t^C}$ with $\kappa^P \geq 0$ the degree of rigidity and $\lambda^P \in [0, 1]$ the indexation on past inflation. Given this price adjustment cost, firms determine their optimal selling price $P_{f,t}$ in order to maximise their expected stream of profits given by:

$$\max_{P_{f,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta)^t (1 - \delta^N)^t \left\{ \left(\frac{P_{f,t}}{P_t^C} - mc_{f,t} - AC_{f,t}^P \right) y_{f,t} \right\}. \quad (1.21)$$

Since the amount of firm-specific output $y_{f,t}$ is demand-determined in response to its relative price $\rho_{f,t} = \frac{P_{f,t}}{P_t^C}$ (Eq.1.2), the optimal scheme policy is:

$$\frac{\rho_{f,t}}{mc_{f,t}} = \frac{\mu_{D,t}}{(1 - AC_{f,t}^P) + \kappa^P \psi_{f,t}^P}, \quad (1.22)$$

where μ_t^D is the desired markup defined in the household preferences ($\mu_t^D = 1 + \frac{1}{\sigma \varepsilon_t^P N_t}$), which depends on the number of varieties due to translog preferences. The term $\psi_{f,t}^P$ is

¹³The value of hiring a new worker $\mu_{f,t}^L$ corresponds to the Lagrange multiplier associated with the law of motion of employment at the firm level (Eq.1.3).

an auxiliary variable that depends on the Rotemberg cost of adjustment.¹⁴ The optimal pricing induces that the price is set as a markup over the real marginal cost of production such as:

$$\mu_{f,t} = \frac{\rho_{f,t}}{mc_{f,t}}. \quad (1.23)$$

Note that in absence of nominal rigidities i.e. $\kappa^P = 0$, the markup in Eq.1.22 is equal to the desired markup $\mu_{f,t} = \mu_{D,t}$. As a consequence, more diversity for consumers leads to lower desired markup for firms.

1.2.2 New entrants

As mentioned before, an entrant must pay a sunk cost, denoted $\phi_{e,t}^E$, to establish its business and producing at the next period. As in [Cacciatore and Fiori \(2016\)](#), we adopt the following form:

$$\phi_{e,t}^E = \frac{f^E}{\varepsilon_t^Z} + f^V v_{e,t}^E. \quad (1.24)$$

As standard practice in the literature, we assume that the technological shock for incumbents also affects new entrants, since new technology leads to lower costs in both sectors.

In expression 1.24, the first term (f^E) represents technological requirements, such as research and development and the cost in terms of goods and services imposed by administrative barriers to market entry. The second term of the entry cost corresponds to the recruitment of workers with $v_{e,t}^E$ vacancies posted by a new firm. The optimal hiring policy of new firms, which starts with no initial workforce, must post as many vacancies as required at time t to reach the same workforce as incumbent ones. Therefore, a new entrant posts:

$$v_{e,t}^E = \frac{(1 - AC_t^E) (1 - \delta^N) (1 - \delta^L) l_{f,t}}{\mathbb{E}_t \{q_{t+1}\}}. \quad (1.25)$$

Then, $(1 - AC_t^E) (1 - \delta^N)$ accounts for the probability of a new entrant succeeding in producing in $t + 1$ and $(1 - \delta^L)$ represents the exogenous probability of the employment

¹⁴More precisely $\psi_{f,t}^P = -\mathbb{E}_t \left\{ (1 - \delta^N) \beta_{t,t+1} \frac{y_{f,t+1} P_{t+1}^C}{y_{f,t} P_t^C} (\pi_{f,t+1} - 1 + \lambda^P (1 - \pi_t)) (\pi_{f,t+1})^2 \right\}$.

relationship succeeding. Note that the probability of finding a worker is in $t + 1$ since vacancies posted by new firms need a time lag to be filled.¹⁵

The cost of entering the market thus depends positively on the incumbent labor force and negatively on the expected probability of finding a worker. The formulation of the entry cost is also further discussed in the sensitivity analysis made in the subsection 4.2.

1.3 Wage setting

We assume that nominal wages are determined through a Nash bargaining process between workers and firms that maximises the joint surplus of an employment relationship. The bargaining solution is determined by the following program:

$$W_{f,t} = \arg \max_{\{W_{f,t}\}} (\mu_{f,t}^W)^{\eta \varepsilon_t^L} (\mu_{f,t}^L)^{1-\eta \varepsilon_t^L}, \quad (1.26)$$

where $\eta \in [0, 1]$ is the fixed bargaining power of workers and ε_t^L is an exogenous shock.¹⁶ The variable $\mu_{f,t}^L$ is the firm's surplus from Eq.1.18 and the surplus of a worker from employment of the firm f^{th} is given by:¹⁷

$$\mu_{f,t}^W = \frac{W_{f,t}}{P_t^C} - b + \mathbb{E}_t \{ \beta_{t,t+1} \mu_{t+1}^W (1 - \delta^N) (1 - \delta^L) (1 - f_{t+1}) \}, \quad (1.27)$$

which corresponds to the net value of being in employment (wage minus transfer to unemployed workers) and the expected continuation value of the job in the next period, accounting for destruction for both markets (labor and product) and less the expected probability of an unemployed worker finding a job. Intuitively, an higher probability to find a job (f_{t+1}) in the future, makes it easier for the worker to find another job (thus reducing the incentive to keep his job of firm f).

¹⁵The assumption of the time lag for hiring people for new entrants and not for incumbents is necessary, otherwise new entrants would pay wages, or some workers would be in unemployment but with no producing activities and no labor compensation.

¹⁶This shock captures some institutional changes in the wage negotiation process such as changes in union participation rate.

¹⁷Due to the value of the firm, which is linear in its employment level, all workers are the same at the margin, and the wage negotiation is between the firm and the marginal worker.

The resulting equilibrium wage from the Nash bargaining reads as follows:

$$\omega_t \mu_{f,t}^L = (1 - \omega_t) \mu_{f,t}^W. \quad (1.28)$$

Due to nominal wage adjustment costs in the expected value of a job for firms (Eq.1.18) denoted by $AC_{f,t}^W = \frac{\kappa^W}{2} \left(\frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W (\pi_{t-1} - 1) \right)^2$ with $\kappa^W \geq 0$ the degree of rigidity and $\lambda^W \in [0, 1]$ the indexation on past inflation, the effective bargaining power of the worker denoted by ω_t is time-varying and reflects the evolution of current and expected wage adjustment costs. This expression is,

$$\omega_t = \frac{\varepsilon_t^L \eta}{\varepsilon_t^L \eta + (1 - \varepsilon_t^L \eta) (1 + AC_{f,t}^W + \kappa^W \psi_{f,t}^W)}, \quad (1.29)$$

where $\psi_{f,t}^W$ is an auxiliary variable that depends on the Rotemberg cost of adjustment, similar in its form to that obtained for the optimal price (Eq.1.22).¹⁸ With adjustment costs, bargaining power becomes state-dependent. During periods of rising wages, $\frac{\partial \omega_t}{\partial W_{f,t}} < 0$, the effective bargaining power of workers declines (respectively when wages are declining, bargaining power increases). Intuitively, when wages decline, workers capture a larger fraction of the joint surplus of the employment relationship, and a smaller fraction when they rise.

Finally, by replacing the marginal value of posting a vacancy (Eq.1.18) and its marginal value (Eq.1.19), the expression of wages is given by:¹⁹

$$\frac{W_{f,t}}{P_t^C} = \omega_t \left(\alpha \frac{m c_{f,t} y_{f,t}}{l_{f,t}} \right) + (1 - \omega_t) b + \mathbb{E}_t \left\{ \frac{\beta_{t,t+1} f^V (1 - \delta^L) (1 - \delta^N)}{q_{t+1}} \left(\omega_t - \frac{(1 - \omega_t) \omega_{t+1}}{(1 - \omega_{t+1})} \left(1 - \frac{f_{t+1}}{(1 - \delta^L) (1 - \delta^N)} \right) \right) \right\}. \quad (1.30)$$

Note that in the absence of distortion due to wage adjustment costs or exogenous shock ($\kappa^W = 0$ and $\varepsilon_t^L = 0$), the real wage is simply an average of the highest wage that the

¹⁸More precisely $\psi_{f,t}^W = -\mathbb{E}_t \left\{ \frac{\pi_{f,t}^W (\pi_{f,t}^W - 1 + \lambda^W (1 - \pi_{t-1}))}{\pi_{t+1}^C} \left(\pi_{f,t+1}^W - 1 + \lambda^W (1 - \pi_t) \right) \left(\pi_{f,t+1}^W \right)^2 \right\}$, where

$\pi_{f,t}^W = \frac{W_{f,t}}{W_{f,t-1}} \pi_t$ is the inflation of nominal wages for workers employed by a producer f .

¹⁹To be precise in the left side of the following equation, we have $\frac{W_{f,t}}{P_t^C} (1 + \omega_t AC_{f,t}^W)$ but at a first linear approximation this term disappears.

firm can pay and the minimum that households accept:

$$\frac{W_{f,t}}{P_t^C} = \eta \left(\alpha \frac{m c_{f,t} y_{f,t}}{l_{f,t}} + \mathbb{E}_t \left\{ \beta_{t,t+1} f^V \frac{f_{t+1}}{q_{t+1}} \right\} \right) + (1 - \eta) b. \quad (1.31)$$

1.4 Authorities

The central bank sets the interest rate following a standard Taylor rule,

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r} \right)^{\rho^R} \left(\left(\frac{\pi_t}{\pi} \right)^{\phi^\pi} \left(\frac{Y_t^C}{Y_{t-1}^C} \right)^{\phi^{\Delta Y}} \right)^{(1-\rho^R)} \varepsilon_t^R, \quad (1.32)$$

where $\rho^R \in [0, 1]$ is the weight according to the past interest rate, $\phi^{\Delta Y}$ the emphasis for GDP growth (where we use Y_t^C the aggregate demand defined below as a proxy for GDP), ϕ^π the parameter for inflation dynamics and ε_t^R corresponding to the exogenous monetary policy shock.

The government finances public spending and compensation for unemployed households by collecting lump-sum taxes T_t from households, as well as issuing one-period bonds B_t . The total amount of public spending G_t is assumed to evolve according to an exogenous process, such that $G_t = \varepsilon_t^G g^Y$ where g^Y is the steady-state ratio of public spending to GDP ($g^Y = G/Y^C$). Thus, the balance sheet of government is given by:

$$\varepsilon_t^G g^Y + (1 - L_t) b + R_{t-1} B_{t-1} = T_t + B_t. \quad (1.33)$$

1.5 Shocks, aggregation and equilibrium condition

In this model, there are eight exogenous shock processes defined by $\varepsilon_t^i = \rho^i \varepsilon_{t-1}^i + \eta_t^i$ for $i = \{Z, E, C, G, I, P, L, R\}$ and where ρ^i are autoregressive roots (AR(1)) of the exogenous variables, η_t^i are standard errors that are mutually independent, serially uncorrelated and normally distributed such that $\eta_t^i \sim \mathcal{N}(0, \sigma^i)$ with σ^i the variance. Price and bargaining shocks are augmented with a moving average (MA(1)) term denoted u^l for $l = \{P, L\}$ as in [Smets and Wouters \(2007\)](#) such that $\varepsilon_t^m = \rho^m \varepsilon_{t-1}^m + \eta_t^m - u^m \eta_{t-1}^m$. Finally, we also

follow the same authors in assuming that the spending shock is affected by productivity innovation by ρ^{GZ} i.e. $\varepsilon_t^G = \rho^G \varepsilon_{t-1}^G + \rho^{GZ} \eta_t^Z + \eta_t^G$.

Aggregating all the agents and varieties in the economy and imposing market clearing for all markets leads to the definition of aggregate labor $L_t = \int_0^{N_t} l_{ft} df$ and capital $K_t = \int_0^{N_t} k_{ft} df$, while aggregate vacancies also depend on the number of new entrants, such that $V_t = N_t v_t + N_{t-1}^E v_{t-1}^E$. With these definitions and the total amount of vacancies posted by new entrants (Eq.1.25), we can write the law of motion of employment at the aggregate level $L_t = (1 - \delta^L) (1 - \delta^N) L_{t-1} + q_t V_t$. The aggregate supply of the economy is obtained in the same way $Y_t = \int_0^{N_t} y_{f,t} df$.

After (i) aggregating all agents and varieties in the economy (ii) imposing market clearing on all markets and (iii) substituting the relevant demand functions, the resource constraint for the economy, also defined as GDP through the demand approach, reads as follows:

$$Y_t^C = C_t + I_t + \Psi(\nu_t) K_t + g^Y \varepsilon_t^G + N_t f^V v_t + N_t^E \phi_t^E, \quad (1.34)$$

which equals the sum of consumption, investment, the cost of using capital, vacancy costs and product creation expenses.²⁰ Using the optimal demand (Eq.1.2) and equalizing supply with demand, we obtain,

$$\rho_t Y_t = Y_t^C. \quad (1.35)$$

Concerning prices, rearranging the translog expenditure function (Eq.1.1) and imposing symmetry among producers, the relative price $\rho_t = \frac{P_t}{P_t^C}$ emerges,

$$\rho_t = \exp \left(- \frac{\tilde{N} - N_t}{2\sigma^P \varepsilon_t^P \tilde{N} N_t} \right), \quad (1.36)$$

which is referred to as the love variety effect in the literature.

Using this price index, we deduct the welfare-based inflation π_t^C ,

$$\frac{\rho_t}{\rho_{t-1}} = \frac{\pi_t}{\pi_t^C}. \quad (1.37)$$

²⁰They also include adjustment costs on wages and prices ($w_t L_t A C_t^W + \rho_t Y_t A C_t^P$). However, at a first order condition approximation, these two rigidities do not affect the GDP.

2 Estimation

We estimate the model using Bayesian techniques as in [Smets and Wouters \(2007\)](#) and [Smets and Wouters \(2003\)](#). In this section, we present the data sources and transformations, before turning out to prior and posterior distributions of the model parameters.

2.1 Data

The model is estimated with Bayesian methods on US quarterly data from 1993Q2 to 2016Q4. Our sample is rather short since samples on business establishments are discontinued as pointed out by [Lewis and Stevens \(2015\)](#). The dataset includes output, consumption, investment, nominal interest rate, inflation, real wage, unemployment and net business formation. The first six data are the same as [Smets and Wouters \(2007\)](#). Output is measured by GDP, consumption by personal consumption expenditures and investment with fixed private investment which abstracts from changes in inventories. Since our period covers the Zero Lower Bond (ZLB hereafter), we use the shadow value provide by [Wu and Xia \(2016\)](#). Inflation is defined as the first log difference of the GDP deflator. Since our model allows for the extensive margin of employment, we use the unemployment rate rather than hours. Finally, Net Business Formation (NBF hereafter) is defined as the ratio between the number of establishment births to establishment deaths from the Bureau of Labor Statistics (BLS hereafter).²¹

The goal of each item of data is to be matched with our theoretical counterpart. In the model, nominal variables are deflated using the welfare price index P_t^C which is unobserved.²² Thus, we stripped the variety effect, i.e. the welfare price index, by multiplying each real variable by P_t^C and dividing by the product price P_t which corresponds to the GDP price deflator. As in [Lewis and Stevens \(2015\)](#), for any real variable a_t in the model, the data-consistent counterpart of any real theoretical variable is given by $a_t^R = \frac{a_t}{\rho_t}$ where ρ_t corresponds to the price index defined in Eq.1.36.

²¹For a complete details of the data sources, we refer to the appendix A

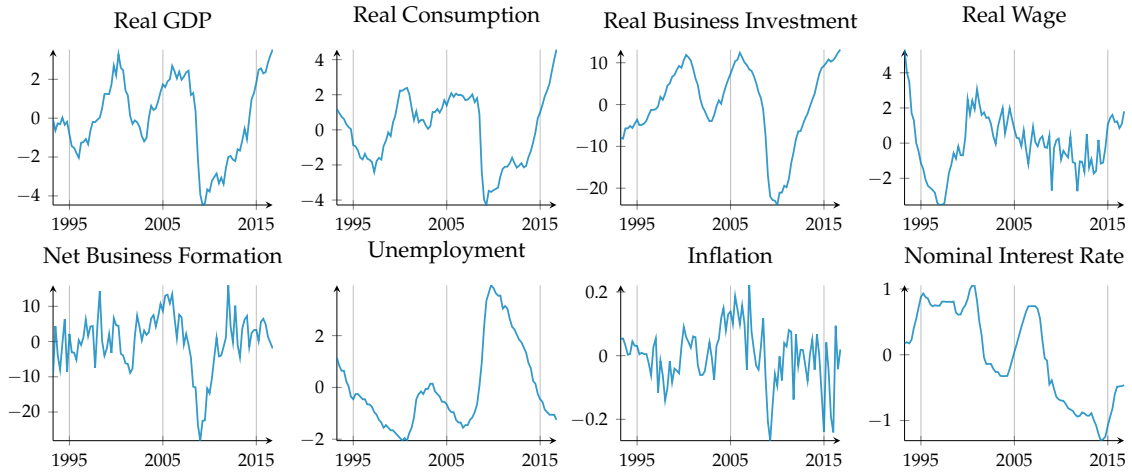
²²The consumer price index computed by the Bureau of Labor Statistics (BLS) does not reflect the welfare product turnover represented by the translog expenditure function.

With $\hat{a}_t = 100 \log\left(\frac{a_t}{\bar{a}}\right)$ which denotes the log-deviation of a variable a_t from its steady-state, the measurement equation reads as follows:

$$\mathbb{Y}_t = \left\{ \hat{Y}_t^R, \hat{C}_t^R, \hat{I}_t^R, \hat{w}_t^R, U_t - U, R_t - R, \hat{\pi}_t, N\hat{B}F_t \right\}. \quad (1.38)$$

where the net business formation is defined as the difference between entry N_t^E and exit $\delta^N (N_t + N_t^E)$. Then, in log deviations we have: $N\hat{B}F_t = (1 - \delta^N) (\hat{N}_t^E - \hat{N}_t)$.

FIGURE 1.1: Observable variables used in the estimation



Notes: Data used in estimation and expressed in percentage deviation from their quadratic trend (except for the unemployment, inflation and the nominal interest rate which is simply demeaned).

In absence of trends in our setup, we neglect the low component of macroeconomic time series and focus on the short term fluctuations.²³ Thus, the series are transformed in order to map non-stationary data to a stationary-model. All nominal variables are deflated with the GDP deflator and aggregate real variables are expressed in *per capita* terms by dividing by the Civilian Non-institutional Population over 16. The series for GDP, consumption, investment, wages and net business formation are taken in log and detrended using a quadratic trend. The other series, namely unemployment, interest rate and inflation, are demeaned by subtracting their respective sample averages. The transformed series used in the estimation are displayed in Fig.1.1.

²³Lewis and Stevens (2015) and Poutineau and Vermandel (2015) used the same approach for a similar fit exercise on firms' entry.

2.2 Calibration and prior distribution of parameters

It is standard practice in empirical macroeconomics to calibrate some parameters which are weakly identified (Smets and Wouters (2003)). For these parameters, they are selected to match long-run averages and key ratios but for which little information is contained in the transformed data we used in the estimation. Our calibration is summarised in Tab.1.1.

TABLE 1.1: Calibrated parameter

Parameter	Value	Parameter	Value
Total labor separation	$\rho = .1$	Spending to GDP ratio	$g^Y = 0.21$
Labor share	$\alpha = 0.67$	Exit rate Producers	$\delta^N = 0.025$
Discount factor	$\beta = .9925$	Labor separation by firms	$\delta^L = 0.0769$
Probability to find a job	$f = 0.7$	Capital depreciation rate	$\delta^K = 0.025$
Matching elasticity vancancies	$\zeta = 0.5$	Probability to find an employee	$q = 0.73$
Fixed barrier entry	$f^E = 0.4$	Share of potential producers	$\frac{N}{\bar{N}} = 0.95$

We use standard values for all the parameters that are conventional in the business cycle literature. These include the share of labor in the Cobb-Douglas production function $\alpha = .67$, the discount factor $\beta = 0.9925$, the steady-state government expenditures to output $g^Y = 0.21$. As in Bilbiie et al. (2012), the exogenous exit rate δ^N has the same value as the capital depreciation rate, with $\delta^N = \delta^K = .025$.

For labor market parameters, we set the elasticity of matches to unemployment, ζ to 0.5 in the range of estimation provided by Petrongolo and Pissarides (2001). The total job separation rate $1 - \rho$ (with $1 - \rho = 1 - (1 - \delta^N)(1 - \delta^L)$) due to product turnover δ^N and exogenous separation by firms δ^L is calibrated to 0.1, matching the average job duration of two and a half years in the US.²⁴ Then, the labor separation into incumbent firms is equal to $\delta^L = 0.0769$. The steady-state job finding probability f is set to 0.7 using the average of the finding rate made by Shimer (2005). Then, we have a steady-state value of unemployment equal to 5% as in Blanchard and Galí (2010) and which is close to the

²⁴Since the exogenous exit rate for firms is $\delta^N = .025$, we have a steady-state value for exogenous separation within firms of 7.69% using $\delta^L = \frac{\rho}{(1 - \delta^N)}$.

average of unemployment rate observed in our sample (6%). Finally, the steady-state job filling rate q is set to 0.73 as in [Den Haan et al. \(2000\)](#).

The last two parameters are the share of potential producers $\frac{N}{N}$ and the fixed barrier upon entry f^E . For the first, we follow [Lewis and Stevens \(2015\)](#) with a share of potential producers equal to 95%. Regarding f^E , as shown in the appendix B, for any positive value of $f^E > 0$, this barrier only influences the steady-state number of goods available in the economy and the price elasticity of the spending share on an individual good σ and have a marginal impact on GDP ratios. Thus, we calibrate this parameter in order to match the price elasticity of the spending share as in [Lewis and Stevens \(2015\)](#), with $\sigma = 0.61$ implying a value for the fixed barrier of $f^E = 0.4$.

Concerning prior distributions, for majority of new Keynesian model parameters such as those related to household utilities (h^C , σ^C), to indexation for prices and wages (λ^P , λ^W), to the Taylor rule (ρ^R , ϕ^π , $\phi^{\Delta Y}$), to rigidities on investment and capital adjustment (φ^I , Ψ) we used the prior distributions chosen by [Smets and Wouters \(2007\)](#). Regarding the endogenous market structure i.e. for adjustment costs on extensive investment (φ^E) and demand elasticity (θ), we used the prior of [Lewis and Stevens \(2015\)](#). For Rotemberg adjustment costs on wages and prices (κ^P and κ^W), we chose a reasonably loose prior using a gamma distribution with mean 50 and standard deviation of 7.5.

For the labor market, two important parameters can be estimated, namely the negotiation power of workers η and the steady-state value of unemployment \tilde{b} . As in [Gertler et al. \(2008\)](#), we define the steady-state value of unemployment as a fraction of the contribution of the worker to the job:

$$\tilde{b} = \frac{b}{\alpha \frac{mcY}{L}}, \quad (1.39)$$

where $\alpha \frac{mcY}{L}$ is the marginal product of labor.²⁵ Thus, we used the same prior as [Gertler et al. \(2008\)](#) for these two parameters.

²⁵However, we do not have the same cost structure as [Gertler et al. \(2008\)](#). In their paper, they used hiring costs rather than vacancy costs. As a consequence, in our model the firms' value of the job (Eq.1.18) does not depend on the discounted savings on adjustment costs. Thus, in [Gertler et al. \(2008\)](#) they have $\tilde{b} = \frac{b}{\alpha \frac{mcY}{L} + \beta \frac{f^V}{2} (x)^2}$ with $\beta \frac{f^V}{2} (x)^2$ the steady-state value of saving on adjustment costs and x the steady-state value of hiring.

2.3 Posterior estimates

In this subsection, we discuss our posterior estimates and contrast them, when possible, with the existing empirical evidence from the literature. Tab.1.2 reports our baseline estimation which summarises the means and 5th and 95th percentiles of the posterior distributions.²⁶

TABLE 1.2: Prior and posterior distributions of structural parameters

Estimated structural parameters		Posterior		
Symbol	Description	Prior (P1,P2)	Mean	[5%, 95%]
σ^C	Consumption utility	$\mathcal{N}(1.50, 0.375)$	2.16	[1.62, 2.66]
h^C	External habit	$\mathcal{B}(0.70, 0.10)$	0.66	[0.57, 0.81]
φ^I	Investment adjustment cost	$\mathcal{N}(4.00, 1.50)$	5.74	[3.91, 7.49]
ψ	Capacity utilisation cost	$\mathcal{B}(0.50, 0.15)$	0.84	[0.75, 0.95]
φ^E	Entry adjustment cost	$\mathcal{N}(4.00, 1.50)$	4.76	[2.96, 6.61]
θ	Demand elasticity	$\mathcal{N}(4.00, 1.50)$	11.18	[9.62, 12.65]
κ^P	Price rigidity	$\mathcal{G}(50.0, 7.50)$	55.32	[44.99, 65.94]
λ^P	Indexation price	$\mathcal{B}(0.50, 0.15)$	0.32	[0.15, 0.49]
κ^W	Wage rigidity	$\mathcal{G}(50.0, 7.50)$	49.01	[37.06, 60.88]
λ^W	Indexation wage	$\mathcal{B}(0.50, 0.15)$	0.63	[0.42, 0.84]
η	Negotiation power household	$\mathcal{B}(0.50, 0.1)$	0.84	[0.75, 0.94]
\bar{b}	Unemployment value	$\mathcal{B}(0.50, 0.1)$	0.80	[0.66, 0.93]
ρ^R	Interest rate smoothing	$\mathcal{B}(0.70, 0.10)$	0.80	[0.75, 0.84]
ϕ^π	Policy inflation	$\mathcal{N}(2, 0.25)$	2.46	[2.13, 2.79]
$\phi^{\Delta Y}$	Policy lagged output	$\mathcal{G}(0.5, 0.25)$	0.27	[0.14, 0.41]

Notes: For the distributions we have : \mathcal{B} , beta; \mathcal{IG} , inverse gamma; \mathcal{N} normal; \mathcal{G} , gamma; P1, the mean and P2 the standard deviation.

The estimated Taylor rule parameters are consistent with existing evidence, with substantial interest rate smoothing ($\rho = 0.80$), a response coefficient on inflation that satisfies the Taylor Principle ($\phi^\pi = 2.46$) and the influence response of the growth rate ($\phi^{\Delta Y} = 0.27$). Concerning wages and price rigidities, they are in line with the estimation of [Smets and Wouters \(2007\)](#) with a higher degree of indexation to past inflation for wages than for prices. Indexation parameters λ^P and λ^W are respectively estimated to 0.32 and 0.63. On a Calvo basis, the Rotemberg price rigidity is associated with a probability of 0.59 to

²⁶Posterior moments are computed using 600,000 draws from the distribution simulated by the Random Walk Metropolis Hastings algorithm using Dynare, see [Adjemian et al. \(2011\)](#) for further information.

reset prices, which corresponds to an average contract duration of approximately two and a half quarters for prices.²⁷

The negotiation power of households (η) and unemployment value (\tilde{b}) are in line with those estimated by [Gertler et al. \(2008\)](#). We have a higher value for negotiation power ($\eta = 0.84$) than the range provided in the literature, which typically lies between 0.5 and 0.7. A higher η weights more the employment value for households than for firms, as a result wages are more sensitive to the shadow value of labor and thus less sensitive to employment. Together with wage rigidities, they confirm that wages are sensitive to movements in productivity for low and medium frequencies and dependent on past value for high frequencies as pointed by [Gertler et al. \(2008\)](#).

Turning to adjustment costs on investment, our results are in line with business cycle models without entry (e.g. [Christiano et al. \(2005\)](#), [Smets and Wouters \(2007\)](#)) with $\varphi^I = 5.74$ and $\psi = 0.85$. For the extensive part, we find a reasonable value for rigidity on entry ($\varphi^E = 4.76$).

We find a higher value for demand elasticity with $\theta = 11.18$, which delivers a steady-state markup of 9.8% and a ratio of entry to GDP of 6.88% ($\frac{N^E \phi^E}{Y^C}$). For the first value, this is in line with the literature without entry (see [Smets and Wouters \(2007\)](#) among others use a value of $\theta = 10$). However, this is almost twice the value found by [Lewis and Stevens \(2015\)](#), this is probably explained by our marginal cost structure that results from matching frictions. Since our entry cost only depends on a fixed component and vacancy posting, a higher value for θ is key to catch the volatility of entry. For the second value, the proportion of entry cost-to-GDP is in the low range of empirical estimates: [Barseghyan and DiCecio \(2011\)](#) estimate two different ratios using the ratio of entry to operating costs and the evolution of firms' productivity over time. The first gives a ratio between entry costs and output per worker is 20.8%. The second is lower at 12.15%. The

²⁷In the Calvo version of the model, we have the coefficient $\frac{(1-\beta\xi^P)(1-\xi^P)}{\xi^P}$ on the output gap in the NKPC with ξ^P the Calvo probability of resetting price. With Rotemberg adjustment costs, this coefficient corresponds to $\frac{\theta-1}{\kappa^P}$. Using this relation to calculate the Calvo lottery, we have the duration of price stickiness with $\frac{1}{1-\xi^P}$. However, as emphasised by [Lewis and Stevens \(2015\)](#), we cannot compute an average price contract duration as this requires a constant population of price setters.

World Bank reports that legal fees to register a business reached up to 1.4% of per capita income in the US for in 2011.

Finally, steady-state value of employment and job creation induced by new entrants are in line with empirical evidence. The share of employment of startups is equal to 2.56% ($\frac{qv^E N^E}{L}$) and close to Haltiwanger et al. (2010).²⁸ The average job creation attributed to new firms ($\frac{N^E v^E}{V}$) is equal to 25.64%, which is in line with Jaimovich and Floetotto (2008).²⁹

TABLE 1.3: Prior and posterior distributions of shock processes

Shocks AR(1), MA(1)			Posterior	
Symbol	Description	Prior (P1,P2)	Mean	[5%, 95%]
ρ^Z	AR - Productivity	$\mathcal{B}(0.50, 0.20)$	0.87	[0.82, 0.92]
ρ^C	AR - Risk premium	$\mathcal{B}(0.50, 0.20)$	0.67	[0.53, 0.82]
ρ^G	AR - Spending	$\mathcal{B}(0.50, 0.20)$	0.93	[0.90, 0.96]
ρ^{GZ}	AR - Productivity-spending	$\mathcal{B}(0.50, 0.20)$	0.52	[0.28, 0.76]
ρ^I	AR - Investment	$\mathcal{B}(0.50, 0.20)$	0.70	[0.62, 0.77]
ρ^E	AR - Entry	$\mathcal{B}(0.50, 0.20)$	0.16	[0.06, 0.25]
ρ^R	AR - Monetary policy	$\mathcal{B}(0.50, 0.20)$	0.60	[0.52, 0.69]
ρ^P	AR - Price markup	$\mathcal{B}(0.50, 0.20)$	0.75	[0.64, 0.87]
ρ^L	AR - Firm's bargaining	$\mathcal{B}(0.50, 0.20)$	0.79	[0.71, 0.87]
u^P	MA - Price	$\mathcal{B}(0.50, 0.20)$	0.52	[0.33, 0.72]
u^L	MA - Firm's bargaining	$\mathcal{B}(0.50, 0.20)$	0.45	[0.27, 0.63]
<i>Innovations</i>				
σ^Z	Productivity	$\mathcal{IG}(0.1, 2)$	0.48	[0.43, 0.54]
σ^C	Risk premium	$\mathcal{IG}(0.1, 2)$	3.42	[2.16, 4.64]
σ^G	Spending	$\mathcal{IG}(0.1, 2)$	2.93	[2.55, 3.31]
σ^I	Investment	$\mathcal{IG}(0.1, 2)$	2.23	[1.79, 2.66]
σ^R	Monetary policy	$\mathcal{IG}(0.1, 2)$	0.10	[0.09, 0.11]
σ^E	Entry	$\mathcal{IG}(0.1, 2)$	7.54	[6.47, 8.56]
σ^P	Price markup	$\mathcal{IG}(0.1, 2)$	0.39	[0.27, 0.50]
σ^L	Bargaining	$\mathcal{IG}(0.1, 2)$	0.18	[0.10, 0.25]

Notes: For the distributions we have : \mathcal{B} , beta; \mathcal{IG} , inverse gamma; \mathcal{N} normal; \mathcal{G} , gamma; P1, the mean and P2 the standard deviation.

For the estimations of parameters for shocks, there are described in the Tab.1.3.

²⁸They found that business startups account for roughly 3 percent of U.S. total employment in any given year between 1992 and 2005.

²⁹Using employment data at the establishment level, they estimate that the average fraction of quarterly job gain that can be explained by the opening of establishments is about 20 percent.

3 Unemployment and markup dynamics

In this section, we analyse the empirical dynamics of employment and markup implied by our estimated model. We start this section by conducting an empirical validation of our model. Then we compare the simulated standard deviations and correlations with those of the data, as well as the ability of our model to reproduce some key labor market variables. Next, we study the propagation mechanism through the Impulse Response Functions (IRF hereafter) and contrast our propagation features with the existing literature. Third, we examine the source of employment dynamics based on variance decomposition and the time path of unemployment conditional to entry shock.

3.1 Empirical fit

One way to assess how the model captures empirical salient features of the data is to compare the volatilities of the model against the data. The tab.1.4 reports the standard deviations of key variables normalised by the standard deviation of the output.

TABLE 1.4: Relative Standard Deviations: Model vs Data

Variable	Y_t	I_t	w_t	π_t	U_t	V_t	θ_t	N_t^E
Data	1	4.81	0.85	0.04	0.80	10.36	21.63	2.79
Model	1	4.31	0.58	0.06	0.88	10.48	22.34	2.67

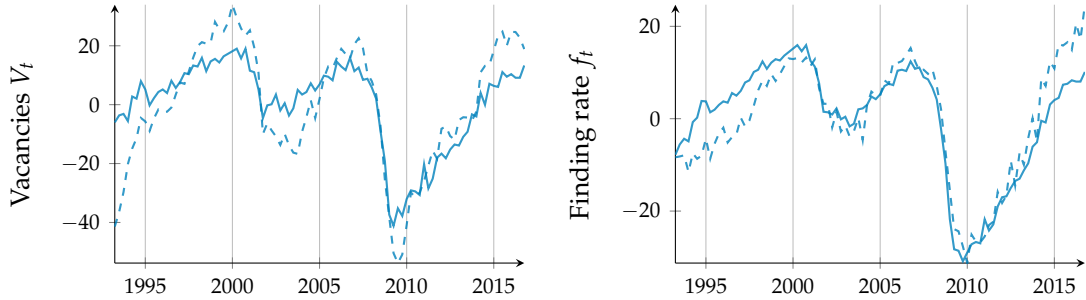
Notes: All standard deviation are normalised by the standard deviation of GDP (Y_t). Except for the labor market tightness (θ_t) which is expressed as the log ratio between unemployment and vacancies and detrended using a quadratic trend, all the others variables are the same as those used in the estimation.

Except for the volatility of wages, the model provides a good fit with the data. In particular, it captures the feature that both unemployment and vacancies are highly volatile and that investment on the intensive margin (I_t) is more volatile than the extensive margin (N_t^E). However, the model overshoots the rigidity of the real wages.

Then we conduct an external validation exercise to assess the reliability of the model in fitting time series that are not used as inputs in the estimation. Such an exercise is of particular interest since it addresses the critique that DSGE models can do a good job at fitting the data in the sample, but have poor performances at replicating the data.

Two important series related to unemployment dynamics are not taken into account in the estimation: vacancies (V_t) and the finding probability (f_t).

FIGURE 1.2: External validation: model simulated (smoothed estimates) versus actual data



Notes: The solid lines plot model simulated series and the dashed line actual data.

In the Fig.1.2, we contrast the model's simulated time series for these two measures against their data counterparts. These two data series are constructed respectively using the methodology of Barnichon (2010) and of Shimer (2007).³⁰ Then, in order to establish a comparison, we take these series in log and detrended using a quadratic trend. Both simulated series are closer to their data counterparts, pointing to a large and persistent decline around the financial crisis.

The model reproduces the negative correlation between vacancies and unemployment i.e. the Beveridge curve (in the data, we have a negative correlation of -0.85 against -0.86 in the model) and generates the strong volatility observed in the data for unemployment and vacancies. Similarly, for the finding rate, we have a negative correlation with unemployment that is relatively close to the data (-0.91 in the data and -0.96 in the model).

Since the model shows good performances in replicating the data, we now turn to the dynamics of unemployment, entry and markup implied by the model using Impulses Responses Functions (IRF hereafter).

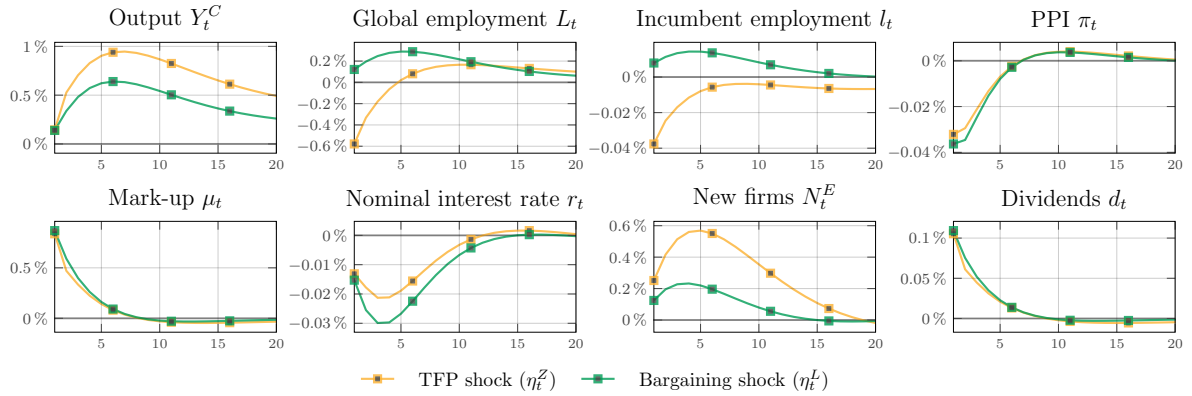
³⁰Since the Job Openings and Labor Turnover Survey (JOLTS) measure of job openings starts from December 2000 and our period from 1984, I used a composite index based on "print" and "online" help wanted index as in Barnichon (2010). The approximation of Shimer (2007) gives the following definition of this probability: $f_t = 1 - \frac{u_{t+1} - u_{t+1}^S}{u_t}$ where u_{t+1}^S corresponds to workers unemployed for less than 5 weeks and u_{t+1} the number of unemployed people.

3.2 Impulse response analysis

All the shocks displayed in this subsection generate an expansion to facilitate the comparison between them.

Supply shocks The Fig.1.3 focuses on two supply shocks, namely TFP (η_t^Z) and the bargaining shock (η_t^L).

FIGURE 1.3: System response to supply shocks.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

A positive technology shock leads to a short-run decline in the total level of employment in line with Galí et al. (2012). As standardly documented in the literature, this shock lowers firms' real marginal cost. As prices are sticky and do not fall by the same amount, this increases markups and profits. Since dividends increase, more entry occurs according to the Euler on shares (Eq.1.8). After a few quarters, the global level of employment is positively affected by the number of entrants. However, the fraction of the labor force employed by incumbents is challenged by the number of firms willing to enter the market and then l_t decreases.

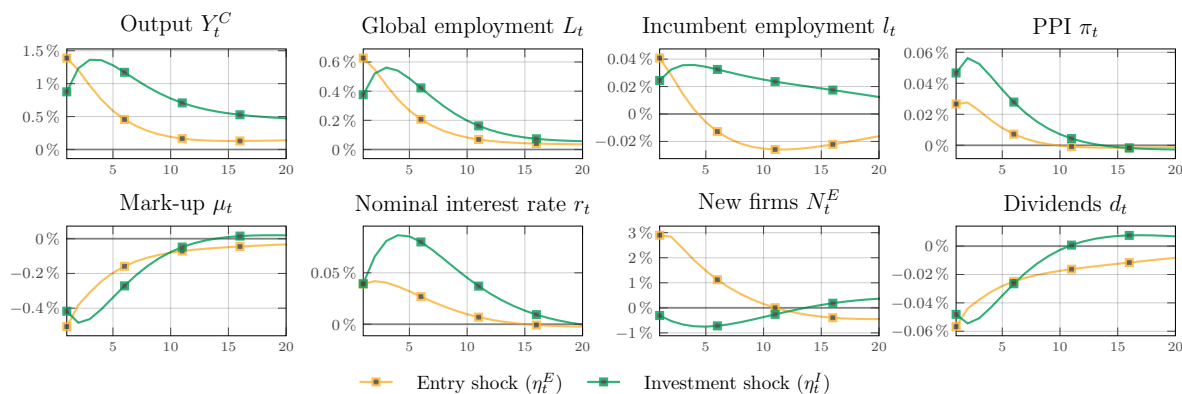
For the negative bargaining power shock, i.e. a decrease in workers bargaining power, employment rises in its extensive and intensive margin. The immediate drop in wage creates an incentive for firms to post more vacancies (Eq. 1.18) by boosting the marginal value of hiring a worker. Unlike the TFP shock, employment rises immediately after the

realisation of the shock. Lower wages reduce marginal costs and boost profits, which leads in turn to the creation of new firms.

Demand shocks We use the “demand shocks” label to shocks that imply a positive movement between output, inflation and real wages. Then, unlike Lewis and Stevens (2015), we include the entry shock as a specific demand shock.³¹

Since the preference shock (η_t^C), the government spending shock (η_t^G) and the investment shock (η_t^I) induce variations in the same direction for all variables (as shown in the Appendix B), the Fig.1.4 depicts the impulse responses of the investment shock and the entry shock (η_t^E).

FIGURE 1.4: System response to demand shocks



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

Concerning the investment shock, the increase in demand increases the marginal production cost, leading to inflation and a reduction in markup. This type of shock leads to a strong crowding-out effect at the extensive margin due to two complementary forces. First, despite the rise in demand, dividends decline and the market value of firms declines synchronously. Second, monetary tightening (i.e. the rise in nominal interest rate in reaction to the demand shock) leads to the same effect on firm value through Eq.1.8. Entry then declines as the prospect of future dividends collapses too. Despite the drop in the

³¹By construction, in their model, the aggregate demand does not depend on the extensive margin of activity, unlike in our model (Eq.1.34). Moreover, our specification shock acting as an investment technological shock as described in the Eq.1.14.

varieties goods, the total level of employment is stimulated via incumbents (l_t), which is enough to compensate the overall effect on total employment.

For the entry shock, we obtain a similar dynamic except for two variables, namely the number of entrants and the incumbent labor force. This shock increases the extensive margin activity and leads to a rise in demand via the definition of GDP (Eq.1.34). Then, as for previous demand shocks, the marginal cost rises. However, this shock acts directly on the free entry condition (Eq.1.9) and, despite the decreases in firm dividends, firm entry increases. For the second difference, the incumbent labor force is ultimately reduced, since the number of entrants reduces the markup and thus discourage firms to hire new workers (Eq.1.18).

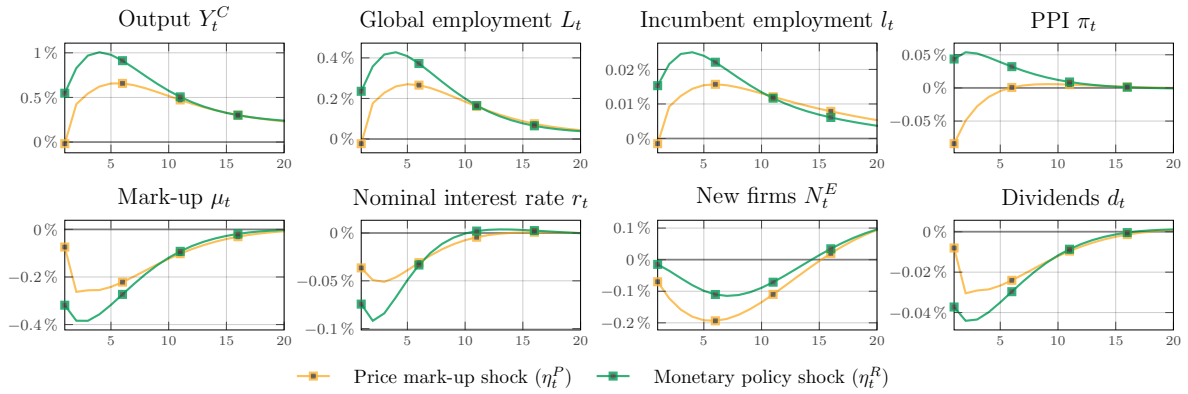
As in Lewis and Stevens (2015), all demand shocks induce countercyclical markups.

Monetary and Price markup shock Fig. 1.5 shows the response of our model to the price markup (η_t^P) and monetary policy (η_t^R) shocks.

An expansionary price markup shock leads to a reduction in prices via the NKPC (Eq. 1.22). The reduction in prices leads to more demand for goods from households which in turn boosts GDP. Since the markup of firms decreases, their dividends adjust accordingly path. As a consequence, firms become less attractive and the number of varieties produced declines. As for demand shocks, employment drops in its extensive part but the overall effect remains positive due to the intensive part.

As already documented in the literature (Bilbiie et al. (2012)), an expansionary monetary policy induces two opposing forces on profit. First, the decline in interest rate increases marginal costs and, with price stickiness, markup decreases. Together, they depress profits. Second, this shock has expansionary effects on aggregate demand that boosts profits. Unlike Bilbiie et al. (2007) and Lewis and Stevens (2015), the first effect dominates for profits. With the drop in profits, entry falls even with the decline of interest rate.

FIGURE 1.5: System response to markup price and monetary policy

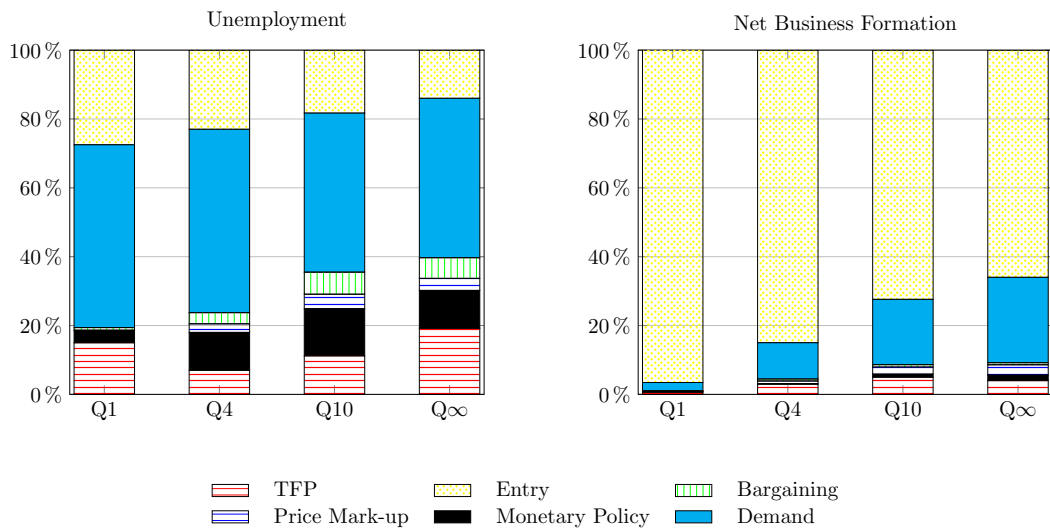


Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

3.3 Unemployment dynamics

In this section, we study the importance of the Net Business Formation to drive unemployment fluctuations. As a first exercise, we decompose the variance of unemployment and NBF at different time horizons. After that, we perform the time path of unemployment conditional to entry shock.

TABLE 1.5: Variance decomposition for unemployment at different horizons



Notes: Variance decomposition at different horizons where Q corresponds to quarters.

Variance decomposition Tab.1.5 reports the forecast error variance decomposition for unemployment and NBF at different time horizons. Starting with the NBF, in the short-to-medium run ($Q10$ represents two years and a half), the NBF is almost exclusively explained by the entry shock and the other shocks explained only one third of the variance at the long run.

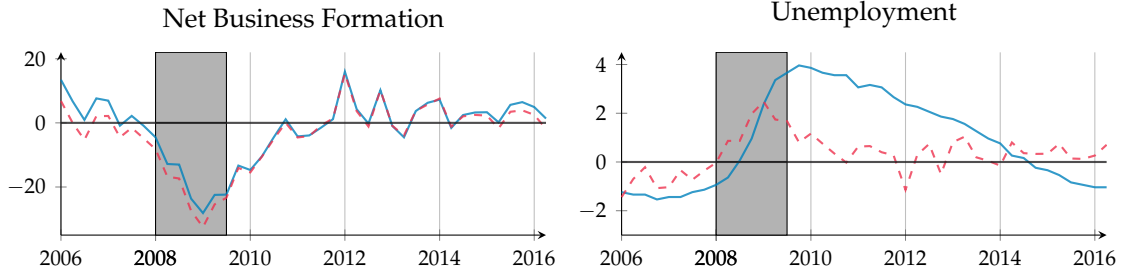
Turning to unemployment, as documented in the literature, demand shocks drive a large fraction for any time horizon considered. Turning to the entry shock, it explains 24% of unemployment in the short run ($Q1$ to $Q4$). This is in line with the result of [Jaimovich and Floetotto \(2008\)](#), where 20% of the cyclical of employment is driven by the NBF. One other interesting aspect regarding the shock decomposition is the minor role of the exogenous bargaining shock in explaining unemployment fluctuations. This is conflicting with [Gertler et al. \(2008\)](#) who find that 10% of the fluctuation in employment are driven by bargaining shocks. In our model, half of the variations in wages is naturally driven by the bargaining shock since it directly appears in its definition (Eq.1.26). However, it hardly affects the fluctuation in unemployment.

Time path of unemployment conditional to entry shock Then we have seen the importance of entry shock (ε_t^E) to drive the variance of unemployment, we perform a counterfactual history on the great recession episode and the following recovery (2006Q1 to 2016Q4). We estimate the counterfactual path of unemployment and the NBF conditional on the entry shock.

The Fig.1.6 displays our exercise. As expected by the variance decomposition (Fig.1.5), almost all declines in the NBF during the Great Recession period and the subsequent increases are attributable to entry shocks. Then, the crash in the NBF is followed by a surge in the unemployment rate as shown on the right-hand side of the Fig.1.6.

However, when the NBF returns to a reasonable volatility (around the 2011 year), the effect on unemployment is limited. Then, the NBF has a crucial impact on the unemployment rate during the Great Recession.

FIGURE 1.6: Counterfactual paths of the NBF and the unemployment rate conditional on the estimated entry shock.



Notes: Each graph shows the actual path used in the sample (blue thick line) and counterfactual path from the model (red dashed line). Actual path are expressed in percentage and in deviation from their mean. The great Recession period is associated with the shaded area.

3.4 Cyclicity of the markup with search and matching frictions in the labor market

An interesting question is to study whether search and matching frictions affect the markup behavior of firms. In our setup, two mechanisms reshape the dynamic of markup with respect to the standard DSGE models to study markups: *i*) The marginal cost (Eq. 1.18) is affected by the value of future hires; *ii*) the elasticity of substitutions for goods is higher than Lewis and Stevens (2015), which implies a desired markup less sensitive to variations in the number of goods.³² Following the same exercise as Lewis and Stevens (2015) and Bilbiie et al. (2012), we examine the unconditional cyclicity of the markup implied by the model at different time horizons (both forward and backward).

Up to the log-linearised form of the markup Eq.1.23 reads as follows:

$$\hat{\mu}_t = \frac{1}{2(\theta - 1)} \left(\hat{N}_t + \left(1 - \frac{\tilde{N}}{N} \right) \hat{\varepsilon}_t^P \right) - \hat{m}_{c_t}. \quad (1.40)$$

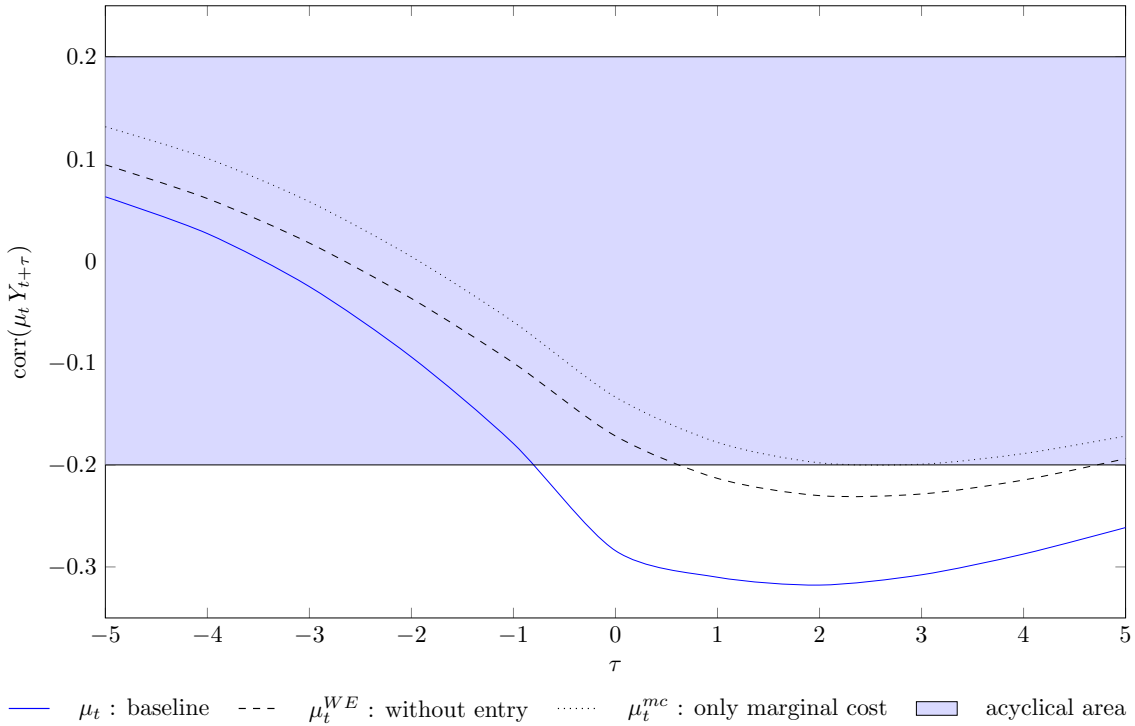
In our setup, markup cyclicity is driven by the number of competitors (\hat{N}_t), the exogenous price markup shock ($\hat{\varepsilon}_t^P$) and the marginal cost (\hat{m}_{c_t}). A striking feature of DSGE

³²At a first order approximation, the desired markup reads : $\hat{\mu}_t^D = -\frac{1}{\theta} (\hat{N}_t - \hat{\varepsilon}_t^P)$. Then with higher elasticity of substitution, we have a less sensitive desired markup to the number of varieties.

models with entry is the presence of the market structure capture by \hat{N}_t , as an additional determinant of markup with respect to standard DSGE model *à la* Smets and Wouters (2007). In order to see how each of individual components affect the cyclicity of markups, we disentangle them into three separate components. First, μ_t corresponds to the markup obtained in the baseline model. Second, μ_t^{WE} is the markup component without the entry mechanism by imposing a clock to infinity entry cost ($\varphi^E \rightarrow \infty$). Finally, letting μ_t^{MC} denote the markup only driven by the marginal cost in absence of any entry and exogenous markup shock i.e. $\hat{\mu}_t = -\hat{m}c_t$.

Fig.1.7 thus displays the means of cross-correlation between the different types of markup and GDP at different leads and lags (i.e. $corr(\mu_t, Y_{t+\tau})$, $corr(\mu_t^{WE}, Y_{t+\tau})$ and $corr(\mu_t^{mc}, Y_{t+\tau})$ for $\tau = -5, \dots, 0, \dots, 5$) and the traditionally acyclical correlation area used in the literature (between 0.2 and -0.2).

FIGURE 1.7: Cyclicity of the markup



The markup generated by the model, taken at the posterior mean, denoted μ_t is countercyclical at all leads ($\tau \geq 0$) and acyclical for lags ($\tau < 0$). This suggests that when markup rises (or decreases), gross domestic product tends to decrease (resp. increase) in

the following periods (1to 5). However, the countercyclical nature of markups is much lower than in previous theoretical model.³³

If we shut down the firm entry devices (μ_t^{WE}), the sign of cyclicality is maintained but is slightly slower than the baseline model. This slight difference is due to the higher degree of substitutions between goods which implies a markup that is less sensitive to entry (higher value of θ). Taking down both the exogenous markup shock and entry results in examining the cyclicality of the marginal cost. In our model, this component is strikingly acyclical, which contrast with the existing literature of Walrasian labor markets (e.g. [Bilbiie et al. \(2012\)](#) and [Lewis and Stevens \(2015\)](#) who find a procyclical markup in the absence of entry and exogenous price mark-up shock i.e. μ_t^{MC}). The presence of search and matching frictions in the labor market, affects the dynamic of markups through the acyclical marginal cost compared to Walrasian labor market, as pointed out by [Krause and Lubik \(2007\)](#).

Then, the combination of those elements (exogenous shock, entry and marginal cost) makes the markup countercyclical. However, endogenous entry has a less order of importance compared to [Lewis and Stevens \(2015\)](#).

4 Sensitivity analysis

In this section, we check the robustness of our results to : *i*) different time series *ii*) alternative hypothesis.

4.1 Alternative data

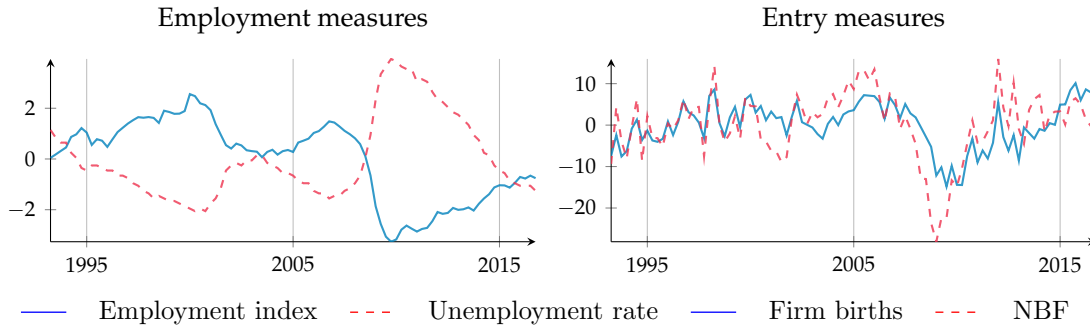
We use two alternative time series to check the robustness of our result. The first series concerns the labor market: we replace our unemployment rate measure in our observable vector by a measure of employment as [Gertler et al. \(2008\)](#). This index is based on the product of hours of all persons in the nonfarm business sector times the ratio of total

³³[Bilbiie et al. \(2012\)](#) finds countercyclical markup for lags (from -5 to -1) between -0.8 and -0.4 and [Lewis and Stevens \(2015\)](#) find -0.4 for all leads and lags.

employment to employment in nonfarm business sector. This time series can affect our results since they exhibit a higher volatility than unemployment.

The second series concerns the measure of entry: in our model, we used the net business formation as observable, defined as the ratio between births and deaths provided by the BLS. However, our setup does not feature endogenous exit, so the utilisation of NBF can potentially bias the estimation by overshooting the role of entry to compensate for the absence of explicit exit mechanism. Then, we estimate our model using only firm births which correspond to N_t^E in our model. The transformation data series used in this sensitivity analysis is plotted in Fig.1.8 with the original data used in the baseline model.³⁴

FIGURE 1.8: Additional data series comparing to data used in the baseline model



Starting with the employment index from [Gertler et al. \(2008\)](#) in Tab.1.8, none of the parameters are significantly affected with respect our baseline version. This is quite remarkable as these two series exhibit different patterns : the volatility of the employment index is higher than the unemployment rate (2.2745 versus 1.6050) and less persistence (0.9458 versus 0.9769).

By considering entry rather than net business formation as depicted in Tab.1.8, the main difference lies in the volatility of the exogenous shock on entry (σ^E). Since this shock is the principal driving force of entry in our model and firm' births is less volatile than the NBF, the volatility of this shock diminishes (from 7.54 in the baseline model to 4.80).³⁵

³⁴The first series of labor is simply demeaned, while the second is detrended using a quadratic trend in order to make a comparison between estimations on a regular basis.

³⁵The volatility of NBF is equal to 8.15 and for births 5.56.

For the rest of the parameters they do not differ from the baseline version, except for demand elasticity which is lower (from $\theta = 11.18$ to $\theta = 9.54$).

TABLE 1.6: Alternative data series

Symbol Description		Posterior distribution: Mean [5%, 95%]		
		Baseline	Employment index	Births
STRUCTURAL PARAMETERS				
σ^C	Consumption utility	2.16 [1.62, 2.66]	2.16 [1.62, 2.66]	2.13 [1.60, 2.64]
h^C	Habit consumption	0.66 [0.57, 0.81]	0.65 [0.57, 0.86]	0.71 [0.58, 0.84]
φ^I	Investment adjustment cost	5.74 [3.91, 7.49]	5.96 [4.15, 7.69]	5.64 [3.96, 7.51]
ψ	Capacity utilisation cost	0.84 [0.75, 0.95]	0.87 [0.79, 0.95]	0.86 [0.77, 0.94]
φ^E	Entry adjustment cost	4.76 [2.96, 6.61]	5.20 [3.36, 7.0]	6.02 [4.27, 7.87]
θ	Demand elasticity	11.18 [9.62, 12.65]	11.05 [9.54, 12.63]	9.54 [7.96, 11.03]
κ^P	Price rigidity	55.32 [44.99, 65.94]	57.10 [47.89, 68.74]	55.35 [45.76, 65.49]
λ^P	Indexation price	0.32 [0.15, 0.49]	0.31 [0.12, 0.43]	0.31 [0.14, 0.48]
κ^W	Wage rigidity	49.01 [37.06, 60.88]	47.25 [35.45, 58.30]	48.84 [36.51, 60.60]
λ^W	Indexation wage	0.63 [0.42, 0.84]	0.61 [0.38, 0.83]	0.62 [0.42, 0.85]
η	Negotiation power household	0.84 [0.75, 0.94]	0.80 [0.69, 0.92]	0.84 [0.75, 0.94]
\bar{b}	Unemployment value	0.80 [0.66, 0.93]	0.84 [0.73, 0.94]	0.80 [0.66, 0.94]
ρ^R	Interest rate smoothing	0.80 [0.75, 0.84]	0.80 [0.76, 0.84]	0.79 [0.75, 0.94]
ϕ^π	Policy inflation	2.46 [2.13, 2.79]	2.45 [2.12, 2.82]	2.44 [2.09, 2.79]
$\phi^{\Delta Y}$	Policy lagged output	0.27 [0.14, 0.41]	0.27 [0.13, 0.40]	0.26 [0.13, 0.40]
AR(1), MA(1)				
ρ^Z	AR - Productivity	0.87 [0.82, 0.92]	0.89 [0.85, 0.93]	0.88 [0.83, 0.92]
ρ^C	AR - Risk premium	0.67 [0.53, 0.82]	0.64 [0.46, 0.82]	0.64 [0.47, 0.80]
ρ^G	AR - Spending	0.93 [0.90, 0.96]	0.92 [0.89, 0.96]	0.93 [0.90, 0.96]
ρ^{GZ}	AR - Productivity-spending	0.52 [0.28, 0.76]	0.52 [0.33, 0.79]	0.57 [0.36, 0.81]
ρ^I	AR - Investment	0.70 [0.62, 0.77]	0.69 [0.61, 0.76]	0.69 [0.62, 0.77]
ρ^E	AR - Entry	0.16 [0.06, 0.25]	0.17 [0.06, 0.27]	0.18 [0.06, 0.28]
ρ^R	AR - Monetary policy	0.60 [0.52, 0.69]	0.60 [0.51, 0.69]	0.60 [0.51, 0.69]
ρ^P	AR - Price markup	0.75 [0.64, 0.87]	0.75 [0.62, 0.88]	0.75 [0.62, 0.87]
ρ^L	AR - Firm's bargaining	0.79 [0.71, 0.87]	0.77 [0.69, 0.86]	0.80 [0.72, 0.88]
u^P	MA - Price	0.52 [0.33, 0.72]	0.49 [0.30, 0.70]	0.52 [0.33, 0.72]
u^L	MA - Firm's bargaining	0.45 [0.27, 0.63]	0.43 [0.25, 0.61]	0.45 [0.25, 0.64]
INNOVATIONS				
σ^Z	Productivity	0.48 [0.43, 0.54]	0.52 [0.45, 0.59]	0.48 [0.42, 0.54]
σ^C	Risk premium	3.42 [2.16, 4.64]	3.84 [2.01, 5.66]	3.72 [2.16, 5.25]
σ^G	Spending	2.93 [2.55, 3.31]	2.94 [2.54, 3.32]	2.58 [2.25, 2.93]
σ^I	Investment	2.23 [1.79, 2.66]	2.20 [1.79, 2.60]	2.24 [1.84, 2.67]
σ^R	Monetary policy	0.10 [0.09, 0.11]	0.10 [0.09, 0.11]	0.09 [0.07, 0.10]
σ^E	Entry	7.54 [6.47, 8.56]	7.66 [6.49, 8.65]	4.80 [4.11, 5.53]
σ^P	Price markup	0.39 [0.27, 0.50]	0.40 [0.27, 0.51]	0.34 [0.22, 0.45]
σ^L	Bargaining	0.18 [0.10, 0.25]	0.19 [0.10, 0.25]	0.17 [0.09, 0.12]

Notes: We use the same distribution for parameters as in the Tab.1.2 and Tab.1.3.

4.2 Entry and translog preferences

A second aspect that we investigate is the role of entry cost function and the functional form of preferences for consumption goods.

In our model, the sunk cost paid upon entry is measured as a combination of a fixed cost (f^E) and a cost for posting vacancies (see Eq.1.24). Since the number of vacancies depends on the labor used by incumbent firms l_t and the marginal value of adding a worker, this cost introduces a direct link between the labor and goods markets. We evaluate this link

by considering $\phi_t^E = \frac{f^E}{\varepsilon_t^Z}$, without the cost of vacancy posting. The underlying assumption is that firms starting their business finds their labor force at no cost. We discuss the estimated parameters that significantly differ from our benchmark model.

The result is presented in the third column of Tab.1.7 as Fixed entry. The wage is more rigid than in the baseline model. We have a higher Rotemberg rigidity (κ^W), lower negotiation power for households (η) and higher unemployment value (\tilde{b}). This result can be explained as follows: since only incumbent firms have a role in the determination of the labor force, wages need to be less volatile to induce higher volatility of unemployment. Conversely, when new entrants play a direct role on the labor force with vacancy postings, the wage does not need to be less volatile.

Another point is to evaluate the role of the *competition effect* in the model. This effect plays a key role in the analysis of markup, since as discussed previously it induces a countercyclical desired markup in the case of increasing entry. However, this effect can also reduce the incentive for incumbent firms to increase their labor force (Eq.1.18), and thus can explain the relatively higher demand elasticity for goods.³⁶ To measure the importance of the *competition effect*, we considered a Constant Elasticity of Substitution (CES hereafter) rather than a translog function as in Bilbiie et al. (2012). This preference specification implies a different price dispersion and markup. The fourth column of Tab.1.7, named CES, shows the result of the estimation. Dropping the competition effect results in a higher degree of price Rotemberg adjustment cost. Since the desired markup in Eq.1.22 becomes independent of entry, higher rigidity for price is needed to account for inflation dynamics. Rigidity for the extensive φ^E margin becomes higher, in line with the estimation of Poutineau and Vermandel (2015) who used CES preferences.

Comparing these three models, our baseline specification is preferred to any other one through a posterior model probability of 1 versus 0 for the others.

Even if our model outperforms the two other specifications from statistical standpoint, we also want to evaluate the cyclical nature of markup along these three sepecifications as in

³⁶A greater competition effect reduces the markup and can act as a disincentive effect for incumbent to hire new workers

TABLE 1.7: Comparison with other model specification

Symbol Description		Posterior distribution: Mean [5%, 95%]		
		Baseline	Fixed entry	CES
STRUCTURAL PARAMETERS				
σ^C	Consumption utility	2.16 [1.62, 2.66]	2.25 [1.72, 2.75]	1.58 [1.01, 2.10]
h^C	Habit consumption	0.66 [0.57, 0.81]	0.66 [0.57, 0.81]	0.89 [0.85, 0.93]
φ^I	Investment adjustment cost	5.74 [3.91, 7.49]	5.80 [4.01, 7.52]	6.86 [5.14, 8.55]
ψ	Capacity utilisation cost	0.84 [0.75, 0.95]	0.84 [0.75, 0.95]	0.85 [0.76, 0.94]
φ^E	Entry adjustment cost	4.76 [2.96, 6.61]	4.43 [2.66, 6.28]	5.03 [3.32, 6.68]
θ	Demand elasticity	11.18 [9.62, 12.65]	10.77 [9.23, 12.15]	11.08 [9.42, 12.54]
κ^P	Price rigidity	55.32 [44.99, 65.94]	55.05 [44.78, 65.64]	66.02 [54.6, 77.98]
λ^P	Indexation price	0.32 [0.15, 0.49]	0.22 [0.06, 0.38]	0.31 [0.13, 0.49]
κ^W	Wage rigidity	49.01 [37.06, 60.88]	54.77 [42.12, 64.98]	50.88 [40.09, 62.34]
λ^W	Indexation wage	0.63 [0.42, 0.84]	0.45 [0.21, 0.70]	0.47 [0.22, 0.72]
η	Negotiation power household	0.84 [0.75, 0.94]	0.79 [0.69, 0.89]	0.84 [0.77, 0.91]
\bar{b}	Unemployment value	0.80 [0.66, 0.93]	0.91 [0.83, 0.95]	0.83 [0.76, 0.91]
ρ^R	Interest rate smoothing	0.80 [0.75, 0.84]	0.79 [0.75, 0.84]	0.81 [0.77, 0.86]
ϕ^π	Policy inflation	2.46 [2.13, 2.79]	2.21 [2.10, 2.77]	2.35 [2.02, 2.71]
$\phi^{\Delta Y}$	Policy lagged output	0.27 [0.14, 0.41]	0.26 [0.13, 0.40]	0.16 [0.06, 0.25]
AR(1), MA(1)				
ρ^Z	AR - Productivity	0.87 [0.82, 0.92]	0.92 [0.87, 0.96]	0.92 [0.90, 0.95]
ρ^C	AR - Risk premium	0.67 [0.53, 0.82]	0.65 [0.43, 0.77]	0.42 [0.28, 0.55]
ρ^G	AR - Spending	0.93 [0.90, 0.96]	0.94 [0.92, 0.97]	0.90 [0.86, 0.94]
ρ^{GZ}	AR - Productivity-spending	0.52 [0.28, 0.76]	0.54 [0.30, 0.76]	0.52 [0.29, 0.75]
ρ^I	AR - Investment	0.70 [0.62, 0.77]	0.68 [0.57, 0.74]	0.68 [0.61, 0.75]
ρ^E	AR - Entry	0.16 [0.06, 0.25]	0.15 [0.05, 0.22]	0.13 [0.04, 0.20]
ρ^R	AR - Monetary policy	0.60 [0.52, 0.69]	0.63 [0.58, 0.69]	0.65 [0.56, 0.74]
ρ^P	AR - Price markup	0.75 [0.64, 0.87]	0.73 [0.62, 0.85]	0.70 [0.59, 0.82]
ρ^L	AR - Firm's bargaining	0.79 [0.71, 0.87]	0.27 [0.13, 0.41]	0.39 [0.18, 0.60]
u^P	MA - Price	0.52 [0.33, 0.72]	0.42 [0.21, 0.59]	0.47 [0.28, 0.65]
u^L	MA - Firm's bargaining	0.45 [0.27, 0.63]	0.43 [0.30, 0.62]	0.51 [0.35, 0.67]
INNOVATIONS				
σ^Z	Productivity	0.48 [0.43, 0.54]	0.47 [0.43, 0.53]	0.53 [0.46, 0.59]
σ^C	Risk premium	3.42 [2.16, 4.64]	2.25 [1.11, 3.34]	7.95 [4.63, 11.08]
σ^G	Spending	2.93 [2.55, 3.31]	2.88 [2.48, 3.27]	2.97 [2.58, 3.36]
σ^I	Investment	2.23 [1.79, 2.66]	2.14 [1.72, 2.54]	2.23 [1.87, 2.58]
σ^R	Monetary policy	0.10 [0.09, 0.11]	0.10 [0.09, 0.11]	0.09 [0.08, 0.11]
σ^E	Entry	7.54 [6.47, 8.56]	7.22 [6.32, 8.24]	7.01 [6.01, 7.91]
σ^P	Price markup	0.39 [0.27, 0.50]	0.34 [0.21, 0.52]	0.41 [0.26, 0.55]
σ^L	Bargaining	0.18 [0.10, 0.25]	0.11 [0.06, 0.19]	0.07 [0.04, 0.11]
MODEL COMPARISON				
Log marginal Data Density		-793.70	-862.35	-808.22
Prior probability		1/3	1/3	1/3
Posterior model probability		1	0	0

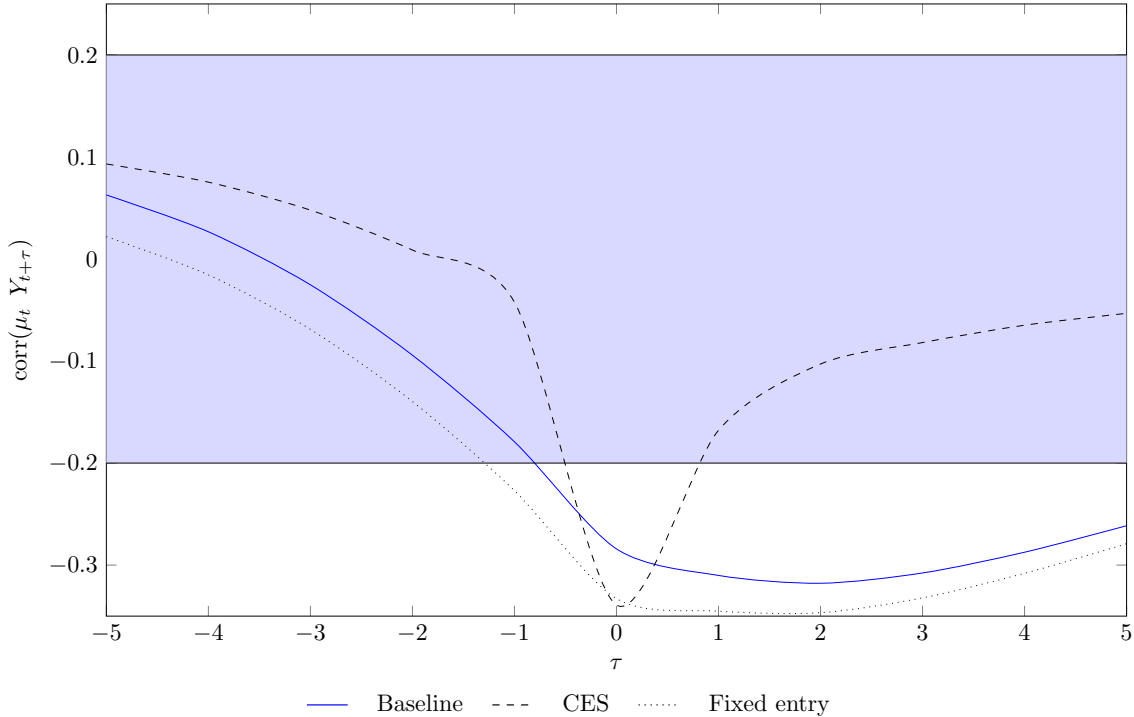
Notes: We use the same distribution for parameters as in the Tab.1.2 and Tab.1.3.

section 3.4. Then, the Fig.1.9 reports the unconditional cyclicity of the markup implied by the three different model specifications at different time horizons (both forward and backward). With respect to the translog specification, CES preferences make the markup less countercyclical with output only at time $t = 0$ and remains acyclical for all leads and lags.

Considering only a fixed sunk cost upon entry without vacancy posting slightly increases the countercyclical nature of the markup for $\tau \geq 0$. This is probably driven by the lower

elasticity between goods obtained in this version which enhances the effect of entry on the markup.

FIGURE 1.9: Cyclicalty of the markup with other specification model.



Conclusion

In this chapter, we have developed and estimated a DSGE model with unemployment and endogenous entry. Using Bayesian econometrics, we have found evidence of the key role of net business formation as an amplifying mechanism for employment dynamics. In particular, our model reveals that even when entry is countercyclical, the employment level for the economy is procyclical. Using search and matching frictions rather than a Walrasian labor market leads to countercyclical markup, even in the absence of entry, due to the acyclical aspect of the marginal cost in our setup.

In the chapter, firms enter the market after paying a sunk entry cost and cost for posting vacancies. For future research, it would be interesting to incorporate capital and wages as determinants of entry. This new setting could have two potential effects. First, it could reconcile our chapter with the traditional technique for modeling entry as in [Lewis](#)

and Stevens (2015) and Bilbie et al. (2007) and lead to other conclusions. Second, these determinants can affect regulation in the goods and labor markets. Since our entry cost is modeled as in Cacciatore and Fiori (2016), changing this cost could affect market (de)regulation policies.

Chapter 2

The role of labor market frictions on mortgage debt dynamics

Introduction¹

Conventional business cycle models featuring a housing market exemplified by [Iacoviello \(2005\)](#) interpret mortgage debt cycles as a macroeconomic response to changes in the future value of durable goods. The underlying debt contract, referred to as collateral constraints, limits the borrowing capacity of an agent to its next period collateral value. Despite its success in policymaking institutions, this conception of mortgage cycles is questionable on two main aspects.

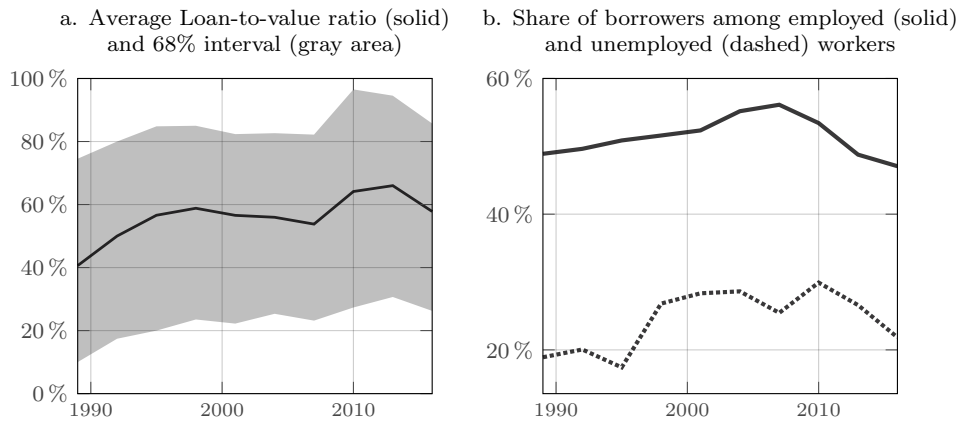
The first aspect is theoretical and concerns the lack of micro-foundation of the financial contract with respect to the borrower's employment situation. In real life situations, financial intermediaries typically review the borrower's ability to make the payments on the loan and avoid potential losses from a future default.² Financial intermediary naturally consider employment as an important criterion in their decision to grant a mortgage, which *de facto* excludes jobseekers from the mortgage market. To illustrate this phenomenon

¹This chapter is based on a paper co-authored with Gauthier VERMANDEL.

²In the literature on the empirical determinants of mortgage default, there is a broad agreement that employment plays a critical role in causing default (*e.g.* [Case et al. \(1995\)](#), [Elul et al. \(2010\)](#) and [Gerardi et al. \(2013\)](#)) among other factors such as house price drop or equity balance.

in the US, Figure 2.1.b shows that the share of borrowers is three times higher among employed workers than jobseekers. In addition, the 2019 SCE housing survey suggests that a decline in home values accounts for 17.1% of all foreclosure decisions, while a rise in mortgage rates accounts for 8.3%. Conversely, job loss and income reduction accounts for 36% and 48% of foreclosure decisions. This evidence conflicts with the current micro-foundation of the collateral constraint model as the latter only consider house price value and interest rates as drivers of borrowing dynamics.

FIGURE 2.1: Housing market characteristics (sources: Survey of Consumer Finance)



The second aspect is empirical: this class of financial frictions typically exhibits poor performances in replicating both (i) Loan-to-Value (LTV) dynamics;³ (ii) and salient business cycle features of mortgage data.⁴ These failures have important implications for the estimation of these models with full information methods as they typically fail at replicating the joint dynamics of house prices and mortgage debt. As a consequence, the usual practice in current state-of-art DSGE models is to: (i) discard mortgage debt liabilities as an observable variable (*e.g.* Neri and Iacoviello (2010) or Guerrieri and Iacoviello (2017)); (ii) arbitrary sweep out the low frequency component of mortgage data using business cycle filters (*e.g.* Gerali et al. (2010)); (iii) include some *ad hoc*

³This debt contract typically imposes a time-invariant loan-to-value ratio, which conflicts with the cyclical change of the LTV ratio in Figure 2.1.a.

⁴To illustrate this limitation, let us consider a simple collateral constraint $d = m.Eq.h$, where the real amount of credit d is limited by a fixed fraction $0 < m < 1$ of future house value, denoted $Eq.h$. Assuming fixed housing stock h , applying logs and differentiating the collateral constraint, then second moment statistics between housing debt and expected house price are theoretically the same. However empirically US data suggest that the autocorrelation of growth real debt is 0.85 vs -0.12 for house price, while 1.09 vs 1.7 for the standard deviations.

persistence mechanism as [Iacoviello \(2015\)](#) that captures a reduced form for some contract persistence as loans are typically not renegotiated on a quarterly basis. The failure of standard collateral constraint models calls for an alternative friction that seriously tackles the empirical relevance of housing models.

As a tractable solution to these concerns, we propose to link the borrowing capacity of households to their employment situation on the labor market. To do so, we enrich the collateral constraint by limiting mortgage granting solely to households in employment. Given the presence of inflows and outflows in employment, the collateral constraint originally depends on employment flows balance. This new collateral constraint referred in this chapter to as the labor-adjusted collateral constraint, introduces a new propagation channel: new matches on the labor market translate into more mortgages (where classical collateral requirements apply), while separation induces an exclusion from financial markets for jobseekers. As a result, the LTV becomes endogenous by responding pro-cyclically to employment cycles.

Our *labor-adjusted* constraint successfully exhibits appealing business cycle features with respect to the canonical setup of [Iacoviello \(2005\)](#). On empirical grounds, our model is able to (i) better account for salient features of financial business cycles, (ii) significantly improve the forecasting performances for most of macroeconomic time series, (iii) be favored by the data according to likelihood ratios. On theoretical grounds, we find that labor market frictions are a key determinant of housing debt dynamics. As a consequence, we show that leakages from the labor to the housing market poses important policy implications for structural reforms and macroprudential policy. In particular, we find that a labor market reform aimed at lowering structural unemployment also leaks to the mortgage market through a surge in mortgages and house prices. As employment rises, the borrowing limit mechanically eases through our labor-adjusted collateral constraint. For macroprudential policy, we find that a loan-to-value tightening affects the labor market through a temporary rise in employment.

Our chapter is connected to the literature that examine the link between labor market fluctuations and mortgage cycles. [Andrés et al. \(2013\)](#) is the closest approach in terms of

the theoretical framework except for the collateral constraint which is simply the expected value of the real estate holdings for borrowers. They find that the response of labor market variables have been substantially affected by the slackening of the LTV ratio in the US in the last twenty years. More precisely, they find that the unemployment is less responsive following a technological shock with lower LTV ratios. [Sterk \(2015\)](#) study the role of house prices on geographical mobility. The author introduces a collateral constraint that depends on the mobility rate and the expected value of real estate holdings. With this setup, he finds that housing price affects the unemployment negatively via the geographical mobility channel. Finally, [Liu et al. \(2016\)](#) documents the relationship between land prices and unemployment. They find an important role of housing shocks in driving unemployment fluctuations. However, in their model constrained household is not present and only firms faced collateral constraint which is the exact opposite of our approach.

Our chapter is organised as follows. The section 1 presents the theoretical framework with our collateral constraint. In the section 2, we present the data that we used and the estimation of the three different models i.e. one for each specific collateral constraint using Bayesian econometrics. The section 3 is dedicated to the empirical performance (RMSE, marginal density, business cycles statistics) of each model and the propagation mechanism implied by each of them. The section 4 discuss the role of different calibration for labor market variables which differs in the literature. Finally, the section 5 investigate how the presence of labor in the collateral constraint affects the obtained from a labor market reform and a macroprudential policy tightening.

1 Theoretical framework

The economy is populated by a continuum of households of unit mass. As in [Kiyotaki and Moore \(1997\)](#), this continuum is composed by patient and impatient households. Impatient households are characterised by a lower discount factor than patient ones such that in equilibrium impatient are net borrowers and patient net lenders. Variables with the superscript P (I) refers to (im)patient households. Following [Andolfatto \(1996\)](#) and

Merz (1995), the family for both types of households provides perfect consumption insurance for its members which allows the latter to have the same consumption level between employed and unemployed family members. Patient households work, consume and accumulate housing and physical capital. Impatient households work, consume and accumulate housing. Due to some underlying frictions in financial markets, borrowers face a binding constraint in the amount of credit they can take.

A key innovation of the model is that collateral requirements depends on the employment status of impatient households. New mortgages are contracted when an impatient household family member finds a job and then classical collateral requirement such as the expected real value of their real estate holdings are applied. For existing mortgage, debt is simply limited not to exceed the amount of the previous period and conditionally to keep the job. This new modeling device establishes a direct link between housing debt and the labor market.

1.1 Labor market

The labor market is subject to matching frictions *à la* Mortensen and Pissarides (1994). For each type of household $j = \{P, I\}$, the hiring process led by firms first starts by a vacancy posting, denoted v_t^j . A vacant position is matched with a job seeker u_t^j through a constant return to the scale matching technology $e_t^j = \psi(v_t^j)^\zeta (u_t^j)^{1-\zeta}$ with $\psi \in [0, 1]$ the efficiency degree of this function and $\zeta \in [0, 1]$ the elasticity of matches with respect to vacancies. As in Gertler et al. (2008), we suppose that unemployed workers who find a match immediately go to work within the period. Regarding the outflow from employment, old matches are destroyed at a constant rate $\delta^L \in [0, 1]$.

Normalising to one the size of the active population, the pool of unemployed workers searching for a job at t is given by the difference between unity and the number of unemployed workers at the end of period $t - 1$:

$$u_t^j = 1 - (1 - \delta^L) l_{t-1}^j. \quad (2.1)$$

Thus, the law of motion of employment is given by:

$$l_t^j = (1 - \delta^L) l_{t-1}^j + e_t^j, \quad (2.2)$$

where e_t^j is the net inflow of employment. This inflow can be expressed in two different ways for the firm or the household. Then for an individual firm, the inflow of new gross hires in t is represented by $e_t^j = q_t^j v_t^j$ while for households by $e_t^j = f_t^j (1 - (1 - \delta^L) l_{t-1}^j)$. The probability that both a firm fills a vacancy and an unemployed worker finds a job are respectively $q_t^j \equiv e_t^j / v_t^j$ and $f_t^j \equiv e_t^j / u_t^j$.

The evolution of employment considering by the firm evolves according to:

$$l_t^j = (1 - \delta^L) l_{t-1}^j + q_t^j v_t^j, \quad (2.3)$$

and for each type of household by:

$$l_t^j = (1 - \delta^L) l_{t-1}^j + f_t^j (1 - (1 - \delta^L) l_{t-1}^j). \quad (2.4)$$

1.2 Households

There is a continuum of measure 1 of agents in each of the two groups of patient and impatient households. As [Neri and Iacoviello \(2010\)](#), the relative size of each group is measured by its wage share, which is assumed to be constant through a unit elasticity of substitution production function. Recall that variables and parameters indexed by I and P denote respectively impatient and patient households, non-indexed variables apply indistinctly to both types of households.

1.2.1 Impatient households

The impatient households maximise the following welfare index:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^I)^t \left\{ (1 - h^C) \log (c_t^I - b c_{t-1}^I) + \varepsilon_t^H j \log (h_t^I) \right\}, \quad (2.5)$$

where β^I is their discount factor, c_t^I is consumption subject to habits $h^C \in [0, 1]$, h_t^I the holdings of housing and j is the consumption weight in life time utility. The term ε_t^H is a shock to housing preferences. This shock can be interpreted as a reduced form source of fluctuations emanating from the productivity changes in the housing sector, or social and institutional changes that shift the demand toward dwellings. Each period, borrowers decide on the optimal amount of nondurable consumption, housing, debt and labor subject to the following budget constraint:

$$c_t^I + q_t^H \Delta h_t^I + r_{t-1} d_{t-1}^I = w_t^I l_t^I + (1 - l_t^I) b^I + d_t^I. \quad (2.6)$$

where Δ is the first difference operator. The left side of the budget constraint is composed by nondurable consumption c_t^I , housing spending Δh_t^I with q_t^H the housing price and one-period housing loan payment d_{t-1}^I at an interest rate r_{t-1} . The right side consists of income with w_t^I the wage per employed worker, b^I the unemployment benefit per unemployed family members and the amount of newly issued loans d_t^I .

In the literature on business cycle models with collateral constraints exemplified by [Kiyotaki and Moore \(1997\)](#) and [Iacoviello \(2005\)](#), impatient households face a borrowing constraint that limits the amount they can borrow d_t^I to a fraction m^I of the expected housing value $\mathbb{E}_t \{q_{t+1}^H\} h_t^I$. The remaining fraction $1 - m^I$ can be interpreted as a down-payment requirements from financial intermediaries. Thus, the collateral constraints read as $d_t^I \leq m^I \mathbb{E}_t \{q_{t+1}^H\} h_t^I$. As a consequence, loans are mainly driven by future house prices. This conception of housing debt cycle is actually at odds with the data as loans typically exhibit more persistence than house prices. As a consequence of this simplistic setup, these models are poorly relevant when they are estimated through full information methods with housing debt as an observable variable. As an alternative to these specifications, we propose to link the borrowing capacity of households to both their collateral and their situation on the labor market. As underlined by [Elul et al. \(2010\)](#), unemployment is an important driver of default on the mortgage market.⁵ To consider these effects into a

⁵In particular, these authors find that when unemployment is high, the mortgage default probability rises simultaneously.

full-fledged macroeconomic model, we assume that only family members of the households who are in employment obtain mortgages from financial intermediaries. Given the presence of inflows and outflows in employment, the collateral constraint directly depends of this employment flows:⁶

$$d_t^I \leq (1 - \delta^L) l_{t-1}^I d_{t-1}^I + e_t^I m^I \varepsilon_t^M \mathbb{E}_t \{ q_{t+1}^H \} h_t^I. \quad (2.7)$$

For the fraction of family members experiencing the separation shock on the labor market, denoted $\delta^L l_{t-1}^I$, they simply cannot pursue the existing mortgage contract. In contrast for the remaining fraction of family members in employment, denoted $(1 - \delta^L) l_{t-1}^I$, they simply roll over their existing mortgage. Regarding inflows in employment, only jobseekers filling a vacancy - denoted e_t^I - are granted new loans by patient households. We also include a structural disturbance, denoted ε_t^M , that captures some exogenous changes in down-payment requirements from financial intermediaries. This shock can be interpreted as reduced form for financial frictions from the supply of assets from banks.

The representative borrower chooses the optimal amount of consumption, housing, debt and labor by maximising his utility (Eq.2.5) subject to his budget constraint (Eq.2.6), his collateral constraint (Eq.2.7) and the flow of labor (Eq.2.4). Thus, the optimal consumption choice gives the marginal utility of consumption denoted by λ_t^I :

$$\lambda_t^I = (1 - h^C) (c_t^I - h^C c_{t-1}^I)^{-1}. \quad (2.8)$$

Letting $\mathbb{E}_t \beta_{t,t+1}^I = \beta^I \mathbb{E}_t \{ \lambda_{t+1}^I \} / \lambda_t^I$ denote the borrower's stochastic discount factor and ϕ_t^I the Lagrangian multiplier on the collateral constraint normalised by the marginal utility of consumption, the Euler condition for borrowers is given by:

$$1 - \phi_t^I = \mathbb{E}_t \beta_{t,t+1}^I [r_t - \phi_{t+1}^I (1 - \delta^L) l_t^I]. \quad (2.9)$$

In this expression, variable ϕ_t^I can be interpreted as the lifetime utility stemming from

⁶We have not included the real interest rate as a determinant of the collateral constraint. However its inclusion has very modest effects on the transmission channels of the model and does not statistically improve the fit of the model.

borrowing for a home purchase. The borrowing constraint introduces a wedge with respect to the patient Euler equation. A rise in borrowing - captured in the Euler equation through $\Delta\phi_t^I > 0$ - implies that the impatient household increases the fraction of his income spent for a home purchase, to the detriment of his current consumption. As a result in log-linearised form of the Euler equation, variations in consumption are negatively linked to changes in the current shadow value of borrowing, ϕ_t^I . This effect of borrowing on consumption is standard in the collateral literature. In contrast, the labor-adjusted collateral constraint also offers a second original effect on the Euler equation that is directly connected to the worker's employment situation. Recall that if the borrower remains employed, he simply rolls over his existing debt contract without further renegotiating with his creditors. The opportunity cost of investing in housing in turn reduces through a rise in current consumption. The magnitude of this effect is positively driven by the borrower's employment rate l_t^I and implies that a rise in the employment rate drives current consumption upward. This positive effect of employment over consumption is usually featured by non-separable utility function such as [King et al. \(1988\)](#).

The first order condition for housing reads as follows:

$$q_t^H = \frac{j\varepsilon_t^H}{h_t^I} \frac{1}{\lambda_t^I} + \mathbb{E}_t \left\{ \beta_{t,t+1}^I q_{t+1}^H + \varepsilon_t^M \phi_t^I f_t^I (1 - (1 - \delta^L) l_{t-1}^I) m^I q_{t+1}^H \right\}. \quad (2.10)$$

This equation determines the housing price q_t^H . The hand right side of this equation is composed of three terms. The first term captures the lifetime utility gain from a marginal unit of housing. The second term is the future gain from reselling the house at the next period, while the third one denotes the lifetime utility gain for matched jobseekers allowed to borrow on financial markets.

Finally, the first-order condition with respect to labor is given by:

$$\mu_t^I = \begin{aligned} & w_t^I - b^I + (1 - \delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^I \mu_{t+1}^I (1 - f_{t+1}^I) \right\} \\ & + (1 - \delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^I \phi_{t+1}^I (d_t^I - d_{t+1}^I) / u_{t+1}^I \right\} \end{aligned}, \quad (2.11)$$

where μ_t^I stands for the marginal utility of a match.⁷ The marginal utility of a match is determined by three terms, two are standard with respect to the matching literature and one is new. The first term is net pecuniary gain of being in employment rather than being unemployed. The second term is the continuation value if the worker remains in employment. In contrast, the third term results from the presence of employment in the collateral constraint. The continuation value of match now includes the roll over of the mortgage contract which increases employment value.

1.2.2 Patient household

Patient household discounts the future more weakly than impatient ones so their discount factor satisfies $\beta^P > \beta^I$.⁸ They maximise the following welfare index:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \{ (1 - h^C) \log (c_t^P - bc_{t-1}^P) + \varepsilon_t^H j \log (h_t^P) \}, \quad (2.12)$$

subject to the budget constraint:

$$c_t^P + q_t^H \Delta h_t^P + I_t + d_t^P + T_t^P + \Phi_K (\Delta K_t) = w_t^P l_t^P + (1 - l_t^P) b^P + r_{t-1} d_{t-1}^P + z_t v_t K_{t-1}, \quad (2.13)$$

where the left side displays spending with consumption (c_t^P), the holdings of housing (h_t^P), investment in physical capital (I_t) subject to some adjustment costs ($\Phi_K (\Delta K_t)$), deposits (d_t^P) and taxes (T_t^P). The right side gathers different sources of income from labor activities (w_t^P if they are employed and b^P otherwise), interest payments on deposits ($r_{t-1} d_{t-1}^P$) and physical capital remuneration z_t at some utilisation rate v_t . Our functional form for physical capital adjustment costs is taken from [Iacoviello \(2015\)](#) and reads as $\Phi_K (\Delta K_t) = \frac{\phi^K}{2K} (K_t - K_{t-1})^2$. The latter allows the model to replicate a hump shape response of investment as suggested by VAR models.

⁷ μ_t^I correspond to the Lagrangian multiplier associated with the labor market law of motion normalised by the marginal utility of consumption.

⁸This restriction on discount factors implies that the Lagrangian multipliers ϕ_t^I on the collateral constraint Eq.2.7 is always positive and thus the constraint holds to equality in the neighborhood of the steady state. Our calibration for the gap between discount factors ensures that $\phi_t^I > 0$ which allows a linear approximation to be accurate.

The law of motion of investment is given by:

$$I_t = \frac{K_t - (1 - \delta_t^K)K_{t-1}}{\varepsilon_t^I}, \quad (2.14)$$

where ε_t^I is an investment shock to the efficiency of investment as in [Smets and Wouters \(2007\)](#) and δ_t^K is the time-varying depreciation of physical capital. As in [Greenwood et al. \(1988\)](#), when the cost of installing new units of physical capital rises, firms prefer to postpone investment and raise the utilisation rate of existing physical capital at the cost of more depreciation.

The representative lenders maximise the welfare index (Eq.68) to choose the optimal amount of consumption, housing, deposits, labor and capital subject to his budget constraint (Eq.2.13) and the flow of labor (Eq.2.4). Then the First Order Condition with respect to c_t^P gives the marginal utility of consumption denoted by (λ_t^P) :

$$\lambda_t^P = (1 - h^C) (c_t^P - h^C c_{t-1}^P)^{-1}. \quad (2.15)$$

Letting $\beta_{t,t+1}^P = \beta^P \mathbb{E}_t \{ \lambda_{t+1}^P \} / \lambda_t^P$ denote the lender's stochastic discount factor, the optimal choice for deposit provides a standard Euler condition for the patient household:

$$\mathbb{E}_t \{ \beta_{t,t+1}^P \} r_t = 1. \quad (2.16)$$

The optimal stock of housing is given by:

$$\frac{1}{\lambda_t^P} \frac{\varepsilon_t^H j}{h_t^P} - q_t^H = -\mathbb{E}_t \{ \beta_{t,t+1}^P q_{t+1}^H \}, \quad (2.17)$$

where the left hand side denotes the current net gain in consumption equivalents from housing purchase.

Letting μ_t^P denote the Lagrangian multiplier associated to the employment law of motion normalised by the marginal utility of consumption, then the marginal utility to have a

new match for patient household is:

$$\mu_t^P = w_t^P - b^P + \mathbb{E}_t \left\{ \beta_{t,t+1}^P \mu_{t+1}^P (1 - \delta^L) (1 - f_{t+1}^P) \right\}, \quad (2.18)$$

compared with the marginal utility of a match for an impatient household (Eq.2.10), this equation is standard in the matching literature with the net pecuniary gain of being in employment rather than being unemployed and the expected future discounted pecuniary gains if the worker remains in employment.

The optimal condition for lenders to provide capital services is:

$$\frac{1}{\varepsilon_t^I} + \Phi'_K(\Delta k_t) = \mathbb{E}_t \left\{ \beta_{t,t+1}^P \left(\frac{(1 - \delta_t^K)}{\varepsilon_{t+1}^I} + z_{t+1} v_{t+1} + \Phi'_K(\Delta k_{t+1}) \right) \right\}. \quad (2.19)$$

The choice for utilisation rate depends on the functional form for the depreciation rate δ_t^K is the same than [Iacoviello \(2015\)](#) and reads as:

$$\delta_t^K = \delta^K + \left(\frac{1}{\beta_P} + 1 - \delta^K \right) \left(\frac{1}{2} \left(\frac{\psi}{1 - \psi} \right) (v_t)^2 + \frac{(1 - 2\psi)}{1 - \psi} v_t + \frac{1}{2} \left(\frac{\psi}{1 - \psi} \right) - 1 \right). \quad (2.20)$$

$\psi \in [0, 1]$ measures the curvature of the utilisation rate function. When $\psi = 1$, δ_t^K stays constant over time i.e. $\delta_t^K = \delta^K$ and with ψ approaching 0 the depreciation rate is very sensitive to the utilisation rate. Considering the budget constraint (Eq.2.13) with the definition of investment (Eq.2.14) and the functional form of the depreciation rate (Eq.2.20), the optimal utilisation rate is:

$$z_t = \left(\frac{1}{\beta_P} + 1 - \delta^K \right) \left(\left(\frac{\psi}{1 - \psi} \right) v_t + \frac{(1 - 2\psi)}{1 - \psi} \right). \quad (2.21)$$

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1.3 Firms

Each competitive producer produce an intermediate good Y_t according to:

$$Y_t = \varepsilon_t^Z L_t^{1-\alpha} (K_t^U)^\alpha, \quad (2.22)$$

where K_t^U is the utilised capital stock ($K_t^U = v_t K_{t-1}$), $L_t = (l_t^P)^\lambda (l_t^I)^{1-\lambda}$ is the total input of labor input used by the firms where λ measures the relative size of lenders, $\alpha \in [0; 1]$ is the part of labor capital in the production and ε_t^Z is the Total Factor Productivity (TFP) shock.⁹

Following [Gertler et al. \(2008\)](#), to hire new workers the firm has to pay quadratic hiring cost $\varepsilon_t^L \kappa^j (x_t^j) l_{t-1}^j$ where $x_t^j = \frac{q_t^j v_t^j}{l_{t-1}^j}$ is the hiring rate and ε_t^L an exogenous shock on hiring new workers.

Then, using the definition of output (Eq.2.22) and the definition of labor composite the problem faced by the representative firm is to choose the optimal amount of labor and vacancies for both type (lenders and borrowers) and the optimal amount of capital to maximise his profit given by:

$$\max_{\{v_t^j, l_t^j, K_t^U\}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{ Y_t - z_t K_t^U - \sum_{j=P,I} \left(w_t^j l_t^j + \varepsilon_t^L \frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j \right) \right\}, \quad (2.23)$$

subject to the labor market law of motion (Eq.2.3) from the firm's perspective.

To obtain the job creation condition for both types of household ($j = \{P, I\}$), we combine the optimal choice of vacancies and labor. Then the job creation condition for patient and impatient households read respectively:

$$\varepsilon_t^L \kappa^P x_t^P = \frac{(1-\alpha)\lambda Y_t}{l_t^P} - w_t^P + \kappa^P \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \beta_{t,t+1}^P \left(\frac{(x_{t+1}^P)^2}{2} + (1-\delta^L) x_{t+1}^P \right) \right\}, \quad (2.24)$$

$$\varepsilon_t^L \kappa^I x_t^I = \frac{(1-\alpha)(1-\lambda) Y_t}{l_t^I} - w_t^I + \kappa^I \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \beta_{t,t+1}^P \left(\frac{(x_{t+1}^I)^2}{2} + (1-\delta^L) x_{t+1}^I \right) \right\}. \quad (2.25)$$

These two equations state that the job creation condition occurs until the marginal cost of hiring reaches the net marginal profit per worker and the expected continuation value.

⁹As experimented by [Iacoviello and Neri \(2010\)](#), a formulation in which labor supply across different households are substitutes are analytically less tractable, since it implies that labor supply by one group will affect total wage income received by the other group, thus creating a complex interplay between borrowing constraints and labor supply decisions.

For installed capital, the first order condition is simply given by:

$$z_t = \alpha Y_t / K_t^U, \quad (2.26)$$

where the rental rate equates the marginal cost of using capital.

1.4 Wage setting

The wage is set according to a Nash bargaining scheme which splits the surplus between workers and employers. For each type j , the Nash bargaining solution is determined by the following program,

$$w_t^j = \arg \max_{\{w_t^j\}} (\mu_t^j)^\eta (\mu_{j,t}^L)^{1-\eta}, \quad (2.27)$$

where $\eta \in [0, 1]$ is the exogenous bargaining power of the worker of type j and $\mu_{j,t}^L$ the marginal value of adding a new worker of type j to the firm's workforce equal to $\mu_{j,t}^L = \kappa^j \varepsilon_t^L x_t^j$.¹⁰

Solving this program for the patient household, we have the following wage:

$$w_t^P = \eta \left(\frac{(1-\alpha)\lambda Y_t}{l_t^P} + \mathbb{E}_t \left\{ \beta_{t,t+1}^P \varepsilon_{t+1}^L \frac{\kappa^P}{2} (x_{t+1}^P)^2 \right\} \right) + (1-\eta) b^P, \quad (2.28)$$

$$+ \eta (1-\delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^P \varepsilon_{t+1}^L \kappa^P x_{t+1}^P f_{t+1}^P \right\}$$

and for impatient :

$$w_t^I = \eta \left(\frac{(1-\alpha)(1-\lambda)Y_t}{l_t^I} + \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \beta_{t,t+1}^P \frac{\kappa^I}{2} (x_{t+1}^I)^2 \right\} \right) + (1-\eta) b^I \quad (2.29)$$

$$+ \eta (1-\delta^L) \kappa^I \mathbb{E}_t \left\{ \varepsilon_{t+1}^L x_{t+1}^I (\beta_{t,t+1}^P - \beta_{t,t+1}^I (1-f_{t+1}^I)) \right\}$$

$$+ (1-\eta) (1-\delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^I \phi_{t+1}^I \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\}$$

The patient wage corresponds to the definition of wages in the search equilibrium when impatient households are not present in the model. This corresponds to a weighted average between the worker value for a firm (i.e. the marginal productivity of labor plus the saving cost of hiring) and the outside option (i.e. the transfer b^P).

¹⁰This equation comes from the FOC with respect to vacancies. See appendix C for more details.

For the impatient wage, the first line is classic as for the patient wage. The second line take into account that impatient household discount the future more heavily than patients. Thus we have a utility gap between lenders and borrowers (i.e. $\beta_{t,t+1}^P - \beta_{t,t+1}^I$). Finally, the last line is due to the integration of the labor market into the borrowing constraint (Eq.2.7). Thus, this term depends positively on the growth of debt and negatively on the anticipation of future unemployment.

1.5 General equilibrium

Market clearing is implied by Walras's law by aggregating the budget constraints of lenders and borrowers. In absence of explicit residential production sector the supply of dwellings is normalised to one as [Iacoviello \(2005\)](#), the market clearing for housing just reads as:

$$h_t^P + h_t^I = 1. \quad (2.30)$$

As a consequence, housing cycle boils down to reallocation effect between impatient and patient households driven by the borrowing constraint. An easing on the borrowing constraint through higher a housing price rises the demand for durable goods h_t^I and mechanically reduces the housing detention for patient households as long as Eq.2.30 holds.

In absence of explicit financial frictions on financial markets, we assume that patient households grant loans to impatient ones at no cost using their own deposits as liabilities, which implies the following equilibrium condition on the mortgage loans:

$$d_t^P = d_t^I. \quad (2.31)$$

Taxes finance unemployment insurance for both types of workers, denoted $(1 - l_t^P) b^P + (1 - l_t^I) b^P$ and government spending G_t . Following the usual practice in modern macroeconomic models, public spending are exogenous, $G_t = g^Y \varepsilon_t^G$ where g^Y is the fixed spending relative to GDP ratio and ε_t^G is an exogenous process allowing the government to

transitory deviate from this fixed ratio. The balance for the government reads as:

$$T_t^P = (1 - l_t^P) b^P + (1 - l_t^I) b^I + g^Y \varepsilon_t^G, \quad (2.32)$$

where T_t^P is the lump-sum tax from patient households.

Then using these relationships, the aggregate GDP is defined as:

$$Y_t = C_t + g^Y \varepsilon_t^G + I_t + \Phi_K (\Delta K_t) + \sum_{j=P,I} \varepsilon_t^L (x_t^j)^2 (l_{t-1}^j)^{-1}, \quad (2.33)$$

where aggregate consumption is given by $C_t = c_t^P + c_t^I$.

2 Data and estimation

The model is estimated using Bayesian methods and quarterly data for the US economy. We estimate the structural parameters and the sequence of shocks following the seminal contributions of [Smets and Wouters \(2007\)](#) and [An and Schorfheide \(2007\)](#). In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method. We solve the model using a linear approximation to the model's policy function, and employ the Kalman filter to form the likelihood function and compute the sequence of errors. For a detailed description, we refer the reader to the original papers.

TABLE 2.1: Description of the estimated models featuring different collateral constraints

Model Symbol	Collateral constraint type	Collateral constraint equation
\mathcal{M}_1	Simple	$d_t^I = \varepsilon_t^M m^I h_t^I \mathbb{E}_t \{q_{t+1}^H\}$
\mathcal{M}_2	Exogenous persistence	$d_t^I = \chi d_{t-1}^I + \varepsilon_t^M (1 - \chi) m^I \mathbb{E}_t h_t^I \mathbb{E}_t \{q_{t+1}^H\}$
\mathcal{M}_3	Labor-adjusted	$d_t^I = (1 - \delta^L) l_{t-1}^I d_{t-1}^I + \varepsilon_t^M m^I e_t^I \mathbb{E}_t \{q_{t+1}^H\} h_t^I$

In this chapter, we compare our labor-sensitive constraint to two different benchmark models that are typically employed in the literature of collateral constraints. [Tab.2.1](#) summarises the three models estimated using the same sample. The first model, referred

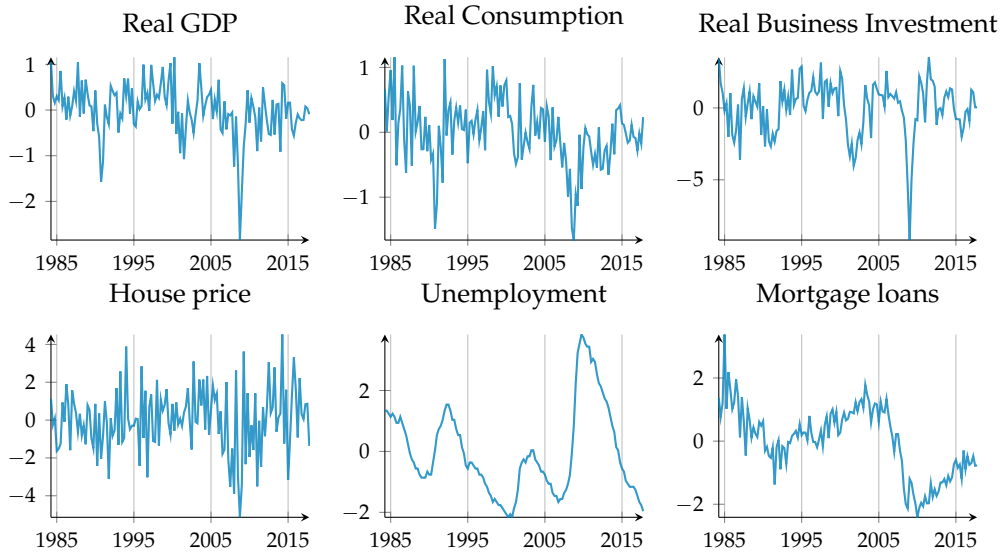
to as 'Simple' in the first row of Tab.2.1, is a model in which the borrowing capacity is bounded by market expectations about future prices of dwellings in the same way as [Iacoviello \(2005\)](#) or [Kiyotaki and Moore \(1997\)](#). Alternatively, we also consider the collateral constraint of [Iacoviello \(2015\)](#) characterised by an *ad hoc* persistence mechanism. In this setup, parameter $\chi \in [0, 1]$ captures the difference between existing mortgage and new mortgage (in proportion $(1 - \chi)$). Given its *ad hoc* nature, we simply call this setup as the 'Exogenous persistence' model. Finally our last model explained in the model section is referred to as the 'Labor-adjusted' model. Since we perform a linear approximation to the policy function of each model, we assume that each constraint holds to equality and we select a calibration which allows the Lagrangian multiplier associated to any of these collateral constraints always to remain positive.

2.1 Data

We fit the DSGE model to US time series data from 1984Q2 to 2017Q4. Following the usual practice, we keep the number of exogenous disturbances (ε_t^Z , ε_t^L , ε_t^H , ε_t^I , ε_t^G and ε_t^M) the same as the number of observable variables in order to obtain the smoothing of filtered disturbances. Our sample includes housing price, gross domestic product, consumption, investment, unemployment rate and loans. Appendix A describes the data sources.

Concerning the transformation of series, the point is to map non-stationary data to a stationary model. Except for the unemployment rate, all other data exhibit a trend and are made stationary in two steps. First, we deflate nominal variables using the GDP deflator as in [Iacoviello \(2015\)](#). Second, data are taken in logs to use a first difference filtering to obtain growth rates. Since we do not consider trends, we demean our sample to make the sample consistent with the measurement equations of our model. The Fig.2.2 plot the transformed series.

FIGURE 2.2: Data used in the estimation



Notes: The vertical axis plots the percentage deviation of the data sample from their average.

2.2 Calibration and prior distributions

Our calibration is reported in Tab.2.2. Parameters which are calibrated are typically those are weakly identified by the data. As a simplifying assumption, most of parameters common between patient and impatient households are symmetric (an exception for discount factors). For the labor market, we fix the steady-state of the finding rate to 70% ($f^j = 0.7$) as in Shimer (2005) and the exogenous separation by δ^L to 0.1 matching the average job duration of two and a half years in the US. Together they lead to a steady-state value of unemployment to 5% as in Blanchard and Galí (2010) and which is close to the average 6% unemployment rate observed in our sample. The filling rate is fixed to 73% ($q^j = 0.73$) in order to have a steady-state tightness in the labor market to be below one. Consistently with labor matching models, we fix the negotiation power of households η to 0.5 and impose the so-called Hosios condition by imposing $1 - \zeta = \eta$, i.e. the elasticity with respect to unemployment in the matching function is equal to the negotiation power of workers. For the replacement rate $\tau_j^B = b^j/w^j$, we use the same calibration as Christiano et al. (2016) with $\tau_j^B = 0.75$.

Turning to the calibration of discount factor, we use the same calibration as Iacoviello and

TABLE 2.2: Calibrated parameter

Parameter	Value	Parameter	Value		
α	Capital share	0.33	β^P	Discount factor lenders	0.9925
δ^K	Capital depreciation rate	0.035	β^I	Discount factor borrowers	0.97
g^Y	Spending-to-GDP ratio	0.21	λ	Share of lenders in technology	0.75
j	Housing preference	0.09	d^I/Y	Mortgage debt-to-GDP	0.50
f	Finding rate	0.70	ξ	Elasticity matching function	0.50
q	Filling rate	0.73	δ^L	Labor separation	0.10
b^j/w^j	SS unemployment benefit	0.75	η	Worker negotiation power	0.50

Neri (2010) with $\beta^P = 0.9925$ and $\beta^I = 0.97$. According to the estimation of Iacoviello (2015) we set the share of lenders to 75% i.e. $\lambda = 0.75$ and the share of capital in the production function to $\alpha = 0.33$.¹¹ As common practice in the literature, we calibrate the depreciation rate of capital to $\delta^K = 0.035$ and the ratio of public spending to GDP $g^Y = .21$.

The housing wealth is 123 percent of annual output ($q^H(h^P+h^I)/(4Y) = 1.23$) following Iacoviello (2005) and lead to a housing preference of $j = 0.09$. However, in other versions of our model, this ratio can vary in function of the value of χ . For $\chi = 0$ (the simple version \mathcal{M}_1) we have a housing wealth of 126% of annual output against 120% for $\chi = 0.95$. Finally, the only parameter which differ between models is the LTV ratio m^I . In the baseline model, the steady-state of debt is $d^I = m^I f^I q^H h^I$ against $d^I = m^I q^H h^I$ for other models. Since the steady-state value of the finding rate is already calibrated, we fix $m^I = 0.9$ in the baseline model leading to a global LTV ($m^I f^I$) of 0.63. As a consequence, we calibrate m^I in other models to 0.63 to have the same debt to output ratio which is 50% closer from the average of mortgage loans to GDP ratio observed in the data.

For the estimation of our structural parameters, we choose the same prior distribution for the utilisation curvature (ψ), the capital adjustment cost (ϕ^K) and the habit consumption (h^C) than Iacoviello (2015). For the exogenous component in the collateral constraint (χ), in our knowledge two articles estimate it. Iacoviello (2015) choose a beta distribution with

¹¹The calibration of the impatient discount factor diverges in the literature, typically between 0.94 and 0.98. However, we choose the same calibration than Iacoviello and Neri (2010) since in our knowledge it is the only paper that estimates a model with lenders and borrowers without considering entrepreneurs as in our model.

a mean of 0.25 and a standard deviation of 0.1 and [Guerrieri and Iacoviello \(2017\)](#) a beta distribution with a mean of 0.75 and a standard deviation of 0.1. Then, we thus consider an average between these two papers by imposing a prior means of 0.5 and a standard deviation of 0.1.

Concerning the prior of shock process we follow [Smets and Wouters \(2007\)](#) where the standard deviations of the shocks are assumed to follow an Inverted Gamma distribution with a means of 0.5 percent and two degrees of freedom and for the persistence of the shock (AR(1)) a Beta distribution with means 0.5 and a standard deviation 0.15.

2.3 Posterior distributions

In this subsection, we discuss our posterior results and contrast them with the results obtained from previous estimates in the literature. The [Tab.2.3](#) summarises means and the 5% and 95% of the posterior distributions for the structural parameters as well as for shock processes.

The estimated degree of habit formation is rather lower compared to other studies ([Smets and Wouters \(2007\)](#) and [Gertler et al. \(2008\)](#) estimate this parameter around 0.7). Traditionally, higher degree of habits formation is a necessary device to account for consumption persistence. However, models featuring a housing market such as [Iacoviello and Neri \(2010\)](#), [Iacoviello \(2015\)](#) and our models generate the desired persistence with a lower habits degree. The intuition behind this result is that the wedge in the Euler equation of impatient households is affected sufficiently to account for this business cycle pattern. Regarding the adjustment costs on investment (ϕ^K) and capital utilisation elasticity (ψ), these are close to the findings of [Iacoviello \(2015\)](#). We also find that there is no clear difference between estimated parameters across the three models.

The estimated value of the *ad hoc* persistence parameter in the second model is higher than in the literature. We guess that this result is obtained by imposing a looser prior to χ , the data are thus more informative and predicts a very high persistence. This highlights the underlying problem of collateral constraint models that fails at capturing the persistence of loans.

TABLE 2.3: Prior and posterior distributions for structural parameters

		Prior(P1,P2)	Posterior Mean [5%, 95%]		
			\mathcal{M}_1	\mathcal{M}_2	\mathcal{M}_3
<u>Shock processes:</u>					
ρ_Z	AR - Productivity	$B(0.50, 0.15)$	0.98 [0.97;0.98]	0.97 [0.97;0.98]	0.97 [0.97;0.98]
ρ_L	AR - Hiring	$B(0.50, 0.15)$	0.89 [0.86;0.92]	0.89 [0.86;0.92]	0.91 [0.89;0.94]
ρ_H	AR - Housing preference	$B(0.50, 0.15)$	0.97 [0.96;0.99]	0.97 [0.95;0.98]	0.96 [0.95;0.98]
ρ_G	AR - Public Spending	$B(0.50, 0.15)$	0.91 [0.89;0.94]	0.91 [0.89;0.94]	0.90 [0.88;0.93]
ρ_I	AR - Investment	$B(0.50, 0.15)$	0.93 [0.89;0.97]	0.93 [0.88;0.97]	0.93 [0.89;0.97]
ρ_M	AR - Loan To Value	$B(0.50, 0.15)$	0.97 [0.95;0.98]	0.82 [0.75;0.90]	0.90 [0.86;0.95]
σ_Z	Std.Dev Productivity	$IG(0.1, 2)$	0.39 [0.34;0.44]	0.39 [0.34;0.44]	0.40 [0.35;0.45]
σ_L	Std.Dev Hiring	$IG(0.1, 2)$	4.89 [4.40;5.38]	4.90 [4.41;5.40]	6.13 [5.51;6.73]
σ_H	Std.Dev Housing preference	$IG(0.1, 2)$	0.07 [0.04;0.09]	0.08 [0.05;0.11]	0.08 [0.05;0.11]
σ_G	Std.Dev Public Spending	$IG(0.1, 2)$	2.37 [2.13;2.61]	2.36 [2.11;2.59]	2.38 [2.14;2.63]
σ_I	Std.Dev Investment	$IG(0.1, 2)$	0.66 [0.52;0.81]	0.64 [0.50;0.77]	0.61 [0.48;0.73]
σ_M	Std.Dev Loan To Value	$IG(0.1, 2)$	2.05 [1.85;2.27]	6.39 [4.52;8.30]	5.45 [4.92;6.01]
<u>Structural parameters:</u>					
h^C	consumption habits	$B(0.50, 0.15)$	0.27 [0.15;0.39]	0.26 [0.14;0.37]	0.24 [0.13;0.34]
ϕ^K	capital adj. cost	$G(1.00, 0.50)$	0.85 [0.21;1.46]	0.89 [0.22;1.51]	0.79 [0.20;1.34]
ψ	utilisation elasticity	$B(0.50, 0.10)$	0.34 [0.20;0.48]	0.33 [0.19;0.47]	0.38 [0.23;0.52]
χ	Collateral persistence	$B(0.50, 0.10)$	-	0.91 [0.89;0.94]	-

Notes: \mathcal{B} , beta; \mathcal{G} , gamma; \mathcal{N} , normal; \mathcal{IG} , inverse gamma type 1; $P1$, prior mean and $P2$ prior standard deviation for all distributions.

Except for the LTV shock ε_t^M , all other sources of disturbance exhibit quite similar persistence and standard deviations across models. The difference regarding the standard deviation of the LTV shock is simply induced by a scale effect because \mathcal{M}_1 the steady state pre-multiplying the shock is much higher than in \mathcal{M}_2 and \mathcal{M}_3 .

3 Empirical implications

The labor-sensitive collateral constraint introduces a new propagation channel from the labor to the housing market. In this section, we investigate the empirical relevance of this channel in a four-step analysis. First, we compare the fit of our three models through a likelihood comparison between models. Second, we compare the business cycle moments to see how models are able to account for salient features of the data. Third, we examine the forecasting performances of each model. Finally, we compare the propagation mechanism by comparing the IRFs between models.

3.1 Fit comparison

To gauge the empirical relevance of employment in directly shaping debt dynamics, the Tab.2.4 reports the (Laplace-approximated) marginal data densities, the posterior odd ratio and probability for each of the three models considered.¹² Since the simple collateral model is the most popular model in this literature, we consider this model as the benchmark to compute the posterior odds ratios and probabilities. To compute the latter, we impose an uninformative prior distribution over models (*i.e.* 1/3 prior probability for each model). In a nutshell, one should favor a model whose data density, posterior odds ratio and model probability are the highest compared to any other model.

Which model best explains the behaviour of the sample?

Our model with labor-adjusted collateral constraints appears to be favored by the data as its marginal data density is the highest. This model is next followed by the exogenous persistence model while the simple model is naturally the last in the ranking. The difference in marginal data densities across models is large enough to validate this ranking.¹³ This is therefore confirmed by posterior odds ratios and model probabilities. Given this evidence, we conclude that our model with labor-adjusted constraints outperforms the two other models.

TABLE 2.4: Models fit comparison

	\mathcal{M}_1 Simple	\mathcal{M}_2 Exo. Persistence	\mathcal{M}_3 Labor-adjusted
Prior probability	1/3	1/3	1/3
Log marginal density	-921.81	-899.87	-849.11
Bayes ratio	1.000000	3.388×10^9	3.735×10^{31}
Posterior probability	0.000000	0.000000	1.000000

¹²We refer to Rabanal and Rubio-Ramírez (2005) for a formal description and discussion of these criteria to compare estimated DSGE models.

¹³The results are statistically strong as the marginal data density difference between \mathcal{M}_3 and \mathcal{M}_2 is 72.69, thus we would need a prior probability ratio (currently this ratio is unity) to be higher than $\exp(72.69) = 3.73 \times 10^{31}$ to alter the ranking.

The likelihood ratio does not allow to clearly understand how one model is able to outperform the other. The next subsections provide further investigations through business cycle moments and forecasting performance comparisons between models.

3.2 Business cycle analysis

To assess the empirical relevance of the model, Tab.2.5 report's key business cycle statistics for observable variables generated by the three models considered, taking for each model the parameters at their posterior means in Tab.2.3. The aims of this exercise is to assess whether these models are able to capture salient features of the data. Observed moments are expressed in terms of a 90% confidence interval to highlight whether a moment generated by the model is not statistically different from its empirical counterpart.

We first start by examining the standard deviations generated by each model. The three models considered exhibit similar quantitative patterns: they all overpredict the volatility of unemployment, while they are doing a good job in replicating the other standard deviations. The only exception lies in the growth rate of mortgages which is best replicated by the *ad hoc* persistence model \mathcal{M}_2 , followed by the labor-adjusted constraint.¹⁴ This result is rather expected as models \mathcal{M}_2 has one additional estimated parameter with respect to the two other competitors, which clearly helps in capturing the dynamic patterns of mortgages.

Regarding the persistence, all the models are well able to replicate the observed correlation of each observable variable, except for mortgage and investment growth rates. For investment, all the models fail at replicating the observed persistence of investment. This puzzling result is explained by the capital cost function which does not generate the desired autocorrelation. For mortgage dynamics, only the simple collateral constraint models fail at capturing the observed persistence of housing debt growth.

Finally, concerning the correlation coefficient with unemployment, all the models succeed in replicating the co-movement with other observed variables except for housing debt. Our

¹⁴This result contrasts with respect to the literature of estimated models with matching. For instance, Lubik (2009) estimates a simple labor matching model on US data and finds that his model with matching over shoot the variance of unemployment.

TABLE 2.5: Business cycle statistics comparison between the three different collateral constraint models

	U_t	ΔY_t	ΔC_t	ΔI_t	Δd_t^I	Δq_t^H
Standard deviations						
Data	[1.30;1.66]	[0.52;0.66]	[0.46;0.58]	[1.67;2.13]	[0.97;1.23]	[1.52;1.93]
\mathcal{M}_1 - Simple	2.33	0.62	0.67	2.15	1.99	1.84
\mathcal{M}_2 - Exo. persistence	2.30	0.62	0.66	2.16	1.15	1.82
\mathcal{M}_3 - Labor-adjusted	2.28	0.61	0.68	2.12	1.67	1.81
Auto-correlation						
Data	[0.97;0.99]	[0.17;0.55]	[0.16;0.54]	[0.44;0.72]	[0.80;0.91]	[-0.33;0.10]
\mathcal{M}_1 - Simple	0.99	0.17	0.26	0.04	0.69	-0.01
\mathcal{M}_2 - Exo. persistence	0.99	0.18	0.25	0.04	0.84	-0.01
\mathcal{M}_3 - Labor-adjusted	0.99	0.17	0.25	0.02	0.90	-0.01
Correlation w/ unemployment						
Data	[1.00;1.00]	[-0.34;0.10]	[-0.42;-0.00]	[-0.33;0.11]	[-0.62;-0.28]	[-0.28;0.16]
\mathcal{M}_1 - Simple	1.00	-0.02	-0.04	0.01	-0.00	-0.02
\mathcal{M}_2 - Exo. persistence	1.00	-0.02	-0.04	0.01	-0.05	-0.02
\mathcal{M}_3 - Labor-adjusted	1.00	0.06	0.03	0.05	-0.28	0.01

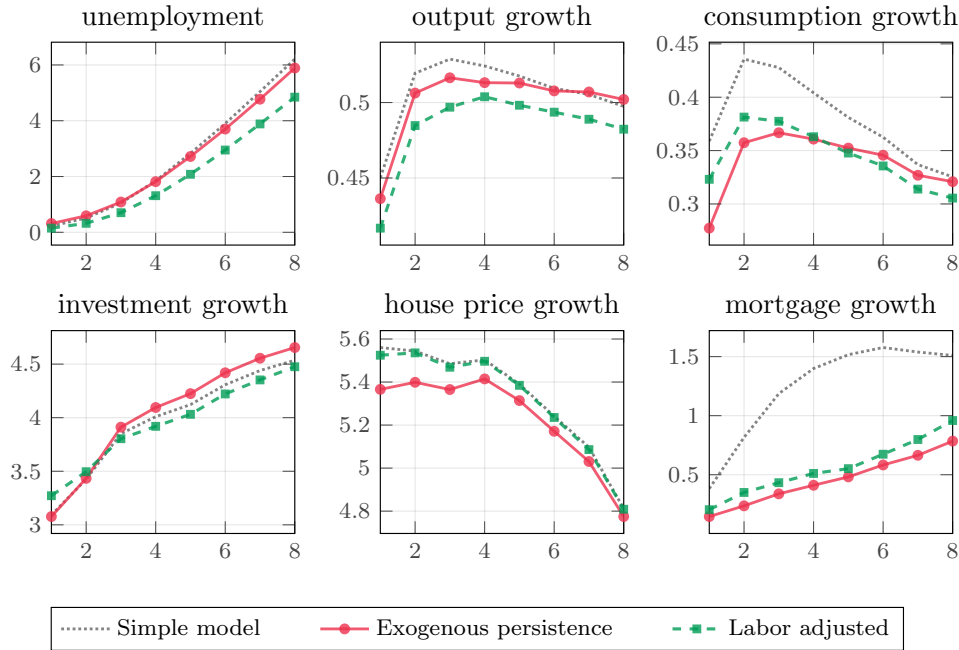
model \mathcal{M}_3 with labor-adjusted borrowing capacity is the only one that successfully captures the important negative correlation link between mortgage and unemployment. A rise in unemployment mechanically terminate the mortgage contracts for workers experiencing the separation. As a consequence, this constraint naturally generate the appropriate correlation magnitude and sign for these two observed variables.

3.3 Forecasting performance

Turning to the forecasting performances, Fig.2.3 displays the out-of-sample root means square errors (RMSE, hereafter) at different forecast horizons (1 to 8 periods) for each observable variable. Our RMSE's are computed on the out-of-sample forecasting on ten years, spanning the period from 2004Q4 to 2014Q4.¹⁵ Our models are estimated each quarter, forecasts are performed using the posterior mode. Therefore, the best model at forecasting is the one that obtains the lowest RMSE.

¹⁵Our forecasts stops at 2014Q4 (*i.e.*, 8 periods before the end our sample) because after this date we would not have the corresponding observable to compute the error distance between the forecast and its realised value.

FIGURE 2.3: Root mean square errors (RMSE) comparison between models featuring different collateral constraints



Our model with labor in the collateral constraint remarkably outperforms the other models for unemployment, output, investment. The results are rather unclear for consumption, as our model \mathcal{M}_3 is doing better than \mathcal{M}_2 in the short run, however, after 5 periods, the forecasting performance between these two models reverses in favor of \mathcal{M}_2 . However for house price growth prediction, the model with exogenous persistence \mathcal{M}_2 clearly outperforms other models, in particular in the very short-run. Mortgage growth clearly illustrates a clear weakness of simple collateral constraints which clearly does not generate the desired level of persistence. In contrast, both models \mathcal{M}_2 and \mathcal{M}_3 are able to capture this persistence either through an ad hoc device in \mathcal{M}_2 or through employment in \mathcal{M}_3 .

3.4 Inspecting the propagation mechanism

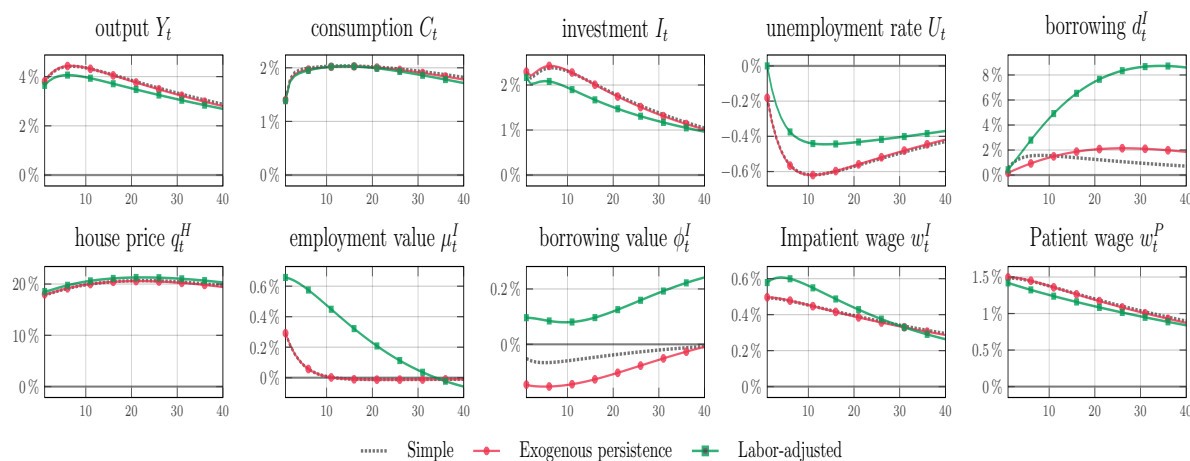
The three models provide different representations of the data, these differences are implied by the propagation mechanisms reshaped according to the type of collateral constraint considered. To illustrate how conditioning mortgage on employment affects the propagation, we contrast the response of our model with the two others. We examine the

response following a productivity shock, this shock is particularly relevant as it is the main driving force of unemployment fluctuations in the RBC literature (*e.g.* Mortensen and Pissarides (1994); Shimer (2005)), and we complement the analysis with a hiring shock to highlight how changes in hiring affect mortgage dynamics.¹⁶ Each model parameters are set to their posterior mean.

A productivity shock

Fig.2.4 displays the response of the three models following an increase in the productivity of firms. The response obtained by the three models are rather in line with matching model literature: a rise in TFP makes labor and capital more productive, which raise the marginal profit from hiring a new worker and then encourage firms to hire more.

FIGURE 2.4: System response to a 1% productivity shock η_t^Z for each estimated models.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

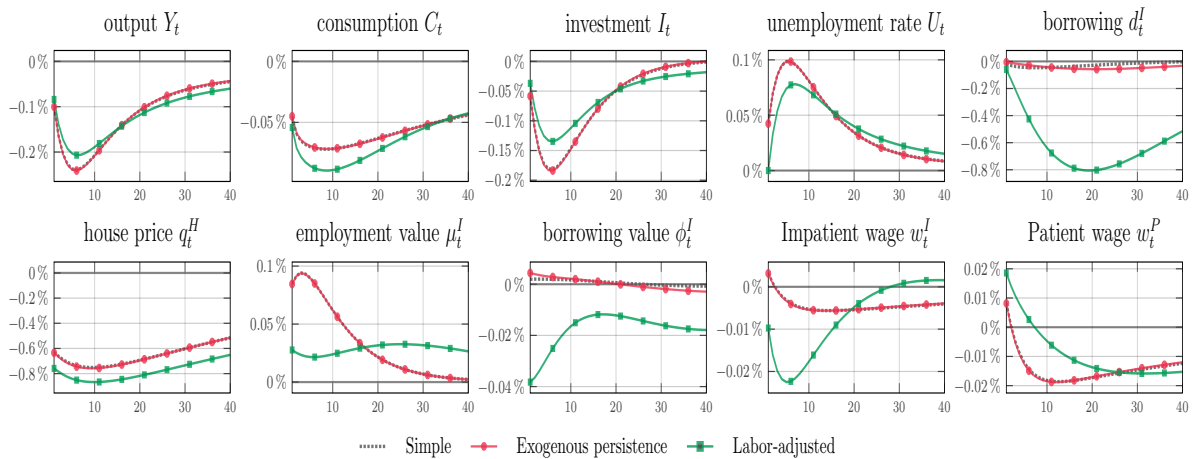
Therefore employment slowly rises as a result of the sluggishness of the matching process. Thus there are no clear difference on output, consumption, investment and housing price. Our model differs in the response of debt and unemployment compared to the two others. For the debt, in our model is four times more responsive than the others through two complementary reasons. The first reason lies in the direct easing of our collateral constraint (Eq.2.7) when there are more matches on the labor market. The value of borrowing

¹⁶In model \mathcal{M}_3 , hiring and productivity shocks respectively drive 62% and 34% of unemployment fluctuations.

rises as a response from employment, which naturally encourages impatient households to borrow more. Then impatient households buy more durable goods by contracting new mortgages, these fuels a temporary re-allocation of housing goods from patient to impatient households. The second reason is indirect and concerns how the labor market respond to housing fluctuations. Our collateral constraint also affects households labor decisions. As a result, the rise in the value of borrowing also increases the value of employment (μ_t^I) as more workers are allowed to borrow. The equilibrium wage also responds positively (Eq.2.29) as a result of the Nash bargaining process.

Hiring shock Fig.2.5 displays the response of the three models following an exogenous increase in the cost of hiring.

FIGURE 2.5: System response to a 1% hiring cost shock η_t^L for each estimated models.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

On the labor market, there are more outflows from separation than inflows from new matching, so employment reduces. For most of real variables, all models exhibit similar responses: when hiring gets costlier, the number of matching decreases, so the workforce also reduces which drives down aggregate production, investment and consumption. However, propagation predictions across models become different on housing and borrowing aspects. For standard models, disturbance within the labor market has no implication for mortgages, as the stock of borrowing remains almost unchanged. In contrast with a labor-sensitive borrowing, there are fewer family members of the impatient who can

get mortgages, so the stock of loans decreases, which further depresses house prices. A remarkable feature of our constraint is the endogenous persistence of borrowing, that is directly connected to the sluggishness of the labor market. This allows our setup to endogenously replicate the desired persistence observed in the data, which is typically overstated by standard models.

4 Discussing the role of labor frictions

Since we have shown that our model is favored by the data through the presence of an endogenous persistence of mortgages determined by the labor market, in this section, we examine how labor market frictions actually shape the propagation. We thus check how the propagation is sensitive to the bargaining power, the separation rate and the share of impatient workers.

Bargaining power. We first start examining the role of the bargaining power of households on sharing the surplus between the firm and the worker. We have not estimated the bargaining power η , but rather have calibrated it as in [Mortensen and Pissarides \(1994\)](#).¹⁷ We thus examine how this parameter affects the housing market by contrasting a situation with a low bargaining power for households versus a high-bargaining power.

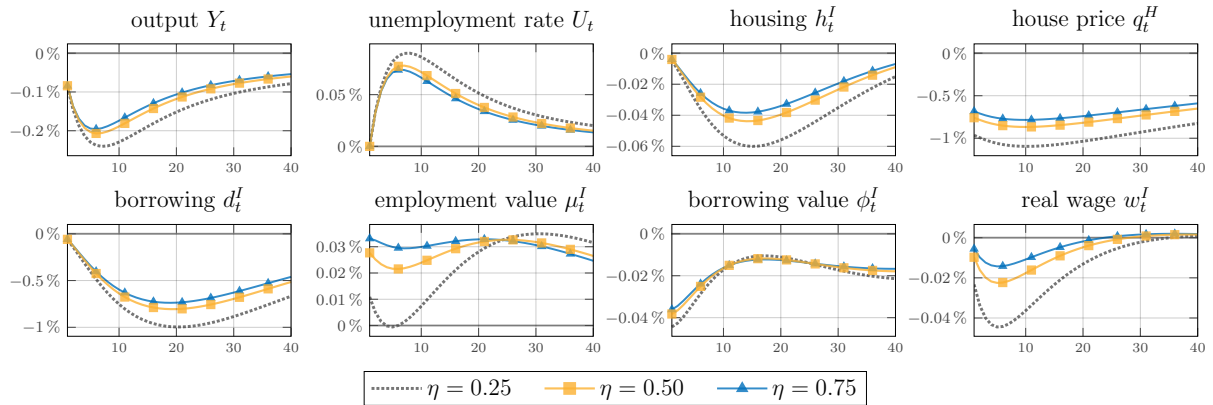
First considering $\eta = 0.75$ which is a rather widespread calibration in the labor matching literature (*e.g.* [Christiano et al. \(2016\)](#); [Gertler et al. \(2008\)](#)). In this case, the employment value is less dependant on the growth of debt and unemployment as described by the wage setting (Eq.2.29). Then, the real wage for impatient household as for the employment value is less affected than the other calibration in their fall. However compared with our calibration *i.e.* $\eta = 0.50$, for all the other variables they have a similar behaviour.

Turning to the case of a low negotiation power calibrated by [Liu et al. \(2016\)](#) which study the relationship between unemployment and land price. The impact on the wage for impatient household is more important. This seems counterintuitive since the impatient

¹⁷Only a model featuring wage rigidities (*e.g.* [Gertler et al. \(2008\)](#)) can provide an estimation of the bargaining power, as using real wages as an observable variable provide enough information on η to identify it accurately.

wage becomes less dependent on the growth debt. However, since the unemployment rate rises more the effect of the debt is more important and compensate the less impact of growth debt in the impatient wage.

FIGURE 2.6: System response to a 1% hiring cost shock η_t^L under three different values for households' bargaining power η .



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

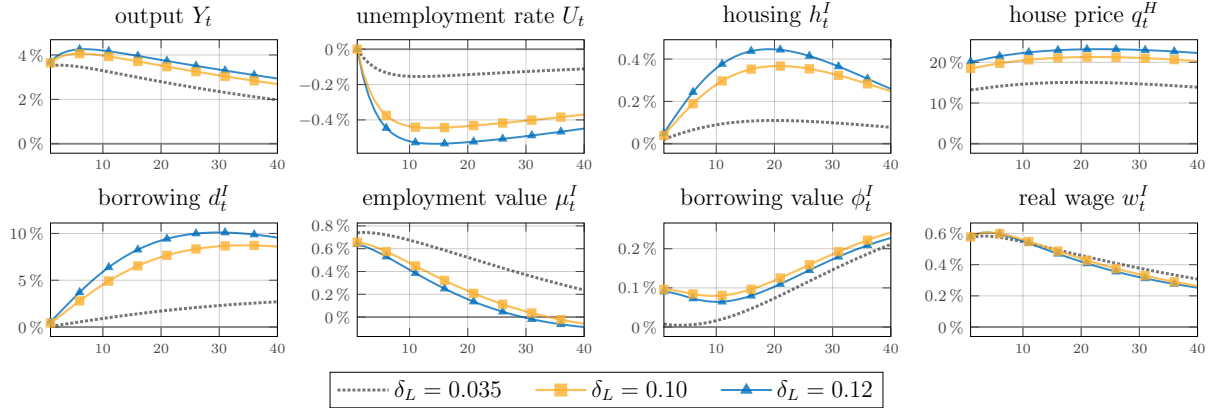
Separation rate. In the literature of matching frictions in the labor market, there are two conflicting approaches for the calibration of the separation rate. The first approach, pioneered by [Shimer \(2005\)](#), is motivated by the microeconomic evidence to impose a low value for the job separation rate with *i.e.* $\delta^L = 0.035$. The second approach aims at matching the macroeconomic unemployment rate by setting a higher value for this parameter, *i.e.* $\delta^L = 0.1$.¹⁸ This calibration features a mean duration for a job in US that is two years and a half.¹⁹ We also consider an even higher value with $\delta^L = 0.12$ as in [Liu et al. \(2016\)](#). Then the Fig.2.7 displays the response of the model to a 1% productivity shock under this three different values of δ^L .

Unemployment is becoming more responsive with the increase in value of the separation rate. Recall that in the deterministic steady-state, the hiring rate is equal to the separation rate, so a higher separation rate induces mechanically a higher hiring rate. Following a productivity shock, firms have less effort to hire new workers when the hiring rate is higher. Since there are more matches when the separation rate is high, borrowing

¹⁸See for instance [Christiano et al. \(2016\)](#).

¹⁹See for example [Den Haan et al. \(2000\)](#), [Blanchard and Galí \(2010\)](#).

FIGURE 2.7: System response to a 1% productivity shock under three different values for the separation rate δ^L .



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

responds accordingly with more mortgage contracted by successful job seekers. This fuels the demand for housing and the price of housing clears by increasing the market value of housing. Thus the separation rate clearly shapes the dynamics of borrowing and housing prices.

The share of impatient workers. We assess how the relative number of borrowers shapes the dynamics of employment and housing. The Fig.2.8 displays the response of the

FIGURE 2.8: System response to a 1% technological shock η_t^Z under three different shares of borrowers.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

three models after a 1% technological shock with different shares of patient households.

With a large proportion of impatient households $\lambda = 0.60$, the unemployment rate is less responsive, as already documented by [Andrés et al. \(2013\)](#). This result is driven here by our borrowing constraint: a higher share of borrowers mechanically induces more borrowing, as borrowing increases the value of employment, the contraction of unemployment is slightly mitigated. Since unemployment is less responsive with a higher fraction of the borrower, the equilibrium wage rises to compensate for the higher value of employment.

5 Policy implications

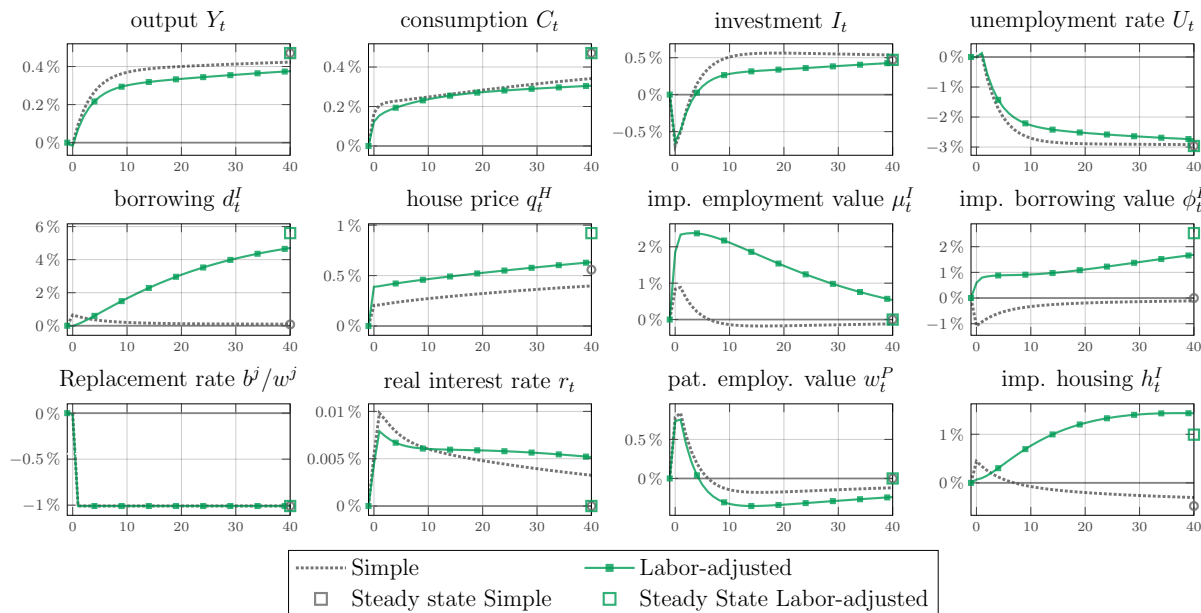
In this section, we investigate how the presence of labor in the collateral constraint affects the obtained from a labor market reform and a macroprudential policy tightening. Since our model with labor-adjusted constraint is favored by the data, then the results obtained from our models are more plausible than those obtained from the standard collateral model.

5.1 Labor market reforms

There is an extensive empirical literature on the macroeconomic effect of labor market reforms in advanced economies which typically focuses on long-term effects of labor market reforms (*e.g.* [Blanchard and Wolfers \(2000\)](#), [Nicoletti and Scarpetta \(2003\)](#), [Bassanini and Duval \(2009\)](#)). This literature has been completed by papers who also evaluate the dynamics effects from reforming the labor market using empirical models (*e.g.* [Bouis et al. \(2012\)](#), [Cette et al. \(2016\)](#) and [Duval and Furceri \(2018\)](#)) or using theoretical ones (*e.g.* [Arpaia et al. \(2007\)](#) and [Cacciatore et al. \(2016\)](#), [Cacciatore and Fiori \(2016\)](#)). There is a broad consensus in this literature about the high gains from reforming the labor market, but none of these papers examine (i) the role of financial frictions in the housing market on the effect of labor market reforms, (ii) the spillover effect of labor market reforms on mortgages. To fill this gap, we investigate the consequences of structural reforms when the housing market matters. [Fig.2.9](#) plots the transition dynamics between two steady states characterised by a permanent decrease by 1% in the replacement rate (b/w). We

also report on the right side the terminal steady state toward which the economy will converge in the long run for each model.

FIGURE 2.9: System response to a 1% decrease in unemployment insurance b^j/w^j in both the simple and the labor-adjusted collateral constraint models.



Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

As already documented in the literature, this reform implies a permanent rise in output and employment. A reduction in employment insurance rises the relative lifetime utility from being employed rather than unemployed, so the worker's employment value μ_t^j increases in response to this structural change in economic fundamentals. As a result, workers are more willing to find a job, there are more the matches as more vacancies are filled. The equilibrium wage clears the labor market through an increase in real wages for both types of workers.

Regarding financial aspects, the reform also leaks to the housing market but in a very different fashion between our two models. Under standard collateral constraints, the effect is rather modest as house prices increase by 0.5% while borrowing exhibits a negligible response. The value of borrowing surprisingly decreases, which translates into a reduction of housing goods after the reform. As a result, patient households are the main winner

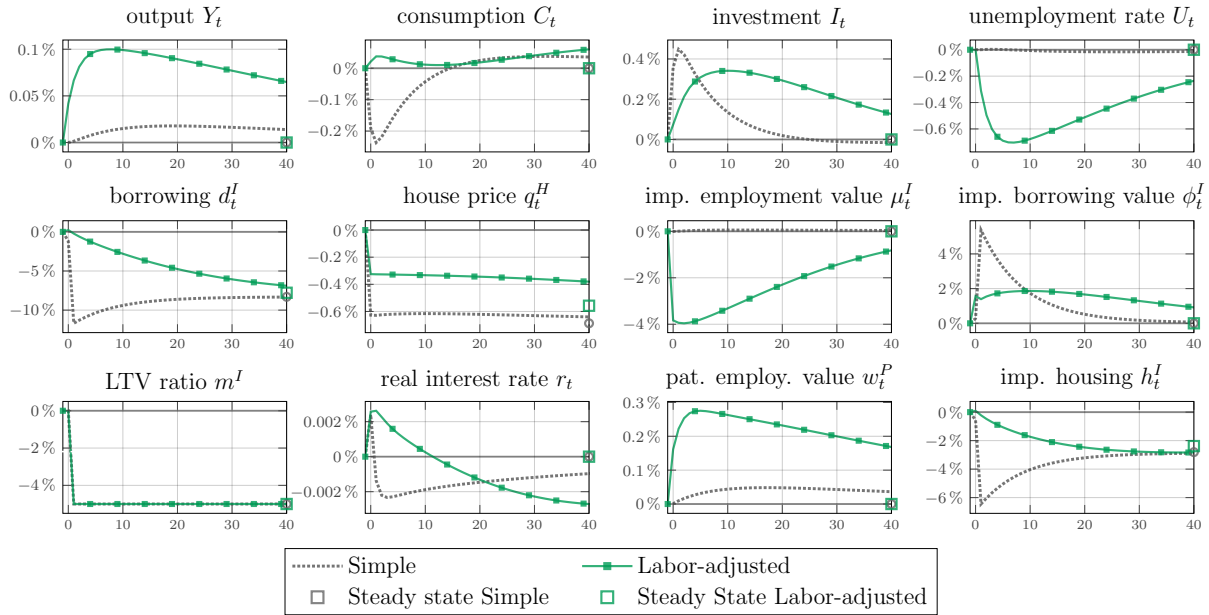
following the reform. In absence of explicit demand channels, long-term changes in the unemployment rate have no important real effects on the mortgage market. Conversely under labor adjusted collateral constraints, the rise in employment allows more family members of the patient household to get mortgages from financial intermediaries, the borrowing constraint eases which materialises through an increase in the borrowing value. Since they can borrow more, the demand for housing goods rises which makes house prices increase up to 1% in the long run. Because more resources are devoted to housing, investment and consumption take more time to reach the terminal steady state. Unlike the simple collateral constraint setup, the impatient household becomes the winner from implementing the reform as the number of dwellings substantially increases by 1% in the terminal steady state.

5.2 Macprudential policy

The model is also amenable for the analysis of macroprudential policy by changing the amount a household can borrow against his housing collateral when a mortgage is granted. Assuming that the Loan-to-Value ratio is determined by US institutional factors (*e.g.* a prudential authority), then we simulate a permanent change in the LTV ratio as in [Chen and Columba \(2016\)](#). [Fig.2.10](#) displays the transition dynamics from permanently reducing the LTV ratio by 5% under the two models of housing.

Both models find similar transition dynamics in line with [Chen and Columba \(2016\)](#): a tightening in mortgage origination reduces house prices, borrowing and the number of houses purchased by the impatient households. Patient households re-allocate their saving toward capital goods which boosts in turn investment. However for the rest of the variables, both models have rather different predictions. Concerning borrowing, macroprudential policy has unrealistic detrimental effects in the simple model as all contracts are renegotiated each quarter. In contrast in our setup, only workers newly matched in the labor market face tighter credit conditions. As a result, borrowing slowly adjusts to these tighter credit conditions at the same speed as inflows and outflows in employment.

FIGURE 2.10: System response to a 5% LTV tightening m^I in both the simple and the labor-adjusted collateral constraint models.



Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

Recall that under a labor-adjusted collateral constraint, house prices positively affect the value of employment. Thus the decline in house prices reduces both the utility gain of being in employment and the wage for impatient households. The labor demand for impatient rises and fuels the rise in employment of impatient households. As new matches are created, our constraint implies a higher demand in housing, which partially dampens the contractionary effect of macroprudential on house prices. To summarise, our model suggests the existence of sizable leakages of macroprudential policy on the labor market. In our setup, macroprudential policy deserts the value of employment and induces a rise in labor demand.

Conclusion

In this chapter, we have proposed to link the borrowing capacity of households to their employment situation on the labor market. Under this setup, new matches on the labor market translate into more mortgages while separation induces an exclusion from financial

markets for jobseekers. As a result, the LTV becomes endogenous by responding procyclically to employment fluctuations. We have shown that this device is empirically relevant and solves the anomalies of the standard collateral constraint model. We have also shown that this constraint poses important implications for economic policy. Structural reforms in the labor market induce more borrowing in the economy, while macroprudential policy tightening induces a pro-cyclical response of output and employment.

Chapter 3

Unemployment dynamics with financial business cycles

Introduction

We continue the ongoing analysis made in the previous chapter by studying the interaction between labor market frictions and financial business cycles. As we have seen before, the previous model is able to solve the anomalies of the standard collateral constraint by made the LTV endogenous to inflows and outflows in the labor market. However, the analysis which considers only financial frictions in the household's side and not for firms explained that the unemployment to be mostly driven by technological shock and an internal shock to the labor market.¹

This is in contrast with the literature studying unemployment fluctuations with financial frictions. In the literature studying this interaction we have two sides. On the one hand, many articles used capital as a collateral requirements *à la* Kiyotaki and Moore (1997) or the financial accelerator *à la* Bernanke et al. (1999) for firms and thus find an amplification mechanism which partially explained the surge in the unemployment rate (e.g Garin (2015), Zhang (2018), Christiano et al. (2011)). In this literature, the

¹The productivity shock drives 62% of the unemployment rate and the hiring shock 34% in the previous chapter.

unemployment rate is then driven by financial frictions. On the other hand, [Liu et al. \(2016\)](#) focus on collateral requirements based on real commercial estate for firms and find a similar amplification channel. Compared to the previous literature, housing movement driving the unemployment rate.

We have three objective in this chapter. The first objective is twofold. On one side, identify which collateral requirements for firms are useful to explain the interaction between labor and financial business cycle. On the other side, to know the main driving sources of unemployment fluctuations : *financial or housing fluctuations ?*

The second objective of this chapter is to reconcile the co-movement between housing price and consumption which is absent in our previous chapter due to the eviction effect.² As pointed by [Mian and Sufi \(2014\)](#), this positive correlation is referred to the *housing net worth channel* plays a key role in the surge of the unemployment rate during the great recession (2007-2009). Thus, for restoring this channel, we consider three types of agents namely household lenders and borrowers, and entrepreneurs as in [Iacoviello \(2015\)](#).³ The consideration of these three types of agent is a novelty in the collateral literature which interacts with unemployment fluctuations. In this literature, we have two opposites side. First, we have those who focus on the interaction between lenders and borrowers and suppose that firms are not subject to housing frictions (e.g [Sterk \(2015\)](#), [Andrés et al. \(2013\)](#)). Second, [Liu et al. \(2016\)](#) focus on lenders (i.e patient household) and constrained firms. Thus, we close the gap between these two sides of the literature by considering lenders, financial constrained households and entrepreneurs in our analysis.

Finally, the last objective of this chapter is to analyse the consequence of labor market deregulation for both collateral constrained agent. As a novelty, we add a long-term approach: macroprudential policy that moves LTV ratio for financial constrained agents have now long-run effect on output and unemployment rate.

²As examine by [Iacoviello \(2005\)](#), when we consider only one financial constrained agent, the movement between housing price and consumption turn to be negative who is at odds with the data (e.g.[Iacoviello \(2011\)](#); [Mian and Sufi \(2014\)](#)).

³For the rest of the chapter, we refer as financial constrained firms as entrepreneurs.

The main findings of this chapter reads as follows. First, we show that a collateral constraint which integrate both capital and real commercial estate for entrepreneurs outperform other specifications in terms of empirical performance. Second, we find that the unemployment rate is driven by both fluctuations in the housing price and the level of debt of entrepreneurs based on a variance decomposition and a counterfactual path of the housing price and the unemployment rate. Third, the integration of lenders with financial constrained households and entrepreneurs restores the *housing net worth channel*. Finally, the policy implications differs from the previous chapter. The deregulation of the labor market conduces to a great increase for the household debt and a substantial rise for the entrepreneurial debt. A tightening LTV ratio for households conduces to positive long-run effect while for entrepreneurs we have the opposite effect.

This chapter is organised as follows. Section 1 presents the theoretical framework with a collateral constraint for entrepreneurs integrating real commercial estate, capital and wage bills in advance. Section 2, introduces the data that we used and the estimation of the three different models i.e. one for each specific collateral constraint. The section 3, is dedicated to the empirical performance (marginal density, business cycles analysis and Impulse Response Function) of each model. The section 4 examines the main sources of the unemployment fluctuation based on a variance decomposition and a counterfactual path of the housing price and the unemployment rate. Finally, the last section 5 investigates how the presence of two financial constrained agents affects the results of a labor market reform and macroprudential policies tightening.

1 Theoretical framework

The economy is populated by a continuum of households of unit mass. As in [Kiyotaki and Moore \(1997\)](#), this continuum is composed by patient and impatient households. Impatient households are characterised by a lower discount rate than patient ones such that in equilibrium impatient are net borrowers and patient net lenders. Variables with the superscript P (I) refers to (im)patient households.

Following [Andolfatto \(1996\)](#); [Merz \(1995\)](#), the family provides perfect consumption insurance for its members which allow the latter to have the same consumption level between employed and unemployment family members. This consumption insurance applies for both types of households. Patient households work, consume and accumulate housing and physical capital. Except for the accumulation of physical capital, impatient households have the same spending. Due to some underlying frictions in financial markets, impatient households face a binding constraint in the amount of credit they can take. As in the chapter II, collateral requirements depends on the employment status of impatient households.⁴ Thus, this modeling device establishes the *first channel* between financial market (in the form of mortgage loans for impatient households) and the labor market. Under this setup, new matches in the labor market translate into the mortgage market through more loans.

The key innovation of the chapter compared to the previous one is to establish a *second channel* between financial market and the labor market by introducing credit-constraint entrepreneurs. As in [Iacoviello \(2015\)](#), entrepreneurs consume and produce goods by hiring both types of household, using real estate and combining capital produced by them and lenders. As for impatient households, they face a binding constraint in their borrowing amount.

We present our theoretical framework as follows. The subsection [1.1](#) describes the labor market with search and matching frictions *à la* [Mortensen and Pissarides \(1994\)](#). The following subsection [1.2](#) reports the main features of both types of household (i.e. lenders and borrowers). The subsection [1.3](#) details entrepreneurs behaviour and how their collateral constraint affects the unemployment dynamics. The subsection [1.4](#) presents the wage setting for both households and highlight the role of credit-constraint entrepreneurs in this process. Finally, the subsection [1.5](#) defines the general equilibrium of the economy.

⁴Recall that new mortgages are contracted when an impatient household family member finds a job and then classical collateral requirement such as the expected value of their real estate holdings are applied. For existing mortgage, debt is simply limited not to exceed the amount of the previous period and conditionally to keep the job.

1.1 Labor market

The labor market is subject to matching frictions *à la* Mortensen and Pissarides (1994). The total input of labor used by the entrepreneur is composed by labor provided by both households and reads as follows:

$$L_t = (l_t^P)^\lambda (l_t^I)^{1-\lambda}, \quad (3.1)$$

where $\lambda \in [0, 1]$ measures the relative size of lenders in the technology of firms. To hire a new worker, the entrepreneur must post a vacancy, denoted v_t^j with $j = \{P, I\}$ corresponding respectively to the patient and impatient household. A vacant position is matched with a job seeker, denoted u_t^j through a constant return to the scale matching technology $m_t^j = \psi (v_t^j)^\zeta (u_t^j)^{1-\zeta}$ with $\psi \in [0, 1]$ the matching efficiency parameter and $\zeta \in [0, 1]$ the elasticity of matches with respect to vacancies. As in Gertler et al. (2008), we suppose that unemployed workers who find a match goes to work immediately within the period. Thus, the pool of unemployed workers searching for a job at t , is given by the difference between unity and the number of unemployed workers at the end of period $t-1$: $(1 - \delta^L)l_{t-1}^j$.⁵ Considering that old matches are destroyed at a constant rate $\delta^L \in [0, 1]$, the pool of job seekers is given by:

$$u_t^j = 1 - (1 - \delta^L) l_{t-1}^j. \quad (3.2)$$

The probability that both a firm fills a vacancy and an unemployed worker finds a job are respectively $q_t^j \equiv m_t^j/v_t^j$ and $f_t^j \equiv m_t^j/u_t^j$. For an individual firm, the inflow of new gross hires in t is represented by $q_t^j v_t^j$ and for household by $f_t^j (1 - (1 - \delta^L) l_{t-1}^j)$.

Thus the evolution of employment from the demand side of the labor market is:

$$l_t^j = (1 - \delta^L) l_{t-1}^j + q_t^j v_t^j, \quad (3.3)$$

⁵All the agent in the economy including lenders and borrowers are normalised to unity.

while for the supply side, it is given by:

$$l_t^j = (1 - \delta^L) l_{t-1}^j + f_t^j (1 - (1 - \delta^L) l_{t-1}^j). \quad (3.4)$$

1.2 Households

Since variables and parameters indexed by I and P denote respectively impatient and patient households, non-indexed variables apply indistinctly to both types of households.

1.2.1 Impatient households

The impatient households maximise the following welfare index:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^I)^t \varepsilon_t^B \{ (1 - h^C) \log (c_t^I - h^C c_{t-1}^I) + \varepsilon_t^H j \log (h_t^I) \}, \quad (3.5)$$

where $\beta^I \in [0, 1)$ is their discount factor, c_t^I is the consumption of nondurable goods subject to external habits $h^C \in [0, 1]$, h_t^I is the holdings of housing and $j \in [0, 1]$ is consumption weight in the household's life time utility. The welfare index (Eq.3.5) is hit by two different shocks : ε_t^B is a preference shock and ε_t^H is an exogenous housing price shock.⁶ Each period, borrowers decide on the optimal amount of nondurable consumption, housing, debt and labor subject to the following budget constraint:

$$c_t^I + q_t^H \Delta h_t^I + r_{t-1} d_{t-1}^I + \xi_t^I = w_t^I l_t^I + (1 - l_t^I) b^I + d_t^I, \quad (3.6)$$

where Δ is the first difference operator. The left-hand side of the budget constraint displays consumption c_t^I , housing spending Δh_t^I with q_t^H the housing price, one period housing loan payment d_{t-1}^I at an interest rate r_{t-1} and adjustment costs stemming from changing the housing stock ξ_t^I . The right-hand side consists of income with w_t^I the wage per employed worker, b^I the compensation per unemployed family members and the amount of newly issued loans d_t^I .

⁶The preference shock ε_t^B can be viewed as a wedge that influences the short-term consumption-saving decision. For the second shock ε_t^H represents an exogenous shift in the household's taste for housing services.

As Sterk (2015), housing is subject to a convex adjustment cost :

$$\xi_t^a = \frac{\Theta^a}{2} h_{t-1}^a q_t^H \left(\frac{h_t^a}{h_{t-1}^a} - 1 \right)^2, \quad (3.7)$$

where $a = \{P, I, E\}$. This cost applies for each agent in the model (households P and I and entrepreneurs E). This formulation helps to replicate a hump shape response of the aggregate housing stock and for housing price. From a microeconomic standpoint, this cost captures some transaction costs from purchasing or selling dwellings.

As in the chapter II, we consider that the fraction of debt that is refinanced depends on the number of new employees in the economy i.e. the probability to find a job times the number of unemployed workers $f_t^I (1 - (1 - \delta^L) l_{t-1}^I)$.⁷ For existing mortgages, only employees who keep their jobs can roll over their existing mortgage contract, denoted $(1 - \delta^L) l_{t-1}^I$. Thus, our collateral constraint reads as follows:

$$d_t^I \leq (1 - \delta^L) l_{t-1}^I d_{t-1}^I + \mathbb{E}_t \{q_{t+1}^H\} \varepsilon_t^M f_t^I (1 - (1 - \delta^L) l_{t-1}^I) m^I h_t^I, \quad (3.8)$$

where ε_t^M is an exogenous shock on the Loan To Value (LTV hereafter) ratio m^I .

The representative borrower chooses the optimal amount of consumption, debt, housing, and labor by maximising his utility (Eq.3.5) subject to his budget constraint (Eq.3.6), his collateral constraint (Eq.3.8) and the flow of labor (Eq.3.4). Thus, the optimal consumption choice gives the marginal utility of consumption denoted by λ_t^I :

$$\lambda_t^I = \varepsilon_t^B (1 - h^C) (c_t^I - h^C c_{t-1}^I)^{-1}. \quad (3.9)$$

Letting $\beta_{t,t+1}^I = \beta^I \mathbb{E}_t \left\{ \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} \frac{(c_t^I - h^C c_{t-1}^I)}{(c_{t+1}^I - h^C c_t^I)} \right\}$ denote the borrower's stochastic discount factor and ϕ_t^I the Lagrangian multiplier on the collateral constraint normalised by the marginal utility of consumption, the Euler condition for borrowers is given by:

$$1 - \phi_t^I = \mathbb{E}_t \beta_{t,t+1}^I [r_t - \phi_{t+1}^I (1 - \delta^L) l_t^I]. \quad (3.10)$$

⁷Or, equivalently the number of new matches in the economy m_t^I .

In this expression, the Lagrangian multiplier ϕ_t^I can be interpreted as the lifetime utility stemming from borrowing for a home purchase. As standard in the collateral literature, this variable introduces a wedge with respect to the patient Euler equation. Then, a rise in the borrowing value i.e. $\Delta\phi_t^I > 0$ implies that the borrowers increase the fraction of his income spent for an home purchase, to the detriment of his current consumption. Due to the specificity of our collateral constraint, a rise in employment rate l_t^I drives current consumption upward.⁸

The first-order condition with respect to housing is given by:⁹

$$q_t^H \left(1 + \Theta^I \frac{\Delta h_t^I}{h_{t-1}^I} \right) = \varepsilon_t^H j \frac{(c_t^I - h^C c_{t-1}^I)}{(1-h^C)h_t^I} + \mathbb{E}_t \left\{ \beta_{t,t+1}^I q_{t+1}^H \left(1 + \Theta^I \frac{h_{t+1}^I}{(h_t^I)^2} \Delta h_{t+1}^I \right) \right\} + \mathbb{E}_t \left\{ q_{t+1}^H \right\} \varepsilon_t^M \phi_t^I m^I f_t^I (1 - (1 - \delta^L) l_{t-1}^I) \quad (3.11)$$

This equation determines the housing price. The first line of the hand right side of this equation is composed by the lifetime utility gain from a marginal unit of housing, plus the future gain from reselling the house at the next period including the expected adjustment cost on housing. The second line of the hand right side is the lifetime utility gain for new employees allowed to borrow on financial markets.

Finally, the first order condition for labor reads as follows:

$$\mu_t^{I,W} = w_t^I - b^I + (1 - \delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^I \mu_{t+1}^{I,W} (1 - f_{t+1}^I) \right\} + (1 - \delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^I \phi_{t+1}^I \frac{(d_t^I - d_{t+1}^I)}{w_{t+1}^I} \right\}, \quad (3.12)$$

where $\mu_t^{I,W}$ stands for the marginal utility of a match.¹⁰ The first line is standard with respect to the matching literature with the net pecuniary gain of being employed (w_t^I) rather than being unemployed (b^I) plus the continuation value if the worker remains in employment. For the second line, due to the collateral constraint, the continuation value of the match includes the roll over of the mortgage contract.

⁸See for more details the previous chapter in the theoretical framework.

⁹For the lack of simplicity the following equation does not include terms which disappear at a first order approximation. See Appendix D for further details.

¹⁰ $\mu_t^{I,W}$ correspond to the Lagrangian multiplier associated with the labor market law of motion normalised by the marginal utility of consumption λ_t^I .

1.2.2 Patient household

Patient household discounts the future more weakly than impatient ones so their discount factor satisfies $\beta^P > \beta^I$.¹¹ They maximise the following welfare index:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \varepsilon_t^B \{ (1 - h^C) \log (c_t^P - h^C c_{t-1}^P) + \varepsilon_t^H j \log (h_t^P) \}, \quad (3.13)$$

subject to the budget constraint:

$$c_t^P + q_t^H \Delta h_t^P + d_t^P + T_t^P + \xi_t^P = w_t^P l_t^P + (1 - l_t^P) b^P + r_{t-1} d_{t-1}^P + \Xi_t^P. \quad (3.14)$$

As for borrowers, the left side displays spending with consumption (c_t^P), housing (h_t^P), deposits (d_t^P), adjustment costs on housing stock (ξ_t^P) and taxes (T_t^P). The right side displays resources from labor (w_t^P if they are employed and otherwise b^P), interest on deposits ($r_{t-1} d_{t-1}^P$) and the net benefits to provide capital services to entrepreneurs (Ξ_t^P). The representative lenders maximise his utility (Eq.3.13) to choose the optimal amount of consumption, deposits, housing and labor subject to his budget constraint (Eq.3.14) and the flow of labor (3.1). Then the First Order Condition with respect to c_t^P gives the marginal utility of consumption denoted by (λ_t^P):

$$\lambda_t^P = \varepsilon_t^B (1 - h^C) (c_t^P - h^C c_{t-1}^P)^{-1}. \quad (3.15)$$

Letting $\beta_{t,t+1}^P = \beta^P \mathbb{E}_t \left\{ \frac{\varepsilon_{t+1}^B (c_t^P - h^C c_{t-1}^P)}{\varepsilon_t^B (c_{t+1}^P - h^C c_t^P)} \right\}$ denote the lenders stochastic discount factor, the optimal choice for deposit provides a standard Euler condition for the patient household:

$$\mathbb{E}_t \{ \beta_{t,t+1}^P \} r_t = 1. \quad (3.16)$$

¹¹This restriction on discount factors implies that the Lagrangian multipliers ϕ_t^I on the collateral constraint is always positive and thus the constraint holds to equality in the neighborhood of the steady-state. Our calibration for the gap between discount factors ensures that $\phi_t^I > 0$ which allows a linear approximation to be accurate. In the same way, the discount factor for entrepreneurs $\beta^P > \beta^E$ satisfies this property for the Lagrangian multiplier ϕ_t^E in the subsection 1.3.

The optimal stock of housing is given by:¹²

$$\varepsilon_t^H j \frac{(c_t^P - h^C c_{t-1}^P)}{(1 - h^C) h_t^P} - q_t^H \left(1 + \Theta^P \frac{\Delta h_t^P}{h_{t-1}^P} \right) = -\mathbb{E}_t \left\{ \beta_{t,t+1}^P q_{t+1}^H \left(1 + \Theta^P \frac{h_{t+1}^P}{(h_t^P)^2} \Delta h_{t+1}^P \right) \right\}. \quad (3.17)$$

where the left-hand side denotes the current net gain in consumption equivalents from housing purchase.

The marginal utility to have a new match for patient household denoted by $\mu_t^{P,W}$ is:¹³

$$\mu_t^{P,W} = w_t^P - b^P + (1 - \delta^L) \mathbb{E}_t \left\{ \mu_{t+1}^{P,W} \beta_{t,t+1}^P (1 - f_{t+1}^P) \right\}. \quad (3.18)$$

In this case, this equation is standard in the matching literature compared with the borrowers one (Eq.3.12).

Finally, concerning capital services, we adopt the same framework as [Iacoviello \(2015\)](#) where Ξ_t^P corresponds to:

$$\Xi_t^P = z_t K_{t-1}^P v_t^P - \left(\frac{K_t^P - (1 - \delta_t^P) K_{t-1}^P}{\varepsilon_t^I} \right) - \frac{\phi^P (K_t^P - K_{t-1}^P)^2}{2 K^P}. \quad (3.19)$$

The first part is the capital services provide to entrepreneurs at the rental rates z_t with K_t^P the physical capital own by patient households and v_t^P the utilisation rate. To have these resources, they need to replace the old capital which is depreciated at δ_t^P and subject to an investment-specific technology shock ε_t^I and pay adjustment costs with $\phi^P > 0$ the cost of changing the stock and K^P the steady state of capital.¹⁴

Then the optimal condition for lenders to provide capital services is :

$$\frac{1}{\varepsilon_t^I} + \phi^P \frac{\Delta K_t^P}{K^P} = \mathbb{E}_t \beta_{t,t+1}^P \left(\frac{(1 - \delta_{t+1}^P)}{\varepsilon_{t+1}^I} + z_{t+1} v_{t+1}^P + \phi^P \frac{\Delta K_{t+1}^P}{K^P} \right). \quad (3.20)$$

¹²For the lack of simplicity the following equation does not include terms which disappear at a first order approximation. See Appendix D for further details.

¹³ $\mu_t^{P,W}$ corresponds to the Lagrangian multiplier associated with the labor market law of motion normalised by the marginal utility of consumption.

¹⁴When a variable appears without its temporal index t , this is its steady-state value.

The choice for the utilisation rate depends on the functional form for the depreciation rate δ_t^P is given by:

$$\delta_t^P = \delta^K + \left(\frac{1}{\beta^P} + 1 - \delta^K \right) \left(\frac{1}{2} \left(\frac{\psi^P}{1 - \psi^P} \right) (v_t^P)^2 + \frac{(1 - 2\psi^P)}{1 - \psi^P} v_t^P + \frac{1}{2} \left(\frac{\psi^P}{1 - \psi^P} \right) - 1 \right). \quad (3.21)$$

The parameter $\psi^P \in [0, 1]$ measures the curvature of the utilisation rate function. When $\psi^P = 1$, δ_t^P stays constant over time and with ψ^P approaching 0 the depreciation rate is very sensitive to the utilisation rate.

Considering the flow of capital services provided by patient households (Eq. 3.19) and the functional form of the depreciation rate (Eq.3.21), the optimal utilisation rate is given by:

$$z_t = \left(\frac{1}{\beta^P} + 1 - \delta^K \right) \left(\left(\frac{\psi^P}{1 - \psi^P} \right) v_t^P + \frac{(1 - 2\psi^P)}{1 - \psi^P} \right). \quad (3.22)$$

1.3 Entrepreneurs

The entrepreneur discounts the future more heavily than patient ones so their discount factor satisfies $\beta^E < \beta^P$. They maximise the following welfare index:¹⁵

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^E)^t (1 - h^C) \log (c_t^E - h^C c_{t-1}^E), \quad (3.23)$$

where c_t^E is their consumption subject to external habit h^C in the same way as households. Each period, entrepreneur decide on the optimal amount of consumption, real estate, capital, debt and labor subject to the following budget constraint:

$$\begin{aligned} c_t^E + q_t^H \Delta h_t^E + \xi_t^E + \sum_{j=P,I} \left(w_t^j l_t^j + \frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j \right) + z_t v_t^P K_{t-1}^P + \Xi_t^E \\ = Y_t + d_t^E - r_{t-1} d_{t-1}^E \end{aligned} \quad (3.24)$$

¹⁵To capture entrepreneurs risk aversion, we assume a log utility welfare for them. As in [Iacoviello \(2005\)](#), we choose this specification more than infinitely lived and risk neutral entrepreneurs or where a constant fraction of entrepreneurs dies each period. See [Carlstrom and Fuerst \(2001\)](#) for more details.

The first line of the budget constraint is composed by consumption of non-durable goods (c_t^E), real estate spending (Δh_t^E) at the housing price (q_t^H) including adjustment costs for changing their housing stock (ξ_t^E), costs associated with the labor force i.e. wage payments ($w_t^j l_t^j$) and quadratic hiring costs as in [Gertler et al. \(2008\)](#) ($\frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j$), renting capital from lenders ($z_t v_t^P K_{t-1}^P$) and finally the net benefits to produce its own capital stock (Ξ_t^E) which for simplicity is described at the end of the subsection.¹⁶

The second line of the budget constraint is composed by the production of the final good (Y_t) and the amount of newly issue loans (d_t^E) net of one period loans payments (d_{t-1}^E) at an interest rate (r_{t-1}). They produce the final good Y_t according to a Cobb-Douglas constant return-to-scale technology that combines labor from both types of household (l_t^P and l_t^I as described in [Eq.3.1](#)), capital provides from lenders ($K_{t-1}^P v_t^P$) and produces by them ($K_{t-1}^E v_t^E$) and finally real estate (h_{t-1}^E). This function reads as follows:

$$Y_t = \varepsilon_t^Z (L_t)^{1-\alpha-\nu} (v_t^E K_{t-1}^E)^{\mu\alpha} (v_t^P K_{t-1}^P)^{(1-\mu)\alpha} (h_{t-1}^E)^\nu, \quad (3.25)$$

where ε_t^Z is the exogenous Total Factor Productivity (TFP hereafter) shock, α is the share of capital used in the production with $\mu \in [0, 1]$ the share of their own capital production (respectively $1 - \mu$ from lenders capital production) and $\nu \in [0, 1]$ the part of real estate.

Turning to the debt contract, entrepreneurs are limited in their capacity to borrow from lenders. They can borrow a fixed part denoted by $m^K \in [0, 1]$ of their own capital stock K_t^E and $m^H \in [0, 1]$ of their real estate stocks h_t^E . As in [Neumeyer and Perri \(2005\)](#), we suppose that the entrepreneur must pay full wages in advance, represented by $m^W \in [0, 1]$. Following [Iacoviello \(2015\)](#), we introduce an *ad hoc* persistence mechanism denoted by $\chi^E \in [0, 1]$. This term allows for slow adjustment over time of the borrowing constraint, to capture the idea that in practice entrepreneurs do not readjust borrowing limits every

¹⁶As common practice in the collateral literature, we use housing price more than the land price for real estate spending. According to [Neri and Iacoviello \(2010\)](#), these two-time series have closely statistical properties.

quarter.¹⁷ Then, the collateral constraint of the entrepreneur reads as:

$$d_t^E \leq \chi^E d_{t-1}^E + (1 - \chi^E) \varepsilon_t^{ME} \left(m^H \frac{\mathbb{E}_t \{q_{t+1}^H\}}{r_t} h_t^E + m^K K_t^E - m^W \sum_{j=P,I} w_{j,t} l_{j,t} \right), \quad (3.26)$$

where ε_t^{ME} is an exogenous shock capturing exogenous changes in the collateralisation technology of financial intermediaries or some changes in the search intensity of firms for new credits.

Thus, the optimal consumption choice gives the marginal utility of consumption denoted by λ_t^E :

$$\lambda_t^E = (1 - h^C) (c_t^E - h^C c_{t-1}^E)^{-1}. \quad (3.27)$$

Letting $\mathbb{E}_t \beta_{t,t+1}^E = \beta^E \mathbb{E}_t \frac{\{\lambda_{t+1}^E\}}{\lambda_t^E}$ denote the entrepreneur's stochastic discount factor and ϕ_t^E the Lagrangian multiplier on the collateral constraint normalised by the marginal utility of consumption, the Euler condition for entrepreneurs is given by:

$$1 - \phi_t^E = \mathbb{E}_t \beta_{t,t+1}^E \{r_t - \chi^E \phi_{t+1}^E\}. \quad (3.28)$$

In a log-linearised form of the Euler equation, variations in consumption are negatively related to changes in the current shadow value of borrowing ϕ_t^E . Then, for $\Delta \phi_t^E > 0$ the entrepreneur increases the fraction of his income spent for estate purchase (h_t^E), physical capital (K_t^E) and labor (l_t^j) to the detriment of his current consumption.

For the optimal choice of commercial estates we have:¹⁸

$$q_t^H \left(1 + \Theta^E \frac{\Delta h_t^E}{h_{t-1}^E} \right) = \mathbb{E}_t \beta_{t,t+1}^E \left\{ \nu \frac{Y_{t+1}}{h_t^E} + q_{t+1}^H \left(1 + \Theta^E \frac{h_{t+1}^E}{(h_t^E)^2} \Delta h_{t+1}^E \right) \right\} + (1 - \chi^E) \varepsilon_t^{ME} \phi_t^E m^H \mathbb{E}_t \{q_{t+1}^H\} (r_t)^{-1}. \quad (3.29)$$

Turning to hiring decisions, firms choose the optimal level of vacancies v_t^j and labor l_t^j for each type of households subject to the employment law of motion (Eq. 3.3). Let $x_t^j = \frac{q_t^j v_t^j}{l_{t-1}^j}$

¹⁷Formally, χ^E captures existing mortgage and $1 - \chi^E$ new mortgages contract by entrepreneurs.

¹⁸For the lack of simplicity the following equation does not include terms which disappear at a first order approximation. See Appendix D for further details.

be the hiring rate, the job creation condition for both types of households is given by:

$$\kappa^P x_t^P = \lambda (1 - \alpha - \nu) \frac{Y_t}{l_t^P} - w_t^P \Omega_t + \kappa^P \mathbb{E}_t \beta_{t,t+1}^E \left\{ .5 (x_{t+1}^P)^2 + (1 - \delta^L) x_{t+1}^P \right\}, \quad (3.30)$$

$$\kappa^I x_t^I = (1 - \lambda) (1 - \alpha - \nu) \frac{Y_t}{l_t^I} - w_t^I \Omega_t + \kappa^I \mathbb{E}_t \beta_{t,t+1}^E \left\{ .5 (x_{t+1}^I)^2 + (1 - \delta^L) x_{t+1}^I \right\}, \quad (3.31)$$

where the first equation concerns lenders and the second borrowers. The term $\Omega_t = (1 + (1 - \chi^E) \varepsilon_t^{ME} \phi_t^E m^W)$ corresponds to the intertemporal wedge induced by the credit constraint.

The optimal amount of physical capital from patient households reads as follows:

$$z_t = \alpha (1 - \mu) \frac{Y_t}{K_{t-1}^P v_t^P}, \quad (3.32)$$

where the rental rate equals the marginal product of using this type of capital.

For their own production of capital denoted by Ξ_t^E in the budget constraint, this corresponds to:

$$\Xi_t^E = \left(\frac{K_t^E - (1 - \delta_t^E) K_{t-1}^E}{\varepsilon_t^I} \right) - \frac{\phi^E (\Delta K_t^E)^2}{2 K^E}, \quad (3.33)$$

where as for lenders they have to pay quadratic adjustment costs to adjust their capital stock with $\phi^E > 0$ the degree of adjustment and the depreciation rate δ_t^E which also depends on the utilisation rate such that:

$$\delta_t^E = \delta^K + \frac{\alpha \mu Y}{K^E} \left(\frac{1}{2} \left(\frac{\psi^E}{1 - \psi^E} \right) (v_t^E)^2 + \frac{(1 - 2\psi^E)}{1 - \psi^E} v_t^E + \frac{1}{2} \left(\frac{\psi^E}{1 - \psi^E} \right) - 1 \right), \quad (3.34)$$

with $\psi^E \in [0, 1]$ the curvature of the utilisation rates function. Then the optimal stock of capital produced by entrepreneurs is given by:

$$\frac{1}{\varepsilon_t^I} + \phi^K \frac{\Delta K_t^E}{K^E} = \mathbb{E}_t \left\{ \beta_{t,t+1}^E \left(\mu \alpha \frac{Y_{t+1}}{K_t^E} + \frac{(1 - \delta_{t+1}^E)}{\varepsilon_{t+1}^I} + \phi^K \frac{\Delta K_{t+1}^E}{K^E} \right) \right\} + \varepsilon_t^{ME} \phi_t^E m^K (1 - \chi^E). \quad (3.35)$$

The optimal utilisation rate is defined as:

$$\frac{\alpha\mu Y_t}{K_{t-1}^E v_t^E} = \frac{\alpha\mu Y}{K^E} \left(\left(\frac{\psi^E}{1-\psi^E} \right) v_t^E + \frac{(1-2\psi^E)}{1-\psi^E} \right). \quad (3.36)$$

Thus, as the first-order conditions show, credit constraints as measured by the multiplier on the borrowing constraint ϕ_t^E introduce a wedge between the cost of factors and their marginal product, thus acting as a tax on the demand for credit and the demand for the factors of production. The wedge is present in the Euler equation (Eq.3.28), in the real estate demand (Eq.3.29), in job creation condition for both types of household (Eq.3.31 and Eq.3.30) and for investment produced by entrepreneurs (Eq.3.35). Note that in all equations the wedge is intertemporal.¹⁹

1.4 Wage setting

The wage is set according to a Nash bargaining scheme which splits the surplus between workers and employers. For each type j , the Nash bargaining solution is determined by the following program,

$$\arg \max_{\{w_{j,t}\}} \left(\mu_t^{j,W} \right)^\eta \left(\mu_t^{j,L} \right)^{1-\eta}, \quad (3.37)$$

where $\eta \in [0, 1]$ is the exogenous bargaining power of workers. The FOC of the Nash bargaining wage is given by:

$$\omega_t \mu_t^{j,L} = (1 - \omega_t) \mu_t^{j,W} \quad (3.38)$$

where ω_t is defined as:

$$\omega_t = \frac{\eta}{\eta + (1 - \eta) (1 + (1 - \chi^E) \varepsilon_t^{ME} \phi_t^E m^W)}. \quad (3.39)$$

After some algebraic manipulation, we obtain the wage for patient household :²⁰

$$w_t^P = \frac{\omega_t^O \left\{ \frac{(1-\alpha-\nu)\lambda Y_t}{l_t^P} + \mathbb{E}_t \beta_{t,t+1}^E \frac{\kappa_P}{2} (x_{t+1}^P)^2 \right\} + (1 - \eta) b^P}{+\kappa^P (1 - \delta^L) \mathbb{E}_t \left\{ x_{t+1}^P [\beta_{t,t+1}^E \omega_t^O - \beta_{t,t+1}^P \omega_{t+1}^O (1 - f_{t+1}^P)] \right\}}. \quad (3.40)$$

¹⁹Compared to a walrasian labor market as in [Iacoviello \(2015\)](#), the wedge in the labor demand is now intertemporal and not intratemporal.

²⁰See appendix D for more details.

where ω_t^O is the endogenous bargaining power due to the influence of the wage bill in advance in the Nash bargaining given by:

$$\omega_t^O = \frac{\eta}{1 + (1 - \chi^E) \varepsilon_t^{ME} \phi_t^E m^W}. \quad (3.41)$$

The hypothesis of the wage bill in advance made the household's bargaining power endogenous (ω_t^O) while keeping fix the entrepreneur ones ($1 - \eta$). During period of rising entrepreneurs borrowing value, $\frac{\partial \omega_t^O}{\partial \phi_t^E} < 0$, the effective bargaining power of workers declines (respectively when the borrowing value rises, bargaining power increases). Intuitively, when the entrepreneur increases the fraction of his income spent for production to the detriment of his own consumption ($\phi_t^E > 0$), workers capture a smaller fraction of the joint surplus of the employment relationship, and a larger fraction when they rise. The same holds for a credit squeeze (ε_t^M). This effect becomes higher when the number of entrepreneurs taking a new mortgage each period is high ($1 - \chi^E$) as for the wage bill in advance parameter (m^W).

Even if the hypothesis of the wage bill in advance is not accurate, the presence of credit constraints for entrepreneurs plays a role in the determination of wages underlined by the stochastic discount factor of entrepreneurs ($\beta_{t,t+1}^E$). Then when entrepreneurs discount the future more heavily i.e. $\beta_{t,t+1}^E$ decreases, the expected continuation value to keep the worker represented by the hiring rate becomes less important in the wage determination. For the second line in Eq.3.40, we have a utility gap between entrepreneurs and lenders adjusted by the endogenous bargaining power (i.e. $\beta_{t,t+1}^E \omega_t^O - \beta_{t,t+1}^P \omega_{t+1}^O$).

Concerning the wage for impatient households, they read as follows:

$$\begin{aligned} \omega_t^O & \left(\frac{(1-\alpha-\nu)(1-\lambda)Y_t}{l_t^I} + \mathbb{E}_t \left\{ \beta_{t,t+1}^E \frac{\kappa_I}{2} (x_{t+1}^I)^2 \right\} \right) + (1 - \eta) b^I \\ w_t^I = & + (1 - \delta^L) \mathbb{E}_t \left\{ x_{t+1}^I (\omega_t^O \beta_{t,t+1}^E - \beta_{t,t+1}^I \omega_{t+1}^O (1 - f_{t+1}^I)) \right\} . \quad (3.42) \\ & + (1 - \delta^L) (1 - \eta) \mathbb{E}_t \beta_{t,t+1}^I \phi_{t+1}^I \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \end{aligned}$$

As for patient ones, credit constraints ($\beta_{t,t+1}^E$) and wage bill in advance (ω_t^O) play the same role in the wage setting. Due to the specificity of our collateral constraint (Eq.3.8), the

third line differs from the patient wage. As in the previous chapter, this term depends positively on the future of debt and negatively on the anticipation of future employment.

1.5 General equilibrium

Market clearing is implied by Walras's law by aggregating the budget constraints of both types of households and entrepreneurs. Due to the absence of explicit residential production as in [Neri and Iacoviello \(2010\)](#), the supply of dwellings is simply normalised to one as in [Iacoviello \(2015\)](#) and thus the housing market clearing reads as follows:

$$h_t^P + h_t^E + h_t^I = 1. \quad (3.43)$$

In the absence of explicit financial frictions on financial markets, we assume that patient households grant loans to impatient and entrepreneurs at no cost using their own deposits as liabilities. Then, the equilibrium condition on the mortgage loans is:

$$d_t^P = d_t^I + d_t^E \quad (3.44)$$

Taxes finance the transfer for both types of workers i.e. $(1 - l_t^P) b^P + (1 - l_t^I) b^I$ and government spending, denoted G_t . As in [Smets and Wouters \(2007\)](#), we suppose that public spending is exogenous, $G_t = g^Y \varepsilon_t^G$ where g^Y is the fixed spending relative to GDP ratio and ε_t^G is an exogenous process allowing the government to transitory deviate from g^Y . Then, the balance sheet for the government reads as follows:

$$T_t^P = (1 - l_t^P) b^P + (1 - l_t^I) b^I + g^Y \varepsilon_t^G, \quad (3.45)$$

where T_t^P is the lump-sum tax collected to patient households (Eq.3.14).

Then using these relationships, the aggregate GDP is defined as:

$$Y_t = C_t + g^Y \varepsilon_t^G + I_t + \sum_{j=P,I} \left(\frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j \right), \quad (3.46)$$

with $I_t = \frac{K_t^P - (1 - \delta_t^P)K_{t-1}^P + K_t^E - (1 - \delta_t^E)K_{t-1}^E}{\varepsilon_t^I}$ the total of investment in physical capital from patient households and entrepreneurs and $C_t = c_t^E + c_t^P + c_t^I$ the aggregate consumption.²¹

2 Data and estimation

As for the previous chapter, we use Bayesian methods as described by [An and Schorfheide \(2007\)](#) and [Smets and Wouters \(2007\)](#) to estimate the model parameters.²²

In this chapter, we compare our collateral constraint to two different benchmark models that are typically employed in the literature of collateral constraints combined with labor market frictions. [Tab.3.1](#) summarises the three models estimated using the same sample.

The first model referred to as *Capital* in the first row of [Tab.3.1](#), refers to a model where the entrepreneur does not use commercial estate in their production function (i.e. $v = 0$) and naturally this component is absent in the borrowing constraint (i.e. $m^H = 0$). We also drop the hypothesis of the wage bill in advance (i.e. $m^W = 0$) and the *ad hoc* persistence parameter à la [Iacoviello \(2015\)](#) (i.e. $\chi^E = 0$).²³ Thus, in this model, we have the collateral constraint the most used in the collateral literature.

In the second model, we restore the wage bill in advance ($m^W = 1$) and the ad hoc persistence mechanism (χ^E). Thus, we simply call this model as *Capital and WB*.

Finally, the last model estimated is the one presented in theoretical framework section. Due to the presence of wage bill in advance, capital, real commercial estimate in the collateral requirements we called them *Both requirements*.

²¹For simplicity we do not make appear the adjustment cost on capital and housing since at a first order approximation they disappear.

²²We use 100000 iterations of the Random Hastings algorithm on three different chains.

²³In our estimation process, we try to integrate the ad-hoc persistence mechanism in the collateral constraint χ^E for this specification in order to have a formal comparison between these different features. However, the introduction of χ^E does not respect the Blanchard-Khan conditions in this setup. However, during the estimation with the wage bill in advance and capital constraint \mathcal{M}_2 the estimation of this parameter is very low as shown by the [Tab.3.3](#). Then, it would be interesting to compare these three features without this parameter in each of them.

Thus, the goal of these three estimations is to identify which is the best approach considering the diversity in the literature for collateral requirements and its consequence for unemployment fluctuations.

TABLE 3.1: Description of the estimated model featuring different estimation procedure for entrepreneur LTV ratio.

Symbol	Collateral assets	Collateral constraint equation
\mathcal{M}_1	Capital	$d_t^E \leq \varepsilon_t^{ME} m^K K_t^E$
\mathcal{M}_2	Capital and WB	$d_t^E \leq \chi^E d_{t-1}^E + (1 - \chi^E) \varepsilon_t^{ME} \left(m^K K_t^E - m^W \sum_{j=P,I} w_t^j l_t^j \right)$
\mathcal{M}_3	Both requirements	$d_t^E \leq \chi^E d_{t-1}^E + (1 - \chi^E) \varepsilon_t^{ME} \left(m^H \mathbb{E}_t \left\{ \frac{q_{t+1}^H}{r_t} \right\} h_t^E + m^K K_t^E - m^W \sum_{j=P,I} w_t^j l_t^j \right)$

2.1 Data

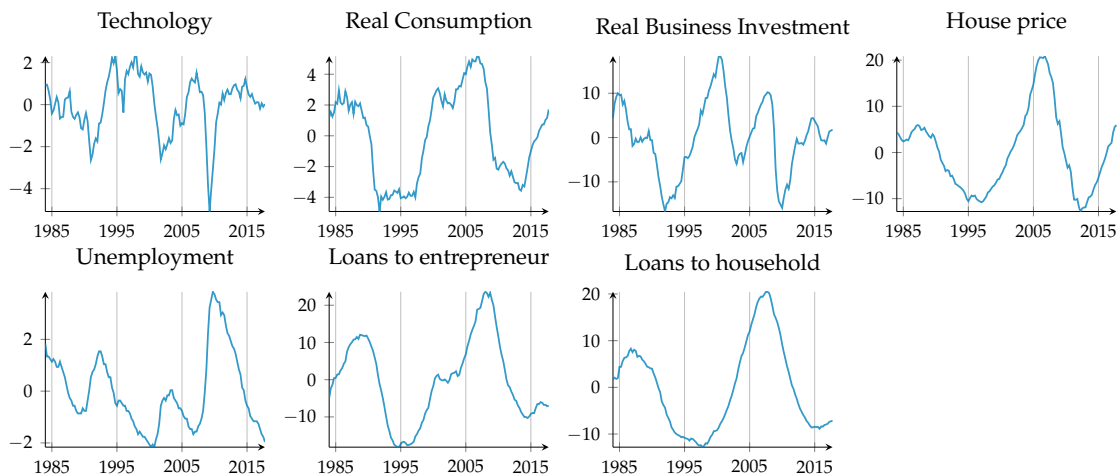
We fit the DSGE model to US time series data from 1984Q1 to 2017Q4. Following usual practice, we use as many shocks as observable variables in order to obtain the smoothing of filtered disturbances. Then, we use seven time series with our seven exogenous shock (ε_t^Z , ε_t^H , ε_t^B , ε_t^I , ε_t^G , ε_t^{ME} and ε_t^M). Our data samples include housing price, total factor productivity, consumption, investment, unemployment rate and loans for both entrepreneurs and households. Our data set is closed from [Iacoviello \(2015\)](#) except for the unemployment rate. Appendix A provides the data sources.

Concerning the transformation of series, the point is to map non-stationary data to a stationary model. Except for unemployment, all data exhibits a trend and are made stationary in two steps. First, we deflated the sample using the GDP deflator. Second, data are taken in logs and we use a quadratic trend to detrend them. The unemployment rate is simply demeaned. The [Fig.3.1](#) plots the transformed series.

2.2 Calibration and prior distributions

Our calibration is reported in [Tab.3.2](#). Calibrated parameters are typically those which are weakly identified. Since the calibration of the discount factor for borrowers and entrepreneurs diverges in the literature, typically between 0.94 and 0.985 we choose a

FIGURE 3.1: Observable variables used in the estimation



Notes: Data used in estimation and expressed in pourcentage deviation from their quadratic trend (except for the unemployment which is simply demeaned). The model parameters are estimating using data from 1990Q1 to 2017Q4. The 1984-1989 period is used to initialise the Kalman filter.

midpoint with $\beta^E = \beta^I = .97$.²⁴ The housing preference parameters is fixed to $j = 0.095$ in order to reach a housing wealth of 130% of annual output ($\frac{q^H(h^P+h^I)}{4Y} = 1.3$) in each of the model considered. However, for the baseline model (\mathcal{M}_3) the steady-state ratio of commercial real estate to annual output ($\frac{q^H h^E}{4Y}$) is sensitive to m^{HE} and in a less magnitude of order to χ^E .²⁵ Then, we compute this steady-state ratio for this estimation.²⁶

For the calibration of the different input used in the production function we set the share of lenders to 75% (i.e. $\lambda = 0.75$) and commercial estate to 3% (i.e. $v = 0.03$) as in [Iacoviello \(2015\)](#). Then for capital we used a standard calibration with $\alpha = 0.3$.²⁷ As common practice in the literature, we calibrate the depreciation rate of capital to $\delta^K = 0.035$ and the ratio of public spending to GDP $g^Y = .19$.

For the labor market, we fix the steady state of the finding rate to 70% ($f = 0.7$) as in [Shimer \(2005\)](#) and the exogenous separation by $\delta^L = 0.1$ to match with the average job

²⁴For example [Iacoviello \(2015\)](#) chooses $\beta^E = \beta^I = .94$, [Garin \(2015\)](#) and [Liu et al. \(2016\)](#) consider $\beta^E = .985$ and [Sterk \(2015\)](#) selects the impatient discount factor to $\beta^I = 0.9899$.

²⁵For example, in the calibrated version with $m^{HE} = .9$ and suppose that $\chi^E = 0$, we have a steady state ratio of 0.74 and for $\chi^E = 1$ we have a steady-state ratio of 0.2425

²⁶This steady-state ratio is naturally absent in the two others collateral specifications (\mathcal{M}_1 and \mathcal{M}_2) since entrepreneurs do not use commercial estate as a factor of production.

²⁷For the two other specifications of the collateral constraint since $v = 0$, we have $\alpha = 0.33$ in order to keep a constant return to scale production function.

duration of two and a half years in the US. Together they lead to a steady-state value of unemployment to 5% as in [Blanchard and Galí \(2010\)](#) and which is closer from the mean observed during the estimation period i.e. 6%. The filling rate is fixed to 73% ($q = 0.73$) in order to have a steady-state value of tightness fewer than one. Since we do not estimate the negotiation power of households (η) and the elasticity of matching with respect to unemployment (ξ), we fix them to 0.5 which is the mean used in the matching literature. For the replacement rate $\tau^B = \frac{b}{w}$, we use the same calibration as [Christiano et al. \(2016\)](#) with $\tau^B = 0.75$.

For the LTV ratio of the borrowers we fix them to 0.9 (m^I) which lead to a global LTV ratio of 0.63 ($m^I f^I$) as in the previous chapter.

Turning to the entrepreneurs LTV ratios, where the specification differs depending on the respective model ([Tab.3.1](#)), we choose for \mathcal{M}_1 and \mathcal{M}_2 a full capital requirements with $m^K = 1$. When the hypothesis of the wage bill in advance is required (i.e. for \mathcal{M}_2 and \mathcal{M}_3), we choose the same calibration than [Neumeier and Perri \(2005\)](#) with $m^W = 1$. Finally, for the model describes in the theoretical section, we choose the same calibration than [Iacoviello \(2015\)](#) with $m^K = m^H = 0.9$.

TABLE 3.2: Calibrated parameter

Parameter	Value	Parameter	Value
Capital share	$\alpha = 0.3$	Discount factor patient	$\beta^P = .9925$
Capital depreciation rate	$\delta^K = 0.035$	Discount factor impatient	$\beta^I = .97$
Spending to GDP ratio	$g^Y = 0.19$	Discount factor entrepreneur	$\beta^E = .97$
Housing preference	$j = 0.095$	Share of lenders	$\lambda = 0.75$
Finding rate	$f = .7$	Elasticity matching function	$\xi = 0.5$
Filling rate	$q = .73$	Labor separation	$\delta^L = 0.1$
Replacement rate	$\tau^B = .75$	Worker negotiation power	$\eta = 0.5$
Loan-to-value ratio impatient	$m^I = 0.9$	Housing share	$\nu = 0.03$

For the estimation of our parameters, we choose the same prior distribution for the utilisation curvature (ψ^E, ψ^P), the capital adjustment cost (ϕ^{KE}, ϕ^{KP}), the collateral persistence for entrepreneurs (χ^E) and the share of entrepreneurial capital in the production function (μ) as [Iacoviello \(2015\)](#). For adjustment costs on housing ($\Theta^P, \Theta^I, \Theta^E$) we choose the

same prior as capital adjustment costs. The habit persistence (h^C) follows the same prior as Smets and Wouters (2007).

Concerning the prior of shock processes we follow the same prior as Iacoviello (2015).

2.3 Posterior estimation

In this subsection, we discuss our estimates and we contrast them with the existing empirical evidence from the existing literature. The Tab.3.3 summarises the mean as well as intervals (5% to 95%) of the posterior distributions for structural parameters as well as for shock processes.

For both estimations, the majority of shocks are estimated to be quite persistent, with autocorrelation coefficients ranging from 0.88 to 0.98 and without clear difference between estimated autoregressive parameters across the three models. Two shocks are not estimated to be quite persistent. The first is the preference shock (ε_t^B) and as pointed by Zhang (2018), there is a trade-off between h^C and ρ^B .²⁸

The second lower parameter is the AR(1) for investment (ρ^I) which differ between the two estimated models without commercial estate (\mathcal{M}_1 and \mathcal{M}_2) and the model with both requirements (\mathcal{M}_3).

An interesting result is the estimation of the structural parameters associated with the capital market which clearly differs across models. More we add collateral requirements and more the capital is subject to rigidity from both side (household and entrepreneur) and also to financial frictions through the rise in the capital share of entrepreneurs (from $\mu = 0.20$ for \mathcal{M}_1 to $\mu = 0.70$ for \mathcal{M}_3).²⁹ For the first result i.e. the rise in capital stiff, this can be explained by the necessity of a less volatile capital in the collateral constraint to match with the volatility of the entrepreneur debt since housing price is highly volatile.³⁰ In the same vein, we have a higher collateral persistence χ^E in the

²⁸When h^C is high (respectively lower), the AR parameters for preference shocks tend to be lower (respectively higher)

²⁹The rise of capital share from entrepreneurs is directly linked to financial frictions as shown by the Eq.3.35.

³⁰The rise of capital stiff can be observed through the utilisation elasticity and capital adjustment cost.

TABLE 3.3: Prior and posterior distributions for structural parameters

			Posterior Mean [5%, 95%]		
			\mathcal{M}_1	\mathcal{M}_2	\mathcal{M}_3
Shock processes:					
ρ_Z	AR - Productivity	$B(0.50, 0.15)$	0.90 [0.87;0.94]	0.88 [0.84;0.92]	0.91 [0.88;0.95]
ρ_B	AR - Preference	$B(0.50, 0.15)$	0.61 [0.53;0.69]	0.61 [0.52;0.70]	0.49 [0.39;0.59]
ρ_H	AR - Housing preference	$B(0.50, 0.15)$	0.97 [0.95;0.99]	0.97 [0.95;0.99]	0.98 [0.97;1.00]
ρ_G	AR - Public Spending	$B(0.50, 0.15)$	0.93 [0.91;0.95]	0.92 [0.90;0.94]	0.93 [0.90;0.95]
ρ_I	AR - Investment	$B(0.50, 0.15)$	0.28 [0.21;0.35]	0.23 [0.14;0.33]	0.43 [0.32;0.54]
ρ_M	AR - LTV IH	$B(0.50, 0.15)$	0.95 [0.93;0.98]	0.94 [0.91;0.97]	0.96 [0.93;0.98]
ρ_{ME}	AR - LTV E	$B(0.50, 0.15)$	0.98 [0.96;0.99]	0.96 [0.94;0.98]	0.92 [0.88;0.95]
σ_Z	Std.Dev Productivity	$IG(0.0025, 0.025)$	0.01 [0.00;0.01]	0.01 [0.00;0.01]	0.01 [0.00;0.01]
σ_B	Std.Dev Preference	$IG(0.0025, 0.025)$	0.05 [0.04;0.07]	0.05 [0.03;0.06]	0.05 [0.04;0.06]
σ_H	Std.Dev Housing preference	$IG(0.05, 0.05)$	0.09 [0.04;0.14]	0.08 [0.04;0.13]	0.07 [0.03;0.11]
σ_G	Std.Dev Public Spending	$IG(0.0025, 0.025)$	0.04 [0.03;0.04]	0.04 [0.03;0.04]	0.03 [0.03;0.04]
σ_I	Std.Dev Investment	$IG(0.0025, 0.025)$	0.01 [0.01;0.01]	0.01 [0.01;0.01]	0.02 [0.01;0.02]
σ_M	Std.Dev LTV IH	$IG(0.0025, 0.025)$	0.04 [0.04;0.05]	0.05 [0.04;0.05]	0.04 [0.04;0.05]
σ_{ME}	Std.Dev LTV E	$IG(0.0025, 0.025)$	0.01 [0.01;0.01]	0.01 [0.01;0.01]	0.01 [0.01;0.02]
Structural parameters:					
h^C	Consumption habits	$B(0.50, 0.15)$	0.90 [0.87;0.92]	0.87 [0.83;0.91]	0.89 [0.86;0.92]
ϕ^E	Capital adj. cost E	$G(1.00, 0.50)$	0.19 [0.10;0.29]	0.43 [0.17;0.69]	0.65 [0.26;1.02]
ϕ^P	Capital adj. cost PH	$G(1.00, 0.50)$	0.39 [0.15;0.60]	1.81 [0.84;2.73]	2.16 [0.91;3.36]
ψ^E	Utilisation elasticity E	$B(0.50, 0.10)$	0.51 [0.35;0.67]	0.69 [0.52;0.86]	0.83 [0.71;0.94]
ψ^P	Utilisation elasticity PH	$B(0.50, 0.10)$	0.82 [0.70;0.94]	0.70 [0.52;0.89]	0.71 [0.53;0.89]
Θ^E	Housing adj. cost E	$G(1.00, 0.50)$	-	-	2.81 [1.56;3.96]
Θ^P	Housing adj. cost PH	$G(1.00, 0.50)$	0.35 [0.23;0.48]	0.45 [0.32;0.58]	0.35 [0.09;0.61]
Θ^I	Housing adj. cost IH	$G(1.00, 0.50)$	0.41 [0.22;0.59]	0.52 [0.32;0.73]	1.32 [0.40;2.21]
μ	Capital share of E	$B(0.50, 0.10)$	0.20 [0.18;0.22]	0.45 [0.39;0.51]	0.70 [0.58;0.82]
χ^E	Collateral persistence E	$B(0.25, 0.10)$	-	0.10 [0.04;0.15]	0.43 [0.34;0.50]
Steady-state:					
g^H	Commercial estate to annual GDP				0.70
ω^O	Endogenous bargaining power		0.5	0.49	0.48

Notes: \mathcal{B} , beta; \mathcal{G} , gamma; \mathcal{N} , normal; \mathcal{IG} , inverse gamma type 1; $P1$, prior mean and $P2$ prior standard deviation for all distributions. E stands for entrepreneurs, IH for impatient households and PH for patient households

model with both collateral requirements to dampen the fickle of the debt. For the second result, i.e. a capital more subject to financial frictions, this highlights the importance of financial frictions in shaping the dynamics of investment.

Since we adopt the same specification for the collateral constraint in the complete model (\mathcal{M}_3) than [Iacoviello \(2015\)](#), we examine the main difference between our estimates. As in his paper, we find higher rigidities for capital markets owned by lenders than entrepreneurs which is normal since the first market is not subject to financial frictions. However, we obtained different results for the share of entrepreneur capital and the exogenous persistence which are respectively higher and lower than their computation (respectively $\mu = 0.46$ and $\chi^E = 0.65$).

For the estimation of shock standard deviation, we do not vary significantly from [Iacoviello \(2015\)](#). Finally, for housing adjustment costs, our results are in line with the calibration exercise of [Sterk \(2015\)](#).

3 Empirical implications

The three collateral constraints that we consider offers different channel from the housing and financial market to the labor market. In this section, we investigate the empirical relevance of each collateral constraint in a three-step analysis. First, we compare the fit of our three models through a likelihood comparison between models. Second, we compare the business cycle moments to see how models are able to account for salient features of the data. Finally, we compare the propagation mechanism by comparing Impulse Response Functions (IRFs hereafter) between models.

3.1 Fit comparison

To gauge the empirical relevance of each collateral constraint in shaping macroeconomic dynamics, the [Tab.3.4](#) reports the (Laplace-approximated) marginal data densities, the posterior ratio and the probability for each of the three estimations considered.³¹ To

³¹As in the previous chapter, we refer to [Rabanal and Rubio-Ramírez \(2005\)](#) for a formal description and discussion of these criteria to compare estimated DSGE model.

TABLE 3.4: Fit comparison between estimation of the model

	\mathcal{M}_1	\mathcal{M}_2	\mathcal{M}_3
	Capital	Capital and WB	Both requirements
Prior probability	1/3	1/3	1/3
Log marginal density	2564.66	2638.52	2671.35
Bayes ratio	1.00000	1.19×10^{32}	2.17×10^{46}
Posterior probability	0.000000	0.000000	1.000000

compute posterior odds ratios and probabilities, we impose an uninformative prior distribution over estimations (i.e. one third prior probability for each estimation). In a nutshell, one should favor of an estimation whose data density, posterior odds ratio and model probability are the higher compared to the other estimations.

Which entrepreneur collateral constraint best explains the behaviour of our sample ?

The model with both requirements appears to be favored by the data as its marginal density is the highest. This specification is next followed by the Capital and Wage bill in advance requirements (\mathcal{M}_2) while the capital requirement is the last in the ranking. The difference in marginal data densities across models is large enough to validate this ranking.³² This is therefore confirmed by posterior odds ratios and model probabilities. Given this evidence, we conclude that the model with a collateral requirement that includes capital, real commercial estate and wage bill in advance outperforms the two other specifications.

However, the likelihood ratio does not allow to clearly understand how one collateral constraint is able to outperform the other. The next subsection provides further investigations through the business cycle moment.

3.2 Business cycles analysis

The Tab.3.5 report's key business cycle statistics for observable variables generated by the three models considering, taking for each model the parameters at their posterior

³²The results are statistically strong as the marginal data density difference between \mathcal{M}_3 and \mathcal{M}_2 is 32.70, thus we would need a prior probability ratio (currently this ratio is unity) to be higher than $\exp(32.70) = 1.72 \times 10^{14}$ to alter this ranking.

TABLE 3.5: Business cycle statistics comparison between the two estimated model

	U_t	TFP_t	C_t	I_t	d_t^E	d_t^I	q_t^H
Standard deviations							
Data	[0.01;0.02]	[0.01;0.02]	[0.03;0.03]	[0.07;0.09]	[0.10;0.13]	[0.09;0.11]	[0.08;0.10]
\mathcal{M}_1	0.01	0.01	0.05	0.09	0.11	0.21	0.10
\mathcal{M}_2	0.01	0.01	0.05	0.08	0.10	0.18	0.09
\mathcal{M}_3	0.01	0.02	0.05	0.10	0.12	0.17	0.11
Auto-correlation							
Data	[0.97;0.99]	[0.88;0.95]	[0.98;0.99]	[0.96;0.98]	[0.99;1.00]	[0.99;1.00]	[0.99;0.99]
\mathcal{M}_1	0.90	0.90	0.98	0.92	0.99	0.99	0.97
\mathcal{M}_2	0.95	0.88	0.97	0.90	0.99	0.99	0.96
\mathcal{M}_3	0.97	0.91	0.99	0.92	0.99	1.00	0.98
Correlation w/ unemployment							
Data	[1.00;1.00]	[-0.55;-0.17]	[-0.60;-0.23]	[-0.75;-0.49]	[-0.61;-0.23]	[-0.52;-0.19]	[-0.46;-0.05]
\mathcal{M}_1	1.00	-0.45	-0.77	-0.35	-0.50	-0.55	-0.39
\mathcal{M}_2	1.00	-0.42	-0.78	-0.33	-0.47	-0.56	-0.36
\mathcal{M}_3	1.00	-0.47	-0.80	-0.73	-0.58	-0.46	-0.43

Notes: \mathcal{M}_1 stands for the model with capital, \mathcal{M}_2 capital and wage bill in advance and \mathcal{M}_3 both requirements.

means in Tab3.3. Observed moments are expressed in terms of a 90% confidence range to highlight whether a moment generated by the model is not statistically different from its empirical counterpart.

Beginning by the standard deviations, both models exhibit good performance to replicate all variables with the exception of the household's debt. All models overestimate this volatility. However, the model with both requirements does a better job than the others.

Turning to auto-correlation, the model with full requirements is capable of replicating the high level of persistence for unemployment highlighting its role in explaining unemployment fluctuations. For the rest of persistence, all models exhibit not significant differences among each of them and are closest from the data.

Finally, concerning the correlation with unemployment, all the models succeed in replicating the correlation with other observed variables except for the consumption where both models overestimate the negative correlation. For investment the model (\mathcal{M}_3) is the only one in the 90% confidence interval.

In the light of results obtained in terms of marginal data densities (Tab.3.4) and those obtained in this subsection, we continue further analysis by comparing the reaction of the three estimated model following three different shocks which mainly drives the economy.

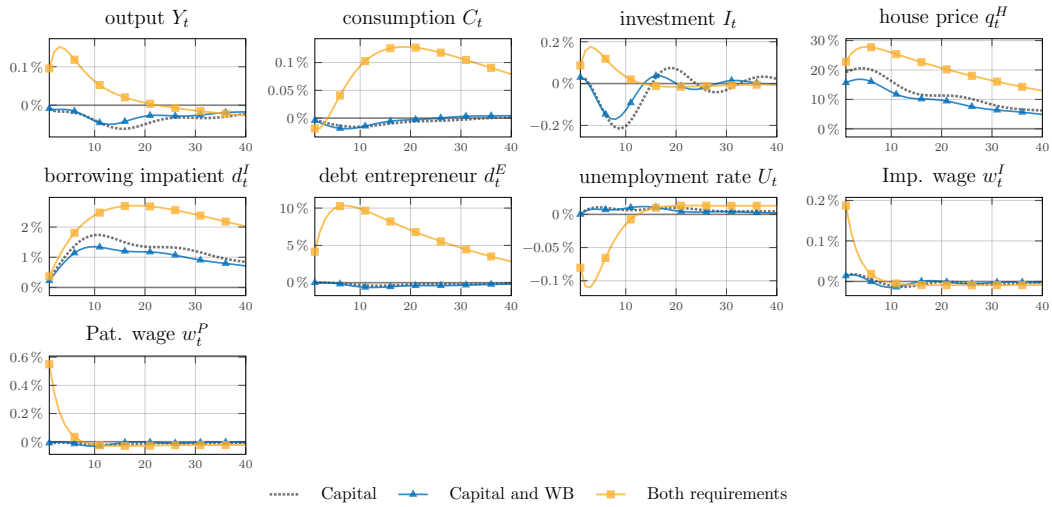
3.3 Inspecting the propagation mechanism

We display the three driving forces of the economy in our model i.e. the IRF following a housing demand shock, a credit squeezes shock for entrepreneurs and finally a TFP shock.

Housing shock The Fig.3.2 displays the response of the three estimated model following a 1% housing demand shock. Starting with the model with real commercial estate presents in the collateral constraint i.e. \mathcal{M}_3 . Two mains channel are at work : i) *the housing net worth channel* ii) *the financial accelerator*.

The first effect is referred to the positive effect on consumption following a rise in the housing prices. As pointed by Iacoviello (2005), considering both collateral-constraint agent (households and entrepreneurs) leads to a positive elasticity between consumption and housing price. Then, the positive effect on consumption partly explained the drop on the unemployment rate through a rise on demand.

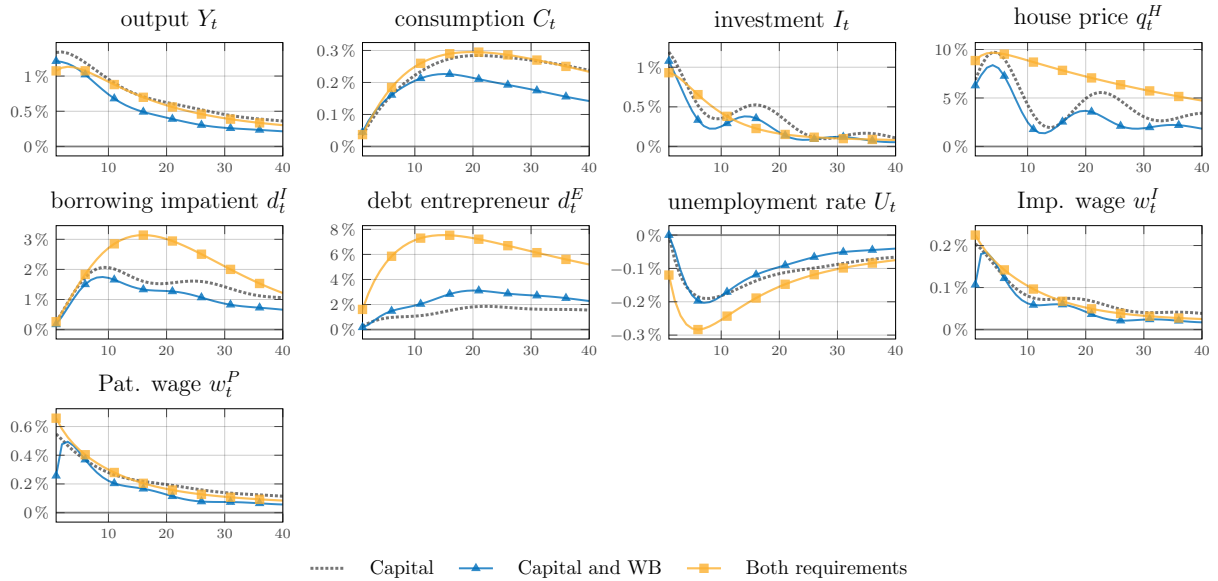
The second effect is induced by the positive effect on the entrepreneurial debt. The increase in the housing price slack entrepreneurs' borrowing capacity (Eq.3.26), which in turn increases their real estate acquisition and investment. As investment growing, the expected marginal value of adding a new worker follows the same path. Despite the growth of debt for impatient households, their wage is more rigid than patient ones. This is due to the difference in terms of utility gap in the second line of the wage determination ($\omega_t^O \beta_{t,t+1}^E - \omega_{t+1}^O \beta_{t,t+1}^I$). However, in both cases (wage for patient and impatient), due to the endogenous bargaining power (ω_t^O) and the presence of credit frictions ($\beta_{t,t+1}^E$), wages are relatively rigid and then does not offset the housing shock on the unemployment rate. Thus, the *financial accelerator* combines with the *wealth effect* decreases significantly the unemployment rate.

FIGURE 3.2: System response to a 1% housing demand shock η_t^H 

Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

Turning to the two other models without commercial estate, we find the opposite result. In these specifications, the rise in the housing price leads to a decline in the consumption. Impatient households purchase housing goods which decrease their consumption. In the other side patient households have more resources on their own and increase their consumption. However, since the impatient household has a higher propensity to consume than the patient one, the total effect on consumption is negative. Then, the housing net worth channel is offset in these two models. The same thing happens for the entrepreneurial debt where the financial accelerator is counteracted since in the two above-mentioned models (\mathcal{M}_1 and \mathcal{M}_2) the commercial estate is absent in the collateral constraint. Thus, as a consequence of the fall in demand, the output and employment level decreases following a housing shock.

TFP shock The Fig.3.3 displays the response of our estimated model following a 1% TFP shock. The response of the model is in line with matching model literature: a rise in TFP makes input more productive, raising the marginal profit from hiring a new worker and then encourages firms to hire more. Therefore, employment slowly rises as a result of the sluggishness of the matching process. As observed for the previous shock, wages are relatively rigid with a higher rigidity for impatient households due to the utility gap.

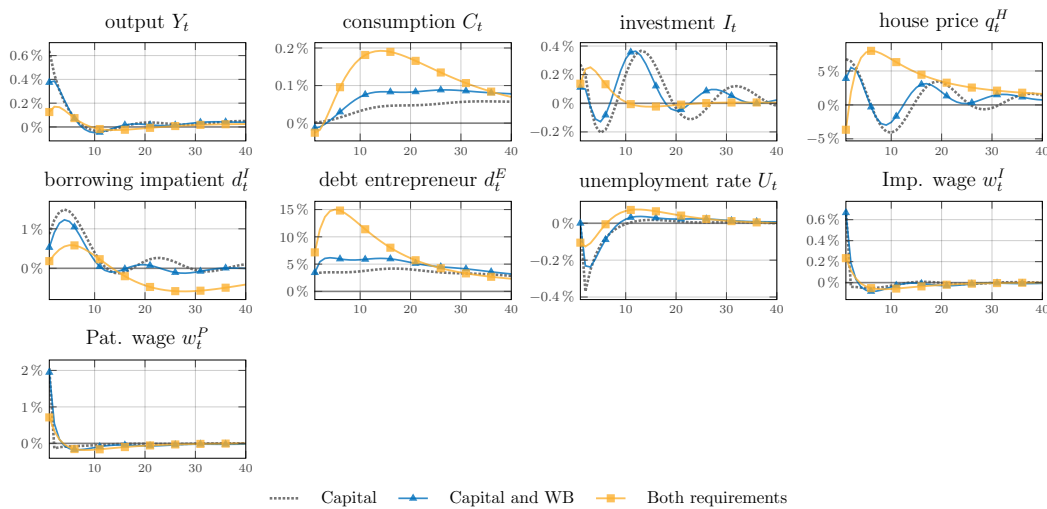
FIGURE 3.3: System response to a 1% TFP shock η_t^Z 

Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

Turning to the comparison between models, the model with both requirements generates a large decline of the unemployment rate comparing to the others. The main reason of this result is the rise of the entrepreneurial debt linked to the housing price. Since entrepreneurs have more assets in this model, the marginal profit from hiring a new worker is higher and then the unemployment rate decreases more.

LTV entrepreneurial shock The Fig.3.4 displays the response of our estimated model following a 1% LTV shock for entrepreneurs. In other words, firms' leverage is exogenously increased and firms are able to borrow a higher fraction of their collateral. As emphasised by [Garin \(2015\)](#), the credit shock does not generate persistent dynamics for output, unemployment and investment. However, in both models, a positive LTV shock conduces to positive effects for these three variables. Since entrepreneurs have access to more funds they invest in their inputs i.e. labor, capital and commercial estate. The response of wages following this shock is in contrast with [Monacelli et al. \(2011\)](#) who find countercyclical wages following an expansion on the borrowing capacity.

To sum up, the TFP shock is the shock which generates the most persistent increase in

FIGURE 3.4: System response to a 1% LTV shock for entrepreneurs η_t^{ME} 

Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

the unemployment rate, following by the housing price shock and finally the LTV shock. However, in both cases the model presented is the one who generates the most persistence and deep impact on employment fluctuations.

Regarding the outcomes presented in this section (Fit comparison, Business cycles statistics and IRF) which compare the three collateral requirements, we continue the analysis between unemployment and financial business cycles based on the model with both requirements.

4 Labor market fluctuations with financial business cycles

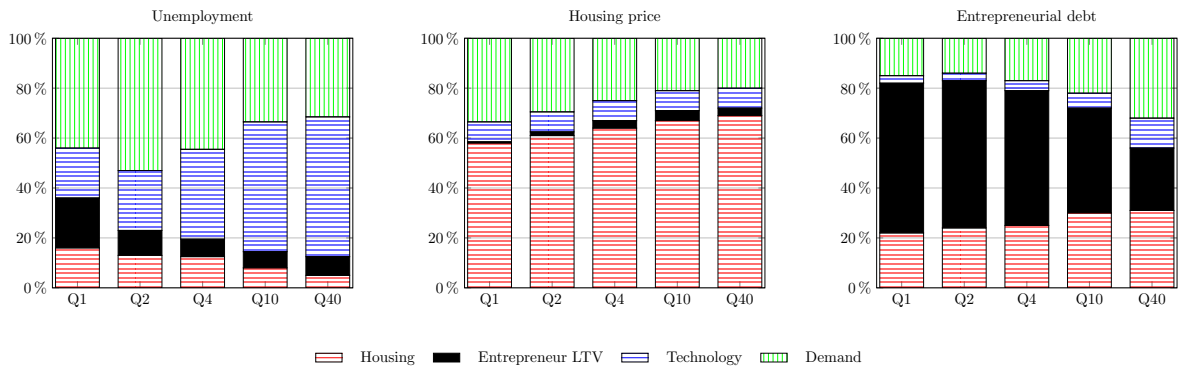
4.1 Variance decomposition

We start this section by studying the variance decomposition at different horizons for unemployment, entrepreneurial debt and housing price. The Fig.3.5 reports this exercise.

Starting with unemployment, accordingly to estimate RBC models the technological shock explained a large part of the variance decomposition for the long run (Q_{40}). However, on impact (Q_1) they have the same weight as the housing shock and the LTV entrepreneur shock. The housing demand shock from 1 quarter to one year explained approximately in equivalent proportion the variance of unemployment (around 16%). This result is in line with the BVAR results of Liu et al. (2016). On the opposite, the LTV shock declines faster over a year and becomes constant until the long run (from 20% at Q_1 to 7% for Q_4 to Q_{40}).

Turning to the variance decomposition of housing price, as in Iacoviello (2015); Liu et al. (2016) housing price is mostly driven by the housing demand shock with 58% for the short run to 69% for the long run.

FIGURE 3.5: Variance decomposition for unemployment, housing price and entrepreneurial debt



Notes: Variance decomposition at different horizons where Q_t corresponds to quarter with t the quarter considered. For the LTV shock we refer to the entrepreneurial LTV shock since the LTV shock for impatient household does not have an impact on this three variables due to the endogenous LTV ratio in their collateral constraint (Eq.3.8). Furthermore, they explain between 0.03% and 0.06% of these three variables considering.

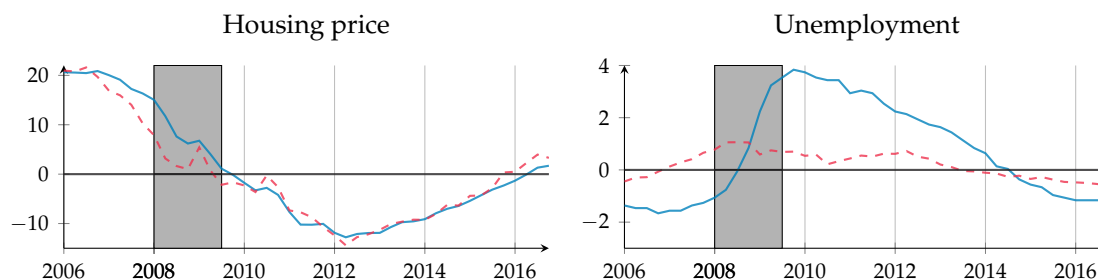
For loans to entrepreneurs, they have more than half of variations explained by the LTV shocks for the short run (over a year i.e. from Q_1 to Q_4). Due to the high value of the real commercial estate in their collateral constraint (m^H), the housing demand shock explained a large part of entrepreneurial debt variance from 22% for the short run to 30% for the long run (Q_{40}).

4.2 The surge in unemployment rate and the housing price

Then we have seen the importance of housing price demand shock (ε_t^H) to drive the variance of unemployment, we perform a counterfactual history on the great recession episode and the following recovery (2006Q1 to 2016Q4).³³ We estimate the counterfactual path of unemployment and the housing price conditional on the housing demand shock.

The Fig.3.6 displays the counterfactual paths. As expected by the variance decomposition (Fig.3.5), almost all declines in the housing price in the Great Recession period and the subsequent increases are attributable to housing demand shocks, with the counterfactual path of housing prices tracking the actual data closely. Then, the crash in housing price is followed by a surge in the unemployment rate.

FIGURE 3.6: Counterfactual paths of the housing price and the unemployment rate conditional on the estimated housing shock.



Notes: Each graph shows the actual path used in the sample (blue thick line) and counterfactual path from the model (\mathcal{M}_3). Actual path are expressed in percentage and in deviation from their mean. The great Recession period is associated with the shaded area.

During the recession period (2008Q1 to 2009Q3) the housing demand shocks generated an increase of the unemployment rate of about 1.7 percentage points. Until the decreases of housing prices stop (close to 2012Q2), this shock limited the recovery of the unemployment rate by inducing a negative impact on the unemployment rate of around 1.6 percentage points. However, when the housing price begins to rise, they have a positive impact on

³³We do not perform a counterfactual path of the entrepreneurial debt and the unemployment rate conditional on the estimated LTV shock for entrepreneurs, since as shown by the variance decomposition, the entrepreneurial debt is also affected by the housing demand shock and is more mixed than the variance of housing price.

the recovery of the unemployment rate.³⁴ This result is in line with the result obtained in the previous section 3.3 where the housing shock has an active role to generate a sharp fluctuation on employment.

5 Policy implications

In this section, we investigate how the presence of labor in both collateral constraints (for impatient households and entrepreneurs) affects the expected results from implementing either a labor market reform or a macroprudential policy. The complete understanding of the transition dynamics induced by the policy reform is very important as the IMF typically provides policy recommendations based on the simulation of these models.

5.1 Labor market reforms

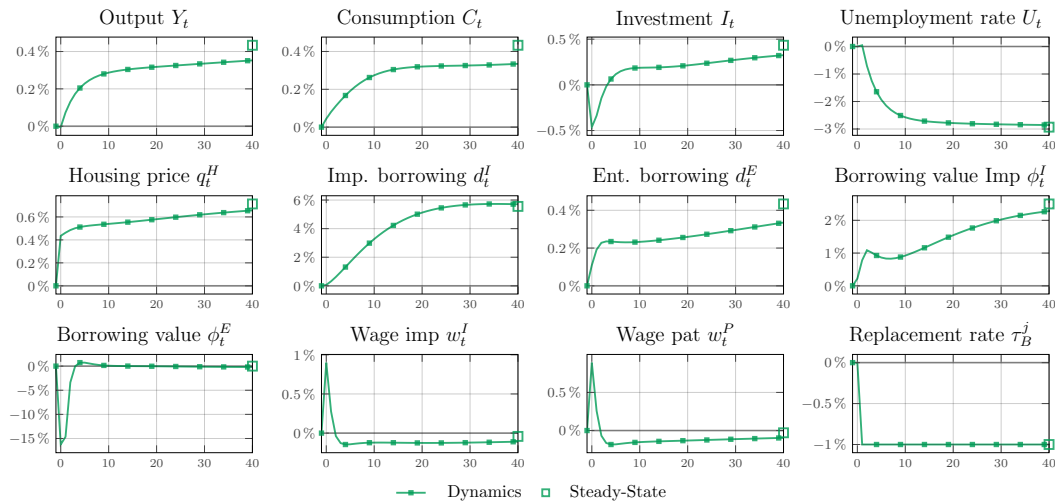
We continue the policy analysis started in the previous chapter. In this chapter, we now consider a new financial friction based on the dynamics implied by the collateral constraint of entrepreneurs. Fig.3.7 plots the transition dynamics implied by a permanent decrease of 1% of the replacement rate (b^j/w^j) for both households. For each variable, we also report on the right side the final steady-state toward the economy will converge in the long run.

In line with the literature (e.g. Bassanini and Duval (2009); Blanchard and Wolfers (2000); Bouis et al. (2012); Cacciatore and Fiori (2016); Cette et al. (2017); Duval and Furceri (2018); Nicoletti and Scarpetta (2003)), this reform implies a permanent rise in output and employment. Since the amount of insurance is lower, the real wage drops and entrepreneurs are more able to create new jobs with lower remuneration.³⁵ Both households experience an increase in their relative lifetime utility from being employed rather than unemployed. As a consequence, the amount of vacancies posted rises, leading to a permanent increase in the number of matches and by extension the finding rate.

³⁴From 2013 to 2016, the housing demand shock generates a decline in the unemployment rate of about 1.2 percentage points.

³⁵We only report the real wage for patient household, since the impatient household wage has the same dynamics.

FIGURE 3.7: System response to a 1% decrease in unemployment insurance for both households b^j/w^j



Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

Turning to financial aspects, as for the previous chapter we find that the impact is deeper for the level of impatient household debt. The presence of labor features in the collateral constraint (Eq.3.8) raises the Loan-To value ratio, thus an increase in the finding rate f^I mechanically slacken the constraint.

Since the rise in employment allows more family members of the patient household to get mortgages from financial intermediaries, the borrowing constraint eases which materialises through an increase in the borrowing value ϕ_t^I . Then, the increase in the borrowing capacity of impatient household leads to more demand for housing goods which makes house prices increase up to 0.8% in the long run.

For entrepreneurs debt, the effect of the labor market reform is more ambiguous. We have two opposing forces on the debt of firms. Starting with the first force, a rise in employment induces a more important wage bill to pay since the decrease in real wage is not enough to compensate the rise of the labor force. Entrepreneurs also experience a lower demand for housing services since the borrowing value decreases. As an opposing

force, housing price and the demand for capital rises. The balance between these two forces in the long run leads the entrepreneurial debt to increase by 0.4%.

Then, in this scenario impatient households is the first winner of the reform as the number of housing stock increases by 1% after the reform. However, patient households and entrepreneurs also benefit from this reform since the level of their consumption rises respectively by 0.6% and 0.4%.

5.2 Macprudential policy

Our model does not limit the analysis to the labor market but can be employed for the analysis of macroprudential policy. Compared to the previous chapter, we can study the consequence of macroprudential policy for two types of financial constrained agents i.e. impatient households and entrepreneurs. Assuming that loan-to-value ratio is determined by US institutional factors (i.e. a prudential authority), then we simulate permanent change for these ratios. However, the transition dynamic is different from the precedent chapter since output and unemployment are now steady-state dependent from LTV ratios due to the introduction of entrepreneurs.³⁶

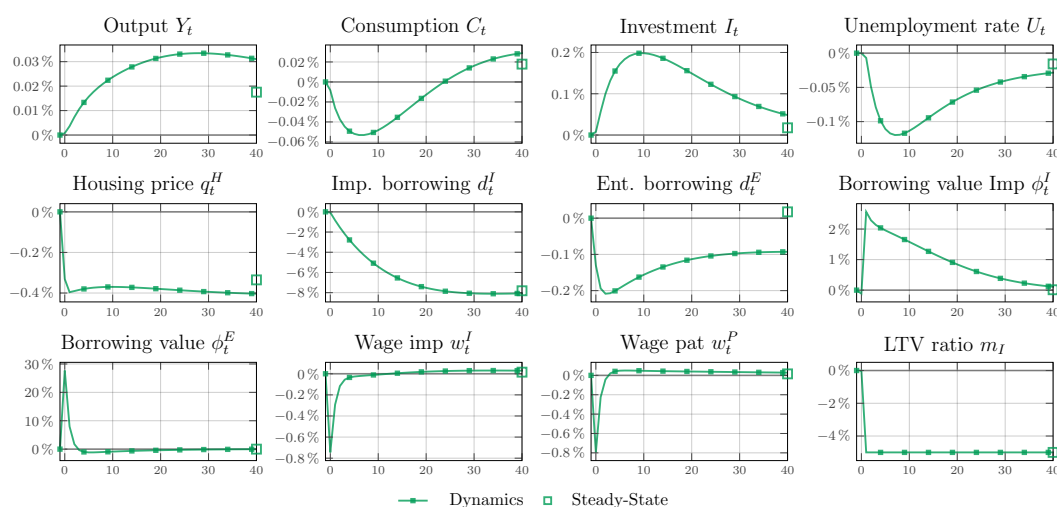
5.2.1 Impatient households

Starting with impatient households, as in [Chen and Columba \(2016\)](#) we simulate a permanent decrease of the LTV ratio denoted by m^I . [Fig.3.8](#) displays the transition dynamics from permanently reducing m^I by 5%. As described by [Chen and Columba \(2016\)](#) this reform conduces to reduce houses prices (q_t^H), borrowing (d_t^I) and the number of houses purchased by the impatient household (h_t^I). Then, the entrepreneur borrowing value (ϕ_t^E) rises since the housing price decreases. As a consequence, entrepreneurs buys commercial estate (h_t^E) and invest in capital (K_t^E) as for the patient household which reallocate their saving toward capital goods (K_t^P). Then, investment is benefiting from this new allocation. Despite the initial drop in consumption, the output rises leading to a reduction in the unemployment rate. Concerning wages, impatient household salary exhibits the same

³⁶See appendix D for further details.

patterns as patient ones with respect to the previous chapter. Since both the level of debt and unemployment rate decrease, this offset the effect of the collateral constraint in the wage determination. In the long-run, they both have a higher wage since the level of output rises. Finally, despite the drop in consumption for patient households, the total consumption of the three agents is positively affected by the rise in borrowers consumption (impatient household and entrepreneur) in the long-run.

FIGURE 3.8: System response to a 5% decrease in LTV ratio for impatient household m^I



Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

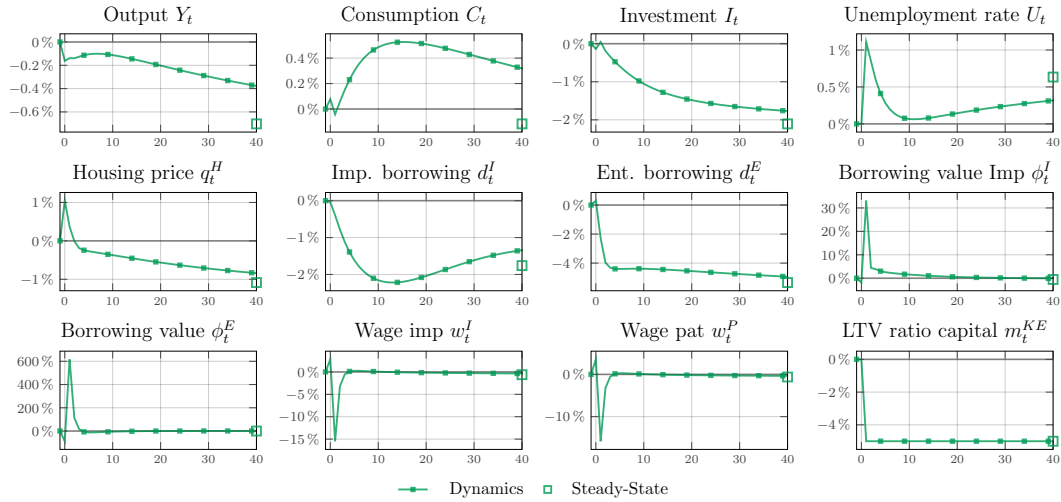
These two reforms could be jointly employed to boost output and employment without generating higher borrowing for impatient households and housing price.

5.2.2 Entrepreneurs

In our knowledge, the literature does not directly explore the possibility that a macroprudential authority changes the LTV ratios for an entrepreneur. However, macroprudential policy has many tools that affect the credit condition of firms. For instance, sectorial capital requirements for banks allow the macroprudential policy to penalise or to ease the credit conditions offered to non-financial corporations. Our LTV policy here can be

interpreted as a shortcut for sectoral capital requirements. A rise in capital requirements with an exposure on firms translates in our model by lower LTV ratios.

FIGURE 3.9: System response to a 5% decrease in the capital LTV ratio for entrepreneurs m^K

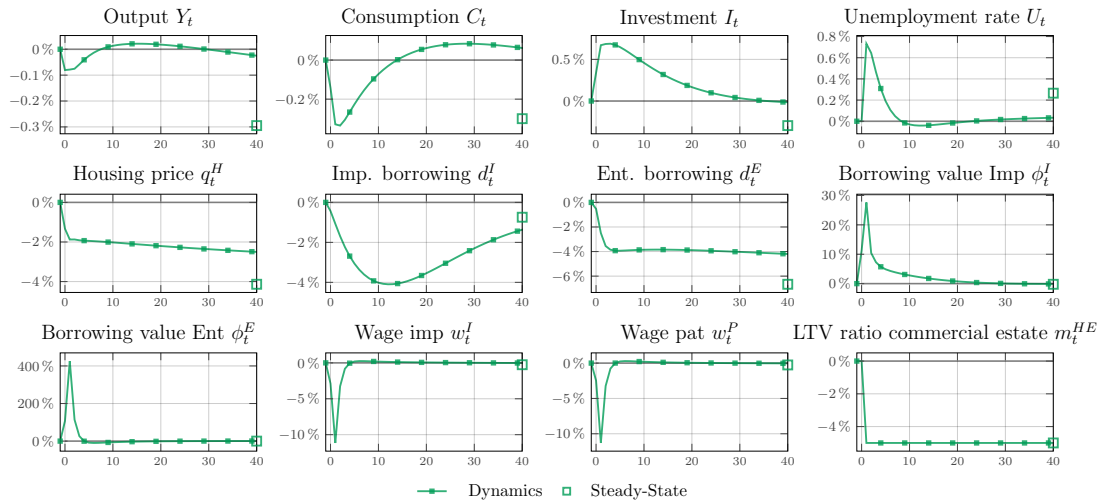


Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

LTV ratio for capital m^K We start by considering a drop in the capacity of the entrepreneur to borrow against their capital stock (m^K). Fig.3.9 displays the transition dynamics implied by a 5% decrease in the capital LTV ratio for entrepreneurs m^K . The first impact is the deep reduction in the level of debt (d_t^E). As a consequence, entrepreneurs decrease their inputs in the production function (capital, labor forces and commercial estate). In response, the housing price falls combined with a rise in the unemployment rate, this leads to a permanent decrease in the impatient borrowing debt. However, both wages remain the same in the long run since the marginal product value of labor remains constant (i.e. $\frac{(1-\alpha-\nu)\lambda Y}{l^P}$ for patient household and $\frac{(1-\alpha-\nu)\lambda Y}{l^I}$ for impatient ones) in the wage setting. Finally, at the long run, output decreases by 0.6%, the amount of debt by 1.8% for impatient households and 5% for entrepreneurial debt while the housing price by 1.1%.

LTV ratio for real estate m^H Turning to the LTV ratio for entrepreneur real estate, the Fig.3.10 plots the system response to a 5% decrease in m^H . The dynamics implied by this ratio are quite similar to thus observed with the LTV ratio for capital but with a lower order of magnitude. The rationale beyond this result is that the total stock of housing is fixed (normalised to one) while the total supply capital stock may vary. However, the consequence on the entrepreneurial debt is higher than the previous reforms since the housing price responds more in the long run (4% for this reform against 1,1% for the previous one).

FIGURE 3.10: System response to a 5% decrease in the real estate LTV ratio for entrepreneurs m^H



Notes: Impulse response are generated using deterministic simulations when parameters are drawn from the mean posterior distribution. System responses are reported in percentage deviations from the initial steady state prior the structural change. In $t = -1$, the model is at the initial steady state, in $t = 0$ the news of a future structural change is released, in $t = 1$ the structural change is effective.

6 Conclusion

In this chapter, we have integrated the mechanism of the previous one and extend the analysis by adding collateral constrained firms in order to have a more complete financial business cycles. The first discovery of our chapter is that an entrepreneur collateral

constraint integrating both capital, real commercial estate and wage bill in advance is empirically relevant compared to the collateral literature linked to the labor market which does not consider these three assets. The second finding is the role of the housing price in the rise of the unemployment rate during the great recession. We have also seen the relative importance of credit squeezes from entrepreneurs as a driving force for the employment. Finally, as for the previous chapter, this framework has important issues for economic policy. A structural reform in the labor induces more borrowing for households and a substantial rise of the entrepreneurial debt. A macro-prudential policy aimed to tighten the LTV ratio for impatient household has from now on positive effect at the long run for output and employment while tightening LTV ratios for entrepreneurs conduces to the opposite effect.

General Conclusion

The aim of this thesis is to shed a light on labor market interactions with firm entry and financial business cycles, by building on the recent theoretical and empirical of DSGE models. The first chapter focuses on labor and firm entry connections, while the two other chapters draw attention to the interactions with financial business cycles.

In the first chapter, we have found evidence of the key role of net business formation as an amplifying mechanism for employment dynamics. In particular our model reveals that the fall in the entry explained a considerable part of the surge in the unemployment rate during the Great Recession. Introducing search and matching frictions, we have studied from a new perspective the cyclicity of the mark-up compared to previous researches which use a Walrasian labor market. We found a less countercyclical markup due to the acyclical aspect of the marginal cost in the DMP framework and a reduced role according to firm entry in the cyclicity of the markup.

In the second chapter, we have linked the borrowing capacity of households to their employment situation on the labor market. With this new microfoundation of the collateral constraint, new matches on the labor market translate into more mortgages, while separation induces an exclusion from financial markets for jobseekers. As a result, the LTV becomes endogenous by responding procyclically to employment fluctuations. We have shown that this device is empirically relevant and solves the anomalies of the standard collateral constraint. This framework has important implications for economic policy. A structural reform in the labor market induces more borrowing and a higher housing price in the economy. However, a macroprudential tightening policy can help reduce these two features.

In the last chapter, we extended the analysis developed in the previous one by integrating collateral constrained firms in order to have a more complete financial business cycle. The first result is that an entrepreneur collateral constraint integrating both capital, real commercial estate and wage bill in advance is empirically relevant compared to the collateral literature associated to the labor market which does not consider these three assets. The second finding is the role of the housing price in the rise of the unemployment rate during the great recession. We have also evidenced the relative importance of credit squeezes from entrepreneurs as a driving force for employment. Finally, as for the previous chapter, this framework raises important issues for economic policy. A structural reform in labor induces more borrowing for households and a substantial rise of entrepreneurial debt. Our approach allows us to reveal that a macroprudential policy aiming to tighten the LTV ratio for borrowers has positive effects in the long run for output and employment, while tightening LTV ratios for entrepreneurs leads to the opposite effect.

Looking forward, our analysis outlines several areas for future research. Starting with the first chapter, a natural extension would be to integrate an entry cost that does not simply depend on the cost of posting vacancy and a fixed component. This modelling device would reconcile our approach with the literature which considers marginal costs as a determinant of entry. Thus, we could explore anew structural reforms in the goods and labor markets with this new cost of entry compared to the previous literature (e.g. [Cacciatore and Fiori \(2016\)](#)).

For the last two chapters, an interesting extension would be to integrate a banking system in order to have a complete financial business cycle. First, we would expect an amplifying role for financial disturbances on unemployment fluctuations. Second, it would allow us to be in line with the recent macroprudential policy literature which imposes sectorial capital requirements for banks to penalise or to ease the credit conditions offered to financial agents.

Appendix A

Data sources

In this appendix, we provide the data sources of all chapters.

- Nominal GDP : Gross Domestic Product, Billions of Dollars, Seasonally Adjusted Annual Rate from the FRED database <https://fred.stlouisfed.org/series/GDP>. Chapter I and II
- Inflation : defines as the log-difference of the Implicit Price Deflator, with Index 2012=100, Seasonally Adjusted from the FRED database <https://fred.stlouisfed.org/series/GDPDEF>. Chapter I
- Unemployment: Civilian Unemployment Rate, in Percent, Seasonally Adjusted from the Fred database <https://fred.stlouisfed.org/series/UNRATE/>, (U_t in all chapters). Chapter I, II and III
- Vacancies: data from [Barnichon \(2010\)](#) which combine job openings from the JOLTS data set, the Help-Wanted Online Advertisement Index published by the Conference Board , and the Help-Wanted Print Advertising Index that was discontinued in October 2008 and it was also constructed by the Conference Board. (<https://sites.google.com/site/regisbarnichon/research>, Composite Help-Wanted Index), (V_t in all chapters).

- Job Finding Probability: we apply the methodology of [Shimer \(2007\)](#) by defining the probability to find a job for an unemployed worker as:

$$f_t = 1 - \frac{u_{t+1} - u_{t+1}^S}{u_t}$$

where u_{t+1}^S correspond to unemployed workers less than 5 weeks (from the Bureau of Labor Statistic, BLS hereafter) and u_{t+1} the number of unemployed people. We convert the obtained serie in a quarterly basis, (f_t in all chapters).

- Labor market tightness : Ratio of vacancies to unemployment defined above.
- Interest rate: the shadow value provides by [Wu and Xia \(2016\)](#) to deal with the Zero Lower Bond. Chapter I.
- Consumption: Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonnaly Adjusted Annual Rate from the FRED database <https://fred.stlouisfed.org/series/PCE>. Chapter I, II and III.
- Wage : the Nonfarm Business Sector: Compensation Per Hour from the FRED database, seasonnaly adjusted (<https://fred.stlouisfed.org/series/COMPNFB>) as a proxy for wage. Chapter I.
- Investment: the Fixed Private Investment, Billions of Dollars, Seasonally Adjusted Annual Rate from the FRED database, (<https://fred.stlouisfed.org/series/FPI>). Chapter I, II, III.
- New firms: Number of establishment births, Total private, Seasonally Adjusted from the BLS (Employment Business Dynamics). Chapter I.
- Establishment deaths: Number of establishment deaths, Total private, Seasonnaly Adjusted from the BLS (Employment Business Dynamics). Chapter I.
- Net Business Formation: define as the ratio between private sector establishment Births and private sector establishment deaths, seasonnaly adjusted (In thousands) from the BLS. Chapter I.

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- Entrepreneurial debt: is the sum of Real Estate Loans, All Commercial Banks equivalent to Nonfinancial business; total mortgages; liability (decomposed in corporate <https://fred.stlouisfed.org/series/MLBSNNCB> and non corporate <https://fred.stlouisfed.org/series/NNBTML#0>) plus Non financial business, other loans and advances, liability (decomposed into corporate <https://fred.stlouisfed.org/series/OLALBSNNCB> and noncorporate <https://fred.stlouisfed.org/series/OLALBSNNB>) and plus Nonfinancial business; depository institution loans (decomposed in corporate <https://fred.stlouisfed.org/series/BLNECLBSNNCB> and non-corporate <https://fred.stlouisfed.org/series/NNBDILNECL>). Chapter III.
 - House price : Census Bureau House Price Index (new one-family houses sold including value of lot). Chapter II and III
 - Loans to household : Household and nonprofit organizations; home mortgages; liability <https://fred.stlouisfed.org/series/HHMSDODNS> plus household and nonprofit organizations; consumer credit, liability <https://fred.stlouisfed.org/series/HCCSDODNS>. Chapter II and III.
 - Total Factor Productivity : Utilization-adjusted quarterly growth rate of TFP constructed by Fernald (2012). Chapter III.

Data in nominal terms are converted using the GDP deflator. Furthermore, in order to create per capita series we divide them by Civilian Noninstitutional Population over 16 from the FRED database <https://fred.stlouisfed.org/series/CNP160V> as in the first chapter.

Appendix B

Appendix for Chapter I

In this appendix, we describe the main features of the chapter I with the complete set of First Order Condition (FOC hereafter), the analytic steady-state and the additional figure for demand shocks.

1 Non-linear model

1.1 Households

1.1.1 Translog preferences

Recall that we use the term “good”, “producer” and “firms” are used interchangeably. Each individual firms produce one differentiated intermediate good indexed by $f \in [0, N_t]$ where N_t is the mass of producers. Firms’ output $y_{f,t}$ are bundled into a final good Y_t^C which is sold at price P_t^C to households. We follow [Feenstra \(2003\)](#) assuming that the final consumption basket Y_t^C a translog form. Translog preferences are characterised by defining the unit expenditure function i.e. the price index P_t^C associated with the preference aggregator. Denote by $P_{f,t}$ the nominal price for the good $f \in [0, N_t]$, the unit

expenditure function on the basket good Y_t^C is:

$$\ln P_t^C = \frac{\tilde{N} - N_t}{2\sigma\varepsilon_t^P \tilde{N}N_t} + \frac{1}{N_t} \int_{f \in N_t} \ln P_{f,t} df + \frac{\sigma\varepsilon_t^P}{2N_t} \int_{f \in N_t} \int_{f' \in N_t} \ln P_{f,t} (\ln P_{f',t} - \ln P_{f,t}) df df' \quad (\text{B.1})$$

where $\sigma > 0$ scales the demand elasticity and ε_t^P the exogenous price-markup shock. This expenditure function gives several properties (see [Lewis and Stevens \(2015\)](#) for a complete demonstration) :

The optimal demand addressed to a firm f is given by:

$$P_t^C Y_t^C = P_{f,t} N_t y_{f,t}, \quad (\text{B.2})$$

The price elasticity of demand is:

$$\theta_{f,t} = 1 + N_t \sigma^P \varepsilon_t^P,$$

and finally by imposing symmetry among producers (such as $P_{ft} = P_{f't}$) and applied exponential to the translog expenditure function, we obtained the price index,

$$\rho_{f,t} = \exp \left(-\frac{\tilde{N} - N_t}{2\sigma^P \varepsilon_t^P \tilde{N}N_t} \right),$$

where $\rho_{f,t} = \frac{P_{f,t}}{P_t^C}$.

1.1.2 Household budget constraint and optimal decisions

As customary in the literature, family members perfectly insure each other against variation in labor income due to employment status, so that there is no ex-post heterogeneity

across individuals (Andolfatto (1996); Merz (1995)). The problem faced by the representative household can be summarised as:

$$\left\{ \begin{array}{l} \max_{C_t, B_t, x_t, N_t^E, K_t, I_t, \nu_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon_t^B \frac{(C_t - h^C C_{t-1})^{1-\sigma^C}}{1-\sigma^C} \right\} \\ s.t. \quad \frac{W_t}{P_t^C} L_t + (1 - L_t) b + r_t^K \nu_t K_t + (1 - \delta^N) (d_t + e_t) (x_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E) \\ \quad \quad \quad = C_t + e_t x_t + \frac{B_t}{P_t^C} + T_t + N_t^E \phi_t^E + \psi(\nu_t) K_t + I_t - R_{t-1} \frac{B_{t-1}}{P_t^C} \\ s.t. \quad N_t = (1 - \delta^N) (N_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E) \\ s.t. \quad K_t = (1 - \delta^K) K_{t-1} + (1 - AC_{t-1}^I) I_t \end{array} \right. , \quad (\text{B.3})$$

where the first line correspond to the utility function of the household (1.4), the second line its budget constraint in real terms (1.5), the third line the law of motion for goods and finally the law of motion of capital (Eq.1.11). The First Order Condition (FOC hereafter) with respect to consumption leads to the marginal utility λ_t^C :

$$\lambda_t^C = \varepsilon_t^B (C_t - h^C C_{t-1})^{-\sigma^C} . \quad (\text{B.4})$$

Combining it with the FOC with respect to B_t leads to the Euler condition on bonds,

$$\frac{\lambda_t^C}{\beta \mathbb{E}_t \{ \lambda_{t+1}^C \}} = \frac{R_t}{\mathbb{E}_t \pi_{t+1}^C} , \quad (\text{B.5})$$

where $\pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$ is the welfare-based inflation.

In the same way, we obtained the Euler condition on share using the FOC with respect to x_t ,

$$e_t = (1 - \delta^N) \mathbb{E}_t \{ \beta_{t,t+1} (d_{t+1} + e_{t+1}) \} , \quad (\text{B.6})$$

where $\beta_{t,t+1} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^C}{\lambda_t^C} \right\}$ is the stochastic discount factor of the representative household.

The free-entry condition is obtained using the FOC with respect to new entrants N_t^E ,

$$\phi_t^E = \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^N) (d_{t+1} + e_{t+1}) \left(1 - \frac{\partial AC_t^E N_t^E}{\partial N_t^E} \right) \right\} - \mathbb{E}_t \left\{ \frac{\lambda_{t+2}^C}{\lambda_t^C} \beta^2 (1 - \delta^N) (d_{t+2} + e_{t+2}) N_{t+1}^E \frac{\partial AC_{t+1}^E}{\partial N_t^E} \right\} , \quad (\text{B.7})$$

and combine it with the firm value equation (Eq.B.6):

$$\phi_t^E = e_t \left(1 - \frac{\partial AC_t^E N_t^E}{\partial N_t^E} \right) - \mathbb{E}_t \left\{ \beta_{t,t+1} e_{t+1} \frac{\partial AC_{t+1}^E N_{t+1}^E}{\partial N_t^E} \right\}, \quad (\text{B.8})$$

as in the text (Eq.1.9). Using the shape of the failure probability $AC_t^E = \frac{\varphi^E}{2} \left(\varepsilon_t^E \frac{N_t^E}{N_{t-1}^E} - 1 \right)^2$, we have $\frac{\partial AC_t^E N_t^E}{\partial N_t^E} = AC_t^E + \varepsilon_t^E \frac{N_t^E}{N_{t-1}^E} \varphi^E \left(\varepsilon_t^E \frac{N_t^E}{N_{t-1}^E} - 1 \right)$ and $\frac{\partial AC_{t+1}^E}{\partial N_t^E} = -\frac{N_{t+1}^E}{(N_t^E)^2} \varphi^E \varepsilon_{t+1}^E \left(\varepsilon_{t+1}^E \frac{N_{t+1}^E}{N_t^E} - 1 \right)$. Then, the entry condition in its complete form reads:

$$\phi_t^E = e_t \left(1 - AC_t^E + \frac{N_t^E}{N_{t-1}^E} \varepsilon_t^E \varphi^E \left(\varepsilon_t^E \frac{N_t^E}{N_{t-1}^E} - 1 \right) \right) + \mathbb{E}_t \left\{ \beta_{t,t+1} e_{t+1} \varepsilon_{t+1}^E \left(\frac{N_{t+1}^E}{N_t^E} \right)^2 \varphi^E \left(\varepsilon_{t+1}^E \frac{N_{t+1}^E}{N_t^E} - 1 \right) \right\}.$$

Turning to capital supply, the representative household choose the optimal amount of capital K_t such that:

$$q_t^K = \mathbb{E}_t \left\{ \beta_{t,t+1} \left(r_{t+1}^K \nu_{t+1} - \psi(\nu_{t+1}) + q_{t+1}^K (1 - \delta^K) \right) \right\}, \quad (\text{B.9})$$

where q_t^K denotes the shadow value of capital i.e. the Lagrange multiplier associated with the capital law of motion and normalised by the marginal utility of consumption¹. This shadow value is defined by the FOC for investment I_t :

$$1 = q_t^K \frac{\partial (1 - AC_t^I) I_t}{\partial I_t} - \mathbb{E}_t \left\{ q_{t+1}^K \beta_{t,t+1} \frac{\partial AC_{t+1}^I I_{t+1}}{\partial I_t} \right\}. \quad (\text{B.10})$$

where $\frac{\partial (1 - AC_t^I) I_t}{\partial I_t} = 1 - AC_t^I - \varepsilon_t^I \frac{I_t}{I_{t-1}} \varphi^I \left(\varepsilon_t^I \frac{I_t}{I_{t-1}} - 1 \right)$ and $\frac{\partial AC_{t+1}^I I_{t+1}}{\partial I_t} = -\varepsilon_{t+1}^I \left(\frac{I_{t+1}}{I_t} \right)^2 \varphi^I \left(\varepsilon_{t+1}^I \frac{I_{t+1}}{I_t} - 1 \right)$. Then in its complete form the shadow value is:

$$1 = q_t^K \left(1 - AC_t^I - \varepsilon_t^I \frac{I_t}{I_{t-1}} \varphi^I \left(\varepsilon_t^I \frac{I_t}{I_{t-1}} - 1 \right) \right) + \mathbb{E}_t \left\{ q_{t+1}^K \beta_{t,t+1} \varepsilon_{t+1}^I \left(\frac{I_{t+1}}{I_t} \right)^2 \varphi^I \left(\varepsilon_{t+1}^I \frac{I_{t+1}}{I_t} - 1 \right) \right\}.$$

Finally, the optimal utilization rate for capital is defined as,

$$\psi'(\nu_t) = r_t^K. \quad (\text{B.11})$$

¹ q_t^K corresponds to the Lagrange multiplier associated with the capital law of motion (Eq.1.11) normalised by the marginal utility of consumption λ_t^C .

Using the definition of the cost for changing the utilisation rate:

$\psi(\nu_t) = \frac{1-\Psi}{\Psi} r^K \left(e^{\frac{\Psi}{1-\Psi}(\nu_t-1)} - 1 \right)$, we can rewrite the optimal utilisation rate as:

$$r^K \left(e^{\frac{\Psi}{1-\Psi}(\nu_t-1)} - 1 \right) = r_t^K.$$

As demonstrated latter, the labor market law of motion from the household perspective writes: $L_t = (1 - \delta^N) (1 - \delta^L) L_{t-1} + f_t (1 - (1 - \delta^N) (1 - \delta^L) L_{t-1})$. Include this equation into the problem faced by the representative household (Eq.B.3) and optimise with respect to L_t , we obtain:

$$\mu_t^W = \frac{W_t}{P_t^C} - b + \mathbb{E}_t \left\{ \beta_{t,t+1} \mu_{t+1}^W (1 - \delta^N) (1 - \delta^L) (1 - f_{t+1}) \right\}. \quad (\text{B.12})$$

which correspond to the surplus from employment in the production sector for the representative household.

1.2 Producers

1.2.1 Cost minimization

Here we consider the maximisation problem solved by producer $f \in [0, N_t]$. In a first step, the firm chooses the optimal level of employment $l_{f,t}$, the number of vacancies to be posted $v_{f,t}$ and the optimal amount of capital ($k_{f,t}^\nu$) subject to the production function and the employment level. Formally, they solve:

$$\begin{cases} \max_{k_{f,t}^\nu, l_{f,t}, v_{f,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t (1 - \delta^N)^t \left\{ \frac{P_{f,t}}{P_t^C} y_{f,t} - (1 + AC_{f,t}^W) \frac{W_{f,t}}{P_t^C} l_{f,t} - r_t^K k_{f,t}^\nu - f^V v_{f,t} \right\} \\ s.t \quad l_{f,t} = (1 - \delta^L) l_{f,t-1} + q_t v_{f,t} \\ s.t \quad y_{f,t} = \varepsilon_t^Z (l_{f,t})^\alpha (k_{f,t}^\nu)^{1-\alpha} \end{cases}, \quad (\text{B.13})$$

The FOC with respect to the workforce is given by:

$$\mu_{f,t}^L = \alpha \frac{mC_{f,t} y_{f,t}}{l_{f,t}} - (1 + AC_{f,t}^W) \frac{W_{f,t}}{P_t^C} + \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^L) (1 - \delta^N) \mu_{f,t+1}^L \right\}, \quad (\text{B.14})$$

where $mc_{f,t}$ is the real marginal cost for the representative firms and $\mu_{f,t}^L$ the marginal utility to get a new worker (mathematically speaking they correspond respectively to the Lagrange multiplier associated with the production function $y_{f,t}$ and the labor market law of motion $l_{f,t}$).

The FOC with respect to vacancies reads,

$$\mu_{f,t}^L = \frac{f^V}{q_t}. \quad (\text{B.15})$$

The FOC with respect to capital is:

$$(1 - \alpha) \frac{mc_{f,t} y_{f,t}}{k_{f,t}^\nu} = r_t^K. \quad (\text{B.16})$$

Given Cobb-Douglas technology and perfect capital mobility, all firms choose the same capital/output ratio and, in turn, the same capital/labor and labor/output ratios. Then the marginal cost is symmetric across firms.

1.2.2 Price setting

In the second step, the firms chooses the price of its product subject to [Rotemberg \(1982\)](#) adjustment cost. More precisely, price adjustment costs are given by:

$$AC_{f,t}^P = \frac{\kappa^P}{2} \left(\frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right)^2,$$

where κ^P is the degree of rigidity and λ^P stands for indexation on past inflation.

Given the real marginal cost ($mc_{f,t}$), real profits can be rewritten as:

$d_{f,t} = \left(\frac{P_{f,t}}{P_t^C} - mc_{f,t} - \frac{P_{f,t}}{P_t^C} AC_{f,t}^P \right) y_{f,t}$. Thus, firms set price $P_{f,t}$ to maximise:

$$\max_{P_{f,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t (1 - \delta^N)^t \left(\frac{P_{f,t}}{P_t^C} - mc_{f,t} - \frac{P_{f,t}}{P_t^C} \frac{\kappa^P}{2} \left(\frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right)^2 \right) y_{f,t}.$$

The FOC of this problem is:

$$\begin{aligned} & \left(\frac{1}{P_t^C} - \frac{1}{P_t^C} AC_{f,t}^P - \frac{P_{f,t}}{P_t^C} \frac{\partial AC_{f,t}^P}{\partial P_{f,t}} \right) y_{f,t} + \left(\frac{P_{f,t}}{P_t^C} - mc_{f,t} - \frac{P_{f,t}}{P_t^C} AC_{f,t}^P \right) \frac{\partial y_{f,t}}{\partial P_{f,t}} \\ & + \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^N) \frac{\partial \left(\frac{P_{f,t+1}}{P_{t+1}^C} - mc_{f,t+1} - \frac{P_{f,t+1}}{P_{t+1}^C} AC_{f,t+1}^P \right) y_{f,t+1}}{\partial P_{f,t}} \right\} = 0. \end{aligned} \quad (\text{B.17})$$

Using the definition of price elasticity for a good f : $\theta_{f,t} = -\frac{\partial y_{f,t}}{\partial P_{f,t}} \frac{P_{f,t}}{y_{f,t}}$, we can rewrite this expression as:

$$\begin{aligned} & (1 - \theta_{f,t}) (1 - AC_{f,t}^P) + \theta_{f,t} \frac{mc_{f,t} P_t^C}{P_{f,t}} \\ & - P_{f,t} \frac{\partial AC_{f,t}^P}{\partial P_{f,t}} - \mathbb{E}_t \left\{ \beta_{t,t+1} \frac{\partial AC_{f,t+1}^P}{\partial P_{f,t}} (1 - \delta^N) \frac{P_{f,t+1} P_t^C}{P_{t+1}^C} \frac{y_{f,t+1}}{y_{f,t}} \right\} = 0. \end{aligned} \quad (\text{B.18})$$

With the markup defined as the real price ($\rho_{f,t} = \frac{P_{f,t}}{P_{C,t}}$) over the marginal cost: $\mu_{f,t} = \frac{\rho_{f,t}}{mc_t}$ and rearrange the previous equation we have:

$$\mu_{f,t} = \frac{\theta_{f,t}}{(\theta_{f,t-1} - 1) (1 - AC_{f,t}^P) + \frac{\partial AC_{f,t}^P}{\partial P_{f,t}} + \mathbb{E}_t \left\{ \frac{\partial AC_{f,t+1}^P}{\partial P_{f,t}} \beta_{t,t+1} (1 - \delta^N) \frac{P_{f,t+1} P_t^C}{P_{t+1}^C} \frac{y_{f,t+1}}{y_{f,t}} \right\}}. \quad (\text{B.19})$$

The FOC of the price adjustment cost in t is given by:

$$\frac{\partial AC_{f,t}^P}{\partial P_{f,t}} = \frac{\kappa^P}{P_{f,t-1}} \left(\frac{P_{f,t}}{P_{f,t-1}} - 1 - \lambda^P (\pi_{t-1} - 1) \right),$$

and in $t + 1$:

$$\frac{\partial AC_{f,t+1}^P}{\partial P_{f,t}} = -\frac{P_{f,t+1}}{(P_{f,t})^2} \kappa^P \left(\frac{P_{f,t+1}}{P_{f,t}} - 1 - \lambda^P (\pi_t - 1) \right).$$

Then, by using derivatives of the adjustment cost at t and $t + 1$ in Eq.B.19, we have:

$$\mu_{f,t} = \frac{\theta_{f,t}}{(\theta_{f,t-1} - 1) (1 - AC_{f,t}^P) + \kappa^P \psi_{f,t}^P}, \quad (\text{B.20})$$

where $\psi_{f,t}^P$ is given by:

$$\psi_{f,t}^P = \frac{\pi_{f,t} (\pi_{f,t} - 1 - \lambda^P (\pi_{t-1} - 1))}{-\mathbb{E}_t \left\{ \frac{y_{f,t+1} P_t^C}{y_{f,t} P_{t+1}^C} \beta_{t,t+1} (1 - \delta^N) (\pi_{f,t+1})^2 (\pi_{f,t+1} - 1 - \lambda^P (\pi_t - 1)) \right\}}, \quad (\text{B.21})$$

with the firm-level inflation $\pi_{f,t} = \frac{P_{f,t}}{P_{f,t-1}}$ and $\pi_t = \frac{P_t}{P_{t-1}}$ the aggregate product inflation.

1.3 Wages determination

Nominal wages are determined through a Nash bargaining scheme between workers and employers who maximise the joint surplus of employment by choosing the nominal wages. Formally, they solve:

$$\arg \max_{W_{f,t}} (\mu_{f,t}^W)^{\varepsilon_t^L \eta} (\mu_{f,t}^L)^{1-\eta \varepsilon_t^L},$$

with $\eta \in [0, 1]$ the negotiation power according to workers associated with an exogenous shock ε_t^L . $\mu_{f,t}^W$ is the employment surplus from the household perspective defined in Eq.B.12. Note that we used the subscript f for $\mu_{f,t}$. Due to constant returns, all workers are the same at the margin and the wage negotiation is between the firm and the marginal worker. Finally, $\mu_{f,t}^L$ is the employment surplus from the producer perspective defined in Eq.B.14.

The FOC with respect to nominal wage $W_{f,t}$ implies the following sharing rule,

$$(1 - \omega_t) \mu_{f,t}^W = \omega_t \mu_{f,t}^L, \quad (\text{B.22})$$

where ω_t is the time-varying negotiation power defined as:

$$\omega_t = \varepsilon_t^N \eta \left(\varepsilon_t^N \eta - (1 - \varepsilon_t^N \eta) \frac{\partial \mu_{f,t}^L}{\partial W_{f,t}} \frac{1}{\frac{\partial \mu_{f,t}^W}{\partial W_{f,t}}} \right)^{-1}.$$

Since the firms is subject to adjustment cost *à la* Rotemberg (1982) for adjusting the nominal wage $W_{f,t}$. Formally, they have to pay:

$$AC_{f,t}^W = \frac{\kappa^W}{2} \left(\frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W (\pi_{t-1} - 1) \right)^2,$$

where $\kappa^W \geq 0$ is the degree of rigidity and $\lambda^W \in [0, 1]$ is the indexation on past inflation π_{t-1} . Derive the employment surplus for the marginal worker with respect to $W_{f,t}$ gives:

$$\frac{\partial \mu_{f,t}^W}{\partial W_{f,t}} = \frac{1}{P_t^C},$$

and for the employment surplus for the producers,

$$\begin{aligned} \frac{\partial \mu_{f,t}^W}{\partial W_{f,t}} &= -\partial \frac{(1 + AC_{f,t}^W)W_{f,t}}{\partial W_{f,t}} \frac{1}{P_t^C} + \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^L) (1 - \delta^N) \frac{\partial \mu_{f,t+1}^L}{\partial W_{f,t}} \right\} \\ \Leftrightarrow &= -(1 + AC_{f,t}^W + W_{f,t} \partial \frac{AC_{f,t}^W}{\partial W_{f,t}}) \frac{1}{P_t^C} - \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^L) (1 - \delta^N) \partial \frac{AC_{f,t+1}^W W_{f,t+1}}{\partial W_{f,t}} \frac{1}{P_{t+1}^C} \right\}, \end{aligned}$$

where $\partial \frac{AC_{f,t}^W}{\partial W_{f,t}} = \frac{1}{W_{f,t-1}} \kappa^W \left(\frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W (\pi_{t-1} - 1) \right)$ and $\partial \frac{AC_{f,t+1}^W}{\partial W_{f,t}} = -\frac{W_{f,t+1}}{(W_{f,t})^2} \kappa^W \left(\frac{W_{f,t+1}}{W_{f,t}} - 1 - \lambda^W (\pi_t - 1) \right)$. Then we can rewrite the FOC of the employment surplus as:

$$\begin{aligned} \frac{\partial \mu_{f,t}^W}{\partial W_{f,t}} &= - \left(1 + AC_{f,t}^W + \frac{W_{f,t}}{W_{f,t-1}} \kappa^W \left(\frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W (\pi_{t-1} - 1) \right) \right) \\ &+ \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^L) (1 - \delta^N) \left(\frac{W_{f,t+1}}{W_{f,t}} \right)^2 \kappa^W \left(\frac{W_{f,t+1}}{W_{f,t}} - 1 - \lambda^W (\pi_t - 1) \right) \frac{P_t^C}{P_{t+1}^C} \right\}. \end{aligned}$$

Then the time-varying negotiation power reads:

$$\omega_t = \frac{\varepsilon_t^N \eta}{(\varepsilon_t^N \eta + (1 - \varepsilon_t^N \eta) (1 + AC_{f,t}^W + \kappa^W \psi_{f,t}^W))},$$

where $\psi_{f,t}^W$ is the auxiliary variable that depends on the Rotemberg adjustment cost used:

$$\psi_{f,t}^W = -\mathbb{E}_t \left\{ \beta_{t,t+1} \frac{(1-\delta^L)(1-\delta^N)}{\pi_{t+1}^C} (\pi_{f,t+1}^W)^2 (\pi_{f,t+1}^W - 1 - \lambda^W (\pi_t - 1)) \right\},$$

where $\pi_{f,t}^W = \frac{w_t}{w_{t-1}} \pi_t$ is the nominal wage inflation rate at firm f .

After presenting the time-varying negotiation power, we turn to the wage setting using the sharing rule (Eq.B.22) and replace with the expression of surplus from both perspectives

(firms in Eq.B.14 and the marginal worker in Eq.B.12) we have:

$$\frac{W_t}{P_t^C} (1 + \omega_t AC_{f,t}^W) = \omega_t \left(\alpha \frac{mc_{f,t} y_{f,t}}{l_{f,t}} + \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^L) (1 - \delta^N) \mu_{f,t+1}^L \right\} \right) + (1 - \omega_t) b - (1 - \omega_t) \mathbb{E}_t \left\{ \beta_{t,t+1} \mu_{f,t+1}^W (1 - \delta^N) (1 - \delta^L) (1 - f_{t+1}) \right\}$$

Since the sharing rule holds in $t + 1$ ($\mathbb{E}_t \mu_{f,t+1}^W = \mathbb{E}_t \frac{\omega_{t+1}}{(1 - \omega_{t+1})} \mu_{f,t+1}^L$), we can rewrite the previous equation as:

$$\frac{W_t}{P_t^C} (1 + \omega_t AC_{f,t}^W) = \omega_t \left(\alpha \frac{mc_{f,t} y_{f,t}}{l_{f,t}} \right) + (1 - \omega_t) b + \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^L) (1 - \delta^N) \mu_{f,t+1}^L \left(\omega_t - \frac{(1 - \omega_t) \omega_{t+1}}{(1 - \omega_{t+1})} (1 - f_{t+1}) \right) \right\},$$

and using the the free-entry condition for vacancies (Eq. B.15) to replace $\mu_{f,t+1}^L$:

$$\frac{W_t}{P_t^C} (1 + \omega_t AC_{f,t}^W) = \omega_t \left(\alpha \frac{mc_{f,t} y_{f,t}}{l_{f,t}} \right) + (1 - \omega_t) b + \mathbb{E}_t \left\{ \beta_{t,t+1} (1 - \delta^L) (1 - \delta^N) \frac{f^V}{q_{t+1}} \left(\omega_t - \frac{(1 - \omega_t) \omega_{t+1}}{(1 - \omega_{t+1})} (1 - f_{t+1}) \right) \right\}.$$

Then in the case of a flexible wage (absence of exogenous shock i.e. ε_t^L and rigidity i.e. $\kappa^W = \lambda^W = 0$), we have:

$$\frac{W_t}{P_t^C} = \eta \left(\alpha \frac{mc_{f,t} y_{f,t}}{l_{f,t}} \right) + (1 - \eta) b + \eta \mathbb{E}_t \left\{ \beta_{t,t+1} \frac{f^V}{q_{t+1}} (1 - \delta^L) (1 - \delta^N) \right\}.$$

1.4 Symetric equilibrium, new entrants and aggregation

1.4.1 Symetric equilibrium

Given Cobb-Douglas technology and perfect capital mobility, all producers choose the same capital/output ratio and, in turn, the same capital/labor and labor/output ratios. As a consequence, the marginal cost is symmetric across firms ($mc_{f,t} = mc_t$). Thus, equilibrium prices and quantities are identical across producers f .

1.4.2 New entrants

To enter the goods market, the representative new entrant e ($e \in [0, N_t^E]$) need to cover the following cost:

$$\phi_{e,t}^E = \frac{f^E}{\varepsilon_t^E} + f^V v_{e,t}^E,$$

where the first part stands for technological requirement subject to an exogenous shock ε_t^E and the second the vacancy posting cost. Since, new firms need to post as many vacancy to reach the size of the labor force of an incumbent f and considering that they only produce in the next period, $v_{e,t}^E$ reads:

$$v_{e,t}^E = \frac{(1 - AC_t^E) (1 - \delta^N) (1 - \delta^L) l_{f,t}}{\mathbb{E}_t \{q_{t+1}\}}.$$

Since all producers choose the same amount of labor $l_{f,t} = l_t$, the cost for entering the market is identical across new entrants. Thus, the firm's value are identical across new entrants i.e. $\phi_{e,t}^E = \phi_t^E$.

1.4.3 Aggregation

Labor market Recall that for an individual firm we have the following employment-level:

$$l_{f,t} = (1 - \delta^L) l_{f,t-1} + q_t v_{f,t}.$$

At the aggregate level, we have $L_t = \int_0^{N_t} l_{f,t} df$ and $V_t = \int_0^{N_t} v_{f,t} + \int_0^{N_{t-1}^E} v_{e,t-1}^E de$. Using the fact that all producers choose the same amount of labor and vacancies (respectively $L_t = N_t l_t$ and $N_t v_t$) and new entrants the same amount of vacancies ($N_{t-1}^E v_{t-1}^E$), we can rewrite the previous equation as:

$$L_t = \frac{N_t}{N_{t-1}} (1 - \delta^L) L_{t-1} + q_t (V_t - N_{t-1}^E v_{t-1}^E).$$

Using the law of motion for variety with $\frac{N_t}{N_{t-1}} = (1 - \delta^N) \left(1 + (1 - AC_{t-1}^E) \frac{N_{t-1}^E}{N_{t-1}}\right)$, we obtain:

$$L_t = (1 - \delta^N) (1 - \delta^L) L_{t-1} + q_t V_t + (1 - AC_{t-1}^E) \frac{N_{t-1}^E}{N_{t-1}} (1 - \delta^N) (1 - \delta^L) L_{t-1} - q_t N_{t-1}^E v_{t-1}^E.$$

Using the total amount of vacancies posted by a new firm $v_t^E = (1 - AC_t^E) \frac{(1 - \delta^L)(1 - \delta^N)L_t}{\mathbb{E}_t\{q_{t+1}\}N_t}$ the last two terms cancel each other out, leading to the aggregate labor law of motion:

$$L_t = (1 - \delta^N) (1 - \delta^L) L_{t-1} + q_t V_t. \quad (\text{B.23})$$

Then it is important that new firms that enter the market only filling vacancy in the next period on the contrary of incumbent. Without this assumption, the aggregate level of employment cannot be read as : $L_t = N_t l_t$ and then we cannot obtained the previous form for aggregate labor law of motion.

Aggregate production The aggregate production of goods Y_t reads as follows:

$$Y_t = \varepsilon_t^Z (L_t)^\alpha (K_t^\nu)^{1-\alpha},$$

where $Y_t = \int_0^{N_t} y_{f,t} df$, $L_t = \int_0^{N_t} l_{f,t} df$ and $K_t^\nu = \int_0^{N_t} k_{f,t}^\nu df$. In a symmetric equilibrium this simplifies to:

$$\Leftrightarrow N_t y_t = \varepsilon_t^Z (L_t)^\alpha (K_t^\nu)^{1-\alpha},$$

with $L_t = N_t l_t$ and $K_t^\nu = N_t k_t^\nu$.

National Income Accounts To define the GDP, we need to aggregate the household budget constraint:

$$\begin{aligned} w_t L_t + (1 - L_t) b + R_{t-1} \frac{B_{t-1}}{P_t^C} + r_t^K \nu_t K_t + (1 - \delta^N) (d_t + e_t) (x_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E) \\ = C_t + e_t x_t + \frac{B_{t+1}}{P_t^C} + T_t + N_t^E \phi_t^E + \psi(\nu_t) K_t + I_t \end{aligned},$$

with $x_t = N_t$ and using the law of motions for firms

($N_t = (1 - \delta^N) (N_{t-1} + (1 - AC_{t-1}^E) N_{t-1}^E)$), we have:

$$\begin{aligned} & \Leftrightarrow w_t L_t + (1 - L_t) b + R_{t-1} \frac{B_t}{P_t^C} + r_t^K \nu_t K_t + N_t d_t \\ & = C_t + \frac{B_{t+1}}{P_t^C} + T_t + N_t^E \phi_t^E + \psi(\nu_t) K_t + I_t \end{aligned}$$

Using the definition of effective capital $K_t^\nu = \nu_t K_t$, the definition of individual profits $d_t = \rho_t y_t - w_t l_t (1 + AC_t^W) - r_t^K k_t^\nu - \rho_t y_t AC_t^P - f^V v_t$, aggregate labor $L_t = n_t l_t$, aggregate capital services $K_t^\nu = n_t k_t^\nu$ and the budget government constraint (Eq.1.33), we have:

$$\rho_t y_t N_t = C_t + g^Y \varepsilon_t^G + \psi(\nu_t) K_{t-1} + I_t + f^V v_t N_t + N_t^E \phi_t^E + w_t L_t AC_t^W + \rho_t y_t N_t AC_t^P. \quad (\text{B.24})$$

where the right side of the equation corresponds to the final consumption basket (Y_t^C) using the optimal demand in Eq.B.2.

Prices Identities Concerning prices, rearranging the translog expenditure function (Eq.1.1) and imposing symmetry among producers, the relative price $\rho_t = \frac{P_t}{P_t^C}$ emerges,

$$\rho_t = \exp \left(- \frac{\tilde{N} - N_t}{2\sigma^P \varepsilon_t^P \tilde{N} N_t} \right). \quad (\text{B.25})$$

Thus, the relative price $\rho_t = \frac{P_t}{P_t^C}$, the product price inflation $\pi_t = \frac{P_t}{P_{t-1}}$ and welfare-base inflation $\pi_t^C = \frac{P_t^C}{P_{t-1}^C}$ can be linked through:

$$\frac{\rho_t}{\rho_{t-1}} = \frac{\pi_t}{\pi_t^C}. \quad (\text{B.26})$$

Since, we have imposing symmetry among producers, the wage nominal inflation rate π_t^W is given by:

$$\pi_t^W = \frac{w_t}{w_{t-1}} \pi_t,$$

with w_t the real wage.

2 Steady state

From the Euler condition, we have the steady state value for interest rate, $r = \frac{1}{\beta} - 1$ and by definition we have $\pi = 1$, $q^K = 1$, $\pi^W = 1$ and $\pi^C = 1$.

Start with the law of motion for aggregate employment described in the general equilibrium conditions but from the household perspective:

$$L = (1 - \delta^N) (1 - \delta^L) L + fU.$$

with $U = 1 - (1 - \delta^N) (1 - \delta^L) L$ the aggregate unemployment rate for job seekers. Using the calibration in Tab.1.1 for δ^N , δ^L and f , we have the steady-state value for labor at the aggregate level:

$$L = \frac{f}{((1 - \delta^N) (1 - \delta^L) f + 1 - (1 - \delta^N) (1 - \delta^L))},$$

and by extension the aggregate unemployment rate for job seekers U . Using the definition of the probability for a firm to find a worker $q = \frac{M}{V}$ and an unemployed worker to find a job $f = \frac{M}{U}$, we obtained the aggregate vacancy such as : $V = \frac{fU}{q}$. Thus, we have the flow of new hires through $fU = qV = M$ where M stands for the matching function. We deduce the matching efficiency on the matching function $m = \frac{M}{VqU^{1-\xi}}$. Note that the steady-state of unemployment rate used in the estimation correspond to $1 - L$ in order to have a SS close from the average mean over the period. Concerning the product market, we start with the dispersion price (Eq.B.25). After some arrangement to make appear the ratio of incumbent to potential producers $\frac{N}{N}$ which is calibrate in the expression of the relative price (Eq.1.36), we have:

$$\rho = e^{(-\frac{1}{2(\theta-1)}(1-\frac{N}{N}))}.$$

Using the definition of markup: $\mu = \frac{\theta}{\theta-1}$, we get the expression of marginal cost using Eq.1.23 $mc = \frac{\rho}{\mu}$.

Using the optimal capital (Eq.1.13), we can obtain the amount of capital which is equivalent to the capital used at the steady state (i.e. $\nu = 1$, $\psi(\nu) = 0$ and $K = K^\nu$),

$$K = \left(\frac{1 - (1 - \delta^K)\beta}{L^\alpha \beta (1 - \alpha) mc} \right)^{-\frac{1}{\alpha}},$$

and by extension the production function ($Y = (L)^\alpha (K)^{1-\alpha}$).

Thus, we define the value of unemployment as in [Gertler et al. \(2008\)](#) by $\tilde{b} = \frac{bL}{m\alpha Y}$. In order to obtain the steady-state value of wage w and the vacancy cost f^V , we use the job creation condition (Eq.1.18 and Eq.B.15) and the equation of Nash bargaining (Eq.1.31) as a system:

$$\begin{cases} \frac{f^V}{q} = \alpha mc \frac{Y}{L} - w + \frac{f^V}{q} \beta (1 - \delta^N) (1 - \delta^L) \\ w = \eta \alpha \frac{mcY}{L} + (1 - \eta) b + \eta \beta \frac{f^V f}{q} (1 - \delta^L) (1 - \delta^N) \end{cases},$$

rearrange the system and make appear the steady-state value of unemployment (\tilde{b}):

$$\begin{cases} f^V = \phi_1 (\alpha mc \frac{Y}{L} - w) \\ w = \alpha \frac{mcY}{L} (\eta + (1 - \eta) \tilde{b}) + \eta \beta f^V \frac{f}{q} (1 - \delta^L) (1 - \delta^N) \end{cases},$$

with $\phi_1 = \frac{q}{(1 - \beta(1 - \delta^N)(1 - \delta^L))}$. Then, replacing the second line in the first gets the steady-state value of posting vacancy:

$$f^V = \frac{\phi_1 \alpha mc \frac{Y}{L} (1 - \eta) (1 - \tilde{b})}{(1 + \phi_1 \eta \beta \frac{f}{q} (1 - \delta^L) (1 - \delta^N))},$$

and then the transfert to unemployed workers is given by : $b = \tilde{b} \frac{m\alpha Y}{L}$.

For the number of producers, we use the Euler condition on equity (Eq.B.6), the equity value (Eq.B.8) and the form of dividends to get the number of incumbent:

$$N = \frac{\left(\frac{\beta(1 - \delta^N)}{(1 - \beta(1 - \delta^N))} \left(1 - \frac{1}{\mu}\right) Y \rho - f^V \frac{(1 - \delta^L)(1 - \delta^N)L}{q} \right)}{f^E}. \quad (\text{B.27})$$

Since new entrants need to post vacancy to reach the same workforce size than incumbent, we have: $v^E = \frac{(1-\delta^N)(1-\delta^L)L}{q^N}$.

After that, we have the number of new entrants $N^E = \frac{\delta^N}{(1-\delta^N)}N$, the maximum of goods that the economy can reach $\tilde{N} = N/.95$ and individual dividends $d = (\rho - mc) \frac{Y}{N}$. Using the definition of optimal demand (Eq.B.2), the capital law of motion and the definition of aggregate demand (Eq.B.24) we get respectively: $Y^C = \rho Y$, $I = \delta^K K$ and $C = (1 - g^Y)Y^C - I - Nf^V v - N^E (f^E + f^V v^E)$.

Now, we show that the proportion of barrier to entry f^E with respect to the equity value e is high in the model whatever the amount of fixed cost specified. We start with the expression of the steady-state number of firms N (Eq.B.27) and make appears $\tau^E = \frac{f^E}{e}$, we have

$$\tau^E = \frac{e}{\left(\frac{\beta(1-\delta^N)}{(1-\beta(1-\delta^N))} \left(1 - \frac{1}{\mu}\right) y\rho - f^V \frac{(1-\delta^L)(1-\delta^N)L}{q} \right)}$$

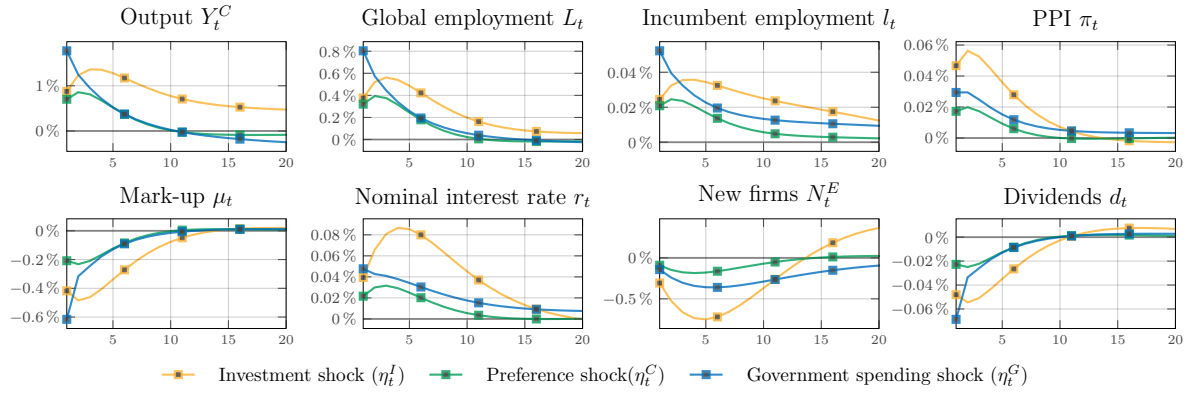
with $e = \beta(1 - \delta^N)(d + e)$ and $d = \left(1 - \frac{1}{\mu}\right)y$, we get :

$$\Leftrightarrow \tau^E = \frac{\frac{\beta(1-\delta^N)}{(1-\beta(1-\delta^N))} \left(1 - \frac{1}{\mu}\right) y\rho}{\left(\frac{\beta(1-\delta^N)}{(1-\beta(1-\delta^N))} \left(1 - \frac{1}{\mu}\right) y\rho - f^V \frac{(1-\delta^L)(1-\delta^N)L}{q} \right)},$$

since N does not appear in the right side of this equation and it is the only variable dependants on f^E , then τ^E is independant of f^E .

3 Additional figure

FIGURE B.1: System response to demand shocks.



Notes: Impulse Response Functions (IRFs) are generated when parameters are drawn from the mean posterior distribution. IRFs are reported in percentage deviations from the deterministic steady state.

Appendix C

Appendix for Chapter II

In this appendix, we describe the main features of the chapter II with the complete set of First Order Condition (FOC hereafter) and the steady-state.

1 Non-linear model

In this section, we present the model in such a way that the collateral constraint for borrowers appears in different forms i.e. our version of the collateral constraint, the exogenous component as in [Iacoviello \(2015\)](#) and the classic version without exogenous components.

1.1 Households

1.1.1 Impatient households

The problem faced by the borrowers can be summarised as:

$$\begin{aligned}
& \max_{\{c_t^I, h_t^I, d_t^I, l_t^I\}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^I)^t \{ (1 - h^C) \log(c_t^I - h^C c_{t-1}^I) + \varepsilon_t^H j \log(h_t^I) \} \\
& \quad s.t. \quad c_t^I + q_t^H (h_t^I - h_{t-1}^I) + r_{t-1} d_{t-1}^I = w_t^I l_t^I + (1 - l_t^I) b_I + d_t^I \\
& \quad \quad s.t. \quad l_t^I = (1 - \delta^L) l_{t-1}^I + f_t^I (1 - (1 - \delta^L) l_{t-1}^I) . \\
& \quad s.t. \quad d_t^I = \begin{cases} a \{ (1 - \delta^L) l_{t-1}^I d_{t-1}^I + \varepsilon_t^M f_t^I (1 - (1 - \delta^L) l_{t-1}^I) m^I \mathbb{E}_t \{ q_{t+1}^H \} h_t^I \} \\ + (1 - a) \{ \chi d_{t-1}^I + (1 - \chi) \varepsilon_t^M m^I \mathbb{E}_t \{ q_{t+1}^H \} h_t^I \} \end{cases} ,
\end{aligned}$$

using the utility function (Eq.2.5), the budget constraint (Eq.2.6) and the labor market law of motion (Eq.2.4). We also use a version of the collateral constraint such that if $a = 0$, we have the exogenous version of the collateral constraints. Moreover, if $a = 0$ and $\chi = 0$ we have the original version of collateral constraint used in [Kiyotaki and Moore \(1997\)](#). We note by λ_t^I the lagrangian mutliplier associated with the budget constraint i.e. the marginal utility of consumption, μ_t^I the Lagrangian multiplier associated to the labor market law of motion normalised by λ_t^I and ϕ_t^I the Lagrangian multiplier associated with the collateral constraint and normalised by λ_t^I . Thus, the FOC with respect to consumption is:

$$\lambda_t^I = \frac{(1 - h^C)}{(c_t^I - h^C c_{t-1}^I)} .$$

As in the text we note by $\beta_{t,t+1}^I = \beta^I \mathbb{E}_t \left(\frac{c_t^I - h^C c_{t-1}^I}{c_{t,t+1}^I - h^C c_t^I} \right)$ the stochastic discount factor of borrowers. The FOC for housing is:

$$\varepsilon_t^H j \frac{(c_t^I - h^C c_{t-1}^I)}{(1 - h^C) h_t^I} = q_t^H - \mathbb{E}_t \{ \beta_{t,t+1}^I q_{t+1}^H \} - \mathbb{E}_t \{ q_{t+1}^H \} \varepsilon_t^M \phi_t^I m^I \begin{cases} a f_t^I (1 - (1 - \delta^L) l_{t-1}^I) \\ + (1 - a) (1 - \chi) \end{cases} . \tag{C.1}$$

For the Euler condition we have:

$$\mathbb{E}_t \{ \beta_{t,t+1}^I \} r_t = 1 - \phi_t^I + \mathbb{E}_t \{ \beta_{t,t+1}^I \phi_{I,t+1} \} \begin{cases} a(1 - \delta^L) l_t^I \\ +(1 - a)\chi \end{cases} . \quad (\text{C.2})$$

Finally, the FOC with respect to n_t^I is:

$$\mu_t^I = \begin{aligned} & w_t^I - b^I + \mathbb{E}_t \{ \beta_{t,t+1}^I \mu_{t+1}^I (1 - \delta^L) (1 - f_{t+1}^I) \} \\ & + \mathbb{E}_t \{ a \beta_{t,t+1}^I \phi_{t+1}^I (1 - \delta^L) (d_t^I - \varepsilon_{t+1}^M f_{t+1}^I m^I q_{t+2}^H h_{t+1}^I) \} . \end{aligned} \quad (\text{C.3})$$

Since the second line depends only on our collateral constraint i.e. when $a = 1$, we can rewrite this using the definition of the debt in $t + 1$:

$$\begin{aligned} d_{t+1}^I &= (1 - \delta^L) l_t^I d_t^I + \varepsilon_{t+1}^M f_{t+1}^I (1 - (1 - \delta^L) l_t^I) m^I q_{t+2}^H h_{t+1}^I, \\ &\Leftrightarrow \frac{d_{t+1}^I - (1 - \delta^L) l_t^I d_t^I}{(1 - (1 - \delta^L) l_t^I)} = \varepsilon_{t+1}^M f_{t+1}^I m^I q_{t+2}^H h_{t+1}^I. \end{aligned}$$

Then replace it in the marginal value of being employed:

$$\mu_t^I = \begin{aligned} & w_t^I - b^I + \mathbb{E}_t \{ \beta_{t,t+1}^I \mu_{t+1}^I (1 - \delta^L) (1 - f_{t+1}^I) \} \\ & + \mathbb{E}_t \left\{ a \beta_{t,t+1}^I \phi_{t+1}^I (1 - \delta^L) \left(\frac{d_t^I - d_{t+1}^I}{(1 - (1 - \delta^L) l_t^I)} \right) \right\} . \end{aligned}$$

Note by $\Delta d_{t+1}^I = d_{t+1}^I - d_t^I$ and using the definition of unemployment we have the same equation as in the text (Eq.2.11):

$$\mu_t^I = \begin{aligned} & w_t^I - b^I + \mathbb{E}_t \{ \beta_{t,t+1}^I \mu_{t+1}^I (1 - \delta^L) (1 - f_{t+1}^I) \} \\ & - \mathbb{E}_t \left\{ a \beta_{t,t+1}^I \phi_{t+1}^I (1 - \delta^L) \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\} . \end{aligned}$$

Then, it is more obvious than our collateral constraint acting on the choice of consumption, housing and labor since in the case of exogenous collateral constraints they only act as a persistence mechanism and not a link between markets.

1.1.2 Patient household

The problem faced by the lenders can be summarised as:

$$\begin{aligned}
& \max_{\{c_t^P, h_t^P, d_t^P, l_t^P, K_t, v_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{ (1 - h^C) \log(c_t^P - h^C c_{t-1}^P) + \varepsilon_t^H j \log(h_t^P) \right\} \\
s.t. \quad & c_t^P + q_t^H (h_t^P - h_{t-1}^P) + d_t^P + T_t^P + \left(\frac{K_t - (1 - \delta_t^K) K_{t-1}}{\varepsilon_t^I} \right) + \frac{\phi^K}{2} \frac{(K_t - K_{t-1})^2}{K} \\
& = w_t^P l_t^P + (1 - l_t^P) b^P + r_{t-1} d_{t-1}^P + z_t K_{t-1} v_t + \Pi_t \\
& s.t. \quad l_t^P = (1 - \delta^L) l_{t-1}^P + f_t^P (1 - (1 - \delta^L) l_{t-1}^P) \\
& \delta_t^K = \delta^K + \left(\frac{1}{\beta^P} + 1 - \delta^K \right) \left(\frac{1}{2} \left(\frac{\psi}{1 - \psi} \right) (v_t)^2 + \frac{(1 - 2\psi)}{1 - \psi} v_t + \frac{1}{2} \left(\frac{\psi}{1 - \psi} \right) - 1 \right)
\end{aligned}$$

using the budget constraint (Eq.2.13), the labor market law of motion (Eq.2.4) and the functional form of the capital depreciation rate (Eq.2.20). Let λ_t^P the Lagrangian multiplier associated with the budget constraint i.e. the marginal utility of consumption and μ_t^P the Lagrangian multiplier associated to the labor market law of motion normalised by λ_t^P . The FOC for consumption is:

$$\lambda_t^P = \frac{(1 - h^C)}{(c_t^P - h^C c_{t-1}^P)}.$$

As in the text, the stochastic discount factor for lenders is denoted by $\beta_{t,t+1}^P = \beta^P \mathbb{E}_t \left(\frac{c_t^P - h^C c_{t-1}^P}{c_{P,t+1} - h^C c_t^P} \right)$. Then the FOC for housing is given by:

$$\varepsilon_t^H j \frac{(c_t^P - h^C c_{t-1}^P)}{(1 - h^C) h_t^P} = q_t^H - \mathbb{E}_t \left\{ \beta_{t,t+1}^P q_{t+1}^H \right\}.$$

The Euler condition is obtained with the FOC for deposits:

$$\mathbb{E}_t \left\{ \beta_{t,t+1}^P \right\} r_t = 1. \quad (C.4)$$

For labor, we have:

$$\mu_t^P = w_t^P - b^P + \mathbb{E}_t \left\{ \mu_{t+1}^P \beta_{t,t+1}^P (1 - \delta^L) (1 - f_{t+1}^P) \right\}. \quad (C.5)$$

Concerning capital services, the FOC with respect to K_t and v_t are respectively:

$$\frac{1}{\varepsilon_t^I} + \phi^K \frac{(K_t - K_{t-1})}{K} = \mathbb{E}_t \left\{ \beta_{t,t+1}^P \left(\frac{(1 - \delta_{t+1}^K)}{\varepsilon_{t+1}^I} + z_{t+1} v_{t+1} + \phi^K \frac{(K_{t+1} - K_t)}{K} \right) \right\}, \quad (\text{C.6})$$

$$z_t = \left(\frac{1}{\beta^P} + 1 - \delta^K \right) \left(\left(\frac{\psi}{1 - \psi} \right) v_t + \frac{(1 - 2\psi)}{1 - \psi} \right). \quad (\text{C.7})$$

1.2 Firms

The representative firm maximise their dividends subject to the labor market law of motion with both types of household:

$$\left\{ \begin{array}{l} \max_{\{v_t^j, l_t^j, K_t^U\}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^P)^t \left\{ Y_t - z_t K_t^U - \sum_{j=P,I} \left(w_t^j l_t^j + \varepsilon_t^L \frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j \right) \right\} \\ \text{s.t. } l_t^j = (1 - \delta^L) l_{t-1}^j + q_t^j v_t^j \end{array} \right. ,$$

We start with the labor force provides by patient household i.e. with $j = P$. Let $x_t^j = \frac{q_t^j v_t^j}{l_{t-1}^j}$ be the hiring rate, the FOC for posting a vacancy is:

$$\mu_{P,t}^L = \varepsilon_t^L \kappa^P x_t^P, \quad (\text{C.8})$$

and the FOC with respect to l_t^P :

$$\mu_{P,t}^L = \frac{(1 - \alpha) \lambda Y_t}{l_t^P} - w_t^P + \mathbb{E}_t \left\{ \beta_{t,t+1}^P \left(\varepsilon_{t+1}^L \frac{\kappa^P}{2} (x_{t+1}^P)^2 + (1 - \delta^L) \mu_{P,t+1}^L \right) \right\}. \quad (\text{C.9})$$

By combining the two FOCs we have the same job creation as in the text (Eq.2.24). For the impatient household i.e. with $j = I$, the FOC for posting a vacancy is:

$$\mu_{I,t}^L = \varepsilon_t^L \kappa^I x_t^I, \quad (\text{C.10})$$

and for l_t^I :

$$\mu_{I,t}^L = \frac{(1 - \alpha) (1 - \lambda) Y_t}{l_t^I} - w_t^I + \mathbb{E}_t \left\{ \beta_{t,t+1}^P \left(\varepsilon_{t+1}^L \frac{\kappa^I}{2} (x_{t+1}^I)^2 + (1 - \delta^L) \mu_{I,t+1}^L \right) \right\}. \quad (\text{C.11})$$

As for lenders, combines the two FOC, we get the job creation as in the text (Eq.2.25).

Turning to the choice of capital, the FOC with respect to K_t^U is :

$$z_t = \alpha \frac{Y_t}{K_t^U}. \quad (\text{C.12})$$

1.3 Wage setting

The period-by-period Nash bargaining implies that the firm and each of its workers determine the wage in period t by solving the following problem:

$$\arg \max_{\{w_t^j\}} (\mu_t^j)^\eta (\mu_{j,t}^L)^{1-\eta},$$

for $j = \{I, P\}$ and $\eta \in [0, 1]$ the power of negotiation for the worker. Whatever the type of worker, the solution of this problem gives:

$$\eta \mu_{j,t}^L = (1 - \eta) \mu_t^j.$$

For the patient household, we use the marginal value of a new match from the firm's perspective i.e. $\mu_{P,t}^L$ (Eq.C.9) and from the household's perspective i.e. μ_t^P (Eq.C.5). Replace it and rearrange to make appear w_t^P :

$$w_t^P = \frac{\eta \left(\frac{(1-\alpha)\lambda Y_t}{l_t^P} + \mathbb{E}_t \left\{ \beta_{t,t+1}^P \left[\varepsilon_{t+1}^L \frac{\kappa^P}{2} (x_{t+1}^P)^2 + (1 - \delta^L) \mu_{P,t+1}^L \right] \right\} \right)}{(1-\eta) (b^P - \mathbb{E}_t \{ \beta_{t,t+1}^P \mu_{t+1}^P (1 - \delta^L) (1 - f_{t+1}^P) \})}.$$

Using the fact that the Nash Bargaining hold in $t + 1$ we can replace μ_{t+1}^P by $\mu_{t+1}^P = \frac{\eta}{(1-\eta)} \mu_{P,t+1}^L$.

$$w_t^P = \frac{\eta \left(\frac{(1-\alpha)\lambda Y_t}{l_t^P} + \varepsilon_{t+1}^L \mathbb{E}_t \left\{ \beta_{t,t+1}^P \frac{\kappa^P}{2} (x_{t+1}^P)^2 \right\} \right) + (1 - \eta) b^P}{+ \eta (1 - \delta^L) \mathbb{E}_t \{ \beta_{t,t+1}^P \mu_{P,t+1}^L f_{t+1}^P \}},$$

and using the FOC with respect to vacancies i.e. $\mu_{P,t}^L = \varepsilon_t^L \kappa^P \frac{q_t^P v_t^P}{l_{t-1}^P}$ (Eq.C.8) and the definition of the hiring rate, we have the same equation as in the text :

$$w_t^P = \eta \left(\frac{(1-\alpha)\lambda Y_t}{l_t^P} + \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \beta_{t,t+1}^P \frac{\kappa^P}{2} (x_{t+1}^P)^2 \right\} \right) + (1-\eta) b^P + \eta (1-\delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^P \varepsilon_{t+1}^L \kappa^P x_{t+1}^P f_{t+1}^P \right\} .$$

Concerning the impatient household, we follow the same step to have the definition of wage. Using the definition of μ_t^I in Eq.C.3 and $\mu_{I,t}^L$ in Eq.C.11, we have:

$$w_t^I = \eta \left(\frac{(1-\alpha)(1-\lambda)Y_t}{l_t^I} + \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \beta_{t,t+1}^P \frac{\kappa^I}{2} (x_{t+1}^I)^2 + \beta_{t,t+1}^P (1-\delta^L) \mu_{I,t+1}^L \right\} \right) + (1-\eta) (b^I - \mathbb{E}_t \left\{ \beta_{t,t+1}^I \mu_{t+1}^I (1-\delta^L) (1-f_{t+1}^I) \right\}) + (1-\eta) \mathbb{E}_t \left\{ a \beta_{t,t+1}^I \phi_{t+1}^I (1-\delta^L) \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\} .$$

Using the $t+1$ relationship, we can replace μ_{t+1}^I by $\mu_{t+1}^I = \frac{\eta}{(1-\eta)} \mu_{I,t+1}^L$:

$$w_t^I = \eta \left(\frac{(1-\alpha)(1-\lambda)Y_t}{l_t^I} + \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \beta_{t,t+1}^P \frac{\kappa^I}{2} (x_{t+1}^I)^2 \right\} \right) + (1-\eta) b^I + \mathbb{E}_t \left\{ \eta \mu_{I,t+1}^L (1-\delta^L) (\beta_{t,t+1}^P - \beta_{t,t+1}^I (1-f_{t+1}^I)) \right\} + (1-\eta) \mathbb{E}_t \left\{ a \beta_{t,t+1}^I \phi_{t+1}^I (1-\delta^L) \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\} .$$

Using $\mu_{I,t}^L = \varepsilon_t^L \kappa^I \frac{q_t^I v_t^I}{l_{t-1}^I}$ we obtain the same equation as in the text:

$$w_t^I = \eta \left(\frac{(1-\alpha)(1-\lambda)Y_t}{l_t^I} + \mathbb{E}_t \left\{ \varepsilon_{t+1}^L \beta_{t,t+1}^P \frac{\kappa^I}{2} (x_{t+1}^I)^2 \right\} \right) + (1-\eta) b^I + \mathbb{E}_t \left\{ \eta \varepsilon_{t+1}^L \kappa^I x_{t+1}^I (1-\delta^L) (\beta_{t,t+1}^P - \beta_{t,t+1}^I (1-f_{t+1}^I)) \right\} + (1-\eta) \mathbb{E}_t \left\{ a \beta_{t,t+1}^I \phi_{t+1}^I (1-\delta^L) \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\} .$$

1.4 General Equilibrium

The total stock of housing is fixed and normalised to one such that $h_t^P + h_t^I = 1$, deposits equals loans i.e. $d_t^I = d_t^P$ and the taxes collected by the government is used to cover the transfer to unemployed people and public spending i.e. $T_t^P = g^Y \varepsilon_t^G + (1-l_t^I) b^I + (1-l_t^P) b^P$. Using these three relationship, we can obtain the same GDP aggregate as in the text.

We start with the budget constraint of patient household:

$$\begin{aligned} c_t^P + q_t^H \Delta h_t^P + d_t^P + T_t^P + \left(\frac{K_t - (1 - \delta_t^K) K_{t-1}}{\varepsilon_t^I} \right) + \frac{\phi^K (K_t - K_{t-1})^2}{2K} \\ = w_t^P l_t^P + (1 - l_t^P) b^P + r_{t-1} d_{t-1}^P + z_t K_{t-1} v_t + \Pi_t \end{aligned}$$

Using the definition of dividends $\Pi_t = Y_t - z_t K_t^U - \sum_{j=P,I} \left(w_t^j l_t^j + \varepsilon_t^L \frac{\kappa_j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j \right)$

and the definition of taxes we have:

$$\begin{aligned} c_t^P + q_t^H \Delta h_t^P + d_t^P + g^Y \varepsilon_t^G + (1 - l_t^I) b^I + \left(\frac{K_t - (1 - \delta_t^K) K_{t-1}}{\varepsilon_t^I} \right) + \frac{\phi^K (K_t - K_{t-1})^2}{2K} \\ = r_{t-1} d_{t-1}^P + Y_t - w_t^I l_t^I - \sum_{j=P,I} \left(\varepsilon_t^L \frac{\kappa_j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j \right) \end{aligned}$$

Turning to the budget constraint of impatient household and using the relationship between deposits and loans :

$$d_t^P - r_{t-1} d_{t-1}^P = c_t^I + q_t^H \Delta h_t^I - w_t^I l_t^I - (1 - l_t^I) b^I.$$

We can use this equation into the previous :

$$\begin{aligned} c_t^P + c_t^I + q_t^H \Delta h_t^I + q_t^H \Delta h_t^P + g^Y \varepsilon_t^G + \left(\frac{K_t - (1 - \delta_t^K) K_{t-1}}{\varepsilon_t^I} \right) + \frac{\phi^K (K_t - K_{t-1})^2}{2K} \\ = Y_t - \sum_{j=P,I} \frac{\kappa_j}{2} \varepsilon_t^L (x_t^j)^2 (l_{t-1}^j)^{-1} \end{aligned}$$

Now using the fact that $h_t^P + h_t^I = 1$, the definition of total consumption $C_t = c_t^P + c_t^I$ and the definition of investment : $I_t = \frac{K_t - (1 - \delta_t^K) K_{t-1}}{\varepsilon_t^I}$ we obtain the same expression:

$$Y_t = C_t + g^Y \varepsilon_t^G + I_t + \Phi_K(\Delta K_t) + \sum_{j=P,I} \frac{\kappa_j}{2} \varepsilon_t^L (x_t^j)^2 (l_{t-1}^j)^{-1}.$$

2 Steady-State

We distinguish between two steady states. First, there is the estimated steady state that results from the set of estimated parameters that determines the long run value of endogenous variables. Some estimated parameters pins down structural parameters, such

as preference parameters, the cost of hiring, etc. However, these parameters are expected not to adjust following a structural reform. We thus define a second steady state, referred to as the policy steady state, which is affecting the key ratios of the economy without affecting preferences and technology parameters.

2.1 Estimated Steady-state

We start the steady-state by the labor market. Since we have set the exogenous rate of destruction (δ^L) and the finding rate (f) for both types, using the law of motion from the household's perspective (Eq.2.4), we get the steady-state value of labor:

$$l^j = \frac{f}{(\delta^L + f(1 - \delta^L))}.$$

Then we have the same level of employment for each type of household $l^P = l^I$ and by extension the total labor used in the production function : $L = (l^P)^\lambda (l^I)^{1-\lambda}$. With the law of motion from the firm's perspective (Eq.2.3) and the probability to fill a vacancy (q) we have $v = \frac{l\delta^L}{q}$ and $x = \frac{qv}{l}$. The matching efficiency parameter is determined using the definition of the finding rate ($f = \frac{\psi(v)^\zeta (1-(1-\delta^L)L)^{1-\zeta}}{(1-(1-\delta^L)L)}$) such that $\psi = \frac{f}{(v)^\zeta (1-(1-\delta^L)L)^{-\zeta}}$. After that we can obtain the total amount of capital and by extension the total output. Recall that the utilisation rate is equal to one at the steady-state ($v = 1$ and $K^U = K$), using the FOC with respect to capital for household (Eq.C.6) and for firms (Eq.C.12) and the definition of output we get:

$$K = \left(\frac{(1 - \beta^P (1 - \delta^K))}{\beta^P \alpha L^{1-\alpha}} \right)^{\frac{1}{\alpha-1}}.$$

Thus, we have $Y = K^\alpha L^{1-\alpha}$, $z = \alpha \frac{Y}{K}$ and $I = \delta^K K$.

After that, we can have the steady-state value of wages (w^j) and the cost to hiring (κ^j) for both types using the job creation condition and the wage setting. Start with the patient

household using the definition of the replacement rate: $\tau_P^B = \frac{b^P}{w^P}$ and rearrange:

$$\begin{cases} \kappa^P (x - \beta^P \frac{1}{2} (x)^2 - \beta^P x (1 - \delta^L)) = \frac{(1-\alpha)\lambda Y}{l^P} - w^P \\ w^P = \frac{\eta}{(1-(1-\eta)\tau_P^B)} \left[\frac{(1-\alpha)\lambda Y}{l^P} + \kappa^P (\beta^P \frac{1}{2} (x)^2 + (1 - \delta^L) \beta^P x f) \right] \end{cases},$$

using $\phi_1 = x - \beta^P \frac{1}{2} (x)^2 - \beta^P x (1 - \delta^L)$ and $\phi_2 = (\beta^P \frac{1}{2} (x)^2 + (1 - \delta^L) \beta^P x f) \frac{\eta}{(1-(1-\eta)\tau_P^B)}$ and put the first equation into the second we have the SS of the hiring cost:

$$\kappa^P = \frac{(1-\alpha)\lambda Y}{(\phi_1 + \phi_2) l^P} \left(1 - \frac{\eta}{(1-(1-\eta)\tau_P^B)} \right),$$

and by extension the SS of the wage.

For impatient household, we proceed in the same way by using the Job creation condition and the wage setting:

$$\begin{cases} \kappa^I x = \frac{(1-\alpha)(1-\lambda)Y}{l^I} - w + \beta^P \frac{\kappa^I}{2} (x)^2 + \beta^P \kappa^I x (1 - \delta^L) \\ \eta \left(\frac{(1-\alpha)(1-\lambda)Y}{l^I} + \beta^P \frac{\kappa^I}{2} (x)^2 \right) + (1 - \eta) b^I \\ w^I = + \eta \kappa^I x (1 - \delta^L) (\beta^P - \beta^I (1 - f)) \\ + (1 - \eta) a \beta^I \phi^I (1 - \delta^L) \frac{\Delta d^I}{u^I} \end{cases}.$$

Note that whatever our endogenous collateral constraint is at work i.e. if $a = 1$, the last line of the equation for wage disappears since $\Delta d^I = 0$. Then, using $\phi_3 = x - \beta^P \frac{1}{2} (x)^2 - \beta^P x (1 - \delta^L)$ and $\phi_4 = (\beta^P \frac{1}{2} (x)^2 + x (1 - \delta^L) (\beta^P - \beta^I (1 - f))) \frac{\eta}{(1-(1-\eta)\tau_I^B)}$ we get the steady-state value of hiring costs and wages for the impatient household:

$$\begin{cases} \kappa^I = \frac{(1-\alpha)(1-\lambda)Y}{l(\phi_3 + \phi_4)} \left(1 - \frac{\eta}{(1-(1-\eta)\tau_I^B)} \right) \\ w^I = \frac{\eta}{(1-(1-\eta)\tau_I^B)} \frac{(1-\alpha)(1-\lambda)Y}{l} + \kappa^I \phi_4 \end{cases}.$$

As we have seen, the collateral constraint choice does not affect the steady-state of labor, capital and product market. However, they can have an impact on two important ratios namely the ratio housing stock to annualised GDP i.e. $g^H = \frac{q^H(h^P + h^I)}{4Y}$ and debt to gdp i.e. $g^D = \frac{d^I}{Y}$.

Using the Euler condition for patient and impatient household (respectively Eq.C.4 and Eq.C.2), we deduce the steady-state value of the interest rate $r = \frac{1}{\beta^P}$ and the Lagrangian multiplier associated to the borrowing constraint :

$$\phi^I = \frac{(1 - \beta^I r)}{(1 - \beta^I (a(1 - \delta^L) l^I + (1 - a)\chi))}.$$

To obtain the level of consumption for impatient household, we start with their budget constraint:

$$c^I = w^I l^I + (1 - l^I) b^I + (1 - r) d^I.$$

Using the steady-state value of d^I :

$$d^I = m^I q^H h^I \left(\frac{af(1 - (1 - \delta^L) l^I) + (1 - a)(1 - \chi)}{(1 - a(1 - \delta^L) l^I - (1 - a)\chi)} \right),$$

and the steady-state value of the housing price from the housing demand for impatient household (Eq.C.1):

$$q^H = j \frac{c^I}{h^I (1 - \beta^I - \phi^I m^I (af(1 - (1 - \delta^L) l^I) + (1 - a)(1 - \chi)))}.$$

Note by $\phi^5 = \left(\frac{j(1-r)m^I(af(1-(1-\delta^L)l^I)+(1-a)(1-\chi))}{(1-\beta^I-\phi^I m^I(af(1-(1-\delta^L)l^I)+(1-a)(1-\chi)))(1-a(1-\delta^L)l^I-(1-a)\chi)} \right)$ and replace the definition of debt and housing price in the budget constraint we have:

$$c^I = w^I l^I + (1 - l^I) b^I + c^I \phi^5.$$

Then, $c^I = \frac{w^I l^I + (1 - l^I) b^I}{(1 - \phi^5)}$. The consumption of patient household is obtained with his budget constraint :

$$c^P = (1 - g^Y) Y - c^I - \frac{\kappa_I}{2} (x)^2 (l^I)^{-1} - \frac{\kappa^P}{2} (x)^2 (l^P)^{-1}.$$

To obtain the steady-state value of h_I we use the relationship between the two stocks of housing : $h_P = 1 - h_I$, the value of housing price and the optimal housing demand for

lenders:

$$h^I = \frac{1}{1 + \frac{c^P(1-\beta^I-\phi^I m^I (af(1-(1-\delta^L)^I)+(1-a)(1-\chi)))}{(1-\beta^P)c^I}}$$

Then we have h^P , q^H and d^I which closed the steady-state.

2.2 Policy steady-state

To introduce the policy steady-state, we take the value of the hiring cost κ^I and κ^P and the parameter for the matching efficiency ψ from the estimated steady-state and relax the calibration of the finding rate for both type of households. We note by $(\tau_j^B)^*$ and $(m^I)^*$ respectively the implementation of the labor market reform and the macroprudential reform. For the lack of simplicity, we only consider the case of the collateral constraint with labor market flows i.e. $a = 1$ in this appendix.

We know the hiring rate for both type of households since by definition it is equal to the exogenous destruction rate i.e. $x^j = \delta^L$, the SS for the interest rate $r = \frac{1}{\beta^P}$ and the SS for the utilisation rate normalised to one $v = 1$. Using the FOC with respect to capital for household (Eq.C.6) and for firms (Eq.C.12) and the definition of output we get the SS value of capital depending on the level of labor :

$$K = \left(\frac{(1 - \beta^P (1 - \delta^K))}{\beta^P \alpha} \right)^{\frac{1}{\alpha-1}} L,$$

we note by $\phi^K = \left(\frac{(1 - \beta^P (1 - \delta^K))}{\beta^P \alpha} \right)^{\frac{1}{\alpha-1}}$ to have $K = \phi^K L$ and by extension the level of output becomes : $Y = L^{-\alpha} \phi^K$ with $L = (l^P)^\lambda (l^I)^{1-\lambda}$.

We start with the job creation condition for both type of households and we use $x^j = \delta^L$ and the previous expression for output:

$$\begin{cases} \kappa^I \delta^L = (1 - \alpha) (1 - \lambda) (l^P)^{-\alpha\lambda} (l^I)^{-\alpha(1-\lambda)-1} \phi^K - w^I + \kappa^I \beta^P \left(\frac{1}{2} (\delta^L)^2 + (1 - \delta^L) \delta^L \right) \\ \kappa^P \delta^L = (1 - \alpha) \lambda (l^P)^{-\alpha\lambda-1} (l^I)^{-\alpha(1-\lambda)} \phi^K - w^P + \kappa^P \beta^P \left(\frac{1}{2} (\delta^L)^2 + (1 - \delta^L) \delta^L \right) \end{cases}$$

We rearrange these two expressions with the parameters that we know and note by $\phi^{I1} = \kappa^I \left(\delta^L - \beta^P \left(\frac{1}{2} (\delta^L)^2 + (1 - \delta^L) \delta^L \right) \right)$ and $\phi^{P1} = \left(\delta^L - \beta^P \left(\frac{1}{2} (\delta^L)^2 + (1 - \delta^L) \delta^L \right) \right)$. Then we have:

$$\Leftrightarrow \begin{cases} \phi^{I1} = (1 - \alpha) (1 - \lambda) (l^P)^{-\alpha\lambda} (l^I)^{-\alpha(1-\lambda)-1} \phi^K - w^I \\ \phi^{P1} = (1 - \alpha) \lambda (l^P)^{-\alpha\lambda-1} (l^I)^{-\alpha(1-\lambda)} \phi^K - w^P \end{cases} \quad (C.13)$$

Now we work with the expression of wages for both types of household:

$$\Leftrightarrow \begin{cases} w^I = \eta \left(\frac{(1-\alpha)(1-\lambda)Y}{l^I} + \beta^P \frac{\kappa^I}{2} (\delta^L)^2 \right) + (1 - \eta) (\tau_I^B)^* w^I \\ \quad + \eta \kappa^I \delta^L (1 - \delta^L) (\beta^P - \beta^I (1 - f^I)) \\ w^P = \eta \left(\frac{(1-\alpha)\lambda Y}{l^P} + \beta^P \frac{\kappa^P}{2} (\delta^L)^2 \right) + (1 - \eta) (\tau_P^B)^* w^P \\ \quad + \eta (1 - \delta^L) \beta^P \kappa^P \delta^L f^P \end{cases}$$

Rearrange these expressions :

$$\Leftrightarrow \begin{cases} w^I = \frac{\eta}{(1-(1-\eta)(\tau_I^B)^*)} \left(\frac{(1-\alpha)(1-\lambda)Y}{l^I} - \kappa^I \delta^L (1 - \delta^L) \beta^I (1 - f^I) \right) \\ \quad + \frac{\eta \kappa^I \delta^L (1 - \delta^L) \beta^P + \eta \beta^P \frac{\kappa^I}{2} (\delta^L)^2}{(1-(1-\eta)(\tau_I^B)^*)} \\ w^P = \frac{\eta}{(1-(1-\eta)(\tau_P^B)^*)} \left(\frac{(1-\alpha)\lambda Y}{l^P} + (1 - \delta^L) \beta^P \kappa^P \delta^L f^P \right) \\ \quad + \frac{\beta^P \frac{\kappa^P}{2} (\delta^L)^2}{(1-(1-\eta)(\tau_P^B)^*)} \end{cases}$$

Note by $\phi^{I2} = \frac{\eta \kappa^I \delta^L (1 - \delta^L) \beta^P + \eta \beta^P \frac{\kappa^I}{2} (\delta^L)^2}{(1-(1-\eta)(\tau_I^B)^*)}$, $\phi^{I3} = \frac{\eta}{(1-(1-\eta)(\tau_I^B)^*)}$, $\phi^{P2} = \frac{\beta^P \frac{\kappa^P}{2} (\delta^L)^2}{(1-(1-\eta)(\tau_P^B)^*)}$ and $\phi^{P3} = \frac{\eta}{(1-(1-\eta)(\tau_P^B)^*)}$ then we have:

$$\Leftrightarrow \begin{cases} w^I = \phi^{I3} \left(\frac{(1-\alpha)(1-\lambda)Y}{l^I} - \kappa^I \delta^L (1 - \delta^L) \beta^I (1 - f^I) \right) + \phi^{I2} \\ w^P = \phi^{P3} \left(\frac{(1-\alpha)\lambda Y}{l^P} + (1 - \delta^L) \beta^P \kappa^P \delta^L f^P \right) + \phi^{P2} \end{cases}$$

Integrating these expressions into JC conditions (C.13), using $f^j = \frac{\delta^L l^j}{(1-(1-\delta^L)l^j)}$ from labor market law of motion and rearrange with the level of output, we have:

$$\Leftrightarrow \begin{cases} \phi^{I1} = (1 - \phi^{I3}) (1 - \alpha) (1 - \lambda) (l^P)^{-\alpha\lambda} (l^I)^{-\alpha(1-\lambda)-1} \phi^K \\ + \phi^{I3} \kappa^I \delta^L (1 - \delta^L) \beta^I \left(1 - \frac{\delta^L l^I}{(1-(1-\delta^L)l^I)}\right) - \phi^{I2} \\ \phi^{P1} = (1 - \phi^{P3}) (1 - \alpha) \lambda (l^P)^{-\alpha\lambda-1} (l^I)^{-\alpha(1-\lambda)} \phi^K \\ - \phi^{P3} (1 - \delta^L) \beta^P \kappa^P \delta^L \frac{\delta^L l^P}{(1-(1-\delta^L)l^P)} - \phi^{P2} \end{cases} .$$

Since we have a system of non-linear system of equation, we use a solver¹ to find the exact value of n^P and n^I from these two equations. The labor for patient and impatient are approximately equal. For example, without implementing the labor market reform we have: $l^P = 0.958904$ and $l^I = 0.958908$ implying a difference of $-4 * 10^{-6}$ between them. In the same way, after the reform we find the same order of difference.

Then, we have the value of the finding rate for both type of households, the unemployment rate, the level of capital (and by extension the investment) and output. To obtain the SS value of the total vacancy posted, we used the definition of the matching function and the labor market law of motion :

$$v^j = \left(\frac{\delta^L l^j}{\psi (1 - (1 - \delta^L)l^j)^{1-\zeta}} \right)^{\frac{1}{\zeta}},$$

and thus the filling rate $q^j = \frac{l^j \delta^L}{v^j}$.

Thus, we can have the new SS for the borrowing value of impatient household ϕ^I :

$$\phi^I = \frac{(1 - \beta^I r)}{(1 - \beta^I (1 - \delta^L) l^I)}.$$

To obtain the level of consumption for impatient household we start with their budget constraint:

$$c^I = w^I l^I + (1 - l^I) b^I + (1 - r) d^I.$$

¹We use the solver optimset from Matlab with 100000 iterations.

Using the steady-state value of d^I :

$$d^I = (m^I)^* q^H h^I f^I,$$

and the steady-state value of the housing price from the housing demand for impatient household (Eq.C.1):

$$q^H = j \frac{c^I}{h^I (1 - \beta^I - \phi^I (m^I)^* (f^I (1 - (1 - \delta^L) l^I)))}.$$

Note by $\phi^5 = \left(\frac{j(1-r)(m^I)^*(f(1-(1-\delta^L)l^I))}{(1-\beta^I-\phi^I(m^I)^*f^I(1-(1-\delta^L)l^I))(1-(1-\delta^L)l^I)} \right)$ and replace the definition of debt and housing price in the budget constraint we have:

$$c^I = w^I l^I + (1 - l^I) b^I + c^I \phi^5.$$

Then, $c^I = \frac{w^I l^I + (1 - l^I) b^I}{(1 - \phi^5)}$. The consumption of patient household is obtained with his budget constraint,

$$c^P = (1 - g^Y) Y - c^I - \frac{\kappa_I}{2} (x)^2 (l^I)^{-1} - \frac{\kappa^P}{2} (x)^2 (l^P)^{-1}.$$

To obtain the steady-state value of h^I we use the relationship between the two stocks of housing : $h^P = 1 - h^I$, the value of housing price and the optimal housing demand for lenders:

$$h^I = \frac{1}{1 + \frac{c^P (1 - \beta^I - \phi^I (m^I)^* (f(1 - (1 - \delta^L) l^I)))}{(1 - \beta^P) c^I}}.$$

Then, we get h^P , q^H and d^I which closed the steady-state.

Thus, a labor market deregulation conduces to a higher level of output, employment (lower wages), debt, housing price, borrowing value and a reallocation effect in the housing market between patient and impatient households (more housing for impatient).

A tightening macroprudential policy affects negatively the housing price, the level of debt in the economy and a reallocation in the housing market between patient and impatient households (more housing for patient).

Appendix D

Appendix for Chapter III

In this appendix, we describe the main features of the chapter III with the complete set of First Order Condition (FOC hereafter) and the steady-state.

1 Non-linear model

In this section, we present the model by considering the borrowing constraint with full requirements i.e. with capital, commercial estate and wage bill in advance.

1.1 Households

1.1.1 Impatient households

The problem faced by the borrowers can be written as:

$$\begin{aligned} \max_{c_t^I, h_t^I, d_t^I, l_t^I} \mathbb{E}_0 \sum_{t=0}^{\infty} \varepsilon_t^B (\beta^I)^t \{ (1 - h^C) \log (c_t^I - h^C c_{t-1}^I) + \varepsilon_t^H j \log (h_t^I) \} \\ \text{s.t. } c_t^I + q_t^H (h_t^I - h_{t-1}^I) + r_{t-1} d_{t-1}^I + \xi_t^I = w_t^I l_t^I + (1 - l_t^I) b^I + d_t^I \\ \text{s.t. } l_t^I = (1 - \delta^L) l_{t-1}^I + f_t^I (1 - (1 - \delta^L) l_{t-1}^I) \\ \text{s.t. } d_t^I = (1 - \delta^L) l_{t-1}^I d_{t-1}^I + \varepsilon_t^M f_t^I (1 - (1 - \delta^L) l_{t-1}^I) m^I \mathbb{E}_t \{ q_{t+1}^H \} h_t^I \end{aligned} \quad , \quad (\text{D.1})$$

Let λ_t^I the Lagrangian multiplier associated with the budget constraint i.e the marginal utility of consumption, the FOC with respect to consumption is:

$$\lambda_t^I = \varepsilon_t^B \frac{(1 - h^C)}{(c_t^I - h^C c_{t-1}^I)}. \quad (\text{D.2})$$

As in the text, we note by $\beta_{t,t+1}^I = \mathbb{E}_t \left\{ \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} \beta_I \left(\frac{c_t^I - h^C c_{t-1}^I}{c_{t+1}^I - h^C c_t^I} \right) \right\}$ the stochastic discount factor of borrowers. The FOC with respect to housing (using the definition of adjustment cost for housing $\xi_t^I = \frac{\Theta^I}{2} h_{t-1}^I q_t^H \left(\frac{h_t^I}{h_{t-1}^I} - 1 \right)^2$) is given by:

$$q_t^H \left(1 + \Theta^I \frac{\Delta h_t^I}{h_{t-1}^I} \right) = \varepsilon_t^H j \frac{(c_t^I - h^C c_{t-1}^I)}{(1 - h^C) h_t^I} + \mathbb{E}_t \left\{ \beta_{t,t+1}^I q_{t+1}^H \left(1 + \Theta^I \frac{h_{t+1}^I}{(h_t^I)^2} \Delta h_{t+1}^I - \frac{\Theta^I}{2} \left(\frac{\Delta h_{t+1}^I}{h_t^I} \right)^2 \right) \right\} + \mathbb{E}_t \left\{ q_{t+1}^H \right\} \varepsilon_t^M \phi_t^I m^I f_t^I (1 - (1 - \delta^L) l_{t-1}^I) \quad (\text{D.3})$$

Since the term $\frac{\Theta^I}{2} \left(\frac{\Delta h_{t+1}^I}{h_t^I} \right)^2$ equals to 0 at a first order approximation, we find the same expression as in the text.

For the Euler condition we have:

$$\mathbb{E}_t \left\{ \beta_{t,t+1}^I \right\} r_t = 1 - \phi_t^I + \mathbb{E}_t \left\{ \beta_{t,t+1}^I \phi_{t+1}^I (1 - \delta^L) l_t^I \right\}, \quad (\text{D.4})$$

with ϕ_t^I the shadow value of the borrowing constraint normalised by the marginal utility of consumption λ_t^I .

Finally, the FOC with respect to l_t^I reads:

$$\mu_t^{I,W} = w_t^I - b^I + \mathbb{E}_t \left\{ \beta_{t,t+1}^I \mu_{t+1}^{I,W} (1 - \delta^L) (1 - f_{t+1}^I) \right\} - (1 - \delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^I \phi_{t+1}^I \Delta d_{t+1}^I / u_{t+1}^I \right\} \quad (\text{D.5})$$

with $\mu_t^{I,W}$ the lagrangian multiplier associated with the labor market law of motion and normalised by the marginal utility of consumption.

1.1.2 Patient household

The problem faced by the lenders can be summarised as follow:

$$\begin{aligned}
 & \max_{c_t^P, h_t^P, d_t^P, l_t^P, K_t^P, v_t^P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_P^t \varepsilon_t^B \left\{ (1 - h^C) \log(c_t^P - h^C c_{t-1}^P) + \varepsilon_t^H j \log(h_t^P) \right\} \\
 & \text{s.t.} \quad c_t^P + q_t^H (h_t^P - h_{t-1}^P) + \xi_t^P + d_t^P + T_t + \left(\frac{K_t^P - (1 - \delta_t^P) K_{t-1}^P}{\varepsilon_t^I} \right) + \frac{\phi^P (K_t^P - K_{t-1}^P)^2}{2 K^P} \\
 & \quad \quad \quad = w_t^P l_t^P + (1 - l_t^P) b^P + r_{t-1} d_{t-1}^P + z_t K_{t-1}^P v_t^P \quad , \quad (\text{D.6}) \\
 & \quad \quad \quad \text{s.t.} \quad l_t^P = (1 - \delta^L) l_{t-1}^P + f_t^P (1 - (1 - \delta^L) l_{t-1}^P) \\
 & \quad \quad \quad \delta_t^P = \delta + \left(\frac{1}{\beta_P} + 1 - \delta \right) \left(\frac{1}{2} \left(\frac{\psi^P}{1 - \psi^P} \right) (v_t^P)^2 + \frac{(1 - 2\psi^P)}{1 - \psi^P} v_t^P + \frac{1}{2} \left(\frac{\psi^P}{1 - \psi^P} \right) - 1 \right)
 \end{aligned}$$

Let λ_t^P the Lagrangian multiplier associated with the budget constraint i.e the marginal utility of consumption, the FOC with respect to consumption is:

$$\lambda_t^P = \varepsilon_t^B \frac{(1 - h^C)}{(c_t^P - h^C c_{t-1}^P)}. \quad (\text{D.7})$$

Note by $\beta_{t,t+1}^P = \beta^P \mathbb{E}_t \left\{ \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} \left(\frac{c_t^P - h^C c_{t-1}^P}{c_{t+1}^P - h^C c_t^P} \right) \right\}$ the stochastic discount factor for lenders, the FOC for housing using the definition of the adjustment cost for housing : $\xi_t^P = \frac{\Theta^P}{2} h_{t-1}^P q_t^H \left(\frac{h_t^P}{h_{t-1}^P} - 1 \right)^2$ is given by:

$$\begin{aligned}
 & \varepsilon_t^H j \frac{(c_t^P - h^C c_{t-1}^P)}{(1 - h^C) h_t^P} - q_t^H \left(1 + \Theta^P \frac{\Delta h_t^P}{h_{t-1}^P} \right) \\
 & = -\mathbb{E}_t \left\{ \beta_{t,t+1}^P q_{t+1}^H \left(1 + \Theta^P \frac{h_{t+1}^P}{(h_t^P)^2} \Delta h_{t+1}^P - \frac{\Theta^P}{2} \left(\frac{\Delta h_{t+1}^P}{h_t^P} \right)^2 \right) \right\}. \quad (\text{D.8})
 \end{aligned}$$

Since the term $\frac{\Theta^P}{2} \left(\frac{\Delta h_{t+1}^P}{h_t^P} \right)^2$ equals to 0 at a first order approximation, we find the same expression as in the text.

The Euler condition is obtained with the FOC for deposit (d_t^P) :

$$\mathbb{E}_t \left\{ \beta_{t,t+1}^P \right\} r_t = 1. \quad (\text{D.9})$$

The marginal utility of be an employee is obtained by derivate with respect to l_t^P :

$$\mu_t^{P,W} = w_t^P - b^P + \mathbb{E}_t \{ \beta_{t,t+1}^P \mu_{t+1}^P (1 - \delta^L) (1 - f_{t+1}^P) \}, \quad (\text{D.10})$$

with $\mu_t^{P,W}$ the Lagrangian multiplier associated with the labor market law of motion and normalised by the marginal utility of consumption.

Finally, concerning capital services, the FOC with respect to capital K_t^P and the utilisation rate v_t^P are respectively:

$$\frac{1}{\varepsilon_t^I} + \phi^K \frac{(K_t^P - K_{t-1}^P)}{K^P} = \mathbb{E}_t \beta_t^P \left(\frac{(1 - \delta_{t+1}^P)}{\varepsilon_{t+1}^I} + \phi^K \frac{(K_{t+1}^P - K_t^P)}{K^P} + z_{t+1} v_{t+1}^P \right), \quad (\text{D.11})$$

$$z_t = \left(\frac{1}{\beta_P} + 1 - \delta_K \right) \left(\left(\frac{\psi^P}{1 - \psi^P} \right) v_t^P + \frac{(1 - 2\psi^P)}{1 - \psi^P} \right). \quad (\text{D.12})$$

1.2 Entrepreneur

The representative entrepreneur maximise his utility subject to his budget constraint, labor market law of motion for both type of household ($j = \{P, I\}$), its collateral constraint and the definition of the capital depreciation rate. Then, the problem faced by the entrepreneur can be summarised as follows:

$$\left\{ \begin{array}{l} \max_{v_t^P, v_t^I, l_t^P, l_t^I, K_{t-1}^P, d_t^E, c_t^E, K_t^E, v_t^E} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_E^t \{ (1 - h^C) \log (c_t^E - h^C c_{t-1}^E) \} \\ s.t \quad c_t^E + \xi_t^E + z_t K_{t-1}^P v_t^P + \sum_{j=P,I} \left(w_t^j l_t^j + \frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j \right) + \frac{K_t^E - (1 - \delta_t^E) K_{t-1}^E}{\varepsilon_t^I} + \frac{\phi^E (\Delta K_t^E)^2}{2 K^E} \\ = \varepsilon_t^Z \left((l_t^P)^\lambda (l_t^I)^{1-\lambda} \right)^{1-\alpha-\nu} (K_{t-1}^P v_t^P)^{(1-\mu)\alpha} (K_{t-1}^E v_t^E)^{\mu\alpha} (h_{t-1}^E)^\nu + d_t^E - r_{t-1} d_{t-1}^E - q_t^H \Delta h_t^E \\ s.t \quad l_t^j = (1 - \delta^L) l_{t-1}^j + q_t^j v_t^j \\ s.t \quad \delta_t^E = \delta^K + \frac{\alpha \mu Y}{K^E} \left(\frac{1}{2} \left(\frac{\psi^E}{1 - \psi^E} \right) (v_t^E)^2 + \frac{(1 - 2\psi^E)}{1 - \psi^E} v_t^E + \frac{1}{2} \left(\frac{\psi^E}{1 - \psi^E} \right) - 1 \right) \\ s.t \quad d_t^E = \chi^E d_{t-1}^E + (1 + \chi^E) \varepsilon_t^{ME} \left(m^H \frac{h_t^E q_{t+1}^H}{r_t} + m^K K_t^E - m^W \sum_{j=P,I} w_t^j l_t^j \right) \\ s.t \quad \xi_t^E = \frac{\Theta^E}{2} h_{t-1}^E q_t^H \left(\frac{h_t^E}{h_{t-1}^E} - 1 \right)^2 \end{array} \right. , \quad (\text{D.13})$$

Euler condition Starting with the FOC with respect to consumption to get the marginal utility of consumption:

$$\lambda_t^E = (1 - h^C) (c_t^E - h^C c_{t-1}^E)^{-1}. \quad (\text{D.14})$$

Letting $\beta_{t,t+1}^E = \beta_E \frac{\mathbb{E}_t \{\lambda_{t+1}^E\}}{\lambda_t^E}$ be the stochastic discount factor the entrepreneur, then FOC with respect to the level of debt for the representative entrepreneur (d_t^E), we have:

$$1 - \phi_t^E + \mathbb{E}_t \{ \beta_{t,t+1}^E \chi^E \phi_{t+1}^E \} = \beta_{t,t+1}^E r_t, \quad (\text{D.15})$$

where ϕ_t^E corresponds to the shadow value of the collateral constraint¹.

Commercial estate Then, the optimal choice of estate used by the representative entrepreneur is obtained by derivate with respect to h_t^E :

$$\begin{aligned} -q_t^H - q_t^H \Theta^E \frac{\Delta h_t^E}{h_{t-1}^E} + \mathbb{E}_t \left\{ \beta_{t,t+1}^E \nu \frac{Y_{t+1}}{h_t^E} + q_{t+1}^H \right\} + (1 - \chi^E) \varepsilon_t^{ME} \phi_t^E m^H \frac{\mathbb{E}_t \{ q_{t+1}^H \}}{r_t} \\ - \mathbb{E}_t \left\{ \beta_{t,t+1}^E q_{t+1}^H \frac{\Theta^E}{2} \left(\frac{\Delta h_{t+1}^E}{h_t^E} \right)^2 \right\} + \mathbb{E}_t \left\{ \beta_{t,t+1}^E q_{t+1}^H \frac{h_{t+1}^E}{(h_t^E)^2} \Delta h_{t+1}^E \right\} = 0. \end{aligned}$$

Rearrange this expression :

$$\begin{aligned} q_t^H \left(1 + \Theta^E \frac{\Delta h_t^E}{h_{t-1}^E} \right) = \mathbb{E}_t \beta_{t,t+1}^E \left\{ \nu \frac{Y_{t+1}}{h_t^E} + q_{t+1}^H \left(1 + \Theta^E \frac{h_{t+1}^E}{(h_t^E)^2} \Delta h_{t+1}^E - \frac{\Theta^E}{2} \left(\frac{\Delta h_{t+1}^E}{h_t^E} \right)^2 \right) \right\} \\ + (1 - \chi^E) \varepsilon_t^{ME} \phi_t^E m^H \mathbb{E}_t \{ q_{t+1}^H \} (r_t)^{-1} \end{aligned} \quad (\text{D.16})$$

Since the term $\frac{\Theta^E}{2} \left(\frac{\Delta h_{t+1}^E}{h_t^E} \right)^2$ equals to 0 at a first order approximation, we find the same expression as in the text (Eq.3.29).

Labor market Starting with the patient household, the optimal choice of vacancies posting (v_t^P) is:

$$\mu_t^{P,L} = \kappa^P x_t^P, \quad (\text{D.17})$$

where $x_t^P = \frac{q_t^P v_t^P}{l_{t-1}^P}$ is the hiring rate for lenders and $\mu_t^{P,L}$ the Lagrangian multiplier associated with the labor market law of motion and normalised by the marginal utility of consumption.

¹Formally, the Lagrangian multiplier associated with the collateral constraint and normalised by the marginal utility of consumption.

Continue with this type of household, the FOC with respect to labor (l_t^P) is:

$$\left\{ \begin{aligned} \mu_t^{P,L} &= \frac{(1-\alpha-\nu)\lambda Y_t}{l_t^P} - w_t^P (1 + \phi_t^E \varepsilon_t^{ME} m^W (1 - \chi^E)) \\ &+ \mathbb{E}_t \left\{ \beta_{t,t+1}^E \left(\frac{\kappa^P}{2} (x_{t+1}^P)^2 + (1 - \delta^L) \mu_{t+1}^{P,L} \right) \right\} \end{aligned} \right. . \quad (\text{D.18})$$

Turning to the impatient household, the optimal choice of vacancies posting (v_t^I):

$$\mu_t^{I,L} = \kappa^I x_t^I, \quad (\text{D.19})$$

with $x_t^I = \frac{q_t^I v_t^I}{l_{t-1}^I}$ the hiring rate for borrowers. Then, the FOC with respect to labor (l_t^I):

$$\left\{ \begin{aligned} \mu_t^{I,L} &= \frac{(1-\alpha-\nu)(1-\lambda)Y_t}{l_t^I} - w_t^I (1 + \phi_t^E \varepsilon_t^{ME} m^W (1 - \chi^E)) \\ &+ \mathbb{E}_t \left\{ \beta_{t,t+1}^E \left(\frac{\kappa^I}{2} (x_{t+1}^I)^2 + (1 - \delta^L) \mu_{t+1}^{I,L} \right) \right\} \end{aligned} \right. . \quad (\text{D.20})$$

Thus, if we combine the FOC for vacancies and labor, we obtained the Job creation condition for both type households as in the text (for impatient combined Eq.D.20 and Eq.D.19 and for patient combined Eq.D.17 and D.18).

Capital market Starting with the capital provided by patient household, the optimal amount (K_{t-1}^P) chosen by the representative entrepreneur is:

$$z_t = \alpha (1 - \mu) \frac{Y_t}{K_{t-1}^P v_t^P}. \quad (\text{D.21})$$

Turning to its own capital production (K_t^E), his optimal choice is given by:

$$\frac{1}{\varepsilon_t^I} + \phi^E \frac{\Delta K_t^E}{K^E} = \mathbb{E}_t \left\{ \beta_{t,t+1}^E \left(\mu \alpha \frac{Y_{t+1}}{K_t^E} + \frac{(1 - \delta_{t+1}^E)}{\varepsilon_{t+1}^I} + \phi^E \frac{\Delta K_{t+1}^E}{K^E} \right) \right\} + (1 - \chi^E) \varepsilon_t^{ME} \phi_t^E m^K. \quad (\text{D.22})$$

Finally, the optimal utilisation rate (v_t^E):

$$\frac{\alpha \mu Y}{K^E} \left(\frac{\psi^E}{1 - \psi^E} v_t^E + \frac{(1 - 2\psi^E)}{1 - \psi^E} \right) = \frac{\mu \alpha Y_t}{v_t^E K_{t-1}^E}. \quad (\text{D.23})$$

1.3 Wage setting

The period-by-period Nash bargaining implies that the firm and each of its workers determine the wage in period t by solving the following problem:

$$\arg \max_{w_t} \left(\mu_t^{j,W} \right)^\eta \left(\mu_t^{j,L} \right)^{1-\eta} \quad (\text{D.24})$$

for $j = \{I, P\}$ depends on the type of household and η is the power of negotiation for the worker. Whatever the type of worker, the solution of this problem gives:

$$\omega_{j,t} \mu_t^{j,L} = (1 - \omega_{j,t}) \mu_t^{j,W} \quad (\text{D.25})$$

where $\omega_{j,t}$ is the endogenous bargaining power due to the presence of wage bill in advance:

$$\omega_{j,t} = \eta \left(\eta - (1 - \eta) \frac{\partial \mu_t^{j,L}}{\partial w_t} \frac{1}{\frac{\partial \mu_t^{j,W}}{\partial w_t}} \right)^{-1}$$

Using $\frac{\partial \mu_t^{j,W}}{\partial w_t} = 1$ and $\frac{\partial \mu_t^{j,L}}{\partial w_t} = -(1 + \phi_t^E \varepsilon_t^{ME} m^W)$ for both household, we have the same endogenous bargaining power:

$$\omega_t = \frac{\eta}{\eta + (1 - \eta) (1 + (1 - \chi^E) \phi_t^E \varepsilon_t^{ME} m^W)}. \quad (\text{D.26})$$

Then using $\omega_{j,t} = \omega_t$, the Nash rule can be rewritten as:

$$\omega_t \mu_t^{j,L} = (1 - \omega_t) \mu_t^{j,W}. \quad (\text{D.27})$$

Lenders wage Using the marginal value of a new match from the firm's perspective ($\mu_t^{P,L}$) and from the household's perspective ($\mu_t^{P,W}$) in the Nash rule (Eq.D.27), we obtained:

$$\frac{w_t^P}{(1 + \omega_t \phi_t^E \varepsilon_t^{ME} m^W (1 - \chi^E))^{-1}} = \omega_t \left\{ \frac{(1 - \alpha - \nu) \lambda Y_t}{l_t^P} + \mathbb{E}_t \left\{ \beta_{t,t+1}^E \frac{\kappa^P}{2} (x_{t+1}^P)^2 \right\} + (1 - \delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^E \mu_{t+1}^{P,L} \right\} \right\} + (1 - \omega_t) b^P - (1 - \omega_t) (1 - \delta^L) \mathbb{E}_t \left\{ \beta_{t,t+1}^P \mu_{t+1}^{P,W} (1 - f_{t+1}^P) \right\}$$

Using the fact that the Nash Bargaining hold in $t + 1$ we can replace $\mu_{t+1}^{P,W}$ by $\mu_{t+1}^{P,W} = \frac{\omega_{t+1}}{(1-\omega_{t+1})} \mu_{t+1}^{P,L}$ using Eq.D.27:

$$\frac{w_t^P}{(1+\omega_t \phi_t^E \varepsilon_t^{ME} m^W (1-\chi^E))^{-1}} = \omega_t \left\{ \frac{(1-\alpha-\nu)\lambda Y_t}{l_t^P} + \mathbb{E}_t \left\{ \beta_{t,t+1}^E \frac{\kappa^P}{2} (x_{t+1}^P)^2 \right\} \right\} + (1-\omega_t) b^P + (1-\delta^L) \mathbb{E}_t \left\{ \mu_{t+1}^{P,L} \left[\beta_{t,t+1}^E \omega_t - \beta_{t,t+1}^P \frac{\omega_{t+1}(1-\omega_t)}{(1-\omega_{t+1})} (1-f_{t+1}^P) \right] \right\}.$$

Finally using the FOC with respect to vacancies i.e. $\mu_t^{P,L} = \kappa^P x_t^P$ (Eq.D.19), we obtained the same equation than in the text :

$$\frac{w_t^P}{(1+\omega_t \phi_t^E \varepsilon_t^{ME} m^W (1-\chi^E))^{-1}} = \omega_t \left\{ \frac{(1-\alpha-\nu)\lambda Y_t}{l_t^P} + \mathbb{E}_t \beta_{t,t+1}^E \frac{\kappa^P}{2} (x_{t+1}^P)^2 \right\} + (1-\omega_t) b^P + \kappa^P (1-\delta^L) \mathbb{E}_t \left\{ x_{t+1}^P \left[\beta_{t,t+1}^E \omega_t - \beta_{t,t+1}^P \frac{\omega_{t+1}(1-\omega_t)}{(1-\omega_{t+1})} (1-f_{t+1}^P) \right] \right\}. \quad (\text{D.28})$$

To simplify this expression, use χ_t as $\chi_t = \phi_t^E \varepsilon_t^{ME} (1-\chi^E) m^W$ and than develop the expression of $(1 + \omega_t \phi_t^E \varepsilon_t^{ME} (1-\chi^E) m^W)$ with the definition of ω_t in Eq.D.26:

$$\begin{aligned} & 1 + \omega_t \phi_t^E \varepsilon_t^{ME} (1-\chi^E) m^W \\ \Leftrightarrow & 1 + \frac{\chi_t \eta}{\eta + (1-\eta)(1+\chi_t)} \\ \Leftrightarrow & \frac{1 + \chi_t}{\eta + (1-\eta)(1+\chi_t)} \end{aligned}$$

At the right side of the expression of the wage (Eq.D.28), we have the endogenous bargaining from the perspective of household (ω_t) and from firms ($1-\omega_t$). Let us start by dividing the household's perspective with our previous equation:

$$\omega_t \frac{(\eta + (1-\eta)(1+\chi_t))}{1 + \chi_t}$$

Using the definition of ω_t we simply have:

$$\Leftrightarrow = \frac{\eta}{1 + \chi_t} \quad (\text{D.29})$$

Now considering the firm's perspective :

$$(1 - \omega_t) \frac{(\eta + (1 - \eta)(1 + \chi_t))}{1 + \chi_t}$$

$$\Leftrightarrow = (1 - \eta) \tag{D.30}$$

Let us simplify $\frac{\omega_{t+1}}{(1 - \omega_{t+1})}$:

$$\Leftrightarrow = \frac{\eta}{\eta + (1 - \eta)(1 + \chi_{t+1})} \left(1 - \frac{\eta}{\eta + (1 - \eta)(1 + \chi_{t+1})} \right)^{-1}$$

$$\Leftrightarrow = \frac{\eta}{\eta + (1 - \eta)(1 + \chi_{t+1})} \left(\frac{(1 - \eta)(1 + \chi_{t+1})}{\eta + (1 - \eta)(1 + \chi_{t+1})} \right)^{-1}$$

$$\Leftrightarrow = \frac{\eta}{(1 - \eta)(1 + \chi_{t+1})} \tag{D.31}$$

Note by $\omega_t^O = \frac{\eta}{1 + \chi_t}$ the influence of wage bill in advance in the determination of the wage.

Using Eq.D.29, Eq.D.30 and Eq.D.31 in the patient's wage we get:

$$w_t^P = \omega_t^O \left\{ \frac{(1 - \alpha - \nu)\lambda Y_t}{l_t^P} + \mathbb{E}_t \beta_{t,t+1}^E \frac{\kappa_P}{2} (x_{t+1}^P)^2 \right\} + (1 - \eta) b^P$$

$$+ \kappa^P (1 - \delta^L) \mathbb{E}_t \left\{ x_{t+1}^P [\beta_{t,t+1}^E \omega_t^O - \beta_{t,t+1}^P \omega_{t+1}^O (1 - f_{t+1}^P)] \right\}.$$

as in the text.

Borrowers wage Concerning the impatient household, we follow the same step to have the definition of wage. Using the definition of $\mu_t^{I,W}$ in Eq.D.5 and $\mu_t^{I,L}$ in Eq.D.20, we have:

$$\frac{w_t^I}{(1 + (1 - \chi^E)\omega_t \phi_t^E \varepsilon_t^{ME} m^W)^{-1}} = \omega_t \left(\frac{(1 - \alpha - \nu)(1 - \lambda)Y_t}{l_t^I} + \mathbb{E}_t \left\{ \beta_{t,t+1}^E \frac{\kappa_I}{2} (x_{t+1}^I)^2 \right\} \right) + (1 - \omega_t) b^I$$

$$+ \mathbb{E}_t \left\{ \omega_t \beta_{t,t+1}^E (1 - \delta^L) \mu_{t+1}^{I,L} - (1 - \omega_t) \beta_{t,t+1}^I \mu_{t+1}^{I,W} (1 - \delta^L) (1 - f_{t+1}^I) \right\}$$

$$+ (1 - \omega_t) \mathbb{E}_t \left\{ \phi_{t+1}^I \beta_{t,t+1}^I (1 - \delta^L) \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\}$$

Using the Nash rule in $t + 1$ as for the lenders wage we have:

$$\frac{w_t^I}{(1 + (1 - \chi^E)\omega_t \phi_t^E \varepsilon_t^{ME} m^W)^{-1}} = \omega_t \left(\frac{(1 - \alpha - \nu)(1 - \lambda)Y_t}{l_t^I} + \mathbb{E}_t \left\{ \beta_{t,t+1}^E \frac{\kappa_I}{2} (x_{t+1}^I)^2 \right\} \right) + (1 - \omega_t) b^I$$

$$+ \mathbb{E}_t \left\{ \omega_t \beta_{t,t+1}^E (1 - \delta^L) \mu_{t+1}^{I,L} - \beta_{t,t+1}^I \frac{(1 - \omega_t)\omega_{t+1}}{(1 - \omega_{t+1})} \mu_{t+1}^{I,L} (1 - \delta^L) (1 - f_{t+1}^I) \right\}$$

$$+ (1 - \omega_t) \mathbb{E}_t \left\{ \phi_{t+1}^I \beta_{t,t+1}^I (1 - \delta^L) \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\}$$

Using $\mu_t^{I,L} = \kappa_I x_t^I$ and the previous development for ω_t to ω_t^O which is independent of the type of households we obtain the same equation as in the text:

$$\begin{aligned} \omega_t^O & \left(\frac{(1-\alpha-\nu)(1-\lambda)Y_t}{l_t^I} + \mathbb{E}_t \beta_{t,t+1}^E \frac{\kappa_I}{2} (x_{t+1}^I)^2 \right) + (1-\eta) b^I \\ w_t^I & = + \mathbb{E}_t \left\{ (1-\delta^L) x_{t+1}^I (\omega_t^O \beta_{t,t+1}^E - \beta_{t,t+1}^I \omega_{t+1}^O (1-f_{t+1}^I)) \right\} \cdot \\ & + (1-\eta) \mathbb{E}_t \left\{ \phi_{t+1}^I \beta_{t,t+1}^I (1-\delta^L) \frac{\Delta d_{t+1}^I}{u_{t+1}^I} \right\} \end{aligned} \quad (\text{D.32})$$

1.4 General Equilibrium

The total stock of housing is fixed and normalised to one such that $h_t^P + h_t^I + h_t^E = 1$, deposits equals loans i.e. $d_t^P = d_t^I + d_t^E$ and the taxes collected by the government used to cover the transfer to unemployed people and public spending i.e. $T_t = g^Y \varepsilon_t^G + (1-l_t^I) b^I + (1-l_t^P) b^P$. Using these three relationships, we can obtain the same GDP aggregate as in the text (Eq.3.46).

We start with the budget constraint of patient household:

$$\begin{aligned} c_t^P + q_t^H \Delta h_t^P + d_t^P + \xi_t^P + T_t + \left(\frac{K_t^P - (1-\delta_t^P) K_{t-1}^P}{\varepsilon_t^I} \right) + \frac{\phi^P (K_t^P - K_{t-1}^P)^2}{2 K^P} \\ = w_t^P l_t^P + (1-l_t^P) b^P + r_{t-1} d_{t-1}^P + z_t K_{t-1}^P v_t^P \end{aligned}$$

Using the balance sheet of government and the equality between deposits and loans we have:

$$\begin{aligned} c_t^P + q_t^H \Delta h_t^P + d_t^I - r_{t-1} d_{t-1}^I + d_t^E - r_{t-1} d_{t-1}^E + \left(\frac{K_t^P - (1-\delta_t^P) K_{t-1}^P}{\varepsilon_t^I} \right) + \frac{\phi^P (\Delta K_t^P)^2}{2 K^P} \\ = w_t^P l_t^P + z_t K_{t-1}^P v_t^P - (1-l_t^I) b^I - g^Y \varepsilon_t^G \end{aligned}$$

Replace $d_t^E - r_{t-1} d_{t-1}^E$ by his definition present in the budget constraint of entrepreneur leads to:

$$\begin{aligned} c_t^P + c_t^E + q_t^H \Delta h_t^P + q_t^H \Delta h_t^E + \xi_t^E + d_t^I - r_{t-1} d_{t-1}^I + I_t + \frac{\phi^P (K_t^P - K_{t-1}^P)^2}{2 K^P} + \frac{\phi^E (K_t^E - K_{t-1}^E)^2}{2 K^E} \\ = Y_t - \sum_{j=P,I} \frac{\kappa_j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j + w_t^I l_t^I - g^Y \varepsilon_t^G - (1-l_t^I) b^I \end{aligned}$$

Then, using $d_t^I - r_{t-1}d_{t-1}^I$ in the budget constraint of the impatient household and the total stock of housing, we obtained:

$$Y_t = c_t^P + c_t^E + c_t^I + g^Y \varepsilon_t^G + I_t + \sum_{j=P,I} \frac{\kappa^j}{2} \left(\frac{q_t^j v_t^j}{l_{t-1}^j} \right)^2 l_{t-1}^j + \frac{\phi^P}{2} \frac{(\Delta K_t^P)^2}{K^P} + \frac{\phi^E}{2} \frac{(\Delta K_t^E)^2}{K^E} + \xi_t^a \quad (D.33)$$

with $a = \{I, P, E\}$. Then at a first order approximation adjustment cost on housing and capital disappear and we find the same GDP equation than in the text.

2 Steady-state

As for the previous chapter, we distinguish between two steady states. First, there is the estimated steady state that results from the set of estimated parameters that determines the long run value of endogenous variables. Some estimated parameters pins down structural parameters such as preference parameters. However, these parameters are expected not to adjust following a structural reform. We thus define a second steady state, referred to as the policy steady state, which is affecting the key ratios of the economy without affecting preferences and technology parameters.

2.1 Estimated steady-state

We start the steady-state by the labor market. Since we have set the exogenous rate of destruction (δ^L) and the finding rate (f) for both types, using the law of motion from the household's perspective, we obtained the steady-state value of labor:

$$l^j = \frac{f}{(\delta^L + f(1 - \delta^L))}.$$

Then we have the same level of employment for each type of household $l^P = l^I$ and by extension the total labor used in the production function : $L = (l^P)^\lambda (l^I)^{1-\lambda}$. With the law of motion from the firm's perspective and the probability to fill a vacancy (q) we have $v = \frac{L\delta^L}{q}$ and $x = \delta^L$. The matching efficiency parameter is determined using the definition of the finding rate ($f = \frac{\psi(v)^\zeta (1 - (1 - \delta^L)L)^{1-\zeta}}{(1 - (1 - \delta^L)L)}$) such that $\psi = \frac{f}{(v)^\zeta (1 - (1 - \delta^L)L)^{-\zeta}}$.

We obtained the SS value for the interest rate, borrowing value for entrepreneurs and borrowers respectively using the Euler equation from lender's perspective, the Euler for entrepreneurs and finally the Euler for impatient households. For capital market recall that at the SS $\delta^E = \delta^K$, $\delta^P = \delta^K$, $v^E = 1$ and $v^P = 1$.

We start by express the level of output in function of the capital for entrepreneurs using the optimal stock of capital (Eq.D.22):

$$Y = \left(\frac{1 - (1 - \chi^E) \phi^E m^K}{\beta^E} - 1 + \delta^K \right) \frac{K^E}{\mu\alpha}.$$

Note by $\phi^{Y1} = \left(\frac{1 - (1 - \chi^E) \phi^E m^K}{\beta^E} - 1 + \delta^K \right) (\mu\alpha)^{-1}$ than $Y = \phi^{Y1} K^E$.

We express the capital stock of household in function of K^E using Eq.D.11 (the optimal decision for patient household) and the previous equation for output:

$$K^P = \frac{\alpha(1 - \mu)}{\left(\frac{1}{\beta^P} - 1 + \delta^K \right)} \phi^{Y1} K^E.$$

Then we note by $\phi^{Y2} = \frac{\alpha(1 - \mu)}{\left(\frac{1}{\beta^P} - 1 + \delta^K \right)} \phi^{Y1}$ to have $K^P = \phi^{Y2} K^E$. Using the definition of output and the previous expressions, we obtain the level of commercial estate in function of the entrepreneurial capital:

$$h^E = \left(\frac{\phi^{Y1}}{L^{1-\alpha-\nu} (\phi^{Y2})^{\alpha(1-\mu)}} \right)^{\frac{1}{\nu}} (K^E)^{\frac{1-\alpha}{\nu}}.$$

Note by $\phi^{Y3} = \left(\frac{\phi^{Y1}}{L^{1-\alpha-\nu} (\phi^{Y2})^{\alpha(1-\mu)}} \right)^{\frac{1}{\nu}}$ we have $h^E = \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}}$. However, since we do not know the SS value of the housing price q^H and we have normalised to one the housing stock $h^E + h^I + h^P = 1$, we need to have the value of h^P , c^P and c^I to compute the level of output (see the optimal condition for commercial estate Eq.D.16).

To do that, we start by rearrange the job creation condition for both types of households:

$$\begin{cases} \kappa^P x^P = \frac{(1-\alpha-\nu)\lambda Y}{l^P} - w^P (1 + \phi^E m^W (1 - \chi^E)) + \beta^E \left(\frac{\kappa^P}{2} (x^P)^2 + (1 - \delta^L) \kappa^P x^P \right) \\ \kappa^I x^I = \frac{(1-\alpha-\nu)(1-\lambda)Y}{l^P} - w^I (1 + \phi^E m^W (1 - \chi^E)) + \beta^E \left(\frac{\kappa^I}{2} (x^I)^2 + (1 - \delta^L) \kappa^I x^I \right) \end{cases}$$

Note by $\phi^{JCP1} = \kappa^P \left(x^P - \beta^E \left(\frac{1}{2} (x^P)^2 + (1 - \delta^L) x^P \right) \right)$
and $\phi^{JCI1} = \kappa^I \left(x^I - \beta^E \left(\frac{1}{2} (x^I)^2 + (1 - \delta^L) x^I \right) \right)$ we have:

$$\Leftrightarrow \begin{cases} \phi^{JCP1} \kappa^P = \frac{(1-\alpha-\nu)\lambda Y}{l^P} - w^P (1 + \phi^E m^W (1 - \chi^E)) \\ \phi^{JCI1} \kappa^I = \frac{(1-\alpha-\nu)(1-\lambda)Y}{l^P} - w^I (1 + \phi^E m^W (1 - \chi^E)) \end{cases} \quad (D.34)$$

Now, we work with the expression of the wage for both types:

$$\begin{cases} w^I = \omega^O \left(\frac{(1-\alpha-\nu)(1-\lambda)Y}{l^I(1-(1-\eta)\tau_I^B)} + \frac{\beta^E \frac{\kappa^I}{2} (x^I)^2 + \kappa^I x^I (1-\delta^L) (\beta^E - \beta^I (1-f))}{(1-(1-\eta)\tau_I^B)} \right) \\ w^P = \omega^O \left(\frac{(1-\alpha-\nu)\lambda Y}{l^P(1-(1-\eta)\tau_P^B)} + \frac{\beta^E \frac{\kappa^P}{2} (x^P)^2 + (1-\delta^L) \kappa^P x^P (\beta^E - \beta^P (1-f))}{(1-(1-\eta)\tau_P^B)} \right) \end{cases} \quad (D.35)$$

and note by $\phi^{WI1} = \frac{\beta^E \frac{1}{2} (x^I)^2 + (1-\delta^L) x^I (\beta^E - \beta^I (1-f))}{(1-(1-\eta)\tau_I^B)}$, $\phi^{WP1} = \frac{\beta^E \frac{1}{2} (x^P)^2 + (1-\delta^L) x^P (\beta^E - \beta^P (1-f))}{(1-(1-\eta)\tau_P^B)}$ and replace with $Y = \phi^{Y1} K^E$ and $\omega^O = \frac{\eta}{1+\phi^E(1-\chi^E)m^W}$:

$$\begin{cases} w^I = \frac{\eta}{1+\phi^E(1-\chi^E)m^W} \left(\frac{(1-\alpha-\nu)(1-\lambda)\phi^{Y1} K^E}{l^I(1-(1-\eta)\tau_I^B)} + \kappa^I \phi^{WI1} \right) \\ w^P = \frac{\eta}{1+\phi^E(1-\chi^E)m^W} \left(\frac{(1-\alpha-\nu)\lambda\phi^{Y1} K^E}{l^P(1-(1-\eta)\tau_P^B)} + \kappa^P \phi^{WP1} \right) \end{cases} \quad (D.36)$$

Replace the expression of wage in the JC conditions (Eq.D.34):

$$\Leftrightarrow \begin{cases} \phi^{JCP1} \kappa^P + \eta \kappa^P \phi^{WP1} = \frac{(1-\alpha-\nu)\lambda\phi^{Y1} K^E}{l^P} - \eta \frac{(1-\alpha-\nu)\lambda\phi^{Y1} K^E}{l^P(1-(1-\eta)\tau_P^B)} \\ \phi^{JCI1} \kappa^I + \eta \kappa^I \phi^{WI1} = \frac{(1-\alpha-\nu)(1-\lambda)\phi^{Y1} K^E}{l^P} - \eta \frac{(1-\alpha-\nu)(1-\lambda)\phi^{Y1} K^E}{l^I(1-(1-\eta)\tau_I^B)} \end{cases}$$

Rearrange the hiring cost in function of K^E :

$$\Leftrightarrow \begin{cases} \kappa^P = K^E \frac{\phi^{Y1}}{(\phi^{JCP1} + \eta\phi^{WP1})} \left(\frac{(1-\alpha-\nu)\lambda}{l^P} - \eta \frac{(1-\alpha-\nu)\lambda}{l^P(1-(1-\eta)\tau_P^B)} \right) \\ \kappa^I = \frac{\phi^{Y1}}{(\phi^{JCI1} + \eta\phi^{WI1})} K^E \left(\frac{(1-\alpha-\nu)(1-\lambda)}{l^P} - \eta \frac{(1-\alpha-\nu)(1-\lambda)}{l^I(1-(1-\eta)\tau_I^B)} \right) \end{cases} .$$

Note by $\phi^{JCP2} = \frac{\phi^{Y1}}{(\phi^{JCP1} + \eta\phi^{WP1})} \left(\frac{(1-\alpha-\nu)\lambda}{l^P} - \eta \frac{(1-\alpha-\nu)\lambda}{l^P(1-(1-\eta)\tau_P^B)} \right)$ and

$\phi^{JCI2} = \frac{\phi^{Y1}}{(\phi^{JCI1} + \eta\phi^{WI1})} \left(\frac{(1-\alpha-\nu)(1-\lambda)}{l^P} - \eta \frac{(1-\alpha-\nu)(1-\lambda)}{l^I(1-(1-\eta)\tau_I^B)} \right)$, we have:

$$\Leftrightarrow \begin{cases} \kappa^P = K^E \phi^{JCP2} \\ \kappa^I = K^E \phi^{JCI2} \end{cases} .$$

Now, we can express the wage in function of K^E using the previous system (Eq.D.36):

$$\begin{cases} w^I = \frac{\eta}{1+\phi^E(1-\chi^E)m^W} \left(\frac{(1-\alpha-\nu)(1-\lambda)\phi^{Y1}}{l^I(1-(1-\eta)\tau_I^B)} K^E + K^E \phi^{JCI2} \phi^{WI1} \right) \\ w^P = \frac{\eta}{1+\phi^E(1-\chi^E)m^W} \left(\frac{(1-\alpha-\nu)\lambda\phi^{Y1}}{l^P(1-(1-\eta)\tau_P^B)} K^E + K^E \phi^{JCP2} \phi^{WP1} \right) \end{cases} .$$

Use $\phi^{WI2} = \frac{\eta}{1+\phi^E(1-\chi^E)m^W} \left(\frac{(1-\alpha-\nu)(1-\lambda)\phi^{Y1}}{l^I(1-(1-\eta)\tau_I^B)} + \phi^{JCI2} \phi^{WI1} \right)$ and

$\phi^{WP2} = \frac{\eta}{1+\phi^E(1-\chi^E)m^W} \left(\frac{(1-\alpha-\nu)\lambda\phi^{Y1}}{l^P(1-(1-\eta)\tau_P^B)} + \phi^{JCP2} \phi^{WP1} \right)$, we have:

$$\Leftrightarrow \begin{cases} w^I = \phi^{WI2} K^E \\ w^P = \phi^{WP2} K^E \end{cases} .$$

Now we can obtain the level of consumption for both agents in function of K^E and q^H .

Start with the flow of budget for entrepreneurs to express c^E in function of these two variables:

$$c^E + zK^P + w^I l^I + w^P l^P + \frac{\kappa^P}{2} (\delta^L)^2 l^P + \frac{\kappa^I}{2} (\delta^L)^2 l^I + \delta^K K^E = Y + d^E (1-r).$$

We need to know the SS value for the debt level for entrepreneur to obtain c^E :

$$d^E = m^H q^H h^E + m^K K^E - w^I l^I - w^P l^P$$

$$\Leftrightarrow d^E = m^H q^H \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}} + m^K K^E - \phi^{WI2} K^E l^I - \phi^{WP2} K^E l^P.$$

Note by $\phi^{CE1} = m^K - \phi^{WI2} l^I - \phi^{WP2} l^P$, we get:

$$\Leftrightarrow d^E = m^H q^H \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}} + K^E \phi^{CE1}. \quad (\text{D.37})$$

Using $w^I = \phi^{WI2} K^E$, $w^P = \phi^{WP2} K^E$, $\kappa^P = K^E \phi^{JCP2}$, $\kappa^I = K^E \phi^{JCI2}$, $z = \frac{\alpha(1-\mu)Y}{K^P}$ and $Y = \phi^{Y1} K^E$, we can continue the development of the level of consumption for entrepreneurs:

$$\begin{cases} c^E + K^E \left\{ (\alpha(1-\mu) - 1) \phi^{Y1} + l(\phi^{WI2} + \phi^{WP2}) + \frac{(\delta^L)^2 l}{2} (\phi^{JCP2} + \phi^{JCI2}) + \delta^K \right\} \\ = d^E (1-r) \end{cases}.$$

Note by $\phi^{CE2} = (\alpha(1-\mu) - 1) \phi^{Y1} + l(\phi^{WI2} + \phi^{WP2}) + \frac{(\delta^L)^2 l}{2} (\phi^{JCP2} + \phi^{JCI2}) + \delta^K$ and replace with the expression of the debt (Eq.D.37) we have:

$$c^E = \left(m^H q^H \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}} \right) (1-r) - K^E (\phi^{CE2} + \phi^{CE1} (1-r)).$$

Note by $\phi^{CE3} = \phi^{CE2} + \phi^{CE1} (1-r)$ then the expression of c^E depends on the steady-state value of the entrepreneurial capital:

$$\Leftrightarrow c^E = \left(m^H q^H \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}} \right) (1-r) - K^E \phi^{CE3}. \quad (\text{D.38})$$

After the level of consumption for entrepreneurs, we search for the level of consumption for impatient households using its flow of budget:

$$c^I = (l^I + (1-l^I)\tau^{BI}) w^I + (1-r)d^I.$$

Using the level of debt for borrowers : $d^I = q^H m^I f^I h^I$ and the expression of $w^I = \phi^{WI2} K^E$:

$$\Leftrightarrow c^I = (l^I + (1 - l^I)\tau^{BI}) \phi^{WI2} K^E + (1 - r)q^H m^I f^I h^I. \quad (\text{D.39})$$

Turning to the level of consumption for patient household (c^P), we use the GDP definition:

$$Y = c^E + c^I + c^P + \delta^K (K^P + K^E) + .5 (\delta^L)^2 l (\kappa^P + \kappa^I).$$

Then using the level of consumption for impatient (Eq.D.39), $K^P = \phi^{Y2} K^E$, $\kappa^P = K^E \phi^{JCP2}$, $\kappa^I = K^E \phi^{JCI2}$ and $Y = \phi^{Y1} K^E$ we have:

$$\begin{cases} K^E \left[\phi^{Y1} - (l^I + (1 - l^I)\tau^{BI}) \phi^{WI2} - \delta^K (\phi^{Y2} + 1) - .5 (\delta^L)^2 l (\phi^{JCP2} + \phi^{JCI2}) \right] \\ = c^E + c^P + (1 - r)q^H m^I f^I h^I \end{cases}.$$

Note by :

$$\phi^{CP1} = \phi^{Y1} - (l^I + (1 - l^I)\tau^{BI}) \phi^{WI2} - \delta^K (\phi^{Y2} + 1) - .5 (\delta^L)^2 l (\phi^{JCP2} + \phi^{JCI2}),$$

and replace c^E using the level of consumption for entrepreneurs (Eq.D.38):

$$\Leftrightarrow c^P = K^E \phi^{CP1} + K^E \phi^{CE3} - (1 - r) \left(m^H q^H \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}} + q^H m^I f^I h^I \right). \quad (\text{D.40})$$

Finally, we are able to solve our steady-state by finding h^I , q^H , K^E using the optimal demand for housing from entrepreneur, impatient and patient household :

$$\begin{cases} q^H = \beta^E \left\{ \nu \frac{Y}{h^E} + q^H \right\} + (1 - \chi^E) \phi^E m^H q^H (r)^{-1} \\ j_{h^I}^I = q^H (1 - \beta^I - \phi^I m^I f^I (1 - (1 - \delta^L) l^I)) \\ j_{h^P}^P = q^H (1 - \beta^P) \end{cases}.$$

Note by $\Psi = (1 - \beta^I - \phi^I m^I f^I (1 - (1 - \delta^L) l^I))$ and using $Y = \phi^{Y1} K^E$, $h^E = \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}}$, the total stock of housing $h^P = 1 - h^E - h^I$, the definition of c^I from Eq.D.39 and the

definition of c^P from Eq.D.40, we get:

$$\Leftrightarrow \begin{cases} q^H = \beta^E \left\{ \nu \frac{\phi^{Y1} K^E}{\phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}}} + q^H \right\} + (1 - \chi^E) \phi^E m^H q^H (r)^{-1} \\ j \frac{(l^I + (1-l^I)\tau^{BI}) \phi^{WI2} K^E + (1-r) q^H m^I f^I h^I}{h^I} = q^H \Psi \\ j \frac{\left(K^E \phi^{CP1} + K^E \phi^{CE3} - (1-r) \left(m^H q^H \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}} + q^H m^I f^I h^I \right) \right)}{1 - \phi^{Y3} (K^E)^{\frac{1-\alpha}{\nu}} - h^I} = q^H (1 - \beta^P) \end{cases} .$$

To solve this system of non-linear equation, we use a solver to find the exact value of K^E , h^I and q^H .² Then, we are able to close the steady-state using all of the previous expressions.

2.2 Policy steady-state

To introduce the policy steady-state, we proceed as in the previous chapter, with the value of the hiring cost κ^I and κ^P , the parameter for the matching efficiency ψ from the estimated steady-state and relax the calibration of the finding rate for both types of households. Since we need all the optimal decisions for each agent to determine the level of output, any macroprudential policy (for entrepreneurs or household borrower) now have a long-run impact as demonstrated in the estimated steady-state.

²We use the solver optimset from Matlab with 100000 iterations.

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TROIS ESSAIS SUR LES FRICTIONS DU MARCHÉ DU TRAVAIL AVEC CRÉATION DE FIRMES ET CYCLES FINANCIERS

Résumé : Durant la grande récession, les interactions entre fluctuations du prix de l'immobilier, du travail et de l'entrée des firmes sur le marché des biens, ont mis en avant l'existence de relations étroites entre ces marchés. Le but de cette thèse est de mettre en lumière les interactions entre le marché du travail et le marché des biens ainsi que des cycles financiers, en utilisant les récents progrès des modèles DSGE. Dans le premier chapitre, nous avons trouvé un fort rôle joué par la création de firmes dans l'amplification des dynamiques de l'emploi. En introduisant le mécanisme du modèle de Diamond-Mortensen-Pissarides sur le marché du travail, nous avons pu étudier sous un nouvel angle les fluctuations du taux de marge des firmes. Comparé aux travaux théoriques utilisant un marché du travail sans frictions, nous avons trouvé un taux de marge moins contracyclique dû au coût marginal acyclique d'un modèle avec frictions. De plus, le rôle accordé à la création de firmes dans la détermination du taux de marge est moins important que dans les papiers précédents. Dans le second chapitre, nous avons lié la capacité d'endettement des ménages avec leur situation sur le marché de l'emploi. Grâce à cette microfondation, les nouveaux arrivants sur le marché du travail entraînent un plus haut niveau de dette immobilière tandis que ceux qui perdent leurs emplois sont exclus du marché du crédit. En conséquence, le ratio LTV devient endogène et répond de manière procyclique aux fluctuations de l'emploi. Nous avons montré que cette modélisation était empiriquement fondée et résout les anomalies d'une contrainte de crédit standard. Dans le dernier chapitre, nous avons étendu l'analyse précédemment effectuée en intégrant des firmes qui s'endettent dans le but d'obtenir un cycle financier plus complet. Le premier résultat est qu'une contrainte de crédit pour les firmes intégrant à la fois les biens immobiliers, le capital et la masse salariale permet de mieux rendre compte des fluctuations sur le marché du travail comparativement aux contraintes n'intégrant qu'une partie de ces trois composantes. Le second résultat met en évidence le rôle des fluctuations immobilières et du crédit sur l'emploi. Les deux derniers chapitres ont d'importantes implications pour les politiques économiques. Une réforme structurelle du marché du travail visant à le déréguler entraîne une forte hausse de la dette immobilière pour les ménages ainsi que du prix de l'immobilier et une augmentation moindre de la dette des firmes. Notre approche révèle qu'une politique macroprudentielle visant à restreindre la capacité d'emprunt des ménages conduit à des effets positifs à long terme pour l'économie tout en limitant les effets sur le marché immobilier (dette et prix). A l'inverse, une politique macroprudentielle visant à réduire l'emprunt des entreprises conduit à l'effet inverse avec des effets négatifs à long terme pour l'économie.

Mots-clés : Modèle DSGE; Réforme structurelle; Cycles financiers; Marché du travail; Marché des biens

THREE ESSAYS ON LABOR MARKET FRICTIONS UNDER FIRM ENTRY AND FINANCIAL BUSINESS CYCLES

Abstract: During the Great Recession, the interactions between housing, labor and entry highlight the existence of narrow propagation channels between these markets. The aim of this thesis is to shed a light on labor market interactions with firm entry and financial business cycles, by building on the recent theoretical and empirical of DSGE models. In the first chapter, we have found evidence of the key role of the net entry as an amplifying mechanism for employment dynamics. Introducing search and matching frictions, we have studied from a new perspective the cyclical nature of the mark-up compared to previous researches that use Walrasian labor market. We found a less countercyclical markup due to the acyclical aspect of the marginal cost in the DMP framework and a reduced role according to firm's entry in the cyclical nature of the markup. In the second chapter, we have linked the borrowing capacity of households to their employment situation on the labor market. With this new microfoundation of the collateral constraint, new matches on the labor market translate into more mortgages, while separation induces an exclusion from financial markets for jobseekers. As a result, the LTV becomes endogenous by responding procyclically to employment fluctuations. We have shown that this device is empirically relevant and solves the anomalies of the standard collateral constraint. In the last chapter, we extend the analysis developed in the previous one by integrating collateral constrained firms in order to have a more complete financial business cycle. The first result is that an entrepreneur collateral constraint integrating capital, real commercial estate and wage bill in advance is empirically relevant compared to the collateral literature associated to the labor market which does not consider these three assets. The second finding is the role of the housing price and credit squeezes in the rise of the unemployment rate during the Great Recession. The last two chapters have important implications for economic policy. A structural deregulation reform in the labor market induces a significant rise in the debt level for households and housing price, combined with a substantial rise of firm debt. Our approach allows us to reveal that a macroprudential policy aiming to tighten the LTV ratio for household borrowers has positive effects in the long run for output and employment, while tightening LTV ratios for entrepreneurs leads to the opposite effect.

Keywords: DSGE model; Structural reforms; Endogenous Entry; Search and matching frictions; Financial business Cycles; Labor market