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Jorge Fernando Victoria Trujillo

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Jorge Fernando Victoria Trujillo. Les problèmes de tournées de véhicules avec des demandes dépendantes du temps dans la logistique humanitaire. Gestion et management. Université de Technologie de Troyes, 2018. Français. NNT : 2018TROY0037 . tel-03589469

HAL Id: tel-03589469

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Thèse
de doctorat
de l'UTT

Jorge Fernando VICTORIA TRUJILLO

**Vehicle Routing Problems
with Time-dependent Demands
in Humanitarian Logistics**

Champ disciplinaire :
Sciences pour l'Ingénieur

2018TROY0037

Année 2018



THESE

pour l'obtention du grade de

DOCTEUR

de l'UNIVERSITE DE TECHNOLOGIE DE TROYES

en SCIENCES POUR L'INGENIEUR

Spécialité : OPTIMISATION ET SURETE DES SYSTEMES

présentée et soutenue par

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le 12 novembre 2018

**Vehicle Routing Problems with Time-dependent Demands
in Humanitarian Logistics**

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Acknowledgments

I would like to express my gratitude to my supervisors Christian Prins and H. Murat Afsar without whom this manuscript with all experiences and learning behind it would not have been possible. Their proper guidance taught me many things to be an excellent researcher. I am indebted to thank them for providing me an opportunity to work with them. It was a great pleasure.

A special thanks goes to Nubia Velasco who put me in contact with my supervisors, put her trust on me and encouraged me throughout.

I am grateful to Nacima Labadie, Christian Artigues, Nicolas Jozefowicz and Juan Carlos Rivera for agreeing to be members of the jury and contributing from their knowledge and experience towards my research.

I would also like to thank my mom and my sister, who despite the distance, have supported me in hard moments during these years. Likewise, thank you very much to those people I met in this great stage of my life with whom I shared many joys and sorrows. Without you this period would not have been the same.

I really appreciate the members of the Laboratoire d'Optimisation des Systèmes Industriels (LOSI) and the administrative team of the UTT who were always there to respond to my queries and helped me make my stay as good as possible.

Finally, I would like to thank the Conseil départemental de l'Aube for its financial support.

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Introduction

Natural disasters killed 69 thousand people on average per year from 2006 to 2015, while nine thousand people died for the same reason in 2016. There is a reduction of almost 87% between these periods. Despite the decrease in the number of deaths due to disasters in 2016, the number of affected people by disasters increased by 50% in 2016 compared with the annual average of the 2006-2015 decade [88].

Additionally, the reinsurance and consulting firm Aon Benfield presents the “2016 Annual Global Climate and Catastrophe Report” [164] which shows that the number of natural disasters in 2016 increased by approximately 22% compared to the annual average of the previous decade.

The above information worries governments, humanitarian organizations, researchers and population in general. Hence, concerned people about this situation are working to find protocols, strategies, plans and/or methods from their areas of knowledge to alleviate and diminish the consequences of these unfortunate and uncontrollable events.

This thesis proposes and develops operations research (OR) models and solution techniques to alleviate and diminish the consequences of the disasters. The Operational Research Society [1] defines OR as “a discipline of applying appropriate analytical methods to help those who run organisations make better decisions”.

The main contributions of this work are:

- Introduce novel vehicle routing problems which consider population displacement
- Propose and implement exact and heuristics methods to solve these problems.

The manuscript contains six chapters. Chapter 1 describes the context and introduces a brief explanation of the disaster management, a comparison between humanitarian and commercial logistics, a conceptualization of the population displacement as main feature of our problems and the definition of the problems treated throughout the thesis.

Chapter 2 reviews the literature on humanitarian logistics showing its contribution to the different phases of the disaster management (mitigation, preparedness, response and

recovery). Moreover, the works that address vehicle routing problems in which one or several parameters change their values as a function of time are also detailed.

Chapter 3 is divided into two sections. The first section focuses on a vehicle routing problem where the demand at each shelter decreases with time at a fixed and known rate, due to population displacement. The second section studies a similar problem with uncertain displacement rates, from a robust optimization point of view. To solve these problems, a mixed integer linear program (MILP) and a branch-and-price algorithm are developed and adapted to each problem. Moreover, a stability analysis is conducted in the first section to evaluate the impact of possible changes on the displacement rates.

Chapter 4 presents a metaheuristic framework with an optimal splitting procedure to solve large instances of the problem treated in the first section of Chapter 3. The framework is a GRASP \times ELS in which any of its variants such as GRASP, ILS, ELS, or GRASP \times ILS can be also executed changing only three parameters.

Chapter 5 investigates a problem in which the demand decreases in some shelters and increases in other ones due to the population displacement. A MILP is proposed for this problem and a two-phase heuristic method based on the GRASP \times ILS is used to solve it.

In Chapter 6, general conclusions and perspectives for future research are given.

Chapter 1

Context and definition of problems

The International Federation of Red Cross and Red Crescent Societies (IFRC) [3] defines a disaster as “sudden, calamitous event that seriously disrupts the functioning of a community or society and causes human, material, and economic or environmental losses that exceed the community’s or society’s ability to cope using its own resources. Though often caused by nature, disasters can have human origins”.

In addition, they argue that the combination of hazards, vulnerability and inability to reduce the potential negative consequences of risks results in disaster.

The vulnerability can be understood as the reduced capacity of an individual or group to anticipate, resist and recover from the impact of a natural or man-made hazard. While the capacity refers to the resources available to these individuals to deal with a threat or to resist the impact of a hazard [4]. A hazard is a threatening event, or probability of occurrence of a potentially damaging phenomenon within a given time period and area [2].

The more than three thousand natural disasters occurred around the world in the last decade leaving around two billion of affected people and seven hundred thousand deaths [87] show the imbalance between vulnerability and hazard with respect to the ability of the population to cope with a threat.

1.1 Disaster management

Archeologists found evidences of past civilizations applying disaster management concepts that guide today’s practices. The Egyptians created the first river control project using a system of over 200 “water wheels”. The Roman emperor Augustus established a formal city-wide firefighting unit, after Rome was nearly destroyed by fire. The Incas lived on the top of mountains as defensive strategy against the enemy attacks. They developed a form of land terracing that not only conserved water in their unpredictable climate but also

protected their crops [52].

According to Coppola [52] and Pearce [160], the term disaster management began to emerge in the mid-twentieth century, when governments elaborate systems of civil defense due to the threat posed by air raids and nuclear attacks.

The main goals of disaster management are to improve response capacity of population against the threats of man and nature, and decrease the degree of vulnerability and hazard to which the population is exposed.

The disaster management is based on four cyclic phases as shown in Figure 1.1.

Mitigation phase, also called disaster risk reduction, refers to a set of plans or mechanisms that are implemented to reduce or eliminate likelihood and/or consequences of a hazard [52]. This makes the population less vulnerable to disasters.

Preparedness phase consists in establishing operations such as data collecting, personal training, and assignment of tasks that must be executed by the people and government before and during a disaster [184]. This empowers the population and helps to ensure an adequate response to disasters.

The previous phases are called as pre-disaster phases because they must be carried out before the disaster strikes a territory. The next two phases concerning the aftermath of a disaster are called post-disaster phases.

Response phase takes place during, and immediately after the disaster occurs. The operations planned in previous phase must be performed efficiently and effectively. The aim is to reduce the number of injuries, deaths, and damage to property and environment.

Recovery phase occurs immediately after the *response phase* and it can takes months or even years after the disaster. During this phase, the efforts are managed with the intention of returning the lives of victims back to a normal state. This involves repairing and reconstructing all damages, not only in terms of infrastructures or environment, but taking also into account psychological health of victims.

According to Cozzolino [54], the *preparedness, response and recovery phases* constitute the humanitarian logistics stream. This stream is essential to have a well structured humanitarian aid, a flexibility in response, and as consequence the minimization of human suffering can be achieved [204].

1.2 Humanitarian and commercial logistics

The humanitarian logistics is also known as relief logistics or emergency logistics [129]. It is defined as “the process of planning, implementing and controlling the efficient, cost-effective flow and storage of goods and materials, as well as related information, from the

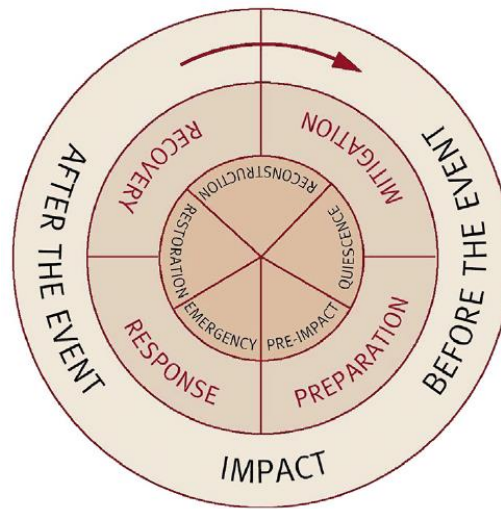


Figure 1.1 – Disaster Management cycle. According to Coppola [52].

point of origin to the point of consumption for the purpose of alleviating the suffering of vulnerable people" [203]. This process in the humanitarian relief chain is responsible for defining the success or failure of humanitarian operations.

Ertem et al. [71] and Van Wassenhove [211] agree that humanitarian logistics is 15 years behind commercial logistics. However, the activities to handle the flow, information, and storage of products from an origin to a destination have to be effective and efficient in both contexts.

Moreover, humanitarian and commercial logistics are quite different. While the latter is focused on financial and classical logistical measures such as cost, time and percentage of fulfilled demand [25], humanitarian logistics is more interested in saving human lives and alleviating human suffering without discrimination [95].

The time plays an important role in both contexts. The difference is the impact it has if the response or lead time is reduced. A life can be saved or the suffering of a person can be diminished in the humanitarian context. In the commercial context, a client is satisfied and the economic benefit is received by the company. This shows that time is more critical in humanitarian logistics than in commercial logistics [177].

The unpredictable demand in terms of timing, location, type and size makes the planning and distribution process more difficult. These data can be known and/or predicted based on historical data more easily in the commercial than in the humanitarian logistics [19]. The required demand depends on the magnitude of the disaster and on the number of people in the affected territory.

The difficulty of forecasting the amount of required products that must be sent to the affected population after a disaster is not the only obstacle at the moment distribution.

Knowing all suppliers in advance is another of them. In the best case, they are partially known. However, if the disaster management cycle is not followed, the suppliers can be totally unknown. Most of the suppliers are donors and they may not appear until or even after the disaster happens. This does not allow to have a complete and efficient distribution plan beforehand.

Humanitarian logistics has a large number of stakeholders compared with commercial logistics. In commercial logistics, they can be easily identified and coordinated. Individual objectives can be grouped and a generalized objective determined. Conversely, the high variety on beliefs and needs among the stakeholders in the humanitarian context causes uncoordinated relations and operations. Most of the time, this variety harms and hinders operations [97]. Some of the stakeholders in humanitarian logistics are the local and international governments, non-governmental organizations, donors, media and affected population.

Resources in commercial logistics are already available and/or easily acquired. While in humanitarian logistics, the required staff might not be available when needed, the capital comes through donations, they might not be available in the right amount or at the right time, and the infrastructure might be damaged by the disaster. For these reasons, the human, capital and/or infrastructure resources are usually scarce [205].

Commercial logistics can measure the performance and invest in improving them. The good performance is rewarded by the market and internal incentives [211]. This does not happen in the humanitarian context due to the lack of reliable information and investment. This does not allow a fast improvement as in commercial logistics, although a return on operating experience can be exploited.

The previous features demonstrate the complexity of humanitarian logistics and illustrate the great challenge faced by people working on this topic.

1.3 Population displacement

Population displacement is a consequence of disasters. The Norwegian Refugee Council and the Internal Displacement Monitoring Centre [5] define the displacement as a forced or obliged movement, evacuation or relocation of individuals or groups of people from their homes or places of habitual residence in order to avoid the threat or impact of a disaster. It can be temporary or permanent, internal or international, or forced or voluntary.

Population displacement can occur before, during and/or after a disaster, depending on the capacity of response and the pre-disaster planning phases of the threatened territory. Three cases of displacement can be considered: when people are (1) evacuated to shelters

where all their basic needs must be satisfied while the territory is made safer and repaired, (2) in shelters and they do not receive aid as quickly as they need and they leave them looking for it outside of the affected territory, and (3) obliged to move directly far away from their homes to other cities or countries because all is completely destroyed or an armed conflict forces them to leave everything. The movement of people in the last two cases may put them in a more precarious situation than if they stay in their place of origin [5].

In addition, Watson et al. [218] and Kouadio et al. [110] underline that such displacements are one of the main risk factors for disease outbreaks after a disaster. These outbreaks, rather than trauma, are the major cause of deaths in emergency and post-conflict situations conclude Hopkins and the IFRC [98].

The contagious diseases associated with the displacement of people are the water-related (access to safe water) and the ones related with crowding [218].

The shelters or camps are controlled places with the capacity to receive a pre-established number of people where they receive the necessary care. The likelihood that contagious diseases appear, increases if the population moves to unknown, uncontrolled and overcrowded places.

In addition, the displacements spread the chaos already generated by the disaster to other territories, adding an extra difficulty for the relief organizations to effectively satisfy the needs of affected people [130].

The above shows the importance of avoiding the displacement of people to places outside of the affected territory. The efficient distribution of first aid to shelters in order to stop the displacement of people is a focus of this work. According to Kouadio et al. [110], the rapid implementation of control measures through re-establishment and improvement of primary healthcare delivery should be given high priority.

1.4 Routing problems with time demand displacements

The capacitated vehicle routing problem with time-dependent demands (CVRP-TDD), the robust CVRP-TDD, and the cumulative CVRP-TDD are introduced in this thesis. These problems contribute to the planning of routes to distribute relief products in the aftermath of a disaster, taking into account population displacements.

When a disaster strikes, people move to shelters (case (1) in Section 1.3) to receive first aid. If they do not receive them as quickly as they need, they tend to leave the shelters looking for aid outside the affected territory (case (2) in Section 1.3). This latter case is an important feature of the proposed problems.

In the CVRP-TDD, people leave the shelters with a fixed and known displacement rate. The number of people who must be served in shelters decreases linearly over time. The objective is to maximize the number of people attended at each shelter, by arriving as quickly as possible with the first aid. Unlike this problem, the robust CVRP-TDD considers that the displacement rates from shelters are uncertain. The cumulative CVRP-TDD considers not only out-going flows but also in-coming ones. Therefore some shelters have a decreasing demand, while the others are the shelters that receive fleeing people. In this latter problem, the objective is to minimize the sum of the arrival times at critical shelters.

In the problems described above, the number of shelters as well as the number of people in them before they begin to flee are fixed and known. The relief products are transported in a number fixed of homogeneous vehicles which start from a single depot. Each vehicle has a maximum tour length to visit shelters and come back to the depot.

The formal definitions of the CVRP-TDD and the robust counterpart are given in Chapter 3, while cumulative CVRP-TDD is defined in Chapter 5.

In the next chapter, the state of the art in humanitarian logistics and vehicle routing problems which consider time-dependent components are detailed.

Chapter 2

State of the art

In the previous chapter, humanitarian logistics is presented as the context of the problems addressed in this thesis. It is also stated how the demand is a time-dependent characteristic of the problems. Therefore, the state of the art is focused on the following two topics: humanitarian logistics and time dependency in vehicle routing problems.

In Section 2.1, the articles that tackle problems in humanitarian logistics are described. Section 2.2 contains contributions on vehicle routing problems that incorporate time-dependent components. Concluding remarks are brought by Section 2.3.

It should be noted that additional research works are referenced in each chapter throughout the thesis. These citations are intended to provide an overview of the solution methods used.

2.1 Humanitarian logistics

Humanitarian logistics (HL) has been extensively discussed in the literature in the last decade. A large number of surveys covering the research in this field have been published between 2006 and 2017. Altay and Green [13] review papers in operations research and management sciences (OR/MS) applied to disaster operations. They give an overview of the articles that were published on this subject from 1980 to 2004. Kovács and Spens [111] are more interested in research works related to logistics operations in disaster relief. They make a parallel between HL and commercial logistics. This analysis is enriched by contributions in both academic and practitioners journals. Simpson and Hancock [189] survey papers in OR applied to emergencies from 1965 to 2007. They extend their research to all types of emergencies, not only those related to disasters. Natarajarathinam et al. [145] focus their review on works in supply chain management in times of crisis. They investigate crisis induced by disasters and by financial and/or

managerial problems. Overstreet et al. [153] describe the complexities that are found in HL and classify the articles based on these complexities and other key elements such as organization personnel, equipment/infrastructure, transporting, information/technology, planning/policies/procedures, and inventory management. Kunz and Reiner [113] analyze and categorize the contributions based on situational factors which are external contextual variables that impact the performance of humanitarian logistics. The factors identified are government, socio-economic, infrastructure, and environmental. Galindo and Batta [80] extend the survey done by Altay and Green [13] considering papers from 2005 until 2011. They also evaluate the most common assumptions in the literature and divide them into three categories: reasonable, limited and unrealistic. The survey of Jabbour et al. [47] is the most recent in humanitarian logistics and supply chain management.

The previous surveys give a broad overview of the state of the art on humanitarian logistics. However, there are other types of surveys dedicated to the following aspects: uncertainty (Liberatore et al. [122], Hoyos et al. [99]), response phase (Anaya-Arenas et al.[15], Özdamar and Ertem [158]), recovery phase (Özdamar and Ertem [158], Çelik [44]), optimization models (Caunhye et al. [42], Habib et al [92]), multicriteria optimization (Gutjahr and Nolz [91]), decision aid models and systems (Ortuño et al. [150]), inventory management (Balcik et al. [23]), relief transportation (de la Torre et al. [58]), and facility location (Boonmee et al. [34]).

In most of the surveys, the most common classification of the HL papers follows the phases of disaster management. This allows a better understanding of the research carried out in this field. For this reason, we decide to divide this section in the same way.

2.1.1 *Mitigation phase*

During the 80's and 90's, the OR/MS research was more focused on this phase than on the others (Altay and Green [13]). The majority of research is in disaster risk assessment and territorial development.

The disaster risk assessment allows to identify and analyze the risks to which a territory is exposed. Besides, it helps to take the appropriate measures to prevent a disaster or reduce its possible effects. Wang et al. [217] evaluate the flood risk in Dongjiang River Basin, China. They introduce a risk assessment method based on a machine learning algorithm called "random forest". The highest risky zones, as well as the most important risk factors, are identified. Norén et al. [148] make a comparison between the flood risk assessment done in the municipalities of Sweden and how the theory states that it should be performed. They find that there is still a big gap between practice and theory on this topic.

The territorial development helps to make better decisions in terms of land use to

avoid disasters as consequences of landslides, earthquakes, mudslides, etc. Saunders and Kilvington [181] describe a risk-based planning approach which guides planners to provide robust decisions around different levels of risk for the regions. Iizuka et al. [103] design three disaster mitigation/prevention structure models for the future urban planning of the 2050s in Nagoya, Japan. Mitchell et al. [136] identify the common post-earthquake land administration functions and challenges and they review it in the cases of Haiti, Nepal, and New Zealand.

2.1.2 *Preparedness phase*

Before a disaster strikes a territory, some activities must be planned to respond quickly to the needs of affected population. Three main activities are identified: the management of inventory and equipment, location of sites to serve victims (meeting points, shelters, temporal hospitals, etc.), and evacuation planning.

In the management of inventory and equipment, the inventory levels and locations of both inventories and resources must be decided. These allow to supply faster the needs of the casualties. Whybark [219] shows the importance and complexity of inventory management in HL. He also makes a comparison between commercial and humanitarian inventory management. He concludes that only a fraction of what has been learned in the commercial context can be replicated in the humanitarian one. Moreover, the needs of humanitarian inventory managers are not very clear and should be further investigated.

The papers related to pre-positioning and management of inventory can be grouped into (1) those in which only the inventory levels have to be determined and (2) those in which both locations of storage facilities and inventory levels have to be optimized.

In the first group, Beamon and Kotleba [27] propose a stochastic inventory control model to determine optimal order quantities and reorder points for a long-term emergency relief response. An inventory policy is investigated which is an extension of the standard (Q,R) inventory policy. On the other hand, Lodree Jr. and Taskin [127] introduce newsvendor variants considering uncertainty in demands and in the time the disaster occurs. The optimal inventory level is obtained and it is compared to the classic newsvendor solution.

In the second group, Balcik and Beamon [21] study a variant of the maximal covering location model to determine the location of several distribution centers (DCs) and the amount of relief supplies to be stored at each DC. The objective is to maximize the total expected demand covered in a set of scenarios. Campbell and Jones [38] provide a single product model to find the optimal positioning of supplies and inventory levels considering risk (probability that the facility is destroyed or inaccessible after the disaster). They seek to minimize the expected total costs. Galindo and Batta [79] tackle a similar problem but they present

a stochastic model to solve it. Paul and MacDonald [159] locate and dimension stocks of medical supplies in case of a hurricane. The goal is to minimize a total expected cost including stock setup costs, storage costs, and external costs like potential fatalities. A stochastic optimization model is solved using a three-step heuristic.

Once the suppliers are located and the inventory levels are optimized, the sites where people should go to receive first aid must be also pre-established. Pérez-Galarce et al. [161] locate a set of shelters to attend the victims of an earthquake. They consider two types of shelters: those which provide basic needs and those with additional medical and psychological care. A mixed integer linear program (MILP) and three variants of it are proposed. The model minimizes the total distance traveled by victims to the shelters. Muggy and Stamm [142] introduce a method that includes dynamic, scenario-based, robust optimization model to choose post-disaster health care facility locations considering accessibility and equity. The objective is to maximize potential spatial accessibility which is measured by a combination of the distance to access to the facility and its capacity of service. Gutjahr and Dzubur [89] implement an exact bi-objective bi-level optimization algorithm to solve the location of DCs considering user equilibria. The objectives are to minimize the opening cost of DCs and the uncovered demand.

In addition to facilities, other resources such as aid personnel and equipment can be pre-positioned. Altay [12] deals with this problem assigning resources to minimize the completion time of a set of tasks in response to a disaster. Huang and Fan [102] allocate multiple service resources to protect transportation infrastructures. They use deterministic, stochastic, and robust optimization to model risk preferences of decision makers considering uncertainty in service availability and accessibility. Minciardi et al. ([134], [135]) develop a mathematical model to determine the optimal resource allocation before and during a disaster. In addition, a decision support system for resource intervention in real time disaster management is proposed.

People should know how to evacuate their homes, workplaces, parks, malls, neighborhoods, towns, cities, etc. An evacuation plan must be established for each of these places. The purpose of the evacuation planning is to identify the most efficient way to move out a group of people from dangerous zones to safe locations while limiting network congestion induced by the chaotic flow of people.

On a small scale, people have to evacuate places by foot such as buildings, stadiums, theme parks, etc. Abdelghany et al. [6] generate a plan to evacuate facilities with multiple exit gates. The plan seeks to minimize the average evacuation time. They use a non-linear mixed integer program and a simulation-based optimization framework. Solmaz and Turgut [192] model the mobility of the pedestrians in disaster areas. They consider the evacuation

of theme parks. The model incorporates mobility decisions as the local knowledge of the theme park and the social interactions among the visitors. Emoto et al. [70] analyze the evacuation of vulnerable people (elderly people, children, pregnant women, disabled or ill people) using tunnels in case of a fire following a major earthquake. They use simulation tools to calculate the evacuation completion time.

Osman and Ram [151] not only find an evacuation plan in a building but also its continuation in the road network to reach a safe place for the evacuees of the building. They solve it as a routing and scheduling evacuation problem using a centralized point-to-point look-back algorithm. The aim is to find evacuation routes minimizing the sum of the evacuation times of each evacuee.

Evacuation plans using the road networks are studied when the affected area is bigger and the travel distances are longer. Pillac et al. [163] generate an evacuation plan from a floodplain in Australia. They use a Column Generation (CG) method which maximizes the number of evacuees reaching safety. Bretschneider and Kimms [37] model the evacuation of an urban area as a pattern-based dynamic network flow minimizing the total evacuation time. A two-stage heuristic approach is implemented. Shahparvari and Abbasi [183] consider the people evacuation in a bushfire. They present a robust stochastic mathematical model to maximize the number of evacuees transported to safety shelters. This model considers road availability and disruptions. The number of evacuees population, time windows, and bushfire propagation are uncertain in this problem. They solve large size instances via a greedy algorithm.

Swamy et al [197] develop a multi-stage approach to evacuate people using the public transportation before a hurricane occurs. The first stage is to determine the pickup points for evacuees, the second is to assign each pickup point to a destination (shelter), the third is to construct the set of routes for the buses visiting the pickup points, and the last one is designed to evaluate the performance of the routes. Goerigk et al. [84] combine the car and buses transportation to evacuate an urban area. They use a multi-criteria MILP model and a genetic algorithm. The objective is to minimize the evacuation time, the risk and the number of used shelters.

The location and routing decisions are also combined by some authors. Ukkusuri and Yushimito [210] study an integrated location-routing problem (LRP). The location of facilities takes into account the possible disruptions in the transportation network. An integer programming model is solved in which only reliable paths are considered. The objective is the maximization of the probability that the relief aid reaches all demand sites. Moshref-Javadi and Lee [141] propose a MILP and two metaheuristics (memetic and recursive granular algorithm) to minimize the sum of the arrival times at the customers. They solve the

cumulative capacitated vehicle routing problem which is explained in the *response phase*. The LRP with multiple objectives is also studied. Rath and Gutjahr [170] minimize the opening cost of facilities, the operational costs, and the uncovered demand. An adaptive epsilon-constraint algorithm is used to determine the Pareto frontier. In addition, a matheuristic composed of a MILP and the variable neighborhood search (VNS) algorithm is developed. Abounacer et al. [7] also implement the epsilon-constraint algorithm to minimize the total transportation duration, the number of personnel required to open and operate the facilities, and the uncovered demand.

Mohammadi et al. [137], Tavana et al. [202], and Chowdhury et al. [48] include inventory decisions in the LRP. The first paper develops a multi-objective stochastic model to determine the location of DCs, the optimal inventory levels, and the optimal commodity flow amount. The objectives are to minimize the expected total costs, maximize total expected demand coverage and minimize the maximum difference between the satisfaction rates of demand nodes. Tavana et al. [202] tackle a multi-echelon humanitarian logistic network. They locate warehouses to handle the inventory of perishable products which are delivered in the response phase. Three objectives are minimized: (1) the costs related with location of warehouses and inventory, (2) the transportation costs, (3) the total operational time. A MILP is proposed and an epsilon-constraint method, a non-dominated sorting genetic algorithm (NSGA-II) and a modified NSGA-II are implemented. Chowdhury et al. [48] introduce an integrated mathematical model to minimize the overall cost. They consider drones and trucks as transportation modes from multiple DCs to demand points. The use of drones to transport relief products is a recent trend in routing problems, as shown in the next phase.

2.1.3 *Response phase*

Immediately after a disaster strikes, the response phase begins and the operations planned in the previous phase must be carried on. In this phase, the relief goods must be sent to the affected areas and the injured people should be attended and/or transported to hospitals.

According to the surveys mentioned above (Kunz and Reiner [113], Galindo and Batta [80], and Jabbour et al. [47]), most of the contributions in HL literature in last years are focused in this phase.

Different optimization criteria are used in the literature in order to better represent the emergency situation. These criteria can be classified into those that pursue *efficiency*, *effectiveness*, or *equity goals* (Gralla et al. [86]). The *efficiency goals* are related to the minimization of operational costs or travel times, the *effectiveness goals* seek to maximize measures of service (e.g., amount of satisfied demand and/or the speed to meet the demand),

and the *equity goals* refer to fairness in the distribution of services among the affected population. Defining the last group of goals has been a difficult task for the researchers due to the different interpretations of the terms “equity” and “fairness”.

The *efficiency goals*, being the same as for the classical VRP applied to commercial distribution, are also used in the humanitarian aid. Berkoune et al. [33] minimize the total duration of all trips performed by a heterogeneous limited fleet. Each vehicle can do multiple trips to distribute several products among the affected population during the day. The duration of a trip is calculated as the round trip time from a DC to a delivery point plus a docking time which depends on the vehicle and the DC plus the loading/unloading time of the relief products. They propose a MILP model and a genetic algorithm to solve the problem. Lu et al. [129] solve a real-time relief distribution problem to minimize the total distribution time considering the risk-averse attitude of the decision makers. In this problem, the travel times, demands and delivery times are uncertain parameters. A rolling horizon-based framework with two modules is developed. The first one is used to predict the demands and delivery times, while the second finds the relief distribution flows. Özdamar [154] and Özdamar and Demir [156] study the delivery of relief products and the transport of the injured people. The first one coordinates helicopter operations to minimize the total mission time. This time is composed of the flight time and the time of loading/unloading commodities and injured people. A planning system is built in which a network flow model is solved and the outputs are taken by a Route Management Procedure to get the helicopter routes and loads. While in Özdamar and Demir [156], a two-phase algorithm is designed to find clusters and vehicle routes to minimize the total travel time considering multiple transportation modes.

Garrido et al. [81] and Sharif and Salari [184] are focused on achieving efficiency in terms of costs rather than time. Garrido et al. [81] consider the cost of holding inventory in the depot, relocating a vehicle and the transport of multiple commodities over a planning horizon during a flood emergency. A stochastic programming model is used to take into account the uncertainty of demand. This model is approximately solved through a scheme of sample average approximation. On the other hand, Sharif and Salari [184] minimize a routing and allocation cost. The latter occurs when a demand point cannot be visited directly by a vehicle and its demand is allocated to another point within a coverage radius. A greedy randomized adaptive search procedure (GRASP) is implemented to determine the vehicle routes to satisfy the demand of visited and covered points.

The authors interested in measures of service from the point of view of the beneficiaries take into account *effectiveness goals* rather than efficiency ones. The effectiveness is defined as the degree to which the requirements of the customer are met (Beamon and Balcik [26]).

For instance, De Angelis et al. [16] maximize the total satisfied demand by scheduling and routing weekly cargo planes to deliver food in Angola. The operation time and the availability of food and parking spaces at depots are considered to build a mathematical model. Their results are compared with the schedules implemented in reality. The same objective function but represented as the minimization of unsatisfied demand is treated by Shen et al. [186] and Özdamar et al. [157]. The first ones address the stochastic vehicle routing problem in which the demand and the travel times are uncertain. They show its applicability in the distribution of medical supplies in natural or man-made disasters. A deterministic model is described, and a chance-constrained programming and robust optimization are used to handle the uncertainty. Moreover, a Tabu Search (TS) is also implemented to solve large instances in reasonable time. Özdamar et al. [157] investigate a dynamic transportation problem which is solved iteratively at given time intervals over a planning horizon. At the end of each interval, the distribution plan is updated with new information such as supply/demand of multiple commodities and availability of transportation modes. The goal is to find the pick-up and delivery schedules for vehicles and the quantities to be loaded and unloaded. A mathematical model and a Lagrangian relaxation based heuristic approach are proposed to solve the problem.

The transportation of both relief products and injured people pursuing *effectiveness goals* is addressed by Yi and Kumar [223] and Najafi et al. [143]. The first ones minimize the unsatisfied demand plus the unserved injured people. Each of these terms is weighted by a given priority. They develop a mathematical model and an ant colony optimization to solve it. In Najafi et al. [143], the number of injured people, hospital capacities, and supply/demand of commodities are uncertain parameters. Moreover, several objectives such as the unserved injured people, the unsatisfied demand, and the number of vehicles used are minimized. A MILP is used to model the problem and a robust optimization method is applied to handle the uncertainty. On the other hand, Dean and Nair [59] are only interested in the transport of injured people. They present a mathematical model to maximize the number of expected survivals in the transportation of injured people. A triage is applied to the victims and a probability of survival is assigned to each one. These victims are sent to the best-adapted hospital (medical staff and medicines available) to meet their needs.

An analysis done by Campbell et al. [39] demonstrates the importance of using *equity goals* in humanitarian contexts instead of those seeking efficiency. Campbell et al. [39], like other authors incorporate the minimization of the sum of arrival times at nodes and the minimization of the maximum arrival time to guarantee equity in their solutions. Talarico et al. [200] solve a similar problem that one presented by Dean and Nair [59], but only critical injured people are transported to the hospital because the others receive help directly in the

field. They build two MILPs and a large neighborhood search (LNS) algorithm to obtain routes that minimize the weighted latest service completion time among the critical and non-critical casualties. This objective is quite similar to the minimization of the maximum arrival time, but service times are included. Ngueveu et al. [146] introduce the cumulative capacitated vehicle routing problem (CCVRP) which is a variant of the CVRP where the objective is to minimize the sum of arrival times at nodes. This objective is equivalent to minimize the average arrival time at nodes which guarantees a more equitable distribution, in terms of time, to the beneficiaries. They propose a MILP, two memetic algorithms and develop new lower bounds for the problem.

The CCVRP is studied by several authors as Ribeiro and Laporte [172] which implement an adaptive LNS heuristic and compare it with the memetic algorithms in [146]. Chen et al. [46] and Ke & Feng [108] propose an iterated local search (ILS) procedure and a two-phase metaheuristic, respectively. A branch-and-cut-and-price algorithm is published by Lysgaard and Wøhlk [131]. Sze et al. [198] solve the classical CCVRP, but also consider the minimization of the maximum arrival time. They implement an adaptive VNS which incorporates a LNS as a diversification strategy.

Some extensions of the CCVRP are also addressed. In Rivera et al. [174], each vehicle can perform more than one trip. They model it as a MILP and a multi-start ILS is proposed to solve it. A single vehicle performing multiple trips is also addressed by Rivera et al. [173]. In this case, two MILPs and an exact algorithm are developed. Moshref-Javadi and Lee [140] assume a heterogeneous fleet to deliver multiple products to satisfy the demand of beneficiaries. Moreover, split delivery is allowed. This means that there may be multiple arrival times to beneficiaries. So, they evaluate two ways to compute the objective function: (1) last delivery time at beneficiary and (2) the arrival time multiply by the quantity delivered and its importance. They propose two MILPs, a hybridization of a VNS and a simulated annealing (SA) as solution methods. In the work done by Flores-Garza et al. [77] not all nodes have to be visited directly by the vehicles. Then, three types of sites are assumed: mandatory (must be visited), unreachable by vehicles, and optional (can be visited to cover unreachable sites within a given distance). Small instances are solved using a MILP, the larger ones with a GRASP.

A different objective function is considered by Balcik [20]. She tackles a selective assessment routing problem in which a minimum coverage rate must be maximized. The coverage rate is given by the number of times that a critical characteristic is evaluated by assessment teams and the total number of sites that carry the characteristic in the network. These characteristics are evaluated to determine the effects of the disaster on the affected region.

In addition to the works that focus solely on *efficiency*, *effectiveness* or *equity goals*, there are also those that make a trade-off between them. For instance, Balcik et al. [22] minimize the routing cost and a penalty cost related to the amount of unsatisfied demand in the same objective function. They introduce the last mile distribution problem (LMDP) which refers to deliver relief supplies from depots to the affected population. A mathematical model is presented to optimize the resource allocation and routing decisions.

Huang et al. [101] also address the LMDP. The three goals are measured separately to analyze the results obtained with each one. The total travel time, the sum of the arrival times at each node multiplied by the amount of demand delivered on it, and three additional metrics which guarantee equity in the distribution are proposed. The first metric seeks to minimize the difference between the maximum and minimum service level over all nodes. The service level of a node is given by the arrival time multiplied by the amount of demand delivered, normalized by its demand. The second metric minimizes the standard deviation of the service level and the third one is the sum of cumulated penalties due to the percentage of unsatisfied demand of each node over each discrete time period.

Shen et al. [185] solve a VRP in which a relief product must be delivered to beneficiaries in a single period by heterogeneous vehicles. The objective is to minimize the unsatisfied demand while the sum of the arrival times guides the construction of complete routes. A planning stage and an operational stage are considered to get a solution to the problem. In the first stage, a stochastic programming model assuming uncertainty in travel times and demands is solved. In the operational stage, the real information is revealed and three recourse strategies with their respective mathematical are implemented.

Holguín-Veras et al. [97] propose a social cost as objective function. This cost is composed of logistics and deprivation costs. The latter cost is the economic value of human suffering resulting from the lack of access to a good or service. They argue that this objective not only take into account the operational costs, but also the service level that must be guaranteed to the affected population. Therefore, *efficiency* and *effectiveness* goals can be achieved at the same time. Gutjahr and Fischer [90] emphasize that the use of this objective can lead to unfair solutions and suggest to extend the deprivation cost by a term proportional to the Gini inequity index to get more equitable solutions.

Rivera-Royero et al. [175] assign a priority score dynamically to the nodes in a multi-period distribution problem. The score is based on accessibility to the node, time that people at nodes have been waiting for relief products, and urgency of each type of relief products. Then, the objective is to minimize the total unsatisfied demand weighted by their corresponding priority scores. A MILP is proposed to solve small instances, while three methods decomposing the problem in several sub-problems are implemented to obtain

solutions to the initial problem.

Lin et al. [125] prioritize critical products to be delivered in the aftermath of a disaster. A multi-objective problem is tackled with the following objective functions: The minimization of the total unsatisfied demand, minimization of the travel time and the minimization of the maximum difference of satisfaction rate between any two nodes. The satisfaction rate is the ratio between the requested demand and the real delivered amounts. It is formulated as a multi-objective integer programming model and two heuristics are introduced to solve it. The first is based on a genetic algorithm, while the second is generated from the decomposition of the original problem.

Tzeng et al.[208] address a multi-objective relief distribution problem. Three objectives are considered: the minimization of total cost, the minimization of total travel time and the maximization of the minimum ratio of the satisfied demand during the planning horizon. In this problem, transfer depots are used as intermediate nodes between the supply and demand nodes. Only the required depots are opened and they serve to transfer the multiple products from large trucks to smaller ones.

Vitoriano et al. [216] use goal programming as a multi-criteria optimization technique to deal with an aid distribution problem. They consider several criteria such as the distribution cost, operation time, equity, priority, reliability, and security. The cost/time criteria are the most common ones, the third is related to the fairness in the amount of relief products distributed, the fourth is to prioritize the satisfaction of demand in specific nodes, the fifth is given by the probability of crossing a road that can suffer a damage during the distribution, and the last criteria is related to the ransack probability in these situations.

The transportation using unmanned aerial vehicle (UAV), also called drones, is receiving increased attention due to their capacities to transports goods quickly, safely, and inexpensively across accessible and inaccessible terrains (Otiede et al. [152]). The last mile distribution problem is solved by Rabta et al. [169] considering this transportation mode. The objective of the proposed MILP is to minimize the total travel distance under payload and energy constraints with recharging stations.

2.1.4 *Recovery phase*

After the disaster, the operations required to return the affected area to normality as quickly as possible must be executed in this phase. The restoration of roads and other infrastructures as well as debris management are the main operations treated in the literature.

The roads can be damaged or blocked by debris in the aftermath of a disaster. This makes some affected areas inaccessible for aid teams and people cannot be evacuated to safe

places either. The accessibility to these isolated areas must be restored as quickly as possible in order to attend the needs of the population. The restoration of roads can be done by repairing them or clearing the debris. Some authors make the distinction between the two approaches although the problems are the same, in the sense that the routes are available after the restoration. These problems fall into the class of arc routing problems (ARPs) because the activities and/or operations to be carried out are on the arcs of the network.

Maya Duque and Sörensen [133] allocate resources to repair a rural road network improving the accessibility of people from towns to cities to satisfy their basic needs after a disaster. They define the accessibility of a town as the travel time required to reach the nearest city. Therefore, the weighted sum of the shortest path for all towns to the nearest cities must be minimized. A GRASP with a variable neighborhood descent (VND) is used to solve the problem. Özdamar et al. [155] generate work plans for a fleet of dozers to perform the debris clearance on the blocked roads. They also seek to minimize the shortest path, but between all pairs of nodes. In addition, the completion time of the debris clearance (makespan) has to be minimized. A MILP is presented and a constructive heuristic with embedded task selection rules is developed. Aksu and Özdamar [11] present a dynamic path based mathematical model to maximize the total weighted earliness of all cleared paths. This objective function is quite similar to the minimization of the makespan because all paths must be completely restored as quickly as possible especially if they have a high priority. In Akbari and Salman [10], multiple vehicles must be synchronized to restore a set of blocked or damaged roads to regain the connectivity of the network. A matheuristic using a MILP relaxation and a local search algorithm is implemented to minimize the maximum cost (time) of the walks. A walk is a time measure by vehicle which consists of the time of traversing the edges, the time of road clearance, and the waiting time for another vehicle to finish its clearance operation in an edge. Kasaei and Salman [107] study two problems for clearing blocked roads. The objective of the first one is to minimize the time to reconnect the road network, while the second one maximizes the total benefit obtained by reconnecting the road network within a time limit. For each problem, they propose a MILP model, a VNS, and a VND.

In Averbakh and Pereira [18], a road network is constructed completely with a crew departing from the depot. The aim is to restore the access to the disconnected sites taking into account the due dates for recovery of the sites. Two problems are solved, one with the objective of minimizing the maximum lateness of the sites and the other with the minimization of the number of tardy sites. Different MILP formulations, heuristics, lower bounds and a branch and bound algorithm are proposed by the authors.

Some authors consider roadway repair and relief distribution in the same problem. Yan

and Shih [222] incorporates two time-space networks into a multi-objective model which integrates these two activities. The objectives used are the minimization of the maximum arc time in the repair time-space network, while the second one is the same but in the relief distribution time-space network. Sahin et al. [178] solve a similar, but they do not use time-space networks. A mathematical model and a heuristic are proposed to minimize the total traveling time plus the total time spent on debris removal operations until all nodes are visited. Sakuraba et al. [179] study the road network accessibility problem and the work-troops scheduling problem. The first consists in finding traversable paths for relief teams to reach the population, and the other generates a repairing schedule to improve access to affected areas. Liberatore et al. [123] propose a hierarchical multi-criteria model where the cost, delivery time, security and reliability are incorporated as criteria. The first level maximizes the total reached demand, the second level minimizes the maximum distance between the criteria and their ideal value and the third level minimizes the sum of the criteria distances.

During the road network restoration, the debris are placed on the sides of the roads. These and other debris must be collected and treated as a second part of the recovery process. Hu and Sheu [100] present a reverse logistics system for post-disaster debris. A multi-objective linear programming model to minimize the operational costs, risk, and psychological costs. In other cases, the debris are reusable or recyclable decreasing the need for reconstruction resources. These materials can be processed in temporary storage sites. In Onan et al. [149], a framework is implemented to determine the location of these sites including planning for the collection and transportation of the debris. A multi-objective optimization model is developed and solved with an NSGA-II. The objectives are the minimization of costs and the minimization of risk from hazardous waste exposure.

Roads are not the only infrastructure that can be damaged during a disaster and need repair, in order to help the affected population. In Xu et al. [147], the electric power system is collapsed. The authors develop a stochastic integer program to determine how to schedule inspections, damage assessment, and repair tasks to restore it. The objective is to minimize the average time each person is without power. A genetic algorithm is implemented. The results are compared with the original schedules of the power company using a discrete event simulation model. Cavdaroglu et al. [43] assume that the essential public services (power, telecommunications, water, and transportation) are damaged. They integrate a restoration and scheduling model to repair the disrupted interdependent infrastructure systems (e.g. power and telecommunications). They develop a MILP and a heuristic to minimize the weighted sum of (1) cost of operating the infrastructures at each period, (2) the cost of unmet demand at each time period, and (3) the cost of restoring the infrastructures.

2.2 Time dependent vehicle routing problems

The time dependent problems include one or several parameters which change their values as a function of time. This allows us to model some problems more realistically. The vehicle routing problems with time-dependent demands belong to this class of problems due to the population displacement which makes the demand in shelters change over time. Research works applied to various time-dependent problems are cited before the few ones dealing with humanitarian logistics.

One of the features which can change with time is the travel time in VRP problems. Beasley [29], Hill and Benton [96], and Malandraki and Daskin [132] are among the pioneers of the time-dependent vehicle routing problem (TDVRP) where the travel time is time dependent. Beasley [29] adapts the saving heuristics. Hill and Benton [96] incorporate this idea for intra-city vehicle scheduling problems arguing that travel speeds (times) change drastically during the day because of congestion in the road network. They propose a modeling approach which is implemented in a commercial courier vehicle scheduling system. Malandraki and Daskin [132] introduce the TDVRP with time windows. A MILP and several heuristics to minimize the total route time (travel time, service time, and waiting time) are presented.

Franceschetti et al. [78], Soysal and Çimen [193], and Ehmke et al. [67] are interested in minimizing the greenhouse gas emissions in VRPs assuming that they are time-dependent. Franceschetti et al. [78] refer to this problem as the time-dependent pollution-routing problem, while Soysal and Çimen [193] called it the green time-dependent VRP. Both not only minimize the emission of gases (fuel cost), but also the wages of drivers. Franceschetti et al. [78] calculate the wages of drivers by two methods and one of them is the same as the one used by Soysal and Çimen [193]. Their works differ in the way that they compute the speed of vehicles on each arc and the amount of emissions. On the other hand, Ehmke et al. [67] consider only the emission of gases in the objective function. They compute the emissions as in Franceschetti et al. [78]. In this calculation, they assume the speed of a vehicle is a random variable.

Another time-dependent problem is motivated by winter gritting. Tagmouti et al. [199] solve this problem in which the service cost is time-dependent. In winter gritting, if the operation is carried out too early or too late, the cost in time and deicing material (i.e. salt) increases. So, the time-dependent service cost is represented by a piecewise linear function which is flat and minimal within a given time interval and linearly increases on both sides of this interval. A VND is used as a solution method.

The perishable food transportation and the garbage collection can also be classified as

time-dependent problems. In first problem, product quality depends on distribution time and deadlines may apply. Coelho and Laporte [50] implement a branch-and-cut (B&C) algorithm for an inventory-routing problem with perishable products. The total profit, to be maximized, includes sales revenue, routing costs, and inventory costs. Rong et al. [176] integrate food quality degradation in a production-distribution planning problem modeled as a MILP. A garbage collection problem where the inconvenience (smell) increases with waste waiting time is investigated by Amponsah and Salhi [14]. They model it as a capacitated arc routing problem (CARP) and implement a constructive heuristic with a look-ahead strategy, followed by a local search.

Erkut and Zhang [66] address the maximum collection problem with time-dependent rewards. In this problem, each node has a reward which decreases linearly in function of time. The aim is to find a tour which maximizes the sum of the rewards collected at the nodes visited. They propose a mathematical programming formulation, a penalty-based greedy heuristic algorithm and a branch and bound algorithm. Ekici and Retharekar [68] study a similar problem, but multiple tours are allowed and a penalty rate that makes the reward decrease faster is included. The multiple tours are performed by several agents. One of the applications of this problem is the marketing representatives of a company who are trying to visit customers to maximize sales. Because of the competitors, the sales may decrease over time. Afsar and Labadie [8] solve a problem as the previous one, but they consider tour length constraints. They implement a column generation and an evolutionary local search (ELS) metaheuristic to obtain lower and upper bounds for the problem. Tang et al. [201] solve a multi-period maintenance scheduling problem in which a single technician must visit a set of customers. Each period is assigned a different reward. A MILP is formulated and a tabu search-based heuristic embedded in an adaptive memory procedure is employed to determine the tasks to perform each day without exceeding the workday duration.

Toumazis and Kwon [206] address a transportation of hazardous materials (hazmat) problem in which the accident probabilities and accident consequences are time-dependent due to traffic conditions. They seek a path and a departure time that minimizes the conditional value-at-risk (CVaR) measure. The probability of an accident is low, but if it happens the consequences for population, environment, and infrastructures can be catastrophic. Li et al. [121] consider time-dependent origin-destination demands over discrete time intervals in a location-routing problem. These demands are determined in the upper level of a scenario-based stochastic bi-level approach. From an origin to a destination (shelter), the demand is calculated as the number of evacuees in a given scenario, multiplied by the cumulative percentage of departed evacuees in the corresponding time interval of a Rayleigh distribution [207]. Ben-Tal et al. [32] tackle a dynamic (multi-period)

emergency response and evacuation traffic assignment problem. In this problem, the demand is uncertain and time-dependent. Robust optimization is applied to handle the uncertainty in the demand. In Özdamar et al. [157], both the demand and supply are time-dependent. They present a dynamic transportation problem in which these parameters are updated throughout the planning horizon. Pérez-Rodríguez and Holguín-Veras [162] introduce three different formulations for the inventory allocation and distribution problem considering the minimization of the time-dependent deprivation cost. Sung and Lee [195] solve an allocation problem of emergency medical resources in a mass casualty incident. Each victim has a survival probability decreasing non-linearly over time. The objective consists in maximizing the sum of survival probabilities of all victims.

2.3 Conclusion

In the state of the art, the research carried out in each of the disaster management phases as well as in the VRPs which involve time-dependent features is presented. This overview shows that our problems can be classified as response phase problems with *effectiveness goals* in the CVRP-TDD and its robust counterpart, and with *equity goals* in the cumulative CVRP-TDD. Moreover, the kind of VRPs with time dependencies that we consider is still seldom investigated in the literature. Indeed, the maximum collection problems with time-dependent rewards (MCP-TDR) described in Section 2.2 are the most similar problems to ours. The rewards that decreases linearly in the nodes are like the demand at (critical) shelters of our problems. In MCP-TDR, the objective is to maximize the profit and it is equivalent to the maximization of the number of people attended in shelters of the CVRP-TDD and robust CVRP-TDD. These objective functions generate that some nodes are not visited prioritizing other ones to receive a greater benefit like in the MCP-TDR. However, the humanitarian context does not allow unvisited shelters because fairness in the distribution must be guarantee. It means that all shelters have to be visited in our problems. On the other hand, MCP-TDR do not consider capacity constraints, while they are taking into account in the problems presented in this thesis.

Unlike the MCP-TDR, the robust CVRP-TDD assume uncertain parameters which is an important feature in the humanitarian context. Finally, the cumulative CVRP-TDD is the one that differs most from the MCP-TDR because our problem deals with both decreasing and increasing demands. Moreover, the optimization criteria is a minimization one which consists of the sum of arrival times at nodes.

The next chapter presents exact methods to solve the deterministic and the robust counterpart of the CVRP-TDD.

Chapter 3

Exact methods for the relief distribution to the critical nodes

In this chapter, the capacitated vehicle routing problem with time-dependent demands (CVRP-TDD) and its robust counterpart are solved using exact methods. For each version, a mixed integer linear programming (MILP) formulation and a branch-and-price (B&P) algorithm are proposed. The exact methods guarantee the optimality of the solution.

The chapter is divided into two main sections: the CVRP-TDD (Section 3.1) and the robust CVRP-TDD (Section 3.2).

Section 3.1 is structured as follows: the CVRP-TDD is formally defined and modeled by a MILP in subsection 3.1.1. The B&P algorithm is developed in subsection 3.1.2. In subsection 3.1.3, an upper bound based on a spanning tree relaxation is proposed. The computational experiments are shown in subsection 3.1.4 and conclusions are given in subsection 3.1.5.

The structure of Section 3.2 is similar to the previous one. The only difference is that it begins introducing the robust optimization technique in subsection 3.2.1. The other subsections are dedicated to the robust CVRP-TDD as follows: the formal definition and description (subsection 3.2.2), the B&P algorithm (subsection 3.2.3), the spanning tree based upper bound (subsection 3.2.4), the computational experiments (subsection 3.2.5) and the conclusions (subsection 3.2.6).

3.1 Capacitated vehicle routing problem with time-dependent demands

In the aftermath of a disaster, the distribution of relief goods must guarantee that the maximum number of affected people at shelters receive aid as soon as possible. In

the capacitated vehicle routing problem with time-dependent demands (CVRP-TDD), the population begins to flee, at a fixed and known rate, from shelters if they do not receive them. The arrival of the relief goods to the shelters stops these displacements. Therefore, the objective is to find the distribution routes in order to attend the maximum number of people in the shelters.

In this section, the exact methods proposed to obtain the optimal distribution plan are detailed.

3.1.1 Mathematical model

The CVRP-TDD is defined on a complete undirected graph $G = (V, E)$, where $V = \{0, \dots, n\}$ is the node-set and E the edge-set. For the sake of simplicity, each edge is replaced by two opposite arcs. Node 0 is the depot while $V' = V \setminus \{0\}$ is the set of shelters. Each shelter has an initial demand (number of people) a_i and a positive displacement rate b_i . This rate is the number of people who flee per time unit, before the arrival of relief aid. Each arc (i, j) has a travel time c_{ij} . The depot has K identical vehicles with capacity Q . Each vehicle can perform a route with a maximum length T_{max} . To simplify, vehicle capacity is expressed as the number of people that can be served.

The aim is to identify a set of feasible routes, beginning and ending at the depot, such that each shelter is visited exactly once and the number of satisfied people is maximized. The following MILP model uses as a basis a CVRP model without vehicle index from Kulkarni and Bhave [112]. Binary variables x_{ij} are equal to 1 if arc (i, j) is used by a vehicle. The other variables are positive real numbers. The vehicle visiting shelter i arrives at time t_i , finds d_i persons to serve, and leaves with a cumulated served demand w_i .

$$\max \sum_{i \in V'} d_i \quad (3.1)$$

$$\sum_{j \in V'} x_{0j} = K \quad (3.2)$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \quad (3.3)$$

$$\sum_{i \in V} x_{ij} = \sum_{i \in V} x_{ji} \quad \forall j \in V' \quad (3.4)$$

$$t_j \geq t_i + c_{ij} - T_{max} \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V' \quad (3.5)$$

$$w_j \geq w_i + d_j - Q \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V' \quad (3.6)$$

$$w_i \leq Q \quad \forall i \in V' \quad (3.7)$$

$$t_i + c_{i0} \leq T_{max} \quad \forall i \in V' \quad (3.8)$$

$$d_i \leq a_i - b_i t_i \quad \forall i \in V' \quad (3.9)$$

$$t_0 = 0 \tag{3.10}$$

$$w_0 = 0 \tag{3.11}$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in E \tag{3.12}$$

$$t_i, d_i, w_i \geq 0 \quad \forall i \in V \tag{3.13}$$

The objective function (3.1) to be maximized is the total number of people attended at shelters. Constraint (3.2) limits the number of vehicles used. Each shelter is served once (3.3). A vehicle arriving at shelter j must leave it (3.4). Constraints (3.5) guarantee the consistency of arrival times and prevent subtours. Vehicle capacities are respected thanks to Equations (3.6) and (3.7). (3.8) are the tour length constraints. The demand satisfied at each shelter is computed in (3.9), it cannot be negative because variable d_i is non-negative. Constraints (3.10) and (3.11) set to zero departure time and cumulated served demand at the depot. Finally, decision variables are defined in Equations (3.12) and (3.13).

If constraints (3.3) become inequalities " \leq ", the total number of aided people can be increased but some shelters may be sacrificed for the benefits of other shelters. As equity and fairness are essential in humanitarian logistics, this cannot be accepted and all shelters must be visited.

3.1.2 Branch-and-price algorithm

The CVRP-TDD formulation can be separated into two problems using the Dantzig-Wolfe decomposition [57]. These two problems are the master and the pricing problem. They are iteratively solved with the column generation (CG) method.

If the CG does not find an integer solution, a branching rule is applied generating a search tree. The branching scheme is explained in subsection 3.1.2.2. At each node in the search tree, a CG is executed. This process is repeated until an optimal solution is found or a time limit is reached. The whole procedure is known as branch-and-price (B&P).

The B&P algorithm is formalized by Desrosiers et al. [62] for time-constrained routing and scheduling problems. Barnhart et al. [24] describe a more general framework to design such solution methods, taking the generalized assignment problem and crew scheduling problem as examples. The rich applications of CG and B&P to various optimization problems are studied by Desaulniers et al. [60]. A tutorial dedicated to CG and B&P for vehicle routing problems can be found in Feillet [73].

B&P applied to humanitarian problems can be seen in Sebbah et al. [182], Sung and Lee [195], Afsar et al. [9] and Jozefowicz [105]. The first reference extends the CVRP with time windows by allowing transshipment and change in transportation mode at intermediate nodes. The second addresses the allocation of emergency medical resources after a mass

casualty incident. While Afsar et al. [9] and Jozefowicz [105] show how the generalized VRP with flexible fleet size and the multivehicle covering tour problem occurs in a humanitarian context, respectively.

3.1.2.1 Column Generation method

The column generation considers a master and a pricing problem. The master problem (MP) is modeled as a set partitioning model which assumes a set P containing all feasible routes for the CVRP-TDD. Each feasible route is associated with a variable (column) in the model. The huge amount of variables in P makes the MP intractable in a reasonable time. For this reason, in the restricted master problem (RMP), P is replaced by a subset of routes P' ($P' \subset P$) where $|P'| \ll |P|$.

Following the Algorithm 3.1, the subset P' is initialized with a set of routes computed by a constructive heuristic (*line 2*). This heuristic is explained in Chapter 4. The CG solves iteratively the linear relaxation of the restricted master problem (LRMP) to get the dual values Π (*lines 4 and 5*). These values are used by the pricing problem to generate new “promising” columns Ω which are included in P' (*lines 6 and 7*). The “promising” columns are those with positive reduced cost for a maximization problem like CVRP-TDD. This is repeated until no more columns Ω are found or a time limit is reached. If the pricing problem finds no “promising” columns, the solution of the LRMP is optimal and provides a high quality upper bound for the complete problem. The MP and the pricing problem are explained in 3.1.2.1.1 and 3.1.2.1.2.

Algorithm 3.1 Column Generation for the CVRP-TDD

- 1: $P' \leftarrow \emptyset$
 - 2: $P' \leftarrow \text{initialSol}()$
 - 3: **repeat**
 - 4: $S \leftarrow \text{LRMP}(P')$
 - 5: $\Pi \leftarrow \text{getDuals}(S)$
 - 6: $\Omega \leftarrow \text{pricing}(\Pi)$
 - 7: $P' \leftarrow P' \cup \Omega$
 - 8: **until** ($\Omega = \emptyset$ or time limit is reached)
-

3.1.2.1.1 Master problem

Let P be the set of all feasible routes, d_p the demand (number of people) satisfied on route p and λ_p a binary variable equal to 1 if route p is selected. The binary parameter

γ_p^i indicates if route p visits shelter i . The master problem is the following set partitioning model:

$$\max \sum_{p \in P} d_p \lambda_p \quad (3.14)$$

$$\sum_{p \in P} \gamma_p^i \lambda_p = 1 \quad \forall i \in V' \quad (3.15)$$

$$\sum_{p \in P} \lambda_p \leq K \quad (3.16)$$

$$\lambda_p \in \{0, 1\} \quad \forall p \in P \quad (3.17)$$

Each shelter is visited once in constraints (3.15) while fleet size is respected in constraint (3.16).

As P is too big, the restricted master problem uses the subset P' . The LRMP is obtained by relaxing the integrity constraints (3.17). From this linear relaxation, the dual variables π_i for each i in V' and π_o associated with constraints (3.15) and (3.16) are extracted.

3.1.2.1.2 Pricing problem

Constraints (3.4)-(3.11) are encoded in the construction of new routes (columns) in the pricing problem. In most VRPs solved by CG, the pricing problem is often a shortest path problem with resources constraints or one of its variants (Irnich and Desaulniers [104]). For instance, Desrochers et al. [61] consider as pricing problem an elementary shortest path problem with resources constraints (ESPPRC) for the VRP with time windows. The ESPPRC is defined and solved by Beasley and Christofides [28] and proved NP-Hard by Dror [63].

The CVRP-TDD being a maximization problem, the pricing problem is an elementary *longest* path problem with resource constraints (ELPPRC) between two copies of the depot (0 and $n + 1$). Each route (column) has an associated reduced cost which is calculated using the dual values obtained from the solution of the LRMP. If the pricing problem finds columns with positive reduced costs, then the LRMP is not yet solved to optimality and the new columns are added to P' .

The reduced cost of a route $p \in P'$ is defined as:

$$\bar{C}_p = d_p - \sum_{i \in V'} \pi_i \gamma_p^i - \pi_o \quad (3.18)$$

As d_p depends on the arrival time t_i to each shelter i in the route, we have:

$$d_p = \sum_{i \in V'} (a_i - b_i t_i) \gamma_p^i \quad (3.19)$$

Substituting d_p by its expression in (3.18) gives Equation (3.20):

$$\bar{C}_p = \sum_{i \in V'} \gamma_p^i (a_i - b_i t_i - \pi_i) - \pi_o \quad (3.20)$$

The ELPPRC is solved using a dynamic programming algorithm. Feillet et al. [74] designed such an algorithm for the ESPPRC. We modify it to handle the maximization objective function and the specific resources of the CVRP-TDD.

Dynamic programming

Dynamic programming builds an optimal solution to a combinatorial optimization problem using the solutions of nested sub-problems. It can be applied to solve pricing problems in B&P algorithms. The principle is to compute for each partial path a label specifying the reduced cost of the path and its resource consumptions, and to eliminate labels which will not led to the optimal solution, with the help of dominance rules.

We define for partial path $p = (0, \dots, i)$ ending at shelter $i \in V'$, a label $l_p = (\bar{C}_p, d_p, t_p, \bar{V}_p^1, \dots, \bar{V}_p^n)$. \bar{C}_p is the reduced cost, d_p is the total served demand, t_p is the arrival time at shelter i , and $\bar{V}_p^i, \forall i \in V'$, a binary parameter equal to 1 if shelter i is visited by path p . To extend the partial path p by an unvisited shelter j , the following feasibility conditions have to be met:

- the demand of j at the time of the visit should be strictly positive ($a_j - b_j(t_p + c_{ij}) > 0$).
- the vehicle capacity should be respected ($d_p + a_j - b_j(t_p + c_{ij}) \leq Q$).
- the vehicle should be able to turn back to the depot before the time limit ($t_p + c_{ij} \leq T_{max} - c_{j(n+1)}$).

The new label $l_{p \oplus j}$ of the extended partial path $p \oplus j$ can be defined as follows:

$$\bar{C}_{p \oplus j} = \bar{C}_p + a_j - b_j(t_p + c_{ij}) - \pi_j \quad (3.21)$$

$$d_{p \oplus j} = d_p + a_j - b_j(t_p + c_{ij}) \quad (3.22)$$

$$t_{p \oplus j} = t_p + c_{ij} \quad (3.23)$$

$$\bar{V}_{p \oplus j}^j = 1, \text{ and } \bar{V}_{p \oplus j}^i = \bar{V}_p^i \quad \forall i \in V' \setminus \{j\} \quad (3.24)$$

The dynamic programming method stops when no more labels can be extended. If the final node $n + 1$ has no label with positive reduced cost, the solution of the last LRMP is optimal and the CG terminates. Otherwise, the routes found are added to P' and another iteration is executed.

Dominance rules

As the dynamic programming method can generate a large number of labels, dominance rules are essential to eliminate unnecessary labels and decrease running time and memory requirement. Two dominance rules are proposed.

Dominance rule 1. *If $p_1 = (0, \dots, i)$ and $p_2 = (0, \dots, i)$ are two distinct paths to shelter i with respective labels l_{p_1} and l_{p_2} , p_1 dominates p_2 if:*

$$t_{p_1} = t_{p_2} \quad (3.25)$$

$$Q - d_{p_1} \geq Q - d_{p_2} \quad (3.26)$$

$$\bar{V}_{p_1}^i \leq \bar{V}_{p_2}^i, \forall i \in V' \quad (3.27)$$

$$\bar{C}_{p_1} \geq \bar{C}_{p_2} \quad (3.28)$$

The dominance principle is based on the fact that p_1 can be extended by any complementary partial path that can extend p_2 and it will always have a greater reduced cost. That is so because p_1 has a greater vehicle capacity and the same duration. Moreover, any shelter visited by p_1 is also visited by p_2 .

It should be noted that condition (3.25) is an equality because if condition (3.29) is considered, the dominance rule is no longer valid.

$$t_{p_1} \leq t_{p_2} \quad (3.29)$$

When $t_{p_1} < t_{p_2}$ and conditions (3.26)-(3.28) are met, it is possible that a partial path p' that is extendible from p_2 , is not from p_1 . This occurs when the arrival times at shelters in p' are smaller satisfying a greater quantity of the demand and the capacity of the vehicle is exceeded. Hence, we propose a second dominance rule that guarantee that p_1 dominates p_2 with an additional condition in case where $t_{p_1} < t_{p_2}$.

Dominance rule 2. *Let $p_1 = (0, \dots, i)$ and $p_2 = (0, \dots, i)$ be two different paths with respective labels l_{p_1} and l_{p_2} , such that conditions (3.26)–(3.28) are satisfied and $t_{p_1} < t_{p_2}$. In that case p_1 dominates p_2 if:*

$$Q - d_{p_1} \geq \sum_{j \in V', j \notin p_2} a_j - b_j(t_{p_1} + c_{ij}) \quad (3.30)$$

The visiting time of any node j , added to p_2 , cannot be smaller than $t_{p_1} + c_{ij}$. Even if we consider the earliest arrival times and the remaining vehicle capacity is sufficient to satisfy all the demand, any partial path p' that can be extended from p_2 , can also be extended from p_1 and $\bar{C}_{p_1 \oplus p'} \geq \bar{C}_{p_2 \oplus p'}$. Therefore, p_1 dominates p_2 .

3.1.2.1.3 Acceleration technique

To speed up the algorithm, the pricing problem is solved heuristically. The conditions (3.26), (3.27) and (3.30) of the dominance rules are not checked and only conditions (3.28) and (3.29) are verified. In this way, dominance tests are faster and eliminate more labels. The CG starts by calling the pricing with acceleration technique (AT) at each iteration. When it is no longer capable of finding paths with positive reduced cost, the exact pricing is executed.

3.1.2.2 Branching scheme

Branching occurs when some variables are fractional. Like in Kallehauge et al. [106], we determine the arc (i, j) such that the flow $\phi_{ij} = \sum_{p \in P' | (i,j) \in p} \lambda_p$ is fractional but closest to 1. A left and a right branch are created following a branching rule. The left branch corresponds to $\phi_{ij} = 1$. So, arc (i, j) is mandatory and the columns in P' that contain arcs (i, k) where $k \neq j$ and (k, j) where $k \neq i$ are eliminated. While in the right branch $\phi_{ij} = 0$ and all the columns where shelter j is visited immediately after shelter i must be discarded from P' .

The CG should be executed again at each node of the B&P tree. The LRMP is solved considering the updated set P' and the pricing algorithm should also respect the branching rule. We implement the exploration of the B&P tree in a depth-first manner.

3.1.3 Spanning tree based relaxation

We present an upper bound for the CVRP-TDD, which helps to measure the solution quality when an upper bound cannot be obtained by the B&P. As our problem is represented by a complete graph where the triangle inequality for c_{ij} is verified, if we relax constraints (3.4) every node is either connected directly to the root (depot node) or to a node which is directly connected to the root. Therefore the height of the tree is 2.

The upper bound is based on the spanning tree with degree constraints which is a NP-hard problem [144], if the same maximum degree is imposed to each node. Here, the tree is directed (rooted) contrary to the classical (undirected) minimum spanning tree problem. The time and capacity constraints ((3.5)-(3.8)) of the original problem are relaxed.

The following MILP is solved to find an upper bound:

$$\max \sum_{i \in V'} d_i \tag{3.31}$$

$$\sum_{i \in V'} x_{0i} \leq K \tag{3.32}$$

$$\sum_{i \in V} x_{ij} = 1 \quad \forall j \in V' \tag{3.33}$$

$$x_{ij} \leq x_{0i} \quad \forall i \in V', \forall j \in V' \quad (3.34)$$

$$d_i \leq a_i - b_i \cdot c_{0i} \cdot x_{0i} \quad \forall i \in V' \quad (3.35)$$

$$d_j \leq a_j - b_j \cdot [(x_{0i} + x_{ij} - 1) \cdot (c_{0i} + c_{ij})] \quad \forall i \in V', \forall j \in V' \quad (3.36)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V, \forall j \in V \quad (3.37)$$

$$d_i \geq 0 \quad \forall i \in V \quad (3.38)$$

The objective function (3.31) maximizes the number of attended people at shelters. Constraint (3.32) is the degree constraint which ensures that at most K vehicles leave the depot. Each shelter must be visited once (3.33). Constraints (3.34) guarantee that any shelter is visited directly from the depot or after an intermediate shelter which is visited immediately after the depot. Note that these constraints do not allow disconnected edges from the tree (connected graph). Constraints (3.35) compute the demand of shelters visited immediately after the depot (height 1) while constraints (3.36) calculate the demand for each shelter not directly connected to the depot (height 2).

3.1.4 Computational experiments

The computational experiments are executed to evaluate and compare the performance of the mathematical model, the CG with/without the acceleration technique (AT), and the branch-and-price algorithm. The instances used are described in subsection 3.1.4.1. The results of the mathematical model are shown in subsection 3.1.4.2. In subsection 3.1.4.3, the dominance rules and the acceleration technique are evaluated executing the column generation method at the root node of the B&P tree. In subsection 3.1.4.4, the results of the B&P algorithm are presented and analyzed. Additionally, a stability analysis of the optimal first aid distribution plan with respect to possible changes in the number of people moving per time unit (displacement rate) from shelters is designed in subsection 3.1.4.5.

3.1.4.1 Implementation and Instances

The B&P algorithm is coded in Java while the MILP and the master problem are solved by IBM ILOG CPLEX Optimization Studio version 12.6.3. The tests are carried out on a 2.70 GHz PC with an Intel Core i7-4800MQ processor, 16 GB of RAM, and Windows 7 Professional.

To the best of our knowledge, the CVRP-TDD has not been addressed previously, therefore no instances are available in the literature. We generate our own instances from seven instances proposed by Christofides et al.[49]. In total, they propose 14 instances for

the CVRP which can be divided into two groups: those that do not include a maximum tour length and service times and those that include them. We use the latter because they are more similar to our problem. The given service time is added to the travel distance between shelter-shelter and shelter-depot pairs.

The seven instances are known in the CVRP literature as CMT6, CMT7, CMT8, CMT9, CMT10, CMT13 and CMT14. Their number of customers is 50, 75, 100, 150, 199, 120 and 100, respectively. The customers of the first five instances are randomly distributed, while customers in the other two are grouped in clusters.

From these instances, two sets I and C of CVRP-TDD instances are generated. Each set contains 42 instances which are created by selecting randomly 10, 15, 20, 25, 30, and 50 shelters from each of the seven instances. The instances are numbered from 1 to 42. Instances 1-6 are taken from CMT6, the next 6 (7-12) instances from CMT7, and so on.

The instances in set I consider that each shelter i has a displacement rate (b_i) randomly generated between $\frac{a_i}{3 \cdot T_{max}}$ and $\frac{a_i}{T_{max}}$. While in set C , the b_i depends on the position of shelter i . If this shelter is located in a region which is strongly struck by the disaster, people will tend to flee faster because they need help urgently. Then, $b_i = (\frac{2 \cdot a_i}{3 \cdot T_{max}}, \frac{a_i}{T_{max}}]$. Otherwise, $b_i = [\frac{a_i}{3 \cdot T_{max}}, \frac{2 \cdot a_i}{3 \cdot T_{max}}]$. A total of six regions are considered in each instance and three of them are strongly struck.

The maximum tour length T_{max} and the capacity Q of the vehicles are given by the seven instances. In these instances, the number of vehicles is not fixed but the minimum number of vehicles in the best-known solutions (BKS) for the CVRP reported in the Uchoa et al. [209] is used to set K .

3.1.4.2 Results of the mathematical model

The results of the MILP on small instances (up to 20 shelters) in I and C are displayed in Table 3.1. The first five columns are parameters of the instances. The lower and upper bounds reported by CPLEX in at most three hours are shown in columns Z_{LB} and Z_{UB} , respectively. The percentage gap ($Gap(\%)$) is calculated as $\frac{Z_{UB} - Z_{LB}}{Z_{LB}} \times 100$. The running time in seconds given by CPLEX is shown in column $Time (s)$ and its minimum, average, and maximum are calculated only by considering instances where an optimal solution is found (shown in bold) within the time limit.

For the sake of simplicity, the instances will be referenced as $\{I/C\}$ -number, e.g., $C-15$ denotes the instance 15 of set C .

The results show that, in both sets, instances up to 10 shelters can be solved to optimally in at most 260 seconds. Instances $C-14$ and $C-20$ are the only ones with 15 shelters for which an optimal solution is reported. Beyond this size, three hours are not enough to find

Table 3.1 – Results of the MILP on set I and C

| Instance | n | K | Q | T_{max} | MILP - set I | | | | MILP - set C | | | |
|----------|-----|-----|-----|-----------|----------------|----------|---------|----------|----------------|----------|---------|----------|
| | | | | | Z_{LB} | Z_{UB} | Gap (%) | Time (s) | Z_{LB} | Z_{UB} | Gap (%) | Time (s) |
| 1 | 10 | 2 | 160 | 200 | 152.922 | 152.922 | 0.00 | 10.07 | 144.808 | 144.808 | 0.00 | 57.17 |
| 2 | 15 | 3 | 160 | 200 | 197.325 | 200.174 | 1.44 | 10800.00 | 178.583 | 184.712 | 3.43 | 10800.00 |
| 3 | 20 | 4 | 160 | 200 | 277.574 | 297.194 | 7.07 | 10800.00 | 258.193 | 277.191 | 7.36 | 10800.00 |
| 7 | 10 | 3 | 140 | 160 | 140.646 | 140.646 | 0.00 | 1.84 | 156.963 | 156.963 | 0.00 | 1.36 |
| 8 | 15 | 3 | 140 | 160 | 241.761 | 252.704 | 4.53 | 10800.00 | 203.249 | 216.772 | 6.65 | 10800.00 |
| 9 | 20 | 4 | 140 | 160 | 305.849 | 329.955 | 7.88 | 10800.00 | 340.811 | 377.753 | 10.84 | 10800.00 |
| 13 | 10 | 2 | 200 | 230 | 101.420 | 101.420 | 0.00 | 64.56 | 140.961 | 140.961 | 0.00 | 11.95 |
| 14 | 15 | 3 | 200 | 230 | 194.186 | 205.713 | 5.94 | 10800.00 | 164.252 | 164.252 | 0.00 | 4449.19 |
| 15 | 20 | 4 | 200 | 230 | 266.095 | 277.737 | 4.38 | 10800.00 | 202.133 | 213.463 | 5.61 | 10800.00 |
| 19 | 10 | 2 | 200 | 200 | 106.228 | 106.228 | 0.00 | 14.83 | 114.446 | 114.446 | 0.00 | 14.05 |
| 20 | 15 | 3 | 200 | 200 | 163.191 | 169.703 | 3.99 | 10800.00 | 209.243 | 209.243 | 0.00 | 2142.13 |
| 21 | 20 | 4 | 200 | 200 | 220.890 | 235.202 | 6.48 | 10800.00 | 240.211 | 250.887 | 4.44 | 10800.00 |
| 25 | 10 | 2 | 200 | 200 | 117.343 | 117.343 | 0.00 | 23.22 | 110.103 | 110.103 | 0.00 | 8.24 |
| 26 | 15 | 3 | 200 | 200 | 193.841 | 199.670 | 3.01 | 10800.00 | 181.894 | 192.709 | 5.95 | 10800.00 |
| 27 | 20 | 4 | 200 | 200 | 294.997 | 313.625 | 6.31 | 10800.00 | 273.355 | 287.751 | 5.27 | 10800.00 |
| 31 | 10 | 2 | 200 | 720 | 98.686 | 98.686 | 0.00 | 89.57 | 74.592 | 74.592 | 0.00 | 50.77 |
| 32 | 15 | 3 | 200 | 720 | 125.279 | 131.411 | 4.89 | 10800.00 | 161.927 | 173.003 | 6.84 | 10800.00 |
| 33 | 20 | 4 | 200 | 720 | 204.776 | 217.479 | 6.20 | 10800.00 | 229.585 | 239.485 | 4.31 | 10800.00 |
| 37 | 10 | 2 | 200 | 1040 | 169.964 | 169.964 | 0.00 | 254.78 | 176.355 | 176.355 | 0.00 | 140.82 |
| 38 | 15 | 3 | 200 | 1040 | 208.147 | 220.227 | 5.80 | 10800.00 | 279.522 | 295.194 | 5.61 | 10800.00 |
| 39 | 20 | 4 | 200 | 1040 | 380.232 | 405.030 | 6.52 | 10800.00 | 335.834 | 356.114 | 6.04 | 10800.00 |
| | | | | | Min. | | 0.00 | 1.84 | Min. | | 0.00 | 1.36 |
| | | | | | Avg. | | 3.54 | 65.55 | Avg. | | 3.44 | 763.96 |
| | | | | | Max. | | 7.88 | 254.78 | Max. | | 10.84 | 4449.19 |

an optimal solution.

The average solution time of set C is greater than that of set I , due to the time spent by CPLEX to solve C -14 and C -20. The average gap (3.54% versus 3.44% for set C) shows that in instances with up to 20 shelters, there is no significant difference between the two sets.

Instances I -9 and C -9 are the hardest instances up to 20 shelters with gaps of 7.88% and 10.84%, respectively.

3.1.4.3 Evaluation of dominance rules and acceleration technique

In this section, we compare the performance of the dominance rules with and without the acceleration technique (AT). Two strategies that combine them are tested. Each strategy is evaluated executing the CG at the root node of the B&P tree.

Tables 3.2 and 3.3 present the results obtained for each strategy in set I and C respectively. Specifically, Tables 3.2a and 3.3a show the results by instance, while the average execution time (in seconds) and the average gap (%) for each group of instances with the same number of nodes are detailed in Tables 3.2b and 3.3b. It should be noted that the average time is calculated considering only the instances for which the CG stops before the

time limit.

If the constructive heuristic does not obtain an initial feasible solution, then dummy routes (columns) are added to the LRMP and penalized in the objective function. If the CG does not converge ($Z_{UB} = "-"$) in the time limit of three hours and dummy routes are not eliminated in the RMP solution, a valid lower bound cannot be shown ($Z_{LB} = "-"$).

The optimality gap is computed as in 3.1.4.2. If $Z_{UB} = "-"$, the upper bound (ST_{UB}) proposed in subsection 3.1.3 is used.

The number of times that the CG calls the pricing procedure and the number of columns generated by it during the corresponding execution time (*Time (s)*) are detailed in columns *Iter.* and *# of Cols.*, respectively.

Without AT

The CG converges and reports a lower bound different from the upper bound ($Z_{LB} \neq Z_{UB}$) in 9 and 7 instances in set *I* and *C* in less than three hours, respectively. An optimal solution is reported in 17 instances of each set. Therefore, a total of 26 and 24 out of 42 instances are solved in set *I* and *C* respectively.

The method is able to solve instances up to 30 nodes in less than one hour. However, only 1 out of 7 instances with this characteristic in set *I* is solved. Conversely, more instances are solved with 25 nodes (5 and 4 out of 7 instances in set *I* and *C*, respectively).

The average execution times in instances with 15, 25, and 30 nodes in set *C* are greater than in set *I*. In addition, the average gap in set *C* is better than in set *I* in all groups of instances, except in instances with 30 and 50 nodes.

Now, the CG using the dominance rules with the acceleration technique will be analyzed.

With AT

The CG solves 29 and 31 out of 42 instances, of which 18 and 20 are optimally solved in set *I* and *C*, respectively. The largest solved instances has 50 nodes (1 out of 7 instances in both sets), 30 nodes (2 and 4 out of 7 instances in set *I* and *C*, respectively) and 25 nodes (6 out of 7 instances in both sets).

The average execution time in all groups of instances in set *C* is greater than in set *I*. However, the average gap in all groups of instances in set *C* is less than in set *I*.

The performance of the CG using the dominance rules is improved with the acceleration technique. This technique allows to find the solution in more instances. For example, the CG without the AT solve 24 instances in set *C*, while with the AT, it solves 31.

When the AT is used, the execution time is reduced considerably. This reduction is approximately 63% and 45% in sets *I* and *C*, respectively. The average gap is also reduced

Table 3.2 – Results of the dominance rules with/without AT for instances in set I

(a) Results by instance

| Instance | n | K | ST_{UB} | Without AT | | | | | | With AT | | | | | |
|----------|-----|-----|-----------|------------|----------|-------|------------|----------|-------------|----------|----------|-------|------------|----------|-------------|
| | | | | Z_{LB} | Z_{UB} | Iter. | # of Cols. | Time (s) | Gap (%) | Z_{LB} | Z_{UB} | Iter. | # of Cols. | Time (s) | Gap (%) |
| I-1 | 10 | 2 | 162.016 | 152.922 | 152.922 | 9 | 533 | 0.96 | 0.00 | 152.922 | 152.922 | 8 | 450 | 0.76 | 0.00 |
| I-2 | 15 | 3 | 208.139 | 197.325 | 197.325 | 23 | 1463 | 3.49 | 0.00 | 197.325 | 197.325 | 23 | 1205 | 1.57 | 0.00 |
| I-3 | 20 | 4 | 295.143 | 277.574 | 277.574 | 40 | 2707 | 76.39 | 0.00 | 277.574 | 277.574 | 31 | 1979 | 6.10 | 0.00 |
| I-4 | 25 | 4 | 321.947 | 296.937 | 297.219 | 48 | 3348 | 2362.20 | 0.09 | 296.937 | 297.219 | 40 | 2449 | 321.33 | 0.09 |
| I-5 | 30 | 4 | 426.074 | 321.792 | - | 32 | 2330 | 10800.00 | 32.41 | 372.257 | - | 53 | 3218 | 10800.00 | 14.46 |
| I-6 | 50 | 6 | 692.314 | - | - | 59 | 4359 | 10800.00 | - | 609.789 | - | 146 | 10573 | 10800.00 | 13.53 |
| I-7 | 10 | 3 | 145.521 | 140.646 | 140.646 | 7 | 262 | 0.74 | 0.00 | 140.646 | 140.646 | 8 | 247 | 0.50 | 0.00 |
| I-8 | 15 | 3 | 260.170 | 241.761 | 242.034 | 11 | 750 | 4.90 | 0.11 | 241.761 | 242.034 | 16 | 671 | 4.05 | 0.11 |
| I-9 | 20 | 4 | 331.400 | 305.825 | 306.495 | 18 | 1159 | 3.90 | 0.22 | 305.757 | 306.495 | 18 | 1028 | 1.59 | 0.24 |
| I-10 | 25 | 5 | 402.554 | 369.996 | 371.121 | 31 | 2172 | 141.68 | 0.30 | 370.370 | 371.121 | 26 | 1715 | 14.35 | 0.20 |
| I-11 | 30 | 5 | 491.984 | 437.336 | 439.501 | 33 | 2195 | 3160.55 | 0.50 | 436.357 | 439.501 | 30 | 1970 | 137.71 | 0.84 |
| I-12 | 50 | 8 | 769.696 | 644.865 | - | 54 | 3975 | 10800.00 | 19.36 | 690.314 | 693.319 | 54 | 3606 | 5148.62 | 0.64 |
| I-13 | 10 | 2 | 110.979 | 101.420 | 101.420 | 10 | 604 | 1.54 | 0.00 | 101.420 | 101.420 | 10 | 515 | 0.84 | 0.00 |
| I-14 | 15 | 3 | 210.387 | 194.157 | 194.227 | 23 | 1504 | 11.51 | 0.04 | 194.157 | 194.227 | 18 | 1068 | 2.35 | 0.04 |
| I-15 | 20 | 4 | 279.672 | 266.656 | 266.682 | 37 | 2558 | 46.76 | 0.01 | 266.656 | 266.682 | 27 | 1864 | 1.81 | 0.01 |
| I-16 | 25 | 4 | 374.678 | 347.132 | 347.132 | 63 | 4521 | 4370.07 | 0.00 | 347.132 | 347.132 | 52 | 3562 | 465.49 | 0.00 |
| I-17 | 30 | 4 | 364.050 | 293.057 | - | 47 | 3451 | 10800.00 | 24.22 | 331.611 | 331.611 | 77 | 5447 | 4285.29 | 0.00 |
| I-18 | 50 | 6 | 565.777 | 446.004 | - | 31 | 2292 | 10800.00 | 26.85 | 510.936 | - | 204 | 14693 | 10800.00 | 10.73 |
| I-19 | 10 | 2 | 111.971 | 106.228 | 106.228 | 11 | 616 | 1.21 | 0.00 | 106.228 | 106.228 | 15 | 646 | 0.29 | 0.00 |
| I-20 | 15 | 3 | 172.258 | 163.191 | 163.191 | 25 | 1696 | 5.05 | 0.00 | 163.191 | 163.191 | 20 | 1235 | 0.52 | 0.00 |
| I-21 | 20 | 4 | 234.762 | 221.109 | 221.146 | 33 | 2327 | 26.54 | 0.02 | 221.109 | 221.146 | 25 | 1589 | 1.19 | 0.02 |
| I-22 | 25 | 4 | 344.449 | 317.436 | 317.436 | 60 | 4126 | 1773.52 | 0.00 | 317.436 | 317.436 | 46 | 2758 | 180.89 | 0.00 |
| I-23 | 30 | 4 | 398.755 | 331.192 | - | 37 | 2717 | 10800.00 | 20.40 | 358.867 | - | 58 | 3572 | 10800.00 | 11.11 |
| I-24 | 50 | 6 | 638.855 | 530.933 | - | 45 | 3303 | 10800.00 | 20.33 | 583.078 | - | 173 | 12517 | 10800.00 | 9.57 |
| I-25 | 10 | 2 | 126.346 | 117.343 | 117.343 | 10 | 535 | 1.04 | 0.00 | 117.343 | 117.343 | 13 | 452 | 0.94 | 0.00 |
| I-26 | 15 | 3 | 203.139 | 193.841 | 193.841 | 23 | 1567 | 3.80 | 0.00 | 193.841 | 193.841 | 22 | 1091 | 1.89 | 0.00 |
| I-27 | 20 | 4 | 313.825 | 294.997 | 294.997 | 24 | 1558 | 12.59 | 0.00 | 294.997 | 294.997 | 24 | 1350 | 2.13 | 0.00 |
| I-28 | 25 | 4 | 347.563 | 321.206 | 322.029 | 60 | 4340 | 641.15 | 0.26 | 321.337 | 322.029 | 55 | 3616 | 97.36 | 0.22 |
| I-29 | 30 | 4 | 421.037 | 348.350 | - | 21 | 1532 | 10800.00 | 20.87 | 370.831 | - | 48 | 2791 | 10800.00 | 13.54 |
| I-30 | 50 | 6 | 795.984 | 624.613 | - | 28 | 2025 | 10800.00 | 27.44 | 707.197 | - | 147 | 9954 | 10800.00 | 12.55 |
| I-31 | 10 | 2 | 106.076 | 98.686 | 98.686 | 14 | 867 | 1.67 | 0.00 | 98.686 | 98.686 | 15 | 695 | 0.79 | 0.00 |
| I-32 | 15 | 3 | 132.085 | 125.279 | 125.279 | 24 | 1518 | 18.82 | 0.00 | 125.279 | 125.279 | 25 | 1436 | 3.35 | 0.00 |
| I-33 | 20 | 4 | 214.765 | 204.776 | 204.776 | 60 | 4270 | 143.12 | 0.00 | 204.776 | 204.776 | 41 | 2676 | 18.10 | 0.00 |
| I-34 | 25 | 4 | 264.271 | 241.497 | - | 119 | 8853 | 10800.00 | 9.43 | 241.827 | 242.968 | 85 | 6091 | 2539.70 | 0.47 |
| I-35 | 30 | 4 | 341.280 | 291.480 | - | 55 | 4053 | 10800.00 | 17.09 | 304.849 | - | 161 | 10418 | 10800.00 | 11.95 |
| I-36 | 50 | 6 | 512.049 | 422.970 | - | 63 | 4691 | 10800.00 | 21.06 | 459.553 | - | 578 | 42926 | 10800.00 | 11.42 |
| I-37 | 10 | 2 | 179.884 | 169.964 | 169.964 | 14 | 921 | 2.97 | 0.00 | 169.964 | 169.964 | 13 | 780 | 1.22 | 0.00 |
| I-38 | 15 | 3 | 218.486 | 208.147 | 208.147 | 45 | 3160 | 82.93 | 0.00 | 208.147 | 208.147 | 32 | 2176 | 7.09 | 0.00 |
| I-39 | 20 | 4 | 397.269 | 378.663 | - | 38 | 2776 | 10800.00 | 4.91 | 380.332 | - | 35 | 2368 | 10800.00 | 4.45 |
| I-40 | 25 | 4 | 462.930 | 406.239 | - | 86 | 6436 | 10800.00 | 13.96 | 430.329 | - | 139 | 9496 | 10800.00 | 7.58 |
| I-41 | 30 | 4 | 478.649 | 412.814 | - | 56 | 4136 | 10800.00 | 15.95 | 433.588 | - | 200 | 14676 | 10800.00 | 10.39 |
| I-42 | 50 | 6 | 869.439 | 748.087 | - | 72 | 5325 | 10800.00 | 16.22 | 771.475 | - | 1318 | 98284 | 10800.00 | 12.70 |

(b) Average results by group of instances

| n | Without AT | | With AT | |
|-----|---------------|--------------|---------------|--------------|
| | Avg. Time (s) | Avg. Gap (%) | Avg. Time (s) | Avg. Gap (%) |
| 10 | 1.45 | 0.00 | 0.76 | 0.00 |
| 15 | 18.64 | 0.02 | 2.97 | 0.02 |
| 20 | 51.55 | 0.74 | 5.15 | 0.67 |
| 25 | 1857.72 | 3.43 | 603.19 | 1.22 |
| 30 | 3160.55 | 18.78 | 2211.50 | 8.90 |
| 50 | - | 21.88 | 5148.62 | 10.16 |

Table 3.3 – Results of the dominance rules with/without AT for instances in set C

(a) Results by instance

| Instance | n | K | ST_{UB} | Without AT | | | | | | With AT | | | | | |
|----------|-----|-----|-----------|------------|----------|-------|------------|----------|-------------|----------|----------|-------|------------|----------|-------------|
| | | | | Z_{LB} | Z_{UB} | Iter. | # of Cols. | Time (s) | Gap (%) | Z_{LB} | Z_{UB} | Iter. | # of Cols. | Time (s) | Gap (%) |
| C-1 | 10 | 2 | 159.315 | 144.808 | 144.808 | 8 | 478 | 0.81 | 0.00 | 144.808 | 144.808 | 11 | 413 | 0.89 | 0.00 |
| C-2 | 15 | 3 | 191.642 | 178.583 | 178.583 | 24 | 1655 | 5.76 | 0.00 | 178.583 | 178.583 | 23 | 1300 | 1.97 | 0.00 |
| C-3 | 20 | 4 | 278.666 | 258.193 | 258.193 | 24 | 1598 | 20.18 | 0.00 | 258.193 | 258.193 | 23 | 1354 | 2.62 | 0.00 |
| C-4 | 25 | 4 | 296.831 | 274.005 | 274.157 | 63 | 4452 | 6314.00 | 0.06 | 273.825 | 274.157 | 48 | 3138 | 502.51 | 0.12 |
| C-5 | 30 | 4 | 418.967 | 330.974 | - | 41 | 3002 | 10800.00 | 26.59 | 375.074 | 375.227 | 57 | 3969 | 8256.06 | 0.04 |
| C-6 | 50 | 6 | 700.770 | - | - | 76 | 5626 | 10800.00 | - | 627.053 | - | 127 | 9192 | 10800.00 | 11.76 |
| C-7 | 10 | 3 | 160.444 | 156.963 | 156.963 | 7 | 307 | 0.54 | 0.00 | 156.963 | 156.963 | 7 | 249 | 0.45 | 0.00 |
| C-8 | 15 | 3 | 222.871 | 203.249 | 203.249 | 12 | 751 | 7.92 | 0.00 | 203.249 | 203.249 | 14 | 552 | 5.08 | 0.00 |
| C-9 | 20 | 4 | 369.326 | 340.869 | 340.995 | 18 | 1059 | 28.65 | 0.04 | 340.869 | 340.995 | 17 | 907 | 3.38 | 0.04 |
| C-10 | 25 | 5 | 369.611 | 344.320 | 344.495 | 24 | 1499 | 46.58 | 0.05 | 344.320 | 344.495 | 20 | 1203 | 5.57 | 0.05 |
| C-11 | 30 | 5 | 463.845 | 405.169 | - | 42 | 3062 | 10800.00 | 14.48 | 398.970 | 408.391 | 38 | 2336 | 5999.11 | 2.36 |
| C-12 | 50 | 8 | 788.719 | 655.963 | - | 35 | 2560 | 10800.00 | 20.24 | 718.280 | 718.774 | 86 | 6035 | 7357.84 | 0.07 |
| C-13 | 10 | 2 | 149.674 | 140.961 | 140.961 | 10 | 561 | 0.90 | 0.00 | 140.961 | 140.961 | 10 | 376 | 1.55 | 0.00 |
| C-14 | 15 | 3 | 172.452 | 164.189 | 164.316 | 22 | 1521 | 6.38 | 0.08 | 164.252 | 164.316 | 22 | 1234 | 1.95 | 0.04 |
| C-15 | 20 | 4 | 213.350 | 202.133 | 202.133 | 28 | 1892 | 39.31 | 0.00 | 202.133 | 202.133 | 27 | 1601 | 7.32 | 0.00 |
| C-16 | 25 | 4 | 335.110 | 309.957 | - | 68 | 5041 | 10800.00 | 8.11 | 310.591 | 310.591 | 57 | 3852 | 456.22 | 0.00 |
| C-17 | 30 | 4 | 394.531 | 320.259 | - | 30 | 2180 | 10800.00 | 23.19 | 348.913 | - | 76 | 4973 | 10800.00 | 13.07 |
| C-18 | 50 | 6 | 671.590 | 544.319 | - | 42 | 3105 | 10800.00 | 23.38 | 616.818 | - | 258 | 18725 | 10800.00 | 8.88 |
| C-19 | 10 | 2 | 123.059 | 114.446 | 114.446 | 9 | 532 | 1.28 | 0.00 | 114.446 | 114.446 | 10 | 384 | 1.05 | 0.00 |
| C-20 | 15 | 3 | 220.500 | 209.243 | 209.243 | 19 | 1284 | 2.32 | 0.00 | 209.243 | 209.243 | 17 | 998 | 1.41 | 0.00 |
| C-21 | 20 | 4 | 252.579 | 240.211 | 240.302 | 32 | 2120 | 18.11 | 0.04 | 240.211 | 240.302 | 26 | 1618 | 1.74 | 0.04 |
| C-22 | 25 | 4 | 399.122 | 360.696 | 361.070 | 33 | 2271 | 959.96 | 0.10 | 360.696 | 361.070 | 36 | 2252 | 122.78 | 0.10 |
| C-23 | 30 | 4 | 411.263 | 360.388 | - | 33 | 2405 | 10800.00 | 14.12 | 372.756 | 372.811 | 58 | 3807 | 5676.07 | 0.01 |
| C-24 | 50 | 6 | 652.330 | - | - | 67 | 4957 | 10800.00 | - | 580.337 | - | 126 | 8340 | 10800.00 | 12.41 |
| C-25 | 10 | 2 | 117.924 | 110.103 | 110.103 | 9 | 393 | 0.65 | 0.00 | 110.103 | 110.103 | 12 | 341 | 1.31 | 0.00 |
| C-26 | 15 | 3 | 197.848 | 181.894 | 181.894 | 19 | 1189 | 8.62 | 0.00 | 181.894 | 181.894 | 17 | 1014 | 2.37 | 0.00 |
| C-27 | 20 | 4 | 289.027 | 273.355 | 273.355 | 24 | 1597 | 11.64 | 0.00 | 273.355 | 273.355 | 23 | 1247 | 2.63 | 0.00 |
| C-28 | 25 | 4 | 322.605 | 296.738 | 296.738 | 55 | 3944 | 2768.08 | 0.00 | 296.738 | 296.738 | 49 | 3098 | 469.03 | 0.00 |
| C-29 | 30 | 4 | 499.570 | 402.112 | - | 25 | 1801 | 10800.00 | 24.24 | 440.906 | 440.906 | 68 | 4504 | 10664.72 | 0.00 |
| C-30 | 50 | 6 | 678.434 | - | - | 77 | 5703 | 10800.00 | - | 603.768 | - | 104 | 7197 | 10800.00 | 12.37 |
| C-31 | 10 | 2 | 79.568 | 74.592 | 74.592 | 14 | 876 | 1.71 | 0.00 | 74.592 | 74.592 | 11 | 606 | 0.96 | 0.00 |
| C-32 | 15 | 3 | 171.905 | 161.927 | 161.927 | 23 | 1765 | 7.25 | 0.00 | 161.927 | 161.927 | 47 | 1907 | 5.14 | 0.00 |
| C-33 | 20 | 4 | 238.773 | 229.489 | 229.745 | 47 | 3387 | 83.10 | 0.11 | 229.621 | 229.745 | 36 | 2330 | 19.44 | 0.05 |
| C-34 | 25 | 4 | 251.999 | 232.622 | - | 85 | 6301 | 10800.00 | 8.33 | 234.725 | 234.725 | 67 | 4612 | 5218.73 | 0.00 |
| C-35 | 30 | 4 | 316.247 | 262.891 | - | 40 | 2925 | 10800.00 | 20.30 | 285.690 | - | 133 | 9308 | 10800.00 | 10.70 |
| C-36 | 50 | 6 | 545.691 | 450.295 | - | 67 | 4978 | 10800.00 | 21.19 | 490.696 | - | 482 | 34969 | 10800.00 | 11.21 |
| C-37 | 10 | 2 | 186.776 | 176.355 | 176.355 | 18 | 1173 | 2.86 | 0.00 | 176.355 | 176.355 | 18 | 961 | 1.88 | 0.00 |
| C-38 | 15 | 3 | 293.448 | 279.522 | 279.522 | 37 | 2587 | 2475.48 | 0.00 | 279.522 | 279.522 | 26 | 1583 | 1562.14 | 0.00 |
| C-39 | 20 | 4 | 349.476 | 335.327 | - | 53 | 3934 | 10800.00 | 4.22 | 336.270 | - | 42 | 2949 | 10800.00 | 3.93 |
| C-40 | 25 | 4 | 499.923 | 440.616 | - | 98 | 7285 | 10800.00 | 13.46 | 467.655 | - | 118 | 8319 | 10800.00 | 6.90 |
| C-41 | 30 | 4 | 490.820 | 408.388 | - | 52 | 3828 | 10800.00 | 20.18 | 432.274 | - | 205 | 14862 | 10800.00 | 13.54 |
| C-42 | 50 | 6 | 871.189 | 738.997 | - | 61 | 4500 | 10800.00 | 17.89 | 778.677 | - | 2136 | 159194 | 10800.00 | 11.88 |

(b) Average results by group of instances

| n | Without AT | | With AT | |
|-----|---------------|--------------|---------------|--------------|
| | Avg. Time (s) | Avg. Gap (%) | Avg. Time (s) | Avg. Gap (%) |
| 10 | 1.25 | 0.00 | 1.16 | 0.00 |
| 15 | 359.10 | 0.01 | 225.72 | 0.01 |
| 20 | 33.50 | 0.63 | 6.19 | 0.58 |
| 25 | 2522.16 | 4.30 | 1123.14 | 1.02 |
| 30 | - | 20.44 | 7648.99 | 5.67 |
| 50 | - | 20.68 | 7357.84 | 9.80 |

Table 3.4 – Results of the B&P on subset I'

| Instance | n | K | Z_{LB} | Z_{UB} | Nodes | Gap (%) | Time (s) |
|----------|-----|-----|----------|----------|-------|---------|----------|
| I-4 | 25 | 4 | 296.937 | 296.937 | 17 | 0.00 | 1841.66 |
| I-8 | 15 | 3 | 241.761 | 241.761 | 3 | 0.00 | 6.07 |
| I-9 | 20 | 4 | 305.849 | 305.849 | 7 | 0.00 | 4.53 |
| I-10 | 25 | 5 | 370.427 | 370.427 | 13 | 0.00 | 74.65 |
| I-11 | 30 | 5 | 437.614 | 437.614 | 67 | 0.00 | 6393.36 |
| I-12 | 50 | 8 | 690.314 | 693.319 | 4 | 0.44 | 10800.00 |
| I-14 | 15 | 3 | 194.186 | 194.186 | 3 | 0.00 | 4.14 |
| I-15 | 20 | 4 | 266.656 | 266.656 | 3 | 0.00 | 3.50 |
| I-21 | 20 | 4 | 221.109 | 221.109 | 3 | 0.00 | 3.79 |
| I-28 | 25 | 4 | 321.517 | 321.517 | 27 | 0.00 | 723.60 |
| I-34 | 25 | 4 | 242.329 | 242.968 | 50 | 0.26 | 10800.00 |
| | | | | | | Min. | 3.50 |
| | | | | | | Avg. | 1006.14 |
| | | | | | | Max. | 6393.36 |

by approximately 35% and 50% in I and C , respectively.

Additionally, the CG with the AT finds a feasible solution on the instances for which the CG without acceleration technique provides no feasible solution.

The previous results demonstrate that the best strategy is using the dominance rules with the AT. In set I , this strategy finds the optimal solution in 18 instances and high quality upper bounds in 11 others. In set C , it obtains 20 optimal solutions and 11 high quality upper bounds.

In the sequel, the B&P algorithm is applied to the instances for which branching is necessary to find the optimal solution.

3.1.4.4 Branch-and-price

The B&P algorithm is run on the instances not solved to optimality but having an upper bound in the previous subsection. We call I' and C' these subsets of I and C . At each node of the B&P tree, the CG using the acceleration technique is executed. The algorithm stops when the optimal solution is found or the time limit of three hours is achieved.

The results of the B&P algorithm are presented in Table 3.4 for subset I' and in Table 3.5 for subset C' . The columns Z_{LB} and Z_{UB} represent the lower and upper bound in the B&P tree, respectively. The $Z_{LB} = Z_{UB}$ if the B&P algorithm gets the optimal solution. The gap ($Gap(\%)$) is calculated as in 3.1.4.2. The number of nodes explored in the B&P tree is provided in column $Nodes$. The running times in seconds are listed in column $Time (s)$.

The minimum, maximum, and average execution times are calculated considering only the instances in which the B&P finds the optimal solution before the time limit.

All instances are solved to optimality in both subsets, except instances $I-12$, $I-34$, $C-5$, $C-11$, $C-12$, and $C-23$ within time limit of three hours. The B&P algorithm is able to solve

Table 3.5 – Results of the B&P on subset C'

| Instance | n | K | Z_{LB} | Z_{UB} | Nodes | Gap (%) | Time (s) |
|----------|-----|-----|----------|----------|-------|-------------|----------|
| C-4 | 25 | 4 | 274.005 | 274.005 | 11 | 0.00 | 2151.01 |
| C-5 | 30 | 4 | 375.074 | 375.227 | 2 | 0.04 | 10800.00 |
| C-9 | 20 | 4 | 340.869 | 340.869 | 3 | 0.00 | 9.62 |
| C-10 | 25 | 5 | 344.320 | 344.320 | 11 | 0.00 | 26.66 |
| C-11 | 30 | 5 | 398.970 | 408.391 | 4 | 2.36 | 10800.00 |
| C-12 | 50 | 8 | 718.280 | 718.774 | 2 | 0.07 | 10800.00 |
| C-14 | 15 | 3 | 164.252 | 164.252 | 5 | 0.00 | 3.12 |
| C-21 | 20 | 4 | 240.211 | 240.211 | 11 | 0.00 | 6.15 |
| C-22 | 25 | 4 | 360.696 | 360.696 | 25 | 0.00 | 773.99 |
| C-23 | 30 | 4 | 372.756 | 372.811 | 2 | 0.01 | 10800.00 |
| C-33 | 20 | 4 | 229.621 | 229.621 | 13 | 0.00 | 143.34 |
| | | | | | | Min. | 3.12 |
| | | | | | | Avg. | 444.84 |
| | | | | | | Max. | 2151.01 |

optimally instances up to 30 shelters in subset I' and up to 25 shelters in subset C' .

Figures 3.1 and 3.2 show the initial population ($t = 0$) and the number of people attended at shelters in the instances where the optimal solution is reported by the CG or the B&P in set I and C , respectively. Moreover, the percentage of people that leave the shelters before the arrival of the relief goods is also presented.

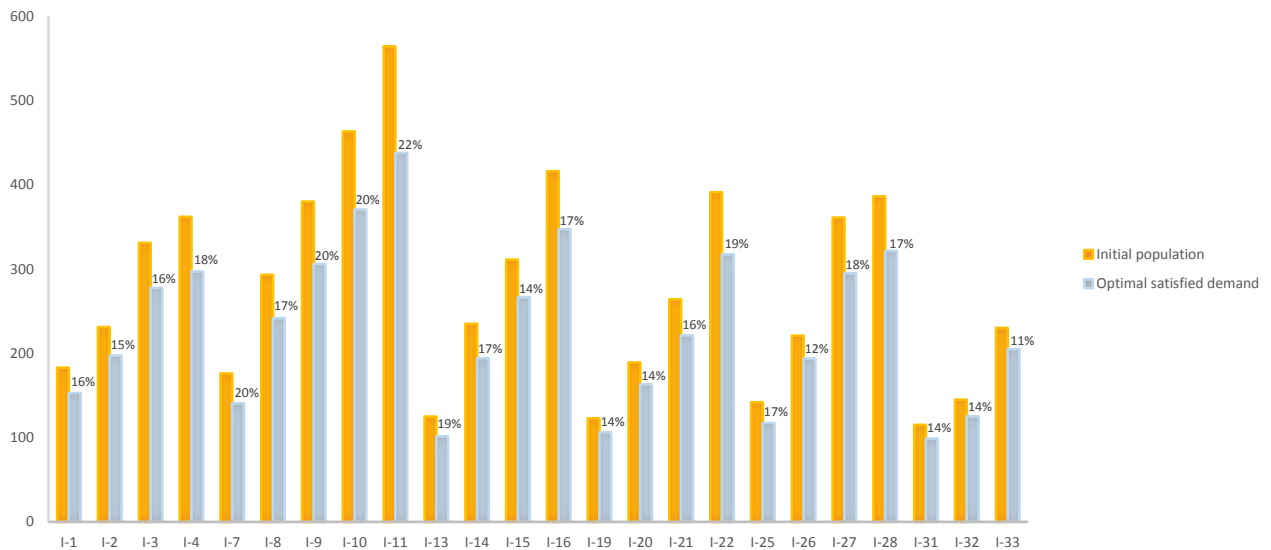
The initial population and the people attended at shelters are 564 and 437 in instance I -11. In this instance, the percentage of people that cannot be served at shelters (22%) is greater than in the other instances of set I . While in instance I -33, only 11% of the initial population does not receive the relief products. In average, the 16.50% of the initial population cannot be satisfied at shelters in set I .

In set C , the maximum and minimum percentage of people that cannot be attended at shelters is 22% and 10% in instances C -19 and C -33 respectively. In average, the 17.10% of the initial population leaves the shelters before the arrival of relief products.

3.1.4.5 Stability analysis

The branch-and-price algorithm can be run using preliminary data in the preparedness phase to provide an initial logistic plan. The optimal routes obtained can be applied on the field at once. However, the situation often evolves rapidly during the chaotic period which follows a disaster. In our case, the number of people leaving a shelter per unit time can increase or decrease compared to the initial estimates. It is always possible to re-run the algorithm with updated data but there can be a problem to re-route the vehicles which have left the depot already. Therefore, it is interesting to evaluate the impact of varying displacement rates on the initial logistic plan.

Three scenarios are considered here: the displacement rate decreases up to 30% at each

Figure 3.1 – Population attended at shelters in the optimal solution in set I

shelter (Scenario 1), increases up to 30% (Scenario 2), or decreases up to 30% at half of the shelters and increases up to 30% at the others (Scenario 3). To appraise the impact of these changes, only the instances where the optimal solution is obtained in previous subsections are modified for each scenario. Like in previous tests, the time limit is set to three hours. The results are presented in Tables 3.6 and 3.7 for set I and C , respectively.

All scenarios of all instances are solved optimally except scenarios 2 and 3 of I -17, scenario 1 and 2 of C -29, and scenario 3 of C -34. In instance I -17, the B&P is not able to find the optimal solution within three hours, but a lower bound (feasible solution) and an upper bound are achieved. In the other two instances, CG does not even converge at the root node. At the end of the time limit with combining already generated columns, a feasible solution for each one is obtained.

In the following, we analyze what happens if the routes of the initial logistic plan are maintained in despite the new displacement rates. In other words, we reevaluate the number of people attended in each route of the initial plan, using the modified displacement rates defined in each scenario.

For each instance of subset I and each scenario, Figure 3.3 displays the percentage gap of the number of people attended in the optimal solution of this scenario (from Table 3.6) to the number of people attended if the initial routes are maintained. The corresponding gaps for subset C are illustrated in Figure 3.4.

In most instances of both subsets, the percentage gap is not null in any scenario. This can be interpreted as the sequence of nodes visited by each route in the optimal solution in the scenario is different to the one found in the preparedness phase (Scenario 0). This

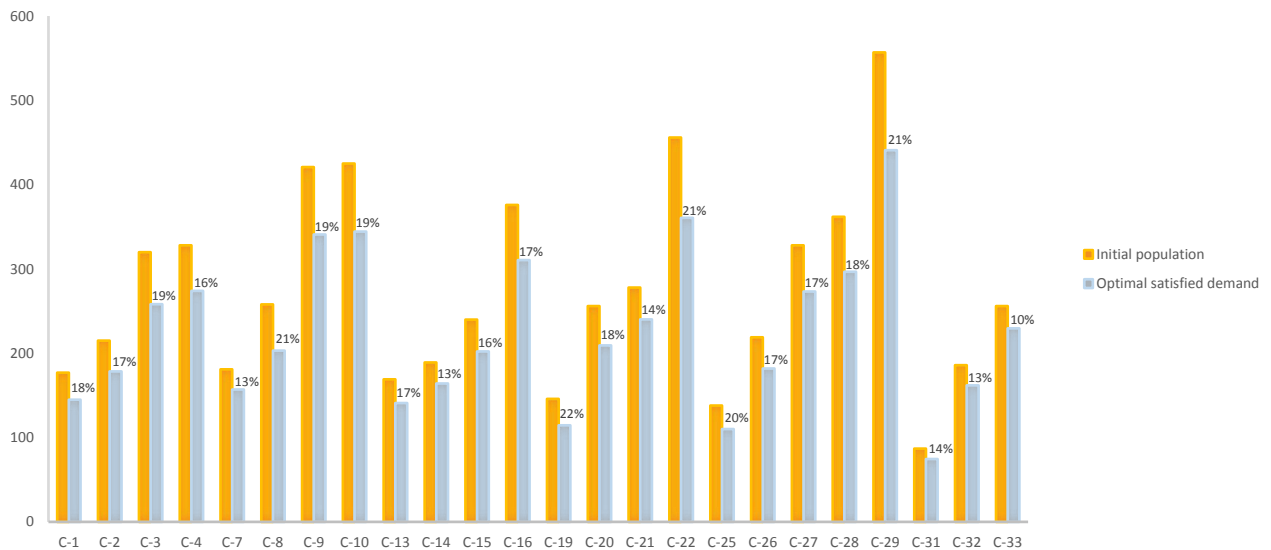


Figure 3.2 – Population attended at shelters in the optimal solution in set C

means that if the displacement rates change, the routes have to change. More precisely, for 22 instances out of 26 in set I and for 23 out of 27 in set C , initial routes should be modified.

The gap in scenario 2 of instances $I-14$ and $C-8$ does not appear because it is negative. It means that the initial logistic plan with the displacement rates of this scenario satisfies more demand than the optimal solution. This happens because most of the visited shelters in the initial plan have a higher demand to satisfy. However, this solution is not allowed in the CVRP-TDD because there is a shelter i where all people flee before the arrival of the vehicle ($d_i = 0$). The equity and fairness must be guaranteed in humanitarian logistics.

In Figure 3.3, the majority of gaps which can be calculated do not exceed 0.5% for all scenarios. The gap is greater in only 3 and 4 out of 26 instances in scenario 2 and 3, respectively. A positive gap means that if we keep the initial logistic plan, less people can be attended compared to the optimal distribution plan of the corresponding scenario. The maximum impact on the logistic plan of an increase (and/or decrease) up to 30% in the fleeing rate is less than 2.5%. In addition, the standard deviations are 0.07%, 0.41%, and 0.48% in the scenario 1, 2, and 3 respectively. The percentage gaps in scenario 1 tend to be more close to its mean (0.03%), while the ones in scenario 3 are spread over a wider range around its mean (0.33%).

In subset C , the maximum gap is smaller, below 1.40% (Figure 3.4). Like in subset I , most gaps are between 0% and 0.5%. Unlike subset I , scenario 2 has a higher standard deviation (0.91%) compared to the other two scenarios (0.12% and 0.26% for scenario 1 and 3 respectively).

Table 3.6 – Solution values for each scenario on the subset of I

| Instance | Scenario 1 | Scenario 2 | Scenario 3 |
|----------|------------|---------------------|---------------------|
| I-1 | 159.319 | 146.022 | 150.903 |
| I-2 | 202.163 | 191.721 | 198.764 |
| I-3 | 283.285 | 268.255 | 275.789 |
| I-4 | 308.653 | 285.546 | 297.947 |
| I-7 | 146.181 | 121.888 | 139.852 |
| I-8 | 251.126 | 218.850 | 235.669 |
| I-9 | 317.960 | 279.620 | 306.895 |
| I-10 | 385.072 | 354.122 | 367.960 |
| I-13 | 104.611 | 91.010 | 102.465 |
| I-14 | 200.655 | 168.105 | 193.306 |
| I-15 | 276.245 | 248.289 | 266.364 |
| I-16 | 358.087 | 338.199 | 351.434 |
| I-17 | 341.840 | (319.905 - 319.988) | (333.874 - 334.664) |
| I-19 | 108.941 | 95.303 | 106.568 |
| I-20 | 168.671 | 148.433 | 162.538 |
| I-21 | 226.197 | 199.718 | 223.796 |
| I-22 | 329.104 | 308.162 | 315.662 |
| I-25 | 121.917 | 109.077 | 119.008 |
| I-26 | 198.946 | 176.783 | 194.427 |
| I-27 | 302.235 | 263.312 | 292.652 |
| I-28 | 330.984 | 310.685 | 327.635 |
| I-31 | 102.266 | 88.045 | 100.278 |
| I-32 | 130.034 | 112.479 | 125.100 |
| I-33 | 209.648 | 190.985 | 202.321 |
| I-37 | 174.335 | 160.058 | 171.069 |
| I-38 | 213.465 | 196.856 | 209.853 |

3.1.5 Concluding remarks

This section introduces a novel problem in humanitarian logistics, the capacitated vehicle routing problem with time-dependent demands (CVRP-TDD). The solution of this problem allows to plan the distribution of first aid considering that people tend to leave the shelters, with a known and fixed displacement rate, if they do not receive the aid as quickly as they need. The objective is to maximize the number of people attended in the shelters.

Two exact methods are developed to solve the CVRP-TDD. A direct resolution of a mathematical model, using a MILP solver, and a branch-and-price (B&P) algorithm.

The CVRP-TDD is modeled as a set partitioning problem for the B&P algorithm. The pricing problem in the column generation is an elementary *longest* path problem with resource constraints adapted to our problem and solved by dynamic programming. Two dominance rules and an acceleration technique are proposed to improve the performance of the algorithm. Small and medium size instances are used to evaluate the B&P algorithm.

A stability analysis is presented to determine the reliability of the optimal routes against changes up to 30% in displacement rates. This analysis shows that if there are variations on

Table 3.7 – Solution values for each scenario on the subset of C

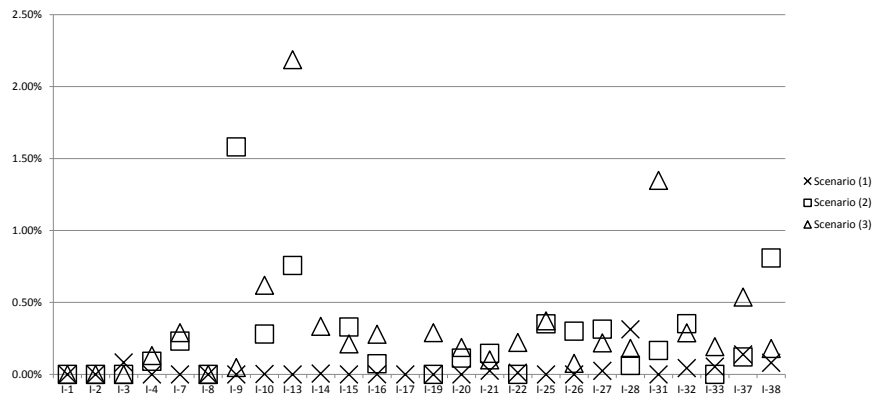
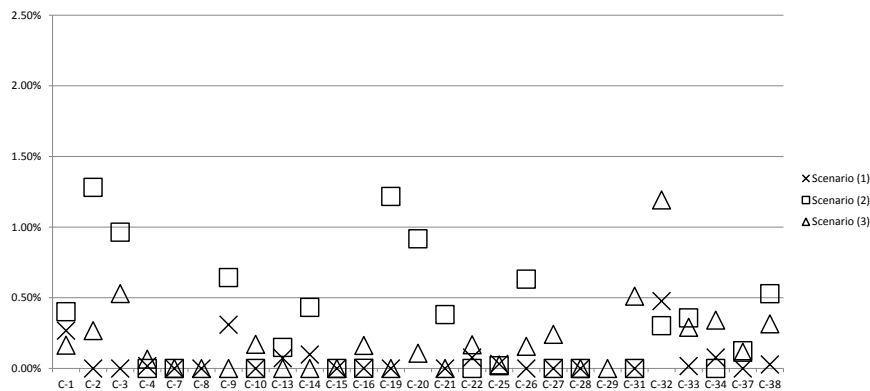
| Instance | Scenario 1 | Scenario 2 | Scenario 3 |
|-----------------|-------------------|-------------------|-------------------|
| C-1 | 150.242 | 129.744 | 148.579 |
| C-2 | 186.448 | 156.473 | 177.912 |
| C-3 | 268.971 | 225.600 | 257.049 |
| C-4 | 282.748 | 264.421 | 277.083 |
| C-7 | 160.515 | 144.372 | 156.057 |
| C-8 | 211.068 | 165.412 | 199.947 |
| C-9 | 355.964 | 299.157 | 343.839 |
| C-10 | 354.893 | 333.133 | 340.309 |
| C-13 | 145.052 | 128.862 | 142.560 |
| C-14 | 169.118 | 151.555 | 162.969 |
| C-15 | 208.356 | 183.793 | 202.795 |
| C-16 | 325.572 | 298.290 | 308.761 |
| C-19 | 120.217 | 97.514 | 114.230 |
| C-20 | 218.162 | 181.882 | 211.625 |
| C-21 | 247.523 | 220.710 | 240.572 |
| C-22 | 374.220 | 348.856 | 361.634 |
| C-25 | 113.895 | 93.093 | 113.387 |
| C-26 | 187.877 | 166.118 | 180.954 |
| C-27 | 283.850 | 241.936 | 276.786 |
| C-28 | 308.967 | 286.344 | 293.119 |
| C-29 | (461.504, -) | (425.152, -) | 442.809 |
| C-31 | 78.050 | 68.009 | 72.385 |
| C-32 | 167.194 | 148.916 | 161.190 |
| C-33 | 235.430 | 216.050 | 229.334 |
| C-34 | 243.412 | 226.925 | (240.156, -) |
| C-37 | 180.858 | 167.248 | 177.744 |
| C-38 | 286.029 | 260.691 | 280.750 |

the displacement rate, the routes may need modifications.

The importance of having a reliable distribution plan in humanitarian logistics leads us to use a solution technique that takes into account the uncertainty and variability of the information. The next section deals with this technique applied to the CVRP-TDD.

3.2 Robust capacitated vehicle routing problem with time-dependent demands

Section 3.1 shows that the distribution plan reliability can be improved when the displacement rates are not deterministic. This section looks for robust routes assuming that

Figure 3.3 – Stability gaps for subset I Figure 3.4 – Stability gaps for subset C

the displacement rate is uncertain. How to handle the uncertainties is the main challenge of disaster management.

This section tackles the robust counterpart of the CVRP-TDD with uncertainty in the displacement rate. The exact methods proposed in Section 3.1 are adapted using robust optimization

3.2.1 Robust Optimization

Robust optimization (RO) is a technique to handle the uncertainty in the problems. It seeks to protect against the worst scenarios that might arise in the future and obtain a solution that is feasible for any realization in a given scenario set. This solution is known as a robust solution.

The uncertain parameters in optimization problems can be expressed by three types of sets: bounded, budget or discrete. In the following we present recent relevant literature on the robust VRP.

3.2.1.1 Bounded set

The uncertain parameters are defined as deviations around nominal values. This provides maximum conservatism against uncertainty for a decision maker. Soyster [194], Ben-Tal and Nemirovski [31], and El Ghaoui et al. [82] introduce it for convex optimization problems where some data are unknown, but bounded by a convex set. Convex hulls, boxes, and ellipsoids are the convex sets most used in the literature. Sungur et al. [196] solve the robust VRP with uncertain demands. They propose a formulation to minimize the transportation costs while satisfying all demands in a given bounded set. They demonstrate that solving it with specific uncertainty set is no more difficult than solving a single deterministic VRP. Cao et al. [40] and Gounaris et al. [85] also consider the uncertainty in demands. The former propose different robust strategies and an improved differential evolutionary algorithm to solve the robust open VRP. They minimize the transportation costs and the unmet demand over all realizations. The latter only consider the transportation costs.

3.2.1.2 Budget set

The budget set approach is more flexible than the previous method. The conservatism of the solution can be controlled with a given parameter Γ which restricts the number of uncertain parameters that can change their value. Eshtehadi et al. [72] study the pollution-routing problem in that way and minimize the fuel consumption (CO₂ emissions) assuming that demand and travel times are uncertain. The travel time is a discrete function defined by non-reducing speed levels. The model decides the speed to traverse each arc in order to reduce CO₂ emissions. Braaten et al. [36] propose a heuristic based on adaptive large neighborhood search to solve the robust VRP with time windows for the maritime shipping industry. The objective is to minimize the travel cost. They assume the sailing legs to be uncertain. Lee et al. [119] propose a column generation (CG) method to solve the robust capacitated VRP with deadlines. The travel times and demands are uncertain.

3.2.1.3 Discrete set

The discrete scenario approach define the uncertain parameters by a finite set of discrete values. Each of them represents a possible scenario. Solano et al. [190] study the robust capacitated VRP with travel times characterized by a set of discrete scenarios. They present a MILP, greedy heuristics, a local search and metaheuristics to solve the problem. Later, Solano et al. [191] not only consider discrete travel times but also discrete demands. They present a bi-objective VRP and they solve it using a multi-objective evolutionary algorithm (MOEA) and the non-dominated sorting genetic algorithm (NSGA-II). Wu et al. [221] also

consider a set of discrete scenarios to define the travel times for the robust VRP and solve it implementing a large neighborhood search.

3.2.1.4 Robust VRP in humanitarian logistics

The humanitarian logistics operations are also affected by uncertainties in disasters. For this reason, robust optimization is a very useful technique in problems raised by the humanitarian context.

Liu et al. [126] propose a mathematical model to solve an allocation and a multi-trip RVRP. They assign personnel and population to affected areas and temporary facilities, both transported by helicopters. The personnel, the population and the travel times are uncertain and described by a budget set. The objective is to minimize the maximum total cost. Zokaei et al. [225] optimize a humanitarian relief chain with three levels (suppliers, relief distribution centers and affected areas). The costs, the demands, and the capacity of relief distribution centers are uncertain and belong to a budget set. These authors minimize the overall cost, while the satisfaction of affected people is maximized by the minimization of the relief commodities shortages. Najafi et al. [143] propose a multi-objective, multi-mode, multi-commodity and multi-period robust model to manage the logistics of commodities and injured people after an earthquake. The unattended injured people, the unsatisfied demands and the number of vehicles used are minimized and hierarchically considered. The demands and supplies are uncertain and characterized by a budget set.

In the next subsection, the robust CVRP-TDD is formally defined and the mathematical model to solve is detailed.

3.2.2 Mathematical model

The robust CVRP-TDD is an extension of the CVRP-TDD (presented in Section 3.1) to handle the uncertainty in the people displacement rate. Instead of leaving the shelters at a fixed and known displacement rate, people leave the shelters at an uncertain rate. This problem contributes to having a reliable route plan for the distribution of first aid in the aftermath of a disaster.

We define a scenario s as a vector (b_1^s, \dots, b_n^s) assigning a specific displacement rate b_i^s to each shelter i . A set R of such scenarios is considered and the satisfied demand, for a given solution Ψ , is redefined as $d_i^s(\Psi) = a_i - b_i^s t_i \forall i \in V'$ which depends on each scenario $s \in R$.

Let $d^s(\Psi) = \sum_{i \in V'} d_i^s(\Psi) \forall s \in R$ be the number of attended people at shelters of solution Ψ on scenario s . Then, it can be represented by a vector $(d^1(\Psi), d^2(\Psi), \dots, d^{|R|}(\Psi))$ for a solution Ψ of the robust CVRP-TDD. The worst solution is the solution Ψ which has the

value $\delta = \min_{s \in R} \{d^s(\Psi)\}$.

The objective is to identify a solution Ψ^* with a set of feasible routes, identical for all scenarios, so that every shelter is visited exactly once and δ is maximized.

The robust CVRP-TDD can be formulated as follows:

$$\max \delta \quad (3.39)$$

$$\sum_{i \in V'} d_i^s \geq \delta \quad \forall s \in R \quad (3.40)$$

$$\sum_{j \in V'} x_{0j} = K \quad (3.41)$$

$$\sum_{i \in V} x_{ij} = \sum_{i \in V} x_{ji} \quad \forall j \in V' \quad (3.42)$$

$$\sum_{i \in V} x_{ij} = 1 \quad \forall j \in V' \quad (3.43)$$

$$w_j^s \geq w_i^s + d_i^s - Q \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V', \forall s \in R \quad (3.44)$$

$$w_i^s \leq Q \quad \forall i \in V', \forall s \in R \quad (3.45)$$

$$t_j \geq t_i + c_{ij} - T_{max} \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V' \quad (3.46)$$

$$t_i + c_{i0} \leq T_{max} \quad \forall i \in V' \quad (3.47)$$

$$d_i^s \leq a_i - b_i^s t_i \quad \forall i \in V', \forall s \in R \quad (3.48)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V, \forall j \in V' \quad (3.49)$$

$$t_i \geq 0 \quad \forall i \in V \quad (3.50)$$

$$d_i^s, w_i^s \geq 0 \quad \forall i \in V, \forall s \in R \quad (3.51)$$

$$\delta \geq 0 \quad (3.52)$$

The objective function (3.39) and constraints (3.40), (3.44), (3.45), (3.48), (3.51) and (3.52) are those that differ from the MILP presented in the subsection 3.1.1 to solve the CVRP-TDD.

The objective (3.39) is to maximize δ which is a bound in constraints (3.40) of the total satisfied demand at shelters for each scenario. These constraints ensure the maximization of the minimum satisfied demand. Constraints (3.44) calculate the amount of product transported until each shelter in each scenario. This cannot exceed the vehicle capacity Q in each scenario (3.45). Constraints (3.48) compute the demand for each scenario that must be satisfied at each shelter, taking into account the displacement of people and the arrival times to each shelter. Constraints (3.51) and (3.52) define the decision variables that depends on each scenario s and the variable of the minimum satisfied demand, respectively.

3.2.3 Branch-and-price algorithm

The Dantzig-Wolfe decomposition is used to separate the MILP formulation in a master and pricing problem. The CG is applied to solved them iteratively.

3.2.3.1 Column Generation

In the CG for the robust CVRP-TDD, the constructive heuristic used in Chapter 4 to find an initial solution is adapted to consider the different scenarios. The calculations that involve b_i , i.e. the demand and the vehicle capacity used, are replaced by b_i^s and computed for each scenario s . The master and pricing problems are explained in 3.2.3.1.1 and 3.2.3.1.2.

3.2.3.1.1 Master problem

The master problem (MP) is modeled as the set partitioning formulation presented in Equations (3.53)-(3.58). The objective function (3.53) corresponds to the one of the robust-CVRP-TDD formulation. Constraints (3.54) and (3.58) are the additional constraints managed by the MP in comparison with the MP for the CVRP-TDD (3.1.2.1.1).

$$\max \delta \tag{3.53}$$

$$\delta - \sum_{p \in P} d_p^s \lambda_p \leq 0 \quad \forall s \in R \tag{3.54}$$

$$\sum_{p \in P} \gamma_p^i \lambda_p = 1 \quad \forall i \in V' \tag{3.55}$$

$$\sum_{p \in P} \lambda_p \leq K \tag{3.56}$$

$$\lambda_p \in \{0, 1\} \quad \forall p \in P \tag{3.57}$$

$$\delta \geq 0 \tag{3.58}$$

The objective function (3.53) and constraints (3.54) ensure that the minimum total satisfied demand for all scenarios is maximized.

Solving the linear relaxation of the restricted master problem (LRMP), the dual variables π_s for each s in R , π_i for each i in V' and π_o associated with constraints (3.54), (3.55) and (3.56) are obtained, respectively. The LRMP is obtained by relaxing the integrity constraints (3.57) and taking only a subset of feasible columns $P' \subset P$.

3.2.3.1.2 Pricing problem

The pricing problem is an elementary *longest* path problem with resource constraints (ELPPRC) that we modify to handle the scenarios of the robust CVRP-TDD. In the robust

case, the reduced cost is computed as:

$$\bar{C}_p = \sum_{i \in V'} \gamma_p^i \left[\sum_{s \in R} (a_i - b_i^s t_i) \pi_s - \pi_i \right] - \pi_o \quad (3.59)$$

It should be noted that the reduced cost does not differ each scenario s . It means that in the dynamic programming algorithm, there is no specific labels for each scenario, but only one that contains all the information like in the pricing problem of the CVRP-TDD (see 3.1.2.1.2).

Dynamic programming

The label for a path p differs from the CVRP-TDD in the fact that the total served demand d_p is replaced by the vector $[d_p^1, \dots, d_p^{|R|}]$ where d_p^s is the total served demand by path p in scenario s . So, let p be a partial path ending at node i with a label $l_p = (\bar{C}_p, t_p, [d_p^1, \dots, d_p^{|R|}], [\bar{V}_p^1, \dots, \bar{V}_p^n])$.

When p is extended with an unvisited node j , the new label of the resulting partial path p' is computed as follows:

$$\bar{C}_{p'} = \bar{C}_p + \sum_{s \in R} (a_j - b_j^s \cdot (t_p + c_{ij})) \pi_s - \pi_j \quad (3.60)$$

$$d_{p'}^s = d_p^s + a_j - b_j^s (t_p + c_{ij}) \quad \forall s \in R \quad (3.61)$$

$$t_{p'} = t_p + c_{ij} \quad (3.62)$$

$$\bar{V}_{p'}^j = 1 \quad \text{and} \quad \bar{V}_{p'}^k = \bar{V}_p^k \quad \forall k \in V' \setminus \{j\} \quad (3.63)$$

The importance of the positive demand of j must be maintained for all scenarios ($a_j - b_j^s (t_p + c_{ij}) > 0 \forall s \in R$). Moreover, the attended demand must fit vehicle capacity in each scenario ($d_{p'}^s \leq Q \forall s \in R$) and the maximum tour length has to be satisfied, including return to the depot ($d_{p'} + c_{j0} \leq T_{max}$).

The dynamic programming algorithm normally stops when no more labels can be extended. If the final node $n + 1$ (dummy depot) has no label with positive reduced cost, the solution of the last LRMP is optimal and the CG terminates. Otherwise, the columns found are added to P' .

Dominance rules

Below, we present how the dominance rules 1 and 2 used in the pricing problem of the CVRP-TDD are adapted to the robust CVRP-TDD.

If $p_1 = (0, \dots, i)$ and $p_2 = (0, \dots, i)$ are two distinct paths to node i with respective labels l_{p_1} and l_{p_2} , p_1 dominates p_2 if:

$$t_{p_1} = t_{p_2} \quad (3.64)$$

$$Q - d_{p_1}^s \geq Q - d_{p_2}^s \quad \forall s \in R \quad (3.65)$$

$$\bar{V}_{p_1}^k \leq \bar{V}_{p_2}^k \quad \forall k \in V' \quad (3.66)$$

$$\bar{C}_{p_1} \geq \bar{C}_{p_2} \quad (3.67)$$

Like in the CVRP-TDD, the dominance rule is no longer valid when condition 3.68 is considered.

$$t_{p_1} \leq t_{p_2} \quad (3.68)$$

However, if $t_{p_1} < t_{p_2}$ and conditions (3.65)-(3.67) are met, p_1 dominates p_2 if:

$$Q - d_{p_1}^s \geq \sum_{j \in V', j \notin p_2} a_j - b_j^s (t_{p_1} + c_{ij}) \quad \forall s \in R \quad (3.69)$$

In other words, if p_1 has a greater reduced cost and consumes as many or fewer resources guaranteeing that the remaining capacity in p_1 is enough to visit all shelters reachable from p_2 in all scenarios, then label l_{p_2} can be deleted.

3.2.3.1.3 Acceleration technique

The pricing with acceleration technique is similar to the one used in the CVRP-TDD. Only conditions (3.67) and (3.68) are kept. The pricing problem considering this acceleration technique is called at each CG iteration until no more paths with positive reduced cost are found, then the exact pricing with all conditions ((3.65)-(3.69) is executed.

3.2.3.2 Branching scheme

The branching scheme is the same used in the Chapter 3. The branching is done on the flow variables x_{ij} of the original problem ((3.39) - (3.52)) and two child-nodes are also created following the same branching rules of the B&P for the CVRP-TDD.

3.2.4 Spanning tree based relaxation

The upper bound based on the spanning tree proposed in subsection 3.1.3 for the CVRP-TDD is also used in the robust version of the problem. The MILP (3.31)-(3.38) is sequentially executed for each scenario $s \in R$ and the minimum value of the objective function (Equation (3.31)) of all scenarios is the upper bound for the robust CVRP-TDD.

3.2.5 Computational experiments

Computational experiments are performed to evaluate the B&P algorithm for the robust CVRP-TDD. In 3.2.5.1, the results on small-medium size instances obtained solving the B&P are described and analyzed.

We use the same computer and the instances as in the computational experiments of Section 3.1. The displacement rate of the shelters in each instance represents a scenario for the robust CVRP-TDD. From this scenario, five other scenarios are generated to a total of $|R| = 6$ scenarios per instance. These scenarios are created decreasing or increasing up to 30% each of the displacement rate at shelters in the original instance.

The tests are executed considering three and six scenarios per instance in set I and C .

3.2.5.1 Branch-and-price

Tables 3.8a and 3.9a present the results of the B&P algorithm per instance considering the three and six scenarios of set I and C for the robust CVRP-TDD, respectively. While the average execution time (in seconds) and the average gap by group of instances with the same number of nodes in set I and C are detailed in Tables 3.8b and 3.9b, respectively. The average time is calculated with the instances solved within the time limit of three hours.

The columns Z_{LB} and Z_{UB} represent the lower and upper bounds at the B&P tree, respectively. If $Z_{UB} = \text{"-"}$, the CG at the root node cannot converge within the time limit of three hours. The number of nodes explored in the B&P tree is provided in column *Nodes*. The column ST_{UB} is the upper bound based on the spanning tree relaxation. This upper bound is used to compute the gap if the CG does not converge in the time limit. The optimal solutions (null gaps) are shown in bold. The running times are listed in column *Time (s)*.

The B&P is able to find the optimal solution for instances up to 30 shelters with 3 and 6 scenarios in set I as well as with 3 scenarios in set C . While only instances up to 25 shelters are solved to optimality with 6 scenarios in set C . In addition, the number of instances solved to optimality is greater with 3 scenarios than with 6 scenarios in both sets (29 vs. 25 instances in set I and 27 vs. 25 in set C).

The algorithm takes less time for solving instances with 3 scenarios in both sets. This can be expected because more you have scenarios, harder it is to dominate labels in the pricing problem. However, the average execution time is greater with 3 scenarios than with 6 scenarios in instances with 25 and 30 shelters due to the additional instances solved to optimality with 3 scenarios in these groups of instances. The average gap is smaller with 3 scenarios than with 6 scenarios in all instance groups.

In Tables 3.10 and 3.11, the initial population (Z_0), the population attended at shelters

Table 3.8 – Results of the B&P for the robust CVRP-TDD on set I

(a) Results by instance

| Instance | n | K | 3 Scenarios | | | | 6 Scenarios | | | | | | | |
|----------|-----|-----|-------------|----------|-------|----------|-------------|---------|----------|----------|-------|----------|-----------|---------|
| | | | Z_{LB} | Z_{UB} | Nodes | Time (s) | ST_{UB} | Gap (%) | Z_{LB} | Z_{LB} | Nodes | Time (s) | ST_{UB} | Gap (%) |
| I-1 | 10 | 2 | 152.922 | 152.922 | 1 | 1.13 | 162.016 | 0.00 | 150.658 | 150.658 | 1 | 0.88 | 160.446 | 0.00 |
| I-2 | 15 | 3 | 196.649 | 196.649 | 1 | 1.71 | 207.142 | 0.00 | 196.093 | 196.093 | 3 | 3.97 | 207.142 | 0.00 |
| I-3 | 20 | 4 | 274.508 | 274.508 | 1 | 7.13 | 292.507 | 0.00 | 274.508 | 274.508 | 1 | 5.42 | 292.507 | 0.00 |
| I-4 | 25 | 4 | 296.937 | 296.937 | 11 | 1677.66 | 321.947 | 0.00 | 296.853 | 296.853 | 17 | 1408.47 | 321.947 | 0.00 |
| I-5 | 30 | 4 | 363.181 | 363.181 | 1 | 6718.03 | 421.397 | 0.00 | 363.181 | - | 1 | 10800.00 | 421.397 | 16.03 |
| I-6 | 50 | 6 | 607.415 | - | 1 | 10800.00 | 689.229 | 13.47 | 604.531 | - | 1 | 10800.00 | 688.703 | 13.92 |
| I-7 | 10 | 3 | 138.949 | 138.949 | 1 | 0.35 | 143.986 | 0.00 | 137.682 | 137.682 | 1 | 0.61 | 142.493 | 0.00 |
| I-8 | 15 | 3 | 241.113 | 241.113 | 5 | 3.25 | 259.394 | 0.00 | 234.605 | 234.605 | 7 | 2.64 | 255.639 | 0.00 |
| I-9 | 20 | 4 | 298.618 | 298.618 | 9 | 6.47 | 326.025 | 0.00 | 298.618 | 298.618 | 3 | 1.01 | 326.025 | 0.00 |
| I-10 | 25 | 5 | 369.462 | 369.462 | 1 | 38.23 | 401.953 | 0.00 | 369.004 | 369.005 | 3 | 17.23 | 401.167 | 0.00 |
| I-11 | 30 | 5 | 433.358 | 433.358 | 19 | 691.60 | 487.140 | 0.00 | 429.352 | 432.587 | 189 | 10800.00 | 487.140 | 0.75 |
| I-12 | 50 | 8 | 685.853 | 689.702 | 4 | 10800.00 | 767.283 | 0.56 | 681.215 | 689.005 | 3 | 10800.00 | 765.760 | 1.14 |
| I-13 | 10 | 2 | 99.538 | 99.538 | 1 | 0.97 | 109.608 | 0.00 | 98.433 | 98.433 | 1 | 0.67 | 108.708 | 0.00 |
| I-14 | 15 | 3 | 193.588 | 193.588 | 9 | 9.58 | 210.311 | 0.00 | 189.266 | 189.266 | 9 | 8.08 | 207.787 | 0.00 |
| I-15 | 20 | 4 | 263.538 | 263.538 | 7 | 11.52 | 277.736 | 0.00 | 263.538 | 263.538 | 7 | 11.40 | 277.736 | 0.00 |
| I-16 | 25 | 4 | 317.132 | 317.132 | 1 | 927.64 | 374.678 | 0.00 | 344.752 | 345.152 | 23 | 10800.00 | 372.880 | 0.12 |
| I-17 | 30 | 4 | 331.611 | 331.611 | 3 | 9991.07 | 363.838 | 0.00 | 330.278 | 330.357 | 4 | 10800.00 | 362.375 | 0.02 |
| I-18 | 50 | 6 | 510.228 | - | 1 | 10800.00 | 565.777 | 10.89 | 508.685 | - | 1 | 10800.00 | 563.108 | 10.70 |
| I-19 | 10 | 2 | 104.147 | 104.147 | 1 | 0.64 | 110.262 | 0.00 | 104.147 | 104.147 | 1 | 0.81 | 110.262 | 0.00 |
| I-20 | 15 | 3 | 161.145 | 161.145 | 3 | 2.23 | 170.283 | 0.00 | 161.145 | 161.145 | 3 | 2.07 | 170.283 | 0.00 |
| I-21 | 20 | 4 | 219.223 | 219.223 | 3 | 5.02 | 232.866 | 0.00 | 219.223 | 219.223 | 3 | 6.82 | 232.866 | 0.00 |
| I-22 | 25 | 4 | 317.436 | 317.436 | 1 | 110.37 | 344.449 | 0.00 | 317.293 | 317.293 | 1 | 188.06 | 343.603 | 0.00 |
| I-23 | 30 | 4 | 358.867 | 358.867 | 1 | 8432.80 | 398.755 | 0.00 | 356.644 | 356.644 | 1 | 3307.00 | 396.406 | 0.00 |
| I-24 | 50 | 6 | 583.943 | - | 1 | 10800.00 | 638.744 | 9.38 | 580.305 | - | 1 | 10800.00 | 636.695 | 9.72 |
| I-25 | 10 | 2 | 116.809 | 116.809 | 5 | 1.95 | 125.889 | 0.00 | 116.407 | 116.407 | 1 | 0.55 | 125.845 | 0.00 |
| I-26 | 15 | 3 | 191.737 | 191.737 | 9 | 6.80 | 201.631 | 0.00 | 191.411 | 191.411 | 25 | 11.08 | 201.631 | 0.00 |
| I-27 | 20 | 4 | 293.916 | 293.916 | 1 | 1.44 | 313.183 | 0.00 | 293.594 | 293.594 | 5 | 3.77 | 312.829 | 0.00 |
| I-28 | 25 | 4 | 319.659 | 319.659 | 5 | 330.97 | 346.512 | 0.00 | 319.659 | 319.659 | 7 | 374.44 | 346.512 | 0.00 |
| I-29 | 30 | 4 | 366.040 | - | 1 | 10800.00 | 417.580 | 14.08 | 366.040 | - | 1 | 10800.00 | 417.580 | 14.08 |
| I-30 | 50 | 6 | 707.197 | - | 1 | 10800.00 | 795.196 | 12.44 | 701.530 | - | 1 | 10800.00 | 795.169 | 13.35 |
| I-31 | 10 | 2 | 97.878 | 97.878 | 1 | 1.06 | 105.529 | 0.00 | 97.759 | 97.759 | 7 | 1.87 | 105.529 | 0.00 |
| I-32 | 15 | 3 | 123.974 | 123.974 | 1 | 1.31 | 131.196 | 0.00 | 123.974 | 123.974 | 1 | 2.47 | 131.196 | 0.00 |
| I-33 | 20 | 4 | 204.776 | 204.776 | 1 | 7.92 | 214.765 | 0.00 | 203.074 | 203.075 | 39 | 170.14 | 213.299 | 0.00 |
| I-34 | 25 | 4 | 241.809 | 242.940 | 3 | 10800.00 | 263.636 | 0.47 | 240.910 | 241.250 | 4 | 10800.00 | 263.031 | 0.14 |
| I-35 | 30 | 4 | 303.961 | - | 1 | 10800.00 | 339.591 | 11.72 | 302.806 | - | 1 | 10800.00 | 339.591 | 12.15 |
| I-36 | 50 | 6 | 459.497 | - | 1 | 10800.00 | 512.049 | 11.44 | 459.152 | - | 1 | 10800.00 | 512.049 | 11.52 |
| I-37 | 10 | 2 | 169.656 | 169.656 | 3 | 2.38 | 179.567 | 0.00 | 168.589 | 168.589 | 39 | 8.66 | 179.234 | 0.00 |
| I-38 | 15 | 3 | 207.811 | 207.811 | 7 | 23.29 | 218.168 | 0.00 | 207.386 | 207.386 | 9 | 29.80 | 218.121 | 0.00 |
| I-39 | 20 | 4 | 380.332 | - | 1 | 10800.00 | 397.269 | 4.45 | 380.332 | - | 1 | 10800.00 | 397.163 | 4.43 |
| I-40 | 25 | 4 | 428.465 | - | 1 | 10800.00 | 460.805 | 7.55 | 423.371 | - | 1 | 10800.00 | 460.805 | 8.84 |
| I-41 | 30 | 4 | 433.329 | - | 1 | 10800.00 | 478.019 | 10.31 | 433.329 | - | 1 | 10800.00 | 478.019 | 10.31 |
| I-42 | 50 | 6 | 771.464 | - | 1 | 10800.00 | 869.439 | 12.70 | 770.090 | - | 1 | 10800.00 | 867.526 | 12.65 |

(b) Average results by group of instances

| n | 3 scenarios | | 6 scenarios | |
|-----|---------------|--------------|---------------|--------------|
| | Avg. Time (s) | Avg. Gap (%) | Avg. Time (s) | Avg. Gap (%) |
| 10 | 1.21 | 0.00 | 2.01 | 0.00 |
| 15 | 6.88 | 0.00 | 8.59 | 0.00 |
| 20 | 6.58 | 0.64 | 33.09 | 0.63 |
| 25 | 616.97 | 1.15 | 497.05 | 1.30 |
| 30 | 6458.38 | 5.16 | 3307.00 | 7.62 |
| 50 | - | 10.13 | - | 10.43 |

Table 3.9 – Results of the B&P for the robust CVRP-TDD on set C

(a) Results by instance

| Instance | n | K | 3 Scenarios | | | | 6 scenarios | | | | | | | |
|----------|-----|-----|-------------|----------|-------|----------|-------------|---------|----------|----------|-------|----------|-----------|---------|
| | | | Z_{LB} | Z_{UB} | Nodes | Time (s) | ST_{UB} | Gap (%) | Z_{LB} | Z_{UB} | Nodes | Time (s) | ST_{UB} | Gap (%) |
| C-1 | 10 | 2 | 142.195 | 142.195 | 1 | 0.89 | 157.750 | 0.00 | 142.195 | 142.195 | 1 | 0.74 | 157.750 | 0.00 |
| C-2 | 15 | 3 | 178.144 | 178.144 | 1 | 1.21 | 191.551 | 0.00 | 178.144 | 178.144 | 1 | 1.63 | 191.551 | 0.00 |
| C-3 | 20 | 4 | 258.193 | 258.193 | 1 | 1.51 | 278.666 | 0.00 | 254.315 | 254.315 | 21 | 9.40 | 276.313 | 0.00 |
| C-4 | 25 | 4 | 274.005 | 274.005 | 13 | 1978.34 | 296.636 | 0.00 | 272.900 | 272.900 | 13 | 1836.43 | 295.942 | 0.00 |
| C-5 | 30 | 4 | 372.777 | 372.888 | 3 | 10800.00 | 418.563 | 0.03 | 369.350 | - | 1 | 10800.00 | 416.323 | 12.72 |
| C-6 | 50 | 6 | 621.855 | - | 1 | 10800.00 | 696.355 | 11.98 | 621.489 | - | 1 | 10800.00 | 696.355 | 12.05 |
| C-7 | 10 | 3 | 154.785 | 154.785 | 1 | 0.15 | 158.925 | 0.00 | 154.785 | 154.785 | 1 | 0.15 | 158.925 | 0.00 |
| C-8 | 15 | 3 | 200.368 | 200.368 | 1 | 0.47 | 222.726 | 0.00 | 200.368 | 200.368 | 1 | 0.51 | 222.726 | 0.00 |
| C-9 | 20 | 4 | 338.306 | 338.306 | 5 | 3.59 | 367.587 | 0.00 | 338.219 | 338.219 | 21 | 6.00 | 367.587 | 0.00 |
| C-10 | 25 | 5 | 344.320 | 344.320 | 7 | 6.76 | 369.611 | 0.00 | 339.800 | 339.800 | 23 | 27.54 | 367.913 | 0.00 |
| C-11 | 30 | 5 | 396.512 | 404.779 | 90 | 10800.00 | 461.726 | 2.08 | 394.579 | 400.148 | 74 | 10800.00 | 458.263 | 1.41 |
| C-12 | 50 | 8 | 714.714 | 716.887 | 2 | 10800.00 | 785.702 | 0.30 | 712.845 | - | 1 | 10800.00 | 785.054 | 10.13 |
| C-13 | 10 | 2 | 140.961 | 140.961 | 1 | 1.09 | 149.674 | 0.00 | 139.197 | 139.197 | 1 | 0.70 | 147.406 | 0.00 |
| C-14 | 15 | 3 | 164.234 | 164.234 | 7 | 2.88 | 172.452 | 0.00 | 163.592 | 163.592 | 11 | 2.60 | 172.399 | 0.00 |
| C-15 | 20 | 4 | 200.535 | 200.535 | 1 | 1.21 | 212.267 | 0.00 | 200.535 | 200.535 | 1 | 1.29 | 212.267 | 0.00 |
| C-16 | 25 | 4 | 306.283 | 306.283 | 1 | 230.88 | 333.128 | 0.00 | 304.845 | 304.845 | 1 | 522.68 | 331.476 | 0.00 |
| C-17 | 30 | 4 | 349.381 | - | 1 | 10800.00 | 394.531 | 12.92 | 349.080 | - | 1 | 10800.00 | 394.531 | 13.02 |
| C-18 | 50 | 6 | 615.636 | - | 1 | 10800.00 | 669.734 | 8.79 | 615.332 | - | 1 | 10800.00 | 669.734 | 8.84 |
| C-19 | 10 | 2 | 112.875 | 112.875 | 1 | 0.91 | 122.748 | 0.00 | 112.875 | 112.875 | 1 | 0.63 | 122.748 | 0.00 |
| C-20 | 15 | 3 | 209.081 | 209.081 | 9 | 4.69 | 220.317 | 0.00 | 208.201 | 208.201 | 1 | 1.46 | 220.189 | 0.00 |
| C-21 | 20 | 4 | 238.545 | 238.545 | 63 | 47.84 | 251.777 | 0.00 | 238.545 | 238.545 | 89 | 61.08 | 251.777 | 0.00 |
| C-22 | 25 | 4 | 357.407 | 357.407 | 1 | 43.07 | 395.501 | 0.00 | 357.407 | 357.407 | 1 | 43.36 | 395.501 | 0.00 |
| C-23 | 30 | 4 | 372.756 | 372.756 | 3 | 5110.66 | 411.263 | 0.00 | 372.316 | 372.811 | 16 | 10800.00 | 411.263 | 0.13 |
| C-24 | 50 | 6 | 577.107 | - | 1 | 10800.00 | 651.427 | 12.88 | 577.888 | - | 1 | 10800.00 | 651.420 | 12.72 |
| C-25 | 10 | 2 | 106.379 | 106.379 | 1 | 4.11 | 115.178 | 0.00 | 106.379 | 106.379 | 1 | 0.83 | 115.178 | 0.00 |
| C-26 | 15 | 3 | 179.997 | 179.997 | 1 | 1.98 | 196.865 | 0.00 | 178.211 | 178.211 | 1 | 1.89 | 195.767 | 0.00 |
| C-27 | 20 | 4 | 273.159 | 273.159 | 3 | 4.12 | 288.900 | 0.00 | 268.744 | 268.744 | 1 | 2.85 | 285.951 | 0.00 |
| C-28 | 25 | 4 | 294.893 | 294.893 | 1 | 85.57 | 320.711 | 0.00 | 294.399 | 294.399 | 1 | 174.41 | 319.990 | 0.00 |
| C-29 | 30 | 4 | 440.906 | - | 1 | 10800.00 | 499.570 | 13.31 | 432.990 | 433.607 | 2 | 10800.00 | 496.807 | 0.14 |
| C-30 | 50 | 6 | 596.924 | - | 1 | 10800.00 | 673.777 | 12.87 | 596.924 | - | 1 | 10800.00 | 673.777 | 12.87 |
| C-31 | 10 | 2 | 73.838 | 73.838 | 5 | 0.61 | 79.282 | 0.00 | 73.157 | 73.157 | 7 | 2.35 | 79.027 | 0.00 |
| C-32 | 15 | 3 | 160.759 | 160.759 | 3 | 3.18 | 171.077 | 0.00 | 159.977 | 159.977 | 35 | 28.78 | 171.076 | 0.00 |
| C-33 | 20 | 4 | 229.468 | 229.468 | 39 | 50.49 | 238.137 | 0.00 | 229.468 | 229.468 | 63 | 54.98 | 238.137 | 0.00 |
| C-34 | 25 | 4 | 233.388 | 233.388 | 17 | 9747.52 | 251.299 | 0.00 | 233.037 | 233.185 | 43 | 10800.00 | 251.299 | 0.06 |
| C-35 | 30 | 4 | 285.358 | - | 1 | 10800.00 | 315.553 | 10.58 | 281.938 | - | 1 | 10800.00 | 313.392 | 11.16 |
| C-36 | 50 | 6 | 490.430 | - | 1 | 10800.00 | 544.239 | 10.97 | 487.672 | - | 1 | 10800.00 | 544.238 | 11.60 |
| C-37 | 10 | 2 | 174.893 | 174.893 | 5 | 0.77 | 186.045 | 0.00 | 174.893 | 174.893 | 5 | 0.73 | 186.045 | 0.00 |
| C-38 | 15 | 3 | 278.077 | 278.077 | 1 | 494.47 | 292.935 | 0.00 | 276.439 | 276.439 | 17 | 5086.64 | 292.713 | 0.00 |
| C-39 | 20 | 4 | 336.270 | - | 1 | 10800.00 | 349.291 | 3.87 | 336.226 | - | 1 | 10800.00 | 349.291 | 3.89 |
| C-40 | 25 | 4 | 466.728 | - | 1 | 10800.00 | 498.292 | 6.76 | 466.209 | - | 1 | 10800.00 | 498.292 | 6.88 |
| C-41 | 30 | 4 | 431.588 | - | 1 | 10800.00 | 489.089 | 13.32 | 430.439 | - | 1 | 10800.00 | 489.089 | 13.63 |
| C-42 | 50 | 6 | 773.868 | - | 1 | 10800.00 | 868.723 | 12.26 | 773.328 | - | 1 | 10800.00 | 868.723 | 12.34 |

(b) Average results by group of instances

| n | 3 scenarios | | 6 scenarios | |
|-----|---------------|--------------|---------------|--------------|
| | Avg. Time (s) | Avg. Gap (%) | Avg. Time (s) | Avg. Gap (%) |
| 10 | 1.22 | 0.00 | 0.88 | 0.00 |
| 15 | 72.70 | 0.00 | 731.93 | 0.00 |
| 20 | 18.13 | 0.55 | 22.60 | 0.56 |
| 25 | 2015.36 | 0.97 | 520.44 | 0.99 |
| 30 | 5110.66 | 7.46 | - | 7.46 |
| 50 | - | 10.01 | - | 11.51 |

Table 3.10 – Comparison between initial and attended population in set I

| Instance | Z_0 | 3 Scenarios | | 6 Scenarios | |
|----------|-------|-------------|---------|-------------|---------|
| | | Z_{LB} | Gap (%) | Z_{LB} | Gap (%) |
| I-1 | 183 | 152.922 | 16.44 | 150.658 | 17.67 |
| I-2 | 231 | 196.649 | 14.87 | 196.093 | 15.11 |
| I-3 | 331 | 274.508 | 17.07 | 274.508 | 17.07 |
| I-4 | 362 | 296.937 | 17.97 | 296.853 | 18.00 |
| I-7 | 176 | 138.949 | 21.05 | 137.682 | 21.77 |
| I-8 | 293 | 241.113 | 17.71 | 234.605 | 19.93 |
| I-9 | 380 | 298.618 | 21.42 | 298.618 | 21.42 |
| I-13 | 125 | 99.538 | 20.37 | 98.433 | 21.25 |
| I-14 | 235 | 193.588 | 17.62 | 189.266 | 19.46 |
| I-15 | 311 | 263.538 | 15.26 | 263.538 | 15.26 |
| I-19 | 123 | 104.147 | 15.33 | 104.147 | 15.33 |
| I-20 | 189 | 161.145 | 14.74 | 161.145 | 14.74 |
| I-21 | 264 | 219.223 | 16.96 | 219.223 | 16.96 |
| I-22 | 391 | 317.436 | 18.81 | 317.293 | 18.85 |
| I-23 | 444 | 358.867 | 19.17 | 356.644 | 19.67 |
| I-25 | 142 | 116.809 | 17.74 | 116.407 | 18.02 |
| I-26 | 221 | 191.737 | 13.24 | 191.411 | 13.39 |
| I-27 | 361 | 293.916 | 18.58 | 293.594 | 18.67 |
| I-28 | 386 | 319.659 | 17.19 | 319.659 | 17.19 |
| I-31 | 115 | 97.878 | 14.89 | 97.759 | 14.99 |
| I-32 | 145 | 123.974 | 14.50 | 123.974 | 14.50 |
| I-37 | 190 | 169.656 | 10.71 | 168.589 | 11.27 |
| I-38 | 230 | 207.811 | 9.65 | 207.386 | 9.83 |
| | | Min. | 9.65 | | 9.83 |
| | | Avg. | 16.58 | | 16.97 |
| | | Max. | 21.42 | | 21.77 |

(Z_{LB}) as well as the percentage of the population that does not receive relief goods with 3 and 6 scenarios in set I and C are shown, respectively. In addition, the gap between the population served at shelters with 3 scenarios and with 6 scenarios is also presented in the last column.

These tables show that the percentage of the initial population that cannot be attended at shelters is between 9.65%-21.42% and 10.30%-22.91% with 3 scenarios in sets I and C respectively. While with 6 scenarios, these ranges are 9.83%-21.77% and 10.36%-22.91% in sets I and C respectively. It should be noted that minimum and maximum percentages increase with respect to the 3 scenarios, except the maximum percentage in set C . This occurs because the 3 scenarios are also included in the 6 scenarios, so the gaps with 6 scenarios are at least as large in all instances.

We can conclude that at least 75% of the initial population will be served in the shelters no matter which of the six scenarios occurs.

3.2.6 Concluding remarks

In the second section of this chapter, the robust counterpart of the CVRP-TDD which considers the uncertainty in the displacement rates is presented. These displacement rates

Table 3.11 – Comparison between initial and attended population in set C

| Instance | Z_0 | 3 Scenarios | | 6 Scenarios | |
|----------|-------|-------------|---------|-------------|---------|
| | | Z_{LB} | Gap (%) | Z_{LB} | Gap (%) |
| C-1 | 177 | 142.195 | 19.66 | 142.195 | 19.66 |
| C-2 | 215 | 178.144 | 17.14 | 178.144 | 17.14 |
| C-3 | 320 | 258.193 | 19.31 | 254.315 | 20.53 |
| C-4 | 328 | 274.005 | 16.46 | 272.900 | 16.80 |
| C-7 | 181 | 154.785 | 14.48 | 154.785 | 14.48 |
| C-8 | 258 | 200.368 | 22.34 | 200.368 | 22.34 |
| C-9 | 421 | 338.306 | 19.64 | 338.219 | 19.66 |
| C-10 | 425 | 344.320 | 18.98 | 339.800 | 20.05 |
| C-13 | 169 | 140.961 | 16.59 | 139.197 | 17.63 |
| C-14 | 189 | 164.234 | 13.10 | 163.592 | 13.44 |
| C-15 | 240 | 200.535 | 16.44 | 200.535 | 16.44 |
| C-16 | 376 | 306.283 | 18.54 | 304.845 | 18.92 |
| C-19 | 146 | 112.875 | 22.69 | 112.875 | 22.69 |
| C-20 | 256 | 209.081 | 18.33 | 208.201 | 18.67 |
| C-21 | 278 | 238.545 | 14.19 | 238.545 | 14.19 |
| C-22 | 456 | 357.407 | 21.62 | 357.407 | 21.62 |
| C-25 | 138 | 106.379 | 22.91 | 106.379 | 22.91 |
| C-26 | 219 | 179.997 | 17.81 | 178.211 | 18.63 |
| C-27 | 328 | 273.159 | 16.72 | 268.744 | 18.07 |
| C-28 | 362 | 294.893 | 18.54 | 294.399 | 18.67 |
| C-31 | 87 | 73.838 | 15.13 | 73.157 | 15.91 |
| C-32 | 186 | 160.759 | 13.57 | 159.977 | 13.99 |
| C-33 | 256 | 229.468 | 10.36 | 229.468 | 10.36 |
| C-37 | 200 | 174.893 | 12.55 | 174.893 | 12.55 |
| C-38 | 310 | 278.077 | 10.30 | 276.439 | 10.83 |
| | | Min. | 10.30 | | 10.36 |
| | | Avg. | 17.10 | | 17.45 |
| | | Max. | 22.91 | | 22.91 |

are described by a set of discrete scenarios.

Two exact methods, a MILP and a B&P algorithm, are proposed to find reliable routes to the problem. Both methods are tested in small and medium size instances (up to 50 shelters) with 3 and 6 scenarios.

The robust CVRP-TDD is modeled as a set partitioning problem in the B&P algorithm. The pricing problem is an elementary *longest* path problem with resource constraint solved by using dynamic programming. Two dominance rules and an acceleration technique are used to speed up the algorithm.

The reliability of the distribution plan in humanitarian logistics is improved using robust optimization.

In this chapter, exact methods are proposed for the CVRP-TDD and its robust counterpart. These methods find optimal solutions on small and medium size instances. However, the execution time increases exponentially making the problem intractable for large size instances.

Next chapter uses the results of this chapter to evaluate the performance of metaheuristics

that are developed to deal with large instances of the CVRP-TDD.

Conferences and publications

This chapter gives as results two presentations:

- Jorge F. Victoria, H. Murat Afsar, and Christian Prins. *Column Generation based heuristic for the Vehicle Routing Problem with Time-Dependent Demand*. 8th IFAC Conference on Manufacturing Modelling, Management & Control 2016. Troyes, France.
- Jorge F. Victoria, H. Murat Afsar, and Christian Prins. *Column Generation Algorithm for the Robust Capacitated Vehicle Routing Problem with Time-Dependent Demands*. 19^{ème} congrès annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision, ROADEF 2018. Lorient, France.

And a published article in the proceedings of the 8th IFAC Conference on Manufacturing Modelling, Management & Control 2016:

- Victoria, J. F., Afsar, H. M. and Prins, C. (2016), 'Column generation based heuristic for the vehicle routing problem with time-dependent demand', IFAC-PapersOnLine 49(12), 526-531.

Chapter 4

Metaheuristic framework for the capacitated vehicle routing problem with time-dependent demands

This chapter presents a metaheuristic framework to solve large size instances in reasonable time for the CVRP-TDD. The framework is a GRASP \times ELS which includes four other metaheuristics as particular cases: GRASP, ILS, ELS and GRASP \times ILS. The variant to execute is selected using three parameters only. An optimal splitting procedure is embedded in the metaheuristics. This procedure obtains an optimal solution for a given sequence of nodes (giant tour).

The chapter is structured as follows: the framework is explained in Section 4.1. The representations of the solution are described in Section 4.2. The objective value of a solution and its penalization are calculated in Section 4.3. The procedure to generate an initial solution is presented in Section 4.4. The pre-computations required to evaluate local search moves in constant time are depicted in Section 4.5. The moves used in the local search are illustrated in Section 4.6. The Section 4.7 gives the optimal splitting procedure which is adapted to the CVRP-TDD. In Section 4.8, perturbation procedures to escape from local optima are developed. A post-optimization procedure to improve the solution found by the metaheuristics is explained in Section 4.9. Computational results are shown in Section 4.10 and concluding remarks are given in Section 4.11.

4.1 General framework

The metaheuristic framework is based on a common local improvement procedure and involves different strategies to explore the solution space. Each strategy can be selected

by changing only 3 parameters. The strategies and their integration in the framework are explained below, assuming an objective function to maximize.

4.1.1 Greedy Randomized Adaptive Search Procedure (GRASP)

GRASP is a multi-start randomized heuristic in two phases. The first phase constructs one initial solution using a randomized heuristic while the second one tries to improve it via a local search procedure. The global best solution over all iterations is reported at the end.

This technique was introduced by Feo and Resende [75]. Several works from 1989 to 2008 on the GRASP are cited by Festa and Resende [76]. These show the large number of combinatorial optimization problems that can be solved by this technique including the VRP. For example, Kontoravdis and Bard [109] used it to solve a VRP with time windows (VRPTW). To generate an initial solution, they iteratively insert one node at a time in accordance with a set of rules. If there is no feasible position to insert it, a new route is created. Then a local search is used to reduce the number of routes and minimize the total distance traveled.

The GRASP is hybridized with other metaheuristics to improve its performance on VRPs. Lin et al. [124] combined GRASP with tabu search (TS) to solve the VRPTW including additional operational constraints. Other hybridizations were made with path relinking [215] [55], variable neighborhood search (VNS) and variable neighborhood descent (VND) [214] [180], iterated local search (ILS) [166] [17] and evolutionary local search (ELS) [65] [118].

The Algorithm 4.1 shows the pseudo-code of the GRASP for a maximization problem. The first and second phase (*lines* 3 and 4) of the GRASP are iteratively executed *nbOfRestarts* times. At each iteration, an initial solution S is generated with a greedy randomized heuristic and it is improved using a local search. After the local search, S is compared with the best global solution S^* . If S is better than S^* , then S becomes the new S^* (*line* 5).

Algorithm 4.1 Greedy Randomized Adaptive Search Procedure (GRASP)

```
1:  $Z(S^*) \leftarrow 0$ 
2: for  $i = 1$  to  $nbOfRestarts$  do
3:    $S \leftarrow initialSol()$ 
4:    $localSearch(S)$ 
5:   if  $Z(S) > Z(S^*)$  then  $S^* \leftarrow S$  end if
6: end for
7: return  $S^*$ 
```

4.1.2 Iterated Local Search (ILS)

Unlike GRASP, ILS explores the solution space from a single initial solution. This exploration is done through a local search and a perturbation procedure which intensify and diversify the search.

ILS was introduced by Lourenço et al. [128] and several authors report its good performance in routing problems.

Hashimoto et al. [94] and Palhazi et al. [56] followed the classical ILS structure to solve VRPs. The first ones address the VRPTW with time-dependent travel times and travel costs, while the others treat the VRP with backhauls. ILS can be combined with other methods to have a broader exploration of the solution space. Morais et al. [139] proposed three heuristics based on ILS for the VRP with cross-docking. Each one incorporates other strategies into the classical ILS structure such as recording a set of elite solutions, a VND as exploration procedure, and an integer linear program for the set partitioning problem as an intensification procedure. Cordeau and Maischberger [53] merged ILS and TS to solve the classical VRP, the periodic VRP, the multi-depot VRP, and the site-dependent VRP. They called it the iterated TS. Silvestrin and Ritt [188] also developed a similar approach for the VRP with multiple compartments.

The classical ILS structure can be seen in Algorithm 4.2. The main loop (*lines* 4–8) executes *nbOfIter* iterations. At each iteration, a *child* is generated applying the perturbation procedure (*line* 5) to S^* . This helps to escape the local optima found by the local search. In a maximization problem, the global best solution S^* is replaced by the *child* if the objective function value of the later is greater (*line* 7).

Algorithm 4.2 Iterated Local Search (ILS)

```
1:  $Z(S^*) \leftarrow 0$ 
2:  $S^* \leftarrow \text{initialSol}()$ 
3:  $\text{localSearch}(S^*)$ 
4: for  $j = 1$  to  $\text{nbOfIter}$  do
5:    $\text{child} \leftarrow \text{perturb}(S^*)$ 
6:    $\text{localSearch}(\text{child})$ 
7:   if  $Z(\text{child}) > Z(S^*)$  then  $S^* \leftarrow \text{child}$  end if
8: end for
9: return  $S^*$ 
```

4.1.3 Evolutionary Local Search (ELS)

ELS is a generalization of ILS introduced by Wolf and Merz [220] for the peer-to-peer network problem. Contrary to ILS, at each iteration (*lines 4–10*) ELS generates several *children* from a solution as shown in the Algorithm 4.3. S^* is perturbed to generate *nbOfChildren* solutions. The local search is applied to each of them and the *bestChild* is saved. If it is better than the global best solution (S^*), the latter is updated (*line 10*). This procedure is repeated *nbOfIter* times.

Most of the researchers using ELS to solve VRPs hybridize it with other metaheuristics or advances techniques. Prins [166] and Duhamel et al. [65] combined ELS with GRASP to solve VRPs. This hybridization is explained in subsection 4.1.4. Zhang et al. [224] tackled the VRP with 3D loading constraints. They used an ELS incorporating a multi-level perturbation strategy to diversify the search trajectory. Recently, Chassaing et al. [45] presented an ELS to the Dial-a-Ride problem. They include a dynamic probability management mechanism during the local search improving the convergence.

Algorithm 4.3 Evolutionary Local Search (ELS)

```
1:  $S^* \leftarrow initialSol()$ 
2:  $localSearch(S^*)$ 
3: for  $j = 1$  to  $nbOfIter$  do
4:    $\bar{Z} \leftarrow 0$  (cost of best child)
5:   for  $k = 1$  to  $nbOfChildren$  do
6:      $child \leftarrow perturb(S^*)$ 
7:      $localSearch(child)$ 
8:     if  $Z(child) > \bar{Z}$  then  $bestChild \leftarrow child$  ;  $\bar{Z} \leftarrow Z(child)$  end if
9:   end for
10:  if  $\bar{Z} > Z(S^*)$  then  $S^* \leftarrow bestChild$  end if
11: end for
12: return  $S^*$ 
```

4.1.4 GRASP×ILS and GRASP×ELS

The hybridization of GRASP with ILS or ELS allows a broader exploration of solution space compared to the basic ILS or ELS.

The idea of the GRASP×ILS and GRASP×ELS is to execute *nbOfRestarts* one ILS or ELS from distinct initial solutions obtained by a randomized constructive heuristic.

Coelho et al [51], Ait Haddadene et al [93], and Avci and Avci [17] applied GRASP×ILS

to VRPs in different contexts. The first authors work in a commercial context considering heterogeneous fleet which can perform multiple trips. The home health care and the humanitarian context are addressed in [93] and in [17], respectively. Avci and Avci [17] solve the traveling repairman problem with profits (TRPP) to maximize the number of people that survive at nodes $\sum_{i \in N} (p_i - t_i)$, where p_i is the people living in node i and t_i is the arrival time. They assume that a person die at each moment of time. This TRPP is slightly different from the CVRP-TDD, because in the first not all nodes must be visited and the profit of all nodes decrease with the same rate.

GRASP×ELS was implemented by Prins [166], Duhamel et al. [65] and Lacomme et al. [118]. Prins [166] introduced it for the classical VRP and the distance-constrained VRP (DVRP) with homogeneous fleet. He showed that this hybridization is faster than most VRP metaheuristics and outperforms several of them. Duhamel et al. [65] solved the vehicle fleet mix problem and the heterogeneous fleet VRP and report very good results on classical instances. They prove the ability of the method to handle all known benchmark in non-homogeneous vehicles problems. Lacomme et al. [118] assume a homogeneous fleet, but the demands of the clients are three-dimensional weighted items. They show that the GRASP×ELS is competitive compared with the best known methods for this problem.

The general framework of GRASP×ELS can be seen in Algorithm 4.4. It reduces to the metaheuristics previously explained by playing on three parameters: *nbOfRestarts*, *nbOfIter* and *nbOfChildren*. Table 4.1 shows their values for each metaheuristic. For example, for ELS *nbOfRestarts* must be equal to one, while *nbOfIter* and *nbOfChildren* must be greater than one.

The precomputations and their updates (*lines* 5 and 12) are explained in 4.5.

Table 4.1 – Parameters to select each metaheuristic

| Parameter Metaheuristic | <i>nbOfRestarts</i> | <i>nbOfIter</i> | <i>nbOfChildren</i> |
|----------------------------|---------------------|-----------------|---------------------|
| GRASP | > 1 | 1 | 1 |
| ILS | 1 | > 1 | 1 |
| ELS | 1 | > 1 | > 1 |
| GRASP×ILS | > 1 | > 1 | 1 |
| GRASP×ELS | > 1 | > 1 | > 1 |

Additionally, a column pool P is created to save all feasible solutions, but these solutions must be transformed into a set of 0-1 columns. This pool P is an input of our post-optimization procedure detailed in Section 4.9.

The components of the general framework are explained in the following sections.

Algorithm 4.4 Metaheuristic Framework

```
1:  $Z^* \leftarrow 0$ 
2:  $P \leftarrow \emptyset$ 
3: for  $i = 1$  to  $nbOfRestarts$  do
4:    $S \leftarrow initialSol()$  {Randomized}
5:    $precomputations(S)$ 
6:    $localSearch(S)$ 
7:    $P \leftarrow P \cup zeroOneColumns(S)$ 
8:   for  $j = 1$  to  $nbOfIter$  do
9:      $\bar{Z} \leftarrow 0$ 
10:    for  $k = 1$  to  $nbOfChildren$  do
11:       $child \leftarrow perturb(S)$ 
12:       $updatePrecomputations(child)$ 
13:       $localSearch(child)$ 
14:       $P \leftarrow P \cup zeroOneColumns(child)$ 
15:      if  $Z(child) > \bar{Z}$  then  $bestChild \leftarrow child$  ;  $\bar{Z} \leftarrow Z(bestChild)$  end if
16:    end for
17:    if  $Z(bestChild) > Z(S)$  then  $S \leftarrow bestChild$  end if
18:  end for
19:  if  $Z(S) > Z^*$  then  $S^* \leftarrow S$  ;  $Z^* \leftarrow Z(S^*)$  end if
20: end for
21:  $setPartitioning(P)$ 
22: return  $S^*$ 
```

4.2 Solution Representation

Two solution representations are used to explore different solution spaces: complete solutions (set of routes) and giant tours. A route is the order in which a subset of shelters is visited by a vehicle. The giant tour is a sequence of n shelters, without any route separator.

Figure 4.1 shows an example of each representation. The complete representation is composed by $(0, E, C, D, 0)$ and $(0, A, B, 0)$. The giant tour is encoded as $(0, E, C, D, A, B, 0)$. Each arc (i, j) has a travel time c_{ij} and each shelter i has its initial population a_i and a displacement rate b_i . The objective function of the set of routes is calculated easily as presented in Section 4.3. For the giant tour, we developed an optimal splitting method for the given order, which transforms the giant tour into feasible routes. The details of this method is given in Section 4.7.

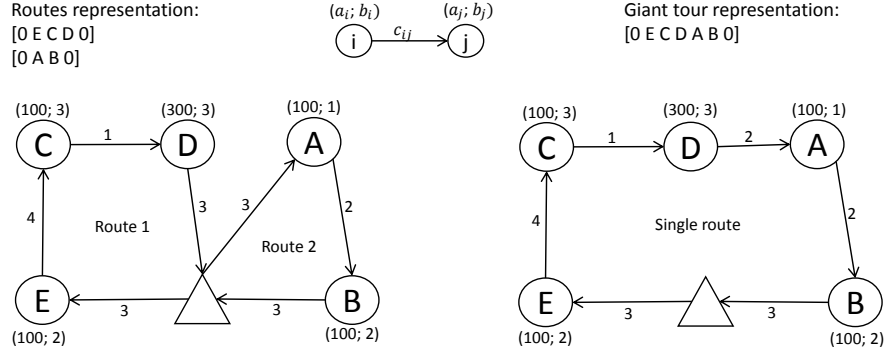


Figure 4.1 – Routes and giant tour representation

4.3 Objective function and penalty calculation

The objective function for a solution S is the total demand satisfied $Z = \sum_{i \in S} a_i - b_i \cdot t_i$.

In the metaheuristic framework, infeasible solutions are allowed but penalized. The penalized objective function of an infeasible solution S is defined by the Equation (4.1).

$$\hat{Z}(S) = Z(S) - \kappa_{demand} \times \mu(S) \quad (4.1)$$

where κ_{demand} is a penalty factor and $\mu(S)$ measures the level of infeasibility. The penalty factor is twice the average initial population: $\kappa_{demand} = 2 \times \frac{\sum_{i \in V'} a_i}{|V'|}$.

Since the infeasibility of a solution is only due to the tour length constraint, $\mu(S)$ is the total overtime of the routes. The length of a route p in solution S is $T_p = \sum_{(i,j) \in p} c_{ij}$. Then, $\mu(S) = \sum_{p \in S} (\max(T_p - T_{max}, 0))$.

4.4 Constructive heuristic

A parallel randomized best insertion procedure is used to find an initial solution for the CVRP-TDD. Iteratively, the procedure chooses one unassigned node from a list U . All possible positions on each route, to insert the node are evaluated and the three best ones are saved. This process is repeated until all nodes are inserted. If no feasible solution can be found, the insertion procedure is restarted. At the end of a given time, if a feasible solution is not yet reached, an infeasible solution S with the largest objective value $\hat{Z}(S)$ is accepted.

The list U can be generated randomly or according to a priority rule: (1) distance, or (2) population. The first one is the distance from depot to shelters, while the second is multiplying this distance by the displacement rate. If any of the priority rules is used, the list can be sorted in a decreasing or increasing order.

A node from the list U must be chosen to evaluate its insertion position. Two different selection criteria are used.

- Consecutive: it follows the order of U .
- Roulette: a list U' is derived from U , where U'_i is the cumulated priority value of nodes U_1 to U_i . A random number r is drawn in $[0, U'(|U'|)]$ and, if r falls between U'_i and U'_{i+1} , then the node corresponding to U'_{i+1} is selected.

The best insertion position is measured following three criteria: the total travel time $T(S)$, the total satisfied demand $Z(S)$, or the ratio $\frac{Z(S)}{T(S)}$. If $T(S)$ is considered, the best positions are those which minimize it. If $Z(S)$ or $\frac{Z(S)}{T(S)}$ is chosen, the best positions are those which maximize these criteria.

If no feasible insertion is found, infeasible insertions are tested. In that case $Z(S)$ is penalized as in Equation (4.1). $T(S)$ is penalized in the same way but with a different penalty factor $\kappa_{time} = (2 \times (\frac{\sum_{(i,j) \in E} c_{ij}}{|E|}))$ equal to twice the average length of an edge (i, j) in the set of edges E of the graph $G = (V, E)$.

In addition, it is possible that when a position is evaluated, the demand of the selected node and/or some of the following nodes already in the route becomes zero due to delayed visits. The humanitarian context of the problem does not allow leaving a node unattended, even if its demand is equal to zero. Because the solution must ensure fairness in the distribution of first aid. So, the nodes with zero demand are allowed in the initial solution.

4.5 Pre-computations

The time-dependent demands complicate the calculations when a sequence of nodes is inserted, removed or inverted by the local search. Silva et al. [187] show for the cumulative capacitated vehicle routing problem (CCVRP) that any move can be expressed by concatenations of node sequences. The operator \oplus represents these concatenations.

By pre-computing and updating some relevant information, we can evaluate concatenation and inversion operators in constant time. Therefore, any move using a fixed number of concatenation and reversing operators can be evaluated in constant time as well. Any sequence of nodes can be represented as $\sigma = \{\sigma(1), \sigma(2), \dots, \sigma(|\sigma|)\}$ where $\sigma(i)$ corresponds to a node in the position i of the sequence σ .

For any sequence of nodes σ , we can define:

- $T(\sigma) = \sum_{i=1}^{|\sigma|-1} c_{\sigma(i), \sigma(i+1)}$ the duration of the sequence σ .

- $\beta(\sigma) = \sum_{i=1}^{|\sigma|} b_{\sigma(i)}$, the total number of people who flee per time unit (total displacement rate) in sequence σ . If a node at position i has no more demand to satisfy due to its position, then $b_{\sigma(i)} = 0$.
- $D(\sigma) = a_{\sigma(1)} + \sum_{i=2}^{|\sigma|} d_{\sigma(i)}$ the total demand to satisfy in sequence σ . $a_{\sigma(1)}$ is the initial demand of node at position 1 and $d_{\sigma(i)}$ is the satisfied demand of each other node at position i in sequence σ , assuming that the sequence begins at node $\sigma(1)$ at time 0.
- $Y(\sigma) = \min(y_{\sigma(i)} \mid i = 1, \dots, |\sigma|)$ the maximum time slack that sequence σ can be delayed while guaranteeing positive demand for each node. $y_{\sigma(i)}$ is calculated as $y_{\sigma(i)} = \max(a_{\sigma(i)}/b_{\sigma(i)} - t_{\sigma(i)}, 0)$. $t_{\sigma(i)}$ is the arrival time at node $\sigma(i)$, assuming a departure at time 0 from $\sigma(1)$. This time slack is checked and it helps to define the feasibility of the moves.

As the problem is highly asymmetrical, the values $\beta(\overleftarrow{\sigma})$, $D(\overleftarrow{\sigma})$, and $Y(\overleftarrow{\sigma})$ must be also pre-computed for the reversal $\overleftarrow{\sigma}$ of each sequence σ .

Let $\sigma = (E, C, D)$ be a sequence of nodes of the route $(0, E, C, D, 0)$ (Figure 4.1). Then, $\sigma(1) = E$, $\sigma(2) = C$, and $\sigma(3) = D$. The respective information of σ are $T(\sigma) = 4 + 1 = 5$, $\beta(\sigma) = 2 + 3 + 3 = 8$, $D(\sigma) = 100 + (88 + 285) = 473$ and $Y(\sigma) = \min(y_E, y_C, y_D)$, where $y_E = \max((100/2) - 0, 0) = 50$, $y_C = \max((100/3) - 4, 0) = 29.33$ and $y_D = \max((300/3) - 5, 0) = 95$. So, $Y(\sigma) = \min(50, 29.33, 95) = 29.33$.

The information of $\overleftarrow{\sigma}$ are $\beta(\overleftarrow{\sigma}) = 3 + 3 + 2 = 8$, $D(\overleftarrow{\sigma}) = 300 + 97 + 90 = 487$, $Y(\overleftarrow{\sigma}) = \min(y_D, y_C, y_E)$, where $y_D = \max((300/3) - 0, 0) = 100$, $y_C = \max((100/3) - 1, 0) = 32.33$ and $y_E = \max((100/2) - 5, 0) = 45$. So, $Y(\overleftarrow{\sigma}) = \min(100, 32.33, 45) = 32.33$.

In the metaheuristic framework, this information is stored in matrices. Each matrix corresponds to the information of T , β , D , and Y . The matrices are initialized with the *precomputations* procedure (*line 5* - Algorithm 4.4). This procedure contains a main loop which scans each route and each node i in the route, while a nested loop inspects each node j from i onward in the same route. So, a node sequence $\sigma = \{i, \dots, j\}$ is constructed and the respective information is calculated. The information is stored in the position (i, j) of each matrix.

For $i = j$, we have $T(\sigma) = 0$, $\beta(\sigma) = \beta(\overleftarrow{\sigma}) = b_j$, $D(\sigma) = D(\overleftarrow{\sigma}) = a_j$ and $Y(\sigma) = Y(\overleftarrow{\sigma}) = a_j/b_j$. When j is incremented, the information for the resulting sequence $\sigma \oplus j$ is computed in $O(1)$ as follows:

$$T(\sigma \oplus j) = T(\sigma) + c_{\sigma(|\sigma|),j} \quad (4.2)$$

$$\beta(\sigma \oplus j) = \beta(\sigma) + b_j \quad (4.3)$$

$$D(\sigma \oplus j) = D(\sigma) + (a_j - b_j(T(\sigma) + c_{\sigma(|\sigma|),j})) \quad (4.4)$$

$$Y(\sigma \oplus j) = \min(Y(\sigma), \max((y_j - T(\sigma) - c_{\sigma(|\sigma|),j}), 0)) \quad (4.5)$$

where $y_j = \max(((a_j/b_j) - 0), 0)$. For the reverse sequence $(\overleftarrow{\sigma \oplus j})$, the node j is added at the beginning and σ is reversed ($\overleftarrow{\sigma}$). The reversal information is computed as:

$$\beta(\overleftarrow{\sigma \oplus j}) = \beta(j \oplus \overleftarrow{\sigma}) = b_j + \beta(\overleftarrow{\sigma}) \quad (4.6)$$

$$D(\overleftarrow{\sigma \oplus j}) = D(j \oplus \overleftarrow{\sigma}) = a_j + D(\overleftarrow{\sigma}) - \beta(\overleftarrow{\sigma})c_{j,\overleftarrow{\sigma}(1)} \quad (4.7)$$

$$Y(\overleftarrow{\sigma \oplus j}) = Y(j \oplus \overleftarrow{\sigma}) = \min(Y(\overleftarrow{\sigma}) - c_{j,\overleftarrow{\sigma}(1)}, \max(y_j, 0)) \quad (4.8)$$

Each time a solution is updated, the *updatePrecomputations* procedure must be executed. This is done after the perturbation of a solution (*line 12* - Algorithm 4.4) and during the local search (detailed in Section 4.6). It is similar to *precomputations*, but restricted to the node sequences that are affected in the modified solution.

Thanks to pre-computations, any two sequence of nodes $\sigma_1 = (\sigma_1(1), \dots, \sigma_1(v))$ and $\sigma_2 = (\sigma_2(1), \dots, \sigma_2(w))$ in a solution can be also concatenated and the information of new sequence $\sigma_1 \oplus \sigma_2 = (\sigma_1(1), \dots, \sigma_1(v), \sigma_2(1), \dots, \sigma_2(w))$ can be also derived in $O(1)$ as follows:

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + c_{\sigma_1(v),\sigma_2(1)} + T(\sigma_2) \quad (4.9)$$

$$\beta(\sigma_1 \oplus \sigma_2) = \beta(\sigma_1) + \beta(\sigma_2) \quad (4.10)$$

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + D(\sigma_2) - \beta(\sigma_2)(T(\sigma_1) + c_{\sigma_1(v),\sigma_2(1)}) \quad (4.11)$$

$$Y(\sigma_1 \oplus \sigma_2) = \min(Y(\sigma_1), \max(Y(\sigma_2) - T(\sigma_1) - c_{\sigma_1(v),\sigma_2(1)}, 0)) \quad (4.12)$$

Equation (4.9) is the duration of $\sigma_1 \oplus \sigma_2$, equal to the sum of the duration of σ_1 and σ_2 plus the distance from the last node in σ_1 to the first node in σ_2 . Equation (4.10) is the total displacement rate of $\sigma_1 \oplus \sigma_2$, equal to the sum of the total number of people who flee per time unit of σ_1 and σ_2 . Equation (4.11) is the total demand to satisfy in $\sigma_1 \oplus \sigma_2$, equal to the sum of the demands of σ_1 and σ_2 , minus the decrease in demand caused by the shift of the starting time in sequence σ_2 . Equation (4.12) is the maximum time that $\sigma_1 \oplus \sigma_2$ can be shifted ensuring that all nodes will have positive demand.

4.6 Local search

This procedure helps to find better solutions from an initial solution by exploring neighborhoods. According to Lenstra [120], one of the challenges is to find efficient neighborhoods that lead to high-quality local optima.

Time dependent behavior of the demand requires a careful mechanism to evaluate the objective function as well as to check the capacity and maximum tour length violations in constant time, during the local search. This mechanism is only possible with the help of some preliminary information, obtained as explained in the previous section. Therefore it is possible to execute the following moves in an efficient way: node relocation, swap 1-1, swap 2-2 and 2-opt. All these moves are applied to a single route (intra-route) and to a pair of routes (inter-routes) looking for the best improvement. The solutions that exceed the maximum tour length (T_{max}) are accepted, but penalized using the Equation 4.1.

Once a best move is executed, the solution and the pre-computed matrices must be updated using the *updatePrecomputations* procedure. Then the best move is determined for the new solution, and so on. The local search stops when the current solution can no longer be improved.

Usually, when an intra-route movement is applied to classical VRP, the verification of capacity feasibility is not necessary. In the CVRP-TDD, the demand at each node varies depending on its position in the route, therefore vehicle capacity must be checked even with the intra-route moves.

For sake of simplicity, the first and last node of a node sequence σ will be referred as $\sigma(f)$ and $\sigma(l)$ in the explanation of each move, respectively. In the case of $|\sigma| = 1$, only σ will be used.

4.6.1 Node relocation

It takes each node of a route and evaluates all possible positions to relocate it.

The inter-route move is presented in Figure 4.2. The node σ_2 that belongs to the route p is relocated between the node sequences σ_4 and σ_5 in the route p' . The concatenations $\sigma_1 \oplus \sigma_3$ and $\sigma_4 \oplus \sigma_2 \oplus \sigma_5$ must be evaluated.

The time duration, the total displacement rate, the total satisfied demand and the maximum shift time of the resulting concatenation of route i are shown in Equations (4.13) - (4.16), respectively. The Equations (4.17)- (4.20) correspond to the values of the resulting concatenation of route p' .

Route p:

$$T(\sigma_1 \oplus \sigma_3) = T(\sigma_1) + c_{\sigma_1(l),\sigma_3(f)} + T(\sigma_3) \quad (4.13)$$

$$\beta(\sigma_1 \oplus \sigma_3) = \beta(\sigma_1) + \beta(\sigma_3) \quad (4.14)$$

$$D(\sigma_1 \oplus \sigma_3) = D(\sigma_1) + D(\sigma_3) - \beta(\sigma_3)(T(\sigma_1) + c_{\sigma_1(l),\sigma_3(f)}) \quad (4.15)$$

$$Y(\sigma_1 \oplus \sigma_3) = \min(Y(\sigma_1), \max(Y(\sigma_3) - T(\sigma_1) - c_{\sigma_1(l),\sigma_3(f)}, 0)) \quad (4.16)$$

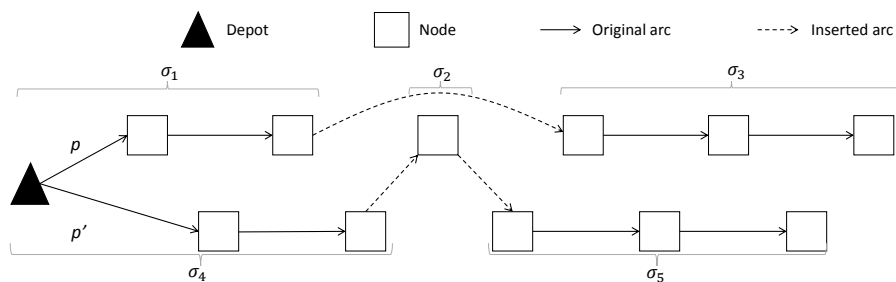


Figure 4.2 – Inter-route node relocation

Route p' :

$$T(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) = T(\sigma_4) + c_{\sigma_4(l),\sigma_2} + T(\sigma_2) + c_{\sigma_2,\sigma_5(f)} + T(\sigma_5) \quad (4.17)$$

$$\beta(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) = \beta(\sigma_4) + \beta(\sigma_2) + \beta(\sigma_5) \quad (4.18)$$

$$D(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) = D(\sigma_4 \oplus \sigma_2) + D(\sigma_5) - \beta(\sigma_5)(T(\sigma_4 \oplus \sigma_2) + c_{\sigma(\sigma_4 \oplus \sigma_2)(l),\sigma_5(f)}) \quad (4.19)$$

$$D(\sigma_4 \oplus \sigma_2) = D(\sigma_4) + D(\sigma_2) - \beta(\sigma_2)(T(\sigma_4) + c_{\sigma_4(l),\sigma_2})$$

$$Y(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) = \min(Y(\sigma_4 \oplus \sigma_2), \max(Y(\sigma_5) - T(\sigma_4 \oplus \sigma_2) - c_{\sigma(\sigma_4 \oplus \sigma_2)(l),\sigma_5(f)}, 0)) \quad (4.20)$$

$$Y(\sigma_4 \oplus \sigma_2) = \min(Y(\sigma_4), \max(Y(\sigma_2) - T(\sigma_4) - c_{\sigma_4(l),\sigma_2}, 0))$$

We can evaluate the change ΔZ in the objective function as follows:

$\Delta Z = D(\sigma_1 \oplus \sigma_3) + D(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) - D(\sigma_1 \oplus \sigma_2 \oplus \sigma_3) - D(\sigma_4 \oplus \sigma_5)$. If $\Delta Z > 0$, this move improves the objective value.

The move is feasible if:

- $D(\sigma_1 \oplus \sigma_3) \leq Q$ and $D(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) \leq Q$
- $T(\sigma_1 \oplus \sigma_3) \leq T_{max} - c_{\sigma_3(l),0}$ and $T(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) \leq T_{max} - c_{\sigma_5(l),0}$

However, the move is discarded if $Y(\sigma_1 \oplus \sigma_3)$ or $Y(\sigma_4 \oplus \sigma_2 \oplus \sigma_5)$ is equal to zero. This can be done because Y is the maximum time that the node sequence can be shifted before the demand of any node in the sequence becomes zero. Then, if $Y = 0$ there is at least one node with demand equal to zero.

4.6.2 Swap 1-1

In this move, two distinct nodes are exchanged. An example of the inter-route move is represented in Figure 4.3 where nodes σ_2 and σ_5 are exchanged. The node σ_2 is placed between the node sequences σ_4 and σ_6 . The node σ_5 is positioned at the position released by σ_2 . The evaluation of concatenations for the route p and route p' is computed with Equations (4.21) - (4.24) and (4.25)-(4.28), respectively.

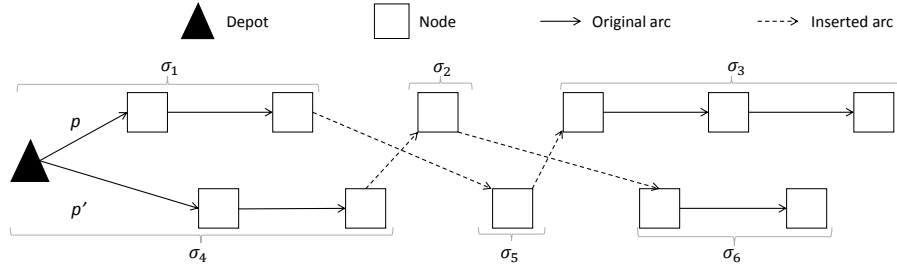


Figure 4.3 – Inter-route swap 1-1

Route p :

$$T(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = T(\sigma_1) + c_{\sigma_1(l), \sigma_5} + T(\sigma_5) + c_{\sigma_5, \sigma_3(f)} + T(\sigma_3) \quad (4.21)$$

$$\beta(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = \beta(\sigma_1) + \beta(\sigma_5) + \beta(\sigma_3) \quad (4.22)$$

$$D(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = D(\sigma_1 \oplus \sigma_5) + D(\sigma_3) - \beta(\sigma_3)(T(\sigma_1 \oplus \sigma_5) + c_{\sigma(\sigma_1 \oplus \sigma_5)(l), \sigma_3(f)}) \quad (4.23)$$

$$D(\sigma_1 \oplus \sigma_5) = D(\sigma_1) + D(\sigma_5) - \beta(\sigma_5)(T(\sigma_1) + c_{\sigma_1(l), \sigma_5})$$

$$Y(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = \min(Y(\sigma_1 \oplus \sigma_5), \max(Y(\sigma_3) - T(\sigma_1 \oplus \sigma_5) - c_{\sigma(\sigma_1 \oplus \sigma_5)(l), \sigma_3(f)}, 0)) \quad (4.24)$$

$$Y(\sigma_1 \oplus \sigma_5) = \min(Y(\sigma_1), \max(Y(\sigma_5) - T(\sigma_1) - c_{\sigma_1(l), \sigma_5}, 0))$$

Route p' :

$$T(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = T(\sigma_4) + c_{\sigma_4(l), \sigma_2} + T(\sigma_2) + c_{\sigma_2, \sigma_6(f)} + T(\sigma_6) \quad (4.25)$$

$$\beta(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = \beta(\sigma_4) + \beta(\sigma_2) + \beta(\sigma_6) \quad (4.26)$$

$$D(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = D(\sigma_4 \oplus \sigma_2) + D(\sigma_6) - \beta(\sigma_6)(T(\sigma_4 \oplus \sigma_2) + c_{\sigma(\sigma_4 \oplus \sigma_2)(l), \sigma_6(f)}) \quad (4.27)$$

$$D(\sigma_4 \oplus \sigma_2) = D(\sigma_4) + D(\sigma_2) - \beta(\sigma_2)(T(\sigma_4) + c_{\sigma_4(l), \sigma_2})$$

$$Y(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = \min(Y(\sigma_4 \oplus \sigma_2), \max(Y(\sigma_6) - T(\sigma_4 \oplus \sigma_2) - c_{\sigma(\sigma_4 \oplus \sigma_2)(l), \sigma_6(f)}, 0)) \quad (4.28)$$

$$Y(\sigma_4 \oplus \sigma_2) = \min(Y(\sigma_4), \max(Y(\sigma_2) - T(\sigma_4) - c_{\sigma_4(l), \sigma_2}, 0))$$

The change in the objective function ΔZ is computed as follows:

$$\Delta Z = D(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) + D(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) - D(\sigma_1 \oplus \sigma_2 \oplus \sigma_3) - D(\sigma_4 \oplus \sigma_5 \oplus \sigma_6).$$

If $\Delta Z > 0$, the objective value is improved. The feasibility of the resulting solution is checked with the following conditions:

- $D(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) \leq Q$ and $D(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) \leq Q$.
- $T(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) \leq T_{max} - c_{\sigma_3(l), 0}$, and $T(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) \leq T_{max} - c_{\sigma_6(l), 0}$

This move is considered, if $Y(\sigma_1 \oplus \sigma_5 \oplus \sigma_3)$ and $Y(\sigma_4 \oplus \sigma_2 \oplus \sigma_6)$ are greater than zero.

4.6.3 Swap 2-2

It is quite similar to the previous swap. Two distinct pairs of consecutive nodes are exchanged. An example of the inter-route move is presented in Figure 4.4. The time duration, the total number of people who flee per time unit, the total satisfied demand and the maximum shift time are evaluated in Equations (4.29)-(4.36), respectively.

In the figure, the node sequence σ_2 moves between the node sequences σ_4 and σ_6 and the node sequence σ_5 between the node sequences σ_1 and σ_3 .

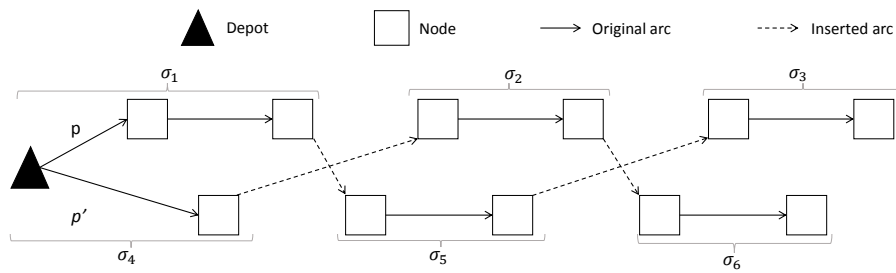


Figure 4.4 – Inter-route swap 2-2

Route p :

$$T(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = T(\sigma_1) + c_{\sigma_1(l),\sigma_5(f)} + T(\sigma_5) + c_{\sigma_5(l),\sigma_3(f)} + T(\sigma_3) \quad (4.29)$$

$$\beta(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = \beta(\sigma_1) + \beta(\sigma_5) + \beta(\sigma_3) \quad (4.30)$$

$$D(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = D(\sigma_1 \oplus \sigma_5) + D(\sigma_3) - \beta(\sigma_3)(T(\sigma_1 \oplus \sigma_5) + c_{\sigma_{(\sigma_1 \oplus \sigma_5)}(l),\sigma_3(f)}) \quad (4.31)$$

$$D(\sigma_1 \oplus \sigma_5) = D(\sigma_1) + D(\sigma_5) - \beta(\sigma_5)(T(\sigma_1) + c_{\sigma_1(l),\sigma_5(f)})$$

$$Y(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) = \min(Y(\sigma_1 \oplus \sigma_5), \max(Y(\sigma_3) - T(\sigma_1 \oplus \sigma_5) - c_{\sigma_{(\sigma_1 \oplus \sigma_5)}(l),\sigma_3(f)}, 0)) \quad (4.32)$$

$$Y(\sigma_1 \oplus \sigma_5) = \min(Y(\sigma_1), \max(Y(\sigma_5) - T(\sigma_1) - c_{\sigma_1(l),\sigma_5(f)}, 0))$$

Route p' :

$$T(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = T(\sigma_4) + c_{\sigma_4(l),\sigma_2(f)} + T(\sigma_2) + c_{\sigma_2(l),\sigma_6(f)} + T(\sigma_6) \quad (4.33)$$

$$\beta(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = \beta(\sigma_4) + \beta(\sigma_2) + \beta(\sigma_6) \quad (4.34)$$

$$D(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = D(\sigma_4 \oplus \sigma_2) + D(\sigma_6) - \beta(\sigma_6)(T(\sigma_4 \oplus \sigma_2) + c_{\sigma_{(\sigma_4 \oplus \sigma_2)}(l),\sigma_6(f)}) \quad (4.35)$$

$$D(\sigma_4 \oplus \sigma_2) = D(\sigma_4) + D(\sigma_2) - \beta(\sigma_2)(T(\sigma_4) + c_{\sigma_4(l),\sigma_2(f)})$$

$$Y(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) = \min(Y(\sigma_4 \oplus \sigma_2), \max(Y(\sigma_6) - T(\sigma_4 \oplus \sigma_2) - c_{\sigma_{(\sigma_4 \oplus \sigma_2)}(l),\sigma_6(f)}, 0)) \quad (4.36)$$

$$Y(\sigma_4 \oplus \sigma_2) = \min(Y(\sigma_4), \max(Y(\sigma_2) - T(\sigma_4) - c_{\sigma_4(l),\sigma_2(f)}, 0))$$

For this move, the change in the objective function is calculated as:

$$\Delta Z = D(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) + D(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) - D(\sigma_1 \oplus \sigma_2 \oplus \sigma_3) - D(\sigma_4 \oplus \sigma_5 \oplus \sigma_6)$$

The feasibility conditions are:

- $D(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) \leq Q$ and $D(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) \leq Q$.
- $T(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) \leq T_{max} - c_{\sigma_3(l),0}$, $T(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) \leq T_{max} - c_{\sigma_6(l),0}$.

The demand in all nodes must be greater than zero. It means that $Y(\sigma_1 \oplus \sigma_5 \oplus \sigma_3) > 0$ and $Y(\sigma_4 \oplus \sigma_2 \oplus \sigma_6) > 0$.

4.6.4 2-Opt

Two distinct and non-consecutive arcs are deleted and the route or routes are reconnected. Unlike the other neighborhoods, the concatenation changes considerably when an intra-route move is applied instead of an inter-routes move.

1. Intra-route move: Figure 4.5 shows this move where the arcs between node sequences σ_1 , σ_2 , and σ_2 , σ_3 are deleted. The node sequence σ_2 is reversed ($\overleftarrow{\sigma_2}$) and reconnected with node sequences σ_1 and σ_3 . The evaluation of this move is done using Equations (4.37)-(4.40).

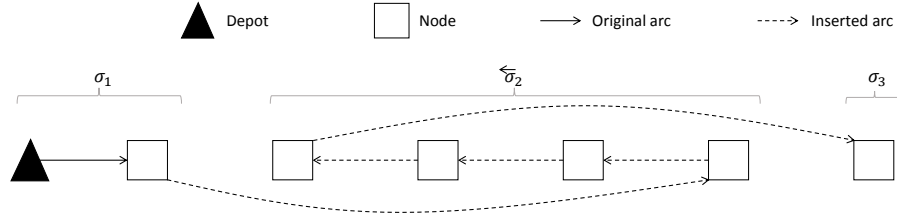


Figure 4.5 – Intra-route 2-Opt

$$T(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) = T(\sigma_1) + c_{\sigma_1(l),\overleftarrow{\sigma_2}(f)} + T(\sigma_3) + c_{\overleftarrow{\sigma_2}(l),\sigma_3} + T(\sigma_3) \quad (4.37)$$

$$\beta(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) = \beta(\sigma_1) + \beta(\overleftarrow{\sigma_2}) + \beta(\sigma_3) \quad (4.38)$$

$$D(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) = D(\sigma_1 \oplus \overleftarrow{\sigma_2}) + D(\sigma_3) - \beta(\sigma_3)(T(\sigma_1 \oplus \overleftarrow{\sigma_2}) + c_{\sigma_1 \oplus \overleftarrow{\sigma_2}(l),\sigma_3}) \quad (4.39)$$

$$D(\sigma_1 \oplus \overleftarrow{\sigma_2}) = D(\sigma_1) + D(\overleftarrow{\sigma_2}) - \beta(\overleftarrow{\sigma_2})(T(\sigma_1) + c_{\sigma_1(l),\overleftarrow{\sigma_2}(f)})$$

$$Y(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) = \min(Y(\sigma_1 \oplus \overleftarrow{\sigma_2}), \max(Y(\sigma_3) - T(\sigma_1 \oplus \overleftarrow{\sigma_2}) - c_{\sigma_1 \oplus \overleftarrow{\sigma_2}(l),\sigma_3}, 0))$$

$$Y(\sigma_1 \oplus \overleftarrow{\sigma_2}) = \min(Y(\sigma_1), \max(Y(\overleftarrow{\sigma_2}) - T(\sigma_1) - c_{\sigma_1(l),\overleftarrow{\sigma_2}(f)}, 0)) \quad (4.40)$$

The change in the objective function (ΔZ) using this move intra-route is calculated as

$$\Delta Z = D(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) - D(\sigma_1 \oplus \sigma_2 \oplus \sigma_3).$$

The move must guarantee that :

- $D(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) \leq Q$.
- $T(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) \leq T_{max} - c_{\sigma_3,0}$

This move can be considered if $Y(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) > 0$.

2. Inter-route move: Two different moves are possible: the inter-route 2-Opt and the inter-route 2-Opt*, shown in Figures 4.6 and 4.7.

The inter-route 2-Opt concatenates the node sequence σ_1 with σ_4 and node sequence σ_3 with σ_2 . Equations (4.41)-(4.48) show how to evaluate the resulting sequences.

The inter-route 2-Opt* corresponds to the concatenation of σ_1 with the reversed node sequence $\overleftarrow{\sigma_3}$ and the concatenation of $\overleftarrow{\sigma_4}$ with σ_2 . Equations (4.49)-(4.56) evaluate this move.

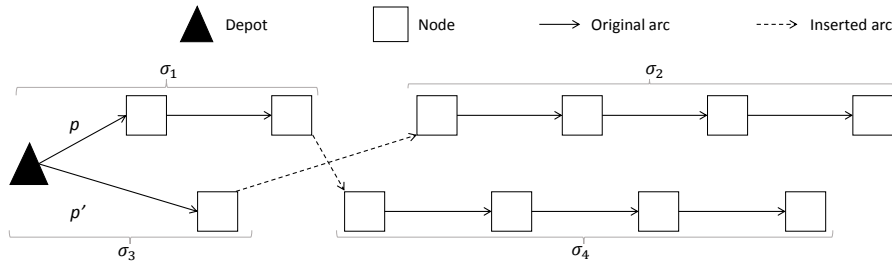


Figure 4.6 – Inter-route 2-Opt

Route p :

$$T(\sigma_1 \oplus \sigma_4) = T(\sigma_1) + c_{\sigma_1(l),\sigma_4(f)} + T(\sigma_4) \quad (4.41)$$

$$\beta(\sigma_1 \oplus \sigma_4) = \beta(\sigma_1) + \beta(\sigma_4) \quad (4.42)$$

$$D(\sigma_1 \oplus \sigma_4) = D(\sigma_1) + D(\sigma_4) - \beta(\sigma_4)(T(\sigma_1) + c_{\sigma_1(l),\sigma_4(f)}) \quad (4.43)$$

$$Y(\sigma_1 \oplus \sigma_4) = \min(Y(\sigma_1), \max(Y(\sigma_4) - T(\sigma_1) - c_{\sigma_1(l),\sigma_4(f)}, 0)) \quad (4.44)$$

Route p' :

$$T(\sigma_3 \oplus \sigma_2) = T(\sigma_3) + c_{\sigma_3(l),\sigma_2(f)} + T(\sigma_2) \quad (4.45)$$

$$\beta(\sigma_3 \oplus \sigma_2) = \beta(\sigma_3) + \beta(\sigma_2) \quad (4.46)$$

$$D(\sigma_3 \oplus \sigma_2) = D(\sigma_3) + D(\sigma_2) - \beta(\sigma_2)(T(\sigma_3) + c_{\sigma_3(l),\sigma_2(f)}) \quad (4.47)$$

$$Y(\sigma_3 \oplus \sigma_2) = \min(Y(\sigma_3), \max(Y(\sigma_2) - T(\sigma_3) - c_{\sigma_3(l), \sigma_2(f)}, 0)) \quad (4.48)$$

The evaluation of the change in the objective function is computed as follows:

$$\Delta Z = D(\sigma_1 \oplus \sigma_4) + D(\sigma_3 \oplus \sigma_2) - D(\sigma_1 \oplus \sigma_2) - D(\sigma_3 \oplus \sigma_4).$$

The feasibility conditions are:

- $D(\sigma_1 \oplus \sigma_4) \leq Q$ and $D(\sigma_3 \oplus \sigma_2) \leq Q$.
- $T(\sigma_1 \oplus \sigma_4) \leq T_{max} - c_{\sigma_4(l), 0}$ and $T(\sigma_3 \oplus \sigma_2) \leq T_{max} - c_{\sigma_2(l), 0}$.

The move must guarantee that the demand at all nodes is greater than zero, so $Y(\sigma_1 \oplus \sigma_4) > 0$ and $Y(\sigma_3 \oplus \sigma_2) > 0$.

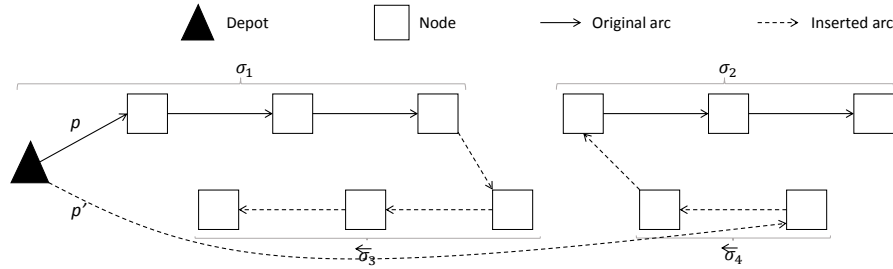


Figure 4.7 – Inter-route 2-Opt*

Route p :

$$T(\sigma_1 \oplus \overleftarrow{\sigma}_3) = T(\sigma_1) + c_{\sigma_1(l), \overleftarrow{\sigma}_3(f)} + T(\overleftarrow{\sigma}_3) \quad (4.49)$$

$$\beta(\sigma_1 \oplus \overleftarrow{\sigma}_3) = \beta(\sigma_1) + \beta(\overleftarrow{\sigma}_3) \quad (4.50)$$

$$D(\sigma_1 \oplus \overleftarrow{\sigma}_3) = D(\sigma_1) + D(\overleftarrow{\sigma}_3) - \beta(\overleftarrow{\sigma}_3)(T(\sigma_1) + c_{\sigma_1(l), \overleftarrow{\sigma}_3(f)}) \quad (4.51)$$

$$Y(\sigma_1 \oplus \overleftarrow{\sigma}_3) = \min(Y(\sigma_1), \max(Y(\overleftarrow{\sigma}_3) - T(\sigma_1) - c_{\sigma_1(l), \overleftarrow{\sigma}_3(f)}, 0)) \quad (4.52)$$

Route p' :

$$T(\overleftarrow{\sigma}_4 \oplus \sigma_2) = T(\overleftarrow{\sigma}_4) + c_{\overleftarrow{\sigma}_4(l), \sigma_2(f)} + T(\sigma_2) \quad (4.53)$$

$$\beta(\overleftarrow{\sigma}_4 \oplus \sigma_2) = \beta(\overleftarrow{\sigma}_4) + \beta(\sigma_2) \quad (4.54)$$

$$D(\overleftarrow{\sigma}_4 \oplus \sigma_2) = D(\overleftarrow{\sigma}_4) + D(\sigma_2) - \beta(\sigma_2)(T(\overleftarrow{\sigma}_4) + c_{\overleftarrow{\sigma}_4(l), \sigma_2(f)}) \quad (4.55)$$

$$Y(\overleftarrow{\sigma}_4 \oplus \sigma_2) = \min(Y(\overleftarrow{\sigma}_4), \max(Y(\sigma_2) - T(\overleftarrow{\sigma}_2) - c_{\overleftarrow{\sigma}_4(l), \sigma_2(f)}, 0)) \quad (4.56)$$

The change in the objective function is:

$$\Delta Z = D(\sigma_1 \oplus \overleftarrow{\sigma}_3) + D(\overleftarrow{\sigma}_4 \oplus \sigma_2) - D(\sigma_1 \oplus \sigma_2) - D(\sigma_3 \oplus \sigma_4).$$

The feasibility conditions are:

- $D(\sigma_1 \oplus \overleftarrow{\delta}_3) \leq Q$ and $D(\overleftarrow{\delta}_4 \oplus \sigma_2) \leq Q$.
- $T(\sigma_1 \oplus \overleftarrow{\delta}_3) \leq T_{max} - c_{\overleftarrow{\delta}_3(l),0}$ and $T(\overleftarrow{\delta}_4 \oplus \sigma_2) \leq T_{max} - c_{\sigma_2(l),0}$

The positive demand in all nodes is checked if $Y(\sigma_1 \oplus \overleftarrow{\delta}_3) > 0$ and $Y(\overleftarrow{\delta}_4 \oplus \sigma_2) > 0$.

4.7 Split procedure

This procedure is based on the second phase of the route-first cluster-second principle introduced by Beasley [30]. In the first phase, a giant tour visiting all nodes is built, by eliminating the time limit and capacity constraint. The second phase is to divide the giant tour considering all constraints to find a solution for the original problem. This is the role of the split procedure. It finds the optimal routing for the given sequence.

Prins [165] introduces a memetic algorithm (MA) which incorporates the split procedure to solve the DVRP where the number of vehicles is a decision variable and the objective is to minimize the total cost of the trips. He shows the flexibility of this procedure mentioning three extensions to other objective functions as the minimization of the total cost and the number of vehicles, the minimization of the operating cost plus the fleet cost considering an unlimited and limited fleet size. The last two extensions refer to the vehicle fleet mix problem (VFMP) and the heterogeneous fleet VRP (HFVRP), respectively. The extension to these problems is treated by Prins [167]. Duhamel et al. [65] solve these problems embedding the split strategies in a hybrid ELS.

Boudia et al. [35], El Fallahi et al. [69], Ngueveu et al. [146], Vidal et al. [213] and Cattaruzza et al. [41] also present MAs that include the split procedure as an optimal splitting procedure. Boudia et al. [35] solve the split delivery VRP (SDVRP) with the homogeneous fleet size as a decision variable. They look for the minimization of the total cost. El Fallahi et al. [69] solve the multi-compartment VRP (MC-VRP) and they consider the homogeneous fleet size and the objective function as in the work of Boudia et al. [35]. Cattaruzza et al. [41] and Vidal et al. [213] minimize the total travel time with a fixed fleet size. The first solve the multi-trip VRP (MTVRP) and the latter deals with the multi-depot and periodic VRP (MPVRP). Ngueveu et al. [146] minimize the sum of arrival times at nodes in the cumulative capacitated VRP (CCVRP).

These authors highlight the good performance of the split procedure embedded in a MA for their VRPs. Montoya et al. [138] and Lacomme et al. [114] underline the advantage of including this procedure in other metaheuristics. The first ones incorporate it in a hybrid metaheuristic which combines an ILS and a heuristic concentration to solve the electric vehicle routing problem with nonlinear charging function. While the latter merge it in a

multi-start path relinking algorithm for the multi-objective VRP with route balancing.

Lacomme et al. ([115], [116]) incorporate the split procedure in the genetic algorithm (GA) and in the MA for the capacitated arc routing problem (CARP). Lacomme et al. [117] also introduce it in the periodic CARP (PCARP). Prins et al. [168] describe how to improve the tour splitting algorithms for the CARP and the capacitated VRP (CVRP). They also present efficient implementations as well as extensions of the split procedure. They include the split procedure into a GRASP and ILS finding them competitive with published metaheuristics.

Additionally, the split procedure is used in problems which involves the VRP as a sub-problem of a whole problem. For example, Duhamel et al. [64] address the capacitated location-routing problem (CLRP). They propose a GRASP, calling an ELS where the split procedure, not only consider the total cost of routes, vehicle capacity and homogeneous fixed fleet size, but also the depot capacity and the cost of open depots.

Vidal [212] presents a technical note where the complexity of the procedure for the CVRP is reduced from $O(nB)$ to $O(n)$. The number of customers is given by n and B is the average number of feasible routes that start with a given customer in the giant tour.

The works cited above prove the good performance and the adaptability of the split procedure into different routing problems.

4.7.1 General structure

The split procedure starts by constructing an auxiliary directed acyclic graph $H = (V, A)$ where V is a set of numbered nodes from 0 (dummy node) to n . The set of arcs A contains all the feasible routes such than an arc (i, j) , $i < j$, represents a route sequence starting from depot, visiting $i + 1$ to j and coming back to the depot. The arc weight w_{ij} is the cost of the route in the real network. Bellman's algorithm is used to find the shortest path from 0 to n in H . This gives as result a set of feasible routes for the original problem and the optimal solution of the given giant tour.

It should be noted that the CVRP-TDD seeks to maximize the satisfied demand. Therefore, a maximum satisfied demand path from 0 to n in H with at most K arcs should be found.

A labeling algorithm is used to find the *longest* path from 0 to n (Algorithm 4.5). A *giantTour* and a *xTmax* parameter are inputs of the algorithm. *xTmax* is a percentage greater or equal to 1 that allows routes to exceed T_{max} until $(T_{max} \times xTmax)$ in H . When $xTmax > 1$, the tour length constraint is relaxed. At the beginning of the algorithm, $xTmax = 1$. Each node i in H corresponds to a node $v(i)$ in the position i of the giant tour.

Each node i in H contains a set of labels L_i . Each label l in L_i store the information of the path followed until node i . The label l is defined as $l = (Z, veh, path)$ where Z is the

Algorithm 4.5 *splitProcedure(giantTour, xTmax)*

```
1: giantTour = ( $v(0), v(1), v(2), \dots, v(n)$ )
2:  $L_0 \leftarrow (0, 0, 0, \{0\})$ 
3: for  $i = 1$  to  $n$  do  $L_i \leftarrow \emptyset$  end for
4: for  $i = 0$  to  $n - 1$  do
5:    $D_{ii} \leftarrow 0$ ;
6:    $j \leftarrow i + 1$ 
7:   repeat
8:     if ( $i + 1 = j$ ) then  $T_{ij} = c_{v(0), v(j)}$ 
9:     else  $T_{ij} = T_{i(j-1)} + c_{v(j-1), v(j)}$  end if
10:     $D_{ij} = D_{i(j-1)} + \max[(a_{v(j)} - (b_{v(j)} \cdot T_{ij})), 0]$ 
11:    if ( $(T_{ij} + c_{v(j), 0} \leq (T_{max} \cdot xTmax))$  and ( $D_{ij} \leq Q$ )) then
12:      for  $l \in L_i$  do
13:        if ( $(veh^l + 1 < K)$  or ( $veh^l + 1 = K$  and  $j = n$ )) then
14:          if  $L_j = \{\emptyset\}$  then
15:            extend( $l, D_{ij}, j$ )
16:          else
17:            oldDomNew  $\leftarrow false$ 
18:            for  $l' \in L_j$  do
19:              newDomOld = dominanceTest( $veh^l + 1, Z^l + D_{ij}, veh^{l'}, Z^{l'}$ )
20:              if newDomOld then
21:                delete( $l'$ )
22:              else
23:                oldDomNew = dominanceTest( $veh^{l'}, Z^{l'}, veh^l + 1, Z^l + D_{ij}$ )
24:                if oldDomNew then break end if
25:              end if
26:            end for
27:            if not oldDomNew then extend( $l, D_{ij}, j$ ) end if
28:          end if
29:        end if
30:      end for
31:    end if
32:     $j \leftarrow j + 1$ 
33:  until ( $j > n$ ) or ( $D_{ij} > Q$ ) or ( $T_{ij} + c_{v(j), 0} > (T_{max} \cdot xTmax)$ )
34: end for
```

total satisfied demand, veh is the total number of vehicles used, $path$ is the set of the visited nodes. The set of labels L_0 in node 0 is initialized setting all the values of a label to 0 and adding the node 0 to $path$. The rest of set of labels $L_i \forall i = 1, \dots, n$ are empty.

Each arc (i, j) in H ($i < j$) represents a route sequence for the CVRP-TDD starting from depot, visiting $v(i+1)$ to $v(j)$ and coming back to the depot. T_{ij} is the route length without considering the time of return from $v(j)$ to the depot and D_{ij} is the total satisfied demand at the visited nodes.

Iteratively, T_{ij} and D_{ij} are computed for each arc (i, j) in H until a maximum satisfied demand path from 0 to n in H is found, or the capacity or the tour length constraint is violated.

Each label l in L_i can be extended to a node j , if the arc (i, j) is feasible, and if the number of vehicles used (veh^l) plus 1 is:

- less than the available vehicles K ($veh^l + 1 < K$), or
- equal to K and the arc (i, j) goes until n ($j = n$).

These conditions ensure that only useful and feasible arcs are generated and the longest path to n contains at most K arcs (vehicles).

The extension of a label l in L_i in a node i to a node j produces a label l' in L_j with the following values:

$$Z^{l'} = Z^l + D_{ij} \tag{4.57}$$

$$veh^{l'} = veh^l + 1 \tag{4.58}$$

$$path^{l'} = path^l \cup \{j\} \tag{4.59}$$

The labeling algorithm enumerates all the possible paths from 0 to n in H . This is time and memory consuming, if all labels are generated and stored. The dominance rules are used to eliminate unnecessary labels keeping only the non-dominated ones. These labels guarantee better solutions than the dominated ones when they are extended along H .

We apply a dominance rule (*dominanceTest*) to check the dominance between two labels (solutions) and to verify if one of them can be discarded. Let l and l' be two distinct labels defined by $(Z^l, veh^l, path^l)$ and $(Z^{l'}, veh^{l'}, path^{l'})$, respectively. Both labels belongs to the same set of labels L_i in node i . Then, l dominates l' , if and only if conditions (4.60) and (4.61) are fulfilled.

$$Z^l \geq Z^{l'} \tag{4.60}$$

$$veh^l \leq veh^{l'} \quad (4.61)$$

The dominance rule means that if the label (solution) l which satisfies more demand with less vehicles than the label l' in the same set of labels L_i , then l dominates l' ($newDomOld = true$). If this happens, the label l' is eliminated. Otherwise, the comparison is also done inversely. If l is a non-dominated label in L_i ($oldDomNew = false$), then it is saved.

At the end of the algorithm, the non-dominated label l in L_n of the last node n with the highest value Z^l is the optimal solution to the longest path problem given the giant tour. The Z^l is the objective value of the solution for the CVRP-TDD and the routes can be extracted from $path^l$.

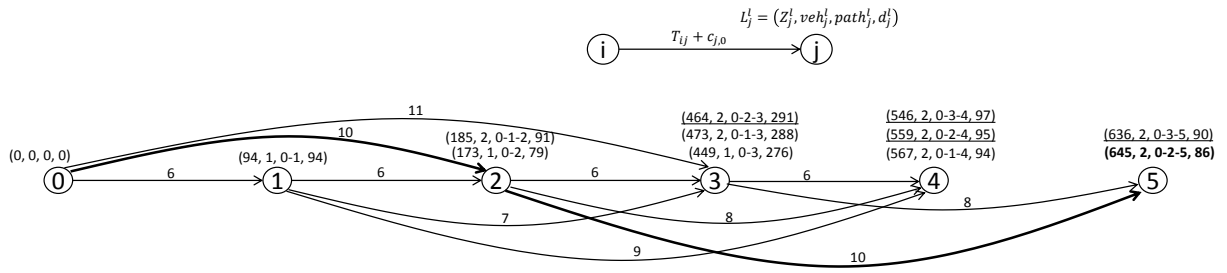
In the following we present an example to explain how the split procedure works in the CVRP-TDD. There are five shelters (A, B, C, D, E) to be attended by 2 vehicles. Each vehicle has a capacity of 500 aid packs and the tour length can not exceed 12 hours. The travel time (in hours) between all pair of shelters is shown in Table 4.2.

Table 4.2 – Travel times for the split example

| From \ To | depot | A | B | C | D | E |
|-----------|-------|---|---|---|---|---|
| depot | - | 3 | 3 | 3 | 3 | 3 |
| A | 3 | - | 2 | 4 | 2 | 6 |
| B | 3 | 2 | - | 2 | 3 | 5 |
| C | 3 | 4 | 2 | - | 1 | 4 |
| D | 3 | 2 | 3 | 1 | - | 3 |
| E | 3 | 6 | 5 | 4 | 3 | - |

Figure 4.1 shows a solution for the CVRP-TDD which is used in this example. Each arc (i, j) has the corresponding travel time c_{ij} and each shelter i has its initial population a_i and the displacement rate b_i . The routes are represented as two sequences of nodes $\sigma_1 = (0, E, C, D)$ and $\sigma_2 = (0, A, B)$. The route length for each route is $T(\sigma_1) + c_{D,0} = 11$ and $T(\sigma_2) + c_{B,0} = 8$. The objective value Z of this solution is $D(\sigma_1) + D(\sigma_2) = 449 + 187 = 636$. It means that 636 people are served at shelters.

The concatenation of these two routes considering that $seq = (1, 2)$ generates a *giantTour* = $(0, E, C, D, A, B)$. It is also shown in Figure 4.1. The *splitProcedure* is applied to this giant tour with a $xTmax = 1$. The auxiliary graph H , the feasible arcs (routes) and the labels L_j of each node j are presented in Figure 4.8. In the example, a variable $d_j^l = a_j - b_j \cdot T_{ij}$ is included to each label l in L_j . This variable is the number of people


 Figure 4.8 – Auxiliary graph H for split

satisfied at shelter j , when the arc (i, j) in H is considered. Then, $L_j^l = (Z_j^l, veh_j^l, path_j^l, d_j^l) \forall j \in L_j$.

H has 6 nodes numbered from 0 to 5. Each arc (i, j) represents a feasible route performed by a vehicle that leaves the depot, visits the nodes $v(i + 1), \dots, v(j)$ of the *giantTour*, and returns to the depot. The total time traveled by a vehicle is given in each arc (i, j) in H . Additionally, each node j in H has the set of labels L_j .

In the example, all the labels are presented, but the underlined labels are dominated ones. Then, they are discarded in the *splitProcedure*. For instance, the node 3 has three labels: $L_3^0 = (449, 1, 0-3, 276)$, $L_3^1 = (473, 2, 0-1-3, 288)$, and $L_3^2 = (464, 2, 0-2-3, 291)$. The label L_3^1 dominates L_3^2 because $veh_3^1 \leq veh_3^2$ and $Z_3^1 \geq Z_3^2$. The label L_3^2 must be eliminated. On the other hand, the arcs $(1,2)$, $(1,3)$, and $(1,4)$ should not be created due to the condition in line 13 of the Algorithm 4.5.

The longest path from 0 to 5 is in bold and represented by the non-dominated label ($L_5^0 = (645, 2, 0-2-5, 86)$) in the last node. This path is composed of arcs $(0,2)$ and $(2,5)$ ($path_5^0 = 0-2-5$). These two arcs correspond to two routes in the solution, depicted in Figure 4.9. The arc $(0,2)$ means that a vehicle leaves the depot, visits the shelters E and C , and returns to the depot. The route length is the 10 hours and 173 people are served. The arc $(2,5)$ refers to the route $(0, D, A, B, 0)$ performed by the second vehicle. It takes 8 hours for the trip and 472 people are served.

The optimal solution of the given *GT* is the *route 1* = $(0, E, C, 0)$ and *route 2* = $(0, D, A, B, 0)$ with a total satisfied demand of 645. This value (Z) is greater than the value of the solution before applying the split procedure. It shows that this procedure can improve the solutions.

It should be noted that if there is no feasible solution for the initial giant tour, a new *giantTour* can be generated concatenating in different ways the routes of the solution. After a number of iterations, the process is restarted and the tour length constraint is relaxed

increasing $xTmax$ by 0.1. If the latter occurs, the arcs with route length up to 13 hours ($12*1.1$) are allowed in H .

4.7.2 Split within the metaheuristic framework

The split procedure can be included into the general framework as follows. Each time that the loop (*lines* 10–16) of Algorithm 4.4 starts with a complete solution S , the *Route-first Cluster-second* method (Algorithm 4.6) is called. This method returns the best set of routes (*splitRoutes*) obtained after splitting. Then, $child \leftarrow splitRoutes$ and the Algorithm 4.4 continues in line 12. It should be noted that if the split procedure is embedded in the framework, a perturbation procedure (explained in Section 4.8) is applied to the giant tour.

The *Route-first Cluster-second* method begins by generating a giant tour (GT) from a solution S and a sequence seq (*line* 3). This sequence gives the order in which the routes belonging to solution S must be concatenated. The initial sequence is following the order of the routes from 1 until the last route ($nRoutes$) in the solution S . This concatenation changes the solution space from the complete solutions to the giant tours.

The main loop (*lines* 4–21) is executed until a feasible or relaxed solution for the CVRP-TDD is found ($splitF = true$). This loop starts constructing a new giant tour GT' using GT swapping $nodesToSwap$ nodes of it (*line* 5). This is the perturbation procedure mentioned above. Each time that GT is perturbed, the variable f increases by one (*line* 6). Then, the *splitProcedure* (Algorithm 4.5) is executed on GT' with a given $xTmax$ (*line* 7). If *splitProcedure* finds a *longest* path until the last node of the auxiliary graph H , the routes for the CVRP-TDD are extracted (*line* 9). Otherwise, if the maximum number of times ($maxNbOfPert$) that GT can be perturbed is achieved: (1) another seq is randomly generated, (2) a new GT from S is built, (3) $xTmax$ increases by 0.1, and (4) variables $nodesToSwap$, f and $xTmax$ are restarted (*lines* 11–14). Note that $xTmax$ is updated when it reaches a maximum value $maxInf$.

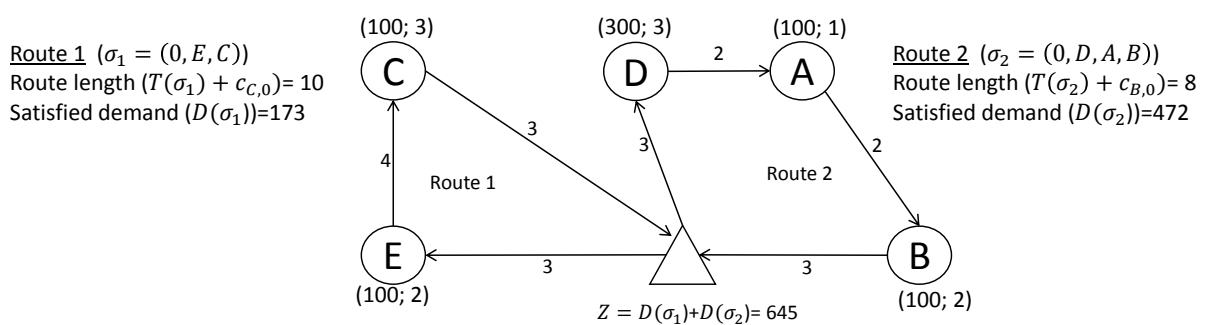


Figure 4.9 – Solution after applying the split procedure

Finally, the iteration terminates updating *nodesToSwap* (lines 16–20). If it is equal to a maximum value (*maxNodesToSwap*), it is restarted. Otherwise it is increased by one.

Algorithm 4.6 *Route-first Cluster-second*

```

1: splitF  $\leftarrow$  false; f  $\leftarrow$  0; nodesToSwap  $\leftarrow$  1; xTmax  $\leftarrow$  1;
2: seq = (1, ..., nRoutes)
3: GT  $\leftarrow$  giantTour(S, seq)
4: while (not splitF) do
5:   GT'  $\leftarrow$  perturb(GT, nodesToSwap)
6:   f  $\leftarrow$  f + 1
7:   splitF  $\leftarrow$  splitProcedure(GT', xTmax)
8:   if (splitF) then
9:     splitRoutes  $\leftarrow$  getRoutesSplit(GT')
10:  else if (f = maxNbOfPert) then
11:    changeSeq(seq)
12:    GT  $\leftarrow$  giantTour(S, seq)
13:    nodesToSwap  $\leftarrow$  0; f  $\leftarrow$  0; xTmax  $\leftarrow$  xTmax + 0.1;
14:    if (xTmax = maxInf) then xTmax  $\leftarrow$  1; end if
15:  end if
16:  if nodesToSwap = maxNodesToSwap then
17:    nodesToSwap  $\leftarrow$  1
18:  else
19:    nodesToSwap  $\leftarrow$  nodesToSwap + 1
20:  end if
21: end while
22: return splitRoutes

```

4.8 Perturbation

A perturbation is a diversification mechanism to escape from local optima and try to explore a different area of the solution space. The perturbation operator cannot be one of the moves used during the local search, otherwise it can be easily undone (repaired).

We propose two perturbation operators. If the split procedure is included into the framework, a perturbation for giant tours (line 5 - Algorithm 4.6) is applied. Otherwise, a perturbation of complete solutions (line 11 - Algorithm 4.4) is called.

The giant tours are perturbed by a dynamic swap move. Given a giant tour, two subsets

$S1$ and $S2$ with equal length ($|S1| = |S2|$) are chosen. Each subset contains distinct and consecutive nodes. The nodes in $S1$ are swapped with nodes in $S2$ generating a different giant tour. The size of subsets is given by *nodesToSwap* in Algorithm 4.6. This is increased by one, if a feasible or relaxed solution is not found by the split procedure.

The routes of a complete solution are perturbed by the node ejection chain procedure, introduced by Glover [83]. He proposes the ejection chain move for the Travelling Salesman Problem (TSP) and later Rego & Roucairol [171] implement a parallel Tabu Search using this move for the VRP.

Figure 4.10 shows the ejection-chain move in three routes of a solution. In each route, a random node is chosen and included into the next route. It is inserted in the best position, i.e. maximizing total satisfied demand. The node removed from the last route is inserted into the first one. A solution that exceeds the maximum tour length is allowed and penalized with Equation (4.1).

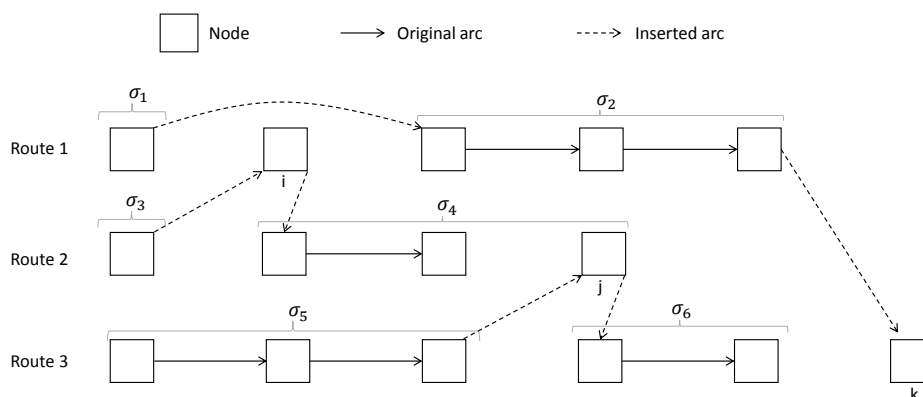


Figure 4.10 – Node-ejection Chain

4.9 Post-optimization procedure

A post-optimization phase is executed at the end of GRASP×ELS to solve a set partitioning model. The model is formulated as follows:

$$\max \sum_{p \in P} d_p \lambda_p \quad (4.62)$$

$$\sum_{p \in P} \gamma_p^i \lambda_p = 1 \quad \forall i \in V' \quad (4.63)$$

$$\sum_{p \in P} \lambda_p \leq K \quad (4.64)$$

$$\lambda_p \geq 0 \quad \forall p \in P \quad (4.65)$$

When a feasible solution is obtained at the end of the local search, each route p is transformed into a zero-one column λ_p and added to a columns pool P . $d_p = \sum_{i \in p} a_i - b_i \cdot t_i^p$ is the sum of the satisfied demand at each shelter i , when a vehicle arrives at time t_i^p in the route p . γ_p^i takes the value of 1 if path $p \in P$ visits node $i \in V'$. Constraints (4.63) impose that each node is visited once and constraint (4.64) guarantee that maximum K routes should be selected.

4.10 Computational experiments

The computational experiments evaluate and compare the performance of the GRASP \times ELS and the other four metaheuristics, with and without the split procedure, to solve the CVRP-TDD. All parameters used in the framework as well as the best combination to the constructive heuristic are presented in subsection 4.10.1. The detailed results of the metaheuristics with/without the split procedure on small-medium and large instances are detailed and analyzed in subsection 4.10.2.

The framework is coded in JAVA and runs on an Intel(R) Core(TM) i7-4800MQ 2.70GHz with 16 GB of RAM running Windows 7 Professional. The set partitioning problem explained in Section 4.9 is solved using CPLEX 12.6.3.

The metaheuristic framework is tested on small-medium (up to 50 shelters) and large (up to 199 shelters) instances. The first ones are the same as the ones used in Chapter 3 while the others are derived from the 7 CVRP instances of Christofides et al. from which the small instances are extracted, except the file known in the CVRP literature as CMT6. This instance belongs to the set of small-medium instances because its size is 50 customers.

The small-medium instances are grouped in two sets (I and C) and each set contains 42 instances numbered from 1 to 42. The number of shelters varies between 10 and 50 in these instances. For the large instances, two sets I and C are also created. The displacement rates at shelters as well as the other parameters are assigned in the same way as for the small-medium instances. They are numbered starting from 43 to 48. For the sake of simplicity, the instances will be referenced as $\{C/I\}$ -number.

4.10.1 Parameter tuning

A constant computing *budget* (number of calls to the local search) is fixed in order to compare the metaheuristics. The reason is that 90% of the running time at least is consumed by the local search. We fix this *budget* to 500. The metaheuristic to execute is configured using the parameters: *nbOfRestarts*, *nbOfIter* and *nbOfChildren*. This values

Table 4.3 – Parameters to select the metaheuristics to execute

| Parameter | <i>nbOfRestarts</i> | <i>nbOfIter</i> | <i>nbOfChildren</i> |
|------------------|---------------------|-----------------|---------------------|
| Metaheuristic | | | |
| GRASP | 500 | 1 | 1 |
| ILS | 1 | 500 | 1 |
| ELS | 1 | 50 | 10 |
| GRASP×ILS | 5 | 100 | 1 |
| GRASP×ELS | 5 | 20 | 5 |

are assigned knowing that $budget = nbOfRestarts \cdot nbOfIter \cdot nbOfChildren$. Table 4.3 shows the value of these parameters to run each metaheuristic.

4.10.1.1 Constructive heuristic

Different components are proposed to generate an initial solution for the CVRP-TDD as explained in Section 4.4. A GRASP with $nbOfRestarts = 10$ using each possible combination of these components is run on the 7 instances $I-6$, $I-43$, $I-44$, ..., $I-48$ to determine the best combination. These instances are the seven Christofides et al. instances for the CVRP, but adapted to the CVRP-TDD.

For each combination, two values are reported. The average level of infeasibility $\mu(S)$ of a solution S per GRASP iteration in all instances and the number of instances where a feasible solution is reported. $\mu(S)$ corresponds to the average overtime of the routes which is explained in Section 4.3. The best combination is the one that finds a feasible solution in more instances. If there is a tie, the one with the lowest $\mu(S)$ is selected.

Table 4.4 shows the results for different combinations of the components (List generation + Node selection + Insertion of node). The combinations which more instances solved as well as the lowest $\mu(S)$ are in bold.

It should be noted that the greedy heuristic cannot find a feasible solution on all instances. However, The combination that will be used in the metaheuristic framework consists in generating the list U with the priority rule (distance) sorted in decreasing order. The unassigned nodes are processed in the order of the list. The total travel time is the measure to determine the best position to insert it.

4.10.1.2 Split procedure

Before the execution of Algorithm 4.6, the values of the parameters ($maxInf$, $maxNodesToSwap$ and $maxNbOfPert$) must be determined. $maxInf$ is the maximum value that $xTmax$ takes allowing routes with an overtime up to $T_{max} \times maxInf$. $maxNodesToSwap$ is the maximum number of nodes that can be swapped in the giant

Table 4.4 – Combinations of the components for the constructive heuristic

| List generation | | Node selection | Insertion of node | Feasible sol. | $\mu(S)$ |
|-----------------------|------------------|----------------|-------------------|---------------|-------------|
| Priority (Distance) | Decreasing order | consecutive | time | 4 | 7.15 |
| | | | demand | 5 | 10.47 |
| | | | demand/time | 5 | 3.94 |
| | | roulette | time | 3 | 10.79 |
| | | | demand | 4 | 9.54 |
| | | | demand/time | 3 | 6.69 |
| | Increasing order | consecutive | time | 5 | 88.01 |
| | | | demand | 4 | 17.37 |
| | | | demand/time | 4 | 55.85 |
| | | roulette | time | 3 | 10.86 |
| | | | demand | 5 | 6.92 |
| | | | demand/time | 5 | 6.05 |
| Priority (Population) | Decreasing order | consecutive | time | 5 | 8.85 |
| | | | demand | 5 | 8.45 |
| | | | demand/time | 4 | 7.26 |
| | | roulette | time | 3 | 10.60 |
| | | | demand | 5 | 7.59 |
| | | | demand/time | 4 | 8.09 |
| | Increasing order | consecutive | time | 5 | 14.38 |
| | | | demand | 3 | 11.28 |
| | | | demand/time | 4 | 11.70 |
| | | roulette | time | 5 | 10.71 |
| | | | demand | 4 | 8.61 |
| | | | demand/time | 4 | 9.47 |
| Randomly | - | consecutive | time | 4 | 10.09 |
| | | | demand | 5 | 7.14 |
| | | | demand/time | 3 | 7.62 |

tour during the perturbation. It limits the value of *nodesToSwap*. *maxNbOfPert* is the maximum number of times that the perturbation is performed on the same giant tour before change it and before relax the tour length constraint.

We assign the following values to each parameter based on the previous analysis. The *maxInf* = 1.5, a route cannot exceed more than $1.5 \times T_{max}$. The *maxNodesToSwap* = 10, if the instance has more than 20 shelters. Otherwise, it is equal to 2. For parameter *maxNbOfPert*, we took $5 \times \text{maxNodesToSwap}$.

These values allow the split procedure to find feasible or relaxed solutions not too far from the feasible region in a short time.

4.10.2 Numerical results

This subsection is divided into two parts: the results on small-medium instances, and on the larger ones.

4.10.2.1 Small-medium instances

Tables (4.5)-(4.9) present the results obtained executing each metaheuristic with and without the split procedure on the small-medium instances in set I and C . GRASP is only run without the split procedure on both sets (Table 4.5) in small-medium instances.

All tables have the same following structure: the name of the instances, the number of nodes (n) and the number of available vehicles (K) are given in the first three columns. The lower bound ($LB_{B\&P}$) found by the B&P algorithm is presented in the next column. The optimal solutions are shown in bold. The results of each metaheuristic with and without the split procedure are detailed in the remaining columns.

Each metaheuristic with and without the split procedure is run 5 times. The maximum (Z_{max}), average (Z_{avg}) and minimum (Z_{min}) number of people attended at shelters and the average execution time (Time (s)) in seconds per run are reported. The percentage gap (Gap (%)) is computed as $\frac{LB_{B\&P} - Z_{max}}{LB_{B\&P}}$.

The optimal solutions obtained by each metaheuristic as well as the solutions that improve the LB are shown in bold.

ELS with split and GRASP×ELS without split find all the optimal solutions reported by the B&P in both sets. Moreover, they obtain the same or a better LB in the instances in which the optimal solution is unknown. They improve the LB in 8 out of 15 instances in set I and 9 out of 13 instances in set C . The average improvement is 0.15% and 0.10% in these sets respectively. In average, ELS with split is faster than GRASP×ELS without split (4.72 vs. 10.93 seconds in set I and 4.54 vs. 11 seconds in set C).

The average execution time is greater using the GRASP than running the other metaheuristics because in some instances the constructive heuristic spends more time at each restart to obtain a feasible or penalized solution. For the same reason, the average time of the GRASP hybridized with the ILS and ELS is also bigger.

The split procedure helps to reduce the average execution time of the metaheuristics, but in some cases it finds one optimum less and/or one improvement less.

Table 4.10 summarizes the number of optimal solutions (# of opt.) found, the improved solution (# of imp.) and the average execution time (Time (s)) by each metaheuristic with and without the split procedure in both sets.

The best metaheuristic to solve the small-medium size instances of the CVRP-TDD is the ELS with split because it finds the maximum number of optimal solutions and improves the LB for most instances in less time.

Table 4.5 – Results of GRASP without split on small-medium instances in set I and C

| Set I | | | | | | | | | | Set C | | | | | | | | | |
|---------------------|-----|-----|-------------|------------|---------------------|------------|----------|---------|----------|---------------------|-----|-------------|------------|------------|---------------------|----------|---------|------|---------|
| GRASP without split | | | | | GRASP without split | | | | | GRASP without split | | | | | GRASP without split | | | | |
| Instance | n | K | $LB_{B\&P}$ | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | Gap (%) | Instance | n | K | $LB_{B\&P}$ | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | Gap (%) | | |
| I-1 | 10 | 2 | 152.92 | 152.92 | 152.92 | 152.92 | 0.55 | 0.00 | C-1 | 10 | 2 | 144.81 | 144.81 | 144.81 | 144.81 | 0.72 | 0.00 | | |
| I-2 | 15 | 3 | 197.33 | 197.33 | 197.33 | 197.33 | 1.54 | 0.00 | C-2 | 15 | 3 | 178.58 | 178.58 | 178.58 | 178.58 | 1.60 | 0.00 | | |
| I-3 | 20 | 4 | 277.57 | 277.57 | 277.57 | 277.57 | 2.38 | 0.00 | C-3 | 20 | 4 | 258.19 | 258.19 | 258.19 | 258.19 | 1.99 | 0.00 | | |
| I-4 | 25 | 4 | 296.94 | 296.94 | 296.94 | 296.94 | 5.00 | 0.00 | C-4 | 25 | 4 | 274.01 | 274.01 | 274.01 | 274.01 | 5.37 | 0.00 | | |
| I-5 | 30 | 4 | 372.26 | 372.26 | 372.26 | 372.26 | 3830.19 | 0.00 | C-5 | 30 | 4 | 375.07 | 375.07 | 375.07 | 375.07 | 56.44 | 0.00 | | |
| I-6 | 50 | 6 | 609.79 | 611.16 | 611.06 | 610.92 | 5025.11 | -0.23 | C-6 | 50 | 6 | 627.05 | 627.03 | 627.02 | 626.98 | 5018.21 | 0.00 | | |
| I-7 | 10 | 3 | 140.65 | 140.65 | 140.65 | 140.65 | 0.65 | 0.00 | C-7 | 10 | 3 | 156.96 | 156.96 | 156.96 | 156.96 | 1.24 | 0.00 | | |
| I-8 | 15 | 3 | 241.76 | 241.76 | 241.73 | 241.72 | 1.75 | 0.00 | C-8 | 15 | 3 | 203.25 | 203.25 | 203.25 | 203.25 | 23.58 | 0.00 | | |
| I-9 | 20 | 4 | 305.85 | 305.85 | 305.85 | 305.85 | 2.58 | 0.00 | C-9 | 20 | 4 | 340.87 | 340.87 | 340.87 | 340.87 | 3.62 | 0.00 | | |
| I-10 | 25 | 5 | 370.43 | 370.43 | 370.43 | 370.43 | 3.48 | 0.00 | C-10 | 25 | 5 | 344.32 | 344.32 | 344.32 | 344.32 | 4.73 | 0.00 | | |
| I-11 | 30 | 5 | 437.61 | 437.34 | 437.34 | 437.34 | 2800.53 | 0.06 | C-11 | 30 | 5 | 405.17 | 405.17 | 405.17 | 405.17 | 4696.68 | 0.00 | | |
| I-12 | 50 | 8 | 690.31 | 691.86 | 691.86 | 691.86 | 5028.05 | -0.23 | C-12 | 50 | 8 | 718.28 | 718.28 | 718.24 | 718.07 | 2179.71 | 0.00 | | |
| I-13 | 10 | 2 | 101.42 | 101.42 | 101.42 | 101.42 | 0.45 | 0.00 | C-13 | 10 | 2 | 140.96 | 140.96 | 140.96 | 140.96 | 0.34 | 0.00 | | |
| I-14 | 15 | 3 | 194.19 | 194.19 | 194.19 | 194.19 | 0.95 | 0.00 | C-14 | 15 | 3 | 164.25 | 164.25 | 164.25 | 164.25 | 1.21 | 0.00 | | |
| I-15 | 20 | 4 | 266.66 | 266.66 | 266.66 | 266.66 | 1.14 | 0.00 | C-15 | 20 | 4 | 202.13 | 202.13 | 202.13 | 202.13 | 2.56 | 0.00 | | |
| I-16 | 25 | 4 | 347.13 | 347.13 | 347.13 | 347.13 | 4.04 | 0.00 | C-16 | 25 | 4 | 310.59 | 310.59 | 310.59 | 310.59 | 5.14 | 0.00 | | |
| I-17 | 30 | 4 | 331.61 | 331.61 | 331.61 | 331.60 | 5.09 | 0.00 | C-17 | 30 | 4 | 348.91 | 349.57 | 349.57 | 349.57 | 7.98 | -0.19 | | |
| I-18 | 50 | 6 | 510.94 | 512.12 | 512.12 | 512.12 | 45.21 | -0.23 | C-18 | 50 | 6 | 616.82 | 618.15 | 618.15 | 618.15 | 36.56 | -0.22 | | |
| I-19 | 10 | 2 | 106.23 | 106.23 | 106.23 | 106.23 | 0.49 | 0.00 | C-19 | 10 | 2 | 114.45 | 114.45 | 114.45 | 114.45 | 0.33 | 0.00 | | |
| I-20 | 15 | 3 | 163.19 | 163.19 | 163.19 | 163.19 | 0.71 | 0.00 | C-20 | 15 | 3 | 209.24 | 209.24 | 209.24 | 209.24 | 0.63 | 0.00 | | |
| I-21 | 20 | 4 | 221.11 | 221.11 | 221.11 | 221.11 | 1.74 | 0.00 | C-21 | 20 | 4 | 240.21 | 240.21 | 240.21 | 240.21 | 3.35 | 0.00 | | |
| I-22 | 25 | 4 | 317.44 | 317.44 | 317.44 | 317.44 | 4.05 | 0.00 | C-22 | 25 | 4 | 360.70 | 360.70 | 360.70 | 360.70 | 5.10 | 0.00 | | |
| I-23 | 30 | 4 | 358.87 | 358.87 | 358.87 | 358.87 | 78.79 | 0.00 | C-23 | 30 | 4 | 372.76 | 372.76 | 372.76 | 372.76 | 24.27 | 0.00 | | |
| I-24 | 50 | 6 | 583.08 | 584.90 | 584.90 | 584.90 | 2731.13 | -0.31 | C-24 | 50 | 6 | 580.34 | 581.09 | 581.09 | 581.09 | 5030.65 | -0.13 | | |
| I-25 | 10 | 2 | 117.34 | 117.34 | 117.34 | 117.34 | 0.48 | 0.00 | C-25 | 10 | 2 | 110.10 | 110.10 | 110.10 | 110.10 | 1.22 | 0.00 | | |
| I-26 | 15 | 3 | 193.84 | 193.84 | 193.84 | 193.84 | 1.34 | 0.00 | C-26 | 15 | 3 | 181.89 | 181.89 | 181.89 | 181.89 | 1.53 | 0.00 | | |
| I-27 | 20 | 4 | 295.00 | 295.00 | 295.00 | 295.00 | 3.39 | 0.00 | C-27 | 20 | 4 | 273.36 | 273.36 | 273.36 | 273.36 | 2.32 | 0.00 | | |
| I-28 | 25 | 4 | 321.52 | 321.52 | 321.52 | 321.52 | 4.60 | 0.00 | C-28 | 25 | 4 | 296.74 | 296.74 | 296.74 | 296.74 | 5.89 | 0.00 | | |
| I-29 | 30 | 4 | 370.83 | 371.09 | 371.08 | 371.04 | 67.43 | -0.07 | C-29 | 30 | 4 | 440.91 | 440.91 | 440.91 | 440.91 | 12.55 | 0.00 | | |
| I-30 | 50 | 6 | 707.20 | 707.08 | 707.07 | 707.05 | 4372.08 | 0.02 | C-30 | 50 | 6 | 603.77 | 605.14 | 603.85 | 602.66 | 5027.27 | -0.23 | | |
| I-31 | 10 | 2 | 98.69 | 98.69 | 98.69 | 98.69 | 0.56 | 0.00 | C-31 | 10 | 2 | 74.59 | 74.59 | 74.59 | 74.59 | 0.72 | 0.00 | | |
| I-32 | 15 | 3 | 125.28 | 125.28 | 125.28 | 125.28 | 1.07 | 0.00 | C-32 | 15 | 3 | 161.93 | 161.93 | 161.93 | 161.93 | 2.07 | 0.00 | | |
| I-33 | 20 | 4 | 204.78 | 204.78 | 204.78 | 204.78 | 3.02 | 0.00 | C-33 | 20 | 4 | 229.62 | 229.62 | 229.62 | 229.62 | 2.79 | 0.00 | | |
| I-34 | 25 | 4 | 242.33 | 242.41 | 242.41 | 242.41 | 4.68 | -0.03 | C-34 | 25 | 4 | 234.73 | 234.73 | 234.73 | 234.73 | 4.69 | 0.00 | | |
| I-35 | 30 | 4 | 304.85 | 304.85 | 304.85 | 304.85 | 7.83 | 0.00 | C-35 | 30 | 4 | 285.69 | 285.93 | 285.93 | 285.93 | 7.91 | -0.08 | | |
| I-36 | 50 | 6 | 459.55 | 460.05 | 460.04 | 460.03 | 34.39 | -0.11 | C-36 | 50 | 6 | 490.70 | 490.75 | 490.75 | 490.75 | 29.80 | -0.01 | | |
| I-37 | 10 | 2 | 169.96 | 169.96 | 169.96 | 169.96 | 0.82 | 0.00 | C-37 | 10 | 2 | 176.36 | 176.36 | 176.36 | 176.36 | 0.79 | 0.00 | | |
| I-38 | 15 | 3 | 208.15 | 208.15 | 208.15 | 208.15 | 1.87 | 0.00 | C-38 | 15 | 3 | 279.52 | 279.52 | 279.52 | 279.52 | 1.18 | 0.00 | | |
| I-39 | 20 | 4 | 380.33 | 380.33 | 380.33 | 380.33 | 3.40 | 0.00 | C-39 | 20 | 4 | 336.27 | 336.27 | 336.27 | 336.27 | 2.39 | 0.00 | | |
| I-40 | 25 | 4 | 430.33 | 430.33 | 430.33 | 430.33 | 4.57 | 0.00 | C-40 | 25 | 4 | 467.66 | 467.66 | 467.66 | 467.66 | 4.20 | 0.00 | | |
| I-41 | 30 | 4 | 433.59 | 433.59 | 433.59 | 433.59 | 8.37 | 0.00 | C-41 | 30 | 4 | 432.34 | 432.34 | 432.34 | 432.34 | 8.02 | -0.02 | | |
| I-42 | 50 | 6 | 771.48 | 771.56 | 771.56 | 771.56 | 30.74 | -0.01 | C-42 | 50 | 6 | 778.68 | 778.74 | 778.73 | 778.71 | 31.40 | -0.01 | | |
| | | | | | | | 0.45 | | | | | | | | | | | Min. | 0.33 |
| | | | | | | | 574.43 | | | | | | | | | | | Avg. | 529.97 |
| | | | | | | | 5028.05 | | | | | | | | | | | Max. | 5030.65 |

Table 4.6 – Detailed results of ILS and GRASP×ILS with/without split on small-medium instances in set I

| Instance | n | K | $L B_{GRSP}$ | Without split | | | | | | With split | | | | | | Without split | | | | | | With split | | | | | |
|----------|-----|-----|--------------|---------------|-----------|-----------|-----------|---------|-----------|---------------|-----------|----------|------------|-----------|-----------|---------------|----------|---------|------------|-----------|-----------|------------|---------|--------|--------|-------|--|
| | | | | ILS | | | GRASP×ILS | | | Without split | | | With split | | | Without split | | | With split | | | | | | | | |
| | | | | Z_{min} | Z_{avg} | Z_{min} | Time (s) | Gap (%) | Z_{min} | Z_{avg} | Z_{min} | Time (s) | Gap (%) | Z_{min} | Z_{avg} | Z_{min} | Time (s) | Gap (%) | Z_{min} | Z_{avg} | Z_{min} | Time (s) | Gap (%) | | | | |
| I-1 | 10 | 2 | 152.92 | 152.92 | 152.92 | 0.57 | 0.00 | 0.00 | 152.92 | 152.92 | 152.92 | 0.47 | 0.00 | 0.00 | 152.92 | 152.92 | 152.92 | 0.56 | 0.00 | 0.00 | 152.92 | 152.92 | 152.92 | 0.46 | 0.00 | 0.00 | |
| I-2 | 15 | 3 | 197.33 | 197.33 | 197.33 | 1.03 | 0.00 | 0.00 | 197.33 | 197.33 | 197.33 | 0.66 | 0.00 | 0.00 | 197.33 | 197.33 | 197.33 | 0.69 | 0.00 | 0.00 | 197.33 | 197.33 | 197.33 | 0.57 | 0.00 | 0.00 | |
| I-3 | 20 | 4 | 277.57 | 277.57 | 277.57 | 0.82 | 0.00 | 0.00 | 277.57 | 277.57 | 277.57 | 1.33 | 0.00 | 0.00 | 277.57 | 277.57 | 277.57 | 2.37 | 0.00 | 0.00 | 277.57 | 277.57 | 277.57 | 0.84 | 0.00 | 0.00 | |
| I-4 | 25 | 4 | 296.94 | 296.94 | 296.94 | 4.30 | 0.00 | 0.00 | 296.94 | 296.94 | 296.94 | 3.23 | 0.00 | 0.00 | 296.94 | 296.94 | 296.94 | 1.72 | 0.00 | 0.00 | 296.94 | 296.94 | 296.94 | 3.36 | 0.00 | 0.00 | |
| I-5 | 30 | 4 | 372.26 | 372.26 | 372.26 | 12.35 | 0.00 | 0.00 | 372.26 | 372.26 | 372.26 | 15.52 | 0.00 | 0.00 | 372.26 | 372.26 | 372.26 | 43.61 | 0.00 | 0.00 | 372.26 | 372.26 | 372.26 | 49.88 | 0.00 | 0.00 | |
| I-6 | 50 | 6 | 609.79 | 611.16 | 610.57 | 608.34 | 32.26 | -0.22 | 611.16 | 610.41 | 608.03 | 28.69 | -0.22 | 611.16 | 611.01 | 610.92 | 76.88 | -0.22 | 610.92 | 610.32 | 609.41 | 73.00 | -0.19 | 610.32 | 609.41 | 73.00 | |
| I-7 | 10 | 3 | 140.65 | 140.65 | 140.65 | 0.63 | 0.00 | 0.00 | 140.65 | 140.65 | 140.65 | 0.43 | 0.00 | 0.00 | 140.65 | 140.65 | 140.65 | 0.59 | 0.00 | 0.00 | 140.65 | 140.65 | 140.65 | 0.50 | 0.00 | 0.00 | |
| I-8 | 15 | 3 | 241.76 | 241.76 | 241.75 | 241.72 | 1.34 | 0.00 | 241.76 | 241.74 | 241.72 | 0.82 | 0.00 | 0.00 | 241.76 | 241.76 | 241.76 | 1.53 | 0.00 | 0.00 | 241.76 | 241.72 | 241.72 | 0.79 | 0.00 | 0.00 | |
| I-9 | 20 | 4 | 305.85 | 305.85 | 305.85 | 2.08 | 0.00 | 0.00 | 305.85 | 305.83 | 305.76 | 1.63 | 0.00 | 0.00 | 305.85 | 305.85 | 305.85 | 2.85 | 0.00 | 0.00 | 305.85 | 305.85 | 305.85 | 1.48 | 0.00 | 0.00 | |
| I-10 | 25 | 5 | 370.43 | 370.43 | 370.43 | 3.29 | 0.00 | 0.00 | 370.43 | 370.40 | 370.38 | 2.61 | 0.00 | 0.00 | 370.43 | 370.43 | 370.43 | 4.13 | 0.00 | 0.00 | 370.43 | 370.43 | 370.43 | 2.96 | 0.00 | 0.00 | |
| I-11 | 30 | 5 | 437.61 | 437.34 | 437.13 | 436.81 | 11.11 | 0.06 | 437.34 | 437.12 | 436.28 | 9.81 | 0.06 | 437.61 | 437.39 | 437.34 | 34.66 | 0.00 | 0.00 | 437.34 | 437.34 | 437.34 | 37.69 | 0.06 | 0.06 | | |
| I-12 | 50 | 8 | 690.31 | 691.86 | 686.37 | 682.90 | 31.14 | -0.22 | 691.86 | 686.77 | 682.01 | 31.58 | -0.22 | 691.86 | 688.82 | 683.98 | 73.91 | -0.22 | 691.86 | 686.53 | 682.17 | 73.40 | -0.22 | 686.53 | 682.17 | 73.40 | |
| I-13 | 10 | 2 | 101.42 | 101.42 | 101.42 | 0.40 | 0.00 | 0.00 | 101.42 | 101.42 | 101.42 | 0.53 | 0.00 | 0.00 | 101.42 | 101.42 | 101.42 | 0.58 | 0.00 | 0.00 | 101.42 | 101.42 | 101.42 | 0.51 | 0.00 | 0.00 | |
| I-14 | 15 | 3 | 194.19 | 194.19 | 194.19 | 0.51 | 0.00 | 0.00 | 194.19 | 194.19 | 194.19 | 0.95 | 0.00 | 0.00 | 194.19 | 194.19 | 194.19 | 0.74 | 0.00 | 0.00 | 194.19 | 194.19 | 194.19 | 0.70 | 0.00 | 0.00 | |
| I-15 | 20 | 4 | 266.66 | 266.66 | 266.66 | 0.77 | 0.00 | 0.00 | 266.66 | 266.66 | 266.66 | 1.32 | 0.00 | 0.00 | 266.66 | 266.66 | 266.66 | 2.02 | 0.00 | 0.00 | 266.66 | 266.66 | 266.66 | 1.80 | 0.00 | 0.00 | |
| I-16 | 25 | 4 | 347.13 | 347.13 | 347.13 | 3.13 | 0.00 | 0.00 | 347.13 | 347.13 | 347.13 | 2.69 | 0.00 | 0.00 | 347.13 | 347.13 | 347.13 | 2.05 | 0.00 | 0.00 | 347.13 | 347.13 | 347.13 | 2.76 | 0.00 | 0.00 | |
| I-17 | 30 | 4 | 331.61 | 331.61 | 331.61 | 331.60 | 6.44 | 0.00 | 331.61 | 331.61 | 331.60 | 4.78 | 0.00 | 0.00 | 331.61 | 331.61 | 331.60 | 6.19 | 0.00 | 0.00 | 331.61 | 331.61 | 331.60 | 5.02 | 0.00 | 0.00 | |
| I-18 | 50 | 6 | 510.94 | 512.12 | 512.01 | 511.58 | 19.13 | -0.23 | 512.12 | 511.99 | 511.45 | 14.70 | -0.23 | 512.12 | 512.09 | 512.04 | 22.32 | -0.23 | 512.12 | 511.98 | 511.45 | 16.88 | -0.23 | 511.98 | 511.45 | 16.88 | |
| I-19 | 10 | 2 | 106.23 | 106.23 | 106.23 | 0.55 | 0.00 | 0.00 | 106.23 | 106.23 | 106.23 | 0.62 | 0.00 | 0.00 | 106.23 | 106.23 | 106.23 | 0.56 | 0.00 | 0.00 | 106.23 | 106.23 | 106.23 | 0.56 | 0.00 | 0.00 | |
| I-20 | 15 | 3 | 163.19 | 163.19 | 163.19 | 163.19 | 0.90 | 0.00 | 163.19 | 163.19 | 163.19 | 1.00 | 0.00 | 0.00 | 163.19 | 163.19 | 163.19 | 0.61 | 0.00 | 0.00 | 163.19 | 163.19 | 163.19 | 0.96 | 0.00 | 0.00 | |
| I-21 | 20 | 4 | 221.11 | 221.11 | 221.11 | 221.11 | 1.73 | 0.00 | 221.11 | 221.11 | 221.11 | 1.83 | 0.00 | 0.00 | 221.11 | 221.11 | 221.11 | 0.92 | 0.00 | 0.00 | 221.11 | 221.11 | 221.11 | 1.18 | 0.00 | 0.00 | |
| I-22 | 25 | 4 | 317.44 | 317.44 | 317.44 | 317.44 | 3.49 | 0.00 | 317.44 | 317.44 | 317.44 | 1.89 | 0.00 | 0.00 | 317.44 | 317.44 | 317.44 | 3.68 | 0.00 | 0.00 | 317.44 | 317.44 | 317.44 | 3.40 | 0.00 | 0.00 | |
| I-23 | 30 | 4 | 358.87 | 358.87 | 358.87 | 358.87 | 5.64 | 0.00 | 358.87 | 358.45 | 357.82 | 2.72 | 0.00 | 0.00 | 358.87 | 358.87 | 358.87 | 5.12 | 0.00 | 0.00 | 358.87 | 358.87 | 358.87 | 5.75 | 0.00 | 0.00 | |
| I-24 | 50 | 6 | 584.90 | 584.87 | 584.84 | 584.84 | 16.41 | -0.31 | 584.90 | 584.74 | 584.51 | 19.55 | -0.31 | 584.90 | 584.90 | 584.90 | 43.89 | -0.31 | 584.90 | 584.89 | 584.87 | 44.95 | -0.31 | 584.89 | 584.87 | 44.95 | |
| I-25 | 10 | 2 | 117.34 | 117.34 | 117.34 | 117.34 | 0.20 | 0.00 | 117.34 | 117.34 | 117.34 | 0.62 | 0.00 | 0.00 | 117.34 | 117.34 | 117.34 | 0.42 | 0.00 | 0.00 | 117.34 | 117.34 | 117.34 | 0.48 | 0.00 | 0.00 | |
| I-26 | 15 | 3 | 193.84 | 193.84 | 193.84 | 193.84 | 0.39 | 0.00 | 193.84 | 193.84 | 193.84 | 0.65 | 0.00 | 0.00 | 193.84 | 193.84 | 193.84 | 0.66 | 0.00 | 0.00 | 193.84 | 193.84 | 193.84 | 0.87 | 0.00 | 0.00 | |
| I-27 | 20 | 4 | 295.00 | 295.00 | 295.00 | 295.00 | 1.99 | 0.00 | 295.00 | 295.00 | 295.00 | 0.59 | 0.00 | 0.00 | 295.00 | 295.00 | 295.00 | 2.33 | 0.00 | 0.00 | 295.00 | 295.00 | 295.00 | 0.78 | 0.00 | 0.00 | |
| I-28 | 25 | 4 | 321.52 | 321.52 | 321.52 | 321.52 | 3.46 | 0.00 | 321.52 | 321.52 | 321.52 | 2.48 | 0.00 | 0.00 | 321.52 | 321.52 | 321.52 | 2.59 | 0.00 | 0.00 | 321.52 | 321.52 | 321.52 | 2.81 | 0.00 | 0.00 | |
| I-29 | 30 | 4 | 371.09 | 371.09 | 371.04 | 371.04 | 5.88 | -0.07 | 371.09 | 371.04 | 371.04 | 4.73 | -0.06 | 371.09 | 371.07 | 371.04 | 7.11 | -0.07 | 371.09 | 371.05 | 371.04 | 6.80 | -0.07 | 371.05 | 371.04 | 6.80 | |
| I-30 | 50 | 6 | 707.20 | 707.20 | 704.65 | 704.01 | 23.16 | 0.00 | 707.20 | 705.61 | 704.01 | 25.45 | 0.00 | 0.00 | 707.20 | 706.32 | 704.01 | 63.45 | 0.00 | 0.00 | 706.99 | 705.59 | 704.30 | 66.40 | 0.03 | 0.03 | |
| I-31 | 10 | 2 | 98.69 | 98.69 | 98.69 | 98.69 | 0.42 | 0.00 | 98.69 | 98.69 | 98.69 | 0.58 | 0.00 | 0.00 | 98.69 | 98.69 | 98.69 | 0.60 | 0.00 | 0.00 | 98.69 | 98.69 | 98.69 | 0.64 | 0.00 | 0.00 | |
| I-32 | 15 | 3 | 125.28 | 125.28 | 125.28 | 125.28 | 0.60 | 0.00 | 125.28 | 125.28 | 125.28 | 0.98 | 0.00 | 0.00 | 125.28 | 125.28 | 125.28 | 1.36 | 0.00 | 0.00 | 125.28 | 125.28 | 125.28 | 0.59 | 0.00 | 0.00 | |
| I-33 | 20 | 4 | 204.78 | 204.78 | 204.78 | 204.78 | 0.68 | 0.00 | 204.78 | 204.78 | 204.78 | 1.57 | 0.00 | 0.00 | 204.78 | 204.78 | 204.78 | 2.30 | 0.00 | 0.00 | 204.78 | 204.78 | 204.78 | 0.67 | 0.00 | 0.00 | |
| I-34 | 25 | 4 | 242.39 | 242.41 | 242.39 | 242.39 | 3.37 | -0.03 | 242.41 | 242.41 | 242.41 | 2.39 | -0.03 | 242.41 | 242.41 | 242.41 | 3.34 | -0.03 | 242.41 | 242.41 | 242.41 | 2.94 | -0.03 | 242.41 | 242.41 | 2.94 | |
| I-35 | 30 | 4 | 304.85 | 304.85 | 304.85 | 304.85 | 4.66 | 0.00 | 304.85 | 304.85 | 304.85 | 2.91 | 0.00 | 0.00 | 304.85 | 304.85 | 304.85 | 5.05 | 0.00 | 0.00 | 304.85 | 304.85 | 304.85 | 3.91 | 0.00 | 0.00 | |
| I-36 | 50 | 6 | 459.55 | 460.05 | 459.98 | 459.89 | 15.89 | -0.11 | 460.05 | 460.05 | 460.05 | 11.11 | -0.11 | 460.05 | 459.99 | 459.79 | 16.11 | -0.11 | 460.05 | 459.99 | 459.78 | 13.49 | -0.11 | 459.99 | 459.78 | 13.49 | |
| I-37 | 10 | 2 | 169.96 | 169.96 | 169.96 | 169.96 | 0.31 | 0.00 | 169.96 | 169.96 | 169.96 | 0.53 | 0.00 | 0.00 | 169.96 | 169.96 | 169.96 | 0.58 | 0.00 | 0.00 | 169.96 | 169.96 | 169.96 | 0.48 | 0.00 | 0.00 | |
| I-38 | 15 | 3 | 208.15 | 208.15 | 208.15 | 208.15 | 0.43 | 0.00 | 208.15 | 208.15 | 208.15 | 0.67 | 0.00 | 0.00 | 208.15 | 208.15 | 208.15 | 0.89 | 0.00 | 0.00 | 208.15 | 208.15 | 208.15 | 0.97 | 0.00 | 0.00 | |
| I-39 | 20 | 4 | 380.33 | 380.33 | 380.33 | 380.33 | 0.73 | 0.00 | 380.33 | 380.32 | 380.29 | 1.17 | 0.00 | 0.00 | 380.33 | 380.33 | 380.33 | 2.11 | 0.00 | 0.00 | 380.33 | 380.33 | 380.33 | 0.83 | 0.00 | 0.00 | |
| I-40 | 25 | 4 | 430.33 | 430.33 | 430.33 | 430.33 | 3.63 | 0.00 | 430.33 | 430.33 | 430.33 | 1.72 | 0.00 | 0.00 | 430.33 | 430.33 | 430.33 | 3.32 | 0.00 | 0.00 | 430.33 | 430.33 | 430.33 | 0.94 | 0.00 | 0.00 | |
| I-41 | 30 | 4 | 433.59 | 433.59 | 433.59 | 433.59 | 4.54 | 0.00 | 433.59 | 433.59 | 433.59 | 3.12 | 0.00 | 0.00 | 433.59 | 433.58 | 433.57 | 5.37 | 0.00 | 0.00 | 433.59 | 433.58 | 433.57 | 2.10 | 0.00 | 0.00 | |
| I-42 | 50 | 6 | 771.48 | 771.56 | 771.56 | 771.56 | 16.00 | -0.01 | 771.56 | 771.56 | 771.56 | 7.99 | -0.01 | 771.56 | 771.56 | 771.56 | 17.50 | -0.01 | 771.56 | 771.46 | 771.08 | 12.20 | -0.01 | 771.46 | 771.08 | 12.20 | |

Table 4.7 – Detailed results of ELS and GRASP×ELS with/without split on small-medium instances in set I

| Instance | n | K | ELS | | | | | | | | | | | GRASP×ELS | | | | | | | | | | |
|----------|-----|-----|---------------|------------|------------|------------|-------------|------------|-------------|------------|------------|-------------|---------------|-------------|------------|------------|-------------|------------|-------------|------------|------------|-------------|---------|--|
| | | | Without split | | | | | With split | | | | | Without split | | | | | With split | | | | | | |
| | | | $LB_{B&P}$ | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | Gap (%) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | Gap (%) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | Gap (%) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | Gap (%) | |
| I-1 | 10 | 2 | 152.92 | 152.92 | 152.92 | 152.92 | 0.47 | 0.00 | 152.92 | 152.92 | 152.92 | 0.48 | 0.00 | 152.92 | 152.92 | 152.92 | 0.55 | 0.00 | 152.92 | 152.92 | 152.92 | 0.52 | 0.00 | |
| I-2 | 15 | 3 | 197.33 | 197.33 | 197.33 | 197.33 | 0.52 | 0.00 | 197.33 | 197.33 | 197.33 | 0.61 | 0.00 | 197.33 | 197.33 | 197.33 | 0.84 | 0.00 | 197.33 | 197.33 | 197.33 | 1.14 | 0.00 | |
| I-3 | 20 | 4 | 277.57 | 277.57 | 277.57 | 277.57 | 1.45 | 0.00 | 277.57 | 277.57 | 277.57 | 1.17 | 0.00 | 277.57 | 277.57 | 277.57 | 2.41 | 0.00 | 277.57 | 277.57 | 277.57 | 1.94 | 0.00 | |
| I-4 | 25 | 4 | 296.94 | 296.94 | 296.94 | 296.94 | 3.84 | 0.00 | 296.94 | 296.94 | 296.94 | 1.21 | 0.00 | 296.94 | 296.94 | 296.94 | 2.31 | 0.00 | 296.94 | 296.94 | 296.94 | 2.93 | 0.00 | |
| I-5 | 30 | 4 | 372.26 | 372.26 | 372.26 | 372.26 | 12.92 | 0.00 | 372.26 | 372.26 | 372.26 | 12.02 | 0.00 | 372.26 | 372.26 | 372.26 | 44.96 | 0.00 | 372.26 | 372.26 | 372.26 | 47.17 | 0.00 | |
| I-6 | 50 | 6 | 609.79 | 611.16 | 610.71 | 608.92 | 17.83 | -0.22 | 611.16 | 608.97 | 608.38 | 27.79 | -0.22 | 611.16 | 611.05 | 610.92 | 75.32 | -0.22 | 610.92 | 610.48 | 608.95 | 71.96 | -0.19 | |
| I-7 | 10 | 3 | 140.65 | 140.65 | 140.65 | 140.65 | 0.27 | 0.00 | 140.65 | 140.65 | 140.65 | 0.46 | 0.00 | 140.65 | 140.65 | 140.65 | 0.52 | 0.00 | 140.65 | 140.65 | 140.65 | 0.53 | 0.00 | |
| I-8 | 15 | 3 | 241.76 | 241.76 | 241.76 | 241.76 | 0.60 | 0.00 | 241.76 | 241.76 | 241.76 | 0.61 | 0.00 | 241.76 | 241.76 | 241.76 | 1.48 | 0.00 | 241.76 | 241.76 | 241.76 | 0.98 | 0.00 | |
| I-9 | 20 | 4 | 305.85 | 305.85 | 305.85 | 305.85 | 0.93 | 0.00 | 305.85 | 305.85 | 305.85 | 0.61 | 0.00 | 305.85 | 305.85 | 305.85 | 2.36 | 0.00 | 305.85 | 305.85 | 305.85 | 1.82 | 0.00 | |
| I-10 | 25 | 5 | 370.43 | 370.43 | 370.43 | 370.43 | 3.72 | 0.00 | 370.43 | 370.41 | 370.38 | 1.00 | 0.00 | 370.43 | 370.43 | 370.43 | 4.23 | 0.00 | 370.43 | 370.43 | 370.43 | 3.26 | 0.00 | |
| I-11 | 30 | 5 | 437.61 | 437.61 | 437.42 | 437.34 | 10.15 | 0.00 | 437.61 | 436.88 | 434.76 | 10.73 | 0.00 | 437.61 | 437.53 | 437.34 | 33.30 | 0.00 | 437.34 | 437.34 | 437.34 | 43.35 | 0.06 | |
| I-12 | 50 | 8 | 690.31 | 691.86 | 685.93 | 681.67 | 30.97 | -0.22 | 691.86 | 687.34 | 684.36 | 29.35 | -0.22 | 691.86 | 691.77 | 691.41 | 72.92 | -0.22 | 683.40 | 682.19 | 681.10 | 74.74 | 1.00 | |
| I-13 | 10 | 2 | 101.42 | 101.42 | 101.42 | 101.42 | 0.24 | 0.00 | 101.42 | 101.42 | 101.42 | 0.55 | 0.00 | 101.42 | 101.42 | 101.42 | 0.51 | 0.00 | 101.42 | 101.42 | 101.42 | 0.61 | 0.00 | |
| I-14 | 15 | 3 | 194.19 | 194.19 | 194.19 | 194.19 | 0.55 | 0.00 | 194.19 | 194.19 | 194.19 | 0.93 | 0.00 | 194.19 | 194.19 | 194.19 | 0.97 | 0.00 | 194.19 | 194.19 | 194.19 | 0.69 | 0.00 | |
| I-15 | 20 | 4 | 266.66 | 266.66 | 266.66 | 266.66 | 1.07 | 0.00 | 266.66 | 266.66 | 266.66 | 0.68 | 0.00 | 266.66 | 266.66 | 266.66 | 2.18 | 0.00 | 266.66 | 266.66 | 266.66 | 0.73 | 0.00 | |
| I-16 | 25 | 4 | 347.13 | 347.13 | 347.13 | 347.13 | 3.15 | 0.00 | 347.13 | 347.13 | 347.13 | 2.60 | 0.00 | 347.13 | 347.13 | 347.13 | 2.34 | 0.00 | 347.13 | 347.13 | 347.13 | 1.64 | 0.00 | |
| I-17 | 30 | 4 | 331.61 | 331.61 | 331.61 | 331.61 | 4.89 | 0.00 | 331.61 | 331.61 | 331.60 | 4.39 | 0.00 | 331.61 | 331.61 | 331.60 | 6.49 | 0.00 | 331.61 | 331.61 | 331.60 | 5.72 | 0.00 | |
| I-18 | 50 | 6 | 510.94 | 512.12 | 512.12 | 512.12 | 22.62 | -0.23 | 512.12 | 511.88 | 511.45 | 13.74 | -0.23 | 512.12 | 512.12 | 512.12 | 17.98 | -0.23 | 512.12 | 511.98 | 511.45 | 18.62 | -0.23 | |
| I-19 | 10 | 2 | 106.23 | 106.23 | 106.23 | 106.23 | 0.69 | 0.00 | 106.23 | 106.23 | 106.23 | 0.19 | 0.00 | 106.23 | 106.23 | 106.23 | 0.52 | 0.00 | 106.23 | 106.23 | 106.23 | 0.34 | 0.00 | |
| I-20 | 15 | 3 | 163.19 | 163.19 | 163.19 | 163.19 | 0.65 | 0.00 | 163.19 | 163.19 | 163.19 | 0.40 | 0.00 | 163.19 | 163.19 | 163.19 | 1.05 | 0.00 | 163.19 | 163.19 | 163.19 | 1.10 | 0.00 | |
| I-21 | 20 | 4 | 221.11 | 221.11 | 221.11 | 221.11 | 1.62 | 0.00 | 221.11 | 221.11 | 221.11 | 0.98 | 0.00 | 221.11 | 221.11 | 221.11 | 2.05 | 0.00 | 221.11 | 221.11 | 221.11 | 2.12 | 0.00 | |
| I-22 | 25 | 4 | 317.44 | 317.44 | 317.44 | 317.44 | 1.66 | 0.00 | 317.44 | 317.44 | 317.44 | 3.34 | 0.00 | 317.44 | 317.44 | 317.44 | 3.52 | 0.00 | 317.44 | 317.44 | 317.44 | 3.03 | 0.00 | |
| I-23 | 30 | 4 | 358.87 | 358.87 | 358.87 | 358.87 | 3.48 | 0.00 | 358.87 | 358.87 | 358.87 | 3.48 | 0.00 | 358.87 | 358.87 | 358.87 | 6.25 | 0.00 | 358.87 | 358.87 | 358.87 | 5.57 | 0.00 | |
| I-24 | 50 | 6 | 583.08 | 584.90 | 584.90 | 584.90 | 22.08 | -0.31 | 584.90 | 584.89 | 584.88 | 16.02 | -0.31 | 584.90 | 584.88 | 584.84 | 40.07 | -0.31 | 584.90 | 584.90 | 584.90 | 51.62 | -0.31 | |
| I-25 | 10 | 2 | 117.34 | 117.34 | 117.34 | 117.34 | 0.28 | 0.00 | 117.34 | 117.34 | 117.34 | 0.51 | 0.00 | 117.34 | 117.34 | 117.34 | 0.51 | 0.00 | 117.34 | 117.34 | 117.34 | 0.63 | 0.00 | |
| I-26 | 15 | 3 | 193.84 | 193.84 | 193.84 | 193.84 | 1.24 | 0.00 | 193.84 | 193.84 | 193.84 | 0.59 | 0.00 | 193.84 | 193.84 | 193.84 | 1.23 | 0.00 | 193.84 | 193.84 | 193.84 | 0.61 | 0.00 | |
| I-27 | 20 | 4 | 295.00 | 295.00 | 295.00 | 295.00 | 2.59 | 0.00 | 295.00 | 295.00 | 295.00 | 0.64 | 0.00 | 295.00 | 295.00 | 295.00 | 2.09 | 0.00 | 295.00 | 295.00 | 295.00 | 1.82 | 0.00 | |
| I-28 | 25 | 4 | 321.52 | 321.52 | 321.52 | 321.52 | 3.66 | 0.00 | 321.52 | 321.52 | 321.52 | 0.93 | 0.00 | 321.52 | 321.52 | 321.52 | 3.31 | 0.00 | 321.52 | 321.52 | 321.52 | 3.07 | 0.00 | |
| I-29 | 30 | 4 | 370.83 | 371.09 | 371.07 | 371.04 | 4.29 | -0.07 | 371.09 | 371.05 | 371.04 | 4.22 | -0.07 | 371.09 | 371.09 | 371.09 | 6.54 | -0.07 | 371.09 | 371.05 | 371.04 | 6.03 | -0.07 | |
| I-30 | 50 | 6 | 707.20 | 704.01 | 704.01 | 704.01 | 26.45 | 0.45 | 707.20 | 706.88 | 705.63 | 22.40 | 0.00 | 707.20 | 707.12 | 707.05 | 65.07 | 0.00 | 707.20 | 704.94 | 702.70 | 61.59 | 0.00 | |
| I-31 | 10 | 2 | 98.69 | 98.69 | 98.69 | 98.69 | 0.27 | 0.00 | 98.69 | 98.69 | 98.69 | 0.59 | 0.00 | 98.69 | 98.69 | 98.69 | 0.44 | 0.00 | 98.69 | 98.69 | 98.69 | 0.49 | 0.00 | |
| I-32 | 15 | 3 | 125.28 | 125.28 | 125.28 | 125.28 | 0.54 | 0.00 | 125.28 | 125.28 | 125.28 | 0.71 | 0.00 | 125.28 | 125.28 | 125.28 | 0.71 | 0.00 | 125.28 | 125.28 | 125.28 | 0.56 | 0.00 | |
| I-33 | 20 | 4 | 204.78 | 204.78 | 204.78 | 204.78 | 2.11 | 0.00 | 204.78 | 204.78 | 204.78 | 0.63 | 0.00 | 204.78 | 204.78 | 204.78 | 1.83 | 0.00 | 204.78 | 204.78 | 204.78 | 0.67 | 0.00 | |
| I-34 | 25 | 4 | 242.33 | 242.41 | 242.41 | 242.41 | 2.66 | -0.03 | 242.41 | 242.41 | 242.41 | 0.76 | -0.03 | 242.41 | 242.41 | 242.41 | 3.23 | -0.03 | 242.41 | 242.41 | 242.41 | 0.83 | -0.03 | |
| I-35 | 30 | 4 | 304.85 | 304.85 | 304.85 | 304.85 | 4.20 | 0.00 | 304.85 | 304.85 | 304.85 | 2.38 | 0.00 | 304.85 | 304.85 | 304.85 | 3.95 | 0.00 | 304.85 | 304.85 | 304.85 | 3.47 | 0.00 | |
| I-36 | 50 | 6 | 459.55 | 460.05 | 459.98 | 459.89 | 15.78 | -0.11 | 460.05 | 459.93 | 459.79 | 12.21 | -0.11 | 460.05 | 460.05 | 460.05 | 17.23 | -0.11 | 460.05 | 459.99 | 459.78 | 13.41 | -0.11 | |
| I-37 | 10 | 2 | 169.96 | 169.96 | 169.96 | 169.96 | 0.23 | 0.00 | 169.96 | 169.96 | 169.96 | 0.49 | 0.00 | 169.96 | 169.96 | 169.96 | 0.48 | 0.00 | 169.96 | 169.96 | 169.96 | 0.50 | 0.00 | |
| I-38 | 15 | 3 | 208.15 | 208.15 | 208.15 | 208.15 | 0.46 | 0.00 | 208.15 | 208.15 | 208.15 | 0.61 | 0.00 | 208.15 | 208.15 | 208.15 | 0.77 | 0.00 | 208.15 | 208.15 | 208.15 | 0.56 | 0.00 | |
| I-39 | 20 | 4 | 380.33 | 380.33 | 380.33 | 380.33 | 0.91 | 0.00 | 380.33 | 380.32 | 380.29 | 1.17 | 0.00 | 380.33 | 380.33 | 380.33 | 1.96 | 0.00 | 380.33 | 380.33 | 380.33 | 0.63 | 0.00 | |
| I-40 | 25 | 4 | 430.33 | 430.33 | 430.33 | 430.33 | 2.98 | 0.00 | 430.33 | 430.33 | 430.33 | 1.96 | 0.00 | 430.33 | 430.33 | 430.33 | 3.06 | 0.00 | 430.33 | 430.33 | 430.33 | 1.89 | 0.00 | |
| I-41 | 30 | 4 | 433.59 | 433.59 | 433.59 | 433.59 | 2.34 | 0.00 | 433.59 | 433.59 | 433.59 | 3.30 | 0.00 | 433.59 | 433.59 | 433.59 | 4.77 | 0.00 | 433.59 | 433.58 | 433.57 | 3.79 | 0.00 | |
| I-42 | 50 | 6 | 771.48 | 771.56 | 771.56 | 771.56 | 12.72 | -0.01 | 771.56 | 771.56 | 771.56 | 9.67 | -0.01 | 771.56 | 771.56 | 771.56 | 16.90 | -0.01 | 771.56 | 771.46 | 771.08 | 11.47 | -0.01 | |
| | | | | | | | Min. | | Min. | | | Min. | | Min. | | | Min. | | Min. | | | Min. | | |
| | | | | | | | Avg. | | Avg. | | | Avg. | | Avg. | | | Avg. | | Avg. | | | Avg. | | |
| | | | | | | | Max. | | Max. | | | Max. | | Max. | | | Max. | | Max. | | | Max. | | |

Table 4.9 – Detailed results of ELS and GRASP×ELS with/without split on small-medium instances in set C

| Instance | n | K | ELS | | | | | | | | | GRASP×ELS | | | | | | | | | | |
|----------|----|---|--------------------|-------------------|-------------------|-------------------|----------|---------|-------------------|-------------------|-------------------|------------|---------|-------------------|-------------------|-------------------|----------|------------|-------------------|-------------------|-------------------|----------|
| | | | Without split | | | With split | | | Without split | | | With split | | | Without split | | | With split | | | | |
| | | | $L B_{\text{B&P}}$ | $Z_{\text{max.}}$ | $Z_{\text{avg.}}$ | $Z_{\text{min.}}$ | Time (s) | Gap (%) | $Z_{\text{max.}}$ | $Z_{\text{avg.}}$ | $Z_{\text{min.}}$ | Time (s) | Gap (%) | $Z_{\text{max.}}$ | $Z_{\text{avg.}}$ | $Z_{\text{min.}}$ | Time (s) | Gap (%) | $Z_{\text{max.}}$ | $Z_{\text{avg.}}$ | $Z_{\text{min.}}$ | Time (s) |
| C-1 | 10 | 2 | 144.81 | 144.81 | 144.81 | 144.81 | 144.36 | 0.43 | 0.00 | 144.81 | 144.81 | 144.81 | 144.81 | 0.49 | 0.00 | 144.81 | 144.80 | 144.76 | 0.53 | 0.00 | | |
| C-2 | 15 | 3 | 178.58 | 178.58 | 178.58 | 178.58 | 178.58 | 1.55 | 0.00 | 178.58 | 178.58 | 178.58 | 178.58 | 1.05 | 0.00 | 178.58 | 178.58 | 178.58 | 0.69 | 0.00 | | |
| C-3 | 20 | 4 | 258.19 | 258.19 | 258.19 | 258.19 | 258.19 | 1.03 | 0.00 | 258.19 | 258.19 | 258.19 | 258.19 | 2.40 | 0.00 | 258.19 | 258.19 | 258.19 | 1.57 | 0.00 | | |
| C-4 | 25 | 4 | 274.01 | 274.01 | 274.01 | 274.01 | 274.01 | 3.38 | 0.00 | 274.01 | 274.01 | 274.01 | 274.01 | 3.62 | 0.00 | 274.01 | 274.01 | 274.01 | 3.11 | 0.00 | | |
| C-5 | 30 | 4 | 375.07 | 375.07 | 375.07 | 375.07 | 375.07 | 5.62 | 0.00 | 375.07 | 375.07 | 375.07 | 375.07 | 6.70 | 0.00 | 375.07 | 375.06 | 374.99 | 6.53 | 0.00 | | |
| C-6 | 50 | 6 | 627.05 | 627.34 | 626.26 | 624.92 | 624.73 | 30.76 | -0.05 | 627.34 | 626.63 | 624.73 | 627.09 | 75.03 | -0.05 | 627.03 | 626.39 | 625.07 | 73.23 | 0.00 | | |
| C-7 | 10 | 3 | 156.96 | 156.96 | 156.96 | 156.96 | 156.96 | 0.31 | 0.00 | 156.96 | 156.96 | 156.96 | 156.96 | 0.54 | 0.00 | 156.96 | 156.96 | 156.96 | 0.48 | 0.00 | | |
| C-8 | 15 | 3 | 203.25 | 203.25 | 203.25 | 203.25 | 203.25 | 0.74 | 0.00 | 203.25 | 203.25 | 203.25 | 203.25 | 1.72 | 0.00 | 203.25 | 203.25 | 203.25 | 0.70 | 0.00 | | |
| C-9 | 20 | 4 | 340.87 | 340.87 | 340.87 | 340.87 | 340.87 | 1.04 | 0.00 | 340.87 | 340.87 | 340.87 | 340.87 | 2.30 | 0.00 | 340.87 | 340.87 | 340.87 | 0.66 | 0.00 | | |
| C-10 | 25 | 5 | 344.32 | 344.32 | 344.32 | 344.32 | 344.32 | 2.50 | 0.00 | 344.32 | 343.96 | 343.10 | 344.32 | 4.27 | 0.00 | 344.32 | 344.24 | 344.13 | 2.58 | 0.00 | | |
| C-11 | 30 | 5 | 405.17 | 405.17 | 404.49 | 401.76 | 401.76 | 13.99 | 0.00 | 405.17 | 404.18 | 401.76 | 405.17 | 50.28 | 0.00 | 405.17 | 403.65 | 402.13 | 55.70 | 0.00 | | |
| C-12 | 50 | 8 | 718.28 | 718.28 | 718.28 | 718.28 | 24.55 | 0.00 | 718.28 | 717.84 | 716.47 | 718.28 | 40.79 | 0.00 | 718.28 | 717.67 | 716.14 | 36.73 | 0.00 | | | |
| C-13 | 10 | 2 | 140.96 | 140.96 | 140.96 | 140.96 | 140.96 | 0.29 | 0.00 | 140.96 | 140.96 | 140.96 | 140.96 | 0.52 | 0.00 | 140.96 | 140.96 | 140.96 | 0.48 | 0.00 | | |
| C-14 | 15 | 3 | 164.25 | 164.25 | 164.25 | 164.25 | 164.25 | 0.40 | 0.00 | 164.25 | 164.25 | 164.25 | 164.25 | 0.42 | 0.00 | 164.25 | 164.25 | 164.25 | 0.56 | 0.00 | | |
| C-15 | 20 | 4 | 202.13 | 202.13 | 202.13 | 202.13 | 202.13 | 0.85 | 0.00 | 202.13 | 202.13 | 202.13 | 202.13 | 1.97 | 0.00 | 202.13 | 202.13 | 202.13 | 0.60 | 0.00 | | |
| C-16 | 25 | 4 | 310.59 | 310.59 | 310.59 | 310.59 | 310.59 | 3.08 | 0.00 | 310.59 | 310.59 | 310.59 | 310.59 | 3.29 | 0.00 | 310.59 | 310.59 | 310.59 | 2.69 | 0.00 | | |
| C-17 | 30 | 4 | 348.91 | 349.57 | 349.57 | 349.57 | 349.57 | 5.92 | -0.19 | 349.57 | 349.17 | 348.68 | 349.57 | 5.86 | -0.19 | 349.57 | 349.57 | 349.57 | 5.23 | -0.19 | | |
| C-18 | 50 | 6 | 616.82 | 618.15 | 618.15 | 618.15 | 18.05 | 0.00 | 618.15 | 618.15 | 618.15 | 618.15 | 20.37 | -0.22 | 618.15 | 618.15 | 618.15 | 12.95 | -0.22 | | | |
| C-19 | 10 | 2 | 114.45 | 114.45 | 114.45 | 114.45 | 114.45 | 0.25 | 0.00 | 114.45 | 114.45 | 114.45 | 114.45 | 0.50 | 0.00 | 114.45 | 114.45 | 114.45 | 0.66 | 0.00 | | |
| C-20 | 15 | 3 | 209.24 | 209.24 | 209.24 | 209.24 | 209.24 | 1.09 | 0.00 | 209.24 | 209.24 | 209.24 | 209.24 | 0.88 | 0.00 | 209.24 | 209.24 | 209.24 | 0.61 | 0.00 | | |
| C-21 | 20 | 4 | 240.21 | 240.21 | 240.21 | 240.21 | 240.21 | 0.91 | 0.00 | 240.21 | 240.21 | 240.21 | 240.21 | 1.79 | 0.00 | 240.21 | 240.21 | 240.21 | 0.63 | 0.00 | | |
| C-22 | 25 | 4 | 360.70 | 360.70 | 360.57 | 360.06 | 360.06 | 4.44 | 0.00 | 360.70 | 360.31 | 360.06 | 360.70 | 4.91 | 0.00 | 360.70 | 360.60 | 360.24 | 3.51 | 0.00 | | |
| C-23 | 30 | 4 | 372.76 | 372.76 | 372.76 | 372.76 | 372.76 | 6.71 | 0.00 | 372.76 | 372.76 | 372.76 | 372.76 | 7.19 | 0.00 | 372.76 | 372.57 | 371.83 | 5.88 | 0.00 | | |
| C-24 | 50 | 6 | 580.34 | 581.09 | 581.09 | 581.09 | 31.04 | 0.00 | 581.09 | 580.81 | 579.93 | 581.09 | 74.62 | -0.13 | 581.09 | 581.08 | 581.05 | 73.44 | -0.13 | | | |
| C-25 | 10 | 2 | 110.10 | 110.10 | 108.86 | 106.99 | 106.99 | 0.42 | 0.00 | 110.10 | 110.10 | 110.10 | 110.10 | 0.53 | 0.00 | 110.10 | 110.10 | 110.10 | 0.55 | 0.00 | | |
| C-26 | 15 | 3 | 181.89 | 181.89 | 181.89 | 181.89 | 181.89 | 0.58 | 0.00 | 181.89 | 181.89 | 181.89 | 181.89 | 1.02 | 0.00 | 181.89 | 181.89 | 181.89 | 0.55 | 0.00 | | |
| C-27 | 20 | 4 | 273.36 | 273.36 | 273.36 | 273.36 | 273.36 | 1.07 | 0.00 | 273.36 | 273.36 | 273.36 | 273.36 | 1.99 | 0.00 | 273.36 | 273.36 | 273.36 | 0.62 | 0.00 | | |
| C-28 | 25 | 4 | 296.74 | 296.74 | 296.74 | 296.74 | 296.74 | 1.74 | 0.00 | 296.74 | 296.74 | 296.74 | 296.74 | 4.29 | 0.00 | 296.74 | 296.74 | 296.74 | 3.24 | 0.00 | | |
| C-29 | 30 | 4 | 440.91 | 440.91 | 440.91 | 440.91 | 440.91 | 4.82 | 0.00 | 440.91 | 440.91 | 440.91 | 440.91 | 7.46 | 0.00 | 440.91 | 440.91 | 440.91 | 5.15 | 0.00 | | |
| C-30 | 50 | 6 | 603.77 | 605.14 | 604.44 | 601.66 | 601.66 | 27.43 | -0.23 | 605.14 | 603.30 | 602.07 | 603.57 | 74.29 | -0.23 | 605.10 | 603.03 | 601.66 | 73.06 | -0.22 | | |
| C-31 | 10 | 2 | 74.59 | 74.59 | 74.59 | 74.59 | 74.59 | 0.26 | 0.00 | 74.59 | 74.59 | 74.59 | 74.59 | 0.46 | 0.00 | 74.59 | 74.59 | 74.59 | 0.55 | 0.00 | | |
| C-32 | 15 | 3 | 161.93 | 161.93 | 161.93 | 161.93 | 161.93 | 0.79 | 0.00 | 161.93 | 161.93 | 161.93 | 161.93 | 0.80 | 0.00 | 161.93 | 161.93 | 161.93 | 0.92 | 0.00 | | |
| C-33 | 20 | 4 | 229.62 | 229.62 | 229.62 | 229.62 | 229.62 | 0.91 | 0.00 | 229.62 | 229.62 | 229.62 | 229.62 | 2.02 | 0.00 | 229.62 | 229.62 | 229.62 | 0.71 | 0.00 | | |
| C-34 | 25 | 4 | 234.73 | 234.73 | 234.73 | 234.73 | 234.73 | 1.67 | 0.00 | 234.73 | 234.73 | 234.73 | 234.73 | 3.24 | 0.00 | 234.73 | 234.73 | 234.73 | 2.12 | 0.00 | | |
| C-35 | 30 | 4 | 285.69 | 285.93 | 285.93 | 285.93 | 285.93 | 2.07 | -0.08 | 285.93 | 285.93 | 285.93 | 285.93 | 5.17 | -0.08 | 285.93 | 285.93 | 285.93 | 4.12 | -0.08 | | |
| C-36 | 50 | 6 | 490.70 | 490.75 | 490.75 | 490.75 | 490.75 | 14.97 | -0.01 | 490.75 | 490.75 | 490.75 | 490.75 | 19.42 | -0.01 | 490.75 | 490.68 | 490.49 | 13.85 | -0.01 | | |
| C-37 | 10 | 2 | 176.36 | 176.36 | 176.36 | 176.36 | 176.36 | 0.26 | 0.00 | 176.36 | 176.36 | 176.36 | 176.36 | 0.50 | 0.00 | 176.36 | 176.36 | 176.36 | 0.54 | 0.00 | | |
| C-38 | 15 | 3 | 279.52 | 279.52 | 279.52 | 279.52 | 279.52 | 0.70 | 0.00 | 279.52 | 279.52 | 279.52 | 279.52 | 1.04 | 0.00 | 279.52 | 279.52 | 279.52 | 0.57 | 0.00 | | |
| C-39 | 20 | 4 | 336.27 | 336.27 | 336.27 | 336.27 | 336.27 | 0.88 | 0.00 | 336.27 | 336.27 | 336.27 | 336.27 | 1.89 | 0.00 | 336.27 | 336.27 | 336.27 | 1.14 | 0.00 | | |
| C-40 | 25 | 4 | 467.66 | 467.66 | 467.66 | 467.66 | 467.66 | 1.94 | 0.00 | 467.66 | 467.66 | 467.66 | 467.66 | 3.24 | 0.00 | 467.66 | 467.66 | 467.66 | 2.25 | 0.00 | | |
| C-41 | 30 | 4 | 432.27 | 432.34 | 432.34 | 432.34 | 432.34 | 3.41 | -0.02 | 432.34 | 432.34 | 432.34 | 432.34 | 4.81 | -0.02 | 432.34 | 432.34 | 432.34 | 3.43 | -0.02 | | |
| C-42 | 50 | 6 | 778.68 | 778.74 | 778.74 | 778.67 | 778.67 | 16.97 | -0.01 | 778.74 | 778.67 | 778.54 | 778.68 | 18.26 | -0.01 | 778.74 | 778.74 | 778.73 | 11.36 | -0.01 | | |
| | | | Min. | 0.25 | | | Min. | 0.43 | | Min. | 0.41 | | Min. | 0.48 | | Min. | 0.48 | | | Min. | 0.48 | |
| | | | Avg. | 5.71 | | | Avg. | 4.54 | | Avg. | 11.00 | | Avg. | 9.87 | | Avg. | 9.87 | | | Avg. | 9.87 | |
| | | | Max. | 31.04 | | | Max. | 27.84 | | Max. | 75.03 | | Max. | 73.44 | | Max. | 73.44 | | | Max. | 73.44 | |

Table 4.10 – Summary of results of metaheuristics with/without split in set I and C

| Metaheuristic | Set I | | | | | | Set C | | | | | |
|------------------|---------------|-----------|----------|------------|-----------|----------|---------------|-----------|----------|------------|-----------|----------|
| | Without split | | | With split | | | Without split | | | With split | | |
| | # of opt. | # of imp. | Time (s) | # of opt. | # of imp. | Time (s) | # of opt. | # of imp. | Time (s) | # of opt. | # of imp. | Time (s) |
| ILS | 26 | 8 | 5.87 | 26 | 8 | 5.21 | 29 | 9 | 5.70 | 29 | 8 | 4.62 |
| ELS | 27 | 8 | 5.48 | 27 | 8 | 4.72 | 29 | 9 | 5.71 | 29 | 9 | 4.54 |
| GRASP×ILS | 27 | 8 | 11.13 | 26 | 8 | 10.60 | 29 | 7 | 10.80 | 29 | 7 | 9.99 |
| GRASP×ELS | 27 | 8 | 10.93 | 26 | 7 | 10.81 | 29 | 9 | 11.00 | 29 | 8 | 9.87 |

4.10.2.2 Large instances

The results on large instances of set I and C as well as the number of nodes (n) and the used vehicles (K) in the instances are detailed in Tables (4.11) and (4.12) respectively. Each metaheuristic is executed 5 times with and without the split procedure, except GRASP. The maximum (Z_{max}), average (Z_{avg}) and minimum (Z_{min}) objective value and the average execution time (Time (s)) in seconds per run are reported.

The best known solution (BKS) using these metaheuristics and the minimum average time for each instance are indicated in boldface.

ELS and ILS are the most competitive metaheuristics for the large instances. In set I , the first without split procedure gets 3 out of 6 BKS s, while the other 2 BKS s. ELS reports two minimum execution times with and without the split procedure. Despite this, the average execution time is less using ILS than ELS, both without split. They find the same number of BKS s (2 out of 6), when the split procedure is included. In set C , the ELS again produces the greatest number of BKS s (4 out of 6). However, ILS with the split procedure solves 4 out of 6 instances faster.

In Tables 4.13 and 4.14, the gaps from the best known solutions are reported.

On average, the quality of the solutions is greater using ILS without split in set I , even if ELS without split finds more optimal solutions. In set C , ELS without split has the lowest average gap and in this case, it also gets more optimal solutions.

The split procedure does not improve the quality of the solutions, but reduces the execution time as in the small-medium instances.

4.11 Concluding remarks

In this chapter we develop a metaheuristic framework including five different metaheuristics as particular cases. These are GRASP, ILS, ELS, GRASP×ILS and GRASP×ELS. Moreover, a splitting procedure which extracts optimally the routes from a giant tour is designed. This allows to alternate between two solution spaces (complete solutions and giant tours). Four intra and inter routes moves are proposed and used in the local search. These

Table 4.11 – Results of metaheuristics with/without split on large instances in set I

| ILS | | | | | | | | | | |
|-----------|-----|-----|----------------|------------|------------|---------------|----------------|------------|------------|---------------|
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| I-43 | 75 | 11 | 1054.45 | 1050.25 | 1043.84 | 87.82 | 1047.89 | 1044.88 | 1043.31 | 71.05 |
| I-44 | 100 | 9 | 1141.20 | 1138.62 | 1134.86 | 163.91 | 1140.96 | 1138.43 | 1135.94 | 128.42 |
| I-45 | 150 | 14 | 1712.18 | 1708.79 | 1702.01 | 525.57 | 1710.23 | 1692.54 | 1628.87 | 334.21 |
| I-46 | 199 | 18 | 2442.48 | 2435.65 | 2419.56 | 1392.16 | 2449.29 | 2424.51 | 2390.27 | 674.66 |
| I-47 | 120 | 11 | 1070.81 | 1068.44 | 1063.63 | 204.26 | 1070.67 | 1067.19 | 1062.98 | 177.28 |
| I-48 | 100 | 11 | 1502.08 | 1502.06 | 1501.96 | 47.81 | 1502.08 | 1501.67 | 1501.10 | 50.65 |
| | | | | | | Min. | | | | 47.81 |
| | | | | | | Avg. | | | | 403.59 |
| | | | | | | Max. | | | | 1392.16 |
| ELS | | | | | | | | | | |
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| I-43 | 75 | 11 | 1054.28 | 1051.59 | 1045.45 | 32.83 | 1052.60 | 1044.81 | 1040.36 | 64.91 |
| I-44 | 100 | 9 | 1141.59 | 1139.75 | 1135.40 | 70.76 | 1141.19 | 1138.76 | 1137.12 | 92.22 |
| I-45 | 150 | 14 | 1708.29 | 1696.60 | 1678.37 | 584.88 | 1712.39 | 1711.79 | 1711.20 | 273.05 |
| I-46 | 190 | 18 | 2440.85 | 2428.63 | 2404.80 | 1548.41 | 2448.47 | 2434.11 | 2405.47 | 686.57 |
| I-47 | 120 | 11 | 1071.02 | 1064.98 | 1059.46 | 395.09 | 1066.83 | 1065.06 | 1062.45 | 179.19 |
| I-48 | 100 | 11 | 1502.08 | 1502.08 | 1502.08 | 160.51 | 1502.08 | 1502.02 | 1501.80 | 47.12 |
| | | | | | | Min. | | | | 32.83 |
| | | | | | | Avg. | | | | 465.41 |
| | | | | | | Max. | | | | 1548.41 |
| GRASPxILS | | | | | | | | | | |
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| I-43 | 75 | 11 | 1049.39 | 1041.87 | 1037.63 | 143.29 | 1042.18 | 1036.81 | 1031.70 | 118.64 |
| I-44 | 100 | 9 | 1138.78 | 1136.23 | 1132.09 | 125.27 | 1134.97 | 1131.94 | 1127.83 | 191.78 |
| I-45 | 150 | 14 | 1689.23 | 1674.00 | 1657.25 | 494.99 | 1691.24 | 1662.10 | 1650.71 | 421.47 |
| I-46 | 199 | 18 | 2400.47 | 2391.17 | 2383.04 | 1456.28 | 2367.59 | 2358.73 | 2353.74 | 983.81 |
| I-47 | 120 | 11 | 1068.89 | 1062.37 | 1057.27 | 173.70 | 1064.31 | 1060.26 | 1057.28 | 322.17 |
| I-48 | 100 | 11 | 1502.08 | 1502.08 | 1502.06 | 55.74 | 1502.08 | 1501.93 | 1501.76 | 72.17 |
| | | | | | | Min. | | | | 55.74 |
| | | | | | | Avg. | | | | 408.21 |
| | | | | | | Max. | | | | 1456.28 |
| GRASPxELS | | | | | | | | | | |
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| I-43 | 75 | 11 | 1050.23 | 1049.07 | 1048.19 | 135.22 | 1046.36 | 1042.52 | 1039.11 | 123.04 |
| I-44 | 100 | 9 | 1138.24 | 1136.62 | 1134.46 | 201.40 | 1138.70 | 1132.88 | 1130.27 | 200.65 |
| I-45 | 150 | 14 | 1685.25 | 1680.66 | 1676.42 | 534.92 | 1673.12 | 1662.27 | 1648.06 | 382.17 |
| I-46 | 199 | 18 | 2410.11 | 2401.54 | 2397.08 | 1283.28 | 2398.35 | 2376.40 | 2357.79 | 951.86 |
| I-47 | 120 | 11 | 1067.99 | 1065.82 | 1063.91 | 472.05 | 1063.31 | 1060.90 | 1058.50 | 311.91 |
| I-48 | 100 | 11 | 1502.08 | 1502.08 | 1502.08 | 137.44 | 1502.08 | 1501.93 | 1501.76 | 67.32 |
| | | | | | | Min. | | | | 135.22 |
| | | | | | | Avg. | | | | 460.72 |
| | | | | | | Max. | | | | 1283.28 |

Table 4.12 – Results of metaheuristics with/without split on large instances in set C

| ILS | | | | | | | | | | |
|-----------|-----|-----|----------------|------------|------------|---------------|----------------|------------|------------|---------------|
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| C-43 | 75 | 11 | 1081.02 | 1077.05 | 1072.35 | 89.29 | 1077.96 | 1076.18 | 1074.63 | 70.49 |
| C-44 | 100 | 9 | 1134.72 | 1133.32 | 1131.03 | 162.11 | 1133.39 | 1129.69 | 1124.95 | 120.36 |
| C-45 | 150 | 14 | 1689.62 | 1673.56 | 1624.96 | 550.46 | 1699.56 | 1689.76 | 1683.71 | 352.98 |
| C-46 | 199 | 18 | 2452.21 | 2445.17 | 2427.02 | 1279.16 | 2446.82 | 2432.51 | 2397.06 | 680.53 |
| C-47 | 120 | 11 | 1060.58 | 1059.67 | 1058.64 | 151.46 | 1060.65 | 1058.40 | 1055.77 | 170.66 |
| C-48 | 100 | 11 | 1468.82 | 1468.74 | 1468.62 | 64.60 | 1468.82 | 1468.45 | 1467.23 | 55.82 |
| | | | | | | Min. | | | | 64.60 |
| | | | | | | Avg. | | | | 382.85 |
| | | | | | | Max. | | | | 1279.16 |
| ELS | | | | | | | | | | |
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| C-43 | 75 | 11 | 1083.25 | 1078.94 | 1074.48 | 36.11 | 1075.68 | 1073.68 | 1071.51 | 68.93 |
| C-44 | 100 | 9 | 1134.86 | 1132.19 | 1129.28 | 141.74 | 1133.98 | 1132.18 | 1129.78 | 125.15 |
| C-45 | 150 | 14 | 1692.04 | 1671.68 | 1638.30 | 600.61 | 1617.36 | 1614.55 | 1611.73 | 391.15 |
| C-46 | 199 | 18 | 2452.78 | 2443.13 | 2436.76 | 1354.86 | 2453.73 | 2441.22 | 2422.64 | 753.81 |
| C-47 | 120 | 11 | 1061.42 | 1059.52 | 1057.03 | 380.96 | 1061.19 | 1059.78 | 1057.21 | 201.57 |
| C-48 | 100 | 11 | 1468.82 | 1468.77 | 1468.74 | 160.69 | 1468.78 | 1468.37 | 1467.29 | 61.79 |
| | | | | | | Min. | | | | 36.11 |
| | | | | | | Avg. | | | | 445.83 |
| | | | | | | Max. | | | | 1354.86 |
| GRASPxILS | | | | | | | | | | |
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| C-43 | 75 | 11 | 1078.55 | 1076.42 | 1073.43 | 115.56 | 1068.71 | 1065.37 | 1063.19 | 121.72 |
| C-44 | 100 | 9 | 1131.13 | 1126.51 | 1123.38 | 145.24 | 1124.48 | 1120.18 | 1113.78 | 195.81 |
| C-45 | 150 | 14 | 1655.41 | 1643.99 | 1620.64 | 620.91 | 1677.80 | 1643.78 | 1625.13 | 451.88 |
| C-46 | 199 | 18 | 2419.33 | 2409.58 | 2396.89 | 1179.99 | 2408.01 | 2392.65 | 2372.29 | 1027.73 |
| C-47 | 120 | 11 | 1057.03 | 1056.09 | 1054.72 | 189.36 | 1058.18 | 1055.40 | 1052.71 | 342.98 |
| C-48 | 100 | 11 | 1468.82 | 1468.74 | 1468.62 | 81.08 | 1468.67 | 1468.27 | 1467.53 | 81.00 |
| | | | | | | Min. | | | | 81.08 |
| | | | | | | Avg. | | | | 388.69 |
| | | | | | | Max. | | | | 1179.99 |
| GRASPxELS | | | | | | | | | | |
| Instance | n | K | Without split | | | | With split | | | |
| | | | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) | $Z_{max.}$ | $Z_{avg.}$ | $Z_{min.}$ | Time (s) |
| C-43 | 75 | 11 | 1078.91 | 1074.49 | 1067.74 | 135.73 | 1071.93 | 1069.52 | 1066.09 | 120.81 |
| C-44 | 100 | 9 | 1131.33 | 1129.03 | 1127.44 | 205.45 | 1127.09 | 1124.00 | 1121.91 | 189.22 |
| C-45 | 150 | 14 | 1673.96 | 1658.80 | 1645.55 | 408.30 | 1652.14 | 1639.29 | 1617.82 | 446.91 |
| C-46 | 199 | 18 | 2419.83 | 2414.06 | 2409.37 | 1647.77 | 2403.79 | 2388.56 | 2375.29 | 943.43 |
| C-47 | 120 | 11 | 1058.94 | 1057.48 | 1055.53 | 488.28 | 1057.69 | 1055.37 | 1052.74 | 325.89 |
| C-48 | 100 | 11 | 1468.82 | 1468.76 | 1468.67 | 185.15 | 1468.67 | 1468.27 | 1467.53 | 77.15 |
| | | | | | | Min. | | | | 135.73 |
| | | | | | | Avg. | | | | 511.78 |
| | | | | | | Max. | | | | 1647.77 |

Table 4.13 – Gaps on large instances for set *I*

| Instance | Set I | | | | | | | |
|----------|---------------|--------------|------------------------|------------------------|--------------|--------------|------------------------|------------------------|
| | Without split | | | | With split | | | |
| | ILS | ELS | GRASP _x ILS | GRASP _x ELS | ILS | ELS | GRASP _x ILS | GRASP _x ELS |
| I-43 | 0.00% | 0.02% | 0.48% | 0.40% | 0.62% | 0.18% | 1.16% | 0.77% |
| I-44 | 0.03% | 0.00% | 0.25% | 0.29% | 0.06% | 0.04% | 0.58% | 0.25% |
| I-45 | 0.01% | 0.24% | 1.35% | 1.58% | 0.13% | 0.00% | 1.24% | 2.29% |
| I-46 | 0.28% | 0.34% | 1.99% | 1.60% | 0.00% | 0.03% | 3.34% | 2.08% |
| I-47 | 0.02% | 0.00% | 0.20% | 0.28% | 0.03% | 0.39% | 0.63% | 0.72% |
| I-48 | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| Avg. Gap | 0.06% | 0.10% | 0.71% | 0.69% | 0.14% | 0.11% | 1.16% | 1.02% |

Table 4.14 – Gaps on large instances for set *C*

| Instance | Set C | | | | | | | |
|----------|---------------|--------------|------------------------|------------------------|--------------|--------------|------------------------|------------------------|
| | Without split | | | | With split | | | |
| | ILS | ELS | GRASP _x ILS | GRASP _x ELS | ILS | ELS | GRASP _x ILS | GRASP _x ELS |
| C-43 | 0.21% | 0.00% | 0.43% | 0.40% | 0.49% | 0.70% | 1.34% | 1.05% |
| C-44 | 0.01% | 0.00% | 0.33% | 0.31% | 0.13% | 0.08% | 0.91% | 0.68% |
| C-45 | 0.58% | 0.44% | 2.60% | 1.51% | 0.00% | 4.84% | 1.28% | 2.79% |
| C-46 | 0.06% | 0.04% | 1.40% | 1.38% | 0.28% | 0.00% | 1.86% | 2.04% |
| C-47 | 0.08% | 0.00% | 0.41% | 0.23% | 0.07% | 0.02% | 0.31% | 0.35% |
| C-48 | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01% | 0.01% |
| Avg. Gap | 0.16% | 0.08% | 0.86% | 0.64% | 0.16% | 0.94% | 0.95% | 1.15% |

moves are evaluated in constant time thanks to the precomputations based on a concatenation operator. On small-medium instances, the metaheuristics improve or find the same lower bounds reported by the B&P algorithm (Chapter 3), and in less time. In the larger ones, ILS and ELS obtain the best results among all metaheuristics. The split procedure decreases the execution times but sometimes solution quality is slightly reduced.

Conferences and Publications

This chapter gives as results one presentation:

- Jorge F. Victoria, H. Murat Afsar, and Christian Prins. *Metaheuristic approaches for the vehicle routing problem with time-dependent demands in humanitarian logistics*. 18^{ème} congrès annuel de la Société Française de Recherche Opérationnelle et d’Aide à la Décision, ROADEF 2017. Metz, France.

And a published article in the proceedings of the 8th International Conference on Computational Logistics, ICCL 2017:

- Jorge F. Victoria, H. Murat Afsar, and Christian Prins. Metaheuristic framework for a disaster logistics problem with time-dependent demands. In Tolga Bektaş, Stefano Coniglio, Antonio Martinez-Sykora, and Stefan Voß, editors, *Computational Logistics*, pages 143-157, Cham, 2017. Springer International Publishing.

Chapter 5

Relief distribution to the critical and non-critical nodes

In this chapter, a variant of the CVRP-TDD referred as the cumulative capacitated vehicle routing problem with time-dependent demands (CCVRP-TDD) is presented. To solve it, a mixed integer linear program (MILP) and a two-phase heuristic method (TPHM) are proposed.

The chapter is structured as follows: the CCVRP-TDD is formally defined and modeled as a MILP in Section 5.1. The two-phase heuristic method (TPHM) is explained in Section 5.2. A lower bound for the problem is proposed in Section 5.3. Results are detailed in Section 5.4 and concluding remarks are given in Section 5.5.

5.1 Problem definition and mathematical model

The cumulative capacitated vehicle routing problem with time-dependent demands (CCVRP-TDD) is a larger scale problem compared to the CVRP-TDD, because in addition to the shelters from which people flee, those where they go are considered.

After a disaster strikes a territory, people go to shelters to receive first aid. These shelters can be critical or non-critical. The first ones are located inside the affected area, while the second ones are outside. Therefore, people in critical shelters tend to flee to the non-critical ones if relief goods do not arrive quickly enough to satisfy their needs.

In CCVRP-TDD, the consequences (mentioned in Chapter 1) caused by the displacements of people can be mitigated by reaching critical shelters as soon as possible, because people stay at these points if they get help. This is why a cumulative objective function (sum of arrival times at critical shelters) is used.

It should be noted that although the objective function seeks to optimize the arrival

times at critical shelters, the time-dependent demand has an impact on the entire network, and it must be taken into account to guarantee feasible solutions to the problem.

The CCVRP-TDD is defined on a complete undirected graph $G = (V, E)$, where $V = \{0\} \cup V_c \cup V_{nc}$ is the node set and E the edge-set. For the sake of simplicity, each edge is replaced by two opposite arcs. Node 0 represents the depot while $V' = V \setminus \{0\}$ is the set of shelters. V_c and V_{nc} are the critical and non-critical shelter sets, respectively. Each shelter has an initial demand (number of people) a_i and each arc (i, j) has a travel time c_{ij} . A displacement rate b_{ij} is also given for the shelter couples such that $i \in V_c$ and $j \in V_{nc}$. This rate corresponds to the number of people per time unit who flee from a critical shelter i to a non-critical one j before the arrival of relief goods. We assume that total demand in the network is constant at any time.

The aim is to identify a set of feasible routes, so that every shelter is visited exactly once and the sum of arrival times at critical shelters is minimized. A fleet of K identical vehicles is available for the relief distribution. A feasible route for the CCVRP-TDD begins and ends at node 0 (depot). The total demand serviced by the route cannot exceed the vehicle capacity Q and the tour length is limited to a maximum value T_{max} .

We propose the following 2-index formulation for the CCVRP-TDD. Binary variables x_{ij} is equal to 1 if arc (i, j) is traversed by a vehicle. The vehicle visiting shelter i arrives at time t_i , finds d_i persons to serve, and leaves with a cumulated served demand w_i .

$$\min \sum_{j \in V_c} t_j \quad (5.1)$$

$$\sum_{j \in V'} x_{0j} = K \quad (5.2)$$

$$\sum_{i \in V} x_{ij} = \sum_{i \in V} x_{ji} \quad \forall j \in V' \quad (5.3)$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \quad (5.4)$$

$$w_j \geq w_i + d_j - Q \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V' \quad (5.5)$$

$$w_i \leq Q \quad \forall i \in V' \quad (5.6)$$

$$w_0 = 0 \quad (5.7)$$

$$t_0 = 0 \quad (5.8)$$

$$t_j \geq t_i + c_{ij} - T_{max} \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V' \quad (5.9)$$

$$t_i + c_{i0} \leq T_{max} \quad \forall i \in V' \quad (5.10)$$

$$d_j \geq a_j - \sum_{i \in V_{nc}} b_{ji} t_j \quad \forall j \in V_c \quad (5.11)$$

$$d_j \geq a_j + \sum_{i \in V_c} b_{ij} t_i \quad \forall j \in V_{nc} \quad (5.12)$$

$$\sum_{j \in V} d_j = \sum_{j \in V} a_j \quad (5.13)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in E \quad (5.14)$$

$$t_i, d_i, w_i \geq 0 \quad \forall i \in V \quad (5.15)$$

The objective function (5.1), to be minimized, is the sum of arrival times at critical shelters. Constraints (5.2) guarantee that K routes are constructed and begin at the depot. Constraints (5.3) specify that a vehicle arriving at shelter i must leave it. Constraints (5.4) ensure that each shelter is served exactly once. Constraints (5.5) calculate the amount of products transported to each shelter. This amount cannot exceed the capacity Q of vehicles (5.6). Constraint (5.7) defines initial load at depot while (5.8) sets the departure time from depot. Constraints (5.9) compute the arrival time at each shelter and prevent subtours. Constraints (5.10) are the tour length constraints. Constraints (5.11)-(5.12) compute the number of people attended at each shelter, taking into account the displacement of people. The first ones (5.11) correspond to the satisfied demand at critical shelters. While the second ones (5.12) calculate the satisfied demand at each non-critical shelter $j \in V_{nc}$, knowing that people flee from critical shelter $i \in V_c$ until aid arrives at time t_i . Note that if a vehicle visits j at time t_j and i is visited later by any vehicle ($t_j < t_i$), the quantity of relief products that are delivered at j corresponds to the number of people in j at time t_j plus future arrivals of unvisited critical shelter i . Constraint (5.13) guarantee that the total demand in the network is constant. Finally, Constraints (5.14) and (5.15) define the decision variables.

5.2 Two phase heuristic method

We propose a two phase heuristic method (TPHM) based on GRASP×ILS and outlined in Algorithm 5.1. This method begins by calling a constructive heuristic (explained in subsection 5.2.2) to generate an initial partial solution S which contains only critical shelters (*line 4*). After this, the precomputations described in subsection 5.2.3 are initialized and a local search (subsection 5.2.4) is applied to S (*lines 5–6*). The ILS (*lines 7–20*) executes *nbOfIter* iterations. Each iteration generates a complete solution S' adding the non-critical shelters (V_{nc}) to a partial solution S (*line 8*) using a parallel randomized insertion procedure (subsection 5.2.5).

If at least one route in S' violates the maximum tour length T_{max} or the capacity Q , the precomputations are updated and the repair procedure (explained in subsection 5.2.6) is executed to fix the infeasibilities (*lines 9–12*).

If a feasible solution S' is found and its objective value $Z(S')$ is less than the objective value of the incumbent solution \bar{Z} , the latter is updated and the partial solution of \bar{Z} is assigned to S (lines 13–16). At the end, S is perturbed using the procedure shown in subsection 5.2.7, the precomputations are updated and the local search is applied to S .

After the ILS, if the objective value of the incumbent solution \bar{Z} is less than the objective value of the global best solution Z^* , the latter is updated.

This algorithm is restarted $nbOfRestart$ times with a different initial partial solution S (loop 2–22) and returns the global best solution S^* .

Algorithm 5.1 Two phase heuristic method

```
1:  $Z^* \leftarrow \infty$ 
2: for  $rest = 1$  to  $nbOfRestarts$  do
3:    $\bar{Z} \leftarrow \infty$ 
4:    $S \leftarrow initialPartialSol(V_c)$ 
5:    $precomputations(S)$ 
6:    $localSearch(S)$ 
7:   for  $iter = 1$  to  $nbOfIter$  do
8:      $S' \leftarrow completeRoutes(S, V_{nc})$ 
9:     if  $S'$  is not feasible then
10:       $updatePrecomputations(S')$ 
11:       $repairProcedure(S')$ 
12:     end if
13:     if ( $S'$  is feasible and  $Z(S') < \bar{Z}$ ) then
14:        $\bar{S} \leftarrow S'$  ;  $\bar{Z} \leftarrow Z(S')$ 
15:        $S \leftarrow extractPartialSol(\bar{S})$ 
16:     end if
17:      $perturb(S)$ 
18:      $updatePrecomputations(S)$ 
19:      $localSearch(S)$ 
20:   end for
21:   if  $\bar{Z} < Z^*$  then  $S^* \leftarrow \bar{S}$  ;  $Z^* \leftarrow \bar{Z}$  end if
22: end for
23: return  $S^*$ 
```

5.2.1 Solution representation

A solution is encoded as a set of routes. Each route indicates the order in which shelters are visited by the vehicles, starting and ending at depot (node 0). The critical shelters are positioned at the beginning of the routes to arrive as quickly as possible to them. This is a design choice to give priority to the total demand satisfied, but of course the price to pay is a possible violation of the tour length constraint.

Figure 5.1 shows an example of the representation. This is composed by two routes: $(0, A, B, C, D, 0)$ and $(0, E, F, G, 0)$. The critical shelters are $A, B, C, E,$ and $F,$ while D and G are the non-critical ones. Each shelter has an initial demand a_i . The arcs traversed by the vehicles are represented by continuous arrows and the displacement of people from one shelter to another by dotted ones. The travel time c_{ij} and the displacement rate b_{ij} are given in their respective arrows.

The arrival time and the satisfied demand at each shelter are shown in Table 5.1. The latter is computed using Equations (5.12) and (5.11) for the non-critical and critical shelters respectively. The objective value Z of the solution is calculated as $Z = \sum_{i \in V_c} t_i = t_A + t_B + t_C + t_E + t_F = 4 + 7 + 9 + 2 + 5 = 27$.

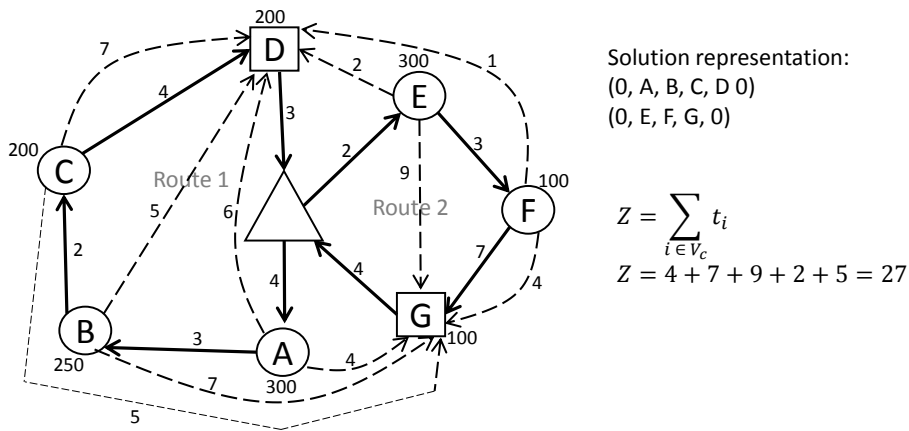


Figure 5.1 – Solution representation for the CCVRP-TDD

Table 5.1 – Arrival times and satisfied demands for Figure 5.1

| Shelter (i) | Arrival time (t) | Satisfied demand (d) |
|-------------|------------------|--|
| A | 4 | $300 - (6(4) + 4(4)) = 260$ |
| B | 7 | $250 - (5(7) + 7(7)) = 166$ |
| C | 9 | $200 - (7(9) + 5(9)) = 92$ |
| D | 13 | $200 + 6(4) + 5(7) + 7(9) + 2(2) + 1(5) = 331$ |
| E | 2 | $300 - (2(2) + 9(2)) = 278$ |
| F | 5 | $100 - (1(5) + 4(5)) = 75$ |
| G | 12 | $100 + 4(4) + 7(7) + 5(9) + 9(2) + 4(5) = 248$ |

5.2.2 Constructive heuristic

A parallel randomized constructive heuristic is used to generate an initial partial solution which includes only critical shelters. The heuristic starts randomly selecting one shelter to be visited. All possible positions are evaluated in each route and the three best ones are saved. The best positions are those that get the minimum sum of arrival times. One of these positions is chosen randomly and the shelter is inserted in it. Then, another unvisited shelter is taken and the process continues until all critical shelters are assigned.

5.2.3 Pre-computations

Computing the variations of the cumulative objective and the time-dependent demand when a sequence of nodes is inserted, removed or inverted by the local search is not trivial. Ngueveu et al. [146] present and exploit the properties of the CCVRP to calculate the variation of the cumulative objective in $O(1)$, using complex cost variation formulas. Silva et al. [187] propose simpler equations to do it for the cumulative traveling salesman problem (as seen in Chapter 4), later extended by Rivera et al. [174] to the multitrip version of the CCVRP. We adapt these equations to the characteristics of the CCVRP-TDD, like the time-dependent demand which is the main difference and the most difficult to handle in the problem. This allows to evaluate the local search moves in constant time as follows.

Any sequence of nodes can be represented as $\sigma = \{\sigma(1), \sigma(2), \dots, \sigma(|\sigma|)\}$ where $\sigma(i)$ corresponds to the node in position i of sequence σ . The following information is defined for any sequence σ :

- $C(\sigma) = \sum_{i=2}^{|\sigma|} t_{\sigma(i)}$, cumulative arrival time at nodes in sequence σ , assuming that the sequence begins at node $\sigma(1)$ at time 0.
- $W(\sigma) = |\sigma|$, number of nodes in sequence σ .
- $T(\sigma) = \sum_{i=1}^{|\sigma|-1} c_{\sigma(i), \sigma(i+1)}$, duration of sequence σ .
- $\beta(\sigma) = \sum_{i=1}^{|\sigma|} b_{\sigma(i)}$, total number of people who flee per time unit (total displacement rate) in sequence σ . The number of people fleeing from a critical node $\sigma(i)$ is computed as $b_{\sigma(i)} = \sum_{j \in V_{nc}} b_{\sigma(i), j}$. If σ is composed by non-critical nodes, $\beta(\sigma) = 0$.
- $D(\sigma) = a_{\sigma(1)} + \sum_{i=2}^{|\sigma|} d_{\sigma(i)}$, total demand to satisfy in sequence σ where $d_{\sigma(i)} = a_{\sigma(i)} - \sum_{j \in V_{nc}} b_{\sigma(i), j} \cdot t_{\sigma(i)}$ assuming that the sequence contains only critical nodes and begins at node $\sigma(1)$ at time 0. If σ has only non-critical nodes, $D(\sigma) = \sum_{i=1}^{|\sigma|} (a_{\sigma(i)} + \sum_{j \in V_c} b_{j, \sigma(i)} \cdot t_j)$.

For all given subsequences σ in a given solution, T , β , and D can be precomputed like for the CVRP-TDD, as explained in Section 4.5. $W(\sigma)$ and $C(\sigma)$ are new information to be calculated for the CCVRP-TDD. In addition, the values $\beta(\overleftarrow{\sigma})$, $D(\overleftarrow{\sigma})$ and $C(\overleftarrow{\sigma})$ must be pre-computed for the reversal $\overleftarrow{\sigma}$ of each sequence σ . They are useful for local search moves like 2-Opt.

For instance, let $\sigma = (A, B, C)$ be a sequence of critical nodes of the route $(0, A, B, C, D, 0)$ (Figure 5.1). Then, $\sigma(1) = A$, $\sigma(2) = B$ and $\sigma(3) = C$. The respective information for σ is $W(\sigma) = 3$, $C(\sigma) = 3+5 = 8$, $T(\sigma) = 3+2 = 5$, $\beta(\sigma) = 10+12+12 = 34$, $D(\sigma) = 300 + 214 + 140 = 654$. Moreover, $C(\overleftarrow{\sigma}) = 2 + 5 = 7$, $\beta(\overleftarrow{\sigma}) = 10 + 12 + 12 = 34$, and $D(\overleftarrow{\sigma}) = 200 + 226 + 250 = 676$.

The matrices to store this information for a given solution are initialized with the *precomputations* procedure (*line 5* - Algorithm 5.1). For every node sequence $\sigma = \{i, \dots, j\}$ the respective information is calculated. The information is stored at the subscripts (i, j) of each matrix.

For $i = j$, we have $W(\sigma) = 1$, $C(\sigma) = 0$, $T(\sigma) = 0$, $D(\sigma) = D(\overleftarrow{\sigma}) = a_j$ and $\beta(\sigma) = \beta(\overleftarrow{\sigma}) = b_i \forall i \in V_c$. When j is incremented, the information for the resulting sequence $\sigma \oplus j$ is computed in $O(1)$ as follows:

$$W(\sigma \oplus j) = W(\sigma) + 1 \quad (5.16)$$

$$T(\sigma \oplus j) = T(\sigma) + c_{\sigma(|\sigma|),j} \quad (5.17)$$

$$C(\sigma \oplus j) = C(\sigma) + T(\sigma) + c_{\sigma(|\sigma|),j} \quad (5.18)$$

$$\beta(\sigma \oplus j) = \beta(\sigma) + b_j \quad (5.19)$$

$$D(\sigma \oplus j) = D(\sigma) + (a_j - b_j(T(\sigma) + c_{\sigma(|\sigma|),j})) \quad (5.20)$$

Equations (5.19) and (5.20) apply only if j is a critical node. Otherwise, $\beta(\sigma \oplus j) = \beta(\sigma)$ and $D(\sigma \oplus j) = D(\sigma) + (a_j + \sum_{k \in V_c} b_{k, \sigma(|\sigma|)} \cdot t_k)$.

For the reverse sequence $(\overleftarrow{\sigma \oplus j})$, the node j is added at the beginning and σ is reversed ($\overleftarrow{\sigma}$). The reversal information is computed as:

$$C(\overleftarrow{\sigma \oplus j}) = C(j \oplus \overleftarrow{\sigma}) = W(\sigma)c_{j, \overleftarrow{\sigma}(1)} + C(\overleftarrow{\sigma}) \quad (5.21)$$

$$\beta(\overleftarrow{\sigma \oplus j}) = \beta(j \oplus \overleftarrow{\sigma}) = b_j + \beta(\overleftarrow{\sigma}) \quad (5.22)$$

$$D(\overleftarrow{\sigma \oplus j}) = D(j \oplus \overleftarrow{\sigma}) = a_j + D(\overleftarrow{\sigma}) - \beta(\overleftarrow{\sigma})c_{j, \overleftarrow{\sigma}(1)} \quad (5.23)$$

Since most local search moves can be expressed as concatenations of sequences, we have to design $O(1)$ formulas based on previous pre-computations. Consider any two sequences of nodes $\sigma_1 = (\sigma_1(1), \dots, \sigma_1(v))$ and $\sigma_2 = (\sigma_2(1), \dots, \sigma_2(w))$ in a solution and their concatenation

$\sigma_1 \oplus \sigma_2 = (\sigma_1(1), \dots, \sigma_1(v), \sigma_2(1), \dots, \sigma_2(w))$. The information for the new sequence can be derived in $O(1)$ as follows:

$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2) \quad (5.24)$$

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + c_{\sigma_1(v), \sigma_2(1)} + T(\sigma_2) \quad (5.25)$$

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(T(\sigma_1) + c_{\sigma_1(v), \sigma_2(1)}) + C(\sigma_2) \quad (5.26)$$

$$\beta(\sigma_1 \oplus \sigma_2) = \beta(\sigma_1) + \beta(\sigma_2) \quad (5.27)$$

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + D(\sigma_2) - \beta(\sigma_2)(T(\sigma_1) + c_{\sigma_1(v), \sigma_2(1)}) \quad (5.28)$$

Equation (5.24) is the total number of nodes in sequence $\sigma_1 \oplus \sigma_2$. Equation (5.25) computes its total duration. Equation (5.26) is the cumulative arrival time at nodes in $\sigma_1 \oplus \sigma_2$. Equation (5.27) computes the displacement rate for $\sigma_1 \oplus \sigma_2$. Equation (5.28) is the total demand of $\sigma_1 \oplus \sigma_2$, equal to the sum of the demands of σ_1 and σ_2 and the demand variation induced by the shift of the starting time in sequence σ_2 .

Similar formulas are possible for the reversal of each sequence, $\overleftarrow{\sigma}$.

5.2.4 Local search

The local search is applied only to critical shelters seeking the best improvement of the objective value. The node relocation and 2-Opt moves used in Section 4.6 are replicated in this chapter, except the inter-route 2-Opt* move. Next, each move is explained, but only the calculations of the W and C values will be detailed. They are derived from Equations (5.24) to (5.28). For T , D , and β , the respective equations used in chapter 4 will be referred to.

For the sake of simplicity, the first and last node of a node sequence σ will be referred as $\sigma(f)$ and $\sigma(l)$ in the explanation of each move, respectively. In the case of $|\sigma| = 1$, only σ will be used.

5.2.4.1 Node Relocation

In this move, all possible positions to relocate a node are evaluated. It is applied to one route (intra-route) or two distinct routes (inter-route). The inter-route variant is presented in Figure 5.2.

The T , β , and D values for the concatenations $(\sigma_1 \oplus \sigma_3)$ and $(\sigma_4 \oplus \sigma_2 \oplus \sigma_5)$ are calculated as in Equations (4.13)-(4.15) and (4.17)-(4.19) respectively. While C and W are computed as follows:

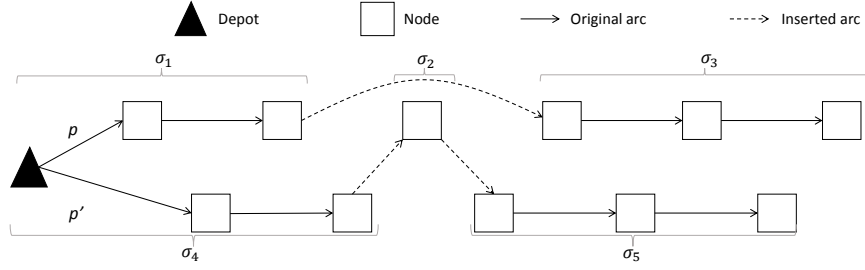


Figure 5.2 – Inter-route node relocation

Route p :

$$W(\sigma_1 \oplus \sigma_3) = W(\sigma_1) + W(\sigma_3) \quad (5.29)$$

$$C(\sigma_1 \oplus \sigma_3) = C(\sigma_1) + W(\sigma_3)(T(\sigma_1) + c_{\sigma_1(l),\sigma_3(f)}) + C(\sigma_3) \quad (5.30)$$

Route p' :

$$W(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) = W(\sigma_4) + W(\sigma_2) + W(\sigma_5) \quad (5.31)$$

$$C(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) = C(\sigma_4 \oplus \sigma_2) + W(\sigma_5)(T(\sigma_4 \oplus \sigma_2) + c_{\sigma_2,\sigma_5(f)}) + C(\sigma_5) \quad (5.32)$$

$$C(\sigma_4 \oplus \sigma_2) = C(\sigma_4) + W(\sigma_2)(T(\sigma_4) + c_{\sigma_4(l),\sigma_2}) + C(\sigma_2)$$

The improvement (variation) of the objective function is measured as:

$$\Delta Z = C(\sigma_1 \oplus \sigma_3) + C(\sigma_4 \oplus \sigma_2 \oplus \sigma_5) - C(\sigma_1 \oplus \sigma_2 \oplus \sigma_3) - C(\sigma_4 \oplus \sigma_5)$$

If $\Delta Z < 0$, this move reduce the objective value and it is feasible if conditions in subsection 4.6.1 are satisfied.

5.2.4.2 Two-Opt

As the intra-route and inter-route moves are too different to be explained in one figure, the first one is shown in Figure 5.3, while the second is illustrated in Figure 5.4.

- *Intra-route move*: the number of nodes W for the concatenation $(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3)$ as well as the evaluation of the cumulative arrival time (C) are detailed in 5.33 and 5.34 respectively. The T , β , and D values are computed as in Equations (4.37)-(4.39).

$$W(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) = W(\sigma_1) + W(\sigma_2) + W(\sigma_3) \quad (5.33)$$

$$\begin{aligned} C(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) &= C(\sigma_1 \oplus \overleftarrow{\sigma_2}) + W(\sigma_3)(T(\sigma_1 \oplus \overleftarrow{\sigma_2}) + c_{\overleftarrow{\sigma_2}(l),\sigma_3(f)}) + C(\sigma_3) \\ C(\sigma_1 \oplus \overleftarrow{\sigma_2}) &= C(\sigma_1) + W(\overleftarrow{\sigma_2})(T(\sigma_1) + c_{\sigma_1(l),\overleftarrow{\sigma_2}(f)}) + C(\overleftarrow{\sigma_2}) \end{aligned} \quad (5.34)$$

The change in the objective function is given by ΔZ which is calculated as:

$$\Delta Z = C(\sigma_1 \oplus \overleftarrow{\sigma_2} \oplus \sigma_3) - C(\sigma_1 \oplus \sigma_2 \oplus \sigma_3)$$

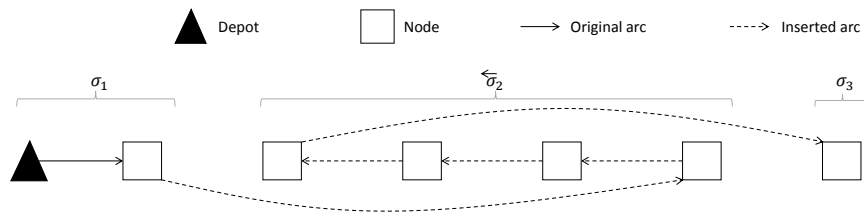


Figure 5.3 – Intra-route 2-Opt

This move is executed, if $\Delta Z < 0$ and the feasibility conditions detailed in subsection 4.6.4 are fulfilled.

- *Inter-route move*: the number of nodes and the cumulative arrival time at shelters for concatenations $(\sigma_1 \oplus \sigma_4)$ and $(\sigma_3 \oplus \sigma_2)$ are evaluated in Equations (5.35)-(5.36) and Equations (5.37)-(5.38) respectively. Equations (4.41)-(4.43) and (4.45)-(4.47) are used to evaluate the T , β , and D for each of the concatenations.

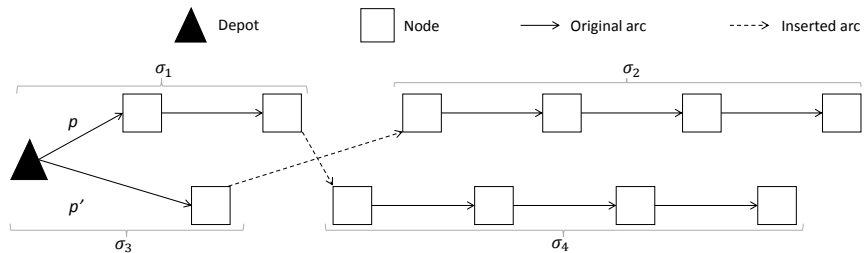


Figure 5.4 – Inter-route 2-Opt

Route p :

$$W(\sigma_1 \oplus \sigma_4) = W(\sigma_1) + W(\sigma_4) \quad (5.35)$$

$$C(\sigma_1 \oplus \sigma_4) = C(\sigma_1) + W(\sigma_4)(T(\sigma_1) + c_{\sigma_1(l), \sigma_4(f)}) + C(\sigma_4) \quad (5.36)$$

Route p' :

$$W(\sigma_3 \oplus \sigma_2) = W(\sigma_3) + W(\sigma_2) \quad (5.37)$$

$$C(\sigma_3 \oplus \sigma_2) = C(\sigma_3) + W(\sigma_2)(T(\sigma_3) + c_{\sigma_3(l), \sigma_2(f)}) + C(\sigma_2) \quad (5.38)$$

The feasibility of this move is validated with conditions in subsection 4.6.4 and the objective value is reduced, if $\Delta Z < 0$:

$$\Delta Z = C(\sigma_1 \oplus \sigma_4) + C(\sigma_3 \oplus \sigma_2) - C(\sigma_1 \oplus \sigma_2) - C(\sigma_3 \oplus \sigma_4)$$

5.2.5 Insertion of non-critical shelters

The non-critical shelters must be inserted after the critical ones. Otherwise, the objective value is increased as the first aid does not arrive as quickly as possible to critical shelters. Consequently, once a partial solution is obtained, the total demand in the non-critical shelters as well as the objective value of the solution are known and fixed.

A parallel randomized best insertion method is used to include the non-critical shelters. This method selects randomly one shelter and all possible positions (after critical shelters) in each route are evaluated to insert it. The best position is the one that minimizes the remaining capacity of the vehicle and the tour length. Nevertheless, if any of these conditions are not fulfilled the following criteria are taken into account:

- If the capacity constraint is violated, the position that minimizes the capacity constraint violation is selected.
- If the capacity constraint is satisfied, the position that minimizes the tour length constraint violation is selected.

This process continues until all non-critical shelters are incorporated in the routes.

5.2.6 Repair procedure

After the insertion of non-critical shelters, the resulting complete solution can be infeasible because the maximum tour length and/or the capacity of the vehicle is exceeded. In this case, a repair procedure which uses the same moves as in the local search (explained in subsection 5.2.4) is executed. These moves are applied to the non-critical shelters, trying to find a feasible solution.

The procedure starts applying the intra and inter-route moves to repair the routes in which the tour length is exceeded. Later, only the inter-route moves are used to fix the routes in which the capacity constraint is violated.

5.2.7 Perturbation

The node ejection chain procedure is used to perturbate the solution in the same manner explained in Section 4.8. In the CCVRP-TDD, only critical shelters in each route are randomly selected to be relocated in the position that minimizes the objective value of next route.

5.3 Spanning tree based relaxation

A lower bound based on the spanning tree relaxation with degree constraints is proposed for the CCVRP-TDD. Relaxing constraints (5.3), each critical shelter is directly connected to the depot or to an intermediate shelter which is visited immediately after the depot. Additionally, the tour length and capacity constraints ((5.9)-(5.10)) of the original problem are also relaxed.

The following mathematical model is solved to find a lower bound:

$$\min \sum_{i \in V_c} t_i \quad (5.39)$$

$$\sum_{i \in V_c} x_{0i} \leq K \quad (5.40)$$

$$\sum_{i \in V_c} x_{ij} = 1 \quad \forall j \in V_c \quad (5.41)$$

$$x_{ij} \leq x_{0i} \quad \forall i \in V_c, \forall j \in V_c \quad (5.42)$$

$$t_i \geq c_{0i} \cdot x_{0i} \quad \forall i \in V_c \quad (5.43)$$

$$t_j \geq (x_{0i} + x_{ij} - 1) \cdot (c_{0i} + c_{ij}) \quad \forall i \in V_c, \forall j \in V_c \quad (5.44)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in V_c, \forall j \in V_c \quad (5.45)$$

$$t_i \geq 0 \quad \forall i \in V_c \quad (5.46)$$

The objective function (5.39) is the minimization of the arrival time at critical shelters. As at most K vehicles leave the depot in the original problem, the degree constraint (5.40) state that at most K edges can be incident to the depot. Constraints (5.41) guarantee that all critical shelters are visited once. These shelters must be visited directly from the depot or after an intermediate shelter which is visited immediately after the depot (5.42) ensuring a tree (connected graph) with all nodes. Constraints (5.43) and (5.10) compute the arrival time to critical shelters visited directly from the depot and from the intermediate shelters respectively. Finally, Constraints (5.45) and (5.46) define the decision variables.

5.4 Computational experiments

The computational experiments evaluate and compare the performance of the mathematical model and the two-phase heuristic method (TPHM). The instances used are presented in subsection 5.4.1 and the numerical results are detailed in subsection 5.4.2.

5.4.1 Implementation and instances

The TPHM is implemented in JAVA and the mathematical models to solve the problem and to find a lower bound are solved using CPLEX 12.6.3. The experiments are carried on an Intel(R) Core(TM) i7-4800MQ 2.70GHz with 16 GB of RAM running Windows 7 Professional. The CCVRP-TDD being a new problem, no instances are available in the literature. Nevertheless, the two set of instances I and C used for the CVRP-TDD are adapted to this problem. It should be remembered that each set (I and C) contains 42 small-medium instances (numbered from 1 to 42) and 6 larger ones (numbered from 43 to 48). The first instances are created by selecting randomly 10, 15, 20, 25, 30 and 50 shelters from each of the seven instances (CMT6, CMT7, CMT8, CMT9, CMT10, CMT13, and CMT14) proposed by Christofides et al. [49] for the CVRP. The latter instances are created with the same number of shelters of the original instances, except the CMT6 which its size is 50 customers and it belongs to the small-medium instances.

In the adapted instances for the CCVRP-TDD, we generate 3 subsets of critical shelters corresponding to 20%, 40% and 60% of $|V'|$ for each instance. These shelters are randomly selected and the displacement rate (b_i) of each one is transformed into $b_{ij} \forall i \in V_c; j \in V_{nc}$. A percentage of b_i is randomly assigned to each non-critical shelter j such that $b_i = \sum_{j \in V_{nc}} b_{ij}$.

The number of vehicles (K) used in the CVRP-TDD is taken as reference value. If a feasible solution is not found, then K is incremented by 1. Conversely, if K is greater than the number of critical shelters (V_c), K is decreased to V_c .

5.4.2 Numerical results

This subsection is divided into results on small-medium instances and on larger ones. The small-medium instances are solved using the MILP and the TPHM, while the larger ones only the TPHM.

5.4.2.1 Small-medium instances

The MILP is solved on the small-medium instances of each set (I and C) considering the different percentage of critical shelters. The detailed results of the used instances are shown in Tables 5.5 and 5.6 for set I and C respectively. Some instances are not tested because we assume that if one instance cannot be solved to optimality in less than one hour, the instances extracted from the same file of the original instances (Christofides et al. [49]) and with larger number of nodes will not be solved within the time limit either.

For each percentage of critical shelters, Table 5.2 summarizes the following results: the average running time (in seconds), the average gap (%), the number of used instances, the

Table 5.2 – Summary of the MILP results on small-medium instances

| | Set I | | | Set C | | |
|---------------------------|--------|--------|--------|-------|--------|--------|
| | 20% | 40% | 60% | 20% | 40% | 60% |
| Average time (s) | 216.46 | 308.21 | 288.52 | 74.16 | 203.19 | 469.68 |
| Average gap (%) | 3.06 | 7.25 | 13.72 | 2.98 | 10.77 | 12.68 |
| # of instances | 32 | 27 | 20 | 29 | 25 | 19 |
| # of feasible sol. | 26 | 25 | 20 | 24 | 24 | 19 |
| # of optimal sol. | 25 | 20 | 13 | 22 | 18 | 12 |

number of instances for which a feasible solution is found and the number of instances solved to optimality.

It should be noted that the average running time is calculated only by considering instances where optimal solution is found and the gap is computed as $\frac{Z_{UB}-Z_{LB}}{Z_{LB}} \times 100$ for each such instance. Z_{LB} and Z_{UB} are the lower and upper bound reported by CPLEX in the time limit of an hour, respectively.

The results show that the number of instances solved to optimality is slightly greater in set I than in set C . These instances contain up to:

- 30 and 20 shelters with 6 and 12 critical ones in set I , respectively.
- 25, 20 and 15 shelters with 5, 8 and 9 critical ones in set C , respectively.

The increase in the number of critical shelters causes that the MILP spends, in average, more time finding the optimal solution in set C . In set I , this only happens in instances up to 40% of critical shelters.

The average gap worsens when the number of critical shelters increases. This goes from 3.06% in instances with 20% of critical shelters to 13.72% in instances with 60% of shelters in set I and from 2.98% to 12.68% in set C .

In the detailed results, it can be seen that the number of vehicles (K) is equal to the number of critical shelters (V_c) in most instances with 20% of critical shelters. In these cases, the optimal solution is to send one vehicle to each critical shelter. However, this is not true in all instances. For example, instances $I-7$ and $C-7$ are infeasible when $K = |V_c|$, due to the tour length constraint.

The lower bound (ST_{LB}) based on the spanning tree relaxation is always greater than the lower bound given by CPLEX for instances not solved in one hour, except in three instances of both sets (I and C) with 60% of critical shelters.

TPHM is run 5 times to solve each instance with a budget of 500 calls to the local search ($nbOfRestarts = 5$ and $nbOfIter = 100$) per run. This time, as we have no problem of

Table 5.3 – Summary of the results of TPHM on small-medium instances

| | Set I | | | Set C | | |
|---------------------------|-------|------|-------|-------|------|-------|
| | 20% | 40% | 60% | 20% | 40% | 60% |
| Average Time (s) | 0.26 | 0.50 | 0.91 | 0.24 | 0.54 | 0.96 |
| Average gap (%) | 0.37 | 9.43 | 28.97 | 0.09 | 9.53 | 28.53 |
| # of instances | 42 | 42 | 42 | 44 | 44 | 44 |
| # of feasible sol. | 42 | 42 | 42 | 42 | 42 | 42 |
| # of optimal sol. | 36 | 20 | 13 | 36 | 19 | 13 |

running time with TPHM, all instances in set I and C are treated. Moreover, two additional instances are considered in set C due to the structure of the optimal solution (which will be explained later) in two instances. A summarized table (Table 5.3) for the results of each set (I and C) and each percentage of critical shelters is also presented. This table has the same structure than the one for MILP results. However, the gap is computed this time as $\frac{Z_{min.} - LB}{LB} \times 100$, where $Z_{min.}$ is the minimum objective value found in the 5 runs, and LB is the maximum value between the lower bound reported by CPLEX (Z_{LB}) and the one calculated using the spanning tree based relaxation (ST_{LB}). The detailed results are shown in Tables 5.7 and 5.8.

TPHM finds the optimal solution in all instances for which this solution is known, except in instances $C-8$ and $C-25$ with $K = 3$ and $K = 2$ respectively. These optimal solutions are not found for two reasons: (1) some non-critical shelters should be visited before the critical ones or (2) the number of critical shelters in each route is not balanced in the optimal solution. Consequently, two additional instances are used ($C-8$ and $C-25$ with $K = 4$ and $K = 3$ respectively). In these cases, TPHM finds the optimal solutions.

In average, the small-medium instances are solved in less than one second in both sets. The average GAPs are 0.37%, 9.43%, and 28.97% with 20%, 40%, and 60% of critical shelters in set I respectively. While in set C are 0.09%, 9.53%, and 28.53%.

5.4.2.2 Large instances

The results obtained by solving the large instances with the TPHM are summarized in Table 5.4. The TPHM is also run 5 times, in each instance, with a budget of 500 calls to the local search ($nbOfRestarts = 5$ and $nbOfIter = 100$) per run.

In these instances, the ST_{LB} is used to compute the GAP and to evaluate the quality of the solutions.

The results show that, in average, instances are solved in less than one minute. Moreover, the instance with 75 nodes and 15 critical shelters is solved to optimality in set I and C .

Table 5.4 – Summary of results of TPHM on large instances

| | Set I | | | Set C | | |
|--------------------|-------|-------|-------|-------|-------|-------|
| | 20% | 40% | 60% | 20% | 40% | 60% |
| Average time (s) | 6.55 | 26.76 | 47.92 | 6.19 | 25.54 | 53.84 |
| Average gap (%) | 1.20 | 30.53 | 66.88 | 0.65 | 31.75 | 67.99 |
| # of instances | 6 | 6 | 6 | 6 | 6 | 6 |
| # of feasible sol. | 6 | 6 | 6 | 6 | 6 | 6 |
| # of optimal sol. | 1 | 0 | 0 | 1 | 0 | 0 |

Tables 5.9a and 5.9b present the detailed results on large instances of each set (I and C respectively). From these tables, we observe that the TPHM is able to provide good solutions on instances up to 199 nodes with 120 critical shelters in less than 180 seconds.

A relative gap with the minimum (Z_{min}), average (Z_{avg}), and maximum (Z_{max}) objective value found in the 5 runs is calculated as $\frac{Z_{max}-Z_{min}}{Z_{avg}} \times 100$ for each instance in both sets. Then, the relative average gaps are computed for instances with 20%, 40%, and 60% of critical shelters in each set. These averages are 0.46%, 0.61%, and 0.27% in set I , respectively. While in set C , they are 0.26%, 0.38%, and 0.28%. In instances with 20% of critical shelters an average equal to zero would be expected due to the small number of critical shelters that should be visited by each vehicle in these instances. Nevertheless, this does not happen because the problem aims not only at optimizing arrival times at critical shelters, but also at finding feasible solutions that visit all shelters.

5.5 Concluding remarks

This chapter presents a variant of the CVRP-TDD in which people flee from a set of critical shelters to other shelters. The outgoing flow will not stop until the arrival of first aid. A cumulative objective function is considered to reduce the displacement of people.

A mixed integer linear program (MILP) and a two-phase heuristic method (TPHM) are proposed to solve this problem. On small-medium instances optimally solved by the MILP, TPHM obtains the same solution if two conditions are met: (1) the number of critical shelters in each route are balanced and (2) they are visited before the non-critical nodes. If one of these conditions is not satisfied, TPHM might not find even a feasible solution.

On large instances, the quality of the solutions found by TPHM is measured with a lower bound. This lower bound is based on a spanning tree relaxation with degree constraints. The results shows the good performance of the method in terms of execution time and optimality gap.

Conferences and publications

This chapter gives as results two presentations:

- Jorge F. Victoria, H. Murat Afsar, and Christian Prins. *Vehicle routing problem with time-dependent demand in humanitarian logistics*. 16^{ème} congrés annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision, ROADEF 2015. Marseille, France.
- Jorge F. Victoria, H. Murat Afsar, and Christian Prins. *Vehicle routing problem with time-dependent demand in humanitarian logistics*. 6th International Conference on Industrial Engineering and Systems Management (IESM) 2015. Seville, Spain.

And a published article in the proceedings of the 6th International Conference on Industrial Engineering and Systems Management (IESM) 2015:

- J. F. Victoria, H. M. Afsar, and C. Prins. *Vehicle routing problem with time-dependent demand in humanitarian logistics*. In 2015 International Conference on Industrial Engineering and Systems Management (IESM), pages 686-694, Oct 2015.

Table 5.5 – MILP results on small-medium instances in set I

| (a) 20% of critical shelters | | | | | | | | | |
|------------------------------|------|-------|-----|------------|----------|----------|----------|--------|--|
| Instance | V' | V_c | K | ST_{Lab} | Z_{LB} | Z_{UB} | Time (s) | Gap(%) | |
| I-1 | 10 | 2 | 2 | 42.52 | 42.52 | 42.52 | 0.65 | 0.00 | |
| I-2 | 15 | 3 | 3 | 57.01 | 57.01 | 57.01 | 0.63 | 0.00 | |
| I-3 | 20 | 4 | 4 | 100.07 | 100.07 | 100.07 | 5.24 | 0.00 | |
| I-4 | 25 | 5 | 4 | 127.00 | - | - | 3600.00 | - | |
| I-7 | 10 | 2 | 3 | 42.56 | 42.56 | 42.56 | 0.51 | 0.00 | |
| I-8 | 15 | 3 | 3 | 66.83 | 66.83 | 66.83 | 5.91 | 0.00 | |
| I-9 | 20 | 4 | 4 | 101.88 | 101.88 | 101.88 | 1990.69 | 0.00 | |
| I-10 | 25 | 5 | 5 | 114.49 | - | - | 3600.00 | - | |
| I-13 | 10 | 2 | 2 | 31.56 | 31.56 | 31.56 | 0.26 | 0.00 | |
| I-14 | 15 | 3 | 3 | 66.14 | 66.14 | 66.14 | 1.54 | 0.00 | |
| I-15 | 20 | 4 | 4 | 108.40 | 108.40 | 108.40 | 3.86 | 0.00 | |
| I-16 | 25 | 5 | 4 | 158.41 | - | - | 3600.00 | - | |
| I-19 | 10 | 2 | 2 | 43.42 | 43.42 | 43.42 | 0.53 | 0.00 | |
| I-20 | 15 | 3 | 3 | 44.92 | 44.92 | 44.92 | 0.32 | 0.00 | |
| I-21 | 20 | 4 | 4 | 133.71 | 133.71 | 133.71 | 6.71 | 0.00 | |
| I-22 | 25 | 5 | 4 | 111.64 | - | - | 3600.00 | - | |
| I-25 | 10 | 2 | 2 | 34.47 | 34.47 | 34.47 | 0.47 | 0.00 | |
| I-26 | 15 | 3 | 3 | 65.49 | 65.49 | 65.49 | 0.62 | 0.00 | |
| I-27 | 20 | 4 | 4 | 92.96 | 92.96 | 92.96 | 13.90 | 0.00 | |
| I-28 | 25 | 5 | 4 | 144.33 | 144.33 | 144.33 | 3297.55 | 0.00 | |
| I-29 | 30 | 6 | 5 | 147.51 | - | - | 3600.00 | - | |
| I-31 | 10 | 2 | 2 | 140.62 | 140.62 | 140.62 | 0.50 | 0.00 | |
| I-32 | 15 | 3 | 3 | 177.54 | 177.54 | 177.54 | 0.18 | 0.00 | |
| I-33 | 20 | 4 | 4 | 151.99 | 151.99 | 151.99 | 0.75 | 0.00 | |
| I-34 | 25 | 5 | 4 | 338.80 | 338.89 | 338.89 | 68.90 | 0.00 | |
| I-35 | 30 | 6 | 4 | 328.94 | 183.15 | 328.94 | 3600.00 | 79.60 | |
| I-37 | 10 | 2 | 2 | 43.26 | 43.26 | 43.26 | 0.50 | 0.00 | |
| I-38 | 15 | 3 | 3 | 125.50 | 125.50 | 125.50 | 0.32 | 0.00 | |
| I-39 | 20 | 4 | 4 | 131.68 | 131.68 | 131.68 | 1.17 | 0.00 | |
| I-40 | 25 | 5 | 4 | 225.10 | 225.10 | 225.10 | 3.56 | 0.00 | |
| I-41 | 30 | 6 | 4 | 346.82 | 346.82 | 346.82 | 6.14 | 0.00 | |
| I-42 | 50 | 10 | 6 | 641.78 | - | - | 3600.00 | - | |
| | | | | | Min. | 0.18 | | 0.00 | |
| | | | | | Avg. | 216.46 | | 3.06 | |
| | | | | | Max. | 3297.55 | | 79.60 | |

| (b) 40% of critical shelters | | | | | | | | | |
|------------------------------|------|-------|-----|------------|----------|----------|----------|--------|--|
| Instance | V' | V_c | K | ST_{Lab} | Z_{LB} | Z_{UB} | Time (s) | Gap(%) | |
| I-1 | 10 | 4 | 2 | 142.34 | 142.34 | 142.34 | 1.60 | 0.00 | |
| I-2 | 15 | 6 | 3 | 170.91 | 170.91 | 170.91 | 6.00 | 0.00 | |
| I-3 | 20 | 8 | 4 | 264.74 | 288.29 | 288.29 | 138.58 | 0.00 | |
| I-4 | 25 | 10 | 4 | 313.12 | - | - | 3600.00 | - | |
| I-7 | 10 | 4 | 3 | 118.38 | 118.38 | 118.38 | 1.25 | 0.00 | |
| I-8 | 15 | 6 | 3 | 147.34 | 153.19 | 153.19 | 3489.41 | 0.00 | |
| I-9 | 20 | 8 | 4 | 255.17 | 231.77 | 267.20 | 3600.00 | 15.29 | |
| I-13 | 10 | 4 | 2 | 111.99 | 124.55 | 124.55 | 0.46 | 0.00 | |
| I-14 | 15 | 6 | 3 | 183.87 | 189.38 | 189.38 | 7.76 | 0.00 | |
| I-15 | 20 | 8 | 4 | 275.96 | 289.57 | 289.57 | 101.57 | 0.00 | |
| I-16 | 25 | 10 | 4 | 360.26 | 270.51 | 437.84 | 3600.00 | 61.86 | |
| I-19 | 10 | 4 | 2 | 118.63 | 118.63 | 118.63 | 0.57 | 0.00 | |
| I-20 | 15 | 6 | 3 | 128.88 | 140.25 | 140.25 | 4.07 | 0.00 | |
| I-21 | 20 | 8 | 4 | 300.36 | 316.58 | 316.58 | 1121.24 | 0.00 | |
| I-22 | 25 | 10 | 4 | 280.27 | - | - | 3600.00 | - | |
| I-25 | 10 | 4 | 2 | 143.70 | 165.30 | 165.30 | 0.96 | 0.00 | |
| I-26 | 15 | 6 | 3 | 168.12 | 181.84 | 181.84 | 11.96 | 0.00 | |
| I-27 | 20 | 8 | 4 | 287.86 | 305.31 | 305.31 | 784.99 | 0.00 | |
| I-28 | 25 | 10 | 4 | 339.03 | 246.97 | 360.49 | 3600.00 | 45.97 | |
| I-31 | 10 | 4 | 2 | 334.37 | 360.76 | 360.77 | 0.41 | 0.00 | |
| I-32 | 15 | 6 | 3 | 430.72 | 440.22 | 440.22 | 21.25 | 0.00 | |
| I-33 | 20 | 8 | 4 | 509.40 | 512.96 | 512.96 | 245.81 | 0.00 | |
| I-34 | 25 | 10 | 4 | 701.57 | 731.33 | 806.82 | 3600.00 | 10.32 | |
| I-37 | 10 | 4 | 2 | 284.29 | 284.29 | 284.29 | 0.32 | 0.00 | |
| I-38 | 15 | 6 | 3 | 469.92 | 491.85 | 491.85 | 17.51 | 0.00 | |
| I-39 | 20 | 8 | 4 | 615.10 | 646.71 | 646.71 | 208.44 | 0.00 | |
| I-40 | 25 | 10 | 4 | 787.65 | 675.07 | 997.81 | 3600.00 | 47.81 | |
| | | | | | Min. | 0.32 | | 0.00 | |
| | | | | | Avg. | 308.21 | | 7.25 | |
| | | | | | Max. | 3489.41 | | 61.86 | |

| (c) 60% of critical shelters | | | | | | | | | |
|------------------------------|------|-------|-----|------------|----------|----------|----------|--------|--|
| Instance | V' | V_c | K | ST_{Lab} | Z_{LB} | Z_{UB} | Time (s) | Gap(%) | |
| I-1 | 10 | 6 | 2 | 202.14 | 250.77 | 250.77 | 5.53 | 0.00 | |
| I-2 | 15 | 9 | 3 | 269.33 | 327.65 | 327.65 | 94.22 | 0.00 | |
| I-3 | 20 | 12 | 4 | 367.78 | 386.52 | 452.13 | 3600.00 | 16.97 | |
| I-7 | 10 | 6 | 3 | 213.16 | 230.82 | 230.82 | 1.72 | 0.00 | |
| I-8 | 15 | 9 | 3 | 271.64 | 238.61 | 342.26 | 3600.00 | 43.44 | |
| I-13 | 10 | 6 | 2 | 193.81 | 277.99 | 277.99 | 8.52 | 0.00 | |
| I-14 | 15 | 9 | 3 | 270.76 | 331.24 | 331.24 | 63.59 | 0.00 | |
| I-15 | 20 | 12 | 4 | 423.86 | 508.15 | 520.67 | 3600.00 | 2.46 | |
| I-19 | 10 | 6 | 2 | 196.14 | 256.06 | 256.06 | 3.73 | 0.00 | |
| I-20 | 15 | 9 | 3 | 238.82 | 308.03 | 308.03 | 127.28 | 0.00 | |
| I-21 | 20 | 12 | 4 | 446.36 | 464.75 | 547.10 | 3600.00 | 17.72 | |
| I-25 | 10 | 6 | 2 | 243.53 | 304.07 | 304.07 | 9.73 | 0.00 | |
| I-26 | 15 | 9 | 3 | 275.73 | 352.95 | 352.95 | 1192.94 | 0.00 | |
| I-27 | 20 | 12 | 4 | 433.92 | 522.05 | 522.05 | 1028.16 | 0.00 | |
| I-28 | 25 | 15 | 4 | 526.36 | 398.14 | 734.97 | 3600.00 | 84.60 | |
| I-31 | 10 | 6 | 2 | 530.88 | 671.56 | 671.56 | 3.92 | 0.00 | |
| I-32 | 15 | 9 | 3 | 752.72 | 925.45 | 925.45 | 1204.90 | 0.00 | |
| I-33 | 20 | 12 | 4 | 933.13 | 751.44 | 1160.15 | 3600.00 | 54.39 | |
| I-37 | 10 | 6 | 2 | 556.95 | 753.21 | 753.21 | 6.54 | 0.00 | |
| I-38 | 15 | 9 | 3 | 796.21 | 734.56 | 1137.60 | 3600.00 | 54.87 | |
| | | | | | Min. | 1.72 | | 0.00 | |
| | | | | | Avg. | 288.52 | | 13.72 | |
| | | | | | Max. | 1204.90 | | 84.60 | |

Table 5.6 – MILP results on small-medium instances in set C

| (a) 20% of critical shelters | | | | | | | | | | | |
|------------------------------|------|-------|-----|-----------|-------------|----------|----------|--------|--|--|--|
| Instance | V' | V_c | K | ST_{LB} | Z_{LB} | Z_{UB} | Time (s) | Gap(%) | | | |
| C-1 | 10 | 2 | 2 | 66.56 | 66.56 | 66.56 | 1.37 | 0.00 | | | |
| C-2 | 15 | 3 | 3 | 48.92 | 48.92 | 48.92 | 1.72 | 0.00 | | | |
| C-3 | 20 | 4 | 4 | 124.12 | 124.12 | 124.12 | 88.62 | 0.00 | | | |
| C-4 | 25 | 5 | 4 | 127.97 | - | - | 3600.00 | - | | | |
| C-7 | 10 | 2 | 3 | 59.69 | 59.69 | 59.69 | 0.09 | 0.00 | | | |
| C-8 | 15 | 3 | 3 | 57.30 | 68.07 | 68.07 | 1247.34 | 0.00 | | | |
| C-9 | 20 | 4 | 4 | 136.66 | - | - | 3600.00 | - | | | |
| C-13 | 10 | 2 | 2 | 60.82 | 60.82 | 60.82 | 0.50 | 0.00 | | | |
| C-14 | 15 | 3 | 3 | 50.61 | 50.61 | 50.61 | 1.48 | 0.00 | | | |
| C-15 | 20 | 4 | 4 | 111.26 | 111.26 | 111.26 | 9.74 | 0.00 | | | |
| C-16 | 25 | 5 | 4 | 118.64 | - | - | 3600.00 | - | | | |
| C-19 | 10 | 2 | 2 | 60.83 | 60.83 | 60.83 | 1.36 | 0.00 | | | |
| C-20 | 15 | 3 | 3 | 49.91 | 49.91 | 49.91 | 7.45 | 0.00 | | | |
| C-21 | 20 | 4 | 4 | 89.84 | 89.84 | 89.84 | 79.84 | 0.00 | | | |
| C-22 | 25 | 5 | 4 | 130.05 | - | - | 3600.00 | - | | | |
| C-25 | 10 | 2 | 2 | 55.15 | 71.78 | 71.78 | 6.28 | 0.00 | | | |
| C-26 | 15 | 3 | 3 | 73.94 | 73.94 | 73.94 | 3.51 | 0.00 | | | |
| C-27 | 20 | 4 | 4 | 140.06 | 140.06 | 140.06 | 47.88 | 0.00 | | | |
| C-28 | 25 | 5 | 4 | 117.50 | - | - | 3600.00 | - | | | |
| C-31 | 10 | 2 | 2 | 52.05 | 52.05 | 52.05 | 0.15 | 0.00 | | | |
| C-32 | 15 | 3 | 3 | 144.05 | 144.05 | 144.05 | 0.12 | 0.00 | | | |
| C-33 | 20 | 4 | 4 | 203.49 | 203.49 | 203.49 | 3.10 | 0.00 | | | |
| C-34 | 25 | 5 | 4 | 240.89 | 240.89 | 240.89 | 117.64 | 0.00 | | | |
| C-35 | 30 | 6 | 4 | 426.69 | 293.19 | 427.37 | 3600.00 | 45.77 | | | |
| C-37 | 10 | 2 | 2 | 41.57 | 41.57 | 41.57 | 0.08 | 0.00 | | | |
| C-38 | 15 | 3 | 3 | 59.89 | 59.89 | 59.89 | 0.73 | 0.00 | | | |
| C-39 | 20 | 4 | 4 | 126.71 | 126.71 | 126.71 | 0.96 | 0.00 | | | |
| C-40 | 25 | 5 | 4 | 242.84 | 242.84 | 242.84 | 11.55 | 0.00 | | | |
| C-41 | 30 | 6 | 4 | 308.57 | 245.51 | 308.57 | 3600.00 | 25.69 | | | |
| | | | | | Min. | | 0.08 | 0.00 | | | |
| | | | | | Avg. | | 74.16 | 2.98 | | | |
| | | | | | Max. | | 1247.34 | 45.77 | | | |

| (b) 40% of critical shelters | | | | | | | | | | | |
|------------------------------|------|-------|-----|-----------|-------------|----------|----------|--------|--|--|--|
| Instance | V' | V_c | K | ST_{LB} | Z_{LB} | Z_{UB} | Time (s) | Gap(%) | | | |
| C-1 | 10 | 4 | 2 | 195.20 | 202.04 | 202.04 | 2.31 | 0.00 | | | |
| C-2 | 15 | 6 | 3 | 161.71 | 174.88 | 174.88 | 6.88 | 0.00 | | | |
| C-3 | 20 | 8 | 4 | 280.94 | 213.89 | 280.94 | 3600.00 | 31.35 | | | |
| C-7 | 10 | 4 | 3 | 101.00 | 101.00 | 101.00 | 0.28 | 0.00 | | | |
| C-8 | 15 | 6 | 3 | 131.83 | 148.35 | 148.35 | 3180.09 | 0.00 | | | |
| C-9 | 20 | 8 | 4 | 273.43 | - | - | 3600.00 | - | | | |
| C-13 | 10 | 4 | 2 | 163.96 | 163.96 | 163.96 | 2.19 | 0.00 | | | |
| C-14 | 15 | 6 | 3 | 173.82 | 180.10 | 180.10 | 8.47 | 0.00 | | | |
| C-15 | 20 | 8 | 4 | 302.76 | 302.76 | 302.76 | 29.14 | 0.00 | | | |
| C-16 | 25 | 10 | 4 | 287.49 | 225.83 | 323.41 | 3600.00 | 43.21 | | | |
| C-19 | 10 | 4 | 2 | 148.21 | 148.21 | 148.21 | 0.53 | 0.00 | | | |
| C-20 | 15 | 6 | 3 | 192.06 | 192.06 | 192.06 | 12.93 | 0.00 | | | |
| C-21 | 20 | 8 | 4 | 232.71 | 236.41 | 236.41 | 69.77 | 0.00 | | | |
| C-22 | 25 | 10 | 4 | 309.09 | 245.28 | 348.00 | 3600.00 | 41.88 | | | |
| C-25 | 10 | 4 | 2 | 153.55 | 166.37 | 166.37 | 1.08 | 0.00 | | | |
| C-26 | 15 | 6 | 3 | 216.41 | 219.82 | 219.82 | 26.79 | 0.00 | | | |
| C-27 | 20 | 8 | 4 | 315.57 | 275.68 | 341.88 | 3600.00 | 24.01 | | | |
| C-31 | 10 | 4 | 2 | 289.17 | 309.70 | 309.70 | 0.26 | 0.00 | | | |
| C-32 | 15 | 6 | 3 | 477.38 | 477.38 | 477.38 | 8.89 | 0.00 | | | |
| C-33 | 20 | 8 | 4 | 601.65 | 614.42 | 614.42 | 139.13 | 0.00 | | | |
| C-34 | 25 | 10 | 4 | 689.85 | 502.50 | 792.69 | 3600.00 | 57.75 | | | |
| C-37 | 10 | 4 | 2 | 328.63 | 328.63 | 328.63 | 0.29 | 0.00 | | | |
| C-38 | 15 | 6 | 3 | 399.65 | 413.90 | 413.90 | 10.09 | 0.00 | | | |
| C-39 | 20 | 8 | 4 | 646.78 | 670.51 | 670.51 | 138.85 | 0.00 | | | |
| C-40 | 25 | 10 | 4 | 858.54 | 658.20 | 1055.14 | 3600.00 | 60.31 | | | |
| | | | | | Min. | | 0.26 | 0.00 | | | |
| | | | | | Avg. | | 203.19 | 10.77 | | | |
| | | | | | Max. | | 3180.09 | 60.31 | | | |

| (c) 60% of critical shelters | | | | | | | | | | | |
|------------------------------|------|-------|-----|-----------|-------------|----------|----------|--------|--|--|--|
| Instance | V' | V_c | K | ST_{LB} | Z_{LB} | Z_{UB} | Time (s) | Gap(%) | | | |
| C-1 | 10 | 6 | 2 | 212.83 | 299.24 | 299.24 | 11.68 | 0.00 | | | |
| C-2 | 15 | 9 | 3 | 275.14 | 351.17 | 351.17 | 385.35 | 0.00 | | | |
| C-3 | 20 | 12 | 4 | 473.88 | 468.08 | 555.53 | 3600.12 | 18.68 | | | |
| C-7 | 10 | 6 | 3 | 196.13 | 199.15 | 199.15 | 0.44 | 0.00 | | | |
| C-8 | 15 | 9 | 3 | 250.24 | 262.94 | 369.44 | 3600.05 | 40.50 | | | |
| C-13 | 10 | 6 | 2 | 293.02 | 363.81 | 363.81 | 11.59 | 0.00 | | | |
| C-14 | 15 | 9 | 3 | 275.42 | 319.31 | 319.31 | 35.30 | 0.00 | | | |
| C-15 | 20 | 12 | 4 | 475.72 | 502.50 | 792.68 | 3600.15 | 57.75 | | | |
| C-19 | 10 | 6 | 2 | 202.7 | 256.74 | 256.74 | 1.25 | 0.00 | | | |
| C-20 | 15 | 9 | 3 | 333.04 | 418.71 | 418.71 | 874.65 | 0.00 | | | |
| C-21 | 20 | 12 | 4 | 385.08 | 415.27 | 448.34 | 3600.01 | 7.96 | | | |
| C-25 | 10 | 6 | 2 | 243.97 | 332.92 | 332.92 | 6.27 | 0.00 | | | |
| C-26 | 15 | 9 | 3 | 325.15 | 402.48 | 402.48 | 1082.55 | 0.00 | | | |
| C-27 | 20 | 12 | 4 | 485.32 | 460.36 | 595.68 | 3600.00 | 29.39 | | | |
| C-31 | 10 | 6 | 2 | 573.43 | 717.44 | 717.44 | 1.60 | 0.00 | | | |
| C-32 | 15 | 9 | 3 | 713.18 | 870.12 | 870.12 | 3221.57 | 0.00 | | | |
| C-33 | 20 | 12 | 4 | 1080.12 | 931.68 | 1296.19 | 3600.02 | 39.12 | | | |
| C-37 | 10 | 6 | 2 | 567.2 | 783.92 | 783.92 | 3.94 | 0.00 | | | |
| C-38 | 15 | 9 | 3 | 798.79 | 771.06 | 1137.75 | 3600.01 | 47.56 | | | |
| | | | | | Min. | | 0.44 | 0.00 | | | |
| | | | | | Avg. | | 469.68 | 12.68 | | | |
| | | | | | Max. | | 3221.57 | 57.75 | | | |

Table 5.7 – Results of TPPM on small-medium instances in set I

| Instance | V' | K | V _c | 20% of critical shelters | | | | | | | 40% of critical shelters | | | | | | | 60% of critical shelters | | | | | | | |
|----------|----|---|----------------|--------------------------|------------------|------------------|------------------|----------|--------|----------------|--------------------------|------------------|------------------|------------------|----------|--------|----------------|--------------------------|------------------|------------------|------------------|----------|--------|--------|--------|
| | | | | LB | Z _{min} | Z _{avg} | Z _{max} | Time (s) | Gap(%) | V _c | LB | Z _{min} | Z _{avg} | Z _{max} | Time (s) | Gap(%) | V _c | LB | Z _{min} | Z _{avg} | Z _{max} | Time (s) | Gap(%) | | |
| I-1 | 10 | 2 | 2 | 42.52 | 42.52 | 42.52 | 42.52 | 0.13 | 0.00 | 0.00 | 4 | 142.34 | 142.34 | 142.34 | 142.34 | 0.06 | 0.00 | 6 | 250.77 | 250.77 | 250.77 | 250.77 | 0.30 | 0.00 | |
| I-2 | 15 | 3 | 3 | 57.01 | 57.01 | 57.01 | 57.01 | 0.16 | 0.00 | 0.00 | 6 | 170.91 | 170.91 | 170.91 | 170.91 | 0.10 | 0.00 | 9 | 327.65 | 327.65 | 327.65 | 327.65 | 0.37 | 0.00 | |
| I-3 | 20 | 4 | 4 | 100.07 | 100.07 | 100.07 | 100.07 | 0.22 | 0.00 | 0.00 | 8 | 288.29 | 288.29 | 288.29 | 288.29 | 0.22 | 0.00 | 12 | 386.52 | 452.13 | 452.13 | 452.13 | 0.45 | 16.97 | |
| I-4 | 25 | 4 | 5 | 127.00 | 127.00 | 127.00 | 127.00 | 0.20 | 0.00 | 0.00 | 10 | 313.12 | 359.06 | 359.06 | 359.06 | 0.23 | 14.67 | 15 | 495.01 | 697.24 | 697.24 | 697.24 | 0.77 | 40.85 | |
| I-5 | 30 | 4 | 6 | 170.42 | 174.12 | 174.12 | 174.12 | 0.47 | 2.17 | 2.17 | 12 | 401.22 | 472.26 | 472.26 | 472.26 | 0.72 | 17.71 | 18 | 595.26 | 891.68 | 891.68 | 891.68 | 0.84 | 49.80 | |
| I-6 | 50 | 7 | 10 | 302.46 | 302.46 | 302.46 | 302.46 | 0.55 | 0.00 | 0.00 | 20 | 635.33 | 728.93 | 728.93 | 728.93 | 1.28 | 14.73 | 30 | 942.82 | 1362.08 | 1362.08 | 1362.08 | 2.88 | 44.47 | |
| I-7 | 10 | 3 | 2 | 42.56 | 42.56 | 42.56 | 42.56 | 0.19 | 0.00 | 0.00 | 4 | 118.38 | 118.38 | 118.38 | 118.38 | 0.27 | 0.00 | 6 | 230.82 | 230.82 | 230.82 | 230.82 | 0.19 | 0.00 | |
| I-8 | 15 | 3 | 3 | 66.83 | 66.83 | 66.83 | 66.83 | 0.07 | 0.00 | 0.00 | 6 | 153.19 | 153.19 | 153.19 | 153.19 | 0.15 | 0.00 | 9 | 271.64 | 342.26 | 342.26 | 342.26 | 0.27 | 26.00 | |
| I-9 | 20 | 4 | 4 | 101.88 | 101.88 | 101.88 | 101.88 | 0.09 | 0.00 | 0.00 | 8 | 235.17 | 267.20 | 267.20 | 267.20 | 0.21 | 4.71 | 12 | 417.55 | 509.88 | 509.88 | 509.88 | 0.40 | 22.11 | |
| I-10 | 25 | 5 | 5 | 114.49 | 114.49 | 114.49 | 114.49 | 0.11 | 0.00 | 0.00 | 10 | 318.55 | 335.26 | 335.26 | 335.26 | 0.24 | 5.25 | 15 | 464.75 | 562.98 | 562.98 | 562.98 | 0.84 | 21.14 | |
| I-11 | 30 | 6 | 6 | 144.18 | 144.18 | 144.18 | 144.18 | 0.14 | 0.00 | 0.00 | 12 | 316.69 | 337.37 | 337.37 | 337.37 | 0.32 | 6.53 | 18 | 519.35 | 622.53 | 622.53 | 622.53 | 0.95 | 19.87 | |
| I-12 | 50 | 9 | 10 | 253.96 | 253.96 | 253.96 | 253.96 | 0.34 | 0.00 | 0.00 | 20 | 585.93 | 620.65 | 620.65 | 620.65 | 1.10 | 5.93 | 30 | 962.22 | 1191.50 | 1191.50 | 1191.50 | 3.14 | 23.83 | |
| I-13 | 10 | 2 | 2 | 31.56 | 31.56 | 31.56 | 31.56 | 0.04 | 0.00 | 0.00 | 4 | 124.55 | 124.55 | 124.55 | 124.55 | 0.20 | 0.00 | 6 | 277.99 | 277.99 | 277.99 | 277.99 | 0.31 | 0.00 | |
| I-14 | 15 | 3 | 3 | 66.14 | 66.14 | 66.14 | 66.14 | 0.08 | 0.00 | 0.00 | 6 | 189.38 | 189.38 | 189.38 | 189.38 | 0.42 | 0.00 | 9 | 331.24 | 331.24 | 332.79 | 335.11 | 0.58 | 0.00 | |
| I-15 | 20 | 4 | 4 | 108.40 | 108.40 | 108.40 | 108.40 | 0.08 | 0.00 | 0.00 | 8 | 289.57 | 289.57 | 289.57 | 289.57 | 0.43 | 0.00 | 12 | 508.15 | 520.67 | 520.67 | 520.67 | 0.52 | 2.46 | |
| I-16 | 25 | 4 | 5 | 158.41 | 158.41 | 158.41 | 158.41 | 0.11 | 0.00 | 0.00 | 10 | 380.26 | 406.06 | 406.06 | 406.06 | 0.31 | 12.71 | 15 | 568.30 | 745.63 | 745.63 | 745.63 | 0.90 | 31.20 | |
| I-17 | 30 | 4 | 6 | 212.09 | 213.03 | 213.03 | 213.03 | 0.62 | 0.44 | 0.44 | 12 | 411.89 | 504.15 | 504.15 | 504.15 | 0.30 | 22.40 | 18 | 612.67 | 916.85 | 916.85 | 916.85 | 0.51 | 49.65 | |
| I-18 | 50 | 6 | 10 | 315.79 | 343.03 | 343.03 | 343.03 | 0.58 | 8.63 | 8.63 | 20 | 703.64 | 874.26 | 874.26 | 874.26 | 1.37 | 24.25 | 30 | 1088.63 | 1601.30 | 1601.30 | 1601.30 | 2.16 | 47.09 | |
| I-19 | 10 | 2 | 2 | 43.42 | 43.42 | 43.42 | 43.42 | 0.23 | 0.00 | 0.00 | 4 | 118.63 | 118.63 | 118.63 | 118.63 | 0.19 | 0.00 | 6 | 256.06 | 256.06 | 256.06 | 256.06 | 0.28 | 0.00 | |
| I-20 | 15 | 3 | 3 | 44.92 | 44.92 | 44.92 | 44.92 | 0.28 | 0.00 | 0.00 | 6 | 140.25 | 140.25 | 140.25 | 140.25 | 0.24 | 0.00 | 9 | 308.03 | 308.03 | 308.03 | 308.03 | 0.53 | 0.00 | |
| I-21 | 20 | 4 | 4 | 133.71 | 133.71 | 133.71 | 133.71 | 0.24 | 0.00 | 0.00 | 8 | 316.58 | 316.58 | 316.58 | 316.58 | 0.31 | 0.00 | 12 | 464.75 | 547.10 | 547.10 | 547.10 | 0.77 | 17.72 | |
| I-22 | 25 | 4 | 5 | 111.64 | 111.64 | 111.64 | 111.64 | 0.38 | 0.00 | 0.00 | 10 | 280.27 | 315.95 | 315.95 | 315.95 | 0.62 | 12.73 | 15 | 485.33 | 664.57 | 664.57 | 664.57 | 0.81 | 36.93 | |
| I-23 | 30 | 5 | 6 | 164.82 | 164.82 | 164.82 | 164.82 | 0.45 | 0.00 | 0.00 | 12 | 400.79 | 426.85 | 426.85 | 426.85 | 0.42 | 6.50 | 18 | 585.53 | 746.84 | 746.84 | 746.84 | 0.58 | 27.55 | |
| I-24 | 50 | 6 | 10 | 284.16 | 291.88 | 291.88 | 291.88 | 1.12 | 2.72 | 2.72 | 20 | 648.66 | 786.96 | 786.96 | 786.96 | 0.99 | 21.32 | 30 | 972.14 | 1509.91 | 1518.56 | 1542.22 | 0.97 | 55.92 | |
| I-25 | 10 | 2 | 2 | 34.47 | 34.47 | 34.47 | 34.47 | 0.06 | 0.00 | 0.00 | 4 | 165.30 | 165.30 | 165.30 | 165.30 | 0.16 | 0.00 | 6 | 304.07 | 304.07 | 304.07 | 304.07 | 0.30 | 0.00 | |
| I-26 | 15 | 3 | 3 | 65.49 | 65.49 | 65.49 | 65.49 | 0.07 | 0.00 | 0.00 | 6 | 181.84 | 181.84 | 181.84 | 181.84 | 0.41 | 0.00 | 9 | 352.95 | 352.95 | 352.95 | 352.95 | 0.46 | 0.00 | |
| I-27 | 20 | 4 | 4 | 92.96 | 92.96 | 92.96 | 92.96 | 0.08 | 0.00 | 0.00 | 8 | 305.31 | 305.31 | 305.31 | 305.31 | 0.53 | 0.00 | 12 | 522.05 | 522.05 | 522.05 | 522.05 | 0.39 | 0.00 | |
| I-28 | 25 | 4 | 5 | 144.33 | 144.33 | 144.33 | 144.33 | 0.13 | 0.00 | 0.00 | 10 | 339.03 | 360.49 | 360.49 | 360.49 | 0.44 | 6.33 | 15 | 526.36 | 716.98 | 716.98 | 716.98 | 0.35 | 36.21 | |
| I-29 | 30 | 5 | 6 | 147.51 | 147.51 | 147.51 | 147.51 | 0.15 | 0.00 | 0.00 | 12 | 372.73 | 424.72 | 424.72 | 424.72 | 0.70 | 13.95 | 18 | 617.20 | 804.67 | 804.67 | 804.67 | 0.43 | 30.37 | |
| I-30 | 50 | 6 | 10 | 283.85 | 287.01 | 287.01 | 287.01 | 0.46 | 1.11 | 1.11 | 20 | 648.44 | 818.48 | 819.89 | 825.54 | 0.76 | 26.22 | 30 | 980.42 | 1530.57 | 1533.85 | 1538.76 | 2.28 | 56.11 | |
| I-31 | 10 | 2 | 2 | 140.62 | 140.62 | 140.62 | 140.62 | 0.10 | 0.00 | 0.00 | 4 | 380.76 | 380.77 | 380.77 | 380.77 | 0.18 | 0.00 | 6 | 671.56 | 671.56 | 671.56 | 671.56 | 0.30 | 0.00 | |
| I-32 | 15 | 3 | 3 | 177.54 | 177.54 | 177.54 | 177.54 | 0.23 | 0.00 | 0.00 | 6 | 440.22 | 440.22 | 440.22 | 440.22 | 0.20 | 0.00 | 9 | 925.46 | 925.46 | 927.44 | 930.88 | 0.44 | 0.00 | |
| I-33 | 20 | 4 | 4 | 151.99 | 151.99 | 151.99 | 151.99 | 0.28 | 0.00 | 0.00 | 8 | 512.96 | 512.96 | 512.96 | 512.96 | 0.25 | 0.00 | 12 | 933.13 | 1160.15 | 1160.15 | 1160.15 | 0.49 | 24.33 | |
| I-34 | 25 | 4 | 5 | 338.89 | 338.89 | 338.89 | 338.89 | 0.29 | 0.00 | 0.00 | 10 | 731.33 | 806.82 | 806.82 | 806.82 | 0.70 | 10.32 | 15 | 1207.41 | 1745.01 | 1745.01 | 1745.01 | 0.57 | 44.53 | |
| I-35 | 30 | 4 | 6 | 328.94 | 328.94 | 328.94 | 328.94 | 0.26 | 0.00 | 0.00 | 12 | 999.76 | 1249.25 | 1249.25 | 1249.25 | 0.71 | 24.95 | 18 | 1469.75 | 2468.19 | 2468.19 | 2468.19 | 0.75 | 67.93 | |
| I-36 | 50 | 6 | 10 | 824.39 | 829.30 | 829.30 | 829.30 | 0.39 | 0.60 | 0.60 | 20 | 1898.48 | 2434.48 | 2434.48 | 2434.48 | 1.44 | 28.23 | 30 | 2715.64 | 4597.56 | 4601.16 | 4608.29 | 3.88 | 69.30 | |
| I-37 | 10 | 2 | 2 | 43.26 | 43.26 | 43.26 | 43.26 | 0.08 | 0.00 | 0.00 | 4 | 284.29 | 284.29 | 284.29 | 284.29 | 0.21 | 0.00 | 6 | 753.21 | 753.21 | 753.21 | 753.21 | 0.34 | 0.00 | |
| I-38 | 15 | 3 | 3 | 125.50 | 125.50 | 125.50 | 125.50 | 0.12 | 0.00 | 0.00 | 6 | 491.84 | 491.85 | 491.85 | 491.85 | 0.21 | 0.00 | 9 | 796.21 | 1135.24 | 1135.24 | 1135.24 | 0.62 | 42.58 | |
| I-39 | 20 | 4 | 4 | 131.68 | 131.68 | 131.68 | 131.68 | 0.10 | 0.00 | 0.00 | 8 | 646.71 | 646.71 | 646.71 | 646.71 | 0.26 | 0.00 | 12 | 1107.80 | 1549.77 | 1549.77 | 1549.77 | 0.46 | 39.90 | |
| I-40 | 25 | 4 | 5 | 225.10 | 225.10 | 225.10 | 225.10 | 0.23 | 0.00 | 0.00 | 10 | 787.65 | 997.81 | 997.81 | 997.81 | 0.80 | 26.68 | 15 | 1400.62 | 2350.85 | 2350.85 | 2350.85 | 1.05 | 67.84 | |
| I-41 | 30 | 4 | 6 | 346.82 | 346.82 | 346.82 | 346.82 | 0.22 | 0.00 | 0.00 | 12 | 1091.64 | 1517.51 | 1517.51 | 1517.51 | 0.41 | 39.01 | 18 | 1821.40 | 3542.42 | 3542.42 | 3542.42 | 1.27 | 94.49 | |
| I-42 | 50 | 6 | 10 | 641.78 | 641.78 | 641.78 | 641.78 | 0.68 | 0.00 | 0.00 | 20 | 1870.24 | 2824.27 | 2824.27 | 2824.27 | 1.84 | 51.01 | 30 | 3079.43 | 6470.45 | 6477.85 | 6492.32 | 3.54 | 110.12 | |
| | | | | Min. | | | | 0.04 | 0.00 | | | Min. | | | | 0.06 | 0.00 | | Min. | | | | | 0.19 | 0.00 |
| | | | | Avg. | | | | 0.26 | 0.37 | | | Avg. | | | | 0.50 | 9.43 | | Avg. | | | | | 0.91 | 28.97 |
| | | | | Max. | | | | 1.12 | 8.63 | | | Max. | | | | 1.84 | 51.01 | | Max. | | | | | 3.88 | 110.12 |

Table 5.9 – Results of TPPM on large instances

| (a) Set I | | | | | | | | | |
|--------------------------|------|-------|-----|---------|-----------|-----------|-----------|----------|--------|
| 20% of critical shelters | | | | | | | | | |
| Instance | V' | V_c | K | LB | Z_{min} | Z_{avg} | Z_{max} | Time (s) | Gap(%) |
| I-43 | 75 | 15 | 13 | 374.16 | 374.16 | 374.27 | 374.46 | 0.61 | 0.00 |
| I-44 | 100 | 20 | 10 | 599.35 | 628.12 | 630.10 | 632.08 | 3.47 | 4.80 |
| I-45 | 150 | 30 | 17 | 829.26 | 835.41 | 837.65 | 840.96 | 7.58 | 0.74 |
| I-46 | 199 | 39 | 22 | 1109.90 | 1114.09 | 1116.64 | 1122.24 | 23.06 | 0.38 |
| I-47 | 120 | 24 | 12 | 1727.11 | 1739.00 | 1743.29 | 1750.93 | 2.67 | 0.69 |
| I-48 | 100 | 20 | 11 | 1402.19 | 1410.78 | 1410.78 | 1410.78 | 1.91 | 0.61 |
| | | | | Min. | | | | 0.61 | 0.00 |
| | | | | Avg. | | | | 6.55 | 1.20 |
| | | | | Max. | | | | 23.06 | 4.80 |
| 40% of critical shelters | | | | | | | | | |
| Instance | V' | V_c | K | LB | Z_{min} | Z_{avg} | Z_{max} | Time (s) | Gap(%) |
| I-43 | 75 | 30 | 13 | 889.69 | 946.57 | 948.96 | 950.76 | 3.00 | 6.39 |
| I-44 | 100 | 40 | 10 | 1327.79 | 1748.28 | 1754.97 | 1778.04 | 9.77 | 31.67 |
| I-45 | 150 | 60 | 17 | 1898.67 | 2276.80 | 2277.70 | 2278.42 | 40.59 | 19.92 |
| I-46 | 199 | 80 | 22 | 2466.27 | 2989.53 | 2991.94 | 2993.89 | 92.93 | 21.22 |
| I-47 | 120 | 48 | 12 | 4329.02 | 6217.48 | 6257.92 | 6300.34 | 9.82 | 43.62 |
| I-48 | 100 | 40 | 11 | 3840.70 | 6159.77 | 6159.86 | 6160.10 | 4.50 | 60.38 |
| | | | | Min. | | | | 3.00 | 6.39 |
| | | | | Avg. | | | | 26.76 | 30.53 |
| | | | | Max. | | | | 92.93 | 60.38 |
| 60% of critical shelters | | | | | | | | | |
| Instance | V' | V_c | K | LB | Z_{min} | Z_{avg} | Z_{max} | Time (s) | Gap(%) |
| I-43 | 75 | 45 | 13 | 1436.17 | 1738.71 | 1738.74 | 1738.88 | 13.19 | 21.07 |
| I-44 | 100 | 60 | 10 | 1980.97 | 3287.63 | 3293.48 | 3304.91 | 16.46 | 65.96 |
| I-45 | 150 | 90 | 17 | 2963.14 | 4308.89 | 4311.51 | 4314.96 | 66.61 | 45.42 |
| I-46 | 199 | 120 | 22 | 3826.39 | 5629.54 | 5636.16 | 5642.46 | 148.34 | 47.12 |
| I-47 | 120 | 72 | 12 | 6517.91 | 12752.15 | 12788.81 | 12833.85 | 23.56 | 95.65 |
| I-48 | 100 | 60 | 11 | 6254.42 | 14138.80 | 14141.68 | 14146.83 | 19.38 | 126.06 |
| | | | | Min. | | | | 13.19 | 21.07 |
| | | | | Avg. | | | | 47.92 | 66.88 |
| | | | | Max. | | | | 148.34 | 126.06 |
| (b) Set C | | | | | | | | | |
| 20% of critical shelters | | | | | | | | | |
| Instance | V' | V_c | K | LB | Z_{min} | Z_{avg} | Z_{max} | Time (s) | Gap(%) |
| C-43 | 75 | 15 | 13 | 386.44 | 386.44 | 386.44 | 386.44 | 0.59 | 0.00 |
| C-44 | 100 | 20 | 10 | 593.80 | 601.39 | 601.39 | 601.39 | 2.05 | 1.28 |
| C-45 | 150 | 30 | 17 | 934.47 | 941.29 | 943.23 | 946.24 | 6.54 | 0.73 |
| C-46 | 199 | 39 | 21 | 1144.67 | 1149.85 | 1151.66 | 1153.48 | 23.45 | 0.45 |
| C-47 | 120 | 24 | 12 | 1560.67 | 1575.68 | 1580.76 | 1587.00 | 3.69 | 0.96 |
| C-48 | 100 | 20 | 11 | 1544.46 | 1551.50 | 1551.50 | 1551.50 | 0.84 | 0.46 |
| | | | | Min. | | | | 0.59 | 0.00 |
| | | | | Avg. | | | | 6.19 | 0.65 |
| | | | | Max. | | | | 23.45 | 1.28 |
| 40% of critical shelters | | | | | | | | | |
| Instance | V' | V_c | K | LB | Z_{min} | Z_{avg} | Z_{max} | Time (s) | Gap(%) |
| C-43 | 75 | 30 | 13 | 865.96 | 920.53 | 921.26 | 922.09 | 5.33 | 6.30 |
| C-44 | 100 | 40 | 10 | 1289.91 | 1697.24 | 1700.47 | 1706.68 | 9.22 | 31.58 |
| C-45 | 150 | 60 | 17 | 1952.80 | 2352.02 | 2354.09 | 2357.88 | 35.08 | 20.44 |
| C-46 | 199 | 80 | 21 | 2557.04 | 3166.07 | 3173.65 | 3181.22 | 86.09 | 23.82 |
| C-47 | 120 | 48 | 12 | 3956.89 | 5868.76 | 5886.63 | 5912.90 | 8.08 | 48.32 |
| C-48 | 100 | 40 | 11 | 3867.48 | 6189.36 | 6190.58 | 6192.83 | 9.46 | 60.04 |
| | | | | Min. | | | | 5.33 | 6.30 |
| | | | | Avg. | | | | 25.54 | 31.75 |
| | | | | Max. | | | | 86.09 | 60.04 |
| 60% of critical shelters | | | | | | | | | |
| Instance | V' | V_c | K | LB | Z_{min} | Z_{avg} | Z_{max} | Time (s) | Gap(%) |
| C-43 | 75 | 45 | 13 | 1392.92 | 1706.69 | 1706.69 | 1706.69 | 10.94 | 22.53 |
| C-44 | 100 | 60 | 10 | 1970.19 | 3296.30 | 3301.41 | 3313.91 | 18.09 | 67.31 |
| C-45 | 150 | 90 | 17 | 3010.14 | 4388.38 | 4393.32 | 4397.35 | 70.54 | 45.79 |
| C-46 | 199 | 120 | 21 | 3930.71 | 5925.89 | 5933.75 | 5948.96 | 173.04 | 50.76 |
| C-47 | 120 | 72 | 12 | 6529.94 | 12771.77 | 12798.67 | 12812.68 | 26.61 | 95.59 |
| C-48 | 100 | 60 | 11 | 6264.90 | 14155.54 | 14163.57 | 14188.81 | 23.79 | 125.95 |
| | | | | Min. | | | | 10.94 | 22.53 |
| | | | | Avg. | | | | 53.84 | 67.99 |
| | | | | Max. | | | | 173.04 | 125.95 |

Chapter 6

General conclusions and future research

This chapter presents some conclusions for this thesis, as well as the future works that could be derived from it.

The disaster management is based on four cyclic phases (mitigation, preparedness, response and recovery) in which the humanitarian logistics plays an important role. The operations research contributes to the improvement of the activities (in each phase) that must be carried out in the humanitarian logistics. While the transportation science is mainly involved in the activities of the response phase. The articles reviewed show how the solutions found from the proposed methods (exacts and heuristics) help making better decisions as well as allowing a greater insight and a good analysis of the situation. In the problems addressed in the literature, several features are considered such as uncertainty in the information, transportation of relief products and/or injured people, and objective functions. The latter can be classified into three groups as those that pursue efficiency, effectiveness, or equity goals.

In this thesis, the capacitated vehicle routing problem with time-dependent demands (CVRP-TDD), the robust CVRP-TDD and the cumulative CVRP-TDD are introduced. These problems arise in the aftermath of the disaster (response phase). The CVRP-TDD and its robust counterpart consider an effectiveness goal, while the cumulative CVRP-TDD considers an equity goal. In the CVRP-TDD, people go directly to shelters to receive first aid, but they flee out (with a known and fixed displacement rate) if they do not receive it as quickly as possible. So, the aim is to find distribution routes to stop the population displacement maximizing the number of people attended at shelters. Unlike the CVRP-TDD, the robust counterpart assumes that the displacement rates are uncertain. The cumulative CVRP-TDD differs from the CVRP-TDD in the fact that emergency of the situation is

privileged and, instead of maximizing the satisfied demand, the sum of the arrival times at shelters is minimized. Moreover, a closed system is studied where the total demand remains constant but the demand at each node varies depending on time.

In Chapter 3, two exact methods are proposed to solve the CVRP-TDD and the robust CVRP-TDD. For each problem, a mixed integer linear program (MILP) and a branch-and-price (B&P) algorithm are implemented. In the B&P algorithms, the master problem is formulated as a set partitioning problem and the pricing problem is an elementary longest path problem with resource constraints (ELPPRC) adapted to each problem. The ELPPRC is solved by using dynamic programming in which two dominance rules and an acceleration technique are applied to improve the performance of the algorithms.

In addition to the exact methods, an upper bound based on a spanning tree relaxation with degree constraints is also presented to provide a good quality upper bound when such a bound cannot be obtained by the column generation in the root node of the branch-and-price tree.

The results of the CVRP-TDD show that the B&P algorithm is able to get the optimal solutions in instances with 30 shelters, while the MILP model obtains the optimal solutions in instances with 15 shelters. In addition, it is observed that more than 75% of the initial population is at their original shelters before the population movement. However, a stability analysis demonstrates that most optimal routes change if the displacement rates vary up to a certain percentage (i.e. 30%).

In the robust CVRP-TDD, robust optimization is used to handle the uncertainty in the displacement rates. These rates are characterized by a set of discrete scenarios. Optimal routes can be computed for instances with 30 shelters considering 3 and 6 scenarios. The robust optimization allows to have a greater reliability of the routes in case the displacement rates vary.

In Chapter 4, a metaheuristic framework is developed to solve large instances of the CVRP-TDD in reasonable time. The framework is a GRASP \times ELS which includes four additional metaheuristics as particular cases: GRASP, ILS, ELS and GRASP \times ILS. Any of them can be executed by changing only three parameters. In addition, a splitting procedure which extracts optimally the routes from a giant tour is designed. This allows to alternate between two solution spaces. Local search moves can be evaluated in constant time. This can be done thanks to non-trivial precomputations based on concatenation and reversing operators which are proposed.

In most small-medium instances, all metaheuristics improve or find the same feasible solution reported by the B&P algorithm, but in less time. However, the best results, in terms of solution quality and time, are achieved by ELS with split procedure. On large

instances, ILS and ELS have the best performance. In these cases, split procedure decreases the execution times but sometimes the solution quality is slightly reduced.

In Chapter 5, a MILP and a two-phase heuristic method (TPHM) based on GRASP \times ILS are implemented to solve the cumulative CVRP-TDD. In addition, a lower bound based on a spanning tree relaxation with degree constraints is used to measure the solution quality on large instances.

In the small-medium instances optimally solved by the MILP, TPHM obtains the same solution if two conditions are met: (1) the number of critical shelters in each route are balanced and (2) they are visited before the non-critical nodes. If one of these conditions is not satisfied, the TPHM might not find even a feasible solution.

On large instances, the results show the good performance of the method in terms of execution time and optimality GAP. However, the method can be improved considering moves that involve both types of shelters from where people flee and where people go outside the affected area.

Some future research axes may include:

- A topic of discussion is the type of objective function that should be optimized in the humanitarian context. It would be interesting to contribute to this debate by proposing a third objective that pursues efficiency (i.e. minimization of total travel time) and make a comparison with the results obtained by each of the three objectives (efficiency, effectiveness, and equity)
- The high degree of uncertainty in this type of emergency situations is one of the biggest challenges that must be faced in the research of this issue. In the short term, the proposed solution methods in which uncertainty is not considered could be extended in order to have a greater insight into the situation. In the long term, uncertainty can be included in other parameters (i.e. availability of road segments) and other techniques to handle uncertainty could also be explored (i.e. stochastic or dynamic optimization).
- Integrating the decisions that must be taken in humanitarian logistics can lead to find solutions more adapted to real situations. For example, it might be interesting to combine the routing decisions of the CVRP-TDD with the location of shelters. In this case, the displacement rates can be associated to the location of shelters in which the most affected zones will have a higher displacement rate because people need faster the first aid.

Résumé

Les problèmes de tournées de véhicules avec des demandes dépendantes du temps dans la logistique humanitaire

Le nombre de décès causés par des catastrophes naturelles en 2016 a diminué de 87% par rapport à la moyenne annuelle de la période entre 2006 et 2015. Cependant, l'augmentation du nombre de personnes touchées et d'occurrences des catastrophes inquiète les gouvernements, les organisations humanitaires, les chercheurs et la population en général.

Cette thèse découle de l'intérêt de proposer et de développer des méthodes issues de la Recherche Opérationnelle (RO) pour réduire les conséquences des catastrophes. La RO est une discipline qui utilise des méthodes analytiques pour aider les organisations à prendre des meilleures décisions.

Le manuscrit est composé de 6 chapitres. Le chapitre 1 décrit le contexte et introduit une brève explication de la gestion des catastrophes ainsi qu'une conceptualisation du déplacement de la population comme principale caractéristique des problèmes traités. Ces derniers sont définis également dans ce chapitre.

Le chapitre 2 présente une revue de la littérature sur la logistique humanitaire dans chacune des phases de la gestion des catastrophes. De plus, les problèmes de tournées de véhicules avec un ou plusieurs paramètres changent en fonction du temps sont aussi détaillés.

Le chapitre 3 est divisé en deux sections. La première partie est centrée sur un problème de tournées de véhicules où la demande de chaque refuge diminue avec le temps à un taux constant et connu en raison du déplacement de la population. La deuxième section étudie un problème similaire, mais en considérant que les taux de déplacement sont inconnus. Pour résoudre ces problèmes, un programme linéaire en nombres entiers (PLNE) et un algorithme

de Branch-and-Price (B&P) sont développés et adoptés pour chaque problème.

Le chapitre 4 présente un cadre métaheuristique avec une procédure de split pour résoudre les instances de grande taille du problème traité dans la première section du chapitre 3. Le cadre est un GRASP×ELS dans lequel une de ses variantes (GRASP, ILS, ELS, et GRASP×ILS) peut être exécutée en changeant seulement trois paramètres.

Le chapitre 5 étudie un problème dans lequel la demande diminue dans certains refuges et augmente dans d'autres. Un PLNE est proposé pour ce problème et une méthode heuristique en deux phases basée sur GRASP×ILS est utilisée pour le résoudre.

Le chapitre 6 présente les conclusions générales et les perspectives de recherche.

Chapitre 1 - Contexte et définition des problèmes

Une catastrophe est définie par la Fédération Internationale des Sociétés de la Croix-Rouge et du Croissant-Rouge comme un événement soudain qui perturbe gravement le fonctionnement d'une communauté ou d'une société et entraîne des pertes humaines, matérielles, économiques et environnementales. Cela peut être causé par la nature ou par l'homme.

La gestion des catastrophes est officiellement née au milieu du XX^{ème} siècle pour améliorer la capacité de la population à gérer les crises face à ce type d'événements. Elle permet de réduire la vulnérabilité et le danger auxquels la population est exposée. Ainsi, elle est composée de quatre phases cycliques, comme illustré sur la Figure 6.1.

La phase d'atténuation se réfère à des plans ou à des mécanismes tels que la formation pour réduire la vulnérabilité des personnes, où les gouvernements et les organisations humanitaires jouent un rôle très important. Lors de la phase de préparation, des opérations ou des stratégies sont prévues avant la catastrophe. Dans cette phase, les expériences passées

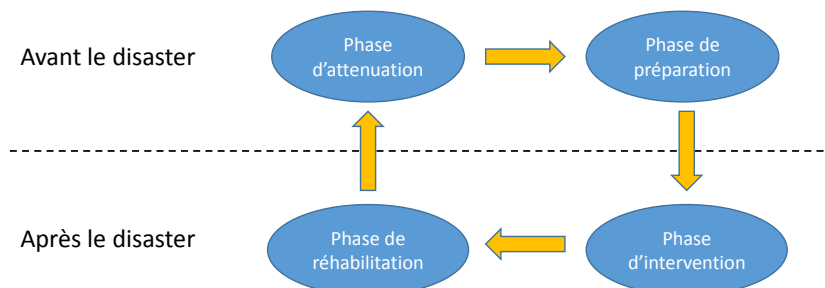


FIGURE 6.1 – Cycle de gestion des catastrophes

doivent être intégrées pour améliorer la réponse aux catastrophes. La phase d'intervention commence immédiatement après qu'une catastrophe ait frappé un territoire. Toutes les opérations prévues dans les phases précédentes doivent être mises en œuvre pour réduire le nombre des personnes impactées. La phase de réhabilitation peut prendre des mois voir des années après la catastrophe et son objectif est d'assurer un retour à la vie normale pour les victimes en terme d'infrastructure et d'environnement, mais aussi en terme psychologique.

La logistique humanitaire intervient principalement au niveaux des phases de préparation, d'intervention et de réhabilitation, permettant de bien structurer l'aide humanitaire et d'obtenir une gestion plus souple afin de minimiser les souffrances de la population.

Logistique humanitaire et logistique commerciale

La logistique humanitaire et la logistique commerciale sont très différentes. La principale différence réside dans le fait que la première s'intéresse à sauver des vies et à minimiser les souffrances des gens, tandis que la seconde, pour la plupart des cas, se concentre sur des objectifs purement économiques tels que la minimisation des coûts de transport et/ou de l'opération. Parmi d'autres différences, on peut citer le degré élevé d'incertitude dans l'information, le grand nombre de parties prenantes ayant des objectifs différents et la grande difficulté à obtenir des ressources disponibles au bon moment.

Déplacement de la population

L'une des conséquences des catastrophes est le déplacement de la population. Il peut avoir lieu avant, pendant et/ou après la catastrophe. Il existe trois types de déplacement : (1) lorsque les personnes sont évacuées vers les refuges pour recevoir les premiers secours, (2) lorsque les personnes sont dans des refuges et ne reçoivent pas d'aide rapidement et décident d'aller chercher l'aide ailleurs et (3) lorsque les personnes sont obligées d'aller ailleurs en dehors de leur ville, car tout est complètement détruit ou un conflit armé les pousse à le faire.

Les déplacements propagent le chaos déjà généré par la catastrophe sur d'autres territoires, compliquant encore le travail des organisations humanitaires pour répondre efficacement aux besoins de l'ensemble de la population touchée.

Problèmes de tournées avec des demandes dépendantes du temps

Le problème de tournées de véhicules avec des demandes dépendantes du temps (Capacitated Vehicle Routing Problem with Time-Dependent Demands "CVRP-TDD"), le CVRP-TDD robuste et le cumulatif CVRP-TDD contribuent à la planification des itinéraires

de distribution des kits d'urgence, immédiatement après que la catastrophe ait frappé un territoire, en tenant compte le déplacement de la population.

Le CVRP-TDD considère que le taux de déplacement des personnes dans les refuges est constant et connu ainsi l'objectif est de maximiser le nombre de personnes desservies dans chaque refuge en apportant l'aide nécessaire le plus rapidement possible. Contrairement au CVRP-TDD, le CVRP-TDD robuste suppose que les taux de déplacement sont incertain. Le cumulatif CVRP-TDD diffère du CVRP-TDD par le fait que les personnes qui fuient les refuges se rendent dans d'autres refuges pour chercher l'aide dont ils ont besoin. Dans ce cas, l'objectif est de minimiser la somme des temps d'arrivée sur l'ensemble des refuges.

Dans ces problèmes, le nombre de refuges ainsi que le nombre de personnes avant qu'elles ne commencent à fuir est fixe et connu. Les kits d'urgence sont transportés dans un nombre déterminé de véhicules homogènes qui quittent le même dépôt. Ainsi, chaque véhicule a un temps maximum pour visiter les refuges et retourner au dépôt.

Chapitre 2 -État de l'art

Dans ce chapitre, la littérature sur la logistique humanitaire et les problèmes de tournées de véhicules dépendants du temps est présentée. Tout au long du manuscrit, un complément à la revue de la littérature, plus axée sur les méthodes de résolution est présenté dans chaque chapitre.

Logistique humanitaire

La logistique humanitaire a été largement discutée au cours des dernières décennies. 8 références bibliographiques proposent un état de l'art générique dans le domaine de la logistique humanitaire, apparues entre 2006 et 2017. De plus, d'autres recherches ont été consacrées à des sujets plus spécifiques tels que : l'incertitude, la phase d'intervention, la phase de réhabilitation, les modèles d'optimisation, l'optimisation multicritère, les systèmes et modèles d'aide à la décision, la gestion des stocks et la localisation des installations.

Dans la plupart des références, la classification la plus courante des articles dans la logistique humanitaire est donnée par les phases de gestion des catastrophes. Par conséquent, nous avons choisi de conserver la même répartition.

Phase d'atténuation

Durant les années 80 et 90, la recherche opérationnelle était plus centrée sur la phase d'atténuation. La plupart des recherches en lien avec cette phase vise l'évaluation des

risques de catastrophe et l'administration foncière. Le premier thème permet d'identifier et d'analyser le risque auquel un territoire est exposé, tandis que le second permet de prendre de meilleures décisions concernant l'utilisation du sol pour éviter les catastrophes telles que les glissements de terrain, les séismes, les coulées de boue, etc.

Phase de préparation

Concernant la phase de préparation, trois activités identifiées doivent être planifiées avant que la catastrophe frappe un territoire : (1) gestion des inventaires et équipements, (2) localisation des sites pour desservir la population affectée (refuges, hôpitaux temporaires, etc.) et (3) la planification de l'évacuation.

Dans la première activité, les niveaux de stocks et la localisation de ceux-ci ainsi que les ressources doivent être décidés. Cela permet de répondre rapidement aux besoins des personnes affectées. Les articles qui se concentrent sur la gestion des stocks sont regroupés en : ceux qui déterminent uniquement les niveaux de stocks et ceux pour lesquels les décisions de niveaux d'inventaire et l'emplacement du lieu de stockage sont optimisés en même temps. Certains des objectifs trouvés dans ces deux groupes sont la minimisation des coûts et la maximisation de la demande couverte.

Pour les problèmes d'emplacement des sites afin d'aider les personnes touchées par la catastrophe, l'objectif est de minimiser la distance parcourue par les victimes vers les refuges, de minimiser les coûts et/ou de minimiser la demande non satisfaite.

Dans la troisième activité, les gens devraient savoir comment évacuer leurs maisons, leurs lieux de travail, les centres commerciaux, leurs villes, etc. Par conséquent, un plan d'évacuation doit être établi pour chacun de ces endroits. Le but est d'identifier les moyens les plus efficaces de déplacer un groupe de personnes des zones dangereuses vers les lieux sûrs, compte tenu de la congestion du réseau causée par le flux chaotique. Les plans d'évacuation peuvent être de petite ou de grande envergure. A petite échelle, les gens doivent évacuer à pied les bâtiments, les stades, les parcs à thème, etc. Alors que sur une grande échelle, d'autres moyens de transport et réseaux de transport (s'il faut utiliser les bateaux, avions, etc.) devraient être utilisés dans le plan d'évacuation. Dans les deux types d'évacuation, le temps d'évacuation total (ou moyen), le risque, le nombre de refuges utilisés et/ou le nombre maximal de personnes évacuées doivent être minimisés.

Certains auteurs combinent plusieurs types et niveaux de décisions, par exemple, combiner l'emplacement des dépôts avec la distribution de kits d'urgence (Location-Routing Problem "LRP"). En outre, il existe des problèmes qui incluent des décisions sur le dimensionnement d'inventaire dans le LRP.

Phase d'intervention

Suivent immédiatement une catastrophe, les interventions d'urgence doivent être réalisées. Par exemple : les kits d'urgence doivent être envoyés dans les zones touchées et les blessés doivent être pris en charge et/ou transportés vers les hôpitaux. Selon les références bibliographiques examinées, la majorité des articles de la logistique humanitaire au cours des dernières années sont au centre de cette phase.

Les différents critères d'optimisation utilisés permettent de mieux représenter la situation d'urgence. Ces critères peuvent être classés en ceux qui poursuivent des objectifs d'efficience, d'efficacité et/ou d'égalité. Les premiers sont liés à la minimisation des coûts opérationnels ou de la distance parcourue. Les secondes cherchent à maximiser les mesures de service telles que l'augmentation de la demande satisfaite et/ou la vitesse à laquelle elle est satisfaite. Les derniers font référence à la répartition équitable des services parmi la population affectée. Ce dernier groupe est difficile à définir en raison des interprétations différentes des termes «égalité» et «impartialité».

Les objectifs d'efficacité sont les mêmes que ceux utilisés dans les problèmes de tournées de véhicules classique appliqué à la distribution commerciale. Certaines études cherchent à minimiser la durée totale du trajet de tous les véhicules, tandis que d'autres travaux minimisent le coût total de l'opération.

Les auteurs intéressés par les mesures de service d'un point de vue bénéficiaire, prennent en compte les objectifs d'efficacité plutôt que d'efficience. La maximisation de la demande satisfaite (ou minimisation de la demande insatisfaite), la minimisation des blessés non suivis et/ou la minimisation des véhicules utilisés sont les critères à optimiser qui garantissent l'efficacité.

Les objectifs de l'égalité sont importants dans le contexte humanitaire car dans ce type de situation d'urgence, il ne faut pas négliger un certain groupe de personnes. Plusieurs auteurs s'intéressent à ce type d'objectifs et en proposent la minimisation de la somme (moyenne) des temps d'arrivées aux nœuds, la minimisation de l'heure d'arrivée maximale et/ou la maximisation du rayon de couverture minimum.

En plus de ce qui précède, il existe des articles qui combinent les différents critères d'optimisation. Par exemple, la minimisation des coûts alors que la demande non satisfaite est pénalisée. D'autres auteurs optimisent chaque critère séparément et les comparent entre eux ou proposent des problèmes multi-objectif qui contiennent la minimisation de la demande non satisfaite, la minimisation du temps de parcours et la minimisation de la différence maximale de taux de satisfaction entre deux nœuds.

Phase de réhabilitation

La restauration des routes et des infrastructures ainsi que la gestion des débris sont les principales opérations traitées dans les références bibliographiques de cette phase.

Les routes peuvent être endommagées ou bloquées par des débris après la catastrophe. Cela rend certaines zones inaccessibles aux équipes de secours et les victimes ne peuvent pas non plus être évacuées vers des lieux sûrs. Par conséquent, les articles qui traitent ce problème s'intéressent à restaurer le plus rapidement possible l'accessibilité aux zones isolées. Ce type de problème est classé comme un problème de tournées de véhicules sur les arcs car les activités et/ou les opérations sont effectuées dans les arcs du réseau. D'autres auteurs résolvent simultanément le problème de la réparation des routes et de la distribution des kits d'urgence.

Pendant le processus de restauration de la route, tous les débris sont placés sur les côtés des routes. Dans la deuxième partie du processus de récupération, les débris doivent être collectés et traités. Ce problème est abordé dans la littérature ainsi que le problème de localisation des sites de stockage de ces débris.

D'autres problèmes ne tiennent pas compte de la restauration des routes, mais de la restauration d'autres infrastructures telles que des services publics essentiels (l'électricité, les télécommunications, l'eau, etc.). Dans ce type de problèmes, on trouve des objectifs tels que : minimiser le temps pendant lequel chaque personne ne reçoit pas le service, minimiser les coûts d'exploitation et/ou minimiser la demande insatisfaite.

Problèmes dépendant du temps

Ces types de problèmes comprennent un ou plusieurs paramètres qui changent en fonction du temps. Cela permet de représenter les problèmes de manière plus réelle. Le principal paramètre dans la littérature des problèmes de tournées de véhicules est le temps de déplacement (vitesse). Ce dernier varie au cours de la journée en raison de la congestion du trafic. De cette idée découlent d'autres problèmes tels que le transport de substances dangereuses, où la probabilité d'accidents et leurs conséquences dépendent du temps, et la réduction des émissions de gaz à effet de serre qui dépendent du temps.

Dans les problèmes abordés dans cette thèse, le paramètre qui change est la demande dans les refuges en raison du déplacement des personnes. Certains articles utilisent le terme « demande dépendante du temps », mais ils se réfèrent davantage au cas où plusieurs périodes de planification sont considérées. D'autre part, le problème de la collection maximale avec des récompenses dépendantes du temps est un problème similaire à ceux présentés dans cette thèse. Cependant, tous les nœuds ne doivent pas être visités et la capacité des véhicules n'est

pas prise en compte dans ce problème.

Chapitre 3 - Méthodes exactes pour la distribution de secours aux nœuds critiques

Dans ce chapitre, le CVRP-TDD et le CVRP-TDD robuste sont résolus en utilisant des méthodes exactes. Pour chaque problème, un PLNE et un algorithme de Branch-and-Price (B&P) sont proposés. Ces méthodes garantissent l'optimalité des solutions.

Capacitated vehicle routing problem with time-dependent demands

Le CVRP considère que la population commence à quitter les refuges à un taux fixe et constant s'ils ne reçoivent pas les premiers secours le plus rapidement possible. Par conséquent, l'objectif est de trouver des routes de distribution pour maximiser le nombre de personnes servies dans les refuges.

Modèle mathématique

Le CVRP-TDD est défini sur un graphe complet non orienté $G = (V, E)$, où $V = \{0, \dots, n\}$ est l'ensemble de nœuds et E est l'ensemble d'arêtes. Pour simplifier, chaque arête est remplacée par deux arcs opposés. Le nœud 0 est le dépôt tandis que $V' = V \setminus \{0\}$ est l'ensemble des refuges. Chaque refuge a une demande initiale (nombre de personnes) a_i et un taux de déplacement positif b_i . Ce taux est le nombre de personnes fuyant par unité de temps avant l'arrivée des kits d'urgence. Chaque arc a un temps de trajet c_{ij} . Le dépôt a K véhicules identiques avec une capacité de Q . Chaque véhicule peut faire un itinéraire d'une longueur maximale T_{max} .

Le modèle suivant PLNE utilise comme base un modèle CVRP sans indice de véhicules. L'idée du modèle est d'identifier un ensemble de routes réalisables qui commencent et finissent dans le dépôt, et qui ne visitent qu'une fois tous les refuges tout en maximisant le nombre de personnes satisfaites dans les refuges. Les variables binaires x_{ij} sont égales à 1 si l'arc (i, j) est utilisé par un véhicule. Le véhicule qui visite le refuge i arrive à l'heure t_i , trouve d_i personnes à servir et le quitte avec une demande satisfaite cumulée w_i .

$$\max \sum_{i \in V'} d_i \tag{6.1}$$

$$\sum_{j \in V'} x_{0j} = K \tag{6.2}$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \quad (6.3)$$

$$\sum_{i \in V} x_{ij} = \sum_{i \in V} x_{ji} \quad \forall j \in V' \quad (6.4)$$

$$t_j \geq t_i + c_{ij} - T_{max} \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V' \quad (6.5)$$

$$w_j \geq w_i + d_j - Q \cdot (1 - x_{ij}) \quad \forall i \in V, \forall j \in V' \quad (6.6)$$

$$w_i \leq Q \quad \forall i \in V' \quad (6.7)$$

$$t_i + c_{i0} \leq T_{max} \quad \forall i \in V' \quad (6.8)$$

$$d_i \leq a_i - b_i t_i \quad \forall i \in V' \quad (6.9)$$

$$t_0 = 0 \quad (6.10)$$

$$w_0 = 0 \quad (6.11)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in E \quad (6.12)$$

$$t_i, d_i, w_i \geq 0 \quad \forall i \in V \quad (6.13)$$

La fonction objectif (6.1) à maximiser est le nombre de personnes servies dans les refuges. La contrainte (6.2) limite le nombre de véhicules utilisés. Chaque refuge est visité une fois (6.3). Un véhicule qui atteint i doit le quitter (6.4). Les contraintes (6.5) garantissent la cohérence de l'heure d'arrivée aux refuges et empêchent les sous-tours. Les capacités des véhicules sont satisfaites grâce aux contraintes (6.6) et (6.7). Les equations (6.8) représentent les contraintes de la longueur maximale autorisée de l'itinéraire. La demande satisfaite dans chaque refuge est calculée en (6.9). Les contraintes (6.10) et (6.11) attribuent la valeur zéro à l'heure de départ et à la demande satisfaite cumulée dans le dépôt. Les variables de décision sont définies dans les equations (6.12) et (6.13).

Branch-and-price

La formulation du CVRP-TDD peut être décomposé par un problème maître et un sous-problème. Ces deux problèmes sont résolus itérativement avec la méthode de génération de colonnes (GC). Si la GC ne trouve pas une solution entière, un schéma de branchement est appliqué et l'arbre B&P est généré. Dans chaque nœud de cet arbre, la GC est exécuté. Ce processus est répété jusqu'à ce qu'une solution optimale soit trouvée ou qu'une limite de temps soit atteinte.

Le problème maître est modélisé en tant que modèle de partitionnement. Dans ce modèle, toutes les routes (colonnes) réalisables sont considérées, mais en raison de la grande taille de cet ensemble de solutions, un problème maître restreint est utilisé ne contenant que les colonnes ayant un coût réduit positif. Le coût réduit de chaque colonne est calculé dans le sous-problème grâce aux variables duales qui sont obtenues en résolvant la relaxation linéaire

du problème maître restreint.

Le coût réduit dépend de la demande satisfaite dans chaque refuge visité et des variables duales associées aux contraintes qui garantissent que chaque refuge est visité une fois (6.3) et que le nombre de routes ne peut pas dépasser le nombre de véhicules disponibles (6.2).

Le sous-problème est un problème de plus long chemin élémentaire avec des contraintes de ressources. Ce problème est résolu en utilisant un algorithme de programmation dynamique. Le principe de cet algorithme est de calculer pour chaque chemin partiel une étiquette spécifiant le coût réduit du chemin et des ressources consommées. Les étiquettes qui ne mènent pas à une solution optimale sont éliminées à l'aide des règles de dominance.

Pour ce problème, deux règles de dominance sont proposées. La première considère qu'un chemin partiel $p_1 = (0, \dots, i)$ domine un chemin partiel différent $p_2 = (0, \dots, i)$ si la longueur des deux chemins est égale, la capacité utilisée du véhicule est plus faible sur p_1 que sur p_2 et que le coût réduit est plus élevé sur p_1 par rapport à p_2 . De plus, tout refuge visité par p_1 est également visité par p_2 . Si tout ce qui précède est rempli, l'étiquette associée à p_2 peut être éliminée.

La deuxième règle de dominance considère le cas où la longueur du p_1 est plus petite, mais les autres conditions continuent à être satisfaites. Dans ce cas, pour que p_1 domine p_2 il faut garantir que la capacité restante du véhicule en p_1 soit supérieure ou égale à la somme des demandes de tous les nœuds, qui peuvent être visités à partir de p_2 , si chacun d'entre eux a été visité immédiatement après le nœud i dans p_1 . De cette façon, l'étiquette associée à p_2 peut être éliminée.

En plus des règles de dominance, une technique d'accélération (AT) est proposée. Cette technique résout de manière heuristique le sous-problème puisque parmi les conditions des règles de dominance, seules les deux conditions suivantes sont à vérifier : (1) que la longueur de p_1 soit inférieure ou égale à p_2 et (2) que le coût réduit de p_1 soit supérieur à celui de p_2 . La GC commence par appeler le sous-problème avec cette technique dans chaque itération. Lorsque aucune colonne avec un coût réduit positif n'est trouvée, le sous-problème exact est exécuté.

Si le sous-problème ne trouve plus de colonne avec un coût réduit positif et que la solution rapportée par la GC n'est pas entière, le schéma de branchement doit être exécuté. Nous considérons un branchement sur les variables de flux (arcs). Cela signifie que pour une branche, un arc est forcé dans la solution, alors que dans l'autre branche il est interdit.

Relaxation basée sur l'arbre couvrant

Une borne supérieure est présentée pour le CVRP-TDD. Cette borne permet de mesurer la qualité de la solution lorsqu'une borne supérieure ne peut pas être obtenue avec le B&P.

La borne est basée sur la relaxation de l'arbre couvrant avec contraintes de degré et elle est obtenue en relâchant les contraintes de capacité et la longueur maximale de la route autorisée.

Évaluations numériques

A notre connaissance, il n'y a pas de jeux de données disponibles dans la littérature pour le CVRP-TDD. Par conséquent, nous générons les instances à partir des instances proposées pour le problème de tournée de véhicules avec contraintes de capacité. Deux ensembles (I et C) avec 42 instances chacune sont créés. Le nombre de refuges par instance varie de 10 à 50.

Le PLNE est testé sur des petites instances (jusqu'à 20 refuges). Les résultats montrent que le modèle est capable de résoudre de façon optimale toutes les instances avec 10 refuges en moins de 260 secondes et 2 instances avec 15 refuges en moins de 4500 secondes.

Les performances des règles de dominance avec et sans l'AT sont comparées en exécutant la génération de colonnes dans le nœud racine de l'arbre B&P. Les Figures 6.2 et 6.3 présentent le temps d'exécution moyen de la GC avec et sans l'AT par groupe d'instances ayant le même nombre de nœuds pour les ensembles I et C respectivement. Les résultats montrent que la performance de la GC est meilleure avec l'AT. Le temps d'exécution moyen est calculé en considérant uniquement les cas où la GC s'arrête avant la limite de temps (3 heures). Les figures 6.2(a) et 6.3(a) montrent qu'avec l'AT, la GC converge plus vite et résout des instances de plus grande taille. De plus, le temps d'exécution et l'écart sont inférieurs dans les deux ensembles lorsque l'AT est utilisée.

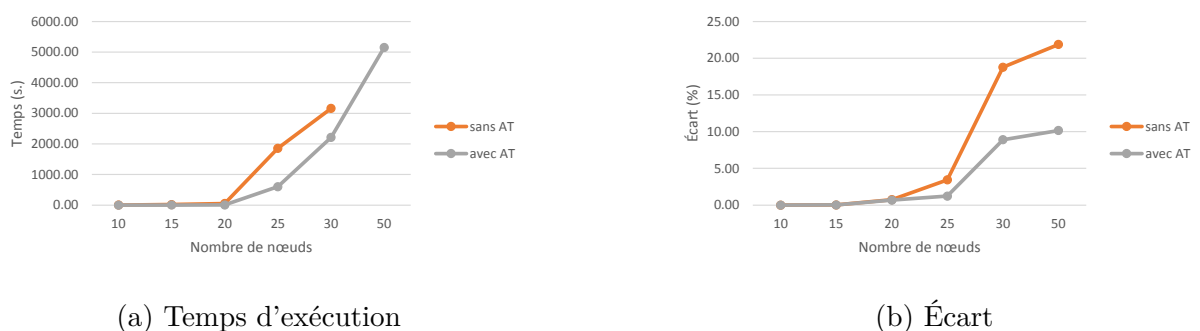


FIGURE 6.2 – Résultats moyens de la GC par groupe d'instances de l'ensemble I

Après avoir identifié que la meilleure stratégie est la GC avec l'AT, l'algorithme de B&P est évalué dans les instances où un écart d'optimalité est constaté. Les résultats montrent que l'algorithme de B&P est capable de résoudre des instances avec 30 et 25 nœuds dans les ensembles I et C respectivement.

D'autre part, le pourcentage de personnes qui ne sont pas traitées dans les refuges par rapport à la population initiale (les victimes présentes dans le refuge juste après la

FIGURE 6.3 – Résultats moyens de la GC par groupe d'instances de l'ensemble C

catastrophe) est calculé dans les cas où la solution optimale est connue. Le maximum, la moyenne et le minimum de ces pourcentages correspondent respectivement à 22%, 17% et 10%.

De plus, une analyse de stabilité est effectuée pour déterminer la fiabilité des solutions optimales par rapport aux changements dans les taux de déplacement allant jusqu'à $\pm 30\%$. Cette analyse montre que les itinéraires peuvent nécessiter des modifications.

L'importance d'avoir des plans de distribution fiables dans la logistique humanitaire nous conduit à prendre en compte l'incertitude et la variabilité de l'information. Ce problème est une extension du CVRP-TDD lequel est abordé dans la section suivante.

CVRP-TDD Robuste

Dans cette section, le CVRP-TDD robuste où les taux de déplacements sont incertains est résolu. Les 2 méthodes exactes utilisées pour résoudre le CVRP-TDD sont adaptées en utilisant l'optimisation robuste.

L'optimisation robuste vise à protéger contre le pire scénario qui pourrait se produire à l'avenir et à obtenir une solution réalisable pour n'importe quelle situation dans un ensemble de scénarios donné. Cette solution est connue comme une solution robuste.

Les paramètres incertains peuvent appartenir à trois types d'ensembles : bornés, budgétaires ou discrets. Dans la première et la seconde, les paramètres sont calculés comme des écarts autour d'une valeur nominale. Cependant, le conservatisme de la seconde peut être contrôlé avec un paramètre Γ . Dans le dernier cas, les paramètres sont définis par un ensemble fini de valeurs discrètes.

Modèle mathématique

Le CVRP-TDD robuste est une extension du CVRP-TDD pour gérer l'incertitude dans le taux de déplacement des personnes. Au lieu de laisser les refuges à un taux fixe et connu

de déplacement, les gens quittent les refuges à un taux inconnu. Ce problème permet d'avoir un plan flexible de distribution de premiers secours immédiatement après la catastrophe.

Les taux de déplacement sont décrits par un ensemble discret de scénarios. Cela signifie que chaque refuge a un taux de déplacement associé pour chaque scénario. Par conséquent, la demande satisfaite dans chaque refuge dépendra également de chaque scénario.

Dans ce cas, l'objectif est d'identifier une solution réalisable qui maximise le nombre minimum de personnes servies dans tous les scénarios.

En comparaison avec le modèle mathématique du CVRP-TDD, une nouvelle variable est introduite et elle doit être maximisée. Cette variable sert de borne à un ensemble de nouvelles restrictions qui garantissent la maximisation de la demande minimale satisfaite. De plus, tous les paramètres et variables qui dépendent des scénarios sont modifiés dans le modèle.

Branch-and-price

Le PLNE proposé dans cette section est également séparé en un problème maître et un sous-problème.

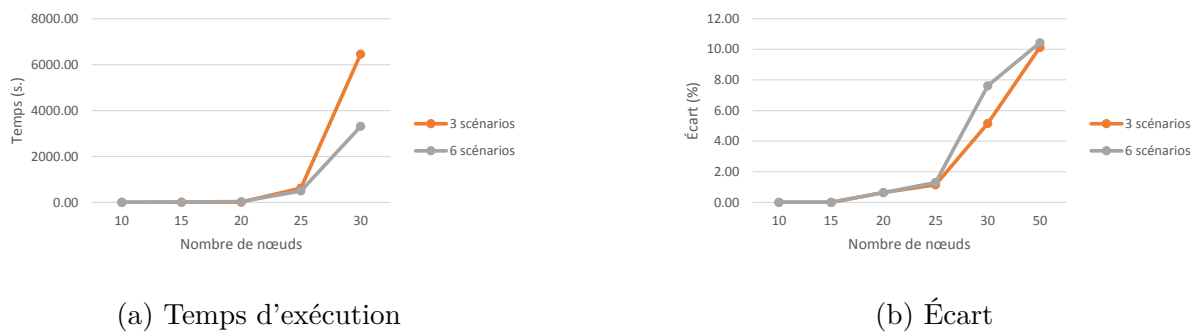
Le problème maître est modélisé comme un modèle de partitionnement. Cela diffère du problème maître du CVRP-TDD dans la fonction objectif et dans les nouvelles contraintes qui garantissent que le nombre minimum de personnes par scénario satisfaites dans les refuges est maximisé. La solution de la relaxation linéaire du problème maître restreint entraîne l'obtention d'une variable duale supplémentaire associée aux nouvelles contraintes.

Le sous-problème est un problème de plus long chemin élémentaire avec des contraintes de ressources (PLCE-RC) qui est modifié pour prendre en compte tous les scénarios du CVRP-TDD robuste. Le coût réduit dépend de la somme des demandes satisfaites dans tous les scénarios de tous les refuges visités et des variables duales.

La programmation dynamique est utilisée pour résoudre le problème du PLCE-RC, mais elle diffère de celle proposée pour le CVRP-TDD dans la structure des étiquettes. Dans ce cas, les ressources consommées doivent être distinguées par chaque scénario.

De plus, les deux règles de dominance proposées dans la section précédente sont adaptées pour que les scénarios soient pris en compte. La première considère qu'un chemin partiel $p_1 = (0, \dots, i)$ domine un chemin partiel différent $p_2 = (0, \dots, i)$ si la longueur des deux chemins est égale, la capacité utilisée du véhicule est plus faible en p_1 qu'en p_2 dans tous les scénarios et que le coût réduit est plus élevé en p_1 qu'en p_2 . De plus, tout refuge visité pour p_1 est également visité pour p_2 . Si toutes les conditions sont satisfaites, l'étiquette associée à p_2 peut être éliminée.

Dans le cas où toutes les conditions sont réunies et que la longueur de p_1 est inférieure à

FIGURE 6.4 – Résultats moyens du B&P par groupe d'instances de l'ensemble I

p_2 , la deuxième règle de dominance est vérifiée. Elle considère que p_1 domine p_2 si la capacité restante du véhicule en p_1 est suffisante pour desservir les refuges visitables à partir de p_2 dans chaque scénario, en leur rendant visite immédiatement après le nœud.

La technique d'accélération (AT) est similaire à celle utilisée dans le CVRP-TDD. Seules les conditions où la longueur de p_1 doit être inférieure ou égale à p_2 ainsi que le coût réduit de p_1 doit être supérieur ou égal à p_2 sont considérées. Le sous-problème avec cette technique est appelé dans chaque itération de la GC jusqu'à ce qu'il n'y ait plus de colonnes avec un coût réduit positif. Ensuite, le sous-problème exact avec toutes les conditions est exécuté.

Le schéma de branchement est appliqué aux variables de flux (arcs) et deux branches sont générées, l'une force l'existence de l'arc et la seconde force son inexistence dans la solution.

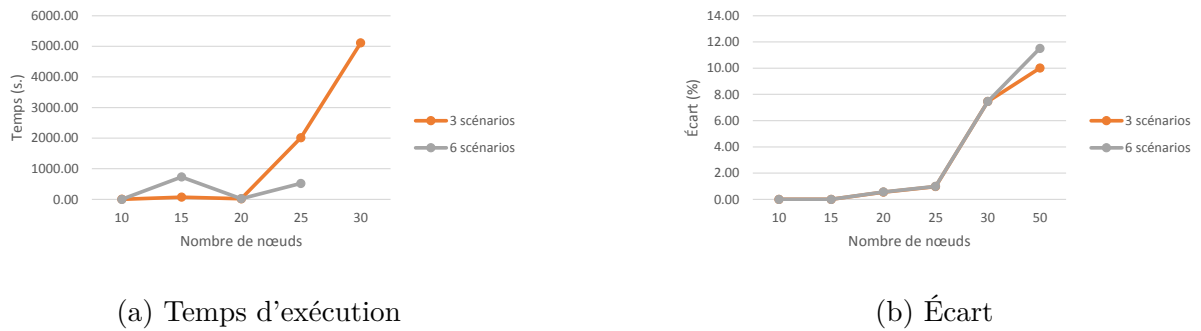
Relaxation basée sur l'arbre couvrant

La borne supérieure basée sur la relaxation de l'arbre couvrant proposé pour le CVRP-TDD est également utilisée pour la version robuste du problème. Le modèle mathématique pour le calculer est séquentiellement exécuté pour chaque scénario et la valeur minimale de la fonction objectif de tous les scénarios est la borne supérieure pour le CVRP-TDD robuste.

Évaluations numériques

Nous utilisons les mêmes instances du CVRP-TDD pour évaluer l'algorithme B&P. Les taux de déplacement dans chaque cas représentent un scénario pour le CVRP-TDD robuste. À partir de ce scénario, 5 scénarios supplémentaires ont été générés en diminuant et en augmentant chaque taux de déplacement jusqu'à 30%. Les tests sont exécutés en considérant 3 et 6 scénarios par instance dans l'ensemble I et C .

Les résultats montrent que le B&P est capable de trouver des solutions optimales dans des instances allant jusqu'à 25 et 30 refuges pour 3 et 6 scénarios dans les deux ensembles. Cependant, plus d'instances avec 25 et 30 refuges sont résolues à l'optimalité avec 3 scénarios

FIGURE 6.5 – Résultats moyens du B&P par groupe d'instances de l'ensemble C

qu'avec 6 scénarios. Le nombre total d'instances résolues à l'optimalité avec 3 et 6 scénarios dans l'ensemble I est de 29 et 25 respectivement sur 42 instances, alors que dans l'ensemble C elles sont de 27 et 25 respectivement.

Les Figures 6.4 et 6.5 montrent le temps et l'écart moyen dans les instances avec le même nombre de nœuds considérant 3 et 6 scénarios dans les ensembles I et C respectivement. Ils prouvent que l'algorithme prend moins de temps à résoudre les instances avec 3 scénarios qu'avec 6 scénarios. Cependant, dans ce groupe, les instances supplémentaires résolues avec 25 et 30 nœuds pour 3 scénarios rendent le temps d'exécution supérieur à celui de 6 scénarios dans ce groupe d'instances. De plus, l'écart moyen est plus faible pour 3 scénarios comparé à 6 scénarios.

D'autre part, si l'on compare le nombre minimum de personnes servies dans les refuges dans les solutions optimales et la population initiale avant que les victimes ne commencent à fuir, on peut conclure qu'au moins 75% de la population initiale serait desservie dans les refuges quel que soit le scénario qui se produit.

Chapitre 4 - Cadre Métaheuristique pour le CVRP-TDD

Ce chapitre présente un cadre métaheuristique (CM) pour résoudre des instances de grande taille dans un délai raisonnable du CVRP-TDD. Le cadre est un GRASP×ELS qui inclut 4 métaheuristicques qui sont des cas particuliers : GRASP, ILS, ELS, et GRASP × ILS. La métaheuristique à exécuter est sélectionnée en utilisant seulement trois paramètres. De plus, une procédure de split est intégrée dans les métaheuristicques. Cette procédure obtient une décomposition optimale en route étant donnée une tournée géante.

TABLE 6.1 – Paramètres pour sélectionner chaque métaheuristique

| Métaheuristique \ Paramètre | $nbRédémarrages$ | $nbItér$ | $nbEnfants$ |
|-----------------------------|------------------|----------|-------------|
| GRASP | > 1 | 1 | 1 |
| ILS | 1 | > 1 | 1 |
| ELS | 1 | > 1 | > 1 |
| GRASP×ILS | > 1 | > 1 | 1 |
| GRASP×ELS | > 1 | > 1 | > 1 |

Cadre général

Le CM est basé sur une procédure de recherche local et comprend différentes stratégies pour explorer l'espace des solutions. Chaque stratégie peut être sélectionnée en changeant seulement 3 paramètres.

Le cadre général du GRASP×ELS peut être vu dans l'algorithme 6.1. Les métaheures-tiques mentionnées ci-dessus peuvent être exécutées en donnant des valeurs aux paramètres : $nbRédémarrages$, $nbItér$ et $nbEnfants$. La Table 6.1 montre les valeurs respectives pour chaque métaheuristique.

Dans le cadre, deux représentations de solution sont utilisées pour explorer différents espaces de solutions : des solutions complètes (ensemble de routes) et des tournées géantes. Une route est l'ordre dans lequel un sous-ensemble de refuges sont visités par un véhicule. Alors que la tournée géante est une séquence avec tous les refuges.

Il est à noter que les solutions non réalisables sont autorisées dans le CM, mais elles sont pénalisées.

Une solution initiale (*ligne 4*) pour le CVRP-TDD est trouvée en utilisant une procédure de meilleure insertion randomisée. S'il n'y a pas d'insertions réalisables, les insertions infaisables sont pris en compte. En outre, il est possible que lors de l'évaluation d'une position, la demande du nœud sélectionné et/ou de n'importe quel nœud successeur sur le reste de l'itinéraire, soit nulle en raison du retard de la visite. Le contexte humanitaire du problème ne permet pas de laisser un nœud sans visite, même si sa demande est égale à zéro. Par conséquent, les nœuds avec une demande nulle sont autorisés dans la solution initiale.

Les demandes dépendantes du temps compliquent les calculs lorsqu'une séquence de nœuds est insérée, supprimée ou inversée par la recherche locale. Nous utilisons quelques précalculs montrant que tout mouvement peut être exprimé par concaténation de séquence de nœuds. Grâce à ces précalculs et à la mise à jour des informations pertinentes (*lignes 5 et 12*), nous pouvons évaluer les mouvements de la recherche locale en temps constant ($O(1)$).

La recherche locale (*lignes 6 et 13*) aide à trouver de meilleures solutions à partir d'une

Algorithm 6.1 Cadre métaheuristique

```

1:  $Z^* \leftarrow 0$ 
2:  $P \leftarrow \emptyset$ 
3: for  $i = 1$  to  $nbRédémarrages$  do
4:    $S \leftarrow solInitiale()$  {Randomisée}
5:    $précomputations(S)$ 
6:    $rechercheLocal(S)$ 
7:    $P \leftarrow P \cup colonnesZéroUn(S)$ 
8:   for  $j = 1$  to  $nbItér$  do
9:      $\bar{Z} \leftarrow 0$ 
10:    for  $k = 1$  to  $nbEnfants$  do
11:       $enfant \leftarrow perturb(S)$ 
12:       $mettreJourPrecomputations(enfant)$ 
13:       $rechercheLocal(enfant)$ 
14:       $P \leftarrow P \cup colonnesZéroUn(enfant)$ 
15:      if  $Z(enfant) > \bar{Z}$  then  $meilleurEnf \leftarrow enfant$  ;  $\bar{Z} \leftarrow Z(meilleurEnf)$  end if
16:    end for
17:    if  $Z(meilleurEnf) > Z(S)$  then  $S \leftarrow meilleurEnf$  end if
18:  end for
19:  if  $Z(S) > Z^*$  then  $S^* \leftarrow S$  ;  $Z^* \leftarrow Z(S^*)$  end if
20: end for
21:  $partitionnement(P)$ 
22: return  $S^*$ 

```

solution initiale à travers l'exploration du voisinage. Quatre mouvements appliqués à une route unique (intra-route) et à un couple de routes (inter-routes) sont utilisés pour rechercher la meilleure amélioration. Ces mouvements sont : re-positionnement d'un sommet, échange 1-1, échange 2-2 et 2-opt.

La procédure de split est basée sur la deuxième phase du principe de route-premier cluster-seconde. Dans la première phase, une tournée géante est construite éliminant les restrictions sur la capacité et la longueur de la route. La deuxième phase consiste à diviser ce tour géant en considérant toutes les restrictions pour trouver une solution réalisable au problème original. C'est le rôle de la procédure de split. En outre, il signale la solution optimale pour la séquence donnée.

Cette procédure peut être incluse dans le cadre général chaque fois que la boucle (lignes 10–16) démarre avec une solution complète. Dans ce cas, la solution complète est transformée

en un tour géant par une concaténation, et elle est perturbée par un mouvement qui sera expliqué plus tard. La perturbation génère une nouvelle tournée géante, qu'il faut diviser pour trouver une solution au CVRP-TDD. Si cela n'est pas possible, le processus se poursuit jusqu'à ce qu'une solution réalisable ou relâchée soit obtenue. Dans la solution relâchée, la longueur maximale de la route autorisée peut être dépassée d'un certain pourcentage.

Une perturbation est un mécanisme de diversification pour échapper à l'optimum local et essayer d'explorer une zone différente de l'espace des solutions. L'opérateur de perturbation ne peut pas être l'un des mouvements utilisés lors de la recherche locale.

Nous proposons deux opérateurs de perturbation. Si la procédure de split est incluse dans le cadre, une perturbation pour les tours géants est appliquée. Sinon, une perturbation des solutions complètes est exécutée (*ligne 11*).

Les tournées géantes sont perturbées par un mouvement d'échange dynamique où deux sous-ensembles de différents nœuds ayant la même cardinalité sont sélectionnés et leurs positions sont échangées. Cela génère un tour géant différent. En outre, la cardinalité des sous-ensembles augmente d'un, si une solution réalisable ou relâchée n'est pas trouvée par la procédure de split.

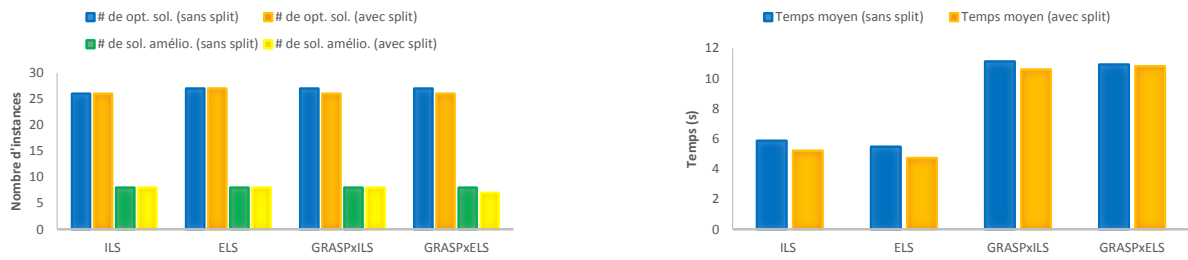
Les solutions complètes sont perturbées par une procédure d'exécution en chaîne des nœuds. Cela consiste à sélectionner un nœud de chaque route et à l'insérer dans la meilleure position de la route suivante à partir de laquelle le nœud a été extrait. Le nœud extrait de la dernière route est inséré dans la première route.

De plus, tout au long du cadre, les solutions CVRP-TDD réalisables sont transformées en colonnes 0-1 et insérées dans un pool de colonnes P (*lignes 7 et 14*). À la fin du cadre (*ligne 21*), un modèle de partitionnement est résolu avec P comme paramètre. Cela sert de procédure de post-optimisation pour essayer de trouver une meilleure solution à celle déjà obtenue par la métaheuristique exécutée.

Évaluations numériques

Les évaluations numériques comparent la performance du GRASP×ELS et des quatre autres métaheuristiques, avec et sans la procédure de split, pour résoudre le CVRP-TDD. Le cadre est testé dans des instances de taille petite-moyenne (jusqu'à 50 refuges) et de grande taille (jusqu'à 200 refuges). Les premiers sont les mêmes que ceux utilisés dans les chapitres précédents et les seconds sont les 7 instances de Christofides et al. [49] où les petites-moyennes instances sont extraites. Dans les grandes instances, les ensembles I et C ont également été créés.

Un budget constant (nombre d'appels à la recherche locale) est fixé à 500 pour pouvoir comparer les métaheuristiques. Les combinaisons pour chaque métaheuristique

FIGURE 6.6 – Résumé des résultats du CM - petites/moyennes instances de l'ensemble I

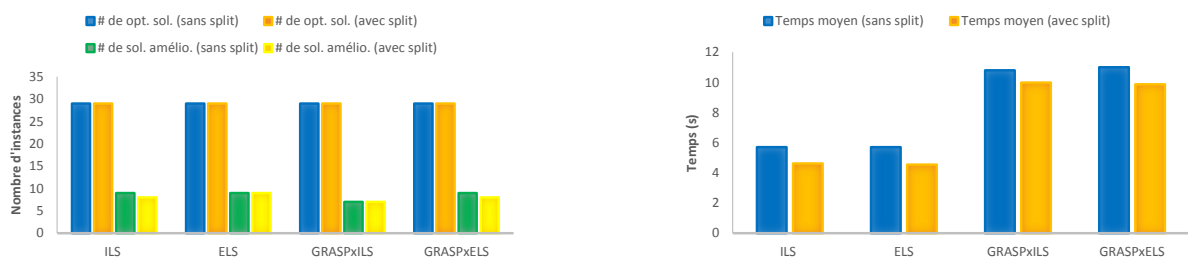
doivent garantir que $budget = nbRédémarrages \cdot nbItér \cdot nbEnfants$. De plus, chaque métaheuristique avec et sans procédure de split est exécutée 5 fois dans tous les tests effectués.

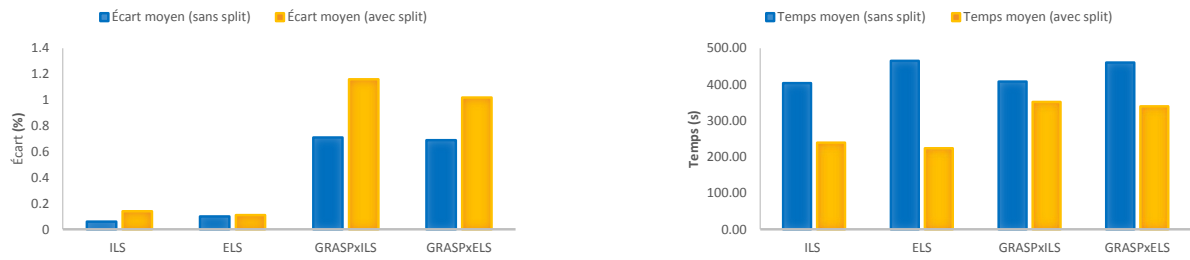
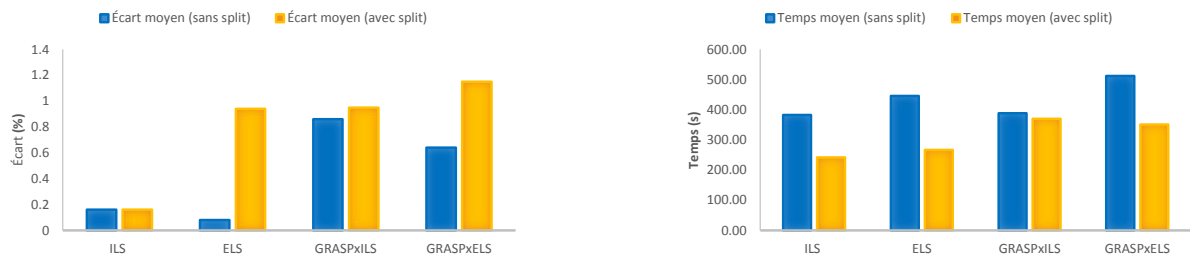
Instances de petite-moyenne taille

Les résultats obtenus montrent que l'ELS avec split et le GRASP×ELS sans split trouvent toutes les solutions optimales rapportées par l'algorithme de B&P dans les deux ensembles d'instances. En outre, ils obtiennent la même ou une meilleure solution réalisable quand la solution optimale est inconnue.

Les Figures 6.6 et 6.7 résument le nombre de solutions optimales trouvées, le nombre de solutions améliorées et le temps d'exécution moyen pour chaque métaheuristique avec et sans la procédure de split dans les ensembles I et C respectivement. Le GRASP est seulement exécuté sans la procédure de split dans les deux ensembles d'instances et n'apparaît donc pas sur les figures.

Dans ces figures on observe qu'en termes de nombre de solutions optimales trouvées, les métaheuristicques qui obtiennent les meilleurs résultats dans les deux ensembles sont l'ELS avec et sans split, le GRASP×ILS sans split et le GRASP×ELS sans split. Cependant, le GRASP×ILS sans split améliore moins les instances que les autres métaheuristicques et l'ELS avec split est, en moyenne, plus rapide que le GRASP×ELS sans split. Par conséquent, la meilleure métaheuristique pour résoudre ces instances de CVRP-TDD est l'ELS avec split car elle trouve le nombre maximum de solutions optimales et améliore la borne inférieure

FIGURE 6.7 – Résumé des résultats du CM - petites/moyennes instances de l'ensemble C

FIGURE 6.8 – Résumé des résultats du CM - grandes instances de l'ensemble I FIGURE 6.9 – Résumé des résultats du CM - grandes instances de l'ensemble C

dans la plupart des instances en moins de temps.

Instances de grande taille

Chaque métaheuristique est exécutée 5 fois avec et sans la procédure de split, sauf le GRASP. Les Figures 6.8 et 6.9 montrent les temps d'exécution et l'écart moyen de chaque métaheuristique dans les instances de l'ensemble I et C respectivement. L'écart est calculé par rapport à la meilleure solution trouvée parmi toutes les métaheuristicues.

L'ELS et l'ILS sont les métaheuristicues les plus compétitives pour les grandes instances. L'écart proche de zéro montre que l'ILS et l'ELS sans split rapportent les meilleures solutions dans la plupart des instances des ensembles I et C respectivement. Il est également prouvé que la procédure de split contribue à la convergence rapide des métaheuristicues, mais la qualité de la solution est affectée.

Chapitre 5 - Distribution de secours aux nœuds critiques et non-critiques

Ce chapitre présente le problème cumulatif de tournées de véhicules avec des demandes dépendantes du temps (Cumulative Capacitated Vehicle Routing Problem with Time-Dependent Demands "CCVRP-TDD") qui est une variante du CVRP-TDD. Un PLNE et une méthode heuristique en deux phases sont proposés pour résoudre cette variante.

Définition du problème et modèle mathématique

Le CCVRP-TDD est un problème qui considère non seulement les refuges (critiques) que les victimes quittent, ainsi que les refuges (non critiques) où ils se rendent pour obtenir l'aide souhaitée.

Lorsqu'une catastrophe frappe un territoire, les gens vont dans les refuges pour recevoir les premiers secours. Ces refuges peuvent être critiques ou non-critiques. Les gens dans les refuges critiques ont tendance à fuir vers des refuges non-critiques si les premiers secours n'arrivent pas rapidement.

Le déplacement de la population affectée entre les refuges devrait être évité car cela génère des variations de la demande dans les refuges, une augmentation du chaos et de la panique déjà causés par la catastrophe, une surpopulation dans les refuges non-critiques et en conséquence une insuffisance des soins apportés en raison de la capacité limitée des véhicules. Cela peut être atténué si les refuges critiques sont desservis le plus vite possible puisque les gens vont rester dans les refuges dès lors qu'ils reçoivent de l'aide.

Dans le CCVRP-TDD, une fonction objectif cumulative (somme des temps d'arrivées aux refuges critiques) est minimisée. Il convient de noter que si la fonction objectif vise à optimiser l'heure d'arrivée dans les refuges, la demande en fonction du temps a un impact sur l'ensemble du réseau et doit être prise en compte pour garantir des solutions réalisables au problème.

La formulation du CCVRP-TDD avec 2 indices diffère de la CVRP-TDD sur 4 aspects : (1) la fonction objectif, (2) un nouvel ensemble (refuges non-critiques), (3) chaque refuge critique a un taux de déplacement vers les refuges non-critiques et (4) l'ensemble de contraintes pour déterminer les demandes qui doivent être satisfaites dans les refuges non-critiques.

Méthode heuristique en deux phases

La méthode heuristique en deux phases (MHDP) est basée sur le GRASP×ILS. La structure générale de cette méthode est présentée dans l'Algorithme 6.2.

La méthode commence par générer une solution partielle initiale avec une heuristique parallèle randomisée constructive (*ligne 4*). Cette solution est partielle car elle ne contient que les refuges critiques. Ensuite, les précalculs sont initialisés et la recherche locale est appliquée à la solution partielle initiale (*ligne 5–6*). Les précalculs servent à évaluer les variations de la fonction objectif cumulative et la demande dépendante du temps en temps constant $O(1)$ dans les mouvements utilisés dans la recherche locale. Ces mouvements sont : re-positionnement d'un sommet et 2-Opt. Les deux mouvements sont réalisés intra-route et

Algorithm 6.2 Methode heuristique en deux phases

```
1:  $Z^* \leftarrow \infty$ 
2: for  $red\acute{e}m = 1$  to  $nbR\acute{e}d\acute{e}marrages$  do
3:    $\bar{Z} \leftarrow \infty$ 
4:    $S \leftarrow solPartielleInitiale(V_c)$ 
5:    $pr\acute{e}computations(S)$ 
6:    $rechercheLocal(S)$ 
7:   for  $it\acute{e}r = 1$  to  $nbIt\acute{e}r$  do
8:      $S' \leftarrow itin\acute{e}rComplets(S, V_{nc})$ 
9:     if  $S'$  is not feasible then
10:        $mettraJourPr\acute{e}computations(S')$ 
11:        $proc\acute{e}dureR\acute{e}paration(S')$ 
12:     end if
13:     if ( $S'$  est r\acute{e}alisable and  $Z(S') < \bar{Z}$ ) then
14:        $\bar{S} \leftarrow S'$  ;  $\bar{Z} \leftarrow Z(S')$ 
15:        $S \leftarrow extraireSolPartielle(\bar{S})$ 
16:     end if
17:      $perturb(S)$ 
18:      $mettraJourPr\acute{e}computations(S)$ 
19:      $rechercheLocal(S)$ 
20:   end for
21:   if  $\bar{Z} < Z^*$  then  $S^* \leftarrow \bar{S}$  ;  $Z^* \leftarrow \bar{Z}$  end if
22: end for
23: return  $S^*$ 
```

inter-route.

Le ILS (*lignes 7–20*) exécute $nbIt\acute{e}r$ itérations. Chaque itération génère une solution complète en ajoutant des refuges non-critiques à une solution partielle (*ligne 8*). Si un itinéraire complet dépasse la longueur maximale de la route ou la capacité du véhicule, une procédure de réparation utilisant les mêmes mouvements de la recherche locale est exécutée (*lignes 9–12*). Cette procédure est uniquement appliquée aux nœuds non-critiques essayant de trouver une solution réalisable.

Si une solution complète réalisable est trouvée et que la fonction objectif est inférieure à celle de la solution en place, celle-ci est mise à jour (*lignes 13–16*). En fin de compte, une solution partielle est perturbée en utilisant la procédure de node-ejection chain et la recherche locale est appliquée à cette solution partielle (*lignes 17–19*). Dans ces procédure,

seuls les nœuds critiques sont sélectionnés.

Après les $nbItér$ itérations sont atteints, si la fonction objectif de la solution en place est inférieure à celle de la meilleure solution obtenue, celle-ci est mise à jour (*ligne 21*).

Cet algorithme est redémarré $nbRédémarrages$ fois avec une solution partielle différente (*boucle 2-22*) et retourne la meilleure solution obtenue.

Relaxation basée sur l'arbre couvrant

Une borne inférieure basée sur la relaxation de l'arbre couvrant avec contraintes de degré est proposée pour le CCVRP-TDD. À cette borne, chaque refuge est directement relié au dépôt ou à un refuge intermédiaire visité immédiatement après le dépôt. De plus, les contraintes de capacité et la longueur maximale de la route du problème original sont relâchées.

Évaluations numériques

Les instances des ensembles I et C utilisées pour le CVRP-TDD sont adaptées à ce problème. Pour chaque instance, nous générons trois sous-ensembles de nœuds critiques correspondant à 20%, 40% et 60% du nombre total de nœuds.

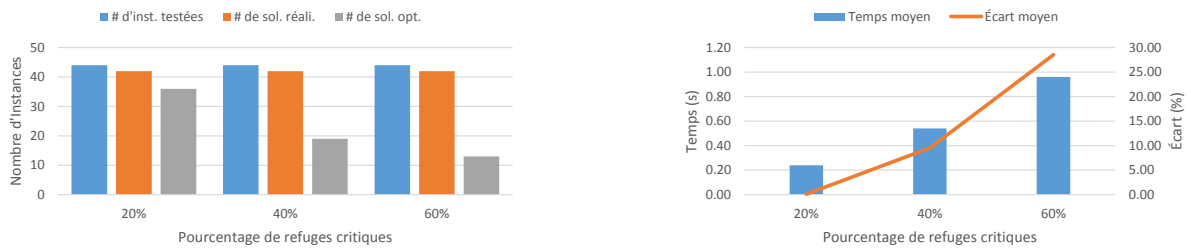
Instances de petite-moyenne taille

Le PLNE est résolu sur des instances de petite et moyenne taille de chaque ensemble (I et C) en considérant les différents pourcentages de nœuds critiques. Les résultats montrent que le nombre d'instances résolues à l'optimalité est un peu plus élevé dans l'ensemble I que dans le C . Dans l'ensemble I , ces instances contiennent jusqu'à 30 et 20 refuges dont 6 et 12 refuges critiques respectivement. Alors que l'ensemble C est composé de 25, 20 et 15 refuges dont 5, 8 et 9 refuges critiques respectivement.

L'augmentation du nombre de refuges critiques génère que le PLNE consacre, en moyenne, plus de temps à trouver la solution optimale dans l'ensemble C . Dans l'ensemble I , cela ne se produit que dans les instances contenant jusqu'à 40% de refuges critiques.

L'écart moyen augmente également lorsque le nombre de refuges critiques augmente. Dans certains cas, le nombre de refuges critiques par route n'est pas équilibré dans la solution optimale.

La borne inférieure proposée est toujours supérieure au niveau signalé par CPLEX dans les cas où une heure n'est pas suffisante pour trouver la solution optimale.

FIGURE 6.10 – Résumé des résultats du MHDP-petites/moyennes instances de l'ensemble I FIGURE 6.11 – Résumé des résultats du MHDP-petites/moyennes instances de l'ensemble C

Le MHDP est exécuté 5 fois pour résoudre chaque instance avec 500 appels de la recherche locale par exécution. Les Figures 6.10 et 6.11 résument les résultats suivants pour chaque ensemble (I et C) : le nombre d'instances utilisées, le nombre d'instances dans lesquelles une solution réalisable est trouvée, le nombre d'instances dans lesquelles la solution optimale est signalée, le temps moyen par exécution (en secondes) et l'écart moyen (%).

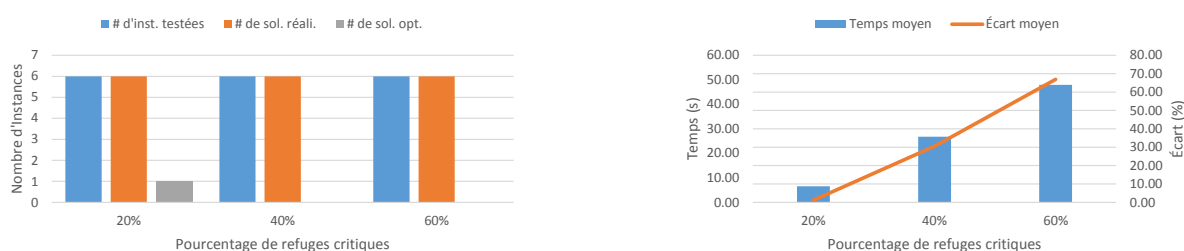
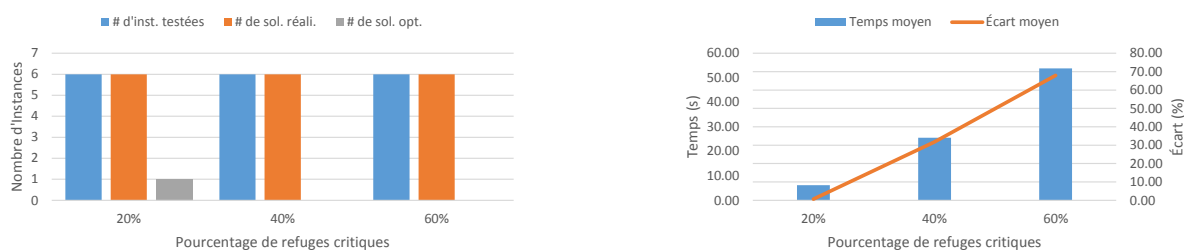
Le MHDP trouve les solutions optimales dans toutes les instances dans lesquelles cette solution est connue, sauf dans deux instances. Dans ces deux instances, certains nœuds non-critiques doivent être visités avant que les nœuds critiques ou le nombre de nœuds critiques par route ne soient équilibrés dans la solution optimale. Pour cette raison, deux instances supplémentaires sont utilisées, augmentant le nombre de véhicules utilisés. Dans ces cas, le MHDP trouve les solutions optimales.

En moyenne, les petites et moyennes instances sont résolues en moins d'une seconde dans les deux ensembles d'instances.

Instances de grande taille

Les Figures 6.12 et 6.13 résument les résultats obtenus en exécutant le MHDP 5 fois dans chaque instance avec 500 appels de la recherche locale. Les résultats montrent que, en moyenne, les instances sont résolues en moins d'une minute. En outre, l'instance avec 75 nœuds et 15 nœuds critiques est résolue à l'optimalité dans les deux ensembles d'instances.

D'un autre côté, l'écart moyen augmente à mesure que le nombre de nœuds critiques

FIGURE 6.12 – Résumé des résultats du MHDP - grandes instances de l'ensemble I FIGURE 6.13 – Résumé des résultats du MHDP - grandes instances de l'ensemble C

augmente. Il convient de noter que l'écart est calculé en fonction de la borne inférieure proposée pour ce problème.

A partir des résultats détaillés, il est observé que le MHDP fournit de bonnes solutions pour les instances avec jusqu'à 199 nœuds, dont 120 nœuds critiques, en moins de 180 secondes.

Chapitre 6 - Conclusions générales et recherches futures

Dans cette thèse, le problème de tournées de véhicules avec demandes dépendantes du temps (CVRP-TDD), le CVRP-TDD robuste et le problème cumulatif CVRP-TDD sont introduits. Ces problèmes surviennent immédiatement après une catastrophe naturelle ou une catastrophe causée par l'homme. Cette période fait partie de la phase d'intervention qui est l'une des quatre phases de la gestion des catastrophes.

Le CVRP-TDD et son homologue robuste considèrent la maximisation de la demande satisfaite dans les refuges (objectif d'efficacité), tandis que le cumulatif CVRP-TDD considère la somme des temps d'arrivée aux refuges (objectif d'égalité). La principale caractéristique de ces problèmes est que la demande dans les refuges varie en fonction du temps. Deux méthodes exactes (PLNE et B&P) pour résoudre les deux premiers problèmes sont proposées, un cadre métaheuristique avec une procédure de split et une méthode heuristique en deux

phases sont implémentés pour résoudre le CVRP-TDD et le cumulatif CVRP-TDD respectivement.

Trois aspects sont pris en compte dans les recherches futures : (1) Le débat sur le type de fonction objectif qui devrait être utilisée dans les problèmes qui se posent dans ce contexte, (2) le défi de faire face au niveau élevé d'incertitude qui existe dans le contexte humanitaire et (3) l'importance d'intégrer les décisions qui doivent être prises dans la logistique humanitaire.

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Doctorat : Optimisation et Sûreté des Systèmes

Année 2018

Les problèmes de tournées de véhicules avec des demandes dépendantes du temps dans la logistique humanitaire

Cette thèse de doctorat introduit le problème de tournées de véhicules avec des demandes dépendantes du temps (CVRP-TDD), le CVRP-TDD robuste et le cumulatif CVRP-TDD dans la logistique humanitaire. Ces problèmes contribuent à la planification de la distribution des kits de secours suite à une catastrophe. Ce problème apparaît lorsque les victimes fuient les refuges en raison du manque d'aide, provoquant la propagation du chaos dans d'autres territoires.

Dans le CVRP-TDD, la distribution doit garantir un nombre maximum de personnes satisfaites dans les refuges avant leur départ. Un Programme Linéaire en Nombres Entiers (PLNE), un algorithme de Branch-and-Price (B&P) et une borne supérieure sont développés. Deux règles de dominance et une technique d'accélération sont également proposées. De plus, un cadre métaheuristique explorant 2 espaces de solution est mis en œuvre, introduisant les méthodes suivantes : GRASP, ILS, ELS, GRASPxEELS et GRASPxILS.

Le CVRP-TDD robuste suppose que le nombre de personnes fuyant les refuges est incertain. Le PLNE et le B&P utilisés précédemment sont alors adaptés pour maximiser le nombre de rescapés servis dans leur refuge, dans le pire des scénarios.

Le CVRP-TDD cumulatif considère le flux sortant des refuges critiques, mais aussi le flux entrant dans les refuges non-critiques. Dans ce cas, l'objectif est de minimiser la somme des temps d'arrivées aux refuges critiques. Un PLNE, une heuristique en deux phases et une borne inférieure sont présentés.

Mots clés : transport, planification – modèles mathématiques – distribution des produits – recherche opérationnelle – secours aux victimes de catastrophes – métaheuristicques.

Vehicle Routing Problems with Time-dependent Demands in Humanitarian Logistics

This PhD dissertation introduces the capacitated vehicle routing problem with time dependent demands (CVRP-TDD), robust CVRP, and cumulative CVRP-TDD in humanitarian logistics. These problems contribute to the planning of routes to distribute relief products in the aftermath of a disaster (natural or man-made), considering population displacements. The latter occur because people flee from shelters, if they do not receive help. The displacements cause a spreading of the chaos to other territories and disease outbreaks.

In the CVRP-TDD, the distribution of goods must guarantee a maximum number of people satisfied at shelters. A mixed integer linear program (MILP), a branch-and-price (B&P) algorithm, and an upper bound are developed. Two dominance rules and an acceleration technique are proposed. In addition, a metaheuristic framework exploring 2 separate solution spaces is implemented. The framework is a GRASPxEELS which includes as particular cases: GRASP, ILS, ELS and GRASPxILS.

The robust CVRP-TDD assumes that the number of people fleeing from shelters is uncertain. The MILP and B&P used for the CVRP-TDD are adapted to maximize the number of people served at shelters, in the worst scenario. An upper bound for this problem is also described.

The cumulative CVRP-TDD considers outgoing flow from critical shelters and incoming flow into non-critical shelters. In this case, the objective is to minimize the sum of the arrival times at critical shelters. A MILP, a two-phase heuristic method, and a lower bound are presented.

Keywords: transportation, planning – mathematical models – physical distribution of goods – operations research – disaster relief – metaheuristiccs.

Thèse réalisée en partenariat entre :



Ecole Doctorale "Sciences pour l'Ingénieur"