



Essays on Firms Production Function, Markups, and the Share of their Income Going to Workers

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Thèse de doctorat



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Essays on Firms Production Function, Markups, and the Share of their Income Going to Workers

Thèse de doctorat de l'Institut Polytechnique de Paris
préparée à Ecole nationale de la statistique et de l'administration économique

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Arthur Bauer

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²*I will always remember the Monday morning (EST) skype sessions with Martin, where after a few words in French (or Spanish) he would make me benefit from his deep knowledge of the literature, and continuous optimism. This was a true pleasure to engage in this intellectual journey with him.*

³*Danial taught me more than the way of conducting careful empirical research. He was patient enough to put my personal growth at the center of the research project when we initiated it together. I am grateful for the thoughtful advice, which remains still very useful in my professional life.*

Résumé en Français

La fonction de production des entreprises lie leur niveau de production à leurs dépenses en facteurs de production. Son estimation est à la fois importante et peu fiable. Importante, parce que des indicateurs clés pour la conception des politiques publiques, tels que le taux de marge, en découlent. Peu fiable, car elle repose sur des hypothèses d'identification. Ce projet étudie les hypothèses qui sous-tendent l'estimation des fonctions de production : à la fois leurs formes fonctionnelles et la flexibilité des facteurs de production. Il s'appuie ensuite sur une estimations des fonctions de production des entreprises françaises, pour évaluer l'évolution de leur taux de marge et de la part du travail dans leur valeur ajoutée au cours des 30 dernières années.

Le chapitre 2 conforte l'hypothèse de flexibilité des facteurs de production: l'ajustement instantané des matières premières ou du travail et l'ajustement retardé du capital. Nous nous appuyons sur l'existence de notches ; des valeurs où les bénéfices après impôt diminuent avec le chiffre d'affaire avant impôt dans le code des impôts français. Après avoir estimé l'élasticité du chiffre d'affaire au taux de taxation des bénéfices, nous montrons que les entreprises qui optimisent ont une plus grande élasticité de production par rapport aux matières premières et une plus faible élasticité de la production par rapport au capital. De même, pour ajuster leur production, les entreprises ont tendance à réduire principalement leurs dépenses en matières premières.

Le chapitre 3 s'appuie sur le résultat du chapitre 2 pour mesurer le taux de marge de toutes les entreprises françaises entre 1984-2016. De Loecker and Warzynski (2012) montrent que la marge d'une entreprise est proportionnelle à l'inverse de la part de revenu de l'un de ses intrants flexibles. Nous analysons l'évolution des marges agrégées en France et documentons que l'augmentation de la concentration est corrélée à une réallocation des parts de marché vers les entreprises à marge élevée. Nous montrons également que l'évolution de la part du travail reflète l'évolution des marges : la réallocation tend à diminuer la part du travail tandis qu'au sein des entreprises, la part du travail augmente. Ces résultats découlent de l'estimation de fonction de production dites translog.

Le choix d'une forme fonctionnelle pour décrire le processus de production est un compromis entre théorie et empirisme. La fonction de production standard est de type Cobb-Douglas mais

impose une élasticité de substitution constante et égale à 1, en contradiction avec la littérature empirique. Les fonctions de production CES ont des élasticités de substitution non unitaires mais constantes au sein de chaque industrie et un ratio d'utilisation des facteurs de production indépendant de la taille de l'entreprise.

Le chapitre 4 questionne le choix d'une forme fonctionnelle pour décrire le processus de production. Il montre que les fonctions de production CES ou Cobb-Douglas ne permettent pas de rendre compte de l'utilisation des technologies de l'information (TIC). En effet nous documentons une augmentation de la demande relative de TIC par rapport aux autres facteurs de production avec la taille des entreprises: ce qui est cohérent avec une fonction de production CES non-homothétique. Nous analysons ensuite comment l'interaction de la baisse des prix des TIC et les caractéristiques non-homothétiques des TIC rationalisent les faits empiriques documentés dans le chapitre 3. (i) comme les grandes entreprises sont plus intensives en TIC, elles bénéficient de manière disproportionnée de la baisse des prix des TIC, ce qui rationalise l'augmentation de la concentration. (ii) comme les grandes entreprises sont plus intensives en TIC dans l'échantillon, elles fonctionnent avec des rendements d'échelle plus faibles et ont donc des parts de bénéfices plus élevées et des parts de travail plus faibles. Cela explique comment l'augmentation de la concentration entraîne une diminution de la part globale du travail. (iii) les statistiques comparatives du modèle prédisent que l'adoption de TIC liée à la baisse de leur prix implique des rendements d'échelle plus élevés et ont donc une part de travail plus importante, ce qui explique la tendance haussière de la part du travail au sein des entreprises (i.e. le fait que pour l'entreprise moyenne la part de ses revenus alloués au facteur travail augmente)

Chapter 1

Introduction

Firms production function link their use of input factors (such as labor, capital, materials, land, information technologies, etc.) to their production level. Production function estimates are at the same time important and untrustworthy. Important, because a large set of key indicators for policy design are derived from those estimates: the measure of aggregate markups, i.e. firms ability to price over marginal cost, and similarly the amount of misallocation in the economy, i.e. the efficiency loss due to the heterogeneity of firms marginal products. Untrustworthy because they require not only identification assumptions that may not hold but also careful firm data processing.

This project studies the assumptions underlying the usual techniques for estimating production functions: both the flexibility of certain production inputs, and the functional forms used to describe these production functions. It then leverages production function estimates, to assess how firms ability to price over marginal income and the share of their income going to workers have evolved over the last 30 years.

The choice of a functional form to describe firms production process is a compromise between theory and empirics. The workhorse production function is the Cobb-Douglas production function with two inputs. This production function imposes a constant (and equal to 1) elasticity of substitution between capital and labor. Recent evidence in the empirical literature has however estimated a micro-elasticity of substitution significantly lower than one ([Oberfield and Raval, 2014a](#)).

While CES production functions allow for non-unit elasticity of substitution, they assume constant elasticity within industry industry and imply that the ratio of input use doesn't depend on firm size.

In [Lashkari et al. \(2019a\)](#), we show that this production function cannot account for IT inputs use in firms. Our point of departure is the introduction of novel datasets that detail micro-level software and hardware investments among French firms, covering a broad set of manufacturing and service industries. We rely on this data to construct stocks of software and hardware capital for French firms between 1995 to 2007. We complement this with detail balance sheet information for the universe of all French firms.

With this source, we can examine the heterogeneity in IT inputs across French firms. We document that the firm-level demand for IT inputs relative to other inputs grows in the firm's scale of operation. This finding is robust to different measures of IT intensity, whether IT is proxied by software or hardware, whether by investment or capital, and whether intensity is measured relative to labor inputs or to non-IT capital. It is also robust to using different measures of firm scale, whether scale is measured by employment, value added, sales, or by more eclectic measures such as the number of plants, the depth of organizational structure, the number of exporting markets, or the number of exported products (the latter two measures only in the sample of exporting firms). We find this relationship to hold across a wide range of industries and classes of firm size, from small firms with just a few workers to large multinationals hiring tens of thousands of workers.

In the same paper, we show that the even more flexible functional forms suggested in [Sato \(1974, 1977\)](#) help rationalizing this empirical fact. Such production functions are non-homothetic, namely production functions for which the relationship between the inputs ratio and the marginal rate of substitution between capital and labor depends on firm size. Such productions are implicitly defined by the following equation :

$$\left(\frac{AK}{Y^\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \left(\frac{BL}{Y^{\gamma+\epsilon}}\right)^{\frac{\sigma-1}{\sigma}} = 1. \quad (1.1)$$

To make sure this functional form is best suited to model firms' production function involving IT inputs, we also estimate this production function.

A large body of work suggests procedures of production function estimation relying on input choices timing that assume a quasi instantaneous adjustment of either material or labor and on the contrary some delay for capital adjustment ([Blundell and Bond, 2000](#); [Levinsohn and Petrin, 2003b](#); [Olley and Pakes, 1996](#); [Akerberg et al., 2015a](#)) Most of these techniques estimate cobb-douglas or translog production functions either in output or value-added.

One issue that prevents us for simply running Ordinary Least Square estimations is that ω_{it} might be observed by the management of the firm and affect the choice of the inputs levels. This creates a spurious correlation between y_{it} and the choice of inputs levels that would bias the

Ordinary Least Squares (OLS) estimator.

Several approaches have been adopted to deal with this endogeneity problem. The most common one relies on a two step procedure: assuming that the amount of used material¹ is increasing in the state productivity ω at all level of capital k and can therefore give an expression of ω_{it} (?). This first step provides an expression for the innovation term. Second step relies on the assumption that capital is fixed and that material or labor on the contrary freely adjust. As a consequence contemporaneous innovation is uncorrelated with *contemporaneous* capital level and *past* labor level, providing two moments conditions that allow identifying firms output elasticities.

In [Bauer and Rotemberg \(2018\)](#) we provide evidence that this assumption on inputs flexibility holds. We rely on the existence of *notches*; values where after-tax profits decrease in before-tax sales in the French tax code. As firms endogenously respond to notches, this leads to excess mass in the firm-size distribution. We study a 1997 policy reform in which the French government implemented a transient tax reform that increased profit taxes by 15% for firms with over 50 million Francs in turnover.

We start by measuring the endogenous response of excess mass in the firm-size distribution: “too many firms” just below the cutoff, and correspondingly “too few” just above². Following a large literature³ we assume that the firm size distribution is “well behaved” in any given year. This implies that firms far away from the tax threshold do not adjust their behavior in response to the policy, and so we can use the distribution (of turnover) of those firms in order to estimate a counterfactual avoidance-free firm size distribution. We compare the avoidance-free counterfactual distribution to the actual firm-size distribution around the cutoff to back out how firms change their behavior. We also use the time-series dimension of the data in order to estimate excess mass by estimating a counterfactual avoidance-free distribution using years when the policy was not in effect.

We find that there were around 150 firms who changed their sales in response to the new tax regime out of roughly one thousand firms concerned. This corresponds to a tax elasticity of sales of 0.16.

We then turn to identifying which type of firms adjust their sales. We do this by studying the ex-ante characteristics of firms below the tax cutoff (as in [Diamond and Persson \(2016\)](#)⁴). If, for

¹Initially [Olley and Pakes \(1996\)](#) m had a parallel reasoning with investment, but such procedure implies heavy data selection

²[Almunia and Lopez-Rodriguez \(2017\)](#); [Bachas and Soto \(2018\)](#); [Garicano et al. \(2016\)](#); [Gourio and Roys \(2014\)](#); [Kleven and Waseem \(2013\)](#); [Liu and Lockwood \(2016\)](#); [Onji \(2009\)](#)

³[Aghion et al. \(2017b\)](#); [Bach \(2015\)](#); [Chetty et al. \(2011\)](#); [Dee et al. \(2016\)](#); [Diamond and Persson \(2016\)](#); [Kleven \(2016\)](#); [Kleven and Waseem \(2013\)](#); [Lardeux \(2018\)](#); [Barbanchon \(2016\)](#); [Saez \(2010\)](#)

⁴[Diamond and Persson \(2016\)](#) suggest considering the tax regulation as a treatment, and firms that engage into tax avoidance as compliers. Since [Abadie \(2003\)](#) it is well known that we the distributions of the characteristics of

instance, firms who normally have high profits adjust their sales more, then we would observe more of those types of firms just below the cutoff than we would expect (for instance, by using their prevalence when the tax cutoff was not in effect).

In addition to finding that high-profit firms are more likely to avoid the tax, we also find that firms with larger adjustment costs are less likely to bunch. We start by classifying firms using their estimated capital adjustment costs (Asker et al., 2014). We show that bunchers are, depending on the years, between 3% and 15% more likely to have capital adjustment cost within the lowest tercile.

We also use this technique to test the fundamental assumption in the production function estimation literature that materials are the most flexible input (Olley and Pakes, 1996; Levinsohn and Petrin, 2003a; Akerberg et al., 2015b) and capital has high adjustment costs. We are able to test that assumption directly, and find that firms who bunch tend to have larger elasticity of output with respect to materials and lower elasticity of output with respect to capital.

We then turn to estimating how firms avoid profit taxes. Firms have many potential margins of adjustment, including affecting their production decisions, prices, and inventories. As in (Diamond and Persson, 2016; Bachas and Soto, 2018; Dee et al., 2016), we note that a similar logic to the bunching estimator can help us back out firm behavior. We compare firms who potentially are distorting their behavior (those close to the cutoff) to those who are likely unaffected (those further away). If firms change some characteristics in order to shrink, then we will see differences in that characteristic in the avoidance. For instance, we find that firms avoid increasing their sales by instead increasing their inventories.

In the data, this shows up as firms in the avoidance region overall having more inventories than would be predicted by the out-of-sample counterfactuals. An RD-type of estimate (comparing firms just below to just above the cutoff) is unlikely to generate the causal mechanisms for tax avoidance since the firms who choose to avoid taxes are ex-ante different along a variety of dimensions. In order to avoid the selection issue, we compare all of the firms who might be affected by avoidance to those who are not.

We find that while firms do lower their production as they lower sales, the primary driver of the avoidance is an increase in change in inventories and capitalized production. This suggests that adjusting production is relatively more costly than either stocking the production or reinvesting it in the production process. Consistent with our findings on adjustment costs, we find that the firms who lower their production do so by adjusting materials but not capital. The changes “add up,” which is consistent with the values we find being real effects of tax avoidance, not just tax evasion through simple misreporting or fraud (Best et al., 2015).

compliers can be identified.

Finally, we ask how firms adjust their remaining production. In particular, which of their inputs do they primarily reduce spending on. We show that declines in production is driven by a decrease in material not capital. This is consistent with the previous finding that firms with larger output elasticities with respect to material have higher propensity to adjust, as well as the central assumption in production function estimation that material is a flexible input and capital is fixed.

In [Bauer and Boussard \(2019\)](#) we leverage the output elasticities with respect to inputs to recover firm level markups of the universe of firms in France over the 1984-2016 period. [De Loecker and Warzynski \(2012\)](#) show that a firm's output elasticity with respect to a flexible input is the coefficient of proportionality in the relationship between markup and this inverse of the share of revenue of the amount spent on this input. To derive this relationship, the authors only assume firms minimize their cost of production and do so by freely adjusting at least one variable input.⁵

We find that the markup of the *typical* firm has decreased over this period, but the reallocation of market shares toward larger firms, with larger markups and lower labor shares, contributed to an increase of the aggregate markup and a decrease of the aggregate labor shares.

Indeed, we document a rise of concentration since the beginning of the 1990s. We show that taking account of this rise of concentration when decomposing the evolution of the labor share is key to understand how the decrease of firm level markup has contributed positively to the evolution of the aggregate labor share in France. This also helps explaining why aggregate labor share in France has remained stable in spite of a significant and negative contribution of reallocation on labor share.

In [Lashkari et al. \(2019a\)](#) we analyze how the interaction of the fall of IT prices and the non-homothetic characteristics of IT inputs help rationalize these empirical facts. First, since larger firms are more IT intensive in the cross-section, they benefit disproportionnally from the fall in IT prices. Their larger pass-through of IT input prices to production prices translates into a larger fall in their production prices when IT input price falls. This first fact helps rationalizing the rise of concentration observed when IT prices have decreased. Similarly, since larger firms are more IT intensive in the cross-section, they operate at lower returns to scale and therefore have higher profit shares and lower labor shares. This explains how the rise in concentration, namely the reallocation of market shares to larger firms drives a decline in aggregate labor shares. Finally, the comparative statistics of the model predicts that the fall of IT prices imply that when firms substitute toward IT they operate at higher returns to scale and therefore tend to increase larger labor share, explaining the positive contribution to aggregate labor share of the within component.

⁵Based on [Bauer and Rotemberg \(2018\)](#), we assume that adjusting capital is costly and that firms can adjust freely materials and labor.

Chapter 2

Tax Avoidance in Firms¹

Corporate tax codes can have *notches*; values where after-tax profits decrease in before-tax sales. Firms endogenously respond to notches, leading to excess mass in the firm-size distribution. We study a 1997 policy reform in which the French government increased profit taxes by 15% for firms with over 50 million Francs in turnover. We use two distinct and complementary approaches to estimate the extent of tax avoidance: (a) using firms far away from (and therefore unlikely to be responsive to) the tax notch in the same year and (b) the entire firm size distribution before the tax reform. Both strategies generate similar results for the extent of tax avoidance. Firms adjust their contemporaneous sales mostly increasing inventories. Declines in production are driven by adjusting materials, not capital.

Keywords: Business Taxes, Tax Evasion, Firm Production

JEL Codes: H25, H26, H32, D24

¹Co-authored with Martin Rotemberg

2.1 Introduction

In France, firms pay additional profit taxes if their turnover is above a certain level. Firms seeking to avoid this additional cost have a variety of options, including misreporting, transfer pricing, and changing “real” behavior (Best et al., 2015; Zucman, 2014; Velayudhan, 2018). Even in the latter category, firms have a variety of options: they can actually produce less (by using fewer inputs), or they can use inventories to shift sales from one year to the next.

We focus our attention on two themes. First, we add additional evidence to a large literature showing that firms’ reported sales respond to increases in taxes. Our second theme is understanding mechanisms. First, we are interested in understanding if there are specific types of firms who are more likely to be affected, and second we are interested in understanding what firms do in order to lower their reported sales.

It is difficult to credibly survey firms on if and how they would have made different choices under counterfactual tax regimes. We use reported firm behavior in order to back out avoidance responses. In particular, we leverage a 1997 tax increase in France, which increased profit taxes for firms with over 50 million Francs in turnover by 15%. As a result, firms with fairly similar sales faced different average (and marginal) tax rates, giving firms incentives to stay just below the threshold relative to just above.

“Bunching” in the firm-size distribution just below the tax notch is *prima facie* evidence of endogenous responses.² We use complementary but distinct approaches to estimate the extent of excess mass: “too many firms” just below the cutoff, and correspondingly “too few” just above. First, following a large literature³ we assume that the firm size distribution is “well behaved” in any given year. This implies that firms far away from the tax threshold do not adjust their behavior in response to the policy, and so we can use the sales distribution of those firms in order to estimate a counterfactual avoidance-free firm size distribution. We compare the avoidance-free counterfactual distribution to the actual firm-size distribution around the cutoff to back out how firms change their behavior. This approach assumes that the firm-size distribution would otherwise be “well-behaved” around the cutoff. In order to validate this assumption, we show that the method neither identifies excess mass in the years before the policy was enacted nor in the years after it was phased out.

A complementary but distinct assumption is to assume that the firm-size distribution (around the

²This approach has been used in a variety of settings, including Kleven and Waseem (2013); Gourio and Roys (2014); Bachas and Soto (2018); Liu and Lockwood (2016); Almunia and Lopez-Rodriguez (2017); Garicano et al. (2016) and Onji (2009).

³For instance, Saez (2010); Chetty et al. (2011); Kleven and Waseem (2013); Bach (2015); Dee et al. (2016); Diamond and Persson (2016); Kleven (2016); Barbanchon (2016); Aghion et al. (2017b) and Lardeux (2018).

region of the tax change) is stationary (Benhabib et al., 2019). This allows us to use the time-series dimension of the data in order to estimate excess mass by estimating a counterfactual avoidance-free distribution using years when the policy was not in effect (Harju et al., 2019), avoiding the need to parametrically estimate the counterfactual distribution in the cross-section (Blomquist et al., 2019).

Using the cross-sectional information we find that there were around 150 firms who changed their sales in response to the new tax regime out of roughly one thousand firms concerned. We find similar results using the time-series information. Following Chetty et al. (2011), this corresponds to a tax elasticity of sales of 0.16.

We then turn to identifying which type of firms adjust their sales. We do this by studying the ex-ante characteristics of firms below the tax cutoff (as in Diamond and Persson 2016). If, for instance, firms who normally have high profits are more responsive to the tax policy, then we would observe relatively more of those types of firms just below the cutoff than we would counterfactually expect (for instance, by using their prevalence when the tax cutoff was not in effect).

In addition to finding that high-profit firms are more likely to avoid the tax, we also find that firms with larger adjustment costs are less likely to bunch. We start by classifying firms using their estimated capital adjustment costs (Asker et al., 2014). We show that bunchers are, depending on the years, between 3% and 15% more likely to have capital adjustment cost within the lowest tercile.

A fundamental assumption in the production function estimation literature is that materials are the most flexible input (Olley and Pakes, 1996; Levinsohn and Petrin, 2003a; Akerberg et al., 2015b) and capital has high adjustment costs. We are able to test that assumption directly, and find that firms who bunch tend to have larger elasticity of output with respect to materials and lower elasticity of output with respect to capital.

We then turn to estimating how firms avoid profit taxes. Firms have many potential margins of adjustment, including affecting their production decisions, prices, and inventories. As in Diamond and Persson (2016); Bachas and Soto (2018), and Dee et al. (2016), we note that a similar logic to the bunching estimator can help us back out firm behavior. We compare firms who potentially are distorting their behavior (those close to the cutoff) to those who are likely unaffected (those further away). If firms change some characteristics in order to shrink, then we will see differences in that characteristic in the avoidance. For instance, we find that firms avoid increasing their sales by instead increasing their inventories. In the data, this shows up as firms in the avoidance region overall having more inventories than would be predicted by the out-of-sample counterfactuals. An RD-type of estimate (comparing firms just below to just above the cutoff) is unlikely to generate the causal mechanisms for tax avoidance since the firms who choose

to avoid taxes are ex-ante different along a variety of dimensions. In order to avoid the selection issue, we compare all of the firms who might be affected by avoidance to those who are not.

We find that while firms do lower their production as they lower sales, the primary driver of the avoidance is an increase in inventories and capitalized production. This suggests that adjusting production is relatively more costly than either stocking the production or reinvesting it in the production process. Consistent with our findings on adjustment costs, we find that the firms who lower their production do so by adjusting materials but not capital. The changes “add up,” which is consistent with the values we find being real effects of tax avoidance, not just tax evasion through simple misreporting or fraud (Best et al., 2015). The importance of inventories (Alessandria et al., 2010, 2011) (Alessandria et al., 2011) (Almunia and Lopez-Rodriguez, 2017) implies that the tax elasticity we estimate is dynamic in nature, as firms shift sales to subsequent tax years.

2.2 Institutional setting and Data

2.2.1 Institutional Setting

As in other countries, entrepreneurs in France choose between two kinds of tax regimes: taxes on income (IR) or corporate income taxes (IS). Around 2/3 of firms, representing 1/5 of aggregate value added, are in the former group. Each firms’ sector determines what category of taxes it pays, with firms with benefits mostly from services (BNC) in one category, and firms in trade and manufacturing activity (BIC) or agricultural activity (BA) choosing between a regular (BRN) or simplified (RSI) setup. Around half of firms, but only around 5% of value added, are in the latter group. Around 90% of aggregate value added comes from firms in the regular tax regime (See Table 2.1 for the exact values).

Over the 1995-2000 period there were several changes in the corporate income tax rates. In 1995-1996 the basic corporate income tax was of 33.33% and firms had to pay a *contribution additionnelle* of 10%⁴ such that the corporate income tax rate was of 36.33%. In 1997-1998, the corporate income tax rate was increased by 15% through a *contribution exceptionnelle* (to 42.16%) for enterprise firms with sales above fifty million Francs (around seven million of Euros).⁵ At the same time the basic marginal tax rate on profit below 200 thousand Francs (around 38 thousand

⁴<https://www.legifrance.gouv.fr/affichTexte.do?cidTexte=JORFTEXT000000737653&categorieLien=id>

⁵Loi no 97-1026 du 10 novembre 1997 portant mesures urgentes à caractère fiscal et financier <https://www.legifrance.gouv.fr/affichTexte.do?cidTexte=JORFTEXT000000185577&categorieLien=id>

Euros) was reduced from 33% to 19% for firms with sales below the same threshold.⁶ In 1999 the *contribution exceptionnelle* rate was lowered to 10%. In 2000 this *contribution exceptionnelle* was removed and the reduced marginal corporate income tax rate on profits below 200 thousand Francs increased to 25%.⁷

The incentives to distort one firm's behavior to avoid the *contribution exceptionnelle* and potentially benefit from the reduced marginal corporate income tax rate were small but significant. Figure 2.1 shows the differential of tax rates above and below the threshold.⁸ This allows us to infer the gains, by profit, to becoming eligible to an exemption of the contribution and to taking advantage of the reduced marginal tax rate. For a firm with profits of 1.5 million Francs (roughly the sample mean), avoiding the tax would save 75 thousand Francs.

2.2.2 Data

We put together three datasets collected by the Direction Générale des Impôts and the French National Institute of Statistics (INSEE): the BRN files, the RSI files as well as the *Enquête sur les liaisons financières* (LiFi). In France, each firm has an identifier, SIREN⁹, which facilitates its interaction with the different administrations. This identifier is present in these three datasets which allows us to merge BRN and RSI files to LiFi, and to follow firms across the years. The BRN files contain balance sheet information contained in tax forms filled by firms affiliated to the BRN regime. Those files have often been used in academic research (see for instance (Caliendo et al., 2018)). Recent work has integrated smaller firms as those affiliated to the RSI regime in their analysis as well (see for instance (Garicano et al., 2016)). The RSI files contain balance sheet information collected from tax forms of firms affiliated to the RSI regime. LiFi contains information on firm conglomerate membership. We use it to restrict our sample to firms that could avoid the additional tax burden, as conglomerate members had to pay the additional tax *contributions* and weren't eligible to the reduced marginal tax rate even with turnover below the threshold.

With the BRN-RSI files we build a panel of firms that spans the 1995-2000 period. This period is well-suited for the analysis of the 1997-1999 reform as it contains two years before its imple-

⁶On November 27 1995, France's Prime Minister Alain Juppé announced a "plan PME pour la France", i.e. a package of reforms that aimed at alleviating credit constraints for SME, fostering their ability to accumulate capital and to settle in urban areas and finally reducing taxes they pay. There is however no evidence that the *contribution exceptionnelle* was part of this announcement

⁷Eligibility to the reduced marginal tax rate and exemption to the *contributions* were conditioned on 75% of firms' share capital owned by physical people and all of share capital paid-up.

⁸Tax rates are the combination of the marginal tax rates, the *contribution additionnelle* and the *contribution exceptionnelle*.

⁹This firm identifier is however not used in this paper to identify firms, in particular we do not try to determine for each firm individually whether it avoids taxes or not.

mentation that we can use as counterfactual and one year after to analyze the persistence of its consequences.¹⁰ The BRN-RSI files contain about 1.6 million firms each year. They cover the universe of firms within the BIC category affiliated to the regular and simplified regimes.¹¹

The BRN-RSI files provide all the relevant characteristics of firms, namely inputs of productions, value-added, turnover, profits, and inventories. We use nonimal values as the eligibility threshold of 50 million Francs was not indexed to inflation. To assess the weight of BRN and RSI regimes in the economy and compare it with the excluded BNC regime (Table 2.1), we use FICUS¹² that also contains aggregated information on BNC firms. For our main analysis we use the BRN-RSI files instead of FICUS as FICUS doesn't provide any information on the type of tax regime firms have opted for (either IR or IS).

A drawback of BRN-RSI files relative to FICUS is that they provide rawer information. As a consequence we make two restrictions to our main sample and clean the dataset. First, we remove extreme values of capital shares above 10. Second, we exclude firms that report negative values of inputs.

We add information on conglomerate membership from the *Enquête sur les liaisons financières* (LiFi), collected by INSEE since 1980 and available every years of the 1995-2000 period. INSEE surveys every year all firms with sales above 393 million Francs, equity portfolio above 7.9 million Francs or with more than 500 employees. Moreover the institute includes in its sample firms that were in the dataset the preceding year or firms that belong to foreign firms.

We further restrict the sample to firms with turnover between 20 million Francs and 100 million Francs, as well as to firms eligible to the tax cut. The BRN-RSI dataset allows us to keep only firms affiliated to the IS regime. Due to data limitation on share capital's ownership -on which the second eligibility criteria applied- we exclude all firms that belong to a conglomerate.

Table 2.2 presents the descriptive statistics. The average output level is 20 million Francs, the average level of turnover is 40 million Francs, material capital and labor average values are respectively 7 million Francs, 90 million Francs and 6 million Francs. The average number of employees is 37. The average profit of firms in our sample is 1.5 million Francs. Firms in our sample are on average twice as large as than the average French firm.

¹⁰In 2001 there were further changes to the tax code preventing us from doing longer follow-up analyses.

¹¹Our dataset does not include firms in the *Bénéfice Non Commercial* regime, who were unaffected by the reform, nor *Bénéfice Agricole* regime firms who tend to be very specific types of firms mostly in the agricultural sector.

¹²FICUS is a production of the French census bureau (INSEE), which confronts the balance sheet information obtained from tax forms collected by the tax office (DGFIP) and gathered in the BRN-RSI files with one of its internal sources of information, the EAE survey.

2.3 Theoretical framework

Firms maximize revenues minus costs, where costs come from production, adjustment, taxes, and potentially inventories. Firms above a certain level of sales pay additional taxes. Firms may not choose to sell everything they produce because of adjustment costs: it is costly to ramp down production (for instance because it is difficult to adjust labor and capital). In order to avoid the higher tax rate, firms may choose instead to hold inventories at the end of the year.¹³ Pushing against this force is that holding inventories is also costly. Given convex inventory and adjustment costs, under some regularity assumptions the optimal choice of the firm is to produce less than the bliss point, but still put some goods in inventory.

Other than considering adjustment costs and inventories, our approach is standard (e.g. [Bachas and Soto \(2018\)](#); [Velayudhan \(2018\)](#)). A firm i who sells s_i units of a good earns profits of

$$R(s_i) - c(q_i) - \gamma_s A(q_i^* - q_i) - \tau(R(s_i) - c(q_i)) + I(q_i - s_i) \quad (2.1)$$

where $R(\cdot)$ is the revenue function, $c(\cdot)$ is the constant-returns-to-scale production cost, $A(\cdot)$ is the twice-differentiable convex adjustment cost coming from producing other than q^* ¹⁴, γ_s is how expensive the adjustment cost is at the sector level, q is the quantity produced τ is the (profits) tax, and $I(\cdot)$ is the twice-differentiable concave inventory net benefit. We solve for optimal firm behavior in two steps: first by showing that for a *given* s , there is an optimal production/inventory decision for the firm, and then solving for s in the profit function.

Given $s < q^*$, a firm seeks to minimize

$$(1 - \tau)c(q_i) + \gamma_s A(q_i^* - q_i) - I(q_i - s_i).$$

Defining $c' = \phi_i$,

$$\gamma_s A'(q_i^* - q_i) = (1 - \tau)\phi_i + I'(q_i - s_i) \quad (2.2)$$

Specifying $A(q - q^*) = 1/2 \cdot A_1(q - q^*)^2$, $c(q) = 1/2 c_1 \cdot q^2$, $I(q - s) = -1/2 \cdot (q - s)^2 + I_2 \cdot (q - s)$,¹⁵ the equation becomes:

¹³For simplicity of exposition, we model the choice of the firm statically: it does not take into account its current production choices on either the bliss point or the ability to draw down future inventories. We think this is reasonable because it is only towards the end of the year that firms discover that they may end up benefiting from adjusting their sales, so long-term adjustments are less relevant. Inventories are an important dynamic consideration for this context, and we use a static reduced-form representation to capture this force.

¹⁴ q^* being the level at which adjustment cost are minimized

¹⁵with I_2 large enough such that for plausible values of $q - s$, I is increasing

$$A_1 \cdot (q^* - q) + A_2 = (1 - \tau)c_1 q + (q - s) + I_2 \quad (2.3)$$

i.e.

$$q = \frac{s - I_2 + A_1 q^* + A_2}{(1 - \tau)c_1 + A_1 + 1}, \quad (2.4)$$

from which, specifying $R(s) = s$ we can derive an expression of profits:

$$\pi(s) = s - c_1 \left(\frac{s - I_2 + A_1 q^* + A_2}{(1 - \tau)c_1 + A_1 + 1} \right)^2 \quad (2.5)$$

which attains its maximum value for

$$\tilde{s} = \frac{((1 - \tau)c_1 + A_1 + 1)^2}{2c_1} - A_2 + I_2 - A_1 q^* \quad (2.6)$$

If there is a jump in the tax schedule at the threshold θ :

$$\begin{cases} \tau = 0 & s_i \leq \theta \\ \tau = .15 & s_i > \theta \end{cases}$$

then some firms will have to make a discrete choice: comparing profits at $s_i = \theta$ to the best choice at $\tau = .15$. Those are the firms for which demand is at least equal to θ and for which the equation

$$\pi(\theta) > \pi(\min(\tilde{s}, s_i)) \quad (2.7)$$

is satisfied.¹⁶ Rearranging the terms and writting $\tilde{s} = \theta + \Delta\theta$ hence $A_2 + A_1 q^* - I_2 = \frac{((1 - \tau)c_1 + A_1 + 1)^2}{2c_1} - \theta - \Delta\theta$ the inequality becomes:

$$\begin{aligned} & (1 - \tau) \left[\theta + \Delta\theta - \frac{(1 + (1 - \tau)c_1 + A_1)^2}{2c_1} \right] \\ & < \theta - \frac{c_1}{(1 + A_1 + (1 - \tau)c_1)^2} \left(\frac{(1 + A_1 + (1 - \tau)c_1)^2}{2c_1} - \Delta\theta \right)^2. \end{aligned} \quad (2.8)$$

As in [Kleven and Waseem \(2013\)](#) there will therefore be a cutoff $\tilde{\theta}_i$ for which a firm would weakly

¹⁶For exposition purposes we then assume that demand is high enough such that $\min(\tilde{s}, s_i) = \tilde{s}$.

prefer to sell θ then anything in $(\theta, \tilde{\theta}_i)$.¹⁷

To be precise $\tilde{\theta}_i$ is defined as:

$$\tilde{\theta}_i = \theta + \tilde{\Delta}\theta \quad (2.9)$$

with

$$\begin{aligned} \tilde{\Delta}\theta = & (1 - \tau) - \left(1 - \frac{\tau}{1 + A_1 + c_1}\right)^2 + \\ & \sqrt{\left[(1 - \tau - \left(1 - \frac{\tau}{1 + A_1 + c_1}\right)^2](1 - \tau - 3\left(1 - \frac{\tau}{1 + A_1 + c_1}\right)^2)^2 - \frac{2\theta c_1}{(1 + A_1 + c_1)^2}\right]} \end{aligned} \quad (2.10)$$

Hence, decreasing A_1 increases $\tilde{\theta}_i$. Besides $\tilde{\theta}_i$ reaches a maximum (that depends of the parameters of profit function), which as in [Kleven and Waseem \(2013\)](#) allows us to bound the dominated region. One way that firms might have lower adjustment costs might be due to idiosyncratic firm-specific features, such as better access to capital markets.

Following [Asker et al. \(2014\)](#), we interpret the dispersion in the marginal product of the inputs of production as a proxy for idiosyncratic adjustment costs (since with no adjustment costs the dispersion would be zero). The production function literature assumes that capital adjustment costs are the largest ones, followed by labor adjustment costs and then material adjustment costs ([Akerberg et al., 2015b](#); [Asker et al., 2014](#); [Levinsohn and Petrin, 2003a](#); [Olley and Pakes, 1996](#)). This assumption implies that firms whose production is relatively more sensitive to materials face in average lower adjustment costs.

2.4 Empirical approach

2.4.1 Discontinuity Estimates

If firms endogenously lower their size in order to avoid the extra taxes, there will be excess mass in the distribution below the threshold and correspondingly too little mass above. We follow the standard approaches in the literature to measure excess mass, [McCrary \(2008\)](#).¹⁸ As a robustness test, we also follow [Cattaneo et al. \(2016\)](#), who have a similar intuition but do not require

¹⁷The cutoff depends on the firm i to the extent that the parameters q_i^* , (A_1, A_2) and (I_1, I_2) are firm specific, depending on the profit function.

¹⁸This approach consists in implementing the local linear density estimator and suggests a bandwidth selection algorithm.

prebinning of the data.

2.4.2 Bunching Estimators

Using Predictions from Distribution away from the Threshold as Counterfactual

An alternative approach to measuring bunching is to use firms far from the threshold. In the region with neither excess nor a dearth of mass, we estimate a fifth-degree polynomial of the firm-sales density. We then estimate the density polynomial in the hold-out avoidance region. The difference between the estimated density and the actual density captures the extent of avoidance. We similarly use the predicted density as a placebo in years with no discontinuous tax threshold. To be precise we estimate the following specification:

$$E[c_j] = e^{(\alpha + \sum_{i=1}^5 \beta_i^{prediction} \cdot (z_j)^i + \sum_{i=r(z_U)-1}^{r(z_U)-1} \gamma_i^{prediction} \cdot 1[j=i])} + \epsilon_j \quad (2.11)$$

where c_j counts the number of firms in bin j . z_j is turnover level in bin j .¹⁹ $r(\cdot)$ is the bin number associated to turnover z . Given that the variable of interest counts the number of firms per year, we follow standard practice in the literature and rely on a Poisson regression. $\beta_i^{prediction}$ is the coefficient of order i of the fifth degree polynomial in turnover. $\gamma_i^{prediction}$ identifies the excess or lack of firms in bin i compared to the counterfactual estimated with the polynomial. z_L is the beginning of the avoidance region and z_U its end.²⁰ We determine z_L by eyeballing the distribution (Figure 2.3) and z_U is determined such that excess bunching,²¹ i.e. the sum of firms in excess below threshold in the avoidance region equals missing mass, i.e. the sum of firms that are missing compared to the counterfactual above threshold in the avoidance region.

Formally we determine z_U as the smallest turnover level such that

$$\hat{M} = \sum_{i=z_T}^{z_U} \hat{c}_i^{cf} - c_i = \sum_{i=z_L}^{z_U-1} c_i - \hat{c}_i^{cf} = \hat{B}, \quad (2.12)$$

where z_T is turnover level at the threshold, i.e. 50 million Francs. The number of firms per bin in the counterfactual distribution is determined from

$$\hat{c}_j^{cf} = e^{(\alpha + \sum_{i=1}^5 \beta_i^{prediction} \cdot (z_j)^i)}. \quad (2.13)$$

¹⁹We take bins of 150 thousand Francs for all the analysis such that one of them start at the threshold.

²⁰When we can't eyeball any avoidance, we estimate bunching to be zero as we can't identify any avoidance region that is necessary to estimate bunching with this methodology.

²¹This is the standard procedure in the literature, see e.g. (Chetty et al., 2011)

To normalize the amount of bunching we estimate the average bunching b_{av} that is defined as the ratio of excess bunching over mean density in the avoidance region below threshold. Empirically we define it as:

$$\hat{b}_{av} = \frac{\hat{B}}{\frac{1}{2} \sum_{i=z_L}^{z_T-1} \hat{c}_j^{cf}} \quad (2.14)$$

Using Past Years as Conterfactual

Another way to investigate the apparition of distortions in the firm-size distribution over time is to compare distributions of firms across times around the threshold. We use distributions in years during which there is no incentive to bunch as counterfactual distributions and compare them to the distribution under the policy.

To be precise we estimate the following specification:

$$c_{jt} = \alpha \cdot Post_t + \sum_{i=r(z_L)}^{r(z_U)} \beta_i^{time-series} \cdot 1[j \in [i, i+2]] 1[i \equiv 0[3]] + \sum_{i=r(z_L)}^{r(z_U)} \gamma_i^{time-series} \cdot 1[j \in [i, i+2]] 1[i \equiv 0[3]] * Post_t + \epsilon_{it}. \quad (2.15)$$

c_{jt} counts the number of firms in bin j in year t . $Post_t$ refers to years 1997-1998 when the 1995-98 period is under study. z_j is turnover level in bin j . Given that the variable of interest counts a number of firms per bin, the natural choice for the estimation is to rely on poisson regression.²²

For the sake of clarity, we only report the coefficients of the interaction terms $\gamma_i^{time-series}$ around the threshold and report exponentiated coefficients in figure 2.4. It shows that there is excess bunching in the 1997-1998 distribution compared to the 1995 and 1996 distributions.²³ there are three positive and significant coefficients below the threshold and three negative and significant coefficients above.

To quantify the size of the distortion we rely on an estimation procedure that looks similar to the one presented in the previous paragraph. We pool 4 consecutive years to increase statistical precision and because we can gather years by the actual incentive level firms face. In particular we gather years 1995 and 1996 where firms face no incentives for avoidance and years 1997 and 1998 where firms above threshold paid 15% more taxes on profit.

The coefficients of the interacted terms ($\gamma_i^{time-series}$) in equation 2.15 allow us to estimate missing

²²In the figures, we show results for groups of three bins.

²³We include 1996 as counterfactual year as there is no evidence of anticipation effect in the cross-section results and because we were unable to find any evidence of announcement of the reform prior to year 1997.

mass and excess bunching. They indicate how many additional firms there is per bin below the cutoff. The product of the exponential of the coefficients of the interacted term $\gamma_i^{time-series}$ and of the dummy variable for bin i tells us by how much we must multiply the number of firms in the excluded bin to estimate the number of firms that are in bin i . Subtracting the number of firms that were in this bin in years during which there was no incentives to bunch gives the number of firms that bunch in this bin. The sum of firms that bunch in each bin below the threshold where the interacted coefficients are significant gives us the amount of excess bunching. We similarly estimate the number of missing firms in bins above the threshold to obtain the missing mass.

We can describe the size of the distortion with formal expressions for excess bunching (\hat{B}) and missing mass (\hat{M}):

$$\begin{aligned}\hat{M} &= \sum_{i=z_T}^{z_U} \hat{c}_{iControl} - \hat{c}_{iTreat} \\ \hat{B} &= \sum_{i=z_L}^{z_T-1} \hat{c}_{iTreat} - \hat{c}_{iControl}\end{aligned}\tag{2.16}$$

where: \hat{c}_{iTreat} refers to the average predicted number of firms per year within bin i during the period of treatment (1997-99) and $\hat{c}_{iControl}$ to the average predicted number of firms per year within this bin during control years.

z_L is the turnover level of the group of bins that precedes the bunch of ones with significant coefficients below threshold.²⁴ In our context, 47,600,000 Francs is the lower end of the valley. This is an advantage of our estimation strategy compared to usual techniques of bunching that eyeball the lower end of the avoidance region.

To back out the upper end of the valley we use prediction from the estimations. We follow [Kleven and Waseem \(2013\)](#) and pin down Z_U by the equality $\hat{M} = \hat{B}$.

²⁴We do not take the smaller level of turnover of this bunch of coefficients because there might be bunchers in the group of bins that precedes it and missing them may misinform us about the characteristics of the bunchers if those that we miss have particular characteristics that would drive a change of the results. On the contrary including non bunchers in the avoidance region does not affect the estimated numbers of bunchers since the coefficient of the interaction term on this group of bins is close to zero. It does not either affect the determination of bunchers characteristics that compare the characteristics of firms below threshold in the avoidance region within years with or without incentives to bunch. Firms below threshold that are not bunching should indeed have the same characteristics in years with and in years without incentives to bunch.

2.4.3 Identifying Bunchers' Characteristics

Conditional on identifying bunching in the firm size distribution, we are also interested in understanding what are the ex-ante characteristics of the firms that avoid taxes. Below, we describe our approach, which builds on [Diamond and Persson \(2016\)](#).

Using Predictions from Distribution away from the Threshold as Counterfactual

In the language of potential outcomes, we can consider the firms below the cutoff in 1995 (before the policy reform) to be “always takers,” and firms above to be both “never takers” and “compliers.” The difference between the observed characteristics of firms above the cutoff and the no-avoidance counterfactual value is due to the compliers leaving (and similarly below the threshold). As a result, we use the difference between observed and counterfactual characteristics of firms to estimate the types of firms who change their size in response to the policy change. In particular, we estimate the characteristics of the bunchers ($\bar{X}^{compliers}$) as the average of the two methods:

$$\begin{aligned} \bar{X}^{compliers} = 0.5 * \left(\frac{N_{down}^{tot}}{N_{down}^{tot} - N_{down}} * \bar{X}^{down_all} - \frac{N_{down}}{N_{down}^{tot} - N_{down}} * \bar{X}^{down} \right) + \\ 0.5 * \left(\frac{N_{up}}{N_{up} - N_{up}^{tot}} * \bar{X}^{up} - \frac{N_{up}^{tot}}{N_{up} - N_{up}^{tot}} * \bar{X}^{up_all} \right), \end{aligned} \quad (2.17)$$

where $\bar{X}^{compliers}$ is defined as the average of the mean values of X for firms that are “missing” above the threshold and for firms that are bunching. \bar{X}^{down_all} (resp. \bar{X}^{up_all}) is the average of characteristic X for firms that are in the avoidance region below (resp. above) threshold. and \bar{X}^{down} (resp. \bar{X}^{up}) is the average of X for firms that would have been in the avoidance region below threshold (resp. above threshold) had there been no avoidance.

\bar{X}^{down} and \bar{X}^{up} are obtained by regressing X on a polynomial of turnover of order 5 for firms outside the avoidance region and predicting levels of X within the avoidance region by extrapolating this relationship. In a sense, we extend the traditional estimation of the number of firms per bin (c_{jt}) to the average characteristics of firms per bins. We are able to predict the characteristics, that *conditional* on its turnover level, the firm would have had had there been no manipulation.

N_{up}^{tot} (resp. N_{down}^{tot}) is the number of firms that fall into the avoidance region above (resp. below) the threshold. They are the number of never takers and the sum of the number of always takers and of compliers. N_{up} (resp. N_{down}) is the number of firms that would have fallen into the

avoidance region above (resp. below) the threshold had there been no avoidance. It is the sum of the number of never takers and the number of compliers (resp. the number of always takers). Note that the parameter $N_{up} - N_{up}^{tot}$ (resp. $N_{down}^{tot} - N_{down}$) identifies the number of compliers which we estimate using the method detailed in the previous section.

We are interested to compare the characteristics of the compliers to the characteristics of the firms that are eligible to bunching i.e. those that would have been above threshold absent avoidance. With [Diamond and Persson \(2016\)](#) notation this means we are interested in estimating:

$$E[\Delta X] = E[X^{compliers}] - E[\bar{X}^{up}]. \quad (2.18)$$

We estimate this raw difference of means as well as a difference of means net of sector fixed effects. In practice, we restrict the sample to firms within the avoidance region. For firms above the threshold, we predict $X_i^{up} = E(X_i | s = s_i, \text{no avoidance})$ the characteristics a firm with the same level of turnover as firm i would have had, had there been no avoidance, filling in the relation between turnover and the characteristics of interest outside the manipulation region. The set of firms above the threshold with the predicted characteristics is used to define a counterfactual population²⁵ with the characteristics of firms above the threshold in absence of avoidance (namely compliers and defiers²⁶).

Then, we estimate for all firms within the avoidance region the firm level counterpart of equation (2.17) i.e. for firms below threshold $X_i^{compliers} = x_i * \frac{N_{down}^{tot} - N_{down}}{N_{down}^{tot} - N_{down}} - \hat{x}_i * \frac{N_{down}}{N_{down}^{tot} - N_{down}}$ and for firms above the threshold $X_i^{compliers} = \hat{x}_i * \frac{N_{up}}{N_{up} - N_{up}^{tot}} - \frac{N_{up}^{tot}}{N_{up} - N_{up}^{tot}} * x_i$, where x_i is the observed characteristics of the firm and $\hat{x}_i = E(X_i | s = s_i, \text{no avoidance})$ is the characteristics a firm with this level of turnover would have had, had there been no avoidance.

To estimate equation (2.18) we simply estimate the difference of means of the $X^{compliers}$ in the observed population and of X^{up} in the counterfactual population.

Technically, to estimate this difference of means, we duplicate the observations corresponding to firms above threshold within the manipulation region and flag the duplicates with a dummy *compliers*. For all²⁷ the initial observations within the manipulation region, *compliers* takes value 1, for the duplicates it takes value 0. We then define X_i as X_i^{up} when *compliers* takes value 0 and $X_i^{compliers}$ when *compliers* takes value 1. Then we estimate the following equation, with

²⁵The size of this counterfactual population is given by the size of defiers. Each firm of this population has the same level of turnover as its observed counterpart.

²⁶If the tax discontinuity is a treatment, then firms that avoid the tax are compliers, those that stay within the manipulation region above the threshold are defiers

²⁷not only those above threshold

and without sector fixed effects:

$$X_i = \eta \cdot compliers_i + \mu_s + \epsilon_i, \quad (2.19)$$

where μ_s indicates sector fixed effects at the 16-sector French classification of industries. The coefficient of interest is η that directly gives us the difference of means between the two populations of interest. We estimate standard errors by bootstrapping the regression.

Using past years as counterfactual

In addition to estimating the counterfactual characteristics of firms in the avoidance region using firms in the same year who are larger, we also can do so using the observed data in the pre-policy years. After the policy change, the compliers move from above to below the cutoff, in order to avoid excess taxation. The difference between the firms above the cutoff before and after the policy change is due to the compliers leaving, and similarly the difference below the cutoff is due to the compliers joining. The expression of bunchers' characteristics become:

$$\begin{aligned} \bar{X}^{compliers} = 0.5 * & \left(\frac{N_{down}^{tot}}{N_{down}^{tot} - N_{down}} * \bar{X}_{Treatment}^{down} - \frac{N_{down}}{N_{down}^{tot} - N_{down}} * \bar{X}_{Control}^{down} \right) + \\ & 0.5 * \left(\frac{N_{up}}{N_{up} - N_{up}^{tot}} * \bar{X}_{Control}^{up} - \frac{N_{up}^{tot}}{N_{up} - N_{up}^{tot}} * \bar{X}_{Treatment}^{up} \right), \end{aligned} \quad (2.20)$$

where $\bar{X}_{Treatment}^{down}$ is the average, when the policy is in place, of the mean values of X for bunching firms (“compliers”) and firms that are naturally present below the threshold absent the policy (“always-takers”). It is obtained by estimating the mean of the characteristic X of interest in the avoidance region below threshold over years with incentives to bunch (treatment years). $\bar{X}_{control}^{down}$ is the mean of the characteristic X of interest in the avoidance region below the threshold over years with no incentives to bunch (control years). It is therefore the mean of the characteristics of interest for the “always takers”.

Similarly $\bar{X}_{Treatment}^{up}$ is the mean of the characteristic of interest X in the avoidance region above threshold over years with incentives to bunch. It is the average level of X for never takers. $\bar{X}_{Control}^{up}$ is the mean of the characteristic of interest X in the avoidance region above the threshold in control years. It is therefore the average of the means of X for never takers and compliers.

N_{up}^{tot} (resp. N_{down}^{tot}) is the number of firms that fall into the avoidance region above (resp. below) the threshold in treatment years. They are the number of never takers and the sum of the number of always takers and of compliers. N_{up} (resp. N_{down}) is the number of firms that fall into the

avoidance region above (resp. below) the threshold in control years. It is the sum of the number of never takers and the number of compliers (resp. the number of always takers).

Our analysis might be subject to a change in the variables due to an underlying trend in their evolution. As a result we are interested in the de-trended variables defined as $\tilde{X} = X - \bar{X}_{below}$ where \bar{X}_{below} is the average of X in the region neighboring the avoidance region below the threshold ($Turnover \in [45000-47600[)$ and during years with no incentives to bunch. We use the region below the part of the avoidance region below the threshold because we are sure there is no compliers in this region in years with no incentives to bunch.

Here we also compare the characteristics of the compliers to the characteristics of the firms that would have been above threshold absent avoidance, which in this case are the firms that are in the avoidance region above threshold in years with no policy. We therefore seek to estimate:

$$E[\Delta\tilde{X}] = E[\tilde{X}^{compliers}] - E[\tilde{X}_{Control}^{up}]. \quad (2.21)$$

$E[\tilde{X}^{compliers}]$ is obtained as the average of the observed characteristics of firms in the avoidance region below and above threshold when or before the policy was implemented, weighted as indicated in equation 2.20. In practice we multiply each characteristic by the appropriate weight²⁸ (and obtain a variable \tilde{X}_i at the firm level). To estimate $E[\tilde{X}_{Control}^{up}]$, we duplicate observations in the avoidance region above threshold in years with no policy. We identify these additional observations with a dummy variable *compliers* that takes value 1 for the initial observations and value 0 for the new observations. When *compliers* takes value 0 we define \tilde{X}_i as firm i characteristics. The average of \tilde{X}_i over firms for which *compliers* take value 0 identifies $E[\tilde{X}_{Control}^{up}]$. To estimate $E[\Delta\tilde{X}]$ we simply estimate η of the following equation with and without fixed effects:

$$\tilde{X}_i = \eta \cdot compliers_i + \mu_s + \epsilon_i. \quad (2.22)$$

2.4.4 Measuring Adjustment Cost and Output Elasticities

One characteristic of firms that we are interested in is the cost of adjustment. First, we can measure adjustment costs using the dispersion of the revenue share of capital within each sector. Optimally, marginal products (the production function elasticity) should equal marginal costs (the revenue share, as in [Asker et al. \(2014\)](#); [Hall \(1988\)](#)). Adjustment costs can prevent equalization, and so the dispersion of the revenue share is a proxy for industry-level adjustment costs. Formally we define adjustment cost as:

²⁸only the multiplication by + or - the inverse of the number of compliers as the numerators of the weights are already captured by the number of firms within the different regions

$$Adjustment\ cost_{it} = SD_{it}(\alpha_i + y_{jt} - k_{jt}), \quad (2.23)$$

where α_i refers to sector i logarithm of output elasticity with respect to capital, y_{jt} refers to firm j log-level of production on year t , k_{jt} refers to firm j log-level of capital on year t and SD is the standard deviation operator.

Second, following a long tradition in production function estimation, e.g. [Levinsohn and Petrin \(2003a\)](#) and [Olley and Pakes \(1996\)](#), we use production function elasticities themselves in order to measure the costs of adjustment. If materials are the most flexible input, the firms for whom the elasticity of output with respect to materials are the highest should have the lowest-cost in quickly adjusting their outputs (and firms with high capital elasticities the highest costs). In order to estimate production function elasticities, we follow the [Wooldridge \(2009\)](#) adaptation of [Levinsohn and Petrin \(2003a\)](#).

2.4.5 Identifying Bunchers' Choices

In addition to measuring the ex-ante characteristics of the bunchers, we are also interested in what they do in order to lower their sales below the threshold. An RD strategy (comparing firms just below to those just above the cutoff) is not appropriate for estimating causal effects in this context. This is for two reasons. First, the firms who are able to (barely) avoid the extra tax may be different than those who do not, and so the RD estimate may suffer from selection bias. Second, conditional on sales the firms who are avoiding the extra tax may behave identically to non-avoiding firms.²⁹ Note that this isn't an issue for ex-ante characteristics, which are not directly affected by firm choices. To estimate firm choices, we compare *all* of the firms in the avoidance region to their counterfactual counterparts.

Using Past Years as Counterfactual

In our setting, in order to compute the average value of the outcome of interest, had there been no avoidance, there is no need to predict the average value of the outcome of interest based on its relationship with the running variable outside the avoidance region. We can simply use the average value of the outcome of interest in years during which there was no incentives to bunch as counterfactual. To make sure our estimate is not driven by temporal changes in the outcome of interest, we detrend the outcome of interest by subtracting from it the mean of the outcome

²⁹Consider the following stark example: sales is a deterministic (continuous) function of materials. As a result, complier firms who choose to lower their sales will have the same materials use as the never-takers, and there would be no discontinuity at the policy cutoff.

of interest in the region neighboring the avoidance region (i.e. the regions just below and just above). Our intent to treat estimate simplifies to :

$$ITT = E(\tilde{Y}|firms\ avoid\ taxes) - E(\tilde{Y}|firms\ don't\ avoid\ taxes) = E_{Avoid}^{Treat}(\tilde{Y}) - E_{Avoid}^{Control}(\tilde{Y}), \quad (2.24)$$

where *Treat* in exponent indicates that the expectation is taken over observation during the treatment years, while *Control* indicates that the expectation is taken over observation in years during which there was no incentives to bunch. \tilde{Y} is the de-trended characteristic of interest.

Using Predictions from Distribution away from the Threshold as Counterfactual

We also follow [Diamond and Persson \(2016\)](#) as a robustness test. Their strategy consists in predicting the value of the outcome of interest for firms in the avoidance region, had there been no avoidance. In that order, we regress the characteristic of interest on a polynomial of turnover of order 5 for firms outside the avoidance region and predict levels of \tilde{Y} within the avoidance region by extrapolating this relationship. As a result the Intent To Treat estimator is estimated as :

$$ITT = E(\tilde{Y}|firms\ avoid\ taxes) - E(\tilde{Y}|firms\ don't\ avoid\ taxes) = E_{Avoid}^{Observed}(\tilde{Y}) - E_{Avoid}^{Predicted}(\tilde{Y}). \quad (2.25)$$

Figure 2.9 gives a visual intuition for the approach. Panel A shows the firm size distribution for 1995. Firms to the left of the cutoff are always takers, since their potential outcome under the tax notch would also be below the notch. To the right of the notch are a mix of compliers and never takers. To the right of the manipulation region are never takers, firms who would not bunch in response to the policy change.

2.5 Empirical analysis

In this section we describe the results of our data analysis. First we show the existence of bunching in the firm size distribution consistent with the theory. We then describe the characteristics of firms who bunch and then the way that they do so.

2.5.1 Excess Mass

Figure 2.2 shows the raw firm size distribution around the tax cutoff. Before the tax reform (in 1995 and 1996) and after (in 2000) there is no visual break in the firm size distribution, but it is clearly visible in 1997-1999. We calculate a counterfactual distribution far away from the cutoff as a solid line. The vertical lines show the excess mass (and under mass) for those years, where the extra mass on the left is equal to the undermass on the right. The avoidance region is fairly symmetric around the threshold. Figure 2.3c shows, just for the years 1997-1998, the observed difference in densities around the cutoff relative to the pre-reform years of 1995-1996.

Table 2.3 presents discontinuity estimates from McCrary (2008) estimation technique which show the bunching only appear in the years with the discrete jumps in the average tax rates.

Table 2.4 shows the size of the avoidance region, where both for the cross sectional and panel estimates we find excess mass of around or above 150 firms just below the increase tax rate. Column (3) shows consistent average bunching estimates using both techniques. Finally, we are able to estimate from this discontinuity the tax elasticity of sales of 0.16, using the *R* package *bunchr* that implements the methodology well described in Chetty et al. (2011).

The size of the avoidance region is not enormous. One important question is when firms realize that it may be profit maximizing to adjust their sales. In Figure 2.5 we show that firms who end up in the avoidance region have a wide range of turnover the previous year. Furthermore, the distribution of previous-year turnover does not shift after the policy, suggesting that it is difficult for firms to predict at the start of the financial year that the tax cutoff will be near their ultimate turnover.

2.5.2 Characteristics of Excess Mass

In this subsection we describe the types of firms who shrink their size in order to avoid paying taxes. First we show differential bunching by profit level. Since the tax rate is on profits, firms who, e.g., have no profits should not be affected by the policy. Consistent with this, we see in the right panel of Figure 2.6 that the lowest profitability firms do not demonstrate excess bunching in any of the years (while in 1997 there is a spike to the left of the cutoff, there is no corresponding valley to the right). For the most profitable firms, however, we do see sales adjustment, consistent with the theory. In Figure 2.7 we use firm profitability in 1995 (instead of in the current year) and find a similar result.

In Figure 2.8 we run the same exercise, but using adjustment costs (as measured using (Asker et al., 2014)). The results are less clean than for profits, but again consistent with theory that firms

with the lowest adjustment costs bunch the most.

Table 2.5 runs the estimation of equation 2.11, and finds consistently that the higher-profit and lower capital adjustment cost firms show more bunching

2.5.3 Characteristics of Compliers

An alternative approach to measuring who bunches is to instead look within the avoidance region (Diamond and Persson, 2016). In Tables 2.6 and 2.7 we report the estimates corresponding to equations 2.22 and 2.19 using respectively predictions from firms away from the threshold and previous years as counterfactual. In the first rows, we extend the results from the previous subsection: compliers are more likely to have low adjustment costs and high profits. Results are robust to the inclusion of region and sector fixed effects as shown in the columns headed with "FE". We can also examine the production function characteristics of the compliers: they are more likely to have a lower capital elasticity and a higher materials elasticity. This is consistent with the oft-stated argument that materials inputs are more flexible (Akerberg et al., 2015b; Asker et al., 2014; Levinsohn and Petrin, 2003a; Olley and Pakes, 1996): the types of firms who find it easier to adjust their sales are those whose output is more responsive to materials. As a robustness check we run the same analysis on the sub sample of firms that have input shares below 1 and for which the sum of the input shares is lower than 2. Tables A.0.1 and A.0.2 show that our results are stable to this restriction.

2.5.4 Behavior of Compliers

We can undertake a similar exercise to show the behavioral changes of the compliers (Tables 2.8 and 2.9 that estimate equations 2.24 and 2.25). Here we consider the entire avoidance region relative to its counterfactual prediction either in the cross section or the panel. Not surprisingly, we see that turnover is lower: this is the direct effect of tax avoidance. Sold production falls by more³⁰ than output does: inventories and capitalized production are higher for the firms adjusting their sales. As a robustness check we run the same analysis on the sub sample of firms that have input shares below 1 and for which the sum of the input shares is lower than 2. Tables A.0.4 and A.0.3 show that our results are stable to this restriction.

The behavior of the compliers is also informative for understanding the extent to which inputs are flexible. Firms who lower their production presumably do so by decreasing their input intensity (instead of by decreasing their TFP, which would not be profit maximizing). This implies that,

³⁰admittedly not significantly

for a given quantity of sales, firms who adjusted inputs in order to avoid the tax should be using relatively *more* of the less flexible inputs, and correspondingly less of the more flexible inputs. In tables 2.11 and 2.10 we find evidence consistent with this. In Table 2.11, using cross-sectional information, we find higher capital/output and lower materials/output ratios for firms in the avoidance region. In Table 2.10, using the panel, we find similar effects for capital although not materials.

Revenue shares are measured with error, so as before we run robustness checks for different levels of cleaning. In Appendix Tables A.0.6 and A.0.5 we show our results are similar when we are less conservative in our data cleaning choices. In particular when instead of restricting to the subsample of firms that have input shares below 1 and for which the sum of the input shares is lower than 2, we focus our analysis to our main sample where the restriction is on having capital shares below 10.

2.6 Discussion

In this paper we describe how firms respond to a new notch in profit taxes. The nature of our setting, the introduction of a new tax above a specific sales threshold, allows us to estimate many important parameters determining firm responses. First, we follow a large literature to estimate the tax elasticity of sales, and find an elasticity of 0.16. This is consistent with a literature which finds large elasticities in developing countries (e.g. Best et al. (2015)) and lower elasticities in Europe Harju et al. (2019).)

Our main contribution is to document mechanisms. First, and not surprisingly, we find that higher-profit firms, who are more affected by profit taxes, are more likely to adjust their reported sales. More fundamentally, we find that measures of adjustment costs used in the literature, such as firms whose production is more responsive to intermediate inputs (Asker et al., 2014), do a good job of predicting which firms distort.

We complement this evidence by showing that as firms lower their production, they mostly do so lowering their material inputs (and not capital), which as far as we know is the first direct evidence for a key assumption in the production function literature on the relative adjustability of inputs (Olley and Pakes, 1996; Levinsohn and Petrin, 2003a; Akerberg et al., 2015b). However, firms mostly do not adjust sales, and instead increase their inventory holdings and push sales to the subsequent year. This is a feature specific to notches, as firms shifting their production over time is only relevant in cases where the marginal tax rate is known to vary over time. In future work, we hope to build on this result to separately identify static versus dynamic tax elasticities.

Tables

Table 2.1: Share of Firms Affiliated to the Different Tax Regimes

	Obs.	Share					VA value	VA share				
	Nb	BNC	BRN	RSI	IS	IR	B Eur	BNC	BRN	RSI	IS	IR
1995	2,034,117	0.19	0.32	0.49	0.33	0.67	611.33	0.05	0.89	0.06	0.78	0.22
1996	2,226,769	0.18	0.31	0.51	0.33	0.67	627.47	0.05	0.88	0.07	0.80	0.20
1997	2,262,301	0.19	0.30	0.51	0.34	0.66	645.21	0.05	0.88	0.07	0.82	0.18
1998	2,297,619	0.19	0.30	0.51	0.35	0.65	719.77	0.05	0.89	0.07	0.84	0.16
1999	2,323,909	0.20	0.30	0.51	0.35	0.65	792.03	0.04	0.89	0.06	0.80	0.20
2000	2,325,726	0.19	0.30	0.50	0.36	0.64	821.33	0.04	0.89	0.06	0.81	0.19

Note: This table present the share of firms affiliated to the different regimes in the economy. Columns 3,4,5 9,10,11 are obtained using FICUS dataset. Columns 6,7,12,13 are obtained using BRN-SI files. BNC firms are in services. Firms can choose between a regular (BRN) or simplified (RSI) setup, and between taxes on income (IR) or corporate income taxes (IS).

Table 2.2: Descriptive statistics

	mean	count	sd
Sample			
Output	20,160	191,511	21,680
Turnover	39,546	203,609	18,870
Profit	1,305	203,609	9,378
Materials	6,999	145,538	10,366
Capital	8,865	199,005	37,887
Wage Bill	5,437	200,754	5,234
Employee	37	203,609	39
All firms			
Output	8,463	8,510,565	1,022,583
Turnover	10,060	10,682,393	925,310
Profit	1,028	10,682,393	277,244
Materials	2,867	5,498,696	207,403
Capital	5,414	9,147,730	796,084
Wage Bill	1,801	6,769,965	70,909
Employee	7	10,682,393	336

Note: The sample is restricted to eligible firms paying a corporate income tax and with turnover between 20 million Francs and 100 million Francs. We also drop observations with capital share larger than 10 and observations with negative inputs values.

Table 2.3: Discontinuity estimates

Raw dataset			
	(McCrary, 2008) estimates	standard errors	(Cattaneo et al., 2016) p-values
1995	.058	(.092)	0.9661
1996	.001	(.076)	0.6937
1997	-.331	(.083)	0.0002
1998	-.598	(.089)	0.0000
1999	-.714	(.114)	0.0002
2000	-.143	(.110)	0.1280
Balanced dataset			
	(McCrary, 2008) point estimates	standard errors	(Cattaneo et al., 2016) p-value
1995	.274	(.210)	0.6291
1996	.110	(.184)	0.3414
1997	-.647	(.193)	0.0384
1998	-.919	(.201)	0.0153
1999	-.617	(.155)	0.0052
2000	-.056	(.150)	0.2822

Note: This table reports discontinuity estimates for the two samples. The balanced dataset is the dataset restricted to the set of firms that have filled tax forms each years of the 1995-2000 period. Column 1 and 2 report the point estimates and standard errors obtained from ([McCrary, 2008](#)) estimation procedure, column (3) reports the p-value.

Table 2.4: Bunching estimators

Panel A: Cross-Section Estimates			
	\hat{B}	\hat{M}	\hat{b}_{av}
1997	49.724	60.947	0.958* (0.633)
1998	114.278	134.700	2.198*** (0.587)
1999	86.100	95.980	1.511*** (0.370)
Panel B: Time-series Estimates			
1997-1998	204.264	225.945	0.415

Note: This table reports the bunching estimators estimated with usual techniques (Panel A) and the bunching estimators obtained from the technique that uses past years as counterfactual (Panel B). \hat{M} is missing mass and \hat{B} excess bunching. \hat{b}_{av} refers to average bunching

Table 2.5: Bunching Estimation by Subgroups

	Capital adjustment cost		Profits		Profits in 1995	
	Top	Bottom	Top	Bottom	Top	Bottom
1997	0	1.017	0.490	0	0.329	0.651
	-	(0.304)***	(0.332)	-	(0.321)	(0.312)**
1998	0.722	0.616	0.879	0	0.832	0.170
	(0.271)***	(0.270)**	(0.210)***	-	(0.248)***	(0.374)
1999	0.376	0.719	0.653	0	0.657	0.288
	(0.228)*	(0.310)**	(0.194)***	-	(0.198)***	(0.392)

Note: This table reports the bunching estimators for different subgroups of firms. Values are zero if we do not estimate any bunching.

Table 2.6: Characteristics of the compliers: Panel

Adjustment cost, Incentives, Ability to bunch						
	Low adjustment cost of capital		Large profit		Large profit in 1995	
	(1)	(2)	(3)	(4)	(5)	(6)
		FE		FE		FE
Compliers	0.0364** (0.0152)	0.0341** (0.0133)	0.00519 (0.0133)	0.00144 (0.0133)	-0.00906 (0.0133)	-0.0123 (0.0103)
Observations	6344	6277	6405	6338	5424	5378
Production function characteristics						
	Large elasticity wrt K		Large elasticity wrt L		Large elasticity wrt M	
[1em] Compliers	-0.0404*** (0.0144)	-0.0387*** (0.0135)	0.00849 (0.0131)	0.00890 (0.0139)	0.0363** (0.0152)	0.0354** (0.0142)
Observations	6344	6277	6344	6277	6344	6277

Note: This table describes characteristics of the bunchers, identified following [Diamond and Persson \(2016\)](#). We compare firms in the adjustment region to firms in the region during years with no incentive to adjust sales. Variables are centered with the mean of the variable in the region just below the avoidance region: Turnover in 45000-47600. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics. Columns (2) (4) and (6) report estimation with region and 16 industry fixed effects. Output elasticities are computed following ([Levinsohn and Petrin, 2003a](#)) estimation procedure. Adjustment cost of capital are calculated following [Asker et al. \(2014\)](#).

Table 2.7: Characteristics of the compliers: Cross-Section

Adjustment cost and Incentives to bunch						
	Low adjustment cost of capital		Large profit		Large profit in 1995	
	(1)	(2)	(3)	(4)	(5)	(6)
		FE		FE		FE
Compliers	0.166*	0.166*	0.486***	0.441***	0.277***	0.241***
	(0.0864)	(0.0936)	(0.0835)	(0.0824)	(0.0705)	(0.0664)
Observations	1326	1326	1334	1334	1178	1178
Production function characteristics						
	Large elasticity wrt K		Large elasticity wrt L		Large elasticity wrt M	
[1em] Compliers	0.0123	0.0520	0.150*	0.126	0.0808	0.0530
	(0.0906)	(0.0943)	(0.0847)	(0.0962)	(0.0892)	(0.0930)
Observations	1326	1326	1326	1326	1326	1326

Note: This table describes characteristics of the bunchers, identified following [Diamond and Persson \(2016\)](#). We compare firms in the adjustment region to what we would predict for their characteristics given those of firms far away from the manipulation region. Variables are centered with the mean of the variable in the region just below the avoidance region: Turnover in 45000-47600. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics. Columns (2) (4) and (6) report estimation with region and 16 industry fixed effects. Output elasticities are computed following ([Levinsohn and Petrin, 2003a](#)) estimation procedure. Adjustment cost of capital are calculated following [Asker et al. \(2014\)](#).

Table 2.8: Consequences of Avoidance on Production Process: panel

	(1)	(2)	(3)	(4)	(5)
	Turnover	Y	Sold production	Change in inventories	Capitalized production
Avoidance	-86.97** (38.71)	-170.3 (275.5)	-329.0 (274.4)	120.7** (53.35)	38.01* (22.23)
Observations	5274	5274	5274	5274	5274

Note: This table describes production choices of the bunchers, identified following [Diamond and Persson \(2016\)](#). We compare firms in the adjustment region to firms in the region during years with no incentive to adjust sales. Variables are centered with the mean of the variable in the region just below the avoidance region: Turnover in 45000-47600. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics.

Table 2.9: Consequences of Avoidance on Production Process: Cross Section

	(1)	(2)	(3)	(4)
	ln Y	ln Sold production	Change in inventories	Capitalized production
Avoidance	0.0937 (0.0723)	0.0839 (0.0771)	92.36 (74.63)	19.02 (44.13)
Observations	2278	2276	2278	2278

Note: This table describes production choices of the bunchers, identified following [Diamond and Persson \(2016\)](#). We compare firms in the adjustment region to what we would predict for their characteristics given those of firms far away from the manipulation region. Variables are centered with the mean of the variable in the region just below the avoidance region: Turnover in 45000-47600. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics.

Table 2.10: Consequences of Avoidance on Input Choices: Panel

	(1) Y	(2) M over Y	(3) L over Y	(4) K over Y
Avoidance	-632.7* (370.2)	0.00766 (0.00562)	0.00388 (0.00463)	0.0311** (0.0151)
Observations	3691	3691	3691	3691

Note: This table describes input choices of the bunchers, identified following [Diamond and Persson \(2016\)](#). We compare firms in the adjustment region to firms in the region during years with no incentive to adjust sales. Variables are centered with the mean of the variable in the region just below the avoidance region: Turnover in 45000-47600. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics.

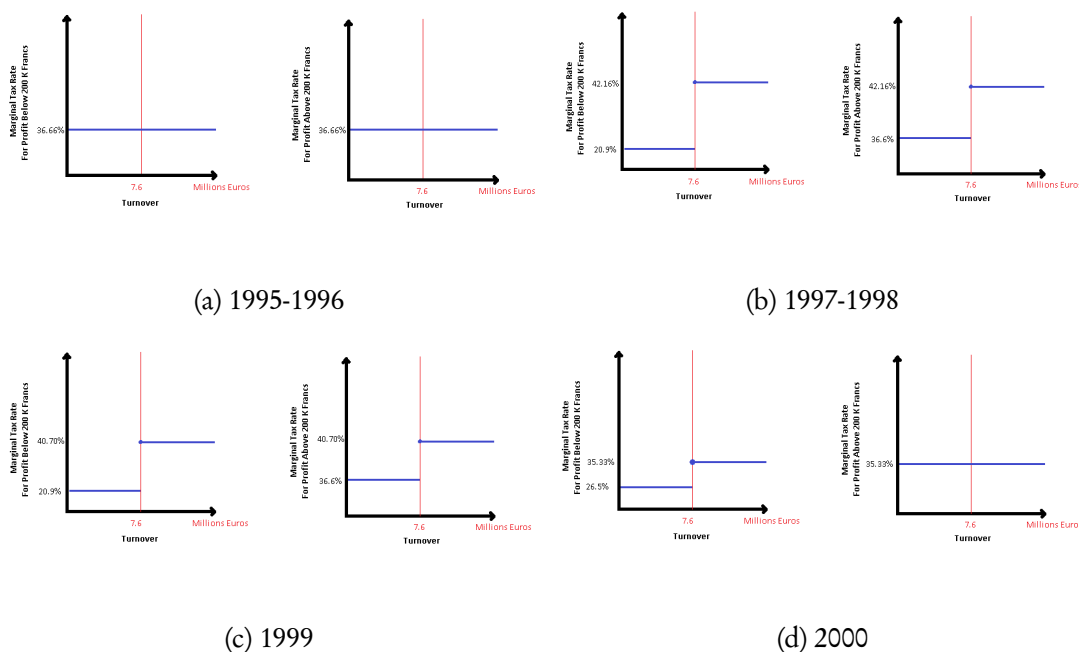
Table 2.11: Consequences of Avoidance on Input Choices: Cross Section

	(1) ln Y	(2) ln M/Y	(3) ln L/Y	(4) ln K/Y
Avoidance	-0.219*** (0.0843)	-0.206*** (0.0743)	0.000148 (0.00963)	0.0832** (0.0337)
Observations	1772	1764	1764	1764

Note: This table describes input choices of the bunchers, identified following [Diamond and Persson \(2016\)](#). We compare firms in the adjustment region to what we would predict for their characteristics given those of firms far away from the manipulation region. Variables are centered with the mean of the variable in the region just below the avoidance region: Turnover in 45000-47600. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics.

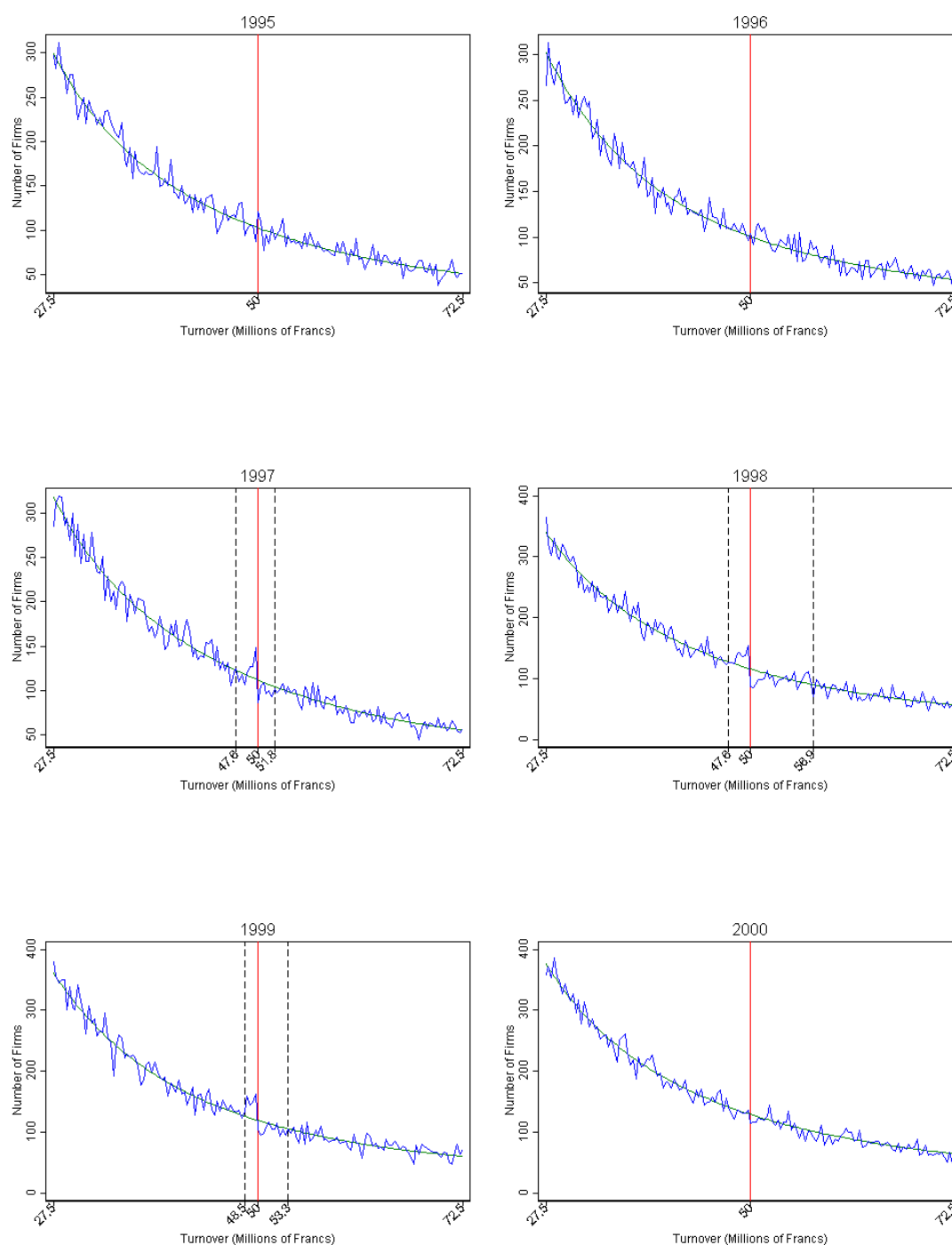
Figures

Figure 2.1: Evolution of the Tax Schedule



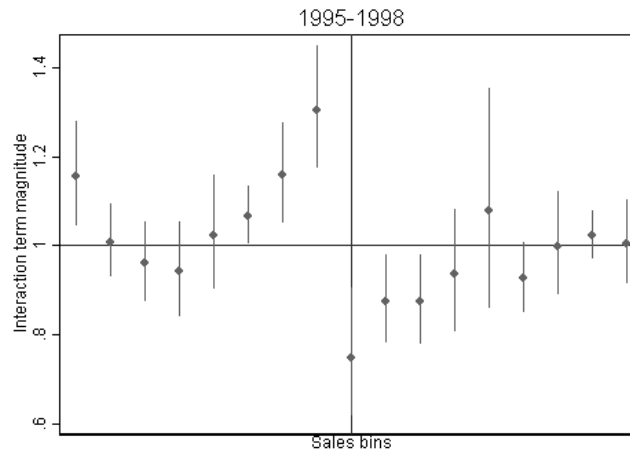
This figure presents the marginal rates of firms eligible to either the reduced corporate income tax or not. The marginal tax rates are the sum of three terms: the *contribution exceptionnelle*, the *contribution additionnelle* and the corporate income tax rate.

Figure 2.2: A transient discontinuity in firms' sales distribution



The distribution of firms with sales between 28 million Francs and 73 million Francs, restricting to firms that are paying corporate income tax and excluding firms that belong to a conglomerate.

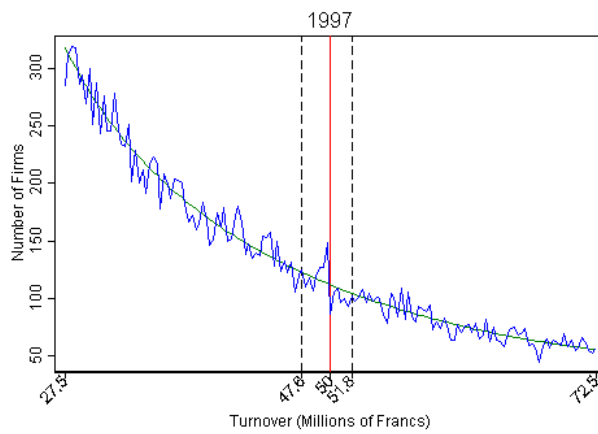
Figure 2.4: Representing bunching compared to counterfactual obtained from years during which there was no incentive to bunch



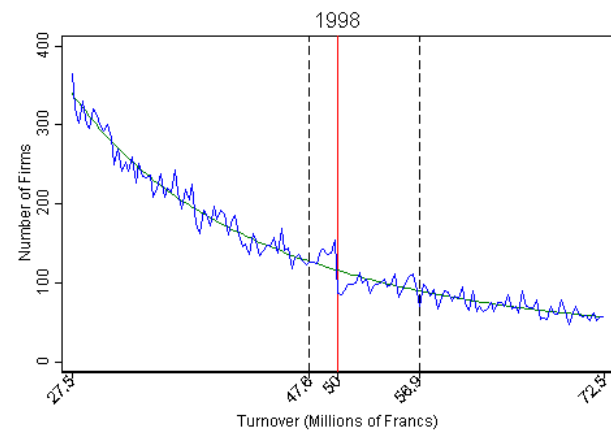
The figure plots the differential number of firms in bins around the threshold in years during which there was incentives to bunch compared to years during which there was no incentives.

Each point comes from an interaction term between the bin indicator and the indicator of incentives. 95% confidence intervals are constructed using robust standard errors clustered at the bin level.

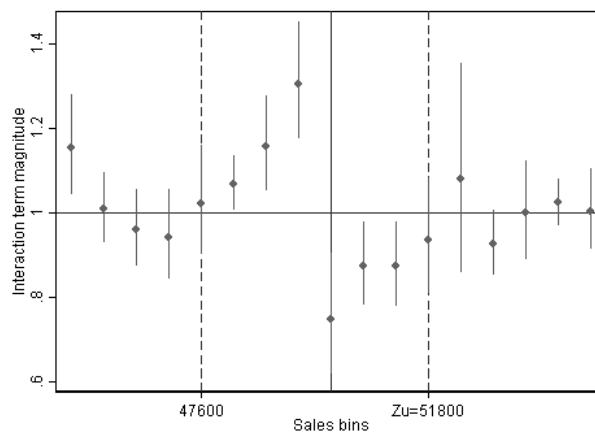
Figure 2.3: Estimating the width of the valley



(a) Cross section 1997



(b) Cross section 1998



(c) Time series 1997-1998

The figure reports the avoidance region. It illustrates the lower end z_L that is determined either eyeballing where the distribution starts being different from the counterfactual (a) and (b) or as the lower end of the group of bins that precede the bunch of coefficients that are significant below threshold (c). z_U is determined from the equality of missing mass and excess bunching as reported in Table 2.4

Figure 2.5: Pdf Conditional on Being in the Avoidance Region the Following Year

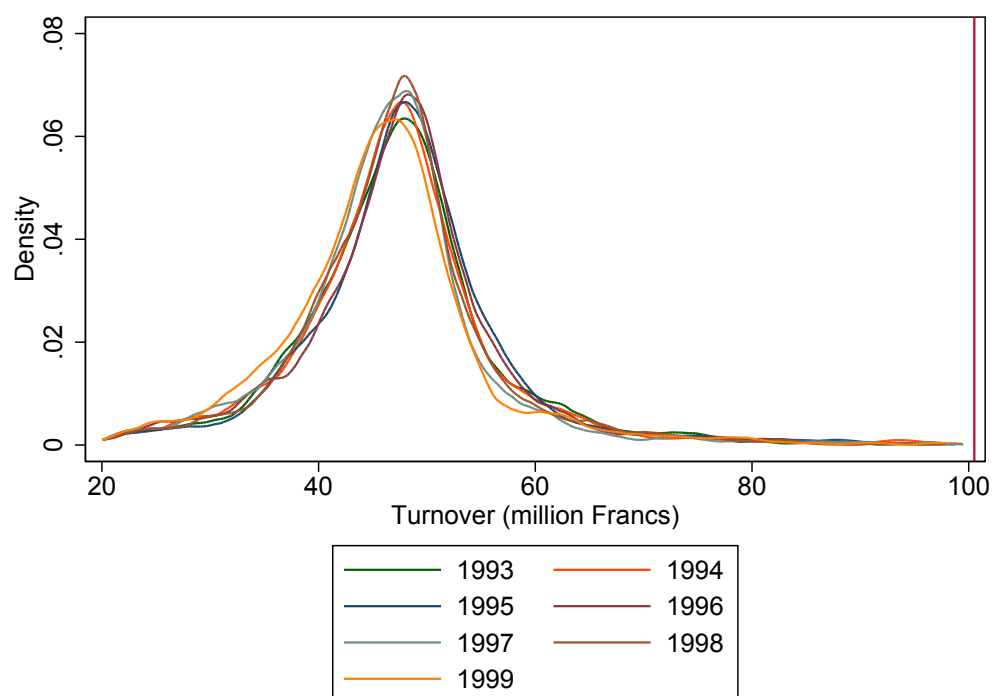
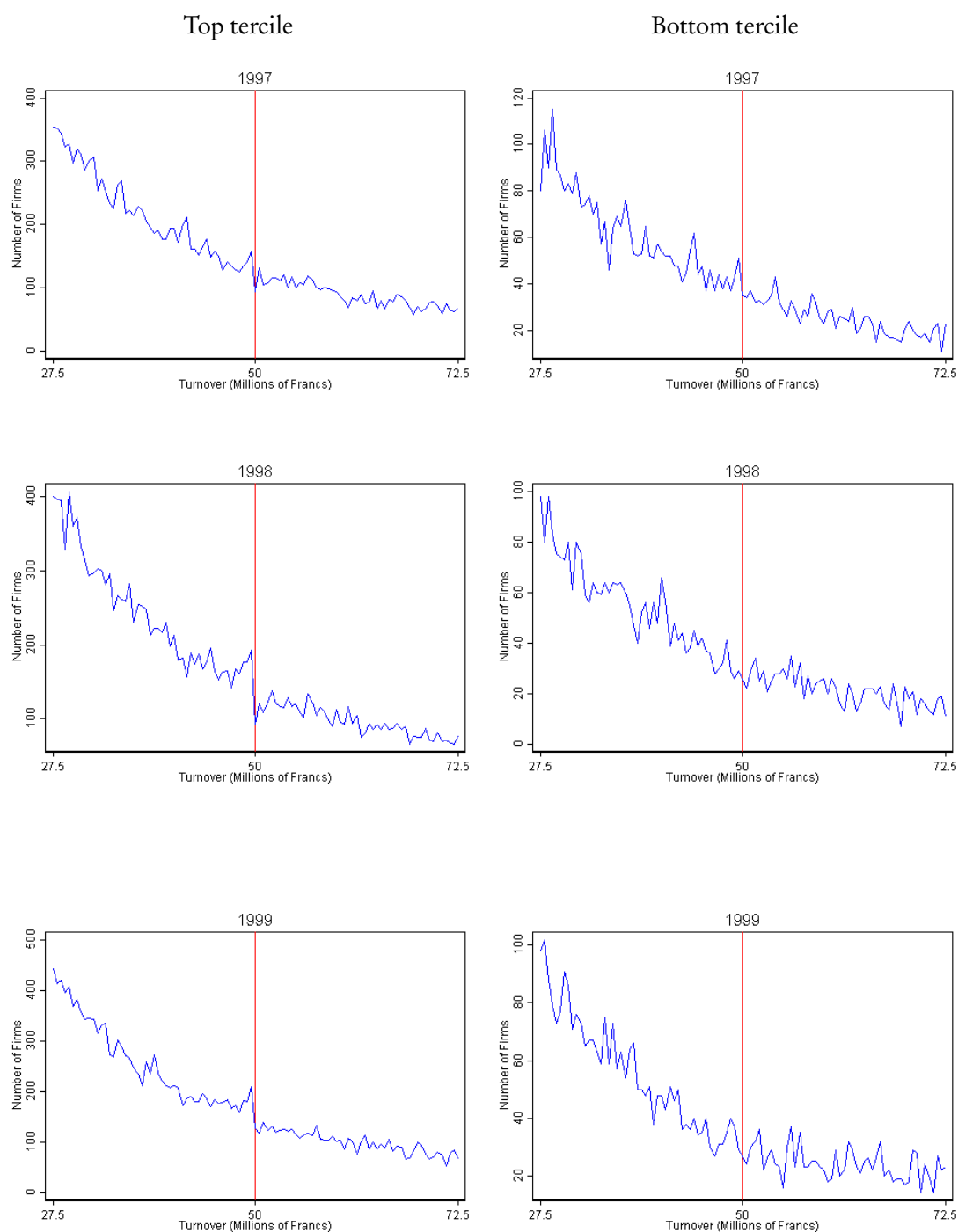
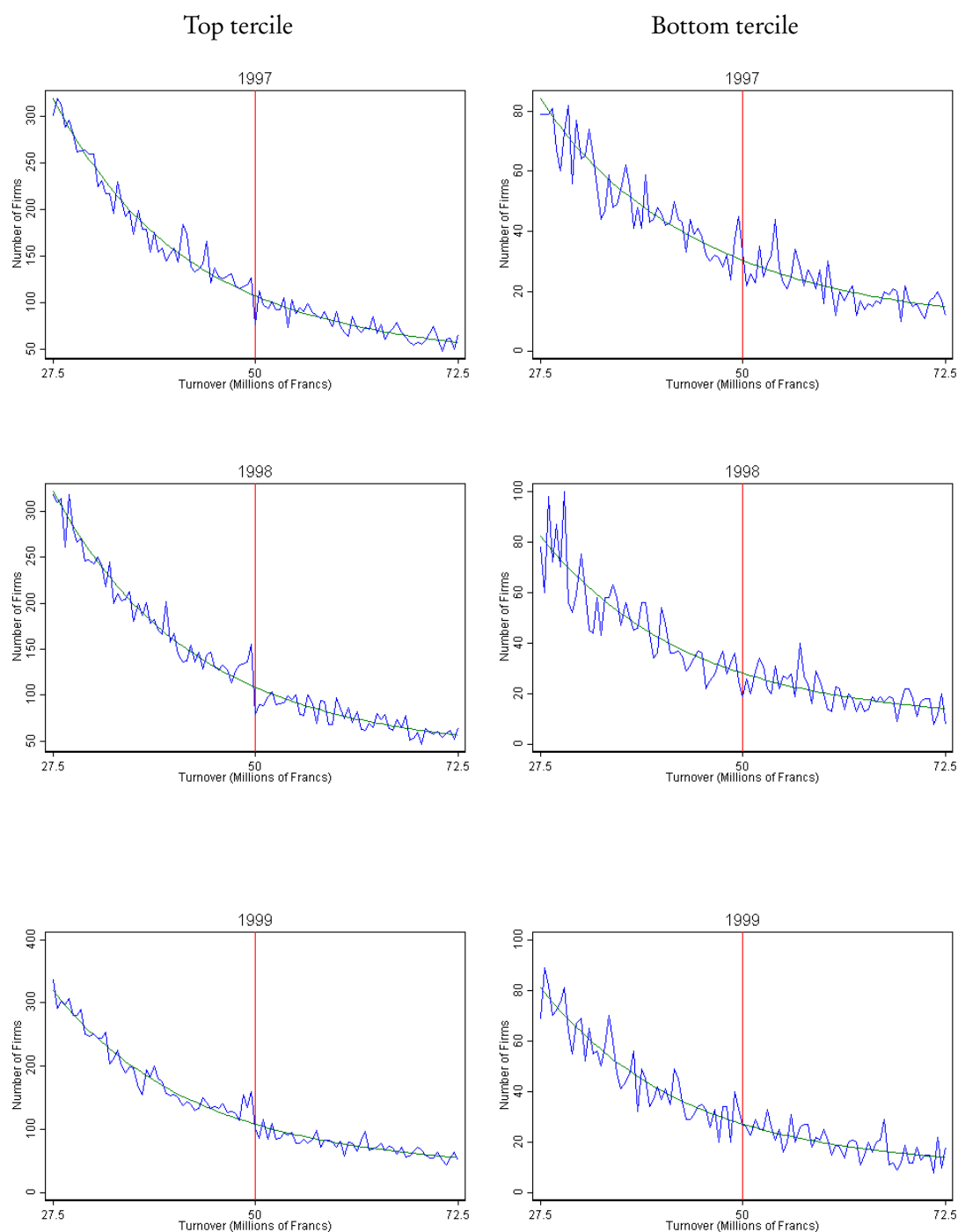


Figure 2.6: Differential bunching by profit level



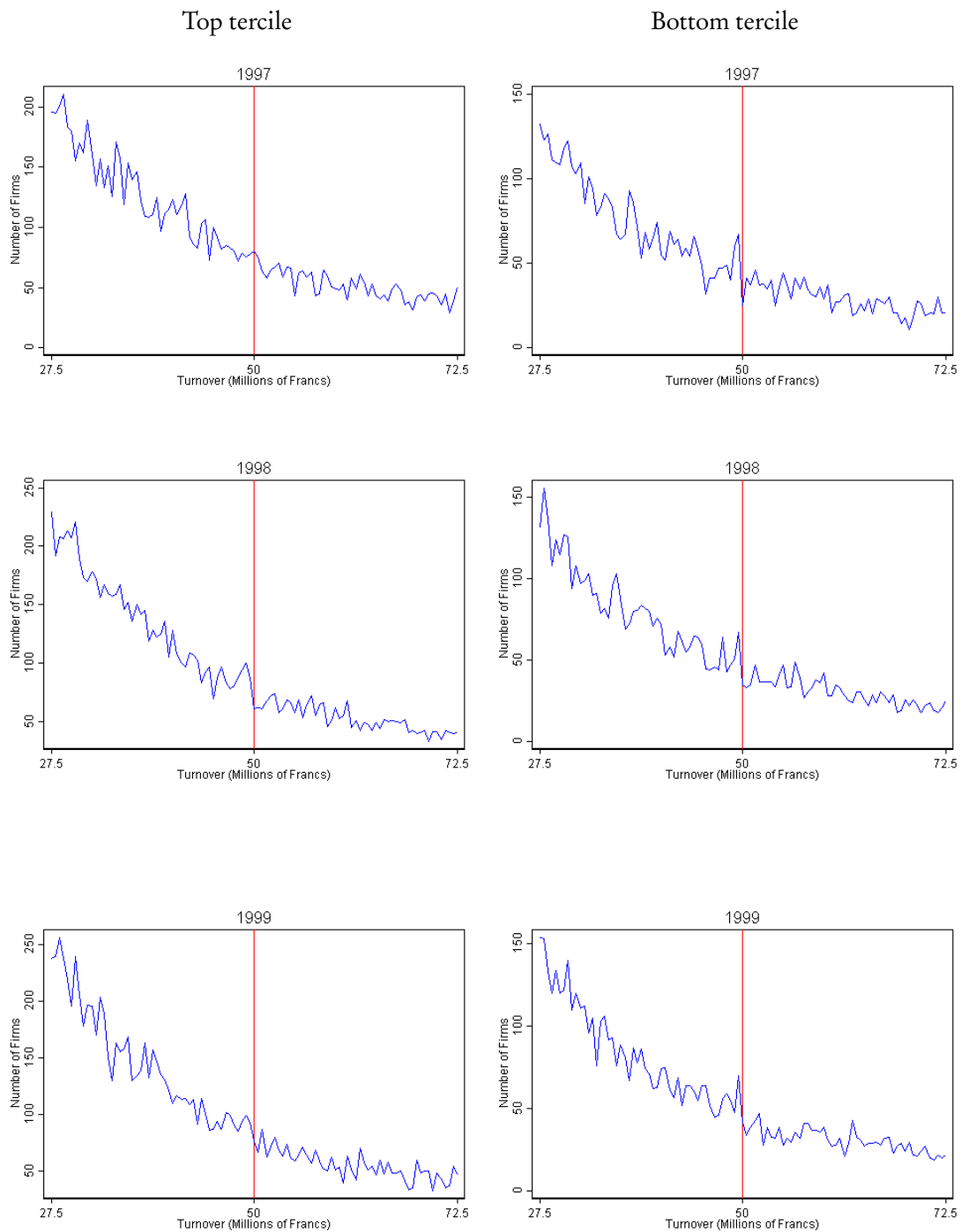
The distribution of firms with sales between 28 million Francs and 73 million Francs, restricting to firms that are paying corporate income tax and excluding firms that belong to a conglomerate.

Figure 2.7: Differential bunching by profit level in 1995



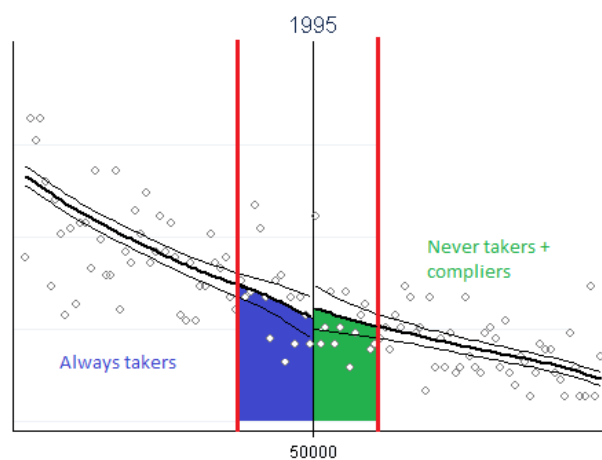
The distribution of firms with sales between 28 million Francs and 73 million Francs, restricting to firms that are paying corporate income tax and excluding firms that belong to a conglomerate, by level of profit.

Figure 2.8: Differential bunching by Adjustment cost

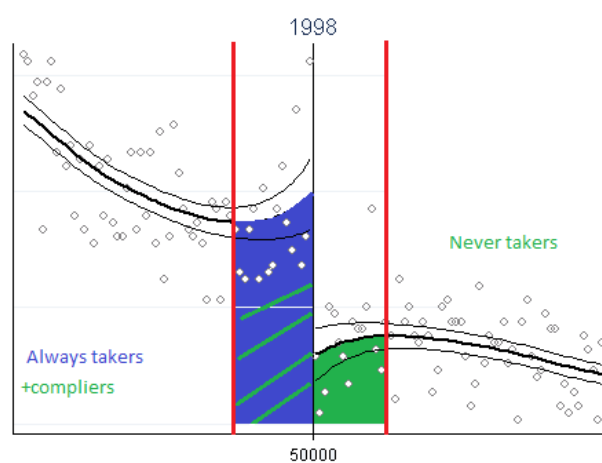


The distribution of firms with sales between 28 million Francs and 73 million Francs, restricting to firms that are paying corporate income tax and excluding firms that belong to a conglomerate, by level of adjustment cost measured as in ([Asker et al., 2014](#)).

Figure 2.9: Assessing the Characteristics of the Bunchers



(a) Control



(b) Treatment

These graphs follow [Diamond and Persson \(2016\)](#). The additional tax can be interpreted as a treatment to which bunching firms are compliers.

Chapter 3

Market Power and Labor Share ¹

Secular trends in market power and labor share have important implications for inequality and allocative efficiency. Studying them requires comprehensive and detailed firm-level data spanning several decades. For that purpose, we leverage a novel and detailed database on the universe of French firms between 1984 and 2016, that we use to document a rise in concentration in France since the beginning of the 1990s. Despite a relative stability of the aggregate labor share, we show that larger firms with lower labor shares have been gaining market shares, especially in industries where concentration increased the most. We rely on markups as proxies of firm-level market power, and on a flexible production function that allows the identification of firm-specific output elasticities and markups. We find that the markup of the *typical* firm has decreased, but the reallocation of market shares toward larger firms contributed to an increase of the aggregate markup. Finally, we show how taking into account reallocation across firms is essential to understand how the aggregate market power evolution has shaped the dynamics of the aggregate labor share in France.

¹Co-authored with Jocelyn Boussard

3.1 Introduction

Large and productive superstar firms have been gaining market shares in many advanced economies, and the rise of their market power has been the focus of attention in many recent works. [De Loecker et al. \(2020b\)](#) have documented an increase in top firm market power in the US that is large enough to have important macroeconomic consequences. They find that the weighted average markup in the United States rose from 21% above marginal cost at the beginning of the 1980s to around 61% now. [Autor et al. \(2020a\)](#) also document a rise of the weighted average markup in the US. [Gutiérrez and Philippon \(2018\)](#) argue that European market are more competitive, and exhibit lower levels of concentration, lower excess profits and lower barriers to entry, which raises the question of whether the secular trends above are specific to the US. We use detailed firm-level administrative data on the universe of French firms to document facts about market power and labor shares in France.

These questions are important for inequality concerns. One of the important macroeconomic implications of a rise of market power is a decline in the aggregate share of income going to workers. Given that there is ample evidence that labor is more evenly distributed than capital ([Garbinti et al., 2018](#); [Piketty et al., 2018](#)) or firm ([Bauer et al., 2018](#)) ownership, a decline in the aggregate labor share is a possible driver of inequality. Important work has shown that the aggregate labor share has indeed been declining in a wide range of countries ([Karabarbounis and Neiman, 2014](#); [Elsby et al., 2013a](#); [Grossman et al., 2017](#)). Using aggregate data, [Barkai \(2017\)](#) shows that both the labor share and capital shares have declined in the United States, while measures of the profit share have increased. Looking more closely at firm-level data, [Autor et al. \(2020a\)](#); [Kehrig and Vincent \(2018\)](#) show that the labor share of the typical firm has actually increased, while the aggregate fall is attributable to reallocation from high to low-labor share firms.

Market power trends have also important but ambiguous consequences for efficiency. As shown by [Baqaee and Farhi \(2020a\)](#), a reallocation of market share to high-markup firms, as in [Autor et al. \(2020a\)](#) increases efficiency, but an increase in markup dispersion, as in [De Loecker et al. \(2020b\)](#) decreases efficiency. Moreover, as shown by [Aghion et al. \(2005\)](#), the sign of the relationship between competition and innovation depends on the initial level of competition, and [Gutiérrez and Philippon \(2017b\)](#) show that lower competition have led firms to under-invest.

Understanding the underlying micro-structural transformations behind these aggregate trends is crucial to identify their possible explanations such as changes in the competitive environment and changes in technology. For instance, [Bonfiglioli et al. \(2019\)](#) and [Panon \(2019a\)](#) show that national firms compete in markets that are increasingly global, which reduces firm-level markups but benefits larger firms, and [Melitz \(2003\)](#) [Mayer et al. \(2014\)](#) show that international compe-

tition causes reallocation toward top producers. Recent work ([Autor et al., 2020a](#); [Van Reenen, 2018](#)) argues that technological change, such as the growth of platform competition in digital markets, may have caused reallocation from small to large firms that could lead to dominance by a small number of firms. [Lashkari et al. \(2019b\)](#) find that the rise of Information Technology has disproportionately benefited larger firms.

We use France as a laboratory to study the link between variations in industry concentration and firm-level outcomes, and provide evidence on the sources of market power variations. France is an interesting case because in contrast to the US, the labor share in France appears to have been stable or increasing over the past decades (see [Figure 3.1](#)).

We document important facts about secular trends in France that are similar to what has been documented for other advanced economies. When we decompose labor share variations in France, we show that there has been an important reallocation of market shares from firms with high labor shares to firms with low labor shares, which tend to be larger. This reallocation is correlated with a rise of industry concentration, measured through a wide range of proxies, from the beginning of the 1990s. However, labor shares have on average increased at all points of distribution, a development that has offset the effect of reallocation and explains why the aggregate labor share in France is broadly stable.

To assess the extent to which firm-level market power dynamics has played a role in explaining the divergence between firm-level labor share in France and the US, as opposed to other explanations like technological change, we estimate firm-level markups and output elasticities using a flexible production function that allows variations in the marginal product of inputs both across firms and time periods. We follow [De Loecker and Warzynski \(2012\)](#) and first estimate firm-level elasticities of value-added to labor and capital, and then recover markups by assuming that firms minimize their costs and that labor is a flexible input. We rely on unique and comprehensive administrative data covering the universe of French firms. This data is produced by the French National Institute of Economics and Statistics (INSEE) and contains standard income and balance sheet information for almost all firms in France from 1984 to 2016.

We find no evidence that the rise in concentration translated into an increase in firm-level market power. We find that there is substantial heterogeneity in markups, and that markups are increasing with firm size. We also find that much of the increase in firm-level labor shares is attributable to decreases in firm-level markups. All in all, high-markup firms gained market shares while the markup of the typical firm decreased, which indicates both an improvement in allocative efficiency and a reduction of the distortive effect of markups. We also show that this reallocation is strongly correlated with the rise in concentration at the industry level.

Our paper contributes to the macroeconomic literature that documents a number of important

secular trends that have recently swept across advanced economies. A number of recent papers have documented growing industry concentration and within-industry dispersion in firm outcomes (Andrews et al., 2016a; Berlingieri et al., 2017b; Song et al., 2018; Card et al., 2013). In parallel, there is a large body of evidence on a global fall in the labor share across many industries (Elsby et al., 2013a; Karabarbounis and Neiman, 2014, 2018; Barkai, 2017; Grossman et al., 2017). We show that concentration and firm-level market power are not necessarily correlated, even at the top, even though at the aggregate level the reallocation of market shares toward high-markup firms contributes to a rise in the aggregate markup. Our results that firm-level markups have decreased, and that reallocation towards high markup firms contribute both to a rise in concentration and rise in aggregate markup are consistent with Autor et al. (2020a) but the decrease in firm-level markups in France is larger, and the reallocation effect does not offset it.² This difference is also consistent with evidence in Gutiérrez and Philippon (2018) that European markets have become more competitive than US markets.

The rest of the paper is organized as follows. Section 3.2 presents our theoretical framework, Section 3.2 presents our strategy for estimating firm-level markups, Section 3.3 presents the data that we use to implement this strategy, Section 3.4 documents important macro and micro facts about labor share and concentration in France, and Section 3.5 presents our results about markups in France. Finally, Section 3.6 concludes.

3.2 Theoretical and Empirical Framework

Theoretical Framework

In this section, we provide a general theoretical framework that allows us map variations in aggregate the labor share to variations in firm-level market power, input elasticities and market shares. Consider an industry with N firms indexed by i . Consistent with a wealth of evidence and in the spirit of canonical models (Melitz, 2003; Hopenhayn, 1992), we assume that firms have heterogeneous exogenous productivity Ω_{it} and have access to a common production technology $\mathcal{Q}(\cdot)$ defined as:

$$Y_{it} = \mathcal{Q}(\Omega_{it}, L_{it}, K_{it}),$$

that they use to produce value-added Y_{it} , using variable labor input L_{it} , and capital stock K_{it} . We assume that adjusting the capital stock is subject to cost $\mathcal{C}_a(\cdot)$, which depends only on the current

²Possible interpretations of these difference are that the market power of French firms is more sensitive to the underlying cause, for instance if French firms are more exposed to globalization or to competition on internet platforms than US firms, or if the productivity gap between top French firms and laggards is not as large as for top US firms.

and previous levels of capital, and crucially not on variable inputs levels. The value function of the firm is:

$$\begin{aligned} \mathcal{V}(\mathbf{Z}_{it}) &= \min_{\mathbf{X}_{it}} \mathcal{C}(\mathbf{X}_{it}, \mathbf{Z}_{it}) + \beta \mathbb{E}[\mathcal{V}(\mathbf{Z}_{it+1})], \\ \text{s.t. } \mathcal{Q}(\Omega_{it}, \mathbf{X}_{it}) &= Y_{it}, \end{aligned}$$

where $\mathcal{C}(\cdot)$ is the total cost of the firm, $\mathbf{X}_{it} = (L_{it}, K_{it})$ refers to inputs, and \mathbf{Z}_{it} to variables that are exogenous to the firm at time t , such as previous year capital stock, productivity and input prices. The Lagrangian associated with the right-hand-side of the Bellman equation is defined as:

$$\begin{aligned} \mathcal{L}(\mathbf{X}_{it}, \xi_{it}, Y_{it}, \mathbf{Z}_{it}) &= W_{it}L_{it} + r_{it}(K_{it} + \mathcal{C}_a(K_{it}, K_{it-1})) + F_{it} \\ &\quad + \beta \mathbb{E}[\mathcal{V}(\mathbf{Z}_{it+1})] - \xi_{it}(\mathcal{Q}(\Omega_{it}, \mathbf{X}_{it}) - Y_{it}), \end{aligned}$$

where W_{it} is the wage, r_{it} is the user cost of capital, F_{it} is an exogenous fixed cost, and ξ_{it} is the Lagrange multiplier. The first-order conditions at the optimal choice of inputs \mathbf{X}_{it}^* and ξ_{it}^* imply that:

$$\nabla \mathcal{L}(\mathbf{X}_{it}^*, \xi_{it}^*, Y_{it}, \mathbf{Z}_{it}) = 0, \quad (3.1)$$

where ∇ denotes the gradient vector of partial derivatives with respect to inputs. Applying equation (3.1) to the flexible labor input yields the following cost-minimization condition linking the wage and marginal product of labor:

$$\frac{\partial \mathcal{L}}{\partial L}(\mathbf{X}_{it}^*, \xi_{it}^*, Y_{it}, \mathbf{Z}_{it}) = W_{it} - \xi_{it}^* \frac{\partial \mathcal{Q}}{\partial L}(\Omega_{it}, \mathbf{X}_{it}^*, \cdot) = 0.$$

The output elasticity with respect to the labor input L , $\theta_{l,it}$, can therefore be expressed at the optimum as:

$$\theta_{l,it} \equiv \frac{L_{it}^*}{Y_{it}} \frac{\partial \mathcal{Q}}{\partial L}(\Omega_{it}, \mathbf{X}_{it}^*) = \frac{1}{\xi_{it}^*} \frac{W_{it} L_{it}^*}{Y_{it}}. \quad (3.2)$$

Using the first order conditions in equation (3.1) to express the optimal choice of inputs \mathbf{X}_{it}^* and ξ_{it}^* as functions of output Y_{it} and exogenous variables \mathbf{Z}_{it} , we derive the optimal total cost as a function of output and exogenous variables:

$$C^*(Y_{it}, \mathbf{Z}_{it}) = \mathcal{C}(\mathbf{X}_{it}^*(Y_{it}, \mathbf{Z}_{it}), \mathbf{Z}_{it}).$$

At the optimum, the Lagrangian is equal to total cost, and from the envelop theorem it follows

that the marginal cost is equal to the Lagrange multiplier ξ_{it}^* :

$$\frac{\partial \mathcal{C}^*}{\partial Y}(Y_{it}, \mathbf{Z}_{it}) = \frac{\partial \mathcal{L}^*}{\partial Y}(Y_{it}, \mathbf{Z}_{it}) = \frac{\partial \mathcal{L}}{\partial Y}(\mathbf{X}_{it}^*, \xi_{it}^*, Y_{it}, \mathbf{Z}_{it}) = \xi_{it}^*.$$

Dropping for simplicity the superscript $*$ to denote optimal variables, we define the markup as the ratio of the output price of the firm P_{it} to the marginal cost:

$$\mu_{it} = \frac{P_{it}}{\xi_{it}}. \quad (3.3)$$

The markup captures the degree of pricing power of the firm, and is a widely used measure of firm-level market power. As noted by [De Loecker and Warzynski \(2012\)](#), this expression is robust to various static price setting models, and does not depend on any particular form of price competition among firms. The markup itself will, however, depend on the specific nature of competition among firms. Moreover, it follows from equations (3.2) and (3.3) that the markup is defined as the elasticity of output with respect to the labor input, divided by the share of this labor costs in total firm revenue, *i.e.* the labor share λ_{it} :

$$\mu_{it} = \theta_{l,it} \frac{P_{it} Y_{it}}{W_{it} L_{it}} \equiv \frac{\theta_{l,it}}{\lambda_{it}}. \quad (3.4)$$

It is important to note that equation (3.4) only applies to inputs that are freely adjustable, at least at the margin. In the case of capital, the relationship between the markup and the output elasticity becomes:

$$\mu_{it} = \frac{\theta_{k,it}}{\lambda_{it}^k (1 + \Delta_{a,it})}, \quad (3.5)$$

where λ_{it}^k is the capital share of revenue, $\theta_{k,it}$ is the output elasticity with respect to capital, and $\Delta_{a,it} = \frac{\partial \mathcal{C}_a}{\partial K}(K_{it}, K_{it-1}) + \frac{\partial \mathcal{C}_a}{\partial K_{-1}}(K_{it+1}, K_{it})$ is the wedge attributable to the adjustment costs. The sign of the wedge is not straightforward and depends on the convexity of the adjustment cost function as well as on expectations of future target stock of capital. [De Loecker and Warzynski \(2012\)](#) show that abstracting from adjustment costs generally results in a negative wedge, and therefore an overestimated markup.³

Another important source of gap between the output elasticity of an input and its share in revenue is when firms are not price-takers on the market for inputs, for instance if the firm has monopsony power in the labor market, or engages in efficient bargaining ([Dobbelaere and Mairesse, 2013](#); [Dobbelaere and Kiyota, 2018](#)). In that case, the relationship between the markup and the output

³See [Doraszelksi and Jaumandreu \(2019\)](#) for a discussion of the sign of adjustment cost wedges.

elasticity becomes:

$$\mu_{it} = \frac{\beta_{l,it}}{\lambda_{it}(1 + \Delta_{m,it})}, \quad (3.6)$$

where the sign of the wedge $\Delta_{m,it}$ depends on the labor market setting. [Dobbelaere and Kiyota \(2018\)](#) show that in the case of efficient bargaining, the wedge is positive, and in the case of monopsony the wedge is negative. We abstract from these two possible wedges but discuss in more detail the implications for our results in section 3.5.

In what follows, we map the aggregate labor share into firm level markups, and the output elasticity of labor. First, we define the aggregate labor share Λ_t as the value-added-weighted average of firm-level labor shares:

$$\Lambda_t \equiv \frac{\sum_i W_{it} L_{it}}{\sum_i P_{it} Y_{it}} = \sum_i S_{it} \lambda_{it},$$

where $S_{it} = \frac{P_{it} Y_{it}}{\sum_i P_{it} Y_{it}}$ is the market share of firm i . From equation (3.4) we know that the labor share is the product of the output elasticity of labor and the inverse markup:

$$\lambda_{it} = \theta_{l,it} \mu_{it}^{-1}. \quad (3.7)$$

We decompose the output elasticity of labor $\theta_{l,it}$ into a component stemming from returns to scale, which tells us how much output expands when all inputs increase proportionally, and a component stemming from the labor intensity of the production process relative to capital:

$$\theta_{l,it} = \underbrace{\theta_{l,it} / (\theta_{l,it} + \theta_{k,it})}_{\text{Labor Intensity}} \underbrace{(\theta_{l,it} + \theta_{k,it})}_{\text{Returns to Scale}} \equiv \alpha_{it} \gamma_{it}, \quad (3.8)$$

noting that when α_{it} is high the production process is intensive in labor relative to capital. It follows from equations (3.2), (3.7), and (3.8) that the aggregate labor share can be expressed as a function of firm level labor intensity, returns to scale, and markups:

$$\Lambda_t = \sum_i S_{it} \alpha_{it} \gamma_{it} \mu_{it}^{-1}. \quad (3.9)$$

We compute the aggregate markup M_t as the value-added weighted average of firm-level markups:

$$M_t \equiv \frac{\sum_i W_{it} L_{it}}{\sum_i P_{it} Y_{it}} = \sum_i S_{it} \mu_{it},$$

In the next section, we describe the estimation procedure we follow to recover estimates of firm-level output elasticities of labor and capital, which together with firm-level labor and market

shares observed in the data, allows us to compute the contribution of markups, labor intensity, and returns to scale to the aggregate labor share.⁴

Estimation Procedure

To recover markup from production data, we rely on equation (3.4). This framework is particularly convenient to analyze the evolution of markups in the long run because it does not require observing consumer-level attributes to estimate demand elasticities. Second, it makes no assumption on firms pricing behavior and competition environment. It only requires two assumptions: firms minimize production cost and freely adjust at least one variable input.

We can directly observe firm-specific input shares in production data. It is not the case for output elasticities with respect to inputs. Because these elasticities can vary across time and firms, we estimate a flexible production function, with a minimum number of parametric restrictions. In what follows, we assume that firms belonging to a particular industry j share the same technology $f_j(\cdot)$, using labor and capital to generate value added. Moreover, we assume that productivity is Hicks-neutral and evolves according to an AR(1) Markov process. For firm i in industry j , our empirical model is given by:

$$\begin{cases} y_{it} = f_j(k_{it}, l_{it}) + \omega_{it} + \epsilon_{it}, \\ \omega_{it} = \rho_{jt} \omega_{it-1} + \eta_{jt} + \nu_{jt}t + \xi_{it}, \end{cases} \quad (3.10)$$

$$(3.11)$$

where y_{it} stands for the logarithm of value added firm i at time t , and l_{it} and k_{it} are the logarithms of employment and capital stock. Productivity ω_{it} is Hicks-neutral, ϵ_{it} is an i.i.d measurement error, and ξ_{it} is the i.i.d innovation to productivity. Steady-state productivity η_{jt} and time trend ν_{jt} are common across firms in industry j in period t .

One issue that prevents us for simply running Ordinary Least Squares (OLS) on equation (3.10) is that we do not observe productivity ω_{it} but firms have information about their productivity when they choose their inputs. ω_{it} is therefore correlated with k_{it} and l_{it} and OLS estimates are biased. In what follows, we make the following standard assumptions regarding the timing of firm decisions:

Assumption 1. (Information Set) *The firm's information set at t , i.e. I_t , includes current and past productivity shocks $\{\omega_{i\tau}\}_{\tau=0}^t$ but does not include future productivity shocks $\{\omega_{i\tau}\}_{\tau=t+1}^{\infty}$. Measurement errors ϵ_{it} satisfy $\mathbb{E}[\epsilon_{it}|I_t] = 0$. The productivity process defined in equation 3.11 is known to firms and stochastically increasing in ω_{it-1}*

⁴We abstract from input-output linkages by considering value-added production function. Baqaee and Farhi (2020a) show that input-output linkages are important for the propagation of productivity shocks, and Grassi (2016) shows that they matter for market power in the case oligopolistic competition.

Assumption 2. (*Input Choices*) Labor and capital inputs used at time t are chosen with information set I_t .

Assumptions (1) and (2) are straightforward: firms do not observe ω_{it} until time t , but the Markov process defines what the firm knows about the distribution of future productivity shocks. The literature on production function estimation often relies on the proxy variable method to produce first a non-parametric estimate of unobserved productivity ω_{it} from observed variables using the assumption that some proxy variable, either investment (Olley and Pakes, 1996) or intermediate input demand (Levinsohn and Petrin, 2003a; Akerberg et al., 2015a), is an invertible function only of other inputs and productivity. However, this approach is not valid if the proxy variable is also a function of some unobserved shock, such as an input cost shock to all inputs, or demand shocks. Let us define intermediate input demand m_{it} as a function of capital, labor, productivity, and some unobserved shock d_{it} :

$$m_{it} = m(\omega_{it}, k_{it}, l_{it}, d_{it}). \quad (3.12)$$

Assuming that this function is invertible in ω_{it} and using equation (3.10), one can write than value added y_{it} is an unknown function of inputs and the unobserved shock:

$$y_{it} = f_j(k_{it}, l_{it}) + \omega(m_{it}, k_{it}, l_{it}, d_{it}) + \epsilon_{1,it} = g(m_{it}, k_{it}, l_{it}, d_{it}) + \epsilon_{1,it}. \quad (3.13)$$

Ignoring the unobserved shock, and under assumption (1) that ϵ_{it} is independant from input choices, we can apply Akerberg et al. (2015a) and obtain a non parametric estimate \hat{g}_{it} of $g(\cdot)$ that is a high-order polynomial in m_{it} , k_{it} , and l_{it} , but not of d_{it} :

$$y_{it} = \hat{g}_{it} + \hat{\epsilon}_{it}, \quad (3.14)$$

where the residuals $\hat{\epsilon}_{it}$ are correlated with d_{it} . In practice, when we apply this procedure, we find that the residuals are not i.i.d. As Doraszelski and Jaumandreu (2019) have recently discussed, d_{it} , as ω_{it} , should also be recognized as an autocorrelated unobservable. If so, the instruments used in the second stage of the proxy variable method are not consistent.

We do not rely on the proxy variable method to estimate equations (3.10) and (3.11). Instead, we rely on an approach to control for unobserved productivity that is usually called dynamic panel estimation (Blundell and Bond, 2000). We use the AR(1) structure of the productivity process to write current value-added as :

$$y_{it} = \rho_{jt} y_{it-1} + \left(f_j(k_{it}, l_{it}) - \rho_{jt} f_j(k_{it-1}, l_{it-1}) \right) + \eta_{jt} - \nu_{jt} t + u_{it},$$

where the composite error $u_{it} = \xi_{it} + \epsilon_{it} - \rho\epsilon_{it-1}$ has by assumptions (1) and (2) a zero mean conditional on information set I_{t-1} . Conditioning on a set of instruments included in I_{t-1} , we estimate the model using non-linear GMM. Our moment conditions can be written as:

$$\mathbb{E}[u_{it}|I_{t-1}] = \mathbb{E}\left[y_{it} - \rho_{jt}y_{it-1} - \left(f_j(k_{it}, l_{it}) - \rho_{jt}f_j(k_{it-1}, l_{it-1})\right) - \eta_{jt} - \nu_{jt}t | I_{t-1}\right] = 0. \quad (3.15)$$

We assume that technology $f_j(\cdot)$ in sector j is a translog production function of capital and labor:

$$f(k_t, l_t) = \beta_{l,jt}l_{it} + \beta_{k,jt}k_{it} + \beta_{ll,jt}l_{it}^2 + \beta_{kk,jt}k_{it}^2 + \beta_{lk,jt}l_{it}k_{it},$$

and we use past values ω_{it-1} , l_{it-1} , m_{it-1} , k_{it-1} and higher order combinations of those terms, a time trend t and a constant as instruments in equation 3.15. From the estimates of the parameters of the production, we compute the firm-level output elasticity of labor and capital for firm i in year t as:

$$\begin{aligned} \theta_{l,it} &= \beta_{l,jt} + 2\beta_{ll,jt}l_{it} + \beta_{lk,jt}k_{it}, \\ \theta_{k,it} &= \beta_{k,jt} + 2\beta_{kk,jt}k_{it} + \beta_{lk,jt}l_{it}. \end{aligned}$$

From equation (3.8), we retrieve firm-level labor intensity and returns to scale.

3.3 Data

To carry out our empirical analysis we rely on several sources of micro data produced by the French Institute of Statistics and Economic Studies (INSEE), covering the universe of French firms spanning the 1984-2016 period. These data are in particular one of the main sources of the elaboration of National Accounts. Our sources are gathered out of the universe of firms' tax forms and provide balance sheet, income, and cost information at the firm level, as well as employment, the industry in which the firm operates, the type of legal entity (micro-firms, sole proprietorship entities, or limited liability companies and corporations) and the tax regime to which it is affiliated (micro-regime, simplified regime, or normal regime).

From 1984 to 2007, we rely on the SUSE sources (*Système Unifié de Statistiques d'Entreprises*), gathering information from firms affiliated to two tax regimes, the BRN regime (*Bénéfice Réel Normal*) and RSI regime (*Régime Simplifié d'Imposition*). These files allow to distinguish between payments to labor, material inputs, other intermediary inputs, and investment, and provide information of the book value of capital of the firm and total employment. Hence, they have been widely used in previous research (di Giovanni et al., 2014; Caliendo et al., 2018).

From 2008, we rely on the ESANE sources (*Élaboration des Statistiques Annuelles d'Entreprises*), that result from the unification of the previous SUSE data with Annual Surveys of Firms that were conducted each year for broad sectors of industries. Because there is some overlap of information between tax forms and surveys, INSEE has a process to reconcile diverging information. To construct our panel of firms we exclude from the post-2008 data firms affiliated to the micro-BIC regime.⁵ Moreover, we restrict our analysis to legal units with a unique and valid identifier number.⁶

We focus on market sectors⁷ and exclude agriculture because our sample does not cover well firms in that sector, which are mostly affiliated to a tax regime that is not included in the micro-BIC, BRN and RSI regimes. We also exclude real estate and finance, because we focus on the production side of value-added distribution among workers and owners of capital and firms. We have 5.7 million firms in our sample, 3.7 million of which have at least one employee. Finally, we rely on industry-level data from KLEMS (Jäger, 2017a) for information on investment and output prices to compute deflated values for value-added and capital stocks.

Overview of the data

Table 3.1 reports the year-by-year total number of observations, as well as aggregate labor costs, value-added, investment, both in level and in share of their aggregate values for the corporate sector in France. There are on average 800 thousand observations per year, accounting for 87% of total labor costs, 84% of total value-added, with little variations over time. Our data only accounts for 68% of total investment in the corporate sector. This is due to the fact that many small firms affiliated to the simplified regime report missing investment. To construct measures of capital input, we use instead the reported book values of the capital stock.

Table 3.2 describes the main variables that we use in our empirical analysis. Our sample of 3.7 million firms with at least one employee spans over 33 years, with 27 millions firm-year observations, average sales is 2.6 M€, average number of employees: 14, and average capital stock: 1.3 M€. This data is highly skewed as the median level of sales is 285 K€, median number of employees is 3, and median capital stock: 76 K€. This reflects the fact that our data is nearly exhaustive and includes many small firms. For firms that report non missing investment, the average reported value is 185 K€, and the median is 4 K€, which also partly reflects the fact that

⁵An extremely simplified regime introduced in 2008 applicable to very small firms, whose total sales do not exceed 170 K€ if the firm operates within the real estate and trade sectors, or 70 K€ otherwise. This regime has been widely used by free-lance workers who do not report any capital nor employment.

⁶A firm is defined as a legal unit with a unique SIREN identifying number. In ESANE, legal units belonging to the same conglomerate are brought together and their accounts are consolidated (Deroyon, 2015). We do not use this information here.

⁷The market sectors are total economy excluding public administrations, healthcare, and education.

investment is lumpy.⁸ The average labor share in our sample, computed as the ratio of the sum of the wage bill and payroll taxes to value-added, is 75%, close to the median at 74%.

Aggregate Labor Share

Figure 3.1 reports the ratio of compensation of employees, including payroll taxes, to total value-added in the macro and micro data, from 1984 to 2016. The aggregate labor share in our sample, is lower than the average firm-level labor share. As discussed below in Section 3.4, larger firms have a lower labor share, which brings down the weighted average labor share. In the sample of firms with at least one employee on which we rely in the rest of the paper, the aggregate labor share decreases from 69.3% in 1984 to 64.7% in 2000, and then increases back to a level close to its initial level, reaching 69.1% in 2016. The aggregate level is on average 67.1% over the period. Aggregate data in principle also includes firms that have no employee, and doing so in our micro data decreases the aggregate level of the labor share by around 1 percentage point: it stands at 66.1% of value-added on average over the period, and has the same U-shaped trajectory. This aggregate pattern differs substantially from the decrease of the labor share in the US, discussed by Autor et al. (2020a); Kehrig and Vincent (2018), while others have argued that France, as many advanced economies, also experienced a secular decrease in the labor share (see e.g. Grossman et al., 2017; Karabarbounis and Neiman, 2014). Because of the U-shaped trajectory of the labor share, both in the micro and macro data, we find that conclusions of a secular decline in France are misguided.

Our sample is limited to incorporated firms in the market economy outside agriculture, real estate, and finance. Despite the fact that there is no available aggregate data for France for this particular sample, the aggregate labor share in our data closely matches the aggregate patterns of the labor share that we can measure on similar spheres of activity, both in levels and in variations.

French National Accounts provide detailed operating accounts for spheres that are larger than our data in various dimensions. Figure 3.1 reports the labor share of the entire corporate sector, including corporations operating in the agriculture, real estate, and finance. Before 2000, the average level of the labor share in the corporate sector, reported by INSEE, is the same as the aggregate labor share in our sample including firms with no employees (65.4%). It starts from a slightly higher level in 1984 (71.6%) than our sample estimate (68.4%) and reaches a slightly lower level in 2000 (63.4% as opposed to 64.1% in our sample). After 2000, however, the corporate labor share rises by 2 percentage points, but the labor share in our sample rises by 4 percentage points.

Figure 3.1 also reports the total labor share (corporate and non-corporate) excluding agriculture, real estate, and finance. The non-corporate sector is mainly composed of self-employed workers

⁸ Average mean firm investment across years is 140 K€ and mean investment is 8 K€.

with few salaried workers. As a result, the total labor share reported by INSEE is lower - on average 61% over the period, against 66.1% in our data with all firms. Nevertheless, after 2000, and despite this difference in levels, the rise of the total labor share measured with the same industry composition as our data matches the 4 percentage point increase that we observe in our data. One possible explanation, as [Cette et al. \(2019\)](#) discuss, is that because the real estate sector has a labor share close to zero, its growing share in total value-added contributes negatively to the aggregate labor share of the corporate sector, especially during the housing boom after 2000.

3.4 Labor Share and Concentration

In this section, we revisit five important facts about concentration and labor shares in the French context. In particular, we find that the rise in concentration in France is associated with an increase in firm-level labor shares, and a reallocation of market shares towards large and low-labor-share firms.

Rise in concentration

Figure 3.2 reports the cumulative change since 1984 in sales weighted average levels of industry concentration indexes, where each index measures concentration of sales at the 3-digit national industry level. The share of sales of the largest 1% or 5% firms in each industry increased sharply on average since 1984, by 9 and 7 percentage points respectively. The concentration ratios, defined as shares of the 4 and 20 largest firms in each industry, followed a different pattern before 1995, but have increased by close to 4 percentage points each on average since 1995.⁹

Overall, we find that concentration ratios and top shares have increased in more than half of the 211 industries since 1995: the median increase of both concentration ratios is 2 percentage points, and the median increases of the top 1% and 5% shares are 4 and 5 percentage points respectively. Figure B.3.1 in the Appendix shows that the average rise in concentration is observed across broad sectors of the economy: the magnitude of the increase is similar in both the manufacturing (3 to 7 percentage points from the lowest point to 2016, depending on the index) and non-manufacturing sector (4 to 10 percentage points).

These results are consistent with evidence across the US and other OECD countries ([CEA, 2016](#); [Autor et al., 2020a](#); [Andrews et al., 2016a](#)). It is important to note that concentration is calculated

⁹The median 3-digit industry has around 900 firms in a given year, but because 25% of the industries have more than 5,000 firms, and 25% have less than 200 firm, the number of firms in the top 1% and 5% differs greatly from one industry to the next. The median size of the 3-digit manufacturing industry is around 500 and the median size of the 3-digit non manufacturing industry is 3,600.

as the share of sales of largest firms in a narrowly defined industry at the national level. It is best to interpret it as a measure of firm dispersion of outcomes rather than firm market power, as the national industry level is not necessarily the relevant market. In fact, Section 3.5 shows that the industry-level correlation between these measures of concentration and the largest firms market power is not significantly positive.

Reallocation of labor shares

We build on Kehrig and Vincent (2018) and decompose the variations of the aggregate labor share to understand whether they are driven by a variations at the firm level or by composition effects. Figure 3.3 reports for each decile of labor share, the value-added-weighted average labor share and the share of industry value added of firms in that decile, in the first and last five years of the sample. To account for industry-specific differences in the joint distributions of labor share and value-added, they are first calculated within each 3-digit industry. These distributions are then averaged across these 211 industries using value-added weights in a given period. The vertical bars illustrate how low labor share firms gained market share in the last 30 years. Firms in the lowest decile of labor share accounted for 12% of their industry value-added before 1990, compared to 16% in after 2010. The rise in industry shares is verified for four out of the five lowest deciles of labor share, while all five highest deciles of labor share accounted for less of industry value-added in 2011-2016 than in 1984-1989. The connected lines illustrate how the raw distribution of labor shares has shifted upwards. The average labor share of each decile is higher in after 2010 than before 1990. Figure B.3.2 in the Appendix shows that these patterns are observed across broad sectors of the economy, in manufacturing as well as non-manufacturing industries.

Figure 3.4 quantifies the contribution of three components to the variation of the aggregate labor share in France: a term accounting for reallocation of value added shares across industries, and the two within-industry components discussed above: within and across quantiles of labor shares.¹⁰ The figure first confirms that reallocation across industries plays only a minor role for aggregate labor share variations. The increase in the value-added shares of low-labor-share firms contributed to an accumulated 5 percentage points decrease of the aggregate labor share since 1984, holding the distribution of labor shares constant from one year to the next. This was offset by the upward shift in the labor share distribution, that contributed to a rise of the aggregate labor share of 5 percentage points, holding the value-added distribution constant.

Figure B.3.3 in the Appendix presents the 1984-2016 cumulative results of the same decomposition for manufacturing and non-manufacturing industries separately. In both macro sectors, as in the whole economy, the reallocation across industries had a negligible impact, the reallocation of

¹⁰The details of decomposition are presented in Appendix B.2.

value-added contributed negatively to the aggregate labor share and the labor share distribution shifted upwards and contributed positively to the aggregate labor share. In manufacturing, the aggregate labor share decreased because the upward shift in the labor share distribution did not offset the reallocation.¹¹

Our decompositions show that the typical industry in France experienced a consistent trend in reallocation of value-added shares towards low-labor-share firms throughout the period, as in the US. The upward shift of the labor share distribution has also been documented in the US manufacturing industry by [Kehrig and Vincent \(2018\)](#), but is less significant than in France. As emphasized by [Kehrig and Vincent \(2018\)](#), this decomposition groups firms into labor shares quantiles, which allows us to compare two static equilibria. It is conceptually distinct from standard within and cross firm decompositions, because it abstracts from the contributions of firm entry and exit. We focus on long term shifts in the joint distribution of labor and value added shares, not on the role of entry nor on the trajectories of specific firms.

Correlation of rise in concentration and reallocation of labor shares

We now ask whether variations in industry concentration are correlated with labor share variations within industries. To that end we estimate the industry-level relationship between long-term changes in concentration and labor share. We run the following regression:

$$\Delta\lambda_{jt} = \phi_\lambda \Delta Conc_{jt} + FE_t + \epsilon_{jt}, \quad (3.16)$$

where $\Delta Conc_{jt}$ is the 10-year change of sector j concentration level, proxied by the top 1% of top 5% share of sales and FE_t is a set of time fixed-effects that control for year-specific shocks and $\Delta\lambda_{jt}$ is the 10 year change in industry j labor share.

Table 3.3 presents the results of these regressions. The first two columns show that long term variation of industry concentration are negatively correlated with long term variation of industry labor shares. This relationship is significant and hold for all proxies of concentration. The first two columns of Table B.4.1 in the Appendix show that this result holds among both manufacturing and non-manufacturing industries. We find that a 10 percentage point rise in concentration is associated with a 0.7 to 1.1 decline in the weighted average labor share of the industry. These results are close to those documented in the US ([Autor et al., 2020a](#)).

Next we ask whether this result is driven by a correlation between the rise in concentration and

¹¹Interestingly, the aggregate labor share decreased in both manufacturing and non-manufacturing but it does not on aggregate. This is because while reallocation across industries within each macro sectors contributed negatively to each macro sectors aggregate labor share, reallocation from manufacturing to non-manufacturing industries contributed positively to the total aggregate labor share.

the shift in value-added shares from high to low labor share. We consider two components of the 10-year change of the labor share: the cross-quantile contribution to the labor share variation discussed in the previous paragraph, and the evolution of the average labor share of the 5% firms with the lowest labor share within each industry. We use these components as dependent variable in regression (3.16).

On the one hand, we find that larger increases in concentration are associated with a more negative contribution of value added share reallocation to the aggregate labor share. All coefficients are negative and significant. Table B.4.1 in the Appendix shows that this result holds among both manufacturing and non-manufacturing industries. On the other hand, we find a positive correlation between change in concentration and change in the average labor share of low labor share firms, defined as firm with labor share in the bottom 5% of their 3-digit industry. These firms are sometimes referred to in the literature as 'hyper-productive' (Kehrig and Vincent, 2018) or 'superstar' firms (Autor et al., 2020a). As we will show next, firms with low labor shares also tend to be larger in our sample. Our result suggests that the negative correlation between labor share and concentration is not driven by the fall in the labor share of these 'superstar' firms. Table B.4.1 in the Appendix reports the results for manufacturing and non-manufacturing industries: the correlation between variations of concentration and variations of the labor share of low-labor-share firms is mostly positive in manufacturing, indicating that the 'superstar' firms in manufacturing today have not only higher market shares but also higher labor shares. Results for non-manufacturing are mixed and vary with the concentration index, but we do not find a negative correlation that is significant at the 5% level.

Labor share and size

To reconcile these facts, we show that there is a negative correlation between labor share and firm size, and that this relationship is monotonic. We run the following regression:

$$\lambda_{it} = FE_{size_{it}} + FE_{jt} + \epsilon_{it}, \quad (3.17)$$

where $FE_{size_{it}}$ is a set of dummies indicating in the size of firm i in industry j in terms of employment categories, FE_{jt} is a set of interacted fixed effects at the 3-digit industry j and year level.

Figure 3.5 presents the results of this regression, considering labor share in value added and in gross output. Relative to 10-20 employee firms, larger firms tend to report lower labor shares even after controlling for industry and year fixed effects. This decreasing relationship is monotonic, at all levels of employment. Labor shares of firms with 50 to 100 employees tend to be 2 percentage

points lower than labor shares of 10 to 20 employees firms of the same industry at the same year. For firms with 2500 to 5000 employees the gap rises to 5 percentage points considering labor share in value added and to 7 percentage points considering labor share in sales.

This decreasing relationship between firm size and labor share helps reconcile our previous results: in industries where concentration increased, the weighted mean labor share decreased and the distribution of labor shares shifted upwards. The reallocation effect due to rising concentration, drives the change in the weighted mean labor share, as larger firms who gain market shares also have lower labor shares.

Firm level trends

So far, we have focused on describing long term shifts in the distribution of firm outcomes, without discussing whether these firms are the same over time. In what follows, we look at the within-firm variation of labor share, for different groups of firm size. We run the following regression:

$$\lambda_{it} = FE_i + \Psi_\lambda t + Control_{it} + \epsilon_{it}, \quad (3.18)$$

where λ_{it} is the labor share of firm i in year t , FE_i is a firm fixed effect, and $Control_{it}$ is either the logarithm of employment or a set of categories of employment size fixed effects. We run this regression on four samples, with different thresholds of employment size, and two panels, an unbalanced panel of firms possibly entering and exiting, and a balanced panel of firms that are present in the sample from 1984 to 2016.

Table 3.4 presents the results of these regressions. We find that average firm experienced a trend increase in labor share of around 0.6 percentage points per year, including controlling for changes in the employment level. Firms that are present in the sample from 1984 to 2016 experienced a trend increase of around 0.25 percentage points per year. If we restrict our sample to firms with more than 50, 100, or 1000 employees, we find results that are similar across all specifications and panels: the average firm of any size experienced a trend increase in labor share of around 0.3 percentage points per year.

3.5 Estimation Results

In this section, we first present the results of our estimation procedure, and then show how aggregate and firm-level markups have evolved in France. We document additional facts about market power and concentration, and how variations in market power have contributed to the aggregate

labor share, compared to other technological factors.

Production function

Table 3.5 reports the results of the non-rolling estimation of the production and Table 3.6 reports the results of the rolling estimation of the production function, for the 27 sectors of our data.¹² Rolling estimations are obtained by first estimating the parameters of the production function $\beta_{jt} \in \{\beta_{l,j}; \beta_{k,j}; \beta_{ll,j}; \beta_{kk,j}; \beta_{lk,j}\}$ in industry j on 11-year rolling window samples, and then averaging for each firm each year the various estimated output elasticities based on samples that include that year:

$$\beta_{jt}^{\text{rolling}} = \frac{1}{11} \sum_{n=-5}^5 \beta_j^{t+n},$$

where β_j^t is the estimated parameter on the sample restricted to years $t - 5$ to $t + 5$. For the first and last five years of our sample, the average is calculated on fewer estimates. Unlike in a Cobb–Douglas production function, output elasticities also vary across firms of the same sector, even in a given year. We report, for the different sectors, the average and standard deviation of the elasticities. We note that a few sectors appear to have negative average capital elasticities or low returns to scale but these are driven by outliers. Tables B.4.2 and B.4.3 in the Appendix report the median output elasticities which are less influenced by outliers. Since the returns to scale vary across firms, it is possible for many firms in a sector to have increasing returns to scale, while the estimate of the industry average returns to scale is close to 1. On average, the output elasticity of labor in our data is 0.74 according to the non-rolling estimates and 0.72 according to the rolling estimates.

Aggregate markup

Figure 3.6 reports the evolutions of the value-added weighted and unweighted average markups across all firms in our sample for both sets of estimates. The unweighted average markup is smaller than the weighted average markup, because firms with larger value-added have on average higher markup. As we will see below, the positive relationship between markup and size holds in our data, with employment as a measure of size.

We find that according to both non-rolling and rolling estimates, the unweighted average markup has decreased. Overall, the unweighted average markup has fallen from 1.3 to 1.0. The value-added weighted markup has decreased from 1.6 to 1.4 according to non-rolling-window estimates.

¹²Estimation is done at the 27 sector level. Each sector includes several 3-digit industries.

When we allow the parameters of the translog production function to vary over time, we report an increase of the unweighted average markup from 1.4 to 1.6.

Figure B.3.4 in the Appendix presents the levels of the unweighted and weighted average markups for firms in the manufacturing and non-manufacturing sectors separately. Patterns are qualitatively similar in both sectors to what they are in the overall economy. We find a decrease in unweighted markups for both non-rolling and rolling estimations both in manufacturing and non-manufacturing sectors, that broadly matches the one observed in the whole economy. Similarly, the variations of the weighted average markup estimates observed in both manufacturing and non-manufacturing, and according to both sets of estimates, quantitatively matches the one observed in the whole economy. These aggregate patterns are therefore neither specific to one sector nor driven by a reallocation from manufacturing to non-manufacturing. Interestingly, our results show that, regardless of the pre-crisis trend, the weighted average markup of manufacturing firms sharply dropped after 2008, and only recovered its pre-crisis level at the end of the period.

Figure B.3.5 in Appendix shows how relying on the proxy-variable method in [Akerberg et al. \(2015a\)](#) (ACF) would have changed our results. The level of the estimated markups differ, because DP estimates of the output elasticity of labor are on average lower than the ACF estimates. Second, in non-rolling estimation, the unweighted markup is not always larger than the weighted markup, suggesting that the increasing relationship between size and markups is not verified. Third, the trend of the average ACF estimated aggregate markup, is significantly different from the average DP estimated aggregate markup. For instance, with rolling estimations, average markup remained however broadly stable around 1.6 according to the ACF estimates.

Markup decomposition

Figure 3.7 shows the decomposition of the aggregate - weighted average - markup into within-markup-quantile and across-markup-quantile components. It shows the importance of controlling for industry and disentangling the respective contributions of variations in value-added shares holding markup constant or in markup holding value-added shares constant to interpret aggregate variation.

The decomposition of the aggregate markup mirrors the decomposition of the aggregate labor shares shows how the within-markup quantile component contributed negatively to the evolution of the aggregate markup, while the cross-quantile contributed positively.¹³ The contribution of reallocation across industries is negligible. Firms with relatively higher markups within narrowly defined industries have been gaining value-added shares, while the typical firm markup has

¹³For ACF, this contribution decreased after 2005.

slightly decreased. Figure B.3.6 in the Appendix shows that the reallocation from low to high markup firms holds both in manufacturing and non-manufacturing separately.

Comparing rolling and non-rolling window estimations shows why accounting for variations over time of the production function parameters matter. Even though both estimations yield qualitatively similar contributions of the within and across terms, the within term dominates with rolling estimations and the cross term dominates with non-rolling estimations. This quantitative differences translate into qualitative difference in the aggregate trends of markups estimated with both estimations.

Markup and concentration

As for the labor share, we ask whether the observed rise in concentration is correlated with markup variations, on aggregate or along the distribution of markups. To that end we estimate the industry-level relationship between long term changes in concentration and the industry aggregate markup, or the contributions to the aggregate variation. We run the following regressions:

$$\Delta\mu_{jt} = \phi_{\mu}\Delta Conc_{jt} + FE_t + \epsilon_{jt}, \quad (3.19)$$

where $\Delta\mu_{jt}$ is the 10-year change of sector j aggregate markup level, or one of its contributions according to the decomposition described in Appendix B.2. The independent variables are the same in equation (3.16).

Table 3.7 reports the results of the estimation of equation (3.19). The first two columns show that there is a positive and significant long-term relationship between the evolution of aggregate markup and the evolution of concentration at the 3-digit industry level. This relationship is significant and hold for all proxies of concentration. It holds both for markups estimated with rolling and non-rolling estimations. The two first columns of Tables B.4.4 and B.4.5 in the Appendix confirm that this positive correlation is separately present for industries belonging to the manufacturing and non-manufacturing sectors.

Next, as for the labor share, we ask whether this result is driven by a correlation between the rise in concentration and the shift in value-added shares from low to high markup firms. The coefficients of the third and fourth columns of Tables 3.7 (and Tables B.4.4 and B.4.5 in the Appendix) are the results of regressions described in equation (3.19), where the dependent variable is the cross-quantile component to the evolution of aggregate markup, while in the last two columns the dependent variable is the within-quantile component of firms high markups, defined as firms with the markup in the top 5 % of their 3-digit industry.

Columns 3 and 4 in all three tables show a positive correlation between the rise in concentration and the cross-quantile component of the evolution of the aggregate markup. As for the labor share, this means that the cross-quantile component contributed more to the rise in markup in those industries that have become more concentrated at the top. Columns 5 and 6 find no robust evidence that a rise in concentration is correlated with increases in top markups. The correlations with variations in the top 1% and 5% shares of sales are not significantly positive, the correlations with variations in the shares of the 4 and 20 largest firms are all negative, and significant at the 5% level for three out of four estimations. These results are also observed on the samples of industries belonging to the manufacturing and non-manufacturing sectors separately, except in non-manufacturing, where the correlation of the average top markup with the top 1% and 5% shares is positive and significant, but small relative to the other components. The fact that top markups are not linked with rises in concentration are consistent with theories (see e.g. [Aghion et al., 2019](#)) where high productivity firms with higher markups, benefit from an external shock more than laggard firms, and expand without increasing their markup. However, it is in contrast with results in the US documented by [De Loecker et al. \(2020b\)](#) where top markups contributed to a third of the overall increase in weighted average markups. However, the authors do not provide evidence that the rise in top firm markups is correlated at the industry level with the reallocation component, or with concentration.

Markup and size

As for the labor share, we investigate whether markups are increasing with firm size to understand the correlation between the growing share of the largest firms in each industry's total sales and the reallocation of market shares towards high markup firms. To that end, we run the following regression:

$$\mu_{it} = FE_{size_{it}} + FE_{jt} + \epsilon_{it}, \quad (3.20)$$

where $FE_{size_{it}}$ is a set of dummies indicating in the size of firm i in industry j in terms of employment categories, FE_{jt} is a set of interacted fixed effects at the 3-digit industry j and year level.

Figure 3.8 reports the results of this regression. In our data, we find that larger firms have higher estimated markups. Firms with more than 5000 employees have, on average, markups larger by 30 percentage points than firms with 10 to 20 employees firms, within the same 3-digit industry, on the same year. This increasing relationship is well observed at all levels of employment, and both for markups obtained with the non-rolling and rolling estimations. The top two panels of Figure

B.3.7 in the Appendix show that this result holds both in manufacturing and non-manufacturing sectors separately, although for larger manufacturing firms the relationship flattens out.

The markup is defined in equation (3.4) as the ratio of the output elasticity of labor to the labor share. It is important to note that because the output elasticity of labor vary across firms, the markup is not perfectly correlated with the labor share, and therefore the positive relationship between a firm's markup and its size does not flow directly from the negative relationship between its labor share and its size that we document in section 3.4. The other four panels of Figure **B.3.7** in the Appendix plot the results of the same regression with labor intensity α_{it} and returns to scale γ_{it} as the dependent variable, and shows that the intensity of labor in the production process decreases with size, while returns to scale increase with firm size.

Markup trends

Before turning to the link between the evolution of markups and labor shares in France, we look at the within-firm variation of markups, for different groups of firm size. We run the following regression:

$$\mu_{it} = FE_i + \Psi_\mu t + Control_{it} + \epsilon_{it}, \quad (3.21)$$

where μ_{it} is firm i markup in year t , FE_i is a firm fixed effect, and $Control_{it}$ is either the logarithm of employment or a set of categories of employment size fixed effects. We run this regression on four samples, with different thresholds of employment size, and two panels, an unbalanced panel of firms possibly entering and exiting, and a balanced panel of firms that are present in the sample from 1984 to 2016.

Table 3.8 reports the results for markups obtained with non-rolling estimation and Table 3.9 for markups obtained with rolling estimation. We find that the average firm experienced a trend decrease in markup of around 1.3 to 1.6 percentage points per year. The decrease in larger firm markups is around 0.8 to 0.9 percentage points per year according to non-rolling estimates, and close to zero according to rolling estimates. For the balanced panel of firms that remain in the sample, markups decreased by 0.6 to 0.9 percentage points per year depending on group of size, according to non-rolling estimates, and close to zero according to rolling estimates. Overall, we find that firm level markups on average decreased, but less so for large and surviving firms. This result indicates that part of the decrease of the within-quantile contribution to aggregate markup is driven by smaller firms and by firm entry and exit.

Link between labor shares and markups

In this section, we circle back to the labor share and ask whether variations in firm-level labor share are mainly driven by markups - i.e. are labor shares increasing because markups are decreasing? - or by technology - i.e. are labor shares increasing because production has become *more* labor intensive?

First, we find that there is a clear negative relationship between firm-level labor shares and markups in France. We run the following regressions:

$$\lambda_{it} = \phi \mu_{it} + FE_{ijt} + \epsilon_{it}, \quad (3.22)$$

where μ_{it} is the markup of firm i in year t , λ_{it} is the labor share, and FE_{ijt} is a set of fixed effect, either industry, or firm-level, and year.

Table 3.10 present the results of these regressions, and show that firms with high markup have low labor shares both across industries and across firms within the same industry. We also find that as a firm markup grows, its labor share decreases. The absolute value of coefficient ϕ is around 0.3 to 0.5 depending on the estimation: as a firm's markup increases 10 percentage points, its labor share decreases by 3 percentage points. Finally, as the coefficient of determination of the regression without fixed effects shows, the heterogeneity of markups explains 45% of the heterogeneity of labor shares across firms. The different panels of the table show that this relationship holds statistically and quantitatively for various groups of size.

To extrapolate these firm-level results to the aggregate economy, we need to keep in mind that there is no such a thing as a representative firm in this context. Recall that equations (3.7) and (3.8) show that at the level of the individual firm, the labor share is the product of labor intensity, returns to scale and the inverse markup:

$$\lambda_{it} = \alpha_{it} \gamma_{it} \mu_{it}^{-1},$$

but this result does not hold at the aggregate level. From equation (3.9), we now decompose variations of the aggregate labor share into contributions from labor intensity, returns to scale, and markups, either by taking the "representative firm" approach and computing the contributions of the weighted averages of each component of the aggregate labor share, therefore ignoring the reallocation between firms ; or alternatively, by isolating the contribution of reallocation and computing the contributions of the unweighted averages of each component. Appendix ?? provides further information on the decomposition.

Figure 3.9 presents the results of the decomposition for the representative firm. The total vari-

ation of the aggregate labor share from 1984 to 2016 is small and positive, and ignoring the role of reallocation, aggregate markups have contributed positively to the aggregate labor share according to the non-rolling estimates, and negatively according to the rolling estimates. This is consistent with evidence above that the conclusions in terms of the variations of the aggregate markup are not the same in both sets of estimates. The sum of the contributions of labor intensity and returns to scale, in other words the contribution of weighted average output elasticity of labor, is negative according to non-rolling estimates and positive according to rolling estimates.

However, taking into account reallocation provides a better picture of underlying determinants of the dynamics of the aggregate labor share in France. Figure 3.10 presents the results of the decomposition isolating the contribution of reallocation. The contribution of reallocation is negative and very significant, as we have already showed in Figures 3.4 and 3.3. In both sets of estimates, firm-level markups have contributed positively to the aggregate labor share, while firm-level returns to scale and labor intensity had a slight negative contribution. Figures B.3.8 and B.3.9 show that this results hold in both the manufacturing and non-manufacturing sectors.

3.6 Conclusion

In this paper, we find no evidence of a rise in market power in France: firm-level markups decreased on average, and the rise in concentration is not correlated with increases in top markups. These facts are however correlated with an important reallocation of market shares towards low-labor share and high-markup firms. Because those firms tend to be larger, this reallocation translates into a rise in concentration.

This reallocation of market shares towards large firms is consistent with a wealth of evidence about the increasing differences between firms (Decker and Haltiwanger, 2013; Haltiwanger et al., 2015; Decker et al., 2015; Andrews et al., 2016b; Karahan and Pugsley, 2016). However, the simultaneous rise in concentration and the relative stability of top firm-level markups raises questions about the interpretation of concentration that goes beyond the French case. One channel than could possibly explain both the reallocation of market shares towards large firms and the within-firm increase in the labor share of income is an increase in winner-take-most competition level, as discussed by Autor et al. (2020a): as consumers become more sensitive to firm prices, more productive and bigger firms gain market share but a given firm market power decreases. The source of this increase in competition could be international competition, as argued by Bonfiglioli et al. (2019), but our results hold across broad sectors of the French economy, including within non manufacturing firms, which suggests that other factors are at play. Technological factors, such as the rise of internet platforms and price comparison websites, may explain why

firm-level market power has decreased.

The textbook explanation of a rise in competition has many predictions that are consistent with evidence provided here. We do not take a stance on the source of market power, and in particular on why there is an increasing relationship between firm size and firm markup: the price elasticity of demand may decrease with quantity, or large firms may be large enough to influence the equilibrium price, and therefore act strategically. However, in both cases, an increase in competition will have offsetting effects on the markup of large firms: holding size constant, it will tend to decrease their markup, but because of reallocation, these firms will grow and their markup will increase. Qualitatively, it is thus possible to observe a rise in top firm markups, as [De Loecker et al. \(2020b\)](#) find for the US, or a stability or decrease, as we find for France.

Tables

Table 3.1: Data Representativeness

	Obs	Labor Costs		Value Added		Investment	
		Total	Share	Total	Share	Total	Share
	(Nb)	(M€)	(%)	(M€)	(%)	(M€)	(%)
1984	532,996	283,772	82	283,772	84	47,202	70
1985	548,669	312,930	84	312,930	87	49,752	68
1986	571,885	332,184	84	332,184	84	57,344	71
1987	592,065	351,970	84	351,970	84	56,737	65
1988	601,927	379,705	83	379,705	82	63,893	65
1989	596,754	413,480	84	413,480	83	73,858	67
1990	647,678	428,452	83	428,452	81	78,487	65
1991	666,606	458,394	84	458,394	83	81,559	64
1992	702,357	471,285	85	471,285	82	81,502	65
1993	734,122	475,615	86	475,615	84	77,745	67
1994	741,347	487,676	86	487,676	84	73,961	63
1995	765,457	510,294	87	510,294	84	79,238	66
1996	796,722	515,257	86	515,257	84	79,844	65
1997	868,408	544,548	88	544,548	85	112,410	90
1998	851,193	575,456	89	575,456	85	90,792	67
1999	852,305	607,464	89	607,464	87	97,016	66
2000	913,683	651,199	89	651,199	87	120,356	74
2001	891,453	672,645	89	672,645	86	120,600	70
2002	925,390	696,835	89	696,835	86	110,329	65
2003	938,783	707,062	89	707,062	85	120,849	71
2004	976,069	739,259	89	739,259	85	121,434	68
2005	991,904	770,758	89	770,758	86	146,352	78
2006	1,040,977	809,623	89	809,623	86	128,399	63
2007	1,058,540	845,743	89	845,743	85	169,717	75
2008	1,022,553	880,096	92	880,096	86	187,424	78
2009	991,614	830,123	90	830,123	84	148,316	70
2010	984,428	864,506	89	864,506	86	150,904	68
2011	947,166	874,459	88	874,459	84	154,229	65
2012	944,272	875,717	88	875,717	83	158,964	66
2013	943,845	882,930	87	882,930	82	174,412	72
2014	937,468	888,054	86	888,054	82	148,430	60
2015	952,305	911,883	86	911,883	81	158,674	62
2016	1,061,582	940,008	87	940,008	82	146,943	55
1984 - 2016	836,137	636,042	87	636,042	84	111,142	68

Note: This table presents the share of aggregate labor costs (including employer social contributions), value-added and investment in the corporate sector in France that our sample accounts for year by year and on average over the whole period. The sample is all firms in the corporate market sectors, excluding agriculture, finance and real

Table 3.2: Summary Statistics

	Obs. (Nb)	Mean	Median	Sd
Sales	27,543,090	2,642.620	284.620	77,556.280
Gross Output	27,517,472	1,818.489	203.735	69,157.491
Value-Added	27,517,472	730.007	111.297	32,121.450
Labor Costs	27,517,428	507.781	81.000	18,092.476
Labor Share	27,334,884	0.751	0.741	0.336
Employment	27,360,292	14.115	3.000	471.567
Intermediary Inputs	27,517,477	1,088.481	80.188	46,270.444
Investment	19,814,136	185.104	4.000	19,200.450
Capital Book Value	27,507,848	1,305.843	76.000	168,002.986

Note: This table presents the main descriptive statistics of firm level outcomes. The sample is all firms in the corporate market sectors, excluding agriculture, finance and real estate, with non zero employment.

Table 3.3: Correlations Between Variations in Industry-Level Concentration and Labor Shares

	Industry Labor Share		Across Labor Share Quantiles		Within Low Labor Share Quantiles	
Labor Share						
Top 1% Share	-0.0777 (0.0123)		-0.0457 (0.0112)		0.0097 (0.0099)	
Top 5% Share		-0.1102 (0.0167)		-0.1288 (0.0150)		0.0092 (0.0135)
Observations	4,666	4,673	4,665	4,660	4,661	4,664
R2	0.0341	0.0347	0.0290	0.0405	0.0281	0.0292
Labor Share						
4 Largest Share	-0.0728 (0.0147)		-0.0602 (0.0133)		0.0772 (0.0119)	
20 Largest Share		-0.1113 (0.0168)		-0.1196 (0.0152)		0.0615 (0.0137)
Observations	4,649	4,648	4,645	4,645	4,651	4,650
R2	0.0320	0.0388	0.0325	0.0401	0.0366	0.0340

Note: Each estimate is the result of OLS estimation at the 3-digit industry with year fixed-effects. The dependent variable in columns "Industry Labor Share" is the long-term change of the industry aggregate labor share, defined as the ratio of the sum of firm level compensation and taxes paid on labor over the sum of firm level value added in that industry. The dependant variable in columns "Across Labor Share Quantiles" and "Within Low Labor Share Quantiles" are the corresponding contributions to the industry aggregate labor share according to the decomposition described in Appendix B.2. The independent variables are the changes of the share of sales of the top 1%, top 5 %, largest 4 and largest 20 firms.

Table 3.4: Firm Level Labor Share Trends

Labor Share	Unbalanced Panel				Balanced Panel	
	Firm FE	Firm x Size FE	Firm FE	Firm FE	Firm x Size FE	Firm FE
No Size Threshold						
Trend	0.0057 (0.0000)	0.0062 (0.0000)	0.0056 (0.0000)	0.0025 (0.0000)	0.0026 (0.0001)	0.0025 (0.0000)
Log Employment			0.0322 (0.0002)			0.0016 (0.0011)
Observations	26,761,933	26,032,310	26,623,375	887,205	880,534	887,201
R2	0.594	0.630	0.598	0.429	0.514	0.429
More than 50 Employees						
Trend	0.0038 (0.0001)	0.0042 (0.0001)	0.0041 (0.0001)	0.0029 (0.0001)	0.0030 (0.0001)	0.0030 (0.0001)
Log Employment			-0.0226 (0.0014)			-0.0175 (0.0036)
Observations	849,448	803,590	818,020	165,843	165,013	165,843
R2	0.608	0.630	0.577	0.513	0.587	0.514
More than 100 Employees						
Trend	0.0037 (0.0001)	0.0039 (0.0001)	0.0040 (0.0001)	0.0029 (0.0002)	0.0030 (0.0002)	0.0030 (0.0002)
Log Employment			-0.0265 (0.0022)			-0.0220 (0.0052)
Observations	434,631	399,285	404,322	95,311	94,916	95,311
R2	0.649	0.642	0.597	0.538	0.602	0.539
More than 1000 Employees						
Trend	0.0032 (0.0004)	0.0033 (0.0005)	0.0033 (0.0004)	0.0028 (0.0006)	0.0030 (0.0006)	0.0029 (0.0006)
Log Employment			-0.0321 (0.0088)			-0.0312 (0.0184)
Observations	56,186	26,560	26,760	9,406	9,383	9,406
R2	0.821	0.719	0.689	0.668	0.716	0.669

Note: Each estimate is the result of OLS estimation of firm level labor share on time trends, for four samples: all firms, firms with more than 50 employees, 100 employees, and 1000 employees; and two panels: all firms or a balanced panel of firms present in the data from 1984 to 2016 (these firms account of 20 to 25 % of total value-added). All regressions include a set of firm-level fixed effect. Columns "Firm x Size FE" also include a set of size category fixed effect. Standard errors are clustered at the firm level.

Table 3.5: Average Output Elasticities, Non Rolling Estimation

	θ_l	θ_k	N		θ_l	θ_k	N
Mining	0.607 (0.048)	0.297 (0.081)	45,698	Gas and electricity	0.677 (0.193)	0.231 (0.169)	22,243
Food products	0.759 (0.053)	0.130 (0.100)	1,277,913	Water supply and waste	0.652 (0.141)	0.183 (0.125)	118,249
Textiles	0.588 (0.136)	0.111 (0.048)	282,598	Construction	0.649 (0.145)	0.057 (0.082)	4,969,117
Wood, paper and printing	0.813 (0.118)	0.041 (0.105)	552,510	Wholesale and retail trade	0.758 (0.171)	0.086 (0.138)	8,502,337
Coke and refined petroleum	0.736 (0.250)	0.323 (0.074)	2,472	Transportation	0.830 (0.151)	0.049 (0.145)	988,348
Chemicals	0.819 (0.059)	0.156 (0.073)	62,567	Accommodation and food services	0.601 (0.151)	0.184 (0.128)	3,076,031
Pharmaceuticals	0.901 (0.344)	0.050 (0.295)	11,657	Publishing and motion pictures	1.033 (0.237)	0.010 (0.214)	309,540
Rubber and plastic products	0.774 (0.150)	0.119 (0.164)	245,896	Telecommunications	1.089 (0.187)	-0.055 (0.213)	25,191
Basic Metals	0.729 (0.131)	0.108 (0.094)	545,742	ICT	0.938 (0.128)	-0.016 (0.135)	324,622
Computers and electronics	0.764 (0.071)	0.104 (0.023)	110,072	Legal, accounting and engineering	0.859 (0.150)	-0.025 (0.144)	1,499,590
Electrical equipments	0.750 (0.026)	0.135 (0.048)	50,476	Scientific research	0.935 (0.242)	0.055 (0.211)	30,461
Machinery and equipments	0.839 (0.071)	0.073 (0.046)	161,603	Advertising and market research	0.998 (0.091)	-0.103 (0.092)	406,636
Transport equipments	0.836 (0.159)	0.115 (0.139)	71,000	Administrative and support	0.746 (0.120)	0.044 (0.157)	1,401,753
Other manufacturing products	0.797 (0.089)	0.008 (0.073)	650,254	Total	0.736 (0.175)	0.078 (0.137)	25,744,576

Note: This table reports the output elasticities from non rolling estimation of the translog production function.

Columns θ_l and θ_k report the average estimated output elasticity with respect to each factor of production for the translog production function for all firms. Column N report the number of observations in each sector. Standard deviations (not standard errors) of the output elasticities are reported in brackets.

Table 3.6: Average Output Elasticities, Rolling Estimation

	θ_l	θ_k	N		θ_l	θ_k	N
Mining	0.611 (0.199)	0.289 (0.162)	45,698	Gas and electricity	0.697 (0.190)	0.236 (0.174)	22,243
Food products	0.754 (0.052)	0.127 (0.104)	1,277,913	Water supply and waste	0.630 (0.178)	0.204 (0.146)	118,249
Textiles	0.553 (0.221)	0.135 (0.157)	282,598	Construction	0.611 (0.175)	0.078 (0.087)	4,969,117
Wood, paper and printing	0.794 (0.110)	0.044 (0.104)	552,510	Wholesale and retail trade	0.762 (0.175)	0.093 (0.145)	8,502,337
Coke and refined petroleum	0.533 (0.391)	0.251 (0.258)	2,472	Transportation	0.840 (0.156)	0.045 (0.148)	988,348
Chemicals	0.806 (0.143)	0.163 (0.122)	62,567	Accommodation and food services	0.592 (0.174)	0.181 (0.133)	3,076,031
Pharmaceuticals	0.898 (0.359)	0.072 (0.286)	11,657	Publishing and motion pictures	1.077 (0.245)	-0.001 (0.215)	309,540
Rubber and plastic products	0.763 (0.159)	0.125 (0.176)	245,896	Telecommunications	1.048 (0.242)	-0.035 (0.217)	25,191
Basic Metals	0.719 (0.128)	0.111 (0.095)	545,742	ICT	0.921 (0.140)	0.002 (0.140)	324,622
Computers and electronics	0.747 (0.084)	0.095 (0.068)	110,072	Legal, accounting and engineering	0.843 (0.164)	-0.020 (0.150)	1,499,590
Electrical equipments	0.766 (0.136)	0.127 (0.101)	50,476	Scientific research	0.856 (0.259)	0.015 (0.230)	30,461
Machinery and equipments	0.808 (0.137)	0.094 (0.069)	161,603	Advertising and market research	0.867 (0.269)	-0.067 (0.140)	406,636
Transport equipments	0.834 (0.180)	0.121 (0.156)	71,000	Administrative and support	0.757 (0.126)	0.039 (0.165)	1,401,753
Other manufacturing products	0.745 (0.129)	0.042 (0.080)	650,254	Total	0.724 (0.193)	0.086 (0.143)	25,744,576

Note: This table reports the output elasticities from rolling estimation of the production function. Columns θ_l and θ_k report the average estimated output elasticity with respect to each factor of production for the translog production function for all firms. Column N report the number of observations in each sector. Standard deviations (not standard errors) of the output elasticities are reported in brackets.

Table 3.7: Correlations Between Variations in Industry-Level Concentration and Markup

	Industry Markup		Across Markup Quantiles		Within High Markup Quantiles	
Markup, Non Rolling						
Top 1% Share	0.1754 (0.0157)		0.1205 (0.0140)		0.0143 (0.0112)	
Top 5% Share		0.2416 (0.0216)		0.2207 (0.0190)		0.0219 (0.0153)
Observations	4,664	4,664	4,670	4,668	4,665	4,669
R2	0.0607	0.0609	0.0269	0.0389	0.0346	0.0355
Markup, Non Rolling						
4 Largest Share	0.1855 (0.0190)		0.1653 (0.0170)		-0.0952 (0.0135)	
20 Largest Share		0.2008 (0.0220)		0.2288 (0.0193)		-0.0823 (0.0157)
Observations	4,649	4,649	4,650	4,650	4,655	4,655
R2	0.0544	0.0532	0.0328	0.0417	0.0451	0.0413
Markup, Rolling						
Top 1% Share	0.2640 (0.0257)		0.0790 (0.0245)		0.0092 (0.0145)	
Top 5% Share		0.3577 (0.0353)		0.1460 (0.0337)		0.0400 (0.0199)
Observations	4,660	4,660	4,654	4,654	4,663	4,663
R2	0.0569	0.0586	0.0120	0.0140	0.0168	0.0177
Markup, Rolling						
4 Largest Share	0.2098 (0.0321)		0.0995 (0.0298)		-0.0536 (0.0175)	
20 Largest Share		0.1702 (0.0372)		0.1101 (0.0346)		-0.0242 (0.0202)
Observations	4,647	4,646	4,644	4,644	4,650	4,650
R2	0.0482	0.0447	0.0108	0.0112	0.0172	0.0173

Note: Each estimate is the result of OLS estimation at the 3-digit industry with year fixed-effects. The dependent variable in columns "Industry Markup" is the long-term change of the industry aggregate markup. The dependant variable in columns "Across Markup Quantiles" and "Within High Markup Quantiles" are the corresponding contributions to the industry aggregate markup according to the decomposition described in Appendix B.2. The independent variables are the changes of the share of sales of the top 1%, top 5 %, largest 4 and largest 20 firms.

Table 3.8: Firm Level Markup Trends, Non Rolling

Markup	Unbalanced Panel			Balanced Panel		
	Firm FE	Firm x Size FE	Firm FE	Firm FE	Firm x Size FE	Firm FE
No Size Threshold						
Trend	-0.0141 (0.0000)	-0.0158 (0.0000)	-0.0142 (0.0000)	-0.0060 (0.0001)	-0.0064 (0.0001)	-0.0062 (0.0001)
Log Employment			0.0198 (0.0004)			0.0650 (0.0025)
Observations	25,092,615	24,535,649	25,092,615	879,223	872,598	879,223
R2	0.617	0.649	0.617	0.556	0.644	0.561
More than 50 Employees						
Trend	-0.0080 (0.0001)	-0.0088 (0.0001)	-0.0087 (0.0001)	-0.0069 (0.0003)	-0.0074 (0.0003)	-0.0075 (0.0003)
Log Employment			0.0631 (0.0027)			0.0621 (0.0066)
Observations	789,696	775,795	789,696	163,698	162,859	163,698
R2	0.676	0.733	0.678	0.630	0.711	0.632
More than 100 Employees						
Trend	-0.0083 (0.0002)	-0.0089 (0.0002)	-0.0089 (0.0002)	-0.0074 (0.0004)	-0.0078 (0.0004)	-0.0078 (0.0004)
Log Employment			0.0617 (0.0041)			0.0562 (0.0099)
Observations	391,061	386,202	391,061	94,072	93,670	94,072
R2	0.694	0.743	0.696	0.652	0.725	0.654
More than 1000 Employees						
Trend	-0.0093 (0.0010)	-0.0096 (0.0011)	-0.0098 (0.0010)	-0.0094 (0.0016)	-0.0102 (0.0016)	-0.0097 (0.0015)
Log Employment			0.0798 (0.0186)			0.0866 (0.0373)
Observations	26,261	26,072	26,261	9,309	9,286	9,309
R2	0.781	0.815	0.782	0.785	0.829	0.787

Note: Each estimate is the result of OLS estimation of markups from non rolling estimates on time trends, for four samples: all firms, firms with more than 50 employees, 100 employees, and 1000 employees; and two panels: all firms or a balanced panel of firms present in the data from 1984 to 2016 (these firms account of 20 to 25 % of total value-added). All regressions include a set of firm-level fixed effect. Columns "Firm x Size FE" also include a set of size category fixed effect. Standard errors are clustered at the firm level.

Table 3.9: Firm Level Markup Trends, Rolling

Markup	Unbalanced Panel			Balanced Panel		
	Firm FE	Firm x Size FE	Firm FE	Firm FE	Firm x Size FE	Firm FE
No Size Threshold						
Trend	-0.0132 (0.0000)	-0.0155 (0.0000)	-0.0134 (0.0000)	-0.0023 (0.0001)	-0.0029 (0.0001)	-0.0025 (0.0001)
Log Employment			0.0264 (0.0004)			0.0641 (0.0026)
Observations	25,092,615	24,535,649	25,092,615	879,223	872,598	879,223
R2	0.616	0.650	0.616	0.514	0.613	0.519
More than 50 Employees						
Trend	-0.0010 (0.0001)	-0.0020 (0.0002)	-0.0018 (0.0002)	0.0007 (0.0003)	0.0001 (0.0003)	0.0002 (0.0003)
Log Employment			0.0645 (0.0029)			0.0641 (0.0071)
Observations	789,696	775,795	789,696	163,698	162,859	163,698
R2	0.661	0.723	0.663	0.594	0.688	0.597
More than 100 Employees						
Trend	-0.0006 (0.0002)	-0.0013 (0.0002)	-0.0012 (0.0002)	0.0011 (0.0004)	0.0006 (0.0004)	0.0007 (0.0004)
Log Employment			0.0646 (0.0045)			0.0590 (0.0110)
Observations	391,061	386,202	391,061	94,072	93,670	94,072
R2	0.675	0.730	0.677	0.615	0.702	0.617
More than 1000 Employees						
Trend	0.0023 (0.0011)	0.0021 (0.0011)	0.0017 (0.0011)	0.0033 (0.0017)	0.0028 (0.0017)	0.0028 (0.0016)
Log Employment			0.0999 (0.0202)			0.1296 (0.0422)
Observations	26,261	26,072	26,261	9,309	9,286	9,309
R2	0.745	0.784	0.747	0.744	0.798	0.747

Note: Each estimate is the result of OLS estimation of markups from rolling estimates on time trends, for four samples: all firms, firms with more than 50 employees, 100 employees, and 1000 employees; and two panels: all firms or a balanced panel of firms present in the data from 1984 to 2016 (these firms account of 20 to 25 % of total value-added). All regressions include a set of firm-level fixed effect. Columns "Firm x Size FE" also include a set of size category fixed effect. Standard errors are clustered at the firm level.

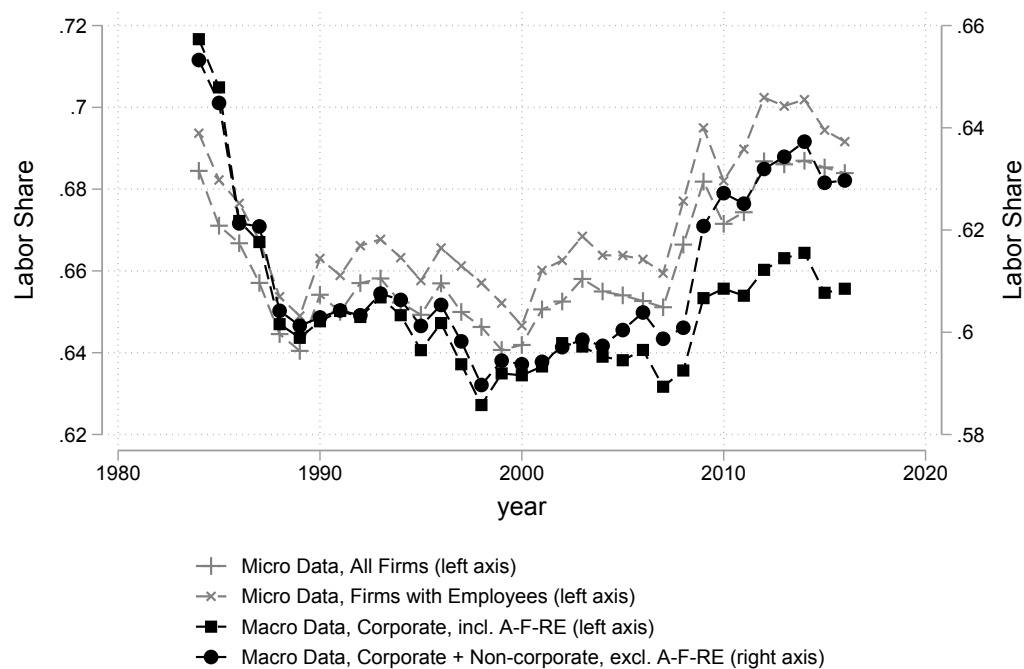
Table 3.10: Correlation between Labor Share and Markup

Labor Share	Non Rolling			Rolling		
	No FE	Industry FE	Firm FE	No FE	Industry FE	Firm FE
No Size Threshold						
Markup	-0.3487 (0.0037)	-0.3713 (0.0024)	-0.3575 (0.0027)	-0.3173 (0.0041)	-0.3520 (0.0022)	-0.3370 (0.0027)
Observations	25,554,561	25,554,533	25,092,587	25,554,561	25,554,533	25,092,587
R2	0.449	0.518	0.772	0.407	0.489	0.761
More than 50 Employees						
Markup	-0.4202 (0.0039)	-0.4460 (0.0034)	-0.4989 (0.0043)	-0.4070 (0.0054)	-0.4351 (0.0035)	-0.4797 (0.0044)
Observations	808,003	807,805	789,488	808,003	807,805	789,488
R2	0.519	0.602	0.816	0.493	0.582	0.805
More than 100 Employees						
Markup	-0.3991 (0.0033)	-0.4268 (0.0039)	-0.4754 (0.0051)	-0.3842 (0.0053)	-0.4163 (0.0041)	-0.4554 (0.0053)
Observations	398,301	398,018	390,768	398,301	398,018	390,768
R2	0.513	0.614	0.825	0.483	0.594	0.814
More than 1000 Employees						
Markup	-0.3320 (0.0079)	-0.3633 (0.0077)	-0.4129 (0.0106)	-0.3270 (0.0050)	-0.3709 (0.0077)	-0.3912 (0.0125)
Observations	26,684	25,305	24,839	26,684	25,305	24,839
R2	0.502	0.721	0.900	0.471	0.710	0.892

Note: Each estimate is the result of OLS estimation of firm level labor share on markups, for four samples: all firms, firms with more than 50 employees, 100 employees, and 1000 employees; and two panels: manufacturing and non manufacturing firms. All columns include year fixed effects. Columns "No FE" include no industry nor firm fixed effect. Columns "Industry FE" include 3-digit industry-level fixed effects. Columns "Firm FE" include firm-level fixed effects. Standard errors are clustered at the 3-digit x year industry level.

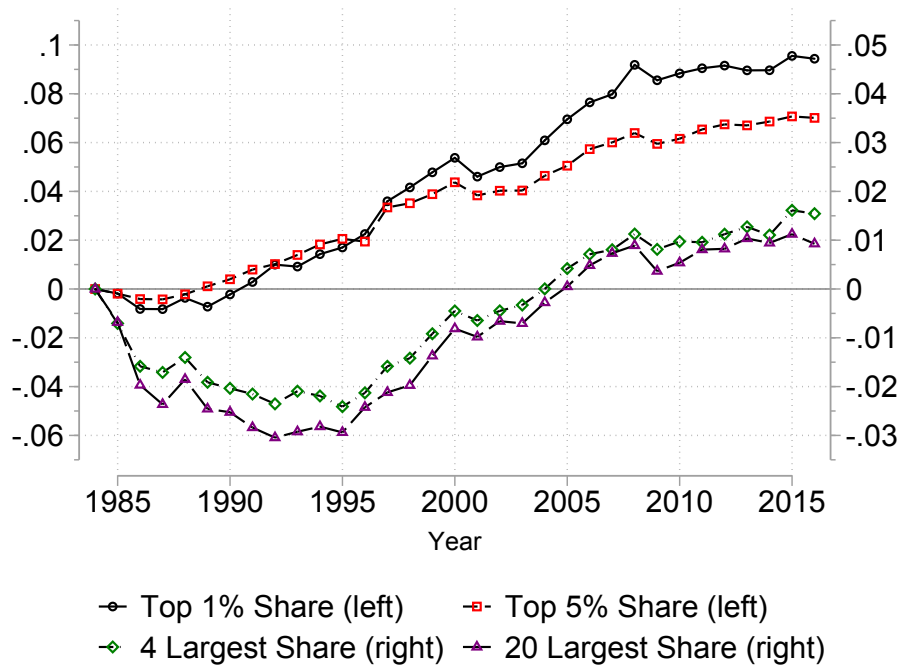
Figures

Figure 3.1: Aggregate Labor Share in France, 1984-2016.



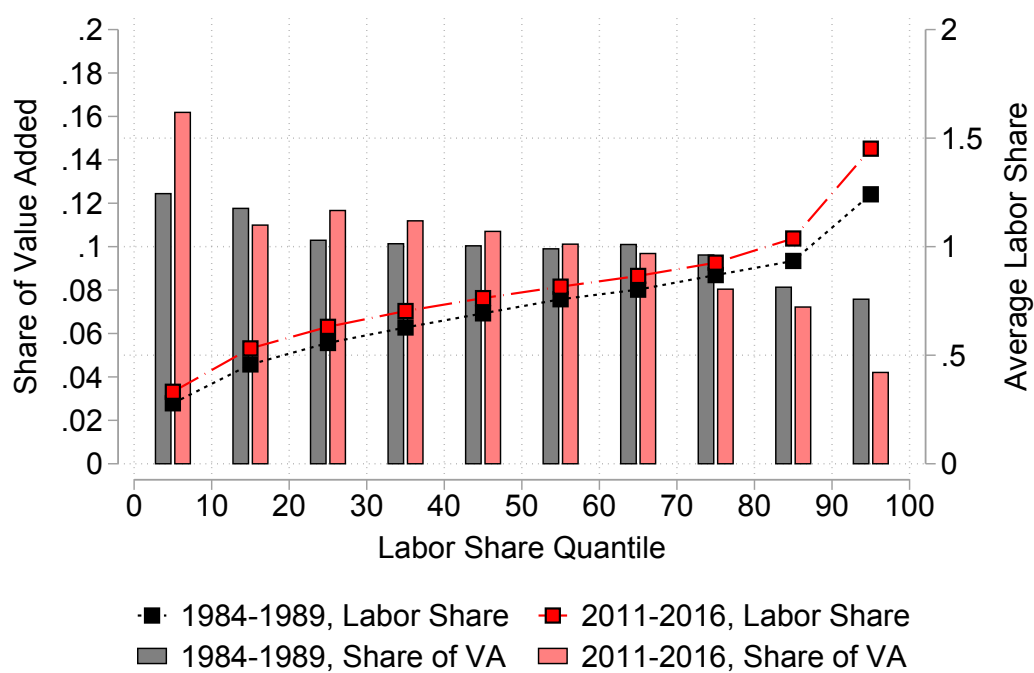
Note: This figure reports the ratio of employee compensation, including payroll taxes, to total value-added in the market sectors in France. See Section 3.3 for details on the different measures.

Figure 3.2: Cumulative Change in Concentration



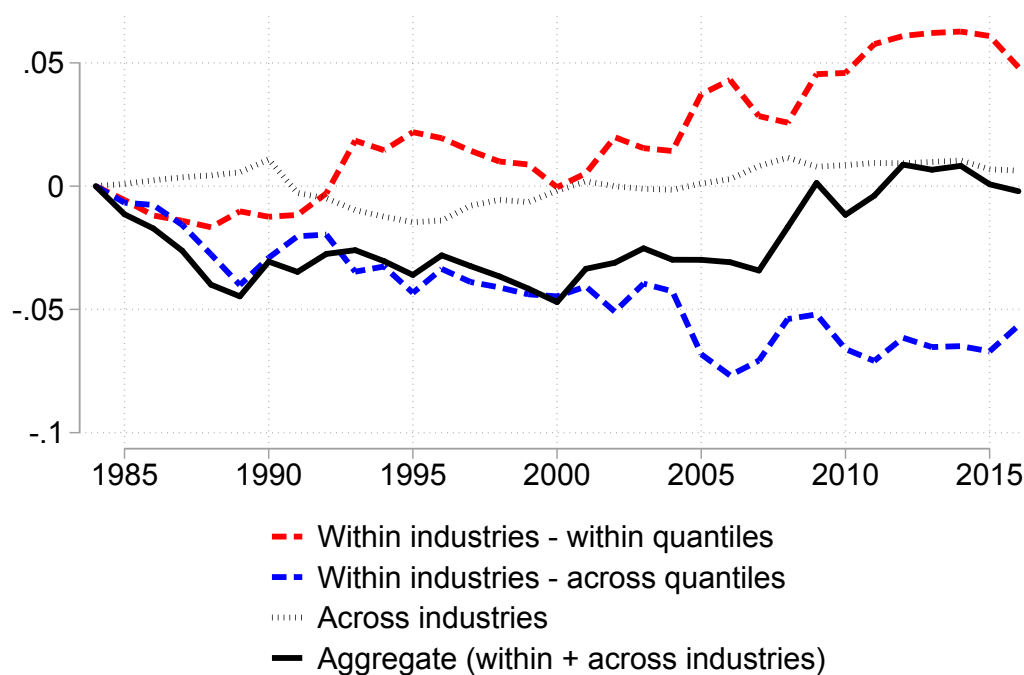
Note: This figure reports the cumulative change of concentration in sales across 3-digit industries. Sample is firms in the market sectors, excluding agriculture, finance and real estate. Industry changes in concentration are weighted by the share of each industry in total sales.

Figure 3.3: Distributions of Labor Shares and Value Added



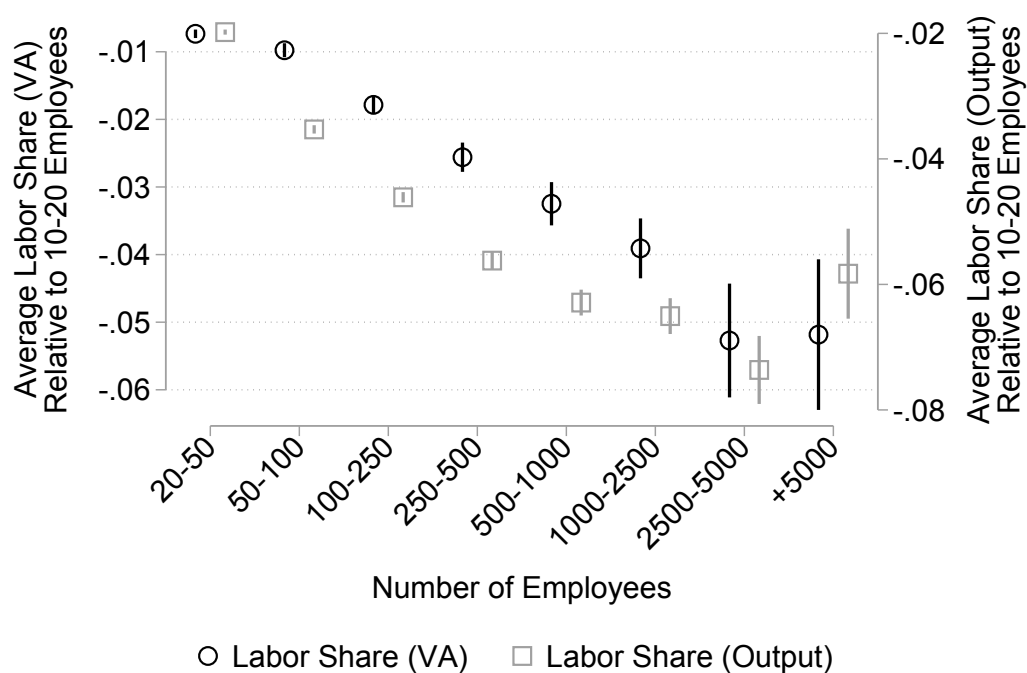
Note: The connected lines (right axis) reflect the raw cross-firm distribution of labor shares. The vertical bars (left axis) reflect the share of industry value added of firms in each unweighted decile of labor share. These distributions are averaged across 3-digit industries using value added weights in a given year, and averaged across 5 year periods.

Figure 3.4: Decomposition of Aggregate Labor Share



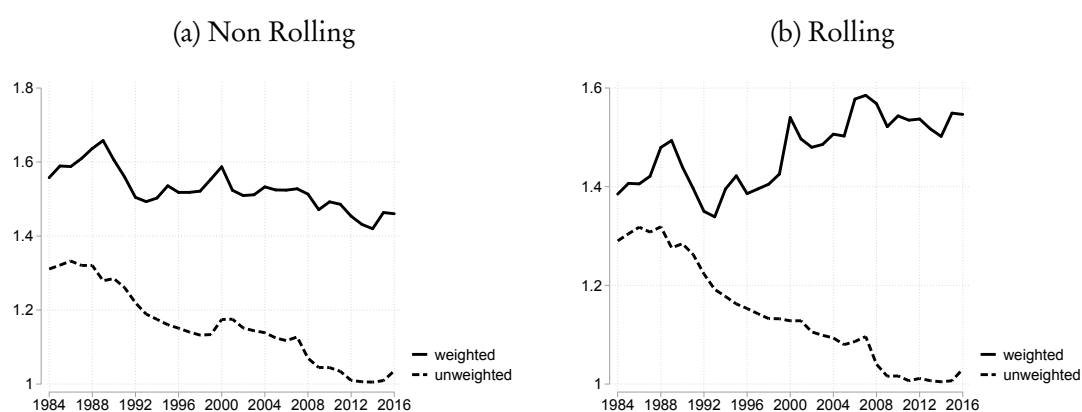
Note: This figure reports the results of decomposition of the aggregate labor share and markup described in Appendix B.2. Quantiles of labor share are calculated each year within 3-digit industries.

Figure 3.5: Labor Share and Size



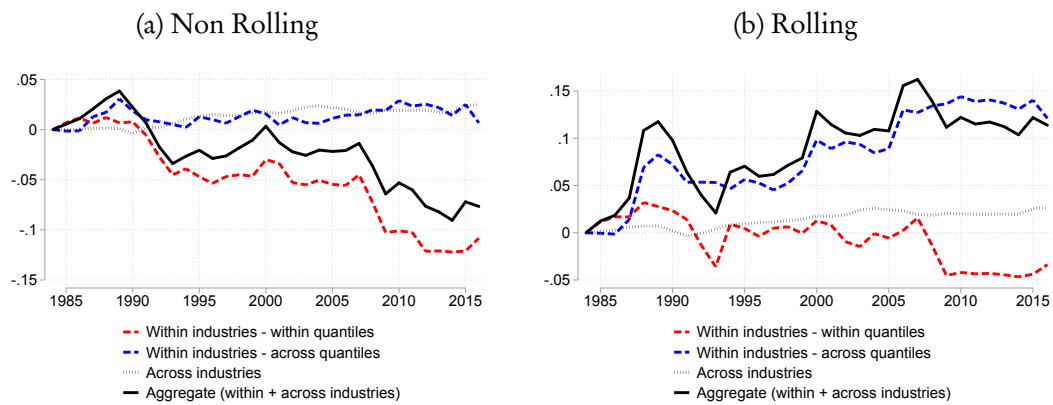
Note: This figure reports the conditional average labor share by firm size, with 99% confidence interval. Averages are conditional on a set of flexible fixed effects constructed from the interaction of 3-digit industry codes and year.

Figure 3.6: Aggregate Markup



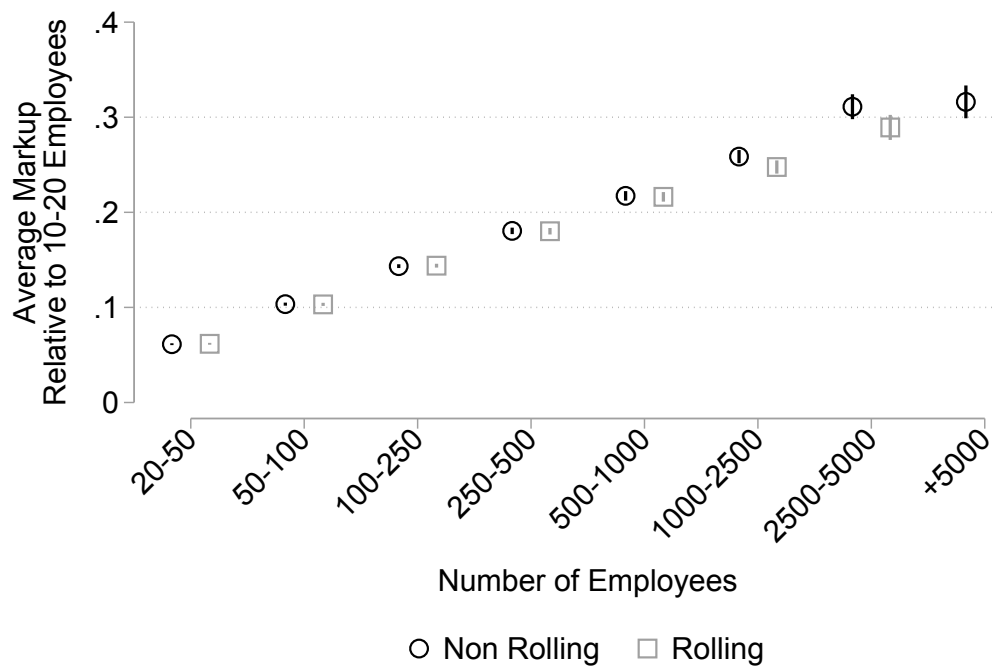
Note: This figures reports the levels of the weighted and unweighted mean markup based on non-rolling and rolling estimation of a translog value-added production function.

Figure 3.7: Decomposition of Aggregate Markup



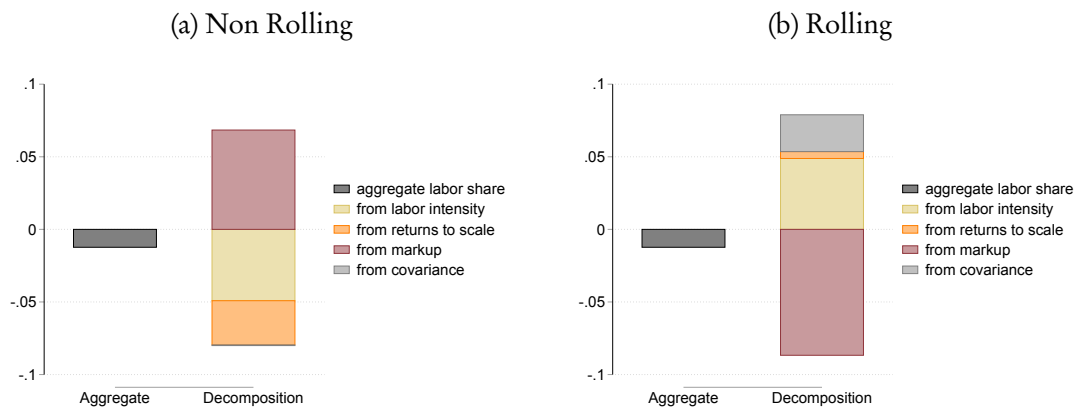
Note: This figures reports the results of decomposition of the aggregate markup described in Appendix B.2. Quantiles of markup are calculated each year within 3-digit industries.

Figure 3.8: Markup and Size



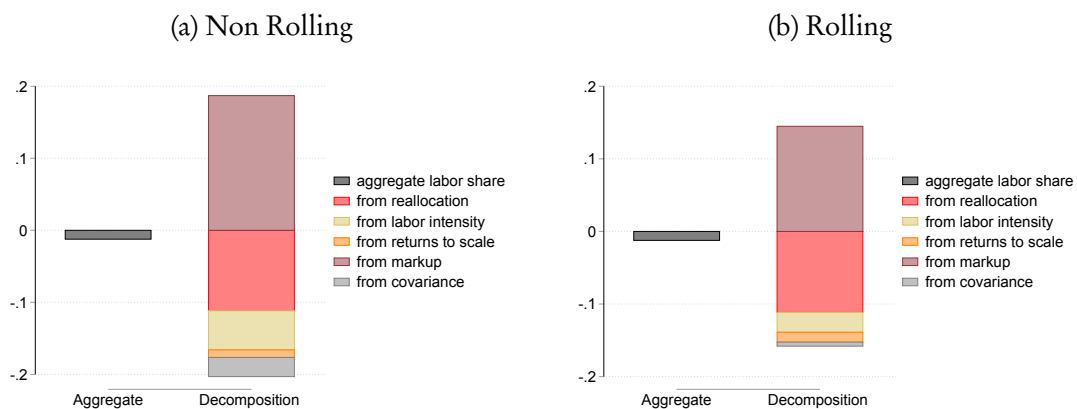
Note: This figure reports the conditional average markup by firm size, with 99% confidence interval. Averages are conditional on a set of flexible fixed effects constructed from the interaction of 3-digit industry codes and year.

Figure 3.9: Contributions to the Evolution of the Aggregate Labor Share, Representative Firm



Note: This figure reports the decomposition of the variation of the labor share of the representative firm from 1984 to 2016, based on translog non-rolling and rolling value-added estimation of the production function. See section 3.5 for detail.

Figure 3.10: Contributions to the Evolution of the Aggregate Labor Share, With Reallocation



Note: This figure reports the decomposition of the variation of the aggregate labor share from 1984 to 2016, including the reallocation term, based on translog non-rolling and rolling value-added estimation of the production function. See section 3.5 for detail.

Chapter 4

IT and Returns to Scale¹

This paper investigates the role of IT in shaping recent trends in market concentration, factor income shares, and market competition. Relying on a novel dataset on hardware and software investments in the universe of French firms, we document a robust within-industry correlation between firm size and the intensity of IT demand. To explain this fact, we argue that the relative marginal product of IT inputs may rise with firm scale, since IT helps firms deal with organizational limits to scale. We propose a general equilibrium model of industry dynamics that features nonhomothetic production functions compatible with this mechanism. Estimating this production function, we identify the nonhomotheticity of IT demand and find an elasticity of substitution between IT and non-IT inputs that falls below unity. Under the estimated model parameters, the cross-sectional predictions of the model match the observed relationship of firm size with IT intensity (positive) and labor share (negative). In addition, in response to the fall in the relative price of IT inputs in post-1990 France, the model can explain about half of both the observed rise in market concentration and the observed market reallocations toward low-labor-share firms.

4.1 Introduction

Advances in Information Technology (IT) have drastically lowered the quality-adjusted prices of computing and information-intensive tools over the past few decades (Byrne and Corrado, 2017). In response, business investment in software and computing equipment has soared, fueling productivity growth at both firm and aggregate levels.² However, investment in IT may have some

¹Co-authored with Danial Lashkari and Jocelyn Boussard

²Between 1997 and 2015, the price of IT relative to machinery investment on average fell by over 65% across 12 richest OECD countries in the EU KLEMS dataset, while the ratio of IT to machinery stocks of capital rose over

conceptually distinct features compared to investments in other forms of productive capital such as machinery, tools, and robots. Whereas machines typically enhance the productivity of firms in performing specific production tasks, IT tools can enhance the coordination and integration of firm processes across many distinct tasks. Consider for instance business management software such as Enterprise Resource Planning (ERP). The ERP systems are tools of organizational planning that standardize information flows across different business divisions such as project, supply chain, and inventory management, as well as procurement, accounting, and customer service.³ Such IT tools enable firms to better cope with organizational complexities of production, both internally and also in relation with their sellers and buyers.⁴ To the extent that such organizational complexities limit the scalability of production for firms, the rise of IT may have important implications for *returns to scale* in production.

In this paper, we provide empirical evidence in favor of the view that IT plays a distinctive role in shaping returns to scale at the firm and the aggregate levels. We further offer a simple theory that allows us to examine the consequences for industry concentration and factor income shares. We document a new fact using micro data from France: a robust, within-industry, cross-sectional correlation between firm size and IT intensity, defined as the ratios of IT to other or total inputs. We show that this fact can be rationalized by a firm-level production function that distinguishes IT from other types of capital, allowing for its marginal product (relative to that of other inputs) to systematically rise with firm size. If organizational efficiency declines with scale but rises with IT intensity, larger firms optimally choose to become more IT intensive. Two important consequences for the rise of IT immediately follow: it 1) raises the returns to scale at the firm level, and 2) disproportionately benefits larger firms and reallocates market shares toward them. We further establish a tight theoretical connection between such nonhomotheticity in IT factor demand and *heterogeneity* in returns to scale across firms. We show that with nonhomothetic factor demand, the degree of returns to scale should vary with firm size. In particular, for the range of the production function parameters we estimate in our data, our model suggests that

100% (the corresponding numbers for the US are 82 and 337 percents, respectively). Empirical work has established that this rise in IT investments has led to strong productivity gains both at the micro (e.g., Brynjolfsson and Yang, 1996; Brynjolfsson and Hitt, 2003; Aral et al., 2006; Draca et al., 2009; Bloom et al., 2012) and macro levels (Oliner and Sichel, 2000; Jorgenson, 2001; Stiroh, 2002; Jorgenson et al., 2005; Stiroh, 2008).

³Since their appearance in the early 1990s, the ERP systems quickly spread across industries such that over 40% of US businesses with revenues over 1 billion dollars already had adopted them by 2001 (Stefanou, 2001). Over the second half of the 90s, ERP vendors witnessed an average annual growth rate of around 38%, with a combined revenue that grew to around 20 billion dollars by the end of the decade (Dorien and Wolf, 2000). The global expenditure on ERPs has continued to grow to over 450 billion dollars per year by 2019 (source: www.statista.com). For a brief history of ERP systems, see Jacobs and Weston (2007).

⁴Productivity in each division of firm activities has in turn benefited from specialized software solutions, e.g., Human Resource Management (HRM), Supply Chain Management (SCM), or Consumer Relation Management (CRM). These tools are commonly integrated within ERP systems to enhance strategic planning and support the decision making processes at the firm level.

returns to scale falls in firm size. Since the degree of returns to scale and the income share of factor payments are positively related, the latter should also fall in firm size.⁵ This is in line with the negative cross-sectional correlation between firm size and labor share in our data (see also [Autor et al., 2020b](#)). In addition, this also implies a second channel through which the rise of IT affects the aggregate returns to scale: as it reallocates market shares toward large and IT intensive firms, it also lowers the aggregate returns to scale.

The dichotomy between within-firm and across-firm effects on returns to scale, and subsequently labor share, finds support in our data. We find divergent patterns between the two components of labor share in France, in line with recent results documented in the case of US (e.g., [Autor et al., 2020b](#); [Kehrig and Vincent, 2017](#)): whereas within-firm labor share rises, the reallocation toward low-labor-share firms contributes negatively in the aggregate. In a calibration of our model, we find that the fall in the relative IT prices observed in the data between 1990 and 2007 can quantitatively explain about half of the observed patterns in labor share and the observed rise in market concentration in the data. We show that nonhomotheticity plays a sizable role in explaining these patterns: aggregating from micro to macro, our model implies a *macro* elasticity of substitution between IT and other factor inputs of around unity, whereas a model with homothetic production functions leads to an elasticity of around 0.75.

Overview of the Paper Empirically, our point of departure is the introduction of novel datasets that detail micro-level software and hardware investments among French firms, covering a broad set of manufacturing and service industries between 1995 to 2007. This is a period that witnessed a substantial and widespread rise in the adoption of IT among French firms. Figure 4.1 shows the series for the user cost of IT equipment (relative to that of non-IT capital) and the ratio of IT equipment capital to non-IT capital services across the entire market economy in France. Between 1990 to 2015, we observe a fall of over 50% in the relative user cost of IT and a sizable shift in the composition of capital of firms toward IT equipment.

We rely on the micro data to construct stocks of software and hardware capital for French firms, and document a positive within-industry relationship between size and IT intensity. This finding is robust to different measures of IT intensity, whether IT is proxied by software or hardware, whether by investment or capital, and whether intensity is measured relative to labor inputs or to non-IT capital. It is also robust to using different measures of firm scale, whether scale is measured by employment, value added, sales, or by more eclectic measures such as the number

⁵In our model, markups are constant and variations in the shares of factor payments in income vis-à-vis profits do not stem from differences in market power. Instead, they correspond to differences in *Ricardian rents*, i.e., the returns to fixed, firm-specific, inimitable factors that distinguish firms from one another. We assume that all other factors receive their marginal products in perfectly competitive markets, leaving the residual value created to the owner of the firm.

of plants, the depth of organizational structure, the number of exporting markets, or the number of exported products (the latter two measures only in the sample of exporting firms). We find this relationship to hold across a wide range of industries and classes of firm size, from small firms with just a few workers to large multinationals hiring thousands of workers.

We further use detailed balance sheet information for the universe of all French firms to revisit the evolution of market concentration and labor share, and the cross sectional relationship between labor share and firm size. Similar to the patterns recently documented in the US and a number of other advanced economies (eg., [Andrews et al., 2016c](#); [Autor et al., 2017, 2020b](#)), we find a sizable rise in industry-level market concentration. In addition, we find that the implied market reallocations toward larger firms have contributed negatively to the evolution of the aggregate labor share (accumulating to a total of around 4 percentage points from 1990 to 2007). In contrast, we find that upward shifts in the distribution of labor share over the same period made a positive within-firm contribution of around the same magnitude to the evolution of the aggregate labor share (similar to patterns found by [Kehrig and Vincent, 2017](#)).

To structurally account for these facts, we construct a general equilibrium model that features monopolistically competitive firms endowed with a nested *Nonhomothetic CES* (nhCES) production function ([Sato, 1974, 1977](#); [Comin et al., 2015](#)). Software and hardware constitute a bundle of IT inputs while labor and non-IT capital constitute a bundle of non-IT inputs. The two bundles are combined using a nhCES production function to produce firm-level output. Relative to the standard CES specification, the nhCES aggregator only adds a *nonhomotheticity parameter* that governs the elasticity of IT intensity with respect to output. We assume that firms are heterogeneous in terms of two productivity states, one factor-symmetric and one IT-biased, and the two evolve over time according to a simple Markovian process.

Nonhomothetic IT demand explains our first micro fact on the cross-sectional correlation between firm size and IT intensity. As mentioned earlier, the model also implies a connection between firm size and returns to scale, which rationalizes our second micro fact on the negative correlation between firm size and labor share. Importantly, the relationship between returns to scale and firm size in the model crucially hinges on the elasticity of substitution between IT and non-IT inputs. In particular, if the elasticity is below unity, nonhomotheticity of IT demand implies that larger firms have both lower returns to scale and lower shares for factor payments in their income.⁶

We aggregate the model and derive the general equilibrium predictions of the model in response to an exogenous fall in the price of IT inputs. We show that the aggregate response can be sum-

⁶In Appendix [C.1](#), we show that this result generalizes beyond the nhCES production function. In particular, subject to mild conditions, if the elasticity of substitution is locally constant and IT demand is locally nonhomothetic, the returns to scale decreases (increases) in firm size if the elasticity of substitution is below (above) unity.

marized in terms of the response of the aggregate IT intensity to this shock. We decompose this response to within-firm and across-firm components, and examine the effect of nonhomothetic IT demand on each of the two components. The within-firm effect raises returns to scale (and factor income shares) for all firms, and the across-firm effect shifts market shares toward larger firms operating with lower returns to scale (and factor income shares).

We bring the model to the data in two steps. First, we employ an identification strategy based on standard timing assumptions to estimate our production function using the panel structure of our micro data. We combine this method with a strategy that relies on shift-share instruments for the price of software relative to wages to estimate the full nhCES production function, including the elasticity of substitution between IT and non-IT inputs (similar to [Oberfield and Raval, 2014b](#)). We find a nonhomotheticity parameter of around 0.4 and reject homotheticity of IT demand in the sample of all industries, the sample of all manufacturing industries, and in samples of more disaggregated industries. In line with the parametric restrictions required by our mechanism, we also find estimates for the elasticity of substitution between IT and non-IT inputs that are below unity: the estimated elasticity is 0.23 (0.17) in the sample of all (manufacturing) industries.⁷ Finally, our estimated parameters implies values for the returns to scale that are very close to unity for the *average* firm, while still leading to sizable variations in the cross section depending on the firm's IT intensity.

Armed with these estimates, we perform a simple calibration of our general equilibrium model to gauge its quantitative success in explaining the evolution of industry concentration and labor share in France. We assume that the fall in the price of IT observed in Figure 4.1 is driven by exogenous technological progress and examine the consequences through the lens of the model. Accordingly, we study the response of the model in moving between two stationary equilibria of the calibrated model, corresponding to the observed trend in the relative price of IT (high initial value in 1990 and low final value in 2007). The calibrated model predicts a rise in market concentration, proxied by the market share of top 1% and 5% of firms, about half of the rise observed in the data. The model also predicts positive within-firm and negative across-firm contributions to the aggregate labor share, both by around 2 percentage points, again accounting for about half of the two components observed in the data. An alternative model calibrated based on a homothetic CES production function generates quantitative responses with about half the magnitude of our model. We conclude that nonhomotheticity of IT demand and the fall in IT prices together help explain a substantial part of the rise in industry concentration and the resulting reallocations toward low-labor-share firms in France.

⁷These numbers are close to recent micro-level estimates of the elasticity of substitution between capital and labor (see, e.g., [Oberfield and Raval, 2014b](#); [Doraszelski and Jaumandreu, 2018](#)).

Prior Literature Our paper contributes to a large literature that has studied the impacts of IT at both the micro and the macro levels.⁸ While in this paper we focus on the aggregate consequences of the nonhomotheticity of IT demand, our results also have important implications for the micro side of this literature. In particular, the fact that the relative IT demand grows in firm size may in fact explain a part of the observed relationship between productivity and IT intensity.

Our paper further contributes to the literature that studies a number of recent secular macroeconomic trends across advanced economies. A number of papers have documented growing industry concentration and within-industry dispersion in firm outcomes (CEA, 2016; Andrews et al., 2016c; Berlingieri et al., 2017a). Indicators of business dynamism, such as the rate of startup formation, appear to be in decline across many advanced economies, particularly in the United States (Decker and Haltiwanger, 2013; Haltiwanger, 2015; Decker et al., 2015; Andrews et al., 2016c; Decker et al., 2016; Karahan and Pugsley, 2016; Gutiérrez and Philippon, 2017a). In parallel, there is a large body of work on a global fall in the labor share across many industries (Elsby et al., 2013b; Karabarbounis and Neiman, 2014; Koh et al., 2015; Barkai, 2020; Grossman et al., 2017).⁹

As discussed above, IT capital has been put forth as a potential explanation for the above trends,¹⁰ and in particular for the fall in labor share through its potential substitution with labor (Karabarbounis and Neiman, 2014; Gaggl and Wright, 2017; Eden and Gaggl, 2018). Our paper reconciles this line of work with available evidence that the micro elasticities of capital-labor substitution fall below unity (Lawrence, 2015), and that market reallocations are responsible for the potential fall in the labor share (Kehrig and Vincent, 2017; Autor et al., 2020b, 2017).¹¹ More recently, Aghion

⁸Beyond the studies cited earlier, recent papers in this line of work rely on exogenous variations in the costs of IT adoption as a strategy for identifying the elasticity of output and productivity with respect to IT (e.g., DeStefano et al., 2014, 2016; Akerman et al., 2015). In two recent papers, Harrigan et al. (e.g., 2016, 2018) have studied the rise of what they refer to as “techies,” the specialized and technically oriented labor inputs that may constitute complementary inputs to IT, in the context of the French labor market. They argue that the shifts in the composition of labor hired by French firms toward techies has played an important role in labor market polarization and skill-biased productivity growth.

⁹Since Karabarbounis and Neiman (2014), a sizable body of work has revisited the evidence on the fall of the labor share to examine the potential explanations, or whether this fact is robust to the relevant details in the construction of the labor share series (e.g., see Elsby et al., 2013b; Koh et al., 2015). In this paper, we focus on the compositional aspects of the evolution of labor share across firms, i.e., the changes within and across firms, rather than the aggregate labor share. In fact, we do not find an aggregate fall in the labor share in France beginning in the 1990s.

¹⁰We note that IT is one among the several mechanisms offered in the literature as the potential drivers of the observed macroeconomic trends. For instance, Crouzet and Eberly (2017, 2018), Gouin-Bonenfant (2018), Martinez (2018), Akcigit and Ates (2019), Hopenhayn et al. (2018), and Liu et al. (2019) have proposed the rise of intangible capital, productivity dispersion, automation, and the decline in knowledge diffusion, population growth, and interest rates as potential channels for these trends, respectively.

¹¹In addition, our framework further allows us to account for potential responses in the aggregate profit share to the rise in the price of IT (Autor et al., 2020b; Barkai, 2020). Note that, in contrast to a number of recent other accounts of the fall of the labor share which focus on market power and markups (e.g., De Loecker et al., 2020a; Baqaee and Farhi, 2020b; Aghion et al., 2019), our model features efficient allocations and therefore our mechanism

et al. (2019), Hsieh and Rossi-Hansberg (2019), and Mariscal (2018) have provided theoretical models that link the rise of IT to the recent secular macroeconomic trends.¹² The mechanisms in these papers share a key feature with ours in connecting the rise of IT to changes in the span and scale of operation of firms. Our paper complements these contributions by providing direct empirical evidence on the micro-level relationship between size and scale. Furthermore, we are able to structurally identify our mechanism in the data, and use the resulting estimates to discipline a quantitative analysis of the connection between the observed fall in the price of IT and the aggregate trends.¹³

In this paper, we focus on identifying how the nonhomotheticity of IT demand at the level of firm production functions may shape the behavior of the aggregate economy. Prior work has identified potential forces that give rise to such nonhomotheticity patterns. For instance, the organizational theories of firm generates nonhomotheticity patterns similar to those uncovered here (Caliendo and Rossi-Hansberg, 2012; Caliendo et al., 2015a,b).¹⁴ In addition, there is some evidence for the effects of IT on the integration and supply-chain management of firms (Fort, 2014; Basco and Mestieri, 2018), which may provide an alternative ground for the higher benefits of IT for larger firms. We discuss further evidence on the relationship between IT and firm in organization in Section 2.6 below.

Outline of the Paper The remainder of this paper is organized as follows. Section 2.2 presents the sources of the data, and Section 4.3 discusses our key empirical facts. Section 2.3 presents the theory, and Sections 4.5.1 and 4.5.2 discuss our identification strategy for estimating the nhCES production function and the calibration results, respectively. Finally, Section 2.6 provides a more in depth discussion of some aspect of our theory and Section 4.7 concludes the paper.

4.2 Data

Our data covers active firms in the corporate sector in France from 1990 to 2007.¹⁵ These firm-level data are collected from surveys and tax records by the French Institute of Statistics (INSEE).

does not involve any changes in the level of allocative distortions.

¹²Relatedly, Autor et al. (2017, 2020b) suggest that IT may have created network effects and facilitated more effective product comparisons for consumers, therefore helping superstar firms gain larger shares of the market.

¹³We also note that our model shares this core mechanism with the results of Basu and Fernald (1997), who study business cycle fluctuations in an environment where producers have heterogeneous returns to scale and where cyclical expansions of output are biased toward firms with higher returns to scale.

¹⁴For a direct application of this theory to the impacts of IT, see Bloom et al. (2014).

¹⁵We restrict our attention to the following sectors: manufacturing, mining, utility, construction, trade, transportation, accommodation and food services, information and communication, and professional services, excluding agriculture, real estate, finance, public administration, education, and health.

The Annual Survey of Firms (EAE) provides information on software investment at the firm level for all firms with more than 20 employees, and a sample of smaller firms. The BRN (normal tax regime) and RSI (simplified tax regime) data provide standard income and employment information for all French firms that have to report to the tax authorities, outside of agriculture. Because reporting obligations vary with the revenues of the firm, firms in the BRN files also report separately their investments in several types of asset, including hardware. Additionally, we rely on the employee-level DADS data and the Customs data for the construction of proxies for the scale and scope of operation of firms. The unique firm identifier SIREN allows us to match these data sources. In addition to these firm-level data, we also rely on aggregate and sectoral series for France from INSEE National Accounts and the KLEMS dataset (Jäger, 2017b).¹⁶ Firms that exclusively report to the RSI tax regime are included in the analysis of the macro trends in France but because they do not report their investments in IT, they are not included in the IT data. Below, we discuss the features of the IT data (see Section C.6 in the Appendix for further details).

IT Data The EAE files contain information on the investment of firms in software (Frouté, 2001). Surveyed firms report total investment in software, as the sum of expenditure on 1) software purchased from outside, 2) software created in-house, and 3) investment made in existing software. The survey further includes a disaggregation of software investment into these three components, for firms operating in some manufacturing, trade and services industries.¹⁷ Our measure of software investment includes all components, and we use the information on the disaggregated components of investment, when available, to ensure that they are compatible with the reported value of total investment in software.

The BRN files report the total investment of firms in office and computing equipment (Barbesol et al., 2008; Chevalier and Luciani, 2017). This component of investment in the BRN data provides, to our knowledge, the closest measure for hardware investments of firms in the universe of French firms, despite the fact that it includes non-investment components such as office furniture. We use this variable as our measure of investment in hardware and as our second indicator of IT investment, acknowledging the potential for measurement error due to the presence of non-IT components.

In the case of firms that report at least once to the BRN tax regime, we construct measures of hardware and other non-IT capital stocks based on the observed investment flow measures, start-

¹⁶The dataset is available online at <http://www.euklems.net>.

¹⁷Firms operating in the food sector also report the sources of funds (internal or external finance) for the investment.

ing from an initial stock at the beginning of 1990.¹⁸ Of those firms, more than a third are also surveyed at least once by the EAE, and we construct measures of their software capital stock following the same procedure. For this procedure, we employ asset-specific price deflators, depreciation rates, and information on the total industry-level stocks in year 1990, for software, hardware, and the other components of capital, based on the KLEMS dataset and the series constructed by INSEE.¹⁹ To ensure that our results are not driven by the initial values of stocks, we discard the first five years of the stock data to focus on the years 1995-2007.

The EAE and BRN files are exhaustive above certain thresholds of size, but still a fraction of firms are missing the values of their investment flows in some periods. Whenever we have no information on an investment flow, because the firm either is missing, reports to the simplified regime, or is not surveyed by EAE, we impute zero investment that year. We use the imputed investment measures only in the construction of our stock measures.²⁰

Measures of IT Intensity We define a number of measures of IT intensity that we use in the next section to document our core facts about the relationship between relative IT demand and firm size. Our first measure, the *IT intensity of labor* for a firm is defined as the ratio of a measure of IT inputs, e.g., investment or stock of software, to a measure of labor inputs, e.g., the firm's number of employees. Similarly, the *IT intensity of capital* for a firm is defined as the ratio of a measure of the firm's IT investment (or stock) to the total capital investment (or stock) of the firm.²¹ Finally, we define two measures of *IT intensity of cost* corresponding to investment and stocks of IT as follows. We define the IT intensity of cost for *investment* as the ratio of investments in software or hardware, to the sum of the firm's wage bill and total investments. The latter corresponds to the accounting notion of total expenditures on production factors. We also use the more economically meaningful definition of IT intensity of cost for *stock* measures as the ratio of the payments to software or hardware divided by the total payments to all production factors, including labor, capital, and IT. Payments to capital factors are computed as the product of the user cost for each type of capital and the nominal value of the corresponding stock.²²

¹⁸Our procedure closely follows that used by Bloom et al. (2012), who construct capital stock measures based on various surveys of IT expenditure in the UK.

¹⁹We use the EU KLEMS (September 2017 release, Jäger, 2017b) to obtain depreciation rates by asset type for France. We rely on the INSEE Annual National Accounts (May 2018 release) for gross fixed capital stocks in current prices and gross fixed capital formation prices. We use these measures at the 38-industry level to construct software, hardware, and non-IT capital stocks depending on the firm's industry. Section C.6.3 in the Appendix provides the complete details of the procedure for construction of capital stock variables and presents the information on price deflators and depreciation rates used.

²⁰See more details in the discussion of our treatment of the missing data in Appendix C.6.

²¹We also compare our results with the total wage bill as an alternative definition of the labor inputs, and the investment or stock of tangibles as an alternative measure of capital inputs.

²²The user cost is the sum of the long-term interest rate on government bonds, the depreciation rate specific to each type of capital, and the expected fall in the price of that type capital, computed as the 3-year moving average of

Summary Statistics Table 4.1 presents the summary statistics of the main variables in our data. The table separately shows the summary statistics for all firms, on the left, and for manufacturing firms, on the right. We have around 15.2 million firm-year observations in the BRN + RSI files from 1990 to 2007, for which we provide standard income statistics. Of those, around 6.2 million observations refer to firms included once in the BRN files from 1995 to 2007, for which we provide statistics on hardware and other non-IT inputs, and around 2.4 million observations refer to firms surveyed at least once by the EAE from 1995 to 2007, for which we provide statistics on software inputs.

The unweighted average value of labor share (across firms) is fairly high and around 74%, similar between manufacturing and non-manufacturing, and does not show strong skewness (since mean and median values are fairly close).²³ Still, the data also suggests substantial heterogeneity in labor share across firms, with a standard error of around 43% in the entire sample. In the next section, we will explore the extent to which the variations in labor share (within industry) are driven by variations in firm size.

4.3 Facts

In this section, we discuss our newly constructed measures of IT intensity and document a number of facts on the heterogeneity in IT demand across firms. We then shift our attention to revisit a number of trends that have been recently uncovered about the evolution of industry concentration and labor share in the US and across other OECD countries (Andrews et al., 2016c; Autor et al., 2020b; Kehrig and Vincent, 2017; Berlingieri et al., 2017a), in the context of the French economy.

4.3.1 IT Intensity and Firm Scale

We begin our analysis of the IT data by investigating the cross-sectional relationship between firm size and the intensity of IT demand across firms. We apply regressions of the form

$$IT\ Intensity_{it} = FE_{Size_{it}} + FE_{kt} + FE_a + FE_c + v_{it}, \quad (4.1)$$

the investment price.

²³The online appendix reports the aggregate labor share in our data, defined as the value-added-weighted average labor share, compared to the aggregate labor share in the corporate sector in France reported by INSEE. We find an average aggregate labor share that is stable around 66% in our data, close to the macroeconomic data value of 64%. Differences are attributable to sectoral composition effects, as the macroeconomic data includes the real estate, finance, and agriculture sectors that our data does not cover. Sectoral data for the corporate sector only (excluding sole proprietorship firms) is not made public by INSEE.

where $IT\ Intensity_{it}$ denotes a measure of the relative demand for IT inputs for a firm i in an industry k at time t , $FE_{Size_{it}}$ denotes the class-size dummy corresponding to the firm- i 's number of employees at time t , and FE_{kt} stands for a flexible set of industry-time fixed effects (at the 3-digit level). In addition, we further add age FE_a and cohort FE_c dummies to control for potential patterns of IT adoption in some specifications.²⁴

Figure 4.2 shows the fixed effects of different size classes for each of our proxies of software and hardware intensity. In all cases, the fixed effects are computed relative to the fixed-effect of the size class corresponding to 0-5 workers. Starting with the software intensity measures (the left column), we find a strong relationship between different measures, both in terms of investment and stock, and the employment size of firms. For instance, a typical firm with more than 5,000 workers has a software investment (stock) intensity of labor close to 200 (300) per worker higher than firms with 0-5 workers. In terms of the IT intensity of capital, a typical firm with more than 5,000 workers has a software investment (stock) intensity 3 percentage points (0.5 percentage points) higher than firms with 0-5 workers. In terms of the IT intensity of cost, a typical firm with more than 5,000 workers has a software investment (stock) intensity 0.3 percentage points (0.2 percentage points) higher than firms with 0-5 workers.

The IT intensity *premium* of larger firms is fairly sizable considering that, as we saw with the statistics provided in Table 4.1, the average software investment (stock) intensity across all firms is around 27 (82) per worker for labor, 2% (0.4%) for capital, and 0.07% (0.06%) for cost. Large firms therefore have a software intensity gap relative to the smallest firms in our data of around 1 to 8 times the average software intensity, depending on the measure.²⁵ We find that the patterns above also broadly hold in the case of hardware. For instance, large firms have hardware investment (stock) intensities of labor that are 250 (1,000) per worker higher than small firms, or 1 to 2 times the average hardware intensity.²⁶

²⁴It is well-known that one cannot jointly identify age, cohort, and year fixed effects due to their collinearity. For this exercise, we apply one of the normalizations suggested by Deaton (2018) and attribute the growth of the dependent variable to year and cohort effects. We then use the age effect to capture fluctuations in the dependent variable that average to zero over the life of the firm. In effect, this consists of rewriting the set of age dummies FE_a as $FE_a^* = FE_a - [(a-1)FE_{a=2} - (a-2)FE_{a=1}]$ and regressing (4.1) excluding all dummies corresponding to the first year, the first cohort, and ages 1 and 2. The results do not change with or without including these cohort/year fixed effects.

²⁵Note that the results include the variations in IT intensity both along the intensive and the extensive margins. The differences in IT investment patterns as a function of firm size also emerge if we only consider the extensive margin. The online appendix shows that the extensive margin of investment grows in firm size for both software and hardware. We also find similar results using the logarithm of the intensity measures, including in Section 4.5.1 when we turn to the estimation of our model, where by construction only the intensive margin is present.

²⁶The main exception is the relative intensity of hardware investment that initially rises but then somewhat falls among the largest firms. We believe this pattern is likely to stem from the fact that our measure of hardware investment includes non-IT related office equipments. The mentioned pattern is largely driven by a group of mid-size firms that report 100% of their total investments in the "office and computing equipment" category, a likely indicator that their investment is in the office and furniture component, rather than IT. When we restrict our analysis

Table 4.6 investigates the relationship between the IT intensity and scale, using firm output (rather than employment) as a proxy for the latter. The tables report the results of regressions like those in Equation (4.1), replacing firm employment dummies with the log of firm sales or the log of value added. The regressions keep the industry/time fixed effects and controls for firm age and cohort of entry. As with the figures, Table 4.6 shows a strong relationship between the scale of operation and the IT intensity of firms. Raising the scale of firm output by a factor of 2 raises its software investment intensity of labor by around 14 per worker (against a mean of 27), of capital by 0.3 percentage points (against a mean of 2%), and cost by 0.03 percentage points (against a mean of 0.07%). The corresponding numbers for the case of hardware investment intensity of labor is 28 per worker (against a mean of 200), of capital by 1 percentage point (against a mean of 11%), and of cost by 0.05 percentage points (against a mean of 0.4%).

For the stock of IT intensity measures, the table shows the same results with the *logarithm* of the measure on the left-hand-side, therefore focusing only on the intensive margin of IT intensity. The coefficients are in the 0.2–0.4 range, suggesting that raising the scale of firm output by a factor of 2 raises its IT stock intensity by about 20% to 40%, with elasticities similar for software and hardware. Columns 3-4 and 7-8 show results for alternative measures of labor and capital inputs, either over the wage bill or tangible capital. The coefficients remain sizable, significant, and comparable with our main measures of intensity in every case.

In addition to measures of firm input and output, we further investigate the relationship between IT intensity and a number of other proxies for the firm's scale of operation. Firms can expand their scales along different margins: they can sell more of the same products to the same markets, they can sell the same products to more markets, or they can sell more products. The BRN data does not provide us with a decomposition of firm sales along these margins. Instead, we rely on customs data that allows us to gain a partial picture of these different margins in the international markets in the sample of exporting firms.

Table 4.7 presents the results of regressions of log stock intensity replacing the size fixed effects in Equation (4.1) with two other measures of the scale of international sales: the number of international markets (destination countries) and the number of exported products. In both cases, there is a positive relationship between the IT intensity of the firm and these proxies of the scale of operations of the firm. On average, exporting to a new market is associated with an increase in IT intensity of around 2% to 3% and exporting a new product with an increase of around 0.5% to 0.8%.

to the sample of 38,410 observations for which we are able to distinguish between computing equipment and non-IT office furniture equipment (firms in the agrifood industry sampled in EAE), computing investment relative to total investment or to hardware investment (computing plus office furniture) is increasing in size (see the online appendix).

As we will see in the next section, we attribute the relationship between firm scale and IT intensity to the organizational demands that stem from more complex patterns of production as firms expand their scale. We rely on DADS data to find suggestive evidence that simple measures of organizational complexity of firms indeed appear to be correlated with IT intensity. Table 4.7 also presents the results of regressions of log stock intensity on the firm's number of plants and the number of occupational layers. The latter measures are constructed from the DADS data following Caliendo et al. (2015a). On average, adding a new plant is associated with an increase in the software (hardware) intensity of firms by 0.15% (0.40%), while adding an occupational layer with an increase of more than 20% (around 10%).

A number of potential issues may complicate the interpretation of our results. First, we may be concerned that small firms face some fixed cost of adopting IT, may not perfectly report their IT investment, or may face different costs of IT compared to large firms in our data. Second, we may be concerned that small firms covered in our data are not representative of all small firms in the French economy. To address these concerns, we show that the relationship between firm size and IT intensity also appears among large firms in our data. Figure 4.2 already shows that the positive relationship between size and IT intensity is fairly consistent across different brackets of size, particularly in the case of software where the issues of selection and measurement error are less pronounced.²⁷ To address this issue more directly, Table 4.8 replicates the results of Table 4.6 on the IT intensity of cost for samples of firms in different brackets of employment size. The results for the IT intensity of software clearly demonstrates that the relationship between size and IT intensity is fairly robust across different brackets of firm size, with the coefficients between 0.2 and 0.3. Again, the results show more variability in the case of hardware data, but the coefficients are still nonmonotonic, first increasing and then decreasing.

In this section, we provided a variety of results to establish a robust positive relationship between firm size and IT intensity in the cross-section of firms. However, we can also observe this relationship *within* firm as well. The online appendix reports the within-firm elasticity of IT stock intensity to output, by replacing the controls for firm age and cohort of entry with firm fixed effects, and finds elasticities in the 0.05-0.4 range. In addition, in Section C.4 of the appendix, we provide a reduced-form identification strategy that relies on export demand shocks, in the sample of exporting firms, to show that we can attribute this relationship to the effect of size on

²⁷ As we already mentioned, the hardware data includes office furniture investments, which biases our measure of hardware intensity upward particularly for middle-size firms for which office capital may constitute the largest share of their capital stock. See footnote 26 for a discussion of how this issue appears in Figure 4.2. With regard to selection, note that the source of our hardware data, includes firms that have voluntarily chosen to file in the BRN tax regime below certain revenue thresholds. It is likely that small firms selecting to file in the BRN regime expect higher future growth, which may also be correlated with currently higher IT intensities. This would make small firms in the BRN sample unsuitable representatives for the sample of all small French firms. In contrast, the source of our software data, the EAE dataset, contains a representative sample for firms with fewer than 20 employees.

IT intensity.

4.3.2 Labor Share and Firm Scale

Let us now examine the cross-sectional relationship between firm size and labor share in the French data, applying a similar strategy as that used above for the case of IT intensity. We run fixed effects regressions similar to those in Equation (4.1), where we replace the left-hand-side variable with labor share LS_{it} , denoting the ratio of the wage bill including payroll taxes to firm value added or sales. This allows us to revisit the patterns that Autor et al. (2020b) have documented in the context of the US data.²⁸ Figure 4.3a presents the resulting fixed effects for different size classes for regressions of labor share in value added and sales, relative to firms between 10-20 workers.²⁹ We find a strong negative relationship between labor share and firm size.

4.3.3 Macro Trends in France

To conclude our facts, we examine the trends in a number of indices of market concentration in France from 1990 to 2007 on the entire BRN + RSI sample that includes all smaller firms. We compute for each 3-digit industry the share of total industry sales accounted for by the top 1%, top 5%, the largest, the top 4 largest, the top 10 largest, and the top 20 largest firms within the industry. We then average these measures across all industries, weighting industries by their share in total sales. Figure 4.4a shows that the top 1% and 5% shares increased by around 8.1 and 6.4 percentage points on average across industries, while the shares of the top 1, 4, 10, and 20 largest firms increased by 2.3 to 4.1 percentage points.³⁰

Second, we look at the evolution of the labor share in France within the same period. Figure 4.4b shows the cumulative change in the aggregate wage bill (including payroll taxes) as a share of aggregate value added for our sample of all BRN and RSI firms. In addition, it shows the contribution to this change of a within-industry (as opposed to cross-industry) component, when we keep industry shares of total value added constant from one period to the next. Over the course of the entire period, the aggregate labor share and the within-industry component have not substan-

²⁸In the case of US data, data on firm-level value added is not available outside the manufacturing sector. Here, we are able to compare the patterns for the labor share measured both relative to sales and value added.

²⁹We limit our analysis to firms with more than 10 employees because many firms with fewer employees in our data have unreasonably small values of labor share. We believe that our proxy for labor payments for these small firms is likely to be downwardly biased, due to the fact that it may not include the bulk of labor payments that goes to the firm's owner.

³⁰The initial value of the weighted averages of the top 1% and 5% share measures across industries in 1990 are 43.4% and 65.0%, respectively. The corresponding numbers for the shares of the top 1, 4, 10, and 20 largest firms range from 14.0% to 41.3% in that year.

tially fallen in our data, with the former remaining around 66% of value added. However, our decomposition also shows that starting in 1995, the typical industry began experiencing a fall in sectoral labor share.

Following the strategy used recently by [Kehrig and Vincent \(2017\)](#) for the US data, we further decompose the average within-industry changes in labor share into that stemming from, first, the shifts in the industry's distribution of firm-level labor shares (keeping shares of firm-level value added constant), and second, the within-industry reallocations in value-added shares of different quantiles of labor share.³¹ Figure 4.4b presents the results of this decomposition and shows that, while for the typical firm the labor share increased by around 3.4 percentage points over the entire period (by 4.3 percentage points until 2006), the reallocations of market shares had a negative contribution of around -3.9 percentage points to the aggregate labor share.

Figure 4.3b shows that the two patterns above are linked. The correlation between 5-year changes in industry-level labor shares and concentration measures is negative, when the former is proxied by each of the measures discussed above. In other words, the labor share has been falling relatively more in those industries where market shares have shifted more toward larger firms.

4.4 Theory

In this section, we provide a theory that rationalizes the empirical facts that we uncovered in the previous section. The core of the theory is an account of firm-level production that interprets the micro facts on the relation between IT intensity and size as evidence for the nonhomotheticity of IT factor demand. We set up the economic environment of the model in Section 4.4.1, introduce the production function in Section 4.4.2, and proceed to discuss the equilibrium of the model and the predictions regarding our micro and macro facts in Section 4.4.3.

4.4.1 Economic Environment

Consumers and Preferences The economy is populated by a unit mass of identical, infinitely-lived consumers, who in each period inelastically supply a unit of (homogenous) labor in the market and earn wage $W_{L,t}$. The consumers choose their consumption to maximize $\sum_t \varrho^t \log Y_t$, where ϱ is the discount factor and Y_t is a standard CES aggregator defined over a continuum of goods $i \in \mathcal{J}_t$ at time t :

$$Y_t = \left(\int_{i \in \mathcal{J}_t} Y_{it}^{\frac{\lambda-1}{\lambda}} di \right)^{\frac{\lambda}{\lambda-1}}. \quad (4.2)$$

³¹The online appendix provides the full details of this decomposition exercise.

Consumers own all production factors, and may additionally invest in an asset comprised of the portfolio of all firms in the economy.

Firms and Production We assume monopolistic competition. Each firm produces a unique good i using a production function that transforms four inputs: labor L_{it} , capital K_{it} , software S_{it} , and hardware H_{it} into output Y_{it} . We assume a nested structure in which the non-IT inputs, labor and capital, and the IT inputs, software and hardware, are first aggregated into bundles of non-IT and IT inputs, respectively. Accordingly, we consider the following relation between inputs and output:

$$Y_{it} = \mathcal{F}(e^{\theta_{it}} X_{N,it}, e^{\theta_{it} + \phi_{it}} X_{I,it}), \quad (4.3)$$

$$X_{N,it} = K_{it}^\alpha L_{it}^{1-\alpha}, \quad (4.4)$$

$$X_{I,it} = S_{it}^\beta H_{it}^{1-\beta}, \quad (4.5)$$

where θ_{it} and ϕ_{it} are factor-symmetric and IT-biased (log) productivity states, respectively, which are heterogeneous across firms. In addition, $X_{N,it}$ is a constant-returns-to-scale Cobb-Douglas bundle of non-IT inputs, capital and labor. Similarly, $X_{I,it}$ is a constant-returns-to-scale Cobb-Douglas bundle of the IT inputs, software and hardware.

The specification of the upper nest of the production function in Equation (4.3), i.e., function $Y = \mathcal{F}(e^\theta X_N, e^{\theta + \phi} X_I)$, lies at the heart of the model. We will present and discuss this specification in detail in Section 4.4.2 below. As we will see, our choice of production function \mathcal{F} is compatible with nonhomothetic IT demand in the form of:

$$\frac{\partial \log(X_I/X_N)}{\partial \log Y} = \eta, \quad \eta > 0, \quad (4.6)$$

which, in line with the empirical patterns we uncovered in the previous section, suggests a stable relationship between firm size and IT intensity.

Firm Exit, Entry, and Dynamics We assume that firms have to expend a fixed cost ψ in units of the bundle of non-IT inputs every period in which they operate. However, they can temporarily shut down in periods in which they are not profitable. In other words, if firms decide to endogenously exit the market, the decision to exit is *reversible*, and therefore does not involve an option value.³² In addition, we assume that in addition firms may receive an exogenous death

³²We have made this assumption merely to maintain the simplicity of our model while 1) preserving an active role for selection and 2) generating a stationary distribution of firm sales with levels of concentration that are in line with the observed data. The option value of operation corresponding to potential irreversibility of endogenous exit

shock with probability δ in each period, in which case they *irreversibly* exit the market.

Each period, potential entrants can enter the industry subject to paying sunk entry costs χ in units of the bundle of non-IT inputs. Once they enter, they draw their productivity pair $\vartheta_{it} \equiv (\theta_{it}, \phi_{it})$ from a distribution F and start operating in that very period. We assume a Markov structure on the evolution of firm-level productivity, whereby the next-period productivity of an active firm i is given by

$$\vartheta_{it} = \mu(\vartheta_{it-1}) + \mathbf{u}_{it}, \quad (4.7)$$

where $\mu(\cdot)$ is the conditional expectation function and $\mathbf{u}_{it} \equiv (u_{\theta,it}, u_{\phi,it})'$ is a vector of zero-mean and normally distributed productivity innovations.

Factor Markets As with labor, the supply of all other factors are also inelastic and exogenous. In particular, let \bar{K} , \bar{S}_t , and \bar{H}_t denote the aggregate stocks of non-IT capital, software, and hardware, respectively, where we allow the aggregate stocks of software and hardware to potentially vary over time. Correspondingly, we define the aggregate stocks of the bundle of non-IT inputs $\bar{X}_N = \bar{K}^\alpha$ and IT inputs $\bar{X}_{I,t} = \bar{S}_t^\beta \bar{H}_t^{1-\beta}$. In addition, we assume that all factors have perfect and national markets. Let $W_{L,t}$, $W_{K,t}$, $W_{S,t}$, and $W_{H,t}$ denote wages and the rental prices of non-IT capital, software, and hardware, respectively. We normalize the price of the bundle of non-IT inputs to unity and let W_t denote the prevailing price of the bundle of IT inputs at time t , that is, we assume:

$$1 \equiv \left(\frac{W_{K,t}}{\alpha} \right)^\alpha \left(\frac{W_{L,t}}{1-\alpha} \right)^{1-\alpha}, \quad W_t \equiv \left(\frac{W_{S,t}}{\beta} \right)^\beta \left(\frac{W_{H,t}}{1-\beta} \right)^{1-\beta}. \quad (4.8)$$

4.4.2 Firm-Level Production Function

Consider a function $Y = \mathcal{F}(e^\theta X_N, e^{\theta+\phi} X_I)$ in Equation (4.3) that is implicitly defined through the constraint

$$\left(\frac{e^\theta X_N}{Y^\gamma} \right)^{\frac{\sigma-1}{\sigma}} + \left(\frac{e^{\theta+\phi} X_I}{Y^{\gamma+\epsilon}} \right)^{\frac{\sigma-1}{\sigma}} = 1. \quad (4.9)$$

We assume that parameters (γ, σ) are positive valued and ϵ satisfies $\epsilon > -\gamma$ to ensure the production function is globally monotonically increasing in both inputs. This production function belongs to the class of *nonhomothetic CES (nhCES) production functions* (Sato, 1974, 1977; Hanoch, 1975; Comin et al., 2015) for reasons that will become evident shortly.³³

would not play an important role in our setting.

³³The general class of nonhomothetic CES preferences may be defined as $(X_N/\mathcal{F}_N(Y))^{1-\frac{1}{\sigma}} + (X_I/\mathcal{F}_I(Y))^{1-\frac{1}{\sigma}} = 1$ for two monotonically increasing functions $\mathcal{F}_N(\cdot)$ and $\mathcal{F}_I(\cdot)$ (see also Hanoch, 1975; Russell and Blackorby, 1981;

Since firms face perfect and frictionless factor markets, they solve the cost minimization problem corresponding to this production function to decide on the allocation of their inputs between IT and non-IT. The following lemma characterizes the cost minimizing solution for this production function.

Lemma 1. *Consider the cost minimization corresponding to the production function in Equation (4.9) where W denotes the relative price of X_I to X_N . The relative factor demand is given by*

$$\frac{W X_I}{X_N} = \left(e^{-\phi} W Y^\epsilon \right)^{1-\sigma}, \quad (4.10)$$

and the cost function (with the price of X_N as the numeraire) is given by

$$C = e^{-\theta} \mathcal{C}(Y; e^{-\phi} W) \equiv e^{-\theta} Y^\gamma \left[1 + \left(e^{-\phi} W Y^\epsilon \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (4.11)$$

The relative factor demand in Equation (4.10) systematically varies in output Y in line with Equation (C.1) with a constant elasticity $\eta \equiv (1 - \sigma)\epsilon$. The nhCES production function above nests the standard homothetic CES production for the case of $\epsilon = 0$. As with the standard CES, a nhCES production function also features a globally constant elasticity of substitution σ between IT and non-IT inputs. For any given value of the elasticity of substitution $\sigma \neq 1$, changing the parameter ϵ allows us to vary the elasticity η of relative IT demand with respect to output. For this reason, we will refer to parameter ϵ as the *nonhomotheticity parameter*.

So far, we have seen that the specification in Equation (4.9) is a flexible generalization of the standard CES production function that allows for a systematic relationship between size and factor intensities. We now turn to the important implications of nonhomotheticity of factor demand for returns to scale at the firm level. We first provide a construction for our production function that helps build some intuition about the underlying forces shaping the connection between nonhomotheticity and firm-level return to scale, in Section 4.4.2.1. We then proceed to study this connection in the case of nhCES production function in Section 4.4.2.2.

4.4.2.1 Firm-Level Production Function and Organizational Complexity

Let us provide a simple foundation for the nhCES production function in Equation (4.9). Following Lucas (1978), we distinguish between technology and organization as two distinct aspects of

Comin et al., 2015). To distinguish the more specific class defined by Equation (4.9), Sato (1977) refers to the class of preferences as *almost-homothetic* nonhomothetic CES preferences. Here, to simplify the exposition, we will follow Hanoch (1975) and Comin et al. (2015) and use the broad term nonhomothetic CES (nhCES) to refer to the particular class defined by Equation (4.9), which further imposes a constant elasticity of relative factor demand with respect to output.

firm productivity. The technology of production is simply the idea (or, blueprint) describing how to transform inputs to the desired output. However, even if firms pursue the same technology, they may still differ in the organizational efficiency with which they implement it. Accordingly, we assume that a firm is characterized by an *organizational input* O as well as its *technological efficiency* A . The output Y of the firm is given by

$$Y = [A \mathcal{X}(O, X_N)]^{\frac{1}{\gamma}}, \quad (4.12)$$

where the parameter $1/\gamma$ controls the degree of returns to scale, and where $\mathcal{X}(\cdot, \cdot)$ is a CES aggregator of the bundle of non-IT inputs X_N and the organizational input O :

$$\mathcal{X}(O, X_N) = \left(O^{\frac{\sigma-1}{\sigma}} + X_N^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \quad (4.13)$$

The key assumption is, while the technology is an exogenous constant $A \equiv e^{\theta}$, the organizational inputs of the firm endogenously varies according to:

$$O = X_I \times Y^{-\epsilon} \times e^{\phi}. \quad (4.14)$$

Equation (4.14) assumes that organizational input O depends on an intrinsic firm-level organizational efficiency e^{ϕ} . In addition, it also allows for diminishing returns to the organizational input as the output Y rises, due to potential increase in organizational complexity. Finally, it allows for the firm to raise its organizational input by adjusting its level of IT inputs X_I .

Combining Equations (4.12)-(4.14), we arrive at our specification of nhCES production functions provided in Equation (4.9). The construction above provides a specific interpretation of the nonhomotheticity of IT demand in Equation (4.10): larger firms optimally choose higher IT intensities in response to the fall in their organizational efficiency due to higher complexities of production. We will provide a more detailed discussion of the foundations of Equations (4.12)-(4.14) in Section 2.6 below.

4.4.2.2 Firm-Level Production Function and Returns to Scale

Let us now examine the returns to scale properties of the production function in Equation (4.9).³⁴ To this end, it proves helpful to define the *IT weight* of the firm as the term in Equation (4.9)

³⁴Note that we abstract away from the fixed operating cost ϕ in this section since our goal is to characterize the properties of the nhCES production function.

involving the IT inputs:

$$\Omega \equiv \left(\frac{e^{\theta+\phi} X_I}{Y^{\gamma+\epsilon}} \right)^{\frac{\sigma-1}{\sigma}} \in [0, 1]. \quad (4.15)$$

To simplify the exposition, throughout this section we consider the parametric restrictions $\sigma < 1$ and $\epsilon \geq 0$.³⁵ These restrictions correspond to the case where technological and organizational efficiency are gross complements, and where the organizational efficiency falls in the scale of output. In this case, the IT weight is *decreasing* in the level of IT inputs X_I for a given level of output Y .

The main index of returns to scale is the *scale elasticity* of the production function, defined as the sum of the output elasticity of all inputs, which captures how the output scales as we proportionally scale all inputs. First, consider a case where IT inputs are exogenous and constant at the firm-level. In this case, the scale elasticity is the output elasticity of the bundle of non-IT, which we can show is given by

$$\frac{X_N \partial Y}{Y \partial X_N} = \frac{1 - \Omega}{\gamma + \epsilon \Omega}, \quad (4.16)$$

and inversely depends on the IT weight of the firm. If we further allow for the firm to adjust its level of IT inputs, we can show that the scale elasticity of the full production function is given by

$$\frac{X_N \partial Y}{Y \partial X_N} + \frac{X_I \partial Y}{Y \partial X_I} = \frac{1}{\gamma + \epsilon \Omega}. \quad (4.17)$$

Even if we scale the IT inputs proportionally to maintain the ratio of IT to non-IT inputs, the output may grow less than proportionally with $X_N^{1/\gamma}$. This is due to the adverse effect of scale Y on organizational efficiency: we need to increase IT more than proportionally with non-IT inputs to compensate this adverse effect.³⁶

In the presence of nonhomotheticity ($\epsilon > 0$), both expressions (4.16) and (4.17) are decreasing in the IT weight Ω and are therefore increasing in the level of the IT inputs X_I for constant output Y . In other words, along an isoquant Y , as we substitute IT inputs X_I for non-IT inputs X_N , IT weight Ω falls and scale elasticity rises. Thus, *adopting IT allows firms to fight the organizational decreasing returns and to raise the output elasticity of all inputs*. The top row of Figure 4.8 compares the isoquants and the output elasticities of two production functions corresponding to two values of the nonhomotheticity parameter: homothetic, $\epsilon = 0$, and nonhomothetic CES, $\epsilon = 0.5$. In the latter case, we can see that the overall size of the vector of output elasticities

³⁵Our micro-level estimates of the two parameters σ and ϵ in Section ?? satisfy these restrictions.

³⁶In particular, in order for the output to grow proportional to $X_N^{1/\gamma}$ as we grow the bundle of non-IT inputs X_N , the bundle of IT inputs needs to grow proportional to $X_N^{1+\epsilon/\gamma}$.

$(X_N \partial Y / Y \partial X_N, X_I \partial Y / Y \partial X_I)$ grows as we move from the low-IT end of each isoquant toward its high-IT end. In contrast, in the homothetic case, the sum of the two components of the vector remains constant along (and across) isoquants. Moreover, in the nonhomothetic case, we can see that as we move along a ray that emanates from the origin, the output elasticity of IT rises relative to the output elasticity of non-IT inputs.

Examining the dual problem of cost minimization allows us to see the same forces from an alternative angle. The proof of Lemma 1 shows that, under cost minimization, the *cost share of IT*, i.e., WX_I/C , is equal to the IT weight defined in Equation (4.15). Since throughout this section we assume flexible inputs and perfectly competitive factor markets, cost minimization holds and we use the same notation Ω to also refer to the IT cost share to save on notation.³⁷ From Equation (4.10), we define a function that gives the cost share of IT as

$$\Omega(e^{-\phi} W Y^\epsilon) \equiv \frac{(e^{-\phi} W Y^\epsilon)^{1-\sigma}}{1 + (e^{-\phi} W Y^\epsilon)^{1-\sigma}}. \quad (4.18)$$

We can show that the cost elasticity \mathcal{E} , defined as the ratio of the marginal to average costs for Equation (4.11), is a linear function of the IT cost share.³⁸

$$\mathcal{E}(e^{-\phi} W Y^\epsilon) \equiv \frac{Y C'(Y; e^{-\phi} W)}{C(Y; e^{-\phi} W)} = \gamma + \epsilon \Omega(e^{-\phi} W Y^\epsilon). \quad (4.19)$$

As Equations (4.19) and (4.17) show, the cost elasticity is the reciprocal of the scale elasticity. Equation (4.18) shows that the differences in firm-level IT cost shares stem either from variations in the effective IT prices $e^{-\phi} W$ or in scales of operation Y . In particular, firms facing higher effective IT prices or firms that operate at larger scales have higher IT cost shares. Equation (4.19) then implies that, all else equal, *larger firms operate at higher levels of cost elasticity and therefore at lower levels of returns to scale*.

Under nonhomotheticity, the effect of the wider access to IT of firms, e.g., through a fall in IT prices, depends on the firms' scales of operation. The bottom row of Figure (4.8) compares the cost function and the cost elasticities for the homothetic and nonhomothetic CES production functions at two levels of the relative price of IT: high ($W = 10$) and low ($W = 0.1$). In the nonhomothetic case, the reduction in costs associated with the fall in IT prices is relatively stronger for larger firms. In addition, the figure shows how *the availability of IT lowers the cost elasticity and therefore raises the returns to scale for all firms*. In contrast, in the homothetic case, the cost

³⁷Note, however, that in the empirical section we relax these assumptions to allow for inflexibility of capital and hardware inputs. In that case, the equality between the IT weight and the IT cost share does *not* hold.

³⁸Note that, if $\gamma < 1$, Equation (4.19) implies a minimum efficient scale given by the level of output Y satisfying $\Omega = (1 - \gamma)/\epsilon$.

reductions do not depend on firm scale and IT does not impact the cost elasticity of firms (here constant at one).

The results above have important implications for the connection between IT, labor share, and industry concentration. Under the assumption of perfect factor markets, factor prices equal marginal products and cost minimization holds. With CES demand and monopolistic competition, firms charge a constant markup of $\frac{\lambda}{\lambda-1}$. This implies that the ratio of firm revenues to variable factor payments is proportional to the firm's cost elasticity:

$$\frac{R}{C} = \frac{P Y}{X_N + W X_I} = \frac{\frac{\lambda}{\lambda-1} C'(Y) \times Y}{C(Y)} = \frac{\lambda}{\lambda-1} \mathcal{E} = \frac{\lambda}{\lambda-1} (\gamma + \epsilon \Omega). \quad (4.20)$$

This simple equality allows us to translate the variations in cost elasticity to variations in the share of labor in the income of the firm. Despite the fact that all firms charge the same markups, Equation (4.20) implies that the share of all factors, including labor, in the income of firms falls with cost elasticity. First, as we saw above, firm-level cost elasticity is linear in the IT cost share, which in turn grows in firm scale due to nonhomotheticity. Second, a fall in IT prices reduces the cost elasticity, raising the share of all factors, including labor, in the income of firms.

Lastly, we also saw that the fall in IT prices disproportionately reduces the costs of larger firms that are more IT intensive, leading to a rise in industry concentration. In Section 2.6 below, we revisit the generality of the results above and contrast our framework with alternatives such as a fixed-cost model of IT adoption.

4.4.3 General Equilibrium

Having studied the properties of the nhCES production function, we now turn to the characterization of the general equilibrium of the model. Our goal is to examine the differences between two stationary equilibria of the model, characterized by an initial aggregate stock of the bundle of IT inputs \bar{X}_I and a larger final stock $\bar{X}'_I > \bar{X}_I$. Correspondingly, the relative factor price of IT falls from an initial value of W to W' , in line with Figure 4.1. For the purpose of this analysis, we study the stationary equilibria along which aggregate variables, including total output Y , price index P , mass of firms N , and the relative price of IT W are all constant. We henceforth drop the time indices to simplify the notation wherever it is clear from the context that the time dimension does not play a crucial role.

4.4.3.1 Stationary Equilibrium

Allocations Across Firms Let us first consider the problem of a firm with productivity state $\vartheta_i \equiv (\theta_i, \phi_i)$ that decides to produce along a stationary equilibrium in which the relative price of IT is W . As usual, monopolistic competition and CES aggregation imply that the firm faces the demand $Y_i = Y(P_i/P)^{-\lambda}$ where the aggregate CES price index for consumers is given by $P = \left(\int_{i \in \mathcal{I}} P_i^{1-\lambda} di\right)^{\frac{1}{1-\lambda}}$. Accordingly, the firm prices its output at constant markup $\frac{\lambda}{\lambda-1}$ over its marginal cost. From Lemma 1, we know that the variable cost of the firm satisfies $C_i = e^{-\theta_i} \mathcal{C}(Y_i; e^{-\phi_i} W)$ and Equation (4.19) allows us to write the marginal cost of the firm.

The following lemma characterizes the firm's choice of output and price.

Lemma 2. *Assume that the elasticity of IT demand with respect to output is positive, i.e., $\eta = \epsilon(1-\sigma) > 0$, that model parameters $(\gamma, \epsilon, \sigma, \lambda)$ satisfy $\gamma > 1 - 1/\lambda$, and that, if $1 < \sigma$, the additional constraint $\sigma < 1 + \frac{4}{\epsilon}(\gamma + \epsilon - 1 + \frac{1}{\lambda})(1 + \frac{\gamma}{\epsilon})$ holds. The optimal output $\tilde{Y}(\vartheta_i)$ of a firm with productivity pair $\vartheta_i \equiv (\theta_i, \phi_i)$ is the unique solution to the following problem*

$$Y_i^{\frac{1}{\zeta}} = \frac{\lambda-1}{\lambda} P Y^{\frac{1}{\lambda}} \Psi(e^{-\phi_i} W Y_i^\epsilon) e^{\theta_i}, \quad (4.21)$$

where we have defined the composite parameter $\zeta \equiv \frac{\lambda}{1+\lambda(\gamma-1)}$ and the function:

$$\Psi(e^{-\phi_i} W Y_i^\epsilon) \equiv \frac{[1 - \Omega(e^{-\phi_i} W Y_i^\epsilon)]^{\frac{1}{1-\sigma}}}{\mathcal{E}(e^{-\phi_i} W Y_i^\epsilon)}, \quad (4.22)$$

with $\Omega(\cdot)$ and $\mathcal{E}(\cdot)$ following Equations (4.18) and (4.19). The optimal output Y_i and the corresponding optimal price $\tilde{P}(\vartheta_i) = P(\tilde{Y}(\vartheta_i)/Y)^{-1/\lambda}$ defined by Equation (4.21) are monotonically increasing and decreasing, respectively, in each of the two firm productivity states θ_i and ϕ_i .³⁹

The conditions for the lemma ensure that the firm-level cost function is everywhere convex.⁴⁰ Under these conditions, the solution to the firm problem is unique and has the same intuitive characteristics as those of the standard models of monopolistic competition (Melitz, 2003). More productive firms charge lower prices and produce larger quantities of output, earn higher revenues, and hire more workers.

³⁹See the proof of this lemma and the next corollary in the online appendix.

⁴⁰Imposing the upper bound on the elasticity of substitution between IT and non-IT inputs ensures the convexity of the cost function. The online appendix characterizes the elasticity of the marginal cost function with respect to the size of output Y . It shows that the convexity of the cost function rises (falls) with the variance of the IT share Ω if $\sigma < 1$ ($1 < \sigma$). When $\sigma < 1$, the conditions $\gamma > 1$ and $\epsilon > 0$ are sufficient to ensure that the marginal cost always exceeds average costs and the cost function is globally convex. When $\sigma > 1$, the cost function may not in general remain globally convex, even if the scale elasticity parameter satisfies $\gamma > 1$.

However, Equation (4.21) also shows important differences compared to the benchmark with a single factor input and CRS production functions. First, potential deviations from CRS change the elasticity of output with respect to factor-symmetric productivity e^{θ_i} from λ to $\zeta = \frac{\lambda}{1+\lambda(\gamma-1)}$. In addition, the expression in the numerator of function $\Psi(\cdot)$ captures variations in average cost as a function of IT prices and shows that, for given level of θ_i , output declines with the IT cost share.⁴¹ Finally, the term in the denominator of function $\Psi(\cdot)$ captures the fact that marginal cost grows relative to average cost as the IT cost share rises, leading to a further decline in output.

Given a tuple of aggregate variables (P, Y, W, N) , Equation (4.21) determines the allocations of output $\tilde{Y}(\vartheta)$ and price $\tilde{P}(\vartheta)$. We can then define functions that characterize the cost share and cost elasticity of a firm with productivity pair ϑ_i as $\Omega_i = \tilde{\Omega}(\vartheta_i) \equiv \Omega(e^{-\phi_i} W \tilde{Y}(\vartheta_i)^\epsilon)$ and $\mathcal{E}_i = \tilde{\mathcal{E}}(\vartheta_i) \equiv \mathcal{E}(e^{-\phi_i} W \tilde{Y}(\vartheta_i)^\epsilon)$. Similarly, we can define a function $\tilde{\Psi}(\vartheta)$ from Equation (4.22) as well as revenue $\tilde{R}(\vartheta) \equiv \tilde{P}(\vartheta) \tilde{Y}(\vartheta)$ and variable cost function $\tilde{C}(\vartheta)$ from Equation (4.11). With these allocations at hand, it is straightforward to characterize the value function of firms, the set \mathcal{J} of productivity states of active firms $\mathcal{J} \equiv \{\vartheta \mid \tilde{R}(\vartheta) - \tilde{C}(\vartheta) \geq \psi\}$, and the pdf $g(\vartheta)$ of the stationary distribution of productivities among active firms. To close the model, we need to clear all markets and apply a free entry condition to pin down the aggregate variables (P, Y, W, N) . The steps involved are standard and we relegate the discussion to the online appendix. The following corollary of Lemma 2 characterizes the distribution of revenues and (variable) factor payments across firms. Let $\bar{R} \equiv PY/N$ and \bar{C} stand for mean revenue and mean variable factor payment across firms, and define the aggregate productivity \bar{Z} as

$$\bar{Z} \equiv \left[\int \int_{\mathcal{J}} (\tilde{\Psi}(\vartheta) e^{\theta})^{\gamma\zeta-1} g(\vartheta) d^2\vartheta \right]^{\frac{1}{\gamma\zeta-1}}. \quad (4.23)$$

Then, the distribution densities of revenue and (variable) factor payments $\Lambda_R(\vartheta) \equiv \tilde{R}(\vartheta) g(\vartheta) / \bar{R}$ and $\Lambda_C(\vartheta) \equiv \tilde{C}(\vartheta) g(\vartheta) / \bar{C}$ across firms satisfy

$$\Lambda_R(\vartheta) = \left(\frac{\tilde{\Psi}(\vartheta) e^{\theta}}{\bar{Z}} \right)^{\gamma\zeta-1} g(\vartheta), \quad \Lambda_C(\vartheta) = \frac{\bar{\mathcal{E}}}{\tilde{\mathcal{E}}(\vartheta)} \Lambda_R(\vartheta), \quad (4.24)$$

where the aggregate cost elasticity $\bar{\mathcal{E}}$ is defined as

$$\bar{\mathcal{E}} \equiv \int \int_{\mathcal{J}} \tilde{\mathcal{E}}(\vartheta) \Lambda_C(\vartheta) d^2\vartheta = \left[\int \int_{\mathcal{J}} \tilde{\mathcal{E}}(\vartheta)^{-1} \Lambda_R(\vartheta) d^2\vartheta \right]^{-1}. \quad (4.25)$$

⁴¹Note that from Equations (4.11) and (4.18), we have $\mathcal{C}(Y; e^{-\phi} W) = Y^\gamma [1 - \Omega(e^{-\phi} W Y^\epsilon)]^{\frac{1}{\sigma-1}}$.

Moreover, the ratio of revenues to (variable) factor payments satisfies $\frac{\bar{R}}{\bar{C}} = \frac{\lambda}{\lambda-1} \bar{\mathcal{E}}$. The corollary shows that the elasticity of market shares with respect to factor symmetric productivity e^θ , for a constant level of IT cost share $\tilde{\Omega}(\vartheta)$, is given by $\gamma\zeta - 1 \equiv \frac{\lambda-1}{1+\lambda(\gamma-1)}$. More importantly, in line with Equation (4.20), we find that the wedge between the share of a firm in aggregate revenues and its share in aggregate (variable) factor payments is proportional to the cost elasticity $\tilde{\mathcal{E}}(\vartheta)$. We also find that the wedge between aggregate revenues PY and aggregate variable factor payments $N\bar{C}$ is the *aggregate cost elasticity*, i.e., the factor-payment-weighted mean of cost elasticities across firms. Finally, Equation (4.19) implies that the aggregate cost elasticity is linear *aggregate IT cost share* $\bar{\Omega}$, i.e., the factor-payment-weighted mean IT cost share $\bar{\Omega}$:

$$\bar{\mathcal{E}} = \gamma + \epsilon \bar{\Omega} \equiv \gamma + \epsilon \int \int_{\mathcal{J}} \tilde{\Omega}(\vartheta) \Lambda_C(\vartheta) d^2 \vartheta. \quad (4.26)$$

Micro Predictions: Cross Sectional Relationships Let us examine the cross-sectional relationships between firm size, IT intensity, and labor share under the stationary distribution g . First, consider the regression coefficient of log ratio of IT to (variable) non-IT inputs on log firm size:

$$\frac{Cov\left(\log\left(\frac{\tilde{\Omega}(\vartheta)}{1-\tilde{\Omega}(\vartheta)}\right), \log \tilde{Y}(\vartheta)\right)}{Var\left(\log \tilde{Y}(\vartheta)\right)} = (1-\sigma)\epsilon - (1-\sigma) \frac{Cov\left(\phi, \log \tilde{Y}(\vartheta)\right)}{Var\left(\log \tilde{Y}(\vartheta)\right)}, \quad (4.27)$$

where the covariances and the variance are defined under the distribution g . This expression shows that the positive correlation between IT intensity and firm size that we documented in Section 4.3 can be driven by two potential mechanisms. The nonhomotheticity in IT factor demand under the condition $\eta = (1-\sigma)\epsilon > 0$, or alternatively, a potentially negative correlation between IT biased productivity ϕ_i and size $\log Y_i$ may both generate this relationship. In Section 4.5.1, we develop a micro-level estimation strategy to separate these two potential channels and identify the parameters ϵ and σ .

Next, consider the relationship between labor share and firm size. The labor share of the firm satisfies

$$LS_i \equiv \frac{W_{L,i} L_i}{P_i Y_i} = (1-\alpha) \frac{\lambda-1}{\lambda} \frac{1-\Omega_i + \frac{\psi}{\Omega_i}}{\gamma + \epsilon \Omega_i}, \quad (4.28)$$

in which, just like Equation (4.16), the cost elasticity $\mathcal{E}_i = \gamma + \epsilon \Omega_i$ appears in the denominator. Then, a regression of labor share on log size yields the coefficient

$$Cov\left(\tilde{LS}(\vartheta), \log \tilde{Y}(\vartheta)\right) \approx -\frac{1-\alpha}{\gamma} \frac{1-\lambda}{\lambda} \left(1 + \frac{\epsilon}{\gamma}\right) Cov\left(\tilde{\Omega}(\vartheta), \log \tilde{Y}(\vartheta)\right),$$

where we have again abstracted away from the fixed costs ψ and have additionally used the approximation $\tilde{\Omega}(\vartheta) \ll 1$. Given the positive correlation between IT share and size in Equation (4.27), the expression above predicts a negative relationship between labor share and firm size in the cross section. A larger share of IT in income implies a factor substitution away from labor and a lower labor share of income. In addition, nonhomotheticity of factor demand in the case of $\epsilon > 0$ implies that the relationship between labor share and firm size is going to be stronger compared to the homothetic case by a factor of ϵ/γ .

4.4.3.2 Aggregation

The stationary general equilibrium of the model defines the aggregate output Y of the economy as a function of the aggregate stocks \bar{X}_I of IT and \bar{X}_N of non-IT inputs. We can show that the aggregate output Y and price P satisfy

$$Y^\gamma = N^{\frac{1}{\gamma\bar{\epsilon}-1}} \left(\frac{\gamma + \epsilon\bar{\Omega}}{1 - \bar{\Omega} + \psi/\bar{C}} \right) \bar{Z} \bar{X}_N^{prod}, \quad P = \frac{\lambda}{\lambda-1} Y^{\gamma-1} N^{-\frac{1}{\gamma\bar{\epsilon}-1}} \bar{Z}^{-1}, \quad (4.29)$$

where \bar{X}_N^{prod} denotes the total non-IT inputs deployed in the production sector, and aggregate productivity \bar{Z} , aggregate IT cost share $\bar{\Omega}$, and mean variable factor payment \bar{C} are defined as in Corollary 4.4.3.1.

We can show that the aggregate profit to cost ratio in the production sector is given by $\frac{1}{1+\psi/\bar{C}} \frac{\lambda}{\lambda-1} \bar{\mathcal{E}} - 1$, and that the labor share in the production sector satisfies:⁴²

$$LS^{prod} = (1 - \alpha) \frac{\lambda - 1}{\lambda} \frac{1 - \bar{\Omega} + \frac{\psi}{\bar{C}}}{\gamma + \epsilon\bar{\Omega}}. \quad (4.30)$$

Equation (4.30) is the aggregate parallel to Equation (4.28), and allows us to draw intuitions about the drivers of aggregate profit and labor shares. As we saw in Corollary 4.4.3.1, the profit share depends on the aggregate cost elasticity $\bar{\mathcal{E}} = \gamma + \epsilon\bar{\Omega}$. When IT cost share and aggregate cost elasticity rise, returns to scale fall and therefore the ratio of revenues to factor payments and the profit share also rise. As we can see in Equation (4.30), this further leads to a fall in the aggregate labor share. In addition, the labor share negatively depends on aggregate IT intensity $\bar{\Omega}$ since a shift of income payments toward IT reduces the income going to the bundle of non-IT inputs.

⁴²We focus on the labor share in the production sector since it is the natural parallel to what we observe in the data. Alternatively, we can assume that the costs of entry are paid in units of a final good, which is produced by competitive firms according to the aggregator in Equation (4.2) and is used both for final good consumption and for the costs of entry. In this case, the labor share in the production sector corresponds to the aggregate labor share in the model.

Since we assumed fixed operation costs are paid in labor and capital, the expressions above also depend on the ratio of fixed to average variable costs ψ/\bar{C} , which we expect to be negligible in typical calibrations of the model.⁴³

Response to the Fall in IT Prices and the Aggregate Elasticity of Substitution Next, we examine the comparative statics of factor income shares with respect to a change in the relative IT price W . Let $d\omega \equiv d \log W$ and consider the comparative statics of the aggregate IT cost share $d\bar{\Omega}/d\omega$. From Equation (4.30), we can write the response of the labor share of income in the production sector to a small IT shock as

$$\frac{1}{1-\alpha} \frac{\lambda}{\lambda-1} \frac{d \log LS^{prod}}{d\omega} \approx -\frac{\gamma + \epsilon}{(\gamma + \epsilon \bar{\Omega})^2} \frac{d\bar{\Omega}}{d\omega} \approx -\frac{1}{\gamma} \left(1 + \frac{\epsilon}{\gamma}\right) \frac{d\bar{\Omega}}{d\omega}, \quad (4.31)$$

where the first and second approximations follow the assumptions that fixed costs are small relative to the average variable costs ($\psi/\bar{C} \ll 1$), and that the aggregate IT cost share is small ($\bar{\Omega} \ll 1$), respectively. As with the benchmark model with homothetic CES production functions ($\epsilon = 0$), the substitution between IT and non-IT inputs implies that the labor share moves in the opposite direction of the response of aggregate IT cost share $d\bar{\Omega}/d\omega$. However, the presence of nonhomotheticity ($\epsilon > 0$) introduces the *endogenous response of returns to scale*, which creates an additional shift in the share of all factor payments in aggregate income. This channel is captured through the response in the aggregate cost elasticity $\bar{\epsilon} \equiv \gamma + \epsilon \bar{\Omega}$ in the denominator of Equation (4.30) and the additional term ϵ/γ in Equation (4.31). For a given response in the IT cost share $d\bar{\Omega}/d\omega$, our model predicts a larger fall in the aggregate labor share in the production sector.

In addition to the relation between aggregate labor and aggregate IT cost shares in Equation (4.31), nonhomotheticity also matters for the response of the aggregate IT cost share, $d\bar{\Omega}/d\omega$. We note that this response is also tied to the *aggregate elasticity of substitution* (in our case, between IT and variable non-IT factors) in production. Defining the aggregate elasticity as $\bar{\sigma} \equiv d \log \left(\frac{1-\bar{\Omega}}{\bar{\Omega}} \right) / d\omega$, we find:

$$\frac{d\bar{\Omega}}{d\omega} = \bar{\Omega} (1 - \bar{\Omega}) (1 - \bar{\sigma}). \quad (4.32)$$

Equations (4.32) and (4.31) together capture the standard result that whether or not the aggregate elasticity $\bar{\sigma}$ exceeds unity is important for the distribution of income across sectors. If it exceeds unity, then $d\bar{\Omega}/d\omega < 0$, and a fall in the price of IT lowers the share of labor in factor payments (e.g., Karabarbounis and Neiman, 2014; Eden and Gaggl, 2018).

⁴³We can make the alternative assumption that the aggregator in Equation (4.2) corresponds to a final good producer, the output of which used both for final good consumption and for the fixed operation costs. Under such a model, the term ψ/\bar{C} drops out of Equation (4.30).

As is well-known, and also emphasized by several recent papers (e.g., [Oberfield and Raval, 2014b](#); [Baqee and Farhi, 2018](#)), the heterogeneity in factor intensities across firms creates a gap between the micro and macro elasticities of substitution. A change in factor prices, not only induces each firm to adjust its factor intensity, but also reallocates factors toward firms that are more or less intensive in that factor. In the remainder of this section, we discuss how both these channels are affected by the presence of nonhomotheticity in factor demand. Later in [Section 4.5.2](#), we will see that these two channels in our model, when calibrated to the French economy, together imply a macro elasticity of substitution $\bar{\sigma}$ that is around unity. This number is compatible with the stability of the aggregate labor share in France ([Figure 4.4b](#)) and much larger than our estimated micro elasticity σ . However, the discussion below should make it apparent that the gap between micro and macro elasticities is endogenous and highly dependent on the characteristics of the economic environment (in this case the French economy).

As already mentioned, we can decompose the response of the IT cost share into within-firm and across-firm effects as

$$\frac{d\bar{\Omega}}{dw} = \underbrace{\int \int \frac{d\tilde{\Omega}(\vartheta)}{dw} \Lambda_C(\vartheta) d^2\vartheta}_{\text{Within-firm Effect}} + \underbrace{\int \int \tilde{\Omega}(\vartheta) \frac{d\Lambda_C(\vartheta)}{dw} d^2\vartheta}_{\text{Across-firm Effect}}, \quad (4.33)$$

where $\Lambda_C(\vartheta)$ is the density of factor payment shares defined by [Equation \(4.24\)](#). Below we present a simplified partial equilibrium analysis that illustrates the main effects of nonhomotheticity on the response of the aggregate IT cost share $d\bar{\Omega}/dw$.

Let us first examine the within-firm response of the IT cost share $\tilde{\Omega}(\vartheta)$. The results of [Appendix C.1.3](#) show that:

$$\left. \frac{\partial \tilde{\Omega}(\vartheta)}{\partial w} \right|_{(P,Y)\text{const.}} = \tilde{\Omega}(\vartheta) (1 - \tilde{\Omega}(\vartheta)) \left[1 - \underbrace{\left(\sigma - (1 - \sigma) \epsilon \left. \frac{\partial \tilde{Y}(\vartheta)}{\partial w} \right|_{(P,Y)\text{const.}} \right)}_{\equiv \tilde{\sigma}(\vartheta)} \right], \quad (4.34)$$

where we have defined an *effective* firm-level elasticity of substitution $\tilde{\sigma}(\vartheta)$, relying on the parallel between [Equations \(4.34\) and \(4.32\)](#).⁴⁴ In addition, we have defined the elasticity of output with respect to the relative price of IT, i.e., $\partial \tilde{Y}(\vartheta)/\partial w \equiv \partial \log \tilde{Y}(\vartheta)/\partial w$. If the fall in IT prices raises the output of a firm, i.e., $\partial \tilde{Y}(\vartheta)/\partial w < 0$, [Equation \(4.34\)](#) implies that nonhomotheticity ($\epsilon > 0$) leads to a higher effective elasticity of substitution in shaping the within-firm effect in

⁴⁴We emphasize that the standard definition of the elasticity of substitution focuses on substitution patterns along a given isoquant. In this sense, as the name nhCES suggests, the elasticity of substitution here remains constant and equal to σ for all firms. The partial derivatives in the definition in [Equation \(4.34\)](#) additionally include the response of the firm output, keeping constant all general equilibrium variables except the relative price of IT.

Equation (4.33). Moving to the across-firm effect, the partial equilibrium response of the density of factor payments $\Lambda_C(\vartheta)$ is also directly tied to the elasticity of firm output through:

$$\left. \frac{\partial \Lambda_C(\vartheta)}{\partial w} \right|_{(P,Y,g)\text{const.}} \approx \frac{\lambda-1}{\lambda} \Lambda_C(\vartheta) \left. \frac{\partial \tilde{y}(\vartheta)}{\partial w} \right|_{(P,Y)\text{const.}}, \quad (4.35)$$

where we have assumed that the IT cost share is small, $\tilde{\Omega}(\vartheta) \ll 1$. Therefore, the effect of nonhomotheticity on the response of output, $\partial \tilde{y}(\vartheta)/\partial w$, determines both the within-firm response (in Equation 4.34) and the across-firm response (in Equation 4.35).

Let us now examine the variations across firms in the elasticity of output with respect to IT prices. To the first order of approximation in IT cost share $\tilde{\Omega}(\vartheta)$, the elasticity is given by:⁴⁵

$$\left. \frac{\partial \tilde{y}(\vartheta)}{\partial w} \right|_{(P,Y)\text{const.}} \approx -\zeta \left(1 + \frac{(1-\sigma)\epsilon}{\gamma} \right) \tilde{\Omega}(\vartheta). \quad (4.36)$$

A fall in the relative price of IT lowers the average cost of each firm proportionally to its IT cost share $\tilde{\Omega}(\vartheta)$, which in turn raises the output with an adjusted demand elasticity $\zeta = \frac{\lambda}{1+\lambda(\gamma-1)}$. But this shock also raises the returns to scale, lowering the cost elasticity and the gap between marginal and average costs. Therefore, the pass-through of a fall in IT prices to the marginal cost of firms is higher due to the effect on their returns to scale. Equation (4.36) shows that this channel raises the elasticity of firm output to IT prices compared to the homothetic case by a factor of $(1-\sigma)\epsilon/\gamma$. Finally, the nonhomotheticity of IT demand and the resulting cross-sectional correlation between size and IT cost share shown in Equation (4.27), in combination with Equation (4.36), predict that the response of output is greater for larger firms. Therefore, the IT shock reallocates market shares toward larger firms and raises market concentration.

The discussion above provided a partial equilibrium analysis that aimed to illustrate the different channels through which the presence of nonhomotheticity of IT demand modifies the aggregate response. To gauge the precise magnitude of this response, however, we need to perform a calibration exercise. We will turn to this question in Section 4.5.2, after estimating the parameters of the production function.

⁴⁵See Appendix C.1.3 for the full expression.

4.5 Estimation and Calibration

In this section, we bring the model developed in Section 4.4 to data and use it to explain the evolution of market concentration and labor share in response to the rise of IT. We first identify the production function of Section 4.4.2 in the micro data and provide evidence for the presence of nonhomotheticity. We then use the resulting estimates to calibrate the general equilibrium model of Section 4.4.3 show that the model can fit the broad patterns of changes in the labor share documented in Section 4.3.

4.5.1 Estimation

In this section, we identify the parameters of the production function of Section 4.4.2 in the micro data, and provide structural evidence for the nonhomotheticity of the IT demand.

Let us set up an empirical counterpart to the economic environment that we defined in the previous section. As before, we define the production function as a function of labor, capital, software, and hardware as $Y_{it} = \mathcal{F}(e^{\theta_{it}} K_{it}^\alpha L_{it}^{1-\alpha}, e^{\theta_{it} + \phi_{it}} S_{it}^\beta H_{it}^{1-\beta})$, where function $\mathcal{F}(\cdot, \cdot)$ is the nonhomothetic CES function defined as before. Aside from the productivity terms, the production function is fully characterized with the tuple of parameters $\varsigma \equiv (\alpha, \beta, \gamma, \epsilon, \sigma)$. Henceforth, we will use the small cap letters to denote the logarithm of each corresponding variable, e.g., $y_{it} \equiv \log Y_{it}$.

We follow the literature on production function estimation and impose constraints on the timing and nature of firm decision making in order to identify the production function. We define the information set of the firm in each period t as follows.

Assumption 3. Let \mathcal{I}_t denote the information set of the firm at time t , which includes the paths of all observables up to time t . We assume that $\theta_{it}, \phi_{it} \in \mathcal{I}_{it}$ and all choices of the firm at time t , including capital, may depend on its information set \mathcal{I}_t .⁴⁶

Next, recall that we imposed a Markov structure on the evolution of productivities in Equation (??). We slightly generalize that process by allowing for a time-trend in the long-run averages of each of the two productivities, and define the vector of productivity state innovations as

$$\mathbf{u}_{it} \equiv \begin{pmatrix} u_{\theta, it} \\ u_{\phi, it} \end{pmatrix} = \begin{pmatrix} \theta_{it} \\ \phi_{it} \end{pmatrix} - \begin{pmatrix} \rho_{\theta\theta} & \rho_{\theta\phi} \\ \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \begin{pmatrix} \theta_{it-1} \\ \phi_{it-1} \end{pmatrix} - \begin{pmatrix} \eta_\theta + \mu_\theta t \\ \eta_\phi + \mu_\phi t \end{pmatrix}. \quad (4.37)$$

⁴⁶Note that based on the accumulation equation that we have used to construct the stocks of capital (Equation C.41 in the appendix), the firm's choices at time t include its investment choices within that period, which is in turn within the information set of the firm at time t .

We further allow for these innovations to have a general distribution but maintain that they are orthogonal to the information set of the firm at time t . Following the literature on dynamic panel data methods, this structure allows us to use the lagged choices of firms as instruments for current inputs, subject to controlling for current productivities. The next assumption summarizes our assumptions about the evolution of productivity states and will be crucial for deriving our moment conditions.

Assumption 4. *The evolution of the productivity states satisfies Markov structure in Equations (4.37) where $\mathbb{E}[\mathbf{u}_{it}|\mathcal{I}_{t-1}] = (0, 0)'$.*

Next, we characterize the potential frictions in firm decision making. Here, we relax our assumptions in the previous section on the frictionless nature of all factor input markets. In particular, we will allow for potential adjustment costs or other frictions in non-IT capital and hardware, but continue to rule out such adjustment costs for labor and software inputs. The resulting setup, summarized by Assumption 5 below, allows us to provide some generality to our account of input market frictions while maintaining one static first order condition in each of the two bundles of IT and non-IT inputs. This will allow us to derive one moment condition based on static cost minimization.

Assumption 5. *Labor and software inputs are flexible and costlessly adjustable.*

We rely on Assumptions 4 and 5 to derive moment conditions that allow us to identify the production function in the presence of two sources of unobserved heterogeneity. Our strategy combines two sets of moment conditions. The first set of moments, which come from the functional form of the production function, corresponds to the assumptions made on the factor-symmetric unobserved productivity state θ_{it} and resembles the moment conditions used in standard production function estimation. The second set comes from cost minimization and corresponds to the assumptions made on the IT-biased productivity. The strategy closely parallels those employed recently by Oberfield and Raval (2014b) and Doraszelski and Jaumandreu (2018) for estimating a homothetic CES production function, which is nested in our functional form if we restrict the model to $\epsilon = 0$.

We next derive expressions that allow us to characterize the two unobserved productivity terms, IT-biased and factor-symmetric, as functions of observed data and the parameters of the production function. Setting up the dynamic program of the firm, we use Assumption 5 to derive the following result.

Lemma 3. *Under Assumption 5 and under general forms of adjustment costs for non-IT capital and*

hardware, the firm's choices of inputs in each period satisfies

$$s_{it} - l_{it} = -\sigma \widehat{w}_{it} + (\sigma - 1)[\phi_{it} - \epsilon y_{it} - \alpha(k_{it} - l_{it}) + (1 - \beta)(h_{it} - s_{it})], \quad (4.38)$$

where we have defined $\widehat{w}_{it} \equiv w_{S,t} - w_{Li,t}$ as the (firm-specific) relative price of software.

Here, we allow the relative price of software to in general vary at the firm level. In practice, depending on the location of the headquarters of a given firm i , we will use a measure of average wage at the corresponding local employment area to construct a firm-specific wage $w_{Li,t}$.⁴⁷ The lemma generalizes the static cost minimization result we used in the previous section to accommodate potential adjustment costs for non-IT capital and hardware (see the proof in Appendix C.2.1 for details). Note that if these two inputs are flexible, then the Cobb-Douglas aggregation of the two bundles of IT and non-IT pins down the (log) factor input ratios $k_{it} - l_{it}$ and $h_{it} - s_{it}$ to be constant in the two parameters α and β , respectively. In this case, we recover the standard cost minimization that we employed in the previous section.

In the presence of adjustment costs, the (log) capital to labor ratio $k_{it} - l_{it}$ and hardware to software ratio $h_{it} - s_{it}$ both vary over time and across firms (even within an industry). Therefore, the optimal choices of software to labor ratio $s_{it} - l_{it}$ depend, not only on relative prices $w_{L,t} - w_{S,t}$, output y_{it} , and IT-biased productivity ϕ_{it} , but also on the capital-to-labor and hardware-to-software ratios.⁴⁸

This result allows us to derive a key moment condition that we will use to identify our core parameter ϵ . Let $\mathbf{D}_{it} \equiv (\widehat{w}_{it}, l_{it}, k_{it}, s_{it}, h_{it}, y_{it})$ denote the vector of all relevant observations for firm i at time t . Accordingly, we define a function $\Phi(\cdot; \cdot)$ of observables and model parameters as

$$\Phi(\mathbf{D}_{it}; \varsigma) = \frac{1}{\sigma - 1} (\sigma \widehat{w}_{it} + s_{it} - l_{it}) + \epsilon y_{it} + \alpha(k_{it} - l_{it}) - (1 - \beta)(h_{it} - s_{it}). \quad (4.39)$$

Comparing Equation (4.39) and the expression in Equation (4.38) shows that function Φ indeed equates with the value of IT-biased productivity ϕ_{it} , conditional on the observed data for a given set of model parameters.

To derive the expression for factor symmetric productivity, we rewrite the production function in terms of the logarithms of firm inputs and outputs as follows

⁴⁷ As explained below, we will rely on this regional variation in wages as part of our strategy for the identification of σ . See Section C.6.4 in the Appendix for further details on the construction of our measures of local wages.

⁴⁸ Note that this result also poses a challenge for alternative reduced-form identification strategies relying solely on demand shocks as potential instruments for output y_{it} . Since output y_{it} may have an impact on capital-to-labor and hardware-to-software ratios, due potentially to adjustment costs or financial constraints, Equation (4.38) shows that we additionally need instruments for the latter two ratios to identify ϵ .

$$y_{it} = \frac{1}{\gamma} \left\{ \frac{\sigma-1}{\sigma} \log \left[e^{\frac{\sigma-1}{\sigma}(\alpha k_{it} + (1-\alpha)l_{it})} + e^{\frac{\sigma-1}{\sigma}(\beta s_{it} + (1-\beta)h_{it} + \phi_{it} - \epsilon y_{it})} \right] + \theta_{it} \right\}.$$

This implies that we can define a function $\Theta(\cdot; \cdot)$ of observables and model parameters

$$\Theta(D_{it}; \varsigma) \equiv \gamma y_{it} + \frac{\sigma}{1-\sigma} \log \left[e^{\frac{\sigma-1}{\sigma}(\alpha k_{it} + (1-\alpha)l_{it})} + e^{\frac{\sigma-1}{\sigma}(\beta s_{it} + (1-\beta)h_{it} - \epsilon y_{it} + \Phi(\widehat{w}_{it}, l_{it}, k_{it}, s_{it}, h_{it}, y_{it}; \varsigma))} \right], \quad (4.40)$$

that equates with the value of factor-symmetric productivity θ_{it} .

Having expressed the unobserved productivity states (θ_{it}, ϕ_{it}) in terms of observed data and model parameters as (Θ, Φ) , we can now substitute them in Equation (4.37) and use Assumption 5 to derive the following moment conditions:

$$\mathbb{E} \left[\begin{pmatrix} \Theta(D_{it}; \varsigma) \\ \Phi(D_{it}; \varsigma) \end{pmatrix} - \begin{pmatrix} \rho_{\theta\theta} & \rho_{\theta\phi} \\ \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Theta(D_{it-1}; \varsigma) \\ \Phi(D_{it-1}; \varsigma) \end{pmatrix} - \begin{pmatrix} \eta_{\theta} + \mu_{\theta}t \\ \eta_{\phi} + \mu_{\phi}t \end{pmatrix} \middle| \mathcal{I}_{t-1} \right] = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (4.41)$$

These moment conditions can in principle identify all of the parameters of the production function. However, the identification of the elasticity of substitution σ in this case does not rely on the variations in the relative price of software \widehat{w}_{it} as we do not have any corresponding instrument.⁴⁹ Therefore, we amend our strategy with additional instruments for the relative price of software, as we will describe below.

We include a final moment condition in our estimation to instrument for variations in relative price of software \widehat{w}_{it} at the regional level. For this purpose, we borrow from Oberfeld and Raval (2014b) and construct shift-share instruments z_{it} for local wages for each firm, using the initial local industrial composition and the change in each industry's wage bill at the national level. These instruments are correlated with current and past relative wages $w_{L,it}$ and $w_{L,it-1}$ and hence with relative prices of software \widehat{w}_{it} and their lagged values \widehat{w}_{it-1} . We integrate these instruments in the same estimation framework by writing the following moment condition

$$\mathbb{E} \left[\Phi(D_{it}; \varsigma) - \begin{pmatrix} \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Theta(D_{it-1}; \varsigma) \\ \Phi(D_{it-1}; \varsigma) \end{pmatrix} - (\eta_{\phi} + \mu_{\phi}t) \middle| z_{it} \right] = 0, \quad (4.42)$$

assuming that the shift-share instruments are orthogonal to innovations in contemporaneous IT-

⁴⁹ Instead, the identification of σ based on Equation (4.38) would merely rely on variations in software-to-hardware $s_{it} - h_{it}$ and capital-to-labor ratios $k_{it} - l_{it}$, instrumented by their corresponding lagged values. Variations in these ratios across firms are driven by likely variations in distortions to the optimal choices of relative software-to-hardware and capital-to-labor ratios due to adjustment costs.

biased productivity. This is a weaker assumption than the typical assumption in the literature, which posits that the instruments are orthogonal to contemporaneous productivity rather than just productivity innovations (see, e.g., [Oberfield and Raval, 2014b](#)).

We can use these system of moment conditions (4.41) and (4.42) to estimate the production function in a nonlinear 2SLS framework, using as instruments the shift-share instruments, the lagged values of labor, capital, software, hardware, as well as lagged productivities $\Theta(D_{it-1}; \varsigma)$ and $\Phi(D_{it-1}; \varsigma)$.

Estimation Results

Table 4.9 presents the estimated parameters of the production function for the sample of all firms and for the sample of manufacturing firms. For comparison, it also presents the estimated values of parameters under two standard production functions nested in our model: a homothetic CES production function, when we restrict the nonhomotheticity parameter to $\epsilon = 0$, and a Cobb-Douglas production function, when we additionally assume $\sigma = 1$. In the second case, the cost minimization equation does not deliver any information for the estimation and we drop it from the framework, leading to a standard dynamic-panel estimation of the Cobb-Douglas production function with four inputs: non-IT capital, labor, software, and hardware.

Focusing first on our core parameter of interest ϵ , we find precise, significant, and sizable positive estimates for this parameter in both samples ($\epsilon = 0.39$ for all industries and $\epsilon = 0.48$ in manufacturing). These estimated values reject the homotheticity of the production function. Importantly, the estimated elasticities of substitution in both samples are below unity ($\sigma = 0.23$ for all industries and $\sigma = 0.17$ in manufacturing).⁵⁰ The combination of these two parameters indeed satisfy the condition proposed in Assumption ??, suggesting that the positive correlations between IT intensity and size that we uncovered are at least partially explained by the presence of nonhomotheticity. Importantly, the estimated elasticities of substitution also imply gross complementarity between IT and non-IT inputs. Despite the fact that we reject the homothetic CES, we still find that the estimated elasticity of substitution under a CES production function is also below 1, although it appears biased downward in both samples.

We find similar estimates for the cost elasticity parameter γ in both samples ($\gamma \approx 0.95$) that imply increasing returns to scale for the firms as their size goes to zero. Importantly, the implication

⁵⁰[Oberfield and Raval \(2014b\)](#) and [Doraszelski and Jaumandreu \(2018\)](#) estimate the micro-level elasticity of substitution between capital and labor using identification strategies that account for potential factor-augmenting productivity shocks, and find values between 0.4 and 0.7. An earlier set of macroeconomic estimates find values slightly higher but still below unity (e.g., [Antràs, 2004](#); [Klump et al., 2007](#); [Chirinko, 2008](#)). We also note that [Karabarbounis and Neiman \(2014\)](#) use an estimation strategy that relies on the cross-sectional variations in the industry-level data that finds a value above 1.

of these estimates for our production function is a systematic relationship between firm size and returns to scale. Recall that cost elasticity is the reciprocal of the scale elasticity. Therefore, the combination of point estimates for γ and ϵ in the sample of all industries imply scale elasticities that range from 0.75 to 1.06 as we move from the largest to the smallest firms (the corresponding values are 0.70 and 1.05 for the sample of manufacturing firms). In contrast, under both CES and Cobb-Douglas production functions, the scale elasticity is constant across all firms. As we would expect, the estimated values of the cost elasticity parameter γ under these restricted models imply scale elasticities within the same range but fairly close to constant return to scale (e.g., in the sample of all industries, 1.01 under CES and 1.00 under Cobb-Douglas). These numbers are in line with most prior micro estimates of the returns to scale, but suggest a potential heterogeneity bias in those standard estimates.

Moving on to the other parameters, the values of the elasticity of non-IT capital and software are precisely estimated and imply lower output elasticities compared to labor and hardware, respectively. While the estimates for the elasticity of non-IT capital α appear fairly robust to misspecification (similar across the three production functions), the estimates for the software elasticity appear more sensitive to the specification. In particular, the Cobb-Douglas estimates are much larger (smaller) than those under the two CES specifications in the sample of all industries (manufacturing). We find fairly sizable autocorrelations ($\rho_{\theta\theta}, \rho_{\phi\phi}$) in the dynamics of the two productivity states implying a high degree of persistence. However, we find very small, and precisely estimated, cross terms in the persistence coefficients ($\rho_{\theta\phi}, \rho_{\phi\theta}$). The last set of results appear fairly robust under all specifications.

Figure 4.9 provides the estimated parameters ϵ , σ and γ across 17 industries at the A38 level of the aggregated NAF classification.⁵¹ These industries span most of the French market economy excluding real estate, finance and agriculture.

First, note that the estimated values of the parameter ϵ are positive and significant for most of the industries. The mean and the median estimate across the industries are 0.86 and 0.57, respectively. Second, estimated values of the parameter σ are never significantly above 1. The mean and the median estimate across the industries are 0.18 and 0.14, respectively. Interestingly enough, our estimated values of the cost elasticity parameter γ are generally close to 1 across industries (the

⁵¹With 38 industries, the A38 level is more detailed than the 1-digit NAF level (10 industries) but more aggregated than the 2-digits NAF level (88 industries). Excluding agriculture, finance, real estate and nonmarket industries, our sample is comprised of 27 industries. Because some of those industries contain fewer than 50 observations in a given year, we construct three pooled industries (ICT, Research and Energy, mining and utilities), for a new total of 22 industries. For five of those industries, our estimation procedure failed to provide reasonable estimates: linear estimations for the accommodation and food services failed to converge, and nonlinear estimations for chemicals and “other manufacturing” industries also failed to converge. Estimations for the construction and ICT industries yield very large point estimates and standard errors for ϵ so we exclude those two industries from our main tables and figures. We are left with 17 industries, 13 of which are included in the manufacturing sector.

mean and medians across industries are 0.94 and 0.97, respectively). This is in line with most available estimates of the cost elasticity that are interpreted as evidence for constant returns to scale at the firm level. In our setting, these very estimates only imply (approximate) constant returns to scale for the lowest IT share firms.

4.5.2 Calibration

In this section, we present the results of a calibration of the general equilibrium model of Section 4.4.3 to investigate its quantitative implications regarding the micro and macro facts documented in Section 4.3. As discussed earlier, we particularly focus on the response of the model to the observed fall in the relative price of IT presented in Figure 4.1. We construct the series for the relative price of IT as the ratio of the price of the bundle of IT inputs to the price of the bundle of non-IT inputs. To aggregate the price of each bundle, we rely on the macro data on average wages and the user costs of software, hardware, and non-IT capital, and use the estimated values of parameters (α, β) from Table 4.9.⁵² The resulting series suggests a fall in the relative price of the bundle of IT inputs from an initial level of $W = 0.0203$ in the beginning of the 1990s to around $W' = 0.0075$ by 2007. As already discussed, our estimation uncovers negligible trends in IT-biased productivity over this period, and therefore we take our *IT shock* to be completely captured by the fall in the relative IT price. Correspondingly, we first calibrate the model at the initial level of relative IT price W , and subsequently examine the equilibrium at the new level of the relative price of IT corresponding to W' .

4.5.3 Calibrated Model Parameters

The estimation results of Section 4.5.1 allow us to determine the values of most model parameters that characterize the production function or the heterogeneity in the productivity states within and across firms. Furthermore, they suggest the following functional form for the joint distribution of the productivity states of entrants:

$$F(\theta, \phi) \equiv \text{TrunPareto}(e^\theta; \xi_o, \underline{\theta}_o, \bar{\theta}_o) \times \mathcal{N}(\phi; \bar{\phi}_o, \kappa_o^2), \quad (4.43)$$

⁵²See the online appendix for a discussion of the construction of the relative price of IT based on the series reported in the French national accounts. We note that the size of the fall in the IT prices that we used here is substantially lower than the values reported in recent work that attempts to improve IT price indices by properly adjusting for quality improvements (see, e.g., Byrne and Corrado, 2017). Since we partially rely on other macro values reported by INSEE, we choose to also rely on their series for the prices of IT inputs for consistency.

where the cumulative distribution function of the truncated Pareto distribution is given by $F(\theta) = (e^{-\xi_o \underline{\theta}_o} - e^{-\xi_o \theta}) / (e^{-\xi_o \underline{\theta}_o} - e^{-\xi_o \bar{\theta}_o})$, and IT-biased productivity state ϕ has a normal distribution with mean $\bar{\phi}_o$ and variance κ_o^2 . Online Appendix provides the details of our calibration strategy, which leads to the model parameters reported in Table 4.10. The table indicates the parameters directly estimated in Section 4.5.1 as “Estimated,” those that are indirectly implied by the estimation results as “Estimation,” and those calibrated based on the values common in the prior work as “Calibrated.” We determine the values of the remaining parameters using a simple parameter search targeting the aggregate IT intensity and two measures of industry concentration observed in the data.

Table 4.11 presents the data moments used in the calibration and their model counterparts. For each moment, the table also indicates the corresponding model parameters that have been calibrated based on that moment. The model closely fits the values of all the moments based on the data. The table further provides a number of untargeted moments, including the aggregate labor share, the unweighted means of the distribution of labor share and IT intensity, and alternative measures of concentration, specifically, the standard deviations of the distributions of log sales and employment. The model further provides a reasonable fit for these untargeted moments.⁵³

We follow the same procedure to calibrate an alternative model with a homothetic CES production function starting from the estimated parameters reported in the second column of Table 4.9 and setting $\epsilon = 0$. Throughout, we compare the results of our model with the nhCES production function against this benchmark to illustrate the consequences of nonhomotheticity in the production function for the model predictions.

4.5.4 Calibration Results

Cross-sectional Patterns Figure 4.5 shows how firm size varies with IT intensity of cost and labor share across firms in the model. In line with the facts we documented in Section 4.3.1, our model predicts a strong positive relationship between firm size and IT intensity. A regression of log IT intensity on log size in the model gives a coefficient of 0.330, which is closely in line with the corresponding estimates reported in panels 1 and 3 of Table 4.6 based on our micro data.

⁵³To compute the value of labor share in Table 4.11 and throughout the rest of this section, we use the value $\alpha = 0.182$ for the capital intensity of the bundle of non-IT inputs estimated in the sample of manufacturing firms and reported in Table 4.9. As shown in Table 4.11, this value provides a close fit between the predictions of the model and the micro and macro data. If we instead use the reported estimate for the sample of all industries, i.e., the value $\alpha = 0.074$, the predicted labor share in the model uniformly shifts up by 13.2%, e.g., from the aggregate labor share of 65.8% reported in Table 4.11 to 74.49%. Note that the parameter α is not used in the calibration of the model and only becomes relevant for computing the predictions regarding labor share. Moreover, changing this value only changes a uniform multiplicative factor $(1 - \alpha)$ in the predictions of labor share and otherwise does not bear on the within versus cross-firm predictions of the model.

In contrast, the benchmark model with homothetic CES production functions produces a small relationship between size and IT intensity.⁵⁴ Regarding the relationship between size and labor share, Figure 4.5b shows that the benchmark model does indeed predict a negative relationship between size and labor share across firms. This is driven by the fact that our model attributes all fixed costs to non-IT factors, and this fixed cost is relatively larger for smaller firms. However, the figure shows that this negative relationship is stronger in our model, due to the relationship between IT intensity and firm size already shown in Figure 4.5a. As we discussed at length in Section 2.3, higher IT intensity among larger firms implies both a higher profit share and a higher income share for IT inputs among these firms.

Heterogeneity in Firm-Level Response to the IT Shock Figure 4.6 shows the responses of labor share and the shares of firms in factor payments to the IT shock as a function of the IT cost share Ω_i . As we saw before, the IT cost share summarizes all the relevant information for the response of the firm. Figure 4.6a compares the response of labor share among different firms between the model with nhCES production functions and the calibrated benchmark model with CES production functions. For firms with small IT cost shares, which constitute the vast majority of the firms in the calibrated model and in the data, the same IT shock generates a larger rise for labor share in our model. As we discussed in Section 4.4.3.2, this is due to the returns to scale channel introduced by nonhomotheticity: lower IT prices allow firms to adopt IT to raise their returns to scale, shifting income from profits to factor payments, and raising the labor share. However, the figure shows that this pattern reverses for firms with large IT cost shares. For these firms, the direct effect of nonhomotheticity highlighted in Equation (4.34) kicks in: since they gain market share and their outputs rise, they also choose to raise their IT intensity, shifting their income from labor to IT. Figure 4.6b compares the reallocations between the two models. As predicted by Equation (4.36), for firms with IT cost shares up to around 25%, which again constitute the vast majority of firms, the nhCES model predicts greater reallocations from firms with low IT cost shares to those with high IT cost shares.

Aggregate Response Table 4.12 presents the response of aggregate variables to the IT shock in the calibrated model. For each aggregate variable, we compare the change observed in the data in the 1990-2007 period with the predicted change caused by the IT shock in our “nhCES” model. The table also presents the change predicted by a benchmark homothetic “CES” model to the same IT shock.

⁵⁴Even if not discernible in Figure 4.5a, a regression of log IT intensity on log size in the benchmark model still produces a small positive coefficient. This positive relationship is driven by the selection channel: firms with higher factor-symmetric productivity θ can remain active with lower levels of ϕ . Since the elasticity of substitution is smaller than unity, this implies that large firms may on average have higher levels of IT cost share.

The first four rows of the table show the main four aggregate variables of the model. In response to the fall in the price of IT, aggregate output rises by around 6% and the price index falls by around 7%.⁵⁵ Examining the same aggregate variables in the model with homothetic CES production function, we find a slightly smaller aggregate output response to the IT shock of around 4%. The difference between the aggregate responses in the two models stems from their different implications for returns to scale, which is endogenous in our model and is exogenous in the benchmark. The IT shock in our model raises the aggregate productivity by raising the returns to scale, especially among larger firms with higher levels of IT intensity.

Comparing the response of the mass of active firms N between the two models, we find diverging predictions: whereas our model predicts a *fall* of over 3% in the mass of active firms, the CES benchmark predicts a *rise*. In contrast with the benchmark model, our model's prediction is in line with the sizable fall observed in the data in the number of firms per worker, by over 14%, over the period.⁵⁶ The difference between the two models is driven by the fact that in our model the benefits of IT disproportionately accrue to large firms, due to the correlation patterns shown in Figure 4.5a. Entrants are on average smaller and therefore face stronger competition, leading to a shift in the allocation of non-IT inputs from the entry to the production sector.

Table 4.12 further presents the response of a number of other aggregate variables that relate to the macro facts in Section 4.3.3. The shares of top 1% and 5% of firms in total sales rise in the data rise by 8.1 and 6.4 percentage points, respectively (see Figure 4.4a). As we should expect based on Figures 4.6b and 4.5a, our model indeed predicts a sizable reallocation from small and low-IT intensity firms to large and high-IT intensity firms, leading to a rise in industry concentration. The rise in the two proxies of industry concentration in our model are 5.4 and 2.7 percentage points (66% and 42% of the observed rise in the data). The benchmark CES model also predicts a rise in concentration, but one that is quantitatively about half as large: 3.0 and 1.4 percentage points (37% and 21% of the observed rise in the data), respectively.

As we saw in Section 4.3.3, the data suggests that the stability of the aggregate labor share masks sizable compositional changes. Table 4.12 shows that a positive contribution of 3.8 percentage points from the within effect is accompanied by a negative contribution of 3.9 percentage points from the reallocation effect.⁵⁷ Our model also predicts a negligible response to the IT shock in the

⁵⁵This result suggests that the fall in IT prices can explain around 20% of the rise in output per worker in France, which rose by around 29% between 1990 and 2007. We do not include this number in Table 4.12 since an important part of the rise in output per worker should be attributable to the aggregate productivity *growth*, which lies outside of our stationary model.

⁵⁶For this result, we rely on the SIRENE dataset, which is distinct from the BRN and RSI datasets. The online appendix reports the evolution of the number of firms per worker in France, provides more details on the SIRENE dataset, and why we prefer this source for measuring the number of firms.

⁵⁷Figure 4.4b shows that the within-firm component of the aggregate labor share shows a sizable drop of around 1 percentage point in the last year of our data, from 2006 to 2007. To avoid this confound, Table 4.12 reports the

labor share of the production sector, along with a positive and negative contribution of around 2.1 percentage points from the within and across-firm effects, respectively (55% of the observed changes in the data). In contrast, the benchmark CES model predicts a reallocation effect of only 1.2 percentage points (30% of the fall in the data). Once again, accounting for nonhomotheticity results in a response to the IT shock that is around twice as large as that of the benchmark CES model.

Table 4.12 also shows that data suggests a negligible change in the aggregate profit share. In the benchmark CES model, the aggregate profit share is exogenous. In our model, the aggregate profit share is endogenous but responds negligibly to the IT shock.⁵⁸

Finally, we extend the analysis by studying the response of the model to IT shocks of different magnitudes. Figure 4.7 compares the responses to the shock examined in Figure 4.6 and Table 4.12 (indicated by the vertical dotted black line) with smaller or larger drops in the relative IT prices. As shown in Table 4.12, our model implies an aggregate elasticity of substitution $\bar{\sigma} = 1.007$, which is very close to 1 and much higher than the micro elasticity of $\sigma = 0.225$. The corresponding value under the benchmark CES model is substantially smaller ($\bar{\sigma} = 0.750$). As a result, Figure 4.7a shows that if the relative IT price fall by up to one order of magnitude from their baseline W , the model behaves close to a Cobb-Douglas aggregate production function. Thus, the aggregate labor share and IT intensity remain fairly stable. In contrast, the lower aggregate elasticity of substitution under the CES model implies that the labor share sharply rises and the IT intensity falls in response to such shocks. For stronger IT shocks, these variables begin to respond even in our model, but the magnitude of their responses remain smaller compared to that generated under the CES benchmark. Figure 4.7b compares the responses of market concentration, showing that the rise in concentration is monotonic and around twice larger in our model compared to the benchmark CES model.⁵⁹

4.6 Discussion

In this section, we first provide a more in depth discussion of our construction of nhCES production functions in Section 4.4.2.1. We then comment on the generality of the relationship between

cumulative change from 1990 to the average between 2006 and 2007. The cumulative change in the within-firm component is 4.3 percentage points by 2006 and falls to 3.3 percentage point to 2007.

⁵⁸By definition, the share of economic profits is 1 minus the sum of the labor and capital shares. The online appendix discusses how we construct our measures of capital share in the French economy that allows us to compute the aggregate profit share. As with the labor share, we also find the aggregate share of capital to be stable in the 1900-2007 period.

⁵⁹The online appendix compares the responses of aggregate output and the mass of active firms, showing that these responses are also monotonic and broadly follow the same patterns discussed in the case of Table 4.12.

nonhomotheticity and returns to scale, following up on the discussions in Section 4.4.2.2. Lastly, we discuss the potential consequences of accounting for variations in market power and markups in our framework.

The Foundations of nhCES Production Function The assumption of *organization* as a firm-specific factor that can potentially generate decreasing returns, in our Equation (4.12), is by no means novel. In his *Principles of Economics*, Marshall already identifies organization as a fourth factor of production, in addition to labor, capital, and land.⁶⁰ Coase (1937) argues that “decreasing returns to the entrepreneur function” is a key factor in determining the boundary of the firm.⁶¹ To justify firm-level decreasing returns, McKenzie (1959) defines a firm-specific *entrepreneurial input*, Lucas (1978) refers to it as the *managerial talent* of the firm’s manager, and Atkeson and Kehoe (2005) interpret it as the *organizational capital* of the firm. Our concept of organizational input is also in line with the concept of the firm-level *economic competencies* that is considered to be a core component of the intangible capital of firms (Haskel and Westlake, 2018).

At least since Williamson (1967), many economic theories have formalized potential grounds for firm-level organizational limits to scale.⁶² These models offer various mechanisms through which the complexities of coordination and communication, both within the firm and also between the firm and its buyers and suppliers, may rise as the firm scale grows. Equation (4.14) offers a stylized account of such organizational limits to scale, inspired by the span-of-control

⁶⁰In addition, Marshall also explains the heterogeneity in the organizational efficiency of businesses through the differences in the business ability of their managers. He then goes on to provide a decomposition of the earnings of a business into three components that distinguishes the returns to organizational inputs:

Finally, we may regard this supply price of business ability in command of capital as composed of three elements. The first is the supply price of capital; the second is the supply price of business ability and energy; and the third is the supply price of that organization by which the appropriate business ability and the requisite capital are brought together. [...] We may call the price of the second taken by itself net earnings of management, and that of the second and third, taken together, gross earnings of management.

⁶¹In line with our assumptions in Equation (4.14), he suggests (Coase, 1937):

First, as a firm gets larger, there may be decreasing returns to the entrepreneur function, that is, the costs of organising additional transactions within the firm may rise. [...] Secondly, it may be that as the transactions which are organised increase, the entrepreneur fails to place the factors of production in the uses where their value is greatest, that is, fails to make the best use of the factors of production.

⁶²Some approaches emphasize the agency issues inherent in delegation and the formation of hierarchies (see Mukherjee, 2012, for a review), while others focus on the complexities of solving allocative decisions within the firm (see Garicano and Van Zandt, 2012). In particular, in theories of knowledge hierarchies the complexity of the process of dealing with production errors changes endogenously with the scale of operation (Garicano, 2000; Garicano and Rossi-Hansberg, 2006; Caliendo and Rossi-Hansberg, 2012). Another alternative follows the approach of Simon (1962) and examines the implications of bounded rationality and the limits to information processing (e.g., Van Zandt and Radner, 2001).

model of Lucas (1978) (nested in our model for the case of $\sigma = 1$).⁶³ The latter focuses solely on the limits to the ability of a manager to supervise production over increasing scales of inputs. In contrast, Equation (4.14) accounts for the organizational limits to the scale in terms of firm output Y . A larger scale of output Y brings about complexities that stem from organizing production tasks over larger scales of inputs and outputs. On the input side, the firm potentially has to find and coordinate with more intermediate suppliers, hire more or different types of workers, procure more machines, and manage across a large set of inputs. On the output side, it has to potentially manage larger inventories, coordinate with more buyers or across more markets, provide support services to a larger set of customers, organize larger scale marketing efforts, and solve larger delivery or distribution problems.

Equation (4.14) further generalizes the benchmark span-of-control model by allowing firms to endogenously deploy IT in order to enhance their organizational efficiency. A sizable body of empirical work has documented the connection between IT and the organizational efficiency of the firm. Brynjolfsson and Hitt (2000) discuss numerous case studies that showcase the effect of IT on the organizational practices of businesses. A variety of firm-level cross sectional studies have documented the complementarity between IT adoption and organizational capital (e.g., Bresnahan et al., 2002; Brynjolfsson and Hitt, 2003; Baker and Hubbard, 2004). In particular, Bartel et al. (2007) use detailed firm-level data on setup times, run times, and inspection times to show how IT improves the efficiency of organizing multiple production processes within the firm. As another example, Bloom et al. (2014) find that the adoption of IT impacts the organization of the firm in terms of span of control over individual workers. In sum, IT allows firms to more effectively apply their technology or core competencies to a larger scale of inputs, across a wider range of activities, and to serve more buyers and markets.

While our model may give rise to decreasing returns to scale at the firm-level, we can still apply the standard replication argument *across* firms to conclude that the aggregate returns to scale exceeds unity. In our theory, a firm is characterized by a firm-specific rival factor, which we call firm-level organization, whose production requires using scarce resources in the form of entry costs.⁶⁴ Thus, we can apply the replication argument in the aggregate: doubling the resources allocated to entry doubles the mass of firms, which then leads to a doubling of output if each firm receives the same level of inputs as before. In line with this argument, we find an aggregate returns to scale that exceeds unity when we calibrate our model to the French data.⁶⁵

⁶³ Appendix C.1.1 provides a derivation of the Lucas span-of-control model in the special case of $\sigma = 1$, i.e., the case where the elasticity of substitution between technology and organization is unity.

⁶⁴ Examples of such startup costs include those associated with creating a new business, initial branding, marketing, and advertising, setting up supply chains, etc.

⁶⁵ In the calibration, we find the aggregate returns to scale of the production sector is around 1.036, while the elasticity of output with respect to the aggregate level of the bundle of non-IT inputs is as high as 1.245. Note that

Alternatives to the nhCES Production Function The core mechanism in our paper is the tight connection between the nonhomotheticity of factor demand and the endogeneity and heterogeneity in returns to scale across firms. In the context of our nhCES production function, Equations (4.17) and (4.19) show that nonhomotheticity of the production function generates a dependence of scale and cost elasticities on firm size. However, this result is more general than the case of nhCES production functions. Importantly, we show in Appendix C.1 that the insight generalizes beyond the current specification, in the sense that it *locally holds for all production functions that satisfy nonhomothetic factor demand*. In other words, once we establish the nonhomotheticity of factor demand, it immediately follows that returns to scale should vary across firms as a function of firm size.

We should emphasize that the account of the relation between IT and returns to scale implied by production function in Equation (4.9) has a close conceptual connection to a model involving a fixed-cost of IT adoption. Appendix C.1.2 lays out a simple model in which adopting IT raises productivity subject to a fixed cost in units of the bundle of non-IT inputs. This model generates both the nonhomothetic IT demand and the negative relationship between size and scale elasticity. This result is in line with the generalized result mentioned above (Appendix C.1) that locally links nonhomothetic IT demand with the relationship between firm size and scale elasticity under any specification of the production function.

Despite the generality of the mechanism, our nhCES specification is particularly well suited, compared to alternatives such as the fixed-cost model, for a quantitative account of the patterns observed in the data. Consider the results of Table 4.8 that show a robust correlation between software intensity and firm size across different brackets of firm size. This finding suggests a specification of relative IT demand along the lines of Equation (C.1) with a value of η that is relatively constant as firm scale changes. The nhCES production function predicts exactly this pattern with $\eta = (1 - \sigma)\epsilon$. In contrast, as the derivations in Section C.1.2 illustrate, the corresponding elasticity in the case of the fixed-cost model falls with the scale of firm output and converges to zero as firm size grows.

Returns to Scale, Market Power, and Aggregate Productivity In our framework, technological advances in information technology reduce the price of IT as a factor input and lead to strong productivity gains at both micro and macro levels. Despite the fact that they are biased toward larger firms and raise market concentration, markets allocate resources efficiently across productive units. In emphasizing the potentially efficient aspects of the recent trends, our paper contrasts with a number of recent contributions that instead focus on their potentially distort-

in our model, due to the presence of the love-of-variety, doubling the mass of firms leads to more than double the original output.

tionary consequences (e.g., [De Loecker et al., 2020a](#); [Baqaee and Farhi, 2020b](#); [Aghion et al., 2019](#)). This line of work interprets the trends in market concentration and labor share through the lens of variations in markups and market power. In contrast, the current paper emphasizes the fact that these trends may in part stem from the nature of technological advances that lower the organizational costs of scale.

Nevertheless, these two accounts of the recent trends are not mutually exclusive. Consider, for instance, using an alternative demand aggregator such as Kimball preferences instead of the CES specification in Equation (4.2). This alternative specification allows for a monotonically increasing relationship between markups and relative firm size (see, e.g., [Edmond et al., 2018](#)). We can rely on the results of Section 4.5.2 to draw insights about the response of this modified model to the rise of IT within and across firms. There, we saw that with the fall in IT prices in our model, due in part to nonhomotheticity in IT demand, the relative output of large firms rise while the relative output of small firms falls. With endogenous markups, the within-firm response in markups becomes heterogeneous across firms, rising for large and falling for small firms.⁶⁶ In addition, the resulting cross-firm reallocations lead to a shift of market shares toward high-markup firms and away from low-markup firms. These patterns are in line with those documented by [De Loecker et al. \(2020a\)](#).

Lastly, we note that our results may have implications for the current approaches to the estimation of markups and their variations across firms following [De Loecker and Warzynski \(2012\)](#). This approach relies on the estimation of output elasticities of a given variable input, e.g., labor in our setting, in order to infer markups. As emphasized recently by [Demirer \(2020\)](#), accounting for factor-augmenting productivity, IT biased productivity in our setting, is important to account for the endogenous variations in output elasticities. In our model, Equation (4.16) suggests that the output elasticity of labor is given by $(1 - \alpha) \frac{1 - \Omega_i}{\gamma + \epsilon \Omega_i}$, which is decreasing in IT cost share Ω_i and therefore firm size. To the extent that the presence of nonhomotheticity ($\epsilon > 0$) intensifies the negative relationship between size and the output elasticity of labor, it may weaken the relationship between size and the implied markup.

4.7 Conclusion

In this paper, we presented novel data on the investment and capital stocks of firms in software and hardware in the universe of French firms. In our data, we found that the intensity of IT

⁶⁶We note, however, that the rise (fall) of markup among large (small) firms in turn curbs the first-order effect of the rise of IT on output. In other words, under preferences with variable price elasticities, the output of large (small) firms rises (falls) less than that under our benchmark with constant price elasticities.

demand strongly and robustly correlates with firm size, using a broad set of different measures of IT intensity and firm size. Moreover, we argued that a production function featuring non-homotheticity of IT factor demand fits this empirical regularity, as well as an observed negative correlation between firm size and labor share. The latter holds assuming an elasticity of substitution between IT and non-IT inputs that falls below unity, and stems from lower degrees of returns to scale predicted by the model for larger firms.

We applied an identification strategy to estimate the production function and find that IT demand is indeed nonhomothetic and that the elasticity of substitution between IT and non-IT is indeed below one. These results imply that the marginal product of IT, relative to the marginal product of non-IT inputs, grows in firm size. We further provided a simple theoretical general equilibrium model of industry dynamics to study the aggregate implications of the firm-level nonhomotheticity of the production function. In particular, we showed that the resulting model predicts, just as we find in the data, that the observed fall in the price of IT results in a reallocation of market shares across firms toward those firms with higher IT intensity and typically larger size and market shares.

Tables

Table 4.1: Summary Statistics

	Source	Obs. (Nb)	All firms			Manufacturing firms			
			Mean	Median	Sd	Obs. (Nb)	Mean	Median	Sd
Sales	BRN + RSI	15,202,967	2,498.8	265	85,056.8	2,422,381	4,171.2	316.9	60,560.2
Value-Added	BRN + RSI	15,202,967	708.3	106	33,071.4	2,422,381	1,271.9	147.1	25,846.5
Number of Employees	BRN + RSI	15,202,967	13.8	3	480.7	2,422,381	23.3	4	177.0
Wage Bill	BRN + RSI	15,202,967	472.4	74	18,404.5	2,422,381	815.2	109	8,105.5
Labor Share (%)	BRN + RSI	15,202,967	74.0	73.0	33.9	2,422,381	74.1	73.0	31.4
Total Investment	BRN	6,166,342	143.1	5	9,880.3	986,722	274.9	12	4,109.5
Total Capital Stock	BRN	6,166,342	1,202.2	88.0	92,297.9	986,722	2,599.2	217.9	30,598.2
Total Cost	BRN	6,166,341	898.1	181.0	33,623.7	986,722	1,578.2	305.9	12,659.3
IT Measures									
Software Investment	EAE	2,435,356	5.9	0	528.2	380,756	14.8	0	290.9
Software Stock	EAE	2,435,356	15.8	0	1,216.2	380,756	41.0	0.7	721.9
Hardware Investment	BRN	6,166,342	6.1	0	405.2	986,722	9.3	0	173.1
Hardware Stock	BRN	6,166,342	24.5	0	1,857.4	986,722	45.8	0	666.0
IT Intensity of Labor									
Software Investment	EAE	2,435,356	27.5	0	167.0	380,756	67.2	0	228.1
Software Stock	EAE	2,435,356	81.7	0	3,214.5	380,756	220.4	20.8	7,825.8
Hardware Investment	BRN	6,166,342	177.5	0	750.0	986,722	114.7	0	460.2
Hardware Stock	BRN	6,166,342	477.4	0	2,435.9	986,722	398.2	0	1,235.5
IT Intensity of Capital									
Software Investment	EAE	1,985,530	21.9	0	1,156.5	353,971	30.1	0	596.7
Software Stock	EAE	2,284,444	3.8	0	22.1	371,701	5.9	0.6	19.1
Hardware Investment	BRN	4,381,031	112.9	0	1,601.1	771,006	71.2	0	1,403.6
Hardware Stock	BRN	5,550,954	39.5	0	128.3	916,263	18.9	0.2	71.5
IT Intensity of Cost									
Software Investment	EAE	2,435,351	0.7	0	4.1	380,756	1.6	0	5.5
Software Stock	EAE	2,435,356	0.6	0	2.6	380,756	1.6	0.2	3.7
Hardware Investment	BRN	6,166,303	3.8	0	20.0	986,716	2.6	0	15.4
Hardware Stock	BRN	6,166,341	2.4	0	7.9	986,722	1.7	0	4.5

Table 4.2: Regressions of IT Intensity on Log Firm Size

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Costs	Costs
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.3650 (0.0031)		0.3115 (0.0031)		0.2779 (0.0032)		0.2842 (0.0032)		0.2996 (0.0030)	
Size (proxied by VA)		0.3458 (0.0033)		0.2933 (0.0033)		0.2899 (0.0034)		0.2980 (0.0035)		0.2834 (0.0033)
Observations	575,594	575,686	575,579	575,676	530,334	530,395	529,045	529,104	575,655	575,755
R2	0.2396	0.2356	0.2281	0.2249	0.2346	0.2341	0.2350	0.2346	0.2314	0.2286
Panel 2: Software (Investment)										
Size (proxied by sales)	20.5010 (0.1070)		0.5034 (0.0029)		4.4246 (0.0277)		5.2615 (0.0348)		0.4286 (0.0025)	
Size (proxied by VA)		21.1063 (0.1131)		0.5191 (0.0030)		4.7405 (0.0293)		5.6491 (0.0368)		0.4409 (0.0026)
Observations	1,145,874	1,146,068	1,145,917	1,146,115	1,127,629	1,127,816	1,117,482	1,117,639	1,146,512	1,146,707
R2	0.0911	0.0896	0.0840	0.0829	0.0820	0.0826	0.0756	0.0762	0.0829	0.0817
Panel 3 : Hardware (Stock)										
Size (proxied by sales)	0.2630 (0.0007)		0.2031 (0.0007)		0.2134 (0.0008)		0.2279 (0.0008)		0.1993 (0.0007)	
Size (proxied by VA)		0.1991 (0.0008)		0.1289 (0.0008)		0.1705 (0.0009)		0.1884 (0.0009)		0.1279 (0.0008)
Observations	2,839,365	2,839,569	2,839,373	2,839,754	2,755,218	2,755,436	2,756,088	2,756,211	2,840,459	2,840,804
R2	0.4188	0.4068	0.3823	0.3718	0.4163	0.4104	0.4491	0.4435	0.3474	0.3367
Panel 4 : Hardware (Investment)										
Size (proxied by sales)	41.1954 (0.1841)		0.8812 (0.0052)		17.1734 (0.0571)		19.7820 (0.0712)		0.7486 (0.0037)	
Size (proxied by VA)		32.2761 (0.1894)		0.5930 (0.0054)		15.7131 (0.0587)		18.2151 (0.0732)		0.5411 (0.0038)
Observations	4,340,454	4,341,159	4,340,014	4,340,853	4,366,163	4,366,860	4,302,290	4,302,802	4,344,803	4,345,554
R2	0.1647	0.1607	0.1386	0.1353	0.1860	0.1826	0.2409	0.2384	0.1478	0.1438

Table 4.3: Regressions of Log IT Intensity of Capital On Measures of Firm Scale

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Cost	Cost
Panel 1 : Software (Stock)										
Number of plants	0.0015 (0.0002)		0.0014 (0.0002)		0.0014 (0.0002)		0.0016 (0.0002)		0.0013 (0.0002)	
Number of occupational layers		0.2623 (0.0047)		0.2230 (0.0047)		0.2558 (0.0049)		0.2603 (0.0050)		0.2250 (0.0047)
Observations	562,858	562,858	562,997	562,997	518,716	518,716	517,470	517,470	563,027	563,027
R2	0.2214	0.2255	0.2143	0.2173	0.2237	0.2276	0.2238	0.2278	0.2186	0.2217
Number of destination countries	0.0276 (0.0004)		0.0243 (0.0004)		0.0225 (0.0004)		0.0232 (0.0004)		0.0238 (0.0004)	
Number of products		0.0065 (0.0002)		0.0059 (0.0002)		0.0054 (0.0002)		0.0056 (0.0002)		0.0057 (0.0002)
Observations	278,803	278,803	279,590	279,590	261,609	261,609	261,144	261,144	279,902	279,902
R2	0.1958	0.1871	0.1887	0.1817	0.1867	0.1806	0.1875	0.1811	0.1921	0.1854
Panel 2: Hardware (Stock)										
Number of plants	0.0040 (0.0001)		0.0040 (0.0001)		0.0036 (0.0001)		0.0040 (0.0001)		0.0039 (0.0001)	
Number of occupational layers		0.0986 (0.0009)		0.0700 (0.0009)		0.1124 (0.0010)		0.1251 (0.0010)		0.0735 (0.0009)
Observations	2,696,655	2,696,655	2,698,300	2,698,300	2,622,236	2,622,236	2,622,998	2,622,998	2,698,872	2,698,872
R2	0.3913	0.3933	0.3632	0.3640	0.4003	0.4027	0.4319	0.4347	0.3275	0.3286
Number of destination countries	0.0337 (0.0002)		0.0299 (0.0002)		0.0258 (0.0002)		0.0272 (0.0002)		0.0291 (0.0002)	
Number of products		0.0084 (0.0001)		0.0078 (0.0001)		0.0068 (0.0001)		0.0071 (0.0001)		0.0076 (0.0001)
Observations	553,427	553,427	555,879	555,879	546,058	546,058	546,478	546,478	555,847	555,847
R2	0.2831	0.2628	0.2533	0.2366	0.3129	0.3033	0.3339	0.3237	0.2226	0.2058

Table 4.4: Regressions of IT Intensity of Cost on Log Firm Size, by Bins of Employment

	IT Intensity of Cost									
	[1; 50[[50; 100[[100; 250[[250; 1000[≥ 1000	[1; 50[[50; 100[[100; 250[[250; 1000[≥ 1000
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.2871 (0.0057)	0.2528 (0.0136)	0.2199 (0.0140)	0.2861 (0.0185)	0.2761 (0.0370)					
Size (proxied by VA)						0.2159 (0.0065)	0.2202 (0.0167)	0.1959 (0.0170)	0.2906 (0.0213)	0.3024 (0.0395)
Observations	379,543	91,406	66,022	30,925	6,375	379,603	91,430	66,030	30,933	6,375
R2	0.2506	0.2132	0.2216	0.2716	0.3715	0.2478	0.2117	0.2202	0.2705	0.3718
Panel 2: Hardware (Stock)										
Size (proxied by sales)	0.0812 (0.0009)	0.7282 (0.0061)	0.5073 (0.0058)	0.2934 (0.0070)	0.1400 (0.0127)					
Size (proxied by VA)						-0.0343 (0.0010)	0.5140 (0.0074)	0.3700 (0.0070)	0.2135 (0.0082)	0.0911 (0.0132)
Observations	2,563,488	132,537	93,307	41,372	8,295	2,563,741	132,577	93,338	41,393	8,295
R2	0.3752	0.3428	0.3847	0.3817	0.4572	0.3735	0.2960	0.3525	0.3647	0.4514

Table 4.5: Regressions of IT Intensity on Log Firm Size, Within Firms

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Costs	Costs
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.2042 (0.0326)		0.1455 (0.0327)		0.3422 (0.0337)		0.3533 (0.0338)		0.1702 (0.0326)	
Size (proxied by VA)		0.2161 (0.0288)		0.1432 (0.0288)		0.3296 (0.0297)		0.3379 (0.0298)		0.1621 (0.0287)
Observations	233,654	233,376	233,507	233,189	221,456	221,319	221,676	221,614	233,548	233,230
R2	0.8361	0.8325	0.8319	0.8281	0.8313	0.8270	0.8311	0.8272	0.8327	0.8288
Panel 2 : Hardware (Stock)										
Size (proxied by sales)	0.2612 (0.0097)		0.1686 (0.0097)		0.3706 (0.0101)		0.3765 (0.0102)		0.1874 (0.0095)	
Size (proxied by VA)		0.1470 (0.0081)		0.0437 (0.0082)		0.2482 (0.0084)		0.2533 (0.0085)		0.0660 (0.0080)
Observations	248,038	249,026	248,995	250,111	244,282	245,286	243,095	244,175	248,466	249,591
R2	0.8689	0.8691	0.8467	0.8466	0.9078	0.9077	0.9177	0.9175	0.8485	0.8493

Table 4.6: Regressions of IT Intensity on Log Firm Size

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Costs	Costs
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.3650 (0.0031)		0.3115 (0.0031)		0.2779 (0.0032)		0.2842 (0.0032)		0.2996 (0.0030)	
Size (proxied by VA)		0.3458 (0.0033)		0.2933 (0.0033)		0.2899 (0.0034)		0.2980 (0.0035)		0.2834 (0.0033)
Observations	575,594	575,686	575,579	575,676	530,334	530,395	529,045	529,104	575,655	575,755
R2	0.2396	0.2356	0.2281	0.2249	0.2346	0.2341	0.2350	0.2346	0.2314	0.2286
Panel 2: Software (Investment)										
Size (proxied by sales)	20.5010 (0.1070)		0.5034 (0.0029)		4.4246 (0.0277)		5.2615 (0.0348)		0.4286 (0.0025)	
Size (proxied by VA)		21.1063 (0.1131)		0.5191 (0.0030)		4.7405 (0.0293)		5.6491 (0.0368)		0.4409 (0.0026)
Observations	1,145,874	1,146,068	1,145,917	1,146,115	1,127,629	1,127,816	1,117,482	1,117,639	1,146,512	1,146,707
R2	0.0911	0.0896	0.0840	0.0829	0.0820	0.0826	0.0756	0.0762	0.0829	0.0817
Panel 3 : Hardware (Stock)										
Size (proxied by sales)	0.2630 (0.0007)		0.2031 (0.0007)		0.2134 (0.0008)		0.2279 (0.0008)		0.1993 (0.0007)	
Size (proxied by VA)		0.1991 (0.0008)		0.1289 (0.0008)		0.1705 (0.0009)		0.1884 (0.0009)		0.1279 (0.0008)
Observations	2,839,365	2,839,569	2,839,373	2,839,754	2,755,218	2,755,436	2,756,088	2,756,211	2,840,459	2,840,804
R2	0.4188	0.4068	0.3823	0.3718	0.4163	0.4104	0.4491	0.4435	0.3474	0.3367
Panel 4 : Hardware (Investment)										
Size (proxied by sales)	41.1954 (0.1841)		0.8812 (0.0052)		17.1734 (0.0571)		19.7820 (0.0712)		0.7486 (0.0037)	
Size (proxied by VA)		32.2761 (0.1894)		0.5930 (0.0054)		15.7131 (0.0587)		18.2151 (0.0732)		0.5411 (0.0038)
Observations	4,340,454	4,341,159	4,340,014	4,340,853	4,366,163	4,366,860	4,302,290	4,302,802	4,344,803	4,345,554
R2	0.1647	0.1607	0.1386	0.1353	0.1860	0.1826	0.2409	0.2384	0.1478	0.1438

Table 4.7: Regressions of Log IT Intensity of Capital On Measures of Firm Scale

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Cost	Cost
Panel 1 : Software (Stock)										
Number of plants	0.0015 (0.0002)		0.0014 (0.0002)		0.0014 (0.0002)		0.0016 (0.0002)		0.0013 (0.0002)	
Number of occupational layers		0.2623 (0.0047)		0.2230 (0.0047)		0.2558 (0.0049)		0.2603 (0.0050)		0.2250 (0.0047)
Observations	562,858	562,858	562,997	562,997	518,716	518,716	517,470	517,470	563,027	563,027
R2	0.2214	0.2255	0.2143	0.2173	0.2237	0.2276	0.2238	0.2278	0.2186	0.2217
Number of destination countries	0.0276 (0.0004)		0.0243 (0.0004)		0.0225 (0.0004)		0.0232 (0.0004)		0.0238 (0.0004)	
Number of products		0.0065 (0.0002)		0.0059 (0.0002)		0.0054 (0.0002)		0.0056 (0.0002)		0.0057 (0.0002)
Observations	278,803	278,803	279,590	279,590	261,609	261,609	261,144	261,144	279,902	279,902
R2	0.1958	0.1871	0.1887	0.1817	0.1867	0.1806	0.1875	0.1811	0.1921	0.1854
Panel 2: Hardware (Stock)										
Number of plants	0.0040 (0.0001)		0.0040 (0.0001)		0.0036 (0.0001)		0.0040 (0.0001)		0.0039 (0.0001)	
Number of occupational layers		0.0986 (0.0009)		0.0700 (0.0009)		0.1124 (0.0010)		0.1251 (0.0010)		0.0735 (0.0009)
Observations	2,696,655	2,696,655	2,698,300	2,698,300	2,622,236	2,622,236	2,622,998	2,622,998	2,698,872	2,698,872
R2	0.3913	0.3933	0.3632	0.3640	0.4003	0.4027	0.4319	0.4347	0.3275	0.3286
Number of destination countries	0.0337 (0.0002)		0.0299 (0.0002)		0.0258 (0.0002)		0.0272 (0.0002)		0.0291 (0.0002)	
Number of products		0.0084 (0.0001)		0.0078 (0.0001)		0.0068 (0.0001)		0.0071 (0.0001)		0.0076 (0.0001)
Observations	553,427	553,427	555,879	555,879	546,058	546,058	546,478	546,478	555,847	555,847
R2	0.2831	0.2628	0.2533	0.2366	0.3129	0.3033	0.3339	0.3237	0.2226	0.2058

Table 4.8: Regressions of IT Intensity of Cost on Log Firm Size, by Bins of Employment

	IT Intensity of Cost									
	[1; 50[[50; 100[[100; 250[[250; 1000[≥ 1000	[1; 50[[50; 100[[100; 250[[250; 1000[≥ 1000
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.2871 (0.0057)	0.2528 (0.0136)	0.2199 (0.0140)	0.2861 (0.0185)	0.2761 (0.0370)					
Size (proxied by VA)						0.2159 (0.0065)	0.2202 (0.0167)	0.1959 (0.0170)	0.2906 (0.0213)	0.3024 (0.0395)
Observations	379,543	91,406	66,022	30,925	6,375	379,603	91,430	66,030	30,933	6,375
R2	0.2506	0.2132	0.2216	0.2716	0.3715	0.2478	0.2117	0.2202	0.2705	0.3718
Panel 2: Hardware (Stock)										
Size (proxied by sales)	0.0812 (0.0009)	0.7282 (0.0061)	0.5073 (0.0058)	0.2934 (0.0070)	0.1400 (0.0127)					
Size (proxied by VA)						-0.0343 (0.0010)	0.5140 (0.0074)	0.3700 (0.0070)	0.2135 (0.0082)	0.0911 (0.0132)
Observations	2,563,488	132,537	93,307	41,372	8,295	2,563,741	132,577	93,338	41,393	8,295
R2	0.3752	0.3428	0.3847	0.3817	0.4572	0.3735	0.2960	0.3525	0.3647	0.4514

Table 4.9: Estimation Results

		All Industries			Manufacturing		
		nhCES	CES	Cobb-Douglas	nhCES	CES	Cobb-Douglas
IT Nonhomotheticity	ϵ	0.389 (0.011)			0.477 (0.019)		
Elasticity of substitution	σ	0.225 (0.012)	0.125 (0.016)		0.171 (0.009)	0.165 (0.013)	
Cost elasticity	γ	0.947 (0.004)	0.978 (0.004)	1.001 (0.004)	0.954 (0.006)	1.014 (0.005)	1.022 (0.006)
Capital elasticity	α	0.074 (0.005)	0.068 (0.005)	0.070 (0.005)	0.182 (0.009)	0.166 (0.008)	0.167 (0.008)
Software elasticity	β	0.113 (0.029)	0.185 (0.023)	0.015 (0.038)	0.120 (0.034)	0.148 (0.028)	0.303 (0.042)
Persistence of θ	$\rho_{\theta\theta}$	0.830 (0.003)	0.825 (0.003)	0.808 (0.003)	0.834 (0.005)	0.804 (0.005)	0.802 (0.005)
Persistence of θ wrt ϕ	$\rho_{\theta\phi}$	-0.010 (0.000)	-0.012 (0.001)		-0.008 (0.001)	-0.014 (0.001)	
Persistence of ϕ wrt θ	$\rho_{\phi\theta}$	-0.072 (0.003)	-0.047 (0.003)		-0.066 (0.004)	-0.048 (0.004)	
Persistence of ϕ	$\rho_{\phi\phi}$	0.898 (0.001)	0.908 (0.001)		0.901 (0.002)	0.919 (0.001)	
Trend for θ	μ_{θ}	0.002 (0.000)	0.002 (0.000)	-0.001 (0.001)	0.004 (0.000)	0.004 (0.000)	0.003 (0.000)
Trend for ϕ	μ_{ϕ}	0.008 (0.001)	0.008 (0.001)		-0.007 (0.001)	-0.003 (0.001)	
Shifter for θ	η_{θ}	0.590 (0.014)	0.626 (0.015)	0.718 (0.013)	0.479 (0.019)	0.658 (0.021)	0.662 (0.018)
Shifter for ϕ	η_{ϕ}	0.677	0.185		0.795	0.306	

Table 4.10: Calibrated Parameters

Model Component	Parameter		Value	
Production Function	IT nonhomotheticity	ϵ	0.39	Estimated
	Cost Elasticity Parameter	γ	0.95	Estimated
	Elasticity of substitution	σ	0.22	Estimated
	Fixed Costs	ψ	0.08	Search
Productivity Process	Persistence of Shocks	$(\rho_{\theta\theta}, \rho_{\phi\phi})$	(0.83, 0.90)	Estimated
	Long-run Mean Productivities	$(\eta_{\theta}, \eta_{\phi})$	(0.59, 0.68)	Estimated
	Variances of Innovations	$(\chi_{\theta}^2, \chi_{\phi}^2)$	(0.09, 0.48)	Estimation
Entry & Exit	Distribution of Entry	$(\xi_o, \bar{\theta}_o)$	(2.82, 8.12)	Estimation
		χ_o^2	2.10	Estimation
		$\underline{\theta}_o$	2.17	Search
		$\bar{\phi}_o$	6.13	Search
	Costs of Entry	χ	0.08	Calibrated
	Exogenous Probability of Exit	δ	0.03	Calibrated
Demand	Elasticity of Substitution	λ	5.00	Calibrated
	Discount Factor	ϱ	0.95	Calibrated

Table 4.11: Calibrated Moments

	Moments	Source	Data	Model	Relevant Parameters
Targeted	Entrant Top %1 θ	EAE (Estimation)	3.46	3.80	ξ_o
	Entrant Top %0.1 θ	EAE (Estimation)	4.28	4.62	ξ_o
	Entrant Highest θ	EAE (Estimation)	8.12	8.12	$\bar{\theta}_o$
	Share of Top 1% of Firms in Sales	BRN+RSI	59.3%	59.9%	$(\psi, \underline{\theta}_o, \bar{\phi}_o)$
	Share of Top 5% of Firms in Sales	BRN+RSI	77.4%	77.0%	$(\psi, \underline{\theta}_o, \bar{\phi}_o)$
	Aggregate IT Intensity	INSEE	3.8%	3.8%	$(\psi, \underline{\theta}_o, \bar{\phi}_o)$
	Mass of Firms (N)	BRN+RSI	0.073	0.073	χ
	Rate of Exit of Large Firms	BRN+RSI	0.031	0.031	δ
Untargeted	Aggregate Labor Share	BRN+RSI	66.2%	65.8%	—
	Unweighted Mean of Labor Share	BRN+RSI	73.5%	73.4%	—
	Unweighted Mean of IT Intensity	EAE	0.2%	0.3%	—
	S.D. of Log Sales	BRN+RSI	1.4	1.3	—
	S.D. of Log Employment	BRN+RSI	1.2	1.3	—

Table 4.12: Calibration Results

Aggregate Variable		CES	nhCES	Data
Change in Price of IT	W	-63.1%	-63.1%	-63.1%
Change in Aggregate Output	Y	+3.7%	+5.8%	—
Change in Price Index	P	-5.4%	-6.9%	—
Change in Mass of Active Firms	N	+3.7%	-3.4%	-14.1%
Change in Share of Top 1% of Firms in Sales	LS^{prod}	+3.0 p.p.	+5.4 p.p.	+8.1 p.p.
Change in Share of Top 5% of Firms in Sales		+1.4 p.p.	+2.7 p.p.	+6.4 p.p.
Change in Labor Share (Production)		+0.6 p.p.	-0.0 p.p.	-0.0 p.p.
<i>Within-Firm Contribution</i>		+1.8 p.p.	+2.1 p.p.	+3.8 p.p.
<i>Reallocation Contribution</i>		-1.2 p.p.	-2.1 p.p.	-3.9 p.p.
Change in Profit Share (Production)		0	+0.0 p.p.	+0.2 p.p.
Aggregate Elasticity of Substitution	$\bar{\sigma}$	0.750	1.007	—

Figures

Figure 4.1: Rise of IT

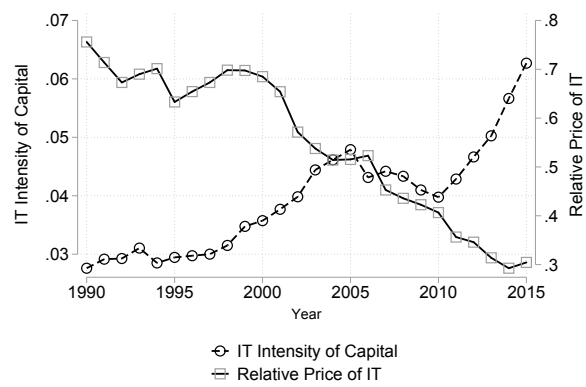


Figure 4.2: Cross-sectional Relationship Between IT and Firm Size

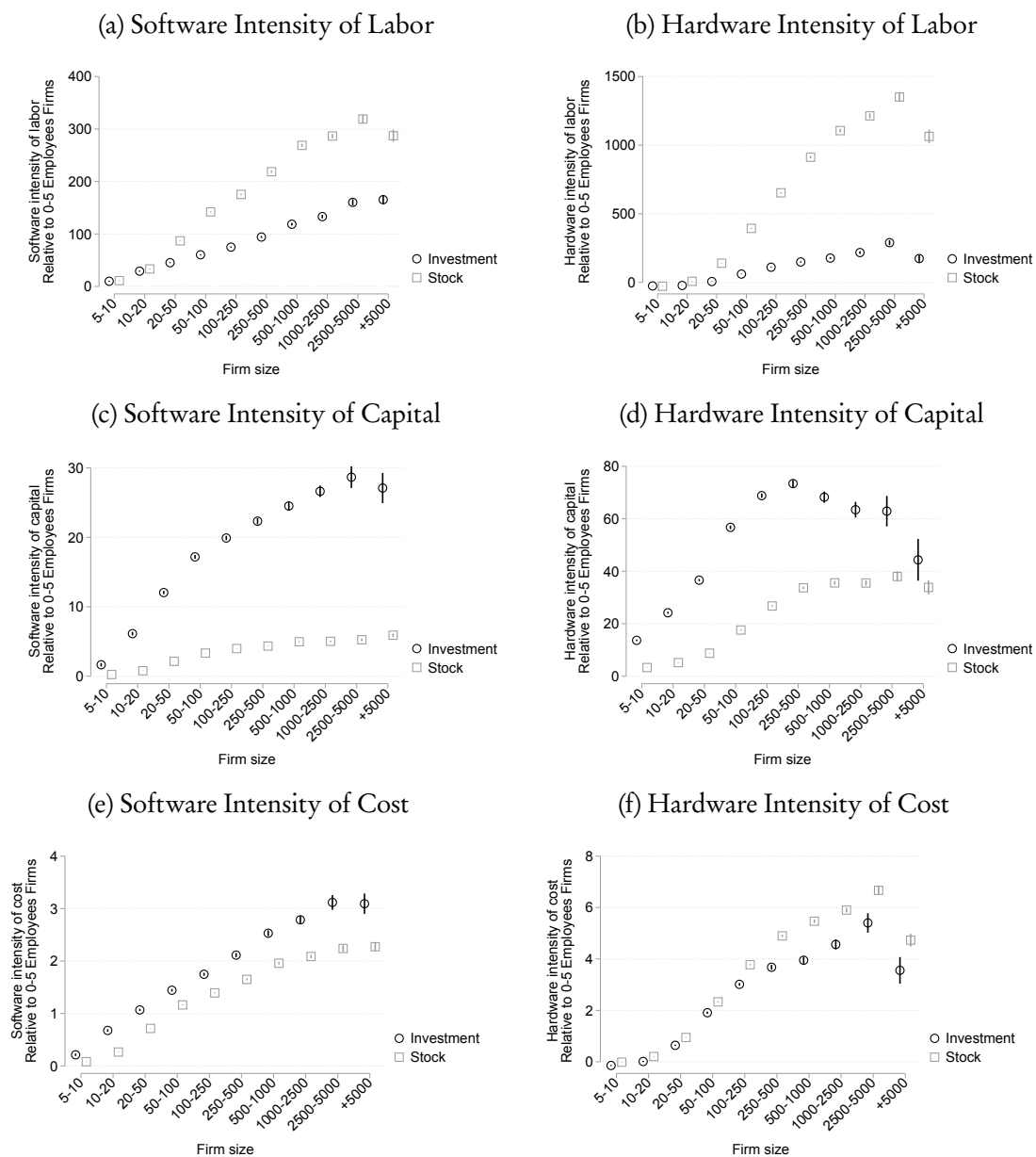


Figure 4.3: Labor Share, Concentration, and Firm Size



Figure 4.4: Macro Trends in France

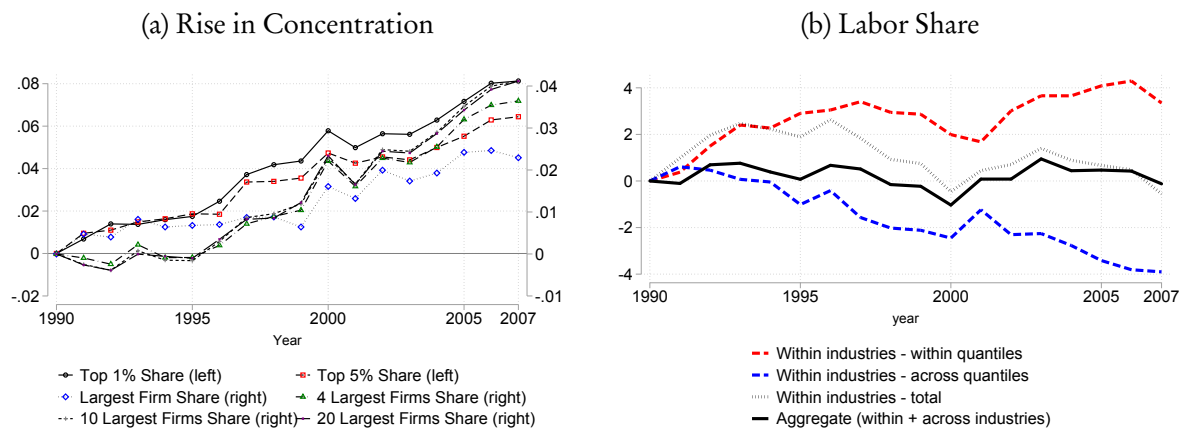


Figure 4.5: Cross-sectional Relationship between Size and IT Intensity/Labor Share

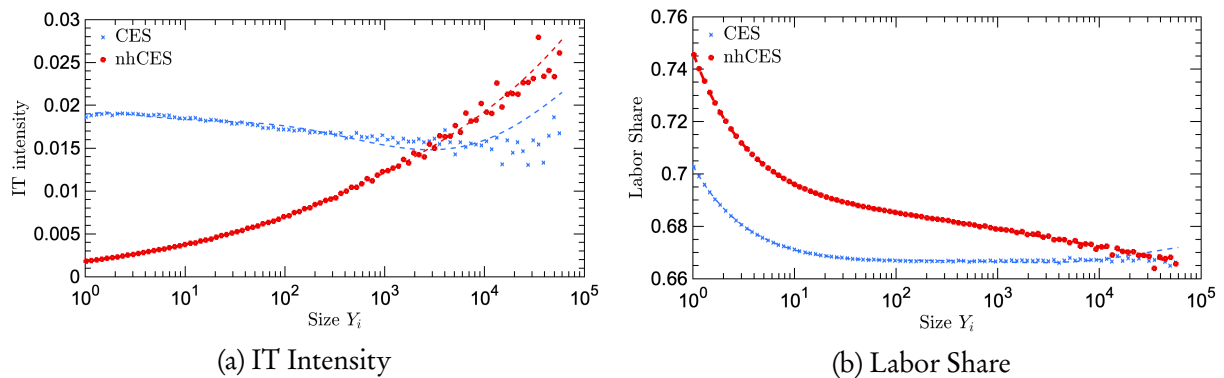


Figure 4.6: Heterogeneity in Responses to the IT Price Shock

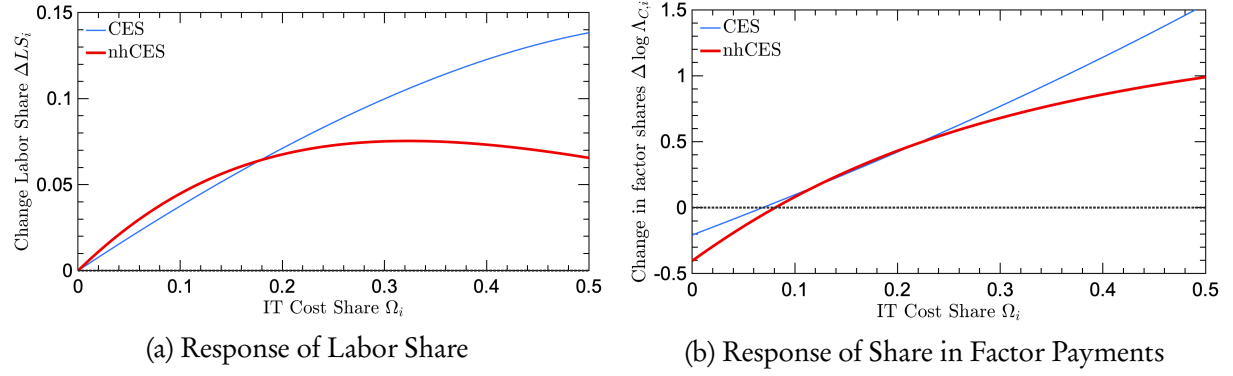


Figure 4.7: Aggregate Responses as a Function of the Size of IT Shock

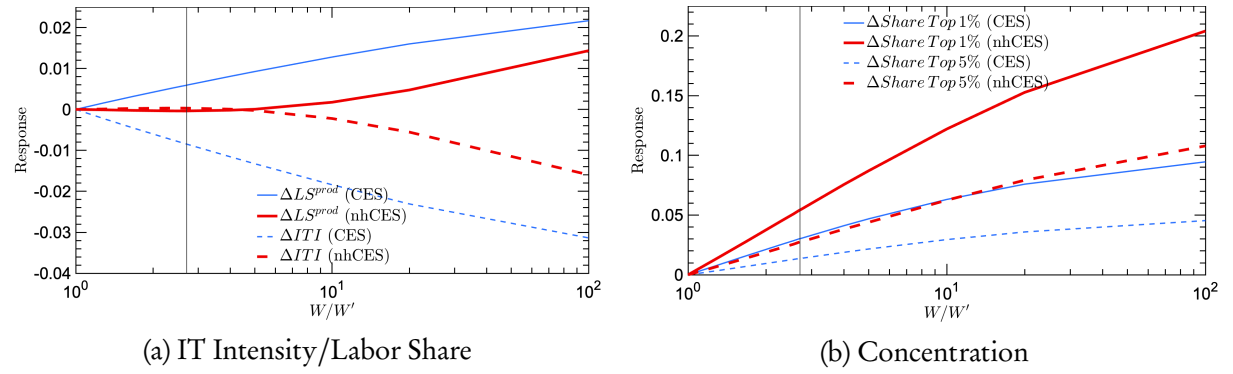


Figure 4.8: Production Isoquants, Output Elasticities, the Cost Function, and Cost Elasticities

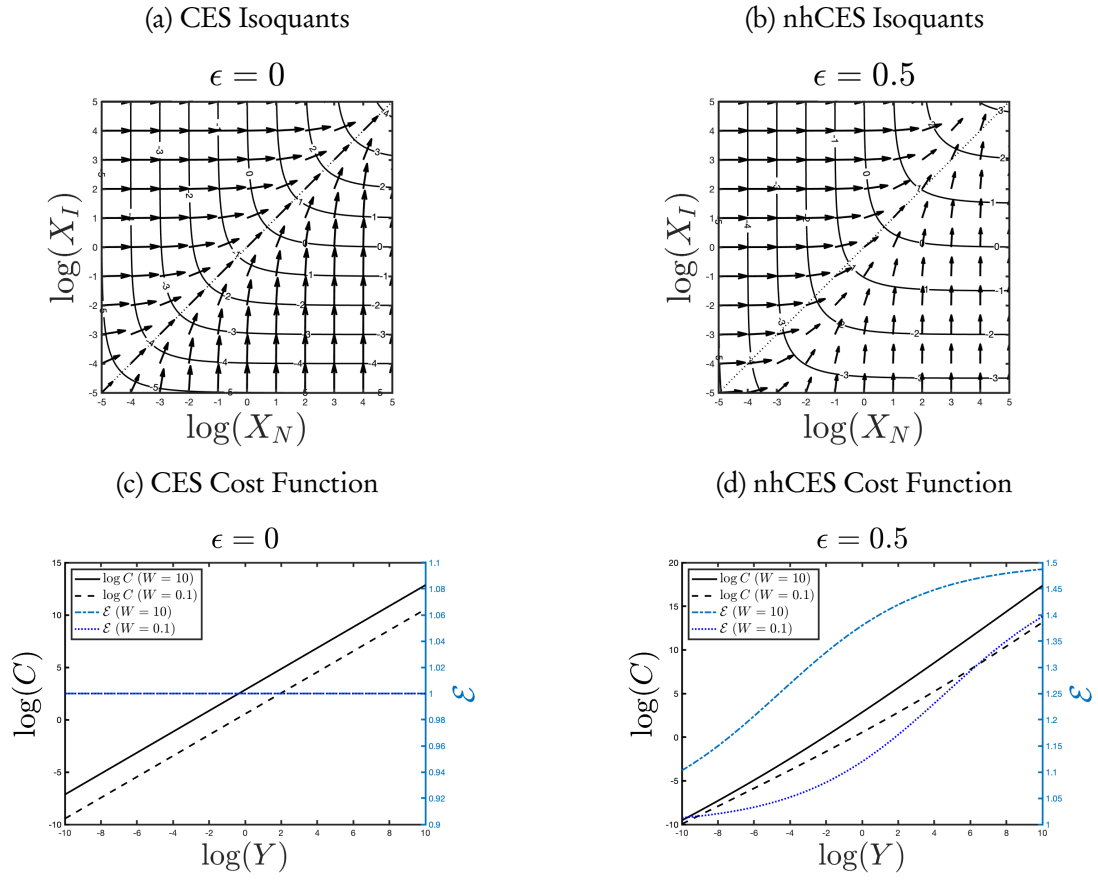
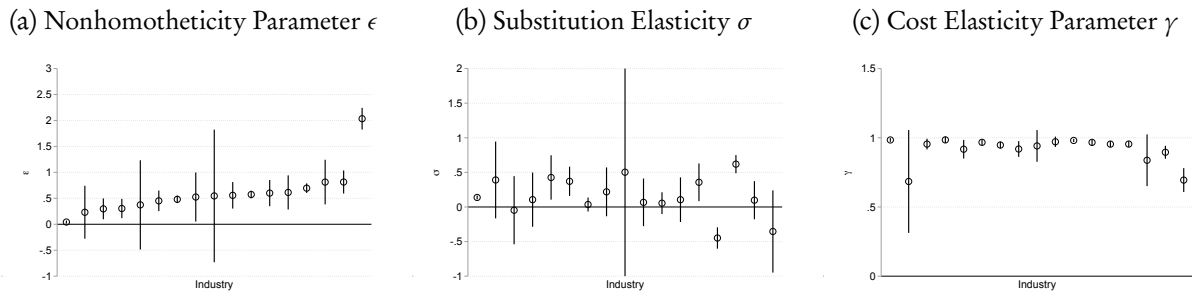


Figure 4.9: Industry-Level Estimates



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Appendix A

Tax Avoidance in Firms

Table A.0.1: Characteristics of the Compliers: Panel

Adjustment cost, Incentives, Ability to bunch						
	Low adjustment cost of capital		Large profit		Large profit in 1995	
	(1)	(2) FE	(3)	(4) FE	(5)	(6) FE
Compliers	0.0495*** (0.0174)	0.0464*** (0.0160)	0.0277* (0.0146)	0.0237* (0.0143)	0.00351 (0.0165)	0.000422 (0.0163)
Observations	4439	4386	4494	4441	3881	3841
Production function characteristics						
	Large elasticity wrt K		Large elasticity wrt L		Large elasticity wrt M	
[1em] Compliers	-0.0526*** (0.0184)	-0.0492*** (0.0187)	-0.00200 (0.0188)	-0.00163 (0.0173)	0.0598*** (0.0185)	0.0570*** (0.0178)
Observations	4439	4386	4439	4386	4439	4386

Note: This table estimates the different characteristics of the bunchers compared to other firms that were eligible to bunching. The sample is restricted to firms without extreme values of input shares. Firms eligible to bunching are the firms that were above threshold in the avoidance region in years during which there was no incentives to bunch. Characteristics of the bunchers are identified adapting [Diamond and Persson \(2016\)](#) technique to the time series setting. Variables are centered with the mean of the variable in the region just below the avoidance region: Turnover in 45000-47600. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics. Columns (2) (4) and (6) report estimation with region and

Table A.0.2: Characteristics of the Compliers: Cross-Section

Adjustment Cost and Incentives to Bunch						
	Low adjustment cost of capital		Large profit		Large profit in 1995	
	(1)	(2)	(3)	(4)	(5)	(6)
		FE		FE		FE
Compliers	0.205** (0.0797)	0.154* (0.0834)	0.270*** (0.0550)	0.222*** (0.0588)	0.181*** (0.0477)	0.147*** (0.0553)
Observations	906	906	912	912	801	801
Production Function Characteristics						
	Large elasticity wrt K		Large elasticity wrt L		Large elasticity wrt M	
[1em] Compliers	-0.0512 (0.0695)	-0.0171 (0.0831)	-0.0118 (0.0789)	-0.0417 (0.0907)	0.191** (0.0752)	0.155* (0.0831)
Observations	906	906	906	906	906	906

Note: This table estimates the different characteristics of the bunchers in 1997 compared to other firms that were eligible to bunching. Firms eligible to bunching are the firms that were above threshold in the avoidance region in years during which there was no incentives to bunch. Characteristics of the bunchers are identified following [Diamond and Persson \(2016\)](#) technique. Standard errors reported in parentheses are obtained by bootstrapping 500 times the test for the difference of characteristics. Column (2) (4) and (6) report estimation with region and 16 industry fixed effects. Output elasticities are computed following ([Levinsohn and Petrin, 2003a](#)) estimation procedure. Adjustment cost of capital is determined from ([Asker et al., 2014](#)) estimation procedure.

Table A.0.3: Consequences of Avoidance on Production Process: panel

	(1)	(2)	(3)	(4)	(5)
	Turnover	Y	Sold production	Change in inventories	Capitalized production
Avoidance	-79.03** (38.88)	-632.7* (370.2)	-815.7** (363.2)	130.7* (72.82)	52.26* (30.60)
Observations	3691	3691	3691	3691	3691

Note: This table estimates bunchers' choices. The sample is restricted to firms without extreme values of material, labor and capital shares nor negative values inputs.

Table A.0.4: Consequences of Avoidance on Production Process: cross section

	(1)	(2)	(3)	(4)
	ln Y	ln Sold Production	Change in inventories	Capitalized production
Avoidance	-0.219*** (0.0843)	-0.227*** (0.0803)	45.51 (53.68)	23.58 (61.68)
Observations	1772	1770	1772	1772

Note: This table estimates bunchers' choices. The sample is restricted to firms without extreme values of material, labor and capital shares nor negative values of inputs and to year 1997.

Table A.0.5: Consequences of Avoidance on Input Choices: Cross-Section

	(1) ln Y	(2) ln M/Y	(3) ln L/Y	(4) ln K/Y
Avoidance	0.0937 (0.0723)	-0.170** (0.0784)	0.0546*** (0.0121)	0.129*** (0.0282)
Observations	2278	2270	2270	2270

Note: This table estimates bunchers' choices. The sample is restricted to firms without extreme values of capital shares nor negative values of inputs and to year 1997.

Table A.0.6: Consequences of Avoidance on Input Choices: Panel

	(1) Y	(2) M over Y	(3) L over Y	(4) K over Y
Avoidance	-170.3 (275.5)	-0.199 (0.384)	0.155 (0.114)	0.137** (0.0550)
Observations	5274	5274	5274	5274

Note: This table estimates bunchers' choices. The sample is restricted to firms without extreme values of capital shares nor negative values of inputs and to year 1997.

Appendix B

Labor Share and Market Power

B.1 Data

Industry codes

Industry classification has changed over the 1985-2016 period. From 1985 to 1993 the classification in vigor was the NAP. It changed to NAF in 1993, to NAF rév. 1 in 2003 and finally to NAF rév. 2 in 2008. There is no one-to-one correspondence between these classifications. As a result we make the choice to map each NAP industry code to its most often associated NAF industry code. Similarly we map each NAF industry code to its most often associated NAF rév. 1 industry code, and each NAF rév. 1 code to its most often association NAF rév. 2. As a result we are able to associate to each firm for each year its industry code in the NAF rév. 2 classification.

Variable definitions

Our data provide information on total sales of goods, services and merchandises, as well as variations in inventory and immobilized production. For inputs, they provide the book value of tangible and intangible capital, the wage bill and payroll taxes, and the cost of materials, merchandise, and other intermediary inputs. All data on sales, cost of inventory variations and cost of inputs are recorded separately for merchandise and other inputs. We follow definitions from the National Accounts and define output as the sum of immobilized production, variations in inventory, and sales excluding the cost of merchandise; and we define intermediary inputs use as the sum of material expenditures minus inventory variations, and other external inputs. These definitions mean that gross output includes the net margin on merchandise sold, not gross sales of merchandise. Importantly, our data also includes in intermediary inputs the cost of purchased external services. Except for employment, our micro data is denominated in current prices, and we do not observe firm-level prices of intermediary and capital inputs, nor output prices. We deflate nominal values of gross output, intermediary inputs, and capital stock at the NA38 sectors level using price indexes for investment and outputs from the September 2018 release of the INSEE Annual National Accounts.

Data cleaning

We exclude micro-firms and profiled enterprises from the 2008-2016 data. Very high or negative observations of labor share that stem from very low or negative value-added observations relative to the firm average across years are replaced with the average labor share of the firm across years. Concentration measures are computed using sales on the entire sample of firms, labor share decomposition and all subsequent analysis are conducted on the sample of firms with at

least one salaried employee. The parameters of the translog production function are estimated using a smaller sample of firms with sales above 1M€, positive value-added, intermediary inputs, and capital. We also exclude from the estimation sample firms with wage, labor productivity, or capital per employee in the top or bottom 0.1%.

B.2 Decomposition

This section details the decomposition method we apply to aggregate labor share and aggregate inverse markups.

Industry level decomposition

Let $k \in \{1, \dots, K\}$ be some industry classification (e.g., 3 digits in micro data), M stands for an aggregate measure (labor share or markup). Also, let S_k and M_k stand respectively for the weight of the industry in total value-added or total sales, and the industry average measure. Define for any variable X :

$$\begin{aligned}\Delta X_t &\equiv X_t - X_{t-1}, \quad \bar{X}_t \equiv \frac{1}{2}(X_t + X_{t-1}), \\ \Delta_T X &\equiv X_T - X_0,\end{aligned}$$

where T is the last period and 0 is the first period. Our first decomposition is:¹

$$\Delta_T M \equiv \underbrace{\sum_{t=1}^T \sum_k \bar{S}_{kt} \Delta M_{kt}}_{\text{within industry}} + \underbrace{\sum_{t=1}^T \sum_k \Delta S_{kt} \bar{M}_{kt}}_{\text{across industries}}. \quad (\text{B.1})$$

This allows us to distinguish the extent to which the aggregate variation in markup or labor is due to a change of industry shares or a within industry variation, irrespective of the sectoral composition of the economy.

Within Industry Decomposition

Next, we focus on changes in the industry-level measure. Our aim is to decompose the changes at the industry level to the changes in the distribution of firm level markup or labor share and the changes in the markup or labor share for the firms of a given quantile. Let y denote firm quantile.

¹This is simply because:

$$\begin{aligned}\Delta(S_t M_t) &= \bar{S}_t \Delta M_t + \Delta S_t \bar{M}_t \\ \Delta_T(SM) &= \sum_{t=1}^T \Delta(S_t M_t).\end{aligned}$$

We can write the industry-level measure as

$$M_{kt} \equiv \int_{\underline{y}}^{\bar{y}} S_{kt}(y) M_{kt}(y) dy, \quad (\text{B.2})$$

where $S_{kt}(y)$ denotes the share of industry- k value added or sales that is in firms of quantile y at time t and $M_{kt}(y)$ denotes the weighted average outcome (labor share or markup) of firms of quantile y in industry k at time t . We can now decompose²

$$\Delta M_{kt} = \underbrace{\int_{\underline{y}}^{\bar{y}} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy}_{\text{Within quantile}} + \underbrace{\int_{\underline{y}}^{\bar{y}} \Delta S_{kt}(y) \bar{M}_{kt}(y) dy}_{\text{Cross quantile}}. \quad (\text{B.4})$$

Top firms are defined as low-labor-share firms or high-markup firms. We further decompose the within quantile component into three components: top firms component (for firms with y in the top/bottom 5%), above/below median component (for firms with y above/below the median but not top firms) and the rest. Let y^* be the threshold of size for being among the top firms, and y' the median size. We can write

$$\int_{\underline{y}}^{\bar{y}} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy = \underbrace{\int_{\underline{y}}^{y'} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy}_{\text{Within Below Median}} + \underbrace{\int_{y'}^{y^*} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy}_{\text{Within Above Median}} + \underbrace{\int_{y^*}^{\bar{y}} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy}_{\text{Within Top Firms}}. \quad (\text{B.5})$$

We now summarize the within-industry component change in aggregate measure into the following components:

1. The cross quantile component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \int_{\underline{y}}^{\bar{y}} \Delta S_{kt}(y) \bar{M}_{kt}(y) dy.$$

²As emphasized by [Kehrig and Vincent \(2018\)](#), this decomposition is conceptually distinct from standard within and cross firm decompositions. Let Ω_{kt} be the set of firms active in time t , and $\bar{\Omega}_{kt}$ be the set of firms common between time t and $t-1$, Ω_{kt}^+ the set of *new* firms at time t , and Ω_{kt}^- the set of firms exiting between time t and $t+1$. We can then write:

$$\Delta M_{kt} \equiv \underbrace{\sum_{i \in \Omega_t^-} \bar{S}_{it} \Delta M_{it}}_{\text{within firm}} + \underbrace{\sum_{i \in \Omega_{kt}^-} \Delta S_{it} \bar{M}_{it}}_{\text{cross firm}} + \underbrace{\left(\sum_{i \in \Omega_{kt}^+} S_{it} M_{it} - \sum_{i \in \Omega_{kt-1}^-} S_{it-1} M_{it-1} \right)}_{\text{net entry}}, \quad (\text{B.3})$$

where again shares are computed within the industry.

2. The within quantile component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \int_{\underline{y}}^{\bar{y}} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy,$$

which can then be further decomposed to

(a) Within top firms component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \int_{y^*}^{\bar{y}} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy,$$

(b) Within middle firms component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \int_{y'}^{y^*} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy,$$

(c) Within bottom firms component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \int_{\underline{y}}^{y'} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy.$$

We can also write the average variations in those three components across all industries, without weighing them to their industry level contributions :

(a) Average top firms component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \frac{\int_{y^*}^{\bar{y}} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy}{\int_{y^*}^{\bar{y}} \bar{S}_{kt}(y) dy}$$

(b) Average middle firms component:

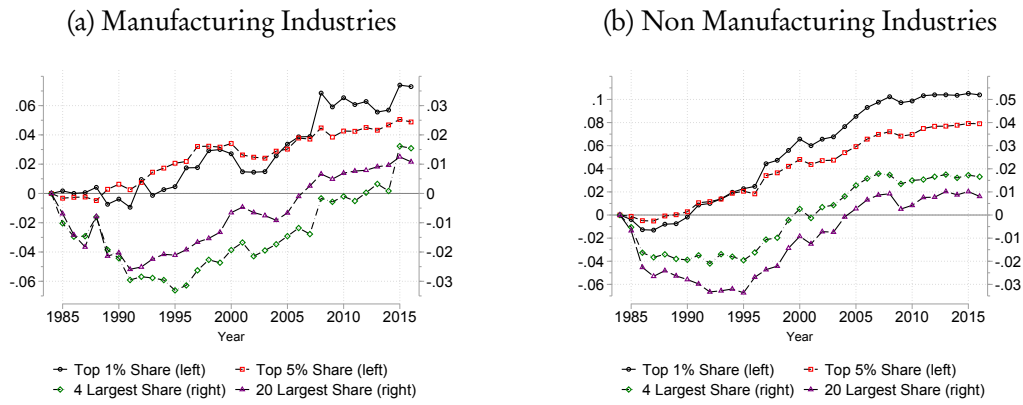
$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \frac{\int_{y'}^{y^*} \bar{S}_{kt}(y) \Delta M_{kt}(y) dy}{\int_{y'}^{y^*} \bar{S}_{kt}(y) dy}$$

(c) Average bottom firms component:

$$\sum_{t=1}^T \sum_k \bar{s}_{kt} \frac{\int_{\underline{y}}^{\underline{y}'} \bar{s}_{kt}(y) \Delta M_{kt}(y) dy}{\int_{\underline{y}}^{\underline{y}'} \bar{s}_{kt}(y) dy}$$

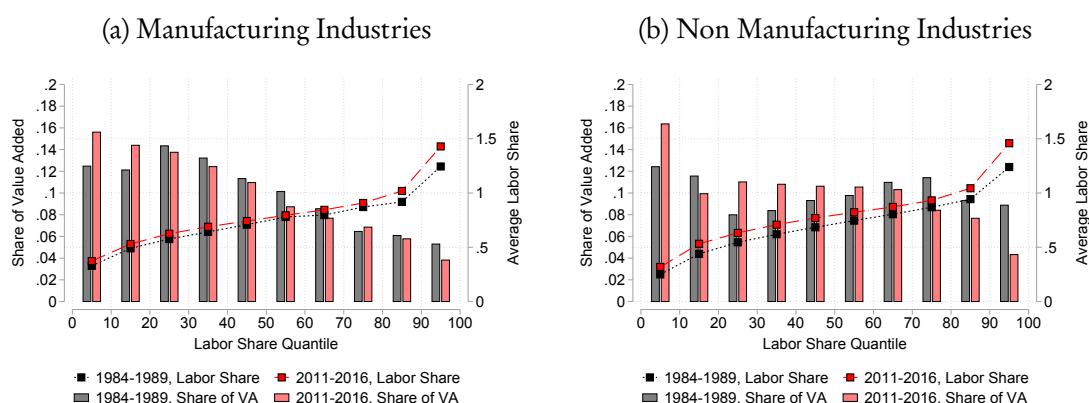
B.3 Figures

Figure B.3.1: Cumulative Change in Concentration



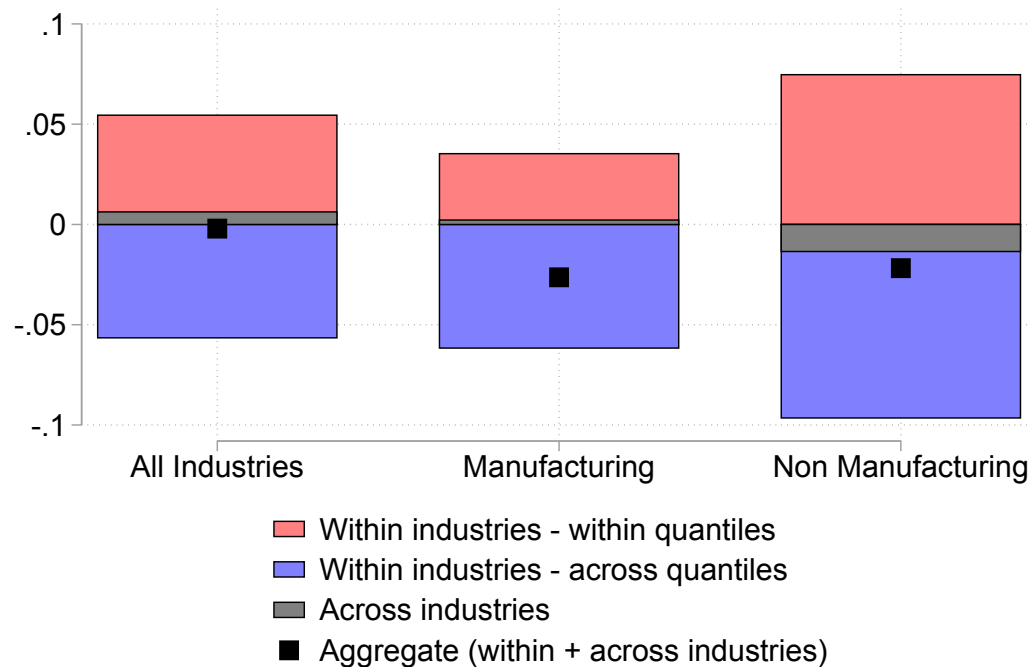
Note: This figure reports the cumulative change of concentration in sales across each 3-digit industry. Sample is firms in the market sectors, excluding agriculture, finance and real estate. Industry changes in concentration are weighted by the share of each industry in total sales the previous year.

Figure B.3.2: Distributions of Labor Shares and Value Added



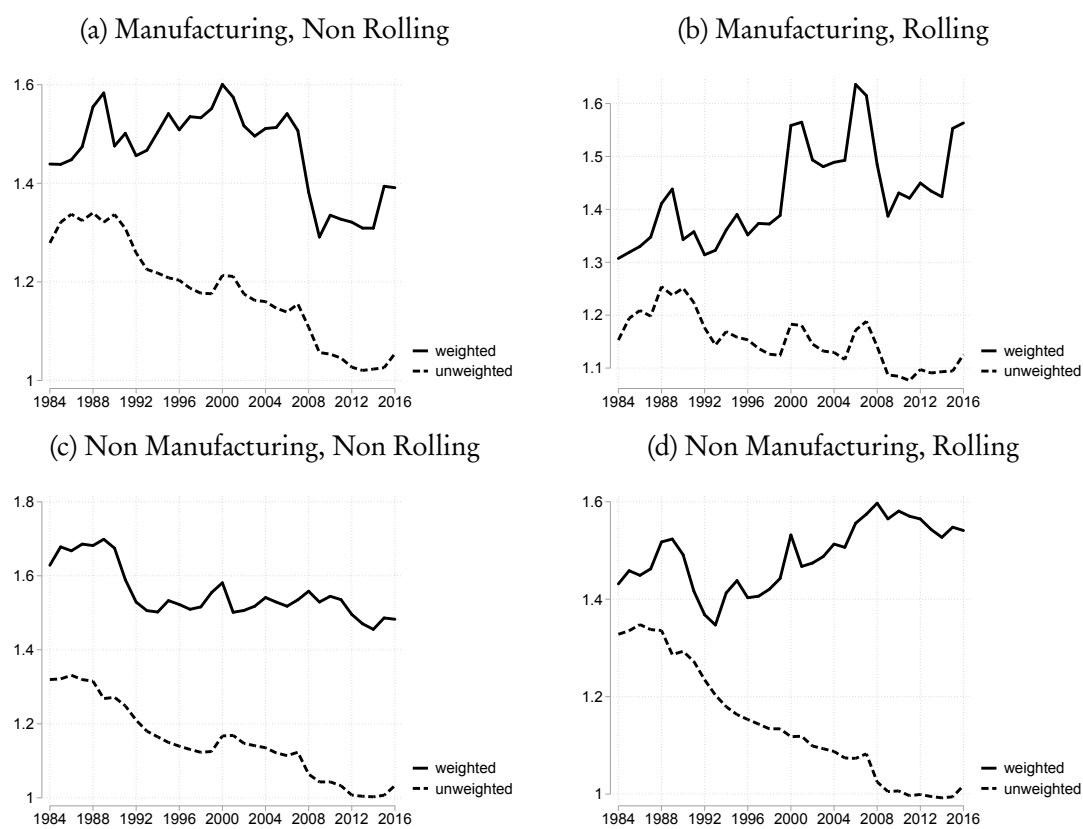
Note: The connected lines (right axis) reflect the raw cross-firm distribution of labor shares. The vertical bars (left axis) reflect the share of industry value added of firms in each unweighted decile of labor share. These distributions are averaged across 3-digit industries using value added weights in a given year, and then average across 5 year periods.

Figure B.3.3: Decomposition of the Cumulative Change in Aggregate Labor Share



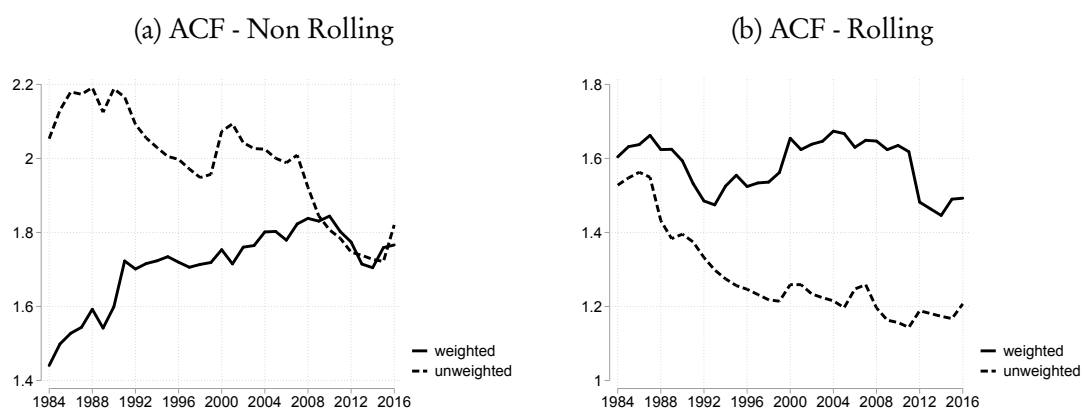
Note: This figures reports the results of decomposition of the aggregate labor share described in Appendix B.2. Quantiles of labor share are calculated each year within 3-digit industries.

Figure B.3.4: Aggregate Markup



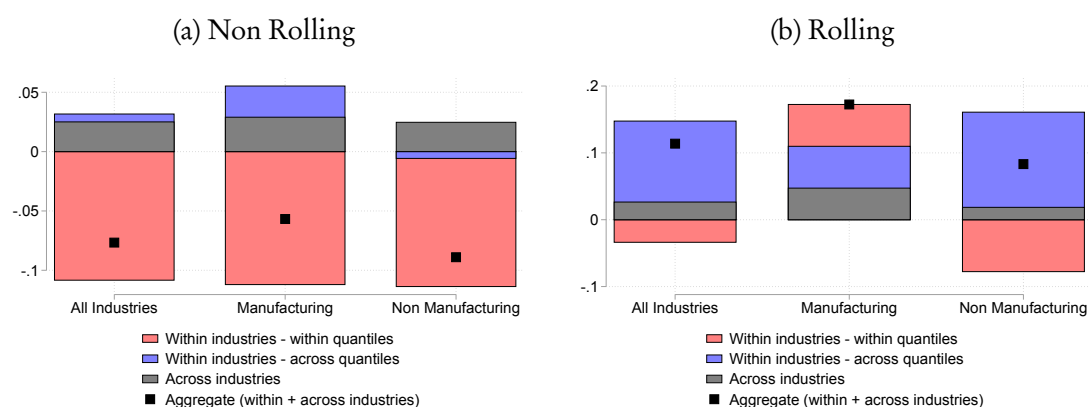
Note: This figures reports the levels of the weighted and unweighted mean markup based on non-rolling and rolling estimation of a translog value-added production function.

Figure B.3.5: Aggregate Markup - ACF



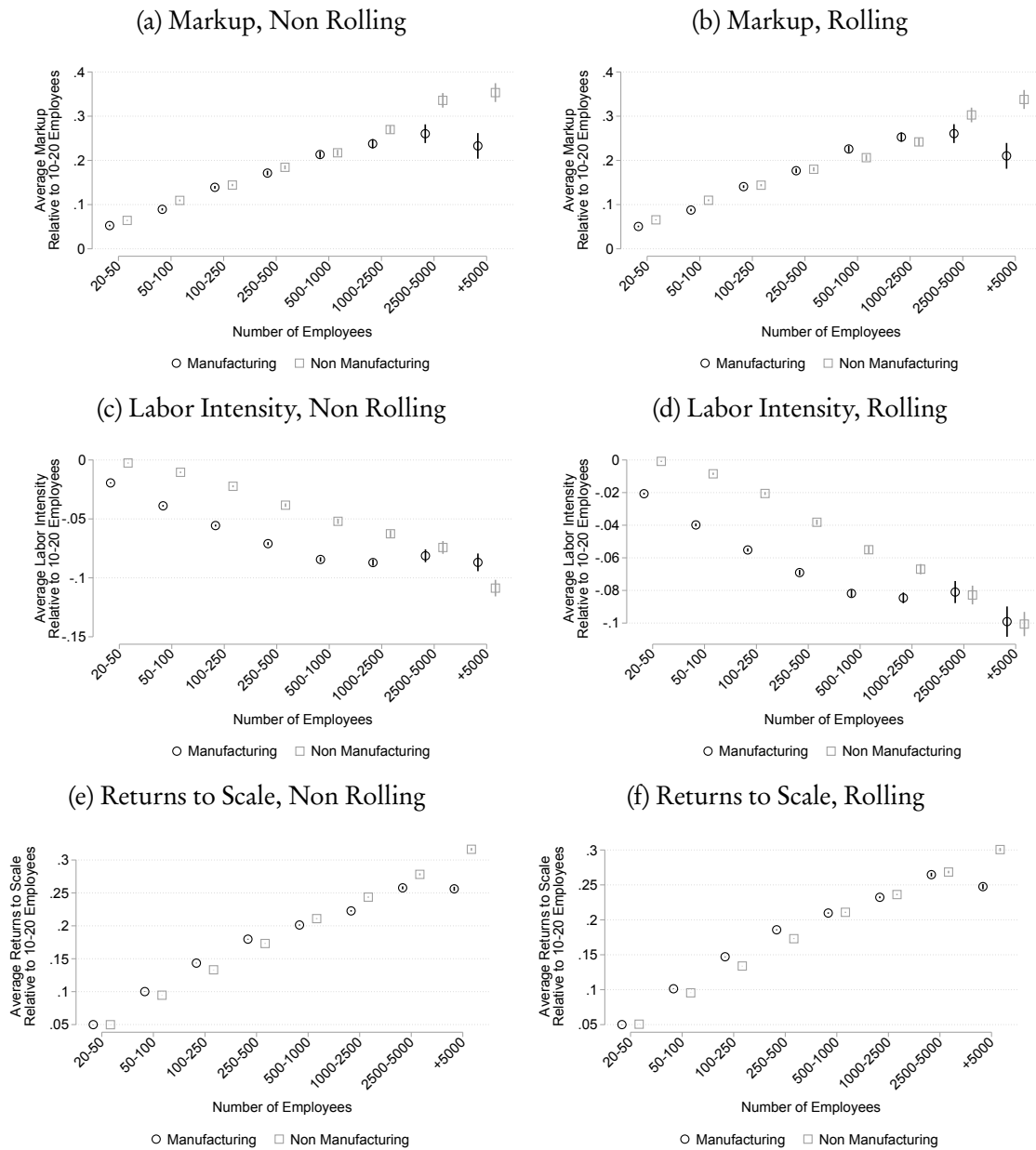
Note: This figures reports the levels of the weighted and unweighted mean markup based on non-rolling and rolling estimation of a translog value-added production function following the ACF procedure.

Figure B.3.6: Decomposition of Aggregate Markup



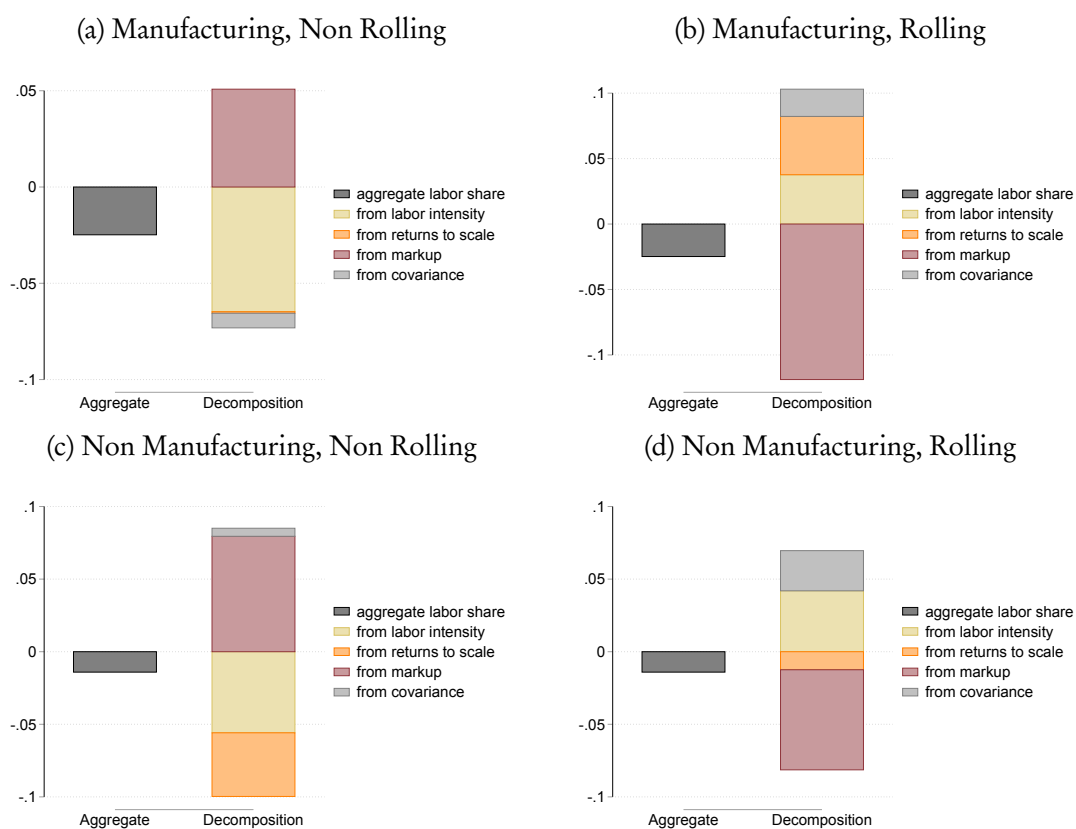
Note: This figures reports the results of decomposition of the aggregate markup described in Appendix B.2. Quantiles of markup are calculated each year within 3-digit industries.

Figure B.3.7: Correlations with Size



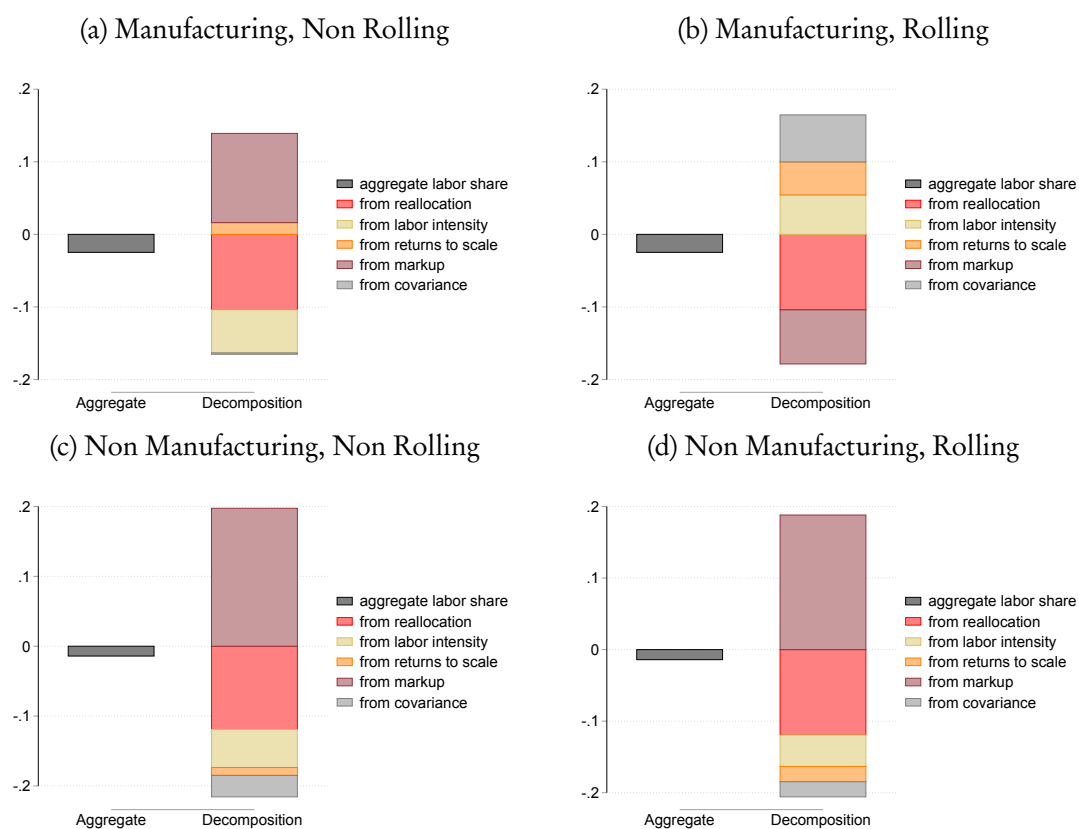
Note: This figure reports the conditional average markup, labor intensity and returns to scale by firm size, with 99% confidence interval. Averages are conditional on a set of flexible fixed effects constructed from the interaction of 3-digit industry codes and year.

Figure B.3.8: Contributions to the Evolution of the Aggregate Labor Share, Representative Firm



Note: This figure reports the decomposition of the variation of the aggregate labor share of the representative firm from 1984 to 2016, based on translog non-rolling and rolling value-added estimation of the production function. See section 3.5 for detail.

Figure B.3.9: Contributions to the Evolution of the Aggregate Labor Share, With Reallocation



Note: This figure reports the decomposition of the variation of the aggregate labor share from 1984 to 2016, including the reallocation term, based on translog non-rolling and rolling value-added estimation of the production function. See section 3.5 for detail.

B.4 Tables

Table B.4.1: Correlations Between Variations in Industry-Level Concentration and Labor Shares

	Industry Labor Share		Across Labor Share Quantiles		Within Low Labor Share Quantiles	
Manufacturing						
Top 1% Share	-0.0694 (0.0188)		-0.0793 (0.0156)		0.0387 (0.0139)	
Top 5% Share		-0.1178 (0.0254)		-0.1355 (0.0214)		0.0663 (0.0190)
Observations	2,131	2,143	2,135	2,143	2,130	2,142
R2	0.0805	0.0828	0.0553	0.0641	0.0622	0.0647
Manufacturing						
4 Largest Share	-0.0955 (0.0232)		-0.0852 (0.0196)		0.0424 (0.0177)	
20 Largest Share		-0.1764 (0.0285)		-0.1796 (0.0239)		0.0008 (0.0215)
Observations	2,121	2,147	2,122	2,148	2,119	2,144
R2	0.0816	0.0879	0.0580	0.0705	0.0618	0.0588
Non Manufacturing						
Top 1% Share	-0.0943 (0.0171)		-0.0458 (0.0161)		-0.0014 (0.0139)	
Top 5% Share		-0.1054 (0.0238)		-0.1448 (0.0223)		-0.0310 (0.0189)
Observations	2,202	2,201	2,202	2,196	2,192	2,190
R2	0.0234	0.0192	0.0298	0.0419	0.0354	0.0422
Non Manufacturing						
4 Largest Share	-0.0601 (0.0194)		-0.0629 (0.0181)		0.0690 (0.0156)	
20 Largest Share		-0.1102 (0.0208)		-0.1196 (0.0198)		0.0597 (0.0171)
Observations	2,172	2,145	2,176	2,150	2,163	2,137
R2	0.0146	0.0272	0.0355	0.0454	0.0448	0.0455

Note: Each estimate is the result of OLS estimation at the 3-digit industry with year fixed-effects. The dependent variable in columns "Industry Labor Share" is the long-term change of the industry aggregate labor share, defined as the ratio of the sum of firm level compensation and taxes paid on labor over the sum of firm level value added in that industry. The dependant variable in columns "Across Labor Share Quantiles" and "Within Low Labor Share Quantiles" are the corresponding contributions to the industry aggregate labor share according to the decomposition described in Appendix B.2. The independent variables are the changes of the share of sales of the top 1%, top 5%, 1 largest, 4 and 20 firms.

Table B.4.2: Median Output Elasticities, Non Rolling Estimation

	θ_l	θ_k	N		θ_l	θ_k	N
Mining	0.606 (0.048)	0.308 (0.081)	45,698	Gas and electricity	0.648 (0.193)	0.276 (0.169)	22,243
Food products	0.754 (0.053)	0.138 (0.100)	1,277,913	Water supply and waste	0.665 (0.141)	0.182 (0.125)	118,249
Textiles	0.569 (0.136)	0.110 (0.048)	282,598	Construction	0.643 (0.145)	0.063 (0.082)	4,969,117
Wood, paper and printing	0.813 (0.118)	0.048 (0.105)	552,510	Wholesale and retail trade	0.752 (0.171)	0.098 (0.138)	8,502,337
Coke and refined petroleum	0.720 (0.250)	0.341 (0.074)	2,472	Transportation	0.822 (0.151)	0.063 (0.145)	988,348
Chemicals	0.821 (0.059)	0.156 (0.073)	62,567	Accommodation and food services	0.590 (0.151)	0.197 (0.128)	3,076,031
Pharmaceuticals	1.013 (0.344)	0.067 (0.295)	11,657	Publishing and motion pictures	1.118 (0.237)	-0.005 (0.214)	309,540
Rubber and plastic products	0.772 (0.150)	0.127 (0.164)	245,896	Telecommunications	1.160 (0.187)	-0.083 (0.213)	25,191
Basic Metals	0.734 (0.131)	0.115 (0.094)	545,742	ICT	0.937 (0.128)	-0.015 (0.135)	324,622
Computers and electronics	0.757 (0.071)	0.104 (0.023)	110,072	Legal, accounting and engineering	0.857 (0.150)	-0.017 (0.144)	1,499,590
Electrical equipments	0.749 (0.026)	0.130 (0.048)	50,476	Scientific research	0.956 (0.242)	0.067 (0.211)	30,461
Machinery and equipments	0.842 (0.071)	0.076 (0.046)	161,603	Advertising and market research	1.006 (0.091)	-0.109 (0.092)	406,636
Transport equipments	0.840 (0.159)	0.122 (0.139)	71,000	Administrative and support	0.737 (0.120)	0.052 (0.157)	1,401,753
Other manufacturing products	0.795 (0.089)	0.015 (0.073)	650,254	Total	0.734 (0.175)	0.086 (0.137)	25,744,576

Note: This table reports the output elasticities from non rolling estimation of the translog production function.

Columns θ_l and θ_k report the median estimated output elasticity with respect to each factor of production for the translog production function for all firms. Column N report the number of observations in each sector. Standard deviations (not standard errors) of the output elasticities are reported in brackets.

Table B.4.3: Median Output Elasticities, Rolling Estimation

	θ_l	θ_k	N		θ_l	θ_k	N
Mining	0.612 (0.199)	0.334 (0.162)	45,698	Gas and electricity	0.688 (0.190)	0.265 (0.174)	22,243
Food products	0.750 (0.052)	0.134 (0.104)	1,277,913	Water supply and waste	0.639 (0.178)	0.211 (0.146)	118,249
Textiles	0.540 (0.221)	0.118 (0.157)	282,598	Construction	0.605 (0.175)	0.084 (0.087)	4,969,117
Wood, paper and printing	0.803 (0.110)	0.050 (0.104)	552,510	Wholesale and retail trade	0.762 (0.175)	0.105 (0.145)	8,502,337
Coke and refined petroleum	0.258 (0.391)	0.433 (0.258)	2,472	Transportation	0.835 (0.156)	0.060 (0.148)	988,348
Chemicals	0.808 (0.143)	0.179 (0.122)	62,567	Accommodation and food services	0.585 (0.174)	0.192 (0.133)	3,076,031
Pharmaceuticals	0.981 (0.359)	0.106 (0.286)	11,657	Publishing and motion pictures	1.163 (0.245)	-0.023 (0.215)	309,540
Rubber and plastic products	0.759 (0.159)	0.136 (0.176)	245,896	Telecommunications	1.111 (0.242)	-0.056 (0.217)	25,191
Basic Metals	0.726 (0.128)	0.117 (0.095)	545,742	ICT	0.921 (0.140)	0.008 (0.140)	324,622
Computers and electronics	0.756 (0.084)	0.091 (0.068)	110,072	Legal, accounting and engineering	0.843 (0.164)	-0.012 (0.150)	1,499,590
Electrical equipments	0.774 (0.136)	0.142 (0.101)	50,476	Scientific research	0.881 (0.259)	0.017 (0.230)	30,461
Machinery and equipments	0.823 (0.137)	0.093 (0.069)	161,603	Advertising and market research	0.887 (0.269)	-0.056 (0.140)	406,636
Transport equipments	0.837 (0.180)	0.131 (0.156)	71,000	Administrative and support	0.753 (0.126)	0.047 (0.165)	1,401,753
Other manufacturing products	0.748 (0.129)	0.051 (0.080)	650,254	Total	0.733 (0.193)	0.096 (0.143)	25,744,576

Note: This table reports the output elasticities from rolling estimation of the production function. Columns θ_l and θ_k report the median estimated output elasticity with respect to each factor of production for the translog production function for all firms. Column N report the number of observations in each sector. Standard deviations (not standard errors) of the output elasticities are reported in brackets.

Table B.4.4: Correlations Between Variations in Industry-Level Concentration and Markup, Non Rolling

	Industry Markup		Across Markup Quantiles		Within High Markup Quantiles	
Manufacturing						
Top 1% Share	0.1759 (0.0242)		0.1480 (0.0202)		-0.0111 (0.0174)	
Top 5% Share		0.3109 (0.0329)		0.2818 (0.0274)		-0.0177 (0.0237)
Observations	2,122	2,131	2,129	2,138	2,125	2,136
R2	0.0893	0.0985	0.0543	0.0768	0.0536	0.0573
Manufacturing						
4 Largest Share	0.2511 (0.0303)		0.2081 (0.0258)		-0.0788 (0.0219)	
20 Largest Share		0.3305 (0.0371)		0.3267 (0.0314)		-0.0433 (0.0268)
Observations	2,112	2,138	2,118	2,144	2,113	2,139
R2	0.0918	0.0944	0.0628	0.0787	0.0580	0.0546
Non Manufacturing						
Top 1% Share	0.2438 (0.0207)		0.1465 (0.0201)		0.0534 (0.0152)	
Top 5% Share		0.2679 (0.0294)		0.2296 (0.0281)		0.0782 (0.0210)
Observations	2,218	2,213	2,211	2,205	2,215	2,212
R2	0.0809	0.0628	0.0311	0.0367	0.0490	0.0499
Non Manufacturing						
4 Largest Share	0.1671 (0.0241)		0.1565 (0.0230)		-0.0541 (0.0175)	
20 Largest Share		0.1891 (0.0267)		0.2202 (0.0251)		-0.0643 (0.0194)
Observations	2,191	2,165	2,185	2,159	2,191	2,165
R2	0.0443	0.0482	0.0286	0.0424	0.0495	0.0507

Note: Each estimate is the result of OLS estimation at the 3-digit industry with year fixed-effects. The dependent variable in columns "Industry Markup" is the long-term change of the industry aggregate markup. The dependant variable in columns "Across Markup Quantiles" and "Within High Markup Quantiles" are the corresponding contributions to the industry aggregate markup according to the decomposition described in Appendix B.2. The independent variables are the changes of the share of sales of the top 1%, top 5 %, largest 4 and largest 20 firms.

Table B.4.5: Correlations Between Variations in Industry-Level Concentration and Markup, Rolling

	Industry Markup		Across Markup Quantiles		Within High Markup Quantiles	
Manufacturing						
Top 1% Share	0.2142 (0.0340)		0.1339 (0.0299)		-0.0136 (0.0217)	
Top 5% Share		0.3852 (0.0471)		0.2882 (0.0417)		0.0279 (0.0297)
Observations	2,135	2,146	2,135	2,145	2,131	2,142
R2	0.0959	0.1068	0.0697	0.0755	0.0704	0.0675
Manufacturing						
4 Largest Share	0.2977 (0.0435)		0.1667 (0.0390)		-0.0237 (0.0274)	
20 Largest Share		0.4232 (0.0531)		0.2449 (0.0476)		0.0518 (0.0334)
Observations	2,126	2,152	2,126	2,152	2,117	2,143
R2	0.1004	0.1067	0.0636	0.0672	0.0632	0.0665
Non Manufacturing						
Top 1% Share	0.3458 (0.0403)		0.0906 (0.0381)		0.0804 (0.0192)	
Top 5% Share		0.3685 (0.0570)		0.0591 (0.0535)		0.0729 (0.0270)
Observations	2,199	2,194	2,183	2,178	2,198	2,193
R2	0.0515	0.0393	0.0406	0.0370	0.0267	0.0216
Non Manufacturing						
4 Largest Share	0.1344 (0.0475)		0.1047 (0.0432)		-0.0756 (0.0223)	
20 Largest Share		0.0327 (0.0533)		0.0968 (0.0484)		-0.1055 (0.0248)
Observations	2,174	2,147	2,156	2,130	2,175	2,149
R2	0.0248	0.0222	0.0422	0.0411	0.0259	0.0278

Note: Each estimate is the result of OLS estimation at the 3-digit industry with year fixed-effects. The dependent variable in columns "Industry Markup" is the long-term change of the industry aggregate markup. The dependant variable in columns "Across Markup Quantiles" and "Within High Markup Quantiles" are the corresponding contributions to the industry aggregate markup according to the decomposition described in Appendix B.2. The independent variables are the changes of the share of sales of the top 1%, top 5 %, largest 4 and largest 20 firms.

Table B.4.6: Correlation between Labor Share and Markup, Non Rolling

Labor Share	Manufacturing			Non Manufacturing		
	No FE	Industry FE	Firm FE	No FE	Industry FE	Firm FE
No Size Threshold						
Markup	-0.3766 (0.0099)	-0.3768 (0.0066)	-0.3614 (0.0087)	-0.3435 (0.0031)	-0.3703 (0.0025)	-0.3567 (0.0028)
Observations	4,189,494	4,189,478	4,133,837	21,365,067	21,365,055	20,958,750
R2	0.568	0.590	0.774	0.429	0.506	0.772
More than 50 Employees						
Markup	-0.4789 (0.0045)	-0.5017 (0.0051)	-0.5483 (0.0060)	-0.4019 (0.0041)	-0.4180 (0.0039)	-0.4678 (0.0054)
Observations	296,014	295,922	291,562	511,989	511,883	497,926
R2	0.566	0.640	0.806	0.523	0.578	0.822
More than 100 Employees						
Markup	-0.4490 (0.0040)	-0.4765 (0.0058)	-0.5204 (0.0071)	-0.3823 (0.0037)	-0.3988 (0.0047)	-0.4420 (0.0066)
Observations	157,602	157,514	155,793	240,699	240,504	234,975
R2	0.551	0.644	0.809	0.527	0.587	0.833
More than 1000 Employees						
Markup	-0.3782 (0.0125)	-0.4376 (0.0148)	-0.4396 (0.0168)	-0.3241 (0.0122)	-0.3387 (0.0084)	-0.3949 (0.0135)
Observations	10,154	9,347	9,238	16,530	15,958	15,601
R2	0.504	0.739	0.871	0.556	0.706	0.911

Note: Each estimate is the result of OLS estimation of firm level labor share on markups, for four samples: all firms, firms with more than 50 employees, 100 employees, and 1000 employees; and two panels: manufacturing and non manufacturing firms. All columns include year fixed effects. Columns "No FE" include no industry nor firm fixed effect. Columns "Industry FE" include 3-digit industry-level fixed effects. Columns "Firm FE" include firm-level fixed effects. Standard errors are clustered at the 3-digit x year industry level.

Table B.4.7: Correlation between Labor Share and Markup, Rolling

Labor Share	Manufacturing			Non Manufacturing		
	No FE	Industry FE	Firm FE	No FE	Industry FE	Firm FE
No Size Threshold						
Markup	-0.3625 (0.0090)	-0.3719 (0.0061)	-0.3566 (0.0081)	-0.3100 (0.0035)	-0.3485 (0.0024)	-0.3338 (0.0029)
Observations	4,189,494	4,189,478	4,133,837	21,365,067	21,365,055	20,958,750
R2	0.532	0.567	0.767	0.387	0.475	0.760
More than 50 Employees						
Markup	-0.4354 (0.0102)	-0.4727 (0.0061)	-0.5062 (0.0071)	-0.4013 (0.0038)	-0.4153 (0.0041)	-0.4617 (0.0054)
Observations	296,014	295,922	291,562	511,989	511,883	497,926
R2	0.506	0.600	0.783	0.511	0.567	0.817
More than 100 Employees						
Markup	-0.4004 (0.0074)	-0.4463 (0.0067)	-0.4754 (0.0083)	-0.3818 (0.0047)	-0.3981 (0.0050)	-0.4390 (0.0067)
Observations	157,602	157,514	155,793	240,699	240,504	234,975
R2	0.486	0.604	0.786	0.511	0.577	0.829
More than 1000 Employees						
Markup	-0.3053 (0.0138)	-0.3912 (0.0188)	-0.3805 (0.0214)	-0.3439 (0.0065)	-0.3624 (0.0077)	-0.4000 (0.0141)
Observations	10,154	9,347	9,238	16,530	15,958	15,601
R2	0.409	0.691	0.852	0.543	0.707	0.909

Note: Each estimate is the result of OLS estimation of firm level labor share on markups, for four samples: all firms, firms with more than 50 employees, 100 employees, and 1000 employees; and two panels: manufacturing and non manufacturing firms. All columns include year fixed effects. Columns "No FE" include no industry nor firm fixed effect. Columns "Industry FE" include 3-digit industry-level fixed effects. Columns "Firm FE" include firm-level fixed effects. Standard errors are clustered at the 3-digit x year industry level.

Appendix C

IT and Returns to Scale

C.1 Nonhomothetic Demand and Returns to Scale

In this section we examine the properties of general production functions that are compatible with nonhomothetic IT demand, that is,

$$\frac{\partial \log(X_I/X_N)}{\partial \log Y} = \eta, \quad \eta > 0. \quad (\text{C.1})$$

The following lemma establishes two properties for the elasticities of substitution and scale of the production functions that give rise to nonhomothetic IT demand with a constant relative output elasticity.

Lemma 4. *Consider a continuous, differentiable, and monotonically increasing production function $Y = \mathcal{F}(X_N, X_I)$ such that the set $\{(X_N, X_I) \geq 0 \mid Y \leq \mathcal{F}(X_N, X_I)\}$ is strictly convex for all Y . Assume that the corresponding cost minimization problem yields factor demand functions satisfying Equation (C.1). Then, along any expansion path in the (X_N, X_I) -space¹ the production function satisfies the following two properties:*

1. *The elasticity of substitution between IT and non-IT inputs is constant everywhere.*
2. *The production function is “not” homogeneous of a constant degree, that is, there is “no” $\epsilon > 0$ such that for all $Z > 0$ and all (X_N, X_I) the production function satisfies $\mathcal{F}(Z X_N, Z X_I) = Z^\epsilon \mathcal{F}(X_N, X_I)$.*

Proof. See the end of the section. □

¹An expansion path is a curve with constant marginal rate of transformation $\mathcal{F}_I/\mathcal{F}_N \equiv (\partial \mathcal{F}/\partial X_I)/(\partial \mathcal{F}/\partial X_N)$.

We can alternatively state the result of Lemma 4 in terms of the properties of the cost function, when we fix relative factor input prices W_I/W_N . The first part of the lemma tells us that the elasticity of substitution $\partial \log(X_I/X_N)/\partial \log(W_I/W_N)$ only depends on relative input prices W_I/W_N , and not output Y . In other words, if we consider in the space of inputs (X_N, X_I) paths characterized by a constant marginal rate of substitution, the normal vectors to this curve form parallel vectors everywhere along the curve.

More importantly, the second part says that the output elasticity of the cost function, which is the reciprocal of the scale elasticity, *varies* with the scale of the firm output Y . In other words, if the IT factor input has a higher output elasticity than the non-IT factor input, when firms change their scale of operation their scale elasticity is bound to change. Specifically, the lemma rules out a constant-returns-to-scale (CRS) production function.

We may naturally wonder whether the scale elasticity rises or falls with output. The next lemma shows that to answer this question we need to impose further structure on the production function. More specifically, the answer to this question hinges on whether the two inputs are gross complements or gross substitutes.

Lemma 5. *Consider a production function \mathcal{F} satisfying the conditions in Lemma 4. Assume, in addition, that the scale elasticity is constant in the limit that either input goes to zero, that is, there is some $\epsilon_k > 0$ such that for all $Z > 0$ we have*

$$\lim_{X_k \rightarrow 0} \frac{\mathcal{F}(Z X_N, Z X_I)}{\mathcal{F}(X_N, X_I)} = Z^{\epsilon_k}, \quad k \in \{N, I\}.$$

Then, if the elasticity of substitution is less (greater) than 1 and nonincreasing (nondecreasing) in the marginal rate of transformation $\mathcal{F}_I/\mathcal{F}_N$, the scale elasticity is monotonically decreasing (increasing) in output Y along a curve with constant marginal rate of transformation.

Proof. See the end of the section. □

Lemma 5 shows that, under fairly mild conditions on the production function, the relationship between scale elasticity and firm size is monotonic and depends on the elasticity of substitution between IT and non-IT inputs. In particular, when the elasticity of substitution is constant, with an elasticity of substitution below 1, larger firms have higher scale elasticities. The opposite is the case with an elasticity of substitution greater than 1. To better illustrate this result, below we will consider two extreme examples: the first being a production function with a zero elasticity of substitution between the two inputs and the second being the polar case with perfect substitutability. We will see that, as the firm size grows, the scale elasticity falls and rises in the first and second examples, respectively.

Example 1. Consider a nonhomothetic Leontief production function $Y = \mathcal{F}(X_N, X_I)$ defined implicitly through

$$Y = \min \{X_N, Y^{-\eta} X_I\}, \quad \eta > 0.$$

In any cost minimizing solution, we have $Y = X_N = X_I^{1/(1+\eta)}$, which implies $X_I/X_N = Y^\eta$ satisfying condition (C.1). Given factor prices (W_N, W_I) , the corresponding cost function is given by $C(W_N, W_I; Y) = W_N Y + W_I Y^{1+\eta}$. Therefore, the output elasticity of costs, which is the reciprocal of the scale elasticity, satisfies

$$\frac{\partial \log C}{\partial \log Y} = 1 + \eta \Omega_I(W_N, W_I; Y),$$

where we have defined $\Omega_I(W_N, W_I; Y) \equiv W_I Y^\eta / (W_N + W_I Y^\eta)$ as the cost share of IT-inputs. This share is monotonically increasing in the level of output Y , implying that the output elasticity of costs is increasing in output.

Example 2. Consider a linear production function $Y = \mathcal{F}(X_N, X_I)$ defined as

$$Y = X_N + Y^\eta X_I, \quad 0 < \eta < 1.$$

Although strictly speaking, this production function does not satisfy condition (C.1), its factor demand and cost functions demonstrate quasi-nonhomothetic behavior. In particular, when the scale of output is small enough to satisfy $W_I/W_N \geq Y^\eta$, we have $X_N = Y$ and $X_I = 0$. On the other hand, when the scale of output is large enough such that $W_I/W_N < Y^\eta$, we have $X_N = 0$ and $X_I = Y^{1-\eta}$. The corresponding cost function is

$$C(W_N, W_I; Y) = \begin{cases} W_N Y, & W_I/W_N \geq Y^\eta, \\ W_I Y^{1-\eta}, & W_I/W_N < Y^\eta. \end{cases}$$

In stark contrast to Example 1, in this case the output elasticity of costs falls as output rises. With $W_I/W_N \geq Y^\eta$, the production function has constant returns to scale whereas for $W_I/W_N < Y^\eta$, the scale elasticity is $1/(1-\eta)$ strictly greater than 1.

The intuition for the result above is as follows. As output rises, Equation (C.1) implies that IT inputs grow faster than non-IT inputs because of their higher output elasticity. The rise in the IT inputs raises the marginal product of the non-IT inputs, which means that we have to further raise the non-IT input in order to again equalize the marginal product and the non-IT factor price. When the elasticity of substitution is below 1, the required rise in the non-IT input is sufficiently high to make the cost function increasingly convex as the output rises. The situation is reverse

when the elasticity of substitution is greater than 1.

C.1.1 Connection to the [Lucas](#)' Span-of-Control Model

[Lucas \(1978\)](#) assumed that managerial productivity is heterogeneous across firms, and the equilibrium size of firms is determined through span-of-control limits. Specifically, he assumed that $X_N = f(K, L)$ is a constant-returns-to-scale aggregator of capital and labor, and the output is given by $Y = B g(X_N)$ where $g(\cdot)$ is a concave function and B is the managerial productivity of the firm. Comparing this specification with our construction in Section 4.4.2.1, consider the case of $\gamma = 1$, Cobb-Douglas functional forms $f(K, L) = K^\alpha L^{1-\alpha}$, and $g(X_N) = X_N^{1-\psi}$.

Next, we provide a mapping between the specification above for the [Lucas](#) model and our construction in Section 4.4.2.1. Without loss of generality, we generalize Equation (4.13) to $\mathcal{X} = \left(\xi^{\frac{1}{\sigma}} X_N^{1-\frac{1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}} O^{1-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$ with coefficients $\xi \in (0, 1)$. Substituting the expression for O in this equation, we find

$$\mathcal{X} = \left(\xi^{\frac{1}{\sigma}} (X_N)^{\frac{\sigma-1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}} (e^\phi X_I Y^{-\epsilon})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \quad (\text{C.2})$$

Now, let us apply the l'Hopital's rule, to find the limit of the expression as $\sigma \rightarrow 1$:

$$\lim_{\sigma \rightarrow 1} \log \frac{\mathcal{X}}{X_N} = \theta + (1-\xi) \left(\phi + \log \left(\frac{X_I}{X_N} \right) - \epsilon \log Y \right) + \omega,$$

where ω is a constant.

Substituting the expression above in Equation (4.12) with $\gamma = 1$, we find:

$$Y = e^{\omega + \frac{\theta + (1-\xi)\phi}{1+(1-\xi)\epsilon}} \times X_I^{\frac{1-\xi}{1+(1-\xi)\epsilon}} \times X_N^{\frac{\xi}{1+(1-\xi)\epsilon}}.$$

Comparing this equation with the Lucas specification $Y = B X_N^{1-\psi}$, we find that the two coincide if we set

$$\begin{aligned} \psi &\equiv \frac{(1-\xi)(1+\epsilon)}{1+(1-\xi)\epsilon}, \\ B &\equiv e^{\omega + \frac{\theta + (1-\xi)\phi}{1+(1-\xi)\epsilon}} \times X_I^{\frac{1-\xi}{1+(1-\xi)\epsilon}}. \end{aligned}$$

This expression is in line with the [Lucas](#) model, assuming that we take the input X_I of each firm to be an exogenous value.

C.1.2 A Fixed-Cost Model of IT

Consider the following fixed-cost-of-IT production function $Y = Z\mathcal{F}_c(X_N, X_I)$ defined as

$$Y = \begin{cases} ZX_N, & X_I = 0, \\ \Delta_I Z \left(\frac{X_I}{\xi}\right)^\xi \left(\frac{X_N - \psi_I}{1-\xi}\right)^{1-\xi}, & X_I > 0, \end{cases}$$

where ψ_I and Δ_I are constants that captures the fixed cost of adopting IT (in units of non-IT inputs) and its corresponding productivity premium, respectively. In order for IT to be adopted by some firms, assume that $\Delta_I > W^\xi$, where W is the relative price of non-IT inputs. We can then show that the cost function is given by

$$\mathcal{C}(Y) = \begin{cases} \frac{Y}{Z}, & Y \leq Y^*, \\ \psi_I + \frac{W^\xi}{\Delta_I} \frac{Y}{Z}, & Y \geq Y^*, \end{cases}$$

where $Y^* = Z\psi_I/(1 - W^\xi/\Delta_I)$, denotes the threshold of firm size above which firms adopt nonzero IT inputs. Accordingly, the share of IT in total costs is given by

$$\Psi = \frac{WX_I}{\mathcal{C}(Y)} = \begin{cases} 0, & Y \leq Y^*, \\ \frac{\xi}{1 + \psi_I \frac{\Delta_I}{W^\xi} \frac{Z}{Y}}, & Y \geq Y^*, \end{cases}$$

which increases from 0 to $\xi > 0$ as the size Y goes from zero to infinity.

We can characterize the output elasticity of relative demand and the elasticity of substitution between IT and non-IT inputs as follows. First, we calculate the the elasticity of relative demand with respect to output:

$$\frac{\partial \log(X_I/X_N)}{\partial \log Y} = \frac{\partial \log}{\partial \log Y} \left(\frac{\xi/W}{1 - \xi + \psi_I \frac{\Delta_I}{W^\xi} \frac{Z}{Y}} \right) = \frac{1}{1 + (1 - \xi) \frac{W^\xi}{\psi_I \Delta_I} \frac{Y}{Z}} > 0.$$

This shows that the IT intensity is increasing in firm size both on the extensive and the intensive margins. However, this elasticity converges to zero as Y goes to infinity. Next, we derive the elasticity of substitution between IT and non-IT inputs

$$-\frac{\partial \log(X_I/X_N)}{\partial \log W} = -\frac{\partial \log}{\partial \log W} \left(\frac{\xi/W}{1 - \xi + \psi_I \frac{\Delta_I}{W^\xi} \frac{Z}{Y}} \right) = 1 - \frac{1}{1 + (1 - \xi) \frac{W^\xi}{\psi_I \Delta_I} \frac{Y}{Z}},$$

which we find to be less than unity.

Finally, let us examine the returns to scale properties of the production function. First, the production function features increasing returns to scale for $Y \geq Y^*$. Second, the scale elasticity is decreasing in size for $Y \geq Y^*$, as its reciprocal the cost elasticity is given by

$$\mathcal{E}(Y) = \frac{\partial \log \mathcal{C}(Y)}{\partial \log Y} = \frac{1}{1 + \psi_I \frac{\Delta_I}{W^\xi} \frac{Z}{Y}},$$

and increases from W^ξ / Δ_I to 1 as Y goes from Y^* to infinity.

C.1.3 Aggregate Elasticity of Substitution

To characterize the two components of Equation (4.33), the following lemma first presents the comparative statics of the IT cost share and output with respect to the relative IT price.

Lemma 6. *The response of IT share and the output of firms with productivity state ϑ are given by*

$$\frac{d\tilde{\Omega}(\vartheta)}{dw} = (1 - \sigma)\tilde{\Omega}(\vartheta)(1 - \tilde{\Omega}(\vartheta)) \frac{\left[1 + \zeta \epsilon \left(\frac{dp}{dw} + \frac{dy}{\lambda dw}\right)\right] \left(1 + \frac{\epsilon}{\gamma} \tilde{\Omega}(\vartheta)\right)}{1 + \frac{\epsilon}{\gamma} \tilde{\Omega}(\vartheta) \left[1 + \zeta \left(\gamma + \eta (1 - \tilde{\Omega}(\vartheta))\right)\right]}, \quad (\text{C.3})$$

$$\frac{d\tilde{y}(\vartheta)}{dw} = -\zeta \frac{\tilde{\Omega}(\vartheta) \left[1 + \frac{\eta}{\gamma} (1 - \tilde{\Omega}(\vartheta))\right] - \left(\frac{dp}{dw} + \frac{dy}{\lambda dw}\right) \left(1 + \frac{\epsilon}{\gamma} \tilde{\Omega}(\vartheta)\right)}{1 + \frac{\epsilon}{\gamma} \tilde{\Omega}(\vartheta) \left[1 + \zeta \left(\gamma + \eta (1 - \tilde{\Omega}(\vartheta))\right)\right]}, \quad (\text{C.4})$$

where we have used the definitions $\eta \equiv (1 - \sigma)\epsilon$, and where dp/dw and dy/dw are the responses of the aggregate price index and output to the change in the relative price of IT.

Lemma 6 shows that the heterogeneity across firms in their responses of both IT intensity and market share only depends on their initial IT intensity $\tilde{\Omega}(\vartheta)$. Due to the CES property of the production function, the response of IT intensity is proportional to $1 - \sigma$, where σ is the elasticity of substitution between IT and non-IT inputs. In addition to this substitution, the response of IT intensity in Equation (C.3) partly depends on the response of the firm's output due to nonhomotheticity: if a firm's output grows, then its IT intensity rises. Equation (C.4) shows that, to the first order of approximation, the pass-through from IT price to the firm prices is in line with Equation (4.36) and proportional to the IT intensity $\tilde{\Omega}(\vartheta)$ of firms. However, for firms with larger IT intensity $\tilde{\Omega}(\vartheta)$, the full response is more complicated since the marginal cost function nonlinearly varies both in IT prices and in the size of the firm.

Using the results of Lemma 6, the following proposition characterizes the within and across firm components of the response of the aggregate IT cost share. Assume that the ratio of fixed to

average costs is small, i.e., $\psi/\bar{C} \ll 1$ and define:

$$\bar{\bar{\Omega}}^n \equiv \int \int \left[\frac{\tilde{\Omega}^n(\vartheta)}{1 + \frac{\epsilon}{\gamma} \tilde{\Omega}(\vartheta) [1 + \zeta (\gamma + \eta (1 - \tilde{\Omega}(\vartheta)))]} \right] \Lambda(\vartheta) d^2 \vartheta, \quad \text{for } 0 \leq n \leq 3, \quad (\text{C.5})$$

Then, the within and across firm effects in Equation (4.33) are given by

$$\text{Within-firm effect} = (1 - \sigma) \left[1 + \zeta \epsilon \left(\frac{dp}{dw} + \frac{dy}{\lambda dw} \right) \right] \left[\bar{\bar{\Omega}} - \bar{\bar{\Omega}}^2 + \frac{\epsilon}{\gamma} (\bar{\bar{\Omega}}^2 - \bar{\bar{\Omega}}^3) \right], \quad (\text{C.6})$$

$$\begin{aligned} \text{Across-firm effect} = & \left[1 + \zeta \epsilon \left(\frac{dp}{dw} + \frac{dy}{\lambda dw} \right) \right] \left[\frac{\lambda - 1}{\lambda \epsilon} \left(\bar{\bar{\Omega}} + \frac{\epsilon}{\gamma} \bar{\bar{\Omega}}^2 - \bar{\bar{\Omega}} \left(\bar{\bar{\Omega}}^0 + \frac{\epsilon}{\gamma} \bar{\bar{\Omega}} \right) \right) \right. \\ & \left. + \frac{\eta}{\gamma} (\bar{\bar{\Omega}}^2 - \bar{\bar{\Omega}}^3 - \bar{\bar{\Omega}} (\bar{\bar{\Omega}} - \bar{\bar{\Omega}}^2)) \right], \end{aligned} \quad (\text{C.7})$$

where the latter term ignores the terms first-order in ψ/\bar{C} . To unpack the different terms in Equations (C.6) and (C.7), let us first examine the special case of homothetic CES production functions. In the case of a homothetic production function, $\epsilon = 0$, the within and across firm effects in Equation (4.33) are given by

$$\text{Within-firm effect} = (1 - \sigma) (\bar{\bar{\Omega}} - \bar{\bar{\Omega}}^2), \quad (\text{C.8})$$

$$\text{Across-firm effect} = -(\gamma \zeta - 1) (\bar{\bar{\Omega}}^2 - \bar{\bar{\Omega}}^2). \quad (\text{C.9})$$

Equations (C.8) and (C.9) show that the signs of the two components critically depend on whether the two elasticities of substitution σ and λ exceed unity. In particular, when $\sigma < 1$ and $\lambda > 1$ the within-firm effect is positive while the across-firm is negative. The equations also generalize the results of Oberfield and Raval (2014b) and Baqaee and Farhi (2018) to a non-CRS case. In particular, they show that if there are increasing returns to scale, $\gamma < 1$, the reallocation response is greater compared to the CRS case, and the reverse holds if there are decreasing returns, $\gamma > 1$. As with the CRS case, the relative size of the two components depends on the dispersion of IT intensity, as captured in the factor-payment-weighted variance of the IT cost share, $\bar{\bar{\Omega}}^2 - \bar{\bar{\Omega}}^2$.

Comparing the results of Corollary C.1.3 with those in the general case in Proposition C.1.3, we first note that Equation (C.5) defines the four moments that account for the nonlinearities in the marginal cost function that stem from the nonhomotheticity. In addition, we find that in the nonhomothetic case, $\epsilon \neq 0$, the general equilibrium effect on the average firm's output affects the response. This effect is captured by the term $dp/dw + dy/\lambda dw$ in Equations (C.6)

and (C.7) and implies a larger response of the aggregate IT intensity to a fall in IT price relative to a CES benchmark if $\eta = (1 - \sigma)\epsilon > 0$. Due to nonhomotheticity of IT demand, a change in factor prices that results in a larger average firm size leads to higher IT intensity for the average firm.

C.2 Derivations and Proofs

C.2.1 Proofs

Lemma 7. *The output elasticity of the marginal costs is given by*

$$\frac{YC''(Y)}{C'(Y)} = \mathcal{E}(Y) \left(1 + (1 - \sigma) \frac{\mathcal{V}(Y)}{\mathcal{E}(Y)^2} \right) - 1, \quad (\text{C.10})$$

where we have suppressed the dependence on factor prices (W_N, W_I) to simplify the expression and have defined $\mathcal{V}(\cdot)$ as the variance of the income elasticity parameters:

$$\mathcal{V}(Y) \equiv \epsilon^2 \Omega(Y) [1 - \Omega(Y)]. \quad (\text{C.11})$$

Proof. We provide the proof for a generalized version of the lemma, for a nonhomothetic CES production function with J inputs $\{X_j\}_{j=1}^J$, where output Y satisfies $\sum_j \left(\frac{X_j}{Y^{\epsilon_j}} \right)^{\frac{\sigma-1}{\sigma}} = 1$. Equation (4.9) corresponds to the case of $\epsilon_N = \gamma$ and $\epsilon_I = \gamma + \epsilon$. The cost function \mathcal{C} then satisfies $\mathcal{C}(Y)^{1-\sigma} = \sum_j \left(Y^{\epsilon_j} W_j \right)^{1-\sigma}$ where W_j is the factor price of input j , and the cost elasticity is given by $\mathcal{E}(Y) = \sum_j \left(\frac{W_j}{\mathcal{C}} Y^{\epsilon_j} \right)^{1-\sigma} \epsilon_j$.

We can write the second order derivative of the cost function as

$$\begin{aligned} \mathcal{C}''(Y) &= \left(\frac{\mathcal{C}}{Y} \mathcal{E} \right)', \\ &= \mathcal{E} \frac{\mathcal{C}}{Y^2} \left(\frac{Y \mathcal{C}'}{\mathcal{C}} - 1 \right) + \frac{\mathcal{C}}{Y} \mathcal{E}', \\ &= \mathcal{E} \frac{\mathcal{C}}{Y^2} \left(\mathcal{E} - 1 + \frac{Y \mathcal{E}'}{\mathcal{E}} \right), \end{aligned} \quad (\text{C.12})$$

where \mathcal{E} is the cost elasticity function defined above. The 2nd order scale elasticity is then given

by

$$\begin{aligned}
\frac{Y\mathcal{E}'}{\mathcal{E}} &= \frac{Y}{\mathcal{E}} \frac{d}{dY} \left(\sum_j \left(\frac{W_j}{\mathcal{C}} Y^{\epsilon_j} \right)^{1-\sigma} \epsilon_j \right), \\
&= \frac{1-\sigma}{\mathcal{E}} \left(\sum_j \left(\frac{W_j}{\mathcal{C}} Y^{\epsilon_j} \right)^{1-\sigma} \epsilon_j \left(\epsilon_j - \frac{Y\mathcal{C}'}{\mathcal{C}} \right) \right), \\
&= \frac{1-\sigma}{\mathcal{E}} \left(\sum_j \Omega_j \epsilon_j^2 - \mathcal{E}^2 \right),
\end{aligned} \tag{C.13}$$

where Ω_j is the factor intensity defined by $\Omega_j \equiv \left(Y^{\epsilon_j} W_j / \mathcal{C} \right)^{1-\sigma}$. We can write the expression within the parentheses as the variance of the parameters $\{\epsilon_j\}_{j=1}^J$ under the distribution implied by factor shares $\{\Omega_j\}_{j=1}^J$, defined as $\mathcal{V}(Y) \equiv \sum_j \Omega_j (Y) [\epsilon_j - \mathcal{E}(Y)]^2$. Note that in the case of $(\epsilon_N = \gamma, \epsilon_I = \gamma + \epsilon)$, this variance expression leads to Equation (C.11). Putting everything together, we find the following expression for the second derivative of the cost function

$$\mathcal{C}''(Y) = \mathcal{E} \frac{\mathcal{C}}{Y^2} \left[\mathcal{E} \left(1 + (1-\sigma) \frac{\mathcal{V}}{\mathcal{E}^2} \right) - 1 \right]. \tag{C.14}$$

□

See Lemma 2.

Proof. First, let us rewrite problem (??) as

$$\Pi_i = \max_{Y_i} \mathcal{R}(Y_i) - e^{-\theta_i} \mathcal{C}(Y_i; e^{-\phi_i} W) - \psi,$$

where $\mathcal{R}(Y_i) \equiv P(Y_i/Y)^{-1/\lambda} Y_i$ is the firm revenue function. The first order condition is given by $\mathcal{R}'(Y_i) - e^{-\theta_i} \mathcal{C}'(Y_i) \leq 0$. The goal is to ensure that for all (θ_i, ϕ_i) , there is a unique Y_i that satisfies this condition with equality.

First, we show that the condition is always satisfied with equality at least for one value of Y_i . Given Assumption ??, we find that

$$\begin{aligned}
\lim_{Y_i \rightarrow 0} \frac{\mathcal{C}'(Y_i)}{Y_i^{\gamma-1}} &= \lim_{Y_i \rightarrow 0} \mathcal{E}_i \left(1 + (e^{-\phi_i} W)^{1-\sigma} Y_i^{\epsilon(1-\sigma)} \right)^{\frac{1}{1-\sigma}} = \gamma, \\
\lim_{Y_i \rightarrow \infty} \frac{\mathcal{C}'(Y_i)}{Y_i^{\gamma+\epsilon-1}} &= \lim_{Y_i \rightarrow \infty} \mathcal{E}_i \left((e^{-\phi_i} W)^{1-\sigma} + Y_i^{-\epsilon(1-\sigma)} \right)^{\frac{1}{1-\sigma}} = (\gamma + \epsilon) e^{-\phi_i} W,
\end{aligned}$$

where we have used the notation $\mathcal{E}_i \equiv \mathcal{E}(Y_i; e^{-\phi_i} W)$. Given the assumption $-\frac{1}{\lambda} < \gamma - 1$, we have the following limits for the first order condition:

$$\mathcal{R}'(Y_i) - e^{-\theta_i} \mathcal{C}'(Y_i; e^{-\phi_i} W) \longrightarrow \begin{cases} P \left(1 - \frac{1}{\lambda}\right) \left(\frac{Y_i}{Y}\right)^{-1/\lambda} - \gamma e^{-\theta_i} Y_i^{\gamma-1} \rightarrow \infty, & Y_i \rightarrow 0, \\ P \left(1 - \frac{1}{\lambda}\right) \left(\frac{Y_i}{Y}\right)^{-1/\lambda} - (\gamma + \epsilon) \frac{W_i}{e^{\theta_i}} Y_i^{\gamma+\epsilon-1} \rightarrow -\infty, & Y_i \rightarrow \infty. \end{cases}$$

herefore, the first order condition has at least one zero for any finite pair (θ_i, ϕ_i) and any finite combinations of (P, Y, W) .

Consider now one such a solution Y_i^* for (θ_i, ϕ_i) and (P, Y, W) satisfying

$$\mathcal{R}'(Y_i^*) - e^{-\theta_i} \mathcal{C}'(Y_i^*; e^{-\phi_i} W) = \left(1 - \frac{1}{\lambda}\right) P Y^\lambda (Y_i^*)^{-\frac{1}{\lambda}} - \mathcal{E}_i \frac{\mathcal{C}(Y_i^*; e^{-\phi_i} W)}{e^{\theta_i} Y_i^*} = 0. \quad (\text{C.15})$$

To ensure that this zero indeed corresponds to a maximum, we need to examine the second order condition. Using Equations (??) and (C.10), we find

$$\begin{aligned} \mathcal{R}''(Y_i^*) - e^{-\theta_i} \mathcal{C}''(Y_i^*; e^{-\phi_i} W) &= -\frac{1}{(Y_i^*)^2} \left\{ \frac{1}{\lambda} \left(1 - \frac{1}{\lambda}\right) P Y^\lambda (Y_i^*)^{1-\frac{1}{\lambda}} + \mathcal{E}_i \frac{\mathcal{C}(Y_i^*; e^{-\phi_i} W)}{e^{\theta_i}} \left[\mathcal{E}_i \left(1 + (1-\sigma) \frac{V_i}{\mathcal{E}_i^2}\right) - 1 \right] \right\} \\ &= -\left(1 - \frac{1}{\lambda}\right) P Y^\lambda (Y_i^*)^{-\frac{1}{\lambda}-1} \left\{ \frac{1}{\lambda} + \mathcal{E}_i \left(1 + (1-\sigma) \frac{V_i}{\mathcal{E}_i^2}\right) - 1 \right\}, \end{aligned}$$

where we have used the notation $V_i \equiv \mathcal{V}(Y_i; e^{-\phi_i} W)$. The expression above leads to the following condition

$$1 - \frac{1}{\lambda} < \mathcal{E}_i \left(1 + (1-\sigma) \frac{V_i}{\mathcal{E}_i^2}\right).$$

The next step is to show that the solution Y_i^* above is unique. We can rewrite Equation (C.15) as

$$\mathcal{C}'(Y_i^*; e^{-\phi_i} W) (Y_i^*)^{1/\lambda} = \left(1 - \frac{1}{\lambda}\right) P Y^\lambda e^{\theta_i}.$$

To ensure that there is only Y_i^* satisfying the equation above, it is sufficient to show that function $\mathcal{L}(Y) \equiv \mathcal{C}'(Y) Y^{\frac{1}{\lambda}}$ is monotonic. Computing the derivative, we find

$$\begin{aligned} \mathcal{L}'(Y) &= \mathcal{C}''(Y) Y^{\frac{1}{\lambda}} + \frac{1}{\lambda Y} \mathcal{C}'(Y) Y^{\frac{1}{\lambda}}, \\ &= \mathcal{C}'(Y) Y^{\frac{1}{\lambda}-1} \left(\frac{1}{\lambda} + \frac{Y \mathcal{C}''}{\mathcal{C}'} \right), \\ &= \mathcal{C}'(Y) Y^{\frac{1}{\lambda}-1} \left(\frac{1}{\lambda} + \mathcal{E} \left(1 + (1-\sigma) \frac{\mathcal{V}}{\mathcal{E}^2}\right) - 1 \right), \end{aligned}$$

where we have used Equation (C.14).

Now, first consider the case of $0 < \sigma < 1$. In this case, one can verify that the restriction $1 - \frac{1}{\lambda} < \gamma$ is sufficient to satisfy all the conditions above required for the existence and uniqueness of the solution. Now define $\epsilon_N \equiv \gamma$ and $\epsilon_I \equiv \gamma + \epsilon$. In the case of $\sigma > 1$, we can bound the term involving the coefficient of variation as

$$\frac{\mathcal{V}}{\mathcal{E}} < \min \left\{ \frac{1}{4} \frac{(\epsilon_N - \epsilon_I)^2}{\epsilon_I}, \epsilon_N - \epsilon_I \right\},$$

where the first term comes from the fact that upper bound on the variance term is given by $\frac{1}{4}(\max\{\epsilon_N, \epsilon_I\} - \min\{\epsilon_N, \epsilon_I\})^2 = \frac{1}{4}(\epsilon_N - \epsilon_I)^2$, and the second term from the fact that $\epsilon_N^2(1 - \Omega) + \epsilon_I^2\Omega \leq \epsilon_N \bar{\epsilon}$. We can then write

$$\begin{aligned} \mathcal{E} - (\sigma - 1) \frac{\mathcal{V}}{\mathcal{E}^2} &> \bar{\epsilon}_i - (\sigma - 1) \min \left\{ \frac{1}{4} \frac{(\epsilon_N - \epsilon_I)^2}{\epsilon_I}, \epsilon_N - \epsilon_I \right\}, \\ &\geq \epsilon_I - (\sigma - 1) \min \left\{ \frac{1}{4} \frac{(\epsilon_N - \epsilon_I)^2}{\epsilon_I}, \epsilon_N - \epsilon_I \right\}. \end{aligned}$$

In this case, one can verify that, in addition to the restriction $1 - \frac{1}{\lambda} < \epsilon_I$, the following restriction is sufficient to satisfy the existence and uniqueness conditions laid out above:

$$(\sigma - 1) \min \left\{ \frac{1}{4} \frac{(\epsilon_N - \epsilon_I)^2}{\epsilon_I}, \epsilon_N - \epsilon_I \right\} < \epsilon_I - \left(1 - \frac{1}{\lambda}\right).$$

Finally, taking the derivative of the first-order condition with respect to e^{θ_i} , we find:

$$\left[\mathcal{R}''(Y_i^*) - \frac{1}{Z} \mathcal{C}''(Y_i^*) \right] \frac{\partial Y_i}{\partial e^{\theta_i}} = -\frac{1}{e^{2\theta_i}} \mathcal{C}'(Y_i).$$

As we saw the second order condition ensures that $\mathcal{R}''(Y_i) - e^{-\theta_i} \mathcal{C}''(Y; e^{-\phi_i} W) < 0$ at the optimum, which implies that $\frac{\partial Y}{\partial e^{\theta}} > 0$. Since the inverse demand function of decreasing, it follows that the optimal price is decreasing in θ_i . Showing the same result for ϕ_i is straightforward. \square

Definition 1. Tuple (W, Y, P, N) and the stationary distribution G together characterize the general equilibrium of the model if it satisfy the following conditions:

1. The probability density of function of the stationary distributions G and G_o satisfy Equations (??) and (??) for all (θ, ϕ) .

2. The CES aggregate the price index satisfies:

$$1 = N \int \int \tilde{P}(\theta, \phi; P, Y, W)^{1-\lambda} g(\theta, \phi) d\theta d\phi. \quad (\text{C.16})$$

3. Factor market clearing conditions for the bundles of non-IT and IT inputs hold:

$$N \left[\psi + \int \int (1 - \tilde{\Omega}(\theta, \phi; P, Y, W)) \tilde{C}(\theta, \phi; P, Y, W) g(\theta, \phi) d\theta d\phi \right] + N_o \delta \chi = \bar{X}_N, \quad (\text{C.17})$$

$$N \int \int \tilde{\Omega}(\theta, \phi; P, Y, W) \tilde{C}(\theta, \phi; P, Y, W) g(\theta, \phi) d\theta d\phi = W \bar{X}_I, \quad (\text{C.18})$$

where we have defined $\bar{X}_N \equiv \bar{K}^\alpha \bar{L}^{1-\alpha}$ and $\bar{X}_I \equiv \bar{S}^\beta \bar{H}^{1-\beta}$ and $N_o \equiv N / \int_{\mathcal{J}} g_o(u, v) du dv$.

4. Free entry condition holds:

$$\chi = \int \int V(\theta, \phi; P, Y, W) f(\theta, \phi) d\theta d\phi. \quad (\text{C.19})$$

Let us start with a cost minimization problem of the firm:

$$\begin{aligned} \min & W_L L_i + W_K K_i + W_S S_i + W_H H_i \\ & Y_i \geq \mathcal{F}(e^{\theta_i + \phi_i} X_{N,i}, e^{\theta_i} X_{I,i}), \\ & X_{N,i} \leq K_i^\alpha L_i^{1-\alpha}, \\ & X_{I,i} \leq S_i^\beta H_i^{1-\beta}. \end{aligned}$$

Using f.o.c.'s for K_i and L_i , we find that $W_L = (1 - \alpha) \mu_{N,i} (K_i / L_i)^\alpha$ and $W_K = \alpha \mu_{N,i} (L_i / K_i)^{1-\alpha}$ where $\mu_{N,i}$ is the Lagrange multiplier corresponding to the constraint on the non-IT input bundle. Applying the normalization $(W_{L,t} / (1 - \alpha))^{1-\alpha} (W_{K,t} / \alpha)^\alpha$, we find that $\mu_{N,i} = 1$ and $X_{N,i} = W_L L_i + W_K K_i$. Since capital to labor ratios are equalized across all firms, we know that $\bar{K} / \bar{L} = K_i / L_i$ and $\bar{X}_N \equiv \int X_{N,i} di = \bar{K}^\alpha \bar{L}^{1-\alpha}$. Applying the same argument to the f.o.c.'s for S_i and H_i , we find that $W_S S_i = \beta W X_{I,i}$, $W_H H_i = (1 - \beta) W X_{I,i}$, and $\bar{X}_I \equiv \int X_{I,i} = \bar{S}^\beta \bar{H}^{1-\beta}$.

We can now write the labor and capital market clearing conditions

$$\begin{aligned} W_L \bar{L} &= (1 - \alpha) \bar{X}_N, \\ W_K \bar{K} &= \alpha \bar{X}_N. \end{aligned}$$

and combine them to find market clearing condition for the non-IT input as:

$$\bar{X}_N = W_L \bar{L} + W_K K, \quad (\text{C.20})$$

$$= N \left[\phi + \int \int \left(1 - \tilde{\Omega}(\theta, \phi; P, Y, W) \right) \tilde{C}(\theta, \phi; P, Y, W) g(\theta, \phi) d\theta d\phi \right] + N_o \delta \chi. \quad (\text{C.21})$$

The market clearing condition for the IT inputs follows similarly.

Equation (C.16) follows from the aggregation of the prices across a mass N of firms with varying productivity states characterized by distribution G and pins down the price index P . The factor market clearing conditions pin down the total output Y and the relative price of IT inputs W . The factor market clearing condition for non-IT inputs (C.17) ensures that the sum of non-IT variable inputs, fixed costs, and costs of entry paid by all active incumbents and entrants equates the total stock of non-IT inputs \bar{X}_N . The term outside the parentheses corresponds to the costs of entry, accounting for the fact that the mass of entrants is given by $\delta \times N_o$ where N_o is the mass of all firms in the economy, satisfying $N = N_o \times \int_{\mathcal{J}} g_o(u, v) du dv$. We can think of this market clearing condition as the resource constraint that determines the size of the economy and the total outputs Y . The IT factor market clearing condition (C.18) then pins down the relative price of IT inputs W . Factor market clearing conditions for the components of the two bundle of IT and non-IT inputs follow from the two upper level factor market clearing conditions. Finally, the free entry condition in Equation (C.19) ensures that the expected value of entry, given by the expression on the right hand side, equates with the costs of entry, paid in units of non-IT inputs. Note that even entrants that end up initially inactive still have non-zero expected net present value due to the fact that future shocks might make them active and therefore profitable.

Derivations for Equations (??) and (??).

First, note that we have $\Omega_i = (Y_i^{\gamma+\epsilon} e^{-\phi_i} W / C_i)^{1-\sigma}$, and therefore:

$$\begin{aligned} \frac{\partial \log \Omega_i}{\partial w} &= (1-\sigma) \left(1 - \frac{\partial c_i}{\partial w} \right) = (1-\sigma)(1-\Omega_i), \\ \frac{\partial \log \Omega_i}{\partial y_i} &= (1-\sigma) \left(\gamma + \epsilon - \frac{\partial c_i}{\partial y_i} \right) = (1-\sigma)(\gamma + \epsilon - \mathcal{E}_i) = (1-\sigma)\epsilon(1-\Omega_i). \end{aligned}$$

where in the first and second lines we have used y_i and c_i to refer to the log of output Y_i and costs C_i , respectively. Recall that we have $C_i = e^{-\theta_i} Y_i^\gamma \left(1 + (Y_i^\epsilon e^{-\phi_i} W)^{1-\sigma} \right)^{1/(1-\sigma)}$, implying $\partial c_i / \partial w = \Omega_i$. We now substitute the expressions above in a first-order approximation of the

change in the cost elasticity \mathcal{E}_i as the result of the log change Δw in the relative price of IT:

$$\begin{aligned}
 \Delta \mathcal{E}_i &\approx \frac{\partial \mathcal{E}_i}{\partial w} \Delta w + \frac{\partial \mathcal{E}_i}{\partial y} \Delta y_i, \\
 &= \epsilon \left(\frac{\partial \Omega_i}{\partial w} \Delta w + \frac{\partial \Omega_i}{\partial y} \Delta y_i \right), \\
 &= \epsilon \Omega_i \left(\frac{\partial \log \Omega_i}{\partial w} \Delta w + \frac{\partial \log \Omega_i}{\partial y} \Delta y_i \right), \\
 &= (1 - \sigma) \epsilon \Omega_i [(1 - \Omega_i) \Delta w + (\gamma + \epsilon - \mathcal{E}_i) \Delta y_i], \\
 &= (1 - \sigma) \epsilon \Omega_i [(1 - \Omega_i) \Delta w + \epsilon (1 - \Omega_i) \Delta y_i],
 \end{aligned}$$

where the last equality leads to Equation (??). Noting that $S_i = C_i g_i / \bar{C}$ then leads to Equation (??).

See Lemma ??.

Proof. Let us write Equation (4.21) as:

$$\left(\frac{Y_i}{Y} \right)^{-\frac{1}{\lambda}} = \frac{\lambda}{\lambda - 1} \frac{C'(Y_i; e^{-\phi_i} W)}{e^{\theta_i} P}.$$

Rewriting the equation in logarithm terms and differentiating, we find:

$$-\frac{1}{\lambda} (\Delta y_i - \Delta y) \approx \frac{\partial mc}{\partial y} \Delta y_i + \frac{\partial mc}{\partial w} \Delta w - \Delta p,$$

where, as before, we have used the small cap letters to denote log functions. This will lead to Equation (??). To derive Equations (??) and (??), we first note that $MC_i = \mathcal{E}_i C_i / Y_i$. It follows that:

$$\begin{aligned}
 \frac{\partial mc_i}{\partial w} &= \frac{\partial c_i}{\partial w} + \frac{\partial \log \mathcal{E}_i}{\partial w}, \\
 &= \Omega_i + \frac{\epsilon}{\mathcal{E}_i} \frac{\partial \Omega_i}{\partial w}, \\
 &= \Omega_i + \frac{\epsilon \Omega_i}{\gamma + \epsilon \Omega_i} \frac{\partial \log \Omega_i}{\partial w}, \\
 &= \Omega_i \left(1 + (1 - \sigma) \epsilon \frac{1 - \Omega_i}{\gamma + \epsilon \Omega_i} \right),
 \end{aligned}$$

where in the last equality we have substituted the expression for $\frac{\partial \log \Omega_i}{\partial w}$ from above. Similarly, we

have:

$$\begin{aligned}
\frac{\partial mc_i}{\partial y_i} &= \frac{\partial c_i}{\partial y_i} - 1 + \frac{\partial \log \mathcal{E}_i}{\partial y_i}, \\
&= \mathcal{E}_i - 1 + \frac{\epsilon}{\mathcal{E}_i} \frac{\partial \Omega_i}{\partial y_i}, \\
&= \gamma - 1 + \Omega_i \epsilon + \frac{\epsilon \Omega_i}{\gamma + \epsilon \Omega_i} \frac{\partial \log \Omega_i}{\partial y_i}, \\
&= \gamma - 1 + \Omega_i \epsilon \left(1 + (1 - \sigma) \epsilon \frac{1 - \Omega_i}{\gamma + \epsilon \Omega_i} \right).
\end{aligned}$$

□

See Lemma 3.

Proof. Consider the problem of the firm that starts period t with stock productivity state variables (θ_{it}, ϕ_{it}) and stocks of non-IT capital, hardware, and software $(K_{it-1}, H_{it-1}, S_{it-1})$ (as expected, we will see that the stock of software S_{it-1} is not a state variable):

$$\begin{aligned}
V_t(\theta_{it}, \phi_{it}, K_{it-1}, H_{it-1}, S_{it-1}) &= \max_{Y_{it}, L_{it}, K_{it}, S_{it}, H_{it}} \mathcal{P}(Y_{it}) Y_{it} - W_t L_{it} - Q_t (S_{it} - S_{it-1} (1 - \delta_s)) \\
&\quad - \mathcal{C}_t^K(K_{it}; K_{it-1}) - \mathcal{C}_t^H(H_{it}; H_{it-1}) + \frac{1}{1 + r_t} \mathbb{E}_{\theta_{it+1}, \phi_{it+1}} [V_{t+1}(\theta_{it+1}, \phi_{it+1}, K_{it}, H_{it}, S_{it})] \\
\text{subject to: } Y_t &= F(e^{\theta_{it}} K_t^\alpha L_t^{1-\alpha}, e^{\phi_{it} + \theta_{it}} S_t^\beta H_t^{1-\beta}),
\end{aligned}$$

where $\mathcal{P}(Y_{it})$ denotes the demand function (potentially downward sloping), δ_s is the depreciation rate of software capital, and \mathcal{C}_t^K and \mathcal{C}_t^H are generic functions that capture potential adjustment costs for dynamic inputs K_{it} and H_{it} , respectively. Based on assumption 5, we assume no adjustment costs for software capital and therefore the firm faces software investment at price Q_t . Conditional on the optimal Y_t , and defining Λ_t to be the Lagrange multiplier corresponding to the constraint on Y_t , we find:

$$Q_t = \Lambda_{it} \frac{\partial Y_{it}}{\partial X_{N,it}} \frac{\partial X_{N,it}}{\partial S_{it}} + \frac{1}{1 + r_t} \mathbb{E}_{\theta_{it+1}, \phi_{it+1}} \left[\frac{\partial}{\partial S_{it}} V_{t+1}(\theta_{it+1}, \phi_{it+1}, K_{it}, H_{it}, S_{it}) | \mathcal{I}_{it} \right].$$

We can also compute the partial derivative of the value function simply as:

$$\frac{\partial}{\partial S_{it}} V_{t+1}(\theta_{it+1}, \phi_{it+1}, K_{it}, H_{it}, S_{it}) = (1 - \delta_s) Q_{t+1}.$$

Therefore, we can define the *user cost of software* (the effective price of software capital) as follows

$$\begin{aligned} W_{S,it} &\equiv Q_t - \frac{1-\delta_S}{1+r_t} Q_{t+1}, \\ &= Q_t \left(1 - \frac{1-\delta_S}{1+r_t} \frac{Q_{t+1}}{Q_t} \right), \\ &= Q_t (1 - R_{S,t}^{-1}), \end{aligned}$$

where $R_{S,t}$ is the required rate of return for software investments.

Based on the derivations above, we can redefine the problem as that of a firm with four state variables $(\theta_{it}, \phi_{it}, K_{it}, H_{it})$ and with:

$$\begin{aligned} V_t(\theta_{it}, \phi_{it}, K_{it-1}, H_{it-1}) &= \max_{Y_{it}, L_{it}, K_{it}, S_{it}, H_{it}} \mathcal{P}(Y_{it}) Y_{it} - W_{L,t} L_{it} - W_{S,t} S_{it} \\ &\quad - \mathcal{C}_t^K(K_{it}; K_{it-1}) - \mathcal{C}_t^H(H_{it}; H_{it-1}) + (1-\delta_t) \mathbb{E}_{\theta_{it+1}, \phi_{it+1}} [V_{t+1}(\theta_{it+1}, \phi_{it+1}, K_{it}, H_{it})] \\ \text{subject to: } Y_{it} &= F(e^{\theta_{it}} K_{it}^\alpha L_{it}^{1-\alpha}, e^{\phi_{it}} e^{\theta_{it}} S_{it}^\beta H_{it}^{1-\beta}). \end{aligned}$$

Let us now look at the first order conditions with respect to the two flexible inputs:

$$W_{L,t} = \Lambda_{it} \frac{\partial Y_{it}}{\partial X_{N,it}} \frac{\partial X_{N,it}}{\partial L_{it}} = \Lambda_{it} \frac{Y_{it}}{X_{N,it}} \frac{1}{\mathcal{E}_{it}} \left(\frac{e^{\theta_{it}} X_{N,it}}{Y_{it}^\gamma} \right)^{1-\frac{1}{\sigma}} (1-\alpha) \frac{X_{N,it}}{L_{it}}, \quad (\text{C.22})$$

$$W_{S,t} = \Lambda_{it} \frac{\partial Y_{it}}{\partial X_{I,it}} \frac{\partial X_{I,it}}{\partial S_{it}} = \Lambda_{it} \frac{Y_{it}}{X_{I,it}} \frac{1}{\mathcal{E}_{it}} \left(\frac{e^{\theta_{it}+\phi_{it}} X_{I,it}}{Y_{it}^{\gamma+\epsilon}} \right)^{1-\frac{1}{\sigma}} \beta \frac{X_{I,it}}{S_{it}}, \quad (\text{C.23})$$

where $W_{L,t}$ and $W_{S,t}$ are the wage rate and the rental price of software, respectively, and Λ_{it} is the marginal cost of production for the firm. First, we divide the two equations Equations (C.22) and (C.23) and rewrite them as

$$\begin{aligned} \frac{W_{S,t}}{W_{L,t}} &= \frac{\beta}{1-\alpha} \frac{L_{it}}{S_{it}} \left(e^{\phi_{it}} \frac{X_{I,it}}{X_{N,it}} (Y_{it}^*)^{\epsilon_N - \epsilon_I} \right)^{1-\frac{1}{\sigma}}, \\ &= \frac{\beta}{1-\alpha} \left(\frac{S_{it}}{L_{it}} \right)^{-\frac{1}{\sigma}} \left(e^{\phi_{it}} \frac{(H_{it}/S_{it})^{1-\beta}}{(K_{it}/L_{it})^\alpha} Y_{it}^\epsilon \right)^{1-\frac{1}{\sigma}}. \end{aligned}$$

We now write this equation in first differences to find:

$$s_{it} - l_{it} = \sigma (w_{L,t} - w_{S,t}) + (\sigma - 1) [\phi_{it} - \epsilon y_{it} - \alpha (k_{it} - l_{it}) + (1-\beta)(h_{it} - s_{it})].$$

□

C.3 Decompositions of the Labor Share

In this section, we will present the details of the decompositions of labor share presented in Section 4.3. These decompositions are done at two levels: first we decompose the aggregate changes in labor share to the within and across industries components. Next, we decompose the industry-level changes in labor share to within and across quantiles components, where quantiles of labor share are specific to each year and each industry.

Let $k \in \{1, \dots, K\}$ be some industry classification (e.g., 3 digits in micro data), Y stands for value added, L for wage bill, and Λ for the labor share defined as L/Y . Also, let S_k and Λ_k stand for the share of the industry in total value added and the industry average labor share, defined as L_k/Y_k , respectively. Define for any variable X :

$$\begin{aligned}\Delta X_t &\equiv X_t - X_{t-1}, \quad \bar{X}_t \equiv \frac{1}{2}(X_t + X_{t-1}), \\ \Delta_T X &\equiv X_T - X_0,\end{aligned}$$

where T is the last period and 0 is the first period. Our first decomposition is:²

$$\Delta_T \Lambda \equiv \underbrace{\sum_{t=1}^T \sum_k \bar{S}_{kt} \Delta \Lambda_{kt}}_{\text{within industry}} + \underbrace{\sum_{t=1}^T \sum_k \Delta S_{kt} \bar{\Lambda}_{kt}}_{\text{across industries}}. \quad (\text{C.24})$$

Next, we focus on changes in the industry-level labor shares. Our aim is to decompose the changes in the industry-level labor share to the shifts in the distribution of firms labor shares and the changes in market shares of firms along the distribution. Let p denote a probability, we can write the industry-level labor share as

$$\Lambda_{kt} \equiv \int_0^1 S_{kt}(p) \Lambda_{kt}(p) dp, \quad (\text{C.25})$$

²This is simply because:

$$\begin{aligned}\Delta(S_t \Lambda_t) &= \bar{S}_t \Delta \Lambda_t + \Delta S_t \bar{\Lambda}_t \\ \Delta_T(S\Lambda) &= \sum_{t=1}^T \Delta(S_t \Lambda_t).\end{aligned}$$

where $\Lambda_{kt}(p)$ is the average labor share of firms between the $(p-1)$ -th and p -th quantile in the labor share distribution in industry- k at time t and $S_{kt}(p)$ denotes their share of industry- k value added at time t . We can now decompose³

$$\Delta\Lambda_{kt} = \underbrace{\int_0^1 \bar{S}_{kt}(p) \Delta\Lambda_{kt}(p) dp}_{\text{Within quantile}} + \underbrace{\int_0^1 \Delta S_{kt}(p) \bar{\Lambda}_{kt}(p) dp}_{\text{Across quantiles}}. \quad (\text{C.27})$$

We now summarize the within-industry component change in aggregate labor share into the following components:

1. The cross quantiles component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \int_0^1 \Delta S_{kt}(p) \bar{\Lambda}_{kt}(p) dp.$$

2. The within quantile component:

$$\sum_{t=1}^T \sum_k \bar{S}_{kt} \int_0^1 \bar{S}_{kt}(p) \Delta\Lambda_{kt}(p) dp.$$

C.4 Within Firm Estimates and Reduced-Form Identification

In this section, we apply a reduced-form identification strategy to estimate the elasticity of relative IT demand to size $\eta \equiv (1-\sigma)\epsilon$. The empirical counterpart of Equation (??) in the main text is

$$x_{I,it} - x_{N,it} = \eta y_{it} - \sigma w_t + (\sigma - 1) \phi_{it}, \quad (\text{C.28})$$

³As emphasized by [Kehrig and Vincent \(2017\)](#), this decomposition is conceptually distinct from standard within and cross firm decompositions. Let Ω_{kt} be the set of firms active in time t , and $\bar{\Omega}_{kt}$ be the set of firms common between time t and $t-1$, Ω_{kt}^+ the set of *new* firms at time t , and Ω_{kt}^- the set of firms exiting between time t and $t+1$. We can then write:

$$\Delta\Lambda_{kt} \equiv \underbrace{\sum_{i \in \Omega_{kt}^-} \bar{S}_{it} \Delta\Lambda_{it}}_{\text{within firm}} + \underbrace{\sum_{i \in \bar{\Omega}_{kt}} \Delta S_{it} \bar{\Lambda}_{it}}_{\text{cross firm}} + \underbrace{\left(\sum_{i \in \Omega_{kt}^+} S_{it} \Lambda_{it} - \sum_{i \in \Omega_{kt-1}^-} S_{it-1} \Lambda_{it-1} \right)}_{\text{net entry}}, \quad (\text{C.26})$$

where again shares are computed within the industry.

where we have used lower case letters to denote the logarithm of the upper case variables, e.g., $x_t \equiv \log(X_t)$ and ϕ_{it} is the IT-biased (log) productivity state. To the extent that ϕ_{it} is correlated with size, , e.g., if firms that are better at using IT also grow larger, then we may find a positive relationship between firm size and IT intensity. In this case the error term in Equation (C.28) is potentially correlated with regressor y_{it} and the OLS estimates of η in Section ?? may possibly be biased.

Table C.4.1: Regressions of IT Intensity on Log Firm Size (Within Firm)

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Costs	Costs
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.2042		0.1455		0.3422		0.3533		0.1702	
	(0.0326)		(0.0327)		(0.0337)		(0.0338)		(0.0326)	
Size (proxied by VA)		0.2161		0.1432		0.3296		0.3379		0.1621
		(0.0288)		(0.0288)		(0.0297)		(0.0298)		(0.0287)
Observations	233,654	233,376	233,507	233,189	221,456	221,319	221,676	221,614	233,548	233,230
R2	0.8361	0.8325	0.8319	0.8281	0.8313	0.8270	0.8311	0.8272	0.8327	0.8288
Panel 2 : Hardware (Stock)										
Size (proxied by sales)	0.2612		0.1686		0.3706		0.3765		0.1874	
	(0.0097)		(0.0097)		(0.0101)		(0.0102)		(0.0095)	
Size (proxied by VA)		0.1470		0.0437		0.2482		0.2533		0.0660
		(0.0081)		(0.0082)		(0.0084)		(0.0085)		(0.0080)
Observations	248,038	249,026	248,995	250,111	244,282	245,286	243,095	244,175	248,466	249,591
R2	0.8689	0.8691	0.8467	0.8466	0.9078	0.9077	0.9177	0.9175	0.8485	0.8493

C.4.1 Within-Firm Estimates

Table C.4.1 reports the results of the following regression:

$$x_{I,it} - x_{N,it} = \eta y_{it} + FE_{kt} + FE_i + v_{it}, \quad (\text{C.29})$$

where FE_i is a firm fixed effect and FE_{kt} stands for the industry-time fixed effects account for the variations in relative price of IT w_t , as well as the industry-time component of the IT-biased productivity ϕ_{it} . The specification allows us to examine the within-firm relationship between firm size and IT intensity. The table shows that the results of Section ?? are *not* driven by a potential confounding cross-sectional relationship between IT-biased productivity and size.

C.4.2 Reduced-Form Identification

The within-firm estimates in Table C.4.1 still leave us with the possibility that the within-firm changes in IT-biased productivity, the residual v_{it} in Equation (C.29) could be correlated with size. To identify the contribution of nonhomotheticity to the correlation between size and IT intensity, we rely on demand shocks to different export destinations of firms as an exogenous source of variation in their expected potential for growth. To the extent that firms take advantage of these opportunities to expand their activities, we should find a first stage effect of the instrument on firm sales and value added. The idea behind this strategy has been used in a number of recent papers (e.g., [Hummels et al., 2014](#); [Mayer et al., 2015](#); [Aghion et al., 2017a](#); [Garin and Silveiro, 2017](#); [Panon, 2019b](#)). As we will see below, our specifications identify the within-firm relationship between size and IT intensity. As a result, our key identification assumption is that the *variations in value of the demand shock measures above are uncorrelated with firm-level residual v_{it} 's in Equation (C.29)*.

Export Demand Shock Instruments We construct the product-destination-level export demand shocks for firm i at time t as

$$ds_{it}^p = \sum_{np} \Lambda_{inp,0} \left(im p_{np,t}^{-FR} - \overline{im p_{p,t}^{-FR}} \right), \quad (\text{C.30})$$

where Λ_{inp} denotes destination- n /product- p share of firm- i exports, $im p_{np,t}^{-FR}$, the destination- n /product- p log import from all countries except France, and $\overline{im p_{p,t}^{-FR}}$ the product-level average value of the log import across all other destinations. With this specification, we avoid including the component of demand in any given product-destination that might be driven by potential productivity shocks to all French exporters.

Data for Export Demand Shocks To construct the instruments, we use the French customs data that provides the value of the exports of firms by destination and product (at the nc8 level) spanning the 1995-2007 period. The data allows us to compute the share of each destination- n /product- p share of firm- i exports (Λ_{inp} in Equation C.30) as the corresponding average for years 1995 and 1996. To build the product-level demand shocks in Equation (C.30), we rely on the COMTRADE bilateral Trade Flows Data, and in particular on the harmonized version of the data provided in the BACI dataset. This dataset includes the values of flows from each exporter to each importing destination as HS6 code product-level.⁴ We use this information to compute for

⁴We use the concordance procedure made available by [Van Beveren et al. \(2012\)](#) to map CN8 products code over time, and to more aggregated HS6 product codes.

Table C.4.2: Summary Statistics: Exporting vs All Firms

	Source	Obs. (Nb)	All firms			Obs. (Nb)	Exporting firms		
			Mean	Median	Sd		Mean	Median	Sd
Sales	BRN + RSI	15,202,967	2,498.8	265	85,056.8	2,544,556	11,400.9	1316	206,743.5
Value-Added	BRN + RSI	15,202,967	708.3	106	33,071.4	2,544,556	3,023.9	371	80,298.8
Number of Employees	BRN + RSI	15,202,967	13.8	3	480.7	2,544,556	50.4	9	1,141.3
Wage Bill	BRN + RSI	15,202,967	472.4	74	18,404.5	2,544,556	1,916.2	284.3	44,228.0
Labor Share (%)	BRN + RSI	15,202,967	74.0	73.0	33.9	2,544,556	78.9	76.6	32.8
Total Investment	BRN	6,166,342	143.1	5	9,880.3	1,665,474	425.2	17.4	18,993.1
Total Capital Stock	BRN	6,166,342	1,202.2	88.0	92,297.9	1,665,474	3,701.7	227.9	177,412.4
Total Cost	BRN	6,166,341	898.1	181.0	33,623.7	1,665,474	2,376.6	370.9	64,183.4
IT Measures									
Software Investment	EAE	2,435,356	5.9	0	528.2	954,970	14.1	0	843.2
Software Stock	EAE	2,435,356	15.8	0	1,216.2	954,970	38.0	0	1,941.7
Hardware Investment	BRN	6,166,342	6.1	0	405.2	1,665,474	18.1	0	778.0
Hardware Stock	BRN	6,166,342	24.5	0	1,857.4	1,665,474	79.0	0.4	3,570.8
IT Intensity of Labor									
Software Investment	EAE	2,435,356	27.5	0	167.0	954,970	53.0	0	227.0
Software Stock	EAE	2,435,356	81.7	0	3,214.5	954,970	160.1	0	4,971.6
Hardware Investment	BRN	6,166,342	177.5	0	750.0	1,665,474	208.4	0	702.1
Hardware Stock	BRN	6,166,342	477.4	0	2,435.9	1,665,474	665.3	33.5	3,011.0
IT Intensity of Capital									
Software Investment	EAE	1,985,530	21.9	0	1,156.5	859,667	31.2	0	1,429.7
Software Stock	EAE	2,284,444	3.8	0	22.1	923,783	5.5	0	21.7
Hardware Investment	BRN	4,381,031	112.9	0	1,601.1	1,383,292	114.1	0	2,810.8
Hardware Stock	BRN	5,550,954	39.5	0	128.3	1,569,517	36.1	1.9	105.0
IT Intensity of Cost									
Software Investment	EAE	2,435,351	0.7	0	4.1	954,970	1.2	0	5.2
Software Stock	EAE	2,435,356	0.6	0	2.6	954,970	1.1	0	3.3
Hardware Investment	BRN	6,166,303	3.8	0	20.0	1,665,472	4.3	0	27.1
Hardware Stock	BRN	6,166,341	2.4	0	7.9	1,665,474	3.0	0.2	7.1

each product in each destination country the sum of all imports from all other countries, leaving out France. We construct the instrument ds_{it}^P for years 1997-2007.

Empirical Specification For the results of this section, we limit our attention to the sample of exporting firms. Table C.4.2 compares the summary statistics of this sample with the sample of all firms. As is well-known, exporting firms are typically larger than other firms. The table shows that they are, in addition, also slightly more IT intensive than average firms (Fort et al., 2017).

Table C.4.3 present the results of applying the following specification in the sample of exporting firms⁵

$$IT Intensity_{it} - \overline{IT Intensity}_i = \eta(y_{it} - \bar{y}_i) + FE_{kt} + v_{it}, \quad (C.31)$$

⁵The results in Table C.4.3 are weighted by each firm's initial share of exports in total sales. Table ?? presents the unweighted results, which are typically much larger in both magnitude of the estimates and the standard errors. Note that the only negative coefficients in Table C.4.3, for the hardware intensity of costs, change sign and are again positive in Table ??.

Table C.4.3: Reduced Form Identification of the Elasticity of IT Intensity on Log Firm Size

	IT Intensity of Labor				IT Intensity of Capital				IT Intensity of Cost	
	Workers	Workers	Wage Bill	Wage Bill	Total	Total	Tangible	Tangible	Costs	Costs
Panel 1 : Software (Stock)										
Size (proxied by sales)	0.5656 (0.2615)		0.3196 (0.2514)		0.4419 (0.2632)		0.5192 (0.2523)		0.5422 (0.2499)	
Size (proxied by VA)		0.8018 (0.3666)		0.4519 (0.3617)		0.7278 (0.3784)		0.8511 (0.3556)		0.6926 (0.3510)
Observations	102,481	101,439	102,803	101,685	98,146	97,067	98,336	97,264	102,939	101,826
Panel 2 : Hardware (Stock)										
Size (proxied by sales)	0.6962 (0.0874)		0.3597 (0.0842)		0.5328 (0.0848)		0.4892 (0.0827)		-0.0341 (0.0793)	
Size (proxied by VA)		0.9066 (0.1430)		0.4598 (0.1287)		0.7853 (0.1246)		0.6865 (0.1162)		-0.0474 (0.1266)
Observations	96,370	95,434	97,066	96,194	97,036	96,218	97,006	96,136	97,021	96,171

where $ITIntensity_{it}$ denotes a measure of the relative demand for IT inputs for a firm i in an industry k at time t , FE_{kt} stands for a flexible set of industry-time fixed effects (at the 3-digit level), y_{it} is the sales or value added of the firm (depending on the specification), and \bar{y}_i is the firm-level mean of log firm size y_{it} . We estimate Equation (C.31) with 2SLS, using the shocks defined in Equations (C.30) as instruments for $y_{it} - \bar{y}_i$. Results in Tables C.4.3 (and ??) are provided with product demand shocks from 1997 to 2007.

The coefficients are positive and significant for the majority of specifications. They are also close in magnitude to, even if larger than, those reported in Table C.4.1 for the within-firm effects. Note, however, that the sample of firms in Table C.4.3 is much smaller, only featuring relatively large exporting firms for which we can construct the instrument.

Connection to Lemma ?? in the Main Text Finally, we should highlight an important concern with the reduced-form estimates reported in Table C.4.3, in light of our results in Section ?? of the main paper. Let us compare our specification in Equation (C.31) with Equation (??) in the main text, which accounts for the potential presence of adjustment costs in the firm's choices of hardware and non-IT capital:

$$s_{it} - l_{it} = -\sigma (w_{S,t} - w_{L,t}) + (1 - \sigma) \epsilon y_{it} + (1 - \sigma) [\alpha (k_{it} - l_{it}) - (1 - \beta)(h_{it} - s_{it})] + (\sigma - 1) \phi_{it}.$$

We notice that Equation (C.31) abstracts away from the potential within-firm variations in capital-to-labor ratio $k_{it} - l_{it}$ and hardware-to-software ratio $h_{it} - s_{it}$ on the right hand side. If our instruments ds_{it}^P is correlated with these variations, and if those are in turn correlated with variations in output y_{it} , then our identification strategy the coefficients reported in may feature a potential

Table C.4.4: Correlations Between Non-Flexible Relative to Flexible Inputs and the Instrument for Size.

	Non IT Capital Stock per Worker	Hardware to Software Stock Ratio
Shift-Share Instrument	-0.0450 (0.0137)	-0.3103 (0.0626)
Observations	76,665	76,665

bias. Table C.4.4 below reports the regression coefficients of the within-firm variations in capital-to-labor ratio $k_{it} - l_{it}$ and hardware-to-software ratio $h_{it} - s_{it}$ on our instrument, which show sizable and significant coefficients. This is one of the reasons that we prefer to rely on our structural estimation strategy in Section ??, rather than the results reported here, for the calibration of our model.

C.5 Estimation

C.5.1 Setting Up the GMM Estimation

Let \mathbf{D} stand for the observed data and $\boldsymbol{\rho} \equiv (\rho_\theta, \rho_\phi)$ for the vector of persistence coefficients. We can write the $2J$ -dimensional vector of moments $\mathbf{g}(\mathbf{D}_{it}; \boldsymbol{\varsigma}, \boldsymbol{\rho})$ as

$$\begin{aligned} g_j(\mathbf{D}_{it}; \boldsymbol{\varsigma}, \boldsymbol{\rho}) &\equiv b_{jit-1} \left(\Theta_{it}(\boldsymbol{\varsigma}) - \rho_{\theta\theta} \Theta_{it-1}(\boldsymbol{\varsigma}) - \rho_{\theta\phi} \Phi_{it-1}(\boldsymbol{\varsigma}) \right), \quad j \leq J, \\ g_j(\mathbf{D}_{it}; \boldsymbol{\varsigma}, \boldsymbol{\rho}) &\equiv b_{jit-1} \left(\Phi_{it}(\boldsymbol{\varsigma}) - \rho_{\phi\theta} \Theta_{it-1}(\boldsymbol{\varsigma}) - \rho_{\phi\phi} \Phi_{it-1}(\boldsymbol{\varsigma}) \right), \quad j \geq J+1, \end{aligned}$$

where $b_{jit-1} \in \mathcal{I}_{it-1}$ is each of the instruments and function Φ defines IT biased productivity as a function of observed data and model parameters

$$\Phi(\mathbf{D}_{it}; \boldsymbol{\varsigma}) = \frac{1}{\sigma - 1} (\sigma \widehat{w}_{it} + s_{it} - l_{it}) + \epsilon y_{it} + \alpha (k_{it} - l_{it}) - (1 - \beta)(h_{it} - s_{it}), \quad (\text{C.32})$$

while function Θ is the corresponding function for factor symmetric productivity:

$$\Theta(\mathbf{D}_{it}; \boldsymbol{\varsigma}) \equiv \gamma y_{it} + \frac{\sigma}{1 - \sigma} \log \left[e^{\frac{\sigma-1}{\sigma} (\alpha k_{it} + (1-\alpha) l_{it})} + e^{\frac{\sigma-1}{\sigma} (\beta s_{it} + (1-\beta) h_{it} - \epsilon y_{it} + \Phi(\widehat{w}_{it}, l_{it}, k_{it}, s_{it}, h_{it}, y_{it}; \boldsymbol{\varsigma}))} \right], \quad (\text{C.33})$$

where $\widehat{w}_{it} \equiv w_{s,t} - w_{Li,t}$ is the relative price of software. User cost of software $w_{s,t}$ has no unit (euros per euros of software capital). Wage $w_{Li,t}$ is in thousand euros per worker, l_{it} is number of workers, and y_{it} , k_{it} , s_{it} , h_{it} are in thousand euros. These units should be reflected in the values used for calibration.

The GMM estimator is then given by

$$(\widehat{\varsigma}, \widehat{\rho}) \equiv \underset{(\varsigma, \rho)}{\operatorname{argmax}} \left[\sum_{it} g(D_{it}; \varsigma, \rho) \right]' \widehat{\Xi} \left[\sum_{it} g(D_{it}; \varsigma, \rho) \right],$$

where $\widehat{\Xi}$ is a $J \times J$ full-rank matrix. Alternatively, we can write

$$g_{it}(\varsigma, \rho) \equiv B'_{it} e_{it}(\varsigma, \rho),$$

where we have defined $b_{it-1} \equiv (b_{1,it-1}, \dots, b_{J,it-1})'$ and

$$B_{it} \equiv \begin{pmatrix} b_{it-1} & 0 \\ 0 & b_{it-1} \end{pmatrix}, \quad e_{it} \equiv \begin{pmatrix} \Theta_{it}(\varsigma) - \rho_{\theta\theta}\Theta_{it-1}(\varsigma) - \rho_{\theta\phi}\Phi_{it-1}(\varsigma) \\ \Phi_{it}(\varsigma) - \rho_{\phi\theta}\Theta_{it-1}(\varsigma) - \rho_{\phi\phi}\Phi_{it-1}(\varsigma) \end{pmatrix}.$$

We use a nonlinear system 2SLS estimator $(\widehat{\varsigma}^0, \widehat{\rho}^0)$, setting

$$\widehat{\Xi} \equiv \left(\frac{1}{NT} \sum_{it} B'_{it} B_{it} \right)^{-1}.$$

C.5.2 Initializing Parameter Estimates

To find reasonable initial parameters for the optimization step in the GMM estimation, we use a cascading series of intuitive simplifications of the model to lead the optimizer to the relevant parts of the parameter space. A key step in this approach is a log-linear approximation of the production function that helps create a bridge between the GMM moment conditions derived from our model with those typically used under the assumption of a Cobb-Douglas production function. More specifically, this approximation gives us a log-linear expression for the factor-symmetric productivity function $\theta_{it} = \widetilde{\Theta}(D_{it}; \varsigma)$ that we use in lieu of Equation (C.35). We will discuss this approximation in Section C.5.3 below.

C.5.3 Log-Linear Approximation for the Factor-Symmetric Productivity Function

Recall the evolution of productivities in Equation (??) and consider a firm i in industry j with a stationary distribution of productivities G_j . The Markov process of productivity states in this industry for each firm converges to a long-run distribution of productivities with industry-level mean values $(\theta_j \equiv \mathbb{E}_{G_j}[\theta], \phi_j \equiv \mathbb{E}_{G_j}[\phi])$. Similarly, let inputs (K_j, L_j, H_j, S_j) be the corresponding mean values of inputs for a firm with the corresponding mean productivity states. Log-linearizing the production function around the average industry level gives us

$$\begin{aligned} (\gamma + \Omega_j \epsilon)(y_{it} - y_j) \approx & \Omega_j [\beta(s_{it} - s_j) + (1 - \beta)(h_{it} - h_j) + (\phi_{it} - \phi_j)] \\ & + (1 - \Omega_j) [\alpha(k_{it} - k_j) + (1 - \alpha)(l_{it} - l_j)] + \theta_{it} - \theta_j, \end{aligned} \quad (\text{C.34})$$

where we have defined $\Omega_j \equiv (e^{\phi_j + \theta_j} X_{I,j} / Y_j^{\gamma + \epsilon})^{1-1/\sigma}$ and, as before, used small cap letter to denote the logarithms of the corresponding variables.

We can rewrite the expression above in a form that resembles the Cobb-Douglas production function:

$$y_{it} \approx \gamma_1 k_{it} + \gamma_2 l_{it} + \gamma_3 s_{it} + \gamma_4 h_{it} + \tilde{\theta}_{it},$$

where we have defined:

$$\begin{aligned} \gamma_1 &\equiv \frac{\alpha(1 - \Omega_j)}{\gamma + \Omega_j \epsilon}, \\ \gamma_2 &\equiv \frac{(1 - \alpha)(1 - \Omega_j)}{\gamma + \Omega_j \epsilon}, \\ \gamma_3 &\equiv \frac{\beta \Omega_j}{\gamma + \Omega_j \epsilon}, \\ \gamma_4 &\equiv \frac{(1 - \beta) \Omega_j}{\gamma + \Omega_j \epsilon}, \\ \tilde{\theta}_{it} &\equiv (\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4) \theta_{it} + (\gamma_3 + \gamma_4) \phi_{it} + \tilde{\theta}_j, \\ \tilde{\theta}_j &\equiv -((\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4) \theta_j + (\gamma_3 + \gamma_4) \phi_j) + [y_j - (\gamma_1 k_j + \gamma_2 l_j + \gamma_3 s_j + \gamma_4 h_j)]. \end{aligned}$$

Let us also define a scaling of the IT-biased productivity and rewrite the cost minimization con-

dition as:

$$\begin{aligned}\tilde{\phi}_{it} &\equiv (\sigma - 1)\phi_{it}, \\ &= s_{it} - l_{it} + \sigma \widehat{w}_t + (\sigma - 1)[\epsilon y_{it} + \alpha(k_{it} - l_{it}) - (1 - \beta)(h_{it} - s_{it})].\end{aligned}$$

Now, we substitute Equations (C.32) and (C.33) with:

$$\tilde{\Theta}(\mathbf{D}_{it}; \gamma) \equiv y_{it} - (\gamma_1 k_{it} + \gamma_2 l_{it} + \gamma_3 s_{it} + \gamma_4 h_{it}), \quad (\text{C.35})$$

$$\tilde{\Phi}(\mathbf{D}_{it}; \sigma, \epsilon, \gamma) \equiv s_{it} - l_{it} + \sigma \widehat{w}_{it} + (\sigma - 1) \left[\epsilon y_{it} + \frac{\gamma_1}{\gamma_1 + \gamma_2} (k_{it} - l_{it}) - \frac{\gamma_4}{\gamma_3 + \gamma_4} (h_{it} - s_{it}) \right]. \quad (\text{C.36})$$

Next, note that we can write:

$$\begin{aligned}\begin{pmatrix} \tilde{\theta}_{it} \\ \tilde{\phi}_{it} \end{pmatrix} &= \mathbf{B} \begin{pmatrix} \theta_{it} \\ \phi_{it} \end{pmatrix} + \begin{pmatrix} \tilde{\theta}_j \\ 0 \end{pmatrix}, \\ &= \mathbf{B} \times \left[\begin{pmatrix} \rho_{\theta\theta} & \rho_{\theta\phi} \\ \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \begin{pmatrix} \theta_{it-1} \\ \phi_{it-1} \end{pmatrix} + \begin{pmatrix} \eta_{\theta} + \mu_{\theta}^t \\ \eta_{\phi} + \mu_{\phi}^t \end{pmatrix} + \begin{pmatrix} u_{\theta,it} \\ u_{\phi,it} \end{pmatrix} \right] + \begin{pmatrix} \tilde{\theta}_j \\ 0 \end{pmatrix}, \\ &= \mathbf{B} \times \left[\begin{pmatrix} \rho_{\theta\theta} & \rho_{\theta\phi} \\ \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \mathbf{B}^{-1} \begin{pmatrix} \tilde{\theta}_{it-1} \\ \tilde{\phi}_{it-1} \end{pmatrix} + \begin{pmatrix} \eta_{\theta} + \mu_{\theta}^t \\ \eta_{\phi} + \mu_{\phi}^t \end{pmatrix} + \begin{pmatrix} u_{\theta,it} \\ u_{\phi,it} \end{pmatrix} \right] + \begin{pmatrix} \tilde{\theta}_j \\ 0 \end{pmatrix}, \\ &= \begin{pmatrix} \rho_{\tilde{\theta}\tilde{\theta}} & \rho_{\tilde{\theta}\tilde{\phi}} \\ \rho_{\tilde{\phi}\tilde{\theta}} & \rho_{\tilde{\phi}\tilde{\phi}} \end{pmatrix} \begin{pmatrix} \tilde{\theta}_{it-1} \\ \tilde{\phi}_{it-1} \end{pmatrix} + \begin{pmatrix} \eta_{\tilde{\theta}} + \mu_{\tilde{\theta}}^t \\ \eta_{\tilde{\phi}} + \mu_{\tilde{\phi}}^t \end{pmatrix} + \begin{pmatrix} u_{\tilde{\theta},it} \\ u_{\tilde{\phi},it} \end{pmatrix},\end{aligned}$$

where we have defined the matrix

$$\mathbf{B} \equiv \begin{pmatrix} \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 & \gamma_3 + \gamma_4 \\ 0 & \sigma - 1 \end{pmatrix},$$

and we have

$$\begin{aligned}\begin{pmatrix} \rho_{\tilde{\theta}\tilde{\theta}} & \rho_{\tilde{\theta}\tilde{\phi}} \\ \rho_{\tilde{\phi}\tilde{\theta}} & \rho_{\tilde{\phi}\tilde{\phi}} \end{pmatrix} &\equiv \mathbf{B} \begin{pmatrix} \rho_{\theta\theta} & \rho_{\theta\phi} \\ \rho_{\phi\theta} & \rho_{\phi\phi} \end{pmatrix} \mathbf{B}^{-1}, \\ \begin{pmatrix} \eta_{\tilde{\theta}} + \mu_{\tilde{\theta}}^t \\ \eta_{\tilde{\phi}} + \mu_{\tilde{\phi}}^t \end{pmatrix} &\equiv \mathbf{B} \begin{pmatrix} \eta_{\theta} + \mu_{\theta}^t \\ \eta_{\phi} + \mu_{\phi}^t \end{pmatrix} + \begin{pmatrix} \tilde{\theta}_j \\ 0 \end{pmatrix}, \\ \begin{pmatrix} u_{\tilde{\theta},it} \\ u_{\tilde{\phi},it} \end{pmatrix} &\equiv \mathbf{B} \begin{pmatrix} u_{\theta,it} \\ u_{\phi,it} \end{pmatrix}.\end{aligned}$$

C.5.4 Algorithm for Solving the Problem

Our initialization procedure is as follows. We first ignore the dynamic nature of the productivity processes and the heterogeneity of software relative price \widehat{w}_{it} and estimate separately the following two simplified versions of Equations (C.35) and (C.36) with linear 2SLS:

$$y_{it} = \gamma_1^0 k_{it} + \gamma_2^0 l_{it} + \gamma_3^0 s_{it} + \gamma_4^0 h_{it} + v_{\theta,it}^0, \quad (\text{C.37})$$

$$s_{it} - l_{it} = \gamma_5^0 y_{it} + \gamma_6^0 (k_{it} - l_{it}) + \gamma_7^0 (s_{it} - h_{it}) + v_{\phi,it}^0. \quad (\text{C.38})$$

We estimate Equation (C.37), instrumenting s_{it} , l_{it} , h_{it} and k_{it} by their lagged values, and Equation (C.38), instrumenting y_{it} , $k_{it} - l_{it}$, and $h_{it} - s_{it}$ by their lagged values. We retrieve the predicted values of the initial residuals $\widetilde{\theta}_{it}^0 = v_{\theta,it}^0$ and $\widetilde{\phi}_{it}^0 = v_{\phi,it}^0$ and estimate by OLS the persistence, trends and shifters from the joint productivity process:

$$\begin{pmatrix} \widetilde{\theta}_{it}^0 \\ \widetilde{\phi}_{it}^0 \end{pmatrix} = \begin{pmatrix} \rho_{\widetilde{\theta}\widetilde{\theta}}^0 & \rho_{\widetilde{\theta}\widetilde{\phi}}^0 \\ \rho_{\widetilde{\phi}\widetilde{\theta}}^0 & \rho_{\widetilde{\phi}\widetilde{\phi}}^0 \end{pmatrix} \begin{pmatrix} \widetilde{\theta}_{it-1}^0 \\ \widetilde{\phi}_{it-1}^0 \end{pmatrix} + \begin{pmatrix} \eta_{\widetilde{\theta}}^0 + \mu_{\widetilde{\theta}}^0 t \\ \eta_{\widetilde{\phi}}^0 + \mu_{\widetilde{\phi}}^0 t \end{pmatrix} + \begin{pmatrix} u_{\widetilde{\theta},it}^0 \\ u_{\widetilde{\phi},it}^0 \end{pmatrix}.$$

Second, we iterate the following step : armed with estimates from step $\tau - 1$, we ρ -differentiate Equations (C.37) and (C.38) :

$$\Delta_{\rho_{\theta\theta}^{\tau-1}} y_{it} - \rho_{\theta\phi}^{\tau-1} \widetilde{\phi}_{it-1}^{\tau-1} = \Delta_{\rho_{\theta\theta}^{\tau-1}} (\gamma_1^\tau k_{it} + \gamma_2^\tau l_{it} + \gamma_3^\tau s_{it} + \gamma_4^\tau h_{it} + v_{\theta,it}^\tau), \quad (\text{C.39})$$

$$\Delta_{\rho_{\phi\phi}^{\tau-1}} (s_{it} - l_{it}) - \Delta_{\rho_{\phi\phi}^{\tau-1}} b_{it}^\tau - \rho_{\phi\theta}^{\tau-1} \widetilde{\theta}_{it-1}^{\tau-1} = \Delta_{\rho_{\phi\phi}^{\tau-1}} (\gamma_5^\tau y_{it} + \gamma_8^\tau [\widehat{w}_{it} + b_{it}^\tau] + v_{\phi,it}^\tau), \quad (\text{C.40})$$

where we define $\Delta_{\rho} x_{it} = x_{it} - \rho x_{it}$ and $b_{it}^\tau = \frac{\gamma_1^\tau}{\gamma_1^\tau + \gamma_2^\tau} (k_{it} - l_{it}) + \frac{\gamma_4^\tau}{\gamma_3^\tau + \gamma_4^\tau} (s_{it} - h_{it})$. We estimate separately Equation (C.39) then Equation (C.40) with linear 2SLS. As in the initial step, we retrieve the predicted values of the initial residuals $\widetilde{\theta}_{it}^\tau = v_{\theta,it}^\tau$ and $\widetilde{\phi}_{it}^\tau = v_{\phi,it}^\tau$ and estimate by OLS the persistence, trends and shifters from the joint productivity process:

$$\begin{pmatrix} \widetilde{\theta}_{it}^\tau \\ \widetilde{\phi}_{it}^\tau \end{pmatrix} = \begin{pmatrix} \rho_{\widetilde{\theta}\widetilde{\theta}}^\tau & \rho_{\widetilde{\theta}\widetilde{\phi}}^\tau \\ \rho_{\widetilde{\phi}\widetilde{\theta}}^\tau & \rho_{\widetilde{\phi}\widetilde{\phi}}^\tau \end{pmatrix} \begin{pmatrix} \widetilde{\theta}_{it-1}^\tau \\ \widetilde{\phi}_{it-1}^\tau \end{pmatrix} + \begin{pmatrix} \eta_{\widetilde{\theta}}^\tau + \mu_{\widetilde{\theta}}^\tau t \\ \eta_{\widetilde{\phi}}^\tau + \mu_{\widetilde{\phi}}^\tau t \end{pmatrix} + \begin{pmatrix} u_{\widetilde{\theta},it}^\tau \\ u_{\widetilde{\phi},it}^\tau \end{pmatrix}.$$

We stop the iterative procedure when the maximum of the absolute differences of all parameters of the productivity process between steps $\tau - 1$ and τ is below 0.01 or the number of iterations

reaches 20. In practice, this procedure converges in less than three steps. In the final step τ_f , we compute the following parameter values:

$$\begin{aligned}\epsilon^{\tau_f} &= \frac{\gamma_5^{\tau_f}}{1+\gamma_8^{\tau_f}}, \\ \sigma^{\tau_f} &= -\gamma_8^{\tau_f}.\end{aligned}$$

Third, we estimate simultaneously Equations (C.35) and (C.36) by two-step GMM using $\gamma_1^{\tau_f}, \gamma_2^{\tau_f}, \gamma_3^{\tau_f}, \gamma_4^{\tau_f}, \epsilon^{\tau_f}, \sigma^{\tau_f}, \rho_{\tilde{\theta}\tilde{\theta}}^{\tau_f}, \rho_{\tilde{\theta}\tilde{\phi}}^{\tau_f}, \rho_{\tilde{\phi}\tilde{\theta}}^{\tau_f}, \rho_{\tilde{\phi}\tilde{\phi}}^{\tau_f}, \eta_{\tilde{\theta}}^{\tau_f}, \mu_{\tilde{\theta}}^{\tau_f}, \eta_{\tilde{\phi}}^{\tau_f}$ and $\mu_{\tilde{\phi}}^{\tau_f}$ as initial guesses. Fourth, we estimate simultaneously Equations (C.33) and (C.32) by two-step GMM results from the third step as an initial guess.⁶

C.6 Appendix: Data

In this section, we provide further details about the sources of data used and describe the procedure we use to merge our different data sources, to clean the resulting dataset of outliers, and to compute the firm level capital stock for each asset-type. We also discuss the construction of local wages and report some summary statistics of IT investment.

C.6.1 Sources of Micro Data

BRN and RSI are our two principal sources of data on firm activity in the universe of French firms. These administrative data are based on tax returns and are available starting 1984. They cover firms affiliated with the two main French tax regimes: BRN (*Bénéfice Réel Normal*) and RSI (*Régime Simplifié d'Imposition*). The BRN is the standard regime and the RSI is a simplified regime intended for small firms. Depending on their domain of activity, firms with revenues above a certain threshold must be affiliated with the BRN regime.⁷ These data provide information on the firm's number of employees, sales, value added, total and tangible investment, year of creation, industry, and location. Information on the disaggregated components of firm investment by asset types, including hardware investment, is available in the BRN files starting in 1989. While we rely on the whole sample of RSI and BRN firms for our measures of concentration and

⁶In both these steps, we use $t, s_{it-1}, l_{it-1}, h_{it-1}, k_{it-1}, \tilde{\theta}_{it-1}$ and $\tilde{\phi}_{it-1}^{\tau}$ as instruments for the first equation, and $t, y_{it-1}, k_{it-1} - l_{it-1}, h_{it-1} - s_{it-1}, \tilde{\theta}_{it-1}^{\tau}, \tilde{\phi}_{it-1}^{\tau}$ and the local wage instrument z_{it} as instruments for the second equation.

⁷In 2007, the thresholds were 763,000 euros if the firm operates in trade or real estate sectors, and 230,000 euros otherwise.

aggregate labor shares, we restrict our analysis of capital and investment to firms that appear at least once in the BRN dataset.

The EAE (*Enquête Annuelle d'Entreprises*) is a survey-based dataset collected every year from 1982 to 2007. The survey is conducted separately for each broad sector of the French economy (trade, transport, construction, manufacturing & utility, agri-food, and services), with some variation in the list of questions asked and the sampling methods used. Overall, the data comprehensively covers medium and large firms, i.e., those with more than 20 employees, and surveys a sample of the smaller ones.⁸ Starting in the 1990s, large firms are surveyed with a more comprehensive questionnaire that includes questions about software investment of firms.⁹ In the EAE files, missing values for software investment are coded as 0. Most of these missing values correspond to the smaller firms, surveyed with the simplified questionnaire which does not include information on investments. We adopt the following strategy to ensure that we distinguish actual zeros from missing data: we impute as missing software investment of firms that report 0 investment and whose employment and sales reported the previous year are below the threshold necessary to be fully surveyed.

We use two additional sources of data. We rely on the employee-level DADS data to find information on the number of plants and the organizational structure of the firm in terms of the occupational mix of employees. We also use the Customs data for information on the number of exported products and destination countries, as additional proxies for the scope of operation of firms.

C.6.2 Data Cleaning

We start with the BRN and RSI files from 1990 to 2007, in which we drop firms that have invalid SIREN using the cross-validation algorithm used to generate SIREN numbers. They correspond to firms whose self-reported SIREN identifiers do not match the SIREN identifiers recorded by INSEE. We then collapse observations that are not unique in terms of SIREN-year. They correspond to firms that appear in both BRN and RSI regimes in the same year. Finally we drop 382,854 observations for which we cannot build industry codes. We restrict our sample to firms that have one or more employees, and that report positive sales, value-added, and wage bill (including taxes on labor). This leaves us with 15,202,967 firm-year observations. We use these data (labelled "BRN+RSI") to compute the decomposition of labor shares, concentration and

⁸The only exception is manufacturing & utility, in which only the large firms are surveyed.

⁹The criterion for inclusion is based on the employment size of the firm at the end of the previous year. This more comprehensive questionnaire has been applied in select sectors starting in 1989, and has been extended to all sectors starting in 1995. For more details on the criteria for coverage in EAE (as well as the other datasets used in the paper).

some calibration moments from Table 4.11.

Using the French unique firm identifier (SIREN), we are able to match the observations from BRN-RSI to observations included in the EAE. For the rest of the paper, we construct measures of capital stock using the procedure described below, restricting our sample to firms that appear at least once in the BRN files.¹⁰ After the construction of capital stocks, initialized at the start of the year 1990, we discard the first five years of data. We then drop observations with negative values added (8.51% of observations), negative wage bill (34 observations), no employee (19.4 % of observations), book capital per employee relative to the industry average that is outside of the 99.99% probability range of a fitted distribution (0.28% of observations) and observations in the top 0.1% for total investment per employee. There are 6,166,342 firm-year observations from 1995 to 2007 in these data (labelled "BRN"), of which 2,435,356 firm-year observations correspond to firms also surveyed at least once by the EAE (labelled "EAE").¹¹

BRN firms are broadly representative of the aggregate French economy: they account for 74.9% of private value-added and 81.9% of private employment. Table 4.1 reports summary statistics on the three samples, BRN + RSI, BRN, and BRN restricted to EAE firms.

To compute our measures of firm scale, we then match with the observations in the DADS and Customs data. Of the 6,166,342 observations in our BRN sample, 5,692,230 are also in DADS, and 1,665,474 correspond to exporting firms. Some firms in the DADS and Customs data are not present in the BRN files. DADS covers all employers with salaried workers, so include non-profits, households as employers, and public employers. Matched DADS observations have on average 36 employees according to the DADS, against less than 10 employees for unmatched firms. Matched customs firms declare total exports of 3.1 million euros on average, against less than 1.7 million euros for unmatched customs firms.

Finally, in the estimation, we further restrict our sample to EAE firms with positive software, hardware, and non-IT capital, positive labor and value added, and for which the location of the firm's headquarter is known. These last restrictions bring the number of observations in our estimation sample to 302,318.

¹⁰RSI only firms have average sales of 102 thousands euros, against 3,848 thousands euros for firms that appear in the BRN files at least once.

¹¹BRN firms that are never surveyed in EAE have average sales of 3,809 thousands euros, against 13,583 thousands euros for surveyed firms.

C.6.3 Building Measures of Capital Stock

To compute capital stock measures, we apply the Perpetual Inventory Method. For each asset-type j , firm i , and year t , we build capital stocks using the following recursive formula:

$$K_{j,i,t} = K_{j,i,t-1}(1 - \delta_j) + \frac{I_{j,t}}{W_{j,t}}, \quad (\text{C.41})$$

where $W_{j,t}$ stands for the price deflator for asset-type j at time t , and δ_j for the depreciation rate in asset-type j . Below, we discuss how we initialize this recursive formula for each asset-type and for each firm. We also discuss where we obtain the information on price deflators and depreciation rates from. Total capital stock is the sum of all asset-types stocks, which allows us to fully take into account the heterogeneity of investment composition across firms instead of using a common price and depreciation rate.

There are 273,181 unique firms in the EAE sample, totaling more than 2 million observations. For 25% of those firms, we do not impute any software investment values as these firms are present in the BRN EAE data every year from their first entry to their exit. Among the remaining firms whose software stocks include some imputed values, more than two third of imputed zeros correspond to firms that typically first appear as small firms in the RSI sample, then as larger firms in the BRN sample, and then large enough to be sampled in EAE. Before the first year in which they appear in the EAE data, these firms have virtually zero software stock and those years are dropped from regressions of log software intensity or from the estimation sample. The remaining cases correspond to firms that are not systematically sampled in EAE even after the first year that they are sampled because they remain close to the threshold of size that determines which firms are exhaustively surveyed by EAE. Similarly, there are 855,492 unique firms in the BRN sample, totaling more than 6 million observations. For 75% of these firms, we do not impute any investment values for the hardware and other non-IT investment data as these firms are present in the BRN data every year from their first entry to their exit.

Imputed values are not used in the regressions corresponding to the IT “investment” intensity of hardware and software in Figure 4.2 and Table 4.6. Moreover, as we restrict our sample to larger firms in Table 4.8, the share of firms whose software stocks include some imputed zeros drops considerably: fewer than 1% of firms have more than one imputed zero. In our estimation sample, the 302,318 observations corresponds to the 64,698 firms that have positive values, hardware, and other non IT capital stocks, and stocks of software larger than 10 euros. For these firms, the imputation method impacts the stocks of the software, hardware, or non-IT capital only if the firm appears in the BRN or EAE data in year $t - 1$ and $t + 1$ but not in year t . In practice, this

impacts fewer than 1% of the firms in the cases of hardware and non-IT capital stocks, and fewer than 10% of the firms in the case of software stock.

Initialization

We initialize the stock of each asset-type in 1989 ($t = 0$) assuming first that in each of the 38 industries s for which aggregate data is available, the ratio of total investment to total stock in our sample is equal to the ratio of investment $\bar{I}_{j,0}^s$ to stock $\bar{K}_{j,0}^s$ in the aggregate data:¹²

$$\sum_{i \in s} K_{j,i,0} = \frac{\sum_{i \in s} I_{j,i,0}}{\bar{I}_{j,0}^s} \times \bar{K}_{j,0}^s, \quad (\text{C.42})$$

where $\frac{\sum_{i \in s} I_{j,i,0}}{\bar{I}_{j,0}^s}$ is typically below 1 (0.469 on average). This allows us to construct an industry-level stock for our sample of firms. Then, we assume that the share of each firm in that industry-level stock is given by the share of the firm average investment across all years in that asset-type $I_{j,i,0}^m$ to the sum of the average investments in that asset-type of all firms in that industry. At year 0, the imputed value of the stock of asset j of firm i in industry s is then given by:

$$K_{j,i,0} = \frac{I_{j,i,0}^m}{\sum_{i \in s} I_{j,i,0}^m} \times \frac{\sum_{i \in s} I_{j,i,0}}{\bar{I}_{j,0}^s} \times \bar{K}_{j,0}^s. \quad (\text{C.43})$$

C.6.4 Measures of Local Wages

As we will discuss in Section 4.5.1, in our identification of the production function we rely on the series for the price of software relative to wages at the local level. The BRN and RSI files contain information about the municipality where the headquarters of the firm are located, as well as the 5-digits industry to which the firm belongs. We use this information to construct measures of average wages by 2-digits industry at the level of local employment area (*Zone d'emploi*).¹³ We further rely on an instrument for the relative price of IT that follows the standard logic of Bartik (1991), relying on local variations on the industrial composition of employment. We compute an instrument capturing the predicted change in the labor demand in each employment area, based on the interaction of the initial composition of the wage bill in each employment area and the change in each industry's employment at the national level.

¹²The aggregate industry levels of stocks and investment are provided by INSEE at the 38 industries level. We use net values of capital at constant replacement cost, which already account for previous years capital depreciation. We report the resulting aggregate capital stocks by broad industries in the online appendix.

¹³There are 364 employment areas, defined in 1990 as geographical units with more than 25,000 workers within which most of the workforce commutes. See the online appendix for details about the construction of local wages.

C.6.5 Overview of the Data

As Table 4.1 shows, the distribution of both types of IT investment is highly skewed, much more so than that of the total investment. The modal firm invests zero in both software and hardware, whereas the average firm invests over 10,700 euros in software and 6,600 euros in hardware annually (conditional on being surveyed). The values of investment are higher in manufacturing compared to other sectors. However, manufacturing firms are on average larger than non-manufacturing firms. As we will see below, the differences between sectors is less pronounced when we examine proxies of software and hardware intensity of firms.

Table ?? reports the intensive and extensive margins of software investment, separately for different classes of firm employment (in year 1996).¹⁴ The likelihood of reporting non-zero investment values is larger among larger firms. Conditional on reporting non-zero investment in software, both the median and mean of the values reported are also greater among larger firms. This relationship appears in all years in our data, e.g., we can observe it in 2006. Examining the data on hardware investment also shows a similar pattern.

As with the investment measures, our constructed measures of capital in Table 4.1 show evidence of skewness. The median firm has zero stocks of software and hardware capital, while the mean values of software and hardware capital stock are around 15,500 euros and 23,300 euros, respectively

In our analysis, we introduce a number of proxies for *IT intensity* of firms. For both software and hardware, and for each of the investment (flow) and the capital (stock) measures, we define two proxies for intensity: first, intensity per worker (where we divide the values by the number of workers), and second, relative intensity (where we divide the values by the respective total value of investment or capital of the firm). As Table 4.1 shows, these intensity measures also inherit the skewness of the original measures: when in per-worker terms, the average investments in software and hardware are 50 and 200 euros, respectively, and those for software and hardware capital stock are 80 euros and 469 euros, respectively. The medians are zero in all four cases. The averages of relative intensity measures are fairly small, with software constituting an average of only 2% (0.4%) and hardware of just above 6% (3.5%) of total investment (capital).

¹⁴The drop in the coverage of the data (as reflected in the number of observations) for firms with less than 20 employees is due to the design of the EAE survey.

Titre: Essais sur les fonctions de production des entreprises, leurs taux de marge et la part du travail dans leur valeur ajoutée

Mots clés: Comportement des entreprises, Part du Travail, Fonctions de production des entreprises

Résumé: La fonction de production des entreprises lie leur niveau de production à leurs dépenses en facteurs de production. Son estimation est à la fois importante et peu fiable. Importante, parce que des indicateurs clés pour la conception des politiques publiques, tels que le taux de marge, en découlent. Peu fiable, car elle repose sur des hypothèses d'identification. Ce projet étudie les hypothèses qui sous-tendent l'estimation des fonctions de production : à la fois leurs formes fonctionnelles et la flexibilité des facteurs de production. Il s'appuie ensuite sur une estimation des fonctions de production des entreprises françaises, pour évaluer l'évolution de leur taux de marge et de la part du travail dans leur valeur ajoutée au cours des 30 dernières années.

Le chapitre 2 conforte l'hypothèse de flexibilité des facteurs de production: l'ajustement instantané des matières premières ou du travail et l'ajustement retardé du capital. Nous nous appuyons sur l'existence de notches ; des valeurs où les bénéfices après impôt diminuent avec le chiffre d'affaire avant impôt dans le code des impôts français. Nous montrons que les entreprises qui optimisent ont une plus grande élasticité de production par rapport aux matières premières et une plus faible élasticité de la production par rapport au capital. De même, pour ajuster leur production, les entreprises ont tendance à réduire principalement leurs dépenses en matières premières.

Le chapitre 3 s'appuie sur le résultat du chapitre 2 pour mesurer le taux de marge de toutes les entreprises françaises entre 1984-2016. De Loecker and Warzynski (2012) montrent que la marge d'une entreprise est proportionnelle à l'inverse de la part de revenu de l'un de ses intrants flexibles. Nous analysons l'évolution des marges

agrégées en France et documentons que l'augmentation de la concentration est corrélée à une réallocation des parts de marché vers les entreprises à marge élevée. Nous montrons également que l'évolution de la part du travail reflète l'évolution des marges : la réallocation tend à diminuer la part du travail tandis qu'au sein des entreprises, la part du travail augmente.

Le chapitre 4 questionne la forme fonctionnelle des fonctions de production. Il montre que les fonctions de production CES ou Cobb-Douglas ne rend pas compte de l'utilisation des technologies de l'information (TIC), puisque nous documentons une augmentation de la demande relative de TIC avec la taille des entreprises: en cohérence avec une fonction de production CES non-homothétique. Nous analysons ensuite comment l'interaction de la baisse des prix des TIC et les caractéristiques non-homothétiques des TIC rationalisent les faits empiriques documentés dans le chapitre 3. (i) comme les grandes entreprises sont plus intensives en TIC, elles bénéficient de manière disproportionnée de la baisse des prix des TIC, ce qui rationalise l'augmentation de la concentration. (ii) comme les grandes entreprises sont plus intensives en TIC dans l'échantillon, elles fonctionnent avec des rendements d'échelle plus faibles et ont donc des parts de bénéfices plus élevées et des parts de travail plus faibles. Cela explique comment l'augmentation de la concentration entraîne une diminution de la part globale du travail. (iii) les statistiques comparatives du modèle prédisent que l'adoption de TIC liée à la baisse de leur prix implique des rendements d'échelle plus élevés et ont donc une part de travail plus importante, ce qui explique la tendance haussière de la part du travail au sein des entreprises.

Title: Essays on Firms Production Function, Markups, and the Share of their Income Going to Workers

Keywords: Firms behaviors, Labor Share, Firms Production Functions

Abstract: Firms production function link their use of input to their production level. Production function estimates are at the same time important and untrustworthy. Important, because key indicators for policy design, such as the measure of aggregate markups, are derived from those estimates. Untrustworthy because they rely on identification assumptions. This project studies the assumptions underlying the usual techniques for estimating production functions: both their functional forms and the often assumed inputs flexibility. It then leverages production function estimates, to assess how firms ability to price over marginal income and the share of their income going to workers have evolved over the last 30 years.

Chapter 2 provides evidence on the input flexibility assumption grounding production function estimation: the quasi instantaneous adjustment of either material or labor and delayed adjustment of capital hold. We rely on the existence of notches; values where after-tax profits decrease in before-tax sales in the French tax code, to identify which type of firms adjust their size in response to a transient notch. We show that firms with larger output elasticities with respect to material and lower output elasticities with respect to capital tend to bunch more. They also tend to adjust their production firms by primarily reduce spending on material, while labor seems to adjust in the same proportion as production.

Chapter 3 leverages evidence on inputs flexibility to recover aggregate markups in France over the 1984-2016 period. De Loecker and Warzynski (2012) show that a firm's markup is inversely proportional to one of its flexible input's share in revenue. We analyze the evolution of ag-

gregate markups and document that the rise of concentration correlates with a reallocation of market share towards high markup firms. We also show that the evolution of the labor share mirrors the evolution of markups: reallocation tends to decrease labor share while within firms, labor share rises.

Chapter 4 discusses production functions' functional form. It shows that standard CES production function cannot account for IT inputs use in firms. With detailed data on software and hardware investments among French firms, we document that the firm-level demand for IT inputs relative to other inputs grows in the firm's scale of operation. Theoretically, a non-homothetic CES production function helps rationalizing this empirical fact. We then analyze how the interaction of the fall of IT prices and the non-homothetic characteristics of IT inputs also help rationalize the empirical facts documented in chapter 3. First, since larger firms are more IT intensive in the cross-section, they benefit disproportionally from the fall in IT prices, rationalizing the rise of concentration. Similarly, since larger firms are more IT intensive in the cross-section, they operate at lower returns to scale and therefore have higher profit shares and lower labor shares. This explains how the rise in concentration drives a decline in aggregate labor shares. Finally, the comparative statistics of the model predicts that the fall of IT prices imply that when firms substitute toward IT they operate at higher returns to scale and therefore tend to have larger labor share, explaining the positive contribution to aggregate labor share of the within component.