



**HAL**  
open science

# Les modèles de la structure par terme des taux d'intérêt, la dette publique et les obligations indexées sur l'inflation

Olga Pakulyak

► **To cite this version:**

Olga Pakulyak. Les modèles de la structure par terme des taux d'intérêt, la dette publique et les obligations indexées sur l'inflation. Gestion et management. Université de Rennes, 2019. Français. NNT : 2019REN1G022 . tel-03612849

**HAL Id: tel-03612849**

**<https://theses.hal.science/tel-03612849>**

Submitted on 18 Mar 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# THESE DE DOCTORAT DE

L'UNIVERSITE DE RENNES 1  
COMUE UNIVERSITE BRETAGNE LOIRE

ECOLE DOCTORALE N° 597  
*Sciences Economiques et Sciences De Gestion*  
Spécialité : « *Sciences de Gestion* »

Par

**Olga PAKULYAK**

**Term structure of interest rates models, nominal government  
debt and inflation-protected securities**

Thèse présentée et soutenue à Rennes, le 4 Décembre 2019  
Unité de recherche : CNRS UMR 6211 CREM

## Rapporteurs avant soutenance :

Delphine LAUTIER      Professeure, Université Paris-Dauphine  
Jean-Paul RENNE      Professeur, HEC Lausanne

## Composition du Jury :

Président :	Jean-Jacques LILTI	Professeur, Université de Rennes 1
Examineurs:	Olesya GRISHCHENKO	Senior Economist, Federal Reserve System
	Delphine LAUTIER	Professeure, Université Paris-Dauphine
	Jean-Paul RENNE	Professeur, HEC Lausanne
Dir. de thèse :	Franck MORAUX	Professeur, Université de Rennes 1



The University of Rennes 1 neither endorses nor censure the opinions expressed in this thesis; these opinions are those of the author.

L'Université de Rennes 1 n'entend donner aucune approbation ni improbation aux opinions émises dans cette thèse; ses opinions doivent être considérées comme propres à leur auteur.



I would like to dedicate this thesis to my parents  
for their love and support.



## Acknowledgements

First of all, I would like to thank my supervisor Professor Franck MORAUX for all his help and guidance over the past four years. He trusted me and gave me a lot of freedom in my research.

My special thanks will go to Olesya GRISHCHENKO for valuable ideas, guidance in my research and interesting discussions we had together.

I am also very grateful to Delphine LAUTIER and Jean-Paul RENNE who accepted to review this thesis, to the examiners Jean-Jacques LILTI and Olesya GRISHCHENKO for their interest in my thesis and for the honor they have conferred upon me by agreeing to be in my thesis comity.

I would like to thank the laboratory CREM (UMR CNRS 6211) for the financial support I have received to present my work at conferences.

Needless to say, I am extremely grateful to all professors and staff members at IGR-IAE and IUT of Saint-Malo for various assistance. I also thank the PhD students at IGR-IAE for good times spent together. They all contribute making this part of my life such a great time.

Last but not least, special thanks to my parents for loving and supporting me all my life. Also, thanks to my husband for being there for me and making me smile.

Thanks for your support and encouragement.

Olga (October 2019)



---

## Abstract

This thesis analyzes the term structure of interest rates, the debt management and inflation-protected securities. The analysis is conducted through three empirical studies. These investigations give some interesting results about government bond markets. Prior to these latter, we provide several theoretical notions of the term structure of interest rates models; the three factors of the yield curve known as the level, the slope and the curvature; the duration measure; the organization of the fixed income securities market; different types of yields as zero-coupon, par yield and forward rates; and inflation. Our first study examines four Nelson-Siegel style yield curve models for fitting the term structure of interest rates on data about government bond prices. The dataset contains bonds issued by four countries in Euro area. We compare these specifications by their in-sample performance to match bond prices and find that the extended Svensson specification performs better overall in bond price calculation. In our second study we construct the French nominal yield curve using all available public data of French nominal Treasury securities with maturities at issuance from 1 to 50 years. The French sovereign bond market has been functioning reasonably well, especially since the launch of the euro, outside of a few episodes as (for instance) the Global Financial Crisis period and the European sovereign crisis period. Our third study investigates real rates on French government bond market using the data on French inflation-protected Treasury securities. Our data set includes both types of such securities, those indexed on the domestic consumer price index and those indexed on the European inflation index. We backcast the five-year five-year forward breakeven inflation rate before the appearance of any inflation-protected securities on the market.

**Keywords:** Term structure of interest rates, fitting the yield curve, Euro area, Svensson model, OTR premium, nominal rates, inflation-protected securities, real rates, inflation.



---

## Résumé

Dans cette thèse, nous analysons la structure par terme des taux d'intérêt, la gestion de dette publique et les obligations indexées sur l'inflation. Pour conduire notre analyse, nous avons effectué trois études empiriques. Ces trois études empiriques donnent des résultats intéressants sur les marchés des obligations d'État. Avant ces derniers, nous avons commencé par présenter plusieurs aspects théoriques tels que les modèles de la structure par terme des taux d'intérêt; les trois facteurs de la courbe des taux qui correspondent au niveau, à la pente et à la courbure; la mesure de la durée; l'organisation du marché des titres à revenu fixe; différents types de rendements tels que zéro-coupon, *par yield* et taux à terme et l'inflation. Notre première étude examine quatre modèles de la structure par terme des taux d'intérêt de type Nelson-Siegel pour ajuster la courbe des taux aux données des prix des obligations d'État. La base de données contient des obligations émises par quatre pays de la zone euro. Nous comparons ces spécifications en fonction de leur capacité à expliquer les prix des obligations observés et constatons que le modèle *extended* Svensson, que nous proposons, offre la meilleure performance sur les données de tous les quatre pays. Dans notre deuxième étude, nous construisons la courbe des taux nominaux française en utilisant toutes les données publiques disponibles sur les obligations émises par le Trésor français avec des échéances au moment de l'émission comprises entre 1 et 50 ans. Le marché des obligations souveraines françaises fonctionne relativement bien, en particulier depuis le lancement de l'euro, en dehors de quelques épisodes comme la crise financière mondiale de 2007-2008 et la période de crise de la dette de la zone Euro. Dans notre dernière étude nous examinons les taux réels du marché des obligations d'État français en utilisant les données sur les titres indexés sur l'inflation et émis par le Trésor français. Les données comprennent les deux types de titres, ceux indexés sur l'inflation domestique et ceux indexés sur l'indice européen d'inflation.

**Mots-clés:** La structure par terme des taux d'intérêt, l'ajustement de la courbe des taux, la zone euro, les obligations d'état, modèle de Svensson, la prime OTR, les taux nominaux, les obligations indexées sur l'inflation, les taux réels, l'inflation.



# Contents

<b>Contents</b>	<b>vi</b>
<b>List of Figures</b>	<b>x</b>
<b>List of Tables</b>	<b>xii</b>
<b>General Introduction</b>	<b>1</b>
<b>1 Fundamental aspects of the term structure of interest rates</b>	<b>10</b>
1.1 Introduction . . . . .	10
1.2 The term structure of interest rates models . . . . .	11
1.2.1 Equilibrium approach . . . . .	12
1.2.2 Arbitrage-free approach . . . . .	14
1.2.3 Functional approach . . . . .	15
1.3 Factors of TSIR . . . . .	18
1.4 The duration . . . . .	20
1.4.1 Risk factors . . . . .	23
1.4.2 Investment strategies . . . . .	24
1.5 Institutional aspects . . . . .	26
1.5.1 Auctions, actors, and secondary markets . . . . .	27
1.5.2 Products, monetary policy, and maturity distribution . . . . .	28
1.5.3 Market supply and demand . . . . .	30
1.6 Types of yields . . . . .	31
1.6.1 Par yield . . . . .	32
1.6.2 Forward yield . . . . .	32

1.7	Government debt and inflation . . . . .	35
1.8	Government bond market illiquidity . . . . .	36
<b>2</b>	<b>Pricing government bonds in Euro area: performance evaluation of term structure interest rate models <sup>1</sup></b>	<b>39</b>
2.1	Introduction . . . . .	39
2.2	Literature review . . . . .	41
2.3	Yield Curve Models . . . . .	43
2.3.1	The NS approach . . . . .	43
2.3.2	Dynamic Svensson approach . . . . .	45
2.3.3	Extended Bjork and Christensen approach . . . . .	45
2.3.4	Extended Svensson approach . . . . .	46
2.4	Empirical methodologies . . . . .	48
2.4.1	Parameter estimation . . . . .	48
2.4.2	Performance evaluation . . . . .	49
2.5	Data Description . . . . .	50
2.6	Results . . . . .	53
2.6.1	In-sample model fit . . . . .	54
2.6.2	Identifying the best model . . . . .	58
2.6.3	A closer look at the fitting errors over the sample period . . . . .	58
2.6.4	A closer look at the par-yield curves . . . . .	64
2.6.5	Correlation . . . . .	64
2.7	Out-of-sample forecast performance . . . . .	66
2.7.1	Forecasting method . . . . .	66
2.7.2	Forecasting results . . . . .	67
2.8	Conclusion . . . . .	69
	<b>Appendices</b>	<b>70</b>
<b>3</b>	<b>The Term Structure of the French Nominal Government Debt <sup>2</sup></b>	<b>89</b>
3.1	Introduction . . . . .	89
3.2	The French government bond market . . . . .	93
3.3	Data . . . . .	96
3.4	Methodology . . . . .	98

3.4.1	Basic concepts . . . . .	99
3.4.2	Svensson methodology . . . . .	101
3.4.3	Filters . . . . .	102
3.4.4	Estimation . . . . .	103
3.5	Results . . . . .	105
3.5.1	Model fit . . . . .	105
3.5.2	The term structures of zero-coupon and forward rates . . .	109
3.5.2.1	Shapes of the yield curve . . . . .	109
3.5.2.2	Dynamics of the yield curve . . . . .	112
3.5.3	Factors of the yield curve . . . . .	113
3.6	Is there any on-the-run premium on the French bond market? . .	116
3.7	A closer look at the pre-euro era . . . . .	121
3.7.1	Model fit . . . . .	122
3.7.2	Noise measure . . . . .	123
3.8	Conclusion . . . . .	123
 <b>Appendices</b>		 <b>126</b>
<b>4</b>	<b>French Inflation-Protected Government Bonds <sup>1</sup></b>	<b>135</b>
4.1	Introduction . . . . .	135
4.2	Literature review . . . . .	138
4.3	Methodology . . . . .	141
4.3.1	About inflation . . . . .	143
4.3.2	Yield mathematics . . . . .	145
4.4	Data description . . . . .	146
4.5	Results . . . . .	149
4.5.1	Fitting the yield curve . . . . .	154
4.5.2	Term structure of real and breakeven rates . . . . .	157
4.6	Backcasting inflation . . . . .	160
4.6.1	Methodology aspect . . . . .	161
4.6.2	Results of backcasting . . . . .	162
4.7	Managerial aspect and discussion . . . . .	164
4.7.1	Asset pricing and real rates . . . . .	165

## CONTENTS

---

4.7.2	Real rates and inflation risk . . . . .	167
4.8	Conclusion . . . . .	168
<b>Appendices</b>		<b>170</b>
	<b>General Conclusion</b>	<b>181</b>
	<b>Résumé</b>	<b>185</b>
<b>Bibliography</b>		<b>195</b>

# List of Figures

1.1	Loadings of Nelson-Siegel model . . . . .	17
2.1	Loadings of extended Svensson model . . . . .	47
2.2	Number of government bonds within the dataset . . . . .	51
2.3	Maturity Distribution of the government bonds . . . . .	52
2.4	Fitting Errors (extended Svensson model) . . . . .	60
2.5	Par yield curve (NS specification) on August 16, 2018 . . . . .	61
2.6	Par yield curve (NS specification) on June 6, 2008 . . . . .	62
2.7	Par yield curve (eSv specification) on June 6, 2008 . . . . .	63
A.8	Fitting Errors (Nelson-Siegel model) . . . . .	81
A.9	Fitting Errors (Svensson model) . . . . .	82
A.10	Fitting Errors (extended Bjork-Christensen model) . . . . .	83
A.11	Par yield curve (Sv specification) on June 6, 2008 . . . . .	84
A.12	Par yield curve (eBC specification) on June 6, 2008 . . . . .	85
3.1	Notional Amount of the French Nominal Debt . . . . .	90
3.2	Maturity Distribution of the BTANs and OATs . . . . .	97
3.3	Fitting Errors . . . . .	106
3.4	Par Yield Curve . . . . .	108
3.5	Time Series of Zero-Coupon Yields . . . . .	112
3.6	Principal Components of the Yield Curve . . . . .	114
3.7	Fleming On-the-run Premium . . . . .	117
3.8	GSW On-the-run Premium . . . . .	120
3.9	Noise Measure of the French Bond Market . . . . .	124
B.10	Maturity-specific Fitting Errors . . . . .	130

## LIST OF FIGURES

---

B.11	Term Structures of the Zero-Coupon Rates and Forward Rates . . .	131
B.12	Principal Component Loadings of Yield Curve . . . . .	132
B.13	Maturity-specific Fitting Errors: Pre-euro Sample . . . . .	133
B.14	Par Yield Curve: Pre-euro Sample Period . . . . .	134
4.1	Maturity Distribution . . . . .	149
4.2	Notional Amount of the French Debt and Number of Bonds . . .	150
4.3	Time Series of Annual Inflation Rates . . . . .	151
4.4	Noise Measure and Duration Cutoff for OAT <sub>i</sub> . . . . .	152
4.5	Noise Measure and Duration Cutoff for OAT <sub>€i</sub> . . . . .	153
4.6	Par Yield Curve for OAT <sub>i</sub> sample . . . . .	155
4.7	Par Yield Curve for OAT <sub>€i</sub> sample . . . . .	156
4.8	Time Series of Zero-Coupon Real Yields for OAT <sub>i</sub> sample . . . . .	157
4.9	Time Series of Zero-Coupon Real Yields for OAT <sub>€i</sub> sample . . . . .	158
4.10	Unconditional Zero-Coupon Real Yields . . . . .	159
4.11	Zero-Coupon Real, Nominal and Breakeven Yields for OAT <sub>i</sub> sample	160
4.12	Zero-Coupon Real, Nominal and Breakeven Yields for OAT <sub>€i</sub> sample	161
4.13	Actual and Fitted Five-to-ten Year Forward Inflation Compensation	163
C.14	Fitting Errors for OAT <sub>i</sub> sample . . . . .	171
C.15	Maturity-specific Fitting Errors for OAT <sub>i</sub> sample . . . . .	172
C.16	Fitting Errors for OAT <sub>€i</sub> sample . . . . .	173
C.17	Maturity-specific Fitting Errors for OAT <sub>€i</sub> sample . . . . .	174
C.18	Term Structure of the Zero-Coupon and Forward Rates for OAT <sub>i</sub> sample . . . . .	175
C.19	Term Structure of the Zero-Coupon and Forward Rates for OAT <sub>€i</sub> sample . . . . .	176
C.20	Time series of 5 year Zero-Coupon Real, Nominal and Breakeven Yields for OAT <sub>i</sub> sample . . . . .	177
C.21	Time series of 5 year Zero-Coupon Real, Nominal and Breakeven Yields for OAT <sub>€i</sub> sample . . . . .	178

# List of Tables

2.1	Summary of term structure models . . . . .	44
2.2	Summary statistics of the model fitting errors . . . . .	57
2.3	Performance analysis . . . . .	59
2.4	Correlation of zero-coupon yields . . . . .	65
A.5	Summary of the Nominal Securities, France . . . . .	71
A.6	Summary of the Nominal Securities, Germany . . . . .	72
A.7	Summary of the Nominal Securities, Italy . . . . .	74
A.8	Summary of the Nominal Securities, Spain . . . . .	76
A.9	Descriptive statistics of estimated parameters . . . . .	77
A.10	Out-of-sample forecasting results . . . . .	79
3.1	Summary Statistics of the Svensson Model Fitting Errors . . . . .	107
3.2	Summary Statistics about zero-coupon and instantaneous rates . . . . .	110
3.3	Correlation . . . . .	111
3.4	Principal Component Decomposition . . . . .	115
3.5	On-the-Run/Off-the-Run Yield Spreads . . . . .	118
B.6	Summary of the Nominal BTAN and OAT Securities, 1987 - 1998	127
B.7	Summary of the Nominal BTAN and OAT Securities, 1999 - 2009	128
B.8	Summary of the Nominal BTAN and OAT Securities, 2010 - 2018	129
4.1	Summary of OAT <sub>i</sub> Securities . . . . .	147
4.2	Summary of OAT <sub>€i</sub> Securities . . . . .	148



# General Introduction

The term structure (spot) of interest rates is the relationship between the yield to maturity of zero-coupon bonds and their time-to-maturity. This relationship is among the most fundamental concepts in finance and is relevant to diverse fields such as portfolio management, pricing of interest rate derivatives, and risk management. This relationship is also exploited by actors far beyond the discipline of finance, such as regulators, economists, policymakers, and journalists. Despite the broad use of the term “structure of interest rates,” which is also called the yield curve, it is not directly observable in the market. Moreover, the yield curve exhibits a wide range of shapes over time, and many theoretical frameworks are trying to explain this phenomenon. However, we still need further analysis on the behavior of the term structure of interest rates, specifically the intricate nexus between interest rates and coupon bond prices.

While modeling or explaining the yield curve, a key challenge is to provide a useful summary of the information at any given point in time. Many types of bonds are traded in the market and, ideally, the model should be parsimonious. Moreover, such a model should be able to reproduce both the historical stylized facts (among those associated with the average shape of the yield curve) and forecast the future level of interest rates. Another key challenge is to provide an effective way to forecast the term structure of interest rates dynamics. In this thesis, we investigate the French government bond market from 1988 to 2018 and address the challenge of understanding the inflation-protected securities that France issued for the first time in 2000.

There are several motivations to study the term structure of interest rates, with

---

the first being the need for pricing fixed-income securities, which are loans made by the investor to the government or a corporate borrower. The second is the need for asset and risk management of bond portfolios. Investors include bonds in their portfolios for different reasons such as income generation, capital preservation and appreciation, and as a hedge against economic slowdowns and downturns. Finally, it is important to analyze the information content, more precisely the ability of the term structure of interest rates to predict recessions. As it is critical to continue to investigate the term structure of interest rates, we conducted three empirical studies to contribute to a better understanding of the latter.

In this thesis, we do the in-sample fit of the yield curve in order to provide smooth functioning for a wide range of maturities. We also attempt to define an optimal method to forecast the dynamics of the term structure of interest rates, as government or sovereign bonds are related to national debt. We study the term structure of the French nominal interest rates using data on government bond prices. The estimated yield curve is expressed as zero-coupon yields, par yields, and forward rates. Inflation-protected securities provide investors financial instruments that are shielded from inflation risks.

The purpose of this thesis is to conduct empirical analysis of the term structure of France's real interest rates using data on inflation-protected securities. In the first stage, we compare different approaches to model this phenomenon by considering four models on a dataset of government bond prices from four eurozone countries. In the second stage, we study the term structure of nominal interest rates in France working with a thirty-year sample period and 179 French government bonds. Finally, we consider the French inflation-protected securities to study the term structure of real interest rates. We construct a dataset with prices of government bonds linked to the domestic and European inflation index (OATi and OAT€I respectively).

This thesis contributes to the understanding of the term structure of interest rates through three essays. The first essay implements different functional approaches to model the yield curve using data on government bond prices. We work with bonds issued by France, Germany, Italy, and Spain. This investigation aims at

---

finding the model that performs best in terms of fitting the term structure. The exercise rests on two aspects: first, in a static environment, where we study the in-sample fitting capacity of the term structure model; second, the ability to provide a good out-of-sample forecast of the dynamics of the term structure of interest rates. The second essay studies nominal interest rates in France, by exploiting a thirty-year sample period from 1988 to 2018. Our investigation concludes the absence of the on-the-run premium on the French government bond market. We also document some significant improvements in the quality of the French government bond market functioning after the introduction of the euro currency. The third essay explores the French inflation-protected securities and the associated term structure of real interest rates. This investigation aims at understanding the interaction between government debt and inflation in the economy of a country, for instance France. Our main contribution is that we can compute the five-year forward five-year inflation compensation rate before the appearance of any inflation-protected securities in the market using the backcasting method. The rest of this thesis is organized as follows.

**The first chapter** describes the analytical framework. It presents some prerequisites for reading the following chapters. It also provides a robust foundation for the topic and delimits the perimeter of the research. Readers who are familiar with these aspects may omit it. In this chapter we present three popular families of term structure models: equilibrium approach, no-arbitrage approach, and dynamic approach. We also present the principal component analysis with three yield curve factors known as the level, slope, and curvature. These factors explain most of the variations in the term structure of interest rates. Duration and convexity, the two standard tools to manage the risk exposure of fixed-income investments, are discussed. We also provide a description of the institutional aspect of the fixed-income securities market, for instance, the actors and products on the market. At the end of the chapter, we present different types of yields as zero-coupon, par yield, and forward rates and discuss government debt management and liquidity in the fixed-income securities markets.

**The second chapter** considers the international aspect of term structure of interest rates, and the choice of approach to model the dynamics. It compares

---

the results provided by four specifications on the data taken from four countries. For instance, [Chan, Karolyi, Longstaff, and Sanders \[1992\]](#) estimate and compare a variety of continuous-time models of the short-term riskless rate. A few articles compare several interest rates models on the dataset containing several countries, but none of them use bond prices. Instead, most of them take zero-coupon yield curves for granted focusing on one country and then analyze the different models. For fixed income managers, macroeconomists, and financial economists it is important to be able to produce an accurate forecast of the term structure of interest rates. Moreover, bond portfolio optimization, pricing of financial assets and their derivatives, as well as risk management, rely heavily on interest rate forecasts. These forecasts are widely used by financial institutions, regulators, and investors to develop macroeconomic scenarios. We propose a forecasting investigation of the term structure of interest rates.

The starting point of our research is the term structure of interest rates model proposed by [Nelson and Siegel \[1987\]](#). For market participants, the main reference for the term structure model is the Nelson-Siegel type approach. The Bank of International Settlements (BIS) reports the list of countries that use the yield-curve fitting methodology and indicates the term structure on interest rate models used by the central banks of each country (see [Ricart, Sicsic, and Jondeau \[2005\]](#)). For most countries, the [Svensson \[1994\]](#) model is the most popular. We compare four different specifications including the Svensson model for extracting the yield curve from observed coupon bond prices. The first two specifications are taken from the literature: Nelson-Siegel model with four parameters and one hump, and the Svensson model with six parameters and two humps. The last two are introduced for the first time in this research and constitute our contribution: a) the extended Bjork-Christensen with six parameters and two humps along with the original Bjork-Christensen model proposed by [Bjork and Christensen \[1997\]](#) with five parameters and some constraints on non-linear parameters; and b) the extended Svensson with seven parameters and two humps.

Let us now consider the eurozone, officially called the euro area, which is a monetary union of 19 of the 28 European Union member states (this number may vary) and has adopted the Euro as the common currency and sole legal tender. The

---

Euro system is the monetary authority of the eurozone. The other nine members of the European Union continue to use their own national currencies, although most of them would be obliged to adopt the euro in the future. From a practical point of view, it is interesting to explore the eurozone. The early academic work on the euro area goes back to 1987 with [Campbell and Clarida \[1987\]](#), who studied the predictability and co-movement of risk premium in the term structure of euro market interest rates. More recently, [Sander and Kleimeier \[2004\]](#) aim at unifying the empirical research on interest rate pass-through in the eurozone. The interest rate pass-through describes how changes in a reference rate (monetary policy, money market, or T-bill rate) are transmitted to bank lending rates.

For our study we collected data on government bond prices for four eurozone countries, namely France, Germany, Italy and Spain over twenty years (from 1999 to 2018). Overall, our dataset of daily prices contains 800 bonds with different coupon rates and maturities. By taking euro-denominated bonds, we avoid any complications related to the exchange rate. We consider *Obligation Assimilable du Trésor (OATs)* for France, *Bundesobligation* and *Bundesanleihen* (or *Bund*) for Germany, *Italy Buoni Poliennali Del Tesoro* for Italy, and *Bonos del Estado* and *Obligaciones del Estado* for Spain.

We explore the structural differences and the relative goodness of fitting the coupon bond prices of functional term structure modelling using the Nelson-Siegel type of model. [Dai and Singleton \[2000\]](#) study similar problems and show theoretically and empirically that some subfamilies of affine term structure models are better suited than others to explain the historical interest rate behavior. [Diebold and Li \[2006\]](#) address the practical problem of forecasting the yield curve and propose autoregressive models for the yield curve factors and estimate the corresponding parameters. They show that their models are consistent with a variety of stylized facts regarding the yield curve. We use autoregressive models to forecast the out-of-sample yield curve in contrast with [Diebold and Li \[2006\]](#) who forecast the yield curve factors. More recently, [Koopman and Wel \[2013\]](#) extend the class of dynamic factor yields curve models by including macroeconomic factors, and conclude that macroeconomic variables can lead to more accurate yield curve forecasts.

---

We propose two criteria to evaluate the performance of each term structure specifications: mean absolute error, and score measure, which is the number of days when the considered model provides the best performance among others. [Christensen and Wei \[2019\]](#) recently used the [Diebold and Li \[2006\]](#) methodology to test the general term structure of models. They developed a new empirical approach with both unobservable factors and factors identified as innovations to the observed macroeconomic variables to test the time-varying risk premiums and arbitrage opportunities. We conduct some forecasting exercises conforming to Diebold and Li. Using the two performance criteria, we determine the best specification, which is used for out-of-sample forecasting. We test the random walk, univariate, and multivariate autoregressive models to forecast the term structure dynamics. [Koo, Vecchia, and Linton \[2019\]](#) recently developed a methodology to estimate an additive nonparametric panel model that is suitable for the pricing of coupon-paying government bonds over different time periods.

**The third chapter** analyzes the term structure of France’s nominal interest rates. It is a very important area of study from an economic and finance perspective. This is the first comprehensive study of all publicly available data on the French nominal debt that encompasses the 30-year period from 1988 to 2018. Recently, markets worldwide faced the reality of negative interest rates, mostly on the short-term end of the yield curve. [Buiter \[2009\]](#) addresses this problem by considering three methods for eliminating the zero lower bound on nominal interest rates and for restoring symmetry to the domain over which a central bank can vary its official policy rate.

[Smith \[2002\]](#) considers data for the United States (U.S.), Canada, the United Kingdom (U.K.), Germany, France, and Japan to investigate the market efficiency by testing for seasonality and cointegration. Correlation analysis shows considerable diversification opportunities for short-term investors. Cointegration tests indicate that several markets share cointegrating vectors increasing the possibilities of using other endogenous bond markets to better predict movements in a market. From a practical point of view, market participants use government bond market data to study many interesting aspects of fixed-income markets. We present two aspects. First, [Abad, Chulia, and Gomez-Puig \[2010\]](#) compare the

---

importance of two sources of systemic risks (global and eurozone) on government bond returns. The results show that the euro markets are less vulnerable to the influence of global risk factors, but more vulnerable to eurozone risk factors. Especially, the markets of countries that decided to stay out of the Monetary Union present a higher vulnerability to external risk factors. Second, [Bernoth, Hagen, and Schuknecht \[2012\]](#) study the bond yield differentials among the euro area government bonds based on a unique dataset of issue spreads in the U.S. and the Eurobond market between 1993 and 2009. Interest differentials between the bonds issued by EU countries and the U.S. contain risk premiums that increase with fiscal imbalances and depend negatively on the issuer’s relative bond market size.

We construct the French nominal yield curve using nominal quotations for securities called *Obligations assimilable du Trésor* (OATs) and *Bons du Trésor à taux fixe et à intérêts Annuels* (BTANs) on a daily frequency. These bonds have maturities at issuance ranging from one to fifty years. Our sample period starts in 1988, includes the launch of the euro in January 1999, and ends in April 2018. Our study methodology relies on [Gürkaynak, Sack, and Wright \[2007\]](#), as their study makes public the Treasury yield curve estimates of the Federal Reserve Board at a daily frequency from 1961 to the present. We use a similar smoothing method to fit the data and we show that the method fits very well. The resulting estimates are used to compute yields and forward rates for any maturity.

We use the noise measure to assess the “quality” of the French market. This measure is proposed by [Hu, Pan, and Wang \[2013\]](#) to capture episodes of liquidity crises of different origins across the financial market. The measure provides information on illiquidity beyond the existing liquidity proxies. Overall, using the noise measure, we find that in the first decade of our sample period, arbitrage opportunities were not infrequent on the OATs market, and the situation improved substantially with the introduction of the euro.

We also study the on-the-run premium for French data. [Vayanos and Weill \[2008\]](#) provide a search-based theory in which assets with identical cash flows can trade at different prices. The authors show that liquidity and specialness explain this

---

phenomenon simultaneously via the short-selling activity. For a particular security, we use the end-of-the-day quotes from Bloomberg. Using these quotes, we compute the spreads between the yield to maturity of the most recently issued bond (called on-the-run security) and the bond that already exists in the market with the same characteristics (called off-the-run security). We find that both the average and median spreads are negative. Moreover, the standard deviations are relatively high for most maturity ranges as well, indicating the absence of on-the-run premium in the French government bond market. [D'Amico, Fan, and Kitsul \[2018\]](#) state that the repo transactions are important on the bond market and they find a positive and significant scarcity premium for the on- and off-the-run Treasuries that persist for approximately three months and is larger in magnitude for short-term securities.

**The fourth chapter** analyzes the term structure of the French real rates. We challenge the well-known approach that the real rates are constant and nominal rates and breakeven change. We find that the inflation compensation remains constant while real rates change considerably. As highlighted in [Barding and Lehnert \[2004\]](#), the U.K. was the first industrialized country to issue index-linked government bonds. The authors study the efficiency of inflation-protected security markets and test the information content of inflation forecasts to develop trading strategies speculating on the movement of breakeven inflation. Their results indicate that the market for the French OATi offers the possibility of excess returns.

There are several types of inflation-indexed securities, such as bonds, swaps, and other derivatives.<sup>1</sup> We use the dataset with market information about inflation-protected bonds issued by the French government. The dataset is divided in two parts. The first one comprises inflation-linked bonds protected from the domestic inflation index. The second is inflation-linked bonds protected from the European inflation index. We fully implement the methodology proposed by [Gürkaynak, Sack, and Wright \[2010\]](#). They work on Treasury Inflation-Protected Securities (TIPS) and show that inflation compensation is not a pure measure of inflation expectations as it also contains the inflation risk premium and liquidity premium

---

<sup>1</sup>see [Deacon, Derry, and Mirfendereski \[2004\]](#) for more details.

---

components. From a practical perspective, TIPS are frequently considered to be a form of risk-free real bonds. [D'Amico, Kim, and Wei \[2018\]](#) show that TIPS yields exceeded risk-free real yields by as much as 100 basis points when TIPS were first issued, and they rose up to 300 basis points during the 2007–2008 financial crisis. The authors explain that this spread predominantly reflects the poor liquidity of TIPS relative to nominal Treasury securities. Applying the [Gürkaynak, Sack, and Wright \[2010\]](#) methodology, we observe that the fitting errors are quite small, so that we obtain a good fit in the class of functional models. [Westerhout and Beetsma \[2019\]](#) make a welfare comparison between the issuance of price-indexed and nominal public debt in the presence of fiscal constraints. They conclude that under a debt constraint, indexed debt is generally preferred, while under a deficit constraint, the results are more mixed.

[Christensen, Lopez, and Shultz \[2017\]](#) study the on-the-run premium of TIPS by studying yield spreads between pairs of TIPS with identical maturities but of separate vintages. After adjusting for differences in coupon rates and values of embedded deflation options, they find a small and positive premium on the more recently issued TIPS, averaging between one and four basis points. It persists even after new and similar TIPS are issued and, hence, is different from the on-the-run phenomenon observed in the nominal Treasury market. Another interesting investigation is suggested by [Grishchenko and Huang \[2013\]](#), who estimate inflation risk premium using a dataset on TIPS prices from 2000 to 2008. The authors find that the inflation risk premium is time-varying and, on average, considerably lower than suggested by various structural models. In our study, we compute the breakeven rates and find that the inflation compensation is stable.

We also backcast the inflation compensation. For this, we find the combination of nominal yields that best proxy a breakeven measure over the period for which we have French inflation-protected securities and compute this proxy over a much longer sample. [Andreasen, Christensen, and Riddell \[2018\]](#) study the liquidity risk in TIPS. They introduce an arbitrage-free term structure model of nominal and real yields and their model relies on the fact that, like most fixed-income securities, the TIPS go into buy-and-hold investing portfolios as time passes. The authors also find a sizable and countercyclical TIPS liquidity premium.

# Chapter 1

## Fundamental aspects of the term structure of interest rates

### 1.1 Introduction

In this chapter and the rest of the thesis, I review the fundamental aspects of the term structure of interest rates (TSIR). First, it is interesting to know the different types of models that can be used to work with the TSIR phenomenon (Section 2). Section 3 presents the so-called pure statistical approach called principal component analysis (PCA). The PCA provides evidence that the variance of the entire term structure can be captured by the variance of only three factors that have important economic explanations as level, slope, and curvature. As the TSIR provide information about what the output is to invest for specific horizons, it is important to discuss about the fixed-income portfolio management strategies. In Section 4, we review some basics of fixed income analytics such as the concept of duration and some major sources of risk for bond investors and conclude with an overview of the spectrum of bond portfolio strategies. We also present some information about institutions that shape the fixed-income market. Precisely, there is some information about products and actors participating in the government bond market (Section 5). The different types of yields are worth considering. Section 6 presents the spot rates, forward rates, and par yield. In the next section, we provide some reflections about government debt management

---

and inflation (Section 7). Section 8 concludes this chapter presenting information about liquidity in fixed-income markets.

## 1.2 The term structure of interest rates models

Over the past four decades, several major developments have been made in the field of term structure modeling; however, no superior model was proposed. In the literature on term structure, all the existing approaches have diverged into three popular families, namely equilibrium models, no-arbitrage models, and dynamic models, and the main objective of these three approaches is to capture and explain the TSIR. First, we briefly present these three fields. In each family of models there are several different approaches, thus each field is presented in the manner of a guided tour. Finally, we mention a purely statistical approach that is also commonly used to define the term structure of interest rates.

- The family of equilibrium models includes two types: (i) affine models, which include general and partial equilibrium models, and (ii) quadratic models. In affine models of the partial equilibrium, we assume that the spot instantaneous interest rate is an affine function of a set of state variables. In the quadratic models, we proceed in the same way, but use the non-linear function of state variables.
- In the family of arbitrage-free models, the absence of an opportunity of arbitrage is central to design the term structure of interest rates. An important goal of this approach is to rely on a perfect fitting at each point in time and the appropriate dynamics of the term structure of interest rates.
- In the family of dynamic models, we have a functional form with several parameters. Studies in this field are based on the work of [Nelson and Siegel \[1987\]](#), who introduced a parsimonious three-factor model that fits the yield curve remarkably well.

---

### 1.2.1 Equilibrium approach

The basic idea for all the approaches in the family is that there is an underlying economy to derive bond prices, this is not so in the case of arbitrage-free models that take bond prices as given. At any point in time, term structure of interest rates is a function of a small set of common state variables. Once the dynamics of the state variables and their risk premiums are specified, the dynamics of the term structure can be determined. Depending on the form of such a function, we can define affine term structure models (ATSMs), if it is a time-invariant linear function, or quadratic term structure models (QTSMs), if it is a second order polynomial function.

The focus on ATSMs extends back to the pathbreaking studies by [Merton \[1973\]](#) and [Vasicek \[1977\]](#) with the partial equilibrium approach followed by [Cox, Ingersoll, and Ross \[1985\]](#) with the general equilibrium approach. For instance, Cox, Ingersoll, and Ross (1985 a, b) consider a state variable while describing the change in production opportunities over time, in other words, the state of technology. Subsequently, they infer from the economy that the instantaneous (spot) interest rates governing bond prices necessarily follow the mean reverting stochastic process:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t,$$

where  $\theta > 0$  is a central location or a long-term mean value;  $\kappa > 0$  is the speed of adjustment (i.e., mean reversing toward  $\theta$ ) and  $\sigma$  is the level of instantaneous volatility of the process. This positivity was viewed as a key feature in favor of this approach, but currently short-term interest rates can be negative. The equation corresponds to a continuous time first-order autoregressive process, where the randomly moving interest rate is elastically pulled toward a central location. In a partial equilibrium approach, the dynamics of the instantaneous (spot) interest rate considered by [Vasicek \[1977\]](#) is:

$$dr_t = \kappa(\theta - r_t)dt + \sigma dW_t,$$

where  $\kappa > 0$  determines the speed of a reversion to the constant mean,  $\theta > 0$ , and

---

$\sigma$  is the level of the instantaneous volatility of the process. [Vasicek \[1977\]](#) was the first to capture mean reversion, which is an essential characteristic of the interest rate. These ATSMs gained in popularity with [Duffie and Kan \[1996\]](#), who generalize this class of models. They clarify assumptions underlying this framework and their approach can be considered as the most general affine term structure model.

[Ang and Piazzesi \[2003\]](#) and [Ang, Dong, and Piazzesi \[2007\]](#) combine ATSMs with some elements of the macroeconomy. [Brandt and Chapman \[2008\]](#) propose a good summary of ATSMs by presenting two important advantages this model has in comparison with other term structure models. The ATSMs provide fulfillment for the absence of arbitrage opportunities for bond prices and also allow for flexible specifications of term premiums and their dynamics. Even though this family of models was proposed almost four decades ago, there are still many questions to deal with. For instance, [Hamilton and Wu \[2014\]](#) investigate ATSMs' testable implications that were not previously explored and [Creal and Wu \[2015\]](#) present new procedures for the maximum likelihood estimation of ATSMs with spanned or unspanned stochastic volatility.

While discussing QTSMs, we assume that the instantaneous spot interest rate is essentially a second-order polynomial of state variables. There are several seminal contributions to this family of models. For instance, [Longstaff \[1989\]](#) presents a double square-root model, [Beaglehole and Tenney \[1991, 1992\]](#) extend a multivariate quadratic model and formulate a univariate quadratic model. [Constantinides \[1992\]](#) describes a squared autoregressive independent variable nominal term structure model. The QTSMs overcome limitations inherent in ATSMs. For example, [Ahn, Dittmar, and Gallant \[2002\]](#) state that QTSMs show better performance than ATSMs in explaining historical bond price behavior in the U.S. Thus, QTSMs assume that the instantaneous (spot) interest rate is defined by the equation

$$r_t = \alpha + \beta' X_t + X_t' \gamma X_t.$$

Here  $\alpha$  is a constant,  $\beta$  is a vector, and  $\gamma$  is a matrix;  $X_t$  is a  $n$ -dimensional state variable, which follows a diffusion process under the risk-neutral probability. This

---

process is typically an Ito process such that

$$dX_t = f(X_t)dt + \rho(X_t)dW_t^Q,$$

where  $W^Q$  is a Wiener process under the risk-neutral probability. Regarding the first equation, we can state that the affine model is a case of a quadratic model, where matrix  $\gamma$  is vanishing. [Chen, Filipovic, and Poor \[2003\]](#) analyze the QTSMs in terms of the Markov chain process. They deduce that no jumps are allowed in the state processes of QTSMs. However, these state processes can incorporate a quadratic potential, which enables the QTSMs to model the default risk. Recently, [Andreasen and Meldrum \[2014\]](#) investigate the term structure models for the U.S. nominal bond yields with QTSMs.

### 1.2.2 Arbitrage-free approach

The previous term structure models typically specify the instantaneous (spot) interest rate as a function of a small set of state variables, which follow a time-homogeneous Markov chain process. Such models have many advantages, but they are generally inconsistent with the observed term structure of bond prices, as pointed out by [Kimmel \[2004\]](#). The no-arbitrage models focus on perfectly fitting the term structure at a point in time to ensure that no arbitrage possibilities exist, a condition which is important for pricing derivatives. Prominent contributions to this family of models include the study by [Ho and Lee \[1986\]](#), where they consider the spot interest rate dynamics as:

$$dr_t = \mu(t)dt + \sigma dW_t,$$

where  $\mu(t)$  is a function of  $t$  deduced from the current term structure of interest rates. This approach is especially important since it was the first to model movements in the entire term structure of interest rates. Next [Hull and White \[1990\]](#) propose to extend Vasicek's model with time-varying parameters. The general specification for the [Hull and White \[1993\]](#) model is given as:

$$dr_t = [\theta(t) - \kappa(t)r_t]dt + \sigma(t)r_t^\beta dW_t.$$

---

The functions  $\theta(t)$ ,  $\kappa(t)$ , and  $\sigma(t)$  are time-varying and can be used to calibrate the model to the current market prices of bonds. Another arbitrage-free approach proposed by [Heath, Jarrow, and Morton \[1992\]](#) is based on the forward rates. The authors extend the Ho and Lee model in three directions as pointed out by [Gibson, Lhabitant, and Talay \[2012\]](#). First, they consider forward rates rather than bond prices as their basic building blocks; second, they allow for continuous trading; and third, they extend the initial one-factor approach to a multiple factor method. [Gombani and Runggaldier \[2013\]](#) proposed an arbitrage-free multifactor term structure models using a theory based on stochastic control.

### 1.2.3 Functional approach

The foundation for dynamic term structure models based on the representation introduced by [Nelson and Siegel \[1987\]](#). The next paragraph briefly presents the main fitting result of this seminal paper. First, we fix the ideas and establish some notations by introducing three key theoretical constructs and the relationships among them: the discount curve, forward curve, and the yield curve. Denote by  $P_t(\tau)$ , the price of a  $\tau$ -period discount bond, which is the present value at the time  $t$  of 1€ that will be received  $\tau$  periods ahead. Denote by  $y_t(\tau)$  the corresponding continuously compounded zero-coupon nominal yield to maturity. The relationship between the discount bond price and the yield-to-maturity is given by

$$P_t(\tau) = e^{-\tau y_t(\tau)}.$$

As this relation is satisfied for all maturities, it provides a relationship between the discount curve and the yield curve. From the discount curve, we can obtain the forward curve

$$f_t(\tau) = -\frac{P'_t(\tau)}{P_t(\tau)}.$$

and the relationship between the yield to maturity and the forward rate is

$$y_t(\tau) = \frac{1}{\tau} \int_0^\tau f_t(u) du.$$

---

Consequently, the zero-coupon yield to maturity is an average of forward rates. Reverting to [Nelson and Siegel \[1987\]](#), the authors introduce a static fitting methodology based on a functional form that is a convenient and parsimonious three-component exponential function. The forward rate curve is

$$f(\tau) = \beta_1 + \beta_2 e^{-\lambda\tau} + \beta_3 \lambda e^{-\lambda\tau}.$$

The corresponding yield curve is presented as

$$y(\tau) = \beta_1 + \beta_2 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right),$$

where  $y(\tau)$  is the interest rate for maturity  $\tau$ . In this functional form we have four parameters:  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  and  $\lambda$ . The  $\lambda$  parameter controls the exponential decay rate. The small values of  $\lambda$  produce slow decay and can better fit the curve at long maturities. The large values of  $\lambda$  produce fast decay and can better fit the curve at short maturities. Parameter  $\lambda$  is assumed to be constant through time. Parameters  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  capture the cross-sectional level, slope and curvature of the yield curve, respectively. [Figure 1.1](#) plots the three factor loadings.

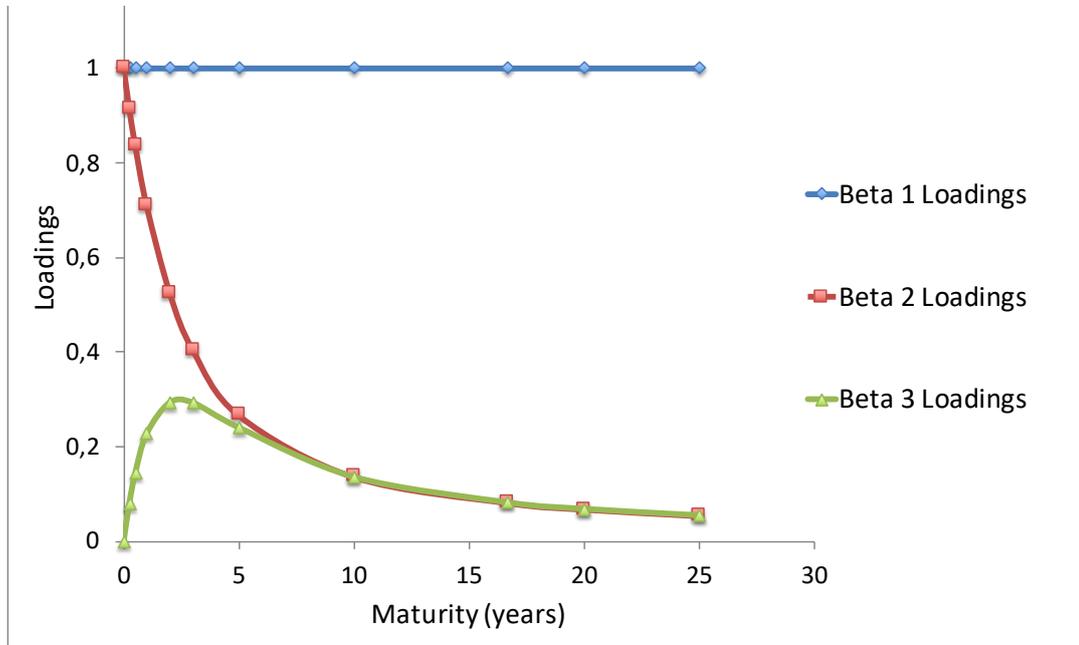
Let us return to the dynamic term structure models. The main idea of these models is to modify the functional form to take into account the time evolution and to explain the term structure of interest rates dynamics. [Diebold and Li \[2006\]](#) were the first to adopt this technique. In fact, [Diebold and Li \[2006\]](#) introduce time-varying parameters and repeat the Nelson and Siegel approach every day/week/month so

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right).$$

The  $\beta$  parameters become the dynamic latent factors. [Diebold and Li \[2006\]](#) logically interpret the time series of parameters  $(\beta_{1t})_t$ ,  $(\beta_{2t})_t$  and  $(\beta_{3t})_t$  as the level, the slope and the curvature factors of the yield curve.

Nelson and Siegel introduce a parsimonious three-factor fitting approach. [Svens-](#)

Figure 1.1: Loadings of Nelson-Siegel model



This figure shows the factor loadings in the three-factor model, where the three factors are  $\beta_1$ ,  $\beta_2$  and  $\beta_3$ . The associated loadings are 1,  $(1 - e^{-\lambda\tau})/\lambda\tau$  and  $(1 - e^{-\lambda\tau})/\lambda\tau - e^{-\lambda\tau}$  where  $\tau$  denoted maturity in years. The  $\lambda$  parameter is fixed and equal to 0.0609.

son [1994] and Bjork and Christensen [1997] subsequently proposed four-factor and five-factor extensions. De Pooter [2007] studies the in-sample and out-of-sample performance of several Nelson-Siegel type models and finds the specification of Svensson to show the best in-sample fit (albeit marginally better than the Bjork-Christensen (BC) specification). Furthermore, the authors reveals that the four-factor BC specification can provide the most accurate interest rate forecasts at various forecasting horizons.

Diebold, Rudebusch, and Aruoba [2006] estimate the parameters of a model and propose an approach to summarize the yield curve latent factors (i.e., level, slope, and curvature) and include observable macroeconomic variables (i.e., real activity, inflation, and monetary policy instrument). The authors find strong evidence of the impact of macro variables on future movements in the yield curve, as well as

---

the evidence of a reverse influence. [Christensen, Diebold, and Rudebusch \[2011\]](#) propose an arbitrage-free Nelson Siegel approach, and we provide certain additional details on their work in this thesis.

[Joslin, Pribsch, and Singleton \[2014\]](#) propose an arbitrage-free dynamic term structure approach, in which bond investment decisions are influenced by inflation risks correlated with the information on the shape of the yield curve. [Diebold and Li \[2006\]](#) emphasize the importance of interpreting the parameters  $\beta_{1t}$ ,  $\beta_{2t}$  and  $\beta_{3t}$  as level, slope, and curvature and also discuss yield curve forecasting. It is known that forecasting the interest rate point is crucial for bond portfolio management. In the literature related to arbitrage-free family of term structure models, there is little information on the dynamics of forecasting. In the family of equilibrium models, it is more common as in [Jong \[200\]](#) or [Dai and Singleton \[2000\]](#) to favor in-sample fitting rather than out-of-sample forecasting. Studies that focus on out-of-sample forecasting of equilibrium models, notably [Duffee \[2002\]](#), conclude that these models forecast poorly. Diebold and Li worked on the data from U.S. Treasuries, and their forecasting results are encouraging, specifically, their models produce year-ahead forecasting that is noticeably more accurate than the standard benchmarks.

### 1.3 Factors of TSIR

Principal component analysis is a common technique applied to interest rate markets to describe yield curve behavior in a parsimonious manner. First, three principal components are frequently identified with the economically meaningful shift, twist, and butterfly moves of the yield curve. As such, they relate intimately to the level, slope, and curvature factors evidenced by [Nelson and Siegel \[1987\]](#) and [Diebold and Li \[2006\]](#). [Litterman and Scheinkman \[1991\]](#) use this pure statistical approach to extract factors (called principal components) from the observed interest rates because such factors may explain the deformations of the yield curve. Moreover, the authors show that there are three factors explaining most of the moves. However, there are certain constraints in the application of

---

this approach. Nevertheless, one may find application of PCA, for example, in the PhD thesis by [Moungala \[2013\]](#).

Principal component analysis helps to identify patterns in data, highlights their similarities and differences, and is a powerful tool for analyzing data. The other main advantage of PCA is that once these patterns are determined in the data, the number of dimensions can be reduced without much loss of information. The PCA model explicitly selects the factors based on their contribution to the total variance of interest rate changes. Consequently, PCA may ease hedging by using only a small number of factors. Factor analysis is a general name denoting a class of statistical procedures that are primarily used for data reduction and summarization. For factor analysis to be efficient, it is important that the sample size used must be appropriate. As a rough guideline, there should be at least four or five times as many observations as there are variables. In PCA, the total variance in the data is considered and the technique is recommended when the primary concern is to determine the minimum number of factors that will account for maximum variance in the data.

[Golub and Tilman \[2000\]](#) propose an excellent overview of the use of PCA in the areas of fixed income risk measurement and management. We divide the advantages of PCA into three categories: risk estimation, risk reporting, and scenario analysis. The ability to parsimoniously describe complex structures is the benefit of using PCA in risk estimation. For the purpose of interest rate risk measurement, the yield curve can be represented as a structure that comprises of key individual rates, viewed as random variables. Risk reporting is simplified because practitioners can see the contributions to portfolio risk from factors that are not postulated a priori but are rather derived from actual market data. There are two main benefits to using PCA for scenario analysis. First, it helps to understand the shape and dynamics of yield curves movement and the benefit is similar to the effect of risk reporting. Second, PCA enables users to describe the joint distribution of the key rates. Therefore, the probability of any particular scenario can be determined, which is important in designing an appropriate reaction to the results of the scenario analysis.

---

Even though PCA is a well-established technique for term structure, few attempts have been made to apply it to estimate the distribution of the joint global term structure. With international markets becoming more integrated, considering the global yield curve is important to portfolio investors. [Phoa \[1999\]](#) briefly refers to the topic of global joint structure behavior and uses PCA to decompose international ten-year bond yields. He concludes that the global shift factor, while it is somewhat visible, does not explain as much movement in the curves as it does in the curve-specific models. [Malava \[1999\]](#) performs direct PCA of global term structure and finds that fourteen are needed to explain 99% of the variability in the joint term structure of the London Interbank Offered Rate (LIBOR) USD, JPY, EUR, and GBP yield curves. [Moraux, Perignon, and Villa \[2002\]](#) analyze international curves applying Common PCA (CPCA). However, CPCA requires common eigenvectors and does not specify a joint distribution of the resulting factors.

## 1.4 The duration

This section introduces the fundamental concepts and terminology for fixed income portfolio management. It reviews the sectors of the bond market, basics of fixed income analytics as the concept of duration and convexity, and the major sources of risk for bond investors, concluding with a description of the spectrum of bond portfolio strategies.

Duration and convexity are two standard tools that are used to manage the risk exposure of fixed-income investments. Duration measures the bond's sensitivity to interest rate changes, while convexity relates to the interaction between a bond's price and its yield as it experiences changes in interest rates. With coupon bonds, investors rely on a metric known as duration to measure a bond's price sensitivity to changes in interest rates. As a coupon bond makes a series of payments over its lifetime, fixed income investors require techniques to measure the average maturity of a bond's promised cash flow, primarily to serve as a summary statistic of the bond's effective maturity. The duration accomplishes this and thereby allows fixed income investors to effectively gauge uncertainty

---

when managing their portfolios.

In 1938, Canadian economist Frederick Robertson Macaulay dubbed the effective maturity concept the “duration” of the bond. He also suggested that this duration must be computed as the weighted average of the times to maturity of each coupon, or principal payment, made by the bond. There are also the so-called dollar duration and modified duration. Dollar duration is one of the several different measurements of bond duration and measures the dollar change in a bond’s value to a change in the market interest rate. Professional bond fund managers use dollar duration as a way of approximating the portfolio’s interest rate risk. As duration measures the sensitivity of a bond’s price to interest rate changes, dollar duration seeks to provide these changes as an actual dollar amount. Modified duration is a formula that expresses the measurable change in the value of a security in response to a change in interest rates. Modified duration follows the concept that interest rates and bond prices move in opposite directions. This formula is used to determine the effect that a 100 basis points (1 percent) change in interest rates will have on the bond’s present value or price.

Modified duration is an extension of the Macaulay duration, which allows investors to measure the sensitivity of a bond to changes in interest rates. In order to calculate modified duration, the Macaulay duration must first be calculated. The Macaulay duration calculates the weighted average time before a bondholder would receive the bond’s cash flows. Conversely, modified duration measures the price sensitivity of a bond when there is a change in the yield-to-maturity. Effective duration is a measure of the duration for bonds with embedded options (e.g., callable bonds). Unlike modified duration and Macaulay duration, effective duration considers fluctuations in the bond’s price movements relative to the changes in the bond’s yield-to-maturity. In other words, the measure considers possible fluctuations in the expected cash flows of a bond. There are three principles of duration that must be considered. First, as maturity increases, duration increases, and the bond becomes more volatile. Second, as a bond’s coupon increases, its duration decreases, and the bond becomes less volatile. Third, as interest rates increase, duration decreases and the bond’s sensitivity to further increases in the interest rate diminishes. However, duration has limitations when used as a mea-

---

sure of interest rate sensitivity. While the statistic calculates a linear relationship between price and yield changes in bonds, in fact, the relationship between the changes in price and yield is convex.

Convexity, a measure of the curvature of the changes in the price or value of a bond in relation to changes in interest rates, addresses this error by measuring the change in duration, as interest rates fluctuate. In general, the higher the coupon, the lower the convexity; however, due to the call feature, callable bonds display negative convexity if yields fall too low, indicating that the duration will decrease when yields decline. Zero-coupon bonds have the highest convexity, where relationships are only valid when the compared bonds have the same duration and yields-to-maturity. In fact, a high convexity bond is more sensitive to changes in interest rates and must consequently witness larger fluctuations in price when interest rates move. The opposite is true of low convexity bonds, where prices do not fluctuate as much when interest rates change. Low-coupon and zero-coupon bonds, which tend to have lower yields, show the highest interest rate volatility. In technical terms, this implies that the modified duration of the bond requires a larger adjustment to keep pace with the higher change in price after the interest rate moves. Lower coupon rates lead to lower yields, and lower yields lead to higher degrees of convexity.

Key rate duration measures how the value of a security or portfolio changes at a specific maturity point along the entirety of the yield curve. When keeping other maturities constant, the key rate duration can be used to measure the sensitivity in a security's price to a 1% change in yield for a specific maturity. Key rate duration is an important concept in estimating the expected changes in value for a bond or portfolio of bonds because it does so when the yield curve shifts in a manner that is not perfectly parallel, which occurs often. Effective duration (another important bond metric discussed above) is an insightful duration measure that also calculates the expected changes in price for a bond or portfolio of bonds given a 1% change in yield, but it is only valid for parallel shifts in the yield curve. Therefore, key rate duration is a valuable metric and is related to effective duration. For example, there are 11 maturities along the Treasury spot rate curve, and a key rate duration may be calculated for each maturity. The

---

sum of all the 11 key rate durations along the portfolio's yield curve equal the effective duration of the portfolio. As discussed later in the thesis, different key rate durations are associated with various fixed-income portfolio strategies such as bullet, barbell, and ladder.

There are several techniques investors can use to protect their portfolios from interest rate risk, many of which involve simple yet effective fixed-income strategies. A bond investment strategy can help to reduce risk or maximize income in a way that is tailored to an individual's risk/return needs. However, it takes time and effort to realize the potential benefits of the bond investment strategies listed below, but if an investor is willing to make the effort and has the patience to see long-term gains, then these strategies can be beneficial. We introduce here some terminologies. The ladder strategy consists of having bonds that mature at different times and investors continually reinvest them. In the barbells strategy, investors invest in a set of bonds that mature in the long- and short-term but not in the medium term. In the bullets strategy, the bonds held are purchased at different times but all have the same target maturity date.

### **1.4.1 Risk factors**

The first risk factor is the change in yield levels, for instance the parallel shift. Another factor affecting investment decisions is the yield curve risk. The three factors (level, slope, and curvature) are systematic so they could be viewed as market risk. The yield curve risk occurs due to changes in the slope or shape of the yield curve. To measure this, investors use the notion of convexity and different key rate duration measures (with corresponding active portfolio strategies, for instance bullet, barbell, and ladder). A well-known risk is the exposure to market volatility. Volatility can be historical, based on past actual prices or yields or expected yields, as indicated by implied volatility of options. Investors have the convexity notion to measure bond responsiveness or sensibility to interest rate movements. For example, a bond trading at higher yield-to-maturity will have lower price volatility. Callable bonds are negatively convex assets, meaning that a portfolio with such bonds is adversely affected by volatility (i.e., more volatility makes less profit). Puttable bonds are assets with positive convexity, meaning

---

that a portfolio with such bonds benefits from volatility (more volatility makes more profit).

A risk is associated with liquidity. Inherently, different securities have different liquidity levels; for instance, government bonds are more liquid than corporate bonds. The liquidity of all securities, particularly riskier securities, decreases during periods of market turmoil (instability). The liquidity risk is typically measured by the difference between the price at which a security can be bought and sold at a point in time, the so-called bid/ask price spread. The liquidity of a security refers to both the marketability and stability of the market price. Marketability means the time it takes to sell a security at its market price. For instance, a registered corporate bond takes less time to sell than a private placement.

### **1.4.2 Investment strategies**

This list summarizes the passive strategies and common active strategies. Active strategies relate to various fixed-income risk factors, and an active fixed-income manager may be active relative to any set of these risk factors, or all of them. Indexing strategy is a passive strategy, that is, the main idea is to replicate all the risk factors in the “index” or benchmark. The only certain way to accomplish this is to buy all the securities in the index in amounts equal to their weight in the index. While this can be easily done in the stock market, say for the S&P 500 (index by buying all 500 stocks in the appropriate amounts), it is difficult to do so in the fixed-income market. For example, the Lehman Aggregate Bond Index is based on approximately 6,000 bonds, many of them quite illiquid. Market timing is another active strategy, and the main idea is to deviate from the duration of the benchmark. If a portfolio has a greater duration than the benchmark, then it outperforms the benchmark during market rallies (i.e., periods of good performance) and underperforms during market contractions (i.e., recessions). A yield curve trade is also an active strategy in this list, where the main idea is to replicate the duration of the benchmark but vary the convexity and yield curve exposure by altering the composition of key rate durations. The change in yield curve slope factor is called flattening and steepening. When the yield curve is

---

flat, the barbells strategy outperforms among others, but when the yield curve is not flat, the bullets strategy outperforms among others.

Next, volatility trades are also an active strategy. The main idea is to use a bond with some embedded options. Volatility increases the benefit to puttable bonds, which is essentially a long call option, and negatively affects the callable bonds, which are short call options. We consider the bullets strategy as the benchmark. Due to the call feature, callable bonds display negative convexity if the yields fall too low, meaning the duration will decrease when yields decrease. Bonds with put options are more positively convex than straight bonds. Hence, an increase in volatility benefits puttable bonds and diminishes the price of callable bonds. In a stable yield curve environment, investors are willing to pay more for high-quality callable bonds compared to straight bonds of similar quality.

Asset allocation or sector trades is another active strategy. The main idea is to change portfolio components, either by taking different types of securities or by changing the weights assigned to each security included in the portfolio. Investors can undertake this strategy at different levels; for instance, deviate from the macro- or micro-sector basic type of portfolios or change the security weighting of a benchmark portfolio. At the macro level, we find several portfolio components among government securities, the so-called Treasuries in the U.S., bonds issued by agencies<sup>1</sup>, corporate bonds, mortgage or asset-backed securities<sup>2</sup> and bonds issued at the municipal level (e.g., it is possible to buy bonds issued by Lyon city in France). On micro-components at a macro-sector level, investors can switch between corporate bonds in the utilities sector and corporate bonds in the industrial sector. To switch the securities trading level, investors can switch allocation between overweight and underweight individual securities in a micro-sector. This type of fixed income portfolio strategy profits from deviations based on option-adjusted spread of sectors, subsectors, and securities. This deviation

---

<sup>1</sup>Agencies are affiliated with, but separate from, the U.S. government. Investors can buy various securities issued by government-sponsored and government-owned corporations that, strictly speaking, are not actually a part of the U.S. government.

<sup>2</sup>Mortgage-backed securities are investment opportunities that are secured by mortgages. It allows investors to benefit from the mortgage business without ever having to buy or sell an actual home loan (real estate loan for mortgage-backed securities and asset for asset-backed securities).

---

is relative to historical averages and fundamental projections and investors can use breakeven spreads (i.e., based on option-adjusted spreads) as the basis for deviations. If option-adjusted spreads become smaller (tightening), this produces some gains to investors, and if option-adjusted spreads become larger (widening), this leads to some losses. An investor must keep this in mind when he decides to attribute new weights for some portfolio securities to profit from the market conditions.

Credit risk allocation is also an active strategy. The main idea is to deviate from the average credit rating of the macro- or micro-sectors and their composites. Credit spreads typically widen (increase) when economic growth is slow or negative, and this (credit spread widening) benefits higher credit rating, and vice versa. When investors hold a risky fixed income security (i.e., an asset with high credit rating) in some sector of the economy and the economic growth of this sector slows down, the situation is profitable for such investors. For instance, the investor can use the spread of the duration as the basis for deviations. With one more active strategy trading, we conclude this section. The main idea is to profit from short-term changes in specific securities based on short-term price discrepancies. Often, this means short-term technical, including short-term supply/demand factors that cause temporary price discrepancies.

## **1.5 Institutional aspects**

The Federal Reserve (Fed) serves as the Treasury's fiscal agent. In this role, it is responsible for the primary dealer relationships, which are used not only for Treasury auctions but also for other open market operations conducted in accordance with the monetary policy. The Federal Reserve plays an important role in the operational aspects of the auction process and payment mechanism (see Subsection 1 of this section). In addition, the Fed is the holder of Treasury securities. It is involved in the purchase and resale of these securities to the secondary market through its open market operations.

---

### 1.5.1 Auctions, actors, and secondary markets

Auctions are the cornerstone of the Treasury's debt management strategy, and their offering amounts are scheduled and announced in advance of the auction date. Bidders in Treasury auctions may be either foreign or domestic and individual or institutional investors, federal, state, or local government entities. Auction bids for Treasury securities may be submitted as noncompetitive or competitive. With a noncompetitive bid, a bidder agrees to accept the discount rate (or yield) determined at auction and is guaranteed to receive the full amount of the bid. With a competitive bid, a bidder specifies the yield that is acceptable and the investor may or not receive the bonds, depending on the competitor's bids.

Primary dealers are securities brokers and dealers who are registered to operate in the government securities market and have trading relationships with the Federal Reserve Bank of New York. Primary dealers are the largest purchasers of Treasury securities sold to the public at auctions. In addition to their role in the auction process, primary dealers also work closely with the Fed to execute its monetary policy. These primary dealers are large financial institutions and the Fed relies on them to act as intermediaries through which Treasury securities are bought and sold and resold in the secondary market to either increase or decrease money supply. They are expected to maintain trading relationships with the Fed's trading desk and provide market information and analysis that may be useful to the Fed in the formulation and implementation of the monetary policy. These primary dealers also use this system to help them meet their liquidity demand by swapping securities with the Fed on an overnight basis. This type of securities lending does not impact the general interest rate or money supply as it does not involve cash but can affect the liquidity premium of the securities traded.

Along with the primary dealers and the Fed, individual investors, other dealers and brokers, private pension and retirement funds, insurance companies, investment funds, and foreign investors (private citizens and government entities) also purchase Treasury securities through the auction process and in the secondary market. Participants in the secondary market play an indirect role in determining

---

the price of Treasury securities. Once the Treasury announces an auction, dealers and market participants start trading securities on a “when issued” basis. This means that once a security has been purchased and issued, it will be immediately resold to the secondary market purchaser. As trading starts in the secondary market before the actual auction takes place, “when issued” market participants effectively determine the yield or discount rate of Treasury securities based on what they are willing to pay.

### **1.5.2 Products, monetary policy, and maturity distribution**

Currently, the U.S. Treasury offers five types of marketable securities: Treasury bills, Treasury notes, Treasury bonds, inflation-protected securities (TIPS), and floating rate notes (FRNs). For instance, in 2015, the Treasury sold securities through 272 different public auctions, and each of them is dedicated to a special issue. Treasury bills, Treasury notes, and Treasury bonds are fixed-income investments issued by the U.S. Department of Treasury. They are the safest investments in the world since the U.S. government guarantees them, and this low risk means they have the lowest interest rates of any fixed-income security. The difference between bills, notes, and bonds are the lengths until maturity. Treasury bills are issued for terms less than a year. Treasury notes are issued for terms of two, three, five, and ten years. Treasury bonds are issued for terms of thirty years to maturity and were reintroduced in February 2006. The uncertainty following the 2008 financial crisis heightened their popularity. In fact, these securities reached record-high demand levels on June 1, 2012. The 10-year Treasury note yield dropped to 1.46 percent, the lowest level in more than 200 years. This was because investors fled to ultra-safe securities in response to the eurozone debt crisis. On July 25, 2012, the yield hit 1.43, a new record low. On July 5, 2016, the yield fell to an intra-day low of 1.375, and these lows had a flattening effect on the Treasury yield curve.

The Treasury also issues Treasury Inflation-Protected Securities in terms of five, ten, and twenty years. They work similarly with regular bonds with the only

---

difference being is that the Treasury Department increases their value if inflation rises. During an inflation or even if inflation is expected, TIPS perform well. In the secondary market, people pay higher valuations for the safety of TIPS if they foresee an inflation. For this reason, TIPS also do well when the dollar value is declining because a declining dollar usually leads to inflation. When TIPS mature, investors receive the highest adjusted principal, which in most instances is never less than the original principal. This provision protects investors against deflation because they will not receive less even if prices drop. However, TIPS are not a great investment when the economy is stable. In fact, TIPS return the flat interest rate on a flat principal when the economy is doing well and is not experiencing much of inflationary pressure. This situation describes the U.S. economy since the 1970s, that is, the last time when double-digit inflation existed.

The U.S. Treasury began issuing floating rate notes (FRNs) in January 2014. Issued for a two-year term, FRNs pay varying amounts of quarterly interest until maturity. Interest payments rise and fall based on discount rates in auctions of 13-week Treasury bills. Thus, an FRN is a debt instrument with a variable interest rate, which is tied to a benchmark rate. Benchmarks include the U.S. Treasury note rate, the Federal Reserve funds rate (known as the Fed funds rate), and the LIBOR. Compared with fixed-rate debt instruments, floaters allow investors to benefit from a rise in interest rates since the rate on the floater adjusts periodically to current market rates. Floaters are usually benchmarked against short-term rates like the Fed funds rate, which is the rate the Federal Reserve Bank sets for short-term borrowing between banks.

The Fed's monetary policy actions can affect interest rates on Treasury securities in the short run. The Fed conducts its monetary policy by setting the federal funds rate, that is, the price at which banks buy and sell reserves on an overnight basis. The level of the federal funds rate is directly related to the supply and demand for bank reserves. Monetary actions by the Fed generally affect short term nominal interest rates (for more details see [Thornton \[1988\]](#)). If the Fed lowers the federal funds rate, resulting in a lower short-term interest rate for banks, long-term interest rates are also likely to decline, although they may not

---

plummet as much or as quickly as observed.

Newly issued Treasury securities, sold to finance the operations of the federal government, are offered at a mix of maturities or horizons in order to satisfy the provisions of the regular and predictable debt management strategy and to minimize interest payments over time. The profile of securities is also important due to its influence on liquidity. In addition, the Treasury must ensure that it has adequate cash balance available to pay federal obligations. Balancing all these objectives leads to a strategy that offers a mix of short- and long-term securities. However, longer term securities generally command higher interest rates. The following chapters will offer some additional information about maturity distributions. Chapter 2 provides this information for several eurozone countries, namely Germany, Italy, and Spain. Chapter 3 proposes maturity distribution for the French nominal government securities, and Chapter 4 offers the same for French inflation-protected securities.

### **1.5.3 Market supply and demand**

Investors examine several key factors when deciding if they should purchase Treasury securities. As with all types of investments, price, expected return, and risk play a role in this process. Treasury securities provide a known and riskless stream of income and offer greater liquidity than other types of fixed income securities. Prices are determined by investors who place a value on Treasury securities based on the characteristics of safety and liquidity afforded by this investment option. As they are also backed by the full faith and credit of the U.S., they are often seen as one of the safest investments available, although investors are not completely immune to losses. Market behavior can also lead to price fluctuations, changes in interest rates, or inflation, which can create some investment risk. The demand for long-term investment opportunities is related to retirement strategies. Despite the current economic conditions and financial market volatility, Treasury securities continue to remain attractive to investors.

---

## 1.6 Types of yields

This section reviews the different types of yields, namely zero-coupon, par yields, and forward rates. First, we discuss zero-coupon yields.

The term structure of the zero-coupon yield curve is a relationship between maturity and interest rates. This notion starts from the basic premise of “time value of money,” that is, a given amount of money today has a value different from the same amount due at a future point in time. An individual willing to invest money today must be compensated in terms of a higher amount in the future. The rate of interest to be paid would vary with the time period that elapses between today (when the principal amount is being foregone) and the future point of time (at which the amount is repaid). A bond can be (or not) issued at par or at least in principle, and it becomes the “premium” or “discount,” consequently the trading process on the market. New bonds are sold in the “primary market” and existing bonds are traded in the “secondary market.” A bond with the coupon rate equal to zero percent is called a zero-coupon bond. A zero curve is a special type of yield curve that maps interest rates on zero-coupon bonds to different maturities across time.

Most fixed income instruments pay to the holder a periodic interest payment, commonly known as the coupon, and an amount due at maturity or the redemption value. Using a technique known as the bootstrap, one could straightaway derive the zero-coupon yield curve (for more details see [Deaves and Parlar \[2000\]](#)). This technique is based on the idea that individual coupon-paying bonds can be viewed as “packages” of pure discount or zero-coupon bonds. This suggests that a bond’s value can be viewed as the present value of future cash flows discounted at the yield to maturity. Or else, it can be viewed as the sum of the values of individual pure discount bonds, each of which is the present value of a cash flow discounted at its own time-specific yield. The classic explication of the bootstrap method is based on the assumption of the existence of a set of perfectly spaced bonds. If such bonds exist, the bootstrap renders the correct yield curve in a straightforward manner; however, such a situation is not observed in the market. Therefore, the problem with this method is that it relies heavily on the existence

---

of suitable data. There are two main drawbacks in this method: illiquidity and missing data points, and these factors are the reason for avoiding the use of the bootstrap technique.

### 1.6.1 Par yield

A par yield curve is a graphical representation of the yields of hypothetical fixed income securities with prices at par. On the par yield curve, the coupon rate equals the yield-to-maturity of the security, which is why the bond trades at par. The par curve gives the yield-to-maturity (YTM) for (coupon-paying) bonds at each maturity: the single discount rate that is used to discount the bond's cash flows to obtain the current market price. In other words, it is the IRR (internal rate of return) versus maturity curve for bonds. Thus, when the YTM equals the bond's coupon rate, then bond sales are at par, and this is called the par curve as it gives the coupon rate that a bond with a given maturity must pay to sell at par. Usually bonds are not available in the market at every maturity (e.g., a bond is not likely to be available with exactly 4.5 years to maturity, and another with exactly 13.5 years to maturity), and the par curve is constructed using whatever maturities are available in the market applying some mathematical technique to interpolate (or extrapolate) to obtain the remaining YTM's. Therefore, different sources may lead to slightly different values for the yields.

### 1.6.2 Forward yield

There are two types of forward rates, the discrete forward rate and instantaneous forward rate. The discrete forward rate, denoted by  $F(t, t + T_1, t + T_1 + T_2)$ , is applied in  $T_1$  years to  $T_2$  tenor rate. One way to consider the forward rate is to fix  $T_2$ , and consider that it is equal to a very small value. By definition this is the instantaneous forward rate. Furthermore,  $T_1$  as can be changed to observe the instantaneous rates in one, two, three, and successive years in the future. Another way to consider the forward rate is to fix  $T_1$ , for example, equal to one year and change  $T_2$ . In this case, a term structure of interest rates one-year ahead is obtained. There is also a special forward rate, known as the five-year

---

forward five-year rate (for more details see [Gürkaynak, Sack, and Wright \[2010\]](#) and Mehrotra and [Mehrotra and Yetman \[2018\]](#)).

A spot interest rate is conventionally associated to a fictitious transaction in a zero-coupon bond that takes place immediately (it corresponds to the spot price). A forward rate, on the other hand, is associated to a fictitious transaction in a zero-coupon bond with terms set to the current rate but that will not take place until a future predetermined date. The forward rate can be calculated from the spot interest rates and vice versa. Forward rates are calculated from the spot rate and adjusted for the cost of carry to determine the future interest rate that equals the total return of a longer-term investment with a roll over short-term investment strategy.

There are two interpretations of forward interest rates. First, a forward rate is the interest rate that makes an investor indifferent to investing between the full investment horizon (one possibility) and part of the investment horizon and rolling over the proceeds for the balance of the investment horizon (another possibility). Second, it is a rate that allows the investor to lock a rate for some future period. Many market participants consider that, by principle, forward rates reflect the market's consensus about future interest rates and forward rates can be used to predict future interest rates. In fact, the interpretation of forward rates, which is the market's consensus of future rates, depends on the theory of the term structure of interest rates that one adheres to. Two major theories are the expectation theory and market segmentation theory. The former theory develops the hypothesis that the level of short-term forward rates (i.e., instantaneous forward rate) is closely related to the market's expectations about future short-term spot interest rates<sup>1</sup>. The latter, expectation theory, has a serious drawback as it neglects the risks inherent in investing in bonds (this view is debatable). The second theory, namely the market segmentation theory, states that there is an imbalance between the supply and demand for funds within a given maturity range. Although market participants may prefer habitats dictated by the nature of their liabilities, investors and borrowers will not be reluctant to shift their investing

---

<sup>1</sup>In our empirical study, we rely on this hypothesis. We construct the instantaneous forward rates and state that these rates represent expected future short-term spot rates.

---

and financing activities out of their preferred maturity sector to take advantage of any imbalance.

Studies have demonstrated that forward rates are not good predictors of future interest rates. However, forward rates can be relevant in deciding between two alternative investments. Specifically, if an investor's expectation about a rate in the future is less than the corresponding forward rate, it would be better to invest today to lock in the forward rate. Cibulka [2015] studies the implied efficiency of forward rate in government bond yields. The author provides a comprehensive view of the market by constructing the yield curve model. The implied yields on government bonds contain considerable information. This study shows low effectiveness of participants' expectations on the government bond market via decomposition of implied interest rates.

As mentioned earlier, there is one specific forward interest rate, which is the forward five-year ahead for five-year rate, and this rate has shown importance for inflation expectations. There are two types of market-based measures for gauging inflation expectations: inflation compensation implied by the difference between the yields on nominal, and the yields on inflation-indexed Treasury bonds. Inflation swap rates are also used to measure inflation expectations. One measure of interest is the five-year forward five-year inflation compensation rate. This five-year forward breakeven inflation rate has been explicitly mentioned by Fed policymakers, and the behavior of this measure is often used to gauge the Fed's inflation-fighting credibility. Policymakers consider this specific measure to assess if and the extent to which near-term inflation pressures work their way into longer-term expectations. The concern here is that such leakages would create a more persistent inflation problem that would be costly to reverse. If the Fed maintains its credibility, then the forward inflation compensation measure should be relatively unresponsive to information about the near-term outlook. However, forward inflation compensation rates cannot simply be understood as inflation expectations because of the presence of an inflation risk premium and liquidity premium. This provides some room for the measure to vary without raising concerns among policymakers. However, risks to the inflation outlook are also important and a large enough upswing would be of concern to a central bank,

---

regardless of whether it was driven by inflation expectations or investors' assessment of considerable upside inflation risks.

## 1.7 Government debt and inflation

Governments need money to operate and borrow money when necessary<sup>1</sup>. A government security is a bond or a type of debt obligation that is issued by a government with the promise of repayment upon the security's maturity date. Government securities are usually issued for two different reasons, the primary reason being raising funds for government expenditures. Also, federal governments issue treasury securities to cover short-falls (deficits) in annual budgets. Additionally, cities will often issue bonds for construction of schools, libraries, stadiums, and other public infrastructure programs. A central bank of a country will sell or buy debt securities for another reason: to control money supply in the economy. To increase money supply, a central bank purchases bonds from banks to inject money into the banking system. Banks can use these funds to provide loans to individuals and businesses. Greater loan activity reduces interest rates and stimulates the economy. If a central bank sells bonds to banks, it takes money out of the financial system, which increases interest rates, reduces demand for loans, and slows the economy. In economics, money supply (or money stock) is the total value of monetary assets available in an economy at a specific time. There is strong empirical evidence of a direct relationship between money-supply growth and long-term price inflation, at least for rapid increase in the amount of money in the economy.

---

<sup>1</sup>A corporation has two options when it comes to raising money without taking a loan. These two options issue corporate bonds or sell shares of a stock. Corporations can indeed raise money by issuing both debt and equity, but issuing bonds or stock shares affect the corporation in different ways. The sale of shares gives investors an implicit share in future profits. Currently, the government has only one option to raise money, that is, issuing bonds. In a magazine article in December 2009 (New York Times) we find a proposition that governments should do something like corporations, and not just rely on debt. Thus, governments could sell a new type of security that commits them to paying shares in national "profit", as measured by the gross domestic product. This proposition can be implemented in the future, but the government is still issuing bonds to raise money.

---

Issue of debt is a political decision, and government institutions hold the authority to issue debt on behalf of the country. In case of the U.S., this institution is the Congress through power granted by the Constitution. If spending exceeds revenues, the Treasury determines the type of debt instruments that are used to finance the borrowing required to fulfill all obligations. The Treasury adheres to three debt management principles, and the first one is to issue debt in a regular and predictable pattern. The second role is to provide transparency in the decision-making process, and finally the third one is to seek continuous improvements in the auction process. During the mid-1970s, the U.S. economy experienced a period of rise in nominal federal budget deficits, which increased debt issuance and disrupted financial markets. Presently, policymakers have improved institutional practices to provide predictability in the debt sell process. As a result, the Treasury was able to raise large amounts of money with a minimal impact on the financial markets. These policies also extended the average maturity of the national debt and produced a better defined yield curve. If the U.S. continues to issue Treasury securities to finance government operations, the Treasury will continue to play a key role in maintaining stability in the financial and credit markets and the U.S. economy.

## **1.8 Government bond market illiquidity**

Trading costs and liquidity are inextricably linked through the bid-ask spread. The cost of trading depends on this bid-ask spread, as well as the duration and frequency of turnover. The uncertainty about the cost of trading creates risk (liquidity risk) and liquidity risk, in turn, gives rise to a risk premium. Although it is challenging to fully capture liquidity risk, it does not defy analysis. Our analysis begins with a list of observations about liquidity. First, and obviously, investors need to be rewarded for liquidity risk. Liquidity or, to be more precise, illiquidity can be viewed as a risk that reduces the flexibility of a portfolio. Liquidity risk should be reflected in the yield spread on a bond relative to a more liquid benchmark: the greater the illiquidity, the wider the spread.

Trading generates costs, and it helps to explore the mechanics and structure of the

---

secondary market. In the bond market, most trades are directed through bond dealers, mainly investment banks, than exchanges or electronic platforms. Bond dealers serve as intermediaries between investors, ready to buy and sell securities in the secondary market. The cost of trading is measured by the bid-ask spread. Most major bond dealers are willing to provide indicative “two-sided” (bid-ask) quotes for all but the most obscure bonds. For example, a dealer may quote a Ford 5-year bond as “80–78, 5-by-10”, indicating that the dealer would be willing to buy \$5 million of the Ford bond at a spread of 80 basis points above the 5-year Treasury, and sell \$10 million of the same bond at a 78-basis-point spread. Clearly, bonds that have narrow bid-ask spreads have good liquidity. Liquidity depends not only on the magnitude of the bid-ask spread but also on the depth of the market, as measured by the number of dealers who are willing to make markets, and by the size that can be transacted near the quoted market. For example, an “80–78, 5-by-5” market quoted by three dealers is more liquid than an “80–78, 1-by-2” market quoted by a single dealer.

When investing in a spread product, we need to be paid for what we know. We know that yields on corporate bonds and other spread products must be high enough to compensate for the cost of trading. Furthermore, we also know that trading costs depend on duration, turnover, and the bid-ask spread. We also demand to be paid for what we do not know. We do not know the frequency of turnover or the magnitude of the bid-ask spread, and face the risk that the bid-ask gap will widen the moment we want to trade in size. We need to be paid for uncertainty. Rational portfolio managers understand that trading is costly and, in effect, trading transfers performance from investors’ portfolios to the bonus pools of bond dealers. Moreover, trading eats into the yield spread on a non-Treasury or some other high-quality benchmark security, that is, it drives a wedge between a bond’s spread and its expected excess return. However, this is not to say that portfolio managers must abandon active portfolio strategies to avoid trading costs. Rather, portfolio managers should merely recognize that the benefits of active strategies must be weighed against the costs of trading.

The relationship between size and liquidity is complicated by the fact that size has several dimensions. In dealer markets, liquidity is often supplied by market

---

makers who not only provide quotes but also take positions. Thus, how far size matters for liquidity hinges on the various economies of scale in market-making. Those managing debt in the face of fiscal surpluses emphasize gross issuance in specific securities by focusing issuance on fewer maturities, holding auctions less frequently, and buying back illiquid issues. However, if important fixed costs are involved in the production of information about the future path of interest rates, the size of the entire market across maturities also matters. Similarly, if there are scale economies in extracting information from order flows, the scale of trading activities may also matter. Size does seem to matter, although it is clearly not the only determinant of liquidity. Judging by the success of the government bond futures markets as well as by bid-ask spreads in the G10 markets, there may be a size threshold that lies around \$100-200 billion. Below this, sustaining a very liquid government bond market may not be easy.

An important but often neglected policy choice is between splitting and lumping various forms of government debt. This choice has several dimensions: few versus many maturities, nominal versus inflation-indexed bonds, and one versus many public-sector obligors. In the case of industrialized countries, there seem to be four maturities of choice: two, five, ten, and thirty years. France, Germany, and the U.S. each conduct regular auctions of straight nominal bonds for only these maturities. Italy and Spain have both opted for five maturities (including 15 years for Spain), while the U.K. issues conventional gilts in mostly 10-year and 30-year maturities. In addition to nominal bonds, the governments of Canada, France, U.K., and U.S. have committed themselves to issuing inflation-indexed bonds.

How will liquidity adjust in the short run to the changing supplies of tradable government debt? In growing markets, increased supply should enhance liquidity and contribute to smooth market functioning if other structural conditions are present. In markets that are already well developed, it may be thought that arbitrage activity would ensure that yields are little affected by declining supply. In practice, however, liquidity requires market-making capital and this capital is allocated based on a forward-looking calculation. Hence, liquidity may anticipate rather than follow the market size.

## Chapter 2

# Pricing government bonds in Euro area: performance evaluation of term structure interest rate models <sup>1</sup>

### 2.1 Introduction

The term structure (spot) of interest rates is a theoretical relationship between the yields to maturity of zero-coupon bonds and their time-to-maturity. Supposedly, this existing yield curve (for short) is a fundamental information from financial markets. Market participants pay attention to the form of the term structure. For example, the slope of the yield curve has proven to be a good proxy for economic growth (see, e.g., [Ang, Piazzesi, and Wei \[2006\]](#)). The term structure of interest rates is also essential to asset and risk management (see, e.g., [Cochrane and Piazzesi \[2008\]](#) and [Diebold, Piazzesi, and Rudebusch \[2005\]](#)). Of course, one can observe a finite set of spot interest rates because zero coupon bonds are relatively rare. However, the yield curve is a theoretical concept that is

---

<sup>1</sup>This study is based on a working paper co-authored with Professor Franck Moraux. These results were presented at the 33rd International Conference of the French finance Association (May 2016) and the 23rd Forecasting financial Markets Conference (May 2016). We thank participants of these conferences for the useful comments and discussion.

---

not directly observable in the market. We can find several term structures of interest rates provided by agencies and central banks for data providers (e.g., Bloomberg), based on which these different term structures are built. The values of most interest rates with fixed tenors given by data providers are model-based information as they are implied by the interpolation technique rather than direct price information. Spot interest rates are more of outputs rather than observed data.

The general concept of this research is to compare the results provided by four specifications on data related to four countries. A few articles compare several models on datasets containing several countries and none of them use bond prices. Instead, most of them take zero coupon yield curves for granted, focus on one country and analyze the different models on such a dataset.

There are several competing functionals here, namely model, approach, specification, and function are used interchangeably to build the term structure (spot) of interest rates, but the literature offers no clear and definitive results about the model to be adopted. Here, we consider four Nelson-Siegel style specifications. The first two models were taken from the literature, that is, [Nelson and Siegel \[1987\]](#), and [Svensson \[1994\]](#). The last two models are new and are explored in this research for the first time. We use government bond prices of four countries in the euro area and find the parameters of each model for each country on a daily frequency. We then deduce the term structures of interest rates during the sample period. The gap between predicted and observed prices provides evidence on how well a specific model describes the reality. This is the in-sample fitting comparison of a range of different term structure models. We collect data on government bond prices denominated in the euro and issued by four eurozone countries: France, Germany, Italy, and Spain. We take several countries in one geographical area to find the best model for each country separately and then provide analysis by comparing the models for all the countries together, irrespective of any currency issues. For this purpose, we study the euro area, which is a good laboratory for such experiments as the currency is the same for all the countries under study. Moreover, comparing various specifications allows us to assess the model risk.

---

Several clarifications deserve to be mentioned. First, we do not work with forward rates, as they are by-products of the fitted functional, and we can compute such rates explicitly. Second, this study is not a strictly speaking yield curve fitting exercise, that is, we do not use any data on bond yields. Instead, we exploit coupon bond prices for calibration. Most studies in fixed income securities work with monthly data, whereas, in this study, we operate with daily prices and relative to these studies (see, e.g., [Diebold and Li \[2006\]](#)), this is a rather high frequency data. Our sample period spans twenty years from January 1999 to December 2018. We exploit more than 515,000 bond prices in our research, all taken from Bloomberg. Third, we calibrate the parameters continually in the spirit of [Diebold and Li \[2006\]](#). The functional defines the interpolation specification. We consider four candidates and our criteria is the ability to match bond prices, and the performance is assessed through various dimensions. For instance, we compute the average pricing errors (in euros) as well as the best relative model. The model comparison is organized in two steps. First, we find the best model for each country separately; second, we provide analysis in the best-model comparison for all the four countries. During the second step, there are several possible results as we attempt to obtain the best model for all the countries, or one best model for each country.

This chapter is organized as follows. Section [2.2](#) reviews the literature on the subject and briefly presents the results obtained in the field of multiple-model comparison and on taking datasets from several countries in the euro area. Section [2.3](#) describes the models. Section [2.4](#) describes the algorithm and presents the multi-criteria description. Section [2.5](#) describes the dataset. Section [2.6](#) deals with the empirical results, and Section [2.7](#) investigates the out-of-sample forecasting of the yield curve dynamics. Section [2.8](#) presents the concluding remarks.

## 2.2 Literature review

[Bliss \[1997\]](#) tests and compares five distinct methods. To estimate the term structure of interest rates in the U.S., the author collects monthly prices of bills, coupon-bearing notes, and bonds and implemented various parametric and non-

---

parametric tests. The model parameters are estimated monthly to build a term structure of interest rates. This term structure for a given month is then used to compute the fitted prices and to assess the pricing errors of Treasury securities. The author uses two criteria for evaluating and comparing the fitted-term structure. The first one is the duration-weighted mean of the absolute fitted-price errors. The second one is the “hit rate”, which is intuitive. [Ioannides \[2003\]](#) compares seven methods for estimating the term structure of interest rates from a daily dataset comprising the prices of U.K. Treasury bills. The author investigates two mainstream approaches: a parsimonious representation relying on an exponential decay term and a spline representation that may be further specified into parametric and nonparametric splines. He runs both in-sample and out-of-sample performance analysis. Based on the obtained residuals, he suggests that the parsimonious specifications perform better than the linear spline counterparts. [Kalev \[2004\]](#) works with two models for curve fitting together with two specifications to estimate the zero-coupon yield curve of Australian treasuries. The dataset contains treasury notes and the Commonwealth Government treasury bonds. The author studies a ten-year sample period containing nineteen monetary policy change decisions made by the Reserve Bank of Australia. All the previous studies used only a single country’s dataset, whereas, in this research, we investigate four countries belonging to a single geographic area.

Some studies in the relevant literature explore the term structure of interest rates of eurozone countries. Most of them pursue different goals, employing distinctively different techniques. For example, [Koukouritakis and Michelis \[2006\]](#) collect the yield curve of the original 15 countries of the eurozone to test the expectation hypothesis of the term structure of interest rates. Using cointegration analysis and common trend techniques, they decompose the dynamics of the term structures into transitory and permanent components. Focusing on central and eastern European countries by using the [Diebold, Rudebusch, and Aruoba \[2006\]](#) dynamic version of the NS model, [Hoffmaister, Roldos, and Tuladhar \[2010\]](#) explore the dynamics of the yield curves and analyze the impact of macro shocks on the term structure of interest rates of these countries. [Sopov and Seidler \[2010\]](#) examine the dynamics of the yield curve of central European countries (i.e., the

---

Czech Republic, Slovakia, Poland, and Hungary). To account for some possible regional latent factors, they propose a state space approach coupled with a Kalman filtering technique.

Finally, studies focusing on monetary policy issues in the eurozone use the term structure of interest rates, as the concept provides important information on the economy for policy-makers. First, the term structure of interest rates may itself be the target of the monetary policy. The level of interest rates has strong implications and significance for the banking sector and industry. Second, the shape of the yield curve (level, slope, and curvature) is known to reveal information about the expectations and preferences of the market participants. Third, any change in the term structure of interest rates following a monetary policy decision may be viewed as a market judgment of that decision. [Estrella and Mishkin \[1995\]](#) examine the relationship of the term structure of interest rates with monetary policy instruments in the U.S. and the eurozone.

## 2.3 Yield Curve Models

In this study, we compare four different specifications for extracting the yield curve from the observed coupon bond prices. The first two specifications we consider are taken from the relevant literature, while the next two are proposed for this research. We denote by  $\tau$  the time to maturity measured in years. [Table 2.1](#) summarizes the key features of the models such as the number of parameters to estimate, and the linear and non-linear loadings for all the models.

### 2.3.1 The NS approach

The dynamic Nelson-Siegel (hereafter DNS) approach is based on the model introduced and developed by [Diebold and Li \[2006\]](#) (hereafter NS) and the yield curve specifications of [Diebold, Rudebusch, and Aruoba \[2006\]](#). Summing up this approach, the instantaneous forward rates are supposed to be correctly described at time  $t$  as:

$$f^{NS}(t, \tau) = \beta_{1t} + \beta_{2t}e^{-\lambda_t\tau} + \beta_{3t}\lambda_t e^{-\lambda_t\tau}. \quad (2.1)$$

---

Table 2.1: Summary of term structure models

Linear Loadings	NS	Sv	eBC	eSv	BC
1	*	*	*	*	*
$\frac{1-e^{-\lambda\tau}}{\lambda\tau}$	*	*	*	*	*
$\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}$	*	*	*	*	*
$\frac{1-e^{-\gamma\tau}}{\gamma\tau}$			*	*	$\gamma = 2 \times \lambda$
$\frac{1-e^{-\gamma\tau}}{\gamma\tau} - e^{-\gamma\tau}$		*		*	
Linear parameters	3	4	4	5	4
Non-linear parameters	1	2	2	2	1

This table summarizes information about term structure models. The abbreviations NS, Sv, eBC, eSv, and BC represent the Nelson-Siegel model, Svensson model, extended Bjork-Christensen model, extended Svensson model, and Bjork-Christensen model, respectively.

It is straightforward to deduce from equation (2.1) the yield to maturity given as:

$$y^{NS}(t, \tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right). \quad (2.2)$$

Here  $\beta_{1t}$ ,  $\beta_{2t}$ ,  $\beta_{3t}$  and  $\lambda_t$  are time-dependent parameters that must be estimated.

The main difference between the original NS specification and the DNS approach is that the structural parameters are time-dependent. As the NS model is widely accepted and used among practitioners, the DNS approach becomes one of the most popular methods to estimate the term structure of interest rates. The DNS approach is attractive because it inherits the precise features of the “static” NS specification. In short, the DNS approach is simple, intuitive, and parsimonious

---

with only four parameters to fit at each period <sup>1</sup>.

### 2.3.2 Dynamic Svensson approach

The dynamic Svensson (hereafter Sv) approach is based on the model introduced and developed by [Svensson \[1994\]](#) specifications for the yield curve. He suggests increasing the flexibility of the seminal “static” NS specification by adding a fourth term with two additional parameters, namely  $\beta_{4t}^{Sv}$  and  $\gamma_t$ . His goal is to improve the curve fitting performance on Swedish data. In the [Svensson \[1994\]](#) specification, the forward curve is described by

$$f^{Sv}(t, \tau) = \beta_{1t} + \beta_{2t}e^{-\lambda_t\tau} + \beta_{3t}\lambda_t e^{-\lambda_t\tau} + \beta_{4t}\gamma_t e^{-\gamma_t\tau} \quad (2.3)$$

and the yield to maturity curve is given as:

$$y^{Sv}(t, \tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau} \right) + \beta_{4t} \left( \frac{1 - e^{-\gamma_t\tau}}{\gamma_t\tau} - e^{-\gamma_t\tau} \right), \quad (2.4)$$

where  $\beta_{1t}$ ,  $\beta_{2t}$ ,  $\beta_{3t}$ ,  $\beta_{4t}$ ,  $\lambda_t$  and  $\gamma_t$  are the six time-dependent parameters to be estimated. By adding a second hump shape to the original specification, perhaps, a better fit can be obtained.

### 2.3.3 Extended Bjork and Christensen approach

We include a couple of dynamic approaches. First, we propose the “extended Bjork & Christensen” (hereafter eBC) model that is inspired by the [Bjork and Christensen \[1997\]](#) specification (hereafter BC). This dynamic version is indeed a modified version, that is, we include a parameter  $\gamma_t$  to the BC approach that defines the position of the second hump. Here, we do not have an additional curvature element, but we retain the additional slope element.

$$f^{eBC}(t, \tau) = \beta_{1t} + \beta_{2t}e^{-\lambda_t\tau} + \beta_{3t}\lambda_t e^{-\lambda_t\tau} + \beta_{4t}e^{-\gamma_t\tau}. \quad (2.5)$$

---

<sup>1</sup>Among the popular competitors, we mention the spline curve fitting approach of [McCulloch \[1971\]](#) and [McCulloch \[1975\]](#).

---

The associated yield to maturity is given as:

$$y^{eBC}(t, \tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right) + \beta_{4t} \left( \frac{1 - e^{-\gamma_t \tau}}{\gamma_t \tau} \right), \quad (2.6)$$

where  $\beta_{1t}$ ,  $\beta_{2t}$ ,  $\beta_{3t}$ ,  $\beta_{4t}$ , and  $\lambda_t$  are the five time-dependent parameters to be estimated. The special case where  $\gamma_t = 2\lambda_t$  corresponds to the original BC specification.

Bjork and Christensen propose to revisit the “static” NS model specification to address some consistency concerns between the shapes of the forward rate curve this specification can provide and the possible dynamics of future interest rates. They suggest including an additional exponential term, that is,  $e^{-2\lambda_t \tau}$  to the NS specification, so that the forward curve is described as:

$$f^{BC}(t, \tau) = \beta_{1t} + \beta_{2t} e^{-\lambda_t \tau} + \beta_{3t} \lambda_t e^{-\lambda_t \tau} + \beta_{4t} e^{-2\lambda_t \tau}. \quad (2.7)$$

The associated yield to maturity is given as:

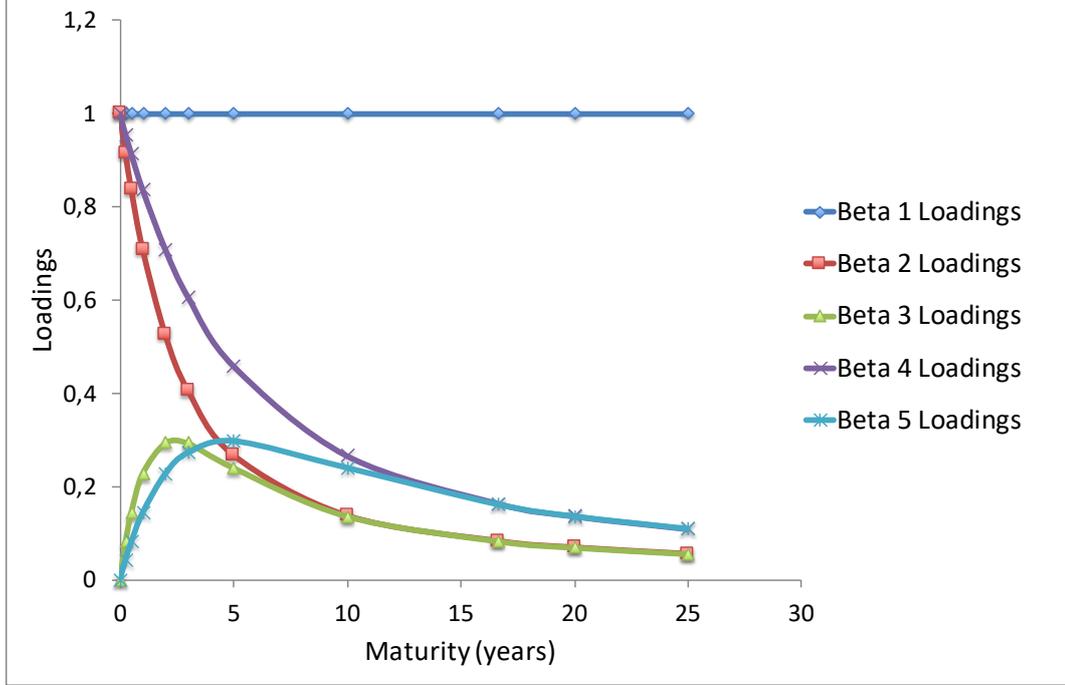
$$y^{BC}(t, \tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right) + \beta_{4t} \left( \frac{1 - e^{-2\lambda_t \tau}}{2\lambda_t \tau} \right), \quad (2.8)$$

where  $\beta_{1t}$ ,  $\beta_{2t}$ ,  $\beta_{3t}$ ,  $\beta_{4t}$ , and  $\lambda_t$  are the five time-dependent parameters to be estimated.

### 2.3.4 Extended Svensson approach

In addition to the eBC approach, we propose an “extended Svensson” (hereafter eSv) model that is inspired from [Svensson \[1994\]](#). This dynamic version is indeed a modified version as we include a parameter  $\beta_{5t}$  associated with  $\gamma_t$  to the Sv approach for additional slope loading. We plot the factor loadings associated

Figure 2.1: Loadings of extended Svensson model



This figure shows the factor loadings in the extended Svensson model where the factors are  $\beta_1, \beta_2, \beta_3, \beta_4$  and  $\beta_5$ . The associated loadings are 1 for  $\beta_1$ ,  $(1 - e^{-\lambda\tau})/\lambda\tau$  for  $\beta_2$ ,  $(1 - e^{-\lambda\tau})/\lambda\tau - e^{-\lambda\tau}$  for  $\beta_3$  where  $\lambda$  parameter is equal to 0.0609,  $(1 - e^{-\gamma\tau})/\gamma\tau$  for  $\beta_4$ , and  $(1 - e^{-\gamma\tau})/\gamma\tau - e^{-\gamma\tau}$  for  $\beta_5$  where  $\gamma$  parameter is equal to 0.03045. The parameter  $\tau$  denotes maturity in years.

with  $\gamma_t$  and  $\lambda$  parameters in Figure 2.1. The forward curve is given as:

$$f^{eSv}(t, \tau) = \beta_{1t} + \beta_{2t}e^{-\lambda_t\tau} + \beta_{3t}\lambda_t e^{-\lambda_t\tau} + \beta_{4t}e^{-\gamma_t\tau} + \beta_{5t}\gamma_t e^{-\gamma_t\tau} \quad (2.9)$$

and the corresponding yield to maturity is given as:

$$y^{eSv}(t, \tau) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau} \right) + \beta_{4t} \left( \frac{1 - e^{-\gamma_t\tau}}{\gamma_t\tau} \right) + \beta_{5t} \left( \frac{1 - e^{-\gamma_t\tau}}{\gamma_t\tau} - e^{-\gamma_t\tau} \right), \quad (2.10)$$

where  $\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t}, \lambda_t$ , and  $\gamma_t$  are the seven time-dependent parameters to be estimated.

---

## 2.4 Empirical methodologies

All the specifications previously discussed allow us to price government coupon bonds. Consequently, the general principle of an empirical fitting methodology is to find parameters so that the prices predicted by the model fit the ones observed in the market as closely as possible.

### 2.4.1 Parameter estimation

Consider at time  $t$ , a set of  $N$  coupon bonds with prices denoted by  $B_i(t)$ , where  $i = 1, \dots, N$ . Each of them promises some payments  $C_{iq}$  at time  $t_{iq}$  where  $q = 1, \dots, K_i$  and  $K_i$  are the number of promised payments. The payments  $C_{iq}$  are typically constant  $C_i$  for  $q$  strictly less than  $K_i$  and  $C_K$  equal to  $C_i + F$ , where  $F$  is the face value (equal to 100). Denoted by  $\Theta$ , the set of parameters and  $y(t, t_{iq}; \Theta)$  the yield to maturity at time  $t$  associated with horizon  $t_{iq}$  provided by the specification under security. Then, the price of the coupon bond at time  $t$  is given by

$$B_i(t, \Theta) = \sum_{q=1}^{K_i} C_{iq} e^{-(t_{iq}-t)y(t, t_{iq}; \Theta)}, \quad i = 1, \dots, N. \quad (2.11)$$

The main goal of our research is to select the model from the list presented in the previous section that provides accuracy in bond price calculation. At time  $t$ , on the market we observe real coupon bond prices  $\hat{B}_i(t)$ . The parameter vector  $\hat{\Theta}_t$  at time  $t$  may be, in principle, chosen to minimize the sum of squared errors between the values predicted by the model and the observed prices.

$$\hat{\Theta}_t = \underset{\Theta}{\operatorname{argmin}} \sum_{i=1}^N \left( B_i(t, \Theta) - \hat{B}_i(t) \right)^2. \quad (2.12)$$

In fact, we consider some potential heterogeneity in the observed errors. More specifically, we weigh the squared errors in (2.12) to minimize the sum of the

---

weighted squared errors at time  $t$

$$SSE(t) = \min_{\Theta} \sum_{i=1}^N p_i(t) \left( B_i(t, \Theta) - \hat{B}_i(t) \right)^2, \quad (2.13)$$

where

$$p_i(t) = \frac{1}{(D_i(t))^2} \quad (2.14)$$

with  $D_i(t)$  the modified duration of the  $i$ th bond captured by

$$D_i(t) = \sum_{q=1}^{K_i} t_{iq} \frac{C_{iq} e^{-(t_{iq}-t)YTM(t)}}{\hat{B}_i(t)}. \quad (2.15)$$

Here  $YTM(t)$  represents the yield-to-maturity for this bond at time  $t$ . The modified duration allows to convert the pricing errors into the fitted yield curve errors. The concept of duration provides a useful method for understanding the relationship between the price and yield-to-maturity of a bond. That is, for a given change in a bond's yield-to-maturity, the change in price will be greater for a longer-term bond than for a shorter-term bond, and duration attempts to quantify this impact. Adding weights in the minimization function, results in a better fit to yield curves (see [Bolder and Streliski \[1999\]](#)).

## 2.4.2 Performance evaluation

The above optimization process to estimate parameters is repeated for our sample period daily. Denote  $M$  as the total number of days in the sample period. Let  $B_i(t, \Theta^*(\cdot))$  be a price of the bond  $i$  calculated for day  $t$  in the framework of one of the models for each country. In order to formulate a criterion to identify the model that describes the real market bond prices with the best accuracy, we define the quantity as:

$$X(\text{country, model}) = \frac{1}{MN} \sum_{t=1}^M \sum_{i=1}^N \left| B_i(t, \Theta^*(\text{model})) - \hat{B}_i(t) \right|, \quad (2.16)$$

where  $|\cdot|$  represents the absolute value.

---

To investigate the robustness of our performance results, we consider a second performance evaluation criteria. Recall that for a given country we have  $N$  bonds. For each specification  $j = 1, 2, 3, 4$  we compute the estimated bond price  $B_i(t, \Theta_j^*)$  at time  $t$ , where  $i = 1, \dots, N$ . The mean absolute error is given by:

$$MAE_j(t) = \frac{1}{N} \sum_{i=1}^N \left| B_i(t, \Theta_j^*) - \hat{B}_i(t) \right|. \quad (2.17)$$

For each day  $t$  of the sample period, we define the specification that produces the smallest mean absolute error. For each model  $j$ , we compute the score as follows:

$$\text{Score}_j = \sum_{t=1}^M I_j(t), \quad (2.18)$$

where  $I_j(t)$  is an indicator that the specification  $j$  performs the best during the day  $t$ .

$$I_j(t) = \begin{cases} 1, & \text{MAE}_j(t) < \text{MAE}_h(t) \quad \forall h \neq j; \\ 0, & \text{otherwise.} \end{cases} \quad (2.19)$$

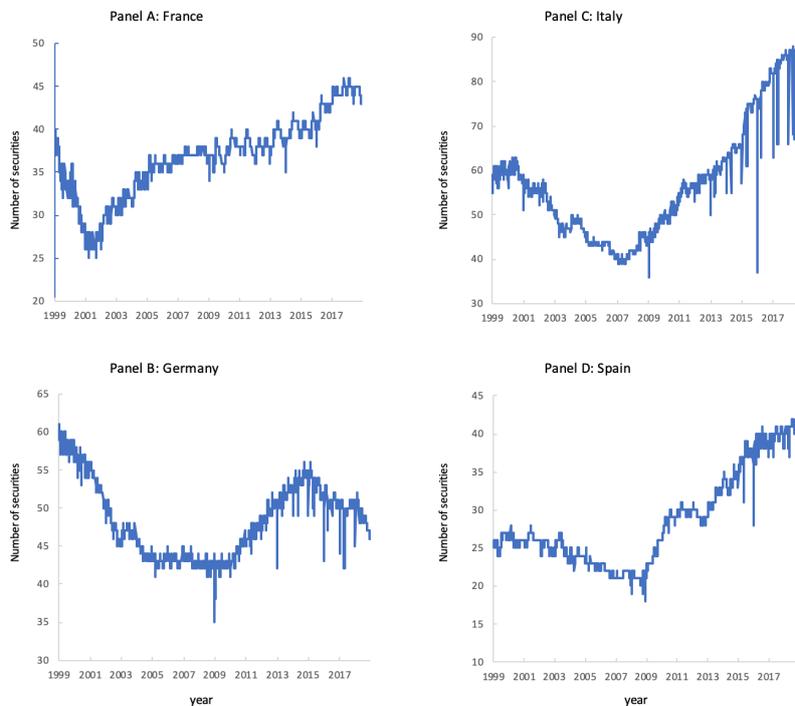
Finally, for one country we obtain the score for each NS term structure specification. This score corresponds to the number of days in the sample period when the given model performs the best among others.

## 2.5 Data Description

We collect data on government bonds for four eurozone countries over a twenty-year (1999–2018) period. The set of daily prices contains 800 different bonds with different coupon rates, maturities, and countries (i.e., France, Germany, Italy, and Spain). We only select the coupon “fixed rate” type and the “bullet” type bonds. We consider only euro-denominated bonds to avoid any problems associated with the exchange rates. Figure 2.2 shows the total number of available securities on a specific day for a given country. It shows that, from 2001, the number of available French government securities increased in the market. For Italy, after decreasing until 2007, the number of bonds increased up to 90 available bonds at the end of the sample period. We can observe that Germany has 60 available government

securities at the beginning of the sample period and this number decreased in the following years. From 2005 to 2011, the number of available German bonds have remained stable. Thereafter, there was an increase; however, since 2015 a decrease is observed in the number of available bonds. This contrasts with France, Italy, and Spain. For Spain, since 2009, the number of available government securities increased almost linearly and, by 2018, there were about 45 bonds in the market.

Figure 2.2: Number of government bonds within the dataset

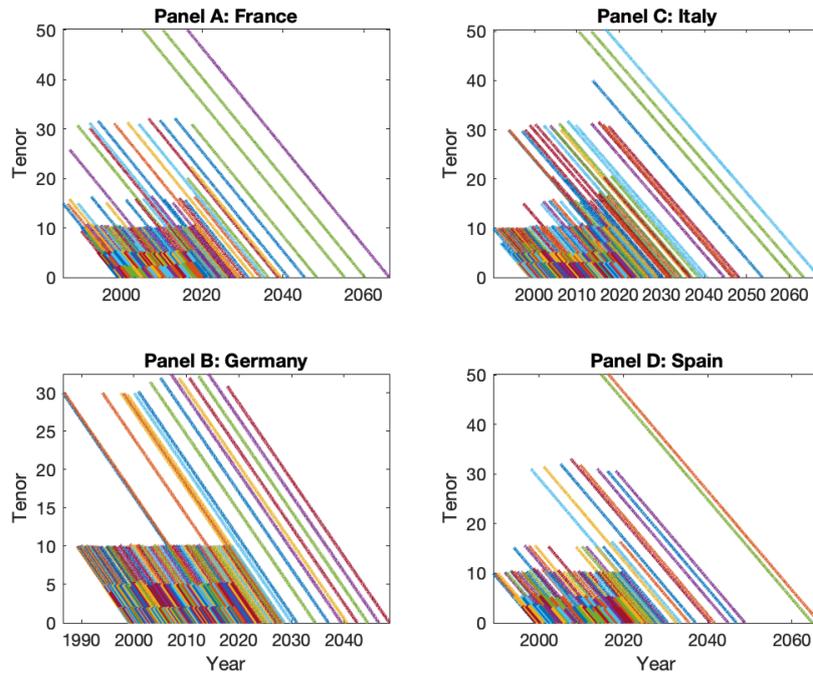


This figure shows the total number of available securities on a particular day for a given country. Sample period: January 4, 1999, to December 28, 2018. Frequency: Daily.

The terms and functioning of the government securities market are not the same in the eurozone. We consider *Obligation Assimilable by Trésor (OATs)* for France, *Bundesobligation* and *Bundesanleihen* (or *Bund*) for Germany, *Italy Buoni Poliennali DelTesoro* for Italy, and *Bonos del Estado* and *Obligaciones del Estado*

for Spain. In Germany, the Federal government issues bonds because there also exist very powerful leaders, but this issuer is not like the centralized French government. All the bonds have yearly coupon rates, and the number of bonds is different for each country. The dataset has 158 bonds for France, 261 bonds for Germany, 258 bonds for Italy, and 123 bonds for Spain. Tables A.5, A.6, A.7 and A.8 in the Appendix provide the complete list of the considered bonds and other details about these bond issues: ISIN code, issue and maturity date, coupon rate and tenor for France, Germany, Italy, and Spain, respectively.

Figure 2.3: Maturity Distribution of the government bonds



This figure shows the maturity structure of the nominal government securities for a given country. The first bond issue for France was in 1985, for Germany in 1986, for Italy in 1990 and for Spain in 1989. Source: Bloomberg.

Figure 2.3 shows the maturity distribution of government securities for a given country. The first bond issue for France was in 1985, for Germany in 1986, for Italy in 1990, and 1989 for Spain. We can clearly see that each country most often issues 10-year bonds. Government securities with time to maturity equal

---

to thirty years at the time of issuance also represent an important part of all the issued bonds. We notice the presence of bonds with tenor less than ten years, five years, and two or three years. In addition, France, Italy, and Spain propose 15-year securities; however, this is not the case for Germany. Unlike France, Italy, and Spain, Germany issues very long-term bonds. France issues these securities with fifty-year tenor, while Spain issues for two years, and Italy issues for three years. Italy has also issued some bonds with a time to maturity of 40 years at issuance.

The first issue to address when dealing with a bond database is to either choose between living bonds only and avoid bonds that are paid back before the end of the period and, therefore, disappear from the database. We used a set of bonds that remain identical over the sample period. Our objective, nevertheless, is to consider as many available bonds as possible. The sample period for all the countries is from January 4, 1999<sup>1</sup> to December 28, 2018.

Alternatively, there is also the possibility of considering yields-to-maturity of zero-coupon bonds provided by Bloomberg. However, for two reasons, we do not consider these available interest rates as suitable for our study. First, zero-coupon bonds are infrequent, consequently, the yield-to-maturity of zero-coupon bonds provided by Bloomberg is deduced from coupon-bearing bonds using a stripping method (e.g., bootstrap), therefore, there is a quality concern. Second, we may not know which model was used to compute the provided interest rates, and this is a supplementary source of problems. The bottom line is to avoid using the provided bond yields.

## 2.6 Results

In this section, we present the results and discuss the model fit. The two performance criteria described earlier provide the method to compare the term structure models and identify the answer that is best to price government bonds in the eurozone area. Finally, we discuss the time series of the fitted errors and the

---

<sup>1</sup>This data corresponds to the launch of the EURO currency.

---

implied par yields for the four countries and the four-term structure specifications to closely study the differences between twenty such cases.

### 2.6.1 In-sample model fit

We compare how different NS style specifications describe the bond prices observed in the market. We take the datasets on government bond prices and compute the parameters, and calculate the bond prices for each model and country. Market practitioners tend to favor a term structure model equipped with several characteristics<sup>1</sup>: it must be flexible, simple, specified appropriately, and realistic. Econometricians would add to this list that a term structure model must provide a good fit to data, while a theoretical economist would also require an equilibrium derivation of the model. Hereafter, we stay close to the market participants' point of view and, among the four specifications, we search for the one that provides the best fit for government securities prices.

We fit all the four term structure models using daily bond prices issued by the four respective countries. For France and Germany Panel A and B of Table A.9 in the appendix provide the descriptive statistics for the estimated set of parameters and Panel C and D give the same information for Italy and Spain. In order to discuss our results, we propose to define one term structure of interest rates specification and observe the differences across the four countries.

- First we describe the Nelson and Siegel model.

Factor  $\beta_1$  corresponds to the yield curve level. For theoretical reasons, essentially, this value is considered as positive. Accordingly, we add a restriction in the parameter space considered by the estimation process. For France and Germany the value is about 4 percent, for Spain it is 5.28 percent, and for Germany the value is 9.22 percent. The non-linear parameter  $\lambda$  defines the hump position. Our parameter estimation procedure keeps it free, unlike most of the previous literature that fixes it in advance. For all the counties, its average value is about 3. The factor  $\beta_2$  is associated with the slope loading and the factor  $\beta_3$  with

---

<sup>1</sup>For more details see [Rogers \[1995\]](#).

---

curvature loading. Both parameters have negative mean values across the sample period and for all the countries.

- Second we describe the Svensson model.

Factor  $\beta_1$  corresponds to the yield curve level. According to the restrictions used in the estimation process, this value must be positive. It is highest for Italy at 4.2 and lowest for Germany at 1.2 percent. There are two non-linear parameters  $\lambda$  and  $\gamma$  that define the position of the hump. For all the four countries, parameter  $\lambda$  is about 2 and defines the position of the first hump. For France and Germany, the  $\gamma$  parameter is about 13 and 19 for Italy and Spain, and it defines the position of the second hump. Factor  $\beta_2$  is associated with the slope loading, factor  $\beta_3$  with curvature loading, and both are related to  $\lambda$  parameter. Parameter  $\beta_3$  value is negative across all the countries; parameter  $\beta_2$  is positive for Germany and Spain and negative for France and Italy. Factor  $\beta_4$  is associated with curvature loading related to the  $\gamma$  parameter. Compared to  $\beta_2$  and  $\beta_3$  parameter values,  $\beta_4$  value is more important and is about 9 for France and Spain and 13 for Germany and Italy.

The two following term structures of interest rate specifications are new in the literature and introduced in this research.

- Next we describe the extended Bjork and Christensen model.

Factor  $\beta_1$  corresponds to the yield curve level. According to restrictions used in the estimation process, this value must be positive. It is lowest for Germany at 2.3 and highest for Italy at 9.4 percent. The two non-linear parameters, namely  $\lambda$  and  $\gamma$ , define the position of humps. For all the four countries, parameters  $\lambda$ , which defines the position of the first hump, is about 3, except in the case of Italy where the value is 5.23. Parameter  $\gamma$ , which defines the position of the second hump, is the smallest value for Italy at 14.2. For France and Germany, the  $\gamma$  parameter has values close to about 16 and, finally, the greatest value is for Spain at 20.46. Factor  $\beta_2$  is associated with the slope loading, factor  $\beta_3$  with curvature loading, and both are related to the  $\lambda$  parameter. We see that for all the four countries, these two parameters are negative. Factor  $\beta_4$  is associated with slope

---

loading and is related to  $\gamma$  parameter. Its value is positive for all the countries, except Italy where  $\beta_4$  is negative.

- Finally, we describe the extended Svensson model.

Factor  $\beta_1$  corresponds to the yield curve level. According to the restrictions used in the estimation process, this value must be positive. It is lowest for Germany at 0.8 and highest for Italy at 9.2 percent. There are two non-linear parameters  $\lambda$  and  $\gamma$  that define the position of humps. For all the four countries,  $\lambda$  is about 3 and defines the position of the first hump, while  $\gamma$  ranges from 12 for France and 18 for Italy. Factor  $\beta_2$  is associated with slope loading, factor  $\beta_3$  with curvature loading, and both are related to the  $\lambda$  parameter. Parameter  $\beta_2$  is positive and  $\beta_3$  is negative for all the four countries. Factor  $\beta_4$  is associated with slope loading, factor  $\beta_5$  with curvature loading, and both are related to the  $\gamma$  parameter. In contrast to  $\beta_2$  and  $\beta_3$ , parameter  $\beta_4$  is negative and  $\beta_5$  is positive across all the four countries.

Table 2.2 presents the descriptive statistics for fitting errors. The measure of the overall fitting error on a particular day is the average of the absolute errors between the predicted and market yields across all the available securities during that day. It is computed by

$$MAE_t = \frac{1}{N_t} \sum_{i=1}^{N_t} \left| \hat{y}(t, i) - y(c, T_i; \hat{\Theta}_t) \right|, \quad (2.20)$$

where  $N_t$  is the number of available government securities for that day. The table shows that the models face difficulties to fit the data for Italy. For this country, the mean fitting error across the entire sample period is about 22–24 basis points, whatever be the specifications. This is confirmed by the median fitting error that is twice smaller and equal to 10 basis points. The mean fitting error for France is about 11 basis points. The median fitting error for France is six times less than its mean value. This suggests that the distribution is highly skewed, as confirmed by the maximum. For these two countries, the range of the average fitting error is quite large compared to Germany and Spain for all the term structures of interest

---

Table 2.2: Summary statistics of the model fitting errors

	Mean	Std. Dev.	Median	Minimum	Maximum
France					
NS	11.720	29.924	2.335	0.672	187.167
Sv	10.682	29.472	1.573	0.553	173.146
eBC	10.743	29.145	1.661	0.585	168.055
eSv	10.484	28.878	1.541	0.553	167.910
Germany					
NS	3.406	3.355	3.082	0.763	10.166
Sv	2.351	6.163	1.768	0.546	8.349
eBC	2.494	3.711	1.961	0.642	8.639
eSv	2.337	8.937	1.759	0.523	7.855
Italy					
NS	24.005	29.338	10.262	1.680	122.359
Sv	22.996	28.161	10.056	1.631	126.002
eBC	23.876	29.463	10.241	1.632	124.815
eSv	22.549	27.166	9.987	1.633	118.957
Spain					
NS	3.977	2.012	3.677	0.744	11.501
Sv	3.339	1.724	3.183	0.570	11.042
eBC	3.496	1.738	3.299	0.579	10.835
eSv	3.202	1.636	3.054	0.566	10.863

This table reports the descriptive statistics (i.e., the mean, standard deviation, median, min and max) of the daily fitting errors for securities issued by the four countries and the four term structure models for the sample period from January 4, 1999, to December 28, 2018.

rate specifications. The model fit for German data is better than the model fit for the Spanish market data.

---

## 2.6.2 Identifying the best model

To identify the best model we use two criteria, which are criteria (2.16) and (2.18). Table 2.3 presents the results. For all the four countries and both criteria, the best model is the extended Svensson model. Panel A of Table 2.3 provides the results of performance analysis with the mean absolute errors. The criterion value for all the four countries, corresponding to the eSv term structure model, is lower than the same values for other models. Of course, the average value taken for all the four countries for each model concludes the same in the last row of Table 2.3. For both fitting criteria, ordering of the four specifications is given as: eSv, Sv, eBC, and NS term structure model. Panel B of Table 2.3 provides the score values calculated by the equation (2.18). For each country and for each model, we calculate the number of days when the given model produces the least mean absolute bond price errors taken for all the government bonds. Yet again, we observe that for all the four countries, the eSv model is the best fit for most days of the sample period.

## 2.6.3 A closer look at the fitting errors over the sample period

This section presents an alternate method to compare the specification. For the given specification, we provide two figures, where one plots the fitted errors and the other plots the implied par yield curve with the observed and fitted yield to maturity for all the available government securities for a specific day of the sample period. The time series of fitting errors provide information about the ability of the term structure model to fit the data for each day of the sample period.

Figure 2.4 plots the time series of fitting errors across all the available securities for a specific day for a given country for the advanced Svensson specifications (Figures A.8 to A.10 plot the time series of the fitting errors for the remaining three specifications and are presented in the appendix). For all the figures, panel A corresponds to France, panel B to Germany, panel C to Italy, and panel D to Spain. There are two different y-axis limits ranging from minimum to maximum

---

Table 2.3: Performance analysis

Panel A: Mean Absolute Errors				
	DNS	Sv	eBC	eSv
France	0.7254	0.5976	0.6013	<b>0.5802</b>
Germany	0.3508	0.1994	0.2177	<b>0.1938</b>
Italy	1.7279	1.6835	1.7221	<b>1.6651</b>
Spain	0.2928	0.2239	0.2319	<b>0.2088</b>
Mean	0.7740	0.6759	0.6931	<b>0.6618</b>

Panel B: Score Measure					
	DNS	Sv	eBC	eSv	Total
France	150	828	442	<b>3912</b>	5332
Germany	119	1041	462	<b>3767</b>	5389
Italy	733	1307	625	<b>2632</b>	5297
Spain	210	911	672	<b>3492</b>	5285

This table shows two performance criteria. The first one is a comparison of the term structure models and defines with NS specification to provide the calculated bond prices that are closest to the observed bond prices via the mean absolute errors. The second performance criterion gives the number of days when a given term structure model produces the smallest absolute error between the observed bond price and model prediction.

fitting errors. In figure 2.4, on Panels A and C, the fitting errors range from zero to 200 basis points; on Panels B and D, the fitting errors range from zero to 20 basis points.

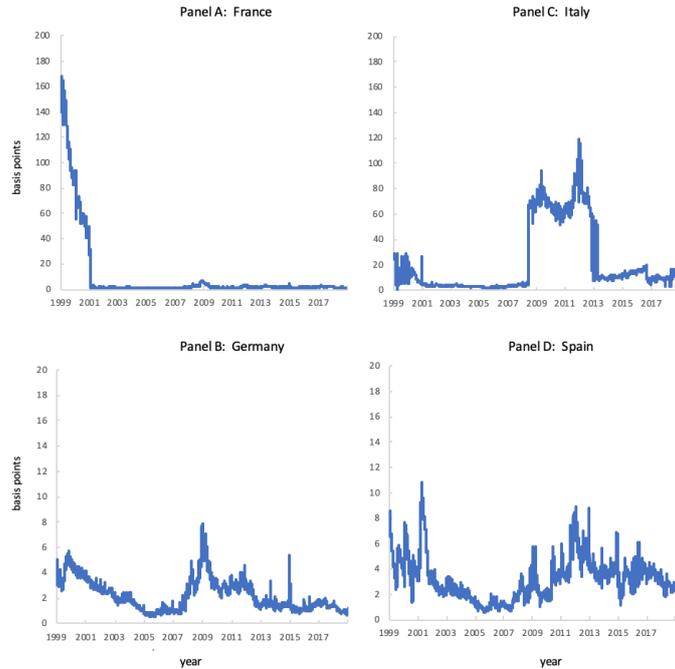
We propose to define one country and compare the four figures across different term structures of interest rate models. In the case of France, the fitting errors are relatively important at the beginning of the sample period. Precisely, this happens over a two-year period from 1999 to 2001. We observe a decrease in the plotted values from about 170 basis points to 1.8 basis points on January 22, 2001.<sup>1</sup> After that date, the fitted errors range from zero to about 3–4 basis points for all the specifications. In the case of Italy, the figure displays large fitting errors during the global financial crisis (2007–2008). The two pics probably reflect some

---

<sup>1</sup>This will be explained in more detail in Chapter 3 with reference to the French market.

---

Figure 2.4: Fitting Errors (extended Svensson model)

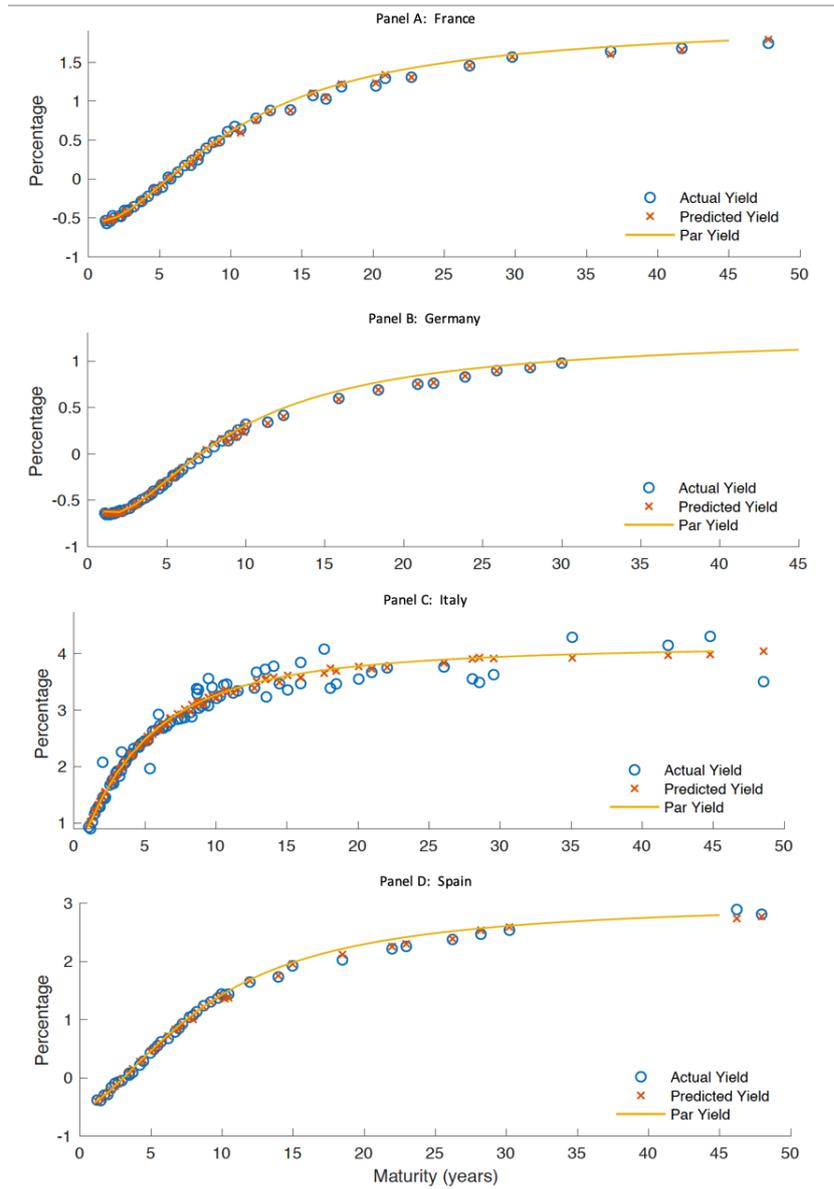


This figure shows the total fitting error implied by the extended Svensson model. The fitting error is computed as the mean absolute error between the predicted and the market yields across all the available securities on a particular day for a given country. The fitting errors are shown in basis points. The sample period: January 4, 1999 to December 28, 2018. Frequency: Daily.

market events. The first one may correspond to the global financial crisis in 2007–2008, and the second to the 2012 European debt crisis.

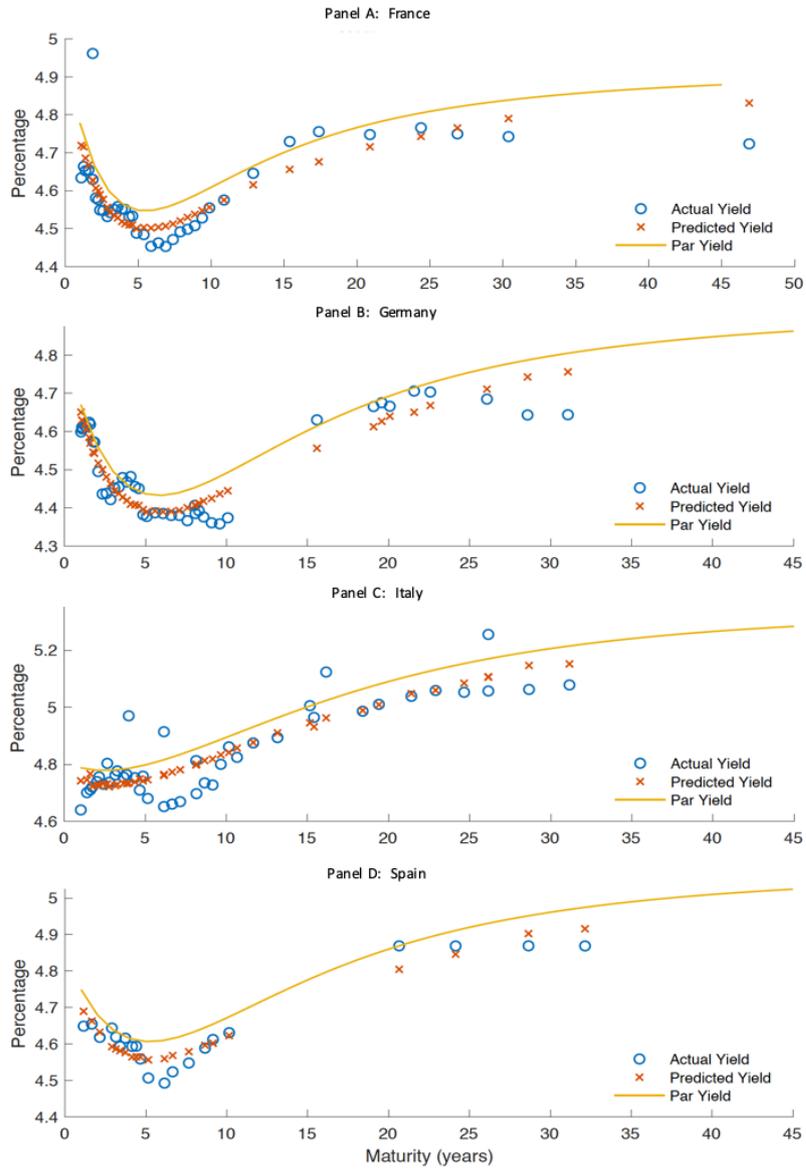
For Germany and Spain, the range of fitting errors varies from 10 times less up to 20 basis points instead of 200 basis points. Germany experiences a decrease in fitting errors from 2000 to 2007, an upsurge associated with the financial crisis in 2007–2008, followed by a decreasing trend in the fitting errors from 2009 onward. Germany was less impacted by the European debt crisis in 2012, while Spain has fitting errors that are more relative than other countries.

Figure 2.5: Par yield curve (NS specification) on August 16, 2018



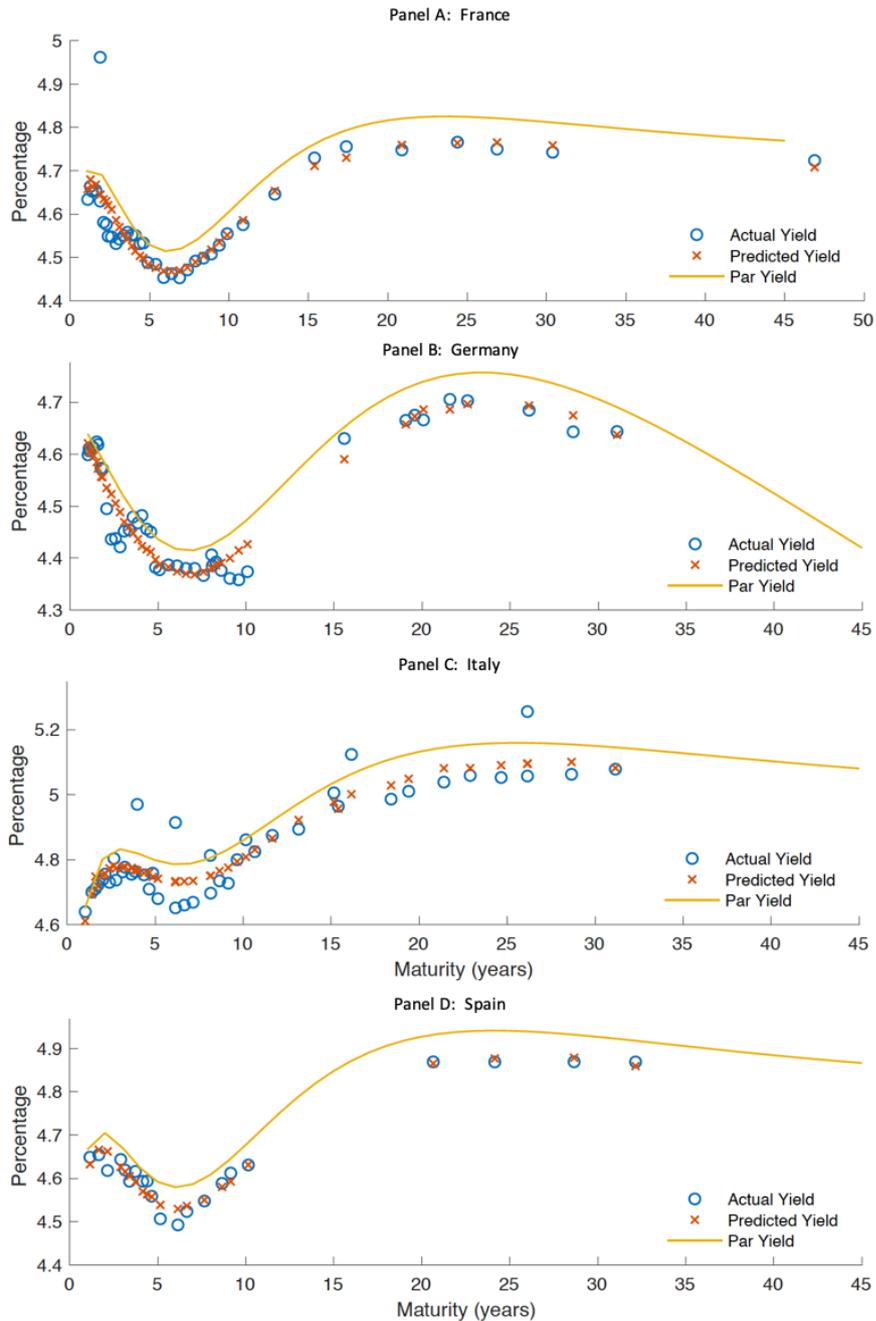
This figure shows the Nelson-Siegel par yield curve and fit of individual securities for August 14, 2002, for a given country during the sample period. The curve is reported in annualized percent.

Figure 2.6: Par yield curve (NS specification) on June 6, 2008



This figure shows the Nelson-Siegel par yield curve and the fit of individual securities as on June 6, 2008, for a given country during the sample period. The curve is reported in annualized percent.

Figure 2.7: Par yield curve (eSv specification) on June 6, 2008



This figure shows the Svensson par yield curve and the fit of individual securities for April 14, 2008, for a given country during the sample period. The curve is reported in annualized percent.

---

#### 2.6.4 A closer look at the par-yield curves

We plot the par yield curve for a given day (August 16, 2018) of the sample period to compare how a term structure specification performs in different countries. Figure 2.5 presents the implied par yield curve and the fit of individual securities with actual and predicted yield-to-maturity for all available government securities using the NS term structure of interest rate model. The flexibility of NS specification with only four parameters is sufficient to provide a good fit on this day. For all the four countries we see the classical shape of the term structure of interest rates, meaning the upward-sloping yield curve.

Second, we choose June 6, 2008, as another day. Figure 2.6 and Figure 2.7 present the implied par yield curve and the fit of individual securities for the NS and eSv models. We see that the NS term structure of interest rate model does not reflect the complexity of the yield-to-maturity values. In contrast the extended Svensson specification with seven parameters is flexible with two possible yield curve humps to adjust to the reality of the selected day (June 6, 2008). We see two yield curve humps in the case of France and Germany. Even more complex shapes of the term structure of interest rates are presented in the case of Italy and Spain. One mode difference between Figure 2.6 and Figure 2.7 is that the NS specification suggests the upward-sloping yield curve for all the countries on June 6, 2008 and, at the same time, the eSv specification provides downward-sloping curves for all the countries.

Figure A.11 and Figure A.12 in the Appendix show the respective implied par yield curve for Sv and eBC specifications for a given country on June 6, 2008. The Svensson term structure of interest rates provides the possibility of defining two humps of the yield curve. We can clearly see the presence of two humps across all the countries. Moreover, this date corresponds to the crisis period; thus, we notice that the model fit is noisy.

#### 2.6.5 Correlation

Table 2.4 reports the correlations among the different rates of maturities. The correlation coefficients vary by country: for France the variation is from 0.615

Table 2.4: Correlation of zero-coupon yields

	3 months	1 year	3 years	5 years	10 years
France					
3 months	1.0000	0.7991	0.6150	0.6227	0.6158
1 year		1.0000	0.9393	0.9175	0.8820
3 years			1.0000	0.9910	0.9581
5 years				1.0000	0.9817
10 years					1.0000
Germany					
3 months	1.0000	0.9058	0.8958	0.8818	0.8457
1 year		1.0000	0.9857	0.9651	0.9180
3 years			1.0000	0.9944	0.9646
5 years				1.0000	0.9859
10 years					1.0000
Italy					
3 months	1.0000	0.6245	0.4333	0.4119	0.3282
1 year		1.0000	0.9232	0.8822	0.7771
3 years			1.0000	0.9914	0.9358
5 years				1.0000	0.9695
10 years					1.0000
Spain					
3 months	1.0000	0.7658	0.6836	0.6498	0.5655
1 year		1.0000	0.9620	0.9171	0.8138
3 years			1.0000	0.9888	0.9289
5 years				1.0000	0.9722
10 years					1.0000

This table reports the correlation of eSv fitted zero-coupon yields for three months, one, three, five, and ten year maturities implied by our sample of government nominal securities issued by France, Germany, Italy, and Spain. The sample period is from January 4, 1999 to December 28, 2018, with daily frequency.

to 0.99; Germany from 0.846 to 0.99; Italy from 0.328 to 0.99; and Spain from 0.565 to 0.99. The correlations are computed for a daily series throughout the sample period. Across all the countries, we see that the correlation between the

---

three-year and five-year zero-coupon rates is the maximum value among other values in the corresponding correlation matrix. In particular, Table 2.4 shows that the less correlated time series of interest rates is the correlation between the three-month and ten-year Italian zero-coupon rates and the value is 0.3282.

## 2.7 Out-of-sample forecast performance

This section proposes a forecasting investigation of the term structure of interest rates in the spirit of Diebold and Li [2006]. An approximation description of the dynamics of the yield curve should not only be able to fit the data appropriately (in-sample). It should also be able to forecast the data (out-of-sample). Caldeira, Moura, and Santos [2016] explore the performance of several forecasts for the yield curve. The main benchmark model adopted in their paper is the random walk (RW) model. Authors consider that, in practice, it is difficult to beat the RW method in terms of out-of-sample forecasting accuracy.

### 2.7.1 Forecasting method

We consider three alternative forecasting methods including the RW, univariate, and multivariate autoregressive specifications.

- Random walk

The  $t + h$ -step-ahead forecast for a yield of maturity  $\tau$  is given by

$$y_{t+h}(\tau) = y_t(\tau) + \varepsilon_t(\tau) \tag{2.21}$$

where

$$\varepsilon_t(\tau) \sim N(0, \sigma^2(\tau)).$$

The RW is the simplest and yet important model in time series forecasting. We assume that during each period, the interest rate for the given maturity  $\tau$  takes a random step away from the previous value, and the steps are independently and identically distributed in size.

- Univariate autoregressive specification

---

A first-order univariate autoregressive model AR(1) allows forecasting the yield for maturity  $\tau$  using the available data for that maturity as

$$y_t(\tau) = \alpha + \beta y_{t-1}(\tau) + \varepsilon_t. \quad (2.22)$$

The forecast for  $h$ -step ahead horizon is obtained as:

$$\hat{y}_{t+h|t}(\tau) = \hat{\alpha} \left( 1 + \hat{\beta} + \hat{\beta}^2 + \dots + \hat{\beta}^{h-1} \right) + \hat{\beta}^h y_t(\tau)$$

where the one-step ahead forecast is produced as  $\hat{y}_{t+1}(\tau) = \hat{\alpha} + \hat{\beta} y_t(\tau)$ .

- Multivariate autoregressive specification

A first-order unrestricted vector autoregressive model VAR(1) can be seen as the extension of the AR(1) model and the estimation model is:

$$y_t = A + B y_{t-1} + \varepsilon_t, \quad (2.23)$$

where  $y_t = (y_t(\tau_1), y_t(\tau_2), \dots, y_t(\tau_N))'$ . The forecast for  $h$ -step ahead horizon is obtained as:

$$\hat{y}_{t+h|t} = \hat{A} \left( I + \hat{B} + \hat{B}^2 + \dots + \hat{B}^{h-1} \right) + \hat{B}^h y_t$$

where the 1-step ahead forecast is produced as  $\hat{y}_{t+1} = \hat{A} + \hat{B} y_t$ . For this model  $N = 5$  since we take five maturities with  $\tau$  equal to three months, one, three, five and ten years; thus, vector  $y_t$  has the dimension of  $5 \times 1$ . Once we estimate a multivariable regression of  $y_t$  on  $y_{t-1}$ , the vector  $A$  has the dimension of  $1 \times 5$ , and matrix  $B$  has the dimension of  $5 \times 5$ .

## 2.7.2 Forecasting results

In this section, we describe our results on the evaluation of forecast combinations for yield curves. first, we provide information on the correlation between the time series of interest rates for the set of maturities and compute popular error metrics to evaluate the out-of-sample forecasts. Given a sample of three out-of-sample forecasts for  $h$ -period ahead forecast horizon, with  $h$  equal to one, six, and twelve

---

months, we compute the root mean squared forecast error (RMSE) and several sample autocorrelation coefficients.

In Table [A.10](#) we report the  $h$ -month-ahead out-of-sample yield curve forecasting results using the advanced Svensson specification. Panels A, B, and C compare three competitors for maturities of quarter, one, three, five, and ten years and forecasting horizons of  $h =$  one, six, and twelve months respectively. The forecast errors at  $t + h$  are defined as

$$y_{t+h}(\tau) - \hat{y}_{t+h|t}(\tau).$$

We present several descriptive statistics for the forecast errors, including mean, standard deviation, root mean squared error (RMSE), and autocorrelations at various displacements. The results for the one-month ahead forecasts are reported in Panel A of Table [A.10](#). The mean forecasting errors are higher in the case of the VAR(1) model and lower in case of the AR(1) for horizons longer than one year. For three-month maturity, the mean forecasting error using RW is equal to 0.0059, and it is less than the mean forecasting error using AR(1) model, which is equal to 0.0093. Time series of forecasting errors are highly autocorrelated in the case of VAR(1) model and less autocorrelated in case of RW and AR(1) competitors.

Results for the six-month and twelve-month ahead forecasts are reported in Panel B of Table [A.10](#) and Panel c of Table [A.10](#), respectively. Since we are keen to understand which forecast model produces the smallest mean forecasting error, the results reported in the last three tables suggest that the AR(1) model produces the best term structure of interest rates forecast. The autocorrelation coefficients have important values in the case of VAR(1) model, indicating that this forecasting model is not applicable.

Our results show that AR(1) provides more accurate forecasts, in fact, the mean forecast errors are lower compared to the two other competitors. Many other studies that consider interest rate forecasting show that it is difficult to consistently outperform RW; thus, this question suggests that further research is required on yield curve forecasting.

---

## 2.8 Conclusion

Despite the widespread use of the term structure of interest rates, it is not clear which specification is the best fit for the data. To answer this question, we compare different specifications to fit the term structure of interest rates for the eurozone four countries based on daily observations. We select a list of government bonds denominated in the euro and issued by eurozone countries. We examine four Nelson-Siegel style models, given that one of them allows us to test the Bjork-Christensen (BC) restriction  $\lambda = 2 \times \gamma$ . The main conclusion emerging from our investigation is that an extended Svensson specification has the best performance to price bonds for the four countries studied.

This result is expected because the eSv model has the largest number of parameters compared to other specifications. We work with data from four eurozone countries. Our empirical results confirm that all the models do a good job in estimating government bond prices.

The main contributions are (1) there are no major differences between the models in terms of performance fit, (2) all the functionals provide a good fit so that the Nelson-Siegel type is appropriate for use, and (3) the best specification is the eSv approach, but the Svensson model is not far from it in terms of performance fit; thus, the conclusion is to use the Svensson term structure model.

# Appendices

Table A.5: Summary of the Nominal Securities, France

ISIN	Coupon	Issue	Maturity	Term	ISIN	Coupon	Issue	Maturity	Term
FR0000570095	10	14/06/1985	27/05/2000	14.95	FR0010163543	3.5	08/02/2005	25/04/2015	10.21
FR0000570178	7.5	25/08/1986	25/07/2001	14.92	FR0107489959	2.25	22/02/2005	12/03/2007	2.05
FR0000570244	8.5	26/01/1987	25/11/2002	15.83	FR0010171975	4	28/02/2005	25/04/2055	50.15
FR0000570780	8.5	25/02/1987	26/12/2012	25.83	FR0010192997	3.75	10/05/2005	25/04/2021	15.96
FR0000570921	8.5	25/01/1989	25/10/2019	30.75	FR0107674006	2.5	21/06/2005	12/07/2010	5.06
FR0000570327	8.25	27/02/1989	27/02/2004	15	FR0010216481	3	12/07/2005	25/10/2015	10.29
FR0000570038	8.125	26/12/1989	25/05/1999	9.41	FR0108197569	2.75	22/11/2005	12/03/2008	2.3
FR0000570053	8.5	25/01/1990	28/03/2000	10.17	FR0108354806	3	24/01/2006	12/01/2011	4.97
FR0000570152	10	26/11/1990	26/02/2001	10.25	FR0010288357	3.25	07/02/2006	25/04/2016	10.21
FR0000570145	9.5	25/01/1991	25/01/2001	10	FR0108847049	3.5	20/06/2006	12/07/2011	5.06
FR0000570194	8.5	10/05/1991	15/03/2002	10.85	FR0109136137	3.5	25/07/2006	12/09/2008	2.14
FR0000570061	9.5	13/06/1991	25/04/2000	8.87	FR0010371401	4	12/09/2006	25/10/2038	32.12
FR0000571085	8.5	27/01/1992	25/04/2023	31.24	FR0010415331	3.75	09/01/2007	25/04/2017	10.29
FR0000571044	8.25	12/02/1992	25/04/2022	30.2	FR0109970386	3.75	23/01/2007	12/01/2012	4.97
FR0000570665	8.5	25/06/1992	25/10/2008	16.33	FR0110979178	4	24/04/2007	12/09/2009	2.39
FR0000570277	8.5	20/07/1992	25/04/2003	10.76	FR0010466938	4.25	09/05/2007	25/10/2023	16.46
FR0000570285	8	27/07/1992	25/04/2003	10.74	FR0110979186	4.5	26/06/2007	12/07/2012	5.05
FR0000570301	6.75	26/04/1993	25/10/2003	10.5	FR0010517417	4.25	11/09/2007	25/10/2017	10.12
FR0000570343	6	11/10/1993	25/04/2004	10.54	FR0113087466	3.75	22/01/2008	12/01/2013	4.97
FR0000570368	5.5	25/11/1993	25/04/2004	10.41	FR0010604983	4	08/04/2008	25/04/2018	10.05
FR0100059486	4.75	14/02/1994	12/04/1999	5.16	FR0113872776	3.75	20/05/2008	12/09/2010	2.31
FR0000571150	6	25/02/1994	25/10/2025	31.66	FR0114683842	4.5	22/07/2008	12/07/2013	4.97
FR0000570228	6.75	25/05/1994	25/04/2002	7.92	FR0010670737	4.25	07/10/2008	25/10/2018	10.05
FR0000570400	6.75	27/06/1994	25/10/2004	10.33	FR0116114978	2.5	27/01/2009	12/01/2014	4.96
FR0000570434	7.5	25/10/1994	25/04/2005	10.5	FR0116843519	1.5	26/05/2009	12/09/2011	2.3
FR0100059502	7	12/01/1995	12/11/1999	4.83	FR0116843535	3	23/06/2009	12/07/2014	5.05
FR0000570467	7.75	25/04/1995	25/10/2005	10.5	FR0010773192	4.5	30/06/2009	25/04/2041	31.82
FR0100059528	7.75	12/07/1995	12/04/2000	4.75	FR0010776161	3.75	07/07/2009	25/10/2019	10.3
FR0100059544	7	11/08/1995	12/10/2000	5.17	FR0117836652	2.5	26/01/2010	15/01/2015	4.97
FR0000570491	7.25	25/10/1995	25/04/2006	10.5	FR0010854182	3.5	09/02/2010	25/04/2020	10.21
FR0000570731	6.5	26/02/1996	25/04/2011	15.16	FR0010870956	4	17/03/2010	25/04/2060	50.11
FR0000570533	6.5	25/04/1996	25/10/2006	10.5	FR0118153370	0.75	25/05/2010	20/09/2012	2.32
FR0000570509	7	23/05/1996	25/04/2006	9.92	FR0118462128	2	22/06/2010	12/07/2015	5.05
FR0100059551	5.75	12/06/1996	12/03/2001	4.75	FR0010916924	3.5	06/07/2010	25/04/2026	15.8
FR0100059577	5.5	12/08/1996	12/10/2001	5.17	FR0010949651	2.5	12/10/2010	25/10/2020	10.04
FR0100059478	5	26/09/1996	16/03/1999	2.47	FR0119105809	2.25	25/01/2011	25/02/2016	5.08
FR0100059569	6	27/12/1996	16/03/2001	4.22	FR0119580019	2	24/05/2011	25/09/2013	2.34
FR0000570574	5.5	23/01/1997	25/04/2007	10.25	FR0011059088	3.25	07/06/2011	25/10/2021	10.38
FR0100059585	4.75	12/03/1997	12/03/2002	5	FR0119580050	2.5	21/06/2011	25/07/2016	5.1
FR0000570590	5.5	10/07/1997	25/10/2007	10.29	FR0011196856	3	07/02/2012	25/04/2022	10.21
FR0100059593	4.5	24/07/1997	12/07/2002	4.97	FR0120473253	1.75	21/02/2012	25/02/2017	5.01
FR0000570632	5.25	15/01/1998	25/04/2008	10.28	FR0120634490	0.75	24/04/2012	25/09/2014	2.42
FR0100059601	4.5	26/02/1998	12/07/2003	5.37	FR0120746609	1	24/07/2012	25/07/2017	5
FR0000571218	5.5	12/03/1998	25/04/2029	31.12	FR0011317783	2.75	11/09/2012	25/10/2027	15.12
FR0100059510	4	26/03/1998	12/01/2000	1.8	FR0011337880	2.25	09/10/2012	25/10/2022	10.04
FR0100059536	4	28/05/1998	12/07/2000	2.12	FR0011394345	1	22/01/2013	25/05/2018	5.34
FR0000571432	4	08/10/1998	25/04/2009	10.55	FR0011452721	0.25	26/03/2013	25/11/2015	2.67
FR0100802273	3.5	28/01/1999	12/07/2004	5.45	FR0011461037	3.25	04/04/2013	25/05/2045	32.14
FR0100877812	3	25/03/1999	12/07/2001	2.3	FR0011486067	1.75	07/05/2013	25/05/2023	10.05
FR0000186199	4	12/05/1999	25/10/2009	10.46	FR0011523257	1	25/06/2013	25/11/2018	5.42
FR0101465831	4	28/10/1999	12/01/2002	2.21	FR0011619436	2.25	12/11/2013	25/05/2024	10.53
FR0101659813	5	27/01/2000	12/07/2005	5.46	FR0011708080	1	28/01/2014	25/05/2019	5.32
FR0000186603	5.5	08/02/2000	25/04/2010	10.21	FR0011857218	0.25	23/04/2014	25/11/2016	2.59
FR0102325695	5	17/08/2000	12/01/2003	2.4	FR0011883966	2.5	06/05/2014	25/05/2030	16.05
FR0000187023	5.5	12/09/2000	25/10/2010	10.12	FR0011962398	1.75	10/06/2014	25/11/2024	10.46
FR0102626779	5	24/10/2000	12/01/2006	5.22	FR0011993179	0.5	24/06/2014	25/11/2019	5.42
FR0000187361	5	06/02/2001	25/10/2016	15.72	FR0012517027	0.5	09/02/2015	25/05/2025	10.29
FR0103230423	4.5	24/04/2001	12/07/2006	5.22	FR0012557957	0	23/02/2015	25/05/2020	5.25
FR0000187635	5.75	12/06/2001	25/10/2032	31.37	FR0012634558	0	23/03/2015	25/02/2018	2.93
FR0000187874	5	11/09/2001	25/10/2011	10.12	FR0012938116	1	07/09/2015	25/11/2025	10.22
FR0103536092	4	25/09/2001	12/01/2004	2.3	FR0012968337	0.25	21/09/2015	25/11/2020	5.18
FR0103840098	3.75	27/11/2001	12/01/2007	5.13	FR0012993103	1.5	05/10/2015	25/05/2031	15.64
FR0000188328	5	12/03/2002	25/04/2012	10.12	FR0013101466	0	25/01/2016	25/02/2019	3.09
FR0104446556	4.75	16/05/2002	12/07/2007	5.16	FR0013131877	0.5	07/03/2016	25/05/2026	10.21
FR0000188690	4.75	10/09/2002	25/10/2012	10.12	FR0013154028	1.75	19/04/2016	25/05/2066	50.1

FR0104756962	3.5	24/09/2002	12/01/2005	2.3	FR0013154044	1.25	19/04/2016	25/05/2036	20.1
FR0105427795	3.5	28/01/2003	12/01/2008	4.96	FR0013157096	0	25/04/2016	25/05/2021	5.08
FR0000188989	4	11/03/2003	25/04/2013	10.12	FR0013200813	0.25	05/09/2016	25/11/2026	10.22
FR0000474413	3.75	02/05/2003	25/04/2010	6.98	FR0013219177	0	21/11/2016	25/05/2022	5.51
FR0000189151	4.25	10/06/2003	25/04/2019	15.87	FR0013232485	0	23/01/2017	25/02/2020	3.09
FR0105760112	3	24/06/2003	12/07/2008	5.05	FR0013234333	1.75	31/01/2017	25/06/2039	22.4
FR0010011130	4	09/09/2003	25/10/2013	10.13	FR0013250560	1	10/04/2017	25/05/2027	10.12
FR0106589437	3.5	22/01/2004	12/01/2009	4.97	FR0013257524	2	25/05/2017	25/05/2048	31
FR0010061242	4	09/03/2004	25/04/2014	10.13	FR0013283686	0	25/09/2017	25/03/2023	5.49
FR0106589445	2.25	23/03/2004	12/03/2006	1.97	FR0013286192	0.75	09/10/2017	25/05/2028	10.63
FR0010070060	4.75	06/04/2004	25/04/2035	31.05	FR0013311016	0	22/01/2018	25/02/2021	3.09
FR0106841887	3.5	22/06/2004	12/07/2009	5.05	FR0013313582	1.25	05/02/2018	25/05/2034	16.3
FR0010112052	4	07/09/2004	25/10/2014	10.13	FR0013341682	0.75	11/06/2018	25/11/2028	10.46
FR0107369672	3	23/11/2004	12/01/2010	5.14	FR0013344751	0	25/06/2018	25/03/2024	5.75

This table shows the sample of the nominal securities issued by France from 1985 to 2018. Column "ISIN" refers to the ISIN number and Column "Coupon" to the coupon rate of the security in percent. Column "Issue" reports the issue date of the bond and Column "Maturity" provides the expiration date. Column "Term" specifies the term-to-maturity at issuance of the security. Source: Bloomberg.

Table A.6: Summary of the Nominal Securities, Germany

ISIN	Coupon	Issue	Maturity	Term	ISIN	Coupon	Issue	Maturity	Term
DE0001134468	6	20/06/1986	20/06/2016	30	DE0001135267	3.75	26/11/2004	04/01/2015	10.11
DE0001134492	5.625	20/09/1986	20/09/2016	30	DE0001137081	2.25	10/12/2004	15/12/2006	2.01
DE0001134708	7	22/02/1989	22/02/1999	10	DE0001135275	4	28/01/2005	04/01/2037	31.93
DE0001134716	7	20/04/1989	20/04/1999	10	DE0001137099	2.5	18/03/2005	23/03/2007	2.01
DE0001134724	6.75	07/07/1989	21/06/1999	9.95	DE0001141463	3.25	01/04/2005	09/04/2010	5.02
DE0001134732	7	12/09/1989	20/09/1999	10.02	DE0001135283	3.25	20/05/2005	04/07/2015	10.12
DE0001134740	7	16/10/1989	20/10/1999	10.01	DE0001137107	2	17/06/2005	15/06/2007	1.99
DE0001134757	7.125	09/11/1989	20/12/1999	10.11	DE0001137115	2.25	16/09/2005	14/09/2007	1.99
DE0001134765	7.25	04/01/1990	20/01/2000	10.04	DE0001141471	2.5	23/09/2005	08/10/2010	5.04
DE0001134773	7.75	09/02/1990	21/02/2000	10.03	DE0001135291	3.5	25/11/2005	04/01/2016	10.11
DE0001134799	8.75	22/05/1990	22/05/2000	10	DE0001137123	2.75	16/12/2005	14/12/2007	1.99
DE0001030005	8.75	10/07/1990	20/07/2000	10.03	DE0001137131	3	10/03/2006	14/03/2008	2.01
DE0001134807	8.5	03/08/1990	21/08/2000	10.05	DE0001141489	3.5	24/03/2006	08/04/2011	5.04
DE0001134815	9	11/10/1990	20/10/2000	10.03	DE0001135309	4	19/05/2006	04/07/2016	10.13
DE0001134823	8.875	07/12/1990	20/12/2000	10.04	DE0001137149	3.25	23/06/2006	13/06/2008	1.97
DE0001134831	9	03/01/1991	22/01/2001	10.05	DE0001137156	3.5	15/09/2006	12/09/2008	1.99
DE0001030013	8.5	12/02/1991	20/02/2001	10.02	DE0001141497	3.5	29/09/2006	14/10/2011	5.04
DE0001134849	8.375	13/05/1991	21/05/2001	10.02	DE0001135317	3.75	17/11/2006	04/01/2017	10.13
DE0001030021	8.75	05/08/1991	20/08/2001	10.04	DE0001137164	3.75	15/12/2006	12/12/2008	1.99
DE0001134856	8.25	11/10/1991	20/09/2001	9.94	DE0001135325	4.25	26/01/2007	04/07/2039	32.44
DE0001030039	8	07/01/1992	21/01/2002	10.04	DE0001137172	3.75	16/03/2007	13/03/2009	1.99
DE0001026508	8	07/05/1992	02/05/2002	9.98	DE0001141505	4	30/03/2007	13/04/2012	5.04
DE0001134864	8	14/07/1992	22/07/2002	10.02	DE0001135333	4.25	25/05/2007	04/07/2017	10.11
DE0001134872	7.25	09/10/1992	21/10/2002	10.03	DE0001137180	4.5	15/06/2007	12/06/2009	1.99
DE0001134880	7.125	05/01/1993	20/12/2002	9.95	DE0001137198	4	14/09/2007	11/09/2009	1.99
DE0001134898	6.75	07/05/1993	22/04/2003	9.96	DE0001141513	4.25	28/09/2007	12/10/2012	5.04
DE0001026516	6.75	21/05/1993	02/05/2003	9.95	DE0001135341	4	16/11/2007	04/01/2018	10.14
DE0001134906	6.5	06/08/1993	15/07/2003	9.94	DE0001137206	4	14/12/2007	11/12/2009	1.99
DE0001134914	6	12/10/1993	15/09/2003	9.92	DE0001137214	3	14/03/2008	12/03/2010	1.99
DE0001134922	6.25	04/01/1994	04/01/2024	30	DE0001141521	3.5	28/03/2008	12/04/2013	5.04
DE0001141109	5.375	15/03/1994	22/02/1999	4.94	DE0001135358	4.25	30/05/2008	04/07/2018	10.09
DE0001141117	6.125	20/06/1994	20/05/1999	4.91	DE0001137222	4.75	13/06/2008	11/06/2010	1.99
DE0001134930	6.75	22/07/1994	15/07/2004	9.98	DE0001135366	4.75	25/07/2008	04/07/2040	31.94
DE0001141125	6.75	15/09/1994	15/09/1999	5	DE0001137230	4	12/09/2008	10/09/2010	1.99
DE0001134955	7.5	11/11/1994	11/11/2004	10	DE0001141539	4	26/09/2008	11/10/2013	5.04
DE0001134963	7.375	03/01/1995	03/01/2005	10	DE0001135374	3.75	14/11/2008	04/01/2019	10.14
DE0001141133	7	13/01/1995	13/01/2000	5	DE0001137248	2.25	12/12/2008	10/12/2010	1.99
DE0001136737	6.875	24/02/1995	24/02/1999	4	DE0001137255	1.25	13/03/2009	11/03/2011	1.99
DE0001141141	6.5	15/03/1995	15/03/2000	5	DE0001141547	2.25	27/03/2009	11/04/2014	5.04
DE0001134971	6.875	12/05/1995	12/05/2005	10	DE0001135382	3.5	22/05/2009	04/07/2019	10.12
DE0001141158	5.875	15/05/1995	15/05/2000	5	DE0001137263	1.5	29/05/2009	10/06/2011	2.03
DE0001136745	5.75	28/05/1995	28/05/1999	4	DE0001137271	1.25	11/09/2009	16/09/2011	2.01
DE0001141166	5.75	22/08/1995	22/08/2000	5	DE0001141554	2.5	25/09/2009	10/10/2014	5.04
DE0001134989	6.5	20/10/1995	14/10/2005	9.98	DE0001135390	3.25	13/11/2009	04/01/2020	10.14

DE0001141174	5.125	21/11/1995	21/11/2000	5	DE0001137289	1.25	20/11/2009	16/12/2011	2.07
DE0001134997	6	08/01/1996	05/01/2006	9.99	DE0001141562	2.5	15/01/2010	27/02/2015	5.12
DE0001135002	6	16/02/1996	16/02/2006	10	DE0001137297	1	19/02/2010	16/03/2012	2.07
DE0001141182	5.25	21/02/1996	21/02/2001	5	DE0001141570	2.25	16/04/2010	10/04/2015	4.98
DE0001135010	6.25	26/04/1996	26/04/2006	10	DE0001135408	3	30/04/2010	04/07/2020	10.18
DE0001141190	5	21/05/1996	21/05/2001	5	DE0001137305	0.5	14/05/2010	15/06/2012	2.09
DE0001141208	5	20/08/1996	20/08/2001	5	DE0001135432	3.25	23/07/2010	04/07/2042	31.95
DE0001141216	4.75	20/11/1996	20/11/2001	5	DE0001137313	0.75	13/08/2010	14/09/2012	2.09
DE0001135028	6	10/01/1997	04/01/2007	9.98	DE0001135416	2.25	20/08/2010	04/09/2020	10.04
DE0001141224	4.5	22/02/1997	22/02/2002	5	DE0001141588	1.75	24/09/2010	09/10/2015	5.04
DE0001136778	3.75	20/03/1997	19/03/1999	2	DE0001137321	1	12/11/2010	14/12/2012	2.09
DE0001135036	6	25/04/1997	04/07/2007	10.19	DE0001135424	2.5	26/11/2010	04/01/2021	10.11
DE0001141232	4.5	17/05/1997	17/05/2002	5	DE0001141596	2	14/01/2011	26/02/2016	5.12
DE0001136786	3.5	20/06/1997	18/06/1999	1.99	DE0001137339	1.5	25/02/2011	15/03/2013	2.05
DE0001135044	6.5	04/07/1997	04/07/2027	30	DE0001141604	2.75	26/04/2011	08/04/2016	4.95
DE0001141240	4.5	19/08/1997	19/08/2002	5	DE0001135440	3.25	29/04/2011	04/07/2021	10.18
DE0001136794	4	26/09/1997	17/09/1999	1.97	DE0001137347	1.75	13/05/2011	14/06/2013	2.09
DE0001141257	5	12/11/1997	12/11/2002	5	DE0001137354	0.75	19/08/2011	13/09/2013	2.07
DE0001136802	4.25	19/12/1997	17/12/1999	1.99	DE0001135457	2.25	26/08/2011	04/09/2021	10.03
DE0001135051	5.25	09/01/1998	04/01/2008	9.98	DE0001141612	1.25	30/09/2011	14/10/2016	5.04
DE0001135069	5.625	23/01/1998	04/01/2028	29.95	DE0001137362	0.25	18/11/2011	13/12/2013	2.07
DE0001141265	4.5	18/02/1998	18/02/2003	5	DE0001135465	2	25/11/2011	04/01/2022	10.11
DE0001136810	4	20/03/1998	17/03/2000	1.99	DE0001141620	0.75	13/01/2012	24/02/2017	5.12
DE0001141273	4.5	19/05/1998	19/05/2003	5	DE0001137370	0.25	24/02/2012	14/03/2014	2.05
DE0001136828	4	26/06/1998	16/06/2000	1.97	DE0001135473	1.75	13/04/2012	04/07/2022	10.22
DE0001135077	4.75	10/07/1998	04/07/2008	9.98	DE0001135481	2.5	27/04/2012	04/07/2044	32.19
DE0001141281	3.75	26/08/1998	26/08/2003	5	DE0001141638	0.5	11/05/2012	07/04/2017	4.91
DE0001136836	3.25	18/09/1998	15/09/2000	1.99	DE0001137388	0	25/05/2012	13/06/2014	2.05
DE0001135085	4.75	09/10/1998	04/07/2028	29.74	DE0001137396	0	24/08/2012	12/09/2014	2.05
DE0001135093	4.125	30/10/1998	04/07/2008	9.68	DE0001135499	1.5	07/09/2012	04/09/2022	9.99
DE0001141299	3.5	11/11/1998	11/11/2003	5	DE0001141646	0.5	14/09/2012	13/10/2017	5.08
DE0001136844	3	18/12/1998	15/12/2000	1.99	DE0001137404	0	16/11/2012	12/12/2014	2.07
DE0001135101	3.75	08/01/1999	04/01/2009	9.99	DE0001141653	0.5	11/01/2013	23/02/2018	5.12
DE0001136851	3	19/03/1999	16/03/2001	1.99	DE0001102309	1.5	18/01/2013	15/02/2023	10.08
DE0001135119	4	26/03/1999	04/07/2009	10.28	DE0001137412	0.25	15/02/2013	13/03/2015	2.07
DE0001141315	3.25	19/05/1999	19/05/2004	5	DE0001141661	0.25	10/05/2013	13/04/2018	4.93
DE0001141307	3.25	21/05/1999	17/02/2004	4.74	DE0001137420	0	17/05/2013	12/06/2015	2.07
DE0001136869	3	18/06/1999	15/06/2001	1.99	DE0001102317	1.5	24/05/2013	15/05/2023	9.97
DE0001135127	4.5	04/07/1999	04/07/2009	10	DE0001137438	0.25	23/08/2013	11/09/2015	2.05
DE0001141331	4.25	25/08/1999	26/11/2004	5.26	DE0001141679	1	06/09/2013	12/10/2018	5.1
DE0001141323	4.125	27/08/1999	27/08/2004	5	DE0001102325	2	13/09/2013	15/08/2023	9.92
DE0001136877	3.5	17/09/1999	14/09/2001	1.99	DE0001137446	0	15/11/2013	11/12/2015	2.07
DE0001135135	5.375	22/10/1999	04/01/2010	10.2	DE0001141687	1	17/01/2014	22/02/2019	5.1
DE0001141349	4.25	17/11/1999	18/02/2005	5.26	DE0001102333	1.75	31/01/2014	15/02/2024	10.04
DE0001136885	4	17/12/1999	14/12/2001	1.99	DE0001137453	0.25	14/02/2014	11/03/2016	2.07
DE0001135143	6.25	21/01/2000	04/01/2030	29.95	DE0001102341	2.5	28/02/2014	15/08/2046	32.46
DE0001141356	5	16/02/2000	20/05/2005	5.26	DE0001141695	0.5	09/05/2014	12/04/2019	4.93
DE0001136893	4.5	17/03/2000	15/03/2002	1.99	DE0001137461	0.25	16/05/2014	10/06/2016	2.07
DE0001135150	5.25	05/05/2000	04/07/2010	10.16	DE0001102358	1.5	23/05/2014	15/05/2024	9.98
DE0001141364	5	17/05/2000	19/08/2005	5.26	DE0001137479	0	22/08/2014	16/09/2016	2.07
DE0001136901	5	16/06/2000	14/06/2002	1.99	DE0001141703	0.25	05/09/2014	11/10/2019	5.1
DE0001141372	5	16/08/2000	17/02/2006	5.51	DE0001102366	1	12/09/2014	15/08/2024	9.92
DE0001136919	5	15/09/2000	13/09/2002	1.99	DE0001137487	0	14/11/2014	16/12/2016	2.09
DE0001135168	5.25	20/10/2000	04/01/2011	10.21	DE0001102374	0.5	16/01/2015	15/02/2025	10.08
DE0001135176	5.5	27/10/2000	04/01/2031	30.19	DE0001141711	0	23/01/2015	17/04/2020	5.23
DE0001136927	4.75	13/12/2000	13/12/2002	2	DE0001137495	0	13/02/2015	10/03/2017	2.07
DE0001141380	4.5	14/02/2001	18/08/2006	5.51	DE0001104602	0	08/05/2015	16/06/2017	2.11
DE0001136935	4.25	16/03/2001	14/03/2003	1.99	DE0001141729	0.25	03/07/2015	16/10/2020	5.29
DE0001135184	5	25/05/2001	04/07/2011	10.11	DE0001102382	1	17/07/2015	15/08/2025	10.08
DE0001136943	4.25	15/06/2001	13/06/2003	1.99	DE0001104610	0	21/08/2015	15/09/2017	2.07
DE0001136950	3.75	14/09/2001	12/09/2003	1.99	DE0001104628	0	20/11/2015	15/12/2017	2.07
DE0001136968	3.5	14/12/2001	12/12/2003	1.99	DE0001102390	0.5	15/01/2016	15/02/2026	10.09
DE0001135192	5	04/01/2002	04/01/2012	10	DE0001141737	0	05/02/2016	09/04/2021	5.17
DE0001141398	4	22/02/2002	16/02/2007	4.98	DE0001104636	0	12/02/2016	16/03/2018	2.09
DE0001136976	4.25	15/03/2002	12/03/2004	1.99	DE0001104644	0	13/05/2016	15/06/2018	2.09
DE0001136984	4	28/06/2002	25/06/2004	1.99	DE0001102408	0	15/07/2016	15/08/2026	10.08
DE0001135200	5	05/07/2002	04/07/2012	10	DE0001141745	0	22/07/2016	08/10/2021	5.21

DE0001141406	4.5	16/08/2002	17/08/2007	5	DE0001104651	0	05/08/2016	14/09/2018	2.11
DE0001141414	4.25	19/08/2002	15/02/2008	5.49	DE0001104669	0	11/11/2016	14/12/2018	2.09
DE0001136992	3.25	27/09/2002	24/09/2004	1.99	DE0001102416	0.25	13/01/2017	15/02/2027	10.09
DE0001137008	3	13/12/2002	10/12/2004	1.99	DE0001141752	0	03/02/2017	08/04/2022	5.17
DE0001135218	4.5	10/01/2003	04/01/2013	9.98	DE0001104677	0	02/03/2017	15/03/2019	2.03
DE0001135226	4.75	31/01/2003	04/07/2034	31.42	DE0001104685	0	25/05/2017	14/06/2019	2.05
DE0001137016	2.5	28/03/2003	18/03/2005	1.97	DE0001141760	0	07/07/2017	07/10/2022	5.25
DE0001141422	3	16/05/2003	11/04/2008	4.91	DE0001102424	0.5	14/07/2017	15/08/2027	10.09
DE0001137024	2	27/06/2003	17/06/2005	1.97	DE0001104693	0	31/08/2017	13/09/2019	2.03
DE0001135234	3.75	04/07/2003	04/07/2013	10	DE0001102432	1.25	22/09/2017	15/08/2048	30.9
DE0001137032	2.5	26/09/2003	16/09/2005	1.97	DE0001104701	0	16/11/2017	13/12/2019	2.07
DE0001141430	3.5	10/10/2003	10/10/2008	5	DE0001102440	0.5	12/01/2018	15/02/2028	10.09
DE0001135242	4.25	31/10/2003	04/01/2014	10.18	DE0001141778	0	02/02/2018	14/04/2023	5.19
DE0001137040	2.75	12/12/2003	16/12/2005	2.01	DE0001104719	0	22/02/2018	13/03/2020	2.05
DE0001141448	3.25	13/02/2004	17/04/2009	5.17	DE0001104727	0	25/05/2018	12/06/2020	2.05
DE0001137057	2	26/03/2004	10/03/2006	1.95	DE0001102457	0.25	13/07/2018	15/08/2028	10.09
DE0001135259	4.25	28/05/2004	04/07/2014	10.1	DE0001141786	0	27/07/2018	13/10/2023	5.21
DE0001137065	2.75	25/06/2004	23/06/2006	1.99	DE0001104735	0	23/08/2018	11/09/2020	2.05
DE0001141455	3.5	27/08/2004	09/10/2009	5.12	DE0001104743	0	15/11/2018	11/12/2020	2.07
DE0001137073	2.5	24/09/2004	22/09/2006	1.99					

This table shows the sample of the nominal securities issued by Germany from 1986 to 2018. Column “ISIN” refers to the ISIN number and Column “Coupon” to the coupon rate of the security in percent. Column “Issue” reports the issue date of the bond and Column “Maturity” provides the expiration date. Column “Term” specifies the term-to-maturity at issuance of the security. Source: Bloomberg.

Table A.7: Summary of the Nominal Securities, Italy

ISIN	Coupon	Issue	Maturity	Term	ISIN	Coupon	Issue	Maturity	Term
XS0015168338	10.75	18/04/1990	18/04/2000	10	IT0003877708	2.5	01/07/2005	15/06/2008	2.96
IT0000126778	12.5	05/03/1991	01/03/2001	9.99	IT0003934657	4	19/10/2005	01/02/2037	31.29
IT0000126794	12	03/06/1991	01/06/2001	10	IT0004009673	3.75	01/02/2006	01/08/2021	15.5
IT0000126836	12	04/09/1991	01/09/2001	9.99	IT0004008121	3	01/02/2006	01/02/2009	3
IT0000126877	12	07/01/1992	01/01/2002	9.98	IT0004019581	3.75	01/03/2006	01/08/2016	10.42
IT0000126885	12	17/01/1992	17/01/1999	7	IT0004026297	3.5	15/03/2006	15/03/2011	5
IT0000366051	12	04/05/1992	01/05/2002	9.99	XS0247541971	4.425	28/03/2006	28/03/2036	30
IT0000366077	12	18/05/1992	18/05/1999	7	IT0004085244	3.75	03/07/2006	15/06/2009	2.95
IT0000366143	12	03/09/1992	01/09/2002	9.99	IT0004112816	3.75	18/09/2006	15/09/2011	4.99
IT0000366234	12	08/01/1993	01/01/2003	9.98	IT0004164775	4	02/01/2007	01/02/2017	10.08
IT0000366325	11.5	03/03/1993	01/03/2003	9.99	IT0004196918	4	01/03/2007	01/03/2010	3
IT0000366424	11	04/06/1993	01/06/2003	9.99	IT0004235559	4.49	05/04/2007	05/04/2027	20
IT0000366515	10	04/08/1993	01/08/2003	9.99	IT0004220627	4	17/04/2007	15/04/2012	5
IT0000366606	9	05/10/1993	01/10/2003	9.99	IT0004254352	4.5	01/08/2007	01/08/2010	3
IT0000366655	9	18/11/1993	01/11/2023	29.95	IT0004273493	4.5	03/09/2007	01/02/2018	10.41
IT0000366721	8.5	22/12/1993	22/12/2023	30	IT0004284334	4.25	15/10/2007	15/10/2012	5
IT0000366713	8.5	22/12/1993	22/12/2003	10	IT0004286966	5	23/10/2007	01/08/2039	31.77
IT0000366762	8.5	05/01/1994	01/01/2004	9.99	IT0004332521	3.75	03/03/2008	01/02/2011	2.92
IT0000366846	8.5	05/04/1994	01/04/2004	9.99	IT0004356843	4.75	16/04/2008	01/08/2023	15.29
IT0000366929	8.5	02/08/1994	01/08/1999	5	IT0004361041	4.5	02/05/2008	01/08/2018	10.25
IT0000366937	8.5	03/08/1994	01/08/2004	10	IT0004365554	4.25	16/05/2008	15/04/2013	4.91
IT0000367083	9.5	03/01/1995	01/12/1999	4.91	IT0004404973	4.25	01/09/2008	01/09/2011	3
IT0000367091	9.5	04/01/1995	01/01/2005	9.99	IT0004423957	4.5	03/11/2008	01/03/2019	10.32
IT0000367174	10.5	02/05/1995	01/04/2005	9.92	IT0004448863	3.75	16/01/2009	15/12/2013	4.91
IT0000367166	10.5	03/05/1995	01/04/2000	4.91	IT0004467483	3	02/03/2009	01/03/2012	3
IT0000367190	8.25	24/05/1995	24/05/2000	5	IT0004489610	4.25	04/05/2009	01/09/2019	10.33
IT0000367281	10.5	02/08/1995	15/07/2000	4.95	XS0431307221	3	29/05/2009	29/11/2013	4.5
IT0000367315	10.5	01/09/1995	01/09/2005	10	IT0004505076	3.5	15/06/2009	01/06/2014	4.96
IT0000367414	10.5	03/11/1995	01/11/2000	5	IT0004508971	2.5	01/07/2009	01/07/2012	3
IT0000367497	9.5	01/02/1996	01/02/2006	10	IT0004513641	5	15/07/2009	01/03/2025	15.63
IT0000367471	9.5	02/02/1996	01/02/1999	3	IT0004532559	5	16/09/2009	01/09/2040	30.96
IT0000367489	9.5	02/02/1996	01/02/2001	5	IT0004536949	4.25	01/10/2009	01/03/2020	10.41
IT0000367612	9.5	03/05/1996	01/05/2001	4.99	IT0004564636	2	04/01/2010	15/12/2012	2.95
IT0000367604	9.5	03/05/1996	15/04/1999	2.95	IT0004568272	3	15/01/2010	15/04/2015	5.25
IT0000367687	8.75	01/07/1996	01/07/2006	10	IT0004594930	4	01/04/2010	01/09/2020	10.42
IT0000367679	8.25	02/07/1996	01/07/2001	5	IT0004612179	2	01/06/2010	01/06/2013	3
IT0000367661	8.25	02/07/1996	01/07/1999	3	XS0515753183	4.85	11/06/2010	11/06/2060	50
IT0000367786	7.75	02/10/1996	15/09/2001	4.95	IT0004615917	3	15/06/2010	15/06/2015	5

IT0000367778	7.5	02/10/1996	01/10/1999	3	IT0004634132	3.75	01/09/2010	01/03/2021	10.5
IT0000367810	7.75	04/11/1996	01/11/2006	9.99	IT0004644735	4.5	29/09/2010	01/03/2026	15.42
IT0000367851	6	03/01/1997	01/01/2000	2.99	IT0004653108	2.25	01/11/2010	01/11/2013	3
IT0000367844	6.25	03/01/1997	01/01/2002	4.99	IT0004656275	3	16/11/2010	01/11/2015	4.96
IT0001086567	7.25	03/02/1997	01/11/2026	29.74	XS0572691979	4.45	23/12/2010	23/12/2021	11
IT0001086559	6.75	03/02/1997	01/02/2007	9.99	XS0595269365	4.45	24/02/2011	24/08/2020	9.5
IT0001092367	6	18/02/1997	15/02/2000	2.99	IT0004695075	4.75	01/03/2011	01/09/2021	10.51
IT0001096491	6.25	04/03/1997	01/03/2002	4.99	IT0004707995	3	01/04/2011	01/04/2014	3
XS0075039114	6	02/04/1997	02/04/2004	7	IT0004712748	3.75	18/04/2011	15/04/2016	4.99
IT0001119509	6	20/05/1997	15/05/2000	2.99	IT0004750409	4.25	01/08/2011	01/07/2014	2.92
NL0000121739	6.125	29/05/1997	29/05/2012	15	IT0004759673	5	01/09/2011	01/03/2022	10.5
IT0001124251	6.25	03/06/1997	15/05/2002	4.95	IT0004761950	4.75	15/09/2011	15/09/2016	5
IT0001132098	6.75	01/07/1997	01/07/2007	10	IT0004780380	6	01/12/2011	15/11/2014	2.96
FR0000572026	5.875	02/07/1997	02/07/2007	10	IT0004793474	4.75	01/02/2012	01/05/2017	5.25
DE0001937209	5.75	10/07/1997	10/07/2007	10	IT0004801541	5.5	01/03/2012	01/09/2022	10.5
IT0001156394	5.75	18/09/1997	15/09/2002	4.99	IT0004805070	2.5	16/03/2012	01/03/2015	2.96
IT0001156386	5.5	18/09/1997	15/09/2000	2.99	IT0004820426	4.75	01/06/2012	01/06/2017	5
IT0001174611	6.5	01/11/1997	01/11/2027	30	IT0004840788	4.5	17/07/2012	15/07/2015	2.99
IT0001170007	6	03/11/1997	01/11/2007	9.99	IT0004848831	5.5	03/09/2012	01/11/2022	10.16
IT0001195491	5	20/01/1998	15/01/2001	2.99	IT0004867070	3.5	01/11/2012	01/11/2017	5
IT0001206066	5	18/02/1998	15/02/2003	4.99	IT0004880990	2.75	17/12/2012	01/12/2015	2.95
IT0001220851	4.5	17/04/1998	15/04/2001	3	IT0004889033	4.75	22/01/2013	01/09/2028	15.61
IT0001224309	5	04/05/1998	01/05/2008	9.99	IT0004898034	4.5	01/03/2013	01/05/2023	10.17
IT0001224283	4.75	05/05/1998	01/05/2003	4.99	IT0004907843	3.5	02/04/2013	01/06/2018	5.16
IT0001239588	4.5	02/07/1998	01/07/2001	3	IT0004917792	2.25	15/04/2013	15/05/2016	3.08
IT0001244638	4.5	17/07/1998	15/07/2003	4.99	IT0004923998	4.75	22/05/2013	01/09/2044	31.28
XS0089766942	8.625	10/08/1998	10/08/2001	3	XS0936805612	4.75	28/05/2013	28/05/2063	50
IT0001260808	4	19/09/1998	01/09/2001	2.95	IT0004953417	4.5	01/08/2013	01/03/2024	10.58
IT0001263844	4	02/10/1998	01/10/2003	5	IT0004957574	3.5	02/09/2013	01/12/2018	5.25
XS0091852243	4	26/10/1998	26/10/2005	7	XS0970703772	5.05	11/09/2013	11/09/2053	40
IT0001273363	4.5	02/11/1998	01/05/2009	10.49	IT0004960826	2.75	16/09/2013	15/11/2016	3.16
IT0001278511	5.25	18/11/1998	01/11/2029	30.95	IT0004966401	3.75	16/10/2013	01/05/2021	7.54
IT0001278503	3.5	18/11/1998	01/11/2001	2.95	IT0004987191	1.5	15/01/2014	15/12/2016	2.92
XS0093227014	7	04/01/1999	04/01/2002	3	IT0004992308	2.5	03/02/2014	01/05/2019	5.24
IT0001305454	3.25	02/02/1999	01/02/2004	5	IT0005001547	3.75	03/03/2014	01/09/2024	10.5
IT0001310363	3	17/02/1999	15/02/2002	3	IT0005023459	1.15	15/05/2014	15/05/2017	3
IT0001326567	3.25	16/04/1999	15/04/2004	5	IT0005024234	3.5	21/05/2014	01/03/2030	15.78
IT0001326575	3	16/04/1999	15/04/2002	3	IT0005028003	2.15	16/06/2014	15/12/2021	7.5
IT0001338612	4.25	01/06/1999	01/11/2009	10.42	IT0005030504	1.5	01/07/2014	01/08/2019	5.08
IT0001344057	3	15/06/1999	15/06/2002	3	IT0005045270	2.5	01/09/2014	01/12/2024	10.25
IT0001352803	4	16/07/1999	15/07/2004	5	IT0005058463	0.75	15/10/2014	15/01/2018	3.25
IT0001376141	3.75	04/10/1999	01/09/2002	2.91	IT0005069395	1.05	01/12/2014	01/12/2019	5
IT0001413936	4.75	05/01/2000	01/07/2005	5.49	IT0005083057	3.25	22/01/2015	01/09/2046	31.61
IT0001423844	4.5	02/02/2000	15/01/2003	2.95	XS1180157544	1.862	02/02/2015	02/02/2028	13
XS0108632018	5.25	10/03/2000	10/03/2005	5	XS1180459395	2.192	02/02/2015	02/02/2032	17
IT0001444378	6	17/03/2000	01/05/2031	31.12	IT0005086886	1.35	16/02/2015	15/04/2022	7.16
IT0001448619	5.5	03/04/2000	01/11/2010	10.58	IT0005090318	1.5	02/03/2015	01/06/2025	10.25
IT0001453262	4.75	18/04/2000	15/04/2003	2.99	XS1199008670	2	05/03/2015	05/09/2032	17.51
IT0001477386	5	16/06/2000	15/06/2003	3	XS1199014306	1.771	05/03/2015	05/03/2029	14
XS0112900351	5.25	19/06/2000	19/06/2003	3	IT0005094088	1.65	24/03/2015	01/03/2032	16.94
IT0001488102	5.25	14/07/2000	15/12/2005	5.42	IT0005106049	0.25	15/04/2015	15/05/2018	3.08
IT0003023550	5.25	18/10/2000	15/10/2003	2.99	IT0005107708	0.7	04/05/2015	01/05/2020	4.99
XS0123431677	4.75	23/01/2001	23/01/2006	5	XS1227831382	1.666	06/05/2015	06/05/2028	13
IT0003074991	5	16/02/2001	15/02/2004	3	XS1236858657	2.127	22/05/2015	22/05/2027	12
IT0003080402	5.25	01/03/2001	01/08/2011	10.42	IT0005127086	2	01/09/2015	01/12/2025	10.25
IT0003088959	4.75	16/03/2001	15/03/2006	5	IT0005135840	1.45	15/09/2015	15/09/2022	7
IT0003101992	4.5	17/04/2001	15/03/2004	2.91	IT0005139099	0.3	15/10/2015	15/10/2018	3
IT0003141741	4.5	02/07/2001	01/07/2004	3	IT0005142143	0.65	02/11/2015	01/11/2020	5
XS0133144898	5.75	25/07/2001	25/07/2016	15	IT0005162828	2.7	09/02/2016	01/03/2047	31.06
IT0003171946	4.5	17/09/2001	01/03/2007	5.45	IT0005170839	1.6	01/03/2016	01/06/2026	10.25
IT0003178446	4	01/10/2001	01/10/2004	3	IT0005172322	0.95	15/03/2016	15/03/2023	7
IT0003190912	5	01/11/2001	01/02/2012	10.25	IT0005175598	0.45	01/04/2016	01/06/2021	5.17
IT0003231146	4	17/01/2002	15/07/2005	3.49	IT0005177271	0.1	15/04/2016	15/04/2019	3
IT0003242747	5.25	14/02/2002	01/08/2017	15.46	IT0005177909	2.25	26/04/2016	01/09/2036	20.35
IT0003248512	4	01/03/2002	01/03/2005	3	XS1413812881	1.913	18/05/2016	18/05/2029	13
IT0003256820	5.75	18/03/2002	01/02/2033	30.88	XS1435990863	1.901	22/06/2016	22/06/2031	15
IT0003271019	5	15/04/2002	15/10/2007	5.5	IT0005210650	1.25	01/08/2016	01/12/2026	10.33

IT0003288864	4.5	15/05/2002	15/05/2005	3	IT0005215246	0.65	15/09/2016	15/10/2023	7.08
IT0003357982	4.75	02/09/2002	01/02/2013	10.42	IT0005216491	0.35	03/10/2016	01/11/2021	5.08
IT0003364566	3.5	16/09/2002	15/09/2005	3	IT0005217390	2.8	11/10/2016	01/03/2067	50.38
IT0003413892	3.5	15/01/2003	15/01/2008	5	XS1505666815	1.448	17/10/2016	17/04/2027	10.5
IT0003424485	2.75	03/02/2003	01/02/2006	3	IT0005217929	0.05	17/10/2016	15/10/2019	2.99
IT0003472336	4.25	02/05/2003	01/08/2013	10.25	IT0005240350	2.45	25/01/2017	01/09/2033	16.6
IT0003477111	2.75	16/05/2003	15/05/2006	3	IT0005240830	2.2	01/02/2017	01/06/2027	10.33
IT0003493258	4.25	25/06/2003	01/02/2019	15.61	IT0005244782	1.2	01/03/2017	01/04/2022	5.08
IT0003522254	2.75	01/09/2003	01/09/2006	3	IT0005246340	1.85	15/03/2017	15/05/2024	7.17
IT0003532097	3.5	17/09/2003	15/09/2008	5	IT0005250946	0.35	18/04/2017	15/06/2020	3.16
IT0003535157	5	24/09/2003	01/08/2034	30.85	IT0005273013	3.45	14/06/2017	01/03/2048	30.71
IT0003611156	2.75	16/01/2004	15/01/2007	3	IT0005274805	2.05	04/07/2017	01/08/2027	10.08
IT0003618383	4.25	02/02/2004	01/08/2014	10.49	IT0005277444	0.9	01/08/2017	01/08/2022	5
IT0003621460	5.125	06/02/2004	31/07/2024	20.48	IT0005282527	1.45	15/09/2017	15/11/2024	7.17
IT0003621445	4.5	06/02/2004	31/07/2014	10.48	IT0005285041	0.2	16/10/2017	15/10/2020	3
IT0003644769	4.5	24/03/2004	01/02/2020	15.86	IT0005321325	2.95	17/01/2018	01/09/2038	20.62
IT0003652077	3	15/04/2004	15/04/2009	5	IT0005323032	2	01/02/2018	01/02/2028	10
IT0003674238	3	01/06/2004	01/06/2007	3	IT0005325946	0.95	01/03/2018	01/03/2023	5
IT0003685093	5.2	06/07/2004	31/07/2034	30.07	IT0005327306	1.45	15/03/2018	15/05/2025	7.17
IT0003719918	4.25	01/09/2004	01/02/2015	10.42	IT0005330961	0.05	16/04/2018	15/04/2021	3
IT0003799597	3	17/01/2005	15/01/2010	4.99	IT0005340929	2.8	01/08/2018	01/12/2028	10.34
IT0003804850	2.75	01/02/2005	01/02/2008	3	IT0005344335	2.45	03/09/2018	01/10/2023	5.08
IT0003844534	3.75	02/05/2005	01/08/2015	10.25	IT0005345183	2.5	17/09/2018	15/11/2025	7.16
IT0003872923	2.75	17/06/2005	15/06/2010	4.99	IT0005348443	2.3	15/10/2018	15/10/2021	3

This table shows the sample of the nominal securities issued by Italy from 1990 to 2018. Column “ISIN” refers to the ISIN number and Column “Coupon” to the coupon rate of the security in percent. Column “Issue” reports the issue date of the bond and Column “Maturity” provides the expiration date. Column “Term” specifies the term-to-maturity at issuance of the security. Source: Bloomberg.

Table A.8: Summary of the Nominal Securities, Spain

ISIN	Coupon	Issue	Maturity	Term	ISIN	Coupon	Issue	Maturity	Term
ES0000011165	10.75	30/01/1989	30/01/1999	10	ES00000121I8	2.75	13/01/2009	30/04/2012	3.29
XS0047970784	9	26/07/1989	26/07/1999	10	ES00000121L2	4.6	10/02/2009	30/07/2019	10.46
ES0000011249	12.25	25/03/1990	25/03/2000	10	ES00000121O6	4.3	02/06/2009	31/10/2019	10.41
ES0000011355	11.3	15/11/1991	15/01/2002	10.17	ES00000121P3	3.3	07/07/2009	31/10/2014	5.32
ES0000011371	10.3	15/04/1992	15/06/2002	10.17	ES00000121S7	4.7	28/09/2009	30/07/2041	31.84
ES0000011413	10.9	15/02/1993	30/08/2003	10.54	ES00000121T5	2.3	06/10/2009	30/04/2013	3.56
ES0000011421	10.5	17/05/1993	30/10/2003	10.45	ES00000122D7	4	20/01/2010	30/04/2020	10.28
ES0000011470	8.2	15/12/1993	28/02/2009	15.21	ES00000122E5	4.65	24/02/2010	30/07/2025	15.43
ES0000011488	8	17/01/1994	30/05/2004	10.37	ES00000122F2	3	09/03/2010	30/04/2015	5.14
ES0000011504	7.4	15/02/1994	30/07/1999	5.45	ES00000122R7	2.5	15/06/2010	31/10/2013	3.38
ES0000011512	10	15/11/1994	28/02/2005	10.29	ES00000122T3	4.85	13/07/2010	31/10/2020	10.3
ES0000011546	10.15	15/09/1995	31/01/2006	10.38	ES00000122X5	3.25	09/11/2010	30/04/2016	5.47
ES0000011538	10.1	15/09/1995	28/02/2001	5.46	ES00000123B9	5.5	24/01/2011	30/04/2021	10.26
ES0000011553	9.4	15/12/1995	30/04/1999	3.37	ES00000123C7	5.9	15/03/2011	30/07/2026	15.38
ES0000011561	8.4	15/03/1996	30/04/2001	5.13	ES00000123D5	3.4	12/04/2011	30/04/2014	3.05
ES0000011579	8.8	15/03/1996	30/04/2006	10.12	ES00000123J2	4.25	06/09/2011	31/10/2016	5.15
ES0000011587	7.8	17/06/1996	31/10/1999	3.37	ES00000123K0	5.85	22/11/2011	31/01/2022	10.19
ES0000011595	8.7	15/07/1996	28/02/2012	15.62	ES00000123L8	4	17/01/2012	30/07/2015	3.53
FR000108656	6.625	31/07/1996	31/07/2006	10	ES00000123P9		25/09/2012	31/10/2015	3.1
ES0000011603	7.9	15/10/1996	28/02/2002	5.37	ES00000123Q7	4.5	13/11/2012	31/01/2018	5.22
ES0000011611	6.75	15/11/1996	15/04/2000	3.41	ES00000123R5	4.75	29/11/2012	30/09/2017	4.84
ES0000011629	7.35	16/12/1996	31/03/2007	10.29	ES00000123T1	2.75	15/01/2013	31/03/2015	2.2
ES0000011660	6.15	15/07/1997	31/01/2013	15.55	ES00000123U9	5.4	29/01/2013	31/01/2023	10
ES0000011637	5	15/07/1997	31/01/2001	3.55	ES00000123W5	3.3	09/04/2013	30/07/2016	3.31
ES0000011645	5.25	15/07/1997	31/01/2003	5.55	ES00000123X3	4.4	21/05/2013	31/10/2023	10.44
ES0000011652	6	15/07/1997	31/01/2008	10.55	ES00000124B7	3.75	09/07/2013	31/10/2018	5.31
ES0000011868	6	15/01/1998	31/01/2029	31.04	ES00000124C5	5.15	16/07/2013	31/10/2028	15.29
ES0000012064	5.15	10/07/1998	30/07/2009	11.06	ES00000124H4	5.15	16/10/2013	31/10/2044	31.04
ES0000012072	4.25	07/08/1998	30/07/2002	3.98	ES00000124I2	2.1	26/11/2013	30/04/2017	3.43
ES0000012080	4.5	10/08/1998	30/07/2004	5.97	ES00000124V5	2.75	14/01/2014	30/04/2019	5.29
ES0000012098	4.75	07/12/1998	30/07/2014	15.64	ES00000124W3	3.8	29/01/2014	30/04/2024	10.25
ES0000012239	4	11/05/1999	31/01/2010	10.73	ES0202762003	3.82	30/01/2014	31/01/2022	8
ES0000012254	3.25	12/07/1999	31/01/2005	5.56	ES0302762127	2.45	30/01/2014	31/10/2018	4.75
ES0000012247	3	13/07/1999	31/01/2003	3.55	ES00000126B2	2.75	20/06/2014	31/10/2024	10.37

ES0000012379	4.95	14/02/2000	30/07/2005	5.46	ES00000126C0	1.4	08/07/2014	31/01/2020	5.57
ES0000012361	4.6	15/02/2000	30/07/2003	3.45	ES00000126D8	4	08/09/2014	31/10/2064	50.15
ES0000012387	5.4	19/09/2000	30/07/2011	10.86	ES00000126V0	0.5	23/09/2014	31/10/2017	3.1
ES0000012411	5.75	23/01/2001	30/07/2032	31.52	ES00000126Z1	1.6	27/01/2015	30/04/2025	10.26
ES0000012437	4.65	12/03/2001	31/10/2004	3.64	ES00000127A2	1.95	04/03/2015	30/07/2030	15.41
ES0000012445	4.8	09/04/2001	31/10/2006	5.56	ES00000127D6	0.25	26/05/2015	30/04/2018	2.93
ES0000012452	5.35	12/06/2001	31/10/2011	10.38	ES00000127G9	2.15	09/06/2015	31/10/2025	10.4
ES0000012783	5.5	11/03/2002	30/07/2017	15.39	ES00000127H7	1.15	16/06/2015	30/07/2020	5.12
ES0000012791	5	14/05/2002	30/07/2012	10.21	ES00000127Z9	1.95	19/01/2016	30/04/2026	10.28
ES0000012825	4.25	09/09/2002	31/10/2007	5.14	ES00000128A0	0.25	26/01/2016	31/01/2019	3.01
ES0000012841	3.2	13/01/2003	31/01/2006	3.05	ES00000128B8	0.75	08/03/2016	30/07/2021	5.39
ES0000012866	4.2	15/04/2003	30/07/2013	10.29	ES00000128C6	2.9	15/03/2016	31/10/2046	30.63
ES0000012882	3.6	19/01/2004	31/01/2009	5.03	ES00000128E2	3.45	18/05/2016	30/07/2066	50.2
ES0000012908	3	11/05/2004	30/07/2007	3.22	ES00000128H5	1.3	26/07/2016	31/10/2026	10.26
ES0000012916	4.4	28/06/2004	31/01/2015	10.59	ES00000128O1	0.4	24/01/2017	30/04/2022	5.26
ES0000012932	4.2	17/01/2005	31/01/2037	32.04	ES00000128P8	1.5	31/01/2017	30/04/2027	10.24
ES00000120E9	3.25	12/04/2005	30/07/2010	5.3	ES00000128Q6	2.35	01/03/2017	30/07/2033	16.41
ES00000120G4	3.15	20/09/2005	31/01/2016	10.36	ES00000128X2	0.05	06/06/2017	31/01/2021	3.66
ES00000120H2	2.9	17/01/2006	31/10/2008	2.79	ES0000012A89	1.45	04/07/2017	31/10/2027	10.32
ES00000120J8	3.8	18/10/2006	31/01/2017	10.29	ES0000012A97	0.45	10/10/2017	31/10/2022	5.06
ES00000120L4	3.9	16/01/2007	31/10/2012	5.79	ES0000012B39	1.4	30/01/2018	30/04/2028	10.25
ES00000120N0	4.9	20/06/2007	30/07/2040	33.11	ES0000012B47	2.7	27/02/2018	31/10/2048	30.67
ES00000120Z4	4.1	15/01/2008	30/04/2011	3.29	ES0000012B62	0.35	22/05/2018	30/07/2023	5.19
ES00000121A5	4.1	19/02/2008	30/07/2018	10.44	ES0000012B88	1.4	03/07/2018	30/07/2028	10.08
ES00000121G2	4.8	16/09/2008	31/01/2024	15.37	ES0000012C46	0.05	09/10/2018	31/10/2021	3.06
ES00000121H0	4.25	07/10/2008	31/01/2014	5.32					

This table shows the sample of the nominal securities issued by Spain from 1989 to 2018. Column “ISIN” refers to the ISIN number and Column “Coupon” to the coupon rate of the security in percent. Column “Issue” reports the issue date of the bond and Column “Maturity” provides the expiration date. Column “Term” specifies the term-to-maturity of the security. Source: Bloomberg.

Table A.9: Descriptive statistics of estimated parameters

	Panel A: France				Panel B: Germany				
	Mean	Std. Dev.	Minimum	Maximum	Mean	Std.Dev.	Minimum	Maximum	
NS					NS				
$\beta_1$	4.81	2.26	1.54	17.60	$\beta_1$	4.14	1.57	0.00	9.81
$\beta_2$	-2.80	3.06	-20.00	19.86	$\beta_2$	-2.40	1.10	-6.31	14.88
$\beta_3$	-4.17	3.88	-20.00	20.00	$\beta_3$	-3.02	2.05	-16.04	20.00
$\lambda$	2.52	1.40	0.10	30.00	$\lambda$	2.86	2.48	0.16	30.00
Sv					Sv				
$\beta_1$	2.71	1.18	0.00	10.41	$\beta_1$	1.20	1.90	0.00	25.00
$\beta_2$	-1.20	3.95	-20.00	20.00	$\beta_2$	0.98	3.60	-20.00	20.00
$\beta_3$	-4.76	10.70	-20.00	20.00	$\beta_3$	-7.12	9.77	-20.00	20.00
$\beta_4$	9.77	8.58	-23.73	43.56	$\beta_4$	13.94	8.94	-125.08	34.33
$\lambda$	2.22	1.45	0.10	12.15	$\lambda$	2.98	1.87	0.10	13.74
$\gamma$	13.05	6.77	0.48	30.00	$\gamma$	13.69	7.07	2.03	30.00
eBC					eBC				
$\beta_1$	3.86	3.26	0.00	25.00	$\beta_1$	2.36	2.57	0.00	10.48
$\beta_2$	-8.67	10.44	-20.00	25.00	$\beta_2$	-5.69	11.59	-20.00	20.00
$\beta_3$	-5.9	6.99	-20.00	25.00	$\beta_3$	-5.55	5.93	-20.00	20.00
$\beta_4$	6.74	11.58	-29.19	26.07	$\beta_4$	5.57	12.02	-26.72	24.84

$\lambda$	3.65	2.70	0.10	26.07	$\lambda$	3.91	2.59	0.10	18.22
$\gamma$	16.35	10.92	0.76	30.00	$\gamma$	16.6	10.47	0.96	30.00
eSv					eSv				
$\beta_1$	2.73	2.84	0.00	25.00	$\beta_1$	0.81	1.53	0.00	14.56
$\beta_2$	5.40	12.36	-20.00	20.00	$\beta_2$	10.02	10.04	-20.00	20.00
$\beta_3$	-5.71	11.08	-20.00	20.00	$\beta_3$	-6.45	11.07	-20.00	19.94
$\beta_4$	-5.86	12.06	-27.63	18.16	$\beta_4$	-8.40	10.43	-22.39	30.65
$\beta_5$	11.55	11.56	-37.25	52.62	$\beta_5$	17.73	9.56	-98.28	37.46
$\lambda$	2.46	1.64	0.10	10.23	$\lambda$	3.07	2.08	0.10	19.78
$\gamma$	12.02	8.59	2.83	30.00	$\gamma$	13.28	7.75	2.88	30.00
Panel C: Italy					Panel D: Spain				
	Mean	Std. Dev.	Minimum	Maximum		Mean	Std.Dev.	Minimum	Maximum
NS					NS				
$\beta_1$	9.22	7.28	2.50	25.00	$\beta_1$	5.28	1.11	2.62	8.44
$\beta_2$	-2.40	7.31	-20.00	7.97	$\beta_2$	-3.26	1.56	-6.30	15.44
$\beta_3$	-3.02	6.34	-20.00	1.58	$\beta_3$	-2.79	2.52	-20.00	6.51
$\lambda$	2.86	4.18	0.36	23.07	$\lambda$	2.88	2.06	0.38	30.00
Sv					Sv				
$\beta_1$	4.20	5.30	0.00	24.61	$\beta_1$	2.72	2.42	0.00	23.49
$\beta_2$	-1.26	9.06	-20.00	20.00	$\beta_2$	0.16	5.29	-20.00	20.00
$\beta_3$	-5.72	12.14	-20.00	20.00	$\beta_3$	-3.70	7.89	-20.00	20.00
$\beta_4$	12.68	16.86	-50.07	78.33	$\beta_4$	9.15	8.47	-51.09	39.09
$\lambda$	2.74	3.20	0.10	20.99	$\lambda$	2.35	1.82	0.10	27.63
$\gamma$	19.14	10.71	2.39	30.00	$\gamma$	18.57	8.32	1.56	30.00
eBC					eBC				
$\beta_1$	9.42	8.76	0.00	25.00	$\beta_1$	4.63	2.29	0.00	19.48
$\beta_2$	-2.86	13.98	-20.00	20.00	$\beta_2$	-4.13	10.55	-20.00	20.00
$\beta_3$	-8.53	9.31	-20.00	20.00	$\beta_3$	-3.94	5.79	-20.00	20.00
$\beta_4$	-2.82	17.62	-45.66	24.16	$\beta_4$	2.21	10.89	-36.29	24.65
$\lambda$	5.23	4.83	0.10	23.84	$\lambda$	3.20	2.73	0.10	21.49
$\gamma$	14.21	10.71	0.10	30.00	$\gamma$	20.46	11.54	1.13	30.00
eSv					eSv				
$\beta_1$	9.23	9.41	0.00	25.00	$\beta_1$	3.81	3.76	0.00	25.00
$\beta_2$	7.52	14.35	-20.00	20.00	$\beta_2$	1.79	14.23	-20.00	20.00
$\beta_3$	-2.07	15.66	-20.00	20.00	$\beta_3$	-5.17	9.58	-20.00	20.00
$\beta_4$	-12.4	19.73	-45.42	18.27	$\beta_4$	-1.81	12.12	-29.35	20.36
$\beta_5$	12.68	19.32	-57.49	68.56	$\beta_5$	5.39	13.68	-58.12	43.11
$\lambda$	2.75	2.74	0.10	30.00	$\lambda$	2.19	1.74	0.10	17.35

---

$\gamma$	18.18	10.83	1.32	30.00	$\gamma$	15.77	9.48	1.87	30.00
----------	-------	-------	------	-------	----------	-------	------	------	-------

---

We fit all four term structure models using daily market data on government securities prices. This table reports descriptive statistics (i.e., the mean, standard deviation, min and max) for all parameters for given country, i.e., Italy (panel C) and Spain (panel D). The sample period from January 4, 1999, to December 28, 2018.

Table A.10: Out-of-sample forecasting results

Maturity ( $\tau$ )	Mean	Std. Dev.	RMSE	$\hat{\rho}(h)$	$\hat{\rho}(h + 12)$
Panel A: 1-month-ahead forecasting results					
RW					
0.25	0.0059	1.7746	1.7744	-0.0563	-0.1082
1	-0.0094	0.4384	0.4385	0.0307	-0.1359
3	-0.0187	0.2021	0.2029	0.1574	0.0053
5	-0.0175	0.1935	0.1943	0.1066	0.0140
10	-0.0172	0.1966	0.1973	0.0939	-0.0374
AR(1)					
0.25	0.0093	1.8077	1.8075	0.3557	0.1573
1	0.0022	0.4362	0.4362	0.0474	-0.1131
3	0.0000	0.2019	0.2019	0.1608	0.0086
5	-0.0003	0.1934	0.1934	0.1095	0.0167
10	0.0002	0.1964	0.1964	0.0977	-0.0340
VAR(1)					
0.25	2.4229	2.7075	3.6331	0.7932	0.4286
1	0.4665	0.6802	0.8248	0.6992	0.6301
3	0.0598	0.2072	0.2156	0.2224	0.0718
5	0.5998	0.4726	0.7636	0.8828	0.8080
10	0.0353	0.2118	0.2147	0.2587	0.1315
Panel B: 6-months-ahead forecasting results					
RW					
0.25	0.0434	3.0195	3.0195	-0.4130	0.2132
1	-0.0636	1.0044	1.0063	-0.1716	-0.0286
3	-0.1154	0.5540	0.5658	-0.1033	-0.0084
5	-0.1072	0.4969	0.5082	-0.1779	-0.0322
10	-0.1091	0.4854	0.4975	-0.1964	-0.0368
AR(1)					
0.25	0.1012	2.5634	2.5651	0.5962	0.5705
1	0.0106	0.9766	0.9766	0.0004	0.1168

---

Maturity ( $\tau$ )	Mean	Std. Dev.	RMSE	$\hat{\rho}(h)$	$\hat{\rho}(h + 12)$
3	-0.0006	0.5491	0.5491	-0.0841	0.0029
5	-0.0011	0.4945	0.4944	-0.1579	-0.0175
10	-0.002	0.4832	0.4831	-0.1717	-0.019
VAR(1)					
0.25	2.4838	2.5828	3.5831	0.6142	0.5772
1	1.4038	1.7412	2.2365	0.9188	0.7814
3	0.3387	0.6003	0.6892	0.1987	0.2170
5	1.9502	1.4506	2.4304	0.9343	0.8847
10	0.1866	0.6333	0.6601	0.4638	0.4815

Panel C: 12-month-ahead forecasting results

RW

0.25	-0.2423	2.7997	2.8099	-0.3831	-0.1173
1	-0.2511	1.1127	1.1406	0.0176	-0.2449
3	-0.2487	0.7517	0.7917	-0.1200	-0.1260
5	-0.2321	0.6433	0.6839	-0.2158	0.0091
10	-0.2433	0.6123	0.6588	-0.3591	0.1841

AR(1)

0.25	-0.1138	2.094	2.0969	0.6336	0.5086
1	-0.0934	1.0383	1.0424	0.2343	-0.0068
3	-0.0158	0.7371	0.7372	-0.0934	-0.0867
5	-0.0166	0.6375	0.6376	-0.1768	0.0510
10	-0.0254	0.6058	0.6063	-0.3145	0.2353

VAR(1)

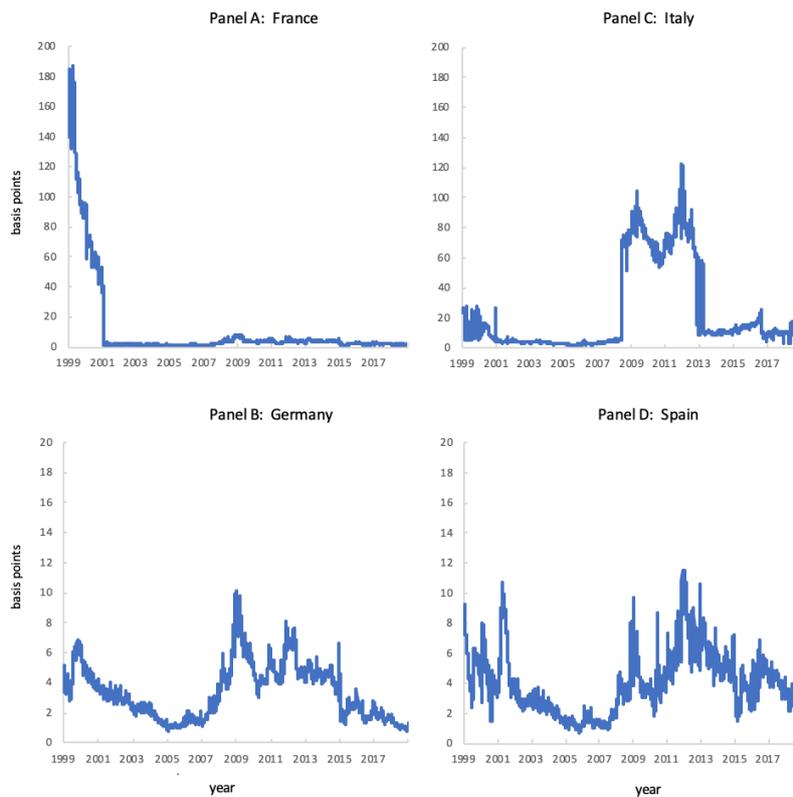
0.25	2.2648	2.0853	3.0782	0.6374	0.5100
1	1.4521	1.7707	2.2899	0.8706	0.6913
3	0.6089	0.8512	1.0465	0.3372	0.2950
5	2.2235	1.6537	2.7709	0.9236	0.8652
10	0.3006	0.9104	0.9587	0.6216	0.7241

---

This table reports the results of out-of-sample  $h$ -month-ahead forecasting using three models,  $h = 1, 6, 12$ , as described in detail in the text. We report the mean, standard deviation and root mean squared errors of the forecast errors, as well as their sample autocorrelation coefficients.

---

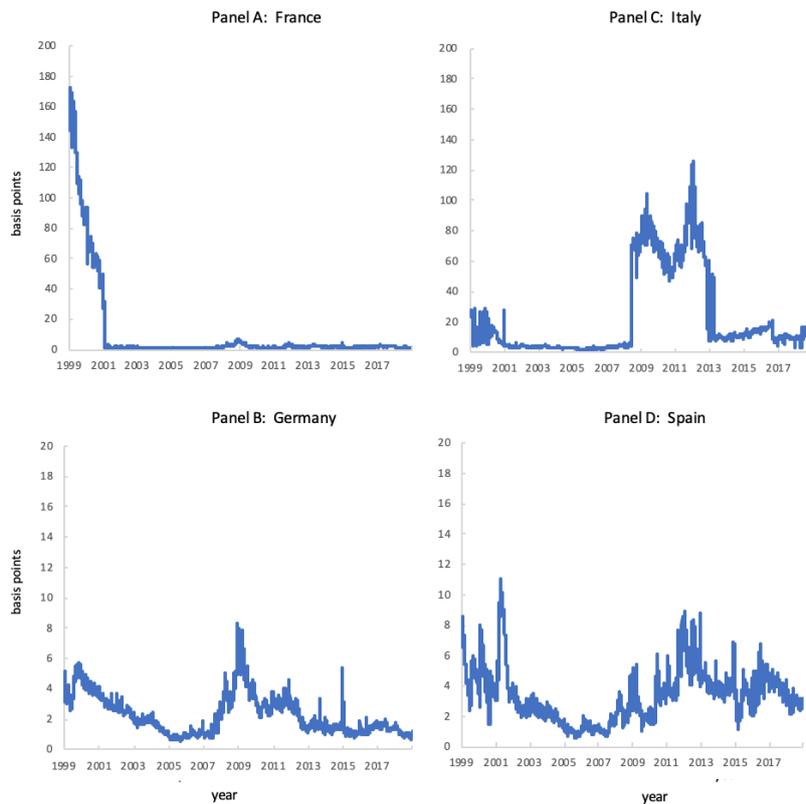
Figure A.8: Fitting Errors (Nelson-Siegel model)



This figure shows the total fitting error implied by the Nelson-Siegel model. The fitting error is computed as the mean absolute error between the predicted and the market yields across all available securities on a particular day for given country. The fitting errors are shown in basis points. Sample period: January 4, 1999, to December 28, 2018. Frequency: Daily.

---

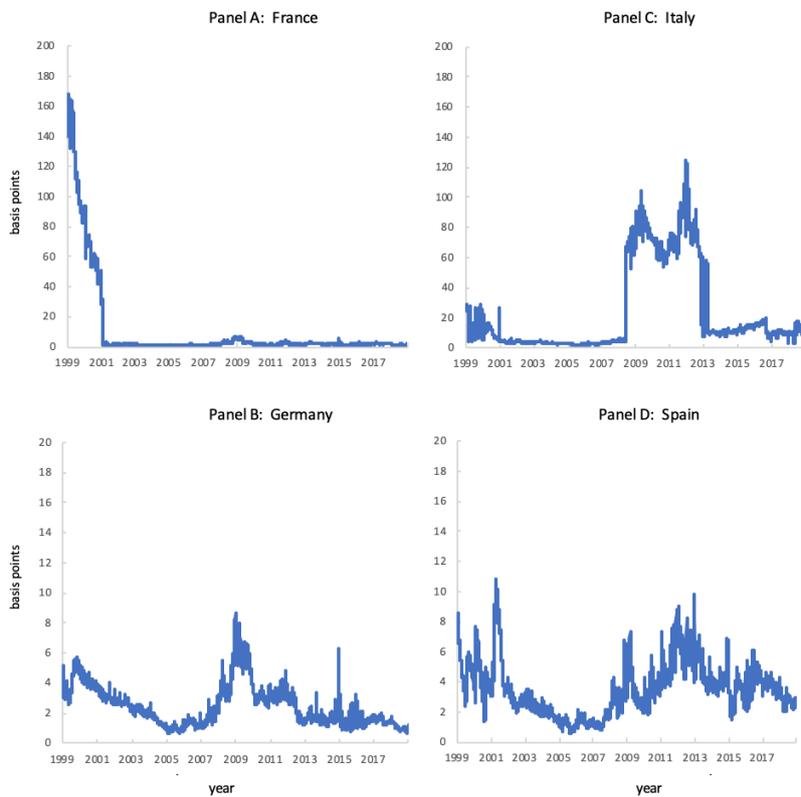
Figure A.9: Fitting Errors (Svensson model)



This figure shows the total fitting error implied by the Svensson model. The fitting error is computed as the mean absolute error between the predicted and the market yields across all available securities on a particular day for a given country. The fitting errors are shown in basis points. Sample period: January 4, 1999, to December 28, 2018. Frequency: Daily.

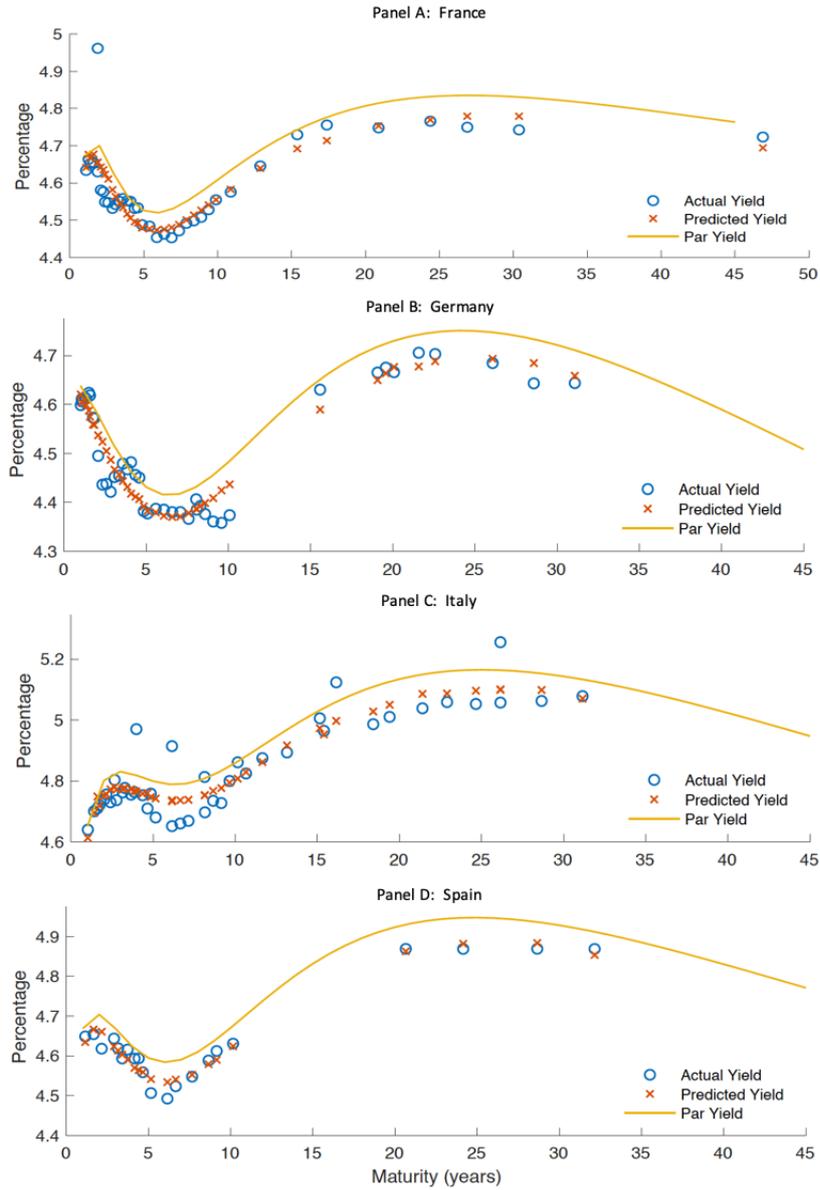
---

Figure A.10: Fitting Errors (extended Bjork-Christensen model)



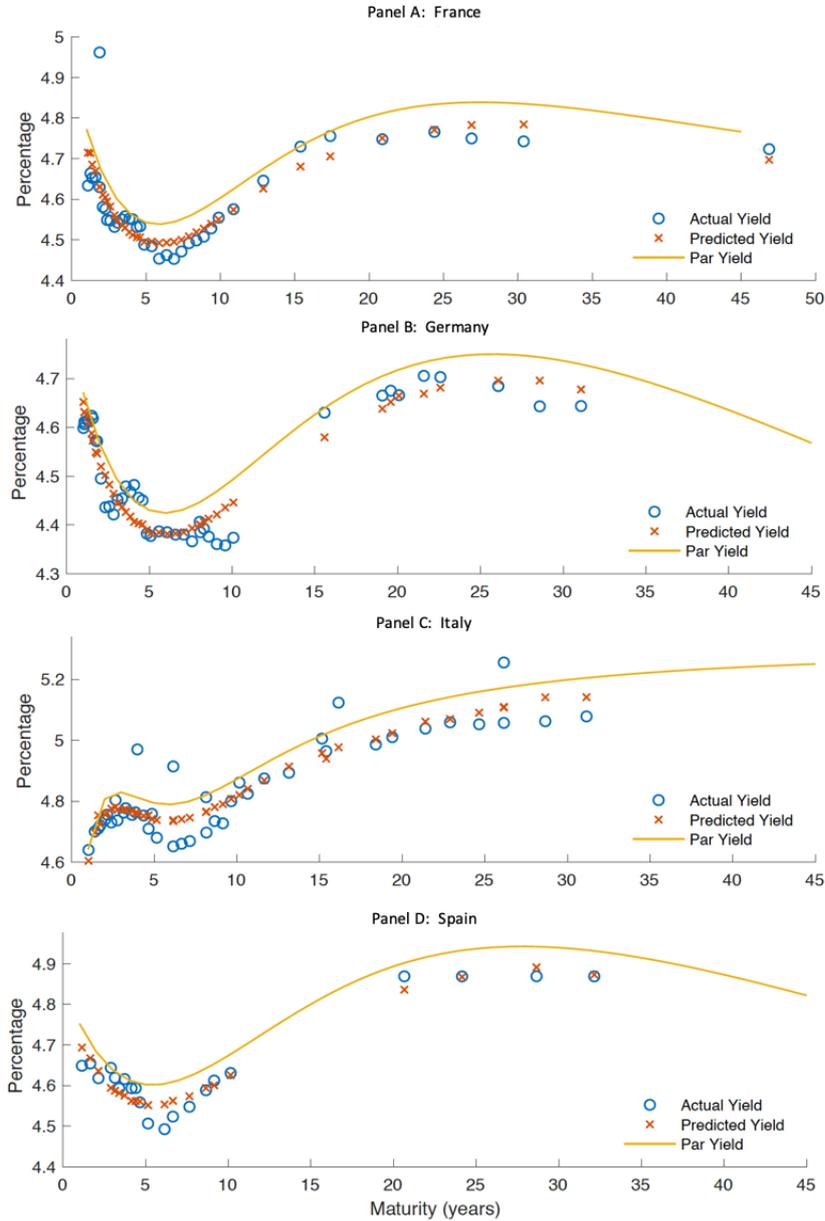
This figure shows the total fitting error implied by the extended Bjork-Christensen model. The fitting error is computed as the mean absolute error between the predicted and the market yields across all available securities on a particular day for a given country. The fitting errors are shown in basis points. Sample period: January 4, 1999 to December 28, 2018. Frequency: Daily.

Figure A.11: Par yield curve (Sv specification) on June 6, 2008



This figure shows Svensson par yield curve and the fit of individual securities for one day of the sample period, June 6, 2008, for a given country. The curve is reported in annualized percent.

Figure A.12: Par yield curve (eBC specification) on June 6, 2008



This figure shows extended Bjork-Christensen par yield curve and the fit of individual securities for one day of the sample period, June 6, 2008, for a given country. The curve is reported in annualized percent.



## Chapter 3

# The Term Structure of the French Nominal Government Debt <sup>1</sup>

### 3.1 Introduction

As in a number of OECD countries, the French sovereign bond debt market was constantly growing over the past few decades. Figure 3.1 plots the year-end notional outstanding amount of the French short-, medium-, and long-term securities since 1993.<sup>2</sup> The market has grown almost sixfold, from about €300 million in 1993 to about €1,700 million in 2018. As of April 2018, the total outstanding amount of the French government negotiable debt securities was €1,725 billion, 92 percent of which was represented by medium- and long-term

---

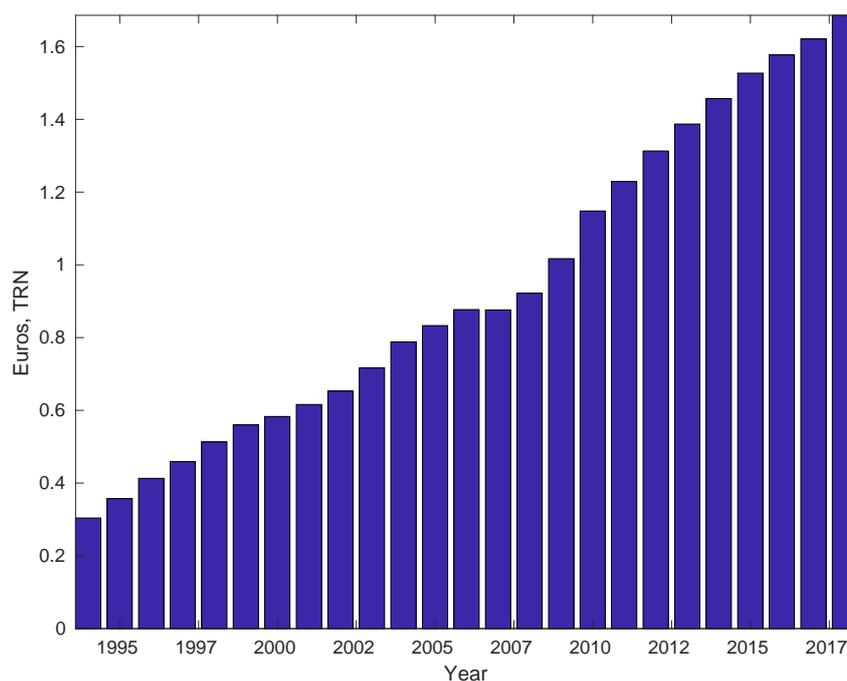
<sup>1</sup> This paper is based on a working paper co-authored with Olesya GRISHCHENKO and Franck MORAUX. These results were presented while the 6th Paris Financial Management Conference in December 2018 and 36th International Conference of the French Finance Association in June 2016. 7th Paris Financial Management Conference in December 2019. We thank participants of these conferences for the useful comments and discussion. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve System.

<sup>2</sup>We did not find outstanding notional amounts for earlier dates, as the first publicly available AFT newsletter was issued in January 1998; the end of 1993 is the earliest outstanding amount reported there. Note also that, in order to plot this graph, we convert into euros the notional amounts of bonds that were issued in French francs prior to the euro launch. The conversion rate of ECU to euros is 1:1.

---

debt securities.<sup>1</sup>

Figure 3.1: Notional Amount of the French Nominal Debt



This figure shows the outstanding notional amount of the French nominal government debt (BTF, BTANs, and OATs) from December 1993 to December 2017. Data are hand-collected and merged from the monthly newsletters released by the Agence France Trésor.

---

<sup>1</sup>Source: <https://www.aft.gouv.fr/files/archives/attachments/26686.pdf>. For general discussions on the budget and the financing policies of the French state, one may consult monthly newsletters (so-called *Bulletins Mensuels*) published by the *Agence France Trésor* (AFT) and working papers provided by Banque de France on its website. Established in February 2001, the AFT is an important institution whose goal is to manage the French government treasury (including a day-to-day perspective), define for the government the debt strategy, control and manage the risk and provide back-office services, provide macroeconomic and financial analysis and expertise, collect and diffuse economic information, and cooperate with international sister organizations. It has been engaged for years in a strategy to refinance the total debt and to benefit from the favorable low financing conditions. Many statistical figures and general comments we present hereafter are based on the October 2017 technical notes of the ECB [2017] and also BFS. See also a recently published report by the OECD [2017].

---

The French public bond market is known to be very liquid: *“The market is also very highly regarded worldwide as a benchmark reference because of the regularly held auctions and fungibility, it is already the second most liquid in the world after its American equivalent.”* [see [Batten, Fetherston, and Szilagyi, 2004](#)]. Despite its size, the French public debt market is not well studied in the academic literature. Because the interest rate is considered to be one of the basic components in both financial economics and macroeconomics, the availability of the historical French yield curve should be an incredibly useful tool for researchers in these areas. Our paper aims to fill this gap.

To our knowledge, we are the first to comprehensively study more than 30 years of all available public data of French government securities prices. To that end, we have constructed the nominal yield curves at a daily frequency during this sample period, thus making it possible to study the evolution of French interest rates in detail.

In particular, we fully implement the [Gürkaynak, Sack, and Wright \[GSW, 2007\]](#) empirical methodology on all available and eligible OATs and BTANs we found on Bloomberg since the first issuance of the OAT on October 8, 1984. Consequently, this research heavily relies on the yield curve fitting methodology of [Svensson \[1994\]](#), our goal is to obtain some reliable estimates of intermediate/long-horizon yields that would reflect fundamentals.<sup>1</sup> We document a number of interesting facts about the French government bond market and French interest rates.

Our first result is related to the recurrent and open question about the existence of the so-called on-the-run premium on government debt markets. This phenomenon refers to the fact that investors are willing to pay a (liquidity) premium for the newly issued government obligations, which, therefore, trade at higher prices relative to the previous most recent issues of the debt. Based on two different empirical strategies, we find no evidence of the on-the-run premium on the French market. Statistically, we find that the French on-the-run premium has been, on average, negligible and within the bounds determined by the model mean absolute

---

<sup>1</sup>Starting with [Ricart and Sicsic \[1995\]](#), the Svensson approach is used by the Banque de France, when it passes the test against the Nelson-Siegel curve fitting model.

---

fitting errors. This is clearly a distinct feature compared with the U.S. nominal Treasury securities market, where various researchers document a sizable on-the-run premium [see [Fleming, 2003](#); [Gauthier and Simonato, 2012](#); [Gürkaynak, Sack, and Wright, 2007](#); [Krishnamurthy, 2002](#); [Pasquariello and Vega, 2009](#); [Warga, 1992](#)]. The absence of such a premium may be explained by the already mentioned deep liquidity of the market, but it is also directly imputable to the existence of so-called *souches*.<sup>1</sup> This result complements the early empirical evidence of [Ejlsing and Sihvonen \[2009\]](#), who find no on-the-run premium in the German sovereign bond market.

Our second result shows the dynamics of the French nominal yield curve. Fitted zero-coupon yields indicate a clear downward trend in interest rates since the Global Financial Crisis (GFC), consistent with declining interest rates in other countries, particularly the United States. Toward the end of our sample, interest rates in France appeared to have reached a zero-lower-bound level. In addition, the slope of the term structure as measured by, for example, the difference in the 10- and 2-year yields has been declining as well. Numerous studies [such as [Ang, Piazzesi, and Wei, 2006](#)] find that the changes in the slope predict the changes in the GDP growth, so our findings may be useful to macroeconomic forecasters in future research.

Our third result is related to the functioning of the French sovereign bond market that has considerably improved since the introduction of the euro, which we relate to the influx of outside investors to the French market. In fact, according to the noise measure of [Hu, Pan, and Wang \[HPW, 2013\]](#), that reflects the (un)availability of arbitrage capital on a market, our findings suggest that the French market development can be decisively separated into pre-euro (1988 to 1998) and post-euro (1999 to 2018) periods in our sample.

The rest of the paper is organized as follows. Section [3.2](#) describes some institutional details of the French government bond market. Section [3.3](#) describes our data set. Section [3.4](#) describes the methodology behind the Svensson curve

---

<sup>1</sup>These different arguments have been confirmed by interviews and informal discussions with market participants in the French government bond market and bond portfolio managers in insurance companies.

---

estimation. Section 3.5 reports the results and investigates the shape and the dynamics of the fitted zero-coupon yield curve. Section 3.6 focuses on the on-the-run premium issue of the French government bond market. Section 3.7 explores the period preceding the euro's 1999 launch. Section 3.8 concludes.

## 3.2 The French government bond market

The French treasury (FT hereafter) has a long history of bond issuance and security innovations as different marketable debt securities have been issued by the French government for years. BTFs, BTANs, and OATs are emblematic acronyms for this market that stand respectively for *Bons du Trésor à Taux Fixe et à intérêt précompté*, *Bons du Trésor à taux fixe et à intérêts Annuels* and *Obligations Assimilables du Trésor*, note that in general these definitions are viewed as almost meaningless. These acronyms are nevertheless useful to discriminate short-, medium-, and long-term securities, respectively.<sup>1</sup> BTFs are standard bills issued on a discount basis and redeemed at par. Their maturity is expressed in weeks, and the most frequently maturities are 13, 26, and 52 weeks. Hence, their initial maturity is equal to or less than one year. In principle, these bonds are issued to manage short-term operations. Both BTANs and OATs are, in general, coupon bonds although there exist some zero-coupon bonds, too. BTANs, first issued on February 11, 1986, were debt securities with initial maturities between 2 and 5 years. However, as of January 1, 2013, the FT stopped issuing BTANs and started issuing only two types of sovereign securities, BTFs and OATs. The last BTAN debt was fully reimbursed on July 25, 2017, and the total BTAN debt amounted to €1188 billion during its period of existence. The goal of stopping the issuance of BTANs under their original name was to simplify the structure of the issued French public debt. Since then, medium-term securities have been renamed as OATs. Currently, all OATs and BTANs are euro-denominated French debt securities. Most of the marketable government debt has a residual maturity of more than one year, and the current average duration, as of April 2018 equals

---

<sup>1</sup>Just like one finds T-bills, T-notes and T-bonds in the United States and Bubills, Schaetze/Bobls and Bunds in Germany.

---

7 years and 288 days.<sup>1</sup>

The term “assimilable” in OATs is technical. It first refers to the fact that these securities are fungible with some vintage bonds with identical characteristics: the same expiration date, the same coupon rate and the same face value.<sup>2</sup> This also means that newly issued bonds blend in with the bond debt issue that contains these vintage bonds. At first sight, this may appear very similar to the U.S. reopening device. However, they are not exactly the same. First, in France, bond debt management relies explicitly on an initial *souche*, which is the very first debt issue that will serve as a matrix for the following ones. The newly issued bonds are so fungible that vintage bond and newborn bonds are effectively indistinguishable, and it is not really appropriate to talk about a new tranche. Second, two mechanisms differ with respect to the usage policy. In France, it is a general way to respond to the demand. In the United States, reopenings are used to manage the short debt squeezes (see the Joint Report on the Government Securities Market, 1992). Third, they differ in terms of issuance features. In the United States, both standard auctions and “tap” issues are used by the Treasury for reopening purposes. In France, new bonds that contribute to an existing *souche* are offered by auction (*adjudication*).

A limited portion of the French public bond debt (€200 million) is adjusted to inflation according to two indexes: the French CPI index (*l'indice des prix à la consommation en France*) and the HICP euro index (*l'indice des prix de la zone Euro*). The first inflation-adjusted French government bond, *Obligations Assimilables du Trésor indexée sur l'indice des prix à la consommation en France (OATi)*, was issued on September 15, 1998. In October 2001, the French government issued the first for *Obligations Assimilables du Trésor indexée sur l'indice*

---

<sup>1</sup> The full list of a given debt can be found in the monthly newsletter of the AFT: <https://www.aft.gouv.fr/files/archives/attachments/26686.pdf>. More information on characteristics of OAT securities can be found here: <https://www.aft.gouv.fr/fr/presentation-oat>.

<sup>2</sup>Two exceptions to this general principle are worth discussing. The first one arises when the newly issued bonds are restricted to certain investors as individuals (that is, *particuliers*). The second one comes from the inclusion, as of January 1, 2013, of some collective action clauses in the debt contract. Now, like all bonds issued in the euro area after January 1, 2013, OATs have some collective action clauses. As a result, they are not entirely fungible with bonds issued prior to this date.

---

*des prix de la zone Euro (OAT€i)*. At last, some bond debts are denominated in other currencies (USD and GBP), but they represent less than 3 percent of the total outstanding amount. These specific segments of the French government bond market are out of the scope of the present research, because they deserve tailored investigations. We do not consider currently these bonds in our dataset for the purposes of this paper.

The French OATs/BTANs bond market experienced a number of different periods since its onset in 1984. During the first period, the bond market was regularly exposed to some political events and institutional changes. From a macroeconomic perspective, the French franc-denominated debt was exposed to the French political risk factor during a number of economic episodes. From a microeconomic perspective, the market increased gradually in credibility and liquidity. As an example of these changes, the *marché à terme international de France* (MATIF) opened in 1986 and proposed a number of interest rate derivatives. Later, the OATs bond stripping was authorized in 1991, which allowed people to make arbitrage between zeros and coupon bonds and one may also refer to floating-rate OATs, called OAT TEC 10 that were first issued on April 9, 1996, and to the regular improvement of the legal and market environment for repos transactions (including the technique of “pension livrée” in 1988, the designation of 20 market makers in 1994 from merging *Spécialistes en Valeurs du Trésor* (SVTs) and *Spécialistes en Pensions sur Valeurs du Trésor* (SPVTs), and the use of the ISMA Master Agreement for repos since the euro launch). The *Agence de la Dette* then renamed *Agence France Trésor* was created on February 8, 2001. All of these innovations contributed to a better-functioning French bond market, making it highly attractive for investors. It is nowadays a very liquid place to invest.

The investor clientele on the French bond market has profoundly changed over the 35 years of the market’s existence. In the 1980s, large investors were mainly French institutional investors sometimes called *zinzins*<sup>1</sup> as well as some large

---

<sup>1</sup>According to Af2i - the Association Française des Investisseurs Institutionnels, institutional investors are investors collecting private funds and they are required to invest a large part of their stake with a long-term perspective: “*Organismes collecteurs de l’épargne qui placent leurs fonds sur les marchés pour leur compte propre ou celui de leurs clients (particuliers, fonds de pensions, assurés,...)*. Ils sont tenus institutionnellement de placer à long terme

---

state-owned companies. By law, *zinzins* are required to be engaged in some long-term strategies such as the buy-and-holds of OAT securities. For their part, insurance companies are concerned by asset-liability management issues. In the 1980s, no one had incentive to actively trade these bonds or even to lend them on the repo market. With the launch of the euro, such political and institutional issues largely disappeared and new outside investors came into the market. In fact, nonresident holdings of French government negotiable debt securities regularly increased from about 15 percent of the total negotiable debt outstanding in early 1998 (about one year before the euro) to almost 30 percent at the end of 2000. Nonresident participation in the market reached its peak of 71 percent in June 2010<sup>1</sup>.

### 3.3 Data

We identify the list of all French government marketable debt securities available on Bloomberg by their ISIN number. We then select all BTANs and OATs with fixed coupons.<sup>2</sup> Each ISIN number refers to a specific issuance of bonds. In the case of OATs, the very first issue of securities with some given characteristics is called a *souche*. *Souches* are therefore uniquely identified by the ISIN number associated to first OATs issued. Later on *souches* can be reopened several times. Actually, in the case of a reopening, each additional issue would have another ISIN, but with no available data on Bloomberg. Indeed, this would be redundant information because, by design, these new OATs are just similar and they can be “assimilated” to the vintage ones. Before the launch of the euro, the straight coupon bonds we collect were denominated at issuance in FF (French francs), and in a very few cases in XEU (or ECU for European Currency Unit). They are denominated in euros starting January 1, 1999. All of them pay annual fixed coupons to bondholders and have neither special nor optional features.

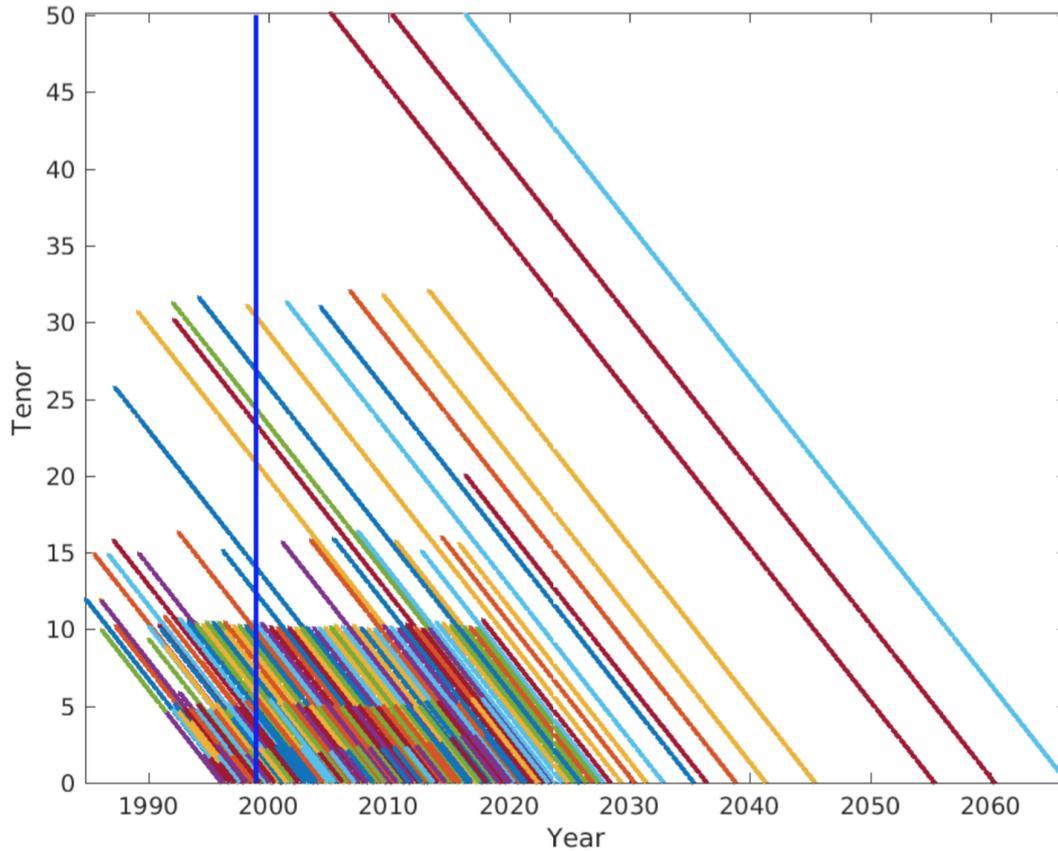
---

*une part importante de leurs ressources. [...] Cette dénomination d'Investisseurs Institutionnels regroupe sous le même vocable des institutions fort diverses (caisses de retraite, institut de prévoyance, compagnies d'assurance, mutuelles, associations, fondations, caisses de congés payés, institutions spéciales...).*”

<sup>1</sup>In a situation where debt is far larger (see Figure 3.1).

<sup>2</sup>Neither OATs nor BTANs are callable. When appropriate, the AFT can try to buy back the debt.

Figure 3.2: Maturity Distribution of the BTANs and OATs



This figure shows the maturity structure of the French nominal government securities, BTANs and OATs issued from 1984 to 2018. The vertical line corresponds to January 1, 1999, the beginning of the euro-area sample. Source: Bloomberg.

We randomly check that the information from Bloomberg is consistent with the information in the AFT monthly newsletters.<sup>12</sup>

<sup>1</sup>These newsletters are available at the AFT's website <https://www.aft.gouv.fr/fr/bulletins-mensuels> since the January 2010 issue. At the start of our project, we manually collected newsletters for earlier years, starting from January 1998, from which we obtained the outstanding notional amount of the OAT market. The January 1998 newsletter contained the OAT notional debt amount that was dated back to December 1993 (start of the sample in Figure 3.1). Unfortunately, currently the AFT site provides newsletters only from January 2010.

<sup>2</sup>It appears that the information recorded by Bloomberg (especially for bonds issued before 1999) does not necessarily match.

---

Ultimately, our data set consists of 179 bonds and daily available (bid) prices for these bonds from July 1, 1987, through April 10, 2018. Thus, our data set contains 300,105 price quotes in total. Tables [B.6](#), [B.7](#), and [B.8](#) in the appendix provide a detailed description of all the individual securities in our sample from 1987 to 1999, 2000 to 2008, and 2009 to 2018, respectively. For a given bond, we provide the following information: the security type (as it can be a BTAN or an OAT before January 1, 2013), the ISIN number, the coupon rate of the security, the first date on which the quotes for the security are available, the expiration date of the security (maturity), the term-to-maturity of the bond, and the total number of available observations for the security. Bond debts mature most frequently at the end of April or October, and the expiration dates of BTANs and OATs have occurred on the 12th and 25th day of a particular month, respectively.

The ranges of time-to-maturities available for estimation over our sample period are plotted in [Figure 3.2](#). Each line represents one security. The date is shown on the horizontal axis and the remaining time-to-maturity is shown on the vertical axis in years. The upper-left point of the line corresponds to the first date for which the quote is available on Bloomberg. The lower-right point of the line corresponds to the bond expiration date. As one can see from this graph, most of the issuance is concentrated in the maturity range of 5 to 10 years. An interesting feature of the OAT market (and different from U.S. Treasury securities) is that there are currently three 50-year (ultra-long) bonds, issued in 2005, 2010, and 2016. The vertical line on the figure corresponds to the January 1, 1999—the first trading day in euros in our sample.

### 3.4 Methodology

In this section, we first define basic concepts that we use in the paper, then introduce [Svensson \[1994\]](#) methodology, describe various filters used for our data set, and discuss our estimation procedure.

---

### 3.4.1 Basic concepts

The first and most basic concept for pricing any fixed-income asset is the discount function or the price of a zero-coupon bond that represents the value at time  $t$  of paying €1 at a future point of time  $T$ . We denote this bond price as  $B(t, T)$ , and it is worth introducing the continuously compounded zero-coupon yield on this bond denoted by  $y(t, T)$ . The zero-coupon bond price and this zero-coupon bond yield are linked via the relationship

$$B(t, T) = \exp[-y(t, T) \times (T - t),]. \quad (3.1)$$

or equivalently

$$y(t, T) = -\frac{1}{T - t} \ln B(t, T). \quad (3.2)$$

Assume now that we observe a number of zero-coupon bond prices. We can then price any coupon-bearing bond. Actually, by using the no-arbitrage argument, the time  $t$ -price of a coupon bond maturing in  $T - t$  years, promising  $N_{c,t}$  identical coupon payments  $c$ , and paying €1 in  $T - t$  years, is given by

$$p(c, t, T) = \sum_{i=1}^{N_{c,t}} c \times B(t, t_i) + B(t, T). \quad (3.3)$$

In this formula,  $t_i$  stands for the  $i$ -th coupon payment date and  $t_{N_t}$  is the last payment date. Therefore,  $t_{N_t} = T$ . Because OATs and BTANs pay annual coupons, the set of payment dates also satisfies  $t_i - t_{i-1} = (t_i - t) - (t_{i-1} - t) = 1$  for all  $i \geq 1$ , that is, two cash flow payments are separated by one year. It is worth noting finally that the face value of French securities is not of course €1, but this issue is straightforward to address.<sup>1</sup> In what follows, one will denote by  $Y$  the yield-to-maturity of the coupon bond;  $Y$  makes the present value of future (annual) cash flows equal to the coupon bond price. And one has

$$p(c, t, T) = \sum_{i=1}^{N_{c,t}} \frac{c}{(1 + Y)^{t_i - t}} + \frac{1}{(1 + Y)^{T - t}}$$

---

<sup>1</sup>In contrast, nominal Treasury securities pay semiannual coupons so the cashflows would be in this case  $c/2$ .

---

It is straightforward to convert this yield to maturity into its continuously compounded counterpart ( $y = \ln(1 + Y)$ ). Another popular way among market participants is to express and quote bond prices in terms of par yields. The par yield over a certain horizon  $T$  is the coupon rate at which a coupon bond security maturing at  $T$  will trade at par. Setting the price of the coupon bond in equation (3.3) to  $p(c, t, T) = \$1$ , we obtain the solution for the coupon rate  $c \equiv y^c(t, T)$ :

$$y^c(t, T) = \frac{1 - B(t, T)}{\sum_{i=1}^{N_t} B(t, t_i)}. \quad (3.4)$$

While zero-coupon yields represent a mathematically simpler concept, market participants usually quote yields to maturity on coupon-bearing bonds and use par yields. We compute both par yields and zero-coupon yields in this paper.

The yield curve can also be expressed in terms of forward rates. A forward rate is the rate that an investor is able to lock in some time in the future by trading zero-coupon bonds of different horizons now. For example, if an investor wishes to lock in a  $m$ -period rate between  $T$  and  $T + m$  years in the future, this forward rate, denoted as  $f(t, T, m)$ , can be obtained as follows:

$$f(t, T, m) = -\frac{1}{m} \ln \frac{P(t, T + m)}{P(t, T)} = \frac{1}{m} ((T + m)y(t, T + m) - Ty(t, T)). \quad (3.5)$$

Taking the limit  $m \rightarrow 0$ , we obtain the instantaneous forward rate  $f(t, T, 0)$ :

$$f(t, T, 0) = \lim_{m \rightarrow 0} f(t, T, m) = y(t, T) + Ty'(t, T) = -\frac{\partial}{\partial T} \ln P(t, T). \quad (3.6)$$

Equation (3.6) essentially means that if the forward rate is above (below) the yield at a certain maturity, then the yield curve is upward (downward) sloping at that maturity. The zero-coupon yield over time  $T - t$  can be thought of as a continuous rollover of the instantaneous forward rate investments and therefore can be expressed as the average of the forward rates over the horizon  $T - t$ :

$$y(t, T) = \frac{1}{T - t} \int_t^T f(t, x, 0) dx. \quad (3.7)$$

It is useful to think of the forward rates rather than yields themselves as describing

---

the yield curves. For example, the 30-year OAT yield can be represented as the average of the one-year forward rates over 30 years. While forward rates at shorter horizons might be influenced by cyclical factors (such as monetary policy expectations), at longer horizons forward rates appear to be reflecting more fundamental factors like changes in the risk attitudes of investors. Zero-coupon yields combine information about these two types of factors in one number, while forward rates disentangle this information.

Finally, we introduce the concept of the modified duration used in our yield curve estimation:

$$D = \frac{D_{Mac}}{1 + Y}, \quad (3.8)$$

where  $Y$  stands for the yield-to-maturity and  $D_{Mac}$  is the Macaulay duration. It is well known that the Macaulay duration is the weighted average of the time (in years) that the investor must wait to receive the cash flows of a coupon bond. It is computed by

$$D_{Mac} = \frac{1}{p(c, t, T)} \sum_{i=1}^{N_{c,t}} (t_i - t) \times c \times B(t, t_i) + (T - t) \times F \times B(t, T).$$

The modified duration is very popular among participants because it connects more explicitly the change in yields to the change in prices, see, e.g., [Martellini, Priaulet, and Priaulet \[2003\]](#) for additional information about duration.

### 3.4.2 Svensson methodology

We broadly follow GSW to fit the nominal (BTAN- and OAT-based) par yield and zero-coupon yield curves using the [Svensson \[1994\]](#) methodology, which may be viewed as an augmented (and therefore more flexible) version of the [Nelson and Siegel \[1987\]](#) approach. The Svensson curve fitting approach relies on the premise that the curve associated with the instantaneous forward rates  $f(t, m, 0)$   $m$  periods ahead at time  $t$  and is correctly described by the following functional form:

$$f(t, m; \Theta) = \beta_0 + \beta_1 \exp\left[-\frac{m}{\tau}\right] + \beta_2 \frac{m}{\tau_1} \exp\left[-\frac{m}{\tau_1}\right] + \beta_3 \frac{m}{\tau_2} \exp\left[-\frac{m}{\tau_2}\right], \quad (3.9)$$

---

where  $\Theta = \{\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2\}$  are six Svensson parameters that need to be estimated. The instantaneous forward rate (3.9) starts at the level  $\beta_0 + \beta_1$  at a horizon zero and eventually converges to  $\beta_0$  as  $m$  approaches infinity. Thus,  $\beta_0 + \beta_1$  and  $\beta_0$  have a natural interpretation of the short rates at the short and long end of the yield curve. The functional form (3.5) is also flexible enough to accommodate two potential humps in the shape of the forward curve (observed, for example, in the U.S. Treasury forward rate curves). The third and fourth terms in the above equation control two humps of the curve, given that the respective parameters  $(\beta_2, \tau_1)$  and  $(\beta_3, \tau_2)$  specify the size and the location of these humps.

Zero-coupon yields are obtained by integrating  $f(t, m; \Theta)$  over the interest rate horizon  $[t, t + m]$  using (3.7) and (3.9):

$$y(t, t + m; \Theta) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} + \beta_2 \left[ \frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} - e^{-\frac{m}{\tau_1}} \right] + \beta_3 \left[ \frac{1 - e^{-\frac{m}{\tau_2}}}{\frac{m}{\tau_2}} - e^{-\frac{m}{\tau_2}} \right]. \quad (3.10)$$

Therefore, for a given set of parameters  $\Theta$ , the Svensson curve (3.9) defines the forward rate curve. From the latter, we can obtain the zero-coupon yields using (3.10) and par yields defined in (3.4). We use zero-coupon yields to price zero-coupon bonds and, consequently, compute the model-implied prices of the OAT coupon securities with a specific coupon rate and a specific maturity date. The next two subsections discuss certain data filters and estimation details.

### 3.4.3 Filters

In fitting the curve, we impose the following filters, following GSW.

1. First, we confine our database only to regular bonds with no special or option features. We therefore exclude floating-rate bonds, inflation-linked bonds, and bonds that were denominated in currencies other than FFs, ECUs, or euros. In addition, following GSW, we exclude STRIPS of OATs known as the “*certificats zéro-coupon fongibles*” that are available on the secondary market.

- 
2. We exclude BTFs from our investigation. As [Duffee \[1996\]](#) points out, with U.S. Treasury bills, it is not clear how innocuous is the use of the BTFs.<sup>1</sup>
  3. We exclude the short-duration securities—that is, all securities with less than 12 months to maturity—to prevent particular institutional details, unrelated to movements reflecting fundamentals, to affect the fit of the curve. For example, some long-term asset (pension or insurance) fund managers tend to sell off shorter-duration bonds in rebalancing their portfolios.<sup>2</sup>
  4. Unlike GSW, we did not exclude the on-the-run bond (that is, the most recently issued bond) and the first off-the-run bond (that is, the most recent bond after the on-the-run bond).<sup>3</sup>

### 3.4.4 Estimation

We collect at time  $t$  a set of observed bond prices  $(\hat{p}_c(t, T_k))_{k=1, \dots, N_{b,t}}$  where  $N_{b,t}$  is the number of coupon bond prices we observe at that time. Observed and model bond prices are related via the following relationship:

$$\hat{p}_c(t, T_k) = p(c, T_k; \Theta_t) + \varepsilon_k, \quad (3.11)$$

where  $\varepsilon_k$  is the error term with zero mean. One assumes that the vector of error terms  $\varepsilon' = (\varepsilon_1, \dots, \varepsilon_{N_{b,t}})$  has a diagonal covariance matrix with possibly different variances on the diagonal. The set of parameters  $\Theta_t$  is estimated by minimizing a weighted sum of squared errors whose weights are the inverses of the squared modified duration  $D$  defined in equation (3.8) for each coupon bond.

---

<sup>1</sup>In this way, we also avoid selecting a particular approach among the (discordant and sometimes debatable) empirical strategies found in the literature to deal with the French short-term debt securities. For example, [Ricart and Sicsic \[1995\]](#) select BTFs, BTANs, and OATs with time-to-maturity larger than one month and one year, respectively (for liquidity concerns), and they force the yield curve to fit exactly the yield of the next-to-repay BTF.

<sup>2</sup>GSW exclude bonds with remaining time to maturity of less than 18 months. We adopted a shorter threshold because initially the OAT market was not sufficiently mature. Using the 18-month threshold would have excluded, in relative terms, considerably more bonds from the model estimation.

<sup>3</sup>In fact, we have fit the curve excluding these two bonds but did not find significant differences. We discuss these findings in detail in Section 3.6.

---

More formally, the solution set satisfies

$$\hat{\Theta}_t = \arg \min_{\Theta_t} \sum_{k=1}^{N_{b,t}} \left[ \frac{\hat{p}_c(t, T_k) - p(c, T_k; \Theta_t)}{D_k} \right]^2 \quad (3.12)$$

where  $D_k$  is the modified duration of the  $k$ -th bond. This particular weighting scheme is an appropriate way to deal with the nonlinear relation between yields and prices (see [Svensson 1994](#); GSW; [Gauthier and Simonato 2012](#)). As explained by GSW (see their footnote 4 on page 2296), this way to proceed avoids converting bond prices into yields and therefore speeds up the calibration exercise. <sup>1</sup>

We place some constraints on the parameters according to their economic meaning. For instance,  $\tau_1$ ,  $\tau_2$ , and  $\beta_0$  are constrained to be positive numbers in our estimation. Note that we do not constrain  $\beta_0 + \beta_1$  to be positive to allow for the short negative rates, a feature prevalent in the advanced economies toward the end of our sample.

We then compute, at a given time  $t$ , mean absolute error (MAE) of the model fit for particular maturity bins. MAE simply averages the differences between the observed yield-to-maturity of the coupon bond and the one predicted by the model:

$$MAE_t(\tau) = \frac{1}{N_t(\tau)} \sum_{k=1}^{N_t(\tau)} \left| \hat{y}(t, k) - y(c, T_k; \hat{\Theta}_t) \right|, \quad (3.13)$$

where  $\tau$  represents the range of maturities over which the MAE has been computed;  $N_t(\tau)$  is the number of different bonds within a particular range  $\tau$ ;  $\hat{y}(t, k)$  is the observed yield-to-maturity of the  $k^{th}$  bond;  $T_k$  is the time-to-maturity of the  $k^{th}$  bond; and  $y(c, T_k; \hat{\Theta}_t)$  is the yield-to-maturity of the  $k^{th}$  bond predicted by the model that makes use of the fitted parameters  $\hat{\Theta}_t$ .

---

<sup>1</sup> Note that some other authors use more standard durations. For example, HPW use the Macauley duration in estimating the curve.

---

## 3.5 Results

In this section we discuss our estimation results—namely, we discuss the model fit and the implied term structure of OAT interest rates.

### 3.5.1 Model fit

Figure 3.3 plots the time series of the overall fitting errors. The measure of the overall fitting error on a particular day is the average of absolute errors between the predicted and market yields across all available securities that day. It is computed by

$$MAE_t = \frac{1}{N_{b,t}} \sum_{i=1}^{N_{b,t}} \left| \hat{y}(t, i) - y(c, T_i; \hat{\Theta}_t) \right|, \quad (3.14)$$

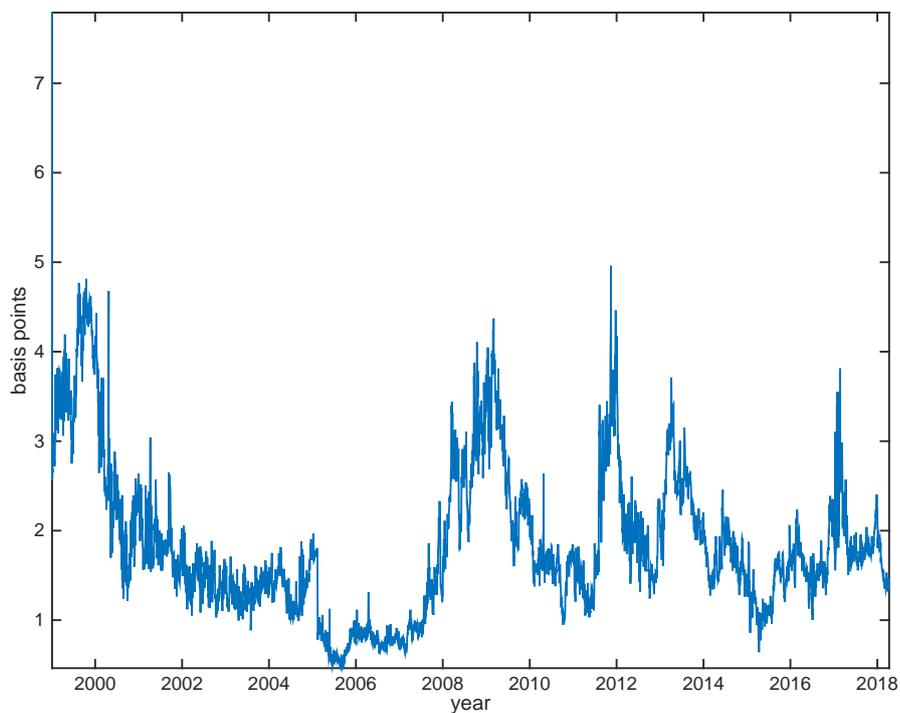
or equivalently,  $MAE_t = \frac{1}{N_{b,t}} \sum_{\tau=1}^{N_{\tau}} N_t(\tau) MAE_t(\tau)$ , where  $N_{\tau}$  is the number of maturity ranges (we also call them bins interchangeably). We show these errors for our benchmark sample, which is the sample period after the euro was introduced, from January 1, 1999, onwards.<sup>1</sup> One can see that the model does a very good job of fitting the cross section of OAT securities with only six parameters. Indeed, pricing errors do not exceed 5 basis points in the post-euro sample. This magnitude is definitely reasonable and consistent with GSW findings of the U.S. Treasury securities yields curve in the post-1980s sample period. In particular, the model fit has been improving from the onset of the euro area until the beginning of the 2007-08 subprime mortgage crisis period and the following 2008-09 GFC period. Then, the model fit has deteriorated temporarily. Consequently, the errors spiked again during the 2011-12 sovereign bond crisis (when, in particular, France has lost its AAA Standard & Poor's rating on January 9, 2012). A possible explanation for such variation throughout the post-euro sample period, consistent with the views of OAT market participants, is that the liquidity and attractiveness of OATs has generally improved over time after the launch of the euro area but deteriorated during the turmoil of the global financial and sovereign bond crises.

---

<sup>1</sup>We discuss the behavior of the model in the pre-euro period in Section 3.7.

---

Figure 3.3: Fitting Errors



This figure shows the total fitting error implied by the [Svensson \[1994\]](#) model. The fitting error is computed as the mean absolute error between the predicted and the market yields across all available BTAN and OAT securities on a particular day. The fitting errors are shown in basis points. Sample period: January 4, 1999, to April 10, 2018. Frequency: Daily.

Figure [B.10](#) in appendix plots the time series of fitting errors [\(3.13\)](#) for six maturity bins: 0 to 2 years, 2 to 5 years, 5 to 10 years, 10 to 20 years, 20 to 30 years, and 30 to 50 years. Interestingly, the fitting error magnitude and behavior differ according to the maturity interval. In particular, fitting errors in the 2 to 5 years maturity range (top-right chart) appear to be notably higher than overall during the U.S. subprime mortgage crisis period, while fitting errors in the 5 to 10 years range indicate the deterioration in the model fit during the 2011-12 sovereign debt crisis in the euro zone. For longer-term securities (the low row of panels in [Figure B.10](#)), the model fit has worsened, particularly during the GFC period.

Table 3.1: Summary Statistics of the Svensson Model Fitting Errors

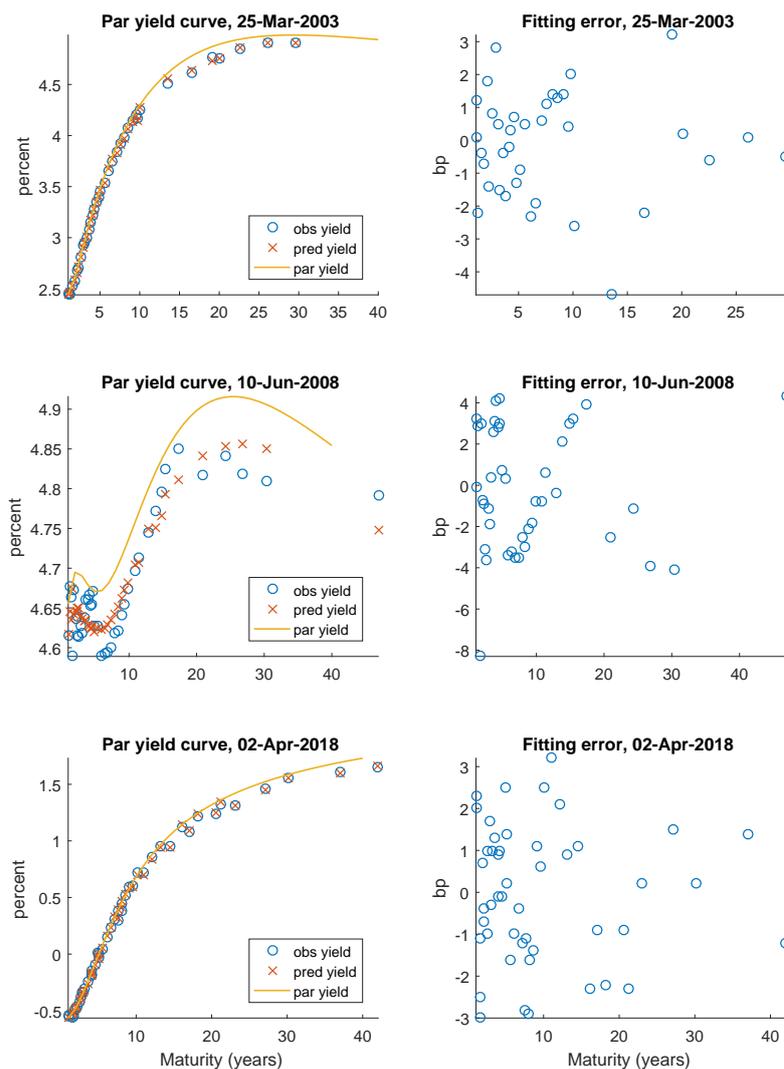
The table reports descriptive statistics of the daily fitting errors for securities in the indicated maturity bins for the full sample period, from January 4, 1988, to April 10, 2018 (Panel A) and for the euro sample period, from January 4, 1999, to April 10, 2018 (Panel B). The fitting errors are mean absolute errors between observed and predicted yields of the [Svensson \[1994\]](#) model. The errors are reported in basis points. Frequency: daily.

	0-2yr	2-5yr	5-10yr	10-20yr	20-30yr	50yr
<u>Panel A: Full sample period</u>						
Mean	2.90	4.74	5.92	4.42	4.66	3.38
Max	29.14	43.54	40.64	30.96	37.60	25.40
Min	0.07	0.32	0.30	0.00	0.01	0.00
Std. Dev.	3.90	6.43	7.27	4.11	6.11	4.59
<u>Panel B: Post-euro sample period</u>						
Mean	1.75	1.50	1.87	2.52	1.97	1.64
Max	14.27	6.02	11.34	9.00	10.84	10.89
Min	0.07	0.32	0.30	0.23	0.11	0.00
Std. Dev.	1.21	0.75	1.04	1.40	1.36	1.46

In addition, we report descriptive statistics of the overall fitting errors and fitting errors for different maturity bins in Table 3.1 for the full sample period (Panel A) and the post-euro sample period (Panel B). The numbers confirm our visual representation in Figures 3.3 and B.10 that fitting errors became smaller in the post-euro period (on average not exceeding 3 basis points) than in the full sample period (on average 5 basis points). While the maximum fitting error has reached 44 basis points in the pre-euro period, it was only 13 basis points in our benchmark period. The fit also became much more stable in the post-euro period: volatility of the fitting errors did not exceed 2 basis points during this period of time, but it was very high before the launch of the euro (as demonstrated by the full-sample average of 8 basis points). Finally, we observe the worst fit in the full sample period in the 5 to 10 years maturity range of OAT securities, while in the post-euro sample we observe it for the 10 to 20-year maturities.

Figure 3.4 shows the estimated Svensson nominal par yield curve on three different dates, which we picked in the three broadly defined periods: on March

Figure 3.4: Par Yield Curve



This figure shows the par yield curve and the fit of individual securities (left charts) along with security-specific fitting errors (right charts) of the securities for three days in our sample: March 25, 2003, June 10, 2008, and April 2, 2018. The curve is reported in annualized percent, the fitting errors are reported in basis points.

---

25, 2003 (during the “before the crisis” period), on June 10, 2008 (during the “crisis” period), and on April 2, 2018 (during the “after the crisis” period). The left-hand side of the figure shows the model-implied par yield curve along with observed (blue round circles) and predicted (red crosses) continuously compounded yields. The predicted yields are computed using parameters that are estimated using BTAN/OAT quotes on the indicated day. The right-hand side of the figure shows security-specific pricing errors computed as differences between observed and predicted yield to maturity. Thus, positive errors correspond to higher observed than predicted yields and, thus, lower observed than predicted bond prices. Therefore, in this case the model overprices bonds relative to observed prices. Alternatively, negative errors correspond to model underpricing relative to observed prices. Overall, we find that before and after the crisis, the range of values for the fitting errors remains relatively narrow, not exceeding 3 basis points in absolute values. However, during the crisis period, the fit of the curve worsened notably, likely reflecting a shortage of arbitrage capital and overall deteriorated market functioning.

### **3.5.2 The term structures of zero-coupon and forward rates**

We then investigate the term structures of zero-coupon and forward rates to document the different shapes and behaviors of the French yield curves we estimated.

#### **3.5.2.1 Shapes of the yield curve**

Table 3.2 reports the summary statistics of the fitted zero-coupon rates and associated instantaneous forward rates implied by the price quotes of BTAN and OAT securities. For six different horizons, it displays the average, maximum, and minimum values; volatility; skewness; kurtosis; and the autoregressive of order 1 coefficient, AR(1). On average, zero-coupon rates increase up to a horizon of 30 years and forward rates increase up to a horizon of 10 years. The volatility of zero-coupon rates is decreasing at short-to-intermediate horizons and then in-

Table 3.2: Summary Statistics about zero-coupon and instantaneous rates

This table reports summary statistics of the [Svensson \[1994\]](#) fitted zero-coupon yields (Panel A) and instantaneous forward rates (Panel B) for 2-, 5-, 7-, 10-, 30-, and 50-year maturities implied by our sample of the nominal BTAN and OAT securities. All statistics are reported in the annualized percent. Sample: January 1, 1999, to April 10, 2018. Frequency: daily.

	2yr	5yr	7yr	10yr	30yr	50yr
<u>Panel A: Zero-coupon rates</u>						
Mean	1.9479	2.5307	2.8724	3.2724	4.0152	3.8687
Max	5.1929	5.2737	5.3698	5.6587	6.3592	6.0439
Min	-0.6854	-0.4390	-0.2281	0.0812	1.0099	1.3050
Std. Dev.	1.7179	1.6415	1.5649	1.4747	1.2721	1.0402
Skewness	0.0230	-0.3257	-0.4725	-0.6002	-0.5388	-0.4773
Kurtosis	1.6567	1.8358	2.0114	2.2388	2.4323	2.5500
AR(1) coeff	0.9997	0.9997	0.9996	0.9996	0.9994	0.9965
<u>Panel B: Forward rates</u>						
Mean	2.3147	3.4560	3.9631	4.3916	4.0310	3.3305
Max	5.3084	5.8455	6.1765	6.6639	6.3982	5.6260
Min	-0.6878	0.0282	0.4885	0.9842	1.4695	1.2821
Std. Dev.	1.7585	1.5151	1.3886	1.2968	1.0249	0.6775
Skewness	-0.2411	-0.7569	-0.8827	-0.8416	-0.1699	0.1479
Kurtosis	1.7420	2.3778	2.7320	2.8636	2.5159	2.9619
AR(1) coeff	0.9995	0.9995	0.9994	0.9993	0.9960	0.9123

creasing. However, the volatility of forward rates is strictly decreasing with the horizon. We find nevertheless that the zero-coupon rate curve has had different shapes over our sample period as shown in Figure [B.11](#) in the appendix, which plots zero-coupon yield curves and instantaneous forward rate curves on the same days as Figure [3.4](#) does—namely, on March 25, 2003, June 10, 2008, and April 2, 2018. The zero-coupon rate curves appear on the left side of the figure and the forward rate curves are on the right side of it. These plots show various shapes yield curve shapes implied by the OATs. For example, the term structure on March 25, 2003, is upward-sloping until about the 30-year maturity point, after which it slopes down. This is a typical behavior of the term structure, as the very long end of the curve is affected by convexity and can be captured by the second hump in the Svensson function ([3.10](#)). Indeed, on March 25, 2003, the

Table 3.3: Correlation

This table reports correlations of Svensson [1994] fitted zero-coupon yields (Panel A) and instantaneous forward rates (Panel B) for 2-, 5-, 7-, 10-, 30-, and 50-year maturities implied by our sample of the nominal BTAN and OAT securities. Sample: January 1, 1999 to April 10, 2018. Frequency: daily.

	2yr	5yr	7yr	10yr	30yr	50yr
<u>Panel A: Zero-coupon rates</u>						
2yr	1.0000	0.9776	0.9547	0.9237	0.8727	0.8469
5yr		1.0000	0.9955	0.9813	0.9414	0.9136
7yr			1.0000	0.9950	0.9637	0.9368
10yr				1.0000	0.9812	0.9559
30yr					1.0000	0.9871
50yr						1.0000
<u>Panel B: Forward rates</u>						
2yr	1.0000	0.9381	0.8872	0.8526	0.8617	0.3777
5yr		1.0000	0.9894	0.9673	0.9061	0.4604
7yr			1.0000	0.9919	0.9113	0.4855
10yr				1.0000	0.9224	0.4907
30yr					1.0000	0.6547
50yr						1.0000

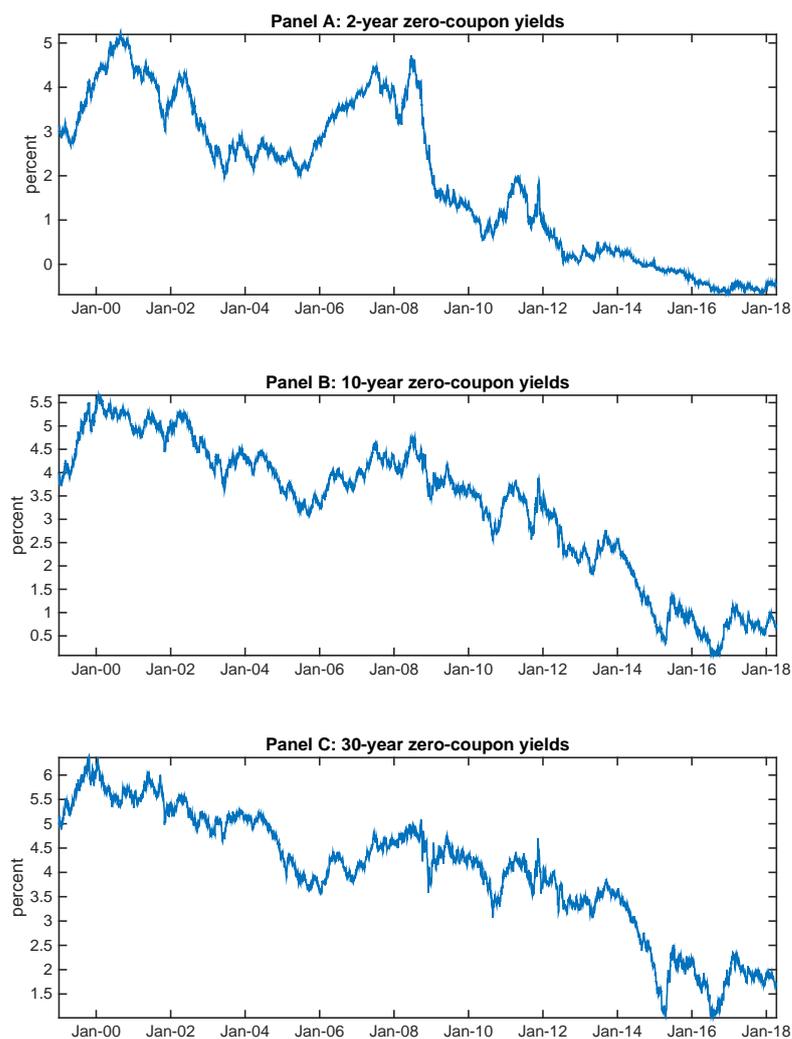
zero-coupon and forward yield curves have only one hump, but they experience changes in the sign of curve convexity (first negative and then positive for very long horizons). In general, the Svensson specification allows the term structure to have two humps and thus the Svensson formulation is more flexible relative to the Nelson-Siegel model.

The yield curve and forward rate curve on June 10, 2008, have two humps. In addition, the term structure for both zero-coupon and forward curves is downward-sloping, likely indicating worsening economic conditions. This in turn supports the widespread view that the financial crisis was indeed global and affected the growth prospects in many advanced economies, including France. Finally, toward the end of our sample, and as indicated by the graph on April 2, 2018, the yield curve becomes upward sloping.

---

### 3.5.2.2 Dynamics of the yield curve

Figure 3.5: Time Series of Zero-Coupon Yields



This figure shows the time series of the [Svensson \[1994\]](#) fitted 2-, 10-, and 30-year zero-coupon yields implied by the price quotes of BTANs and OATs from January 1, 1999, to April 10, 2018, at daily frequency.

Figure 3.5 plots 2-, 10-, and 30-year zero-coupon yields in our sample, from

---

1988 to 2018. It is obvious that the movements in the rates at these three tenors are highly correlated, although not perfectly correlated. Table 3.3 reports correlations among rates of different maturities, which vary from about 0.85 to about 0.99. The correlations are computed for daily series in the post-euro sample. In particular, the table shows that the correlations between the 2- and 10-year, 10- and 30-year, and 2- and 30-year zero-coupon rates are 0.92, 0.98, and 0.85, respectively. Figure 3.5 also indicates that all series declined following the peak of the GFC. In the beginning of 2015, the 2-year yields reached the zero-lower bound and declined further down into negative territories from then on, supporting the trend of declining and negative interest rates in other advanced economies in Europe.<sup>1</sup>

Turning to specific maturities, in our sample period the 2-year yield stayed in the range of 3 percent prior to the GFC period. It shortly reached five percent around 2001. The 2-year yield rose during the pre-crisis period in 2006 and 2007 and then started declining almost monotonically from about 4 percent level. Later in our sample, the 2-year yield increased shortly in 2010 and then declined sharply again around the 2011-12 sovereign debt crisis in Europe. The 10- and 30-year yields also declined similarly to the 2-year yield starting from about 4 or 5 percent levels depending on the maturity. At the end of the sample period, the 10- and 30-year zero-coupon rates had values of around 1 and 2 percent, respectively. We leave it to further research to investigate to what extent the decline in OAT-implied rates was due to the decline in expected short-term rates, the decline in term premiums, or both.

### 3.5.3 Factors of the yield curve

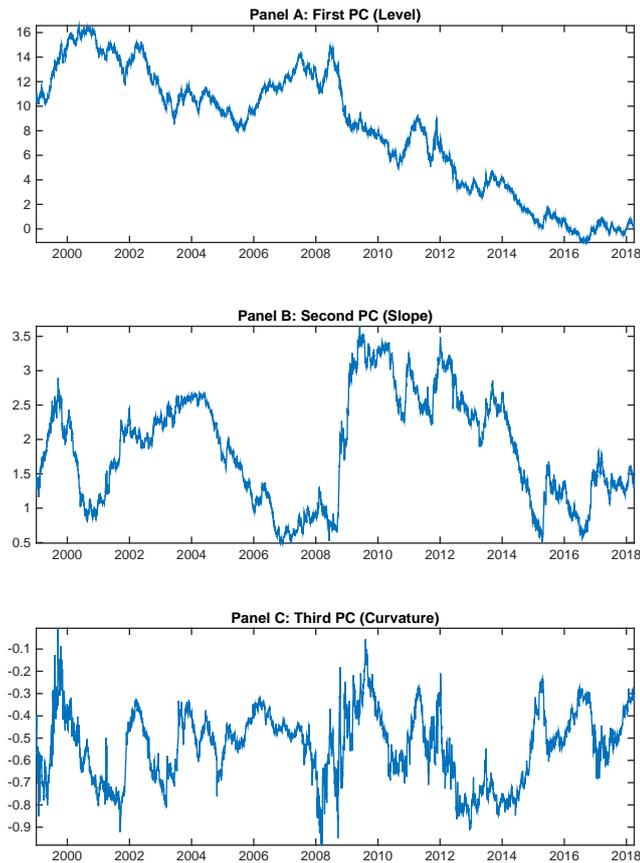
We investigate the dynamics of the yield curve by running a principal component analysis (PCA). It is widely known that most variations in U.S. Treasury yields can be explained by a few factors—namely, the first three principal components of the yield curve, loosely labeled as the level, slope, and curvature factors [see Bliss,

---

<sup>1</sup>Our OAT-implied zero-coupon yields and those available at the ECB website have correlations close to 1 and thus have also shown similar trends, although ECB reported yields were slightly higher during the sovereign financial crisis.

---

Figure 3.6: Principal Components of the Yield Curve



This figure shows time series of the first three principal components of the French zero-coupon rate curve. The principal component analysis used the zero-coupon yields of maturities from 1 to 10 years. Sample period: January 1, 1999, to April 10, 2018. Frequency: Daily.

1997; Litterman and Scheinkman, 1991]. We also derive the principal components from 1- to 10-year zero-coupon OAT yields in the 1999-2018 sample. According to Table 3.4, the first principal component explains 97.56 percent of the variation in OAT yields, and the second one explains 2.34 percent. Naturally, the rest of the yield curve variation is explained by the third and higher-order principal

---

components. Note that the level of a rate curve can be proxied by computing the arithmetic average of available interest rates (we select yields with maturities from 1 to 30 years). The slope can be proxied by the spread between a long-term rate and a short-term rate (and we select for this the 2- and 30-year yields). The curvature can be proxied by multiplying a middle-term yield by 2 and then by subtracting from the result the sum of a short-term yield and a long-term yield (we select the 5-, 18-, and 30-year yields). Factors constructed in this way yield a similar conclusion.

Table 3.4: Principal Component Decomposition

This table reports the percent of variance in [Svensson \[1994\]](#) fitted zero-coupon yields explained by the first three principal components. Full Sample: January 4, 1988, to April 10, 2018; Euro Sample: January 1, 1999, to April 10, 2018. Frequency: daily.

PC	Full Sample	Euro Sample
PC1	0.9572	0.9754
PC2	0.0395	0.0236
PC3	0.0030	0.0010

Figure [B.12](#) in the appendix plots the factor loadings for the principal components with non-normalized variance (Panel A) and the unit variance (Panel B). From the figure it is obvious that the first principal component is essentially a level factor because the yields at all maturities load similarly on this factor (the blue curve is roughly flat across maturities), and that the second principal component essentially captures the slope factor (red curve) because the loadings on short- and long-term maturities have different signs and magnitudes, while the relationship between loadings and maturities remains monotonic. Finally, the curvature factor is close to zero at all maturities (yellow curve on Panel B).

Figure [3.6](#) shows times series for the first three principal components. First, the level factor shows that, on average, the downward trend in French interest rates is consistent with declines in other advanced economies. Second, the level factor became slightly negative at the end of this period. This is in line with declining 2-, 10-, and 30-year yields, shown on Figure [3.5](#). Third, the slope factor also shows

---

significant variations. In the United States, the slope factor is known to be an important predictor of future GDP growth. In particular, the decline in the slope corresponds to the flattening of the yield curve that is found to be associated with a slowdown in future economic activity [Ang, Piazzesi, and Wei, 2006]. Finally, the curvature factor is also highly time-varying, although this variation appears to be on a much smaller scale in absolute values. Thus, the third factor appears to be a relatively less important factor given that it explains less than 1 percent of the variation in yields in our sample.

### 3.6 Is there any on-the-run premium on the French bond market?

In this section we explore the phenomenon of the on-the-run premium on the French government bond market. The phenomenon refers to the fact that, in general, newly issued bonds are sought by market participants and therefore are more expensive than other bonds with similar time-to-maturity characteristics that were issued earlier. The existence of the sizeable on-the-run, or liquidity, premium has been well established on the U.S. nominal Treasury securities market (see, Fleming 2003; Pasquariello and Vega 2009, and GSW). To assess how large the on-the-run premium could be on the French market, we use two approaches: the approach of Fleming [2003] and then the GSW approach.

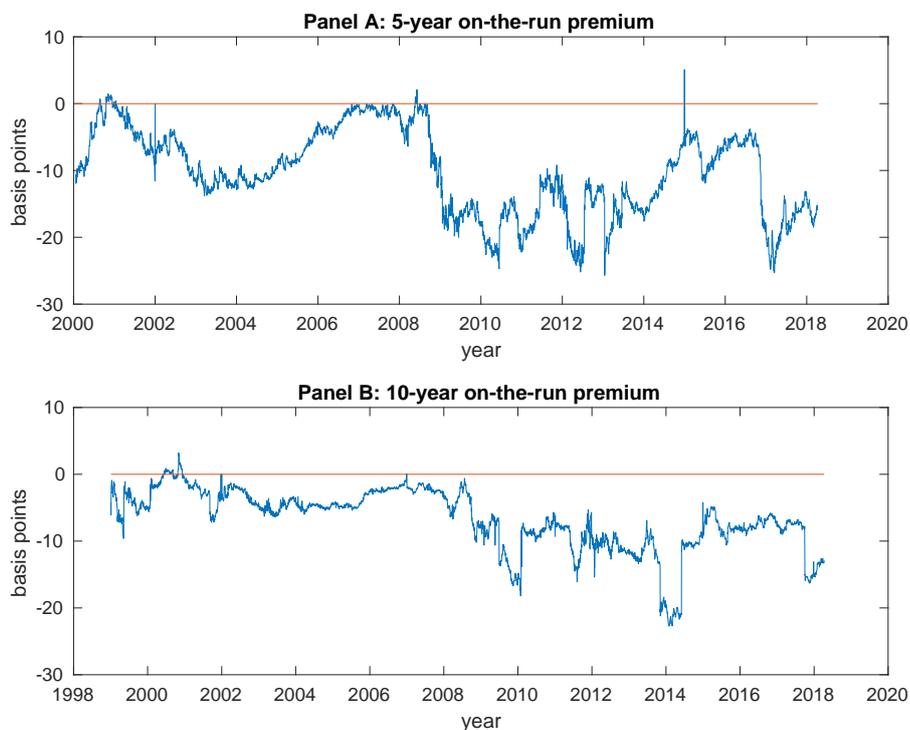
We start with the definition of the premium. The most recently issued security is called the “on-the-run” security and the one issued right before it is called the “first off-the-run” security. Thus, we define the spread between the on-the-run security and the first off-the-run security for a particular maturity  $n$  as:

$$OTR_{t,n} = y_{t,n}^{\text{off-the-run}} - y_{t,n}^{\text{on-the-run}}. \quad (3.15)$$

The spread in (3.15) is expected to be positive when market participants seek the on-the-run security that would be traded, therefore, at a higher price (and a lower yield) than the first off-the-run security, everything else being equal.

---

Figure 3.7: Fleming On-the-run Premium



This figure shows the time series of the on-the-run/off-the-run yield spreads for securities in the indicated maturity bins for the post-euro sample period. The spreads are calculated as in Fleming 2003 as the differences between the end-of-day yields of the on-the-run and the first off-the-run BTAN or OAT securities. The differences are reported in basis points. Sample period: January 1, 1999, to April 10, 2018. Frequency: Daily.

Figure 3.7 plots the time series of the on-the-run premium for the 5- and 10-year maturity bonds. Following Fleming [2003] methodology, we define 6 categories of bonds according to their time to maturity at issuance (less than 5 years, around 5 years, around 10 years, around 15 years, around 30 years and 50 years). The on-the-run premium is calculated as the difference in yield for the off-the-run and on-the-run securities. Once there is a new security issue, we define this newly issued bond as on-the-run. The bond that was recently on-the-run becomes off-the-run. During the period of time between two issues, on-the-run and off-the-run securities remain the same. Once there is a new issue, we define a new pair of on-the-run and off-the-run securities.

Table 3.5: On-the-Run/Off-the-Run Yield Spreads

The table reports descriptive statistics of the on-the-run/off-the-run yield spreads for securities in the indicated maturity bins for the full sample period (Panel A) and the post-euro sample period (Panel B). The spreads are calculated as in Fleming [2003] as the differences between the end-of-day yields of the on-the-run and the first off-the-run BTAN or OAT securities. The differences are reported in basis points. Frequency: daily. Source: Bloomberg.

Maturity bin, yrs	Mean	Median	St.Dev
Panel A: Full sample period: March 8, 1988 - April 10, 2018			
1 - 4	-7.22	-11.95	55.23
4 - 6	-9.80	-8.70	17.47
8 - 12	-5.07	-5.20	15.62
15 - 16	-17.44	-17.10	9.07
20 - 32	-1.05	-1.40	20.97
Panel B: Euro-area period: January 4, 1999 - April 10, 2018			
1 - 4	-14.49	-13.40	35.78
4 - 6	-10.17	-10.10	6.30
8 - 12	-6.76	-6.10	4.52
15 - 16	-17.07	-16.40	9.62
20 - 32	-3.90	-1.70	10.99

We report in Table 3.5 the average and median in these spreads and their associated standard deviation. We report these statistics per maturity ranges. For this table, we compute the spreads using the Bloomberg end-of-the-day quotes for a particular security. We find that both average and median spreads are negative. The standard deviations are relatively high for most of maturity ranges as well, suggesting the absence of the on-the-run premium on the French government bond market.

The drawback of this finding is that the yields compared in equation (3.15) may have (slightly) different maturities, meaning that the first off-the-run security tends to have a shorter duration. This difference in duration may distort the on-the-run-off-the-run spread.<sup>1</sup> To address this concern, we also compute the on-the-run premium following the GSW approach, where the observed yield-to-maturity

<sup>1</sup>See also GSW on this point.

---

of the on-the-run security is compared to the predicted yield-to-maturity of a so-called *synthetic* bond with identical characteristics (expiration date, coupon rate, and coupon frequency).

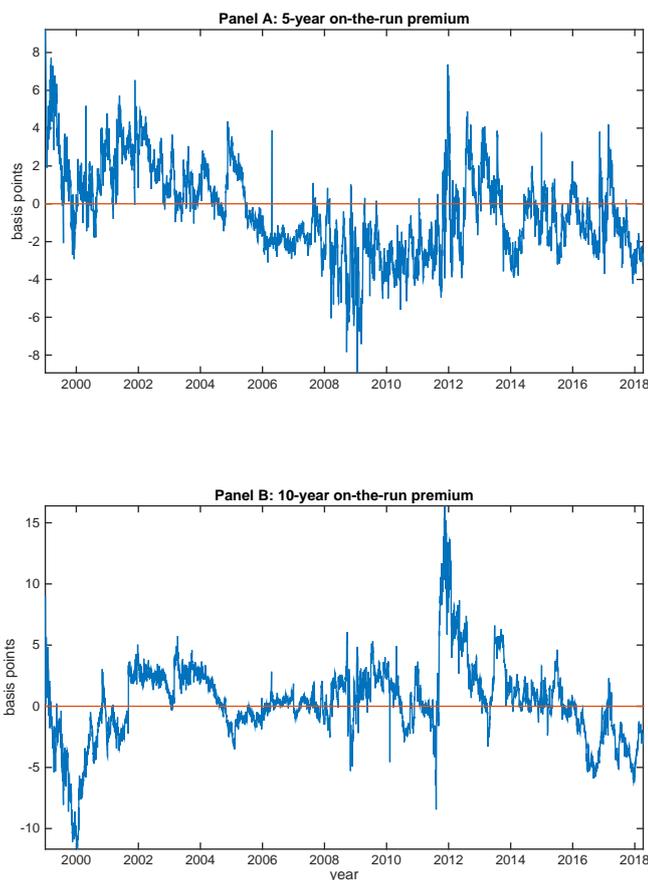
To fit the yield curve for this purpose, we exclude from the cross section of available bonds both the on-the-run and the first off-the-run bonds from each maturity range provided in Table 3.5. We refit the Svensson par yield curve, obtain the predicted price of the synthetic security, compare to the observed prices of the security with the same characteristics and compute the on-the-run premium as the difference between the two.

Figure 3.8 plots the time series of the on-the-run premium for the 5- and 10-year maturity bonds. We compute the premium for the current on-the-run bond, then for the next on-the-run bond when the new bond is issued, and so on. Thus, we obtain the time series of the on-the-run premium, as we rollover the on-the-run securities in our sample. Therefore, the on-the-run premium is not related to a particular bond, but only to a particular-maturity security. As it is obvious from Panel A, the 5-year premium varies between negative 8 basis points and positive 6 basis points, but most of the time it does not exceed 3 to 4 basis points (in the absolute value). The order of the magnitude of the 5-year on-the-run premium broadly corresponds to the fitting error magnitude of that same maturity (see the top-right panel of Figure B.10 in the appendix). During the GFC, the premium briefly reached 6 to 7 basis points; however, during the European sovereign debt crisis the premium appeared to be even slightly negative.

As Panel B shows, the 10-year on-the-run premium has also been hovering within the 5 basis points band, but it also reached briefly 15 basis points at the time of the GFC. However, according to Table 3.5, the premium remains small on average. Our findings are in contrast to the numbers reported by GSW for the U.S. on-the-run premium that was as high as 30 to 40 basis points during several periods—for example, in the early 2000s—but appeared to have declined toward the end of its sample in 2006 to about 10 basis points. However, our results appear to be in line with some evidence of the absence of on-the-run premium in the German sovereign bond market [Ejsing and Sihvonen, 2009]. These authors

---

Figure 3.8: GSW On-the-run Premium



This figure shows the time series of the 5-year (Panel A) and 10-year (Panel B) on-the-run premiums of the BTAN and OAT securities, respectively, following GSW methodology using synthetic bond. Sample period: January 1, 1999, to April 10, 2018. Frequency: Daily.

relate it to the existence of a mature futures market and the set of deliverable bonds in the futures contracts. In the French OAT market, this explanation potentially differs as the French OAT market has employed a futures market to which OATs could be delivered. The MATIF has proposed for years a “contrat notionnel”. Moreover, anecdotal evidence suggests that OATs (and BTANs when relevant) were essentially perceived by investors as commodities. More recently, in

---

2012 and 2013, the EUREX exchange launched the Euro-OAT Futures (FOATs) and the Mid-Term Euro-OAT Futures (FOAMs). The very active repo market as well as the French OAT-based futures market can, at least in part, explain the attractiveness of all, not only the most recently issued bonds. However, these two markets are far too recent to explain the absence of the on-the-run premium entirely.

Overall our findings of the negligible on-the-run premium motivates us to keep recently issued bonds in the cross section of OAT securities for our benchmark curve estimation.

### 3.7 A closer look at the pre-euro era

In this section we report some results related to the period preceding the launch of the euro. We would like to emphasize some observable and significant differences between the periods preceding and following January 1, 1999. We examine the functioning/illiquidity of the French BTAN/OAT bond market through the lens of the noise measure of introduced by HPW. Their proxy for illiquidity is defined as the root mean squared error between the market yields  $\hat{y}(t, i)$  and model-implied yields  $y_p(c, T_i; \hat{\Theta}_t)$ :

$$\text{Noise}_t = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left( \hat{y}(t, i) - y_p(c, T_i; \hat{\Theta}_t) \right)^2}, \quad (3.16)$$

where  $N_t$  is the number of considered bond securities on day  $t$ . The idea behind the *noise* measure (3.16) is that it indicates the availability of arbitrage capital on the bond market (or, on a different closely related market). When the arbitrage capital is abundant, arbitrage opportunities disappear quickly, so the prices converge quickly to the fundamentals, and observed prices are relatively close to the predicted prices estimated by the arbitrage-free model than in the periods when there is a shortage of such capital. Therefore, an increase in the noise measure indicates deterioration in market functioning conditions; conversely, a decrease reflects an improvement in market functioning conditions. This measure has been

---

used widely by various researchers as a proxy for the liquidity measure in the U.S. Treasury securities market [see [Andreasen, Christensen, and Riddell, 2017](#); [Grishchenko and Huang, 2013](#); [Malkhozov, Mueller, Vedolin, and Venter, 2017](#)]. Naturally, the noise measure (3.16) and the mean absolute error measure (3.13) are closely related.

### 3.7.1 Model fit

Similar to Figure B.10, Figure B.13 in the appendix shows the MAEs (3.13) in the pre-euro period per maturity bins for our model (3.7). There are blank spaces in the period from 1988 to 1994 (for the 0 to 2 years range) and from 1988 to 1992 (for the 2 to 5 years range), as the OAT market was at its early stage of development. At that time, there were no OATs with remaining maturities of less than 5 years, and also there were no shorter term OATs issued at that time. At that time, pricing errors were sometimes as high as 20 to 40 basis points depending on the maturity. In comparison, GSW report that the average absolute errors were quite high in their early part of their sample period, ranging from 40 to 80 basis points across different maturity ranges.

Similar to Section 3.5, we also demonstrate the model fit in the pre-euro area for several days. Figure B.14 in the appendix shows the estimated Svensson nominal par yield curve on two different dates preceding the euro launch—January 4, 1988, and September 20, 1995—and also in the beginning of the sample following the Euro launch, January 5, 1999. The left-hand side of the figure shows the model-implied par yield curve along with observed (blue round circles) and predicted (red crosses) yields on these three days. The predicted yields to maturity are computed by using parameters that are estimated on that day. The right-hand side of the figure shows security-specific pricing errors. The two upper graphs that are associated with January 4, 1988, highlight the fitting consequence of using only a few securities whose maturities are concentrated around the 10-year tenor. One of the securities appears to be especially poorly priced (with the pricing errors of negative 40 basis points).<sup>1</sup> With the exception of this security,

---

<sup>1</sup>Informal interviews of practitioners confirm that the arbitrage opportunities were not infrequent at that time.

---

the pricing errors ranged between zero and 10 basis points. As the middle charts that correspond to September 20, 1995, show, in the mid 1990s the French market appeared to have been far more developed than in the late 1980s, as the number of available bond price quotes were larger and the maturity of these bonds was more diverse.<sup>1</sup> However, the range of pricing errors was, though smaller, still quite large, at between negative 10 to positive 15 basis points. Finally, the bottom charts present the yield curve and curve fit on January 5, 1999. Although the fitting errors still ranged from negative 10 to positive 15 basis points, it is interesting to observe the yield curve in the very beginning of the euro period.

### 3.7.2 Noise measure

We also assess and compare the quality of the functioning of the French market before and after the launch of the euro using the noise measure (3.16). Figure 3.9 demonstrates quantitatively how the market functioning improved after the launch of the euro. The noise measure fluctuated between 5 and 20 basis points before 1999 and reached 35 basis points at certain times in the pre-euro period. Upon the introduction of the euro, the noise measure plummeted almost instantaneously to levels around or below 5 basis points. Thus, the large “noise” values before 1999 can indicate mispricing and, therefore, the existence of arbitrage opportunities. The volatility of the noise measure can be indicative of some arbitrage activities. It is interesting to note that the noise measure never exceeded 35 basis points, suggesting that, in general, the fitting ability of the Svensson model is reasonable (as it was illustrated by the discussion of Figure B.14 in the previous subsection).

## 3.8 Conclusion

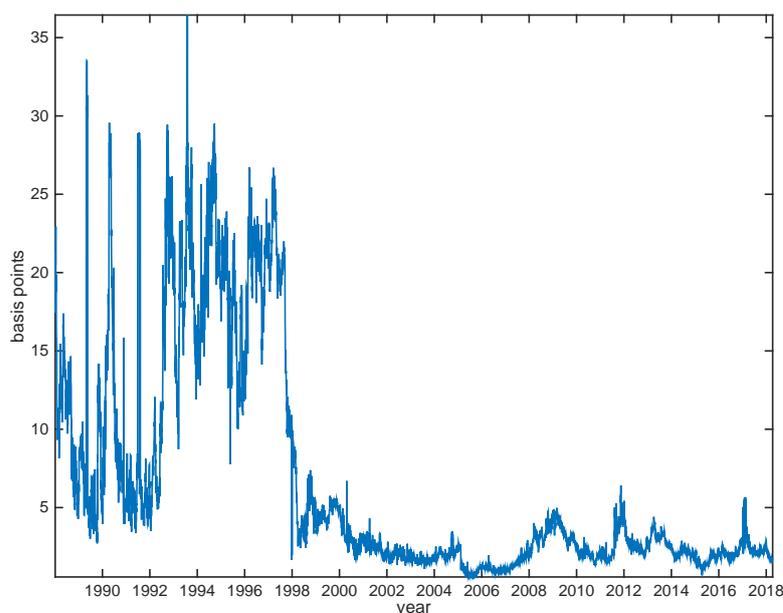
Our study is the first comprehensive study of all publicly available data of the French nominal debt that encompasses the 30-year period from 1988 to 2018. In particular, we construct the French nominal yield curve using quotes of the French nominal government bond securities called OATs and BTANs, at a daily

---

<sup>1</sup>We showed a usual day at that time period for the middle charts on Figure B.14.

---

Figure 3.9: Noise Measure of the French Bond Market



This figure shows the noise measure, which is computed as the root mean squared error between the observed and predicted yields across all available OAT/BTAN securities on a particular day. Frequency: daily. Sample period: January 4, 1988, to April 10, 2018.

frequency. These bonds have maturities at issuance ranging from 1 to 50 years. Our sample period starts in 1988, includes the launch of the euro in January 1999 and ends in April 2018. We use the Svensson smoothing method to interpolate the curve and manage to fit the curve quite well.

Overall, we find that in the first decade, the arbitrage opportunities were not infrequent on the OAT market, but that the situation improved substantially since the euro introduction. Since then, the market functioned reasonably well outside of a few episodes—notably, the GFC period and the European sovereign crisis period. We also find that, in sharp contrast to the U.S. nominal Treasury securities market, on-the-run securities have, on average, a negligible liquidity premium. Lastly, we provide evidence that, similar to other developed economies,

---

French interest rates have been declining since the GFC, and the slope of the French yield curve declined as well, potentially signaling some downside risks to the growth of the French economy.

Our results and available yield curve data should be valuable to monetary policy-makers as well as financial and macroeconomic researchers of European fixed income markets. We plan to update our results regularly.

# Appendices

Table B.6: Summary of the Nominal BTAN and OAT Securities, 1987 - 1998

This table shows the sample of the nominal BTAN and OAT securities issued from 1987 to 1998. Column "Type" refers to the type of the security (BTAN or OAT), Column "ISIN" to the ISIN number and Column "Coupon" to the coupon rate of the security in percent. Column "First Qte" reports the first date on which the quote for the security becomes available, Column "Maturity" provides the expiration date, Column "Term" specifies the term-to-maturity of the bond and the "Obs" column reports the total number of available observations for the security. Frequency of the data: daily. Source: Bloomberg.

Type	ISIN	Coupon	First Qte	Maturity	Term	Obs	Type	ISIN	Coupon	First Qte	Maturity	Term	Obs
OAT	FR0000041410	11.6	01/07/1987	08/10/1996	9.27	1313	BTAN	FR0100059486	4.75	04/01/1994	12/04/1999	5.27	1372
OAT	FR0000570095	10	01/07/1987	27/05/2000	12.91	2772	BTAN	XB000A113817	4.5	04/01/1994	12/05/1996	2.35	614
OAT	FR0000043705	9.9	01/07/1987	13/12/1997	10.45	2614	BTAN	FR0100059478	5	10/01/1994	16/03/1999	5.18	1304
OAT	FR0000100257	9.7	01/07/1987	30/01/1996	8.58	2203	OAT	FR0000571150	6	24/01/1994	25/10/2025	31.75	5312
OAT	FR0000100240	9.8	01/07/1987	13/12/1997	10.45	2637	OAT	FR0000570228	6.75	29/04/1994	25/04/2002	7.99	1564
OAT	FR0000570780	8.5	01/07/1987	26/12/2012	25.49	6587	OAT	FR0000570400	6.75	26/05/1994	25/10/2004	10.42	2717
OAT	FR0000102469	8.5	01/07/1987	25/06/1997	9.98	2570	BTAN	FR0100059502	7	06/07/1994	12/11/1999	5.35	1397
OAT	FR0000110488	9.5	03/03/1988	25/06/1998	10.31	2664	BTAN	XB000A114096	6.5	06/07/1994	12/10/1996	2.27	593
OAT	FR0000570327	8.25	01/07/1988	27/02/2004	15.66	3900	OAT	FR0000570434	7.5	26/09/1994	25/04/2005	10.58	2760
OAT	FR0000570921	8.5	06/01/1989	25/10/2019	30.8	7862	BTAN	FR0100059528	7.75	06/01/1995	12/04/2000	5.26	1374
OAT	FR0000570038	8.125	06/01/1989	25/05/1999	10.38	2689	BTAN	XB000A114468	7.5	23/02/1995	16/03/1997	2.06	536
OAT	FR0000114308	8.5	27/04/1989	12/05/1997	8.04	2087	BTAN	XB000A114476	7.25	07/03/1995	12/08/1997	2.43	634
OAT	FR0000570178	7.5	31/07/1989	25/07/2001	11.98	1876	OAT	FR0000570467	7.75	28/03/1995	25/10/2005	10.58	2760
OAT	FR0000570053	8.5	04/01/1990	28/03/2000	10.23	2659	BTAN	FR0100059544	7	11/07/1995	12/10/2000	5.26	1371
OAT	FR0000570061	9.5	12/04/1990	25/04/2000	10.04	2264	OAT	FR0000570491	7.25	27/09/1995	25/04/2006	10.58	2758
OAT	FR0000570152	10	06/11/1990	26/02/2001	10.31	2180	BTAN	FR0100000365	5.75	10/11/1995	12/03/1998	2.34	606
OAT	FR0000570145	9.5	03/01/1991	25/01/2001	10.06	2619	BTAN	FR0100059551	5.75	08/01/1996	12/03/2001	5.17	1350
BTAN	XB000A112181	9	08/01/1991	12/02/1996	5.1	1325	BTAN	FR0100059569	6	09/01/1996	16/03/2001	5.18	1130
OAT	FR0000570194	8.5	24/04/1991	15/03/2002	10.89	2349	OAT	FR0000570731	6.5	25/01/1996	25/04/2011	15.25	3976
BTAN	XB000A112413	8.5	19/07/1991	12/11/1996	5.32	1384	OAT	FR0000570533	6.5	27/03/1996	25/10/2006	10.58	2759
OAT	FR0000571085	8.5	31/12/1991	25/04/2023	31.32	6851	OAT	FR0000570509	7	08/05/1996	25/04/2006	9.96	2123
BTAN	XB000A112728	8.5	13/01/1992	12/03/1997	5.16	1345	BTAN	FR0100024795	4.5	18/06/1996	12/10/1998	2.32	596
OAT	FR0000571044	8.25	22/01/1992	25/04/2022	30.26	6569	BTAN	FR0100059577	5.5	08/07/1996	12/10/2001	5.26	1373
OAT	FR0000570665	8.5	02/06/1992	25/10/2008	16.4	4278	OAT	FR0000570574	5.5	17/12/1996	25/04/2007	10.35	2700
OAT	FR0000570277	8.5	24/06/1992	25/04/2003	10.83	2827	BTAN	FR0100059585	4.75	24/01/1997	12/03/2002	5.13	1317
BTAN	XB000A113007	8.5	08/07/1992	12/11/1997	5.35	1391	OAT	FR0000570590	5.5	01/07/1997	25/10/2007	10.32	2691
BTAN	XB000A113270	8	31/12/1992	12/05/1998	5.36	1394	BTAN	FR0100015967	4.5	15/07/1997	12/07/2002	4.99	381
OAT	FR0000570285	8	03/02/1993	25/04/2003	10.22	1688	BTAN	FR0100059593	4.5	16/07/1997	12/07/2002	4.99	1303
BTAN	XB000A113346	7.25	24/02/1993	16/03/1998	5.05	1312	BTAN	FR0100059510	4	18/09/1997	12/01/2000	2.32	603
OAT	FR0000570301	6.75	30/03/1993	25/10/2003	10.57	2756	OAT	FR0000570632	5.25	07/01/1998	25/04/2008	10.3	2686
OAT	FR0000194995	6	25/06/1993	16/07/1997	4.06	550	BTAN	FR0100059601	4.5	23/02/1998	12/07/2003	5.38	1405
BTAN	XB000A113528	5.75	25/06/1993	12/11/1998	5.38	1383	BTAN	FR0100033242	4.5	24/02/1998	12/07/2003	5.38	223
OAT	FR0000570368	5.5	26/10/1993	25/04/2004	10.5	2738	OAT	FR0000571218	5.5	26/02/1998	25/04/2029	31.16	5245
OAT	FR0000570343	6	29/10/1993	25/04/2004	10.49	2355	BTAN	FR0100034208	4	08/04/1998	12/07/2000	2.26	191
							BTAN	FR0100059536	4	20/05/1998	12/07/2000	2.15	560
							OAT	FR0000571432	4	01/10/1998	25/04/2009	10.57	2755

Table B.7: Summary of the Nominal BTAN and OAT Securities, 1999 - 2009

This table shows the sample of the nominal BTAN and OAT securities issued from 1999 to 2009. Column "Type" refers to the type of the security (BTAN or OAT), Column "ISIN" to the ISIN number and Column "Coupon" to the coupon rate of the security in percent. Column "First Qte" reports the first date on which the quote for the security becomes available, Column "Maturity" provides the expiration date, Column "Term" specifies the term-to-maturity of the bond and the "Obs" column reports the total number of available observations for the security. Frequency of the data: daily. Source: Bloomberg.

Type	ISIN	Coupon	First Qte	Maturity	Term	Obs	Type	ISIN	Coupon	First Qte	Maturity	Term	Obs
BTAN	FR0100802273	3.5	21/01/1999	12/07/2004	5.47	1429	OAT	FR0010163543	3.5	01/02/2005	25/04/2015	10.23	2667
BTAN	FR0100877812	3	18/03/1999	12/07/2001	2.32	606	BTAN	FR0107489959	2.25	15/02/2005	12/03/2007	2.07	539
OAT	FR0000186199	4	04/05/1999	25/10/2009	10.48	2733	OAT	FR0010171975	4	24/02/2005	25/04/2055	50.16	3421
BTAN	FR0101465831	4	20/10/1999	12/01/2002	2.23	576	OAT	FR0010192997	3.75	03/05/2005	25/04/2021	15.98	3375
BTAN	FR0101659813	5	19/01/2000	12/07/2005	5.48	1431	BTAN	FR0107674006	2.5	14/06/2005	12/07/2010	5.08	1324
OAT	FR0000186603	5.5	27/01/2000	25/04/2010	10.24	2671	OAT	FR0010216481	3	07/07/2005	25/10/2015	10.3	2685
BTAN	FR0102325695	5	17/08/2000	12/01/2003	2.4	627	BTAN	FR0108197569	2.75	17/11/2005	12/03/2008	2.32	604
OAT	FR0000187023	5.5	07/09/2000	25/10/2010	10.13	2641	BTAN	FR0108354806	3	17/01/2006	12/01/2011	4.99	1301
BTAN	FR0102626779	5	18/10/2000	12/01/2006	5.23	1368	OAT	FR0010288357	3.25	30/01/2006	25/04/2016	10.23	2665
OAT	FR0000187361	5	16/01/2001	25/10/2016	15.77	4112	BTAN	FR0108847049	3.5	09/06/2006	12/07/2011	5.09	1328
BTAN	FR0103230423	4.5	20/04/2001	12/07/2006	5.23	1363	BTAN	FR0109136137	3.5	19/07/2006	12/09/2008	2.15	562
OAT	FR0000187635	5.75	07/06/2001	25/10/2032	31.38	4391	OAT	FR0010371401	4	30/08/2006	25/10/2038	32.15	3029
OAT	FR0000187874	5	06/09/2001	25/10/2011	10.13	2640	OAT	FR0010415331	3.75	29/12/2006	25/04/2017	10.32	2688
BTAN	FR0103536092	4	17/09/2001	12/01/2004	2.32	605	BTAN	FR0109970386	3.75	15/01/2007	12/01/2012	4.99	1305
BTAN	FR0103840098	3.75	20/11/2001	12/01/2007	5.14	1342	BTAN	FR0110979178	4	17/04/2007	12/09/2009	2.41	629
OAT	FR0000188328	5	01/03/2002	25/04/2012	10.15	2647	OAT	FR0010466938	4.25	30/04/2007	25/10/2023	16.49	2856
OAT	FR0000570244	8.5	25/03/2002	25/11/2002	0.67	97	BTAN	FR0110979186	4.5	18/06/2007	12/07/2012	5.07	1324
BTAN	FR0104446556	4.75	13/05/2002	12/07/2007	5.16	1348	OAT	FR0010517417	4.25	03/09/2007	25/10/2017	10.14	2645
OAT	FR0000188690	4.75	02/09/2002	25/10/2012	10.15	2647	BTAN	FR0113087466	3.75	14/01/2008	12/01/2013	5.00	1304
BTAN	FR0104756962	3.5	12/09/2002	12/01/2005	2.34	610	OAT	FR0010604983	4	01/04/2008	25/04/2018	10.06	2615
BTAN	FR0105427795	3.5	15/01/2003	12/01/2008	4.99	1303	BTAN	FR0113872776	3.75	12/05/2008	12/09/2010	2.34	610
OAT	FR0000188989	4	28/02/2003	25/04/2013	10.15	2648	BTAN	FR0114683842	4.5	15/07/2008	12/07/2013	4.99	1303
OAT	FR0000189151	4.25	02/06/2003	25/04/2019	15.9	3876	OAT	FR0010670737	4.25	01/10/2008	25/10/2018	10.06	2484
BTAN	FR0105760112	3	16/06/2003	12/07/2008	5.07	1325	BTAN	FR0116114978	2.5	20/01/2009	12/01/2014	4.98	1298
OAT	FR0010011130	4	27/08/2003	25/10/2013	10.16	2651	BTAN	FR0116843519	1.5	18/05/2009	12/09/2011	2.32	608
BTAN	FR0106589437	3.5	16/01/2004	12/01/2009	4.99	1301	BTAN	FR0116843535	3	17/06/2009	12/07/2014	5.07	1323
OAT	FR0010061242	4	24/02/2004	25/04/2014	10.17	2652	OAT	FR0010773192	4.5	23/06/2009	25/04/2041	31.84	2295
BTAN	FR0106589445	2.25	12/03/2004	12/03/2006	2.00	521	OAT	FR0010776161	3.75	30/06/2009	25/10/2019	10.32	2290
OAT	FR0010070060	4.75	23/03/2004	25/04/2035	31.09	3660							
BTAN	FR0106841887	3.5	11/06/2004	12/07/2009	5.08	1326							
OAT	FR0010112052	4	26/08/2004	25/10/2014	10.16	2650							
BTAN	FR0107369672	3	17/11/2004	12/01/2010	5.15	1344							

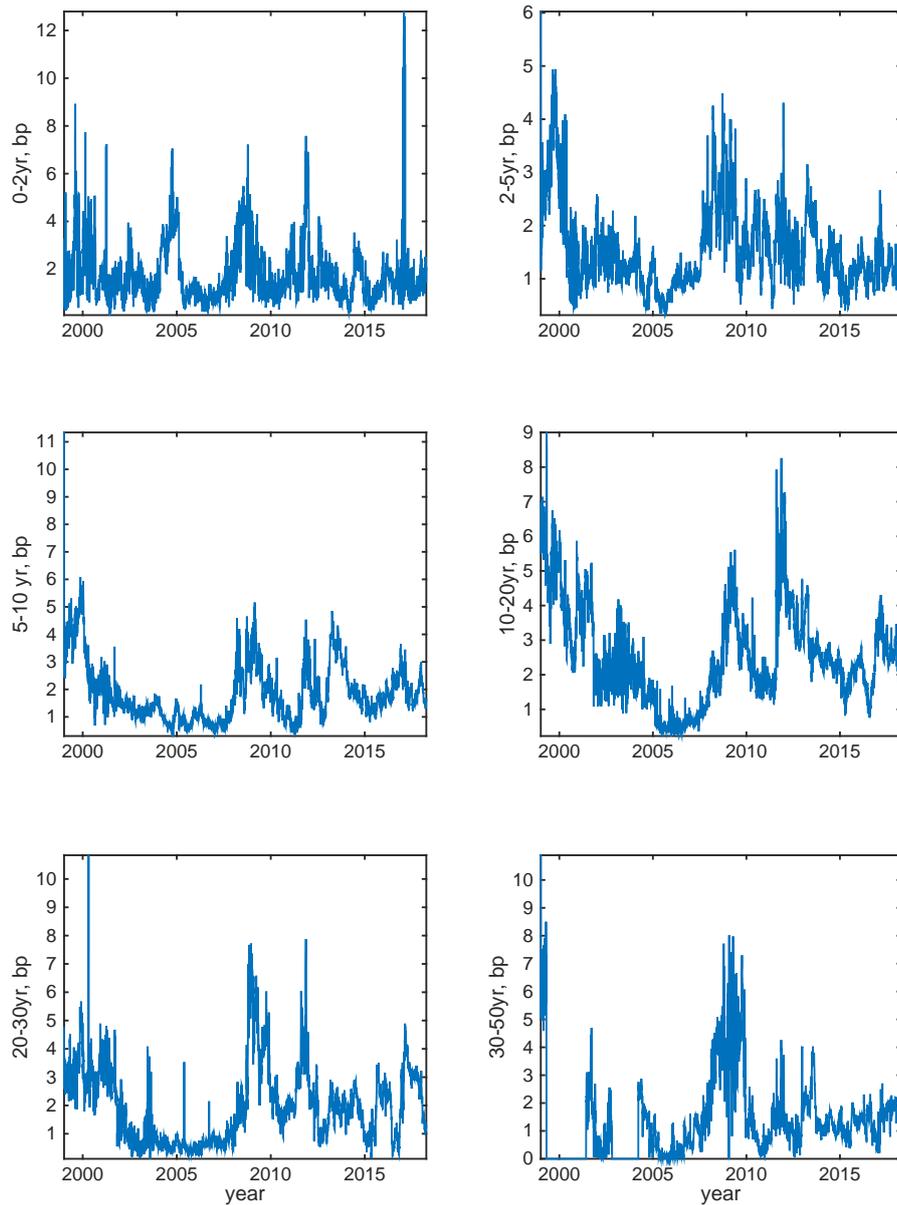
Table B.8: Summary of the Nominal BTAN and OAT Securities, 2010 - 2018

This table shows the sample of the nominal BTAN and OAT securities issued from 2010 to 2018. Column "Type" refers to the type of the security (BTAN or OAT), Column "ISIN" to the ISIN number and Column "Coupon" to the coupon rate of the security in percent. Column "First Qte" reports the first date on which the quote for the security becomes available, Column "Maturity" provides the expiration date, Column "Term" specifies the term-to-maturity of the bond and the "Obs" column reports the total number of available observations for the security. Frequency of the data: daily. Source: Bloomberg.

Type	ISIN	Coupon	First Qte	Maturity	Term	Obs	Type	ISIN	Coupon	First Qte	Maturity	Term	Obs
BTAN	FR0117836652	2.5	19/01/2010	15/01/2015	4.99	1302	OAT	FR0013101466	0	19/01/2016	25/02/2019	3.1	581
OAT	FR0010854182	3.5	02/02/2010	25/04/2020	10.23	2133	OAT	FR0013131877	0.5	01/03/2016	25/05/2026	10.23	551
OAT	FR0010870956	4	11/03/2010	25/04/2060	50.12	2105	OAT	FR0013154044	1.75	13/04/2016	25/05/2036	20.11	520
BTAN	FR0118153370	0.75	18/05/2010	20/09/2012	2.34	612	OAT	FR0013154028	1.25	13/04/2016	25/05/2066	50.11	520
BTAN	FR0118462128	2	15/06/2010	12/07/2015	5.07	1323	OAT	FR0013157096	0	20/04/2016	25/05/2021	5.1	515
OAT	FR0010916924	3.5	28/06/2010	25/04/2026	15.82	2031	OAT	FR0013200813	0.25	30/08/2016	25/11/2026	10.24	412
OAT	FR0010949651	2.5	05/10/2010	25/10/2020	10.06	1960	OAT	FR0013219177	0	15/11/2016	25/05/2022	5.52	366
BTAN	FR0119105809	2.25	18/01/2011	25/02/2016	5.1	1328	OAT	FR0013232485	0	17/01/2017	25/02/2020	3.1	321
BTAN	FR0119580019	2	18/05/2011	25/09/2013	2.36	615	OAT	FR0013234333	1.75	25/01/2017	25/06/2039	22.41	315
OAT	FR0011059088	3.25	31/05/2011	25/10/2021	10.4	1790	OAT	FR0013250560	1	04/04/2017	25/05/2027	10.14	266
BTAN	FR0119580050	2.5	14/06/2011	25/07/2016	5.11	1331	OAT	FR0013257524	2	17/05/2017	25/05/2048	31.02	235
OAT	FR0011196856	3	31/01/2012	25/04/2022	10.23	1615	OAT	FR0013283686	0	19/09/2017	25/03/2023	5.51	146
BTAN	FR0120473253	1.75	15/02/2012	25/02/2017	5.03	1310	OAT	FR0013286192	0.75	02/10/2017	25/05/2028	10.64	137
BTAN	FR0120634490	0.75	17/04/2012	25/09/2014	2.44	637	OAT	FR0013311016	0	16/01/2018	25/02/2021	3.11	61
BTAN	FR0120746609	1	17/07/2012	25/07/2017	5.02	1310	OAT	FR0013313582	1.25	29/01/2018	25/05/2034	16.32	51
OAT	FR0011317783	2.75	03/09/2012	25/10/2027	15.14	1462							
OAT	FR0011337880	2.25	02/10/2012	25/10/2022	10.06	1441							
OAT	FR0011394345	1	15/01/2013	25/05/2018	5.36	1366							
OAT	FR0011452721	0.25	20/03/2013	25/11/2015	2.68	700							
OAT	FR0011461037	3.25	27/03/2013	25/05/2045	32.16	1315							
OAT	FR0011486067	1.75	02/05/2013	25/05/2023	10.06	1289							
OAT	FR0011523257	1	19/06/2013	25/11/2018	5.43	1255							
OAT	FR0011619436	2.25	05/11/2013	25/05/2024	10.55	1156							
OAT	FR0011708080	1	21/01/2014	25/05/2019	5.34	1101							
OAT	FR0011857218	0.25	15/04/2014	25/11/2016	2.61	679							
OAT	FR0011883966	2.5	28/04/2014	25/05/2030	16.07	1032							
OAT	FR0011962398	1.75	03/06/2014	25/11/2024	10.48	1006							
OAT	FR0011993179	0.5	17/06/2014	25/11/2019	5.44	994							
OAT	FR0012517027	0.5	03/02/2015	25/05/2025	10.31	831							
OAT	FR0012557957	0	17/02/2015	25/05/2020	5.27	821							
OAT	FR0012634558	0	09/06/2015	25/02/2018	2.72	20							
OAT	FR0012938116	1	01/09/2015	25/11/2025	10.23	681							
OAT	FR0012968337	0.25	15/09/2015	25/11/2020	5.2	671							
OAT	FR0012993103	1.5	29/09/2015	25/05/2031	15.65	661							

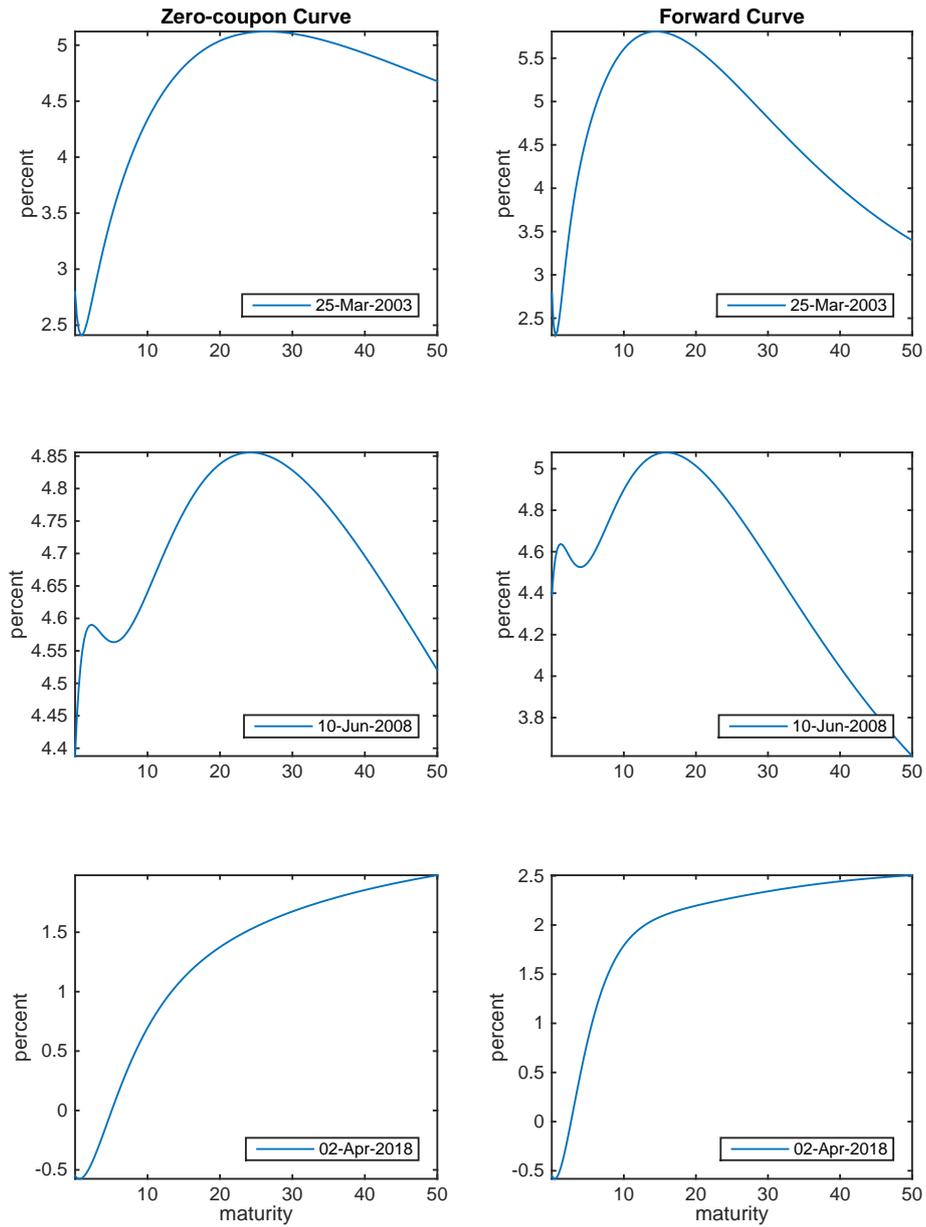
---

Figure B.10: Maturity-specific Fitting Errors



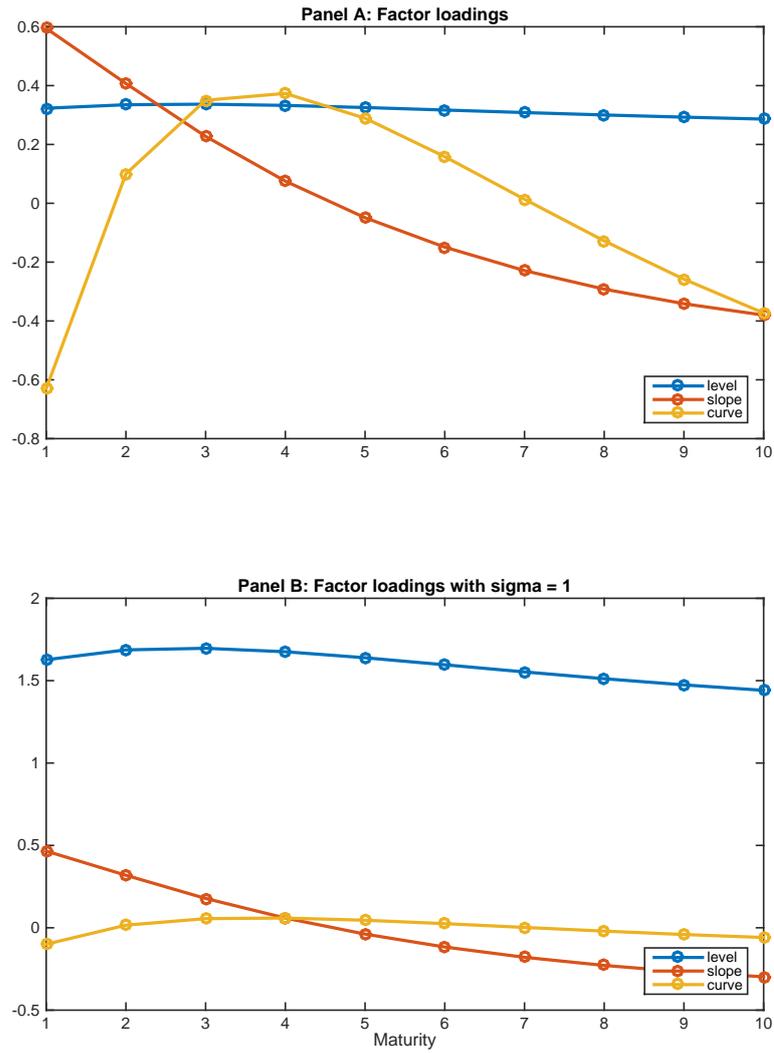
This figure shows the fitting errors of the [Svensson \[1994\]](#) model implied by the BTANs and OATs. The fitting error is computed as the mean absolute error between the predicted and the market prices in a certain maturity bin. We report the errors for six maturity bins: 0-2-year, 2-5-year, 5-10-year, 10-20-year, 20-30-year, and 30-50-year bin. The fitting errors are shown in basis points. Sample period: January 4, 1999, to April 10, 2018. Frequency: Daily.

Figure B.11: Term Structures of the Zero-Coupon Rates and Forward Rates



This figure shows [Svensson \[1994\]](#) zero-coupon yield and instantaneous forward rate term structures on three days in our sample: March 25, 2003, June 10, 2008, and April 2, 2018.

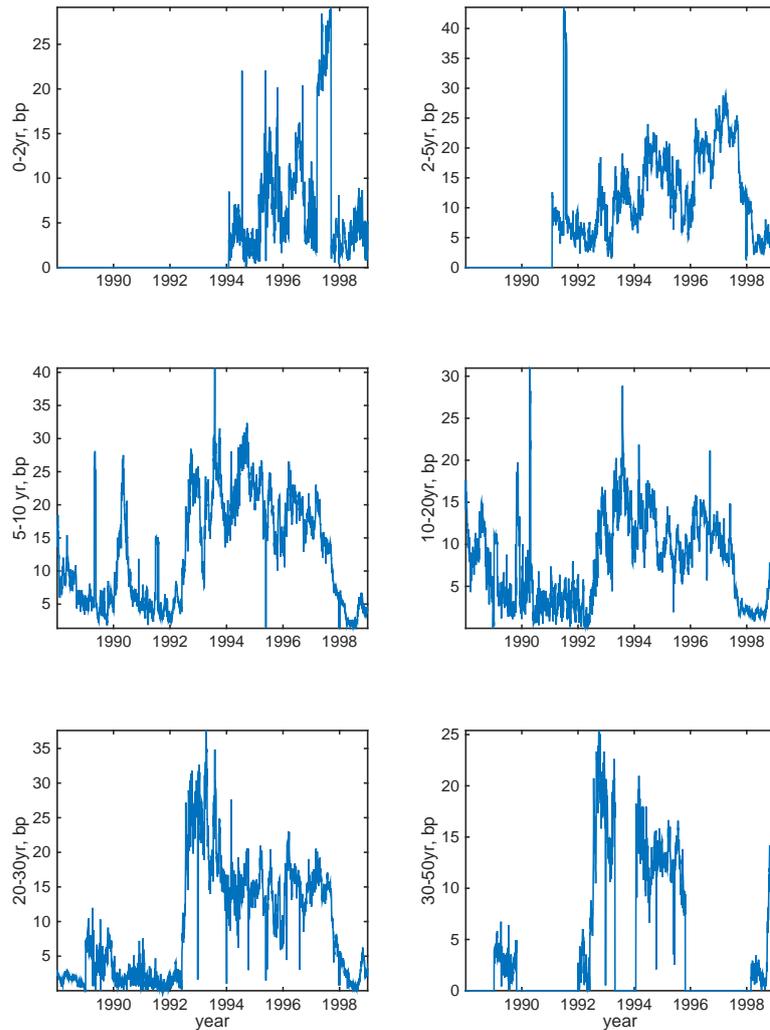
Figure B.12: Principal Component Loadings of Yield Curve



This figure shows the loadings of the first three principal components for the non-normalized variance (top chart) and normalized variance (bottom chart) cases. The principal component analysis used the zero-coupon yields of maturities from 1 to 10 years. Sample period: January 1, 1999, to April 10, 2018. Frequency: Daily.

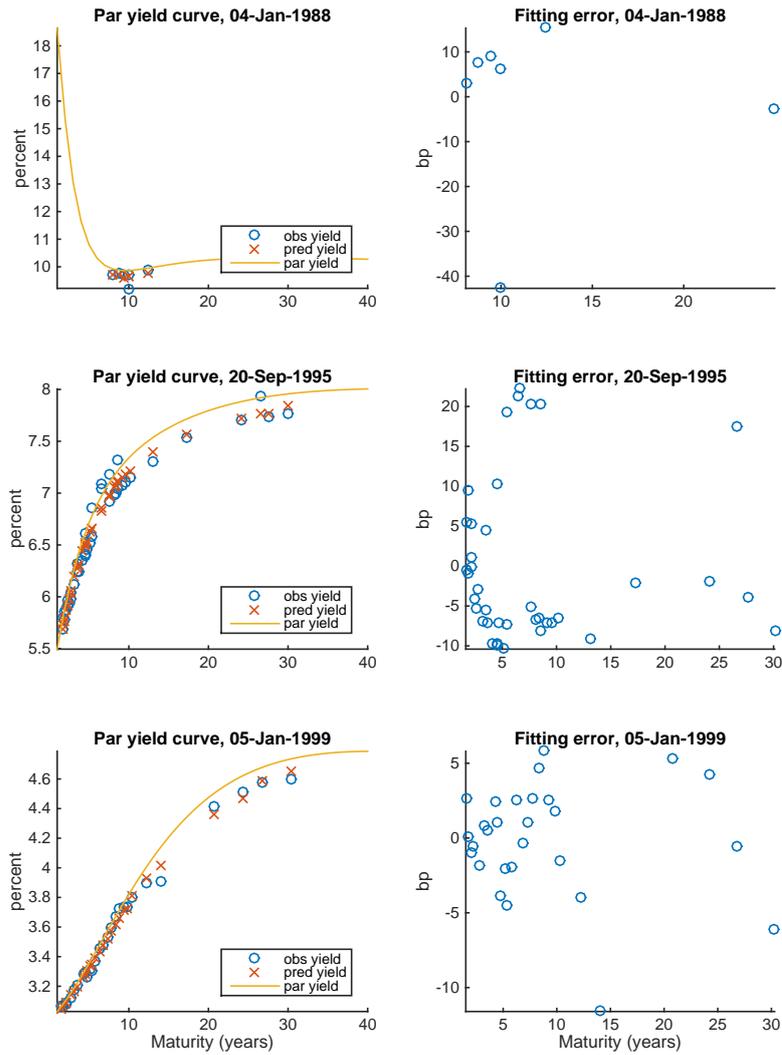
---

Figure B.13: Maturity-specific Fitting Errors: Pre-euro Sample



This figure shows the fitting errors of the [Svensson \[1994\]](#) model implied by the BTANs and OATs computed as the mean absolute error between the predicted and the market prices in a certain maturity range. We report the errors for six maturity ranges: 0 to 2 years, 2 to 5 years, 5 to 10 years, 10 to 20 years, 20 to 30 years, and 30 to 50 years. The start of the sample in the charts depends on the selected maturity range. For all charts shown, the sample ends on December 30, 1998. The fitting errors are shown in basis points. Frequency: Daily.

Figure B.14: Par Yield Curve: Pre-euro Sample Period



This figure shows the par yield curve and the fit of individual securities (left-hand side charts) along with security-specific fitting errors (right-hand side charts) in two days in the pre-euro-area period—January 4, 1988, and September 20, 1995—and following the onset of the euro area, January 5, 1999. The curve is reported in annualized percent, the fitting errors are reported in basis points.

# Chapter 4

## French Inflation-Protected Government Bonds <sup>1</sup>

### 4.1 Introduction

A real interest rate is the interest that takes into account inflation. In an economic system, inflation results in the increase of consumer prices and the decrease of debt prices. The sum of real interest and inflation rates is the nominal interest rate, according to the well-known Fisher equation. It is agreed that the inflation rate must be positive to maintain the country's economic system. There is even a point of view that without positive inflation there cannot be economic growth, but this is not true — economic growth comes from scientific progress. The fundamental task of a central bank is to preserve the value of the currency. Today, many central banks across the world use inflation targeting — a monetary policy regime when there is an explicit target inflation rate. Also, inflation targeting can provide maximum economic growth, optimal employment, and exchange-rate and financial stability. Inflation targeting was pioneered in New Zealand in 1990, Canada in 1991, and the United Kingdom in 1992. For example, Japan, the United States, and some other countries have an inflation rate target of 2%. In

---

<sup>1</sup> This chapter is based on a working paper co-authored with Olesya GRISHCHENKO and Franck MORAUX. These results will be presented at the 7th Paris Financial Management Conference in December 2019. The opinions expressed in this chapter are those of the authors and do not necessarily reflect the views of the Federal Reserve System.

---

the euro area, the target is below 2% for the 2017–19 period.

The term structure of real interest is the subject of this chapter. We overview the real interest rates in the French government debt market. The study of inflation compensation using data on inflation-protected securities issued by the French Treasury is addressed in this research. With [Gürkaynak, Sack, and Wright \[2010\]](#) methodology, we also study the spread between nominal and real yields—the so-called breakeven inflation rate. The term structure of real interest rates and related inflation-protected securities are of interest in academic literature. [Chen, Liu, and Cheng \[2010\]](#) take a multifactor, modified quadratic term structure model to study inflation risk and the term structure of inflation risk premia in the U.S. market. [Grishchenko and Huang \[2013\]](#) also study the inflation risk premium. Authors obtain inflation risk premium estimates by using a simple and easy-to-implement method that takes into account the impact of the indexation lag on real yields and the liquidity adjustment of the real yields. [Fleckenstein, Longstaff, and Lustig \[2014\]](#) study the relative pricing of nominal and inflation-protected securities. A simple no-arbitrage argument places a strong restriction on the relation between the prices of these securities. Authors show that this no-arbitrage relation is frequently violated in markets, and that the mispricing can exceed \$20 per \$100 notional amount. Thus, while there is literature about U.S. Treasury Inflation-Protected Securities (TIPS), there are not many articles that study the French market (for TIPS, see [D’Amico, Kim, and Wei \[2014\]](#) and [Chang \[2019\]](#); for the French market, see [Kita and Tortorice \[2018\]](#)). Our work aims to fill this gap and investigate all available public data of French inflation-protected securities. Our database covers securities from their inception launch.

Inflation-indexed securities are designed to help protect borrowers and investors from changes in the general level of prices in the real economy. The United Kingdom, Australia, and Canada first issued such securities in 1981, 1985, and 1991, respectively. Countries in the euro area also introduced inflation-protected securities. In particular, France issued inflation-protected securities in 1998. Usually, such bonds are indexed to a domestic consumer price index (CPI), but they can also be indexed to other inflation indices, such as wholesale prices, average earn-

---

ings, or the GDP deflator. While indexed bonds aim to provide investors with a certain real return, it is not exactly the case in practice due to several reasons. The first fundamental reason is that whatever the given inflation index, it is only an approximation to any individual investor's particular consumption basket. The second reason is that there is a lag between the relevant period for which an index value is computed and the date when its value is published, making it impossible to provide continuously and instantaneously price index series. The third reason is taxes. Despite these imperfections, inflation-protected securities still offer a high degree of protection against unexpected inflation. For completeness, it is worth mentioning that there is also a parallel market for inflation derivatives that has evolved quite rapidly; for example, in the largest inflation-linked derivatives market for contracts linked to the euro-area Harmonised Index of Consumer Prices excluding tobacco (HICP excluding tobacco, henceforth HICP), turnover is estimated to be 25% of the turnover of the corresponding OAT€i market (see [Deacon, Derry, and Mirfendereski \[2004\]](#)).

The biggest issuer in the world of inflation-protected securities is the United States. Its outstanding value in March 2019 was 1.421 trillion USD. The United States issues two types of inflation-protected securities: TIPS and inflation-indexed savings bonds for domestic retail purposes. Both securities are issued by the U.S. Treasury and linked to the U.S. CPI. The other important inflation-linked market is the United Kingdom, which has two types of inflation-protected securities: index-linked gilts issued by the U.K. Debt Management Office and index-linked savings certificates issued by National Savings and Investments for domestic retail purposes. Both securities are related to the retail price index. Among European countries, France and Italy have the biggest markets for inflation-protected securities. A particularity of these two markets is that there are securities related to two indices: the domestic inflation index and the HICP. In the case of France, there are OATi and OAT€i. For Italy, there are BTP Italia linked to the Italian CPI and BTP€i related to the HICP. Other countries in the euro area take only one index to issue inflation-protected securities. Sweden issues inflation-protected securities related to its domestic index only. Germany and Spain issue inflation-protected securities related to the HICP only. This is

---

the same situation for countries around the world. In Canada, Australia, Russia, Hong Kong, India, Japan, and Mexico, only the domestic inflation index is used to issue inflation-protected securities. Brazil has two indices, IPCA and IGP-M, but both are domestic indices with different methods to compute them.

We first fit the term structure of real rates. This research heavily relies on the yield-curve-fitting methodology of [Svensson \[1994\]](#). We obtain some reliable estimates of intermediate and long-horizon yields that would reflect fundamentals. We obtain the day-to-day evaluation and different shapes of the term structure of real rates. We also obtain the dynamics of the French real yield curve. We repeat the fitting exercise for each day, so for a period we have the day-to-day evaluation of the term structure. [D’Amico, Kim, and Wei \[2014\]](#) study the “true” dynamic model, meaning that they take a vector with three latent variables to explain the real yields, expected inflation, and nominal yields. This approach is not covered by our research. Nevertheless, we document different shapes of the term structure of real rates according to different situations on the market. Finally, we compute some breakeven rates for inflation.

The rest of this chapter is organized as follows. Section [4.2](#) describes some institutional details of the French government debt linked to the inflation index. Section [4.3](#) describes the methodology to deal with inflation-linked bonds. Section [4.4](#) describes our data set with French inflation-protected securities. Next there is a section devoted to result description with several subsections in it. Section [4.5](#) reports the results and investigates the duration cutoff issue using the noise measure. Subsection [4.5.1](#) focuses on the shape and the dynamics of the fitted zero-coupon yield curve. Subsection [4.5.2](#) explores the breakeven rates. Section [4.8](#) concludes.

## 4.2 Literature review

The academic literature assumes that the real interest rate is constant, yet empirical estimates for the real interest rate show that this is not true. [Ang, Bekaert, and Wei \[2008\]](#) establish a comprehensive set of stylized facts about real rates, expected inflation, and inflation risk premium. They study the U.S. market and

---

document that the term structure of real rates has a flat shape of around 1.3%, with a slight hump at around the one-year maturity mark. Inflation expectations play an important role in policymaking and in research on asset pricing. [Chernov and Mueller \[2012\]](#) propose a dynamic macrofinance model that incorporates the behavior of inflation, real activity, nominal yields, and survey-based forecasts of inflation. They find that model-based inflation expectations are driven by inflation, output, and one latent factor. There is also literature about different aspects of inflation—for example, inflation expectations. [Buraschi and Jiltsov \[2005\]](#) study the ability of a general equilibrium model to explain deviations from the expectations hypothesis of interest rates. They estimate the structural parameters of the economy using panel data on U.S. Treasury bonds and find that the inflation risk premium plays an important role in explaining deviations from the expectations hypothesis of interest rates. [Grishchenko, Mouabbi, and Renne \[2016\]](#) investigate the joint estimation of inflation expectations in the United States and the euro area. They exploit surveys of professional forecasters to fit the first two moments of future inflation rates and find that, since 2010, inflation expectations decreased in both economies. In addition, over the sample period, the United States displayed larger inflation uncertainty relative to the euro area.

Claiming that splines are preferable to other popular methodologies, such as the [Nelson and Siegel \[1987\]](#) model, because they are more stable when the number of bonds is small, [Pericoli \[2014\]](#) uses a spline methodology to estimate the real term structure for the euro area implied by French index-linked bonds. [Christenses, Lopez, and Shultz \[2017\]](#) study whether the U.S. market of inflation-protected securities displays the on-the-run premium, which is related to the situation when the most recently issued security trades at a price above those of more seasoned but otherwise comparable securities. They document a small, positive premium on recently issued TIPS that averages between 1 and 4 basis points. [Fleming and Krishnan \[2012\]](#) investigate the microstructure of the U.S. market of inflation-protected securities and use some high-frequency data to analyze announcement effects. They find that price volatility spikes at the time of a major announcement and also document the existence of the on-the-run premium. They suggest that

---

trading activity and quote incidence may be better cross-sectional measures of liquidity in the TIPS market than bid-ask spreads or quoted depth. Ermolov [2017] uses inflation-linked government bonds prices data from nine countries to study the market-implied real yields. He finds that the unconditional real yield curves are upward-sloping and that, across countries, real yields are strongly positively correlated while liquidity premium.

Inflation-protected securities were introduced by the French Treasury in September 1998. The face value of such bonds is adjusted for inflation over time according to the nonseasonally adjusted CPI. A price index is a measure of the proportionate changes in a set of prices over time. The CPI measures changes in the prices of goods and services that households consume. Inflation-linked bonds issued by the French Treasury are called OATi, which stands for *Obligations Assimilables au Trésor*, and these bonds are indexed to the domestic CPI. The CPI is the instrument used to measure inflation. The first generation of indices dates back to 1914. In January 2016, the reference year in the CPI changed to 2015. The previous index values were calculated with 1998 as the reference year. Several years later, in October 2001, after the issuance of the first OAT, the French Treasury proposed the first inflation-protected security, OAT€i, linked to the HICP. In the euro area, consumer price inflation is measured by the HICP, which measures the change over time in the prices of consumer goods and services acquired, used, or paid for by euro-area households.<sup>1</sup> Both OATi and OAT€i make annual interest payments, which are a fixed percentage of the inflation-adjusted principal. The value of the paid coupon is equal to the multiplication of the coupon rate, the face value, and the indexation coefficient. The indexation coefficient is the ratio of today's inflation level to the reference inflation level. Today's inflation level is calculated as linear interpolation between the index value three months ago and the value two months ago. The reference level of inflation is the inflation level of some given year. The reference year is 2015 for both the CPI and HICP.

Overall, CPIs most often contain seasonal patterns. Seasonal price movements are

---

<sup>1</sup>The term “harmonized” means that all the countries in the European Union follow the same methodology. This ensures that the data for one country can be compared with the data for another.

---

intra-year changes occurring to a similar extent in successive years. Some seasonal adjustments are therefore needed to correct for the regular movements in the time series that occur every year during the same period. Traditionally, statistical institutes do not calculate price indices in a seasonally adjusted format. The European Central Bank (ECB) started to compile seasonally adjusted euro-area HICPs in 2000. In 2016, the ECB established in its monthly economic bulletin that the seasonal fluctuations in the euro area have become more pronounced over time, in particular due to the gradual harmonization of statistical concepts and methods related to prices that exhibit seasonality (see [ECB \[2016\]](#)). [Ejsing, Garcia, and Werner \[2007\]](#) claim that accounting for the seasonality in consumer prices is an important issue both for the correct pricing of inflation-linked bonds and for extracting breakeven interest rates. They give two examples of seasonality in the euro area. First, the January price level is below the trend level of prices due to the winter sale prices taken into account in the calculations. Second, in contrast, index price levels in the second quarter of the year are above the general trend level of prices. In this work we do not take into account the seasonal adjustment. The bottom line from the literature is that the shorter the maturity of the bond, the stronger the impact of seasonality. We eliminate bonds with short time to maturity from the estimation process. Therefore, we suppose that our results are not affected by the impact of seasonality. There is another reason that affects our decision. Our data set has two samples—one related to French inflation and one related to euro-area inflation. While seasonally adjusted HICP is widely discussed, the situation with seasonally adjusted CPI in France is not clear.

### 4.3 Methodology

We follow the [Gürkaynak, Sack, and Wright \[2010\]](#) methodology in which the U.S. TIPS yield curve is estimated using a simple and parsimonious approach. The methodology is quite effective at capturing the general shape of the yield curve while smoothing through idiosyncratic variation in the yields of individual inflation-protected securities.

---

Under [Svensson \[1994\]](#) parametrization,  $m$ -period continuously compounded zero-coupon yield at time  $t$  is

$$y(t, t + m; \Theta) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} + \beta_2 \left[ \frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} - e^{-\frac{m}{\tau_1}} \right] + \beta_3 \left[ \frac{1 - e^{-\frac{m}{\tau_2}}}{\frac{m}{\tau_2}} - e^{-\frac{m}{\tau_2}} \right] \quad (4.1)$$

where  $\Theta = \{\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2\}$  are six parameters that need to be estimated. We estimate these parameters by minimizing the sum of squared deviations between observed and predicted bond prices weighted by the inverse bond duration. We take BID quotes from Bloomberg for bond prices, although some authors take MID quotes (see, for example, [Ermolov 2017](#)).

Given the prices of inflation-linked bonds, we fit [Svensson \[1994\]](#) yield curves to construct zero-coupon yields. There is a possibility to use [Nelson and Siegel \[1987\]](#) model with only four parameters. The tradeoff is that either we have fewer parameters and thus need a smaller number of available bonds to accomplish the fitting exercise, or we use a more flexible model to accommodate two potential humps in the shape of the zero coupon and forward yield curve but need more available bonds. We choose the [Svensson \[1994\]](#) methodology instead of [Nelson and Siegel \[1987\]](#) for its flexibility.

We denote the price of a zero-coupon bond that represents the value at time  $t$  of paying €1 at a future point of time  $t + m$  as  $B(t, t + m)$ :

$$B(t, t + m) = \exp[-y(t, t + m; \Theta) \times m]. \quad (4.2)$$

We assume a coupon bond maturing in  $m$  periods of time, promising  $N_{c,t}$  identical coupon payments  $c$  and paying some face value  $F$  at the maturity. The price of such a coupon bond at time  $t$  can be written as

$$p(c, t, t + m) = \sum_{i=1}^{N_{c,t}} c \times B(t, t_i) + F \times B(t, t + m). \quad (4.3)$$

---

In this formula,  $t_i$  stands for the  $i$  –  $th$  coupon payment date and  $t_{N_t}$  is the last payment date. We can express the coupon bond price in terms of continuously compounded zero-coupon yield as

$$p(c, t, t + m) = \sum_{i=1}^{N_{c,t}} c \times \exp[-y(t, t_i; \Theta) \times (t_i - t)] + FV \times \exp[-y(t, t + m; \Theta) \times m]. \quad (4.4)$$

Prices of inflation-protected securities are observed in the market. We use these prices to estimate parameters by minimizing the sum of squared deviations between observed and predicted bond prices weighted by the inverse bond duration. The concept of the modified duration used in our yield curve estimation is

$$D = \frac{D_{Mac}}{1 + Y}, \quad (4.5)$$

where  $Y$  stands for the yield-to-maturity and  $D_{Mac}$  is the Macaulay duration. The Macaulay duration is computed by

$$D_{Mac} = \frac{1}{p(c, t, T)} \sum_{i=1}^{N_{c,t}} (t_i - t) \times c \times B(t, t_i) + (T - t) \times F \times B(t, T).$$

The Macaulay duration is very popular among participants because it connects more explicitly the change in yields to the change in prices; see, for example, [Martellini, Priaulet, and Priaulet \[2003\]](#) for additional information about duration.

### 4.3.1 About inflation

The index factor is used to adjust the cash flows of inflation-linked bonds for inflation, and it expresses the change in the related index between two dates. The index factor is calculated as the ratio between the “reference index,” meaning the index value for a given date, and the “base index,” meaning the historical index value, for the bond. The base index is determined when the bond is issued and it never changes. The reference index is its value on a given date. The

---

CPI and HICP are updated once a month and published in the middle of the following month. The reference index for the first day of every month is equal to the corresponding CPI or HICP three months earlier. The reference index for all other days in the month is calculated by linear interpolation between the two index values corresponding to one in the beginning of the current month and one in the beginning of the next month:

$$\text{Index factor} = \frac{\text{Reference index}}{\text{Base index}}$$

For example, suppose the settlement date is April 17, 2015, for a bond. The reference index for April 1, 2015, is 115.13, corresponding to the January 2015 CPI value. The reference index for May 1, 2015, is 115.87, corresponding to the February 2015 CPI value. On April 17, 2015, we can compute the index by linear interpolation between 115.13 and 115.87; the result is 115.52467 ( $=115.13+(115.87-115.13)\times(17-1)/30$ ). Assume the bond was issued on July 25, 2010. We can compute the inflation reference for this date using two values (the reference index on July 1, 2010, and on August 1, 2010). These two values are 109.58 and 109.71 given the CPI for April and May, respectively. The result of this calculation is 109.68065, which gives the reference index value for July 25, 2010. The base index for the given bond stays fixed during the bond's life. For a bond that was issued on July 25, 2010, the base index is 109.68065, and now one can calculate the index factor on April 17, 2015. To compute the index factor, just divide 115.52467 by 109.68065.

The coupon amount to be disbursed on the coupon day is calculated by multiplying the index factor by the “real” coupon, which gives us the nominal coupon expressed in percentage terms. To find its value, multiply it by the face value:

$$\text{Coupon} = \text{Reference coupon (in \%)} \times \text{Index factor}$$

$$\text{Coupon amount} = \text{Coupon (in\%)} \times \text{Reference face value}$$

The amount to be disbursed on the maturity date (excluding the last coupon) is calculated by multiplying the face value by the index factor. All French-linked bonds have deflation protection, which means that the index factor on the matu-

---

rity date cannot be less than 1:

$$\text{Redemption amount} = \text{Reference face value} \times \max[\text{index factor}; 1]$$

To calculate the settlement amount, one must first calculate the price, which is done by multiplying the index factor by the sum of all future real cash flows discounted by the real yield. Based on this price, one can calculate the clean price by subtracting the accrued interest and rounding the result to three decimal points. The settlement amount is calculated by adding back the accrued interest on the clean price and then multiplying it by the face value.

### 4.3.2 Yield mathematics

Another popular way for market participants to express and quote bond prices is in terms of par yields. The par yield over a certain horizon  $T$  is the coupon rate at which a coupon bond security maturing at  $T$  will trade at par. Setting the price of the coupon bond in equation (4.3) and  $F$  to 1 €, we obtain the solution for the coupon rate  $c \equiv y^c(t, T)$ :

$$y^c(t, T) = \frac{1 - B(t, T)}{\sum_{i=1}^{N_t} B(t, t_i)}. \quad (4.6)$$

While zero-coupon yields represent a mathematically simpler concept, market participants usually quote yields to maturity on coupon-bearing bonds and use par yields. We compute both par yields and zero-coupon yields in this chapter.

The yield curve can also be expressed in terms of forward rates. A forward rate is the rate that an investor is able to lock in some time in the future by trading zero-coupon bonds of different horizons now. For example, if an investor wishes to lock in a  $m$ -period rate between  $T$  and  $T + m$  years in the future, this forward rate, denoted as  $f(t, T, m)$ , can be obtained as:

$$f(t, T, m) = -\frac{1}{m} \ln \frac{B(t, T + m)}{B(t, T)} = \frac{1}{m} ((T + m)y(t, T + m) - Ty(t, T)). \quad (4.7)$$

---

Taking the limit  $m \rightarrow 0$ , we obtain the instantaneous forward rate  $f(t, T, 0)$ :

$$f(t, T, 0) = \lim_{m \rightarrow 0} f(t, T, m) = y(t, T) + Ty'(t, T) = -\frac{\partial}{\partial T} \ln B(t, T). \quad (4.8)$$

The Svensson curve fitting approach relies on the idea that the curve associated with the instantaneous forward rate  $f(t, m, 0)$   $m$  periods ahead at time  $t$  and is correctly described by the following functional form:

$$f(t, m; \Theta) = \beta_0 + \beta_1 \exp\left[-\frac{m}{\tau}\right] + \beta_2 \frac{m}{\tau_1} \exp\left[-\frac{m}{\tau_1}\right] + \beta_3 \frac{m}{\tau_2} \exp\left[-\frac{m}{\tau_2}\right]. \quad (4.9)$$

## 4.4 Data description

Securities related to inflation were proposed on the French bond market starting from 1998. The first time the French Treasury issued an OAT indexed to the French CPI was on September 15, 1998. Another innovation took place in October 2001, with the issuance of the first OAT indexed to the euro-area price index.

We identify all indexed bonds, and in our data set there are 24 securities with 13 bonds linked to euro-area inflation and 11 bonds linked to French inflation. Tables 4.1 and 4.2 provide a detailed description of all the individual securities in our sample related to the French CPI and to the euro-area price index, respectively. For a given bond, we provide the following information: the ISIN number, the security type (OAT<sub>i</sub> if indexed to the French CPI or OAT€<sub>i</sub> if indexed to the euro-area price index), the issue date of the security, the coupon rate, the expiration date of the security (maturity), the term to maturity of the bond at the issuance, and the total number of available observations for the security. We collect observed bond prices on a daily basis. We take the end-of-day BID prices. In total, we have about 25,000 bond prices for securities indexed to the French CPI and about 28,000 bond prices for securities indexed to the euro-area CPI.

We observe that coupon rates were more important for bonds issued in the nineties with a 3% value. Recent bonds have much lower coupon rates, with 0.1% values for securities issued in 2012, 2014, 2016, and 2017. Almost all bond debts mature on July 25, with the exception of three securities that mature on March

Table 4.1: Summary of OATi Securities

This table shows our sample of the inflation-linked (related to the CPI) Obligation Assimilables du Trésor (OATi) securities. Source: Bloomberg.

ISIN	Type	Issue	Coupon	Maturity	Term	Obs
FR0000571424	OATi	29/09/1998	3	25/07/2009	10.82	2736
FR0000186413	OATi	25/07/1999	3.4	25/07/2029	30.00	4859
FR0000188955	OATi	11/02/2003	2.5	25/07/2013	10.45	2733
FR0010094375	OATi	22/06/2004	1.6	25/07/2011	7.09	1854
FR0010235176	OATi	20/09/2005	1	25/07/2017	11.84	3096
FR0010585901	OATi	25/07/2007	2.1	25/07/2023	16.00	2837
FR0010850032	OATi	25/07/2009	1.3	25/07/2019	10.00	2334
FR0119105791	OATi	25/01/2011	0.45	25/07/2016	5.50	1435
FR0011347046	OATi	25/07/2012	0.1	25/07/2021	9.00	1620
FR0012558310	OATi	01/03/2014	0.1	01/03/2025	11.00	1009
FR0013238268	OATi	01/03/2016	0.1	01/03/2028	12.00	490

1. Issuance dates vary more compared to maturity dates, but July 25 is still the most frequent issuance date. We notice that the French Treasury issued securities indexed to the euro-area CPI yearly from 2009 to 2013. Concerning the term to maturity of the bond at the issuance, we have several observations. For OATi bonds, this feature has an average value of 12.15 years, and its minimum and maximum values are 5.50 years and 30 years, respectively. For OAT€i bonds, this feature has an average value of 16.26 years, and its minimum and maximum values are 4.25 years and 34 years, respectively. It is worth noting that on the nominal bond market, one can find ultra-long OAT bonds with term to maturity at the issuance equal to 50 years.

The ranges of time-to-maturities available for estimation over our sample period are plotted in panel A of Figure 4.1 for securities indexed to the French CPI and panel B for securities indexed to the euro-area CPI. Each line represents one security. The date is shown on the horizontal axis and the remaining time to maturity is shown on the vertical axis in years. The upper-left point of the line corresponds to the issue date. The lower-right point of the line corresponds to the bond expiration date. As previously mentioned, we have data for about 11

Table 4.2: Summary of OAT€i Securities

This table shows our sample of the inflation-linked (related to the HICP) Obligation Assimilables du Trésor (OAT€i) securities. Source: Bloomberg.

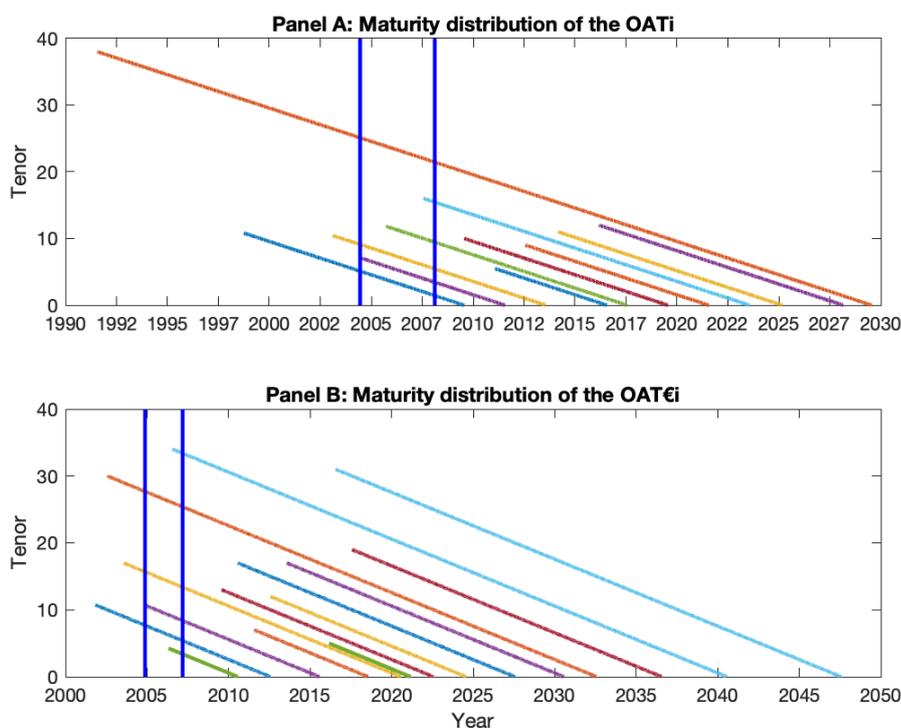
ISIN	Type	Issue	Coupon	Maturity	Term	Obs
FR0000188013	OAT€i	31/10/2001	3	25/07/2012	10.73	2793
FR0000188799	OAT€i	25/07/2002	3.15	25/07/2032	30.00	4215
FR0010050559	OAT€i	25/07/2003	2.25	25/07/2020	17.00	3902
FR0010135525	OAT€i	23/11/2004	1.6	25/07/2015	10.67	2787
FR0108664055	OAT€i	25/04/2006	1.25	25/07/2010	4.25	1111
FR0010447367	OAT€i	25/07/2006	1.8	25/07/2040	34.00	3079
FR0010899765	OAT€i	25/07/2009	1.1	25/07/2022	13.00	2250
FR0011008705	OAT€i	25/07/2010	1.85	25/07/2027	17.00	2057
FR0011237643	OAT€i	25/07/2011	0.25	25/07/2018	7.00	1636
FR0011427848	OAT€i	25/07/2012	0.25	25/07/2024	12.00	1530
FR0011982776	OAT€i	25/07/2013	0.7	25/07/2030	17.00	1188
FR0013140035	OAT€i	01/03/2016	0.1	01/03/2021	5.00	730
FR0013209871	OAT€i	25/07/2016	0.1	25/07/2047	31.00	588
FR0013327491	OAT€i	25/07/2017	0.1	25/07/2036	19.00	198

OATi securities and 14 OAT€i securities. We also can see that the first bond indexed to the French CPI was issued in 1998, and the first bond indexed to the euro-area price index was issued in 2001.

Figure 4.2 plots the year-end notional outstanding amount and the number of securities of the French real securities indexed to the French CPI (panel A) and indexed to the euro-area price index (panel B). Values in blue represent the number of bonds on the market, and the corresponding axis is on the left side. Values in red represent the notional outstanding amount of French real government debt, and the corresponding axis is on the right side. We see that the volume of French debt related to the HICP has stable growth. At the same time, the French debt related to the CPI had stable growth until 2008 and since then it remains on the same level.

Figure 4.3 plots the historical inflation values for both indices. We present these values from 1999 to 2018. Data on ICP existed well before 1999; for instance, it

Figure 4.1: Maturity Distribution



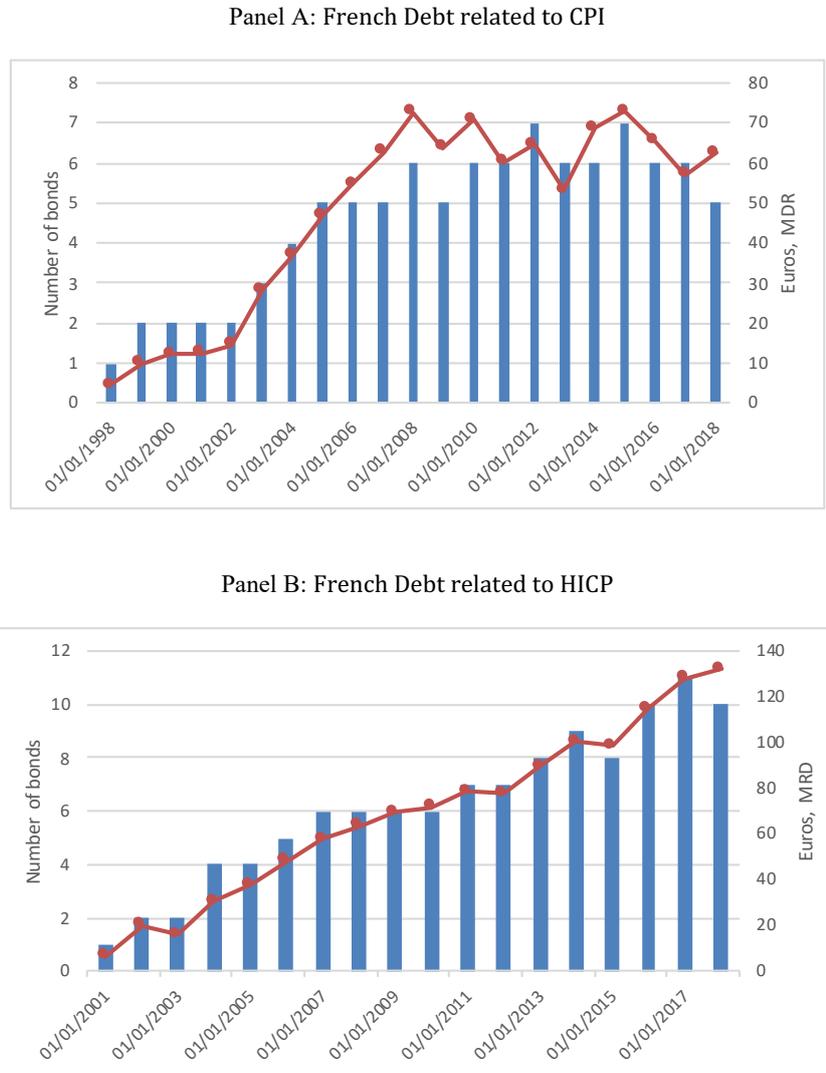
This figure shows the maturity structure of the French real government securities related to the CPI issued from 1998 to 2018 (i.e., the OATi) and to the HICP issued from 2001 to 2018 (i.e., the OAT€i). Source: Bloomberg.

is available from January 1956. Data on the HICP are available from the moment of creation of the European Union—i.e., from January 1, 1999. It is relevant to report historical inflation values measured by two indices during the same period. The correlation between these two time series is 99.36%.

## 4.5 Results

When a bond is close to its maturity date, its price becomes special and it no longer reflects the situation on the market. It is known from the literature that to estimate parameters, one needs to eliminate from the estimation bonds with short time to maturity (see [Sarig and Warga \[1989\]](#)). This duration cutoff can

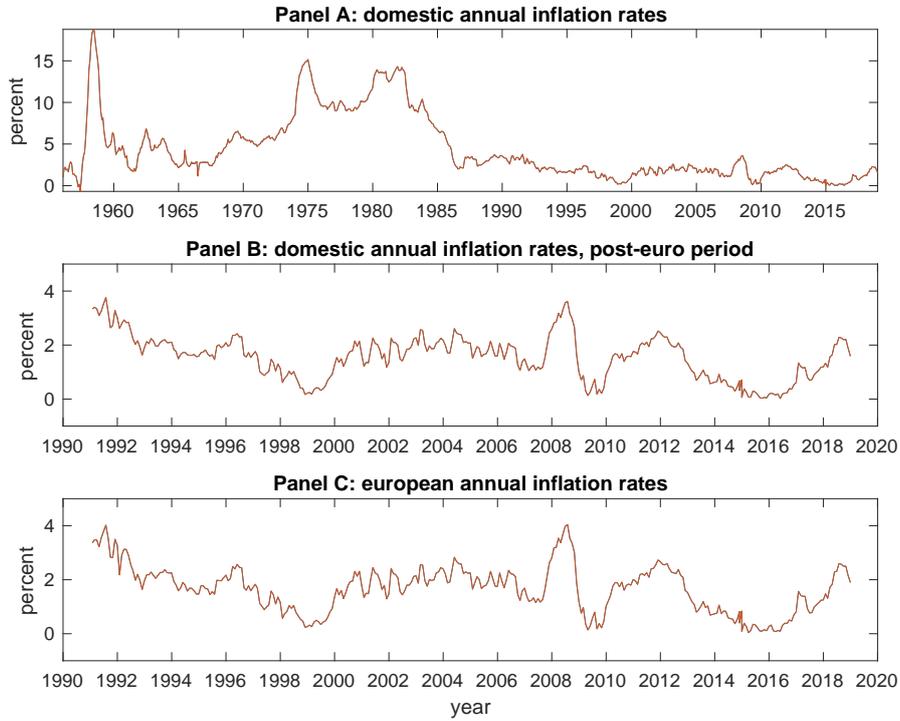
Figure 4.2: Notional Amount of the French Debt and Number of Bonds



This figure shows the outstanding amount of the French real government debt (OATi and OAT€i) on the right-hand scale and the number of available inflation-protected government bonds on the market on the left-hand scale. Data are hand-collected and merged from the monthly newsletters released by the *Agence France Trésor*.

have different sizes. It is common in the literature to take the duration cutoff equal to one year. To find the optimal size, we propose to use the noise measure. This is a market-wide liquidity measure proposed in HuPanWang. To compute

Figure 4.3: Time Series of Annual Inflation Rates



This figure shows historical inflation rates for the CPI and HICP. Panels A and B report French annual inflation rates starting from their first observation in January 1956 and starting from January 1999, which corresponds to the introduction of €currency. Panel C shows European annual inflation rates from January 1999.

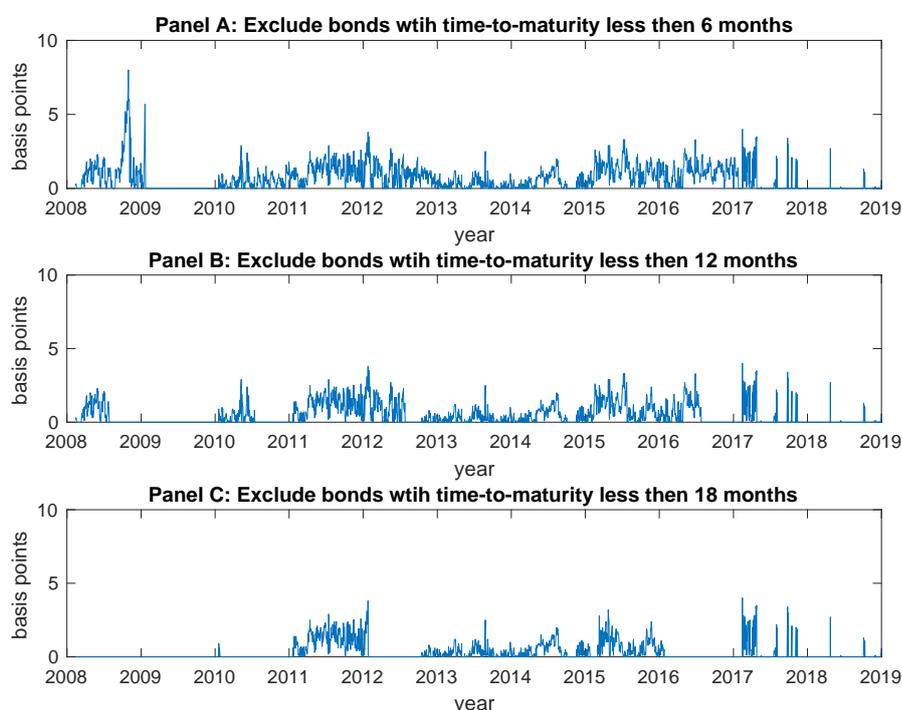
the noise measure, we take the root mean squared distance between the market yields and the model-implied yields:

$$\text{Noise}_t = \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left( \hat{y}(t, i) - y^p(c, T_i; \hat{\Theta}_t) \right)^2}, \quad (4.10)$$

This measure is related to illiquidity on the market. As discussed in [Ang, Papanikolaou, and Westerfield \[2014\]](#), most asset classes are illiquid, in the sense that trading is infrequent. They precise that most assets are characterized by long periods between trades, low turnover, and difficulties to find counterparts.

These difficulties are present in most markets with the exception of “plain vanilla” fixed-income securities and public equities. Even within the fixed-income securities there are subclasses that are illiquid. It is fairly obvious that bonds with short time to maturity are illiquid on the market. Thus, it is necessary to set up the relevant duration cutoff.

Figure 4.4: Noise Measure and Duration Cutoff for OATi

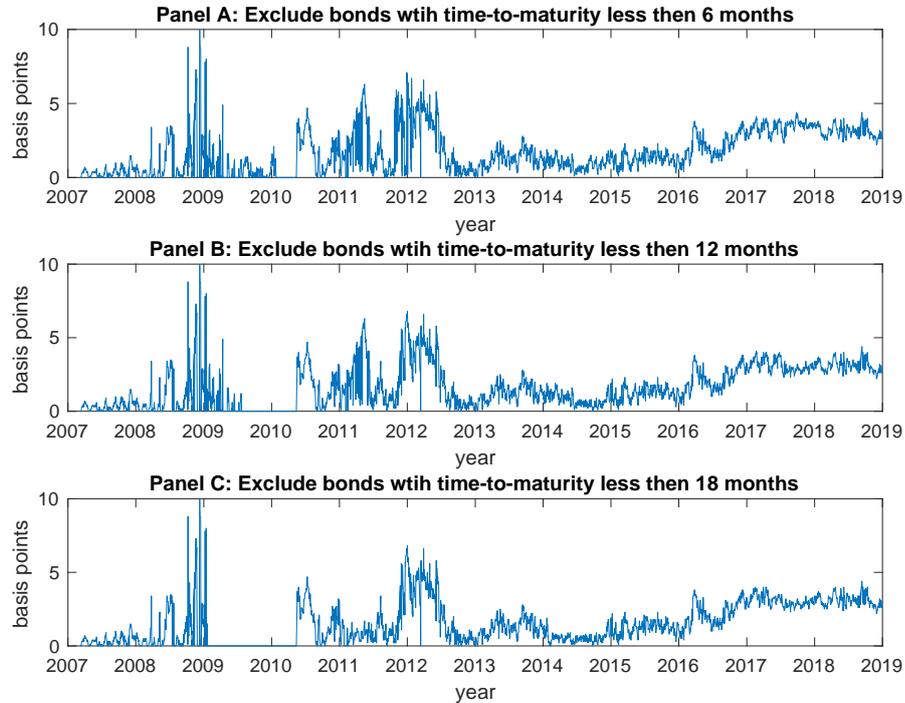


This figure shows the time series of the noise measure, which is computed as the root mean squared error between the observed and predicted yields across all available OATi with a different duration cutoff. Sample period: February 13, 2008, to December 31, 2018. Frequency: daily.

Using the noise measure, we try to establish the best duration cutoff to consider. We use several duration cutoff sizes. First, we exclude bonds with less than 6 months to maturity; second, we exclude bonds with less than 12 months to maturity; finally, we exclude bonds with less than 18 months to maturity. Figures 4.4 and 4.5 plot the noise measure on a daily frequency for securities indexed to the French CPI and indexed to the euro-area price index, respectively. Each figure

---

Figure 4.5: Noise Measure and Duration Cutoff for OAT€i



This figure shows the time series of the noise measure, which is computed as the root mean squared error between the observed and predicted yields across all available OAT€i with a different duration cutoff. Sample period: March 7, 2007, to December 31, 2018. Frequency: daily.

has three panels that correspond to three duration cutoff sizes. It is worth noting that there is no such securities on the French real bond market. Moreover, we treat OATi and OAT€i securities separately. The Svensson term structure model has six parameters; thus, to accomplish the yield curve fitting exercise, we need to have at least six available bonds. With an 18-month duration cutoff, we often find ourselves with less than six available securities. In this case, it is impossible to realize the parameters estimation; thus, there is no noise measure. Periods on the figures when the noise measure is zero correspond to this situation.

Let's compare three panels on one figure. One can conclude that there is no significant difference between the duration cutoffs. Thus, we deduce that it is

---

still important to exclude bonds with short time to maturity from the fitting exercise, but there is no clear evidence for one particular size of the duration cutoff. For the following steps in this chapter, we set the duration cutoff at 12 months. This is a compromise between a small duration cutoff size, when we leave securities with particular prices in the parameter estimation, and an 18-month duration cutoff, when we exclude so many bonds that finally we do not have enough to accomplish the yield curve fitting exercise.

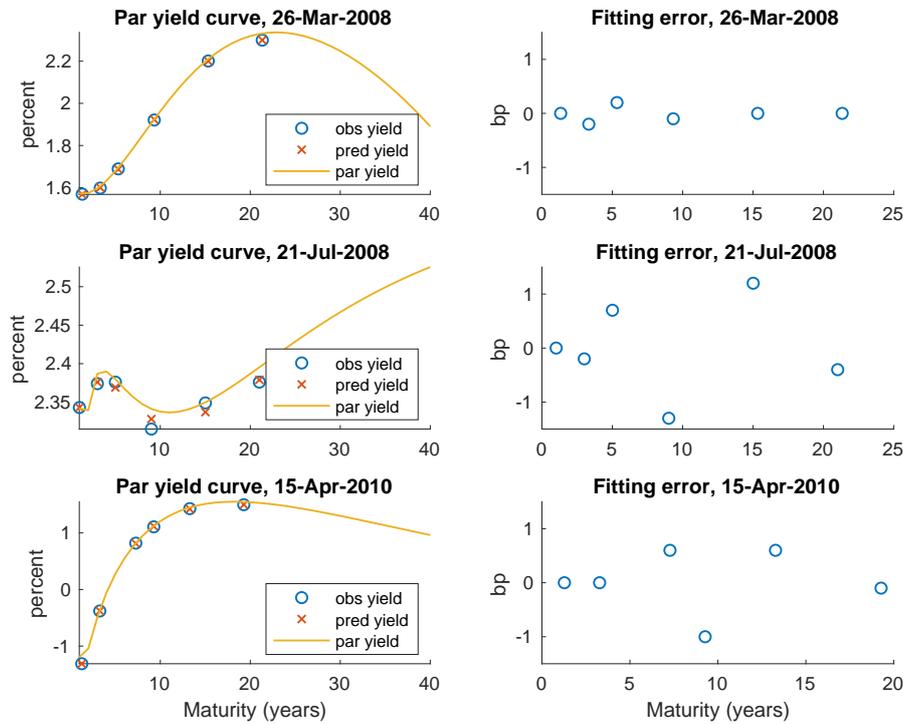
### 4.5.1 Fitting the yield curve

To fit the French term structure of real interest rates, we use a duration cutoff of one year. Figures C.14 and C.16 plot the time series of the overall fitting errors for securities indexed to the French CPI and indexed to the euro-area price index, respectively. The measure of the overall fitting error on a particular day is the average of absolute errors between the predicted and market yields across all available securities that day. Figures C.15 and C.17 plot the time series of fitting errors for four maturity bins for securities indexed to the French CPI and indexed to the euro-area price index, respectively. We fix four maturity bins as follows: 0 to 5 years, 5 to 10 years, 10 to 20 years, and 20 to 40 years.

Figures C.14 and C.16 have the same limits on the horizontal axis, from 0 to 10 basis points, to point out that the model fit for the OAT<sub>i</sub> sample is better than for the OAT<sub>€i</sub> sample. The total error for securities related to the French CPI does not exceed 3 basis points during all sample period. While the total error for securities related to the euro-area price index goes up to 10 basis points during 2009, it is about 6 basis points during 2012, which corresponds to the 2012 sovereign bond crisis. Starting from 2015, the total fitting error for OAT<sub>€i</sub> securities has a rising trend and a value between 2 and 3 basis points at the end of 2018.

Figure C.15 shows that almost all OAT<sub>i</sub> securities have time to maturity of less than 20 years. There are not enough bonds to accomplish the fitting exercise from the middle of 2008 to the beginning of 2010. The maturity bin of 5 to 10 years contains the most important values for errors. Figure C.17 shows that OAT<sub>€i</sub>

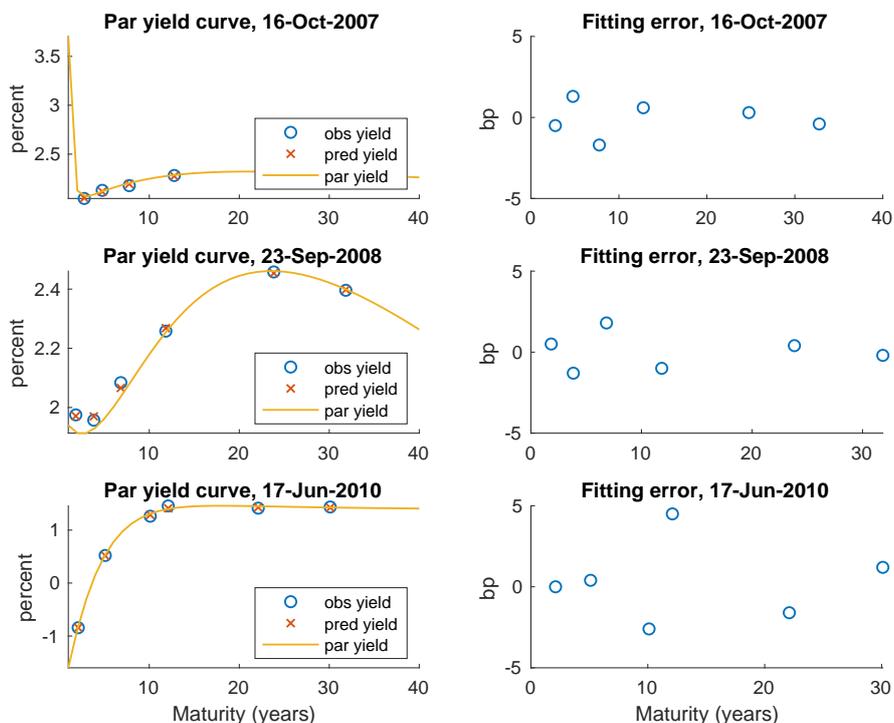
Figure 4.6: Par Yield Curve for OATi sample



This figure shows the par yield curve and the fit of individual OATi securities (left-hand side charts) along with security-specific fitting errors (right-hand side charts) in three days across the sample period: March 26, 2008, July 21, 2008 and April 15, 2010. The curve is reported in annualized percent, the fitting errors are reported in basis points.

securities are present in all maturity bins, even the last one. For all maturity bins, we observe the same pattern for the errors. There are important error values in 2009, which can be related to the 2008 Global Financial Crisis (GFC). There are not enough bonds to accomplish the fitting exercise from the middle of 2009 to the middle of 2010. The next period is from the middle of 2010 to the beginning of 2012, where we see errors with values up to 5 basis points. Then errors are more present according to the sovereign bond crisis in 2012. We have perfect fit and, as a consequence, small values for errors in all maturity bins during the period from the middle of 2012 to the beginning of 2016. After this point, the behavior in error values becomes different in each maturity bin.

Figure 4.7: Par Yield Curve for OAT€i sample

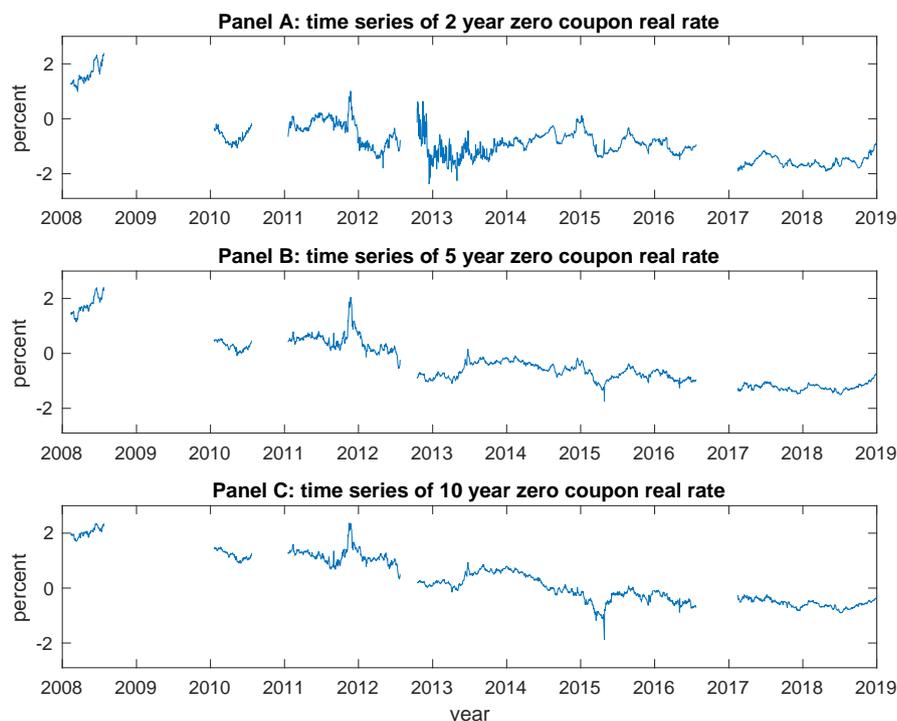


This figure shows the par yield curve and the fit of individual OAT€i securities (left-hand side charts) along with security-specific fitting errors (right-hand side charts) in three days across the sample period: October 16, 2007, September 23, 2008 and June 17, 2010. The curve is reported in annualized percent, the fitting errors are reported in basis points.

Figure 4.6 shows the estimated Svensson nominal par yield curve for OATi securities on three different dates, which we picked from the three broadly defined periods: March 26, 2008, July 21, 2008, and April 15, 2010. Figure 4.7 shows the estimated Svensson nominal par yield curve for OAT€i securities on the three different dates, which we picked from the three broadly defined periods: October 16, 2007, September 22, 2008, and June 17, 2010. The left-hand side of these figures shows the model-implied par yield curve along with observed (blue round circles) and predicted (red crosses) continuously compounded yields. The predicted yields are computed using parameters that are estimated using bond quotes on the indicated day. For Figure 4.6 we take securities indexed to the

---

Figure 4.8: Time Series of Zero-Coupon Real Yields for OATi sample



This figure shows the time series of the [Svensson \[1994\]](#) fitted 2-, 5-, and 10-year zero-coupon real yields implied by the price quotes of OATi securities from February 13, 2008, to April 10, 2018, at daily frequency.

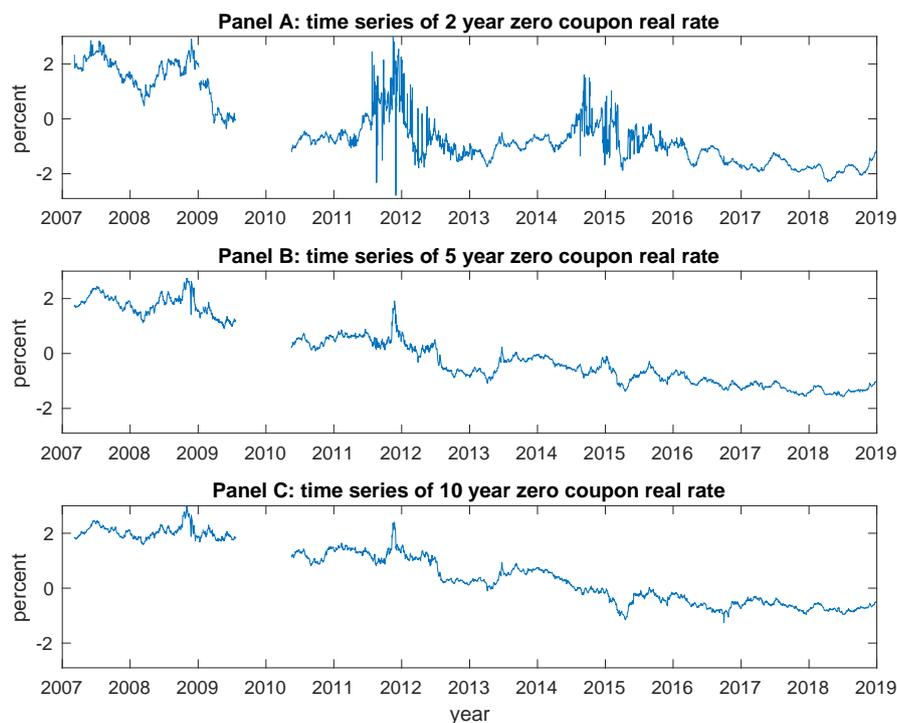
French CPI, and for Figure 4.7 we take securities indexed to the euro-area price index. The right-hand side of these figures shows security-specific pricing errors computed as differences between observed and predicted yield to maturity.

#### 4.5.2 Term structure of real and breakeven rates

We find nevertheless that the zero-coupon rate curve has had different shapes over our sample period as shown in Figure C.18 for the OATi sample and in Figure C.19 for the OAT€i sample. These figures plot zero-coupon yield curves and instantaneous forward rate curves on the same days as Figures 4.6 and 4.7 respectively do—namely, on March 26, 2008, July 21, 2008, and April 15, 2010, for securities indexed to the French CPI, and on October 16, 2007, September

---

Figure 4.9: Time Series of Zero-Coupon Real Yields for OAT€i sample



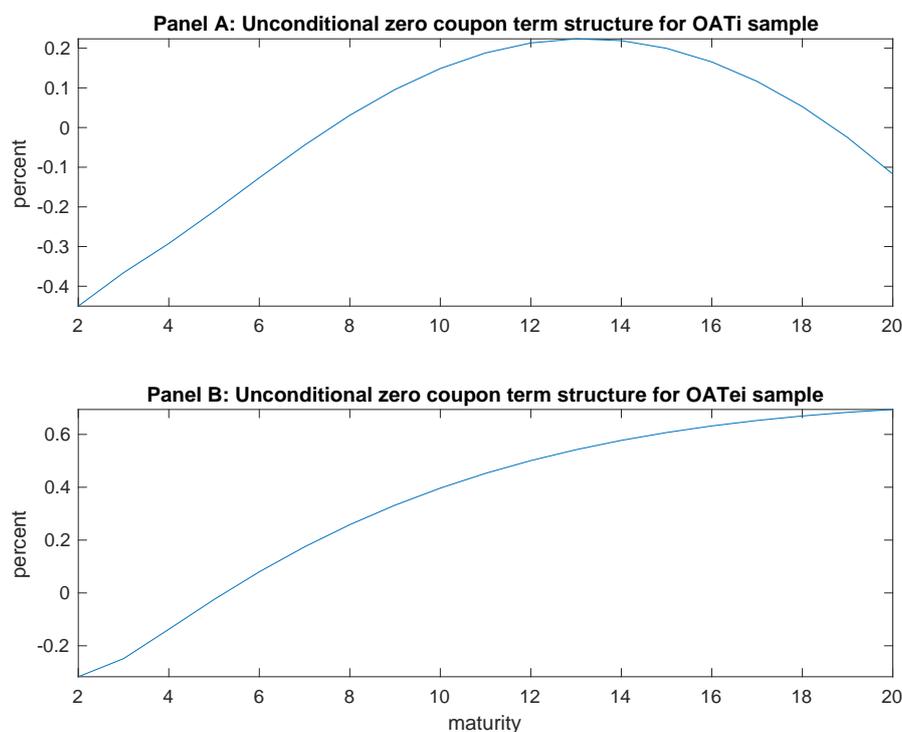
This figure shows the time series of the [Svensson \[1994\]](#) fitted 2-, 5-, and 10-year zero-coupon real yields implied by the price quotes of OAT€i securities from March 7, 2007, to April 10, 2018, at daily frequency.

23, 2008, and June 17, 2010, for securities indexed to the euro-area index. The zero-coupon rate curves appear on the left side of the figure, and the forward rate curves are on the right side of the figure. These plots show that various shapes yield curve shapes implied by the OATi and OAT€i.

Figures 4.8 and 4.9 plot time series of 2-, 5-, and 10-year zero-coupon real yields in our OAT sample and OAT€i sample, respectively. For Figure 4.8, the sample period is from February 13, 2008, to December 31, 2018. For Figure 4.9, the sample period is from March 7, 2007, to December 31, 2018. Periods when the zero-coupon rate is zero on these figures correspond to periods when there is not enough securities to accomplish the fitting exercise; thus, there are no parameters to compute the zero-coupon rate.

---

Figure 4.10: Unconditional Zero-Coupon Real Yields



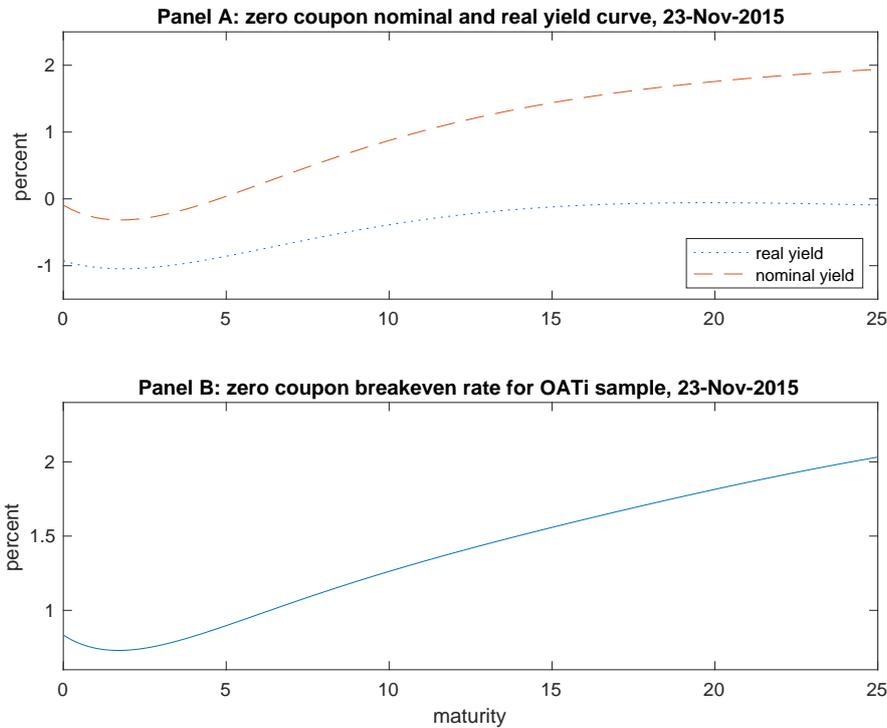
This figure shows the unconditional zero-coupon real yield curve. These values are the simple average of all the yields across the sample period. Panel A reports the unconditional zero-coupon real yield curve for the OATi sample and Panel B for the OAT€i sample.

Figure 4.10 plots the unconditional real term structure. This term structure was computed as the mean value in each horizon point across the sample period.

Figures 4.11 and 4.12 plot zero-coupon yields for breakeven rates. We can see the same nominal zero-coupon yield curve on November 23, 2015, on both figures. The breakeven rate is the difference between the nominal and the real rate on one day for a given maturity. For Figure 4.11, we plot the real zero-coupon yield curve for the OATi sample. For Figure 4.12, we plot real zero-coupon yield curve for the OAT€i sample. Figures C.20 and C.21 plot time series of 5-year zero-coupon yields for breakeven rates. We see that it is on the 2% level.

---

Figure 4.11: Zero-Coupon Real, Nominal and Breakeven Yields for OATi sample



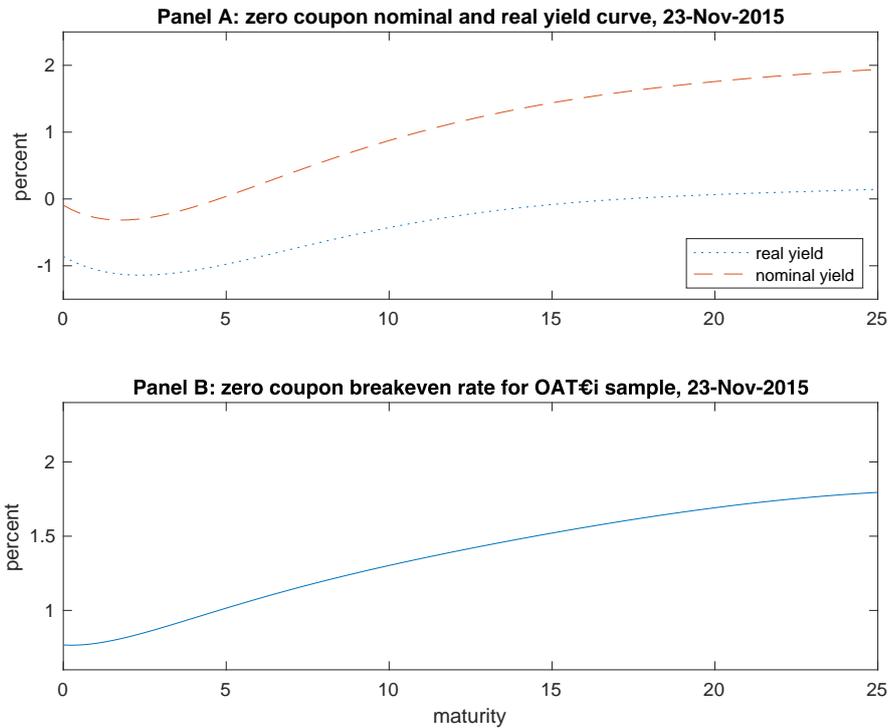
This figure shows zero-coupon yields for the OATi sample. Panel A reports the nominal and real yield curve on a specific date, November 23, 2015. Panel B reports the breakeven rate for the same day of the sample period calculated as the difference between real and nominal rates.

## 4.6 Backcasting inflation

In this section we propose a proxy for the five-year forward, five-year inflation compensation rate and conduct a backcasting exercise. While forecasting involves predicting the future based on current trend analysis, backcasting approaches the challenge of discussing the future from the opposite direction. More specifically, forecasting means making statements regarding the future based on explicit or implicit assumptions from the present situation and observed trends. On the other hand, backcasting is a strategic problem-solving framework, searching for the answer of how to reach specified outcomes in the future.

---

Figure 4.12: Zero-Coupon Real, Nominal and Breakeven Yields for OAT€i sample



This figure shows zero-coupon yields for the OAT€i sample. Panel A reports the nominal and real yield curve on a specific date, November 23, 2015. Panel B reports the breakeven rate for the same day of the sample period calculated as the difference between real and nominal rates.

#### 4.6.1 Methodology aspect

Inflation compensation is the difference between the nominal and real interest rate. There are longer sample periods for nominal rates on the French government securities market. When inflation-linked securities were issued around 2000 in France, it became possible to construct the real yields. Knowing that inflation compensation is the difference between real and nominal rates, one can compute or estimate inflation compensation starting only from 2000. To overcome this issue, we present a backcasting inflation compensation exercise. We notice that the relationship between nominal yields and inflation compensation tends to be stable over years. For our exercise, we estimate this relationship with a

---

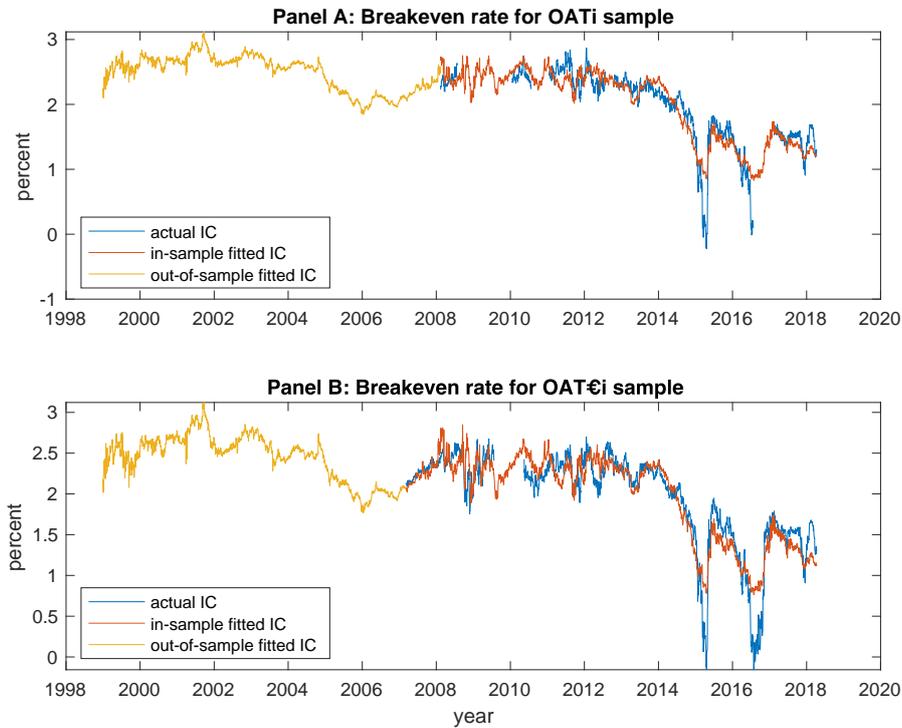
multiple regression tool by regressing the inflation compensation on three principal components of nominal rates. Once this is done, we produce fitted inflation compensation for a period when nominal rates are available. This gives us the period with actual inflation compensation, equal to the period when real rates are available (i.e., from 2007 to 2018 for the EU sample, and from 2008 to 2018 for the FR sample), and the larger period with fitted inflation compensation, equal to the period when nominal rates are available (i.e., from 1999 to 2018 for both samples).

We follow the [Gürkaynak, Sack, and Wright \[2010\]](#) methodology to conduct the backcasting exercise with inflation compensation. They suggest that three principal components of nominal yields provide the best proxy for a particular breakeven measure. In the literature, these factors are called the level, the slope, and the curvature, but we can also interpret them with a macroeconomic explanation. We pick the five-year forward, five-year inflation compensation rate for a particular breakeven measure. We conduct the multiple regression with the five-year forward, five-year inflation compensation rate as the variable to explain and three principal components of nominal yields as three explanatory variables. There are two data samples—one with OATi securities and one with OAT€i securities. We find that the  $R$ -squared is 85% for the OATi sample and 84.84% for the OAT€i sample. These values are high enough to indicate an important relationship between the nominal and breakeven rates. [Figures 4.13](#) plots the five-year forward, five-year breakeven rate for the OATi sample (panel A) and for the OAT€i sample (panel B). Blue indicates actual inflation compensation—i.e., calculated as the difference between the nominal and real rates. Gray and orange indicate the fitted five-year forward five-year inflation compensation rate, which we compute using regression parameters and the available three principal components of nominal yield curve back to 1999—i.e., the post-euro period.

## 4.6.2 Results of backcasting

When we use the Svensson model, we need to have at least six available quotes to produce the parameter estimation. Starting from the first issuance of linked

Figure 4.13: Actual and Fitted Five-to-ten Year Forward Inflation Compensation



This figure shows the actual and fitted five-year forward five-year inflation compensation rate. We follow the [Gürkaynak, Sack, and Wright \[2010\]](#) methodology to conduct the backcasting investigation. Panel A reports the result for the OATi sample with out-of-sample period from January 4, 1998, to February 13, 2008. Panel B reports the results for the OAT€i sample with the out-of-sample period from January 4, 1998, to March 7, 2007.

securities in the French government bond market, the first moment when there is enough data points for parameter estimation is February 13, 2008, for the OATi sample and March 7, 2007, for the OAT€i sample. Due to the fact that sometimes there are not enough security issues and that there are those that arrive to maturity, we can easily drop below the minimum number of available data points—i.e., six quotes. This is the reason why one can see the empty spaces in the time series of the actual five-year forward, five-year inflation compensation rate (blue). This is not the case for fitted rates; thus, we note the absence of empty spaces on the orange and red lines.

Fitted inflation compensation for the OATi sample stays between 2.5% and 3%

---

through the end of 2004. Then we observe the drop to quite a low level—i.e., below 2% in the beginning of 2006. This rate returns to its comfortable level, 2.5%, in February 2008, where it stays until August 2012. Then we observe a fall so dramatic that one can say that it is a regime-changing moment. The fall is most certainly related to the GFC in 2012. There are also two drops before the five-year forward, five-year inflation compensation rate arrives at a 1.5% level at the end of 2018.

To resume the figure description above, we propose that the five-year forward, five-year inflation compensation rate was approximately at a 2.5% level until the GFC, and then this rate changed its regime to a new comfortable level of 1.5%. Once the inflation compensation rate is 2.5% or 1.5%, we say that the nominal rates are higher than the real rates for this value. When the breakeven rate is negative, it gives us the information about instability on the market. The actual breakeven rate goes below the zero level twice: March–April 2015 and June–October 2016. These dates were detected when the inflation rate hit the 1% level. When the actual inflation compensation rate goes below its comfortable value and, even more, goes below the zero level, we see that the fitted inflation compensation rate remains at more reasonable values.

The time series of five-year forward, five-year breakeven rate for the OAT<sub>i</sub> and OAT<sub>€i</sub> samples are quite similar. Indeed, the correlation coefficient between the two actual rates is 98.6%. The correlation coefficient between the two fitted rates is 99.5%, meaning that all the descriptions above for the OAT<sub>i</sub> sample are also relevant for the OAT<sub>€i</sub> sample.

## 4.7 Managerial aspect and discussion

In this section we discuss how our results can be used in the future. We already documented our results in previous sections and now there are two motivations. First, we show that our results are in line with the existing literature. Second, we point out the utility of investigation on the real interest rate topic—in other words, how our results can be used in future research. We will document the asset pricing case and then the risk-management aspect.

---

### 4.7.1 Asset pricing and real rates

Boyd, Levine, and Smith [2001] find a significant and economically important negative relationship between inflation and the ability of the financial sector to allocate resources effectively. Even a predictable increase in the rate of inflation impacts both banking-sector developments and equity market activity. With empirical investigation, they document that this negative relationship is nonlinear. They present two important findings. The first finding is if inflation rates exceed 15%, it decreases the performance for the whole financial sector of the economy. The second finding shows the difference between low-inflation and high-inflation countries. In low-inflation economies, more inflation does not mean greater nominal equity returns, while in high-inflation countries, marginal increases in inflation increase nominal stock returns in a one-for-one proportion.

These relationships are true for any economy. Huybens and Smith [1999] resume some known relationships between inflation and market performance. There are three strongly positively correlated values: (1) real activity, (2) the volume of bank lending activity, and (3) the volume of trading in equity markets. There are also two strongly negatively correlated values: (1) inflation and financial market activity in the long run, and (2) inflation and the real rate of return on equity. Huybens and Smith highlight a critical role of banks and secondary capital markets in the allocative function of the financial sector of economy. They propose a monetary growth model such that model predictions are consistent with empirical observations about inflation, finance, and long-run real activity.

One can also investigate the relationship between expected inflation and money growth. Stulz [1986] proposes an equilibrium model to explain that negative relationship between expected real returns on common stocks and money growth. The decrease in real wealth leads to an increase in expected inflation, a decrease in real interest rates, and, as a consequence, a decrease in the expected real rate of return of the market portfolio. Again we see the empirical evidence of the effect of a change in expected inflation on the cross-sectional distribution of asset returns. The model proposed is consistent with a negative relation between stock returns and inflation and presents clear evidence that assets that have positive

---

covariance with expected inflation have lower expected returns.

[Stehle \[1977\]](#) tests some hypotheses about national and international pricing of assets. He investigates asset pricing with an integrated world capital market model when there were no barriers to international capital flows. He discusses also a model of segmented capital markets when financial transactions were not possible on the international level. His problematics is whether a valuation model assuming no barriers to international capital flows predicts rates of return better than a model that assumes complete market segmentation. This study is important for international portfolio investment management. The investor's portfolio decision only depends upon the real rates of return, which are identical for all investors, regardless of the currency area in which they live. Stehle explains that exchange-rate changes reflect different inflation rates (i.e., different monetary policies) in the case of a single commodity world.

To return to the importance of inflation in the economy and the implications of real rates, [Harvey \[1988\]](#) investigates a linear relation between expected returns and expected consumption growth. To forecast the consumption growth, he suggests the use of an expected real term structure. This proposition is based on strong empirical evidence between the 1970s and 1980s. Moreover, two alternative measures: lagged consumption growth and lagged stock returns, which contain less information to forecast the consumption growth. Thus, Harvey concludes that the real term structure has more forecasting power than the leading commercial econometric models.

Finally, [Bakshi and Chen \[1996\]](#) study the fact that everything is linked in an economy with a tractable monetary asset pricing model. They list several variables—for instance, price level, inflation, asset prices, and real and nominal interest rates. All of these variables have to be determined simultaneously and in relation to each other in monetary economics. Bakshi and Chen relate each of the dependent entities to the underlying real and monetary variables. They find that the process followed by the real term structure is independent of that followed by its nominal counterpart. Our results are in line with this result.

---

## 4.7.2 Real rates and inflation risk

Inflation-hedging portfolio strategies aim to build a portfolio that offers protection against inflation. Both individuals and institutional investors aim to preserve the purchasing power of savings. Developed countries tend to apply the expansionary monetary policies that seek to expand money supply to encourage economic growth or combat inflation through stimulus packages and liquidity injections into money markets. This situation is even more true after the subprime crisis in 2007 and 2008. In the case of emerging markets, there is a dilemma. They can commit to stabilizing either the exchange rate or domestic output, but not both. The central banks in emerging economics are no longer focusing on combating inflation. A country has monetary policy autonomy if its central bank has the freedom to make changes to the country's money supply, therefore allowing us to use that tool to impact the country's economy. Greater monetary autonomy is associated with a higher level of inflation, while greater exchange-rate stability and greater financial openness could lower the inflation rate.

In principle, only inflation-linked bonds provide protection from uncertainty about real interest rates and inflation. But not all developing countries issue inflation-linked bonds, and these markets are still narrow and less liquid compared to their nominal counterparts. Moreover, the current low-yield regime makes it more challenging to obtain high real returns. These concerns raise the interest of reconsidering how to build a portfolio that protects investors from inflation risk. [Piazzesi and Schneider \[2009\]](#) document that the Great Inflation led to a portfolio shift by making housing more attractive than equity. The starting point of their research was a 20% shift away from equity and into real estate during the 1970s. This was related to the surprising surge of inflation. They explore three different channels through which inflation expectations can induce negative co-movement of stock and house prices. Their quantitative analysis suggests that both inflation and growth expectations were relevant for asset prices and household positions in the 1970s.

[Barr and Campbell \[1997\]](#) study expected future real interest rates and inflation rates from observed prices of U.K. government nominal and index-linked bonds.

---

They suggest that expected real interest rates and inflation follow simple time-series processes whose parameters can be estimated from the cross-section of bond prices. Barr and Campbell find that the extracted inflation expectations forecast actual future inflation more accurately than nominal yields do. They also find that the estimated real interest rate is highly variable at short horizons but comparatively stable at long horizons. Finally, they document that changes in real rates and expected inflation are strongly negatively correlated at short horizons but not at long horizons.

To conclude, we return to the analysis of the corporate financing decision in the case of debt and not equity. A firm can issue debt of different maturities and, given the decision to use debt, each time the firm contemplates borrowing to meet its need for capital it faces a decision regarding the term to maturity for its debt. The debt maturity decision involves a consideration of both cost and risk elements as shown in [Morris \[1976\]](#), which explores the effects of bond maturity upon the variance of net income. This is one dimension of the risk associated with different maturity policies. Firms can deal with this risk following a hedging policy when the maturity of the debt is approximately equal to the life of the asset. By matching debt maturity to asset life, it is expected that the cash flows generated by the asset will be sufficient to service and retire the debt by the end of the asset's life. Debt of maturity shorter than asset life is considered more risky since there is some possibility the asset will not have generated sufficient cash flows by the maturity date to retire the debt. Debt of a maturity longer than the asset life is considered risky due to the uncertainty of the source and volume of the cash flows that are necessary to service the debt after the asset is retired.

## 4.8 Conclusion

The term structure of real interest is the subject of this chapter. The observation that inflation has the potential to greatly affect investment outcomes is our main motivation. French-linked government bonds—so-called inflation-protected securities—were first issued at the beginning of the 21st century. This study follows

---

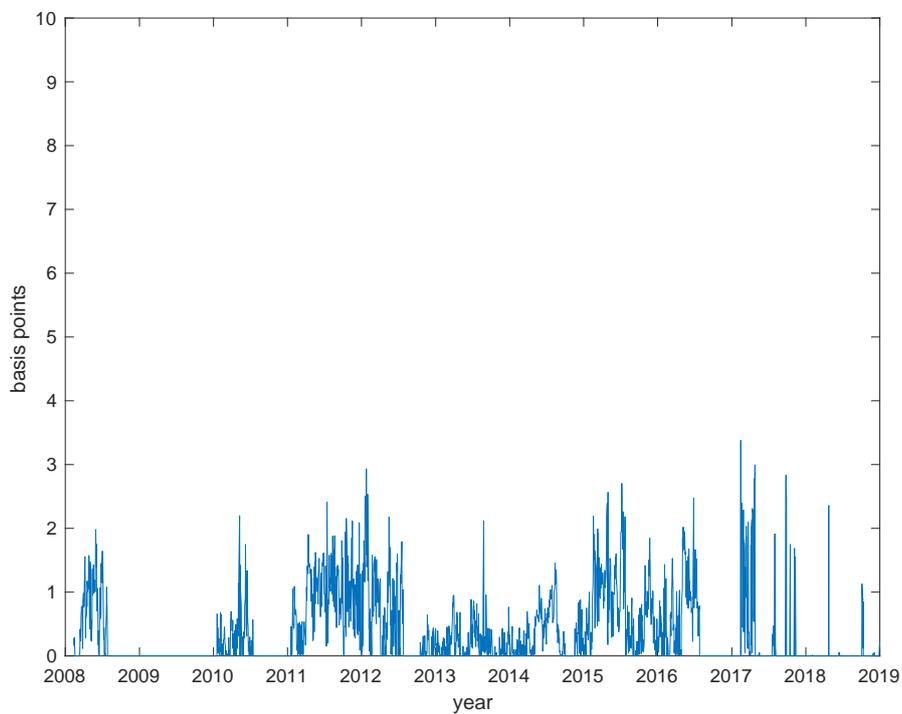
the [Gürkaynak, Sack, and Wright \[2010\]](#) methodology. On the French inflation-protected securities market, we find securities related to two inflation indices. There are securities linked to (1) domestic inflation rates, or the CPI, and to (2) European inflation rates, or the HICP. Throughout our study we split the data sets into two broad categories: OATi and OAT€i markets.

Our first result is the good fitting of the term structure of real rates with the [Svensson \[1994\]](#) model. We find that this model does a very good job in reflecting the reality of the market. Our second result is the day-to-day evaluation and different shapes of the term structure of real rates. We do not use any dynamic approach for real rates. Our static model with parameter estimation for each day in the sample period gives us the time series of the studied phenomenon. Thus, we document the time series of zero-coupon, par yield, and forward rates. And our results are in line with [Bakshi and Chen \[1996\]](#). We propose the valuable proxy for the five-year forward, five-year inflation compensation rate. In future research we plan to incorporate the seasonal adjustment to see how it can impact the breakeven rate estimation.

# Appendices

---

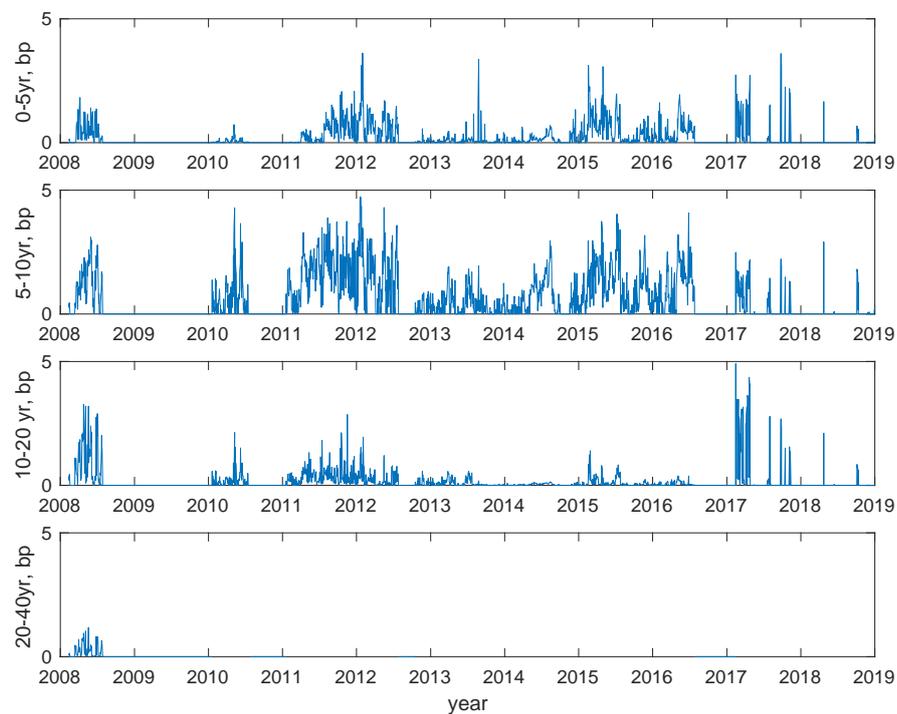
Figure C.14: Fitting Errors for OATi sample



This figure shows the total fitting error implied by the [Svensson \[1994\]](#) model. The fitting error is computed as the mean absolute error between the predicted and the market yields across all available OATi securities on a particular day. The fitting errors are shown in basis points. Sample period: February 13, 2008, to December 31, 2018. Frequency: Daily.

---

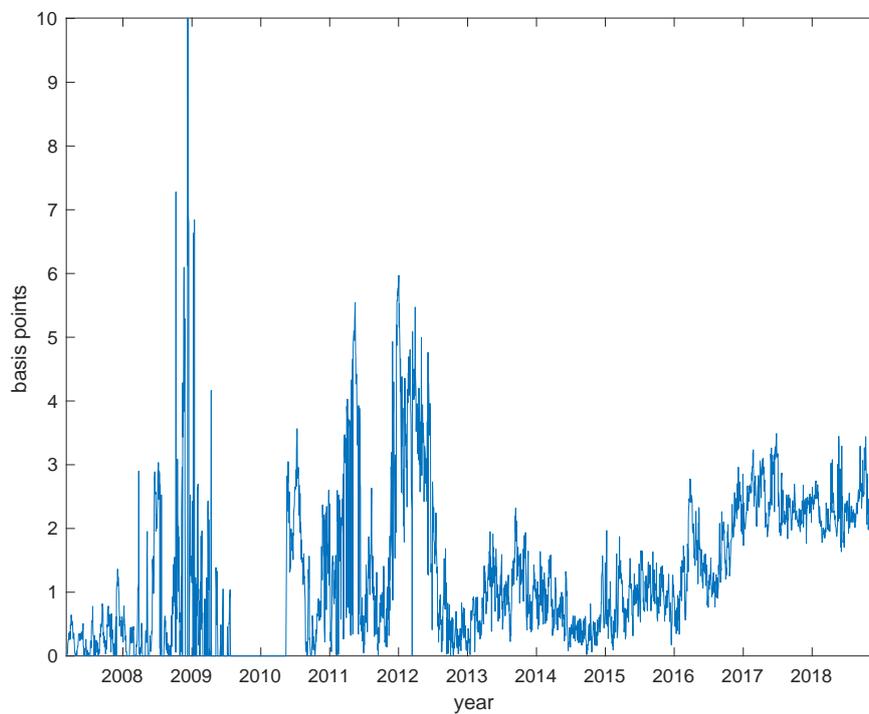
Figure C.15: Maturity-specific Fitting Errors for OATi sample



This figure shows the fitting errors of the [Svensson \[1994\]](#) model implied by the OATi securities. The fitting error is computed as the mean absolute error between the predicted and the market prices in a certain maturity bin. We report the errors for four maturity bins: 0-5-year, 5-10-year, 10-20-year, 20-40-year bin. The fitting errors are shown in basis points. Sample period: February 13, 2008, to December 31, 2018. Frequency: Daily.

---

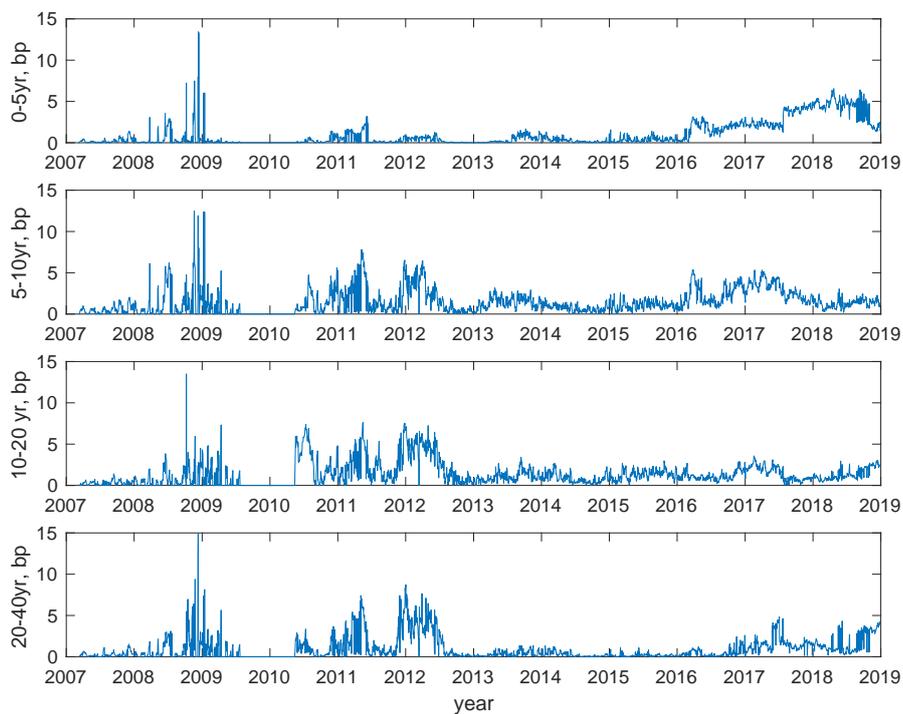
Figure C.16: Fitting Errors for OAT€i sample



This figure shows the total fitting error implied by the [Svensson \[1994\]](#) model. The fitting error is computed as the mean absolute error between the predicted and the market yields across all available OAT€i securities on a particular day. The fitting errors are shown in basis points. Sample period: March 7, 2007, to December 31, 2018. Frequency: Daily.

---

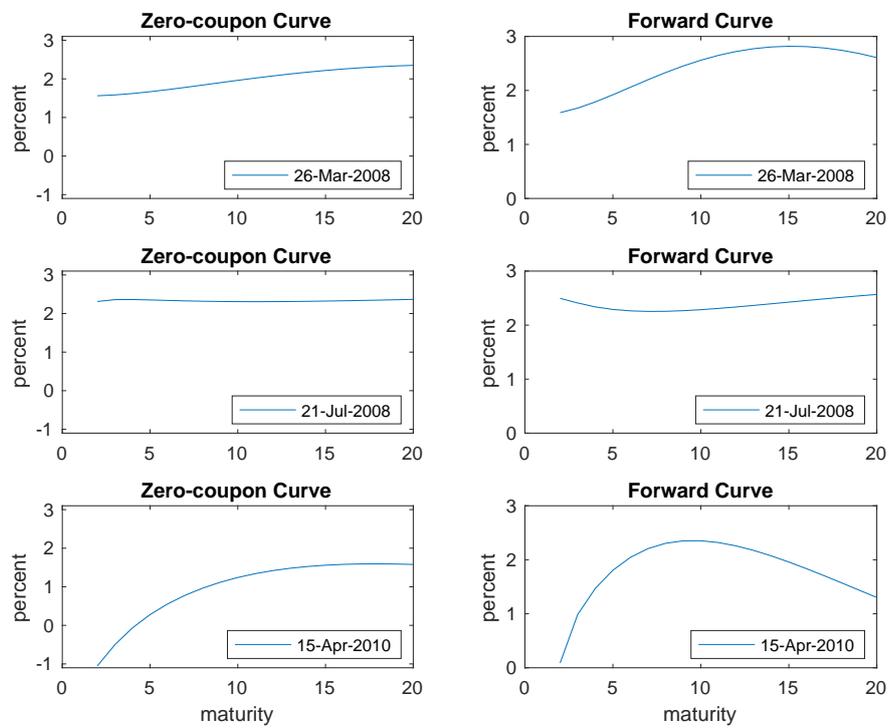
Figure C.17: Maturity-specific Fitting Errors for OAT€i sample



This figure shows the fitting errors of the [Svensson \[1994\]](#) model implied by the OAT€i securities. The fitting error is computed as the mean absolute error between the predicted and the market prices in a certain maturity bin. We report the errors for four maturity bins: 0-5-year, 5-10-year, 10-20-year, 20-40-year bin. The fitting errors are shown in basis points. Sample period: March 7, 2007, to December 31, 2018. Frequency: Daily.

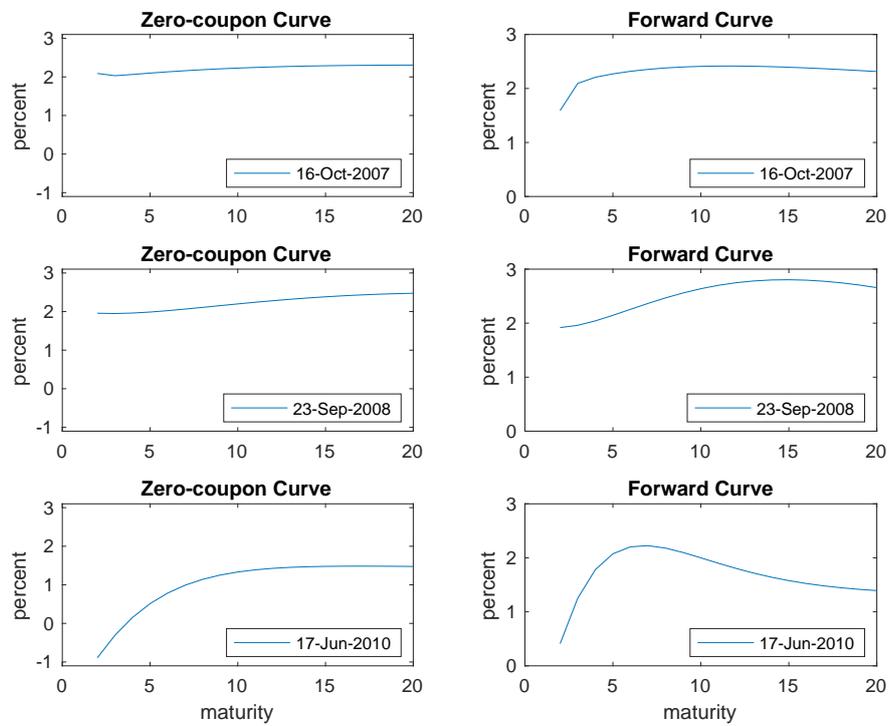
---

Figure C.18: Term Structure of the Zero-Coupon and Forward Rates for OATi sample



This figure shows [Svensson \[1994\]](#) zero-coupon yield and instantaneous forward rate term structures on three days in the OATi sample: March 26, 2008, July 21, 2008, and April 15, 2010.

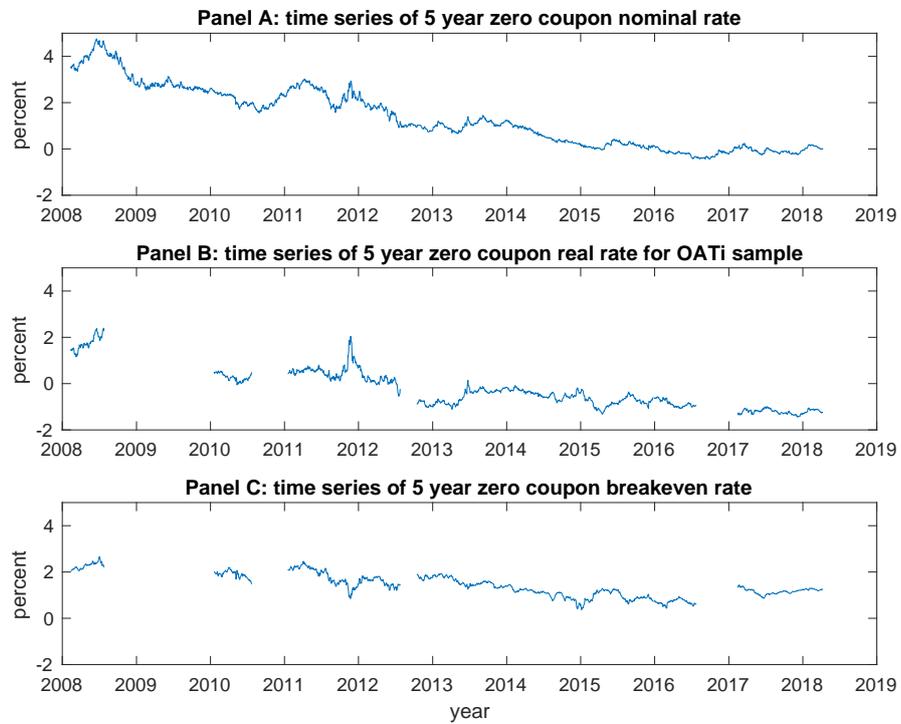
Figure C.19: Term Structure of the Zero-Coupon and Forward Rates for OAT€i sample



This figure shows [Svensson \[1994\]](#) zero-coupon yield and instantaneous forward rate term structures on three days in the OAT€i sample: October 16, 2007, September 23, 2008, and June 17, 2010.

---

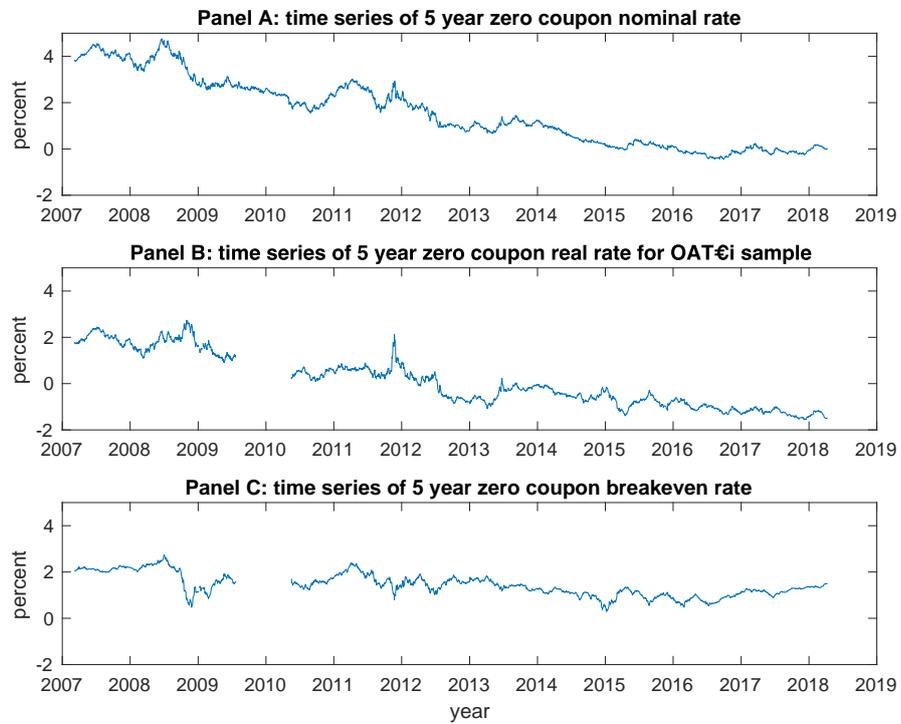
Figure C.20: Time series of 5 year Zero-Coupon Real, Nominal and Breakeven Yields for OATi sample



This figure shows the time series of the Svensson (1994) fitted 5-year zero-coupon nominal, real and breakeven yields implied by the price quotes of OATi securities from February 13, 2008, to December 31, 2018, at daily frequency.

---

Figure C.21: Time series of 5 year Zero-Coupon Real, Nominal and Breakeven Yields for OAT€i sample



This figure shows the time series of the Svensson (1994) fitted 5-year zero-coupon nominal, real and breakeven yields implied by the price quotes of OAT€i securities from March 7, 2007, to December 31, 2018, at daily frequency.



# General Conclusion

The domestic bond market is critical to the economy and financial system for the following reasons. First, sovereign debt issued by either the central bank or the federal government plays a major role in the development of the credit market, which is safer than debt instruments issued by private parties. Second, the yield on sovereign debt serves as the baseline from which all other debt instruments can be priced by adding appropriate risk premium (e.g., liquidity) and term premium to the underlying pure interest rate. Third, high-quality securities aid in market development by providing quality collateral to secure financial transactions. Finally, a well-developed domestic bond market helps the government to finance its fiscal deficit in a non-inflationary way. In this thesis we present three in-depth empirical studies, all related to the government bond markets.

Prior to this, we present the introductory chapter, which is the basis for all the investigations that follow. Chapter 1 presents several theoretical aspects that are necessary for future empirical studies. First, it is interesting to know the type of models that can be used to work with term structure of interest rates (TSIR) phenomenon. We also present a pure statistical approach called principal component analysis (PCA). Applying PCA, it is possible to explain the dynamics of the entire term structure with only three factors, which in addition have precise interpretation as the level, slope, and curvature. As the TSIR provides information on what is the output to invest for a specific horizon, it is important to discuss the duration measure. We also present information about the different institutions regarding TSIR, precisely, information about the products and actors in the government bond market. There are different types of yields that

---

we can construct. The next section presents the spot rates, forward rates, and par yield curves. In Chapter 1 conclusion, we present information about debt management and the problem of inflation, and the dynamics of government bond market liquidity.

Chapter 2 examines several Nelson-Siegel style yield curve models for fitting the term structure of interest rates. During the last four decades, no superior model was clearly identified. We collect data on government bond prices issued by the four eurozone countries. Unlike previous research, we do not consider available interest rates as suitable. We restrict ourselves to the limits of the Nelson Siegel class of term structure models in order to consider four specifications with the same structure. We compare these specifications by their in-sample performance to match bond prices. We find that the extended Svensson specification performance is overall suitable to calculate bond prices. Our result is robust to several criteria to compare all the four competitors.

In Chapter 3 we construct the French nominal yield curve using available public data on the maturities of French nominal Treasury securities at issuance from one to fifty years. Our investigation period starts in 1984, includes the advent of the euro in January 1999, and concludes in April 2018. The analysis of fitting errors shows that the Svensson model fits the data appropriately. The French sovereign bond market has been functioning reasonably well, especially since the launch of the euro, outside of a few episodes such as the global financial crisis and the European sovereign debt crisis. In sharp contrast to the nominal U.S. Treasury securities market, on-the-run securities have, on average, nil or negligible liquidity premium. Both the level and slope of the French zero-coupon rates have been on a decline since the financial crisis.

Chapter 4 investigates the real rates in the French government bond market. The French Treasury has been issuing inflation-linked debt since the start of the 21st century. This chapter provides a comprehensive overview of the market by constructing the real yield curve. We use the Svensson model to compute the implied real zero coupon, real par yield, and real forward rates. Our dataset includes both French government securities OATi (indexed on the domestic consumer price in-

---

dex in France), first issued on September 15, 1998, and the French government securities OAT€(indexed to the euro area price index) that were issued in October 2001. We use the noise measure to detect the reasonable duration cut off for government bonds. First, the calibration results show good fit by the Svensson model. Second, we backcast the five-year forward five-year breakeven inflation rate before the appearance of any inflation-protected securities on the market.

These three empirical studies provide interesting results about the term structure of French interest rates, debt management, and government bond markets. This work is subject to limitations which, at the same time, suggest potentially promising avenues for future research. We have left some open questions when various term structures are similarly considered. Finally, this thesis focuses on understanding the nominal and real interest rates and fitting the term structure of interest rates. Future research can investigate the pricing of related derivatives and risk management issues. However, these topics require different modeling technologies.

Investors and economists strongly believe that the shape of the yield curve reflects the conditions for monetary policy and the market's future expectation about interest rates. In other words, understanding the term structure of interest rates is important because it integrates the market's anticipation of future events by offering a complete schedule of interest rates across time. The various models of the term structure provide us ways to derive this information and predict how the changes in the underlying variables will affect the yield curve. In conclusion, it is hoped that this thesis is of interest to the reader and will encourage future research in the field.



# Résumé

La structure par terme des taux d'intérêt est une relation entre les rendements à l'échéance des obligations à zéro-coupon et leur maturité respective. Cette relation et son contenu informatif font partie des concepts les plus fondamentaux de la finance. Elle est essentielle pour de nombreux domaines tels que la gestion de portefeuille, les options sur taux pour la détermination des prix et la gestion des risques. Elle est également exploitée par des acteurs bien au-delà du strict domaine de la finance, tels que les régulateurs, les économistes et même les journalistes. Malgré la large utilisation de la structure par terme des taux d'intérêt (appelée aussi courbe des taux), cette relation n'est pas directement observable sur le marché. De plus, la structure par terme des taux d'intérêt peut admettre de différentes formes au fil du temps. De nombreux cadres théoriques tentent d'expliquer ce phénomène. Aujourd'hui, il reste encore beaucoup à faire pour étudier le comportement de la structure par terme des taux d'intérêt et plus précisément le lien complexe entre les taux d'intérêt et les prix des obligations à coupon.

Il y a plusieurs motivations pour étudier la structure par terme des taux d'intérêt. La première est qu'elle est nécessaire pour déterminer le prix des titres à revenu fixe. Les titres à revenu fixe sont des prêts consentis par un investisseur à un emprunteur (gouvernement ou entreprise). La deuxième motivation est la nécessité de gérer les actifs et les risques des portefeuilles obligataires. Les investisseurs incluent des obligations dans leurs portefeuilles pour différentes raisons, notamment la génération de revenus, la préservation et la plus-value du capital et la protec-

---

tion contre le ralentissement économique. Enfin, le contenu de l'information et, plus précisément, la capacité de la structure par terme des taux d'intérêt à prévoir les récessions. Il est donc important de poursuivre les recherches empiriques sur la structure par terme des taux d'intérêt.

Cette thèse contribue à la compréhension de la structure par terme des taux d'intérêt à travers trois essais. Le premier met en œuvre plusieurs modèles de structure par terme de taux d'intérêt sur les données d'obligations d'état émises par quatre pays de la zone euro. Cette enquête vise à déterminer quel modèle est le plus performant. Cette question a deux aspects. Premièrement, en environnement statique, lorsque nous étudions la capacité d'adaptation du modèle de structure par terme. Deuxièmement, la capacité de fournir une bonne prévision de la structure par terme des taux d'intérêt. Le deuxième essai étudie la structure par terme des taux d'intérêt nominaux en France, en exploitant une période d'échantillonnage de 30 ans. Notre enquête montre que la prime *on-the-run* (*OTR*) est absente du marché obligataire français. Nous documentons également quelques améliorations significatives de la qualité du marché des obligations d'état français après l'introduction de l'euro. Le troisième essai analyse les titres français protégés contre l'inflation et la structure par terme des taux d'intérêt réels. Cette enquête vise à comprendre l'interaction entre la dette publique et l'inflation dans l'économie d'un pays, par exemple la France. Nous obtenons principalement deux observations. D'abord le modèle que nous utilisons se montre performant pour capter les prix de titres français protégés contre l'inflation domestique et européenne. Deuxièmement, nous avons réussi à calculer le *five-year forward five-year breakeven rate* avec la méthode de *backcasting* sur une période avant l'apparition de tout titre protégé contre l'inflation sur le marché.

Notre travail est divisé en quatre chapitres. Le **premier chapitre** est un chapitre introductif. Il présente le cadre d'études dans lequel se déroulera notre travail et constitue un point de départ pour les enquêtes suivantes. Il présente plusieurs aspects théoriques à rappeler pour les études empiriques futures. Tout d'abord, il est intéressant de savoir quel type de modèle existe pour expliquer la structure par terme des taux d'intérêt. Dans la famille des modèles d'équilibre, il existe, d'une part, des modèles affines, qui incluent des approches d'équilibre

---

général et partiel, et, d'autre part, des modèles - quadratiques. Dans les modèles affines du type à équilibre partiel, nous supposons que le taux zéro-coupon est une fonction affine d'un ensemble de variables d'état. Dans les modèles quadratiques, nous procédons de la même manière, en utilisant toutefois la fonction non linéaire des variables d'état. Dans la famille des modèles dits sans arbitrage, l'absence d'opportunité d'arbitrage est essentielle pour concevoir la structure par terme des taux d'intérêt. L'un des objectifs de cette approche est de s'appuyer sur un ajustement parfait à chaque instant, puis sur la dynamique appropriée de la structure par terme des taux d'intérêt. Dans la famille des modèles dynamiques, nous avons une forme fonctionnelle avec plusieurs paramètres. Les articles de recherche dans ce domaine sont basés sur les travaux de [Nelson and Siegel \[1987\]](#) qui ont présenté un modèle parcimonieux à trois facteurs qui s'avère parfaitement épouser la courbe des taux. Nous y présentons également une approche purement statistique appelée l'analyse des composantes principales (ACP). Avec ACP, on peut expliquer la dynamique de la structure de terme entière avec seulement 3 facteurs qui, en plus, ont une interprétation intéressante comme le niveau, la pente et la courbure de la courbe des taux.

Comme la structure par terme des taux d'intérêt nous donne des informations sur le rendement prévu à l'échéance spécifique, il est important de parler de la mesure de la duration. La duration et la convexité sont deux outils standard utilisés pour gérer l'exposition au risque pour des investissements à revenu fixe. La duration mesure la sensibilité de l'obligation aux variations de taux d'intérêt. Nous présentons également des informations sur les institutions qu'on trouve sur les marchés des titres à revenu fixe. Précisément des informations sur les produits et les acteurs sur le marché des obligations d'état. Sur le marché des obligations, le marché primaire est le lieu où les dettes arrivent en premier. Il existe des mécanismes particuliers pour vendre ces dettes, appelées vente aux enchères. Mais une fois qu'une action ou une obligation émise sur marché a été achetée par un investisseur, nous avons affaire à un marché secondaire. Un marché secondaire est le lieu de rencontre d'investisseurs désireux de vendre et d'investisseurs désireux d'acheter. Ce processus donne lieu à un cours déterminé par le niveau de l'offre et de la demande à un moment donné. Il existe différents

---

types de courbes de taux que nous pouvons construire, notamment les taux zéro-coupon, les taux à terme et *par yield*. Nous présentons quelques informations sur l'aspect gestion de la dette avec les problèmes d'inflation. Dans la conclusion du premier chapitre de cette thèse, nous présentons l'aspect liquidité sur le marché des obligations d'état.

**Le deuxième chapitre** compare la performance de quatre modèles de structure par terme des taux d'intérêt sur des données relatives à quatre pays dans la zone euro: la France, l'Allemagne, l'Italy et l'Espagne. Peu d'articles comparent plusieurs modèles de taux d'intérêt sur des ensembles de données contenant plusieurs pays. Presque aucun d'entre eux n'utilise les prix des obligations. Pour les gestionnaires de titres à revenu fixe, les macroéconomistes et les économistes financiers, il est très important de pouvoir établir une prévision précise de la structure par terme des taux d'intérêt. De plus, l'optimisation du portefeuille obligataire, l'estimation du prix des actifs financiers et de leurs dérivés, ainsi que la gestion des risques, reposent largement sur les prévisions de taux d'intérêt. Nous proposons une étude prévisionnelle de la structure par terme des taux d'intérêt.

Le modèle de structure par terme de taux d'intérêt proposé par [Nelson and Siegel \[1987\]](#) constitue le point de départ de nos recherches. Pour les participants au marché, les modèles de type Nelson Siegel sont les modèles de référence. La *Bank of International Settings (BIS)* présente la liste des pays qui utilisent la méthode d'ajustement de la courbe des taux et, pour chaque pays, elle indique le modèle de la structure par terme des taux d'intérêt utilisés par les banques centrales. Pour la majorité des pays, le modèle de [Svensson \[1994\]](#) est le plus populaire. Dans le deuxième chapitre de cette thèse, nous comparons quatre modèles de la structure par terme des taux d'intérêt pour extraire la courbe des taux à partir des prix des obligations à coupon observés. Les deux premiers modèles que nous considérons sont pris de la littérature: (1) Nelson-Siegel à 4 paramètres et une bosse possible; (2) Svensson avec 6 paramètres et deux bosses possibles. Les deux derniers sont nouveaux et introduits dans cette recherche: (3) *extended* Bjork-Christensen avec 6 paramètres et deux bosses possibles ainsi que le Bjork-Christensen original proposé par [Bjork and Christensen \[1997\]](#) avec 5 paramètres et une contrainte sur les paramètres non linéaires; (4) *extended* Svensson avec 7 paramètres et deux

---

bosses. Pour évaluer les performances de chaque modèle de structure par terme des taux d'intérêt, nous proposons deux critères. Le premier est l'erreur absolue moyenne. La seconde donne le nombre de jours où le modèle donné a fourni les meilleures performances parmi les autres. Ainsi, nous pouvons trouver un modèle qui décrit les prix réels des obligations sur le marché avec la meilleure précision.

Nous collectons des données sur les obligations d'état de quatre pays de la zone euro: la France, l'Allemagne, l'Italie et l'Espagne sur une période d'environ 20 ans, de 1999 à 2018. Notre ensemble de données de prix quotidiens contient au total 800 obligations assorties de taux de coupon et d'échéances différents. En prenant des obligations libellées en euros, nous évitons toute complication liée au taux de change. Nous comparons la manière dont quatre modèles de structure par terme des taux d'intérêt de type Nelson Siegel arrivent à décrire les prix des obligations d'état observés sur le marché. Nous prenons des données sur les prix des obligations d'état et calculons des paramètres. Ensuite, nous calculons les prix des obligations pour chaque modèle et chaque pays. La principale conclusion de notre enquête est qu'un modèle de structure par terme des taux d'intérêt *extended* Svensson offre les meilleures performances en matière de prix des obligations pour tous les quatre pays. Les erreurs d'ajustement montrent que tous les modèles rencontrent des difficultés pour s'ajuster aux données italiennes. Pour ce pays, l'erreur d'ajustement moyenne sur l'ensemble de la période d'échantillonnage est d'environ 22-24 points de base, quels que soient les modèles de structure par terme des taux d'intérêt. Pour comparer, l'erreur d'ajustement moyenne pour la France est d'environ 11 points de base. Pour l'Italie et la France, la marge d'erreur d'ajustement moyenne est assez large par rapport à l'Allemagne et à l'Espagne. L'ajustement du modèle aux données allemandes est mieux par rapport aux données du marché espagnol.

Une description de la dynamique de la courbe des taux ne devrait pas seulement permettre d'ajuster très bien les données (dans l'échantillon). Il devrait également être capable de prévoir les données (hors échantillon). Nous proposons une étude prévisionnelle de la structure par terme des taux d'intérêt dans l'esprit de [Diebold and Li \[2006\]](#). Les auteurs abordent le problème pratique de la prévision de la dynamique de courbe des taux. Nous considérons trois méthodes

---

de prévision alternatives, notamment les spécifications de marche aléatoire, les spécifications autorégressives univariées et multivariées. Nos résultats montrent que la spécification autorégressive univariée donne une prévision plus précise, à savoir que les erreurs de prévision moyennes sont plus petites que celles de deux autres concurrents.

**Le troisième chapitre** examine la structure par terme des taux d'intérêt nominaux français. C'est un domaine d'étude très important du point de vue financier et économique. Notre étude est la première étude exhaustive de toutes les données publiques disponibles sur la dette nominale française couvrant la période de 30 ans allant de 1988 à 2018. Récemment, les marchés du monde entier ont été confrontés à la réalité avec des taux d'intérêt négatifs, principalement à court terme de la courbe des taux. Nous construisons la courbe des taux nominaux français en utilisant des cotations des titres nominaux français appelés "Obligation Assimilable du Trésor" (OAT) et "Bons du Trésor à taux fixe et Intersets Annuels" (BTAN) à une fréquence quotidienne. Ces obligations ont des échéances à l'émission allant de 1 à 50 ans. Notre période d'échantillonnage commence en 1988 et inclut le lancement de la devise euro en janvier 1999 et se termine en avril 2018. La méthodologie de notre article repose sur [Gürkaynak, Sack, and Wright \[2007\]](#). Ces auteurs comblent ce vide en publiant quotidiennement les estimations de la courbe des taux du Trésor de la Réserve fédérale américaine de 1961 à nos jours. Nous utilisons une méthode de lissage similaire pour ajuster les données et nous montrons que cet ajustement est très bon. Les estimations résultantes sont utilisées pour calculer les taux à terme pour n'importe quel horizon.

Le terme "assimilable" dans les OAT est technique. Il fait référence au fait que ces titres sont fongibles avec des obligations anciennes aux caractéristiques identiques: même maturité, même taux nominal et même valeur nominale. Cela signifie également que les obligations nouvellement émises se mélangent avec l'émission de dette qui contient ces obligations anciennes. À première vue, cela peut paraître très semblable au dispositif de réouverture de la dette américaine. Cependant, ils ne sont pas exactement les mêmes. En France la gestion de la dette obligataire repose explicitement sur une première souche, qui est la toute première émission de dette qui servira de matrice pour les émissions suivantes. Les obli-

---

gations récemment émises sont si fongibles que les obligations anciennes et les nouvellement émises sont effectivement impossibles à distinguer. Par conséquent il n'est pas vraiment approprié de parler d'une nouvelle tranche.

Les principaux résultats du troisième chapitre sont les suivants. Nous exploitons la mesure de bruit pour évaluer la "qualité" du marché français. Cette mesure est proposée par [Hu, Pan, and Wang \[2013\]](#) pour saisir les épisodes de crise de liquidité d'origines différentes sur le marché financier. Il fournit des informations sur l'illiquidité au-delà des procurations de liquidité existantes. Globalement, au moyen de la mesure du bruit, nous constatons que, durant la première décennie de notre période d'échantillonnage, les opportunités d'arbitrage n'étaient pas rares sur le marché des OAT, mais que la situation s'était considérablement améliorée depuis l'introduction de l'euro.

Nous étudions également la prime *on-the-run* (*OTR*) sur les données françaises. [Vayanos and Weill \[2008\]](#) proposent une théorie fondée sur la recherche dans laquelle des actifs dotés de flux de trésorerie identiques peuvent être négociés à des prix différents. Les auteurs montrent que la liquidité et les particularités expliquent ce phénomène simultanément via l'activité de vente à découvert. Pour un titre financier particulier, nous utilisons le cours de clôture fourni par Bloomberg. En utilisant ces cotations, nous calculons les écarts entre le rendement à échéance de la dernière obligation émise (appelé *on-the-run security*) et l'obligation qui existe déjà sur le marché avec les mêmes caractéristiques (appelé *off-the-run security*). Nous constatons que les écarts moyens et médians sont négatifs. De plus, les écarts-types sont relativement élevés pour la plupart des fourchettes de maturité, ce qui suggère l'absence de prime *OTR* sur le marché des obligations d'état françaises.

**Le quatrième et dernier chapitre** analyse les obligations protégées de l'inflation et examine la structure par terme des taux réels français. Nous contestons l'approche bien connue selon laquelle les taux réels sont constants et les taux nominaux évoluent dans le temps. Nous constatons que la différence entre les taux réels et les taux nominaux (qui correspond à la compensation de l'inflation) reste constante et que les taux réels varient dans le temps. La particu-

---

larité de nos recherches est que nous travaillons avec deux échantillons différents dans l'ensemble de données de marché sur les titres protégés contre l'inflation émis par le gouvernement français. L'ensemble de données est divisé en deux parties, la première comprenant les obligations indexées sur l'inflation domestique La seconde comprend les obligations indexées sur l'inflation européenne. Nous mettons pleinement en œuvre la méthodologie proposée par [Gürkaynak, Sack, and Wright \[2010\]](#). En appliquant cette méthodologie, nous observons que les erreurs d'ajustement sont assez petites, et ça nous montre la qualité du modèle à expliquer les données.

Notre premier résultat est le bon ajustement du modèle de la structure par terme des taux d'intérêt sur les taux réels français qui sont donnés implicitement par les prix des obligations indexées sur l'inflation. Cette recherche s'appuie sur la méthodologie de calcul de la courbe des taux de [Svensson \[1994\]](#) et nous trouverons que ce modèle arrive à très bien décrire la réalité du marché. Nous obtenons des estimations fiables de taux réels intermédiaires et à long terme. Nous répétons chaque jour l'exercice d'ajustement et nous avons donc une évaluation quotidienne de la forme de la structure par terme des taux d'intérêt.

Le *breakeven rate*, aussi appelé point mort de l'inflation, représente la différence de rendement entre le taux nominal (le rendement à l'échéance d'une obligation classique) et le taux réel (le rendement à l'échéance d'une obligation de même émetteur et avec même échéance, mais indexée sur l'inflation). Parmi nos résultats, on calcule le *breakeven rate* et on peut dire que cette valeur ne varie pas beaucoup dans le temps. Nous faisons également un exercice de *backcasting* sur les valeurs de point mort de l'inflation dans le temps. Pour ce faire, nous trouvons la combinaison de taux nominaux qui reflète le mieux les recherches sur la période pour laquelle nous avons des données sur les titres français protégés contre l'inflation, puis nous calculons ces valeurs sur un échantillon beaucoup plus long.

En conclusion on peut préciser qu'une contribution a été apportée à l'étude de la structure par terme des taux d'intérêt avec trois études empiriques approfondies portant toutes sur les marchés des obligations d'état. Ces trois études empiriques

---

donnent des résultats intéressants sur la structure par terme des taux d'intérêt français, de la gestion de la dette et des marchés des obligations d'état. Ce travail est soumis à certaines limitations qui suggèrent en même temps des pistes potentiellement prometteuses pour des recherches futures. Nous avons laissé ouvertes quelques questions lorsque différentes structures de termes sont considérées de la même manière. Enfin, cette thèse porte sur la compréhension des taux d'intérêt nominaux et réels et l'ajustement de la structure par terme des taux d'intérêt sur les données de prix d'obligations.

**Mots-clés:** La structure par terme des taux d'intérêt, l'ajustement de la courbe des taux, la zone euro, les obligations d'état, modèle de Svensson, OTR premium, les taux nominaux, les obligations indexées sur l'inflation, les taux réels, l'inflation.



# Bibliography

- Abad, P., H. Chulia, and M. Gomez-Puig, 2010, “EMU and European government bond market integration,” *Journal of Banking and Finance*, 34(12), 2851–2860. [6](#)
- Ahn, D. H., R. F. Dittmar, and A. R. Gallant, 2002, “Quadratic Term Structure Models: Theory and Evidence,” *Review of Financial Studies*, 15(1), 243–288. [13](#)
- Andreasen, M., and A. Meldrum, 2014, “Dynamic term structure models: The best way to enforce the zero lower bound,” CREATES Research Paper 2014-47. [14](#)
- Andreasen, M. M., J. H. Christensen, and S. Riddell, 2018, “The TIPS liquidity premium,” Federal Reserve Bank of San Francisco Working Paper. [9](#)
- Andreasen, M. M., J. H. E. Christensen, and S. Riddell, 2017, “The TIPS liquidity premium,” Federal Reserve Bank at San Francisco, Working paper 2017-11. [122](#)
- Ang, A., G. Bekaert, and M. Wei, 2008, “The term structure of real rates and expected inflation,” *Journal of Finance*, 63, 797–849. [138](#)
- Ang, A., S. Dong, and M. Piazzesi, 2007, “No-arbitrage Taylor rules,” National Bureau of Economic Research (No. w13448). [13](#)
- Ang, A., D. Papanikolaou, and M. Westerfield, 2014, “Portfolio Choice with Illiquid Assets,” *Management Science*, 60(11). [151](#)
- Ang, A., and M. Piazzesi, 2003, “A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables,” *Journal of Monetary Economics*, 50, 745–787. [13](#)
- Ang, A., M. Piazzesi, and M. Wei, 2006, “What does the yield curve tell us about GDP growth?,” *Journal of Econometrics*, 131(1–2), 359–403. [39](#), [92](#), [116](#)
- Bakshi, G. S., and Z. Chen, 1996, “Inflation, asset prices, and the term structure of interest rates in monetary economics,” *Review of Financial Studies*, 9, 241–275. [166](#), [169](#)

- Barding, F., and T. Lehnert, 2004, “European inflation-indexed government debt security markets,” *Journal of Portfolio Management*, 30(4), 226–238. [8](#)
- Barr, D. G., and J. Y. Campbell, 1997, “Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices,” *Journal of Monetary Economics*, 39, 361–383. [167](#)
- Batten, J., T. Fetherston, and P. Szilagyi, 2004, *European fixed income markets*. Wiley. [91](#)
- Beaglehole, D., and M. Tenney, 1991, “General Solutions of Some Interest Rate-Contingent Claim Pricing Equations,” *Journal of Fixed Income*, 1, 69–83. [13](#)
- , 1992, “A Nonlinear Equilibrium Model of the Term Structure of Interest Rates: Corrections and Additions,” *Journal of Financial Economics*, 32, 345–454. [13](#)
- Bernoth, K., J. Hagen, and L. Schuknecht, 2012, “Sovereign risk premiums in the European government bond market,” *Journal of International Money and Finance*, 31(5), 975–995. [7](#)
- Bjork, T., and B. Christensen, 1997, “Interest Rate Dynamics and Consistent Forward Rate Curves,” *Mathematical Finance*, 9, 197–231. [4](#), [17](#), [45](#), [188](#)
- Bliss, R., 1997, “Testing term structure estimation models,” *Advances in Futures and Options Research*, 9, 197–231. [41](#), [113](#)
- Bolder, D. J., and D. Streliski, 1999, “Yield Curve Modelling at the Bank of Canada,” Technical Reports 84, Bank of Canada. [49](#)
- Boyd, J. H., R. Levine, and B. D. Smith, 2001, “The impact of inflation on financial sector performance,” *Journal of Monetary Economics*, 47, 221–248. [165](#)
- Brandt, M., and D. Chapman, 2008, *Affine Term Structure Models*. The New Palgrave Dictionary of Economics. [13](#)
- Buiter, W. H., 2009, “Negative nominal interest rates: Three ways to overcome the zero lower bound,” *The North American Journal of Economic and Finance*, 20(3), 213–238. [6](#)
- Buraschi, A., and A. Jiltsov, 2005, “Inflation risk premia and the expectation hypothesis,” *Journal of Financial Economics*, 75, 429–490. [139](#)
- Caldeira, J. F., G. V. Moura, and A. A. Santos, 2016, “Predicting the yield curve using forecast combinations,” *Computational Statistics and Data Analysis*, 100, 79–98. [66](#)

## BIBLIOGRAPHY

---

- Campbell, J. Y., and R. H. Clarida, 1987, “The term structure of euro market interest rates: An empirical investigation,” *Journal of Monetary Economics*, 19(1), 25–44. [5](#)
- Chan, K. C., G. A. Karolyi, F. A. Longstaff, and A. B. Sanders, 1992, “An Empirical Comparison of Alternative Models of the Short-Term Interest Rate,” *Journal of Finance*, 47(3), 1209–1227. [4](#)
- Chang, H., 2019, “Oil and Inflation Compensation: Evidence from Treasury Inflation-Protected Securities Prices,” Available at SSRN 3449811. [136](#)
- Chen, L., D. Filipovic, and H. V. Poor, 2003, “Quadratic term structure models for risk-free and defaultable rates,” *Mathematical Finance*, 14, 515–536. [14](#)
- Chen, R., B. Liu, and X. Cheng, 2010, “Pricing the term structure of inflation risk premia: Theory and evidence from TIPS,” *Journal of Empirical Finance*, 17(4), 702–721. [136](#)
- Chernov, M., and P. Mueller, 2012, “The term structure of inflation expectations,” *Journal of Financial Economics*, 106, 367–394. [139](#)
- Christensen, B. J., and M. Wei, 2019, “An asset pricing approach to testing general term structure models,” *Journal of Financial Economics*, forthcoming. [6](#)
- Christensen, J., F. Diebold, and G. Rudebusch, 2011, “The affine arbitrage-free class of Nelson-Siegel term structure models,” *Journal of Econometrics*, 164, 4–20. [18](#)
- Christensen, J. H., J. A. Lopez, and P. Shultz, 2017, “Is There an On-the-Run Premium in TIPS?,” Federal Reserve Bank of San Francisco Working Paper. [9](#)
- Christenses, J., J. Lopez, and P. Shultz, 2017, “Is There an On-the-Run Premium in TIPS?,” Federal Reserve Bank of San Francisco Working Paper No. 2017-10. [139](#)
- Cibulka, J., 2015, “Implied efficiency of forward rates in the yields of government bonds,” *Procedia Economics and Finance*, 25, 192–199. [34](#)
- Cochrane, J., and M. Piazzessi, 2008, “Decomposing the Yield Curve,” Working paper. [39](#)
- Constantinides, G., 1992, “A Theory of the Nominal Structure of Interest Rates,” *Review of Financial Studies*, 5, 531–552. [13](#)
- Cox, J., J. Ingersoll, and S. Ross, 1985, “A theory of the term structure of interest rates,” *Econometrica*, 53, 385–407. [12](#)

- Creal, D., and J. Wu, 2015, “Estimation of affine term structure models with spanned or unspanned stochastic volatility,” *Journal of Econometrics*, 185(1), 60–81. [13](#)
- Dai, Q., and K. Singleton, 2000, “Specification analysis of affine term structure models,” *Journal of Finance*, 55, 1943–1978. [5](#), [18](#)
- D’Amico, S., D. Kim, and M. Wei, 2018, “Tips from TIPS: the informational content of Treasury Inflation-Protected Security prices,” *Journal of Financial and Quantitative*, 53(1), 395–436. [9](#)
- D’Amico, S., D. H. Kim, and M. Wei, 2014, “Tips from TIPS: The informational content of Treasury Inflation-Protected Security prices,” Working paper 2014-24r, Federal Reserve Board, Washington, D.C. [136](#), [138](#)
- De Pooter, M., 2007, “Examining the Nelson-Siegel Class of Term Structure Models,” Tinbergen Institute Discussion Paper. No. 2007-043/4. [17](#)
- Deacon, M., A. Derry, and D. Mirfendereski, 2004, *Inflation-indexed securities: bonds, swaps and other derivatives*. John Wiley & Sons. [8](#), [137](#)
- Deaves, R., and M. Parlar, 2000, “A generalized bootstrap method to determine the yield curve,” *Applied Mathematical Finance*, 7(4), 257–270. [31](#)
- Diebold, F., and C. Li, 2006, “Forecasting the term structure of government bond yields,” *Journal of Econometrics*, 130, 337–364. [5](#), [6](#), [16](#), [18](#), [41](#), [43](#), [66](#), [189](#)
- Diebold, F., M. Piazzesi, and G. Rudebusch, 2005, “Modeling Bond Yields in Finance and Macroeconomics,” NBER Working Paper. [39](#)
- Diebold, F., G. Rudebusch, and S. Aruoba, 2006, “The macroeconomy and the yield curve: a dynamic latent factor approach,” *Journal of Econometrics*, 131, 309–338. [17](#), [42](#), [43](#)
- Duffee, G., 1996, “Idiosyncratic variation of Treasury bill yields,” *Journal of Finance*, 51(2). [103](#)
- Duffee, G., 2002, “Term premia and interest rate forecasts in affine models,” *Journal of Finance*, 57, 405–443. [18](#)
- Duffie, D., and R. Kan, 1996, “A yield-factor model of interest rates,” *Mathematical Finance*, 6, 379–406. [13](#)
- D’Amico, S., R. Fan, and Y. Kitsul, 2018, “The Scarcity Value of Treasury Collateral: Repo-Market Effects of Security-Specific Supply and Demand Factors,” *Journal of Financial and Quantitative Analysis*, 53(5), 2103–2129. [8](#)

- ECB, 2016, “Economic and monetary developments,” ECB Economic Bulletin, issue 8. [141](#)
- , 2017, “Debt securities issuance and service by EU governments,” Monthly Report of the European Central Bank, available at <http://sdw.ecb.europa.eu/reports.do?node=1000004059>. [90](#)
- Ejsing, J. W., J. A. Garcia, and T. Werner, 2007, “The Term Structure of Euro Area Break-Even Inflation Rates: The Impact of Seasonality (November 2007),” ECB Working Paper No. 830. Available at SSRN: <https://ssrn.com/abstract=1028713>. [141](#)
- Ejsing, J. W., and J. Sihvonen, 2009, “Liquidity premia in German government bonds,” ECB Working paper series No. 1081. [92](#), [119](#)
- Ermolov, A., 2017, “International Real Yields (July 7, 2017),” Paris December 2017 Finance Meeting EUROFIDAI - AFFI. [140](#), [142](#)
- Estrella, A., and F. Mishkin, 1995, “The term structure of interest rates and its role in monetary policy for the European Central Bank,” Federal Reserve Bank of New York, Research Paper No. 9526. [43](#)
- Fleckenstein, M., F. A. Longstaff, and H. Lustig, 2014, “The TIPS-Treasury bond puzzle,” *Journal of Finance*, 69(5), 2151–2197. [136](#)
- Fleming, M., 2003, “Measuring Treasury market liquidity,” *FRBNY Economic Policy Review*. [92](#), [116](#), [117](#), [118](#)
- Fleming, M., and N. Krishnan, 2012, “The Microstructure of the TIPS Market,” FRBNY Economic Policy Review March 2012. [139](#)
- Gauthier, G., and J.-G. Simonato, 2012, “Linearized Nelson–Siegel and Svensson models for the estimation of spot interest rates,” *European Journal of Operational Research*, 219(2), 442–451. [92](#), [104](#)
- Gibson, R., L. S. Lhabitant, and D. Talay, 2012, “Modelling the term structure of interest rates: a review of the literature,” *Foundations and Trends in Finance*, 5(1-2). [15](#)
- Golub, B. W., and L. M. Tilman, 2000, *Risk Management: Approaches for Fixed Income Markets*. Wiley. [19](#)
- Gombani, A., and W. Runggaldier, 2013, “Arbitrage-free multifactor term structure models: a theory based on stochastic control,” *Mathematical Finance*, 23(4), 659–686. [15](#)

- Grishchenko, O., S. Mouabbi, and J.-P. Renne, 2016, “Measuring Inflation Anchoring and Uncertainty: A US and Euro Area Comparison,” *Journal of Money, Credit, and Banking*, forthcoming. [139](#)
- Grishchenko, O. V., and J.-Z. Huang, 2013, “The inflation risk premium: Evidence from the TIPS market,” *Journal of Fixed Income*, 22(4), 5–30. [9](#), [122](#), [136](#)
- Gürkaynak, R., B. Sack, and J. H. Wright, 2007, “The U.S. Treasury Yield Curve: 1961 to the Present,” *Journal of Monetary Economics*, 54, 2291–2304. [7](#), [91](#), [190](#)
- , 2010, “The TIPS Yield Curve and Inflation Compensation,” *American Economic Journal: Macroeconomics*, 2(1), 70–92. [8](#), [9](#), [33](#), [136](#), [141](#), [162](#), [163](#), [169](#), [192](#)
- Hamilton, J., and J. Wu, 2014, “Testable implications of affine term structure models,” *Journal of Econometrics*, 178(2), 231–242. [13](#)
- Harvey, C. R., 1988, “The real term structure and consumption growth,” *Journal of Financial Economics*, 22, 305–333. [166](#)
- Heath, D., R. Jarrow, and A. Morton, 1992, “Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation,” *Econometrica*, 60, 77–105. [15](#)
- Ho, T., and S. Lee, 1986, “Term structure movements and pricing interest rate contingent claims,” *Journal of Finance*, 41, 27–45. [14](#)
- Hoffmaister, A., J. Roldos, and A. Tuladhar, 2010, “Yield Curve Dynamics and Spillovers in Central and Eastern European Countries,” IMF Working Papers. [42](#)
- Hu, X., J. Pan, and J. Wang, 2013, “Noise as information for illiquidity,” *Journal of Finance*, 68(6), 2341–2382. [7](#), [92](#), [191](#)
- Hull, J., and A. White, 1993, “The pricing of options on interest rate caps and floors using the Hull-White model,” *Journal of Financial Engineering*, 2(3), 287–296. [14](#)
- Hull, J. C., and A. White, 1990, “Pricing Interest-Rate-Derivative Securities,” *The Review of Financial Studies*, 3, 573–592. [14](#)
- Huybens, E., and B. D. Smith, 1999, “Inflation, financial markets and long-run real activity,” *Journal of Monetary Economics*, 43, 283–315. [165](#)
- Ioannides, M., 2003, “A comparison of yield curve estimation techniques using UK data,” *Journal of Banking and Finance*, 27, 1–26. [42](#)

- Jong, F. D., 200, “Time series and cross section information in affine term structure models,” *Journal of Business and Economic Statistics*, 18, 300–314. 18
- Joslin, S., M. Pribsch, and K. J. Singleton, 2014, “Risk premiums in dynamic term structure models with unspanned macro risks,” *Journal of Finance*, 69(3), 1197–1233. 18
- Kalev, P. S., 2004, “Estimating and interpreting zero coupon and forward rates: Australia, 1992-2001,” Available at SSRN 495702. 42
- Kimmel, R. L., 2004, “Modeling the term structure of interest rates: A new approach,” *Journal of Financial Economics*, 72(1), 143–183. 14
- Kita, A., and D. L. Tortorice, 2018, “Can Risk Models Extract Inflation Expectations from Financial Market Data? Evidence from the Inflation Protected Securities of Six Countries,” Working paper. 136
- Koo, B., D. L. Vecchia, and O. B. Linton, 2019, “Estimation of a Nonparametric model for Bond Prices from Cross-section and Time series Information,” Working Paper and available at SSRN: [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=3341344](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3341344). 6
- Koopman, S. J., and M. Wel, 2013, “Forecasting the US term structure of interest rates using a macroeconomic smooth dynamic factor model,” *International Journal of Forecasting*, 29(4), 676–694. 5
- Koukouritakis, M., and L. Michelis, 2006, “The Term Structure of Interest Rates in the European Union,” Working Paper. 42
- Krishnamurthy, A., 2002, “The bond/old-bond spread,” *Journal of Financial Economics*, 66, 463–506. 92
- Litterman, R., and J. Scheinkman, 1991, “Common factors affecting bond returns,” *Journal of Fixed Income*, 1, 54–61. 18, 114
- Longstaff, F. A., 1989, “A non linear general equilibrium model of the term structure of interest rates,” *Journal of Financial Economics*, 23, 195–224. 13
- Malava, A., 1999, “Principal Component Analysis on Term Structure of Interest Rates,” *Helsinki University of Technology Department of Engineering Physics and Mathematics Working Paper*. 20
- Malkhozov, A., P. Mueller, A. Vedolin, and G. Venter, 2017, “International illiquidity,” Federal Reserve Board International Finance Discussion Paper No. 1201. 122
- Martellini, L., P. Priaulet, and S. Priaulet, 2003, *Fixed-income securities: Valuation, risk management and portfolio strategies*. Wiley. 101, 143

## BIBLIOGRAPHY

---

- McCulloch, J., 1971, “Measuring the Term Structure of Interest Rates,” *Journal of Business*, 44, 19–31. [45](#)
- , 1975, “The Tax-Adjusted Yield Curve,” *Journal of Finance*, 30, 811–830. [45](#)
- Mehrotra, A., and J. Yetman, 2018, “Decaying expectations: what inflation forecasts tell us about the anchoring of inflation expectations,” *International Journal of Central Banking*, 14(5), 55–101. [33](#)
- Merton, R. C., 1973, “Theory of Rational Option Pricing,” *Bell Journal of Economics and Management Science*, 4(1), 141–183. [12](#)
- Moraux, F., C. Perignon, and C. Villa, 2002, “Common Factors in International Bond Returns Revisited: A Common Principal Component Approach,” Available at SSRN: <https://ssrn.com/abstract=302086> or <http://dx.doi.org/10.2139/ssrn.302086>. [20](#)
- Morris, J. R., 1976, “On corporate debt maturity strategies,” *Journal of Finance*, 31, 29–37. [168](#)
- Moungala, W. P., 2013, “Modelisation et Gestion sur les marches obligataires souverains,” PhD thesis, University Rennes 1. [19](#)
- Nelson, C. R., and A. F. Siegel, 1987, “Parsimonious modeling of the yield curves,” *Journal of Business*, 60, 473–489. [4](#), [11](#), [15](#), [16](#), [18](#), [40](#), [101](#), [139](#), [142](#), [187](#), [188](#)
- OECD, 2017, “OECD Sovereign Borrowing Outlook,” . [90](#)
- Pasquariello, P., and C. Vega, 2009, “The on-the-run liquidity phenomenon,” *Journal of Financial Economics*, 92, 1–24. [92](#), [116](#)
- Pericoli, M., 2014, “Real Term Structure and Inflation Compensation in the Euro Area,” *International Journal of Central Banking*, 10(1), 1–42. [139](#)
- Phoa, W., 1999, “Estimating credit spread risk using extreme value theory,” *Journal of Portfolio Management*, 25(3), 69–73. [20](#)
- Piazzesi, M., and M. Schneider, 2009, “Inflation and the price of real assets,” Federal Reserve Bank of Minneapolis, Research Departement. [167](#)
- Ricart, R., and P. Sicsic, 1995, “Estimation d’une structure par terme des taux d’intérêt sur données françaises,” *Bulletin de la Banque de France*, 22, 117–129. [91](#), [103](#)

## BIBLIOGRAPHY

---

- Ricart, R., P. Sicsic, and E. Jondeau, 2005, “Zero-coupon yield curves: technical documentation,” BIS working paper 25, Available at SSRN: <https://ssrn.com/abstract=1188514> or <http://dx.doi.org/10.2139/ssrn.1188514>. 4
- Rogers, L., 1995, “Which model for term-structure of interest rates should one use?,” *IMA Volumes in Mathematics and Its Applications*, 65, 93–116. 54
- Sander, H., and S. Kleimeier, 2004, “Convergence in euro-zone retail banking? What interest rate pass-through tells us about monetary policy transmission, competition and integration,” *Journal of International Money and Finance*, 23(3), 461–492. 5
- Sarig, O., and A. Warga, 1989, “Bond Price Data and Bond Market Liquidity,” *Journal of Financial and Quantitative Analysis*, 24(3), 367–378. 149
- Smith, K. L., 2002, “Government Bond Market Seasonality, Diversification, and Cointegration: International Evidence,” *Journal of Financial Research*, 25(2), 203–221. 6
- Sopov, B., and J. Seidler, 2010, “Yield curve dynamics: Regional common factor model,” IES Working Paper, 17/2010. 42
- Stehle, R., 1977, “An empirical test of alternative hypothesis of national and international pricing of risky assets,” *Journal of Finance*, 32, 493–502. 166
- Stulz, R. M., 1986, “Asset Pricing and Expected Inflation,” *Journal of Finance*, 41, 209–223. 165
- Svensson, L. E. O., 1994, “Estimating and interpreting forward rates: Sweden 1992-1994,” National Bureau of Economic Research working paper 4871. 4, 16, 40, 45, 46, 91, 98, 101, 104, 106, 107, 110, 111, 112, 115, 130, 131, 133, 138, 142, 157, 158, 169, 171, 172, 173, 174, 175, 176, 188, 192
- Thornton, D. L., 1988, “The effect of monetary policy on short-term interest rates,” *Federal Reserve Bank of St. Louis Review*, 70(3), 53–72. 29
- Treasury, SEC, and FRB, 1992, “Joint report on the government securities market,” Published by the Department of Treasury and Securities Exchange Commission and Board of Governors of the Federal Reserve System, available at <https://www.treasury.gov/resource-center/fin-mkts/Documents/gsr92rpt.pdf>. 94
- Vasicek, O., 1977, “An equilibrium characterisation of the term structure,” *Journal of Financial Economics*, 5, 177–188. 12, 13
- Vayanos, D., and P.-O. Weill, 2008, “A search-based theory of the on-the-run phenomenon,” *Journal of Finance*, 63(3), 1361–1398. 7, 191

## BIBLIOGRAPHY

---

- Warga, A., 1992, “Bond returns, liquidity, and missing data,” *Journal of Financial and Quantitative Analysis*, 27, 605–617. [92](#)
- Westerhout, E., and R. Beetsma, 2019, “A comparison of nominal and indexed debt under fiscal constraints,” *Journal of International Money and Finance*, 91, 177–194. [9](#)

**Titre :** Les modèles de la structure par terme des taux d'intérêt, la dette publique et les obligations indexées sur l'inflation

**Mots clés :** La courbe des taux, modèle de Svensson, les taux nominaux et réels.

**Résumé :** Cette thèse présente trois études empiriques afin d'étudier la structure par terme des taux d'intérêt. Ces trois enquêtes empiriques donnent des résultats intéressants sur la structure par terme des taux d'intérêt français, de la gestion de la dette publique et des marchés des obligations d'état. La thèse présente des aspects théoriques tels que les modèles de la structure par terme des taux d'intérêt, les trois facteurs de la courbe des taux qui correspondent au niveau, à la pente et à la courbure, la mesure de la durée, l'organisation du marché des titres à revenu fixe, différents types de rendements et enfin la notion de l'inflation. La thèse examine quatre modèles de la structure par terme des taux d'intérêt de type Nelson-Siegel pour ajuster la

courbe des taux sur les données de marché avec les prix des obligations d'état. Les données contiennent des obligations émises par quatre pays de la zone euro. La thèse construit la courbe des taux nominaux française en utilisant toutes les données publiques disponibles sur les obligations émises par le Trésor français avec les échéances au moment de l'émission de 1 à 50 ans. Enfin, la thèse examine les taux réels du marché des obligations d'état français en utilisant les données sur les titres indexées sur l'inflation et émis par le Trésor français. Une contribution a été apportée à l'étude de la structure par terme des taux d'intérêt avec trois études empiriques approfondies portant toutes sur les marchés des obligations d'état.

**Title :** Term structure of interest rates models, nominal government debt and inflation-protected securities

**Keywords :** Fitting the yield curve, Svensson model, OTR premium, nominal and real rates

**Abstract :** This thesis investigates the term structure of interest rates via three empirical studies. These three empirical investigations give some interesting results about the term structure of French interest rates, debt management and government bond markets. This thesis provides several theoretical aspects of term structure of interest rates models, the three factors of the yield curve known as the level, the slope and the curvature, the duration measure, the organization of the fixed income securities market, different types of yields as zero-coupon, par yield and forward rates and finally the inflation. This thesis examines four Nelson-Siegel style yield curve models for fitting the term structure of interest rates on data about government bond prices. The dataset contains

bonds issued by four countries in Euro area. This thesis constructs the French nominal yield curve using all available public data of French nominal Treasury securities of maturities at issuance from 1 to 50 years. Finally, this thesis investigates real rates on French government bond market using the data on French inflation-protected Treasury securities. This study provides a comprehensive view on the market by the construction of real yield curve. Our data set includes both types of such securities, those indexed on the domestic consumer price index and on the European inflation index. A contribution was made to the understanding the term structure of interest rates with three in-depth empirical studies, all dealing with government bond markets.