

Architectured poroelastic metamaterials for sound attenuation enhancement in the low frequency range: application to silencers and partition panels

Ke Li

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Ke Li. Architectured poroelastic metamaterials for sound attenuation enhancement in the low frequency range : application to silencers and partition panels. Acoustics [physics.class-ph]. Université de Technologie de Compiègne, 2021. English. NNT : 2021COMP2628 . tel-03614598

HAL Id: tel-03614598 https://theses.hal.science/tel-03614598

Submitted on 21 Mar 2022

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Par Ke Ll

Architectured poroelastic metamaterials for sound attenuation enhancement in the low frequency range: application to silencers and partition panels

Thèse présentée pour l'obtention du grade de Docteur de l'UTC



Soutenue le 5 juillet 2021 **Spécialité** : Acoustique et Vibrations : Unité de recherche en Mécanique - Laboratoire Roberval (FRE UTC - CNRS 2012) D2628





THÈSE DE DOCTORAT

PRÉSENTÉE À

Université de technologie de Compiègne

PAR

Ke LI

POUR OBTENIR LE GRADE DE DOCTEUR DE L'UNIVERSITE DE TECHNOLOGIE DE COMPIEGNE

Spécialité : Acoustique et Vibrations

Architectured poroelastic metamaterials for sound attenuation enhancement in the low frequency range: application to silencers and partition panels

Soutenue le 5 Juillet 2021 devant le jury composé de :

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Abstract

People exposed to noise pollution may suffer from a wide range of physical and psychological disorders. To reduce the noise level, porous materials are widely used because of their light weight and excellent sound absorption capacity. However the sound attenuation in the low frequency range is still a serious challenge, especially if one considers easy manufacturing and low cost constraints for industrial applications.

To face this issue, a new metamaterial family obtained by shaping a homogeneous poroelastic material is proposed. It allows to enhance the acoustic efficiency thanks to skeleton resonances that are easily tuned without increasing the bulk density and sacrificing space.

Two configurations are addressed experimentally and numerically: the sound attenuation in a duct and the sound transmission through a partition. In the case of the sound attenuation in a duct, the metamaterial is a poroelastic lamella network, that provides an enhanced sound attenuation around its first bending structural resonance. In the case of sound transmission through a finite size panel or an infinite double panel partition, the metamaterial is a network of poroelastic clamped beams, that provides an enhanced sound insulation when tuned to the first mode of the finite size plate or to the mass-spring-mass resonance of the double panel partition. For both configurations, numerical models accounting for the periodicity of the structure allow to efficiently analyze the dissipation mechanisms and the effect of several parameters, like mass ratio, Young's modulus and air flow resistivity. The double panel partition with embedded resonators is also investigated by transfer matrix method for which effective parameters of the host panel are derived. Dynamic behaviors are observed on the effective density and modulus of the panel.

Keywords: metamaterial, metaporous, poroelastic material, finite element method, periodic model, transfer matrix method, duct acoustics, silencer, sound transmission loss, single and double panel partitions.

Résumé

La pollution sonore a des conséquences notables sur la santé des populations exposées. Pour réduire le niveau sonore, les matériaux poreux sont largement utilisés en raison de leur faible masse et de leur excellente propriété d'absorption acoustique. Cependant, l'atténuation du son en basses fréquences reste un défi majeur, surtout si l'on considère les contraintes de fabrication et de coût pour les applications industrielles.

Dans cette étude, une nouvelle famille de métamatériaux obtenus par en donnant une forme particulière à un matériau poroélastique homogène est proposée. L'amélioration des propriétés acoustiques s'appuie sur des résonances du squelette facilement ajustées sans ajouter de masse ou compromettre l'espace.

Deux configurations sont étudiées expérimentalement et numériquement : l'atténuation du son en conduit et l'isolation acoustique à travers une paroi. Pour l'application en conduit, le métamatériau est un réseau de lamelles poroélastiques, qui procure une atténuation acoustique supplémentaire autour de sa première résonance de flexion. Dans le cas de l'isolation acoustique d'une paroi de taille finie ou de double paroi, le métamatériau est un réseau de poutres poroélastiques, accordé sur le premier mode de la paroi ou sur la résonance masse-ressort-masse de la cloison double. Pour les deux configurations, un modèle numérique prenant en compte la périodicité de la structure du matériau permet d'analyser efficacement la dissipation et l'effet de plusieurs paramètres, comme le rapport de masses, le module d'Young et la résistivité du passage de l'air. La double cloison, intégrant des résonateurs élastiques, est également étudiée par la méthode des matrices de transfert avec laquelle les paramètres effectifs du panneau sont dérivés. Des comportements dynamiques sont observés sur la densité et la rigidité de flexion effective du panneau.

Mots-clés: métamatériau, matériau poroélastique, méthode des éléments finis, modèle periodique, méthode des matrices de transfert, acoustique en conduit, silencieux, isolation acoustique, paroi simple et double.

Acknowledgements

This is a memorable and precious experience for me to seek my Ph.D. degree in the Roberval laboratory at the University of Technology of Compiègne from 2017 to 2021.

Firstly, I would like to appreciate my supervisor Nicolas DAUCHEZ for proposing this fantastic Ph.D. project, leading me into the acoustic field, and guiding me to finish the thesis. He always encouraged and offered me wise suggestions to face and come over the troubles in science and life.

I would like to thank my supervisor Benoit NENNIG, who taught me the codes, equations, and physics, with his patience and professionalism. His research attitude and thinking pattern deeply touch and influence me.

I would like to express my thanks to Emmanuel PERREY-DEBAIN, for patiently teaching me the acoustic basics and inspiring me by giving physical explanations.

I would like to give my sincere to Félix FOUCART and Yorick BOUT DE L'EPINE for the duct measurement setup, sample preparation, and technical support. I also would like to appreciate Thomas BOUTIN for manufacturing my experimental samples.

Meanwhile, I would like to acknowledge my thesis reviewers Elker DECKERS and Jean-Philippe GROBY for their carefully reading, wonderful suggestions to improve my manuscript quality.

I also would show my appreciation to Noureddine ATALLA, Jean-Baptiste CASIMIR, Jean-Daniel CHAZOT, and Morvan OUISSE for accepting to be examiners of my defense, as well as providing their pertinent arguments on this project.

I also would like to thank all the current and graduated colleagues in the lab for their supports, kindness, and sharing their scientific opinions without conservation and accompanying me during daily life.

I would like to appreciate China Scholarship Council (CSC) for offering me this opportunity to study in France.

Finally, I would like to express my deepest gratitude to my parents Mr. Huaxi LI and Ms. Julian LIU for their great love and unconditional support throughout my study career, and my girlfriend Kaidi PENG for her love and accompany for these years.

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Chapter 1

Introduction

This chapter gives the context of this work, an overview of sound absorbing techniques and presents the outline of the manuscript.

1.1 Noise pollution

The noise is usually defined as 'unwanted or intrusive sound' [139] like the noise from neighbors and the barking of pets. Noise problem drastically increases after human-being entered industrialized society due to population growth, urbanization, globalization of transportation networks and expansion of resource extractions [110, 49] as depicted in Fig.1.1(a). In daily life, the source of noise nuisance generally contains aircraft, road traffic, railways, building infrastructure, industrial machinery and households [22].

The negative effects of noise on citizens are divided into two categories: pervasive physiological effect and psychological impact. The noise pollution can cause annoyance, fatigue, sleep disturbance, less efficiency and hearing loss. In addition it could increase incidence of physiological diseases such as constriction of blood vessels, increased heart-rate, increased



Figure 1.1: (a) Noise pollution and change over time in the county of Stockholm [16]; (b) Effectiveness of active and passive noise methods [81].



Figure 1.2: (a) Diagram of a porous cross section[132]; (b) The scanning electron microscopy image of a foam [46].

blood pressure, muscle tightness and dilation of eye pupils [22]. Hence, it is highlighted its urgency by stating that the noise is a growing health problem and without enough attention, which probably results in hazardous conditions [2].

There are numerous and relatively effective noise reduction treatments. They are generally classified into two aspects: passive and active noise control. The active noise control method needs to apply external energy into the system to attenuate the original noise based on the Young's interference principle of sound wave. The concept is given by Lueg (1936) in the patent of acoustic-oscillation elimination [100]. Over the last several decades, the active method have been developed rapidly as mature and development of digital signal processing and large scale integrated circuit technology. Now, many advanced and effective control algorithms have been reported and widely applied into automobiles, aircraft and electric appliance industries worldwide [68]. The active noise control effectively makes amends for the poor sound absorption efficiency in low-frequency band of traditional passive noise treatments as depicted in Fig.1.1(b). Moreover it is relatively easy to design or modify for the targeted frequency regions, conveniently install and less effective on performance of the host. The most popular application is the noise-canceling headphones.

Sound absorbing material is the most common passive noise control solution especially to mitigate the mid and high-frequency noise. It can be classified into two types: porous absorption materials and resonant absorption materials [165]. The porous sound absorption materials mitigate most of the sound energy impinging them and reflect very little part by transferring it into heat. Hence, the porous sound absorption material is found very helpful to reduce the effect of noise and widely used on sound sources, paths and receivers [5].

A porous material is a heterogeneous media constituted by a relatively motionless or elastic solid, saturated by a liquid or a gas. According to the availability of the external fluid, like air, to go through the pores, porous material can be divided into *closed* and *open pores* [5] as shown in Fig.1.2. The "closed" pore is isolated from its neighbors and influences the



Figure 1.3: (a): Classification of ordinary porous materials [27]; (b): Illustration of sound power dissipation mechanisms in pores [128].

macroscopic bulk modulus and thermal conductivity of materials. In contrast, the "open" pores produce significant effect on the sound absorption properties of material because of flow and fluid-solid interface interactions like in the boundary layer.

There are two kinds of commonly used porous materials in industries as shown in Fig.1.3(a): foam with cellular structure and fibrous. When the porous material is exposed to the sound field, the fluid on porous surface or in pores will be forced to oscillate. Due to the existence of viscosity and thermal conductivity in the fluid phase and viscoelasticity in the skeleton, part of the sound energy is converted into heat during the motion of two phases. At high frequencies, the viscous and thermal dissipation mainly occur in the vicinity of the solid surface, in the the viscous and thermal boundary layers. In addition, for viscoelastic poroelastic materials, another part of the energy attenuation is due to the damping effect of solid structure, as illustrated in Fig.1.3(b).

The most common foams used as a sound absorption absorber are polyurethane and melamine foams. In order to cope with strict environmental requirements, novel foams have been developed such as metal foams which exhibit high stiffness, light weight, great fire resistance and waterproof. Fiber materials can be classified as natural and synthetic [5, 148]. The natural fiber is mainly generated from vegetable, animal or some minerals. The manufactured fibrous materials made from minerals and polymers are widely applied to sound absorption and thermal isolation. However, it is worth to mention that these porous materials especially instantaneous foam spray also have a negative impact on public health and environment during the production and installation.

Sound propagation in porous layer has been first described through empirical models.

These models are mainly used to calculate the surface impedance and wave propagation constant. One of the most popular approach to calculate the absorption properties of porous layer is the empirical model by Delany-Bazley [38] improved by Miki [109], since it only requires the airflow resistivity. Later, the emergence of the semi-phenomenological model, like the so-called Johnson-Champoux-Allard (JCA) [73, 29] and its extension by Lafarge [78], offers precise prediction of porous sound absorption but requires more parameters describing the pores shape, e.g. the porosity, the viscous and thermal characteristic length, the tortuosity. The sound absorption efficiency of porous materials depends not only on the properties of the porous medium but also on its size and its shape. For instance, the surface impedance of a finite size porous layer of thickness L backed by a rigid wall is

$$Z = i(\rho_p c_p) \cot k_p L, \tag{1.1}$$

where $i = \sqrt{-1}$, and c_p , ρ_p stand for the sound speed and the density in the porous respectively. The term $\cot k_p L$ accounts for successive reflexions between the layer interface and the rigid wall. Maximal absorption arises when the impedance Z is equal to the air impedance, i.e., when there is no reflexion. For such configuration, this happens close to the quarter-wave resonance [70] (and its multiple). The particle velocity at the porous surface reaches a maximum value at the quarter-wave resonant frequency, that induces the first maximum on the sound absorption curve of a porous layer.

In order to limit the impedance mismatch at the air porous interface, a standard approach is to create a graded media by shaping the air-porous interface. Bolton and Kang [74] investigated the sound absorption performance of shaped foam whose interface is arbitrarily oriented with respect to global coordinate axis. They show that the optimizing geometrical parameter like wedge angle can improve the sound absorption at low frequency in a waveguide. This kind of material can also be described by layered double porosity media [121, 8]. Reference [66] adopted an inverted wedge shape melamine foam backed with acoustically hard wall to form partially linear phase shift porous metamaterial. This provides a better sound absorption properties in wide a range of frequencies than uniform melamine layers with the same weight.

The attenuation of low-frequency sound by a porous layer is still a challenge since the dynamic of dissipative system is almost governed by the rules of linear response, which indicates that the frictional force and fluxes to be both linearly proportional to the rates. Therefore, the dissipated power is quadratic in rates and results in the inherently weak absorption of low-frequency for isotropic porous layer [108].

1.2 Resonant sound absorbers

To improve sound absorption at low frequency, it is required to free the wavelength/thickness relation to create a low frequency resonator. One way is to achieve it by partitioning parts of the fluid. The best example of this is the well-known Helmholtz resonator, where the partition allow to change arbitrarily the fluid local compressibility in order to reach compact system with arbitrary low resonance frequency (if losses are neglected). Such systems are used since many years as scatters or sound absorbers [65, 151]. Other approach based on membranes or plate are also often used to create sub-wavelength resonators. Resonances are also associated to phase shift, which are one of the corner stone of metamaterials.

1.2.1 Acoustic metamaterials and their properties

Metamaterials are generally defined as precisely designed artificial structures containing subwavelength resonators [123, 47, 39, 130], which show unusual properties [102, 30] or outperform traditional materials in some aspects. Studies on acoustic metamaterials was originally motivated by parallel research in electromagnetics which made it possible to obtain theoretical solution to the invisibility cloak, superlens and other problems believed unsolvable. The use of the metamaterial provides a potential solution to face less sound attenuation efficiency of the traditional acoustic treatments in the low frequency range. Liu *et al.* [97] experimentally investigated the first locally resonant sonic material as a prototype of acoustic metamaterial and demonstrated that it has negative effective density.

The acoustic band structure with periodic acoustic lattice [23, 144] was developed in the last twenty years and has shown some interesting properties such as wave manipulation and high sound absorption [77, 45, 168]. In fact acoustic metamaterial can be categorized according to two mechanisms: Bragg scattering and local resonance. The homogenized effective parameters like effective density and modulus can almost represent the physical mechanism within the acoustic metamaterial. These two parameters can determine the propagation characteristics of acoustic waves in the medium. The nature of the resonance, monopolar or dipolar, imposes which quantity become negative.

Both effective parameters are positive in common materials and exclusively rely on the material composition and its microstructure. When the resonators are introduced into a host material to enhance the acoustic-host interaction, the effective parameters possibly become negative within its resonant regions[94, 45] as shown in Fig.1.4. The effective mass usually expresses the inertial characteristic of a common material and the effective modulus reflects the resistance of the object being deformed. The effective density for an artificial acoustic



Figure 1.4: Acoustic metamaterial with various parameters [164]. (a): Ordinary materials usually present positive density and modulus; (b): The metamaterial consisting of metallic spheres, coated with layer of silicon rubber exhibits negative mass density [97]; (c): Acoustic metamaterial consisting of acoustic waveguide and Helmholtz resonators shows negative bulk modulus [45]; (d): A metamaterial integrating membrane and Helmholtz resonators reports double negative parameters [85].



Figure 1.5: (a): Simulated effective bulk modulus for one-dimensional subwavelength Helmholtz resonators [45]; (b): Real and imaginary parts of effective density for a sonic crystal structure [98].

medium made with local resonators can be obtained by [107, 98]

$$\rho_e V = M_0 + \frac{m\omega_0^2}{\omega_0^2 - \omega^2},$$
(1.2)

where ρ_e is the effective density, V is the total volume, ω_0 corresponds to the resonant frequency of resonators, ω is the angular frequency, M_0 and m are the mass of the host and the resonators section respectively. In Fig.1.5(a), the real and imaginary parts of effective density have a negative sign at frequencies near the resonance. The negative effective density indicates that the objects move toward the left with the input force to the right.

When the damping of the system is negligible, the effective bulk modulus K_e of a Helmholtz resonator which is used to built up a periodical daisy-chained Helmholtz resonators can be obtained by [45]

$$\frac{1}{K_e} = \frac{1}{K} \left(1 - \frac{\omega_0^2}{\omega^2} \right) \tag{1.3}$$

where K is the bulk modulus of air, and ω_0 is the resonant frequency. The negative effective bulk modulus can be observed in Fig.1.5(b) on two frequency regions, which exactly corresponds to two stop bands of the sonic crystal. The negative effective bulk modulus states that the material expands with exerted pressure. Such properties can also be found on flexural waves in plates and beams [163] (more details in Chap. 4). Most of early studies on acoustic metamaterial, neglect losses. The systematic modeling of viscothermal dissipation mechanisms and how to exploit them to maximize sound absorption yield to the metaporous and metasurface field.

1.2.2 Metasurface

The noise in the low-frequency range has a highly penetrating ability and slowly dissipate during the propagation, which makes it difficult to deal with it in a limited space. The bulky and heavy porous absorption materials are required to absorb noise in the low frequency range which breaks down the thickness constraint during the application. The presence of metamaterial provides a novel approach to design an efficient acoustic absorber with a sub-wavelength dimension.

Many works have been performed by combining resonant and scattering phenomena with the commonly viscous and thermal dissipation, see for instance [59, 115, 80, 21, 56, 57, 79], to overcome the intrinsic limitation of porous in the low frequency range. Practical realization of resonators are generally made with rigid or elastic inclusions, membrane or plate, Helmholtz resonator, by space coiling-up of quarter wavelength resonator. Another approach is to create slow wave channel. All approaches are detailed below. These metamaterials can be regarded as metaporous which aim to integrate the dissipative properties of a porous matrix in medium- and high-frequency regions and the excellent sound absorption of acoustic or elastic resonators.

Membrane-type metamaterials achieve high absorption around their resonance frequencies due to the flapping motion of the metallic platelets since the high energy density regions in the membrane structure decorated with platelets couple minimally with radiation modes. In addition, they also exhibit near-total reflection at the anti-resonance frequencies, which is resulted from that the structure is definitely decoupled from the acoustic wave at the antiresonance frequency[108, 98]. The membrane resonator presents good sound absorption performance in several selective resonance frequencies. Furthermore, the lightweight nature of membrane can be used to design and fabricate lightweight noise treatments to battle lowfrequency sound which has been a longstanding challenge in industry. Conversely, the acoustic insulation performance of membrane is very sensitive to the tension applied on it. The tension can change over time and with external environmental conditions such as temperature and humidity. Consequently, it is hard to tune the target frequency by modifying the tension. The hybridization of porous matrix and membrane resonator building-block elements has been studied and relatively enhances the sound absorption in the low frequency range of the porous host [1, 89].

In *Helmholtz resonators*, dissipation occurs by viscous friction nearby the neck or by thermal losses in the cavity. Several predictive models has been built to estimate its resonance frequency. One of the first patented works employing Helmholtz resonator as a resonant acoustic protection was proposed in [19] to attenuate low frequency noise level inside payload fairings of launch vehicles. Fang *et al.* [45] proposed an analytical expression of effective bulk modulus for the metamaterial consisting of an array of Helmholtz resonators loading on



Figure 1.6: (a): A sample of Helmholtz resonators embedded into porous; (b) The absorption coefficient of relative configuration (black curve) and associated homogeneous sheet (dashed curve)[57]

an acoustic duct. Boutin [21] proposed an analytic model through homogenization method to describe the sound propagation in a porous matrix with inner Helmholtz resonators and the associated negative compressibility. In the meantime, Groby *et al.* [57] investigated, numerically and experimentally, the effect of Helmholtz resonator tuning on sound absorption performance when it is embedded in a porous layer. As shown in Fig.1.6, it was pointed out that the enhancement is large when resonance frequency is higher than the Biot frequency, i.e., the transition frequency from viscous to inertial regimes. Integrating the resonator induces Helmholtz resonance and trapped mode between the resonators and rigid wall. Griffiths *et al.* [54] presented the theoretical and experimental results of porogranular media made out of rigid or elastic shells and Helmholtz resonators. Using elastic shell allows to lower the resonator resonance frequency and to increase the density of states.

Latest works in this fields focus on the design of resonators and how to adjust the losses to reach perfect absorption. This is achieved by the mechanism of *critical coupling* whereby the leakage rate of energy out of the resonator and its inherent losses are properly balanced [131]. This yields to deep sub-wavelength absorbers [69] (λ /88). Several resonators can be used to enlarge the bandwidth leading to rainbow trapping absorber even in transmission problems [71]. The use of porous material is not mandatory as long as the neck of the Helmholtz resonator provides a sufficient amount of dissipation. Obviously, such approach can be extended to other kind of resonators.

To save space, *space coiling-up metamaterial* was investigated and presents some highlighting properties such as negative refraction and sub-wavelength sound absorption [93]. The idea consists in coiling up Fabry-Pérot (FP) channels which offers a easily tunable bandwidth by adjusting the folding number and the absorber thickness [150]. Subsequently, similar principles applied to porous layer were investigated through periodically and appropriately arranged rigid partitions into a hard-backed porous layer [159]. The introduction of rigid walls



Figure 1.7: (a): The lamella network in free field; (b): The absorption coefficient of lamella network measurement setup in free field; (c): The absorption coefficient of lamella under different oblique incidence [34].

in a porous layer shifts down the first thickness resonance mode and yields multiple thickness resonances with higher absorption peaks. Another example is proposed in [20] where different helicoidal cavities are filled with 3D printed porous.

The last category evoked here are *Poroelastic resonators*. Christensen *et al.* [31] presented a structured melamine porous material which shows complete absorption of sound with a broadband response for any direction of incident wave. Lowering porous ratio makes the system more absorptive compared to a fully filled one. Changing this ratio allows to adapt the surface impedance of the architectured porous layer and can be related to weak contrast case in double porosity porous media [121]. It is noteworthy that the targeted impedance generally depends on the incidence angle.

Recently, Dauchez *et al.* [34] investigated the sound absorption performance of a poroelastic lamella network experimentally and numerically as shown in Fig.1.7. An enhancement is observed for oblique incidence due to the excitation of skeleton modes. Changing the lamella shape, allows to control bending resonance frequencies, their quality factor and their coupling with the air while keeping global broadband absorption.

1.3 Outline of the thesis

While various kind of metaporous has been developed and succeed in improving sound absorption compared to traditional solutions, they are built up on the expense of increasing the complexity of the system. In this dissertation, we will focus on the sound absorption and transmission in the low frequency range by taking advantage of the skeleton motion of a poroelastic material as shown in [34].

Unlike other metaporous using inclusions, the lamellar poroelastic metamaterial only needs some reshaping (cuts, air gaps, \dots) to tune solid born resonances. The native diphasic nature of the poroelastic lamella provides a natural way to combine the properties of the sound

absorbing material and vibration control. In other word, the aim here is to operate a shift of the metaporous from inclusion to architecture. The following application to duct acoustics and to transmission through panels will be proposed.

The dissertation is organized as follows.

In chapter 2, the conceptual and theoretical backgrounds, which are necessary to understand the main contents of the thesis, are reminded. In detail, the derivation of Helmholtz equation to describe the acoustic propagation in fluid without flow is given. Then, the Biot-Allard and equivalent fluid theories are presented to explain the wave propagation in dissipative porous media. Dissipation mechanisms and computation methods are finally introduced.

In chapter 3, the sound attenuation of an anisotropic metamaterial in a duct is investigated numerically and experimentally. Subsequently, a simplified model based on pseudo-periodic boundary conditions is built up to carry out the parametric study and optimization of its sound attenuation.

In chapter 4, the effective parameters like density and modulus of a host thin panel with embedded resonators is obtained based on the TMM method. The effect of resonators on the sound transmission loss of an infinite double thin panel is studied based on TMM model. A parametric study is then conducted.

In chapter 5, the lamellar poroelastic metamaterial concept is moved to a panel to face the drop of sound insulation close to panel resonances. The sound insulation performance of a finite size single panel is studied through both experimental and numerical approaches. Then, this concept is extended to infinite double panel, in order to improve its sound insulation efficiency at the well-known mass-spring-mass resonance, using a periodic numerical model.

The last chapter of the manuscript presents the general conclusions and gives perspectives for this work.

Chapter 2

Acoustic wave propagation in air and porous media

This chapter primarily introduces the acoustic wave propagation in a free fluid and guided waves in a duct (without flow). Then, sepcial attention is paid on the Biot-Allard equation describing the wave propagation on an isotropic poroelastic material in terms of macroscopic quantities. Finally, explanation and formulation of the dissipation mechanisms in poroelastic media is detailed. This chapter refers to several Ph.D theses [122, 133, 116, 12, 88, 120, 33] and books [24, 4, 25, 44] in the field.

2.1 Acoustic waves propagation

This subsection presents the derivation of Helmholtz equation for sound propagation and its application in duct.

2.1.1 Fundamental equations of acoustic in perfect gas

Let us consider a homogeneous perfect fluid at rest with reversible adiabatic transformation. Fundamental equations of acoustics can be deduced from conservation law and constitutive equations. Conservation of mass reads,

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0, \qquad (2.1)$$

where ρ is the fluid density, **v** is the particle velocity, and the convective derivative $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$. The second equation is the Newton's second law, which involves velocity **v** and pressure p

$$\rho \frac{D\mathbf{v}}{Dt} + \nabla p = 0. \tag{2.2}$$

Energy conservation, expressed with the entropy, is given under the assumption of negligible viscous effect and thermal transfer in fluid. This yields,

$$\frac{Ds}{Dt} = 0. \tag{2.3}$$

Then, it is necessary to introduce the state equation which relate the pressure p to other independent variables like entropy s and mass density ρ

$$p = p\left(\rho, s\right),\tag{2.4}$$

which can be rewritten by

$$dp = c_0^2 d\rho + \beta ds, \qquad (2.5)$$

where $c_0^2 = \gamma \frac{p_0}{\rho_0} = \frac{\partial p}{\partial \rho}\Big|_s$ is defined as the adiabatic acoustic sound speed, $\beta = \frac{\partial p}{\partial s}\Big|_{\rho}$ is the isentropic dilatation coefficient of the fluid, and γ is the ratio of specific heat at constant pressure and volume. For a homogeneous fluid, Eq. (2.3) implies that the second right hand side term vanishes and the pressure only depends on the density $dp = c_0^2 d\rho$. Eq. (2.4) closes the system of equations (2.1)-(2.3). In order to obtain the linear wave equation, the pressure and density are separated into two components

$$p = p_0 + p',$$
 (2.6a)

$$\rho = \rho_0 + \rho', \tag{2.6b}$$

$$\mathbf{v} = \mathbf{0} + \mathbf{v}'. \tag{2.6c}$$

The first component is the reference value, which is assumed constant in space and time, while the second component is the *acoustic* fluctuation, which is much smaller than the reference value as shown in Fig. 2.1.



Figure 2.1: Pressure field perturbation.

The Taylor expansion of Eq. (2.4) can be written as

$$p = p(\rho_0, s_0) + \frac{\partial p}{\partial \rho} \rho' + \frac{1}{2} \left(\frac{\partial^2 p}{\partial \rho^2} \right) \rho'^2 + \cdots .$$
(2.7)

Substituting Eq. (2.7) into Eq. (2.6) with the assumption that only the first order terms are considered leads to

$$p' = c_0^2 \rho'. (2.8)$$

Substituting Eq. (2.8) into equations of mass conservation and Newton's second law and keeping only first order perturbation, yields

$$\frac{1}{c_0^2}\frac{\partial p'}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}' = 0, \qquad (2.9a)$$

$$\rho_0 \frac{\partial \mathbf{v}'}{\partial t} + \nabla p' = 0. \tag{2.9b}$$

Taking divergence operator of Eq. (2.9) (b), and differentiating Eq. (2.9) (a) with respect to time, and eliminating the velocity variable allows to derive the wave equation

$$\Delta p' - \frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} = 0, \qquad (2.10)$$

which describes the relation of spatial and temporal variation of pressure fluctuation.

In addition, the Euler's equation (linearized Newton's second law for perfect fluid) is retained,

$$\rho_0 \frac{\partial \mathbf{v}'}{\partial t} = -\nabla p', \qquad (2.11)$$



Figure 2.2: A rectangular duct with of cross-section S.

which shows that acceleration of fluid is proportional to the pressure gradient. Fourier transform of Eq. (2.10) gives the Helmholtz equation

$$\Delta p' + k^2 p' = 0, (2.12)$$

where $k = \omega/c_0$ is the wavenumber. For sake of clarity, ' will be removed and p and v will denotes the acoustic perturbations in the following. In free field, the solution of Helmholtz Eq.(2.12) can be found in separable coordinate system. In cartesian coordinate system, the solutions are plane waves $e^{i\mathbf{k}\cdot\mathbf{x}}$, where the wavevector k must satisfy $\|\mathbf{k}\| = k$.

2.1.2 Sound propagation in a duct

When the acoustic waves propagate in a semi-infinite medium like duct of cross-section S, the acoustic pressure should satisfy the Helmholtz equation and boundary conditions. For a rigid duct, the boundary conditions reflect that the normal velocity of particles vanishes at the duct wall. In this section, we present application to a rectangular duct, as illustrated in Fig. 2.2.

The Helmholtz equation can be rewritten in the cartesian coordinate system by

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} + k^2 p = 0.$$
(2.13)

The pressure p(x, y, z) can be written as $p = p_x(x)p_y(y)p_z(z)e^{-i\omega t}$ through separation of variables and substituted into Eq. (2.13). One obtains

$$\frac{\partial^2 p_i}{\partial x_i^2} + k_i^2 p_i = 0 \quad \text{where} \quad i = x, y, z, \tag{2.14}$$

where k_i is the wavenumber along the direction *i*. Helmholtz equation implies that $\sum_{i=1}^{3} k_i^2 = k^2$. Along *z*, solution are propagative and contra-propagative waves. Along *x* and *y*, the

boundary conditions read,

$$\frac{\partial p_x(x)}{\partial x} = 0 \quad \text{at} \quad x = 0 \text{ and } l_x,$$
(2.15a)

$$\frac{\partial p_y(y)}{\partial y} = 0 \quad \text{at} \quad y = 0 \text{ and } l_y.$$
 (2.15b)

which imply that the solutions of wave in x and y directions are standing waves and can be expressed as $\cos(k_x x)$ and $\cos(k_y y)$ respectively. Then, the general solution of wave propagation in a rigid duct is the sum of all the transverse modes

$$p = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn}^{+} \phi_{mn}(x, y) e^{ik_{mn}z} + A_{m,n}^{-} \phi_{mn}(x, y) e^{-ik_{mn}z}.$$
 (2.16)

where A_{mn}^+ and A_{mn}^- are the amplitude fraction of each modes for forward and backward waves. The transverse modes ϕ_{mn} can be expressed as,

$$\phi_{mn}(x,y) = \cos(k_{x,m}x)\cos(k_{y,n}y)$$
(2.17)

with

$$k_{x,m} = \frac{m\pi}{l_x}, \quad k_{y,n} = \frac{n\pi}{l_y}, \quad \text{and} \quad k_z = \sqrt{k^2 - k_{x,m}^2 - k_{y,n}^2}.$$
 (2.18)

When k_z is an imaginary number, the corresponding mode is evanescent and rapidly attenuates, while when k_z is real, the mode is propagative. For each duct mode, there is a cut-off frequency given by $f_{mn} = \frac{c_0}{2\pi} \sqrt{k_{x,m}^2 + k_{y,n}^2}$. The modes (m, n) becomes propagative when $f > f_{mn}$.

The transverse mode, satisfies orthogonality relation

$$\int_{S} \phi_{mn}^{*} \phi_{ij} \, \mathrm{d}S = \delta_{mn,ij},\tag{2.19}$$

with δ the Kronecker symbol.

2.2 Sound propagation in porous material

This subsection briefly introduce the principal models for sound propagation in a porous material. Quasi static behavior of flow though porous material is well described by Darcy's Law (1856), who introduces the permeability concept. The model proposed by Zwikker and Kosten [25] (1949) is usually considered as the starting point of acoustical porous material modeling. They introduced effective density and compressibility by averaging the solution of Stokes flow in a straight circular pore. Another significant contribution of their work is the definition of the decoupling frequency above which the acoustic propagation in fluid does not induce a wave in the solid phase. Then, the famous sound propagation model within a fluid-saturated porous material was introduced by Biot (1956) [14]. The Representative Elementary Volume (REV) is built up at a macroscopic scale where a homogenized description of the superposed solid and fluid phases with interactions can be derived. Biot subsequently pointed out the existence of three propagative waves in the media: a slow and fast compression waves propagating in both phases, as well as a shear wave originated from the skeleton.

Delany and Bazley (1970) [38] proposed an empirical model for fibrous material based on experimental results. It only depends on two parameters, the flow resistivity and the frequency, and can predict the general tendency of acoustic properties of fibrous materials. Hence, it is still widely used among engineers and scientists and an updated version has been proposed by Miki [109].

Over the past decades, Biot model was further developed on two main topics dealing with viscous and thermal effects considering the fluid as air. For more general pore shape, Johnson *et al* (1987) [73] developed an effective density formula dependent on frequency by taking into account the viscous effect. In parallel, Champoux and Allard (1991) [29] give the definition of effective bulk modulus of fluid saturating in pores by considering thermal effect dominated by a additional parameter such as the thermal characteristic length. These two features rise to the Johnson-Champoux-Allard (JCA) model that is widely used now. Then, the Biot model accounting for JCA model is referred as the Biot-Allard theory [4].

2.2.1 General assumptions

Poroelastic model

The poroelastic model of Biot-Allard is based on the Biot theory that proposed constitutive relation of fluid-saturated porous media to describe the elastic wave propagation in it by means of a Lagrangian formulation. The basic assumption for the Biot-Allard theory are summarized below [4, 14]:

- the heterogeneous medium is isotropic and the porosity is uniform throughout;
- the wavelength of interest is much larger than the characteristic dimension of the porous microstructure such as diameter of fibrous and pores;
- the deformation of both solid and fluid phase are assumed to be small. Hence the constitutive equation and dissipation force are linear; the strain energy, kinetic energy and dissipation potentials are quadratic in respect of variables;
- most of pores are open and interaction between fluid and closed pores is ignored;
- there is no mean flow and fluid phase is at rest;
- the viscous and thermal effect are independent [25];
- the displacement and pressure are defined on each points in the porous media;
- the solid phase of media is elastic.

2.2.2 Biot-Allard model

In this subsection, we present the sound propagation in a poroelastic material from the Biot theory [14] to extended models reported by Allard and Atalla[4].

Stress-strain relation

The stress-strain relation of the fluid and solid phase of isotropic material in the Biot theory are given by these two constitutive equations

$$\sigma_{ij}^s = [A\theta^s + Q\theta^f]\delta_{ij} + 2Ne_{ij}^s, \qquad (2.20a)$$

$$\sigma_{ij}^f = \left(Q\theta^s + R\theta^f\right)\delta_{ij},\tag{2.20b}$$

where θ^s and θ^f are the solid and fluid volumetric strains that can be respectively obtained from divergence of displacement of the solid phase **u** and of the fluid phase **U**

$$\theta^s = \nabla \cdot \mathbf{u} \text{ and } \theta^f = \nabla \cdot \mathbf{U}.$$
 (2.21)

and with

$$Q = K_f(1 - \phi),$$
 (2.22a)

$$A = \frac{(1-\phi)^2}{\phi} K_f - \frac{2}{3}N + K_b, \qquad (2.22b)$$

$$R = \phi K_f, \tag{2.22c}$$

$$K_b = (2/3) * N * (1+\nu)/(1-2*\nu).$$
(2.22d)

The parameter Q accounts for the coupling effect between the volume change of the solid and that of the fluid [14]. A corresponds to the first Lamé coefficient and N is the shear modulus of the skeleton. R is the effective bulk modulus of the fluid, i.e., an estimation of the pressure on the fluid required to force a further amount of fluid into the aggregate while the total volume stay constant [7, 14, 3]. Biot and Willis proposed the principle of an experimental method to determine the static parameter Q and R [13]. ϕ is the porosity of the material which is only related to open pores. K_f is the bulk modulus of the fluid which depends on frequency because it accounts for the thermal exchange at the wall of pores. ε^s_{ij} is the strain of solid phase

$$\varepsilon_{ij}^{s} = \begin{bmatrix} \varepsilon_{x} & \gamma_{z} & \gamma_{y} \\ \gamma_{z} & \varepsilon_{y} & \gamma_{x} \\ \gamma_{y} & \gamma_{x} & \varepsilon_{z} \end{bmatrix}, \qquad (2.23)$$

where

$$\varepsilon_i = \frac{\partial u_i}{\partial x_i},\tag{2.24a}$$

$$\gamma_i = \frac{1}{2} \left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} \right).$$
(2.24b)

Equations of motion

Once these constitutive equation established, the motion equations for both phases are obtained using Lagrangian approach. The viscous dissipation is then introduced by Biot assuming Poiseuille flow for the relative flow through

$$\frac{\partial \sigma_{ij}^s}{\partial x_j} = \rho_{11} \frac{\partial^2 u_i}{\partial t^2} + \rho_{12} \frac{\partial^2 U_i}{\partial t^2} + b \frac{\partial}{\partial t} \left(u_i - U_i \right), \qquad (2.25a)$$

$$\frac{\partial \sigma_{ij}^{f}}{\partial x_{i}} = \rho_{22} \frac{\partial^{2} U_{i}}{\partial t^{2}} + \rho_{12} \frac{\partial^{2} u_{i}}{\partial t^{2}} + b \frac{\partial}{\partial t} \left(U_{i} - u_{i} \right), \qquad (2.25b)$$

where the apparent density are,

$$\rho_1 = (1 - \phi)\rho_s,$$
(2.26a)

$$\rho_{12} = -\phi \rho_0(\alpha_\infty - 1),$$
(2.26b)

$$\rho_{11} = \rho_1 - \rho_{12} \,, \tag{2.26c}$$

$$\rho_{22} = \phi \rho_0 - \rho_{12}. \tag{2.26d}$$

Here, ρ_{11} is the total apparent density of the skeleton moving in the fluid. ρ_{12} is the opposite of the apparent density due to the inertial coupling of both phases. Finally, ρ_{22} is the total effective mass of fluid by adding inertial term from skeleton. These effective densities are defined from the density of the skeleton material ρ_s and the density of the fluid ρ_0 and related to the high frequency limit of the tortuosity α_{∞} . This parameter accounts for the geometric irregularity of the pore network (see Fig. 2.3).

Beside, the final term on the right-hand side of the Eq. (2.25a) is related to viscous dissipation between the two phases that is linearly related to relative velocity of skeleton and fluid. For a non viscous fluid, b = 0. For a viscous fluid, Biot gave the static value estimation by using permeability coefficient in Darcy's law or the airflow resistivity σ . Thus, b is a constant. To



Figure 2.3: Diagram of tortuosity concept $\alpha_{\infty} = \frac{\ell}{AB}$ from [116].

enhance the model behavior when the boundary layer becomes smaller than the pore size [14], correction can be added and b becomes frequency dependent. This frequency dependance can be estimated by

$$b(\omega) = \sigma \phi^2 G(\omega), \qquad (2.27)$$

where $G(\omega)$ is a frequency-dependent function, which will be discussed later using the Johnson-Champoux-Allard model (see Eq. (2.49)). If time harmonic version of (2.25a) are considered, it is common to recast inertial and viscous effect in effective complex density, such as

$$\tilde{\rho}_{11} = \rho_{11} + \frac{\mathrm{i}b(\omega)}{\omega},\tag{2.28a}$$

$$\tilde{\rho}_{12} = \rho_{12} - \frac{\mathrm{i}b(\omega)}{\omega},\tag{2.28b}$$

$$\tilde{\rho}_{22} = \rho_{22} + \frac{\mathrm{i}b(\omega)}{\omega}.$$
(2.28c)

For a sake of clarity, $\tilde{}$ will be removed in the following and all densities are assumed to be complex valued. Other equivalent formulation of Biot model have been proposed, see [4, Chap. 6] for a complete review. It is worth mentioning that the (\mathbf{u} , p) [6] formulation (Eq.3.4) will be used here for FEM computation.

Wave equation

Substituting stress-stain relations Eq. (2.21), Eq. (2.24), and Eq. (2.20) into Eq. (2.25) yields the wave equation

$$-\omega^2(\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U}) = (A+N)\nabla\nabla\cdot\mathbf{u} + N\nabla^2\mathbf{u} + Q\nabla\nabla\cdot\mathbf{U}, \qquad (2.29a)$$

$$-\omega^2(\rho_{22}\mathbf{U} + \rho_{12}\mathbf{u}) = R\nabla\nabla\cdot\mathbf{U} + Q\nabla\nabla\cdot\mathbf{u}.$$
(2.29b)

Assuming the porous material is isotropic, the longitudinal and transversal wave equations can be obtained by using scalar and vector displacement potentials respectively, as in the case of an elastic solid. Firstly two scalar displacement potentials ψ_s and ψ_f for the skeleton and fluid are introduced [4, chap. 6]

$$\mathbf{u} = \nabla \psi_s, \tag{2.30a}$$

$$\mathbf{U} = \nabla \psi_f. \tag{2.30b}$$

Substitution Eq. (2.30) into Eq. (2.29) yields

$$-\omega^2 \mathbf{M}^{-1} \boldsymbol{\rho} \boldsymbol{\psi} = \nabla^2 \boldsymbol{\psi}, \qquad (2.31a)$$

where ψ , ρ , and M are given respectively

$$\boldsymbol{\psi} = \begin{bmatrix} \psi_s, \psi_f \end{bmatrix}^T, \tag{2.32a}$$

$$\boldsymbol{\rho} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{12} & \rho_{22} \end{bmatrix}, \qquad (2.32b)$$

$$\mathbf{M} = \begin{bmatrix} P & Q \\ Q & R \end{bmatrix}.$$
(2.32c)

In the above equations, P = A + 2N. Eq. (2.31) is an eigenvalue problem where the wavenumber δ is the eigenvalue and eigenvector is ψ

$$\delta_1^2 = \frac{\omega^2}{2(PR - Q^2)} \left[P\rho_{22} + R\rho_{11} - 2Q\rho_{12} - \sqrt{\Delta} \right], \qquad (2.33a)$$

$$\delta_2^2 = \frac{\omega^2}{2(PR - Q^2)} \left[P\rho_{22} + R\rho_{11} - 2Q\rho_{12} + \sqrt{\Delta} \right], \qquad (2.33b)$$

where

$$\Delta = [P\rho_{22} + R\rho_{11} - 2Q\rho_{12}]^2 - 4(PR - Q^2)(\rho_{11}\rho_{22} - \rho_{12}^2).$$
(2.34)

Eq. (2.33) indicates that two kinds of compressional waves are obtained with different wave velocities in both phases. In addition, the wave preferentially propagating in solid or fluid can be estimated through coefficient $\mu = \frac{\psi_f}{\psi_s}$, such as

$$\mu_i = \frac{P\delta_i^2 - \omega^2 \rho_{11}}{\omega^2 \rho_{12} - Q\delta_i^2}, \quad \text{with } i = 1, 2.$$
(2.35)

In the same way, the wave equation for rotational wave component is obtained by using vector potentials. The displacement equals the curl operator of these potentials. Only one

shear wave is derived and its wavenumber δ_3 is given by

$$\delta_3^2 = \frac{\omega^2}{N} \left(\frac{\rho_{11}\rho_{22} - \rho_{12}^2}{\rho_{22}} \right). \tag{2.36}$$

Effect of viscous dissipation

In Eq. (2.25), the parameter b is introduced to account for the viscous effect resulting from relative motion air versus skeleton. It is generally determined by the static airflow resistivity σ and the dynamic parameter $G(\omega)$. σ is defined by the ratio of the pressure differential across a sample of the material to normal flow velocity through the material and can be estimated experimentally from

$$\sigma = (p_2 - p_1)/Vh, \tag{2.37}$$

where V is the mean flow per unit area of material and h is the thickness of the sample.

Johnson *et al.* [73] introduced the concept of dynamic tortuosity combining the effect of inertial coupling and viscous dissipation. For an ideal fluid, whose properties are real valued and frequency independent, the definition of dynamic tortuosity is given by the modified Euler's equation,

$$\tilde{\alpha}(\omega)\rho_f \frac{\partial v}{\partial t} = -\nabla p, \qquad (2.38)$$

where $\rho_f = \rho_0$ is the density of the fluid in the pores. Meanwhile, the dynamic permeability was proposed by analogy with steady-state definition in the modified Darcy equation,

$$\phi v = -\frac{\hat{k}(\omega)}{\mu} \nabla p, \qquad (2.39)$$

where μ is the fluid viscosity. The static limit of the permeability k_0 , as defined in Darcy's law, is related to the airflow resistivity σ is

$$k_0 = \frac{\mu}{\sigma}.\tag{2.40}$$

The dynamic permeability and tortuosity are related by

$$\tilde{\alpha}(\omega) = \frac{\mathrm{i}\mu\phi}{\tilde{k}(\omega)\omega\rho_f}.$$
(2.41)

Towards low frequencies, the dynamic permeability and tortuosity collapse to their static value

$$\lim_{\omega \to 0} \tilde{k}(\omega) = k_0, \quad \lim_{\omega \to 0} \tilde{\alpha}(\omega) = \frac{i\mu\phi}{k_0\rho_f\omega}.$$
(2.42)

For high frequencies, both quantities were estimated, assuming Stokes boundary layer, as

$$\lim_{\omega \to \infty} \tilde{k}(\omega) = \frac{\mathrm{i}\mu\phi}{\alpha_{\infty}\rho_{f}\omega} \left[1 - \left(\frac{\mathrm{i}\mu}{\rho_{f}\omega}\right)^{\frac{1}{2}} \frac{2}{\Lambda} \right], \qquad (2.43a)$$

$$\lim_{\omega \to \infty} \tilde{\alpha}(\omega) = \alpha_{\infty} \left[1 + \left(\frac{\mathrm{i}\mu}{\rho_f \omega} \right)^{\frac{1}{2}} \frac{2}{\Lambda} \right], \qquad (2.43b)$$

introducing a new paramater, called characteristic viscous length Λ , defined by

$$\frac{2}{\Lambda} = \frac{\int v^2(\mathbf{r}_w) \, \mathrm{d}A}{\int v^2(\mathbf{r}) \, \mathrm{d}V}.$$
(2.44)

The numerator of Eq. (2.44) is the integration of the velocity $v(\mathbf{r}_w)$ of a inviscid fluid over the walls of the pores. The denominator is the integration of this velocity over the pores volume. It only depends on geometrical property of the skeleton and is independent of frequency.

Then, a new compact and simple expression of dynamic tortousity was proposed by Johnson *et al.* [73], which covers most of the properties reported above and is function of the four parameters α_{∞} , k_0 , Λ , and ϕ

$$\tilde{\alpha}(\omega) = \alpha_{\infty} + \frac{\mathrm{i}\mu\phi}{\omega\rho_f k_0} F(\omega), \qquad (2.45)$$

where

$$F(\omega) = \left\{ 1 - i \frac{4\alpha_{\infty}^2 k_0^2 \rho_f \omega}{\mu \Lambda^2 \phi^2} \right\}^{\frac{1}{2}}.$$
 (2.46)

The limitation of $F(\omega)$ for high and low frequency regions insures that $\alpha(\omega)$ satisfies Eq. (2.43) reminded here

$$F(0) = 1$$
 and $\lim_{\omega \to \infty} = \frac{2k_0 \alpha_\infty}{\Lambda \phi} \left[\frac{-i\omega \rho_f}{\mu}\right]^{\frac{1}{2}}$. (2.47)

Finally, the dynamic mass density [73], with dependent-frequency properties, is defined by

$$\rho_e(\omega) = \rho_f \tilde{\alpha}(\omega) = \alpha_\infty \rho_f \left[1 - \frac{1}{\mathrm{i}\hat{\omega}} G(\omega) \right], \qquad (2.48)$$

with

$$G(\omega) = \sqrt{1 - \mathrm{i}\hat{\omega}\frac{M}{2}},\tag{2.49}$$

and $\hat{\omega}$ is a dimensionless factor given by

$$\hat{\omega} = \frac{\omega \alpha_{\infty} \rho_f}{\phi \sigma}.$$
(2.50)

The airflow resistivity σ is used instead of permeability k_0 by substituting Eq. (2.40) into

Eq. (2.46) and the shape factor $M = \frac{8\alpha_{\infty}\mu}{\phi\Lambda^2\sigma}$ (1 for a cylindrical shape pore). The parameter b in Eq. (2.27) accounting for viscous dissipation can be written as [122]

$$b(\omega) = \sigma \phi^2 \sqrt{1 - i\frac{\omega}{H}} \quad \text{with} \quad H = \frac{\sigma^2 \Lambda^2 \phi^2}{4\alpha_{\infty}^2 \mu \rho_f}.$$
 (2.51)

This modified coefficient will be used in effective densities appearing in the Biot's equations (2.28).

Effect of thermal dissipation

The thermal dissipation is mainly governed the widest pores in the porous material [29]. Lafarge *et al.* [78] proposed to treat thermal dissipation in an analogue of viscous dissipation, introducing the thermal permeability k' defined by

$$\phi\langle\tau\rangle = \frac{k'(\omega)}{\kappa}\frac{\partial}{\partial t}\langle p\rangle, \qquad (2.52)$$

where k' is the dynamical thermal permeability, $\langle \tau \rangle$ is the macroscopic excess temperature in air, and κ is the thermal conduction coefficient.

The semi-analytical formulation of the frequency dependent parameter $k'(\omega)$ was obtained by following the similar approach as for the dynamic viscous permeability $k(\omega)$. This yields,

$$k'(\omega) = \frac{k'_0}{(1 - (M'/2)i\tilde{\omega}')^{\frac{1}{2}} - i\tilde{\omega}'},$$
(2.53)

with the dimensionless shape factor $M' = \frac{8k'_0}{\phi\Lambda'^2}$ and the reduced frequency $\tilde{\omega}' = \frac{\omega}{\nu'}\frac{k'_0}{\phi}$. Here, k'_0 is a real valued constant corresponding to the low frequency limit of $k'(\omega)$. Λ' is a characteristic length associated with the temperature effects present in the thermal Stokes layer at high frequency. This is a purely geometrical parameter, defined as

$$\Lambda' = 2 \frac{\int \mathrm{d}V}{\int \mathrm{d}A}.$$
(2.54)

This parameter corresponds to the effective hydraulic radius of the pores. Normally the viscous characteristic length Λ is smaller than thermal ones Λ' because of the weighting by the fluid velocity which is higher in small size pores as shown in Fig. 2.4.

Champoux *et al.* finally introduced the expression of the effective bulk modulus related to ϕ and Λ'

$$\tilde{K}_f = \frac{\gamma P_0}{\gamma - (\gamma - 1) \left[1 - \frac{8\mu}{i\Lambda'^2 P r \omega \rho_f} \sqrt{1 - i\frac{\Lambda'^2 P r \rho_f \omega}{16\mu}}\right]^{-1}},$$
(2.55)

where P_0 is the atmospheric pressure ($P_0 = 1013$ hPa), γ is the fluid specific heat ratio and Pr is the Prandtl number. Tilde symbol on variables state for complex value and frequency dependence. This effective bulk modulus will be used in R and Q modulus in the consistitutive law (2.20).



Figure 2.4: Illustration of viscous and thermal characteristic lengths relation.

Dissipation mechanism in poroelastic material

In the following, the computation of dissipated powers related to structural, viscous and thermal losses are presented. Dazel *et al.* [35] derived the expression of dissipated power and stored energy inside elastic porous domain for several formulations of the Biot's equations. We present here those based on (\mathbf{u}, p) formulation using similar formalism as (\mathbf{u}, \mathbf{U}) . The derivation stems from the kinetic energy theorem and the first law of thermodynamic [119]. The kinetic energy theorem presents that for actual or virtual velocity field, the sum of external power P_{ext} , inertia power $\frac{DK}{Dt}$, and internal power P_{int} equals zero giving

$$P_{ext} = P_{int} + \frac{DK}{Dt},$$
(2.56)

where K is the kinetic energy. The first law of thermodynamic states that the material derivative of total energy $\frac{DE^{tot}}{Dt}$ always equals the sum of external heat supply Q and work rate P_{ext} added in the expression of

$$\frac{DE^{tot}}{Dt} = Q + P_{ext}.$$
(2.57)

The dissipation power can be obtained by integrating Eq. (2.56) and (2.57) over a time cycle in the porous domain. Three kinds of dissipation powers, due to viscous (*vsic*), thermal

(the) and to viscoelastic losses in the frame (str) are defined as

$$W_{str} = \pi \int_{\Omega} \hat{\boldsymbol{\sigma}}_{i}^{s}(\mathbf{u}_{0}) : \boldsymbol{\varepsilon}(\mathbf{u}_{0}^{*}) \,\mathrm{d}\Omega, \qquad (2.58)$$

$$W_{visc} = -\pi\omega^2 \int_{\Omega} \underbrace{\Im(\tilde{\rho}) |\mathbf{u}_0|^2}_{\text{In solid phase}} - \underbrace{\Im\left(\frac{\phi^2}{\rho_{22}\omega^4}\right) |\nabla p_0|^2}_{\text{In fluid phase}} + \underbrace{\frac{2}{\omega^2} \Im\left(\frac{\phi}{\tilde{\alpha}}\right) \Re(\mathbf{u}_0^* \nabla p_0)}_{\text{Interaction of phases}} \, \mathrm{d}\Omega, \quad (2.59)$$

$$W_{the} = \frac{\phi^2 \pi R_i}{|R|^2} \int_{\Omega} |p_0|^2 \, \mathrm{d}\Omega.$$
 (2.60)

Here, $\tilde{\rho} = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}$ and we recall the definition of the dynamic tortuosity $\tilde{\alpha} = \rho_{22}/\rho_2$ using definition from (2.26a) and (2.28). In these expressions, \Im and \Re refer to imaginary and real part of the quantities in bracket, the star superscript represents the complex conjugate. The subscript $_0$ represents the complex amplitude $\mathbf{u} = \mathbf{u}_0 e^{-i\omega t}$. In order to make a distinction between the conservative and dissipative parts of the stress tensor, $\boldsymbol{\sigma}_i^s$ and $\boldsymbol{\sigma}_r^s$ are introduced and computed respectively with the imaginary part and real part of 'elastic' constant.

The structural dissipation is due to the deformation of the viscoelastic skeleton phase [167] and is proportional to viscoelastic properties of the solid phase, i.e., the loss factor. The viscous dissipation derives from the interaction between the two phases and the air viscosity. It is function of the imaginary part of the effective densities [14, 73]. The thermal dissipation results from heat gradient in the fluid phase, and is function of the imaginary part of the fluid bulk moduls K_f [78] involved in R.

2.2.3 Equivalent fluid model

In some case, the Biot's model can be simplified to an equivalent fluid, where only one kind of wave can propagate instead of three. Generally, it can be done when the skeleton is considered as motionless, i.g., the rigid frame model, or when the skeleton is considered as infinitly soft, like the limp model. Such approach may be useful to save computational time or to find analytic solution. From a historical point of view, equivalent fluid model is older [25] than the Biot model but can also be derived from it. Some empirical model, like Delany and Bazeley model, are also presented.

Rigid frame model

When the skeleton is motionless or its motion is negligible when compared to the fluid displacement, the rigid frame model is a good alternative. This hypothesis is valid above the decoupling frequency, as stated by Zwikker and Kosten [25],

$$f_d = \frac{1}{2\pi} \left(\frac{\phi^2 \sigma}{\rho_1} \right). \tag{2.61}$$

This assumption is all the more valid as the material has heavy skeleton and small airflow resistivity. When the frequencies is higher than f_d , the visco-inertial coupling between solid and fluid phases is so weak that an acoustic wave propagating in the fluid phase would not induce a sufficient force to excite the skeleton. The most important ratio is σ/ρ_1 for judging the visco-inertial coupling of the two phases [125]. However, this analysis must be nuanced close to frame resonances.

The rigid skeleton model can be derived from Biot model assuming that the displacement u of the solid phase equals zero in Eq. (2.29). Then, Eq. (2.29)(b), recast in the Helmholtz equation, and is written in term of pore pressure p

$$K_f \Delta p + \omega^2 \rho_e p = 0. \tag{2.62}$$

Here, K_f is the equivalent bulk modulus of the fluid accounting for thermal dissipation and $\rho_e = \hat{\rho}_{22}/\phi$ is the equivalent density of fluid accounting for viscous dissipation. The waves propagates with the wavenumber

$$k_{rf} = \omega \sqrt{\frac{\rho_e}{K_f}}.$$
(2.63)

The effective density and bulk modulus are usually obtained from the Johnson-Champoux-Allard model using Eq. (2.48) and (2.55) respectively.

Limp model

Some soft porous material like fibrous materials have a skeleton bulk modulus smaller than the fluid phase bulk modulus and the rigid frame model is no more suitable. The limp model makes the assumption that the solid phase has no bulk modulus $\hat{P} = 0$. Applying this condition in the wave equation Eq. (2.29) and combining Eq. (2.29a) - $\frac{Q}{B} \times$ Eq. (2.29b) gives

$$-\omega^2 \frac{\rho_{12}}{\phi} \Gamma \mathbf{u} - \omega^2 \frac{\rho_{22}}{\phi} \gamma \mathbf{U} = \hat{P} \Delta \mathbf{u}, \qquad (2.64)$$

with

$$\hat{P} = P - \frac{Q^2}{R},\tag{2.65a}$$

$$\Gamma = \phi \left(\frac{\rho_{11}}{\rho_{12}} - \frac{Q}{R} \right), \qquad (2.65b)$$

$$\gamma = \phi \left(\frac{\rho_{12}}{\rho_{22}} - \frac{Q}{R} \right), \qquad (2.65c)$$

where \hat{P} is the bulk modulus of the solid phase *in vaccum* and γ is the notation given by Attala *et al.* [129] in the mixed pressure-displacement formulation Eq.3.4. When the skeleton of the

porous material is 'soft' ($\hat{P} = 0$), the displacement between the two phases is given by

$$\mathbf{u} = \frac{\rho_{22}}{\rho_{12}} \frac{\gamma}{\Gamma} \mathbf{U}.$$
 (2.66)

Substituting the solid displacement \mathbf{u} into Eq. (2.29b) introduces the new wave equation for the fluid phase

$$K_f \Delta \mathbf{U} + \omega^2 \rho_{limp} \mathbf{U} = 0, \qquad (2.67)$$

with

$$\rho_{limp} = \frac{B}{A}\rho_e \quad , \quad A = \left(1 - \frac{Q\rho_{22}\gamma}{R\rho_{12}\Gamma}\right) \quad \text{and} \quad B = \left(1 - \frac{\gamma}{\Gamma}\right),$$
(2.68)

where ρ_e is the equivalent density of the fluid as in Eq. (2.48). It implies that the solution of wave equation involves only one compressional wave, characterized by the wavenumber

$$k_{limp}^2 = \frac{B}{A}k_{rf}^2,\tag{2.69}$$

where k_{rf} is the wavenumber of the rigid frame model. The limp model is less restrictive than the rigid frame model since it takes into account the inertia of the solid phase in its modified effective density ρ_{limp} and can be an alternative to The Biot's model when the frame born wave has weak influence [9, 41], and the mass of the skeleton should not be ignored like in transmission loss problems.

Delany-Bazley model

Delany and Bazley [38] proposed one of the first empirical model by determining expression of the acoustic impedance Z_e and wavenumber k_e for fibrous materials with motionless skeleton. The model is derived from impedance measurement of many fibrous materials. The quantities Z_e and k_e are functions of the angular frequency ω and the air flow resistivity σ of the porous material as

$$Z_e = \rho_0 c_0 [1 + 0.057 X^{-0.754} + i0.087 X^{-0.732}], \qquad (2.70a)$$

$$k_e = \frac{\omega}{c_0} [1 + 0.0978 X^{-0.700} + i0.189 X^{-0.595}], \qquad (2.70b)$$

where ρ_0 and c_0 are the density and speed of sound in air. The dimensionless quantity X is

$$X = \rho_0 f / \sigma. \tag{2.71}$$

The main drawback of this model is that the porosity of material has to be close to 1.0 and it is only valid for 0.01 < X < 1.0. Further improvements of this model has been proposed by other authors [10, 48, 109].

2.3 Conclusion

In this part, we presented wave propagation in air and porous media. The Biot model allows to account for viscous, thermal and viscoelastic dissipations. It will be used in the following to study architectured poroloelatic metamaterials.

Chapter 3

Poroelastic lamellar metamaterial for sound attenuation in a rectangular duct

This chapter corresponds to the paper published in **Applied Acoustics** journal [91], dealing with the sound attenuation in a duct by means of a sound absorbing material. Common porous materials produce a broadband sound attenuation but lack of efficiency in the low frequency range. To overcome this limitation, the poroelastic material is shaped onto a lamellar network to take benefit of its structural resonances. The metamaterial is easy to manufacture and tune to the targeted frequency. The numerical results considering the global system or accounting for the geometerical periodicity are both compared to experimental results. The effect of geometrical and physical parameters, like the dimensions of the lamellas, Young's modulus, and air flow resistivity is investigated.

3.1 Introduction

The control of low frequency noise remains a challenge for the automotive, aircraft and building industries. In many instances, the noise is produced by various airflow systems and is transmitted in ducts which act as acoustic waveguides.

The mitigation of these noise disturbances is usually accomplished using passive treatments by either using acoustic liners which consists in treating the wall of the duct adequately or by inserting dissipative splitter silencers in the duct. In the latter case, silencers are made with fibrous materials such as rock wool or glass fibre and are thus less efficient at low frequency and for best sound attenuation, it is normally required that the thickness of the treatment should be of the same order as the acoustic wavelength [76, 11, 117]. Traditional acoustic liners, made of a resistive perforated plate coupled to a quarter-wavelength resonator, somewhat suffer from the same limitation as low frequency performances are typically limited by the cavity depth. The geometric structure of these liners, which consist of a periodic arrangement of acoustic resonators, makes the direct ancestor of acoustic metasurfaces or metamaterials which have experienced a continuous and robust development for the last twenty years. Because these materials have effective dynamic quantities with negative values, like negative mass density [162] and bulk modulus [45, 63], that cannot be observed in natural materials, they break the traditional design rules for acoustic treatment.

The corner stone of these materials relies on subwavelength resonators and its associated phase shift. Here, we can mention quarter-wavelength and space-coiling structures [92, 93, 118], Helmholtz resonators [26, 158, 58, 63] or membranes [162, 131]. The geometrical configuration of the resonators network can also play a crucial role, like in slow sound channels [83, 72]. Another major ingredient, which must be considered in order to design efficient sound absorbing materials, is the losses. Viscothermal losses in porous materials [4] have been combined with the inclusion of small resonators in the so-called metaporous materials [21, 55, 159, 160, 54], showing improvement at low frequency while keeping broadband absorption. Perfect absorption can also be achieved by the mechanism of critical coupling whereby the leakage rate of energy out of the resonator and its inherent losses are properly balanced [131].

Recently, Christensen *et al.* [31] proposed a structured material fabricated out of porous lamellas backed by a reflecting support. The increase in dissipation is explained by the fact that sound is trapped more efficiently than for a homogeneous porous layer. It is reported that complete absorption of sound within a two octave band can be obtained though the concept is not optimal within the long wavelength regime and at grazing incidence. Dauchez *et al.* [34] studied sound absorption of a large scale poroelastic lamella network under oblique incidence in free field. Results show that this type of structured material, which bears similarity with [31], permits to gain extra absorption in the low frequency range by taking advantage of the

resonance of the elastic frame of the foam. This particular effect, which is often ignored in the literature, is indeed not studied in [31].

Silencers and sound attenuation in waveguides by metamaterial have been less investigated than for panels dedicated to acoustic room corrections. The grazing incidence of sound waves which, depending on the duct dimensions, may propagate in a multimode context makes the design of appropriate acoustic treatments more complex and less understood.

Generally the best attenuation is obtained when two guided modes of the silencer are close to merge [145, 157, 127]. This can be achieve by exploiting Fano resonances [156, 51] or the high tunability of metamaterial [157]. Another lever is to take advantage of poroelastic frame *elastic* resonances. Their strong impact on the sound attenuation have been shown in [114, 112, 67] in the poroelastic silencer.

The present chapter is in the wake of previous work published by the authors [114, 112, 34] on the role of the frame elasticity for the passive sound control. More precisely, our aim is to devise and investigate a new duct silencer concept made of a specific arrangement of lamellas, here made of melamine foam, inserted in a rectangular duct. The lamella bending motion provides a natural sub-wavelength resonator, without any other inclusions. The interest of this configuration rely on its simple manufacturing, its high tunability and its broadband efficiency.

The chapter is organized as follows. First, the experimental setup and material properties are described. Two configurations with different orientations of the lamellas, i.e., parallel or perpendicular to the duct axis, are investigated and compared to the homogeneous case.

Experimental results are given in terms of the Transmission Losses (TL) of the silencer and are compared with numerical simulations obtained either from a full model, i.e., a 3D finite element model (FEM) of the whole silencer, or from a simplified and idealized periodic FEM model. This simplified model is then used to carry out a parametric study in order to identify the effect of different geometrical parameters as well as physical parameters such as the airflow resistivity and Young elastic modulus of the poroelastic material. Taking advantage of the modal description given by periodic model, the paper ends with a discussion which highlights the fact that best attenuation are nearly-optimal when modes are close to veering. This phenomena which was established earlier by Tester [145] for locally reacting liners is also described in recent papers [157, 127].



Figure 3.1: Poroelastic lamellar metamaterial.



Figure 3.2: Orientations of the lamellar material; (a): homogeneous; (b): parallel; (c): perpendicular.

Porosity	Airflow resistivity	Viscous length	Thermal length	Tortuosity	Density	Young's modulus	Loss factor	Poisson ratio
ϕ	σ	Λ	Λ'	α_{∞}	ρ_1	E	η	ν
	$(Nm^{-4}s)$	(µm)	(µm)		kg.m ⁻³	kPa		
0.982	7920	132.6	149.9	1	6.11	120	0.075	0

Table 3.1: Properties of the poroelastic material [34].

3.2 Experimental approach

3.2.1 Description of the poroelastic lamellar metamaterial

The poroelastic lamellar metamaterial shown in Fig. 3.1 is made up of several melamine foam strips which properties are given in Table 3.1. Each lamella has a nominal thickness $h_1 = 25$ mm along the y-axis, a width $w_1 = 15$ mm and a length of 200 mm. The air gap between two lamellas is 5 mm. There are ten parallel strips glued on a 20 cm \times 20 cm stiff plate. Four samples are fabricated and arranged with different orientations. The two configurations investigated, parallel and perpendicular, are shown in Fig. 3.2. In the parallel arrangement, with 10 lamellas, each lamella is parallel to the duct axis, which is also the direction of the incident pressure field. In the perpendicular arrangement, with 20 lamellas, each lamella is perpendicular to the duct axis. In all cases, the sample covers both bottom and top surfaces of the duct over a 40 cm length in the z-direction (see Fig. 3.3).

3.2.2 Experimental setup

The experimental set-up used to measure the Transmission Loss (TL) of the lined duct is also shown. The test bench has been designed for the acoustic multi-modal characterization of a test section in presence of a low Mach number flow within the frequency band [200 Hz–3.5 kHz]. In this work, we consider the no flow case and only the incident plane wave is accounted for. The duct has a rectangular section of $0.2 \text{ m} \times 0.1 \text{ m}$ with an anechoic termination at both ends. The scattering matrix, which contains the modal reflection and transmission coefficients is measured using a multi-source method described in [147]. Note that the symmetry of each configuration tested prevents the existence of the first transverse duct acoustic mode so in the frequency range of interest, here up to 1500 Hz, only the plane wave mode is allowed to propagate in the rigid duct.

3.2.3 Results

In order to illustrate the influence of the orientation and of the filling fraction, parallel and perpendicular orientations are compared to an homogeneous layer made of the same material. These results are given in Fig. 3.4. The TL obtained with the homogeneous layer exhibits a peak around 850 Hz. Attenuation peaks can be related to the cutoff frequency of Lamb-like waves. Because of the motionless boundary condition on the bottom, all elastic modes in



Figure 3.3: Experimental setup for transmission loss measurement (a) and lamellas orientations (b-d).



Figure 3.4: Comparison of experimental results for parallel (- -), perpendicular(__) and homogeneous layer (...).

the slab problem have a cutoff frequency, whereas classical Lamb waves are associated with pressure release conditions on both sides. Exact solutions for fully coupled systems can be found numerically in [17, 114]. It is sufficient in the present analysis to remind the following approximation which holds if the coupling with the fluid is neglected [17]:

$$f_{i,m} \approx (2m+1)\frac{c_i}{4h_1}$$
, for $i = s, l,$ (3.1)

where $c_s = \sqrt{\frac{E/2(1+\nu)}{\rho_1}}$ and $c_l = \sqrt{\frac{E(1-\nu)/[(\nu+1)(1-2\nu)]}{\rho_1}}$ are the in vacuo shear and longitudinal bulk velocities, respectively. These cutoff frequencies correspond to quarter wavelength (or its multiple) resonances of the shear and compression wave of an homogeneous layer, as mentioned in [34] for absorbing panels.

The parallel configuration exhibits the same trend, with a lower TL due to the air gaps that increase the macro-porosity of the sample.

The perpendicular configuration exhibits also a peak, but at a lower frequency around 440 Hz, resulting from the excitation of the first bending resonance of the lamella [34]. Based on a simple cantilever beam model, its first frequency can be approximated by

$$f_b \approx 0.56 \frac{w_1}{h_1^2} \sqrt{\frac{E}{12\,\rho_1}}.$$
 (3.2)

3.3 Numerical models

3.3.1 Model of the silencer

In order to further interpret experimental results and analyze absorption mechanisms taking place in the porous material, the wave propagation in the silencer is computed numerically using Finite Element Method. Typical FEM meshes used in our calculations are illustrated in Fig. 3.5 showing the poroelastic domain Ω_p and the air domain Ω_a for the three configurations. The dimensions of the duct and the silencer are the same as the experimental one.

In the air domain of density ρ_0 and sound speed c_0 , the acoustic pressure p obeys the Helmholtz equation

$$\Delta p + k_0^2 p = 0, \tag{3.3}$$

where $k_0 = \omega/c_0$ is the wavenumber (time dependence $e^{-i\omega t}$ is considered here). On the rigid wall, the acoustic normal velocity vanishes. In the porous domain Ω_p , the classical mixed (\mathbf{u}, p_p) formulation [4, Chap. 13] is used as it allows to reduce the number of degrees of freedom and permits to ease the treatment of the transmission conditions at the air-porous interface. The formulation is reminded here:

$$\nabla \cdot \hat{\boldsymbol{\sigma}}^{s}(\mathbf{u}) + \omega^{2} \rho \, \mathbf{u} + \gamma \, \nabla p_{p} = 0, \qquad (3.4a)$$

$$\Delta p_p + \omega^2 \frac{\rho_{22}}{R} p_p - \omega^2 \frac{\rho_{22}}{\phi^2} \gamma \,\nabla \cdot \mathbf{u} = 0.$$
(3.4b)

Here, p_p is the pore pressure, ϕ is porosity of the porous material, $\gamma = \phi \left(\frac{\rho_{12}}{\rho_{22}} - \frac{Q}{R}\right)$ and $\rho = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}$. Coefficient *R* is the effective bulk modulus of the fluid phase and takes into account the thermal dissipation, *Q* couples the two phases by volumic dilatation. The effective density coefficient ρ_{11} and ρ_{22} , respectively for the solid phase and the fluid phase, and the coupling density coefficient ρ_{12} are complex-valued, and their imaginary part takes into account viscous losses. The first two terms in (3.4a) and in (3.4b) describe respectively the dynamics of the elastic skeleton and equivalent fluid. The last term in both equations couples the two phases. The *in vacuo* stress tensor $\hat{\sigma}^s$ is given by

$$\hat{\boldsymbol{\sigma}}^{s}(\mathbf{u}) = \mathbf{I}\left(K_{b} - \frac{2}{3}N\right)\nabla\cdot\mathbf{u} + 2N\boldsymbol{\varepsilon}^{s}(\mathbf{u}).$$
(3.5)

Here, K_b is the complex dynamic bulk modulus of the frame, N is the shear modulus and includes the structural damping. All these coefficients are related to the poroelastic structural parameters (see Table 3.1) by the Johnson-Champoux-Allard model and can be found in Ref. [4, Chap. 6]. At the interface between the fluid and the porous material, the coupling conditions impose the continuity of normal displacement, of the pressure, and of the normal



Figure 3.5: Meshes and boundary conditions for the three tested configurations: homogeneous (a), parallel (b), perpendicular (c).

stress (see for instance [4, Chap. 13]). Since the poroelastic material is clamped, the skeleton and the normal fluid displacements vanish on the duct wall. Radiation conditions at both ends of the duct and the incident pressure field have been implemented using the Dirichlet-to-Neumann (DtN) map [61] using the expansion of the pressure in terms of duct acoustic modes (see Section 3.6.1). Duct modes are also used to compute the The transmission loss (TL), defined as the ratio between the transmitted and incident power.

For completeness, the final form for the weak formulation is reminded in the section 3.6.2. Note that the pressure, in the air and in the pore, and solid displacements are discretized using Lagrange quadratic finite elements. In all cases, the mesh size was chosen to ensure a good trade-off between accuracy and computational time.

3.3.2 Periodic model

Here, we shall use the periodic structure of the silencer and consider a single periodic cell. The two configurations, parallel and perpendicular, are depicted in Fig. 3.6. A typical cell of dimensions w, d and h is periodic in the x and z directions whereas rigid wall conditions are imposed at y = 0, h and, by invoking symmetry reasons, we take h=5 cm which corresponds to the half-width of the rectangular duct. The dimension d is arbitrary here and we take d =6 cm. Dimensions of the porous material are $w_1 \times h_1 \times d$ (recall that experimental values are $w_1=1.5$ cm and $h_1=2.5$ cm). The periodic model not only allows to diminish drastically the computational burden but also provides a new insight into the absorbing properties of the silencer with a view of optimisation, this will be exploited in Section 4.



Figure 3.6: Periodic cell model; (a): perpendicular case; (b): parallel case

The starting point stems from the Bloch theorem that states that all fluctuations, i.e., pressure and solid displacements, call it X, can be written as [32]

$$X(\mathbf{x}) = \hat{X}(\mathbf{x})e^{i\mathbf{k}_B \cdot \mathbf{x}}, \text{ for } \mathbf{x} = [x, y, z]^t,$$
(3.6)

where \hat{X} is a periodic function with $\hat{X}(\mathbf{x}+\mathbf{d})=\hat{X}(\mathbf{x})$ where $\mathbf{d} = [w, 0, d]^t$ and \mathbf{k}_B is the Bloch wavevector which can also be expressed as $\mathbf{k}_B = k_B \boldsymbol{\kappa}$ with unit vector $\boldsymbol{\kappa} = [\kappa_x, \kappa_y, \kappa_z]^t$ and the norm $k_B = ||\mathbf{k}_B||$. Thus, Bloch waves are **d**-periodic functions modulated by plane waves involving the Bloch wavevector. The real part of \mathbf{k}_B is the phase change across the cell, and more importantly, the imaginary part is related to the wave attenuation. Each configuration, i.e. parallel and perpendicular, is easily obtained by simply setting $\boldsymbol{\kappa} = [0, 0, 1]^t$ or $\boldsymbol{\kappa} = [1, 0, 0]^t$, respectively.

To solve the periodic problem with the FEM, the sytem of equations need to be rewritten for the periodic field \hat{X} . This can be done in a systematic way, from the original equations Eq. (3.3) and Eq. (3.4). For instance, in the air domain Ω_a , using the expansion (3.6) yields the new wave equation

$$\Delta \hat{p} + 2\mathrm{i}k_B \boldsymbol{\kappa} \cdot \nabla \hat{p} + \left(k_B^2 - k_0^2\right) \hat{p} = 0.$$
(3.7)

The wave equation in the poroelastic domain can be derived and a similar manner. The associated weak formulation is presented in section 3.6.2. Finally, the problem takes the form of a quadratic eigenvalue problem with eigenvalue k_B ,

$$[\mathbf{K}_0(\omega) + k_B \mathbf{K}_1(\omega) + k_B^2 \mathbf{K}_2(\omega)] \hat{\mathbf{X}} = 0,$$
(3.8)

where $\hat{\mathbf{X}}$ contains the FEM unknowns and \mathbf{K}_i (i = 0, 1, 2) are FEM matrices detailed in section 3.6.2. The computation of Eq. (3.8) is performed after transformation into a generalized eigenvalue problem and the latter is solved using standard sparse solver libraries. The TL of the silencer is estimated, by assuming that (i) the lowest attenuated Bloch wave should provide a fair description of the wave field (both in the air and poroelastic domains) as it propagates in the silencer and (ii) the reflected waves at the entrance of the silencer can be neglected, and thus

$$TL \approx 8.68 \operatorname{Im}(k_B^0) L, \tag{3.9}$$

where k_B^0 corresponds to the eigenvalue of Eq. (3.8) with smallest imaginary part and L = 0.4 m is the length of the silencer.

3.3.3 Comparison with experimental results

The TL of the three configurations are compared in Fig. 3.7 showing good agreement between numerical, here using the full FEM model, and experimental results. A convenient way to determine more precisely the influence of the solid frame is to investigate the relative contribution of the different dissipation mechanisms involved in the sound attenuation. Three mechanisms are considered: viscous, thermal and structural [40, 35]. Viscous loss is due to the viscosity of the air and the relative movement air-skeleton in the porous material. Thermal dissipation is due to the heat exchange between the air and the skeleton. Structural dissipation is due to the viscoelasticity of the skeleton and its strain energy. Results are presented for the three configurations in Fig. 3.9. Generally, through the whole range of frequency, the viscous dissipation is dominating and has similar trend. Thermal dissipation is the second important attenuation mechanism. Structural dissipation is generally smaller except close to the frame resonances, around 900 Hz for the parallel and homogeneous configurations where the first shear resonance occurs. For the perpendicular configuration, the structural dissipation reaches its maximum around 450 Hz which corresponds to the first bending mode of the lamellas and is more pronounced. Thermal dissipation is found to increase steadily with frequency and is less affected by the frame resonance.

Clearly, the additional enhancement in sound attenuation due to the frame resonance, which cannot exist with rigid frame porous materials, offers an interesting alternative for low frequency noise control. The resonance frequency of the first bending mode of the poroelastic lamella is simply determined by its dimensions, Young's modulus and density, thus allowing to design tailored solutions as shown in Ref. [34]. To have a better understanding of the physical mechanisms taking place, it is instructive to illustrate the horizontal displacement field of the skeleton, see Fig. 3.8, when the skeleton of the silencer resonates. For the parallel configuration, one sees the shear motion along one lamella, and for the perpendicular configuration, the



Figure 3.7: Comparison of experimental (black dashed line) and numerical (black solid line) results for the three configurations: (a): homogeneous, (b): parallel and (c): perpendicular



Figure 3.8: Displacement fields (color and motion) in the poroelastic material: (a) parallel configuration at shear resonance around 900 Hz; (b) perpendicular configuration at bending resonance around 450 Hz. Normalized color scale.

bending motion of each lamella. The color allows to see the phase shift and the wavelength close to the acoustic wavelength that is 38 cm at 900 Hz and 75 cm at 450 Hz.

Finally, using the periodic model, Bloch wave attenuations calculated from Eq. (3.9) are also shown in Fig. 3.10. Comparisons with the full model show very good agreements and the skeleton resonances are well captured. Small discrepancies stem mainly from the reflected waves at the entrance of the silencer which are not taken into account in the simplified model. In addition, the reduced periodic FEM model allows thinner mesh that better capture the skeleton motion.

3.4 Parametric study

In this section, we shall benefit from the last observations and analyze the efficiency of the silencer with the periodic model, thus avoiding the computational burden of the full model. Our aim is to conduct a parametric study and show the influence of some parameters, geometrical and physical which can be gathered in two groups. First, we study parameters related to the bending resonance, that is the Young's modulus, the lamella's width, and the structural damping. Then, we study the influence of air-skeleton coupling parameters, characterized by the air gap width and the air flow resistivity.



Figure 3.9: Contribution of viscous (blue dashed line), thermal (black dash-dotted line) and structural dissipation (red dashed line) for the three configurations: (a) homogeneous layer, (b) parallel and (c) perpendicular.



Figure 3.10: Comparison of full and periodic model based on hypothesis of elastic skeleton; parallel:(a); perpendicular :(b).



Figure 3.11: Comparison of TL performance for lamella network with different modulus from 30 000 to 240 000 Pa. For parallel (a) and perpendicular (b) configurations.

3.4.1 Bending resonance related parameters

Young modulus

Fig. 3.11 shows the effect of Young's modulus on the attenuation for both parallel (a) and perpendicular configurations (b). The nominal value is E = 120 kPa. Resonance peaks, which can be anticipated using the approximation Eq. (3.1) and (3.2) for parallel and perpendicular configurations respectively, is proportional to the square root of the Young's modulus E as can be observed. The loss factor is constant in the analysis so the attenuation also increases with E near the resonance.

Width of lamella

The width of lamella w_1 (see Fig. 3.6) is modified from 6 mm to its nominal value 15 mm, keeping the air-gap $w - w_1$ constant. The results are shown in Fig. 3.12. For the parallel



Figure 3.12: Attenuation for different lamella width $w_1 = [6 \text{ mm}, 9 \text{ mm}, 12 \text{ mm}, 15 \text{ mm}]$. parallel (a); perpendicular (b).

configuration (Fig. 3.12a), the location of the peak is not affected as expected from the theoretical estimate Eq. (3.1). However, the attenuation increases with w_1 since the filling fraction of the porous liner increases and tends to behave like the homogeneous configuration. For the perpendicular configuration (see Fig. 3.12b), the frequency of the peak increases almost linearly with w_1 , according to Eq. (3.2). This means that it is possible to match a specific frequency by merely choosing the width of the lamella. Of course reducing the width has a negative impact on the attenuation but this can be partly limited by reducing the width of air gap as shown later.

Loss factor

The effect of the loss factor is shown in Fig. 3.13. Reducing the loss factor tends to produce a higher peak of attenuation followed by a more pronounced dip. This is observed for both configurations. A too large loss factor will reduce the motion of the lamella at the resonance and therefore diminishe the associated additional dissipation.

3.4.2 Air-skeleton coupling related parameters

The effective coupling bewteen the air and the porous frame plays an important role in the sound attenuation of the lamella network. The effect of air gap width and air flow resistivity, which is the most influential coupling parameter in the low frequency range as it is related to viscous dissipation, are now investigated.

Air gap width

We study the effect of the air gap $w - w_1$ ranging from 1 mm to 10 mm, keeping the lamella width constant $w_1 = 15$ mm. Reducing the air gap increases the volume ratio of sound



Figure 3.13: Attenuation of lamella network under different loss factors (LF) [0.01, 0.10, 0.20]. (a): parallel; (b): perpendicular.



Figure 3.14: Attenuation of lamella network for different airgap width [1 mm, 4 mm, 7 mm, 10 mm]. (a): parallel; (b): perpendicular.

absorbing material. This can be observed in Fig. 3.14 in both configurations. Note that the air gap width has a stronger impact at the resonance for the parallel configuration, showing a variation of 8 dB whereas effects on the perpendicular configuration are more moderate. This can be explained by the different coupling mechanism between the movement of air and the porous frame also by the fact that the bending resonance occurs at a lower frequency.

Airflow resistivity

The airflow resistivity is known to be the most important parameter governing the acoustic dissipation in the low frequency range [4]. In the specific case of a silencer made up of a number of parallel splitters, there exists an optimal value for the resistivity as explained in [117] for splitters silencer. In the present work, this fact is also observed in Fig. 3.15, showing the attenuation of silencer for different values ranging from 5 000 Nm⁻⁴s to a very high value



Figure 3.15: Comparison of TL performance of lamella network under different resistivity from 5 000 to 320 000 Nm⁻⁴s. (a): parallel; (b): perpendicular.

of 320 000 Nm⁻⁴s. In the parallel configuration, which somewhat bears resemblance with the configuration considered in [117], the attenuation at the resonance shows strong variations, up to 20 dB. Here, the maximum value, above 25 dB, has been found to be nearly optimal for the fixed geometry of the lamellas grating. For the perpendicular configuration, the attenuation at the first bending resonance increases with σ from 7 dB to 22 dB. In this configuration, the optimal value at the peak can reach a higher value at the detriment of a poorer attenuation over a larger frequency spectrum.

3.4.3 Towards an optimized configuration

The parametric study made earlier permits a physical interpretation of the different mechanisms related to the acoustic attenuation. It is also instructive to address the problem from a mathematical point of view by observing that the quadratic eigenvalue problem of Eq. (3.8) is non-Hermitian due the dissipative nature of the media. In the context of guiding waves, it has been observed in many instances that optimal modal attenuation occurs when two eigenvalues, here the Bloch wavenumber, are nearly coalescing. The interested reader can refer to the seminal paper of Tester [145] for locally reacting materials and [127, 157] for rigid frame porous material and metamaterial having periodic structures. In the context of structural dynamics, this phenomenon is also known as *veering* [52].

This is well illustrated for the perpendicular configuration in Fig. 3.16 where the evolution of the first four eigenvalues with respect to frequency are shown. It can be observed that maximal attenuation arises when the two least attenuated modes have wavenumbers which are getting closer in the complex plane. This happens around 400 Hz and also, to a least extent, around 1500 Hz. One can also observe that 2 modes are nearly coalescing around 1100 Hz but with no effect on the TL since it does not affect the least attenuated mode. These situations



Figure 3.16: Bloch wavenumber on perpendicular case when $\sigma = 320000 \text{ Nm}^{-4}\text{s}$

correspond to strong modal interactions which are very sensitive to various parameters, geometrical and physical, especially those driving the coupling between the elastic frame and the fluid (see sec. 3.4.2) or influencing the frame resonances frequency (see sec.3.4.1). Finding optimized configurations could be achieved via parametric studies, as illustrated in this work, or by using optimization methods based on Exceptional point (EP) location as proposed in [113, 127].

3.5 Conclusion

The sound attenuation of a silencer consisting of a lamella network made with melamine foam inserted in rectangular duct is investigated numerically and experimentally. Two arrangements, parallel and perpendicular to the duct axis, are considered in the study. Results indicate that the perpendicular configuration yields strong low frequency sound attenuation peaks due to the excitation of bending modes whereas, in the parallel configuration, attenuation peaks are found to result from shear waves resonances across the width of the lamella. Though acoustic

attenuation is mainly due to viscous dissipation as expected, the relative contribution from structural dissipation plays a noticeable role near the resonance.

The periodic structure of the silencer is exploited in order to devise a simplified numerical model which not only allows to diminish drastically the computational burden but also allows a more detailed analysis of the nature of the waves which propagate in the silencer. Because reflected waves at the entrance of the silencer can be neglected, this periodic model has been shown to be a reliable predictive tool showing good agreement with the full model and experimental results. A parametric study has been conducted in order to identify the effect of different geometrical parameters, i.e., dimensions of the lamella and air gap, as well as physical parameters, i.e. resistivity, elastic modulus of the porous foam and the loss factor, on the sound attenuation in the silencer. It is shown that resonance frequencies associated with peaks of attenuation, given by approximate formulas (Eq. (3.1) and (3.2)), are proportional to the square root of the Young's modulus, and width of the lamella for the perpendicular configuration.

The interest for this new type of silencer concept relies mainly on the fact it can be designed using relatively simple manufacturing processes without any other kind of inclusions. It is highly tunable and permits lower frequency attenuation, thanks to sub-wavelength resonances, whilst keeping its absorbing efficiency in the medium frequency range. It is thought that the concept could be investigated further by mixing several lamellas width or length in order to extend its efficiency in the low frequency regime.

3.6 Appendix

3.6.1 Rigid duct modes

In the rigid duct, modal expansion is used in the DtN and to compute the TL. For instance, the transmitted pressure field reads

$$p = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn}^{t} \phi_{mn}(x, y) e^{ik_{z,mn}z},$$
(3.10)

where A_{mn}^t , represents the amplitude of the mode (m, n). The upstream pressure is a combination of the incident and the reflected waves. For a rectangular duct, the orthonormal modal shape is given by

$$\phi_{mn} = \Lambda_{mn} \cos(k_{x,m} x) \cos(k_{y,n} y), \qquad (3.11)$$

where $k_{x,m} = m\pi/L_x$, $k_{y,n} = n\pi/L_y$, $k_{z,mn} = \sqrt{k_0^2 - k_{x,m}^2 - k_{y,n}^2}$ are the wavenumbers in the *x*, *y* and *z* directions respectively and Λ_{mn} is the modal norm [111]. Once, the pressure field *p* is known from the FEM computation, the amplitude of each transmitted mode can be

recovered using orthogonality relation

$$A_{mn}^t = \int_{\Gamma_o} p \,\phi_{mn} \,\mathrm{d}\Gamma. \tag{3.12}$$

The transmission loss can be obtained from

$$TL = -10\log_{10}\frac{I_t}{I_i},$$
(3.13)

where I_i and I_t are the intensity of the incident and transmitted sound in duct respectively. The first is known and the later reads

$$I_t = \sum_{m}^{M} \sum_{n=0}^{N} \frac{k_{z,mn} |A^t|_{mn}^2}{2\omega\rho_0},$$
(3.14)

where the sum is limited to the propagating modes.

3.6.2 Weak formulation

In the air domain, applying the standard weighted residual scheme to the Helmholtz Eq. (3.3) yields, after integrating by parts,

$$-\int_{\Omega_a} \nabla p^* \cdot \nabla p \,\mathrm{d}\Omega + k_0^2 \int_{\Omega_a} p^* p \,\mathrm{d}\Omega + \int_{\Gamma_a} p^* \frac{\partial p}{\partial n} \mathrm{d}\Gamma = 0, \qquad (3.15)$$

where p^* stands for the weighting function. In the poroelastic domain governed by the Biot's Eqs. (3.4) expressed with (**u**, p), the same approach leads to [4, Chap. 13]

$$\int_{\Omega_{p}} \hat{\boldsymbol{\sigma}}^{s}(\mathbf{u}) : \boldsymbol{\varepsilon}^{s}(\mathbf{u}^{*}) \, \mathrm{d}\Omega - \omega^{2} \int_{\Omega_{p}} \tilde{\rho} \mathbf{u} \cdot \mathbf{u}^{*} \, \mathrm{d}\Omega + \int_{\Omega_{p}} \left[\frac{\phi^{2}}{\omega^{2} \tilde{\rho}_{22}} \nabla p_{p} \cdot \nabla p_{p}^{*} - \frac{\phi^{2}}{\tilde{R}} p_{p} \, p_{p}^{*} \right] \, \mathrm{d}\Omega \\ - \int_{\Omega_{p}} (\tilde{\gamma} + \phi') (\nabla p_{p}^{*} \cdot \mathbf{u} + \nabla p_{p} \cdot \mathbf{u}^{*}) \, \mathrm{d}\Omega - \int_{\Omega_{p}} \phi' \left(p_{p}^{*} \nabla \cdot \mathbf{u} + p_{p} \nabla \cdot \mathbf{u}^{*} \right) \, \mathrm{d}\Omega \\ - \int_{\Gamma_{p}} \boldsymbol{\sigma}^{t} \, \mathbf{n} \cdot \mathbf{u}^{*} \, \mathrm{d}\Gamma - \int_{\Gamma_{p}} \phi \left(\mathbf{U} - \mathbf{u} \right) \cdot \mathbf{n} \, p_{p}^{*} \, \mathrm{d}\Gamma = 0 \,. \quad (3.16)$$

with $\phi' = \phi \left(1 + \frac{Q}{R}\right)$.

3.6.3 Weak formulation for Bloch waves computation

We propose here a convenient systematic way to obtained the weak form of periodic part from the standard week formulation of each domain given in (3.15) and (3.16). The basic idea is to transform each differential operator to account for the Bloch decomposition given in Eq. (3.6). This approach can be managed automatically by picking all the combinations once the operator involving the pressure p or p_p and frame displacement **u** are expressed with the periodic fields \hat{p} , \hat{p}_p and \hat{u} . The advantage of this approach is to keep boundary terms unchanged which is convenient to applied standard boundary conditions like rigid wall. For instance gradient, divergence and vector gradient yield

$$\nabla p = \left[\nabla \hat{p} + \mathrm{i}\hat{p}k_B\boldsymbol{\kappa}\right]e^{\mathrm{i}k_B\boldsymbol{\kappa}\cdot\mathbf{x}},\tag{3.17a}$$

$$\nabla \cdot \mathbf{u} = \left[\nabla \cdot \hat{\mathbf{u}} + \mathrm{i}k_B \boldsymbol{\kappa} \cdot \hat{\mathbf{u}}\right] e^{\mathrm{i}k_B \boldsymbol{\kappa} \cdot \mathbf{x}},\tag{3.17b}$$

$$\nabla \hat{\mathbf{u}} = \left[\nabla \mathbf{u} + \mathrm{i}k_B \mathbf{u} \,\boldsymbol{\kappa}^{\mathrm{t}} \right] \mathrm{e}^{\mathrm{i}k_B \boldsymbol{\kappa} \cdot \mathbf{x}}. \tag{3.17c}$$

The strain tensor, using the Voight formalism

$$\epsilon(\mathbf{u}) = \left[\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, 2\epsilon_{xy}, 2\epsilon_{yz}, 2\epsilon_{xz}\right]^{\mathrm{t}}, \qquad (3.18)$$

now reads

$$\epsilon(\mathbf{u}) = [\mathbf{B}_0 + k_B \mathbf{B}_1] \,\hat{\mathbf{u}} e^{ik_B \boldsymbol{\kappa} \cdot \mathbf{x}},\tag{3.19a}$$

where \mathbf{B}_0 and \mathbf{B}_1

$$\mathbf{B}_{0} = \begin{bmatrix} \partial_{x} & 0 & 0 \\ 0 & \partial_{y} & 0 \\ 0 & 0 & \partial_{z} \\ \partial_{y} & \partial_{x} & 0 \\ 0 & \partial_{z} & \partial_{y} \\ \partial_{z} & 0 & \partial_{x} \end{bmatrix} \quad \text{and} \quad \mathbf{B}_{1} = \mathbf{i} \begin{bmatrix} \kappa_{x} & 0 & 0 \\ 0 & \kappa_{y} & 0 \\ 0 & 0 & \kappa_{z} \\ \kappa_{y} & \kappa_{x} & 0 \\ 0 & \kappa_{z} & \kappa_{y} \\ \kappa_{z} & 0 & \kappa_{x} \end{bmatrix}. \quad (3.20a)$$

In the air domain Ω_a , this yields for volumic terms

$$a_0(\hat{p}^*, \hat{p}) + k_B + a_1(\hat{p}^*, \hat{p}) + k_B^2 a_2(\hat{p}^*, \hat{p}) = 0, \qquad (3.21)$$

with the operators

$$a_{0}(\hat{p}^{*},\hat{p}) = -\int_{\Omega_{a}} \nabla \hat{p}^{*} \cdot \nabla \hat{p} \, \mathrm{d}\Omega + k_{a}^{2} \int_{\Omega_{a}} \hat{p}^{*} \hat{p} \, \mathrm{d}\Omega,$$

$$a_{1}(\hat{p}^{*},\hat{p}) = \mathrm{i} \int_{\Omega_{a}} \left(-\nabla \hat{p}^{*} \cdot (\boldsymbol{\kappa} \hat{p}) + (\boldsymbol{\kappa} \hat{p}^{*}) \cdot \nabla \hat{p} \right) \, \mathrm{d}\Omega,$$

$$a_{2}(\hat{p}^{*},\hat{p}) = -\int_{\Omega_{a}} \hat{p}^{*} \hat{p} \, \mathrm{d}\Omega.$$

In the poroelastic domain Ω_p , this yields for volumic terms

$$b_0(\hat{p}_p^*, \hat{\mathbf{u}}^*, \hat{p}_p, \hat{\mathbf{u}}) + k_B b_1(\hat{p}_p^*, \hat{\mathbf{u}}^*, \hat{p}_p, \hat{\mathbf{u}}) + k_B^2 b_2(\hat{p}_p^*, \hat{\mathbf{u}}^*, \hat{p}_p, \hat{\mathbf{u}}) = 0, \quad (3.22)$$

with the operators

$$\begin{split} b_{0}(\hat{p}_{p}^{*},\hat{\mathbf{u}}^{*},\hat{p}_{p},\hat{\mathbf{u}}) &= \int_{\Omega_{p}} \hat{\mathbf{u}}^{*t} \left(\mathbf{B}_{0}^{t}\mathbf{D}\mathbf{B}_{0}\right) \hat{\mathbf{u}} \mathrm{d}\Omega - \int_{\Omega_{p}} \tilde{\rho}\omega^{2}\hat{\mathbf{u}}^{*} \cdot \hat{\mathbf{u}} \mathrm{d}\Omega \\ &+ \int_{\Omega_{p}} \left(\frac{\phi^{2}}{\omega^{2}\rho_{22}}\right) \nabla \hat{p}_{p}^{*} \cdot \nabla \hat{p}_{p} \mathrm{d}\Omega - \int_{\Omega_{p}} \frac{\phi^{2}}{R} \hat{p}_{p}^{*} \hat{p}_{p} \mathrm{d}\Omega \\ &- \int_{\Omega_{p}} \left(\gamma + \phi'\right) \left(\nabla \hat{p}_{p}^{*} \cdot \hat{\mathbf{u}} + \hat{\mathbf{u}}^{*} \cdot \nabla \hat{p}_{p}\right) \mathrm{d}\Omega \\ &- \int_{\Omega_{p}} \phi' \left(\hat{p}_{p}^{*} \nabla \cdot \hat{\mathbf{u}} + \nabla \cdot \hat{\mathbf{u}}^{*} \hat{p}_{p}\right) \mathrm{d}\Omega \\ &- \int_{\Omega_{p}} \phi' \left(\hat{p}_{p}^{*} \nabla \cdot \hat{\mathbf{u}} + \nabla \cdot \hat{\mathbf{u}}^{*} \hat{p}_{p}\right) \mathrm{d}\Omega \\ &+ \int_{\Omega_{p}} \frac{\phi^{2}}{\omega^{2}\rho_{22}} \left(\nabla \hat{p}_{p}^{*} \cdot \boldsymbol{\kappa} \mathrm{i} \hat{p}_{p} - \hat{p}_{p}^{*} \mathrm{i} \boldsymbol{\kappa} \cdot \nabla \hat{p}_{p}\right) \mathrm{d}\Omega \\ &- \left(\gamma + \phi'\right) \int_{\Omega_{p}} -\hat{p}_{p}^{*} \mathrm{i} \boldsymbol{\kappa} \cdot \hat{\mathbf{u}} + \hat{\mathbf{u}}^{*} \cdot \boldsymbol{\kappa} \mathrm{i} \hat{p}_{p} \mathrm{d}\Omega \\ &- \phi' \int_{\Omega_{p}} \hat{p}_{p}^{*} \mathrm{i} \boldsymbol{\kappa} \cdot \hat{\mathbf{u}} - \hat{\mathbf{u}}^{*} \cdot \boldsymbol{\kappa} \mathrm{i} \hat{p}_{p} \mathrm{d}\Omega , \\ b_{2}(\hat{p}_{p}^{*}, \hat{\mathbf{u}}^{*}, \hat{p}_{p}, \hat{\mathbf{u}}) = - \int_{\Omega_{p}} \hat{\mathbf{u}}^{*t} \mathbf{B}_{1}^{t} \mathbf{D} \mathbf{B}_{1} \hat{\mathbf{u}} \mathrm{d}\Omega \\ &+ \int_{\Omega_{p}} \left(\frac{\phi^{2}}{\omega^{2}\rho_{22}}\right) \hat{p}_{p}^{*} (\boldsymbol{\kappa} \cdot \boldsymbol{\kappa}) \hat{p}_{p} \mathrm{d}\Omega . \end{split}$$

Once the boundary conditions are applied, the discrete operator (3.22) and (3.23a) are assembled and coupled with the condition given in sec. 3.3, the quadratic eigenvalue problem of the whole model (3.8) can be found.
Chapter 4

Sound transmission loss of the infinite double panel with resonators

This chapter addresses the sound transmission loss (TL) of a double panel partition with embedded resonators. First, the typical TL of a finite size single panel and infinite double panel partitions are recalled to identify some issues at low frequency. Then, the transfer matrix method (TMM) is used to predict the sound insulation performance of an infinite double panel with embedded resonators. The effective parameters are derived by TMM method thank to a new approach. The effect of the main parameters on TL are then investigated.

4.1 Flexural waves

The flexural wave propagation in a plate is briefly discussed. Bending waves, or flexural waves, propagate as shown in Fig. 4.1(a). The bending wave motion is due to the rotation of the cross section around the neutral axis for beams and the rotation of the line normal to the mid plane around the axis transverse to the direction of wave propagation for plates [166]. Among various wave types, the bending wave is of greatest importance in the interaction process between plate and air, since it involves large displacement and surfaces that can efficiently disturb an adjacent fluid [44].

Bending wave are dispersive. The phase velocity in a thin plate (or a beam) is not constant with respect to the frequency and reads

$$c_b = \sqrt{\omega} \cdot \sqrt[4]{\frac{D}{\rho_s}},\tag{4.1}$$

where D is the bending stiffness and ρ_s is the surface density of plate. The wave speed in air is higher than in a plate until the critical frequency

$$f_c = \frac{c_0^2}{2\pi} \left(\frac{\rho_s}{D}\right)^{\frac{1}{2}},$$
(4.2)

where the speed of bending wave in the plate equals those of the acoustic wave in the surrounding air. The flexural wavenumber is given by

$$k_f = \sqrt{\omega} \cdot \left(\frac{\rho_s}{D}\right)^{\frac{1}{4}}.$$
(4.3)

It is worth noting that with Euler-Bernoulli beam model or Kirshoff plate model, the solution are a propagative wave, with the wavenumber k_f , and an evanescent wave, with the wavenumber ik_f .

4.1.1 Single wall

The most usual indicator for sound insulation performance of partitions is the *sound transmission loss* which is the ratio between incident sound power W_i and transmitted sound power W_t , expressed in decibels. For a finite size panel, a first dip on the sound transmission loss curve occurs at the first bending mode of the panel as shown in Fig. 4.1(b). This frequency is given for a simply supported rectangular plate by,

$$f_{11} = \pi^2 \left(\frac{1}{a^2} + \frac{1}{b^2}\right) \sqrt{\frac{D}{\rho_s}},$$
(4.4)



Figure 4.1: (a): Deformation pattern of bending wave in a flat plate; (b) Typical sound transmission loss curve of an isotropic thin panel [60].

where a and b are the dimensions of the plate. For other boundary conditions [15], similar expressions can be found. As for infinite panel, a second dip occurs at the coincidence frequency, given by

$$f_{\rm coin} = \frac{f_c}{\sin^2 \theta},\tag{4.5}$$

where θ is the angle with respect to the normal vector of the plate of the incident wave. At coincidence frequency, the wavelength of the incident wave projected onto the plate equals the wavelength of the bending wave. At normal incidence f_{coin} tends to infinity and when θ tends to $\pi/2$, f_{coin} tends to f_c .

The sound transmission loss of isotropic and finite size single panel shown in Fig. 4.1(b) can be divided into several regions:

- At frequencies below the first bending resonance frequency f_{11} , the transmission loss performance is determined by the stiffness of the panel. The slope of TL curve is -6 dB per doubling of frequency.
- Above this frequency and below the critical frequency, the sound insulation performance is linearly related to the surface density of the panel. The transmission loss increases with 6 dB when doubling the frequency or the surface density: this corresponds to the *mass law*. In this range, the transmission loss decreases when the incident angle θ increases. Structural vibration resonances occur in this region, but the radiation efficiency of these modes is lower than the first one [60].
- Above the coincidence frequency, the transmission loss is also mainly controlled by the stiffness of the structure. The increase in transmission loss is of the order of 18 dB per doubling of frequency [60, 44].



Figure 4.2: (a): A infinite double panel partition filled with porous material; (b): The sound transmission loss curve of a double panel with (blue lines) and without (black lines) porous material under normal or oblique ($\theta = \pi/3$) incidence.

• When the frequency is close to any dips, the depth of the dip is controlled by the damping of the structure.

4.1.2 Double wall

The double or triple partition as shown in Fig. 4.2(a) is widely used in buildings and industries since it presents more effective sound isolation performance in the mid or high frequencies compared to the single panel of the same mass. In addition, the double panel also outperforms thermal insulation and structure stiffness of a single panel. Fig. 4.2 (b) presents the transmission loss performance of a double wall under normal and oblique incidence. Generally, in the low frequency, it has two characteristic regions below or above the mass-spring-mass resonance. At the mass-spring-mass frequency, the double panel acts like two masses coupled by an air cavity acting as a spring. When there is no porous filling the air cavity, this resonance frequency shifts as incidence angle varies as presented in Fig. 4.2(b) and can be determined from [44, Chap. 5]

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{\gamma P_0}{h \cos \theta_1} \frac{\rho_1 + \rho_2}{\rho_1 \rho_2}},$$
(4.6)

where P_0 and γ denote as the atmospheric pressure of air and the heat capacity ratio respectively, h is the distance between the two panels, and ρ_1 and ρ_2 are the surface density of each panel. When frequencies are lower than f_0 , acoustic insulation properties of the double panel system are identical to that of single panel having the same mass. Above f_0 , also called the decoupling frequency, the two panels vibrates out of phase and the second panel displacement decreases with frequency. In this region, the transmission loss increases by 18 dB per octave.

Porous layer When a porous material is introduced in the cavity of the double panel partition, the sound transmission loss is enhanced for several reasons as illustrated in Fig. 4.2 (b):

- At normal incidence, it is seen that the mass-spring-mass frequency is smaller compared to the bare double panel, allowing a decoupling at a lower frequency. This is related to the bulk modulus of the equivalent fluid $\tilde{K}_f \approx P_0$ that is smaller than $K_0 = \gamma P_0$ of the air, with P_0 the atmospheric pressure and $\gamma = 1.4$.
- The mass-spring-mass frequency is less sensitive to the incidence angle, because the speed of sound is smaller in the porous material, and is almost independent on the incidence angle θ_2 according to Snell-Descartes law. This allows to keep the decoupling frequency close to f_0 even in diffuse field.
- The dips corresponding to the mass-spring-mass resonance, coincidence frequency and acoustic resonances within the cavity are attenuated because of the introduction of damping in the cavity.

Resonators To overcome the lack of sound insulation around resonant frequencies, local resonators can be introduced on the host panel [146]. Local resonators are porous layer with mass inclusions [75, 64] or cantilever absorbers [62] tuned to match the structural resonant frequency of the host panel. Acoustic and elastic resonators are also introduced into the cavity inside the double panel to improve its insulation performance [82, 42]. Due to the periodicity of the resonators implementation, band gaps where flexural waves can not propagate are observed. Resonators can change the panel dynamics providing a way to overcome some insulation issues around a given frequency band.

4.2 Modeling of thin panel with local resonators

In the following section, we will assume that the plate can be modelled as an Euler-Bernoulli beam. This assumption is valid for an infinite system with continuous and homogeneous distribution of resonators or when the period of the resonator is small in comparison with the wavelength. In particular, the lumped model [53, 149, 36], standard in the metamaterial community for such configuration, will be compared to a transfer matrix method (TMM) approach to find effective parameters. We present here a preliminary work on this topic.



Figure 4.3: Double panel with resonators in cavity

4.2.1 Lumped model

Firstly the lumped model is recalled here. The main assumption behind this approach is the long wavelength limit, i.e., the phase shift on the unit cell is neglected and the resonator will be model as a continuous distribution. This approach works for plate, or for beam,, but for conciseness results will be presented for beams. The extension to plate is straightly forward by changing the linear density by surface density and the bending stiffness conveniently, e.g. D = EI or $D = \frac{Eh^3}{12(1-\nu^2)}$ for beam and plate respectively.

The beam with a mass-spring resonator is depicted in Fig. 4.3. The local equation of motion is given by [149, 53],

$$D\frac{\partial^4 w(x,t)}{\partial x^4} + \rho h \frac{\partial^2 w(x,t)}{\partial t^2} = F,$$
(4.7)

$$F = -nk_1(w(x,t) - u(t)), (4.8)$$

$$m_1 \ddot{u}(t) = -k_1(u(t) - w(x, t)), \tag{4.9}$$

where n = 1/d is the number of resonators per unit length and d is the spatial period. The two displacement variables u and w are expressed in the form of harmonic time dependence $w(x,t) = We^{i(\mu_a x - \omega t)}$, $u = Ue^{-i\omega t}$ with μ_a the projection of wavenumber on the x direction. These equations can be written in a matrix form

$$\begin{bmatrix} \mu_a^4 D - \rho h \omega^2 + n k_1 & -k_1 n \\ -k_1 & k_1 - m_1 \omega^2 \end{bmatrix} \begin{cases} W \\ U \end{cases} = \mathbf{0}.$$
 (4.10)

To obtain a nontrivial solution $[W, U]^T$, the matrix determinant should equal zero. The corresponding expansion of the determinant is,

$$(-\rho h m_1)\omega^4 + (\mu_a^4 D m_1 + n m_1 k_1 + \rho h k_1)\omega^2 - (\mu_a^4 D k_1) = 0.$$
(4.11)

To solve the dispersion equation, there are two options: i) set a real value for ω and find $\mu_a(\omega)$; ii) set μ_a real and find the associated $\omega_i(\mu_a)$. The first approach corresponds generally to



Figure 4.4: Comparison of dispersion curves obtained by solving for μ_a or for ω for a one resonator per period d = 40 mm.

forced problems with an incident wave and is generally more easy to manage with dissipation. This two strategies are illustrated in Fig. 4.4 and yields to the same solutions in the passing band. It is worth noting that $\mu_a(\omega)$ yield to complex wavenumber in the band gap. Because lumped model is based on a continuous distribution of resonators, the Bragg band gap is not present and cannot be observed.

The effective density can be obtained by factoring out all terms proportional to w in the equation of dispersion,

$$\rho_e = \rho - \frac{n}{S} \frac{m_1 k_1}{k_1 - \omega^2 m_1}.$$
(4.12)

Using this expression, the dispersion equation can be recast into an homogeneous beam, with ρ_e instead of ρ . Thus, the flexural wavenumber reads,

$$\mu_a = \left(\frac{\rho_e S \omega^2}{D}\right)^{1/4},\tag{4.13}$$

and is formally identical to (4.3). This approach can be generalized in a straight forward manner for a plate.

This approach can also be applied to other kind of resonators (see for instance Fig. 4.11 or [135]), but all the analysis have to be done again. The TMM is a more versatile approach, although more difficult to apply to plate, especially with anisotropy. Expressions can also be generalized with different resonators.

4.2.2 TMM model

Transfer matrix method is ideal to build composite structures from standard components. The mass-spring resonator depicted in Fig. 4.3 can be obtained by multiplying beam matrix and a junction matrix containing the resonator, since the state vector is continuous at each interface. TMM has been successfully applied to a wide category of 1D structures [141, 140, 28], metamaterials [163, 95] and has a long history in multilayer porous material modeling [4]. A key property of TMM in periodic context is that their eigenvalue and eigenvector correspond to Bloch waves. It means that an eigenvector propagates without being scattered in the lattice.

With this approach, the position of the resonators inside the unit cell is now taken into account and dispersion relation (Bloch waves) can be solved outside the long wavelength assumption. We will follow the approach given in [18].

The first step is to recall the transfer matrix of an homogeneous beam section of length l. The motion equation of Euler-Benoulli beam reads

$$\frac{\partial^4 w(x,t)}{\partial x^4} D + \rho S \frac{\partial^2 w(x,t)}{\partial t^2} = 0, \qquad (4.14)$$

where D = EI is the bending stiffness of the beam, S is the beam cross-section, E is the Young's modulus, and I is the second moment of area, equal to $bh^3/12$ for $b \times h$ rectangular cross section.

The general solution of Eq. (4.14) can be represented as

$$w(x,t) = (A_1 \cosh(\mu x) + A_2 \sinh(\mu x) + A_3 \cos(\mu x) + A_4 \sin(\mu x)) e^{-i\omega t}, \qquad (4.15)$$

with $\mu = \left(\frac{\rho S \omega^2}{D}\right)^{1/4}$ and the integration constants A_i $(i = 1, \dots, 4)$.

The next step is to recast (4.14) as a system of first order ODE. This is done by using the angular deflection $\phi(x) = w'(x)$, bending moment M(x) = -EIw''(x) and shearing force F(x) at location x. The explicit expression of them can be derived from the displacement w(x), respectively proportional to w', w'' and w'''

$$\begin{cases} w(x) = A_1 \cosh(\mu x) + A_2 \sinh(\mu x) + A_3 \cos(\mu x) + A_4 \sin(\mu x), \\ \phi(x) = w'(x) = \mu A_1 \sinh(\mu x) + \mu A_2 \cosh(\mu x) - \mu A_3 \sin(\mu x) + \mu A_4 \cos(\mu x), \\ M(x) = -Dw''(x) = -D(\mu^2 A_1 \cosh(\mu x) + \mu^2 A_2 \sinh(\mu x) - \mu^2 A_3 \cos(\mu x) - \mu^2 A_4 \sin(\mu x)) \\ F(x) = -Dw'''(x) = -D(\mu^3 A_1 \sinh(\mu x) + \mu^3 A_2 \cosh(\mu x) + \mu^3 A_3 \sin(\mu x) - \mu^3 A_4 \cos(\mu x)) \end{cases}$$

It can be rewritten in matrix form,

$$\mathbf{q} = \mathbf{N}\mathbf{c},\tag{4.16}$$

where the state vector \mathbf{q} and wave amplitude vector \mathbf{c} (independent of the position) are

$$\mathbf{q} = \left[w(x), \phi(x), M(x), F(x)\right]^t, \tag{4.17}$$

$$\mathbf{c} = \left[A_1, A_2, A_3, A_4\right]^{\iota}, \tag{4.18}$$

and **N** is function of x coordinate and writes

-

$$\mathbf{N} = \begin{bmatrix} \cosh(\mu x) & \sinh(\mu x) & \cos(\mu x) & \sin(\mu x) \\ \mu \sinh(\mu x) & \mu \cosh(\mu x) & -\mu \sin(\mu x) & \mu \cos(\mu x) \\ -D\mu^2 \cosh(\mu x) & -D\mu^2 \sinh(\mu x) & D\mu^2 \cos(\mu x) & D\mu^2 \sin(\mu x) \\ -D\mu^3 \sinh(\mu x) & -D\mu^3 \cosh(\mu x) & -D\mu^3 \sin(\mu x) & D\mu^3 \cos(\mu x) \end{bmatrix}.$$
 (4.19)

Making use of boundary conditions at p1 (x = 0) and p2 (x = l) points as depicted in Fig. 4.3, the corresponding state vectors $\mathbf{q}_{x=0}$ and $\mathbf{q}_{x=l}$ in these two points can be written as,

$$\mathbf{q}_0 = \mathbf{N}(0)\mathbf{c},\tag{4.20}$$

$$\mathbf{q}_l = \mathbf{N}(l)\mathbf{c}.\tag{4.21}$$

Then, the wave amplitude vector **c** can be expressed as

$$\mathbf{c} = \mathbf{N}(0)^{-1} \mathbf{q}_0, \tag{4.22}$$

or as

$$\mathbf{c} = \mathbf{N}(l)^{-1}\mathbf{q}_l,\tag{4.23}$$

when these matrices are invertible. Using both expressions for c yields the in-between state vectors at the two positions

$$\mathbf{q}_l = \mathbf{R}\mathbf{q}_0,\tag{4.24}$$

where $\mathbf{R} = \mathbf{N}(l)^{-1}\mathbf{N}(0)$ is the so-called transfer matrix. Closed form expression can be found [18] and finally, it reads

$$\mathbf{R} = \begin{bmatrix} Q & \frac{T}{\mu} & -\frac{U}{\mu^2 D} & -\frac{V}{\mu^3 D} \\ \mu V & Q & -\frac{T}{\mu D} & -\frac{U}{\mu^2 D} \\ -\mu^2 DU & -\mu DV & Q & \frac{T}{\mu} \\ -\mu^3 DT & -\mu^2 DU & \mu V & Q \end{bmatrix}.$$
 (4.25)

The Krylov-Rayleigh functions Q, T, U, V are defined as

$$Q(\mu l) = \frac{1}{2} \left[\cosh(\mu l) + \cos(\mu l) \right],$$
(4.26)

$$T(\mu l) = \frac{1}{2} \left[\sinh(\mu l) + \sin(\mu l) \right],$$
(4.27)

$$U(\mu l) = \frac{1}{2} \left[\cosh(\mu l) - \cos(\mu l) \right], \tag{4.28}$$

$$V(\mu l) = \frac{1}{2} \left[\sinh(\mu l) - \sin(\mu l) \right].$$
 (4.29)

It is worth mentioning that D is the only *free* parameter in this matrix, although we could expect two, since there are two modes. This comes from the Euler-Bernoulli model assumptions.

Based on the continuities of displacement, rotation, bending moment, and shearing force, the state vectors located at points p2 and p3 are linked by

$$\mathbf{q}_2 = \mathbf{E}\mathbf{q}_3,\tag{4.30}$$

where matrix \mathbf{E} is the point-transfer matrix. When there is no mass-spring system, \mathbf{E} is the identity matrix. When a mass-spring system is located at point p3, the relative equilibrium equation of shearing force is modified by [126, 149]

$$F(p_2) = F(p_3) + \frac{m_1 k_1 \omega^2 w}{m_1 \omega^2 - k_1},$$
(4.31)

where k_1 is the stiffness of the spring and m_1 is the mass as depicted in Fig. 4.3. Then, the transfer matrix between points p2 and p3 is obtained as

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{m_1 k_1 \omega^2}{m_1 \omega^2 - k_1} & 0 & 0 & 1 \end{bmatrix}.$$
 (4.32)

In this model because the distance between points p1, p2 and p3, p4 as depicted in Fig. 4.3 are the same, the two corresponding transfer matrices \mathbf{R}_2 and \mathbf{R}_1 are the identical. Then, the global equation is obtained as

$$\mathbf{q}_1 = \mathbf{R}_1 \mathbf{E} \mathbf{R}_2 \mathbf{q}_4. \tag{4.33}$$

In the following, we call $\mathbf{R}_g = \mathbf{R}_1 \mathbf{E} \mathbf{R}_2$ the global transfer matrix. As depicted in Fig. 4.3, the points p1 and p4 are located on the left and right boundary of unit cell of the beam. Taking accordance to Bloch theorem, the vector **q** in point p1 is equal to the one in point 4 multiplying



Figure 4.5: Validation of R matrix for dispersion relation of beams.

Property	Density	Young's modulus	Loss factor	Poisson ratio	Thickness	Periodicity
	$ ho_1$	E	η	ν	h	d
	$kg.m^{-3}$	MPa	-	-	mm	mm
Panel	400	180	0	0.33	10	40

Table 4.1: Homogeneous properties of the plate.

a phase shift

$$\mathbf{q}_1 = e^{\mathbf{i}k_B d} \mathbf{q}_4,\tag{4.34}$$

with d = 2l an k_B the Bloch wavenumber. Substitution of Eq. (4.34) into Eq. (4.33) yields an eigenvalue problem with the eigenvalue is $\lambda = e^{ik_B d}$,

$$\left(\mathbf{R}_{g}-\lambda\mathbf{I}\right)\mathbf{q}_{4}=0.\tag{4.35}$$

Thus, the corresponding dispersion behavior for the plate with or without resonator can be obtained.

The homogeneous beam TMM has been validated in Fig. 4.5 by comparing the Bloch wavenumbers to Eq. (4.3). The relative properties of the host beam and embedded resonators are listed in Table 4.1 and 4.2. For the beam with added resonators, the Bloch wavenumber based on TMM is shown in Fig. 4.6. A small bandgap is obtained between 333 Hz and 342 Hz. It is consistent with the reference results from the lumped model given in Eq. (4.13). This agreement validates the composite TMM.

Property	m_1 (g)	k_1 (N/m)	m_2 (g)	k ₂ (N/m)
value	9.68e-2	423	9.68e-2	350

Table 4.2: Homogeneous properties of the mass-spring resonator; the imaginary part of stiffness represents mass ratio.



Figure 4.6: Comparaison between the wavenumbers obtained using μ_a from the Lumped model(4.13), the TMM Bloch mode k_B (4.35) and the effective bending wavenumber μ_e (4.40).

Effective parameters

The knowledge of effective parameters could be useful to derive frequency dependent parameters to classical vibroacoustic formulas and to get a new physical insight on the problem. Meanwhile, they can provide some effective guidance to design the metamaterial with desired properties [152]. Nonetheless, obtaining such parameters is not straight forward and not always unique, depending on how the effective parameters are defined. In 1D (scalar) acoustic, like in duct, a classical approach for obtaining effective parameters of porous material is to use the transfer matrix [137]. The wavenumber is then deduced from its trace and the effective impedance is obtained by the ratio of extra diagonal terms. For beams, using similar approach is appealing but there are two kinds of waves in the beam that makes the situation more complex. However, transfer matrix and their spectral properties have been used to identify effective stiffness of periodic beams assembly in static regime (truss) [141, 140, 50]. In this section, we will present a first attempt and propose some possible enhancements.

The trace T_g of the global transfer matrix \mathbf{R}_g is related to its eigenvalues

$$\sum_{i=1}^{4} \lambda_i = e^{ik_{B1}d} + e^{ik_{B2}d} + e^{ik_{B3}d} + e^{ik_{B4}d} = 2\left(\cos(k_{B1}d) + \cos(k_{B2}d)\right) = T_g.$$
 (4.36)

Indeed, due to the reciprocity, each Bloch wavenumber comes by pair $\pm k_{Bi}$ (i = 1, 2). The trace has a similar form as the homogeneous beam transfer matrix \mathbf{R}_e given in (4.25)

$$T_e = 2\left(\cosh(\mu_e d) + \cos(\mu_e d)\right),\tag{4.37}$$

excepted that only one wavenumber μ_e is involved. This is due to a degeneracy of the Bernoulli beam model. Using a more general beam model may lead to a better approach when looking for effective parameters; this will be let for further works.

Keeping this in the mind, we can expect that obtaining effective parameters from the Euler-Bernoulli transfer matrix entries will work as long as all Bloch wavenumbers satisfy the condition

$$k_{B1}^4 \approx k_{B2}^4 \approx k_{B3}^4 \approx k_{B4}^4, \tag{4.38}$$

as for an Euler-Bernoulli beam. The Bloch wavenumber are plotted in Fig. 4.7 and $(k_B)_i^4$ are given in Fig. 4.8. It can be observed that this assumption is well satisfied in the vicinity of the hybridization gap. It can be justified mathematically, because the resonators affect *locally* the beam effective parameters (the lumped model works) without changing the nature of the dispersion equation. However, in the Bragg gap, as the destructive interferences affect only the propagative waves, this assumption breaks down and two wavenumbers are required [105]. Under this form, the homogenization scheme based on Euler-Bernoulli beam cannot be extended at higher frequency when $k_B \frac{d}{\pi} \approx 1$. This should be possible with the Rayleigh beam



Figure 4.7: Symmetry of the four Bloch wavenumbers in the vicinity of the hybridization gap.



Figure 4.8: Modulus of $(k_B)_i^4$.

model, where propagative and evanescent wave can be uncoupled by taking into account the inertia.

In order to analyze wave attenuation and estimate the sound insulation properties through the panel, the effective surface mass density ρ_e , wavenumber μ_e and bending stiffness D_e of the host plate with embedded resonators can be figured out by

1

$$D_e = E_e I_e = -\frac{\mathbf{R}_g[1,2]}{\mathbf{R}_g[2,3]},$$
(4.39a)

$$\rho_e = \frac{\mu_e^4 D_e}{S\omega^2}.$$
(4.39b)



Figure 4.9: Real (black) and imaginary (blue) part effective parameters deduced from TMM homogenization and compared to results from the lumped model.

To get the effective wavenumber μ_e , since there is not obvious inverse function, we solve numerically

$$2\left(\cosh(\mu_e d) + \cos(\mu_e d)\right) = \operatorname{Tr} \mathbf{R}_g,\tag{4.40}$$

using Newton-Raphson scheme by starting at an averaged value based on Bloch wavenumber $\sqrt[4]{\frac{1}{4}\sum_{i=1}^{4}k_{Bi}^4}$. Because the initial guess is close to the solution, no convergence problem have been found.

A comparison between the wavenumbers obtained using μ_a from the Lumped model (4.13), the TMM Bloch mode k_B (4.35) and the effective bending wavenumber (4.40) is presented in Fig. 4.6. A good agreement is obtained by the three approaches in the vicinity of the hybridization gap.

The gap corresponds to the bandwith where the effective density real part (see Fig. 4.9a) becomes negative in a narrow frequency band after 320 Hz. In the present form of the homogenization scheme, only the Bloch wave allow to predict the Bragg band gap. Note that with the selected light resonator, this band gap is narrow.

In Fig. 4.9, the effective density and the effective bending stiffness are compared with those given by the lumped model. A good agreement is obtained, showing the interest of the TMM homogenization scheme. The effective density present strong fluctuation $\operatorname{Re} \rho_e / \rho > 2$, whereas the effective bending stiffness remains almost constant, ie $0.9 < \operatorname{Re} D_e / D < 1.1$, even if some artifacts are present. These artifacts are due to the *discrete* periodicity since lumped model assume continuous distribution of resonators and tend to zero when $d \to 0$.

Because these effects are related to resonances, they are very sensitive to damping. Results are presented in Fig. 4.10 for several loss factor values. It can observe that this aspect will remain a key parameter for practical achievements proposed in the next chapter. Effective parameters for three different resonator damping are compared. When the damping is near



Figure 4.10: Effect of resonant damping on the dynamic properties of the beam: (a) Density; (b) Effective bending stiffness.



Figure 4.11: Different kind of resonator arrangements to build meta-beam.

zero, the density of the plate increases to unbounded positive values at resonance frequencies. Then, it suddenly drops to negative infinity and remains negative within the band gap. When the damping increases from 0.001 to 0.01 and 0.05, the denominator in Eq. (4.13) does not vanish and the effective density is less affected by the resonator. The effective bending stiffness remains almost constant, especially when damping increases. For very small damping, μ_e spreads from k_B close to singularity, explaining the stronger fluctuation of D_e .

Other kind of resonators

For other kind of resonators, as described in Fig. 4.11, similar approaches can be used. As long as the long wavelength assumption is valid, lumped parameter and TMM can be used to find the dispersion relation. It is noteworthy that the second configuration [135], affecting the inertia cannot be homogenized in the framework of Euler-Bernoulli beam since the wavenumbers satisfy a bi-quadratic equation and do not degenerate anymore. Again, Rayleigh beam will be required. However, the Bloch wave can be easily computed from the TMM, using the

following matrix to take the resonator into account:

$$\left[\mathbf{E}_{\theta}\right] = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & \frac{-I_{1}k_{\theta}\omega^{2}}{I_{1}\omega^{2} - k_{\theta}} & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix},$$
(4.41)

with the inertia $I_1 = m_1 \ell^2$. The last configuration should produce a negative dynamic bending stiffness, and work is on going on this topic. As for the negative density, this configuration can be recast into Euler-Bernoulli model.

4.3 Sound insulation performance

The aim of this section is to couple the *meta*-beam (or plate) to ambient fluid and to evaluate the sound insulation of the double panel. The proposed approach is to use TMM to create a complete vibroacoustic model as depicted in Fig. 4.12. In particular, the number and the tuning of resonators will be investigated.



Figure 4.12: Layered structure.

4.3.1 Transmission loss by TMM

Assuming that layers are laterally infinite and each media is isotropic and homogeneous, we can use the transfer matrix method to model the fluid and the panel. For an acoustic fluid layer, the vector has two parameters: pressure and normal velocity, perpendicular to the layer surface. When the system is composed of multiple layers, a global matrix can be obtained by multiplying the matrix of each layer and an interface matrix between two neighboring layers based on the continuity of sound pressure and velocity. In addition, the projection of

the wavenumber for each wave in each layer should be equal to the x component $k_0 \sin \theta$ of the incident wave in free field. When the global matrix of the system is known, the acoustic properties of the system like absorption coefficient and transmission loss can be obtained.

The acoustic fluid layer of thickness d_2 , as depicted in Fig. 4.12, is described by the state vectors $\{p, v_z\}_{p3}$ and $\{p, v_z\}_{p4}$ and the transfer matrix **T** such as

$$\begin{cases} p \\ v_z \end{cases}_{p3} = \begin{bmatrix} \cos k_3 d_2 & i \frac{Z_c}{\cos \theta} \sin k_3 d_2 \\ i \frac{\cos \theta}{Z_c} \sin k_3 d_2 & \cos k_3 d_2 \end{bmatrix} \begin{cases} p \\ v_3 \end{cases}_{p4},$$
(4.42)

where Z_c refers to the characteristic impedance of the acoustic fluid layer between two plates and $k_3 = (k^2 - k_0^2 \sin \theta^2)^{1/2}$ is the z component of wavenumber in it.

For a thin elastic layers as shown in Fig. 4.12, its transfer matrix can be greatly simplified. The normal stresses and the normal velocity of the thin plate is chosen to express the mechanic field in a point p1, p2, p5 and p6 of the plate. The vector in point p1 can be expressed as

$$\begin{cases} \sigma_{zz} \\ v_z \end{cases}_{p1} = \begin{bmatrix} 1 & -Z_s(\omega) \\ 0 & 1 \end{bmatrix} \begin{cases} \sigma_{zz} \\ v_z \end{cases}_{p2},$$
(4.43)

where Z_s is the mechanical impedance of the panel, and can be obtained from

$$Z_s(\omega) = -i\omega m \left(1 - \frac{Dk_t^4}{\omega^2 m}\right),\tag{4.44}$$

where D_e and $\rho_e h$ are the bending stiffness and the surface density of plates. $k_t = k_0 \sin \theta$ is the trace wavenumber in plate. In the following, the two plates are treated as thin Kirchhoff plate to simplify the model and computation.



Figure 4.13: TL comparison of TMM and lumped model, for double panel structure with embedded resonator when loss factor $\eta = 0.05$.

On the coupling surfaces between the thin plate and air layers located at points p2 and p3, the continuity of normal velocity and stress can be written by

$$v_z(p2) = v_z(p3),$$
 (4.45)
 $\sigma_{zz}(p2) = -p(p3).$

It can be rewritten in matrix form

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{cases} \sigma_{zz} \\ v_z \end{cases}_{p2} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{cases} p \\ v_z \end{cases}_{p3}.$$
 (4.46)

Now, the global matrix is obtained by assembling all of the interfaces and layers matrices. Combining the global matrix and boundary conditions allows to derive the acoustic performance of the multilayer system [4, Section 11.5]. The TL performance of the double panel structure is estimated with equivalent parameters derived from TMM and lumped model. Fig. 4.13 shows a good agreement of the two approaches.

4.3.2 Single kind of resonator

We now present some TL results when all resonators are tuned on the same frequency. The corresponding parameters of the panel and resonators are presented in Table 4.1 and 4.2.

Fig. 4.14 clearly shows that the transmission loss of the double panel with resonators gives an improvement of more than 30 dB at decoupling frequency. This corresponds to the



Figure 4.14: Transmission loss of infinite double panel with or without resonators ($\eta = 0.05$)

hybridization band gap observed on the dispersion curve in Fig. 4.10. However, two dips are now present apart this peak, leading to locally worse sound insulation than bare panel. Nervertheless, the overall minimum TL jumps from 0 to 5 dB. At higher and lower frequencies, the transmission loss is identical.

A parametric study in term of resonators loss factor, mass ratio of resonators to host panel, and resonator distribution density is now proposed. Fig. 4.15 compares TL for several loss factors. It can be seen that the TL increases at the mass-spring-mass resonance frequency when the loss factor decreases. However, with more losses, the TL dips are less stringent.

Fig. 4.16(a) shows the influence of the mass ratio between resonator over host plate. For the comparison convenience, the resonance frequency of the resonator is kept constant by modifying the stiffness k_1 accordingly. As expected, the TL increases with the mass ratio. In addition, the improvement bandwidth also increases as mass ratio increases.

4.3.3 Multiple resonators

Now, two resonators with different natural frequencies $f_1 = 302$ Hz and $f_2 = 332$ Hz (see Table. 4.2) are embedded on the double panel, with the same added mass $m_1 = m_2 = m_0/2$. The global matrix for the beam can be obtained from

$$\mathbf{R}_g = \mathbf{R}_1 \mathbf{E}_1 \mathbf{E}_2 \mathbf{R}_2 \tag{4.47}$$

where \mathbf{E}_1 and \mathbf{E}_2 are the matrix of each resonator. Effective parameters can then be deduced, using the TMM homogenization scheme. The effective wavenumber is shown in Fig. 4.17



Figure 4.15: Influence of the resonator loss factor on the transmission loss of the double panel.



Figure 4.16: Influence of the mass ratio of resonators over the host panel on the transmission loss of the double panel.



Figure 4.17: Dipersion relation of host panel with two types of resonators.

where a larger bandgap is observed compared to the one from the former configuration.

Fig. 4.18 shows the TL when two different resonators are embedded. Two peaks are observed at frequencies f_1 and f_2 . The loss factor has again a major effect as seen in Fig. 4.19: the minimal TL value in the vicinity of the resonant region increases from 3 dB to 10 dB when η increases from 0.01 to 0.1.

Fig. 4.20 presents the TL of the double panel with resonators for the different mass ratio. It is seen that the mass ratio rather drives the peak value than the minimal TL value.

4.4 Conclusion

The effect of resonators on the sound transmission loss of the infinite double panel is studied analytically. A predictive model based on TMM method is proposed to estimate Bloch waves and the effective parameters of a host panel with embedded resonators. This model is applicable when only one direction of propagation is considered. For other situations, plane wave extension or finite element method can be used to obtained the dispersion equation [155].

It is shown that this approach works well as long as the evanescent and propagative waves wavenumber have the same modulus. TMM model is validated since the dispersion relation of bare panel or panel with resonators obtained from TMM model are precisely consistent with that from the lumped model. The negative density is located in a narrow band after the resonant frequency and the corresponding effective bulk modulus is almost constant although fluctuation arise close to the singularity of the effective density. An interesting way of improvement



Figure 4.18: Transmission loss of the double panel with two different resonators. Loss factor of the resonators is 0.05.



Figure 4.19: Effect of the resonator loss factor on the transmission loss of the double panel.



Figure 4.20: Effect of the resonator mass ratio on the transmission loss of the double panel.

is to work with Rayleigh beam model to allow two distinct wavenumbers. The sound transmission loss of the double panel structure with periodically arranged resonators is predicted by making use of effective parameters from TMM model. The results indicate that the resonators can significantly enhance the sound insulation of a double panel around its mass-spring-mass resonance frequencies. However, around this frequency, two side dips appear. The enhancement performance is very sensitive to the mass ratio and resonators damping. A broadband enhancement can be achieved by making use of resonators tuned at different frequencies.

In the next chapter practical realization of such concept will be proposed based on poroelastic beams.

Chapter 5

Sound insulation enhancement of partitions through poroelastic resonators

This chapter corresponds to the paper entitled **Poroelastic resonant metamaterial to face** sound insulation issues of the single and double panel partitions and submitted to Journal of Sound and Vibration. Single or double panel partitions suffer from poor sound insulation in the vicinity of their first bending or mass-spring-mass resonances respectively. Based on the concept presented in chapter 4, the use of embedded poroelastic resonators made out of thermocompressed foam is now studied. First, experimental results obtained for a finite size panel in a duct are presented. Dissipation mechanisms are analyzed using a numerical model. Then, the performance of the infinite double panel partitions with embedded poroelastic resonators is studied on the basis of a periodic numerical model. Finally, a parametric study focusing on mass ratio and incident angle is carried out to show optimization rules.

5.1 Introduction

To save energy and resources during manufacturing and life cycle, compact and lightweight structures are increasingly sought after, especially in the construction and transport fields.

However, acoustic insulation of infinite single panel is mainly driven by the *mass law*, which states the heavier the better. When dealing with finite size panels [44], the first resonance of the structure induces a severe dip in the sound insulation. Similar behavior is also observed on curved panels, close to the ring frequency [43, 42]. Below this frequency, the transmission loss (TL) is controlled by the panel stiffness and decreases by 6 dB per octave. Close to the resonances, the TL is minimal and controlled by the damping. Above the first resonances, infinite panel behavior is recovered: the TL is mass controlled up to the coincidence frequency [44]. The coincidence phenomenon appears when the trace of the incident acoustic wavenumber is equal to the free bending wavenumber leading to poor sound insulation.

Double wall partitions are set up in practical applications to overcome the mass law: two panels are separated by an air gap, usually fed with a sound absorbing material. This structure exhibits a mass-spring-mass resonance, beyond which the two panels are decoupled and outperform single partition with the same mass. However, close to this resonant frequency, the partition is nearly transparent. Note that the inner porous layer reduces the concequences of air-gap acoustics resonances and allows to keep the mass-spring-mass resonance quasiindependant of the incident angle, allowing an efficient decoupling even in diffuse field.

Another way to enhance the TL is to add resonators to change the effective mass [142, 43] of the panel or the effective stiffness of the core [104]. Such approach has been recently revisited and extended through the metamaterial paradigm allowing better integrated solutions. Metamaterial and metasurfaces are designed with subwavelength resonators often arranged in periodic structures to control wave propagation and stop bands. Early works for sound insulation were based on bulk materials [134, 84] or Lamb waves, and rapidly spread to flexural wave manipulation in beams and plates [136, 163, 124, 154]. In the meantime, metamaterials have been widely used in acoustics and especially to improve sound absorption [58, 161] (see the references therein).

Since sound attenuation by panels can present dips at localized frequencies, metamaterials are good candidates to face these limitations. Many works have proposed locally resonant metamaterials to enhance sound insulation index of infinite panel close to coincidence [99], to create stop bands [138], or tackle finite panel resonances [96] or curved panels ring frequency [42]. For double panels, the mass-spring-mass resonance have been tackle by i) changing the core *effective dynamical stiffness* with Helmholtz resonators [104, 143, 82] ii) changing the *effective dynamical mass*, without [36] and with foam core [75, 143, 37, 96, 99]. The metamaterial efficiency is mainly driven by the mass ratio between the resonators and the bare plate, and by the tuning staggering [155]. Generally, an added mass less than 10% can greatly enhanced the TL. When subwavelength resonators are used, the effective parameters can be used in standard formulas [99, 36, 37] to simplify the design procedure.

Combining such approach with porous or poroleastic materials [75, 96, 37] is of great interest since viscothermal losses improve the sound absorption and the impedance mismatch [96, 86] limiting the TL dips especially in diffuse field. Practical realization of passive resonators are made with an array of cylindrical dots [124, 99], stepped resonators composed of a soft material with a heavy cap [138], a cantilever beam [155, 153], eventually 3D-printed systems [42, 36, 37], dual-beam resonators [101] or using the added mass and the elasticity of the foam core [75]. It is noteworthy that for double panel, the resonator can be added inside the cavity [75, 99] for compactness and robustness.

The aim of this paper is to propose a new kind of poroelastic resonator to enhance the TL close to finite size panel resonances or mass-spring-mass resonance of the infinite double panel. The resonators are made with poroelastic beams lying on spacers, providing a simple and easy to manufacture solution. Especially for double panel, where a porous layer is generally present, the solution just consists as reshaping this porous layer: the proposed approach do not require any additional system. The native diphasic nature of the poroelastic lamella provides a natural way to combine the properties of the sound absorbing material and vibration control. Similar arrangements of poroelastic lamellas have proven their potential for sound absorption [31, 34] and attenuation in silencers [91].

The paper is organized as follows. First, the sound transmission loss of a finite size single panel covered by a poroelastic lamellar metamaterial is studied experimentally in a rectangular duct under normal incidence. Dissipation mechanisms are further analyzed by finite element method (FEM). Then, an infinite double partition is investigated by the mean of a periodic finite element model. Finally, a parametric study dealing with incident angle and mass ratio is addressed to conclude on the poroelastic lamellar metamaterial performance.

5.2 Experimental investigation

The finite size single panel is firstly investigated experimentally to illustrate how poroelastic resonators can control the sound transmission dip. The TL is estimated in a duct allowing accurate measurements at low frequency.

5.2.1 Test bench for transmission loss measurement

The test bench is a rectangular duct of section 100 mm \times 200 mm as shown in Fig. 5.1. It has been designed for acoustic multimodal characterization under a low Mach number flow within the frequency band [200 Hz- 3.5 kHz] accounting for 10 propagative modes[147]. The



Figure 5.1: Experimental set-up. (a) Description of the multimodal test bench; (b) Plate without poroelastic beams; (3) Plate located on the measurement section before closing the duct.

first cut-off frequency of the duct is 857 Hz. The multimodal scattering matrix, which contains the modal reflection and transmission coefficients, is measured using a multi-source method described in reference [147] without flow. Only the plane wave contribution is considered in our experiment.

The tested panel is clamped at the interface between a rigid wooden duct and the microphone section as shown in Fig. 5.1(c). Broadband noise is chosen as excitation signal and two measurements are repeated using three source positions (1, 3, 7) for each configuration to increase the measurement quality.

5.2.2 Description of the single panel

The dimensions of the panel are chosen to fit in the test bench cross-section. Since the first cut-off frequency of the bench is 857 Hz, we designed the single panel so that its first mode is far below. A rather light structure has been chosen to keep a mass ratio poroelastic material over panel around 10%. Hence, the host structure is made out of 2 juxtaposed plates of foam core cardboard having a thickness of 3 mm each and 90 mm \times 190 mm size. This structure is bonded on a soft and thin cardboard sheet of size 126 mm \times 226 mm to be clamped between two duct sections. In this manner, the distance between all edges of foam core cardboard plate and duct wall is 5 mm: this allows having a reproducible boundary condition around the

Porosity	Airflow resistivity	Viscous length	Thermal length	Tortuosity	Density	Young's modulus	Loss factor	Poisson ratio
φ	σ	Λ	Λ'	α_{∞}	ρ_1	E	η	ν
	$(Nm^{-4}s)$	(µm)	(µm)		$(kg.m^{-3})$	(kPa)		
0.967	41 222	58	65	1.05	24.2	121.4	0.102	0.21

Table 5.1: Properties of the thermocompressed poroelastic material measured in Roberval laboratory at UTC as described in [87].

sample. The mass of the foam core cardboard per unit area is 0.858 kg.m^{-2} . The bare panel first resonance is around 300 Hz as shown in Fig. 5.4.

5.2.3 Poroelastic metamaterial

The metamaterial is made of an array of poroelastic beams lying on spacers. This configuration will ensure a good coupling with the transverse motion of the panel. In order to control the first mode of the panel, the resonators have to be tuned at the same frequency.

Assuming an elastic behavior, an estimation of the first resonance frequencies of the clamped beam, are given by

$$f_i = \frac{\lambda_i^2}{2\pi\ell^2} \sqrt{\frac{EI}{\rho_1 S}},\tag{5.1}$$

where $\lambda_1 \approx 4.73$, $\lambda_2 \approx 7.85$ and $\lambda_i \approx (2i+1)\frac{\pi}{2}$ for i > 3 [15]. *E* is the Young's modulus, ρ_1 is the mass density, $I = bh^3/12$ is the second moment of area, ℓ is the beam length and S = bh is the cross section area. It has been observed, that for sparse array of resonators, rather resistive and dense foam were more suitable. Hence, thermocompressed melamine foam, with a compression rate of 2.9 [88, 87] has been chosen. The mechanical and acoustical properties of this poroelastic material are listed in Tab. 5.1.

Once the material is set, the tuning can be done by changing ℓ and the added mass can be independently set by changing the width b. Each poroelastic beam has the same length as host structure, i.e., 190 mm, and is bonded on several rigid spacers made out of 3 mm thick foam core cardboard as shown in Fig. 5.3(left side). The clamped-clamped beams resonator network is depicted in Fig. 5.3(right side). The thickness of the beam section is h = 10.4 mm while its width b, can be 10 mm or 15 mm.

Firstly, we estimate the Young's modulus of the poroelastic material from the frequency response of a single beam impacted at its center, using a Scanning Polytec PSV-500 Laser Doppler Vibrometer (Fig. 5.2)[34]. The estimated Young's modulus for the 100 mm beam is 121.4 kPa according to Eq.(5.1).

The two configurations depicted in Fig. 5.3(a,c), named P1 and P2, are designed to investigate the effect of beam length on TL. Five or six spacers are placed on the plate with the same periodicity, so that we have 20 beams B1 and 25 beams B2 for P1 and P2 configurations respectively. Note that, for P1 configuration, the width of central spacer is 10 mm and not 5



Figure 5.2: (a) Poroelastic resonators used for Young's modulus estimation; (b) Frequency response of the 100 mm length beam.

mm for symmetrical arrangement. All the beams are bonded on the spacers using double-face tape.

P3 and P4 configurations shown in Fig. 5.3(e,g) are designed to achieve a broadband effect by mixing resonators of various lengths. In P3 case, combination of eight beams B1 and ten beams B2 are used to produce multi-modal locally resonator as provided in Fig. 5.3(f,h). Shorter spacers are used to realize this configuration and the width of porous beam is 15 mm. For P4 configuration, five kinds of the poroelastic resonators B1-5 are distributed symmetrically on the plate surface. Two long spacers are located obliquely on the host plate to linearly change the resonator length. The distance between two poroelastic beams is 10 mm in all tested configurations.

	B1	B2	B3	B4	B5	
Length of beam ℓ (mm)	41.5	32	36.5	45.5	50	mass ratio
P1 [Fig. 5.3(b)]	5×4					7.8 %
P2 [Fig. 5.3(d)]		5×5				7.8 %
P3 [Fig. 5.3(f)]	2×4	2×5				9.4%
P4 [Fig. 5.3(h)]	1×4	7.8 %				

The poroelastic network covers approximately 55% of the panel for P1, P2, P4 configurations and 67% for P3 arrangement.

Table 5.2: Resonators dimensions and number per configuration. The mass ratio represents the resonator mass over the mass sum of the host structure and cardboard sheet.



Figure 5.3: Spacers (left) and resonators (right) arrangement on the panel for several configurations: P1(a-b), P2 (c-d), P3 (e-f), P4 (g-h).



Figure 5.4: Measured TL of finite size panel with and without poroelastic metamaterial at normal incidence for the different configurations. The black solid line on (b) shows the theoretical slope for mass law (+6 dB per octave).

5.2.4 Experimental results

The TL measurements are presented in Fig. 5.4. The configurations with resonators (red solid line) are compared to the bare plate with only the spacers (blue dashed line). It is noteworthy that the artifact at 857 Hz is due to the cut-off frequency of the duct.

In all curves, the TL first decreases (stiffness controlled zone) until the first resonance of the plate and then increases roughly by 6 dB/octave, which is in line with the mass law (see the slope drawn in Fig. 5.4(b)). The level at 100 Hz is related to the stiffness of the plate, the spacers, and the mounting conditions: it varies from 15 dB to 17 dB among the configurations. The lower value is obtained for the P1 configuration, and as a consequence, the frequency of the dip is also the lowest, i.e., 250 Hz. In addition, the dip for the configuration P2, having the highest level at 100 Hz, is reached at 295 Hz. For each configuration P_i , adding the resonators does not affect the mounting conditions, since the level at 100 Hz varies less than 1 dB.

However, the addition of the poroelastic resonators involves a clear increase of the TL in the region of the dip between 250 Hz and 350 Hz. As for a tuned mass damper [106, 103], when tuned on the panel first mode, the resonators almost cancel the motion of host structure at this frequency. However, two new coupled modes appear around the tuning frequency leading to new less stringent dips because of damping.

Reducing the length the beam from 41.5 mm (P1) to 32 mm (P2) shifts the peak from 270 Hz to 300 Hz. The value of the improvement is between 9 dB (P1) and 12 dB (P2). By covering the second face of the bare plate, an extra 3 dB is obtained. The effect of the mass ratio between the host structure and the resonator is well known [90] and covering both sides allow to double it. The higher amplitude of the peak for the configuration P2 when compared to P1, may be explained by a lower dispersion on the resonators manufacturing.

Fig. 5.4(c,d) shows that the use of different resonators (P3 and P4) increase the bandwidth of the metamaterial. Again doubling the treatment increases the effect by 2 to 3 dB.

Finally, we note that the metamaterial also affects, in a lesser extent, the region of the second mode of the bare plate being around 500 Hz.

Strong effect have been obtained. This validate the efficiency of such poroelastic resonators, although the device is not fully optimized. To do so, a numerical model is needed.

5.3 Numerical analysis of the finite size single panel

To improve the physical understanding of the tested configurations, we model the TL of the finite size panel lined by the poroelastic resonators. The geometry is that of the P1 configuration, which is tested experimentally.



Figure 5.5: (a) Schematic view of the host plate, surrounding layer and spacers. (b) Structure with poroelastic resonators on spacers. (c) Back side of the host structure. (d) Acoustic boundary conditions in the rectangular duct.

As shown in Fig.5.5 (a) and (c), the host panel is composed of two components. The host structure has dimensions of 190 mm \times 90 mm \times 6 mm. We consider homogeneous properties equivalent to the two layers foam core cardboard used for the experiment (see Tab. 5.3). The host structure is clamped through a surrounding 3 mm thick layer whose properties are adjusted according to the first experimental resonance frequency.

5.3.1 Finite element model

The global vibroacoustic problem is solved in the frequency domain (convention $e^{-i\omega t}$) using the finite element method.

In the inviscid air domain, the acoustic pressure p is governed by Helmholtz equation. On the duct walls, the acoustic normal velocity vanishes. The radiation conditions at both ends of the duct and the incident pressure field have been implemented using the Dirichlet-to-Neumann (DtN) map [61] using the expansion of the pressure in terms of duct acoustic modes [91]. The transmitted and reflected sound power can be derived from modal amplitude and the TL can be obtained.

In the poroleastic domain Ω_p , the classical mixed (\mathbf{u}, p_p) formulation [4, Chap. 13] is used with the Johnson-Champoux-Allard model. All foam parameters are given in Tab. 5.1. At the interface between the fluid and the porous material, the coupling conditions impose the continuity of normal displacement, the pressure, and normal stress (see for instance [4, Chap. 13]).

Name	Density	Young's modulus	Loss factor	Poisson ratio
	$ ho_1$	E	η	ν
	$kg.m^{-3}$	MPa	-	-
Foam core cardboard	143	180	0.075	0.33
Surrounding layer	800	2.0	0.075	0.20

Table 5.3: Homogeneous properties of the foam core cardboard and surrounding layer.

In the plate, linear 3D elasto-dynamic model is employed and viscoelastic losses are taken into account though a complex Young's modulus. Since the plate is clamped on the internal surface of the duct, all displacement components vanish on that boundary. The continuity of the displacement and normal stress is imposed on the interfaces between the elastic and air domains and the poroelastic domain.

Note that the pressure and solid displacements are discretized using quadratic tetrahedral Lagrange finite elements. In all cases, the mesh size was chosen to ensure a good trade-off between accuracy and computational time.

5.3.2 Numerical results

Fig. 5.6(a) presents the TL under normal incidence of the structure with and without resonators. The bare plate has its first resonance frequency around 254 Hz and the second one around 400 Hz. Adding resonators shows a strong improvement of 8 dB around the first dip, due to the excitation of the first bending resonance of poroelastic beams. As observed in Fig. 5.6(c, d), the displacement of the beams is higher at the center of the plate, following the first mode shape of the bare plate (see mode shape in Fig. 5.6(b). The displacement of the resonators is around 10 times higher than that of the plate.

5.3.3 Power balance

The balance between transmitted, reflected, and dissipated powers versus frequency are shown for the bare structure in Fig. 5.7(a) and with resonators in Fig. 5.7(b). The reflected and transmitted powers are calculated through modal decomposition at the duct ends [11], while the total dissipated power is deduced from energy conservation. At low frequency, the incident sound is mainly reflected. The reflection decreases until the region of the first mode (1,1) of the bare plate. At this frequency, the dynamic stiffness of the plate tends to 0 and the amplitude of the plate displacement strongly increases. Most of the incident power is transmitted but limited by viscoelastic losses. At higher frequency, similar trends can be observed, but the second TL dip is less pronounced because of weaker coupling with the incident field. At higher frequency, the reflected power increases again due to the inertia effect.

When the metamaterial is added, the resonators mitigate the plate displacement at 254 Hz, which reflects back most of the incident power. The global losses are also increased, and



Figure 5.6: (a) Predicted TL of the plate with and without poroelastic resonators. (b) Displacement amplitude at 254 Hz of the bare plate (c) and of the plate with resonators. (d) Deformed mesh of resonators and plate at 254 Hz.

their distribution is given in Fig. 5.8(a). It is seen that the dissipation within the resonators is dominating only at the beams resonances.

The distribution of the dissipated power within the poroelastic resonators, in terms of viscous, thermal, and structural, i.e., viscoelastic, dissipations [35, 41] is now analyzed. Fig. 5.8(b) presents the dissipated power in the logarithmic scale of each components over the [20 - 1000]Hz frequency band. Here, structural dissipations dominates close to the beams resonances. Elsewhere, thermal dissipation dominates. Viscous dissipation follows, in a lesser extend, the structural dissipation.

5.4 Numerical analysis of the infinite double panel

This section addresses, by mean of a numerical model, the infinite double panel known for exhibiting a sound transmission loss dip at the mass-spring-mass resonance, also called decoupling frequency. This frequency can be estimated by [44]

$$f_r = \frac{1}{2\pi} \sqrt{\frac{K}{h \cos^2 \theta} \frac{m_1 + m_2}{m_1 m_2}},$$
(5.2)

where K is the bulk modulus of the fluid layer of thickness h, θ is the incidence angle ($\theta = 0$ correspond to normal incidence), m_1 and m_2 are surface densities of each panel. In the adiabatic regime, $K = \gamma P_0$ where P_0 denotes the atmospheric pressure and γ is the ratio of specific heats. Insulation benefit will arise above this frequency, that should be as low as


Figure 5.7: Fraction distribution of three different sound powers: reflected (red dotted line), transmitted (green solid line) and dissipated (black dotted line); (a): bare structure; (b): structure with resonators.



Figure 5.8: (a) Distribution of dissipated power in poroelastic and viscoelastic domains ; (b) Distribution of viscous (black dotted line), thermal (blue dash-dotted line) and structural (red dotted line) dissipation in the poroelastic domain.

possible. However, f_r increases with the incident angle, except if the air gap is filled with a porous material (see Fig. 5.9(b)). Indeed, the sound velocity in the porous material c_e , represented by an equivalent fluid [4], is smaller than the sound velocity c_0 in the air. So, the incident angle θ_e in the porous layer can be estimated according to the Snell-Descartes law, $\sin \theta_e = c_e/c_0 \sin \theta$. For instance, for f = 470 Hz the ratio of sound speed in the porous material over that in the air is around 0.25, which indicates the θ_e is 12.5° for $\theta = 60^\circ$. Filling the double panel with a porous material will have several advantages:

- in diffuse filed, the incident angle within the porous layer will stay close to the normal incidence, allowing an efficient decoupling,
- the porous layer will impose an isothermal regime instead of an adiabatic regime in the low frequency, reducing the bulk modulus from γP₀ to P₀: the decoupling frequency will be lowered by 20% (see Fig. 5.9(b)). This will occur if no solid transmission occurs between the two panels through a the skeleton: keeping a small air gap between the porous layer and one panel is thus recommended.
- this will also mitigate acoustic resonances between the two panels.

The poroelastic resonators aims at combining these features while reducing the TL dip at the decoupling frequency. A numerical approach, taking advantage of the periodicity of the system, is now developed.

5.4.1 Periodic finite element model

Wave propagation in an infinite periodic media excited by a plane wave can be handled making use of Floquet-Bloch theorem to diminish the computation cost. We use the same approach as Chapter 3. Only a unit cell with pseudo-periodic boundary conditions on all lateral sides is

considered as shown in Fig. 5.11(a). Geometrical periodicity vector l is $(l_x, l_y, 0)$. In the unit cell, the fluid, poroelastic, and the elastic domains are discretized using quadratic Lagrange tetrahedral finite element.

Incident plane wave P_I is impinging on the top surface of the air domain with wavenumber

$$\mathbf{k} = -k_a \left(\sin\theta \cdot \cos\varphi, \sin\theta \cdot \sin\varphi, \cos\theta\right),\tag{5.3}$$

where θ is the inclination and φ is the azimuth. The homogeneous plates are independent of φ , but because of the resonators orientation, the lined panel present slight anisotropy at low frequency.

The radiation condition of the scattered field in the top and bottom air domains is implemented with a 'Dirichlet to Neumann' map based on Floquet modes decomposition[34]. The amplitude and power of the transmitted wave P_T can be recovered by projection on the Floquet modes. The transmission coefficient at a specific angle is $\tau(\theta) = P_T(\theta)/P_I(\theta)$. Under diffuse field excitation, the transmission coefficient τ_d is obtained by integrating τ over the incident angles as following

$$\tau_d = \frac{1}{\pi} \frac{\int_0^{\pi} \int_0^{\theta_{lim}} \tau(\theta, \varphi) \sin \theta \cos \theta \, \mathrm{d}\theta \, \mathrm{d}\varphi}{\int_0^{\theta_{lim}} \sin \theta \cos \theta \, \mathrm{d}\theta}.$$
(5.4)

In the present study, $\theta_{lim} = 78^{\circ}$ for diffuse field transmission [4].

5.4.2 Comparison with TMM code

Firstly, the periodic FEM model is validated with the transfer matrix method (TMM) using Maine3A code [4, Chap. 11]. A double panel fully or partially filled up by a homogeneous poroelastic layer is considered as shown in Fig. 5.9. The elastic plates have a thickness of 10 mm each and are 16 mm apart. The porous layer is 10 mm or 16 mm thick in the two cases respectively. The properties of both media are given in Tab. 5.1 and 5.3.

Fig. 5.9(a) shows the TL curves under normal incidence. It is seen that the results from FEM model perfectly match TMM results for both conditions. Fig. 5.9(b) illustrate the effect of the incidence angle and cavity filling. Without porous layer, it is seen that the decoupling frequency increase with the incident angle. With the porous layer, the frequency at normal incidence is lowered, and the incidence angle shifts the decoupling frequency in a smaller extent.

5.4.3 Infinite double panels with poroelastic resonators

The first configuration, depicted in Fig. 5.10, is composed of the two plates of 6 mm foam core cardboard and 70 mm apart. The mass-spring-mass resonance is set around 260 Hz and the panel surface density is 1.43 kg.m^{-2} . The poroelastic resonator B1 (see Table. 5.2) is applied



Figure 5.9: (a) Comparison of FEM computation with TMM model for two configurations; (b) Impact of the incidence angle on the transmission loss dip frequency with and without porous material filling.



Figure 5.10: The effect of poro-elastic resonator B1 on sound transmission of double panel system predicted by its primitive FEM model.

on both plates inside the cavity as shown in Fig. 5.10. It is seen that the TL at normal incidence increases up to 15 dB in the region of the dip.

In the second configuration, the distance between the two panels is reduced from 70 mm to 16 mm to increase the ratio of the porous material volume to the air volume up to 42%. Only one panel is covered by the poroelastic resonator as shown in Fig. 5.11(a)) and the porous beams are separated from each others by a 5 mm air gap. To keep the mass-spring-mass frequency be small, the plate surface density is increased to 4 kg.m⁻².

Fig. 5.11(b) shows the TL at normal incidence for different mass ratio between the porous and whole structure. Ratio from 2% to 8% are considered. To keep the same beam resonance frequency, the density and Young's modulus of the poroelastic material are increased together. It is shown that the amplitude of TL enhancement increases from 12 dB to 25 dB in the region of the mass-spring-mass dip around f = 300 Hz, which is in line with previous works [99]. In comparison with the homogeneous layer having the same volume as that of poroelastic resonators (equivalent thickness of 6.6 mm) and seperated by the same distance from the plate (3mm), the 2% configuration increases the minimal TL from 3 dB to 8 dB. Note that increasing the mass ratio will change the frequency of the two dips: the resonators should be tuned accordingly for optimization.

Fig. 5.11(c) shows the effect of the incidence angle for the 8% mass ratio configuration. The frequency of the peak related to the poroelastic resonators is independent of the incidence angle, unlike the dip location that is shifted from 270 Hz at normal incidence to 400 Hz at $\theta = 80^{\circ}$. Note that the maximal TL in the dip region is obtained for an angle $\theta = 30^{\circ}$.



Figure 5.11: Double panel with poroelastic resonators: (a) perodic cell geometry; (b) effect of the resonator versus panel mass ratio on the normal incidence TL, (c) effect of the incident angle θ on the TL; (d) comparison of the TL in diffuse field for the double panel filled by air (blue dotted line), by a poroelastic homogeneous layer (black line), or loaded by poroelastic resonators (red line).

Finally, Fig. 5.11(d) compares the TL in diffuse field for several configurations. Without any porous material, the TL slope raising after the normal incidence decoupling frequency is not observed because of the shift toward higher frequencies of this frequency with the incidence angle according to Eq. (5.2). Partially filling the gap with porous material reduces drastically this effect, because of the lower sound celerity in the porous layer, that is around 80 m.s^{-1} at 300 Hz. This effect still holds with the poroelastic metamaterial that combine the local increase due the interaction of its structural resonance with the host panel and the decoupling of the two panels, occuring close to the normal incidence decoupling frequency, whatever the angle incidence is.

5.4.4 Power balance

The dissipation analysis is carried out for the double panel. The fraction distribution of the dissipated, transmistted, and reflected sound powers are presented for the bare double panel in Fig. 5.12(a) and with resonators in Fig. 5.12(b) for an added mass ratio of 8%. The reflected and transmitted powers are obtained by the Bloch modal decomposition at the input and output ends respectively. For the bare double panel, the sound power is almost perfectly reflected by the double panel excepted around f = 300 Hz that corresponds to the TL dip occurring at the decoupling frequency f_r . Due to the resonance, the sound power is almost perfectly transmitted and only the dissipation in the cavity can limit it. When the resonators are added to the double panel, two dips are observed on the reflected fraction which corresponds to the two dips below and above the TL enhancement peak as shown in Fig. 5.11(b). At the first dip frequency, most of the sound power is dissipated through the porous resonators while most of the sound is still reflected at the second dip frequency. The TL peak does not correspond to the dissipation peak but to the strong reflection of the incident wave.

The dissipated power in both domains, i.e., panels and poroelastic resonators, are shown in Fig. 5.13(a) around the decoupling frequency. The highest mitigated power in both domains corresponds to the first TL dip. Most of the dissipation occurs within the poroelastic resonators since the panels have a nearly rigid body motion in this regime.

Fig. 5.13(b) presents the distribution of the dissipated power within the poroelastic resonators. The three dissipation components are maximal around the decoupling frequency and the structural dissipation dominates in this region. Unlike the finite size single panel (see Fig. 5.7), the thermal dissipation is also maximal in this region, since the pressure amplitude in the cavity between the two panels is maximal at the mass-spring-mass resonance.



Figure 5.12: Fraction distribution of sound powers: reflected (red dotted line), transmitted (green solid line) and dissipated (black dotted line). (a) bare structure; (b) structure with resonators.



Figure 5.13: (a) Distribution of dissipated power in poroelastic and viscoelastic domains; (b) Distribution of viscous (black dotted line), thermal (blue dash-dotted line) and structural (red dotted line) dissipation in poroelastic domain.

5.5 Conclusion

In this study, a poroelastic metamaterial is proposed to improve the acoustic insulation performance of either a finite size single panel or an infinite double panel in the region of their first dip. The metamaterial is composed of poroelastic beam resonators whose first structural resonance is tuned according to the first mode of the structure. This frequency can be simply adjusted by the geometry of the poroelastic beam. The concept has been illustrated experimentally using compressed melamine foam giving a suitable balance between the quality factor of the beam resonance and dissipation. A local increase of more than 10 dB has been achieved experimentally and can be spread with multi resonators. A numerical analysis using a finite element model, together with periodic assumption for the infinite double wall, has been performed. It is shown that structural dissipation dominates in the region of metamaterial resonances, followed by viscous dissipation and thermal dissipation only in the double panel partition. Moreover, the porous nature of the resonant metamaterial allows to keep the decoupling effect of the double wall configuration, independently of the incident angle. Finally, it can be considered for transport applications where weight is a major issue.

5.6 Appendix: (\mathbf{u}, p_p) formulation

In the poroelastic domain, the acoustic fields are described by the (\mathbf{u}, p_p) formulation of the Biot model [4, Chap. 13],

$$\nabla \cdot \hat{\boldsymbol{\sigma}}^{s}(\mathbf{u}) + \omega^{2} \rho \, \mathbf{u} + \gamma \, \nabla p_{p} = 0, \qquad (5.5a)$$

$$\Delta p_p + \omega^2 \frac{\rho_{22}}{R} p_p - \omega^2 \frac{\rho_{22}}{\phi^2} \gamma \,\nabla \cdot \mathbf{u} = 0.$$
(5.5b)

Here, p_p is the pore pressure, ϕ is porosity of the porous material, $\gamma = \phi \left(\frac{\rho_{12}}{\rho_{22}} - \frac{Q}{R}\right)$ and $\rho = \rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}$. Coefficient *R* is the effective bulk modulus of the fluid phase and takes into account the thermal dissipation, *Q* couples the two phases by volumic dilatation. The effective densities ρ_{11} and ρ_{22} , respectively for the solid phase and the fluid phase, and the coupling density ρ_{12} are complex-valued, and their imaginary part takes into account viscous losses. The first two terms in Eq.(5.5a) and in (5.5b) describe respectively the dynamics of the elastic skeleton and that of the equivalent fluid. The last term in both equations couples the two phases. The *in vacuo* stress tensor $\hat{\sigma}^s$ is given by

$$\boldsymbol{\sigma}(\mathbf{u}) = \mathbf{I}\left(K_b - \frac{2}{3}N\right)\nabla\cdot\mathbf{u} + 2N\boldsymbol{\varepsilon}^s(\mathbf{u}).$$
(5.6)

Here, K_b is the complex dynamic bulk modulus of the frame, N is the shear modulus and includes the structural damping.

In the plate, the acoustic perturbation of the displacement \mathbf{u} fulfill the three dimensional elasto-dynamic equations,

$$\nabla \cdot \hat{\boldsymbol{\sigma}}^s(\mathbf{u}) + \omega^2 \rho \, \mathbf{u} = 0, \tag{5.7}$$

with the standard Hooke stress-strain relations,

$$\boldsymbol{\sigma}(\mathbf{u}) = \lambda \mathbf{I} \nabla \cdot \mathbf{u} + 2\mu \boldsymbol{\varepsilon}^s(\mathbf{u}), \tag{5.8}$$

where the quantities λ and μ are the Lamé coefficients.

Conclusions and Perspectives

The main goal of this thesis is to investigate and optimize the sound attenuation in the low frequency range, by means of a metamaterial built by shaping an homogeneous poroelastic material. Two designs have been proposed to tackle sound attenuation in duct and in single and double panel partitions. It has been shown numerically and experimentally that theses metamaterials outperform an homogeneous poroelastic layer at low frequency without compromising the high sound absorption properties in medium- and high- frequency ranges. This metamaterial family is characterized by an easy manufacturing, an easy tuning, a light weight as well as a low cost.

After a general introduction in chapter 1 and a presentation of the theoretical background in chapter 2, chapter 3 investigates the sound attenuation in duct lined by a poroelastic lamella network. First, an experimental approach is proposed using a rectangular duct under plane wave regime. Two configurations are proposed and labeled as *parallel* and *perpendicular* regarding the lamellas orientation with respect to the duct axis. A strong attenuation increment is obtained due to the bending resonance of the lamella network in the perpendicular configuration. The attenuation enhancement is mainly due to viscous and structural dissipations boosted by the lamella resonance that can be primarily tuned by approximated formulae. A numerical model taking advantage of the periodicity of the lamella network allows a deep inspection of wave propagation in the system. A parametric study indicates that the bending resonance frequency can be easily tuned by modifying the geometrical properties, as well as by adjusting the physical parameters, like the airflow resistivity, the Young's modulus or the density of poroelastic material. In the wavenumber space, we show that the attenuation enhancement is linked to the avoided or unavoided crossing of the two least attenuated Bloch modes. The optimal attenuation can be related to the presence of exceptional points in the parametric plane.

In chapter 4, the effect of resonators on the sound transmission loss (TL) of the infinite double panel partition is studied analytically. A predictive model based on the transfer matrix method (TMM) is proposed to estimate the effective parameters of the thin panel with embedded resonators and compared to analytical models assuming continuous resonator distribution. This approach allows to easily account for metamaterial in standard vibroacoustic softwares or formulae. It has been shown that when the resonators motion is along the transverse direction

of the plate, it yields to a negative effective density. Other configurations have been proposed to alter other beam parameters. The transmission loss of such double panel indicates that the resonators can significantly enhance the sound insulation performance around the massspring-mass resonance frequency. Increasing the loss factor and integrating different types of resonator will broaden the effect and reduce the dips around each resonance frequency.

In chapter 5, the concept presented in chapter 4 is implemented using poroelastic beams made out of thermocompressed foam. A significant TL increment is observed experimentally around the bending resonance dip of a clamped plate. The enhancement bandwidth can be widden when multi-type of resonators are used. A numerical model allows to analyze the dissipation mechanisms: the structural damping dominates while viscous dissipation dominated in Chap. 3. Then, the sound insulation performance of the infinite double panel with embedded poroelatic beams is studied on the basis of a periodic finite element model. It is shown that the porous nature of the metamaterial allows to keep the decoupling effect of the double panel independently of the incident angle, in addition to the sound insulation enhancement around the decoupling frequency. The dissipation in the poroelastic domain is also significantly improved around the bending resonance of the resonators.

Perspectives

Future works about lamella network could focus on the optimization of the proposed design. It could be done in a straight forward manner by coupling black box optimization algorithms to the developed model. Another interesting approach could be to push a step further the connexion between the bending resonance and the location of exceptional points in parameter space. It has be shown in chapter 3 for duct attenuation but it could also be applied to the double panel systems.

Concerning sound insulation of partitions, in our experiment, the host panel is a light weight foam cardboard which is not commonly used in public transport or building industries. More realistic host panels could be used to design and test the efficiency of poroelastic resonators. Taking into account this weight, but also the manufacturing constraints may lead to new designs, grounded on the same physical principle.

Finally, at long term, working on homogenization or other model reduction techniques will be of great interest. Indeed, during this work, testing many configurations through 3D computation tool-chain, was cumbersome. The transfer matrix approach provides an interesting framework common to acoustic and vibration analysis. The approach sketched in chapter 4 could be extended. The large spread of such architectured materials could be done only if clear design and tuning methods are available.

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