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de doctorat
de l'UTT

Duc Huy NGUYEN

# Study on Biomass Supply Chain Planning and Inventory Control of Perishable Products

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*pour l'obtention du grade de*

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*présentée et soutenue par*

**Duc Huy NGUYEN**

*le 23 janvier 2019*

# Study on Biomass Supply Chain Planning and Inventory Control of Perishable Products

## JURY

| | | |
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Secondly, I would like to express my special thanks to M. Stéphane DAUZERE-PERES and M. Aziz MOUKRIM for accepting to review my Ph.D. thesis. I would also like to thank Mme Evren SAHIN, M. Christian PRINS and M. Zheng WANG for agreeing to examine this thesis. Their valuable remarks and constructive comments have helped us to improve the presentation and the quality of this manuscript.

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**Abstract**

In the last decades, the use of renewable energy sources reduced the effect of global warming. Biomass is a promising energy resource to produce heat, electricity, and biofuel. An efficient supply chain design and optimal inventory management could contribute to reduce biofuel prices significantly and improve competitiveness with fossil fuel.

This thesis addresses two crucial problems: biomass supply chain optimization and inventory control for a perishable product (biomass). The former is devoted to an issue of supplier selection and operation planning in biomass supply chains under uncertainty. This problem is formulated as a deterministic (MILP) model and a two-stage stochastic programming model. The deterministic model is solved by using a MIP solver GUROBI. An enhanced L-shaped decomposition method is developed to find an optimal solution for the stochastic model.

The second deals with a stochastic inventory problem of a perishable product with uncertainty in both supply and demand. After demonstrating its optimal inventory policy is an order-to-level policy, a Lagrangian relaxation based algorithm is developed to quickly find a near-optimal solution of the problem. The stochastic inventory problem is, then, extended to a product with a fixed lifetime. The Conditional scenario method is developed to solve approximately this problem.

**Keywords:** Operations research, Combinatorial Optimization, Business logistics, Biomass, Inventory Control, Stochastic Programming, Uncertainty, Heuristic.

**Résumé**

Au cours des dernières décennies, l'utilisation de sources d'énergie renouvelables a réduit les effets du réchauffement climatique. La biomasse est une ressource énergétique prometteuse pour la production de la chaleur, de l'électricité et des biocarburants. Une conception efficace de la chaîne d'approvisionnement et une gestion optimale des stocks permettent à réduire significativement les prix des biocarburants et à améliorer la compétitivité contre des combustibles fossiles.

Cette thèse aborde deux problèmes cruciaux : l'optimisation d'une chaîne logistique et la gestion des stocks périssables (biomasse). Le premier problème est consacré à la sélection des fournisseurs et la planification des opérations dans une chaîne logistique en biomasse. Ce problème est formulé sous forme de modèle déterministe (MILP) et de modèle stochastique à deux étapes. Le premier modèle est résolu de manière optimale par le solveur GUROBI. Une méthode de décomposition « L-shaped » est développée pour traiter le deuxième modèle.

Le second problème consiste à la gestion des stocks d'un produit périssable sous incertitudes d'approvisionnement et de demande. Après avoir démontré que sa politique de gestion des stocks est une politique "order-up-to level", un algorithme basé sur la relaxation lagrangienne est développé pour trouver rapidement une solution quasi-optimale du problème. Ensuite, ce problème est étendu pour un produit à durée de vie fixe. La méthode « Conditional scenario » est développée pour résoudre approximativement ce problème.

**Mots-clés :** Recherche opérationnelle, Optimisation combinatoire, Logistique, Biomasse, Gestion des stocks, Programmation stochastique, Incertitude, Heuristique.

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# Abbreviations

| | |
|---|---|
| **LP** | Linear Programming |
| **IP** | Integer Programming |
| **MILP** | Mixed Integer Linear Programming |
| **DEP** | Deterministic Equivalent Programming |
| **SP** | Stochastic Programming |
| **DP** | Dynamic Programming |
| **FIFO** | First In First Out |
| **LIFO** | Last In First Out |
| **MLS** | Multi-cut L-Shaped method |
| **ERD** | Enhanced and Regularized decomposition L-shaped method |
| **RD** | Regularized Decomposition method |
| **VSS** | Value of the Stochastic Solution |
| **LR** | Lagrangian Relaxation |
| **CS** | Conditional Scenarios method |
| **SAA** | Sample Average Approximation method |
| **GIS** | Geographic Information System |
| **EOQ** | Economic Order Quantity |
| **EPQ** | Economic Production Quantity |

# Chapter 1

# Introduction

## 1.1 Context

For many centuries, the global economy has relied mainly on fossil energy including coal, oil and natural gas to provide goods and services. Nevertheless, these sources are unsustainable and likely to exhaust in the next several decades. In addition, the adoption of fossil fuel causes greenhouse emission, which is one of the major causes of global warming severely affecting human life and increasing natural disasters (Shafiee and Topal [2009]). Therefore, renewable energy would play a significant role in the energy transition and appear to be a very promising solution to sustainable development.

According to the report Ren21 [2018], renewable energy accounts for 20.5% of global energy consumption in 2016 and allows to create an additional 10.3 million of jobs in this field. As depicted in Figure 1.1, traditional biomass provides 7.8% of global energy consumption whereas modern renewables generate 10.4%. As for the total renewable sources, 0.9% arises from biofuels, 4.1% from heat energy, and 1.7% from wind, solar, geothermal, biomass and ocean power. In 2017, global bioelectricity generation increases at a rate of 11%, to 555 TWh and global biofuels production increases at around 2% to 143 billion liters.

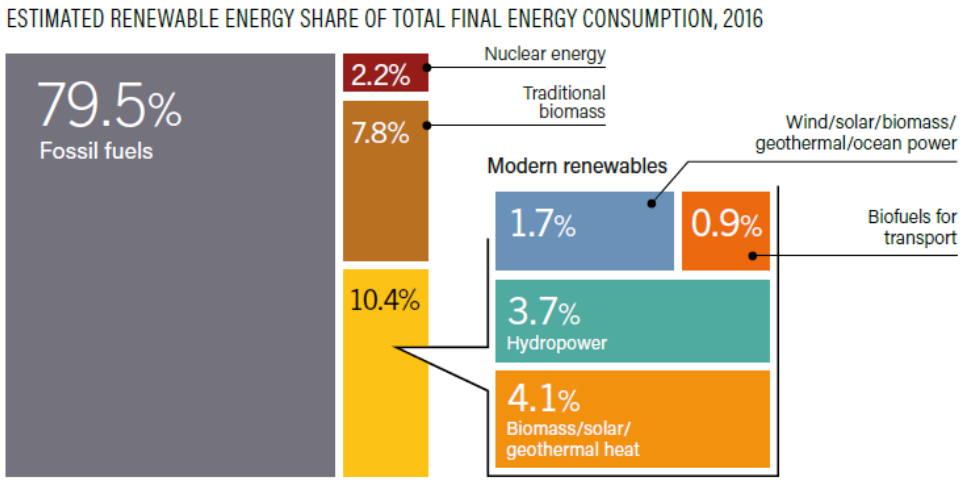


Figure 1.1: Share of renewable energy in global energy consumption, 2016

Bioenergy could be generated from various sources such as wood, agricultural products, animal and plant wastes. In contrast to wind and solar power, biomass can be stored in existing infrastructures in response to various demand according to Kaut et al. [2015]. Although biomass is a relatively low-cost raw material, costs associated with logistic activities might lead to a significant increase in total cost. The study of Ekşioğlu et al. [2009] pointed out that issues related to low energy density, high harvest, and transportation costs of biomass could have a substantial impact on the final production price. A number of studies affirmed that an efficient supply chain design and management could contribute to lowering the biofuel price significantly. Nowadays, many efforts have been devoted to deal with challenges in design and management of biomass supply chains to

improve competitiveness and increase the bioenergy segment in total global energy consumption. In other words, the success or failure of a bioenergy project mainly depends on the management of logistics costs as well as the design of logistics networks.

The sustainability of a biomass supply chain also depends on the uncertainty management considering feedstock supply, bioenergy demand, and price. Several researchers confirm that biomass storage facilities in a biorefinery site could reduce the dependency of feedstock seasonality e.g. Andersson et al. [2002]. In practice, biomass supply and demand are unlikely matching due to feedstock seasonality and demand variation. Consequently, storage of biomass becomes urgent at certain intermediate facilities. Such storage facilities allow collecting enough biomass feedstock during low demand periods to meet the peak demand. An alternative is to increase biofuel production during low-cost periods as a buffer storage for peak demand periods.

## 1.2 The problems studied in this thesis

The biomass supply chain is quite complex and faces uncertainty in several aspects such as supply, biomass quality, production yield and demand. The reliability of feedstock supply considerably affects the efficiency of a biomass supply chain, so building a stable relationship with biomass suppliers is highly important to assure a steady supply of high-quality biomass at low prices. The partnership established with suppliers should be integrated with tactical planning to cope with uncertain environments.

Besides, one of the top priorities in biomass supply chain management at the tactical level is to determine the optimal inventory policy. An effective inventory policy noticeably contributes to reducing system costs. Unlike conventional products, biomass is characterized by degradation during storage. Therefore, optimization of inventory policy for each stock in a biomass supply chain should consider perishable nature of biomass.

In this thesis, we focus on the management of a biomass supply chain against the variations of feedstock supply at strategic and tactical levels. We aim to address issues related to the following questions:

- How to select suppliers to stabilize the feedstock supply and design an optimal operation plan for biomass transportation and biofuel production? Could we determine an optimal solution integrating the two mentioned objectives?
- How to find an optimal inventory policy in a biomass supply chain taking account of the degradation of biomass under environmental uncertainty?

The primary objective is to develop optimization models and algorithms to find an optimal solution for the stated problems. The study should take the degradable nature of biomass and all constraints related to biomass logistics activities into consideration. For the first problem, optimization models are required to evaluate the economic impact

of the feedstock supply and operation plan on a biomass supply chain at the strategic and tactical levels. For the second problem, a model needs to be proposed to capture the entire characteristics and constraints of biomass at the tactical level. Additionally, an effective algorithm should be developed to find an optimal/near-optimal solution in a reasonable computational time.

## 1.3 The contributions of the thesis

The objective of this thesis is to address the inventory management challenges in a biomass supply chain. In particular, the biomass supply chain management and perishable inventory control problems are examined, taking into account the uncertainty in different aspects. The contributions of this study are summarized as follows:

Firstly, we study a mixed integer linear programming model for a biomass supply chain management. This model has a flexible structure which allows capturing most logistic activities in transportation, operation planning and supplier selection decision in a biomass supply chain. A numerical study is conducted to evaluate the economic impact of the supplier selection and operation planning in a biomass supply chain.

Secondly, we propose a two-stage stochastic programming model for biomass supply chain planning. As a stochastic optimization problem, it is difficult to solve the model optimally due to large number of integer variables. Therefore, we develop an enhanced and regularized L-shaped decomposition method for solving our model optimally in a reasonable time. The algorithm based on Bender decomposition. The numerical results show that the algorithm can find an optimal solution in a reasonable computation time while a commercial solver cannot for large instances. Besides, we also evaluate some critical factors that affect the total system cost and the structure of supplier selection to better understand the system behaviors. This study can be used as a decision support tool for both supplier selection and operations planning of a biomass supply chain under uncertainty.

Thirdly, we propose two stochastic inventory models for a perishable product under uncertainty in both demand and supply. The study is relevant to inventory management of each stock in a biomass supply chain.

For a product with a constant deterioration rate, we discover several fundamental properties of the proposed model and demonstrate that the optimal inventory policy is an order-up-to level. Nevertheless, a considerable computational effort is required to find the optimal solution. For this reason, an algorithm for finding the near-optimal solution of the model is developed. The solution approach is based on the scenario optimization and the Lagrangian Relaxation. A numerical study shows that the proposed solution approach could find near-optimal inventory policies with the expected total cost less than 1% of deviation from the optimal expected total cost on average. Besides, several important factors are also evaluated through sensibility analysis to provide a better understand-

ing of the system.

For a product with a fixed lifetime, we propose a stochastic model that could be applied to determine the timing and quantity of order in each period. For solving the model, we adopt the two approaches, the Conditional scenarios (CS) and the Sample Average Approximation (SAA). The results show that the CS approach could provide a better solution in a shorter computational time than the SAA approach.

The results of the research work of this thesis have been published in international journals or proceedings of international conferences as given below.

**Journal articles**

1. **Duc Huy Nguyen**, Haoxun Chen (2018). *"Supplier selection and operation planning in biomass supply chains with supply uncertainty"*, Computers & Chemical Engineering, Jul 2018, DOI: *10.1016/j.compchemeng.2018.07.012*

2. **Duc Huy Nguyen**, Haoxun Chen, *"Inventory Control of Perishable Product with Uncertain Demand and Supply: Optimal policy and Algorithm"*, 2018 (under review).

3. **Duc Huy Nguyen**, Haoxun Chen. *"Optimization of a perishable inventory system with both stochastic demand and supply: Comparison of two scenario approaches"*, 2018 (under review).

**Conference papers**

1. **Duc Huy Nguyen**, Haoxun Chen, and Nengmin Wang. *"Modeling and optimization of biomass supply chain with two types of feedstock suppliers"*. In Proceedings of the 7th International Conference on Industrial Engineering and Systems Management IESM 2017, pages 610–615, Oct 2017, Saarbrücken, Germany.

2. **Duc Huy Nguyen**, Haoxun Chen. *"Optimization of a perishable inventory system with both stochastic demand and supply: Comparison of two scenario approaches"*, the 17th International Conference on Operational Research KOI2018, Sep 2018, Zadar, Croatia

## 1.4 Structure of the thesis

The rest of this thesis is organized as follows. Chapter 2 provides a general review of the literature considered in this work. We introduce some key concepts and focus on supply chain planning models and inventory models available in the literature. The chapter motives the need of considering the integration of supplier selection into the framework of operational and transportation planning for biomass supply chain management. In addition, this literature review also indicates the requirement of considering perishable inventory model with uncertainty in both demand and supply.

Chapter 3 presents an optimization model for biomass supply chains. We consider the problem of selecting suppliers under specific constraints. Supplier selection has an important impact on the stability of biomass supply chain, minimizing shortage costs and satisfying biofuel demand for end-use. Besides, the integration of supplier selection into the framework of operational and transportation planning helps to enhance the relevance of selection.

Chapter 4 extends the previous biomass supply chain model by considering environmental uncertainty. A two-stage stochastic programming model for biomass supply chain planning is proposed. An enhanced and regularized L-shaped decomposition method is developed for solving the model optimally in a reasonable time. The effectiveness of the solution method is proved by a numerical study.

Due to the degradable nature of biomass, it can be considered as a perishable product with a constant deterioration rate per unit time. Chapter 5 presents a stochastic perishable inventory model for a product with a constant deterioration rate. In this model, the demand and supply are both stochastic. An algorithm based on Lagrangian relaxation is developed to find a near-optimal solution.

In Chapter 6, this previous perishable inventory model is extended for a product with fixed lifetime and uncertainty under both supply and demand. We also develop a scenario-based optimization approach based on Conditional Scenarios (CS) approach to solve this challenging problem.

Finally, the last chapter summarizes the work of this thesis and draws some conclusions based on the results obtained. Besides, research perspectives are provided in this chapter.

# Chapter 2

# State of the art

*“ “Life throws challenges and every challenge comes with rainbows and lights to conquer it.” ”*

Amit Ray, World Peace: The Voice of a Mountain Bird

## Contents


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**1.2 The problems studied in this thesis** . . . . . **17**

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## 2.1 Introduction

In this chapter, we first address characteristics and challenging issues related to biomass supply chain management and perishable inventory management. Then, we attempt to identify supply chain models and inventory models available in the literature and related to our research topics in this thesis. This analysis shows the abundance of models and approaches to deal with various decision-making problems in the two research domains. After identifying the research gaps in the existing works, we propose some research directions to bridge these gaps.

## 2.2 Biomass supply chains management

In this subsection, a brief overview of biomass supply chains is provided, but a more detailed explanation can be found in Ba et al. [2016]; Melis et al. [2018]; Shabani et al. [2013]; Zandi Atashbar et al. [2017].

This subsection is organized as follows: Subsection 2.2.1 describes the main characteristics of biomass supply chains. Subsection 2.2.2 presents the structure, activities and challenges of biomass supply chains. A literature review on optimization models of biomass supply chain is given in Subsection 2.2.3.

### 2.2.1 Biomass supply chains

Biomass is any biological material produced on the planet through the process of photosynthesis. Biofuels refer to the fuels produced from biomass and bioenergy is the energy generated from biofuels [Allen et al., 1998; Ba et al., 2016; Zandi Atashbar et al., 2017]. As an extensive resource, biofuels/bioenergy could be produced from forestry and agricultural resources, animal excrement, industrial and municipal biodegradable waste according to An et al. [2011]. The research of Gouveia et al. [2018]; Pérez et al. [2017] classified the feedstock, biofuels and the related production processes into four generations. Figure 2.1 shows the main type of biomass feedstock in each biofuel generation.

- In first-generation, biofuels are produced from food crops (corn, sugarcane, or sweet sorghum) through conventional technologies such as abstracting oils and fermentation. Up to now, only the first generation of biofuels has reached the industrial stage. However, the main issue with the first generation biofuels is their competition with the global food security that raised the debate about their actual benefit over the last two decades.

- The second generation biofuels are in the industrial take-off phase. They use lignocellulosic biomass such as wood, organic waste, food crop waste and specific biomass crops. Unlike the first generation, they can eliminate the principal problem of the production of the first generation biofuels because they use the biomass

that is not suitable to be used as food.However, the progress of lignocellulosic biofuels is slower than expected in the past decade due to the difficulty in improving refinery technologies for biomass [Gouveia et al., 2018].

- The third generation of biofuels utilized algae as a feedstock. Through photosynthesis of CO2, algaculture could produce a large quantity of biomass and then extract for biofuel production, for example, biodiesel from algae cells is very promising for future energy production [Aro, 2016]. According to Gouveia et al. [2018], the microalgal biomass is considered a low-cost, high-energy and entirely renewable feedstock. It has the potential to produce more energy per acre than oleaginous crops (first generation) and other raw materials (second generation). However, their possibility of scaling up has not been yet proved at this time because of high processing costs and complicated production processes.

- The fourth generation biofuels will be produced from raw materials that are essentially inexhaustible, cheap and widely available through artificial photosynthesis using solar biofuel production technologies.

According to Olabi [2010], the main difference between these generations is only the change of the feedstock sources, but the characteristics of the biofuel may remain unchangeable. In our study, we focus on the management of a lignocellulosic biomass supply chain against the variations of feedstock supply.

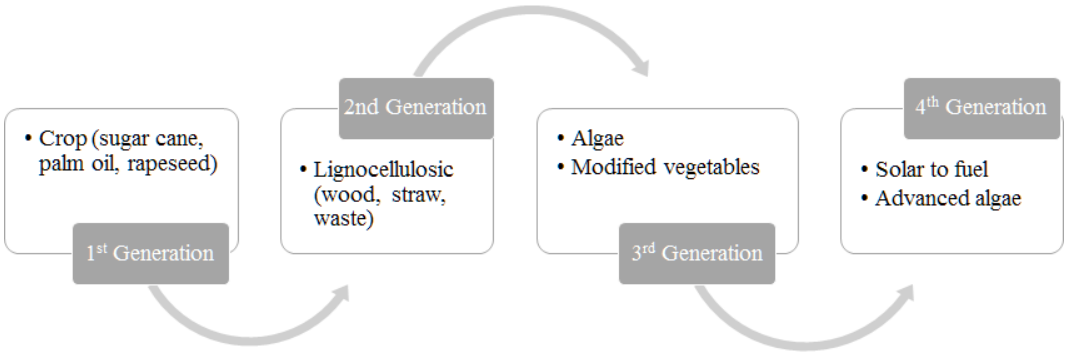


Figure 2.1: Biomass feedstock in each biofuel generation

### 2.2.2 Structure, activities and challenges of biomass supply chains

In biomass supply chains, there are various actors such as farmers, preprocessing facilities, biorefineries, transporters, final clients. Each actor plays a different role in supply chains, and its performance depends on the network design, planning, and operational activities. Figure 2.2 shows all actors and the corresponding activity logistics such as harvest, storage transportation, and distribution for bioenergy production.

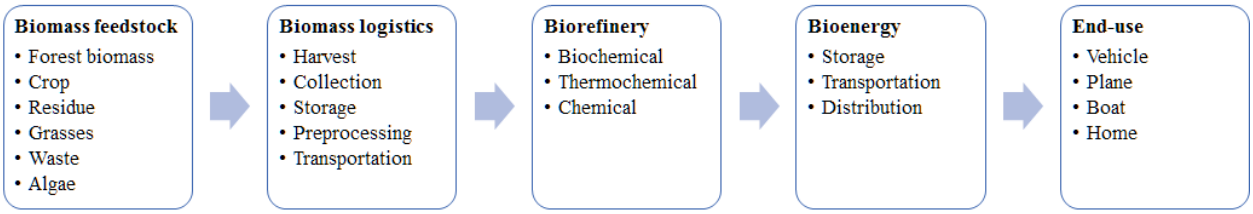


Figure 2.2: Examples of a bioenergy supply chain

In comparison with industrial supply chains, biomass supply chains have many different features regarding their structures and characteristics. Their structures could be naturally classified according to logistics activities required by supplying biomass from farmers to production sites and delivering biofuel from the production sites to service stations (gas stations). These activities include ground preparation and plantation, cultivation, harvesting, handling, storage, conversion, transportation and utilization of the biofuel at service stations according to Ba et al. [2016]. The characteristics of these activities in biomass supply chains are underlined as follows:

i) **Harvesting activities** usually occur in fields on a limited time window with a material loss of 10-20%. A limited group of machines such as combine harvesters is employed when the crop is ready. Sambra et al. [2008] distinguished three modes of harvest: multi-pass, single-pass, and whole-crop harvesting. A combine harvester is used for harvesting wheat, corn, and rapeseed in a multi-pass method. Grain, straw, and chaff are separated and stored in different places. In single-pass harvesting, grain and straw are harvested at the same time. Compared to a multi-pass collection, this mode is faster but requires more powerful and expensive equipment. In the latter one, the whole crop is cut and collected without separating its different components. There are four ways (baling, loafing, dry chop, wet chop) to collect biomass before storage and transportation to a biorefinery [Forsberg, 2000]. The appropriate method for biomass collection also depends on the nature of biomass and the desired moisture level.

ii) **Storage** is necessary to synchronize the biomass production calendar with the biofuel production plan. Storage can occur in the fields, in the farms, in centralized storage sites, or before the processes in a biorefinery. It is important to note that storage is a buffer between the harvesting windows of the different crops and their consumptions by the biorefineries. Suitable storage could reduce dry matter loss and protect biomass against the indoor/outdoor condition. Ebadian et al. [2013] affirmed that the total storage capacity required in a biomass supply chain should be much larger than in the traditional one to meet the same biofuel demand all over the year. Unlike fossil energy, biomass decays during storage, at a rate estimated to be 1-2% of the material stocked per month under the ambient storage, according to Rentizelas et al. [2009].

iii) **Preprocessing**, also called pretreatment allows to improve the quality of preservation and handling and to reduce transportation costs by increasing density and reducing degradations. There are many type of pretreatments such as ensilaging, pelletisation, torrefaction, pyrolysis. These pretreatments aim to reduce moisture, stabilize products and increase its calorific value and yield solid uniform products.

iv) Several **transport modes** can be used to deliver biomass feedstock such as road, rail transportation. However, road transport is often the best solution if fields have limited accessibility. Railways can be used if distances are large enough [Rentizelas et al., 2009].

Different types of uncertainty present in almost all logistics activities of a biomass supply chain, not only arise from external environments, but also from the supply chain itself Bot et al. [2015]. Firstly, the feedstock supply is always seasonal and depends on weather conditions and harvest time. In addition, a plant's location has a significant impact on the sustainability of feedstock supply thanks to geographical dispersion. Due to the heterogeneous and low-density characteristics, pretreatment operations are essential to homogenize and increase biomass density that may help us to reduce transportation costs of biomass significantly. Moreover, the fluctuation of purchase price and biofuel demand affects the efficiency of a supply chain network eventually. In some cases, it is hard to find an optimal network design because of uncertainty (fluctuation) in price and demand. The efficiency of a biomass supply chain is also affected by long-term contracts with suppliers, transportation and local distribution infrastructure, conversion technology and policies of the government according to Ba et al. [2016]; Gold and Seuring [2011].

### 2.2.3 Optimization models for biomass supply chain management

Mathematical programming and optimization techniques have been employed to design and manage biomass supply chains considering their different components and various decisions. Iakovou et al. [2010]; Mula et al. [2010]; Shabani et al. [2013] categorized decision-making in supply chain management into three levels according to the degree of importance and the planning horizon such as strategic, tactical and operational decisions. Figure 2.3 illustrates three decision levels for a biomass supply chain.

#### Strategic decision models

Most part of the considered publication focused on the design and management of biomass supply chains at the strategic level (55.8% according to the overview in Melis et al. [2018]). The decision-making at this level involves long-term decisions such as selecting production technologies, biomass preprocessing techniques, determining the facilities'capacity and location, the optimal configuration of logistics network, establishing long-term contracts with suppliers.

Samsatli et al. [2015] proposed a biomass supply chain model to evaluate the economic and environmental impacts under different scenarios and constraints concerning

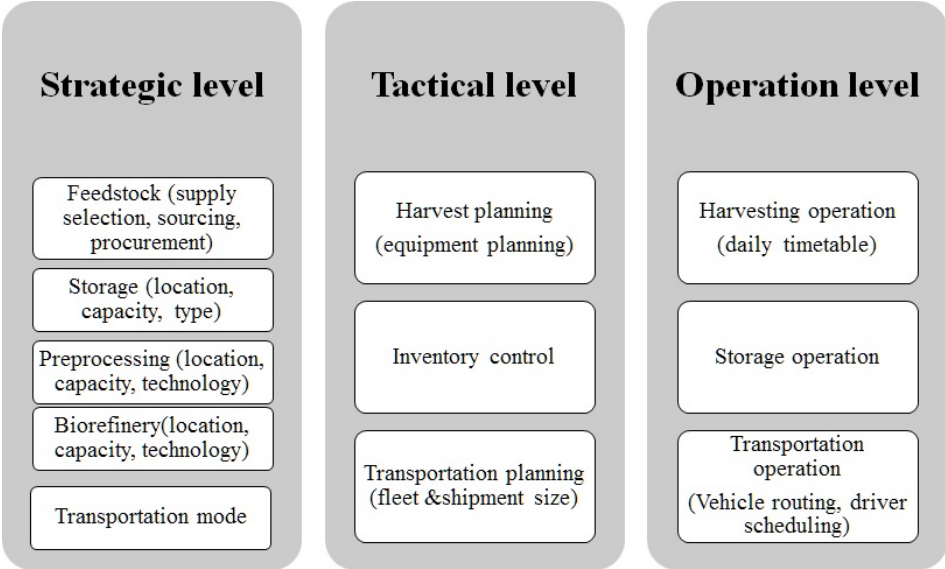


Figure 2.3: Decision levels in biomass supply chains

optimal resources and technologies selected. In Hombach et al. [2016], a mixed-integer linear programming (MILP) model was proposed to design a second generation biofuel supply chain in a German region under European biofuel regulations. Woo et al. [2016] proposed a MILP model for designing a supply chain network against fluctuations of biomass availability and hydrogen demand. The impact of biomass quality-related costs on operational costs in a biofuel supply chain was investigated in Castillo-Villar et al. [2016].

In the research of Woo et al. [2016], the authors proposed a mixed integer linear programming (MILP) model which can be used to design the optimal supply chain network to manage logistics operations against fluctuations of biomass availability and hydrogen demand. The research of Samsatli et al. [2015] developed a MILP model called Biomass Value Chain Model (BVCM) which takes account of the economic and environmental impacts associated such as: optimal use of land, biomass resources and technologies with respect to different objectives, scenarios and constraints. Castillo-Villar et al. [2016] proposed a bioefuel supply chain model that minimizes operational costs and the biomass quality-related costs. A MILP model in Rudi et al. [2017] aimed to identify location, technology, and capacity investment in a biomass supply chain by considering the compromise between economic scale and technology range. A case study is presented for a biomass value chain design in the Upper Rhine Region.

Geographic information system (GIS) is a powerful visualization tool that allows capture, store, manipulate, analyze, manage, and present spatial or geographic data. In biomass supply chain management, it could support decision-makers to estimate biomass feedstock availability, transportation cost and evaluate land use and natural obstacles. Recently, Zareei [2018] developed a GIS-based model to evaluate biogas production from livestock manure and rural household waste in Iran. Similarly, Zyadin et al. [2018] used

mapping and survey data from farmers to assess the potental exploitation of biomass in central Poland. Using a remote sensing high resolution data and GIS tools, Setyawidati et al. [2018] confirmed the high possibility to exploit brown seaweeds in Ekas Bay (Indonesia) as a biomass feedstock. Based on open source GIS and DBMS (Database Management System) applications, Tommasi et al. [2018] analyzed the technical, economic and environmental aspects and identified the optimal power plant location in Friuli Venezia Giulia Region, Italia. It is possible to compute the optimal objective function in the GIS-based models through a classical programming language or in the GIS script language such as Zhang et al. [2011].

#### Tactical decision models

The tactical decisions level in biomass supply chain are those related to medium-term decisions such as production planning, logistics planning (number of vehicles and workers), transportation mode and definition of safety stock level. Few studies integrated activities in a biomass supply chain at the tactical decision level such as Atashbar et al. [2018]; Ba et al. [2015]; Marques et al. [2018]; Morales-Chávez et al. [2016]; Sosa et al. [2015].

Ba et al. [2015] studied a biomass supply chain with multi-feedstock supply with the objective of minimizing the total logistic cost of the system on a regional basis. The MILP model could determines the optimal number of harvesting machine, the fleet size of trucks for transportation and the amount of each type of biomass harvested. Morales-Chávez et al. [2016] proposed a mixed-integer linear programming model to minimize the total system costs including costs of resources for harvesting and loading, shortage cost and transportation cost. The model determines the quantity of machines and workers to meet the biofuel plant requirements. Sosa et al. [2015] designed a linear programming tool to manage production planning for a wood biomass supply chain in Ireland. The study focus on the impact of moisture content and truck configurations on logistic planning. The results indicate that using 6-axle trucks could reduce 14.8% of truckloads and a 12.3% of haulage costs in comparison to using 5-axle trucks.

Marques et al. [2018] studied a tactical biomass supply planning problem with synchronization between chipping and transportation at the roadside. The authors also consider the constraints related to the variation of moisture content in storage. The numerical study shows that efficient management of the variation of moisture content in storage could improve until 20% of the supplier's profit. A MILP model in Atashbar et al. [2018] was developed to optimize a multi-period and multi-biomass supply chain with several biorefineries at tactical level. Han et al. [2018] aimed to optimize biomass extraction, transportation, conversion and product production by considering a multi-period, multi-commodity, multi-echelon supply chain problem. The MLIP model is solved by using a genetic algorithm.

### Operational decision models

The operational level corresponds to short-term decisions, weekly and daily, which can be considered as a decomposed activities from tactical level. In biomass supply chain management, the operation levels include logistics operations (harvesting, collection and handling) in a given days, and biomass transportation (vehicle routing problems). There are few mathematical models for biomass supply chains management at operational level.

Bochtis et al. [2013] studied the planning of the biomass collection operations with the objective of minimizing the total completion time of the whole activities in all fields. The case study in Thessaly, Greece involved the collection (baling–loading) of cotton residues from dispersed fields for the production of bioenergy. The numerical research shows the optimal schedule could reduce 9.8% of the total operations time in comparison with the one planned from experience of operations manager. Orfanou et al. [2013] continued the work presented in Bochtis et al. [2013] by considering a large number of machines per task type and introducing an economic objective. The meta-heuristic approach that based on the greedy heuristic and the "Tabu search" is proposed to solve the model.

Gracia et al. [2014] studied a mixed integer programming model for the biomass collection problem. The model is formulated as an application of the classical vehicle routing problem with the objective to minimize the total traveled distance in the field. A hybrid approach based on genetic algorithms and local search methods is presented to solve a real case study. Torjai and Kruzslicz [2016] proposed a mixed integer programming model for the truck scheduling problem in a real-life herbaceous biomass supply chain around Pécs, Hungary. In Li et al. [2017], a new concept, called "distance potential" was proposed, which could be used to optimize biomass logistics networks. Matindi et al. [2018] studied the harvesting and transport operations in a lignocellulosic supply chain and proposed the constraint programming model to optimize the delivery and collection times in the transportation network. An adapted Limited Discrepancy Search (LDS) was introduced to find the near optimal solution of the proposed model.

Some authors applied the GIS-based model to find the optimal operational plan such as Hoefnagels et al. [2014]; Montgomery et al. [2016]. A geo-spatial based model was developed by Montgomery et al. [2016] to generate an operational work plan that can be used on industrial timberlands. Based on the GIS framework, Hoefnagels et al. [2014] assessed lignocellulosic feedstock supply systems at the operational level such as the impact of overseas shipping routes and inter-modal transportation on the delivered costs and greenhouse gas emissions.

Some authors adopted a statistical method to optimize the bale collection activities. Igathinathane et al. [2016]; Subhashree et al. [2017] developed a statistical-based approach to find a better management of bale collection logistics and evaluate the field parameters on logistics activities. The study concludes that the "field middle" (that followed the best geometric median method) was an easy and practical method for locating field stacks.

Most studies at operational level do not consider uncertainty and environmental aspects in their mathematical models. A review paper of Malladi and Sowlati [2017] suggested that the future researches should take into account the environmental impact in the objective function or uncertainties in truck routing and scheduling models.

In addition, these mathematical models can be classified into two categories: deterministic and stochastic. Most studies ignored uncertainty by only considering deterministic models. A bibliographic analysis in Pérez et al. [2017] shows that 75% of studies which focus on economic and environmental criteria adopt deterministic models. We present in this following sections the deterministic and stochastic models.

#### Deterministic optimization models

Deterministic optimization models have a considerable majority of application in biomass supply chains. It is useful for "what-if" scenarios where decision-makers can observe the outcomes of their decisions in different inputs/conditions. With “what-if ” analysis, the sensitivity of optimal solutions can be easy to estimate as the value of the key parameters varies from their original values. In most studies, the critical parameters could be the prices, demands, production capacity and product availabilities.

In deterministic optimization models for biomass supply chains, the simplest way is to apply linear programming to simulate logistics activities (e.g., harvest, purchase, production, and transportation) by continuous variables. All constraints and objective function are linear. Besides, as pointed by Ba et al. [2016], mixed integer linear programming models are broadly used for supply chain network design or biomass logistics planning (e.g., number of vehicles, facility location, and selection of production capacity). In the previous decades, the development of the commercial solvers allows for solving many complex problems in a reasonable time. Most authors use a commercial solver (e.g., CPLEX, AIMMS, AMPL, GORUBI, LINGO, and XPRESS) to solve MILP models thanks to its powerful capabilities.

In some cases, it is hard to find an optimal solution by commercial solvers because the CPU time overgrows as a function of model size and a number of integer variables. That is why researchers have developed their algorithms based on Lagrangian relaxation, genetic, and metaheuristic approaches. However, metaheuristic has rarely appeared in several applications.

#### Stochastic optimization models

Based only on the expected value, deterministic models are not able to well capture events in the tails of the distributions of random variables. This disadvantage could lead to a negative impact on the performance of the models. For example, some scenarios with a low probability of occurrence might have a high impact on the total cost/profit of a system. So, stochastic models may be more appropriate. According to Shabani and Sowlati

[2016], this approach is adequate and useful when the probability distribution of uncertain parameter can be assessed and then decomposed into a set of scenarios. However, stochastic models are not conventional in biomass supply chain management because they are tough to be solved optimally and to guarantee constraint feasibility for all considered scenarios. According to Sahinidis [2004], there are three principal approaches to cope with uncertainty: stochastic programming (recourse models, robust stochastic programming, and probabilistic models), fuzzy programming (flexible and possibilistic programming), and stochastic dynamic programming.

An the literature for biomass supply chains, most mathematical models are deterministic and only few stochastic models exist. As reported in Table 2.1, these stochastic models are usually based on two-stage stochastic programming with recourse to design supply chains at the strategic level (selecting production technologies, determining the location and capacity of each facility). Awudu and Zhang [2013] studied a biofuel supply chain for biorefineries already placed and producing biofuels under demand and price uncertainties in North Dakota. They applied Bender decomposition algorithm and Monte Carlo method to approximate the value of second-stage based on a set of scenarios. Osmani and Zhang [2013] consider a similar problem but with objective to determine a location of biorefineries under more uncertainties (supply, demand, and prices). The solution technique is identical to those previously mentioned.

Kim et al. [2011] also study a similar problem but with objective to determine location and size of two types of conversion facilities with the high level of uncertainty in supply amounts, market demands, market prices, and processing technologies. This model is implemented on the commercial software GAMS and uses the CPLEX solver. Chen and Fan [2012] studied the same problem but consider separately two major sources of uncertainties, feedstock supply and fuel demand. They applied Progressive hedging algorithm to solve this model optimize a biomass supply chain under uncertainties in biofuel demand and feedstock supply. Dal-Mas et al. [2011] investigated economic performances and risk on investment of the entire biomass-based ethanol supply chain in Northern Italy. The authors proposed a multi-echelon mixed Integer Linear Program (MILP) modeling framework and solved the model by using a solver CPLEX. Similar to Osmani and Zhang [2013],Kaut et al. [2015] want to determine the location of facility and flow of biomass in a supply chain with uncertainty in demand, supply and prices. They proposed a hybrid stochastic programming-robust optimization model and solved this MLIP model by using FICO™ Xpress Optimizer.

With similar objective as Kim et al. [2011], a two-stage chance-constrained stochastic programming model for a biofuel supply chain network was presented in Quddus et al. [2018]. This study considered the uncertainties due to feedstock seasonality due to sudden fluctuation or total unavailability in biomass supply arises. Besides, a joint chance-constraint related to a percentage of biomass supply from municipal solid waste at a certain probability was introduced. A Sample Average Approximation algorithm was devel-

oped to solve the mathematical model.

Most works only consider road transportation, except for the study of Marufuzzaman et al. [2014] focused on selecting transportation mode (rented truck, facility-owned trucks, and pipelines) and facility location of a biodiesel supply chain with uncertainties in feedstock and technology development. They investigate the impact of carbon regulatory mechanisms on the design and management of a supply chain. They proposed an algorithm combining Lagrangian relaxation and L-shaped method to solve this model.

Most researchers study supply chains for biofuel production whereas Shabani and Sowlati [2016] investigate a forest-based biomass supply chain for a power-plant with uncertainty in biomass quality and biomass availability. This model is solved using the AIMMS software and CPLEX solver. Almost works mentioned above consider biomass supply chains at only one period such as Awudu and Zhang [2013]; Chen and Fan [2012]; Kim et al. [2011]; Marufuzzaman et al. [2014].

There exist different methods to solve a two-stage stochastic programming model such as L-shaped algorithm in Awudu and Zhang [2013]; Marufuzzaman et al. [2014], Progressive Hedging algorithm in Chen and Fan [2012], Genetic algorithm in Mirkouei et al. [2017]. Some authors used a commercial solver to find an optimal solution of a deterministic equivalent linear program model such as Kaut et al. [2015]; Kim et al. [2011]; Osmani and Zhang [2013]; Shabani and Sowlati [2016]; Shabani et al. [2014].

Table 2.1: Stochastic optimization models for biomass supply chains

| Ref | Uncertainty | Decision variable | Decision level | Objective | Model & Solution method |
|---|---|---|---|---|---|
| Kim et al. [2011] | Biomass price, yields and unit production cost, demands | Network flows, Qties used to produce energy in CU, facility locations (binary) | S | Max total profit | 2-stage stochastic MIP, GAMS- CPLEX, Scenario-based Monte Carlo method |
| Chen and Fan [2012] | Feedstock supply and fuel demand | Network flows, Biorefinerie and terminal locations (binary) | S | Min total expected system cost | 2-stage stochastic MIP, Progressive hedging algorithm |
| Osmani and Zhang [2013] | Biomass availability, price and demands | Network flows, BR locations (binary) | S | Max expected profit | 2-stage stochastic MIP, Aggregated scenarios, GAMS-XPRESS |
| Dal-Mas et al. [2011] | Corn cost, ethanol sale price | Network flows, Corn cost, ethanol selling price, Transport mode selection (binary), BR location, size and technology choice (binary) | S | Max expected NPV | Stochastic MIP, Aggregated scenarios, GAMS + CPLEX |
| Kaut et al. [2015] | Demand, supply and prices | Location of facility (binary) and flow of biomass | S + T | Max overall profit | Hybrid stochastic programming-robust optimization model, FICO™Xpress Optimizer |
| Quddus et al. [2018] | Feedstock supply | Facility location decisions (binary) and flow of biomass | S | Min total expected system cost | 2-stage chance-constrained stochastic programming model, Combined Sample Average Approximation |
| Marufuzzaman et al. [2014] | Biomass supply and technology | Location (binary) and transportation decisions | S | Minimize costs and GHG emissions | 2-stage stochastic programming model, Lagrangian relaxation and L-shaped method |
| Shabani and Sowlati [2016] | Biomass quality and biomass availability | Amount of biomass to purchase, store and consume from each supplier in each month, and whether or not to generate the surplus load | T | Max total profit | Hybrid multi-stage stochastic, programming-robust model, AIMMS- CPLEX |
| Shabani et al. [2014] | Biomass availability | Amount of biomass to be purchased, stored and consumed from each supplier | T | Max total profit | 2-stage stochastic optimization model, AIMMS- CPLEX |
| Mirkouei et al. [2017] | biomass quality and accessibility rates | Network flows (amount of biomass and biofuel to be transported) | T | Min total annual cost | Stochastic model with multi-criteria decision making framework, Support vector machine (SVM), Genetic algorithm and life cycle assessment |

(*) S: Strategic, T: Tactical, O: Operational, NPV: Net Profit Value

### Multi-objective models

In the biomass supply chain optimization, most of the research focuses on minimizing the total cost or maximizing the profit of the whole system. In the last decades, the trend of integrating environmental and social objectives into the mathematical model is gradually becoming more popular. This approach has received much attention from the scientific community as it is suitable for the sustainable development of renewable energy in general and biofuels in particular.

Due to the ability to deal with different and competitive objectives (aspects), the multi-objective programming approach has been applied widely in the natural resources management. For example, the design of a biomass supply chain network may consider simultaneously multiple criteria related to energy sufficiency, environmental effects, and socio-economic impacts. This approach is usually applied for multi-products, single period models with several objectives such as maximizing job creation, reducing greenhouse gas emissions, and minimizing total system cost. Dealing with the conflicts between objectives, most works often use Pareto-optimality to provide several compromise solutions. As an example, Grigoroudis et al. [2014] proposed a multi-objective programming model to find the optimal design of a biomass supply chain network with objective of maximizing efficiency and minimizing the total costs. A RDEA (Recursive Data Envelopment Analysis) algorithm have been developed to provide the optimal solution with high convergence speed. Similarly, Liu et al. [2014] proposed a MILP model for biofuel supply chains with multiple production pathways with economic, energy, and environmental objectives. The model was solved using the $\epsilon$-constraint method.

Many works use the $\epsilon$-constraint method to solve multi-objective models and provide a set of Pareto-optimal solutions. In the $\epsilon$-constraint method, the problem is reformulated as a single objective problem by keeping one selecting objective and transforming other objectives into additional constraints by using a specified $\epsilon$ scalar values. As an example, Cambero and Sowlati [2016] focused on maximizing the social benefit, net present value and minimizing greenhouse gas emission of a forest based biorefinery supply chain. The authors formulated a multi-objective MILP model and developed a improve $\epsilon$-constraint method to solve this model. Roni et al. [2017] attempted to minimize total system costs and total emissions CO2 in the supply chain while maximizing social objective (number of created jobs). The MILP model is solved by the augmented $\epsilon$-constraint method using CPLEX in C++. A case study is conducted with data from the Midwest region of the USA. Recently, Wheeler et al. [2018] studied the multi-objective design of biomass supply chains for bioethanol from sugar cane production in Argentina. The authors combined the mixed-integer programming with multi-attribute decision-making tools to identify a unique Pareto solution of the multi-objective problem. Wang et al. [2018] studied a reliable multi-period supply chain model against feedstock seasonality and time-varying disruption risks. A case study in Hubei, China was studied to validate the model and solved

by using Knitro Solver in AMPL.

There are very few studies developed a mathematical model under uncertainty to find the trade-off between conflicting objectives such as social, economic, and environmental aspect. Mirkouei et al. [2017] proposed a multi-criteria decision-making framework to assess the economic and environmental aspects of a mixed biomass supply chain. The model considers uncertainties in biomass quality and accessibility rates. The authors employed a support vector machine technique to predict the pattern of uncertainty parameters. Then, a stochastic optimization model incorporated with uncertainties are formulated and solved by a genetic algorithm. By applying fuzzy multi-objective programming, Tsao et al. [2018] studied the supply chain networks design under uncertain environments (e.g., demand, capacity, and costs). Three objective functions are considered in the model such as the minimization of total costs, environmental impact, and maximization of social benefits. They proposed an interactive fuzzy approach to provide solutions with the compromise of the objective functions for decision-makers.

## 2.3 Perishable inventory management

As mentioned in the previous subsection, biomass deteriorates with a constant rate estimated at 1% of the material stocked per month under the ambient storage Rentizelas et al. [2009]. Thus, biomass can be considered a perishable product. In fact, perishable product refers to an item which loses its value over time until it eventually becomes worthless. Typical examples of perishable products include fresh products, blood cells, chemicals, photographic films, drugs and other pharmaceutical products.

Effective inventory management allows companies to respond quickly to the change in market demand and develop a flexible production system to improve their competitiveness. Multi-echelon models are usually adopted for inventory management when considering a system with multi locations and multi-facilities. However, for perishable products, single-echelon models remain essential and could provide a primary principle to make multi-echelon models more realistic and relevant according to Duong et al. [2018].

The goal of this subsection is to provide and analyze the perishable inventory models in various dimensions such as its evolution, scope, demand, shelf life, replenishment policy, modeling techniques, and research gaps. We focus not only on the mathematical details but also on the assumption and feature of perishable inventory models (Subsections 2.3.1- 2.3.2). An overview of the solution approaches is outlined in subsection 2.3.3.

### 2.3.1 Characteristics of perishable inventory problems

According to Silver [2008], there are many possible objectives should be taken into considerations in inventory management such as cost minimization, profit maximiza-

tion, maximization of return rate on stock investment, improving flexibility to cope with an uncertain, minimizing political conflicts regarding the competing interests within the organization... Most studies have focused on only the first and second objective. There are various possible constraints including supplier constraints (minimum order sizes, maximum order quantities, restrictions on replenishment times), customer service levels and internal constraints (storage, budget, workforce and workload limitations). Generally, decision-makers who seek to satisfy customer demand at the minimal cost must consider two fundamental decisions: the size and timing of each inventory replenishment order. In most inventory models, it is assumed that products can be stored indefinitely to meet future demands. However, the effects of perishability cannot be ignored for certain types of products, which may become partially or completely unsuitable for consumption as time passes.

Different from that of a product with unlimited shelf life, the structure of the optimal replenishment policy for a perishable product is usually more complex: the replenishment quantity depends on the shelf life categories of the current inventories, as well as on all outstanding orders, uncertain demand, supply, and price. Directly speaking, inventory policies consist of determining time and quantity for replenishment. According to Duong et al. [2018], many characteristics such as demand distribution, stock issuing, product lifetime, replenishment lead time, and shortage situations should be accounted into the models for dealing with different business situations. The fundamental inventory policies are given as follows:

- (s, S) is a minimum/maximum inventory policy., when the inventory falls to a minimum level s, an order is placed to bring the inventory back to the maximum level S. The order size is quantity (S – s). This model provides significant flexibility.

- (s, nQ) or (r, nQ) whenever the inventory falls below the level s or r, an order quantity n times a predetermined Q (order quantity) is placed where n is a multiple equal to or greater than 1. This policy is suitable for ordering in batches.

- (S – 1, S), an order is placed immediately whenever a demand occurs for one or more units of an item. This model is preferred when lead times are zero and ordering cost is low.

- (T, S) is the periodic review order up to level policy. In each period, the decision-maker will request a replenishment quantity that will restore the on-hand inventory to a target, S. The order size is quantity (S – the on-hand inventory). This policy is simple for real world practices.

Due to the limited lifetime of products, the stock issuing policy is essential and has a significant impact not only on the system cost but also on the satisfaction of customers. In perishable inventory management, there are two most commonly used stock issuing

policies: First-In-First-Out (FIFO) and Last-In-Fist-Out (LIFO). In FIFO, the product arrives in warehouse first will be delivered to customers early in the order of arrival; in LIFO, the product that has mostly come in the warehouse will be given to customers first. In reality, customers prefer LIFO to FIFO policies regarding the freshest products. According to Parlar et al. [2011], the FIFO is better than the LIFO when considering the combination of all operational costs. However, the situation is reversed when the holding cost is high, or the purchase cost is low. Most research uses the FIFO policy because this policy is suitable for reducing the waste of the expiration products logically.

In addition, an inventory problem with fixed lifetime is more complicated than the one with stochastic lifetime due to the high-dimension problem and inability to apply the Markovian property to describe the stock on-hand [Olsson and Tydesjö, 2010].

### 2.3.2 Classification of perishable inventory models

To reflect the real-life issues, many researchers integrated several related factors into the inventory models such as credit and different payment problems; inflation and time value for money; investment, promotion and budget constraint; customer satisfaction...In this subsection, we discuss some essential modeling elements that are popular in perishable inventory management.

Many companies offer customer deteriorated items at a discount price to boost sales and increase profit. Moreover, the relevant discount price policy could allow companies to reduce the loss due to deterioration items by enhancing sales. Such policy is applicable for some products in supermarkets, e.g., milk, yogurt, and bread are always sold at a discount price when they are very close to their expiration date. Such price discounts and order policy are presented in the works of Banerjee and Agrawal [2017]; Chung et al. [2015]; Pal [2016]. In recent years, many studies focused on the investment of preservation technology to improve the storage condition for deteriorated products such as Mishra et al. [2017, 2018]; Pal et al. [2017]; Singh et al. [2016a]; Tayal et al. [2016]; Yang et al. [2015].

Since the inflation crises in the 1970s and early 80s, many authors realize the importance of taking consideration inflation into account, to make the models more realistic. A model in Buzacott [1975] is the first model with inflation. Until now, various models have explored different aspects from inflation. The readers can find the recent models in Chakrabarty et al. [2017]; Shah and Vaghela [2018]; Tayal et al. [2016].

To increase the partnership, it is common that suppliers usually offer a retailer a permissible delay period. During this period, there would be no interest charged if the full payment is made. The period is so-called, the permissible delay period. The first EOQ (Economic Order Quantity) model with a permissible delay period is provided by Goyal [1985]. Recently, Mishra et al. [2018] determined the optimal joint ordering, pricing, and preservation technology investment policies for the inventory model with deteriorated items under the permissible delay in payments. Chan and Hsu [2018] provided an EOQ

model under payment conditions that combinate the good-faith prepayment, the cash payment, and the delayed credit payment. The impact of inflation, perishability and trade credit policy is taken into consideration in the study of Tiwari et al. [2016]. Mohanty et al. [2018] study the joint effect of preservation technology investment and trade-credit policy on a stochastic perishable inventory.

Service level constraint is one of the most useful techniques for integrating customer satisfaction into the inventory models. There are several studies focus on finding the order policy under service level constraints. Such problem are addressed in the works of Belo-Filho et al. [2015]; Hendrix et al. [2012, 2015, 2018]; Pauls-Worm et al. [2015, 2016]. These authors proposed approximation approaches to find the near-optimal solution for stochastic inventory models. By considering the service requirement under random yield and uncertain demand, Belo-Filho et al. [2015] proposed a perishable inventory model of coordination with revenue sharing contract. However, that paper only characterizes the optimal policy of one-period inventory model.

In the following, perishable inventory models are classified according to the product life time, demand and supply characteristics. A more detailed classification can be found in Bakker et al. [2012]; Chaudhary et al. [2018]; Duong et al. [2018]; Janssen et al. [2016]; Karaesmen et al. [2011]; Nahmias [1982].

#### Product lifetime

According to Bakker et al. [2012]; Janssen et al. [2016], the limited shelf life products are classified into two categories: deterministic lifetime products and random lifetime products which may have a probabilistic distributed lifetime (e.g., Weibull), constant, known or unknown deterioration rate. In case of fixed lifetime, a product becomes unusable and must be discarded when its lifetime are achieved. In case of random lifetime products, one cannot know exactly the lifetime of items in advance. As an example, vegetables and fruits could be considered as stochastic lifetime products because they have an unknown lifetime and their deterioration rate depend on the external environmental condition such as temperature, storage condition.

Evidently, an inventory model applying to products with limited lifetime is more complicated than one applying to products with unlimited lifetime. This is due to the fact that dimension and state of inventory grows up as a function of a lifetime and number of periods. As mentioned by Schmidt and Nahmias [1985], the perishable inventory problem under either continuous or periodic review systems is challenging to find an optimal policy when the lifetime is more than two periods. Many researchers turned their attention to developing heuristic approaches.

Kouki et al. [2013] focused on verifying the effectiveness of existing (r, Q) inventory systems, in which products have fixed lifetime and positive order lead time. A periodic review inventory model of fixed lifetime product with deterministic demand and supply

disruptions over an infinite horizon is studied in Atan and Rousseau [2016]. Chen et al. [2014] generalized an inventory model for a perishable product with fixed time life and positive lead time over a finite horizon to allow coordinated pricing, inventory control and disposal decisions. A study of Dobson et al. [2017] is focused on an inventory model of a perishable product with age-dependent demand rate and introduces a simple EOQ to calculate approximately the optimal order cycle. Lately, Kouki et al. [2018] studied an inventory model with dual-sourcing and the perishable items have a fixed or exponential lifetime. They recommend that the dual-sourcing could reduce substantial cost and the percentage of cost reduction is dependent on the lifetime distribution, e.g, the shorter the mean lifetime is, the higher the cost reduction.

According to Goyal and Giri [2001], it is likely that an inventory model for a perishable product with a random lifetime is more difficult than the one with a fixed lifetime. The random lifetime problem attracts a lot of attention in the last decades. Particularly, Liu et al. [2018] studied the purchase and inventory policy for a perishable seasonal agricultural product. They formulated this problem as a multi-period dynamic programming model. The optimal purchase and inventory retrieval policy are characterized based on the sell-down-to policy and the corresponding minimal storage level. The optimal inventory period, cycle length and investment amount in preservation techniques are studied in the work of Pal et al. [2017]. They formulated an inventory model for non-instantaneously deteriorating items with constant demand, and the deterioration rate could be reduced by investing in preservation technology. The optimum order quantity and reorder intervals for a periodic review inventory model with non-instantaneous deteriorating items are provided by Pal and Samanta [2018]. In this model, the pre-deterioration period is a random variable and the uniform demand rate during the pre-deterioration period is greater than that in the post-deterioration period.

Inventory models become extremely complex when considering a positive order lead time. By way of illustration, Baron et al. [2017] investigated a continuous review (s, S) model of perishable products under stochastic in both lead times and demand. They developed a heuristic approach to finding the optimal re-order level, s, and order-up-to level, S to minimize the total cost. Chao et al. [2017] proposed an approximation algorithm for perishable inventory systems with positive lead times and limited ordering capacities. A novel transient unit-matching mechanism is provided to find the dynamical match between the supply and demand units.

#### Demand

Customer demand plays a vital role in an inventory management because the better demand planning, the more profit companies can reach. According to Duong et al. [2018]; Goyal and Giri [2001]; Janssen et al. [2016], there are two categories: deterministic and stochastic demand.

*Deterministic demand*

In deterministic problems, demand is known with certainty. The assumption of deterministic demand could simplify the problem and allows to provide optimal ordering policies such as EOQ (Economic Order Quantity) and EPQ (Economic Production Quantity) [Pahl and Voß, 2014]. However, this could not reflect customer demand accurately in practice. The deterministic demand can be also divided into sub-categories as follows: uniform demand, time-vary demand, stock-dependent demand, price-dependent demand,freshness dependent demand.

In inventory models with stock-dependent demand, the assumption is based on the observation that customer attention arises from a large stock of products in the supermarket. This problem becomes more complex in case of perishable products [Goyal and Giri, 2001]. As an example, Singh et al. [2016b] developed a perishable inventory model with stock-dependent and seasonal pattern demand over finite time horizon. The study illustrated that the customer demand decreases when the product is close to the end of the season and the stock level is lower. In the same year, Singh et al. [2016a] provided an optimal economic order quantity for perishable inventory model in consideration of stock-dependent demand with trade credit period and preservation technology.

In reality, many products have fluctuation in the demand rate such as electronic goods, fashionable clothes, etc. In the growth stage, the customers demand increases rapidly and continuously remain at the maturity phase then it will gradually decrease after the emergence of competitive products which affects consumer's preference. Another possibility that could be considered in this sub-category is that the age of production may have some impact on the customer demand. In fact, the customer always prefers a fresh product to the old one. For these products, inventory models with time/age-dependent demand may be plausible and relevant such as the works of Chowdhury et al. [2016]; Demirag et al. [2017]; Dobson et al. [2017]; Tayal et al. [2016]. Lately, Demirag et al. [2017] proposed an adaptive EOQ solution for managing a perishable inventory with remaining lifetime dependent demand. They show that by integrating the freshness-dependent demand into ordering decision, retailers may improve their total profit significantly. Chowdhury et al. [2016] introduced a perishable inventory model with time-quadratic demand and partial backlogging. Based on this model, the authors also investigate the effect of inflation and time value of money. Tayal et al. [2016] study an integrated perishable inventory policy in a two-echelon supply chain, comprising a vendor and a retailer. In this model, set up costs and ordering costs are considered to be time dependent; trade credit period is allowed and preservation technology can be invested with the objective of reducing the deterioration rate of a product.

Other sub-category of deterministic demand is based on the inter-dependent between price and demand. In the market, almost all items are price sensitive because the selling price affects demand directly. There are many studies explore this feature of inventory management. Recently, Lu et al. [2016] studied a continuous-review inventory model for

perishable items with inventory-level-dependent demand. The authors developed the optimal joint dynamic pricing and replenishment policy based on Pontryagin's maximum principle to maximize the total profit. Cheng et al. [2017] proposed a joint pricing and inventory model for perishable products with price-dependent demand and the return of a product after purchasing. This study focus on finding the optimal sales period and selling price to maximizing total profit. The numerical study suggests that by reducing the order cancel rate, a retailer could improve sales and profits simultaneously.

In recent times, many researchers combine different types of demand patterns in their studies. The combination allows to improve the relevant of demand pattern in real situation such as the works of Banerjee and Agrawal [2017]; Chen et al. [2016]; Mishra et al. [2017]; Shah et al. [2018]. In practice, the retail always offers a discount to boost sales for the items which begin to deteriorate because the lower freshness could lead to the lower demand at the same price. By considering the effect of freshness and selling price on the customer demand, Banerjee and Agrawal [2017] proposed an optimal discounting and ordering policies to maximize the total profit. They suggested that decision-makers should offer more discount when selling price rises, but they could still experience a small loss in their benefit.

For the fresh product, Chen et al. [2016] noted that the customer demand depends on the freshness and displayed volume of the product on the shelf space. The author proposed an economic order quantity model with freshness-and-stock dependent demand to maximize the total annual profit. The decision variables related to the optimal level of shelf space size, replenishment cycle time, and ending inventory level. With the similar ideal, Mishra et al. [2017] investigated the impact of the preservation technology investment on a perishable inventory model. In this model, the demand rate depends on the stock level and selling price. They focus on determining the optimal selling price, ordering frequency and storage technology investment to maximize the total profit. An inventory model for returned/reworked items with price-sensitive as well as stock-dependent demand is proposed by Shah et al. [2018]. The authors suggested an optimal economic production quantity for maximizing the total profit with respect to production rate, cycle time and retail price.

*Stochastic demand*

In many real situations, the demand for perishable products not only depends on external factors such as trends, technology, customer's preference but also internal factors pricing, marketing, etc. So the demand should not be considered as deterministic because it is far from many realistic situations. Stochastic models are useful when demand pattern is uncertain or unknown. In many studies, demand function is assumed to follow a probability distribution such as normal, random log-normal exponential, etc. For example, Kouki et al. [2015] studied an (r, Q) inventory system for perishable products under stochastic demand, constant lifetime and constant lead time. In this model, demand is assumed to follow the Gamma or Gaussian distribution. They developed an approxi-

mation algorithm to estimate the parameters r and Q that minimize the total system cost.

Most research uses the Poisson distribution to depict the stochastic demand. As an example, Alizadeh et al. [2014] proposed an inventory system with Poisson demand, positive lead time and an (S – 1, S) ordering policy. The authors developed an enumeration algorithm to find an optimal solution. Baron et al. [2017] studied an inventory model with two cases of demand distribution (Poisson and compound Poisson with general demand sizes) and positive lead time.

A renewal process is a generalization of the Poisson process that has an interesting mathematical structure and can be used to make the inventory model more realistic. Therefore, demand arrival could be modeled as a renewal process where the times between two successive demands are independent and identically distributed such as Gürler and Özkaya [2008]. Lately, Boxma et al. [2016] extended the classical EOQ model by taking into account a random environment and perishability. Demand is alternated between two random state (high and low demand period) according to a continuous-time semi-Markov process. The high and low demand period are both represented by compound Poisson processes with different parameters. The study of Haijema and Minner [2015] dealt with the case of double stochastic Poisson demands, positive lead time, partial backorder, perishable items, and emergency replenishment. They demonstrate the stationary distributions of the age of items is unique.

A lot of studies applied the rolling horizon approach in the literature of inventory management. Hendrix et al. [2012] study an inventory control problem for a perishable product with a non-stationary random demand and a service level constraint. Recently, Janssen et al. [2018] investigated the impact of closing days constraint on waste quantity and total costs. They formulated a stochastic perishable inventory model for multi-items with a fixed lifetime includes total stock capacity constraints, a positive lead time, a periodic inventory control, a target customer service level and mixed FIFO and LIFO issuing policies under a non-stationary random demand. The study shows that closing days constraint could reduce on average 1.77% of the outdating quantity and 0.29% of total costs in grocery stores. Ferreira et al. [2018] developed a inventory model of perishable goods with stochastic demand and stochastic donations for long term humanitarian operations, using Markov Decision Process.

The companies are likely to deal with a stochastic non-stationary demand when products have a short lifetime and demand depends on the season and customer buying pattern. So, the non-stationary stochastic demand should be taken into account because it could make the model more realistic [Chaudhary et al., 2018]. In these models, demand is modeled as a random variable following a probability distribution and statistical parameters changes from one period to the other. A number of research developed a stochastic programming inventory model for a perishable product with non-stationary stochastic demand and a service level constraint such as [Alcoba et al., 2015; Hendrix et al., 2012, 2015; Pauls-Worm et al., 2015]. The study in Gutierrez-Alcoba et al. [2016] is focused on

an inventory model of a perishable product with non-stationary stochastic demand and introduces two heuristics extending Silver's one to solve the model quickly. A randomized proportional-balancing policy (RPB) for a stochastic inventory model of a fixed-lifetime product with setup costs is introduced in Zhang et al. [2016].

To better represent the real-life situation, Chaudhary et al. [2018] recommended that the researchers should focus sharply on the stochastic perishable inventory and multi-item approaches because these approaches would be a valuable contribution to theory and practice.

**Supply**

According to Tajbakhsh et al. [2007], one of the top inventory management challenges that need more attention is the supply uncertainty. Such uncertainty can be found in three aspects as follows:

- Supply lead time
- Supply quantity
- Purchase price.

Uncertainty in supply quantity can be classified into three categories: random yield (the quality ordered and the amount received are not the same), random supply availability and random supply capacity (supplier's capacity is variable).

The importance of the research on perishable inventory management with capacity constraints is emphasized in Bakker et al. [2012]; Janssen et al. [2016]. However, models with limited capacity impose real challenges in practice and require innovative heuristic policies. The study in Belo-Filho et al. [2015] analyzes a perishable inventory model of coordination with revenue sharing contract and service requirement under random yield and uncertain demand. However, that paper only characterizes the optimal policy of one-period inventory model. A periodic review inventory model of fixed lifetime product with deterministic demand and supply disruptions over an infinite horizon is studied in Atan and Rousseau [2016]. By using a base-stock policy, they concluded that companies should focus on reducing the duration of supply disruptions instead of trying to prevent them.

One of the top inventory management challenges that needs more attention is supply uncertainty. Such uncertainty can be found in three aspects: uncertainty in supply lead time (Janakiraman et al. [2013]; Muthuraman et al. [2014]), uncertainty in supply quantity (Ciarallo et al. [1994]; Jakšič [2016]; Wang and Gerchak [1996]) and uncertainty in purchase price (Chen et al. [2015]; Gaur et al. [2015]; Tajbakhsh et al. [2007]). Uncertainty in supply quantity can be classified into three categories: random yield (the quantity ordered and the quantity received are not the same), random supplier availability and random capacity (supplier's capacity is variable). The first inventory model for a single prod-

uct with random demand and random supply in multi-period or infinite-horizon was introduced in Ciarallo et al. [1994]. An inventory model of single product with random yield, random capacity and uncertain demand is studied in Wang and Gerchak [1996], whereas inventory models of single product with stochastic supply capacity and stochastic demand are studied in Jakšič [2016].

### 2.3.3 Modeling approaches for perishable inventory problems

In the literature for perishable inventory models, various mathematical programming models are applied to analyze the whole system under different realistic constraints. The conventional modeling techniques can be used such as dynamic programming, mixed integer nonlinear/linear programming, fuzzy programming, Markov decision process...

Since the 70s, many works used the Markovian process to formulate perishable inventory models. The early works are the inventory models of Brodheim et al. [1975]; Chazan and Gal [1977]; Schmidt and Nahmias [1985]. Some studies considered different types of demand to adapt the models close to the real-life situation such as Baron et al. [2017]; Boxma et al. [2016]; Gürler and Özkaya [2008]; Haijema and Minner [2015]. In recent times, Ferreira et al. [2018] studied an inventory management model for perishable items with stochastic demand and stochastic donations. The products have deterministic deterioration rates. The objective function is to to minimize the average operational costs for a continuous aid operation in a humanitarian crisis environment. Soujanya et al. [2015] considered a perishable inventory model with service interruptions, retrial demands, and negative customers, using the (s, S) policy. Demands are simulated by the Poisson process and the life time is assumed to be exponential. By definition, a positive customer joins the system whereas a negative customer could hinder other's coming into the system. Besides, negative customers could be viruses or commands that delete some transaction in a computer network or a database. Amirthakodi et al. [2015] studied a perishable inventory system with service facility and feedback customers.

When the customer demand is stochastic, dynamic programming can be useful. In the article of Smith [2000], the application of dynamic programming for inventory management problems has been highlighted. It is evident that dynamic programming has suitable properties for modeling the recurrent relationship in inventory problem and allows to prove that there may exist an optimal control policy for the inventory management. Some authors have used this ideal for infinite-horizon or infinite horizon models with a discount factor such as Belo-Filho et al. [2015]; Haijema et al. [2017]; Hendrix et al. [2012, 2015, 2018]; Pauls-Worm et al. [2015]. However, it is difficult to solve the recursive equations in many instances, due to the "curse of dimensionality" or the complexity of the model. That is why many researchers develop and evaluate heuristics for inventory management. Some researchers used stochastic dynamic programming for blood management at blood banks such as Haijema et al. [2017]. The authors suggested that an SDP

approach could be most useful in combination with simulation techniques. Similarly, Hendrix et al. [2018] compared different order policies for an inventory control problem for a perishable product with a maximum fixed shelf life. The system is reviewed periodically under non-stationary demand, fixed ordering cost and a service level (chance) constraint.

Some mixed integer linear programming models could be found in Claassen et al. [2016]; Dillon et al. [2017]; Gunpinar and Centeno [2015]; Pauls-Worm et al. [2015]. Gunpinar and Centeno [2015] presented a stochastic integer programming models to minimize the total cost, shortage and wastage levels of blood inventory. In their models, the uncertainties related to demand and a crossmatch-to-transfusion ratio. Claassen et al. [2016] used mixed integer linear programming to solve a planning problem in the food processing industry. They noted that optimal production schedules change significantly when product decay is considered. Based on two-stage stochastic programming, Dillon et al. [2017] studied the blood inventory under uncertainty in demand. In this model, they consider multiple types of blood, perishability, lead times, and periodic review policy.

Many researchers have studied inventory under fuzzy circumstances such as Chen and Ouyang [2006]; Halim et al. [2008]; Hsiao et al. [2017]; Katagiri and Ishii [2002]; Shaikh et al. [2018]. As an example, Hsiao et al. [2017] developed a perishable inventory model for ready-to-eat food under fuzzy environment by combining the traditional deterioration model and quality prediction model. The storage temperature fluctuation of products is simulated by fuzzy temperature. Lately, Shaikh et al. [2018] proposed a Quantum-behaved particle swarm optimization algorithm to solve a fuzzy inventory model for a deteriorating item with the permissible delay in payments. In this paper, the demand depends on selling price and the advertisement frequency.

In recent decades, machine learning has become a popular approach and also widely used in many fields. Among the machine learning methods, reinforcement learning is considered appropriate to solve sequential decision-making problems in the changing environment. Reinforcement learning allows the use of the past knowledge to make future choices. Recently, reinforcement learning has been employed to solve different problems in inventory management. This method has been implemented to find near-optimal ordering policies in an inventory system. The study in Kara and Dogan [2017] proposes a reinforcement learning method to deal with a perishable inventory system under the random demand and deterministic lead time.

## 2.4 Modeling and solution approaches for stochastic optimization

This subsection presents relevant approaches used to cope with optimization under uncertainty such as stochastic dynamic programming and stochastic programming ap-

proaches. In this thesis, these approaches are mainly applied to solve biomass supply chain and inventory control/management problems with stochastic supply and demand. The two-stage stochastic programming model with recourse is presented in the following subsection and decomposition methods are explained in Section 2.4.2.

### 2.4.1 Two-stage stochastic optimization approach

In a two-stage stochastic programming model with recourse, the decision variables are divided into those of two different stages. In the first stage, the decision maker chooses decisions $x$ associated with a cost $c^{\mathrm{T}}x$ before knowing the actual realization of random parameters $\omega$. In the second stage, an action associated with vector $y$ (recourse action) can be taken to correct the adverse effects that result from the first stage decision $x$. The joint objective function of two-stage recourse model is to minimize the first stage cost and the expected cost of the second stage.

In most applications, the first stage is often related to strategic decisions such as facilities' capacities and locations whereas the second stage usually refers to operational or tactical decisions such as production schedule, inventory control, transportation planning.

The conventional formulation of a two-stage stochastic linear program with fixed recourse is given as follows:

$$\begin{aligned} \min \quad & c^{\mathrm{T}}x + \mathbb{E}_{\omega}[\mathrm{Q}(x,\omega)] \\ \text{s.t} \quad & \mathrm{A}x = b, \quad x \geq 0 \end{aligned} \tag{2.1}$$

where:

$$\begin{aligned} & \mathrm{Q}(x,\omega) = \min_{y} q_s^{\mathrm{T}} y \\ \text{s.t} \quad & \mathrm{W}y = h_{\omega} - \mathrm{T}_{\omega}x \\ & y \geq 0 \end{aligned}$$

and

- $\omega$ is the random vector defined on $\Omega$
- $x \in \mathbb{R}^{n_1}$ is the first stage decision variable. This means that decision $x$ must be made before observing outcome ("here and now").
- $c \in \mathbb{R}^{n_1}$ is fixed cost associated with decision $x$.
- $\mathrm{A} \in \mathbb{R}^{m_1 \times n_1}$ , $b \in \mathbb{R}^{m_1}$ are fixed matrix and vector that related to restrictions constraints on $x$ at the first stage.

- $y \in \mathbb{R}^{n_2}$ is the second stage decision variable. Decision $y$ is made after event occurs and was observed. Obviously, decisions $y$ depend on $x$.
- $q_\omega \in \mathbb{R}^{n_2}$ is the cost associated with the second stage decision $y$.
- $\mathrm{T}_\omega \in \mathbb{R}^{m_2 \times n_1}$ is called technology matrix, is given $\forall s \in \mathrm{S}$
- $\mathrm{Q}(x, \omega)$ be the objective function value of the second stage problem
- $h_\omega \in \mathbb{R}^{m_2}$ is the right hand-side in y restriction constraints for all $\omega in \Omega$
- $\mathrm{W} \in \mathbb{R}^{m_2 \times n_2}$ is the recourse matrix. We assume here that W is fixed (fixed recourse)

Let S denotes the set of all realization of the random variable $\omega$, called scenarios. Let $s = 1, 2, \ldots, |\mathrm{S}|$ index of all possible scenarios. Each scenario $s$ is associated with a probability of occurrence $p_s$ and the sum of the probabilities for all scenarios equal to 1. Let $y_s$ be the second-stage decision variables for each scenario $s$. $\mathrm{T}_s$ is the matrix related to the first stage decision variables $x$. $\mathrm{W}_s$ is a recourse matrix, $q_s$ and $h_s$ are two vectors associated with scenario $s$ in the second stage model. The objective of this problem is to minimize the cost of the first stage ($c^{\mathrm{T}} x$) and the expected cost over all scenarios of the second stage $\sum_{s \in \mathrm{S}} p_s q_s^{\mathrm{T}} y_s$.

The general model (2.1) can be expanded as a deterministic equivalent model.

$$
\begin{array}{llllll}
\min & c^{\mathrm{T}} x + & p_1 q_1^{\mathrm{T}} y_1 + & p_2 q_2^{\mathrm{T}} y_2 + & \cdots + & p_s q_s^{\mathrm{T}} y_s \\
s.t & \mathrm{A}x & & & & = b \rightarrow \text{first-stage problem} \\
 & \left.\begin{array}{llllll}
\mathrm{T}_1 x + & \mathrm{W}_1 y_1 & & & & = h_1 \\
\mathrm{T}_2 x + & & \mathrm{W}_2 y_2 & & & = h_2 \\
\vdots & & & \ddots & & \vdots \\
\mathrm{T}_s x + & & & & \mathrm{W}_s y_s & = h_s \\
x \geq 0, & y_s \geq 0 & \forall s \in \mathrm{S} & & &
\end{array}\right\} \text{Scenario subproblem}
\end{array}
\tag{2.2}
$$

If the number of scenarios considered is small, the deterministic equivalent model could be solved by any commercial solver. The number of scenarios is determined by using a statistical method Linderoth et al. [2006]; Shapiro [2000] to obtain solutions within specific confidence intervals for a desired level of accuracy. This method is very effective for scenario reduction, particularly for large-scale problems. Besides, the two-stage problem 2.2 has a desirable structure which can be exploited in the Benders' framework to solve these problems. For any further and detailed information of the Benders' Principle, the interested reader can easily refer to Appendix B. By extending this decomposition method

in stochastic programming, Birge and Louveaux [1988]; Van Slyke and Wets [1969] introduced the L-shaped algorithm that decompose the model 2.2 into a first-stage problem and a number of scenarios sub-problem.

### 2.4.2 Decomposition methods

#### L-shaped algorithm

Special block (L-shaped) structure of two-stage stochastic linear program with fixed recourse allows to apply L-shaped algorithm to find a solution Van Slyke and Wets [1969]. This is a popular method for solving stochastic programming models. Similar to Bender decomposition, the basic idea of L-shaped method is to first solve the model with those constraints that do not include the second stage variables to obtain the values of first stage decisions. Then solving all the scenario sub-problems with first stage decisions found in the previous step to obtain the optimal values of the second stage decisions. In addition, the L-shaped method allows to construct convex estimation of the recourse function and the optimality Bender cuts can be added to the first-stage problem in each iteration.

The master problem (or the first stage problem) is given by:

$$\begin{aligned} \min_{x,\theta} \quad & c^{\mathrm{T}} x + \theta \\ \text{s.t} \quad & \theta \geq \mathrm{D}^k x + d^k \qquad k = 1,2...\mathrm{K} \\ & 0 \geq \mathrm{E}^k x + e^k \quad k = 1,2...\mathrm{K} \\ & \mathrm{A}x = b, x \in \mathrm{X} \end{aligned} \tag{2.3}$$

where $k$ denotes the $k^{th}$ iteration, K is the number of iterations so far. The first and second constraints in the master problem refer to the optimality and feasibility cuts, respectively. The subproblem for each scenario $s$ is given as follows and is solved with the values $x$ given by the solution of the first-stage problem:

$$\begin{aligned} \min_{y_s} \quad & q_s^{\mathrm{T}} y_s \\ \text{s.t} \quad & \mathrm{W} y_s = h_s - \mathrm{T}_s x \\ & y_s \geq 0 \end{aligned} \tag{2.4}$$

In each iteration, the first-stage problem and the subproblem for each scenario $s$ are linked through optimality cuts. These optimality cuts are generated from the dual of the subproblem. The coefficients $\mathrm{D}^k$ and $d^k$ of the optimality cut are given as follows:

$$\begin{aligned} \mathrm{D}^k &= \sum_{s \in \mathrm{S}} p_s \pi_{k,s}^{\mathrm{T}} \mathrm{T}_s \quad \forall s \in \mathrm{S}, \ \ k = 1,2...\mathrm{K} \\ d^k &= \sum_{s \in \mathrm{S}} p_s \pi_{k,s}^{\mathrm{T}} h_s \quad \forall s \in \mathrm{S}, \ \ k = 1,2...\mathrm{K} \end{aligned} \tag{2.5}$$

where $\pi_s$ are the optimal dual vector of the subproblem for each scenario $s \in S$.

Note that feasibility cuts Birge and Louveaux [1988] can be add to the master problem to deal with infeasible subproblems. If the subproblem is infeasible for some scenario $s$, a dual extreme ray $\mu_{k,s}$ can be obtained and generate the feasibility cut $0 \geq E^k\varphi + e^l \quad \forall l = 1,2...L$ to the master problem. The coefficients are given as follows:

$$\begin{aligned} E^l &= \mu_{l,s}^T T_s \quad \forall l = 1,2...L \\ e^l &= \mu_{l,s}^T h_s \quad \forall l = 1,2...L \end{aligned} \tag{2.6}$$

The standard L-shaped algorithm only generate one cut per iteration to the master problem. Thus, it might need many iterations and running time to reach the optimality gap, $\epsilon$ for large-scale problem.

**Multi-cut L-shaped algorithm**

To speed up the L-shaped algorithm, Birge and Louveaux [1988] showed that the problem of calculating the expected value in the second stage problem could be decomposed by scenarios $s$ and multiple cuts as many as the number of scenarios can be generated. These cuts allow to reduce the solution space of the first stage variables in each iteration. The multi-cut L-shaped algorithm takes a fewer number of iterations to reach an optimal solution in comparison with the corresponding L-shaped algorithm, but each iteration may take a longer computational time in solving a large number of scenario subproblems.

Now, let's introduce new variables $\theta_s, s \in S$ in the solution framework. These variables provide an link between the first-stage problem 2.7 and the scenario subproblem 2.4. $\theta_s$ can be interpreted as an approximation of $Q(\varphi, s)$. In this multi-cut version, the master problem (or the first stage problem) is given by:

$$\begin{aligned} \min_{x,\theta} \quad & c^T x + \sum_{s \in S} p_s \theta_s \\ \text{s.t} \quad & \theta_s \geq D_s^k x + d_s^k \qquad k = 1,2...K \\ & 0 \geq E_s^l x + e_s^l \quad \forall l = 1,2...L \\ & Ax = b, x \in X \end{aligned} \tag{2.7}$$

The subproblem for each scenario $s$ is also formulated as the one in 2.4. In each iteration, the first-stage problem and the subproblem for each scenario $s$ are linked through optimality cuts and feasibility cuts. By solving the dual of the subproblem 2.4, we obtain the optimal dual variable $\pi_s$ for each scenario and then compute the coefficient $D_s^k$ and $d_s^k$ as follows:

$$\begin{aligned} D_s^k &= \pi_{k,s}^T T_s \quad \forall s \in S, \ k = 1,2...K \\ d_s^k &= \pi_{k,s}^T h_s \quad \forall s \in S, \ k = 1,2...K \end{aligned} \tag{2.8}$$

If there is at least one infeasible subproblem under scenario $s$, a dual extreme ray $\mu_{l,s}$ can be obtained and generate the feasibility cut $0 \geq E^k\varphi + e^k \ \forall l = 1,2...L$ and add to the master problem. The coefficients $E_s^l$ and $e_s^l$ of a feasibility cut are given as follows:

$$\begin{aligned} E^l &= \mu_{l,s}^T T_s \quad \forall l = 1,2...l \\ e^l &= \mu_{l,s}^T h_s \quad \forall l = 1,2...L \end{aligned} \tag{2.9}$$

The framework for the multi-cut L-shaped algorithm is very similar to the standard one. It is outlined as follows:

(i) In each iteration, the first-stage problem is solved and the dual of subproblem 2.4 is solved as many times as the number of scenarios $s \in S$

(ii) By solving the first-stage problem, the values of the first stage decision variables $\varphi$ are determined and the value of function $Q(x,s)$ at each scenario $s$ in the second stage is approximated by $\theta_s$.

(iii) By solving the dual of subproblem 2.4, the expected value of the second stage problem for all scenarios can be found for the given value of the first stage variables found in the previous step.

(iv) In each iteration, after solving the dual of the scenario subproblem, multiple cuts of type with the number equal to that of scenarios are added to the first-stage problem.

## 2.5 Conclusion

From the analysis of recent literature on biomass supply chain management, we observe that many models are used to formulate problems at different decision levels at different objectives to meet various decision-making requirements. Almost all works are based on deterministic models, whereas the use of stochastic approach is quite rare although it can reflect better real-life situations than the previous one. To find an optimal solution of the models, most authors used a commercial solver such as CPLEX, GUROBI, GAMS, and XPRESS while the rest developed their algorithms such as Lagrangian relaxation, Bender decomposition. In addition, we discover that no previous work addresses the issue of supplier selection for better stabilizing feedstock supply in biomass supply chains. In fact, stabilizing feedstock supply would contribute to the sustainability of a biomass supply chain and allows to improve competitiveness with fossil energy.

Thus, in Chapters 3 and 4, we address the supplier selection and operation planning problem in biomass supply chains. These two issues are analyzed by considering two approaches: deterministic and stochastic. The first approach is suitable for the "what-if" analysis where decision-makers already know the inputs and want to evaluate the outcome of their decisions. This model is then extended to consider under environmental

uncertainty by applying a two-stage stochastic programming approach in Chapter 4. An improved L-shaped algorithm is developed to solve the stochastic model.

An overview of perishable inventory management has identified the important factors such as demand, supply, uncertainty, price, deterioration rate, shelf life, replenishment policy, etc. It is clear that the scientific community did not consider all of these factors at the same time. For example, uncertainty in both supply and demand can be easily found in the real world, but only few works considered this kind of uncertainty. Most of models proposed to combinate stochastic demand with one or more specific factors such as discount price, credit, and different payment problems; inflation and time value for money; investment, promotion and budget constraint; customer satisfaction to represent real-life issues. From the analysis, we discover that the study of inventory models for perishable products under both stochastic demand and stochastic supply is essential for the inventory management of biomass supply chains. In such inventory management, some available information may be not precise due to environmental uncertainty. So it is logical to develop appropriate methods such as fuzzy, stochastic and dynamic approaches to capture uncertainty in a perishable inventory problem.

Thus, a perishable inventory model under both stochastic demand and supply will be presented in Chapter 5 based on dynamic programming. In Chapter 5, the product considered is assumed to have a constant deterioration rate. We then develop a solution method based on the scenario optimization approach and Lagrangian relaxation to find a near-optimal solution. In the last chapter, we extend this model to a product with a fixed lifetime.

# Chapter 3

# Modeling and optimization of biomass supply chain with two types of feedstock suppliers

*" Knowledge is an unending adventure at the edge of uncertainty. "*

Jacob Bronowski

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## 3.1 Introduction

This chapter presents a mixed-integer linear approach for a lignocellulosic biomass supply chain planning including different logistics activities such as: feedstock supply, preprocessing, storage, transportation and biofuel production.

According to Li et al. [2011], lignocellulosic biomass such as corn, wheat and switchgrass can be used as feedstock and is interchangeable with minimal effect on biorefinery production costs and bioethanol yields. The multi-biomass approach could be a potential solution, which uses a mix of feedstock (corn, wheat and switchgrass) harvested/collected in different time period. Stabilization of feedstock has a critical impact on the sustainability of a biomass supply chain. To stabilize the feedstock supply, it is essential to integrate supplier selection decision with operational planning. In other words, the better understanding of the operational plan, the better supplier selection decision in this problem.

To the best of our knowledge, none of these studies present a multi-period supply chain model to cope with the supplier selection and the design of operation planning. For this reason, we present a multi-period mixed integer programming model that can be applied to capture most activities and supplier selection decision in biomass supply chain. Our model has a flexible structure which allows for the modeling of value chains supplier selection and different logistic activities (transportation and operation planning). We also evaluated the economic impact of the supplier selection and operation planning on a biomass supply chain.

This chapter is organized as follows: In Section 3.2, the problem and its model are introduced. A numerical study is presented and its results are analyzed in Section 3.3. Section 3.4 concludes this chapter with some remarks.

## 3.2 Problem description and model formulation

### 3.2.1 Problem description

In this chapter, we study a biomass supply chain as shown in Figure 3.1 and develop a deterministic mixed integer linear programming (MILP) model to support decision-makers to observe the outcomes of their decisions in different inputs/conditions. Our study focused on how to select feedstock suppliers and establish a mid/long-term contract with them to ensure the stabilization of feedstock supply. Moreover, numerous constraints should be respected such as feedstock availability, biomass degradations, resource capacities, and refinery demands.

In the model, the production infrastructure such as pretreatment facilities and biorefineries have been located. We assume that biofuel producers seek to establish a long-term relationship with certain local/regional suppliers in order to stabilize the feedstock supply. The unit purchase price and the minimum delivery quantity are fixed under a

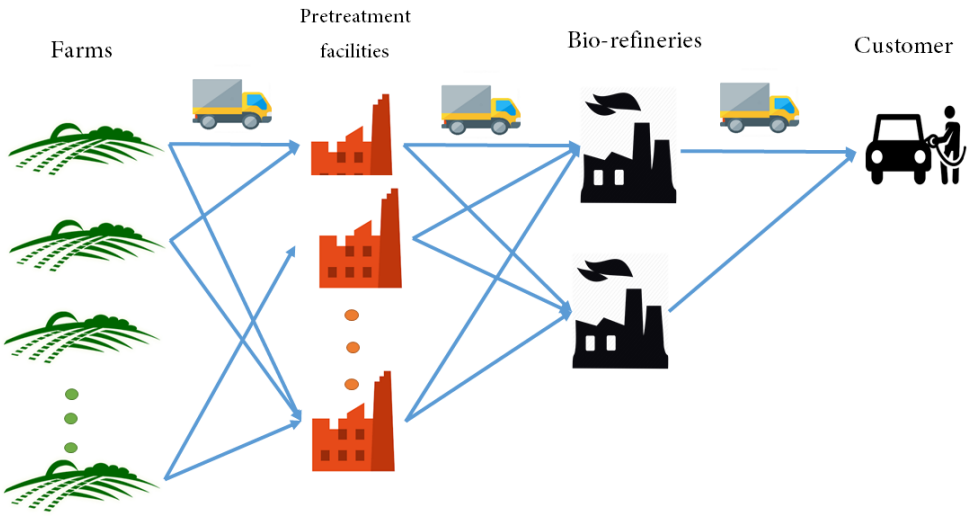


Figure 3.1: A biomass supply chain

long-term contract during a planning horizon. The unit transportation cost per trip from local/regional suppliers to biorefineries is pre-computed based on their distances and fixed costs associated with trucks.

The logistics activities occurred in each period are presented as follows:

- In each period, biomass is transported from contracted suppliers to preprocessing facilities, then to biorefineries.
- In each conversion facility, the supplied biomass feedstock is pre-processed and then converted into biofuel or be kept as inventory at the biorefinery.
- Biomass can be bought from international/national markets to meet part of biofuel demand only at a reasonable price of biomass.
- After satisfying the biofuel demand in each period, any excess volume of biofuel is kept as inventory. A shortage cost is charged for any unit of unmet biofuel demand.
- Biomass preprocessed/biofuel produced is limited by biomass preprocessing/processing capacity and also by storage capacity in each facility.
- Many lignocellulosic crops can be used lignocellulosic biomass (corn, wheat, and switchgrass...) can be used as feedstock and is interchangeable in biofuel production because their compositions are very similar according to Li et al. [2011].

### 3.2.2 Model formulation

In the following, we formulate a MILP model that considers the suppliers selection with a mid/long-term contract, inventory management and transportation planning. We consider a planning horizon of T periods, each period could be a week, a month or a

Table 3.1: List of parameters and variables

| | |
|---|---|
| **Set** | |
| T | Set of periods $t$ |
| I | Set of suppliers $i$ |
| P | Set of preprocessing facilities $p$ |
| B | Set of biorefineries $b$ |
| A | Set of all arcs in the biomass supply chain A = (I × P) ∩ (P × B) ∩ (M × B) |
| **Parameters** | |
| $avail^{\mathrm{I}}(i,t)$ | Amount of biomass (tons) available at supplier $i$ in period $t$ |
| $cost\mathrm{A}^{\mathrm{I}}(i)$ | Unit purchase cost of biomass from supplier $i$ (\$/ton) |
| $\mathrm{Q}_{min}(i)$ | Minimum supply quantity (tons) under contract with supplier $i$ during planning horizon T |
| $mcost(i)$ | Fixed cost for establishing a contract with supplier $i$ |
| | *International/national market* |
| $avail^{\mathrm{M}}(m,t)$ | Amount of biomass (tons) available at market $m$ in period $t$ |
| $cost\mathrm{A}^{\mathrm{M}}(m)$ | Unit purchase cost of biomass from market $m$ (\$/ton) |
| | *Preprocessing site* |
| $\Gamma_p^{max}(p)$ | Maximum amount of biomass (tons) that could be preprocessed at preprocessing facility $p$ |
| $\Gamma_p^{min}(p)$ | Minimum amount of biomass (tons) that could be preprocessed at preprocessing facility $p$ |
| $\mathrm{W}^{max}(p)$ | Maximum biomass storage capacity (tons) at preprocessing facility $p$ in each period |
| $\mathrm{W}^{min}(p)$ | Minimum biomass storage capacity (tons) at preprocessing facility $p$ in each period |
| $hcost\mathrm{P}(p)$ | Biomass inventory holding cost at preprocessing facility $p$ (\$/ton) |
| $tcost\mathrm{P}(p)$ | Biomass preprocessing cost of preprocessing facility $p$ (\$/ton) |
| $\lambda^{\mathrm{P}}(p)$ | Ratio of biomass storage loss at preprocessing facility $p$ in each period |
| $\eta^{\mathrm{P}}(p)$ | Preprocessing rate at preprocessing facility $p$ in each period |
| | *Biorefinery site:* |
| $\Gamma_b^{max}(b)$ | Maximum amount of biomass (tons) could be processed by biorefinery $b$ in each period |
| $\Gamma_b^{min}(b)$ | Minimum amount of biomass (tons) could be processed by biorefinery $b$ in each period |
| $\mathrm{V}_{av}^{max}(b)$ | Maximum biomass storage capacity (tons) of biorefinery $b$ in each period |
| $\mathrm{V}_{av}^{mmin}(b)$ | Minimum biomass storage capacity(tons)of biorefinery $b$ in each period |
| $\mathrm{V}_{ap}^{max}(b)$ | Maximum biofuel storage capacity(litres) of biorefinery $b$ in each period |
| $\mathrm{V}_{ap}^{mmin}(b)$ | Minimum biofuel storage capacity (litres) of biorefinery $b$ in each period |
| $pcost(b)$ | Unit penalty cost for unmet biofuel demand at biorefinery $b$ (\$/tons) |
| $tcost\mathrm{B}(b)$ | Biofuel production cost at biorefinery $b$ (\$/tons) |
| $hcost\mathrm{B}_{av}(b)$ | Unit inventory holding cost of biomass at biorefinery site $b$ in each period (\$ / ton) |
| $hcost_{ap}^{\mathrm{B}}(b)$ | Unit inventory holding cost of biofuel at biorefinery $b$ in each period (\$/ ton) |
| $\eta^{\mathrm{B}}(b)$ | Biofuel yield rate of lignocellulosic biomass for biorefinery $b$ (litre/ton) |
| $\lambda^{\mathrm{B}}(b)$ | Ratio of biomass storage loss at biorefinery $b$ in each period |
| $demand(b,t)$ | Demand of biofuel(litre) at biorefinery $b$ in period $t$ |
| | *Transportation*: |
| $\mathrm{C}ap\mathrm{V}(i,j)$ | Transportation capacity from node $i$ to node $j$ (tons/truck) |
| $\mathrm{V}cost(i,j)$ | Unit transportation cost of each trip of a truck from node $i$ to node $j$ (\$ /trip) |
| **Decision variables** | |
| | *Continuous variables* |
| $x(i,j,t)$ | Amount of biomass transported from node $i$ to node $j$ in each period $t$ |
| $\mathrm{W}(p,t)$ | Biomass inventory level (tons) at preprocessing facility $p$ in period $t$ |
| $\mathrm{V}_{av}^{\mathrm{B}}(b,t)$ | Inventory level of biomass at biorefinery $b$ in period $t$ |
| $\mathrm{V}_{ap}^{\mathrm{B}}(b,t)$ | Inventory level of biofuel at biorefinery $b$ in period $t$ |
| $\mathrm{Q}^{\mathrm{P}}(p,t)$ | Amount of biomass preprocessed (tons) at preprocessing facility $p$ in period $t$ |
| $z(b,t)$ | Amount of biomass required (tons) to produce biofuel at refinery $b$ in period $t$ |
| $short(b,t)$ | Shortage of biofuel (liters) at biorefinery $b$ in period $t$ |
| | *Integer variables* |
| $nbr\mathrm{T}(i,j,t)$ | Number of truck trips from node $i$ to node $j$ in period $t$ |
| | *Binary variables* |
| $\varphi(i)$ | Equal to 1 if a contract between refinery $b$ and supplier $i$ is established, 0 otherwise |

trimester. Let I be a set of biomass suppliers, P be a set of pretraitement facilities and B be a set of biorefineries.

Let $\varphi = \{\varphi(i), i \in \mathrm{I}\}$ be the decision variables related to mid/longterm contracts between the biorefineries and suppliers, $\varphi(i) = 1$ if a long term contract is established with

supplier $i$, 0 otherwise. The vector $mcost = \{mcost(i), i \in \mathrm{I}\}$ includes fixed cost $mcost(i)$ for establishing a contract with supplier $i$.

Denote $x(i, j, t, s)$ as the flow of biomass on arc $(i, j) \in \mathrm{A}$ in period $t$. Such an arc $(i, j)$ could be arc $(i, p)$ or $(p, b)$ that refers to an arc from supplier $i \in \mathrm{I}$ to preprocessing facility $p \in \mathrm{P}$ or from preprocessing facility $p \in \mathrm{P}$ to biorefinery $b$. Let $pre(n)$, $suc(n)$ denote, respectively, the set of predecessor and the set of successors of a given node $n$. For example, $suc(i), i \in \mathrm{I}$ indicates a set of pretreatment facilities to which biomass feedstock is delivered from farm $i$; $pre(p), p \in \mathrm{P}$ indicates a set of farms that supply biomass feedstock to pretreatment facility $p$; $suc(p), p \in \mathrm{P}$ indicates a set of biorefineries to which biomass is delivered from pretreatment facility $p$.

Let $cost\mathrm{A}^{\mathrm{I}}(i)$, $cost\mathrm{A}^{\mathrm{M}}(m)$ be unit purchase price of biomass from supplier $i$ and from market $m$, respectively; $tcost\mathrm{P}(p)$ be unit biomass preprocessing cost at pretreatment facility $p$; $hcost\mathrm{B}_{av}(b)$, $hcost^{\mathrm{B}}_{ap}(b)$ be unit inventory holding costs of biomass and biofuel at biorefinery site $b$ in each period, respectively; $\eta^{\mathrm{B}}(b)$ be the biofuel yield rate and $\lambda^{\mathrm{B}}(b)$ be the ratio of biomass storage loss at biorefinery $b$ in each period.

For convenience, all notations used in the model formulation are summarized in Table 3.1. All continuous decision variables are non-negative. The minimization of the different costs of the biomass supply chain can be modeled by the mixed linear program detailed as follows:

**Objective function:**

$$
\begin{aligned}
\min \quad z = & \sum_{t\in\mathrm{T}} \sum_{(i,j)\in\mathrm{A}} nbr\mathrm{T}(i,j,t) * \mathrm{V}cost(i,j) \\
& + \sum_{t\in\mathrm{T}} \sum_{p\in\mathrm{P}} \eta^{\mathrm{P}} \mathrm{Q}^{\mathrm{P}}(p,t)\, tcost\mathrm{P}(p) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} short(b,t)\, pcost\mathrm{B}(b) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} \eta^{\mathrm{B}} z(b,t)\, cost\mathrm{B}(\mathrm{B}) \\
& + \sum_{t\in\mathrm{T}} \sum_{p\in\mathrm{P}} hcost^{\mathrm{P}}(p)\mathrm{W}(p,t) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} hcost^{\mathrm{B}}_{av}(b)\mathrm{V}_{av}(b,t) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} hcost^{\mathrm{B}}_{ap}(b)\mathrm{V}_{ap} \\
& + \sum_{t\in\mathrm{T}} \sum_{i\in\mathrm{I}} \mathrm{Q}^{\mathrm{I}}(i,t) * cost\mathrm{A}^{\mathrm{I}}(i) \\
& + \sum_{t\in\mathrm{T}} \sum_{m\in\mathrm{M}} \sum_{b\in\mathrm{B}} x(m,b,t) * cost\mathrm{A}^{\mathrm{M}}(m) \\
& + \sum_{i\in\mathrm{I}} \phi(i) * mcost(i) \qquad (3.1)
\end{aligned}
$$

The objective function (3.1) is to minimize the total cost, composed of nine costs: transportation costs (1st term), pre-traitement costs (2nd term), shortage costs (3rd term),

biofuel conversion costs (4th terms) holding costs(5-7th terms), purchase costs from suppliers and markets (8-9th terms) and fixed cost for establishing a contract with suppliers (last term).

**Constraints on suppliers**

$$\mathrm{Q}^{\mathrm{I}}(i,t) = \sum_{p \in suc(i)} x(i,p,t) \qquad \forall t \in \mathrm{T}, i \in \mathrm{I} \tag{3.2}$$

$$\mathrm{Q}^{\mathrm{I}}(i,t) \leq avail^{\mathrm{I}}(i,t).\varphi(i) \qquad \forall t \in \mathrm{T}, i \in \mathrm{I} \tag{3.3}$$

$$\mathrm{Q}_{min}(i)\varphi(i) \leq \sum_{t \in \mathrm{T}} \mathrm{Q}^{\mathrm{I}}(i,t) \qquad \forall i \in \mathrm{I} \tag{3.4}$$

Equation (3.2) defines the total amount of feedstock is transported from supplier $i$ to pretreatment facilities in each period. Note that the decision variable $\mathrm{Q}^{\mathrm{I}}(i,t)$ is redundant but it is introduced for improving the readability of the model.

Inequality (3.3) states that there is no feedstock supply unless there is a mid/long-term contract between supplier $i$ and biorefinery $b$. This inequality also ensures that the total purchase amount must not exceed the available amount in each period. Inequality (3.4) ensures the minimum supply quantity under a contract between a supplier $i$ and the biorefinery $b$ during planning horizon T.

**Constraints on market**

$$\sum_{b \in \mathrm{B}} x(m,b,t) \leq avail^{\mathrm{M}}(m,t) \qquad \forall t \in \mathrm{T}, m \in \mathrm{M} \tag{3.5}$$

Inequality (3.5) shows that the purchase amount of biomass is also limited by the market's supply availability.

**Constraints on preprocessing sites**

$$\mathrm{Q}^{\mathrm{P}}(p,t) = \sum_{(i) \in pre(p)} x(i,p,t) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P} \tag{3.6}$$

$$\mathrm{W}(p,t) = \lambda^{\mathrm{P}}(p)\mathrm{W}(p,t-1) + \eta^{\mathrm{P}}\mathrm{Q}^{\mathrm{P}}(p,t) - \sum_{(b) \in suc(p)} x(p,b,t) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P} \tag{3.7}$$

$$\Gamma_p^{min}(p) \leq \mathrm{Q}^{\mathrm{P}}(p,t) \leq \Gamma_p^{max}(p) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P} \tag{3.8}$$

$$\mathrm{W}^{min}(p) \leq \mathrm{W}(p,t) \leq \mathrm{W}^{max}(p) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P} \tag{3.9}$$

Equation (3.6) defines the total amount of biomass is transported from pretreatment facility $p$ to refineries in each period. Note that the decision variable $\mathrm{Q}^{\mathrm{P}}(p,t)$ is redundant but it is introduced for improving the readability of the model. Constraint (3.7) is the inventory balance constraints for biomass storage at preprocessing facility. Constraint (3.8) ensures the capacity production of each pre-processing plant.

**Constrains on refinery sites**

$$\begin{aligned} \mathrm{V}_{av}(b,t) &= \lambda^{\mathrm{B}}(b)\mathrm{V}_{av}(b,t-1) + \sum_{(p)\in pre(b)} x(p,b,t) \\ &+ \sum_{m\in \mathrm{M}} x(m,b,t) - z(b,t) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \qquad (3.10) \\ \mathrm{V}_{ap}(b,t) &= \mathrm{V}_{ap}(b,t-1) + \eta^{\mathrm{B}} z(b,t) \\ &+ short(b,t) - demand(b,t) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \qquad (3.11) \\ \Gamma_b^{max}(b) &\le z(b,t) \le \Gamma_b^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \qquad (3.12) \\ \mathrm{V}_{av}^{min}(b) &\le \mathrm{V}_{av}^{\mathrm{B}}(b,t) \le \mathrm{V}_{av}^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \qquad (3.13) \\ \mathrm{V}_{ap}^{min}(b) &\le \mathrm{V}_{ap}^{\mathrm{B}}(b,t) \le \mathrm{V}_{ap}^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \qquad (3.14) \end{aligned}$$

Constraints (3.10) and (3.11) are the inventory balance constraints for biomass and biofuel storage at biorefinery, respectively. These constraints ensure that no more biomass or biofuel is delivered or processed at a location than available amount in stock. Constraint (3.11) ensures that biofuel demand is satisfied as much as possible, and unsatisfied demand is lost. Shortage cost is incurred for any unit lost.

The capacity production of each refinery plant is respected via constraint (3.12). Constraints (3.13) and (3.14) show inventory capacity for biomass and biofuel at refinery site, respectively.

**Constraints on transportation**

$$x(i,j,t) \le nbr\mathrm{T}(i,j,t) * \mathrm{C}ap\mathrm{V}(i,j) \qquad (3.15)$$

Constraint (3.15) shows the number of trips required for transporting a given amount of biomass from node $i$ to node $j$ in each period. The total flows should not be greater than the required number of vehicles in anytime period.

This model allows the user to formulate complex biomass supply chains by adding a new nodes (suppliers, preprocessing/refinery plants) and arcs linking these nodes.

## 3.3 Numerical study

In this section, a numerical study is conducted to evaluate the economic aspect and some critical factors of the biomass supply chain. This model is implemented in Python 3.5 on a HP computer with Intel Core i5-4210M CPU 2.66 GHz and 8.0 GB RAM. All MILP problems were solved by a solver GUROBI 6.5 with default parameters.

### 3.3.1 Data generation

To demonstrate the use of the biomass supply chain model, a simple case study is carried out. In the baseline case, we consider a biomass supply chain with one refinery,

two pretreatment site, 100 supplier candidates over 12 periods. Each period in this study corresponds to one month. The biofuel producer seeks to sign a contract with several local feedstock suppliers.

Concerning suppliers, amount of biomass available in each supplier can be estimated from a land area for cultivation and biomass yield. We also use parameter data from Osmani and Zhang [2013] and other sources such as literature reviews and research articles to validate our model.Contracts with suppliers are characterized by fixed cost, purchase price and minimum quantity to supply. Purchase price varies from 35 to 37 $/ton. The minimum supply quantity varies from 20 000 to 100 000 tons. Fixed cost for establishing a contract varies from $20 000 to $30 000. We precomputed transportation cost per trip based on distances, variable transportation cost, and fixed cost.

All 100 candidates of supplier selection are located within 50 km radius of the biorefinery. The amount of biomass available in each supplier is estimated from land availability according to Osmani and Zhang [2013, 2014]. For each supplier, biomass yield is generated from uniform distribution U[3.5,4.5]. A ratio of biomass storage loss varies from 1 to 3% per month (worst case). The biorefinery plant has a production capacity of 190 MLPY with a unit production cost of 0.2$/ liter. The bioethanol yield rate of lignocellulosic biomass is about 313 liters/ton. The unit biofuel storage cost and unit biomass storage cost at biorefinery is 0.227 $/liter and 0.9 $/ ton, respectively. The unit shortage cost is 1.06 $/liter.

The dataset is summarized in Table 3.2. Based on historical data (19981-2005) in Osmani and Zhang [2013], we generate annual biofuel demand randomly from Normal probability distribution N(2133,4138) and truncated on the interval [2006,2280]. The model size increases when the numbers of periods, suppliers, pretraitement facilities and biorefineries increase. For testing the model with different instances, we adjust biofuel demand according to the scalability of feedstock suppliers.

Table 3.2: Data of model parameters

| Input parameters | Value |
|---|---|
| Bioethanol yield rate (litre/tonne) | 313.00 |
| Penalty cost for unmet bioethanol demand ($/liter) | 1.06 |
| Bioethanol production cost ($/liter) | 0.20 |
| Maximum annual production capacity of biorefinery(MLPY) | 380.00 |
| Minimum annual production capacity of biorefinery(MLPY) | 190.00 |
| Preprocessing cost at preprocessing facility ($/ton) | 13.39 |
| Inventory cost for biomass in time period t ($/ton) | 0.90 |
| Ratio of biomass storage loss in time period | 0.01 |
| Ratio of biomass storage loss in time period (%) | 1-3 % |
| Maximum annual pretraitement capacity (tons) | 2 000 000.00 |
| Maximum biomass storage capacity (tons) of biorefinery | 3 500 000.00 |
| Maximum biomass storage capacity (tons) of pretraitement facility | 3 500 000.00 |
| Yield biomass feedstocks (tons/hecta) | Uniform[3.5, 4.5] |
| Unit biomass purchase price from suppliers ($/ton) | 35-37 |

### 3.3.2 Analysis of the solution

In this section, we analyze our numerical results. The optimal value of total cost in the supply chain is $181 281 119 when a biomass supply chain includes one refinery, two pretreatment site and 100 supplier candidates over 12 time periods. For this instance, the whole model has in fact 3 808 constraints, 2 532 continuous variables and 1 324 integer variables. The running time is reasonable: an optimal solution is obtained in less than 5 seconds (1.78 seconds).

Fig 3.2 represents the distribution of costs in the biomass supply chain over planning horizon T= 12. It is evident that transportation costs play an essential role in the biomass supply chain with approximatively 25% of total system costs. This result shows the importance of transportation operation in biomass supply chains. Holding costs is only 2.3% because in this study the amount of biomass available for biofuel production is higher than the demand, so the storage of a significant amount of biomass at biorefinery seems to be not critical.

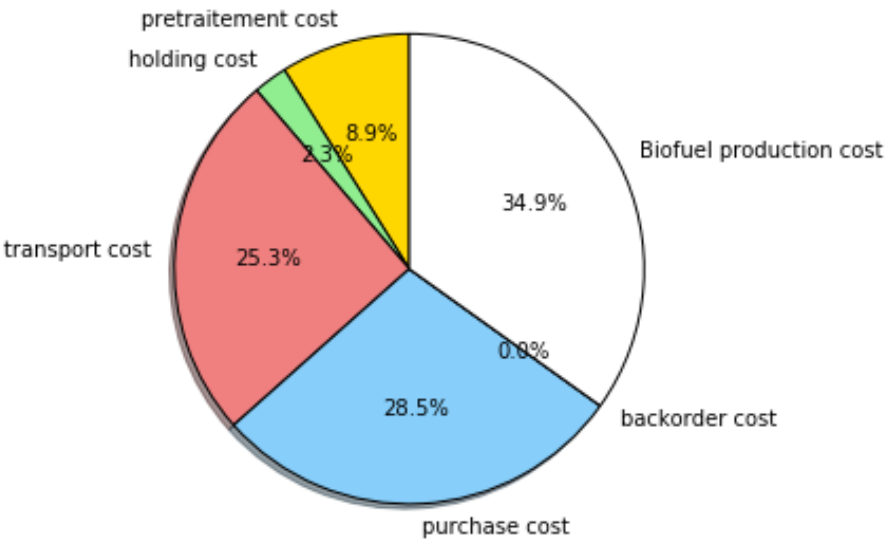


Figure 3.2: Distribution of cost in supply chain

One important aspect of the model is the running time which depends on the number of periods, suppliers, pretreatment facilities and biorefineries. We further tested this model for different instances when I= 10, 20, 50, 100, 150, 200, 250, 300, 350, 400; P= 1, 2, 5 and T= 12. The problem size and execution time of these instances are summarized in Table 3.3. The results illustrate that this model can be solved quickly and exactly with a commercial solver.

### 3.3.3 Sensitivity analysis

We then study the impacts of some critical parameters (purchase price from a market, minimum supply quantity and fixed contracted cost) on the total system cost. As these

Table 3.3: Problem size and solution performance

| | | | | | Problem size | | | |
|---|---|---|---|---|---|---|---|---|
| **N°** | **I** | **P** | **B** | **T** | **Integer** | **Continous** | **Constraint** | **CPU(s)** |
| 1 | 10 | 1 | 1 | 12 | 142 | 336 | 442 | 0.04 |
| 2 | 20 | 1 | 1 | 12 | 272 | 576 | 812 | 0.14 |
| 3 | 50 | 1 | 1 | 12 | 662 | 1296 | 1922 | 0.15 |
| 4 | 100 | 1 | 1 | 12 | 1312 | 2496 | 3772 | 0.52 |
| 5 | 200 | 1 | 1 | 12 | 2612 | 4896 | 7472 | 3.26 |
| 6 | 10 | 2 | 1 | 12 | 154 | 372 | 478 | 0.03 |
| 7 | 20 | 2 | 1 | 12 | 284 | 612 | 848 | 0.07 |
| 8 | 50 | 2 | 1 | 12 | 674 | 1332 | 1958 | 0.85 |
| 9 | 100 | 2 | 1 | 12 | 1324 | 2532 | 3808 | 1.78 |
| 10 | 200 | 2 | 1 | 12 | 2624 | 4932 | 7508 | 5.44 |
| 11 | 10 | 5 | 1 | 12 | 190 | 480 | 586 | 0.05 |
| 12 | 20 | 5 | 1 | 12 | 320 | 720 | 956 | 0.07 |
| 13 | 50 | 5 | 1 | 12 | 710 | 1440 | 2066 | 0.13 |
| 14 | 100 | 5 | 1 | 12 | 1360 | 2640 | 3916 | 0.37 |
| 15 | 150 | 5 | 1 | 12 | 2010 | 3840 | 5766 | 0.80 |
| 16 | 200 | 5 | 1 | 12 | 2660 | 5040 | 7616 | 0.84 |
| 17 | 250 | 5 | 2 | 12 | 3310 | 6240 | 9466 | 1.46 |
| 18 | 300 | 5 | 2 | 12 | 3960 | 7440 | 11316 | 2.01 |
| 19 | 350 | 5 | 2 | 12 | 4610 | 8640 | 13166 | 9.58 |
| 20 | 400 | 5 | 2 | 12 | 5260 | 9840 | 15016 | 12.84 |

parameters have significant impacts on feedstock supply, it is essential to learn how they affect the optimal expected total cost of biomass supply chain and the total number of contracted suppliers.

### Effect of purchase price

Figure 3.3 shows the total system cost and the number of contracted suppliers under different values of the purchase price that varies between 90% and 130 % of its base value. The horizontal axis represents the various purchase price of biomass from an international or regional market. The bar graphs with the height measured by the vertical axis on the left correspond to the total costs of this supply chain under different values of the purchase price. The dotted line with the value of each point measured by the vertical axis on the right indicates the number of contracted suppliers at each possible value of the purchase price considered.

We observe that the total system cost is less sensitive to the variation of the market price. In fact, the optimal total system cost increases slightly from $ 179 828 032 to $ 181 281 119 (less than 1%) when the market price varies from 90% to 100% of its base value. Then, it remains nearly constant when the market price is set from 100% to 130% of its basic level. This relationship confirms that when the market price increases too high, decision-makers have less interest in purchasing an additional quantity of biomass from markets to meet biofuel demand. In this case, they would intend to establish more long-

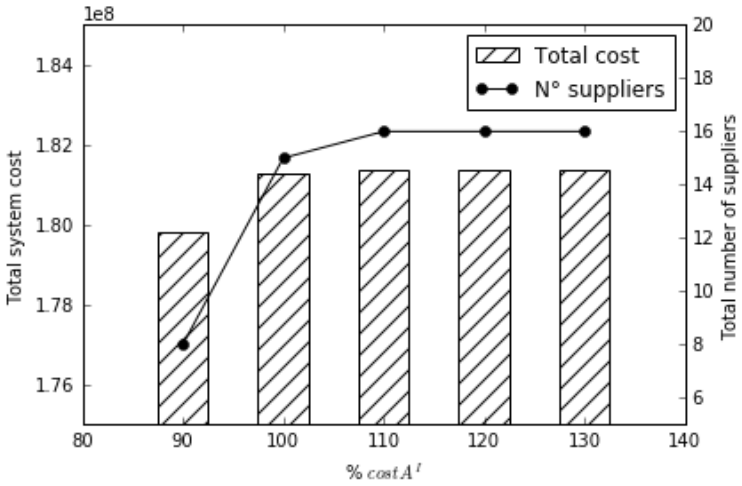


Figure 3.3: Distribution of cost in supply chain

term supply contracts to ensure the stability of feedstock supply. This explains why the number of contracted suppliers increases when the market price grows.

**Effect of fixed cost**

Figure 3.4 shows the effect of fixed cost on the biomass supply chain in term of the total system cost and the number of contracted suppliers. The horizontal axis represents the fixed cost varying from 50% to 400% in comparison to its base value. The bar graphs with the height measured by the vertical axis on the left hand present the total expected system costs under different values of the fixed cost. The dotted line with the value of each point measured by the vertical axis on the right hand indicates the number of contracted suppliers at each possible value of the fixed cost.

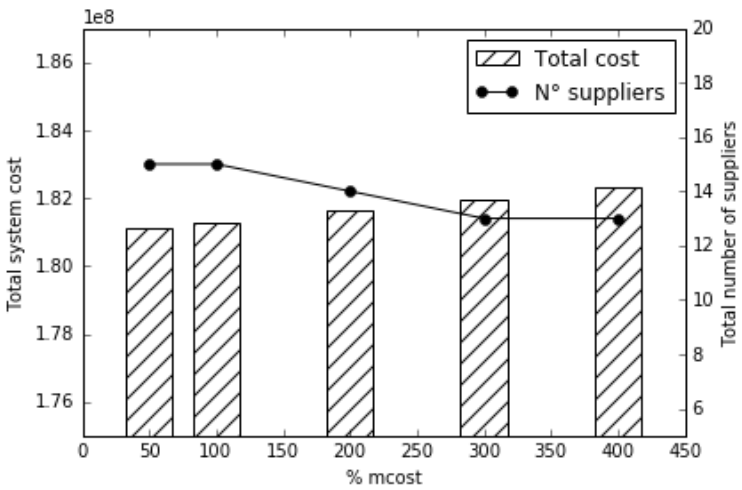


Figure 3.4: Distribution of cost in supply chain

This figure shows that the increase in fixed cost may lead to higher total system cost and might hinder the establishment of long-term contracts. However, it is a slight effect on the overall system cost and the total number of contracted suppliers.

**Effect of the minimum supply quantity of suppliers**

Figure 3.5 presents the impacts of a contracted supply quantity on the total system cost of biomass supply chain and the number of contracted suppliers. The horizontal axis represents the various contracted quantity delivered from 60% to 180% in comparison with its base value. The bar graphs with the height measured by the vertical axis on the left hand show the total expected system costs under different values of the minimum supply quantity. The dotted line with the value of each point measured by the vertical axis on the right hand indicates the number of contracted suppliers at each possible value of the minimum supply quantity.

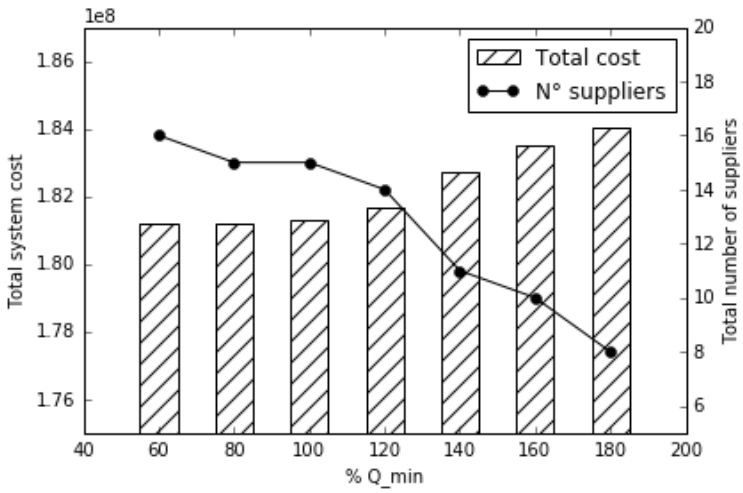


Figure 3.5: Distribution of cost in supply chain

The figure shows that the minimum supply quantity and the total system cost are proportional to each other; however, the increase of minimum supply quantity is not as much as the one of total system cost. Besides, the total number of contracted suppliers decreases as the supply quantity increases. In this case, decision-maker would require additional quantity from market to satisfy the final demand but only at a reasonable market price.

## 3.4 Conclusion

This chapter presents a mixed integer linear programming (MILP) model for minimizing the total system costs in a biomass supply chain. The model integrates all supply chain logistics such as transportation plan, purchase activities, inventory control, and production to provide an optimal solution. The case presented has been solved optimally with different instances in reasonable computation time. Moreover, we have analyzed the impacts of some critical parameters on the optimal expected cost of the system and supplier selection.

This chapter could provide a decision-making support tool for biomass supply chain management for a "what-if" scenarios where decision markers want to evaluate the outcomes from their decisions with different inputs. In the next chapter, we extended our model into stochastic environments.

The work in this chapter was presented at the 7th International Conference on Industrial Engineering and Systems Management in Saarbrücken, Germany [Duc-Huy et al., 2017].

# Chapter 4

# Supplier selection and Operation planning in biomass supply chain with supply uncertainty

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## 4.1 Introduction

In the previous chapter, we have developed a deterministic model to provide the supplier selection and operation planning decisions in a biomass supply chain. This deterministic approach is only appropriate when all parameters are precise. In real-life situations, we often encounter the presence of uncertainty in biomass feedstock supply because its availability and yield depend on seasonality (weather condition, temperature) and harvest time. Therefore, this uncertainty is a critical factor that must be accounted for because it has a substantial impact on the sustainability of a biomass supply chain.

To the best of our knowledge, the study of supplier selection in a biomass supply chain was rare in the literature. There is no work considering strategical supplier selection along with tactical planning under uncertain environments although the two types de decisions are interdependent. For this reason, we present a stochastic programming model that can be used to capture most activities in a biomass supply chain and the uncertainty in feedstock supply. The main contributions of this chapter include:

Firstly, we propose a two-stage stochastic programming model for biomass supply chain management. In the model, the first stage decision is to select suppliers who provide long/mid term feedstock supply. The second-stage decisions determine the amount of biomass to purchase, to transport, to pre-process and the volume of bioethanol to produce in each period.

Secondly, we develop an enhanced and regularized L-shaped decomposition method to solve the stochastic programming model with a given number of scenarios. The Monte Carlo sampling approach is used to find the required number of scenarios for ensuring that the exact value of an output of the model is located within a confidence interval with a given probability.

Thirdly, we conduct a numerical study to evaluate the performance of the proposed algorithm. The numerical results show that the algorithm can find an optimal solution in a reasonable computation time while a commercial solver cannot for large instances. Sensitivity analysis is performed to investigate the effect of some critical parameters on the optimal expected cost and suppliers selection.

This chapter is organized as follows: Section 4.2 describes the problem studied and its model are introduced. We describe the solution method for the model in Section 4.3. A numerical study is presented, and its results are analyzed in Section 4.4. Section 4.5 concludes this chapter with some remarks for future research.

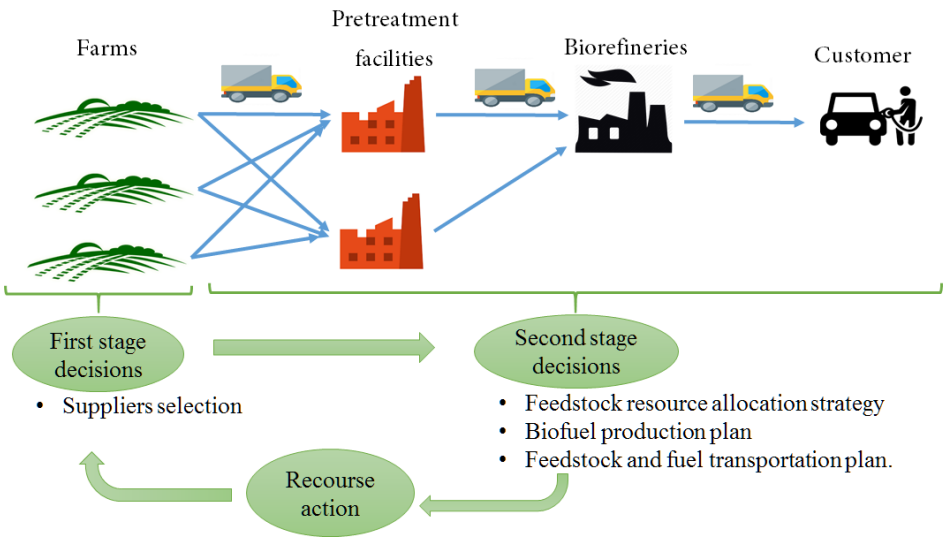


Figure 4.1: A two stage stochastic programming model with recourse for a biomass supply chain

## 4.2 Problem description and model formulation

### 4.2.1 Problem description

This chapter studies a biomass supply chain under uncertainty in supply as shown in Figure 4.1. Biorefineries seek to establish a long-term relationship with certain local/regional suppliers to stabilize its feedstock supply. Under a mid/long-term contract, the unit purchase price and the minimum supply quantity are predetermined. The logistics activities occurred in each period are presented as follows:

- Biorefineries can buy biomass from international/national markets (if available) to meet part of biofuel demand only at a reasonable price of biomass.
- In each period, biomass is transported from contracted suppliers to preprocessing facilities, then to biorefineries. In each conversion facility, the supplied biomass feedstock is pre-processed and then converted into biofuel or be kept as inventory at the biorefinery.
- After satisfying the biofuel demand in each period, any excess volume of biofuel is kept as inventory. In case of shortfall in biofuel requirement, a relatively high shortage cost is charged for any unmet biofuel demand.
- Biomass preprocessed/biofuel produced is limited by biomass preprocessing/processing capacity and also by storage capacity in each facility. During storage, biomass has a ratio of loss about 1-2% per month.

We assume that the production infrastructure such as pretreatment facilities and biorefineries have been located. The unit transportation costs from local/regional suppliers to biorefineries are pre-calculated based on their distances and fixed costs are associated.

In the following, we introduce a two-stage stochastic programming model to tackle the supplier selection and operation planning in a biomass supply chain. The general framework of the two-stage stochastic programming model is given in the next subsection.

### 4.2.2 Mathematical formulation

In this section, we formulate a two-stage stochastic programming model that considers the selection of suppliers with a mid/long-term contract, inventory management and transportation planning in a biomass supply chain. The first-stage decision variables include only binary variables which determine the choice of suppliers with long/mid-term contracts. The second-stage variables include the amount of biomass to be purchased from suppliers, the amount of biomass feedstock to be transported to biorefinery, the amount of biomass to be used to transform into bioethanol, the volume of unmet bioethanol, the inventory of biomass and bioethanol in each period (Figure 4.1).

We consider a planning horizon of T periods, each period corresponds to a month. Let I be a set of biomass suppliers, P be a set of pretreatment facilities, B be a set of biorefineries, S be a set of stochastic scenarios and A be the set of arcs in the biomass supply chain A. An arc $(i, j) \in \mathrm{A}$ stands for a path with biomass flow from node $i$ to node $j$ in the supply chain network considered. A node may be a supplier, a pretreatment facility or a biorefinery. Let $pre(n)$, $suc(n)$ denote, respectively, the set of predecessor and the set of successors of a given node $n$. For example, $suc(i), i \in \mathrm{I}$ indicates a set of pretreatment facilities to which biomass feedstock is delivered from farm $i$; $pre(p), p \in \mathrm{P}$ indicates a set of farms that supply biomass feedstock to pretreatment facility $p$; $suc(p), p \in \mathrm{P}$ indicates a set of biorefineries to which biomass is delivered from pretreatment facility $p$.

The first-stage and second stage model are presented as follows:

$$\textbf{(SP):} \quad \min_{\varphi} \left\{ mcost^{\mathrm{T}}\varphi + \sum_{s\in \mathrm{S}} p_s \mathrm{Q}(\varphi, s) \mid \varphi \in \{0,1\} \right\} \tag{4.1}$$

$$\text{where:} \quad \mathrm{Q}(\varphi, s) = \min_{y_s} \left\{ f(\varphi, s, y_s) \mid \text{subject to (4.4)} - \text{(4.19)} \right\} \tag{4.2}$$

where $\varphi = \{\varphi(i), i \in \mathrm{I}\}$ are the first stage decision variables related to mid/longterm contracts between the biorefineries and suppliers, $\varphi(i) = 1$ if a long term contract is established with supplier $i$, 0 otherwise. The vector $mcost = \{mcost(i), i \in \mathrm{I}\}$ includes fixed cost $mcost(i)$ for establishing a contract with supplier $i$, which is independent of scenario $s$. The function $\mathrm{Q}(\varphi, s)$ refers to the cost function of the second stage problem for each scenario $s$, in which the first stage decisions, $\varphi$ are already made.

For convenience, all notations used in the model formulation are summarized in Table 4.1. All continuous decision variables are non-negative. The constraints (4.4 - 4.19) will be presented in the next part of this section. The different components of the second stage total cost $f(\varphi, s, y_s)$ under scenario $s$ are given below; where $y_s$ contains all second-stage

Table 4.1: List of parameters and variables

| | |
|---|---|
| **Set** | |
| T | Set of periods $t$ |
| I | Set of suppliers $i$ |
| P | Set of preprocessing facilities $p$ |
| B | Set of biorefineries $b$ |
| M | Set of markets $m$ |
| A | Set of all arcs in the biomass supply chain A = (I × P) ∩ (P × B) ∩ (M × B) |
| S | Set of scenarios $s$ |
| **Deterministic parameters** | |
| | *Local/Regional suppliers* |
| $mcost(i)$ | Fixed cost for establishing a contract with supplier $i$ |
| $cost\mathrm{A}^{\mathrm{I}}(i)$ | Unit purchase price of biomass from supplier $i$ (\$/ton) |
| $q_{min}(i)$ | Minimum supply quantity of biomass (tons) under contract with supplier $i$ during planning horizon T |
| | *International/national market* |
| $avail^{\mathrm{M}}(m,t)$ | Amount of biomass (tons) available at market $m$ in period $t$ |
| $cost\mathrm{A}^{\mathrm{M}}(m)$ | Unit purchase price of biomass from market $m$ (\$/ton) |
| | *Preprocessing site* |
| $\gamma_p^{max}(p)$ | Maximum amount of biomass (tons) that could be preprocessed at preprocessing facility $p$ |
| $\gamma_p^{min}(p)$ | Minimum amount of biomass (tons) that could be preprocessed at preprocessing facility $p$ |
| $w^{max}(p)$ | Maximum biomass storage capacity (tons) at preprocessing facility $p$ in each period |
| $w^{min}(p)$ | Minimum biomass storage capacity (tons) at preprocessing facility $p$ in each period |
| $w^{init}(p)$ | Initial stock of biomass (tons) at preprocessing facility $p$ |
| $hcost\mathrm{P}(p)$ | Unit holding cost at preprocessing facility $p$ (\$/ton) |
| $tcost\mathrm{P}(p)$ | Unit biomass preprocessing cost of preprocessing facility $p$ (\$/ton) |
| $\lambda^{\mathrm{P}}(p)$ | Ratio of biomass storage loss at preprocessing facility $p$ in each period |
| $\eta^{\mathrm{P}}(p)$ | Preprocessing rate at preprocessing facility $p$ in each period |
| | *Biorefinery site* |
| $\gamma_b^{max}(b)$ | Maximum amount of biomass (tons) that could be processed by biorefinery $b$ in each period |
| $\gamma_b^{min}(b)$ | Minimum amount of biomass (tons) that could be processed by biorefinery $b$ in each period |
| $\mathrm{V}_{av}^{init}(b)$ | Initial stock of biomass (tons) of biorefinery $b$ |
| $\mathrm{V}_{av}^{max}(b)$ | Maximum biomass storage capacity (tons) of biorefinery $b$ in each period |
| $\mathrm{V}_{av}^{min}(b)$ | Minimum biomass storage capacity(tons)of biorefinery $b$ in each period |
| $\mathrm{V}_{ap}^{init}(b)$ | Initial stock of biofuel (litres) of biorefinery $b$ |
| $\mathrm{V}_{ap}^{max}(b)$ | Maximum biofuel storage capacity(litres) of biorefinery $b$ in each period |
| $\mathrm{V}_{ap}^{min}(b)$ | Minimum biofuel storage capacity (litres) of biorefinery $b$ in each period |
| $pcost(b)$ | Unit shortage cost for unmet biofuel demand at biorefinery $b$ (\$/tons) |
| $tcost\mathrm{B}(b)$ | Unit biofuel production cost at biorefinery $b$ (\$/tons) |
| $hcost\mathrm{B}_{av}(b)$ | Unit inventory holding cost of biomass at biorefinery site $b$ in each period (\$ / ton) |
| $hcost_{ap}^{\mathrm{B}}(b)$ | Unit inventory holding cost of biofuel at biorefinery $b$ in each period (\$/ ton) |
| $\eta^{\mathrm{B}}(b)$ | Biofuel yield rate of biomass for biorefinery $b$ (litre/ton) |
| $\lambda^{\mathrm{B}}(b)$ | Ratio of biomass storage loss at biorefinery $b$ in each period |
| $demand(b,t)$ | Demand of biofuel(litre) at biorefinery $b$ in period $t$ |
| | *Transportation*: |
| $\mathrm{V}cost(i,j)$ | Unit transportation cost from node $i$ to node $j$ (\$ /tons) |
| **Stochastic parameters** | |
| $avail^{\mathrm{I}}(i,t,s)$ | Amount of biomass (tons) available at supplier $i$ in period $t$ under scenario $s$ |
| **Decision variables** | *Continuous variables* |
| $x(i,j,t,s)$ | A flow of biomass on arc $(i,j) \in$ A in period $t$ under scenario $s$, originally from node $i$ to node $j$ |
| $w(p,t,s)$ | Biomass inventory level (tons) at preprocessing facility $p$ in period $t$ under scenario $s$ |
| $\mathrm{V}_{av}(b,t,s)$ | Inventory level of biomass at biorefinery $b$ in period $t$ under scenario $s$ |
| $\mathrm{V}_{ap}(b,t,s)$ | Inventory level of biofuel at biorefinery $b$ in period $t$ under scenario $s$ |
| $q^{\mathrm{I}}(i,t,s)$ | Total amount of biomass transported from supplier $i$ in period $t$ under scenario $s$ |
| $q^{\mathrm{P}}(p,t,s)$ | Total amount of biomass transported to preprocessing facility $p$ in period $t$ under scenario $s$ |
| $z(b,t,s)$ | Amount of biomass required (tons) to produce biofuel at refinery $b$ in period $t$ under scenario $s$ |
| $short(b,t,s)$ | Shortage of biofuel (litres) at biorefinery $b$ in period $t$ under scenario $s$ |
| | *Binary variables* |
| $\varphi(i)$ | Equal to 1 if a mid/long-term contract between the biorefineries and supplier $i$ is established, 0 otherwise |
| | *Dual variables* |
| $\pi_{(.)}$ | Dual variables associated with the constraints (5)-(20) for problem **(P2)**. For readability, each dual variable is written in bracket at the right hand of the corresponding constraint in model **(P2)** |

decision variables, $y_s = (x, w, \mathrm{V}_{av}, \mathrm{V}_{ap}, q^{\mathrm{P}}, z, short)$.

$$
\begin{aligned}
f(\varphi, s, y_s) = & \sum_{t\in\mathrm{T}} \sum_{(i,j)\in\mathrm{A}} x(i,j,t,s) * \mathrm{V}cost(i,j) \\
& + \sum_{t\in\mathrm{T}} \sum_{p\in\mathrm{P}} \eta^{\mathrm{P}} * q^{\mathrm{P}}(p,t,s) * tcost\mathrm{P}(p) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} short(b,t,s) * pcost(b) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} \eta^{\mathrm{B}} * z(b,t,s) * tcost\mathrm{B}(\mathrm{B}) \\
& + \sum_{t\in\mathrm{T}} \sum_{p\in\mathrm{P}} hcost^{\mathrm{P}}(p) w(p,t,s) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} hcost^{\mathrm{B}}_{av}(b) \mathrm{V}_{av}(b,t,s) \\
& + \sum_{t\in\mathrm{T}} \sum_{b\in\mathrm{B}} hcost^{\mathrm{B}}_{ap}(b) \mathrm{V}_{ap}(b,t,s) \\
& + \sum_{t\in\mathrm{T}} \sum_{i\in\mathrm{I}} q^{\mathrm{I}}(i,t,s) * cost\mathrm{A}^{\mathrm{I}}(i) \\
& + \sum_{t\in\mathrm{T}} \sum_{m\in\mathrm{M}} \sum_{b\in\mathrm{B}} x(m,b,t,s) * cost\mathrm{A}^{\mathrm{M}}(m)
\end{aligned} \tag{4.3}
$$

where $x(i,j,t,s)$ is the flow of biomass on arc $(i,j) \in \mathrm{A}$ in period $t$ under scenario $s$. Such an arc $(i,j)$ could be arc $(i,p)$ or $(p,b)$ that refers to an arc from supplier $i \in \mathrm{I}$ to preprocessing facility $p \in \mathrm{P}$ or from preprocessing facility $p \in \mathrm{P}$ to biorefinery $b$. $w(p,t,s)$ is the inventory level at preprocessing facility $p$ in period $t$ under scenario $s$.

The variables $\mathrm{V}_{av}(b,t,s)$, $\mathrm{V}_{ap}(b,t,s)$ are the inventory level of biomass and the inventory level of biofuel at biorefinery $b$ in period $t$ under scenario $s$, respectively. $z(b,t,s)$, $short(b,t,s)$ are the amount of biomass used to produce biofuel and the shortage of biofuel at biorefinery $b$ in period $t$ under scenario $s$, respectively. $q^{\mathrm{I}}(i,t,s)$ is the total amount of biomass transported from supplier $i$ to the pretreatment facilities in period $t$ under scenario $s$. $q^{\mathrm{P}}(p,t,s)$ is the total amount of biomass transported to pretreatment facility $p$ in period $t$ under scenario $s$. Note that the decision variables $q^{\mathrm{I}}(i,t,s)$ and $q^{\mathrm{P}}(p,t,s)$ are redundant but they are introduced for the readability of the model. $cost\mathrm{A}^{\mathrm{I}}(i)$, $cost\mathrm{A}^{\mathrm{M}}(m)$ are unit purchase price of biomass from supplier $i$ and from market $m$, respectively; $tcost\mathrm{P}(p)$ is unit biomass preprocessing cost at pretreatment facility $p$; $hcost\mathrm{B}_{av}(b)$, $hcost^{\mathrm{B}}_{ap}(b)$ are unit inventory holding costs of biomass and biofuel at biorefinery site $b$ in each period, respectively; $\eta^{\mathrm{B}}(b)$ is the biofuel yield rate and $\lambda^{\mathrm{B}}(b)$ is the ratio of biomass storage loss at biorefinery $b$ in each period.

The objective function is to minimize the total cost of the supply chain which includes the first stage costs and the expected second-stage costs. The first stage costs comprise the total costs for establishing contracts with local/regional suppliers. Equation (4.3) gives the second stage costs $\mathrm{Q}(\varphi, s)$ for each scenario $s$, which comprise nine costs: transportation costs (1st term), pretreatment costs (2nd term), shortage costs (3rd term), biofuel

conversion costs (4th term ), holding costs (5th-7th terms), and purchase costs (9th-10th terms).

**Constraints on suppliers**

$$q^{\mathrm{I}}(i,t,s) = \sum_{p \in suc(i)} x(i,p,t,s) \qquad \forall t \in \mathrm{T}, i \in \mathrm{I}, s \in \mathrm{S} \qquad [\pi^1_{i,t,s}] \qquad (4.4)$$

$$q^{\mathrm{I}}(i,t,s) \leq avail^{\mathrm{I}}(i,t,s).\varphi(i) \qquad \forall t \in \mathrm{T}, i \in \mathrm{I}, s \in \mathrm{S} \qquad [\pi^2_{i,t,s}] \qquad (4.5)$$

$$\sum_{t \in \mathrm{T}} q^{\mathrm{I}}(i,t,s) \geq q_{min}(i).\varphi(i) \qquad \forall i \in \mathrm{I}, s \in \mathrm{S} \qquad [\pi^3_{i,s}] \qquad (4.6)$$

Equation (4.4) defines the total amount of feedstock transported from supplier $i$ to the pretreatment facilities in each period under scenario $s$.

Inequality (4.5) states that there is no feedstock supply from supplier $i$ unless there is a mid/long-term contract between the supplier and biorefinery $b$. This inequality also ensures that the total purchase amount does not exceed the available amount of the supplier in each period under scenario $s$.

Inequality (4.6) specifies the minimum supply quantity under a contract between a supplier $i$ and the biorefinery $b$ during planning horizon T.

**Constraints on market**

$$\sum_{b \in \mathrm{B}} x(m,b,t,s) \leq avail^{\mathrm{M}}(m,t) \qquad \forall t \in \mathrm{T}, m \in \mathrm{M} \qquad [\pi^4_{m,t,s}] \qquad (4.7)$$

Inequality (4.7) ensures that the purchase quantity of biomass is also limited by the market's supply availability in each scenario.

**Constraints on preprocessing sites**

$$q^{\mathrm{P}}(p,t,s) = \sum_{i \in pre(p)} x(i,p,t,s) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^5_{p,t,s}] \qquad (4.8)$$

$$\begin{aligned} w(p,t,s) &= (1-\lambda^{\mathrm{P}}(p))\, w(p,t-1,s) + \eta^{\mathrm{P}} q^{\mathrm{P}}(p,t,s) \\ &\quad - \sum_{b \in suc(p)} x(p,b,t,s) \qquad \forall t \geq 2, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^6_{p,t,s}] \qquad (4.9) \end{aligned}$$

$$\begin{aligned} w(p,1,s) &= (1-\lambda^{\mathrm{P}}(p))\, w^{init}(p) + \eta^{\mathrm{P}} q^{\mathrm{P}}(p,1,s) \\ &\quad - \sum_{b \in suc(p)} x(p,b,1,s) \qquad \forall p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^7_{p,s}] \qquad (4.10) \end{aligned}$$

$$\gamma^{min}_p(p) \leq q^{\mathrm{P}}(p,t,s) \leq \gamma^{max}_p(p) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^{8a/b}_{p,t,s}] \qquad (4.11)$$

$$w^{min}(p) \leq w(p,t,s) \leq w^{max}(p) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^{9a/b}_{p,t,s}] \qquad (4.12)$$

Equation (4.8) defines the total amount of biomass transported from suppliers to pretreatment facility $p$ in each period under scenario $s$. Equation (4.9) and (4.10) ensure the inventory balance for each stock at preprocessing facility under scenario $s$. These constraints ensure that no more biomass is delivered from or processed at a location than its available amount in stock. The capacity of each preprocessing facility and its inventory

capacity are given by constraints (4.11) and (4.12), respectively.

**Constrains on refinery sites**

$$\begin{aligned}
\mathrm{V}_{av}(b,t,s) &= (1-\lambda^{\mathrm{B}}(b))\mathrm{V}_{av}(b,t-1,s) - z(b,t,s) \\
&\quad + \sum_{p\in pre(b)} x(p,b,t,s) + \sum_{m\in \mathrm{M}} x(m,b,t,s) \qquad \forall t\geq 2, b\in \mathrm{B}, s\in \mathrm{S} \qquad [\pi^{10}_{b,t,s}] \quad (4.13)\\
\mathrm{V}_{av}(b,1,s) &= (1-\lambda^{\mathrm{B}}(b))\mathrm{V}^{init}_{av}(b) - z(b,1,s) \\
&\quad + \sum_{p\in pre(b)} x(p,b,1,s) + \sum_{m\in \mathrm{M}} x(m,b,1,s) \qquad \forall b\in \mathrm{B}, s\in \mathrm{S} \qquad [\pi^{11}_{b,s}] \quad (4.14)\\
\mathrm{V}_{ap}(b,t,s) &= \mathrm{V}_{ap}(b,t-1,s) + \eta^{\mathrm{B}} z(b,t,s) \\
&\quad - demand(b,t) + short(b,t,s) \qquad \forall t\geq 2, b\in \mathrm{B}, s\in \mathrm{S} \qquad [\pi^{12}_{b,t,s}] \quad (4.15)\\
\mathrm{V}_{ap}(b,1,s) &= \mathrm{V}^{init}_{ap}(b) + \eta^{\mathrm{B}} z(b,1,s) \\
&\quad - demand(b,1) + short(b,1,s) \qquad \forall b\in \mathrm{B}, s\in \mathrm{S} \qquad [\pi^{13}_{b,s}] \quad (4.16)\\
\gamma^{min}_{b}(b) &\leq z(b,t,s) \leq \gamma^{max}_{b}(b) \qquad \forall t\in \mathrm{T}, b\in \mathrm{B}, s\in \mathrm{S} \qquad [\pi^{14a/b}_{b,t,s}] \quad (4.17)\\
\mathrm{V}^{min}_{av}(b) &\leq \mathrm{V}_{av}(b,t,s) \leq \mathrm{V}^{max}_{av}(b) \qquad \forall t\in \mathrm{T}, b\in \mathrm{B}, s\in \mathrm{S} \qquad [\pi^{15a/b}_{b,t,s}] \quad (4.18)\\
\mathrm{V}^{min}_{ap}(b) &\leq \mathrm{V}_{ap}(b,t,s) \leq \mathrm{V}^{max}_{ap}(b) \qquad \forall t\in \mathrm{T}, b\in \mathrm{B}, s\in \mathrm{S} \qquad [\pi^{16a/b}_{b,t,s}] \quad (4.19)
\end{aligned}$$

Equations (4.13 -4.16) are the inventory balance constraints for each biomass or biofuel stock at biorefinery $b$, respectively. These constraints ensure that no more biomass or biofuel is delivered from or processed at a location than its available amount in stock. They also ensure that biofuel demand is satisfied as much as possible, and any unsatisfied demand is lost. The shortage cost is charged for any unit lost.

Equation (4.18) and (4.19) are inventory capacity constraints for biomass and biofuel stock at biorefinery $b$. The conversion capacity of each refinery facility is given by constraints (4.17).

## 4.3 Solution approach

In this section, the stochastic programming model (SP) is transformed into a deterministic equivalent model (DEP) by applying the scenario approach. The second model can be solved by a commercial solver like CPLEX ou GUROBI. However, if the number of scenarios is large, the resolution of the model requires an enormous memory and computational effort. Therefore, we propose in this section an enhanced and regularized decomposition (ERD) method for the model to overcome the computational difficulties.

### 4.3.1 Scenario-based approach and deterministic equivalent model

We reformulate the model **(SP)** as a deterministic mixed integer linear programming (MILP) model as follows:

$$\textbf{(DEP)} \quad \min_{x, y_s} \quad \left( mcost^{\mathrm{T}} \varphi + \sum_{s \in \mathrm{S}} p_s f(\varphi, s, y_s) \right)$$
$$s.t. \quad \varphi(i) \in \{0, 1\} \quad \forall i \in \mathrm{I}$$
$$y_s \quad \text{subject to constraints (4.4)-(4.19)}$$

This (DEP) model could be solved by a commercial solver like CPLEX or GUROBI. However, for an instance considering a large number of scenarios, the size of the model is too large to be solved by such a solver due to the memory requirement of a computer. For example, in case of an instance with I = 60, P = 6, T = 12 and S = 1000 scenarios, the coefficient matrix of the DEP model has 1.680.000 rows and 1.716.060 columns. A computer uses 12 bytes to store each non-zero element of this matrix. Thus, it requires at least $1.716.060x1.680.000 \times 12/10^9 \approx 240$ GB to store the matrix. So, to solve this instance, the PC should have at least 240GB of RAM. Such high memory requirement could not be met by most PCs. For this reason, we develop a stochastic decomposition method to solve the model in a reasonable time with much less memory requirement.

### 4.3.2 Multi-cut L-shaped algorithm

The scenario-based stochastic programming approach allows to capture the uncertainty in an approximate way with the precision depending on the number of scenarios considered. The size of the model can grow dramatically due to the consideration of a large number of scenarios. By extended a Bender decomposition in stochastic programming, Van Slyke and Wets [1969] introduced an iterative method, namely the L-shaped algorithm. To speed up this algorithm, Birge and Louveaux [1988] showed that the problem of calculating the expected value in the second stage problem could be decomposed by scenarios $s$ and multiple cuts as many as the number of scenarios can be generated. These cuts allow to reduce the solution space of the first stage variables in each iteration. The multi-cut L-shaped algorithm takes a fewer number of iterations to reach an optimal solution in comparison with the corresponding L-shaped algorithm, but each iteration may take a longer computational time in solving a large number of scenario subproblems.

We now introduce a new variable vector $\theta = (\theta_s, s \in \mathrm{S})$ that provides an link between the first-stage problem **(P1)** and the scenario subproblem **(P2)**. $\theta_s$ can be interpreted as

an approximation of $Q(\varphi, s)$. The first-stage problem **(P1)** is thus given by:

$$\begin{aligned} \textbf{(P1)} \quad & \min_{\varphi,\theta} \quad mcost^{\mathrm{T}}\varphi + \sum_{s\in \mathrm{S}} p_s\theta_s \\ s.t. \quad & \theta_s \geq d_s^k\varphi + e_s^k \quad \forall s \in \mathrm{S}, \;\; k = 1,2...\mathrm{K} \\ & \varphi(i) \in \{0,1\} \quad \forall i \in \mathrm{I} \end{aligned} \tag{4.20}$$

where $k$ denotes the $k^{th}$ iteration, K is the number of iterations so far. In the first iteration, the first-stage problem is solved without considering the second stage variables and their constraints. Then, the subproblem for each scenario $s \in \mathrm{S}$ is formulated as follows and is solved with the values of the first-stage variables, $\varphi$ given by the solution of the first-stage problem **(P1)**.

$$\begin{aligned} \textbf{(P2)} \quad & \min_{y_s} \quad f(\varphi, s, y_s) \\ s.t. \quad & \text{constraints} \quad (4.4) - (4.19) \end{aligned}$$

In each iteration, the first-stage problem and the subproblem for each scenario $s$ are linked through optimality cuts. These cuts (4.20) are generated from the dual of the subproblem **(P2)**. The coefficients $d_s^k$ and $e_s^k$ of the optimality cut are given as follows:

$$\begin{aligned} d_s^k &= \pi_{k,s}^{\mathrm{T}}\mathrm{T}_s \quad \forall s \in \mathrm{S}, \;\; k = 1,2...\mathrm{K} \\ e_s^k &= \pi_{k,s}^{\mathrm{T}}h_s \quad \forall s \in \mathrm{S}, \;\; k = 1,2...\mathrm{K} \end{aligned} \tag{4.21}$$

where $\pi_s = \{\pi_s^i\} \quad (\forall i = 1...16)$ is the optimal dual vector of the subproblem **(P2)** for each scenario $s \in \mathrm{S}$. $h_s$ is the vector associated with scenario $s$ and $\mathrm{T}_s$ is the matrix related to the first stage decision variables $\varphi$ in the corresponding model (2.1).

By solving the dual of the subproblem (P2), we obtain the optimal dual variable $\pi_s$ for

each scenario and then compute the coefficient $d_s^k$ and $e_s^k$ as follows:

$$
\begin{aligned}
d_s^k &= \sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} avail^{\mathrm{I}}(i,t,s)\pi^2_{i,t,s} + \sum_{i\in \mathrm{I}} q_{min}(i)\pi^3_{i,s} \\
e_s^k &= \sum_{t\in \mathrm{T}}\sum_{m\in \mathrm{M}} avail^{\mathrm{M}}(m,t,s)\pi^4_{m,t,s} + \sum_{p\in \mathrm{P}} (1-\lambda^{\mathrm{P}}(p))\, w^{init}(p)\pi^7_{p,s} \\
&+ \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{max}(p)\pi^{8a}_{p,t,s} + \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{min}(p)\pi^{8b}_{p,t,s} \\
&+ \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} w^{max}(p)\pi^{9a}_{p,t,s} + \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} w^{min}(p)\pi^{9b}_{p,t,s} \\
&+ \sum_{b\in \mathrm{B}} (1-\lambda^{\mathrm{B}}(b))\mathrm{V}^{init}_{av}(\mathrm{B})\pi^{11}_{b,s} - \sum_{t\geq 2\in \mathrm{T}}\sum_{b\in \mathrm{B}} demand(b,t)\pi^{12}_{b,t,s} \\
&+ \sum_{b\in \mathrm{B}} \left[\mathrm{V}^{init}_{ap}(\mathrm{B}) - demand(b,1)\right]\pi^{13}_{b,s} + \sum_{t\in \mathrm{T}}\sum_{b\in \mathrm{B}} \gamma_b^{max}(b)\pi^{14a}_{b,t,s} \\
&+ \sum_{t\in \mathrm{T}}\sum_{b\in \mathrm{B}} \gamma_b^{min}(b)\pi^{14b}_{b,t,s} + \sum_{t\in \mathrm{T}}\sum_{b\in \mathrm{B}} \mathrm{V}^{max}_{av}(b)\pi^{15a}_{b,t,s} \\
&+ \sum_{t\in \mathrm{T}}\sum_{b\in \mathrm{B}} \mathrm{V}^{min}_{av}(b)\pi^{15b}_{b,t,s} + \sum_{t\in \mathrm{T}}\sum_{b\in \mathrm{B}} \mathrm{V}^{max}_{ap}(b)\pi^{16a}_{b,t,s} \\
&+ \sum_{t\in \mathrm{T}}\sum_{b\in \mathrm{B}} \mathrm{V}^{min}_{ap}(b)\pi^{16b}_{b,t,s}
\end{aligned}
$$

**Algorithm 4.1:** : Multi-cut L-shaped algorithm

**Initialization**: Set $k = 1$, LB $= -\infty$, UB $= \infty$.

**Step 1**: In each iteration $k$, solve the first stage problem **(P1)** with all the optimality cuts generated at all previous iterations. Denote the optimal objective value of the problem as $\alpha^k$ and the optimal solution of the first stage decision variables $\varphi$ as $\varphi^k$.

If $\alpha^k \geq$ LB, set LB $= \alpha^k$

**Step 2**: Solve all the dual of subproblems of **(P2)** with the values of first stage decision variables fixed as $\varphi = \varphi^k$ and obtain the optimal dual vector for each scenario $s \in \mathrm{S}$ and .

Set $\beta^k = \sum_{i\in \mathrm{I}} \varphi^k(i) * mcost(i) + \sum_{s\in \mathrm{S}} p_s\theta_s$.

If $\beta^k \leq$ UB then update UB $= \beta^k$

**Step 3**: If (UB − LB)/UB $\leq \epsilon$, stop and get an optimal solution. Otherwise, go to Step 4.

**Step 4**: Add the optimality cuts to the first-stage problem. Set $k = k+1$, and go to Step 1.

This multi-cut L-shaped algorithm is presented in pseudo-code in Algorithm 4.1. It is outlined as follows:

(i) In each iteration, the first-stage problem is solved and the dual of subproblem **(P2)** is solved as many times as the number of scenarios $s \in \mathrm{S}$

(ii) By solving the first-stage problem, the values of the first stage decision variables $\varphi$ are determined and the value of function $\mathrm{Q}(\varphi, s)$ at each scenario $s$ in the second stage is approximated by $\theta_s$.

(iii) By solving the dual of subproblem **(P2)**, the expected value of the second stage problem for all scenarios can be found for the given value of the first stage variables found in the previous step.

(iv) In each iteration, after solving the dual of the scenario subproblem, multiple cuts of type (4.20) with the number equal to that of scenarios are added to the first-stage problem.

### 4.3.3 Enhanced and regularized decomposition approach

#### Regularized decomposition

In general, the classical L-shaped method has two drawbacks: (1) At initial iterations; cuts are often inefficient; (2) At final iterations, cuts become degenerately. To avoid these drawbacks, Ruszczynski and Swietanowski [1997] introduced a regularized decomposition method that combines a multi-cut approach with adding a quadratic regularized term in the objective function. We have adapted this approach to our problem. The regularized decomposition approach is given in Algorithm 4.2.

In addition, we have also applied several acceleration techniques to improve the convergence of the regularized decomposition algorithm. The valid inequalities are added to the regularized first-stage problem **(RD)** involved. These valid inequalities are described in the next section.

**Algorithm 4.2:** : Regularized decomposition algorithm

**Initialization:** Set $k = 0$, LB $= -\infty$ and UB $= \infty$. Select $a^1$ as a feasible solution.
**Step 1:** Set $k = k + 1$. Solve the regularized first-stage problem

$$\textbf{(RD)} \quad \min_{\varphi,\theta} \quad mcost^{\mathrm{T}}\varphi + \sum_{s\in \mathrm{S}} p_s\theta_s + \frac{1}{2}\|\varphi - a^k\|^2$$
$$s.t. \quad \theta_s \geq e_s^k\varphi + d_s^k \quad \forall s \in \mathrm{S}, \quad k = 1,2...\mathrm{K}$$
$$\varphi(i) \in \{0,1\} \quad \forall i \in \mathrm{I}$$

Let $(\varphi^k, \theta^k)$ be an optimal solution of the (RD) problem at iteration $k$, where $\varphi^k = (\varphi^k(i), i \in \mathrm{T})$ is a vector of $\varphi$.

If $(\mathrm{UB} - \mathrm{LB})/\mathrm{UB} \leq \epsilon$, stop and get an optimal solution. Otherwise, go to Step 2
**Step 2:** Solve the dual of subproblem (P2) for all $s \in \mathrm{S}$. Compute an optimality cut (4.20) as before.
**Step 3:** If $mcost^{\mathrm{T}}\varphi^k + \sum_{s\in \mathrm{S}} p_s\mathrm{Q}(\varphi, s) \leq mcost^{\mathrm{T}} a^k + \sum_{s\in \mathrm{S}} p_s\mathrm{Q}(a^k, s)$
then update UB $= mcost^{\mathrm{T}}\varphi^k + \sum_{s\in \mathrm{S}} p_s\mathrm{Q}(\varphi, s)$ and set $a^{k+1} = \varphi^k$, go to Step 1.
Else, $a^{k+1} = a^k$, go to Step 1.

**Valid inequalities**

The first-stage problem only includes the binary constraints of $\varphi$ and a few cuts. In some initial iteration, that may lead to the fact that very few suppliers are selected and that correspond to a small objective function value (lower bound). In addition, for such small number of suppliers selected, there is a little demand can be met in the scenario sub-problems. Consequently, it causes high shortage costs and leads to a high value of upper bound.

To avoid these inefficiencies in the initial iterations, it is necessary to integrate more information obtained from subproblems into the first stage problem to improve more significantly the quality of lower and upper bound in each iteration. From the nature of the biomass supply chain problem, valid inequalities may be added to the first-stage problem to improve the convergence rate and produce high-quality solutions.

From inequalities (4.4), (4.6) and (4.11), we obtain the following valid inequalities:

$$\sum_{i\in \mathrm{I}} q_{min}(i)\varphi(i) \leq \sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} q^{\mathrm{I}}(i,t,s) = \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} q^{\mathrm{P}}(p,t,s) \leq \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{max}(p)$$
$$\Longrightarrow \sum_{i\in \mathrm{I}} q_{min}(i)\varphi(i) \leq \mathrm{T}\sum_{p\in \mathrm{P}} \gamma_p^{max}(p) \quad \forall s \in \mathrm{S} \tag{4.22}$$

These valid inequalities (4.22) ensure the total minimum supply quantity from suppliers selected that are not surpassed a maximal amount biomass processed at preprocessing facility.

We also have:

$$\sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} avail^{\mathrm{I}}(i,t,s)\varphi(i) \geq \sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} q^{\mathrm{I}}(i,t,s) \geq \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{max}(p) \geq \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{min}(p)$$
$$\Longrightarrow \sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} avail^{\mathrm{I}}(i,t,s)\varphi(i) \geq \mathrm{T}\sum_{p\in \mathrm{P}} \gamma_p^{min}(p) \quad \forall s \in \mathrm{S} \tag{4.23}$$

These valid inequalities (4.23) ensure that the suppliers selected are capable of satisfying at least a minimum amount biomass processed at preprocessing facility.

Besides, adding these valid inequalities (4.22) and (4.23) not only reduce the feasible region of the first-stage problem but also avoids the infeasibility of the scenario subproblems in each iteration.

**Knapsack inequalities**

Let UB be the current upper bound in the iteration considered. By combining the optimal cut: $\theta_s \geq e_s^k \varphi + d_s^k$ and $\mathrm{UB} \geq mcost^{\mathrm{T}}\varphi + \sum_{p\in \mathrm{P}} p_s\theta_s$, we have:

$$\begin{aligned}
&\mathrm{UB} \geq mcost^{\mathrm{T}}\varphi + \sum_{p\in \mathrm{P}} p_s\theta_s \geq mcost^{\mathrm{T}}\varphi + \sum_{s\in \mathrm{S}} p_s(e_s^k\varphi + d_s^k) \\
\Longrightarrow &\big(mcost + \sum_{s\in \mathrm{S}} p_s e_s^k\big)^{\mathrm{T}}\varphi \leq \mathrm{UB} - \sum_{s\in \mathrm{S}} p_s d_s^k
\end{aligned}$$

As $\varphi$ are binary variables, we can rounded down two sides of the above inequality and obtain a following valid inequality follows:

$$\lfloor \big(mcost + \sum_{s\in \mathrm{S}} p_s e_s^k\big)^{\mathrm{T}} \rfloor \varphi \leq \lfloor \mathrm{UB} - \sum_{s\in \mathrm{S}} p_s d_s^k \rfloor \tag{4.24}$$

Where $\lfloor a \rfloor$ is a number rounded down from a given number $a$. When an upper bound $ub_s$ is available, adding a knapsack inequality (4.24) along with the optimality cut (4.20) could tighten the first-stage problem and lead to a good quality solution in each iteration.

**Specific optimality cut**

Laporte and Louveaux [1993] defined an optimality cut for the general L-shaped algorithm with the binary first-stage decision variable. This specific optimality cut can be added to accelerate the L-shaped algorithm when the first-stage variables are binary variables. We reformulated the specific optimality cut for the multi-cut L-shaped algorithm as follows:

$$\sum_{s\in \mathrm{S}} p_s\theta_s \geq (q_s - \mathrm{L})\Big(\sum_{i\in\tau} \varphi(i) - \sum_{i\notin\tau} \varphi(i) - |\tau| + 1\Big) + \mathrm{L} \tag{4.25}$$

where: $q_s = \sum_{s\in \mathrm{S}} p_s \mathrm{Q}(\varphi, s)$ is the corresponding recourse function value in each iteration. L is a lower bound satisfying: $\mathrm{L} \leq min_{\varphi}\{\sum_{s\in \mathrm{S}} p_s \mathrm{Q}(\varphi, s)\}\ \forall\varphi$. The set $\tau$ is defined as the set of all selected suppliers $i$: $\tau = \{i \mid \varphi(i) = 1\}$ and $|\tau|$ denote the cardinality of set $\tau$.

### 4.3.4 Determination of the number of scenarios by Monte Carlo Sampling

To approximate the distribution of a random event, we use a Monte Carlo sampling approach to generate the scenarios which allow to reduce the model size according to Linderoth et al. [2006]; Shapiro [2000]. Each scenario is assigned with the same probability, and the sum of the probabilities of all scenarios is equal to 1. Using a statistical method in Shapiro and Homem-de Mello [1998], we are able to determine the minimum number of scenarios required to obtain a solution within a confidence interval for a given level of confidence $\alpha$. The idea of this approach is based on the theory of probability to find a

relationship between reducing the number of scenarios and the reliability of the obtained solution. This method is based on the measure of the confidence interval of the expected total cost. The Monte Carlo sampling variance estimator $\sigma(n)$ is given as follows:

$$\sigma(n) = \sqrt{\frac{\sum_{s=1}^{n}(\mathbb{E}[totalcost] - totalcost_s)^2}{n-1}} \tag{4.26}$$

where $n$ is the number of scenarios used to estimate the Monte Carlo sampling variance, and $totalcost_s$ is the total system cost under scenario $s$. The minimum number of scenarios N required can be calculated by:

$$\mathrm{N} = \left(\frac{z_{\alpha/2}\sigma(n)}{\mathrm{H}}\right)^2 \tag{4.27}$$

where H is a given confidence interval and $\alpha$ is a given level of confidence. The value $z_{\alpha/2}$ is determined by $\mathrm{P}r(z \leq z_{\alpha/2}) = 1 - \alpha/2$, where $z \sim \mathrm{N}(0,1)$. In summary, the procedure for determining the minimum number of scenarios is given as follows:

(i) Solve the stochastic programming model with a small number of scenarios $n$ (E.g $n$= 10-100).

(ii) Estimate the value of sampling estimator $\sigma(n)$.

(iii) Determine the number of scenarios N required for a given confidence interval H and level of confidence $\alpha$.

## 4.4 Numerical study

In this section, numerical studies are conducted to evaluate the performance of the proposed algorithm and demonstrate the effectiveness of our approach. The algorithm is implemented in Python 3.5 on an HP computer with Intel Core i5-4210M CPU 2.66 GHz and 8.0 GB RAM. All linear programming (LP) problems are solved by using commercial solver GUROBI 6.5. According to the Benchmarks of Optimization Software [Mittelmann, 2018], GUROBI performs better than CPLEX on the MIPLIB2010 benchmark set. That is why we chose GUROBI as MILP solver in our numerical study.

### 4.4.1 Data generation

In the numerical study, we use data from Osmani and Zhang [2013, 2014]; Zhang et al. [2012] to make our tested instances more realistic. The dataset is summarized in Table 4.2. We consider a biomass supply chain over one year-horizon divided into 12 periods. Miscanthus is used as biomass raw material, and it is harvested from March to April. In our study, the first period, $t = 1$ corresponds to March. The model data assumptions are presented as follows:

1. One period in this study corresponds to one month. The biorefinery seeks to sign a contract with several local feedstock suppliers. The initial inventory quantity required to commence operations is 25 000 tons of biomass for a biofuel facility.

2. The installed biorefinery has a production capacity of 190 MLPY with a unit production cost of 0.2$/ liter. The bioethanol yield rate of lignocellulosic biomass is about 313 liters/ton. The unit biofuel storage cost and unit biomass storage cost at biorefinery are 0.227 $/liter and 0.9 $/ton , respectively. The unit shortage cost is 1.06 $/liter.

3. All 50 potential suppliers are located within 100km radius of the biorefinery.

4. The amount of biomass available in each supplier is estimated from land availability Osmani and Zhang [2013, 2014] and miscanthus yield (from 10 to 14 ton/hecta according to Clifton-Brown et al. [2007]). For each supplier, its miscanthus yield is generated from uniform distribution $U[10,14]$.

5. The pretreatment facility has a production capacity of 2 Mton/year with unit production cost of 13.94 $/ton and its transformation yield rate is 0.75. Unit biomass storage cost at pretreatment facility is 1.125 $/ton.

6. The ratio of biomass storage loss at pretreatment and at biorefinery are 3% and 1% respectively.

7. Biofuel demand is generated from Normal probability distribution $N(2133,4138)$ and truncated in the interval $[2006,2280]$ based on historical data (1981-2005) in Osmani and Zhang [2013]. The monthly biofuel demand is also adjusted to the production capacity of the biorefinery.

8. Contracts with suppliers are characterized by fixed cost, purchase price and minimum quantity to supply. Purchase price varies from 42 to 45 $/ton. The minimum supply quantity varies from 20 000 to 30 000 tons. Fixed cost for establishing a contract varies from $20 000 to $30 000.

9. Biomass in market has a purchase price 55 $/ton. However, the distance from a market to the biorefinery varies from 150 to 200km.

The objective of this section is to study the impact of uncertain supply on the strategic decision made. This uncertainty is incorporated into the model by considering a set of possible scenarios. The number of scenarios should be large enough to capture the entire range of the probability distribution of feedstock supply.

Table 4.2: Data of model parameters

| Input parameters | Value |
|---|---|
| Bioethanol yield (litre/tonne) | 313 |
| Unit shortage cost for ummet bioethanol demand ($/litre) | 1.06 |
| Unit bioethanol production cost ($/litre) | 0,200 |
| Unit biomass storage cost at biorefinery ($/litre) | 0.9 |
| Unit biofuel storage cost ($/ton ) | 0.228 |
| Annual production capacity of biorefinery(MLPY) | 190 |
| Unit preprocessing cost at preprocessing facility ($/tonne) | 13.94 |
| Unit biomass storage cost at pretreatment facility ($/tonne) | 1.125 |
| Transformation yield rate at pretreatment facility i | 75 |
| Ratio of biomass storage loss in time period (%) | 1-3% |
| Annual production capacity of pretraitement facility (Mton) | 2.00 |
| Unit biomass purchase price from market ($/ton) | 55 |
| Unit biomass purchase price from suppliers ($/ton) | 42-45 |
| Minimum quantitity to supply from each supplier (tons/year) | 20 000-30 000 |
| Fixed cost for establishing a contract ($) | 20 000-30 000 |
| Demand of biofuel (MLPY) | $N(2133, 4138)$ on $[2016, 2280]$ |
| Yield biomass feedstocks (tons/hecta) | Uniform[10, 14] |

Table 4.3: Problem size of the deterministic equivalent model

| I | P | B | T | No. Of scenarios | No. Of binary variables | No. Of continuous variables |
|---|---|---|---|---|---|---|
| 40 | 4 | 1 | 12 | 50 | 50 | 58 200 |
| 40 | 4 | 1 | 12 | 100 | 100 | 116 400 |
| 40 | 4 | 1 | 12 | 1000 | 100 | 1 164 000 |
| 50 | 5 | 1 | 12 | 50 | 50 | 72 000 |
| 50 | 5 | 1 | 12 | 100 | 100 | 144 000 |
| 50 | 5 | 1 | 12 | 1000 | 100 | 1 440 000 |
| 60 | 6 | 1 | 12 | 50 | 50 | 85 800 |
| 60 | 6 | 1 | 12 | 100 | 100 | 171 600 |
| 60 | 6 | 1 | 12 | 1000 | 100 | 1 716 000 |

### 4.4.2 Performance evaluation of the solution algorithm

In this section, we describe our computational experiments on the algorithm proposed in the previous section when solving instances of different sizes. The computation time for solving the model depends on the number of periods, suppliers, preprocessing facilities, biorefineries, and scenarios. We tested this model for instances with I = 40, 50, 60; P = 4, 5, 6; T = 12 and number of scenarios S = 50, 100, 1000. The dimension of the deterministic equivalent problem for these instances is presented in Table 4.3. It shows that the size of the stochastic programming model increases rapidly as a function of the number of scenarios, suppliers and pretreatment sites.

Table 4.4 summarizes the size, the number of scenarios and the execution time of the solver GUROBI, the multi-cut L-Shaped method (MLS) and the enhanced and regularized decomposition L-shaped method (ERD) for these instances. We fixed the stop-

ping condition of the algorithm as follows: the optimality gap $\epsilon \leq 0.01(\%)$ or CPU time $\geq$ $time_{max}$=1800s or the number of iterations $\geq iteration_{max} = 1000$, where the optimality gap is calculated by: $\epsilon(\%) = 100 * (\text{UB} - \text{LB}/\text{UB})$. We also generated 10 samples of biofuel demand for each instance, denoted by $d_i$ ($\forall i = 1...10$). As the experimental results indicate, the (ERD) method and GUROBI constantly outperforms by a factor of 2 the standard multi-cut L-Shaped method (MLS). On average, the (ERD) method is 1.01, 1.21, 1.16, 1.20, 1.20 and 1.32 times faster than GUROBI for all instances studied, respectively (6 sub-tables in the 1st and 2nd lines of Table 4.4). Therefore, the (ERD) method can be used to solve the considered problem because it can find an optimal solution in a shorter computational time.

For large instances, the size of the model is too large to be solved by solver GUROBI due to the limited memory of our PC. Three sub-tables in the third line of Table 4.4 show that the (ERD) algorithm still performed well for large instances because the second stage problem is decomposed into multiple subproblems with reasonable size.

By comparing the last two columns, we observe that the added valid inequalities allow improving the convergence speed of the solution method significantly. In fact, adding the valid inequalities presented in Section 4.3.2- 4.3.4 could reduce 8.8-30.3% of the computation time in comparison with the RD method (the classic regularization approach without adding valid inequalities). This result proves the relevance of adding these inequalities in the solution method.

### 4.4.3 Analysis of the solution

This section analyzes our numerical results. Figure 4.2 shows the cost distribution of the base case for the biomass supply chain over planning horizon T= 12. The optimal value of the total expected cost of the supply chain is $114 271 335 when the biomass supply chain includes 1 biorefinery, 1 pretreatment site, 50 suppliers over 12 time periods.

The biofuel production cost and feedstock purchase cost are the primary cost drivers that account for nearly 39.5 % and 34.8% of the total system costs respectively. The transportation cost has an essential role in the biomass supply chain with approximatively 14.7% of total system cost. This result shows the importance of transportation operation in biomass supply chains. The holding cost is only 2.9%, so in this case, the storage of a significant amount of biomass at biorefinery seems not critical.

Table 4.4: Comparison of solution approaches

| Problem size | | | | | | CPU time | | | |
|---|---|---|---|---|---|---|---|---|---|
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 40 | 4 | 1 | 12 | 50 | $d_1$ | 2.24 | 4.47 | 2.46 | 2.09 |
| 40 | 4 | 1 | 12 | 50 | $d_2$ | 2.29 | 5.53 | 4.38 | 3.51 |
| 40 | 4 | 1 | 12 | 50 | $d_3$ | 2.27 | 4.35 | 2.56 | 2.21 |
| 40 | 4 | 1 | 12 | 50 | $d_4$ | 2.32 | 4.38 | 2.49 | 2.07 |
| 40 | 4 | 1 | 12 | 50 | $d_5$ | 2.42 | 6.16 | 4.38 | 3.61 |
| 40 | 4 | 1 | 12 | 50 | $d_6$ | 2.31 | 4.36 | 2.26 | 1.79 |
| 40 | 4 | 1 | 12 | 50 | $d_7$ | 2.43 | 4.37 | 2.27 | 1.85 |
| 40 | 4 | 1 | 12 | 50 | $d_8$ | 2.44 | 4.39 | 2.43 | 2.01 |
| 40 | 4 | 1 | 12 | 50 | $d_9$ | 2.30 | 6.67 | 2.34 | 2.07 |
| 40 | 4 | 1 | 12 | 50 | $d_{10}$ | 2.31 | 4.83 | 2.27 | 1.88 |
| **Average** | | | | | | **2.33** | **4.95** | **2.78** | **2.31** |
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 40 | 4 | 1 | 12 | 100 | $d_1$ | 4.75 | 8.82 | 4.75 | 4.05 |
| 40 | 4 | 1 | 12 | 100 | $d_2$ | 4.84 | 8.84 | 4.92 | 4.43 |
| 40 | 4 | 1 | 12 | 100 | $d_3$ | 4.91 | 8.86 | 4.52 | 3.79 |
| 40 | 4 | 1 | 12 | 100 | $d_4$ | 5.10 | 8.76 | 4.61 | 4.24 |
| 40 | 4 | 1 | 12 | 100 | $d_5$ | 4.89 | 8.82 | 4.63 | 3.88 |
| 40 | 4 | 1 | 12 | 100 | $d_6$ | 4.99 | 8.85 | 4.52 | 3.90 |
| 40 | 4 | 1 | 12 | 100 | $d_7$ | 4.86 | 8.77 | 4.57 | 4.11 |
| 40 | 4 | 1 | 12 | 100 | $d_8$ | 4.76 | 8.93 | 4.72 | 3.94 |
| 40 | 4 | 1 | 12 | 100 | $d_9$ | 4.82 | 8.81 | 4.50 | 3.83 |
| 40 | 4 | 1 | 12 | 100 | $d_{10}$ | 5.08 | 9.03 | 5.02 | 4.35 |
| **Average** | | | | | | **4.90** | **8.85** | **4.68** | **4.05** |
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 40 | 4 | 1 | 12 | 1000 | $d_1$ | OOM | 88.34 | 51.23 | 50.21 |
| 40 | 4 | 1 | 12 | 1000 | $d_2$ | OOM | 90.66 | 54.76 | 48.46 |
| 40 | 4 | 1 | 12 | 1000 | $d_3$ | OOM | 89.24 | 60.40 | 43.27 |
| 40 | 4 | 1 | 12 | 1000 | $d_4$ | OOM | 152.32 | 67.64 | 51.28 |
| 40 | 4 | 1 | 12 | 1000 | $d_5$ | OOM | 103.24 | 54.74 | 67.24 |
| 40 | 4 | 1 | 12 | 1000 | $d_6$ | OOM | 102.47 | 57.34 | 55.69 |
| 40 | 4 | 1 | 12 | 1000 | $d_7$ | OOM | 104.47 | 54.90 | 45.58 |
| 40 | 4 | 1 | 12 | 1000 | $d_8$ | OOM | 98.17 | 53.38 | 48.48 |
| 40 | 4 | 1 | 12 | 1000 | $d_9$ | OOM | 93.81 | 60.51 | 44.55 |
| 40 | 4 | 1 | 12 | 1000 | $d_10$ | OOM | 90.04 | 53.19 | 46.48 |
| **Average** | | | | | | | **101.28** | **56.81** | **50.12** |

| Problem size | | | | | | CPU time | | | |
|---|---|---|---|---|---|---|---|---|---|
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 50 | 5 | 1 | 12 | 50 | $d_1$ | 2.99 | 7.18 | 3.52 | 2.83 |
| 50 | 5 | 1 | 12 | 50 | $d_2$ | 3.04 | 7.75 | 5.08 | 4.11 |
| 50 | 5 | 1 | 12 | 50 | $d_3$ | 3.18 | 7.67 | 5.66 | 4.17 |
| 50 | 5 | 1 | 12 | 50 | $d_4$ | 3.21 | 5.19 | 3.24 | 2.29 |
| 50 | 5 | 1 | 12 | 50 | $d_5$ | 3.22 | 5.22 | 2.71 | 2.49 |
| 50 | 5 | 1 | 12 | 50 | $d_6$ | 3.29 | 7.78 | 2.68 | 2.27 |
| 50 | 5 | 1 | 12 | 50 | $d_7$ | 2.97 | 5.24 | 2.77 | 2.29 |
| 50 | 5 | 1 | 12 | 50 | $d_8$ | 3.08 | 5.13 | 2.74 | 2.16 |
| 50 | 5 | 1 | 12 | 50 | $d_9$ | 3.11 | 5.10 | 2.63 | 2.10 |
| 50 | 5 | 1 | 12 | 50 | $d_{10}$ | 3.15 | 5.13 | 2.65 | 2.11 |
| **Average** | | | | | | **3.12** | **6.14** | **3.37** | **2.68** |
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 50 | 5 | 1 | 12 | 100 | $d_1$ | 6.56 | 15.58 | 9.76 | 7.56 |
| 50 | 5 | 1 | 12 | 100 | $d_2$ | 6.57 | 10.20 | 5.11 | 4.13 |
| 50 | 5 | 1 | 12 | 100 | $d_3$ | 6.88 | 15.66 | 6.19 | 5.03 |
| 50 | 5 | 1 | 12 | 100 | $d_4$ | 6.64 | 10.43 | 5.47 | 4.50 |
| 50 | 5 | 1 | 12 | 100 | $d_5$ | 6.42 | 15.38 | 9.93 | 8.17 |
| 50 | 5 | 1 | 12 | 100 | $d_6$ | 6.58 | 10.38 | 5.21 | 4.27 |
| 50 | 5 | 1 | 12 | 100 | $d_7$ | 6.53 | 15.44 | 9.85 | 8.06 |
| 50 | 5 | 1 | 12 | 100 | $d_8$ | 6.70 | 10.40 | 5.24 | 4.39 |
| 50 | 5 | 1 | 12 | 100 | $d_9$ | 6.66 | 10.24 | 5.13 | 4.32 |
| 50 | 5 | 1 | 12 | 100 | $d_{10}$ | 6.27 | 10.34 | 5.35 | 4.44 |
| **Average** | | | | | | **6.58** | **12.41** | **6.72** | **5.49** |
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 50 | 5 | 1 | 12 | 1000 | $d_1$ | OOM | 126.97 | 65.53 | 62.81 |
| 50 | 5 | 1 | 12 | 1000 | $d_2$ | OOM | 190.58 | 77.51 | 77.30 |
| 50 | 5 | 1 | 12 | 1000 | $d_3$ | OOM | 189.96 | 118.39 | 103.34 |
| 50 | 5 | 1 | 12 | 1000 | $d_4$ | OOM | 183.64 | 72.89 | 69.58 |
| 50 | 5 | 1 | 12 | 1000 | $d_5$ | OOM | 126.61 | 72.30 | 68.57 |
| 50 | 5 | 1 | 12 | 1000 | $d_6$ | OOM | 191.83 | 86.76 | 72.63 |
| 50 | 5 | 1 | 12 | 1000 | $d_7$ | OOM | 190.03 | 116.46 | 108.27 |
| 50 | 5 | 1 | 12 | 1000 | $d_8$ | OOM | 188.73 | 115.86 | 110.90 |
| 50 | 5 | 1 | 12 | 1000 | $d_9$ | OOM | 190.80 | 67.58 | 55.73 |
| 50 | 5 | 1 | 12 | 1000 | $d_{10}$ | OOM | 128.39 | 70.18 | 64.31 |
| **Average** | | | | | | | **170.75** | **86.35** | **79.34** |

| Problem size | | | | | | CPU time | | | |
|---|---|---|---|---|---|---|---|---|---|
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 60 | 6 | 1 | 12 | 50 | $d_1$ | 3.77 | 9.96 | 3.94 | 3.32 |
| 60 | 6 | 1 | 12 | 50 | $d_2$ | 3.86 | 6.45 | 3.68 | 3.27 |
| 60 | 6 | 1 | 12 | 50 | $d_3$ | 3.82 | 9.52 | 4.22 | 3.55 |
| 60 | 6 | 1 | 12 | 50 | $d_4$ | 3.87 | 6.53 | 3.46 | 2.90 |
| 60 | 6 | 1 | 12 | 50 | $d_5$ | 3.93 | 6.51 | 3.91 | 3.22 |
| 60 | 6 | 1 | 12 | 50 | $d_6$ | 3.85 | 6.55 | 3.83 | 3.30 |
| 60 | 6 | 1 | 12 | 50 | $d_7$ | 3.80 | 6.66 | 3.57 | 3.05 |
| 60 | 6 | 1 | 12 | 50 | $d_8$ | 3.77 | 6.48 | 3.90 | 3.23 |
| 60 | 6 | 1 | 12 | 50 | $d_9$ | 3.75 | 6.63 | 3.60 | 2.96 |
| 60 | 6 | 1 | 12 | 50 | $d_{10}$ | 3.74 | 6.53 | 3.44 | 2.90 |
| **Average** | | | | | | **3.82** | **7.18** | **3.76** | **3.17** |
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 60 | 6 | 1 | 12 | 100 | $d_1$ | 7.86 | 13.04 | 7.16 | 6.09 |
| 60 | 6 | 1 | 12 | 100 | $d_2$ | 7.97 | 13.05 | 7.87 | 6.36 |
| 60 | 6 | 1 | 12 | 100 | $d_3$ | 8.20 | 12.88 | 7.13 | 6.03 |
| 60 | 6 | 1 | 12 | 100 | $d_4$ | 8.12 | 12.83 | 6.75 | 5.44 |
| 60 | 6 | 1 | 12 | 100 | $d_5$ | 8.09 | 13.06 | 6.83 | 5.97 |
| 60 | 6 | 1 | 12 | 100 | $d_6$ | 8.17 | 13.04 | 6.52 | 5.49 |
| 60 | 6 | 1 | 12 | 100 | $d_7$ | 7.98 | 13.07 | 6.70 | 5.60 |
| 60 | 6 | 1 | 12 | 100 | $d_8$ | 8.17 | 13.05 | 10.48 | 8.47 |
| 60 | 6 | 1 | 12 | 100 | $d_9$ | 7.97 | 20.10 | 6.63 | 5.91 |
| 60 | 6 | 1 | 12 | 100 | $d_{10}$ | 8.14 | 12.94 | 6.77 | 5.74 |
| **Average** | | | | | | **8.07** | **13.71** | **7.28** | **6.11** |
| **I** | **P** | **B** | **T** | **S** | **D** | **Gurobi** | **MLS** | **RD** | **ERD** |
| 60 | 6 | 1 | 12 | 1000 | $d_1$ | OOM | 227.08 | 150.31 | 111.68 |
| 60 | 6 | 1 | 12 | 1000 | $d_2$ | OOM | 229.65 | 159.33 | 110.65 |
| 60 | 6 | 1 | 12 | 1000 | $d_3$ | OOM | 231.56 | 109.85 | 95.94 |
| 60 | 6 | 1 | 12 | 1000 | $d_4$ | OOM | 322.03 | 200.92 | 123.39 |
| 60 | 6 | 1 | 12 | 1000 | $d_5$ | OOM | 243.87 | 162.12 | 123.67 |
| 60 | 6 | 1 | 12 | 1000 | $d_6$ | OOM | 240.04 | 162.35 | 116.21 |
| 60 | 6 | 1 | 12 | 1000 | $d_7$ | OOM | 231.45 | 161.97 | 126.01 |
| 60 | 6 | 1 | 12 | 1000 | $d_8$ | OOM | 323.20 | 168.89 | 127.85 |
| 60 | 6 | 1 | 12 | 1000 | $d_9$ | OOM | 247.60 | 80.47 | 84.95 |
| 60 | 6 | 1 | 12 | 1000 | $d_{10}$ | OOM | 227.30 | 89.19 | 88.38 |
| **Average** | | | | | | | **252.38** | **144.54** | **110.87** |

(*)OOM: out of memory

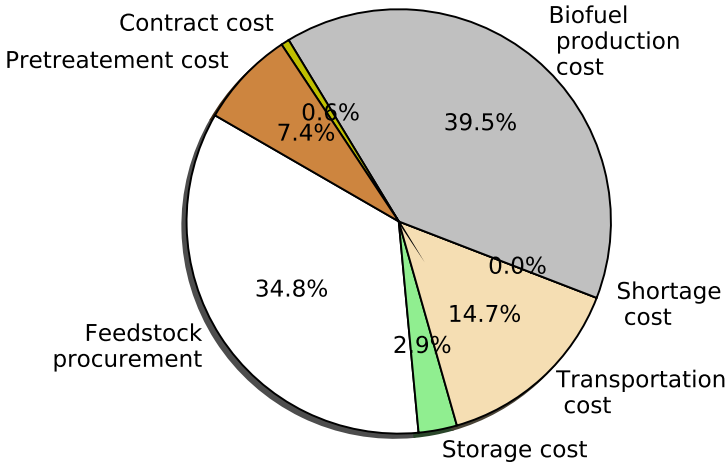


Figure 4.2: Cost distribution for the entire supply chain

Figure 4.3 shows the vital role of biomass storage facility in a biorefinery site. Due to seasonality and short harvest time of biomass, we must collect enough biomass feedstock during the harvest season and store it later in order to cope with supply variability and to satisfy biofuel demand over the whole planning horizon. That explains why the inventory level of biomass during the harvest season (period $t = 1,2,3$) is higher than that in other periods. The biomass storage facility provides a buffer for the biomass supply chain to protect against the fluctuations of supply and demand.

### 4.4.4 Value of the stochastic solution

In a deterministic approach, the decision-maker may assume expected supply availability based on average yield (12 tons/hectare) and seek for optimal contracts with suppliers (farmer) according to these supply availability. In other words, we formulate a deterministic model by using expected values of biomass supply availability. Based on the solution of the model found, we fix the values of the first-stage variables in formulation **(SP)** and then solve the model.

This approach represents the expected value solution as we mentioned before, but it might lead to unfavorable consequences. Here, as shown in Table 4.5, using the expected value solution could provide in the planning horizon the annual total cost of \$ 124 088 777. This total cost is 7.9% higher than the one resulting from the corresponding stochastic programming model, \$ 114 271 335. The choice of suppliers would affect the expected total cost strongly under the stochastic environments. We can consider that the value, VSS = 124088777 − 114271335 = \$9817442, represents the possible gain by taking account of the random variations of biomass supply. This value is called the value of the stochastic solution (VSS) according to Birge and Louveaux [2011].

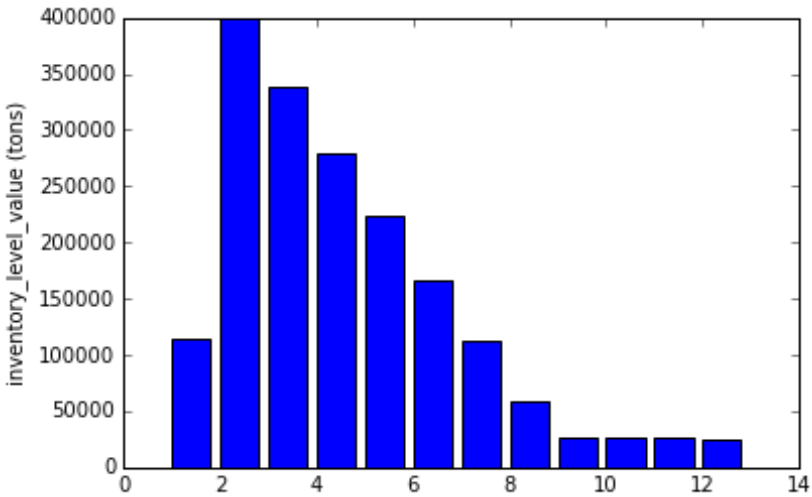


Figure 4.3: Evolution of biomass inventory level at biorefinery

Table 4.5: Comparison of results from stochastic program

| Solution | Total costs ($) |
|---|---|
| Deterministic | 124 088 777 |
| Stochastic | 114 271 335 |
| VSS | 9 817 442 |
| VSS (%) | 7.9% |

### 4.4.5 Sensitivity analysis

We investigated the impacts of some critical parameters such as purchase price from a market, minimum supply quantity and fixed contracted cost on the expected total system cost since they have significant impacts on feedstock supply. A sensitivity analysis is conducted to learn how purchase price, minimum quantity to supply and fixed contracted cost impact the optimal expected total cost of biomass supply chain and the total number of contracted suppliers.

#### Effect of purchase price

Figure 4.4 shows the total system cost and the number of contracted suppliers under different values of the purchase price which ranges between 70% and 140 % of its base value. The horizontal axis represents the various purchase price of biomass from an international/regional market. The bar graphs with the height measured by the vertical axis on the left correspond to the total costs of this supply chain under different values of the purchase price. The dotted line with the value of each point measured by the vertical axis on the right indicates the number of contracted suppliers at each possible value of the purchase price considered. As the market price increases, the optimal total system cost increases slightly from $ 112 685 030 to $ 114 410 480 (less than 2%), and then it remains constant at $ 114 435 526 when the market price is set at 120% or higher than its basic level. This confirms that when the market price increases too high, decision-

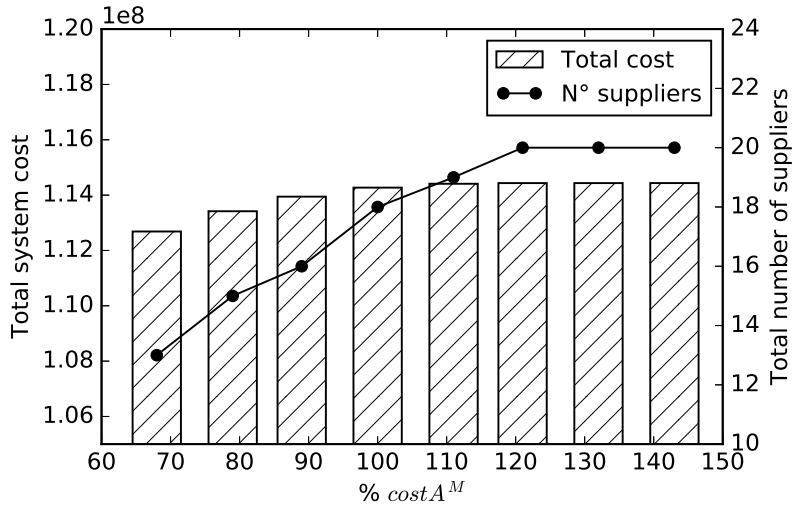


Figure 4.4: Impact of unit purchase price of biomass from market

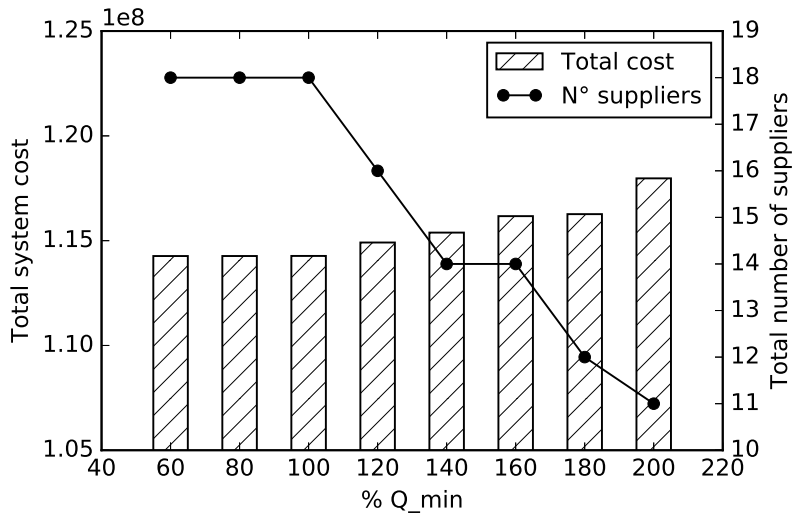


Figure 4.5: Impact of minimum contracted quantity of biomass

makers have less interest in purchasing an additional quantity of biomass from markets to meet biofuel demand. In this case, they would intend to establish more long-term supply contracts to ensure the stability of feedstock supply. This explains why the number of contracted suppliers increases when the market price grows. This result suggests that to achieve the overall system efficiency, a balance between purchase price under long-term contracts and the one from market supply need to be considered. In the long term, the quantity and price of biomass supplied from suppliers would be guaranteed by long-term contracts whereas the price and availability of biomass from markets vary highly due to external conditions such as weather.

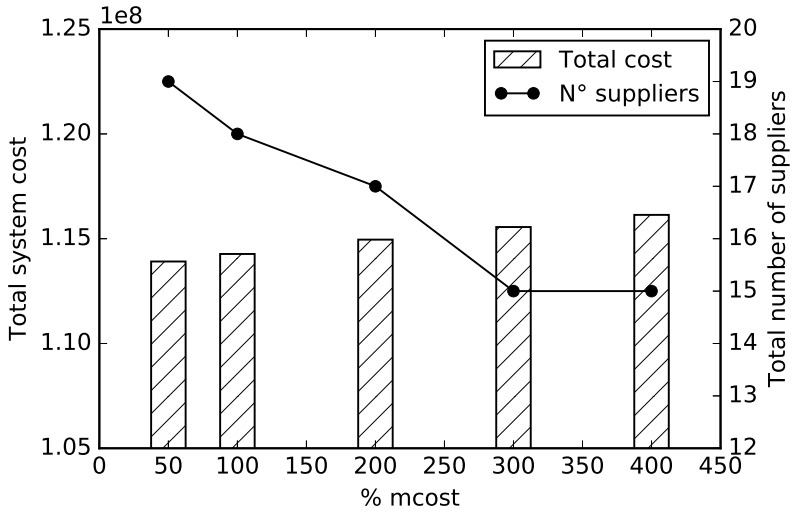


Figure 4.6: Impact of fixed cost for establish contracts

**Effect of fixed cost**

Factors such as volatile market price, uncertain yield and limited land availability would contribute the difficulty in estimating a fixed purchase price and the minimum supply quantity in long-term contracts. This explains the motivation we conduct a study of the effect of fixed cost on the biomass supply chain. In Figure 4.6, the horizontal axis represents the fixed cost varying from 50% to 400% in comparison to its base value. The bar graphs with the height measured by the vertical axis on the left hand present the total expected system costs under different values of the fixed cost. The dotted line with the value of each point measured by the vertical axis on the right hand indicates the number of contracted suppliers at each possible value of the fixed cost. The result indicates that the increase of fixed cost slightly affects the total system cost and the total number of contracted suppliers. This also reveals the fact that the increase in fixed cost could lead to higher total system cost and might hinder the establishment of long-term contracts.

**Effect of the minimum supply quantity of suppliers**

Figure 4.5 presents the impacts of a contracted supply quantity on the total system cost of biomass supply chain and the number of contracted suppliers. The horizontal axis represents the various contracted quantity delivered from 60% to 200% in comparison with its base value. The bar graphs with the height measured by the vertical axis on the left hand show the total expected system costs under different values of the minimum supply quantity. The dotted line with the value of each point measured by the vertical axis on the right hand indicates the number of contracted suppliers at each possible value of the minimum supply quantity. This result shows that with the increase in minimum supply quantity, the total system cost will increase; however, this increase is not as significant as the increase in supply quantity. On the other hand, the total number of contracted

suppliers decreases as the supply quantity increases. In this case, decision-maker would require additional quantity from market to satisfy the final demand but only at a reasonable market price.

## 4.5 Conclusions

This chapter proposes a mathematical model to tackle the supplier selection and operation planning problem in biomass supply chains to support decision-makers facing uncertainty of biomass feedstock supply. The objective is to minimize the total system cost of a biomass supply chain. The approach is based on the two-stage stochastic programming model that is able to capture uncertainty in biomass availability. An enhanced and regularized L-shaped method is proposed to solve the stochastic model. This technique allows us to decompose a high dimensional stochastic model into subproblems of reasonable size. These sub-problems could be solved on a personal computer with limited memory. Moreover, our proposed method could find an optimal solution faster than the standard L-shaped decomposition method and commercial MILP solver Gurobi. Besides, we have analyzed the impacts of critical parameters on the optimal expected cost of the system and supplier selection.

These works in this chapter have been presented in the following publication:

**Duc Huy Nguyen**, Haoxun Chen (2018). *"Supplier selection and operation planning in biomass supply chains with supply uncertainty"*, Computers & Chemical Engineering

# Chapter 5

# Optimal policy and algorithm for a perishable inventory system with uncertainty in both Demand and Supply

*" Knowledge is an unending adventure at the edge of uncertainty. "*

Jacob Bronowski

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## 5.1 Introduction

This chapter presents an inventory model for perishable products under both stochastic demand and stochastic supply. Such problem appears in biomass supply chains. As mentioned before, the ratio of biomass degradation during storage approximate 1-2% per month under the ambient storage Rentizelas et al. [2009]. Besides, feedstock supply is uncertain due to weather condition, insect population. So, it is essential to study a perishable inventory model under two types of uncertainty: demand uncertainty and supply uncertainty.

In this chapter, we propose a stochastic inventory model which can be considered an extension of two inventory models: a model with perishable inventory and infinite supply capacity Shah and Jaiswal [1977], and a model with non-perishable inventory and uncertain supply capacity Ciarallo et al. [1994], by combining the two features "inventory perishability" and "supply uncertainty". Also, we consider inventory holding, backorder costs and purchase costs whereas Ciarallo et al. [1994] only considers inventory holding costs and backorder costs. For the two models in Ciarallo et al. [1994]; Shah and Jaiswal [1977], their optimal inventory policies have been proven to be order-up-to policies or base-stock policies. However, *perishability may destroy the simple base-stock structure of optimal policies for discrete review models without fixed ordering costs in the absence of perishability* according to Karaesmen et al. [2011]. In fact, we tried to transform our model into a model without decay but failed. Therefore, it is necessary to study the structure of optimal inventory policy of this extended model.

To the best of our knowledge, only few articles dealing with perishable product, stochastic demand, and stochastic supply were published. For this reason, we study a periodic review perishable inventory model with multi-periods under both stochastic demand and stochastic supply.

The stochastic inventory model is quite complicated. Although the optimal inventory policy of the model can be found by a dynamic programming algorithm, it is very time-consuming. For this reason, instead of finding the optimal inventory policy, we try to find a high-quality near-optimal policy quickly. Birge and Louveaux [2011] suggests a solution approach, called a scenario-based stochastic framework. A scenario is a realization for the stochastic variables. By constructing a large number of scenarios, we can transform conventional stochastic programming model into a mixed integer linear programming (MILP) model by replacing the stochastic parameters by the values taken in the corresponding scenario. One advantage of this approach is that we can use a commercial solver to solve the MILP model if the number of scenarios is small. However, to well approximate the stochastic model, the number of scenarios should be taken large enough, which leads to a MILP model of very large size whose resolution by using a commercial solver requires a lot of memory and is very time-consuming.

Lagrangian relaxation is a very popular approach not only for linear optimization but

also for nonlinear optimization such as the research of Barahona and Anbil [2000]; Lemaréchal [1989, 2001]; Shor [2012], this method is useful when a problem can be solved easily if few complicated constraints are relaxed. By solving the so-called Lagrangian dual, the optimal solution or a near-optimal solution of the original problem may be found under certain conditions. For these two reasons, we combine the scenario-based optimization and Lagrangian relaxation to find a near-optimal solution of the stochastic inventory problem considered in this chapter.

The main contributions of this chapter include:

(i) Introduce a periodic review perishable inventory model with uncertainty in both demand and supply where a part of inventory is degraded/deteriorated at a constant rate from one period to the next;

(ii) Understanding some crucial properties of the model and demonstrate that its optimal inventory policy is an order-up-to level policy;

(iii) Develop a solution method that combines scenario-based optimization and Lagrangian relaxation to find a near-optimal solution (order-up-to levels) for the original problem. The proposed method is applicable to problems with a large number of scenarios;

(iv) Conduct a numerical study which shows that the proposed solution approach can find near-optimal inventory policies with the expected total cost less than 1% of deviation from the optimal expected total cost on average;

This chapter is organized as follows. In Section 5.2, the problem and its model are introduced. The properties of the model and its optimal inventory policy are then presented in Section 5.3. Lagrangian relaxation based algorithm for finding optimal inventory polices and a numerical study are then presented in Section 5.4 and 5.5. Section 5.6 concludes this chapter with some remarks for future research.

## 5.2 Problem description and model formulation

### 5.2.1 Problem description

In this section, we present an inventory model with a single item, single-stock and a time horizon of $n$-periods. The product we consider is perishable, and its deterioration rate is constant, denoted by $(1-\lambda)$, where $\lambda$ is a parameter between 0 and 1. Let $s_t$ be the inventory level at the beginning of period $t$ and before eventual placement of an order of quantity $q_t$ in this period. The stochastic demand in each period occurs independently. The demand in period $t$ is represented by a non negative random variable $\mathrm{D}_t$ with known probability density function $f_t$ and cumulative distribution function $\mathrm{F}_t$.

The supply of the stock is also stochastic. The stochastic supply capacities in $n$-periods, which are independent and distributed random variables. The supply in period $t$ is denoted by $y_t$, which has probability density function $g_t$ and cumulative distribution func-

tion $G_t$. We assume that the lead time of the stock is zero, unsatisfied demand is backlogged, and the inventory level is reviewed periodically. The objective of this problem is to minimize the total expected (discounted) cost which includes three types of costs: inventory holding costs, backorder costs and purchasing costs over the planning horizon of $n$ periods.

The order of occurred events in each period is assumed as follows. At the beginning of the period, the inventory level is reviewed considering the product deterioration that occurred in the previous period. Based on the inventory policy used, an order is then placed if necessary. After that, the demand and the supply in this period are observed and customer demand is satisfied as much as possible, and unsatisfied demand is back-ordered. In the end, holding costs are charged according to the remaining inventory which will be carried over from the current period to the next. At the same time, backorder costs are incurred in case of negative inventory level. The holding and backorder costs are determined by the inventory level at the end of the period which depends on both customer demand and supply capacity.

The stochastic perishable inventory model is presented as follows:

$$\textbf{(SP)} \quad \min_{q_t} \quad \sum_{t=1}^{T} \alpha^t \phi_t(s_t, q_t) \tag{5.1}$$

$$s.t \quad s_{t+1} = \lambda s_t^+ - s_t^- + min(q_t, y_t) - D_t \tag{5.2}$$

$$s_t^+ = \max(0, s_t) \tag{5.3}$$

$$s_t^- = \max(0, -s_t) \tag{5.4}$$

where the function $\phi_t(s_t, q_t)$ is the expected single period cost which includes the expected inventory holding, backorder and purchasing cost, given as follows. Let $c$, $p$, $h$ be purchasing cost, backorder cost and holding cost per unit in each period, respectively. Let $\alpha$ represents the discount factor per period $(0 < \alpha < 1)$.

$$\begin{aligned}\phi_t(s_t, q_t) = \mathop{\mathbb{E}}_{(D_t, y_t)} \Big[ & c(\min\{q_t, y_t\}) + h(\lambda s_t^+ - s_t^- + \min(q_t, y_t) - D_t)^+ \\ & + p(D_t - \min\{q_t, y_t\} - \lambda s_t^+ + s_t^-)^+ \Big]\end{aligned} \tag{5.5}$$

Equation 5.3 illustrates the relationship between the inventory level at the beginning of period $t$ and that at the beginning of period $t$+1 under product perishability effect.

Since we assume that unsatisfied demand is backlogged, the inventory level $s_t$ may be negative when there is a backlogged demand. In this case, $s_t < 0$, the on-hand inventory is zero, and only $-s_t^- = s_t$ is carried over to the next period. In case of positive inventory level $s_t$, the on-hand inventory is also $s_t$, part of it, i.e., $\lambda s_t$, is carried out over to the next period after removing $(1-\lambda)s_t$, the quantity perished during the period $t$.

### 5.2.2 Dynamic programming model

In this section, we formulate a stochastic dynamic programming model that considers uncertainty in both supply and demand. Let $V^*_{t,n}(s_t)$ denote the optimal total expected (discounted) cost from period $t$ to $n$ using the optimal policy, given the inventory level $s_t$ in period $t$. The optimal cost $V^*_{t,n}(s_t)$ can be calculated recursively by:

$$V^*_{t,n}(s_t) = \min_{q_t} V_{t,n}(s_t, q_t) \tag{5.6}$$
$$\text{where} \quad V_{t,n}(s_t, q_t) = \phi_t(s_t, q_t) + \underset{(D_t, y_t)}{\alpha \mathbb{E}} V^*_{t+1,n}(s_{t+1})$$

In the equation above, $V_{t,n}(s_t, q_t)$ corresponds to the total expected (discounted) cost from period $t$ to $n$, obtained by using the optimal policy for period $t+1$ through $n$, when the starting inventory level is $s_t$ and the order quantity is $q_t$ in period $t$. By definition, $V_{n+1,n}(.) \equiv 0$. The function $\phi_t(s_t, q_t)$ is a function of the starting inventory $s_t$, as well as $q_t$. From Equation (5), it can be rewritten in the following form:

$$\begin{aligned}
\phi(s_t, q_t) =& ((1 - G_t(q_t)) \int_{\lambda s_t^+ - s_t^- + q_t}^{+\infty} \Big[(c-p)q_t + p(D - \lambda s_t + s_t^-)\Big] f_t(D)\, dD \\
&+ ((1 - G_t(q_t)) \int_0^{\lambda s_t^+ - s_t^- + q_t} \Big[(c+h)q_t + h(\lambda s_t - s_t^- - D)\Big] f_t(D)\, dD \\
&+ \int_0^{q_t} \int_{\lambda s_t^+ - s_t^- + y}^{+\infty} \Big[(c-p)y + p(D - \lambda s_t + s_t^-)\Big] f_t(D) g_t(y)\, dD dy \\
&+ \int_0^{q_t} \int_0^{\lambda s_t^+ - s_t^- + y} \Big[(c+h)y + h(\lambda s_t - s_t^- - D)\Big] f_t(D) g_t(y)\, dD dy
\end{aligned} \tag{5.7}$$

## 5.3 Properties of the model

In this section, we study the model's proprieties and demonstrate the function $V_{t,n}(s_t, q_t)$ is quasi-convex. As a result, the optimal inventory policy of the model is an order-up-to level policy.

### 5.3.1 Single period total cost

The first derivative of $\phi(s_n, q_n)$, with respect to the order quantity $q_n$, is as follows

$$\frac{\partial \phi(s_n, q_n)}{\partial q_n} = \big[1 - G_n(q_n)\big]\Big[c - p + (h+p)F_n(\lambda s_n^+ - s_n^- + q_n)\Big] \tag{5.8}$$

and the second derivative:

$$\begin{aligned}
\frac{\partial^2 \phi(s_n, q_n)}{\partial q_n^2} =& ((1 - G_n(q_n))(h+p) f\Big[\lambda s_n^+ - s_n^- + q_n\Big] \\
&- g_n(q_n)\Big((h+p)F_n(\lambda s_n^+ - s_n^- + q_n) - p + c\Big)
\end{aligned} \tag{5.9}$$

$$\frac{\partial \phi(s_n, q_n)}{\partial q_n} = 0 \Leftrightarrow q_n^* = \mathrm{F}_n^{-1}(\frac{p-c}{h+p}) - \lambda s_n^+ + s_n^- \tag{5.10}$$

When $q_n \leq q_n^*$, we have:

$$\begin{aligned} & q_n \leq \mathrm{F}_n^{-1}(\frac{p-c}{h+p}) - \lambda s_n^+ + s_n^- \\ & \Rightarrow \Big((h+p)\mathrm{F}_n(-\lambda s_n^+ + s_n^- + q_n) - p + c\Big) \leq 0 \end{aligned}$$

In this case, the first right-hand term of Equation (5.9) is always non-negative and its second term is negative. Thus, $\frac{\partial^2}{\partial q_n^2}\phi(s_n, q_n) \geq 0$ then the function $\phi(s_n, q_n)$ is convex in $[0, q_n^*]$.

When $q_n > q_n^*$, we have: $\Big((h+p)\mathrm{F}(-\lambda s_n^+ + s_n^- + q_n) - p + c\Big) > 0$. Thus, the right hand of Equation (5.8) is positive, then the function $\phi(s_n, q_n)$ is increasing in $(q_n^*, +\infty)$.

So, the function $\phi(s_n, q_n)$ has a global minimum at $q_n = q_n^*$. Graphically, the function $\phi(s_n, q_n)$ has a quasi-convex form as shown in Figure 5.1. For the single-period case, it is clear that the order quantity only depends on customer demand, the deterioration rate, and the initial stock level.

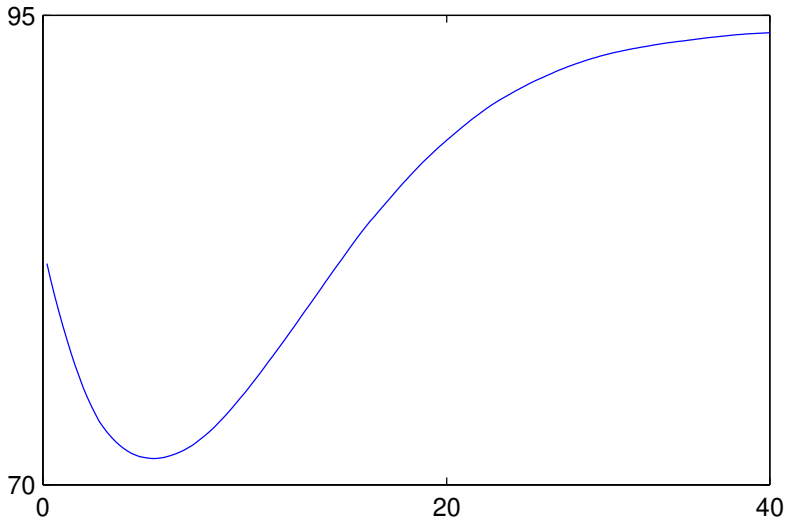


Figure 5.1: Quasi convex function

### 5.3.2 Multi-period expected cost

Now, we consider the $n$-periods model. The total expected (discounted) cost from periods $t$ to $n$ when the initial stock level is $s_t$ in period $t$ and the optimal policy is applied

from period $t$+1 to $n$, is given as follows:

$$\begin{aligned} \mathrm{V}_{t,n}(s_t,q_t) &= \phi(s_t,q_t) + \alpha \int_0^{q_t}\int_0^{+\infty} \mathrm{V}^*_{t+1,n}(\lambda s_t^+ - s_t^- + y - \mathrm{D}) f_t(\mathrm{D}) g_t(y)\, d\mathrm{D} dy \\ &+ \alpha((1-\mathrm{G}_t(q_t)) \int_0^{+\infty} \mathrm{V}^*_{t+1,n}(\lambda s_t^+ - s_t^- + q_t - \mathrm{D}) f_t(\mathrm{D})\, d\mathrm{D} \end{aligned} \tag{5.11}$$

The following expressions give the first and second partial derivatives of total expected cost $\mathrm{V}_{t,n}(s_t,q_t)$, with respect to the order quantity $q_t$:

$$\frac{\partial \mathrm{V}_{t,n}(s_t,q_t)}{\partial q_t} = \left[1-\mathrm{G}_t(q_t)\right]\varphi_{t,n}(s_t,q_t) \triangleq \Psi_{t,n}(s_t,q_t) \tag{5.12}$$

$$\begin{aligned} \text{where} \quad \varphi_{t,n}(s_t,q_t) &\triangleq \alpha \int_0^{+\infty} \frac{\partial}{\partial q_t} \mathrm{V}^*_{t+1,n}(\lambda s_t^+ - s_t^- + q_t - \mathrm{D}) f_t(\mathrm{D}) d\mathrm{D} \\ &+ c - p + (h+p)\mathrm{F}_t(\lambda s_t^+ - s_t^- + q_t) \end{aligned} \tag{5.13}$$

Note that $q_t = q_t^*$ is a solution of the equation: $\Psi_{t,n}(s_t,q_t) = 0$, which implies:

$$\Psi_{t,n}(s_t,q_t^*) = 0 \Leftrightarrow \frac{\partial \mathrm{V}_{t,n}(s_t,q_t)}{\partial q_t}|_{q_t=q_t^*} = 0 \Leftrightarrow \varphi_{t,n}(s_t,q_t^*) = 0 \tag{5.14}$$

**Proposition 1** $\frac{\partial^2}{\partial s_t^2}\mathrm{V}_{t,n}(s_t,q_t^*) \geq 0 \quad \forall s_t$

**Proof**: The following expressions give the first and second partial derivatives of total expected cost $\mathrm{V}_{t,n}(s_t,q_t)$, with respect to the inventory level $s_t$:

$$\frac{\partial \mathrm{V}_{t,n}(s_t,q_t)}{\partial s_t} = \begin{cases} \frac{\partial}{\partial s_t}\phi(s_t,q_t) + \alpha \frac{\partial}{\partial s_t}\int_0^{q_t}\int_0^{+\infty} \mathrm{V}^*_{t+1,n}(\lambda s_t + y - \mathrm{D}) f_t(\mathrm{D}) g_t(y)\, d\mathrm{D} dy \\ +\alpha((1-\mathrm{G}_t(q_t)) \frac{\partial}{\partial s_t}\int_0^{+\infty} \mathrm{V}^*_{t+1,n}(\lambda s_t + q_t - \mathrm{D}) f_t(\mathrm{D})\, d\mathrm{D}, \quad \text{if } s_t \geq 0 \\ \\ \frac{\partial}{\partial s_t}\phi(s_t,q_t) + \alpha \frac{\partial}{\partial s_t}\int_0^{q_t}\int_0^{+\infty} \mathrm{V}^*_{t+1,n}(s_t + y - \mathrm{D}) f_t(\mathrm{D}) g_t(y)\, d\mathrm{D} dy \\ +\alpha((1-\mathrm{G}_t(q_t)) \frac{\partial}{\partial s_t}\int_0^{+\infty} \mathrm{V}^*_{t+1,n}(s_t + q_t - \mathrm{D}) f_t(\mathrm{D})\, d\mathrm{D}, \quad \text{if } s_t < 0 \end{cases} \tag{5.15}$$

$$\frac{\partial^2 \mathrm{V}_{t,n}(s_t,q_t^*)}{\partial s_t^2} = \begin{cases} \lambda^2(h+p)\int_0^{q_t^*} f_t(\lambda s_t + y) g_t(y) d(y) \\ +\alpha \int_0^{+\infty}\int_0^{q_t^*} \frac{\partial^2}{\partial s_t^2}\mathrm{V}^*_{t+1,n}(\lambda s_t + y - \mathrm{D}) f_t(\mathrm{D}) d\mathrm{D} g_t(y) dy, \quad \text{if } s_t \geq 0 \\ \\ (h+p)\int_0^{q_t^*} f_t(s_t + y) g_t(y) d(y) \\ +\alpha \int_0^{+\infty}\int_0^{q_t^*} \frac{\partial^2}{\partial s_t^2}\mathrm{V}^*_{t+1,n}(s_t + y - \mathrm{D}) f_t(\mathrm{D}) d\mathrm{D} g_t(y) dy, \quad \text{if } s_t < 0 \end{cases} \tag{5.16}$$

The proposition can be proved by induction. For $t = n$, the equation above can be written as follows:

$$\frac{\partial^2 V_{n,n}(s_n, q_n^*)}{\partial s_n^2} = \begin{cases} \lambda^2 (h+p) \int_0^{q_n^*} f_t(\lambda s_n + y) g_t(y) dy \geq 0, & \text{if } s_n \geq 0 \\ (h+p) \int_0^{q_n^*} f_t(s_n + y) g_t(y) dy \geq 0, & \text{if } s_n < 0 \end{cases} \tag{5.17}$$

Note that $V_{n+1,n}(.) = 0$. Thus, Proposition 1 is true when $t = n$. Suppose that Proposition 1 is true for every period from $t$+1 to $n$. We can prove that Proposition 1 is also true for period $t$ as follows:

$$\frac{\partial^2 V_{t,n}(s_t, q_t^*)}{\partial s_t^2} = \begin{cases} \lambda^2 \frac{\partial^2}{\partial s_{t+1}^2} V_{t+1,n}^*(s_{t+1}, q_{t+1}^*) \geq 0, & \text{if } s_t \geq 0 \\ \frac{\partial^2}{\partial s_{t+1}^2} V_{t+1,n}^*(s_{t+1}, q_{t+1}^*) \geq 0, & \text{if } s_t < 0 \end{cases}$$

From the assumption of the induction, we obtain $\frac{\partial^2}{\partial s_{t+1}^2} V_{t+1,n}^*(s_{t+1}, q_{t+1}^*) \geq 0$. So, $\frac{\partial^2}{\partial s_{t+1}^2} V_{t+1,n}(s_{t+1})$ is non negative for any $t$. Therefore, the second term of Equation (5.16) is also non negative. As a result, we obtain $\frac{\partial^2}{\partial s_t^2} V_{t,n}(s_t, q_t^*) \geq 0$. Then, this proposition is true for every period $t$.

**Proposition 2** $\varphi_{t,n}(s_t, q_t)$ *is an increasing function in* $q_t$.

**Proof**: In Equation (5.13), we can see that $F_t(\lambda s_t^+ - s_t^- + q_t)$ is an increasing function, so it remains to demonstrate $\frac{\partial}{\partial q_t} V_{t+1,n}^*(\lambda s_t^+ - s_t^- + q_t - D)$ is also an increasing function. To simplify the notation, let $\Omega_2(s_t, q_t) = \lambda s_t^+ - s_t^- + q_t - D = s_{t+1}$. Thanks to partial derivative properties, we obtain the following equation:

$$\begin{aligned} &\frac{\partial}{\partial q_t} V_{t+1,n}^*(\lambda s_t^+ - s_t^- + q_t - D) = \frac{\partial}{\partial q_t} V_{t+1,n}^*(\Omega_2(s_t, q_t)) \\ &= \frac{\partial V_{t+1,n}^*(\Omega_2(s_t, q_t))}{\partial \Omega_2(s_t, q_t)} \frac{\partial \Omega_2(s_t, q_t)}{\partial q_t} = \frac{\partial V_{t+1,n}^*(\Omega_2(s_t, q_t))}{\partial \Omega_2(s_t, q_t)} \end{aligned} \tag{5.18}$$

The right hand side of Equation (5.18) is equal to $\frac{\partial}{\partial s_{t+1}} V_{t+1,n}^*(s_{t+1})$. Then the left hand side of Equation (5.18) is an increasing function in $q_t$ because $\frac{\partial}{\partial s_{t+1}} V_{t+1,n}^*(s_{t+1})$ is increasing due to Proposition 1. Note that: $V_{t+1,n}(s_{t+1}, q_{t+1}^*) = V_{t+1,n}^*(s_{t+1})$.

**Proposition 3** $V_{t,n}(s_t, q_t)$ *has a global minimum at* $q_t = q_t^*$

**Proof**: The second derivative $V_{t,n}(s_t, q_t^*)$ with respect to the order quality $q_t$ is given as follows:

$$\begin{aligned} \frac{\partial^2 V_{t,n}(s_t, q_t)}{\partial q_t} &= \frac{\partial \Psi(s_t, q_t)}{\partial q_t} \\ &= \left[1 - G_t(q_t)\right] \theta_{t,n}(s_t, q_t) - g_t(q_t) \varphi_{t,n}(s_t, q_t) \end{aligned} \tag{5.19}$$

$$\begin{aligned}
\text{Let}: \theta_{t,n}(s_t, q_t) &\triangleq \frac{\partial \varphi_{t,n}(s_t, q_t)}{\partial q_t} \\
&= \alpha \int_0^{+\infty} \frac{\partial^2}{\partial q_t^2} \mathrm{V}^*_{t+1,n}(\lambda s_t^+ - s_t^- + q_t - \mathrm{D}) f_t(\mathrm{D}) d\mathrm{D} \\
&+ (h+p) f_t(\lambda s_t^+ - s_t^- + q_t) \qquad (5.20)
\end{aligned}$$

When $q_t \leq q_t^*$, the function $\theta_{t,n}(s_t, q_t)$ is always no negative and $\varphi_{t,n}(s_t, q_t)$ is an increasing function (Proposition 2), thus $\varphi_{t,n}(s_t, q_t) \leqslant \varphi_{t,n}(s_t, q_t^*) = 0$. Therefore, $\frac{\partial^2}{\partial q_t^2} \mathrm{V}_{t,n}(s_t, q_t)$ is non negative when $q_t \leq q_t^*$. In other words, $\mathrm{V}(s_t, q_t)$ is convex in $(-\infty, q_t^*]$.

When $q_t \geq q_t^*$, Equation (5.12) is non negative because $1 - \mathrm{G}_t(q_t) \geq 0$ and $\varphi_{t,n}(s_t, q_t)$ is an increasing function, thus $\varphi_{t,n}(s_t, q_t) \geq \varphi_{t,n}(s_t, q_t^*) = 0$. In other words, $\mathrm{V}_t(s_t, q_t)$ is increasing in $[q_t^*, +\infty)$. So the function $\mathrm{V}_{t,n}(s_t, q_t)$ has a global minimum at $q_t = q_t^*$. Graphically, the function $\mathrm{V}_{t,n}(s_t, q_t)$ has a quasi-convex form. Note that $q_t^*$ is a solution of equation $\Psi_{t,n}(s_t, q_t) = 0$.

**Proposition 4** *The optimal policy for the n-periods model is an order-up-to level policy.*

**Proof**: Differentiating $\Psi_{t,n}(s_t, q_t^*)$ with respect to the inventory level, $s_t$, and applying Equation (5.15), we have the following equation:

$$\frac{d\Psi_{t,n}(s_t, q_t^*)}{ds_t} = \begin{cases} (\lambda + \frac{dq_t^*}{ds_t})\big[1 - \mathrm{G}_t(q_t^*)\big]\Big(\alpha \int_0^{+\infty} \frac{\partial^2}{\partial q_t^2} \mathrm{V}^*_{t+1,n}(\lambda s_t + q_t - \mathrm{D}) f_t(\mathrm{D}) d\mathrm{D} + \lambda(h+p) f_t(\lambda s_t + q_t)\Big) \\ -\frac{dq_t^*}{ds_t} g_t(q_t^*) \varphi_{t,n}(s_t, q_t^*), & \text{if } s_t \geq 0 \\ \\ (1 + \frac{dq_t^*}{ds_t})\big[1 - \mathrm{G}_t(q_t^*)\big]\Big(\alpha \int_0^{+\infty} \frac{\partial^2}{\partial q_t^2} \mathrm{V}^*_{t+1,n}(s_t + q_t - \mathrm{D}) f_t(\mathrm{D}) d\mathrm{D} + (h+p) f_t(s_t + q_t)\Big) \\ -\frac{dq_t^*}{ds_t} g_t(q_t^*) \varphi_{t,n}(s_t, q_t^*), & \text{if } s_t < 0 \end{cases} \qquad (5.21)$$

In Equation (5.21), we observe that: $1 - \mathrm{G}_t(q_t^*) > 0$ and the next term, $(\alpha \int (.) f_t(\mathrm{D}) d\mathrm{D} + \lambda(h+p) f_t(.))$ is nonnegative due to Proposition 1. The second term of Equation (5.21) is zero at $q_t^*$ because $\varphi_{t,n}(s_t, q_t^*) = 0$, according to the first-order condition provided by Equation (5.14).

As a result, Equation (5.21) is zero if:

$$\begin{cases} (\lambda + \frac{dq_t}{ds_t})|_{q_t = q_t^*} = 0 \Longrightarrow q_t^* + \lambda s_t = consant, & \text{if } s_t \geq 0 \\ (1 + \frac{dq_t}{ds_t})|_{q_t = q_t^*} = 0 \Longrightarrow q_t^* + s_t = constant, & \text{if } s_t < 0 \end{cases} \qquad (5.22)$$

In both cases, the order-up-to level (the inventory level after placing an order) does not depend on $s_t$, so the optimal inventory policy is an order-up-to level policy.

## 5.4 Solution approach

Dynamic programming is a useful tool which supports multi-stage decision-making processes, but its drawback is the curse of dimensionality. For the studied problem with high dimension, it is hard to apply this approach directly. That is why we develop a new solution approach to overcome the curse of dimensionality in the inventory model of the problem.

In this section, we first introduce scenario-based stochastic optimization approach and then transform the (SP) model into a deterministic equivalent model (DEP). The scenario-based stochastic programming approach allows to capture the uncertainty approximately, but its drawback is computational burden related to high dimensional data. The size of the model can grow dramatically due to the consideration of a large number of scenarios. To reduce computational burden, we proposed a Lagrangian relaxation based algorithm to find a near-optimal solution.

### 5.4.1 Deterministic equivalent model

Based on scenario approach, we reformulate the stochastic model in Section 5.2.1 as a mixed integer linear programming (MILP) model. Let $\Omega$ be a set of scenarios, $\omega \in \Omega$ be the index of a scenario (a possible realization of random variables) and its probability of occurrence is denoted by $p_\omega$. The index $\omega$ is added to decision variables related to the considered scenario $\omega$. The mathematic expectation $\mathbb{E}(.)$ in the objective function (5.1) is replaced by $\sum_\omega p_\omega(.)$

Let $s_{t,\omega}$ be the inventory level at the beginning of period $t$ under scenario $\omega$. Denote $\mathrm{L}_t$ as an order-up to level in period $t$. Our objective is to find this level, $\mathrm{L}_t$, by solving the following (DEP) model. Thanks to Equation (5.22), the order quantity, $q_{t,\omega}$, in each period $t$ under scenario $\omega$ is defined by:

$$q_{t,\omega} = \min\{\mathrm{L}_t - \lambda s^+_{t,\omega} + s^-_{t,\omega}, y_{t,\omega}\} \tag{5.23}$$

We introduce auxiliary variable, $\beta_{t,\omega}$ as follows:

$$\beta_{t,\omega} = \begin{cases} 1 & \text{if } \mathrm{L}_t - \lambda s^+_{t,\omega} + s^-_{t,\omega} \leq y_{t,\omega} \\ 0 & \text{otherwise} \end{cases}$$

The deterministic equivalent (DEP) model is given as follows:

$$\textbf{(DEP)} \quad \min \sum_{t\in T} \alpha^t \Big( \sum_{\omega\in\Omega} p_\omega (c q_{t,\omega} + h s^+_{t,\omega} + p s^-_{t,\omega}) \Big) \tag{5.24}$$

$$\text{s.t} \quad L_t - \lambda s^+_{t,\omega} + s^-_{t,\omega} - (1-\beta_{t,\omega})M \le q_{t,\omega} \le L_t - \lambda s^+_{t,\omega} + s^-_{t,\omega} \tag{5.25}$$

$$y_{t,\omega} - \beta_{t,\omega} M \le q_{t,\omega} \le y_{t,\omega} \tag{5.26}$$

$$s_{t+1,\omega} = \lambda s^+_{t,\omega} - s^-_{t,\omega} + q_{t,\omega} - D_{t,\omega} \tag{5.27}$$

$$s_{t,\omega} = s^+_{t,\omega} - s^-_{t,\omega} \tag{5.28}$$

$$s^+_{t,\omega} \ge s_{t,\omega} \tag{5.29}$$

$$s^-_{t,\omega} \ge -s_{t,\omega} \tag{5.30}$$

$$\beta_{t,\omega} \in \{0,1\}, s_{t,\omega} \in \mathbb{R}, s^+_{t,\omega} \ge 0, s^-_{t,\omega} \ge 0, q_{t,\omega} \ge 0, L_t \ge 0 \tag{5.31}$$

M: a big number

Equation (5.27) ensures the inventory balance constraint as Equation (5.2) in the original model. Equations (5.3)-(5.4) are linearized by constraints (5.28)-(5.30). Equation (5.23) is linearized by constraints (5.25-5.26). That can be explained as follows:

- If $\beta_{t,\omega} = 1$, we have $q_{t,\omega} = L_t - \lambda s^+_{t,\omega} + s^-_{t,\omega}$ and $q_{t,\omega} \le y_{t,\omega}$ due to (5.25-5.26).
- If $\beta_{t,\omega} = 0$, we have $q_{t,\omega} \le L_t - \lambda s^+_{t,\omega} + s^-_{t,\omega}$ and $q_{t,\omega} = y_{t,\omega}$ due to (5.25-5.26).

The two cases of $\beta_{t,\omega}$ imply that the constraint (5.23) is well ensured by the constraints (5.25-5.26) in the deterministic equivalent model.

### 5.4.2 Lagrangian relaxation approach

The (DEP) model could be solved by a commercial solver like CPLEX or GUROBI. However, it is too time consuming for the solvers to solve optimally large instances of the model. For this reason, we develop a Lagrangian relaxation based-method to find a near-optimal solution in a reasonable time. Lagrangian relaxation was introduced by Held and Karp [1970, 1971]; Held et al. [1974] and then this method has been widely used in combinatorial optimization. The Lagrangian relaxation is one of the techniques for generating lower bounds for solving combinatorial problems.

In this section, we propose a Lagrangian relaxation based algorithm for solving the (DEP) model. We decomposes the DEP model across scenarios and partitions the model into manageable sub-models. Our method used the volume algorithm of Barahona and Anbil [2000] for dual optimization. The volume method is an extension of the sub-gradient algorithm and very similar to the bundle method [Lemaréchal, 1989] but it could avoid the extra computational cost for solving a quadratic problem in each iteration. Solving the scenario sub-models defined for all $\omega \in \Omega$ will give us different order-quantity, $q_{t,\omega}$ which depend on the order-up-to levels $L_t$ and the scenario corresponding corresponded. To

combine the scenario-dependent solutions of the sub-models to an implementable solution of the original problem in Lagrangian relaxation framework, we replace $L_t$ by $L_{t,\omega}$ and introduce the following condition:

$$L_{t,\omega} = L_{t,\omega'} \quad \forall \omega, \omega' \in \Omega \text{ and } \omega \neq \omega' \tag{5.32}$$

We then use Lagrange multipliers to relax the above equalities. Let $\pi = (\pi_{t,\omega})\ \forall t \in T, \omega \in \Omega$ be a vector of Lagrange multipliers associated with the constraint (5.32). For the readability of the solution approach, we introduce an intermediate variable, $\delta_{t,\omega}$ which can be interpreted as the violation of the constraint (5.32).

$$\delta_{t,\omega} = L_{t,\omega} - L_{t,\omega'} \quad \forall \omega, \omega' \in \Omega \text{ and } \omega \neq \omega' \tag{5.33}$$

Therefore, the Lagrangian relaxed program (LRP) can be formulated as follows. It could provide a lower bound on the optimal objective value to the deterministic equivalent (DEP) model.

$$\textbf{(LRP)} \quad \min \sum_{t\in T} \alpha^t \Big( \sum_{\omega\in\Omega} p_\omega (cq_{t,\omega} + hs^+_{t,\omega} + ps^-_{t,\omega}) \Big) + \sum_{t\in T} \sum_{\omega\in\Omega} \pi_{t,\omega}\delta_{t,\omega} \tag{5.34}$$

$$\text{s.t} \quad L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} - (1-\beta_{t,\omega})M \leq q_{t,\omega} \leq L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} \tag{5.35}$$

$$\text{constraint}(5.26) - (5.31)$$

From the solution $L_{t,\omega}$ found by solving the LRP, we can transform it into a feasible solution, $L_t$ for the original problem by scenarios aggregation as follows. This feasible solution can provide an upper bound of the optimal objective value of the (DEP) model.

$$L_t = \sum_{\omega\in\Omega} p_\omega L_{t,\omega} \quad \forall t \in T \tag{5.36}$$

It is important to find the values for the Lagrange multipliers, $\pi$ that give the maximum lower bound. In other words, we need to find a lower bound that is as close as possible to the optimal objective value of the (DEP) model. This leads to the following Lagrangian dual problem:

$$\max_{\pi\in\mathbb{R}} \{\textbf{LRP}(\pi)\}$$
$$= \max_{\pi\in\mathbb{R}} \left\{ \begin{array}{l} \min \sum_{t\in T} \alpha^t \Big( \sum_{\omega\in\Omega} p_\omega (cq_{t,\omega} + hs^+_{t,\omega} + ps^-_{t,\omega}) \Big) + \sum_{t\in T} \sum_{\omega\in\Omega} \pi_{t,\omega}\delta_{t,\omega} \\ \text{s.t} \\ L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} - (1-\beta_{t,\omega})M \leq q_{t,\omega} \leq L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} \\ \text{constraint} \quad (5.26)\text{-}(5.31) \end{array} \right\}$$

The sub-gradient method is often used to solve a Lagrangian dual problem. According to Barahona and Anbil [2000], this method has a low computational burden, but it

converges to the optimal solution of the dual problem too slowly or even does not converge. The volume method maintains the same low computational burden per iteration, but it has a good convergence performance. Besides, the volume method is faster than the bundle method because it avoids the extra computational burden for solving a quadratic problem in each iteration. That is why we use the volume method to solve our Lagrangian dual program.

The Lagrangian relaxation algorithm with the volume method for dual optimization is presented in Algorithm 5.1.

**Algorithm 5.1:** : Lagrangian relaxation algorithm

Set $\pi \leftarrow \bar{\pi}$ and the iteration index $k \leftarrow 0$.
Solve the LRP with $\pi = \bar{\pi}$ to obtain solution $\bar{\mathrm{L}}_{t,\omega}$ and a lower bound $\bar{lb} \leftarrow \mathrm{LRP}(\bar{\pi})$.
Transform the value of $\bar{\mathrm{L}}_{t,\omega}$ into a feasible solution $\mathrm{L}_t$ for the (DEP) model by using Equation (5.36). Then, construct a feasible solution $\mathrm{L}_t$ of the (DEP) model with the corresponding upper bound denote by $ub$.
**while** *new iteration* **do**
  Compute $\bar{\delta}_{t,\omega} = \bar{\mathrm{L}}_{t,\omega} - \bar{\mathrm{L}}_{t,\omega'} \quad \forall \omega, \omega' \in \Omega$ and $\omega \neq \omega'$;
  Update the Lagrange multiplier as follows:

$$\pi^k \leftarrow \bar{\pi} + f \frac{ub - \bar{lb}}{\sum_{t\in \mathrm{T}} \sum_{\omega\in\Omega} \bar{\delta}^2_{t,\omega}} \bar{\delta}_{t,\omega} \tag{5.37}$$

  where $f(ub - \bar{lb})/(\sum_{t\in\mathrm{T}} \sum_{\omega\in\Omega} \bar{\delta}^2_{t,\omega})$ is a step size, $f$ is a number between 0 and 2, and $ub$ is an upper bound for the optimal value of the (DEP) model;
  Solve the LRP with $\pi = \pi^k$ to obtain solution $\mathrm{L}^k_{t,\omega}$ and a lower bound $lb^k \leftarrow \mathrm{LRP}(\pi^k)$;
  Update the solution: $\bar{\mathrm{L}}_{t,\omega} \leftarrow \zeta \mathrm{L}^k_{t,\omega} + (1-\zeta)\bar{\mathrm{L}}_{t,\omega}$;
  Transform the value of $\bar{\mathrm{L}}_{t,\omega}$ into a feasible solution $\mathrm{L}_t$ for the (DEP) model by using Equation (5.36). Then, construct a feasible solution $\mathrm{L}_t$ of the (DEP) model with the corresponding upper bound denote by $ub^k$;
  Update $ub \leftarrow \max(ub, ub^k)$;
  **if** $\bar{lb} < lb^k$ **then**
    Update $\bar{\pi} \leftarrow \pi^k$ and $\bar{lb} \leftarrow lb^k$;
  $k \leftarrow k+1$;

This algorithm is stopped when the relative gap, $(ub - \bar{lb})/ub$ is less than a certain threshold or a maximal number of iterations is reached.

In the following, we explain how to update the value of coefficients $f$ and $\zeta$ in each iteration of the Lagrangian relaxation algorithm. The value of $f$ in Equation (5.37) is modified in each iteration according to whether it is a red, yellow, or green iteration. In a red iteration, there is no improvement on the lower bound (i.e $\bar{lb} \geq lb^k$). A number of consecutive

red iterations suggests the need for a smaller step-size, i.e the value of $f$ should be reduced (multiplied by a positive number less than 1). Otherwise, $d = \sum_{\omega\in\Omega}\sum_{t\in T}\bar{\delta}_{t,\omega}\delta^k_{t,\omega}$ is computed. The value $d \leq 0$ means that a longer step in the direction of $\bar{\delta} = (\bar{\delta}_{t,\omega})\ \forall t \in T, \omega \in \Omega$ would give a smaller value for $LRP(\pi^k)$. A yellow iteration is defined as an iteration with no improvement on lower bound and the value $d \leq 0$. If $d \geq 0$, we have a green iteration which suggests the requirement of a larger step-size, i.e., the value of $f$ should be multiplied by a positive number greater than 1). In our numerical study, the value $f$ is updated as follows.

$$f^k = \begin{cases} 1.1f & \text{if k-th iteration is green iteration} \\ 0.66f & \text{after 5 consecutive red iterations} \\ f^{k-1} & \text{Otherwise} \end{cases}$$

According to Barahona and Anbil [2000], $\zeta$ is set in each iteration as the value that minimizes $\sqrt{\sum_{\omega\in\Omega}\sum_{t\in T}[\zeta\delta^k_{t,\omega} + (1-\zeta)\bar{\delta}_{t,\omega}]^2}$ where $\zeta \in [\zeta_{min}, \zeta_{max}]$. Therefore, the value of $\zeta$ is given by:

$$\zeta = \begin{cases} \zeta_{min} & \text{if} \quad \zeta_{opt} \leq \zeta_{min} \\ \zeta_{opt} & \text{if} \quad \zeta_{opt} \in [\zeta_{min}, \zeta_{max}] \\ \zeta_{max} & \text{if} \quad \zeta_{opt} \geq \zeta_{max} \end{cases}$$

$$\text{where } \zeta_{opt} = \frac{\sum_{t\in T}\sum_{\omega\in\Omega}\bar{\delta}_{t,\omega}(\bar{\delta}_{t,\omega} - \delta^k_{t,\omega})}{\sum_{t\in T}\sum_{\omega\in\Omega}(\bar{\delta}_{t,\omega} - \delta^k_{t,\omega})^2}$$

## 5.5 Numerical study

In this section, a numerical study is conducted to evaluate the performance and demonstrate the effectiveness of our Lagrangian relaxation algorithm. This approach is implemented in Python 3.5 on a HP computer with Intel Core i5-4210M CPU 2.66 GHz and 8.0 GB RAM. All MILP problems were solved by a solver GUROBI 6.5 with default parameters. According to the Benchmarks of Optimization Software of Mittelmann [2018], GUROBI performs better than CPLEX on the MIPLIB2010 benchmark set. That is why we chose GUROBI as MILP solver in our numerical study.

### 5.5.1 Parameters setting

In the numerical study, we use the data from Broekmeulen and Van Donselaar [2009]; Pauls-Worm et al. [2015] for average demand, coefficient of variation ($CV = \sigma/\mu$), number of time periods. The holding cost $h$ is set to 5. The unit purchase cost $c$ is varied from 30 to 40. The unit backorder cost $p$ is determined from the ratio $p/(p+h)$, which is varied between 95% and 97.5%. The number of periods and scenarios are $T = 7$ and $|\Omega| = 200$. By applying a statistical method in Shapiro [2000], we observed that the stochastic program-

ming model requires 200 scenarios for an instance with 7 periods to achieve an approximatively optimal solution with probability 95% within 1% deviation from the true optimal solution.

According to Rentizelas et al. [2009], the degradation of biomass is estimated about 1% on average under the ambient storage condition. Therefore, we vary the deterioration rate $(1-\lambda)$ from 1% to 3% (extreme case). All parameter values are given in Table 5.1. We also consider three cases of demand and supply distribution as follows:

(i) The mean demand in each period is computed by $\mu_{D_t} = f_t.\mu_D$ where the demand pattern in each period is $f_t = \{0.12, 0.13, 0.13, 0.16, 0.18, 0.18, 0.10\}$, and the value of mean demand over 7 periods $\mu_D$ is taken as $[12, 13, 13, 16, 18, 18, 10]$. The stochastic supply follows a stationary normal distribution with $\mu_{S_t} = 16$ and $\sigma_{S_t} = 0.2\mu_{S_t}$ for all $t = 1...7$.

(ii) Demand and supply follow a stationary normal distribution with $\mu_{D_t} = 13$, $\sigma_{D_t} = 3.5$, $\mu_{S_t} = 15$ and $\sigma_{S_t} = 0.2\mu_{S_t}$ for all $t = 1...7$.

(iii) Demand and supply follow a stationary normal distribution with $\mu_{D_t} = 14$, $\sigma_{D_t} = 0.25\mu_{D_t}$, $\mu_{S_t} = 15$ and $\sigma_{S_t} = 0.2\mu_{S_t}$ for all $t = 1...7$.

The parameters setting of the LR algorithm is given as follows. We fix the optimality gap as $\epsilon \leq 0.01\%$, maximal number of iterations as $iteration_{max} = 500$ and maximal CPU time as $time_{max} = 3600$s. The algorithm also stops if the upper bound cannot be improved after 50 consecutive iterations. In each green iteration, the coefficient $f$ is multiplied by 1.1. After a sequence of 5 consecutive red iterations, $f$ will be multiplied by 0.66. We set $\zeta_{min} = 0.3$ and $\zeta_{max} = 0.8$.

Table 5.1: Parameter values for the numerical study

| Input parameters | Range |
|---|---|
| Coefficient of variation $CV_D = \sigma_D/\mu_D$ | [0.20 , 0.25] |
| Coefficient of variation $CV_S = \sigma_S/\mu_S$ | 0.20 |
| Backorder cost ratio $p/(p+h)$ | [0.95, 0.975] |
| Holding cost $h$ | [5] |
| Purchase cost, $c$ | [30, 40] |
| Deterioration rate, $(1-\lambda)$ | [1%, 3%] |
| Discount factor, $\alpha$ | [0.96, 0.98] |

### 5.5.2 Performance evaluation of the solution algorithm

To compare the performance of our solution approach, we also ran a dynamic programming (DP) algorithm ([Bellmam, 1957, Chapter III.3]) to find the optimal order-up-to level policy for the multi-period perishable inventory model. One issue that may affect the accuracy of the results of the DP algorithm needs to be addressed. In each period, the

Table 5.2: Performance of the LR algorithm in case a

| | | | | | **LR method** | | **DP method** | | |
|---|---|---|---|---|---|---|---|---|---|
| **c** | **p** | **h** | α | λ | **CPU(s)** | **E-LR** | **CPU(s)** | **DP** | **Gap** |
| 40 | 100 | 5 | 0.96 | 0.97 | 166.31 | 3792.24 | 5463.00 | 3787.15 | 0.13% |
| 40 | 100 | 5 | 0.98 | 0.97 | 201.50 | 4023.11 | 5228.27 | 4012.88 | 0.25% |
| 40 | 100 | 5 | 0.96 | 0.99 | 199.17 | 3734.97 | 5301.20 | 3729.39 | 0.15% |
| 40 | 100 | 5 | 0.98 | 0.99 | 215.42 | 3958.55 | 5149.86 | 3948.82 | 0.25% |
| 40 | 200 | 5 | 0.96 | 0.97 | 195.36 | 4065.29 | 5155.27 | 3990.89 | 1.86% |
| 40 | 200 | 5 | 0.98 | 0.97 | 234.94 | 4273.45 | 5182.28 | 4220.89 | 1.25% |
| 40 | 200 | 5 | 0.96 | 0.99 | 388.29 | 3939.26 | 5146.70 | 3910.90 | 0.73% |
| 40 | 200 | 5 | 0.98 | 0.99 | 217.81 | 4177.95 | 5157.06 | 4133.24 | 1.08% |
| 50 | 100 | 5 | 0.96 | 0.97 | 197.71 | 4624.92 | 5159.33 | 4592.93 | 0.70% |
| 50 | 100 | 5 | 0.98 | 0.97 | 203.13 | 4879.23 | 5136.91 | 4868.01 | 0.23% |
| 50 | 100 | 5 | 0.96 | 0.99 | 201.45 | 4544.56 | 5139.81 | 4525.84 | 0.41% |
| 50 | 100 | 5 | 0.98 | 0.99 | 207.78 | 4809.46 | 5139.49 | 4792.93 | 0.34% |
| 50 | 200 | 5 | 0.96 | 0.97 | 181.45 | 4955.46 | 5134.46 | 4821.47 | 2.78% |
| 50 | 200 | 5 | 0.98 | 0.97 | 225.10 | 5146.83 | 5130.03 | 5098.38 | 0.95% |
| 50 | 200 | 5 | 0.96 | 0.99 | 211.12 | 4748.26 | 5130.63 | 4728.81 | 0.41% |
| 50 | 200 | 5 | 0.98 | 0.99 | 208.01 | 5034.91 | 5138.43 | 4996.62 | 0.77% |
| **Average** | | | | | **215.91** | | **5180.80** | | **0.77%** |

expected total cost $\mathop{\mathbb{E}}_{(D_t,y_t)} V^*_{t+1,n}(s_{t+1})$ is calculated using a numerical integration method. However, the numerical integration may introduce some error in the cost. To make the integration accurate enough, its step size should be taken as small as possible, i.e., the interval of integration must be small enough. In our implementation of the algorithm, the problem is solved at different levels of discretization,i.e., using different step sizes from large to small for the numerical integration, until no significant change in the cost can be observed.

To evaluate the quality of the solutions obtained by the LR algorithm, we compute the E-LR which is the expected objective value of the stochastic inventory model at the solution obtained by our Lagrangian relaxation approach. Note that for each instance, the (DEP) model includes 3 807 continuous, 1400 binary variables, and 7 007 constraints.

Table 5.2, 5.3 and 5.4 give the computational time and the expected objective value of the solution obtained by LR and DP algorithm, respectively. The last column shows the relative gap between the two solution approaches (E-LR and DP). As we can see, the LR algorithm has a much shorter CPU time than DP algorithm on average (215.91s vs 5180.80s in case a, 187.83s vs 5476.63s in case b and 172.59s vs 5399.91s in case c). In terms of expected cost, the average gap of the LR solution from the DP solution is only 0.77% , 0.70% and 0.50% in three cases, respectively. The numerical study demonstrates that the LR approach can be obtain a high-quality near-optimal solution in a reasonable time and thus more practical than the DP approach.

Table 5.3: Performance of the LR algorithm in case b

| | | | | | **LR method** | | **DP method** | | |
|---|---|---|---|---|---|---|---|---|---|
| **c** | **p** | **h** | α | λ | **CPU(s)** | **E-LR** | **CPU(s)** | **DP** | **Gap** |
| 40 | 100 | 5 | 0.96 | 0.97 | 162.27 | 3331.37 | 5463.00 | 3316.79 | 0.44% |
| 40 | 100 | 5 | 0.98 | 0.97 | 181.42 | 3527.55 | 5472.82 | 3517.36 | 0.29% |
| 40 | 100 | 5 | 0.96 | 0.99 | 186.09 | 3297.49 | 5480.86 | 3277.14 | 0.62% |
| 40 | 100 | 5 | 0.98 | 0.99 | 191.07 | 3485.77 | 5474.70 | 3473.88 | 0.34% |
| 40 | 200 | 5 | 0.96 | 0.97 | 177.89 | 3525.09 | 5476.79 | 3477.96 | 1.35% |
| 40 | 200 | 5 | 0.98 | 0.97 | 189.15 | 3745.13 | 5470.18 | 3683.81 | 1.66% |
| 40 | 200 | 5 | 0.96 | 0.99 | 189.78 | 3437.06 | 5476.65 | 3426.16 | 0.32% |
| 40 | 200 | 5 | 0.98 | 0.99 | 201.63 | 3667.29 | 5481.99 | 3627.36 | 1.10% |
| 50 | 100 | 5 | 0.96 | 0.97 | 178.00 | 4057.46 | 5489.69 | 4037.91 | 0.48% |
| 50 | 100 | 5 | 0.98 | 0.97 | 185.39 | 4294.05 | 5477.86 | 4282.11 | 0.28% |
| 50 | 100 | 5 | 0.96 | 0.99 | 191.71 | 4022.92 | 5472.96 | 3990.68 | 0.81% |
| 50 | 100 | 5 | 0.98 | 0.99 | 203.08 | 4249.43 | 5486.43 | 4230.02 | 0.46% |
| 50 | 200 | 5 | 0.96 | 0.97 | 179.80 | 4244.37 | 5478.14 | 4223.15 | 0.50% |
| 50 | 200 | 5 | 0.98 | 0.97 | 189.87 | 4526.88 | 5473.96 | 4472.82 | 1.21% |
| 50 | 200 | 5 | 0.96 | 0.99 | 190.09 | 4178.69 | 5477.39 | 4161.80 | 0.41% |
| 50 | 200 | 5 | 0.98 | 0.99 | 207.98 | 4443.27 | 5472.69 | 4405.73 | 0.85% |
| **Average** | | | | | **187.83** | | **5476.63** | | **0.70%** |

Table 5.4: Performance of the LR algorithm in case c

| | | | | | **LR method** | | **DP method** | | |
|---|---|---|---|---|---|---|---|---|---|
| **c** | **p** | **h** | α | λ | **CPU(s)** | **E-LR** | **CPU(s)** | **DP** | **Gap** |
| 40 | 100 | 5 | 0.96 | 0.97 | 144.86 | 3666.22 | 5413.41 | 3651.44 | 0.40% |
| 40 | 100 | 5 | 0.98 | 0.97 | 159.23 | 3878.07 | 5400.99 | 3871.24 | 0.18% |
| 40 | 100 | 5 | 0.96 | 0.99 | 166.72 | 3613.40 | 5397.55 | 3604.50 | 0.25% |
| 40 | 100 | 5 | 0.98 | 0.99 | 178.00 | 3822.41 | 5401.94 | 3819.79 | 0.07% |
| 40 | 200 | 5 | 0.96 | 0.97 | 159.76 | 3933.00 | 5394.93 | 3898.08 | 0.90% |
| 40 | 200 | 5 | 0.98 | 0.97 | 177.87 | 4164.19 | 5397.44 | 4133.60 | 0.74% |
| 40 | 200 | 5 | 0.96 | 0.99 | 176.09 | 3843.55 | 5402.22 | 3831.49 | 0.31% |
| 40 | 200 | 5 | 0.98 | 0.99 | 182.61 | 4090.81 | 5398.43 | 4060.84 | 0.74% |
| 50 | 100 | 5 | 0.96 | 0.97 | 165.11 | 4442.42 | 5402.30 | 4432.73 | 0.22% |
| 50 | 100 | 5 | 0.98 | 0.97 | 173.30 | 4709.94 | 5397.96 | 4698.94 | 0.23% |
| 50 | 100 | 5 | 0.96 | 0.99 | 180.34 | 4391.54 | 5400.17 | 4377.90 | 0.31% |
| 50 | 100 | 5 | 0.98 | 0.99 | 188.62 | 4659.73 | 5393.06 | 4638.59 | 0.46% |
| 50 | 200 | 5 | 0.96 | 0.97 | 163.72 | 4719.05 | 5390.70 | 4701.33 | 0.38% |
| 50 | 200 | 5 | 0.98 | 0.97 | 177.95 | 5089.67 | 5399.97 | 4983.42 | 2.13% |
| 50 | 200 | 5 | 0.96 | 0.99 | 182.22 | 4635.81 | 5403.49 | 4625.36 | 0.23% |
| 50 | 200 | 5 | 0.98 | 0.99 | 185.02 | 4924.90 | 5403.99 | 4900.30 | 0.50% |
| **Average** | | | | | **172.59** | | **5399.91** | | **0.50%** |

## 5.6 Conclusion

In this chapter, we proposed a stochastic inventory model for a perishable product under stochastic environments and developed an algorithm that combines scenario-based optimization and Lagrangian relaxation to solve approximatively the model. A computational study was conducted with data from the literature to evaluate the near-optimal solution obtained by the algorithm by comparing it to the optimal solution found by dynamic programming. Our results indicated that the LR solution is usually very close to the optimal solution. The relevance of our solution approach is thus proved. Our proposed solution approach can assist managers to make ordering decisions under fluctuation in demand and supply.

# Chapter 6

# Modeling and optimization for an inventory problem of a fixed lifetime product under uncertainties

> *" "Il n'est pas certain que tout soit incertain. "*
>
> — Blaise Pascal, Pascal's Pensees

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## 6.1 Introduction

In the previous chapter, we have studied an inventory problem for a product with constant deterioration rate under uncertainty in both supply and demand. As highlighted in Chapter 2, some perishable products continuously deteriorate whereas the others have fixed lifetime. To the best of our knowledge, very few works study the inventory control problem for a product with a fixed lifetime under uncertainty in both supply and demand. Therefore, the mentioned problem should be taken into account.

In this chapter, we extend a previous model to a perishable product with a fixed lifetime under both stochastic demand and stochastic supply. Based on the product's remaining lifetime and some useful historical information related to demand and supply, we develop a practical approach to help retail managers generating an effective inventory replenishment plan. The essential characteristics of the inventory model are perishability, stochastic demand, stochastic supply, and multi-period. The main contributions of this chapter include:

Firstly, we propose a stochastic programming model for a perishable inventory system with fixed life shelf under both stochastic demand and stochastic supply in a rolling horizon framework.

Secondly, we formulate a mixed integer linear programming (MILP) model equivalent to the original problem using the Conditional Scenarios (CS) approach.

Thirdly, we conduct a numerical study to evaluate the performance of the proposed algorithm. The results show that these approaches can find a high-quality solutions with statistical performance guaranteed in a reasonable computation time. Sensitivity analysis is conducted to examine the effect of some critical parameters on the total system cost.

This remainder of this chapter is organized as follows: in Section 6.2, the problem and its model are introduced. Then, we introduce the solution approaches and solution evaluation in Sections 6.3 and 6.4. A numerical study is performed, and its results are analyzed in Section 6.5. Section 6.6 concludes this chapter with a brief resume and perspectives.

## 6.2 Problem description and model formulation

### 6.2.1 Problem description

In this chapter, we consider a multi-period inventory problem for a perishable product with fixed shelf life. For simplicity, we assume that company uses the first in first out (FIFO) withdraw policy. In this paper, the order of events occurred in each period is given as follows:

(i) Fresh items arrive at the beginning of each period. Then, the inventory level of each age is updated. The expired items are discarded.

(ii) Based on the inventory policy used, an order then placed if necessary. After that, the demand and supply in this period are observed, and customer demand is satisfied as much as possible, and unsatisfied demand is lost (lost sale).

(iii) At the end of each period, holding costs or shortage costs are charged based on the remaining inventory which will be carried over from the current period to the next. The out-dating costs are also charged for expired items.

### 6.2.2 Model formulation

In the following, we formulate a stochastic programming model for the inventory optimization problem of perishable product considered. The product is perishable, and its age is denoted by $i \in \{1, ..., M\}$. Let $I_t^i$ denote the inventory level of the product which has age $i$ at the end of period $t \in \{1, ..., T\}$. It is convenient to denote $I_t^1$ as fresh items of the product in stock at the end of period $t \in \{1, ..., T\}$. In the first stage, the decision variable includes the timing of ordering $y_t$ and the ordering quantity $O_t$ in each period $t \in \{1, ..., T\}$. The second-stage variables include the supply quantity received $q_t$, inventory level $I_{t,\omega}^i$ for each age $i$, lost sales $B_t$ and outdated items $I_t^M$ at the end of period $t$. In the second stage, the recourse actions are made to correct the negative effects due to the first stage decision (the timing of ordering). In the model, all continuous decision variables are non-negative.

It is assumed that demand $D_t$ and supply $S_t$ in each period $t$ are multi-normally distributed random variables with known distribution $N(\mu, \Sigma)$ where $\mu$ is a vector of mean of $\{D_t, S_t\}$ and $\Sigma$ is a covariance matrix which gives the correlation between the supply $S_t$ and the demand $D_t$.

For convenience, all notations used in the model formulation are summarized as follows.

**Indices:**

- $i$: Age index, $i \in \{1, ..., M\}$ with M being the fixed shelf life
- $t$: Period index $t \in \{1, ..., T\}$ with T being the time horizon

**Deterministic Parameters**

- $a$: Ordering cost for each order placed
- $v$: Purchasing cost for each unit of product
- $h$: Holding cost for each unit of product and each period in stock
- $p$: Shortage cost charged for each unit of unmet demand at the end of period $t$
- $w$: Waste/out-dating cost for each unit of products expired
- $I_0^i$ : Initial inventory, for all age $i = 1...M$

**Stochastic Parameters**

- $\mathrm{D}_t$: Random demand in period $t$
- $\mathrm{S}_t$: Random supply capacity in period $t$

**Decision variables:**

- $\mathrm{I}_t^i$: Inventory level of age $i \in \{1,...,\mathrm{M}\}$ at the end of period $t$. Note that $\mathrm{I}_t^1$ refers to fresh items in stock and $\mathrm{I}_t^{\mathrm{M}}$ refers to wasted/outdated items at the end of period $t$.
- $q_t$: Supply quantity received in period $t$
- $y_t$: Timing of ordering in period $t$ (binary variable), $y_t = 1$ if an order is placed in period $t$ and $y_t = 0$ otherwise.
- $\mathrm{O}_t$: Order quantity in period $t$
- $\mathrm{B}_t$: Unsatisfied demand (lost sales) in period $t$

The stochastic programming model for an inventory of perishable product is given as follows:

$$\textbf{(SP)} \quad \min \sum_{t=1}^{\mathrm{T}} \mathbb{E}_{\mathrm{D}_t,\mathrm{S}_t}\Big[ a y_t + u q_t + h(\sum_{i=1}^{\mathrm{M}-1} \mathrm{I}_t^i) + p\mathrm{B}_t + w\mathrm{I}_t^{\mathrm{M}}\Big] \tag{6.1}$$

s.t

$$y_t = \begin{cases} 1, & if \quad \mathrm{O}_t > 0 \\ 0, & otherwise \end{cases} \qquad t = 1,...\mathrm{T} \tag{6.2}$$

$$\mathrm{Y}_t^i = (\mathrm{D}_t - \sum_{j=i}^{\mathrm{M}-1} \mathrm{I}_{t-1}^j)^+ \quad i = 1...\mathrm{M},\ t = 1,...\mathrm{T} \tag{6.3}$$

$$\mathrm{I}_t^i = (\mathrm{I}_{t-1}^{i-1} - \mathrm{Y}_t^i)^+, \quad t = 1,...\mathrm{T},\ i = 2...\mathrm{M} \tag{6.4}$$

$$\mathrm{I}_t^1 = (q_t - \mathrm{Y}_t^1)^+, \quad t = 1,...\mathrm{T} \tag{6.5}$$

$$\mathrm{B}_t = \sum_{i=1}^{\mathrm{M}-1} \mathrm{I}_{t-1}^i - \sum_{i=1}^{\mathrm{M}} \mathrm{I}_t^i + q_t - \mathrm{D}_t, \quad t = 1,...\mathrm{T} \tag{6.6}$$

$$q_t = min\{\mathrm{S}_t, \mathrm{O}_t\}, \quad t = 1,...\mathrm{T} \tag{6.7}$$

$$\mathrm{I}_t^i \geq 0, \quad i = 1...\mathrm{M},\ t = 1,...\mathrm{T} \tag{6.8}$$

$$q_t, \mathrm{B}_t \geq 0, \quad t = 1,...\mathrm{T} \tag{6.9}$$

$$y_t \in \{0,1\}, \quad t = 1,...\mathrm{T} \tag{6.10}$$

where: $x^+ = max\{0, x\}$

The objective (6.1) is to minimize the total expected cost which includes five types of costs: ordering, purchasing, inventory holding, lost sales and out-dating cost over the planning horizon of T periods.

In Equation (6.2), the binary variable $y_t = 1$ only if there is an order quantity $O_t$ in period $t$. Equation (6.3) shows the variable $Y^i_{t,}$ as the remaining demand after withdrawing all products having ages from $i$ to M using FIFO withdraw policy. Note that the decision variables $Y^i_{t,}$ are redundant, but they are introduced for the readability of the model. Equation (6.4) and (6.5) give the transition of inventory from the actual period to the next. In Equations (6.3) and (6.4), we implicitly assume that for the first period, $t = 1$, the variables $I^i_{t-1}$ are replaced by the predetermined parameters $I^i_0$ (initial inventory). Equation (6.6) shows the inventory balance constraints between period $t$ and $t+1$. This equation also determines that lost sale, $B_t$, which only occurs when the supply quantity received, $q_t$, and the inventory of all age $i$ cannot meet demand, $D_t$. Equation (6.7) ensures that the quantity received is always limited by the supply $S_t$.

## 6.3 Solution approach

The stochastic programming model is quite complicated, and it requires an enormous memory and computational effort to find the optimal solution. For this reason, we try to provide a high-quality solution quickly instead of finding the optimal solution because solving. By applying a scenario-based stochastic framework Birge and Louveaux [2011], we can transform a stochastic programming model into a mixed integer linear programming (MILP) model by replacing the stochastic parameters by the values taken in the corresponding scenario. However, the number of scenarios should be considered significant enough to approximate the stochastic model well. So, it could lead to a MILP model having enormous size whose resolution requires a lot of memory and is very time-consuming.

The Sample Average Approximation (SAA) method is a conventional approach for stochastic optimization such as the research of Kim et al. [2015]; Kleywegt et al. [2002]; Pagnoncelli et al. [2009]. This method allows to reduce the number of scenarios considered and find the near-optimal solution. Based on conditional expectation discretization, Beltran-Royo [2017] proves that Conditional Scenarios (CS) method might provide a better solution than SAA method. That is why we develop a scenario-based optimization approach for solving our model based on Conditional Scenarios (CS) approach.

In this section, the original model (SP) is transformed into an equivalent MILP model by applying the scenario approach. This solution approach is outlined in Figure 6.1. The Conditional Scenarios method allows to generate a set of scenarios from given continuous random vector by applying a conditional expectation discretization. The CS approach will be compared with the Sample Average Approximation (SAA) method in the next section.

### 6.3.1 Scenario-based approach and deterministic equivalent model

Based on the scenario-based stochastic programming approach, we reformulate the model **(SP)** into a mixed integer linear programming (MILP) model. This approach allows

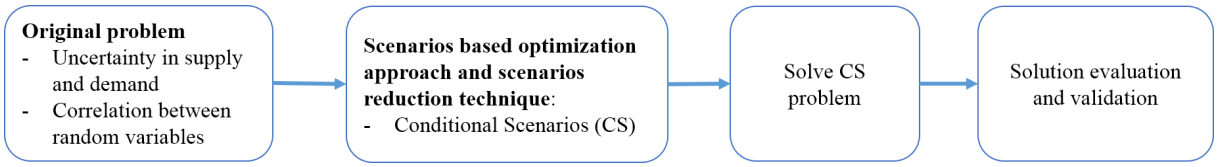



Figure 6.1: Solution approach

to capture the uncertainty in an approximate way with the precision depending on the number of scenarios considered.

Let $\omega \in \Omega$ be the index of a scenario (a possible realization of random variables) and its probability of occurrence is $p_\omega$. Note that the index pair for conditional scenario $\{r, e\}$ and its corresponding probability are replaced by scenario $\omega$ and $p_\omega$ in this section. The mathematic expectation $\mathbb{E}(.)$ in the objective function (6.1) can be replaced by $\sum_\omega p_\omega[.]$. Then, the nonlinear objective function (6.1 is transformed into the deterministic objective function (6.11). The deterministic equivalent MILP model is given as follows:

$$\min \sum_{t=1}^{T} a y_t + \sum_{\omega\in\Omega} \sum_{t=1}^{T} p_\omega \Big[ u q_{t,\omega} + h(\sum_{i=1}^{M-1} \mathrm{I}^i_{t,\omega}) + p\mathrm{B}_{t,\omega} + w\mathrm{I}^{\mathrm{M}}_{t,\omega} \Big] \tag{6.11}$$

$$s.t. \quad \text{Constraints} \quad (6.12) - (6.21)$$

The index $\omega$ is added to each two-stage decision variables related to the scenario $\omega$ except for the ordering quantity $\mathrm{O}_t$. The notation of the first-stage decision variables $y_t$ remains as before. The ordering quantity $\mathrm{O}_t$ does not appear explicitly in the objective function but implicitly through the realization of random variables such demand $\mathrm{D}_{t,\omega}$ and supply $\mathrm{S}_{t,\omega}$.

In the following subsections, we linearize the nonlinear constraints (6.2)-(6.6) of the original problem (SP). Note that H is a big number.

### Ordering constraints

Constraint (6.2) can be linearized as follows:

$$0 \le \mathrm{O}_t \le \mathrm{H} y_t \quad t = 1, \ldots \mathrm{T} \tag{6.12}$$

### Constraints on $\mathrm{Y}^i_{t,\omega}$

Equation (6.3) can be linearized as follows:

$$\mathrm{D}_{t,\omega} - \sum_{j=i}^{M-1} \mathrm{I}^j_{t-1,\omega} \le \mathrm{Y}^i_{t,\omega} \le (\mathrm{D}_{t,\omega} - \sum_{j=i}^{M-1} \mathrm{I}^j_{t-1,\omega}) + (1 - \alpha^i_{t,\omega})\mathrm{H} \tag{6.13}$$

$$0 \le \mathrm{Y}^i_{t,\omega} \le \alpha^i_{t,\omega}\mathrm{H} \tag{6.14}$$

$$\alpha^i_{t,\omega} \in \{0, 1\} \tag{6.15}$$

That can be explained as follows:

- If $\alpha^i_{t,\omega} = 1$, we have $Y^i_{t,\omega} = (D_{t,\omega} - \sum_{j=i}^{M-1} I^j_{t-1,\omega}) \geq 0$ due to constraints (6.13) and (6.14).
- If $\alpha^i_{t,\omega} = 0$, we have $Y^i_{t,\omega} = 0$ and $(D_{t,\omega} - \sum_{j=i}^{M-1} I^j_{t-1,\omega}) \leq 0$ due to constraints (6.13) and (6.14).

The two cases of $\alpha^i_{t,\omega}$ imply that the constraint (3) can be replaced by the constraints (6.13 - 6.15) in the deterministic equivalent model.

**FIFO constraints**

Now, the constraints (6.4) and (6.5) can be linearized as follows:

$$I^i_{t,\omega} \geq (I^{i-1}_{t-1,\omega} - Y^i_{t,\omega}) \quad \forall t = 1,...T,\ i = 2...M \tag{6.16}$$

$$I^1_{t,\omega} \geq (q_{t,\omega} - Y^1_{t,\omega}) \quad \forall t = 1,...T \tag{6.17}$$

Since the coefficients related to $I^i_{t,\omega} i = 1,...M$ in the objective function (6.1) to minimize are positive.

**Unsatisfied demand**

Constraint (6.6) ensure the inventory balance between period $t$ and $t+1$. Lost sale, $B_{t,\omega}$, occurs when the demand $D_{t,\omega}$ can not be satisfied in period $t$ under scenario $\omega$. The constraints are given by:

$$B_{t,\omega} = \sum_{i=1}^{M-1} I^i_{t-1,\omega} - \sum_{i=1}^{M} I^i_{t,\omega} + q_{t,\omega} - D_{t,\omega}, \quad t = 1,...T \tag{6.18}$$

**Random supply capacity**

Constraint (6.7) specifies that the quantity received $q_{t,\omega}$ is the minimum between order quantity $O_t$ and a realization of supply capacity $S_{t,\omega}$ in period $t$ under scenario $\omega$. That constraint can be linearized as follows:

$$S_{t,\omega} - (1 - \gamma_{t,\omega})H \leq q_{t,\omega} \leq S_{t,\omega} \tag{6.19}$$

$$O_t - \gamma_{t,\omega}H \leq q_{t,\omega} \leq O_t \tag{6.20}$$

$$\gamma_{t,\omega} \in \{0,1\} \tag{6.21}$$

H: a big number

That can be explained as follows:

- If $\gamma_{t,\omega} = 1$, we have $q_{t,\omega} = S_{t,\omega}$ because $S_{t,\omega}$ is not only an upper bound but also a lower bound of $q_{t,\omega}$ due to Constraint(6.19). We then have $q_{t,\omega} = S_{t,\omega} \leq O_t$ due to Constraint (6.20). Then $q_{t,\omega} = min\{S_{t,\omega}, O_t\}$.

- If $\gamma_{t,\omega} = 0$. we have $q_{t,\omega} = O_t$ due to Constraint (6.20) and $q_{t,\omega} = O_t \leq S_{t,\omega}$ due to Constraint (6.19). So, $q_{t,\omega} = min\{S_{t,\omega}, O_t\}$

Then in both cases, we have $q_{t,\omega} = \min\{S_{t,\omega}, O_t\}$.

### 6.3.2 Conditional scenarios approach

Since the computational complexity for solving the stochastic programming problem increases with the number of scenarios, some techniques to reduce the number of representative scenarios that should be considered: moment matching methods, the Sample Average Approximation (SAA) method, approaches based on probability metrics among others. According to Beltran-Royo [2017], the Conditional Scenarios (CS) problem allows to deal with uncertainty in shorter computational time, and it could yield a better solution. Also, the CS approach can be seen as an aggregation method, in which, aggregation weights are given by conditional probability function of the random parameters. It is also possible to compute conditional scenarios directly from a given continuous random vector by applying a conditional expectation discretization. The notation for the conditional scenarios method is given as follows:

**Indices:**

- E: number of discrete realizations of random variable considered
- $e$: index of realization of random variables, $e = 1...E$
- $r$: Component index of random vectors, $r = 1...R$
- $\{r, e\}$: Index pair for conditional scenarios

**Random vectors:**

- $\xi$: random vectors including R components $(\xi_1, ...\xi_R)^T$
- $\xi_r$: $r^{th}$ component of random vector $\xi$
- $\tilde{\xi}_{r,e}$: realization of random variable $\xi_r$ with $e = 1, ...E$
- $\{\tilde{p}_{r,e}\}$: the corresponding probability of the realization $\tilde{\xi}_{r,e}$
- $\hat{\xi}^{r,e}$: Conditional scenario or realization given at the end of the CS procedure.
- $\{\hat{p}^{r,e}\}$: the corresponding probability of realization $\hat{\xi}^{r,e}$

We summarize the conditional scenarios method Beltran-Royo [2017] in the following procedure:

1. Given a vector of random variables $\xi = (\xi_1, ...\xi_R)^T$ with the index $r = 1...R$ and $\xi \sim N_R(\mu, \Sigma)$. For each random variable $\xi_r$ defined in a given interval $L_r$, we discretize it and obtain finite realizations $\tilde{\xi}_{r,e}$, $e = 1, ...E_r$ with the corresponding probabilities $\{\tilde{p}_{r,e}\}$. $E_r$ is the number of discrete realizations in the given interval $L_r = [a_r, b_r]$. For example, $L_r = [\mu_r - 4\sigma_r, \mu_r + 4\sigma_r]$.

   (i) Divide the interval $L_r$ into $E_r$ sub-intervals of equal length $L_{r,e} = [a_{r,e}, b_{r,e}]$

   (ii) For each $L_{r,e}$, compute the discrete realizations, $\tilde{\xi}_{r,e}$ and their corresponding probabilities, $\tilde{p}_{r,e}$ as follows:

$$\tilde{\xi}_{r,e} = \mathbb{E}[\xi_r \mid \xi_r \in L_{r,e}] = \mu_r + \sigma_r \frac{pdf(\alpha_{r,e}) - pdf(\beta_{r,e})}{cdf(\alpha_{r,e}) - cdf(\beta_{r,e})}$$
$$\tilde{p}_{r,e} = \mathbb{P}(\xi_r \in L_{r,e}) = cdf(\alpha_{r,e}) - cdf(\beta_{r,e})$$

   where $\alpha_{r,e} = (a_{r,e} - \mu_r)/\sigma_r$ and $\beta_{r,e} = (b_{r,e} - \mu_r)/\sigma_r$. $pdf$ and $cdf$ are the probability density function and the cumulative distribution function, respectively, of a standard normal variable. $\mu_r$ and $\sigma_r$ are the mean and the standard deviation of the random variable $\xi_r$

2. We compute the conditional scenarios $\hat{\xi}^{r,e}$ and the corresponding probabilities $\{\hat{p}^{r,e}\}$ by using the discrete realizations, $\tilde{\xi}_{r,e}$ found in step 1 as follows:

$$\hat{\xi}^{r,e} = \mathbb{E}[\xi \mid \tilde{\xi}_{r,e}] = \mu + \frac{\tilde{\xi}_{r,e} - \mu_r}{\sigma_r^2} \Sigma_r$$
$$\hat{p}^{r,e} = \tilde{p}_{r,e}$$

   The total number of conditional scenarios is: $\sum_{r=1}^{R} E_r$ where $E_r$ is the number discrete realizations of $\xi_r$ into $\tilde{\xi}_r$.

The above procedure can be summarized by the following procedure: $\xi \rightarrow \{\tilde{\xi}_{r,e}\} \rightarrow \{\hat{\xi}^{r,e}\}$. With this method, we can formulate the MILP problem with its conditional expectation scenarios.

## 6.4 Solution evaluation

To evaluate the solution quality, we compare our approach with the most common approach, Sample Average Approximation (SAA) method. In this technique, the expected objective function of the stochastic problem is approximated by an average sample estimate derived from a random sample. The main differences between SAA and CS approach are the ways to approximate the random vector $\xi$ and their associated probabilities (equal probability versus conditional probability). Besides, only one sampling is made in CS ap-

proach, but a large number of samplings is necessary to ensure the quality of solution in SAA approach.

Some reduction techniques such as Latin hypercube sampling [Diwekar and David, 2015; McKay et al., 1979] has been proposed to increase the efficiency of the SAA method. According to Diwekar and David [2015], Latin Hypercube Sampling can estimate more precisely the distribution function and improve computational efficiency.

### 6.4.1 Sample Average Approximation

The sample average approximation (SAA) method Kleywegt et al. [2002] is an approach for solving stochastic optimization problems by using Monte Carlo simulation. In this technique, the expected objective function of the stochastic problem is approximated by an average sample estimate derived from a random sample. In the SAA algorithm, a sample set $\{\xi_1, \xi_2, ....\xi_R\}$ of R scenarios is randomly generated from $\Omega$ total scenarios. In other words, this sample set is a subset of $\Omega$ whole scenarios. Each scenario occurs with equal probability $p_\xi = 1/R$. The expected value of the objective function $\mathbb{E}(.))$ is approximated by a set of scenarios considered. Then, the general two-stage stochastic programming model can be rewritten as follows:

$$Min \quad \left\{cx + \frac{1}{R}\sum_{r=1}^{R} Q(x,\xi_r)\right\} \tag{6.22}$$

$$s.t$$

$$Ax = b, \quad x \geq 0 \tag{6.23}$$

$$T_r x + W_r y_r = h_r \tag{6.24}$$

$$y_r \geq 0, \quad r = 1...R$$

Where the vector $x$ is the first-stage decision variables and $y_r$ is the second-stage decision variables for each scenario $r$. The objective is to minimize the cost of the first stage $(c^T x)$ and the expected cost over all scenarios in the second stage $\frac{1}{R}\sum_{r=1}^{R} Q(x,\xi_r)$. $T_s$ is the matrix related to the first stage decision variables $x$. $W_r$ is a recourse matrix, $q_r$ and $h_r$ are two vectors associated with scenario $r$ in the model of the second stage.

The SAA method then generates A independent sample tests, each containing R scenarios, solving the associated problems, and obtain objective values $z_R^1; z_R^2; ...; z_R^A$ and candidate solutions $\hat{x}^1; \hat{x}^2; ...; \hat{x}^A$ are obtained. The average of the A optimal values of the problems is given by:

$$\bar{z}_R = \frac{1}{R}\sum_{a=1}^{A} z_R^a \tag{6.25}$$

It is well known that $\mathbb{E}(\bar{z}_R) \leq z^*$, where $z^*$ is the optimal objective value of the original problem. Thus, we can obtain a lower bound, $z^*$ by estimating $\mathbb{E}(\bar{z}_R)$ according to the

formulation in (6.25). This estimation (6.25) is an unbiased of $\mathbb{E}(\bar{z}_{\mathrm{R}})$ and therefore is a statistical lower bound to $z_*$.

For any feasible solution $\hat{x} \in \mathrm{X}$, the upper bound of $z$ can be obtained by using total scenarios $\Omega$:

$$z_{\Omega}(\hat{x}) = c\hat{x} + \frac{1}{\Omega}\sum_{a=1}^{\Omega} \mathrm{Q}(\hat{x}, \xi^{a}) \tag{6.26}$$

Note that the above procedure produces up to A different candidate solutions. It is natural to take $\hat{x}^*$ as one of the optimal solutions $\hat{x}^1; \hat{x}^2; ...; \hat{x}^{\mathrm{A}}$ of the A problems which has the smallest estimated objective value, that is,

$$\hat{x}^* \in arg\ \ min\Big\{z_{\Omega}(\hat{x}) \mid \hat{x} \in [\hat{x}^1; \hat{x}^2; ...; \hat{x}^{\mathrm{A}}]\Big\} \tag{6.27}$$

The above procedure is stopped when the relative gap, $(\mid z_{\Omega}(\hat{x}^*) - \bar{z}_{\mathrm{R}} \mid / \bar{z}_{\mathrm{R}})$ or the variance of the expected solution $\mathrm{Var}(z_{\Omega}(\hat{x}))$ is less than a certain threshold or a maximal number of iterations is reached.

### 6.4.2 Latin hypercube sampling

According to Diwekar and David [2015], Latin Hypercube Sampling (LHS) can estimate more precisely the distribution function and improve computational efficiency. In LHS method, the range of each uncertain parameter is subdivided into non-overlapping intervals of equal probability. One value from each interval is selected randomly with the probability distribution in the interval. Iman and Conover [1982]; Pebesma and Heuvelink [1999]; Zhang and Pinder [2003] show that Latin hypercube sampling (LHS) combined with Cholesky decomposition method allows to cover better the distributions of random input variables and obtain the desired correlations between different input random variables via covariance matrix. For this reason, we applied this technique to generate the whole scenarios in the SAA method.

## 6.5 Numerical study

In this section, numerical studies are conducted to evaluate the performance of the proposed algorithm and demonstrate the effectiveness of our approach. All MILP problems related to CS and SAA approach were solved by Gurobi 6.5 [Gurobi Optimization, 2016] with default parameters on a HP computer with Intel Core i5-4210M CPU 2.66 GHz and 8.0 GB RAM. According to the Benchmarks of Optimization Software [Mittelmann, 2018], GUROBI performs better than CPLEX on the MIPLIB2010 benchmark set. That is why we chose GUROBI as MILP solver in our numerical study.

Table 6.1: Parameter values for the simulation experiment

| Input parameters | Level |
|---|---|
| Product lifetime M | [2, 3] |
| Mean week demand $\mu_D$ | [200] |
| Coefficient of variation $CV_D = \sigma_D/\mu_D$ | 0.25 |
| Coefficient of variation $CV_S = \sigma_S/\mu_S$ | 0.20 |
| Correlation between random variables | [0.5] |
| Out-dating cost ratio $w/v$ | [0.5, 1.0] |
| Lost sales cost ratio $p/(v+p+h)$ | [0.90, 0.95, 0.99] |
| Holding cost ratio $h/v$ | [0.002, 0,004] |
| Ordering cost for each order placed, $a$ | [$250v$, $500v$] |

### 6.5.1 Data generation

In the numerical study, we use the data from Broekmeulen and Van Donselaar [2009]; Pauls-Worm et al. [2016] for the fixed maximum shelf life, average demand, coefficient of variation (CV = $\sigma/\mu$), length of time period. All parameter values are given in Table 6.1. Following Broekmeulen and Van Donselaar [2009], we normalize all cost parameters on the purchasing cost $v$, which is set to 1. The unit holding cost ratio $h/v$ is varied from 0.02 to 0.04. The unit lost sales cost $p$ is determined from the lost sales cost ratio $p/(v+p+h)$, which is varied between 90% and 99%. The unit out-dating cost ratio, $w/v$ is varied between 0 and 1. The fixed ordering cost for each order placed is varies from $250v$ to $500v$. The maximum shelf life is M= 2 or 3. The number of periods is T =7. The mean demand for each day is computed by $\mu_{D_t} = f_t.\mu_D$ where demand pattern is $f_t = \{0.12, 0.13, 0.13, 0.16, 0.18, 0.18, 0.10\}$, and the value of mean demand in each periods ,$\mu_{D_t} = [240, 260, 260, 320, 360, 360, 200]$. Supply follows a stationary normal distribution with $\mu_{S_t} = 320$ and $\sigma_{S_t} = 0.2\mu_{S_t}$ for all $t = 1...7$.

The main focus of our work is to study the performance of the proposed approaches and the impacts of some key parameters of the studied model on its optimal cost. This uncertainty is incorporated into the model by considering a set of possible scenarios. The number of scenarios should be large enough to capture the entire range of the probability distribution of supply and demand.

### 6.5.2 Performance evaluation of the solution algorithms

In this section, we present our computational experiments on the algorithms proposed in the previous section for solving instances. To formulate the CS problem, we apply Section 4.1 to calculate the conditional scenarios that approximate the normal random vector which includes $\mu = \{\mu_{D_t}, \mu_{S_t}\}$ and $\sigma = \{\sigma_{D_t}, \sigma_{S_t}\}$. For T=7, we have R=14 random parameters, E=12 discretization points and $L_r = [\mu_r - 4\sigma_r, \mu_r + 4\sigma_r]$. The choice of E=12 can be explained as follows: we considered the (MILP) problem with different discretization level such as E= 5,6,7..100. We observe that the scenarios optimal solution has very similar quality compared with others for the discretization with E $\geq$ 12.

In fact, the SAA and CS approaches try to solve the original problem approximately with an intention to reduce the computational burden. However, it is hard to assess the quality of the optimal SAA and CS solutions by using the optimality gap because we can not find the optimal solution of the original problem. In this case, we assess the E-SAA and E-CS value which are the expected value of the objective function of the original problem obtained at the SAA solution and CS solution, respectively. To calculate the E-SAA, E-CS, we generated 1000 000 scenarios randomly sampled from a multivariate normal distribution of $\xi = \{D_t, S_t\}$, $t = 1...T$ to estimate approximately the value of E-SAA and E-CS.

Table 6.2 and 6.3 present the computational time and solution obtained by CS and SAA approach. All instances include 4 368 continuous variables, 3 192 binary variables, and 16 471 constraints. The last column shows the relative gap between the two solutions found by CS and SAA approach (E-CS and E-SAA). Based on the inferential statistical analysis, the obtained solutions, E-SAA, could achieve an approximatively optimal solution with probability 95% within 0.1% deviation from the true optimal solution. As we can see, we can see that the CS approach performed efficiently for instances of different sizes because the E-CS value is always less than the E-SAA value with 0.07% and 0.09% in average for M=2 and 3 respectively. Besides, the CS approach has much shorter CPU times (35.37s versus 284.91s and 35.41s versus 317.08s respectively). The numerical study suggests that the CS approach has a good balance between the solution quality and the computational time.

### 6.5.3 Analysis of the solution

Figure 6.2a shows the cost distribution of the base case for the perishable inventory system over planning horizon T= 5. We consider a perishable product with the fixed shelf life M=2. The ordering cost is \$30. The unit holding cost, unit shortage cost, and unit wastage cost are \$2, \$15, \$3 respectively. In this study, we assume demand and supply are normal variables. Their parameters over T periods are presented as follows:

The optimal value of the total expected cost of CS problem is \$41 740,98. As we can see, the purchase cost and shortage cost are the primary cost drivers that account for nearly 71% and 27% of the total system costs respectively. The holding cost is only 1%, so in this case, the storage of a significant amount of product seems not critical. This result shows how the most important of inventory policy on the total system cost because it affects the purchased quantity and unmet demand strongly.

$$\mu_D = [860, 810, 760, 570, 620]$$
$$\mu_S = [520, 550, 860, 940, 760]$$
$$\sigma_D^2 = [29584, 26244, 23104, 12996, 15376]$$
$$\sigma_S^2 = [10816, 12100, 29584, 35344, 23104]$$

Table 6.2: Performance of the solution algorithm in case of M = 2

| | | | | | | CS approach | | SAA approach | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Instances** | **M** | **a** | **h** | **p** | **w** | **CPUs** | **E-CS** | **CPUs** | **E-SAA** | **%** |
| 1 | 2 | 250 | 0.02 | 10 | 0.5 | 41.70 | 4 292.30 | 412.22 | 4 296.62 | 0.10% |
| 2 | 2 | 250 | 0.02 | 10 | 1.0 | 46.02 | 4 314.66 | 416.08 | 4 320.61 | 0.14% |
| 3 | 2 | 250 | 0.02 | 20 | 0.5 | 37.08 | 4 624.11 | 277.45 | 4 627.32 | 0.07% |
| 4 | 2 | 250 | 0.02 | 20 | 1.0 | 42.38 | 4 650.57 | 278.18 | 4 651.37 | 0.02% |
| 5 | 2 | 250 | 0.02 | 100 | 0.5 | 21.74 | 7 168.08 | 124.52 | 7 165.30 | 0.04% |
| 6 | 2 | 250 | 0.02 | 100 | 1.0 | 22.70 | 7 187.49 | 124.44 | 7 189.40 | 0.03% |
| 7 | 2 | 250 | 0.04 | 10 | 0.5 | 50.63 | 4 310.32 | 402.67 | 4 316.87 | 0.15% |
| 8 | 2 | 250 | 0.04 | 10 | 1.0 | 50.46 | 4 335.91 | 404.42 | 4 340.85 | 0.11% |
| 9 | 2 | 250 | 0.04 | 20 | 0.5 | 44.42 | 4 643.95 | 284.44 | 4 648.22 | 0.09% |
| 10 | 2 | 250 | 0.04 | 20 | 1.0 | 37.32 | 4 672.40 | 284.28 | 4 672.30 | 0.00% |
| 11 | 2 | 250 | 0.04 | 100 | 0.5 | 20.13 | 7 190.01 | 131.28 | 7 187.21 | 0.04% |
| 12 | 2 | 250 | 0.04 | 100 | 1.0 | 21.31 | 7 208.03 | 137.75 | 7 211.31 | 0.05% |
| 13 | 2 | 500 | 0.02 | 10 | 0.5 | 40.72 | 6 045.13 | 493.51 | 6 046.23 | 0.02% |
| 14 | 2 | 500 | 0.02 | 10 | 1.0 | 45.74 | 6 065.13 | 471.14 | 6 070.21 | 0.08% |
| 15 | 2 | 500 | 0.02 | 20 | 0.5 | 35.96 | 6 376.24 | 281.32 | 6 376.38 | 0.00% |
| 16 | 2 | 500 | 0.02 | 20 | 1.0 | 40.82 | 6 398.31 | 281.08 | 6 400.45 | 0.03% |
| 17 | 2 | 500 | 0.02 | 100 | 0.5 | 23.83 | 8 917.29 | 125.89 | 8 917.31 | 0.00% |
| 18 | 2 | 500 | 0.02 | 100 | 1.0 | 18.01 | 8 959.10 | 126.31 | 8 941.42 | 0.20% |
| 19 | 2 | 500 | 0.04 | 10 | 0.5 | 44.07 | 6 061.58 | 472.26 | 6 066.87 | 0.09% |
| 20 | 2 | 500 | 0.04 | 10 | 1.0 | 50.49 | 6 084.54 | 478.42 | 6 090.85 | 0.10% |
| 21 | 2 | 500 | 0.04 | 20 | 0.5 | 35.37 | 6 396.94 | 284.28 | 6 398.14 | 0.02% |
| 22 | 2 | 500 | 0.04 | 20 | 1.0 | 35.50 | 6 420.17 | 283.06 | 6 422.20 | 0.03% |
| 23 | 2 | 500 | 0.04 | 100 | 0.5 | 21.37 | 8 929.61 | 131.20 | 8 939.28 | 0.11% |
| 24 | 2 | 500 | 0.04 | 100 | 1.0 | 21.18 | 8 951.67 | 131.63 | 8 963.38 | 0.13% |
| **Mean** | | | | | | **35.37** | | **284.91** | | **0.07%** |

### 6.5.4 Sensitivity analysis

We investigated the impacts of some critical parameters such as purchase price, unit set up cost, unit shortage cost on the expected total system cost.

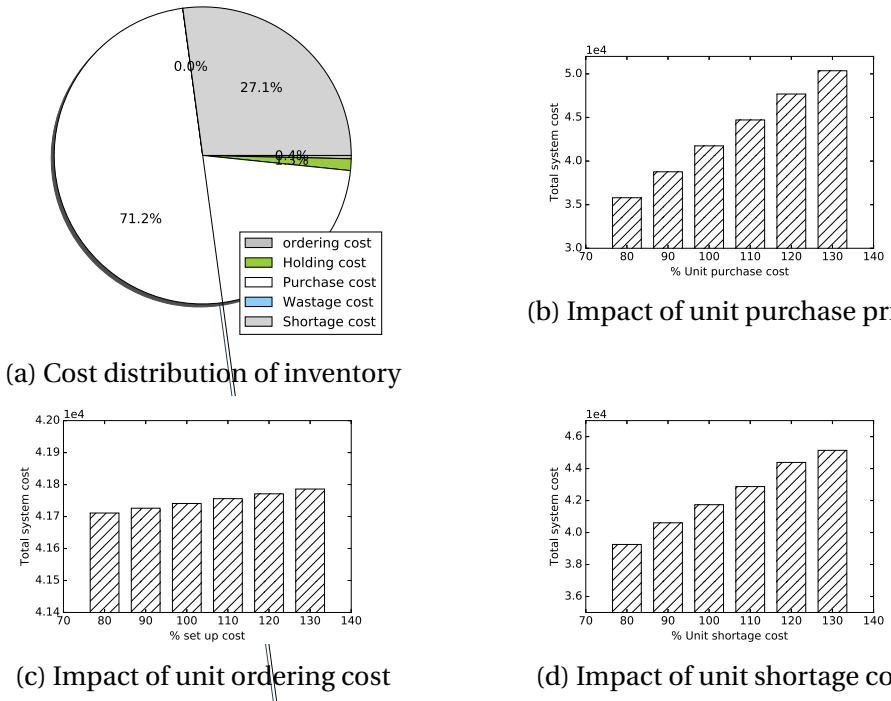
Figure 6.2b presents the effect of the unit purchase price on the total cost of perishable inventory. The horizontal axis represents the various unit purchase price from 80% to 130% in comparison with its base value. The vertical axis shows the total expected system costs under different values of the unit purchase price. This result indicates that with the increase in purchase price, the total system cost will increase rapidly. This confirms the fact that buying cost plays the most crucial role in this system with 72% of total cost.

Whereas total system cost also grows up slightly as the function of setup cost (Figure 6.2c), however, this increases is not as significant as the increase in setup cost (from 80% to 130% in comparison with its base value).

Due to uncertainties in demand and supply, the total system cost seems to be sensitive due to unit shortage cost. Figure 6.2d shows of the unit purchase price on the total cost of perishable inventory. The horizontal axis represents the various unit shortage cost from 80% to 130% in comparison with its base value. The vertical axis shows the total expected system costs under different values of unit shortage cost.

Table 6.3: Performance of the solution algorithm in case of M = 3

| | | | | | | **CS approach** | | **SAA approach** | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Instances** | **M** | **a** | **h** | **p** | **w** | **CPUs** | **E-CS** | **CPUs** | **E-SAA** | **%** |
| 25 | 3 | 250 | 0.02 | 10 | 0.5 | 55.84 | 4 266.77 | 445.58 | 4 273.36 | 0.15% |
| 26 | 3 | 250 | 0.02 | 10 | 1.0 | 54.97 | 4 267.57 | 444.91 | 4 274.58 | 0.16% |
| 27 | 3 | 250 | 0.02 | 20 | 0.5 | 47.55 | 4 604.56 | 292.38 | 4 602.29 | 0.05% |
| 28 | 3 | 250 | 0.02 | 20 | 1.0 | 51.79 | 4 598.56 | 292.78 | 4 603.51 | 0.11% |
| 29 | 3 | 250 | 0.02 | 100 | 0.5 | 22.27 | 7 118.65 | 159.28 | 7 115.32 | 0.05% |
| 30 | 3 | 250 | 0.02 | 100 | 1.0 | 23.20 | 7 114.27 | 159.04 | 7 116.54 | 0.03% |
| 31 | 3 | 250 | 0.04 | 10 | 0.5 | 45.14 | 4 289.65 | 451.54 | 4 297.92 | 0.19% |
| 32 | 3 | 250 | 0.04 | 10 | 1.0 | 46.21 | 4 292.36 | 453.91 | 4 299.14 | 0.16% |
| 33 | 3 | 250 | 0.04 | 20 | 0.5 | 32.27 | 4 623.46 | 322.44 | 4 622.92 | 0.01% |
| 34 | 3 | 250 | 0.04 | 20 | 1.0 | 31.60 | 4 624.95 | 320.81 | 4 624.14 | 0.02% |
| 35 | 3 | 250 | 0.04 | 100 | 0.5 | 19.76 | 7 141.74 | 161.24 | 7 139.52 | 0.03% |
| 36 | 3 | 250 | 0.04 | 100 | 1.0 | 20.74 | 7 143.25 | 162.05 | 7 140.74 | 0.04% |
| 37 | 3 | 500 | 0.02 | 10 | 0.5 | 48.60 | 6 017.28 | 491.71 | 6 023.72 | 0.11% |
| 38 | 3 | 500 | 0.02 | 10 | 1.0 | 42.23 | 6 018.43 | 490.53 | 6 024.94 | 0.11% |
| 39 | 3 | 500 | 0.02 | 20 | 0.5 | 32.26 | 6 349.65 | 315.73 | 6 349.74 | 0.00% |
| 40 | 3 | 500 | 0.02 | 20 | 1.0 | 34.53 | 6 348.71 | 313.72 | 6 350.96 | 0.04% |
| 41 | 3 | 500 | 0.02 | 100 | 0.5 | 23.70 | 8 874.10 | 161.54 | 8 867.33 | 0.08% |
| 42 | 3 | 500 | 0.02 | 100 | 1.0 | 18.09 | 8 843.12 | 161.43 | 8 868.56 | 0.29% |
| 43 | 3 | 500 | 0.04 | 10 | 0.5 | 43.59 | 6 041.52 | 514.15 | 6 046.52 | 0.08% |
| 44 | 3 | 500 | 0.04 | 10 | 1.0 | 46.65 | 6 042.33 | 516.14 | 6 047.75 | 0.09% |
| 45 | 3 | 500 | 0.04 | 20 | 0.5 | 34.15 | 6 371.62 | 324.92 | 6 378.35 | 0.11% |
| 46 | 3 | 500 | 0.04 | 20 | 1.0 | 35.77 | 6 378.56 | 324.59 | 6 379.57 | 0.02% |
| 47 | 3 | 500 | 0.04 | 100 | 0.5 | 19.39 | 8 881.60 | 165.02 | 8 891.59 | 0.11% |
| 48 | 3 | 500 | 0.04 | 100 | 1.0 | 19.65 | 8 871.92 | 164.53 | 8 892.81 | 0.23% |
| **Mean** | | | | | | **35.41** | | **317.08** | | **0.09%** |




(a) Cost distribution of inventory

(b) Impact of unit purchase price

(c) Impact of unit ordering cost

(d) Impact of unit shortage cost

Figure 6.2: Analysis of the solution

## 6.6 Conclusion

In this chapter, we proposed a stochastic model for minimizing the total system cost of a perishable inventory under stochastic environments. We proposed a CS method to solve approximatively the stochastic programming model. The CS approach appears a better choice in comparison with the SAA approach since the former can provide a better quality solution in a shorter computation time. The CS approach seems practical since the computation of the conditional scenarios is comprehensible and straightforward. It requires only a set of samplings generated from conditional expectations of random parameters.

The work in this chapter has been submitted to the Croatian Operational Research Review (CRORR) journal.

# Conclusions and Perspectives

The transition from traditional fuel to biofuel has raised a number of management challenges in biomass supply chains. The efficient design and management of the supply chain could enhance competitiveness and promote the biofuel to become more common in real life. In recent decades, the field of biomass supply chain management has attracted the interest of several researchers in response to the grave threat from the global climate change and the global warming.

The primary objective of this thesis is to propose optimization models and solution methods for biomass supply chain management and biomass inventory management with different hierarchical decision levels.

State of the art in Section 2.2 shows that there has not been any work on biomass supply chain management at strategic level considering the supplier selection along with tactical planning under uncertain environments although the two types of decisions are interdependent. At tactical level, biomass inventory management could be considered as a perishable inventory problem. A literature review in Section 2.3 affirms that very few works study perishable inventory system with both stochastic supply and stochastic demand. Therefore, we propose mathematical models to solve problems at different management decision levels. In this thesis, the biomass supply chain management is approached by different directions, depending on the different decision levels and mathematical model.

The biomass supply chain models with integrated supplier selection and production planning are presented in Chapters 3 and 4. In Chapter 3, the deterministic model is used to support decision-makers when the input is predetermined. In the next chapter, the biomass supply chain model is extended in the stochastic environment, which makes the solution more robust compared to the deterministic approach in Chapter 3. According to the numerical study in Section 4.4.4, this stochastic approach could provide an optimal solution which allows reducing 7.9% of the expected total cost in comparison with the corresponding deterministic programming model. The choice of suppliers would highly affect the expected total cost under the stochastic environments.

In chapter 3, we present a mixed integer linear programming (MILP) model for minimizing the total system cost in a biomass supply chain. The decision variables in this model are related to the supplier selection and logistics activities (transportation plan, purchase activities, inventory control, and production). The case study shows that the

model can be solved optimally with different instances in reasonable computation time. Moreover, the impacts of several parameters on the expected optimal cost of the system and supplier selection are presented. The deterministic approach presented in this chapter is appropriate when decision markers want to evaluate the outcomes from their decisions with different inputs.

In Chapter 4, we propose a two-stage stochastic programming model for biomass supply chain management under uncertain supply. To solve the stochastic model, a commercial solver requires a large memory (RAM), which could not be met by most PCs. For this reason, we develop an improved version of L-shaped decomposition method to solve the model in a reasonable time with much less memory requirement. The Monte Carlo sampling approach is applied to find the required number of scenarios for ensuring that the exact value of an output of the model is located within a confidence interval with a given probability. The numerical results show the effectiveness of the proposed algorithm.

Perishable inventory problems are discussed in Chapters 5 and 6. Both models address the central problem in the context of stochastic supply and stochastic demand. The only difference between the two models is related to the nature of a product. Chapter 5 is devoted to an inventory problem for a product having similar characteristics as biomass (deterioration rate in each period is a constant). Chapter 6 solves an inventory problem for a product with a fixed lifetime. Due to the different characteristics of the products, the corresponding algorithms have been developed to provide near-optimal solutions that can be applied in many real-life situations.

In Chapter 5, we propose a periodic review perishable inventory model with uncertainty in both demand and supply. A part of inventory is degraded/deteriorated at a constant rate from one period to the next. We also delivery several crucial properties of the model and demonstrate that its optimal inventory policy is an order-up-to level policy. A solution method that is based on a combination of scenario-based optimization and Lagrangian relaxation is developed. The Lagrangian relaxation algorithm with the volume method for dual optimization could provide a near-optimal solution with high-quality. A numerical study shows that the proposed method is applicable to problems with a large number of scenarios because it can find near-optimal inventory policies with less than 1% of deviation from the optimal expected total cost in reasonable computational time

In Chapter 6, we propose a stochastic programming model for a perishable inventory system with fixed life shelf under both stochastic demand and stochastic supply in a rolling horizon framework. In our solution method, the original model is transformed into an equivalent MILP model by applying the scenario approach based on Conditional Scenarios method. This method allows generating a set of scenarios from given continuous random vector by applying a conditional expectation discretization. The CS approach is compared with the Sample Average Approximation (SAA) method in the numerical study. The results suggest that the CS approach could be a good choice in comparison with the SAA approach in term of quality solution and CPU time. Sensitivity analysis is conducted

to evaluate the effect of several parameters on the total system cost.

Overall, all works in this thesis are original but our research still has some inevitable limitations as follows: 1) In the stochastic programming model for biomass supply chain, biofuel demand is assumed to be constant, but it is actually stochastic in the real-life situation; 2) In the perishable inventory control model, lead-time is supposed to be zero, but it can be positive (non-negligible).

The work achieved during this thesis opens several perspectives for future research by following the two axes: biomass supply chain management and perishable inventory control.

First, stochastic biofuel demand and several options for setting up contracts with suppliers should be considered in a biomass supply chain model. This integration would allow the model to become more realistic. Considerable terms for setting up contracts between suppliers and biorefinery could include: quality requirements (water content, contamination...), agreements on minimum delivery volumes, indemnification from delivery promise in case of adverse weather and crop condition, regulations for future price changes, payment condition. These options also enhance the flexibility of biofuel production and improve the stability of biomass feedstock supply. Therefore, the mentioned problem may be more complicated because of a large number of integer variables which, as a result, is necessary to develop an effective algorithm to find an optimal solution.

Besides, another possibility for future research is to integrate the environmental and social impact into the objective function. A multi-objective model should be developed to find the trade-off between the conflict objectives: maximization of profit/reduction of total supply chain costs, minimization of greenhouse gas emissions, and maximization of jobs creation.

Second, the future study of the perishable inventory control should include a positive lead-time. It may be interested to consider the joint of several key topics and the environmental uncertainty (price, quality) in a perishable inventory problem. The key topic for future research could be as follows: credit and different payment problems; inflation and time value for money; investment, promotion and budget constraint; customer satisfaction.

On the other hand, we also could integrate risk criteria (Value-at-risk, Conditional value-at-risk, Mean absolute deviation) into the actual perishable inventory model. This integration could improve the robustness of the solution and help the model be more realistic.

# Annexe A

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# A.1 Introduction

## A.1.1 Contexte

L'émission des gaz à effet de serre représente une part des principales causes du réchauffement de la planète qui affecte gravement à la vie humaine et l'augmentation des catastrophes naturelles (Shafiee and Topal [2009]). Par conséquent, les énergies renouvelables joueraient un rôle important dans la transition énergétique et seraient une solution très prometteuse au développement durable.

La bioénergie pourrait être générée à partir de la matière organique telle que le bois, les produits agricoles, les déchets animaux et végétaux. Ils sont des matières premières moins chères, mais les coûts élevés associés aux activités logistiques pourraient entraîner une augmentation significative du prix de biocarburant. Le succès ou l'échec d'un projet de bioénergie dépend notamment de la gestion des activités logistiques ainsi que de la conception du système.

La durabilité d'une chaîne d'approvisionnement en biomasse dépend également de la gestion des incertitudes en tenant compte de l'approvisionnement en matières premières, de la demande en bioénergie et du prix. Dans la pratique, le déséquilibre entre l'approvisionnement et la demande sont inévitables à cause de la saisonnalité de biomasse et de la variation de la demande. Par conséquent, le stockage de grandes quantités de biomasse est essentielle afin de mitiger ce déséquilibre.

## A.1.2 Problèmatique

La gestion d'une chaîne d'approvisionnement en biomasse est assez complexe à cause de la présence des incertitudes d'approvisionnement, de qualité de la biomasse, de rendement de la production et de demande. La fiabilité de l'approvisionnement en matières premières affecte considérablement l'efficacité d'une chaîne d'approvisionnement en biomasse. Au niveau stratégique, une relation fiable avec les fournisseurs est essentielle pour maintenir un flux d'approvisionnement constant en biomasse de bonne qualité. Cette relation devrait être intégrée à la planification de production à long terme pour mitiger des défis dans l'environnement incertain.

Au niveau tactique, la gestion d'une chaîne logistique en biomasse nécessite une politique de gestion des stocks efficaces pour réduire les coûts du système entier de manière significative. Contrairement aux produits conventionnels, la biomasse est détériorée par un taux constant pendant sa période de stockage. Par conséquent, l'identification d'une politique de gestion des stocks optimales est essentielle pour répondre aux besoins des clients et améliorer la compétitivité de cette source d'énergie.

Dans cette thèse, nous nous intéressons à la gestion d'une chaîne d'approvisionnement de biomasse aux niveaux stratégiques et tactiques.

### A.1.3 Contributions

Le but de cette thèse est de développer des modèles d'optimisation et des méthodes de résolution pour trouver des solutions optimales aux problèmes considérés. Les modèles d'optimisation devraient prendre en compte toutes les caractéristiques de biomasse et les contraintes liées aux activités de logistique de biomasse.

Concernant la gestion d'une chaîne logistique en biomasse, nous développons un modèle flexible pour capturer la plupart des activités logistiques et la sélection des fournisseurs dans une chaîne logistique en biomasse.

Ensuite, nous étendons le modèle précédant en considérant des incertitudes dans le système d'approvisionnement. Un modèle de programmation stochastique en deux étages est proposée. Une méthode de résolution est développée en se basant sur la méthode de décomposition de L-shaped. Cette méthode permet de résoudre de manière optimale les grandes instances du problème dans un temps de calcul raisonnable.

Concernant la gestion des stocks d'un produit périssable (biomasse), nous proposons deux modèles de gestion des stocks de produits périssables avec demandes et capacité d'approvisionnement aléatoires. L'étude est pertinente pour la gestion des stocks dans une chaîne d'approvisionnement en biomasse.

Pour un produit périssable avec un taux de détérioration constant, nous découvrons les propriétés fondamentales du modèle proposé et démontrons que la politique optimale est « Order-up to level ». Une approche basée sur la méthode de scénarios et la méthode de relaxation lagrangienne est développée. Cette approche nous permet de trouver une solution approchée de bonne qualité du problème étudié. Une expérimentation numérique montre que notre approche permet de trouver une solution très proche de l'optimum avec l'écart de 1% en termes de l'espérance du coût total par rapport à la solution optimale.

Pour un produit à durée de vie fixe, nous proposons un modèle stochastique dont la résolution est basée sur les approches, Conditional scenarios (CS) et Sample Average Approximation (SAA). Une étude numérique montre que l'intérêt de l'approche CS à la fois en terme de la qualité des solutions et des temps de calcul.

### A.1.4 Organisation

Dans ce premier chapitre, nous soulignons la motivation et le contexte de la thèse. Le chapitre 2 présente une revue de la littérature générale concernant la gestion d'une chaîne logistique en biomasse et la gestion des stocks périssables. Nous examinons les travaux existants et identifions la manque de recherches clés pour positionner notre recherche.

Le chapitre 3 présente le modèle d'optimisation proposé pour une chaîne d'approvisionnement en biomasse. Nous considérons le problème de l'intégration de la sélection des fournisseurs dans le cadre de la planification des opérations. La sélection des fournisseurs est essentielle pour influer de manière significative sur la stabilité de la chaîne

d'approvisionnement, en minimisant les coûts de rupture et en satisfaisant la demande de biocarburant.

Dans le chapitre 4, nous étendons le modèle précédent à l'incertitude d'approvisionnement. Un modèle de programmation stochastique en deux étapes pour la gestion d'une chaîne logistique en biomasse. Le reste du chapitre présente une approche de résolution basée sur la méthode de décomposition de L-shaped. Cette approche nous permet de résoudre ce problème de manière optimale dans un temps de calcul raisonnable. Notre méthode de résolution est testée numériquement pour montrer son efficacité.

Le chapitre 5 présente un modèle de gestion des stocks d'un produit périssable dont le taux de détérioration est constant (par exemple : biomasse). Dans ce modèle, l'approvisionnement et la demande sont stochastiques. Une approche de résolution basée sur la méthode de relaxation lagrangienne est développée. Une étude numérique montre que notre approche permet de trouver une solution approchée de bonne qualité dans un temps de calcule raisonnable.

Dans le chapitre 6, nous étendons le modèle précédent pour un produit à durée de vie fixe avec demandes et capacité d'approvisionnement aléatoires à la fois. Nous développons une approche de résolution basée sur les approches CS (Conditional Scenarios) et SAA (Sample Average Approximation).

Enfin, la dernière partie résume les résultats de nos travaux et tire ainsi des conclusions. De plus, la perspective et les travaux futurs sont exposés dans cette partie.

## A.2 Etat de l'art

Dans ce chapitre, nous abordons des défis, des caractéristiques et des problèmes liés à la gestion d'une chaîne logistique en biomasse et à la gestion des stocks périssables. Ensuite, nous avons cherché à identifier le manque de recherches clés pour positionner notre recherche.

### A.2.1 Chaîne d'approvisionnement en biomasse

La biomasse représente l'ensemble de la matière organique d'origine végétale ou animale. Le biocarburant et la bioénergie pourraient être générés à partir de la biomasse. Dans les chaînes d'approvisionnement de la biomasse, il existe différents acteurs tels que les agriculteurs, les installations de prétraitement, les bioraffineries, les transporteurs et les clients finaux. Chaque acteur joue un rôle différent dans les chaînes d'approvisionnement et ses performances dépendent de la conception du système, de la planification opérationnelle.

En effet, des chaînes d'approvisionnement en biomasse procèdent un nombre des caractéristiques différentes concernant leurs structures et leurs caractéristiques. Leurs structures pourraient être classées naturellement en fonction des activités logistiques requises. Ces activités comprennent la préparation du sol et la plantation, la culture, la récolte, la manutention, le stockage, la conversion, le transport et l'utilisation du biocarburant dans les stations-service, selon Ba et al. [2016].

De plus, l'incertitude se trouve dans plupart activités logistiques d'une chaîne d'approvisionnement en biomasse. L'approvisionnement en biomasse est toujours saisonnier et dépend de condition météorologique et de la période de récolte. En outre, la fluctuation du prix d'achat et de la demande affecte l'efficacité d'une chaîne d'approvisionnement. L'efficacité d'une chaîne d'approvisionnement en biomasse est également affectée par la fluctuation du prix d'achat et de la demande, des contrats à long terme avec fournisseurs, l'infrastructure et réseaux de distribution locale, la technologie de conversion et la politique du gouvernement, selon Ba et al. [2016]; Gold and Seuring [2011].

#### Modèles d'optimisation de la gestion d'une chaîne d'approvisionnement en biomasse

Des modèles de programmation mathématique pour la gestion d'une chaîne logistique en biomasse peuvent être classées en deux catégories : déterministes et stochastiques. Une analyse bibliographique de Pérez et al. [2017] montre que 75% des études sont basées sur l'approche déterministe.

**Modèles d'optimisation déterministe :**

Le modèle de programmation linéaire à nombres entiers mixtes est largement utilisé pour déterminer la conception optimale d'une chaîne d'approvisionnement considérée. Un solveur commercial (CPLEX, AIMMS, AMPL, GORUBI, LINGO et XPRESS) peut

être utilisé pour résoudre le modèle MILP. Certains chercheurs ont développé des algorithmes basés sur la méthode de relaxation lagrangienne, la méthode génétique et méta-heuristique. Cependant, la méta-heuristique est rarement apparue dans plusieurs applications.

Selon Melis et al. [2018], 55,8% de publications concentrent sur la conception d'une chaîne logistique en biomasse au niveau stratégique. La décision à ce niveau concerne la sélection de technologie de production, la technique de prétraitement, la capacité et de l'emplacement des installations, la configuration optimale du réseau logistique, l'établissement des contrats avec fournisseurs, par exemple Castillo-Villar et al. [2016]; Hombach et al. [2016]; Samsatli et al. [2015]; Woo et al. [2016].

Au niveau tactique, la décision est liée à la planification des opérations, la planification logistique (nombre de véhicules et de travailleurs), le mode de transport et l'identification du niveau de stock de sécurité. Il n'y a pas beaucoup d'études effectuées à ce niveau tel que les travaux de Atashbar et al. [2018]; Ba et al. [2015]; Marques et al. [2018]; Morales-Chávez et al. [2016]; Sosa et al. [2015].

Le niveau opérationnel comprend les opérations logistiques (récolte, collecte et manutention) et le transport de la biomasse (problèmes d'acheminement des véhicules) au cours d'une journée. Il existe très peu de modèles mathématiques à ce niveau. Au niveau opérationnel, la plupart des études néglige souvent des incertitudes et des aspects environnementaux dans leurs modèles, selon Malladi and Sowlati [2017].

**Modèles d'optimisation stochastique**

Le modèle stochastique peut être plus approprié dans le cas des événements aléatoires survenue. Il existe trois approches principales pour faire face à l'incertitude : la programmation stochastique (modèles de recours, programmation stochastique robuste et modèles probabilistes), la programmation floue et la programmation dynamique stochastique.

Grace à cette analyse de la littérature, on peut trouver quelques modèles stochastiques existant qui sont basées sur la programmation stochastique en deux étages pour trouver une conception optimale de la chaîne logistique considérée au niveau stratégique, par exemple [Awudu and Zhang, 2013; Chen and Fan, 2012; Dal-Mas et al., 2011; Kaut et al., 2015; Kim et al., 2011; Osmani and Zhang, 2013; Quddus et al., 2018].

Pour résoudre ces modèles stochastiques, plusieurs techniques sont proposées telles que la méthode de L-shaped [Awudu and Zhang, 2013; Marufuzzaman et al., 2014], la méthode de Progressive Hedging [Chen and Fan, 2012], l'algorithme génétique [Mirkouei et al., 2017]. Les autres utilisent un solveur commercial pour résoudre un modèle linéaire équivalent déterministe [Kaut et al., 2015; Kim et al., 2011; Osmani and Zhang, 2013; Shabani and Sowlati, 2016; Shabani et al., 2014].

**Modèle multi-objectif**

Dans dernières décennies, la programmation multi-objectif a été largement appliquée dans la gestion d'une chaîne d'approvisionnement en biomasse en intégrant des aspects

environnementaux et sociaux (la création d'emplois, la réduction des émissions de gaz à effet de serre et la réduction du coût total du système). La plupart des travaux utilisent la méthode ϵ-contrainte pour déterminer la frontière d'efficacité de Pareto [Cambero and Sowlati, 2016; Grigoroudis et al., 2014; Liu et al., 2014; Roni et al., 2017; Wang et al., 2018; Wheeler et al., 2018]. Très peu d'études considèrent des objectifs contradictoires sous incertitudes, par exemple des travaux dans [Mirkouei et al., 2017; Tsao et al., 2018].

### A.2.2 Gestion des stocks périssables

La biomasse peut être considérée comme un produit périssable à un taux de détérioration constant. Le produit périssable est un article qui perd sa valeur au fil du temps jusqu'à ce qu'il devienne sans valeur, par exemple des produits frais, des produits sanguins, des produits chimiques et autres produits pharmaceutiques.

#### Caractéristiques de la gestion des stocks périssables

Contrairement à un produit conventionnel, la gestion des stocks d'un produit périssable est généralement plus complexe parce que la quantité de réapprovisionnement dépend des stocks physiques, de la commande en cours, de l'incertaine de demande, d'approvisionnement et de prix.

Il existe deux politiques d'extrait les plus utilisées : le premier entré, le premier sorti (FIFO) et le dernier entré, le dernier sorti (LIFO). Selon Parlar et al. [2011], le FIFO est meilleur que le LIFO pour la combinaison de tous les coûts opérationnels. Cependant, la situation est inversée lorsque le coût de stockage est élevé ou que le coût d'achat est faible. La plupart des recherches utilisent la politique FIFO, car elle est appropriée pour réduire le gaspillage des produits d'expiration.

Selon Olsson and Tydesjö [2010], la gestion des stocks d'un produit à durée de vie fixe est plus compliqué que celui à durée de vie stochastique à cause de la dimension élevée et de l'impossibilité d'appliquer la propriété markovienne pour modéliser le stock disponible.

#### Classification des modèles de gestion des stocks périssables

Des modèles de gestion des stocks périssables peuvent être classés par la durée de vie d'un produit, la caractéristique de la demande et de l'approvisionnement.

**Durée de vie** : Des produits périssables peuvent être classés en deux catégories : les produits à durée de vie déterministe et les produits à durée de vie aléatoire. Selon Schmidt and Nahmias [1985], le problème de gestion des stocks est difficile de trouver une politique optimale pour un produit dont la durée de vie est supérieure à deux périodes. C'est la raison pour laquelle plusieurs chercheurs se sont concentrés de développer des méthodes heuristiques, par example Atan and Rousseau [2016]; Chen et al. [2014]; Dobson

et al. [2017]; Kouki et al. [2018, 2013]. Le problème de la durée de vie aléatoire attire beaucoup d'attention au cours des dernières décennies Liu et al. [2018]; Pal et al. [2017]; Pal and Samanta [2018]. Le problème de gestion des stocks périssables devient extrêmement complexes en considérant un délai de commande positif.

**Demande** : L'hypothèse de la demande déterministe pourrait simplifier le problème et permettre de trouver une politique optimale telles que la quantité d'ordre économique (EOQ) et la quantité de production économique (EPQ) [Pahl and Voß, 2014]. La demande déterministe peut également être divisée en sous-catégories : demande uniforme, demande variable dans le temps, demande dépendante du niveau de stock, demande dépendante du prix, demande dépendante de la fraîcheur.

Le modèle stochastique est très utile lorsque la demande est incertaine. Dans plusieurs études, la demande suit une distribution de probabilité (normale, exponentielle log-normale aléatoire, etc). La plupart des recherches utilisent la distribution de Poisson pour décrire la demande stochastique [Alizadeh et al., 2014; Baron et al., 2017]. L'arrivée de la demande pourrait être modélisée comme un processus de renouvellement où les délais entre deux demandes successives sont indépendants et distribués de manière identique, tels que [Boxma et al., 2016; Gürler and Özkaya, 2008; Haijema and Minner, 2015]. Certain recherches ont développé un modèle stochastique pour la gestion des stocks d'un produit périssable avec demande stochastique non stationnaire et une contrainte de niveau de service, par exemple [Alcoba et al., 2015; Hendrix et al., 2012, 2015; Pauls-Worm et al., 2015].

**Supply** : L'incertitude d'approvisionnement est l'un des défis dans la gestion des stocks périssables. Cette incertitude peut être trouvée sous trois aspects : le délai d'approvisionnement [Janakiraman et al., 2013; Muthuraman et al., 2014], la quantité d'approvisionnement [Chen et al., 2015; Gaur et al., 2015]) et le prix d'achat [Chen et al., 2015; Gaur et al., 2015; Tajbakhsh et al., 2007]. L'incertitude de la quantité d'approvisionnement peut être classée en trois catégories : le rendement aléatoire, la disponibilité aléatoire et la capacité aléatoire.

**Sujets clés :** Un nombreux chercheurs considèrent des différents sujets dans les modèles des stocks périssables, tels que le crédit et les différents problèmes de paiement, l'inflation et le rapport qualité-prix, l'investissement, la promotion et la contrainte budgétaire, la satisfaction du client...

Lorsque la date d'expiration est proche, une réduction de prix d'un produit périssable permet de stimuler la vente et d'augmenter le bénéfice de société. De plus, cette politique de tarification pourrait permettre aux sociétés de réduire les pertes dues à la détérioration. Ce type de problème est présenté dans les travaux de Banerjee and Agrawal [2017]; Chung et al. [2015]; Pal [2016]. Ces dernières années, le problème d'investissement de technologies de préservation est attiré l'attention de plusieurs rechercher comme Mishra et al. [2017, 2018]; Singh et al. [2016a]; Tayal et al. [2016]; Yang et al. [2015]. De plus, l'inflation peut être considérée pour rendre le modèle de gestion des stocks plus réaliste, par

example Chakrabarty et al. [2017]; Shah and Vaghela [2018]; Tayal et al. [2016].

Les autres travaux se portent sur le délai autorisé de paiement dans la gestion des stocks périssables. Récemment, Mishra et al. [2018] détermine une politique optimale conjointe pour l'investissement de technologie de préservation, la tarification et la commande dans un délai de paiement autorisés. L'intégration du niveau de service est très utile pour traiter la satisfaction du client dans la gestion des stocks périssables, par exemple les travaux de Belo-Filho et al. [2015]; Hendrix et al. [2012, 2015, 2018]; Pauls-Worm et al. [2015, 2016].

**Modélisation et méthode de résolution dans la gestion des stocks périssables**

En passant en revue la littérature de gestion des stocks périssables, nous constatons qu'il y a différents modèles mathématiques utilisés pour analyser le système considéré sous différentes contraintes réalistes telles que la programmation dynamique, la programmation non linéaire/linéaire mixte en nombres entiers, la programmation floue, le processus de décision de Markovienne.

Depuis les années 70, il y a beaucoup de travaux basés sur le processus de Markov pour formuler des modèles de gestion des stocks périssables Baron et al. [2017]; Boxma et al. [2016]; Gürler and Özkaya [2008]; Haijema and Minner [2015]. La programmation dynamique possède des propriétés appropriées pour modéliser la relation récurrente de niveau de stock par exemple Belo-Filho et al. [2015]; Haijema et al. [2017]; Hendrix et al. [2015, 2018]; Pauls-Worm et al. [2015]. Donc, la méthode heuristique est développée afin de résoudre les équations récursives à cause de la complexité et de la dimension élévée du problème.

Certains modèles de programmation linéaire en nombres entiers mixtes peuvent être trouvés dans les travaux de Claassen et al. [2016]; Dillon et al. [2017]; Gunpinar and Centeno [2015]; Pauls-Worm et al. [2015]. Cetains chercheurs utilisent la programmation floues comme les travaux de Chen and Ouyang [2006]; Halim et al. [2008]; Hsiao et al. [2017]; Katagiri and Ishii [2002]; Shaikh et al. [2018]. Certain travaux ont utilisé la méthode de machine learning pour gérer un stock périssable sous la demande aléatoire et le délai positif, par example Kara and Dogan [2017].

### A.2.3 Modélisation et méthode de résolution

Dans cette sous-section, nous présentons le principal de la programmation stochastique à deux étapes et sa méthode de résolution commune.

Dans un modèle de programmation stochastique à deux étapes avec recours, les variables de décision sont divisées en deux étapes différentes. Les décisions dans la première étape ont pris avant de connaître la réalisation réelle des paramètres aléatoires. Dans la deuxième étape, une action de recours peut être effectuée pour corriger les effets négatifs en provenant des décisions dans la première étape. La fonction objectif consiste à

minimiser le coût de la première étape et l'espérance du coût de la deuxième étape. Dans la plupart des applications, la première étape est souvent liée aux décisions stratégiques telles que la capacité et l'emplacement des installations, tandis que la deuxième étape concerne généralement les décisions opérationnelles et tactiques (plan de production, gestion des stocks, planification des transports).

Si le nombre de scénarios considéré est faible, le modèle équivalent déterministe pourrait être résolu par un solveur commercial. Le nombre de scénarios peut être déterminé en utilisant une méthode statistique Linderoth et al. [2006]; Shapiro [2000] afin d'obtenir des solutions approchées dans un intervalle de confiance avec une tolérance considérée. Cette méthode est très efficace pour réduire la taille des problèmes considérés.

La structure en blocs spéciaux du modèle de programmation stochastique en deux étages permet d'appliquer un algorithme de L-shaped pour trouver une solution optimale Van Slyke and Wets [1969]. Comme la décomposition de Bender, son idée consiste à résoudre d'abord le modèle avec les contraintes qui n'incluent pas les variables dans le 2e étage pour obtenir les valeurs des décisions de premier étage. Ensuite, les sous-problèmes du deuxième étage sont résolus sachant qu'on connait les décisions de premier étage grâce à l'étape précédente.

De plus, la méthode de L-shaped permet de construire une estimation convexe de la fonction de recours et les coupes d'optimalité peuvent être ajoutées au problème de premier étage à chaque itération. À chaque itération, le problème de première étape et le sous-problème de deuxième étage sont reliés par des coupes d'optimalité. Ces coupes sont générées à partir de problème dual des sous-problèmes. Notez que les coupes de faisabilité Birge and Louveaux [1988] peuvent être ajoutées au problème de première étape pour éviter des sous-problèmes infaisable. L'algorithme standard ne génère qu'une seule coupe par itération dans le problème de première étape. Pour accélérer cet algorithme, Birge and Louveaux [1988] a montré que le problème de deuxième étage pourrait être décomposé par scénarios et de multiples coupes peuvent être générés.

### A.2.4 Conclusions

Dans cette section, nous avons passé une revue de littérature pour la gestion de la chaîne logistique en biomasse ainsi que la gestion des stocks périssables. Grace à cette analyse, nous avons pu noter deux orientations majeures dans notre recherche.

La première orientation est l'intégration de la sélection des fournisseurs à la planification des opérations dans une chaîne logistique en biomasse. De plus, la présence de l'incertitude doit être prise en compte pour rendre notre modèle plus réaliste.

La seconde orientation est dévouée à la gestion des stocks d'un produit périssable sous incertitudes d'approvisionnement et de demande. Cette étude est essentielle pour la gestion d'un stock dans la chaîne d'approvisionnement en biomasse.

## A.3 Modélisation et optimisation de la chaîne d'approvisionnement en biomasse avec deux types de fournisseurs de matières premières

Cette section présente une programmation linéaire mixte en nombres entiers (MILP) pour la résolution d'un problème de gestion de la chaîne d'approvisionnement en biomasse. Nous nous intéressons à la sélection des fournisseurs et à l'établissement du contrat avec eux pour assurer la stabilisation de l'approvisionnement en biomasse. Selon l'état de l'art, aucune étude ne considère la sélection des fournisseurs et la planification simultanément dans la gestion d'une chaîne d'approvisionnement en biomasse.

### A.3.1 Description du problème

Nous considérons une chaîne logistique en biomasse comme celle présentée dans le Figure 3.1. On suppose que les infrastructures de production telles que les installations de prétraitement et les bioraffineries ont été localisées. Les producteurs de biocarburants cherchent à établir une relation à long terme avec certains fournisseurs locaux/régionaux afin de stabiliser l'approvisionnement en biomasse. Le prix d'achat unitaire et la quantité minimum de livraison sont fixés dans le cadre d'un contrat au cours d'un horizon de planification. Le coût de transport unitaire par trajet est calculé au préalable en fonction des distances et des coûts fixes associés aux camions.

Les activités logistiques sont les suivants. À chaque période, la biomasse est transportée des fournisseurs sous contrat aux installations de prétraitement, puis aux bioraffineries. Dans chaque installation de conversion, la biomasse est prétraitée, puis transformée en biocarburant ou conservée en stock à la bioraffinerie. De plus, la biomasse peut être achetée sur les marchés internationaux ou nationaux pour satifaire une partie de la demande de biocarburant à un prix raisonnable de la biomasse. Après avoir satisfait à la demande de biocarburant, le reste de biocarburant est conservé en stock. Un coût en rupture s'est chargé pour la partie de la demande non satisfaite. Le prétraitement de la biomasse et la production de biocarburant sont limitées par la capacité de prétraitement / production et par la capacité de stockage de chaque installation.

### A.3.2 Modèle mathématique

Soit T l'ensemble des périodes de l'horizon de planification T dont chaque période est un mois. Soit I un ensemble de fournisseurs de biomasse, P un ensemble d'installations de prétraitement et B un ensemble de bioraffineries. Soit $x(i, j, t, s)$ flux de biomasse sur l'arc $(i, j) \in$ A à la période $t$. Un tel arc $(i, j)$ pourrait être un arc $(i, p)$ ou $(p, b)$ qui représente un arc du fournisseur $i\ en$I à la facilité de prétraitement $p \in$ P ou de l'installation de prétraitement $p \in$ P à la bioraffinerie $b$. Soit $pre(n)$, $suc(n)$, respectivement, l'ensemble

des prédécesseurs et l'ensemble des successeurs d'un noeud donné $n$. Les notations utilisées sont résumées dans le tableau 3.1. Toutes les variables de décision continues sont non négatives. La formulation du problème est presenté au-dessous :

**Fonction objectif**

$$\begin{aligned}
\min \quad z = & \sum_{t\in \mathrm{T}} \sum_{(i,j)\in \mathrm{A}} nbr\mathrm{T}(i,j,t)\mathrm{V}cost(i,j) + \sum_{t\in \mathrm{T}} \sum_{p\in \mathrm{P}} \eta^{\mathrm{P}} \mathrm{Q}^{\mathrm{P}}(p,t) tcost\mathrm{P}(p) \\
& + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} short(b,t) pcost\mathrm{B}(b) + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} \eta^{\mathrm{B}} z(b,t) cost\mathrm{B}(\mathrm{B}) \\
& + \sum_{t\in \mathrm{T}} \sum_{p\in \mathrm{P}} hcost^{\mathrm{P}}(p)\mathrm{W}(p,t) + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} hcost^{\mathrm{B}}_{av}(b)\mathrm{V}_{av}(b,t) \\
& + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} hcost^{\mathrm{B}}_{ap}(b)\mathrm{V}_{ap} + \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{I}} \mathrm{Q}^{\mathrm{I}}(i,t) cost\mathrm{A}^{\mathrm{I}}(i) \\
& + \sum_{t\in \mathrm{T}} \sum_{m\in \mathrm{M}} \sum_{b\in \mathrm{B}} x(m,b,t) cost\mathrm{A}^{\mathrm{M}}(m) + \sum_{i\in \mathrm{I}} \phi(i) * mcost(i) \qquad \text{(A.1)}
\end{aligned}$$

La fonction objectif (A.1) est de minimiser le coût total qui compose les différents coûts individuels : coûts de transport (1er terme), coûts de prétraitement (2ème terme), coûts de rupture (3ème), coûts de conversion en biocarburants (4ème terme), coûts de stockage (5-7ème terme), coûts d'achat (8-9ème terme) et coûts fixes pour l'établissement d'un contrat avec les fournisseurs (dernier terme).

**Contraintes**

$$\mathrm{Q}^{\mathrm{I}}(i,t) = \sum_{p\in suc(i)} x(i,p,t) \qquad \forall t\in \mathrm{T}, i\in \mathrm{I} \qquad \text{(A.2)}$$

$$\mathrm{Q}^{\mathrm{I}}(i,t) \le avail^{\mathrm{I}}(i,t).\varphi(i) \qquad \forall t\in \mathrm{T}, i\in \mathrm{I} \qquad \text{(A.3)}$$

$$\mathrm{Q}_{min}(i)\varphi(i) \le \sum_{t\in \mathrm{T}} \mathrm{Q}^{\mathrm{I}}(i,t) \qquad \forall i\in \mathrm{I} \qquad \text{(A.4)}$$

$$\sum_{b\in \mathrm{B}} x(m,b,t) \le avail^{\mathrm{M}}(m,t) \qquad \forall t\in \mathrm{T}, m\in \mathrm{M} \qquad \text{(A.5)}$$

$$\mathrm{Q}^{\mathrm{P}}(p,t) = \sum_{(i)\in pre(p)} x(i,p,t) \qquad \forall t\in \mathrm{T}, p\in \mathrm{P} \qquad \text{(A.6)}$$

$$\begin{aligned}
\mathrm{W}(p,t) = & \lambda^{\mathrm{P}}(p)\mathrm{W}(p,t-1) + \eta^{\mathrm{P}}\mathrm{Q}^{\mathrm{P}}(p,t) \\
& - \sum_{b\in suc(p)} x(p,b,t) \qquad \forall t\in \mathrm{T}, p\in \mathrm{P} \qquad \text{(A.7)}
\end{aligned}$$

$$\Gamma^{min}_{p}(p) \le \mathrm{Q}^{\mathrm{P}}(p,t) \le \Gamma^{max}_{p}(p) \qquad \forall t\in \mathrm{T}, p\in \mathrm{P} \qquad \text{(A.8)}$$

$$\mathrm{W}^{min}(p) \le \mathrm{W}(p,t) \le \mathrm{W}^{max}(p) \qquad \forall t\in \mathrm{T}, p\in \mathrm{P} \qquad \text{(A.9)}$$

$$\begin{aligned}
\mathrm{V}_{av}(b,t) = & \lambda^{\mathrm{B}}(b)\mathrm{V}_{av}(b,t-1) + \sum_{(p)\in pre(b)} x(p,b,t) \\
& + \sum_{m\in \mathrm{M}} x(m,b,t) - z(b,t) \qquad \forall t\in \mathrm{T}, b\in \mathrm{B} \qquad \text{(A.10)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{V}_{ap}(b,t) = & \mathrm{V}_{ap}(b,t-1) + \eta^{\mathrm{B}} z(b,t) \\
& + short(b,t) - demand(b,t) \qquad \forall t\in \mathrm{T}, b\in \mathrm{B} \qquad \text{(A.11)}
\end{aligned}$$

$$\Gamma_b^{max}(b) \leq z(b,t) \leq \Gamma_b^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \tag{A.12}$$

$$\mathrm{V}_{av}^{min}(b) \leq \mathrm{V}_{av}^{\mathrm{B}}(b,t) \leq \mathrm{V}_{av}^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \tag{A.13}$$

$$\mathrm{V}_{ap}^{min}(b) \leq \mathrm{V}_{ap}^{\mathrm{B}}(b,t) \leq \mathrm{V}_{ap}^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B} \tag{A.14}$$

$$x(i,j,t) \leq nbr\mathrm{T}(i,j,t) * \mathrm{C}ap\mathrm{V}(i,j) \tag{A.15}$$

L'équation (A.2) définit la quantité totale de matières premières acheminées du fournisseur $i$ aux installations de prétraitement. L'inégalité (A.3) indique qu'il n'y a pas d'approvisionnement de biomasse sauf s'il existe un contrat entre le fournisseur $i$ et le bioraffinerie $b$. La quantité minimale de livraison selon le contrat d'approvisionnement est exprimée par la contrainte (A.4).

La contrainte (A.5) exprime que la quantité d'achat de biomasse est également limitée par la disponibilité de marché. L'équation (A.6) définit la quantité totale de biomasse acheminée des installations de prétraitement $p$ aux bioraffineries à chaque période. La contrainte (A.7) assure l'équilibre des flux d'entrée et des flux de sortie dans les installations de prétraitement.

La capacité de production de l'installation de prétraitement et de bioraffinerie est respectée via les contraintes (A.8 - A.12). Les contraintes (A.10) et (A.11) représentent l'équilibre des stocks en biomasse et en biocarburant dans chaque bioraffinerie. La contrainte (A.11) assure que la demande en biocarburant est satisfaite autant que possible. Les contraintes (A.13) et (A.14) indiquent respectivement la capacité de stockage en biomasse et en biocarburant dans chaque bioraffinerie. La contrainte (A.15) représente le nombre de tourné requis pour la livraison de biomasse à chaque période. Le total des flux ne doit pas dépasser le nombre de véhicules requis.

### A.3.3 Étude numérique

Ce modèle est implémenté en langue de programmation Python 3.5 sur un ordinateur HP Intel Core i5-4210M de 8,0 Gb RAM. Tous les problèmes MILP ont été résolus par le solveur GUROBI 6.5 avec les paramètres par défaut.

#### Génération de données d'entrée

Une étude numérique est effectuée pour évaluer l'aspect économique et certains facteurs critiques de la chaîne d'approvisionnement en biomasse. Dans le cas de référence, nous considérons une chaîne d'approvisionnement en biomasse avec une raffinerie, deux sites de prétraitement, les 100 fournisseurs potentiels sur 12 périodes. Chaque période de cette étude correspond à un mois. Le producteur de biocarburants cherche à signer un contrat avec plusieurs fournisseurs locaux de matières premières. La quantité disponible de biomasse de chaque fournisseur peut être estimée en fonction de sa surface cultivée et son rendement de récolte.

Nous utilisons les données de Osmani and Zhang [2013] pour valider notre modèle. Un contrat avec fournisseur se caractérise par un coût fixe, un prix d'achat unitaire et une quantité minimale d'approvisionnement en biomasse à fournir. Le prix d'achat varie de 35 à 37 $/tonne. La quantité minimum d'approvisionnement varie entre 20 000 et 100 000 tonne. Le coût fixe pour l'établissement d'un contrat varie entre 20 000 et 30 000 dollar.

Le coût unitaire de transport par trajet est calculé en fonction des distances, des coûts variables et des coûts fixes liées au transport. Les 100 candidats de fournisseurs se situent dans un rayon de 50 km de la bioraffinerie. La quantité disponible de biomasse est estimée en fonction de la disponibilité des terres selon Osmani and Zhang [2013, 2014] et le rendement de la biomasse en suivant la distribution uniforme $U[3,5,4,5]$. Un ratio de perte pour la biomasse en stockage varie de 1 à 3% par mois (dans le pire des cas).

La capacité de production de bioraffinerie est de 190 MLPY. Le coût de production unitaire est de 0,2 $/litre. Le taux de transformation en bioéthanol à partir de la biomasse est 313 litres/tonne. Le coût unitaire de stockage des biocarburants et celui de la biomasse dans la bioraffinerie sont respectivement de 0,227 $/litre et 0,9 $/tonne. Le coût unitaire de rupture est de 1,06$/litre. Grâce à la base de données historiques (19981-2005) de Osmani and Zhang [2013], la demande est modélisée aléatoirement à l'aide de la distribution normale $N(2133,4138)$ et tronquée dans l'intervalle $[2006,2280]$ .

### Résultats numériques

Dans le cas de référence, la solution optimale du coût total est de $ 181 281 119. Pour cette instance, le modèle contient 3 808 contraintes, 2 532 variables continues et 1 324 variables entières. Le temps de calcul est moins de 5 secondes (1,78 secondes). La figure 3.2 représente la répartition des coûts dans la chaîne d'approvisionnement de la biomasse sur l'horizon de planification T = 12. Il est évident que les coûts de transport jouent un rôle essentiel dans la chaîne d'approvisionnement en biomasse avec environ 25% du coût total du système.

Nous avons également testé ce modèle pour différentes instances lorsque I = 10, 20, 50, 100, 150, 200, 250, 300, 350, 400 ; P = 1, 2, 5 et T = 12. La taille du problème et la durée d'exécution de ces instances sont résumées dans le tableau 3.3. Ces résultats montrent que ce modèle peut être résolu rapidement et exactement avec un solveur commercial.

### Analyse de sensibilité

Certains paramètres tels que prix d'achat sur un marché, quantité minimum d'approvisionnement et coût contractuel fixe ont un impact significativement sur le coût total optimal du système et le nombre total de fournisseurs sous contrat.

La figure 3.3 représente le coût total du système est moins sensible à la variation du prix du marché, qui varie entre 90 % et 130 % de sa valeur de base. Lorsque le prix du marché devient trop cher, e décideur hésite à acheter une quantité supplémentaire de

biomasse sur les marchés pour satisfaire la totalité de la demande de biocarburants. Dans ce cas, il préfère établir plus de contrats d'approvisionnement. Cela explique pourquoi le nombre de fournisseurs sous contrat augmente lorsque le prix du marché augmente.

La figure 3.4 montre qu'une augmentation des coûts fixes de l'établissement de contrats entraînerai un coût total du système très élevé et entraverai l'établissement de contrats à long terme. Cependant, il s'agit d'un léger effet sur le coût global du système et le nombre total de fournisseurs sous contrat.

La figure 3.5 indique la proportionne faible entre la quantité minimum d'approvisionnement et le coût total du système. Cependant, le nombre total de fournisseurs sous contrat diminue à mesure que la quantité d'approvisionnement augmente. Dans ce cas, on a besoin d'une quantité supplémentaire en provenant du marché pour satisfaire la demande finale si et seulement si le prix d'achat est raisonnable.

## A.4 Sélection des fournisseurs et planification dans la chaîne d'approvisionnement en biomasse sous incertitude

Dans cette section, nous étendons le problème précédent en considérant l'incertitude d'approvisionnement. Nous présentons au-dessous un modèle de programmation stochastique en deux étages pour aborder la sélection des fournisseurs et la planification des opérations dans une chaîne logistique en biomasse.

### A.4.1 Mathematical formulation of the problem

Dans cette section, nous formulons un modèle de programmation stochastique en deux étages en prenant en compte la sélection des fournisseurs et la planification des opérations de façon simultanée. Les variables de décision de la première étape comprennent uniquement les variables binaires qui déterminent le choix des fournisseurs ayant des contrats à long ou moyen terme. Les variables de la deuxième étape comprennent la quantité de biomasse à acheter auprès des fournisseurs, la quantité de biomasse à transporter à la bioraffinerie, la quantité de biomasse à utiliser pour la transformation en bioéthanol, le volume de bioéthanol non satisfait, le niveau de stock en biomasse et bioéthanol à chaque période. Le modèle mathématique est présenté au-dessous :

$$\textbf{(SP) :} \quad \min_{\varphi} \left\{ mcost^{\mathrm{T}}\varphi + \sum_{s\in \mathrm{S}} p_s \mathrm{Q}(\varphi, s) \mid \varphi \in \{0,1\} \right\} \tag{A.16}$$

$$\text{where :} \quad \mathrm{Q}(\varphi, s) = \min_{y_s} \left\{ f(\varphi, s, y_s) \mid \text{subject to (A.19)} - \text{(A.34)} \right\} \tag{A.17}$$

où $\varphi = \{\varphi(i), i \in \mathrm{I}\}$ sont les variables de décision de la première étape relatives aux contrats entre les bioraffineries et les fournisseurs, $\varphi(i) = 1$ si le contrat est établi avec le fournisseur $i$, 0 sinon. Le vecteur $mcost = \{mcost(i), i \in \mathrm{I}\}$ inclut le coût fixe $mcost(i)$ pour

établir un contrat avec le fournisseur $i$. La fonction $Q(\varphi, s)$ est la fonction de coût de sous-problème de deuxième étape pour chaque scénario $s$, dans lequel les décisions de la première étape, $\varphi$, ont déjà été prises.

Toutes les notations utilisées sont résumées dans le tableau 4.1. Toutes les variables de décision continues sont non négatives. Les différentes composantes du coût total de la deuxième étape $f(\varphi, s, y_s)$ pour chaque scénario $s$ sont indiquées ci-dessous; où $y_s$ contient toutes les variables de décision de seconde étape, $y_s = (x, w, \mathrm{V}_{av}, \mathrm{V}_{ap}, q^{\mathrm{P}}, z, short)$.

$$\begin{aligned} f(\varphi, s, y_s) = & \sum_{t\in \mathrm{T}} \sum_{(i,j)\in \mathrm{A}} x(i,j,t,s) * \mathrm{V}cost(i,j) + \sum_{t\in \mathrm{T}} \sum_{p\in \mathrm{P}} \eta^{\mathrm{P}} * q^{\mathrm{P}}(p,t,s) * tcost\mathrm{P}(p) \\ & + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} short(b,t,s) * pcost(b) + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} \eta^{\mathrm{B}} * z(b,t,s) * tcost\mathrm{B}(\mathrm{B}) \\ & + \sum_{t\in \mathrm{T}} \sum_{p\in \mathrm{P}} hcost^{\mathrm{P}}(p) w(p,t,s) + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} hcost^{\mathrm{B}}_{av}(b) \mathrm{V}_{av}(b,t,s) \\ & + \sum_{t\in \mathrm{T}} \sum_{b\in \mathrm{B}} hcost^{\mathrm{B}}_{ap}(b) \mathrm{V}_{ap}(b,t,s) + \sum_{t\in \mathrm{T}} \sum_{i\in \mathrm{I}} q^{\mathrm{I}}(i,t,s) * cost\mathrm{A}^{\mathrm{I}}(i) \\ & + \sum_{t\in \mathrm{T}} \sum_{m\in \mathrm{M}} \sum_{b\in \mathrm{B}} x(m,b,t,s) * cost\mathrm{A}^{\mathrm{M}}(m) \end{aligned} \tag{A.18}$$

où $x(i, j, t, s)$ est le flux de biomasse sur l'arc $(i, j) \in \mathrm{A}$ à la période $t$ dans le scénario $s$. Un tel arc $(i, j)$ pourrait être un arc $(i, p)$ ou $(p, b)$ qui fait référence à un arc du fournisseur $i \in \mathrm{I}$ à la facilité de prétraitement $p \in \mathrm{P}$ ou de l'installation de prétraitement $p \in \mathrm{P}$ à la bio-raffinerie $b$.

La fonction objectif est de minimiser le coût total de la chaîne d'approvisionnement, qui comprend les coûts de la première étape et l'espérance des coûts de deuxième étape. Les coûts de la première étape comprennent les coûts totaux liés à l'établissement de contrats avec fournisseurs. L'équation (A.18) présente les coûts de la deuxième étape $Q(\varphi, s)$ pour chaque scénario $s$, qui comprend neuf coûts : coûts de transport (1er terme), coûts de prétraitement (2ème terme), coûts de rupture (3ème terme), coûts de conversion en biocarburant (4ème terme), coûts de stockage (5ème-7ème termes) et coûts d'achat (9ème-10ème termes).

**Constraints on suppliers**

$$q^{\mathrm{I}}(i,t,s) = \sum_{p\in suc(i)} x(i,p,t,s) \qquad \forall t \in \mathrm{T}, i \in \mathrm{I}, s \in \mathrm{S} \qquad [\pi^1_{i,t,s}] \tag{A.19}$$

$$q^{\mathrm{I}}(i,t,s) \le avail^{\mathrm{I}}(i,t,s).\varphi(i) \qquad \forall t \in \mathrm{T}, i \in \mathrm{I}, s \in \mathrm{S} \qquad [\pi^2_{i,t,s}] \tag{A.20}$$

$$\sum_{t\in \mathrm{T}} q^{\mathrm{I}}(i,t,s) \ge q_{min}(i).\varphi(i) \qquad \forall i \in \mathrm{I}, s \in \mathrm{S} \qquad [\pi^3_{i,s}] \tag{A.21}$$

L'équation (4.4) définit la quantité totale de matières premières transportées du fournisseur $i$ aux installations de prétraitement dans chaque période du scénario $s$. L'inégalité (A.20) indique qu'il n'y a pas d'approvisionnement en biomasse en provenant du fournisseur $i$ sauf s'il existe un contrat avec fournisseur $i$. L'inégalité (A.21) assure que la quantité

minimum d'approvisionnement dans un contrat entre un fournisseur $i$ et la bioraffinerie $b$ est respecté.

**Constraints on market**

$$\sum_{b\in \mathrm{B}} x(m,b,t,s) \leq avail^{\mathrm{M}}(m,t) \qquad \forall t \in \mathrm{T}, m \in \mathrm{M} \qquad [\pi^4_{m,t,s}] \qquad (A.22)$$

L'inégalité (A.22) assure que la quantité d'achat en biomasse est limitée par la disponibilité de biomasse sur le marché dans chaque scénario.

**Constraints on preprocessing sites**

$$q^{\mathrm{P}}(p,t,s) = \sum_{i\in pre(p)} x(i,p,t,s) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^5_{p,t,s}] \qquad (A.23)$$

$$\begin{aligned} w(p,t,s) &= (1-\lambda^{\mathrm{P}}(p))\, w(p,t-1,s) + \eta^{\mathrm{P}} q^{\mathrm{P}}(p,t,s) \\ &\quad - \sum_{b\in suc(p)} x(p,b,t,s) \qquad \forall t \geq 2, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^6_{p,t,s}] \qquad (A.24) \end{aligned}$$

$$\begin{aligned} w(p,1,s) &= (1-\lambda^{\mathrm{P}}(p))\, w^{init}(p) + \eta^{\mathrm{P}} q^{\mathrm{P}}(p,1,s) \\ &\quad - \sum_{b\in suc(p)} x(p,b,1,s) \qquad \forall p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^7_{p,s}] \qquad (A.25) \end{aligned}$$

$$\gamma^{min}_p(p) \leq q^{\mathrm{P}}(p,t,s) \leq \gamma^{max}_p(p) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^{8a/b}_{p,t,s}] \qquad (A.26)$$

$$w^{min}(p) \leq w(p,t,s) \leq w^{max}(p) \qquad \forall t \in \mathrm{T}, p \in \mathrm{P}, s \in \mathrm{S} \qquad [\pi^{9a/b}_{p,t,s}] \qquad (A.27)$$

L'équation (A.23) présente la quantité totale de biomasse transportée des fournisseurs aux installations de prétraitement $p$ à chaque période dans le scénario $s$. L'équation (A.24) et (A.25) sont la contrainte de balance de chaque stock dans l'installation de prétraitement dans le scénario $s$. La capacité de prétraitement et sa capacité de stockage sont présentées par des contraintes (A.26) et (A.27), respectivement.

**Constrains on refinery sites**

$$\begin{aligned} \mathrm{V}_{av}(b,t,s) &= (1-\lambda^{\mathrm{B}}(b))\mathrm{V}_{av}(b,t-1,s) - z(b,t,s) \\ &\quad + \sum_{p\in pre(b)} x(p,b,t,s) + \sum_{m\in \mathrm{M}} x(m,b,t,s) \qquad \forall t \geq 2, b \in \mathrm{B}, s \in \mathrm{S} \qquad [\pi^{10}_{b,t,s}] \qquad (A.28) \end{aligned}$$

$$\begin{aligned} \mathrm{V}_{av}(b,1,s) &= (1-\lambda^{\mathrm{B}}(b))\mathrm{V}^{init}_{av}(b) - z(b,1,s) \\ &\quad + \sum_{p\in pre(b)} x(p,b,1,s) + \sum_{m\in \mathrm{M}} x(m,b,1,s) \qquad \forall b \in \mathrm{B}, s \in \mathrm{S} \qquad [\pi^{11}_{b,s}] \qquad (A.29) \end{aligned}$$

$$\begin{aligned} \mathrm{V}_{ap}(b,t,s) &= \mathrm{V}_{ap}(b,t-1,s) + \eta^{\mathrm{B}} z(b,t,s) \\ &\quad - demand(b,t) + short(b,t,s) \qquad \forall t \geq 2, b \in \mathrm{B}, s \in \mathrm{S} \qquad [\pi^{12}_{b,t,s}] \qquad (A.30) \end{aligned}$$

$$\begin{aligned} \mathrm{V}_{ap}(b,1,s) &= \mathrm{V}^{init}_{ap}(b) + \eta^{\mathrm{B}} z(b,1,s) \\ &\quad - demand(b,1) + short(b,1,s) \qquad \forall b \in \mathrm{B}, s \in \mathrm{S} \qquad [\pi^{13}_{b,s}] \qquad (A.31) \end{aligned}$$

$$\gamma_b^{min}(b) \leq z(b,t,s) \leq \gamma_b^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B}, s \in \mathrm{S} \qquad [\pi_{b,t,s}^{14a/b}] \tag{A.32}$$

$$\mathrm{V}_{av}^{min}(b) \leq \mathrm{V}_{av}(b,t,s) \leq \mathrm{V}_{av}^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B}, s \in \mathrm{S} \qquad [\pi_{b,t,s}^{15a/b}] \tag{A.33}$$

$$\mathrm{V}_{ap}^{min}(b) \leq \mathrm{V}_{ap}(b,t,s) \leq \mathrm{V}_{ap}^{max}(b) \qquad \forall t \in \mathrm{T}, b \in \mathrm{B}, s \in \mathrm{S} \qquad [\pi_{b,t,s}^{16a/b}] \tag{A.34}$$

Les équations (A.28 -A.31) sont les contraintes de balance des stocks en biomasse et en biocarburant à la bio-raffinerie $b$, respectivement. Les équations (A.33) and (A.34) sont des contraintes de capacité de stockage en biomasse et en biocarburant à la bioraffinerie $b$. La capacité de conversion de chaque bio-raffinerie est donnée par des contraintes (A.32).

### A.4.2 Approches de résolution

Dans cette section, le modèle de programmation stochastique (SP) est transformé en un modèle équivalent déterministe (DEP) en appliquant l'approche de scénarios. Le second modèle peut être résolu par un solveur commercial comme CPLEX ou GUROBI. Cependant, si le nombre de scénarios est élevé, la résolution du modèle nécessite une énorme quantité de mémoire et d'effort de calcul. Par conséquent, nous proposons dans cette sous-section une méthode de décomposition (ERD) pour surmonter les difficultés de calcul.

#### Modèle déterministe équivalent

Nous transformons le modèle original en un modèle déterministe de programmation linéaire en nombres entiers (MILP) au dessous :

$$\begin{aligned} \textbf{(DEP)} \quad & \min_{x,y_s} \quad \Big( mcost^{\mathrm{T}}\varphi + \sum_{s\in \mathrm{S}} p_s f(\varphi, s, y_s) \Big) \\ & s.t. \quad \varphi(i) \in \{0,1\} \quad \forall i \in \mathrm{I} \\ & \qquad y_s \quad \text{subject to constraints (A.19)-(A.34)} \end{aligned}$$

#### Méthode de résolution

En général, la méthode de L-shaped ont deux inconvénients : (1) lors d'itérations initiales; les coupures sont souvent inefficaces; (2) Lors des dernières itérations, les coupes deviennent dégénérés. Ruszczynski and Swietanowski [1997] propose une méthode de décomposition régularisée pour éviter les deux inconvénients de la méthode de L-shaped. Le concept de cette méthode est similaire à celui de la méthode de L-shaped multi-coupes mais on ajoute un terme régularisé quadratique dans la fonction objectif . Nous avons adapté cette approche à notre problème.

Dans notre algorithme (ERD), nous ajoutons des coupes dans le problème de premier étage pour accélérer la vitesse de convergence de l'algorithme. Ils permettent de réduire

la région faisable du problème de premier étage et évite l'infaisabilité des sous-problèmes de deuxième étage dans chaque itération. Les coupes sont présentées au-dessous :

$$\sum_{i\in \mathrm{I}} q_{min}(i)\varphi(i) \leq \sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} q^{\mathrm{I}}(i,t,s) = \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} q^{\mathrm{P}}(p,t,s) \leq \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{max}(p)$$
$$\Longrightarrow \sum_{i\in \mathrm{I}} q_{min}(i)\varphi(i) \leq \mathrm{T}\sum_{p\in \mathrm{P}} \gamma_p^{max}(p) \quad \forall s \in \mathrm{S} \tag{A.35}$$

Ces inégalités au-dessus assurent que la quantité totale minimale fournie par les fournisseurs sélectionnés ne dépasse pas la quantité maximale de biomasse traitée dans les installations de prétraitement.

$$\sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} avail^{\mathrm{I}}(i,t,s)\varphi(i) \geq \sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} q^{\mathrm{I}}(i,t,s) \geq \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{max}(p) \geq \sum_{t\in \mathrm{T}}\sum_{p\in \mathrm{P}} \gamma_p^{min}(p)$$
$$\Longrightarrow \sum_{t\in \mathrm{T}}\sum_{i\in \mathrm{I}} avail^{\mathrm{I}}(i,t,s)\varphi(i) \geq \mathrm{T}\sum_{p\in \mathrm{P}} \gamma_p^{min}(p) \quad \forall s \in \mathrm{S} \tag{A.36}$$

Les coupes au-dessus garantissent que les fournisseurs sélectionnés sont capables de satisfaire au moins une quantité minimale de biomasse traitée dans une installation de prétraitement.

**Coupes de Knapsack :** Soit UB la borne supérieure actuelle dans l'itération considérée. En combinant la coupe d'optimalité : $\theta_s \geq e_s^k\varphi + d_s^k$ et $\mathrm{UB} \geq mcost^{\mathrm{T}}\varphi + \sum_{p\in \mathrm{P}} p_s\theta_s$, on obtient :

$$\mathrm{UB} \geq mcost^{\mathrm{T}}\varphi + \sum_{p\in \mathrm{P}} p_s\theta_s \geq mcost^{\mathrm{T}}\varphi + \sum_{s\in \mathrm{S}} p_s(e_s^k\varphi + d_s^k)$$
$$\Longrightarrow \big(mcost + \sum_{s\in \mathrm{S}} p_s e_s^k\big)^{\mathrm{T}}\varphi \leq \mathrm{UB} - \sum_{s\in \mathrm{S}} p_s d_s^k$$

Comme $\varphi$ sont des variables binaires, nous pouvons arrondir les deux côtés de l'inégalité ci-dessus et obtenir une inégalité valide suivante :

$$\lfloor \big(mcost + \sum_{s\in \mathrm{S}} p_s e_s^k\big)^{\mathrm{T}} \rfloor \varphi \leq \lfloor \mathrm{UB} - \sum_{s\in \mathrm{S}} p_s d_s^k \rfloor \tag{A.37}$$

Où $\lfloor a \rfloor$ est un nombre arrondi d'un nombre donné $a$. Quand une limite supérieure $ub_s$ est disponible, on peut ajouter une coupe (A.37) pour resserrer l'espace de solution du problème de la première étape.

**Coupes d'optimalité spécifique :** Selon Laporte and Louveaux [1993], une coupe d'optimalité spécifique permet d'accélérer notre algorithme lorsque les variables du premier étage sont des variables binaires. Nous avons reformulé cette coupe pour notre algo-

rithme comme suit :

$$\sum_{s\in S} p_s\theta_s \geq (q_s - L)\Big(\sum_{i\in\tau}\varphi(i) - \sum_{i\notin\tau}\varphi(i) - |\tau| + 1\Big) + L \tag{A.38}$$

Où $q_s = \sum_{s\in S} p_s Q(\varphi, s)$ est la valeur de la fonction de recours correspondante à chaque itération. L est une borne inférieure satisfaisant :$L \leq min_{\varphi}\{\sum_{s\in S} p_s Q(\varphi, s)\}\ \forall\varphi$. L'ensemble $\tau$ est un ensemble des fournisseurs sélectionnées $i$ : $\tau = \{i \mid \varphi(i) = 1\}$ et $|\tau|$ est la cardinalité de l'ensemble $\tau$.

### A.4.3 Étude numérique

Dans l'étude numérique, nous utilisons des données de Osmani and Zhang [2013, 2014]; Zhang et al. [2012] pour rendre nos instances testées plus réalistes. Les données sont résumées dans le tableau 4.2.

#### Evaluation des performances de méthode de résolution

Le tableau 4.4 résume la taille, le nombre de scénarios et le temps d'exécution du solveur GUROBI, de la version "multi-coupes" de L-shaped (MLS) et de la méthode ERD pour instances différentes. Nous observons que la méthode (ERD) et GUROBI sont deux fois plus rapide que la méthode MLS. En moyenne, la méthode (ERD) est respectivement 1,01, 1,21, 1,16, 1,20, 1,20 et 1,32 fois plus rapide que GUROBI pour toutes les instances. Par conséquent, la méthode ERD peut être utilisée pour résoudre le problème considéré, car elle permet de trouver une solution optimale en un temps de calcul plus court.

Les trois sous-tables de la troisième ligne de la table 4.5 montrent que l'algorithme (ERD) fonctionne toujours bien pour les grandes instances. Le solveur GUROBI ne peut pas résoudre ces grandes tailles du modèle en raison de la mémoire limitée de notre PC. De plus, les deux dernières colonnes prouvent la pertinence d'ajouter ces inégalités dans la méthode de résolution. Les coupes permettent d'améliorer considérablement la vitesse de convergence de la méthode de résolution. En fait, l'ajout des coupes pourrait réduire de 8,8 à 30,3% du temps de calcul par rapport à la méthode classique RD (sans ajouter des coupes).

#### Analyse de la solution

La figure 4.2 montre la répartition des coûts du scénario de base pour une chaîne d'approvisionnement en biomasse sur l'horizon de planification T = 12. La valeur optimale du coût total attendu de la chaîne d'approvisionnement est de $114 271 335. Le coût de production des biocarburants et le coût d'achat des matières premières sont les principaux facteurs de coût qui représentent respectivement près de 39,5% et 34,8% du coût total du système. Le coût du transport joue un rôle essentiel dans la chaîne d'approvisionnement

de la biomasse, avec environ 14,7% du coût total du système. Ce résultat montre l'importance des opérations de transport dans les chaînes d'approvisionnement en biomasse. Le coût de possession n'est que de 2,9%, de sorte que dans ce cas, le stockage d'une quantité importante de biomasse à la bioraffinerie ne semble pas critique.

**Valeur de la solution stochastique**

En réalité, la disponibilité de biomasse peut être estimée par le rendement moyen (12 tonnes/hectare) et puis chercher à identifier des contrats avec les fournisseurs. Cette approche est une approche déterministe qui base sur la valeur moyenne de la disponibilité de biomasse mais elle pourrait avoir des conséquences défavorables. Le tableau 4.5 indique que l'approche stochastique permet de réduire de 7,9% l'espérance du coût total annuel en comparaison de celle obtenue par l'approche déterministe. On peut considérer que la valeur VSS = 124088777 − 114271335 = $9817442 représente le gain possible en tenant compte de l'incertitude d'approvisionnement en biomasse. Cette valeur s'appelle la valeur de la solution stochastique (VSS).

**Analyse de sensibilité**

Dans cette sous-section, nous avons examiné les impacts de certains paramètres critiques sur l'espérance du coût total du système.

**Effet du prix d'achat :** Selon la figure 4.4, lorsque le prix du marché augmente trop, les décideurs ont moins intérêt à acheter une quantité supplémentaire de biomasse sur les marchés pour répondre à la demande de biocarburants. Dans ce cas, ils auraient l'intention d'établir des contrats d'approvisionnement à plus long terme pour assurer la stabilité de l'approvisionnement en matières premières. Cela explique pourquoi le nombre de fournisseurs sous contrat augmente lorsque le prix du marché augmente. Ce résultat suggère que pour atteindre l'efficacité globale du système, il convient d'établir un équilibre entre le prix d'achat dans le cadre de contrats à long terme et celui du marché.

**Effet du coût fixe :** Nous effectuons une étude de l'effet du coût fixe sur la chaîne d'approvisionnement de la biomasse. Dans la figure 4.6, le résultat indique que l'augmentation du coût fixe affecte légèrement le coût total du système et le nombre total de fournisseurs sous contrat. Cela montre également que l'augmentation des coûts fixes pourrait entraîner une augmentation du coût total du système et entraver l'établissement de contrats à long terme.

**Effet de la quantité minimum d'approvisionnement des fournisseurs :** Figure 4.5 montre que l'augmentation de la quantité minimum d'approvisionnement entraine une augmentation légèrement du coût total du système. En outre, le nombre total de fournisseurs sous contrat diminue quand la quantité d'approvisionnement augmente. Dans ce cas, le décideur exigerait une quantité supplémentaire du marché pour satisfaire la demande finale, mais uniquement à un prix raisonnable du marché.

## A.5 Gestion de stocks périssables sous incertitudes : Politique optimal et algorithme

Ce chapitre présente un modèle stochastique de gestion des stocks pour un produit périssable sous incertitudes. Une partie des stocks est dégradée à un constant taux dune période à l'autre. Ce problème apparaît dans les chaînes d'approvisionnement en biomasse. En fait, le taux de dégradation de biomasse en stockage est d'environ 1 à 2% par mois Rentizelas et al. [2009].

### A.5.1 Description du problème et formulation du modèle

#### Description du problème

Considérons un modèle de gestion des stocks pour un seul produit, un seul stock à l'horizon de planification de $n$ -périodes. Ce produit est périssable et son taux de détérioration est constant, noté $(1-\lambda)$, où $0 < \lambda < 1$. Soit $s_t$ le niveau de stock au début de la période $t$ et avant de lancer une quantité commandée $q_t$ dans cette période.

La demande dans la période $t$ est représentée par la variable aléatoire non négative $\mathrm{D}_t$ dont la fonction de densité de probabilité $f_t$ et la fonction de distribution cumulative $\mathrm{F}_t$ sont connues. Les demandes dans toutes les périodes sont indépendantes.

Les capacités d'approvisionnement dans $n$ -périodes, qui sont des variables aléatoires et indépendantes, notée par $y_t$, dont la fonction de densité de probabilité $g_t$ et la fonction de distribution cumulative $\mathrm{G}_t$ sont connues. Supposons que le délai de livraison est nul, la demande non satisfaite est mise en attente et le niveau de stock est revu périodiquement.

L'objectif de ce problème est de minimiser l'espérance mathématique du coût total, qui comprend trois types de coûts : coûts de stockage, coûts de rupture et coûts d'achat sur l'horizon de planification de $n$ périodes.

L'ordre des événements survenus dans chaque période est supposé comme suit.

- Au début de la période, le niveau de stock est examiné en tenant compte de la détérioration du produit survenue dans la période précédente.

- Selon la politique de gestion des stocks choisies, une commande est alors placée si nécessaire. Ensuite, la demande de client est satisfaite autant que possible et la demande non satisfaite est mise en attente.

- Un coût de stockage est chargé en fonction du stock disponible à la fin de cette période. Un coût de rupture est chargé en cas de niveau de stock négatif à la fin de chaque période.

Le modèle de gestion des stocks périssables est présenté sous la forme suivante :

$$\textbf{(SP)} \quad \min_{q_t} \quad \sum_{t=1}^{\mathrm{T}} \alpha^t \phi_t(s_t, q_t) \tag{A.39}$$

$$s.t \quad s_{t+1} = \lambda s_t^+ - s_t^- + min(q_t, y_t) - \mathrm{D}_t \tag{A.40}$$

$$s_t^+ = \max(0, s_t) \tag{A.41}$$

$$s_t^- = \max(0, -s_t) \tag{A.42}$$

où la fonction $\phi_t(s_t, q_t)$ est l'espérance mathématique du coût pour une seul période qui inclut l'espérance mathématique du coût de stockage, du coût de rupture et du coût d'achat. Soit *c, p,* textit h le coût d'achat, celui de rupture et celui de stockage par unité, respectivement. Soit α le facteur de discount ($0 < \ alpha < 1$).

$$\phi_t(s_t, q_t) = \mathop{\mathbb{E}}_{(\mathrm{D}_t, y_t)} \Big[ c(\min\{q_t, y_t\}) + h(\lambda s_t^+ - s_t^- + \min(q_t, y_t) - \mathrm{D}_t)^+ + p(\mathrm{D}_t - \min\{q_t, y_t\} - \lambda s_t^+ + s_t^-)^+ \Big] \tag{A.43}$$

L'équation A.41 représente la relation entre le niveau de stocks au début de la période *t* et celle au début de la période *t+1* en considérant la périssabilité du produit.

Le niveau de stock $s_t$ peut être négatif en cas où la demande est en attente. Dans ce cas, $s_t < 0$, le stock disponible est zéro, et seul $-s_t^- = s_t$ est reporté à la période suivante. Au cas d'un niveau de stocks positif $s_t$, le stock disponible est égal à $s_t$. Donc, une partie de stocks disponible $(1-\lambda)s_t$, aura péri pendant la période $t$.

### Modèle de programmation dynamique et propriétés du modèle

Dans cette section, nous formulons un modèle de programmation dynamique stochastique tenant en compte l'incertitude à la fois dans l'approvisionnement et la demande. Soit $\mathrm{V}^*_{t,n}(s_t)$ la valeur optimale de l'espérance mathématique du coût total de la période $t$ à $n$ en utilisant la politique optimale sachant que le niveau de stock $s_t$ à la période $t$ est connu. Le coût optimal $\mathrm{V}^*_{t,n}(s_t)$ peut être calculé de manière récursive comme suit :

$$\mathrm{V}^*_{t,n}(s_t) = \min_{q_t} \mathrm{V}_{t,n}(s_t, q_t) \tag{A.44}$$

$$\text{where} \quad \mathrm{V}_{t,n}(s_t, q_t) = \phi_t(s_t, q_t) + \alpha \mathop{\mathbb{E}}_{(\mathrm{D}_t, y_t)} \mathrm{V}^*_{t+1,n}(s_{t+1})$$

Dans l'équation au-dessus, $\mathrm{V}_{t,n}(s_t, q_t)$ correspond à la valeur du coût total cumulée entre la période $t$ et $n$, obtenu en utilisant la politique optimale pour les périodes $t+1$ à $n$, sachant que le niveau de stock $s_t$ et la quantité de commande $q_t$ sont connus. Par définition, $\mathrm{V}_{n+1,n}(.) \equiv 0$. La fonction $\phi_t(s_t, q_t)$ est une fonction du stock initial $s_t$, ainsi que la quantité de commande $q_t$.

Les propriétés du modèle se présentent comme suit :

1. $\frac{\partial^2}{\partial s_t^2} \mathrm{V}_{t,n}(s_t, q_t^*) \geq 0 \quad \forall s_t$

2. $\varphi_{t,n}(s_t, q_t)$ est une function croissante en $q_t$.

3. $\mathrm{V}_{t,n}(s_t, q_t)$ a un minimum global en $q_t = q_t^*$

4. Le politique optimal est une politique « order-up-to level ».

### A.5.2 Approche de résolution

Pour trouver les niveaux de recomplètement optimaux, nous constatons qu'il est difficile d'appliquer directement l'approche de programmation dynamique due à la grand dimension du problème. C'est pourquoi nous développons une approche de résolution qui est une combinaison entre l'approche de scénarios et la méthode de relaxation lagrangienne. L'approche de scénarios permet de capturer l'incertitude de manière approximative, mais la taille du modèle peut augmenter considérablement en fonction du nombre de scénarios considérée. Pour surmonter ce défi, nous développons un algorithme basé sur la relaxation lagrangienne pour trouver une solution approchée de bonne qualité.

#### Modèle équivalent déterministe

Dans cette sous-section, nous transformons le modèle stochastique (A.39) au modèle de programmation linéaire à nombres entiers mixtes (MILP) en utilisant de l'approche basée sur scenarios. Soit $\Omega$ un ensemble de scénarios, $\omega \in \Omega$ l'indice d'un scénario et sa probabilité d'occurrence est notée par $p_\omega$. L'index $\omega$ est ajouté aux variables de décision liées au scénario considéré $\omega$. L'espérance mathématique $\mathbb{E}(.)$ dans la fonction objectif ( ref modèle original) est remplacée par $\sum_\omega p_\omega(.)$

Soit $s_{t,\omega}$ le niveau de stock au début de la période textit t dans le scénario $\omega$. Soit $\mathrm{L}_t$ le niveau de recomplètement dans la période $t$. Notre objectif est de trouver ce niveau, $\mathrm{L}_t$, en résolvant le modèle suivant (DEP). La quantité de commande, $q_{t,\omega}$, est définie par :

$$q_{t,\omega} = \min\{\mathrm{L}_t - \lambda s_{t,\omega}^+ + s_{t,\omega}^-, y_{t,\omega}\} \tag{A.45}$$

La variable auxiliaire $\beta_{t,\omega}$ est définie comme suit :

$$\beta_{t,\omega} = \begin{cases} 1 & \text{if } \mathrm{L}_t - \lambda s_{t,\omega}^+ + s_{t,\omega}^- \leq y_{t,\omega} \\ 0 & \text{otherwise} \end{cases}$$

Le modèle équivalent déterministe (DEP) est le suivant :

$$\textbf{(DEP)} \quad \min \sum_{t\in \mathrm{T}} \alpha^t \Big( \sum_{\omega\in\Omega} p_\omega (cq_{t,\omega} + hs^+_{t,\omega} + ps^-_{t,\omega}) \Big) \tag{A.46}$$

s.t

$$\mathrm{L}_t - \lambda s^+_{t,\omega} + s^-_{t,\omega} - (1-\beta_{t,\omega})\mathrm{M} \le q_{t,\omega} \le \mathrm{L}_t - \lambda s^+_{t,\omega} + s^-_{t,\omega} \tag{A.47}$$

$$y_{t,\omega} - \beta_{t,\omega}\mathrm{M} \le q_{t,\omega} \le y_{t,\omega} \tag{A.48}$$

$$s_{t+1,\omega} = \lambda s^+_{t,\omega} - s^-_{t,\omega} + q_{t,\omega} - \mathrm{D}_{t,\omega} \tag{A.49}$$

$$s_{t,\omega} = s^+_{t,\omega} - s^-_{t,\omega} \tag{A.50}$$

$$s^+_{t,\omega} \ge s_{t,\omega} \tag{A.51}$$

$$s^-_{t,\omega} \ge -s_{t,\omega} \tag{A.52}$$

$$\beta_{t,\omega} \in \{0,1\} \tag{A.53}$$

$$s_{t,\omega} \in \mathbb{R}, s^+_{t,\omega} \ge 0, s^-_{t,\omega} \ge 0, q_{t,\omega} \ge 0, \mathrm{L}_t \ge 0 \tag{A.54}$$

L'équation (A.49) représente l'équilibre des stocks, similaire à l'équation (A.40) dans le modèle d'origine.

Les équations (A.41) - (A.42) sont linéarisées par des contraintes (A.50) - (A.52).

L'équation (A.45) est linéarisée par des contraintes (A.47 - A.48). Notez que M est un grand nombre.

### Relaxation Lagrangienne

Le modèle (DEP) pourrait être résolu par un solveur commercial mais son temp de calcul devient plus important pour les instances de grande taille. Dans cette sous-section, nous proposons un algorithme basé sur la méthode de relaxation lagrangienne.

Nous relaxons $\mathrm{L}_t$ par $\mathrm{L}_{t,\omega}$ en introduisant la contrainte suivante :

$$\mathrm{L}_{t,\omega} = \mathrm{L}_{t,\omega'} \quad \forall \omega, \omega' \in \Omega \text{ and } \omega \neq \omega' \tag{A.55}$$

Nous définissons une variable intermédiaire, $\delta_{t,\omega}$ comme la violation de la contrainte (A.55). Soit $\pi = (\pi_{t,\omega})\ \forall t \in \mathrm{T}, \omega \in \Omega$ un vecteur de multiplicateurs de Lagrange associé à la contrainte (A.55).

$$\delta_{t,\omega} = \mathrm{L}_{t,\omega} - \mathrm{L}_{t,\omega'} \quad \forall \omega, \omega' \in \Omega \text{ and } \omega \neq \omega' \tag{A.56}$$

Donc, la programmation de relaxation lagrangien (LRP) est présentée au-dessous. Elle

permet de trouver une borne inférieure pour le modèle (DEP).

$$\textbf{(LRP)} \quad \min \sum_{t\in T} \alpha^t \Big( \sum_{\omega\in\Omega} p_\omega (cq_{t,\omega} + hs^+_{t,\omega} + ps^-_{t,\omega}) \Big) + \sum_{t\in T} \sum_{\omega\in\Omega} \pi_{t,\omega}\delta_{t,\omega} \tag{A.57}$$

$$\text{s.t}$$

$$L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} - (1-\beta_{t,\omega})M \le q_{t,\omega} \le L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} \tag{A.58}$$

$$\text{constraint} \quad (A.48) - (A.54)$$

À partir des solutions $L_{t,\omega}, \omega \in \Omega$ obtenue par la résolution du problème de relaxation lagrangien (LRP), nous pouvons la transformer en une solution réalisable, $L_t$ pour le problème originale par une agrégation de scénarios au-dessous. Cette solution faisable $L_t$ permet de fournir une borne supérieure de la valeur objective optimale du modèle (DEP).

$$L_t = \sum_{\omega\in\Omega} p_\omega L_{t,\omega} \quad \forall t \in T \tag{A.59}$$

Il est nécessaire de déterminer les multiplicateurs de Lagrange, $\pi$ qui maximisent la borne inférieure. Autrement dit, nous devons chercher une borne inférieure aussi proche que possible de la valeur objective optimale du modèle (DEP). Cela conduit au problème dual lagrangien suivant :

$$\max_{\pi\in\mathbb{R}} \{\textbf{LRP}(\pi)\}$$

$$= \max_{\pi\in\mathbb{R}} \left\{ \begin{array}{l} \min \sum_{t\in T} \alpha^t \Big( \sum_{\omega\in\Omega} p_\omega (cq_{t,\omega} + hs^+_{t,\omega} + ps^-_{t,\omega}) \Big) + \sum_{t\in T} \sum_{\omega\in\Omega} \pi_{t,\omega}\delta_{t,\omega} \\ \text{s.t} \\ L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} - (1-\beta_{t,\omega})M \le q_{t,\omega} \le L_{t,\omega} - \lambda s^+_{t,\omega} + s^-_{t,\omega} \\ \text{constraint (5.26)-(5.31)} \end{array} \right\}$$

La méthode du sous-gradient est souvent utilisée pour résoudre le problème dual lagrangien mais elle converge très lente ou même ne converge pas. Nous utilisons la méthode de volume pour la résolution du problème dual lagrangien. Selon Barahona and Anbil [2000], la méthode de volume est une extension de l'algorithme de sous-gradient et très similaire à la méthode de bundle [Lemaréchal, 1989] mais son temps de calcul est moins important que les autres méthodes. C'est pourquoi nous utilisons la méthode du volume pour résoudre le problème dual lagrangien. Notre algorithme est détaillé dans l'Algorithme 5.1

### A.5.3 Expériences numériques

Dans cette sous-section, nous effectuons une étude numérique pour évaluer la performance et démontrer l'efficacité de notre algorithme. Cet algorithme est implémenté en Python 3.5 sur un ordinateur HP Intel Core i5-4210M de 8 Gb RAM.

**Paramétrage**

Dans l'étude numérique, nous utilisons les données présentées dans Broekmeulen and Van Donselaar [2009]; Pauls-Worm et al. [2015] telles que la demande moyenne, le coefficient de variation (CV = $\sigma/\mu$) et le nombre de périodes. Le coût de stockage unitaire $h$ est fixé à 5. Le coût d'achat unitaire $c$ varie de 30 à 40. Le coût de rupture par unité $p$ est déterminé à partir du ratio $p/(p+h)$. Le nombre de périodes et celui de scénarios sont respectivement T = 7 et $|\Omega| = 200$. Basant sur la méthode statistique de Shapiro [2000], nous estimons que l'instance inclut 200 scénarios permet d'obtenir une solution très proche de l'optimum avec l'écart de 1% dans l'intervalle de confiance à 95% par rapport à la solution optimale. Toutes les valeurs de paramètre sont présentées dans le tableau 5.1. Les trois cas concernant la lois de distribution de la demande et de l'approvisionnement sont les suivantes :

a) La demande suit une distribution normale non-stationnaire avec $\mu_{D_t} = f_t.\mu_D$ où $f_t = \{0.12, 0.13, 0.13, 0.16, 0.18, 0.18, 0.10\}$, et $\mu_D$ est pris dans la liste $[12, 13, 13, 16, 18, 18, 10]$. L'approvisionnement suit une distribution normale stationnaire avec $\mu_{S_t} = 16$ et $\sigma_{S_t} = 0.2\mu_{S_t} \forall t = 1...7$.

b) La demande et l'approvisionnement suivent une distribution normale stationnaire avec les paramètres suivante $\mu_{D_t} = 13$, $\sigma_{D_t} = 3.5$, $\mu_{S_t} = 15$ et $\sigma_{S_t} = 0.2\mu_{S_t} \forall t = 1...7$.

c) La demande et l'approvisionnement suivent une distribution normale stationnaire avec les paramètres suivante $\mu_{D_t} = 14$, $\sigma_{D_t} = 0.25\mu_{D_t}$, $\mu_{S_t} = 15$ et $\sigma_{S_t} = 0.2\mu_{S_t} \forall t = 1...7$.

Concernant, le paramétrage de l'algorithme LR, nous fixons le seuil d'optimalité $\epsilon \leq 0.01\%$, le nombre d'itérations maximal $iteration_{max} = 500$ et le temps de calcul maximal $time_{max} = 3600$s. L'algorithme s'arrête également lorsque la borne supérieure ne peut pas être améliorée après 50 itérations consécutives.

**Evaluation des performances de l'algorithme**

Dans cette sous-section, nous comparons la performance de notre solution avec avec l'approche de programmation dynamique ([Bellmam, 1957, Chapter III.3]). Les tables 5.2, 5.3 et 5.4 représentent le temps de calcul et l'écart relatif entre les deux approches (E-LR et DP). En fait, E-LR est la valeur objective attendue du modèle stochastique obtenue en utilisant la solution de l'approche de relaxation lagrangienne. Pour chaque instance, le modèle (DEP) comprend 3 807 variables continues, 1400 variables binaires et 7 007 contraintes.

Nous observons que le temps de calcul de l'algorithme LR est plus court que celui de l'algorithme DP (215,91s vs 5180,80s dans le cas a, 187,83s vs 5476,63s dans le cas b et 172,59s vs 5399,91s dans le cas c). De plus, l'écart moyen entre la solution LR et celle de DP n'est que de 0,77 %, 0,70 % et 0,50 % respectivement dans les trois cas considérés. L'étude numérique montre que l'approche LR permet de trouver une solution de haute qualité en temps de calcul raisonnable et donc il est convient pour une application pratique.

## A.6 Modélisation et optimisation de la gestion des stocks d'un produit à durée de vie fixe sous incertitudes

La réduction du gaspillage provenant de la périssabilité des produits, est l'une des défies majeures de plusieurs secteurs industriels concernant les produits frais, les produits sanguins, la viande, les produits chimiques, les matériaux composites et les produits pharmaceutiques. À notre connaissance, il y a très peu d'études de recherche portées sur la gestion des stocks d'un produit périssable, avec l'approvisionnement aléatoire et la demande aléatoire dans un horizon de planification glissant. Pour cette raison, nous étudions un modèle de gestion des stocks d'un produit périssable à durée de vie fixe. Les caractéristiques essentielles de ce modèle sont : périssabilité, la demande aléatoire, l'approvisionnement aléatoire et multi-périodes.

### A.6.1 Description du problème et formulation du modèle

#### Description du problème

Dans cette sous-section, nous considérons un problème multi-périodes de gestion des stocks d'un produit périssable à durée de vie fixe dans un horizon de planification glissant. Pour simplifier, nous supposons que la méthode du premier entré, premier sorti (FIFO) est employée pour la méthode de rangement. Supposons que l'ordre des événements survenus au cours de chaque période est le suivant :

i) Les articles frais arrivent au début de chaque période. Ensuite, le niveau de stock est mis à jour selon la durée de vie restant de chaque article. Les articles expirés sont rejetés.

ii) Selon la politique de gestion des stocks utilisée, une commande est alors lancée si nécessaire. Puis, la demande des clients est satisfaite autant que possible et la demande non satisfaite est perdue ; iii) À la fin de chaque période, le coût de stockage et le coût de rupture sont chargés en fonction du stock disponible. Le coût de gaspillage est également estimé pour tous les articles expirés.

#### Formulation du modèle

Dans cette sous-section, nous formulons un modèle de programmation stochastique pour le problème de gestion des stocks d'un produit périssable considéré. Le produit est périssable et son âge est noté par l'indice $i \in \{1, ..., \mathrm{M}\}$. Soit $\mathrm{I}_t^i$ le niveau de stocks concernant des articles ayant l'âge $i$ à la fin de la période $t \in \{1, ..., \mathrm{T}\}$. Il convient de noter $\mathrm{I}_t^1$ comme le niveau de stocks pour des articles frais à la fin de la période $t \in \{1, ..., \mathrm{T}\}$.

La première temps consistes la décision de lancement d'un commande au période $t$. La variable binaire $y_t$ est égal à 1 si la quantité de commande $\mathrm{O}_t$ est lancé à la période $t \in \{1, ..., \mathrm{T}\}$, sinon 0. Les variables de la deuxième étape comprennent la quantité d'approvisionnement reçue $q_t$, le niveau de stock $\mathrm{I}_{t,\omega}^i$ pour chaque âge $i$, la demande perdue

$B_t$ et des articles expirées $I_t^M$. Dans un deuxième temps, les recours sont prises pour atténuer les effets négatifs de la décision prise en première temps. Toutes les variables de décision continues sont non négatives.

Supposons que $D_t$ et $S_t$ sont des variables aléatoires qui suivent une distribution normal multidimensionnelle connue $N(\mu, \Sigma)$ où $\mu$ est un vecteur de moyenne de $\{D_t, S_t\}$ et $\Sigma$ est une matrice de covariance qui donne la corrélation entre l'offre $S_t$ et la demande $D_t$. Notons que $x^+ = \max\{0, x\}$.

Le modèle de programmation stochastique pour la gestion des stocks d'un produit périssable est le suivant :

$$\textbf{(SP)} \quad \min \sum_{t=1}^{T} \mathbb{E}_{D_t,S_t}\Big[ a y_t + u q_t + h(\sum_{i=1}^{M-1} I_t^i) + p B_t + w I_t^M \Big] \tag{A.60}$$

s.t

$$y_t = \begin{cases} 1, & \text{if} \quad O_t > 0 \\ 0, & \text{sinon} \end{cases} \qquad t = 1, \ldots T \tag{A.61}$$

$$Y_t^i = (D_t - \sum_{j=i}^{M-1} I_{t-1}^j)^+ \quad i = 1 \ldots M, \; t = 1, \ldots T \tag{A.62}$$

$$I_t^i = (I_{t-1}^{i-1} - Y_t^i)^+, \quad t = 1, \ldots T, \; i = 2 \ldots M \tag{A.63}$$

$$I_t^1 = (q_t - Y_t^1)^+, \quad t = 1, \ldots T \tag{A.64}$$

$$B_t = \sum_{i=1}^{M-1} I_{t-1}^i - \sum_{i=1}^{M} I_t^i + q_t - D_t, \quad t = 1, \ldots T \tag{A.65}$$

$$q_t = \min\{S_t, O_t\}, \quad t = 1, \ldots T \tag{A.66}$$

$$I_t^i \geq 0, \quad i = 1 \ldots M, \; t = 1, \ldots T \tag{A.67}$$

$$q_t, B_t \geq 0, y_t \in \{0, 1\} \quad t = 1, \ldots T \tag{A.68}$$

L'objectif (A.60) est de minimiser l'espérance mathématique du coût total, qui comprend cinq types de coûts : coût de passasion de commande, coût d'achat, coût de stockage, coût de rupture et coût de gaspillage sur l'horizon de planification de T.

Equation (A.61) indique que la variable binaire $y_t = 1$ seulement s'il y a une quantité de commande $O_t$ est lancé à la période $t$, sinon 0. L'équation (A.62) représente que la variable $Y_{t,}^i$ est la demande restante après avoir retiré tous articles ayant l'âge compris entre $i$ et M en utilisant la méthode de FIFO. L'équation ( ref 5.Equaton 4A) et (A.64) présentent la transition du stocks de la période actuelle à la période suivante. Concernant les équations (A.62) et (A.63), nous supposons implicitement que pour la période, $t = 1$, les variables $I_{t-1}^i$ sont remplacés par les paramètres prédéterminés $I_0^i$ (stocks initial).

L'équation (A.65) montre les contraintes de balance des stocks entre période $t$ et $t+1$. Cette équation détermine également la demande perdue, $B_t$, dans le cas où la quantité de commande reçue, $q_t$, et le stock de tous les âges $i$ ne peuvent pas satisfaire la demande,

$D_t$. L'équation (A.66) assure que la quantité de commande reçue ne dépasse pas la capacité d'approvisionnement $S_t$.

### A.6.2 Approache de résolution

Dans cette sous-section, le modèle original (SP) est transformé en un modèle MILP équivalent en appliquant l'approche de scénario. La méthode d'approximation de la moyenne de l'échantillon (SAA) est une méthode conventionnelle pour l'optimisation stochastique. L'étude de Beltran-Royo [2017] montre que la méthode des scénarios conditionnels (CS) peut trouver une meilleure solution que cela obtenue par la méthode SAA. C'est pourquoi nous développons une approche de résolution en basant sur la méthode de scénarios conditionnels.

#### Deterministic equivalent model

Dans cette sous-section, nous reformulons le modèle **(SP)** en un modèle de programmation linéaire à nombres entiers mixtes (MILP). Cette approche permet de capturer l'incertitude de manière approximative avec la précision en fonction du nombre de scénarios considérés. Soit $\omega \in \Omega$ l'index d'un scénario et sa probabilité d'occurrence est $p_{omega}$. L'espérance mathématique $\mathbb{E}(.)$ peut être remplacée par $\sum_\omega p_\omega[.]$. Ensuite, la fonction objective non linéaire (A.60) se transforme en fonction objectif déterministe (A.69).

La fonction objectif du modèle MILP équivalent déterministe est donné comme suit :

$$\min \sum_{t=1}^{T} a y_t + \sum_{\omega\in\Omega}\sum_{t=1}^{T} p_\omega \Big[ u q_{t,\omega} + h(\sum_{i=1}^{M-1} I^i_{t,\omega}) + p B_{t,\omega} + w I^M_{t,\omega} \Big] \tag{A.69}$$

$$s.t. \quad \text{Constraints} \quad (A.70) - (A.79)$$

L'index $\omega$ est ajouté à chaque variable de décision en deux étapes liée au scénario $\omega$ sauf la variable $O_t$ et $y_t$. La quantité d'ordre $O_t$ n'apparaît pas explicitement dans la fonction objectif mais implicitement à travers la réalisation de variables aléatoires $D_{t,\omega}$ et $S_{t,\omega}$. En suite, nous linéarisons les contraintes (A.61) - (A.65) du problème original (SP). Soit H est un grand nombre. Les contraintes du problème déterministe équivalent sont présentées au-dessous :

$$0 \le O_t \le H y_t \quad t = 1, \ldots T \tag{A.70}$$

$$D_{t,\omega} - \sum_{j=i}^{M-1} I^j_{t-1,\omega} \le Y^i_{t,\omega} \le (D_{t,\omega} - \sum_{j=i}^{M-1} I^j_{t-1,\omega}) + (1 - \alpha^i_{t,\omega})H \tag{A.71}$$

$$0 \le Y^i_{t,\omega} \le \alpha^i_{t,\omega} H \tag{A.72}$$

$$\alpha^i_{t,\omega} \in \{0, 1\} \tag{A.73}$$

$$I^i_{t,\omega} \ge (I^{i-1}_{t-1,\omega} - Y^i_{t,\omega}) \quad \forall t = 1, \ldots T,\ i = 2 \ldots M \tag{A.74}$$

$$\mathrm{I}^1_{t,\omega} \geq (q_{t,\omega} - \mathrm{Y}^1_{t,\omega}) \quad \forall t = 1, \ldots \mathrm{T} \tag{A.75}$$

$$\mathrm{B}_{t,\omega} = \sum_{i=1}^{\mathrm{M}-1} \mathrm{I}^i_{t-1,\omega} - \sum_{i=1}^{\mathrm{M}} \mathrm{I}^i_{t,\omega} + q_{t,\omega} - \mathrm{D}_{t,\omega}, \quad t = 1, \ldots \mathrm{T} \tag{A.76}$$

$$\mathrm{S}_{t,\omega} - (1 - \gamma_{t,\omega})\mathrm{H} \leq q_{t,\omega} \leq \mathrm{S}_{t,\omega} \tag{A.77}$$

$$\mathrm{O}_t - \gamma_{t,\omega}\mathrm{H} \leq q_{t,\omega} \leq \mathrm{O}_t \tag{A.78}$$

$$\gamma_{t,\omega} \in \{0, 1\} \tag{A.79}$$

### Conditional scenarios approach

Afin de réduire la complexité du problème de programmation, nous considérons certaines techniques telles que la méthode de « moment matching », la méthode d'approximation de la moyenne de l'échantillon (SAA) et les autres basées sur métriques de probabilité.

Selon Beltran-Royo [2017], la méthode de scénarios conditionnels (CS) permet de gérer l'incertitude dans un temps de calcul plus court et pourrait apporter une meilleure solution. De plus, cette méthode peut être considérée comme une méthode d'agrégation dans laquelle les poids correspondants sont calculés en fonction de la probabilité conditionnelle des paramètres aléatoires. Il est également possible de calculer des scénarios conditionnels directement à partir d'un vecteur aléatoire continu donné en appliquant une discrétisation conditionnelle.

### Evaluation des solutions

Pour évaluer la qualité de la solution, nous comparons notre méthode de résolution avec la méthode de l'approximation de la moyenne de l'échantillon (Sample Average Approximation). Dans la méthode de SAA, la fonction objectif attendue du problème stochastique est approximée par une estimation d'échantillon moyenne tirée à partir d'un échantillon aléatoire.

La différence principale entre les approches SAA et CS concerne la manière d'approximer le vecteur aléatoire et leurs probabilités associées (probabilité égaux versus probabilité conditionnelle). En outre, un seul échantillonnage est utilisé dans la méthode de CS, mais un grand nombre d'échantillonnages est nécessaire pour garantir la qualité de solution dans la méthode SAA.

De plus, l'utilisation de techniques de réduction telles que l'échantillonnage en hypercube latin Diwekar and David [2015]; McKay et al. [1979] pourrait accroître l'efficacité de la méthode SAA. Pour cette raison, nous avons appliqué cette technique pour générer l'ensemble des échantillons dans la méthode SAA.

### A.6.3 Etude numérique

Dans cette sous-section, nous effectuons l'étude numérique dans laquelle les problèmes MILP liés à la méthode CS et SAA ont été résolus par le solveur Gurobi 6.5 avec des paramètres par défaut sur un ordinateur HP Intel Core i5-4210M 8GB RAM.

#### Paramétrage

Dans l'étude numérique, nous utilisons les données de Broekmeulen and Van Donselaar [2009]; Pauls-Worm et al. [2016] telles que la durée de vie maximale, la demande moyenne, le coefficient de variation (CV = $\sigma/\mu$), le nombre de période. Les paramètres de coût sont normalisé par rapport au coût d'achat unitaire $v$, qui est fixé à 1. Le ratio de coût de stockage unitaire $h/v$ varie de 0,02 à 0,04. Le coût de rupture unitaire $p$ est déterminé à partir du ratio $p/(v+p+h)$, qui varie entre 90 % et 99 %. Le ratio de coût de gaspillage unitaire, $w/v$, varie entre 0 et 1. Le coût fixe de commande varie de $250v$ à $500v$. La durée de vie maximale est de M = 2 ou 3. Le nombre de périodes est de T = 7. La demande moyenne est calculée par $\mu_{D_t} = f_t.\mu_D$ où coefficient saisonner est $f_t = \{0.12, 0.13, 0.13, 0.16, 0.18, 0.18, 0.10\}$ et la demande moyenne $\mu_{D_t}$ est pris dans la liste $[240, 260, 260, 320, 360, 360, 200]$. L'approvisionnement suit une distribution normale stationnaire avec $\mu_{S_t} = 320$ et $\sigma_{S_t} = 0.2\mu_{S_t}$ pour tous $t = 1...7$.

#### Evaluation de la performance de l'approche de résolution

Pour évaluer la performance des solutions, nous comparons notre méthode avec la méthode SAA en terme de la valeur attendue et du temps de calcule. Les valeurs E-SAA et E-CS sont des valeurs attendues de la fonction objectif du problème d'origine obtenues respectivement en utilisant la solution obtenue par la méthode SAA et celle de CS. Nous générons de manière aléatoire 1000 000 scénarios à partir de la distribution normale multivariée du vecteur aléatoire $\xi = \{D_t, S_t\}$, $t = 1...T$ pour estimer la valeur de E-SAA et E-CS.

Le tableau 6.2 présente le temps de calcul et la solution obtenue par l'approche CS et SAA. Toutes les instances comprennent 4 368 variables continues, 3 192 variables binaires et 16 471 contraintes. La dernière colonne montre l'écart relatif entre E-CS et E-SAA. Par une analyse statistique, la méthode de SAA peut obtenir une solution très proche de l'optimum dans un intervalle de confiance de 95% avec un écart de 0,1% en termes de l'espérance du coût total par rapport à la solution optimale.

Nous constatons que la qualité de solution des deux méthodes sont similaire, car l'écart rélatif entre eux est très petit (0,07 % et 0,09 % en moyenne dans le case M = 2 et 3, respectivement). De plus, notre approche CS possède le temps de calcul moins important que la méthode SAA (35,37s contre 284,91s et 35,41s contre 317,08s respectivement). L'étude numérique suggère que l'approche CS présente un bon équilibre entre la qualité de solution et le temps de calcul.

**Analyse de la solution**

Dans cette sous-section, nous considérons un produit périssable avec la durée de vie fixe M = 2 sur l'horizon de planification T = 5. Le coût fixe de commande est de $ 30. Le coût de stockage unitaire, le coût de rupture unitaire et le coût de gaspillage unitaire sont respectivement de $ 2, $ 15, $ 3. Supposons que la capcité d'approvisionnement et la demande sont des variables normales. Leurs paramètres sur des périodes T sont présentés comme suit :

$\mu_D = [860, 810, 760, 570, 620]$ ;

$\mu_S = [520, 550, 860, 940, 760]$ ;

$\sigma_D^2 = [29584, 26244, 23104, 12996, 15376]$ ;

$\sigma_S^2 = [10816, 12100, 29584, 35344, 23104]$.

La figure 6.2a présente la distribution des coûts taux du système. La valeur optimale du coût total attendu obtenue par la méthode de CS est de 41 740,98 dollars. Nous observons que le coût d'achat et le coût de rupture représentent respectivement de 71% et 27% du coût total du système. Le coût de stockage est d'environ 1%. Dans ce cas, le stockage d'une quantité importante de produit ne semble pas critique.

Ensuite, nous étudions les impacts de certains paramètres critiques sur le coût total du système. La figure 6.2b présente l'effet du prix d'achat unitaire sur le coût total. Ce résultat indique qu'avec l'augmentation du prix d'achat, le coût total du système augmentera rapidement. Cela confirme le fait que le coût d'achat unitaire joue le rôle le plus important dans ce système avec 72% du coût total.

Le coût total du système augmente légèrement en fonction du coût fixe (Figure 6.2c), cette augmentation est moins importante par rapport à l'augmentation du coût fixe (de 80% à 130%). La figure 6.2d montre le changement du coût total des stocks lorsqu'on varie le coût unitaire de rupture. A cause de la présence des incertitudes, le coût total du système semble être sensible au changement du coût unitaire de rupture .

## A.7 Conclusions et Perspectives

La transition du carburant traditionnel au biocarburant a soulevé un certain nombre de problèmes de gestion dans les chaînes d'approvisionnement de la biomasse. La conception optimale et la gestion efficaces de la chaîne d'approvisionnement pourraient renforcer la compétitivité et promouvoir le biocarburant pour qu'il devienne plus courant dans la vie quotidienne.

L'objectif principal de cette thèse est de proposer des modèles d'optimisation et des méthodes de résolution pour la gestion de la chaîne d'approvisionnement en biomasse et la gestion des stocks en biomasse.

Tout d'abord, l'état de l'art a souligné le manque de recherche sur un problème de gestion d'une chaîne d'approvisionnement et des stocks périssables. Par cette analyse, nous pouvons positionner notre recherche dans cette thèse en considérant des différents aspects du problème.

En premier lieu, nous avons étudié la gestion d'une chaîne d'approvisionnement en biomasse. Dans la section 3, nous avons présenté un modèle (MILP) pour minimiser le coût total d'une chaîne d'approvisionnement de la biomasse. Dans ce modèle, les variables de décision sont liées à la sélection des fournisseurs et aux activités logistiques. L'étude de cas montre que ce modèle peut être résolu de manière optimale dans un temps de calcul raisonnable par un solveur commercial.

Ensuite, dans la section 4, nous avons étendu le problème précèdent sous incertitudes en formulant un modèle de programmation stochastique à deux étapes. A la recherche d'une méthode de résolution plus rapide, une méthode de la famille de décomposition de L-shaped a été développée. Le résultat numérique prouve une excellente performance de notre méthode par rapport au solveur commercial Gurobi. De plus, la sous-section 4.4.4 indique que l'approche stochastique pourrait constituer une solution optimale permettant de réduire de 7,9% du coût total par rapport à l'approche déterministe. Il montre que le choix des fournisseurs affecterait fortement le coût total attendu dans les environnements stochastiques

En second lieu, nous avons cherché à identifier la politique optimale de la gestion des stocks d'un produit périssable. Nous avons proposé deux modèles dans les chapitres 5 et 6. Une seule différence entre les deux modèles concerne la nature d'un produit considéré. Le chapitre 5 est destinée à la gestion des stocks pour un produit ayant des caractéristiques similaires à la biomasse (le taux constant de détérioration). Le chapitre 6 adresse au produit ayant une durée de vie fixée.

Dans le chapitre 5, nous proposons un modèle stochastique de gestion des stocks périssables avec demande et capacité d'approvisionnement aléatoires. Ce produit considéré s'est dégradé par un taux constant. Nous avons démontré que la politique optimale est une politique « Oder-up-to-level ». Puis, nous avons développé un algorithme basant sur la relaxation lagrangienne permettant de trouver une solution approchée de bonne qua-

lité. L'étude numérique montre que notre méthode est capable de résoudre les problèmes de grande taille avec un écart de l'espérance du coût total plus petite (<1%).

Le chapitre 6 présente un modèle stochastique de la gestion des stocks d'un produit à durée de vie fixe sur un horizon glissant. La demande et la capacité d'approvisionnement sont aléatoires. Nous avons développé une méthode de résolution en basant l'approche par scénario et la méthode des scénarios conditionnels (CS). L'étude numérique montre que l'approche CS est plus performant que la méthode SAA en termes de la qualité de solution et du temps de calcule. Une analyse de sensibilité est effectuée pour évaluer l'impact de certains paramètres sur le coût total du système.

Avant de terminer cette thèse, nous présentons certaines orientations des recherches selon chaque contribution.

Dans la gestion d'une chaine d'approvisionnement en biomasse, nous pouvons considérer la demande stochastique de biocarburants et certaines options au contrat d'approvisionnent telles que la qualité requise (teneur en eau, contamination...), les volumes de livraison minimaux, l'indemnisation contre la promesse de livraison en cas d'intempéries et de conditions de récolte, les réglementations pour les changements de prix futurs, la condition de paiement. Cette intégration permet de rendre le modèle plus réaliste

Autre possibilité est d'intégrer l'aspect environnemental et social dans la fonction objectif. Puis, un modèle multi-objectif pourrait être développé pour trouver le compromis entre les objectifs contradictoires : maximisation du profit / réduction des coûts totaux, minimisation des émissions de gaz à effet de serre et maximisation de la création d'emplois.

Il nous faut encore poursuivre l'étude sur la gestion des stocks périssables avec le délai positif. De plus, il est intéressant d'envisager d'autres incertitudes comme le prix, la qualité. Les sujets de recherche futurs pourraient être les suivants : le crédit et le mode de paiement, l'inflation, l'investissement, la promotion et la contrainte budgétaire, la satisfaction du client.

Nous pouvons également intégrer un des critères de risque ( la valeur à risque, la valeur à risque conditionnelle, l'écart absolu moyen) afin d'améliorer la robustesse de la solution obtenue et de rendre notre modèle plus réaliste.

# Annexe B

# Benders decomposition Principle

Benders decomposition technique is a popular method for solving certain large-scale linear optimization problems. Consider the following problem :

$$\begin{aligned} \min_{x,y} \quad & c^{\mathrm{T}} x + f^{\mathrm{T}} y \\ s.t \quad & \mathrm{A}x + \mathrm{D}y \geq b \\ & y \in \mathrm{Y}, x \geq 0 \end{aligned} \tag{B.1}$$

Suppose that the problem becomes easy to solve if the "complicating" variables y are fixed. The idea of Benders' decomposition is to partition the problem into two problems, one called the master problem which contains the complicating variables $y$, the second called the subproblem that contains the variables $x$. For a fixed value of y in the original problem B.1, the resulting model to solve is given by :

$$\begin{aligned} \min_{x,y} \quad & c^{\mathrm{T}} x \\ s.t \quad & \mathrm{A}x \geq b - \mathrm{D}\bar{y} \\ & x \geq 0 \end{aligned} \tag{B.2}$$

The problem B.1 can be reformulated as :

$$\min_{y \in \mathrm{Y}} \left[ f^{\mathrm{T}} y + \min_{x \geq 0} \{ c^{\mathrm{T}} x \mid \mathrm{A}x \geq b - \mathrm{D}y \} \right] \tag{B.3}$$

Let u be the dual variable associated with $\mathrm{A}x \geq b - \mathrm{D}\bar{y}$. The dual of the subproblem B.2

is given by :

$$\begin{aligned} \max_{u} \quad & (b - \mathrm{D}\bar{y})^{\mathrm{T}} u \\ s.t \quad & \mathrm{A}^{\mathrm{T}} u \leq c \\ & u \geq 0 \end{aligned} \tag{B.4}$$

The feasible region of the dual problem B.4 does not depend on $y$. Assuming that this feasible region in the problem B.4 is non-empty, we can find all extreme points $u_j$ and directions $u_l$ based on the fixed $\bar{y}$ that maximizes the value of objective function $(b - \mathrm{D}\bar{y})^{\mathrm{T}} u_j$. Let $z = \max\left\{(b - \mathrm{D}\bar{y})^{\mathrm{T}} u_j, j = 1, \ldots, \mathrm{Q}\right\}$. The dual subproblem B.4 can be reformulated as follows :

$$\begin{aligned} \min_{y} \quad & z \\ s.t \quad & z \geq (b - \mathrm{D}\bar{y})^{\mathrm{T}} u_j, \qquad j = 1, \ldots, \mathrm{Q} \\ & 0 \geq (b - \mathrm{D}\bar{y})^{\mathrm{T}} u_l, \qquad l = 1, \ldots, \mathrm{L} \end{aligned} \tag{B.5}$$

The reformulation of the original problem B.1 in term of z and y-variables can be obtained by replaced $c^{\mathrm{T}} x$ with $z$. Thus the master problem which has the form :

$$(\mathrm{MP}) \min \quad f^{\mathrm{T}} y + z$$

$$s.t \quad z \geq (b - \mathrm{D}y)^{\mathrm{T}} u_j, \qquad j = 1, \ldots, \mathrm{Q} \tag{B.6}$$

$$0 \geq (b - \mathrm{D}y)^{\mathrm{T}} u_l, \qquad l = 1, \ldots, \mathrm{L} \tag{B.7}$$

$$y \in \mathrm{Y}, z \quad \text{unrestricted} \tag{B.8}$$

The Benders Decomposition algorithm is given as follows :

**Algorithm B.1:** : Bender decomposition algorithm

**Initialization** : Set $\bar{y}$ initial feasible solution, $\mathrm{LB} = -\infty$, $\mathrm{UB} = \infty$.
**while** $\mathrm{UB} - \mathrm{LB}) / \mathrm{UB} \geq \epsilon$ **do**
  Solve subproblem with fixed variable $\bar{y}$, $\max_u \left\{(b - \mathrm{D}\bar{y})^{\mathrm{T}} u \mid \mathrm{A}^{\mathrm{T}} u \leq c, u \geq 0\right\}$
  **if** *Unbounded* **then**
    Get unbounded ray $\bar{u}$
    Add cut $(b - \mathrm{D}y)^{\mathrm{T}} \bar{u} \leq 0$ to master problem (MP)
  **else**
    Get extreme point $\bar{u}$ and add cut $z \geq (b - \mathrm{D}y)^{\mathrm{T}} \bar{u}$ to master problem (MP)
    Update upper bound, $\mathrm{UB} = \min\left\{\mathrm{UB}, f^{\mathrm{T}} \bar{y} + (b - \mathrm{D}\bar{y}])\, \bar{u}\right\}$
  Solve master problem (MP), $\min_{y,z} \left\{f^{\mathrm{T}} y + z \mid \text{cuts}, y \in \mathrm{Y}, z \quad \text{unrestricted}\right\}$ and get $\bar{y}$ and a lower bound $\mathrm{LB} = f^{\mathrm{T}} \bar{y} + \bar{z}$

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## Doctorat : Optimisation et Sûreté des Systèmes

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## Étude sur la planification de la chaîne logistique en biomasse et la gestion des stocks de produits périssables


Au cours des dernières décennies, l'utilisation de sources d'énergie renouvelables a réduit les effets du réchauffement climatique. La biomasse est une ressource énergétique prometteuse pour la production de la chaleur, de l'électricité et des biocarburants. Une conception efficace de la chaîne d'approvisionnement et une gestion optimale des stocks permettent à réduire significativement les prix des biocarburants et à améliorer la compétitivité contre des combustibles fossiles.

Cette thèse aborde deux problèmes cruciaux : l'optimisation d'une chaîne logistique et la gestion des stocks périssables (biomasse). Le premier problème est consacré à la sélection des fournisseurs et la planification des opérations dans une chaîne logistique en biomasse. Ce problème est formulé sous forme de modèle déterministe (MILP) et de modèle stochastique à deux étapes. Le premier modèle est résolu de manière optimale par le solveur GUROBI. Une méthode de décomposition « *L-shaped* » est développée pour traiter le deuxième modèle.

Le second problème consiste à la gestion des stocks d'un produit périssable sous incertitudes d'approvisionnement et de demande. Après avoir démontré que sa politique de gestion des stocks est une politique "*order-up-to level*", un algorithme basé sur la relaxation lagrangienne est développé pour trouver rapidement une solution quasi-optimale du problème. Ensuite, ce problème est étendu pour un produit à durée de vie fixe. La méthode « Conditional scenario » est développée pour résoudre approximativement ce problème.



Mots clés : recherche opérationnelle – optimisation combinatoire – logistique (organisation) – gestion des stocks – programmation stochastique – incertitude – biomasse.


## Study on Biomass Supply Chain Planning and Inventory Control of Perishable Products


In the last decades, the use of renewable energy sources reduced the effect of global warming. Biomass is a promising energy resource to produce heat, electricity, and biofuel. An efficient supply chain design and optimal inventory management could contribute to reduce biofuel prices significantly and improve competitiveness with fossil fuel.

This thesis addresses two crucial problems: biomass supply chain optimization and inventory control for a perishable product (biomass). The former is devoted to an issue of supplier selection and operation planning in biomass supply chains under uncertainty. This problem is formulated as a deterministic (MILP) model and a two-stage stochastic programming model. The deterministic model is solved by using a MIP solver GUROBI. An enhanced L-shaped decomposition method is developed to find an optimal solution for the stochastic model.

The second deals with a stochastic inventory problem of a perishable product with uncertainty in both supply and demand. After demonstrating its optimal inventory policy is an order-to-level policy, a Lagrangian relaxation based algorithm is developed to quickly find a near-optimal solution of the problem. The stochastic inventory problem is, then, extended to a product with a fixed lifetime. The Conditional scenario method is developed to solve approximately this problem.



Keywords: operations research – combinatorial optimization – business logistics – inventory control – stochastic programming – uncertainty – biomass.