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# Experimental validation of an information theoretic model of cognitive effort

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# Empirical Validation of an Information Theoretic Model of Cognitive Effort

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the Requirements for the Degree of

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University of Bordeaux

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# Validation Expérimentale d'un Modèle de l'Effort Cognitif Basé sur la Théorie de l'Information

Lam Sze Ying

Thèse présentée pour obtenir le grade de

Doctorat en Neurosciences

à l'Ecole Doctorale Sciences de la Vie et de la Santé  
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# Abstract

The sensation of effort, from an evolutionary point of view, could be understood as a mechanism for signalling the expenditure of scarce resources and which allows their efficient allocation. Understanding the decision making processes that are involved in effort allocation is crucial if one is to gain insight into human behaviour.

One type of effort that is observed and reported in humans, and is the central subject of this thesis, is cognitive effort. Although there is still no general consensus over the true nature of the resources that cognitive effort was developed to safeguard, its aversiveness and involvement in decision-making are widely agreed upon. The principle of least action, entailing the minimisation of effort, provides a rational account for seemingly sub-optimal behaviours.

Nevertheless, there are major obstacles to overcome in studying cognitive effort, many of which are associated with complications and biases associated with the measurement of subjective experiences. In response to these limitations, some recent work has focused instead on the influence that these subjective experiences have over observable, free choices of engagement. Notably, a neuroeconomic approach was employed to establish preference functions that express cognitive effort costs and task rewards in a common currency.

Following this line of research, an information theoretic model of cognitive effort is proposed in this thesis work. The motivation for such a model is three-fold.

Firstly, the mathematical framework of information theory provides a natural common currency, that is information, for quantifying task difficulty, engagement and performance. This could provide a more direct interpretation of the relationship between task demand, effort expenditure and associated gains.

Secondly, information theoretic measures derived from first principles set bounds on the information rate associated with automatic and controlled behaviours.

Lastly, information theory provides the common framework in which the interpretation of cognitive effort can be linked to well-established theories regarding computational efficiency in the brain such as efficient coding and/or predictive coding theorems.

In this thesis work, a series of experiments were designed to validate the proposed model of cognitive effort. The main task used in these experiments is a continuous visual-motor tracking task with joystick control. In the first study, information theoretic measures representing information rate of the feed-back (controlled) and feed-forward (automatic) processing of the signal were derived from first principles and were validated through simulated tracking data from a linear quadratic regulator (LQR) model. These

measures were subsequently applied to real tracking data to gain insight of their engagement in the task in terms of real-time information processing rate.

The second study aims at investigating and comparing the effect that different task attributes, including signal speed, predictability and joystick delay have on feed-back and feed-forward information rate, as well as on performance.

The third and fourth studies were dual-task experiments designed to investigate cross-task interactions in information rate and to infer global limits in the brain in terms of computational resources.

Lastly, a model is built by modifying an intermittent controller to include an information bottleneck objective to provide a normative account of the cost/value trade-off in human tracking performance. This model is then applied to behavioral data to study the principles of allocation of information rate and the optimality of human motor control.

**Keywords:** cognitive effort, information theory, visuo-motor tracking, Dual-task

# Résumé

La sensation d'effort, d'un point de vue évolutif, peut être comprise comme un mécanisme qui permet de signaler la dépense de ressources rares et qui rend possible leur allocation efficace. Il est essentiel de comprendre les processus décisionnels qui interviennent dans l'allocation de l'effort si l'on veut mieux comprendre le comportement humain.

Un type d'effort observé chez les humains, et qui est le sujet central de cette thèse, est l'effort cognitif. Bien qu'il n'y ait toujours pas de consensus général sur la véritable nature des ressources que l'effort cognitif serait chargé de protéger, son aversivité et son implication dans la prise de décision sont largement reconnues. Le principe de moindre action, qui implique la minimisation de l'effort, fournit une explication rationnelle de comportements apparemment sous-optimaux.

Néanmoins, il existe des obstacles majeurs à surmonter dans l'étude de l'effort cognitif, dont beaucoup sont liés aux complications et aux biais associés à la mesure des expériences subjectives. En réponse à ces limitations, certains travaux récents se sont plutôt concentrés sur l'influence que ces expériences subjectives ont sur les choix d'engagement observables. Une approche neuroéconomique a notamment été utilisée pour établir des fonctions de préférence qui expriment les coûts de l'effort cognitif et les récompenses de la tâche dans une monnaie commune.

En suivant cette ligne de recherche, un modèle théorique de l'information de l'effort cognitif est proposé dans ce travail de thèse. La motivation d'un tel modèle est triple.

Premièrement, le cadre mathématique de la théorie de l'information fournit une monnaie commune naturelle, à savoir l'information, pour quantifier la difficulté de la tâche, l'engagement et la performance. Cela permet une interprétation plus directe de la relation entre la demande de la tâche, la dépense d'effort et les gains associés.

Deuxièmement, les mesures théoriques de l'information dérivées de principes premiers fixent des limites au taux d'information associé aux comportements automatiques et contrôlés. Enfin, la théorie de l'information fournit le cadre commun dans lequel l'interprétation de l'effort cognitif peut être liée à des théories bien établies concernant l'efficacité computationnelle dans le cerveau, comme les théorèmes de codage efficace et/ou de codage prédictif.

Dans ce travail de thèse, une série d'expériences a été conçue pour valider le modèle proposé de l'effort cognitif. La tâche principale utilisée dans ces expériences est une tâche de suivi visuo-moteur continu avec contrôle par joystick. Dans la première étude, des mesures théoriques de l'information représentant le taux d'information du traitement



feed-back (contrôlé) et feed-forward (automatique) du signal ont été dérivées à partir des premiers principes et ont été validées par des données de suivi simulées à partir d'un modèle de régulateur linéaire quadratique (LQR). Ces mesures ont ensuite été appliquées à des données de suivi réelles afin de mieux comprendre leur engagement dans la tâche en termes de taux de traitement de l'information en temps réel.

La deuxième étude vise à examiner et à comparer l'effet de différents attributs de la tâche, notamment la vitesse du signal, la prévisibilité et le retard du joystick, sur le taux d'information en feed-back et en feed-forward, ainsi que sur la performance.

Les troisième et quatrième études sont des expériences à double tâche conçues pour étudier les interactions entre les tâches sur le taux d'information et pour déduire les limites globales du cerveau en termes de ressources computationnelles.

Enfin, un modèle est construit en modifiant un contrôleur intermittent pour inclure un objectif d'information bottleneck afin de fournir un compte rendu normatif du compromis coût/valeur dans les performances de suivi humain. Ce modèle est ensuite appliqué à des données comportementales pour étudier les principes d'allocation du taux d'information et l'optimalité du contrôle moteur humain.

**Mots clés : effort cognitif, théorie de l'information, suivi visuo-moteur, double tâche**

# Résumé substantiel

Cette thèse a pour but de comprendre la perception de l'effort cognitif chez l'homme. L'étude de phénomènes subjectifs tels que la sensation d'effort peut parfois devenir illusoire si la question de recherche est mal définie. Tout comme l'expérience subjective commune de la couleur rouge peut être attribuée à la lumière de longueurs d'onde comprises entre 620 et 750 nm qui frappe la rétine, nous cherchons à dévoiler le mécanisme qui donne lieu à la sensation d'effort cognitif. En passant en revue les concepts les plus étroitement liés à l'effort cognitif, on s'aperçoit que la fonction de contrôle cognitif est largement appliquée pour comprendre le mécanisme informatique et biologique qui soutient l'effort cognitif. Cependant, ce cadre ne permet pas une quantification précise de la quantité d'effort ou de contrôle dans un large éventail de processus cognitifs. Nous nous tournons donc vers la formulation du coût de l'information des processus cognitifs dans le cadre de la théorie de l'information. Cette formulation suit la formalisation de la cognition en termes de proportionnalité bornée et postule que le cerveau est une machine d'inférence qui met activement à jour ses modèles internes avec les entrées sensorielles en fonction des croyances antérieures. Ce cadre donne un coût de calcul associé à chaque mise à jour qui constitue la base du coût de l'information d'un processus cognitif. Une autre mesure d'information pertinente d'un processus cognitif est identifiée comme l'information prédictive. Complétant le coût de l'information, elle devrait donner le transfert total d'information d'un processus cognitif. Il est proposé que l'information FB soit liée au traitement de l'information en temps réel, qui nécessite beaucoup de ressources, et que l'information FF représente le traitement automatique de l'information, qui est considérablement plus efficace. Par conséquent, il est également proposé que l'information FB, mais pas l'information FF, soit étroitement liée à la perception de l'effort cognitif.

Les mesures d'information proposées sont dérivées et testées empiriquement dans une tâche de suivi visuo-moteur. Comme on l'a supposé, la mesure FB augmente considérablement à mesure que les signaux de poursuite deviennent plus complexes, tandis que l'information FF augmente avec la prévisibilité du signal. L'effet dominant de la prévisibilité du signal sur les informations FB et FF est confirmé dans une deuxième expérience en isolant de l'effet de la vitesse du signal. Une autre manipulation expérimentale est conçue pour influencer spécifiquement la qualité de l'information prédictive dans la performance de suivi en prolongeant les délais visuo-moteurs. En réponse aux délais prolongés, les résultats montrent une baisse significative de l'information FF. Dans l'ensemble, nos résultats montrent de manière convaincante que lorsque les contingences statistiques

des signaux ou la qualité de la prédiction changent, les réponses changeantes correspondantes du traitement de l'information concernant le signal sont reflétées par les informations FB et FF.

Certaines des théories les plus courantes sur l'effort cognitif font état d'une ressource cognitive limitée, de nature non spécifiée, qui est si rare et si précieuse que le phénomène de l'effort subjectif s'est développé au cours de l'évolution pour faciliter l'allocation de cette ressource. Afin d'étudier dans quelle mesure notre vitesse de traitement de l'information est limitée, nous utilisons un paradigme à double tâche et analysons les effets inter-tâches. Dans la première expérience consistant à combiner la tâche de suivi de la VM avec une tâche de N-back auditif, les performances des deux tâches chutent lorsque la difficulté de l'autre tâche augmente. Ceci est cohérent avec l'effet d'interférence inter-tâches observé dans les études de contrôle cognitif. L'analyse du coût de l'information de la tâche de suivi VM montre également une baisse considérable lorsque la tâche N-back concomitante est plus exigeante. Cela pourrait laisser penser que la tâche N-back a augmenté la tension dans les ressources cognitives et a effectivement diminué la capacité de traitement de l'information de la tâche de suivi de la VM, entraînant une baisse de la mesure FB. Cependant, une deuxième expérience où la tâche N-back est remplacée par la tâche de Hick fournit un regard plus détaillé sur l'interaction du taux de traitement de l'information des deux tâches impliquées. La tâche de Hick et la tâche de suivi VM montrent toutes deux une baisse de performance lorsque l'autre tâche est plus exigeante, ce qui démontre une fois de plus une interférence inter-tâches dans la performance. Cependant, l'analyse du taux d'information par seconde dans la tâche de Hick s'avère constante, même lorsqu'elle est confrontée à une demande accrue dans la tâche simultanée de suivi de la VM. Le taux d'information par seconde de la tâche de suivi des VM s'avère également exempt de l'influence de la tâche de Hick. Ce taux d'information (bits/s) est basé sur une estimation du temps d'engagement de la tâche VM, en supposant que les sujets ne s'engagent pas dans la tâche de suivi lorsqu'ils répondent à la tâche de Hick”

$$\textit{Temps d'engagement de la tâche} = \textit{Durée de l'essai} - \textit{TR total de Hick}$$

Il s'agit d'un résultat intéressant car il montre que, bien qu'il y ait une interférence entre les tâches sur la performance, les taux d'information par seconde pour les deux tâches restent constants. La conception de la première expérience à double tâche avec la tâche N-back ne permet pas la même analyse; mais même si c'était le cas, ce temps d'engagement de la tâche VM est toujours au mieux une limite supérieure du temps d'engagement réel, rendant le taux d'information correspondant par seconde une limite inférieure du taux.

l'exception de la première expérience, toutes les expériences ont inclus des mesures de la demande ou de l'effort perçu pour la tâche. En général, les demandes mentales autodéclarées, l'évitement des essais et la dilatation de la pupille corrigée par rapport à la ligne de base pendant les essais sont corrélés positivement avec au moins certaines conditions de tâches qui impliquent également un coût d'information plus élevé, telles qu'une complexité accrue du signal, une vitesse plus faible, une tâche N-back plus difficile

et une tâche de Hick plus difficile. Ces résultats soutiennent principalement l'hypothèse selon laquelle le coût de l'information est étroitement lié à l'effort cognitif. Cependant, une exception à ces résultats est la condition de retard moteur ajouté. Les informations FB et FF chutent en réponse à des délais plus longs, mais ils sont également signalés comme étant plus exigeants mentalement, plus susceptibles d'être évités à l'avenir et associés à une dilatation accrue des pupilles, signe d'un effort accru. Si l'on considère que les taux d'information par seconde peuvent être constants mais que la perception de la demande ou de l'effort ne l'est pas, cela suggère que la perception de l'effort est plus susceptible d'être influencée par le montant total du coût de l'information dans un essai plutôt que par la mesure du taux par seconde. Les simulations du modèle IC semblent également soutenir cette hypothèse puisqu'il est montré que si le taux d'encodage est inchangé (en raison de valeurs  $\beta$  et VMD constantes, par exemple), l'information FB totale d'un essai peut encore être modifiée par l'engagement plus fréquent du contrôle.

Les résultats de l'expérience en double tâche révèlent une pièce potentiellement manquante dans la compréhension du lien entre le coût de l'information et l'effort cognitif, à savoir le temps d'engagement réel. Pour combler cette lacune, nous explorons la possibilité de modéliser la performance de suivi avec un contrôleur intermittent et essayons de déduire l'engagement dans la tâche par la mesure de la fréquence de contrôle fournie par le modèle. En comparant les données de simulation avec les phénomènes observés lors des expériences, on constate que la fréquence de contrôle explique bien la demande de tâche perdue et l'effort perdu. Cette interprétation est logique si l'on considère que l'objectif final de la perception de l'effort cognitif est de motiver un changement de comportement pour parvenir à une meilleure allocation des ressources. Si le taux d'information n'est pas quelque chose que l'on peut modifier par la volonté, alors il n'y a pas de raison pratique pour que l'effort cognitif le signale. En revanche, la fréquence de contrôle ou le temps d'engagement est quelque chose qui peut être modifié volontairement, et serait donc une cause plus probable de la sensation d'effort. Pour résumer les théories et les expériences présentées, la prévisibilité semble être l'élément récurrent qui réduit l'effort. Non seulement les signaux prévisibles sont systématiquement jugés moins exigeants ou moins pénibles, mais le modèle IC montre également une baisse significative de la fréquence de contrôle si les prédictions utilisées sont fiables. D'un point de vue théorique, la qualité de la prédiction est associée à l'optimalité des priors/représentations utilisés dans un processus cognitif. L'utilisation de priors sous-optimaux entraîne un coût d'information supplémentaire. Le lien entre ce coût et l'effort perdu réside dans les ressources métaboliques ou informatiques limitées que le traitement de l'information est censé puiser. Ces hypothèses, lorsqu'elles sont complétées par la notion d'un taux constant de gain d'information, comme le suggèrent nos résultats expérimentaux, pointent vers une ressource plus directement quantifiable : le temps. L'étude actuelle se concentre principalement sur les tâches de suivi visuomoteur, ce qui fournit une grande quantité de données dans un contexte de tâche écologique. Cependant, comme on peut le constater, l'estimation du temps d'engagement pourrait devoir s'appuyer sur une modélisation mathématique, si tant est que cela soit possible. Les

mesures FB et FF présentées ici pourraient potentiellement être dérivées pour d'autres tâches discrètes ou continues, tant que la prévisibilité de la tâche peut être bien manipulée et estimée. L'application de ces mesures devrait être explorée dans des études futures.

Un réseau VIB profond est utilisé dans cette étude pour générer des simulations et des prédictions pour le modèle IC. Il ne s'agit que d'un modèle préliminaire et de nombreuses explorations sont encore nécessaires. Pour commencer, nous pourrions élargir la dimension d'entrée pour inclure un historique plus long de signaux et entraîner les modèles avec différentes valeurs  $\beta$ . Ensuite, nous pourrions analyser les variables latentes du modèle pour essayer de visualiser quelle serait la manière optimale de compresser les signaux entrants compte tenu des contraintes de ressources, par exemple le sous-échantillonnage ou l'élimination de l'historique plus ancien. Bien que cela ne représente évidemment pas nécessairement la façon dont les humains forment réellement la représentation, cela pourrait fournir certaines hypothèses qui pourraient être testées par des expériences. Une autre amélioration du modèle serait d'incorporer un modèle d'espace d'état dans le décodeur du réseau VIB profond, de sorte que les représentations codées soient décodées en paramètres d'un modèle d'espace d'état, par exemple un modèle autorégressif. Cela permettrait au réseau VIB profond de générer des prédictions de signaux futurs de longueur arbitraire. Comme pour le contrôleur intermittent, les travaux futurs pourraient également essayer de mettre en œuvre des caractéristiques de double tâche dans la tâche en imposant des contraintes supplémentaires sur les paramètres de la période réfractaire psychologique ou même en changeant les valeurs  $\beta$  dynamiquement tout au long du procès en fonction des demandes de double tâche en temps réel. De plus, le modèle actuel omet intentionnellement le système neuromusculaire (SNM) dans un souci de simplicité. Mais les recherches futures devraient également explorer cette partie du modèle et étudier sa contribution en termes d'information, à la fois en coût d'information et en information prédictive, à la performance globale de suivi. À la lumière de ce qui précède, les expériences portant sur le sens du suivi semblent appropriées pour dissocier la contribution des composantes cognitives, par exemple l'encodage des entrées et la réalisation de prédictions, de celle des composantes motrices. Les recherches futures pourraient viser à étudier un modèle plus élaboré de contrôle intermittent. Les récompenses externes et le retour d'information explicite sur les performances ont tous deux un rôle central à jouer dans les problèmes de prise de décision chez l'homme. Les études futures devraient viser à utiliser les mesures FB et FF comme outils pour révéler les changements dans les processus d'information sous l'influence des récompenses et du feedback. L'incorporation de récompenses externes redéfinirait aussi essentiellement l'objectif d'optimisation du modèle de CI, dont le comportement peut alors être analysé en fonction de ces variables.

# Declaration

I declare that this thesis has been composed solely by myself and that it has not been submitted, in whole or in part, in any previous application for a degree. Except where states otherwise by reference or acknowledgment, the work presented is entirely my own. Parts of this work have been published under the title ‘Information rate in humans during visuomotor tracking’, in the journal *Entropy* (2021), 23(2):228.



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# Chapter 1

## Introduction

### 1.1 What is cognitive effort?

Cognitive effort, taken here in the sense of a subjective phenomenon, is associated with the engagement in many different cognitive activities. Leading theories of cognitive effort assume that subjective experience is developed to motivate adaptive behaviours in organisms (Damasio, 1999; Tooby et al., 2008). Central to the emergence of the sensation of cognitive effort is the survival need to guard precious resources that allow for information processing in the brain. The sensation of effort can therefore be interpreted as a signal to help facilitate optimal allocation and preservation of these resources. The nature of these resources is still hotly debated, although in recent years the ‘computational account’ (e.g. attention) is gaining more traction than the ‘metabolic account’ (e.g. glucose) (Gailliot and Baumeister, 2007).

Although cognitive effort is mostly found to be aversive, some might actively seek it. (Cacioppo and Petty, 1982). Using the Need for Cognition Scale, Cacioppo et al. (1996) found that an individual’s tendency to engage in and enjoy effortful cognitive activities is linked to higher academic achievements and better performance in a handful of other cognitive tasks. Research has also found that subjects perceive cognitive effort as a cost and take that into account when making decisions (Kool et al., 2010; Manohar et al., 2015; Westbrook et al., 2013). Last but not least, a maladaptive perception of cognitive effort was identified in some mental illnesses, such as depression (Hammar et al., 2011; Hartlage et al., 1993; Zakzanis et al., 1998) and schizophrenia (Fervaha et al., 2013; Gold et al., 2013, 2015).

Understanding cognitive effort could therefore potentially help us gain insight on both the normal and abnormal human decision-making mechanisms and to reveal maladaptive behaviour or biases. Given its importance in influencing decision-making at so many levels, there is great interest in identifying the factors that give rise to the perception of cognitive effort. However, its study, like that of other subjective experiences, can prove to be non-trivial. One of the challenges is the close yet potentially misleading correspondance between cognitive effort with many other attributes, such as task difficulty, motivation,

attention, performance, etc. Although these might affect or even interact with cognitive effort, they are not equivalent. In their review paper, Westbrook and Braver (2015) have clearly laid out the relationships between cognitive effort and these attributes. For example, task difficulty was proposed by some to be a determinant of effort, however their uncoupling can be shown in ‘data-limited’ tasks (Norman and Bobrow, 1975) in which performance is actually constrained by data quality instead of cognitive resources. In other words, the perception of difficulty would represent the degree of achievability taking into account all limiting factors, both controllable and uncontrollable. Therefore, one could rate a task - such as solving the Poincaré conjecture - to be incredibly difficult but not at all effortful to engage in, because there is little a person could do (e.g. assuming this person is me).

## 1.2 Cognitive control

Of all the concepts that link to cognitive effort, cognitive control might be the hardest one to differentiate. It is defined as the process that influences behaviours by incorporating goals or plans, distinguishing controlled behaviours from automatic ones. Cognitive control is sometimes referred to as executive control and it is involved in many cognitive functions, such as inhibition and planning. Its engagement is also instrumental in allocating attention and working memory, making it almost omnipresent in all types of cognitive activities we engage in every day. Although it is argued that they are not redundant concepts (Westbrook and Braver, 2015; Kurzban et al., 2013), there is an apparent conceptual overlap in theories regarding cognitive control and effort. Not only are tasks that involve cognitive control characteristically rated as more effortful by subjects (Dixon and Christoff, 2012; Kool et al., 2010; Westbrook et al., 2013), many theories also point to cognitive control as a source of constraint to human cognitive capability, hypothesising that it is tied to some underlying limited resources, similar to the connotation given to cognitive effort (Tyler et al., 1979; Longo and Barrett, 2010; Westbrook and Braver, 2015; Kurzban et al., 2013).

Indeed, if one is to accept the theory that cognitive effort is a sensation that is developed to guard some limited sources needed for cognitive activities, understanding the kinds of constraints that our cognitive system is subject to might be a good starting point. In this regard, cognitive control is found to be instrumental in the study of constraints on multitasking capability.

Studies on these constraints usually focus on the observed drop in performance of individual tasks when they are executed with other concurrent tasks. However, it is important to note that not all tasks are constrained in the same way. For instance, one could easily walk, eat, listen to music and pay attention to their surroundings all at the same time without any difficulty or feeling any effort. While this certainly demonstrates our extraordinary multitasking capability, these are not the tasks that we will be focussing on. The discussion on multitasking constraints here is specifically aimed at tasks that necessarily suffer from performance drop when performed simultaneously. By definition, these are called control-dependent tasks and it is implied that their execution requires cognitive control (Shiffrin and Schneider, 1977). This is an important distinction because most theories on multitasking capability point to cognitive control as the source of constraints.

As the most restrictive set of theories regarding multitasking constraints, structural bottleneck theories postulate that there is a centralised, limited resource that all tasks require for their performance but cannot be used by more than one task at a time (Pashler, 1994; Broadbent, 1957; De Jong, 1993; Welford, 1967; Keele and Boies, 1973). Unitary resource theories relax the no-sharing assumption and allow this resource to be divided between tasks, but they hold on to the assumption of a central resource (Kahneman, 1973; Norman and Bobrow, 1975; Tombu and Jollicœur, 2003). Multiple-resource theories (Göthe et al., 2016; Koch, 2009), on the other hand, propose that, instead of a centralised,

limited resource, there are specialised or local resources accommodating the demands of different tasks. There are still variations regarding whether or not these local resources can be shared by more than one task at a time. But all in all, multiple-resource theories provide a less restrictive framework for describing multitasking phenomena and are comparatively more successful in explaining experimental multitasking data, especially regarding the increased level of interference as a function of tasks' overlap (Göthe et al., 2016; Koch, 2009). Figure 1.1 from Musslick and Cohen (2021)'s paper provides a general overview of these theories.

Limited resources appear to be a common theme in theorising cognitive control and constraints on multitasking performance. Associating this with the proposal that the sensation of cognitive effort is developed to signal depletion of precious resources, one could postulate that the resources needed for cognitive control are guarded by sensation of cognitive effort, thus directly coupling the two concepts. Without any assumption of the underlying mechanism involved, some neuroeconomic studies implicate the role of cognitive effort in decision-making by drawing a direct correspondence between the execution of cognitive control and the perception of effort (Kool and Botvinick, 2014; Kool et al., 2010), inferring the latter from the observed aversiveness towards control-dependent tasks.

Diverging from the line of theories that ascribe constraints in multitasking capability to limited resources for cognitive control, Musslick and Cohen (2021) propose a more definite view that links these constraints to the sharing of representations between tasks. When tasks that share representations are being executed simultaneously, this will cause interference and therefore lead to impairment in performance. In this framework, cognitive control is postulated to be a response preventing simultaneous execution and minimising interference. From the outlook, this theory still relies on some shared resources (whatever form of biological or computational implementation a representation takes in the brain) and control, but what sets it apart is the motivation. The theories that are fundamentally built on limited resources have failed to provide any principled explanations for why such limitations should even exist in the first place, or what the biological advantage the mechanism of relying on a central resource is. The development of the theory of shared representation, on the other hand, is motivated by a trade-off between cognitive flexibility and stability in the cognitive system. Cognitive flexibility refers to the capacity to adapt task goals in face of a changing environment and cognitive stability is characterised by one's resistance to distraction and maintenance of a task goal. While studies in semantic cognition and machine learning both suggest that task flexibility is supported by shared representations (Rogers et al., 2004; Hinton, 1990; Saxe et al., 2019; Frankland and Greene, 2020), cognitive stability is made possible by separating and dedicating representations to different tasks. The emergence of shared representations is therefore interpreted as the cognitive system's bias towards flexibility. The bias towards cognitive flexibility is linked to the bias towards high learning efficacy as well. In behavioural experiments, it is very common that researchers would separate the learning stage of a task from its

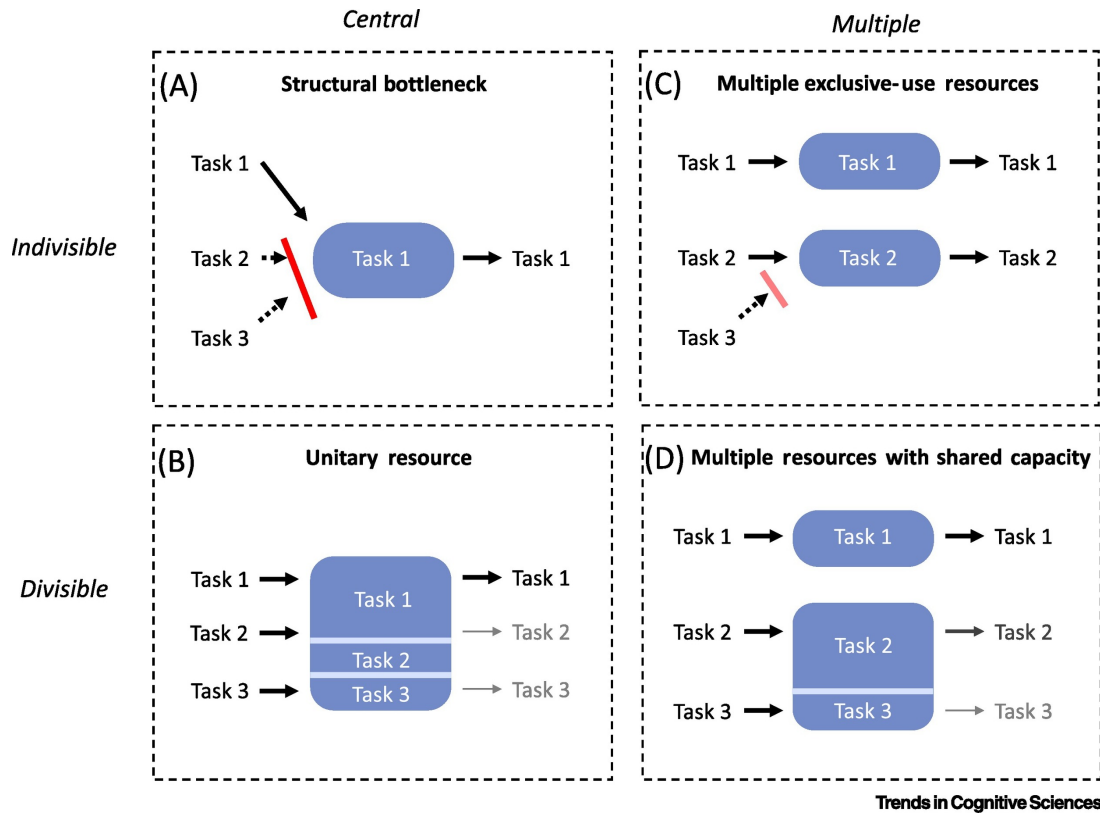


Figure 1.1: **Classification of resources in theories of human multitasking.** (Muslick and Cohen, 2021): **Figure 1.**Theories differ according to whether they assume that tasks require the same, central resource, or local task-specific resources (central versus multiple) and the way in which those resources can be allocated (indivisible or divisible). (A) Structural bottleneck. A central resource constitutes a bottleneck in that it is required for execution of all tasks and can operate only one of those at a time; if the resource is engaged by one task, it causes a delay in the processing of others. (B) Unitary resource. Tasks rely on a unitary centralised resource, but it can be allocated to multiple tasks at the same time; task interference occurs if the demands of those tasks exceed the available capacity of the unitary resource. (C) Multiple exclusive-use resources. Tasks rely on local, task-specific resources, each of which can only be used for one task at a time; interference arises if two tasks make simultaneous use of the same resource. (D) Multiple resources with shared capacity. Local, task-specific resources can be shared; interference arises if the capacity of a local resource is exceeded by the number of tasks using it at the same time.

testing stage, in which performance is supposed to have reached its peak. This practice highlights how learning and performing are treated as different cognitive processes, potentially motivated by different goals. Learning a new task requires rapid adaptation to new task rules and context. It often requires subjects to flexibly reassign values to different information, constantly updating its goal until it aligns with the context. Once a task



is learned, however, a different type of response is expected of the subjects to maximise the outcome. To be able to perform well in a given context, subjects should fixate on the task goal and effectively ignore all irrelevant information and any distraction from the goal. Learning and performing therefore demonstrate the need for cognitive flexibility and stability respectively. The bias towards cognitive flexibility might therefore imply a bias in learning as well.

The hypothesis of shared representations frees the interpretation of multitasking constraints from a direct limitation of resources and recasts it into a more dynamic problem involving a trade-off between cognitive flexibility and cognitive stability. Considering the role of cognitive effort in facilitating limiting resources, which under this framework are the shared representations, cognitive effort would be postulated to signal the extent to which representations are unspecific or sharable, thus their potential to cause interference. Interestingly, this will once again mean coupling effort and control since interference is mitigated by cognitive control.

By revealing the constraints our cognitive system is subjected to, the need for a mechanism to gate precious resources is well motivated. One potential mechanism for the perception of cognitive effort is thus in which effort is directly related to the exertion of cognitive control. However, while this may facilitate the allocation of some cognitive resources, it is unclear how this can be generalised beyond the interpretation of cognitive control. How can cognitive effort be interpreted in a wider framework where it signals generally the engagement in resource-intensive cognitive processes? Following the publication of ‘A Mathematical Theory of Communication’ by Shannon (1948), neuroscience, along with many other scientific fields, has found new perspectives in old problems. Inspired by Shannon’s information theory, Attneave (1954) and Barlow et al. (1961) proposed the efficient coding hypothesis to formalise a mechanism in which neurons are optimised to maximise information transfer in the brain. Leveraging the information theoretic framework to attempt to make sense of brain processes provides tremendous potential for generalising theories across different cognitive processes.

## 1.3 Efficient coding theorem

### 1.3.1 Information theory primer

This section will begin with a primer on information theory, where all the related terms and concepts will be formalised.

Information theory is developed as a mathematical framework to quantify communication of information (Cover and Thomas, 2006). In this framework, information is measured with entropy, which is a measure of uncertainty about a random variable (RV). For simplicity, all RVs here will be assumed to be discrete. Let  $X$  be a discrete RV, with probability mass function  $p(x)$ . Its entropy is defined as:

$$H(X) = - \sum_x p(x) \log p(x) \quad (1.1)$$

which represents the amount of ‘surprise’ contained on average by the RV. The higher the entropy, the higher the surprise, the more the information.

The same concept can be extended to cover the definitions of joint and conditional entropy:

$$H(X, Y) = - \sum_x \sum_y p(x, y) \log p(x, y) \quad (1.2)$$

$$H(Y|X) = - \sum_x p(x) H(Y|X = x) \quad (1.3)$$

$$= - \sum_x p(x) \sum_y p(y|x) \log p(y|x) \quad (1.4)$$

$$= - \sum_x \sum_y p(x, y) \log p(y|x) \quad (1.5)$$

$$= -\mathbb{E} \log p(Y|X) \quad (1.6)$$

When considering two different distributions, e.g.  $p(x)$  and  $q(x)$ , a relevant measure is relative entropy  $DL(p||q)$ , which represents the distance between them. It is interpreted as the inefficiency of assuming the distribution of a RV is  $q(x)$  while it really is  $p(x)$ .

$$DL(p||q) = \sum_x p(x) \log \frac{p(x)}{q(x)} \quad (1.7)$$

This will give rise to one of the most used information theoretic measures, the mutual information, which quantifies the amount of information one RV, e.g.  $X$ , contains about another, e.g.  $Y$ . This can be expressed as the relative entropy  $DL(p(x, y)||p(x)p(y))$ . When RVs are independent of each other, their joint is equal to the product of their

marginals:  $p(x, y) = p(x)p(y)$ . In this case,  $DL(p(x, y)||p(x)p(y))$  is zero and that shows that independent RVs do not share information. However, if these RVs are not independent, the equality in this equation does not hold and the inefficiency of representing the joint probability assuming they are independent is the amount of information they share, the mutual information:

$$I(X; Y) = \sum_x \sum_y p(x, y) \log \frac{p(x, y)}{p(x)p(y)} \quad (1.8)$$

$$= DL(p(x, y)||p(x)p(y)) \quad (1.9)$$

Mutual information can also be interpreted as the reduction in uncertainty about one RV given the knowledge of the other:

$$I(X; Y) = H(Y) - H(Y|X) \quad (1.10)$$

$$= H(X) - H(X|Y) \quad (1.11)$$

In information theory, information transmission is studied through channels, which are defined as a mapping of inputs  $X$  to an output  $Y$ , and is represented by a probability transition matrix  $p(y|x)$ . Channel capacity is a concept that defines the upper limit of how much information can go through the channel and is defined as:

$$C = \max_{p(x)} I(X; Y) \quad (1.12)$$

where the maximum is taken over all possible  $p(x)$ .

### 1.3.2 Efficient coding

Considering the brain as an information processing machine, cognitive processes can be formalised as information channels that encode inputs, and produce outputs. The efficient coding hypothesis, simply put, is a theory that concerns how neurons are organised in the brain to facilitate efficient information transfer. The basis of the hypothesis was built trying to understand the processing of sensory stimuli from the environment and is motivated by two main observations. The first is the observation that sensory stimuli are highly redundant given their spatial and temporal interdependence in the environment. The second observation concerns the apparent physical constraints of the brain as an information processing machine. This is amusingly demonstrated by Attneave (1954) when he contrasts the  $10^{1,200,000}$  possible retinal configurations (given by 4 million cones) to the number of neurons that could fit into a cubic light year, which is a mere  $10^{54}$ . These two observations strongly motivate the possibility that our perceptual system is developed to adapt to redundancy in sensory inputs. It is proposed that the major function of perception should be to strip away some of the redundancy of stimulation so as to arrive at a more economical description or encoding of the incoming information. This idea is

referred to as the redundancy reducing hypothesis, and Barlow et al. (1961) was the first to formalise this neural coding objective using information theory. Redundancy, in the information theory framework, refers to a specific measure regarding the usage of a channel and is defined over an information rate measure. Information rate is the average entropy per symbol. For a single variable  $X$  with information rate  $r$ , the absolute redundancy is defined as:

$$D = R - r \quad (1.13)$$

where  $R$  is the maximum rate this variable can theoretically attain.

Relative redundancy is defined as the ratio between the absolute redundancy with respect to the maximum rate:

$$\text{Relative redundancy} = \frac{D}{R} \quad (1.14)$$

The opposite to redundancy is efficiency and is expressed as:

$$\text{Efficiency} = 1 - \frac{D}{R} = \frac{r}{R} \quad (1.15)$$

For measuring redundancy between 2 RVs, the absolute redundancy is:

$$D_{2RVs} = C - I(X; Y) \quad (1.16)$$

where  $C$  is the channel capacity.

The corresponding relative redundancy and efficiency are therefore:

$$\text{Relative redundancy}_{2RVs} = \frac{D}{C} \quad (1.17)$$

$$\text{Efficiency}_{2RVs} = 1 - \frac{D}{C} = \frac{I(X; Y)}{C} \quad (1.18)$$

In this formulation a redundancy-minimising coding is equivalent to an efficiency-maximising coding. For a fixed channel capacity  $C$ , the goal is to maximise  $I(X; Y)$ .

Given a well-defined and motivated objective, how can such a coding scheme be identified? In fact, ever since the introduction of information theory, there has been a lot of effort in discovering ‘optimal coding schemes’. Obviously, this will give rise to a myriad of schemes depending on how optimality is defined in each case. A coding scheme that is the most relevant to Barlow’s redundancy reduction hypothesis is Huffman coding, presented in the paper ‘A Method for the Construction of Minimum Redundancy Codes’ by Huffman (1952). The main idea of Huffman coding is to assign codewords of different lengths to inputs depending on their frequency of occurrence so that the most common inputs are represented by the shortest codewords and rarest inputs the longest. This coding scheme will therefore give rise to a set of codewords with minimum expected codeword length.

Applying this concept to neural coding, it is realised that the neural coding scheme that would give rise to the shortest expected codeword length, i.e. the fewest expected nerve impulses, will be one that is designed according to the probability of inputs (Barlow et al., 1961).

In conclusion, the efficient coding hypothesis postulates that in order to produce economical descriptions (defined by the number of neuronal spikes needed) of the rich environment, our sensory system should be organised in a way that is representative of the environment that we are in. This implies that if the brain's coding scheme is anywhere near optimal in terms of efficiency, it should possess some knowledge or representations about the environment that we interact with. In their seminal paper, Simoncelli and Olshausen (2001) have reviewed a wealth of studies in the visual system that reported evidence in support of the efficient coding hypothesis. For instance, it is found that the contrast-response function of the fly visual system correlates with the contrasts found in the environment of the fly (Laughlin, 1981). In the study of colour vision, it is also found that our cones demonstrate spectral sensitivities that have specific filtering properties which could facilitate solving the surface reflectance estimation problem of stimuli found in the natural environment (Maloney, 1986).

## 1.4 Information cost of cognitive processes

By formalising cognitive processes within the information theoretic framework, efficient coding hypothesis provides the theoretical foundations for many highly influential theories regarding the organisation principles of the brain (Olshausen and Field, 1997; Lewicki, 2002; Zhaoping, 2006; Doi et al., 2012; Li, 2002). In order to understand behaviours and subjective phenomena, such as cognitive effort, we will consider theories at a higher level of analysis that involve decision-making or conscious cognitive processes. One such theory is the bounded rationality formalisation of cognition (Friston, 2010; Kingma and Welling, 2013; Ortega and Braun, 2013; Tishby et al., 2000; Tkačik and Bialek, 2016). In this framework, it is proposed that cognitive processes trade off performance with computational costs. Formulations of computational cost from different fields of research are found to be rather coherent and all point to some measures of divergence between an initial belief and an updated belief obtained after processing new data. Adopting a probabilistic approach, the initial belief is usually modelled with a prior probability distribution and the updated belief with a posterior probability distribution, both over the same variable of interest. The amount of information it takes to update a prior to the posterior is the relative entropy, or Kullback-Leibler (KL) divergence, between the two probability distributions (Mattsson and Weibull, 2002). Cognitive activity can be considered as the information process that refines an initial assumption to a more accurate belief using observed data. Under this framework, the concept of cognitive effort could be closely related to the information cost of the process involved. Here, the formulation of this information cost will first be elaborated on some of the examples from the Zenon et al. (2019) paper.

### 1.4.1 Formulation of information cost

Consider a simple perception and response system that uses an internal representation  $x'$  to encode the occurrence of sensory data input  $x$ , to produce actions  $y$ , that is  $x \rightarrow x'$  and  $x' \rightarrow y$ .

The information cost of this two-step cognitive process is formally defined as the KL divergence (relative entropy, Eq.1.7) between the prior distribution of the internal representation  $p(x')$  and its posterior given the observation  $p(x'|x)$  plus the KL divergence between the prior of action  $p(y)$  and its posterior given the internal representation  $p(y|x')$ :

$$Cost = KL(p(x'|x)||p_0(x')) + KL(p(y|x')||p_0(y)) \quad (1.19)$$

where  $p_0$  is a prior distribution.

The information cost of such a system can also be interpreted as the reduction of entropy of the internal representation  $x'$  after observing the input  $x$ , and the reduction in entropy of the output  $y$  given the internal representation  $x'$ . This formulation of the information cost allows it to be dependent on both the internal representation  $x'$  and

the output  $y$ , therefore lending itself as an analytic tool to understand the demands and processes of different tasks in terms of information.

For instance, this framework can be applied to understand the information demand when learning a new task. Engaging in a novel task is usually characterised by one's lack of knowledge about its statistical properties, e.g.  $p(x')$  or  $p(y)$ . Without task-relevant knowledge, prior distributions would usually be assumed to be some uninformative distributions, like a uniform distribution. The process of tuning the prior to an optimal prior distribution is called learning and its cost is defined as the extra information cost incurred for using a suboptimal prior. Once the priors have become optimal, the learning cost vanishes.

$$\text{Learning cost} = KL(p_{opt}(x')||p_0(x')) + KL(p_{opt}(y)||p_0(y)) \quad (1.20)$$

For simplicity, let us discuss an example based on a perceptual process here, i.e.  $x \rightarrow x'$ , but the same idea can be extended to the action selection process, i.e.  $x' \rightarrow y$ , as well as processes at other levels. The meaning of learning cost can be demonstrated by considering the familiarisation of first-letter frequencies in English words. When one has first learned all the letters in the English alphabet, without knowing any English words, one might believe that all letters are equally likely to be the first letter of a word. This is represented by the uniform distribution over all letters, shown in grey in Figure 1.2 ( $p_0(\text{letters}) = p_{uniform}(\text{letters})$ ). However, over time, with exposure to many new words, one's belief of first-letter frequencies would change and might resemble more the true probability distribution, shown in colour in the barplot ( $p_{opt}(\text{letters})$ ). As mentioned above, KL divergence can be used to quantify the difference between probability distributions. Therefore, the learning cost of first-letter frequencies in English can be expressed as:

$$\text{Perceptual learning cost} = KL(p_{opt}(\text{letters})||p_0(\text{letters})) \quad (1.21)$$

Once a prior is well learned, possibly through training over time, the prior is considered to be optimal with respect to the specific task. However, it should be noted that the optimality of a prior is defined by how closely it follows the true task statistics. If the occurrence of a variable is truly random, in the sense that all possibilities are equally likely, the optimal prior would actually follow a uniform distribution. Although it might not be considered 'informative', it is indeed optimal. For tasks with this type of variables, it is theoretically impossible to obtain an information prior. Another task feature that could make informative priors hard to obtain is a huge amount of possibilities over the variables. The broader the range a probability distribution has to cover, the thinner it spreads, thus the lower the probability it informs. These are all factors that would contribute to a large difference between priors and posteriors, therefore to a higher information cost. Creative tasks usually require one to produce something out of a sea of possibility, such as creative writing (produce some words out of all words), painting (some colours out of all colours)

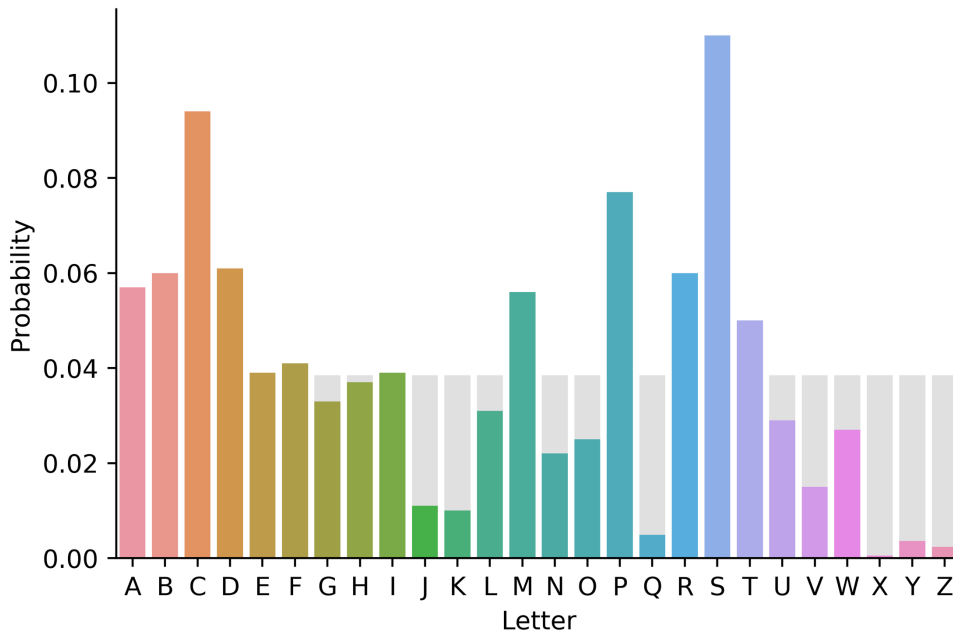


Figure 1.2: **Probability distribution of first-letter in English words.** Non-informative prior of first-letter frequencies,  $p_{uniform}(letters)$  in grey and true distribution of first-letter frequencies  $p_{opt}(letters)$  in colour.

or composing (some notes out of all notes). These are all tasks that would be proposed to have rather high information cost under the current framework and they are also found to be associated with higher cognitive effort (Hess and Polt, 1964; Kellogg, 1987; Marshall, 2002; Westbrook and Braver, 2015).

### 1.4.2 Information cost of using shared representation

Facing the diversity of tasks one has to perform to survive in the environment, sometimes the natural solution is to apply prior knowledge from other similar tasks rather than treating every new task as an independent or distinct task to be learned from scratch. Indeed, extra information cost incurred in the learning process starting without any assumption (using an uninformative prior) could be prohibitive and the need for fast learning is hypothesised as a cognitive priority for survival in the dynamic and unpredictable environment (Musslick and Cohen, 2021). Faster learning, i.e. to reach a reasonable level of performance in a shorter time, is achieved through sharing representations. Uninformative representations (e.g.: uniform priors) are replaced by some semi-informative representations that could potentially be used by many different tasks. Obviously, the brain is subjected to physiological and computational limitations imposed by the biological implementation of these information processes. Metabolic resources, number of neurons and



possible number of simultaneous network activations in the brain are all factors that eventually would constrain the number of tasks for which we could maintain high and accurate priors in the brain. Sharing priors between tasks can therefore also be considered as a natural consequence of this constraint. However, sharing priors/representations between tasks could mean that these representations are non-specific, i.e. suboptimal, to all of the tasks involved, implying in most cases their inference costs are not optimal. Moreover, if two tasks have incongruent goals and share representations, this might incur an even higher information cost, making this practice counter-productive. This might also be related to the cause of some cognitive bias such as apophenia or patternicity, the tendency to perceive meaningful patterns when there is none (Fyfe et al., 2008), and the effort required to ‘unsee’ the illusion. The extra cost of using a counter-productive prior can be formulated in a similar way as the learning cost (Eq.1.20), but instead of starting from just an uninformative prior (e.g.: uniform probability distribution), one would start with a counterproductive prior, defined as a probability distribution that is biased towards an inappropriate response for the task concerned. The resulting information cost will therefore be even larger than when one is just learning a new task without any assumptions at all.

This obstacle to fast learning posed by the cost of incongruent task goals in a shared representation scenario may be mitigated by considering an additional layer in the cognitive process, one which helps disambiguate the context  $T$  of the task. Adding more processing will naturally lead to an extra contribution to the total information cost:

$$\text{Context cost} = KL(p(y|x', T) || p_{opt}(y|x')) \quad (1.22)$$

The formulation of information cost therefore demonstrates the trade-off between learning cost and processing efficiency of sharing representations, not dissimilar to the one found between cognitive flexibility and stability in cognitive control (Musslick and Cohen, 2021).

Another perspective on this trade-off is provided by considering the minimising of long-term information cost of cognitive processes. Whether or not a prior should be specialised for a task should depend on how likely and often the novel task will re-occur in the future according to one’s belief. If the new task is projected to be the new norm, it might be worth paying the learning cost for a task-specific prior since it will likely help minimise information cost in the long term. However, if these new phenomena are believed to be rare, it might be more economical to use an unspecific (shared) prior and pay a higher (with respect to optimal), but one-off inference cost rather than paying the learning cost for obtaining an optimal prior that might seldom be used again in the future.

When stimuli or some task contingencies occur repeatedly, a prior will be gradually trained to represent these frequently occurring events more efficiently. In fact, with training, a general representation can lose its generality and become more task-specific and eventually reach optimality with respect to the task (Genewein et al., 2015; Tishby et al.,

2000). The process of developing a deep prior resonates with that of the separation of representations between tasks, which was proposed to be underpinned by repeated practice. It is interesting to note that the diminishing of information cost nicely coincides with that of control in the description of the emergence of automatic behaviour.

## 1.5 Empirical adaptation of the information cost model

The efficient coding theorem states that internal representations in the brain might be organised in such a way as to reduce processing of redundant information. In light of this, the formulation of information processing cost provides a framework to potentially evaluate the efficiency of cognitive processes in terms of information. This leads to the prediction that perceived effort might be closely related to the notion of information cost of cognitive processes. We designed a series of experiments involving a visuo-motor tracking task to investigate the relationship between task statistics, contingencies and task predictability with information cost, as well as how this information cost might impact perceived effort.

In fact, already back in the 60s', Crossman (1960) conducted a study on human information capacity when performing visuo-motor tracking tasks. However, in his formulation of the information rate during said task, the contribution of prediction was not formally defined or isolated from the tracking performance. The function of prediction is particularly important when one considers the fact that the inherent delays in information relays in the brain (Foxy and Simpson, 2002) necessarily render the perceived sensory inputs outdated with respect to the actual current state of the event (Carlton, 1981; Wolpert et al., 2001). The role of prediction in performing a visuo-motor task regarding a moving target is strongly supported by literature, such as studies of how humans intercept objects (Soechting et al., 2009; Brenner and Smeets, 2015; Dessing et al., 2009) or perform tracking when targets are occluded (Mrotek and Soechting, 2007; Zago et al., 2020). All these studies suggest that subjects are using observed information to anticipate future locations of the moving target. More specifically for a visuo-motor tracking task, evidence of prediction is mostly found in studies of sinusoidal wave tracking performance, in which most subjects manage to reduce the lag of the cursor with respect to the moving target to much below the visuo-motor delay (Viviani and Mounoud, 1990; Brenner and Smeets, 2015; Day and Lyon, 2000; Franklin and Wolpert, 2008; Saunders and Knill, 2005), indicating the involvement of anticipation or prediction in some form. Other studies aim to investigate the mechanism and degree of involvement of prediction (Parker et al., 2021; Drop et al., 2013): the current study is part of this line of research inasmuch as it attempts to identify and quantify the contribution of a predictive component of visuo-motor tracking performance. In particular, this is done by applying the framework of information cost as introduced above. Specifically, a link is proposed between task predictability, both in terms of signal and actions, and prior information. It is argued that the task-relevant information contained by a prior is predictive information: the more predictive information there is in the inputs, the more informative the priors can be, which will allow for information transfer at low cost. While priors might be hard to estimate, one can develop measures of predictive information from the task to approximate the relevance of the priors involved. We will derive one such measure for the visuo-motor tracking task, termed feedforward information. On the other hand, for information that is not captured

by the prior distributions, a feedback information measure is derived, which yields a lower bound for the information cost in such a process. Both feedback and feedforward measures are computed and compared in a series of experiments involving the same visuo-motor tracking task.

### 1.5.1 Priors over dynamic input data

Depending on the cognitive process involved, different probability distributions would be used as prior beliefs. For instance, a conditional probability distribution can be used to encode dynamic, rather than static, task-relevant information, e.g.  $p(x'_t|x'_{t-1})$  instead of just  $p(x'_t)$ , and  $p(y_t|y_{t-1})$  instead of just  $p(y_t)$ . This modification will allow the prior distributions to store information about the dynamics of the inputs and outputs, making it possible for the system to perform more sophisticated tasks without much information cost. Examples of conditional probability distributions as priors would be the encoding of sinusoidal target trajectories at input level and sinusoidal arm movements at output level. Sinusoidal trajectories are found everywhere in the natural environment, such as observing swings and pendulum movements, sea waves on the shore, etc. Given their prevalence in the natural environment, it is very likely that our cognitive system has developed an efficient representation to encode them, according to the efficient coding hypothesis. Sinusoidal trajectories are characterised and defined by their dynamics, i.e. their evolution through time. Therefore, instead of using the probability distribution  $p(x'_t)$  of possible spatial locations a sinusoidal trajectory would occupy at one particular time point as a representation, the dynamic signal is best encoded by the conditional probability distribution over current location given the two previous locations  $p(x'_t|x'_{t-1}, x'_{t-2})$ . This could potentially encode all information contained in sinusoidal trajectories since these can be expressed as second-order autoregressive processes:  $x_t = 2\cos(2\pi f)x_{t-1} - x_{t-2}$ . Following the same logic, a representation of sinusoidal arm movement, i.e. the sequence of applications of force needed to be exerted to produce a sinusoidal trajectory, could be encoded the same way. Since  $F = ma$ , this force is proportional to the second derivative of the sine wave:  $f''(x) = \frac{d}{dx}(\frac{d}{dx}\sin x) = -\sin x$ , which is also a sine wave.

### 1.5.2 Information cost and predictive information

An informative prior is defined as one that contains task-relevant information. Continuing with the example established on the encoding of sinusoidal inputs and outputs, let us consider a system that is trying to produce a tracking response to sinusoidal inputs. The system concerned consists of a perceptual and an action selection process.

For the perceptual process, the informative prior of the inputs would be a conditional probability distribution of current signals given previous ones,  $p(x'_t|x'_{t-1}, x'_{t-2})$  that fits the sinusoidal signal statistics. If one is observing an object assuming it follows sinusoidal movements, the perceptual information cost will then be:

$$\textit{Perceptual cost} = KL(p(x'_t|x'_{t-1}, x'_{t-2}, x_t) || p(x'_t|x'_{t-1}, x'_{t-2})) \quad (1.23)$$

If the observed input  $x_t$  was indeed following a sinusoidal trajectory, this cost will be small.

As for the action selection process, an informative prior of motor response would similarly be represented as a conditional probability distribution of the selected action given previous actions,  $p(y_t|y_{t-1}, y_{t-2})$ , that fits the sinusoidal movement statistics. The information cost incurred in the process of selecting a response given the internal representation of input  $x'_t$  is given by:

$$\textit{Action selection cost} = KL(p(y_t|y_{t-1}, y_{t-2}, x'_t) || p(y_t|y_{t-1}, y_{t-2})) \quad (1.24)$$

If the corresponding action  $y_t$  to the given input representation  $x'_t$  follows the sinusoidal movement dynamics as encoded in the prior, the information cost will be small.

With regard to the whole process from perception to action selection, the total information transferred can be quantified by the mutual information between visual inputs and motor response,  $I(x; y)$ . From this example, it can be seen that the priors of both inputs and responses contain useful information that could allow information to flow through these processes:  $x \rightarrow x'$  and  $x' \rightarrow y$  with minimum information cost, demonstrating an idealised system with very little information costs yet very high total information transfer.

The predictive information can be defined as the prior's contribution to total information transfer. The complement of that contribution is the information cost of the process.

$$\textit{Total information transfer} = \textit{Predictive information} + \textit{Information cost} \quad (1.25)$$

The derivation of information theoretic measures to approximate these quantities would be the main subject of the first study.

### 1.5.3 Hierarchical structure of cognitive processes

Internal representations can be shared in cognitive processes of different levels, as supported by an abundance of literature (Lashley et al., 1951; Miller et al., 1960; Botvinick and Plaut, 2004; Schneider and Logan, 2006; Zacks et al., 2007). The prior belief of sinusoidal movements of objects would be an example of sharing representation at lower visual perception. The so-called executive functions such as attention and working memory would be examples of higher level representation sharing. Sometimes tasks that seem to involve non-overlapping information processes, such as visual and auditory perception, might still suffer from cross-task interference due to resource sharing at a higher level, e.g.: attention and working memory (Kahneman, 1973; Pashler, 1994; Broadbent, 1957;

De Jong, 1993; Welford, 1967; Keele and Boies, 1973; Tombu and Jolicœur, 2003). Depending on the number of levels in a cognitive process that is sharing resources, the effect of interference could scale up drastically (Alon et al., 2017). This would correspond to the accumulation of information cost along the sequential cognitive processes.

## 1.6 Biological implementation of cognitive effort in the brain

Besides seeking a rational account for the emergence of cognitive effort, another fundamental question concerns its biological implementation in the brain. Due to their close correspondence, studies of cognitive effort are often linked to that of cognitive control. Among these studies, the theory of expected value of control proposed by Shenhav et al. (2013) is of particular relevance. Their theory, following the line of research that poses control as an optimisation problem (Bogacz et al., 2006; Dayan, 2012; Hazy et al., 2007; O’Reilly and Frank, 2006; Todd et al., 2008; Yu et al., 2009), defines cognitive control as the mechanism responsible for leading to reward-maximising behaviours. Together with the assumption that cognitive control is inherently costly (Kool and Botvinick, 2014; Kool et al., 2010), it naturally leads to the suggestion that allocation of control is based on a cost-benefit analysis that tries to balance the potential rewards with the incurred cost, giving rise to the formulation of an expected value of control, EVC, (Shenhav et al., 2013):

$$EVC(\text{signal}, \text{state}) = \left[ \sum_i P(\text{outcome}_i | \text{signal}, \text{state}) \times \text{Value}(\text{outcome}_i) \right] - \text{Cost}(\text{signal})$$

where signal is a specific control signal and state is the current situation. It is further proposed that the dorsal anterior cingulate cortex (dACC) plays the essential role of estimating EVC for different control signals and consequently provides information to downstream structures like the inferior prefrontal cortex (IPFC) to bias their execution of control functions (Banich, 2009; Cavanagh et al., 2009; Kerns et al., 2004; Kouneiher et al., 2009; MacDonald et al., 2000; O’Reilly, 2010; Ridderinkhof et al., 2007; Rothé et al., 2011; Venkatraman and Huettel, 2012). Moreover, other findings suggest dACC responds to both exertion and prospective demand of physical effort of actions (Crosson et al., 2009; Hillman and Bilkey, 2010, 2012; Walton and Cohen, 2007; Kennerley et al., 2011; Cowen et al., 2012), implying dACC’s potential for the encoding of the overall value of actions.

Further studies explicitly disambiguate cognitive effort from control. Instead of directly interpreting the execution of control as effortful, they employ a cognitive effort-discounting paradigm (Westbrook et al., 2013) to quantify the subjective value of cognitive effort (Westbrook and Braver, 2015). A recent study (Westbrook et al., 2020) has found evidence that an increase in striatal dopamine levels could bias the benefit over the cost of cognitive effort, leading to a boost in cognitive control. This mechanism of neurochemical modulation can be incorporated into the network view proposed by the EVC theory through the dACC projections to both ventral and dorsomedial striatum (Choi et al., 2012; Haber and Knutson, 2010). The balance of benefit/cost of cognitive effort associated with control might be altered in the striatum by the increased dopamine level there. This effect is then relayed to the dACC where the EVC is computed taking into account other factors. The EVC will then subsequently signal the related structures such as those in the prefrontal cortex that are responsible for the actual execution of control.

# Chapter 2

## Information rate in humans during visuomotor tracking

### 2.1 Introduction

Our living environment is rich with stimuli, some of which are crucial in guiding our decisions. Imagine walking into a room full of people: each face, each moving object and each voice in the room are in competition for our cognitive resources. Our brain deals with this overwhelming computational demand by selecting information through attentional mechanisms (Cohen and Maunsell, 2009) and by using efficient coding, explaining away predictable data and transmitting only the prediction error, that is the sensory evidence that cannot be predicted from other sources or earlier inputs (Smith and Lewicki, 2006). According to this view, cognitive resources (e.g., metabolic rate of neurons or information capacity usage) would be dedicated to processing surprising inputs while predictable data would be virtually free to encode (Zenon et al., 2019).

Surprisingly, despite the consensual view of the brain as an information processing machine, few attempts have been made to quantify the amount of information being processed by it, beside the pioneering work described below. One of the reasons for this may be inherent to the technical difficulty of separating predictable from novel information in ecological tasks. The present study aims at filling this gap by applying information theoretic measures to a visuomotor tracking task.

#### 2.1.1 Information Processing Rate in Humans

Just a few years after the publication of Shannon's seminal paper on information theory (Shannon, 1948), several studies attempted to apply this novel theory to estimate the information processing capacity of the human brain. In 1952, Hick (Hick, 1952) compared subjects' reaction times in a simple forced choice task while varying the number of discrete choices available to them. He observed that the reaction time varied linearly with the logarithm of the number of choices in the task. This result, later coined Hick's law,



implies that there is a constant rate of information gain for this task. This is important because it suggests, counter to intuition, that information processing rate does not vary as a function of task difficulty.

In Hick’s task, the focus was on the decision process and the motor component was assumed constant across conditions. To address this issue, Fitts (1954) designed a movement-amplitude control task to estimate information rate with respect to the speed and accuracy of the movement. He first quantified the ‘difficulty’ of reaching a target in information-theoretic terms; then, by dividing this quantity of information by the time it took the subject to attain the target, he obtained an index of performance in units of bits per second, an analogue of information gain in Hick’s task. Fitts found this rate of the human visuomotor-proprioceptive channel to be relatively constant across a range of task conditions (see Gori et al. (2018) for a more recent discussion on this matter).

### 2.1.2 Pursuit-Tracking Task and its Feedforward Component

In these early attempts at measuring information processing capacity in humans, both Hick and Fitts used simple task designs that involved discrete decisions or movements in each trial. While these might be simpler to study, they do not necessarily provide a good representation of the tasks with which we are faced most often in day-to-day life. To extend the study of human information rate beyond the discrete-task context, Crossman (1960) chose to study a pursuit-tracking task. In this experiment, Crossman used an apparatus consisting of a variable-speed velodyne, which drove a piece of paper showing the target course, and a vertical handwheel which subjects used to track it. Importantly, although Crossman’s paradigm involved predictions, a crucial addition for studying information processing during skilled movement (Poulton, 1957), the information rate was computed simply as the mutual information between the course and the tracking after correcting for the lag between them, without dissociating the respective contributions of the predictive and error-correcting components. In line with the motor control literature (Yeo et al., 2016; Maeda et al., 2018), these components will be referred to as the feedforward and feedback components respectively. Recent studies have provided evidence in support of the existence of such predictive (feedforward) components during target-tracking in humans. Drop et al. (2013) compared three models of tracking on human tracking data and found that the model containing a feedforward component fit their data best. The same authors (Drop et al., 2016) also tested the effect of the predictability of the target signal on predictive control and found that the degree of reliance on feedforward control is proportional to signal predictability.

To our knowledge, no effort has yet been made to disentangle prediction in a pursuit-tracking task from the more physiologically relevant (Trujillo, 2019) real-time processing of prediction errors, which we will refer to as the feedback component. Feedforward components, on the other hand, can produce accurate motor responses that are not based on real-time information processing of sensory inputs, but rather on a read-out from the

internal model, when faced with predictable data. The present study thus sets out to investigate specifically the role of feedback components of information processing and to leverage the tools of information theory to provide a quantitative description of the real-time information processing rate of human in this visuomotor task.

## 2.2 Results

Our aim is to study the visuomotor channel that receives visual inputs and generates motor outputs in a one-dimensional visuomotor tracking task with targets of variable predictability (Fig 2.1).

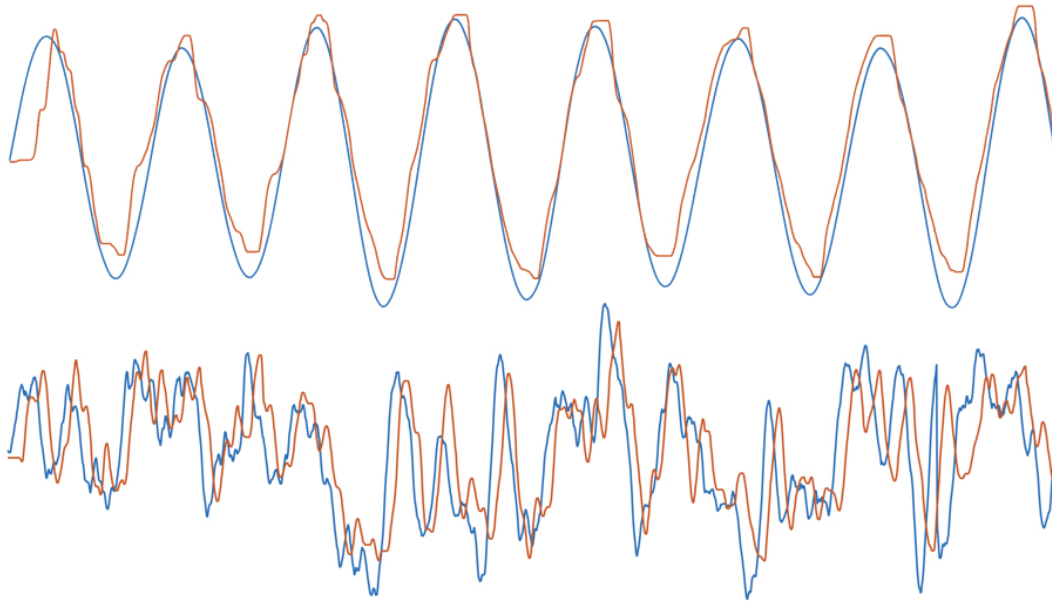


Figure 2.1: **Example tracking data.** Example experimental data showing the x-coordinates of target signal (blue) and tracking response (orange) for condition 1 (top; most predictable condition) and condition 4 (bottom; least predictable condition).

### 2.2.1 Background

To give some background to our information-theoretic measures, we start by revisiting the definition of entropy, mutual information, entropy rate, as well as the interpretation of transfer entropy as the rate of information transmission of a channel.

Entropy is the basic quantity we use to measure information. Defined for a random variable  $X$  with probability  $p(X)$ , it is given by:

$$H(X) = - \sum p(X) \log p(X) \quad (2.1)$$

A channel that takes  $X$  as input and gives  $Y$  as output is characterized by a conditional probability function that determines the transition from  $X$  to  $Y$ . The rate at which information is processed through such a channel is given by the mutual information between  $X$  and  $Y$ :

$$\begin{aligned}
I(X; Y) &= \sum p(X, Y) \log \frac{p(X, Y)}{p(X)p(Y)} \\
&= H(X) - H(X|Y) \\
&= H(Y) - H(Y|X)
\end{aligned} \tag{2.2}$$

Mutual information provides insights about the static relationship between two random variables. In order to quantify the dynamics, or causality, of the relationship between multiple random processes, one must consider transition, rather than static, probabilities, which leads to the definition of entropy rate (for a single variable) and transfer entropy (for the interaction of two systems) (Schreiber, 2000).

Entropy rate measures the rate of growth of entropy of a sequence, that is how much novel information each new sample provides. For a sequence  $\mathcal{X}$  of  $n$  random variables, entropy rate is given by:

$$\begin{aligned}
H(\mathcal{X}) &= \lim_{n \rightarrow \infty} \frac{1}{n} H(X_1, X_2, \dots, X_n) \\
&= \lim_{n \rightarrow \infty} H(X_n | X_{n-1}, X_{n-2}, \dots, X_1) \\
&\leq H(X_n)
\end{aligned} \tag{2.3}$$

when the limits exist. It can be interpreted as the entropy per symbol in the sequence or as the conditional entropy of the last random variable given all the previous ones. For stationary processes, it is proven that both limits exist and that they are equal. Entropy rate is of particular interest to the current study because the continuous visual target movement in the experimental task was constructed as a sequence of target positions presented on the screen. Therefore, the entropy rate of the target position quantifies all the information there is to know about target position, and which could be potentially transferred to tracking response.

The last inequality in Equation 2.3 follows from the property of conditioning, which can never increase the entropy of a random variable; the equality is attained for a sequence of independent and identically distributed (i.i.d.) variables, since in that case  $X_n$  is independent of the preceding symbols and conditioning on them does not reduce the entropy. However, in our study, successive target positions are correlated. We would therefore expect the sequence to have an entropy rate that is smaller than the entropy of the target position,  $H(\mathcal{X}) < H(X_n)$ . In other words, there is less uncertainty associated with a target position that follows the sequence than one that is randomly drawn at any given time point.

Transfer entropy, representing the information processed with respect to each new element of the input sequence, is defined as the conditional mutual information between the last output and previous inputs, given the history of the outputs:

$$I(Y_{n+d}; X_n^{(l)} | Y_n^{(k)}) \tag{2.4}$$

where one defines the notation  $X_n^{(l)} = (X_n, \dots, X_{n-l+1})$ . Parameters  $l$  and  $k$  determine the depth of past values one uses to encode the history of  $X$  and  $Y$ , respectively, while  $d$  represents a time difference between  $X$  and  $Y$ , assuming the information transfer is not instantaneous. Our analysis, detailed in the next section, allows us to identify specific choices of  $d$ ,  $k$  and  $l$  to compute the information transfer from signal  $X$  to tracking  $Y$ .

## 2.2.2 Definition of Measures

### Basic assumptions

Adopting a model-free approach, we did not make any specific assumptions on the mechanism involved in producing the observed tracking performance. For our analysis, we rely on two key properties of information sharing and transmission in the system.

The first one is an effective time delay. The feedback channel, while engaging in real-time information processing, suffers from a non-reducible time delay in producing motor outputs with respect to the visual input signals. This time delay is referred to as the visuomotor delay (VMD) is a consequence of the physical constraints of the visuomotor system.

The second key property of the system is its autocorrelation. The signals used in the current study were constructed by passing white noise through a sinusoidal filter (order 2). By altering the parameters of the filter we could control the amount of noise that passed through, thus the predictability of the signal. However, regardless of the predictability, the autocorrelation in the target signals was always limited to second order, due to their sinusoidal nature.

### Feedback component information content

Using the above properties, we were able to fix the free parameters in the transfer entropy formula in Equation 2.4, thus tailoring it to the quantification of the information rate of the feedback component, as desired. Given that the expected delay of information transfer from signal  $X$  to tracking  $Y$  is the VMD and that the target is a second-order autocorrelated signal, we set the delay  $d$  between the two processes to be VMD and the depths  $l$  and  $k$  to be 2. To ensure independence between successive samples, we further modified the transfer entropy term by conditioning it on  $Y_{t-1}$ , thus obtaining the following feedback component measure:

$$I_{FB} = I(Y_t; \{X_{t-VMD}, X_{t-VMD-1}\} | \{Y_{t-VMD}, Y_{t-VMD-1}, Y_{t-1}\}) \quad (2.5)$$

### Total information and feedforward component

The total information shared between signal  $X$  and tracking  $Y$  has either of two origins: it arises via feedback information transfer with a non-reducible time delay ( $X_{t-VMD}, X_{t-VMD-1} \rightarrow$

$Y_t$ ), or it is due to prediction ( $X_t, X_{t-1} \rightarrow Y_t$ ). This allowed us to compute it as the joint mutual information:

$$I_{total} = I(Y_t; \{X_t, X_{t-1}, X_{t-VMD}, X_{t-VMD-1}\}) \quad (2.6)$$

This term represents the expected value of total shared information between  $X$  and  $Y$ . Deducting the feedback component from it yields the average information attributable to the feedforward component:

$$I_{FF} = I_{total} - I_{FB} \quad (2.7)$$

### 2.2.3 Validation through Model Simulations

In order to validate the quantities derived above, we built a mathematical model of the task based on optimal control theory (Mulder et al., 2017) (see Fig 2.2A and Method), in which we could manipulate and measure directly the involvement of the feedback component and therefore provide a ‘true’ value against which to compare our  $I_{FB}$  and  $I_{FF}$  measures.

We formulated the visuomotor tracking task as a linear state space model with quadratic regulation cost. We generated the target and joystick dynamics with a set of linear differential equations, which were also included in the transition matrix  $A$  of the model (i.e., the model had perfect knowledge of the target and joystick dynamics). State representation  $s$  also included the error between target and joystick coordinates and the regulation objective was to minimise the value of this state element. All state representations were updated at each time step by means of a Kalman filter, on the basis of a novel observation  $x$  corresponding to the position of the joystick and the target, acquired  $VMD$  timesteps before. Optimisation of the control variable  $u$  was obtained with a model predictive controller, as described in the Methods section.

#### Validation of $I_{FB}$ and $I_{FF}$

To establish the ground truth for the feedback measure,  $T_{FB}$ , we took advantage of the linearity of the Kalman filter to directly quantify the information transfer through the feedback pathway in the model by computing the mutual information between observation  $x$  and state estimates  $s$  at the Kalman filter level. Given the Gaussian distribution of both the observation  $x$  and state estimates  $s$ , the mutual information can be expressed as:

$$T_{FB} = I(s_t; x_t) = \frac{1}{2} * \log\left(\frac{|C\Sigma C' + R'|}{|R|}\right) \quad (2.8)$$

with  $\Sigma$  being the state covariance matrix,  $C$  the state-to-observation matrix, and  $R$  the observation noise.

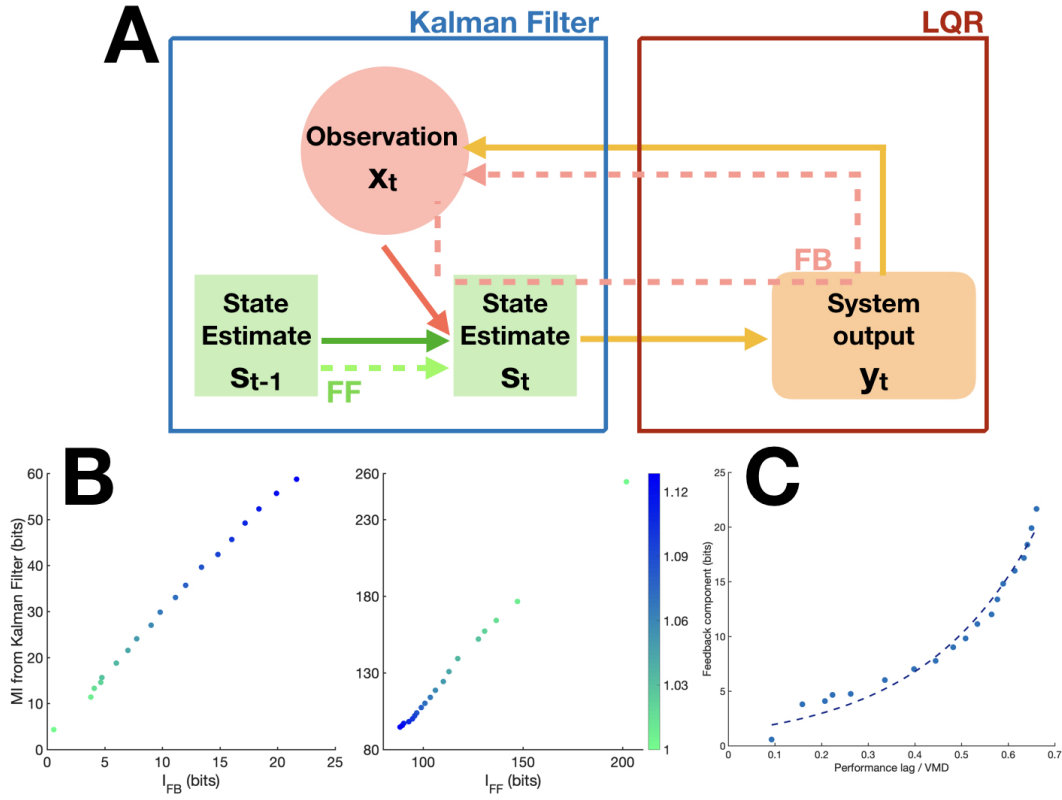


Figure 2.2: **Simulation design and results.** (A) Schematic of Linear Quadratic Regulator model of the visuomotor tracking task. (B) Correlation between true feedback measure  $T_{FB}$  and proposed measure  $I_{FB}$  from model data,  $R = 0.999$ . (left) Correlation between true feedforward measure  $T_{FF}$  and proposed measure  $I_{FF}$  from model data,  $R = 0.999$ . (right) Color code indicates the value of the noise parameter used to generate the signal (see Methods). Larger values correspond to higher complexity in signals, thus less predictable. (C) Relationship between  $I_{FB}$  and performance lag/visuomotor delay (VMD) ratio. An exponential function  $\frac{PL}{VMD} = a \exp(bI_{FB})$  was fitted on the data, with  $PL$  the performance lag and  $VMD$  the visuo-motor delay. The R squared of the fit was 0.98.  $a = 1.172$  (95% confidence interval: 0.815 – 1.53),  $b = 4.282$  (3.779 - 4.785).

The feedforward component, on the other hand, is formulated as the mutual information between state estimates at two successive time points  $t$  and  $t - 1$ :

$$T_{FF} = I(s_{t-1}; s_t) = \frac{1}{2} * \log\left(\frac{|\Sigma|}{|Q|}\right) \quad (2.9)$$

with  $Q$  being the process noise.  $T_{FB}$  and  $T_{FF}$  were computed for sets of simulation data generated by the model using different predictability levels of input signals, while all other parameters were kept constant. It is important to stress that  $T_{FB}$  and  $T_{FF}$  provide an upper bound on the actual mutual information between inputs and outputs because they are concerned only with state representations at the Kalman filter level and do not take

into account the potential loss of information through filtering at the level of the linear quadratic regulator.

We then computed the proposed information theoretic measures  $I_{FB}$  and  $I_{FF}$  on the same data using Gaussian copula estimation (see Methods) and obtained the correlation of the two measures with their respective ground truth values across different predictability levels. Figure 2.2B shows the high correlation between  $T_{FB}$  and  $I_{FB}$  ( $R^2=0.999$ ), and that of  $T_{FF}$  and  $I_{FF}$  ( $R^2=0.999$ ), attesting to the validity of the proposed measures in quantifying component-specific information.

### Validation of the Estimate for VMD

While VMD can be directly extracted from the model for simulation data, there is no way to access it directly in real experimental data. We therefore needed to establish an estimate of VMD that could be applied to experimental data. A candidate for such an estimate was the peak latency of the transfer entropy from signal  $X$  to tracking  $Y$ ,  $TE_{X \rightarrow Y}$ . Since the feedback component is delayed by VMD, the transfer entropy should peak at t-VMD. To evaluate the correspondence of this candidate measure to the true VMD, we generated simulated data corresponding to true VMD values from 9 to 19 frames while all other parameters were kept constant. Notably, the effective VMD of the simulation data was determined by the sum of the visual and motor delay parameters with an additional delay that was inherent to the joystick mechanism and which depended on the parameters of its state space representation (i.e., spring, mass and damping coefficients). Therefore, here again, we were seeking a correlation rather than a strict equality between inferred and reference values. The comparison showed perfect correlation between the peak latency of  $I_{FB}$  and the VMD actually implemented in the model ( $R^2 = 1$ ), validating this estimate of VMD from data.

### Relationship between feedback component and performance lag

The effect of prediction on tracking performance is two-fold: first, it provides a cognitively efficient way to encode the target signal, thus saving cognitive resources; second, it compensates for VMD by allowing subjects to act in advance, which contributes to a reduction in performance lag (that is, the lag corresponding to maximum cross-correlation between target and tracking). When prediction fails, we would expect the feedback component to take up more information load to maintain performance level. Due to the irreducible VMD of the feedback component, the more it is involved, the more performance lag will tend to VMD. We looked at our simulation data to confirm this effect by observing the relationship between the ratio of performance lag to VMD and the feedback component measure (Fig 2.2C). We found a strong exponential relationship between the two variables.

## 2.2.4 Experimental Results



### Identifying VMD from experimental tracking data

To be able to compute  $I_{FB}$  and  $I_{FF}$ , one must first know the VMD of the system. Having confirmed that the peak latency of transfer entropy  $TE_{X \rightarrow Y}$  corresponds perfectly with VMD in simulation data, we computed the trial-averaged peak latencies of  $TE_{X \rightarrow Y}$  for each subject from their performance in the most complex condition to obtain an estimate for each subject's VMD (Fig 2.3A). Our results showed that VMD lay between 14 and 16 frames (about 230 to 270ms) for 10 out of the 11 subjects, while one subject showed a VMD of around 380 ms.

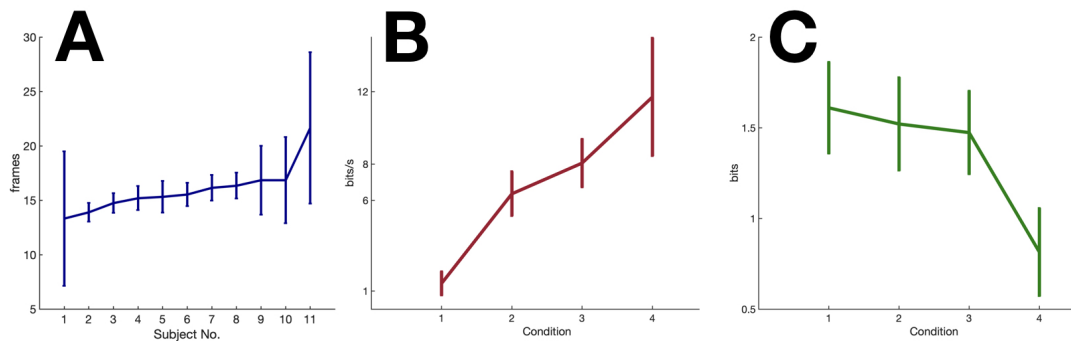


Figure 2.3: **Experimental results.** (A) VMD of individual subjects (sorted in increasing order). Error bars represent the standard deviation across trials. (B) Average real time information processing rate per second across subjects (C) Average  $I_{FF}$  across subjects for different conditions

### Feedback information rate

Using subject-specific VMD, we computed the feedback component  $I_{FB}$  using equation 2.5. Results showed that feedback information transfer increased with the complexity of the signal ( $F(3,40) = 34.9$ ,  $p < 0.0001$ ). Post hoc Tukey HSD test indicated that condition 1 and condition 4 were significantly different from all other conditions while the difference between condition 2 and 3 did not reach significance.

Since  $I_{FB}$  represents information rate per sample, one can obtain subject-specific information processing rates per second by multiplying  $I_{FB}$  by the number of samples in one second. With a frame rate of 60Hz, we have concluded that the subjects' real-time information processing rate lies between 1 to 12 bits/s, depending on the complexity of the signal (Fig 2.3B).

### Feedforward component and predictability of signal

The feedforward component measure  $I_{FF}$  cannot be interpreted as an information transfer rate per unit of time because, unlike  $I_{FB}$ , it is not an independent measure between

successive samples. However, it can still be compared across conditions to help us gain insight about the role of prediction with regards to signals of different predictability. We observed a clearly opposite trend relative to that of the feedback component. As predictability of signals decreased,  $I_{FF}$  also decreased,  $F(3,40) = 30.7$ ,  $p < 0.0001$ . Post hoc Tukey HSD test once again indicated only condition 1 and condition 4 were significantly different from all other conditions (Fig 2.3C).

## 2.3 Discussion

In the current study we have proposed an original information-theoretic approach to evaluate the computational demand of sensorimotor tasks. One of our key contributions was to obtain a decomposition of the total mutual information between inputs and outputs, tailored to dissociate the contribution of real-time processing of prediction errors (referred to as the feedback component) from that attributable to internal predictions (feedforward component).

This approach affords us the opportunity to quantify the information rate of sensorimotor tasks, and hence, to study the information capacity of sensorimotor systems. We hypothesize that the feedback component is a better marker of the amount of cognitive resources required by the task than the total mutual information. Indeed, in a communication channel in which both encoder and decoder are aware of the autocorrelation of the data  $X$ , predictability can be leveraged to decrease information rate by encoding only the data that is not already predicted by the conditional probability  $P(X_t|X_{t-1,t-2,\dots,1}, \mathcal{M})$  implemented in the decoder/encoder, achieving entropy encoding, i.e., a code length that is equal to  $\mathcal{H}(P(X_t|X_{t-1,t-2,\dots,1}))$  (Cover and Thomas, 2006). This part of the data that cannot be predicted corresponds to the feedback component in the present study. In predictive coding models (Rao and Ballard, 1999), such optimization of encoding through prediction can be understood in terms of firing rate of prediction error neurons. When inputs are perfectly predictable, these neurons would not fire at all, thereby leading to low metabolic costs.

### 2.3.1 Information Processing Rate in Humans

When applying the discussed measures to a visual tracking task, we found that, whereas the feedforward information increases with predictability of signals, the information rate of the feedback component decreased with predictability.

Our results thus imply that, in our task, information processing rate adapted to signal predictability, in apparent contradiction with Hick's law. This suggests that the engagement of cognitive resources in the task was balanced against performance goals, in agreement with cost-benefit models of effort (Kool et al., 2010; Westbrook and Braver, 2015; Shenhav et al., 2017). Participants would thus invest cognitive resources in proportion to their impact on performance. In the case of predictable targets, investing more resources would have only minimal effect on performance, justifying to maintain a low information rate. In contrast, when predictability is low, performance depends more on information rate, explaining larger rates across subjects in this condition. Future studies will determine in more details the nature of this rate-performance trade-off.

### 2.3.2 Information-Theoretic Approach to Evaluating Tracking Performance

VMD is an important property of subjects' sensorimotor system, however its direct estimation from tracking data poses some challenges. Ideally, VMD should correspond to the performance lag in a situation where subjects track completely unpredictable signals, i.e., white noise. However, such a signal has too many high frequency components, which subjects are unable to track, making this approach infeasible in practice. We therefore proposed and validated a model-free solution to estimate VMD in tracking tasks. We found VMD values between 230 to 270 ms, in agreement with previous literature on human visuomotor reaction time (Hülzdünker et al., 2019). We observed sizeable inter-personal differences in VMD but, within subject, VMD varied little across conditions.

A common outcome measure in tracking experiments is the time lag between tracking response and signal (Miall et al., 1985; Foulkes and Miall, 2000; Ballard et al., 2001; Bormann et al., 2004). While this lag by itself is a good indicator of performance, normalizing it with respect to VMD highlighted an interesting relationship to the feedback component. In particular,  $\log(\frac{\text{PerfLag}}{\text{VMD}})$  has an approximately linear relationship with the FB component. The combination of VMD and real-time information processing rate thus provide a more complete picture of subjects' performance in a tracking task.

Its model-free nature, coupled with state-of-the-art methods for information estimation, grant our approach enough flexibility for generalizing it to more complex tasks, such as to accommodate higher-dimensional target/tracking spaces (Lee et al., 2020) or delay-embeddings of random processes (Takagi et al., 2020).

### 2.3.3 Limitations

A major advantage of the information-theoretic approach is that it is model-free, and thus requires few assumptions. In the present study, we relied only on the following postulates. First, we assumed that the visuomotor system can be viewed as a constant communication channel that takes visual input  $X$  and gives motor output  $Y$ , related through the conditional  $p(Y|X)$ , which is constant over time within subjects and conditions. Second, we assumed that the VMD was constant over time. Were these assumptions incorrect, our measure would still provide a valid average of the actual information rate. Third, the method used to measure mutual information, namely the Gaussian copula, relies on the assumption of normally distributed dependency structure between variables. This makes our estimate a lower-bound to the true mutual information value, since the Gaussian distribution has the highest entropy among all distributions. Fourth, in our measures, for the sake of simplicity, we have conflated the joystick visual input with the motor output. However, in reality, motor noise is added to the motor output such that the joystick position can differ from the intended one (Hamilton et al., 2004). Therefore, the variable  $Y$  used in our formulae, which corresponds to the joystick cursor position on the screen, does not really represent the motor output but rather motor output corrupted by noise.

This simplification leads to a potential underestimation of the true feedback component that is insensitive to variations in motor noise. This should be addressed in future work.

Another limitation of the present work pertains to the resolution of the VMD estimate. Given the discrete nature of the computerized visuomotor tracking task, we can only measure subjects' VMD up to the resolution that is allowed by the frame rate of the experimental display. With the 60 Hz display system we used in the experiment, the resolution of the VMD is around 17 ms. Future studies can improve the experimental design to allow for higher resolution for studying the tracking performance.

Finally, it is worth mentioning that our proposed measures are tailored to an order-2 target tracking task. A target with a more complex autocorrelation structure would require adapting the mutual information formulae by adding higher-order terms.

## 2.4 Conclusions

We have presented here a method allowing us to separate feedback and feedforward information rates of a visuomotor tracking task and have shown that both components are influenced by the predictability of signals. We argue that our proposed measure of the feedback component should provide a more relevant measure of task difficulty, cognitive demand and associated metabolic costs than a non-discriminative total information transfer measure. Future studies should aim at comparing this measure with currently existing metrics of cognitive effort and metabolic demands.

## 2.5 Materials and Methods

### 2.5.1 Participants

We recruited 11 right-handed subjects (2 males) aged between 20–28 years old from the local university network. They all have normal or corrected to normal vision. All participants have given their consent in written form. The experiment lasted around 1 hour and all subjects were compensated equally for their time.

### 2.5.2 Experimental Design

The visuomotor task employed for the current experiment was a one-dimensional target tracking task. A vertical bar (3.3 mm wide and 66.5 mm tall) was presented as the visual target on the screen (1024x1280) and subjects were asked to follow the movement of the target with a triangular cursor (6.6 mm wide and 13.2 mm tall), which they controlled through a joystick. The target was programmed to move only along the horizontal plane, so subjects were instructed to constrain their joystick movement to this plane during the task, which they could easily achieve by letting the joystick lean on the front end of its pad while moving it sideways. To prevent subjects from cancelling the target movement on the screen by head or eye movements, subjects were instructed to place their heads on a fixed headstand attached to the table to stabilise their head positions and they are instructed to fixate at a center crosshair during all trials. In addition, we have installed an Eyelink 1000 + eye tracker (SR Research Ltd., Kanata, Ontario, Canada) to monitor their eye movements to ensure compliance to the task instruction. The trajectory of the visual target  $y$  was pre-programmed by passing white noise  $\theta$  through a sinusoidal filter:  $a_1x_t = \theta_t + a_2x_{t-1} + x_{t-2}$ , with  $a_2 = -2\sqrt{a_1}\cos(\frac{\pi}{100})$ . We could manipulate the predictability of the signal by altering the parameter  $a_1$  of the filter controlling the amount of noise that passed through. Fig 2.1 shows example signal and tracking from condition 1 and 4. Since the target trajectory was pre-programmed, it was completely independent of the subjects' response during the task. Horizontal joystick movement was registered as the main output response. We further registered vertical joystick movement and discarded trials during which subjects failed to keep to the required plane.

### 2.5.3 Mutual Information Estimation Using Gaussian Copula

There exist many different methods for estimating mutual information. For the current study, we found the Gaussian copula method to be the most appropriate for our data (Ince et al., 2017). Compared to a classic binning or k-nearest-neighbour methods (Kraskov et al., 2004) for mutual information estimation, the copula-based method is less subject to sampling bias and it does not require any assumption regarding the distribution of the random variable.

A copula is a multi-dimensional cumulative distribution function (CDF) for which the marginal distributions of all variables are uniformly distributed over the interval  $[0, 1]$ .

For a multivariate random vector  $(X_1, X_2, \dots, X_d)$  with continuous CDFs  $F_i(x) = P(X_i \leq x)$ , one can apply the probability integral transform (Casella and Berger, 2002) to obtain uniformly distributed marginals over the interval  $[0, 1]$ :

$$(U_1, U_2, \dots, U_d) = (F_1(X_1), F_2(X_2), \dots, F_d(X_d)) \quad (2.10)$$

Using the uniformly distributed marginals, we can define a copula:

$$C(u_1, u_2, \dots, u_d) = P(U_1 \leq u_1, U_2 \leq u_2, \dots, U_d \leq u_d) \quad (2.11)$$

Sklar's theorem (1959) states that every multivariate CDF of a random vector can be expressed in terms of its marginals and a unique copula, if the marginals are continuous.

$$F(x_1, x_2, \dots, x_d) = C(F_1(x_1), F_2(x_2), \dots, F_d(x_d)) \quad (2.12)$$

The theorem has the implication that one could separate the dependency structure of a multivariate distribution from its marginals. The copula is the part of the density function that retains all dependencies between variables, and is independent from individual marginal distributions. It was shown that the mutual information between the random variables equals the negative entropy of their corresponding copula (Jenison and Reale, 2004). This implies that mutual information, like copula, does not depend on individual marginal distributions but rather depends only on the interaction between variables.

Using the characteristics of the copula and its link to mutual information, we can now estimate MI by computing the corresponding copula density of the random variables. For a faster estimation of the copula entropy, the marginals are transformed to standard Gaussian variables. Since copula entropy is independent of individual marginal distributions, this transformation should not affect the result. However, this transformation will allow the application of the parametric Gaussian model for MI estimation using covariance matrices and joint covariance matrix of the random variables  $(X, Y)$ .

$$I(X; Y) = \frac{1}{2} \log \left[ \frac{|\Sigma_X| |\Sigma_Y|}{|\Sigma_{XY}|} \right] \quad (2.13)$$

To obtain a bias-corrected measure, we compute and remove the estimation bias of  $\ln|\Sigma|$  using a known analytical solution (Misra et al., 2005; Ince et al., 2017):

$$bias = k \ln 2 + \sum_{i=1}^k \psi \left( \frac{N-i}{2} \right) \quad (2.14)$$

where  $k$  is the dimensionality of the data and  $\psi$  is the digamma function.

### 2.5.4 Linear-Quadratic Regulator model

The state space model takes observation vector  $x$  as input, incorporating both joystick and visual target positions, and models their dynamics through internal state  $s$ , transition matrix  $A$ , motor output  $u$  and control matrix  $B$ :

$$\begin{aligned} s_t &= As_{t-1} + Bu_{t-1} \\ x_t &= Cs_t \end{aligned} \quad (2.15)$$

and we define the cost function to be:

$$J = s_{t+N}^T Q s_{t+N} + \sum_{k=0}^{N-1} (s_{t+k}^T Q s_{t+k} + u_{t+k}^T R u_{t+k}) \quad (2.16)$$

with  $N$  being the control horizon. The matrix  $A$  was composed of the delayed joystick spring-mass system  $A_j$  and delayed target dynamics  $A_s$ :

$$A = \begin{pmatrix} A_s & \dots & 0 \\ & A_j & \dots & 0 \\ 0 \dots 1 & 0 \dots -1 & 0 & \dots & 0 \end{pmatrix} \quad (2.17)$$

with the number of zeros on the last line depending on the visual and motor delays. The transition matrix for the spring-mass system is a 2x2 matrix, which is augmented to account for the visual and motor delays:

$$A_j = \begin{pmatrix} 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & & \ddots & \\ 0 & \dots & 1 & 0 & \dots \\ 0 & \dots & 1 & 1 & \dots \\ 0 & \dots & -0.01 & 0.8 & 1 & \dots \\ 0 & \dots & & & & \ddots \\ 0 & \dots & & & & & 1 \\ 0 & \dots & & & & & 0 \end{pmatrix} \quad (2.18)$$

with the number of leading and ending ones depending on the visual and motor delays, respectively. The delayed target dynamics is represented as

$$A_s = \begin{pmatrix} 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ 0 & & \ddots & \\ 0 & \dots & C_1^0 & C_2^0 \\ 0 & \dots & A_{1,1}^0 & A_{1,2}^0 \\ 0 & \dots & A_{2,1}^0 & A_{2,2}^0 \end{pmatrix} \quad (2.19)$$



with  $A$  and  $C$  corresponding to the matrices of the actual state space representation of the target signal (eq. 2.15). The control sequence that minimizes the cost function at time  $t$  is:

$$u_t = -H_t^{-1}F_t s_t \quad (2.20)$$

where  $F_t = \hat{A}^T Q \bar{C}$  and  $H_t$  is the state to observation matrix,

$$H_t = \bar{C}^T Q \bar{C} + \bar{R} \quad (2.21)$$

Augmented matrices  $\hat{A}$ ,  $\bar{C}$  and  $\bar{R}$  were defined as follows:

$$\hat{A} = \begin{pmatrix} A \\ A^2 \\ \vdots \\ A^N \end{pmatrix} \quad \bar{C} = \begin{pmatrix} B & & & \\ AB & B & & \\ A^2 B & & \ddots & \\ \vdots & & & \\ A^{N-1} B & \dots & & B \end{pmatrix} \quad \bar{R} = \begin{pmatrix} R & & \\ & \ddots & \\ & & R \end{pmatrix} \quad (2.22)$$

At every time step, the state vector was updated with new observation data by means of a Kalman filter.

# Chapter 3

## Information rate variations and perceived mental demand in visuo-motor tracking task

### 3.1 Introduction

Recall from Section 1.4 that in a two-step cognitive process involving a perceptual process and an action selection process, the total information cost is the sum of two KL divergence terms, each representing the information processed to achieve an update from a prior to a posterior belief:

$$Cost = KL(p(x'|x)||p_0(x')) + KL(p(y|x')||p_0(y)) \quad (3.1)$$

Complementing this information cost with the predictive information, one should obtain the total information transfer of this whole process. As mentioned before, the more informative the priors are, the smaller the information cost it incurs in the process. Predictive information is representative of how informative these priors are. Specifically, it was mentioned that for a second order autoregressive perceptual process, the prior can be expressed as  $p(x'_t|x'_{t-1}, x'_{t-2})$ . The same applies to the prior of an autoregressive action selection process:  $p(y_t|y_{t-1}, y_{t-2})$ . If these priors are used by subjects in the visuo-motor tracking task, predictive information will reflect how informative these priors are with respect to the task. Complementing the information cost, it will yield the total information transfer of the process.(eq. 1.25)

As mentioned before, the main goal of the first study was to identify and validate information measures that could approximate information cost and predictive information in a cognitive process. The derivation of feedback (FB) and feedforward (FF) information measures presented in the previous study can help discern and meaningfully quantify the cognitively demanding portion of information processing, as opposed to the more automatic processes. It is therefore proposed that FB and FF measures should be considered

as an empirical approximation of the information cost and predictive information respectively. Moreover, it is proposed that FB, but not FF measure, could inform the perceived cognitive effort of the task. In the current study, more properties of these measures will be explored.

Specifically, experimental conditions are designed to isolate the effect of signal predictability from that of signal speed, given that these two properties are closely coupled in the previous experiment. Secondly, tracking performance in response to added motor delay is analysed and used to test the theory concerning changes in prediction quality. Lastly, subjects' self-reported perceived task demands are compared against different task manipulations to gain insight on what factors constitute a cognitively demanding task for the subjects.

### 3.1.1 Speed vs predictability

Both the experimental results and model simulations from the first study convincingly showed that feedback information increases and feedforward information decreases as the visual signals become less predictable. While these are not surprising results, we aim at complementing these findings by discounting the potentially confounding influence of signal speed in order to isolate the effect attributable exclusively to prediction.

Speed and predictability of signal usually go hand-in-hand but their implications on the statistics structure could be very different. Changes in signal predictability directly affect the upper-bound of task-relevant information a prior can represent. As postulated by the current framework, these changes should be reflected in the FF information measure. Unlike signal predictability, signal speed might have arbitrary effects on the statistical distributions of task-relevant variables. In light of this, extra caution is taken to create task conditions to decorrelate these two properties so as to rule out the possibility that the observed changes in FB and FF information were due to some systematic yet irrelevant effect of signal speed.

### 3.1.2 Changing prediction horizon

As presented in the previous study, the predictability of a signal can be changed by controlling the amount of noise in the signal. This is precisely the mechanism used to generate signals of different predictability in the previous and current study. One of the objectives of this study is to explore other factors that could influence quality of prediction and to observe whether FF information varies in a manner predicted by the current framework. One factor that could influence the quality of prediction without having to change the signal involved is the prediction horizon. Prediction horizon is defined as how far ahead in the future the prediction is made. For an autocorrelated noisy signal, like those used in the previous and current experiments, it is observed that the quality of optimal prediction decreases as prediction horizon increases. This effect is due to accumulation of noise, and is demonstrated by training second-order autocorrelated (AR2)

models to predict a given signal (shown in blue, right panel of Figure 3.1) at different prediction horizons. The quality of prediction is assessed with innovation, defined as the difference between the model prediction and actual target it aims at predicting.

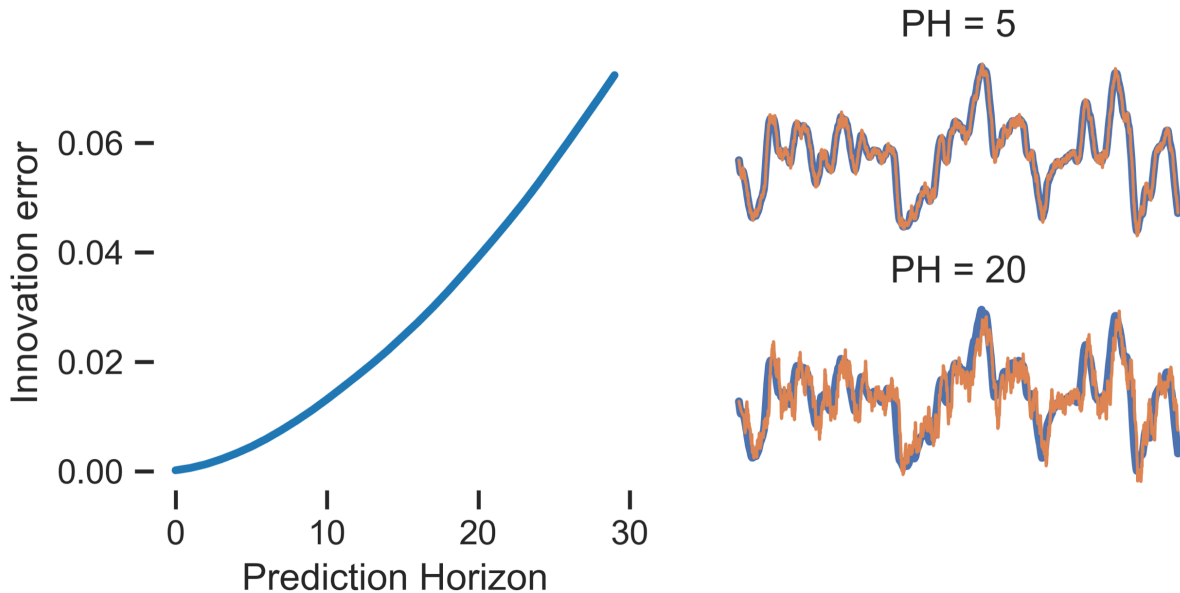


Figure 3.1: **Quality of prediction drops as prediction horizon increases.** (Left) Relationship between average amount of innovation errors and the prediction horizon of the model. (Top right) Prediction of an AR2 model (orange) overlaid on the signal it is predicting (blue) with prediction horizon set at 5 time points ahead. (Bottom right) Same as top but with prediction horizon set at 20 time points ahead.

Applying this observed effect of prediction horizon on quality of prediction in the current experimental setting, it is postulated that the quality of prediction used by the subjects will decrease if they tried to predict farther ahead in the future, i.e. longer prediction horizon.

In a visuo-motor tracking task, it is assumed that subjects do use prediction to reduce tracking error by anticipating where the signal will be in the future. This is shown in many studies, especially for sinusoidal signal tracking (Viviani and Mounoud, 1990; Brenner and Smeets, 2015; Day and Lyon, 2000; Franklin and Wolpert, 2008; Saunders and Knill, 2005). The choice of their prediction horizon should match their visual motor delays (VMD) if the goal is to cancel the lag between the target and the cursor. VMD is the inherent delay in motor response in respect to the visual stimuli (Carlton, 1981; Smith and Bowen, 1980; Smith et al., 1970). Since this mainly arises from the neurological and physical system, it is generally rather constant within subjects and might not be easily manipulated. However, instead of trying to change the innate delay of the human visual-motor system, which could be very difficult to do, if at all possible, one could manipulate the perceived VMD by adding a delay to the presentation of the cursor subjects were

controlling. Subjects' motor responses are coupled to the cursor locations on the screen, and this is the feedback they rely on understanding their motor outputs. If a delay is added to this updating of cursor location, subjects will perceive their VMD to be longer. By manipulating the amount of this extra delay added to the update of cursor location, we control the minimum horizon needed to generate predictions that could lead to an on-time tracking response. There is a wealth of studies on tracking performance with delayed feedback, they in general suggest that tracking performance deteriorates as the delay grows and support the role of prediction in maintaining performance (Smith and Bowen, 1980; Stepp, 2009; Foulkes and Miall, 2000; Miall and Jackson, 2006; Rohde et al., 2014). In particular, Vercher and Gauthier (1992)'s study also provided evidence of a changing prediction horizon as the authors showed that in some conditions with delayed feedback, subjects' arm movement would be leading the presented target. Interestingly, they also found that this effect is restricted to feedback delay below 300ms. Beyond this threshold, subjects' arm movement would always lag behind the target. These results seem to also hint at a limit as to how far the prediction horizon can stretch, which is coherent to what is proposed here. The longer the added delay, the farther ahead in the future subjects need to predict. As shown from the example above, with a longer prediction horizon, the quality of prediction is expected to drop. The experimental manipulation of added motor delay would therefore provide an opportunity to test how FF information measure responds to this drop in predictive information.

### 3.1.3 Perceived task demands

Last but not least, in order to investigate how subjects perceive the mental demand of different task conditions, a questionnaire is included at the end of trials to survey subjects' self-reported level of mental and physical demand of the trial. By studying the correspondence between different task manipulations, such as signal speed, predictability and added motor delay with the subjects' perceived mental demand of the task, one can understand what are the factors that make a task mentally demanding for the subjects. As mentioned in Chapter 1, task difficulty and effort are not redundant concepts, it is therefore important to note that the factors that contribute to the perceived task difficulty need not to be the same influencers behind perceived effort spent in the task. Nevertheless, the questionnaire could reveal task features that subjects would deem as requiring effort to succeed, regardless of whether they expend the required level of effort at the end. This could still help gain insight into the mechanism in which cognitive effort is perceived.

Pupil dilation is one of the physiological measures that was developed over the years to investigate the perception of effort. It was reported that perception of both physical and mental effort is associated with an increase in pupil diameter (Zénon et al., 2014; Zenon et al., 2019). Using pupillometric data and the task demand measures, the correspondence between this physiological measure and the self-reported levels of task demand is also explored.

## 3.2 Methods

### 3.2.1 Participants

Thirteen right-handed subjects (two males) of age between 19 to 24 years old were included in this study. The experiment is approximately one hour long and contains 75 25-second long trials. Each participant is reimbursed 10 euros for an hour at the end of the experiment.

### 3.2.2 Experimental design and procedure

Three types of manipulations are introduced in this experiment, they are signal predictability, signal speed, and added motor delay (AMD). Like the signals presented in the first experiment, the signals in this experiment are also generated by processing white noise through a sinusoidal filter:  $a_1x_t = \theta_t + a_2x_{t-1} + x_{t-2}$ , with  $a_2 = -2\sqrt{a_1} \cos(\frac{\pi}{100})$ . The complexity of the signal is controlled by the parameter  $a_1$ . We first generated signals of 3 different levels of complexity using parameter values 1, 1.05 and 1.2. Then, to create signals that allow us to explore the effects of speed and predictability separately, the most complex level of signals are duplicated and slowed down and the most predictable level signals are duplicated and sped up to create 2 new conditions such as slow but unpredictable signals and fast but predictable signals. There are eventually in total 5 groups of different signal predictability and speed. Each group of signals generated is tested for 3 different AMDs: 17ms (corresponding to 1 frame of a 60Hz display, so no added delays), 150ms (9 frames) and 280ms (17 frames). There are therefore in total 5 (groups) x 3 (AMD) = 15 different conditions in this experiment. There are 5 trials for each condition, making up a total of 75 trials in an experiment.

The instruction given to subjects in this visuo-motor tracking task is to aim at placing the cursor on the moving target as best as they could. They exert control on the cursor by means of a joystick. Throughout the whole experiment, subjects have to keep their head still on the head stand anchored to the desk. They are also instructed to fixate on the crosshair at the centre of the screen during the trials. Both measures were taken to minimise head and eye movement that could compromise pupil diameter recording.

The experiment is split into blocks of 9-10 trials and trials are presented in a random order. We use different colours of the cursor to provide cues for different AMDs. Orange represents the long delay, blue medium and green no added delay. But regardless of the colour cues or task conditions, subjects' goal in this experiment is the same, that is to try to place the cursor on the moving target. The purpose for a colour cue for the AMDs is to make it unambiguous from the beginning of trial when the VMD will be longer than usual so as to allow subjects to adopt their strategies accordingly.

At the end of some of the trials, subjects are presented with two selected questions from the NASA Task Load Index questionnaire (Hart and Staveland, 1988) on perceived level of mental and physical demands of the trial they have just done. The complete questionnaire

is first shown to the subjects before the experiment so as to ensure their understanding of the subtle differences between the different items surveyed by the questionnaire, including but not limited to the differences between mental demand, physical demand, effort and performance. After the target and cursor both disappear from the screen after the trial, the first question is presented with a ruler scale at the bottom of the screen. Subjects can then respond to the question by placing the cursor on the scale using the joystick and then validate the chosen location with a key press.

## 3.3 Data analysis and results

### 3.3.1 Signal property measures

In the current experiment, signal predictability and speed are defined by sample entropy and average absolute derivative of signal positions, respectively. Sample entropy is a method of measuring randomness in the signal (Pincus, 1991; Richman and Moorman, 2000; Delgado-Bonal and Marshak, 2019) by counting and comparing ‘match’ against ‘possible’ pairs identified in the signal. Sample entropy measure for a signal or process with  $N$  data points is defined for a chosen length of template ( $m \leq N$ ) and tolerance measure  $r$ . The signal is first broken down into templates of length  $m$  (with overlapping). For each template, its distances with respect to all other templates are measured by a predefined distance function and a ‘possible’ pair is identified when this distance is within the tolerance  $r$ . When a ‘possible’ pair is identified, it signifies that this pair of templates are similar to each other, as far as  $m$  data points are concerned. For each ‘possible’ pair of templates, the one subsequent time point following each of them ( $m + 1$ ) are also compared. If this is once again below the tolerance  $r$ , this pair will be further identified as a ‘match’ pair. The sample entropy is then computed as a function of the ratio between the sum of all ‘match’ pairs  $A^{m(r)}$  and the sum of all ‘possible’ pairs  $B^{m(r)}$  :

$$SampEn(m, r) = \lim_{N \rightarrow \infty} -\log \frac{A^{m(r)}}{B^{m(r)}} \quad (3.2)$$

$$SampEn(m, r, N) = -\log \frac{A^{m(r)}}{B^{m(r)}} \quad (3.3)$$

The correlation matrix plot in Figure 3.2 sums up the experimental design regarding the properties of the visual signal. The colour represents the five sets of different sinusoidal parameter/speeding factor combinations used to produce these signals.



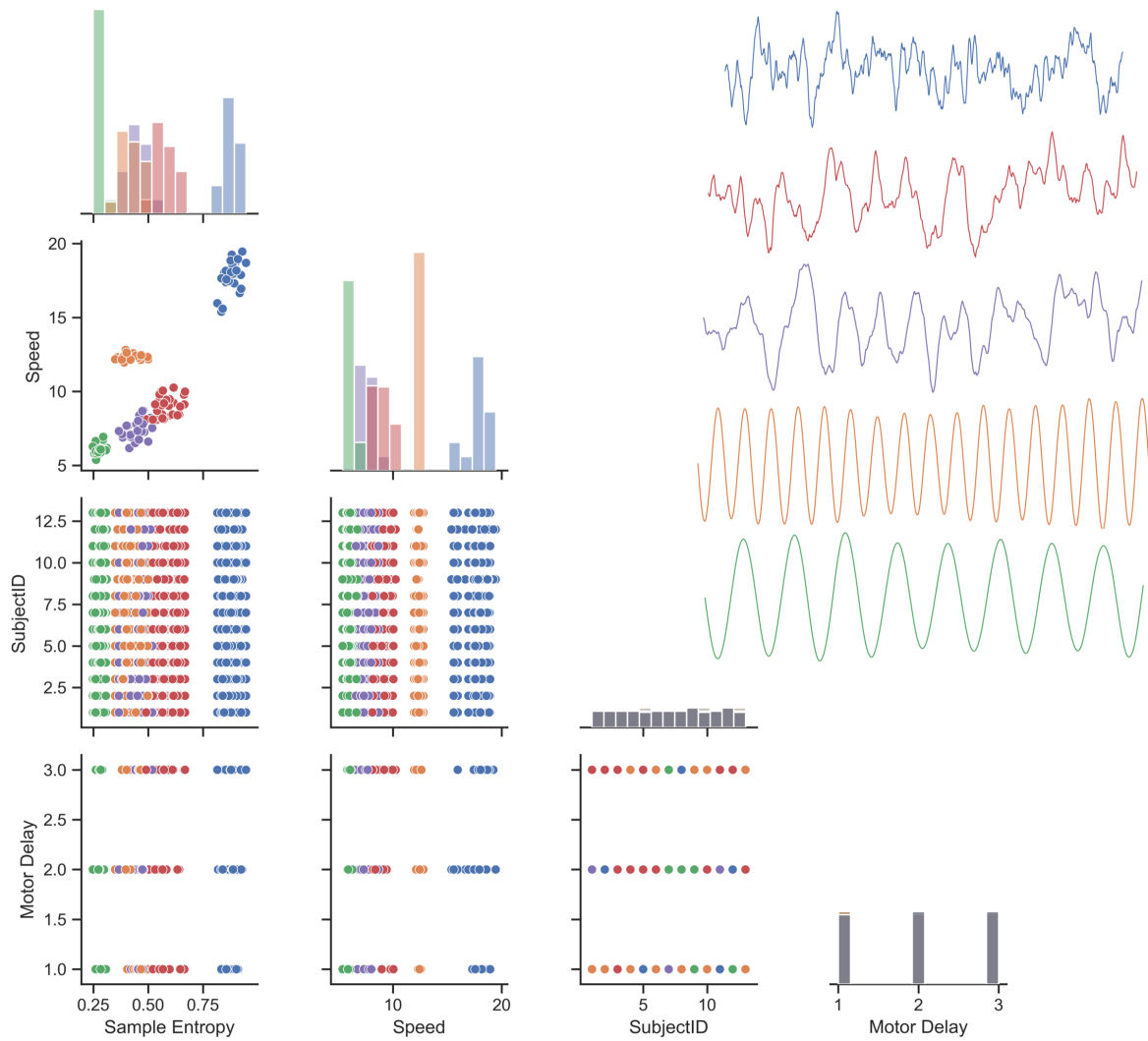


Figure 3.2: **Experimental conditions pairplot with example signals.** Each colour represents one of the 5 different configurations of parameters that generate the data. (Top right) Example signals for each speed/predictability configuration.

### 3.3.2 Information measures

FB and FF information measures will be estimated by means of Gaussian copulae (Ince et al., 2017) and used for comparison in the current experiment. Recall from the previous chapter 2 (Lam and Zénon, 2021), FB information is defined as:

$$I_{FB} = I(Y_t; \{X_{t-VMD}, X_{t-VMD-1}\} | \{Y_{t-VMD}, Y_{t-VMD-1}, Y_{t-1}\}) \quad (3.4)$$

Total information is defined as:

$$I_{total} = I(Y_t; \{X_t, X_{t-1}, X_{t-VMD}, X_{t-VMD-1}\}) \quad (3.5)$$

And FF information can be found by:

$$I_{FF} = I_{total} - I_{FB} \quad (3.6)$$

### 3.3.3 Signal predictability and speed predicts feedback information rate

In the current experiment, added motor delays are implemented by adding a delay to the display of the cursor location on screen. The limitation to this approach is that subjects could ignore the cursor location on the screen and choose to rely on the proprioceptive feedback from the control of the joystick to gauge and guide their tracking performance. To see whether that is in fact the case, one can compare the performance delays in different AMD conditions for the same type of signal, i.e. similar predictability and speed. If subjects were ignoring the actual cursor location shown, we should see that, when deducting the AMDs from their total tracking delays with respect to the signal, it should be constant for all different AMD conditions. However, performance delays less than the linear increase of AMDs will mean that subjects were actively trying to incorporate a longer prediction horizon in their performance to account for the extra delays in the cursor display.

Figure 3.3 (top) shows the location of the cursor (light blue) overlaid on that of the moving target (dark blue) during a trial of no added motor delay. It can be seen that the subject is quite successful in catching up with the moving target, cancelling their visual motor delay.

For trials with  $AMD > 0$ , if subjects are ignoring the cursor location on screen, and relying on just the joystick configuration to perform the tracking task, one would expect to observe performance such as this simulation shown in the bottom panel of Figure 3.3, with the cursor being constantly lagged behind the moving target.

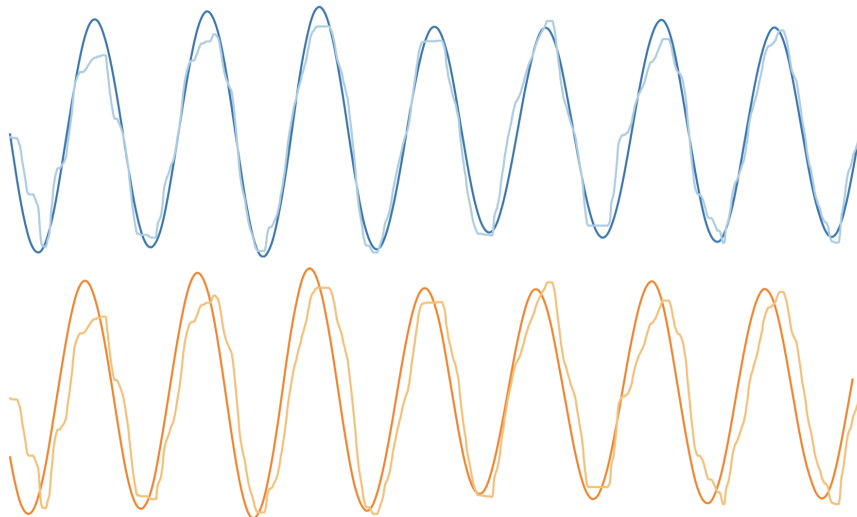


Figure 3.3: **Simulated performance delay.** (Top) A subject's performance at  $AMD = 0$  condition. (Bottom) Simulated performance for an  $AMD = 17$  condition by shifting the performance by 17 frames.

Looking at the subjects' performance delays, especially for the more predictable signals, it is observed that subjects do not demonstrate performance delays that are as long as the one shown above. Figure 3.4 shows the real tracking performances of a subject for AMDs set at 9 and 17 frames respectively. It is obvious that this subject was still succeeding in cancelling the total VMD given the added motor delay, signifying the adoption of a longer prediction horizon.

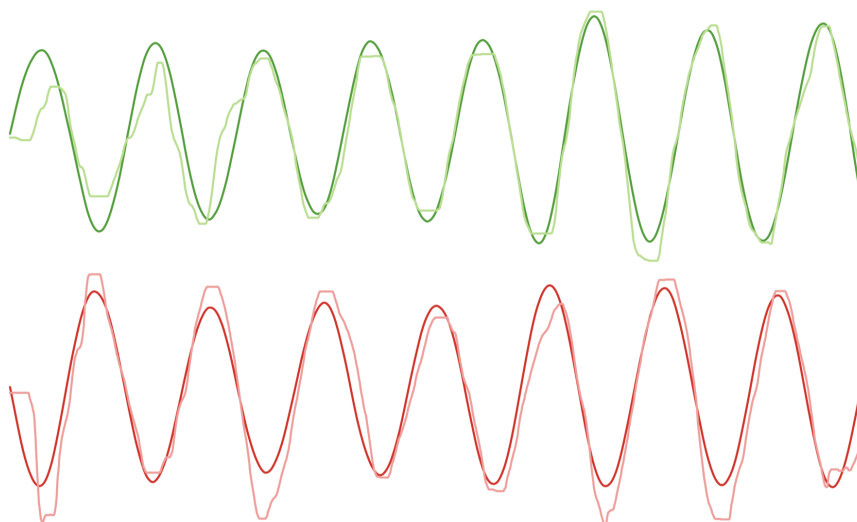


Figure 3.4: **Example performance for AMD=9 and AMD=17.** (Top) A subject's performance at AMD=9 condition. (Bottom) A subject's performance at AMD=17 condition.

Even with the added motor delays, most subjects still manage to catch up with the moving target and eliminate the VMDs. This, however, will prove to be more difficult to achieve as the predictability in the signal decreases. Figure 3.5 shows a systematic comparison of the performance delays for different added motor delay conditions. Some simulations of performance delays (shown in grey) are generated by shifting the performance delays observed in the no added motor delay condition by exactly the amount of the extra motor delays (Figure 3.3, bottom panel). This is to demonstrate what performance delays could be like if subjects do not adopt their prediction horizons to account for the added motor delays.

By comparing the actual performance delays (in colour) to that of the simulated ones (in grey), one can gauge how much subjects have managed to reduce their performance delays by adopting the appropriate prediction horizon. It is clear that for conditions that are not very predictable to start with, the added motor delays in the conditions are translated almost linearly to the performance delays, just like in the simulated ones. This could mean one of two things: first, subjects do not alter their prediction horizons when signal predictability is low; or, second, subjects might do alter their prediction horizon, but it still fails at reducing the performance lags.

These results provide some evidence that subjects have indeed used a longer prediction horizon in response to added motor delays, meaning they are adjusting their predictions in response to a longer VMD, at least for the most predictable signals. The analysis regarding the corresponding FF and FB information rate is presented in the following section.

### 3.3.4 FB and FF information as functions of task manipulations

To compare the effect of different task manipulations, a generalised linear mixed model (GLMM) was applied to the data to try to predict feedback and feedforward information rate using task variables as fixed effects and individual subjects as random effect. The following model is chosen by comparing its Akaike information criterion (AIC, Akaike (1973)) value to a model that excludes subjects as random effect.

$$Feedback \sim 1 + Motor\ Delay + Sample\ Entropy + Speed + (1|Subject)$$

All predictor values are standardised so that the coefficients can be compared. The variance inflation factors (VIFs) are computed for all the predictors and the values are shown in the table 3.1.

Sample Entropy	Signal Speed	Subject ID	Motor Delay
3.329	3.329	1.000	1.000

Table 3.1: VIFs of condition variables.

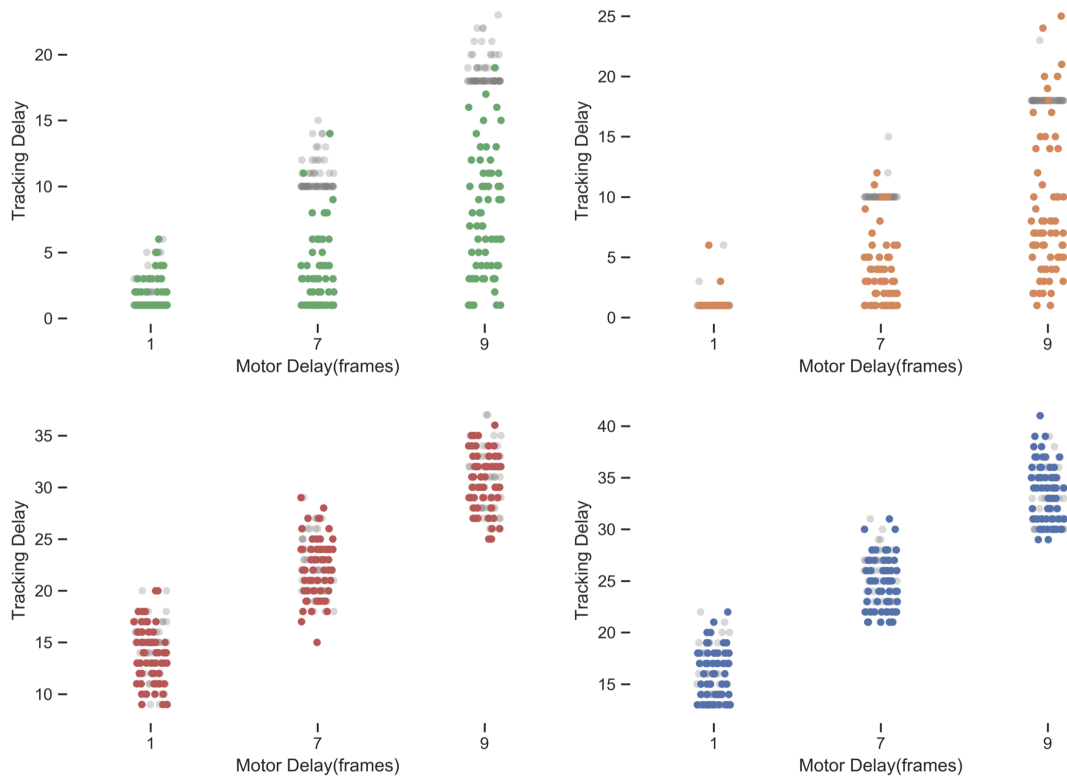


Figure 3.5: **Distributions of performance delays observed for tracking different signals and different AMD** (Top) Actual performance delays (green and orange) of the two most predictable conditions overlaid on simulated performance delays in grey. (Bottom) The same data shown for the two most unpredictable conditions. Please refer to figure3.2 for example signals of these conditions.

As a rule of thumb, a VIF value up to 5 is interpreted as an indicator of moderate correlation between predictors, which does not preclude the inclusion of the variable in the model.

The GLMM results (Figure 3.6) showed that feedback information is best predicted by sample entropy of signal, shown by its highest coefficient value,  $\beta = 0.084$ ,  $F(1, 987) = 1269.8$ ,  $p = 0$ . The positive coefficient suggests that signals with higher sample entropy, i.e.: more unpredictable signals, leads to higher feedback information rate. This is coherent to findings presented in the previous study, suggesting a higher information cost is incurred for processing of noisier signals. On the other hand, signal speed and added motor delay are also found to be significant predictors of feedback information rate, speed:  $\beta = -0.040$ ,  $F(1, 987) = 298.7$ ,  $p = 0$ ; AMD:  $\beta = -0.01$ ,  $F(1, 987) = 112.9$ ,  $p = 0$ . Despite being positively correlated to sample entropy, signal speed has an opposite effect than that of signal entropy on feedback information rate. Although not as strong as sample entropy in terms of coefficient value, both speed and AMD have very robust effects.

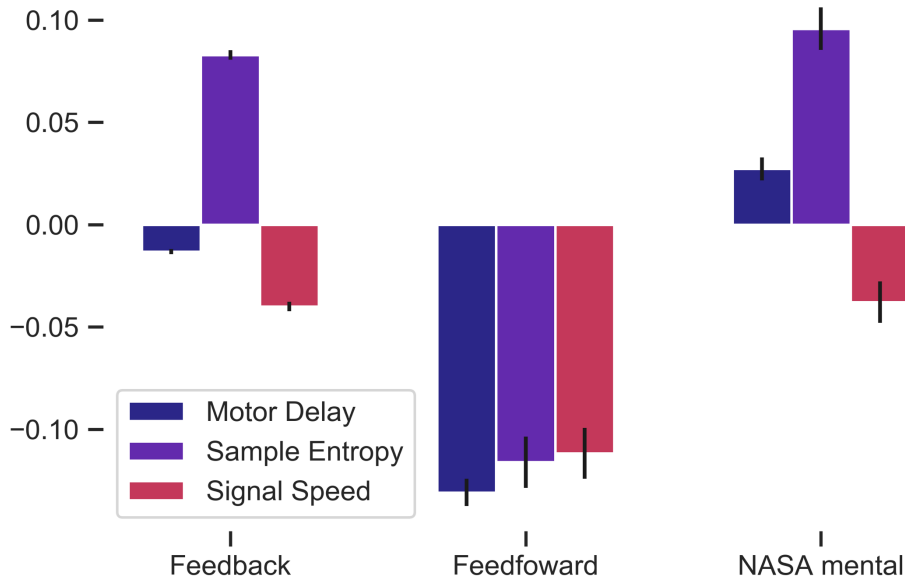


Figure 3.6: GLMM coefficients on normalised data for predicting FB, FF and NASA mental demand rating with task conditions.

Using the same method, it is shown that FF information also varies as a function of all task manipulations.

$$\text{Feedforward} \sim 1 + \text{Motor Delay} + \text{Sample Entropy} + \text{Speed} + (1|\text{Subject})$$

Results show coherence to previous findings in which FF information decreases with signal complexity, further strengthening the proposal that FF information is representative of predictive information,  $\beta = -0.116$ ,  $F(1, 992) = 86.07$ ,  $p = 0$ . Moreover, added motor delay significantly decreases FF information,  $\beta = -0.131$ ,  $F(1, 992) = 384.4$ ,  $p = 0$ . (Figure 3.7) This could be explained by the fact that the quality of prediction is compromised due to a lengthened prediction horizon. This effect is observed for all conditions with signals varying in predictability and speed. Signal speed is also found to be negatively correlated with FF information, an effect similar to that observed between speed and FB information,  $\beta = -0.112$ ,  $F(1, 992) = 80.18$ ,  $p = 0$ .

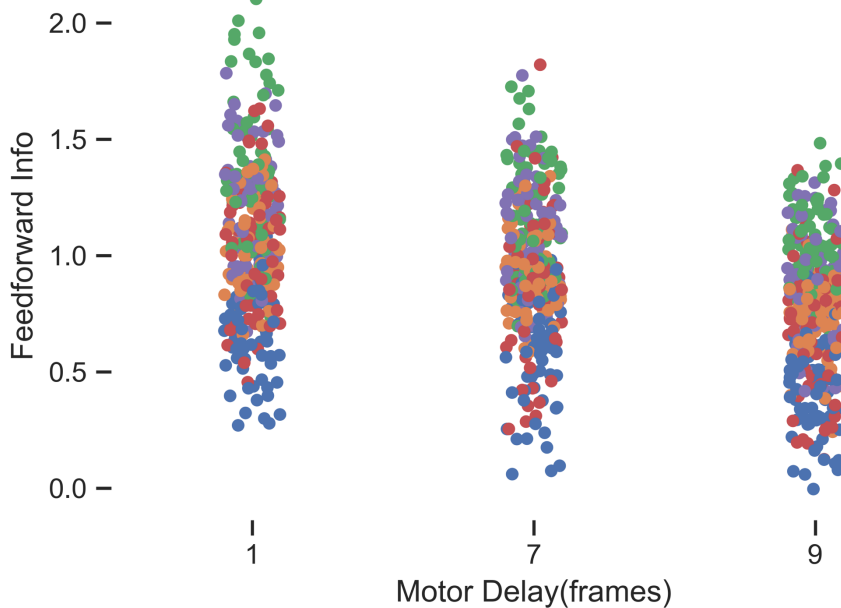


Figure 3.7: **Feedforward information in different AMD conditions.** Different colours code for the different signal conditions.

### 3.3.5 Perceived mental demand

To investigate what task properties are perceived as mentally demanding, another GLMM is run to predict subjects' reported mental demand using signal speed, sample entropy and added motor delay as fixed effect predictors and individual subject as random effect predictor:

$$NASA\ mental \sim 1 + Motor\ Delay + Sample\ Entropy + Speed + (1|Subject)$$

The VIFs for all the predictors are shown in Table 3.1 since these are the same predictors as those used for feedback information. Results show that while all task properties seem to influence perception of task demand, sample entropy is by far the strongest predictor,  $\beta = 0.097, F(1, 572) = 83.82, p = 0$ , (Figure 3.6). Perceived mental demand seems to increase with both increase in sample entropy and added motor delay,  $AMD : \beta = 0.027, F(1, 572) = 23.34, p = 0$ . However, similar to its effect on FB and FF information rate, signal speed seems to be negatively correlated with perceived mental demand,  $\beta = -0.038, F(1, 572) = 13.78, p = .0002$ . Subjects seem to think faster signals are less demanding. Analysis shows that baseline corrected pupil dilation during trial is positively correlated with perceived mental demand, which suggests that subjects are also experiencing increased effort in response to higher task demand.  $\beta = 0.16543, F(1, 562) = 7.3773, p = 0.007$ .

Since both sample entropy and added motor delays are proposed to influence quality of prediction, we test for their interaction effect on reported mental demands to try to lay out a link between perceived effort and prediction:

$$NASA\ mental \sim 1 + Sample\ entropy \times Motor\ delay + Speed + (1|Subject)$$

Results indeed reveal a significant interaction effect,  $F(1, 571) = 8.78, p = 0.003$ , indicating in face of added motor delays, rise in reported mental demands are higher for more predictable signals than unpredictable ones (Figure 3.8). This indeed might be related to the fact that for predictable signals, the effect of added motor delays on quality of prediction is more substantial given the higher initial values compared to unpredictable signals.

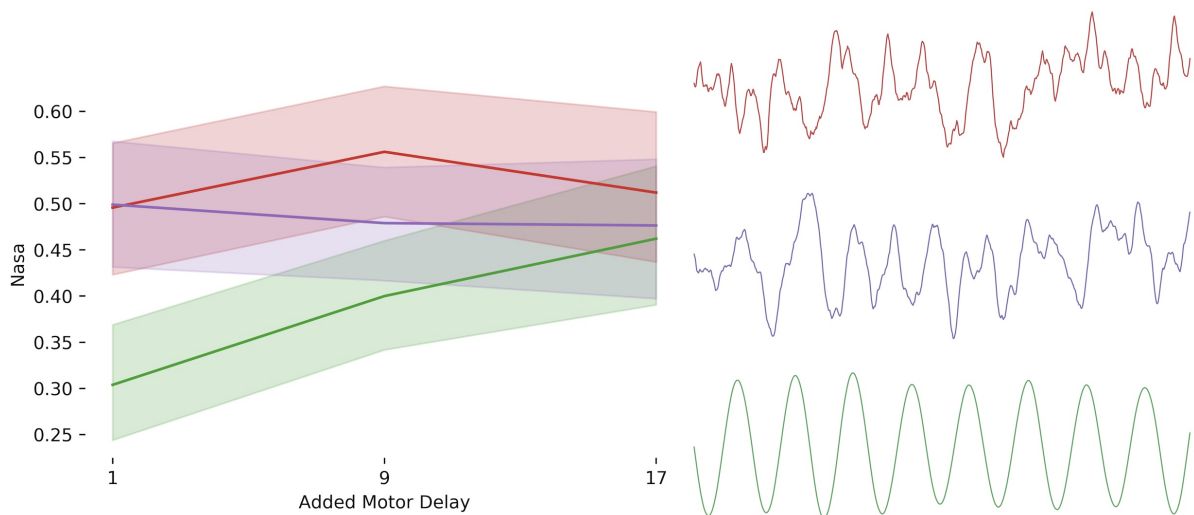


Figure 3.8: **NASA mental demand ratings of conditions differ in signal predictability and added motor delay.** (Left) Subjects' average NASA mental demand ratings of signals of 3 conditions across different AMDs. (Right) Examples of the colour-coded conditions. Here only the conditions with non-altered speed are shown for the sake of clarity. However, the reported data analysis includes data from all conditions.



### 3.4 Conclusion

In this experiment, the role of signal predictability in influencing FB and FF information is reconfirmed through strong evidence shown in experimental data. To reiterate, FB and FF information measures are the empirical approximation of information cost and predictive information in a cognitive process. Therefore, the experimental data so far have shown that signal predictability dictates information cost and it does so by effectively limiting the theoretical upper bound of predictive information in a cognitive process. The amount of predictive information used in a cognitive process is also susceptible to any other factors that could affect quality of prediction. Using the added motor delay paradigm, it is demonstrated that subjects do adapt their prediction horizons to accommodate extra VMD and subsequently sustain a decline in their FF information. It is worth mentioning that when observing the performance delays in all conditions with high sample entropy (less predictable signals), it would almost seem like predictive information is not in use at all because subjects are in general incapable of cancelling any VMD. However, not only are the FF information measures in these conditions shown to be non-zero, they are also shown to decrease with added motor delays. This result emphasises the inadequacy of using only tracking latency or squared errors to quantify tracking performance. Another objective of this study is to isolate the effect of signal predictability from that of signal speed regarding FB and FF information. These two properties, while being positively correlated with each other, are found to be exerting opposite effects on both FB and FF information. The relationship between signal predictability and FB and FF information has been discussed in detail in Chapter 2 and it is instrumental in validating these information measures. Signal speed and its effect, however, might require some new insights to make sense of. One possible explanation that fits in the current framework could be that the increase in speed encourages subjects to increase their error tolerance, a standard on which they rely on to regulate and exert control over their cognitive process. A higher error tolerance would then lead to a lower FB information rate. This could also help explain why a higher speed is associated with a decreased self-reported mental demand.

Lastly, through the NASA-TLX questionnaire, links are established between perceived mental demand and some of the task conditions. As expected, subjects are sensitive to manipulation regarding the statistics and dynamic of the signals. Increase in sample entropy of signal is found to be the dominant factor that increases the self-reported mental demand of a trial. As a reminder, high sample entropy is also associated with decrease in predictive information and increase in information cost, as signalled by its correlation with FF and FB information respectively. This highlights how FB, but not FF, information might signal perception of task demands. Another task condition that is reported to be demanding is AMD. To perform the task with added motor delays, subjects are required to change their prediction horizon from a habitual one that accounts for their normal visual-motor delay, to a novel one that also accounts for the extra delay imposed on them in the trial. The overriding of habitual prediction horizon might require extra cognitive

control (Shah and Oppenheimer, 2008), or learning cost, as formulated in Eq.1.20, that is not captured by the FB information in this task. This might in turn explain the increase the perceived mental demand of added motor delay trials despite FB information decreases with increased AMDs.



# Chapter 4

## Information processing rate transferability in dual-task

### 4.1 Introduction

The studies presented thus far focus on demonstrating how FB and FF information vary with signal-related task manipulations and results are in agreement with the hypothesis that FB information is an empirical approximation of the information cost of a cognitive process whereas FF represents the predictive information. In the current experiment, cross-task interference on performance, and more importantly, on information cost and predictive information are studied in a dual-task paradigm. Through the study on cross-task interference effects, we aim at investigating potential correspondance between cognitive control and information cost.

#### 4.1.1 Cognitive control and information processing

A task is defined as being dependent on cognitive control if it cannot be executed simultaneously with another control-dependent task without sustaining performance decline (Posner et al., 2004; Shiffrin and Schneider, 1977). Cross-task interference is therefore instrumental in defining cognitive control. The dependency on control of a visuo-motor tracking task will therefore be put to test in a dual-task paradigm involving an N-back (NB) task, a proven control-dependent task (Kirchner, 1958; Novick et al., 2013; Drollette et al., 2012). A performance decline in the visuo-motor task would indicate that it is indeed control-dependent. Moreover, we will investigate the relationship between signal predictability and the degree of reliance on control in the task.

Besides cross-interference on performance, special attention will be paid to the cross-interference on FB and FF information of the visuo-motor tracking task. It should be made clear that while FB information is proposed as an approximation of the information cost of the cognitive process responsible for the visuo-motor tracking task, it is only part of the total information cost required for performing a dual-task trial. Both the N-back

task and potentially the switching between the tracking and N-back task would incur their own information cost that is not shown in the FB measure. Having said that, although the analysis of the FB and FF measures of the tracking task could only yield a partial picture of the dual-task performance, it might still provide important insights regarding resources allocation in face of changes of task demands.

### 4.1.2 Allocation of limited information processing capacity

The analysis of cross-task interference in performance would usually show a trade-off between tasks in a dual-task paradigm. However, it should first be stated clearly that performance and information cost are not equivalent measures. For instance, in the current experiment, total information transfer would be a closer approximation of performance than information cost. As mentioned in the section 1.4.1, total information transfer is the sum of information cost and predictive information. Information cost and information transfer, will only be equal if predictive information is proved to not exist. Analysis of cross-task interference in information cost is therefore an important distinction from that of performance. The derivation of FB and FF information would allow one to infer directly the cross-task interference with respect to information cost and to build hypotheses regarding the allocation of information processing capacity in face of dual-task demands.

As in the previous study, a survey is included to probe subjects' self-reported perceived mental demand of the task. A new survey over trial preference is also included to inquire subjects' willingness to repeat these trials.

## 4.2 Methods

### 4.2.1 Participants

This experiment has recruited twenty-one healthy right-handed subjects between 19-24 years old, of which 5 are male. Each participant came in for two hours of experiment in two consecutive days (one hour each day) and was remunerated 10 euros for each hour of their time.

### 4.2.2 Experimental design and procedure

The N-back task is chosen as the control-dependent task to be performed simultaneously with the visual-motor tracking task in this experiment. N-back task is known to be a task that requires attention and working memory to succeed (Kirchner, 1958; Kane et al., 2007; Jaeggi et al., 2010). In an N-back task, subjects are presented with a sequence of stimuli, and their task is to report whether or not the currently shown stimulus is identical to the Nth previous stimulus. An example of a sequence of stimulus and correct response of a 2-back task is as followed:

Stimulus	J	U	H	U	K	D	K	F	S	F	S
Response	-	-	0	1	0	0	1	0	0	1	1

The higher N is, the harder the task becomes as it requires more working memory (Kane et al., 2007; Jaeggi et al., 2010). In the current experiment, the N-back task stimuli are presented as auditory signals and subjects are instructed to respond with key presses. The use of auditory signals is to minimise its interference with the visual signals presented in the visuo-motor tracking task. To further reduce interference with the motor component during visuomotor-tracking, participants are not asked to respond to each letter presented during the trial. Rather, they are trained to respond only to the last letter presented before the trial's end. However, to ensure that this does not encourage participants to intentionally neglect the early stages of each N-back trial, the trial lengths are varied unpredictably and very short trials (with length N) are included as 'catch trials', to test whether participants are actually paying attention and are doing the task as instructed. Moreover, in an attempt to prevent participants from postponing the processing of the task until prompted, a tight deadline for response is introduced at the end of each trial. Once the visual target on the screen disappears, participants have 1.1s to key in their response, where the last letter plays the role of the target for the N-back task. The auditory N-back task consists of two levels of difficulty, modulated by varying N. The easy level is 0-back, in which participants only need to identify whether the letter is an 'A'. The hard level is maxN-back, with maxN corresponding to the participant's highest achievable N-back level, which is identified in the training session on the first day. During the training, subjects are instructed to complete blocks of 10 N-back trials starting from a block of 0-back trials. For each block of N-back trials, if the subjects

achieve an accuracy of at least 90% for the block, they will be presented with a block of  $(N+1)$ -back trials subsequently. Alternatively, if their accuracy for the block is between 60 and 90%, they will repeat another block of the same level. If their accuracy is below 60%, they will be presented with a block of  $(N-1)$ -back trials. The maximum difficulty is set at 5-back, meaning that even if subjects manage to reach 90% accuracy for a block of 5-back trials, they will only be asked to repeat another 5-back block instead of going to a 6-back block. Eventually, when a subject has done 3 blocks of trials of the same difficulty  $N$ , the training will be terminated and this  $N$  is defined as the  $\text{maxN}$  of the subject. After finding out the subjects' maximum achievable  $N$ -back levels ( $\text{maxN}$ ), they are presented with a few blocks of dual-task trials to get familiarised with the setting.

The visuomotor tracking task is the same as described in the previous experiments. For this experiment, there are only two difficulty levels of signal predictability, easy and hard, corresponding to parameter values 1 and 1.2.

The instruction for dual-task trials is simply to track a vertical bar moving on the screen as best as they could while concurrently also to engage in an auditory  $N$ -back task with keyboard response. Subjects are trained to associate the colour of the cursor in the visuomotor tracking task to the difficulty of the  $N$ -back task. For instance, when the cursor is blue, they should know that they are engaging in the  $\text{maxN}$ -back task whereas when it is green, it is a 0-back task. There are in total 96 trials in the experiment, evenly distributed among different combinations of conditions.

Similar to previous studies, Eyelink camera is employed to record pupil size for the whole duration of the experiment. To ensure the quality of the pupil size data, participants are asked to place their head on a fixed headstand and are instructed to fixate at a cross-hair at the centre of the screen during tracking. At the end of some trials, subjects are presented with the NASA-TLX questionnaire on subjective mental demand. Subjects are asked to assess, specifically for the trial they just completed, their perceived level of physical demand, mental demand, temporal demand, performance, effort and frustration. They respond by placing the cursor on a horizontal scale to indicate the appropriate level for each question.

Last but not least, a 'shopping' option is introduced at the end of some of the trials to probe participants' preference for different types of trials. At the beginning of the experiment, subjects are told that they will be asked to repeat some of the trials during the last 10 minutes of the experiment. They are also told that they can fill a cart with trials they prefer to repeat. Therefore, after some trials, subjects are presented with an 'add this trial to cart' option and an 'add a random trial to cart' option to allow them to decide whether or not they would like to repeat that particular trial. By imposing a fixed time for the repeating session, subjects should lose incentives in basing their choices on the length of trials and this might in turn allow for their choices to be more sensitive to other task conditions, such as difficulties in  $N$ -back task or VM task. This shopping option is designed only for the purpose of surveying their preference among trials of different conditions and they do not actually repeat any of the trials at the end.

### 4.3 Data analysis and results

In the analysis of the previous study, sample entropy replaces VM condition as a more accurate measure for signal predictability in face of influences brought by different signal speeds. However, in the current experiment, signal speed is no longer manipulated and is constant throughout each VM condition. For the sake of clarity, categorical VM conditions will be used instead of sample entropy to represent signal predictability. It is verified by extra analysis that replacing VM conditions by sample entropy does not cause any differences to the results presented below.

#### 4.3.1 Cross-task interference on performance

Following the common practice of dual-task analysis (Kahneman, 1973; Wickens, 1991), the performances of the individual tasks in a dual-task paradigm are compared. Performance in the visuo-motor tracking task is measured by the mean squared errors (MSE) between target and cursor locations on screen. The effects of task conditions on MSE are analysed by means of a generalised linear mixed model (GLMM). Several GLMMs are tested and compared based on the Akaike information criterion (AIC):

$$MSE \sim 1 + VM \text{ con} + NB \text{ con} + (1|Subject)$$

$$MSE \sim 1 + VM \text{ con} \times NB \text{ con} + (1|Subject)$$

$$MSE \sim 1 + VM \text{ con} + NB \text{ con}$$

$$MSE \sim 1 + VM \text{ con} \times NB \text{ con}$$

For the analysis of MSE, it is found that the GLMM using task conditions as fixed effects and subjects as random effect has the minimum AIC value, and is thus chosen:

$$MSE \sim 1 + VM \text{ con} + NB \text{ con} + (1|Subject)$$

Note that this procedure is repeated for all analysis and results will only be presented for the chosen model. GLMM result shows that both VM,  $\beta = 1.84$ ,  $F(1, 2010) = 14275$ ,  $p = 0$ , and NB conditions,  $\beta = 0.038$ ,  $F(1, 2010) = 6.017$ ,  $p = 0.014$ , are significant predictors of MSE (Figure 4.1, left, 4.2), suggesting a cross-task interference in performance of VM task by NB task, confirming the VM task as a control-dependent task.



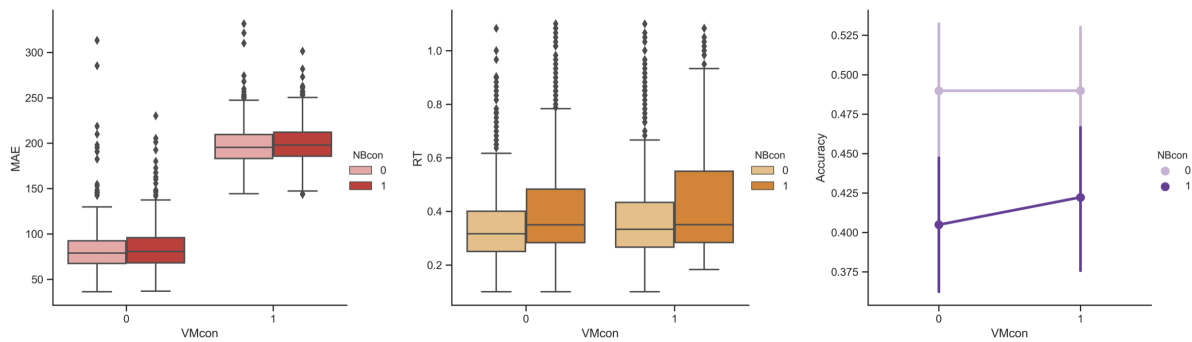


Figure 4.1: **Dual-task performance.** (Left) Mean absolute errors of tracking performance. (Middle) Reaction time at N-back task. (Right) Accuracy at N-back task,

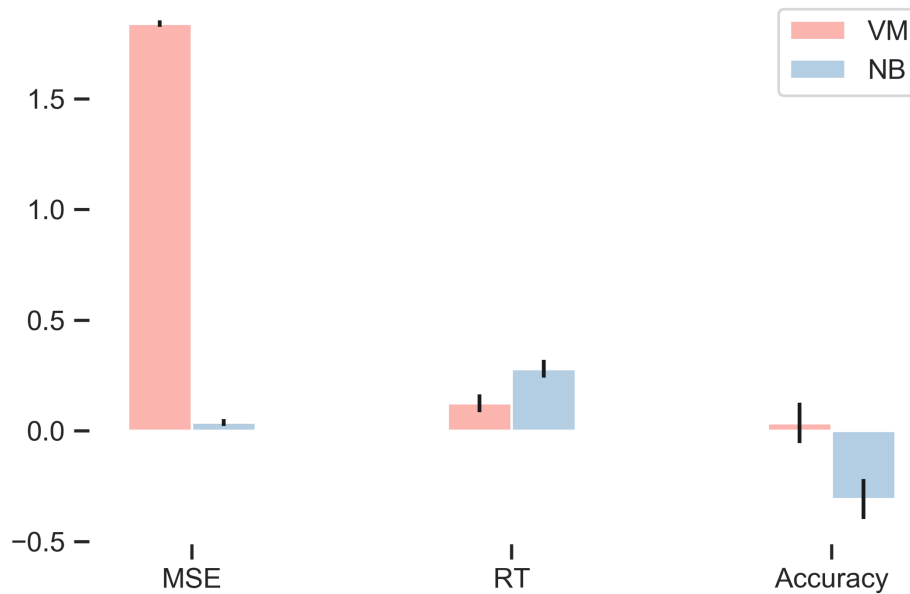


Figure 4.2: **GLMM results on dual-task performance.** Coefficient values of normalised variables predicting MSE, reaction time and accuracy of dual-task performance.

On the other hand, a reciprocal influence from VM task on NB task performance is found.

$$RT \sim 1 + VM \text{ con} + NB \text{ con} + (1|Subject)$$

$$Accuracy \sim 1 + VM \text{ con} + NB \text{ con}$$

N-back task performance is measured using reaction time and accuracy (Figure 4.1, middle and right) and results show that while NB task difficulty dominates the in-

fluence over both reaction time,  $\beta = 0.28, F(1, 1963) = 47.32, p = 0$ , and accuracy,  $\beta = -0.076, F(1, 1976) = 11.71, p < 0.001$ , harder VM tasks appear to also significantly increase NB task reaction times,  $\beta = 0.124, F(1, 1963) = 9.323, p = 0.002$ . NB accuracy, on the other hand, does not seem to be affected by VM task difficulty,  $\beta = 0.008, F(1, 1976) = 0.152, p = 0.696$ . This asymmetric influence of the visuo-motor tracking task difficulty on N-back task reaction time and accuracy could imply a different choice of speed/accuracy tradeoff in face of dual-task demands.

### 4.3.2 Cross-task interference on FB/FF information

Designed as a more cognitively demanding task, the maxN-back compared to the 0-back task requires more engagement of working memory, thus a higher cognitive load to the subject who is trying to perform the dual-task experiment. Even though the task is designed in a way that the modality of the two tasks should not overlap, there might still be a common information processing bottleneck for the two processes that could cause performance tradeoff when one of the tasks is more cognitively demanding. In this section, FB and FF information will be used to get a more detailed look into the changes in information cost and predictive information in the cognitive process involving visuo-motor tracking task during dual-task performance. Specifically, the effects of VM and NB task difficulty on FB and FF are investigated.

A GLMM is fitted to the data to predict FB information using both NB task difficulty, VM task difficulty and their interaction as fixed effects and subjects as random effect:

$$Feedback \sim 1 + VM\ con \times NB\ con + (1|Subject)$$

Results show that FB information is predicted by VM task difficulty,  $\beta = 1.692, F(1, 1916) = 2555, p = 0$ , as well as an interaction between VM and NB difficulty,  $\beta = -0.122, F(1, 1916) = 6.58, p = 0.010$ , (Figure 4.3). Feedback information rate is higher when VM task signals are unpredictable regardless of NB conditions. A difference in FB information between NB conditions can also be observed when signals are unpredictable.

Feedforward information approximates the predictive information in the cognitive process. Using experimental data, the sensitivity of feedforward information to task difficulty is investigated.

$$Feedforward \sim 1 + VM\ con + NB\ con + (1|Subject)$$

Results show that both harder VM and NB task conditions are significant predictors of lower FF information, VM:  $\beta = -1.72, F(1, 2013) = 8264, p = 0$ ; N-Back:  $\beta = -0.111, F(1, 2013) = 34.04, p = 0$ . (Figure 4.3)

Analysis of FB and FF information shows that these measures are first and foremost dictated by the predictability of the signals involved. As for the effect of NB task difficulty, it is shown that maxN-back does put pressure on both information transfer and predictive information of the VM task.

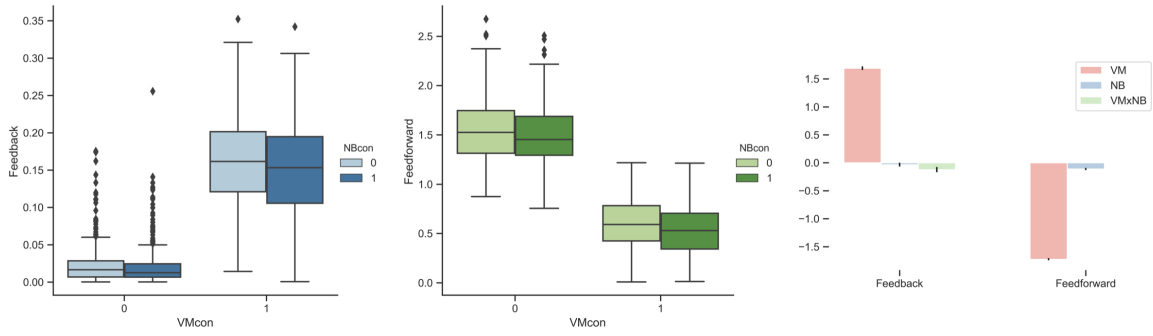


Figure 4.3: **FB and FF of VM task.** FB (Left) and FF (Middle) information measured in VM task. (Right) Coefficient values of normalised variables predicting FB and FF information of VM task.

### 4.3.3 Perceived mental demand and task conditions

In the current experiment, the maxN-back task is designed to be the ‘heavyweight’ cognitive task to load the cognitive system whenever it is involved, so that its effect on VM tasks can be studied. By surveying subjects’ perception of mental demand of the task, it is verified that this control has been successfully implemented. (Figure 4.4, left) A GLMM with interaction is chosen over a model without, based on its lower AIC:

$$NASA\ Mental \sim 1 + VM\ con \times NB\ con + (1|Subject)$$

N-back task condition is the most powerful predictor of perceived mental demand of the trial, given its highest coefficient value,  $\beta = 0.765$ ,  $F(1, 165) = 23.7$ ,  $p = 0$ , (Figure 4.5). However, although with smaller magnitude, visuo-motor task condition also affects subjects’ perception of mental demand,  $\beta = 0.326$ ,  $F(1, 165) = 4.29$ ,  $p = 0.039$ .

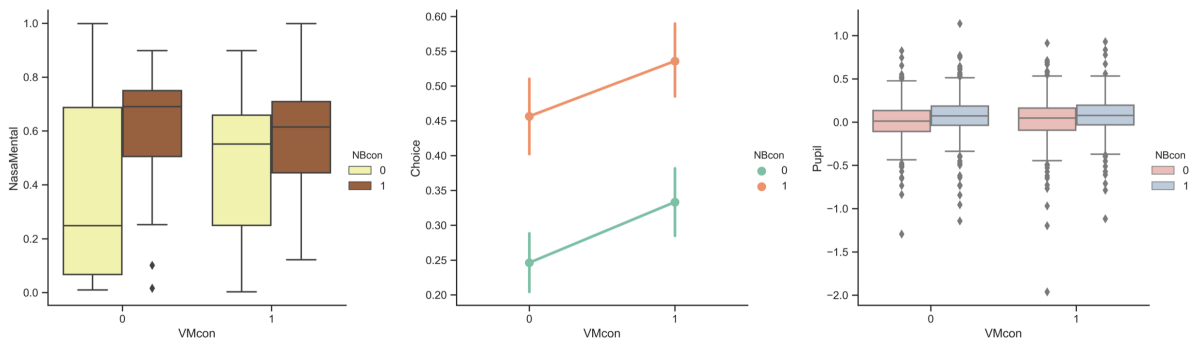


Figure 4.4: **Subjective and physiological measures.** (Left) Subjects’ self-reported mental demand of trials. (Middle) Proportion of trials subjects chose to avoid in the future (Right) Average baseline-corrected pupil dilation during trial.

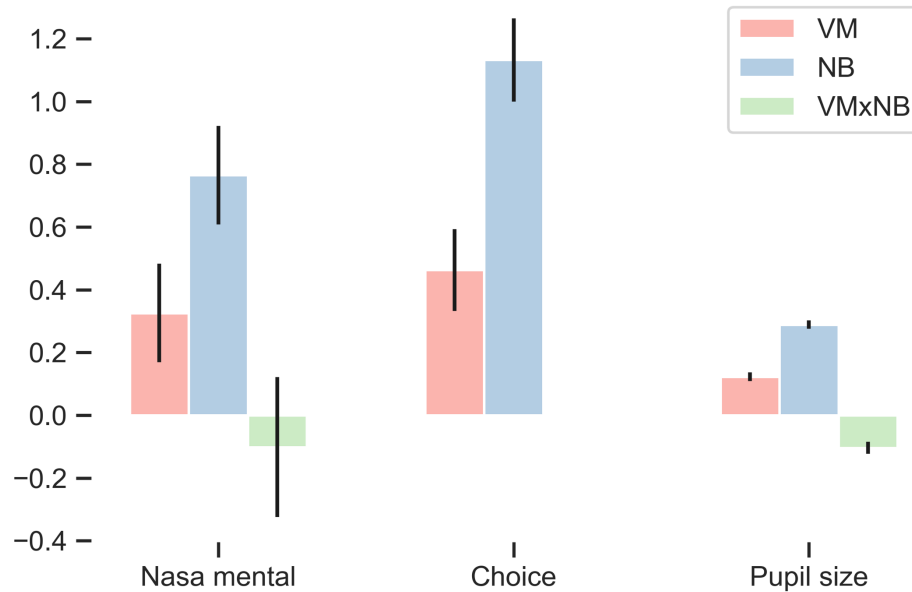


Figure 4.5: **GLMM results on subjective and physiological measures.** GLMM coefficients for predicting NASA-TLX mental demand rating, choice to avoid and baseline-corrected pupil dilation during trial.

#### 4.3.4 Trial preference and taks conditions

Besides the NASA-TLX questionnaire, subjects are also asked to show their preference towards different trials by indicating whether or not they would like to repeat these trials. A GLMM is used to predict their binary choices (0 is to repeat, 1 is to avoid) using task conditions.

$$\textit{Choose avoid} \sim 1 + \textit{VM con} + \textit{NB con} + (1|\textit{Subject})$$

Results show a similar trend as that seen in perceived mental demand. (Figure 4.5) Subjects are most reluctant to repeat trials with hard N-back tasks,  $\beta = 1.13$ ,  $F(1, 1327) = 72.68$ ,  $p = 0$ , but they also tend to avoid trials with unpredictable signals,  $\beta = 0.463$ ,  $F(1, 1327) = 12.67$ ,  $p = .0004$ . This avoidance is coherent to their perception of mental demand of these trials.

#### 4.3.5 Physiological measure of effort

Lastly, the association between pupil dilation and task conditions is tested to see whether the physiological response is coherent to the self-reported perception of mental demand

and task avoidance.

$$\text{Pupil size} \sim 1 + \text{VM con} \times \text{NB con} + (1|\text{Subject})$$

Results show that pupil dilation shows a coherent trend with that of perceived mental demand and trial avoidance, suggesting these elements might be linked, N-back:  $\beta = 0.289, F(1, 1916) = 23.88, p = 0$ ; VM:  $\beta = 0.123, F(1, 1916) = 4.300, p = 0.038$ , (Figure 4.5).

### 4.3.6 Simpson's paradox

When FB/FF correlation is tested in combined data, they are shown to be negatively correlated with each other (Figure 4.6, left). A negative correlation between FB and FF information might hint at the possibility that one has a suppressing effect over the other. However, when this data is analysed in separate group analysis, the trend is reversed, showing a positive correlation between FB and FF information (Figure 4.6, right).

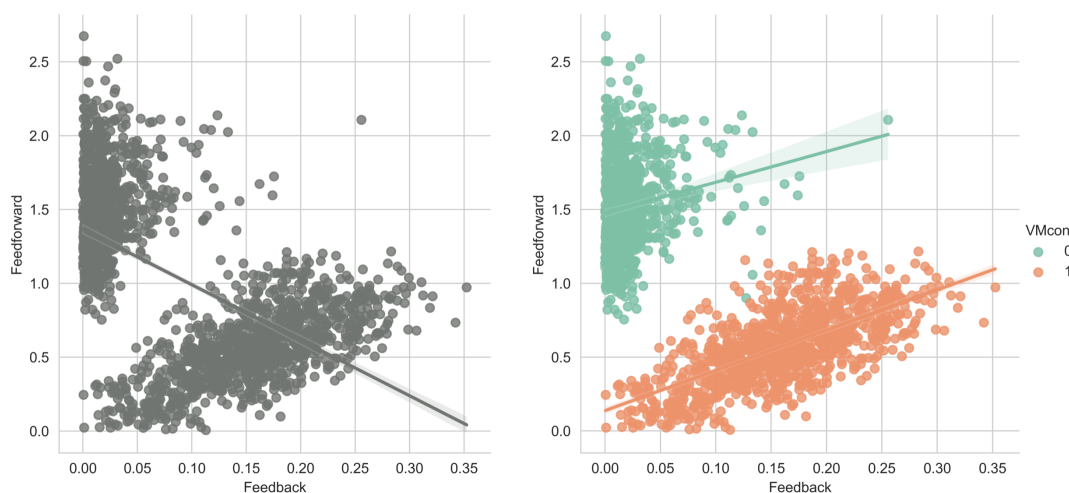


Figure 4.6: **FB/FF correlation showing Simpson's paradox.** (Left) Combined-group analysis of FB and FF data, showing negative regression line. (Right) Separate-group analysis showing positive regression line for each group.

The mismatch in trends observed in combined versus separate group analysis is described as the Simpson's paradox (Simpson, 1951; Pearson, 1899; Yule, 1903; Blyth, 1972; Good and Mittal, 1987). In face of this anomaly, one must be careful when interpreting the data to understand the mechanisms behind FB and FF components. The negative correlation observed in combined data can be explained by the fact that FB and FF information vary in opposite directions with signal predictability when compared to each other. FB information increases, whereas FF decreases with signal predictability. However, within constant signal predictability conditions, FB and FF information rate actually

show a positive correlation. These results suggest the existence of a common bottleneck for FB and FF performance. One hypothesised bottleneck could be the input encoding process. When resource constraints increase, input encoding would deteriorate, translating into a loss of FB information. However, the performance of the seemingly effortless predictive mechanism might also be affected by the input encoding rate and fidelity, since that will change the quality of the internal representation of signals on which the predictive mechanism rely to generate predictions. Therefore, a drop in encoding rate might be the underlying cause for the positive correlation between FB and FF information. We reanalysed the data from the motor delay experiment and found the same result. (Figure 4.7)

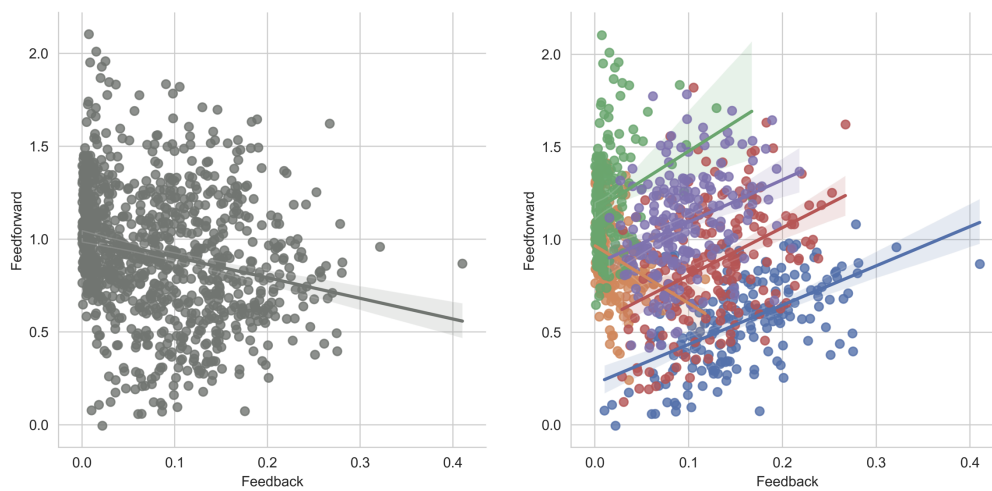


Figure 4.7: **FB/FF correlation showing Simpson's paradox in motor delay experiment.** (Left) Combined-group analysis of FB and FF data, showing negative regression line. (Right) Separate-group analysis showing different regression lines for each group.

## 4.4 Interim conclusion

Dual-task paradigms are used very often to study cognitive control. However, the majority of these studies focus on analysing task performance and use its drop to infer strength of interference. The current experiment tries to complement these studies by providing a means to directly quantify the information processes involved. Using FB and FF measures derived from previous studies, it is found that information cost in VM tasks decreases when the NB task is more demanding. This is perhaps not too surprising even though the two tasks involved are seemingly non-overlapping in terms of modality, these tasks might still share higher cognitive processes in the brain such as attention and working memory. The effect from sharing cognitive processes will then limit information processing capacities in the cognitive processes downstream, directly influencing information cost (Musslick et al., 2016; Petri et al., 2021; Alon et al., 2017). Interestingly, predictive information is also found to decrease when NB tasks are demanding. This seems to suggest that predictive information is also sensitive to resource constraints in cognitive systems, implying the support of predictive information could also be relying on some common cognitive processes as the information cost.

The mechanism with which predictions are generated from a given input could be considered as quite automatic and it does not require a lot of cognitive resources, making its performance insensitive to resource constraint. However, the eventual quality of prediction does not only depend on the prediction generating mechanism, but also on the mechanism of input encoding. Input encoding is a process that is sensitive to resource constraint and suboptimal input encoding would increase noise in input representation and could therefore lead to worse predictions even if the prediction generating mechanism remained unchanged. This effect can be modelled by the information bottleneck method and it will be the focus of Chapter 5.

This experiment demonstrates the effect of cognitive resource constraint on visuo-motor task performance in terms of both feedback and feedforward information. The NASA-TLX questionnaire reveals that subjects do perceive N-back tasks with maxN trials as more mentally demanding. Coherent to this perceived mental demand, they also show a decrease in feedback and feedforward information in the visuo-motor tracking task. This drop in information could be explained by the tension on cognitive resources caused by the mentally demanding N-back task that subjects were trying to perform simultaneously. Interestingly, there is also a performance tradeoff in the other direction in which subjects' N-back task reaction times are slower in trials with a more complex tracking signal. The performance drop in the N-back task is not a general one since there is no change in accuracy is observed. This implies that in face of increased cognitive demand, subjects choose to prioritise resources to maintain accuracy over reaction time, so that they only sustain performance impairment in reaction times. This is a reasonable choice given that there is no extra reward for faster responses as long as subjects could answer correctly within the given deadline. In the trial preference choices subjects demonstrate avoidance

towards trials with difficult N-back conditions or with unpredictable tracking signals, or both, implying these task conditions might be perceived as effortful. Last but not least, the physiological measure of effort used in this experiment, i.e. pupil dilation, is shown to also vary with these task conditions. In conclusion, feedback and feedforward information provide a way to measure cross-task interference in the dual-task context and results show that even though the two tasks involved are not employing the same modality of processing, there is still interference. This suggests there might be a global limitation of cognitive resources in the brain. Moreover, these results show a link between FB information, task demand and subjective effort, both implied by task avoidance and the physiological measure. Especially, it is shown that increases in reported task demand and subjective effort are linked to an increase in FB, but decrease in FF information. This is coherent to the initial hypothesis that information cost, represented by FB measure, is linked to task demand and potentially perceived effort.



## 4.5 Dual-task with Hick's task

In the previous experiment, the difficult N-back task acts as a 'dead weight' to the cognitive system to allow probing its effect on the visuo-motor tracking task whose information transfer can be quantified. Although a reciprocal effect of the visuo-motor task on the N-back task performance is also observed, there is no measure of information transfer in the N-back task. In the following experiment, the working-memory-dependent N-back task is replaced by the Hick's task (1952), whose information rate can be computed using the stimuli and response distributions, allowing one to investigate the cross-task interference in information theoretic terms. Unlike the visuo-motor tracking task, the Hick's task stimuli presentation is designed in a way that it provides no predictable component, so the information rate measured from the Hick's task is equivalent to the FB information.

### 4.5.1 Hick's law

Hick's Law is supported by a wealth of literature since its introduction (Teichner and Krebs, 1974). Among these studies, stimulus-response latency are analysed in many different domains, such as visual stimuli to verbal responses (Alluisi, 1965) and tactual stimulation to motor response (Broadbent and Broadbent, 1987). The robustness of Hick's Law has also led to its wide application from guiding user-interface design (Abowd et al., 2014), to benchmarking performance of neural network models (Bogacz, 2007). However, there are also instances in which Hick's Law is violated. One notable example is found in saccades directed to visual target (Kveraga et al., 2002). In an experiment designed to compare the compliance to Hick's Law, three different types of responses to visual targets are studied: manual key-press, pro-saccade (look at the target) and anti-saccade (look to the opposite direction of the target). By varying the target set size, it is found that while manual key-press and ant-saccadic responses comply with Hick's Law, showing response times increase in proportion to the logarithm of the target set size, pro-saccadic responses latency shows no difference across conditions. These results are interpreted as an effect of overtraining on pro-saccadic response to the target, which makes it immune to Hick's effect. But later, another group of researchers have conducted similar experiments on both monkeys and humans and actually observed an anti-Hick's effect in which saccadic latency is found to be oppositely correlated with target set size (Lawrence et al., 2008). To explain this effect, the authors suggest that inhibition is required to fixate a target and the strength of inhibition will actually increase as the target set size decrease because the propensity to a particular location increases. This inhibition effect will then lead to a longer response latency found in the experimental data. These studies show that even though Hick's Law is in generally a robust effect, it cannot always be taken for granted. Therefore, the first step in the analysis of the current experiment will be to verify the compliance to Hick's Law by checking whether there is a linear relationship between the logarithm of number of choices and reaction time. Then this relationship is quantified to obtain the information rate for Hick's task. According to Hick's law, there is a constant

gain of information, implying the information processing rate in the brain is constant and insensitive to task demand. By comparing this information rate in the Hick's task across different visuo-motor tracking conditions, the constant rate hypothesis is put to test.

#### **4.5.2 Cross-task interference on FB information**

Similar to previous experiments, signals with different predictability are used as a way to manipulate visuo-motor task demand. One of the main goals of this study is to investigate whether the information rate in the tracking task would be interfered by Hick's task information rate. From the N-back dual task experiment, it is found that N-back task difficulty incur a significant interference effect on the FB information rate of the VM task. The current experimental design will allow for a more detailed look into cross-task interference, or the lack thereof, with information measures on both tasks involved.

#### **4.5.3 Perceived mental demand, subjective task avoidance, physiological effort and information rate**

To study the relationship between task conditions and perceived mental demand, task avoidance and physiological effort, categorical comparison between different conditions is carried out as in previous experiments. However, the current experiment also allows for the computation of the total information processed in the whole trial, across the two tasks. All these different methods will be employed with the aim of building an information theoretic account of cognitive effort and gaining insight into how subjects distribute the scarce cognitive resources during the task.

## 4.6 Methods

### 4.6.1 Participants

This study recruited twenty healthy right-handed subjects between 18 and 50 years old, of which 5 are male. The experiment takes approximately one hour and each subject is remunerated 10 euros for their time.

### 4.6.2 Experimental design and procedure

Similar to the previous experiment, this experiment requires subjects to perform two tasks simultaneously. The first task is a visuo-motor tracking task in which subjects aim at placing a cursor they control with a joystick as best as they could onto the moving target. There are in total 3 difficulty levels for this task and they are controlled by the predictability of the moving target. The parameters used are the same as in Section 3.2.2. The second task of this experiment is an auditory Hick's task. In a Hick's task, subjects will hear a stimulus and they have to press a key that corresponds to that stimulus they perceive. The difficulty of a Hick's task is controlled by the possible number of choices in that condition. In this particular experiment, subjects are instructed to press a key corresponding to the number they hear. There are 3 conditions, each differs in its maximum possible number in the stimulus set, either 1, 2 or 4. This task condition is cued by the colour of the cursor used during the visuo-motor tracking task. Unlike the previous experiment where the subjects are asked to respond at the end of the trial for the auditory task, subjects have to respond to each auditory stimulus presented during the trial. The inter-stimulus intervals are randomly drawn from a uniform distribution between 60 to 150 frames, corresponding to 1 to 2.5 seconds. The total number of Hick's stimulus in a single trial therefore ranges between 12 and 16. Theoretically there is no explicit deadline for the response to the Hick's stimulus besides the implied deadline imposed by the inter-stimulus intervals. If subjects failed to respond before the presentation of the next Hick's stimulus, it will be considered as a miss.

There are in total 72 trials evenly spread between combinations of conditions of the two tasks. Similar to previous experiments, this study includes the NASA-TLX questionnaire on subjective mental demand in half of the trials, choices regarding trial preference in another half and pupillometric measurements in all of the trials.

## 4.7 Data analysis and results

### 4.7.1 Hick's rate

It is first tested whether subjects' performance in Hick's task demonstrates the relationship between number of choices and reaction time as stated in Hick's law. Two different ways of computing the information rate of the task are compared. The first approach is the one proposed in the original study (Hick, 1952), by taking the logarithm of the number of choices +1, and correlating that with the reaction time, excluding all wrongly answered trials. This result is shown in Figure 4.8. Each subplot shows an estimation of the information gain function of an individual subject. It can be seen that for most subjects the regression line fits the data with little residuals, suggesting a good fit. This regression line also shows a positive relationship between logarithm of number of choices +1 and reaction time, consistent with that predicted by Hick's law. The slopes of the regression functions are extracted to represent subjects' information rates in Hick's task.

The second approach to Hick's task information rate is to directly compute the mutual information between Hick's stimuli and subjects' responses and correlate that with the overall reaction time, including wrongly answered trials. However, to avoid zero information in the first Hick's condition, where number of choice is 1 and entropy is null, an extra target is added to each condition to signify the absence of signal as one of the possible targets. Therefore, the entropy of the signals in the first Hick's condition, which normally would only have one target and therefore no information (0 bit) now will have two targets and the information is 1 bit. It is further assumed that subjects have perfect performance regarding the detection of signal absence. This will allow one to replicate the (No. of choice + 1) logic in the original Hick's rate computation in this mutual information approach. The results are shown in Figure 4.9. This approach allows the integration of both speed and accuracy into the computation of information rate and could therefore provide a more complete picture in terms of Hick's task performance.

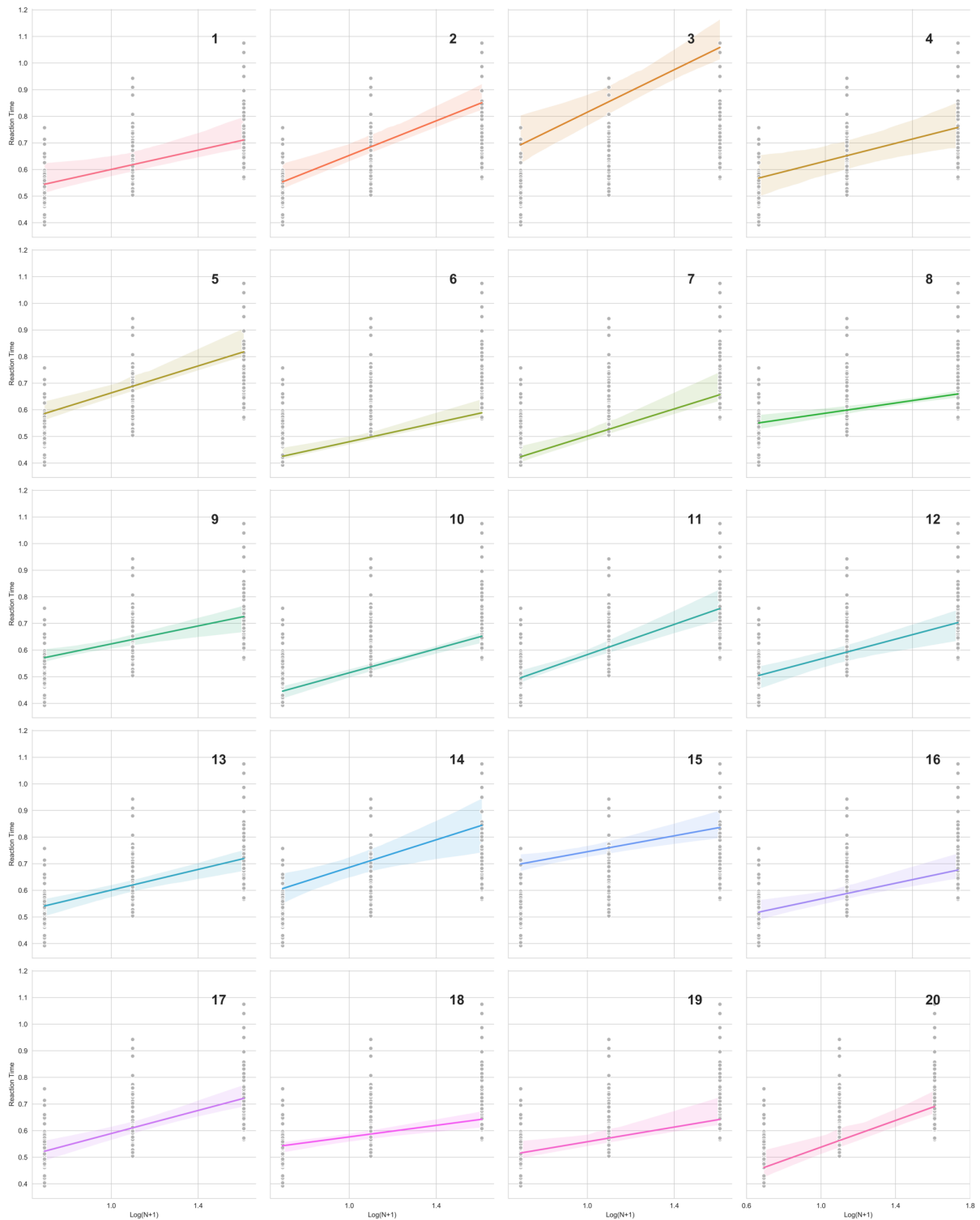


Figure 4.8: **Individual computation of Hick's rate using Hick's original method.** Each plot shows a linear regression for reaction time using logarithm of number of choices +1. All subjects' data are plotted in grey in the background.

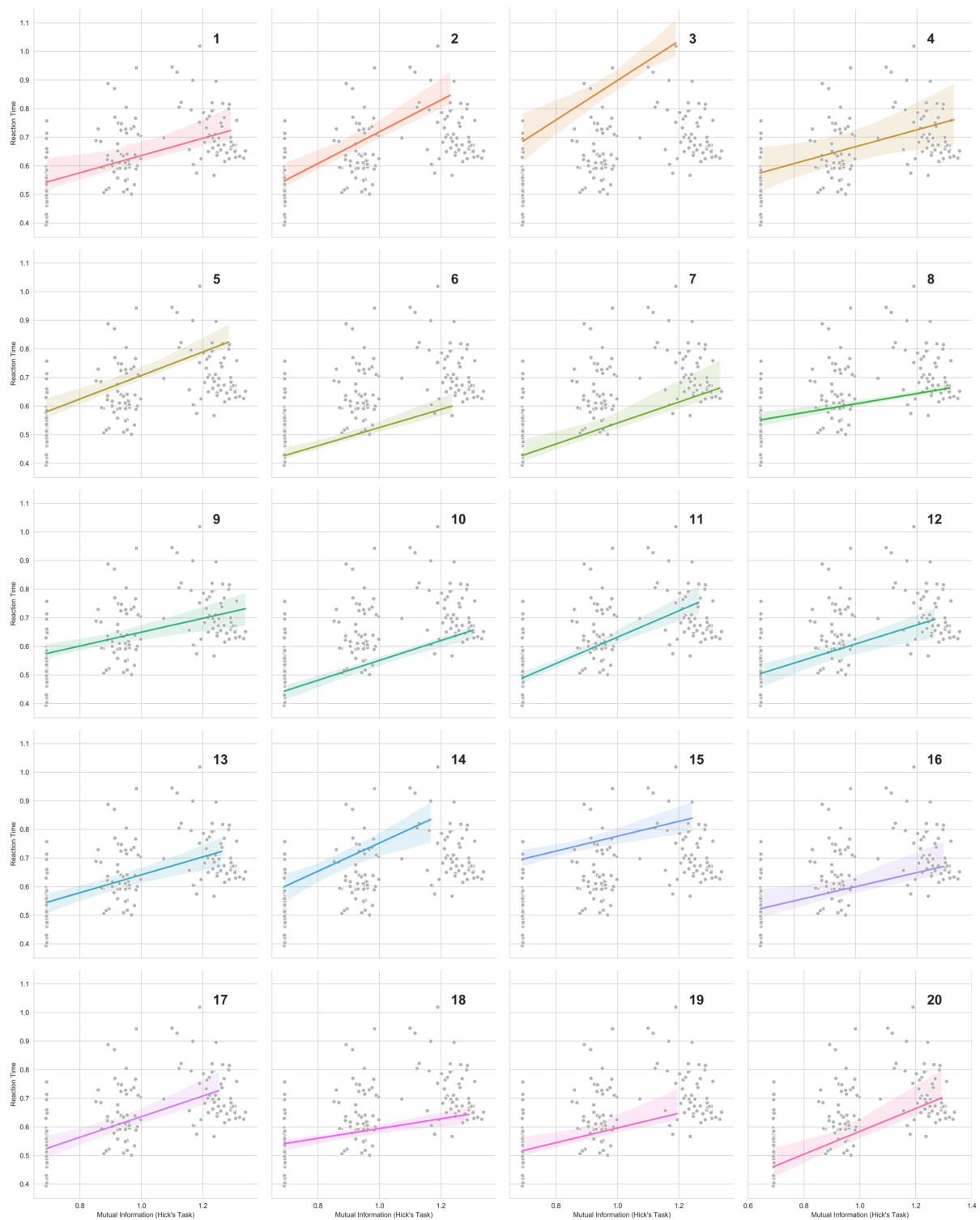


Figure 4.9: **Individual computation of Hick's rate using MI method.** Each plot shows a linear regression for reaction time using  $I(X;Y)$ . All subjects' data are plotted in grey in the background.

The slopes computed from these two approaches represent the rate of information gain,

since reaction time is on the y-axis, this can be interpreted as the increase in reaction time with respect to a unit increase in information processed. The steeper the slope, the slower the information process. Comparing the distributions of the slopes computed from the two approaches (Figure 4.10, left), it can be seen that the MI method gives rise to a higher slope value. This makes sense because in the MI method, errors are also taken into account, even if reaction time would remain constant (which it most likely does not), this will lead to a smaller amount of information being processed, thus the steeper slope.

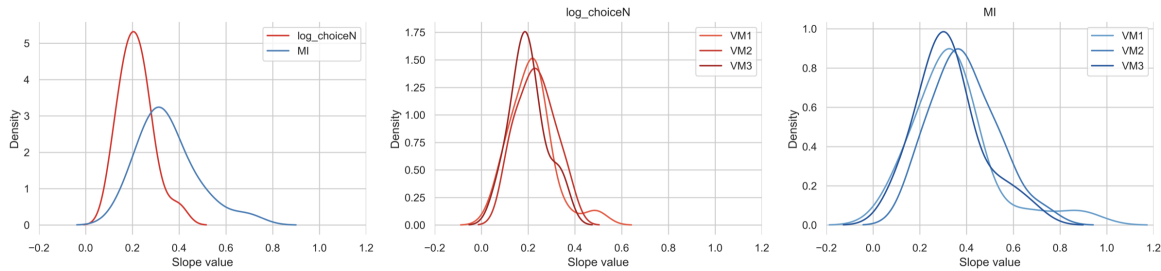


Figure 4.10: **Distributions of slopes of Hick's function.** (Left) Comparison of computed slopes between the two different approaches to Hick's rate. (Middle) Comparison of slopes computed using the log method, divided by VM conditions. (Right) Comparison of slopes computed using the MI method, divided by VM conditions.

To investigate whether there is cross-task interference on Hick's task information rate, we compared it across different visuo-motor conditions (Figure 4.10, middle and right).

$$Hick's\ rate_{log} \sim 1 + VM\ con + (1|Subject)$$

$$Hick's\ rate_{MI} \sim 1 + VM\ con + (1|Subject)$$

Statistical analysis shows that regardless of the approach used to compute the information rates, they do not differ across the visuo-motor tracking conditions. ( $Hick's\ rate_{log}, p = 0.62$ ;  $Hick's\ rate_{MI}, p = 0.45$ )

This implies that the increased task demand in the visuo-motor task does not interfere with the information rates in Hick's task. However, further analysis shows that both accuracy (Hick  $\times$  VM:  $\beta = -0.010, F(1, 1436) = 7.95, p = 0.0049$ ) and reaction time (Hick:  $\beta = 0.33, F(1, 1386) = 146.5, p = 0$ ; VM:  $\beta = -0.065, F(1, 1386) = 5.65, p = 0.018$ ) are lower when visuo-motor tracking conditions are more difficult. (Figure 4.11)

$$Accuracy \sim 1 + VM\ con \times Hick\ con + (1|Subject)$$

$$Reaction\ time \sim 1 + VM\ con + Hick\ con + (1|Subject)$$

This suggests that while the information rate per second remains the same, subjects might have chosen to reduce their engagement in the Hick's task by opting for a high error/short reaction time trade off. In other words, by choosing to process less information, subjects show a drop in performance but not in information rate.

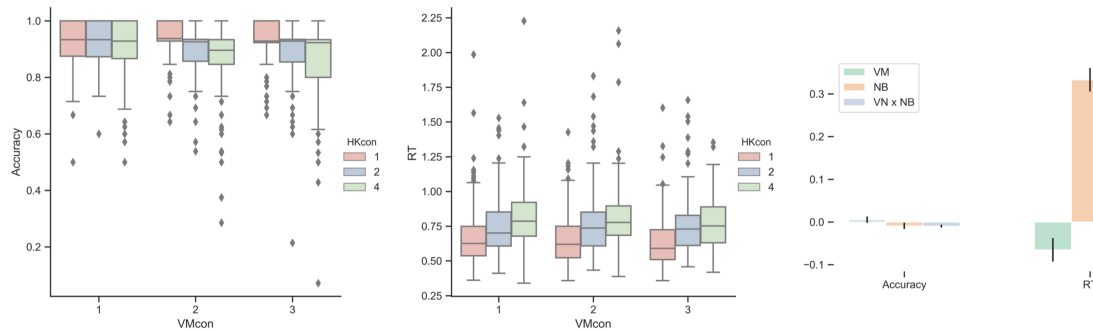


Figure 4.11: **Accuracy and reaction time in Hick's task.** (Left) Subjects' average Hick's task accuracy in a trial. (Middle) Average Hick's task reaction time in a trial. (Right) GLMM coefficients of predicting accuracy and reaction time with task conditions.

#### 4.7.2 Visuo-motor task information rate

When analysing the feedback and feedforward information of the VM task, once again a positive correlation between the two is found within conditions. But when VM conditions are combined, FB and FF information show an opposite trend, demonstrating the Simpson's paradox. (Figure 4.12) This effect has already been observed and explained in previous experiments.

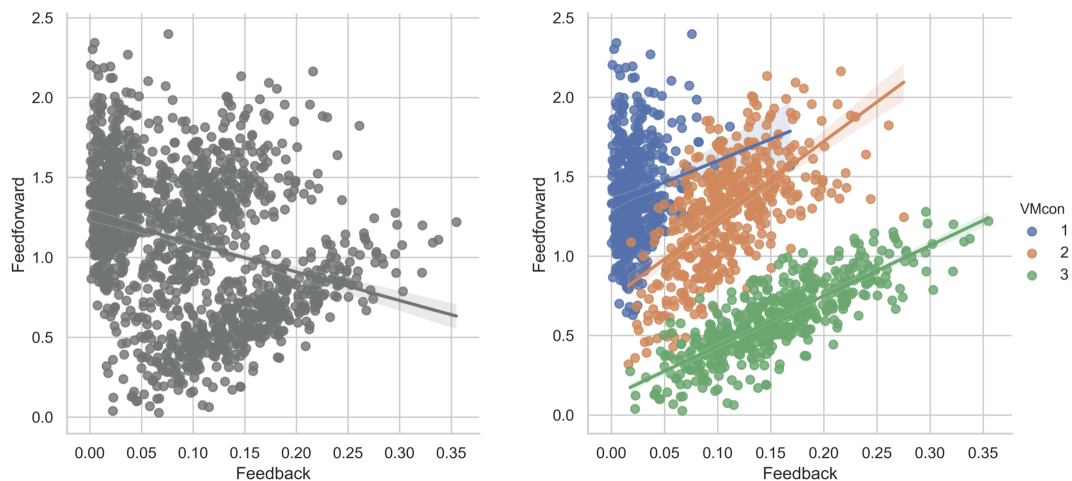


Figure 4.12: **FB/FF correlation showing Simpson's paradox in Dual task with Hick's task.** (Left) Combined-group analysis of FB and FF data, showing negative regression line. (Right) Separate-group analysis showing different regression lines for each group.

We investigated the changes in feedback and feedforward information as a function of Hick's task demand. Since the inter-stimulus intervals used in each trial are randomised,



different trials would end up having different numbers of Hick’s stimuli and this might contribute to the total information demand of Hick’s task. The number of Hick’s task stimuli in each trial is therefore added as a predictor for the feedback and feedforward information and the corresponding model is tested and chosen by AIC.

$$\text{Feedback} \sim 1 + \text{VM con} + \text{Hick con} + \text{Hick N} + (1|\text{Subject})$$

$$\text{Feedforward} \sim 1 + \text{VM con} + \text{Hick con} + \text{Hick N} + (1|\text{Subject})$$

Besides the apparent VM condition effect on both FB and FF information, VM on FB:  $\beta = 0.930, F(1, 1433) = 2965, p = 0$ ; VM on FF:  $\beta = -0.860, F(1, 1436) = 2253.6, p = 0$ , results also show that both feedback and feedforward information decrease significantly in response to an increased Hick’s task demand, indicating a cross-task interference in information processing rate, Hick on FB:  $\beta = -0.144, F(1, 1433) = 71.80, p = 0$ ; Hick on FF:  $\beta = -0.161, F(1, 1436) = 79.47, p = 0$ . Moreover we show that both FB and FF information decrease with increased number of Hick’s task stimuli, Hick N on FB:  $\beta = -0.050, F(1, 1433) = 11.75, p < .001$ ; Hick N on FF:  $\beta = -0.041, F(1, 1436) = 7.059, p = .008$ .

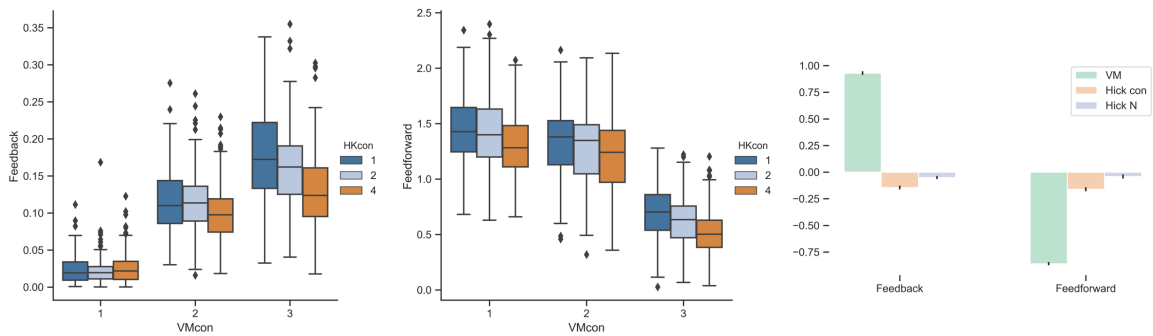


Figure 4.13: **FB and FF of VM task in Dual task experiment with Hick’s task.** FB (Left) and FF (Middle) information measured in VM task. (Right) Coefficient values of normalised variables predicting FB and FF information of VM task.

### 4.7.3 Perceived task demand and total information

Regarding the perceived task demand, it is first tested how well VM and Hick’s task conditions can predict subjective mental demand, trial preference and pupil dilation. Results show that self-reported perceived mental demand of the trials is significantly predicted by both VM and Hick’s conditions, with more difficult conditions in either being perceived as more mentally demanding, VM:  $\beta = 0.332, F(1, 717) = 128.3, p = 0$ ; Hick:  $\beta = 0.211, F(1, 717) = 51.94, p = 0$ , (Figure 4.14, left, 4.15).

$$\text{NASA mental} \sim 1 + \text{VM con} + \text{Hick con} + (1|\text{Subject})$$

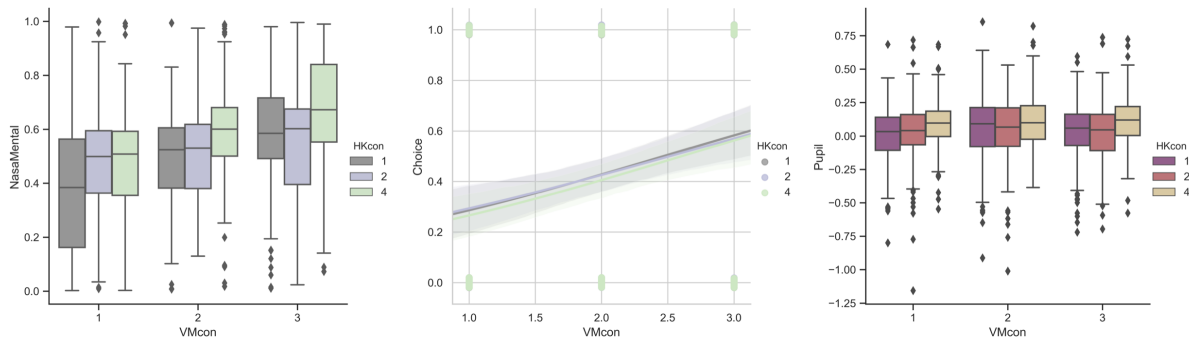


Figure 4.14: **Subjective and physiological measures in dual-task experiment with Hick's task.** (Left) Subjects' self-reported mental demand of trials. (Middle) Proportion of trials subjects chose to avoid in the future (Right) Average baseline-corrected pupil dilation during trial.

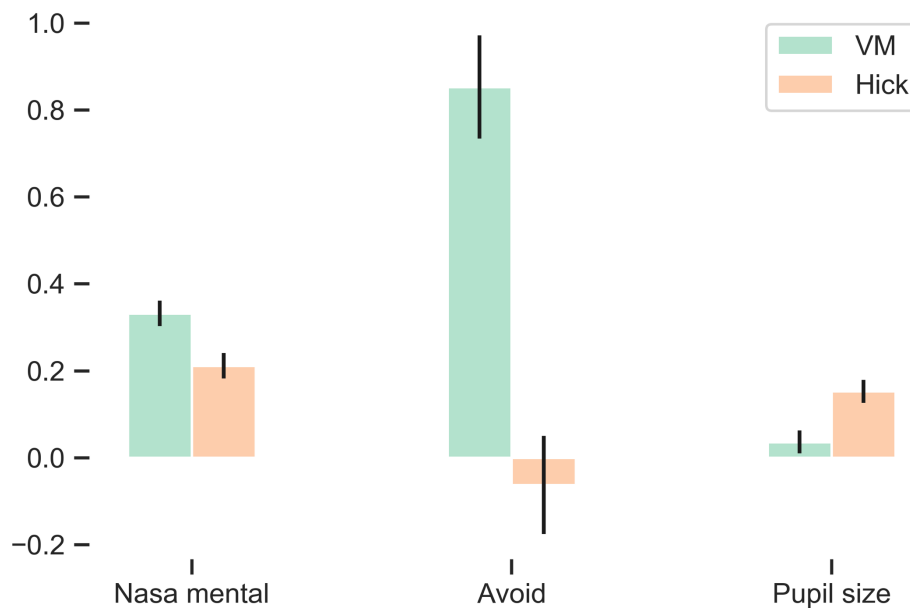


Figure 4.15: **GLMM results of subjective and physiological measures in dual-task experiment with Hick's task.** Coefficient values for GLMMs predicting NASA-TLX mental demand rating, choice to avoid trial in the future and baseline-corrected pupil dilation during trial.

As for trial preferences, a GLMM predicting subjects' choice to repeat either the same (0) or a random trial (1) shows that only VM condition can predict this choice, indicating that subjects only tend to avoid difficult VM trials, VM:  $\beta = 0.853$ ,  $F(1, 711) = 51.29$ ,  $p =$

0; Hick:  $\beta = -0.06$ ,  $F(1, 711) = 0.313$ ,  $p = 0.57$ , (Figure 4.14, middle, 4.15).

$$\text{Choose avoid} \sim 1 + \text{VM con} + \text{Hick con} + (1|\text{Subject})$$

Analysis on pupil dilation shows the opposite tale from that of trial avoidance. Results suggest that subjects' pupil dilation is only predicted by Hick's condition and show no correlation with VM difficulty, VM:  $\beta = 0.036$ ,  $F(1, 1437) = 1.89$ ,  $p = 0.17$ ; Hick:  $\beta = 0.153$ ,  $F(1, 1437) = 33.95$ ,  $p = 0$ , (Figure 4.14, right, 4.15).

$$\text{Pupil size} \sim 1 + \text{VM con} + \text{Hick con} + (1|\text{Subject})$$

To summarise the findings on perception on task demands, it seems that NASA-TLX questionnaire, trial preference survey and pupil dilation fail to show agreement on perceived difficulty, task avoidance and physiological response to effort.

As an attempt to reconcile these differences and to get hold of a more complete picture in terms of information processing in this dual-task context, the total FB information of a trial is considered. After all, if subjects are making choices regarding the whole trial, one potential factor that they take into consideration may be the total amount of FB information processed. The total FB information in each trial is therefore computed by summing up the amount of feedback information in visuo-motor tracking task with that from the Hick's task. Then, its relationship with all the above measures on task demand is re-evaluated.

$$\text{NASA/Avoid/Pupil} \sim 1 + \text{Total FB} + (1|\text{Subject})$$

Results show that this total amount of FB information processed in a trial significantly correlates with subjective mental demand,  $\beta = 0.002$ ,  $F(1, 717) = 64.97$ ,  $p = 0$ , trial avoidance,  $\beta = 0.005$ ,  $F(1, 710) = 32.05$ ,  $p = 0$ , and pupil dilation,  $\beta = 0.0005$ ,  $F(1, 1435) = 7.812$ ,  $p = 0.005$ , suggesting that total FB information could be closely related to the subjective perception of cognitive effort.

#### 4.7.4 Task-switching account

Many studies have suggested that humans could not really perform two tasks simultaneously and the best they could do is to switch between tasks quickly enough to maintain performance of both tasks (Rogers and Monsell, 1995; Schneider and Logan, 2006; Mayr and Keele, 2000; Alport et al., 1994). The analysis in this study shows that subjects' feedback information during the visuomotor tracking task declines when performing the demanding version of Hick's task concurrently. Recall from Chapter 2 that FB information rate is computed with the assumption that the observed tracking response data are sampled from a stochastic process. FB information rate is therefore a measure of the property of the stochastic process, i.e. the entire tracking response in a trial, and it can be interpreted as the total amount of FB information averaged over the number of frames in the course of a trial (bits/frame). Therefore, to get the total amount of FB information

over a trial, we can simply multiply the FB information rate per frame by the number of frames in a trial (bits). This can also be used to compute an information rate per second measure. If we assume subjects are engaging continuously throughout the trial, one only needs to divide the total amount of FB information by the length of the trial in seconds (bits/s) (Figure 4.16, left). On the other hand, if one is to apply the task-switching hypothesis to the current experiment, it would mean that when subjects are engaging in the Hick’s task, they are not engaging in the VM task. With less engagement time as the denominator, the new FB information rate per second will then increase (Figure 4.16, right). The new engagement time can be computed as the length of the whole trial deducted by the sum of Hick’s task response times in the trial. The new FB rate measure is compared across conditions to see if it still varies with Hick’s task conditions. If it does not, it would imply that the feedback information rate per second for VM tasks, under task-switching hypothesis, is constant, just like Hick’s task information rate. And the observed decrease in total amount of FB information with higher Hick’s task demand can be explained by lower engagement time.

$$\text{New FB} \sim 1 + \text{VM con} + \text{Hick con} + \text{Hick N} + (1|\text{Subject})$$

Results show that once corrected for the engagement time, the effect of Hick’s task difficulty on FB information rate vanishes, indicating a constant FB information rate throughout the same VM condition while the VM condition effects remain, VM:  $\beta = 0.761, F(1, 1433) = 2867, p = 0$ ; Hick:  $\beta = -0.022, F(1, 1433) = 2.54, p = 0.11$ ; Hick N:  $\beta = -0.004, F(1, 1433) = 0.092, p = 0.76$ . This can add as new evidence that subjects might not be engaging in both tasks at the same time, but instead, were switching between the two tasks. This also shows that given the same signal predictability, the gain of information indeed is constant, as suggested by Hick, even in a continuous tracking task. Across different VM conditions, however, the total FB information remains largely different and dependent on signal predictability.

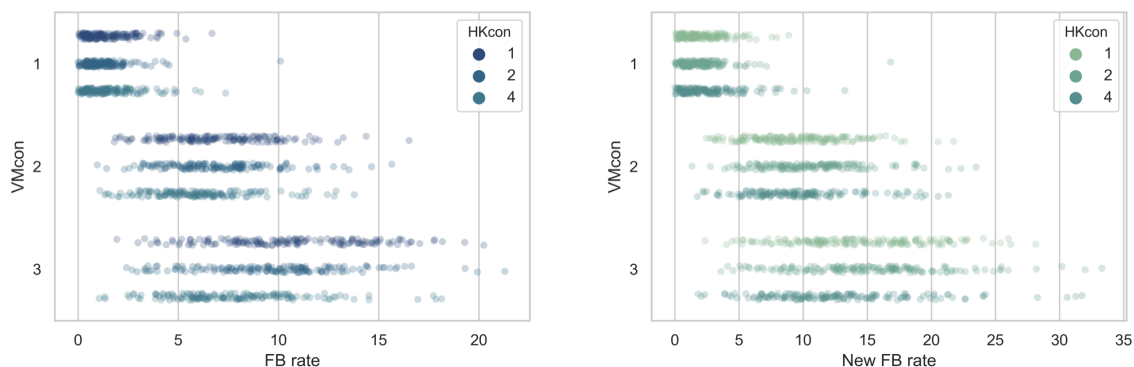


Figure 4.16: **Comparison between old and new FB information rate per second.** (Left) FB information rate (bits/s) assuming engagement every frame. (Right) New FB information rate (bits/s) assuming non-overlapping engagement with Hick’s task.

## 4.8 Conclusion

The main objective of this study is to investigate the cross-task interference in terms of FB information rate in a dual-task paradigm. FB and FF information measures established from the previous study are used in the current analysis. The first dual-task experiment with an N-back task has shown that besides the cross-task interference effect found in general task performance, measured with MSE, reaction time and accuracy, the information measures also reveal a decline in both information cost and predictive information, implying a decrease in information processing rate in the tracking task. The second experiment replaces the N-back task with the Hick's task to study the potential cross-task interference in terms of information processing capacity. The information processing rate in Hick's task is computed by fitting Hick's Law into the data. And since there is no predictable element in the Hick's task used in this experiment, the total information rate is equivalent to the FB information for Hick's task, and is generally referred to as Hick's task rate.

Two approaches to computing Hick's rate are shown and compared. They are then both shown to remain constant with respect to VM task demands through a simultaneous decrease in accuracy and reaction time. This result therefore suggests a decline in task performance but not information rate per second in the Hick's task. Then, initial analysis shows that FB rates in VM tasks decrease with higher Hick's task demands, implying a cross-task interference on information rate of the tracking task. However, in further analysis a new FB information rate measure for VM tasks is derived by assuming a task-switching mechanism in dual-task performance. Under this assumption, it emerges that FB information rate per second in VM tasks might also be constant and free of interference from Hick's task. These results suggest that the mechanism with which subjects exert control over the engagement in information processes might be restricted to varying their engagement time, but not the actual information rate per second. On the other hand, summarising the results regarding task demand and effort measures, experiment 1 shows high coherence between task mental demands, subjective avoidance and pupil dilation with respect to task manipulations. Subjects do find trials with maxN-back condition more demanding, and tend to choose to avoid it and show higher pupil dilation when they engage in them. These are all evidence for maxN-back task being a cognitive demanding and effortful task for the subjects. In experiment 2, results with respect to task conditions show that subjects find both unpredictable signals and more choices in Hick's task to be more mentally demanding, but they only seem to show avoidance towards unpredictable signals. Pupil dilation, on the other hand, shows the opposite effect from that of avoidance. Facing this inconsistency between task mental demands and perceived effort measures, we replace task conditions with total amount of FB information processed in the trial from both tasks as a predictor. Results show that higher total FB information in a trial indeed is associated with higher reported task demand, higher tendency to avoid the trial and higher pupil dilation during engagement. This suggests that while task conditions are

somewhat informative about these subjective phenomena, total FB information might be more relevant in predicting and interpreting perceived demand and effort.

The inclusion of Hick's task has provided some insights into potential interaction of the information processing capacity of the two tasks involved in a dual-task experiment. Results suggest that the information processing rate per second in both tasks might actually be free of each other's interference. However, the current experimental design cannot help clarify whether the difference in FB rates between VM tasks is caused by direct modulation of information processing rate or engagement time, or both. To that end, a model of visuomotor tracking will be explored by combining information bottleneck method, which allows for altering information processes as a function of resource constraint, and an intermittent controller, which provides a way to approximate task engagement time. These concepts and details of the model will be the subject of the Chapter 5.



# Chapter 5

## Intermittent controller with information bottleneck objective

### 5.1 Introduction

In a series of experiments, we have demonstrated that signal predictability in a visuo-motor tracking task could influence both feedback and feedforward information rate. Feedback information rate increases with the noise level of the signal and feedforward information rate increases with the level of autocorrelation in the signal. While these two components show opposite trends with respect to signal predictability, they are shown to be positively correlated with each other. To understand the mechanism that could give rise to the kind of phenomena observed in our experiments, we would employ the information bottleneck method (Tishby et al., 2000) to analyse the process of compressing inputs into representations and to infer from it some predicted outcomes. Our experimental data also show other interesting phenomena that deserve further investigation. For instance, in the added motor delay experiment, it is found that subjects found trials with longer added delay to be more effortful but their FB information actually decreases in those trials. This seems contradictory to the hypothesis that effort should be related to the information cost, as represented by the FB information and it hints at a potential mechanism that might be overlooked, such as engagement time. In order to analyse the tracking performance with higher temporal precision, we employed a dual-task paradigm with Hick's task in which the engagement of the two tasks can be quantified both in time and in terms of information rate. Interestingly, the results initially seem to suggest a more demanding Hick's task would decrease the VM task's FB information rate in the same trial. But when we apply the task-switching assumption and correct for the engagement time of the VM task in the trial, it is shown that this new VM task FB information rate is not impacted by Hick's task, implicating a constant FB information rate that is free of the influence from a second concurrent task. This result highlights the importance of considering engagement time when studying information rate. In the study of Hick's task, for example, the reaction time is generally an accurate enough measure that signals engage-



ment time. However, for the VM task, there is no such measure. Even by subtracting the Hick's task engagement time from the trial under the task switching assumption, this will still give only an upper bound for the VM task engagement time at best. In cases where there is no second task at all, this will not even be possible. Therefore, in order to get a proxy for the level of engagement in the VM tracking task, we now turn to computational models of human control. We discuss the role and potential mechanism behind a feedback and a feedforward component in human visuo-motor tracking performance and propose to model our experimental data with an intermittent controller model whose performance relies on discrete instances of control. The frequency of these control instances can act as a proxy for engagement in the task. Moreover, to incorporate elements of resource constraints, we include an information bottleneck objective in the model to gain insights into how information processing capacity could restrain the feedback and feedforward information rate and to provide a rational account of subjects' behaviour and perceived cognitive effort.

## 5.2 Models of human control in visuo-motor tracking

Besides the more general and vast interest in understanding all sorts of human behaviour and performance, such as perception or motor skills, there has been special interest in understanding human performance in visuo-motor tracking. Especially following the invention of many machines and instruments that require human operators, the study of human tracking behaviour has enjoyed a lot of research focus and many theories have since been developed to provide explanations for how humans achieve their level of tracking performance (Wiener, 1961; McRuer and Jex, 1967). These theories in turn have brought forward the improvement of a lot of operating designs in the automobile (McRuer and Weir, 1969; Mulder et al., 2004) and aerospace industry (McRuer and Krendel, 1974; McRuer and Ashkenas, 1962; Hess, 1997). As of today, the study of human visuo-motor tracking behaviours continues to be instrumental in developing advanced robotics (Goodrich and Boer, 2003; Tseng et al., 2005) and brain-computer interfaces (Taylor et al., 2002; Velliste et al., 2008; Musallam et al., 2004; Mashat et al., 2017; Cunningham et al., 2011). Human tracking is a very complex behaviour that could easily involve a dozen interconnected neural mechanisms (Chase et al., 2011; Benedetto et al., 2021; Stavisky et al., 2017; Ames et al., 2014; Archambault et al., 2015; Churchland et al., 2012; Cisek and Kalaska, 2010; Dum and Strick, 2002; Druckmann and Chklovskii, 2012) making it very difficult to model. Instead, some researchers have tried to focus on simpler tasks in order to reduce the model of human tracking to a manageable one. An example of such a simple task is the compensatory tracking task on completely random signals (McRuer and Jex, 1967). While there exists models that seem to provide a sufficiently good explanation for human performance in this task (McRuer and Krendel, 1974; Potter and Singhose, 2014), compensatory tracking of completely random signals is rather far from an ecological task that truly represents the visuo-motor challenges we are put to in everyday life. One of

the main factors that was omitted from this task is humans' ability to form and make use of predictions in guiding their behaviours. There has been evidence showing that human tracking performance is much better for predictable signals than for unpredictable ones, even if the frequency and bandwidth of the signals are the same (Levison et al., 1969; Poulton, 1952; Pew et al., 1967; Poulton, 1957; Noble et al., 1966; Trumbo et al., 1965; Drop et al., 2016, 2013; Laurence et al., 2014; Drop et al., 2018). We have also shown from our experiments that regardless of the speed of the signal, signal sample entropy, which is a measure of signal predictability, dictates tracking performance in terms of total information transferred (Lam and Z  non, 2021). To understand how humans use predictions in tracking tasks, one must consider the internal representation they use when performing such a task. This internal representation should inform the subject of the dynamics of the target and would be used to adapt their feedback and feedforward control to balance control effort and performance. One extreme of using prediction in tracking could be in the case when subjects' internal representation fully captures the system dynamics, and therefore allows them to develop a purely open loop control that does not require any feedback. An example would be when subjects have learnt a signal as they track and even when the target display is switched off, they would still manage to reproduce the target without any external inputs.

### 5.2.1 Continuous OPF control

The relevant realistic tasks of visuo-motor tracking usually lie between the two extremes of pure feedback and pure feedforward models. To bring these two elements together, we adopt an approach that applies optimisation and estimation theory, to build state-space models of modern human control. The basic assumptions of such a theory include the near-optimality of human control, subject to limitations and constraints. Kleinman et al. introduced the influential continuous-time observer, predictor, feedback (OPF) control which accounts for prediction and delays, among other features of human tracking performance.

The goal of this model is to gain control over a dynamic system consisting of neuromuscular system (NMS) dynamics, target system dynamics and external disturbances. These systems could be considered as one unified system that takes a single input  $u(t)$  and gives a single output  $y(t)$ . The output of this dynamical system is presented to subjects usually through a display, and the control task itself is reflected in the human's choice of a control input  $u(t)$  back into this system that would minimise a predefined cost function  $J(u)$ . This cost function could be a quadratic cost function that is set to achieve the usual tracking goal, i.e. minimising mean squared errors, as well as to account for the physiological limitations of the neuromuscular system through regulating control rate, i.e. minimising motor costs. The formulation of the cost function is essential to solving the tracking problem. For instance, a recent study reveals that the model of minimising motor costs is not sufficient in explaining bimanual tracking behaviour because it overlooks

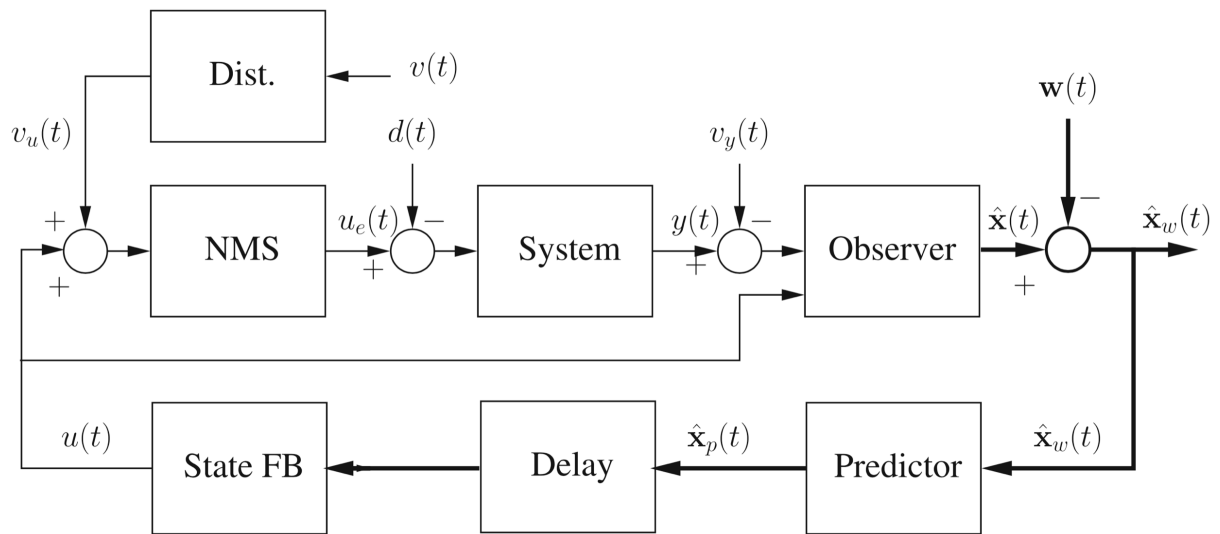


Figure 5.1: **Schematic of an observer, predictor, feedback control model.** (Gawthrop et al. (2011), Fig. 1) The thin arrows represent scalar signals and the thick arrows represent vector signals. The block labelled NMS is a linear model of the neuro-muscular dynamics with input  $u(t)$ . System is the linear external controlled system driven by the externally observed control signal  $u_e$  and disturbance  $d$ , and with output  $y$  and associated measurement noise  $v_y$ . The input disturbance  $v_u$  is modelled as the output of the block labelled Dist. and driven by the external signal  $v$ . The block labelled Delay is a pure time delay of  $t_d$  which accounts for the various delays in the human controller. The block labelled Observer gives an estimate  $\hat{x}$  of the state  $x$  of the composite NMS and System (and, optionally, the Dist.) blocks. The predictor provides an estimate of the future state error  $\hat{x}_p(t)$  the delayed version of which is multiplied by the feedback gain vector  $k$  (block State FB) to give the feedback control signal  $u$ . This figure is based on Kleinman (1970), Fig. 2.

how hand coordination in space is exploited (Mathew et al., 2020). The OPF controller consists of three main elements starting from a Kalman estimator (Kalman, 1960) that generates an optimal estimate of the delayed state. Acting as an optimal state observer (Anderson and Moore, 2005), the Kalman filter models how humans deduce system states from noisy observations (Figure 5.2). This estimate is then passed to a state predictor, which is used by humans to compensate for the inherent time delay in the system. Lastly, an optimal state feedback will generate control signals with respect to the predicted future state and this will be taken by the dynamical system as inputs, closing the loop of the controller model. While the OPF control has enjoyed great success in modelling human tracking performance (Kleinman, 1970; Baron et al., 1970; McRuer, 1980), some open questions regarding human-specific characters remain, such as the existence of a psychological refractory period (PRP; Telford (1931); Welford (1967)) or hypotheses regarding competing resources (McLeod, 1977; Navon and Miller, 2002). The intermittent controller

model was later developed in response to these limitations.

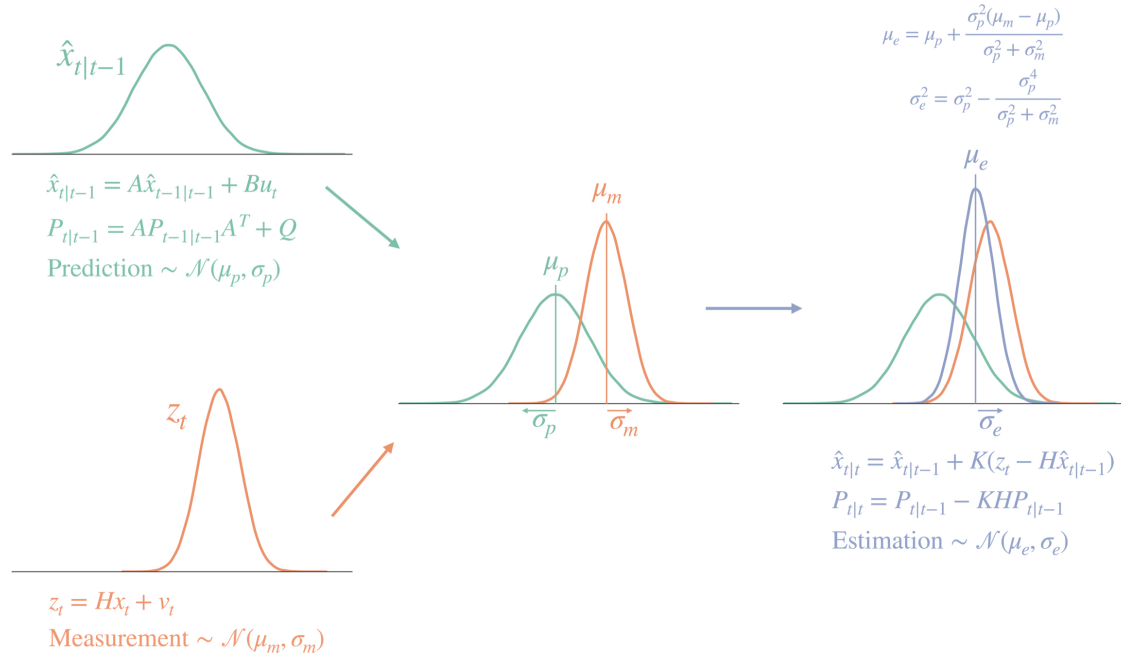


Figure 5.2: **Kalman filter as optimal estimator.** By combining imperfect predictions (green) and noisy measurements (orange), Kalman filter can provide optimal state estimate (blue). By assuming all distributions are Gaussian, the mean and variance of the Gaussian function of optimal state estimate can be readily computed as the product of the Gaussian functions representing predicted and observed states.  $\hat{x}_{t|t-1}$  is the predicted state, it is a function of state transition matrix  $A$ , previous estimate  $\hat{x}_{t-1|t-1}$ , control matrix  $B$  and control input  $u_t$ .  $P_{t|t-1}$  is the variance of the predicted state and it is also a function of the transition matrix  $A$ , as well as the process noise covariance matrix  $Q$ .  $z_t$  is the measured state and it is a function of the transformation matrix  $H$  which is a mapping between state and measurement domain, and  $v_t$ , the zero mean Gaussian measurement noise.  $K$  in the estimation equations represents the Kalman gain, and be derived as  $P_{t|t-1}H^T(H P_{t|t-1}H^T + R)^{-1}$ . In this illustrated example, measurement and predicted states are assumed to be in the same domain already, therefore the solution to the optimal state estimate is particularly straight-forward and they are shown on the top right corner.

### 5.2.2 Intermittent controller

Pioneered by Neilson et al. (1988), the intermittent controller is defined as a sequence of open-loop trajectories determined by intermittent feedback. Intermittent control does not just seamlessly combine the feedback and feedforward components of tracking, it also

provides an account for time delays in systems, such as that in humans. Compared to the classic continuous-time observer, predictor, feedback (OPF) control, the intermittent controller provides solutions to the mechanisms of discrete, preprogrammed, ballistic control, psychological refractory period (Vince, 1948; Navas and Stark, 1968; Telford, 1931), single-channel hypothesis (Smith, 1967; Welford, 1967) and competing resources (McLeod, 1977; Navon and Miller, 2002). The main framework of an intermittent control was described by Gawthrop and others in several papers (Ronco, 1999; Gawthrop et al., 2011; Gawthrop, 2004; Gawthrop and Wang, 2009, 2006, 2007). The intermittent control model shown in Figure 5.3 is built on a well-established OPF controller model by Kleinman (1970). It shares a lot of common components with the OPF model, including the neuromuscular system (NMS), Disturbance, System, Observer, Predictor and State FB components. However, unlike the OPF model whose state feedback was driven by the close-loop optimal state estimate and prediction, state feedback control in an intermittent controller is driven by the open loop state estimate provided by the hold state. The intermittent controller is event-driven (Gawthrop and Wang, 2009), and an event detector would continuously monitor the difference between the hold state and closed-loop observer states. When the difference exceeds a certain set threshold, it would trigger the intermittent feedback loop and would reset the hold state based on the estimated system state generated by the observer.

One can impose a lower bound on the intermittent interval to mimic the limitations found in the human biological system, such as the psychological refractory period and single processor bottleneck. This lower bound, together with the choice of an error threshold, would determine the degree and extent to which control is exerted in this model (Gawthrop et al., 2011). This design highlights how the intermittent controller captures some main characters of human behaviour in tracking, namely discrete, preprogrammed open-loop trajectories (Novak et al., 2002; Ben-Itzhak and Karniel, 2008; Karniel and Inbar, 1997; Barto et al., 1999), temporal refractory periods and triggered responses (Gawthrop et al., 2011). Most importantly, it is believed that the intermittent controller model provides a more natural setting for implementation regarding dual-task scenarios, assuming subjects do not really do two tasks at the same time but, rather, simply switch back and forth between tasks (Rogers and Monsell, 1995; Schneider and Logan, 2006).

The intermittent controller provides the perfect basis of a computational model for human tracking performance. As mentioned before, this model requires an internal model for state estimation and prediction. Instead of directly assuming subjects' full knowledge of the target dynamics, here we propose an internal representation of target signals that is inferred by the subjects through encoding of input signals. The quality of this internal representation is subject to resource constraints and would bring direct consequence to the quality of prediction and thus the overall tracking performance and amount of effort incurred. In the following section we will discuss how internal representation and prediction relate to the encoding of external stimuli through the information bottleneck method.

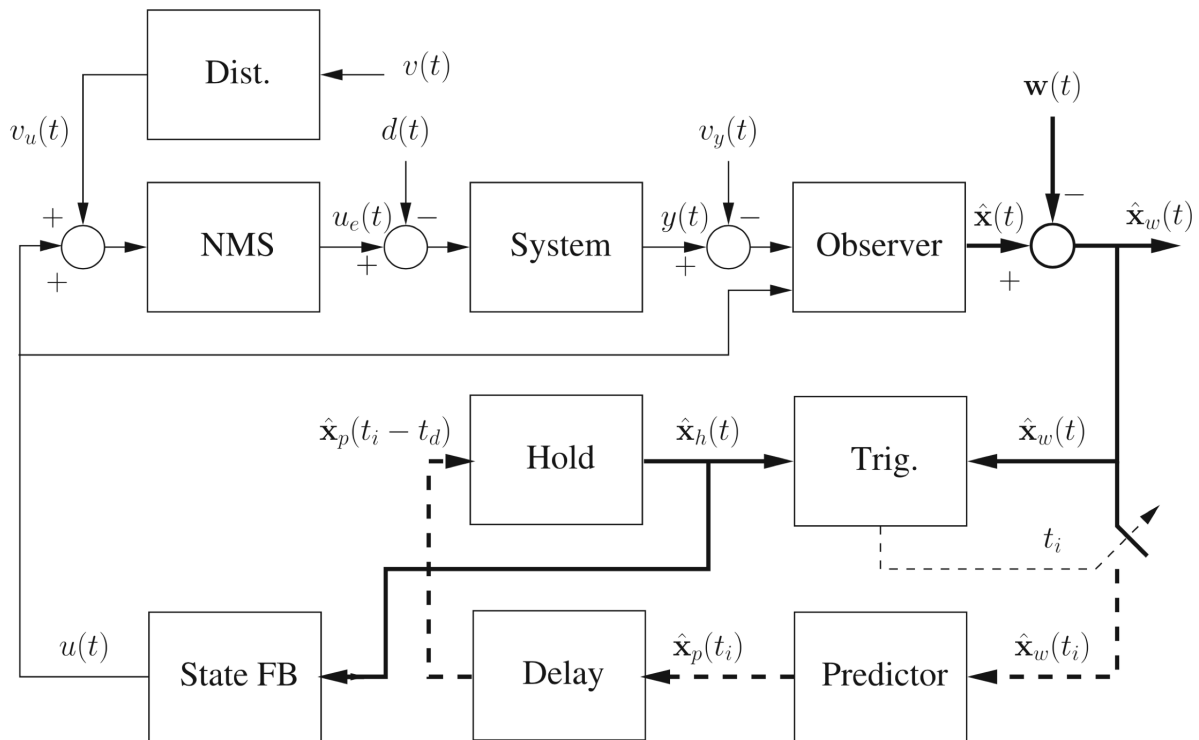


Figure 5.3: **Schematic of an intermittent controller model.** (Gawthrop et al. (2011), Fig. 2) This diagram has blocks in common with those of the OPF of Figure 5.1: NMS, Dist., System, Observer, Predictor and State FB which have the same function; the continuous-time Predictor block of Figure 5.1 is replaced by the much simpler intermittent version here. There are three new elements: a sampling element which samples  $\hat{x}_w$  at discrete times  $t_i$ ; the block labelled Hold, the system-matched hold, which provides the continuous-time input to the State FB block and and the event detector block labelled Trig. which provides the trigger for the sampling times  $t_i$ . The dashed lines represent sampled signals defined only at the sample instants  $t_i$ .

Just like the OPF model, intermittent control also requires internal models for state estimation and prediction. Not only is there growing evidence for the physiological basis of internal models found in cerebellum (Wolpert et al., 1998; Dean and Porrill, 2008; Miall et al., 2007; Miall and King, 2008), internal models will also be the key link to an information theoretic model of control in tracking.

### 5.3 Internal representations in control models

In the context of a visuo-motor tracking task, the goal of this task is to place the cursor on the constantly moving target. Given the inevitable delays of the neuromuscular system (Wolpert and Flanagan, 2001), if one only relies on reacting to visual stimuli as they appear, the cursor will always lag behind the moving target and the goal of the task is not optimally fulfilled. Alternatively, one could try to construct an internal model that represents the dynamics of the moving target and use that to produce predictions of the future location of the target (Vaziri et al., 2006; Saunders and Knill, 2005). These predictions can then guide one's tracking behaviour, allowing one to initiate actions to move the cursor to a certain location before they could even perceive the target in that location. A successful deployment of this predictive mechanism could effectively eliminate the visuo-motor delays and therefore optimise tracking performance.

To apply the concepts of an information theoretic account of cognition cost in a visuo-motor tracking task, we consider the following information processes:  $x_{t-\delta} \rightarrow x'_{t-\delta} \rightarrow \hat{x}_t \rightarrow y_t$

Where  $x_{t-\delta}$  is the external visual inputs,  $x'_{t-\delta}$  is its internal representation,  $\hat{x}_t$  is the internal prediction of the future inputs and  $y_t$  is the tracking response. The information cost involved in each step of this process will be discussed in detail.

#### 5.3.1 Perceptual process

$x_{t-\delta} \rightarrow x'_{t-\delta}$  represents the perceptual process of using external inputs  $x$  at time  $t - \delta$  to update one's prior belief on this input. This corresponds to the 'observer' in the OPF and intermittent controller model, where it is usually formulated as a Kalman filter that provides optimal delayed state estimates from noisy data. The information cost of this process can be expressed as:

$$\text{Perceptual cost} = KL(p(x'_{t-\delta}|x'_{t-\delta-1}, x_{t-\delta})||p(x'_{t-\delta}|x'_{t-\delta-1})) \quad (5.1)$$

#### 5.3.2 Predictive process

$x'_{t-\delta} \rightarrow \hat{x}_t$  represents the predictive process of updating one's prediction on future signals using the internal representation of current signals. This corresponds to the predictor of the OPF and intermittent controller model and it has information cost:

$$\text{Prediction cost} = KL(p(\hat{x}_t|\hat{x}_{t-1}, x'_{t-\delta})||p(\hat{x}_t|\hat{x}_{t-1})) \quad (5.2)$$

A good knowledge/strong belief on the signal dynamics is represented by an optimal prior over the conditional probability between signals at different time points,  $p(\hat{x}_t|\hat{x}_{t-1})$ . A lack of such knowledge, on the other hand, will give a weak prior and the influence of  $x'_{t-\delta}$  on the prediction of future signals will be larger, translated to a larger cost.

### 5.3.3 Action selection

$\hat{x}_t \rightarrow y_t$  represents the action selection process of choosing an appropriate action with respect to the task goal. Its correspondence in the control models is the state feedback control.

$$\text{Action selection cost} = KL(p(y_t|y_{t-1}, \hat{x}_t)||p(y_t|y_{t-1})) \quad (5.3)$$

It is assumed that the action selection is biased towards some internal dynamics already. This is shown as a prior belief represented by the conditional probability distribution over the next action given previous actions,  $p(y_t|y_{t-1})$ . For instance, this could be interpreted as some learned neuromuscular dynamics. The prediction of future signals  $\hat{x}_t$  would affect the inference of this action selection process.

The success of any of the above-mentioned information processes depends on both signal-related and system-related factors. One of the fundamental factors that determines the predictive component of any of these processes is the amount of task-relevant information the priors contain. This has been discussed in detail in the introduction and feedforward information was proposed to approximate the informativeness of priors in information processes.

$$FF = Total - FB \quad (5.4)$$

$$= I(Y_t; \{X_{t-\delta}, X_t\}) - I(Y_t; X_{t-\delta}|Y_{t-\delta}) \quad (5.5)$$

$$\leq I(Y_t|Y_{t-\delta}) \quad (5.6)$$

The upper bound for a feedforward component is the inherent predictability of the signal. Signal predictability is a general term that can be applied to describe the extent to which a signal can be predicted and is closely related, but not limited, to concepts like autocorrelation and periodicity. In some contexts, predictability can also arise from past experience and memory. This upper bound can be interpreted as the maximal informativeness a prior can contain. Another limiting factor of an information process is related to its biological implementation. The brain, like all biological systems, is subject to energetic constraints imposed by availability of metabolic resources, as well as to functional constraints imposed by physiological structure. These can be interpreted as resource constraints that limit information flow and can be understood through rate distortion theory and the information bottleneck, as described in the following (Zenon et al., 2019).



## 5.4 Performance trade-off

### 5.4.1 Rate-distortion theory

Physical implementations of information channels are subject to constraints, putting a limit to a channel's capacity. Therefore, more often than not, representations of inputs are imperfect. This is particularly true for encoding continuous random variables since the description of an arbitrary real number requires an infinite number of information units. The relationship between information rate and representation quality is formalised in rate-distortion theory. For a given source input to be encoded, a distortion measure can be defined to quantify the distance between the input variable and its representation. The tools of rate-distortion theory then provide the answer to a very important question in the information process involved in mapping the input to its representation, that is: how much distortion is achievable for a given information rate, or what is the minimum information rate needed to achieve a certain level of distortion? Figure 5.4 shows an example of a rate-distortion function. In reality, any rate-distortion combination is possible along and above the rate-distortion curve (grey area) while the rate-distortion function demonstrates the theoretical lower bound on information rate to achieve a given distortion. Resource constraints can be implemented as upper bounds on information rate. Using the rate-distortion function, one can then find a solution to the minimum distortion expected given an optimal encoding scheme.

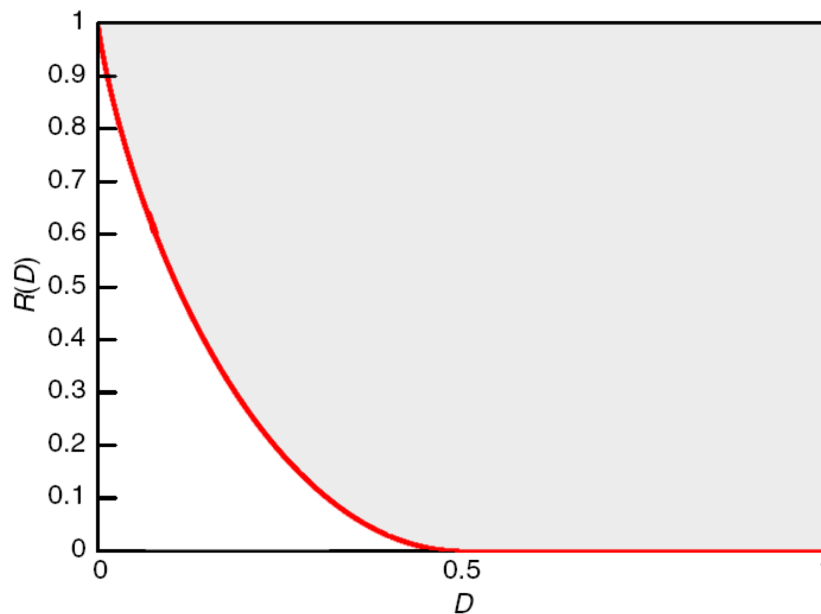


Figure 5.4: **Example of rate-distortion function.** The y-intercept is the information required for a lossless representation of the input. The x-intercept is the minimum distortion if the channel capacity is null.

### 5.4.2 Information bottleneck method

For multi-step information processes, the representations of inputs might just be a means to achieving the task goals, instead of being the goals themselves. In these cases, a distortion measure designed to evaluate the representation only by its accuracy with respect to the inputs (data compression) might not provide insights into its influence on the actual task goal (meaningful information). To tackle this problem, Tishby and colleagues (2000) have proposed the information bottleneck method, a generalisation of rate-distortion theory. It allows one to define and to use a ‘relevance’ variable to formulate a constrained optimization problem whose solution will give an information curve, analogous to a rate-distortion curve, showing the relationship between data compression and meaningful information. The formalisation of the information bottleneck method considers the information process:  $X \rightarrow X'$  and  $X' \rightarrow Y$ . While the data compression goal remains the same, that is to compress the signal  $X$  as much as possible in  $X'$ , another goal is defined so as to maximise the amount of information representation  $X'$  can capture about a relevant variable  $Y$ , this amount of information is defined by  $I(X'; Y)$ . Combining this with the data compression objective, one should obtain the information bottleneck objective to be maximised:

$$L[p(x'|x)] = I(X'; Y) - \beta I(X'; X) \quad (5.7)$$

where  $\beta \geq 0$ .

This is a constrained optimisation problem that aims at finding the encoding scheme  $p(x'|x)$  that would maximise the RHS terms.  $\beta$  as a non-negative constant is the Lagrange multiplier attached to the constrained meaningful information. When  $\beta = 0$ , it will give rise to a lossless compression  $X'$ , allowing it to be maximally predictive of  $Y$ . On the other hand, as  $\beta \rightarrow +\infty$ , it will give rise to a maximally compressed representation of  $X$ , such as mapping all  $X$  to a single point.

## 5.5 Results

### 5.5.1 Deep variational information bottleneck

In the context of visuo-motor tracking, we can apply the information bottleneck method to help gain insight on the performance of an information process under resource constraints. In particular, we consider the perceptual and predictive processes:  $x_{t-\delta} \rightarrow x'_{t-\delta}$  and  $x'_{t-\delta} \rightarrow \hat{x}_t$ , with the information bottleneck objective defined as the constrained optimization problem:

$$L[p(x'_{t-\delta}|x_{t-\delta})] = I(X'_{t-\delta}; \hat{X}_t) - \beta I(X'_{t-\delta}; X_{t-\delta}) \quad (5.8)$$

Tishby and colleagues (2000) have shown that an exact formal solution exists for this problem and it can be solved by self-consistent iterative algorithms such as the Blahut-Arimoto algorithm. However, these methods usually require the random variables to have certain properties, such as being discrete (Tishby et al., 2000) or jointly Gaussian (Chechik et al., 2003). To cope with the multi-dimensional and continuous signal and tracking data in the current study, we adopted a variational inference approach (Alemi et al., 2016) to compute a lower bound on the information bottleneck objective function (Agakov, 2004; Achille and Soatto, 2016). The key to this approach is the reparameterization trick (Kingma and Welling, 2013) which, when combined with Monte Carlo sampling method, could provide an unbiased estimate of the objective function's gradient. Applying stochastic descent on this gradient would then lead to optimisation of the objective function. The conditional probability distributions of our high-dimensional and continuous data can then be parameterised using deep neural networks and fit with the variational information bottleneck method. The main elements of the implementation of a deep variational information bottleneck (deep VIB) model will be introduced here by substituting our experimental variables into the framework provided by Alemi et al. (2016). The full detail and derivation can be found in their original paper.

Our deep VIB model concerns three main variables,  $X_{t-\delta}$ ,  $\hat{X}_t$  and  $X'_{t-\delta}$ , corresponding to observed signal, prediction of future signal and representation of observed signal respectively. Note that each variable can be multi-dimensional and their joint distribution can be written as:

$$p(X_{t-\delta}, \hat{X}_t, X'_{t-\delta}) = p(X'_{t-\delta}|\hat{X}_t, X_{t-\delta})p(\hat{X}_t|X_{t-\delta})p(X_{t-\delta}) \quad (5.9)$$

by assuming the Markov properties:  $\hat{X}_t \leftrightarrow X_{t-\delta} \leftrightarrow X'_{t-\delta}$ , which will give the conditional independence between representation  $X'_{t-\delta}$  and target variable  $\hat{X}_t$ :

$$p(X'_{t-\delta}|X_{t-\delta}, \hat{X}_t) = p(X'_{t-\delta}|X_{t-\delta}) \quad (5.10)$$

By letting  $q(\hat{X}_t|X'_{t-\delta})$  be a variational approximation to  $p(\hat{X}_t|X'_{t-\delta})$  and the fact that KL divergence is always positive, the lower bound of the first term in the IB objective

function 5.8,  $I(X'_{t-\delta}; \hat{X}_t)$ , which represents the predictive objective, can be derived as:

$$I(X'_{t-\delta}; \hat{X}_t) = \int d\hat{x}_t dx'_{t-\delta} p(\hat{x}_t, x'_{t-\delta}) \log \frac{p(\hat{x}_t, x'_{t-\delta})}{p(\hat{x}_t)p(x'_{t-\delta})} \quad (5.11)$$

$$= \int d\hat{x}_t dx'_{t-\delta} p(\hat{x}_t, x'_{t-\delta}) \log \frac{p(\hat{x}_t|x'_{t-\delta})}{p(\hat{x}_t)} \quad (5.12)$$

$$\geq \int d\hat{x}_t dx'_{t-\delta} p(\hat{x}_t, x'_{t-\delta}) \log \frac{q(\hat{x}_t|x'_{t-\delta})}{p(\hat{x}_t)} \quad (5.13)$$

$$= \int d\hat{x}_t dx'_{t-\delta} p(\hat{x}_t, x'_{t-\delta}) \log p(\hat{x}_t|x'_{t-\delta}) + H(\hat{X}_t) \quad (5.14)$$

As for the second term in 5.8,  $\beta I(X'_{t-\delta}; X_{t-\delta})$  which represents the compression objective, we let  $r(x'_{t-\delta})$  be a variational approximation to the marginal distribution  $p(x'_{t-\delta})$  and derive the upper bound of the compression objective term as:

$$I(X'_{t-\delta}; X_{t-\delta}) = \int dx_{t-\delta} dx'_{t-\delta} p(x_{t-\delta}, x'_{t-\delta}) \log \frac{p(x'_{t-\delta}|x_{t-\delta})}{p(x'_{t-\delta})} \quad (5.15)$$

$$\leq \int dx_{t-\delta} dx'_{t-\delta} p(x_{t-\delta})p(x'_{t-\delta}|x_{t-\delta}) \log \frac{p(x'_{t-\delta}|x_{t-\delta})}{r(x'_{t-\delta})} \quad (5.16)$$

These bounds can then be combined to give a lower bound  $L$  to the whole IB objective:

$$\begin{aligned} I(X'_{t-\delta}; \hat{X}_t) - \beta I(X'_{t-\delta}; X_{t-\delta}) &\geq \int dx_{t-\delta} d\hat{x}_t dx'_{t-\delta} p(x_{t-\delta}) p(\hat{x}_t|x_{t-\delta}) p(x'_{t-\delta}|x_{t-\delta}) \log q(\hat{x}_t|x'_{t-\delta}) \\ &\quad - \beta \int dx_{t-\delta} dx'_{t-\delta} p(x_{t-\delta})p(x'_{t-\delta}|x_{t-\delta}) \log \frac{p(x'_{t-\delta}|x_{t-\delta})}{r(x'_{t-\delta})} \end{aligned} \quad (5.17)$$

$p(x_{t-\delta}, \hat{x}_t)$  can be approximated by empirical data, so the lower bound  $L$  becomes:

$$L \approx \frac{1}{N} \sum_{n=1}^N \left[ \int dx'_{t-\delta} p(x'_{t-\delta}|x_{t-\delta_n}) - \beta p(x'_{t-\delta}|x_{t-\delta_n}) \log \frac{p(x'_{t-\delta}|x_{t-\delta_n})}{r(x'_{t-\delta})} \right] \quad (5.18)$$

We can use a deep neural network  $f_e$  as an encoder that takes observations  $x_{t-\delta_n}$  as inputs and outputs mean  $\mu$  and covariance matrix  $\Sigma$  of its representations  $x'_{t-\delta}$ . By assuming normality, the encoder  $p(x'_{t-\delta}|x_{t-\delta_n})$  is now  $\mathcal{N}(x'_{t-\delta}|f_e^\mu(x_{t-\delta_n}), f_e^\Sigma(x_{t-\delta_n}))$ . It can then allow  $p(x'_{t-\delta}|x_{t-\delta_n}) dx'_{t-\delta}$  to be reparameterised (Kingma and Welling, 2013) into  $p(\theta) d(\theta)$ , giving rise to the final objective function to be minimised:

$$\frac{1}{N} \sum_{n=1}^N \mathbb{E}_{\theta \sim p(\theta)} [-\log q(\hat{x}_t|f(x_{t-\delta_n}, \theta))] + \beta KL [p(X'_{t-\delta}|x_{t-\delta_n})||r(X'_{t-\delta})] \quad (5.19)$$

Note that this function can now be optimised using stochastic gradient descent and the corresponding optimal encoder can be obtained.

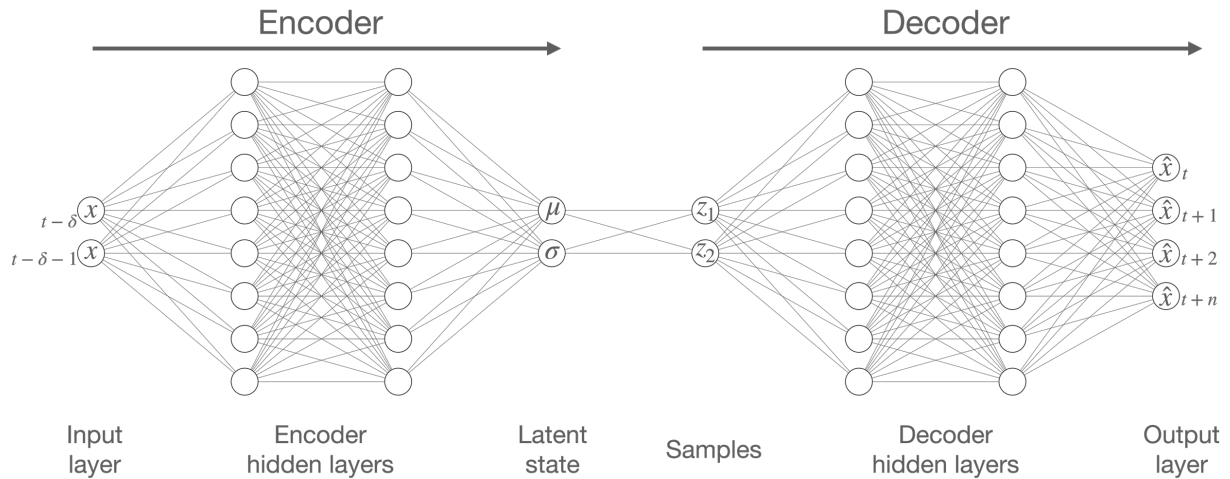


Figure 5.5: **Schematic of the deep neural network employed (Not to scale).** The model comprises an encoder that takes observed signals  $x$  as input and a decoder that outputs predictions  $\hat{x}$  of future signals.

The encoder will produce a representation of the inputs, and the KL divergence between the representation and the input  $KL(x'_{t-\delta}||x_{t-\delta})$  will be used to represent the compression loss, corresponding to the second part of 5.19. The decoder, on the other hand, will be trained to make predictions of the future signals. The negative log probability of the decoder  $-\log p(\hat{x}_t|x'_{t-\delta})$  will be computed as a measure of the prediction loss, corresponding to the first part of 5.19. The total loss to be minimised in the training of this network will then be the sum of the compression loss, multiplied by the constant  $\beta$  that controls resource constraints, and prediction loss:

$$Loss = Prediction\ loss + \beta\ Compression\ loss \quad (5.20)$$

Note that the non-negative constant  $\beta$  is put in front of the compression loss, meaning that the higher the  $\beta$  value, the more the signal will be compressed.

Using Tensorflow, we construct and train the deep VIB models to predict signals of different predictability. The encoder  $p(x'_{t-\delta}|x_{t-\delta})$  is programmed as a deep neural network and is connected to a decoder  $q(\hat{x}_t|x'_{t-\delta})$ , which is another deep neural network. The entire network is then trained using the Adam optimizer (Kingma and Ba, 2014) for 700 epochs with learning rate 0.01. This training process is repeated 9 times for each signal predictability (x3) and different beta values (x12). Information curves are obtained for each type of signal and are shown in Figure 5.6.

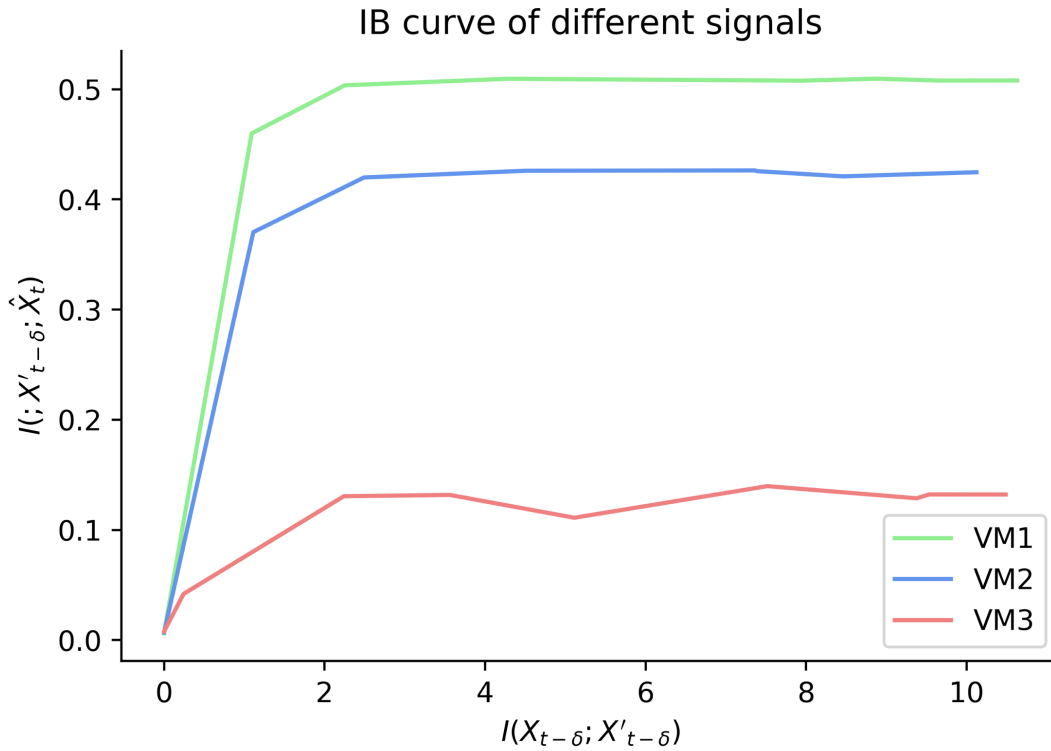


Figure 5.6: **Information curves of signals of different predictability.** These information curves demonstrate the positive correlation between encoding rate (x-axis) and predictive information (y-axis).

$I(X_{t-\delta}; X'_{t-\delta})$  is the encoding rate, which is the amount of information the representation contains about the signal and  $I(X'_{t-\delta}; \hat{X}_t)$  accounts for how well predictions can be made with the representation. First of all, it is observed that as the encoding rate increases, predictive power also increases. Predictive information shows different upper bounds depending on the signals. The more predictable the signal is, the higher this bound will be. Notably, for more predictable signals, a lower encoding rate is needed to achieve high predictive power. Lastly, it is noted that encoding rate beyond a certain point, for instance at around 3 bits, does not bring extra predictive information anymore.

From the information bottleneck results, the relationship between encoding rate and predictive power of representation is demonstrated by the information curves. It is observed that signal predictability remains a dominant factor in deciding the relationship and upper bounds of these qualities. In the next section, these trained optimal encoders will be incorporated into a controller model in order to simulate tracking data that could help unveil the potential mechanism behind some phenomena we observe from the experimental data.

### 5.5.2 Intermittent controller repredicting model

There are studies that apply the concept of resource constraints to control systems in which tradeoffs between information rate and cost regarding the control optimisation problem are rigorously derived and solved for (Kostina and Hassibi, 2019, 2018). Among these studies, one of the more relevant work is by Sabag et al. (2020), in which they study the trade-off between the cost of a linear quadratic Gaussian control problem and directed information, which in some way is similar to the FB measures derived in this study. While these theoretical works are insightful and fundamental to the tracking problem, we will first focus on building a minimum working model that could simulate human tracking behaviours.

From the comparison between OPF model and intermittent controller (IC), it is established that IC provides a more appropriate model for human performance because it allows the model to account for discrete control, psychological refractory periods and triggered responses (Gawthrop et al., 2011). Specifically in an IC model, both encoding rate and frequency of control are very important in determining tracking performance. High encoding rate would increase both FB and FF information at each encoding and predicting instance, as established from the results above. High frequency of control, on the other hand, would mean to constantly update predictions with the latest observed signals. This would minimise the horizons used in making predictions, thus making them more precise. However, both achieving a high encoding rate and a high frequency of control could be cognitively demanding and subjects might make decisions to economise their effort too. For instance, control occurrences in an IC model are event-triggered. In the current formulation, these events are defined as when the absolute errors between signal and tracking exceed a certain error threshold. In other words, by adjusting the error threshold, subjects could decide on the trade-off between performance and the corresponding level of engagement or associated level of effort. Besides the FB and FF information measure, simulating tracking responses with the IC model can therefore also allow comparison of frequency of control across different task conditions to gain insight into subjects' engagement and potentially their perceived effort too. To incorporate the resource constraints into the tracking model, predictions based on different information bottleneck objectives are also applied in the IC models. In order to expand the prediction horizon without compromising trainability of the network, the deep VIB network is trained, with a specified  $\beta$ , to encode 2 time points (e.g:  $x_1, x_2$ ) and output only 20 future time points ( $x_{1+\delta}, \dots, x_{20+\delta}$ ). An AR2 function is then fitted to the deep VIB network outputs to help extrapolate the prediction for as long as the trial needs. The effect of changing  $\beta$  values, error thresholds and VMD on tracking behaviour in terms of FB, FF information and frequency of control will be explored.

For the sake of simplicity, the current formulation of the model is deterministic. We adopt the notation:  $X_t^{-(k)} = (X_t, X_{t-1}, \dots, X_{t-k+1})$  where  $t$  corresponds to the time label of the variable and  $k$  represents the depth of past values this variable contains. For time-forward sequences, we define  $X_t^{+(k)} = (X_t, X_{t+1}, \dots, X_{t+k-1})$ , where  $t$  is time label and  $k$

represents the depth of future values this variable contains.  $k$  in both cases is therefore also representative of the dimension of the variable.  $\Delta$  is the visual motor delay and this will determine the horizon of the prediction.  $\beta$  is the constant in the information bottleneck objective in 5.19 and it controls the degree of data compression, and thus quality of prediction too.  $\bar{\epsilon}$  is the error threshold that will help determine when control instances are triggered.  $C$  is the count of control instances in a trial.  $\psi$  is the psychological refractory period, which represents a control deadzone after each control instance is triggered.

The run of the model can be summarised as followed:

1. At time  $t - \Delta$ , signal  $x_{t-\Delta}^{-(2)}$  is observed.
2. A prediction of where the signal will be at  $t$  is generated by the deep VIB model with compression target set by  $\beta$ . The model will first encode  $x_{t-\Delta}^{-(2)}$ , with a pre-trained optimal encoder of the corresponding  $\beta$ . The output of the deep VIB model is the prediction of future signals,  $\hat{x}_t^{+(N)}$ ,  $N$  represents as many time points there are left until the end of the trial.
3. At time  $t$ , a tracking response  $y_t$  is generated according to the prediction made in step 2. For simplicity, we do not include an NMS here and directly apply the prediction of future signals as the tracking response:  $y_t = \hat{x}_t$ .
4. The absolute difference  $\epsilon$  between signal  $x$  and tracking  $y$  is computed and checked at each time point.
5. At any time point  $a$ :
  - If  $\epsilon < \bar{\epsilon}$ :  
No control instance will be triggered,  $y_{a+\Delta} = \hat{x}_{a+\Delta}$ , where  $\hat{x}$  is generated from the last control instance.
  - If  $\epsilon \geq \bar{\epsilon}$  and  $(a - t_{last\ control}) < \psi$ :  
A control instance will be triggered,  $C += 1$ . Steps 1 to 3 will be repeated, so that  $x_a^{-(2)}$  is encoded and predictions  $\hat{x}_{a+\Delta}^{+(N)}$  are produced. At time  $a + \Delta$ , tracking response will be produced according to this prediction:  $y_{a+\Delta} = \hat{x}_{a+\Delta}$ .

This model will generate a tracking response whose FB and FF information can be computed and analysed. Moreover, this simulation will produce the frequency of control measure, which corresponds to the number of times control instances are triggered in a trial. This is a very useful measure in understanding engagement and related effort of the tracking behaviour.

### Resource constraint and signal predictability

The information bottleneck method predicts that prediction quality should deteriorate as signals are highly compressed (low  $\beta$  value). This is tested by comparing FF information corresponding to intermittent controller simulations generated with different  $\beta$  values



across different VM conditions. Also, our experimental data has consistently shown that signal predictability is the dominant factor in determining FF information. We will also aim at confirming this with simulation data. Besides the usual information measures, the frequency of control from different model configurations will also be compared and its relationship with FB information will also be studied.

We first observe that FF information, representative of the quality of prediction used in the tracking, decreases significantly with respect to drop in signal predictability, consistent with the same robust effect observed in experimental data. On the other hand, it is shown that FF information also decreases as  $\beta$  values increase. The results are coherent to the hypothesis that decreases in  $\beta$  values would allow for higher encoding rate, and thus better quality of prediction.

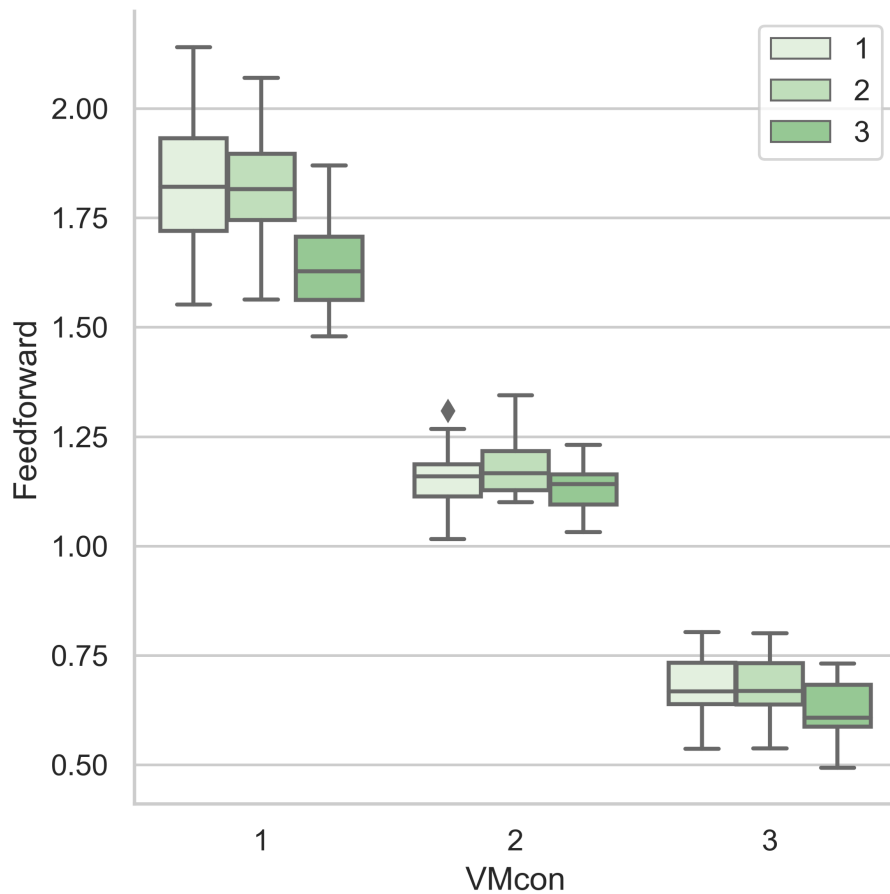


Figure 5.7: **FF information of intermittent controller simulated data.** FF information of tracking performance generated by intermittent controller model with different  $\beta$  values for different signals.

Next, we will look into FB information and the frequency of control in a trial. Firstly, it is observed that the frequency of control is higher for more unpredictable signals. (Figure

5.8, left) This is most likely due to low quality of prediction in unpredictable conditions. High frequency of control could thus also be the cause for the higher FB information found in these conditions (Figure 5.8, right), consistent with the interpretation of a more reactive type of tracking when signals are not predictable.

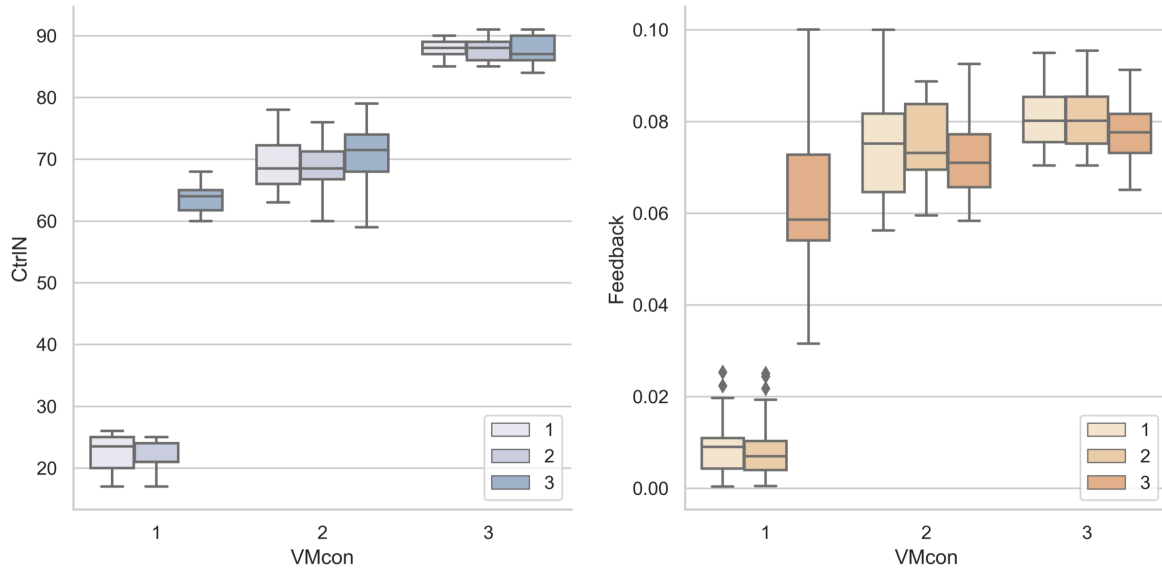


Figure 5.8: **Frequency of control and FB information of intermittent controller simulated data.** Average frequency of control (left) and FB information (right) of tracking performance generated by intermittent controller model with different  $\beta$  values for different signals.

Then, we would like to draw special attention to the condition with the most predictable signals (VM1) and the most compressed ( $\beta 3$ ) representation. It can be seen that there is a significant jump in frequency of control, as well as FB information. These phenomena are illustrated with two example simulations (VM1,  $\beta 3$ ) and (VM1,  $\beta 2$ ) shown in Figure 5.9.

Comparing the two tracking responses of the same signal, it is obvious that the quality of prediction in the second condition is superior thanks to the lower compression rate of the signal (lower  $\beta$  value). As a result, it gives rise to a much smoother tracking consisting of a low frequency of control. For the result in blue, it is observed that the compression rate has reached the point where prediction quality is heavily compromised, so much so that it has caused a drastic change of strategy from reliance on prediction to reliance on control. This change of strategy is shown by the significant jump of both frequency of control and FB information. Such jumps are not observed for the other VM conditions because the baselines of prediction quality regardless of compression rate, are already low due to the low signal predictability.

Such low predictability of the signals has also led to reduced variability within the

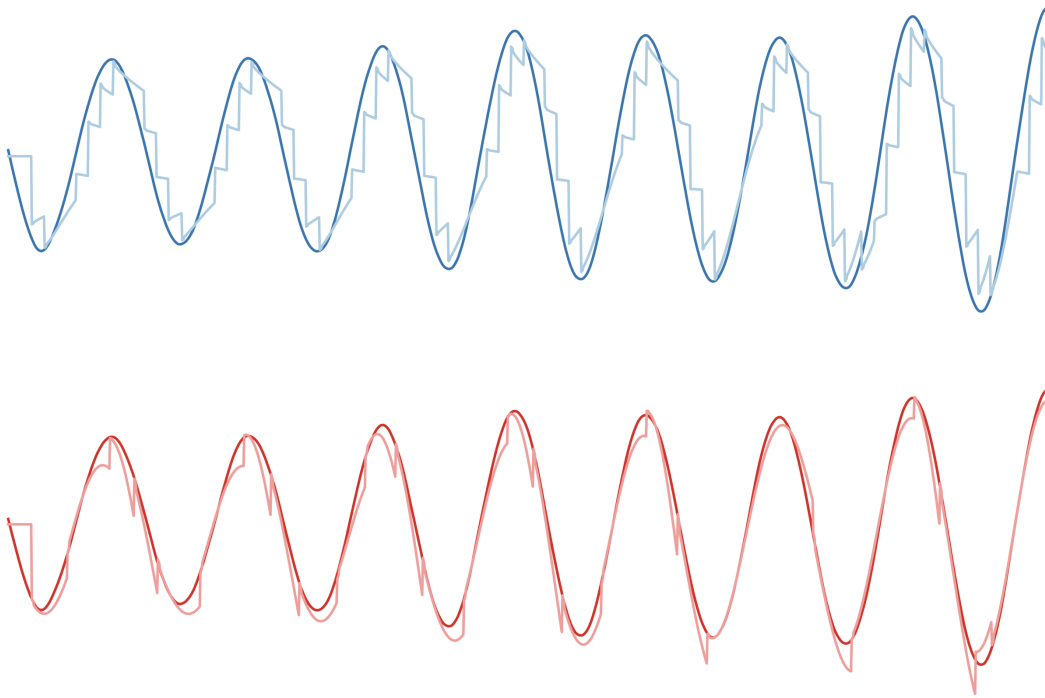


Figure 5.9: **Example data from intermittent controller simulation.** (Top) Simulation generated for most predictable signal with  $\beta = 0.01$ . (Bottom) Simulation data for the same signal with  $\beta = 0.001$ .

same VM condition. FB information, nevertheless, seems to show a decreasing trend with respect to increase in  $\beta$  values, as predicted by the relationship between  $\beta$  and encoding rate in the information bottleneck method.

It is interesting to note that, judging from the frequency of control in the simulated data, it would require as much as 4 times the frequency of control for tracking an unpredictable signal (VM3) compared to a predictable one (VM1). This might help explain subjects' higher perceived effort for conditions with unpredictable signals. These findings help link the measured FB/FF information in a trial to the intermittency of event-triggered control. This provides a rational account of measured information cost and perceived effort in terms of frequency of control. It also acts as the necessary mechanism underlying the emergence of Simpson's paradox of FB/FF information, both observed in experimental and simulation data (Figure 5.10).

### Error threshold

As mentioned before, one way engagement can be altered in an IC is by adjusting the error threshold  $\bar{\epsilon}$  in the model. In a real experiment, error threshold could be an experimental manipulation that is implemented through some performance feedback. In this simulation, different error thresholds  $\bar{\epsilon}$  are implemented and the corresponding simulated behaviours

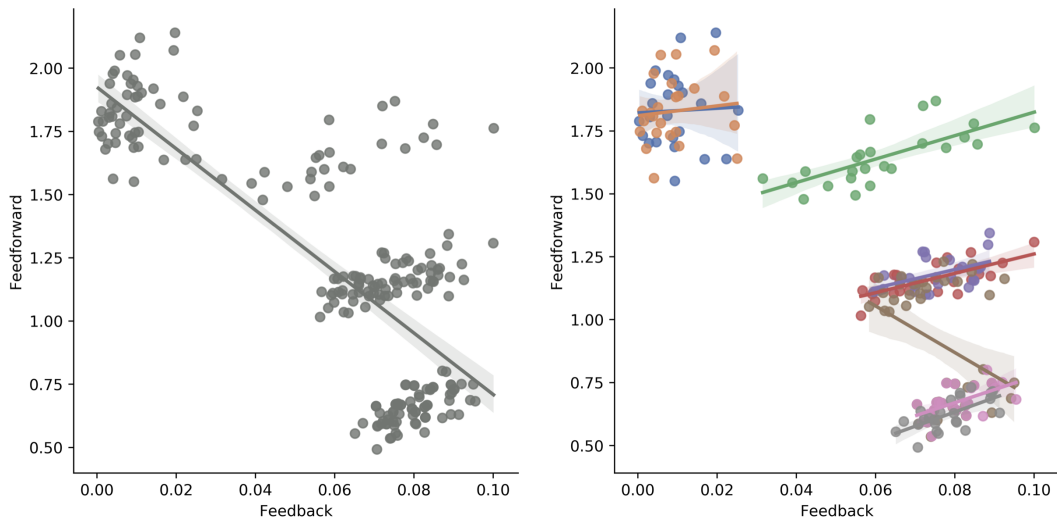


Figure 5.10: **Demonstration of the Simpson's paradox from simulation data.** (Left) FB/FF correlation of combined data. (Right) FB/FF correlations of group specific data.

are analysed. It is briefly mentioned in Chapter 3 that it is possible that subjects increase their error threshold when faced with signals with higher speed, resulting in drop in both FB and FF information, and also a lower self-reported perceived mental demand. This hypothesis will be tested using the IC model. More generally, tracking performances for different  $\bar{\epsilon}$  are compared in terms of FB and FF information while mental demand/effort related attributes are inferred by frequency of control  $C$ . Ten different  $\bar{\epsilon}$  of value between 0.1 to 1 with increment 0.1 are tested for two VM conditions (1 and 2) separately. All parameters, including VMD  $\Delta$ , PRP  $\psi$  and  $\beta$  are all held constant for all conditions. Since  $\beta$  is held constant throughout, the main differences in FB and FF information measured in the trials can be attributed to frequency of control  $C$ . We first observe that when  $\bar{\epsilon}$  increases, MSE naturally increases in all cases, corresponding to worse tracking performance (Figure 5.11, purple). As  $\bar{\epsilon}$  increases, it is observed that  $C$  monotonically decreases (Figure 5.11, blue), regardless of VM conditions. Control instances are triggered by tracking errors exceeding the error threshold, as error thresholds get higher, these control instances will also become rarer. Since VM1 signals are very predictable, the frequency of control is much lower overall. It can be seen that even when  $\bar{\epsilon} = 1.0$ , the frequency of control for VM2 is still 4 times as much as in VM1. The effect of this decrease in frequency of control is also seen to cause a decrease in FB information in VM2. The same effect in FB information for VM1 is hard to detect because of the generally small contribution of FB information to the performance in VM1 (Figure 5.11, red). For both VM conditions, a clear trend of decreasing FF information can be seen with respect to increased error threshold and decreasing frequency of control (Figure 5.11, green). As control instances are rarer, it means the average horizon of a prediction will get longer

and therefore the average quality of prediction in a trial will decrease. As hypothesised, by increasing the error threshold, frequency of control would decrease and in turn would lead to drop in FF and FB (depending on VM condition) information. Using frequency of control to infer task engagement or effort, an increase in error threshold could indeed provide a possible explanation of the observed decrease in both FB and FF information, as well as a lower self-reported mental demand in the experiment reported in Chapter 3.

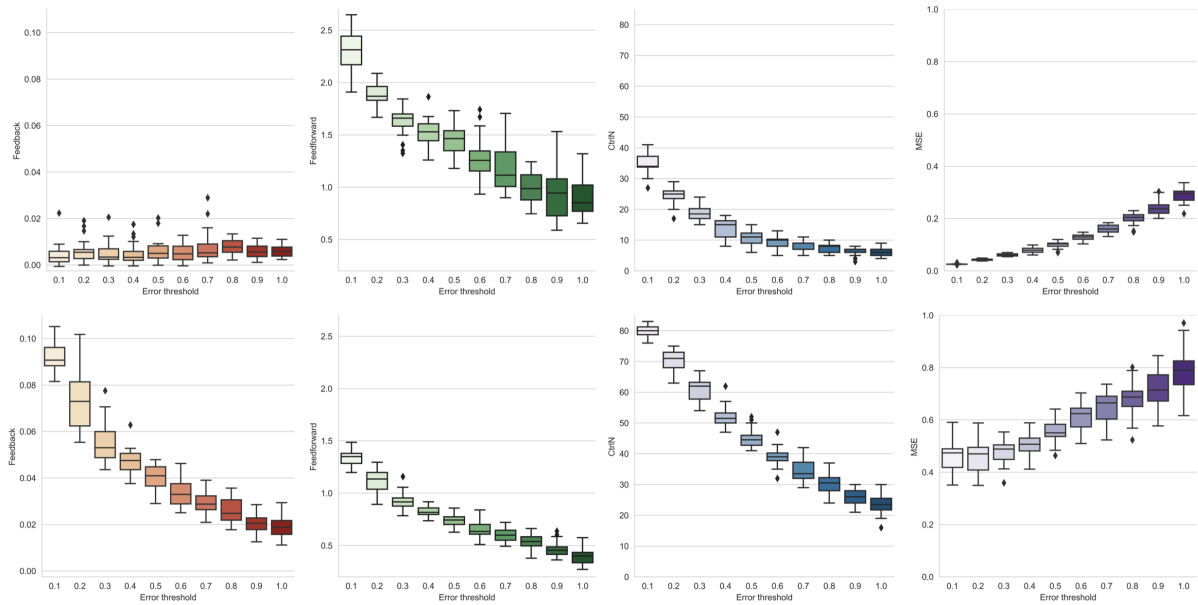


Figure 5.11: **FB, FF information, frequency of control and MSE change with error threshold.** (Top 4) Simulation results for signals of VM1. (Bottom 4) Simulation results for signals of VM2.

## VMD

One of the experimental manipulations applied in the experiment presented in Chapter 3 is the added motor delay conditions. The goal of this is to investigate the effect on prediction quality and overall tracking performance. Our results showed convincingly that increasing VMD did lead to worse prediction quality and tracking performance overall. It is also found that subjects find conditions with added motor delay more mentally demanding and the physiological effort measure (pupil dilation) also shows that they feel more effortful in trials with added motor delay. Our experimental data has shown that FB information also drops as AMD increases, making it negatively correlated with perceived effort measure. We therefore could not provide evidence to support the hypothesis that increase in FB information is the potential cause of the higher perceived effort. To reconcile this difference, we will now use the IC to simulate data with added motor delay, to explore whether frequency of control could help understand better the measured effort in that experiment.

VMDs ranging from 17 to 32 frames with increments of 3 frames are applied to the IC model to generate simulated tracking data for 2 VM conditions (1 and 2) separately. All other parameters, including  $\beta$ ,  $\psi$  and  $\bar{\epsilon}$  are kept constant. Although  $\beta$  is not altered throughout, increasing  $\Delta$  will mean to increase the horizon of the predictions used, therefore would decrease prediction quality. This is first tested by checking the FF information of the simulation with respect to increasing VMD. It is found that FF information for both VM conditions decrease significantly as VMD increases, (Figure 5.12, green). This is consistent with experimental data and theoretical hypotheses. Then, we look at the frequency of control in response to the increased VMD to gain insight into the potential effort required for the trial. Results show that frequency of control increases significantly in response to the decrease in FF information in both VM conditions (Figure 5.12, blue). The effect on frequency of control is more apparent on VM1 since it certainly has more room for increment compared to VM2. This increase provides a logical explanation as to why subjects might feel more effortful for added motor delay conditions. On the same note, our experimental results also demonstrate a significant interaction effect of signal predictability and added motor delay on self-reported task mental demands (Figure 3.8), confirming a pattern akin to that of frequency of control (Figure 3.8). Remarkably, despite the increase in frequency of control, FB information in VM2 still decreases (Figure 5.12, red) and general performance also decreases (increase in MSE, Figure 5.12, purple). The FB and FF information, as well as the implied engagement/effort by the frequency of control are all coherent to the experimental findings in Chapter 3, hinting at a potentially pivotal role frequency of control plays in giving rise to the perception of effort.

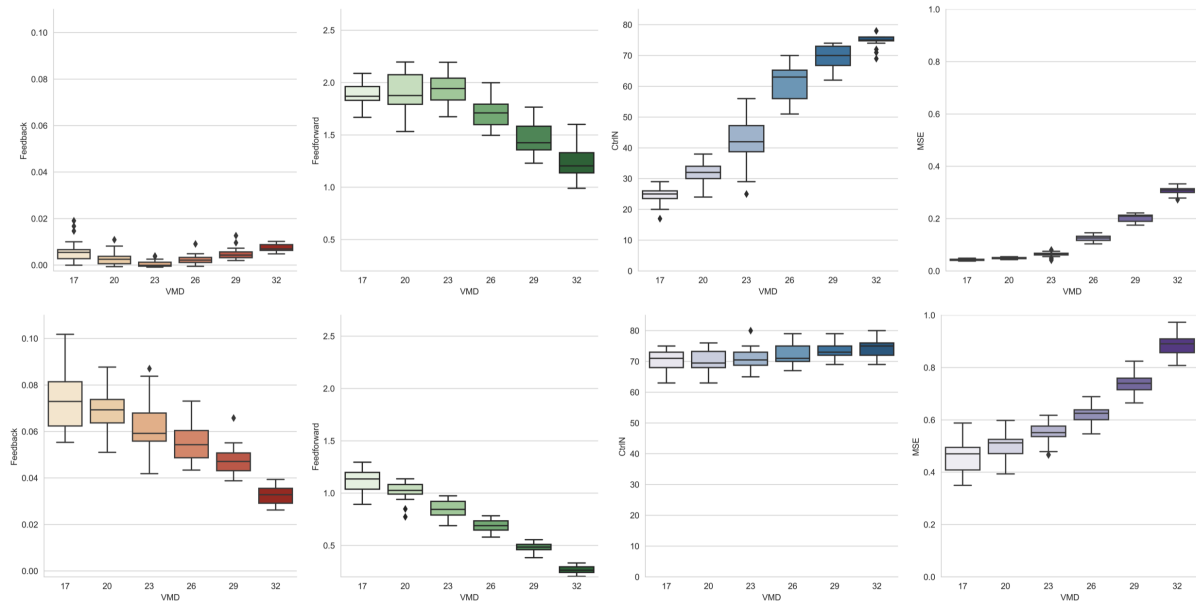


Figure 5.12: **FB, FF information, frequency of control and MSE change with VMD.** (Top 4) Simulation results for signals of VM1. (Bottom 4) Simulation results for signals of VM2.

It is very important to note the relationship between FF information and frequency of control in the current simulation is the opposite of the one found in the previous simulation with increased error threshold. In the previous simulation, it was found that they are positively correlated and in the current one they are shown to be negatively correlated. The key to understanding these differences is to lay out the causal relationship between prediction quality and frequency of control. Normally, it is the quality of prediction that causes changes in frequency of control. As prediction quality drops, it is more likely that the tracking response will exceed the error threshold, thus triggering control instances. That is precisely what is observed in the current simulation. However, sometimes, this relationship could in turn reverse, like in the previous simulation. In the previous simulation, all predictions are generated with the same parameters, so they do not differ in quality to begin with. However, as tolerance for errors gets higher, control instances will become rarer and prediction horizon will in turn grow longer and quality lower. This drop in quality of prediction is therefore caused by lack of control and the lack of control is not caused by drop in quality of prediction.

## 5.6 Conclusion

In the field of human control studies, the observer, predictor, and feedback control model has enjoyed a lot of success in producing smooth and realistic tracking performance for some types of tracking tasks. However, theoretically, these types of models fail to capture characteristics of human tracking such as discrete control and the existence of a psychological refractory period. Intermittent controller is later developed to tackle these limitations in the original OPF model. In the current experiment, a model is built by combining an intermittent controller with predictions made by a deep variational information bottleneck network. The deep VIB network comprises an optimal encoder and a decoder to generate predictions of future signals from some observed signals, given a certain resource constraint. By varying the resource constraint in the deep VIB network and the parameters in the intermittent controller model, we obtain simulation data that helps gain insight into the potential underlying mechanism of human visuomotor tracking performance in terms of information cost, predictive information, and frequency of control. The deep VIB network results demonstrate that low encoding rate of incoming signals would lead to low outcome predictive information. Moreover, the maximum predictive information, and the corresponding minimum encoding rate to achieve that, are constrained by the signal predictability. Applying the trained deep VIB network into an intermittent controller to provide predictions during the tracking performance, it is found that high signal predictability is associated with high FF and low FB information, just as observed from the real experimental data. High signal predictability is also associated with lower frequency of control, consistent with the low reported mental demand and subjective effort for tracking these signals. Changing the error threshold in the intermittent controller model leads to a consistent drop of frequency of control, implying reduced effort. Performance in terms of FB and FF information also drops due to lack of correction by control. Lastly, increasing VMD in the intermittent controller is shown to significantly decrease FF information, hinting at a decrease in quality of prediction. An increase in frequency of control is observed in response to this, which in turn could help explain the increased perceived effort reported in the added motor delay conditions from previous experiment. The deep VIB network could potentially provide many insights into the relationship between external inputs and internal representations with respect to a specific goal. For instance, one could observe the changes in representations and performance by varying the depth of history of incoming signals that are encoded. In the current model the depth is set at 2 because the signals are produced by an AR2 process and it is more economical for training purposes. Incorporating a longer history would almost certainly increase the quality of prediction, but it can only be done with a constrained encoding rate. It might be interesting to analyse how the optimal representations of long history of incoming signals change qualitatively as a function of increased resource constraint. This might be insightful for understanding what are the important elements in a signal to be encoded for making good predictions. On the other hand, more research can also be done



on finding a more efficient predictive function that can be implemented more readily and flexibly in the intermittent controller. Currently, the intermittent controller relies on a pre-trained deep VIB network on a limited range of  $\beta$  values. Ideally, there should be a way to control resource constraints in a more continuous and seamless fashion while running the controller model. This would allow one to study the possibility and potential effect of modulating encoding rate during the course of a trial. For the sake of clarity and simplicity, the current model did not include the consideration of the neuromuscular system dynamics although this system could potentially have a significant role in influencing visuomotor tracking performance. Future studies should aim at incorporating that into the model, for instance as a filter function that transforms predictions to smoother motor outputs. This could help fill the gap in the current study on topics such as motor cost and motor noise.

# Chapter 6

## Conclusion

This thesis sets out to gain understanding on the perception of cognitive effort in humans. The study of subjective phenomena such as sensation of effort could sometimes get illusory if the research question is ill-defined. Just as the common subjective experience of the colour red can be attributed to light of wavelengths between 620 to 750 nm hitting the retina, we seek to unveil the mechanism that gives rise to the sensation of cognitive effort. Reviewing concepts that are most closely related to cognitive effort, it is realised that the function of cognitive control is widely applied to understand both the computational and biological mechanism behind cognitive effort. However, this framework does not allow for precise quantification of the amount of ‘effort’ or ‘control’ in a broad range of cognitive processes. We therefore turn to the formulation of information cost of cognitive processes within the information theoretic framework. This formulation follows the bounded rationality formalisation of cognition and postulates the brain as an inference machine that actively updates its internal models with sensory inputs with respect to prior beliefs. This framework yields a computational cost associated with each update which forms the basis of the information cost of a cognitive process. Another relevant information measure of a cognitive process is identified as the predictive information. Complementing the information cost, it should give the total information transfer of a cognitive process. It is proposed that FB information is related to the resource-intensive real time information processing and FF information represents the automatic processing of information that is considerably much more efficient. Therefore, it is further proposed that FB, but not FF information should be closely related to the perception of cognitive effort.

### 6.1 Information measures validation in a visuo-motor tracking task

The proposed information measures are derived and tested empirically in a visual-motor tracking task. As hypothesised, FB measure increases dramatically as tracking signals become more complex while FF information increases with signal predictability. The

dominant effect of signal predictability on FB and FF information is further confirmed in a second experiment by isolating it from the effect of signal speed. Another experimental manipulation is designed to specifically influence the quality of predictive information in the tracking performance by prolonging visual-motor delays. In response to the prolonged delays, results show a significant drop in FF information. Taken together, our results convincingly show that as the statistical contingencies of the signals or quality of prediction change, the corresponding changing responses of information processing regarding the signal are reflected through the FB and FF information.

## 6.2 Cross-task interference on information cost

Some of the most common theories of cognitive effort point to a limited cognitive resource with unspecified nature that is so scarce and precious that the phenomenon of subjective effort is developed evolutionarily to facilitate the allocation of this resource. To investigate the extent to which our information processing rate is limited, we employ a dual-task paradigm and analyse the cross-task effects. In the first experiment consisting of the VM tracking task with an auditory N-back task, the performance of both tasks dropped as the difficulty of the other task increased. This is coherent to the cross-task interference effect observed in cognitive control studies. The analysis on the information cost of the VM tracking task also shows a considerable drop when the concurrent N-back task is more demanding. This might invite the interpretation that the N-back task has increased tension in cognitive resources and effectively decreased the information processing capacity of the VM tracking task, causing a drop in FB measure. However, a second experiment where the N-back task is replaced by Hick's task provides a more detailed look into the interaction of information processing rate of the two tasks involved. Both Hick's task and VM tracking task show performance drop when the other task is more demanding, once again demonstrating a cross-task interference in performance. However, the analysis of information rate per second in Hick's task is shown to be constant, even when faced with increased demand in the concurrent VM tracking task. The information rate per second of the VM tracking task is also found to be free of the Hick task's influence. This information rate (bits/s) is based on an estimation of VM task engagement time assuming subjects are not engaging in the tracking task when they are responding to the Hick's task:

$$VM \text{ task engagement time} = \text{Trial length} - \text{Total Hick RT}$$

This is an interesting result as it shows that while there is a cross-task interference on performance, the information rates per second for both tasks remain constant. The design of the first dual-task experiment with N-back task does not allow for the same analysis; but even if it does, this VM task engagement time is still at best an upper bound for the true engagement time, rendering the corresponding information rate per second a lower bound of the rate.

## 6.3 Cognitive demand and effort measures

With the exception of the first experiment, all experiments have included some measures of perceived task demand or effort. In general, self-reported mental demands, avoidance of trials, and baseline-corrected pupil dilation during trials are found to be positively correlated with at least some task conditions that also invoke higher information cost, such as increased signal complexity, lower speed, harder N-back task and harder Hick's task. These results mostly support the hypothesis that information cost is closely related to cognitive effort. However, one exception to these findings is the added motor delay condition. Both FB and FF information drop in response to longer delays, but they are also reported to be more mentally demanding, more likely to be avoided in the future and associated with increased pupil dilation, signalling increased effort. Considering information rates per second might be constant but perception of demands or effort is not, this suggests that the perception of effort is more likely to be contrived by the total amount of information cost within a trial rather than the rate per second measure. The IC model simulations also seem to support this hypothesis since it is shown that while the encoding rate is unchanged (due to constant  $\beta$  values and VMD, for instance), the total FB information of a trial can still be changed by the more frequent engagement of control.

## 6.4 Engagement and effort

The findings from the dual-task experiment reveal a potentially missing piece in understanding the link between information cost and cognitive effort, that is the actual engagement time. To fill this gap, we explore the possibility of modelling tracking performance with an intermittent controller and try to infer task engagement by the frequency of control measure provided by the model. By comparing simulation data with observed phenomena from the experiments, it is found that frequency of control explains well the perceived task demand and perceived effort. This interpretation makes sense from the point of view that the end-goal of the perception of cognitive effort is to motivate behaviour change to achieve better allocation of resources. If information rate is not something one can alter by will, then there is no practical purpose for cognitive effort to be signalling it. On the other hand, frequency of control or engagement time is something that can be voluntarily modified, and therefore would be a more probable cause for the sensation of effort.

Summarising from the theories and experiments presented, predictability seems to be the recurrent element that reduces effort. Not only are predictable signals consistently rated as less demanding or effortful, the IC model also shows a significant drop in frequency of control if predictions used are reliable. From a theoretical point of view, quality of prediction is associated with optimality of priors/representations used in a cognitive process. The usage of suboptimal priors incur extra information cost. The link between this cost and perceived effort lies in the limited metabolic or computational resources that information processing supposedly exhausts. These assumptions, when supplemented with the

notion of a constant rate of information gain, as suggested by our experimental results, point to a resource that is more directly quantifiable — time.

The current study mainly focuses on visuomotor tracking tasks which provides a wealth of data in an ecological task setting. However, as one can see, the estimation of engagement time might need to rely on mathematical modelling, if possible at all. The FB and FF measures presented here could potentially be derived for other discrete or continuous tasks as long as task predictability can be well manipulated and estimated. The application of these measures should be explored in future studies.

## 6.5 Future directions

A deep VIB network is used in this study to generate simulations and predictions for the IC model. This is only a preliminary model and many explorations are still needed. As a starter, we could expand the input dimension to include a longer history of signals and train the models with different  $\beta$  values. Then, we could analyse the latent variables of the model to try to visualise what would be the optimal way to compress the incoming signals given resource constraints, e.g. downsampling or dumping of older history. While this obviously does not necessarily represent how humans actually form representation, it might provide some hypotheses that could be tested by experiments. Another improvement of the model would be to incorporate a state space model in the decoder of the deep VIB network, so that the encoded representations would be decoded to parameters of a state space model, e.g. an autoregressive model. This would allow the deep VIB network to generate predictions of future signals of arbitrary length.

As for the intermittent controller, future work could also try to implement dual-task features in the task by imposing extra constraints on the psychological refractory period parameters or even by changing  $\beta$  values dynamically throughout the trial depending on real-time dual-task demands. Moreover, the current model intentionally omits the neuromuscular system (NMS) for the sake of simplicity. But future research should also explore this part of the model and investigate its contribution in terms of information, both in information cost and in predictive information, to the overall tracking performance. In light of this, experiments concerning handedness of tracking seem fitting for the purpose of dissociating the contribution of cognitive, e.g. encoding inputs, making predictions, from that of motor components (Mathew et al., 2020). Future research could aim at investigating a more elaborate model of intermittent control.

External rewards and explicit performance feedback both have pivotal roles to play in human decision-making problems. Future studies should aim at using FB and FF measures as tools to reveal changes in information processes under the influence of rewards and feedback. The incorporation of external rewards would also essentially redefine the optimisation objective of the IC model, whose behaviour can then be analysed as a function of these variables.

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