Three essays in financial economics : climate risk and the transition to a low carbon economy

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General Introduction

The most recent Intergovernmental Panel on Climate Change (IPCC) report (Masson-Delmotte et al. [2021]) has reminded us once again of the fragility of our ecosystem. In essence, it concludes that climate change already impacts all regions of the world, and that any increase in temperature will lead to further disruptions that could materialize in diverse ways. Extreme weather events are bound to become more frequent, and billions of people will have to bear the consequences of the likely exit from the Holocene geological epoch. However, climate change is not the only process that is threatening the sustainability of our species on planet Earth.

To apprehend this global environmental issue, it is helpful to think in terms of planetary boundaries, as defined in the seminal contribution of Rockström et al. [2009]. According to this framework, nine environmental dimensions govern the stability of our planet, climate change being only one of them. It is important to note, however, that all these dimensions are interconnected. Hence, losing the fight against climate change would precipitate being in a worse state along most (if not all) of these dimensions. As shown in figure 0.1, the threshold that represents the safe operating space had already been exceeded in at least four of them in 2015, namely biogeochemical flows, land-system change, biosphere integrity, and climate change. However, stratospheric ozone depletion seemed to be particularly under control. This is a striking example of how coordinated public policies can prevent us from crossing thresholds that could induce irreversible consequences. In light of the risks associated with the loss of the ozone layer that protects humans from harmful ultraviolet light wavelengths, politicians from around the world agreed on banning the use of most of the ozone-depleting substances at the Montreal Protocol. At the time of writing this thesis, all countries belonging to the United Nations have ratified the protocol and we have returned to the safe operating space with respect to stratospheric ozone depletion. This example of a successful coordinated action gives hope to face the environmental challenges ahead, and many other inspiring examples could be found at the local level.

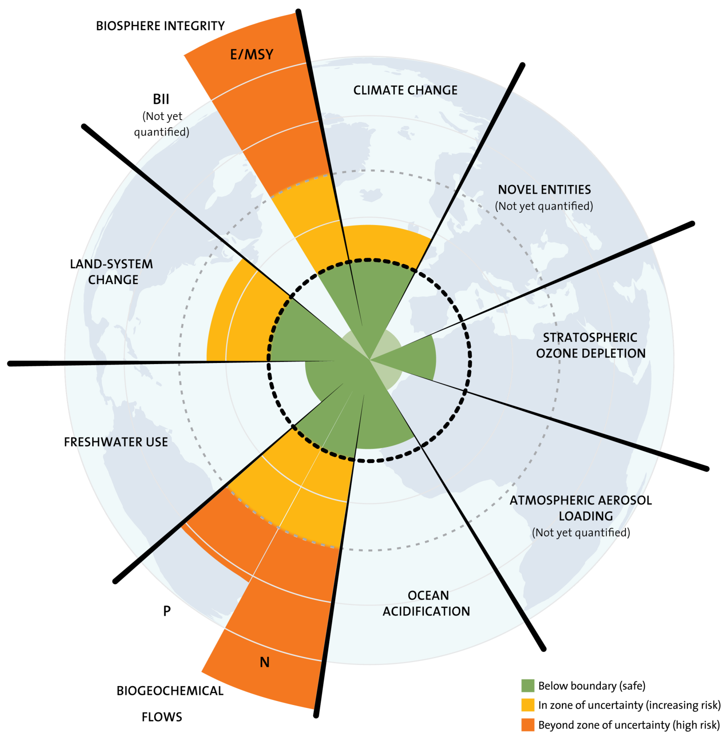


FIGURE 0.1. Planetary boundaries - Lokrantz/Azote based on Steffen et al. [2015]

Although economic activity is likely the common cause of movement toward many of these boundaries, macroeconomic policies will only be able to mitigate human impact on some of them. By implementing policies that will favor a low carbon production system, governments hope to be able to curb the temperature curve and return to the safe operating space along the climate change boundary. In practice, much of the efforts in terms of macroeconomic policies have been focused on the implementation of a price on carbon emissions.

In many areas of the world (*e.g.* Europe, Canada, California, China), governments have opted for a market cap and trade system to limit greenhouse gases (GHG hereafter) emissions of companies operating within their borders. This type of measure paves the way for a more comprehensive approach to climate change mitigation. It is thus natural to investigate other policies (alongside fiscal policy) that could be implemented to foster the transition to a cleaner economy. Related to this question, it is also important to assess the potential negative side effects of putting a price on carbon in order to try and offset them.

Macro-financial institutions, such as central banks or macroprudential authorities, are becoming more concerned and involved in the push for climate change mitigation. The creation of the Network for Greening the Financial System, back in December 2017, materialized the willingness of central banks and financial authorities from around the world to acknowledge and monitor more closely climate risk and its impact on the financial system. Since its inception, the network has added 75 members and now wields the strength of 83 macro-financial institutions.

A few months ago, the European Central Bank (ECB) unveiled the results of its 18-months strategic review. Among the conclusions of the review, two particularly stand out. First, the ECB presented a *climate change action plan* "to incorporate climate change considerations into [its] policy framework". This is additional proof that monetary policy is now ready to work together with fiscal policy, in order to tackle the daunting climate change challenge. Second, the ECB also decided to adopt a symmetric inflation target around 2% over the medium-term and gave up the "below but near 2%" previous objective that was more constraining. This decision gives more flexibility to central bank operations, but also reflects the challenges it faced following the Great Financial Crisis. Since 2008, inflation in the euro area has consistently fell short of the ECB target, sometimes even flirting with deflation. A common view is that the decline in the natural rate of interest prevented the central bank to conduct accommodative monetary policy and stimulate inflation. This explanation laid the ground for large scale asset purchases, but these measures did not allow inflation and interest rates to pick up substantially and get back to long-term averages.

The low rate environment, much like the climate change challenge, can be seen both as a curse and as an opportunity. In the case of near zero interest rates, it is a curse as it implies a higher risk of deflation and lower flexibility for the central bank. On the other hand, it allows government to run deficits without risking a debt crisis and firms to enjoy favorable financing conditions, which proved very convenient during the peak of the Covid crisis. With respect to climate change, the curse is the looming environmental disaster associated with inaction in the way we use our planet's resources, and the opportunity lies in the benefits we could reap from building a more resilient production system based on sustainable technologies. These two challenging environment are also related, since low rates imply a lower discount factor of future damages to the economy, as shown by Bauer and Rudebusch [2021]. In terms of policies, an important question tackled in this thesis is the efficiency of the carbon price in steering growth in sustainable technologies, and the role of macro-financial policies in incentivizing investment in the research and development of these technologies.

The goal of this Ph.D. thesis is to provide a framework for policy-oriented analysis in times of changing climate and low interest rates, with an emphasis on macro-financial policies. This thesis covers the three following topics: i) the natural rate of interest in the context of the effective lower bound on nominal rates ii) the side effects of a market for carbon permits and the benefits of interactions with other polices iii) the role of endogenous growth in sustainable technologies in the transition to a low carbon economy.

Scientific Background

This thesis builds on two main strands of literature, namely macro-finance and macro-environmental economics. Although these two strands share a common macroeconomic component, they also differ largely in the issues they seek to address and the tools they employ. Another common factor is the impressive development these two fields have experienced in recent years.

The evolution of macroeconomic business cycles models. The birth of the business cycle literature can be traced back to Lucas [1978] and Kydland and Prescott [1982]. Following Lucas critique (Lucas [1976]), which called for a radical change in the way macroeconomic policies were studied, Lucas built the first business cycle analysis model with microfoundations. The main idea was to incorporate the forward looking nature of economic agents in order to infer their reactions to policy changes.[1] This gave rise to the first wave of the business cycle literature, often referred to as Real Business Cycle (RBC) models. In these types of models, economic fluctuations are assumed to come from exogenous changes in productivity. Optimizing agents will in turn react rationally to these unexpected surprises, leading to changes in aggregate macroeconomic variables.

Given that RBC models are based on the classical economics theory, it was not long before critics from other economic schools of thought started to call into question the validity of their conclusions. One particular feature of these models was rapidly challenged by New-Keynesian (NK) economists: the flexibility of prices. By assuming that firms could not change their prices in each period, Rotemberg [1982] and Calvo [1983] both introduced sticky prices to business cycle models, allowing for nominal changes to have an implication on agents' decisions. This led to the concept of the NK Phillips curve (NKPC),[2] which relates current inflation to inflation expectations and the response of output to inflation.

[1] In the context of asset pricing for Lucas [1978] and economic fluctuations for Kydland and Prescott [1982].

[2] It was first implemented in business cycle models by Roberts [1995].

Shortly after, Taylor [1993] formulated the idea of a Taylor rule, which can be defined as the reaction function of the central bank. It relates the level of the nominal rate of interest to variations in output and prices. The Taylor rule, together with the NKPC and the Investment-Saving (IS) curve, forms the three-equation NK model that can be found in Clarida et al. [1999]. This reduced-form model summarized the NK economics applied to business cycles and has been widely used for monetary policy analysis ever since.

At the beginning of the 21st century, the evolution turned to the size of the models and the methods used to solve them. In an attempt to better replicate data, models started to grow more complicated[3] and the number of equations involved sharply increased. Applying Bayesian techniques to the estimation of parameters and shocks, dynamic stochastic general equilibrium (DSGE) models became more efficient at matching observed macroeconomic series. This new generation of business cycle models, exemplified by Smets and Wouters [2003] and Christiano et al. [2005], delivered better empirical performances than standard econometric tools in matching data. As a consequence, it became the workhorse model for many public institutions, such as central banks or governments.

The type of model developed in this thesis owe much to the evolution of the business cycle literature. Throughout the three chapters, the core of the framework is often borrowed from Smets and Wouters [2003], itself standing on the shoulders of other giants. Two of the chapters also rely on Bayesian estimation, as defined in An and Schorfheide [2007], to infer structural shocks and parameters. However, we depart from the standard DSGE models of the beginning of the 21st century by including both financial frictions and the environmental externality.

The Great Financial Crisis and the rise of macro-finance. The Great Financial Crisis was not only an important macroeconomic shock, but also an electroshock to the macroeconomic profession. Indeed, DSGE models became widely criticized for their

[3] Incorporating various features such as capital adjustment costs, variable capacity utilization rate, habit formation, and several new shocks.

inability to predict the crisis and deliver suitable policy recommendations. In retrospect, it is not surprising since these models did not feature any representation of the financial sector. Thus, they were completely unable to account for the interaction between the financial sphere and the real economy. The natural way forward was to improve existing models in order to be able to analyze issues related to financial intermediation and financial innovation within the business cycles framework.

Models of financial intermediation were brought back to the forefront and their key mechanisms incorporated into state of the art DSGE models. The financial accelerator, developed by Bernanke et al. [1999], was adapted to take into account unconventional monetary policy and led to the model of Gertler and Karadi [2011]. Merging financial frictions with modern monetary tools within medium-scale business cycles models allowed for a better understanding of the nexus between finance and the real economy.

At the same time, macro-finance was faced with a new challenge: an extended period of extremely low nominal rates. In order to explain the secular decline in interest rates, significant research focused on the estimation and drivers of the natural rate of interest.[4] In parallel, Wu and Xia [2016] developed the concept of the shadow rate of interest that allows for the measurement of the "true" impact of monetary policy on financing conditions when the effective lower bound on nominal rate is binding. Other authors, such as Guerrieri and Iacoviello [2015], focused on the methods to take into account this new occasionally binding constraint on the nominal rate set by the central bank. These advancements over the standard NK models have been instrumental in guiding policy makers during the extraordinary period that followed the Great Financial Crisis.

The three chapters of this thesis build on recent macro-finance literature by featuring a role for the financial sector as well as acknowledging the current low rate environment. A motivation for conducting this research, however, was the intuition that macro-finance models lacked a key component to guide policy makers in today's world: the impact of

[4] A detailed explanation of the concept, as well as an extensive literature review on the topic, can be found in section 1.1.

climate change and the risks associated with it.

Climate change and the macro-environmental view. Although the acceleration of climate change has led to a tremendous amount of research on this topic in recent years, the incorporation of climate dynamics in macro modeling can be traced back to the 1970s. The impressive early work of Nordhaus, bringing geophysical dynamics into economics, culminated in the development of the first Dynamic Integrated model of Climate and the Economy (DICE) (Nordhaus [1992]), which paved the way for an entirely new field of Integrated Assessment Models (IAMs) featuring growth theory components. By linking production and the carbon cycle, these models provide estimates of the social cost of carbon (SCC). This theoretical cost, which can be seen as the present cost of potential future damages from climate change to the society, provides an idea of the price that would be put on carbon dioxide (CO_2) emissions by a social planner. However, there is a lively debate around the calibration of the parameters that drive the SCC. Stern [2008] argues that the marginal abatement cost is a more reliable indicator when it comes to designing carbon policies. Regarding the SCC, he believes that the discount rate used by Nordhaus is too high and proposes a lower one to favor next generations. Discounting future damages less aggressively implies a higher estimation of the SCC. Weitzman [2012] shows that the damage function[5] used in DICE models is biased, as it doesn't take into account extreme tail events. His specification leads to a higher SCC and more aggressive policy recommendations. In the same spirit, Dietz and Stern [2015] advocate for a convex damage function that factors tipping-points in climate dynamics. This approach allows one to account for irreversible chain of events that could be triggered by climate dynamics and leads to a SCC that is higher than the one computed by Nordhaus, but lower than the one computed by Weitzman.

Although IAMs offer a unique insight into the nexus between climate and growth, these models usually feature a very stylized representation of agents' behaviors. As outlined

[5]The function that relates a rise in temperature to a deterioration in output.

in Stern and Stiglitz [2021], IAMs provide a reductive picture of the complexity of our economic systems. By incorporating the interaction between production, emissions, and climate into the business cycle literature, Heutel [2012] created a new path for the macro-environment literature. With DSGE models being more flexible, it is easier to include various components in order to address questions that could not be tackled by IAMs. In the same spirit, Golosov et al. [2014] provided an assessment of the optimal carbon policy using a general equilibrium model. Interest in environmental DSGE (E-DSGE) models picked up recently and this type of model is now used to study a wide range of questions related to climate change.

The three chapters of this thesis rely on different specifications of E-DSGE models. We add to this literature by also considering the impact of financial frictions. The second chapter of this thesis was actually the first article to incorporate both balance-sheet constrained financial intermediaries and an environmental externality within a state of the art DSGE model. The main idea was that, as much as macro-finance models lacked the environmental dimension, macro-environment models lacked the financial dimension. By bridging the gap between these two sub fields of economics, the goal was to provide a framework to analyze environment-related issues relevant for macro-financial authorities.

Contributions of the Thesis

Building on the literature detailed above, we bring various contributions to the burgeoning field of macro-finance under environmental constraints.

On theoretical grounds, this Ph.D. thesis offers three main contributions. The first contribution concerns a general equilibrium interpretation of the nexus between economics, climate, and finance. The framework developed paves the way for a more comprehensive approach allowing to highlight the role of the financial sector in the transition to a low carbon economy. Another theoretical contribution highlights how a market for carbon pricing designed to implement a gradual drop in emissions affects both welfare and risk premia. With respect to the abatement technology, we develop a model of endogenous growth that is able to account for the development of new sustainable technologies lowering the cost of firms' abatement. This is instrumental in understanding the nexus between carbon price, growth in green technologies, and the level of abatement.

On methodological grounds, we propose a new way of estimating the natural rate of interest allowing to take into account the effect of unconventional monetary policies when assessing the monetary policy stance. The shadow natural rate of interest (SNRI) is particularly useful when the effective lower bound on the nominal rate of the central bank is binding. Another methodological contribution is the computation of pathways consistent with the transition to a low carbon economy and featuring both deterministic trends and stochastic process. Using the extended path algorithm of Adjemian and Juillard [2013], we derive credible transition pathways that exhibit some level of uncertainty at the business cycle frequency.

On applied grounds, we provide both an estimation of the SNRI and an analysis of its drivers, including financial and environmental factors. With respect to mitigating the side effects of the carbon policy, we assess the role of two macro-financial policies: macroprudential and monetary. We also quantify the effect of "green quantitative easing' programs compared to standard quantitative easing (QE) programs. The last applied contribution is related to finding ways to steer growth in green technologies. Using the endogenous growth framework we developed, we assess how public policies could favor investment in green entrepreneurs that will favor the emergence of cheaper abatement technologies.

Finally, there are also empirical contributions. We set up a difference-in-difference estimator to analyze the consequences of the implementation of a market for carbon permits. We also disentangle the links between carbon price, the level of emissions, and the level of abatement using a panel regression on the Eurozone (EZ).

Main Results

In the first chapter, we rely on Bayesian estimation to find the time path and drivers of the SNRI. We show that financial factors play a substantial role in fluctuations of the SNRI, along with standard supply and demand factors. Environmental factors, however, are found to have a negligible effect on the path of the SNRI, whether through emissions shocks, or through the implementation of a carbon price.

Analyzing the monetary policy stance through the lens of the shadow interest gap, we find that there were two periods of extended accommodative monetary policy: the years leading to the Great Financial Crisis and the period 2012-2018. On the one hand, it suggests that loose monetary policy might be partly to blame for the financial imbalances that gave rise to the subprime crisis. On the other hand, it shows that central bankers provided favorable financing conditions in the aftermath of the Great Financial Crisis, and that they can not be blamed for the slack in the economy observed during these years.

Overall, the analysis conducted in this chapter suggests that the SNRI is an indicator that could be used in practice by central banks. It is particularly useful since it incorporates more information and provides a different view than standard estimations of the natural rate of interest, especially when the zero lower bound is binding.

In the second chapter, we develop a macro-finance E-DSGE model with both endogenously-constrained financial intermediaries and heterogeneous firms. We then use the model to assess the effects of various policies and their interactions on carbon emissions.

We find that a fiscal instrument of about 350€ per ton of carbon is needed to be aligned with the net-zero target by 2030. However, the actual implementation of this second-best instrument induces two inefficiencies. The first inefficiency is linked to the need of an increasingly higher price of carbon (compared to the optimal) to meet the EU targets. This decoupling generates a growing welfare loss. To address this wedge, we show that a green macroprudential policy aiming at reducing climate-related financial risk is also efficient in partially offsetting the welfare loss, while reaching the emissions target. The second inefficiency is related to the market design of the environmental fiscal policy in the EU. Uncertainty in the carbon price resulting from the ETS is shown to affect firms' marginal costs and thus to alter risk premia. We find that QE rules that react to changes in risk premia are able to completely offset movements in spread levels and volatility, allowing for a smooth transmission of monetary policy.

Turning to QE programs, we find that macroprudential policy is needed to provide an incentive to central banks to engage in green QE. Choosing between brown and green QE then implies a trade-off between higher output and lower emissions. This trade-off would disappear in the event that the green sector grows enough to be as large as or larger than the brown sector. More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero.

In the last chapter, we first conduct an empirical analysis on the role of the ETS in emissions reduction within the EZ using a diff-in-diff analysis, with the US as the control area. We find that the cap and trade EU system contributed significantly to emissions reduction. We then rely on a panel data set on the EZ to assess the impacts of fiscal environmental policies and macro-financial policies on green innovation. We find that both policies play an important and significant role in boosting green innovation. However, we also find that above a certain threshold, the carbon price has a negative effect on green innovation.

Second, we develop a dynamic general equilibrium model based on the empirical evidence, to assess the role fiscal and macro-financial policies can play both in the long-run and in the short-run.

We use a reduced form model to get the long-run transition pathways toward the net-zero transition and find that making abatement technology available and cheap coupled with an optimal environmental policy is the most efficient tool (from a welfare perspective) in achieving climate goals. Relying solely on a carbon price could reach the same target, but comes with higher welfare costs.

Finally, we use a full fledged model incorporating both endogenous green innovation growth and financial intermediaries to quantitatively estimate trends on output and green innovation. We then assess the role subsidies, macroprudential policies, and QE, could play in boosting green innovation. We show that these three policies differently affect the path of the trend growth in green innovation, but that they have the same pro-cyclical dynamics. In addition, we show that financial subsidies are more effective than macroprudential and QE rules in reaching the net-zero while ensuring a lower carbon price over time. This leads us to conclude that policy makers could optimally foster growth in projects that enable cheaper and more effective abatement by giving incentives to financial intermediaries and entrepreneurs.

Chapter 1

The Natural Rate of Interest in the 21st Century

A previous version of this chapter was presented at the International Macroeconomics Workshop in Rennes, Theories and Methods in Macroeconomics conference in Paris, Computing in Economics and Finance conference in Milano, and Computational and Financial Econometrics conference in Pisa.

1.1 Introduction

At the beginning of the 21st century, the concept of the natural rate of interest made an impressive comeback in both academic and policy economic circles. Sometimes referred to as the equilibrium real rate, the indicator first theorized by Wicksell [1898] was redefined by Woodford [2003] in a business cycle framework as the interest rate that allows an economy to reach its potential, consistent with a stable inflation. It determines the level of interest rates that would be necessary to maximize output by clearing the market between saving and investment, in the absence of any friction. This interest rate has become a leading indicator in determining whether monetary policy stance was accommodative enough, in particular in the context of the zero lower bound on nominal rates. By comparing the effective real rate with its natural counterpart, monetary policymakers are able to determine whether the stance of monetary policy is too tight or too loose.[6] In parallel, the concept of shadow rate has gained momentum in recent years. This indicator, inferred from yield curves, allows economists to assess the true impact of unconventional monetary policies on interest rates relevant for the real economy. It is particularly useful when the nominal rate set by the central bank is constrained by the zero lower bound. Associated with the natural rate, it could thus be an effective way to assess the monetary policy stance when rates are abnormally low.

In this context, a debate rage constantly among economists about the driving factors beyond low interest rates. On the one hand, the secular stagnation hypothesis, pioneered by Summers [2015], explains the decline in real rates as a result of structural factors acting on both the supply and demand side of savings. The main consequence of this misallocation on the saving market is a structural decline in aggregate demand and interest rates. On the other hand, Rogoff [2015] and Borio [2017] explains this downward pressure on interest rates

[6] For a concrete illustration of the natural rate as an indicator employed by policymakers, see Yellen [2015] for instance. Beyond being a straightforward indicator, it can also be incorporated as a target in Taylor [1993] rules.

through financial factors.[7] According to these authors, the financial nature of the great depression has caused a very long-lasting damage to the economy. Since financial cycles are more persistent than standard business cycles, financial imbalances require more time than expected to clear, and by so interest rates to normalize. In particular, Borio [2017] suggests to incorporate financial factors in the estimation of the natural rate, and expect the latter to be above zero and considerably higher than the secular stagnation hypothesis suggests. Thus, the puzzle regarding the natural interest rate lies in the identification of its driving forces that would validate any of the financial drag or the secular stagnation hypothesis. Furthermore, recent studies suggest that there could be a nexus between climate change and interest rates. Bauer and Rudebusch [2021] show that the drop in the equilibrium real rate results in a higher social cost of carbon, as future damages from climate change are discounted with a lower rate than before. On the other hand, Benmir et al. [2020] find that climate risk reduces the level of the natural rate, increasing the probability of hitting the effective lower bound on nominal rates. Cantelmo [2020] confirms this result when investigating the role of disaster risk, from an ex-ante perspective. Taken together, these findings suggest that there could be a reinforcement loop between the modeling of climate change damages and the impact it could have on interest rates.

Given the current debate on the natural rate in a low interest rate environment, we set up and estimate over the period 1995:I:2019:IV a DSGE model for the US economy that allows to examine the role of structural and financial factors as suggested by Borio [2017], but also takes into account environmental factors. The originality of our approach is threefold. First, we extend the workhorse model of Smets and Wouters [2007] by including credit frictions a la Gertler and Karadi [2011] in order to disentangle standard business cycles from their financial counterpart, and an environmental externality to take into account emissions shocks. Second, we also disentangle the contribution of short run versus long drivers of the natural rates by including and estimating stochastic trends on the labor-

[7] Rogoff [2015] refers to debt supercycles while Borio [2017] to a financial drag. These two theories share many aspects.

productivity and on the flow of emissions. Third, we estimate the model using a series of the shadow rate rather than the nominal interest rate, which offers several advantages over the standard method, such as accounting for unconventional monetary policy and providing more accurate policy recommendations when the zero lower bound is binding.

Our analysis is related to a growing body of estimation methods of the natural interest rate, that can be divided in three different strands of methodology. The fist one estimates semi-structural models using the Kalman filter, as introduced by Laubach and Williams [2003], where the natural rate is assumed to depend on the trend growth rate of the economy as well as on unobserved components. They find that the natural interest rate has fallen sharply since the start of the Great Recession and this drop was generalized to most of developed economies.[8] The second methodology group uses pure econometric methods to estimate the natural rate of interest. Using a time-varying parameter vector autoregressive model, Lubik and Matthes [2015] show that the natural rate has recently been above its effective counterpart, which means that the monetary policy is too loose. Yi and Zhang [2016] also find evidence of a structural decline of real interest rates in numerous countries and attribute this decline to a reduction in investment demand rather than to a saving rise. Finally, the last strand of methods relies on New Keynesian DSGE models estimated using Bayesian techniques.[9] Justiniano and Primiceri [2010] and Barsky et al. [2014] estimate a standard New-Keynesian model to measure the natural rate of interest for the US economy. Curdia [2015] and Cúrdia et al. [2015] enrich this environment by including the role of forward guidance, while Gerali and Neri [2017] disentangle the role of short term versus long term effects on the natural interest rate through the introduction of trends. In addition, Del Negro et al. [2017] interestingly extend the Smets-Wouters model to include a financial accelerator mechanism, and use inflation expectations and the

[8]See Mesonnier and Renne [2007], Hamilton et al. [2015] or Fries et al. [2016] for other estimation exercises on the natural rate through semi-structural models.

[9]By disabling markups shocks and nominal rigidities, any New-Keynesian model can characterize the natural dynamics of an economy through the determination of the rate of interest that makes the economy reach its potential.

treasury yield as observable in their fit exercise.

Our work is strongly connected to Del Negro et al. [2017] as we both feature trends and financial frictions in our analysis. However, we differ in including an environmental component, with firms emitting greenhouse gases as they produce. Our analysis also includes both financial and environmental observable variables in addition to six standard macroeconomic variables. This approach allows us to estimate quantitatively the impact of financial and environmental factors on the variations of the natural rate of interest over the business cycle and in the long run, which has not been done before.

The main contribution of the paper is to provide an estimation of the shadow natural of interest, an indicator that is better suited for monetary policy analysis when the effective lower bound is binding than standard estimation of the natural rate of interest. We then investigate the driving forces of the SNRI and show that financial factors play a substantial role in fluctuations of this indicator, along with standard supply and demand factors. This result tends to validate the thesis put forward by Borio [2017]. Environmental factors, however, are found to have a negligible effect on the path of the SNRI, whether through emissions shocks, or through the implementation of a carbon price. Regarding ex-post monetary policy analysis, we find two periods of extended accommodative monetary policy and discuss their implications.

The remainder of the paper is organized as follows. In section 1.2, we develop a New-Keynesian model *à la* Smets and Wouters [2007] with financial frictions *à la* Gertler and Karadi [2011] and an environmental component *à la* Heutel [2012]. Section 1.3 presents the estimation strategy. Section 1.4 presents the results of the estimation and drivers of the SNRI, as well as monetary policy implications. Finally, section 1.5 concludes.

1.2 The model

In this section, we develop a standard NK model with both nominal and real rigidities enhanced with financial frictions and an environmental externality coming from the production of firms.

1.2.1 Households

There is a continuum of identical households indexed by $j \in (0,1)$. At each period households supply labor, consume and save. They have two choices to save: either lending their money to the government, or making a deposit at a financial intermediary that will finance firms. In each household, there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households can't lend their money to a financial intermediary owned by one of their members. Members who are workers supply labor and return their salaries to the household they belong to.

Agents can switch between the two occupations over time. There is a fraction f of agents who are bankers and a probability θ that a banker stays banker in the next period. Thus, $(1-f)\theta$ bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to the household, which will use it as start-up funds for the new banker.

Households solve the following maximization problem:

$$\max_{\{C_{jt},H_{jt},B_{jt+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\varepsilon^B_{t+i} \frac{(C_{jt+i} - hC_{jt+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} H^{1+\varphi}_{jt+i} \right], \tag{1.1}$$

$$s.t. C_{jt} = w^h_t H_{jt} + \Pi_{jt} + T_{jt} + R_t B_{jt} - B_{jt+1}, \tag{1.2}$$

where $\beta \in (0,1)$ is the discount factor, parameters σ, $\varphi > 0$ shape the utility function of the j^{th} household associated to risk consumption C_{jt} and hours worked H_{jt}. The consumption index C_{jt} is subject to external habits with degree $h \in [0;1)$ while $\chi > 0$ is a shift parameter allowing me to pin down the steady state amount of hours worked. Labor supply is remunerated at real desired wage w^h_t that will be negotiated by unions, Π_{jt} are dividends from the ownership of firms (both financial and non-financial) and T_{jt} are lump sum taxes. As we assume that intermediary deposits and government bonds are one period bonds, $R_{jt}B_{jt}$ are interests received on bonds held and B_{jt+1} are bonds acquired. Household consumption preferences are affected by a shock ε_t^B affecting the intertemporal allocation of consumption following an $AR(1)$ shock process: $\log(\varepsilon_t^B) = \rho_B \log(\varepsilon_{t-1}^B) + \sigma_B \eta_t^B, \quad$ with $\eta_t^B \sim \mathcal{N}(0,1)$.

Solving the first order conditions, we get the labor supply equation:

$$\varrho_t w_t^h = \chi H_{jt}^{\varphi}, \tag{1.3}$$

where ϱ_t is the marginal utility of consumption:

$$\varrho_t = \varepsilon_t^B (C_{jt} - hC_{jt-1})^{-\sigma} - \beta h E_t \left\{ \varepsilon_{t+1}^B (C_{jt+1} - hC_{jt})^{-\sigma} \right\}, \tag{1.4}$$

$$\text{and} \quad \beta E_t \Lambda_{t,t+1} R_{t+1} = 1, \tag{1.5}$$

$$\text{with} \quad \Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}. \tag{1.6}$$

1.2.2 Unions

Households delegate the wage negotiation process to unions. Households provide differentiated labor types, sold by labor unions to perfectly competitive labor packers who assemble them in a CES aggregator and sell the homogeneous labor to intermediate firms.[10] Unions negotiate the real margin between the real desired wage of households w_t^h and the real marginal product of labor W_t/P_t. Using a Calvo wage nominal rigidity device, each period a random fraction θ_W of unions is unable to re-negotiate a new wage. Assuming that the trade union is able to modify its wage with a probability $(1-\theta_W)$ the jth union

[10] Labor packers are perfectly competitive and maximize profits, $W_t H_t^d - \int_0^1 W_{jt} H_{jt} \mathrm{d}j$, under their packing technology constraint, $H_t = [\int_0^1 (H_{jt}{}^{(\epsilon_W-1)/\epsilon_W})\mathrm{d}j]^{\epsilon_W/(\epsilon_W-1)}$. Here, W_t is the nominal wage, H_t^d is the labor demand and $\epsilon_W > 1$ is a substitution parameter. The first order condition which determines the optimal demand for the j^{th} labor type is, $H_{jt} = (W_{jt}/W_t)^{-\epsilon_W} H_t^d$, $\forall j$. Thus the aggregate wage index of all labor types in the economy emerges from the zero-profit condition: $W_{jt} = [\int_0^1 (W_{jt}{}^{1-\epsilon_W})\mathrm{d}j]^{1/(1-\epsilon_W)}$.

chooses the nominal optimal wage W_t^* to maximize its expected sum of profits:

$$\max_{\{W_t^*\}} E_t \sum_{s=0}^{\infty} (\beta\theta_W)^s \Lambda_{t,t+s} \left[\frac{W_{jt}^*}{P_t} \prod_{k=1}^{s} \pi_{t+k-1}^{\xi_W} \bar{\pi}^{(1-\xi_w)} - \varepsilon_{t+s}^W w_{t+s}^h \right] H_{jt+s}, \tag{1.7}$$

subject to the downward sloping demand constraint from labor packers:

$$H_{jt+s} = \left(\frac{W_{jt}^*}{W_{t+s}} \prod_{k=1}^{s} \pi_{t+k-1}^{\xi_W} \bar{\pi}^{(1-\xi_w)} \right)^{-\epsilon_w} H_t^d,$$

where ε_{t+s}^W is an ad-hoc wage-push shock to the real wage equation which captures exogenous fluctuations in the wage margin negotiated by unions and affects in turn the productivity of the economy. It follows an ARMA process, $\log(\varepsilon_t^W) = \rho_W \log(\varepsilon_{t-1}^W) + \sigma_W \left(\eta_t^W - u_W \eta_{t-1}^W \right)$,with $\eta_t^W \sim \mathcal{N}(0,1)$, where $\rho_W \in [0,1)$ is the AR term and $u_W \in [0,1)$ the MA one. The latter captures high frequency fluctuations in the variations of the wage inflation rate.

1.2.3 Production sector

The production sector is made of two kinds of firms: intermediate goods firms and retail firms. Intermediate goods firms produce differentiated types of intermediates goods that are bought and packed by retail firms into an homogeneous good sold to households.

1.2.3.1 Intermediate goods firms

At the end of the period, intermediate goods firms acquire capital K_{it+1} from capital producing firms. They finance this acquisition by issuing claims S_{it} that they sell to financial intermediaries. Firms price each claim at the price of a unit of capital and issue as much claims as they buy capital. we assume no frictions in the process of obtaining funds. Thus, we have the following equality:

$$Q_t K_{it+1} = Q_t S_{it}. \tag{1.8}$$

Intermediate goods firms use this capital to produce in the next period and are able to resell it on the open market. As we don't consider adjustment costs, the firm's capital choice problem is always static. Unlike households, bankers have perfect information on firms and no problem enforcing payments. Hence, firms face no capital constraint in obtaining funds but are indirectly subject to the capital constraint faced by financial intermediaries.

The production function reads as follows:

$$Y_{it} = d(T_t^o)(U_t \varepsilon_t^K K_{it})^{\alpha} \left(\Gamma_t^y H_{it}^d\right)^{1-\alpha}. \tag{1.9}$$

Here, U_t is the utilization rate of capital affecting the services of effective capital K_{it}^E, H_t^d labor demand and where $\alpha \in [0,1]$ is the effective capital share. $d(T_t^o)$ is a convex polynomial function of order 2 displaying the temperature level ($d(T_t^o) = ae^{-(bT_t^{o2})}$), with $(a,b) \in \mathbb{R}^2$, which is borrowed from Nordhaus and Moffat [2017]. We will show in the next section how production in turn affects climate dynamics. As in Gertler and Karadi [2011], we introduce a shock on the stock of physical capital capturing exogenous variations in the quality of capital, where $\log(\varepsilon_t^K) = \rho_K \log(\varepsilon_{t-1}^K) + \sigma_K \eta_t^K$,with $\eta_t^K \sim \mathcal{N}(0,1)$. In addition, we introduce a time-varying labor-augmenting trend γ_t^y on labor, featuring a stochastic growth rate in the economy. The stochastic trend is determined by:

$$\frac{\Gamma_t^y}{\Gamma_{t-1}^y} = \gamma_t^y = \bar{\gamma}^y \varepsilon_t^A, \tag{1.10}$$

where $\bar{\gamma}^y \geq 0$ is the gross growth rate of the economy that is estimated in the fit exercise, the latter is affected by persistent technology shock ε_t^A defined by $\log(\varepsilon_t^A) = \rho_A \log(\varepsilon_{t-1}^A) + \sigma_A \eta_t^A$, with $\eta_t^A \sim \mathcal{N}(0,1)$. A positive realization of η_t^A thus features a permanent increase in the growth rate of the economy through a rise in labor productivity.

Letting P_t^m be the real price of intermediate goods, the representative firm maximizes the expected stream of profits under the supply constraint (equation (1.9)) and the funding constraint (equation (1.8)):

$$\max_{\{Y_{it},H_{it}^d,K_{it+1},U_t\}} E_t\Bigg\{\sum_{s=0}^{\infty} P_{it+s}^m Y_{it+s} - w_{t+s}H_{it+s}^d + \left(Q_{t+1+s} - \delta(U_{t+s})\right)\varepsilon_t^K K_{it}$$
$$- R_{t+s}^k Q_{t-1+s} S_{t-1+s}\Bigg\},$$
$$s.t. \quad Y_{it} = d(T_t^o)(U_t\varepsilon_t^K K_{it})^{\alpha}\left(\Gamma_t^y H_{it}^d\right)^{1-\alpha},$$
$$with \quad Q_t K_{it+1} = Q_t S_{it}.$$

Here, $\delta_t(U_t)$ is the time-varying depreciation rate. It is given by, $\delta\left(U_t\right) = \delta_c + \frac{b}{1+\zeta}U_t^{1+\zeta}$, where $\zeta \geq 0$ is the utilization rate elasticity and $\delta_c \in [0,1]$ is the depreciation rate parameter of the real business cycle literature.[11]

We assume that the replacement price of used capital is also fixed and at unity to get the following utilization rate, labor demand and rate of return of physical capital:

$$\delta'(U_t) = \Psi_t\alpha\frac{Y_t + \gamma_t\Phi\bar{Y}_i}{K_t^E U_t}, \tag{1.11}$$

$$w_t = \Psi_t(1-\alpha)\frac{Y_t + \gamma_t\Phi\bar{Y}_i}{H_t}, \tag{1.12}$$

$$R_t^k = \left[\Psi_t\alpha\frac{Y_t + \gamma_t\Phi\bar{Y}_i}{\varepsilon_t^K K_t} + Q_t\left(1 - \delta(U_t)\right)\right]\frac{\varepsilon_t^K}{Q_{t-1}}, \tag{1.13}$$

$$\Psi_t = P_t^m, \tag{1.14}$$

with Ψ_t the Lagrange multiplier on the production constraint.

1.2.3.2 Retail firms

A continuum of f differentiated retail firms produce final output according to a CES function: $Y_t = \left[\int_0^1 Y_{ft}^{(\epsilon_P-1)/\epsilon_P}\mathrm{d}f\right]^{\epsilon_P/(\epsilon_P-1)}$, where Y_{ft} is output by retailer. From cost minimization by users of final output, we get: $Y_{ft} = \left(P_{ft}/P_t\right)^{-\epsilon_P}Y_t$, and $P_t = \left[\int_0^1 P_{ft}^{1-\epsilon_P}\mathrm{d}f\right]^{1/(1-\epsilon_P)}$.

[11] Assuming that $\delta\left(\bar{U}\right)$ is calibrated and with $\bar{U} = 1$, we compute in the de-trended steady state the following parameters: $b = \bar{P}^m\alpha\left(1+\Phi\right)\bar{Y}/K$ and $\delta_c = \delta\left(\bar{U}\right) - b/\left(1+\zeta\right)$.

The role of retail firms is simply to re-package output produced by intermediate firms. As they use one unit of intermediate output to produce one unit of final output, the marginal cost is equal to the intermediate output price P_{mt}. We add nominal rigidities as in Christiano et al. [2005]: there is a probability $1-\theta_P$ that a firm is able to freely adjust its price. Otherwise, it can only index it to the lagged inflation. Retail firms thus choose the optimal reset price P_t^* according to the following maximization problem:

$$\max_{\{P_t^*\}} E_t \sum_{i=0}^{\infty} (\theta_P \beta)^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} \pi_{t+k-1}^{\xi_p} \bar{\pi}^{(1-\xi_p)} - \varepsilon_{t+i}^P P_{t+i}^m \right] Y_{ft+i}, \tag{1.15}$$

where $\pi_t = P_t/P_{t-1}$ is the rate of inflation from $t-i$ to t and ε_t^P is an *ad-hoc* cost-push shock to the inflation equation following an AR(1) process which captures exogenous inflation pressures. As for wages, the price-push shock follows an ARMA process, $\log(\varepsilon_t^P) = \rho_P \log(\varepsilon_{t-1}^P) + \sigma_P \left(\eta_t^P - u_P \eta_{t-1}^P\right)$,with $\eta_t^P \sim \mathcal{N}(0,1)$, where $\rho_P \in [0,1)$ is the AR term and $u_P \in [0,1)$ the MA one.

The optimal price P_t^* is given by the following sum:

$$E_t \sum_{i=0}^{\infty} (\theta_P \beta)^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} \pi_{t+k-1}^{\xi_p} \bar{\pi}^{(1-\xi_p)} - \frac{\epsilon_P}{\epsilon_P - 1} \varepsilon_{t+i}^P P_{t+i}^m \right] Y_{ft+i} = 0. \tag{1.16}$$

1.2.4 Emissions and Climate

As shown in the previous section, the level of the temperature will impact the production function. On the other hand, emissions are also a consequence of intermediate firms' production. The emissions level of each intermediate firm is assumed to depend on two carbon intensity parameters ϑ_1^o and ϑ_2^o:

$$E_{it} = \Gamma_t^e \vartheta_1^o Y_{it}^{1-\vartheta_2^o}. \tag{1.17}$$

We introduce a time-varying trend γ_t^e on emissions, which accounts for changes in carbon intensity over time. The stochastic trend is determined by:

$$\frac{\Gamma_t^e}{\Gamma_{t-1}^e} = \gamma_t^e = \bar{\gamma}^e \varepsilon_t^E, \tag{1.18}$$

where $\bar{\gamma}^e \leq 0$ is the gross growth rate of the carbon intensity that is estimated in the fit exercise. This trend can be affected by shocks ε_t^E defined by $\log(\varepsilon_t^E) = \rho_E \log(\varepsilon_{t-1}^E) + \sigma_E \eta_t^E$, with $\eta_t^E \sim \mathcal{N}(0,1)$. A negative realization of η_t^E thus features a permanent decrease in the growth rate of carbon intensity. This allows to capture technological advances in the abatement technology in a stylized way. Cumulative emissions end up in the atmosphere and give rise to a law of motion for the carbon stock:

$$X_t = (1-\gamma_d)X_{t-1} + E_t + E^*, \tag{1.19}$$

where E_t is the aggregate flow of emissions at time t ($\int_0^1 E_{it} di$) and γ_d is the decay rate. E^* represents the rest of the world emissions. The global temperature T_t^o is then linearly proportional to the level of cumulative emissions as argued by Dietz and Venmans [2019]:

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \tag{1.20}$$

with υ_1^o and υ_2^o chosen following Dietz and Venmans [2019]. The higher the temperature, the bigger the damages to the production function, as exhibited in equation (1.9).

1.2.5 Capital producing firms

We assume that households own capital producing firms and receive any profits. These firms buy capital from intermediate goods firms at the end of period t and then repair depreciated capital and build new capital. They then sell both the new and re-furbished capital. As we showed earlier, the value of a unit of new capital is Q_t. We suppose that there are flow adjustment costs associated with producing new capital. Then, capital producing firms are facing the following maximization problem:

$$\max_{\{I_t^n\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s}-1) I_{t+s}^n - \frac{\kappa}{2} \left(\varepsilon_{t+s}^I \frac{I_{t+s}^n + \bar{I}_{t+s}}{I_{t+s-1}^n + \bar{I}_{t-1+s}} - \gamma_{t+s} \right)^2 (I_{t+s}^n + \bar{I}_{t+s}) \right\} \tag{1.21}$$

$$\text{with} \quad I_t^n = I_t - \delta(U_t)\varepsilon_t^K K_t \tag{1.22}$$

where I_t^n and I_t are respectively net and gross capital created, $\bar{I}_t = \bar{I} A_t$ is the steady state investment including a trend and $\delta(U_t)\varepsilon_t^K K_t$ is the quantity of re-furbished capital. We differ from Gertler and Karadi [2011] by including a shock ε_t^I as in Smets and Wouters [2003] to captures exogenous variations in the cost of producing physical capital in the economy. The latter shock follows $\log(\varepsilon_t^I) = \rho_I \log(\varepsilon_{t-1}^I) + \sigma_I \left(\eta_t^I - u_I \eta_{t-1}^I \right)$, with $\eta_t^I \sim \mathcal{N}(0,1)$. Thus, we get the following value for Q_t:

$$Q_t = 1 + \frac{\kappa}{2} (\iota_t - \gamma_t)^2 + \kappa \varepsilon_t^I (\iota_t - \gamma_t) \iota_t - \beta \varepsilon_t^I E_t \left\{ \Lambda_{t,t+1} \kappa \frac{(\iota_{t+1} - \gamma_{t+1})}{\iota_{t+1}^3} \frac{I_{t+1}^n + \bar{I}}{I_t^n + \bar{I}} \right\}. \tag{1.23}$$

where $\iota_t = \varepsilon_t^I (I_{nt}^n + \bar{I} A_t) / \left(I_{t-1}^n + \bar{I} A_t \right)$.

1.2.6 Financial intermediaries

As explained before, households save by making deposits to financial intermediaries. Financial intermediaries will, in turn, offer loans to non-financial firms. For the sake of simplicity, they represent the whole banking sector and can be thought as universal banks.

A financial intermediary balance sheet can be depicted as:

$$Q_t S_t = N_t + B_t, \tag{1.24}$$

where S_t is the quantity of financial claims banks own on non-financial firms and Q_t is their relative price. N_t is financial intermediaries' net worth and B_{t+1} the deposits obtained from households. Another way to read this equation is to see the left-hand side as the assets of financial intermediaries and the right-hand side as their liabilities, N_t being their equity capital and B_{t+1} their debt. Over time, financial intermediaries' equity capital evolves as the difference between return earned on financial claims hold R^k_{t+1} and interests paid to household R_t:

$$N_{t+1} = (R^k_{t+1} - R_t)Q_t S_t + R_t N_t \tag{1.25}$$

Thus, there is a fixed part in the growth in equity, which is the riskless rate of return. The variable part depends on the risk premium $R^k_{t+1} - R_t$ as well as on the total quantity of assets held by the financial intermediary.

Financial intermediaries will maximize equity on an infinite horizon, yielding the following objective function:

$$V_t^B = E_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1-\theta_B) \theta_B^{i-1} N_{t+1+i} \right\}, \tag{1.26}$$

where θ_B is the probability of a bank exiting the market. To avoid financial intermediaries to grow indefinitely, we introduce a moral hazard problem. At the beginning of each period, bankers can divert a fraction $\lambda \in (0,1]$ of invested funds back to the household they belong to. In this case, depositors can force them into bankruptcy but will only be able to recover the remaining $1-\lambda$. Therefore, the following incentive constraint must be respected:

$$V_t \geq \lambda Q_t S_t. \tag{1.27}$$

Concretely, it means that the expected gain of staying a banker is superior or equal to the gain realized when a banker diverts funds. We guess that the value function is linear of the form $V_t = \Gamma_t N_t$ so we can rewrite V_t as:

$$V_t = \max_{S_t} E_t \left\{ \beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \right\}, \tag{1.28}$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t$. Maximization subject to constraint (1.27) yields the following first order and slackness conditions:

$$\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R^k_{t+1} - R_t) \right\} = \nu_t \lambda, \tag{1.29}$$

$$\nu_t \left[\Gamma_t N_t - \lambda Q_t S_t \right] = 0, \tag{1.30}$$

where ν_t is the multiplier for constraint (1.27). Finally, we can rewrite the value function to find Γ_t:

$$\begin{aligned} V_t &= \nu_t \lambda Q_t S_t + \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \right\} \\ \Gamma_t N_t &= \nu_t \Gamma_t N_t + \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \right\} \\ \Gamma_t &= \frac{1}{1 - \nu_t} \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_t \right\}. \end{aligned} \tag{1.31}$$

We close this part of the model with the aggregate law of motion for the net worth of financial intermediaries $N_t = N^s_t + N^e_t$. Surviving bankers carry on with the following net worth:

$$N^s_t = \theta_B [(R^k_t - R_{t-1}) Q_{t-1} S_{t-1} + R_{t-1}] N_{t-1}, \tag{1.32}$$

while entering bankers are endowed with a fraction of the beginning of period net worth N_{t-1}:

$$N^e_t = \omega N_{t-1}. \tag{1.33}$$

Overall, the law of motion for the net worth reads as:

$$N_t = \theta_B (R^k_t - R_{t-1}) Q_{t-1} S_{t-1} + (\theta_B R_{t-1} + \omega) N_{t-1}, \tag{1.34}$$

with $\omega \in [0; 1)$ the proportion of funds transferred to entering bankers.

1.2.7 Authorities

1.2.7.1 Government

We assume that government consumption is a fixed proportion of current output and subject to a government consumption shock : $G_t = \frac{G}{Y} Y_t e_t^G$. Capital evolves according to the following law of motion: $K_{t+1} = \varepsilon_t^K K_t + I_t^n$ and government finances its expenditures thanks to lump sum taxes:

$$G_t = T_t. \tag{1.35}$$

1.2.7.2 Monetary policy

The central bank follows a simple Taylor [1993] rule to set the interest rate:

$$i_t - \bar{\imath} = \rho_c \left(i_{t-1} - \bar{\imath}\right) + (1 - \rho_c) \left[\phi_\pi \left(\pi_t - \bar{\pi}\right) + \phi_y \left(Y_t - \tilde{Y}_t\right) A_t^{-1}\right] + \varepsilon_t^R, \tag{1.36}$$

where $\bar{\imath}$ is the steady state of the nominal rate i_t, $\rho_c \in [0, 1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, ϕ_y is the output gap stance penalizing deviations of detrended output from its natural counterpart $\tilde{Y}_t$. Parameter ρ_c is a the monetary policy smoothing coefficient, , $\phi_{\Delta y}$ is the growth gap target and ε_t^R is an exogenous shock to monetary policy that follows an $AR(1)$ shock process: $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \sigma_R \eta_t^R$,with $\eta_t^R \sim \mathcal{N}(0, 1)$.

Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t = R_t E_t \left\{\pi_{t+1}\right\}. \tag{1.37}$$

1.2.8 Aggregation and market equilibrium

The general equilibrium of the model is set as follows. After (i) aggregating all agents and varieties in the economy, (ii) imposing market clearing for all markets, (iii) substituting the relevant demand functions, we get the general equilibrium conditions of the model.

Output is composed of consumption, investment and government consumption:

$$Y_t = \Delta_t^P \left[C_t + I_t + (\iota_t - 1)^2 \left(I_{nt} + \bar{I} \right) + G_t \right]. \tag{1.38}$$

where $\Delta_t^P = \int_0^1 (P_{ft}/P_t)^{-\epsilon_P} \mathrm{d}f$ denotes the price dispersion term, which is induced by the assumed nature of price stickiness.

In addition, the labor market clears when the following condition holds:

$$H_t = \Delta_t^W H_t^d, \tag{1.39}$$

where the wage dispersion terms is given by $\Delta_t^W = \int_0^1 (W_{jt}/W_t)^{-\epsilon_W} \mathrm{d}j$.

From the law of large numbers, the following relations for the evolution of the price and wage levels emerge:

$$P_t = \left[(1-\theta_P)(P_t^*)^{1-\epsilon_P} + \theta_P (\pi_{t-1}^{\xi_p} P_{t-1})^{1-\epsilon_P} \right]^{1/(1-\epsilon_P)}. \tag{1.40}$$

$$W_t = \left[(1-\theta_W)(W_t^*)^{1-\epsilon_W} + \theta_W (\pi_{t-1}^{\xi_W} W_{t-1})^{1-\epsilon_W} \right]^{1/(1-\epsilon_W)}. \tag{1.41}$$

1.2.9 The natural allocation

Even though the natural rate of interest is not directly observable, it is possible to compute it from any New Keynesian model by developing a parallel version of the model with no nominal rigidities as shown in Woodford [2003]. In our setup, these rigidities are driven by Calvo devices for both prices and wages and their respective markups shocks. In absence of these nominal rigidities, the economy reaches its full employment level with stable inflation. This equilibrium constitutes a first best allocation that monetary policy seeks to reach by adjusting the nominal rate in normal times, and possibly the size of its balance sheet when the nominal rate is constrained by the effective lower bound.

As Barsky et al. [2014] and Del Negro et al. [2017], we can compute the real natural rate $\tilde{R}_t$ from the Euler equation of the natural allocation:

$$\beta E_t \tilde{\Lambda}_{t,t+1} \tilde{R}_t = 1, \tag{1.42}$$

For comparison purposes with the nominal rate, the real natural rate is taken in nominal terms by including the expected inflation:

$$\tilde{\imath}_t = \tilde{R}_t E_t \left\{ \pi_{t+1} \right\}. \tag{1.43}$$

Wu and Zhang [2019] show that estimating a standard NK model with a series of shadow rate removes the challenge imposed by the zero lower bound, while keeping the properties of the model. Thus, we use the same procedure as Barsky et al. [2014] or Del Negro et al. [2017] to estimate the shadow natural rate of interest.

1.3 Calibration and estimation strategy

1.3.1 Calibration

Table 1.1 summarizes the calibration. All parameters related to the business cycle are set according to the estimation performed in Smets and Wouters [2007]. I set $\bar{L}$=1/3 for the steady state share of hours worked per day; $\bar{U}$, the utilization rate is normalized one; $\delta\left(\bar{U}\right)$=0.025, the depreciation rate of physical capital; α=0.18, the capital share in the technology of firms; G/Y=0.18, the ratio of public spending to GDP; $\theta_p = 0.66$ and $\theta_w = 0.70$ for the Calvo probabilities; $\xi_p = 0.24$ and $\xi_w = 0.58$ for the indexation to previous prices/wages; $\epsilon_P = \epsilon_W = 10$ for the elasticity of substitution between goods and labor types.

Turning to parameters related to the banking sector, we borrow calibration from Gertler and Karadi [2011] with $\omega = 0.002$ the start-up funds to new bankers and $\theta = 0.972$ the survival rate of bankers in the next period. We set the parameter $\lambda = 0.404$ to pin down the aggregate common equity tier 1 capital ratio in the US.[12] Parameters of the monetary policy rule are taken from Smets and Wouters [2007]: the persistence $\rho_c = 0.81$, the reaction to inflation $\phi_\pi = 2.04$ and the reaction to the output gap $\phi_y = 0.08$.

Regarding the environmental part, we calibrate the damage function according to Dietz and Stern [2015] with $d_1 = 0.04$ and $d_2 = 1$. The global temperature parameters υ_1° and υ_2° are set following Dietz and Venmans [2019] to pin down the "initial pulse-adjustment timescale" of the climate system. We set the global level of carbon in the atmosphere of 840 gigatons at the steady state, yielding a value of 0.045 for ϑ_1. We use the carbon intensity parameter $\vartheta_2 = 0.96$ to match the observed ratio of emissions to output for the USA at 25%.[13] The rest of the world's emissions parameter $E^* = 0.183$ so that the USA account for approximately 15% of total emissions. The decay rate of emissions δ_x is set at 0.21%.

1.3.2 Estimation Strategy

The model is estimated using Bayesian methods and quarterly data for the US economy. We estimate the structural parameters and the sequence of shocks following the seminal contributions of Smets and Wouters [2007] and An and Schorfheide [2007]. In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method. In the following fit exercise, we solve the model using a linear approximation to the policy function, and employ the Kalman filter to form the likelihood function. For a detailed description, we refer the reader to the original papers.

[12] The value of the capital ratio (12.2%) is taken from the FED stress test results of December 2020.

[13] We compute the emissions to output ratio as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from the World Bank.

1.3.2.1 Data and measurement equations

The Bayesian estimation relies on US quarterly data over the sample period 1995:I to 2019:IV. Therefore, each observable variable is composed of 100 observations. The data set includes 8 times series: output, consumption, investment, inflation, shadow rate,[14] risk premium, and the flow of emissions. The risk premium refers to the series of spread between BAA-rated corporate bonds and the federal funds rate provided by the FRB of St. Louis. Overall, our sample includes six standard series in the business cycle literature, one financial series and one environmental series. The goal is to include series that have not been considered in previous articles estimating the natural rate of interest, in order to uncover potential new drivers of this indicator.

Concerning the transformation of the series, the point is to map non-stationary data to a stationary model. The variables are made stationary in three steps. First, they are divided by the working age population. Second, they are divided by the GDP deflator price index. Third, they are taken in logs and we use a first difference filtering to obtain growth rates. The corresponding vector of observable is given by:

$$\begin{aligned}\Theta_t^{\text{obs}} = 100 \times [\ & \Delta \log Y_t^{\text{obs}}, \quad \Delta \log C_t^{\text{obs}}, \quad \Delta \log I_t^{\text{obs}}, \quad \log \pi_t^{\text{obs}}, \\ & \Delta \log w_t^{\text{obs}}, \quad i_t^{\text{obs}}, \quad s_t^{\text{obs}}, \quad \Delta \log E_t^{\text{obs}}\ ]', \end{aligned} \tag{1.44}$$

where Y_t^{obs}, C_t^{obs}, I_t^{obs}, π_t^{obs} are respectively the real per capita production, consumption, investment and inflation; while w_t^{obs} is the real wage, i_t^{obs} the quarterly shadow rate, s_t^{obs} the risk premium and E_t^{obs} the flow of emissions.

Regarding the model, the introduction of stochastic trends on productivity and emissions makes my endogenous variables non-stationary in steady state. However, the solution method used here implies a local approximation around a fixed point, thus requiring us to

[14] We use the fed funds rate in normal times and the series of Wu and Xia [2016] when the zero lower bound is binding.

rewrite the model in a de-trended fashion.[15] The corresponding measurement equations are given by:

$$\Theta_t = 100 \times [\ \log\left(\gamma_t^y \frac{Y_t}{Y_{t-1}}\right),\ \ \log\left(\gamma_t^y \frac{C_t}{C_{t-1}}\right),\ \ \log\left(\gamma_t^y \frac{I_t}{I_{t-1}}\right),\ \ \log(\pi_t),$$
$$\log\left(\gamma_t^y \frac{w_t}{w_{t-1}}\right),\ \ i_t,\ \ s_t,\ \ \log\left(\gamma_t^{y(1-\upsilon_2^o)} \gamma_t^e \frac{E_t}{E_{t-1}}\right)\ ]', \quad (1.45)$$

We capture the information contained in the mean of the sample through the steady state of our measurement equations which are different from zero.

1.3.2.2 Prior distributions

The rest of the parameters are estimated using Bayesian methods. Table 1.2 reports the prior (and posterior) distributions of the parameters for the US economy.[16] Most of our prior distributions are either relatively uninformative or consistent with previous works involving Bayesian estimations such as Smets and Wouters [2007]. For ARMA terms, $(\bar{\pi} - 1) \times 100$, $(\beta^{-1} - 1) \times 100$ and $(\bar{\gamma}^y - 1) \times 100$, our priors are directly borrowed from Smets and Wouters [2007]. For the standard deviation of shocks, we impose an inverse gamma distribution of type 2 as in Christiano et al. [2014] with prior inputs close to Smets and Wouters [2007] with mean 0.1 and standard deviation 0.5. Regarding the estimation of the trend on emissions $(\bar{\gamma}^e - 1) \times 100$, we want to give as less information as possible. Since this trend is not commonly estimated, we choose a normal distribution centered on 0 and let the data speak for itself.

[15] A complete description of the balanced growth path can be found in section 1.B.2.

[16] The posterior distribution combines the likelihood function with prior information. To calculate the posterior distribution to evaluate the marginal likelihood of the model, the Metropolis-Hastings algorithm is employed. I compute the posterior moments of the parameters using a total generated sample of 80,000, discarding the first 8,000, and based on eight parallel chains. The scale factor was set in order to deliver acceptance rates close to 23%. Convergence was assessed by means of the multivariate convergence statistics taken from Brooks and Gelman [1998]. I estimate the model using the Dynare package from Adjemian et al. [2011].

1.3.2.3 Posterior distributions

In addition to the prior distributions, table 1.2 reports the estimation results that summarize the means and the 5th and 95th percentiles of the posterior distributions. According to table 1.2, the data were fairly informative, as their posterior distributions did not stay very close to their priors.

While our estimates of the standard parameters are in line with the business cycle literature for the US economy as in Smets and Wouters [2007] or Christiano et al. [2014], several observations are worth making. In particular, we find that the trend on emissions is slightly negative (-0.08% annually), while the trend on productivity is consistent with data (roughly 2% annually). The estimation is thus able to account for the decoupling between output and emissions that was witnessed over the studied period.

1.4 Exercises

1.4.1 Time Path and Drivers of the Natural Rate of Interest

Figure 1.1 displays the estimation of the shadow natural rate of interest along with 95% confidence intervals.[17] A first interesting finding is that it did not go below zero following the great financial crisis, except maybe very shortly in 2011. In most estimation performed with DSGE models, the natural rate is found to drop far below zero and stay in negative territory for a prolonged period of time. Using a shadow rate model, however, the estimation is more consistent with the long term trend in the NRI found in semi-structural models, and is actually very similar to the estimation obtained in Lubik and Matthes [2015][18] through a time varying parameters VAR.

Turning to the drivers of the SNRI, figure 1.2 show the historical decomposition by

[17] To compute the confidence intervals, I randomly draw 5 000 samples for parameters and shock values from the 100 000 generated during the estimation procedure. I then simulate 5 000 paths for the SNRI and compute the standard deviation for each period.

[18] The main difference is the extra volatility induced by business cycle components in my exercise.

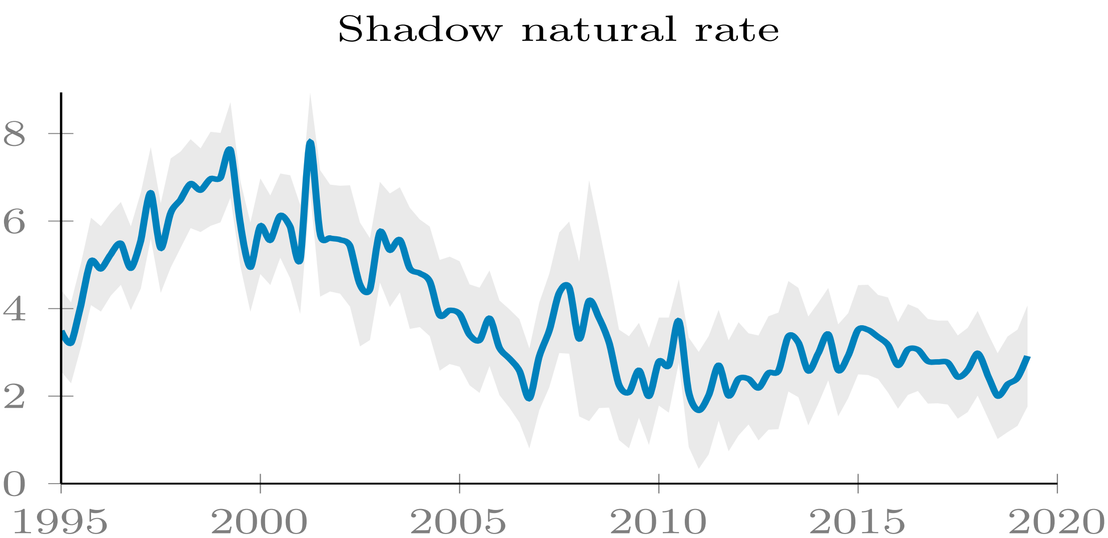


FIGURE 1.1. Time path of the shadow natural rate of interest with 95% confidence intervals

type of shock. Financial factors are found to be a main driver of the SNRI, along with standard supply and demand factors. The value of the stock of capital seems to have mixed effect on the level of the SNRI over the period studied. One plausible explanation is that rising asset prices tended to increase the level of the SNRI in normal times, but had the opposite effect when assets were notoriously overvalued (years leading to the financial crisis and between 2016 and 2020). This can be thought as periods of irrational exuberance, when agents tend to invest in financial markets more than usual. It leads to a disequilibrium between savings and investment that pushes the SNRI lower. The negative effect of financial factors is partly offset by a positive contribution of demand factors, driven by rising firms' investment. Firms are able to absorb some of the excess savings, even though the net effect on the SNRI is ultimately negative.

The leading role of financial factors in the SNRI variations is confirmed by the mean variance decomposition over several horizons displayed in figure 1.3. It shows the structural

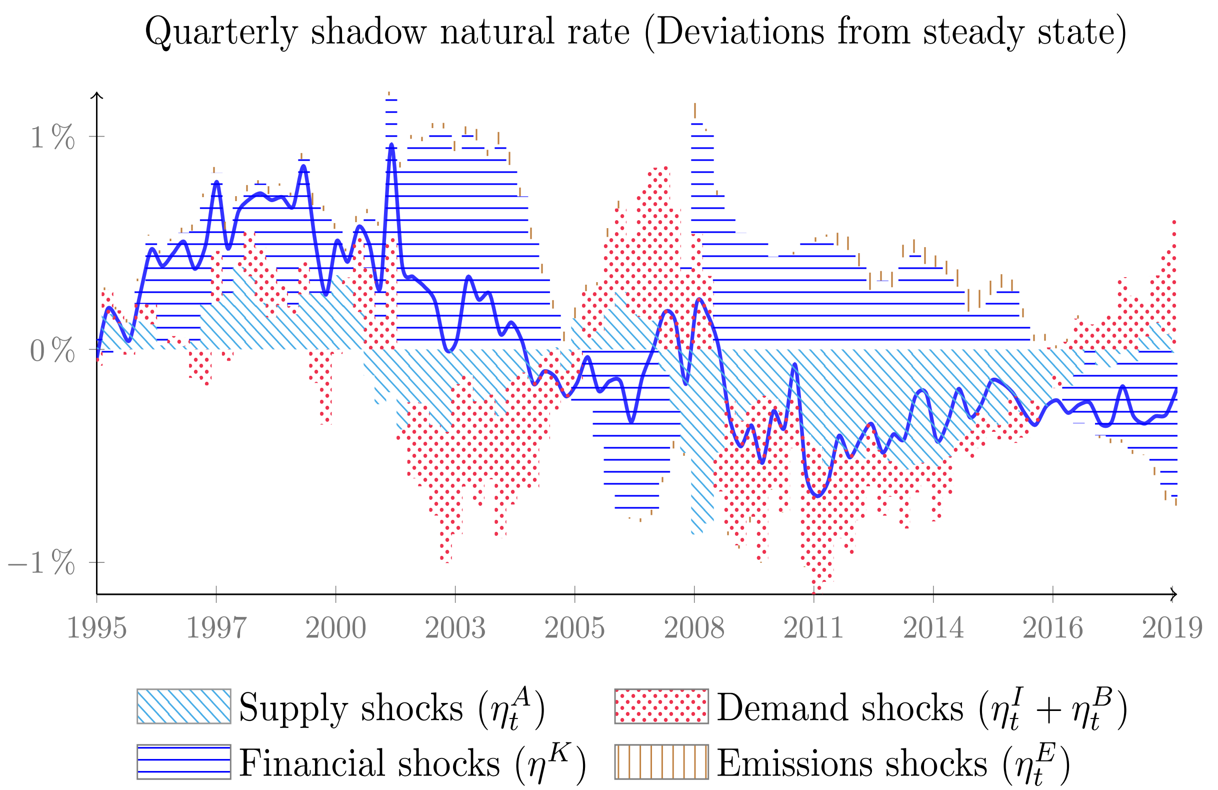


FIGURE 1.2. Historical decomposition of the natural rate of interest by type of shock

role of financial shocks in shaping the path of this indicator, accounting for roughly 40% of the unconditional variance. In the very short run, most of the variance in the SNRI can be explained by demand shocks, which is consistent with economic theory. As we extend the horizon studied, however, supply shocks tend to become more important, at the expense of demand shocks. This result is in line with the assumption underlying the estimation of the NRI through semi-structural models, that the long run trend in the equilibrium real rate can be explained by changes in productivity.

Taken together, results in this section suggest that financial factors are a key driver of fluctuations in the SNRI. However, the increase in the price of financial assets following

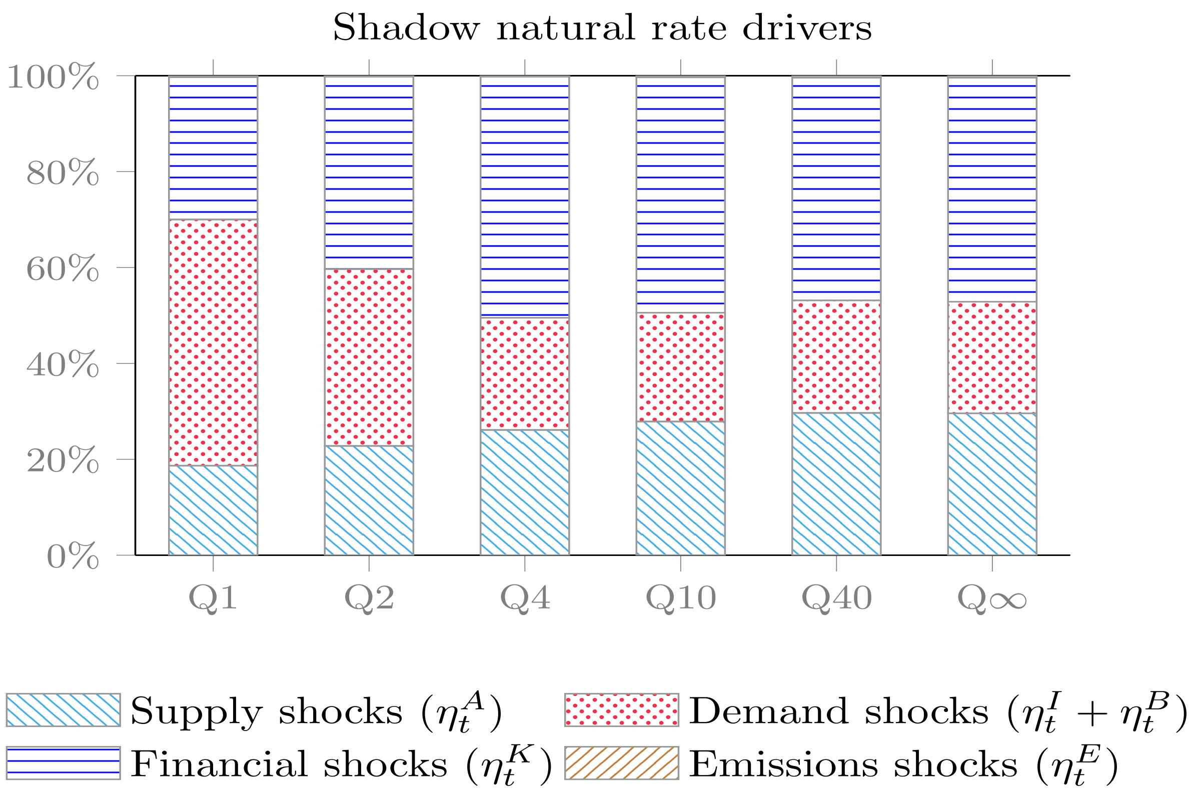


FIGURE 1.3. Posterior mean variance decomposition of the natural rate of interest by type of shock

the great financial crisis seems to have contributed positively to the level of this indicator. This finding confirms the intuition of Borio [2017] as the introduction of financial shocks changed the estimation of the NRI and implied a higher level than in other estimations performed with DSGE models.

1.4.2 Implications for Monetary Policy Analysis

Figure 1.4 shows the interest rate gap, computed as the difference between the shadow rate and the shadow natural rate plus inflation. If this gap is negative, it means that, taking into account both conventional and unconventional measures, monetary policy has been accommodative. The shadow interest rate gap, unlike the standard interest gap, allows for monetary policy stance analysis even when the zero lower bound on the nominal interest rate is binding, since it takes into account unconventional monetary policy.

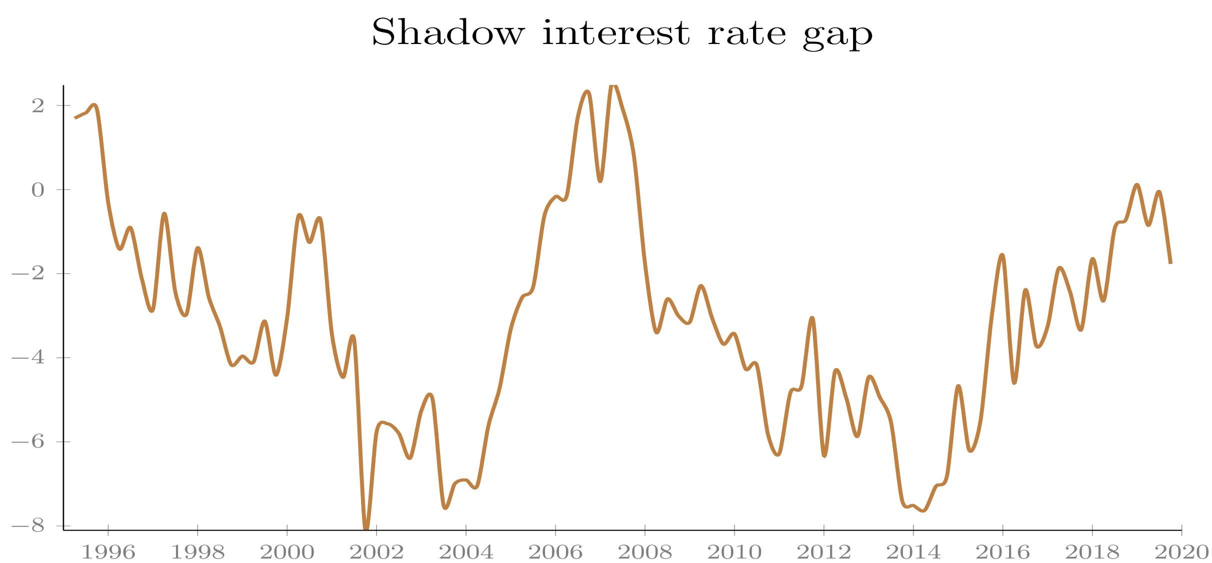


FIGURE 1.4. Monetary policy stance

Our estimation shows that monetary policy has been accommodative over most of the sample studied, except at the start of the great financial crisis and in recent months. Interestingly, monetary conditions were particularly favorable in the early 2000s, which is an explanation often put forward to explain the various financial follies that led to the subprime crisis. According to the shadow interest rate gap, 2012-2018 is another example of highly accommodative monetary policy. The implication for economic analysis is that, if we were in a form of secular stagnation, central bankers did everything they could to prevent it. By letting substantially grow its balance sheet, the Federal Reserve maintained favorable financing conditions for most of the past decade. The reason for slack in the economy is thus possibly to be found on the fiscal policy side, but it would require further work to assert.

1.4.3 Carbon Price and the Natural Rate of Interest

The impact of carbon price on the natural rate of interest is still understudied, and thus misunderstood. The question is to know how a carbon pricing scheme consistent with climate goals would alter the path of the equilibrium real rate. In order to have an idea of the consequences of implementing environmental fiscal policies on the path of the natural rate, we conduct a counterfactual analysis where the US authorities would have implemented a fixed carbon price of 45$ per ton of CO_2 over the period studied.

1.4.3.1 Model Modifications

To perform this exercise, we need to augment the model with a fiscal authority that will impose a carbon price on emissions generated by firms' production. The intermediate firms' maximization problem becomes:

$$\begin{aligned}
\max_{\left\{Y_{it},H^d_{it},K_{it+1},U_t,\mu_{it}\right\}} E_t \Bigg\{ \sum_{s=0}^{\infty} & P^m_{it+s}Y_{it+s} - w_{t+s}H^d_{it+s} + \left(Q_{t+1+s} - \delta(U_{t+s})\right)\varepsilon^K_t K_{it} \\
& - R^k_{t+s}Q_{t-1+s}S_{t-1+s} - f(\mu_{it})Y_{it} - \tau_t E_{it} \Bigg\}, \\
& s.t. \quad Y_{it} = d(T^o_t)(U_t\varepsilon^K_t K_{it})^{\alpha}\left(\Gamma^y_t H^d_{it}\right)^{1-\alpha}, \\
& with \quad Q_t K_{it+1} = Q_t S_{it}, \\
& and \quad E_{it} = \Gamma^e_t(1-\mu_{it})\vartheta^o_1 Y_{it}^{1-\vartheta^o_2}.
\end{aligned}$$

where $f(\mu_{it})Y_{it}$[19] is the cost of the abatement effort for each firm and $\tau_t E_{it}$ is the real price of carbon for each firm. Taking into account the abatement effort, equation (1.17)

[19] The abatement technology reads as follows: $f(\mu_{it}) = \theta_1\mu_t^{\theta_2}$ and parameters are calibrated according to Heutel [2012].

becomes:

$$E_{it} = \Gamma_t^e (1 - \mu_{it}) \vartheta_1^o Y_{it}^{1-\vartheta_2^o}. \tag{1.46}$$

Plugging equation (1.46) in the maximization problem leads to the following level of abatement with respect to the real price of carbon:

$$\tau_t = \theta_1 \theta_2 \mu_{it}^{\theta_2 - 1} \frac{Y_{it}}{E_{it}} (1 - \mu_{it}) \tag{1.47}$$

The marginal cost of firms is also modified and becomes:

$$P_{it}^m = \Psi_{it} + \tau_t (1 - \vartheta_2^o) \frac{E_{it}}{Y_{it}} + \theta_1 \mu_{it}^{\theta_2} \tag{1.48}$$

where Ψ_{it} is the marginal cost without any carbon policy. Total marginal cost now features expenses related to the environmental externality. Finally, the aggregate government budget constraint and the aggregate resource constraint are modified as follows:

$$G_t = T_t + \frac{\tau_t}{P_t} E_t \tag{1.49}$$

$$Y_t = \Delta_t^P \left[C_t + I_t + (\iota_t - 1)^2 \left(I_{nt} + \bar{I} \right) + G_t + f(\mu_t) Y_t \right]. \tag{1.50}$$

1.4.3.2 Results

Figure 1.5 displays the counterfactual path of the SNRI when a flat 45$ price on carbon is implemented, along with the previously estimated SNRI without any carbon price. The main result is that, although the estimation is slightly different, the introduction of a carbon price does not drastically change the behavior of the SNRI. Thus, it indicates that not taking into account environmental factors is not a major flaw of standard estimations of the NRI. This is also consistent with the fact that shocks to emissions did not have a sizeable impact on the path of the SNRI shown in the previous section.

This result, however, differs from Benmir et al. [2020], who find that the implementation

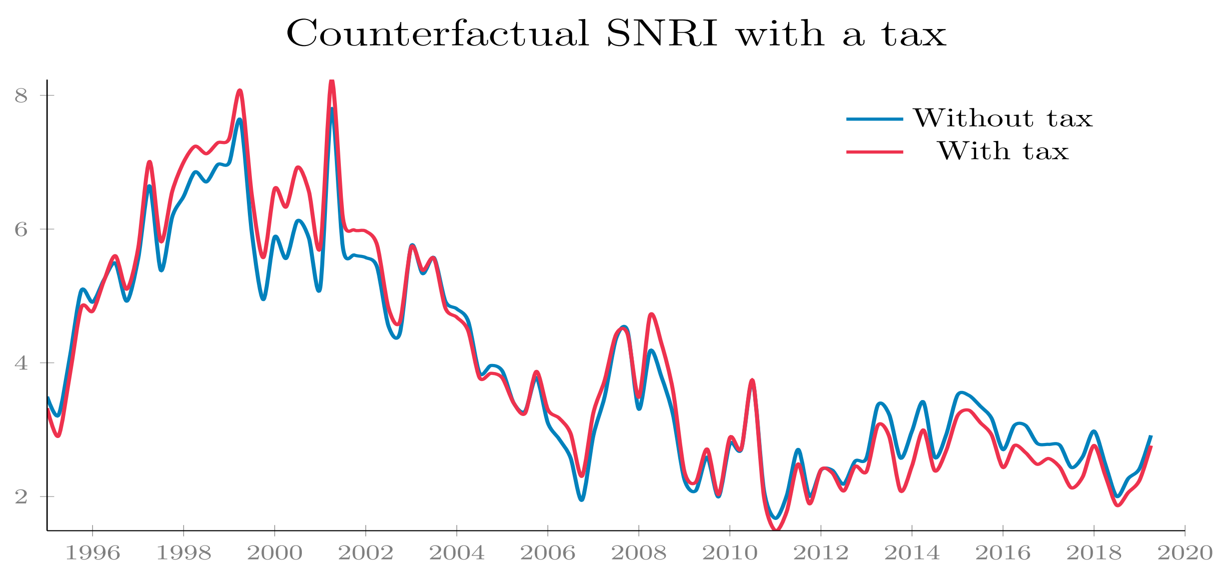


FIGURE 1.5. Counterfactual impact of a fixed 45$ price on carbon on the path of the shadow natural rate

of an optimal price for carbon leads to a reduction in the expected mean of the natural rate. This can explained by the fact that we do not rely on the same model, nor on the same solution method. While they use a RBC model and look at the real rate, we compute the path of the SNRI in a way that is more consistent with the literature on the natural rate of interest. In addition, we rely on standard perturbation methods to solve the model, while they employ an inversion filter.

Another complementary exercise would be to assess the impact of a time-varying carbon price on the fluctuations of the SNRI. Benmir and Roman [2020] show that variations in the carbon price leads to volatility in risk premia, and it would be interesting to see if this is also true for the SNRI. However, this question is left for further research.

1.5 Conclusion

In this paper, we introduced a new indicator to assess the monetary policy stance when the effective lower bound on nominal interest rates is binding: the shadow natural rate of interest. Using a series of the shadow rate, we rely on Bayesian estimation to find the time path and drivers of the SNRI. We showed that financial factors play a substantial role in fluctuations of the SNRI, along with standard supply and demand factors. Environmental factors, however, are found to have a negligible effect on the path of the SNRI, whether through emissions shocks, or through the implementation of a carbon price.

Analyzing the monetary policy stance through the lens of the shadow interest gap, we found that there were two periods of extended accommodative monetary policy: the years leading to the great financial crisis and the period 2012-2018. On the one hand, it suggests that loose monetary policy might be partly to blame for the rising financial imbalances that gave rise to the subprime crisis. On the other hand, it shows that central bankers provided favorable financing conditions in the aftermath of the great financial crisis, and that they can not be blamed for the slack in the economy observed during these years.

Overall, the analysis conducted in this article suggests that the shadow natural of interest is an indicator that could be used in practice by central banks. It is particularly useful since it incorporates more information and provides a different view than standard estimations of the natural rate of interest, particularly when the zero lower bound is binding.

Appendices

1.A Tables and Figures

TABLE 1.1
Calibrated parameters.

Parameter	Interpretation	Value	Source
Business Cycle			
σ	Intertemporal elasticity	1.38	Smets and Wouters [2007]
φ	Inverse Frisch elasticity	1.83	Smets and Wouters [2007]
h	Habits in consumption	0.71	Smets and Wouters [2007]
Φ	Fixed cost in production	1.60	Smets and Wouters [2007]
θ_w	Calvo wages probability	0.70	Smets and Wouters [2007]
ξ_w	Calvo wages indexation	0.58	Smets and Wouters [2007]
θ_p	Calvo probability	0.66	Smets and Wouters [2007]
ξ_p	Calvo indexation	0.24	Smets and Wouters [2007]
α	Effective capital share	0.19	Smets and Wouters [2007]
ϵ_P, ϵ_W	Elasticity of substitution	10	Smets and Wouters [2007]
$\frac{G}{Y}$	Proportion of government expenditures	0.18	Smets and Wouters [2007]
$\delta(\bar{U})$	Steady state depreciation rate	0.025	Smets and Wouters [2007]
κ	Elasticity of adjustment cost	5.74	Smets and Wouters [2007]
ζ	Elasticity of capital utilization	0.54	Smets and Wouters [2007]
Banking Sector			
λ	Regulatory constraint	0.404	Matching US banks capital ratio
ω	Proportional transfer to the new bankers	0.002	Gertler and Karadi [2011]
θ	Survival rate of the bankers	0.972	Gertler and Karadi [2011]
ρ_c	MP rule persistence	0.81	Smets and Wouters [2007]
ϕ_π	MP rule reaction to inflation	2.04	Smets and Wouters [2007]
ϕ_y	MP rule reaction to output gap	0.08	Smets and Wouters [2007]
Climate			
E^*	Rest of the world emissions	0.183	Matching the share of the USA
ϑ_1	Carbon Intensity 1	0.045	Matching the global stock of carbon
ϑ_2	Carbon Intensity 2	0.96	Matching the carbon intensity of the USA
d_1	Damage function 1	0.04	Dietz and Stern [2015]
d_2	Damage function 2	1.00	Dietz and Stern [2015]
υ_1°	Temperature reaction 1	0.500	Dietz and Venmans [2019]
υ_2°	Temperature reaction 2	0.00125	Dietz and Venmans [2019]
δ_x	Decay rate of emissions	0.0021	Heutel [2012]

TABLE 1.2
Prior and posterior distributions - Shock processes and trends.

		Prior distributions			Posterior distribution	
		Shape	Mean	Std.	Mean [5%:95%]	
Shock Process						
Productivity sd	σ_A	$\mathcal{IG}$	0.1	0.5	0.40	*[0.34:0.47]*
Preference sd	σ_B	$\mathcal{IG}$	0.1	0.5	0.10	*[0.01:0.22]*
Investment sd	σ_I	$\mathcal{IG}$	0.1	0.5	4.97	*[3.37:6.53]*
Prices sd	σ_P	$\mathcal{IG}$	0.1	0.5	0.18	*[0.15:0.21]*
Wages sd	σ_W	$\mathcal{IG}$	0.1	0.5	1.48	*[1.25:1.70]*
Monetary policy sd	σ_R	$\mathcal{IG}$	0.1	0.5	0.13	*[0.11:0.15]*
Capital quality sd	σ_K	$\mathcal{IG}$	0.1	0.5	1.28	*[1.12:1.44]*
Emissions sd	σ_E	$\mathcal{IG}$	0.1	0.5	0.02	*[0.01:0.02]*
Productivity AR	ρ_A	$\mathcal{B}$	0.5	0.2	0.87	*[0.84:0.90]*
Preference AR	ρ_B	$\mathcal{B}$	0.5	0.2	0.48	*[0.15:0.81]*
Investment AR	ρ_I	$\mathcal{B}$	0.5	0.2	0.90	*[0.86:0.94]*
Prices AR	ρ_P	$\mathcal{B}$	0.5	0.2	0.87	*[0.79:0.94]*
Wages AR	ρ_W	$\mathcal{B}$	0.5	0.2	0.97	*[0.95:0.99]*
Monetary policy AR	ρ_R	$\mathcal{B}$	0.5	0.2	0.75	*[0.68:0.81]*
Capital quality AR	ρ_K	$\mathcal{B}$	0.5	0.2	0.82	*[0.81:0.84]*
Emissions AR	ρ_E	$\mathcal{B}$	0.5	0.2	0.65	*[0.53:0.78]*
Price MA	u_P	$\mathcal{B}$	0.5	0.2	0.25	*[0.11:0.37]*
Wage MA	u_W	$\mathcal{B}$	0.5	0.2	0.74	*[0.64:0.85]*
Trends						
Discount factor	$\left(\beta^{-1}-1\right)\times 100$	$\mathcal{G}$	0.5	0.25	0.20	*[0.06:0.33]*
Inflation rate	$(\bar{\pi}-1)\times 100$	$\mathcal{N}$	0	0.2	0.16	*[0.05:0.27]*
Emissions growth rate	$(\bar{\gamma^e}-1)\times 100$	$\mathcal{N}$	0	0.25	-0.02	*[-0.03:-0.01]*
Productivity growth rate	$(\bar{\gamma^y}-1)\times 100$	$\mathcal{G}$	0.5	0.1	0.52	*[0.39:0.66]*
Marginal log-likelihood					-354.25	

Notes: The column entitled "Shape" indicates the prior distributions using the following acronyms: N describes a normal distribution, G a Gamma distribution, B a Beta distribution, and IG an Inverse Gamma distribution, type 2.

1.B Model Equations

1.B.1 Equilibrium Equations

$$\varrho_t = \varepsilon_t^B (C_t - hC_{t-1})^{-\sigma} + \beta h \varepsilon_{t+1}^B (C_{t+1} - hC_t)^{-\sigma} \tag{1.1}$$

$$\varrho_t w_t^h = \chi H_t^{\varphi} \tag{1.2}$$

$$\beta \Lambda_{t,t+1} R_{t+1} = 1 \tag{1.3}$$

$$\Lambda_{t,t+1} = \frac{\varrho_{t+1}}{\varrho_t} \tag{1.4}$$

$$q_{t,1}^w = H_t^d \left(\frac{w_t}{w_t^*}\right)^{\epsilon_w} w_t^* + \beta \theta_w \Lambda_{t,t+1} \left(\frac{w_t^* \pi_t^{\xi_w}}{w_{t+1}^* \pi_{t+1}} \bar{\pi}^{(1-\xi_w)}\right)^{1-\epsilon_w} q_{t+1,1}^w \tag{1.5}$$

$$q_{t,2}^w = \varepsilon_t^W \frac{\epsilon_w}{\epsilon_w - 1} H_t^d w_t^h \left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} + \beta \theta_w \Lambda_{t,t+1} \left(\frac{w_t^* \pi_t^{\xi_w}}{w_{t+1}^* \pi_{t+1}} \bar{\pi}^{(1-\xi_w)}\right)^{-\epsilon_w} q_{t+1,2}^w \tag{1.6}$$

$$q_{t,1}^w = q_{t,2}^w \tag{1.7}$$

$$w_t^{(1-\epsilon_w)} = \theta_w \left(w_{t-1} \frac{\pi_{t-1}^{\xi_w}}{\pi_t} \bar{\pi}^{(1-\xi_w)}\right)^{(1-\epsilon_w)} + (1-\theta_w) w_t^{*,(1-\epsilon_w)} \tag{1.8}$$

$$D_t^w = \theta_w D_{t-1}^w \left(\frac{w_{t-1}}{w_t}\right)^{-\epsilon_w} \left(\frac{\pi_t}{\pi_{t-1}^{\xi_w}} \bar{\pi}^{(\xi_w - 1)}\right)^{\epsilon_w} + (1-\theta_w) \left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} \tag{1.9}$$

$$Y_t^m = \varepsilon_t^A d(T_t^o)(U_t \varepsilon_t^K K_t)^{\alpha} \left(\Gamma_t^y H_t^d\right)^{1-\alpha} \tag{1.10}$$

$$w_t = P_t^m (1-\alpha) \frac{Y_t}{H_{it}^d} \tag{1.11}$$

$$R_{t+1}^k = \frac{P_{t+1}^m \alpha \frac{Y_{t+1}^m}{K_{t+1}} + (Q_{t+1} - \delta(U_{t+1}) \varepsilon_t^K}{Q_t} \tag{1.12}$$

$$P_t^m \alpha \frac{Y_t^m}{U_t} = \delta'(U_t) \varepsilon_t^K K_t \tag{1.13}$$

$$\delta\left(U_t\right) = \delta_c + \frac{b}{1+\zeta} U_t^{1+\zeta} \tag{1.14}$$

$$I_t^n = I_t - \delta(U_t)\varepsilon_t^K K_t \tag{1.15}$$

$$K_{t+1} = \epsilon_t^K K_t + I_t^n \tag{1.16}$$

$$Q_t = 1 + \frac{\kappa}{2}(\iota_t - \gamma_t)^2 + \kappa\varepsilon_t^I(\iota_t - \gamma_t)\iota_t - \beta\varepsilon_t^I E_t\left\{\Lambda_{t,t+1}\kappa\frac{(\iota_{t+1} - \gamma_{t+1})}{\iota_{t+1}^3}\frac{I_{t+1}^n + \bar{I}}{I_t^n + \bar{I}}\right\} \tag{1.17}$$

$$E_t = \varepsilon_t^E \Gamma_t^e \vartheta_1^o Y_t^{1-\vartheta_2^o} \tag{1.18}$$

$$X_t = (1 - \gamma_d)X_{t-1} + E_t + E^* \tag{1.19}$$

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o \tag{1.20}$$

$$Q_t K_{t+1}\varepsilon_{t+1}^K = N_t + B_t \tag{1.21}$$

$$\Omega_t = 1 - \theta_B + \theta_B \Gamma_t \tag{1.22}$$

$$\Gamma_t = \frac{1}{1 - \nu_t}\beta\Lambda_{t,t+1}\Omega_{t+1}R_t \tag{1.23}$$

$$\beta\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^k - R_t) = \lambda\nu_t \tag{1.24}$$

$$N_t = \theta_B(R_t^k - R_{t-1})Q_{t-1}K_t\varepsilon_t^K + (\theta_B R_{t-1} + \omega)N_{t-1} \tag{1.25}$$

$$\nu_t(\Gamma_t N_t - \lambda Q_t K_{t+1}\varepsilon_{t+1}^K) = 0 \tag{1.26}$$

$$\pi_t^* = \varepsilon_t^P \frac{\epsilon_p}{\epsilon_p - 1}\frac{q_{t,1}^p}{q_{t,2}^p}\pi_t \tag{1.27}$$

$$q_{t,1}^p = P_t^m Y_t + \theta_p\beta\Lambda_{t,t+1}\pi_{t+1}^{\epsilon_p}(\bar{\pi}^{(1-\xi_p)}\pi_t^{\xi_p})^{-\epsilon_p}q_{t+1,1}^p \tag{1.28}$$

$$q_{t,2}^p = Y_t + \theta_p\beta\Lambda_{t,t+1}\pi_{t+1}^{(\epsilon_p-1)}(\bar{\pi}^{(1-\xi_p)}\pi_t^{\xi_p})^{(1-\epsilon_p)}q_{t+1,2}^p \tag{1.29}$$

$$\pi_t^{(1-\epsilon_p)} = \theta_p(\bar{\pi}^{(1-\xi_p)}\pi_{t-1}^{\xi_p})^{(1-\epsilon_p)} + (1 - \theta_p)\pi_t^{*,(1-\epsilon_p)} \tag{1.30}$$

$$D_t^p = \theta_p D_{t-1}^p(\bar{\pi}^{(\xi_p-1)}\pi_{t-1}^{-\xi_p}\pi_t)^{\epsilon_p} + (1 - \theta_p)\left[\frac{1 - \theta_p(\bar{\pi}^{(1-\xi_p)}\pi_{t-1}^{\xi_p})^{(1-\theta_p)}\pi_t^{(\theta_p-1)})}{(1 - \theta_p)}\right]^{-\epsilon_p/(1-\theta_p)} \tag{1.31}$$

$$i_t = \rho_c i_{t-1} + (1 - \rho_c)\left[\phi^\pi(\pi_t - \bar{\pi}) + \phi^y(Y_t - \bar{Y}) + \bar{i}\right] + \varepsilon_t^R \tag{1.32}$$

$$i_t = R_t\pi_{t+1} \tag{1.33}$$

$$Y_t^m = Y_t D_t^p \tag{1.34}$$

$$H_t = H_t^d D_t^w \tag{1.35}$$

$$Y_t = C_t + I_t + (\iota_t - 1)^2(I_t^n + \bar{I}) + \varepsilon_t^G\frac{\bar{g}}{\bar{y}}Y_t \tag{1.36}$$

$$\varepsilon_t^A = \rho_A \varepsilon_{t-1}^A + \sigma_A \eta_t^A \tag{1.37}$$

$$\varepsilon_t^E = \rho_E \varepsilon_{t-1}^E + \sigma_E \eta_t^E \tag{1.38}$$

$$\varepsilon_t^K = \rho_K \varepsilon_{t-1}^K + \sigma_K \eta_t^K \varepsilon_t^P = \rho_P \varepsilon_{t-1}^P + \sigma_P \eta_t^P \tag{1.39}$$

$$\varepsilon_t^I = \rho_I \varepsilon_{t-1}^I + \sigma_I \eta_t^I \tag{1.40}$$

$$\varepsilon_t^W = \rho_W \varepsilon_{t-1}^W + \sigma_W \eta_t^W \tag{1.41}$$

$$\varepsilon_t^B = \rho_B \varepsilon_{t-1}^B + \sigma_B \eta_t^B \tag{1.42}$$

$$\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \sigma_R \eta_t^R \tag{1.43}$$

1.B.2 Balanced Growth Path Methodology

The growth rate of Γ_t^y determines the growth rate of the economy along the balanced growth path. This growth rate is denoted by γ_t^y, where $\gamma_t^y = \Gamma_t^y / \Gamma_{t-1}^y$ Stationary variables are denoted with tilde. For example, in the growing economy output is denoted by Y_t. De-trended output is thus obtained by dividing output in the growing economy by the level of growth progress:

$$\tilde{Y}_t = \frac{Y_t}{\Gamma_t^y}. \tag{1.1}$$

Emissions, which we denote by E_t, in the growing economy are given as follows:

$$E_t = \varepsilon_t^E \Gamma_t^e \vartheta_1^o Y_t^{1-\vartheta_2^o}, \tag{1.2}$$

where $\Gamma_t^e = \gamma_t^e \Gamma_{t-1}^e$ defines the growth rate of emissions.

Thus, in the de-trended economy, emissions law of motion reads as following:

$$\tilde{E}_t = \varepsilon_t^E \vartheta_1^o \tilde{Y}_t^{1-\vartheta_2^o} \tag{1.3}$$

where:

$$\tilde{E}_t = \frac{E_t}{\Gamma_t^e (\Gamma_t^y)^{1-\vartheta_2^o}} \tag{1.4}$$

The stock of emissions in the atmosphere is denoted by X_t, while the temperature is called T_t^o in the growing economy:

$$X_t = (1-\gamma_d)X_{t-1} + E_t + E^* \tag{1.5}$$

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o. \tag{1.6}$$

The de-trended X_t and T_t^o read as follows:

$$\tilde{X}_t = (1-\gamma_d)(\gamma_t^y)^{\upsilon_2^o - 1}(\gamma_t^e)^{-1}\tilde{X}_{t-1} + \tilde{E}_t + E^* \tag{1.7}$$

$$\gamma_t^e(\gamma_t^y)^{1-\vartheta_2^o}\tilde{T}_t^o = \upsilon_1^o\left(\upsilon_2^o \tilde{X}_{t-1} - \tilde{T}_{t-1}^o\right) + \tilde{T}_{t-1}^o \tag{1.8}$$

where:

$$\tilde{X}_t = \frac{X_t}{\Gamma_t^e (\Gamma_t^y)^{1-\vartheta_2^o}} \tag{1.9}$$

$$\tilde{T}_t^o = \frac{T_t^o}{\Gamma_t^e (\Gamma_t^y)^{1-\vartheta_2^o}} \tag{1.10}$$

De-trending equation (1.36) gives:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + (\iota_t - 1)^2(\tilde{I}_t^n + \bar{I}) + \varepsilon_t^G \frac{\bar{g}}{\bar{y}}\tilde{Y}_t. \tag{1.11}$$

It implies that $\tilde{C}_t = C_t/\Gamma_t^y$ and $\tilde{I}_t = I_t/\Gamma_t^y$. Equation (1.15) and equation (1.16) also give $\tilde{I}_t^n = I_t^n/\Gamma_t^y$ and $\tilde{K}_t = K_t/\Gamma_t^y$. From equation (1.34), we know that $\tilde{Y}_t^m = Y_t^m/\Gamma_t^y$, D_t^p being stationary by construction. Noting that the damage function is made stationary by the adjustment in equation (1.10), and that the utilization rate and the hours worked are stationary by construction we get the de-trended production function:

$$\tilde{Y}_t = \varepsilon_t^A d(\tilde{T}_t^o)(U_t \varepsilon_t^K \tilde{K}_t)^\alpha \left(H_t^d\right)^{1-\alpha} \tag{1.12}$$

The capital accumulation equation in the growing economy is:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{1.13}$$

In the de-trended economy, we thus have that:

$$\gamma_t^y \tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \tilde{I}_t \tag{1.14}$$

with both capital and investment de-trended variables read as: $\tilde{K}_t = \frac{K_t}{\Gamma_t}$ and$\tilde{I}_t = \frac{I_t}{\Gamma_t}$, respectively. The wage as shown in the model section reads as following:

$$w_t = (1-\alpha)\Psi_t \frac{Y_t}{H_t^d} \tag{1.15}$$

The de-trended wages[20] will therefore read as:

$$\tilde{w}_t = (1-\alpha)\Psi_t \frac{\tilde{Y}_t}{H_t^d} \tag{1.16}$$

with the de-trended wage $\tilde{w}_t$ reads as $w_t = \frac{W_t}{\Gamma_t}$.

Turning to the households, de-trending equation (1.1) leads to:

$$\tilde{\lambda}_t = \varepsilon_t^B(\tilde{C}_t - \frac{h}{\gamma_t^y}\tilde{C}_{t-1})^{-\sigma} + \beta h \varepsilon_{t+1}^B(\gamma_{t+1}^y \tilde{C}_{t+1} - hC_t)^{-\sigma}, \tag{1.17}$$

with $\tilde{\lambda}_t = \lambda_t \Gamma_t^{y,\sigma}$. Equation (1.4) then becomes:

$$\tilde{\Lambda}_{t,t+1} = \frac{\tilde{\varrho}_{t+1}}{\tilde{\varrho}_t}\gamma_{t+1}^{y,-\sigma}, \tag{1.18}$$

[20]I note that Ψ_t the labor/capital share is a stationary variable. The same can be noticed for the returns on capital R_t^k, the total marginal cost P_t^m, and abatement μ_t

and equation (1.2) gives $\tilde{w}_t^h = w_t^h \Gamma_t^{y,-\sigma}$.

Turning to financial intermediaries, the de-trended balance sheet equation reads as:

$$Q_t \tilde{K}_{t+1} \varepsilon_{t+1}^K = \tilde{N}_t + \tilde{B}_t, \tag{1.19}$$

with $\tilde{N}_t = N_t \Gamma_t^y$ and $\tilde{B}_t = B_t \Gamma_t^y$. Thus, the net worth law of motion equation (1.25) is modified as follows:

$$\gamma_t^y \tilde{N}_t = \theta_B (R_t^k - R_{t-1}) Q_{t-1} \tilde{K}_t \varepsilon_t^K + (\theta_B R_{t-1} + \omega) \tilde{N}_{t-1}. \tag{1.20}$$

The other equations are left unchanged.

Chapter 2

Policy Interaction and the Transition to Clean Technology

This chapter was presented at the Climate Risk Workshop of the Federal Reserve Bank of San Francisco, the Annual Meetings of the AEA, CEBRA and EAERE, the Annual Conference of the MIT Golub Center for Finance and Policy, the Annual Conference of the Money Macro and Finance Society, the 1st Sustainable Macro Conference, as well as the Paris-Dauphine Economics Ph.D. Workshop and Economic Department Seminar. It has also been published as a working paper at the Grantham Research Institute (LSE). It is a joint work with Ghassane Benmir (LSE).

2.1 Introduction

Climate change has shifted from a fringe issue to a worldwide emergency. Our understanding of the phenomenon and our willingness to act have developed significantly, in part paralleling the ways in which climate change is being experienced around the globe. It has become a hot topic where academics, industry, and lay people alike are finding common ground. As such, growing academic awareness is leading to important literature in the domain. The implementation of a strategy for the substantial reduction of greenhouse gases (GHG) at the global level has become a major priority. Since the Rio Conference in 1992, a debate has raged in academic and political circles over the growth-environmental trade-off. Discussions focus on the means by which economic activities could align with environmental concerns instead of being hindered by assumed mutual exclusivity. In practice, especially in the short and medium terms, however, financial and economic activity on one side, and environmental policy on the other, are in tension. A need for both medium/long and short-term policies aimed at bridging the gap between environmental sustainability and economic efficiency, as well as addressing financial stability, are in dire need, in order to foster economic transition. Of special concern are climate actions that may strongly impact macroeconomic activity, given the potentially high added cost of GHG offsetting. With the substantial effects of climate actions on the overall economy, a growing body of research from the field of macroeconomics and macro-finance, among others, are now tackling these issues.

In this paper, we study the implication of setting a market for carbon permits to meet the net-zero target (in the European Union (EU), this corresponds to an emission reduction objective of 55 percent by 2030 compared to the 1990 level). To de-carbonize the economy, the price of carbon is expected to rise sharply, as the welfare maximizing optimal policy is shown not be sufficient (Golosov et al. [2014] and Hassler et al. [2020]). This could potentially lead to both welfare distortions in the long run and financial disruptions in the short run (depending on the market structure and price volatility). A framework seeking a better integration of macro-finance and environment would allow, on one hand, for a better understanding of carbon mitigation pricing policies as well as their impacts on different macro aggregates including consumer welfare, which is shown to be significantly impacted and differs depending on the carbon pricing policy market design in place (Sager [2019]). On the other hand, this framework would also allow for investigating the linkages and impacts of the climate externality on financial aggregates such as the natural rate of interest and the risk premium (Benmir et al. [2020] and Bauer and Rudebusch [2021]). In our quantitative analysis, we take the EU net-zero policy as given and investigate how macro-financial policies could interact with it.

This paper is tightly linked to three strands of literature that address macro-environmental issues and the role of macro-financial authorities.

The first strand focuses on long-term analysis of the nexus between climate policies and the macroeconomy and can be traced back to the early work of Nordhaus [1991]. A wide range of literature of integrated assessment models (IAMs) extended the framework developed by Nordhaus to account for uncertainty in climate dynamics and damages (see Stern [2008], Weitzman [2012], and Dietz and Stern [2015], among others). Golosov et al. [2014] use a dynamic stochastic general equilibrium (DSGE) model to show that the optimal carbon price is not impacted by future uncertainty. They also find that following the optimal policy would not allow for global warming to be kept well below 2°C over a 50 years horizon. This is consistent with our simulations, which show that the price of carbon needs to rise well above its optimal counterpart to set the Euro Area (EA) on the net-zero path. While Golosov et al. [2014] compute transition pathways resulting from the implementation of an optimal carbon price policy, we instead consider the carbon price resulting from the European Trading System (ETS) cap policy. In the same spirit of our work, Hassler et al. [2020] investigate several sub-optimal policy scenarios using a multi-country IAM. These scenarios, however, are not designed to represent current carbon policies in the European Union (EU) and IAMs do not feature a role for the financial system. In a recent paper, Van der Ploeg et al. [2020] study the financial consequences of climate risk with respect to portfolio choices. Although our article shares similar components with the latter, we differ by explicitly modeling financial intermediaries. Carattini et al. [2021] and Diluiso et al. [2020] also build environmental DSGE (E-DSGE) models with financial frictions, yet they do not account for trend growth and uncertainty around the level of TFP and carbon price in their long-term simulations, both of which are featured in our analysis. Furthermore, they both simulate transition pathways as a response to exogenous shocks, rather than using deterministic simulations. However, similar to Carattini et al. [2021], we consider macroprudential policy as a long-term tool that can be used to shape banks' balance sheets in order to contain climate risk rather than a short-term tool to address financial shocks (Diluiso et al. [2020]). With respect to the literature on long-term transition pathways, our simulations feature both deterministic trends and uncertainty on the level of TFP, as well as on the carbon price. While Cai and Lontzek [2019] also perform long-term transitions with uncertainty around the trend of TFP and climate damages, we focus on TFP and the price of carbon as we consider a shorter horizon. In addition, we use a Newton-based method to compute the solution where Cai and Lontzek [2019] use value function iteration. We also provide a dynamic analysis of welfare, which allows us to study the benefits of macroprudential policy along the transition to the net-zero target.

The second strand of literature relevant to our work focuses on business cycle implications of environmental policies. Angelopoulos et al. [2010], Fischer and Springborn [2011], Heutel [2012], among others,[21] paved the way for business cycle analysis under an environmental externality. The main focus of these papers is to assess the efficiency of different environmental policies. In recent months, papers such as Diluiso et al. [2020] or Carattini et al. [2021] incorporated a financial sector in order to study the role of monetary and macroprudential policies in the fight against climate change. Our short-term analysis is

[21] E.g. Bosetti et al. [2014], Annicchiarico and Di Dio [2015], and Dissou and Karnizova [2016]. For an extensive literature review distinguishing between the long-term and business cycle environmental macroeconomics, respectively, please refer to Schubert [2018].

tangentially related to these two papers. In our framework, however, the monetary authority intervenes to correct a distortion in risk premia stemming from carbon price volatility, which we estimate based on observed ETS futures price data. The role of the central bank thus arises endogenously from the transmission of carbon price shocks to financial variables through the marginal cost of firms, while Diluiso et al. [2020] explore the benefits of both monetary and macroprudential policies in response to an exogenous shock to the quality of brown assets.

Finally, this paper is also linked to a strand of literature assessing central banks' large-scale asset purchases (LSAP) programs, and especially the so-called green quantitative easing (green QE). In the wake of the Great Financial Crisis, Gertler and Karadi [2011] provided a framework to study the impact of central banks' LSAP programs in response to a shock to the quality of capital. With respect to green QE, Ferrari and Nispi Landi [2021] investigate the impact of a series of positive unexpected shocks to the central bank's holdings of green bonds to simulate an assets purchase program. We differ by considering that LSAP programs are expected by agents, as central banks communicate about them beforehand. We also consider two types of green LSAP programs (transitory and permanent) and the interaction between them and pre-announced macroprudential policy.

Our modeling device borrows components from several macroeconomic types of models. We first build on the canonical versions of New Keynesian (NK) models such as Woodford [2003], Smets and Wouters [2003] or Christiano et al. [2005] to derive the core of our economy.[22] Second, we add environmental components as in Nordhaus [2008], Heutel [2012], and Dietz and Venmans [2019], which allow for the analysis of the dynamics of the economy under the presence of the CO_2 externality. However, as opposed to Heutel [2012], we differentiate between green and brown firms instead of using one sole representation for firms, thus borrowing from the multi-sector literature (Carvalho and Nechio [2016]

[22]Note that for simplicity we abstract from wages rigidities and labor disutility.

among others[23]). Finally, we include balance sheet constrained financial intermediaries as in Gertler and Karadi [2011]. Given that we introduce a macroprudential authority that can alter this constraint, we also draw on Pietrunti [2017].

As we will consider monetary policy, we only focus on the EA. We perform medium/long-term simulations both for transition pathways to meet the net-zero target and for LSAP programs along the transition to net-zero. As for business cycle simulations, we rely on second order impulse responses to analyze the impact of the ETS carbon price shock on macro-financial aggregates. The novelty of our approach is that our transition pathways feature both long-run deterministic growth rates (i.e. labor augmenting technology and carbon cap policy) and stochastic components around these trends. This allows us to compute confidence intervals for our variables of interest using Monte-Carlo simulations. Furthermore, we rely on the simulated method of moments (SMM) to estimate key structural parameters and match the EA macroeconomic, financial, and environmental empirical data.

Our main theoretical result highlights the inefficiencies stemming from the EU ETS design. In the long term we show that, as the cap policy diverges from the optimal social cost of carbon (SCC), the loss on welfare increases, whereas, in the short term the ETS market design induces volatility in the carbon price that distorts risk premia.

On applied grounds, our contribution is to propose tools to mitigate these inefficiencies. Using numerical simulations, we find that an instrument that deviates from the optimal policy (SCC), such as the ETS, is needed to meet the net-zero target. However, this induces a substantial cost in terms of welfare (3 percent consumption equivalent). To ease the welfare burden, we show that a sectoral risk-weight (*i.e.* climate risk-weight) macroprudential policy is able to reduce the wedge gap, without imposing infeasible regulatory weights on assets held by financial intermediaries and jeopardizing financial stability. In particular,

[23] We note that a substantial literature referred to as "directed technical progress" uses two sectors (green and dirty) to investigate the transition to a green economy and impacts of different environmental policies. See, for example, Smulders and De Nooij [2003], Grimaud and Rouge [2008], Di Maria and Valente [2008], Acemoglu et al. [2012], Aghion et al. [2016], Acemoglu et al. [2019].

a sectoral macroprudential policy favorable to the green sector boosts green capital and output, inducing a gain in welfare, compared to the sub-optimal policy economy without macroprudential policy, as the green sector is less sensitive to the rise in carbon price. With respect to the distortion on risk premia, we show that short-term monetary policy instruments (*i.e* QE rules) are able to restore the equilibrium in the financial markets. Thus, macroprudential and monetary policies could play an important role in offsetting the negative effects stemming from the implementation of a market for carbon permits. Finally, we investigate the role of asset purchase programs over the net-zero transition and find that central banks would have an incentive to tilt their portfolio of assets toward the green sector when macroprudential policy takes into account climate risk. More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero emissions.

Our actual findings could be further reinforced if we were to see an increase in the share of the green sector, as illustrated in our simulated transition in figure 2.2 and figure 2.3, and as argued in the work of Acemoglu et al. [2016], where the focus is on the long-term transition strategies.

This paper is organized as follows: section 2 presents the model, section 3 explains the solution method, section 4 discusses the results, and section 5 concludes.

2.2 The Model

Using the NK-DSGE framework as a foundation, the present paper investigates the potential role of fiscal policy, central bank unconventional monetary policy, and macroprudential policy, in mitigating climate change impacts on macroeconomic and financial aggregates. We first model our two-sector economy following Carvalho and Nechio [2016]. Then, we incorporate the environmental component following Nordhaus [2008], Heutel [2012], and Dietz and Venmans [2019], among others. Finally, we model financial intermediaries drawing on Gertler and Karadi [2011].

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \ldots \infty)$. The production sectors produce two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms via financial intermediaries. Public authorities decide on the fiscal and environmental policy, the central bank decides on the monetary policy, and the financial authority sets the macroprudential policy.

2.2.1 The Household

At each period, the representative household supplies labor inelastically to the two sectors of our economy (i.e green and brown sectors denoted by $k \in \{g, b\}$[24]), while they also consume and save. Households can either lend their money to the government or to financial intermediaries, who will in turn leverage and finance firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households cannot lend their money to a financial intermediary owned by one of their members. Household members who are workers supply labor and return their salaries to the household to which they belong.

Agents can switch between the two occupations over time. There is a fraction f of agents who are bankers and a probability θ_B that a banker remains a banker in the next period. Thus, $(1-f)\theta_B$ bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to households, which will use them as start-up funds for new bankers.

Households solve the following maximization problem:

$$\max_{\{C_t, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} \right] \tag{2.1}$$

[24] Where ‘g’ refers to the green sector and ‘b’ to the brown sector.

s.t.

$$C_t + B_{t+1} = \sum_k g(\varkappa)\left(W_{t,k}L_{t,k} + \Pi_{t,k}\right) + \Pi_t^T + T_t + R_t B_t, \tag{2.2}$$

where $\beta \in (0,1)$ is the discount factor and σ shapes the utility function of the representative household associated with risk consumption C_t. The consumption index C_t is subject to external habits with degree $h \in [0;1)$. Labor supply $L_{t,k}$[25] in each sector is remunerated at nominal wage $W_{t,k}$. Note that the sector share for the green g is $g(\varkappa) = \varkappa$ and $(1-\varkappa)$ for the brown sector b. $\Pi_{t,k}$ are profits from the ownership of firms, while Π_t^T are profits from the ownership of financial intermediaries and capital producing firms. T_t is lump sum taxes. As we assume that intermediaries deposits and government bonds are one period bonds, $R_t B_t$ is interest received on bonds held and B_{t+1} is bonds acquired.

Solving the first order conditions and denoting ϱ_t as the marginal utility of consumption, the consumption/saving equations are:

$$\varrho_t = \left(C_t - hC_{t-1}\right)^{-\sigma} - \beta h E_t\left\{\left(C_{t+1} - hC_t\right)^{-\sigma}\right\}, \tag{2.3}$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \tag{2.4}$$

with $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$ the expected variation in the marginal utility of consumption.

2.2.2 The Firms

2.2.2.1 The Final Firms

Using the multi-sector framework from Carvalho and Nechio [2016], and under non-perfect competition, we assume that production comprises two sectors. Our representative final firms produce a final good $Y_{t,k}$ in these two competitive sectors. Using no more than capital and labor to produce the intermediate good Y_{jt} (where $j \in (0,1)$ is the continuum

[25] We note that inelastic labor $L_{t,k} = \bar{L_k}$, where $\bar{L_k}$ is the steady state level of labor in each sector.

of intermediate goods firms), intermediate firms supply the final sectors. In other words, the "bundling" of intermediate goods within the two sectors leads to a final good. The final economy good is a constant elasticity of substitution aggregate of the two sectors:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}} Y_{t,g}^{1-\frac{1}{\theta}} + (1-\varkappa)^{\frac{1}{\theta}} Y_{t,b}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{2.5}$$

with $\theta \in (1, \infty)$ the elasticity of substitution between the two sectors, and $\varkappa$ the weight of each sector. The final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t subject to the intermediate goods j in each of the two sectors k at prices $P_{jt,k}$:

$$\max_{Y_{jt}} \Pi_t^{\text{Final}} = P_t Y_t - \varkappa \int_0^1 P_{jt,g} Y_{jt,g} dj - (1-\varkappa) \int_0^1 P_{jt,b} Y_{jt,b} dj, \tag{2.6}$$

where the aggregation of green and brown firms reads as:

$$Y_{t,k} = \int_0^1 \left(Y_{jt,k}^{1-\frac{1}{\theta_k}} \right)^{\frac{1}{1-\frac{1}{\theta_k}}}. \tag{2.7}$$

However, while we assume a constant elasticity of substitution between the final sectors, we consider a different elasticity of substitution θ_k between differentiated intermediate goods within each sector. As the goods of the two sectors entail different costs, a different elasticity of substitution is considered. This assumption, which shapes the marginal cost structure, is based both on theoretical work of Tucker [2010] as well as on the empirical findings of Chan et al. [2013] and Chegut et al. [2019], where it is found that green projects entail higher marginal cost (7-13 percent higher costs for green projects in the construction industry compared to non green projects depending on the 'greenness' of the project, and 5-7 percent higher costs in the cement and iron & steel sectors, respectively).

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_t. \tag{2.8}$$

Under perfect competition and free entry, the price of the final good is denoted P_t, while the price $P_{t,k}$ is the price index of sector-k intermediate goods. Finally, the price $P_{jt,k}$ is the price charged by firm j from sector k.

Prices of final aggregate goods and for each sector are given by:

$$P_t = \left(\varkappa P_{t,g}^{1-\theta} + (1-\varkappa) P_{t,b}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{2.9}$$

$$P_{t,k} = \left(\int_0^1 P_{jt,k}^{1-\theta_k} dj\right)^{\frac{1}{1-\theta_k}}. \tag{2.10}$$

2.2.2.2 The Intermediate Firms

Our economy is composed of two categories of firms: i) green firms, which are environmentally-friendly and ii) brown firms with a higher emission intensity. The representative firms j in each sector k of the modeled economy uses capital $K_{t,k}$ and labor $L_{t,k}$ to produce the intermediate good. In our framework, firms' productivity is subject to climate dynamics. As presented in Golosov et al. [2014] real business cycle model, the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externality deteriorates the environment and alters production possibilities for firms. However, we differ from Golosov et al. [2014] by incorporating damages from the stock of emissions through the level of temperature as follows:

$$Y_{jt,k} = \varepsilon_t^{A_k} d(T_t^o) K_{jt,k}^{\alpha} (\Gamma_t L_{jt,k})^{1-\alpha},\ \alpha \in (0,1), \tag{2.11}$$

where Γ_t is the economy growth trend and $d(T_t^o)$ a convex function relating the temperature level to a deterioration in output ($d(T_t^o) = ae^{-\frac{b}{\Gamma_t{}^2} T_t^{o2}}$), with $(a,b) \in \mathbb{R}^2$, which is borrowed from Nordhaus and Moffat [2017]. As highlighted by Benhabib et al. [1991], Jaimovich and Rebelo [2009], and Queralto [2020], the business cycle literature typically features preferences and/or production functions with $\Gamma_t = 1$ for all t. Within a business cycle framework, we usually assume no long-run growth. However, as we are also interested in the transition pathways, our economy features a growth trend Γ_t different than 1 in hours worked. Therefore, we introduce ${\Gamma_t}^2$ to the damage sensitivity parameter b, such that $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}}$. The goal is to ensure the existence of a balanced growth path without a loss of generality, as over the studied period $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}} \approx ae^{-bT_t^{o2}}$. In addition, the growth rate of Γ_t, which determines the growth rate of economy, is set exogenously to γ^Y where $\Gamma_t = \gamma^Y \Gamma_{t-1}$. Furthermore, α is the standard elasticity of output with respect to capital, and $\varepsilon_t^{A_k}$ is a sector-specific technology shock that follows an $AR(1)$ process: $\varepsilon_t^{A_k} = \rho_{A_k}\varepsilon_{t-1}^{A_k} + \sigma_{A_k}\eta_t^{A_k}$, with $\eta_t^{A_k} \sim \mathcal{N}(0,1)$.

Global temperature $d(T_t^o)$ is linearly proportional to the level of the emission stock, which in turn is proportional to cumulative emissions as argued by Dietz and Venmans [2019]:[26,27]

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \tag{2.12}$$

with υ_1^o and υ_2^o chosen following Dietz and Venmans [2019].

Furthermore, the carbon emissions stock X_t follows a law of motion:

$$X_t = (1-\gamma_d)X_{t-1} + E_t + E_t^*, \tag{2.13}$$

where $E_t = \sum_k g(\varkappa)\int_0^1 E_{jt,k}dj$ is the aggregate flow of emissions from both the green and brown firms at time t and γ_d is the decay rate. $E_t^* = E^*\Gamma_t$ represents the rest of the world

[26] To allow for convergence in the auto-regressive law of motion for the stock of emissions process (shown in equation (2.13)) we slightly depart from the transient climate response to cumulative carbon emissions theory and set $\gamma_d \neq 0$. However, we choose γ_d sufficiently low such that $X_t \approx X_0 + \sum_{i=0}^t (E_i + E_i^*)$.

[27] We note that while differences on climate dynamics and damages modeling over the long horizon (whether à la Golosov et al. [2014], à la Nordhaus [2017], or à la Dietz and Venmans [2019], among others) induce consequent impacts on macroeconomic aggregate equilibriums, over the business cycle horizon (and under equivalent calibrations), these modeling specifications do not induce significant impacts on macroeconomic aggregate equilibriums.

emissions and is used to pin down the actual steady state level of the stock of emission in the atmosphere. We assume that the rest of the world's emissions grow at the same rate as the domestic GDP over the period studied.

The emissions level is shaped by a non-linear abatement technology $\mu_{jt,k}$ that allows firms to reduce their emissions inflows:

$$E_{jt,k} = (1 - \mu_{jt,k})\varphi_k Y_{jt,k}. \tag{2.14}$$

Emissions $E_{jt,k}$ at firm level are proportional to the production $Y_{jt,k}$ with φ_k the fraction of emissions to output in each sector.[28] Also, emissions could be reduced at the firm level through an abatement effort $\mu_{jt,k}$. The firms are allowed to invest in an abatement technology, but it represents an extra cost.

We model the direct abatement effort costs as follows:

$$Z_{jt,k} = f(\mu_{jt,k})Y_{jt,k}, \tag{2.15}$$

where

$$f(\mu_{jt,k}) = \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}},\ \theta_1 > 0,\, \theta_2 > 1, \tag{2.16}$$

with $\theta_{1,k}$ and $\theta_{2,k}$ the cost efficiency of abatement parameters for each sector.

Thus, profits of our representative intermediate firms in each sector $\Pi_{jt,k}$ will be impacted by the presence of the environmental externality. Revenues are the value of intermediate goods $Y_{jt,k}$, while costs arise from: i) wages $W_{t,k}$ (paid to the labor force $L_{jt,k}$), ii) rents $R^K_{t,k}$ (on capital $K_{jt,k}$), iii) abatement investments $f(\mu_{jt,k})$, and iv) the cost of re-

[28]Contrary to Cai and Lontzek [2019], we consider $\varphi_{t,k} = \varphi_k$ constant overtime and calibrate it using Euro Area emissions to GDP data, as we focus on shorter time horizons (less than 50 years).

leasing carbon in the atmosphere $\tau_{et,k}E_{jt,k}$ (i.e. the carbon price paid to the government).

$$\begin{aligned}\Pi_{jt,k} &= \frac{P_{jt,k}}{P_t}Y_{jt,k} - W_{t,k}L_{jt,k} - R^K_{t,k}K_{jt,k} - \theta_{1,k}\mu^{\theta_{2,k}}_{jt,k}Y_{jt,k} - \tau_{et,k}E_{jt,k} \\ &= \left(\frac{P_{jt,k}}{P_t} - MC_{t,k}\right)Y_{jt,k},\end{aligned} \tag{2.17}$$

As firms are not free to update prices each period, they first choose inputs so as to minimize costs, given a price, subject to the demand constraint.

The cost-minimization problem yields the marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital, labor, abatement, and production level, respectively:

$$R^K_{t,k} = \alpha\Psi_{jt,k}\frac{Y_{jt,k}}{K_{jt,k}}, \tag{2.18}$$

$$W^K_{t,k} = (1-\alpha)\Psi_{jt,k}\frac{Y_{jt,k}}{L_{jt,k}}, \tag{2.19}$$

$$\tau_{et,k} = \frac{\theta_{1,k}\theta_{2,k}}{\varphi_k}\mu^{\theta_{2,k}-1}_{jt,k}, \tag{2.20}$$

$$MC_{jt,k} = MC_{t,k} = \Psi_{t,k} + \theta_{1,k}\mu^{\theta_{2,k}}_{t,k} + \tau_{et,k}(1-\mu_{t,k})\varphi_k, \tag{2.21}$$

where $\Psi_{jt,k} = \Psi_{t,k}$[29] is the marginal cost component related to the same capital-labor ratio all firms from each sector choose. This marginal cost component is common to all intermediate firms, but differs across sectors.

Equation (2.20) is the optimal condition on abatement: abating CO_2 emissions is optimal when its marginal gain equals its marginal cost. This equation highlights the key role of the carbon price in shaping firms' decisions. In addition, abatement efforts $\mu_{t,k}$ are common to all firms of the same sector, as the environmental cost is also common to all firms of the same sector. Furthermore, as the impact of the environmental externality is not internalize by firms (i.e. they take X_t and T^o_t as given), the shadow value of the

[29] $\Psi_{jt,k} = \Psi_{t,k} = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{1}{\varepsilon^{A,k}_t d(T^o_t)}\left(W_{t,k}\right)^{1-\alpha}\left(R^K_{t,k}\right)^\alpha$.

environmental externality is zero.

The total marginal cost captures both abatement and emissions costs as shown above in equation (2.21). Note that in the case of the laissez-faire scenario, $MC_{t,k} = \Psi_{t,k}$, as the firms are not subject to emissions and abatement constraints.

In addition, monopolistic firms engage in a price setting à la Rotemberg.[30] Price update is subject to an adjustment cost given by $\Delta^P_{jt,k} = \frac{\theta^P}{2}\left(\frac{P_{jt,k}}{P_{jt-1,k}} - 1\right)^2$. Thus, profit maximization subject to the demand from final firms reads as follows:

$$\max_{P_{jt,k}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left(\Pi_{jt+i,k} - \Delta^P_{jt+i,k} Y_{t+i}\right) \tag{2.22}$$

$$\text{s.t. } Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_t,$$

where $\beta^i \Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor, or as commonly called in the macro-finance literature, the pricing kernel.

The NK Philips Curve pricing equation for each sector is as follows:

$$\theta^P \pi_{t,k}(\pi_{t,k} - 1) = \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} \left(\frac{P_{t,k}}{P_t}(1-\theta_k) + \theta_k MC_{t,k}\right) + E_t\left\{M_{t,t+1}\frac{Y_{t+1}}{Y_t}\theta^P \pi_{t+1,k}(\pi_{t+1,k} - 1)\right\}, \tag{2.23}$$

with sectoral inflation $\pi_{t,k} = P_{t,k}/P_{t-1,k}$.

The aggregate inflation $\pi_t = \frac{P_t}{P_{t-1}}$ reads as:

$$\pi_t = \left(\varkappa^{\frac{1}{\theta}} \frac{P_{t-1,g}}{P_{t-1}} \pi_{t,g}^{1-\frac{1}{\theta}} + (1-\varkappa)^{\frac{1}{\theta}} \frac{P_{t-1,b}}{P_{t-1}} \pi_{t,b}^{1-\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\theta}}}. \tag{2.24}$$

In addition, please note that the j-index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t, will make the same decisions.

[30] As a robustness exercise we set price stickiness à la Calvo (online appendix) and find similar results.

2.2.2.3 Capital Producing Firms

We assume that households own capital producing firms and receive profits. Capital producing firms buy specific types of capital from intermediate goods firms at the end of period t, repair depreciated capital, and create new capital. They then sell both the new and re-furbished capital. The relative price of a unit of capital is $Q_{t,g}$ for green and $Q_{t,b}$ for brown. We suppose that there are flow adjustment costs associated with producing new capital as in Jermann [1998]. Accordingly, capital producing firms face the following maximization problem:

$$\max_{\{I_{t,k}\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s,k} - 1) I_{t+s,k} - f_k(.)(I_{t+s,k}) \right\} \tag{2.25}$$

$$\text{with} \quad I^n_{t,k} = I_{t,k} - \delta K_{t,k}, \tag{2.26}$$

$$K_{t+1,k} = K_{t,k} + I^n_{t,k}, \tag{2.27}$$

$$\text{and} \quad f_k(.) = \frac{\eta_i}{2} \left(\frac{I_{t,k}}{I_{t-1,k}} - \theta^I \right)^2, \tag{2.28}$$

where $I^n_{t,k}$ and $I_{t,k}$ are net and gross capital created, respectively. $\delta K_{t,k}$ is the quantity of re-furbished capital, and η_i the inverse elasticity of net investment to the price of capital.[31] Thus, we get the following value for $Q_{t,k}$:

$$Q_{t,k} = 1 + f_k(.) + f'_k(.) \left(\frac{I_{t,k}}{I_{t-1,k}} \right) - \beta E_t \left\{ \Lambda_{t,t+1} f'_k(.) \left(\frac{I_{t+1,k}}{I_{t,k}} \right)^2 \right\}. \tag{2.29}$$

2.2.3 Financial Intermediaries

We augment the setup of Gertler and Karadi [2011] to allow financial intermediaries to invest in both green and carbon-intensive firms. We also modify the incentive constraint to provide a realistic implementation of macroprudential policy through regulatory risk-

[31] The term θ^I is set such that the over the balanced growth path ($f_k \left(\frac{i_{t,k}}{i_{t-1,k}} \right) = 0$), where $i_{t,k}$ is the de-trended net investment.

weights on loans.

A representative bank's balance sheet can be depicted as:

$$Q_{t,g}S_{t,g} + Q_{t,b}S_{t,b} = N_t + B_t, \tag{2.30}$$

where $S_{t,g}$ and $S_{t,b}$ are financial claims on green and brown firms and $Q_{t,g}$ and $Q_{t,b}$ their respective relative price. Note that $S_{t,k} = K_{t,k}$, as firms from both sectors do not face frictions when requesting financing. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, the banks' equity capital evolves as follows:

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,b}Q_{t-1,b}S_{t-1,b} - R_tB_{t-1}, \tag{2.31}$$

$$N_t = (R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,b} - R_t)Q_{t-1,b}S_{t-1,b} + R_tN_{t-1}, \tag{2.32}$$

where $R_{t,k} = \frac{R^K_{t,k} - (Q_{t,k} - \delta)}{Q_{t-1,k}}$ denotes the gross rate of return on a unit of the bank's assets from $t-1$ to t for sector k.[32]

The goal of a financial intermediary is to maximize its equity over time. Thus, we can write the following objective function:

$$V_t = E_t \left\{ \sum_{i=1}^{\infty} (\Delta\beta)^i \Lambda_{t,t+i}(1-\theta_B)\theta_B^{i-1} N_{t+i} \right\}, \tag{2.33}$$

with $(1-\theta_B)$ the exogenous probability of going out of business for a banker and Δ a parameter accounting for the subjective discount factor of bankers.[33] We introduce a regulator in charge of the supervision of financial intermediaries. Drawing on Pietrunti [2017], we assume that the regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their

[32]Note that the depreciated capital has a value of one as adjustment costs only apply to net investment.
[33]This parameter allows us to perfectly match financial steady state data for the EA.

relative risk:

$$V_t \geq \lambda(\lambda_g Q_{t,g} S_{t,g} + \lambda_b Q_{t,b} S_{t,b}), \tag{2.34}$$

with λ the risk-weight on loans and λ_g and λ_b sectoral specific weights that can be applied to loans for green and/or brown firms. The regulator can modify these weights, altering the constraint weighing on banks and thus the allocation of loans between sectors. In our baseline version of the model, however, we consider the case where λ_g and λ_b are both equal to one, and we calibrate λ and other banks-related parameters to match the capital ratio of banks in the Euro Area as well as risk premia levels. We guess that the value function is linear of the form $V_t = \Gamma_t^B N_t$ so we can rewrite V_t as:

$$V_t = \max_{S_{t,g}, S_{t,b}} E_t \left\{ \Delta\beta\Lambda_{t,t+1}\Omega_{t+1}N_{t+1} \right\}, \tag{2.35}$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t^B$. Maximization subject to the regulatory constraint (2.34) yields the following first order and slackness conditions:

$$\Delta\beta E_t \left\{ \Lambda_{t,t+1}\Omega_{t+1}(R_{t+1,k} - R_{t+1}) \right\} = \nu_t \lambda_k \lambda, \tag{2.36}$$

$$\nu_t \left[\Gamma_t^B N_t - \lambda(\lambda_g Q_{t,g} S_{t,g} + \lambda_b Q_{t,b} S_{t,b}) \right] = 0, \tag{2.37}$$

where ν_t is the multiplier for constraint (2.34). One interesting result is that we get:

$$N_t \geq \Xi_t(\lambda_g Q_{t,g} S_{t,g} + \lambda_b Q_{t,b} S_{t,b}), \tag{2.38}$$

where $\Xi_t = \lambda/\Gamma_t^B$ is the regulatory capital requirement for banks and λ_g and λ_b represent potential rewards or penalties on the weights required by the regulator on green and brown loans, respectively.[34] Finally, we rewrite the value function to find Γ_t:

$$\begin{aligned} V_t &= \lambda\nu_t(\lambda_g Q_{t,g}S_{t,g} + \lambda_d Q_{t,b}S_{t,b}) + \Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}N_t\right\} \\ \Gamma_t^B N_t &= \nu_t\Gamma_t^B N_t + \Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_t N_t\right\} \\ \Gamma_t^B &= \frac{1}{1-\nu_t}\Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}\right\}. \end{aligned} \tag{2.39}$$

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B[(R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,b} - R_t)Q_{t-1,b}S_{t-1,b}] + (\theta_B R_t + \omega)N_{t-1}, \tag{2.40}$$

with $\omega \in [0;1)$ the proportion of funds transferred to entering bankers.

2.2.4 Public Authorities

2.2.4.1 Central Bank

The central bank follows a simple Taylor [1993] rule to set the interest rate:

$$i_t - \bar{\imath} = \rho_c\left(i_{t-1} - \bar{\imath}\right) + (1-\rho_c)\left[\phi_\pi\left(\pi_t - \bar{\pi}\right) + \phi_y\left(Y_t - Y_{t-1}\right)\right], \tag{2.41}$$

where $\bar{\imath}$ is the steady state of the nominal rate i_t, $\rho_c \in [0,1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, ϕ_y is the output gap stance penalizing deviations of output from its previous period level Y_{t-1}. Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t = R_t E_t\left\{\pi_{t+1}\right\}. \tag{2.42}$$

[34] For instance, if $\lambda_g < \lambda_b$ banks will need to hold less capital for loans they grant to green firms compared to brown firms. Note that the actual capital ratio thus also depends on the risk-weights assigned to each asset, consistent with Basel III framework.

We match the observed level of nominal interest rate using the simulated method of moments with the German 10-year Bund as an observable.[35] The estimation leads to a steady sate value of about 1% annually over the sample period. This drastically limits the scope of conventional monetary policy, as the central bank can not set its nominal interest rate below zero.[36]

In addition to setting the nominal interest rate, the central bank conducts open market operations. Within our framework, it will be able to buy and sell assets that are otherwise held by financial intermediaries. We will explain in section 2.2.7 how public financial intermediation (i.e. QE) works in this model.

2.2.4.2 Government

The government sets a budget constraint according to the following rule:

$$T_t + \tau_{et} E_t + RP_{t,g}\psi_{t,g}K_{t,g} + RP_{t,d}\psi_{t,b}K_{t,b} = G_t, \tag{2.43}$$

with public expenditure G_t finding its source from taxes T_t, revenues from the price of carbon $\tau_{et}E_t$ and from public financial intermediation on both green and brown firms $RP_{t,g}\psi_{t,g}K_{t,g}$ and $RP_{t,b}\psi_{t,d}K_{t,b}$ (with $RP_{t,k}$ the spread between each sector's risky rate and the riskless rate, also referred to as risk premia). Government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t. \tag{2.44}$$

2.2.5 Normalization and Aggregation

Factors and goods markets clear as follows. First, the market-clearing conditions for aggregate capital and investment in the two sector economy read as: $K_t = \sum_k g(\varkappa) \int_0^1 K_{jt,k} dj$ and $I_t = \sum_k g(\varkappa) \int_0^1 I_{jt,k} dj$, respectively. Second, global aggregate emissions and aggre-

[35] At the steady state, inflation is normalized to 1, so that $i_t = R_t$.

[36] Since we do not model banks' holding of reserves at the central bank.

gate emissions cost are two weighted sums of sectoral emissions $E_t = \sum_k g(\varkappa) \int_0^1 E_{jt,k} dj$, and sectoral emissions cost $Z_t = \sum_k g(\varkappa) \int_0^1 Z_{jt,k} dj$, respectively. Finally, the resource constraint of the economy features capital adjustment and abatement costs:

$$Y_t = C_t + G_t + I_t + \sum_k g(\varkappa)[f_k(.)(I_{t,k})] + \sum_k g(\varkappa)\Delta^P_{t,k} Y_t + Z_t. \tag{2.45}$$

2.2.6 Climate Externality and Financial-Economics Inefficiencies

Retrieving the optimal allocation where the environmental cost is internalized by the central planner requires setting the carbon price in the decentralized equilibrium equals to the social cost of carbon found in the centralized problem. To keep the framework tractable and without a loss of generality, we solve the centralized problem for households and firms, given an allocation of investment, capital, financial intermediaries net worth and deposit as these do not enter the social cost of carbon derivation.[37]

2.2.6.1 Competitive Equilibrium

To pin down the optimal carbon policy, we solve for the Competitive Equilibrium (CE*). The CE* in this economy is defined as follows:

Definition 2.2.1 *A competitive equilibrium consists of an allocation* $\{C_t, K_{t,k}, E_{t,k}, X_t, T^o_t\}$, *a set of prices* $\{P_t, P_{t,k}, R_t, R^k_{t,k}, W_{t,k}\}$ *and a set of policies* $\{\tau_{et,k}, T_t, B_{t+1}\}$ *such that:*

- *the allocation solves the consumers' and firms' problems given prices and policies,*
- *the government budget constraint is satisfied in every period,*
- *temperature change satisfies the carbon cycle constraint in every period, and*
- *markets clear.*

[37] We can easily show that adding financial intermediaries as well as capital producing firms to the constraints of the centralized problem does not change change the SCC derivation.

Result 1 *The optimal solution sets the carbon price policy $\tau_{et,k}$ as an optimal policy $\tau^*_{et,k}$, which maximizes total welfare in equation (2.1):*[38]

$$\tau^*_{et,k} = g(\varkappa) SCC_t. \tag{2.46}$$

with SCC_t the social cost of carbon:

$$SCC_t = \eta\beta\frac{\lambda_{t+1}}{\lambda_t}SCC_{t+1} + (\upsilon_1^o\upsilon_2^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S^T_{t+1}, \tag{2.47}$$

and with

$$\S^T_t = (1-\upsilon_1^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S^T_{t+1} - \sum_k \Psi_{t,k}\varepsilon_t^{A,k}\frac{\partial d(T_t^o)}{\partial T_t^o}K^\alpha_{t,k}(\Gamma_t L_{t,k})^{1-\alpha}. \tag{2.48}$$

2.2.6.2 Departing from the Competitive Equilibrium to Meet Climate Goals

Definition 2.2.2 *Public authorities, however, do not optimally set the carbon price as highlighted in definition 2. In the EU, public authorities target a level of emissions that is consistent with their objective of a 55% emissions reduction by 2030. In practice, this means gradually increasing the cost of carbon through the reduction of emissions quotas distributed to firms within specific sectors. We model this situation by assuming that the cap set by the fiscal authority follows a decreasing trend, implying a growing price of carbon. The resulting carbon price can then be hit by exogenous shocks, to account in a 'stylized' way for price fluctuations on the ETS market:*

$$E_t = Cap_t \tag{2.49}$$

[38] The full derivation of the CE* can be found in the technical appendix

with $Cap_t = Cap/\Gamma_t^{Cap}$. *Equivalently, a cap on emissions translates to a price of carbon such that:*

$$\tau_{et,k} = Carbon\ Price_t, \tag{2.50}$$

where $Carbon\ Price_t = \varepsilon_t^{\tau}\Gamma_t^{Price}\ Carbon\ Price$. *In this case,* Γ_t^{Price} *is a trend on the carbon price that is proportional to the trend on the cap* Γ_t^{Cap} *and is consistent with the desired emissions reduction implemented through the cap policy.* ε_t^{τ} *represents the ETS price shock.*[39]

This stylized representation of the implementation of a permit market allows us to find theoretical fiscal pathways consistent with the EU climate objectives.

2.2.6.3 Welfare Distortion

Definition 2.2.3 *The welfare distortion arises when there is a difference between the optimal environmental policy and the targeted policy consistent with the EU objectives:*

$$\tau^*_{et,k} \neq \tau_{et,k} \tag{2.51}$$

When $\tau_{et,k}$ *moves away from* $\tau^*_{et,k}$, *the loss in welfare grows:*[40]

$$\Delta_{\{\tau-\tau^*\}}\,Welfare < 0 \tag{2.52}$$

where the welfare could be decomposed as follows:

$$\begin{aligned} Wedge_{C_k} \propto & (1-g)\varepsilon_t^{A,k}(\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^{\alpha} - d(T_t^o)^* K_{t,k}^{\alpha}{}^*) - (f(K_{t,k}) - f(K_{t,k})^*) \\ & - ((\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^{\alpha} f(\mu_{t,k}) - d(T_t^o)^*)K_{t,k}^{\alpha}{}^* f(\mu_{t,k})^*) \end{aligned}$$

[39] In our setup, carbon prices variations at the business cycle frequency are mainly driven by exogenous market forces. While sudden changes in abatement efficiency (i.e. the abatement cost) could in theory be a source of carbon price volatility, we abstract from considering this mechanism as there is a lack of empirical evidence and data availability (at the business cycle frequency) on abatement costs.

[40] A full decomposition of the welfare effect is presented in the online appendix.

Proposition 2.2.1 *Macroprudential climate risk-weights loosening the constraint on bank lending to the green sector can reduce the welfare loss on consumption, while addressing climate-related financial risk.*[41]

Implementing a higher policy rate compared to an optimal policy clearly decreases damages from temperature to production $d(T_t^o) < d(T_t^o)^*$. However, abatement is costlier under the higher policy rate. This results in a loss of welfare, but prevents potential climate risks in the future that are not internalized by firms. The climate risk-weights macroprudential policy, which will lower (increase) the capital requirement for green (brown) assets, will in turn trigger a rise (decrease) in green (brown) firms' capital. As green firms are less subject to the carbon price, the increase in the relative size of the green sector in total output will lead to a welfare gain.

2.2.6.4 Risk Premium Wedge

Volatility in risk premia $\mathrm{RP}_{t,k}$, defined as the difference between expected returns on risky assets $R_{t,k}$ and the return on the riskless asset R_t, could alter monetary policy transmission (Doh et al. [2015]).

Definition 2.2.4 *When the carbon price is set through a market for carbon permits, it induces price uncertainty that is detrimental to firms. Ultimately, it affects the marginal cost of firms as well as the price of capital, and leads to movements in risk premia. In the case of a positive carbon price shock, the marginal cost of firms increases as they are now subject to higher CO_2 prices. This in turn could raise the risk premium:*[42]

$$RP_{t,k} = R_{t,k} - R_t \tag{2.53}$$

$$= f(\Psi_{t,k}, Y_{t,k}, K_{t,k}, Q_{t,k}) - R_t \tag{2.54}$$

[41] As detailed in section 2.2.7 and shown in figure 2.4, macroprudential policy arises as a tool to mitigate climate risk to the financial sector. While primarily intended to ensure financial stability, it also dampens the welfare effect of an increasing carbon price.

[42] The impact is symmetric in the case of a negative carbon price shock. Furthermore, whether the shock is positive or negative, it implies higher volatility for the marginal cost and the risk premium.

Proposition 2.2.2 *Volatility in risk premia stemming from carbon price fluctuations could potentially distort the functioning of monetary policy operations. Short-term monetary policies (QE rules that react to changes in risk premia) can prevent this situation and ensure financial stability.*

The risky rate reacts to changes coming both from the firms' side and the financial side. In this case, the goal is to cut the link between the rise of the marginal cost (triggered by an increase in the carbon price) and the impact on the risk premium. One way to do so is to act on the financial side to compress the risk premium. Similar to models where a rise in risk premia comes from an exogenous shock on the quality of capital (e.g. crisis simulation in Gertler and Karadi [2011]), the central bank is able to offset this effect by intervening in the loan market.

2.2.7 Set of Policies

Environmental Policy

When acting optimally, the decentralized planner would set the environmental policy as shown in result 1 ($\tau^*_{et,k}$ is set equal to the social cost of carbon $g(\varkappa)SCC_{t,k}$). However, as highlighted in the previous section, the EU authorities deviate from the optimal policy and set the environmental policy to be consistent with their net-zero emissions reduction objective ($\tau_{et,k} \neq \tau^*_{et,k}$).

Sectoral Macroprudential Weights

There is a macroprudential authority with the ability to alter the regulatory constraint weighing on banks (equation (2.34)) by modifying risk-weights on loans.

Environmental, social, and corporate governance (ESG) criteria are increasingly valued by both investors and authorities. As these criteria are also gaining importance in firms' credit ratings (Escrig-Olmedo et al. [2019] and Carbone et al. [2021]), it will likely impact banks' portfolio allocation. On the regulatory side, macroprudential authorities are starting to assess how they could consider climate risk within their frameworks. Recently, the Basel Committee [2021] issued a press release stating that "The Committee is taking a holistic approach to addressing climate-related financial risks to the global banking system. This includes the assessment and consideration of disclosure, supervisory and regulatory measures." Within our framework, this would mean that firms with a low carbon intensity would carry a lower risk-weight in the RWA methodology, while carbon-intensive firms would carry a higher risk-weight. In our view, there are two means by which this could materialize. Either ESG criteria would become so important in standard credit ratings such that it could lead to environmentally friendly firms getting a higher rating, and thus a lower risk-weight in banks' regulatory constraint. For instance, a green firm could see its rating upgraded from BBB+ to A-, implying a 25 percent drop in the risk-weight associated with this firm in banks' regulatory capital constraint. On the other hand, a carbon-intensive firm could see its rating downgraded from BBB- to BB+, implying a 25 percent increase in the risk-weight associated with this firm.[43] In this case, this change in the importance of ESG criteria in credit ratings would endogenously transmit to macroprudential policy, and ultimately to banks' portfolio allocation. Another possibility would be that macroprudential authorities apply an additional risk-weight related to the carbon intensity of firms. It could for instance multiply the risk-weight related to the credit rating of a firm by a climate risk-weight related to the environmental performance of a firm. In our setup, implementing climate risk-weights in the spirit of Basel III, would mean decreasing λ_g by 25 percent (i.e. $\lambda_g = 0.75$) and increasing λ_b by 25 percent (i.e. $\lambda_b = 1.25$).[44] This will loosen (tighten) the regulatory constraint on banks with respect to the green (brown) sector, triggering an increase (decrease) in loans to green (brown) firms. In addition to

[43] Please refer to the high-level summary of Basel III reforms (Basel Committee [2017]) for a detailed description of the RWA methodology.

[44] We consider this to be our baseline scenario, where both green and brown bonds held by financial intermediaries are mainly at the lower rank of investment grade bonds (i.e. BBB+ to BBB-). We also investigate other cases in our robustness exercises, where climate risk-weights applied are higher.

addressing climate-related financial risk, it would also support the transition to a greener economy.

Quantitative Easing

QE in this model can be both a short-term or a medium/long-term instrument. In the short term, the central bank can purchase or sell bonds as part of open market operations to ensure the smooth transmission of monetary policy. In this case, we model it as a QE rule, in the spirit of Gertler and Karadi [2011]. We will show quantitatively how QE rules targeting risk premia can offset the inefficiency stemming from the uncertainty over the carbon price. In the long term, the central bank can also implement LSAP programs, where it decides to buy a predefined portion of assets over a determined period of time. Much like the Corporate Sector Purchase Program in the EA, the central bank has the ability to finance non-financial firms in order to reduce corporate spread, steer private investment, and ultimately keep inflation within range of its target. In a complementary exercise, we will assess how green LSAP programs differ from conventional brown LSAP programs.

Then for each type of firm k we now have:

$$Q_{t,k}S_{t,k} = Q_{t,k}S_{pt,k} + Q_{t,k}S_{gt,k}, \tag{2.55}$$

with $Q_{t,k}S_{gt,k}$ the total real value of loans to firms of type k held by the central bank. $Q_{t,k}S_{pt,k}$ is the total real value of loans to firms of type k held by financial intermediaries, as defined in section 2.2.3. As in Gertler and Karadi [2011], we model this intervention by assuming that the central bank holds a portion $\psi_{t,k}$ of total loans to non-financial firms belonging to each sector:[45]

$$Q_{t,k}S_{gt,k} = \psi_{t,k}Q_{t,k}S_{t,k}. \tag{2.56}$$

[45]For simplicity, we abstract from monitoring costs.

To address the inefficiency stemming from carbon price uncertainty, we will assume that, for each sector, the central bank follows a counter-cyclical credit policy rule that reacts to the variations in the expected spread ($E_t\{\mathrm{RP}_{t+1,k}\} = E_t\{R_{t+1,k} - R_{t+1}\}$) in order to decide the share of assets $\psi_{t,k}$ it holds. This rule is defined as follows:

$$\psi_{t,k} = \phi_k^s(E_t\{\mathrm{RP}_{t+1,k}\} - \bar{\mathrm{RP}}_k). \tag{2.57}$$

Note that in our baseline model $\psi_{t,k} = 0$ so that the central bank allows financial intermediaries to be the sole source of financing for firms.

2.3 Solution Method

2.3.1 Balanced Growth Path

In our economy, the labor-augmenting technology grows at rate Γ_t. As a number of variables (e.g. output, emissions, investment, ...) will not be stationary, we need to de-trend the model.[46] In the appendix subsection 3.B.5 we present the de-trended economy, where all variables are stationary along an existing balanced growth path. The variables of our economy growing at the same rate Γ_t include: output per capita $Y_{t,k}$, investment per capita $I_{t,k}$, consumption per capita C_t, government spending G_t, lump sum taxes T_t, capital per capita $K_{t,k}$, emissions $E_{t,k}$, abatement costs $Z_{t,k}$, stock of emissions X_t, temperature T_t^o, debt to households B_t, net worth N_t, and the banks' value function V_t^B.

2.3.2 Model Solving and Methods

To solve for the medium/long-run pathway scenarios, we use the extended path algorithm, which allows us to integrate both deterministic trends and stochastic shocks. This approach maintains the ability of deterministic methods to provide accurate accounts of

[46] This is also necessary to estimate our key structural parameters using the SMM.

non-linearities, while usual local approximation techniques do not perform as well under the presence of such non-linearities (Adjemian and Juillard [2013]). Furthermore, we account for uncertainty and compute confidence intervals along the net-zero transition pathways. We rely on the Monte Carlo method and simulate 2000 series for both stochastic shocks (i.e labor-augmenting technology and carbon price shocks) around their deterministic trends. As for addressing short-term business cycle implications of the ETS price volatility, we use second-order perturbation methods as they are usually performed in the macro-finance literature to retrieve impulse response functions.

2.3.3 Data and Fitting Strategy

As we will study the role of the central bank and macroprudential authority, we calibrate and estimate the model on the EA, even though the environmental ETS policy is set at the EU level. This is without a loss of generality, since all countries in the EA are members of the EU.

In order to best fit our model to real data,[47] we rely on the SMM (Duffie and Singleton [1993]) to estimate key structural parameters of our economy (table 2.4). In the spirit of Jermann [1998] we match the first and second moments of: output growth, investment growth, and consumption to output growth. As we are also interested in the financial and environmental sectors, we match the first moments of the real riskless and risky rates, the capital ratio of banks, the emission to output ratio, the global stock of carbon, and the ETS price level (at the beginning of 2021), as well as the difference between green and brown firms' marginal costs. We estimate the following key structural parameters: $\{\eta_{A_k}$, ρ_{A_k}, $\frac{\bar{g}}{\bar{y}}$, η_i, β, γ_Y, h, α, δ, θ_g, θ_d, E^*, φ_k, Carbon Price, λ, $\omega\}$, using the Metropolis–Hastings algorithm for the Markov Chain Monte Carlo over 5 chains of 2000 draws. The remaining parameters are calibrated and their values are reported in table 3.8, table 3.9, and table 3.10.

[47]For macro-finance data, we match first and second moments using EA data between 2000 and 2020. All data sources are summarized in table 2.5.

2.3.3.1 Calibration

For parameters related to business cycle theory, their calibration is standard: the share of hours worked per day is set at one third in each sector and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2, as argued by Stern [2008] and Weitzman [2007].

Regarding environmental components, we calibrate the damage function according to Nordhaus and Moffat [2017].[48] The global temperature parameters υ_1^o and υ_2^o are set following Dietz and Venmans [2019] to pin down the 'initial pulse-adjustment timescale' of the climate system.[49] We use sectoral data made available by the Transition Pathway Initiative to set the share of the green sector $\varkappa$ at 30 percent.[50] Abatement parameters $\theta_{b,1}$, $\theta_{b,2}$, and $\theta_{g,2}$, which pin down the abatement costs for each sector, are set as in Heutel [2012]. We then proceed to set $\theta_{g,1}$ to match the drop in emissions induced by the introduction of the carbon price policy in the EA. More precisely, we retrieve the value of $\theta_{g,1}$ in such a way so as to be consistent with a reduction of emissions of 14.3 percent between 2009 and 2020,[51] which is associated with an increase in the carbon price from 0 to 30 euro (the price of ETS at the end of 2020). In our model, this leads to a value of $\theta_{g,1}$ of 0.02, which means that the abatement technology is cheaper in the green sector. The decay rate of emissions δ_x is set at 0.21 percent as in Heutel [2012].

As for the financial parameters, we set the probability of remaining a banker θ_B at 0.98, meaning that 2 percent of bankers default every quarter, which is slightly less than in Gertler and Karadi [2011]. Δ is a parameter that introduces a different discount factor in the bankers' objective function relative to households and is set to 0.99. This implies that bankers are slightly more impatient than households. Finally, the monetary rule

[48] We perform a sensitivity analysis using values from Dietz and Stern [2015] and Weitzman [2012] in the next section.

[49] We also perform a sensitivity analysis for υ_2^o.

[50] What we consider green in our model is a sector with a carbon performance that allows for an emission target aligned with the Paris Agreement of 2 degrees Celsius or below.

[51] We remove the first and last years of data.

parameters are set as in Smets and Wouters [2003].

Regarding the carbon price shock, we calibrate the standard deviation using ETS data (futures prices). We find a standard deviation of about 0.18 on a quarterly basis.

2.3.3.2 Estimation

Parameters estimated through the SMM are reported in table 2.4, while the empirical moments matched are reported in table 2.5. Although we only rely on a shock to the labor-augmenting technology, all parameters are well identified and the model is able to match empirical moments for the EA.

More precisely, the depreciation rate of physical capital is estimated at 2.5 percent in quarterly terms, the government spending to GDP ratio at 28 percent, and the capital intensity in the production function α at 0.33. All these estimates are quite standard within the macroeconomic literature. The inverse elasticity of net investment to the price of capital η_i is estimated at 1.7354, in line with the value chosen by Gertler and Karadi [2011]. The parameter b, which allows us to pin down the discount factor, is set at 0.02. This ensures that we match the steady state real interest rate of about 1 percent (the mean rate of 10-year German Bund over the sampled period). Habits in consumption are found to be rather low (0.22) compared to the estimated value of Smets and Wouters [2003].

To replicate the global level of carbon stock in the atmosphere (i.e. 840 gigatons), the level of the rest of the world's emissions E^* is estimated at 3.37. Furthermore, as argued by De Haas and Popov [2019], CO_2 emissions intensity differs largely between sectors and industries. We use carbon intensity parameters φ_b and φ_g to match the observed ratio of emissions to output for the EA, which is at 21 percent.[52] Assuming that the carbon intensity in the green sector is approximately one third of what it is in the brown sector, we find that $\varphi_b = 0.29$ and $\varphi_g = 0.09$.

The value of θ_d, the brown firms' marginal cost parameter, is set as in Smets and

[52] We compute this value as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from Eurostat.

Wouters [2003] to replicate the mean markup and marginal cost levels observed in the economy. On the other hand, θ_g is estimated to match the green marginal cost, which is—as argued by Chan et al. [2013] and Chegut et al. [2019]—6 percent higher than the brown firms' marginal cost.

The parameter shaping the leverage of banks $\bar{\lambda}$ is estimated at 0.0176 to generate a spread of 80 basis points between risky and riskless assets, consistent with Fender et al. [2019]. The authors also find that the spread between green and brown bonds recently disappeared. Thus, we target the same steady state for R_g and R_d.[53] The proportional transfer to entering bankers ω is found to be around 0.006, allowing us to match a capital ratio of approximately 14.4 percent in the EA.

Finally, for the TFP shock, standard deviation and persistence are estimated at 0.006 and 0.78, which are both in line with previous estimates of Smets and Wouters [2003] for the EA.

2.4 Quantitative Analysis

In the EU, the carbon price resulting from the ETS cap policy is subject to high volatility. We use ETS futures weekly prices to retrieve the mean standard deviation over the period, before converting it to a quarterly level. We then set the standard deviation of the ETS carbon price σ_{ETS} to this value for all pathway simulations and exercises we conduct.

With respect to the long-term inefficiency (i.e. the welfare loss), we perform stochastic transition pathway simulations,[54] where we include stochastic shocks on both the price of carbon and the TFP around their respective deterministic growth rate. We perform 2000 Monte Carlo simulations to construct 95 percent confidence intervals around the

[53] This is also in line with recent findings of Flammer [2021] with respect to the so called "Greenium" puzzle (i.e. $R_g < R_d$). In this paper, she finds no evidence for the existence of a Greenium.

[54] We compare two scenarios: a) the carbon policy is consistent with the net-zero objective and b) the carbon policy is consistent with the optimal social cost of carbon.

deterministic trends for both the output and the carbon price needed to achieve the net-zero pledge. We then investigate the role that green macroprudential policy—which favors the green sector over the brown sector—could play in mitigating the welfare wedge, while ensuring financial stability.

Turning to the short-term inefficiency (i.e. risk premia distortion), we perform stochastic simulations to investigate the impulse responses to a shock to the price of carbon on risk premia and inflation, and highlight how the central bank could take into account this type of transition risk within its framework.

2.4.1 Fiscal Environmental Policy Scenario

The goal of this section is to present and analyze theoretical fiscal pathways consistent with the EU objective for 2030.[55] We first find the trajectory of the carbon price that leads to the desired reduction in emissions (i.e. a 55 percent emissions reduction relative to the 1990 level, which corresponds to a 33 percent reduction relative to the 2020 level). We then highlight the impact of sub-optimal carbon pricing policies on welfare.

2.4.1.1 Growth, carbon price, and the EU objectives

Figure 2.5 shows carbon price trajectories (according to two different growth scenarios) consistent with being on track for achieving the net-zero objective in the EU. The blue dashed line is the central scenario with a growth trend of 0.8 percent, corresponding to the average real growth rate per capita in the EA from 2000 to 2020. The orange dotted line is a scenario with a more optimistic growth trend of 1.2 percent. We also add stochastic components drawn from random disturbances to the TFP and the carbon price. The shaded blue and orange areas are 95 percent confidence intervals retrieved over the 2000 Monte Carlo draws. This allows us to account for uncertainty in output growth and the

[55] In this section, as the main focus is long-term transition pathways, we do not consider nominal rigidities in prices.

carbon pricing trajectory.[56] Depending on the growth scenario, reducing emissions by 55 percent compared to 1990 level would require a mean carbon price between 350€ and 375€ per ton of CO_2. Accounting for uncertainty, the price is found to fluctuate between 200€ and 500€, meaning that the target could be either undershot or overshot. Note that this large confidence interval is computed assuming that future volatility can be inferred from past volatility. However, EU countries are considering measures to reduce price fluctuations in the ETS market,[57] which could lead to a lower standard deviation in the future. This exercise provides evidence that such measures are needed if the EU authorities want to improve their ability to meet their emission reduction objective. Furthermore, we also find that the price of carbon needs to follow the growth of output to be able to shrink the flow of emissions to the desired level. It is worth noting, however, that our model takes the abatement technology as given. With improvements in technology, the EU could reach the same target with a lower carbon price, but the mechanisms to trigger this improvement in the abatement technology are left for further research.

Figure 2.6 uses the central growth scenario (i.e. 0.8 percent growth rate) to compare the net-zero trajectory with a carbon market that exhibits uncertainty (blue solid line and shaded area) and a market that yields a completely deterministic carbon price (purple dotted line and shaded area). This is similar to comparing a cap policy with a tax policy. We find that a carbon tax like system, where volatility is controlled, would allow for reaching the net-zero objective with certainty. However, a cap and trade policy ensures that emissions reduction take place efficiently, as firms are able to trade permits while a tax system imposes a fixed reduction in emissions to all firms. In addition, Karp and Traeger [2018] show that, when considering a stock pollutant, a cap market guarantees efficiency gains (compared to a tax system) when the economy is subject to technology shocks that

[56] Where trend growth in output and carbon prices are anticipated, but shocks can distort these deterministic processes in the short run.

[57] A carbon price floor has been implemented in the Netherlands and is currently under consideration in Germany. The EU Market Stability Reserve was also introduced to regain some control over the carbon price.

shift the marginal abatement cost curve and the social cost of carbon.

The ambitious net-zero goal would have several implications on output and consumption alike. In figure 2.7, we show that uncertainty in carbon pricing does not significantly alter consumption pathways and therefore does not alter the welfare, as shown in the case of the certainty equivalence in Golosov et al. [2014]. Carbon price shocks do not propagate to the households as, on one hand, the stochastic discount factor—which is the central part in asset pricing and consumption smoothing mechanisms—is not directly impacted by the carbon pricing, and, on the other hand, the relative risk aversion is set different to 1 (the log utility case). In our setup, climate risk is not directly captured within the utility function, restraining the carbon price shock from propagating to consumption and welfare.[58] As such, we run deterministic transition pathway simulations instead of stochastic transition pathway simulations for the remaining welfare analysis.

2.4.1.2 Welfare implications

The first two plots in figure 2.8 display the trajectory of the environmental policy consistent with the EU objective compared to the optimal environmental policy for both output and emissions. The optimal policy (i.e. setting the carbon price equals to the SCC) trajectory is not able to meet the net-zero pledge. The carbon price needed to achieve net-zero is found to be significantly higher than the SCC, thus altering the welfare pathway. Several key factors are in play. First, the fact that the environmental externality is a slow moving variable pushes the social planner to further its intervention at a late stage when the stock of carbon has significantly accumulated, and has become a major threat. Second, the absence of tipping points, which would force the social planner to account for uncertainty over the climate damages, would obligate the social planner to increase its actions by increasing the SCC (Dietz et al. [2021]). Third, the household utility objective function does not capture the effects of climate change directly, which would impact the

[58] While integrating climate risk as a dis-utility would allow for carbon price shocks to propagate to the welfare, we do not model it in this paper and leave it for future research.

SCC (Barrage [2020] and Benmir et al. [2020]).[59] Finally, in recent work, Cai and Lontzek [2019], Traeger [2021], and Van den Bremer and Van der Ploeg [2021] both show that accounting for uncertainty in climate dynamics could increase the inherent level of the SCC. This increase in the carbon price, which would be welfare enhancing in our framework, is still, however, not sufficient to meet the net-zero emissions reduction goal. We show that the price difference between the optimal SCC and the net-zero ETS induced carbon price needed to reach the target (the "Extra Carbon Price") is about 300€ higher by the end of 2030. While we do not explicitly model tipping points in the damage function, we perform a sensitivity analysis both on the climate damages specification and climate dynamics.

As reported in our sensitivity analysis (table 2.6), the optimal price of carbon depends on the specification of damages. We find carbon prices between 31.2€ to 144.1€ for different calibrations found within the literature. Furthermore, in the spirit of Traeger [2021], we perform a sensitivity analysis over the parameter υ_2^o, which drives the climate dynamics for temperature. We show that for a higher value of υ_2^o, temperature by 2030 could double, but the implied SCC (under both Nordhaus and Dietz damage specifications) would still be insufficient to obtain the desired emission reduction to be on track for net-zero by 2030. Under the Weitzman specification, we find that setting the carbon price equals to the SCC would lead to a 45 percent emissions reduction by 2030, which is higher than the EU objective. However, the carbon price that would be able to achieve such an objective is significantly high (846.65 €), thus suggesting major issues in terms of implementation. Therefore, for the remainder of the paper, we set the climate damage parameter "b" *à la* Nordhaus and υ_2^o to the baseline value as in Dietz and Venmans [2019], as these are the closest to the ETS price at the start of January 2021 for all three estimates.

The two red plots in figure 2.8 show that the welfare loss increases over time as the extra carbon price continues to rise to about 300€. This deviation of the ETS carbon price from the SCC introduces a distortion with respect to the optimal allocation. By

[59] Benmir et al. [2020] show that the SCC increases when households account for the externality within their utility function ($u_{xc} \neq 0$).

2030, the household looses about 3 percent in consumption equivalent (CE) compared to the optimal case. We will see in the next section that this effect can be partially offset by sectoral macroprudential risk-weights.

2.4.1.3 Introducing Macroprudential Policy

To reduce the welfare gap induced by the sub-optimal policy, we investigate the role macroprudential policy could play. We present transition pathway scenarios where the macroprudential authority varies regulatory risk-weights on loans granted to the green and the brown sectors by banks. While there is not yet such a policy in the EU, regulators are increasingly taking into account climate risk (see section 2.2.7).

In figure 2.9, we present two net-zero emissions reduction scenarios: i) the scenario where macroprudential policy is neutral (i.e. $\lambda_g = 1$ and $\lambda_b = 1$) in blue, and ii) the scenario where a green macroprudential policy is implemented by the regulator in green (i.e. $\lambda_g \underset{t\rightarrow 2030}{\longrightarrow} 0.75$ linearly, while $\lambda_b \underset{t\rightarrow 2030}{\longrightarrow} 1.25$). We show that favoring the green sector over the brown sector in banks' regulatory constraint leads to an increase in the green capital (8.3 percent) and a decrease in the brown capital (4.8 percent) by the end of 2030, with respect to the scenario where risk-weights are left unchanged. The implementation of green macroprudential policy thus amplifies the rise (drop) in green (brown) capital induced by the rising carbon price along the transition. Compared to the neutral macroprudential policy case, increasing the capital stock in the green sector reduces the welfare loss (of about 1 percent CE). Intuitively, the increasing carbon price triggers a substitution between brown and green production, as the green sector is less emission intensive. Favoring the green sector in the RWA policy reinforces this substitution effect by tilting investments toward the green sector, leading to an increase in output.

In figure 2.10, we investigate the case where the macroprudential authority favors the brown sector over the green sector to avoid a disorderly transition. The goal would be to attenuate the impact of the rising carbon price on the brown sector, as the current share of the brown sector is higher than the share of green sector (70 and 30 percent respectively). The brown macroprudential policy is displayed in brown (i.e. $\lambda_g \underset{t\to 2030}{\longrightarrow} 1.25$ linearly, while $\lambda_b \underset{t\to 2030}{\longrightarrow} 0.75$). With sectoral shares held constant, this policy would lead to a lower welfare loss by the end of 2030 than in the case of the green macroprudential policy. The RWA policy reduces the substitution effect stemming from the environmental fiscal policy. At the aggregate level, the need for investment is lower, as the substitution effect is weaker than when macroprudential policy favors the green sector. Although output decreases relative to the green macroprudential policy scenario, welfare improves as investment spending is proportionally lower.

In figure 2.11, we compare green and brown macroprudential policies, while assuming that the share of the green sector in the economy increases from 30 percent to 50 percent by the end of 2030.[60] With an increasing share of the green sector,[61] both types of macroprudential policies induce a substitution effect between the two sectors, which otherwise would not arise in the case of brown macroprudential policy (as shown previously in figure 2.10). In this case, green macroprudential policy is able to close the welfare wedge by the end of 2030. Two main factors are at play. First, as the share of the green sector grows, required investments in abatement decrease, thus increasing consumption. Second, green macroprudential policy induces lower investment costs in green capital, which at the aggregate level boosts consumption. Along the transition to a greener economy, favoring green firms in banks' capital requirements rules would ease the welfare burden on households, by lowering transition costs for firms. However, the main challenge would be to identify green firms in practice. As highlighted in Ehlers et al. [2020], there is a need for a 'green label' at the firm-level for companies committed to the net-zero transition, as opposed to the current project-based green labels.

As a robustness exercise, we also report in table 2.7 the steady state impacts of various

[60] These results are further reinforced if the increase in the share of the green sector is greater than 50 percent.

[61] In this setting, we exogenously change the share of the green sector over the 10 year transition period. One could endogenously model this shift in the share of the green sector. We leave this for future work.

macroprudential policy settings. We investigate several risk-weights combinations, where macroprudential policy is conducted as a one off. We consider a carbon price of about 300€ (the net-zero implied price by 2030). We then compare three scenarios: i) the model following the optimal policy ii) the model with a carbon price consistent with the net-zero target and no macroprudential policy iii) the model with a carbon price consistent with the net-zero target and various macroprudential policies. The robustness exercise shows that, the more the macroprudential authority decreases the risk-weight on green loans (while increasing the risk-weight on brown loans), the smaller the consumption loss is compared to the optimal. It would be possible to completely offset the consumption loss, but it would require drastic changes in risk-weight, which could threaten financial stability.

2.4.2 Risk Premia Stabilization

To offset the distortion of risk premia stemming from carbon price volatility, we assess the effectiveness of short-term QE rules set by the central bank.

The simulation reported in figure 2.12 presents the responses of risk premia to a positive shock to the carbon price level. We first show how risk premia react to the volatility in the ETS market. As the EU decided to implement its environmental fiscal policy through carbon permits, there is an inherent variance in the price of carbon.[62] Estimating the standard deviation of the shock on the ETS series and simulating the model allow us to analyze how these unexpected variations in the carbon price could affect firms and banks. The blue line shows the reaction of risk premia in both the green and brown sectors following a positive shock on the carbon price. The shock leads to an increase in risk premia of about 10 basis points annually. This rise in risk premia could lead to financial instability and thus distortion in the transmission of monetary policy. To restore the equilibrium in risk premia, monetary policy could rely on quantitative easing rules (as a 'fire-fighting' tool), which would react to changes in the level of the risk premium. As

[62] Table 2.8 displays the moments of risk premia, marginal costs, and inflation for both sectors following a positive shock on carbon prices.

such, the central bank would have the ability to substitute to financial intermediaries in financing either green or brown firms. This intervention will lead to a temporary increase in the central bank balance sheet.

More specifically, we compare two scenarios: i) a model where the central bank does not implement QE rules, ii) a model where the central bank implements QE rules with various degrees of reaction. We show that the increase in spreads could be offset by an increase in asset purchases, where the intensity of the reaction of the central bank is represented by the parameter ϕ_k^s. For instance, asset purchases of about 0.23 percent (annually) of total assets within each sector (i.e. $\phi_k^s = 0.5$) are sufficient to almost completely offset the induced distortion in risk premia.[63] The mechanism at play here is the same as in the case of exogenous financial shocks on risk premia, except that the initial rise in risk premia is triggered by the shock on the carbon price and its subsequent effect on firms' marginal costs. Compared to the financial crisis simulation in Gertler and Karadi [2011], our carbon price shock triggers a reaction of risk premia that is smaller, but the magnitude of the intervention of the central bank is proportionally similar. By stepping in to directly lend to firms, the central bank is able to restore the equilibrium on the loans market and avoid potential negative effects coming from the rise of spreads. Table 2.8 confirms that the variance of risk premia is significantly reduced in the presence of QE rules. With respect to sectoral inflation, we find that central bank intervention increases inflation, though the magnitude is very small (less than 0.02 percent annually). Thus, a trade-off appears between financial stabilization and inflation control. However, in our framework, the benefits of mitigating the impact of the carbon price shock on risk premia seem to outweigh the inflationary consequences of asset purchases.

[63] We also plot the case where $\phi_k^s = 5$ and $\phi_k^s = 0.05$. We show that when the central bank purchases about 0.27 percent of both green and brown assets annually, it is able to completely offset the rise in risk premia, while a purchase of about 0.15 percent annually reduces the impact on risk premia by about half.

2.5 Asset Purchase Program Scenario – LSAP

To shed some light on the interest of tilting central banks portfolio toward green bonds, we simulate both transitory and permanent LSAP programs run by the central bank under two macroprudential policy scenarios. In the first case, the macroprudential authority implements climate-risk weights along the transition, while in the second case risk-weights are held constant.

2.5.1 Transitory LSAP

The first scenario studied is a transitory LSAP program where the central bank gradually increases the size of its balance sheet to hold around 8 percent of either green or brown total assets by 2028. Asset purchases are then reversed and holdings return to zero in approximately two years. As LSAP programs are announced by central banks before being implemented, we rely on perfect foresight simulations.

Figure 2.13 shows the impact of both green and brown transitory LSAP programs along the transition.[64] The main result is that there is no incentive for a central bank to purchase green rather than brown bonds as part of a LSAP program, since both programs lead to the exact same results. The reason is that green and brown bonds are seen as perfectly substitutable by banks. In this case, if the central bank favors one of the sectors in its asset purchases, the effect is completely offset by the reaction of financial intermediaries. An interesting point to note is that both green and brown transitory LSAP programs allow central banks to postpone the impact of the rising carbon price on brown capital and output by loosening the constraint on banks. If the transition to a low-carbon economy were to take place in a disorderly fashion, such LSAP programs could delay the potential negative impacts the transition might have on stranded assets.

Figure 2.14 shows how a transitory LSAP program focused on green bonds would

[64] As in the previous section, the carbon price is assumed to increase to reach the EU climate goals and trend growth is assumed to be 0.8 percent annually.

interact with a sectoral macroprudential policy favoring the green sector. In this exercise, asset purchases are similar to those in the previous exercise, but the risk-weight on green loans is lowered along the transition, while the risk-weight on brown loans is gradually increased. Breaking the perfect substitution between green and brown assets allows to boost green sector capital and output compared to when macroprudential policy stays neutral over the period studied.[65] Overall, this leads to a positive effect on aggregate capital and output that disappears at the end of the simulation, as the central bank unwinds its asset purchases. Thus, a transitory green LSAP program coupled with a macroprudential policy favoring the green sector exacerbates the effect of the transition induced by the rise in the carbon price, which leads to a slightly better emission to output ratio.

2.5.2 Permanent LSAP

The second scenario studied is a permanent LSAP program where the central bank gradually increases the size of its balance sheet to hold around 8 percent of either green or brown total assets by 2028 and keeps this proportion constant from 2028 on.

Figure 2.15 displays the reaction of selected variables to both green and brown permanent LSAP programs along the transition. The results are quantitatively similar to the case of a transitory LSAP, except at the end of the simulation, where brown permanent LSAP seem to be more effective than transitory LSAP to mitigate the loss in brown capital and output associated with a decarbonization of the economy.

Figure 2.16 shows how a permanent LSAP program focused on green bonds would interact with a sectoral macroprudential policy favoring the green sector. The interaction of the two policies gives the best results in terms of accompanying the transition to a greener economy. Compared to the case where asset purchases were transitory, a permanent LSAP program yields an effect on capital, output, and emissions that is long-lasting. Overall, the emission to output ratio is lower, since green output rises sharply while brown output

[65] Similarly, Ferrari and Nispi Landi [2021] break the perfect substitutability by introducing a quadratic cost related to the holding of green bonds by banks.

decreases over the period studied. It is also important to keep in mind that results presented in this section could be further reinforced if we were to witness an increase in the share of the green sector over the transition, as exemplified in the previous section.

2.6 Conclusion

We develop a DSGE model with both endogenously-constrained financial intermediaries and heterogeneous firms. We then use the model to assess the implications of setting an environmental policy consistent with the net-zero target using a cap system.

We find that a price of about 350€ per ton of carbon is needed to be aligned with the net-zero target. However, the actual implementation of this price induces two inefficiencies. The first inefficiency is linked to the need of an increasingly higher price of carbon (compared to the optimal SCC) to meet the EU targets. This decoupling generates a growing welfare loss. To address this wedge, we show that a RWA policy favoring the green sector (i.e. green macroprudential policy) is efficient in partially offsetting the welfare loss while reaching the emissions target. Furthermore, green macroprudential would allow the regulator to address climate-related financial risk.

The second inefficiency is related to the market design of the environmental fiscal policy in the EU area. The present volatility in the ETS is shown to affect firms' marginal costs and thus to alter risk premia. We find that QE rules that react to changes in risk premia are able to completely offset movements in spread levels and volatility, allowing for a smooth transmission of monetary policy, while not significantly impacting inflation.

Turning to LSAP programs, we find that macroprudential policy is needed to provide an incentive to central banks to engage in both transitory and permanent green QE. However, permanent LSAP programs yields an effect on capital, output, and emissions that is long-lasting compared to transitory LSAP programs.

More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero.

In particular, we find that green macroprudential policy strengthen the substitution effect between the two sectors, which is triggered by the environmental fiscal policy. While this result is obtained with a constant share of the green sector ($\varkappa$), increasing $\varkappa$ along the transition reinforces our findings. Intuitively, making the green sector predominant (figure 2.2 and figure 2.3), would not only decrease substantially emissions, which in turn decreases the environmental policy cost (i.e. the carbon price), it would also help achieve the sought-after decoupling of emissions and output. The emissions to output ratio $E_Y = E/Y$ falls almost linearly with an increase in the green sector share and leads to lower level of carbon price.

Many extensions could be conducted using our framework. In particular, we think that further research could be devoted to the impact of non-linearities within the financial sector on the dynamics of the model and to the role that endogenous TFP could play in fostering the emergence of greener output growth. We also believe it could be fruitful to examine how to capture the environmental quality on the welfare of households in more direct ways than in existing models.

Appendices

2.A Appendix: Tables

TABLE 2.1
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Standard Macro Parameters		
σ	Risk aversion	2
$\varkappa$	% of Green firms in the economy	30
θ	Price elasticity	5
ξ	Price stickiness (Calvo parameter)	2/3
θ^P	Price stickiness (Rotemberg parameter)	$\frac{(\theta-1)\xi}{(1-\xi)(1-\xi\bar{\beta})}$
$\bar{L}$	Labor supply	1/3

TABLE 2.2
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Environmental Parameters		
γ_d	CO_2 natural abatement	0.0021
$\theta_{1,g}$	Abatement cost parameter for sector G	0.02
$\theta_{2,g}$	Abatement cost parameter for sector G	2.7
$\theta_{1,b}$	Abatement cost parameter for sector B	0.05
$\theta_{2,b}$	Abatement cost parameter for sector B	2.7
υ_1^o	Temperature parameter	0.5
υ_2^o	Temperature parameter	0.00125
a	Damage function parameter	1.004
b	Damage function parameter	0.02

TABLE 2.3
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Banking Parameters		
Δ	Parameter impacting the discount factor of bankers	0.99
θ_B	Probability of staying a banker	0.98
ρ_c	Smoothing monetary rule coefficient	0.8
ϕ_y	Output policy parameter	0.2
ϕ_Π	Inflation policy parameter	1.5

TABLE 2.4
Estimated Parameters

		Estimation	
Parameters		Mean	Standard Deviation
Standard Macro Parameters			
$\sigma_{A_{t,k}}$	Output shock standard deviation	0.0063361	7.2574e-06
$\rho_{A_{t,k}}$	Output shock persistence	0.76907	8.3156e-06
$\bar{g}/\bar{y}$	Public spending share in output	0.28503	1.9099e-05
η_i	Capital adjustment cost	1.7354	7.2439e-06
$1/(1+b/100)$	Discount factor	0.027254	6.4961e-06
$1+\gamma_Y/100$	Economy growth rate	0.21907	3.0773e-07
h	habits	0.22278	1.3859e-05
α	Capital intensity	0.34202	4.8802e-07
δ	Depreciation rate of capital	0.024995	1.5241e-07
θ_g	Price elasticity in sector G	11	6.1805e-06
θ_b	Price elasticity in sector B	7.0206	4.3802e-06
Environmental Parameters			
E^*	Rest of the world emissions	3.3666	3.0327e-06
φ_b	Emissions-to-output ratio in sector B	0.2849	1.5072e-06
Carbon Price	Carbon price level	0.0099078	4.5392e-06
Banking Parameters			
λ	Risk weight on loans	0.17618	5.9887e-06
ω	Proportional transfer to the entering bankers	0.006353	2.4101e-06

TABLE 2.5
Model moments compared to observed data (Euro Zone)

Target	Model	Data	Source
Macro Aggregates:			
Output Growth Volatility	0.0065	0.0066	Eurostat
Investment Growth Volatility	0.030	0.030	Eurostat
Consumption to output Growth Volatility	0.0047	0.0048	Eurostat
Mean Output Growth	0.0022	0.0023	Eurostat
Mean Investment Growth	0.0021	0.0023	Eurostat
Consumption to Output Ratio (%)	0.57	0.53	Eurostat
Government Spending to Output Ratio (%)	0.28	0.24	Eurostat
Marginal Cost of the Brown Sector (Normalized)	1	1	Chegut et al. [2019]
Marginal Cost of the Green Sector (6% higher than 'B')	1.06	1.06	Chegut et al. [2019]
Financial Aggregates:			
Risk-less Bond Mean Return (annualized)	1.07	1.08%	ECB
Green Bonds Risk Premium (annualized)	0.80%	0.80%	Fender et al. [2019]
Brown Bonds Risk Premium (annualized)	0.80%	0.80%	Fender et al. [2019]
Banks' Capital Ratio (Equity as a % of RWA)	14.39%	14.40%	ECB
Environmental Aggregates:			
Global Level of Carbon Stock (GtC)	839	839	USDA
Emissions to Output Ratio (kCO_2 per $ of output)	0.21	0.21	Global Carbon Project/FRED
ETS Price (January 2021) in €	30	30	Bloomberg

TABLE 2.6

Sensitivity of the optimal carbon price to climate damages and dynamics

	Nordhaus		Dietz		Weitzman	
	$\upsilon_2^o = 0.00125$	$\upsilon_2^o = 0.0025$	$\upsilon_2^o = 0.00125$	$\upsilon_2^o = 0.0025$	$\upsilon_2^o = 0.00125$	$\upsilon_2^o = 0.0025$
Emissions Reduction (in%)	-	15%	5%	28%	15%	45%
Social Cost of Carbon (in €)	31.2	144.12	65.94	333.53	144.12	846.65
Temperature T^o (in Celsius)	1.06	2.07	1.05	2.04	1.03	2

Notes: The figures reported in the table show the sensitivity of the optimal price of carbon, temperature, and net-zero goal of 55 percent emissions reduction by 2030, to different levels of calibration of: i) the damage function (parameter "b"), and ii) the climate dynamics (parameter "υ_2^o"). With respect to the damage function, $b = 0.01$ corresponds to Nordhaus and Moffat [2017], $b = 0.02$ corresponds to Dietz and Stern [2015], and $b = 0.04$ corresponds to Weitzman [2012]. For the climate dynamics, $\upsilon_2^o = 0.00125$ corresponds to baseline case with $T^o < 1.1C$ by 2030, and $\upsilon_2^o = 0.0025$ corresponds to case with $T^o < 2.1C$ by 2030.

TABLE 2.7
Steady state values

	Optimal Policy	ETS Policy	ETS and Macropru		
			$\lambda_g = 0.75$	$\lambda_g = 0.5$	$\lambda_g = 0.25$
			$\lambda_b = 1.25$	$\lambda_b = 1.5$	$\lambda_b = 1.75$
	(1)	(2)	(3)	(4)	(5)
Consumption	1.2419	1.2372	1.2387	1.2402	1.2418
Aggregate Output	2.1139	2.1029	2.1019	2.1013	2.1011
Green Output	1.0937	1.0937	1.1012	1.1111	1.1213
Brown Output	1.06	1.0515	1.0425	1.0337	1.0251
Emissions to Output	0.2183	0.1569	0.1569	0.1569	0.1569
Green Sector Emissions	0.1034	0.0747	0.0754	0.0760	0.0767
Brown Sector Emissions	0.2876	0.2049	0.2032	0.2014	0.1998
Green Capital Stock	11.4318	11.3383	11.6359	11.9468	12.2717
Brown Capital Stock	10.4235	10.1552	9.9001	9.6554	9.4207
Green Real Rate	1.0045	1.0045	1.004	1.0035	1.003
Brown Real Rate	1.0045	1.0045	1.005	1.0055	1.006
ETS Price (in euros)	31.2	300	303	304	306
Carbon Cost as % of GDP in Green Sector	0.3278	0.5122	0.5122	0.5122	0.5122
Carbon Cost as % of GDP in Brown Sector	0.7650	1.4580	1.4580	1.4580	1.4580

Notes: The first column is the economy subject to an optimal carbon price. The second column is the economy subject to a carbon price consistent with the EU climate goals for 2030 (i.e. ETS cap net-zero objective), and the three last columns feature both a carbon price consistent with the EU climate goals for 2030 and an intervention of the macroprudential authority. We show how the economy responds to different risk-weight requirements related to climate risk exposure of firms. For instance the baseline scenario presents the case where an upgrade in the rating of the green bonds of the asset class BBB+ to A- and the downgrade in the rating of the brown bonds of the asset class BBB+ to BBB- (i.e. $\lambda_g = 0.75$ and $\lambda_b = 1.25$). The two other cases: i) with $\lambda_g = 0.5$ and $\lambda_b = 1.5$, and ii) with $\lambda_g = 0.25$ and $\lambda_b = 1.75$, represent a higher cut in the risk-weight associated with climate risk exposure (i.e. a higher upgrade and downgrade in the ratings).

TABLE 2.8

Risk premia volatility under the carbon price shock

	Baseline Model		**Model with QE Rules (ϕ_k^s=5)**	
	Mean	Standard Deviation	Mean	Standard Deviation
EP_g	0.1989	0.02	0.1989	0.0003
EP_b	0.1989	0.02	0.1989	0.0003
MC_g	0.9091	0.0001	0.9091	0.0003
MC_b	0.8571	0.0001	0.8571	0.0003
Q_g	1.0000	0.0002	1.0000	0.0001
Q_b	1.0000	0.0002	1.0000	0.0001
π_g	1.0000	0.0000	1.0000	0.0001
π_b	1.0000	0.0000	1.0000	0.0001

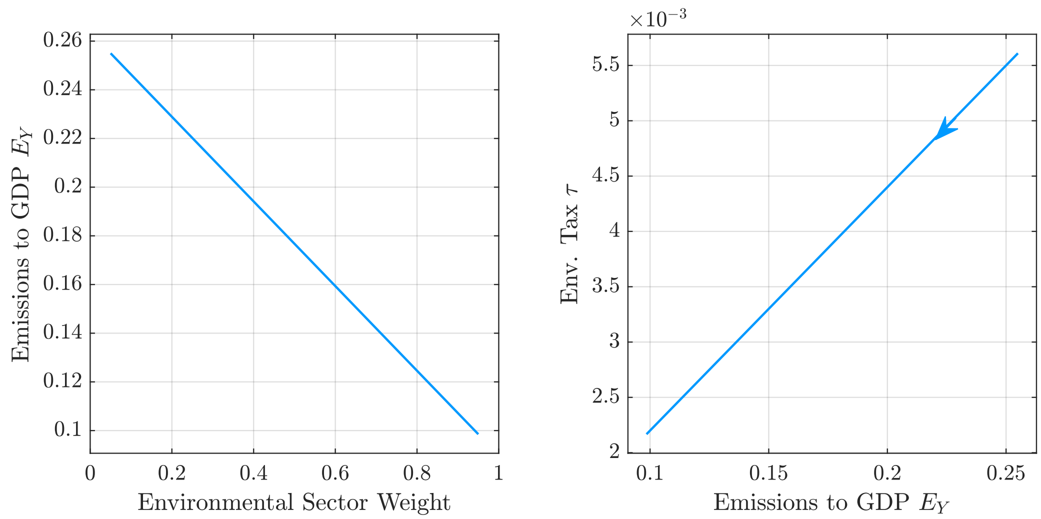
Notes: The figures reported in the table show the first and second moments of selected variables following a positive carbon price shock. The baseline model refers to the model with the ETS carbon price. The model with QE rules incorporates a reaction of the central bank to deviations in risk premia from their respective steady state.

2.B Appendix: Figures

FIGURE 2.1. ETS Price in Euros per Ton of CO_2

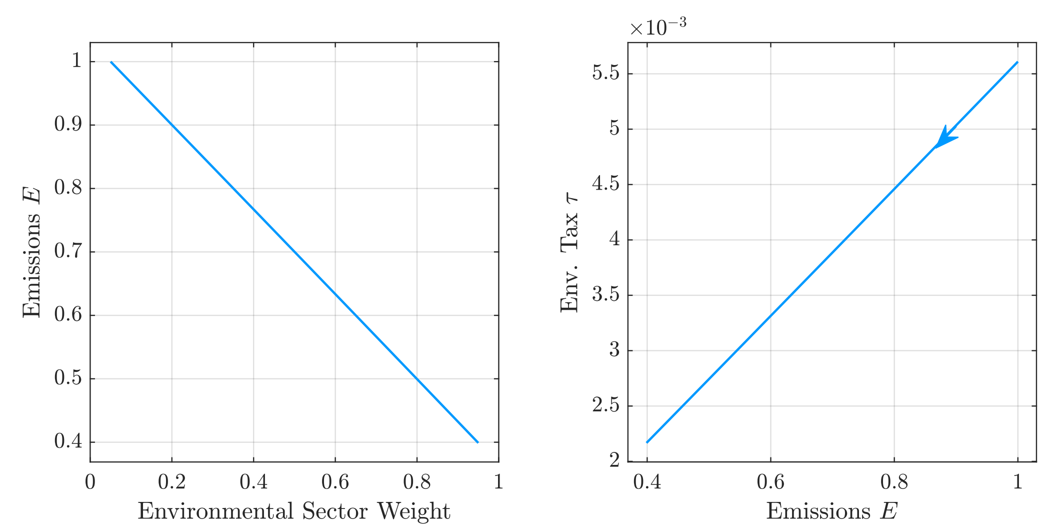
Notes: The figure displays the spot price of carbon permits traded within the ETS in euros per ton of CO_2. (Source: Bloomberg)

FIGURE 2.2. Share of the green sector, carbon intensity, and the environmental policy



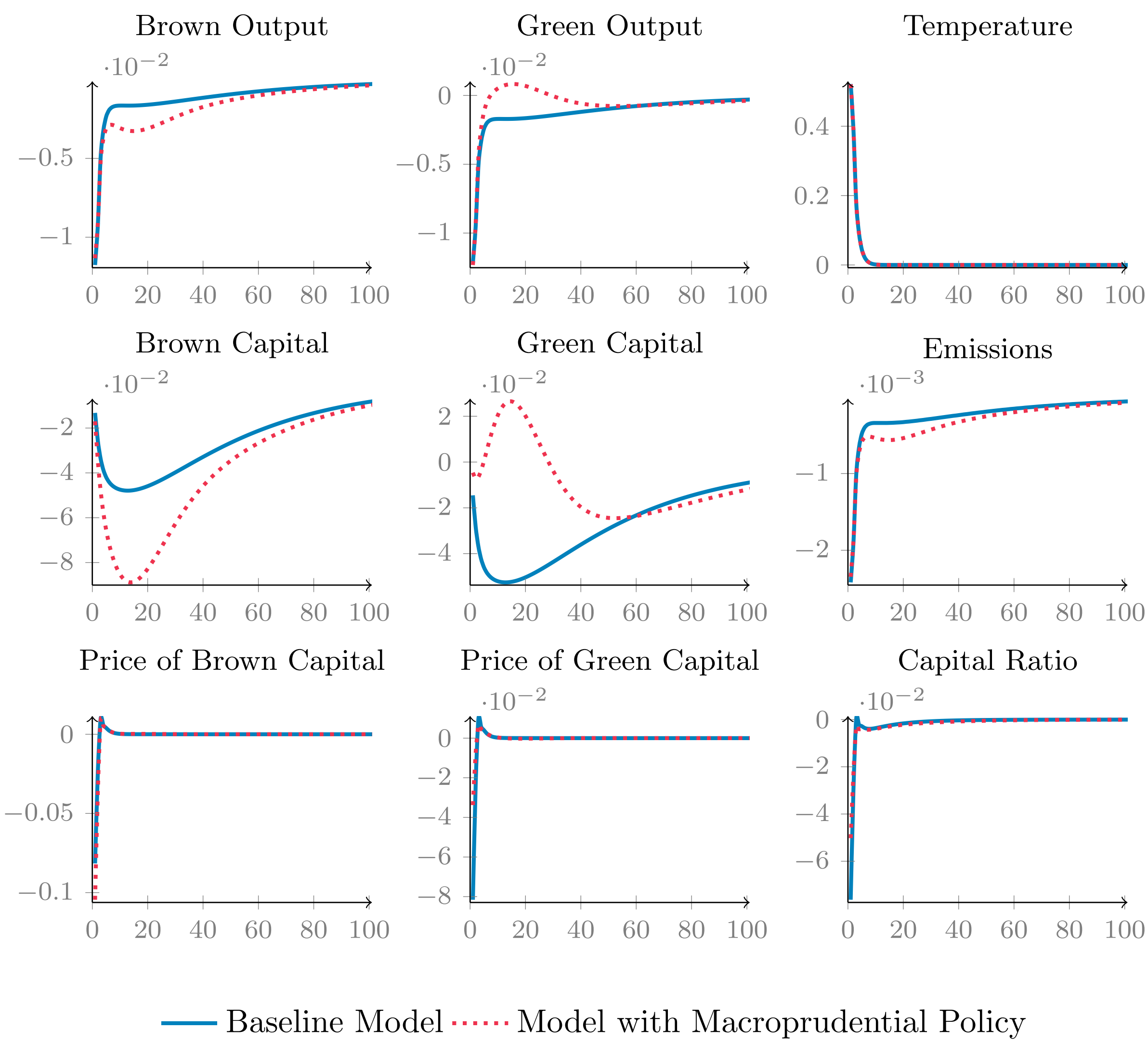
Notes: The graph on the left reports the interaction between emissions to output and the size of the green sector. The right graph reports how a change in the weight of the green sector drives the carbon price, through a decrease in the emissions to output ratio.

FIGURE 2.3. Share of the green sector, emission levels (normalized to one), and the environmental policy



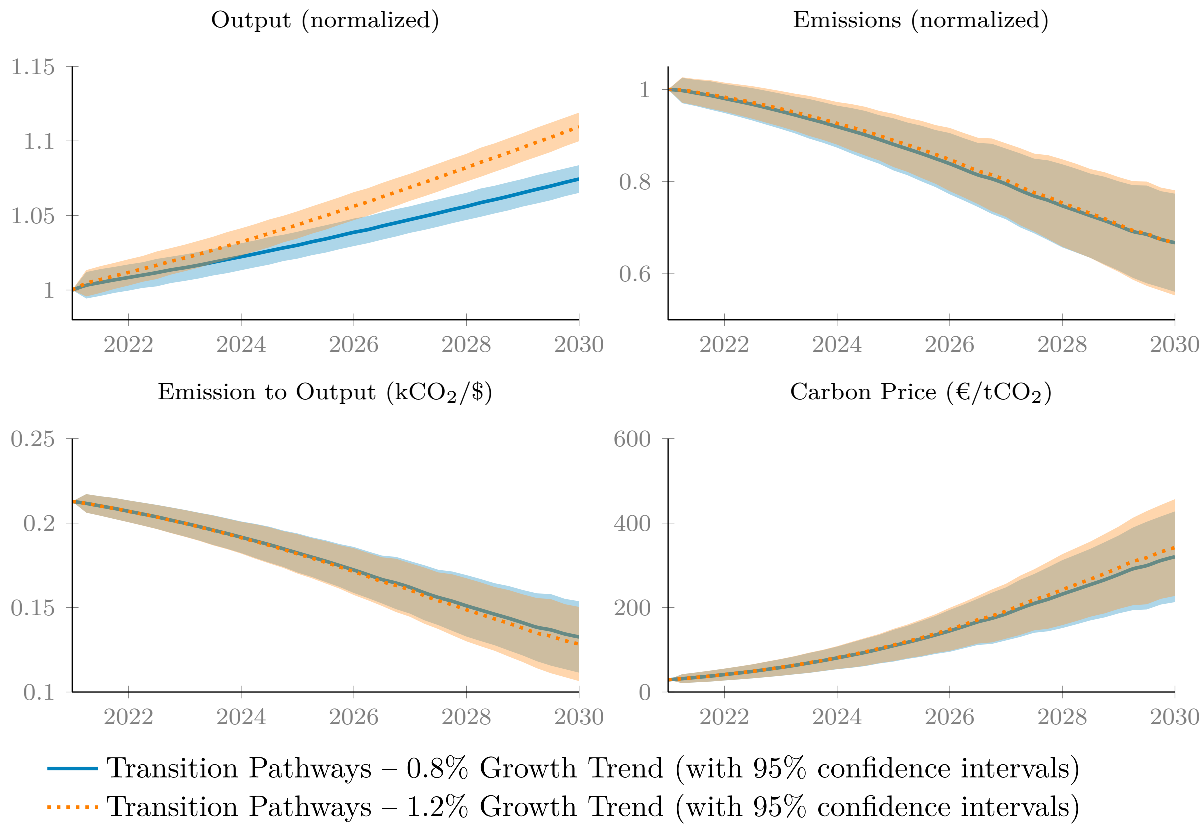
Notes: The graph on the left reports the interaction between emissions and the share of the green sector. The right graph reports how the share of the green sector shapes the carbon price.

FIGURE 2.4. Financial stability and climate risk



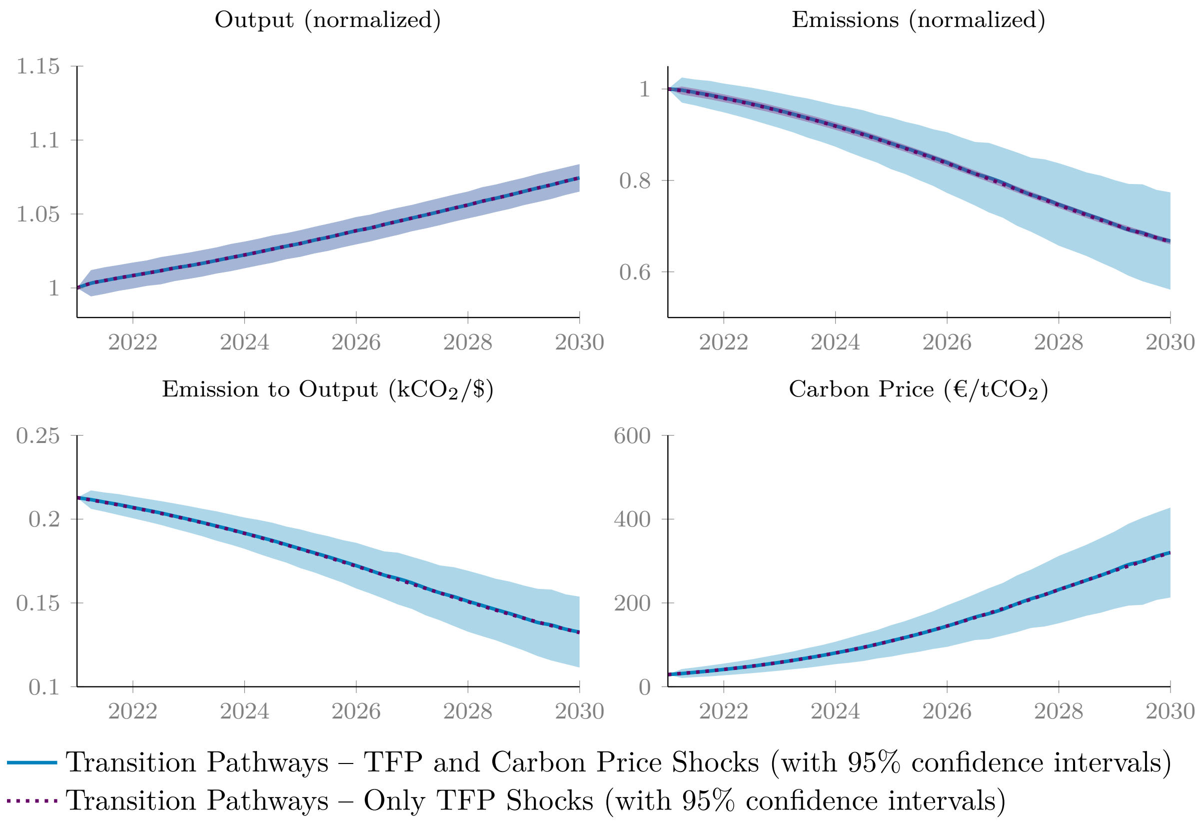
Notes: The figure shows the effect of a 0.5°C increase in the level of temperature, with and without macroprudential policy. In the baseline scenario, there is no sectoral macroprudential policy, which means $\lambda_b = \lambda_g = 1$. To illustrate the impact of green macroprudential policy on climate-related financial risk, we multiply/divide climate risk weights by a factor of 2, which means $\lambda_b = 2$ and $\lambda_g = 0.5$. Green macroprudential policy reduces the impact of a temperature increase on the global capital ratio by providing an incentive to banks to hold more green assets. The results are presented as percentage deviations from the steady state over quarterly periods.

FIGURE 2.5. Net-zero transition pathways with two different growth assumptions



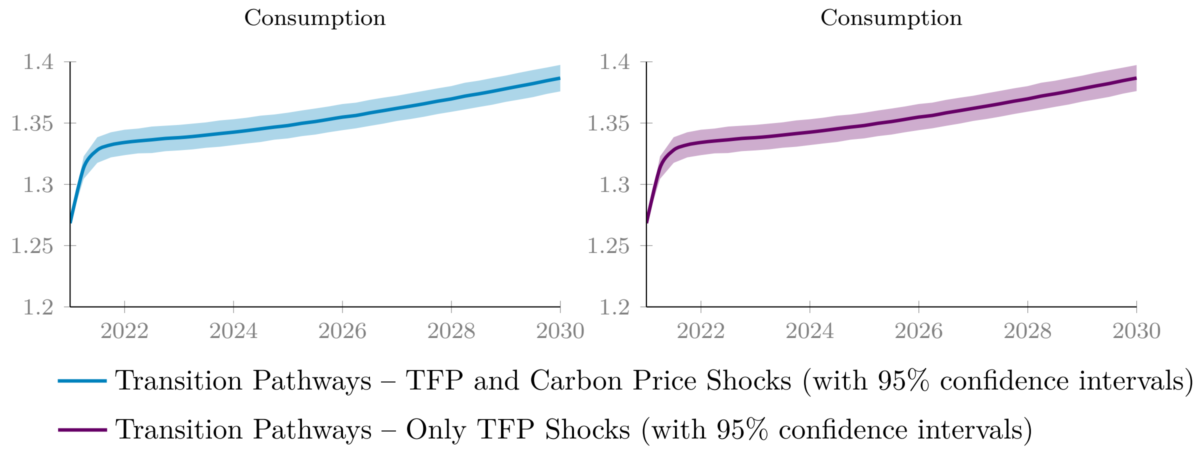
Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to two different growth scenarios. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%), while the orange dotted line corresponds to a more optimistic scenario in line with long term EZ trends (1.2%). The shaded blue and orange areas correspond to 95 percent confidence intervals for each scenario.

FIGURE 2.6. Net-zero transition pathways with and without carbon price uncertainty



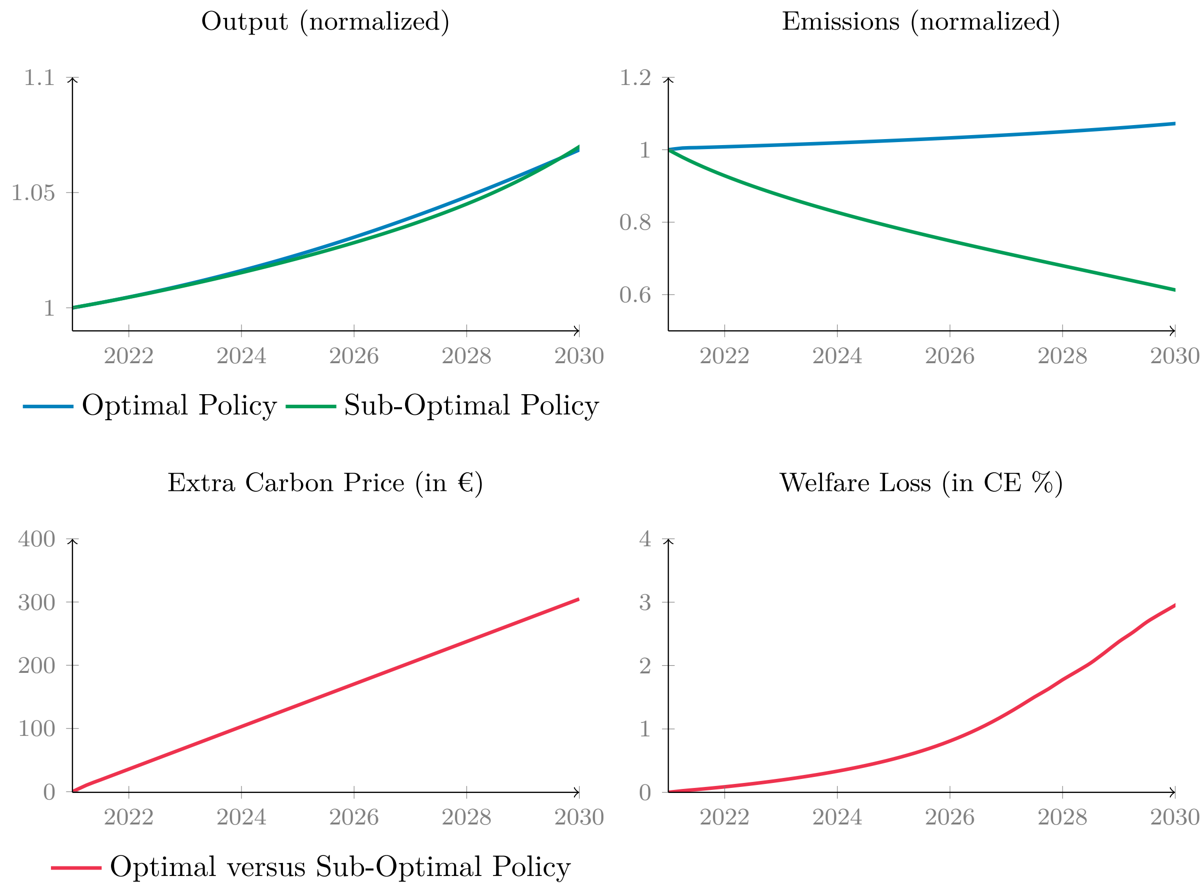
Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to the 0.8% growth scenario, where the carbon price is subject to carbon price volatility (i.e. carbon price shocks) and where the carbon price is not subject to carbon price volatility. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%) where the carbon price is subject to uncertainty, while the purple dotted line corresponds to the case where the carbon price is not subject to uncertainty. The shaded blue and purple areas correspond to the 95 percent confidence intervals for each scenario. Please note that for both scenarios output is subject to TFP shocks consistent with the past 20 years in the EZ.

FIGURE 2.7. Consumption pathways and carbon price uncertainty



Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to the 0.8% growth scenario, where in one case the economy features carbon price volatility (i.e. carbon price shocks) and where in the other case the price of carbon is not subject to carbon price volatility. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%) where the carbon price is subject to uncertainty, while the purple line corresponds to the case where carbon price is not subject to uncertainty. The shaded blue and purple areas correspond to the 95 percent confidence intervals for each scenario. Please note that for both scenarios output is subject to TFP shocks consistent with the past 20 years in the EZ.

FIGURE 2.8. Transition pathways: optimal versus net-zero

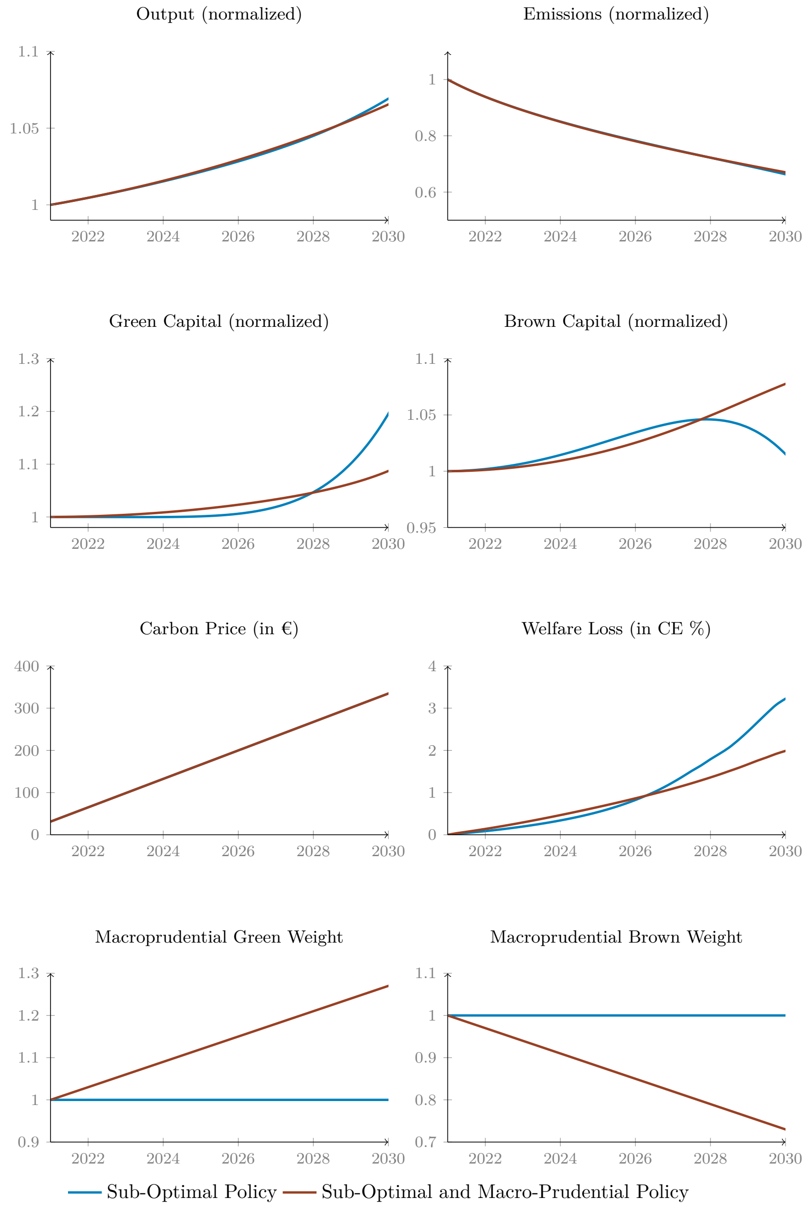


Notes: The figure compares the pathway consistent with the optimal carbon price (the social cost of carbon) to the net-zero ETS cap policy pathway. The blue line corresponds to the social planner choice, while the green dotted line corresponds to a pathway consistent with a reduction of emissions of 33 percent by 2030 (55 percent compared to 1990 level). The red lines show both the difference in carbon price and the welfare loss, between the optimal and sub-optimal policy (ETS inherent price). More specifically, the red graph on the left shows the trajectory of the extra carbon price, which is the carbon price consistent with the net-zero ETS cap policy minus the optimal price of the social planner. The graph on the right shows the welfare loss in consumption equivalent (CE), which is the difference between the welfare implied by the pathway of the social planner and the welfare implied by the pathway consistent with the net-zero objective.

FIGURE 2.9. Transition pathways (net-zero) with and without green macroprudential policy

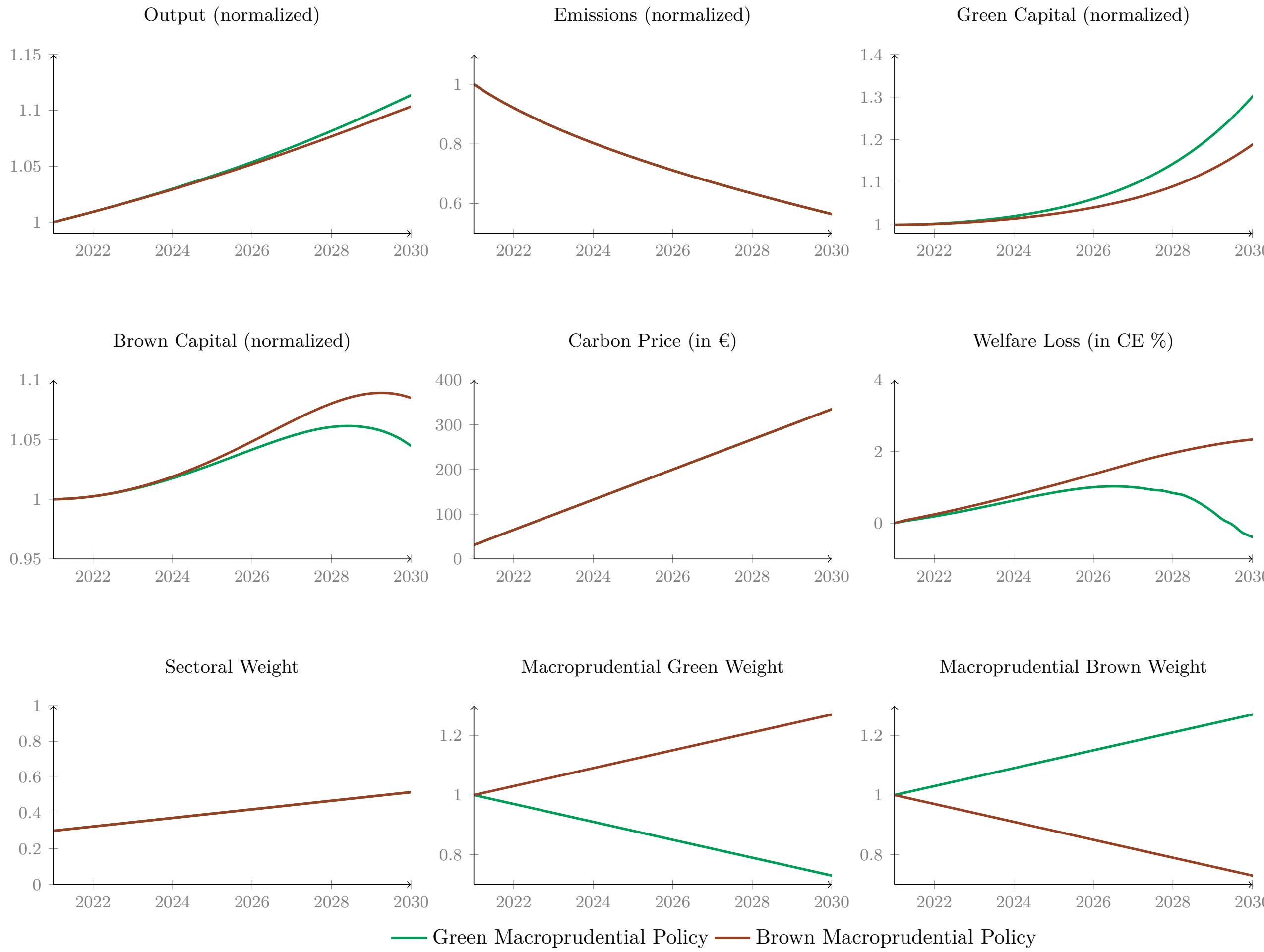
<u>Notes:</u> The figure compares a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.75$ and $\lambda_b \rightarrow 1.25$) .

FIGURE 2.10. Transition pathways (net-zero) with and without brown macroprudential policy



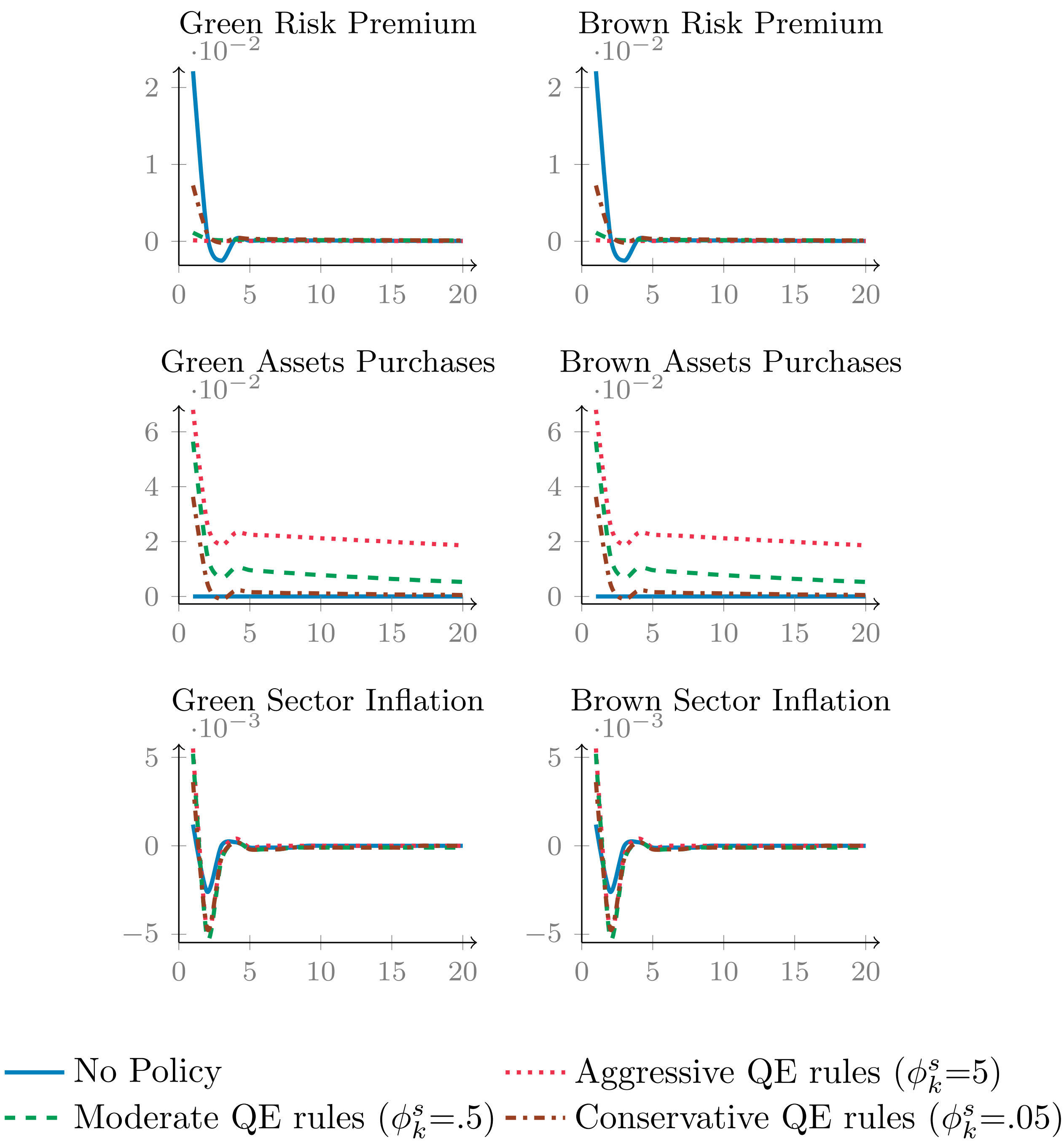
Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy favors the brown sector over the green and where it stays neutral. The blue line corresponds to the neutral case ($\lambda_g = 1$ and $\lambda_b = 1$) and the brown line corresponds to the case where the macroprudential authority favors the brown sector ($\lambda_g \to 1.25$ and $\lambda_b \to 0.75$).

FIGURE 2.11. Transition pathways (net-zero) with macroprudential policy and an increase in the green sector share



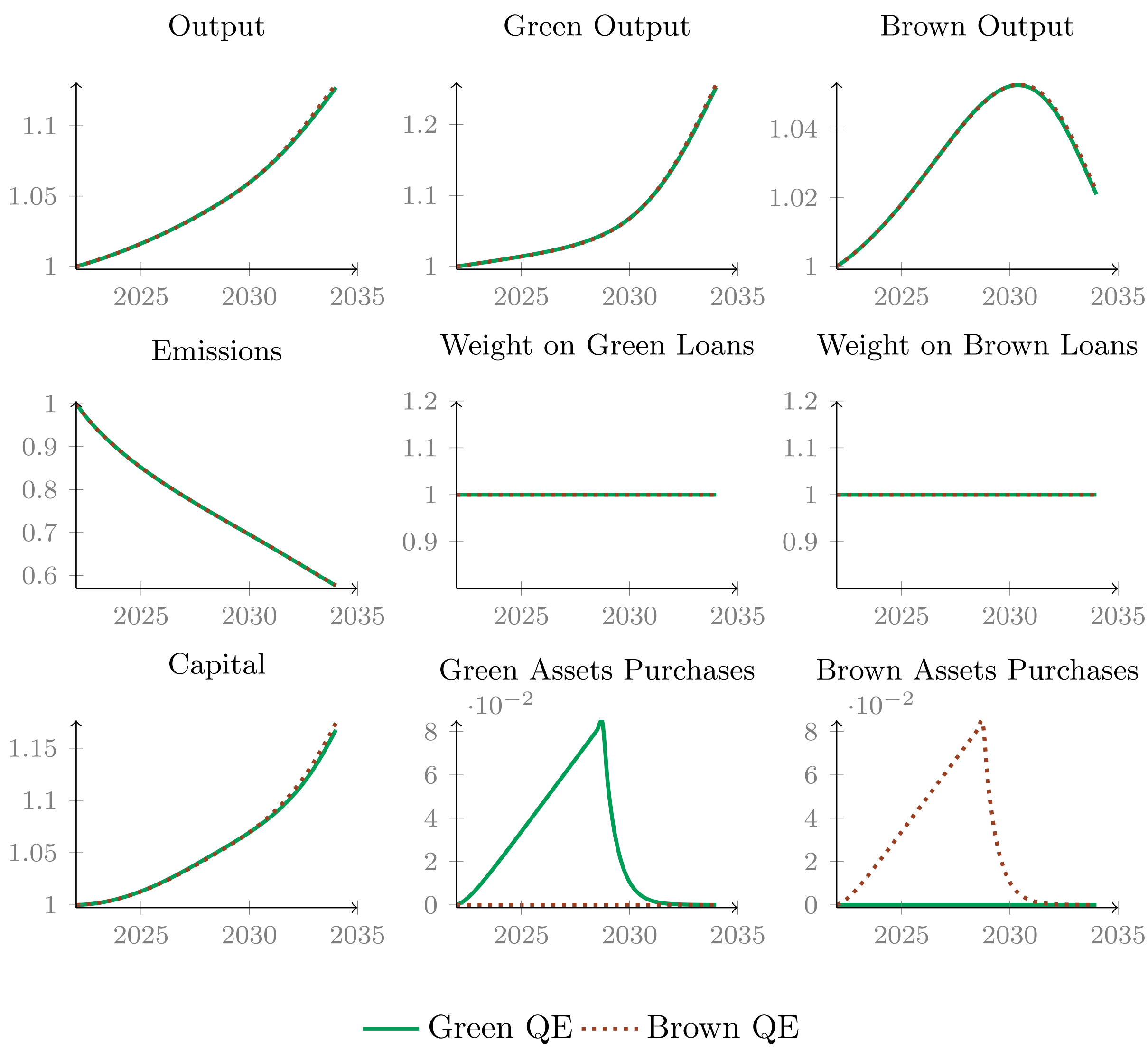
<u>Notes:</u> The figure compares a pathway consistent with the net-zero objective where the share of the green sector increases overtime ($\varkappa \rightarrow 50\%$) and where a macroprudential policy: i) takes into account climate risk, and ii) favors the brown sector over the green. The brown line corresponds to the case where the brown sector is favored over the green ($\lambda_g = 1.25$ and $\lambda_b = 0.75$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.75$ and $\lambda_b \rightarrow 1.25$) .

FIGURE 2.12. Responses to a positive carbon price shock (ε_t^τ). (The Rotemberg Case)



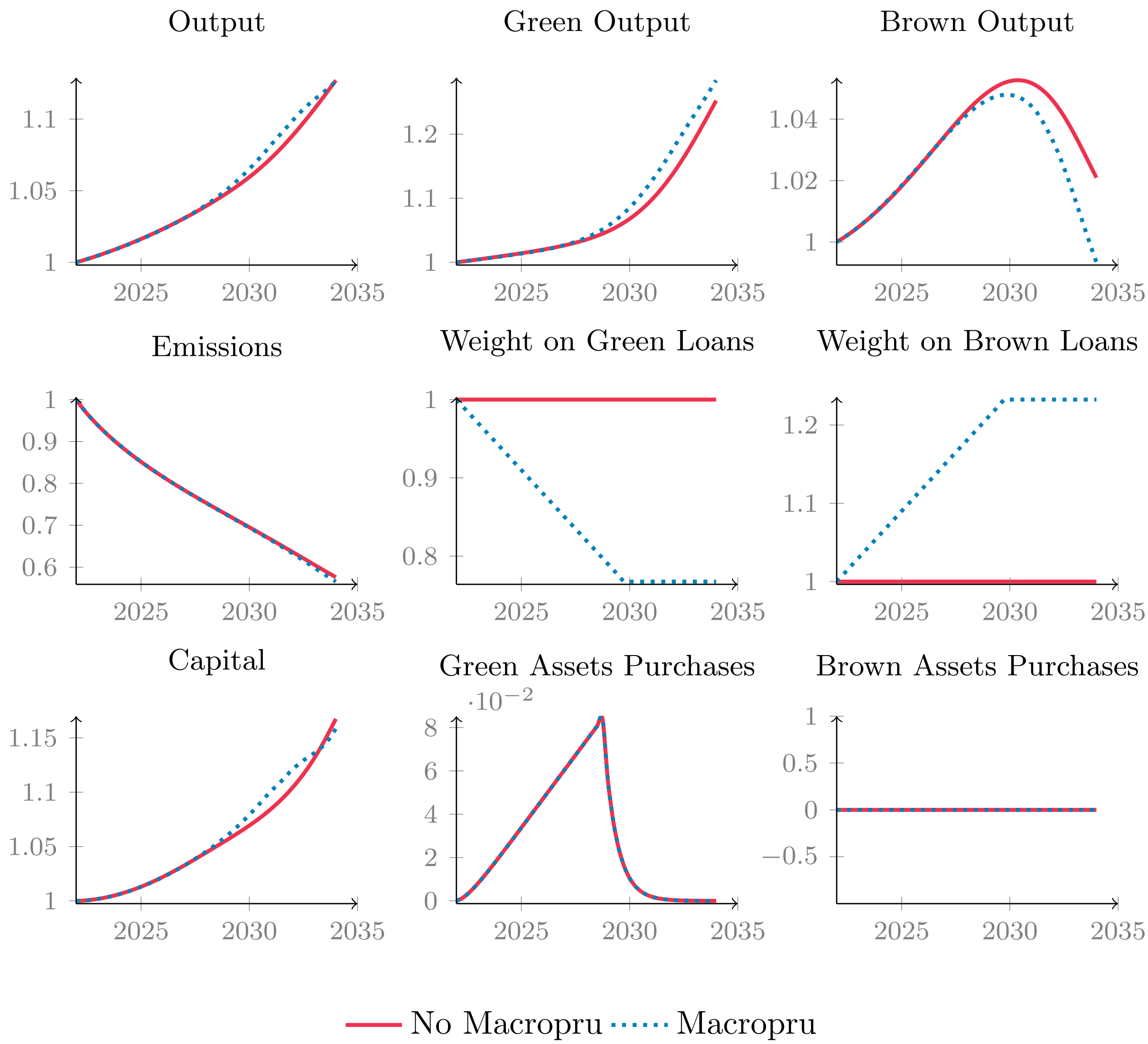
Notes: The figure shows the effect of a positive carbon price shock (ε_t^τ) calibrated on the ETS data on selected variables, with and without QE policy rules. The results are presented as percentage deviations from the steady state over quarterly periods.

FIGURE 2.13. Effect of transitory green and brown asset purchase programs



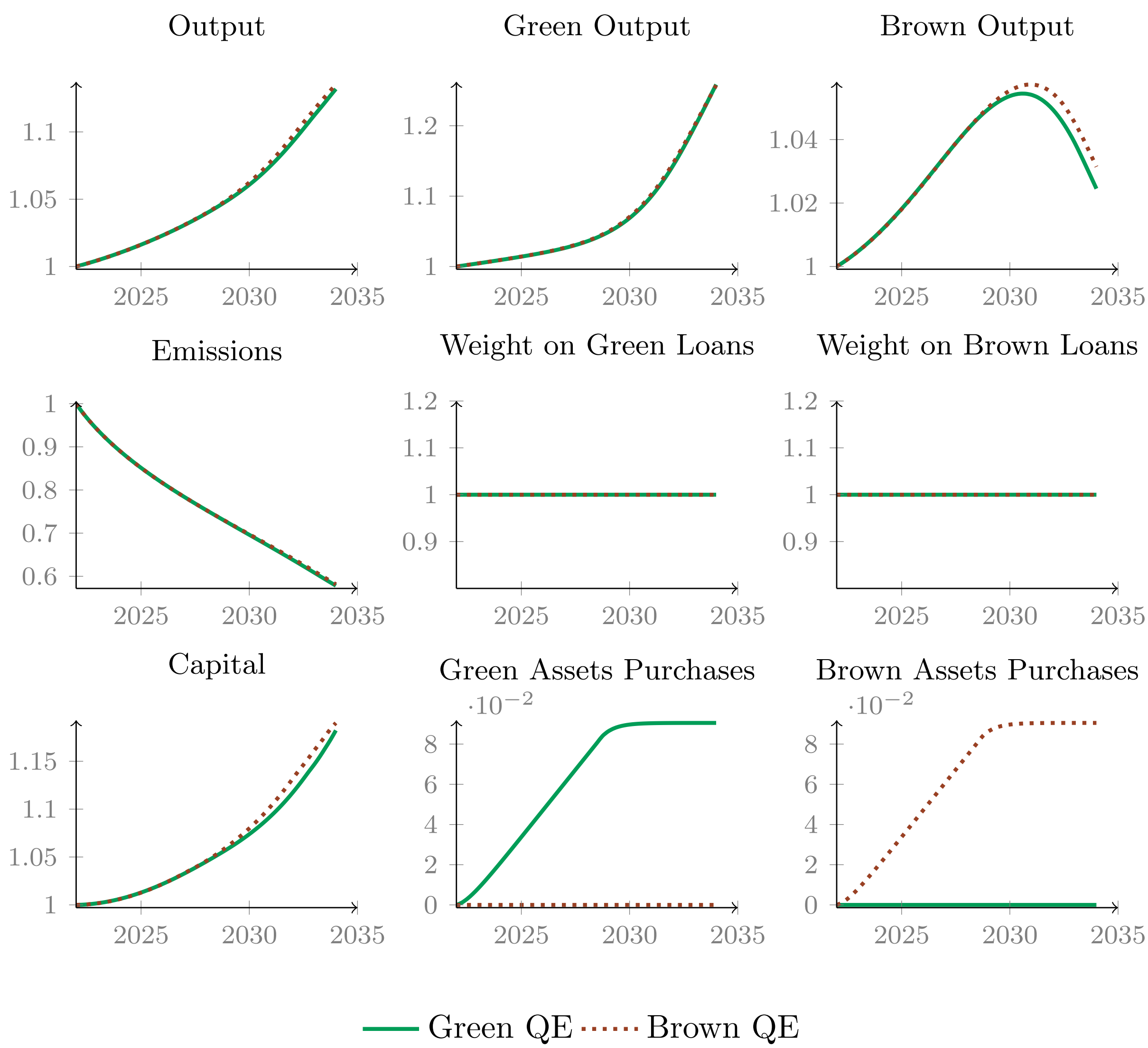
Notes: The figure shows the effect of transitory green and brown asset purchase programs (of about 9% of total asset in the economy) on a selection of variables, where the central bank stops purchasing bonds by 2028.

FIGURE 2.14. Effect of a transitory green asset purchase program with and without green macroprudential policy



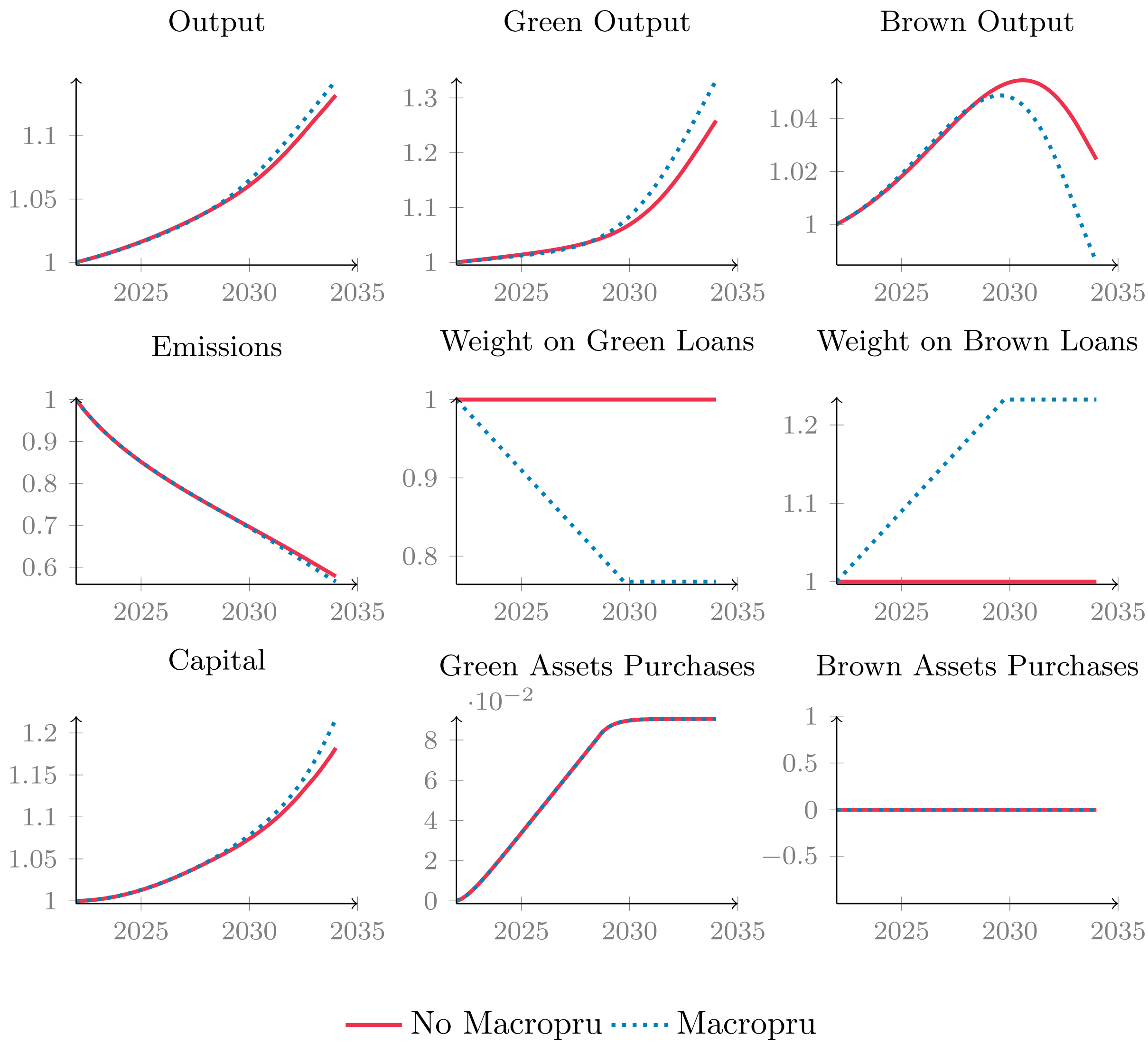
Notes: The figure shows the effect of transitory green asset purchase program (of about 9% of total asset in the economy) on a selection of variables, where the central bank stops purchasing bonds by 2028. In blue, the macroprudential authority sets a green macroprudential policy as presented in the previous section, while in red, it remains neutral.

FIGURE 2.15. Effect of permanent green and brown asset purchase programs



Notes: The figure shows the effect of permanent (where the central bank keeps the share of asset constant at about 9% of total assets in the economy) green and brown asset purchase programs on a selection of variables.

FIGURE 2.16. Effect of a permanent green asset purchase program with and without green macroprudential policy



Notes: The figure shows the effect of a permanent (where the central bank keeps the share of asset constant at about 9% of total assets in the economy) green asset purchase program on a selection of variables. In blue, the macroprudential authority sets a green macroprudential policy as presented in the previous section, while in red, it remains neutral.

Chapter 3

Endogenous Abatement Technology

This chapter will be presented in the poster session of the Annual AEA Meeting. It is a joint work with Ghassane Benmir (LSE).

3.1 Introduction

In recent years, monetary policy makers have become increasingly concerned by the challenges posed by climate change. As a step toward more actions, the European Central Bank (ECB) decided, after an 18-months review of its monetary policy strategy, to monitor more closely climate risk and the consequences it could have on financial stability and monetary policy transmission. For the time being, however, fiscal policy has been the main instrument to mitigate present and future damages from climate change.

While carbon pricing is the major tool used in climate mitigation policies nowadays, this policy is not a free lunch as it induces unintended effects. In Europe, Canada, and China, as well as elsewhere, governments have opted for a market cap and trade system instead of a targeted price to facilitate the attainment of desired emissions level reductions. As this market design is not optimal from a welfare perspective and is subject to market volatility and business cycle fluctuations, a number of inefficiencies arise (e.g. welfare losses and risk premium distortions as highlighted in Benmir and Roman [2020]). In order to address the inefficiencies induced by such a carbon market design, finding ways to steer green innovation without solely relying on increasing carbon pricing becomes a major priority.

The goal of this paper is twofold. First, we seek to empirically investigate the different linkages between carbon emissions, fiscal carbon policies, and green innovation. Second, we want to shed light on how fiscal and financial policies could help steer some of the main drivers that contribute to the next zero carbon emissions transition. To do so, we build a quantitative model to address the evidence and provide a framework that allows for analyzing the role of various green innovation policies in the transition to a low carbon economy.

With respect to the first goal, we rely on empirical data on the Eurozone (EZ), the US, and a panel of the 19 EZ countries. We find that a fraction of emissions reduction is accounted for by carbon pricing policies (e.g. the European Trading System (ETS)), and show that carbon pricing might not always steer green innovation, which in turn is a major contributor to emissions reduction. Furthermore, macro-financial factors (e.g. long-term loans) are found to play a significant, positive role in boosting green innovation.

Regarding the second goal of the paper, the model introduces two modifications to the standard real business cycle economy: i) it explicitly accounts for the process of endogenous green innovation that lowers the cost of abatement; ii) it includes an agency friction in financial markets that may disrupt the financing of investments in innovation à la Queralto [2020]. Endogenous green innovation financed by the banking sector allow for substantial emissions reduction by triggering higher levels of abatement, without having to rely on increasingly higher levels of carbon pricing.

In the spirit of Romer [1990], Acemoglu et al. [2012], and Anzoategui et al. [2019], we introduce sustained growth in green R&D arising from an endogenously expanding variety of green technologies. Green entrepreneurs invest in projects that could lead to an improvement of the green technology, but lacks the funds to finance the necessary expenditures. When it is successful, the green technology allows firms to abate at a cheaper cost, which in turn lower emissions. To obtain funds, our green firms borrow from banks. The outcome from green innovation efforts consists of novel varieties of abatement technologies, which are then used by firms.

The main quantitative application of our model is to explore the EZ net-zero transition pathways, as well as business cycle fluctuations, under the presence of green innovation boosting policies (i.e. fiscal, monetary, and macroprudential). Three main reasons justify the focus on the EZ. First, the ETS carbon pricing market is the most advanced environmental fiscal policy in the world. Second, the European Union (EU) global strategy in emissions reduction is moving toward finding ways in which green innovation could be steered more efficiently. Finally, the availability of data allows for running both empirical exercises and counterfactual scenarios.

Using a Real Business Cycle (RBC) framework as a foundation, the present paper builds on Heutel [2012], Fischer and Springborn [2011], and Golosov et al. [2014], among others, to account for the effect of the environmental externality on the economy, while also following Gertler and Karadi [2011] to model financial intermediaries. The novelty of the model is that we introduce green innovators in the spirit of Romer [1990], Comin and Gertler [2006], and Acemoglu et al. [2012]. The main divergences of our paper with this literature are that: i) endogenous growth in green R&D directly impacts the abatement technology by making it cheaper, thus triggering higher abatement levels, ii) green innovators need to obtain funds from financial intermediaries to set up projects as in Anzoategui et al. [2019] and Queralto [2020], and iii) we estimate the model trends and endogenous growth structural parameters using data on global and green patents.

The paper is divided into three main sections: i) an empirical analysis on the linkages between carbon pricing, green R&D, and macro-financial factors; ii) a transition pathway analysis using a reduced form model; and iii) an analysis of output and green innovation trends as well as net-zero pathways, using a full fledged estimated model with both financial intermediaries and an endogenously-determined abatement technology.

3.2 Motivational Evidence: Emission, Carbon Pricing, and Green Innovation

3.2.1 Data

Data used[66] in this section were obtained from the ECB Statistical Data Warehouse, Eurostat database, the University of Oxford ourworldindata.org database, FRED database, OECD database, European Patent Office (EPO) database, and the European Environment Agency.[67] The data set includes series from all 19 EZ countries, the EZ aggregate, as well as the US, with data spanning from the first quarter (Q1) of year 2000 to the last quarter

[66] All data were either extracted directly on a quarterly basis or transformed from a monthly frequency to a quarterly frequency.

[67] For a detailed list of data used and treatment, please refer to the appendix, section 3.A.1.

(Q4) of year 2019.

Table 3.1 presents the descriptive statistics for the data set we use in our first analysis (i.e. the difference-in-difference between the EZ and the US). First, we ensure that all macro data are end of the date quarterly, and in millions of currency. We transform the emissions and population data to per million. After operating this harmonization, we compute the deflated growth rate for all data. Finally, we add 4 and 8 lags[68] to the green patents, as this represents the time for the green innovation to be adopted by firms.

Table 3.2 presents the descriptive statistics for the data set we use in our second analysis (i.e. the Panel OLS on the EZ 19 countries). We use the same macro variables, however, this time we focus on the 19 EZ countries. We also add green patents data, the ETS price data, and the long-term loans granted by the financial sector to domestic non-financial corporations. As in the first case, we add lags (4, 8, and 12) to the ETS carbon price and to the long-term loans, as this represents the time for both fiscal policies and fund availability to impact green innovation.

3.2.2 Carbon Pricing and Emission Reduction: EZ–US Difference-in-Difference Analysis

The empirical evidence on the role of fiscal carbon policies on emissions reduction is found to be significantly different depending on the market structure and design of the fiscal policy. As highlighted by Sumner et al. [2011], Meckling et al. [2017], Haites [2018], and Best et al. [2020], it is challenging to disentangle the effects of carbon pricing from those of other climate and energy policies (Somanathan et al. [2014] , Narassimhan et al. [2018]). Yet, to date, there isn't a clear consensus on the effectiveness of carbon pricing, where, on one hand, case studies in North America (both British Colombia and California) show that carbon pricing had a significant impact on emissions reduction (Murray and Rivers [2015] and Martin and Saikawa [2017]), while on the other hand, Bel and Joseph [2015]

[68]Where 4 lags is 1 year and 8 lags are 2 years.

as well as Haites [2018], when looking at the EU, don't find that ETS carbon pricing has contributed as much as it did in the US in terms of emissions reduction.

The emission carbon pricing is one difference among many between the socio-economic policies of the EZ and the US. The two major economic areas are among the three biggest contributors to the world CO_2 emissions. Although both pledged to significantly reduce their emissions levels, the carbon policies and market design of the two economic areas are significantly different. First, we conduct an empirical analysis to assess the efficiency of the ETS carbon market. To do so, we compare the situation between the EZ and the US using a difference-in-difference technique. We focus on the third phase of the ETS (2013–2020), as this phase saw the introduction of new rules governing the free allocations of emissions allowances given to energy-intensive industries.

The nature of our data set and research question—which explores the impacts of a public policy (in this case the introduction of ETS carbon policy) on emissions reduction—suggests a comparison between the pre and post policy implementation of phase three in order to assess the effectiveness of the policy. Thus, if a control could be found that would allow us to capture other policies that could also affect emissions reduction that are not directly related to the policy we are analyzing, then difference-in-difference would be an accurate method. Our first choice was the US, as there is no major carbon policy system in place and comparable socio-economic, demographic, and technological advancements attributes. Looking at the EZ and the US, we first check the socio-economic and demographic data summarized in table 3.1. It shows that both economic areas are highly similar for the selected attributes. Then, we test for the trends on emissions for both areas in order to assess the assumption of parallel trends before the policy (ETS 3rd phase (2013)) and to determine if the difference in the trends after the policy holds. Figure 3.1 displays clearly the validity of a Diff-in-Diff approach for our control variable and treatment.

To estimate the impact of the ETS price on emissions reduction, we use a regression model where we compare the average changes in emissions between two economic areas. Furthermore, we use the Newley-West estimator for robust standard errors to avoid the auto-correlation stemming from the spline we operated on the emissions when transforming the frequency of the data to quarterly:

$$ln(E_i) = \alpha + \beta_1 Policy_i + \beta_2 Treatment_i + \beta_3 (Treatment_i \times Policy_i) + \sum_i \beta_i X_i + error_i \tag{3.1}$$

As shown in table 3.3, we first find that the carbon ETS played a significant role in emissions reduction in the EZ as compared to the US. The results are also quite consistent when adding, changing, and/or substituting controls. We find that the coefficient of interest (the diff-in-diff estimator) falls between -.07 and -.19, thus suggesting that the ETS contributed to between 7 to 19 percent of emissions reduction in the EZ.

We also, confirm that green innovations achieved through an increase in green patents contribute to decreasing emissions levels. The results are also significantly consistent whether we consider a 1-year lag or a 2-year lag for green patents to materialize.

However, we don't find any significant impacts of oil prices between the two areas, nor do we conclude on a significant role of government spending or investment on emissions reduction.

We find that the trade balance for goods plays a significant role in reducing emissions, thus suggesting that the ETS carbon pricing didn't have any significant leakage outcomes during the studied period. As for services, we cannot conclude that they play a significant role in emissions reduction, nor in emissions increase. These findings are in line with Dechezleprêtre et al. [2019] where they find no evidence that the EU ETS has led to carbon leakages. It is also supported by Venmans et al. [2020] who show that carbon pricing didn't have linkages that impacted trade.

3.2.3 Green Innovation: EZ Panel OLS Analysis

Turning now to the assessment of the impacts of both fiscal and macro-financial variables on green innovation (i.e. green patents), we use a pool of panel data from the 19 EZ countries. The focus of our analysis is on the fiscal (ETS carbon pricing) and financial (long-term credits to non-financial firms) impacts on green innovation. Unfortunately, due to scarcity of data on green subsidies for the EZ, we are unable to clearly show the impact of such policies on green patenting. However, different studies (e.g. Bai et al. [2019]) show the positive and significant impact of such fiscal tools in facilitating green innovation.

Previous papers (such as Acemoglu et al. [2012] and Aghion et al. [2016]) use panel data to assess the impact of carbon policies (via subsidies or taxes) on fuel prices and green innovation. Acemoglu et al. [2019] relies on diff-in-diff between the US and the EU to assess shell gas discovery and its impact on patents and green innovation. However, these studies do not capture the impact of macro-financial variables on R&D. The originality of our approach is to investigate both fiscal and macro-financial drivers of green innovation.

Understanding the role macro-financial variables could play in steering green innovation is instrumental in designing macro-financial policies aiming to foster investment in green technologies. In this second part of our empirical assessment, we conduct a panel regression analysis to investigate the role long-term loans play in boosting green patents. We start our analysis from Q1 of 2008 to Q4 of 2019 in order to have a balanced panel sample for all the EZ 19 countries, as data on the ETS carbon pricing are only available from 2008. Then, we regress series of green patents for each of the EZ countries on both the ETS prices and long-term loans, as well as on a number of macro controls, and time and country fixed effects.

$$GreenPatent_{i,t} = \beta_1 ETS_{i,t} + \beta_2 FI_{i,t} + \sum_i \beta_i X_{i,t} + T_t + State_i + error_{i,t} \tag{3.2}$$

Results displayed in table 3.4 suggest both a significant and positive role of the ETS price system as well as the long-term bank lending in boosting green innovation. The results are consistently significant as we run robustness checks with different timing lags for both the ETS carbon price and long-term loans.

Output is found to play an important role, suggesting that the stronger the economic growth, the higher the levels of green innovation. This is inline with the finding of Song et al. [2015], where green innovation benefits from the positive spillovers of economic growth.

Table 3.5 shows that, although carbon pricing is found to have played a significant role in steering green innovation over the 11-year period in the EZ, it might have negative effects on green innovation above a certain threshold (for prices higher than 15 euros). This result is confirmed with higher pricing cutoffs (i.e. prices higher than 20 euros and 25 euros). The robustness checks in table 3.6 and table 3.7 confirm that the above results remain largely significant and unchanged when considering different lags for both carbon pricing and long-term loans to non-financial firms.

3.3 General Framework

In this section we present a standard endogenous growth model enhanced with an environmental externality à la Heutel [2012] and the possibility of emission abatement for firms. We assume that this abatement technology can be improved exogenously. The goal is to check whether the model is able to replicate the empirical finding presented above, and perform a forecast simulation for the EZ. In the next section, we will show how it is possible to endogenize the cost and efficiency of the abatement technology.

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \ldots \infty)$. The production sector produces two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms. Public authorities decide on the fiscal and environmental policy.

3.3.1 The Household

The household maximization problem reads:

$$\max_{\{C_t,I_t,K_{t+1},L_t,B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right], \tag{3.3}$$

s.t.

$$C_t + B_{t+1} + I_t + f(K_t, I_t) = W_t L_t + Ws_{t,s} \bar{Ls}^s + T_t + R_t B_t + R_t^K K_t \tag{3.4}$$

$$K_{t+1} = (1-\delta) K_t + I_t \tag{3.5}$$

where $\beta \in (0,1)$ is the discount factor, parameters σ, $\varphi > 0$ shape the utility function of the representative household associated with risk consumption C_t, and labor L_t. The consumption index C_t is subject to external habits with degree $h \in [0;1)$ while $\chi > 0$ is a shift parameter allowing us to pin down the steady state amount of hours worked. Labor supply L_t is remunerated at real wage W_t. As we assume that government bonds are one period bonds, $R_t B_t$ is interest received on bonds held and B_{t+1} is bonds acquired. Households also choose the level of investment I_t and lend capital K_t at a return rate R_t^K. Adjustment costs $f(K_t, I_t) = \frac{\gamma_I}{2}(\frac{I_t}{K_t} - \delta)^2 I_t$ allow for capital building time, as in Christiano et al. [2005]. $\bar{Ls}^s$ is the inelastic labor supply to the R&D sector remunerated at real wage $Ws_{t,s}$. Note that firms do not reverse profits back to households. These profits will instead be revenues for entrepreneurs, as shown in the next section.

The first order conditions read:[69]

[69]We note ϱ_t^C and ϱ_t^K the Lagrange multipliers associated with budget and capital constraints, respectively.

$$\varrho_t^C = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ (C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{3.6}$$

$$\varrho_t^C = \chi \frac{L_t^{\varphi}}{W_t}, \tag{3.7}$$

$$1 = \beta E_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\}, \tag{3.8}$$

$$\varrho_t^C = \frac{\varrho_t^K}{1 + f_I(.)}, \tag{3.9}$$

$$\varrho_t^K = \beta E_t \{ (1-\delta) \varrho_{t+1}^K + \varrho_{t+1}^C (R_{t+1}^K - f_K(.)) \}, \tag{3.10}$$

where the stochastic discount factor (i.e. the expected variation in marginal utility of consumption) reads as follows $\Lambda_{t-1,t} = \frac{\varrho_t^C}{\varrho_{t-1}^C}$.

3.3.2 R&D Entrepreneurs

As in Comin and Gertler [2006] entrepreneurs are an unbounded mass of prospective innovators with the ability to introduce new varieties of intermediates in each period. Each entrepreneur use resources to create a new project $RD_{t,s}$. Both new projects $RD_{t,s}$ and existing varieties $A_{t,s}$ face the risk of an exogenous exit shock $(1 - \phi_{RD,s})$. This process is meant to capture in a simple way the life-cycle dynamics of firms. Note that we also consider that entrepreneurs are not using energy heavy output, thus emitting zero CO_2 emissions. The evolution of the aggregate stock of innovations $A_{t,s}$ reads as follows:

$$A_{t+1,s} = \phi_{RD,s}(A_{t,s} + RD_{t,s}), \tag{3.11}$$

Entrepreneurs are able to produce new varieties by employing materials and skilled workers as inputs, according to the following production function:

$$RD_{t,s} = N_{t,s}^{\eta_s} (A_{t,s} Ls_{t,s})^{1-\eta}, \; \eta_s \in (0,1), \tag{3.12}$$

where $N_{t,s}$ is the amount of materials used (in units of final output) and $Ls_{t,s}$ is the number of skilled workers hired. Once the variety created, entrepreneurs lend it to monopolist firms in exchange for patent exclusivity. The monopolists then manufacture the new good and reverse profits Π_t (as shown in equation (3.29)) back to the entrepreneurs. Furthermore, as in Romer [1990], in order to generate endogenous growth, the entrepreneurs production function captures the externality of the aggregate level of knowledge $A_{t,s}$.

The entrepreneurs problem will read as follows:

$$\max_{\{RD_{t,s},N_{t,s},Ls_{t,s}\}} E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left[\Pi_t RD_{t+i,s} - (N_{t+i,s} + Ws_{t+i,s} Ls_{t+i,s})\right] \tag{3.13}$$

s.t.

$$RD_{t+i,s} = N_{t+i,s}^{\eta_s} (A_{t+i,s} Ls_{t+i,s})^{1-\eta}. \tag{3.14}$$

The first order conditions read:

$$1 = MC_t^{RD,s} \eta_s N_{t,s}^{\eta_s - 1} (A_{t,s} Ls_{t,s})^{1-\eta_s}, \tag{3.15}$$

$$Ws_{t,s} = MC_t^{RD,s} (1-\eta_s) A_{t,s} N_{t,s}^{\eta_s} (A_{t,s} Ls_{t,s})^{-\eta_s}, \tag{3.16}$$

$$\Pi_t = MC_t^{RD,s}, \tag{3.17}$$

where $MC_t^{RD,s}$ the Lagrange multiplier associated to the production constraint. Entrepreneurs equalize their marginal cost to the profit they receive form the the firms and are subject to the inelastic supply of skilled labor $Ls_{t,s} = \bar{Ls}^s$).

3.3.3 The Firms

3.3.3.1 The Final Firms

The final good is produced by a competitive sector, which uses the different varieties of intermediates produced by entrepreneurs as inputs, yielding the following production function:

$$Y_t = \int_0^{A_{t,s}} \left(Y_{jt}^{1-\frac{1}{\theta}} dj \right)^{\frac{1}{1-\frac{1}{\theta}}}. \tag{3.18}$$

Final firms are looking for profit maximization at a given price P_t, subject to the intermediate goods j with prices P_{jt}:

$$P_t = \left(\int_0^{A_{t,s}} P_{jt}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{3.19}$$

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t. \tag{3.20}$$

3.3.3.2 The Intermediate Firms

Contrary to the standard RBC framework, representative firms (indexed by j) of the modeled economy seek face a trade-off between the desired level of abatement level and the environmental policy level, in addition to the usual capital and labor trade-off.

As the environmental externality is a global phenomena, firms do not internalize its impacts, thus, they incur the externality costs as the social planner or government imposes an environmental policy in order to fix the market failure. Setting an environmental policy then pushes firms to optimally choose a level of abatement to maximize their profit. Following Heutel [2012], the environmental externality enters the Cobb-Douglas production function of the firms, through a damage function linked to the level of temperature à la

Nordhaus and Moffat [2017] as follows:

$$Y_{jt} = \varepsilon_t^A d(T_t^o) K_{jt}^{\alpha} L_{jt}^{1-\alpha},\ \alpha \in (0,1), \tag{3.21}$$

where $d(T_t^o)$ is a convex polynomial function of order 2 displaying the temperature level ($d(T_t^o) = ae^{-(bT_t^{o2})}$), with $(a{,}b) \in \mathbb{R}^2$, which is borrowed from Nordhaus and Moffat [2017]. ε_t^A is an exogenous technology shock that follows an AR(1) shock process: $\log(\varepsilon_t^A) = \rho_A \log(\varepsilon_{t-1}^A) + \sigma_A \eta_t^A$, with $\eta_t^A \sim \mathcal{N}(0,1)$.

As argued by Dietz and Venmans [2019], global temperature $d(T_t^o)$ is assumed to be linearly proportional to the level of cumulative emissions:

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o. \tag{3.22}$$

Furthermore, the carbon emissions stock X_t follows a law of motion:

$$X_t = (1-\gamma_d)X_{t-1} + E_t + E^*, \tag{3.23}$$

where E_t is the aggregate flow of emissions at time t ($\int_0^1 E_{jt}dj$) and γ_d is the decay rate. E^* represents the rest of the world emissions and is used to pin down the actual steady state level of the stock of emission in the atmosphere.

The emissions level is modeled by a nonlinear technology (i.e. abatement technology μ) that allows for reducing the inflow of emissions:

$$E_{jt} = (1-\mu_{jt})\vartheta Y_{jt}. \tag{3.24}$$

The emissions E_{jt} at firm level are proportional to the production Y_{jt} with ϑ the carbon intensity parameter. Contrary to Cai and Lontzek [2019], we consider $\vartheta_t = \vartheta$ constant overtime and calibrate it using Euro Area emission to GDP levels, as in our model, we capture the effects of green R&D directly through the abatement cost.

Furthermore, we allow for emissions reduction at the firm level through an abatement effort μ_{jt}. When firms decide on abatement efforts, they incur a technology cost:

$$Z_{jt} = f(\mu_{jt})Y_{jt}, \tag{3.25}$$

where

$$f(\mu_{jt}) = g(\theta_t^1)\mu_{jt}^{\theta_2},\ \theta_2 > 1, \tag{3.26}$$

and

$$g(\theta_t^1) = \frac{\theta_1}{\Gamma_t^{\theta_1}\epsilon_t^{\theta_1}},\ \theta_1 > 0, \tag{3.27}$$

with θ_1 and θ_2 representing the cost efficiency of abatement parameters. In this section, we assume that the cost function of abatement $g(\theta_t^1)$ follows an exogenous trend $\Gamma_t^{\theta_1}$ and can be hit by a random shock $\epsilon_t^{\theta_1}$.[70] The goal is to capture exogenously the impact of improvements in green technology that we will concretely model in the next section. This will result in a decrease in abatement costs that will allow for substantially higher levels of abatement μ_{jt}.

A decrease in $g(\theta_t^1)$ triggers a drop in the marginal cost of abatement, which we define as:

$$MC_\mu = \frac{f(\mu_{jt})'}{\mu_{jt}} \tag{3.28}$$

Thus, the profits of our representative intermediate firms Π_{jt} will be affected by the presence of the environmental externality. The revenues are the real value of intermediate goods Y_{jt}, while the costs arise from wages W_t (paid to the labor force L_{jt}), investment in capital K_{jt} (with returns R_t^K), abatement μ_{jt} (the firms are facing), and the price of emissions E_{jt} associated with the environmental policy.

$$\Pi_{jt} = \frac{P_{jt}}{P_t}Y_{jt} - W_tL_{jt} - R_t^KK_{jt} - g(\theta_t^1)\mu_{jt}^{\theta_2}Y_{jt} - \tau_{et}E_{jt} \tag{3.29}$$

[70] $\epsilon_t^{\theta_1}$ follows an $AR(1)$ shock process: $\log(\varepsilon_t^{\theta_1}) = \rho_{\theta_1}\log(\varepsilon_{t-1}^{\theta_1}) + \sigma_{\theta_1}\eta_t^{\theta_1}$, with $\eta_t^{\theta_1} \sim \mathcal{N}(0,1)$.

The cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital, labor, as well as the abatement, respectively:

$$R_t^K = \alpha \Psi_{jt,k} \frac{Y_{jt}}{K_{jt}}, \tag{3.30}$$

$$W_t = (1-\alpha) \Psi_{jt,k} \frac{Y_{jt}}{L_{jt}}, \tag{3.31}$$

$$\tau_{et} = \frac{g(\theta_t^1)\theta_2}{\upsilon} \mu_{jt}^{\theta_2 - 1}. \tag{3.32}$$

The first two equation equation Equation (3.30) and (3.31) are the standard optimal choice of capital and labor, with $\Psi_{jt} = \Psi_t$ the marginal cost component related to the same capital-labor ratio all firms choose. This marginal cost component is common to all intermediate firms. When capturing the CO_2 externality firms face an additional trade-off (equation (3.32)) between paying the environmental policy τ_t or incurring abatement cost related to the abatement levels they chose μ_t.[71] This last optimality condition highlights the key role of the carbon price dynamics in shaping the abatement level of firms.

We can now rewrite the firm problem as following:

$$\Pi_{jt} = \left(\frac{P_{jt}}{P_t} - MC_t^f \right) Y_{jt}, \tag{3.33}$$

where,

$$MC_{jt}^f = MC_t^f = \Psi_t + g(\theta_t^1)\mu_{jt}^{\theta_2} + \tau_{et}(1-\mu_t)\varphi, \tag{3.34}$$

The total marginal cost captures both abatement and emissions costs. Note that in the case of the laissez-faire scenario, $MC_t^f = \Psi_t$ as the firms are not subject to emissions and abatement constraints.

The aggregate production function of the intermediate firms will now features the

[71] In addition, both the environmental policy τ_t and abatement effort μ_t are common to all firms, as the environmental cost, which firms are subject to, is constant.

measure A_t. Using both the Cobb-Douglas production form (3.21) and the final firms production equation (3.18), we can rewrite the production function as following:

$$Y_t = A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^\alpha L_t^{1-\alpha}. \tag{3.35}$$

The firm profit maximization with respect to output and prices, yields the following pricing rule:[72]

$$MC_t^f = \frac{P_{jt}}{P_t} \frac{\theta - 1}{\theta} \tag{3.36}$$

Each intermediate producer sets its price equals to a constant markup over the marginal cost. Finally, the profits equation will also capture the measure $A_{t,s}$ and can be presented as following:[73]

$$\Pi_t = \frac{1}{\theta} \frac{Y_t}{A_{t,s}}. \tag{3.37}$$

3.3.4 Government

Government levies a lump sum tax and sets an environmental policy to finance its spending as following:

$$T_t + \tau_{et} E_t = G_t, \tag{3.38}$$

with the public expenditure G_t, taxes T_t, and revenue from emissions tax $\tau_{et} E_t$. The government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t. \tag{3.39}$$

[72]With $\frac{P_{jt}}{P_t} = 1$, as we abstract frome price stickiness.

[73]For the full mathematical derivations please refer to the appendix.

3.3.5 The environmental policy

Competitive Equilibrium

To pin down the optimal policy,[74] we solve for the Competitive Equilibrium ("CE"). The CE in this economy is defined as follows:

Definition 3.3.1 *A competitive equilibrium consists of an allocation* $\{C_t, L_t, K_{t+1}, E_t, X_t, T_t^o\}$, *a set of prices* $\{P_t, R_t, R_t^K, W_t\}$ *and a set of policies* $\{\tau_t, T_t, B_{t+1}\}$ *such that*

- *the allocations solve the consumers', firms' problems given prices and policies,*
- *the government budget constraint is satisfied in every period,*
- *temperature change satisfies the carbon cycle constraint in every period, and*
- *markets clear.*

Definition 3.3.2 *The optimal solution sets the carbon price* τ_t *as an optimal policy* τ_t^*, *which maximizes the total welfare in equation (3.3):*

$$\tau_t^* = SCC_t. \tag{3.40}$$

with SCC_t *the social cost of carbon:*

$$SCC_t = \eta\beta\frac{\lambda_{t+1}}{\lambda_t}SCC_{t+1} + (\upsilon_1^o\upsilon_2^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T, \tag{3.41}$$

and with,

$$\S_t^T = (1-\upsilon_1^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T - \sum_k \Psi_t\varepsilon_t^A\frac{\partial d(T_t^o)}{\partial T_t^o}K_{t-1}^{\alpha}L_t^{1-\alpha} \tag{3.42}$$

[74] As we consider a closed economy, we assume that cooperation takes place in such a way to avoid free-riding and potential carbon leakages. This is achieved by setting E^* to a constant.

Departing from the Competitive Equilibrium to Meet Climate Goals

Definition 3.3.3 *The public authorities, however, do not always optimally set the carbon policy. For instance, in the EU area, public authorities target an emissions level that is consistent with their objective of a 55% emissions reduction by 2030. As in Benmir and Roman [2020] we model this situation by assuming that the cap on emissions implies a specific carbon price that can be hit by exogenous shocks and which also incorporates an endogenous trend:*

$$\tau_t = \Gamma_t^\tau \, Carbon\ Price.^{75} \tag{3.43}$$

where $\Gamma_t^\tau = \gamma^\tau \varepsilon_t^\tau \Gamma_{t-1}^\tau$ *is the stochastic growth rate of the tax which allows to reduce emissions to be aligned with the cap policy, and where* ε_t^τ *the stochastic AR(1) shock on tax that represents the market volatility of the ETS system.*

This stylized representation of the implementation of a permit market allows us to find theoretical fiscal pathways consistent with the EU climate objectives. That said, the targeted CO_2 level/price is assumed to be constant at the business cycle frequency.

3.3.6 Normalization and Aggregation

In equilibrium, factors and goods markets clear as shown below. First, the market-clearing conditions for aggregate capital, investment, labor, and wages, read as: $A_t K_t = \int_0^1 K_{jt} dj$, $I_t = \int_0^1 I_{jt} dj$, $A_t L_t = \int_0^1 L_{jt} dj$, and $W_t = \int_0^1 W_{jt} dj$. Similarly, global aggregate emissions and aggregate emissions cost reads as: $E_t = \int_0^1 E_{jt} dj$, and emissions cost $Z_t = \int_0^1 Z_{jt} dj$, respectively. Finally, the resource constraint of the economy reads as follows:

$$Y_t = C_t + G_t + I_t + N_{t,s} + f(.) I_t + Z_t. \tag{3.44}$$

[75] Although the policy used in the EU is E_t = Cap Policy, it analogous to set τ_t = Carbon Price that would allow for decreasing emissions to match the cap.

3.3.7 Transition Pathways with Exogenous Abatement Technology

3.3.7.1 Calibration

Calibrated parameters for the standard endogenous growth model are reported in table 3.8 and table 3.9. For parameters related to business cycle theory, their calibration is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio at 40 percent,[76] the share of hours worked per day at 0.33 for firms and 0.15 for entrepreneurs, and the capital share in the production function α at 0.3. The inverse elasticity of net investment to the price of capital γ_I is set at 1.728 as in Gertler and Karadi [2011] and the coefficient of relative risk aversion σ in the utility function at 2, as argued by Stern [2008] and Weitzman [2007]. We set the discount factor at 0.9975 to get a steady state real interest rate of 1 percent. This choice is motivated by the low interest rate environment witnessed in recent years.

Regarding the environmental part, we calibrate the damage function according to Dietz and Stern [2015]. The global temperature parameters υ_1^o and υ_2^o are set following Dietz and Venmans [2019] to pin down the "initial pulse-adjustment timescale" of the climate system. The level of the remainder of the world's emissions E^* is set at 1.59 in order to replicate the global level of carbon in the atmosphere of 840 gigatons. We use the carbon intensity parameter ϑ to match the observed ratio of emissions to output for the Euro Area (EA) at 21%.[77] The abatement parameters θ_1 and θ_2 are taken from Heutel [2012]. The decay rate of emissions δ_x is set at 0.21 percent. Finally, the firms' marginal cost parameter θ is set to 11.

[76] We match the level of the Euro Area.

[77] We compute the emissions to output ratio as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from FRED.

3.3.7.2 Transition Pathways Simulations

In order to solve for the medium/long-run pathways scenarios, we use the extended path algorithm (Adjemian and Juillard [2013]), which allows for both integrating deterministic trends and stochastic shocks, as it is shown in Benmir and Roman [2020].

The goal of this section is to find and analyze a theoretical pathway consistent with the objective of the EU for 2030 under the presence of i) a targeted carbon price policy, ii) an exogenously growing green technology, and iii) an optimal policy coupled with an exogenously growing green technology.

We thus find the trajectory of the output, the marginal cost of abatement, and the carbon price, that leads to a desired reduction in emissions (55 percent relative to the level of 1990). We then highlight the main differences between relying solely on a carbon policy or solely on an abatement technology, versus using an optimal policy which maximizes the welfare (but would alone fails to attain the 55 percent emissions reduction desired) coupled with an abatement technology that is increasing over time.

Figure 3.2 shows what carbon price and/or reduction in abatement costs trajectories would be needed to be on track for achieving the net-zero target in the EZ, assuming a growth trend of 0.8 percent.[78] We also add a stochastic shock process to TFP, that we calibrate according to the estimation in Smets and Wouters [2003]. This allows us to simulate a realistic transition scenario, where the trend in growth is anticipated, but shocks can distort this deterministic process in the short run. The blue dashed line is a scenario where we build a counterfactual highlighting the pathway if an optimal policy is set and coupled with decreasing marginal abatement costs. The green solid line is a scenario where green technology—coupled with a fixed tax rate—is the only long-run driver of emissions reduction. Finally the dotted red line corresponds to the scenario where the targeted environmental policy (e.g. EU ETS cap system) is the only instrument used to mitigate the climate externality and keeps the economy on track for achieving the desired level of

[78]The average real growth rate per capita in the EZ area from 2000 to 2020

emissions reduction. Relying on a targeted tax alone, requires high levels of carbon price to be on target for net-zero by 2050, and induces a higher output loss than both other scenarios where green innovation is boosted to allow for lower marginal cost of abatement, which in turn triggers higher abatement levels. We find that either fixing the environmental policy at a targeted level and allowing for green innovation to boost abatement levels, or using an optimal fiscal policy coupled with green innovation are more efficient in keeping higher levels of output than just relying on a carbon fiscal policy alone. It suggests that an optimal policy with green innovation boosting is the optimal choice from a welfare perspective.

These results comforts our empirical finding, as both fiscal environmental policy and green innovation growth are major contributors to significant emissions reduction. In addition, higher fiscal carbon prices are also shown to negatively impacts the costs of abatement.

3.4 Introducing Endogenous Green Technology

In this section, we introduce green entrepreneurs who produce innovations in the abatement technology. An improvement in green technologies will, in turn, reduce the cost of abatement for firms. However, green innovators will need to rely on loans from banks to start new projects. Thus, we also show how financial intermediaries are modeled. Finally, we propose a set of policies that could help fostering green innovations. The goal is to show how public policies could ultimately impact the abatement efficiency.

3.4.1 Household

The budget constraint of households is modified to display wages for skilled labor employed by green entrepreneurs, as well as profits from the ownership of financial intermediaries.

$$C_t + B_{t+1} + I_t + f(K_t, I_t) = W_t L_t + Ws_{t,g}\bar{Ls}^g + Ws_{t,s}\bar{Ls}^s + \Pi_t + T_t + R_t^K K_t + R_t B_t \quad (3.45)$$

where $\bar{Ls}^g$ is the inelastic labor supply to green entrepreneurs associated with wage $Ws_{t,g}$. This small change does not have any impact on the first order conditions shown in section 3.3.1.

3.4.2 Green Innovators

Similarly to the R&D entrepreneurs presented in section 3.3.2, we follow Comin and Gertler [2006] and introduce an unbounded mass of prospective green innovators with the ability to improve the abatement technology. However, we differ from their set up insofar as we consider that the innovators are green R&D creators that allows for improving the abatement efficiency via a reduction in abatement costs ($g(\theta_t^1)$). Each green innovator use resources to create a new project $RD_{t,g}$. Both new projects $RD_{t,g}$ and existing technologies $A_{t,g}$ face the risk of an exogenous exit shock $(1 - \phi_{RD,g})$. Similarly to the R&D entrepreneurs, we assume that green innovators do not emit CO_2 while developing new technologies.

Our innovators or research and development centers need to obtain funding from banks to finance entry. Here the idea is that financial intermediaries are the economic entities with the expertise and knowledge when evaluating and monitoring green entrepreneurial projects.

The total number of green technologies in operation at any given time t is denoted by $A_{t,g}$, while the green projects $RD_{t,g}$ are the number of new technologies in process in period t. Accordingly, the evolution of the aggregate stock of green innovations, $A_{t,g}$, is given by:

$$A_{t+1,g} = \phi_{RD,g}(A_{t,g} + RD_{t,g}), \quad (3.46)$$

To be more specific, each green innovator can produce a new potential technology by employing materials and skilled workers as inputs, according to the following production function:

$$RD_{t,g} = N_{t,g}^{\eta_g}(A_{t,g}Ls_{t,g})^{1-\eta},\ \eta_g \in (0,1), \tag{3.47}$$

where $N_{t,g}$ is the amount of materials used (in units of final output) and $Ls_{t,g}$ is the number of skilled workers hired. $A_{t,g}$ denotes the aggregate green technological level of the economy, which as explained below is equal to the total number of technologies in operation. Similarly to the R&D entrepreneurs, the innovators production function captures the externality of the aggregate level of knowledge $A_{t,g}$, which allows for generating endogenous growth.[79]

Once the technology created, entrepreneurs lend it to monopolist firms in exchange for patent exclusivity. The monopolists then use these technologies to lower their abatement cost and pay a rent Z_t corresponding to abatement costs to the green innovators.

As in Queralto [2020], we assume that green entrepreneurs can borrow to face the entry cost without any friction. More specifically, when seeking funding, our innovators can emit a financial intermediaries security which is perfectly contingent on the success of the green project. However, as in Gertler and Karadi [2011], banks do face frictions relative to their leverage ratio, as we will show in the next section. As long as the innovation does not become obsolete, the underlying securities pay in each future period. If the innovation becomes obsolete, then the payoff is zero. We denote the price of one unit of these securities $Q_{t,e}$.

The green innovators optimize over the revenues from selling securities subject to the inherent costs of developing the innovation by using materials N_t and paying wages Ws_t

[79] For simplicity, we consider that spillovers on the green innovation only originate from the green technological level $A_{t,g}$.

to the skilled labor Ls_t. The maximization problem reads as follows:

$$\max_{\{RD_{t,g},N_{t,g},Ls_{t,g}\}} E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left[Q_{t+i,e} RD_{t+i,g} - (N_{t+i,g} + Ws_{t+i,g} Ls_{t+i,g})) \right] \tag{3.48}$$

s.t.

$$RD_{t+i,g} = N_{t+i,g}^{\eta_g} (A_{t+i,g} Ls_{t+i,g})^{1-\eta_g} \tag{3.49}$$

The first order condition reads (denoting $MC_t^{RD,g}$ the Lagrange multiplier associated to the production constraint):

$$1 = MC_t^{RD,g} \eta_g N_{t,g}^{\eta_g - 1} (A_{t,g} Ls_{t,g})^{1-\eta_g}, \tag{3.50}$$

$$Ws_{t,g} = MC_t^{RD,g} (1-\eta_g) A_{t,g} N_{t,g}^{\eta_g} (A_{t,g} Ls_{t,g})^{-\eta_g}, \tag{3.51}$$

$$Q_{t,e} = MC_t^{RD,g}. \tag{3.52}$$

Using these first order conditions[80] and equation (3.47), we can rewrite the price of the inherent security $Q_{t,e}$ in terms of the marginal cost components as following:[81]

$$Q_{t,e} = MC_t^{RD,g} = \frac{1}{\eta_g} \left(\frac{1}{\bar{Ls}^g} \right)^{\frac{1-\eta_g}{\eta_g}} \left(\frac{RD_{t,g}^N}{A_{t,g}} \right)^{\frac{1-\eta_g}{\eta_g}}, \tag{3.53}$$

Contrary to the previous section, where the cost of abatement was driven by an exogenous process, the cost function of abatement is now steered by endogenous green technological changes. Thus, green innovators projects will ultimately lead to higher abatement and lower emissions. The equation (3.27) now reads:

$$f(\mu_t) = \left(\int_0^{A_{t,g}} f(\mu_{jt})^{\frac{1}{\theta_3}} dj \right)^{\theta_3} \tag{3.54}$$

[80] With $Ws_{t,g} = \frac{1-\eta_g}{\eta_g} \frac{N_{t,g}}{A_{t,g} Ls_{t,g}}$

[81] We also use the market clearing condition for skilled labor: $Ls_{t,g} = \bar{Ls}^g$.

Thus,

$$g(\theta_t^1) = \theta_1 A_{t,g}^{-\theta_3},\ \theta_1 > 0 \text{ and } \theta_3 > 0, \tag{3.55}$$

where θ_3 is now the elasticity of the cost of abatement with respect to the green technology.

3.4.3 Financial Intermediaries

A representative financial intermediary make use of deposits from households as well as its own net worth to leverage and invest in green entrepreneurs. We model this part following Gertler and Karadi [2011]. We can write the representative bank's balance sheet as:

$$Q_{t,e} S_{t,e} = N_t + B_t, \tag{3.56}$$

where $S_{t,e}$ are financial claims on green innovators and $Q_{t,e}$ their relative price. Note that market clearing implies that $S_{t,e} = A_{t,g} + RD_{t,g}$, as assets held by banks must match the total number of existing green technologies. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, banks' retained earnings evolve as follows:

$$N_t = R_{t,e} Q_{t-1,e} S_{t-1,e} - R_t B_{t-1}, \tag{3.57}$$

$$N_t = (R_{t,e} - R_t) Q_{t-1,e} S_{t-1,e} + R_t N_{t-1}, \tag{3.58}$$

where $R_{t,e}$ denotes the gross rate of return on a unit of the bank's claims on green innovators:

$$R_{e,t} = \frac{\phi_{RD_g}(Z_t + Q_{t,e})}{Q_{t-1,e}}. \tag{3.59}$$

Financial intermediaries will maximize equity on an infinite horizon, yielding the following objective function:

$$V_t^B = E_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1-\theta_B) \theta_B^{i-1} N_{t+1+i} \right\}, \tag{3.60}$$

where θ_B is the probability of a bank exiting the market. The constraint on banks arise from the existence of a supervisory regulator. Drawing on Pietrunti [2017], we assume that this regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t^B \geq \lambda Q_{t,e} S_{t,e}. \tag{3.61}$$

In this simplified setup, banks only hold one asset, so the regulator will set a value for λ in order to target a specific capital ratio for banks. By modifying this parameter, the macroprudential authority will be able to tighten or relax the constraint on banks, which will impact the number of entrepreneurial projects the financial sector can fund. In our baseline model, we will calibrate λ to match the capital ratio of European banks at the steady state. We guess that the value function is linear of the form $V_t = \Gamma_t^B N_t$ so we can rewrite V_t^B as:

$$V_t^B = \max_{S_{t,e}} E_t \left\{ \beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \right\}, \tag{3.62}$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t^B$. Maximization subject to constraint (3.61) yields the following first order and slackness conditions:

$$\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,e} - R_{t+1}) \right\} = \nu_t \lambda, \tag{3.63}$$

$$\nu_t \left[\Gamma_t^B N_t - \lambda Q_{t,e} S_{t,e} \right] = 0, \tag{3.64}$$

where ν_t is the multiplier for constraint (3.61). We can thus write the capital ratio as $\Xi_t = \lambda/\Gamma_t^B$. Finally, we rewrite the value function to find Γ_t^B:

$$\begin{aligned}
V_t^B &= \nu_t \lambda Q_{t,e} S_{t,e} + \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} N_t \right\} \\
\Gamma_t^B N_t &= \nu_t \Gamma_t^B N_t + \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \right\} \\
\Gamma_t^B &= \frac{1}{1-\nu_t} \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \right\}.
\end{aligned} \tag{3.65}$$

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B (R_{t,e} - R_t) Q_{t-1,e} S_{t-1,e} + (\theta_B R_t + \omega) N_{t-1}, \tag{3.66}$$

with $\omega \in [0;1)$ the proportion of funds transferred to entering bankers.

3.4.4 Carbon Policy and Green Innovation

As argued in the section above on the model equilibrium, many economies rely on a permit-market-based instrument instead of an optimal carbon price (e.g. the ETS in the EU and the carbon permit markets in Canada in California (US)). Thus, in order to reach the Paris Agreement objective of the net-zero emissions by 2050, such carbon pricing strategy requires carbon prices to constantly increase, which in turn incentivizes firms to engage in continuously higher abatement efforts. However, investing in abatement technologies is costly and has a number of consequences such as welfare losses as shown in Benmir and Roman [2020]. Steering green innovation via other tools besides carbon pricing would be less welfare distortionary. Incentivizing green innovation that lowers the cost of abatement, however, might prove difficult if the price of carbon increases substantially and in places where no green abatement technology is yet available.

Definition 3.4.1 *A government, when relying on a carbon permit market solely to tackle* *the climate externality, sets a carbon cap:*

$$E_t = Cap_t \tag{3.67}$$

which inherently determines a carbon price level τ_{et}:

$$\tau_{et} = Carbon\ Price_t. \tag{3.68}$$

where Cap_t is the path of the cap on emissions consistent with the net-zero objective, and $Carbon\ Price_t$ the inherent carbon price associated with this objective. To reach the net-zero target, the price is expected to steadily increase in order to match the expected decrease in the cap.

However, under the presence of endogenous green innovation that contributes to lowering the cost of abatement, the social planner is not limited anymore in terms of tools it could use, and is able to rely on both a carbon price τ_{et} and the green technologies $A_{t,g}$:

Definition 3.4.2 *To decrease emissions, firms engage in higher abatement efforts:*

$$\mu_t = 1 - \frac{Cap_t}{\upsilon Y_t} \tag{3.69}$$

with $\Delta\left(\frac{Cap_t}{\upsilon Y_t}\right) < 0$. Otherwise, the optimal social cost of carbon presented in the initial exogenous framework would be able to achieve the target. Therefore, the carbon price, as defined in equation (3.32), is driven by two instruments, namely, i) the environmental cap Cap_t and ii) the green technologies $A_{t,g}$:

$$Carbon\ Price_t = \theta_1\theta_2\frac{\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1}}{\upsilon}A_{t,g}^{-\theta_3} \tag{3.70}$$

Effectively, when $Cap_t = \upsilon Y_t = \bar{E}_t$[82] *(i.e. a laissez-faire economy)*

$$\min(Carbon\ Price_t) = 0 \tag{3.71}$$

And when $Cap_t = 0 \Rightarrow \mu_t = 1$ *(i.e. a net-zero objective)*

$$\max(Carbon\ Price_t) = \theta_1\theta_2\frac{1}{\upsilon}A_{t,g}^{-\theta_3} \tag{3.72}$$

Definition 3.4.3 *When it is impossible to implement an optimal policy*[83] $\tau_{et} > SCC_t$, *public authorities insure a specific carbon price by setting a cap on emissions. However, the design and trajectory of the cap policy* Cap_t *could also have indirect consequences on green innovation. Depending on the cap policy implemented, this could have the opposite effect. That is, instead of increasing the total cost of abatement for firms* $Z_t = f(\mu_t)Y_t$, *the loss in output could translate to a lower total cost of abatement* Z_t. *This decrease in* Z_t *would reduce banks' investments in green projects. Ultimately, it would lead to slower green innovation and a lower growth rate of* $A_{t,g}$.[84]

$$\frac{\partial Z_t}{\partial Cap_t} = \frac{\partial f(\mu_t)}{\partial Cap_t}Y_t + \frac{\partial Y_t}{\partial Cap_t}f(\mu_t) \tag{3.73}$$

$$= \left(\frac{\partial f(\mu_t)}{\partial Cap_t} + \frac{1}{d(T_t^o)}\frac{\partial d(T_t^o)}{\partial Cap_t}f(\mu_t)\right)Y_t \tag{3.74}$$

Thus, there exists a Cap_t^* *for a level of* $A_{t,g}$,[85] *such that* $\frac{\partial f(\mu_t)}{\partial Cap_t} + \frac{1}{d(T_t^o)}\frac{\partial d(T_t^o)}{\partial Cap_t}f(\mu_t) = 0$.

Corollary 3.4.1 $\Delta Cap_t < 0 \Rightarrow \Delta Z_t > 0 \Rightarrow \Delta R_{t,e} > 0 \Rightarrow \Delta RD_{t,g} > 0$

An increase in the carbon price (i.e. a decrease in the cap), triggers more abatement, which

[82] $\bar{E}_t$ the steady state level of emissions at each period t

[83] Implementing an optimal policy requires major institutional constraints and carbon pricing monitoring, which cannot be achieved with the current public institutions (Delpla and Gollier [2019]).

[84] As shown in Appendix section 3.B.4, note that, as we will get closer to the end of transition to net-zero, the high level of $A_{t,g}$ will imply a decreasing cost Z_t. Through a feedback loop, this will make investment in green projects less interesting for banks and the growth rate of green technologies will be lower.

[85] Note that Z_t is concave in our case.

in turn increases the cost of abatement $Z_t = f(\mu_t)Y_t$, as firms would equate their marginal benefit from investing in abatement to the carbon price. This increase in Z_t imply a higher rate of return on entrepreneurs equity $R_{e,t}$ as entrepreneurs' profits are reversed to banks. The higher the profitability of entrepreneurs, the more banks would direct investment toward green projects, which would spur green innovation $A_{t,g}$.

Corollary 3.4.2 $\Delta Cap_t << 0 \Rightarrow \Delta Z_t < 0 \Rightarrow \Delta R_{t,e} < 0 \Rightarrow \Delta RD_{t,g} < 0$
A significant change in the cap policy design might not always result in an increase of the cost of abatement $Z_t = f(\mu_t)Y_t$. Although a rise in the carbon price increases $f(\mu_t)$ on one hand, it also decreases profits Π_t and output Y_t on the other hand. There exist a point where the decease in output Y_t is superior to the increase in $f(\mu_t)$, which results in a decrease in Z_t. This would in turn lower the rate of return $R_{t,e}$, which would contribute negatively to green innovation.

Proposition 3.4.1 *To ensure we meet the net-zero target with a decreasing cap on emissions, while trying to mitigate the effect on welfare of a rising carbon price, we investigate three macro-financial tools that could foster green innovation: i) the fiscal authority uses revenues from carbon pricing policy to subsidize green innovators; ii) the macroprudential authority adapt its capital requirement to give an incentive to financial intermediaries to invest in green entrepreneurs' equity, thus generating a greater number of successful green technologies; and iii) the central bank engages in an asset purchase program aiming to ease funding conditions for the green innovation sector.*

i) Fiscal Policy

As presented in the model section, the government finances its government spending as follows:

$$T_t + (1 - \bar{s})\tau_t E_t = G_t, \tag{3.75}$$

with the public expenditure G_t finding its source from taxes T_t and revenues from the carbon tax $\tau_t E_t$.

In this setting we will consider the possibility for the government to divert part $\bar{s}$ or all of the environmental policy revenues back to the green innovators (if $\bar{s} = 0$ no subsidy is diverted to the green innovators). In this case, subsidies would raise profits of green entrepreneurs and ultimately be reversed to banks as interest:

$$R_{e,t} = \frac{\phi_{RD}(Z_t + Q_{t,e} + \bar{s}\tau_t E_t)}{Q_{t-1,e}}. \tag{3.76}$$

ii) Macroprudential Authority

As detailed in section 3.4.3, the macroprudential authority imposes a capital constraint on banks modeled through the parameter λ that pins down the steady state capital ratio. In a more sophisticated model, claims on green entrepreneurs could be one of several assets held by banks. In this case, different weights could be applied to different assets, and the regulator could favor a specific sector.[86] Our setup is without loss of generality, since modifying λ in our model is similar to modifying the weight on loans to entrepreneurs in a model with several assets, keeping all other weights constant.

Furthermore, we also allow the macroprudential authority to react to changes in the stock of emissions. By doing so, the macroprudential authority is able to steer credit to the green entrepreneurs when emissions flow of CO_2 in the atmosphere is going far away from its steady state. The macroprudential rule in this setting will read as follows:

$$\lambda_t = 1 - \lambda(E_t - \bar{E}) \tag{3.77}$$

iii) Quantitative Easing

In the previous sections, we introduced the link between the financial sectors and the development of green technologies. We also laid the ground for policy intervention through the existence of a macroprudential authority. As recently put forward in the monetary

[86] See Benmir and Roman [2020].

policy strategy review of the ECB, central banks have a role to play in the fight against climate change. Whether on inflation stabilization or financial stability grounds, there is a growing understanding of risks arising from global warming and potential room for central bank intervention. We now introduce a central bank that can substitute for financial intermediaries in financing green entrepreneurs. Thus, total claims on entrepreneurs are split between government and private holdings:

$$Q_{t,e}S_{t,e} = Q_{pt,e}S_{pt,e} + Q_{gt,e}S_{gt,e}, \tag{3.78}$$

with $Q_{gt,e}S_{gt,e}$ the total real value of loans to entrepreneurs held by the central bank. $Q_{pt,e}S_{pt,e}$ is the total real value of loans to firms of type k held by financial intermediaries as defined in section 3.4.3. The central bank decides in every period to hold a portion $\psi_{t,e}$ of total loans to green entrepreneurs:

$$Q_{gt,e}S_{gt,e} = \psi_{t,e}Q_{t,e}S_{t,e}. \tag{3.79}$$

We assume that the central bank reacts to deviations of carbon emissions from their targeted level (i.e. steady state at the business cycle frequency) in order to decide the share of assets $\psi_{t,e}$ it holds. This rule reads as follows:

$$\psi_{t,e} = \phi^s(E_t - \bar{E}), \tag{3.80}$$

where the reaction parameter ϕ^s is set at 10.[87] Note that in our baseline model $\bar{\psi}_{t,e} > 0$ in order to account for the fact that the ECB keeps a substantial share of private assets in its portfolio.

[87] This corresponds to a maximum 12% of total green asset purchased over the sample period, and is aligned with Gertler and Karadi [2011].

3.4.5 Normalization and Aggregation

When introducing green innovators, the resource constraint of the economy is modified as follows:

$$Y_t = C_t + G_t + I_t + N_{t,g} + N_{t,s} + f(.)I_t. \tag{3.81}$$

3.5 The Balanced Growth Path

From the empirical data on global patents, green patents, and output, both green investment $N_{t,g}$ and global R&D investments $N_{t,s}$ are found to have higher trend growth than output. This empirical finding requires us to balance the growth rates of the green and global R&D investments on the supply side of the resource constraint of our economy to ensure balanced growth. Thus, to allow for a balanced growth path, we introduce investment-specific trends à la Greenwood et al. [1997] that we denote as $V_{t,g} = \gamma_g^V V_{t-1,g}$ and $V_{t,s} = \gamma_s^V V_{t-1,s}$, where γ_g^V and γ_s^V are constant growth rates. These investment goods $N_{t,g}$ and $N_{t,s}$ are produced from final goods by means of a linear technology, whereby $\frac{1}{V_{t,g}}$ and $\frac{1}{V_{t,s}}$ units of final goods yield one unit of investment goods, respectively.[88]

Furthermore, the non-linear climate damages within the production function does not allow for a balanced growth path when considered as the following: $d(T_t^o) = ae^{-bT_t^{o2}}$). To allow for a balanced growth path trajectory, we show that over the period horizon we consider for our estimation (2000-2020), the low growth rate Γ_t had a small to no effect on the damage function dynamics $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}} \approx d(T_t^o) = ae^{-bT_t^{o2}}$. Capturing the growth rate of the economy within the damage function allows for simplifying the de-trended form of the damage function without a loss of generality, given that over the period sample of our estimation, climate damages that are corrected for the economy growth rate Γ_t are not significantly different from climate damages that are not corrected for the economy growth rate. In addition, given that both climate is defined as the average change over the past

[88] The slope of this investment-specific trend crucially appears in the measurement equation of the model and is estimated.

30 years, and that the stock of emissions is a slow moving variable, our 20 year sample period allows us to consider the damage function as a de-trended equation, which allows for reconciling the balanced growth path.

Our economy presents three sources of permanent growth: i) an endogenous source of growth $A_{t,s}$, ii) two exogenous sources of growth $V_{t,s}$ and $V_{t,g}$, and iii) a fourth endogenous source of green innovation growth $A_{t,g}$ which impacts the efficiency of abatement. Having these different sources of growth requires that we de-trend our model as a number of variables (e.g. output, emissions, investment, ...) will not be stationary. In the appendix section 3.B.5 we present the de-trended economy. The aggregate variables of our economy,[89] include: output per capita Y_t, investment per capita I_t, consumption per capita C_t, government spending G_t, lump sum taxes T_t, capital per capita K_{t-1}, emissions E_t, abatement costs $Z_t/V_{t,g}$, green investment expenditures $N_{t,g}/V_{t,g}$, global R&D investment expenditures $N_{t,s}/V_{t,s}$, stock of emissions X_t, Temperature T_t^o, R&D varieties per capita $RD_{t,s}$, and green innovation varieties per capita $RD_{t,g}$, wages W_t, skilled labor wages $W_{t,s}$, relative price of financial claims $Q_{t,e}$, debt to households B_t, net worth N_t, and the banks value function V_t^B, and all grow at the same rate Γ_t, which reads as the following:

$$\Gamma_t = A_{t,s}^{\frac{1}{(\theta-1)(1-\alpha)}} \tag{3.82}$$

where $\Gamma_t = \gamma_t^Y \Gamma_{t-1}$, the stock growth of R&D $A_{t,s}$ is $\gamma_t^{A_s} = \frac{A_{t,s}}{A_{t-1,s}}$, and the stock growth of green innovation $A_{t,g}$ is $\gamma_t^{A_g} = \frac{A_{t,g}}{A_{t-1,g}}$.

[89]Along the balanced growth path.

3.6 Quantitative Analysis

3.6.1 Calibration and Estimation

3.6.1.1 Data and Measurement Equations

The model is estimated using Bayesian methods and EZ quarterly data over the sample time period 2000Q1 to 2019Q4. Data are taken from both Eurostat and the European Patent Office. We focus on the period between 2000 and 2019, as the decoupling between emissions and output started to be more significant in the 2000s. Furthermore, empirical data also support this strategy, since investment in de-carbonized technologies started to exhibit a trend at the same time.

In order to estimate the key shocks and parameters of our model, we start by making our four series (output, emissions, R&D and green innovation expenditures, which we proxy via patents numbers) stationary. We first divide the sample by the working age population. Second, data are taken in logs and we then use a first difference filtering to obtain growth rates. Finally, we use the GDP price index to deflate all nominal variables.

To measure the empirical contribution of endogenous growth in green and standard technologies, we follow Vermandel [2019] and use a cost-based approach. As there is no data available for quarterly investment in both green technologies and global R&D, we use the number of patents filed to proxy expenditures.

Measurement equations are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \\ \text{Real Per Capita R\&D Expenditure Growth} \\ \text{Real Per Capita Green Innovation Expenditure Growth} \end{bmatrix} = \begin{bmatrix} \log \gamma_t^Y + \Delta \log (\tilde{y}_t) \\ \log \gamma_t^Y + \Delta \log (\tilde{e}_t) \\ \log(\gamma_t^Y / \gamma_s^V) + \Delta \log (\tilde{n}_{t,s}) \\ \log(\gamma_t^Y / \gamma_g^V) + \Delta \log (\tilde{n}_{t,g}) \end{bmatrix}, \tag{3.83}$$

where tilde denote de-trended variables.[90]

3.6.1.2 Calibration and Prior Distribution

As the main objective of our paper is to assess trends in R&D and green innovation growth, all standard macro-finance and environmental parameters are calibrated from the literature. The calibration values for the standard macro block and the environmental components are reported in table 3.8 and table 3.9. Table 3.10 reports the calibration of financial parameters related to the full model. We set the probability of remaining a banker θ_B at 0.972 as in Gertler and Karadi [2011]. We find the values of the proportional transfer to the entering banker ω and the regulatory parameter λ to approximately match both the debt to equity ratio[91] and the capital ratio in the EA. Because we only model loans to entrepreneurs, that are seen to carry a high level of risk, we assume that the regulator applies a 150% weight[92] to such assets before multiplying it by the theoretical capital requirement for banks of 10.5%. This yields an effective capital ratio of 15.75% in our baseline model.

For the remaining set of parameters and shocks, we rely on Bayesian methods. In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method (MCMC). In the following fit exercise, we solve the model using a linear approximation to the policy function, and employ the Kalman filter to form the likelihood function. Table 3.11 summarizes the prior—as well as the posterior—distributions of the structural parameters for the U.S. economy. As in Smets and Wouters [2003] the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while the standard deviation of shocks follow an inverse gamma distribution with mean 0.001 and

[90] The balanced growth path of the model can be found in the appendix.

[91] We compute the debt to equity ratio by taking the sum of the debt to equity ratios of the 19 EZ countries, weighted by their relative shares in total banks assets, using data from Eurostat and the ECB.

[92] Corresponding to the highest weight possible for corporate loans according to Basel III regulation.

standard deviation of 0.005.

The output growth rate γ_y and green innovation growth rate γ_{A_g} are estimated using a prior standard deviation of a gamma distribution with mean 0.05 and 0.01, respectively, while we use a beta distribution with mean 0.125 and 0.15 for the investment share in R&D η_s and green innovations η_g. Finally, the exogenous R&D and green innovation investment growth rates γ_{V_s} and γ_{V_g} are estimated using a normal distribution with means 1 and standard deviations of 0.2.

3.6.1.3 Posterior Distribution

In addition to prior distributions, table 3.11 reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel MCMC chains of 20,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor so as to match an average acceptance rate close to 25-30 percent for each chain.

Results of the posterior distributions for each estimated parameter are listed in table 3.11. It is clear from table 3.11 that the data were informative, as the shape of the posterior distributions differs from the priors. Results for structural shocks parameters that are common with Smets and Wouters [2003] are in line with the values they find. Regarding investment elasticities η_k with $k \in \{s, g\}$, our values are close to Queralto [2020]. As for the endogenous and exogenous trends, our estimates are consistent with the observed empirical output and green innovation investment growth rates.

3.6.2 Endogenous Trends

In this section, we first discuss the results of our estimation of endogenous growth trends in output and green innovation. We then perform a counterfactual exercise to assess the relevance of policies aiming at boosting the growth trend in green innovation.

3.6.2.1 Estimated Trends

Figure 3.3 and figure 3.4 display the estimated trends in output and green technology, respectively. Those two trends are highly correlated,[93] but the trend on green innovation is approximately twice as high as the trend on output. This can explain the decoupling between emissions and output witnessed over the studied period. The trend on green innovation also exhibits more volatility at the business cycle frequency, which is consistent with the fact that the green technology sector is less mature than standard R&D.

3.6.2.2 Incentive Policies for Green Innovation

Now that we have retrieved the time path of the two endogenous trends, we perform counterfactual exercises by retrospectively implementing public policies designed to affect the behavior of green entrepreneurs and trigger a higher growth in green innovation.

Tax, Subsidies and Green Innovation

Our first counterfactual exercise is to implement a subsidy scheme as defined in section 3.4.4. By reversing revenues from the carbon tax to green entrepreneurs, the goal is to foster investment in green technologies. Figure 3.5 shows the time path of the trend on green innovation when the tax levied through the carbon permit market is turned into subsidies for green entrepreneurs compared to the baseline model where the revenues from the tax simply finance government spending. The subsidy policy would have worked very well from 2004 to 2011, by raising the trend growth on green innovation by 0.1% to 0.3%. The effect is much more diffuse, however, after the year 2012. This can be explained by the fact that the ratio of emissions to output started to decline around this time, implying lower revenues from the carbon tax and, hence, lower subsidies for green innovators.

[93] This is not surprising, since the model features a spillover effect from the global technology to the green technology.

Macruprudential Policy and Green Innovation

In our second counterfactual exercise, we implement a macroprudential policy rule that reacts to deviations of the emissions level of carbon from its steady state. Figure 3.6 displays the time path of the trend on green innovation when the macroprudential policy is active compared to the baseline model. The idea here is to give an incentive to banks to lend more freely to the green entrepreneurs when the emissions flow of CO_2 is too high. To do so, the macroprudential authority lowers λ_t, following the macroprudential rule specified in section 3.4.4. This implies a decrease in the capital ratio of banks, but also more funds available to green entrepreneurs to start new projects. The policy is effective in steering new projects and the green technology from 2004 to 2011. The reason is that, emissions increased in the first 7 years due to the inefficiencies of the ETS system phase 1 and 2 which were still experimental. The macroprudential authority reacting to deviations from the de-trended steady state got increasingly worried about emissions dynamics and progressively loosened the capital requirements on banks, leading to the launch of more new projects by green entrepreneurs.

Interestingly, the two public policies studied here seem to be achieving similar results but with different magnitudes. In the counterfactual research we conduct, we don't consider an optimal design of the macroprudential rule which could react differently to changes in emissions in order to maximize the growth in green technologies. One could imagine a rule where the financial authority only reacts to an increase in emissions ($E_t - \bar{E} > 0$) for instance. We leave this work on optimal macroprudential rules for future research.

QE Policy and Green Innovation

In our third counterfactual exercise, we implement a QE policy rule that reacts to deviations of the emissions level of carbon from its steady state, similarly to the macroprudential rule. Figure 3.7 displays the time path of the trend on green innovation when the QE policy is active compared to the baseline model. In this scenario, the aim is to allow the central bank to directly fund green entrepreneurs, which would boost the number of green projects and ultimately lead to a higher growth in green technologies. Just like the macroprudential policy, the QE rule is not set optimally, with respect to growth in green technologies. We find that this policy is very similar to the macroprudential policy and acts pro-cyclically as the subsidy policy. The explanation for this pro-cyclicality feature lies in the way we model innovation in green technologies. As shown in section 3.4.4, a higher total cost of abatement leads to higher profits for entrepreneurs and triggers more growth in green technologies. Periods of high emissions (compared to steady state) also imply a higher abatement cost for firms, as we consider the carbon price constant. Thus, the policies we consider will only reinforce this effect, by incentivizing banks to lend more to green entrepreneurs when profits in this sector are already rising.

3.6.3 Transition Pathways with Endogenous Abatement Technology

In this section, we characterize the dynamics of the economy when considering the net-zero pathway consistent with the objective of the EU for 2050 ($E_t = Cap_t$) under the presence of i) a fiscal subsidy scheme where 70% of the environmental revenues are reversed to the financial intermediaries to incentive higher investments in green technologies, ii) a permanent macroprudential policy, which lowers the capital constraint on financial intermediaries by 30%, thus allowing them to increase investments in green entrepreneurs, and iii) an asset purchase program where the central bank buys around 1% of total claims on green entrepreneurs per year. We use the estimated values of the structural parameters to replicate the growth rates in productivity and green technologies of the EZ economy. Furthermore, as we are unable to estimate the elasticity θ_3 of abatement costs $f(\mu_t)$ to green technology $A_{t,g}$ due to data unavailability, we consider three different cases that corresponds to three different values of $\theta_3 \in (0, 1)$.

Figure 3.8 shows the dynamics of our key variables (output, emissions, carbon price, marginal abatement cost, green technology, and global R&D) under a net-zero scenario. The carbon price is significantly driven by the elasticity θ_3. The scenario where $\theta_3 = 1$ (the blue line) is the most optimal in terms of welfare, as the price of carbon is constantly decreasing, which is not the case when $\theta_3 = .7$ and $\theta_3 = .3$. With a higher theta, the output growth rate is also higher as profits are less impacted negatively by the carbon price. This impact on profits in turn lowers the global R&D investments and level. Turning to innovation in green technologies, a higher elasticity lowers the marginal cost of abatement, which leads to a lower carbon price to meet the emissions reduction goal. We note that a scenario where $\theta_3 = 1$ is highly unlikely as carbon prices are increasing nowadays, suggesting that $\theta_3 < 1$.[94]

Figure 3.9, figure 3.10, and figure 3.11 display the counterfactual exercises where the public authorities implement either a fiscal, macroprudential, or monetary policy. Since the level of θ_3 is highly uncertain, we show the transition paths for the 3 values considered above. Focusing, however, on the the case where $\theta_3 = .3$ (the most conservative case), a financial fiscal subsidy, which reverses 70% of the carbon policy revenues to green innovators, is found to be the most effective in steering both growth in green technologies as well as global R&D. The macroprudential policy and QE policy both act as carbon price stabilizers (a lower increase in the first half of the 30 years than the subsidy case). In all scenarios, the carbon price increases in the first 15 years, until the technology is mature enough to trigger higher abatement without having to raise the price on carbon as explained in section 3.4.4.

3.7 Conclusion

In this article, we first conduct an empirical analysis on the role of the ETS in emissions reduction within the EZ using a diff-in-diff analysis, with the US as the control area. We

[94] Further research could be done to investigate the elasticity of θ_3 of abatement cost to green technologies to better characterize the economy dynamics.

find that the cap and trade EU system contributed significantly to emissions reduction. We then rely on a panel data set on the EZ to assess the impacts of fiscal environmental policies and long-term bank lending on green innovation. We find that both the environmental policy and the availability of funds play an important and significant role in boosting green innovation. However, we also find that above a certain threshold, the carbon price has a negative effect on green innovation.

Second, we develop a dynamic general equilibrium model based on the empirical evidence to assess the role fiscal and macro-financial policies can play both in the long-run and in the short-run.

We use a reduced form model to get the long-run transition pathways toward the net-zero transition and find that making abatement technology available and cheap coupled with an optimal environmental policy is the most efficient tool (from a welfare perspective) in achieving climate goals. Relying solely on a carbon price could reach the same target, but comes with higher welfare costs.

Finally, we use a full fledged model incorporating both endogenous green innovation growth and financial intermediaries to quantitatively estimate trends on output and green innovation. We then assess the role subsidies, macroprudential policies, and QE, could play in boosting green innovation. We show that these three policies differently affect the path of the trend growth in green innovation, but that they have the same pro-cyclical dynamics. In addition, we show that financial subsidies are more effective than macroprudential and QE rules in reaching the net-zero while ensuring a lower carbon price over time. This leads us to conclude that policy makers could optimally foster growth in projects that enable cheaper and more effective abatement by giving incentives to financial intermediaries and entrepreneurs. In the context of the fight against climate change, and keeping in mind the ambitious goals that it requires, these findings represent both a glimmer of hope and a call for more action.

Appendices

3.A Empirical Part

3.A.1 Data Sources

The data used[95] in this section were obtained from following sources:

- "Long-term loans granted by the financial sector to domestic non-financial corporation" were extracted from the ECB Statistical Data Warehouse.
- All EZ macro data (output, consumption, government spending, investment, export of goods, export of services, import of goods, import of services, taxes on goods, subsidies on goods) were obtained from the Eurostat database.
- EZ and US Emission data were obtained from the University of Oxford ourworldindata.org database.[96]
- All US macro data are obtained through Fred database.
- Both US and EZ area and countries deflators, as well as crude oil price are extracted from Fred database.
- Quarterly population for all samples are obtained from the OECD database.
- 'Green Patent' data are extracted from the European Patent Office (EPO) database.[97]
- ETS carbon price data are obtained from the European Environment Agency.

[95]All data used were either extracted directly on a quarterly basis or transformed from a monthly frequency to a quarterly frequency.

[96]The only data available for the EZ countries are yearly aggregates. We use a spline to transform yearly emission data to quarterly frequency in order to have a balanced dataset.

[97]Data on green patents are selected through the new search filter introduced by the EPO: "cpc = y02", which allows for identifying patents with green applicability.

3.A.2 Empirical Results

FIGURE 3.1. Parallel Trends Hypothesis

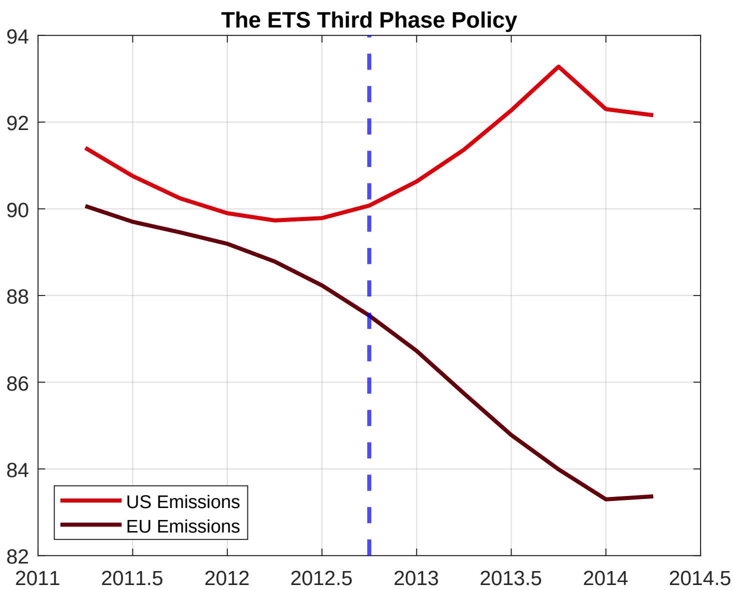


TABLE 3.1
Descriptive Statistics EZ and US.

Variable	Obs	Mean	Std. Dev.	Min	Max
The EZ					
GDP in (Million of Currency)	80	2361.266	344.8842	1725.153	3013.108
Emissions in $GTCO_2$	80	2646960	252602.9	2172255	2985500
Population	80	333.7475	6.509189	319.8963	342.2888
Deflator	80	92.7957	7.953556	77.50166	105.901
Green Patents	80	3111.275	679.2269	1139	4309
Oil Price	80	51.71275	19.41105	20.9	95.61
Gov Spending	80	484.9802	79.18786	336.0314	621.4932
Household Consumption	80	1270.093	167.5075	948.4206	1565.283
Gross capital formation	80	511.4903	70.16437	400.4561	690.9054
Exports of good	80	729.8737	173.5684	450.8578	1028.471
Exports of services	80	245.521	84.31034	127.6664	423.7959
Imports of good	80	681.3173	150.7942	433.557	941.4877
Imports of services	80	230.6274	77.03146	126.3731	414.0665
The US					
GDP in (Million of Currency)	80	3819.033	821.707	2500.714	5436.849
Emissions in $GTCO_2$	80	5668626	340328.9	4634741	6139822
Population	80	306.8736	14.38066	280.4759	327.2556
Deflator	80	95.2847	10.36493	77.396	112.95
Green Patents	80	6782.775	2355.937	2496	10575
Oil Price	80	62.41812	26.59745	19.96	139.96
Gov Spending	80	830.0017	215.3109	466.8755	1204.646
Household Consumption	80	2581.976	568.6365	1653.4	3689.8
Gross capital formation	80	198.5823	38.74323	145.228	281.3773
Exports of good	80	257.3163	73.25903	140.6633	356.7225
Exports of services	80	118.5654	42.93334	54.34583	184.0133
Imports of good	80	402.8038	96.94235	225.3292	538.12
Imports of services	80	84.67429	24.1856	43.94917	124.3208

TABLE 3.2
Descriptive Statistics EZ aggregate.

Variable	Obs	Mean	Std. Dev.	Min	Max
The aggregate EZ					
GDP in (Million of Currency)	864	140894.1	207878.3	1509.7	872335
Population	856	19233.6	24911.43	486	83145
ETS price	846	11.63263	6.868454	3.8696	27.13354
Deflator	864	98.48927	5.351341	80.69107	115.0133
Green Patents	864	202.1134	485.9407	0	2672
Oil Price	864	61.73833	16.73311	30.26	95.61
Gross capital formation	864	29631.05	43870.17	80	189979
Long-term loans	862	196488.1	254660.6	2612.26	920094

TABLE 3.3
ETS Price Impact on Emissions: EZ-US Difference-in-Difference Regression

ln(Emissions per capita) (quarterly)	(1)	(2)	(3)	(4)	(5)	(6)
Policy (Q1 2013)	-0.0614**	-0.0111	0.0186	0.0649***	0.0496**	-0.0170
	(0.0309)	(0.0261)	(0.0276)	(0.0166)	(0.0198)	(0.0350)
Treatment (EZ)	-1.369***	-1.230***	-1.269***	-1.300***	-1.160***	-1.727***
	(0.0861)	(0.0986)	(0.0947)	(0.0741)	(0.0673)	(0.253)
Diff-in-diff Estimator	-0.0730***	-0.112***	-0.121***	-0.191***	-0.137***	-0.0932**
	(0.0276)	(0.0225)	(0.0229)	(0.0255)	(0.0266)	(0.0420)
ln(GDP per capita)	-1.032***	-0.534***	-0.581***	-1.150***	-0.895***	
	(0.168)	(0.202)	(0.187)	(0.184)	(0.152)	
ln(R&D Green) 4 lags		-0.178***				
		(0.0366)				
ln(R&D Green) 8 lags			-0.205***		-0.194***	-0.0957***
			(0.0371)		(0.0377)	(0.0336)
Trade Balance (Goods)				-0.105***	-0.120***	-0.0757***
				(0.0165)	(0.0233)	(0.0276)
Trade Balance (Services)				-0.277***	0.0430	0.168
				(0.0468)	(0.0727)	(0.103)
ln(Oil Price)					-0.00104	0.00745
					(0.0114)	(0.0112)
ln(Consumption per capita)						-1.009***
						(0.335)
ln(Gov Spending per capita)						-0.322
						(0.212)
ln(Investment per capita)						0.127
						(0.111)
Constant	9.159***	10.00***	10.03***	8.947***	9.520***	6.908***
	(0.129)	(0.208)	(0.184)	(0.166)	(0.200)	(0.560)
Observations	160	152	144	160	144	144

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

TABLE 3.4
Green Innovation Drivers: Panel OLS Regression

Green R&D	(1)	(2)	(3)
ETS Price Level (1 year lag)	22.65*		
	(12.92)		
Long-term Loan (1 year lag)	0.0801***		
	(0.0149)		
ETS Price Level (2 years lag)		7.882*	
		(4.167)	
Long-term Loan (2 years lag)		0.0990***	
		(0.0140)	
ETS Price Level (3 years lag)			7.761**
			(3.724)
Long-term Loan (3 years lag)			0.112***
			(0.0140)
GDP per capita	1.502***	1.474***	1.442***
	(0.290)	(0.350)	(0.422)
Constant	-772.8**	-392.9***	-389.4***
	(339.0)	(119.8)	(119.9)
Observations	772	700	628
R-squared	0.969	0.970	0.968
Time fixed effect	Y	Y	Y
Country fixed effect	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: The regression features both time and countries fixed effects that are not reported for simplicity.

TABLE 3.5

Green Innovation Drivers: Panel OLS Regression - Thresholds Effects

Green R&D	(1)	(2)	(3)	(4)	(5)
ETS Price > 5	9.351				
	(27.77)				
ETS Price > 10		13.84			
		(30.19)			
ETS Price > 15			-142.7*		
			(82.42)		
ETS Price > 20				-142.7*	
				(82.42)	
ETS Price > 25					-105.0*
					(58.73)
Long-term Loan (1 year lag)	0.0781***	0.0781***	0.0781***	0.0781***	0.0781***
	(0.0146)	(0.0146)	(0.0146)	(0.0146)	(0.0146)
GDP per capita	1.566***	1.566***	1.566***	1.566***	1.566***
	(0.292)	(0.292)	(0.292)	(0.292)	(0.292)
Constant	-172.2***	-176.7***	-162.8***	-162.8***	-162.8***
	(38.05)	(41.19)	(46.63)	(46.63)	(46.63)
Observations	790	790	790	790	790
R-squared	0.968	0.968	0.968	0.968	0.968
Time fixed effect	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: The regression features both time and countries fixed effects that are not reported for simplicity.

TABLE 3.6

Green Innovation Drivers: Panel OLS Regression - Robustness A

Green R&D	(1)	(2)	(3)	(4)	(5)
ETS Price > 5	16.87				
	(23.13)				
ETS Price > 10		20.79			
		(26.25)			
ETS Price > 15			-150.1*		
			(82.27)		
ETS Price > 20				-150.1*	
				(82.27)	
ETS Price > 25					-111.4*
					(56.91)
Long-term Loan (2 year lag)	0.0972***	0.0972***	0.0972***	0.0972***	0.0972***
	(0.0141)	(0.0141)	(0.0141)	(0.0141)	(0.0141)
GDP per capita	1.539***	1.539***	1.539***	1.539***	1.539***
	(0.343)	(0.343)	(0.343)	(0.343)	(0.343)
Constant	-203.2***	-207.2***	-186.4***	-186.4***	-186.4***
	(40.35)	(43.74)	(43.33)	(43.33)	(43.33)
Observations	718	718	718	718	718
R-squared	0.969	0.969	0.969	0.969	0.969
Time fixed effect	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: Although we do not present the time and countries fixed effects (for simplicity), the regression capture both time and countries fixed effects.

TABLE 3.7

Green Innovation Drivers: Panel OLS Regression - Robustness B

Green R&D	(1)	(2)	(3)	(4)	(5)
ETS Price > 5	16.84				
	(19.71)				
ETS Price > 10		14.38			
		(23.02)			
ETS Price > 15			11.77		
			(26.14)		
ETS Price > 20				-146.1*	
				(80.79)	
ETS Price > 25					-108.5*
					(55.39)
Long-term Loan (3 year lag)	0.109***	0.109***	0.109***	0.109***	0.109***
	(0.0143)	(0.0143)	(0.0143)	(0.0143)	(0.0143)
GDP per capita	1.455***	1.455***	1.455***	1.455***	1.455***
	(0.409)	(0.409)	(0.409)	(0.409)	(0.409)
Constant	-214.4***	-212.0***	-209.3***	-197.6***	-197.6***
	(47.27)	(48.76)	(51.85)	(46.72)	(46.72)
Observations	646	646	646	646	646
R-squared	0.968	0.968	0.968	0.968	0.968
Time fixed effect	Y	Y	Y	Y	Y
Country fixed effect	Y	Y	Y	Y	Y

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Notes: Although we do not present the time and countries fixed effects (for simplicity), the regression capture both time and countries fixed effects.

3.B Model Part

3.B.1 Calibration

TABLE 3.8
Standard parameter values (quarterly basis)

	Calibrated parameters	Values
β	Discount factor	0.9975
α	Capital share	0.33
δ	Depreciation rate of capital	0.025
h	Habits formation parameter	0.8
σ	Risk aversion	2
φ	Disutility of labor	1
θ	Price elasticity	11
$\bar{L}$	Labor supply	0.33
$\bar{L}_s$	Labor supply	0.15
$\bar{g}/\bar{y}$	Public spending share in output	0.4

TABLE 3.9
Environmental and Entrepreneurs parameter values (quarterly basis)

	Calibrated parameters	Values
η	Material share	.125
a	Damage function parameter	1.004
b	Damage function parameter	0.02
υ_1^o	Temperature parameter	0.5
υ_2^o	Temperature parameter	0.00125
E^*	Emissions from the rest of the world	1.59
ϑ	Carbon intensity	0.287
δ_x	CO_2 natural abatement	0.0021
θ_1	Abatement cost parameter	0.05
θ_2	Abatement cost parameter	2.7
θ_3	Abatement cost parameter	-0.6

TABLE 3.10
Financial parameter values (quarterly basis)

	Calibrated parameters	Values
γ_I	Capital adjustment cost	1.728
ω	Proportional transfer to the entering bankers	0.008
λ	Steady state risk weight on loans	0.43
θ_B	Probability of staying a banker	0.98

TABLE 3.11

Prior and Posterior distributions of structural parameters

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean *[0.050;0.950]*
Shock processes:					
Std. productivity	σ_A	$\mathcal{IG}_1$	0.001	0.005	0.0061 *[0.0050 ; 0.0071]*
Std. emission	σ_E	$\mathcal{IG}_1$	0.001	0.005	0.0082 *[0.0070 ; 0.0093]*
Std. R&D	σ_{A_s}	$\mathcal{IG}_1$	0.001	0.005	0.0352 *[0.0307 ; 0.0401]*
Std. green innovation	σ_{A_g}	$\mathcal{IG}_1$	0.001	0.005	0.0451 *0.0392 ; 0.0512]*
AR(1) productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.9641 *[0.9349 ; 0.9934]*
AR(1) emission	ρ_E	$\mathcal{B}$	0.50	0.20	0.9796*[0.9636 ; 0.9983]*
AR(1) R&D	ρ_{A_s}	$\mathcal{B}$	0.50	0.20	0.5456 *[0.3704 ; 0.7129]*
AR(1) green innovation	ρ_{A_g}	$\mathcal{B}$	0.50	0.20	0.9237 *[0.8509 ; 0.9832]*
Endogenous growth parameters:					
Trend slope	$\gamma_y - 1$	$\mathcal{G}$	0.005	0.001	0.0043*[0.0029 ; 0.0058]*
Green innovation trend slope	$\gamma_{A_g} - 1$	$\mathcal{G}$	0.01	0.002	0.0100 *[0.0067 ; 0.0132]*
R&D investment exogenous trend	γ_{V_s}	$\mathcal{N}$	1	0.20	1.0020 *[1.0011 ; 1.0027]*
Green investment exogenous trend	γ_{V_g}	$\mathcal{N}$	1	0.20	1.0097 *[0.9951 ; 1.0276]*
R&D investment elasticity	η_g	$\mathcal{B}$	0.15	0.20	0.0721 *[0.0001 ; 0.1501]*
Green investment elasticity	η_s	$\mathcal{B}$	0.125	0.20	0.1088 *[0.0001 ; 0.2170]*
Log-marginal data density					666.668864

Notes: $\mathcal{B}$ denotes the Beta, $\mathcal{IG}_1$ the Inverse Gamma (type 1), $\mathcal{N}$ the Normal, and $\mathcal{G}$ the Gamma distribution.

TABLE 3.12
Steady state values

	Baseline	Macroprudential	Subsidies	QE
Output	0.8318	0.8330	0.8401	0.8318
Consumption	0.3776	0.3781	0.3813	0.3776
Emissions	0.1749	0.1749	0.1750	0.1749
Emissions to Output	0.2102	0.2100	0.2083	0.2102
Overall Technology	1	1.0102	1.0720	1
Green Projects	0.1055	0.1065	0.1130	0.1055
Abatement Cost	0.0536	0.0535	0.0523	0.0536
Abatement Share	0.2675	0.2685	0.2742	0.2675
Tax in Euros	28.50	28.50	28.50	28.50
Entrepreneurs' Profits	0.0756	0.0750	0.0756	0.0756
Entrepreneurs' Risk Premium	0.0029	0.0020	0.0029	0.0029
Banks' Capital Ratio	0.1581	0.1107	0.1581	0.1581

3.B.2 Figures

FIGURE 3.2. Net-Zero Transition Pathways - 2030

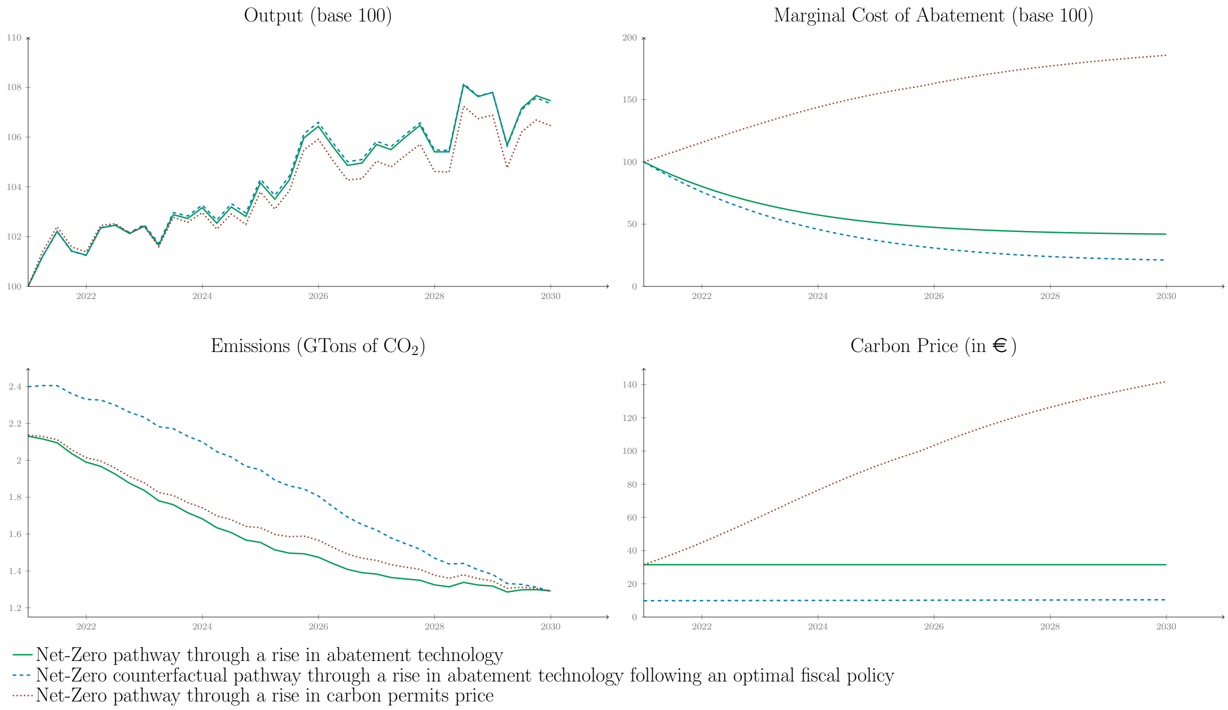


FIGURE 3.3. The Economy Trend Growth Rate (in %).

FIGURE 3.4. The Green Innovation Trend Growth Rate (in %).

FIGURE 3.5. Counterfactual Subsidy Exercise.

FIGURE 3.6. Counterfactual Macroprudential Exercise.

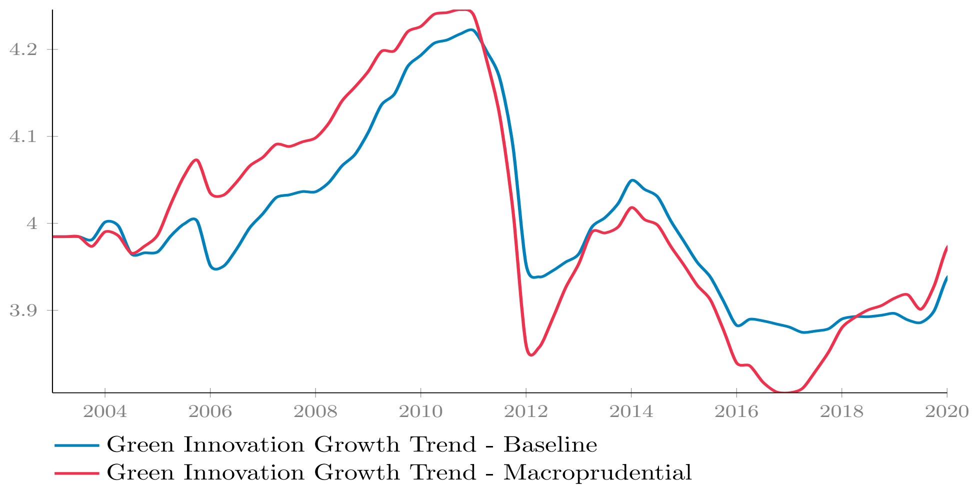


FIGURE 3.7. Counterfactual QE Exercise.

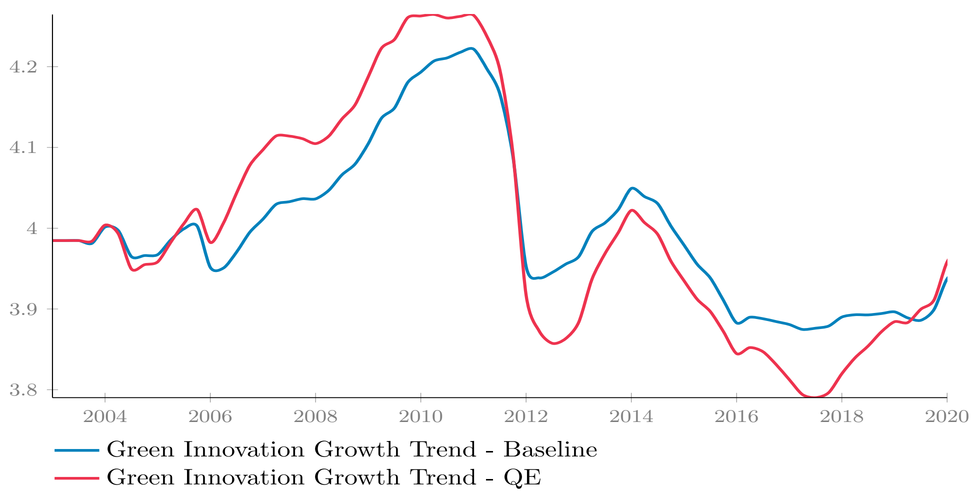


FIGURE 3.8. The Net-Zero Transition Pathway Under Different Abatement to Green Technology Elasticities θ_3.

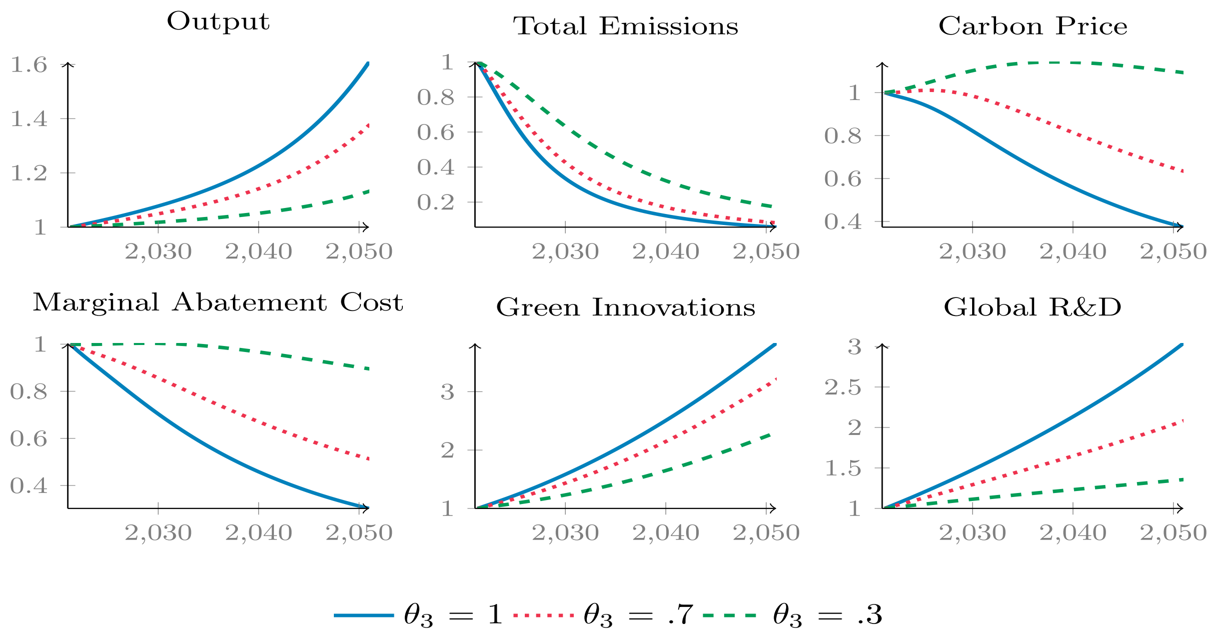


FIGURE 3.9. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = 1$).

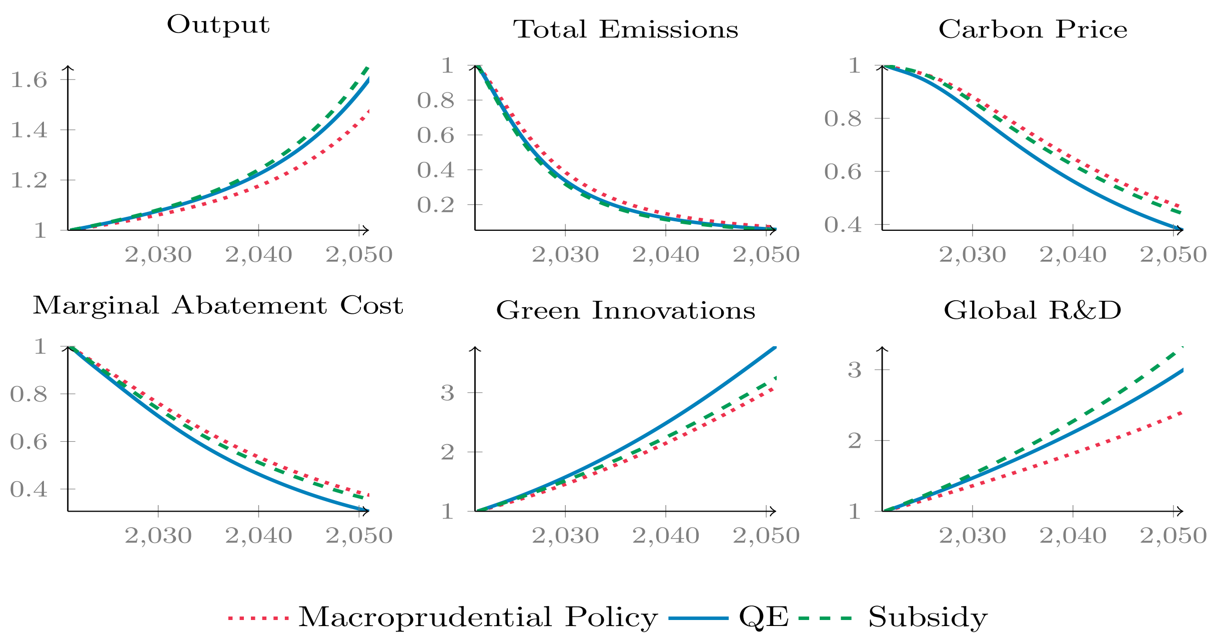


FIGURE 3.10. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = .7$).

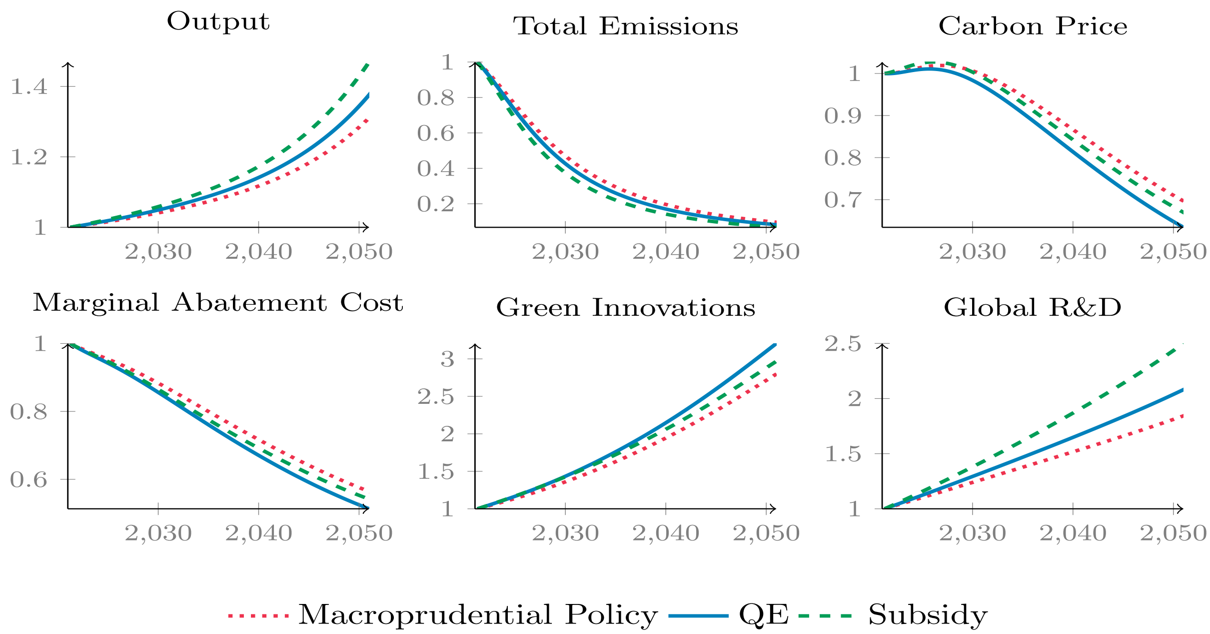
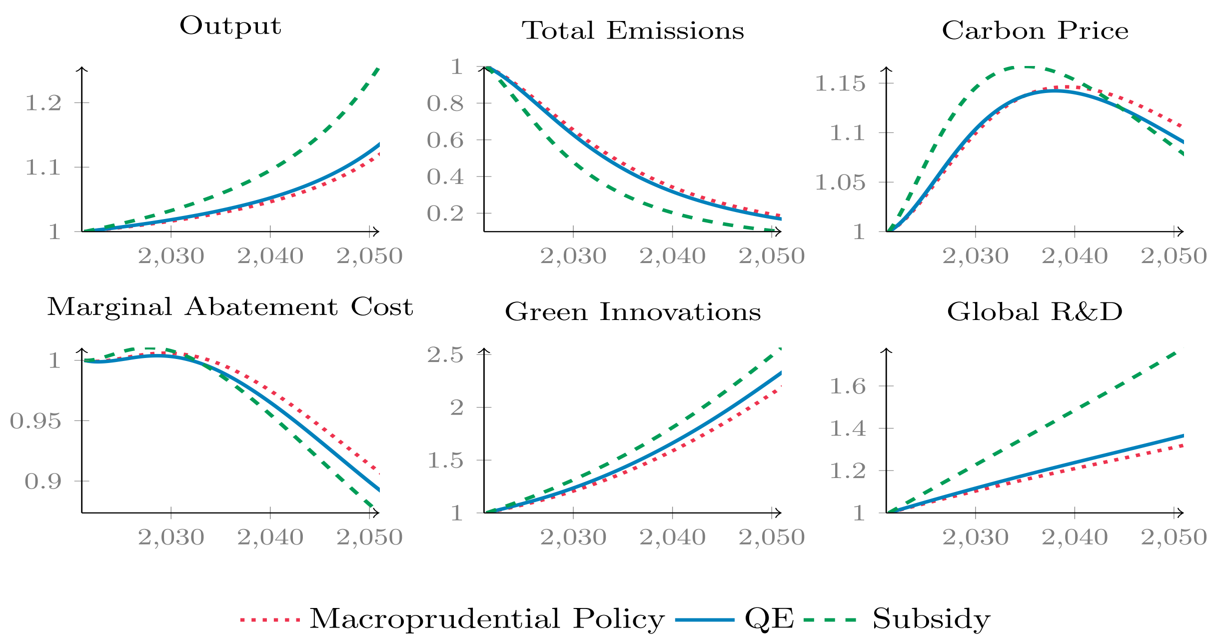


FIGURE 3.11. The Net-Zero Transition Pathway Under The Three Macro-Financial Policies (with $\theta_3 = .3$).



3.B.3 Model Equilibrium

3.B.3.1 The Social Planner Solution

The planners social problem for the households reads as following:[98]

$$
\begin{aligned}
&\max E_t \sum_{i=0}^{\infty} \beta^i \Bigg(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \\
&+ \lambda_t (W_t L_t + W_t^s \bar{L}s + W_t L_t + R_t^K K_t + \Pi_t + T_t + R_t B_t - C_t - I_t - B_{t+1}) \\
&+ \lambda_t \varrho_t^C ((1-\delta) K_t + I_t - K_{t+1}) \\
&+ \lambda_t q_t (Y_t - W_t L_t - R_t^K K_t - f(\mu_t) Y_t - \Pi_t) \\
&+ \lambda_t \Psi_t (d(T_t^o) K_t^\alpha L_t^{1-\alpha} - Y_t) \\
&+ \lambda_t \S_t^X (X_t - \eta X_{t-1} - E_t - E^*) \\
&+ \lambda_t \S_t^T (T_t^o - \upsilon_1^o (\upsilon_2^o X_{t-1} - T_{t-1}^o) - T_{t-1}^o) \\
&+ \lambda_t \S_t^E (E_t - (1-\mu_t) \varphi_t Y_t) \Bigg),
\end{aligned}
$$

where the Social Cost of Carbon SCC_t is $\S_t^X$, and Ψ_t the marginal cost component related to the firms problem.

The first order conditions determining the SCC_t are the ones with respect to T_t^o, X_t, E_t, μ_t and Π_t:

[98] Please note that the social planner problem is not impacted by the financial intermediaries nor by the R&D entrepreneurs or the green innovators.

$$\lambda_t \S_t^T = \beta(1 - \upsilon_1^o)\lambda_{t+1}\S_{t+1}^T - \lambda_t \Psi_t \varepsilon_t^A \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t-1}^\alpha L_t^{1-\alpha} \tag{3.1}$$

$$\lambda_t \S_t^X = \beta(\upsilon_1^o \upsilon_2^o)\lambda_{t+1}\S_{t+1}^T + \beta\eta\lambda_{t+1}\S_{t+1}^X \tag{3.2}$$

$$\lambda_t \S_{t,k}^E = g(\varkappa)\lambda_t \S_t^X \tag{3.3}$$

$$\lambda_t q_{t,k} f'(\mu_{t,k}) = \varphi_{t,k}\lambda_t \S_{t,k}^E \tag{3.4}$$

$$\lambda_t = \lambda_t q_{t,k}. \tag{3.5}$$

Rearranging these FOCs we obtain the following SCC_t:

$$\S_t^T = (1 - \upsilon_1^o)\Lambda_{t,t+1}\S_{t+1}^T - \sum_k \Psi_{t,k} \frac{\partial d(T_t^o)}{\partial T_t^o} K_t^\alpha L_t^{1-\alpha} \tag{3.6}$$

$$\S_t^X = (\upsilon_1^o \upsilon_2^o)\Lambda_{t,t+1}\S_{t+1}^T + \eta\Lambda_{t,t+1}\S_{t+1}^X \tag{3.7}$$

$$\S_t^E = g(\varkappa)\S_t^X \tag{3.8}$$

$$f'(\mu_t) = \varphi_t \S_t^E \tag{3.9}$$

The competitive equilibrium problem for the firms reads as following:

$$\begin{aligned}
&\max E_t \sum_{i=0}^{\infty} \Bigg(\left(\frac{P_{jt}}{P_t} Y_t - W_t L_t - R_t^K K_t - f(\mu_t) Y_t - \tau_t E_t - \Pi_t \right) \\
&+ \lambda_t \Psi_t (d(T_t^o) K_{t-1}^\alpha L_t^{1-\alpha} - Y_t) \\
&+ \lambda_t \S_t^F (E_t - (1 - \mu_t)\varphi_t Y_t) \Bigg)
\end{aligned}$$

The first order conditions determining the tax rate τ_t are the ones with respect to E_t and

μ_t:

$$\S_t^F = \tau_t \tag{3.10}$$

$$f'(\mu_t) = \S_t^F \varphi_t \tag{3.11}$$

Thus, from both the household and firm FOCs, we get:

$$\S_t^F = \tau_t \tag{3.12}$$

$$\S_t^F = \S_t^E \tag{3.13}$$

$$f'(\mu_t) = \S_t^E \varphi_t \tag{3.14}$$

$$\S_t^T = (1 - \upsilon_1^o)\Lambda_{t,t+1}\S_{t+1}^T - \Psi_t \frac{\partial d(T_t^o)}{\partial T_t^o} K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{3.15}$$

$$\S_t^X = (\upsilon_1^o \upsilon_2^o)\Lambda_{t,t+1}\S_{t+1}^T + \eta\Lambda_{t,t+1}\S_{t+1}^X \tag{3.16}$$

$$\S_t^E = \S_t^X \tag{3.17}$$

3.B.3.2 The Firms

The firm maximization of profits reads:

$$\Pi_{jt} = \max_{P_{jt}, Y_{jt}} \left(\frac{P_{jt}}{P_t} - MC_t^f \right) Y_{jt}, \tag{3.18}$$

s.t.

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t. \tag{3.19}$$

The first order condition yields:

$$\frac{P_{jt}}{P_t} = \frac{\theta}{\theta - 1} MC_t^f \tag{3.20}$$

Now using the pricing equation $P_t = (\int_0^{A_{t,s}} P_{jt}^{1-\theta} dj)^{\frac{1}{1-\theta}}$ we get:

$$\frac{P_{jt}}{P_t} = A_{t,s}^{\frac{1}{\theta-1}} \tag{3.21}$$

Thus, we can rewrite the first order condition as:

$$\frac{\theta}{\theta-1} MC_t^f = A_{t,s}^{\frac{1}{\theta-1}}. \tag{3.22}$$

Therefore,

$$\Pi_{jt} = \left(\frac{P_{jt}}{P_t} - MC_t^f\right) Y_{jt}, \tag{3.23}$$

$$= \frac{1}{\theta} \frac{Y_t}{A_{t,s}} \tag{3.24}$$

Turning now to the Cobb-Douglas production function, we use the inputs market-clearing conditions $\int_0^{A_{t,s}} L_{jt} dj = A_{t,s} L_t$ and $\int_0^{A_{t,s}} K_{jt} dj = A_{t,s} K_t$ to retrieve the final form of the production function:

$$Y_t = A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^{\alpha} L_t^{1-\alpha}. \tag{3.25}$$

The rest of the first order condition remains similar to the ones presented in the reduced form model.

3.B.3.3 The Households, Innovators, and Financial intermediaries

For the household, the entrepreneurs, and the banking sector, all equilibrium equations are presented in the core text.

3.B.4 Carbon Cap and Green Innovation

By substituting the environmental cap policy equation ($E_t = Cap_t$) into the emissions flow equation ($E_t = (1-\mu_t)\upsilon Y_t$), we get:

$$\mu_t = 1 - \frac{Cap_t}{\upsilon Y_t} \tag{3.26}$$

Using the FOC on abatement equation (3.32):

$$\text{Carbon Price}_t = \theta_1\theta_2 \frac{\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1}}{\upsilon} A_{t,g}^{-\theta_3} \tag{3.27}$$

We see that the carbon price could be steered by either Cap_t and/or $A_{t,g}$.[99] It is then clear that when:

$\Delta A_{t,g}^{\theta_3} > \Delta\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1} \Rightarrow$ Carbon Price$_t$ decrease.

While when:

$\Delta A_{t,g}^{\theta_3} < \Delta\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2 - 1} \Rightarrow$ Carbon Price$_t$ increase.

Turning now to the abatement cost, we have:

$$f(\mu_t) = \theta_1 \left(1 - \frac{\text{Cap}_t}{\upsilon Y_t}\right)^{\theta_2} A_{t,g}^{-\theta_3} \tag{3.28}$$

Likewise, when:

$\Delta A_{t,g}^{\theta_3} > \Delta\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2} \Rightarrow$ the per unit abatement cost decrease.

While when:

$\Delta A_{t,g}^{\theta_3} < \Delta\left(1 - \frac{Cap_t}{\upsilon Y_t}\right)^{\theta_2} \Rightarrow$ the per unit abatement cost increase.

As the total abatement cost $Z_t = f(\mu_t)Y_t$ enters the banks returns equation $R_{t,e} = \frac{\phi_{RD_g}(Z_t + Q_{t,e})}{Q_{t-1,e}}$, a drop in Z_t would reduce the returns $R_{e,t}$. In turn, the decrease in $R_{e,t}$ gives less incentives to financial intermediaries to finance green equity innovators, which end up

[99] The changes on Y_t being very small over the business cycle with respect to climate damages, we don't focus on their effects on carbon prices.

decreasing their overall number of innovations $A_{t,g}$.

3.B.5 Balanced Growth Path Equilibrium

The growth rate of Γ_t determines the growth rate of the economy along the balanced growth path. This growth rate is denoted by γ_t^Y, where:

$$\Gamma_t = \gamma_t^Y \Gamma_{t-1} \tag{3.29}$$

Stationary variables are denoted by lower case letters, whereas variables that are growing are denoted by capital letters. For example, in the growing economy output is denoted by Y_t. De-trended output is thus obtained by dividing output in the growing economy by the level of growth progress:

$$y_t = \frac{Y_t}{\Gamma_t} \tag{3.30}$$

Emissions, which we denote by E_t, in the growing economy are given as follows:

$$E_t = (1 - \mu_t)\upsilon Y_t \tag{3.31}$$

where υ the elasticity of emissions to output.

Thus, in the de-trended economy, emissions law of motion reads as following:

$$e_t = (1 - \mu_t)\upsilon y_t \tag{3.32}$$

where:

$$e_t = \frac{E_t}{\Gamma_t} \tag{3.33}$$

The stock of emissions in the atmosphere is denoted by X_t, while the temperature is called T_t^o in the growing economy:

$$X_t = (1-\gamma_d)X_{t-1} + E_t \tag{3.34}$$
$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \tag{3.35}$$

where $(1-\gamma_d)$ the decay rate.

The de-trended X_t and T_t^o read as following:

$$x_t = \frac{(1-\gamma_d)}{\gamma_t^Y} x_{t-1} + e_t \tag{3.36}$$
$$\gamma_t^Y t_t^o = \upsilon_1^o(\upsilon_2^o x_{t-1} - t_{t-1}^o) + t_{t-1}^o \tag{3.37}$$

where:

$$x_t = \frac{X_t}{\Gamma_t} \tag{3.38}$$
$$t_t^o = \frac{T_t^o}{\Gamma_t} \tag{3.39}$$
$$\tag{3.40}$$

In the growing economy, with the above growth progress, the production function is as follows:

$$Y_t = \varepsilon_t^A A_{t,s}^{\frac{1}{\theta-1}} d(T_t^o) K_t^\alpha L_t^{1-\alpha} \tag{3.41}$$

where labour L_t and the technology shock ε_t^A are stationary variables. Furthermore, the climate damage function captures the growth rate Γ_t such that $d(T_t^o) = ae^{-b\left(\frac{T_t^o}{\Gamma_t}\right)^2}$. Capturing the growth rate of the economy within the damage function allows us to simplify the de-trended form of the damage function without a loss of generality.

De-trending the production function, gives the following:

$$\Gamma_t y_t = \varepsilon_t^A A_{t,s}^{\frac{1}{\theta-1}} d(t_t^o) \Gamma_t^\alpha k_t^\alpha {L_t}^{1-\alpha} \tag{3.42}$$

Thus, the growth rate of the economy will satisfy:

$$\Gamma_t = A_{t,s}^{\frac{1}{\theta-1}} \Gamma_t^\alpha, \tag{3.43}$$

with the de-trended production function:

$$y_t = \varepsilon_t^A d(t_t^o) k_t^\alpha {L_t}^{1-\alpha} \tag{3.44}$$

Rewriting the equation (3.43), we retrieve the growth rate of the economy:

$$\Gamma_t = A_{t,s}^{\frac{1}{(\theta-1)(1-\alpha)}} \tag{3.45}$$

The capital-accumulation equation in the growing economy is:

$$K_t = (1-\delta)K_{t-1} + I_{t-1} \tag{3.46}$$

In the de-trended economy, we thus have:

$$k_t = {\gamma_t^Y}^{-1}[(1-\delta)k_{t-1} + i_{t-1}] \tag{3.47}$$

with both capital and investment de-trended variables reading as: $k_t = \frac{K_t}{\Gamma_t}$ and$i_t = \frac{I_t}{\Gamma_t}$, respectively. The wage as shown in the model section reads as following:

$$W_t = (1-\alpha)\Psi_t \frac{Y_t}{L_t} \tag{3.48}$$

The de-trended wages[100] will therefore read as:

$$w_t = (1-\alpha)\Psi_t \frac{y_t}{L_t} \tag{3.49}$$

with the de-trended wage w_t reads as $w_t = \frac{W_t}{\Gamma_t}$.

Moving to the endogenous growth components of our economy, both $A_{t,s}$ and $RD_{t,s}$, as well as $A_{t,g}$ and $RD_{t,g}$ grow at similar rates $A_{t,s}$ and $A_{t,g}$, respectively. The law of motion for both the adjusted global R&D entrepreneurs ($\gamma_{A_{t,g}} = \frac{A_{t,g}}{A_{t-1,g}}$ and $\tilde{RD}_{t,g} = \frac{RD_{t,g}}{A_{t,g}}$) and green innovators ($\gamma_{A_{t,s}} = \frac{A_{t,s}}{A_{t-1,s}}$ and $\tilde{RD}_{t,s} = \frac{RD_{t,s}}{A_{t,s}}$) reads as:

$$\gamma_{A_{t,s}} = \phi_{RD,s}(1+\tilde{RD}_{t-1,s}), \tag{3.50}$$

$$\gamma_{A_{t,g}} = \phi_{RD,g}(1+\tilde{RD}_{t-1,g}), \tag{3.51}$$

With these new forms of technology growth rates, we can derive the de-trended expression for initial investment $N_{t,s} = \eta RD_{t,s} MC_t^{RD,s}$ and skilled labour wages $W_{t,s} = (1-\eta) MC_t^{RD,s} \frac{RD_{t,s}}{L_s^s}$:

$$n_{t,s} = \eta_s \tilde{RD}_{t,s} MC_t^{RD,s} \tag{3.52}$$

$$w_{t,s} = (1-\eta_s) MC_t^{RD,s} \frac{\tilde{RD}_{t,s}}{L_s^s} \tag{3.53}$$

where $N_{t,s} = n_{t,s}A_{t,s}$ and $W_{t,s} = w_{t,s}A_{t,s}$.

To insure a balanced growth path within the economy, we added the two exogenous growth rates $V_{t,g}$ and $V_{t,s}$, which will impact the specific investment on green and R&D expenditures, respectively, in order to ensure stationarity. As explained in the balanced growth path section in the core of this paper, the growth rates of green and global R&D expenditures have been increasing faster than output. As such, to capture this trend in the

[100] We note, that Ψ_t the labour/capital share is a stationnary variable. The same can be noticed for the returns on capital R_t^k, the total marginal cost MC_t, abatement μ_t, and the environmental policy τ_t.

expenditure side of GDP and still satisfy the supply side ratios in output, we introduce the common investment-specific trends $V_{t,s}$ and $V_{t,g}$,[101] which grow at gross rates $\gamma_s^V = \frac{V_{t,s}}{V_{t-1,s}}$ and $\gamma_g^V = \frac{V_{t,g}}{V_{t-1,g}}$. Therefore, the economy's resource constraint reads as:

$$y_t = c_t + i_t + g_t + V_{t,g}n_{t,g} + V_{t,s}n_{t,s} \tag{3.54}$$

where the de-trended variables read as: consumption $c_t = \frac{C_t}{\Gamma_t}$, investment $i_t = \frac{I_t}{\Gamma_t}$, government spending $g_t = \frac{G_t}{\Gamma_t}$, and both $n_{t,s} = \frac{N_{t,s}}{A_{t,s}}$ the initial investment overall technologies and $n_{t,g} = \frac{N_{t,g}}{A_{t,g}}$.

Now that we have the expression for investment specific expenditures for the global R&D sector, we can easily derive the de-trended expression for the aggregate firms' profits Π_t, which will be subject to the same exogenous growth rate defined above:

$$MC_t^{RD,s} = \Pi_t \tag{3.55}$$

$$\Pi_t = \frac{1}{\theta}\frac{Y_t}{A_{t,s}} \tag{3.56}$$

As the marginal cost for R&D is stationnary, the profits would be as well. However, the output over endogenous growth grows at a slightly different growth than the BGP growth rate Γ_t. We thus add the same exogenous growth rate $V_{t,s}$ as follows:

$$\Pi_t V_{t,s} = \frac{1}{\theta} y_t \tag{3.57}$$

[101] Using the fact that $A_{t,s}/V_{t,s} = \Gamma_t$ and $A_{t,g}/V_{t,g} = \Gamma_t$ we get the following economy investment specific growth rate which satisfies the BGP $V_{t,s} = A_{t,s}^{1-\frac{1}{(\theta_P-1)(1-\alpha)}}$ and $V_{t,g} = A_{t,g}A_{t,s}^{-\frac{1}{(\theta_P-1)(1-\alpha)}}$.

Similarly, the green innovation sector de-trending reads as the following:

$$n_{t,g} = \eta_g \tilde{RD}_{t,g} MC_t^{RD,g} \tag{3.58}$$

$$w_{t,g} = (1-\eta_g) MC_t^{RD,g} \frac{\tilde{RD}_{t,g}}{L_s^g} \tag{3.59}$$

$$MC_t^{RD,s} = Q_{t,e} \tag{3.60}$$

with, $N_{t,g} = n_{t,g} A_{t,g}$ and $W_{t,g} = w_{t,g} A_{t,g}$.

The abatement cost Z_t, which is impacted by the level of green innovation in the economy $A_{t,g}$ reads as:

$$Z_t = \theta_1 A_{t,g}^{-\theta_3} \mu_t^{\theta_2} Y_t \tag{3.61}$$

When de-trended, it reads as:

$$Z_t V_{t,z} = \theta_1 \mu_t^{\theta_2} y_t \tag{3.62}$$

where $V_{t,z} = A_{t,g}^{\theta_3} / \Gamma_t$ is the exogenous growth rate which acts to correct for the balanced growth path such that $\gamma_z^V = V_{t,z}/V_{t-1,z}$. Notice that for $\theta_3 = 1$, we retrieve the same BGP exogenous correcting growth rate $V_{t,g}$.

The lump sum taxes T_t grow at the growth rate of the economy Γ_t:

$$g_t = t_t + \tau_t e_t \tag{3.63}$$

with $T_t = t_t \Gamma_t$.

Moving now to the household's maximization of utility problem, the de-trended lifetime welfare maximization reads:

$$\max E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(\Gamma_{t+i} c_{t+i} - h \Gamma_{t+i-1} c_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \tag{3.64}$$

Thus, rewriting the above equation by denoting $\beta_t = \beta(\Gamma_t)^{1-\sigma}$ and $h_t = h(\gamma_t^Y)^{-1}$ we get:

$$\max E_t \sum_{i=0}^{\infty} \beta_t^i \left[\frac{(c_{t+i} - h_t c_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \tag{3.65}$$

Moving now to the financial intermediaries, the de-trended balance sheet reads as:

$$Q_{t,e} \tilde{S}_{t,e} = n_t + b_t \tag{3.66}$$

where the net worth $n_t = \frac{N_t}{A_{t,g}}$ grows at Γ_t, the debt to households $b_t = \frac{B_t}{A_{t,g}}$ grows at $A_{t,g}$, and assets held by the bank $(S_{t,e} = A_{t,g} + RD_{t,g})$ grow at $A_{t,g}$ and satisfy $\tilde{S}_{t,e} = S_{t,e}/A_{t,g} = 1 + \tilde{RD}_{t,g}$.

The de-trended retained earnings reads as:

$$n_t = (\gamma_{t,g}^V)^{-1} (R_{t,e} Q_{t-1,e} \tilde{S}_{t-1,e}) - R_t b_{t-1}) \tag{3.67}$$

The value function de-trended value $V_t^B = v_t^B A_{t,g}$ reads as:[102]

$$v_t^B = maxE_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1-\theta_B) \theta_B^{i-1} n_{t+1+i} \right\} \tag{3.68}$$

[102]We note that Γ_t^B is both stationary as $V_t^B = \Gamma_t^B N_t$. In addition, as Γ_t^B is stationary, Ω_t, and ν_t are also stationary.

Conclusion

This Ph.D. thesis provides a framework to study the nexus between macroeconomics, finance, and the environment. This framework is then used to study how macro-financial policies could consider adapting their role, in light of the climate change mitigation challenge. Equipped with this new tool, we offer several theoretical, methodological, applied, and empirical contributions.

On theoretical grounds, besides the development of a model tailored for macro-financial analysis subject to an environmental constraint, this thesis offers two main contributions. The first theoretical contribution highlights how a market for carbon pricing designed to implement a gradual drop in emissions affects both welfare and risk premia. We show that implementing a carbon policy consistent with EU climate goals is welfare-distortionary, as the necessary carbon price to reach the net-zero target is way above the optimal carbon price recommended by the social planner. Regarding risk premia, we prove that uncertainty in the carbon price resulting from the ETS affects firms' marginal costs, which in turn generates volatility in risk premia. The second theoretical contribution is related to modeling of the abatement technology through an endogenous process. We provide a theoretical framework that is able to account for the complexity of the links between carbon price, investment in green technologies, and the abatement technology used by firms.

On methodological grounds, we propose a new way of estimating the natural rate of interest allowing to take into account the effect of unconventional monetary policies when assessing the monetary policy stance. The shadow natural rate of interest is particularly useful when the effective lower bound on the nominal rate of the central bank is binding. Another methodological contribution is the computation of pathways consistent with the transition to a low carbon economy and featuring both deterministic trends and stochastic processes. Using the extended path algorithm of Adjemian and Juillard [2013], we derive credible transition pathways that exhibit some level of uncertainty at the business cycle frequency.

On applied grounds, we provide both an estimation of the SNRI and an analysis of its drivers, including financial and environmental factors. We show that financial factors play a substantial role in fluctuations of the SNRI, along with standard supply and demand factors. Environmental factors, however, are found to have a negligible effect on the path of the SNRI, whether through emissions shocks, or through the implementation of a carbon price. Another applied contribution arises from the need to mitigate the inefficiencies induced by the market for carbon permits. To address the wedge on welfare, we show that green macroprudential policy is efficient in partially offsetting the welfare loss, while reaching the emissions target and improving financial stability. With respect to risk premia volatility induced by the uncertainty on the carbon price, we find that QE rules that react to changes in risk premia deviations are able to completely offset movements in spread levels and volatility, allowing for a smooth transmission of monetary policy. We also quantify the effect of "green QE' programs compared to standard QE programs. We find that applying sectoral macroprudential weights, as it would break the perfect substitutability between green and brown assets, provides an incentive to central banks to engage in green QE. Choosing between brown and green QE then implies a trade-off between higher output and lower emissions. The last applied contribution is related to finding ways to steer innovation in green technologies. We show that financial subsidies are more effective than macroprudential and QE rules in reaching the net-zero target while ensuring a lower carbon price over time.

Finally, there are also empirical contributions. Using a difference-in-difference regression to analyze the consequences of the implementation of a market for carbon permits, we find that the third phase of the cap and trade ETS system contributed significantly to emissions reduction. We also disentangle the links between carbon price, the level of emissions, and the level of abatement using a panel regression on the Eurozone. We show that long-term loans play an important and significant role in boosting green innovation. However, above a certain threshold, the carbon price is found to have a negative effect on green innovation.

We believe that empirical results and tools provided in this thesis are relevant for policy makers and that it will contribute to the global search for policies enabling a smooth transition to a low carbon economy. However, it is helpful to keep in mind that the actual implementation of such measures would entail various challenges and significant execution costs. To illustrate this, we note that the distinction between green and brown bonds is not as straightforward as we model it in this thesis. As argued by Ehlers et al. [2020], it would be instructive to determine which firms are seriously committed to the net-zero target rather than simply identifying green projects. Another concern would be the measurement of the costs associated with an increased financial stability risk related to loosening the capital requirements on a specific sector. This kind of analysis, however, would be carried on by involved institutions before implementing such policies. Finally, there are also growing concerns about the distributional impact of carbon policies.

With this in mind, an agenda for further research in the continuity of this thesis would involve adding heterogeneous households to the macro-finance E-DSGE framework. Policies aiming at mitigating climate change likely affect differently households across the income distribution, as hinted by Sager [2021]. Modeling these effects would be of great help to understand the potential unintended consequences of moving toward net-zero.

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RÉSUMÉ

Cette thèse développe un cadre d'analyse pour étudier les liens entre macroéconomie, finance et environnement. Ce cadre est ensuite utilisé pour évaluer la façon dont les politiques macro-financières pourraient participer à la lutte contre le changement climatique. Équipés de ce nouvel outil, nous apportons plusieurs contributions théoriques, méthodologiques, appliquées et empiriques.

Nous mettons d'abord en lumière l'impact du changement climatique sur la mesure du taux d'intérêt naturel et proposons un nouvel indicateur mieux adapté lorsque la politique monétaire est contrainte par la borne inférieure effective sur les taux nominaux. Nous étudions ensuite les effets secondaires d'un marché de permis de carbone. Nous montrons que le niveau de prix du carbone requis pour atteindre les objectifs climatiques implique une perte de bien-être qui peut être atténuée par la mise en œuvre de poids sectoriels dans les exigences de capital des banques favorables aux secteurs les moins intensifs en carbone. Le design d'un marché de permis de carbone implique également une incertitude sur le prix du carbone qui conduit à une volatilité des primes de risque. Cette volatilité peut être supprimée par l'implémentation de règles d'assouplissement quantitatif. Enfin, nous fournissons des résultats empiriques sur les interactions entre le prix du carbone, le niveau des émissions et la technologie de réduction des émissions. Nous montrons que la troisième phase de l'ETS a contribué de manière significative à la réduction des émissions. Nous trouvons également que les prêts à long terme jouent un rôle important et significatif dans la stimulation de l'innovation verte. Cependant, au-delà d'un certain seuil, le prix du carbone a un effet négatif sur l'innovation verte. Nous construisons ensuite un modèle avec croissance endogène de la productivité et de la technologie d'abattement qui réplique les résultats empiriques et nous l'utilisons pour montrer comment les politiques publiques pourraient orienter la croissance vers les technologies vertes, allégeant ainsi le fardeau des entreprises lié à l'augmentation du prix du carbone. Nous calculons également des trajectoires de transition compatibles avec l'objectif net-zéro qui illustrent la façon dont la technologie d'abattement endogène affecte la dynamique de réduction des émissions.

MOTS CLÉS

Politiques Macro-financières, Trajectoires de Mitigation Climatique, Environnement de Taux Bas, Technologie d'Abattement.

ABSTRACT

This thesis provides a framework to study the nexus between macroeconomics, finance, and the environment. This framework is then used to assess how macro-financial policies could take part in climate change mitigation. Equipped with this new tool, we provide several theoretical, methodological, applied, and empirical contributions.

We first shed light on the impact of climate change on the measurement of the natural rate of interest and propose a new indicator that is better suited when monetary policy is constrained by the effective lower bound on nominal rates. We then study the side effects of a market for carbon permits. We show that the level of carbon price required to achieve climate goals imply a loss in welfare that can be mitigated through the implementation of sector-specific weights in the capital requirements of banks favorable to less carbon intensive sectors. The design of a cap and trade policy also implies uncertainty on the price of carbon that leads to volatility in risk premia. This volatility can be offset by means of quantitative easing rules. Finally, we provide empirical evidence on the interactions between the carbon price, the level of emissions, and the abatement technology. We find that the third phase of the ETS contributed significantly to emissions reduction. We also show that long-term loans play an important and significant role in boosting green innovation. However, above a certain threshold, the carbon price is found to have a negative effect on green innovation. We then proceed to build a model with endogenous growth in productivity and abatement technology to match the evidence. The endogenous abatement technology model is used to show how public policies could steer growth in green technologies, therefore easing the burden on firms related to the rise in the carbon price. We also compute transition pathways consistent with the net-zero target, which illustrate how the endogenous abatement technology affects dynamics in emissions reduction.

KEYWORDS

Macro-financial Policies, Climate Mitigation Pathways, Low Rates Environment, Abatement Technology.