



Novel concepts for theoretical and applied problems of electromagnetism. Part 1: Absolute representation of the fields in the spacetime map. Part 2: Low-volume optical antennas for bolometer applications

Marina Yakovleva

► To cite this version:

Marina Yakovleva. Novel concepts for theoretical and applied problems of electromagnetism. Part 1: Absolute representation of the fields in the spacetime map. Part 2: Low-volume optical antennas for bolometer applications. Electromagnetism. Université Paris-Saclay, 2021. English. NNT: 2021UPASP129 . tel-03679218

HAL Id: tel-03679218

<https://theses.hal.science/tel-03679218>

Submitted on 26 May 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Novel concepts for theoretical and applied
problems of electromagnetism.
Part 1: Absolute representation of the fields
in the spacetime map.
Part 2: Low-volume optical antennas
for bolometer applications.

Thèse de doctorat de l'université Paris-Saclay

Ecole doctorale n°572 Ondes et Matière (EDOM)

Spécialité de doctorat : Physique

Unité de recherche : Université Paris-Saclay, CNRS,

Centre de Nanosciences et de Nanotechnologies, 91120, Palaiseau, France

Graduate School : Physique. Référent : Faculté des sciences d'Orsay

**Thèse présentée et soutenue à Paris-Saclay,
le 9 Décembre 2021, par**

Marina YAKOVLEVA

Composition du Jury

Jean-Jacques GREFFET

Professeur, IOGS, Université Paris-Saclay

Président

Jonathan GRATUS

eq. MCF HDR, Physics Dept., Lancaster University

Rapporteur & Examineur

Agnès MAÎTRE

Professeure, INSP, Paris-Sorbonne

Rapporteuse & Examinatrice

Renaud BACHELOT

Professeur, L2n, Université de Technologie de Troyes

Examineur

Direction de la thèse

Jean-Luc PELOUARD

Directeur de Recherche, C2N, CNRS

Directeur de thèse

Fabrice PARDO

Chargé de Recherche, C2N, CNRS

Co-encadrant

Nicolas BOUDOU

Ingénieur R&D, LYNRED

Invité

Titre: Nouveaux concepts pour les problèmes théoriques et appliqués de l'électromagnétisme.

Partie 1 : Représentation absolue des champs dans la carte de l'espace-temps.

Partie 2 : Antennes optiques de faible volume pour applications bolométriques.

Mots clés: topologie de l'espace-temps, invariants de Lorentz, champ électromagnétique, nanoantenne optique, absorbeur infrarouge, bolomètre

Résumé: Ce travail se compose de deux parties qui, bien que très différentes l'une de l'autre, appartiennent toutes deux au même domaine : l'électromagnétisme et ses problèmes modernes. Dans la première partie, une géométrisation absolue du champ électromagnétique dans l'espace-temps est présentée, remettant en cause l'affirmation selon laquelle la seule façon de présenter le champ électromagnétique est l'abstraction. Il est démontré qu'avec la topologie absolue de l'espace-temps, deux invariants scalaires suffisent à décrire entièrement le champ électromagnétique. Une attention particulière est accordée au cas d'un champ invariant selon une direction et polarisé, dont la topologie peut être représentée dans l'espace-temps 3D par des lignes (spaghettis électriques), les deux invariants scalaires étant alors réduits à un seul. Contrairement aux lignes du monde des particules, qui sont toujours de type temps, ces lignes transcendent la frontière entre l'espace et le temps. Cette approche du champ électromagnétique ouvre de nouvelles perspectives tant

pour l'enseignement que pour la recherche, notamment l'analyse des propriétés topologiques de ces structures absolues dans l'espace-temps. La seconde partie est consacrée à un problème plus appliqué, celui du développement de nano-absorbeurs qui pourraient devenir la base d'une nouvelle génération de bolomètres. Dans ce travail, il est démontré que grâce aux propriétés exceptionnelles des antennes plasmoniques (absorption totale et fort confinement du champ électromagnétique), il est possible de concevoir des absorbeurs infrarouges dont le volume est fortement réduit (plusieurs ordres de grandeur) par rapport à ceux utilisés dans les microbolomètres actuels. Ces résultats ouvrent la voie à une rupture technologique majeure qui, en réduisant fortement la capacité thermique de l'absorbant, conduit à un nouveau compromis sensibilité - vitesse pour les bolomètres dont les performances devraient alors être proches de celles des photodétecteurs refroidis.

Title: Novel concepts for theoretical and applied problems of electromagnetism.

Part 1: Absolute representation of the fields in the spacetime map.

Part 2: Low-volume optical antennas for bolometer applications.

Keywords: spacetime topology, Lorentz invariants, electromagnetic field, optical nanoantenna, infrared absorber, bolometer

Abstract: This work consists of two parts which, although very different from each other, both belong to the same field: electromagnetism and its modern problems. In the first part, an absolute geometrization of the electromagnetic field in spacetime is presented, challenging the claim that the only way to present electromagnetic field is abstraction. It is demonstrated that with the absolute topology in spacetime, two scalar invariants are sufficient to describe the electromagnetic field. Particular attention is paid to the case of y -invariant p -polarised field, the topology of which can be represented in 3D spacetime by lines (electric spaghetti), the two scalar invariants then being reduced to one. Unlike the world lines of particles, which are always time-like, these lines transcend the boundary between space and time. This new approach of the electromagnetic field opens up new perspectives both for teaching and for research, in particular the

analysis of the topological properties of these absolute structures in spacetime.

The second part is dedicated to a more applied problem, that of the development of nano-absorbers which could become the basis for a new generation of bolometers. In this work, it is demonstrated that thanks to the exceptional properties of plasmonic antennas (total absorption and strong confinement of the electromagnetic field), it is possible to design infrared absorbers whose volume is greatly reduced (several orders of magnitude) compared to those used in current microbolometers. These results pave the way for a major technological breakthrough which, by greatly reducing the thermal capacity of the absorbent, leads to a new sensitivity - speed compromise for bolometers whose performance should then be close to that of cooled photo-detectors.

*To my family,
who supported and developed my passion for the science from an early age;
and to my fiancé,
whose invaluable encouragement inspired me in the most difficult moments.*

Acknowledgments

I would like to express my sincere gratitude to my supervisors, Prof. Jean-Luc Pelouard and Dr. Fabrice Pardo, for their invaluable pieces of advice, constant support, and patience throughout this work. I am extremely grateful that they welcomed me on as a student and that they always believed in me. Their knowledge and plentiful experience encouraged me during the time of my academic research. Without them, it would not be possible to progress at such a fast pace and to bring my work to such a high level. It was a pleasure and an honor for me to have you as my supervisors.

I am thankful to all the members of the jury, who were kind enough to evaluate this work. In particular, I express my gratitude to Prof. Agnès Maître and Dr. Jonathan Gratus, who kindly agreed to review this work. Their encouraging words and thoughtful, detailed comments were very important and helpful to me. I also thank Prof. Jean-Jacques Greffet, Prof. Renaud Bachelot, and Dr. Nicolas Boudou, who so generously took time to participate in the jury and engage in a good and profound discussion.

I would like to thank all my friends from C2N, whose kind help and support have made my study and life in France a wonderful time. I am grateful to all members of the NOE team, who graciously helped me in the laboratory and beyond.

I would especially like to thank the team at the NanoElectroMagnetics laboratory, who made a significant contribution to the beginning of my professional development. I greatly appreciate the kind assistance from my former supervisor Dr. Konstantin Batrakov, Prof. Polina Kuzhir, and Prof. Sergey Maksimenko throughout my scientific path. I wish to extend my special thanks to Prof. Victor Tikhomirov for his wise and inspiring pieces of advice.

Last, but not least, my warm and heartfelt thanks go to my family. I am thankful and indebted to my mother and father, my grandmother and my brother for their unwavering support and trust in me. I am grateful to my fiancé, who has been by my side with me these years. Thank you for your love, encouragement and support throughout the entire thesis process and on a daily basis.

Contents

Résumé français	v
General introduction	1
1 On the way to the absolute description of the electromagnetic field	5
1.1 Introduction	5
1.2 Differential approach of electromagnetism (Maxwell's equations)	10
1.3 Integral formulation of electromagnetism	15
1.4 Generalisation of the Finite Integration Technique	19
1.5 Conclusion	20
2 Absolute description of polarized EM field, the y-invariant, p-polarized case	21
2.1 Introduction	21
2.2 The spaghetti object	22
2.2.1 The 1-form ϕ_e	23
2.2.2 The ZEFS surfaces	25
2.2.3 The electrical spaghetti object	25
2.2.4 ϕ_e , a natural but limited measure	26
2.2.5 The η characteristic parameter, a new EM field invariant	26
2.2.6 Computation of electromagnetic field components from electric spaghetti	27
2.2.7 Direct computation of ZEFSs	31
2.3 Three illustrative examples	32
2.3.1 Electrostatic cylinder	32
2.3.2 Interference of two plane waves	34
2.3.3 Extraordinary transmission in resonant slits	35
2.4 Detailed analysis of the interference of a plane wave and an evanescent wave	38
2.5 Conclusion	43
3 The absolute description of EM field in 4D spacetime	45
3.1 Introduction	45
3.2 Elementary construction of the unique surface pair	46
3.3 Another approach, involving zero-flux surfaces	48
3.4 Approach of electromagnetism by 2-forms in the literature	49
3.4.1 Presentation of the electromagnetic field with the differential forms	50
3.4.2 Our approach	55

3.5	The unique decomposition in 4D spacetime	56
3.6	Electromagnetic spacetime map	57
3.7	Examples	61
3.7.1	Electrostatic cylinder	61
3.7.2	Two plane waves interference	62
3.8	Conclusion	63
4	Conclusion and Outlooks	65
5	Plasmonic structures for efficient absorption	69
5.1	Introduction	69
5.2	Plasmonic structures	72
5.3	Long wave photodetection	76
5.3.1	Current technology	79
5.3.2	Nanoplasmonic devices	82
5.4	Structures at $\lambda/4$	84
5.4.1	A standing wave above the mirror	84
5.4.2	Configurations	85
5.5	Thermal properties and mass vs volume	86
5.6	The problem we aimed to solve	87
5.7	Review on the following chapters	88
6	Metal-insulator-metal structures	93
6.1	Introduction	93
6.2	The MIM mode	96
6.3	1D MIM antennas for bolometer applications	97
6.3.1	Introduction	97
6.3.2	An array of 1D MIM antennas on the mirror	100
6.3.3	1D MIM array distanced from the mirror	105
6.3.4	Optimization of an 1D MIM array at $\lambda/4$ above the mirror	108
6.3.5	Conclusion	125
6.4	2D MIM antennas for bolometers applications	126
6.4.1	Introduction	126
6.4.2	Geometries	127
6.4.3	Metals	137
6.4.4	The metal thickness	145
6.4.5	Cross vs Rectangle	156
6.4.6	Comparison of 2D and 1D periodic arrays	164
6.5	Conclusion	166
7	Insulator-metal-insulator structures	169
7.1	Introduction	169
7.2	The studied structure	171
7.3	The IMI mode	172
7.4	The antenna geometry	176
7.4.1	The metal width	176
7.4.2	The metal thickness	178
7.4.3	The cross-section	178

7.5	The optical response	181
7.6	The dipole approximation	184
7.6.1	Introduction	184
7.6.2	Theory	185
7.6.3	Validation of approach	189
7.7	Experimental validation	191
7.8	Conclusion	197
8	Conclusion and Outlooks	201
	Appendices	207
A	Legal and natural units	207
B	Different languages of electromagnetism	211
C	Metric free approach	219
C.1	Introduction	219
C.2	1-forms and 2-forms	220
C.3	Bivector	224
C.4	General case of n -form	227
C.5	Differential forms and tensors	227
C.6	Hodge dual operator	228
C.7	Differentiation and integration	229
C.8	Clifford algebra	230
D	Electromagnetism as 4D-static magnetic and electric flux equations	233
D.1	Conventional flux in 3D	233
D.2	Integral formulation of electromagnetism	234
D.3	Maxwell's equations from 4D static flux equations	234
E	Riesz demonstration	241
F	S-parameters computation	245
F.1	S -matrix	245
F.2	S-parameters in COMSOL Multiphysics	246
F.3	Analytical extrapolation of simulation results	246
G	The critical coupling for MIM structures	249
G.1	Critical coupling parameters for 1D MIM placed at $\lambda/4$ above the mirror	249
G.2	The Poynting vector behaviour in the case of the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror	251
G.3	The volume density of 1D periodic MIM resonators placed at $\lambda/4$ above the mirror	254
G.4	Critical coupling data for different geometries for 2D MIM	255
G.5	A rectangular half-MIM structure on the perfect mirror	256
G.6	Field maps for complete 2D MIM with different geometries	262
G.7	Critical coupling data for 2D MIM made of different metals	266
G.8	The fitting of the electric current density in 2D MIM made of different metals	267
G.9	The critical coupling for 2D MIM arrays of rectangular geometry, placed at $\lambda/4$ above the mirror	269
H	The critical coupling for IMI structures	273
H.1	Coupling with the incident wave	273

H.2	The fitting of the electric current density in IMI structures	275
H.3	Angular dependence IMI	276
H.4	The volume density of IMI structures	277
Bibliography		279
List of Publications		289

Résumé français

Ce travail se compose de deux parties qui, bien que très différentes l'une de l'autre, appartiennent toutes deux au même domaine : l'électromagnétisme et ses problèmes modernes.

Dans la première partie, une géométrisation absolue du champ électromagnétique dans l'espace-temps est présentée, remettant en cause l'affirmation selon laquelle la seule façon de présenter le champ électromagnétique est l'abstraction. Cette approche du champ électromagnétique ouvre de nouvelles perspectives tant pour l'enseignement que pour la recherche, notamment en considérant les propriétés topologiques de ces structures absolues dans l'espace-temps.

La première partie commence avec le Chapitre 1, où la formulation intégrale des équations de Maxwell basée sur les flux magnétiques et électriques sur des surfaces 2D orientées arbitrairement (espace-temps ou espace-temps) est présentée pour l'espace-temps 4D. Contrairement aux champs électrique ou magnétique qui dépendent d'un choix de référentiel, les flux sur les surfaces sont des scalaires absolus (invariants par la transformation Lorentz). Cependant, le choix des surfaces 2D (maillage de l'espace-temps 4D) est arbitraire. Le but de ce travail est de prouver qu'il existe une topologie absolue du champ électromagnétique dans l'espace-temps. Complétée par deux invariants scalaires, cette topologie décrit entièrement le champ électromagnétique.

Dans le Chapitre 2, le cas d'un problème y -invariant p -polarisé est considéré en raison de son avantage à pouvoir être représenté dans un espace-temps 3D. Dans ce cas, la topologie du champ électromagnétique est représentée de manière absolue par des lignes d'événements que nous nommons spaghettis électriques, et les deux invariants scalaires sont réduits à un. Un nouvel invariant scalaire, le paramètre caractéristique η , a été trouvé pour ces lignes. Leur topologie est présentée pour trois exemples illustratifs : deux cylindres électrostatiques, une interférence de deux ondes planes se propageant en sens inverse, et un réseau de fentes résonantes. Ce dernier cas est particulièrement remarquable, car dans la région d'entonnoir, les spaghettis électriques

présentent des motifs en spirale bien qu'il n'y ait pas de charge en leur centre. Il est démontré qu'ils apparaissent dans l'interférence d'une onde plane incidente et d'une onde évanescente.

Le Chapitre 3 présente le cas général du concept de décomposition unique du flux dans l'espace-temps 4D. Comme l'a démontré Riesz en 1959, la décomposition en deux bivecteurs orthogonaux (l'un de type temps-espace, l'autre de type espace-temps) dans l'espace-temps de Minkowski 4D est unique. Ainsi, la carte électromagnétique absolue (réalisée, par exemple, avec des triangles orientés portant les flux magnétiques et électriques) basée sur des paires de segments infinitésimaux complètement orthogonaux de surfaces 2D, chacun contenant les valeurs absolues des flux électriques et magnétiques, peut être définie d'une manière unique, indépendamment de tout référentiel. Dans certains cas, ces éléments de surface définissent une surface unique, que l'on appelle une lasagne. Cette partie se termine par le Chapitre 4 qui résume les principaux résultats.

La seconde partie est dédiée à un problème plus appliqué : le développement de nano-absorbeurs d'énergie électromagnétique à l'usage des bolomètres. Dans ce travail, nous montrons que les propriétés exceptionnelles des antennes plasmoniques (absorption totale dans des volumes nanométriques) permettent de concevoir des absorbeurs infrarouges dont le volume est plusieurs ordres de grandeur plus petit que celui des micro-bolomètres actuels. Cette réduction drastique de la capacité thermique permet, en déplaçant le compromis sensibilité vitesse, de concevoir une nouvelle classe de bolomètres dont les performances devraient alors être proches de celles des photo-détecteurs refroidis.

Le chapitre 5 donne un aperçu de l'état de l'art des micro-bolomètres et présente le potentiel de bolomètres utilisant des structures plasmoniques comme absorbeurs. Celles-ci, de dimensions inférieures à la longueur d'onde, permettent à la fois un fort confinement du champ électromagnétique et une absorption quasi-totale à la résonance.

Le chapitre 6 est dédié à l'étude de l'absorption dans les réseaux (1D et 2D) de structures métal isolant métal (MIM) placés à un quart de longueur d'onde d'un miroir pour compatibilité avec le procédé de fabrication des micro-bolomètres actuels. Malgré ces conditions inhabituelles d'excitation (champ magnétique nul) les conditions de couplage critique ont été démontrées, correspondant à un mode, des conditions de couplage et une concentration du flux d'énergie spécifiques. Une recherche des absorbeurs de plus petit volume a été menée pour différents métaux et différentes géométries. Enfin, les antennes en forme de croix et de rectangle ont été décrites comme l'assemblage de deux antennes $\lambda/4$ séparées par un conducteur parfait.

Le chapitre 7 montre que, dans le cas d'une excitation à $\lambda/4$ au-dessus d'un miroir, les réseaux d'antennes isolant-métal-isolant (IMI) sont mieux adaptées aux applications bolométriques que les antennes MIM. Dans ce cas, le mode excité est de type dipôle, ce qui a permis d'en faire une description analytique complète. Les paramètres permettant de contrôler la réponse du réseau IMI et son absorption ont été définis. Enfin, l'absorption totale prédite par les simulations numériques a été validée expérimentalement.

Enfin, le chapitre 8 résume les résultats de la deuxième partie de cette thèse, et fournit quelques perspectives pour les développements futurs.

General introduction

This work consists of two parts which, although distant from each other, belong to the same field: electromagnetism and its modern problems.

In the first part, an absolute geometrization of the electromagnetic field in spacetime is presented. Although attempts to geometrize the electromagnetic field were made earlier, starting with Maxwell's tiny wheels and vortices, and then with field lines. Unfortunately, these lines depend on the choice of a reference frame. Now it is commonly accepted that the only way to present the electromagnetic field is the abstract way. This work challenges this claim, demonstrating that the electromagnetic world can be fully described by absolute structures in the 4D spacetime. The first step is to consider electric and magnetic fluxes on arbitrary 2D oriented surfaces (space-space or time-space) in 4D spacetime. These fluxes are absolute (Lorentz invariant) scalar values, as opposed to values such as the strength of the electric or magnetic field, which depend on the choice of a reference frame. However, at this step, the choice of 2D surfaces (meshing of 4D spacetime) is arbitrary. In this work it is shown that it is possible to go further in the search of absolute by defining an absolute topology of the electromagnetic field in spacetime. With this topology, two scalar invariants fully describe this field.

The case of y -invariant p -polarised field which has the advantage of being able to be represented in 3D spacetime is studied in detail. The topology is represented by lines (electric spaghetti) and the two scalar invariants are reduced to one. Unlike the world lines of particles, which are always time-like, these lines transcend the boundary between space and time. This new approach of the electromagnetic field opens up new perspectives both for teaching and for research, especially considering the topological properties of these absolute structures in spacetime.

The second part is devoted to a more applied problem, that of the development of nano-

absorbers which could become the basis for a new generation of bolometer devices. In this work, it is demonstrated that thanks to the exceptional properties of plasmonic antennas (total absorption and high confinement of the electromagnetic field), it is possible to design IR absorbers with a greatly reduced volume (several orders of magnitude) compare to those used in current microbolometers. This technological breakthrough, which makes it possible to greatly reduce the thermal capacity of the absorbent, leads to a new sensitivity - speed compromise for bolometers whose performance should be close to that of cooled photo-detectors.

This work is organized as follows. The first part begins with Chapter 1, where the evolution of presentations of the Maxwell equations is overviewed briefly, including their integral formulation for 4D spacetime. It is based on the magnetic and electric fluxes on oriented 2D surfaces, either space-space or time-space. While the concept of flux on a space-space surface is common, and indeed the official name of the $\mathbf{B}$ vector is the density of magnetic flux, and $\mathbf{D}$ is the density of electric flux (on space-space surfaces), it is not common to call the electric field strength $\mathbf{E}$ as the magnetic flux density on time-space surfaces, the magnetic field strength $\mathbf{H}$ as the electric flux density on time-space surfaces. The generalisation of the flux concept in 4D spacetime is a key point for building an absolute presentation of the electromagnetic field.

In Chapter 2, the case of an y -invariant p -polarised problem is considered. In this case, the electromagnetic field is presented with absolute objects in 3D spacetime. These objects called electric spaghetti (ES) and zero-electric-flux surface (ZEFS) are solid, concrete line and surface of events in a 3D spacetime map, correspondingly. They can be successively time-like, null-like, or space-like, encompassing the boundary between space and time. ESs are characterised by three invariant values: the interval length, the electric flux values ϕ_e , and a new quantity, the characteristic parameter η . Three examples illustrating ES-ZEFS topology are considered: two electrostatic cylinders, an interference of two counterpropagating plane waves, and a y -invariant resonant slit system. The third case is particularly noteworthy. In the funneling region ESs exhibit spiral patterns although there is no charge at their centers. These special topological structures are studied in detail. It is demonstrated that they are endless logarithmic spirals, which appear in the interference of an incident plane wave and an evanescent wave.

In Chapter 3 the general case of the unique flux decomposition concept in 4D spacetime is presented. Although a clue to this decomposition can be inspired by means of a classical electromagnetic problem, the strict proof is based on the two bivector decomposition in 4D Minkowski spacetime which was provided by Riesz in 1959. The existence of such decomposition

is the root of the geometrical absolute representation of the electromagnetic field in 4D spacetime. It is shown that at any event of 4D spacetime, two completely orthogonal infinitesimal segments of 2D surfaces (L_1 and L_2), each containing absolute values of the electric and magnetic fluxes, can be defined in a unique way. One of these surface segments belongs to the time-space type, and the other is the space-space type, with a particular exception where the two surfaces contain the same line of light, and are null-space type. In this way, the absolute electromagnetic map (made, for example, with orientated triangles with the magnetic and the electric fluxes on them) does not need any frame of reference. Following an arbitrary tangent curve, each of the infinitesimal surfaces L_1 or L_2 can be united into infinitely narrow tagliatelle ribbon. In some cases, these ribbons constructed from any different paths define a unique surface, which are called a lasagna.

This part ends with Chapter 4 that summarizes the main results and presents future perspectives for the absolute electromagnetic field presentation.

The second part of the work begins with Chapter 5. This chapter provides an overview of the state of the art of microbolometers and presents how the use of plasmonic structures as absorbers would allow the design of a new generation of bolometers. Having subwavelength dimensions, plasmonic particles can provide light concentration and strong field confinement, and as a result can absorb all the incident energy in tiny volumes. The considerably reduced volume of absorbers, compared to that of modern industrial structures, allows to shift the fundamental trade-off of the bolometer between sensitivity (the level of noise) and time response towards a faster and more sensitive behavior. While operating at room temperature, these nanobolometers would have performances closer to those of cooled photo-detectors.

Chapter 6 is devoted to the study of MIM (metal-insulator-metal) structures as absorbers. In this chapter, arrays of MIM antennas are placed in unusual conditions for these structures – the node of the magnetic field of the standing wave at the distance $\lambda/4$ above a mirror. This is the condition set by our industrial partners to keep compatibility with the current fabrication process. In this condition the physical behaviour of these antennas, such as the coupling with an incident wave and the energy funnel effect towards the antenna, changes their character. These two aspects are studied for 1D and 2D periodic MIM arrays.

It is found that, in the case of a 1D MIM array placed at $\lambda/4$ above a mirror, the energy funnel is formed by the electric and magnetic fields of the mode, unlike the case of the classical MIM configuration for which the lower metal layer acts as a mirror. In the case of a 2D MIM array placed at $\lambda/4$ above a mirror, the funnel effect is due to the electric field of the standing

wave and the magnetic field of the mode. Additionally, the most suitable metal and geometry for the smallest absorbers are defined. It is shown that rectangle and cross shaped antennas can be described by two $\lambda/4$ antennas separated by a perfect conductor (instead of the usual $\lambda/2$ antenna). The thickness of the conductor is zero in the case of the rectangle while it is finite in the case of the cross. The application of this model opens perspectives to reconsider the description of the MIM mode and for a more in-depth study of complex MIM structures.

In Chapter 7, it is demonstrated that in the case of IMI (insulator-metal-insulator) antennas placed at the distance $\lambda/4$ above a mirror, the volume density of absorbers can be even lower than that of 2D MIM arrays. The parameters to control the response of the IMI array and its losses were defined. The excited mode in this case is similar to a dipole-like mode. As the IMI structures are found to be the best suited to the objectives of this work, an experimental validation of the total absorption is provided for an IMI array placed at about $\lambda/4$ above a mirror. Good agreement with the numerical simulations is shown when additional losses were introduced to the metal. It is due to additional scatterings for electrons in the metal layer due to its ultra-small thickness (i.e. smaller than the electron mean free path) and imperfections in the fabrication of this thin layer.

Finally, Chapter 8 summarises the results of the second part of this thesis, and provides some prospects for future developments.

Chapter 1

On the way to the absolute description of the electromagnetic field

1.1.

Introduction

As a result of the first relativistic revolution brought about by Giordano Bruno and Galileo Galilei [1], the position and speed of an object, be it a butterfly, a cannonball or a planet, are no longer regarded as absolute physical quantities, bound to the Earth's frame of reference. Instead, these quantities are treated as relativistic, differing from one frame of reference to another, and all Galilean inertial frames of reference are assumed to be equivalent, with the same physical laws.

The physical quantities of the electromagnetic world, at first described by Maxwell [2], were based on physical quantities that depended on the chosen frame of reference. These quantities are now regarded as the components of the tensorial fields $F_{ij} = (-\mathbf{E}, \mathbf{B})$ and $\mathcal{G}_{ij} = (\mathbf{H}, \mathbf{D})$ [3, 4]. Since the second relativistic revolution in 1905, these electromagnetic field components have been regarded as fully relativistic physical quantities, as well as space and time [5]. Time is no longer an absolute quantity, independent of the chosen frame of reference, but is combined with space coordinates in Lorentz transformations, just as electric and magnetic field components are combined into the electromagnetic field.

The tensorial formulation of Maxwell's equations $\partial_i \mathcal{F}^{ij} = 0$ (first group, with $\mathcal{F}$ the dual of F) and $\partial_i G^{ij} = J^j$ (second group in natural units with $c = 1$, and G the dual of $\mathcal{G}$, J the 4-vector current) is referred to as covariant formulation. Maxwell's equations are independent of the

chosen frame of reference while coordinates and field components are relative to it. This raises the question of whether it is possible to express electromagnetic laws using invariant quantities only. Poincaré was the first to find mathematical invariants under the Lorentz transformations [6]. The spacetime interval computed between two spacetime events is the most famous of these. Poincaré also described invariants of the electromagnetic field. The original formulation was written in a somewhat cryptic form between equations (7) and (8) of §5 of Ref. [6]. Using modern formulation, the electromagnetic quantities $\frac{1}{2}F^{ij}F_{ij} = B^2 - E^2$, $\frac{-1}{2}\mathcal{G}^{ij}\mathcal{G}_{ij} = H^2 - D^2$, $\frac{-1}{4}\mathcal{F}^{ij}F_{ij} = \mathbf{E} \cdot \mathbf{B}$, and $\frac{-1}{4}G^{ij}\mathcal{G}_{ij} = \mathbf{H} \cdot \mathbf{D}$ are invariants defined at every spacetime event [3, 4].

Poincaré’s mathematical insights and Einstein’s groundbreaking discovery, that time is not absolute [7], were brought together by Minkowski in 1908 [8, 9]. He declared that space and time should be considered as a spacetime continuum: *"From now onwards space by itself and time by itself will recede completely to become mere shadows and only a type of union of the two will still stand independently on its own"* [10]. The conceptual simplicity of Minkowskian spacetime is largely underestimated [11, 12]. The adjective *invariant*, meaning “invariant across all Lorentz transformations” or “has the same value regardless of the inertial frame of reference” is commonly found in the literature and in textbooks. And yet the adjective *absolute*, meaning “which can be written in The absolute common Minkowski’s Spacetime Map (MSM) ¹” is simpler. It is also more general, remaining valid even when spacetime is not flat.

Just as geometric problems can be solved on a sheet of paper, without defining any axes, so simple 2D spacetime problems can be tackled using absolute values, on a sheet of paper in which the definition of a reference frame ($\mathbf{t}, \mathbf{z}$) is arbitrary and nonessential. The simplification can best be grasped through the following analogy: the distance that is written in kilometers between two cities on a road map represents an absolute value – the length – and the characteristic of a solid object – the road.

In Minkowskian geometry, the possibility of rotating a geometric construction is lost, but special directions appear: the two null directions in 2D spacetime, and the conical family of null directions (the *light cone*) in 4D spacetime. The other main novelty of Minkowskian geometry is the Pythagorean formula that features a minus sign instead of a plus sign if one segment is timelike and the other is spacelike and perpendicular to the first segment [14].

Similarly the proper time between two events on a particle’s worldline is an absolute value – the proper time – of a solid object – the worldline. This approach is much simpler than

¹the term *spacetime map* is sometimes used by other authors for the MSM equipped with 4 axes e.g.[13]

considering the time of a hypothetical clock that is insensitive to accelerations and bound to the particle, with a sequence of Lorentz transformations [15].

The proper time is an example of 1D absolute quantity in the MSM. Poincaré's invariants $E^2 - B^2$ and $\mathbf{E} \cdot \mathbf{B}$ are examples of 0D absolute quantities that can be written for any event in the MSM. 2D absolute quantities can be written from electromagnetic field components F_{ij} : considering an infinitesimal oriented parallelogram based on two vectors $d\mathbf{u}$ and $d\mathbf{v}$ ² of components du^i and dv^i , the flux $d\Phi_m = \frac{1}{2}F_{ij}du^i dv^j$ on the surface $dS = d\mathbf{u} \wedge d\mathbf{v}$ is a scalar, the result of a tensor contraction. So it is invariant under Lorentz transformations [16, 17]. $\Phi_m = \frac{1}{2} \int_{\Sigma} d\Phi_m$ is then an absolute quantity that can be written as a characteristic of any given surface Σ in the MSM. For a space-space infinitesimal surface (e.g. $dydz \mathbf{y} \wedge \mathbf{z}$), this value is the magnetic flux $B_x dydz$ on the surface. Hence, it is natural to call $\Phi_m = \frac{1}{2} \int_{\Sigma} F_{ij} dS^{ij}$ the magnetic flux on the oriented surface Σ . It should be noted that electric field components are the density of magnetic flux on time-space surfaces. For example, E_x corresponds to the magnetic flux $E_x dxdt$ on the oriented surface $dxdt \mathbf{x} \wedge \mathbf{t}$. Similarly, it is natural to call $\Phi_e = \frac{1}{2} \int_{\Sigma} G_{ij} dS^{ij}$ the electric flux on the surface Σ .

The simplification offered by the MSM is best illustrated by how Maxwell's equations can be expressed in terms of absolute magnetic Φ_m and electric Φ_e fluxes: the first group $\nabla \cdot \mathbf{B} = 0$, $\nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0$ of differential equations is equivalent to the elementary law stating that the magnetic flux on a closed surface in spacetime is equal to zero, while the second group $\nabla \cdot \mathbf{D} = \rho$, $\nabla \times \mathbf{H} - \partial_t \mathbf{D} = \mathbf{j}$ is equivalent to the law stating that the electric flux on a closed surface in spacetime is equal to the number of enclosed charges [18].

There have been several attempts to connect the electromagnetic world with a physical, concrete model. Maxwell himself considered that his attempt to construct a sort of machine made up of tiny wheels and vortices [19] was too complicated and ultimately unsuccessful. Furthermore, the concept of field lines does not resist Galilean or Lorentz invariance. It is now commonly assumed that the abstract field notion is the only way of dealing with electromagnetism [20, 21].

Another interesting approach is the theory of differential forms, which manipulates geometric objects and describes the electromagnetic field in frame-free way. Although 3D visualisations are quite successful ([17, 22, 23]), visualisation in 4D spacetime using "tubes of flux" [17] is a dead end. Between modern books, we know only Gratus paper [24], who abandoned the concept of tubes for 4D spacetime. The more detailed description needs the mathematical introduction

²In this work, the "d" letter does not mean the exterior differentiation, for which the symbol $\tilde{d}$ is used, it is just used to emphasise infinitesimal quantities.

and described in Chapter 3.

In the present work, using geometrical approach, we demonstrate that the electromagnetic world can be fully described by absolute objects in the 4D MSM. This work begins with the postulation of the integral Maxwell's equations, which are based on fluxes on oriented 2D surfaces. These surfaces can be not only space-space, but also time-space. Later, it will be demonstrated that at any event of 4D spacetime, two infinitesimal segments of 2D surfaces L_1 and L_2 , each containing absolute values of the electric and magnetic fluxes, can be defined in unique way. These two infinitesimal surfaces are completely orthogonal. Their orientation, which is absolute, corresponds to 4 degrees of freedom in 4D spacetime. Considering, for example, the two electric flux densities (on L_1 and L_2), we have, in an absolute way, the six degrees of freedom of the tensor $F_{ij} = (-\mathbf{E}, \mathbf{B})$.

This is new concept for absolute representation of electromagnetic fields in spacetime. This representation is independent from any frame of reference. Around the same event, two independent physicists will write in the spacetime map the same topology (L_1 and L_2 orientation) and the same densities of flux.

Following an arbitrary tangent curve, each of the infinitesimal surfaces L_1 or L_2 can be united into infinitely narrow tagliatelle ribbon. In some cases, the ribbons constructed from any different paths define a unique surface, which are called **lasagna** here.

As the 4D spacetime map is difficult to visualize, one chapter is dedicated to an analysis of the electromagnetic field of $\mathbf{y}$ -invariant, p -polarized (the direction of the magnetic field coincides with $\mathbf{y}$) problems in spacetime. In this case, two lasagne sets L_1 and L_2 can be constructed. L_1 lasagne contain only electric flux and are invariant in $\mathbf{y}$ direction. Thus, it is enough to consider only a map of electric lasagna surfaces cut (at any $y = \text{Const}$), which we call **electric "spaghetti"** (ES). Inheriting the properties of lasagna, electric spaghetti (ES) are solid, concrete lines of events in 3D spacetime map. The the second lasagne L_2 contain only magnetic flux, thus called the Zero Electric Flux Surfaces, and are perpendicular to ESs and $\mathbf{y}$ direction. ESs can be successively time-like, null-like or space-like, encompassing the boundary between space and time. They are characterised by three invariant values: the interval length, the electric flux values ϕ_e , and a new quantity, the characteristic parameter η . All these quantities are absolute (frame independent) characteristics of any ES segment in the 3D MSM. Contrary to the interval length, which have to be defined separately for time-like and space-like parts, the electric flux value ϕ_e and the characteristic parameter η are defined continuously along any ES segment. The

(absolute) ESs map contains all the information about the electromagnetic field. It permits to compute the (relative) field components in any reference frame.

ES and ZEFS topology exhibits specific patterns that permits to visualize, for example, plane waves, wave interferences. A detailed analysis of ES and ZEFS topology opens up a new field of research.

The remain part is organized as follows. In the rest of this Chapter the different formulations of Maxwell's equations are overviewed briefly (Section 1.2), including their integral formulation for 4D spacetime (Section 1.3). Magnetic and electric fluxes on any surface in 4D are the key concepts of this formulation. In Section 1.4 it is shown that this general flux concept generalizes the Finite Integration Technique with complete freedom of spacetime meshing.

In Chapter 2 the ES and ZEFS objects are introduced (Section 2.2), and the way to construct them from classical field components is presented. It is shown, that the electric flux is a natural ES measure, and the η parameter is introduced. After that, a few illustrative examples are considered in Section 2.3. First (Section 2.3.1), a simple 2D $\mathbf{y}$ - and $\mathbf{t}$ -invariant electrostatic problem is described. Here, the ESs are simply the equipotential lines, as computed in the frame of reference bound to the electrodes. Second (Section 2.3.2), the interference of two counterpropagating plane waves is considered. This is a 2D $\mathbf{x}$ - and $\mathbf{y}$ -invariant case, with all the peculiarities of Minkowskian geometry. Plane wave regions correspond to straight ESs, oriented in the light cone, with sinusoidal density. The interference region is particularly interesting. In this region, the ESs exhibit loop patterns that transcend the boundary between space and time.

Finally (Section 2.3.3), the third example, a 3D $\mathbf{y}$ -invariant resonant slit system is studied. This problem converges together the cases considered in the previous section. The plane wave regions are similar to those in the previous 2D case. There is a further similarity in the slit region, where two plane waves interfere. The evanescent field region where funneling occurs is particularly noteworthy. In this region ESs exhibit spiral patterns although there is no charge at their centers. This special topological structure is studied in Section 2.4. It is demonstrated that they appears in the interference of an incident plane wave and an evanescent wave. The spirals are Bernoulli's spirals, fractal curves that never end.

Chapter 3 begins with qualitative explanations that can helps to understand the unique flux decomposition concept without the use of additional mathematics. Section 3.2 provides a clue to this decomposition by means of a classical electromagnetic problem. It is shown that a pair of completely orthogonal surfaces appears when finding the reference frame where the electric

and the magnetic fields are parallel. Then, in Section 3.3 it is demonstrated, that in the the Euclidean 4D space a decomposition of two bivectors is not unique, but similar demonstration cannot be provided for the Minkowski spacetime. Section 3.4 provides the retrospective of the presentation of the electromagnetic field with differential forms. The fundamental result of uniqueness of a bivector decomposition in 4D is introduced in Section 3.5. The existence of such decomposition is the root of the geometrical absolute representation of the electromagnetic field in 4D spacetime, presented in Section 3.6. After that, in Section 3.7 some examples, considered in Chapter 2 in 3D spacetime, are reconsidered for 4D spacetime.

1.2.

Differential approach of electromagnetism (Maxwell's equations)

In this Section different approaches of electromagnetism are presented, all starting from differential equations. The introduction of differential forms permits to obtain two integral equations discussed in Section 1.3. These equations, being the fundamental principles of electromagnetism in spacetime, can be described without any high level mathematics. Meanwhile, in this Section the other approaches, presented electromagnetism, are discussed.

The EM field and Maxwell's equations naturally appear in the classical field theory as a gauge field under the condition of the local invariance of complex (scalar, vector or spinor) fields with respect to isotropic rotations [25]. On the other hand, in classical electrodynamics Maxwell's equations are either postulated or derived from the experience as a generalization of experimental facts.

At first, Maxwell's equations were presented in 1865 as a set of Eqs. 1.2 – 1.7 [2], employing the vector-potential $\mathbf{A}$ (called as the electromagnetic momentum) and the electric potential ϕ . As original notation is complicated to understand, the equations are presented here using modern notations (the authentic names are saved)

A. Equations of Total Currents

$$j_{total\ x} = j_x + \frac{dD_x}{dt}, \quad j_{total\ y} = j_y + \frac{dD_y}{dt}, \quad j_{total\ z} = j_z + \frac{dD_z}{dt} \quad (1.1)$$

B. Equations of Magnetic Force

$$\mu H_x = \frac{dA_z}{dy} - \frac{dA_y}{dz}, \quad \mu H_y = \frac{dA_x}{dz} - \frac{dA_z}{dx}, \quad \mu H_z = \frac{dA_y}{dx} - \frac{dA_x}{dy} \quad (1.2)$$

C. Equations of Currents

$$\begin{aligned}\frac{dH_z}{dy} - \frac{dH_y}{dz} &= 4\pi(j_x + \frac{dD_x}{dt}), \\ \frac{dH_x}{dz} - \frac{dH_z}{dx} &= 4\pi(j_y + \frac{dD_y}{dt}), \\ \frac{dH_y}{dx} - \frac{dH_x}{dy} &= 4\pi(j_z + \frac{dD_z}{dt})\end{aligned}\tag{1.3}$$

D. Equations of Electromotive Force

$$\begin{aligned}E_x &= \mu(H_z v_y - H_y v_z) - \frac{dA_x}{dt} - \frac{d\phi}{dx}, \\ E_y &= \mu(H_x v_z - H_z v_x) - \frac{dA_y}{dt} - \frac{d\phi}{dy}, \\ E_z &= \mu(H_y v_x - H_x v_y) - \frac{dA_z}{dt} - \frac{d\phi}{dz}\end{aligned}\tag{1.4}$$

F. Equations of Electric Elasticity

$$E_x = \frac{1}{\epsilon} D_x, \quad E_y = \frac{1}{\epsilon} D_y, \quad E_z = \frac{1}{\epsilon} D_z\tag{1.5}$$

G. Equation of Free Electricity

$$\frac{dj_x}{dx} + \frac{dj_y}{dy} + \frac{dj_z}{dz} - \frac{d\rho}{dt} = 0\tag{1.6}$$

H. Equation of Continuity

$$\frac{dj_x}{dx} + \frac{dj_y}{dy} + \frac{dj_z}{dz} - \rho = 0.\tag{1.7}$$

Here ρ is the free charge density, $\mathbf{j}$ is free current, $\mathbf{v}$ is a velocity of a short straight conductor, ϵ and μ are the electric and magnetic permeability, correspondingly. The vectors of the electromagnetic field: $\mathbf{E}$ is the electric field strength (called the electromotive force in [2]), $\mathbf{D}$ is the electric displacement, and $\mathbf{H}$ is the magnetic field strength (the magnetic intensity in [2]), correspondingly. Also in literature one may see names the "magnetic flux density" for $\mathbf{B}$ and the "electric flux density" for $\mathbf{D}$ (for example, in [23], and officially in the International System of Units Brochure [26]).

Being inspired by Hamilton's invention of the quaternions, Maxwell represented his equations in terms of quaternions in [27]. However, this presentation was much harder to understand, and was even seen "unnatural". In 1884 - 1885, using modern vector notation (also advocated by J.W. Gibbs), Oliver Heaviside rewrote Maxwell's equations in more compact form that had become standard [28]. In Heaviside's book [29] one may find the following set of equations

$$\nabla \cdot \mathbf{B} = \sigma, \quad (1.8)$$

$$\nabla \times \mathbf{E} = -\mathbf{G} - \frac{\partial \mathbf{B}}{\partial t}, \quad (1.9)$$

$$\nabla \cdot \mathbf{D} = \rho, \quad (1.10)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (1.11)$$

where σ is the magnetic charge density, and $\mathbf{G}$ is the magnetic current. Although Heaviside used the magnetic charge for the sake of symmetry, he was not sure in its existence. He wrote "The idea of a magnetic current is a very useful one, nevertheless". At the present time, the existence of a magnetic charge has not been proven, but this concept is helpful for some engineering calculations.

The Heaviside's formulation of Maxwell's equations can be divided into two independent groups. The first group includes Eq. 1.8 and Eq. 1.9, using $\mathbf{E}$ and $\mathbf{B}$. Often it is associated with the electromagnetic force (as the Lorentz force is built from $\mathbf{E}$ and $\mathbf{B}$). The second group includes Eq. 1.10 and Eq. 1.11, using $\mathbf{H}$ and $\mathbf{D}$. In literature it is associated with sources [22]. In a vacuum, two groups of Maxwell's equations are connected by constitutive equations $\mathbf{B} = \mu_0 \mathbf{H}$ and $\mathbf{E} = \mu_0 c^2 \mathbf{D}$. Further, unless otherwise specified, natural units with $c = 1$ are used. It is interesting to note, that in Poincaré paper [6] c was equal to 1, but in Minkowski one it was not. In the same time, as it was stated by Minkowski in [9], being non-separated, space and time constitute 4D spacetime. From this point of view, $c = 1$ makes space and time equal terms. In Minkowski spacetime, where space and time are considering on an equal footing, the electric and magnetic field are treated as parts of one electromagnetic tensors (the choice of signs is based on the Minkowski metric tensor $(+ - - -)$)

$$\mathcal{F}^{ij} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{pmatrix}, \quad (1.12)$$

and

$$G^{ij} = \begin{pmatrix} 0 & D_x & D_y & D_z \\ -D_x & 0 & H_z & -H_y \\ -D_y & -H_z & 0 & H_x \\ -D_z & H_y & -H_x & 0 \end{pmatrix}. \quad (1.13)$$

Tensor notation simplifies the symmetrical set of Maxwell's equations written by Heaviside, and reduces their number to two:

$$\begin{aligned} \partial_j \mathcal{F}^{ij} &= 0, \\ \partial_j G^{ij} &= J^i, \end{aligned} \quad (1.14)$$

where $J = (\rho, \mathbf{j})$ is the four-current density. The first tensor equations corresponds to the Eq. 1.8 and Eq. 1.9, and the second tensor equation – to Eq. 1.10 and Eq. 1.11. In tensor notation, constitutive equations are

$$\mathcal{F}^{ij} = \frac{1}{2} \mu_0 \epsilon^{ijkl} G_{kl}, \quad (1.15)$$

where ϵ^{ijkl} is the antisymmetric unit tensor of rank four.

There is another way to present Maxwell's equation that can be found in famous textbooks like Landau [30]. This is not-symmetric presentation including the hypothesis that there is no magnetic charge. It is based on the vector-potential $\mathbf{A}$

$$F_{ik} = \partial A_k / \partial x^i - \partial A_i / \partial x^k. \quad (1.16)$$

The vector-potential is useful to define a Lagrangian, thus this approach is one of the most popular ways to present Maxwell's equations. The components of tensor F are

$$F_{ij} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}, \quad (1.17)$$

In this formulation the first group of Maxwell's equations appears naturally, first as

$$\partial_{[i} F_{ij]} = 0. \quad (1.18)$$

Here the square brackets $[]$ indicate the indices antisymmetrizations. This equation corresponds to four equations where $[i, j, k]$ equal to $[0, 2, 3]$, $[0, 3, 1]$, $[0, 1, 2]$, $[1, 2, 3]$. Defining $\mathcal{F}$ like

$$\mathcal{F}^{ij} = \frac{1}{2}\epsilon^{ijkl}F_{kl} \quad (1.19)$$

the first equation of Eq. 1.14 can be obtained.

The second equation of the antisymmetric set is obtained with the aid of the principle of the least action using the same tensor [30]

$$\partial_j F^{ij} = \mu_0 J^i. \quad (1.20)$$

This equation corresponds to the second equation of Eq. 1.14 with $F^{ij} = \mu_0 G^{ij}$.

The tensor presentation has two advantages. It is laconic, and it doesn't need the introduction of special operators like divergence or rotational. Both approaches with vectors or components of tensors suppose that a frame of reference is defined. There are mathematical objects that are independent of any frame of reference which corresponds to antisymmetrical tensors of rank two: differential 2-forms.

A 2-form is a linear functional, which computes a scalar value from an infinitesimal 2D oriented surface. The mathematical introduction to differential forms is provided in Appendix C. It provides the basis theory of the exterior algebra, including most necessary definitions, the basic concepts and properties of this theory, and their connection with better known mathematical approaches in physics (vectors and tensors). It can be useful for readers who do not yet familiar with this approach.

Here is only a brief introduction to Maxwell's equations with differential forms is given, while a more detailed description can be found in Appendix B. Electrodynamics can be presented using the differential forms approach, since F_{ij} and $\mathcal{G}_{ij}$ are antisymmetric. In 4D spacetime, the canonical basis of 2-forms is made of 6 elements $e^{ij} = e^i \wedge e^j$ ($i, j = [0, 3]$), where $\{e^i\}$ is the canonical basis of 1-forms (covectors). Then, the two electromagnetic 2-forms can be presented in this basis, employing components of electromagnetic tensors F_{ij} and $\mathcal{G}_{ij}$

$$\tilde{\Phi}_m = \frac{1}{2}F_{ij}e^i \wedge e^j \quad (1.21)$$

$$\tilde{\Phi}_e = \frac{1}{2}\mathcal{G}_{ij}e^i \wedge e^j. \quad (1.22)$$

They are explicitly written as

$$\tilde{\Phi}_m = E_x e^1 \wedge e^0 + E_y e^2 \wedge e^0 + E_z e^3 \wedge e^0 + B_x e^2 \wedge e^3 + B_y e^3 \wedge e^1 + B_z e^1 \wedge e^2, \quad (1.23)$$

$$\tilde{\Phi}_e = H_x e^0 \wedge e^1 + H_y e^0 \wedge e^1 + H_z e^0 \wedge e^3 + D_x e^2 \wedge e^3 + D_y e^3 \wedge e^1 + D_z e^1 \wedge e^2. \quad (1.24)$$

Thus, in this notation, two groups of Maxwell's equations take on a new form

$$\tilde{d}\tilde{\Phi}_m = 0 \quad (1.25)$$

$$\tilde{d}\tilde{\Phi}_e = \tilde{J} \quad (1.26)$$

where $\tilde{d}$ is the exterior derivative operator, that constructs a 3-form from a 2-form, and $\tilde{J}$ is a 3-form electric current density. The exterior derivative of each term $F_{ij}e^i \wedge e^j$ is equal to $\partial_k F_{ij}e^k \wedge e^i \wedge e^j$. One of advantages of this presentation is that the form of Maxwell's equations do not change in any system of coordinates. They are valid without modification, whether presented in rectangular or curvilinear coordinates [22]. But in general case the relation between $\tilde{\Phi}_m$ and $\tilde{\Phi}_e$ depends on the metric.

More detailed discussion, devoted to this presentation in literature is presented in Chapter 3, Section 3.4. In the next Section it is shown that the integral formulation of these 2-form differential equations has a very simple interpretation, that can be used as postulate of the electromagnetism.

1.3.

Integral formulation of electromagnetism

As pointed out by Cartan [31], the integral formulation of electromagnetism doesn't need to suppose that partial derivatives of the field do exist, and it is valid in curved spacetime, with no assumption concerning its affine connection. Only constitutive equations need a metric to connect two different fluxes. This formulation is considered below, first from tensorial approach. Next a new nomenclature, extending to spacetime the notions of magnetic and electric fluxes, is introduced.

The names the magnetic flux density for $\mathbf{B}$ and the electric flux density for $\mathbf{D}$ are now well established [26]. We propose to add "on space-space" surfaces to these denominations. The traditional definition of flux *across a surface* seems natural in 3D space. This is because there is only one direction to complement the 2D surface. In 4D this is no longer possible. The general

concept of flux "on an oriented surface" need to be used instead. Then the electric field strength $\mathbf{E}$ has to be called the magnetic flux density on space-time surfaces, and the magnetic field strength $\mathbf{H}$ has to be called the electric flux density on time-space surfaces (see Fig. 1.1). This apparent paradox that electric field is a magnetic flux and thad magnetic field is a magnetic flux comes from the fact that the names for the fields were given on the basis of experimental work, and only later human has learned that the electric and magnetic fields are connected.

The result of the 2-form $\tilde{\Phi}_m$ and $\tilde{\Phi}_e$ computed on an infinitesimal ³ area $d\mathbf{S} = dS^{ij}\mathbf{e}_i \wedge \mathbf{e}_j$ in 4D ⁴ are the scalars equal to the tensorial product between the tensors F_{ij} and $\mathcal{G}_{ij}$ with this surface, which is also an antisymmetric tensor:

$$d\Phi_m = \frac{1}{2}F_{ij}dS^{ij}, \quad (1.27)$$

and

$$d\Phi_e = \frac{1}{2}\mathcal{G}_{ij}dS^{ij}. \quad (1.28)$$

If the elementary infinitesimal surface $d\mathbf{S}$ is space-space, for example $dS^{23} = dydz$ (strictly, it is $\mathbf{e}_2 \wedge \mathbf{e}_3$), we compute $d\Phi_m = B_x dydz$. It is clearly a magnetic flux, that justify our usage of the subscript m . For the elementary space-time surface $dS^{10} = dxdt$ (strictly, it is $\mathbf{e}_1 \wedge \mathbf{e}_0$), we compute $d\Phi_m = E_x dxdt$. The electric field strength is clearly a magnetic field density on space-time surfaces. Symmetrically, on the surface $dS^{23} = dydz$, $d\Phi_e = D_x dydz$ is clearly an electric flux, that justify our usage of the subscript e , and for the elementary time-space surface $dS^{01} = dt dx$ (strictly, it is $\mathbf{e}_0 \wedge \mathbf{e}_1$, opposite to dS^{10}), we compute $d\Phi_e = H_x dt dx$. The magnetic field strength is clearly an electric field density on time-space surfaces. The full set of correspondence between the oriented segments and the components of the tensors is presented in Table 1.1.

Table 1.1: Magnetic and electric flux on the basis surfaces in spacetime.

Surface	$\mathbf{x} \wedge \mathbf{y}$	$\mathbf{x} \wedge \mathbf{z}$	$\mathbf{y} \wedge \mathbf{z}$	$\mathbf{x} \wedge \mathbf{t}$	$\mathbf{y} \wedge \mathbf{t}$	$\mathbf{z} \wedge \mathbf{t}$
$\Phi_m(\text{surface})$	B_z	B_y	B_x	E_x	E_y	E_z
Surface	$\mathbf{x} \wedge \mathbf{y}$	$\mathbf{x} \wedge \mathbf{z}$	$\mathbf{y} \wedge \mathbf{z}$	$\mathbf{t} \wedge \mathbf{x}$	$\mathbf{t} \wedge \mathbf{y}$	$\mathbf{t} \wedge \mathbf{z}$
$\Phi_e(\text{surface})$	D_z	D_y	D_x	H_x	H_y	H_z

³Here and below, the "d" letter does not mean the exterior differentiation, for which the symbol $\tilde{d}$ is used, it is just used to emphasise infinitesimal quantities.

⁴The bivector conception in 4D is considered in Appendix C, Section C.3.

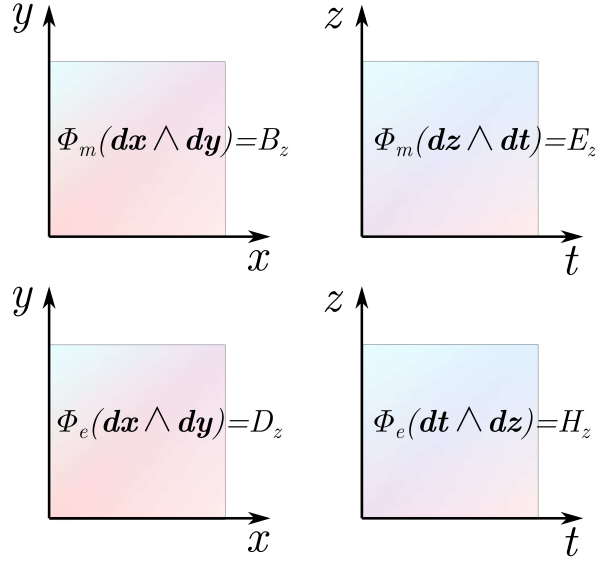


Figure 1.1: On the spatial surfaces the magnetic flux density is $\mathbf{B}$ field, and the electric flux density is the $\mathbf{D}$ field. But on space-time surfaces the magnetic flux density is the $\mathbf{E}$ field, and the electric flux density is $\mathbf{H}$ field.

On an arbitrary 2D oriented surface Σ , the magnetic flux can be computed as

$$\Phi_m(\Sigma) = \frac{1}{2} \int_{\Sigma} F_{ij} dS^{ij}, \quad (1.29)$$

and the electric flux as

$$\Phi_e(\Sigma) = \frac{1}{2} \int_{\Sigma} \mathcal{G}_{ij} dS^{ij}. \quad (1.30)$$

And, using the Stokes' theorem

$$\int_M \tilde{d}\omega = \int_{\partial M} \omega, \quad (1.31)$$

Maxwell's equations (Eqs. 1.25, 1.26) can be rewritten as

$$\oint_{\partial V} d\Phi_m = 0 \quad (1.32)$$

$$\oint_{\partial V} d\Phi_e = n_e \quad (1.33)$$

where $n_e = \int_V \tilde{J}$ is the charge contained in any ⁵ 3D volume V enclosed by the surface ∂V .

These two equation can be directly formulated as the postulates.

1. Any 2D oriented surface of 4D spacetime can be labelled with two scalar quantities, its magnetic flux Φ_m and its electric flux Φ_e .

⁵in 4D there is an infinite number of 3D volumes enclosed by 2D surface.

2. The magnetic flux on any closed surface is zero

$$\Phi_m(\text{closed surface}) = 0. \quad (1.34)$$

3. The electric flux on any closed surface is equal to the number of charges in any 3D volume enclosed by this surface

$$\Phi_e(\text{closed surface}) = n. \quad (1.35)$$

As seen from this equation, the unit of electric flux is the natural unit of the electric charge. Also, in 4D spacetime electric charge is no longer just a point, but a time-like line in spacetime. The lines of charge never end, but two equal charges with opposite sign $+q$ and $-q$ can annihilate.⁶ Then the number of charges n in Eq. 1.35 is in fact the number of intersections between the charges worldlines and the arbitrary 3D volume enclosed by the surface, on which flux is considered.

Two simple postulate of Eq. 1.34 and Eq. 1.35 together with charges considered as lines in spacetime provide Maxwell's equations. If a spatial volume is considered at a fixed time, Eq. 1.34 and Eq. 1.35 are clearly the Gauss's law for the magnetic flux density $\mathbf{B}$ (Eq. 1.8) and the Gauss's law for the electric flux density $\mathbf{D}$ (Eq. 1.10). When considering tyz volume at fixed x , the x component of vectorial equations Eq. 1.9 - Eq. 1.11 are obtained. Detailed discussion and derivation of equations involving curls from flux equations are given in Appendix D.

The physics of electromagnetic fields is contained in the constitutive equation $F_{ij} = \mu_0 G_{ij}$ (see Eq. 31 in Appendix D), or $\tilde{\Phi}_m = \mu_0 \star \tilde{\Phi}_e$. Another way to express the relation between the 2-forms with Hodge product $\star$ is to say that on two infinitesimal surfaces of same values, fully perpendicular, the flux Φ_m on the first and the flux Φ_e on the second are connected by the equation:

$$\frac{\Phi_m}{q_m} = 2\alpha \frac{\Phi_e}{q_e}, \quad (1.36)$$

where $q_m = h/q_e$, h is the Planck constant, $q_e = e$ is the elementary charge, and α is the fine-structure constant.

In this way, considering the electric field strength $\mathbf{E}$ as the magnetic flux density on space-time surface segments, and the magnetic field strength $\mathbf{H}$ as the electric flux density on time-space surface segments, the flux concept can be extended in spacetime. Extended conception of fluxes allows to present Maxwell's equations in simple integral formulation Eq. 1.34 and

⁶It means that algebraic sign of $+q$ and $-q$ define the orientation of worldlines.

Eq. 1.35. Together with constitutive relation, this concept allows direct computations of the electromagnetic field.

1.4.
**Generalisation of the Finite Integration
 Technique**

The integral equations Eq. 1.34 and Eq. 1.35 have been obtained by integration of differential forms, which are high level tools. In Appendix D it is shown that using these simple equations as postulates the classical formulation of Maxwell's equations can be obtained.

Instead of solving differential Maxwell's equations, the flux equations can be used directly to compute electromagnetic problems. This technique is already known as the Finite Integration Technique [32, 33]. But in fact, this technique can be generalized to an arbitrary meshing of spacetime, instead of a meshing of only space.

From the mesh, a set of 2D oriented surfaces have to be constructed in spacetime. The electromagnetic variables to be determined are magnetic and electric fluxes ϕ_m and ϕ_e , on 2D surfaces. The two fundamental equations Eq. 1.34 and Eq. 1.35 are just "the sum of the magnetic flux values on a system constructing a closed surface is 0" and "the sum of electric flux values is equal to the number of charges included in any volume enclosed by a surface". Thus, the flux equations can be written exactly in a computer program as a discrete sum of flux values. This "exact formulation" was noted by Tonti [34] but his exact, "topological formulation" of Maxwell's equation was restricted to spatial part.

By contrast, the constitutive equation Eq. 1.36 cannot be written exactly in a computer. Constitutive equation is connecting the density of flux (the fields). A model is needed, we have to write an approximation that will connect magnetic and electric fluxes. A example of model is the polynomial approximation used by Portier *et al* to compute harmonic 2D modal problems [35].

As said above, the equations Eq. 1.34 and Eq. 1.35, the "Maxwell's equations" can be written in an absolute way, with no approximation. It is also valid in the case of curved space, when gravitation is present. In contrast, constitutive equation needs a metric to be computed.

Conclusion

To summarize, an integral, discrete, absolute and exact formulation of Maxwell's equations was given in spacetime. It was constructed from a classical tensorial approach, from the differential forms approach, and also from a new fundamental approach of electromagnetism. The fundamental concept is the magnetic flux and the electric flux on a oriented arbitrary surface. The surface can be space-space, time-space, null-space, or any of these types on a part and any other type on another part.

Maxwell's equations are now simple algebraic equations declaring that if surface segments are combined to form a closed surface, the sum of the magnetic fluxes is zero, and the sum of electric fluxes is equal to the number of electric charges. The form of surfaces is arbitrary, and Maxwell's equations formulation is independent of the choice of a frame of reference.

If a frame of reference is chosen, elementary surfaces are $\mathbf{t} \wedge \mathbf{x}$, $\mathbf{t} \wedge \mathbf{y}$, ..., $\mathbf{y} \wedge \mathbf{z}$, and density of magnetic flux component B_x is the magnetic flux on elementary surface $\mathbf{y} \wedge \mathbf{z}$, etc. It is the common concept for the magnetic flux density $\mathbf{B}$. We have added that the electric field strength component E_x is also a magnetic flux density, but on space-time elementary surface $\mathbf{x} \wedge \mathbf{t}$. Similarly, the density of the electric flux component D_x is the electric flux on elementary surface $\mathbf{y} \wedge \mathbf{z}$, while the magnetic field strength component H_x is an electric flux density on time-space elementary surface $\mathbf{t} \wedge \mathbf{x}$. In this formulation, the constitutive equation connects the flux densities on completely perpendicular surfaces, for example B_x on $\mathbf{y} \wedge \mathbf{z}$ and H_x on $\mathbf{t} \wedge \mathbf{x}$.

In the two next chapters we go one step further. Rather than simply constructing a map of the electromagnetic world by computing electric and magnetic fluxes on arbitrary surfaces, it is shown that the natural, unique and absolute structures, that support these fluxes, can be defined in spacetime. At first, in Chapter 2, the particular $\mathbf{y}$ -invariant p -polarized problems are considered. In this case, the unique supporting flux objects are lines in 3D spacetime, we call them the electric spaghettis. The generalization to 4D spacetime is presented in Chapter 3. It is shown that under some conditions these specific structures with flux are 2D static surfaces in 4D spacetime, the electromagnetic lasagna.

Absolute description of polarized EM field, the y -invariant, p-polarized case

2.1.

Introduction

The aim of the first part of this thesis is to show that the map of “static” objects that characterizes completely the electromagnetic world can be built in 4D spacetime. Any element of a these object carries two scalar electromagnetic values, its electric and magnetic flux. In the next Chapter it will be shown that under some conditions these objects form 2D surfaces that transcend the boundary between time and space. These surfaces are called electromagnetic lasagne. In 4D spacetime there are two of them, and they are completely orthogonal. In a given reference frame (t, x, y, z) , an oriented surface defines 4 scalars (the 1st vector: 3 components, the 2nd vector: 2 two independent components , minus one rotation angle in the vector plane), and with the two scalar electromagnetic values provided by a lasagna we get the 6 components of the fields $\mathbf{E} = \mathbf{D}$ and $\mathbf{B} = \mathbf{H}$ in the vacuum.

It is not easy to visualize two-dimensional surfaces in a four-dimensional space. This is the reason why in this Chapter a reduction in the invariant case according to y , supplemented by the TM polarized problem, i.e. with the magnetic field oriented according to y , is presented. The first simplification, associated with polarization leads to the fact that the lasagne are divided into a purely electric and purely magnetic. The second simplification – the invariance in y direction – means the cutting of the electric lasagne by a plane $y = \text{Constant}$ constructs lines in 3D space-time (t, x, z) , which we call electric spaghetti ES. Just like the lines of the universe of an

object are "hard" lines, the ES are "rigid" objects in the space-time map. As it is demonstrated below, any piece of ES between two events e_1 and e_2 is bearer of a scalar electromagnetic value, $\Phi_e(e_1, e_2)$ or better $\eta(e_1, e_2)$. In a given (t, x, z) repository, the direction of the ES at a given event defines 2 scalars, and the electromagnetic value, associated with it, defines a third scalar. Thus, the three components of the electromagnetic field, $H_y = B_y$ and $D_x = E_x, D_z = E_z$, can be calculated.

The ES object is presented in Section 2.2. It is demonstrated, that ESs uniquely define an absolute 3D MSM, completed by surfaces of zero electric flux. While electromagnetic field components change from one reference to another, electromagnetic measures along ES are invariants. It is shown, that electromagnetic field components can be straightforwardly derived in an arbitrary frame of reference (set of perpendicular unit vectors in the MSM), from ESs and Zero Electric Flux Surfaces map.

In Section 2.3, three illustrative examples of static map, constructed with ES are presented. At first, the 2D $\mathbf{y}$ - and $\mathbf{t}$ -invariant problem of two oppositely charged cylinders is considered in Section 2.3.1. In this case ES are potential circles. Second, the 2D $\mathbf{x}$ - and $\mathbf{y}$ -invariant case of interference of two counterpropagating plane waves is described in Section 2.3.2. In the regions of the plane wave ESs are simply straight lines with the sinusoidal density. In the interference region, ESs have a form of concentrated loops. Finally, a 3D $\mathbf{y}$ -invariant resonant slit system is considered in Section 2.3.3. This problem include three regions: a plane wave region far from the slits, the wave interference inside the slits, and the region of evanescent waves near the slits, where funneling occurs. While in the first two regions, the ESs patterns repeat the previously demonstrated cases, the last region is especially notable, as there ESs exhibit special topological whirl pattern. In Section 2.4 it is demonstrated, that interference of the plane wave and the evanescent wave creates similar whirl patterns, which are, in fact, Bernoulli's *Spira mirabilis*, that never ends.

2.2.

The spaghetti object

ES are lines of events in 3D MSM. In contrast to the worldlines of physical objects, which are only timelike, ESs transcend the boundary between time and space directions and can be timelike, spacelike, and even lightlike, changing from one to the other. Being lasagna cuts for the $\mathbf{y}$ -invariant, H_y polarized problem, they are absolute objects, containing all the information

about the electromagnetic world. This Section begins with the introduction of 1-form $\tilde{\phi}_e$, the particular case of 2-form $\tilde{\Phi}_e$ for the $\mathbf{y}$ -invariant, H_y polarized problem. Computed between two events, the scalar value $\phi_e = \tilde{\phi}_e(ds)$ is an invariant. The Lorentz invariance of this value is demonstrated. Besides ES, in 3D MSM there are other objects, surfaces of Zero Electric Flux (ZEFS). Electric flux is zero on any segment of this surface. ZEFS complete 3D MSM, and are useful for the extraction of the electric fields components.

In Section 2.2.3, the ES object recipe is provided. After that, measures, that characterise EM field along ES are discussed. A new EM field invariant η , arising in the ES construction, is discussed in Section 2.2.5. After that, direct computation of EM field components from ES, using ϕ_e , η , and ZEFS are presented in Section 2.2.6. Finally, direct computation of ZEFS is introduced in Section 2.2.7.

2.2.1 The 1-form ϕ_e

In the general case, there are six independent components of electric flux (Section 1.3). As this Chapter is restricted to systems that are invariant in the spatial $\mathbf{y}$ -direction, with a magnetic field parallel to the $\mathbf{y}$ -direction, only three electric flux terms remain non-zero:

$$\begin{aligned}\tilde{\Phi}_e(\mathbf{t} \wedge \mathbf{y}) &= H_y \\ \tilde{\Phi}_e(\mathbf{x} \wedge \mathbf{y}) &= D_z \\ \tilde{\Phi}_e(\mathbf{z} \wedge \mathbf{y}) &= -D_x.\end{aligned}\tag{2.1}$$

Taking into account the common factor $\mathbf{y}$, the 2-form $\tilde{\Phi}_e$ can be reduced to a one form with

$$\begin{aligned}\tilde{\phi}_e(\mathbf{t}) &= H_y \\ \tilde{\phi}_e(\mathbf{x}) &= D_z \\ \tilde{\phi}_e(\mathbf{z}) &= -D_x.\end{aligned}\tag{2.2}$$

Thus, the electric flux (the general form in Eq. (1.22)) on the surface $(dt\mathbf{t} + dx\mathbf{x} + dz\mathbf{z}) \wedge \mathbf{y}$ can be regarded in 3D spacetime $(\mathbf{t}, \mathbf{x}, \mathbf{z})$ as the 1-form:

$$\tilde{\phi}_e(ds) = \tilde{\Phi}_e(ds \wedge \mathbf{y}) = H_y dt + D_z dx - D_x dz.\tag{2.3}$$

If we consider an arbitrary pair of events in this 3D spacetime, separated by an infinitesimal

vector $d\mathbf{s} = dt\mathbf{t} + dx\mathbf{x} + dz\mathbf{z}$, the squared interval $ds^2 = dt^2 - dx^2 - dz^2$ computed between these two events is a well known invariant [6]. As demonstrated below, the electric flux value $d\phi_e = \tilde{\phi}_e(ds)$, computed between these two events (Eq. (2.3)), is also an invariant. In other words, $d\phi_e$ does not depend on the particular choice of a reference frame: in any the other frame of reference $(\mathbf{t}', \mathbf{x}', \mathbf{z}')$ the electric flux value $d\phi_e$ between the same events remains the same.

ϕ_e as a Lorentz-invariant measure of intervals

The electric flux ϕ_e (Eq. 2.3) is the value of a 1-form on an infinitesimal interval ds . Hence it is a scalar value. As such, it is absolute, it doesn't depend on the frame of reference choice.

However, here it is demonstrated that Eq. 2.3 is invariant under Lorentz transformations. As the invariant y -direction should be maintained in spacetime, only frames moving in the (x, z) plane therefore should be considered. For the sake of simplicity, we consider a frame (t', x', z') moving in the $\mathbf{x}$ direction with velocity v ($c = 1$). The old coordinates are expressed from the new ones as:

$$\begin{aligned} t &= \gamma(t' + vx') \\ x &= \gamma(x' + vt') \\ z &= z', \end{aligned}$$

with $\gamma = (1 - v^2)^{-1/2}$.

The Lorentz transforms for the fields components of the y -invariant, p -polarized problem are:

$$\begin{aligned} H_y &= \gamma(H_y' - vD_z') \\ D_z &= \gamma(D_z' - vH_y') \\ D_x &= D_x'. \end{aligned}$$

Substitution of the coordinates and the electromagnetic field components in the Eq. (2.3) provides

$$\begin{aligned} d\phi_e &= \gamma(H_y' - vD_z')\gamma(dt' + vdx') + \\ &\quad \gamma(D_z' - vH_y')\gamma(dx' + vdt') - D_x'dz' \\ &= \gamma^2(1 - v^2)(H_y'dt' + D_z'dx') - D_x'dz' \\ &= H_y'dt' + D_z'dx' - D_x'dz', \end{aligned}$$

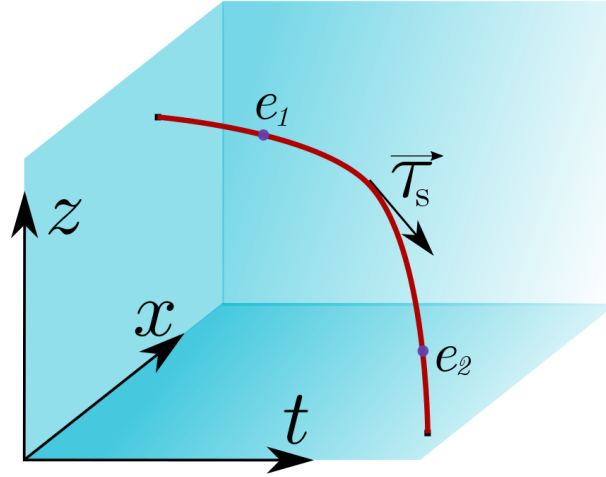


Figure 2.1: Electric spaghetti defined by the tangent vector τ_S .

which is clearly an invariant formula for $d\phi_e$. The same computation can be done for a frame moving in the z -direction, or in any direction in the (x, z) plane.

2.2.2 The ZEFS surfaces

There are three directions along which starting from any event e_1 the electric flux $d\phi_e$ computed with Eq. (2.3) is zero. These three directions are not independent, and actually define a 2D surface, the Zero Electric-Flux Surface (ZEFS) containing the event e_1 . This surface can be regarded as a solid object drawn in the 3D MSM independently of the chosen frame of reference.

2.2.3 The electrical spaghetti object

The vector perpendicular (in the Minkowskian sense) to the ZEFS surface can be computed as $\tau_S = (H_y, -D_z, D_x)$. Although the components of this vector can differ according to the frame of reference, the direction it determines is absolute in spacetime, and does not depend on the choice of frame of reference.

Starting from a given event e_1 in 3D spacetime, we can construct in an absolute way a line, the electric spaghetti (ES) $S(e_1)$ with τ_S as a tangent vector (Fig. 2.1). The construction of ESs does not depend on the chosen frame of reference: if two events e_1, e_2 are connected by a specific ES S ($S = S(e_1) = S(e_2)$) constructed from a given frame of reference (system of axes), they will be connected by the same ES S if another frame of reference is chosen to construct $S(e_1)$.

Contrary to the worldlines of physical objects, which are always timelike, ESs transcend the boundary between time and space directions and can be timelike, spacelike, and even lightlike,

changing from one to the other. Examples of this very special topological property of ESs are provided in Section 2.3.

In two next subsections, it is shown that the electric flux ϕ_e corresponds to a measure on the ES. In order to overcome a limitation of this measure (it is null on null lines), we define a new measure, in characteristic parameter η which is an absolute, monotonic parameter.

2.2.4 ϕ_e , a natural but limited measure

If a given ES $S(e_1, e_2)$, constructed between two events e_1 and e_2 , is timelike, it is characterized by its time length. If it is spacelike, then it is characterized by its spatial length. If some parts of the ES are timelike and others are spacelike, the total interval cannot be characterized by any coherent interval value. By contrast, the electric flux $\phi_e(S(e_1, e_2))$ is well defined for this ES as it is for any other. This flux is an algebraic value, expressed in units of charge per y unit of length, independently of any reference system in the 3D y -invariant spacetime.

However, when electric field is equal to magnetic field (for example in the case of a single plane wave), ES are null-like. The interval ds^2 and the electric flux $\phi_e(ds)$ are both zero. In next section, a new absolute (i.e. Lorentz-invariant) measure, the ES characteristic parameter η , is introduced.

2.2.5 The η characteristic parameter, a new EM field invariant

The simplest effective construction of an ES uses the parametric equation

$$d\mathbf{s} = \tau_S d\eta = (H_y d\eta, -D_z d\eta, D_x d\eta), \quad (2.4)$$

with η as the parameter. This parameter, which initially looks like a simple mathematical auxiliary tool, is actually a new invariant¹, which we refer to as the ES *characteristic parameter*. The invariant nature of $d\eta$ is easy to demonstrate, by considering the Eq. (2.5) obtained by combining Eqs. (2.3) and (2.4):

$$d\phi_e = (H^2 - D^2)d\eta. \quad (2.5)$$

In this equation, $d\phi_e$ is invariant and $H^2 - D^2$ is the well known Poincaré invariant G_α^α [3, 6], hence $d\eta$ is invariant.

From a topological point of view, a set of ESs can be constructed to cover any region of the

¹Strictly speaking, the invariants are the η differences

3D spacetime. Furthermore, an ES segment can be labeled with three quantities: 1) its interval length (though only if the segment is purely timelike or spacelike), 2) its electric flux, which has a definite algebraic value (examples of how ESs transcend the time and space distinction are given in the next section), 3) its characteristic parameter. The particular case where ESs are contained in the light cone requires some attention, as $H^2 - D^2 = 0$ as well as $ds = 0$ and $d\phi_e = 0$. By contrast, $d\eta$ is not equal to zero. This gives the characteristic ES parameter η a very important role. As shown in Section 2.2.6, the ESs with η labels contain all the electromagnetic information.

Technically, an ES can be constructed straightforwardly with η and ϕ_e scales, by solving the system of differential equations (2.4) and (2.5). Moreover, in the absence of an electric charge, the ϕ_e scale on two ESs can be synchronized from ZEFS, as demonstrated in Section 2.2.6. Although the ES recipe is technically based on field components that are specific to the chosen frame of reference, neither the ES topology nor the η or ϕ_e values depend on this specific reference frame. In other words, if two physicists decide to independently draw the 3D map of ESs around a given event, they will obtain the same map, and the ESs will have milestones featuring the same η or ϕ_e quantities. This map is therefore absolute and unique. Furthermore, given the possibility of computing the electromagnetic field components directly from the ES map (Section 2.2.6), this map is not only absolute and unique, but also contains a complete description of the y -invariant p -polarized electromagnetic world.

2.2.6 Computation of electromagnetic field components from electric spaghetti

An ES is a line in the spacetime map which connects a set of events regardless of the choice of coordinate system. In contrast, electromagnetic field components are relative to the choice of an inertial frame of reference $(O, \mathbf{t}, \mathbf{x}, \mathbf{z})$. In any frame of reference, the components of the electric and magnetic fields can be extracted from the ES map in several ways, provided below.

Field computation from η ES measure

The components of the electromagnetic field can be obtained from the equation (2.4) as follows:

$$H_y = \frac{ds_t}{d\eta}, D_z = -\frac{ds_x}{d\eta}, D_x = \frac{ds_z}{d\eta}, \quad (2.6)$$

where (ds_t, ds_x, ds_z) are the components of the vector tangent to the ES.

The smaller the field, the larger the η value on an ES. This may seem rather counterintuitive,

but the characteristic ES parameter η is really an absolute, extensive physical property of ESs, measured as the 3D volume per charge.

Field computation from ϕ_e ES measure

The electromagnetic field components can be computed from a more natural quantity, the electric flux value $d\phi_e$, except null-like ESs segments. First, let us consider Fig. 2.2 (a). An infinitesimal ES segment OE is presented in green, having an electric flux value $d\phi_e$. According to the ES definition, the flux value on any segment that is perpendicular (in the sense of the Minkowski product) to the ES direction is zero. ES segments define the ZEFS plane, plotted in red in Fig. 2.2 (a).

Under a condition where there is no electric charge, the electric flux value on any closed 2D surface of 4D MSM is zero. Transposed to considered p -polarized 3D case, the electric flux value on *any* closed line is zero. From triangles EOT , EOX , EOZ , it is clear that the segments OT , OX , OZ , and OI (Fig. 2.2 (a)) contain the same electric flux value $d\phi_e$. The points T , X and Z are chosen as the intersections of the perpendicular plane, with Ot , Ox and Oz axes. Construction of the point I is explained as follows. In the case presented in Fig. 2.2 (a), ξ is the unit vector in the direction of the electric field, and OI is defined as the direction perpendicular to them, with ζ as the unit vector in this direction. The ES segment OE can therefore be considered in 2D (t, ζ) spacetime (inset in Fig. 2.2 (a)). I is also the intersection of the XZ and ET lines. As the electric flux value is zero on any closed lines in the absence of an enclosed electric charge, and EI belongs to ZEFS (and thus contains zero electric flux), the electric flux value $d\phi_e$ on OI equals the electric flux value on the ES segment OE . In this case, the only component of $\mathbf{D}$ is $D_\xi = d\phi_e / |OI|$. The coordinates of the point I can easily be computed as

$$\left(0, \frac{\frac{1}{|OX|}}{\frac{1}{|OX|^2} + \frac{1}{|OZ|^2}}, \frac{\frac{1}{|OZ|}}{\frac{1}{|OX|^2} + \frac{1}{|OZ|^2}} \right),$$

giving the equality $\frac{d\phi_e^2}{|OI|^2} = \frac{d\phi_e^2}{|OZ|^2} + \frac{d\phi_e^2}{|OX|^2}$. Herewith the formula $D_\xi^2 = D_x^2 + D_z^2$ stands as expected.

Using the formulae $d\phi_e = H_y |OT| = D_z |OX| = -D_x |OZ|$ (Eq. (2.3)) the components of the electromagnetic field can be computed in the $(\mathbf{t}, \mathbf{x}, \mathbf{z})$ frame of reference as

$$H_y = \frac{d\phi_e}{|OT|}, \quad D_z = \frac{d\phi_e}{|OX|}, \quad D_x = -\frac{d\phi_e}{|OZ|}. \quad (2.7)$$

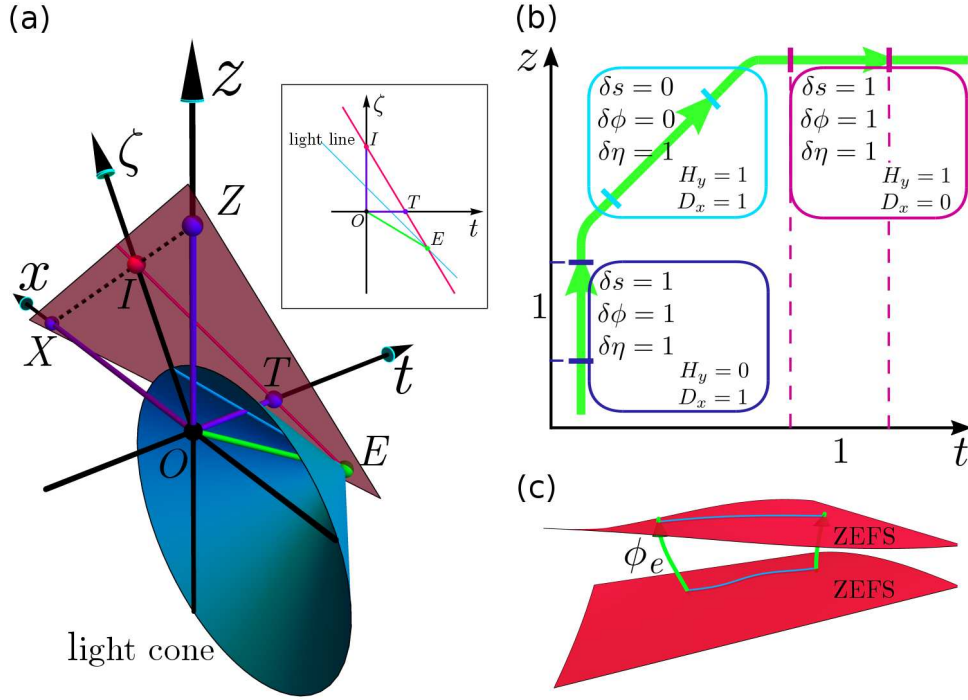


Figure 2.2: (a) ES segment OE (green line) and ZEFS plane (dark red) containing the E, I, T, X, Z events. All the segments OI, OT, OX, OZ contain the same electric flux value. The segments OT, OX, OZ correspond to coordinate axes. I is the intersection of XZ and ET . (b) ES line (green) with spacelike, null-like and timelike parts. For the spacelike (dark-purple frame) and timelike (purple frame) parts, both equations (2.6) and (2.7) can be used to compute field components. When the ES is contained in the light cone (light-blue frame), only the characteristic parameter (Eq. (2.6)) can be used. (c) If no charge is present, any green line connecting two ZEFS carries the same electric flux value ϕ_e , meaning that a potential value can be attributed to each ZEFS.

Once a reference system has been chosen, the components of the fields can be directly computed from the map using Eq. (2.7). As demonstrated in Fig. 2.2 (b), it can be used for both time-like and space-like segments. Let us consider the inset of Fig. 2.2 (a) as an example of a situation where an ES is not aligned along a time or space axis, unlike the case in Fig. 2.2 (b). In the inset, the ES is timelike and the ZEFS line is spacelike. The OT segment is smaller than the OI segment, which corresponds to a higher value for the magnetic field $H = H_y$ than for the electric field $D = D_x$. This is true in all the reference frames, as $H^2 - D^2$ is an invariant. As indicated earlier, a procedure using electric flux milestones (Eq. (2.7)) seems more natural if it uses η milestones. However, unlike the Eq. (2.6), it fails to yield the electromagnetic field components when the ES is null-like (see Fig. 2.2 (b), light-blue zone).

Field computation from ZEFS

A more pictorial way of obtaining electromagnetic field components is to draw ZEFSs in the MSM. By definition, ZEFSs are the surfaces perpendicular to the ESs (in the Minkowskian

sense). In spacetime regions with no charges, the electric flux ϕ_e measured between two ZEFSS is identical, regardless of the choice of starting and finishing points on each ZEFSS (Fig. 2.2 (c)). A value (electric flux potential) can thus be attributed to each ZEFSS, and ϕ_e can be computed as a difference in this potential. If a $(O, \mathbf{t}, \mathbf{x}, \mathbf{z})$ frame of reference is chosen, the components of the EM fields are the ratio of $d\phi_e$ over the distances dt , dx and dz between two ZEFSS in the directions $\mathbf{t}, \mathbf{x}, \mathbf{z}$ with the appropriate signs:

$$H_y = \frac{\partial \phi_e}{\partial t}, \quad D_z = \frac{\partial \phi_e}{\partial x}, \quad D_x = -\frac{\partial \phi_e}{\partial z}. \quad (2.8)$$

Although Eq. (2.8) is similar to Eq. (2.7), its form allows the problem of ES contained in the light cone to be solved. When an ES segment is null-like, it contains a zero electric flux, meaning that the equation (2.7) cannot be used. Moreover, null-like ES segments cannot connect two ZEFSSs because they are contained inside a ZEFSS. This is a peculiarity of Minkowskian geometry, where a null-line is perpendicular to itself. As a consequence, potential values cannot be attributed to ZEFSSs directly from this region. However, potential values can be attributed to these ZEFSSs by computation in adjacent regions where ES are timelike or spacelike (Fig. 2.3(a)). Hence the $d\phi_e$ value between two infinitely closer ZEFSSs can always be determined, and Eq. (2.8) is fully applicable to the computation of the electromagnetic field components.

In the presence of electric charges, equipotential values cannot be assigned to ZEFSSs, but the concept of ZEFSSs remains valid, as does the difference in potential value $d\phi_e$ between two nearby ZEFSSs. Hence in general, an absolute representation of the $\mathbf{y}$ -invariant, p -polarized electromagnetic world is given by the ZEFSS map and the electric flux difference between them. Let us consider the simplest case involving electric charges: a linear, constant density of charge along the $\mathbf{y}$ -direction. The worldline of each charge is a straight line along the $O\mathbf{t}$ axis, and represents all the charges in the 3D MSM $(\mathbf{t}, \mathbf{x}, \mathbf{z})$. ES in this case are concentric circles centred around the $O\mathbf{t}$ line. Every ES circle contains the same electric flux value, equal to the linear charge density λ . ZEFSSs in this case are radial half-planes, starting on the $O\mathbf{t}$ charge line (Fig. 2.3(b)). The difference in electric flux ϕ_e between two ZEFSSs can be defined, as being equal to the electric flux ϕ_e on any ES part cut by these two ZEFSSs (Fig. 2.3(c)).

An example of computation of field components from spaghetti structures

To illustrate how precise this computation can be, let us consider an arbitrary ES, depicted in Fig. 2.4(a), with the characteristic parameter η . This spaghetti is obtained in the slit problem

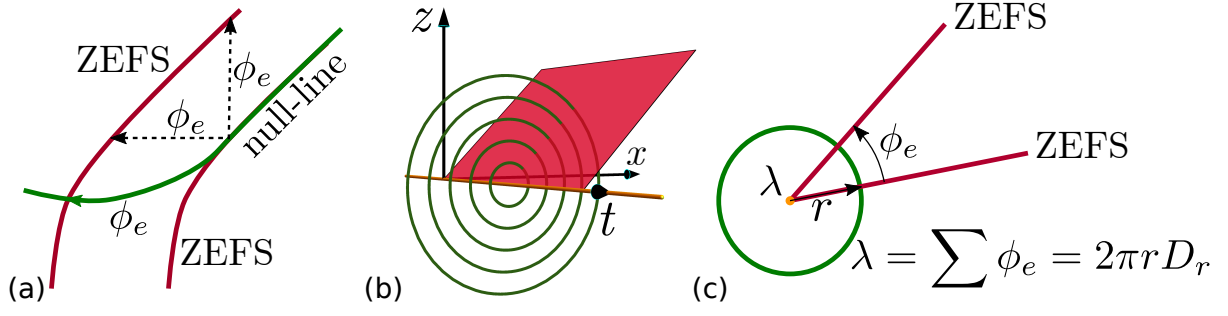


Figure 2.3: (a) Two ZEFs (red) are never linked by an ES where they are null-like (top), but the difference in potential ϕ_e between them can be obtained from other domains where they are connected by an ES (green, bottom). (b) Electron density along the y axis in (t, x, z) spacetime is represented by a straight red line running in the t -direction which gives rise to a ZEF (red plane). In this case, the ESs take the form of concentric green circles. (c) Electric flux value along one of the ES in (b).

from Section 2.3.3. Using Eq. (2.6), the field components are obtained and compared with the field components calculated from Maxwell's equations. The coincidence, shown in Fig. 2.4, is perfect.

2.2.7 Direct computation of ZEFs

The computation of y -invariant p -polarized electromagnetic problems with no charge can be considerably simplified by considering the ϕ_e potential and Eq. (2.8) with the curl equation $\partial_t B_y - \partial_x E_z + \partial_z E_x = 0$. In a vacuum, the resulting equation is just the d'Alembertian equal to zero: $(\partial_t^2 - \partial_x^2 - \partial_z^2) \phi_e = 0$ with the normal derivative at perfect conductor boundaries being equal to zero.

In conclusion, for the y -invariant, H_y polarized problem, a static map of ESs and/or ZEFs with ϕ_e or η milestones can be constructed. Independently of any frame of reference, this map contains all the information about the electromagnetic world. A frame of reference is no more than the choice of three Minkowski-perpendicular unit vectors (t, x, z) in the MSM, and electromagnetic field components are no more than quantities relative to this particular choice of axes in the MSM. They can be straightforwardly computed either from the ESs with their milestones η (Eq. (2.6)) or ϕ_e (Eq. (2.7)), or from the difference in flux between two ZEFs (Eq. (2.8)). In the following section, three different examples illustrating this *absolute* description of the electromagnetic world are provided: the electrostatic coaxial capacitor, the interference of two plane waves, and extraordinary transmission in resonant slits.

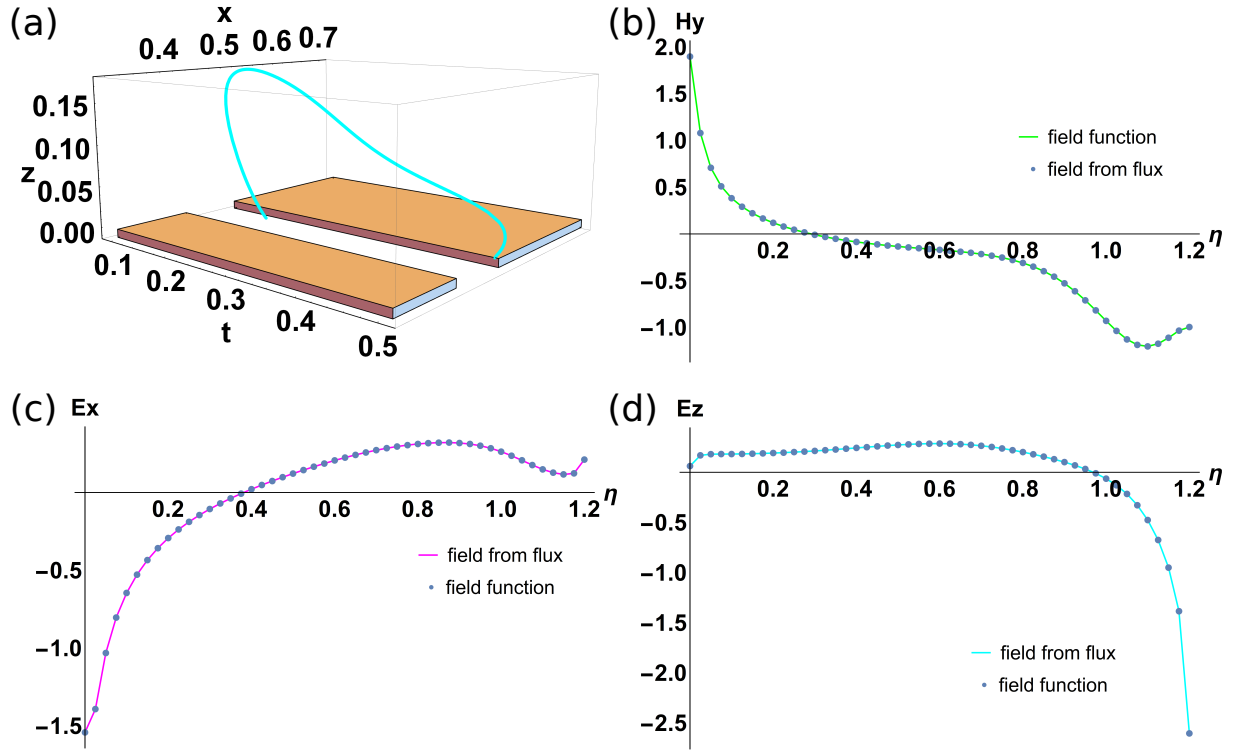


Figure 2.4: (a) Spacetime topology of an ES; (b) magnetic and (c,d) electric field components along it. The original values of the field components (solid curves) are compared with those calculated from the characteristic ES parameter η and ES tangent vector (dots).

2.3.

Three illustrative examples

In this Section, a unique topology of ES and ZEFS maps of three y -invariant, p polarized cases are studied in detail. The first two cases – the electrostatic cylinder (Section 2.3.1) and the interference of two planes waves (Section 2.3.2) – can be analyzed in 2D plane. In addition to the invariance in y direction, the t dimension in the first case is also removed, while in the second example the direction x is excluded. In the third example, the extraordinary transmission in resonant slits is considered in 3D spacetime (Section 2.3.3).

2.3.1 Electrostatic cylinder

First, the electrostatic problem of two oppositely charged co-centred cylinders is considered (Fig. 2.5 (a)) – a similar problem to the one described at the end of Section 2.2.6 (Fig. 2.3). As this problem is invariant in y and t directions, a cut can be made at any t and y (for example, $t = 0$ and at $y = 0$). The solution of this problem is well-known: in the reference frame connected with the system, magnetic field is equal to zero, and electric field in the gap between

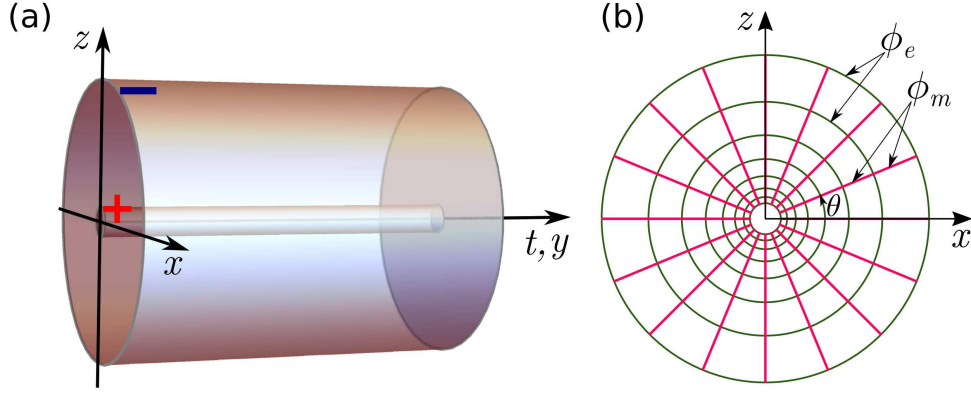


Figure 2.5: (a) Electrostatic problem with two oppositely charged co-centered cylinders. (b) ESs (in green) and ZEFS (in red) cut for the given problem

two cylinders decreases inversely to the distance to the center.

2D map for this problem is presented in Fig. 2.5 (b) where the ESs are shown in green and the ZEFSs in red. In accordance with the ES definition, the green lines are perpendicular to the red lines. At first glance, this is nothing more and nothing less than a picture of electric field strength lines (red lines) and equipotential lines (green lines). However, the two are fundamentally different. In a different frame of reference, a magnetic field would appear, and both the electric field and the equipotential lines would be changed. Instead, the ESs and ZEFSs are absolute (i.e. defined independently of any frame of reference). They yield both magnetic and electric field components, depending on the choice of frame of reference (Eqs. (2.6), (2.7) and (2.8)). In Fig. 2.5 every green ES segment contains the same electric flux $\phi_e = \int_{\theta_1}^{\theta_2} r D_r d\theta = r D_r \delta\theta$. The total value of the electric flux over every ES circle is equal to the linear density of charge λ on the internal cylinder. Here, radial red lines correspond to ZEFSs, and they can also be seen as magnetic spaghettis with a constant magnetic flux $\phi_m = \int_{r_1}^{r_2} E_r dr$ on each red segment. Each green arc in the figure contains the same electric flux ϕ_e , and each red segment contains the same magnetic flux ϕ_m . Both fluxes are connected by the constitutive equation in a vacuum ($D_r = E_r$): $\phi_e = a\phi_m$ with $a = \frac{r d\theta}{dr}$. This can form the basis of a new way of solving Laplace's equation: to build a solution, we need to plot a mesh made up of red and green segments, topologically similar to Fig. 2.5(b). The segments must be mutually orthogonal, and their length ratio must be a constant.

2.3.2 Interference of two plane waves

In our second example, we consider a system that is no longer invariant in the $\mathbf{t}$ -direction, formed by the interference of two counterpropagating waves in a vacuum $E_x = Y(t - z) \sin(2\pi(t - z)) + Y(t + z) \sin(2\pi(t + z))$, where Y is the Heaviside step function and the wavelength Λ is equal to 1. This problem is invariant in $\mathbf{y}$ - and $\mathbf{x}$ -directions, so the MSM can be represented by a 2D time-space $(\mathbf{t}, \mathbf{z})$ plane where the peculiarities of Minkowskian geometry can be observed.

This 2D spacetime is illustrated in Fig. 2.6 where, as before, the ESs are pictured in green, and the ZEFSs in red. In the top and bottom lefthand corners of Fig. 2.6(a), the depicted events are beyond the interference of the two counterpropagating planes waves. The ESs of each plane wave are straight lines running along the null directions $\mathbf{t} - \mathbf{z}$ and $\mathbf{t} + \mathbf{z}$. This case is represented in Fig. 2.2(b). In the special case of plane waves, the red and green lines are superimposed, and the electric flux is measured not on the ESs themselves, but between the ZEFS lines. In Fig. 2.6, the electric flux between two adjacent green lines is constant everywhere, and equal to $\frac{1}{10\pi}$.

The wave interference is pictured in the central and righthand parts of Fig. 2.6(a) and in Fig. 2.6(b) by concentric rounded squares. The ZEFS lines have the same rounded square patterns, but with the time and space shift of $\lambda/4$. Given the definition of ESs and ZEFSs, all the green and red lines are perpendicular. This is obvious in Fig. 2.5(b), which corresponds to space-space Euclidean geometry. By contrast, in Fig. 2.6, Minkowskian time-space geometry should be used, where null lines are perpendicular to themselves. As the flux on line segments is constant, the shorter the segments, the larger the field. The flux orientation is indicated in the figure by periodically alternating blue arrows. The centers of the rounded squares correspond to zero field events.

At the x, y planes of maximum interference ($z = 0, 0.5, 1$, etc.), the ES tangents are parallel to the Oz axis. Therefore, in accordance with Fig. 2.2(b), the field is purely electric at any value of t . For $z = -0.25, 0.25, 0.75$, and so on, the ES tangents are parallel to the Ot axis. The field is therefore purely magnetic. Between the two, there are events where the ESs are null-like. At these events, the green and red lines are tangent, and the electric and magnetic fields are equal, as in Fig. 2.2(b) in the light-blue zone.

As observed in the electrostatic example (Section 2.3.1), the ES map, which was constructed using a special frame of reference $(\mathbf{t}, \mathbf{z})$, is in fact independent of it and any other frame of reference. Conversely, once a frame of reference has been chosen, the electric and magnetic field

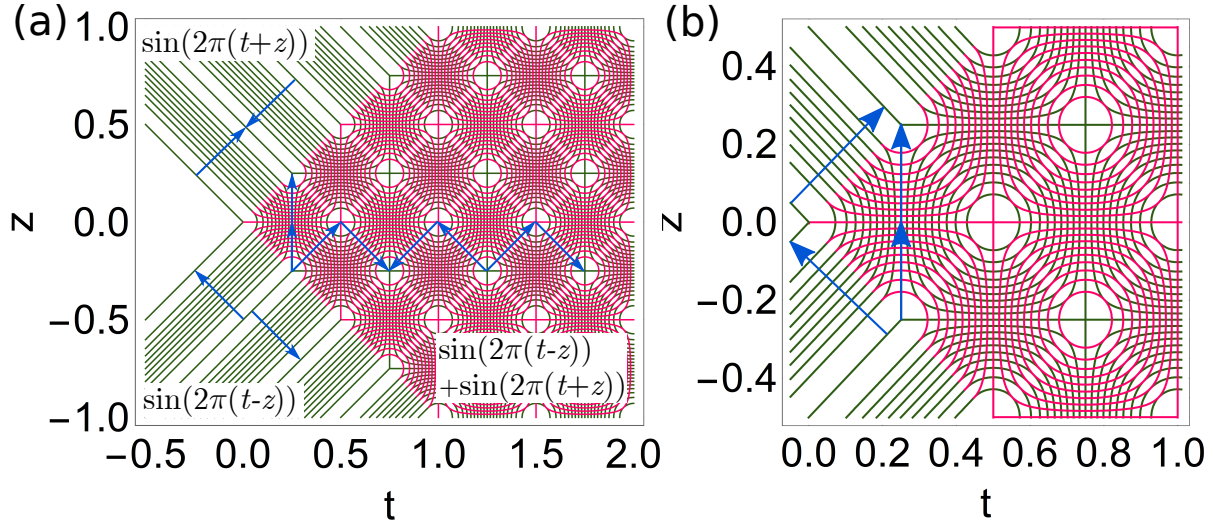


Figure 2.6: (a) Interference of two plane waves. Green lines are ESs, red lines are ZEFs. In the plane-wave region, these lines coincide. There is a constant flux of $1/10\pi$ between two red lines. Blue arrows indicate an oriented electric flux equal to $1/\pi$. (b) Zoomed part of the zone of interference.

components can be directly extracted from the map. The observed topological pattern for the case of interference of two plane wave appeared any time when interference is observed, and in the following sections an example of it will be given.

2.3.3 Extraordinary transmission in resonant slits

In this third example, the system is only invariant in the $\mathbf{y}$ -direction, and no longer invariant in the $\mathbf{x}$ -direction. The ESs are now curves plotted in 3D spacetime, with new topological behavior. We consider a periodic array of slits, made into a perfect conductor material. The slits are parallel to the $\mathbf{y}$ -direction (Fig. 2.7(a)), and excited at normal incidence by a p -polarized plane wave (magnetic field along the $\mathbf{y}$ -direction). The period of the grating is $L = 0.5$, that is, half the wavelength $\Lambda = 1$ ($c = 1$), so all the diffracted waves are evanescent. The slit width is arbitrarily chosen as $w = 1/6$. When the slit height h is adjusted to $h = h_0 + n\Lambda/2$ with $h_0 = 0.417$, n is an arbitrary integer, there is a Fabry-Perot resonance with total transmission, and no reflection. The value of h_0 is slightly smaller than $\Lambda/2$, as the phase of the internal reflection is not exactly π . We chose $h = 0.917$, in order to have a clear view of the interference pattern inside the slits.

The system is only studied in the resonance condition, in order to suppress the reflected wave. The 100% transmission phenomenon is depicted in Fig. 2.7(b), where the energy of the incident wave funnels through the grating [36]. This funneling can be directly attributed to magneto-

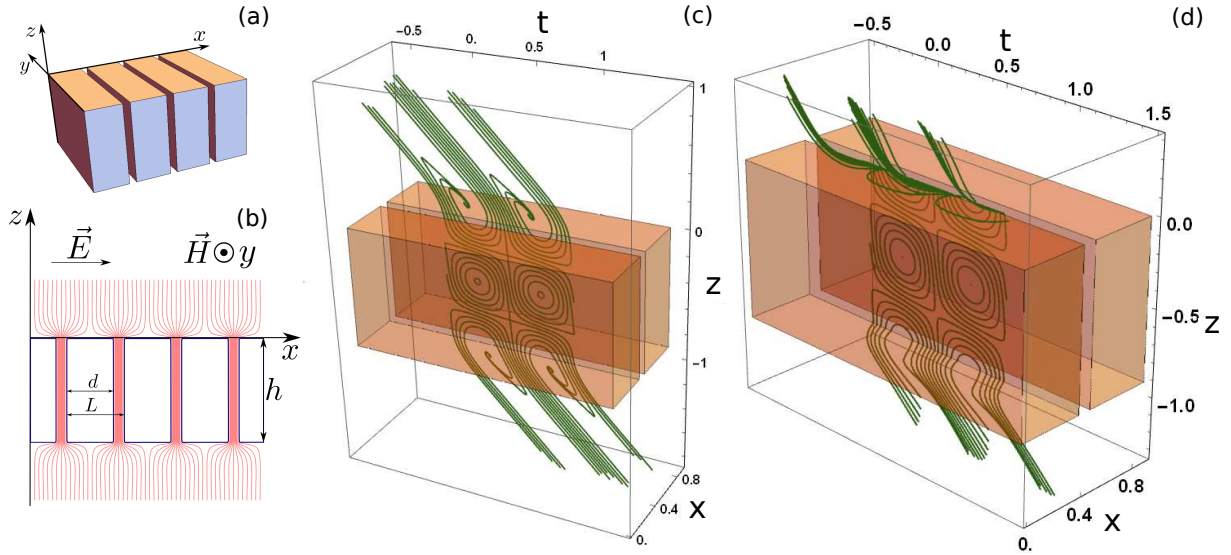


Figure 2.7: (a) Slit grating, invariable in y -direction, made in a perfect conductor. (b) Time-averaged Poynting vector lines in the resonant case, H -polarized normally incident monochromatic wave. ESs (c) in the central plane of a slit, and (d) for x -shifted case.

electric interference between the incident and evanescent fields [37]. Several ESs are plotted in the 3D spacetime map in Figures 2.7 (c) and (d). The median plane is a special location in this system where the E_z component is equal to zero. As a consequence, the ZEFSs are perpendicular to this plane and all the ESs are located in it, as seen in Figs. 2.7(c). Figure 2.3.1 shows one period of ESs (green) and ZEFSs (red) in such a way that they partially overlap. The details of the two patterns are visible, but these patterns fill the entire plane tOz . The electric flux is constant between two neighboring ZEFS lines (bold red) inside the slit. The density of these lines in the t -direction therefore corresponds to the H_y field component, and the density in the z -direction corresponds to the D_x field component. Some characteristics of the ESs are evident, and the plots in Fig. 2.2(b) and Fig. 2.6 are similar, whether they are inside the slit or far from the slits. The field is purely electric when the ESs (green lines) are parallel to the z -direction, and purely magnetic when they are parallel to the t -direction. Owing to the resonance inside the slit, the electric flux density in this region is six times higher than it is in the plane-wave region. This is clearly depicted in Fig. 2.8: in the slit region, there are six ZEFS lines over a half-period in time, whereas in the top part of the figure (plane-wave region), there is only one ZEFS bold line. As we saw in Fig. 2.6, lines beginning in the vacuum are straight null-like ESs corresponding to simple plane waves. The central part is slightly distorted because the bottom-to-top wave amplitude is smaller than the top-to-bottom one. Another slight difference between Fig. 2.6 and Fig. 2.8 is the presence of two ESs per period crossing the structure in

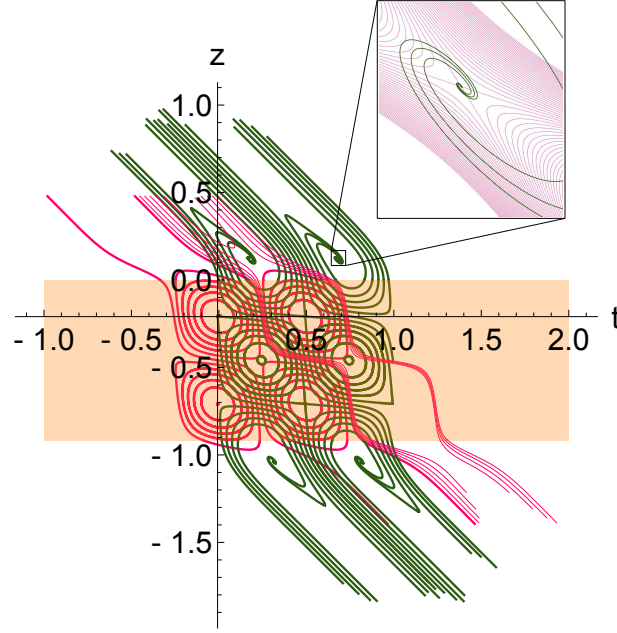


Figure 2.8: 2D view of the ES (green) and ZEFS (red) lines on the median plane for one time period. The orange rectangle corresponds to the slit area. There is a constant electric flux between two thick red ZEFS lines. The flux is 6 times smaller between two thin red lines. Inset: zoom of the whirl zone, with even smaller flux step between ZEFS lines.

the $\mathbf{z}$ -direction. Most ESs entering the slits exit from the same side (top or bottom) of the grating, making a U-turn. Thus, ESs are clearly independent of any past-future direction, or the distinction between time and space.

New topological structures can be seen in the top and bottom parts of the slit grating. These regions are known to give rise to magneto-electric interference between the evanescent field and the incident wave, at the origin of the energy funneling observed in Fig. 2.7 (b) [37]. Whirls observed in the median plane of the slit are very similar to those resulting from the interference of a plane wave with an evanescent wave, discussed in detail in Section 2.4. They are centered on a zero field event, and have the fractal behavior of a stretched logarithmic spiral. They seem to contradict the principle that tubes of flux with no charge never end[17]. They do not, indeed: logarithmic spirals never end. In Fig. 2.7(d), we can see slightly x -shifted ESs abruptly exiting the whirl in the $\mathbf{x}$ -direction. Starting from different x coordinates, an intricate topology is depicted in Fig. 2.9. Some of the ESs touch and even enter the slit region, then either turn around or enter the slit and exit at the bottom. Others enter the gap and return to the free space.

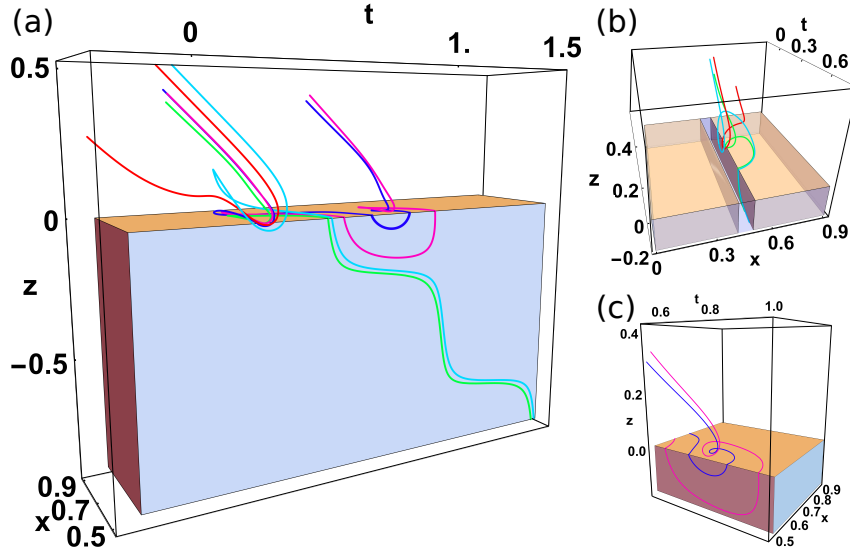


Figure 2.9: The different behaviour of a selection of five out-of-central-plane ESs illustrates that topological analysis of ES should be a new field of research.

2.4.

Detailed analysis of the interference of a plane wave and an evanescent wave

In the problem of the extraordinary transmission in resonant slits (Section 2.3.3) the interesting whirl pattern appears near the slits, in the area of the evanescent waves. At first look, it seems contradictory with the common knowledge that the lines of flux never end if there is no charges [17]. In this Section, the origin of the whirl patterns is considered.

The field in the whirl zone is the superposition of the incident field and of the diffracted field due to the periodic slit structure. As the period of the grating have been chosen smaller than the wavelength, for a normal incident wave the diffracted orders are all evanescent. The height of the slits have been adjusted (close to $\lambda/2$) in the way that there is no reflection (close to $\lambda/2$) so that the reflected amplitude for the far field is zero. In this Section, the following approximation is chosen: only the pair of evanescent waves corresponding to the orders -1 and $+1$ of the diffracted waves is taken into account. They have the same amplitude and build a standing wave.

In order to simplify the writing equations, here the wavelength is set as $\Lambda = 2\pi$. For an incident plane wave of amplitude $\cos(z + t)$, the evanescent wave corresponding to the standing wave for the first diffracted order of the grating with the period $L = 0.5\Lambda$ has the amplitude $a \exp(-\sqrt{3}z) \sin(t) \cos(2x)$. Here $a = 1.09122$ (numerically computed) for the slit grating con-

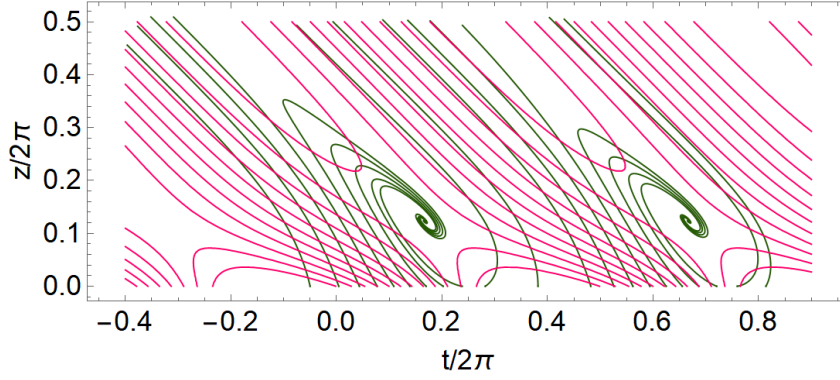


Figure 2.10: ESs (green) and ZEFS-lines (red) in the case of interference of the plane wave and two evanescent ones (with equal amplitudes and opposite propagation directions along x axis.).

sidered. The wave vector component for the z direction is $2\pi\sqrt{-\frac{1}{\Lambda^2} + \frac{1}{L^2}} = \sqrt{3}$. At the surface $z = 0$, the two waves are in quadrature. The flux and field components are then:

$$\phi_e = \sin(z + t) - a \exp(-\sqrt{3}z) \cos t \cos 2x \quad (2.9)$$

$$H_y = \cos(z + t) + a \exp(-\sqrt{3}z) \sin t \cos 2x \quad (2.10)$$

$$D_x = -\cos(z + t) - a\sqrt{3} \exp(-\sqrt{3}z) \cos t \cos 2x \quad (2.11)$$

$$D_z = 2a \exp(-\sqrt{3}z) \cos t \sin 2x \quad (2.12)$$

The ES-ZEFS pattern of this interference at $x = 0$ is plotted as green and red lines in Fig. 2.10. The pattern of this simplified model is very similar to the one observed in Fig. 2.8. The whirls are centered on no-field points $H_y = E_x = 0$, corresponding to equations:

$$\tan t_0 = \sqrt{3} \quad (2.13)$$

$$\frac{\cos z_0}{\sqrt{3}} - \sin z_0 + a \exp(-\sqrt{3}z_0) = 0 \quad (2.14)$$

with solution $(t_0 = \pi/3, z_0 = 0.7736)$.

The asymptotic character of the whirls can be obtained by a linear approximation ($t = t_0 + t', x = x', z = z_0 + z'$) of the field around the point $(t_0, 0, z_0)$:

$$\begin{pmatrix} H_y \\ D_x \end{pmatrix} = \lambda_i A \begin{pmatrix} t' \\ z' \end{pmatrix} \quad (2.15)$$

$$D_z = \lambda_x x' \quad (2.16)$$

with

$$A = \begin{pmatrix} -2\alpha + \beta & -\beta \\ \beta & \beta \end{pmatrix} \quad (2.17)$$

$$\lambda_i = \sqrt{(3 \sin z_0 + \frac{\cos z_0}{\sqrt{3}})(\sin z_0 - \frac{\cos z_0}{\sqrt{3}})} \quad (2.18)$$

$$\lambda_i \alpha = \sin z_0 + \frac{\cos z_0}{\sqrt{3}} \quad (2.19)$$

$$\lambda_i \beta = 2 \sin z_0 \quad (2.20)$$

$$\lambda_x = \lambda_i(-2\alpha + 2\beta). \quad (2.21)$$

The factor λ_i which will appear later as the angular frequency for ES η parameter gives for the moment the relation

$$\beta^2 = 1 + \alpha^2. \quad (2.22)$$

The ES equation (2.4) becomes:

$$\frac{d}{d\eta} \begin{pmatrix} t' \\ z' \end{pmatrix} = \lambda_i A \begin{pmatrix} t' \\ z' \end{pmatrix} \quad (2.23)$$

$$\frac{dx'}{d\eta} = -\lambda_x x. \quad (2.24)$$

The solution of this system of differential equations is

$$\begin{pmatrix} t'(\eta) \\ z'(\eta) \end{pmatrix} = e^{\lambda_i \eta A} \begin{pmatrix} t'(0) \\ z'(0) \end{pmatrix} \quad (2.25)$$

$$x'(\eta) = e^{-\lambda_x \eta} x'(0) \quad (2.26)$$

where the matrix exponential can be obtained from the eigenvalues $\lambda_{\pm}$ and eigenvectors matrix V :

$$\exp(\lambda_i \eta A) = V \begin{pmatrix} \exp(\lambda_+ \eta) & 0 \\ 0 & \exp(\lambda_- \eta) \end{pmatrix} V^{-1} \quad (2.27)$$

where

$$\lambda_i A V = V \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}. \quad (2.28)$$

Explicitly

$$\lambda_{\pm} = \lambda_r \pm i\lambda_i \quad (2.29)$$

with

$$\lambda_r = (\beta - \alpha)\lambda_i = \lambda_x/2 \quad (2.30)$$

and

$$V = \begin{pmatrix} -\alpha + i & -\alpha - i \\ \beta & \beta \end{pmatrix} \quad (2.31)$$

$$V^{-1} = \frac{1}{2\beta} \begin{pmatrix} -i\beta & 1 - i\alpha \\ i\beta & 1 + i\alpha \end{pmatrix}. \quad (2.32)$$

Equation (2.27) can be rewritten as

$$\exp(\lambda_i \eta A) = e^{\lambda_r \eta} \left(\cos(\lambda_i \eta) + \sin(\lambda_i \eta) V \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} V^{-1} \right).$$

Calculating

$$V \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} V^{-1}$$

and considering the starting point $t'(0) = 0, x'(0) = x_s, z'(0) = z_s$, we obtain

$$\begin{pmatrix} t'(\eta) \\ z'(\eta) \end{pmatrix} = z_s e^{\lambda_r \eta} \begin{pmatrix} 0 & -\beta \\ 1 & \alpha \end{pmatrix} \begin{pmatrix} \cos \lambda_i \eta \\ \sin \lambda_i \eta \end{pmatrix} \quad (2.33)$$

$$x'(\eta) = x_s e^{-2\lambda_r \eta} \quad (2.34)$$

The equation (2.33) is an affine transformation of the logarithmic spiral:

$$\begin{pmatrix} t''(\eta) \\ z''(\eta) \end{pmatrix} = e^{\lambda_r \eta} \begin{pmatrix} \cos \lambda_i \eta \\ \sin \lambda_i \eta \end{pmatrix} \quad (2.35)$$

This curve, named *spira mirabilis* by Bernoulli three centuries ago, is a spiral that never ends, the first fractal curve ever studied. For each loop toward the center, corresponding to a decrease in parameter η equal to $-2\pi/\lambda_i$, the radius is decreased by a factor of $\exp(-2\pi\lambda_r/\lambda_i) = 0.12$. The spaghettis starting from a non-zero $x = x_s$ value have the same (t', z') behaviour, spiralling

toward ($t' = 0, z' = 0$), while they are escaping exponentially in x direction, with a factor $\exp(4\pi\lambda_r/\lambda_i) = (1/0.12)^2$ for each loop.

Despite this topological curiosity, the electromagnetic field around the zero-field point has a linear behaviour, as expected. Indeed, the absolute *characteristic parameter* η we have introduced has a constant value of $2\pi/\lambda_i$ for each loop. As the temporal and spatial sizes of the loop are proportional to the distance to the center of the whirl, the field amplitudes (Eq. (2.6)) are proportional to this distance.

The ES plotted (in an absolute way) in the spacetime time map with the η parameter scale written on them are fully describing the field components of an y -invariant, p -polarized problem. Calculating the field from these spiralling ES, which seems counter-intuitive or even erroneous for a field in a vacuum without any charge can be illuminating. The field components are given by equations (2.6), and using Eqs. (2.33, 2.34), we calculate them as

$$\begin{pmatrix} H_y \\ D_x \end{pmatrix} = \frac{d}{d\eta} \begin{pmatrix} t' \\ z' \end{pmatrix} \quad (2.36)$$

$$D_z = -\frac{dx'}{d\eta} \quad (2.37)$$

The component D_z is simple to calculate. We obtain $\lambda_x x_s e^{-\lambda_x \eta} = \lambda_x x'$ which is exactly the expected value, given by Eq. (2.16). The two other components are

$$\begin{pmatrix} H_y \\ D_x \end{pmatrix} = z_s e^{\lambda_r \eta} \begin{pmatrix} -\lambda_i \beta & -\lambda_r \beta \\ \lambda_r + \lambda_i \alpha & \lambda_r \alpha - \lambda_i \end{pmatrix} \begin{pmatrix} \cos \lambda_i \eta \\ \sin \lambda_i \eta \end{pmatrix} \quad (2.38)$$

or

$$\begin{pmatrix} H_y \\ D_x \end{pmatrix} = z_s e^{\lambda_r \eta} \lambda_i \beta \begin{pmatrix} -1 & \alpha - \beta \\ 1 & \alpha - \beta \end{pmatrix} \begin{pmatrix} \cos \lambda_i \eta \\ \sin \lambda_i \eta \end{pmatrix} \quad (2.39)$$

Inverting Eq. (2.33) we obtain

$$z_s e^{\lambda_r \eta} \begin{pmatrix} \cos \lambda_i \eta \\ \sin \lambda_i \eta \end{pmatrix} = \frac{1}{\beta} \begin{pmatrix} \alpha & \beta \\ -1 & 0 \end{pmatrix} \begin{pmatrix} t' \\ z' \end{pmatrix}, \quad (2.40)$$

and eventually

$$\begin{pmatrix} H_y \\ D_x \end{pmatrix} = \lambda_i \begin{pmatrix} -2\alpha + \beta & -\beta \\ \beta & \beta \end{pmatrix} \begin{pmatrix} t' \\ z' \end{pmatrix}, \quad (2.41)$$

which is indeed the expected field, given by Eq. (2.15).

To summarize, the observed ESs whirl pattern corresponds to the case of the interference of the incoming plane wave and the evanescent wave. ESs whirls are centered on a zero field event, and have the fractal behavior of a stretched logarithmic spiral that never ends. The parameters of the spiral are defined by the parameters of the interfered waves.

2.5.

Conclusion

In this Chapter it was demonstrated, that contrary to common opinion, the electromagnetic field can be presented in absolute way. The static 3D map for the $\mathbf{y}$ -invariant, H_y polarized problem can be constructed with ESs, lines of events, supplemented with special measures, and/or ZEFSs. ES lines arise as the cut of more general lasagna surfaces in the invariant polarized case. As ES and ZEFS construction does not depend on a reference frame, they can be considered as static rigid objects in 3D MSM. Unlike physical objects, they can transcend the boundary between time and space, and be timelike, spacelike, and even lightlike, changing from one to the other. Such map containing all the information about the electromagnetic world.

ESs are labeled with three invariant scalar measures. A purely timelike or spacelike segment can be characterized by its interval length. The electric flux ϕ_e is the ES measure, which characterises the electromagnetic field. Being the result of action of a 1-form on an infinitesimal interval $d\mathbf{s}$, the electric flux ϕ_e is a scalar absolute value. However, in case of null-like ES, this measure cannot be applied. Thus, a new electromagnetic invariant η is introduced. Any ES segment between two events e_1 and e_2 is characterized of a scalar electromagnetic value $\eta(e_1, e_2)$, which is not zero even in the light cone region. The ESs with η labels contain all the electromagnetic information.

In the absence of electric charges, ZEFS can be labeled with the electric flux potential values, and ϕ_e value is a difference in this potential. ZEFS start on electric charges, thus equipotential values no more can be assigned to ZEFSs in the presence of electric charges. However, the concept of ZEFSs remains valid, as does the difference in potential value $d\phi_e$ between two nearby ZEFSs. Thus, an absolute representation of the $\mathbf{y}$ -invariant, p -polarized electromagnetic world can be given just by the ZEFS map and the electric flux difference between them.

In this way, electromagnetic map is ES and/or ZEFS and measures on them are the absolute map, independent from any reference frame. It contains all the information about electromag-

netic field. It was shown that components of the electromagnetic field can be computed from ES or ZEFS in any reference frame. In this way, a frame of reference is just the choice of three Minkowski-perpendicular unit vectors in the MSM, and electromagnetic field components are no more than quantities relative to this particular choice of axes in the MSM.

Although the ES recipe is technically based on field components, ESs can be constructed straightforwardly with η and ϕ_e scales. The ES topology, the η and ϕ_e values do not depend on a specific reference frame. If two events e_1, e_2 are connected by a specific ES S ($S = S(e_1) = S(e_2)$) constructed from a given frame of reference (system of axes), they are connected by the same ES S in another frame of reference chosen to construct $S(e_1)$. Thus, two physicists would independently build the same 3D map of ESs around a given event, and the ESs will have milestones featuring the same η or ϕ_e quantities. Therefore this map is absolute and unique.

ES and ZEFS demonstrates unique topology. Some characteristic patterns were shown for the specific cases of ES and ZEFS maps. In the presence of the electric charge density, ESs are concentric circles around charge, while ZEFSs start on electric charges. In the considered case of the electrostatic cylinders, ES-ZEFS pattern is similar to well-known picture of the equipotential lines and the electric strength lines. However, the ES-ZEFS map is absolute. Every segment of ES between two ZEFS (and any ZEFS segment between two ES) contains the same electric (magnetic) flux. In this way, the behavior of the electromagnetic field is clear from the plot.

Plane wave have a specific presentation in the ES-ZEFS topology: lines (ES and ZEFS coincide) with the sinusoidal density. The interference of two plane waves have the characteristic pattern as well. ESs and ZEFS cuts have a form of concentrated loops. They appear as well in the third example, the periodic slits problem. In this problem, the interesting whirl pattern appear in the area of the evanescent waves. It was shown, that the same pattern appear in the case of the interference of the plane wave with the evanescent wave. Linear approximation for the fields around the whirl provides the ES solution in the form of the logarithmic spiral, Bernoulli's spira mirabilis. This spiral has the fractal behaviour and never ends. The whirl parameters are defined by the parameters of the interfered waves.

As ZEFS is a particular case of lasagne, and ES is cut of another one, it is clear that for these particular problems, lasagna can be presented in 4D. The next Chapter is devoted to general case of 4D spacetime, where unique decomposition of two-form at any event provide a map of solid objects with flux values on them. After that, the presented above examples are reconsidered from lasagna point of view.

The absolute description of EM field in 4D spacetime

3.1.

Introduction

In the previous Chapter it was shown that in the spacetime map, restricted to the y -invariant p -polarized case, it is possible to construct "absolute" electromagnetic objects: electric spaghetti (ES) and Zero Electric Flux Surfaces (ZEFS). Regardless of the reference frame, which may have been chosen to build them, these objects connect the events of space-time in a fixed way. This Chapter is devoted to the generalization to any electromagnetic field in the four-dimensional spacetime.

The generalisation of the concrete map of ES, which was a set of oriented linear segments with the electric flux values on them, is now a set of pairs of 2D surface pieces. Each of them are labeled with electric and magnetic flux values.

At any event, the direction of spaghetti (its tangent) was unique and absolute (independent on a choice of the reference frame). Similarly, at any event, the orientation of the pair of surfaces is unique and absolute, and the two surfaces of each pair are completely orthogonal. For example, one surface is $\mathbf{t} \wedge \mathbf{x}$, and the second one is $\mathbf{y} \wedge \mathbf{z}$. The origin of such decomposition is based on the uniqueness of the decomposition of the electromagnetic two forms into a pair of simple two-form.

As far as we know, the mathematical proof of the uniqueness of such decomposition in 4D spacetime is mentioned only in the Riesz course [38], given more than 60 years ago in 1959.

And on the contrary, several later works, be it Misner [17], Lounesto [39], or even indirectly Landau [30], suggest that decomposition is not unique. As it will be shown below (Section 3.3), this uniqueness is specific to the space-time signature $(+ - - -)$ or $(- + + +)$.

Riesz demonstrated the uniqueness of the decomposition in a few lines, using Clifford's algebra [38]. The complete demonstration is provided in Appendix E. The demonstration of the uniqueness needs this advanced Clifford's algebra. However, the explicit calculation of the decomposition can be derived from a classical problem of electromagnetism (Landau, [30] (§25)), presented in Section 3.2, – the search for reference frames where the electric field and the magnetic field are parallel. An attempt to give an intuitive approach of the uniqueness, involving surfaces with zero flux, is presented in Section 3.3. It is demonstrated that this heuristic way is a dead end, because in 4D Euclidean space such bivectors decomposition is clearly not unique. The uniqueness indeed is the special feature of 4D spacetime.

In order to highlight the distinction between the presented approach and the common ways of geometric representation of the electromagnetic field with differential forms, an overview of them is provided in Section 3.4. Contrary to this work, none of them gives an absolute unique geometrical representation of the electromagnetic field. Then, in Section 3.5 unique decomposition with two bivectors is discussed in a general 4D case.

Finally, the explicit construction of unique surface pairs from the Riesz theorem is given in Section 3.6. After that (Section 3.7), the examples considered in Chapter 2, are presented in 4D spacetime, showing that **y** extension of ES and ZEFS themselves correspond to pair of two continuous solid surfaces, the electric and magnetic lasagna objects.

3.2.

Elementary construction of the unique surface pair

In this Section, the introduction to a concept of the unique pair of surfaces at any event of 4D spacetime is provided. A rigorous proof of the unicity of the decomposition of the electromagnetic two-form as the sum of two fully orthogonal simple two-forms was first given by Riesz, using Clifford algebra (see Appendix. E). However, a clue to this decomposition can be inspired by a classical electromagnetic problem. A pair of completely orthogonal surfaces appears when finding the reference frame where the electric and the magnetic fields are parallel.

The following problem is presented in [30] (§25): excepted if **E** and **H** are equal in module

and mutually perpendicular, it is possible to determine a frame of reference where the electric and magnetic fields are parallel. The speed and direction of the new frame of reference are given by the equation

$$\frac{\mathbf{v}}{1 + v^2} = \frac{\mathbf{E} \times \mathbf{H}}{\mathbf{E}^2 + \mathbf{H}^2}. \quad (3.1)$$

Using a notation $\boldsymbol{\sigma} = \frac{\mathbf{S}}{\mathcal{E}}$ with $\mathbf{S} = \mathbf{E} \times \mathbf{H}$, and $\mathcal{E} = \frac{\mathbf{E}^2 + \mathbf{H}^2}{2}$, the solution is

$$\mathbf{v} = \frac{1 - \sqrt{1 - \sigma^2}}{\sigma^2} \boldsymbol{\sigma}. \quad (3.2)$$

Let us consider the obtained result in more details. Let $\mathbf{z}$ be the direction of the fields at a given event in the new frame of reference. The only components of the field are E_z, B_z, H_z, D_z . D_z is the electric flux density on the surface $\mathbf{x} \wedge \mathbf{y}$ and H_z is the electric flux density on the surface $\mathbf{t} \wedge \mathbf{z}$. At the same time, B_z is the magnetic flux density on the surface $\mathbf{x} \wedge \mathbf{y}$, and E_z is the magnetic flux density on the surface $\mathbf{z} \wedge \mathbf{t} = -\mathbf{t} \wedge \mathbf{z}$. It can be observed that the two surfaces $\mathbf{x} \wedge \mathbf{y}$ and $\mathbf{t} \wedge \mathbf{z}$ are completely perpendicular one to another. Only they contain the electric and magnetic flux (each of them), and the flux is zero on the four remaining surfaces that are simply perpendicular to them: $\mathbf{t} \wedge \mathbf{x}, \mathbf{t} \wedge \mathbf{y}, \mathbf{y} \wedge \mathbf{z}$, and $\mathbf{x} \wedge \mathbf{z}$. It is also zero on any combination of them.

This is a construction of the decomposition of the magnetic flux two-form and of the electric flux two-form as the sum of two fully orthogonal simple two-forms:

$$\tilde{\Phi}_m = E_z \tilde{\mathbf{z}} \wedge \tilde{\mathbf{t}} + B_z \tilde{\mathbf{x}} \wedge \tilde{\mathbf{y}} \quad (3.3)$$

$$\tilde{\Phi}_e = H_z \tilde{\mathbf{t}} \wedge \tilde{\mathbf{z}} + D_z \tilde{\mathbf{x}} \wedge \tilde{\mathbf{y}}, \quad (3.4)$$

but this is not a proof that the decomposition is unique. As said by Landau ([30]), the solution of Eq. 3.2 is not unique in the term of reference frame as the electric and magnetic fields remain parallel in any frame of reference moving in the field direction $\mathbf{z}$. However, all these new reference frames correspond to the same two-form decomposition (Eqs. 3.3 - 3.4). Any new frame of reference $\mathbf{t}', \mathbf{z}'$ gives the unit bivector $\mathbf{t}' \wedge \mathbf{z}' = \mathbf{t} \wedge \mathbf{z}$, it is a fundamental property of the Lorentz's transformation. This is not yet a proof but the clue of uniqueness of the two-form decomposition.

In the following sections other approaches considering the uniqueness of the decomposition are provided.

3.3.

Another approach, involving zero-flux surfaces

In this Section, an attempt to understand the uniqueness of two-form decomposition is provided. It is based on an idea of four bivectors with zero flux on them in 4D spacetime. Thus, only two remain bivectors has non-zero flux values. However, this attempt is clearly bad considering that it will be equivalently valid in Euclidean 4D space while a simple counter-example shows two different decompositions.

An infinitesimal piece of a surface (regarded as a simple bivector), which can be considered as flat due to its small size, has two orthogonal directions in 4D spacetime. For the infinitesimal bivector $\mathbf{x} \wedge \mathbf{y}$, based on two non-parallel vectors $\mathbf{x}$ and $\mathbf{y}$ (Fig. 3.1 (a)), there are two orthogonal directions: $\mathbf{t}$ and $\mathbf{z}$. Let the value of the flux (electric or magnetic) on this bivector be $d\Phi_1$. Rotation of this bivector along the direction $\mathbf{y}$ towards the direction $\mathbf{z}$ eventually leads to the fact that the bivector will be flipped (see Fig. 3.1 (b) and (c)). It is important to note that here the term rotation means usual euclidean rotation based on $\sin \alpha$ and $\cos \alpha$ functions, but not the hyperbolic rotation. As a result, the bivector $\mathbf{x} \wedge \mathbf{y}$ changes by $-\mathbf{x} \wedge \mathbf{y}$. Thus, the flux on this surface changes to $-d\Phi_1$. This change of sign means that somewhere between the start and end positions there is such a position of the bivector that the flux on it is zero.

There are four ways to rotate the $\mathbf{x} \wedge \mathbf{y}$ bivector in 4D, which form four orthogonal surfaces: along $\mathbf{x}$ towards $\mathbf{t}$, along $\mathbf{x}$ towards $\mathbf{z}$, along $\mathbf{y}$ towards $\mathbf{t}$, along $\mathbf{y}$ towards $\mathbf{z}$. Thus, in 4D spacetime four simple infinitesimal bivectors with zero flux around a point can be found. As in general case any bivector in 4D can be decomposed on a basis of six simple bivectors, we expect that it is possible to find two fully perpendicular simple bivectors (with the electric and magnetic fluxes on them), orthogonal to these four bivectors with zero flux.

Unfortunately, this heuristic approach is flawed. Indeed, in euclidean space such decomposition is not unique. Let us suppose that zero flux surfaces are $\mathbf{t} \wedge \mathbf{y}$, $\mathbf{t} \wedge \mathbf{z}$, $\mathbf{x} \wedge \mathbf{y}$, $\mathbf{x} \wedge \mathbf{z}$, and the bivector with fluxes (consisted from two simple bivectors) is $\mathbf{b} = \mathbf{t} \wedge \mathbf{x} + \mathbf{y} \wedge \mathbf{z}$. In 4D euclidean space this decomposition is clearly not unique, as it can be presented as $\mathbf{b} = (\mathbf{ct} + \mathbf{sy}) \wedge (\mathbf{cx} + \mathbf{sz}) + (\mathbf{st} - \mathbf{cy}) \wedge (\mathbf{sx} - \mathbf{cz})$ for any simultaneous rotation of basis vectors in $\mathbf{t} \wedge \mathbf{y}$ and $\mathbf{x} \wedge \mathbf{z}$ planes [39]. However, the 4D spacetime case is very special, as the decomposition is really unique. One can observe that $\mathbf{t} \wedge \mathbf{y}$ is a time-space plane, thus transformations

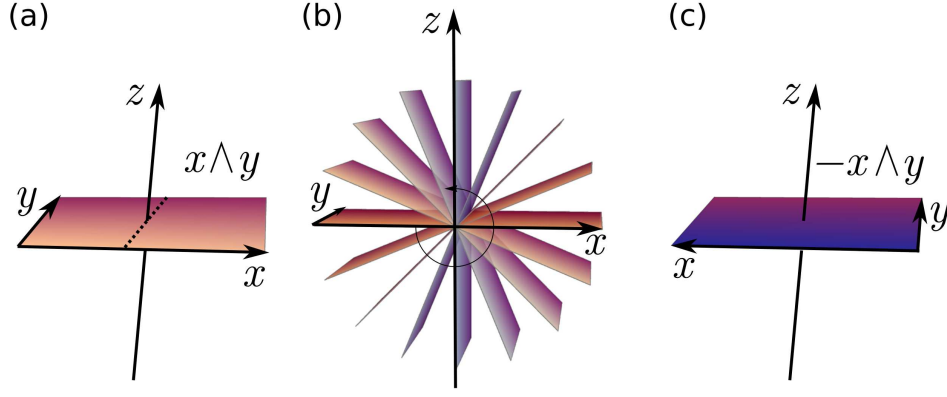


Figure 3.1: To the description of bivectors with flux in 4D: (a) A bivector $\mathbf{x} \wedge \mathbf{y}$, and one of the orthogonal directions $\mathbf{z}$. (b) The rotation of the bivector $\mathbf{x} \wedge \mathbf{y}$ along the direction $\mathbf{y}$ towards the direction $\mathbf{z}$. (c) The flipped bivector $\mathbf{x} \wedge \mathbf{y}$ is a bivector $-\mathbf{x} \wedge \mathbf{y}$.

$(cht + shy)$ and $(sht + chy)$ that keep orthonormality are different from euclidean rotations. As a result, other orthogonal pair of simple bivectors cannot be found using this approach.

Presented above demonstration is only qualitative. Indeed, the rigorous mathematical proof of the existence of such a decomposition in 4D spacetime, and its uniqueness, was proved by Riesz in [38]. However, this decomposition was not mentioned after the publication of the lectures to E.F. Bolinder in 1993. The mention of the electromagnetic canonical decomposition is also appeared in the famous book "Gravitation" by Ch.W. Misner, K.S. Thorne and J.K. Wheeler [17], but authors did not consider it as unique. Further in this Chapter both of these decompositions will be discusses in details. However, this presentation requires a brief mathematical introduction, presented in Appendix C. Also, it is important to highlight the differences between the geometrical approach presented in this work and the classical presentation of the electromagnetic field with the differential forms. The next Section is devoted the review of the presentation of the electromagnetism by 2-forms in literature.

3.4.

Approach of electromagnetism by 2-forms in the literature

In order to highlight the advantageous features of the approach presented in this work, it is initially useful to consider the results presented in the literature on this topic. In this Section, the brief review of classical differential form approach for electromagnetism is presented. Special attention is paid to the illustration of the electromagnetic fields by means of this approach, and the difference with the approach presented in this work is discussed. For a complete under-

standing of the presented results, the mathematical introduction to the exterior algebra and the Clifford algebra, presented in Appendix C, should be revised at first. Section C.2, providing the most basic information, may be especially useful.

3.4.1 Presentation of the electromagnetic field with the differential forms

Differential forms have been found to be extremely useful in electromagnetism [23]. The physical quantities as fields, currents, and charge densities can be integrated over lines, areas, volumes and time, thus they can be represented by differential forms [40]. They even give more elegant form for the equations of electromagnetics [22]. One of the advantages of exterior algebra over conventional vector or tensor calculus is that it provides a way to deal with problems of electrodynamics in a frame-free manner, thus equations stay unchanged [17, 22].

Although applications of exterior calculus to electromagnetism were known before [41, 42, 43], its became widely known thanks to "Gravitation" by Charles W. Misner, Kip S. Thorne, and John A. Wheeler [17], also known as MTW. Although main part of the book is devoted to spacetime and gravity, there are detailed introduction to the exterior algebra (Chapters 2-4) and its application to electromagnetics in spacetime (Chapter 4).

Authors consider a widest list of questions, including description of the Maxwell equations with 2-forms, geometrical version of Lorentz force law, Stokes' theorem, stress-energy tensor and conservation laws. MWT consider electromagnetic field in the form which is classical now (corresponds to the form presented in Eq. 1.21)

$$\begin{aligned}\tilde{\mathbf{F}} &= \frac{1}{2}F_{\alpha\beta}d\tilde{\mathbf{x}}^\alpha \wedge d\tilde{\mathbf{x}}^\beta = \\ &E_x d\tilde{\mathbf{x}} \wedge d\tilde{\mathbf{t}} + E_y d\tilde{\mathbf{y}} \wedge d\tilde{\mathbf{t}} + E_z d\tilde{\mathbf{z}} \wedge d\tilde{\mathbf{t}} + B_x d\tilde{\mathbf{y}} \wedge d\tilde{\mathbf{z}} + B_y d\tilde{\mathbf{z}} \wedge d\tilde{\mathbf{x}} + B_z d\tilde{\mathbf{x}} \wedge d\tilde{\mathbf{y}},\end{aligned}\quad (3.5)$$

where $\tilde{\mathbf{F}}$ is a 2-form of the electromagnetic field, $F_{\alpha\beta} = (-\mathbf{E}, \mathbf{B})$ is components of electromagnetic tensor [3], and the wedge product is a basis of two-forms of a given Lorentz frame. In this way, by means of $\tilde{\mathbf{F}}$ electromagnetic field can be visualized by a tubular structure, which can turn and twist in spacetime. As 2-form is a linear functional which transform any orientated 2-surface in a scalar, the meaning of $\tilde{\mathbf{F}}$ is also

$$\int_{surface} \tilde{\mathbf{F}} = N, \quad (3.6)$$

where N is a number of tubes cut.

The Maxwell's equations in this approach are

$$\begin{aligned} \tilde{d}\tilde{F} &= 0 \\ \tilde{d} \star \tilde{F} &= 4\pi \star \tilde{J}. \end{aligned} \tag{3.7}$$

Here $\tilde{J}$ is 1-form of the charge-current in 4D. The meaning of the first equation have sense that the tubes of $\tilde{F}$ newer end. The second one means that the number of tubes of $\star\tilde{F}$ that end in an elementary volume, is equal to the the amount of electric charge in the volume [17] (p. 105). It is worth to note that the star operator remains the same for vacuum, and if materials are presented, it includes their parameters ϵ and μ . The example of these two-forms presentation as tube of flux ("a honeycomb-like structure") is shown in Fig. 3.2. It is worth noting that each equation of 3.7 is independent of the system of coordinates (both space and time), but the relation between $\tilde{F}$ and $\star\tilde{F}$ depends on the metric in general case. In vacuum the relation $\tilde{F} \longrightarrow \star\tilde{F}$ and the Minkowski metric are invariant under the Lorentz transformation [22].

It is important to note that although tube of flux in 3D is useful for two-form understanding concept, it is not well defined in 4D. For example, for one simple two-form $\tilde{t} \wedge \tilde{x}$, there are two sets of tubes (similar to presented in Fig. 3.2), along $\mathbf{y}$ and $\mathbf{z}$, or any their mix, in the same time. In this way, it is no more a conventional tube, not even a set of events of the spacetime. However, for electromagnetic two-form $\tilde{F}$ tube presentation is even more complex. As it is explained in Section 3.5, the sum from Eq. 3.5 can be simplified to just two fully orthogonal terms. Each of them can be presented with "a honeycomb-like structure". But, as it is explained in Box 4.2 of [17], they should be considered only as a sum. This way does not construct geometrical structure i.e. sets of events in space-time. In Section 3.6 it will be shown that instead of tubes, the electromagnetic field can be described as two orthogonal oriented 2D surfaces in 4D specetime, in a unique, well-defined way.

Thus, most illustrations of Maxwell's equations, involving 1- and 2-forms, are discussed only in 3D. It is presented, for example, in well-known Deschamps's paper [22]. 1-forms $\tilde{E}$ and $\tilde{H}$ corresponds to electric and magnetic fields strength, and 2-forms $\tilde{D}$ and $\tilde{B}$ to electric and magnetic induction vectors, correspondingly [22].

$$\tilde{E} = E_x d\tilde{x} + E_y d\tilde{y} + E_z d\tilde{z}, \tag{3.8}$$

$$\tilde{D} = D_x d\tilde{y} \wedge d\tilde{z} + D_y d\tilde{z} \wedge d\tilde{x} + D_z d\tilde{x} \wedge d\tilde{y}, \tag{3.9}$$

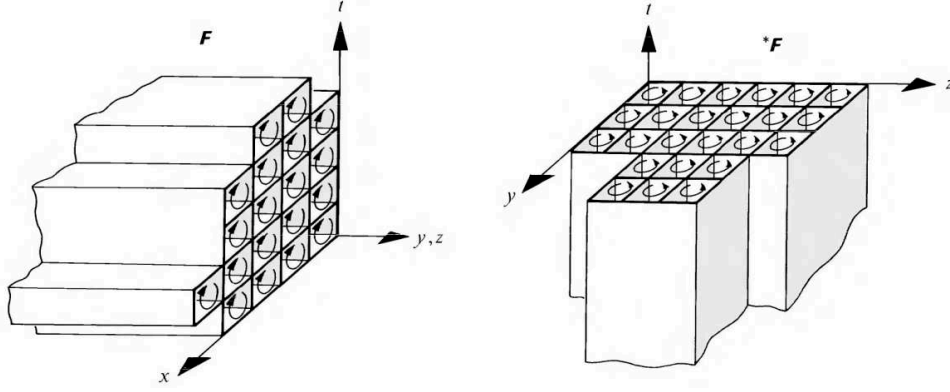


Figure 3.2: Presentation of electric field by 2-form $\tilde{\mathbf{F}} = E_x d\tilde{\mathbf{x}} \wedge d\tilde{\mathbf{t}}$ and its dual $\star\tilde{\mathbf{F}} = E_x d\tilde{\mathbf{y}} \wedge d\tilde{\mathbf{z}}$ ($\mathbf{F}$ and $\star\mathbf{F}$ in the notation of the authors) from [17]

where $\{E_i\}$ and $\{D_i\}$ are the components of the corresponding vectors [22, 23]. Thus, with the differential form notation we immediately see the distinction of forms $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{D}}$, which demonstrate the different geometrical representation and physical properties. It is not obvious (and often not understood by students) in the case of vector notation for $\mathbf{E}$ and $\mathbf{D}$ [23]. For example, the one-form $\tilde{\mathbf{E}}$ can be considered as a linear operator which produces the work done by $\tilde{\mathbf{E}}$ on a unit charge when applied to a displacement vector $\delta\mathbf{r}$. The result of this operation is a scalar.

Then electric field can be plotted as a set of planes, as it is presented in Fig. 3.3 (a). This and other illustrations of electromagnetic field in 3D in Fig. 3.3 comes from Warnik's paper [23]. While the relationship between an electric current and the 1-form $\tilde{\mathbf{H}}$ (Fig. 3.3 (b)), as well as a point charge and the 2-form $\tilde{\mathbf{D}}$ (Fig. 3.3 (c)), are quite simple, the graphical presentation for the Stokes theorem should be considered in detail. In the differential forms notation, it is written as

$$\tilde{\alpha}|\partial\mathbf{D} = \tilde{d}\tilde{\alpha}|D, \quad (3.10)$$

where $\tilde{\alpha}$ is a p -form and $\mathbf{D}$ is a $(p+1)$ integration domain with a boundary $\partial\mathbf{D}$, and the vertical bar sign for inner product [22].

Graphical illustration of Eq. 3.10 is provided in the Fig. 3.3 (d) and (e), in the case when $\tilde{\alpha}$ is an one-form. The number of surfaces $\tilde{\alpha}$ which are pierced by the path $\partial\mathbf{D}$ is equal to the number of tubes of the two-form $\tilde{d}\tilde{\alpha}$ that come through the surface $\mathbf{D}$ limited by this path. All pictures in 3.3 present simplest conceptions in 3D space, which are reasonable for the acquaintance with the approach, but it did not receive significant development. In addition, authors often identify time as a separate coordinate. However, since space and time are not separable, it prevents the

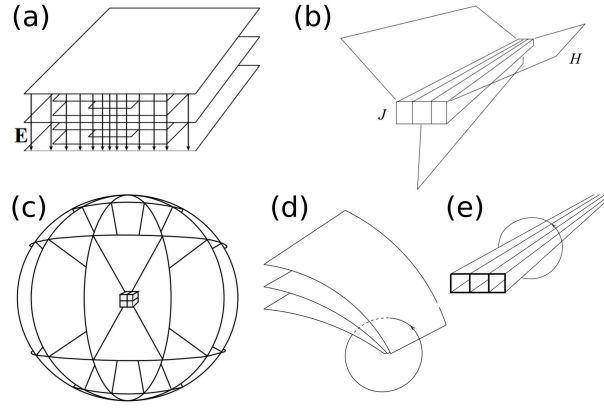


Figure 3.3: Illustration of electromagnetic field in 3D using differential forms (all from [23]). (a) One-form $\tilde{\mathbf{E}}$ for a nonconservative field (compare with vectors). (b) An electric current, presented by tubes $\tilde{\mathbf{J}}$, produce a magnetic field, presented by surfaces $\tilde{\mathbf{H}}$. (c) A point charge, presented as three-form by cubes, generates electric flux density $\tilde{\mathbf{D}}$, presented by radial tubes (two-form). For the Stokes theorem for one-form $\tilde{\alpha}$: (d) The loop $\partial\tilde{\mathbf{D}}$ pierces the $\tilde{\alpha}$ surfaces, and (e) the tubes $d\tilde{\alpha}$ go through a surface $\tilde{\mathbf{D}}$ having the loop boundary $\partial\tilde{\mathbf{D}}$.

creation of full-fledged diagrams.

In the middle of the last century physicists were motivated with differential forms approach, for example, in 1963 Flanders wrote "Physicists are beginning to realize its usefulness; perhaps, it will soon make its way into engineering" [41]. Nowadays, it does not have a significant importance in the continuum of general physics. About ten years ago the advantages of Clifford geometric algebra were briefly discussed in the "Nature physics" [44], and David Hestenes in his biographical article [45], published in 2017, expressed hope that time of geometric algebra will come. However, we can distinguish just a few authors, who use this approach in physics nowadays. The introduction to the differential forms and applications to electromagnetic fields its description including medium and boundaries, Maxwell's equations and electromagnetic waves are presented in the books [40, 46] written by Lindell. Helh and Obukhov in [47] considered applications of the differential forms approach to the electrodynamics in a vacuum and in a matter for spacetime. The conservation laws, Maxwell-Lorentz theory, propagation of electromagnetic waves, reciprocity, constitutive laws, and moving continua are also regarded.

Some modern authors also use geometrical (based on the Clifford calculus) approach to electrodynamics. The most useful examples are textbooks of Lounesto [39], Arthur [48], Doran and Lasenby [49]. Another modern work, discussing the application of the Clifford's geometric algebra in the field of electrical engineering is Chappell's paper [50]. Authors considered a few problems: dipoles, the EM potential, EM wave, the Lienard-Winner potential, and the complex power in circuits analysis, as well as well as a short review of electromagnetism with Clifford's

notation. Undoubtedly, these works present a new way to deal with electromagnetism and provide additional physical insides at the well-known problems.

It should also be emphasized that in many works on exterior calculus the nice tools having a great potential is used for description of electromagnetic problems. But only high-level mathematical physicists can understand to the full extent the essence of all expressions, while mainly physicists are even not familiar with this conception. While the other valuable part of the approach of differential forms which is available to most people – the geometrical interpretation – has been almost completely forgotten. Among modern works, apart from the [23, 51], we know only the paper [24] of Gratus, who moves in the same direction as we do, believing that the geometrical illustration of physical laws can be much easier for a wide audience.

The difference between an approach described in [24] and more tradition illustration of the differential forms, presented in the section C.2, is as follows: in [24] p -form on n -manifold is presented through $n - p$ submanifolds, thus the dimension of the presentation decreases as p increases. In this case, for example, a two-form in 3D is presented by the lines, not by the tubes, created by intersected planes. Some examples involving this approach for electromagnetic laws are displayed in Fig. 3.4. According to the idea " $n - p$ ", in 3D the electric current density being two form is presented as line (1D manifold), the one-form $\tilde{\mathbf{H}}$ – as surfaces, and two-form $\tilde{\mathbf{B}}$ – as circles for the vacuum or similar constitutive relations (Fig. 3.4 (a)). The electromagnetic two-forms $\tilde{\mathbf{F}}$ in blue and $\star\tilde{\mathbf{F}}$ from Eq. 3.5 for a vacuum in 4D spacetime are presented in Fig. 3.4 (b) in blue and in red, correspondingly. They are connected by the Hodge star, and their orthogonality is presented with black right angle. The Lorentz law in 4D is presented in Fig. 3.4 (c), where the force is orthogonal to to the both two form $\tilde{\mathbf{F}}$ from 3.5 and the 4-velocity of the particle.

Some applications of Maxwell's equations are also demonstrated in in [24]. In Fig. 3.4 (d) the integral formulation of Faraday's law of induction for a loop on the plane $z = 0$ using two-form $\tilde{\mathbf{F}}$ is presented. As $\tilde{\mathbf{F}}$ is the 2-form, in 3D it is shown as lines (in blue). It demonstrates a change in both the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. The change in the $\mathbf{B}$ field corresponds to the difference between the number of lines entering the bottom disk (four) and leaving the top one (two). This difference is equal to the number of lines leaving the volume through the side surface (two, corresponds to the change in the electric field), which is the integral value having a meaning of the electromotive force [24]. In Fig. 3.4 (e) similarly the integral formulation of Maxwell-Amperes law for the loop in the $z = 0$ plane using two-form $\star\tilde{\mathbf{F}}$ (in red) is presented.

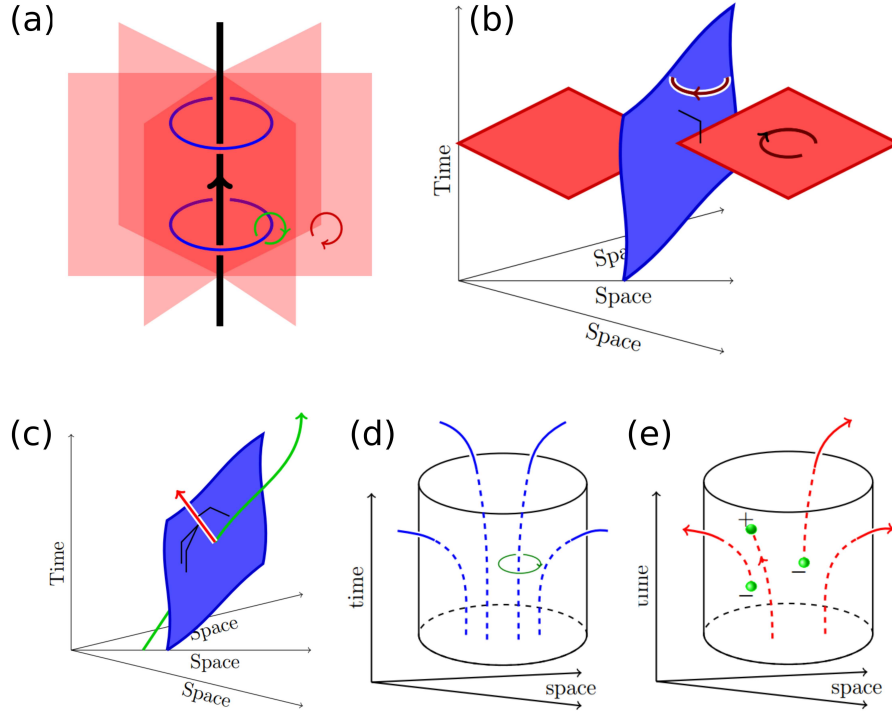


Figure 3.4: Illustrations for electromagnetics laws from [24]. (a) Ampere's law: the constant line current in black (two-form) and the magnetic field strength in red (one-form), and the magnetic induction in blue (two-form). (b) The constitutive relations in the vacuum for $\vec{F}$ in blue and $\star\vec{F}$ in red (orthogonal surfaces). (c) The Lorentz force law on a particle: the worldline of the particle in green, the two form $\vec{F}$ in blue, and the force is in red. (d) The integral formulation of the Faraday's law of induction for $z = 0$. The two-form $\vec{F}$ for a plane $z = 0$ is presented with blue lines. (e) The integral formulation of Maxwell-Ampere law for the $z = 0$. The two-form $\star\vec{F}$ is presented with red lines and electric charges in green (all for a plane $z = 0$).

Red lines start or finish at charges whose worldlines intersect plane $z = 0$ in the green points. Now the difference between the number of lines entering through the bottom surface and leaving the cylinder through the top (corresponds to the magnetic field) and side surfaces (corresponds to the electric field) equals the sum of the charges inside the cylinder.

3.4.2 Our approach

Without detracting from the merits of other authors whose contribution is huge and very useful, the flux geometrization approach from this work is significantly different from others presented earlier, although the same mathematical basis is used. In most works using the differential forms or geometrical algebra approach for electromagnetism, space and time are separated. They are mainly focused on the mathematical description, not aiming attention on geometrical representation, while it is the powerful aspect of these approaches.

In this work, time and space are non-separable, and the unique decomposition into two

simple bivectors with fluxes in 4D spacetime is the key idea of our approach. We are not just trying to depict the electric or magnetic field using a new mathematical tool, but go beyond the usual framework, creating a new concept for electromagnetic field and constructing an absolute electromagnetic spacetime map. It can provide simple concrete models for electromagnetism, allowing a deeper understanding of some effects.

3.5.

The unique decomposition in 4D space-time

In Section 3.2 it was demonstrated, that an electromagnetic two-form as $\tilde{\Phi}_m$ (Eq. 1.21, Section 1.2, or Eq. 3.5, Section 3.4), written in the general case as the sum of six simple two-forms

$$\begin{aligned} \tilde{\Phi}_m = & E_x d\tilde{x} \wedge d\tilde{t} + E_y d\tilde{y} \wedge d\tilde{t} + E_z d\tilde{z} \wedge d\tilde{t} \\ & + B_x d\tilde{y} \wedge d\tilde{z} + B_y d\tilde{z} \wedge d\tilde{x} + B_z d\tilde{x} \wedge d\tilde{z}, \end{aligned} \quad (3.11)$$

can be written as a sum of two simple two-forms:

$$\tilde{F} = E'_z d\tilde{z}' \wedge d\tilde{t}' + B'_z d\tilde{x}' \wedge d\tilde{y}'. \quad (3.12)$$

In the book *Gravitation* [17], a similar problem is considered as well (p. 122, Exercise 4.1). Authors conclude, that only two wedge products needed to represent the generic local field, and call this decomposition "canonical representation".

It was shown in Section 3.2 that Lorentz's transforms along the axis z' provide a decomposition

$$\tilde{F} = E'_z d\tilde{z}'' \wedge d\tilde{t}'' + B'_z d\tilde{x}'' \wedge d\tilde{y}'', \quad (3.13)$$

with the same E'_z, B'_z field components, and the same $d\tilde{z}'' \wedge d\tilde{t}'' = d\tilde{z}' \wedge d\tilde{t}'$ and $d\tilde{x}'' \wedge d\tilde{y}'' = d\tilde{x}' \wedge d\tilde{y}'$ two-forms. In Exercise 4.2, [17], (p.122) authors discussed that for some "canonical transformations" different one-form products are obtained, giving the same two-forms. Their conclusion is that "the decomposition of a six-wedge-product object into two wedge products, miraculous though it seems, is actually far from unique" (Box 4.2 about the concept of "honeycomb-like structure").

Lately, some authors, like Lounesto ([39]) insist on the fact that the decomposition is not unique in 4D Euclidean space (see Section 3.3).

However, about sixty years ago, Riesz gave a prove of the uniqueness of the two-forms decomposition in 4D spacetime [38] (p. 206): "A nonsimple, nonlight-like bivector can, in a unique way, be decomposed into the sum of a time-like and a space-like bivector"¹. This theorem was proved using the Clifford algebra (see Appendix E). The decomposition is

$$\Phi = a\mathbf{T} + b\mathbf{S}. \quad (3.14)$$

Here $\mathbf{T}$ is a time-space simple two-form and $\mathbf{S}$ is a space-space simple two-form. The two are completely orthogonal, a and b are real numbers. Riesz said that this gives a proof of "the theorem of Sommerfeld", probably a reference to his *Lectures on Theoretical Physics* [52, §26 D].

We do not have any explanation why the miraculous uniqueness, specific to the space-time metrics with signature $(+ - - -)$ or $(- + + +)$ (then more miraculous than the miracle described in Box 4.2 of the book [17]), remains unknown as far as we know. Even Pertti Lounesto, who was editing Riesz's book[38], only presented non-uniqueness of the decomposition in the 4D Euclidean space.

In a recent book [53, p. 79], the authors demonstrate (not using the Clifford algebra and not citing Riesz) that an anti-symmetric tensor in 4D can be decomposed into two simple bivectors in a unique way if the orthogonality condition is added. However, the question of uniqueness seems not clear, as in p. 328 it is written "Moreover, the electromagnetic tensor can be decomposed (*even if not uniquely*, see Chap. 2) in the sum of two orthogonal bivectors". Meanwhile, Stazi in [54] gave the explicit unique determination of a decomposition of the electromagnetic tensor into a sum of two completely bivectors.

The uniqueness of the decomposition of electromagnetic two-forms as a sum of two fully perpendicular simple two-forms is the root of the absolute, unique geometrical representation of the electromagnetic field, as shown in the next Section.

3.6.

Electromagnetic spacetime map

As it was discussed in Chapter 1, the electric and magnetic flux can be defined on any surface in 4D spacetime, and they are enough to describe the electromagnetic field. In Chapter

¹Of the vector space or of the dual form-space

2 it was demonstrated, that unique absolute map, built of ESs and their milestones contains all information about electromagnetic field in the case of the $\mathbf{y}$ -invariant, p -polarized problem. In the general case an absolute electromagnetic map is built with the decomposition to two bivectors with flux on them, connected in the infinitesimal ribbons. Sometimes they can be connected into continuous surface, called lasagna. Similar to presented in literature approach (Eq. 3.5), the description of the map is based on two two-forms.

Electromagnetic two-forms $\tilde{\Phi}_m$ and $\tilde{\Phi}_e$ (Eq. 1.21 and Eq. 1.22), discussed in Chapter 1, are used for building of the flux decomposition. As discussed before, each them include 6 components in 4D space-time, and to construct them, tensors $F_{ij} = (-\mathbf{E}, \mathbf{B})$ and $\mathcal{G}_{ij} = (\mathbf{H}, \mathbf{D})$ are used. The two-form associated with the electric flux is

$$\begin{aligned}\tilde{\Phi}_e = & H_x \tilde{\mathbf{t}} \wedge \tilde{\mathbf{x}} + H_y \tilde{\mathbf{t}} \wedge \tilde{\mathbf{y}} + H_z \tilde{\mathbf{t}} \wedge \tilde{\mathbf{z}} + \\ & D_x \tilde{\mathbf{y}} \wedge \tilde{\mathbf{z}} + D_y \tilde{\mathbf{z}} \wedge \tilde{\mathbf{x}} + D_z \tilde{\mathbf{x}} \wedge \tilde{\mathbf{y}}.\end{aligned}\tag{3.15}$$

The two-form associated with the magnetic flux in 4D is

$$\begin{aligned}\tilde{\Phi}_m = & E_x \tilde{\mathbf{x}} \wedge \tilde{\mathbf{t}} + E_y \tilde{\mathbf{y}} \wedge \tilde{\mathbf{t}} + E_z \tilde{\mathbf{z}} \wedge \tilde{\mathbf{t}} + \\ & B_x \tilde{\mathbf{y}} \wedge \tilde{\mathbf{z}} + B_y \tilde{\mathbf{z}} \wedge \tilde{\mathbf{x}} + B_z \tilde{\mathbf{x}} \wedge \tilde{\mathbf{y}}.\end{aligned}\tag{3.16}$$

Here $\tilde{\mathbf{t}}, \tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}$ is an basis of one-forms.

The result of action of two forms $\tilde{\Phi}_e$ and $\tilde{\Phi}_m$ on a bivector is a number, an invariant value. Thus, these values are *absolute* (independent of any frame of reference). The infinitesimal magnetic and electric fluxes on an infinitesimal surface, defined as the bivector $d\mathbf{S} = d\mathbf{u} \wedge d\mathbf{v}$ with $d\mathbf{u} = du^i \mathbf{e}_i$ and $d\mathbf{v} = dv^j \mathbf{e}_j$ are

$$d\Phi_m = \tilde{\Phi}_m(d\mathbf{S}) = F_{ij} du^i dv^j \tag{3.17}$$

$$d\Phi_e = \tilde{\Phi}_e(d\mathbf{S}) = \mathcal{G}_{ij} du^i dv^j \tag{3.18}$$

Again, the unbold "d" letter do not mean the differentiation, but has sense of infinitesimal value.

As it was already discussed in Sections 3.5, the 6 terms decomposition Eq. 3.15 and Eq. 3.16 can be written in a unique way (except the zero-case when the electric and magnetic fields are equal and orthogonal) as the sum of two simple 2-forms

$$\tilde{\Phi}_e = H' \tilde{\mathbf{t}}' \wedge \tilde{\mathbf{z}}' + D' \tilde{\mathbf{x}}' \wedge \tilde{\mathbf{y}}', \tag{3.19}$$

$$\tilde{\Phi}_m = E' \tilde{\mathbf{z}}' \wedge \tilde{\mathbf{t}}' + B' \tilde{\mathbf{x}}' \wedge \tilde{\mathbf{y}}'. \quad (3.20)$$

In this way, at any event in 4D spacetime the pair of simple unit two-forms $\tilde{\mathbf{T}} = \tilde{\mathbf{t}}' \wedge \tilde{\mathbf{z}}'$ and $\tilde{\mathbf{S}} = \tilde{\mathbf{x}}' \wedge \tilde{\mathbf{y}}'$ can be constructed in a unique way. They permit to construct two fully orthogonal infinitesimal surfaces $\varepsilon_t \mathbf{t}' \wedge \mathbf{z}'$ and $\varepsilon_s \mathbf{x}' \wedge \mathbf{y}'$, characterized by magnetic fluxes $-\varepsilon_t E'$ and $\varepsilon_s B'$, and by electric fluxes $\varepsilon_t H'$ and $\varepsilon_s D'$.

The constitutive equation in a vacuum (Eq. 1.36) links the density of the magnetic flux E' and B' with the density of the electric flux H' and D' : $E' = 2\alpha D'$ and $B' = 2\alpha H'$ (Appendix A). As the fully orthogonal two-forms $\tilde{\mathbf{t}}' \wedge \tilde{\mathbf{z}}'$ and $\tilde{\mathbf{x}}' \wedge \tilde{\mathbf{y}}'$ are unique, and the tangent plane of the two infinitesimal surfaces defined by unit bivectors $\mathbf{T} = \mathbf{t}' \wedge \mathbf{z}'$ and $\mathbf{S} = \mathbf{x}' \wedge \mathbf{y}'$ are unique. They construct a unique, absolute topology of the electromagnetic field connecting each event with two sets of infinitely close neighbors. This topology with two flux densities (either E' on $\mathbf{T}$ or D' on $\mathbf{S}$, and H' on $\mathbf{T}$ or B' on $\mathbf{S}$) fully describe the electromagnetic field. Indeed, if $(\mathbf{t}, \mathbf{x}, \mathbf{y}, \mathbf{z})$ frame of reference is chosen, then expanding Eq. 3.19 and Eq. 3.20 in the basis of two-forms $\tilde{\mathbf{t}} \wedge \tilde{\mathbf{x}}, \tilde{\mathbf{t}} \wedge \tilde{\mathbf{y}}, \tilde{\mathbf{t}} \wedge \tilde{\mathbf{z}}, \tilde{\mathbf{y}} \wedge \tilde{\mathbf{z}}, \tilde{\mathbf{z}} \wedge \tilde{\mathbf{x}}, \tilde{\mathbf{x}} \wedge \tilde{\mathbf{y}}$, equations, gives components of fields by identification with Eq. 3.15 and Eq. 3.16.

Now we can think about connecting all these infinitesimal surfaces 1 and 2. If we consider one of these two surfaces, we can follow a tangent line, and construct an infinitely narrow ribbon connecting 1 and 2, a kind of spacetime "tagliatelle". In some cases, the ribbons constructed from any different paths can be joined, defining a unique surface containing the starting event. This surface forms a kind of electromagnetic "lasagna" (Fig. 3.5). Since the decomposition consists of two simple bivectors at any event, there are two families of lasagna surfaces, the L_1 and L_2 surfaces.

In the null-space special case, the L_1 and L_2 surfaces are null-space, with the same $\mathbf{t} + \mathbf{z}$ null direction which is orthogonal to itself. That corresponds to the case when $\mathbf{E} \cdot \mathbf{B} = 0$ and $E = B$. Remembering that $\mathbf{E} \cdot \mathbf{B}$ and $E^2 - B^2$ are Lorentz invariant, $\mathbf{E}$ and $\mathbf{B}$, being orthogonal in one reference, are orthogonal in all of them [30]. In this case, the Riesz decomposition is not unique. However, we do the hypothesis that they can be constructed by continuity from adjacent regions. It is done, for example, in the following Section 3.7.2, Fig. 2.6. There are events of cylindrical patterns where lasagna are null-space, but they connect time-space and space-space pieces. In the next Section two examples of lasagna structures in 4D spacetime are constructed from the ES-ZEFS results of Chapter 2.

In this way, a lasagna, each piece of surface of which is supplemented by fluxes values written on it, constructs a special map in 4D spacetime. Using it, the components of the electromagnetic field can be extracted in any reference frame. Thus, this map provide all the information about the electromagnetic field.

Components of electric and magnetic field in any reference frame can be found for any pair of unique bivector decomposition with flux values on it. This concrete representation of the map near a given event P can be presented with nodes Q_1, Q_2, Q_3, Q_4 , allowing to construct two small completely orthogonal triangles PQ_1Q_2 and PQ_3Q_4 . They corresponds to bivectors $\alpha = (PQ_1 \wedge PQ_2)/2$ and $\beta = (PQ_3 \wedge PQ_4)/2$ with flux values on them. For example, the magnetic flux $\tilde{\Phi}_m(\alpha) = \Phi_{m1}$ and $\tilde{\Phi}_m(\beta) = \Phi_{m2}$. In the same time, the magnetic flux on four orthogonal to α and β bivectors $PQ_1 \wedge PQ_3, PQ_1 \wedge PQ_4, PQ_2 \wedge PQ_3, PQ_2 \wedge PQ_4$ is zero.

When an orthonormal frame of reference $(t, x, y, z) = \mathbf{e}_i$ is defined, the four vectors PQ_j are $PQ_j = \varepsilon_i^j \mathbf{e}_i$, and each bivector is linear combination of six basis bivectors $\mathbf{e}_0 \wedge \mathbf{e}_1$, etc. Then, for example, $\alpha \wedge \beta = (1/2)(\alpha^0 \beta^1 - \alpha^1 \beta^0) \mathbf{e}_0 \wedge \mathbf{e}_1 + \dots + (1/2)(\alpha^2 \beta^3 - \alpha^3 \beta^2) \mathbf{e}_2 \wedge \mathbf{e}_3$. All coefficients α_i and β_i are defined. Then, the magnetic two-form $\tilde{\Phi}_m = E_x \tilde{\mathbf{x}} \wedge \tilde{\mathbf{t}} + \dots + B_z \tilde{\mathbf{x}} \wedge \tilde{\mathbf{y}}$ can be applied on each of six bivectors, with non-zero result for α and β (Φ_{m1} and Φ_{m1} correspondingly), and zero for others. This provide the system of linear equations where unknown variables are $\mathbf{E}$ and $\mathbf{B}$ components in a given reference frame. Solution of this system provides the electromagnetic field components in a given reference.

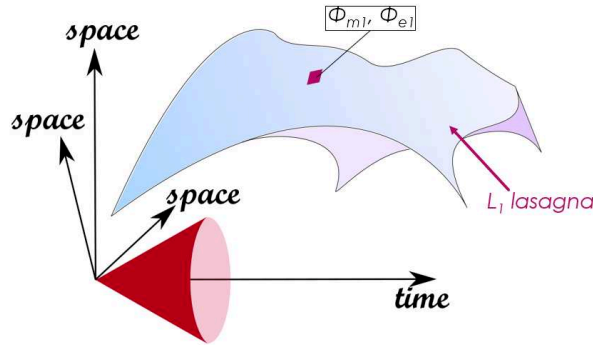


Figure 3.5: Schematic representation of a lasagna in 4D MSM: a surface in 4D which contains the electric and magnetic flux values.

Although the theoretical basis for electromagnetic 4D spacetime map is presented here, this work is just a very introduction of the presentation, which can be developed significantly. In particular, the topology of geometrical structures, supported the fluxes, can become new research area, and provide insights into understanding some electromagnetic problems. Below

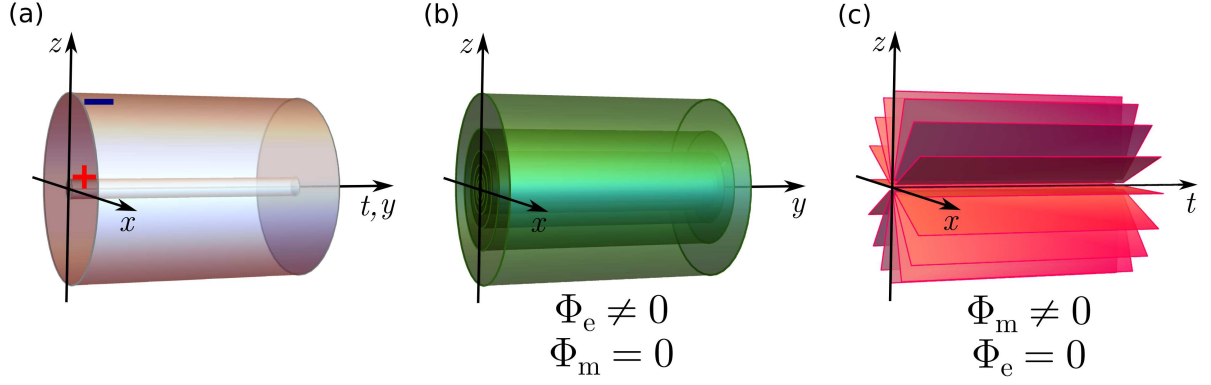


Figure 3.6: (a) Electrostatic problem with two oppositely charged cylinders. (b) The electric lasagna and (c) the magnetic lasagna (ZEFS) for the electrostatic problem.

in this Chapter only simplest lasagna examples are considered.

3.7.

Examples

It was shown that there is an absolute geometrical representation of the electromagnetic field in 4D spacetime. To completely describe the electromagnetic field at an event with a given approximation a pair of small and fully orthogonal surfaces containing this event can be defined, both labeled with an electric flux and a magnetic flux values², in a unique way. Here these surfaces will be constructed as lasagna surfaces for two cases, the electrostatic cylinder and the two wave interference.

In these two cases of y -invariant p -polarized problems, one lasagna will be obtained by extending the ES (defined in Chapter 2) in the $\mathbf{y}$ direction. This lasagna contains only electric flux. The ZEFS is the second one, containing only magnetic flux. In these special, polarized problems we choose to call the two lasagnas as electric and magnetic.

3.7.1 Electrostatic cylinder

At first, let us describe in 4D spacetime the electrostatic cylinder problem from Section 2.3.1 as lasagna surfaces. A schematic presentation of the system is shown in Fig 3.6 (a). The first lasagna, obtained by ES lines extension in the $\mathbf{y}$ direction. In fact they are just the equipotential surfaces between the cylindrical plates, as shown in Fig 3.6 (b). However, contrary to equipotential surfaces that is defined only in the special frame of reference of the electrodes,

²Due to constitutive equation these four values reduce to two.

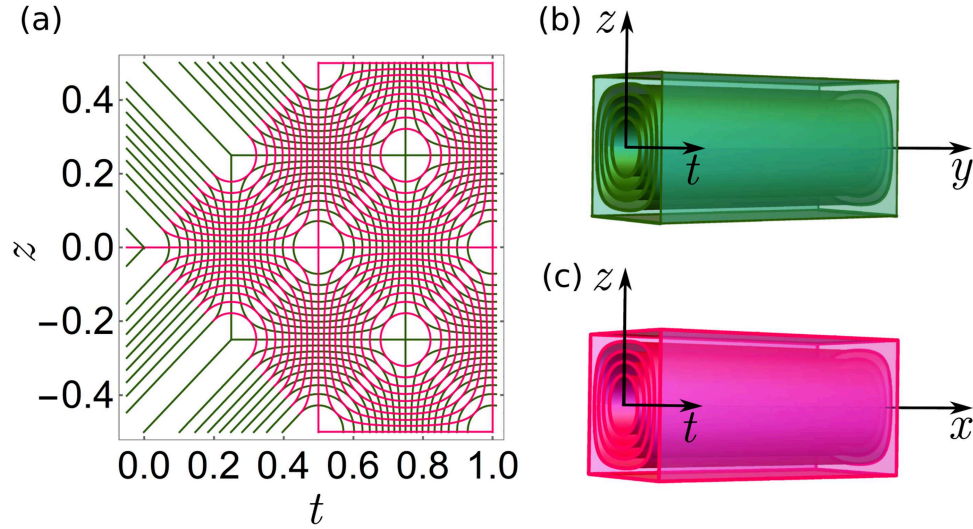


Figure 3.7: (a) Interference of two plane waves. Green lines are ESs, the cut of the first lasagna, and red lines are ZEFs, the cut of the second lasagna. In the plane-wave region, these lines coincide. Schematic representation of the electric (b) and magnetic (c) lasagnas in the interference region.

this lasagna does not depend on the any frame of reference, as they are based on the unique decomposition from Eq. 3.14. The second lasagnas are the ZEFs planes are defined in in Section 2.3.1 (shown in Fig 3.6 (c)). These lasagnas contain only magnetic flux.

As expected, two lasagna surfaces are fully orthogonal. The first one (green) is space-space and the second one (red) is time-space. The density of the electric flux on the space-space green lasagna is the radial vector $\mathbf{D}$. The density of the magnetic flux on a space-time surface is the radial vector $\mathbf{E}$. The two densities are connected by the constitutive equation in a vacuum $E = 2\alpha D$ in natural units (Eq. 1.36).

Additionally, similarly to the 2D case, pieces of green cylinders (different layers) between two ZEFs contains the same electric flux Φ_e , and each piece of red surface between two cylinders contains the same magnetic flux Φ_m . Thus, a character of the electromagnetic field behaviour is seen directly from lasagne map.

3.7.2 Two plane waves interference

Another 2D problem, which can be described in 4D by lasagna surfaces, is the interference of two plane waves (Fig 3.7 (a)), presented in Section 2.3.2. Again, the electric lasagna is obtained as the extension of ESs in the $\mathbf{y}$ direction, as shown in Fig 3.7 (b), and the magnetic lasagna is ZEFs surfaces (Fig 3.7 (c)). The two lasagne are completely orthogonal at any event.

In contrast to the previous example, here both red and green lasagna have space-space and time-space parts. These parts are connected by y - (for green lasagnas) or x - (for red lasagnas)

lines of events, where the lasagna tangents surfaces are null-space, the null-direction being $\mathbf{t} + \mathbf{z}$ or $\mathbf{t} - \mathbf{z}$. At these events the electric and magnetic field modules are equal. It can be observed that the two simple bivector decomposition when electric and magnetic field are orthogonal with same module are indeed defined by continuity, in a unique way (see Section 3.5).

Again, field components can be obtained from the lasagne. For example, the electric flux on the segment of the green surface, which is parallel to the yz plane (for example, at $t = 0.25$), $\tilde{\Phi}_e(\mathbf{y} \wedge \mathbf{z}) = D_x$. On the orthogonal to it segment of the red plane, the magnetic flux is $\tilde{\Phi}_m(\mathbf{x} \wedge \mathbf{t}) = E_x$. On the segment of the green surface, parallel to the ty plane (for example, at $z = 0.25$), the electric flux $\tilde{\Phi}_e(\mathbf{t} \wedge \mathbf{y}) = H_y$. And on the orthogonal segment of the red surface, the magnetic flux is $\tilde{\Phi}_m(\mathbf{x} \wedge \mathbf{z}) = B_y$. Again, the flux components on two orthogonal lasagna pieces are connected by the constitutive equation in a vacuum (Eq. 1.36).

These two considered problems include only invariant polarized problems, while in more general case lasagna surfaces can be more complex. Contrary to field maps, lasagna maps are unique, independent on the choice of the reference frame. The same lasagna map will be obtained from any reference. And, reversely, the components of the electromagnetic field can be obtained in any reference from lasagne and flux density values on them (or, for a given precision, flux values on pieces of lasagna).

The same procedure made for these two cases (green electric lasagnas as y - extension of ES and red magnetic lasagnas as ZEFS) can be done for the interference of homogeneous and evanescent waves, described in Section 2.3.3. It will show stunning fractal-like topological structures of the lasagnas in 4D spacetime, in $x = 0$ region.

3.8.

Conclusion

In this Chapter, the generalisation of the absolute electromagnetic map in the 4D MSM was provided. As it was proved by Riesz, the electric or magnetic two-forms can be expressed as a sum of two fully orthogonal simple two-forms in 4D spacetime, in a unique way. Then, the density of electric and magnetic fluxes are zero on any combination of the four simple bivectors perpendicular to the two bivectors corresponding to these two-forms. These bivectors with non-zero fluxes can be united into two families of 2D ribbons called tagliatelle. In some cases, the ribbons constructed from any different paths can be joined, defining a unique surface called lasagna. Similar to described in Chapter 2 ESs, these surfaces are concrete absolute objects in

MSM, independent on any reference. In a given event, one of lasagna belongs to the time-space type, and the other is the space-space type, with a particular exception where the two surfaces contain the same line of light, and are null-space type.

Instead of considering the fields as one or two sets of 6 components $(E_x, E_y, E_z, B_x, B_y, B_z)$, $(H_x, H_y, H_z, D_x, D_y, D_z)$ depending on the frame of reference, the electromagnetic field is presented as an absolute topology of two fully orthogonal simple bivectors and the magnetic and electric density of flux on them. Considering the vacuum constitutive equation, there is 6 degrees of freedom for the electromagnetic field. Indeed, the bivectors orientation in 4D spacetime corresponds to 4 degrees of freedom, and the scalar density of flux on them correspond to the 2 remaining degrees of freedom.

The concrete electromagnetic map (made, for example, with triangles with the magnetic and the electric fluxes on them) does not need any frame of reference. If a frame of reference is chosen, the electromagnetic field components can be extracted from this map.

Two y -invariant p -polarized examples from Chapter 2 were reconsidered, and lasagna families were presented. The fully orthogonal at any event lasagna surfaces were described. With the example of the interference it was demonstrated that a lasagna surface can include both space-space and time-space segments, transcending the distinction between space and time.

Beyond the simplest problems presented in this work, the topology of absolute electromagnetic objects can become a new field of research, which possibly could provide new insights.

Conclusion and Outlooks

Conceptions, presented in this work, clearly contradict the general idea that the electromagnetic field can only be presented in an abstract way [20, 21]. Attempts to represent the electromagnetic field in a geometrical way existed long before this work. In the middle of the last century differential forms inspired many scientists. The famous book "Gravitation" was published in 1973, and among other things, it popularized the geometrical idea of flux tubes based on "a honeycomb-like structure". But the tube of flux conception is well-defined only in 3D, and cannot be maintained in the 4D spacetime. A way is to split time, but that corresponds to the choice of a special frame of reference, making such approach incomplete.

However, as it was proved by Riesz, the electric or magnetic two-forms can be expressed as a sum of two fully orthogonal simple two-forms in 4D spacetime, in a unique way. It was proved by means of a Clifford algebra, and, as it was demonstrated, the existence of such a decomposition is a property of 4D spacetime, while in 4D Euclidean space this decomposition is not unique. Unfortunately, although Riesz demonstration was provided in 1959, nowadays it seems forgotten. A clue to this decomposition can be inspired by the classical electromagnetic problem of finding the reference frame where the electric and the magnetic fields are parallel. As result, a pair of completely orthogonal surfaces (which can be assigned by flux values) appears.

Instead of considering the electromagnetic field as sets of 6 components in a given frame of reference, and considering the electric and magnetic field separately, as well as time from space, the electromagnetic field can be presented as an absolute topology of two fully orthogonal simple bivectors and the magnetic and electric density of flux on each of them. The density of flux are scalar, absolute values, independent on the reference frame.

The magnetic density of flux on space-space surfaces as $\mathbf{y} \wedge \mathbf{z}$, $\mathbf{z} \wedge \mathbf{x}$, and $\mathbf{x} \wedge \mathbf{y}$ give the

B_x , B_y , and B_z of the $\mathbf{B}$ vector. It is natural but not common to say that the components E_x , E_y , and E_z of the *electric* field strength vector $\mathbf{E}$ are the *magnetic* density of flux on space-time surfaces $\mathbf{x} \wedge \mathbf{t}$, $\mathbf{y} \wedge \mathbf{t}$, and $\mathbf{z} \wedge \mathbf{t}$. Similarly, the electric density of flux on space-space surfaces as $\mathbf{y} \wedge \mathbf{z}$, $\mathbf{z} \wedge \mathbf{x}$, and $\mathbf{x} \wedge \mathbf{y}$ give the D_x , D_y , and D_z of the $\mathbf{D}$ vector. Again, it is natural to say that the components H_x , H_y , and H_z of the *magnetic* field strength vector $\mathbf{H}$ are the *electric* density of flux on space-time surfaces $\mathbf{t} \wedge \mathbf{x}$, $\mathbf{t} \wedge \mathbf{y}$, and $\mathbf{t} \wedge \mathbf{z}$.

Additionally, the constitutive equation connects the magnetic flux density on one surface to the electric flux density on its completely perpendicular surface with the factor $2\alpha \approx 2/137$ in natural units.

The conception of flux on a surface together with uniqueness of two bivectors decomposition in 4D spacetime provide the fundamental result. The electromagnetic field in 4D spacetime has an absolute geometrical representation at any event. It is completely described in a unique way with a given approximation by a pair of small and fully orthogonal surfaces L_1 and L_2 containing this event, respectively labeled with an electric fluxes Φ_{e1} , Φ_{e2} and a magnetic fluxes Φ_{m1} , Φ_{m2} . Due to constitutive equation these four flux values reduce to two. These pairs of surfaces can be one time-space, the other space-space, or both null-space, sharing the same null-line.

Taking into account the vacuum constitutive equation, there are 6 degrees of freedom for the electromagnetic field. In this work, the topological part ((L_1, L_2) orientation in 4D spacetime) corresponds to 4 degrees of freedom, and the scalar density of flux on them correspond to the 2 remaining degrees of freedom. The density of electric and magnetic fluxes are zero on any combination of the four surface perpendicular to L_1 and L_2 .

Following an arbitrary tangent curve, each of the infinitesimal surfaces L_1 or L_2 can be united into infinitely narrow tagliatelle ribbon. In some cases, the ribbons constructed from any different paths define a unique lasagna surface. Since the decomposition consists of two simple bivectors at any event, there are two families of lasagna surfaces. These surfaces are concrete absolute objects in MSM, independent on any reference.

It is geometrical static presentation of field: contrary to field maps, the lasagna map is unique, independent on the choice of the reference frame. If a lasagne containing a given event is determined from the field components in a given frame of reference, the same lasagna will be obtained from field components in another frame of reference. And, reversely, the components of the electromagnetic field can be obtained in any frame of reference from lasagne and flux density values on them.

Two simple examples of lasagne were given to show their topology: the electrostatic coaxial cylinders case, and the interference of two plane waves. It was demonstrated that a lasagna surface can include both time-space and space-space segments, transcending the distinction between space and time.

As a presentation in 4D is a challenging task, we focused on cases where the electromagnetic map can be depicted in 3D spacetime: the $\mathbf{y}$ -invariant p -polarized problems. With these conditions L_1 lasagne contain only electric flux and are invariant in $\mathbf{y}$ direction. They can be described by curves (electric spaghettis ESs) in $(\mathbf{t}, \mathbf{x}, \mathbf{z})$ spacetime. The lasagne L_2 contain only magnetic flux (the Zero Electric Flux Surfaces) and are perpendicular to ESs and $\mathbf{y}$ direction. Contrary to the worldlines of physical objects, which are always timelike, ESs and ZEFs transcend the limit between space and time. Any ES segment is characterized by two electromagnetic invariants: the electric flux ϕ_e , and the characteristic ES parameter η , a new concept. The electric flux is attached not only to ESs, but also to ordered pairs of ZEFs. The ESs and/or ZEFs, complete with flux or η milestones, form a complete representation of the electromagnetic world in the 3D MSM: when three orthogonal directions $\mathbf{t}, \mathbf{x}, \mathbf{z}$ (i.e. inertial frame of reference) are chosen, the field components H_y, D_z, D_x can be directly computed.

Three increasingly complex illustrative examples were analyzed. First, we studied the case of two oppositely charged electrostatic cylinders, this is a (x, z) space-space problem. The ES and ZEF lines correspond to the potential and electric strength lines in the frame of reference bound to the electrodes. However, ESs and ZEFs are frame-of-reference-free. The ES-ZEF map approach offers a new way of solving the Laplace equation, by solely considering orthogonality and a constant length ratio.

The second example was the interference of two plane waves, a (t, z) problem. Out of the interference, in the plane wave regions, the ES-ZEFs are the same null-lines. The interference region is made up of concentric rounded square patterns. The plot is easily interpreted, with regions of pure magnetic or pure electric field. As in the first example, all the ES and ZEF lines are perpendicular to each other, with the peculiarity of Minkowskian spacetime that null lines are perpendicular to themselves.

The third studied case is a resonant subwavelength slit grating, a fully 3D (t, x, z) spacetime problem. There are two regions which are similar to the previous cases: the plane waves far from the grating and two-wave interference in the slit, far from the upper and lower boundaries. The funneling region, where diffracted evanescent waves play a major role, is particularly interesting,

with new topological features for both ESs and cut of ZEFSs. As it was demonstrated, the topological behaviour of the ES whirls can be explained by the interference of the incident plane wave and the first-order evanescent wave. A linear approximation for the fields around a whirl provides the ES solution in the form of a stretched logarithmic spiral, the Bernoulli's spira mirabilis. This spiral has a fractal behaviour and never ends. Its parameters are defined by the parameters of the interfered waves. The relevance of the η parameter was demonstrated with this example. In particular, it was shown, that for each loop there is a constant η parameter variation.

An ES map is the simplest way of representing the electromagnetic world, owing to the 1D character of ESs. Furthermore, ESs can be easily computed from the magnetic and electric field components in an arbitrary frame of reference. Conversely, a ZEFS map can be directly computed from a scalar d'Alembertian equation. In any case, the magnetic and electric field components can be straightforwardly extracted from the unique, absolute, and static map of ESs or ZEFSs.

This work offers a radical change in the conceptual approach to the electromagnetic world. There are three possible consequences. The first one is philosophical and educational. Instead of considering differential equations (Maxwell's equations) involving field components relating to an arbitrary frame of reference, an integral form can be considered, involving absolute quantities (magnetic flux Φ_m and electric flux Φ_e) on arbitrary 2D spacetime surfaces: 1) the magnetic flux is zero on a closed surface; 2) the electric flux on a closed surface is equal to the number of enclosed electric charges; and 3) in a vacuum, Φ_m is proportional to Φ_e on two fully perpendicular identical surfaces. These simple rules make it possible to construct solid, absolute objects in spacetime, independently of any frame of reference. These objects transcend the limit between space and time, and contain all the information about the electromagnetic world.

Second, new numerical techniques can be developed using these concepts. For 2D static problems, we suggest a technique for solving Laplace's (or the 2D d'Alembertian) equation. For y -invariant problems, the six field components can be replaced by two potentials ϕ_e ¹ and ϕ_m in the 3D spacetime map. Third, the topological analysis of these solid, absolute objects, basic components of the electromagnetic world opens up a new field of research.

¹more precisely, difference of potentials if charges are present

Plasmonic structures for efficient absorption

5.1.

Introduction

Since 1957, when Ritchie first reported the plasma losses of electrons incident on a metal film [55], the collective electron motion in metals has received considerable attention [56]. Primarily named for the guiding of confined light using metallic nanostructures, plasmonics has extended far beyond the original concept and has become a branch of optical science [57]. The position of plasmonics among other optical sciences is presented in Fig. 5.1 (a). Here, the dashed line presents the limit where the scale of the structures is equal to the working wavelength. While atomic and solid state physics concentrate on structures of the atomic scale, and molecular chemistry focuses on atomic structures which size are less than 10 nm , plasmonic structures and optical metamaterials are mostly larger in size than natural science objects. Hence, occupying a large vacant area in the diagram, these artificially structured objects are complementary to those of the natural sciences [57].

Plasmonic structures display enhanced properties, being capable of modify the spectral, polarization and phase characteristics of incident wave [58]. They can confine light to dimensions much smaller than the wavelength in free-space, having the ability of strong field enhancement [59, 60, 61], selective frequency response [62], and design flexibility based on geometry rather than materials used [63]. As a result, these structures paved the basis for a new class of elements applied for refractive index sensing [64], cloaking [65], solar cells [66], optical biosensing [67, 68, 69], nonlinear optics [70], spectrophotometry [71, 72], surface-enhanced Raman scattering [73], photocatalysis [74, 75, 76], mechanical thermal sensors [77], gas sensing [78], etc.

Absorption in a small volume compared to a wavelength volume can be useful for wide variety of applications [64, 63]. Plasmon resonators are one possible way of achieving this [79, 63]. A short list of important properties for plasmonic resonators in comparison with other materials is presented in Fig. 5.1 (b). It can be seen that plasmonic resonators can underlie the operation of fast tunable devices. Their only major drawback is their high optical losses in the metal parts. However, the concept of perfect absorbers based on plasmonic effects has more recently initiated a new research area where losses became an advantage [64]. In this work, it is demonstrated that this disadvantage can become a good opportunity for the design of a novel generation of infrared detectors.

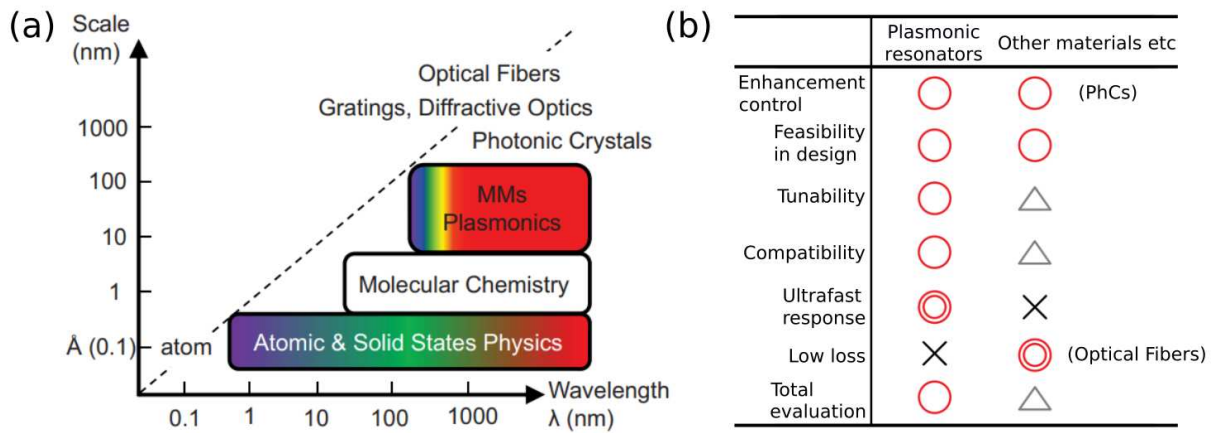


Figure 5.1: (a) A diagram for optical science: scale versus wavelength. The scale stands for the period or characteristic size of objects. (b) Classification for advantages and disadvantages for plasmonic resonators in compare with other materials. In the diagram, double circle means highly suitable, open circles designate possible properties, triangles – difficult to obtain but not impossible, crosses – impractical or almost impossible. PhCs stands for photonic crystals. Pictures from [57].

In recent years, low-cost infrared detectors have been developed for non-military applications: firefighting, aircraft aid, drivers aid, industrial process monitoring, border surveillance, night vision, portable mine detection, community services, search and rescue, law enforcement, etc. [80]. Quantum detectors offer higher sensitivity and speed, however they must be cooled to liquid nitrogen temperatures and cost orders of magnitude more than thermal detectors. Although thermal detectors, and bolometers in particular, are becoming more and more sophisticated, their price has been significantly reduced in recent decades thanks to a manufacturing process allowing massively parallel production. Thus, they are popular for many applications. However, from the point of view of physics, over the past decades, there have been no significant changes in the principles of operation of microbolometers. The pixel size of commercially available devices reduces, providing better resolution but increasing the noise. Meanwhile, the devices with

better characteristics are desirable [80, 81].

In this context, plasmonic structures can form a remarkable basis for a new generation of thermal detectors. As they allow light absorption in volumes much smaller than the wavelength, the thermal capacity of a pixel can be drastically reduced compared to those of current bolometers. As a result, the speed of such detectors could be much higher than that of traditional structures, it could also make it possible to improve the sensitivity of the device, while keeping operation at room temperature. Moreover, the electrical and magnetic properties of a plasmonic structure can be tuned by the design of the structure, thus the operating wavelength, the quality factor and, more generally, the spectral shape of the optical response can be chosen by design.

In this work, we focus on designing the smallest possible plasmonic absorber for infrared applications. As shown below, the main criterion for optimizing these structures is their volume. The configuration of the structure must imperatively satisfy one condition: the absorbing structure must be placed at the distance $\lambda/4$ above the mirror. This resonant configuration enhances the electric field in the structure region, avoids transmission, thus is widely used for traditional bolometers. It will be shown, that optical antennas can be effectively coupled under these conditions to absorb all the incident radiation, some of them occupying only a few percent of the period area.

The rest of this Chapter is organized as follows. First, the operating principle and possible designs of plasmonic structures are introduced in Section 5.2. Great attention is paid to plasmonic absorbing structures used in the infrared range. Then, in Section 5.3, the classic bolometer structure and its working principles are considered. Modern detectors design based on plasmonic structures, presented in the literature, are also reviewed in this section. After that, in Section 5.4, resonant configurations including a mirror are discussed in detail. In Section 5.5, the thermal properties of different materials, used in plasmonics, are considered. It is shown that the values of volumetric specific heat capacity for the most popular plasmonic metals are very close, while specific heat capacities for these materials can be very different. In this way, the most accurate optimisation criterion for the absorber is its volume rather than its mass. Section 5.6 presents the general purpose of this work. Finally, in Section 5.7, a review of the other chapters of this manuscript is provided.

5.2.**Plasmonic structures**

In this Section, a brief review of plasmonic structures is provided. The main working principle of these structures is considered, after that, some examples of metal nanoparticles and their optical response are presented. Particular attention is paid to absorbing metal structures. While there is a wide variety of metal-insulator-metal structures for absorption, a more common application of insulator-metal-insulator structures is as waveguides. However, there are a number of works focused on absorbers based on such structures.

The operation principle of plasmonic structures is based on the resonant interaction of electromagnetic radiation with free electrons of metal at the metal-dielectric interface. As a result, collective electron oscillations, called surface plasmon polaritons (SPP), arise as a propagating wave at a plane interface (Fig. 5.2 (a)). This wave is evanescent in both media, confining them at the interface and making it a waveguide. The amplitude of these oscillations extends further into the dielectric region in comparison with the metal. For the excitation of SPP, an additional momentum to the wave vector should be provided either by periodical patterning or by evanescent coupling of radiation into the metal [82, 83]. A propagation length of SPP is defined by metal absorption, as well as its thickness and interface roughness [83].

As materials for plasmonic structures in IR range, noble metals such as gold and silver are the best known. They demonstrate plasmonic behavior in both the visible and near-infrared domains. Other metals utilized in plasmonics are copper, aluminum, gallium, indium, cobalt, palladium, platinum, nickel, and titanium [84]. As dielectric, silicon, oxides (for example, SiO_2 and Al_2O_3 , phononic materials such as GaN, SrTiO_3 , SiC, GaP, and many others are used [84]. Also graphene, investigated as long wavelength plasmonic material due to its unique band structure and very high charge mobility [85, 86], is widely studied for plasmonic applications [84, 87].

Light can be coupled with metal nanoparticles. The resonant wavelength in this case depends on the shape, size, and composition of the nanoparticle (Fig. 5.2 (b)). A single subwavelength particle can provide field enhancements of 10 - 100 [61]. For instance, spherical nanoparticles exhibit a dipolar plasmonic resonance. Fig. 5.2 (c) shows that an increase of the particle aspect ratio redshifts the resonance wavelength. The extinction cross section and the distributions of the electric field norm in the case of spherical and rod shapes are presented in the top line in

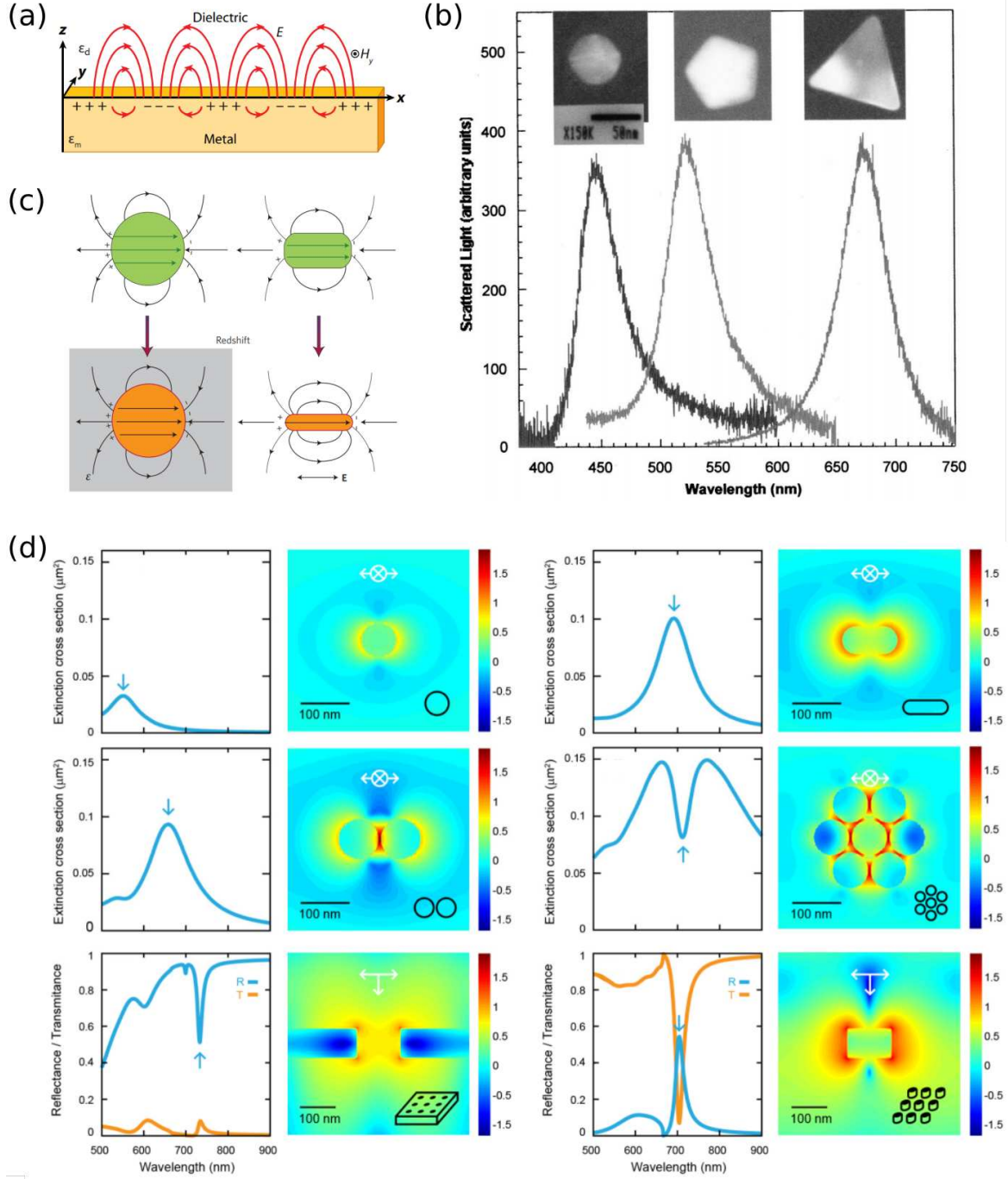


Figure 5.2: (a) Surface plasmon polaritons at a flat metal-dielectric interface [83]. (b) Typical spectra of individual silver nanoparticles. A TEM image of the corresponding particle is presented above its spectrum [88]. (c) Examples of influence of geometry and materials on electrostatic resonances of deep-subwavelength metal nanostructures. Increase of the dielectric constant of the surrounding media causes redshift. Increase of the aspect ratio of a particle leads to redshift as well. Minus and plus signs are for the regions of the high and low electron density [61]. (d) Spectral response and distribution of the norm of the electric field for typical nanophotonic structures, placed in water (refractive index 1.33). The colorbar is in the logarithmic scale. The top left: gold sphere of the 80 nm diameter; the top right: gold nanorod of the 80 nm diameter and 120 nm length; the middle left: gold spheres dimer, a sphere diameter is 80 nm, and the distance between spheres is 5 nm; the middle right: a gold spheres heptamer with a sphere diameter 80 nm and the 5 nm distance between spheres; the bottom left: a 80 nm-thick gold film with periodic array of 120 nm diameter holes; the bottom right: a periodic array of 80 nm-thick gold disks with the diameter 120 nm [69].

Fig. 5.2 (d). Also the dielectric constant of a surrounding media affects the resonance wavelength, and its increase leads to the redshift too [61].

Besides individual particles, a pair or group of particles can be coupled with an electromagnetic wave, as shown in Fig. 5.2 (d). In this case, the electromagnetic field is concentrated in the gap between particles, and Fano resonances originate from the interference of coupled modes. Besides, light can be coupled with a periodic or non-periodic array of particles, as well as with a metal film with array of holes in it [69].

Plasmonic structures composed of metallic elements inserted in a dielectric material form a particularly effective absorbing medium because both the electrical and the magnetic properties of this so-called metamaterial can be tuned by structure design [92]. Effective absorption can be provided by metal-insulator-metal (MIM) plasmonic metamaterials. Having a small absorber volume and being easy to fabricate, they also provide tunability of both the absorption wavelength and the operation mode by geometrical control. Due to their high performance and design flexibility, MIM structures are widely studied for a broad frequency range [63].

Basically, a MIM structure consists of three layers: a bottom flat layer, a thin dielectric layer (or periodic patches), and one or few metal patches, or periodic array of them. Examples of different MIM structures are presented in Fig. 5.3. Fig. 5.3 (a) shows schematic presentation of a conventional plasmonic MIM structures with 2D and 1D periodic patches. It can be seen, that a dielectric layer can be a flat homogeneous layer or also can be structured. A detailed review of the MIM mode is presented in Section 6.2.

The geometric parameters of the MIM structure define the central wavelength λ_{res} of the resonance peak. This is mainly defined by the length of a metal top patch and to a lesser extend by the optical index of the materials and the dielectric thickness. A resonance appears when a round trip of the wave along the resonator corresponds to a multiple phase shift of 2π . This phase shift is due to the wave propagation and to the reflection at the ends of the resonator. The effective index of the mode n_{eff} corresponds to the propagation of this mode. It is defined by the optical index of the dielectric disturbed by the penetration of the field into the metal [93, 94]. As a first approximation, the phase shift at the reflections can be assumed to be equal to 2π . The resonance wavelength and the length of the structure are then related by the effective index to approximately $\lambda_{res} \approx 2n_{eff}l$. The effective index of the mode is also affected by the width and thickness of the metal, as demonstrated in the following chapters.

Thus, if a few metal patches of different sizes are placed in the same period, the absorption

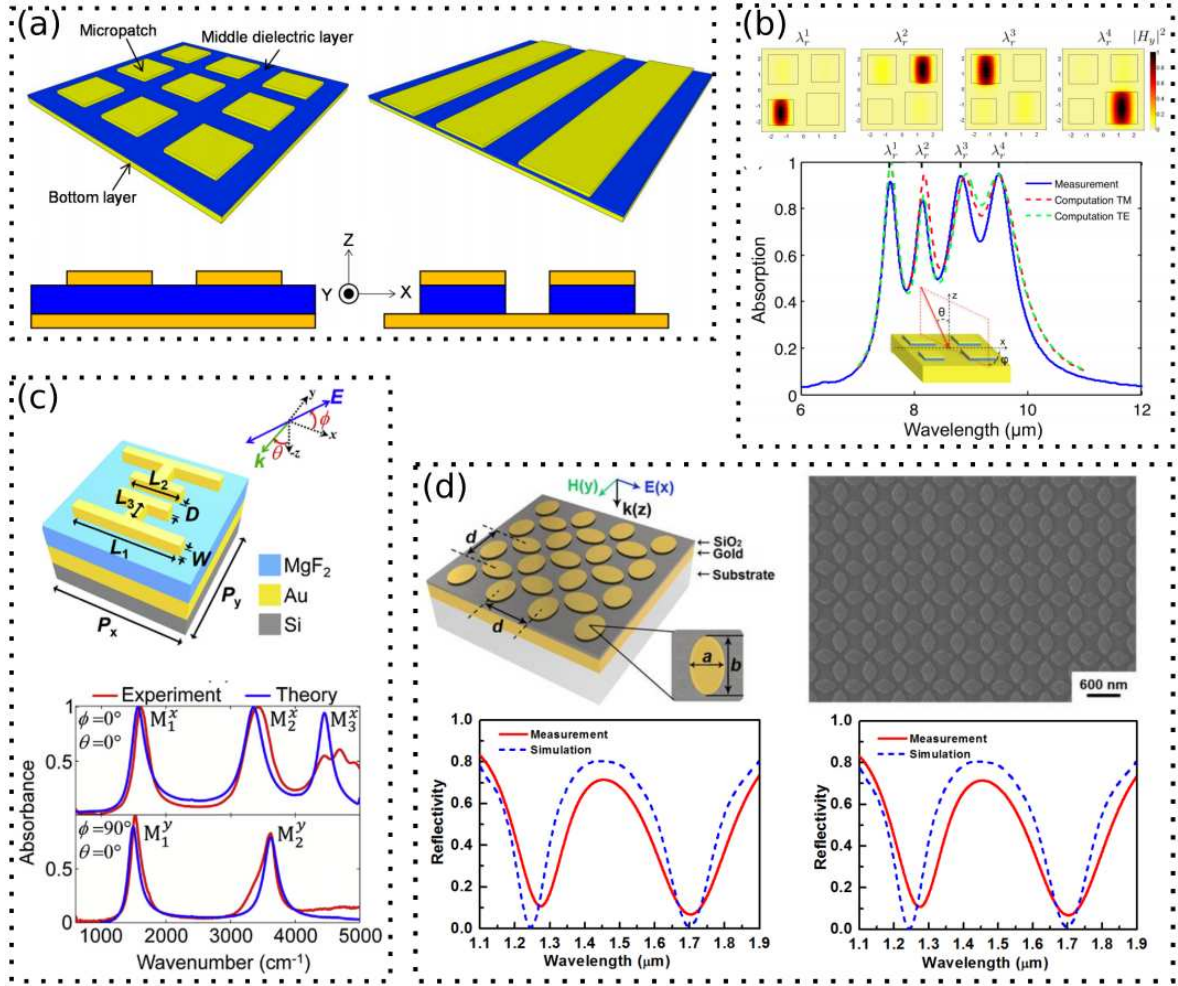


Figure 5.3: (a) A schematic presentation of 2D MIM (the top left) and 1D (the top right) MIM structures [63]. (b) A multi-band 2D MIM structure for the infrared range of wavelength. The structure parameters: the spacer thickness $h_{ZnS} = 290$ nm, the top gold layer thickness $h_{Au} = 50$ nm, the period $p = 5.3$ μ m. Widths of the MIM resonators are $w_1 = 1.64$ μ m, $w_2 = 1.78$ μ m, $w_3 = 1.91$ μ m, and $w_4 = 2.07$ μ m [89]. (c) Dual-band MIM absorber working at the short infrared frequencies. Incidence and polarization angles of the exciting plane wave are shown at the top of the scheme. Measured and simulated absorbance spectra under normal incident x- and y-polarized light sources are presented at the bottom for the structure with parameters $L_1 = 1800$ nm, $L_2 = 800$ nm, $L_3 = 500$ nm, $W = 100$ nm, $D = 200$ nm, and $P_x = P_y = 2100$ nm [90]. (e) A dual-band perfect plasmonic absorber MIM structure: a schematic presentation at the top left and a SEM image at the top right. The thickness of the top gold layer is 20 nm, the bottom gold layer thickness is 150 nm, the thickness of SiO₂ dielectric spacer is 30 nm. Presented at the bottom spectra correspond the TM (on the left) and TE (on the right) polarizations, both are at the normal incidence. Spectra correspond to the structure with following parameters: period along both x and y directions $d = 600$ nm, the minor and major axes of the elliptical gold nanodisks are $a = 240$ nm and $b = 360$ nm, respectively [91].

spectra can show either a wide band or several bands (Fig. 5.3 (b)). As it shown in Fig. 5.3 (c) and (d), more complicated patch geometry can also provide a multi-band response.

Besides MIM structures, there are insulator-metal-insulator (IMI) structures, which operational principle is also based on plasmonic effects. In the presence of material loss, a simple structure, consisting of three infinite plane-parallel layers (not structured) of limited thicknesses, can support two types of modes: symmetric long propagation SPP and anti-symmetric one [95].

Thus, IMI structures are known for waveguide applications. Structuring of the metal layer allows managing the structure response. The examples of two waveguide IMI structures are presented Fig. 5.4 (a) and (b). Fig. 5.4 (c) shows a silver branched nanowire which can serve as a controllable router and multiplexer in integrated plasmonic circuits due to its ability to route light of different wavelength into different branches. As a result, it can emit light from the corresponding wire ends [96].

For the three IMI structures mentioned above, as in general for all plasmonic structures, absorption in metal is considered to be the main drawback [64]. But in this work it is no longer disadvantage but a fundamental property. For the design of new detectors, the opposite situation of a rapidly dissipating mode becomes the central point of research. Conventional thermal detectors employing a uniform layer as an absorber in the infrared domain have limited absorption efficiency, as illustrated by the central graph in Fig. 5.4 (f) showing the absorbance of a uniform layer of titanium-gold at the $6\mu m$ wavelength) [97]. Oppositely, a structured layer (array of antennas) can demonstrate a much more efficient absorption at the chosen wavelength due to the critical coupling with the incident wave. In addition, its volume density will be reduced compared to that of the uniform layer. Some examples of IMI absorbent structures are presented in Fig. 5.4 (d) - (f). These structures can be in the form of periodic 1D or 2D metal patches on a membrane or slits in a metal film. However, a single metallic layer can absorb at most half of the incident radiation [87].

Thus, due to the specific physical properties of metals, an electromagnetic wave can be excited at the metal - dielectric interface. It can be used for concentration of the strong electromagnetic field in sub-wavelength volume, and for efficient absorption of radiation in the visible and IR ranges. MIM structures (1-port absorber) were found as promising absorbers, while IMI antennas (2-port absorber) are capable of absorbing about half of the incident radiation. In contrast, IMI antennas have a smaller volume than MIMs, which is an important parameter for the design of sensitive thermal detectors as shown below.

5.3.

Long wave photodetection

In this section both classical and modern conceptions for optical detectors are consider, mainly focusing on bolometers structures for the infrared (IR) wavelengths. In the infrared domain, there are three main wavelength ranges of interest: $1.2-1.8\mu m$ (short wavelength infrared,

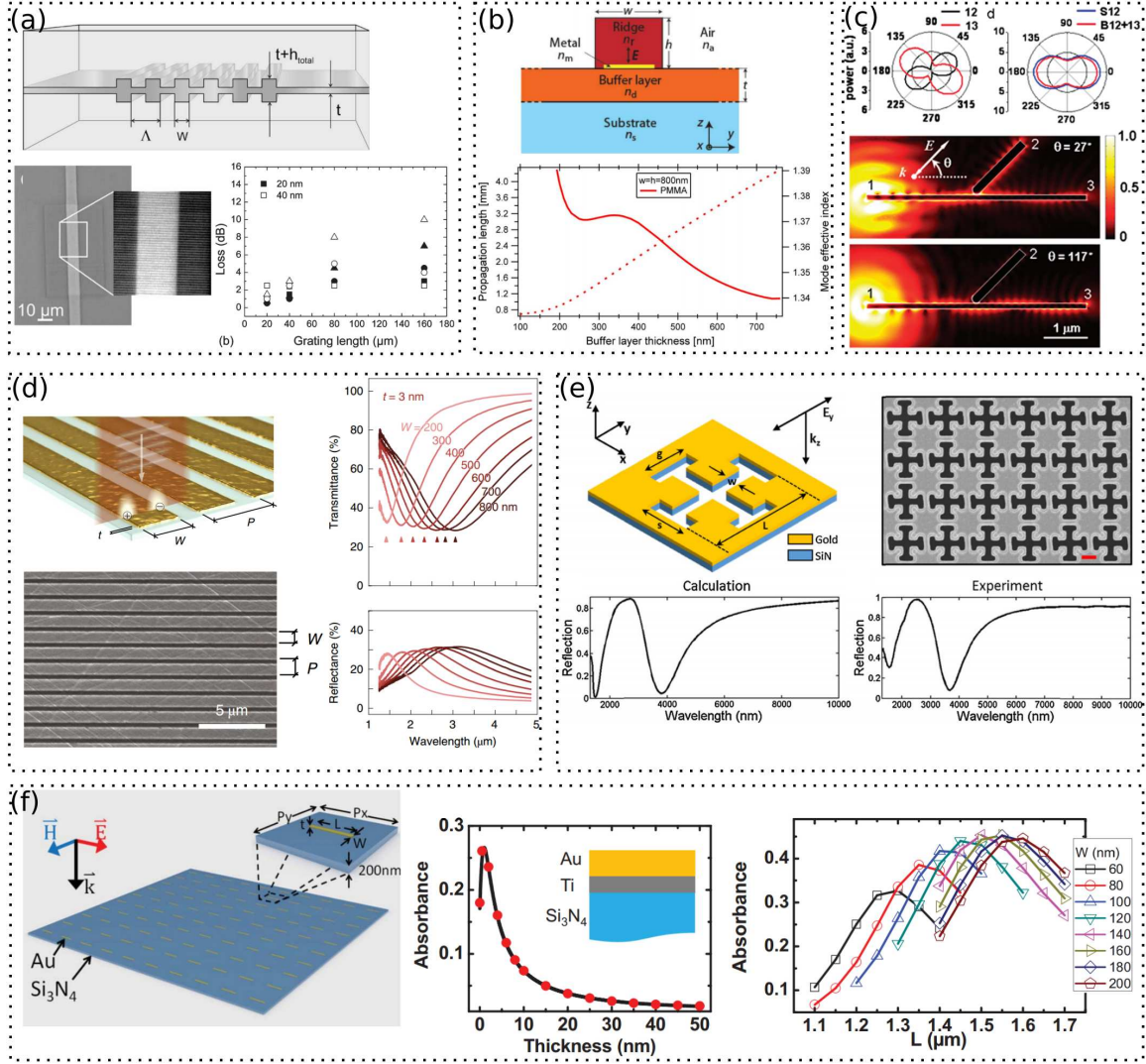


Figure 5.4: (a) A periodic thickness modulation of thin metal stripes embedded in a dielectric for long-range SPP for the telecom wavelength around 1.55 μm . The gold thickness is varied from 10 nm up to 60 nm, a characteristic grating period is about 500 nm [98]. (b) A waveguide for SPP over millimeter distances with tight mode confinement at the telecom wavelengths of about 1.55 μm . The gold strip thickness is 15 nm, its width is 500 nm, the buffer layer is PMMA, a square Benzocyclobutene ridge has the 800 nm side [99]. (c) Numerical simulation results for an excited by Gaussian beam silver nanowire branch, which works as controllable plasmon router. The length of a smaller wire is 1.5 μm and its diameter is 200 nm, the length of a longer wire is 5 μm , and its the diameter is 120 nm. The top left diagram presents angular polarisation (θ) dependence of emission power at terminals of long (red) and short wire (black). The top right diagram shows emission power S_{12} (blue) from a single wire of same dimension as the long wire, compared to the total emission power $B_{12} + 13$ (red) from both terminals in the branched structure. The color maps shows the norm of the electric field at the maxima (top) and minima (bottom) emission on the short wire [96]. (d) Schematic presentation and SEM image of gold nanoribbons of a width w , the period is $1.5w$, and the gold thickness is 3 nm. The array is fabricated on CaF_2 substrate. Measured spectra are presented in the right [100]. (e) An array of periodic holes in a 50 nm gold film, standing on a 100 nm freestanding silicon nitride membrane. The structure parameters are: $s = 600\text{nm}$, $g = 470\text{nm}$, $w = 200\text{nm}$, $L = 1150\text{nm}$, and the array period $P = 1400\text{nm}$. The scanning electron microscopy image of the aperture system is presented in the left. The calculated and measured reflected spectra are presented in the bottom [101]. (f) An absorber based on gold nanostrip antennas array on the 200 nm thick silicon nitride membrane. The gold thickness is 20 nm. A design wavelength is 6 μm . The middle plot shows the dependence of absorption on the gold thickness for the uniform gold film, and the right plot present absorption values for the nanostrip array with different width w and length L [97].

SWIR) $3 - 5 \mu m$ (middle wavelength, MWIR) and $8 - 14 \mu m$ (long wavelength, LWIR) due to the atmosphere transparency [102, 80]. This last IR domain is of particular interest because, according to Planck's law, materials at room temperature (around $T = 300 K$) have a black body emission around $10 \mu m$ [80]. Consequently imaging, including long distance, is possible in the atmosphere allowing applications such as security, remote monitoring and healthcare.

The majority of optical detectors for LWIR range can be divided into two large groups, based on their physical operating principle: photon (or quantum) detectors and thermal detectors. The photon detectors are based on the change in electronic density distribution, due to interaction of incident radiation with bound-to-lattice electrons, or electrons of impurity atoms, or free electrons. As a result, electrical output signal is detected. For example, intersubband absorption due to transitions between quantum levels is the operating principle of quantum well infrared photodetectors (QWIP).

In the second group of detectors are the thermal detectors. Their operating principle was invented by the American astronomer Samuel Pierpont Langley. From 1878 to 1880 he sought to create a device for measuring radiation that was more sensitive than the thermopiles he had available. In 1880 he designed and used a new instrument, the bolometer, and for the first time measured the energy distribution of the solar spectrum by coupling his bolometer to a diffraction grating [103]. The bolometer is based on the absorption of incident radiation by the absorber which heats up. At least one of the physical properties of the thermometer placed in intimate thermal contact with the absorber is modified. It is the measurement of this modification which allows the measurement of the incident flux. A change in electrical resistance, thermoelectric or pyroelectric effects can be used for this measurement [104]. This fundamental concept was used to design the first micro-bolometers for infrared cameras in the late 70's [105], but was not published until the early 90's [105, 106]. Since then, progress has been focused on technological modifications for improved performance (sensitivity, speed, spatial resolution) and reduced prices.

For modern optical detectors there is always a trade-off between better characteristics and price. The photon detectors have high sensitivity, high quantum efficiency, a selective wavelength response, and high speed. HgCdTe ternary alloy, InSb/III-Vs are the most common photon detector materials. But this materials must be cooled down to $70 - 90 K$ for the long waves IR (LWIR) range thus are quite expensive [80].

As it was mentioned by Kinch in [107]: "the ultimate cost reduction for an IR system will only

be achieved by the room temperature operation of depletion-current limited arrays with pixel densities that are fully consistent with background - and diffraction-limited performance due to the system optics". That is the reason why the uncooled thermal detectors are in higher demand. Although they are slower and less sensitive, they have much lower cost without need for cooling and exotic materials [80, 108]. At present, the microbolometer detectors are manufactured in volumes much larger than all other IR array technologies. Being the dominant uncooled IR detector technology, microbolometers had more than 95 % of the market in 2010 [80].

However, the use of plasmonic structures could considerably change this trade-off. Their unique properties, in particular the concentration of the electromagnetic fields in very tiny volumes, may be at the heart of a new class of detectors with characteristics approaching those of cooled detectors, while maintaining operation at the room temperature. The rest of this Section is dedicated to the state of the art of bolometers. It is organized as follows. First, the principle of operation of the bolometer is introduced, and some examples of modern devices are provided. After that, some examples of nanoplasmonic devices, applied for bolometer applications, are presented.

5.3.1 Current technology

The principle of operation of all thermal detectors is based on the absorbing element, characterized by its heat capacity C . It converts incident electromagnetic radiation of power P into heat. To improve the corresponding temperature rise, this element is thermally isolated from the rest of the world. It can be considered as connected to a heat sink at temperature T_S via a thermal conductance G (schematic representation in Fig. 5.5 (a)). Due to the absorbed energy, the temperature of the heated element T_B rises over time at rate $dT_B/dt = P/C$ and approaches the limit value $T_B = T_S + P/G$ with the thermal time constant $\tau = C/G$. Once the radiation is turned off, this temperature relaxed back to its initial value with the same time constant τ [104]. The exploded schematic presentation of a microbolometer is presented in Fig. 5.5 (b).

The core of state-of-the-art microbolometers is mainly composed of two parts : a thin metallic film [110, 111], acting as an absorber, and an amorphous silicon film (α -Si) or vanadium oxide (VO_x) for the thermometer function [80]. The majority of commercially available devices consist of a thermally insulated absorbent membrane covering almost the entire pixel. Fig. 5.6 shows some commercially available structures based on α -Si and VO_x . Modern microbolometers can be classified in two types: single and double layer. For the first type, the pixel design involves

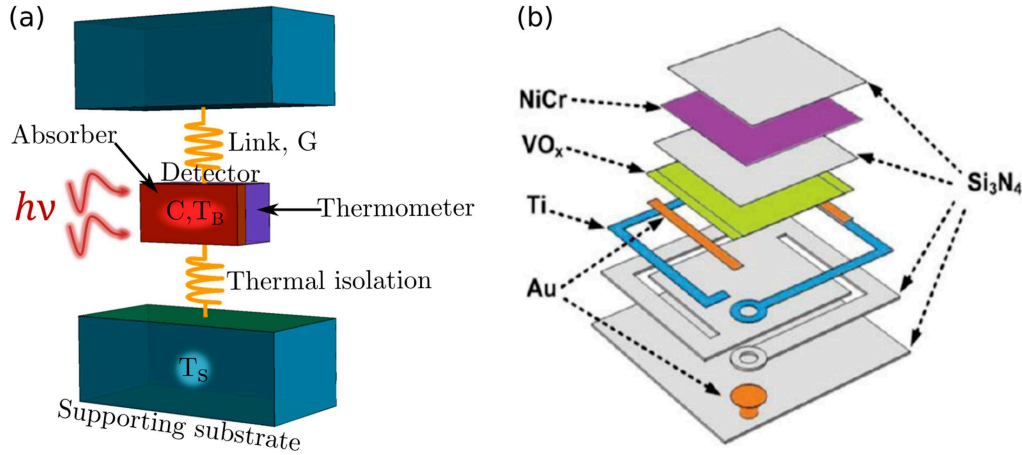


Figure 5.5: (a) A schematic presentation of a thermal detector (inspired by [80]). (b) A schema of VO_x microbolometer from [109]. A pixel size is $25 \times 25 \mu\text{m}^2$, the thickness of the active thermometer V_xO_y layer is 95 nm, placed on a 400 nm thick Si₃N₄ layer which is air-suspended above a silicon substrate employing two 6 μm diameter gold cylindrical pillars. Two titanium electrodes are used for electrical bias and readout connection. A 5 – nm-thin metal film of NiCr, sandwiched between two 50 – nm-layers of Si₃N₄, works as optical absorber. For electrical insulation, the silicon substrate is covered with silicon dioxide. A thin gold film is placed on SiO₂, and a thin chromium layer is used for adhesion between the gold and SiO₂. The gold layer functions as an optical cavity mirror in the microbolometer. There is an 1.5 μm air-gap between the mirror and suspended microbolometer structure.

a resonant cavity, made by an absorbing material layer suspended above a reflecting metal layer [80]. Usually, the length of the cavity is $\lambda/4$. This design aims to amplify the absorption of the incident IR radiation by the absorbent placed at the anti-node of the electric field, since the absorption varies as the electric field squared. Also, the gap created and maintained under vacuum works as a thermal isolation. A double-layer microbolometers are composed of a space-filling metal/dielectric sandwich layers designed to capture a maximum amount of incoming radiant heat energy [80]. This improves the efficiency of the old concept but makes the structure much more complicated to fabricate. This is the opposite process to the goal of this work: to improve the performance of the bolometer while keeping its structure as simple as possible.

For all microbolometer structures, the fundamental compromise lies between their sensitivity and response time. One of the most important characteristics of photo-detectors, the noise equivalent difference temperature (NEDT) represents the minimum temperature difference that a device resolves. It is proportional to the conductance of the thermal conductance G , and the thermal time response $\tau = C/G$ is inversely proportional to this value. As a result, increasing of sensitivity (i.e. decreasing NEDT) results in increased response time, and conversely.

Nowadays, the main improvement of bolometer technology is related to the reduction of the pixel size. This allows to fabricate a higher amount of devices on the same wafer, and thus – to reduce the price of a device. The pixel size has already reached the wavelength limit in the

LWIR range and tends to become twice smaller [80]. New difficulties then emerge: diffraction effects as well as an increase in noise in $1/f$. Additionally, the thermal isolation of the absorbent membrane becomes more challenging: there is less room for isolation arms.

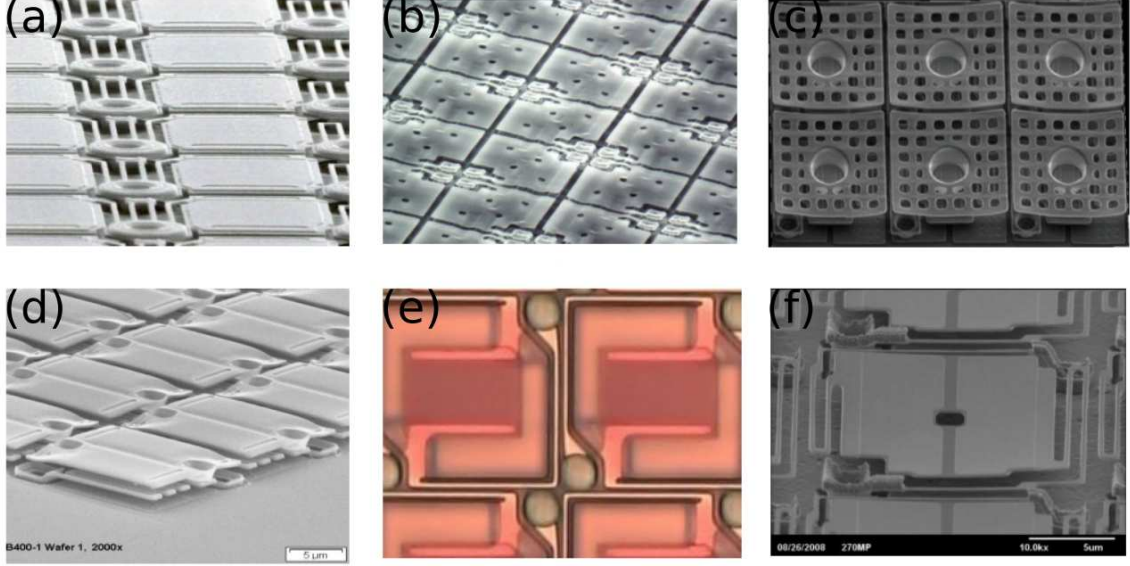


Figure 5.6: Examples of commercial bolometer design. Scanning electron microscope images for (a), (c), (d), (e) VO_x bolometers, (b) $\alpha\text{-Si}$ bolometer, (f) $\alpha\text{-Si}/\alpha\text{-SiGe}$ bolometer [80].

In this way, the bolometer concept has not changed significantly since it was originally designed in the early 90s. The spectacular progress that has been made in terms of performance has been achieved through technological improvements. We can think that they are now reaching their limit, which would explain the slowdown observed in recent years in the progression of performance. Future imaging systems in the LWIR domain require higher pixel sensitivity and improved pixel matrix functionality (spectral sensitivity and / or polarization) for multispectral sensors, while decreasing the unit price of the detector [81]. Additionally, video applications that require a frame rate of at least 30 fps, require a reduction in response time. All of these constraints make the sensitivity - response time compromise hitherto used obsolete and require the entire design of bolometers to be reconsidered.

Plasmonic structures can become an alternative that breaks the fundamental limitation of traditional microbolometers. Being of sub-wavelength size, they allow to fabricate miniature absorbing structures which, while absorbing all incident light, leave enough space for thermal isolation. Their volume can be much smaller than that of the absorbing layers currently used, allowing the thermal capacitance to be drastically reduced and paving the way for another (and better) sensitivity - speed compromise.

5.3.2 Nanoplasmonic devices

The use of plasmonic sub-wavelength structures can break the compromise established for traditional bolometers. Much smaller than the wavelength, plasmonic structures offer significant reduction of the thermal capacitance, thermal conductance, and thermal fluctuation noise, providing advanced responsivities and reduced noise floor [111]. They also make it possible to structure the spectral response of the absorption (spectral width, multi-bands, etc.) This can be done at two levels: on the one hand by modifying the geometric parameters of the optical antennas and on the other hand by associating several different antennas in the same period. In addition, the separate measurement of the temperature of different antennas gives access to a spectral resolution, opening up perspectives for "colored" images. Absorbers based on metasurfaces can be tuned in real time, thus when integrated with a detector, a single detection element could be used for time multiplexed multispectral sensing" [111].

During the last decade, some attempts to employ plasmonic structures ability to collect light energy in small volume were realized for bolometric applications. In the design of the structures of Fig. 5.7 (a) and (b), the current flow is localized in the bolometric material. In the device of Fig. 5.7 (a), due to the optical losses in the metal, the energy absorbed at the resonance is concentrated in the top and bottom gold layers, and is converted into heat by the Joule effect. The generated heat is then transmitted by conduction to the α -Si thermometer film via a thin film of Si_3N_4 . This generates a change of the electrical resistance of the film of α -Si, the measurement of which allows that of the incident IR flux [112]. In structure of Fig 5.7 (b) the sensing layer is made of a 300 nm thick amorphous $\text{Si}_{0.344}\text{Ge}_{0.602}\text{O}_{0.054}$ film.

Bolometers, based on a plasmonic antenna, also can be polarization sensitive. For example, a device in Fig. 5.7 (c) consists of two orthogonal slot antennas. Each antenna is loaded with two 200 nm thick nickel microbolometers at its edges. As a result, this detector is sensitive to polarisation of the incident IR radiation and have a wide dynamic range. In the device, shown in Fig. 5.7 (d) a resistive and fast micro-bolometer is placed between two three-steps tapered dipole nano-antennas. Matching the impedance between the nanoantenna and the microbolometer allows to reduce return losses by a factor of 650 %, opening the route towards an efficient energy transfer between load elements and resonant nanoantennas [115]. The design of the structure, presented in Fig. 5.7 (e), is based on the idea of simplification of device manufacture, making it cheaper. One material is used as an optical absorbent, another for the thermometer. Both

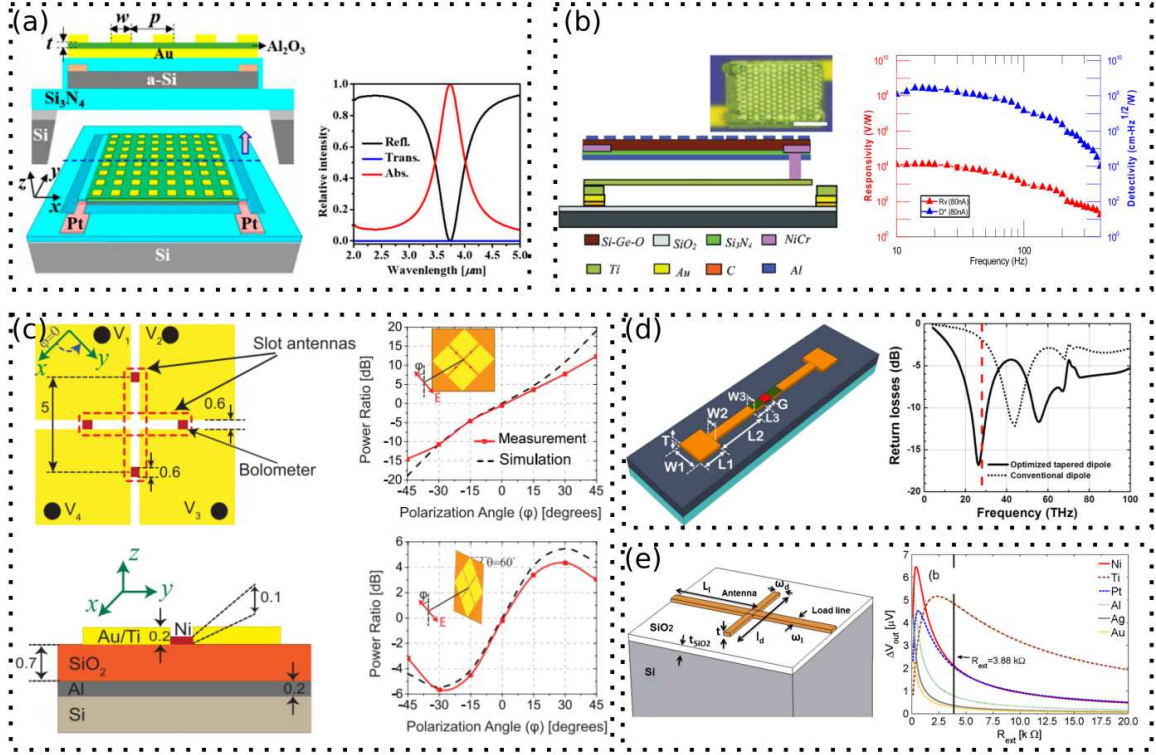


Figure 5.7: (a) A wavelength-selective detector built of a 2D MIM absorber coupled to a thermally isolated a -Si bolometer. Parameters of the structure: period $p = 1.2 \mu\text{m}$, the thickness of Al_2O_3 (dielectric) $t = 0.045 \mu\text{m}$, the side of top square metal patches $w = 0.72 \mu\text{m}$, its thickness is 70 nm , the bottom gold film thickness is 100 nm . A 100 nm -thick Si_3N_4 layer is placed in between the metal film and the bolometer for electrical isolation [113]. (b) A bolometer structure designed for long wave IR ($8 - 14 \mu\text{m}$), consisting of 75 nm -thick aluminum disks arranged on a Hexagonal Close-Packed lattice in direct contact with the top of the 300 nm -thick amorphous $\text{Si}_{0.344}\text{Ge}_{0.602}\text{O}_{0.054}$ film (a sensing layer). The thickness of an isolation/insulation layer of Si_3N_4 with is 50 nm . Thickness of the chromium and gold layers are 50 nm and 150 nm , respectively. Titanium bridge of 200 nm thickness support the rest of the layers [114]. (c) An infrared polarization sensing sub-wavelength detector, consisting of two orthogonal slot antennas, each loaded with two microbolometers at its edges. The operation wavelength is $10.6 \mu\text{m}$. In the right simulation and measurement results of the polarization sensing IR detector for various incidence angles are presented [112]. (d) A schematic representation of the three-steps tapered dipole Ti antenna coupled to a nano-bolometer element, working at the $10.6 \mu\text{m}$ wavelength. The nano-bolometer is freestanding-bridge-like on the dipole elements. The antenna lying on a $1 \mu\text{m}$ SiO_2 layer and semi-infinite Si substrate. Parameters of the structure $L_1 = 273 \text{ nm}$, $L_2 = 563 \text{ nm}$, $L_3 = 162 \text{ nm}$, $W_1 = 292 \text{ nm}$, $W_2 = 103 \text{ nm}$, $W_3 = 97 \text{ nm}$, $T = 42 \text{ nm}$ and $G = 103 \text{ nm}$ [115]. (e) An optical antennas as a distributed bolometer and the load lines perpendicular to it. The length L of the load line is $15 \mu\text{m}$, its width $w_l = 0.2 \mu\text{m}$, its thickness $t = 0.1 \mu\text{m}$. Characteristic parameters for dipole antenna: the length is $2.2 \mu\text{m}$, the width is $0.2 \mu\text{m}$, the thickness is $0.1 \mu\text{m}$. The thickness of the SiO_2 layer is also $0.2 \mu\text{m}$. In the right, the device response for different materials is presented as a function of the external circuit resistance R_{ext} [62].

the resonant antenna and the load line are made of the same metal. The incident IR radiation induces electric current on the optical antenna, and its temperature increases due to Joule effect. This varies the resistance, thus whole antenna structure works as a bolometric element [62].

Due to their property to collect and concentrate light energy, plasmonic resonant structures can be employed for the design of a new generation of efficient thermal detectors. Sub-wavelength plasmonic elements allow smaller thermal capacitance, resulting in increased sensitivity, and

decreased response time and noise floor. However, most of the plasmonic structures presented for bolometric applications have a too complicated design only valid for a lab concept, which is not manufactured industrially. In this work, we focus on exploring the most simple and efficient design for IR absorbers for bolometric applications.

5.4. Structures at $\lambda/4$

As it was mentioned in Section 5.3.1, the traditional bolometer design includes a resonance cavity with a mirror in the bottom. In this section, an example of a standing wave, which arose due to the interaction of incident and reflected waves from the mirror, is considered. The anti-node of the electric field occurs for this wave at the distance $n\lambda/4$ from the mirror, where n is a natural number. That is the reason for choosing the length of the resonant cavity equals $\lambda/4$. Then, classical absorption structures based on the resonance at $\lambda/4$ are discussed.

5.4.1 A standing wave above the mirror

The presence of the mirror on the path of the plane wave $\exp(-ikz - i\omega t)$ leads to the appearance of a standing wave. The field maps in the case when the wave vector is directed orthogonal to the mirror are shown in Fig. 5.8. Both electric and magnetic fields of the standing wave have a sinusoidal behavior, so that the maxima (anti-node) and minima (node) of the amplitude alternate with the distance $\lambda/4$. The electric field on the mirror is zero, and its anti-nodes are at $(2n + 1) \cdot \lambda/4$ from the mirror (n is a natural number). Oppositely, for the magnetic field the first maxima on the mirror, and at $(2n + 1) \cdot \lambda/4$ there are its nodes. Thus, bulk absorbers are placed at $\lambda/4$ above the mirror where the electric field is maximum.

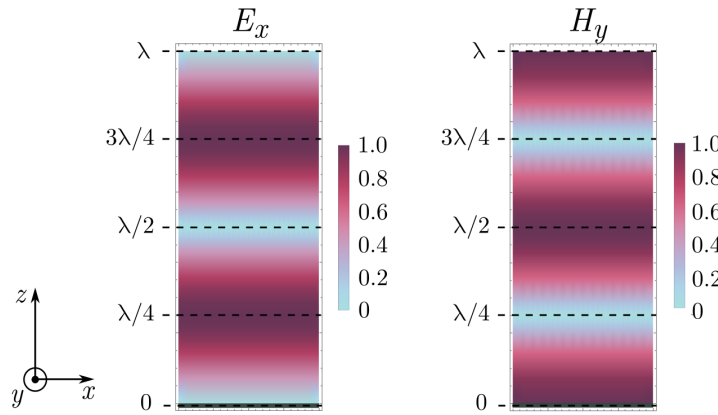


Figure 5.8: The field maps for the standing wave. The mirror is placed at the level of 0. On the left - the norm of the electric field, on the right - the norm of the magnetic field.

5.4.2 Configurations

Maximum of the electric field for the standing wave at the distance $\lambda/4$ provides maximum absorption in the bulk. Structures, employing this effect, can be classified into three categories [63]. These configurations are widely used in microwave range for a long time [116]. But it is important to note, that metals behaviour is different for the microwave and IR ranges. While the permittivity for the microwaves tends to infinity, for the optical range the real part of the permittivity is about minus thousand (or less), and imaginary part is about a few hundreds (or less). Thus, using technological solutions from different ranges, it is important to adapt them appropriately.

The schematic presentation of structures design, involving the $\lambda/4$ electric resonance, is presented in Fig. 5.9. The Salisbury screen construction (Fig. 5.9 (a)) is based on the idea of the placement of an absorption layer above a reflecting surface. A quarter of the wavelength distance between them is aimed to maximal absorption, and the sheet resistance of the layer should be equal to the vacuum impedance. This configuration is widely used in radio frequency engineering and optics [108].

The design of the Jaumann absorber (Fig. 5.9 (c)) is include two or more purely restive sheets distanced at a quarter of a wavelength in front of each other. In the case of a circuit analog absorber (Fig. 5.9 (e)) the periodically patterned top metal layer is separated by a quarter-wavelength thickness dielectric spacer from the bottom homogeneous metal layer. Resent metamaterials structures often employ the conception of the circuit analog absorber for efficient absorption [63].

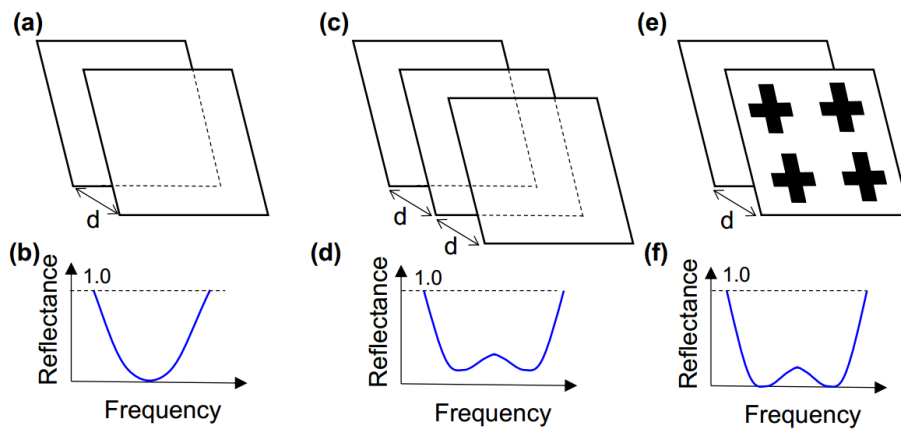


Figure 5.9: Schematic presentation and characteristic response of resonant absorbers: (a,b) the Salisbury screen; (c,d) the Jaumann absorber; and (e,f) the circuit analog absorber. The symbol d stands for the quarter-wavelength gap [63].

In this way, the $\lambda/4$ resonance can be applied for different types of design and is widely used for different frequency ranges. As it was said in Section 5.3.1, the traditional bolometers construction also includes a quarter-wavelength gap. But it is also can be met for the modern prototypes of thermal detectors. For example, in work [108] the Si-nanomembrane-based thermoelectric bolometers for long-wave IR detection are studied.

5.5.**Thermal properties and mass vs volume**

In this work, gold was chosen as the metal for the absorbing structures. However, other well-studied metals with different optical properties also can be used. In this work, four materials were selected for comparison: gold, silver, aluminum, and copper, as the optical response of these materials varies considerably. In addition, as one of the objectives of this work is to define a perfect absorbent with the smallest possible volume, silicon was chosen as a dielectric material for its optical index in the target wavelength region: a high real part (high confinement) and its very low imaginary part (negligible absorption). Before analyzing the response of these structures, it is worth to determine the criterion by which the most optimal structure will be selected. For a bolometer structure an absorber should be connected with a thermometer. Temperature measurements defines the intensity of absorbed electromagnetic energy. Thus, thermal properties of the structure is very important.

The structures from different materials can be compared by masses. For example, gold is the most heavy metal from considered metals (Table 5.1). Meanwhile, as it can be seen from the third column of Table 5.1, its specific heat capacity is smaller than one for other metals. In the same time, the volumetric heat capacity values are very close for three metals, and only copper has higher value.

The specific heat capacity for bulk materials arises from lattice vibrations, and thus it depends on the number of atoms in the volume. A consequence of this is the Dulong-Petit law, which states that the heat capacity per one mole of a chemical element is almost the same for all bulk and equals $3R$ (the universal gas constant). It also worth to note, that the volumes of atoms differ much less than their mass, thus one mole of different chemical elements can have a similar volume, but a large difference in the masses. Indeed, the atomic volume of three metals except copper is very close, while their atomic masses vary significantly (Table 5.1). Also, the number of freedoms of atoms for all four metals is approximately the same. Thus, the volumetric

Table 5.1: Thermal properties of studied materials

Material	Density	Specific Heat Capacity	Vol. s.h.c.	Degrees of freedom	Atomic mass	Atomic volume
	ρ	c_p	ρc_p	N	m_A	m_A/ρ
	g/cm^3	$J/(g\ K)$	$J/(cm^3\ K)$	number	g/mol	cm^3/mol
Gold	19.30	0.129	2.49	6.11	196.97	10.19
Silver	10.5	0.233	2.45	6.10	107.87	10.27
Aluminum	2.699	0.897	2.42	5.82	26.982	10.00
Copper	8.92	0.385	3.43	5.88	63.546	7.12
Silicon	2.33	0.75	1.75	4.76	28.085	12.05

specific heat capacity for three materials are very close, and copper differs a bit.

For silicon, its atomic volume is a bit larger, than one for metals, while the volumetric specific heat capacity is a bit less than 30 % smaller than one for metals. But as these values are comparable (and the difference is smaller than for the specific heat capacity), at the first approximation the full volume of the structure becoming a good parameter to compare and structure optimization. Thus, in this work, the discussion and comparison is based on volumes instead of mass.

5.6.

The problem we aimed to solve

In this Section, the main aim of the second part of the present work is discussed. As mentioned above, since the design of the bolometer was presented (about thirty years ago), it has only undergone quantitative improvements with no qualitative change in its structure. The most significant progress has certainly been the reduction in pixel size. Now, this approach is reaching its limit. Indeed, the design of smaller pixels comes up against the fundamental barrier of the diffraction (the pixel size being close to the wavelength) and the more technological one of the organisation of the thermal insulation while keeping as much space as possible for the absorbent. Moreover, current applications demand detectors that are both more sensitive and faster. It appears that it is time to revisit the founding concepts of bolometers to propose a qualitatively new structure. This work is a contribution to this great project, considering the plasmonic structures capable of causing this break in concept. Although there are some attempts to create plasmonic-based bolometers, nothing has been designed for an industrial fabrication, only lab concepts were presented. However, it is indeed a very promising direction. Being of

subwavelength sizes, plasmonic structures have a small (compared to wavelength) volume and thus – a small heat capacity. This can help to shift the compromise between the time response and the sensitivity into another level, with both better sensitivity and speed.

This work aims to solve only a part of a more complex problem of creating a new type of bolometer based on plasmonic effects. It focuses only on the first part of the problem: to show that an array of ultra-small plasmonic structures, occupying a small percentage of the pixel area, can efficiently absorb incident LWIR radiation. The second stage of this problem – a new concept of temperature measurement – is outside the scope of consideration, is reserved for a future study.

We assume that the use of plasmonic structures will allow to decrease the thermal capacitance. For this we will find a design with the smallest volume of metal possible with absorption of 100 %. Besides, structure miniaturisation also allows to decrease the mechanical support of the absorber, and offers more space for the isolation system.

In this approach, our industrial partner asked to place the plasmonic absorbers at a distance equal to $\lambda_{res}/4$ above a mirror in order to introduce as few modifications as possible in the existing technology. This distance corresponds to a maximum absorption in the volume of the absorbent membrane used in current micro-bolometers. As will be seen, this distance is not very suitable for certain plasmonic structures, the specific design of plasmonic antennas will therefore be required. On the other hand, the presence of a mirror makes it possible to suppress the transmission and to enhance the electric field in the region of the structure.

In this way, we are looking for the simplest plasmonic structure, which, being placed a quarter of the resonant wavelength above the mirror, has the smallest possible volume and efficiently absorbs incoming LWIR radiation.

5.7.

Review on the following chapters

In this Section, a brief review of the following chapters is presented.

The study starts with MIM structures because their modes of operation have been widely studied. However, as in the case under consideration, they are placed in an unusual location above a mirror, at a node of the magnetic field of a standing wave, their properties and the coupling process need to be reconsidered. As the structures are placed in front of a mirror, and the main objective of this study is to obtain the smallest as possible volume for structures in the

critical coupling conditions, only MIM with all structured layers are considered below. In the beginning of Chapter 6, the introduction to MIM structures and their properties is presented (Section 6.1). As the comparison with the fundamental MIM mode arises several times in the present work, this mode and its main properties are presented in Section 6.2). Then, the arrays of MIM structures, periodic only in one direction of the plane and invariant in the other, so called 1D MIM, are considered in Section 6.3.

First, the simpler problem of a MIM array placed on a perfect mirror is considered in Section 6.3. It is shown, that the energy funnel in this case, corresponding to the standard MIM configuration, is provided by the magnetic field of the standing wave and the electric field of the mode. Being optimised on the mirror, 1D MIM array, moved to the node of the magnetic field at $\lambda_{res}/4$ above the mirror, has a weak response with almost no absorption (Section 6.3.3).

However, as demonstrated in Section 6.3.4, the 1D MIM array having a sufficiently thick dielectric layer can be critically coupled at the distance $\lambda_{res}/4$ above the mirror. But the process of energy funneling is no longer that of MIM mode. The structure then shows another behaviour. Indeed, the funnel effect is, in this case, based on the magnetic field of the mode (instead of the magnetic field of the standing wave for the MIM mode). Design rules providing critical coupling conditions, i.e. the balance between incoming energy and non-radiative losses, are found for 1D arrays above the mirror, and some of their optical properties are presented. However, it is observed that this type of structure does not make it possible to significantly reduce the volume density of the absorbers.

Thus, periodic 2D arrays of MIM structures, limited in both directions, are considered at the distance $\lambda_{res}/4$ (Section 6.4). Limiting the structure in the former invariant direction significantly decreases the volume of the metal and provides a new control parameter – width. It is demonstrated that these structures can be critically coupled being placed in the node of the magnetic field above the mirror.

To define the shape providing the minimal volume, four simple geometries of 2D periodic MIM are examined in Section 6.4.2. It is shown that, for a given metal thickness, the width of the structure defines the optical losses in the metal, and, consequently, the critical coupling. The excited mode differs significantly from those of 1D MIMs because edge effects modify the distribution of the field, and new components of the magnetic field are excited, corresponding to a more complex distribution of the current density. It is established that the cross-shaped (resp. rectangle) geometry is optimal (i.e. has the smallest volume) for a non-sensitive (resp.

sensitive) optical response to the polarization. The influence of the metal properties is discussed in Section 6.4.3. It is shown that the distribution of the electric current which is responsible for optical losses in the metal, i.e. for critical coupling, is defined as the complex balance between skin depth and conductivity. Gold is the metal allowing the design of the structure of smaller total volume and lower thermal capacity.

The details of the critical coupling for an array of 2D MIM placed at the distance $\lambda_{res}/4$ above the mirror and the influence of the metal thickness on it are considered in Section 6.4.4. It is shown that the coupling is provided by the electric field of the standing wave and the magnetic field of the mode. As a result, a significant part of the energy comes from above and below the structure and is absorbed at the air-metal interfaces. In addition, a flow of energy through lateral planes appears. Consequently, the electric current density in the metal bars has a more complex distribution than that of 1D MIM. This leads to a change in the character of the critical coupling curve compared to that of the MIM mode when the structure is placed in the anti-node of the magnetic field.

In Section 6.4.5 two geometries of 2D MIM resonators, rectangle and cross, are compared under critical coupling conditions. Having the same length, these structures show a similar response for linear polarisation. However, compared to the rectangular antennas, the cross-shaped resonators show a blue-shift of the resonance as well as a slightly lower maximum absorption. It is demonstrated that these differences can be explained by modelling the structures by two $\lambda/4$ antennas separated by a conductive plane. The thickness of the conductive plane is finite for cross-shaped antennas, zero for rectangles.

Finally, in Section 6.4.6 it is shown that indeed, due to a different coupling process, at the critical coupling the volume density of 2D MIMs is significantly reduced (tens of times) compared to that of 1D MIMs. Although promising results have been found for MIM structures, the presence of the mirror allows to simplify the design, and to study structures with only one layer of metal.

In Chapter 7, periodic arrays of IMI structures are studied for the absorption applications. Due to the presence of a mirror, a metal antenna array supported by a membrane, all placed in a vacuum, can be critically coupled with incident radiation. First, Section 7.2 provides a geometric description of a periodic IMI array placed at the distance $\lambda_{res}/4$ above the mirror. Then, in Section 7.3, the excited mode in the structure at the critical coupling, is analyzed. As for MIM structures in the same conditions, the electric field of the mode has a dipole-like distribution,

while the magnetic field is excited at the side edges, with a maximum near mid-length. It is demonstrated that the coupling is provided by the electric field of the standing wave and the magnetic field of the mode.

In Section 7.4, the influence of the geometric parameters of the metal bar on the optical response of an array of IMI antennas is presented. It has been observed that the antenna width and thickness define the effective index of the mode and that their ratio influences the optical losses. The connection of the critical coupling conditions with the non-radiative losses is provided. The optical response of an array of IMI antennas at the distance $\lambda_{res}/4$ above the mirror is provided in Section 7.5. They have a large manufacturing tolerance on the distance separating them from the mirror and, in the case of the rectangular geometry, show high sensitivity to the polarisation of the incident radiation, demonstrating a high tolerance to the incident angle when the electric field of the wave is orientated along it, but having no absorption when the electric field has the orthogonal direction. It is also demonstrated that IMI arrays placed at $\lambda_{res}/4$ above the mirror have the smallest volume density of absorbers, less than 1 nm , among considered structures in this work. It can be considered as the most promising absorber for the design of the next generation of bolometers.

The simple design of IMI arrays allows to be modeled as a point dipole array, as presented in Section 7.6. The dipole approximation can predict the absorbance spectra by using the polarizability of a single antenna, extracted from numerical computations. This makes it possible to avoid several full numerical simulations which require long calculation times in the 3D cases. Finally, the experimental validation of the quasi-total absorption for the IMI arrays placed at $\lambda/4$ above the mirror is presented in Section 7.7. The experimental part is performed for MWIR spectral domain, while the antennas are held above the mirror by a dielectric layer of thickness $\lambda/(4n)$. A good agreement between the experimental data and the results of the numerical simulation is obtained.

In Chapter 8 the conclusion of this part and outlooks are presented.

Metal-insulator-metal structures

6.1.

Introduction

Metal-insulator-metal (MIM) structures are one of the widely studied plasmonic-based nanostructures [63, 117]. Simple in design, these structures can effectively confine light at the sub-wavelength scale [63, 118]. In the general case, the MIM structures consist of an upper array of periodic micropatches, of an intermediate dielectric layer/patches, and of a homogeneous lower layer, acting as a reflector [63]. Such a construction allows to generate a strong localized surface plasmon resonance [63]. This resonance has a magnetic nature [119], and each metal-insulator-metal stack in such an array can be considered as a magnetic "atom" [120, 121].

Although different modes can be excited in a MIM structure [122], this work focuses on the fundamental resonance of MIM: magnetic dipole mode resonance [119]. Having no cutoff, this resonance allows to create ultra small resonators, whose volumes are much lower than the diffraction limit $(\lambda/2n)^3$ [119]. Additionally, MIM resonators provide a strong field enhancement [123, 124]. Due to their high performance [63] and tunability [119], MIM structures have already been studied in a wide variety of contexts [63, 117], such as enhanced light emission [125], light absorption [126, 127, 128], thermal IR sensors [129, 130], mechanical thermal sensors [77], sensors of refractive index [131, 64], optical switches [132], color pixels [133], surface-enhanced spectroscopy [134], solar cells [66], and gas sensing [78, 135].

In particular, due to their magnetic response, arrays of MIM resonators are effective for applications where high absorption is required [119]. This property can be useful for a new generation of bolometer structures. However, traditional MIM structures with a thick continuous metal layer (few skin depths) have too high thermal capacitance. Thus, to decrease the metal

volume, the lower metal layer must also be structured (in the same way as the upper metal layer). On the other hand, such a structure can absorb a maximum of 50 % of the incident radiation [87, 136]. The use of a mirror below the structure can allow this limit to be exceeded, by eliminating the transmitted flux. The MIM structures can then interact effectively with the incident radiation and absorb it completely.

Configurations with a mirror are usual for currently manufactured micro-bolometers. In this case, a homogeneous absorbing layer is placed at the distance $\lambda/4$ above the mirror that corresponds to one of the anti-nodes of the electric field (see Section 5.4.1 about the standing waves). To keep its fabrication process unchanged, our industrial partner wishes to position the MIM absorbers at the same distance from the mirror as in the case of bulk absorbers. Thus, the MIM structure is located in a node of the magnetic field (see Section 5.4.1). Since the MIM mode behaves like a magnetic meta-atom, it is not coupled to the standing wave. In this chapter the discussion is centered on the question: is it possible to design a MIM resonator allowing a critical coupling with a standing wave when it is placed in a node of its magnetic field?

This Chapter begins with the description of the fundamental MIM mode and its main properties (Section 6.2). In the following sections, the complex resonators, consisting of all-structured-MIMs (i.e. the three layers M, I and M are structured in the same way) and a perfect mirror, are studied. The conditions necessary to obtain complete absorption for this complex resonator are discussed. First, the simple case of 1D periodic arrays of infinite MIM in the other direction, is studied in Section 6.3. Before considering systems at the distance $\lambda_{res}/4$ above the mirror, a MIM array placed on a perfect mirror is considered (Section 6.3). In this case the funneling mechanism is based on the magnetic field of the standing wave. Thus, being moved to the node of the magnetic field (at $\lambda_{res}/4$ above the mirror), this MIM array demonstrates a weak response (Section 6.3.3).

However, as shown in Section 6.3.4, the 1D periodic MIM array at the distance $\lambda_{res}/4$ above the mirror can be critically coupled for sufficiently thick dielectric layers. This is a more complex system compared to the case discussed above, because it includes two resonators: an array of MIM structures and a cavity between the MIM structures and the mirror. The funneling mechanism in this case is different from that observed for the MIM mode: it is based on the magnetic field of the mode instead of the magnetic field of the standing wave. The critical coupling conditions (i.e. the balance between the incoming energy and the absorption) are reached by two design rules: the thickness of the dielectric layer must be increased as either the

period increases or the thickness of the metal is reduced. It is therefore possible to use 1D MIM structures located at $\lambda/4$ above a mirror to fully absorb the incident radiation, but this type of structure does not make it possible to significantly reduce the volume density of the absorbers.

In order to overcome this limitation, 2D periodic MIM arrays are considered in Section 6.4. Limiting these structures in the direction in which 1D MIMs were invariant provides a new control parameter – width. In Section 6.4.2 four simple geometries of 2D periodic MIM are studied. It is shown that, for a given metal thickness, the width of the structure defines the optical losses in the metal thus the total response of the structure. Optimal structures for absorption application is then defined. In addition, the excited modes of 2D MIMs is discussed. They differ significantly from those of 1D MIMs because edge effects modify the distribution of the field, and new components of the magnetic field are excited, corresponding to a more complex distribution of the current density.

The response of the structures made of four different metals is discussed in Section 6.4.3. It is shown that the distribution of the electric current responsible for the optical losses in the metal and thus for the critical coupling is defined as the complex balance between skin depth and conductivity. Gold is defined as the metal allowing the design of the structure of smaller total volume and lower thermal capacity. It is, among the metals studied here, the best candidate.

The critical coupling for the 2D periodic MIM array placed at the distance $\lambda_{res}/4$ above the mirror is considered in detail in Section 6.4.4. It is shown that in this case the coupling is provided by the electric field of the standing wave and the magnetic field of the mode. In this way, a significant part of the energy comes from above and below the structure and is absorbed at the air-metal interface. Besides, a flow of energy through lateral planes appears. Thus, the electric current density has a more complex distribution than that of 1D MIM. This also leads to a change in the character of the critical coupling curve. The l/p ratio increases with dielectric thickness, and for thinner metal layers the curve shifts to the left, towards smaller dielectric thickness, which is opposite to the 1D MIM. As a result, critical coupling conditions have been achieved for a metal thickness of 5 nm and a reduced volume.

In Section 6.4.5 two geometries of 2D MIM resonators are compared under critical coupling conditions: rectangle-shaped structures for a response sensitive to polarization and cross-shaped for a response insensitive to polarization. Although they have a similar response for linear polarisation, the observed differences can be explained by modeling the cross-shaped antennas with two $\lambda/4$ antennas separated by a conductive plane, instead of one $\lambda/2$ antenna for the

rectangular antenna. Finally, in Section 6.4.6 it is shown that indeed, due to different coupling process, 2D MIM arrays have ten times less absorber densities than 1D MIMs.

6.2.

The MIM mode

In this Section the basic principles of the fundamental MIM mode, having the magnetic dipole resonance, are considered.

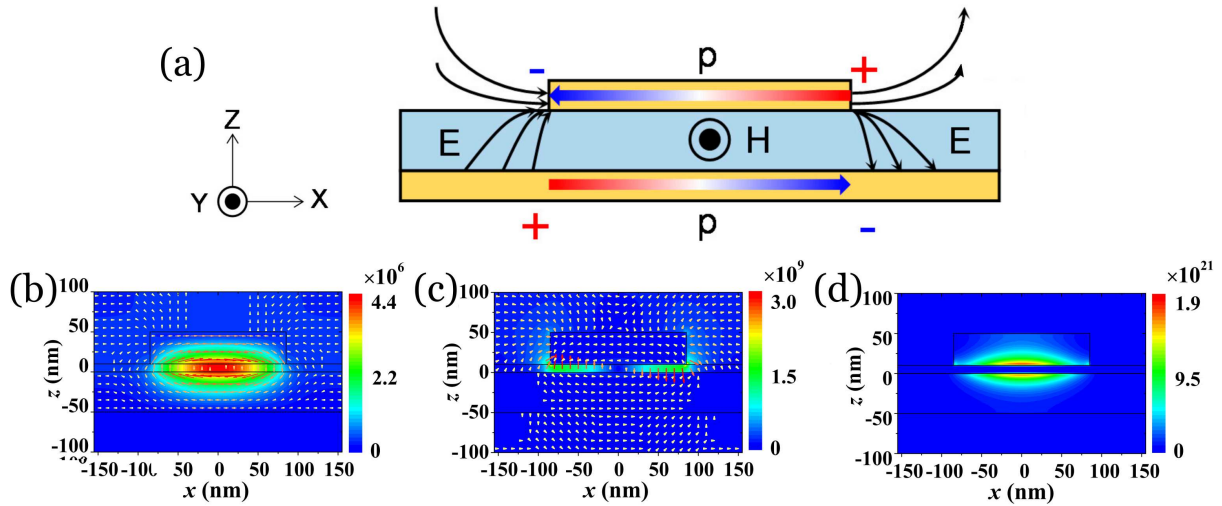


Figure 6.1: An illustration for the operating principle of MIM structure. The letter E stands for the electric displacement vector, H for the magnetic field, and p stands for the electrical current. (b-d) Calculated results: the magnetic field map (H_y), (c) the electric field map (E_x and E_z), arrows stand for the electric field polarization, and (d) the power dissipation. (a) is reprinted from [63], (b-d) from [126].

MIM structures contain three layers, as it is shown in Fig. 6.1 (a). Usually the bottom metal layer is a flat plane. The middle layer is a dielectric spacer, which can be structured or not, and the top layer is periodic metal patches. These patches can have different shapes (rectangular, ellipse, etc.). It can be a single patch [137], or a periodic in one or two dimensions array. In the general case, this does not particularly affect the mode excited in such a structure, as it will be shown below. The thickness of metal layers can be different compare the metal skin depth, however often it is supposed to be more than twice thicker than the skin depth for the operation wavelength. The dielectric thickness can be more than 50 times thinner that the operation wavelength corresponding to a strong mode confinement [63].

If the geometrical and materials parameters are matched correctly, an incident wave is coupled with the structure. In this case, the schematic representation of the excited mode is presented in Fig. 6.1 (a). When the electric field of the incident wave has the E_x component,

between the edges of the top metal patch the dipole electric resonance establishes. In both metal layers a pair of anti-parallel currents is induced, and strong magnetic field is excited inside the resonator.

The fields characteristics obtained in [126] from numerical simulations for a MIM structure with periodic square patches as the top metal are presented in Fig. 6.1 (b) - (d). In Fig. 6.1 (b) the amplitude H_y of the magnetic field is presented with color map, and arrows show the electric displacement field. It can be seen clearly that the stronger magnetic field is excited in the center of resonator, and electric field behavior is dipole-like, with the high amplitude of the electric displacement in metal on the inner side of the resonator. The amplitudes E_x and E_z of the electric field presented in the color map In Fig. 6.1 (c), and the electric field polarisation is presented by arrows. The stronger electric field of the mode is excited in the dielectric closer to the edges of the resonator, with the largest amplitude at the edges of the resonator. Besides, the electric field vectors are oppositely directed at the edges of the structure. Finally, Fig. 6.1 (d) represents resistive heating. This heating occurs in metal layers due to induced currents. The map of electric current repeats one for heating, and corresponds to the magnetic field map. The strongest electrical current is induced at the inner metal-dielectric interfaces in the center of MIM.

The resonant wavelength for the fundamental MIM mode is determined by the size of the metal patch and the effective index of the mode. The last depends on the dielectric permittivity of the insulator (gap) material and its thickness [118, 94]. This mode is also characterized by high absorption for the wide range of the incident angles (almost independent) at the resonant wavelength [118].

6.3.

1D MIM antennas for bolometer applications

6.3.1 Introduction

In this Section, the total absorption conditions of the complex resonator consisting of 1D-periodic MIM arrays and a perfect mirror are studied. A schematic presentation of the 1D MIM is shown in Fig. 6.2 (the mirror is not presented). These structures are periodic in one of the spatial directions (x), and invariant in another (y). Unlike common MIM geometry, the bottom metal

layer is structured in the same way that the two other layers. In the case of invariant structure, the MIM mode can be excited by p -polarized radiation, i.e. by the plane wave with the electric field in xOz plane and the magnetic field along y for the structure presented in Fig. 6.2. The presence of the mirror guarantees that there is no transmitted flux, and the structure can absorb all incident electromagnetic radiation.

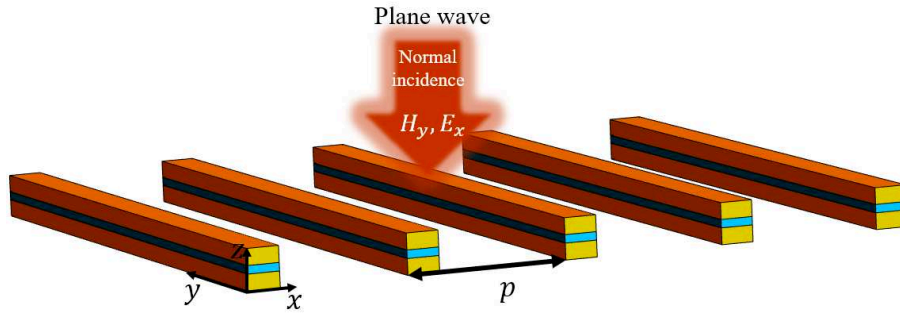


Figure 6.2: A periodic array of MIM antennas translation invariant along the y -direction. All three layers repeat the same pattern.

In this work, the interaction of the electromagnetic field with plasmonic structures is studied by means of analytical models and numerical simulations. The latter were performed with COMSOL Multiphysics, a commercially available finite-element-method software. For the sake of example, here and in what follows geometrical parameters of MIM structures are optimized for the wavelength of $10\ \mu m$. However, the optimization can be done in the same way at any wavelength.

Gold and silicon were selected as materials for the studied MIM structures. But, as it will be demonstrated below, silver, copper and aluminum are suitable for absorption applications as well (Section 6.4.3). In other words, to obtain results similar to those presented in this work, the metal must be characterized by a permittivity having a large and negative real part and an imaginary part of the order of 10^3 .

Silicon was chosen due to its high optical index, because it affects the effective index of the mode and, therefore, the length of the resonator for a given wavelength. This is indeed an important point since we are looking for the smallest resonator. As the main aim of this chapter is to understand the behaviour of the resonant structures, it is desirable to work on a simplify model. Thus, the imaginary part of the silicon is neglected. In this way, the observed absorption arise only due to the metal. To simplify the optimization process and to separate the influence of geometrical parameters and material properties, dielectric permittivity values are considered constant and equal $\varepsilon = -3792 + i825$ for gold [138] and $\varepsilon = 11.56$ for silicon [139].

The skin depth for chosen metal parameters is about 25.7 nm accordingly to the equation [140]

$$\delta = \lambda / (2\pi\kappa). \quad (6.1)$$

Here λ is the wavelength, and the absorption coefficient κ is connected with the complex dielectric function $\varepsilon = \varepsilon_1 + i\varepsilon_2$ as $n_r + i\kappa = \sqrt{\varepsilon_1 + i\varepsilon_2}$. It can be expressed as [140]

$$\kappa = \sqrt{-\frac{\varepsilon_1}{2} + \frac{1}{2}\sqrt{\varepsilon_1^2 + \varepsilon_2^2}}. \quad (6.2)$$

Computed skin depth value corresponds to the gold skin depth values for this wavelength region from the literature [141, 142].

Here and below, the absorption spectra are computed from the Scattering parameters (S-parameters). For high frequencies range, scattering parameters are defined in terms of the electric field on the port [143]. The detailed explanation is provided in Appendix F, Section F.2. Here let us only indicate that the time-average reflection coefficient is computed on the port, which create excitation, and calculated as $R = Abs(S_{11})^2$, where S_{11} is the complex-valued normalized amplitude, corresponding to the reflected wave, detected by the port. If there is no mirror in the system, the time-average transmission coefficient is computed as $T = Abs(S_{21})^2$, where S_{21} is the complex-valued normalized amplitude, corresponding to the electromagnetic energy, transmitted from the Port 1 to the Port 2, for a given wavelength.

The case when for some wavelength the absorption peak appears for a given structure is called the coupling. The geometrical parameters, for which the full absorption is achieved at this wavelength, are called the critical coupling conditions. Under the critical coupling, the energy of an incident wave is totally absorbed by the structure. In this case, the non-radiative and radiative damping rates are perfectly matched [144].

The rest of this Section is organized as follows. At first, the configuration when a MIM array is placed on the mirror, i.e. in the anti-node of magnetic field, is considered in Section 6.3.2. This is the classical way to excite the MIM mode and achieve the critical coupling conditions. It is shown that in this configuration the funneling process is provided by an external magnetic field. Thus, when such a structure is placed in a magnetic field node, it has a much weaker response (Section 6.3.3).

However, in Section 6.3.4 it is demonstrated that the 1D periodic array of MIMs placed at

the distance $\lambda/4$ above the mirror can be optimized and critically coupled. In this case, the excited mode looks similar to the MIM mode, but the funneling process changes. This leads to unexpected behaviour when parameters of the structure change. The thickness of the dielectric layer at the critical coupling increases when the metal thickness decreases (for the constant incident flux). This behaviour is considered in detail, and a basic explanation is provided. Finally, the volume density of 1D MIM absorbers at $\lambda_{res}/4$ above the mirror and some of their optical properties are presented.

6.3.2 An array of 1D MIM antennas on the mirror

In this Section, the critical coupling for an array of the symmetrical 1D MIM antennas, placed on the perfect mirror, is discussed. The procedure for finding the geometrical parameters corresponding to the total absorption of the incident flux in this system is provided. It is demonstrated that this structure exhibits the MIM mode described above. After that, the nature of the energy funneling, connected with the external magnetic field, is demonstrated.

The critical coupling

As the MIM-mode resonance has a magnetic nature, it is worth starting with a case where the antennas array is placed in the region of the strong magnetic field. Let us consider an incident plane wave at normal incidence $\exp(-ikz - i\omega t)$. The wave is linearly-polarized with the magnetic field orientated along the translation-invariant direction of the structure (y -axis). As the array is invariant in the y direction, it is enough to consider a 2D model with the periodic boundary conditions (the Floquet periodicity) on the sides (Fig. 6.3 (a)).

To reach the critical coupling conditions, the following procedure is applied. There are a few parameters for optimizing a 1D MIM structure (assuming that materials permittivities are fixed): the metal thickness and the thickness of the dielectric layer, the length of the structure and the period of the array. At first, we fix all of the antenna parameters except the structure length l which is adjusted to reach the desired resonant wavelength (as mentioned in Section 5.2, the length defines the resonant wavelength). The example of the influence of the structure length on the resonant wavelength is presented in Fig. 6.3 (b): a longer length corresponds to a longer resonant wavelength. At this stage, the absorption can take any value, only the peak position is important. The next step is to vary the period value until the maximum absorption is reached. Then, it may be necessary to slightly change the length (and the period again) to achieve the

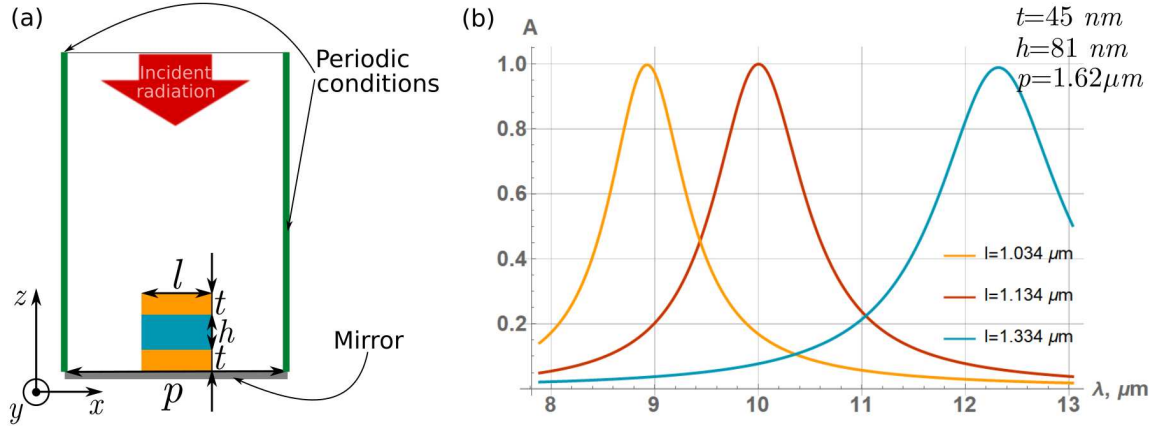


Figure 6.3: 1D MIM antenna deposited on the perfect mirror. (a) A schematic presentation of the numerical model for the simulated array of 1D MIM. A periodicity in x direction is described with periodic conditions on the left and right sides (green lines). The perfect mirror at the bottom is presented in gray. (b) The spectral response of systems with 1D MIM from the picture (a), having the same parameters except for the length of the antenna, in the case of the normal incidence. The parameters: $t = 45 \text{ nm}$, $h = 81 \text{ nm}$, and $p = 1.62 \mu\text{m}$. The length $l = 1.134 \mu\text{m}$ corresponds to the resonant wavelength $10 \mu\text{m}$.

desired value of the central wavelength. If the other parameters (the metal and the dielectric spacer thicknesses) were selected fortunately, the critical coupling will be achieved. Otherwise, for finding critical coupling parameters, these thickness values should be changed one after another, and the length and period should be corrected again.

For example, for an array of 1D MIM with the 35 nm dielectric thickness and the 45 nm metal thickness, the maximum achievable absorption is about 0.9 for the resonant wavelength $10 \mu\text{m}$. In this case, the length of the structure is $0.917 \mu\text{m}$ and the period is $0.98 \mu\text{m}$. Further reduction of the period is necessary to achieve the critical coupling conditions. In this case, the procedure leads to an unrealistic solution where the period is smaller than the MIM length. However, if the dielectric thickness is increased, for example, for 81 nm , as in Fig. 6.3 (b), the critical coupling (red curve) conditions can be found. In this case, the length of the structure is $1.134 \mu\text{m}$ and the period is $1.62 \mu\text{m}$.

The physical behaviour of this system at the resonant wavelength $10 \mu\text{m}$ is close to a classical MIM structure. For a wide range of incident angles, absorption value stays higher than 0.9 (Fig. 6.4 (b)). As it is shown in Fig. 6.4 (c) and (d), both the electric and magnetic field norms are similar to the classic MIM mode, described in Fig. 6.1. The maximum of the electric field is close to the side edges, while the maximum of the magnetic field is at the center of the structure. In the top part of the figure, the distribution of the Poynting vector is uniform (as expected from a plane wave), while near the structure it is strongly perturbed: arrows point towards the ends of the structure. As it can be seen in the distribution of arrows, the absorption for the

considered structure occurs inside the MIM structure, since the long arrows, presented inside the structure, are pointed towards the metal, but they are absent in the corresponding area above the metal layer.

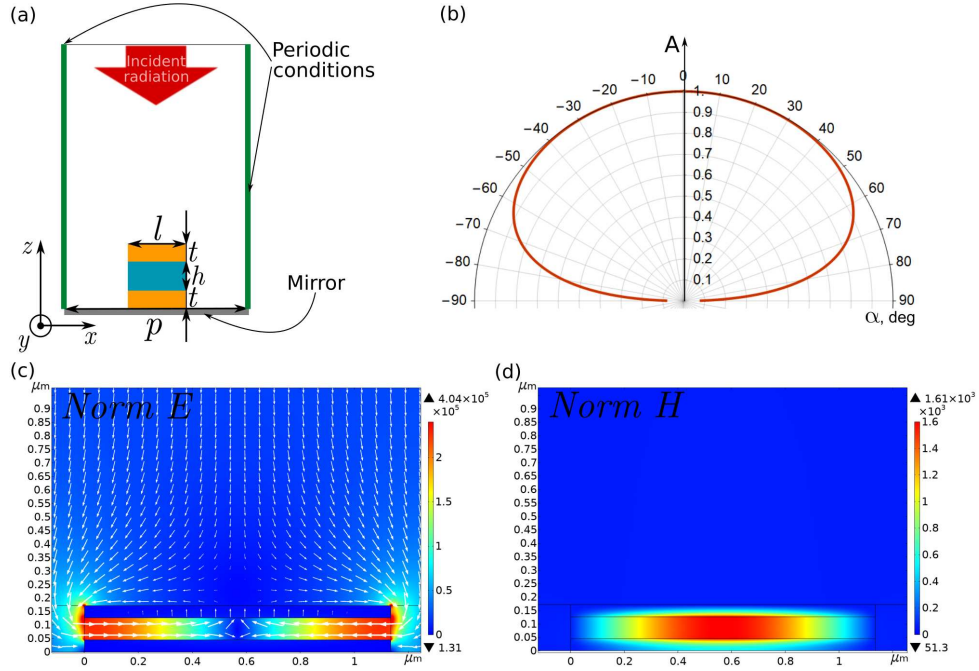


Figure 6.4: The physical behaviour of the 1D MIM structure with the parameters $p = 1.62 \mu m$, $l = 1.134 \mu m$, $t = 45 nm$, $h = 81 nm$ at the resonant wavelength $10 \mu m$. (a) A schematic presentation of the numerical model. (b) The angular dependence of the absorption at the resonant wavelength. The angle is measured from the normal. The color maps of the norm of the electric (c) and the norm of the magnetic (d) fields with the minimum value in blue, and the maximum value in red. The maximum and minimum values are written at the top and the bottom of the scale bar near the black triangles. The arrow map in picture (c) is the map of the time average Poynting vector, where the arrow lengths are proportional to the log of their amplitude.

The funneling mechanism

The energy funneling process can be explained in the following way. The field around the structure can be described as the sum of the external field, i.e. the standing wave, and the scattered field, called the field of the mode in the region close to the structure. This separation will help to indicate the nature of the funneling process. For this study, another numerical model (using Comsol Multiphysics) was created. In this model, two MIM structures are placed in the field of a standing wave, such as it results from the interference between the incident wave and the wave reflected by the mirror (assumed to be perfect). The electric and magnetic fields of the standing wave are shown in Fig. 6.4 (a) and (b), correspondingly. The bottom of the top MIM is located at a node of the electric field of the standing wave, thus at an anti-node of its magnetic field. This case is identical to that of the structure on the mirror, described above.

But it allows to effectively separate the field of the standing wave (a plane wave reflected by the mirror) and the scattered field. The periodic boundary conditions are applied on the sides (thus, a periodic array is studied), and the system is studied at the normal incidence. The fields of the mode are presented in Fig. 6.4 (c) and (d), they correspond to the MIM mode.¹

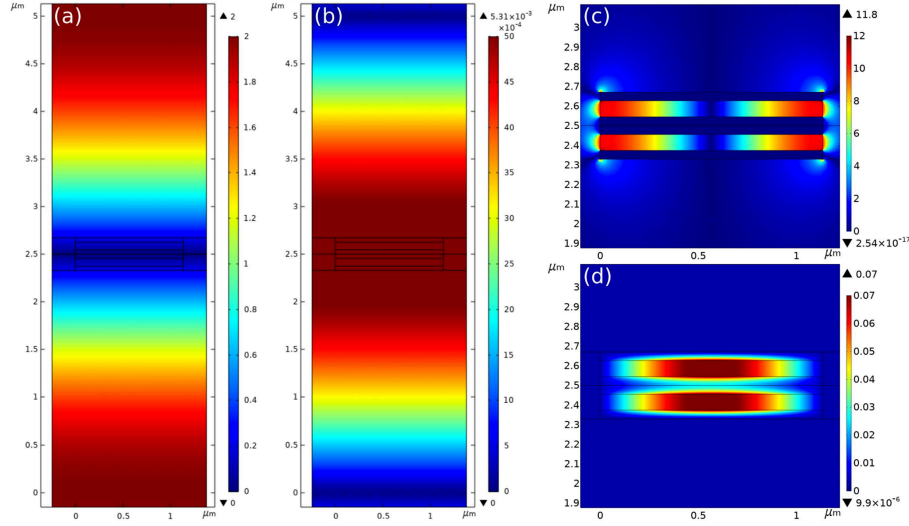


Figure 6.5: Explaining of the funneling process. The total field can be broken down into two components: the standing wave field and the scattered field. Thus, instead of the mirror, two MIM structures are placed at the node of the electric field (and the anti-node of the magnetic field). The norm of (a) the electric field $\mathbf{E}_w = E_w \mathbf{x}$ and (b) the magnetic field $H_w = H_w \mathbf{y}$ of standing wave. The norm of (c) the electric field E_m and (d) the magnetic field H_m of the scattered field for the structure with parameters $p = 1.62 \mu$, $l = 1.134 \mu\text{m}$, $t = 45 \text{ nm}$, $h = 81 \text{ nm}$ at the resonant wavelength $10 \mu\text{m}$.

Then, the time average Poynting vector around the structure can be presented as the sum of four terms, corresponding to field separation of the standing wave and the mode, and their mix:

$$\mathbf{S} = \mathbf{E}_m \times \mathbf{H}_m^* + \mathbf{E}_m \times \mathbf{H}_w^* + \mathbf{E}_w \times \mathbf{H}_m^* + \mathbf{E}_w \times \mathbf{H}_w^*. \quad (6.3)$$

Here the index w corresponds to the field of the standing wave, and the index m to the field of the mode. At first, the last term of the sum in Eq. 6.3 is zero, as it refers to the standing wave, which does not carry any energy.

Three other components of the sum in Eq. 6.3 are presented in Fig. 6.6. Color maps there present the absolute value of the corresponding Poynting vector, and arrows indicate its direction and value as well (in the logarithmic scale). The top MIM structure is discussed, as for the bottom one all processes are the same, but in the opposite direction relative to the vertical direction. The same color scale is chosen for all pictures in Fig. 6.6, as a result, the color map

¹Unless otherwise specified, the field maps in this work are maps of a total field. If such a separation takes place, this is indicated as in this paragraph.

for $\mathbf{E}_m \times \mathbf{H}_w^*$ is saturated. This term has its maximum value at the sides of the structure, near the metal corners and at the air-silicon interfaces, decreasing towards the center of the structure. A decrease in the length of the arrows also shows the decay in the amplitude.

It can be seen that $\mathbf{E}_m \times \mathbf{H}_w^*$ (Fig. 6.6 (c)) is the term, responsible for the energy funneling inside the structure. It also has the highest amplitude, as the magnetic field of the standing wave is maximum in this region, and the electric field of the mode provides high value at the MIM edges. As the S_x component of the Poynting vector is responsible for the power that goes into the structure, thus it must be constructed from the magnetic field in $\mathbf{y}$ direction (due to polarisation there is only $\mathbf{y}$ component of the magnetic field), and thus the electric field must be in $\mathbf{z}$ direction. As the standing wave (and the normally incident wave in the initial problem) has only the component in $\mathbf{x}$ direction, E_z can be only provided by the mode.

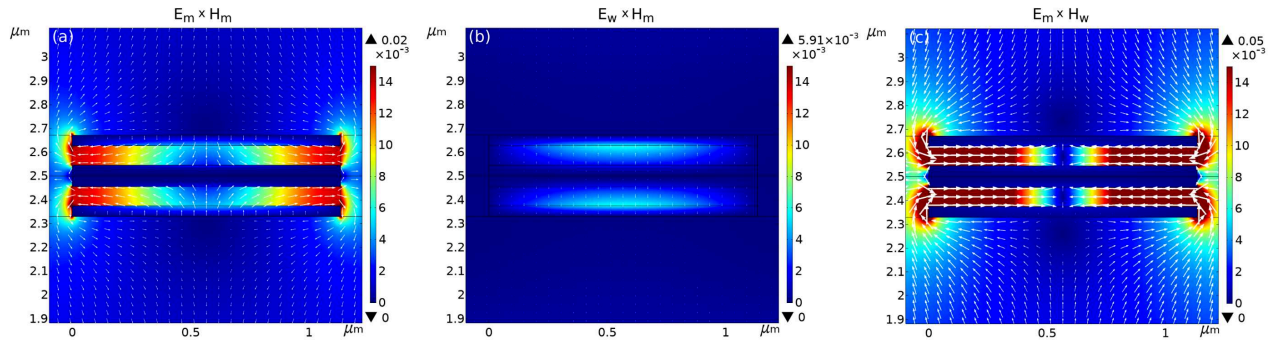


Figure 6.6: The color and arrow maps of the terms of the total Poynting vector: (a) $\mathbf{E}_m \times \mathbf{H}_m^*$, (b) $\mathbf{E}_w \times \mathbf{H}_m^*$, and (c) $\mathbf{E}_m \times \mathbf{H}_w^*$ at the resonant wavelength $10\mu\text{m}$ for the 1D MIM structure having the parameters $p = 1.62\mu$, $l = 1.134\mu\text{m}$, $t = 45\text{nm}$, $h = 81\text{nm}$. The color map displays the norm of the corresponding term. The arrows show its direction, and the length of the arrows corresponds to the vector amplitude in the logarithmic scale. The scales are the same for all three plots.

Meanwhile, the $\mathbf{E}_m \times \mathbf{H}_m^*$ (Fig. 6.6 (a)) term removes some energy from the inner part of the MIM structure, and recycles it along the top border of the structure, providing some absorption in the top layer of metal. It also directs energy to the bottom metal layer inside the structure. The $\mathbf{E}_w \times \mathbf{H}_m^*$ (Fig. 6.6 (b)) term provides energy moving only in the $\mathbf{z}$ direction, since $\mathbf{E}_w$ has only the E_x component. As on the mirror this component is zero, it becomes stronger only near the top metal-dielectric interface, and directs the energy to the top metal layer.

Reaching the critical coupling implies the total absorption of the incoming energy. The absorption mechanism for this mode is as follows. The incoming into resonator energy is directed to metal layers. The electric current is excited on the inner metal-dielectric interfaces of MIM. As it was shown in Fig. 6.1, being proportional to the magnetic field amplitude, the electric current density is strongest in the center of MIM. Thus, according to the Joule's law, this is a

region of strongest absorption.

To summarize, an array of 1D MIM, being placed on the mirror (the anti-node of the magnetic field of the standing wave), demonstrates behaviour similar to the classical MIM mode. The energy funneling in this case is provided by the magnetic field of the standing wave, what is typical for such artificial meta-atoms.

6.3.3 1D MIM array distanced from the mirror

In this Section, an introduction of a more complicated system, an infinite 1D periodic array of MIM, placed at some distance above the perfect mirror, is provided. The difference of this system from one, considered in the Section 6.3.2, is discussed, and particular case, when this distance is $\lambda_{res}/4$ above the mirror, is presented.

As it was shown above, 1D MIM antenna array can be critically coupled to the incident wave when located at the anti-node of the magnetic field. In this case, the MIM in the array behaves as the classical MIM structure, presented in Section 6.2.

When an array is moved from the mirror, as shown in Fig. 6.7 (a), the system becomes more complicated. Now it consists of two resonators: a MIM structure and the resonator, formed in between the MIM and the mirror (the Fabry-Pérot resonator). The behaviour of this system changes with the location of the structure with respect to the magnetic field. In the anti-nodes of the magnetic field, i.e. at distances $n \times \lambda/2$ from the mirror, where n is a natural number, the behavior, similar to one, presented in Section 6.3.2, is expected. The only difference can come from the interaction of the evanescent field with the mirror.

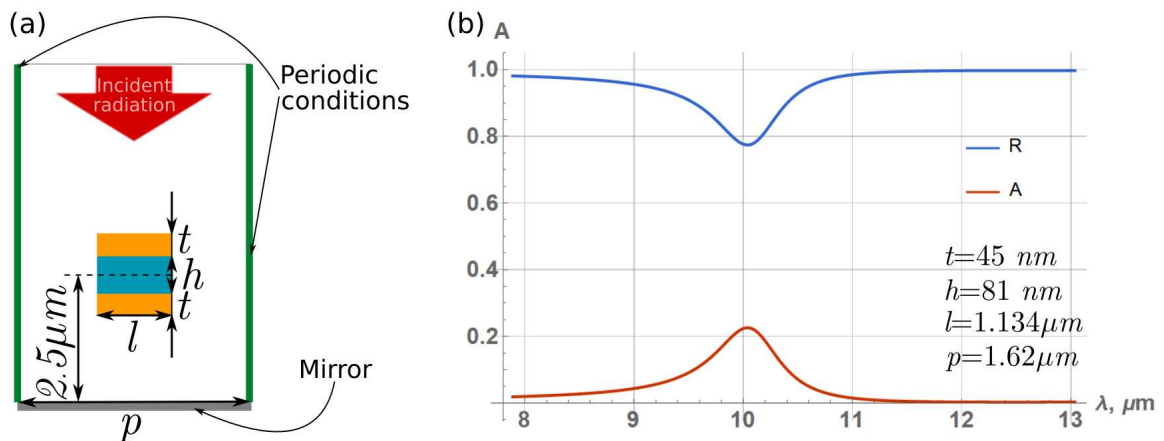


Figure 6.7: The 1D MIM antenna array optimized for the total absorption at $10\mu\text{m}$ being placed on the mirror. (a) The numerical model for the array, placed at the distance $2.5\mu\text{m}$ from the mirror. The periodicity is modeled by means of the periodic boundary condition. (b) The response of the structure, optimized on the mirror at $10\mu\text{m}$, when it is placed at the distance $2.5\mu\text{m}$ above the mirror. The parameters of the structure are specified on the plot.

Regardless of the MIM position, the work of such an absorber differs from classical bulk absorbers. The last show best results at the distance $(2n + 1)\lambda/4$ (with $n \in \mathbb{N}$) above a mirror, since their absorption is proportional to E^2 , and there is an anti-node (i.e. a maximum) of the electric field at these locations (a standing wave). As a result, a bulk absorber and a MIM resonator present completely opposite absorption behaviors: for the former, the absorption is maximum when the electric field is maximum, for the latter – when the magnetic field is maximum.

The first case corresponds to the microbolometer configuration for which the absorption is located into a thin absorbing layer located at $\lambda/4$ above a mirror. Replacing this absorbing layer by MIM antennas would imply placing these antennas at a distance $\lambda/2$ from the mirror. For practical reasons (i.e. keeping the same fabrication process), our industrial partner demanded to keep the same distance from the mirror as in the case of bulk absorbers. **In this way, we approached the key question of this chapter: is it possible to design MIM resonators showing a full absorption being located in the region where the magnetic field of the stationary wave is equal to zero?**

If a MIM structure, optimized on the mirror, is moved at a distance $\lambda_{res}/4$ from a mirror, the structure becomes strongly undercoupled. The magnetic field of the standing wave $\mathbf{H}_w$, which was responsible for the funneling on the mirror, is zero or almost zero in this region. Thus, the funnel effect is no longer effective for this structure. Indeed, Fig. 6.7 (b) shows that the structure optimized on the mirror (from Fig. 6.3 (b)) produces a weak response being placed at the distance $\lambda/4$ above the mirror. The small absorption peak can be explained only due to the magnetic field of the mode. Thus, for the structure placed at $\lambda/4$ from the mirror, the magnetic field of the mode should be tuned.

Let us consider the dependence of the MIM absorption as a function of its distance to the mirror. The optical response of a complex resonator, consisting of the 1D MIM array and a cavity between the array and the mirror, has been computed as a function of the distance d between the MIM antenna and the mirror by the analytical method employing S parameters (see Section F.3, Appendix F). The absorption, presented in Fig. 6.8 (b), demonstrates a periodic behavior with a period equal to $\lambda/2$. As expected from the above discussion, absorption maxima are observed for the distances d from the mirror which are close to $n \times \lambda/2$, i.e. at the locations of the anti-nodes of the magnetic field of the standing wave. In the same way, the minima of absorption are located at distances $d \approx (2k + 1) \times \lambda/4$, i.e. at the locations of the nodes of the

magnetic field.

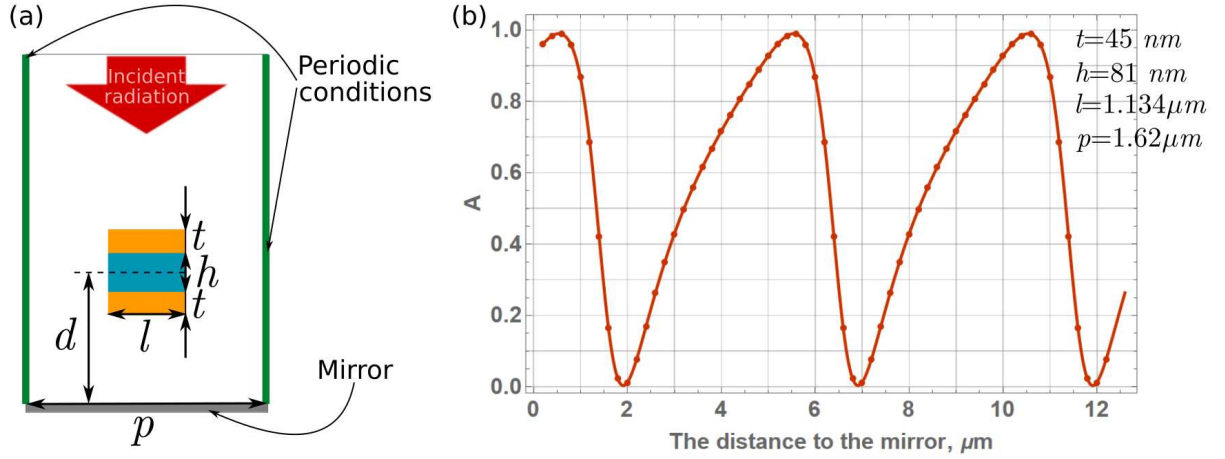


Figure 6.8: (a) A schematic presentation of the numerical model. (b) Absorption at the wavelength $10 \mu\text{m}$ in dependence on the distance d to the mirror for the array of 1D MIM antenna optimized for the total absorption at this wavelength being placed on the mirror. The solid line is for the analytical computation with S-parameters, and the dots are for the COMSOL simulations.

However, the observed maximum of absorption (about 0.98) is a bit less than 1. This difference comes from the fact that the MIM on the mirror benefits from the additional contributions of the evanescent waves reflected by the mirror. This is not the case for the distanced array because of the attenuation of the evanescent waves. Consequently, to reach the critical coupling conditions, the MIM antenna must be designed in two different ways depending on whether they are on the mirror or far away (i.e at a long distance compared to the decay of the evanescent waves). In the second case, it should be optimized directly at the chosen distance. The nature of the small shift in the maximum and minimum position with respect to $d = \lambda/2$ and $d = \lambda/4$, respectively, may be due to the contribution of evanescent waves to the optimization of the structure on the mirror too.

To summarize this Section, the behaviour of plasmonic structures is dramatically different from the behavior of bulk absorbers. While homogeneous absorbers demonstrated the best efficiency in the anti-node of the electric field, the MIM resonators are excited in the anti-node of the magnetic field. Being placed in a node of the magnetic field, a MIM structure, optimized on the mirror, demonstrated a weak absorption value. The funneling process in this case arises from only the magnetic field of the mode. Consequently, the best way to reach a better coupling at the distance $\lambda_{res}/4$ above the mirror is to optimize the structure at this distance. The next Section addresses the key question for this chapter: is it possible to achieve the critical coupling for an array of 1D MIM structures located at $\lambda_{res}/4$ from a perfect mirror?

6.3.4 Optimization of an 1D MIM array at $\lambda/4$ above the mirror

In this Section, it is demonstrated that 1D MIM structures can be critically coupled at the distance $\lambda/4$ above the perfect mirror. This is a more complex system compare to the case discussed in Section 6.3.2, since it includes two resonators: an array of MIM structures and a cavity formed by the MIM array and the mirror. As in this case the array is placed in the node of the magnetic field, the energy funneling inside the structure is provided by the magnetic field of the mode. Due to change in the energy funneling, the parameters for the critical coupling are changed too: for the same metal thickness and the period values, the thickness of the dielectric layer at the critical coupling is a few times higher compare to the case of 1D MIM on the mirror. It is also shown that the decrease of the thickness of the metal layer shifts the critical coupling curve to the right, towards higher dielectric thicknesses, due to the increase in the resistance of the metal layer. It is found that at a constant value of the period, the critical coupling is maintained for thinner metal layers when the product of the effective metal thickness and the thickness of the dielectric layer $t_{\text{eff}} \times h$ is kept constant. In the end of the Section, the volume density of the absorbers and some optical properties of 1D MIM structures at the distance $\lambda/4$ above the mirror are presented.

The vacuum wavelength $\lambda_{\text{res}} = 10 \mu\text{m}$ is chosen again as the central wavelength for the optimization, thus the array of the studied MIM structures is placed at the distance $d = 2.5 \mu\text{m}$ from the mirror.

The critical coupled 1D MIM at $\lambda/4$ above the mirror

The schematic presentation of the system used for computations is shown in Fig. 6.9 (a). As it was mentioned in the previous section, for an effective optimization, parameters of the structure should be adjusted at the fixed distance above the mirror. Fig. 6.9 (b) shows that the MIM length determines the resonant wavelength in the same way as it was describe above. The critical coupling conditions at λ_{res} for the structure of the metal thickness $t = 45 \text{ nm}$ were found for the following parameters: $l = 1.317 \mu\text{m}$, $p = 1.883 \mu\text{m}$, and $h = 314 \text{ nm}$. It can be seen that for the same ratio $l/p = 0.7$ the thickness of the dielectric is almost four times higher compare to the case of the array on the mirror. To understand this difference and the coupling mechanism for the structure placed at $\lambda_{\text{res}}/4$ above the mirror, let us consider the behaviour of this structure at the resonant wavelength.

The field maps and the currents distribution at the critical coupling for the 1D MIM array at

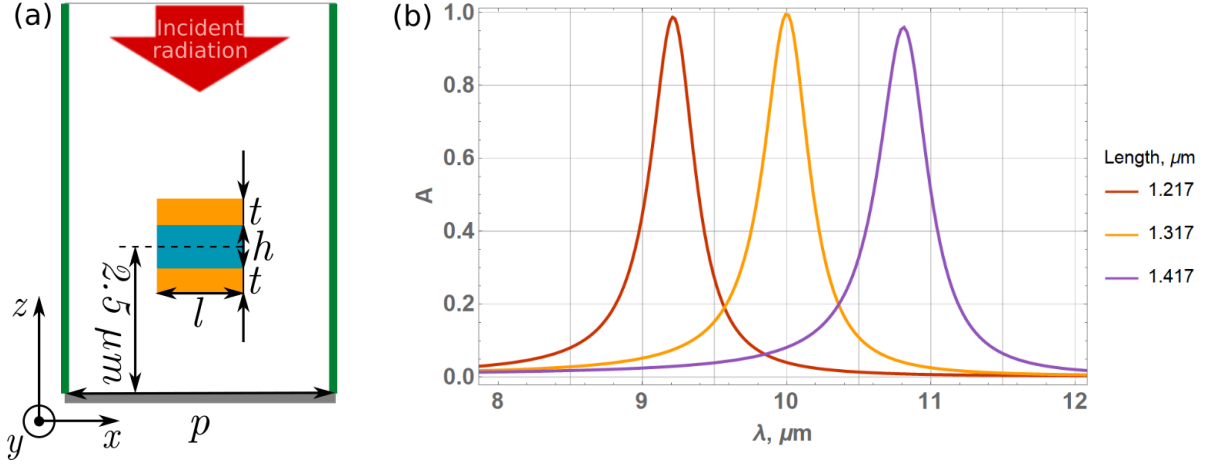


Figure 6.9: (a) A schematic presentation of the simulation model of a 1D MIM structure at the distance $\lambda_{res}/4$ above the mirror. (b) The absorption spectra for the structures with different parameters of the length while other parameters are fixed: $p = 1.883 \mu\text{m}$, $t = 45 \text{ nm}$, $h = 314 \text{ nm}$.

the distance $\lambda_{res}/4$ above the mirror (the yellow curve from Fig 6.9) are presented in Fig. 6.10. As it can be seen, the observed mode looks similar to the classical MIM mode: the E_z component is maximum close to the edges and equal to zero at the center, and the magnetic field has its maximum at the center of the structure. However, unlike the case of the classical MIM mode, the norm of the electric field is not symmetrical with respect to the horizontal line in the middle of the structure.

The Poynting vector is depicted in Fig. 6.10 (a) by the arrow map (in logarithmic scale). It shows different behavior in the upper and lower parts of the structure. This asymmetry is explained in detail below. Since at the critical coupling there is no reflection, there is only the incident plane wave in the top air region, and the Poynting vector is presented by the uniform array directed along z . Below the structure the standing wave still exists, thus there is no energy flux.

Inside the MIM structure, the Poynting vector is symmetric. It is directed towards the centers of metallic parts in the middle region of the structure, and its vertical component is zero on the external horizontal interfaces. This means that the strongest absorption occurs at the center of the metallic parts. It is validated by the current distribution, shown in Fig. 6.10 (d): the strongest electric current density is found at the inner side in the center of the metallic layers. This is the typical MIM-mode behaviour as well.

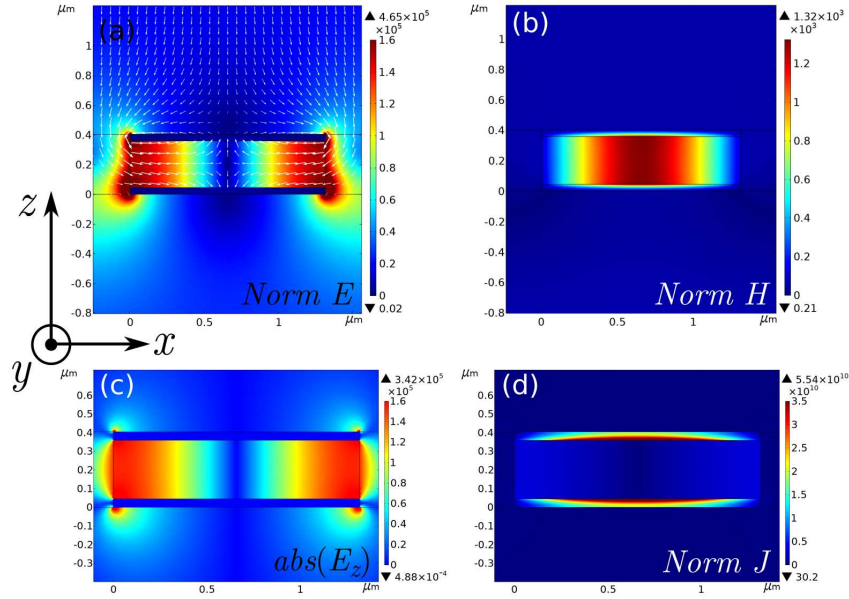


Figure 6.10: The field maps for the 1D MIM structure having the parameters $p = 1.883 \mu\text{m}$, $l = 1.317 \mu\text{m}$, $t = 45 \text{ nm}$, and $h = 314 \text{ nm}$ at the resonant wavelength of $10 \mu\text{m}$ placed at the distance $\lambda_{\text{res}}/4$ from the mirror: (a) the color map for the norm of the total electric field and the arrow map for the time average Poynting vector (in the logarithmic scale); (b) the norm of the total magnetic field; (c) the absolute value of z -component of the electric field; (d) the norm of the electric current density.

The funneling mechanism

To understand the funneling mechanism, let us again separate the field components from the standing wave (occurring as a result of the reflection of the incident wave on the perfect mirror in the initial model) and from the scattered field. There is no contradiction in considering the background field as a standing wave for the critical coupling conditions. In this approach the scattered far field cancels the reflected wave, thus no reflection can be detected.

To model the studied system, two identical 1D MIM arrays (an array is introduced by the periodic conditions) are placed at the distance $\lambda_{\text{res}}/2$, measured between the centers of the MIM structures. The centers of the MIM structures are located in the opposite anti-nodes of electric field (same amplitude but opposite signs) of the standing wave (Fig. 6.11 (a)), thus, also in nodes of the magnetic field Fig. 6.11 (b)). Therefore, this model completely corresponds to the case of the 1D MIM array at the distance $\lambda_{\text{res}}/4$ above the mirror. The field of the mode, presented in Fig. 6.11 (c) and (d), are similar to the MIM mode.

Using this field separation, let us consider each term of the Poynting vector from the decomposition of Eq. 6.3. It can be seen from Fig. 6.12 that the behavior of the first three terms differs from the case of the MIM array on the mirror (the last term does not provide energy move as it corresponds to the standing wave). Oppositely to the previous case, now $\mathbf{E}_m \times \mathbf{H}_m^*$ (Eq. 6.3

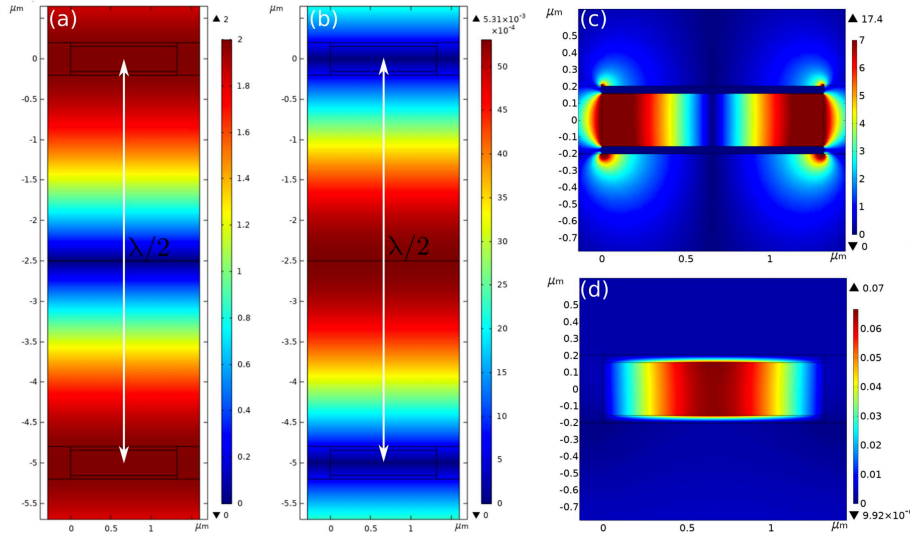


Figure 6.11: To the explanation of the funneling process. The total field can be separated into the field of the standing wave and the scattered field. Thus, instead of the mirror, two MIM structures placed at the anti-nodes of the electric field (and the nodes of the magnetic field). The norm of (a) the electric field $\mathbf{E}_w = E_w \mathbf{x}$ and (b) the magnetic field $\mathbf{H}_w = H_w \mathbf{y}$ of standing wave. The norm of (c) the electric field $\mathbf{E}_m$ and (d) the magnetic field $\mathbf{H}_m$ of the scattered field for the structure with parameters $p = 1.883 \mu\text{m}$, $l = 1.317 \mu\text{m}$, $t = 45 \text{ nm}$, and $h = 314 \text{ nm}$ at the resonant wavelength $10 \mu\text{m}$.

(a)) is the main term responsible for the energy funneling into the structure. The other term having the Poynting direction mainly along the $\mathbf{x}$ -axis, is $\mathbf{E}_m \times \mathbf{H}_w^*$ (Eq. 6.3 (c)). Its color map, providing the absolute value of the Poynting vector amplitude, has horizontal and vertical zero values lines in the middle of the structure (Fig. 6.12 (c)). These lines correspond to the zero line of the magnetic field $\mathbf{H}_w$ (Fig. 6.11 (b)) and the electric field $\mathbf{E}_m$ (Fig. 6.11 (c)). In the upper part of the structure, $\mathbf{E}_m \times \mathbf{H}_w^*$ provides the energy funneling into MIM, while in the bottom part arrows point outwards from the structure. This behavior occurs due to a change in sign of the magnetic field of the standing wave. It was proved by the numerical computation that the integral energy flux for this term through the lateral air-dielectric interface is almost zero. But $\mathbf{E}_m \times \mathbf{H}_w^*$ causes an asymmetry in the total Poynting vector funneling, seen in Fig. 6.10 (a): the longer arrows in the upper part and shorter in the bottom one. Inside the structure, $\mathbf{E}_m \times \mathbf{H}_m^*$ and $\mathbf{E}_w \times \mathbf{H}_m^*$ direct energy to the top and bottom metal layers, correspondingly. This is connected with the strong magnetic field of the mode in this area.

Thus, due to location in the node of the magnetic field of the standing wave fields instead of its anti-node (considered above), in the case, when the array of 1D MIM is placed at the distance $\lambda_{res}/4$ from the mirror, the funneling process is different from the case of the array on the mirror. The MIM array placed above the mirror is at the node of the external magnetic field. Thus, magnetic component for the Poynting vector is mainly provided by the field of the

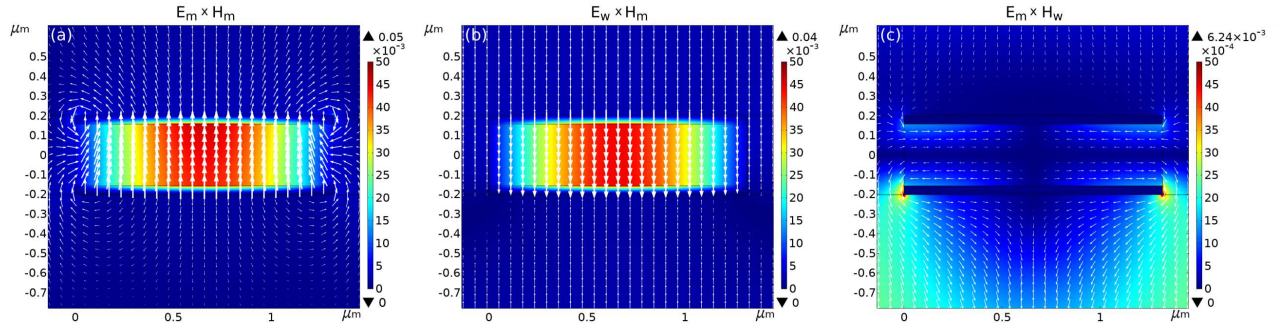


Figure 6.12: The color and arrow maps of the terms of the total Poynting vector: (a) $\mathbf{E}_m \times \mathbf{H}_m^*$, (b) $\mathbf{E}_w \times \mathbf{H}_m^*$, and (c) $\mathbf{E}_m \times \mathbf{H}_w^*$ at the resonant wavelength $10 \mu\text{m}$ for the 1D MIM structure having the parameters $p = 1.883 \mu\text{m}$, $l = 1.317 \mu\text{m}$, $t = 45 \text{ nm}$, and $h = 314 \text{ nm}$. The color map corresponds to the norm of the corresponding term. The arrows show its direction, and the length of the arrows corresponds to the vector amplitude in the logarithmic scale. For the picture (c) the length of arrows is multiplied by 5 compare to (a) and (b).

mode. It was demonstrated that the funneling is provided by the magnetic field of the mode. In the same time, the magnetic field in the center of the structure is connected with absorption, and thus defines the critical coupling. As a result, the magnetic field of the mode defines the both processes. Such behaviour changes the response of the array of 1D MIM at the distance $\lambda_{res}/4$ above the mirror compared to the array placed on the mirror.

The system behaviour

The specific funneling mechanism for the MIM array in the node of the magnetic field introduces an increase in thickness of the dielectric as compared with the MIM deposited on the mirror. To explain this behavior, let us study the critical coupling conditions in dependence on the parameters of the array.

As it was shown before, the length of the MIM defines the resonant wavelength, which is fixed at $10 \mu\text{m}$ in this Chapter. The period influences the coupling: this value defines the incident power, and thus either a system is under-coupled, over-coupled, or at the critical coupling. Thus, there are two parameters left for changes: the thicknesses of the dielectric and metal layers. Let us focus on a few different thicknesses of the metal layers, 15, 45, and 100 nm that correspond to three different cases: thinner, intermediate and thicker than the gold skin depth ($\approx 26 \text{ nm}$). The critical coupling conditions were found for such arrays above the mirror for different dielectric thicknesses. These parameters are presented in dependence on the dielectric thickness in Fig. 6.13, where every point in the plot corresponds to the critical coupling conditions. The tables with these values are presented in Section G.1 of Appendix G. Two main effects were

observed: the decrease of the l/p ratio when the thickness of the dielectric layer is increased, and the shift of the critical coupling curve towards thicker dielectric layers when the metal thickness is decreased.

In Fig. 6.13 (a) the sets of points, corresponding to the critical coupling conditions, are connected by solid interpolation curves. For all metal thicknesses a similar behaviour for l/p in dependence on the dielectric thickness is observed: the ratio l/p increases as the dielectric thickness decreases. To explain this type of behaviour, let us consider how the length and period of the MIM depends on the dielectric layer thickness.

For 1D MIM with constant metal thickness and almost constant length, increasing dielectric thickness influences its radiative losses, thus to keep critical coupling the period should be increased too [145]. For the standard MIM mode (i.e. a MIM structure on a mirror), the period increases linearly with the dielectric thickness when the latter is greater than a few nanometers (see [94]). As it is discussed in the following sub-section, the incident far-field power is proportional to the period p , as carried by a plane wave:

$$P_{in} = \eta p, \quad (6.4)$$

In all presented in this Section computations the amplitude of the incident plane wave (or the flux density) is considered the same. As shown in Section G.2 of Appendix G (Fig. 13), at the critical coupling conditions, the power entering a MIM structure through its air / dielectric interfaces is linearly proportional to the thickness of the dielectric layer ($P'_{in} = \beta h$, the value for both interfaces). Since at the critical coupling conditions all the incident energy is funneled into the structure through these interfaces, $P_{in} = P'_{in}$, and $p \propto h$. Indeed the numerically calculated values of the period of the 1D MIM arrays under the critical coupling conditions (the points in figure 6.13 (b)) are located on the lines $p_{app}(h)$ (solid lines).

On the other hand, for the MIM mode the effective index of the mode decreases when the dielectric thickness increases. This law can be approximated by $n_{eff}(h) = n_0 \sqrt{1 + 2(\delta/h)}$, where n_0 is the bulk index of the dielectric material, δ is the skin depth and h is the dielectric thickness [93]. Fig. 6.13 (c) presents a similar kind of dependence. The points, obtained numerically for the critical coupling conditions, are connected by the interpolation curves for the approximated lengths of the structure l_{app} . It is computed as $l_{app}(h) = \lambda_{res}/(2n_{eff}(h))$, and for each curves parameters n_0 and δ are found to provide a better fit (these values are indicated in the

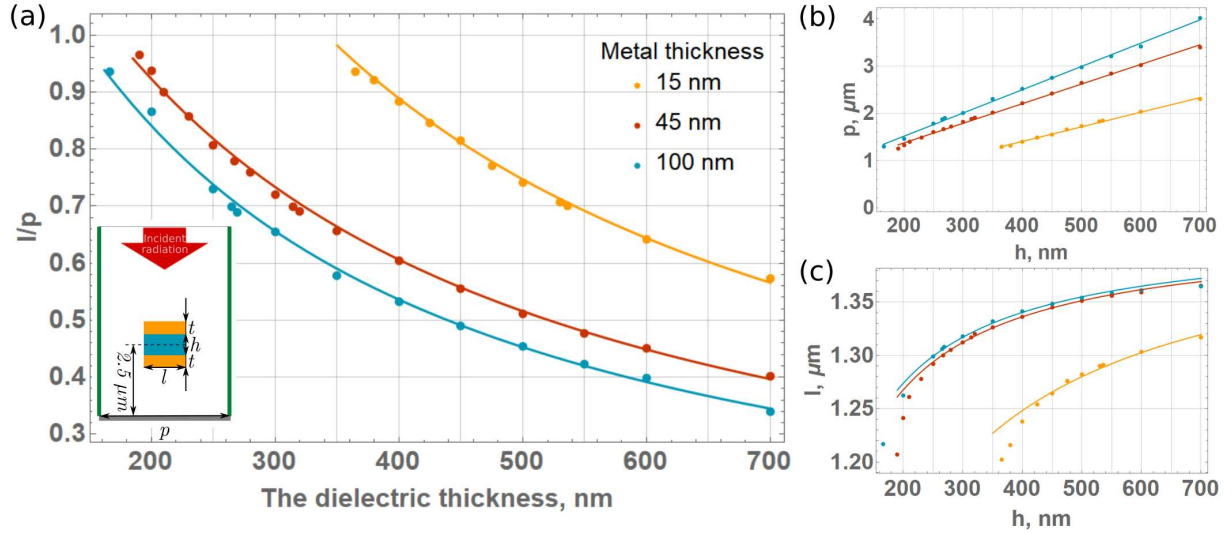


Figure 6.13: The dependence of 1D MIM array parameters at the critical coupling conditions for $\lambda_{res} = 10 \mu\text{m}$ on the dielectric thickness h for three different metal thicknesses. The blue color represents structures with 100 nm thick metal layer, red – for 45 nm thick metal layer, and yellow – for 15 nm thick metal layer. All arrays are placed at the distance of $\lambda_{res}/4$ from the mirror. (a) The critical coupling curves in dependence of the spacer thickness. Points show results of the numerical simulation, the solid lines are interpolation curves, obtained as $l_{app}(h)/p_{app}(h)$. (b) Points: the period values for the critical coupling condition in dependence on the dielectric thickness. It can be seen that the set of points for every metal thickness can be approximated with high accuracy as linear functions p_{apps} , plotted as solid lines. For the blue line $p_{apps}(h) = 529.361 + 4.928h$, for the red line $p_{apps}(h) = 542.383 + 4.156h$, the yellow line $p_{apps}(h) = 166.903 + 3.093h$ (all return values in nm). (c) Points: the MIM length values for the critical coupling condition in dependence on the dielectric thickness. The solid curves are approximation functions $l_{app}(h)$. For the blue curve $n_0 = 3.525$, $\delta = 24 \text{ nm}$, for the red curve $n_0 = 3.528$, $\delta = 25 \text{ nm}$, for the yellow curve $n_0 = 3.48$, $\delta = 65 \text{ nm}$.

figure caption). It can be seen that points for the two thicker metal layers almost coincide, and the fitting parameters are close. Although in this case the two fitting values n_0 and δ are different from the parameters used in the numerical simulations, this difference is less than four percent for the optical index and less than eight and four percent for the skin depth in the case of the corresponding blue and red curves. Oppositely, the yellow points are shifted significantly from others, and the fitting parameter δ more than twice higher than the gold skin depth. This case corresponds to metal layers which are thinner than a skin depth. The fields can pass the layer in this case, consequently, its behaviour requires a corrected approximation (the model for n_{eff} was created for thicker metals), and the effective mathematical parameter has no physical meaning. Also, for all curves, points corresponding the thinnest dielectric are not fitted well because they are corresponds to high values l/p , and thus – to another mode (discussed in detail below).

As a result, keeping the resonant wavelength constant leads to an increase of the MIM length when the dielectric thickness increases. The approximation curves in Fig. 6.13 (a) are plotted as

$l_{\text{app}}/p_{\text{app}}$, using corresponding fitting parameters. Since the period grows faster than the MIM length, the curves decrease with increasing of the dielectric layer thickness. For the smallest dielectric thicknesses this decrease is slower than the $1/h$, and for higher thicknesses it becomes close to it.

For the highest values l/p this approximation does not work fine, as the character of the mode changes. There is a special region for the critical coupling curves, when l/p is close to 1. For this region, the interaction with neighboring elements increases, and the mode type changes. When l/p ratio is high, the array of MIM resonators forms two continuous parallel metal films drilled by vertical slits. As a result, the whole system behaves as a Salisbury screen where the array of resonators plays the role of the screen located at $\lambda/4$ above the mirror. This screen works as a Helmholtz filter without vertical walls, providing a resonant transmission. Extraordinary transmission in optical Helmholtz resonators for considered wavelength region is presented, for example, in [146]. It is demonstrated there, that in such conditions the other mode, characterised by the strong electric field in the narrow gap and the strong magnetic field in the cavity, appears. However, this Section is focused on the MIM mode, described above. Additionally, later it will be shown that this configuration cannot improve the bolometer behavior.

The second effect, observed in Fig. 6.13 (a), is the decrease of the metal thickness when the dielectric thickness increases while the ratio l/p is constant. One could expect the opposite behaviour: assuming the metal absorption is reduced when its thickness is reduced, the field on the metal layer should be increased to keep the critical coupling conditions. But Fig. 6.13 (a) shows the opposite trend. This paradox is explained in the following subsection.

The metal thickness

In the case when the 1D MIM array is placed at a node of the external magnetic field, the magnetic field of the mode is responsible for the both the absorption and the funneling mechanism. This specific situation leads to the unexpected result described above.

Let us now consider in detail how the behaviour of the mode, appearing in this case, depends on the metal thickness. For this study, the critical coupling conditions for the structures with the fixed ratio $l/p = 0.7$ were found at the resonant wavelength $\lambda_{\text{res}} = 10 \mu\text{m}$ for different thicknesses of metal and dielectric. The results are presented in Table 7 in Appendix G. Below in the sub-section, only the structures at critical coupling conditions presented in this table are discussed.

Before beginning the discussion, an important note should be made. The thicknesses considered vary from 15 nm to 100 nm , thus some of them are thinner than the skin depth ($\delta = 25.7\text{ nm}$), and some of them are several times thicker. Thus the fields can path through the thinnest metal layers, in the thickest, the distribution of the field, as well as the electric current density, is not homogeneous but varies according to an exponential law. To facilitate the treatment of absorption in the metal layers despite this strong variation of the field, an effective thickness of metal is introduced below: thickness of metal showing the same absorption if the field was constant.

Fig. 6.14 (a) shows the variation of the electric current density along an axis orthogonal to the faces of a metal layer of 100 nm . This dependency is obtained at the center of the structure as indicated by the white line in the color map (insert in the figure). The points represent values, extracted from the numerical computation, while the solid curve is a fitting curve for the exponential law $j(z) = a \exp(-z/\delta)$ where z is the distance to the internal metal - dielectric interface and δ the skin depth. The value of the skin depth is that of gold ($\delta = 25.7\text{ nm}$). a is only fitting parameter. Its fitted value is $a = (2.94 \pm 0.01)10^6$. It can be seen that, indeed, the electric current density decrease exponentially with the distance to the internal metal - dielectric interface. It is useful to note that the field along the outer face of the metal layer was considered negligible compared to that along the inner face. This explains the small difference observed in the 10 nm closest to this interface.

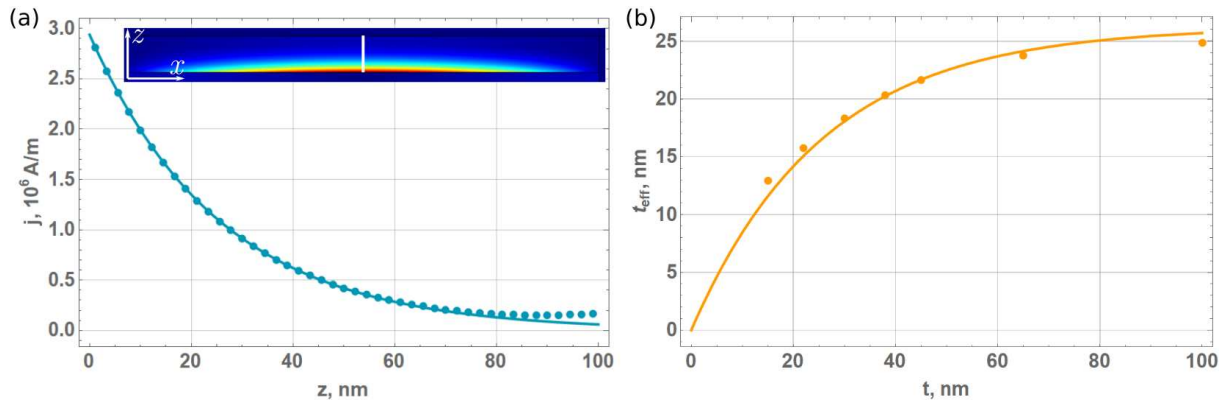


Figure 6.14: (a) The distribution of the electrical current density along the metal layer thickness at the middle of the critically coupled structure with 100 nm metal layer, at the resonant wavelength $10\text{ }\mu\text{m}$. The line, along which the electric current density is presented, is showed in white in the inserted color map. Points present extracted from the numerical computation values, and the solid curve is a fitting curve for the exponential law. The inserted color map shows the distribution of the electric current density in the top layer of the MIM structure with the 100 nm thick metal layer. (b) The dependence of the effective thickness on the geometrical thickness. Point show the result, computed numerically in simulations, while the solid curve shows the fitted effective thickness according to Eq. 6.7.

Now let us introduce the effective thickness of metal $t_{\text{eff}}(t)$, defined as the thickness of metal demonstrating the same total induced current than that induced in the metal layer of thickness t if the current density was constant. The total current I_{tot} induced in the metal layer of thickness t is given by:

$$I_{\text{tot}} = \int_0^t I_0 \exp(-z/\delta) dz \quad (6.5)$$

where I_0 is the current density at the metal - dielectric interface that is proportional to $1/\sqrt{h}$, as observed in the previous sections. After integration it writes:

$$I_{\text{tot}} = I_0 \delta (1 - \exp(-t/\delta)) = I_0 t_{\text{eff}} \quad (6.6)$$

Thus the effective thickness of metal $t_{\text{eff}}(t)$ is defined by

$$t_{\text{eff}}(t) = \delta (1 - \exp(-t/\delta)). \quad (6.7)$$

Indeed, the solid curve showing this dependence fits well the numerical results, represented by the points in Fig. 6.14 (b). These values were computed by numerical integration of the electric current density in Comsol Multiphysics.

Fig. 6.15 shows the dependence of the dielectric thickness on the effective thickness of the metal t_{eff} for 1D-MIM at the critical coupling conditions for a filling factor $l/p = 0.7$ and at the resonant wavelength $\lambda_{\text{res}} = 10 \mu\text{m}$. As it was observed in the previous sub-section, the thickness of the dielectric layer must decrease as the thickness of the metal increases to maintain the critical coupling conditions.

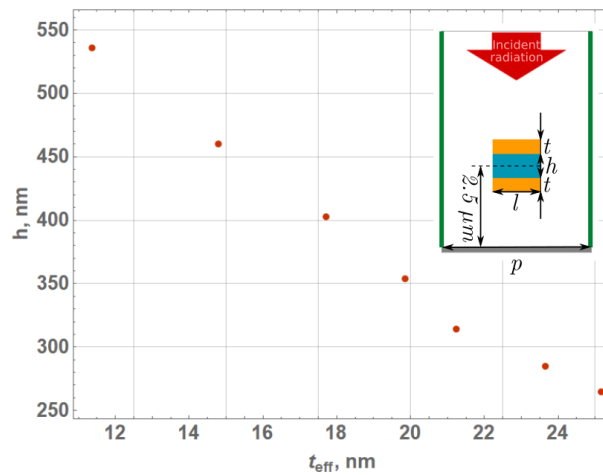


Figure 6.15: The dependence of the dielectric thickness h on the effective metal thickness t_{eff} at the critical coupling conditions for the resonant wavelength $\lambda_{\text{res}} = 10 \mu\text{m}$ for arrays of 1D MIM with the ratio $l/p \simeq 0.7$, placed at the distance $\lambda_{\text{res}}/4$ above the perfect mirror.

The observed dependence between the dielectric and metal thicknesses can be explained as follows. The critical coupling can be separated in two basic parts: the funneling mechanism and the absorption mechanism. The nature of the first one was described in the previous sub-section. In the considered 1D-MIM, absorption takes place only in the metal layers by the Joule effect. Critical coupling means that all the incident energy is absorbed. Since the period of the 1D-MIM arrays studied here varies very little from one array to another, it can be considered as constant, as well as the incoming power P_{in} in each 1D-MIM (Eq. 6.4).

As it was demonstrated in the previous sub-section, all the power absorbed in the 1D-MIM enters the structure by its sides and is described by the $\mathbf{E}_m \times \mathbf{H}_m^*$ term of the Poynting vector [3]. For the air-dielectric interface on one side, the incoming power

$$P_{in} \propto \int_0^h \mathbf{E}_m \times \mathbf{H}_m^* dz. \quad (6.8)$$

As P_{in} is constant for all these structures, i.e. P_{in} does not depend on h , and as $\mathbf{E}_m$ and $\mathbf{H}_m$ have the same dependence on h , then $\mathbf{E}_m$ and $\mathbf{H}_m$ are both proportional to $1/\sqrt{h}$:

$$P_{in} \propto \left(\frac{1}{\sqrt{h}} \right)^2 h \quad (6.9)$$

The power dissipated P_d by the Joule effect is equal to the product of the square of the electric current I by the resistance R , $P_d = RI^2$. Moreover, the electric current is proportional to the induced magnetic field, thus $P_d \propto RH^2$. For the metal layer, R is proportional to $1/t_{\text{eff}}$ (the resistance of a homogeneous conductor is inversely proportional to its section), and $P_d \propto 1/(h t_{\text{eff}})$. Since all these structures are kept at the critical coupling conditions, $P_d = P_{in} = \text{Const}$, thus the product $h \times t_{\text{eff}}$ is constant.

As a result, when the metal thickness is reduced, the optical losses increase, since they are inversely proportional to t_{eff} . Thus, to keep the critical coupling conditions, the fields must be reduced by the same ratio. Numerical simulations validate this conclusion.

The solid fitting line, presented in Fig. 6.16 (a), shows a linear dependence of the thickness of the dielectric layer on the inverted effective metal thickness $h = a/(t_{\text{eff}}) + b$. It has parameters $a = 7193 \pm 452$ and $b = -17 \pm 23$. Indeed, this line fits the numerical results from Fig. 6.15. The exception is the point for the smallest thickness: for thickness twice thinner than the skin depth, the energy flux can go through the metal layer (also proved by field maps), which is not taken into account in the simple model, described above.

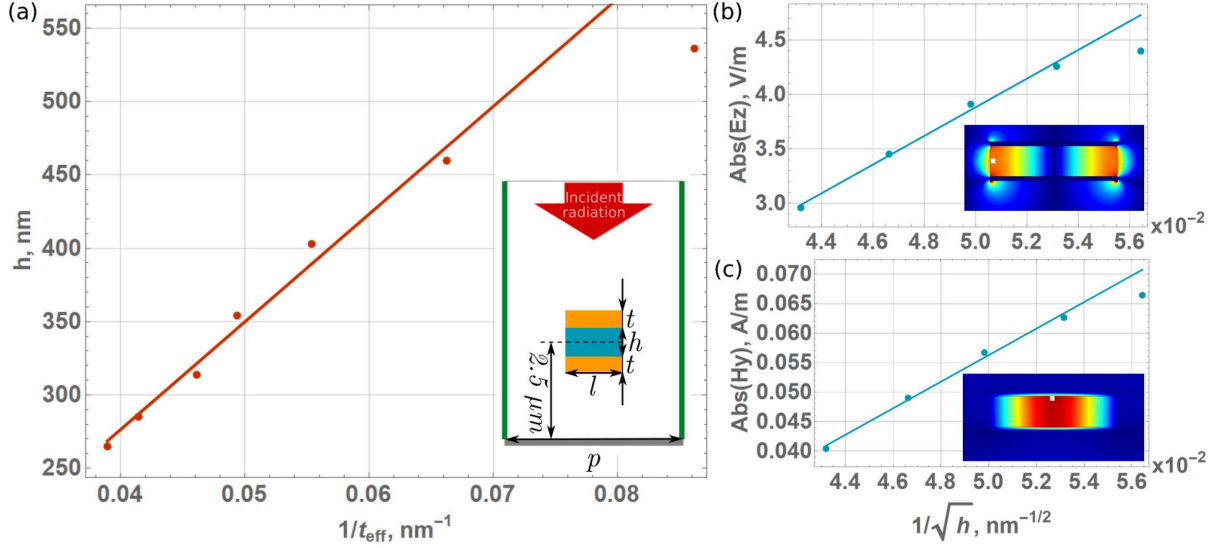


Figure 6.16: (a) Points: the dependence of the dielectric thickness h on the metal thickness t for the critical coupling conditions at the resonant wavelength $\lambda_{\text{res}} = 10 \mu\text{m}$ for arrays of 1D MIM with the ratio $l/p \simeq 0.7$, placed at the distance $\lambda_{\text{res}}/4$ above the perfect mirror. The interpolation red line fits points, obtained from numerical simulations. The dependence of the absolute values of (b) z -component of the electric field of the mode and (c) y -component of the magnetic field of the mode on the dielectric thickness at λ_{res} . Color maps in the inset are for corresponding field maps for one of the structures. The probe areas are marked with white rectangles.

The $1/\sqrt{h}$ dependency is observed on both electric and magnetic field maps. The amplitudes of the fields at the critical coupling when the metal layer thickness is changed are presented in Figs. 6.16 (b) and (c). They show that the electric and magnetic fields of the mode increase proportionally to $1/\sqrt{h}$ when the critical coupling conditions are maintained. For this plot the electric field values were extracted at the center of the side dielectric-air interface and the magnetic field – in the center of the inner top metal-dielectric interface (both probe areas are marked by a white rectangle).² Despite the small discrepancy observed for the largest $1/\sqrt{h}$ values (i.e. the thickest metal layers), the model can be considered to fit well the numerical results. Thus, thinner metal layers, having the highest optical losses, require weaker fields for the critical coupling. Hence, it is important to note that the way, how the metal absorbs in plasmonics, is different from the way for the bulk dielectrics. A thinner metal layer, having a smaller effective cross-section, demonstrates higher resistance.

However, it should be noted that such behaviour for fields is only valid for the case of a constant value of the period, while the metal thickness changes. This is the equivalent of a horizontal move (as l changes insignificantly) in Fig. 6.13, keeping the l/p ratio constant. We can notice that there are two other particular ways of moving on this map: vertical (i.e. keeping

²It can be assumed that the h dependency found at the center of the MIM is maintained up to its side edges.

h constant), and along one of the critical coupling curves (i.e. keeping t constant).

In the first case, where the thickness of the dielectric is constant, the thickness of the metal layers and the period change to maintain the balance between the incident energy and the resistive losses (the critical coupling). The period changes significantly (more than 70% increase between the two extreme metal thicknesses) while the length varies much less (about 5% between the same extremes). As a first approximation, the length is therefore considered to be constant. As observed previously, the small variations of the MIM length correspond to small variations of the effective mode index, here due to changes in the metal thickness.

According to the continuum form of Ohm's law, the dissipated power can be written $P_d \propto \sigma \frac{l}{t_{eff}} E_{mx}^2$, where σ and l are constant. In the far field, the potential difference over a period due to the electric field of the incident wave is written $V_{ff} = \int_0^p E_{in} dl = E_{in} p$ where E_{in} is constant (incident plane wave). The potential difference on the air / dielectric interface of the MIM is $V_{nf} = \int_0^h E_{mz} dz = E_m h$, where E_{mz} is assumed constant along these interfaces. Since the potential difference is zero along metal surfaces and periodic boundaries, $V_{ff} = 2V_{nf}$. So $E_{mz} \propto p$ since h is constant. Due to the proportionality of the field components of the MIM mode, E_{mx} can be also considered proportional to p . In this way, $P_d \propto \frac{p^2}{t_{eff}}$.

The incident power P_{in} is proportional to the period (Eq. 6.4). As the MIM arrays are in the critical coupling conditions, $P_{in} = P_d$. Thus, $p \propto \frac{p^2}{t_{eff}}$, or $p \propto t_{eff}$. Indeed, as shown in Fig. 11 of Appendix G for the few structures considered in Fig. 6.13, the period p varies linearly with the effective metal thickness t_{eff} . In this way, indeed, the decrease in the metal thickness leads to a decrease in the period as observed in Fig. 6.13.

In the second case, the metal thickness is constant while other parameters change. Since the amplitude of an incident plane wave is constant, the incident power P_{in} is proportional to the period p (Eq. 6.4). As the critical coupling curve is considered, each point of which corresponds to a total absorption, as it was discussed above, $P_{in} = P_d$, and thus $P_d \propto p$. In the previous subsection it is demonstrated that the period p is linearly proportional to the dielectric thickness h (Fig. 6.13). Consequently, the dissipated power P_d is also linearly proportional to h . The resistance of the metal layer can be, in a first approximation, considered constant because the thickness of each metal patch is kept constant and its length changes insignificantly compared to p and h . In this way, the increase of the dissipated power is only provided by the increase of the fields of the mode. As $P_d \propto R H_m^2$, the magnetic field of the mode $H_m \propto \sqrt{h}$, as well as other components of the field of the mode due to their proportionality.

Although the fields of the mode are proportional to $\sqrt{h}$, there is no contradiction between this dependency and the linear dependency of P_{in} on h , where P_{in} is given by Eq. 6.8. The energy flux entering the 1D MIMs is provided by the component $P_x = E_{mz} \cdot H_{my}$ of the time-averaged Poynting vector, where E_{mz} and H_{my} have complex amplitudes. It writes: $P_x = 0.5Re(E_{mz})Re(H_{my}) + Im(E_{mz})Im(H_{my})$. Each of the two terms of P_x , computed at the middle of the air-silicon side interface, depends linearly on h . After integrating from 0 to h , the two terms therefore depend as a quadratic function on h . However their terms in h^2 are equal in absolute value but opposite in sign, thus they cancel each other (see Section G.2 of Appendix G). As a result, the power funneled into the structure trough the side air-silicon interfaces is linearly proportional to the dielectric thickness h , or the period p , as it should be. In the case of the constant period and changing metal thickness both terms of P_x are independent of the dielectric thickness h (Section G.2 of Appendix G).

To summarize, it is observed that to maintain the critical coupling when the thickness of the metal layer decreases and the period is kept constant (constant incident power), it is necessary to increase the thickness of the dielectric layer. This is due to the increase in the resistance of the metal layer which varies inversely with its effective thickness t_{eff} where t_{eff} depends on the skin depth. The power dissipated therein can be kept equal to the incident power if the field in the resonator is reduced by increasing the thickness h of the dielectric layer. The critical coupling is maintained when the product $t_{eff} \times h$ is kept constant. When the metal thickness t is kept constant, an increase in the dielectric thickness h requires the increase in the period p (i.e. the increase of the incident power) to keep the critical coupling. In this case, the fields of the mode grow as $\sqrt{h}$. When the dielectric thickness is kept constant, a reduction in the metal thickness (i.e. an increase in optical losses) results in an increase of the period (an increase of the incident power) to maintain the critical coupling conditions. In this case, the period increases linearly with the metal effective thickness: $p \propto t_{eff}$.

Volume

As it was discuss above, none of the MIM arrays considered at $\lambda/4$ above the mirror are optimal: they either have too thick a metal or have too thick a dielectric layer. It makes the volume of the absorber too high compared to that of an array placed on the mirror.

To compare structures with different metal thicknesses, discussed above, the total³ volume

³Metal and dielectric.

density $\omega_V = l \cdot (2t + h)/p$ is used. In this way, both the density of resonators and the volume are taken into account. Fig. 6.17 provides the dependence of ω_V on the dielectric thickness h for the MIM arrays at the critical coupling conditions presented in Fig. 6.13. The points present results of numerical simulations (Section G.1 of Appendix G), while the solid curves are obtained from the curves of Fig. 6.13 by multiplying by $(2t + h)$.

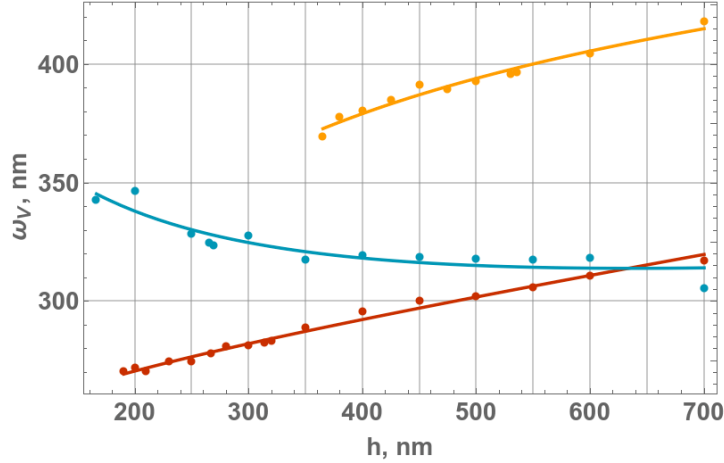


Figure 6.17: The dependence of the total volume density of the resonators ω_V on the dielectric thickness h . Every point corresponds to the critically coupled 1D MIM array at the resonant wavelength $\lambda_{res} = 10 \mu\text{m}$ placed at the distance $\lambda_{res}/4$ above the mirror. The blue points are for the structures with 100 nm thick metal layer, the red – for 45 nm thick metal layers, and yellow ones – for 15 nm thick metal layers.

It can be noted that there are two different types of behavior. For the arrays with the metal thicknesses smaller or comparable to the skin depth (yellow and red curves), the lowest volume density values correspond to the thinnest dielectrics (and to the highest l/p ratios). For structures with a metal thickness greater than a few skin depths (blue curve), the volume density is higher for the thinnest dielectrics (and the highest l/p ratios) and almost constant for the thickest dielectric layers. This difference is due to the high metal thickness: as there are two metal layers with a thickness of 100 nm, which significantly affects the total volume values, especially in the left part of the plot.

This difference appears in the following way. The form of the volume density ω_V is based on the critical coupling curves $l/p(h)$. As the l/p ratio has the form $a/(b + h) + c$, the volume density $\omega_V = a(2t + h)/(b + h) + c(2t + h)$, i.e. ω_V is the sum of a few functions, depending on h and t . While for t this dependence is linear, for h they are linear, hyperbolic and $1/(1 + 1/h)$ terms. Depending on the ratio between t , h , and the coefficients a , b , and c , the resulting curve can take different forms.

To prove it, let us consider the curve $f(x) = a(\tau + x)/(b + x) + c(\tau + x)$. It can be presented

as $f(x) = a\tau/(b+x) + a/(1+b/x) + c(\tau+x)$. As x grows, the third term simply moves the curves upward along the ordinate axis. This can affect the curve shape depending on the coefficients ratio. However, the main difference in curves shape is defined by the first two terms.

In order to simplify the analyses, let us consider the function $f_1(x) = \tau/(b+x) + 1/(1+b/x)$. This function has three different shapes, depending on the ratio between τ and b . When $\tau = b$, $f_1(x) = 1$. When $\tau < b$, $f_1(x)$ increases, as the second term increases faster than the second decreases. And, oppositely, when $\tau > b$, $f_1(x)$ decreases, as the first term decreases faster than the second one increases. The Fig. 6.18 show all three situations for the particular case, when $b = 0.5$. Physically, such behaviour means that there exists a critical thickness $t_{cr} = b/2 \approx 56 \text{ nm}$ for all three curves, which defines the form of the dependence of ω_V on the dielectric thickness. As two curves correspond to metal thicknesses smaller than t_{cr} , they have one shape, and the third – corresponding to the metal thickness higher than t_{cr} – has a different behaviour. Plots, presenting ω_V as sum of three terms, are provided in Fig. 16, Section G.3 of Appendix G.

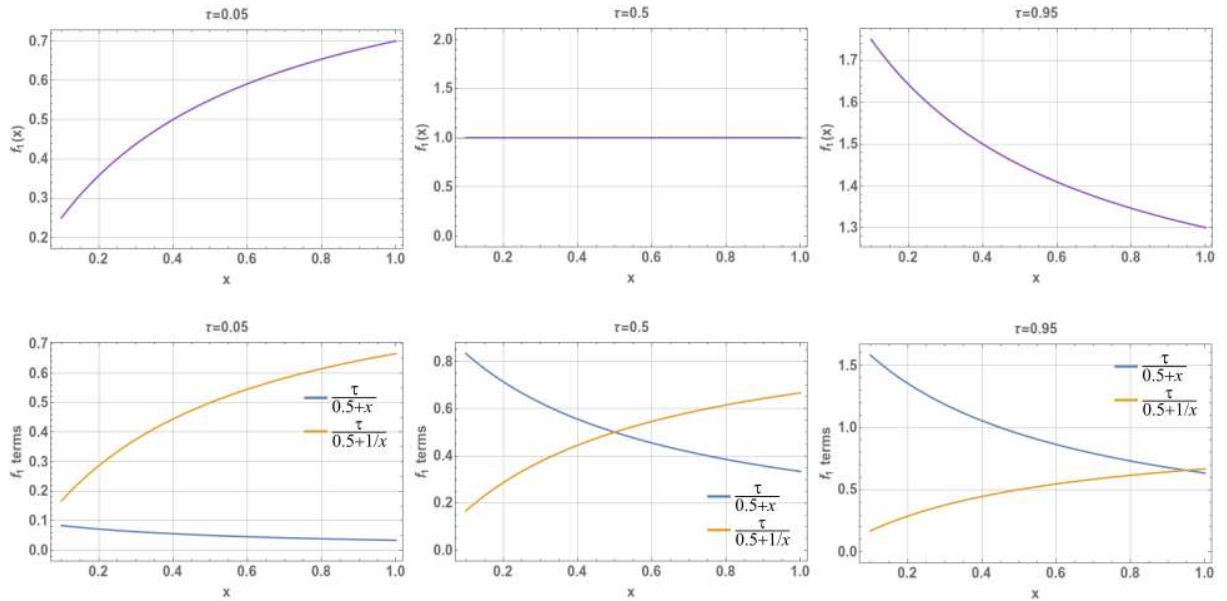


Figure 6.18: The behaviour of the function $f_1(x) = \tau/(b+x) + 1/(1+b/x)$ (the top row) and each of its terms (the bottom row) when $b = 0.5$ and τ is smaller than b , equal to b , and higher than b .

It can be seen from Fig. 6.17 that among the considered structures, the smallest value of the total volume density is found on the red curve ($t = 45 \text{ nm}$) for the thinnest dielectric layers. However, in these conditions the resonators cover all the surface. Thus, there is no heat concentration in a limited area. The use of this kind of antenna would lead to heating almost the entire surface of the device, which is the opposite of our goal: to concentrate the heat at the smallest possible volume. On the other hand, for thicker dielectric layers ($600 - 700 \text{ nm}$),

the volume density is minimal for the thicker metal layers. The heat concentration in this case is better ($l/p \approx 0.4$), but as the arrays are continued in one direction, this result is far from optimal with a coverage rate of 40%. Also, as curves are quite flat, thus the minimum volume density for the 1D-MIM can be considered to be around 300 nm .

It is necessary to note that, in general, such a configuration (i.e. at $\lambda/4$ above a mirror) is not as favorable as the one where the resonators are either on the mirror or at $\lambda_{res}/2$. Indeed, the best way to increase the radiative coupling for this type of structure is to move the structure into a region of a stronger incident magnetic field.

Optical properties

The funneling process in the case of 1D MIM are placed at the distance $\lambda_{res}/4$ from the mirror is different from the case, when array is placed on the mirror. This leads to changes in the electromagnetic response. In this sub-section some useful for application properties for such systems are presented. At first, the angular dependence, presented in Fig. 6.19, differs from the previous case (Fig. 6.4). The absorption between -10° and 10° is almost perfect, and it exceed 90 % for the angles in the range $-20^\circ - 20^\circ$. For higher angle values, the absorption decreases drastically. This difference can appear because of the cavity between the array and the mirror. Such selective dependence on the incident angle can be useful for some applications.

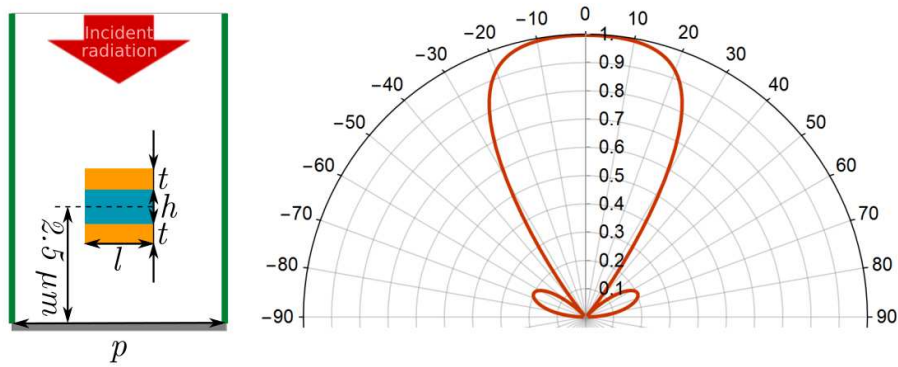


Figure 6.19: Absorption vs the incident angle at the wavelength $10 \mu\text{m}$ for the array of 1D MIM with parameters $p = 1.883 \mu\text{m}$, $l = 1.317 \mu\text{m}$, $t = 45 \text{ nm}$, $h = 314 \text{ nm}$ placed at the distance $2.5 \mu\text{m}$ above the mirror. The angle is measured from the normal.

The 1D MIM array, optimized at the node of magnetic field, demonstrates periodic behavior when moved relative to the mirror. This dependence is presented in Fig. 6.20 (b) at wavelength equal to $10 \mu\text{m}$. It can be seen that the curve is periodic with the period $\lambda_{res}/2 = 5 \mu\text{m}$, but its shape differs from the case of the array optimized on the mirror. The large asymmetry of this curve may be connected with the asymmetry of the Poynting vector caused by $\mathbf{E}_m \times \mathbf{H}_w$.

Also, at the range of distances from $2.13\ \mu\text{m}$ to $2.75\ \mu\text{m}$ the structure optimized at a distance of $2.5\ \mu\text{m}$ absorbs more than 90 % of the radiation. It can be useful for absorption applications.

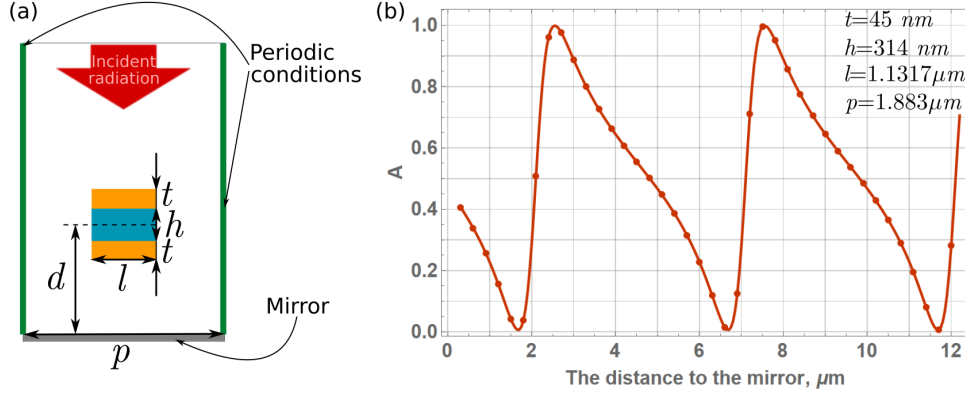


Figure 6.20: The response of the 1D MIM structure, optimized for $\lambda_{res} = 10\ \mu\text{m}$ at the distance $2.5\ \mu\text{m}$ from the mirror, in dependence on the distance d to the mirror at λ_{res} . The parameters of the structure: $p = 1.883\ \mu\text{m}$, $l = 1.1317\ \mu\text{m}$, $t = 45\ \text{nm}$, and $h = 314\ \text{nm}$. The solid line is for the analytical computation with S-parameters, and the dots are for the COMSOL simulations.

6.3.5 Conclusion

To summarize, an array of 1D MIM placed at the node of the magnetic field at the distance $\lambda_{res}/4$ above the mirror can be critically coupled for different metal thickness. The mode in this case seems similar to the classic MIM mode. However, in this case, the critical coupling is very specific. The funneling process in the case of 1D MIM are placed at the distance $\lambda_{res}/4$ from the mirror is different from the case, when the array is placed on the mirror. For the array on the mirror the funneling is provided by the magnetic field of the standing wave, while for the array at the $\lambda_{res}/4$ – by the magnetic field of the mode. The magnetic field of the mode is also involved in the absorption inside a MIM. Due to the characteristic distribution of the magnetic field of the MIM mode, which is weak at the side edges and strong in the center, special conditions are required for the critical coupling. the dielectric layer thickness increases compare to an array of 1D MIM on the mirror (for the same l/p). Even more, the decrease of the metal thickness because of the growth of its resistance requires the increase of the dielectric thickness. Consequently, optimized structures have or thicker metal layers or the high ratio l/p , or involve thick dielectric spacer. All these options are not close enough to the aim of structure miniaturization.

Trying to overcome these limits, in the following sections 2D periodic MIM structures are considered. The limitation in the direction of invariance opens up a new parameter to control the response of the structure, and the edge effects bring change in the field distribution and

energy absorption.

6.4.

2D MIM antennas for bolometers applications

6.4.1 Introduction

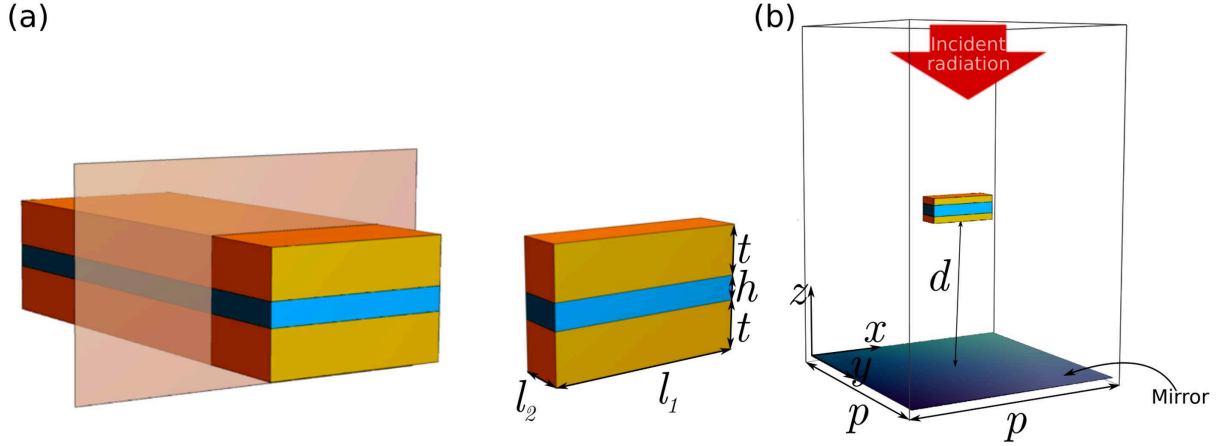


Figure 6.21: 2D MIM structure. (a) A transition from 1D MIM to 2D MIM and parameters of a MIM structure. (b) The schematic presentation of a unit cell of 2D periodic array of MIM structures.

In Section 6.3, it is demonstrated that arrays of 1D-MIM structures can be critically coupled being placed at the distance $\lambda_{res}/4$ above the mirror. However, they are too large in volume (mainly due to the dielectric layer) to make a breakthrough in bolometer applications. In this Section, a way, breaking the invariance of the structure to decrease its volume, is presented. As shown in Fig. 6.21 (a), the 1D MIM structure can be limited in the invariant direction, introducing periodicity along this direction, which creates a periodic 2D array of MIM structures. In this Chapter, the resonance conditions of this reduced volume structure is studied. It is useful to note that it has an additional parameter – the width l_2 – to access the critical coupling conditions.

The schematic representation of the system, considered further in this Chapter, is presented in Fig. 6.21 (b). Again, the chosen resonant wavelength is $\lambda_{res} = 10 \mu\text{m}$, and an array of structures is placed at the distance $d = \lambda_{res}/4 = 2.5 \mu\text{m}$ above the mirror. The response of studied arrays is obtained with numerical simulation. There a periodic port that produces the incident wave and receives the reflected waves is used. It is situated at least one wavelength above the structure. The periodic boundary conditions are applied on the sides to take into

account the array periodicity. In most computations a plane wave incidents normally on the array of MIMs.

The rest of the Section is organized as follows. First, in Section 6.4.2, four simple geometries of MIM in 2D periodic arrays are studied in order to define the influence of the structure shape on its response and to identify the geometry that provides the smallest volume. Second, in Section 6.4.3, the influence of four different metals on optical losses and the critical coupling conditions is studied, and the optimal metal for absorber applications among them is determined. After that, in Section 6.4.4, 2D MIM structures having a gold layer of various thicknesses are studied. The excited mode at the critical coupling conditions is considered. It is demonstrated that due to the distribution of fields inside the MIM and the specificity of the funnel effect⁴, the critical coupling curve exhibits a behaviour different from that of standard MIMs. Then, some optical properties for structures with different metal thicknesses are presented.

The difference between cross and rectangle shaped geometries is explained in Section 6.4.5. Although the response of these structures with the same parameters is close, for the cross-shaped structure the resonance peak is blueshifted with slightly different characteristics. To explain this difference, a new approach, where a $\lambda/2$ antenna is replaced by two $\lambda/4$ antennas, separated by a conductive plane, is proposed. The application of this model provide an explanation for all the differences observed between the two geometries . Finally, in Section 6.4.6 the comparison of 2D and 1D arrays of MIM with is provided. The density of absorbers for rectangle and cross geometries for 2D MIM is given. It is show that, indeed, this value is much smaller for 2D MIM.

6.4.2 Geometries

In this Section, 2D MIM structures with different geometries are considered: circle, square, rectangle, and cross. The metal layer thickness of 35 nm was chosen for all geometries, because in the previous section it is found that intermediate metal thicknesses are optimal⁵ for bolometer applications. The $10\text{ }\mu\text{m}$ resonant wavelength is kept. Below, influence of a geometrical shape on a structure response is studied. Four various structures at the critical coupling are compared from the geometrical parameters point of view, then the modes of the resonator are analyzed under these conditions. Based on the presented results, an optimal geometry is defined.

⁴In the context of 2D periodic MIM arrays funneling is the concentration of incident flux from the period surface p^2 on a small structure, having a surface of a few percent of p^2 .

⁵in the following sections this result will be revised for 2D MIM

General behaviour: half structures

Before presenting the study made on 2D MIM structures located at $\lambda/4$ above a mirror (i.e. the three MIM layers are structured), the critical coupling of standard 2D MIM resonators are studied in this sub-section. They are composed of a thick and infinite perfect metallic layer (M) on which the two structured layers are deposited: dielectric (I) and metallic (M). The results for the full system, completed by the mirror, is presented in the following sub-section.

A perfect electrical conductor is used for the bottom metallic layer (Fig. 6.22 (a)). Thus the studied structure is equivalent to a MIM twice as thick located at an anti-node of the electric field (e.g. at $\lambda/2$) of the standing wave generated by a mirror. This reduction in thickness together with the absence of fixed-length space between the structures and the mirror reduces significantly the computation time of the 3D simulations. This simplified approach makes it possible to distinguish the critical coupling conditions between the different geometries studied.

The schematic presentation of half-cut MIM structures with different geometries, placed on the mirror is shown in Fig. 6.22 (a). In computations, the electric component of the incident field was directed along the x axis: along the longest side of the rectangle MIMs, and along one of the sides for square and cross-shaped ones. For all of them, the critical coupling conditions were found for the $10\ \mu\text{m}$ resonant wavelength and for various dielectric thicknesses. The parameter values leading to the critical coupling for different geometries are presented in Appendix G, Section G.4. Periodic boundary conditions are along the Ox and Oy axes. The period p_x and p_y are equal in both directions.

Critical coupling curves in dependence on the dielectric thickness for periodic arrays of half-cut structures on the mirror are presented in Fig. 6.22 (b). The metal thickness for all curves is equal to $35\ \text{nm}$. Two different behaviors can be observed: disk and square shaped MIMs on the one hand and cross and rectangle shaped MIMs on the other. For the same l/p ratio, the dielectric thickness at the critical coupling for the former is smaller than for the later. For example, for $l/p \approx 0.6$, disk-shaped and squared-shaped MIMs have a dielectric thickness of about three times smaller than that of the rectangles and crosses.

The observed behaviour can be explained in the following way. All curves in Fig. 6.22 (b) are presented for the critical coupling conditions. Similarly, as it was described in Section 6.3.4, the critical coupling can be presented as the balance of the incoming power P_{in} and the dissipated power P_d .

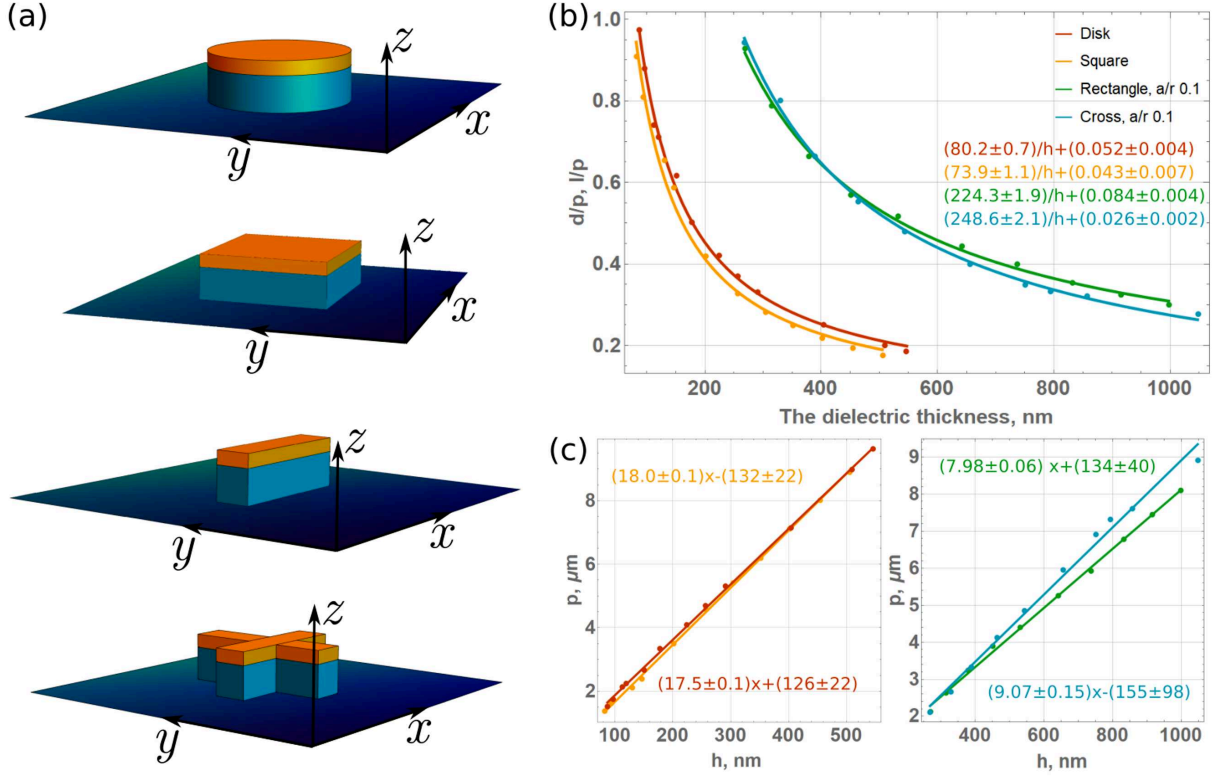


Figure 6.22: (a) Considered geometries of half cut 2D MIM structures: disk, square, rectangle and cross on the perfect mirror. For the rectangle and rectangles of the cross, the sides aspect ratio is 0.1. (b) Critical coupling curves at the resonant wavelength $10 \mu\text{m}$ for the four geometries, fitting numerical results (points), presented in Section G.4. (c) The critical coupling conditions in according with Eq. 6.10 (solid curves) and numerical results (dots) for 2D periodic MIM (half-structures) with different geometries of top patches. Line equations and deviations for the points are indicated by corresponding colors. The color legend is the same as in picture (b).

In this way, the dissipated power $P_d = RI^2$, where $R = \frac{l}{\sigma A}$ is the resistance and $I = jA$ the current. Since the current density $j = \sigma E$, the dissipated power writes $P_d = \sigma l_1 A E^2$. As the metal thickness is kept constant for all structures studied in this sub-section, the cross-section A is only a function of the metal width, thus $A \propto l_2$.

in the structures considered in this sub-section, similar to the classical MIM, the electric field E is proportional to the thickness of the dielectric layer h . As it is demonstrated in Section G.5 of Appendix G, this statement is true in the case of classical 1D MIM mode. In the considered case of the metallic patch with limitation in both directions, the field decomposition is more complicated (Section G.5). However, this is still a modified MIM mode, and the assumption that $E \propto h$ in the considered case is proved both in Section G.5 and by the results presented below.

As $E \propto h$, and all the structures are made with the same material, the dissipated power $P_d \propto l_1 l_2 h^2$. Moreover, the incoming power P_{in} in the 3D case is proportional to p^2 . For the

critical coupling, equilibrium must be reached, thus $l_1 l_2 h^2 \propto p^2$. As a result, at the critical coupling the condition

$$p \propto h \sqrt{l_1 l_2} \quad (6.10)$$

should be satisfied. The linear dependence of the period on the dielectric thickness is shown for the 1D MIM structures for example, in [94]⁶.

In this way, according to Eq. 6.10, the critical coupling curves for structures of different geometries depend inversely on the dielectric thickness ($l_1/p \propto 1/h$). Indeed, Fig. 6.22 (c) shows, for all geometries, an excellent agreement with a linear behavior. Thus, the critical coupling curves (solid lines) in Fig. 6.22 (b) are fitted to the numerical results (points) with a function proportional to $1/h$.

Equation 6.10 also makes it possible to explain the difference in behavior observed between the two groups of curves in figure 6.22 (b). The filling factor l/p is proportional to the $1/(l^2 h)$ for square-shaped patches, and to $1/(d^2 h)$ for disk-shaped ones. For rectangular and cross-shaped patches the ratio l_1/p is proportional to $\sqrt{l_1}/(\sqrt{l_2} h)$. For example, if the dielectric thickness h is fixed, the ratio l_1/p is proportional to $\sqrt{l_1}/\sqrt{l_2}$, i.e. 3.16 for a rectangle of aspect ratio $l_2/l_1 = 0.1$ and 1 for a square. For another example, if the l/p ratio is fixed, the dielectric thickness h is proportional to the ratio $\sqrt{l_1}/\sqrt{l_2}$, so structures with a smaller l_2 must have a thicker dielectric layer.

In this way, a change in the aspect ratio leads to a shift of the critical coupling curves. Although in this comparison the patch lengths are not exactly the same due to the change in the effective mode index, this simple model correctly predicts the separation of critical coupling curves into two groups based on their aspect ratio. As a result, rectangle-shaped and cross-shaped geometries have a higher l/p ratio than disks and squares.

In terms of the simple model described above, the shift of the critical coupling curves towards large dielectric thicknesses as the aspect ratio increases, can also be explained as follows. As the dissipated power P_d is proportional to the width, the structure of greater width (and of constant length) can provide the same losses for a smaller amplitude of the electric field. As the later depends linearly on the dielectric thickness, this means that a larger structure can provide the same losses for a smaller dielectric thickness (while the other parameters are kept constant).

⁶Also, the linear dependence of the period on the dielectric thickness can be explained in the term of the radiative and non-radiative losses. As the metal parameters (for one geometry) change not significantly compared to the period and the dielectric thickness, the non-radiative decay is change slightly. In the same time, radiative decay depends linearly on the dielectric thickness and inversely – on the period [145]. Thus, for increased linearly dielectric the period also should be increased linearly.

However, it should be noted that, as the electric current decomposition in the metal patch is nonlinear (see Section G.6 and Section G.8 of Appendix G), significant variations of the width and/or thickness of the patch would modify the resistance in a more complex way.

To estimate the influence of the resonator geometry on the thermal capacitance, the volume of the dielectric and the metal must be considered for the same period value. Since all the resonators are at the critical coupling conditions and the period is the same in both directions, the same power is absorbed for equal period values. As it was demonstrated in Section 5.5, the volumetric specific heat capacities of gold and silicon are not very different, thus the total volume of the resonator (dielectric and metal) should be used for this estimate.

Fig. 6.23 shows the dependence of the total volume for the four considered geometries in the case of half-structures (i.e. a metallic layer and a half-thickness of dielectric on a perfect mirror). It can be seen that the dependence of the total volume on the period is linear. The points present the results of the numerical simulations, and the solid lines the best fit of a linear function to these results. If for simplicity, we consider the rectangular shape, its total volume is given by $V = l_1 l_2 t + l_1 l_2 h$. As the first term of the sum is much smaller than the second, $V \approx l_1 l_2 h$ (the first term will be considered separately later). Together with Eq. 6.10, $V \propto \sqrt{l_1 l_2} p$. As an increase in the length and width is not significant compared to the increase of the period, in a first approximation the total volume depends linearly on the period value. The inset graph shows the dependence of the metal volume on the period at the critical coupling. It can be seen indeed, that the change in the volume of metal remains low when the parameters l_1 and l_2 increase (increase of p) for a given metal thickness t .

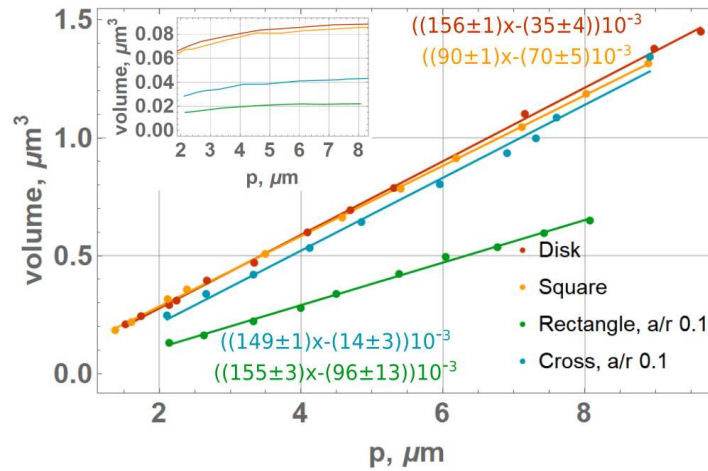


Figure 6.23: The volume of the structures critical coupled at the resonant wavelength $10 \mu\text{m}$. Solid lines corresponds to the linear fit for the total volume, and dots shows results of numerical computations. The line equations and standard deviation for the fitting lines are indicated in corresponding colors. The inset shows the volume of metal uniquely (one layer).

As the only significant impact in the total volume is provided by the dielectric, structures with smaller widths have smaller dielectric thicknesses at the same period value (i.e. to absorb the same incident power). This can be observed in Fig. 6.23: for the same absorbed power, the total volume for the rectangular geometry is twice smaller than for the other geometries. While for disc and square shaped patches higher volume is due to higher width, for cross shaped patch there are just two bars.

However, rectangular geometry can be useful only for applications, where polarization sensitive absorption is required. For non-polarized problems, the cross-shaped geometry is the one with the smallest volume. Since the presented results are for half structures, for the whole structure the difference between the cross and other two geometries is doubled.

Summing up, the geometrical shape of patches of 2D MIM influences its response due to the change in optical losses. Although disc and square shaped geometries require a thinner dielectric layer for the same l/p ratio, rectangle and cross shaped geometries, due to smaller side aspect ratios, employ smaller volumes for dissipation an equal power. The difference in thickness of the dielectric layer observed between the different geometries to achieve the critical coupling conditions is mainly due to the width of the patches which modify the optical losses. As the structures having the smaller volumes are best suited for bolometric applications, a cross-shpaed geometry is preferred for polarization insensitive detection and a rectangular geometry (with almost twice smaller volume) for polarization sensitive detection. However, all these structures have significant dielectric thicknesses (hundreds of nm) and are linearly dependent on the period, which leads to a total volume that is all the greater that the period is large. Below the mode at the critical coupling and the optical response of complete structures of different geometry are considered.

Complete structures at $\lambda/4$ above the mirror

In this Section, unlike the discussion above, 2D arrays of *complete* MIM structures at $\lambda_{res}/4$ above the perfect mirror are considered ($\lambda_{res} = 10\ \mu m$). The same geometric shapes as those presented in the subsection above, are compared. The modes excited under the critical coupling conditions are studied and are compared to those observed in the corresponding half-structures of the sub-section above.

For the critical coupling of an array of complete MIMs at $2.5\ \mu m$ above the mirror⁷, the

⁷Measured from the center of MIM structures.

geometric parameters are different from those of the half-structures studied above (presented in Section G.4). The values of the parameters are presented in Table 6.1 for a fixed dielectric thickness $h = 1.2 \mu m$, thus the structures are placed in the same external fields. This value has been chosen to avoid too high l/p ratios for the rectangle and cross shaped geometries.

In this way, 2D periodic arrays of MIM structures can be critically coupled by being placed at the distance $\lambda_{res}/4$ above the mirror. There are similarities and differences compare to the case, considered above. The key difference between the complete structures at the $\lambda_{res}/4$ and the half-structures on the mirror is the way how a structure is excited. The structures placed on the mirror are excited by the external magnetic field, because they are in the anti-node of the magnetic field of the standing wave. In this case, the total fields on the metal at the center of the metal bar are proportional to h .

On the contrary, the complete structures at $\lambda_{res}/4$ above the mirror are located at the node of the external magnetic field. In this case the only magnetic field providing the coupling is the magnetic field of the mode, which has two components and a complex distribution around the structure compare to the magnetic field of the standing wave. As a result, the critical coupling is provided in more complex way and the dependencies of the period and the total field on the dielectric thickness change its character (shown in Section 6.4.4): they decrease with the growth of the dielectric thickness (keeping the critical coupling). Thus, the critical coupling conditions are different from that used for standard MIMs, and fundamental physical explanation for this case is a matter for further study. However, some aspects of its behaviour, including funneling process and the optical properties are presented in Section 6.4.4, while the behavior in the case of different geometries is considered here.

The main difference observed between the geometries presented in the above subsection, remains present for the complete structures. The only difference between them is again connected with the different widths and thus to the resistances of the metal patches. The critical coupling can be described as a balance between the incoming and dissipated powers by a simple model comparable to that provided in the previous subsection. As $P_d \propto \sigma l_1 A E^2$, and $P_{in} \propto p^2$, at the same dielectric thickness ⁸ the disk and square geometry can absorb more incidence power. Thus, the period value is higher than that of the other two geometries.

The parameters of the complete 2D MIM arrays can be compared to those of half-cut MIM

⁸In this sub-section, the dependence on the dielectric thickness is not considered. This dependence can change the shape of the critical coupling curve, but a similar relative shift of the curves is expected, as the only important parameter for the critical coupling for considered case is the width of a MIM structure.

with doubled dielectric thicknesses. For the same resonant wavelength, the length (diameter) of complete structures at $\lambda_{res}/4$ above the mirror changes insignificantly compared to those of half structures. On the other hand, the periods are almost twice as small for the rectangle and cross-shaped geometries, and almost three times – for disks and squares, compared to half-structures⁹, and thus – the absorbed power. This difference is caused by the change in coupling between these two configurations, associated with a change in the fields of the standing wave.

Moreover, as it was already been mentioned, similar to the half-structures, the disk and square-shaped geometries require, for the same dielectric thickness, a ratio l/p smaller than those of the rectangles and cross, because a higher width results in higher losses and thus in a higher period. However, a high value (compared to the rectangular geometry) of the side aspect ratio leads to significant increase in the total volume. For the parameters, presented in Table 6.1, the incident power between two groups of geometries differs by approximately a factor of 1.5, while the dielectric volumes of the disks and squares are more than twice that of the cross, and about five times that of the rectangle. This makes it possible to apply the conclusion, established for half-structures, to the complete structures.

It can also be noted that for the complete 2D MIM at $\lambda_{res}/4$ above the mirror, the dielectric thickness is a few times higher than that of the 1D arrays having the same ratio l/p (Section 6.3.4).

Table 6.1: Critical coupling parameters at the resonant wavelength $10\ \mu m$ for 2D MIM structures with different geometries, which centers are located at the distance $2.5\ \mu m$ from the mirror. For all the structures the dielectric thickness $h = 1.2\ \mu m$ and the metal thickness $t = 35\ nm$.

Geometry	Side length / diameter $l_1, \mu m$	Side length $l_2, \mu m$	Period $p, \mu m$	l_1/p
Disk	1.822	1.822	3.500	0.521
Square	1.586	1.586	3.500	0.453
Rectangle	2.255	0.226	2.800	0.805
Cross	2.295	0.230	2.850	0.805

The width of the MIMs also influences the field distributions, although the fundamental aspects of the coupling are the same for all geometries, as all arrays are placed at $\lambda_{res}/4$ above the mirror. Field maps of the MIM structures of the studied geometries, under critical coupling conditions, are presented in Appendix G, Section G.6. For all geometries the excited mode looks

⁹In fact, the dielectric thickness $600\ \mu m$ for the disk and square half-structures would require the period value higher than resonant wavelength. In this case, new diffraction orders will appear and the response can be changed, that is not considered in this Chapter. Thus, this comparison is relative, only in order to highlight the significant change in the period for critical coupling.

like the dipole mode in the x direction (along the longest side of the structures)¹⁰, similar to the MIM mode presented in Section 6.3.4. Moreover, additional components of electric field are excited along the edges in the y direction.

Also, due to the limitation in the y direction, the behaviour of the magnetic field changes dramatically¹¹ compare to 1D MIMs. It is inhomogeneous along the y direction, with a stronger excitation at the edges (the H_z component). The width (l_2 is in the y direction) has a significant impact here: for geometries with an aspect ratio equal of 1, the magnetic field is mostly excited on the sides in the y direction, while for the narrow structures the fields on the left and on the right do not decay fast enough in the y direction to be completely separated. Thus they overlap for small widths¹². As a result, the magnetic field along narrow structures is similar to that of a wire carrying an electric current. The magnetic field therefore also appears at the top and bottom (air-metal interfaces) of the structures, unlike 1D MIMs. In addition, this field overlaps the standard magnetic field of the MIM mode in xOz plane, making total field not symmetrical along the z direction. Thus, the decomposition of the magnetic field in this plane is different depending on whether it is a question of wide or narrow structures.

The limitation in the y direction also leads to a considerable change in the electric current density¹³. Unlike the case of 1D MIMs where the electric current is only excited near the internal metal-dielectric interfaces, for 2D MIMs the electric current is also excited from the external metal-air interfaces. The distribution of the current spread over a larger surface for the disks and the squares explains the particular behavior of their l/p ratio for the same dielectric thickness.

The time-average Poynting vector behavior also changed compared to the 1D MIM arrays. While for 1D MIMs the energy only funnels inside the structure through the side slits (the S_x component of the Poynting vector), for 2D MIMs the energy can also pass through the other sides (the S_y component of the Poynting vector), and from the top and the bottom in the case of the rectangular and cross geometries. The Poynting vector behavior for 2D MIMs is discussed in detail in the following sections.

Finally, let us consider some difference in the optical properties. Fig. 6.24 (a) shows the absorption spectra for 2D periodic array of MIM placed at $\lambda_{res}/4$ above the mirror, which parameters are indicated in Table 6.1. It can be seen that, again, similar to Fig. 6.22, the curves are divided on two groups. The structures with cross and rectangle geometries provide

¹⁰Fig. 20 of Appendix G

¹¹Fig. 20 of Appendix G

¹²Fig. 22 of Appendix G

¹³Fig. 23 of Appendix G

the response with a wider spectral width compare to the cross and the rectangle. The main difference can be connected with different structure widths l_2 , as the thickness of the metal for all the structures is 35 nm . Structures with higher width provide higher losses having higher area of the electric current distribution. As a results, structures with higher dissipation provide more narrow spectra.

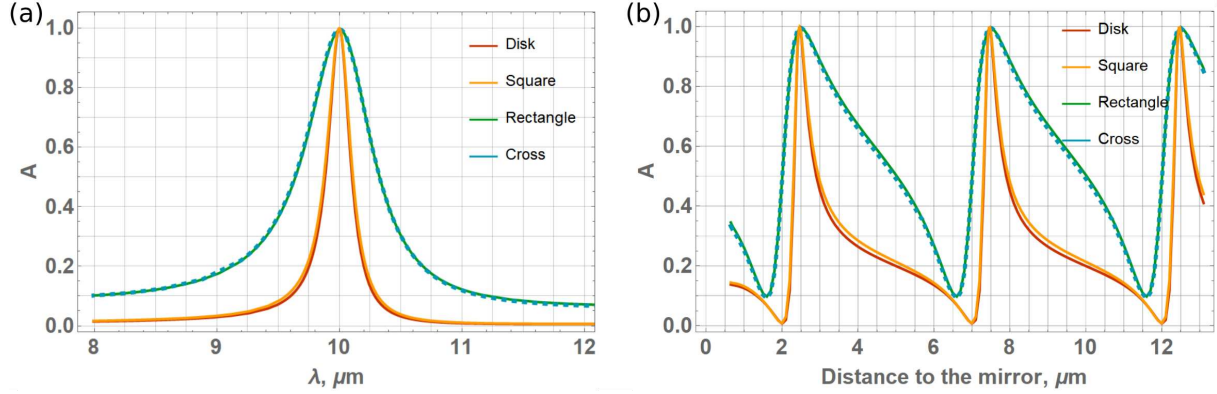


Figure 6.24: The response of periodic 2D arrays for MIM structure with four different geometries, placed at the distance $2.5\text{ }\mu\text{m}$ from the mirror (from a structure center). All structures are optimized for the critical coupling at the resonant wavelength $10\text{ }\mu\text{m}$, being placed at the distance $2.5\text{ }\mu\text{m}$ from the mirror. The geometric parameters of the structures are provided in Table 6.1. (a) The dependence of the absorption on the wavelength. (b) The dependence of the absorption on the distance from the mirror, counted from the center of the structures at the resonant wavelength.

Fig. 6.24 (b) shows the dependence of the absorption on the distance to the mirror at the resonant wavelengths $10\text{ }\mu\text{m}$ for 2D periodic arrays of MIM with different geometries. The distance is measured from the center of the structures, thus the curves begin from non-zero value. Again, similar to the above result, the curves are divided into two groups. Response of the cross and rectangle shaped MIM is similar to that of the 1D periodic MIM array in Fig. 6.20. By contrast disk and square shaped MIMs are much more sensitive to distance change in the distance range $(\lambda/4 - \lambda/2)$. This division into two groups can be assigned to different resistance structures, while their asymmetry in shape can be related to the change in sign of the magnetic field of the standing wave, and thus the $\mathbf{E}_m \times \mathbf{H}_w$ term. However, the full analysis of the shape of the curves remains outside the scope of this work, since it requires a detailed analysis of the operation of a coupled system of two resonators (MIM and Fabry - P  rot).

It is however interesting to note that at the distance $\lambda_{res}/2 = 5\text{ }\mu\text{m}$ the response of structures with the disk and square geometries is very weak, although they are placed in the anti-node of the external magnetic field. This is caused by the fact that the period for the critical coupling at the $\lambda_{res}/4$ is significantly smaller compared to the classical MIM mode, observed for half

structures. Thus, being excited by the external magnetic field, the array of structures become over-coupled.

To summarize, the mode, obtained for all geometries, is similar to the 1D MIM modes in the x direction (the direction of the electric field of the incident wave). However, due to the limitation in the y direction, more complicated fields and current decomposition appear. They are stronger on the sides of a structure, and for rectangular and cross geometries the magnetic field is excited above and below it. The behaviour of considered geometries can be divided into two groups, in dependence on the width of the structure, and thus, optical losses. Demonstrated properties of both groups can be useful for different applications. However, all structures have large dielectric thicknesses which contribute significantly in the total structure volume. For a better absorber in a bolometer it is desirable to reduce its volume. In Section 6.4.4 it is demonstrated that the critical coupling conditions can be achieved for structures with smaller thicknesses. However, at first, let us consider the influence of the metal optical properties on the critical coupling of 2D periodic arrays of MIM structures.

6.4.3 Metals

In this Section, the response of 2D periodic array of half MIM structures on the mirror is analyzed for four different metals, while the dielectric is the same – silicon. The critical coupling is considered, and influence of physical characteristics on the optical losses and the critical coupling is discussed. The absorption mechanism is considered in detail. It is shown that the difference in total volumes for structures is not significant, although gold and copper demonstrate better results. However, for the considered mode whatever the metal, the volume of the dielectric significantly exceeds the volume of the metal. In the same time, the thermal response does not differ significantly with the nature of the metals, except for copper which has a slightly higher thermal capacity.

Critical coupling

Again, in this Section, to save computation time, the half-cut structures are considered. The cross geometry was chosen as the reference geometry due to its versatility. The aspect ratio of its bar is 0.1, and metal layer thickness is about 35 nm . The comparison is done for four metals: gold, silver, copper and aluminum. Their permittivities at the resonant wavelength $\lambda_{res} = 10\text{ }\mu\text{m}$ are presented in the Table 6.2. Both the real and imaginary parts of the permittivities vary

significantly from metal to metal. Gold has the smallest imaginary part of the permittivity, thus it can be considered as the most "ideal" metal, while that of aluminum is about five times higher.

Table 6.2: Optical properties of studied materials at the wavelength $10\ \mu\text{m}$

Material	Gold	Silver	Aluminum	Copper	Silicon
Permittivity ε	$-3792 + 825i$	$-4666 + 1382i$	$-6764 + 4299i$	$-4825 + 1072i$	11.56
Skin depth, nm	25.7	23	18.5	22.8	–
Conductivity, MS/m	$1.38 + 6.32i$	$2.30 + 7.78i$	$7.17 + 11.28i$	$1.79 + 8.05i$	–

The critical coupling conditions of the half-structures were found for the different studied metals and for different dielectric thicknesses. The computations focused on an area of balance between the ratio l_1/p and the dielectric thickness: too high a value of l_1/p leads to another mode, too high values of the dielectric thickness are not useful for the intended purpose. Thus, the intermediate zone is considered. The geometric parameters of the critically coupled structures are grouped together in Tables 11–14 of Appendix G, Section G.7. The critical coupling curves for the different metals are presented in Fig. 6.25. The dots show the sets of values found for the critical coupling conditions, and the solid lines have been interpolated using the $l_1/p \propto 1/h$ dependency shown in Section 6.4.2. A good agreement is observed between the points and the curves.

It can be seen that for the same ratio l_1/p , a greater dielectric thickness is required for aluminum, and a smaller – for copper. The observed difference is linked to the difference in the permittivity of the metals. It determines optical losses through electrical conductivity at optical frequencies and skin depth. Both of them are presented in Table 6.2 at the considered wavelength $10\ \mu\text{m}$. The electrical conductivity is computed from permittivity as $-i\omega\varepsilon_0\varepsilon$ [147], and the skin depth is computed with Eq. 6.1.

However, direct comparison of permittivity, conductivity or skin depth does not allow explain the order of the critical coupling curves. Indeed, the optical response of a structure includes in a more complex way both the effects due conductivity and skin depth, both described by the real and imaginary parts of the permittivity. Additionally, as mentioned in Section 6.4.2, the excited mode at the critical coupling conditions is different for a 2D MIM from that of an infinite 1D MIM. The distribution of the current density in the metal is then more complex. In

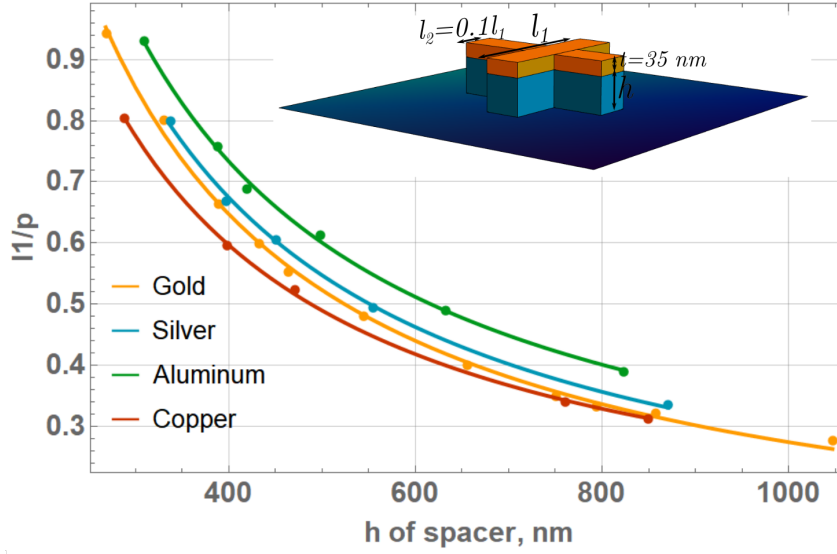


Figure 6.25: Properties of the cross half cut structures on the mirror constructed with different metals, critically coupled at the resonant wavelength $10\ \mu\text{m}$. Critical coupling curves, a schematic representation of the structure is inserted.

the following sub-section, it is shown how these factors lead to the observed order of the critical coupling curves.

Optical losses

To study the difference in behaviour of 2D MIM made from different metals let us consider structures at the same l_1/p ratio and with the same geometrical cross-section. The study is performed for rectangular geometry because of a simpler electric current density distribution (a detailed explanation of the similarity of the cross and rectangle geometry for polarized radiation is presented in Section 6.4.5). Parameters for the critical coupling at $\lambda_{res} = 10\ \mu\text{m}$ are presented in Table 6.3.

The width and thickness of all the metal layers are the same for all structures, thus they all have the same geometric cross-section. As critical coupling conditions are found for the same resonant wavelength, the structures have roughly the same length (see Table 6.3). The only small variations we can see are due to the change in the effective index of the mode provided by the change of the dielectric thickness. Since it is a second order effect (the relative variations are less than 2%), the length will be considered constant.

It can be seen, that the distribution of the structures by the dielectric thickness in Table 6.3 is the same as in the figure 6.25: highest for aluminium and lowest for copper. On the other hand, their order is not directly in accordance with the order of the metal skin depth or the

Table 6.3: Critical coupling parameters for the resonant wavelength $\lambda_{res} = 10\mu m$ for rectangle half-structures computed with different metals; and the electric current trough the cross-section (normalized on the highest value), calculated for this conditions.

Metal	Period $p, \mu m$	Side length, $l_1, \mu m$	Side length, $l_2, \mu m$	l_1/p	Dielectric thick- ness, nm	Gold thick- ness, nm	I_{cr}^2
Aluminum	3.86	2.358	0.25	0.611	488	35	0.64
Silver	3.81	2.253	0.25	0.605	452	35	0.77
Gold	3.65	2.191	0.25	0.600	430	35	0.78
Copper	3.698	2.202	0.25	0.595	398	35	1.0

electric conductivity (Table 6.2). This influence can be taken into account with the effective cross-section (similar to Section 6.3.4). However, due to the complex field distribution in the metal bar, the comparison here is limited to the total excited currents in the metal.

As discussed in Section 6.4.2, the dissipation for such structures is due to losses by the Joule effect. The power dissipated in the metal writes: $P_d = 0.5V Re(\mathbf{E}\mathbf{j}^*)$ [3], where the star symbol denotes the complex conjugation, and V is the volume. As the distribution of both the electric current density and the electric field is inhomogeneous (Fig. 23 of Appendix G) and the length of the structures is almost constant, the study is here limited to the middle section (i.e. orthogonally to the length l_1 of the metal bar). As shown below, this is enough to explain the observed effects. Thus, the dissipated power writes $P_d \propto \int Re(\mathbf{E}\mathbf{j}^*)dA$, where A is the geometrical cross-section. Applying the Ohm's law it comes, $P_d \propto \int Re(1/\sigma)|\mathbf{j}|^2dA$, or

$$P_d \propto Re(\sigma)/abs(\sigma)^2 \int |\mathbf{j}|^2dA. \quad (6.11)$$

In the discussion below, $I_{cr}^2 = \int |\mathbf{j}|^2dA$. It is also should be noted that in the previous sections (for example, Section 6.4.2), only structures made of gold are considered. Thus, the term $Re(\sigma)/abs(\sigma)^2$ was not explicitly mentioned, since it is the same for all structures, while the geometric parameters were different. In this sub-section, the opposite situation is presented: the geometric parameters are the same, but the electrical conductivity varies and plays a key role.

For the 2D arrays presented in Table 6.3, the periods, and therefore the incident fluxes per structure, are almost constant, as is the absorbed power per structure (the critical coupling is considered). The geometrical section being the same, the dissipated power is defined only by the electric current and the metal conductivity. To define I_{cr} , consider the distribution of the electric current density in the metal bar under the critical coupling conditions.

For the considered mode, the electric current is orientated along the length of the structure, or $\mathbf{j} = j_y \mathbf{y}$. The distribution of the current density $abs(j)$ at the surface of the metal bar is shown in Fig. 6.26 (a). The strongest current is excited at the edges of the structure. As shown in Section 6.4.4 for 2D MIM structures, the magnetic field of the mode is excited in two directions: along the width of the metal bar (i.e. above and below) and along its thickness (i.e. at the sides) while the electric field is directed along the longest side. Thus, the electric current which is excited by the component E_y of the electric field, is directed in the direction y and decreases exponentially from the faces of the metal bar: $\exp(-t/\delta)$, where t is the distance to the excited surface and δ the metal skin depth. Thus, the electric current density through a cross-section of the metal bar is proportional to the sum of two exponential terms in the directions x and z .

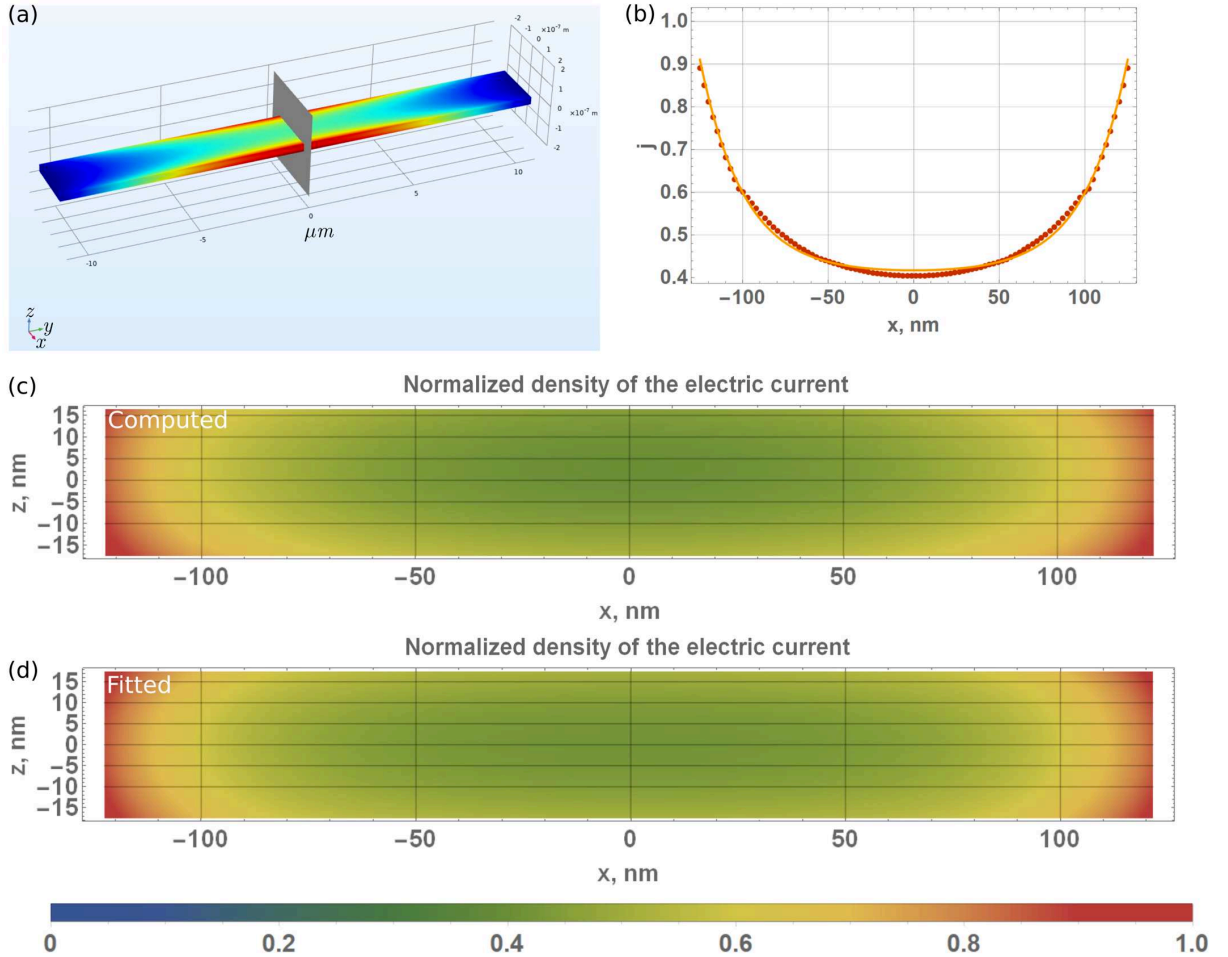


Figure 6.26: The distribution of the electric current density (the absolute value) in the metal part of half MIM on the mirror. (a) The distribution on the metal bar surface. The gray plane shows cut at the middle of the structure. (b) Extracted and fitted distribution of the normalized absolute value electric current density in the cut plane in the center of the thickness ($z = 0$). (c) The normalized absolute value of the electric current density in the middle plane (shown in picture (a)), extracted from numerical simulations. (d) The fitted map of the normalized absolute value of the electric current density in the middle plane (shown in picture (a)).

Let us consider the distribution of the absolute value of electric current density (Fig. 6.26 (a), color scale) in the plane xOz in the middle of the metal bar (shown as a grey plane in Fig. 6.26 (a)), where the current density is strongest. This distribution (normalized) for the gold bar is shown in Fig. 6.26 (c). It can be seen that in the central part the current is minimal, and the highest values are located in the corners of the bar. This distribution, which varies in two directions, can be described as:

$$j(x, z) = a(\exp(-x/\delta) + \exp(x/\delta)) + b(\exp(-z/\delta) + \exp(z/\delta)). \quad (6.12)$$

Here $j(x, z)$ is the absolute value of the electric current density through the cross-section. It writes as the sum of two independent currents induced in the $\mathbf{y}$ direction. a and b are the two fitting parameters. The sum consists of four terms, because the magnetic field is excited both at the top and bottom, and on the left and right sides of the metal bar. Also, to simplify the equation, the center of the metal bar is placed at the point $(x_0 = 0, z_0 = 0)$. In this way, the edges of the metal bar are located at $-l_2/2$ and $l_2/2$ in the x direction, and at $-t_2/2$ and $t_2/2$ in the z direction. Thus, an additional term, including one of these values, is present in each exponential term in order to satisfy the boundary conditions. However, in every pair of exponential functions this term is the same, thus, for simplicity, it is included in the coefficient a or b . The fitting parameters for all the metals considered are presented in Section G.8 of Appendix G, as well as the detailed description of the computation procedure.

The result of the fit for the middle line (at $z = 0$) of the section plane is presented in Fig. 6.26 (b). The points, corresponding to the normalized absolute values of the electric densities, extracted from the numerical simulation, are in good agreement with the fitting curve from Eq. 6.12. The fitting result for complete geometrical section is shown in Fig. 6.26 (d). It agrees well with the map of values extracted from numerical calculations, shown in Fig. 6.26 (c). Fine agreement was demonstrated for the other three metals. Thus, the electric current, excited by the E_y component of the electric field, is maximum at each metal - dielectric interface and decreases exponentially towards depth with a characteristic decay length equal to the skin depth.

It is also important to note that in the central part of the metal the electric current density is not equal to zero (Fig. 6.26 (c)). The side surfaces of the metal bar support the maximum current excitation. As the distance between these two surfaces is much greater than the skin

depth (more than 10 times) one would expect a negligible current at the center of the bar. We observe that this is not the case: the current at the center is about 40% of its value at the surface. This is due to the contribution of the other two faces where the excitation although weaker than that of the lateral faces brings a significant contribution because the thickness of the bar is close to the skin depth. Likewise, this contribution to the current is almost constant in the direction z . This is described in the right part of Eq. 6.12, related to factor b . This effect can be significant in the absorption process: while the maximum current density j_{max} is excited at the left and right ends of the bar over a width of about δ , this constant current density j_c is excited over all the entire thickness of the bar with a width of about ten δ . Thus, although its amplitude is smaller, it can provide even higher dissipation in the metal bar.

The analytical description of the electric current density (Eq.6.12) along the directions x and z , allows to calculate the total current I_{cr}^2 , responsible for the optical losses, through a cross-section of the metal bar (direction y). The results of the calculations are presented in the right column of the Table 6.3, after having been normalized to the largest value, that of copper. It can be seen that the obtained values correspond to the order of the critical coupling curves (Fig. 6.25), in ascending order: aluminum, silver, gold, and copper. Likewise, $Re(\sigma)/abs(\sigma)^2$ values are ordered in reverse order, with a higher value for aluminum and a lower value for copper. As a result, according to the Eq. 6.11, multiplying I_{cr}^2 by the factor $Re(\sigma)/abs(\sigma)^2$ provides similar values of the dissipated power P_d for all the metals. These values differ by a few percent, proportionally to inequality of periods.

Thus, although all the structures have a metal bar of the same geometric section, the optical losses therein depend on the metal conductivity and on the skin depth in a complex way. The electrical conductivity is included not only explicitly in the dissipated power P_d (Eq.6.11) but also through the current density ($j = \sigma E$) whose distribution is strongly inhomogeneous due to the skin depth effect.

To illustrate this complexity, consider the case of aluminum. Among the metals studied here, aluminum has the highest absolute value of the electric conductivity which tends to decrease the resistance of the metal bar (i.e. the dissipated power). It has also the smallest skin depth which tends to increase this resistance. Due to these two opposite effects, the resistance provided by aluminum is higher than that of the other studied metals. Thus to keep the critical coupling conditions, for a given dielectric thickness (i.e. given fields) the period must have the smallest value (i.e. the highest ratio l_1/p). The best compromise between conductivity and skin depth is

found for copper. Despite copper-based structures do not exhibit the highest conductivity nor the largest skin depth, they reach the critical coupling conditions for the largest period when the dielectric thickness is fixed (Fig. 6.25).

To summarize, the nature of the used metal acts on the conditions of the critical coupling by two opposite effects, the skin depth and the finite conductivity of the metal. The complex spatial distribution of the fields makes this dependence strongly nonlinear. Thus, the losses in structure are determined not by the conductivity or the skin depth in a direct way, but by their complex balance. Consequently, the critical coupling conditions for a fixed l/p ratio are reached for the smallest dielectric thickness with copper and the highest with aluminum (among the metals studied).

Thermal properties

As discussed above (for example, Section 5.5 of Chapter 5), from a thermal point of view, the most correct way to compare response of structures is to consider their volumes. Fig. 6.27 (a) shows the total volumes of structures made of different metals at the critical coupling for the same value of the period (i.e. the same absorbed power). As the comparison is again for the same period, the structures absorb the same amount of energy. It can be seen, that similar to 1D MIM structures (Section 6.3.4), the volume of the silicon is significantly higher than the volume of metals.

For the structure made with aluminum, the highest volume values is required, although no significant difference with other metals was observed. The smallest one – for the copper and gold. The volumes of three metal (except aluminum) have similar volume values. In this way, again, an optimisation of the dielectric volume is significant problem for this kind of structures.

In Section 5.5 it was shown that the volumetric specific heat capacity for copper is higher compare to other materials. Thus, it is useful to compare thermal (heat) capacities for critically coupled structures. They are presented separately for metals and dielectric in Fig 6.27 (b). Again, they are compared for the same period, thus comparison is made for the same incident power. It can be seen, that gold, silver and aluminum (metals) in structures absorb almost the same energy amount in order to raise their temperature by one degree. In the most suitable range of the period ($5 - 7 \mu m$), which avoids diffraction orders at oblique incidence, this value is about $0.1 pJ/K$. Copper structures require higher energy to increase their temperature by one degree, the heat capacity value for these structures exceeds the others by about 1.4 times due

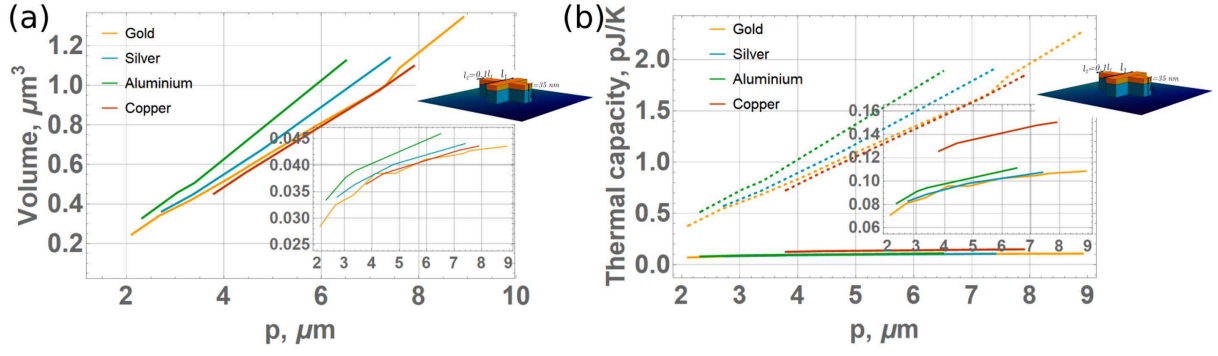


Figure 6.27: Properties of the cross half cut structures on the mirror constructed with different metals, critically coupled at the resonant wavelength $10\mu\text{m}$. (a) Dependence of the total volumes of structures made from different metals and silicon on the period for the critical coupling case. In insert: volumes of metals on the period for the critical coupling case. (b) Thermal capacity in dependence on the period: solid lines for metals and dashed lines for the corresponding dielectric layers.

to higher volumetric specific heat.

In the same time, due to much higher volume, the contribution of the dielectric layer to the thermal capacity significantly exceeds (about ten times) one for the metals. This difference is a consequence of high dielectric volumes (the lines are similar to ones in Fig 6.27 (a)). Thus, the difference between chosen means is insignificant in this situation. From this point of view, again gold and copper are an optimal choice.

It is also important to note that for bolometric applications in addition to the sensitivity of the absorber, the effective increase in temperature when absorbing energy, there is also another important characteristic - the time response. Usually, these values are inversely related. Although this issue is not the subject of discussion in this work, this quantity should be investigated to create an effective device. Thus, the high value of dielectric volume influence this characteristics too.

In this way, although gold, silver, aluminum and copper have a significantly different permittivities, the total volumes of the structures do not differ significantly due to a high dielectric volume. Also, it has been shown, that critically coupled structures, made from this metals have similar thermal response, and gold is optimal among considered metals.

6.4.4 The metal thickness

In this Section, structures having a gold layer of various thicknesses are studied. Considered complete MIM structures are placed at the distance $2.5\mu\text{m}$ above the mirror, this distance is measured from the center of the structure. In order to simplify the model, the rectangle shaped

geometry is chosen in this Section. It is demonstrated that the critical coupling curve exhibits a behaviour different from that of standard MIMs or even 1D MIM arrays at $\lambda_{res}/4$ above the mirror. This is the consequence of different mechanisms both for the funnel effect and for the distribution of fields inside the MIM. Although the fundamental physical explanation is for further study, the excited mode at the critical coupling conditions for the thinnest structure is considered and conclusions on energy absorption is provided. Finally, some optical properties for structures with different metal thicknesses are presented.

Critical coupled 2D periodic MIM arrays with different metal thicknesses

The skin depth of gold at the wavelength of $10\mu m$ is about $26nm$. To find the optimum thickness, three thicknesses close to the skin depth were studied: $5nm$, $20nm$, and $35nm$. The parameters for the critical coupling at $10\mu m$ for these structures are presented in the Table 6.4. For the two highest metal thicknesses, a critical coupling is possible for the same thickness of the dielectric layer, thus a comparison with the same dielectric thickness is possible. The thickness $2.7\mu m$ was chosen for this. If the center of a 2D MIM structure with such a metal thickness is placed in the node of the magnetic field, the dielectric thickness becomes higher than $0.1\lambda_{res}$. The critical coupling parameters for the two structures at the resonant wavelength $\lambda_{res} = 10\mu m$ are as follows. For the structure with the metal thickness $t = 35nm$: the period is $p = 9.6\mu m$, the length $l = 2.80\mu m$, the width $l_2 = 280nm$. For the structure with the metal thickness $t = 20nm$: $p = 5.5\mu m$, $l = 2.88\mu m$, $l_2 = 280nm$. This result is analogous to the case of 1D MIM arrays at $\lambda_{res}/4$ above the mirror with different metal thicknesses, which is shown in Fig. 6.13 of Section 6.3.4: for the same dielectric thickness, a smaller metal thickness requires a higher ratio l_1/p for critical coupling.

Table 6.4: Critical coupling parameters at the resonance wavelength $10\mu m$ for 2D MIM rectangle structures with ratio $l_2/l_1 \approx 0.1$ for different metal thickness

Gold thickness t , nm	Dielectric thickness h , nm	Period p , μm	Side length l_1 , μm	Side length l_2 , μm	l_1/p
35	2700	9.6	2.800	280	0.292
20	2700	5.5	2.880	288	0.523
20	2300	8.02	2.811	281	0.35
20	1998	9.6	2.726	273	0.284
5	300	8.3	3.050	305	0.367

However, for the structure with the metal thickness $t = 20 \text{ nm}$ the behaviour is opposite to that observed for 1D MIM arrays. Here, as shown in the Table 6.4, the decrease in the dielectric thickness leads to the decrease in the ratio l/p . As a result, for the same period value, as in the case of the gold thickness of 35 nm , the dielectric thickness is significantly reduced. A further reduction of the spacer would lead to a further increase of the period, and thus to the appearance of new diffraction orders. But a further reduction of the metal thickness allows the thickness of the dielectric to be reduced even further, as shown for the structure with a metal thickness of 5 nm . The l_1/p ratio for this structure is about the same than that of the structure having a dielectric layer thickness about seven times greater (line 3 of the Table 6.4. The length of the structure is increased compared to MIM having a metal layer of 20 nm , however this difference is not significant (less than 4%).

The reduction in dielectric thickness induced by reduction in metal thickness to maintain critical coupling conditions is the opposite behavior to that observed for 1D MIMs. Indeed, all the critical coupling curves of Fig. 6.13 of Section 6.3.4 verifies the relation $l/p \propto 1/h$, i.e., an increase of the dielectric thickness h leads to an increase in the period¹⁴. In the case of 2D MIM, the dependence is the opposite: the period depends inversely on the dielectric thickness. Let us consider this effect in more detail in the following sub-section.

The critical coupling of a 2D periodic MIM array placed at $\lambda/4$ above the mirror

The critical coupling curve for the MIM array having a metal thickness of 20 nm is presented in Fig. 6.28 (a). The points represent the critical coupling conditions found in the numerical calculation (presented in Table 16 of Section G.9, Appendix G), and the solid line is the fit curve, having the quadratic dependence indicated in the figure caption. It can be seen, indeed, that the increase of the dielectric thickness leads to an increase in the ratio l/p . Thus, as the length l_1 does not change significantly, this behaviour is connected mainly with the period decrease. The dependence of the period on the dielectric thickness at the critical coupling is presented in Fig. 6.28 (b), where the results of numerical simulations are fitted by inverse quadratic function.

Taking into account the dependence $P_{in} \propto p^2$, the decrease in the period with the increase in the dielectric layer thickness to maintain the critical coupling, means that the thicker the dielectric, the weaker the absorbed power. Indeed, as shown in Fig. 6.28 (c), the norm of the electric field at the central point of the metal-silicon interface of the upper metal layer decreases

¹⁴This conclusion concerns structures maintained at the critical coupling conditions with the same metal thickness t .

with the growth of the silicon thickness h . As the geometric parameters of the metallic layers do not change significantly, the absorbed power $P_d \propto \sigma l_1 A E^2$ decreases as the thickness of the dielectric increases.

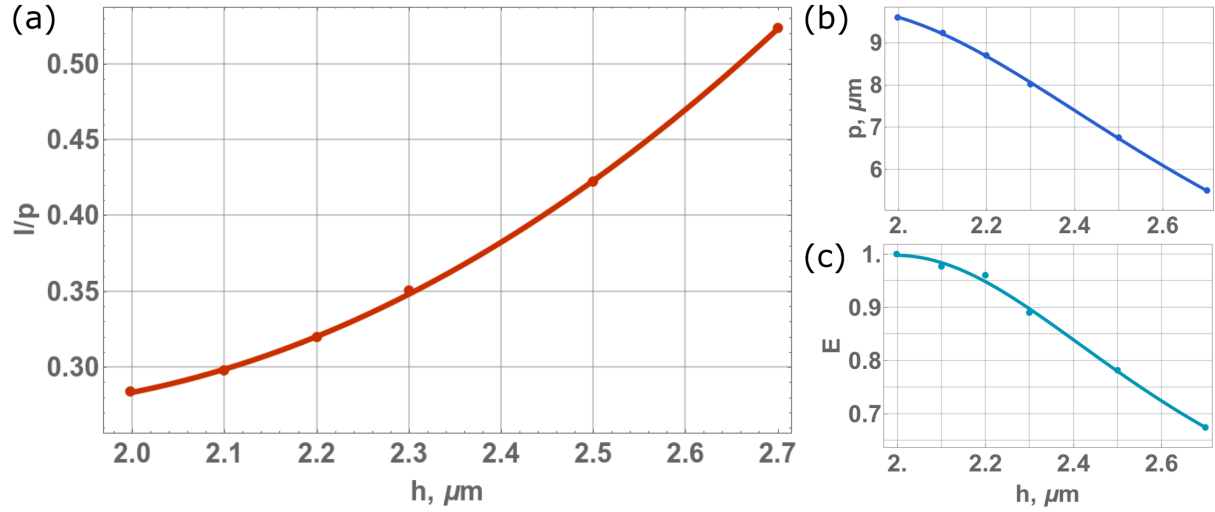


Figure 6.28: The critical coupling for the 2D periodic MIM array of the 20 nm metal thickness placed at the distance $\lambda/4$ above the mirror. For all plots points shows the numerical results for the critical coupling conditions, and solid lines are fitting curves. (a) The critical coupling curve. The fitting curve has dependence $ax^2 + bx + c$, where $a = 0.32 \pm 0.01$, $b = -1.15 \pm 0.03$, $c = 1.30 \pm 0.04$. (b) The period p vs the dielectric thickness h , the fitting curve is $a/(x^2 + bx + c) + d$ with $a = 8.99 \pm 0.20$, $b = -3.70 \pm 0.02$, $c = 4.34 \pm 0.02$. (c) The norm of the electric field at the center of the metal-silicon interface of the top metal layer. The fitting curve is $a/(x^2 + bx + c) + d$ with $a = 0.34 \pm 0.16$, $b = -3.99 \pm 0.06$, $c = 4.50 \pm 0.09$, $d = 0.34 \pm 0.1$.

This significantly different behaviour from that observed with 1D MIMS is the consequence of the limitation of the 2D MIMs in the transverse direction together with a better adaptation to the magnetic field node of the standing wave. The first leads to the significant amplification of the magnetic field of the mode at the 2D MIM edges, which, as demonstrated below, is responsible for the energy funneling to the structure. With the second, the magnetic field node corresponds to the anti-node of the electric field, which allows a different funneling process than those of standard 2D MIMs (where the magnetic field of the standing wave participate in the energy funneling, see Section G.5 of Appendix G). As a result, there is a specific behavior for 2D MIMs placed at $\lambda/4$ above the mirror.

As indicated in Table 6.4, the decrease in the metal thickness is one of the consequences of this specific behavior, as discussed below. The l/p dependence on the dielectric thickness h (Fig. 6.28 (a)) can be supplemented by the data in Table 6.4. Numerical values for two other thickness indicate a shift of the critical coupling curve on the left for smaller metal thicknesses (i.e. towards smaller dielectric thicknesses), and on the right - for the thicker metal layers.

However, as for some structures, the thickness of the dielectric is about $2\,\mu m$ or even more, the lower metal layer is $1.5\,\mu m$ away from the mirror, or even less. Thus, perturbations can come from interactions between the evanescent fields and the mirror or from the influence of the non-zero magnetic field of the standing wave.

Nevertheless, as this mode can be excited in the case of the a silicon layer of only $300\,nm$ (field maps for it, presented in the following sub-section, are similar for ones for thicker structures, presented in Section G.6 of Appendix G), the most significant impact should be connected with the mode fields. In the following sub-section, this excited mode is considered in detail.

The excited mode

As observed above, 2D periodic MIM arrays demonstrate different behaviour than 1D periodic MIM arrays, both being placed at $\lambda_{res}/4$ from the mirror. This behavior is also different from that of standard 2D periodic MIM arrays, i.e. placed on a mirror. Indeed, its critical coupling curve has another type of dependence on the dielectric thickness h , and when the metal thickness is decreased, h should be decreased to keep the critical coupling conditions. This difference comes from another mode excitation. Here, the physical behaviour of 2D MIM above the mirror at the critical coupling is considered in more detail. As structures with a thinner dielectric layer is better suited for potential bolometric applications, the 2D periodic array with metal thickness of $5\,nm$ was chosen for this study. The other parameters are listed in Table 6.4. The thinner dielectric layer also provides another advantage for the mode study: the lower layer of the structure is as far away as possible from the mirror (compared to the other 2D MIM arrays studied above and having thicker metal layers), decreasing thus the interaction of the evanescent field with the mirror.

In general, the absorption of an optical antenna can be divided into two steps: (1) the way in which the energy is collected by the structure (funneling effect) and (2) the way in which that radiative energy is transformed into heat within the structure (optical losses). To study the first step, consider the field maps around a MIM structure (of the array), placed at $\lambda/4$ above the mirror and at critical coupling conditions. The maps of the total fields as well as the map of the time average Poynting vector are placed in Fig. 6.29 for different sections. They are sections of the structure through its center in different directions, as shown in Fig. 6.30 (a). The structure is denoted by a rectangular frame in the maps.

As already mentioned, the main difference from the 1D infinite MIM is the excitation of

more complex fields of the mode. In particular, the magnetic field of the mode now also has a H_z component, and is excited around each metal layer (also outside the structure) in the same way as the magnetic field around a wire carrying an electric current. As shown in Fig. 6.30 (b) for the considered structure, the magnetic field of the mode has both H_y and H_z components, while the magnetic field of the standing wave is close to zero in this region.

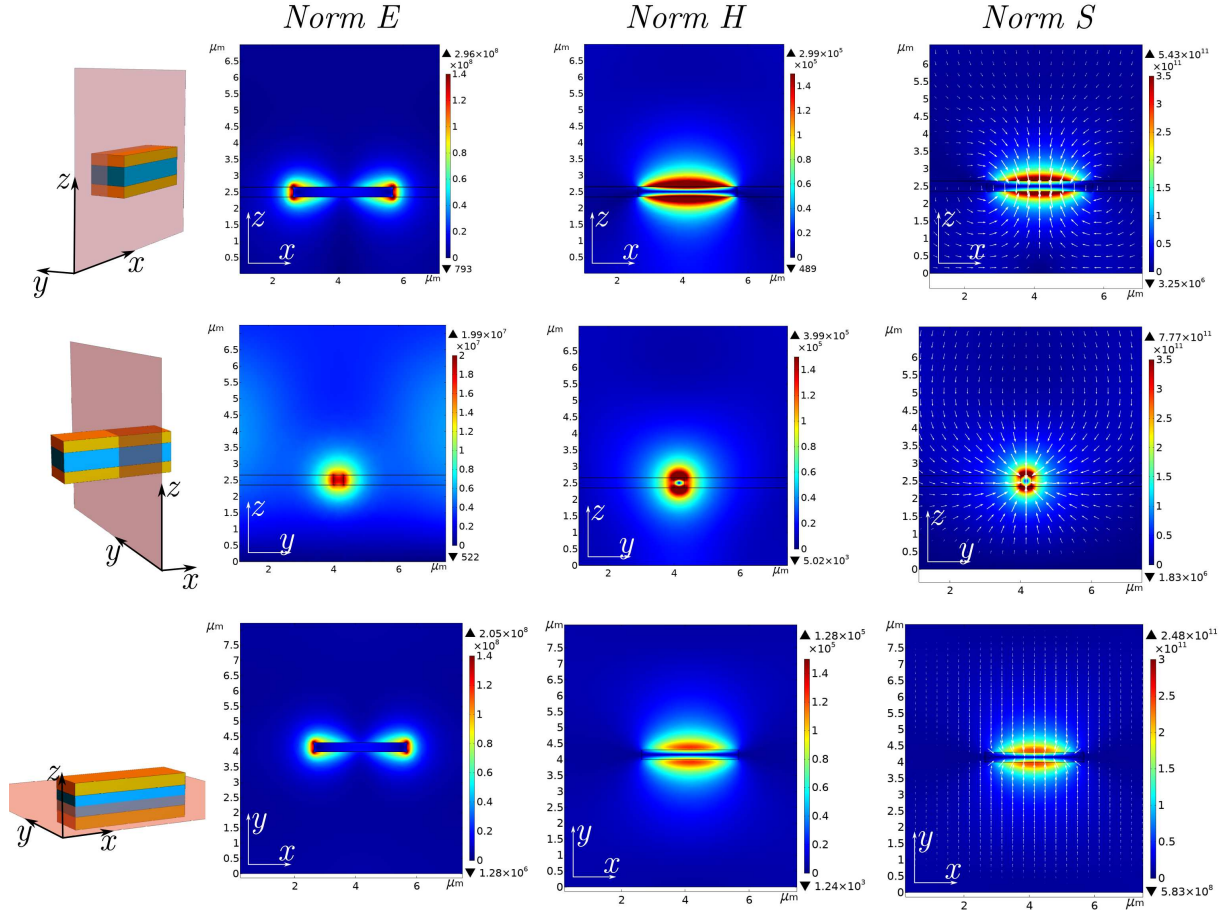


Figure 6.29: Maps of the norm of the total fields (the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$) and the time-average Poynting vector $\mathbf{S}$ for different cross-sections, shown in the left column. The structure is at the critical coupling, being placed at the $\lambda_{res}/4$. White arrows represent the Poynting vector in the logarithmic scale.

The electric field $\mathbf{E}$ (Fig. 6.29, left column) and the magnetic field $\mathbf{H}$ (Fig. 6.29, center column) defined the behaviour of the Poynting vector $\mathbf{S}$ (Fig. 6.29, right column). The color of the maps presents the norm of the vectors while the white arrows indicate their direction and amplitude in logarithmic scale. It can be seen, that unlike a 1D MIM array, the energy comes directly from above and below the structure, and the absorption appears on all sides of metal layers (similar to classical 2D MIM arrays and unlike to 1D MIM arrays). Indeed, the numerical integration of the Poynting vector in the simulation shows that about 65 % of energy reaches

the metallic layer on one of its external faces and is absorbed at the air-metal interfaces. The remainder (about 35 %) reaches the metal via one of its faces internal to the resonator and is absorbed at the silicon-metal interfaces. It can also be seen that there is almost no energy funnel effect in the x direction (along the length of the rectangle): about one percent (of 35 %) of the incident energy comes in this direction, while the rest (34 %) reaches the air-silicon interfaces following the y direction (xOz planes).

A more detailed description of the Poynting vector, showing the terms due to the fields of the standing wave on the one hand and the mode on the other hand (in the same way, as in Eq. 6.3), is provided in Fig. 24 – 26 of Section G.9, Appendix G. It can be seen that for the both effects, the energy coming from the top and the bottom of the structure as well as funnel effect in the y direction, are created by the product of the magnetic field of the mode by the electric field of the standing wave.

It is also important to note that the thickness of the metal layer, which is much smaller than the skin depth (5 nm and 26 nm, correspondingly), influences the mode behaviour. As shown in Fig. 22 of Section G.6 (Appendix G), in the case of a metal thickness of 35 nm, a significant funnel effect is observed in the x direction, similar to the MIM 1D mode. Numerical integration of the time-average Poynting vector shows similar energy flows entering the structure in the x and y directions. According to the numerical simulations, the funnel effect of the energy flow in the x direction is excited by the mode fields (Fig. 27 of Section G.9). It provides the structure with about 23 % of the incident energy, while about 26 % enters in the y direction, the rest passing through the top and the bottom of the structure. Thus, the significant decrease of the metal thickness influences the funnel process. However, this influence has a minor role, as the absorption also takes place at the upper and lower air-metal interfaces, and at the internal interfaces also due to the funnel effect in the y direction.

In this way, in the case of a 2D MIM periodic array above the mirror, the energy flux comes directly on the upper and lower metal surfaces and, by a funnel effect, on the side air-dielectric interfaces. This behavior, different from that of an infinite 1D MIM array, appears to be due to a more complex excitation of the magnetic field of the mode, which is strongest at the sides of the structure.

A more complex behaviour of the fields and of the Poynting vector also leads to a more complex decomposition of the electric current density (Fig 6.30 (c)¹⁵) compared to 1D MIM.

¹⁵also Fig. 23 of Section G.6, Appendix G

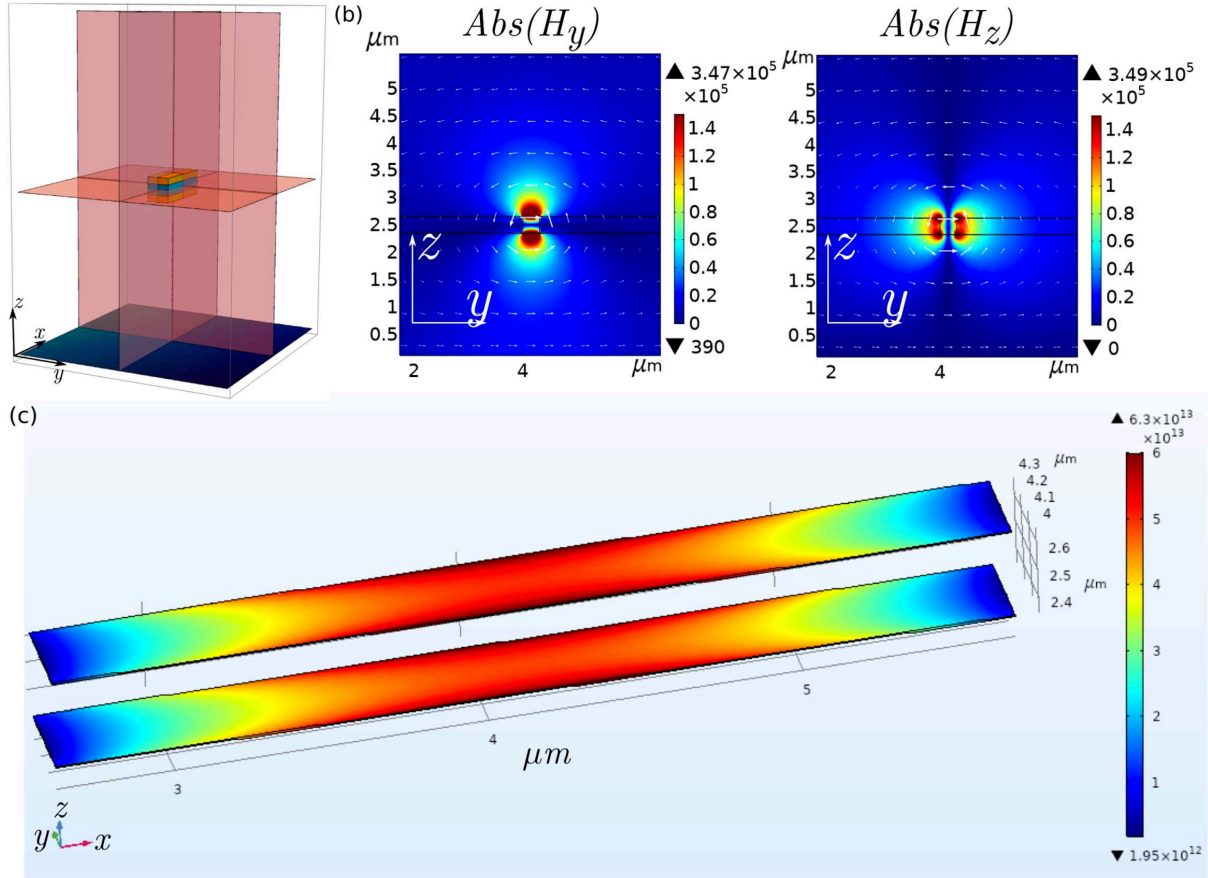


Figure 6.30: (a) A schematic presentation of the cross-section planes, which cut the structure in the middle in different directions, for field maps, presented in Fig. 6.29. (b) The field map of the components of the magnetic field H_y and H_z . White arrows represent the magnetic field direction. (c) The distribution of the electric field density on the metal layers of MIM, placed at the $\lambda_{res}/4$ at the critical coupling.

A strong electric current is excited at the upper and lower air-metal interfaces (a significant part of the incident energy gets there), and at the sides of the structure, in accordance with the magnetic field of the mode. Moreover, this current density decomposition is similar to that of classical 2D MIM, discussed in Section 6.4.3. Despite some differences on the funnel mechanism, in both cases the funnel effect of the energy in the direction y (i.e. through the xOz air-silicon interfaces) is due to the contribution of the electric field of the standing wave.

To summarize, the distribution of the mode fields is more complex for the 2D periodic MIM arrays than for 1D periodic arrays infinite MIM, in both cases structures are placed at the distance $\lambda_{res}/4$ above the mirror. As a result of the limitation in the y direction, the magnetic field and the electric current density are stronger on the sides of the structure, near its mid-length, and for rectangular and cross geometries the magnetic field is excited above and below the structure. As a result, the Poynting vector behaviour and the absorption mechanism are also more complex. Part of energy comes from above and below the structure is absorbed

in the top and bottom interfaces, and the funnel effect of energy in the y direction appears. Additionally, in the case of 2D MIM, the electric field of the standing wave forms the Poynting vector, directing the energy to the structure interfaces, while for 1D MIM all energy absorption is due to funneling, formed by both the electric and magnetic fields of the mode. All mentioned leads to a different critical coupling curve and different behaviour when the metal thickness changes. Opposite to 1D MIM, a reduction in metal thickness induces a reduction in dielectric thickness to maintain critical coupling conditions.

The optical response of 2D periodic MIM arrays with different metal thicknesses

Some optical properties of 2D MIM structures with different metal thickness are considered here. At first, let us consider the absorption spectra (Fig. 6.31) for structures with different thicknesses, which parameters are presented in Table 6.4. The red curve corresponds to the structure with metal thickness of $t = 5 \text{ nm}$, the other parameters being $p = 8.3 \mu\text{m}$, $l = 3.05 \mu\text{m}$, $l_2 = 305 \text{ nm}$, $h = 300 \text{ nm}$; the purple curve to a metal thickness of $t = 20 \text{ nm}$ with $p = 5.5 \mu\text{m}$, $l = 2.88 \mu\text{m}$, $l_2 = 280 \text{ nm}$, $h = 2.7 \mu\text{m}$; and the blue curve to a metal thickness of $t = 35 \text{ nm}$ with $p = 9.6 \mu\text{m}$, $l = 2.80 \mu\text{m}$, $l_2 = 280 \text{ nm}$, $h = 2.7 \mu\text{m}$.

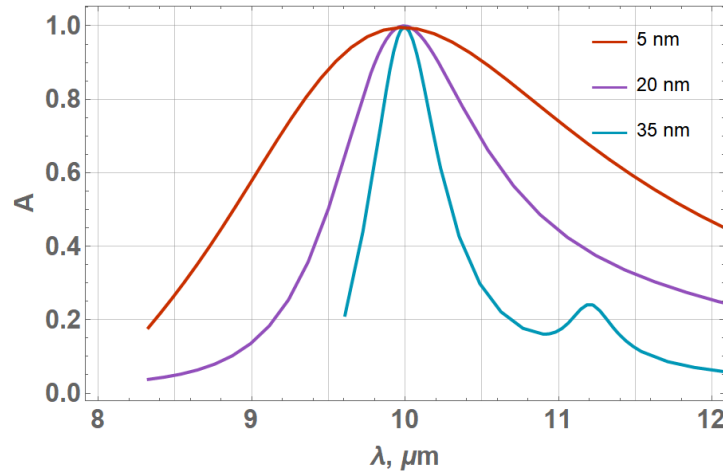


Figure 6.31: Absorption spectra for critically coupled 2D periodic arrays of MIM with different metal thickness, placed at the distance $2.5 \mu\text{m}$ from the center of the structure to the mirror, at the normal incidence. Red corresponds to the structure with the metal thickness $t = 5 \text{ nm}$, purple – with $t = 20 \text{ nm}$, blue – with $t = 35 \text{ nm}$. Lines break at points $\lambda = p$.

From Fig. 6.31 it can be seen that the decrease in the cross section of the metal layer leads to an increase in the width of the absorption spectrum. The widths of the considered structures are relatively close (maximum variation $< 8\%$) while the thickness of the metal layer increases significantly (up to 7 times). The structure with the smallest thickness ($t = 5 \text{ nm}$) has the

widest spectrum, and an increase in this thickness leads to a decrease in the spectrum width. This effect is connected with the change in the optical losses. An increase in the metal thickness results in a non-linear increase in the effective section of the metal bar in accordance with the distribution of the electric current, reflecting the skin effect (Section 6.4.3). As a result, the resistance of the metal bar decreases as well as the resistive losses. The other parameters must then be adjusted to maintain the critical coupling conditions. In the end, an increase in the quality factor of the resonance is obtained.

A similar effect is expected with an increase in the width of the structure as it also defines the effective cross-section (also non-linearly). This explains why the response of the square-shaped geometry is narrower than the rectangle-shaped one in Fig. 6.24.

From this point of view, absorbers with different metal section (the thickness and the width) can serve different aims: rather the wide or narrow absorption band is necessary. Also, a few structures with different characteristics can be placed in one period to cover the wide range of wavelengths. However, from the practical point of view, the change of the width is simpler compare to change the thickness. It allows to fabricate a few different arrays on the same substrate and in the same period (to cover a wider spectral width). The influence of the metal width is discussed more detailed in Chapter 7 for IMI structures.

It also can be noted that for the thicker metal a new peak, connected with interaction of the evanescent field with the mirror, arises. This can be overcome with the decrease of the period due to change of the dielectric thickness.

The dependence on the distance from the mirror for tree structures with different metal thicknesses at the resonant wavelength is presented in Fig. 6.32 (a). The dependence for the 5 nm metal structure has a periodic character, and at a minimum it drops to zero, thus it resembles the 1D MIM case, although it is more symmetric and has wider range of high absorption. For the structure with 20 nm metal thickness the dependence on the distance to the mirror is periodic as well, but the minimum of absorption is about 0.3. The reason for this behavior is the high distance between the two metal plates. As a result of a significant distance (higher than $\lambda_{res}/4$), one of the metal layers is always in a high electric or non-zero magnetic external fields, which means that effectively each of the layers can operate as a separate antenna.

At the same time, the dependence for a structure with 35 nm metal thickness has two different regions. The structure is critically coupled at the distance 2.5 μm from the mirror, with strong absorption at adjacent distances. However, this peak is not repeated, but is replaced by a peak

with absorption at the level 0.25. And the pattern from the range $5.5 - 10.5 \mu m$ is repeated further. This difference is explained by the fact that the evanescent mode takes part in the creation of the critical coupling. As the distance from the center of the structure to the mirror is $2.7 \mu m$, the distance from the bottom of bottom metal to the mirror is $1.315 \mu m$. Thus, the amplitude of the evanescent mode can be strong enough, and thus be reflected from the mirror and change the field compositions around the bottom metal. At large distances, the field of the evanescent mode can no longer reach the mirror, and the field pattern changes significantly. Therefore, at short distances, the structure absorbs a significant part of the radiation, while at longer distances only a small part. Although the structure with the metal thickness of $20 nm$ is placed at the same distance from the mirror, the absorption curve for it remains periodic. There is no contradiction, it only means that in this case the amplitude of the evanescent mode is not strong enough to reach the mirror and make a significant contribution to the fields composition.

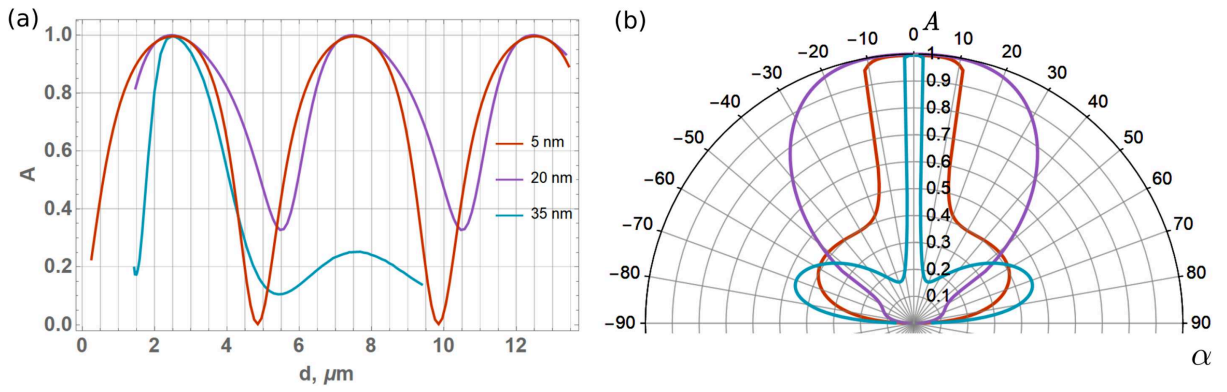


Figure 6.32: Absorption spectra for structures critically coupled at the wavelength $10 \mu m$, placed at the distance $2.5 \mu m$ from the center of the structure to the mirror: (a) in dependence on the distance from the mirror, and (b) in dependence on the incident angle. Red corresponds to the structure with the metal thickness $t = 5 nm$, purple – with $t = 20 nm$, blue – with $t = 35 nm$.

Angular dependencies for three critically coupled structures with different thicknesses at the resonant wavelength $10 \mu m$ are presented in Fig. 6.32 (b). For all structures computations were performed for the case of TE polarisation. For this polarisation the electric field has only the E_x component (i.e. if directed along longer side of the structure), while the incidence plane is the yOz plane (i.e. orthogonal to the longer side of the rectangle). Thus the electric field component of the incidence wave is kept the same, while its magnetic field has changing H_y and H_z components. All three structures demonstrate different dependencies. Mainly it is connected with the period values, which leads to the appearance of a new diffraction order. In the case of the structure with $20 nm$ metal thickness, the period is $5.5 \mu m$, and the high absorption value is

observed in the range $-30^\circ - 30^\circ$. For two other structures the range of high absorption angles is smaller due to higher period values, with sharp decreases on both sides.

To summarize, as a result of the different mode, arising at the critical coupling, the behaviour of 2D MIM differs from an infinite 1D MIM. In the case of 2D MIM structures, a more complex field of the mode is exited. That leads to change to the energy funneling character and to a more complex electric current density distribution. As a result, the critical coupling curve is changed, and keeping the critical coupling the dielectric thickness decreases when the metal thickness decrease. Thus, the optimal 2D MIM structure has both a thin metal and a thin dielectric layer. At the same time, the width and the metal thickness allow influence the response and the effective index, allowing design structures with different characteristics.

6.4.5 Cross vs Rectangle

In the previous sections, it is demonstrated that the critical coupling conditions are possible for various thicknesses of both the metal and the dielectric layers. In this Section, two types of MIM geometry are considered, those giving the smallest volumes: rectangle-shaped and cross-shaped MIMs. The rectangle geometry makes it possible to design MIMs with smaller volumes but exhibits sensitivity to polarization. The cross-shaped geometry provides an insensitive response to polarization but its volume is approximately twice as large. As can be seen from Fig. 6.22, the geometric parameters for the critical coupling are not very different from one structure to another. In both cases, the incident wave is polarized with the electric field along one of the metal bars. In these conditions, let us consider the modes of these geometries at the critical coupling conditions in detail and explain observed differences in the responses.

For this comparison, let us consider the 2D MIM with a thickness of 5 nm of the metal layer. The parameter of the rectangular structure are: the period $p = 8.3\text{ }\mu\text{m}$, the length $l = 3.05\text{ }\mu\text{m}$, the width $l_2 = 305\text{ nm}$, the dielectric thickness $h = 300\text{ nm}$, the distance from the mirror to the center of the structure is $2.5\text{ }\mu\text{m}$. The cross-shaped structure consists of two identical orthogonal rectangles with all the same parameters as those of a rectangular geometry structure. These two rectangles intersect at their center. The absorption spectra of these two structures are presented in Fig. 6.33. Compared to the rectangular structure, the cross-shaped structure has a resonance shifted by about 250 nm towards the low wavelengths. Its spectral width (FWHM) is enlarged by about 17% and its absorption at the resonance is slightly decreased by about one percent. To understand the reasons for this behavior, let us regard field maps.

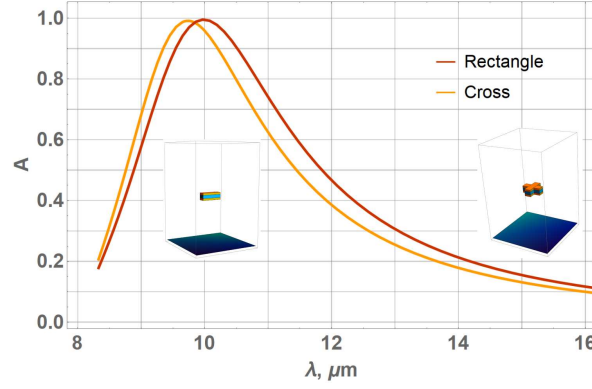


Figure 6.33: The absorption spectra for the rectangle and the cross structures with the same parameters: $p = 8.3 \mu\text{m}$, $l_1 = 3.05 \mu\text{m}$, $l_2 = 305 \text{ nm}$, $t = 5 \text{ nm}$, $h = 300 \text{ nm}$. The distance from the mirror to the center of the structure is $2.5 \mu\text{m}$.

The electric field maps in various cross sections are presented in Fig. 6.34 for the two structures considered. They look a lot similar. The mode behaves like an electric dipole, and it is the same for the upper and the lower metal layers. A difference for the Poynting vector plotted in the top view is observed. It is due to the presence of the second (orthogonal) bar in the case of the cross geometry. While there is no difference observable on the side view.

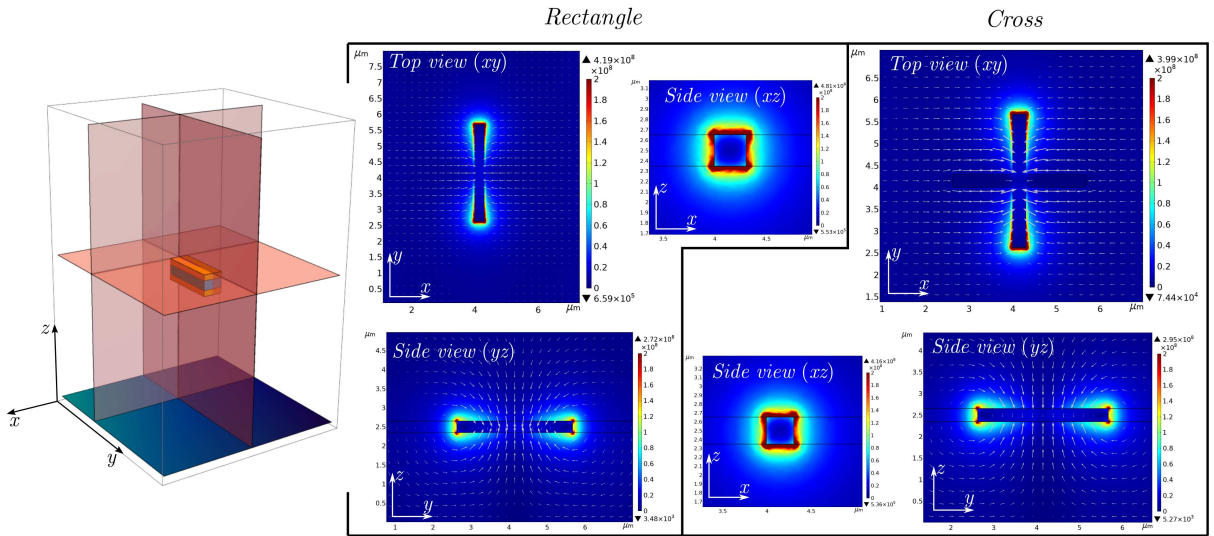


Figure 6.34: On the left: schematic presentation of the cross-sections, for for which the color maps of the field on the right are plotted. The plane xOz is at the edge of the structure, yOz at its middle, and xOy is at the top of the metal layer. The maps present the norm of the electric field vector and the time-average Poynting vector in the case of the rectangle and the cross structures with the same parameters: $p = 8.3 \mu\text{m}$, $l = 3.05 \mu\text{m}$, $l_2 = 305 \text{ nm}$, $t = 5 \text{ nm}$, $h = 300 \text{ nm}$. The distance from the mirror to the center of the structure is $2.5 \mu\text{m}$. All maps are presented at the resonant wavelength: $10 \mu\text{m}$ for the rectangle geometry and $9.75 \mu\text{m}$ for the cross geometry. The sections planes are presented on the left: the xOy plain on the top level of the top metal, the yOz plane at the center of the structure, the xOz plane at the center of the rectangular structure and shifted to the $w/2$ for the cross structure (i.e. the edge of the second rectangle). The color and arrow scales are the same for all the maps.

The magnetic field maps for the rectangular and the cross geometries are presented in

Fig. 6.35 (a). On the top view maps, it can be seen that in the case of rectangular geometry, the strongest magnetic field is concentrated along the longer edges, with a maximum in the center. For the cross-shaped geometry, this pattern is broken: the magnetic field in the area of the intersection of the two rectangles forming the cross is greatly reduced. This behaviour is associated with the electric current in the metal layers (Fig. 6.35 (c)). Due to the presence of the second rectangle, the electrical current broadens, covering in this zone of intersection, a width greater than that of the rectangles l_2 . Thus, although the total current can be conserved, its density decreases and the distribution of the magnetic field changes. The side views also shows the magnetic field decline in the area of intersection for the cross geometry. Meanwhile, the Poynting vector maps are similar.

The break observed in the field pattern for the cross-shaped geometry helps to explain the changes in the absorption peak (Fig. 6.33) compared to that of the rectangle: the blue shift, reduced absorption and narrower spectral response.

The considered rectangular-shaped structure is a $\lambda/2$ antenna. Any complex field of evanescent waves can be presented as an effective wave, propagating along the structure in both directions. This wave is reflected on the free edges of the structure with the effective phase ϕ around π . Thus, the condition for the resonance can be presenter as:

$$2n_{\text{eff}}kl + 2\phi = 2m\pi, \quad (6.13)$$

where $k = 2\pi/\lambda$ is the wave vector and m is a natural number. This equation expresses the relation between the resonance wavelength and the structure length, presented in Chapter 5 and applied to 1D MIMs: $\lambda_{res} \approx 2n_{\text{eff}}l$.

In the case of rectangular geometry, this $\lambda/2$ antenna can be presented as two $\lambda/4$ antennas connected by their ends. The phase shift due to the reflection at the free edges of the structure remains $\phi_1 \approx \pi$ while that at the connection of two antennas is $\phi_2 = 0$. This second reflection can be materialized by an infinitely thin film of perfect conductor placed at the mid-length of the structure. For each of these half-bars ($\lambda/4$ antenna of the length $l_1/2$) the condition for the resonance of Eq. 6.13 becomes

$$2n_{\text{eff}}k\frac{l_1}{2} + \phi_1 + \phi_2 = 2m\pi. \quad (6.14)$$

In the case of cross-shaped geometry, these antennas are separated from each other. Indeed,

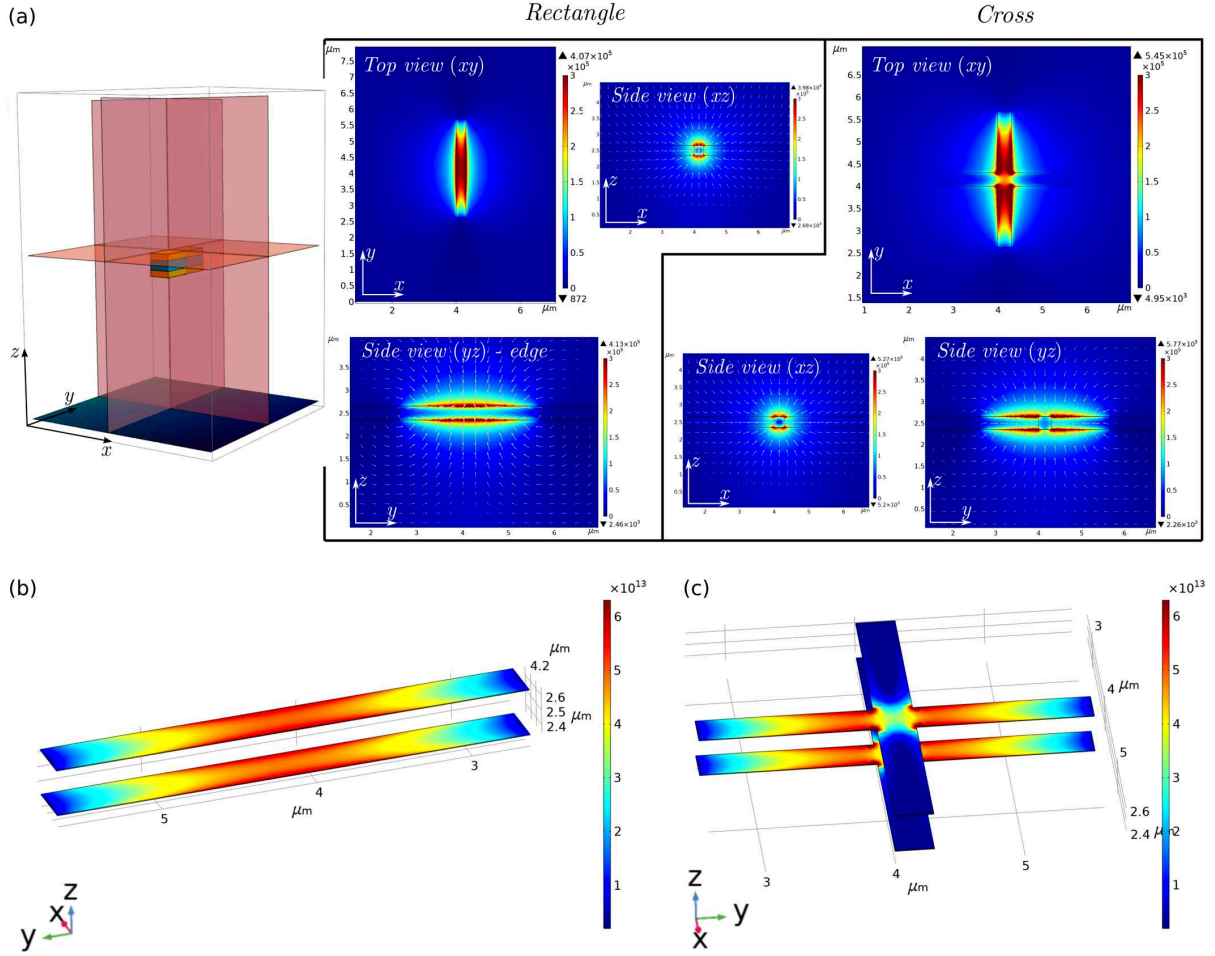


Figure 6.35: The maps of the norm of the magnetic field vector and the time-average Poynting vector (a), and the electric current density ((b) and (c)) for the metal parts in the case of the rectangle and the cross structures with the same parameters: $p = 8.3 \mu\text{m}$, $l = 3.05 \mu\text{m}$, $w = 305 \text{ nm}$, $t = 5 \text{ nm}$, $h = 300 \text{ nm}$. The distance from the mirror to the center of the structure is $2.5 \mu\text{m}$. The schematic presentation on the left of the plot (a) shows the sections for the color maps: xOz at the middle of the structure for rectangle and in the plane at the edge of the second rectangle for the cross, yOz at its edge, and xOz at the top of the metal. All maps are presented at the resonant wavelength: $10 \mu\text{m}$ for the rectangle geometry and $9.75 \mu\text{m}$ for the cross geometry. The sections planes are presented on the left: the xOy plain on the top level of the top metal, the yOz plane at the side edge of the structure, the xOz plane at the edge of the structure. The color and arrow scales are the same for all the maps.

a significant decay is observed in center of structure in the maps of fields and electric current. Let us assume that this field gap can be modeled by introducing a finite thickness of perfect conductor at the mid-length of the structure (as shown in Fig. 6.38 (a)). As for the rectangle-shaped antenna, the evanescent field propagates along the main metal half-bar and is reflected by the perfect conductor with a phase shift $\phi_2 = 0$.

In this model, the length of a single $\lambda/4$ antenna is reduced compared to that of the rectangle, which introduces a blue shift of the resonance (Eq. 6.14). Indeed, as shown in Fig. 6.36 and in Fig. 6.37 (a), increasing the width w of the orthogonal (to the main bar along which the electric

field is directed) bar blueshifts the resonance. From the field maps, it is clear that fields which in the main half-bar are comparable to those observed in the rectangle, penetrate slightly in the overlap region of the two bars constituting the cross. Thus, the reflection of the wave does not appear exactly at the edge of the transverse bar but somewhere in the area where the two bars overlaps.

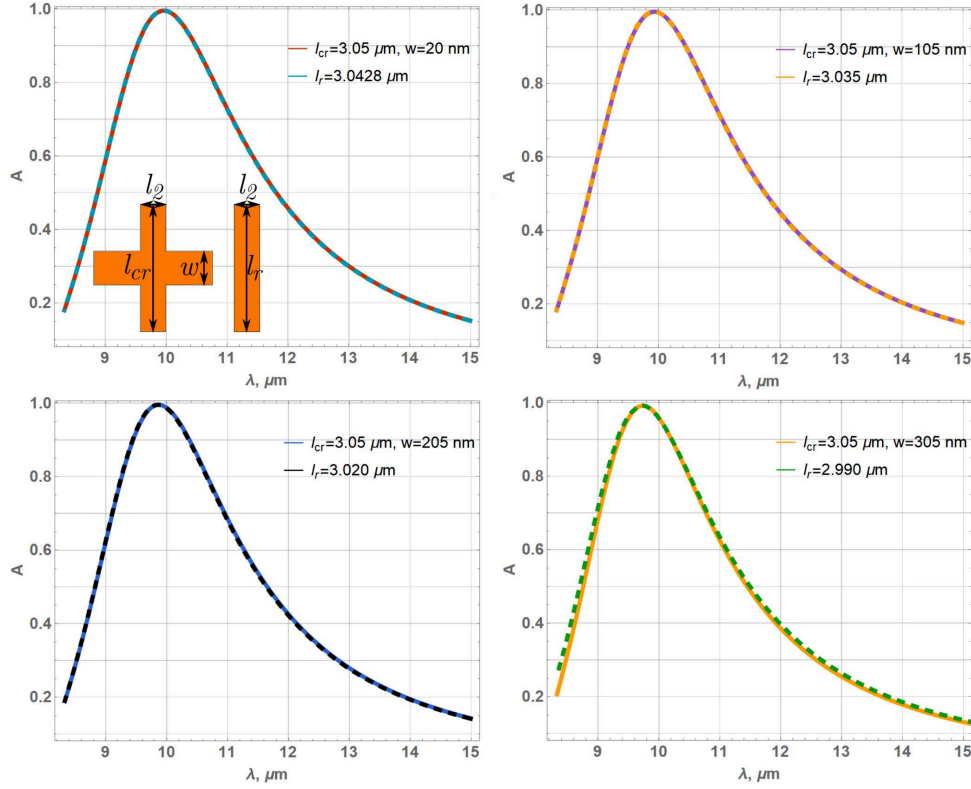


Figure 6.36: Absorption spectra of rectangular and cross-shaped antennas. The length of rectangular antenna l_r was modified to correspond to a response of cross-shape structures. The other parameters were not changed, they are the same for all spectra: $p = 8.3 \mu\text{m}$, $l_2 = 305 \text{ nm}$, $t = 5 \text{ nm}$, $h = 300 \text{ nm}$. The distance from the mirror to the center of the structure is $2.5 \mu\text{m}$.

To confirm this observation, the absorption spectrum of rectangular antennas was fitted to that of cross-shaped antennas using only one free parameter: the length of the antenna. It is important to note that the spectra thus found coincide perfectly, a few examples of cross and rectangle geometry are shown in Fig. 6.36, there is only exception for $w = 305 \text{ nm}$, where small difference in the spectral width is observed. The lengths of the structures with rectangular shape l_r are shorter than one for the structures of the cross shape ($l_{cr} = 3.05 \mu\text{m}$). Presented spectra confirm the hypothesis of an operation with two shorter $\lambda/4$ antennas. The difference $l_{cr} - l_r$ between the lengths of the rectangular and cross structures having the same peak position while the other parameters¹⁶ are identical, is plotted Fig. 6.37 (b) (blue points) as a function of the

¹⁶Except the width of the transverse bar of the cross

width w of the transverse cross bar. This dependence has a linear character and is fitted by the blue line. In the same plot, a green line shows the w values. The position of the blue line is below the green one, this allow to conclude that the length of the $\lambda/4$ antenna of the cross-shaped antenna is longer than just $(l_1 - w)/2$. The reflection, therefore, occurs inside the overlap region of the two crossing bars.

In fact, according to Eq. 6.14, the observed shift in figure 6.37 (b) could also be due to a non-zero reflection phase shift ϕ_2 . The discussion that follows would not be changed. The simplicity of the model (one single parameter) as well as the excellent agreement of the spectra with those of the shortened rectangles (Fig. 6.36) suggests that this approach is preferable.

Using the notation from Fig. 6.37 (a), the effective length l' of the antenna $\lambda/4$ writes:

$$l' = \frac{l_1}{2} - \frac{w}{2} + \delta w. \quad (6.15)$$

Here, δw characterizes the depth of the field penetration into the region of the overlap of two bars and indicates the position of the reflection plane.

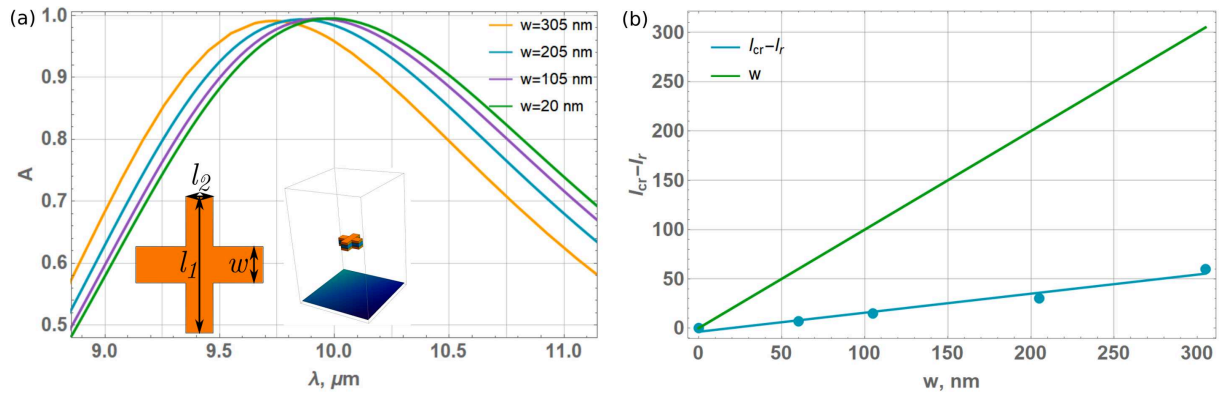


Figure 6.37: (a) The absorption spectra for the cross structures with different width of the orthogonal bar. Other parameters: $p = 8.3 \mu\text{m}$, $w = 305 \text{ nm}$, $t = 5 \text{ nm}$, $h = 300 \text{ nm}$. (b) The difference of the length of the rectangular (l_{1r}) and cross (l_{1c}) structures, providing the absorption peak at the same resonant wavelength (different for each point). The other parameters are the same, taken from the plot (a). Points present direct result from the numerical simulations. The blue fitting line is $ax + b$, $a = 0.39 \pm 0.02$, $b = -3.64 \pm 1.78$. The green line shows function $w/2$, and is used to indicate which difference would be observed in the case of reflection on the edge of the orthogonal cross bar.

In this way, the change in the length of the antenna $\lambda/4$ in the case of a cross, compared to that of the rectangle, is

$$\delta l = l' - \frac{l_1}{2} = -\frac{w}{2} + \delta w. \quad (6.16)$$

The schematic presentation for this case is shown in Fig. 6.38 (a). Then, introducing the modified length in Eq. 6.14, the resonant conditions for the resonant wavelength $\lambda_r = \lambda_0 - \delta\lambda$ (where λ_0

is the resonant wavelength in the case of the rectangular geometry, $\delta l = 0$) is

$$\frac{4n_{\text{eff}}\pi}{\lambda_0 - \delta\lambda} \left(\frac{l_1}{2} + \delta l \right) + \phi_1 + \phi_2 = 2m\pi, \quad (6.17)$$

or, in a first approximation $\phi_1 \approx \pi$ and $\phi_2 \approx 0$,

$$4n_{\text{eff}} \left(\frac{l_1}{2} + \delta l \right) = \lambda_0 - \delta\lambda. \quad (6.18)$$

Finally, the shift of the resonant wavelength is given by:

$$\delta\lambda = -4n_{\text{eff}}\delta l = 2n_{\text{eff}}w - 4n_{\text{eff}}\delta w. \quad (6.19)$$

As a result, the shift of the resonant wavelength depends on two components. The first term (proportional to w) introduces a linear shift of the resonant wavelength. The second term δw is also a function of w . It should be noted that for widths higher than l_2 , the field map becomes more complex and another mode is excited. Indeed, the upper limit of the width is $w = l_1$ for which the antenna becomes a square. The mode is then very different from that of the rectangle (see Section 6.4.2). For the intermediate cases where $l_2 < w < l_1$, a complex mixture of between these two extremes is expected, the study of which stays outside the scope of this work. Thus, the discussion below is limited by the case $w < l_2$.

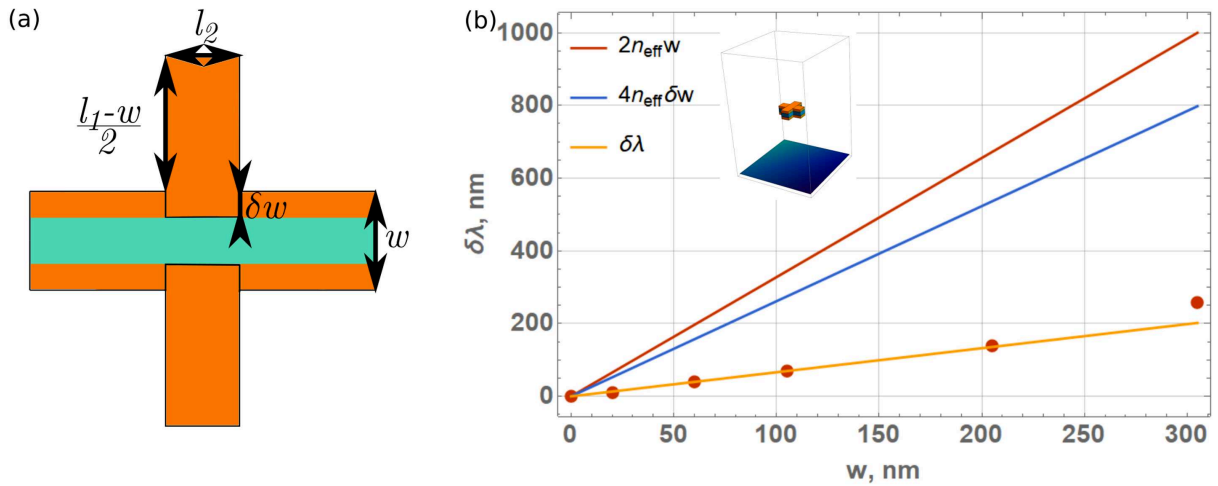


Figure 6.38: (a) the schematic presentation of the cross MIM structure (top view), the central part of which is occupied by the perfect conducting layer. In this way, the part of the two bar overlap region is excluded from the consideration, and the length of one $\lambda/4$ antenna is $l' = \frac{l_1}{2} - \frac{w}{2} + \delta w$. (b) The wavelength shift for the cross-shaped antenna with different width of the orthogonal bar. The red $2n_{\text{eff}}w$, the blue line $aw + b$, $a = 2.60 \pm 0.01$ and $b = 1.46 \pm 1.44$, yellow is the sum of both lines $\delta\lambda$.

Fig. 6.38 (b) shows the dependence of the shift of the resonant wavelength $\delta\lambda$ on the width w of the transverse rectangle. In the considered case $\lambda_0 = 10\ \mu m$, and the other parameters are $p = 8.3\ \mu m$, $l_1 = 3.05\ \mu m$, $l_2 = 305\ nm$, $t = 5\ nm$, $h = 300\ nm$. The red points in the plot are results of numerical computations and the solid lines are built according to the model of Eq. 6.19. The effective index of the mode $n_{\text{eff}} \approx 1.64$ is found for the structure of the rectangular geometry and, as the dielectric and metal thicknesses are kept constant for all geometries, this value is also applied for the cross geometry.

From Fig. 6.38 (b) it can be seen that the red dots demonstrate linear behaviour. The red line in the graph shows the first linear term $2n_{\text{eff}}w$. The blue line is a fitted line for the expression $4n_{\text{eff}}\delta w = 2n_{\text{eff}}w - \delta\lambda$, and, the yellow line is their difference (i.e. $\delta\lambda = 2n_{\text{eff}}w - 4n_{\text{eff}}\delta w$). It can be seen that the yellow line ($\delta\lambda$) fit perfectly all points except the last one, for which the mode starts to change its character. In this way, the penetration depth δw of the field into the overlap region increases with the growth of the w , but remains smaller than $w/2$. This result is consistent with the difference between the lengths of rectangles and crosses, presented in Fig. 6.37. For example, for the width $w = 150\ nm$ the value δw , found from Fig. 6.38 (b), is about $59.7\ nm$. Thus, the length of the rectangular antenna, corresponding to this cross-shaped antenna, should be $w - 2 \cdot \delta w \approx 31\ nm$ shorter compare to the cross-shaped one. Indeed, this value is close to that, presenter by a blue line from Fig. 6.37 (b) for this width.

The considered model makes it possible to explain the three differences observed between cross-shaped and rectangle-shaped geometries. The overlap region of two orthogonal bars of the cross-shaped structure leads to the decrease of the effective length of the $\lambda/4$ antennas compare to the rectangular geometry. As a result, the resonant conditions change, decreasing the resonant wavelength (blueshift). In the overlap region, the energy flow feeding the optical losses in the metal is greatly reduced as it can be seen in Fig. 6.35. The critical coupling condition is no longer satisfied. As a result, the absorption decrease compared to the rectangular structure (Fig. 6.33), and the width of the spectral response decreases, as for any under-coupled resonator.

In the considered model, only the change of the width of the transverse bar is considered, while the centers of the two intersecting bars have remained unmoved. In the light of considered model, a shift of one of the centers with respect to the other would transform the symmetric system of two equivalent $\lambda/4$ antennas into an asymmetric system of two antennas of different length (i.e. with different resonant wavelength). Moreover, this shift also leads to the excitation of the orthogonal metal bar due to the asymmetry of the electrical currents. This more complex

case has not been studied in this work despite the interest in the questions it poses. This will be the subject of further study.

To summarize, the 2D MIM modes of the most compact geometries among those considered – rectangle and cross – were compared. The response of these structures with the same parameters is indeed very close. The only difference appears at the intersection of two rectangular bars for the cross geometry. In this region the actual width of the resonator is much wider leading to a strong reduction of the electric current density (i.e. the electric field). As a result, the resonance peak of the cross is blueshifted with different characteristics. To explain this difference, a new approach for the MIM mode is presented. In this approach, the $\lambda/2$ antenna is replaced by the two antennas $\lambda/4$, separated by a conductive plane. While these two descriptions give the same result for the rectangle-shaped antenna, this is not the case for the cross-shaped antenna for which the actual length of each half MIM resonator is reduced, leading to a blue shift. For width of the transverse bar smaller than that of the main bar, the resonance shifts linearly with the width, corresponding to an increase of the width of the separating conductive plane. The application of this model makes it possible to explain all the differences observed between the two geometries and opens perspectives for reconsidering the MIM mode approach and for a more in-depth study of complex MIM structures.

6.4.6 Comparison of 2D and 1D periodic arrays

In this Section, rectangle and cross geometries for 2D MIM arrays are compared to 1D arrays of MIM, in both cases arrays are placed at $\lambda_{res}/4$ above the mirror.

The absorption spectrum of a 1D MIM array was calculated (the green curve in Fig. 6.39) for the same parameters as those of rectangular-shaped 2D MIMs at critical coupling (Fig. 6.33: $t = 5\text{ nm}$, $h = 300\text{ nm}$ dielectric, $l_1 = 3.05\text{ }\mu\text{m}$, $p = 8.3\text{ }\mu\text{m}$). The maximum value of absorption is low, and the system is definitely under-coupled. It is possible to optimize it by reducing the period, while keeping the same thicknesses of metal (5 nm) and dielectric (300 nm). The optimized absorption curve is presented in Fig. 6.39 in violet. In this case, the length l_1 is $0.99\text{ }\mu\text{m}$, and the period is $1.05\text{ }\mu\text{m}$. The maximum of absorption is about 52%, what is far from a critical coupling. The system is still under coupled, and a significant increase in the ratio l/p would be necessary to obtain the critical coupling conditions, but this is no longer possible because the period is almost equal to the length.

This example demonstrates the difference between 1D and 2D arrays of MIM: due to limita-

tion in the invariance direction, the magnetic field becomes non-homogeneous in the y -direction. In 2D MIM, the highest current density occurs at the sides of a structure, providing greater absorption than in 1D MIM. Thus, critical coupling conditions can be achieved with 2D MIMs having a finer metal and dielectric as well as a lower l_1/p ratio than 1D MIMs.

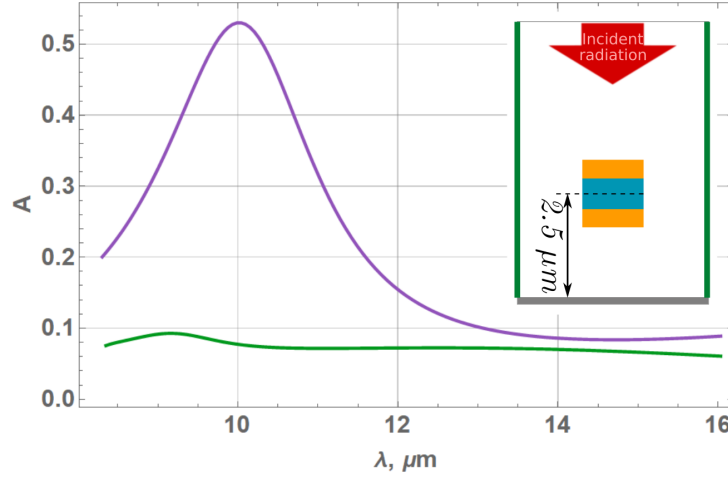


Figure 6.39: The absorption spectra for the 1D MIM structure, placed at the distance $2.5\mu\text{m}$ above the mirror. The metal thickness is 5 nm , and the dielectric spacer thickness is 300 nm . The green curve represent the array of 1D MIM with the length $l_1 = 3.05\mu\text{m}$ and the period $p = 8.3\mu\text{m}$, for the violet – $l_1 = 0.99\mu\text{m}$, and $p = 1.05\mu\text{m}$.

The total volume density can be compared for 1D MIMs and 2D MIMs. In Section 6.3.4, the minimum total volume density was found ($\omega_V \approx 270\text{ nm}$) for the 1D periodic MIM arrays. For comparison, let us compute this value for the rectangular and cross geometries. As mentioned, for a rectangular 2D MIM, the total volume density is $\omega_V = (l_1 \cdot l_2/p^2) \cdot (2t + h)$. For the rectangular structure considered in this Section, $\omega_V \approx 4.2\text{ nm}$. For the cross geometry this value is calculated as $\omega_V = ((2l_1 \cdot l_2 - l_2^2/p^2) \cdot (2t + h))$, and for the considered geometry (the resonant wavelength is shifted a bit from $10\mu\text{m}$) $\omega_V \approx 8\text{ nm}$. In the same time, the minimal density value ω_V for the infinite 1D MIM array at the distance $\lambda_{res}/4$ above the mirror is about 270 nm . Thus, 2D MIMs allow heat to be concentrated in much smaller volumes than 1D MIMs: 33 times for cross-shaped MIM and 64 times for rectangle shaped MIM. This is useful, in particular, for bolometers applications.

In conclusion, 2D MIMs allow to reach the critical coupling conditions for much smaller volumes than 1D MIM due to the structured field along the y direction. In this way, the array of 2D MIM allow to concentrate the generated heat in much smaller volumes than 1D MIM, which is best suited for bolometric applications.

Conclusion

In this Chapter, the possibility of using MIM structures, placed at unusual location – at the node of the magnetic field – for absorption applications has been considered. It has been found that an array of MIM, all of whose layers are structured, can be critically coupled to the incident wave although placed at the distance $\lambda_{res}/4$ above the mirror.

In the case of a 1D periodic array of infinite MIM, the excited mode at the critical coupling is similar to the classic MIM mode. If this 1D MIM array is placed at $\lambda_{res}/4$ above the mirror, the funnel effect directing a flow of energy to the resonator is provided by the magnetic and electric fields of the excited MIM mode, unlike the classic MIM configuration, where the coupling is provided by the magnetic field of the standing wave and the electric field of the mode. This specific condition of the critical coupling leads to a shift of the critical coupling curve towards the highest dielectric thicknesses when the metal thickness is reduced. This is connected with the increase in the resistance of the metal layer which varies inversely with its effective thickness t_{eff} . However, the volume density for 1D MIM structures at the critical coupling is far from the set objective: for the smallest volume densities, the ratio l/p is high, i.e. the heating is distributed over all the surface, and for the smallest l/p the volume density is higher than 300 nm , which is far from desirable values.

2D periodic MIMs placed at $\lambda/4$ above the mirror demonstrate much smaller values of the volume density compare to 1D MIM. It is possible to achieve due to a different mode, arising at the critical coupling, which appears as a result of the limitation in the lateral direction. Contrary to the magnetic field of the 1D MIM mode, which is invariant in this direction, this 2D MIM mode is characterized by two components of the magnetic field, with the strongest values at the sides of the metal patch near its mid-length. The electric field of the mode has a dipole-like distribution along the length of the structure.

In turns, the change in the field leads to modification of the energy funneling character and to a more complex electric current density distribution compare to 1D MIM. In the 2D MIM case the coupling is mostly provided by the electric field of the standing wave and the magnetic field of the mode. In this case, a significant part of the energy comes from above and below the structure and is absorbed in the air-metal interface. Additionally, a funnel effect of energy in the y direction (trough planes built from the length and the thickness) appears. Consequently, the

electric current density has more complex compare to 1D MIM decomposition. In the particular case of a rectangular-shaped structure, the electric current distribution in each metal layer can be described with four exponential terms, with the maximum at the edges at the mid-length. For other geometries this tendency is saved, although distribution is more complex.

As a result, the shape of the critical coupling curve is different for 2D MIM placed at $\lambda_{res}/4$ above the mirror. Keeping the critical coupling, the period decreases when the dielectric thickness increases. Moreover, for thinner metal layers this curve is shifted in left, towards smaller dielectric thickness, which is opposite to the 1D MIM. Thus, for 2D MIM the structure it is possible to achieve the critical coupling for the metal thickness of 5 nm and the 300 nm thick silicon layer. As a result, much smaller compare to 1D volume density is possible to achieve for 2D MIM, 4.2 nm for rectangular structure and about 8 nm for the cross one.

Being a complex function of the skin depth and the conductivity, the electric current decomposition defines losses in the metal. It was found, that among structures made of four popular metals, suitable for absorption applications (gold, silver, aluminum, and copper), having a difference in both parameters, gold one exhibits a bit smaller total volume and thermal capacity, thus can be considered as optimal among studied structures. However, as the thickness of the dielectric layer at the critical coupling is significantly higher than the metal layer, it provides the main impact on the structure volume, and the difference in metals is not significant compare to it.

The width of the structure allow influence the response and optical losses, allowing design structures with different characteristics. The study of four simple MIM shapes (square, disk, rectangle ($a.r. = 0.1$) and cross ($a.r. = 0.1$)) shows division into two groups, depending on the width. As the metal thickness was kept the same, for the same dielectric thickness only the width influences a structure response due to the change in optical losses. Thus, the same l/p ratio requires a smaller dielectric thickness for the disk and square geometry. At the same time, the total volume of the structure is smaller for the rectangle and cross geometries, thus, they are defined as the most optimal for bolometric applications: the rectangular MIM is for the problem where polarisation is essential, and cross MIM – for non-polarized radiation.

At the same polarisation along the length, the cross-shaped and rectangular structures show a similar response, but for the same length, the resonant peak of the cross-shaped structure is blueshifted relative to that of the rectangle. Also, its absorption is slightly decreased. This difference can be explained by modeling the cross-shaped structure by two $\lambda/4$ antennas, sep-

arated by a conductive plane, instead of the $\lambda/2$ antenna of the rectangular structure. The application of this model makes it possible to explain all the differences observed between these two geometries and opens perspectives for reconsidering the description of the MIM mode and for a more in-depth study of complex MIM structures.

2D MIM structures with a metal thickness greater or equal to the skin depth as well as a dielectric thickness higher than $0.1\lambda_{res}/4$ were observed. For such large distances between the metal bars, the coupling of the surface plasmons carried by these interfaces becomes very weak. This suggests that the structure can be simplified. As it operates in the presence of the mirror, it is therefore possible to use the layer which absorbs only half of the incident radiation. Thus, it is reasonable to consider structures with only one metal layer: insulator-metal-insulator (IMI) resonators. Their analysis, when placed at the distance $\lambda_{res}/4$ above the mirror is presented in the following section.

Insulator-metal-insulator structures

7.1.

Introduction

In this Chapter, some results, both numerical and experimental, illustrating that the Insulator-Metal-Insulator (IMI) structure has a potential to be a good resonator for bolometer applications are presented. Beyond these preliminary results, a complete study will be necessary to grasp the concept of physics involved in these structures.

In Chapter 6, it was demonstrated that 2D MIM structures can be critically coupled to the incident wave when located at about $\lambda/4$ above a mirror. In this case, the structure can be much smaller than the wavelength, with a dielectric layer 300 nm thick and metal layers 5 nm thick. However, the largest contribution to the volume of these structures comes from the dielectric layer. Since a structure of smaller dielectric volume would present a greater potential for bolometers, it should be asked whether another type of resonator would be better suited for this kind of application.

On the other hand, it is shown in chapter 6 that the same field and current distributions appear on the two metallic layers of the MIM structure. It is also shown that the dielectric thickness for some structures can be as large as $0.3\lambda_{res}/n$ at the critical coupling. In this case, each layer can function independently, as a pair of separate resonators. It is well known that the maximum absorption of a single thin film in air is 50% and that the introduction of a mirror, transforming this two-port system into a single-port system, allows to overcome this limitation [87]. In this case, a single metal layer should be sufficient. That is to say, an insulator-metal-insulator (IMI) structure placed in front of a mirror should be able to be critically coupled to an incidence wave.

An IMI structure is only composed of a single metal layer surrounded by dielectric. Its geometry allows it to behave like an electric dipole, like one of the metal bars of MIM. This dipole can be excited by the electric field of the incident wave. In this case, the maximum of the electric field of a standing wave formed by the mirror is the optimal position to couple these electric dipoles to the incident wave. Thus, the metal volume can be decreased by a factor of two compared to that of an MIM, and the structure becomes easier to fabricate. In addition, the condition imposed by our industrial partner to keep the structures at a distance $\lambda_{res}/4$ above the mirror, corresponds to maximum excitation of the IMI antenna, unlike the case of MIMs.

In this Chapter, the optical properties of these optical antennas with a single metal layer are explored. Similarly to the previous Chapter, the resonators are placed at the distance $\lambda/4$ above the mirror, i.e. at the maximum of the electric field of a standing wave formed by an incident wave reflected by the mirror. As stated earlier previously for MIM structures, the response of cross-shaped geometry is insensitive to polarization while that of rectangle ones is very sensitive to it. Also, the cross-shaped geometry provides an optical response similar to that of the rectangular geometry provided the light is polarized along one of the axes of the cross. Thus, in this Chapter, for the sake of simplicity, only rectangular geometry is considered. The metal slab is completed with a dielectric membrane that mechanically supports the metal part above the mirror, and also provides additional parameters to adjust the optical response of the structure.

This Chapter is organized as follows. First, a description of the geometry of an IMI array at the distance $\lambda_{res}/4$ above the mirror is presented in Section 7.2. Then, the excited mode in this structure is analyzed under the critical coupling conditions in Section 7.3. It is shown that, similar to MIM structures under similar conditions, the electric field of the mode has a dipole-like distribution along the length of the metal bar, while the magnetic field is excited at the side edges, with the maximum near the mid-length. The coupling is provided by the electric field of the standing wave and the magnetic field of the mode. After that, the influence of geometric parameters on the optical response of an array of IMI antennas is considered in Section 7.4. It is shown that the width and thickness of the antenna define the effective index of the mode, as well as the influence of section of the metal on the non-radiative losses in the antenna and the conditions of the critical coupling.

The optical response of an array of IMI antennas at the distance $\lambda_{res}/4$ above the mirror is presented in Section 7.5. The IMI arrays demonstrate a large manufacturing tolerance which

separate them from the mirror: the absorption being almost total in a band approximately one micrometer wide. The antennas of rectangular geometry, which are sensitive to the polarisation of the incident radiation, demonstrate a high tolerance to the incident angle if the electric field is orientated along it. They have no significant absorption when the electric field has the orthogonal direction. It is also shown that IMI arrays placed at $\lambda_{res}/4$ above the mirror has the smallest volume density among considered structures. The simplicity of IMI arrays also allows to model them as a point dipole array, as presented in Section 7.6. The dipole approximation predicts their absorbance spectra using the polarizability of a single antenna, which avoids time consuming full-wave simulations.

Finally, the experimental validation of the quasi-total absorption for IMI arrays placed at $\lambda/4$ above the mirror is presented in Section 7.7. A good agreement between the experimental data and numerical simulation is shown.

7.2.

The studied structure

In this Section, the geometry of the studied IMI arrays is described. All other results, unless otherwise specified, are provided for this type of structure.

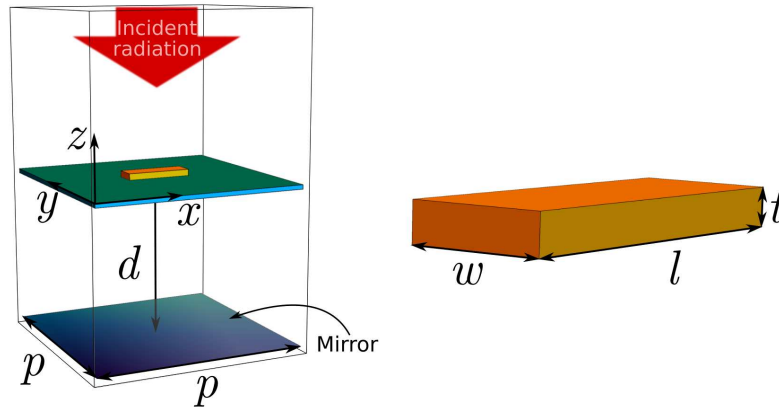


Figure 7.1: The considered in this Chapter IMI structure, which is a rectangular metal bar on a dielectric membrane, placed above a mirror. In the right the measures of metal rectangle are presented.

Typically, an IMI structure consists of three layers of materials: a metal layer interposed between two dielectrics (insulator). In this Chapter the same materials as in Chapter 6 are chosen. The top dielectric layer is air (or vacuum), the metal one is gold ($\varepsilon = -3792 + i825$ at $10\mu m$), and the lower dielectric layer is composed of a silicon membrane ($\varepsilon = 11.56$ at $10\mu m$) for the mechanical support and an air layer (or vacuum). Again, constant dielectric values are important for a rapid search for the optimum response, as well as for separating the influence

of geometric parameters from material properties on the response of the structure.

The schematic presentation of the IMI structure is shown in Fig. 7.1. The structure is placed at the distance d above the mirror (a perfect electric conductor), and this distance is measured from the lower edge of the dielectric. In numerical computations, the thickness of the membrane is 300 nm , as for this value a flat membrane of a large area can be produced [148].

The system is periodic, with the period p equals in both directions. The metal part is a rectangular slab of thickness t , width w , and length l . The membrane is considered as a plane-parallel plate of thickness h which covers the entire illuminated surface. The incident wave comes from the top. All numerical simulations were performed with the multiphysics simulation software COMSOL Multiphysics. In these simulations, a port is placed at a distance of one wavelength above the studied structure. For most sections the structure is optimized for the wavelength $10\text{ }\mu\text{m}$.

7.3.

The IMI mode

In this Section, the fundamental mode of the IMI structure is analyzed at the critical coupling conditions under normal incidence, and physics of the coupling process is described on the base of the Poynting vector. Without limiting its scope, the study is started for a structure with a metal thickness of 20 nm , after which other thicknesses will be considered. For a silicon membrane thickness equal to $h = 300\text{ nm}$, the critical coupling conditions were found for a period $p = 3.18\text{ }\mu\text{m}$, a length of the structure $l = 1.8\text{ }\mu\text{m}$ and a width $w = 200\text{ nm}$. The absorption spectra for this structure is presented in Fig. 7.2.

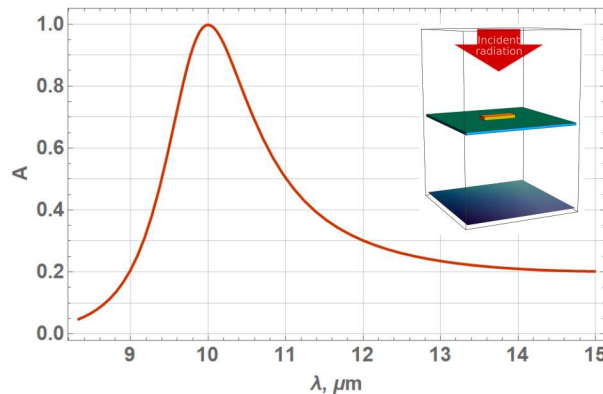


Figure 7.2: The dependence of the absorption on the wavelength for the IMI structure with the parameters $p = 3.18\text{ }\mu\text{m}$, $l = 1.8\text{ }\mu\text{m}$, $w = 200\text{ nm}$, $t = 20\text{ nm}$, $h = 300\text{ nm}$. The distance to the mirror is $2.5\text{ }\mu\text{m}$.

To consider field maps, let us specify the planes which are used for their presentations. They

are shown in Fig. 7.3 with respect to the metal bar¹, and their choice is connected with the specificity of the fields. The orientation of the electric and magnetic fields of the incident wave are also indicated in Fig. 7.3.

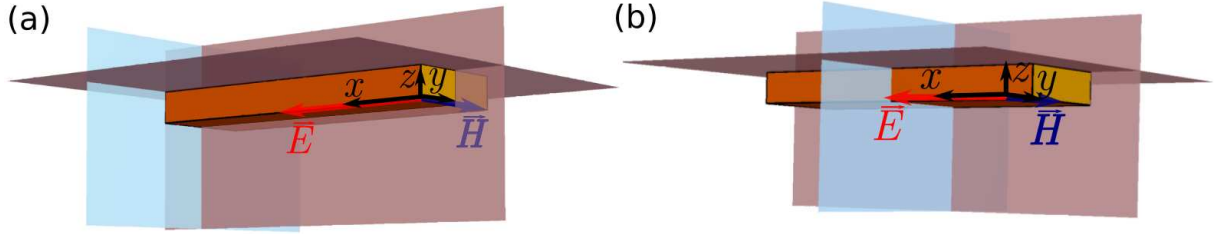


Figure 7.3: Planes, for which field maps are shown in Fig. 7.4. (a) Planes is for the electric field maps: the xOy plane on the top of the metal, xOz plane at the middle of the metal bar and yOz plane at this edge. (b) Planes is for the magnetic field maps: the xOy plane on the top, xOz at the edge, and yOz at the middle of the metal bar. On both plots, a red arrow indicates the direction of the electric field of the incident radiation, and the blue one - of the magnetic one.

The field maps at critical coupling are presented in Fig. 7.4. It is worth to mention that for the side views, the metallic area is represented by a thin line while the membrane is drawn as a rectangle of the 300 nm thickness (from 0 to $0.3\text{ }\mu\text{m}$ on the x -axis). In the maps, it contains a few vertical lines, which are the result of the technical solution designed with the aim to build a mesh physically suitable for numerical computations. As a result, the membrane is divided into sectors, as shown on one of the maps.

The distribution of the electric field norm and the magnetic field norm at the metal level is similar to that of 2D MIM structures. The electric field of the mode is dipolar in shape with maxima at the ends of the metal bar. The magnetic field is maximum along the longest edges of the metal bar. Similar to 2D MIM structures, the flow of the Poynting vector is directed towards the center of the structure in the top views (xOy plane), and towards the metal layer in the side views (xOz and yOz planes).

It can also be noticed that in the side views of the second and third rows of Fig. 7.4, that the fields are asymmetrical with respect to the plane of the metal patch. They have stronger amplitudes deeper in the dielectric membrane than in the air. However, as demonstrated in Section H.2 of Appendix H, it does not introduce significant asymmetry in the electric current distribution in the metal.

The Poynting vector represents the energy flows thus the regions of the metal bar towards which a maximum of Poynting vector points are those where the absorption is maximum. Indeed,

¹These field maps are presented for the complete system with the membrane and the mirror. To make the figure 7.3 more readable only the metal bar is presented.

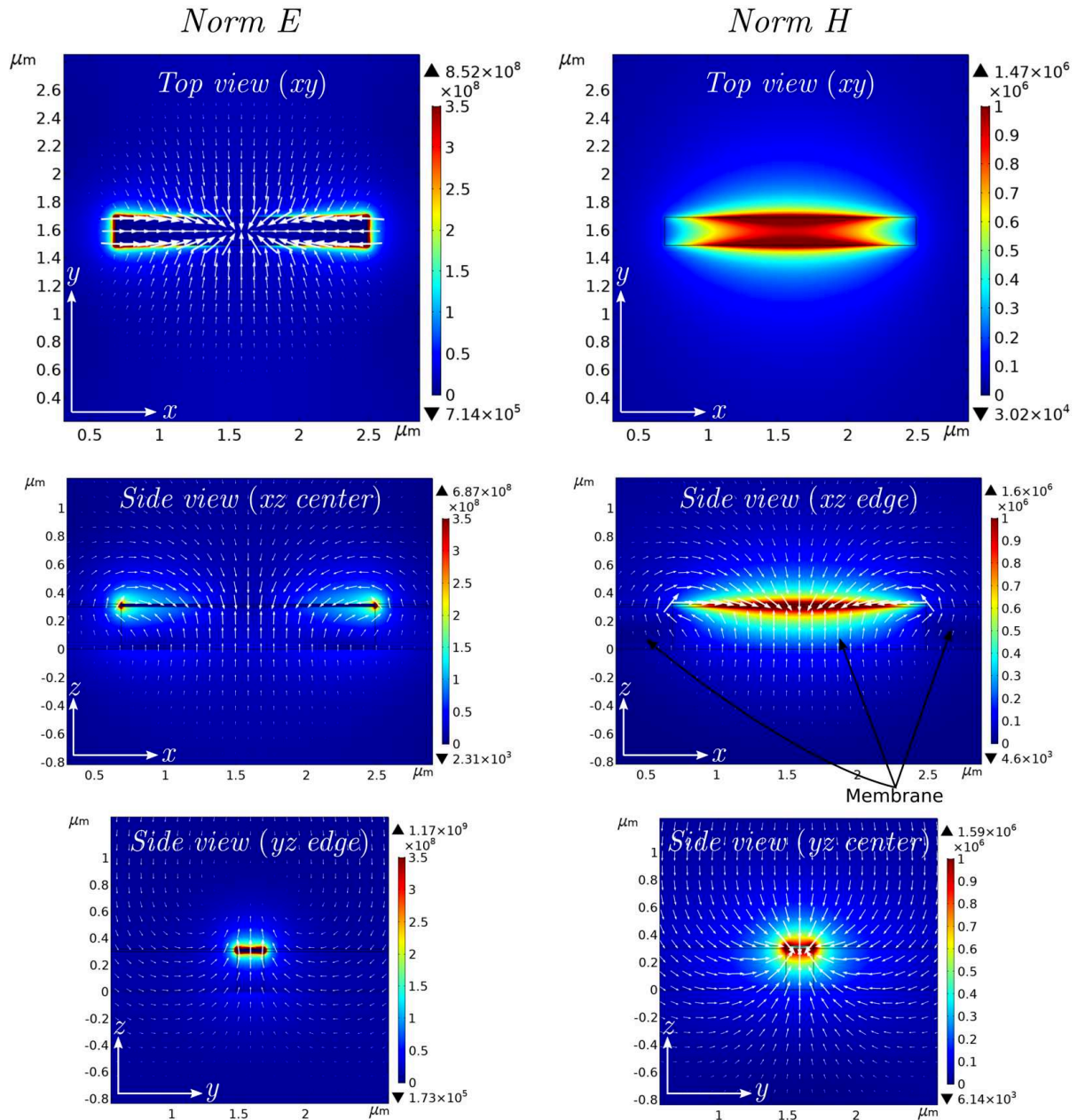


Figure 7.4: Field maps of the norms of the electric and the magnetic fields on the different sections for the structures with parameters $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, $t = 20 \text{ nm}$, $h = 300 \text{ nm}$, placed at the distance $2.5 \mu\text{m}$ from the mirror. The white arrows are for the Poynting vector, which is presented in logarithmic scale. The maps of the Poynting vector for the top and side (xz) and xz views of the electric field have the same scale, while for two remaining (yz) maps the same arrow length corresponds to the 2 times smaller amplitude.

as shown in the current density map on the metal (Fig. 7.5), the strongest electric currents occur on the lateral edges of the long sides of the metal bar, as observed above for 2D MIMs. Furthermore, the distribution in the direction of the thickness of the metal seems to be uniform for this thin metal layer ($t = 20\text{ nm}$).

The electric field of the incident wave plays an important role in the coupling process. As

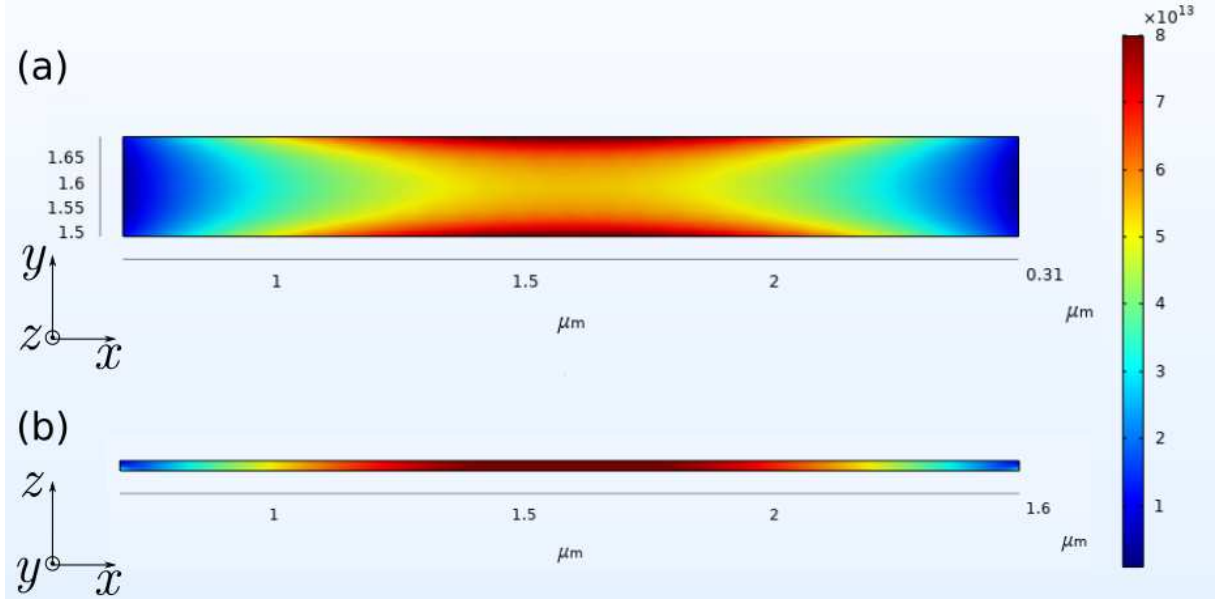


Figure 7.5: The current density on the surface of an IMI antenna in the critical coupled array with parameters $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, $t = 20 \text{ nm}$, $h = 300 \text{ nm}$, placed at the distance $2.5 \mu\text{m}$ from the mirror. (a) The top and (b) the side views.

shown in Fig. 28 of Section H.1, Appendix H, the energy flow to the antenna is provided by the mixed component of the Poynting vector $\mathbf{E}_i \times \mathbf{H}_m$, where $\mathbf{E}_i$ is the electric field of the standing wave, and $\mathbf{H}_m$ is the magnetic field of the mode. The energy arriving in the metal by its upper surface (air-metal interface) and its lower surface (metal-silicon interface) is given by $P_z = E_{ix}H_{my}$, while the component P_y , describing the flow of energy arriving by the lateral surfaces (xOz planes), is written E_{ix} and H_{mz} .

Numerical integration in the COMSOL simulation for the structure with parameters $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, $t = 20 \text{ nm}$, $h = 300 \text{ nm}$ shows that under critical coupling conditions, about 45% of the incident energy comes into the metal trough the top air-dielectric interface, and about 41% trough the bottom metal-silicon interface. The rest of the energy (14%) passes through the side faces of the metal xOz (equal amount for both sides).

To summarize, the IMI array can be critically coupled when placed at the distance $\lambda_{res}/4$ from a mirror. The excited mode in this case looks like an electric dipole mode, with the maximum of the electric field at the antenna ends (in respect to the length), while the maximum of the magnetic field is on the sides at the mid-length. The electric current decomposition on the metal bar seems similar to that observed for MIM antenna array at the same distance above the mirror. The location of the IMI antenna in the anti-node of the electric field of the standing wave is crucial for the coupling process, as it is provided by the electric field of the incident wave

and the magnetic field of the mode. The influence of the geometric parameters of the metal bar on the critical coupling is presented in the following Section.

7.4.
The antenna geometry

In this Section, the critical coupling conditions for the studied IMI structure, placed at the distance $2.5\,\mu\text{m}$ above the mirror (Fig. 7.1) are considered in detail. The influence of the geometrical parameters of the IMI antenna on the array response and losses in it are discussed.

The critical coupling arises when the incident energy is equal to the non-radiative losses. Since the permittivity of the membrane has been chosen as a real number and the mirror is considered perfect, absorption only occurs due to the heating of the metal. Thus, the metal bar defines the non-radiative losses. In this way, setting the parameters to achieve critical coupling relies on two processes: (i) changing the parameters of the metal bar to control losses and (ii) changing the period to control the incident power per antenna. In this way, balancing these two values allows to maximize the absorption and, in some situations, to obtain the critical coupling.

As the length l of the structure defines the resonant wavelength (similar to Chapter 6), only two geometric parameters of metal bar (its width w and its thickness t) remain free to reach the critical coupling conditions at the given wavelength. Below it is shown than they modify both the resonant wavelength and the Q factor. The distance from the mirror to the bottom of the membrane is fixed in this Section, $d = 2.5\,\mu\text{m}$, as the resonance wavelength $\lambda_{res} = 10\,\mu\text{m}$.

7.4.1 The metal width

At first, let us consider the influence of the width. Fig. 7.6 (a) shows the absorption spectra for three structures of different widths, while the other parameters are unchanged: $p = 3.18\,\mu\text{m}$, $l = 1.8\,\mu\text{m}$, $t = 20\,\text{nm}$, $h = 300\,\text{nm}$. It can be seen that as the width w increases, the resonant wavelength blueshifts. That indicates that the effective mode index decreases as the width of the antenna increases.

Oppositely, the Q factor increases as the width of the metal increases. As the period remains the same, i.e. the incident power is constant, only the non-radiative losses are modified when the width w changes. As the width $w = 200\,\text{nm}$ corresponds to the critical coupling conditions, the other two spectra correspond to the under-coupled and over-coupled cases. For the under-coupled case ($w = 150\,\text{nm}$), the non-radiative losses are larger than the incident power therefore

the Q factor is smaller than that at critical coupling. For the over-coupled case ($w = 250 \text{ nm}$), it is the reverse. Also, it is worth to note that for a higher width, the asymmetry level on the right side of the absorption spectrum is smaller than for structures with a smaller width.

Structures of different width can be critically coupled at the same wavelength. To do this, as a first step, the antenna length l must be adjusted to match the desired wavelength. The length of structure of greater width must be increased, since the effective index of the mode decreases as the width increases. After that, the period must be tuned to recover the critical coupling conditions. As the smaller width structure is under-coupled, its period should be reduced. On the other hand, that of the structure of greater width must be enlarged. The parameters for the critical coupling of the three structures at the $10 \mu\text{m}$ resonant wavelength are shown in Table 7.1. The corresponding absorption spectra are plotted in Fig. 7.6 (b). The quality factor of these structures goes from 5 for the blue curve to 7.3 for the purple one. Thus, there is a way to deal with the Q factor of the IMI structure.

Table 7.1: Critical coupling parameters at the resonance wavelength $10 \mu\text{m}$ for IMI rectangle structures with different metal widths. The membrane thickness for all structures is $h = 300 \text{ nm}$.

Period $p, \mu\text{m}$	Side length $l, \mu\text{m}$	Side width $w, \mu\text{m}$	Gold thickness t, nm	Q factor
2.80	1.77	150	20	5.0
3.18	1.80	200	20	6.4
3.4	1.839	250	20	7.3

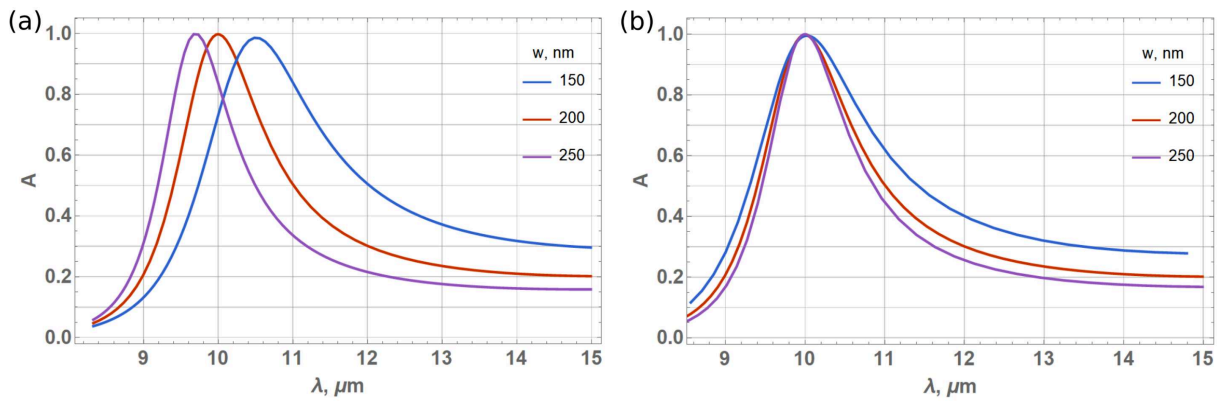


Figure 7.6: (a) Absorption spectra for the structures of different widths, but the same other parameters: $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $t = 20 \text{ nm}$, $h = 300 \text{ nm}$. (b) Absorption spectra for the critically coupled at the $10 \mu\text{m}$ wavelength structures with different width. Blue – $w = 150 \text{ nm}$, red – $w = 200 \text{ nm}$, purple – $w = 250 \text{ nm}$. Other geometrical parameters, corresponding to a given width, are provided in Table 7.1. The distance from the mirror is $2.5 \mu\text{m}$ for both figures.

7.4.2 The metal thickness

The metal thickness also affects the coupling by changing both the resonant wavelength and the Q factor. The absorption spectra of three structures with different metal thicknesses while the other parameters are the same, are presented in Fig. 7.7 (a). The increase of the metal thickness leads to blueshift. Thus the increase of the metal thickness leads to the decrease of the effective index of the mode. Meanwhile, as the metal thickness increases, the Q factor also increases: $Q = 6.4$ for $t = 20\text{ nm}$, $Q = 13.4$ for $t = 40\text{ nm}$, $Q = 22.9$ for $t = 80\text{ nm}$. However, as the thickness increases, but the incident power is retained, the coupling decreases. Critical coupling at a given wavelength can be achieved for structures of different thickness by varying the length and period, as above for the case of different widths of the metal bar.

7.4.3 The cross-section

As the width and thickness of the metal bar affect both the effective index of the mode and the absorbed power, it is interesting to consider the cases where the width w and the thickness t vary while keeping constant the section $w \cdot t$ of the metal bar. Critical coupling conditions were found for three different IMI structures at the resonant wavelength $10\text{ }\mu\text{m}$. Their parameters are given in Table 7.2, and their absorption spectra are presented in Fig. 7.7 (b). While the period remains virtually unchanged, the length increases simultaneously with the metal thickness to keep the critical coupling at the same wavelength. This means that the effective index of the mode decreases. Keeping the constant cross section in the considered case is provided by a decrease of the width and increase of the metal thickness. While the first process leads to the effective index growth (Fig. 7.6 (a)), the second one – to its decrease (Fig. 7.7 (a)). Thus, for considered structures the effective index of the mode varies mainly with the antenna thickness.

The optical losses in the antennas are almost constant, as the period values giving a critical coupling are very close. The Q factor increases a bit for thicker structures. The spectral width (FWHM) increases with the metal thickness when the latter is smaller than the skin depth. This dependence decreases when the thickness increases and saturates for thicknesses of a few skin depths. This variation can be related to decomposition of the electric current density according to the geometric parameters of the metal bar, as discussed below. Also, the absorption level in the right part of the spectrum is the same for the two thicker structures but a little higher for the thinner.

The current density maps on the metal bars of the structures discussed above are presented

Table 7.2: Critical coupling parameters at the resonance wavelength $10\ \mu\text{m}$ for IMI rectangle structures with different metal thicknesses. for all structures in the Table the membrane thickness $h = 300\ \text{nm}$.

Metal thickness t, nm	Metal width w, nm	Metal length $l, \mu\text{m}$	Period $p, \mu\text{m}$	Q factor
20	200	1.80	3.18	6.42
40	100	1.878	3.18	7.63
80	50	2.02	3.20	7.82

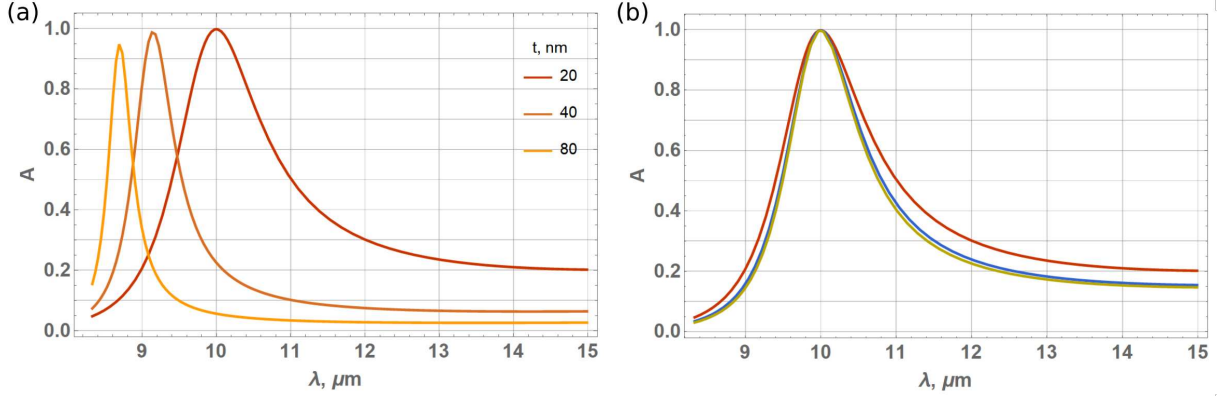


Figure 7.7: (a) Absorption spectra for the structures with different metal thicknesses and the same other parameters $p = 3.18\ \mu\text{m}$, $l = 1.8\ \mu\text{m}$, $w = 200\ \text{nm}$, $h = 300\ \text{nm}$. (b) Absorption spectra for the critically coupled at $10\ \mu\text{m}$ IMI structures with the same metal bar cross section. The red curve correspond to the structure with $t = 20\ \text{nm}$, the blue one – with $t = 40\ \text{nm}$, and the yellow one – with $t = 80\ \text{nm}$. Other parameters are indicated in Table 7.2. The distance from the mirror is $2.5\ \mu\text{m}$.

in Fig. 7.8. It can be seen that for the thicker metal bars, a current density gradient appears along the vertical direction (z). It is seen more clearly for the structure with the thickness $80\ \text{nm}$. This current distribution corresponds to the exponential decay of the field in metal, similar to that described for MIM structures in Section 6.4.3.

The distribution of the electric current density of IMI structures Fig. 7.8 is similar to that of MIM structures (Fig. 6.26). Thus, the similar approach to losses considering the electric current density at the middle plane can be applied. The electric current density $j(y, z)$ (directed along x) at the middle section ($x = l/2$) can be described as the sum of four components, two of them are along the width of the metal bar, and the other two – along the thickness of the metal bar (as in Eq. 6.12). Although the field distribution at the upper and lower metal interfaces (with respect to the direction z) is observed to be unsymmetrical (Fig. 7.4), the difference in current densities is not significant (see Fig. 30 in Section H.2, Appendix H), thus $j(y, z)$ can be described as

$$j(y, z) = a(\exp(-y/\delta) + \exp(y/\delta)) + b(\exp(-z/\delta) + \exp(z/\delta)). \quad (7.1)$$

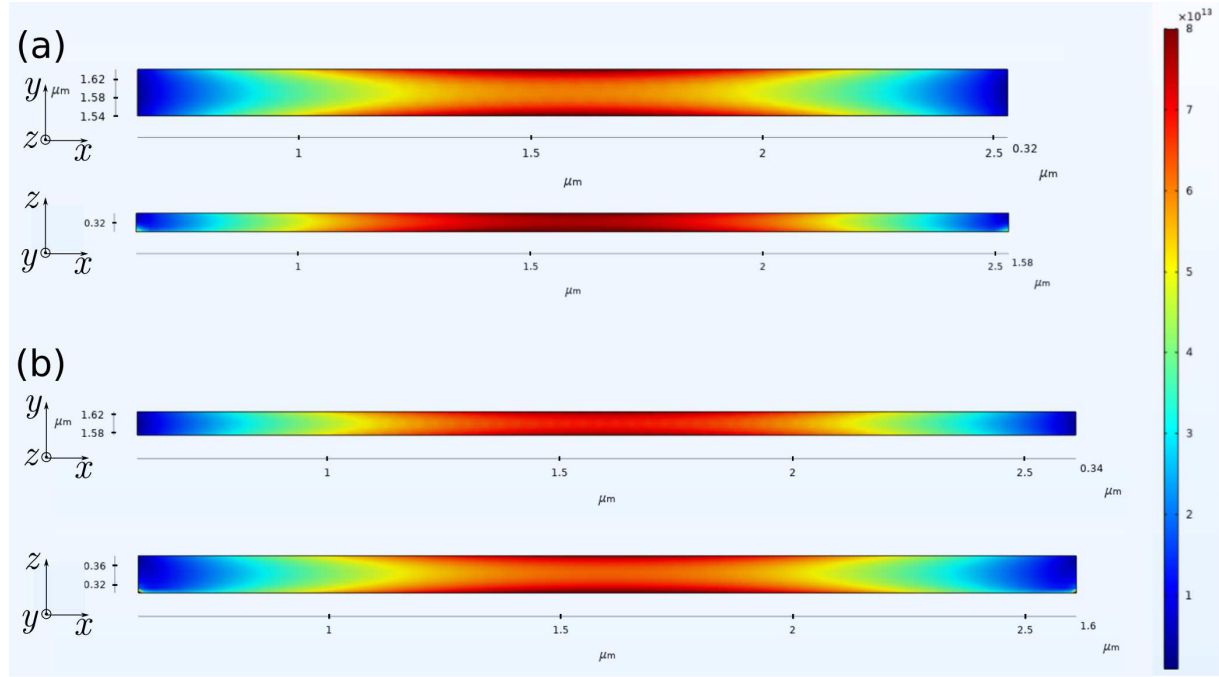


Figure 7.8: The electric current density maps on the surface of an IMI antenna with the metal thickness (a) 40 nm and (b) 80 nm in critically coupled arrays placed at the distance $\lambda_{res}/4$ above the mirror. Other parameters are indicated in Table 7.2. Top maps in (a) and (b) are for top surfaces, and bottom – a right side surfaces of the antennas.

Then, similarly as it was done in Section 6.4.3 of Chapter 6, the losses in structure can be estimated using the current through the middle cross-section $I_{cr}^2 = \int |\mathbf{j}(y, z)|^2 dA$ (Eq. 6.11 of Chapter 6). The integration here is over area A with $y = (-w/2, w/2)$, and $z = -t/2, t/2$.

All the structures considered in this section with different geometric cross-sections (Table 7.1 and Table 7.2, the membrane thickness is the same) can be compared using I_{cr}^2 . The total current values were calculated for five considered structures (presented in Section H.2 of Appendix H).

As the structures are at the critical coupling conditions, all incident energy is absorbed. Thus, the absorbed power is equal to the incident power, which is proportional to p^2 . Consequently, according to Eq. 6.11, p^2 should be proportional to I_{cr}^2 . Because structures have different length l , which defines the array density, it is included into the comparison. The dependence of the structure density l/p^2 on the square of the total current is presented in Fig. 7.9. The points show the result of the numerical computation, and the solid curve is the fit of the inverse dependence $a/(b + x)$. Although the fit is not perfect, it validates the variation of the ratio l/p^2 with the square of the total current.

In summary, the studied mode is affected by both the width and thickness of the metal bar. The modification of one of this parameters induces two different behaviors: 1) the dissipated

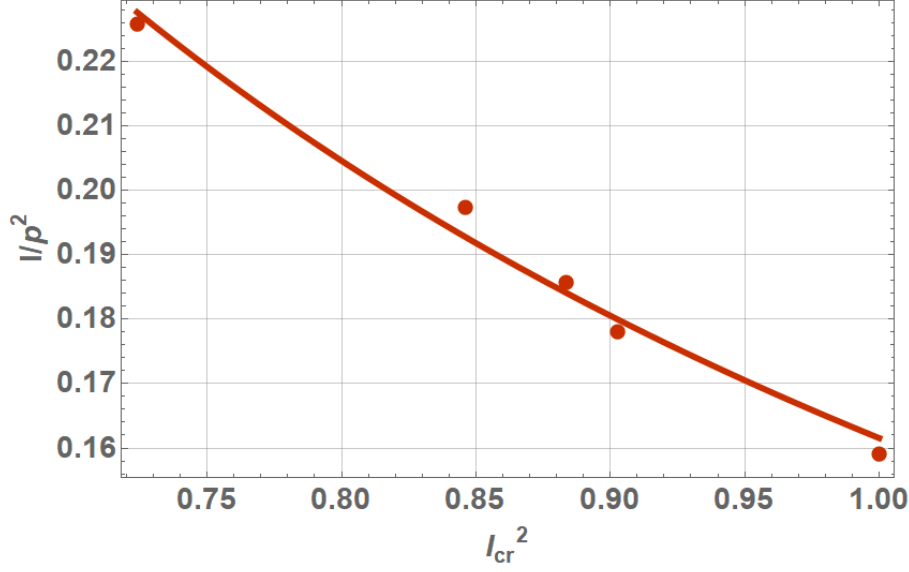


Figure 7.9: The dependence of the filling factor on the electric current through the middle cross-section I_{cr}^2 . Points show the result of numerical computations. The solid line is a fitting curve $a/(x + b)$, where $a = 0.154 \pm 0.005$ and $b = -0.048 \pm 0.028$.

power, and 2) the effective index of the mode. The first property is due to the skin effect on the current density distribution: a decreasing exponential from the metal surface. For the second, the increase of the width or the thickness of the metal leads to a decrease of the effective index of the mode and therefore a resonance blueshift.

The effective index of the mode is also connected with the thickness of the membrane (because of the asymmetry in the distribution of the fields). However, the influence of the membrane, as well as the physical explanation of the asymmetric response of the structure (which can be caused by a superposition of geometrical factors and by the membrane) remain the subject of further research. Meanwhile, some optical properties of the structures under consideration, useful for any absorption application, are presented in the next Section.

7.5.

The optical response

IMI structures can be critically coupled when located at $\lambda_{res}/4$ above the mirror, and the influence of their parameters on their optical response is discussed above. In this Section, the dependencies of the absorption on the distance to the mirror and on the incidence angle are considered.

Since the excited mode of the IMI structure has some similarities with that of 2D MIMs, the absorption dependence on the distance to the mirror is expected to be similar. Indeed, this

dependency for the IMI structure, presented in Fig. 7.10 (a), demonstrates a periodic behaviour of the period $5\mu m$. The maximum absorption is at the distance $2.5\mu m$, and the minimum around $5\mu m$. Moreover, absorption greater than 90% is observed in the range of approximately $1.3\mu m$ (from 1.7 to $3\mu m$), which means that the control of this distance is associated with a large manufacturing tolerance.

The angular dependence for the IMI array is considered for two incident planes: yOz , which is orthogonal to the length of the metal bar (along x), and xOz , along the metal bar length, as shown in inset of Fig. 7.10 (b). As the structure is excited by the electric field of the incident wave, a strong absorption is expected for the TE (transverse electric) polarisation with respect to the incident plane yOz , and for the TM (transverse magnetic) polarisation with respect to the incident plane xOz , because in the two cases the electric field of the wave is orientated along the metal bar. Indeed, the absorption for the two preferential polarizations mentioned above (Fig. 7.10 (b)) is total for the small incident angles. However their angular dependence is different for higher angles (i.e. $> 30^\circ$).

The angular dependence of the TE polarisation with respect to the incident plane yOz (blue line) has a wide range of high absorption values: more than 90% between -60° and 60° . This can be explained by the fact that, for this polarization, the component of the electric field along the long side of the metal bar does not depend on the incident angle. On the other hand, the decreasing component of the magnetic field along the width of the metal rectangle does not have such a strong effect on the excitation mode until the angle of incidence is significant. Under the TM polarisation with respect to the incident plane xOz (red line), the magnetic field is constant and aligned along the short side of the metal bar. On the other hand, the electric field sees its component E_x along the long side of the metal bar vary with the incident angle. Thus when the incidence angle is small the absorption is strong but when it increases the component E_x weakens (around 42°), the absorption decreases significantly (in comparison with other polarization). These two dependencies are connected with the way an IMI structure is coupled to the incidence wave, described in Section 7.3. As the electric field along the metal bar has a key role, the blue curve (the E_x component of the electric field is constant) shows greater absorption tolerance at the incident angle than the red curve for which this field component varies with the incident angle. It is however important to note that the decrease in absorption does not vary like the cosine of the incidence angle which means that second order effects modify this effect: the decrease in the magnetic field for the TM polarization, the component E_z for

the TE.

The absorption for two other polarisations, namely the TE polarisation with respect to the incident plane xOz and for TM polarisation with respect to the incident plane yOz , is close to zero at all incident angles (shown in Fig. 31, Section H.3 of Appendix H). For both of them the electric field of the incident wave is directed along the short side of the metal bar. Thus the structure is not excited because for the mode excitation, the electric field along the long side of the metal bar is needed. Thus, the considered geometry of the metal bar is sensitive to the polarisation of the incident radiation. For applications insensitive to the incident polarization, another geometry must be used, for example the cross-shaped one. As it was shown in Section 6.4.5, the geometric parameters of the cross-shaped IMI antenna can be found by a slight change from those of the rectangle antenna.

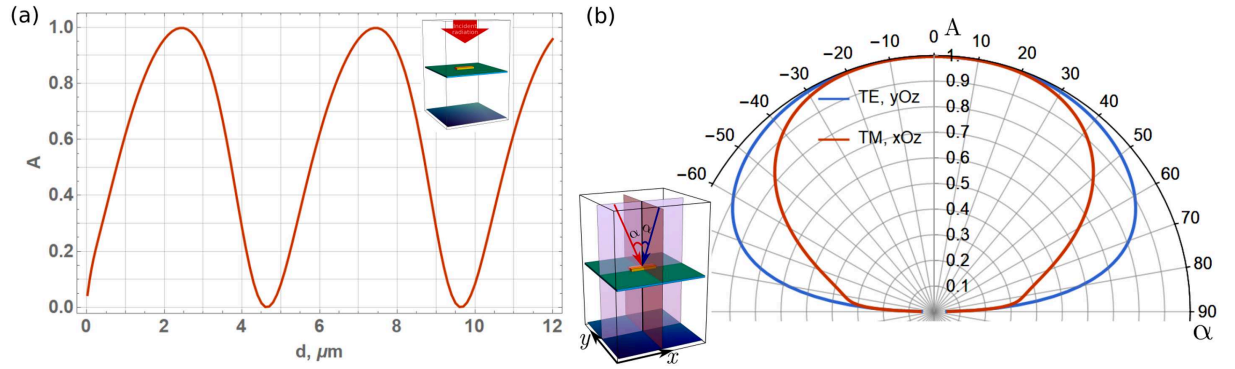


Figure 7.10: The dependence of absorption of IMI structure on (a) distance to the mirror, and (b) the incidence angle. The polarisation and corresponding to it incidence planes for the angular dependence are indicate in the legend. The incidence planes in relation to structure orientation are shown in the inserted schema: the yOz plane is orthogonal to the structure length, and the xOz plane is orientated along it. Parameters of the structure $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, $t = 20 \text{ nm}$, $h = 300 \text{ nm}$. The distance to the mirror is $2.5 \mu\text{m}$.

Finally, the volumes of the IMI and MIM structures at the critical coupling at the resonant wavelength $10 \mu\text{m}$ (all placed at $\lambda_{res}/4$ above the mirror) can be compared. As in Chapter 6, where MIM structures were considered, a membrane, that mechanically holds the structures, was not taken into account, it is not considered in this Section either. The discussion of the membrane impact is a separate thermal problem (for example, [149]), which is out of the scope of this work. Indeed, its heating is not a homogeneous process, having a gradient between the small area heated by an antenna and the edges of the membrane being at room temperature. Additionally, the reduced area of the IMI antenna compared to that of the period allows the design of a membrane more complex than a simple homogeneous membrane, for example a mesh

of narrow ribbons. In this way, the membrane is not considered in

The volume density for IMI antennas is computed as $\omega_V = l \cdot w \cdot t/p^2$. For the structures, presented in Table 7.1 and Table 7.2, the volume densities are in the range $0.68 - 0.80 \text{ nm}$ (presented in Table 18 of Section H.4, Appendix H), which is at least five times less than values obtained for MIM structures with the metal thickness 5 nm . Additionally, these structures are easier to fabricate than the multi-layer MIM structures. Thus, IMI structures are the most optimal structures for bolometer applications among considered. They have small area (about 4% of that of the period), thus a few antennas can be placed in the same period, leaving enough space for isolation arms.

To summarize, the IMI antennas are the structures, among those studied in this work, which have the lowest heated volume. They occupy a small part of the period area (about 4%) and are easier to fabricate than the MIM structures. This is the reason why they were chosen for the experimental validation of the computation results (Section 7.7). These structures demonstrate a large manufacturing tolerance on the distance separating them from the mirror, the absorption is higher than 90% in a band approximately one micrometer wide. Rectangular geometry IMI structures are extremely sensitive to the polarisation of the incident radiation, showing a high absorption tolerance to the incident angle when the electric field is along the metal but demonstrating no absorption when the electric field has the orthogonal direction. This is an intrinsic property connected with its coupling process. The simplicity of a IMI array also allows to model it as a dipole array, as demonstrated in the following Section.

7.6.

The dipole approximation

7.6.1 Introduction

When S-parameters of an array of nano-antennas in vacuum are calculated numerically, reflection and absorption from the same array over the mirror can be found analytically for different distances to the mirror (Section F.3 of Appendix F). It allows one to significantly reduce the time spent on full-wave numerical simulations. Apart from the distance to the mirror, the inter-element distance in a nano-antenna array is another important parameter influencing the absorbance spectra. In order to account for this parameter, a more elaborated analytical model (than the one presented in Section F.3) is required.

In this Section, the dipole approximation is developed to estimate the mutual interactions

in nano-antenna arrays over the mirror and analytically calculate absorbance spectra for different inter-element distances. The dipole approximation is well-known in electromagnetism and was successfully applied to scattering calculations from arbitrarily-shaped targets [150], to metasurface (*dense* nano-antenna arrays) absorbers [136, 151] and many more. Often, dense (in comparison to the wavelength) nano-antenna arrays are considered in the literature, what allows one to introduce approximations in calculation of mutual interaction. In this work, no condition is imposed on the inter-element distance and the calculations are performed rigorously.

In what follows IMI nano-antennas are considered, see Fig. 7.11(a), since their response can be well-approximated by simply accounting for the electric dipole moment. An array of IMI nano-antennas can be thus approximated by an array of point electric dipoles. MIM nano-antennas are additionally characterized by the magnetic dipole moment, what would require a bit more efforts to write down equations.

7.6.2 Theory

In this Section, a retrieval procedure of the electric polarizability of a single dipole scatterer is given by analysing the field scattered by a regular array of identical dipoles placed above the mirror. Using the retrieved polarizability, the response of the dipole array can be calculated for different lattice parameters and distances to the mirror.

Scattering by a regular electric dipole array in vacuum

In this subsection a regular array of identical point electric dipoles at the distance h above the mirror (perfect electric conductor) is considered. A schematics of the system is shown in Fig. 7.11 (b). A dipole is characterized by its polarizability α . An incident plane wave plus its reflection from the mirror constitute the external field, which excites the dipole moment $\mathbf{p}_{nm}$ in the nm th dipole.

The electric field $\mathbf{E}(\mathbf{r})$ scattered by a single dipole can be calculated with the help of the Green's function [3]

$$\mathbf{E}(\mathbf{r}) = iZk \left(1 + \frac{\nabla \otimes \nabla}{k^2} \right) \mathbf{A}(\mathbf{r}), \quad \mathbf{A}(\mathbf{r}) = \frac{-i\omega \mathbf{p}}{4\pi} \frac{e^{ik|\mathbf{r}-\mathbf{r}_{00}|}}{|\mathbf{r}-\mathbf{r}_{00}|}. \quad (7.2)$$

It is assumed that the dipole is placed at the position $\mathbf{r}_{00}$ in vacuum and has the dipole moment $\mathbf{p}$. In this equation $Z = \sqrt{\mu_0/\varepsilon_0}$ and $k = \omega\sqrt{\varepsilon_0\mu_0}$ are respectively the intrinsic impedance and the wavenumber in vacuum, ∇ is the vector operator nabla and $\otimes$ stands for the tensor

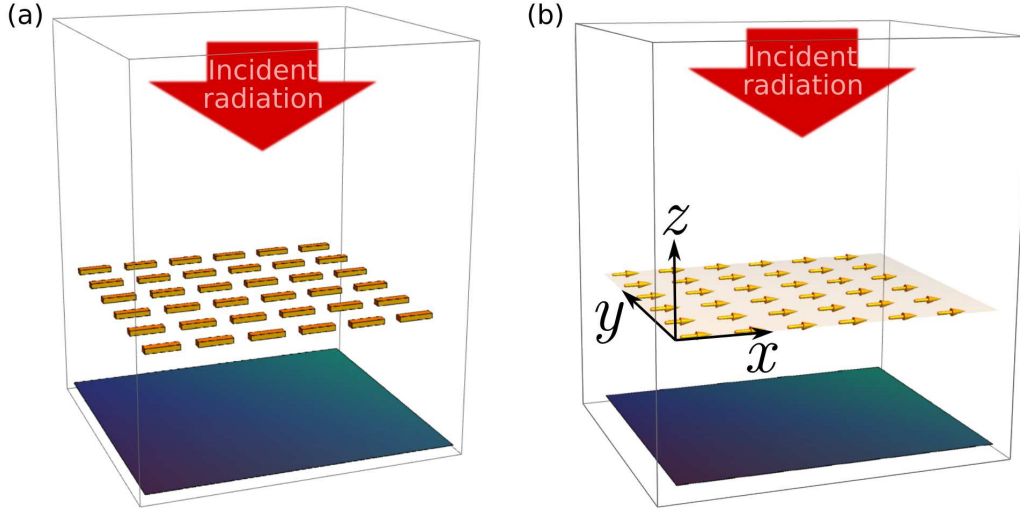


Figure 7.11: A periodic array of (a) IMI nano-antennas (gold bars) and (b) point dipoles over the mirror under plane-wave illumination.

product, $\mathbf{A}(\mathbf{r})$ is the vector potential. The scattered magnetic field $\mathbf{H}(\mathbf{r})$ can be found from the Maxwell's equations.

The vector potential created by the regular array of dipoles can be expressed as the series of the individual contributions

$$\mathbf{A}(\mathbf{r}) = \sum_{n,m=-\infty}^{+\infty} \frac{-i\omega \mathbf{p}_{nm}}{4\pi} \frac{e^{ik|\mathbf{r}-\mathbf{r}_{nm}|}}{|\mathbf{r}-\mathbf{r}_{nm}|}, \quad (7.3)$$

where $\mathbf{r}_{nm} = an\mathbf{x} + bmy$ is the position of the nm th dipole, $\mathbf{x}$ and $\mathbf{y}$ are the two of three basis vectors of the Cartesian coordinate system. Since an array of identical dipoles illuminated by a plane wave is considered, $\mathbf{p}_{nm} = \mathbf{p} \exp(i\mathbf{k}^{(i)} \cdot \mathbf{r}_{nm})$. The series (7.3) can be transformed into the series of plane waves by means of the Poisson formula [152]

$$\sum_{n,m=-\infty}^{+\infty} f(an, bm) = \sum_{p,q=-\infty}^{+\infty} \frac{1}{ab} \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv f(u, v) e^{i\frac{2\pi p}{a}u} e^{i\frac{2\pi q}{b}v} \quad (7.4)$$

and the Fourier transform of the Green's function

$$\int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv e^{ipu} e^{iqv} \frac{e^{ik\sqrt{u^2+v^2+z^2}}}{\sqrt{u^2+v^2+z^2}} = 2\pi \frac{e^{i|z|\sqrt{k^2-p^2-q^2}}}{\sqrt{k^2-p^2-q^2}}, \quad (7.5)$$

the branch of the square root is chosen such that $\Im(\sqrt{k^2-p^2-q^2}) < 0$. Here a and b are period values in the x and y directions, correspondingly. By applying Eqs. (7.4) and (7.5) to Eq. (7.3)

one arrives at the following plane-wave decomposition of the scattered field

$$\mathbf{A}(\mathbf{r}) = \frac{\omega \mathbf{p}}{2ab} \sum_{n,m=-\infty}^{+\infty} \frac{1}{\beta_{nm}} e^{i\tau_{nm}\mathbf{r}} e^{i|z|\beta_{nm}}, \quad (7.6)$$

where $\tau_{nm} = \left(k_x + \frac{2\pi n}{a}\right) \mathbf{x} + \left(k_y + \frac{2\pi m}{b}\right) \mathbf{y}$ and $\beta_{nm} = \sqrt{k^2 - \tau_{nm}^2}$. The corresponding electric field is calculated from the vector potential following Eq. (7.2)

$$\mathbf{E}(\mathbf{r}) = -\frac{1}{2iab\epsilon_0} \sum_{n,m=-\infty}^{+\infty} \frac{k^2 \mathbf{p} - (\mathbf{p} \mathbf{k}_{nm}) \mathbf{k}_{nm}}{\beta_{nm}} e^{i\mathbf{k}_{nm}\mathbf{r}}, \quad (7.7)$$

where $\mathbf{k}_{nm}\mathbf{r} = \tau_{nm}\mathbf{r} + z\beta_{nm}|z|$. As follows from this equation, the array of dipoles scatters the incident wave symmetrically in the upper and lower half-spaces. The incident wave $\mathbf{E}_0 e^{i(k_x x + k_y y - \beta_{00} z)}$ and the scattered wave propagating in the direction opposite to $\mathbf{z}$ are reflected from the mirror and contribute to the total reflected field in the upper half-space

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= (-\hat{I} + \mathbf{z} \otimes \mathbf{z}) \mathbf{E}_0 e^{2i\beta_{00}h} e^{i\mathbf{k}_{00}\mathbf{r}} \\ &- \sum_{n,m=-\infty}^{+\infty} \frac{k^2 \mathbf{p} - (\mathbf{p} \mathbf{k}_{nm}) \mathbf{k}_{nm}}{2iab\epsilon_0 \beta_{nm}} (1 - e^{2i\beta_{nm}h}) e^{i\mathbf{k}_{nm}\mathbf{r}}. \end{aligned} \quad (7.8)$$

Here $\hat{I}$ and $\mathbf{z} \otimes \mathbf{z}$ are projection operators on the plane $x0y$ and z -axis, respectively.

Polarizability retrieval

This subsection describes a procedure of the polarizability retrieval from the reflected field. Consider a particular situation when the array is composed of dipoles polarizable only along $\mathbf{x}$ direction $\mathbf{p} = p\mathbf{x}$ and has the inter-element distance such that $a < \lambda$ and $b < \lambda$. The array is illuminated by a normally-incident plane wave with the electric field $\mathbf{E}_0 = E_0\mathbf{x}$. Under these conditions, the specularly reflected wave is the only propagating wave and has the electric field $R\mathbf{x}$ found from Eq. (7.8)

$$R = -E_0 e^{2i\beta_{00}h} - p \frac{k^2 (1 - e^{2i\beta_{00}h})}{2iab\epsilon_0 \beta_{00}}. \quad (7.9)$$

This equation can be used to extract the dipole moment p from the complex amplitude of the reflected wave R (S_{11} in COMSOL computations accurate to the phase due to the distance between the array and the port). In its turn the dipole moment depends on the polarizability and the electric field at the position of the given dipole (local electric field). The local electric field is the sum of the external field and the scattered field by all dipoles in the array (apart

from the zeroth) at the position of the zeroth dipole $\mathbf{r}_{00} = \mathbf{0}$. The dipole moment can be found from the following equation

$$p = \alpha(E_{ext} + E_{sct}), \quad (7.10)$$

where external field E_{ext} is the incident wave plus its reflection from the mirror

$$E_{ext} = (1 - e^{2i\beta h})E_0 \quad (7.11)$$

and the scattered field is calculated from Eqs. (7.2), (7.3) and (7.8)

$$\begin{aligned} E_{sct} = & \frac{p}{4\pi\epsilon_0} \left(k^2 + \frac{\partial^2}{\partial x^2} \right) \sum_{\substack{n,m=-\infty \\ n,m \neq 0}}^{+\infty} \frac{e^{ik|\mathbf{r}-\mathbf{r}_{nm}|}}{|\mathbf{r}-\mathbf{r}_{nm}|} \Big|_{\mathbf{r}=\mathbf{0}} \\ & + p \sum_{n,m=-\infty}^{+\infty} \frac{k^2 - (\mathbf{x}\boldsymbol{\tau}_{nm})^2}{2iab\epsilon_0\beta_{nm}} e^{2i\beta_{nm}h}. \end{aligned} \quad (7.12)$$

From Eq. (7.8) only the part of the field scattered by the dipoles and reflected from the mirror is taken (corresponding to the second term in the parentheses under the summation sign). Importantly, the local field cannot be calculated directly from Eq. (7.8), since the series diverges when $\mathbf{r} = \mathbf{0}$. Derivations made in Ref. [153] can be used to accelerate numerical calculations of the scattered field (7.12).

Equations (7.10), (7.11) and (7.12) allow one to retrieve the polarizability of the dipole from the complex amplitude of the specularly reflected wave

$$\alpha = \frac{p}{E_{ext} + p\beta}, \quad (7.13)$$

where $\beta = E_{sct}/p$ is the interaction constant [136] that depends only on the inter-dipole distance and the distance to the mirror as follows from Eq. (7.12).

On the other hand, when α is known, the dipole moment can be calculated as follows

$$p = E_{ext} \left(\frac{1}{\alpha} - \beta \right)^{-1}. \quad (7.14)$$

In this way, the polarizability of a single dipole is obtained from the field reflected from a regular array of identical dipoles placed above the mirror. Since the polarizability is a characteristic of a single dipole which does not depend on the inter-dipole distance and the distance to the mirror, absorbance spectra for different values of lattice parameters and distances to the

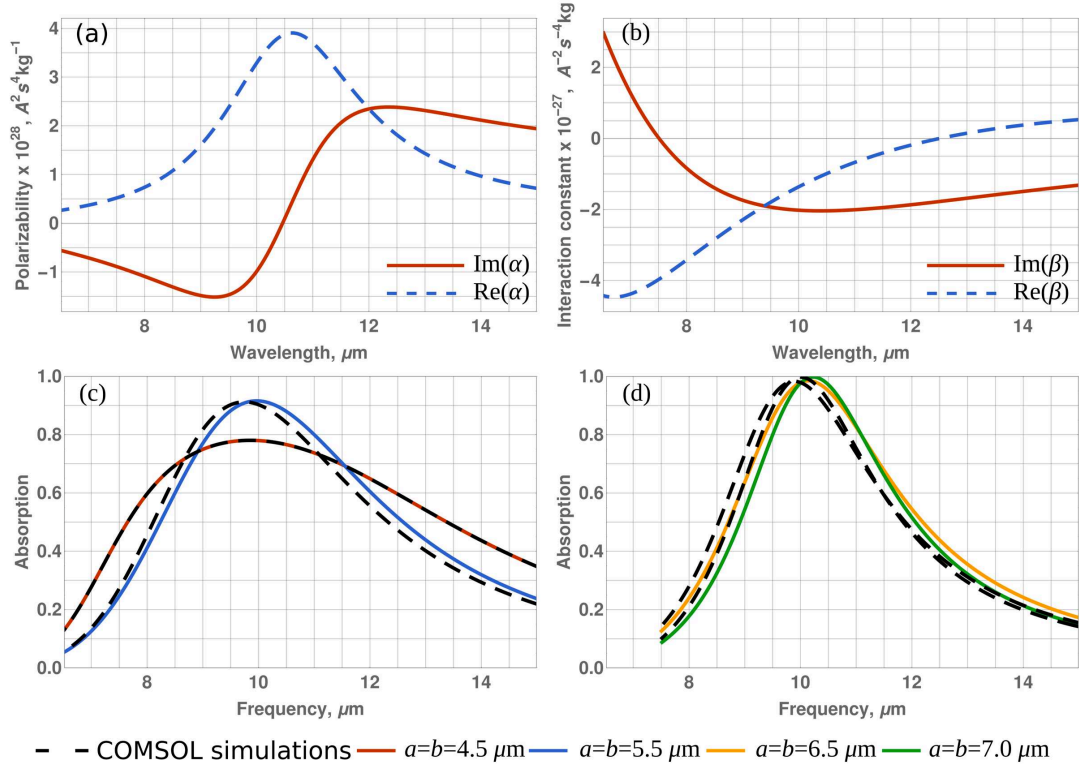


Figure 7.12: (a) The retrieved from a numerical simulation polarizability α of a single gold-bar nano-antenna constituting a periodic array over the mirror with the period values $a = b = 4.5 \mu\text{m}$, $h = 2.5 \mu\text{m}$. (b) The interaction constant β calculated for the array over the mirror with $a = b = 4.5 \mu\text{m}$, $h = 2.5 \mu\text{m}$. (c)-(d) The absorbance spectra for different period values of the array calculated by means of the analytical model (solid curves) and compared to COMSOL Multiphysics full-wave simulations (dotted curves).

mirror can be estimated by means of analytical formulas. In the following section predictions of the analytical model and full-wave numerical simulations are compared.

7.6.3 Validation of approach

In this Section, the approach developed in Section 7.6.2 is applied to retrieve the polarizability of a nano-antenna from the specular reflection from an array of identical nano-antennas. The specular reflection is calculated numerically in the frequency range of interest for a given set of parameters of the array: the inter-antenna distances and the distance to the mirror. After that, the reversed procedure is used to extrapolate the retrieved polarizability on another set of parameters of the array.

As a concrete example, a periodic array of simple metallic nano-antennas (gold bars) above the mirror as shown in Fig. 7.11(a) is considered and the following step-by-step procedure is used:

1. A full-wave numerical simulation is performed for given inter-antenna distances a and b , and the height h above the mirror;

2. Complex amplitude of the specular reflection is recorded in the frequency range of interest;
3. By means of Eq. (7.13), the polarizability of a single nano-antenna is calculated in the frequency range;
4. In the last step the polarizability is used to calculate the absorbance spectra for arbitrary parameters of a , b and h without performing time consuming full-wave simulations, to that end Eqs. (7.14) and (7.9) are used.

Figure 7.12(a) demonstrates the frequency dependence of a single dipole polarizability retrieved from the specular reflection of a plane wave impinging an array of nano-antennas above the mirror at the normal incidence. The array has a square lattice with $a = b = 4.5 \mu\text{m}$, the distance to the mirror h is $2.5 \mu\text{m}$. The mutual coupling between nano-antennas in the array as well as with the mirror is described by the single interaction constant β calculated analytically from Eq. (7.12) and shown on the panel (b) of Fig. 7.12 for the given set of the parameters. When changing either a , b or h , the interaction constant has to be recalculated. Meanwhile, the retrieved polarizability does not depend on these parameters, and together with β it is used to estimate absorbance spectra for various lattice parameters and distances to the mirror. In Fig. 7.12(c) and (d) it is used to find the lattice parameters corresponding to the condition of the critical coupling, i.e. the total absorption. It is seen that when increasing the inter-antenna distance the maximum absorption also increases until the critical coupling is reached for $a = b = 7 \mu\text{m}$. The observed tendency has been validated with COMSOL full-wave simulations.

In comparison to another analytical method, developed in Section F.3 of Appendix F, the dipole approximation can be used not only to account for the reflection from the mirror, but also to change the distance between antennas. Furthermore, the dipole approximation takes into account the near-field inter-dipole interaction (i.e., through the evanescent modes), what allows one to compute the response of the array at distances closer to the mirror (although there is a limitation related to the size of an antenna) in comparison to the simplest case of the single propagating mode.

To summarize, the point dipole approximation can be successfully employed for predicting the absorbance spectra of nano-antennas arrays while avoiding time consuming full-wave simulations. Being the simplest model of a nano-antenna, it can be further improved by taking into account the finite size of nano-antennas and more complex current distribution. The method of moments can be used to that end, see, e.g., Ref. [154].

7.7.

Experimental validation

This Section is dedicated to the experimental validation of the quasi-total absorption demonstrated above for IMI antennas placed at $\lambda/4$ above a mirror. These antennas are difficult to fabricate for at least two reasons. First, they are supported by a thin membrane (300 nm thick), and second, they operate at wavelengths around $10\ \mu\text{m}$ which requires the use of a transparent dielectric material at these wavelengths, silicon or germanium for example. Developing such a process would have taken too long. Thus, in accordance with the laboratory facilities, the wavelength was changed to MWIR, and the design of the structure was modified. In this way, the goal of this experimental validation being focused on the antenna behavior rather than on the materials, it was chosen to simplify the structure.

Design of the test structures

The simplified structure consists of a mirror overlaid with a $\lambda/4n$ thick dielectric layer, on which the IMI antenna arrays are deposited. The mirror is a thick layer of gold. The dielectric is a 700 nm thick layer of SiO_2 . The working wavelength is in the range 3 to $5\ \mu\text{m}$, where SiO_2 is highly transparent. The unit cell of an IMI array is shown schematically in Fig. 7.13 (a). At the center of the 2D period of width p , there is a rectangular parallelepiped of metal (orange bar) whose thickness is t , length l , and width w . The SiO_2 layer (in blue) covers the whole surface, as well as the bottom mirror (in grey).

Fabrication process

The fabrication process begins with a Ti/Au (20/200 nm) deposition (e-evaporation process) on a silicon substrate. Then, the SiO_2 layer is deposited by a PECVD process. A thickness of 700 nm has been measured by ellipsometry with the expected stoichiometry. Finally, several arrays ($100 \times 100\ \mu\text{m}^2$, Fig. 7.13 (c)) of IMI antenna are drawn by electron beam lithography (Fig. 7.13 (b)), followed by a Ti/Au (1/10 nm) deposition and lift-off. The grainy structure of the SiO_2 layer observed by electron microscopy photography (Fig. 7.13 (b)) is mainly due to the difference in charge effects between the volume of the grains and their boundaries. Since the amplitude of the corresponding roughness is much less than the wavelength, their impact on the optical response should be very small, although they may influence the antenna shape.

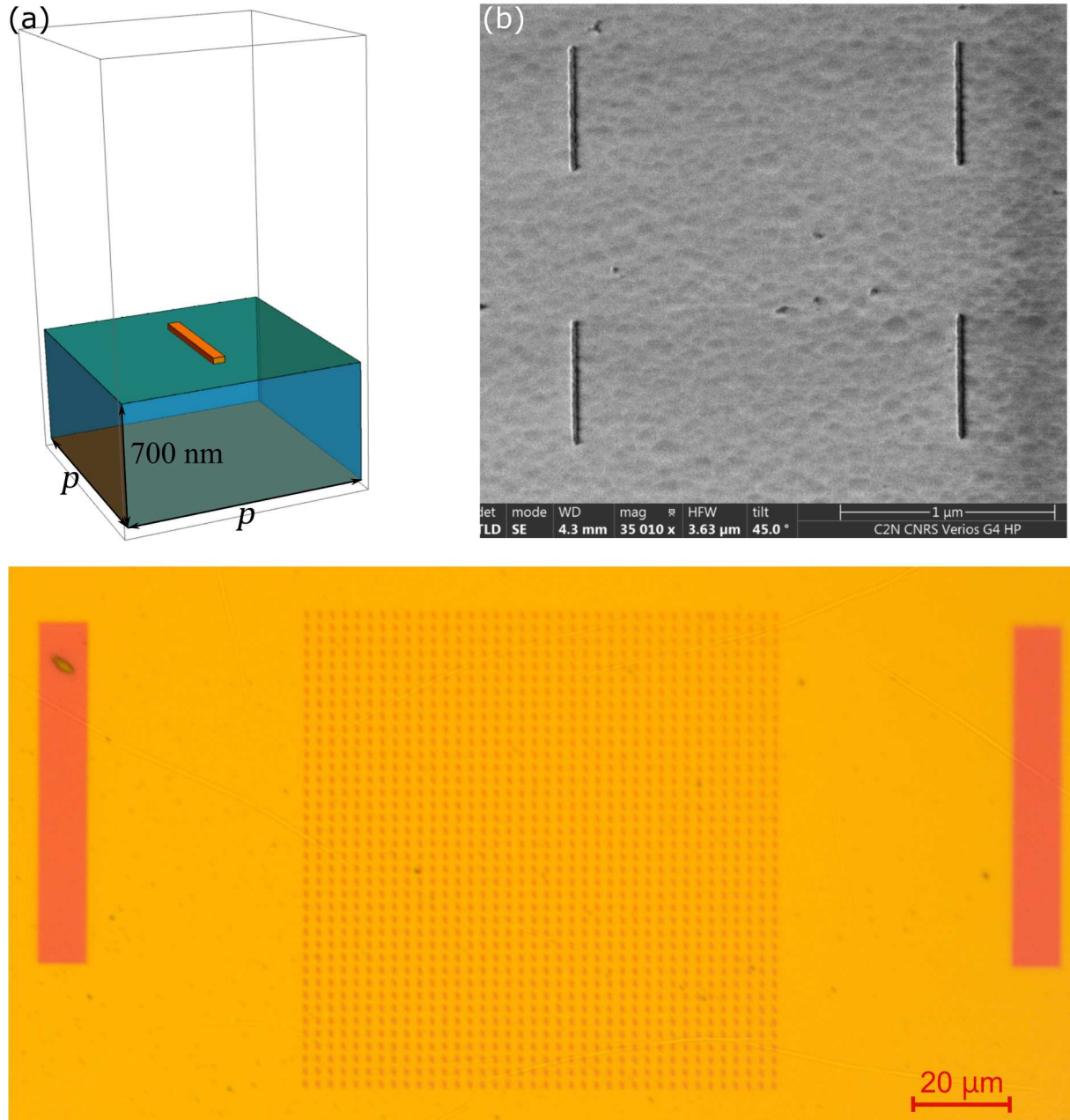


Figure 7.13: (a) The schematic presentation of a unit cell of the IMI array. A gold parallelepiped is placed on the 700 nm thick SiO₂ layer, backed by a thick gold layer. (b) A scanning electron microscopy photography for one of nano-antenna arrays. The top view of a few metal patches on a top of SiO₂ surface. (c) An optical microscopy photography, showing an array of metal patches on the top of the SiO₂ layer. The wide metal rectangles on both sides of the array are used to indicate the array area.

The surface of arrays of IMI antenna are much larger than the resonant wavelength (λ_{res} between 3 and 5.5 μm). Each of these arrays corresponds to a different set of the geometrical parameters of the IMI antennas, with the exception of the thickness of the gold patches, which is fixed as $t = 10 \mu\text{m}$. These geometrical parameters (length l and width w) have been carefully measured by SEM observations. All these measures were calibrated by the period of the IMI

arrays, leading to an uncertainty of less than 1%.

Optical characterization

The reflection spectral response of all arrays was measured in the $3 - 5 \mu m$ wavelength range, by a Fourier-Transform InfraRed (FTIR) spectrometer via an optical microscope, equipped with a Cassegrain objective so that the setup remains achromatic. As a result, the measured reflection corresponds to the sum of the reflections for incident angles allowed by the objective, here between 12° and 24° for the all azimuths.

The reflection spectrum of the IMI array with parameters $p = 2.5 \mu m$, $l = 1.219 \mu m$, $w = 235 nm$ is presented in Fig. 7.14 (a), blue line. This spectrum $R(\lambda)$ can be fitted with $R(\lambda) = 1 - L(\lambda)$ where the absorption is modeled by a Lorentzian function $L(\lambda)$:

$$L(\lambda) = L_0 + \frac{2A}{\pi(4(\lambda - \lambda_0)^2/(\gamma) + \gamma)}. \quad (7.15)$$

The best fit obtained over the interval $(3.70 \mu m - 5.56 \mu m)$ is presented by the red curve in Fig. 7.14 (a). The fitted parameters are $L_0 = 0.94074 \pm 0.0035$, $\lambda_0 = (4.6091 \pm 0.0003) \mu m$, $A = -2.132 \pm 0.015$, $\gamma = (1.5166 \pm 0.0056) \mu m$. Two gray zones in the plot Fig. 7.14 (a) correspond to two absorption windows due to (1) O-H links in the SiO_2 layer (observed empirically on many samples) and (2) CO_2 in the atmosphere for the shortest and middle wavelengths, respectively. These two spectral domains are excluded from the fit, as well as that of the unexpected behaviour in the region of the short wavelengths. The later will be commented on below.

At the resonance, the minimum of reflection is about 0.046, which is 95.4% absorption, as the bottom gold layer is a thick enough to avoid any transmission. Thus this array experimentally demonstrates an almost total absorption, although an IMI antenna only covers 4.6% of the unit cell area.

Numerical simulations

The optical response of these arrays of IMI antennas were also calculated using Comsol Multiphysics. In these simulations, the unit cell of the IMI array is limited by two pairs of periodic boundary conditions, modeling the Floquet periodicity in the directions x and y . At the bottom, the thick gold layer is modeled by an impedance boundary condition, characterised by the gold permittivity. The optical index of SiO_2 is taken from the literature [155]. For the modeling of

gold, its permittivity was calculated with the Drude formulation:

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\Gamma\omega}, \quad (7.16)$$

where ω_p is the electron plasma frequency, Γ the electron relaxation rate, and ω the radiation frequency. In a first approach, the values of these coefficients are taken from the literature: $\omega_p = 12.9 \cdot 10^{15}$ rad/s, and $\Gamma/\omega_p = 0.0055$ for evaporated gold [156]. These values allowed an excellent agreement of the calculated reflection spectra with those routinely measured on thick gold optical resonators fabricated in the same equipment.

As it was shown in Section 7.5, rectangular shaped antennas interact with TE polarisation when the incidence plane is orthogonal to the length of the antenna, and to TM polarisation when the incidence plane is along the length of the antenna. Consequently, to simulate the experimental illumination conditions (angle of incidence varying from 12° to 24° and electric field parallel to the antenna for all azimuths), numerical simulations were performed for two orthogonal incidence planes. One of these planes is along the antenna length (the plane is longitudinal and the wave is excited with TM polarisation) and the other plane is orthogonal to the antenna length (the plane is transverse and the wave is excited with TE polarisation). Those simulations were performed for three angles of incidence $\theta_{in} = 12^\circ, 18^\circ, 24^\circ$ in each plane. The total spectral response is then calculated as the average of the responses to these six conditions.

The spectrum, computed for the gold parameters from the literature, is presented in Fig. 7.14 (b) together with the experimental response. It can be seen that calculated spectrum does not fit the experimental curve. Although the resonance wavelengths almost coincide, the calculated absorption is significantly smaller than the measured one. Likewise, the calculated full width at half maximum (FWHM) is smaller than that measured. The origin of peaks on the left side of the computed spectrum is discussed below.

The discrepancy between the measured and computed spectra may be due to the following reasons. As the thickness ($t = 10$ nm) of the metal layer is thinner than the mean free path of electrons in gold ($\lambda_e = 38$ nm) [157], the free surface at its upper limit and the interface with the SiO₂ layer at its lower limit, introduce an additional scattering process for the electron of the metal. This mechanism must be taken into account in the effective permittivity of the metal. This effect, which has been extensively studied for metal nanoparticles (see for example the reference [158]) is significant for nanoparticles with a diameter of less than about 10 nm.

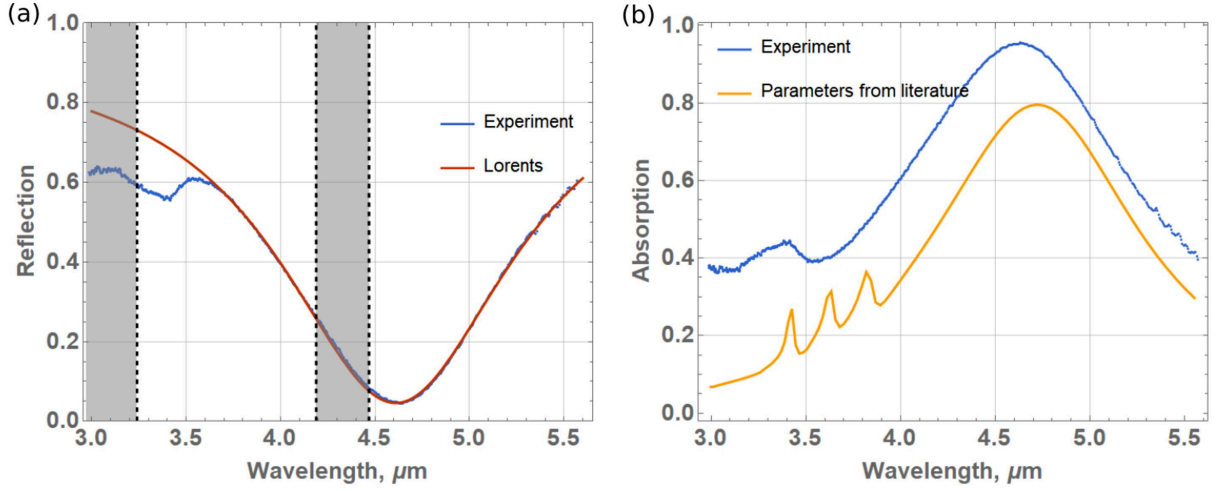


Figure 7.14: (a) The measured reflection spectrum (blue line) for the IMI array with parameters $p = 2.5 \mu\text{m}$, $l = 1.219 \mu\text{m}$, $w = 235 \text{ nm}$, and $t = 10 \text{ nm}$ placed above the mirror, and its Lorentz fit (red line). Gray zones corresponds for two absorption bands. (b) The absorption spectrum (blue line) obtained from the measured reflection spectrum (picture (a)) as $1 - R$, and the result of the numerical simulation for the same array with the gold parameters, taken from literature.

Additionally, irregularities in the deposited metal layer, such as grains or thickness fluctuations, may introduce local variations of the metal permittivity. Finally, 1 nm Ti layer, added at the Au - SiO_2 interface to improve its adhesion, can introduce additional changes in the optical response. In addition, it is likely that this layer of titanium was fully oxidized during the manufacturing process. However, the modeling of this oxidized material, which should require an additional specific study, remains outside the scope of this work. Here, we will be content to extract from the experimental data an effective value of relaxation rate Γ_{eff} of the electrons in the gold layer, noting that it takes into account all the effects mentioned above.

Taking all these effects into account, a fitting procedure of the calculated spectrum to that measured is carried out with three free parameters: the electron relaxation rate Γ_{eff} , the metal thickness t , and the length of the structure l . The best agreement (Fig. 7.15 (a), the purple line) is obtained for $\Gamma_{\text{eff}}/\omega_p = 0.0101 \pm 0.0004$, $l_{\text{eff}} = (1.194 \pm 0.008) \mu\text{m}$, and $t_{\text{eff}} = (10.0 \pm 0.3) \text{ nm}$. The greatest difference from the initial values is observed for the electron relaxation rate with an increase of 82% compared to the value of the literature. This result is in good agreement with those published on nanoparticles and confirms the role of the surface/interface for very thin metal layers (i.e. with thickness less than the electron mean free path). Also, the variation in length is about 2%, that is to say close to its measurement error, while the metal thickness is equal to its experimental value.

The numerical results allow to explain the accidental behaviour observed on the experimental

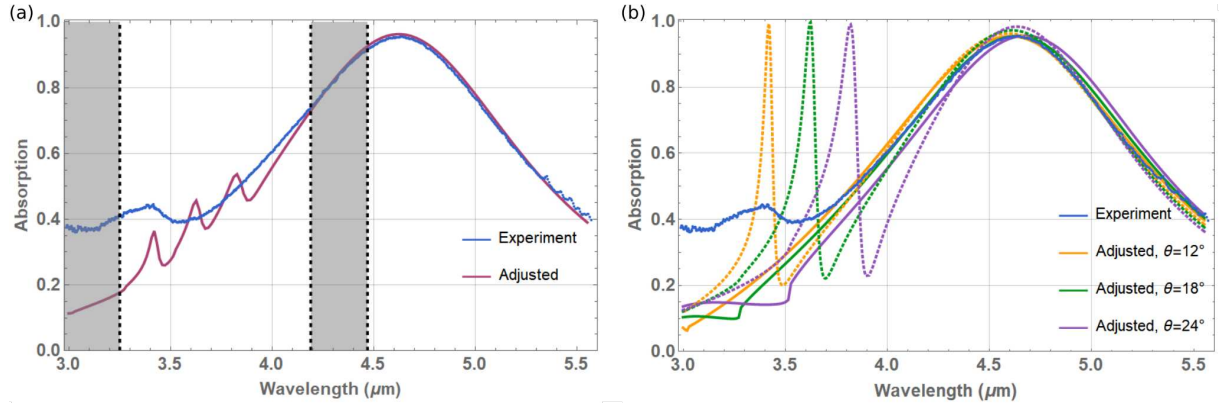


Figure 7.15: The measured reflection spectrum (blue line) for the IMI array, and computed spectra (purple line) with adjusted parameters, each point of which is the mean algebraic value of six spectra from numerical simulations for two incident planes and three incident angles. (b) The measured reflection spectrum (blue line) and these six spectra separately. The solid lines are for the transverse incident plane (orientated orthogonal to the antenna length) and TE polarisation and dashed lines for the longitudinal incident plane (orientated along the antenna length) and TM polarisation.

curve at short wavelengths. Figure 7.15 (b) shows the calculated response for the six chosen configurations: two incident planes and three incident angles. For the longitudinal plane of incidence, the antennas are excited by the TM polarisation. For sufficiently short wavelengths, this polarization excites a guided mode resonance in the SiO₂ layer, represented by sharp peaks plotted by dashed lines in Fig. 7.15 (b). Due to the refractive index of silicon dioxide and the oblique incidence, the first order of diffraction is excited in the SiO₂ layer. This works as a waveguide evacuating part of the corresponding energy away from the IMI antenna. As in the simulation the array is infinite, this energy does not leave the SiO₂ layer, it therefore appears as a sharp absorption peak. This explanation is confirmed by the different peaks, corresponding to different angles of incidence.

As the experimental spectrum was measured over a continuous range of incidence angles, instead of a few peaks calculated for a few angles, it shows a plateau spread over a range of wavelengths. As the measurement is made for all azimuths, the response is averaged over an infinity of incidence planes. The corresponding absorption therefore does not reach the 100% calculated for a single plane but a smaller value. It is represented by a purple curve in Fig. 7.15 (a), where, after summation for two planes and three incident angles, there are a few small peaks.

In the case of the transverse incident plane, the antennas absorb the TE-polarised radiation. In this case, a waveguide mode is not excited in SiO₂ layer, due to different reflection conditions on dielectric interface compared to TM case [159]. However, due to the absence of the peaks

on the left side, these spectra show clearly the appearance of the minus first diffractive order, arising for the shortest wavelength. Indeed, according to the equation connecting the incident angle θ with the wavelength λ and the period d

$$\frac{\lambda}{d} - \sin \theta = 1, \quad (7.17)$$

this order appears at the wavelength $3.52 \mu m$ for the 24° , at $3.27 \mu m$ for the 18° , and at $3.02 \mu m$ for the 12° for the considered period $p = 2.5 \mu m$. This diffractive order appears also for the second incident plane, changing slightly the left slope of the left peak.

To end the discussion on the plateau at the experimental spectrum, the presence of the absorption by SiO_2 , indicated as a grey zone in Fig. 7.15 (a) should be taken into account. This absorption may be the reason for the discrepancy observed at short wavelengths between the computed and measured spectra.

To summarize, the IMI geometry with a mirror has been found to be an effective absorber in the infrared wavelength range. Numerical computations were validated by the experiment, where the rectangular antennas were distanced from the mirror by a SiO_2 layer. Covering about 4% of the period area, the antenna absorb almost all incident radiation. The comparison of the experimental data and simulation results shows that the losses in metal layer are higher than what is predicted by literature. This is due to the ultra-small thickness of this layer and possible manufacturing imperfections. The obtained results open up promising perspectives for ultra-small bolometers, operating at room temperature and having better sensitivity and shorter response time.

7.8.

Conclusion

In this Chapter it is demonstrated that an array of IMI structures can be efficiently coupled to incident radiation being when placed at the distance $\lambda/4$ above the mirror. These metallic antennas absorb all the incident radiation while occupying only a few percent of the period area. In these structures, the mode excited at the critical coupling, is a dipole-like mode with a maximum of the electric field at ends of the metal bar, and a maximum of the magnetic field on its sides at the mid-length. This configuration is similar to that observed on one of the metal layers of an MIM structure placed under the same conditions. In both cases, the coupling is mainly provided by the electric field of the standing wave and the magnetic field of the mode.

The geometric parameters of the IMI antenna makes it possible to manage its optical response. While the length of the metal antenna is associated, in the first order, with the resonance wavelength, its width and thickness define the effective index of the mode as well as the non-radiative losses. As with the MIM structures, the losses in the structure depend in a complex way on the skin depth and the conductivity of the metal. The current distribution in the antenna bar can be described as the sum of four exponential terms. This allows to evaluate the losses in structures through different sections, and to define the critical coupling conditions. It is also demonstrated that for the same geometric section, the optical losses depend on the aspect ratio of this section.

It was also shown that IMI arrays demonstrate a large manufacturing tolerance on the distance separating them from the mirror, having high absorption values ($> 90\%$) in a band approximately one micrometer wide around $\lambda_{res}/4$. Due to specificity of the coupling process, IMI antennas of rectangular geometry are extremely sensitive to the polarisation of the incident radiation, showing a high absorption tolerance to the incident angle when the electric field is orientated along the antenna but demonstrating no absorption when the electric field have the orthogonal direction.

The simplicity of IMI arrays as well as the dipole-like mode at the critical coupling conditions allows to model them as a point dipole array. This model can predict their absorbance spectra by a rapid calculation, thus avoiding the long calculation times of the full-wave simulations. It is enough to extract the response of a single antenna from one numerical simulation to predict its response in the case of different periods or distances to the mirror. Taking into account the distribution of currents in the metal bar observed at the critical coupling, the dipole model can be made more precise, when an antenna is presented as a sum of currents instead a point dipole. This will be the subject of a latter study.

Finally, the quasi-total absorption was demonstrated for IMI antennas placed at a distance of about $\lambda/(4n)$ above a mirror in the MWIR spectral domain ($3.5 - 5.5 \mu m$). The experimental sample consists of a layer of SiO_2 deposited on a thick layer of gold. on which is deposited a periodic array of $10 nm$ tjick rectangular gold antennas. It shows a maximum absorption of about 95%. Good agreement with the numerical simulation results was found for an electron relaxation frequency of about $\Gamma/\omega_p = 0.010$, which is significantly higher than the literature value $\Gamma/\omega_p = 0.0055$. This correction corresponds to an increase in losses due to a metal layer thickness smaller than the electron mean free path in the metal, as well as to imperfections linked

to the fabrication process (grains, variable thickness, etc.). This encouraging result validates numerical simulations, provided in the present work, and confirms that IMI antenna structures are efficient absorbers.

Compared to the other structures studied in this work under the same conditions, IMI arrays have the smallest volume density, less than 1 nm . This result opens up prospects for the creation of a new class of thermal sensors. Their response can be predefined with its geometric parameters, and their low density allows the use of few antennas in the same period, creating multi-color devices. Thus, IMI arrays are promising structures for absorption applications, in particular, bolometers. A small antenna volume means a reduction in thermal capacity which can be used to revisit the sensitivity - rapidity compromise of the bolometer design, allowing better sensitivity (increase in thermal resistance) and faster response (reduction of the product resistance - capacity). However, this technology needs some further development, in particular a thermometer suitable for IMI antennas.

Conclusion and Outlooks

The microbolometer concept has not changed significantly since it was presented in the early 90s. The most significant progress has certainly been the reduction in pixel size and increasing picture resolution. However, this technology is now reaching its limit, and in recent years a slowdown in the progression of performance is observed. The design of smaller pixels comes up against the fundamental barrier of light diffraction, as well as technical difficulties in the organization of the thermal isolation parts and the absorber. Additionally, the compromise between the sensitivity and response time is the integral feature for all microbolometers. Thus, a reduction in pixel size tends to decrease its sensitivity. Meanwhile, future imaging systems in the LWIR domain require improved pixel sensitivity and multispectral sensing without loss of operating speed.

The use of plasmonic structures as an absorber part could overcome the barrier in the development of thermal detectors and shift the compromise between sensitivity and speed towards new, better values. Being smaller than the wavelength, plasmonic structures concentrate the electromagnetic fields in tiny volumes and provide efficient absorption of the electromagnetic radiation. They significantly reduce thermal capacitance, paving the way for a better sensitivity - speed compromise, while their miniature size leaves enough area for thermal isolation. Thus, a device based on plasmonic structure, maintaining operation at room temperature, could have characteristics approaching those of cooled detectors. Besides, the response of these sub-wavelength structures can be tuned by design, promising some useful detector functionality, such as selectivity or polarisation sensing. Therefore, in the present work plasmonic structures are proposed as the heart of a new conception for thermal detectors. Although there are few attempts to create plasmonic-based bolometers, only lab concepts were presented, while nothing

has been designed for industrial fabrication.

In the present work two types of plasmonic structures were examined: metal-insulator-metal (MIM) structures with all structured layers and insulator-metal-insulator (IMI) structures, both placed above a mirror. The presence of the mirror and the placement of the absorbers at a distance equal to $\lambda_{res}/4$ above it is an a priori constraint of this study in order to introduce as few modifications as possible in their manufacturing compared to that of current microbolometers. This distance corresponds to maximum absorption in the volume of the absorbent membrane. On the other hand, the presence of a mirror suppresses the transmission and enhances the electric field in the region of the structure.

It is demonstrated that the two types of studied structures can be critically coupled by being placed at $\lambda_{res}/4$ above the mirror. A priori, this distance is not well suited for MIM structures since they function as a magnetic dipole, also called a magnetic meta-atom. Their usual operating configuration assumes in particular the implementation of the structure at the maximum of the magnetic field. However, it is shown in this work that MIM structures can absorb the total incident radiation when placed at $\lambda_{res}/4$ above the mirror. The corresponding behavior, different of that of "classic" MIM, is reported and studied.

It is observed, that the excited MIM mode in the $\lambda_{res}/4$ configuration with a mirror is similar to the fundamental MIM mode but the processes of funneling energy to the antenna differ. Indeed, the only way to build the Poynting vector, directing electromagnetic energy towards the MIM antenna, is with the magnetic field of the mode, whereas it is with the incident magnetic field in the "classical" case.

In the case of 1D periodic MIM, the energy flow entering inside the structure is due to both magnetic and electric field of the mode, while the decomposition of the fields and currents are similar (although not perfectly) to those for the standard configuration. It is found that the critical coupling curve of this specific configuration for 1D MIM shifts towards higher dielectric thicknesses with decreasing the thickness of the metal layers. This is connected with the increase in the resistance of the metal layer which varies inversely with the effective metal thickness t_{eff} . As a result, the volume density for 1D MIM structures is far from values desirable for absorber applications. The smallest values are obtained for the highest l/p ratios, the heating is then not concentrated on small area. For smaller l/p values, the volume density of MIMs is higher than 300 nm , that is far from the objectives of this study.

2D periodic MIM structures in the same $\lambda_{res}/4$ configuration with a mirror, however, can

overcome this limit due to a more complex field distribution than that of 1D MIMs: a dipole-like electric field along the length of the structure and an intense magnetic field at the edges of the structure in the direction orthogonal to the length, with the maximum at the mid-length. As a result, in the 2D MIM case the coupling is mainly provided by the electric field of the standing wave and the magnetic field of the mode, and the behavior of the Poynting vector, directing the energy flux to the structure, differs from that of 1D MIMs. Indeed, a significant part of the energy comes from above and below the structure and is absorbed at the air-metal interfaces. Also, a funnel effect of energy appears in the direction of the structure limitation (that which is infinite on 1D MIMs).

Consequently, the electric current density of the excited mode is more complex than that of 1D MIMs. For the simplest case of a rectangular-shaped structure, the electric current distribution in each metal layer can be presented as the sum of four exponential terms. For other geometries this tendency is maintained, although distribution is more complex.

It is shown that the geometry of the 2D MIM structure and the nature of the materials can influence the optical losses through the electric current decomposition and thus the critical coupling. In particular, the geometric shape of 2D MIM patches at the critical coupling conditions influences its response due to the change in optical losses, connected with the width of the structure. Four simple geometries and four metals with different permittivities are studied. Among studied geometries the rectangular and cross-shaped structures show the lowest antenna volumes for, respectively, polarised and non-polarized radiation.

The spatial distribution of electric current density is a function of a complex balance between the skin depth and conductivity of the chosen metal. At critical coupling conditions, the dielectric layer having a thickness significantly greater than that of the metal layer, provides the main contribution to the volume of the structure. However, structures made of gold have a bit smaller volume and thermal capacity, thus they can be considered optimal among studied structures.

Particular attention was paid to the behavior differences between the rectangular and cross geometries. For a polarization along the length of the metal bar, the two structures have a similar response but the cross-shaped structure shows on the one hand a resonance peak shifted towards blue (its length is equal to that of the rectangle) and on the other hand a slightly lower absorption maximum. These differences are explained by the modeling of MIM antennas by two $\lambda/4$ antennas separated by a perfect conductive plane. The thickness of the conductor is finite

in the case of the cross-shaped antenna, it is zero in that of the rectangle. This model explains all the differences observed between the two geometries and opens perspectives for reconsidering the MIM mode and for a more in-depth study of complex MIM structures.

The different coupling processes also change the critical coupling curve because in the case of 2D MIMs the period varies inversely with the thickness of the dielectric layer. In this case, the decrease of the metal layer thickness shifts the critical coupling curve towards thinner dielectric layers. As a result, it makes it possible to achieve the critical coupling for thicknesses of 5 nm of metal and 300 nm of silicon. Thus, a much smaller volume density than that of 1D MIM is achieved for 2D MIMs: 4.2 nm for rectangles and about 8 nm for the crosses.

As demonstrated in literature, the presence of mirror allows complete absorption for systems with a single absorbent layer. In this work, it is shown that an array of IMI antennas at the distance $\lambda_{res}/4$ above the mirror, can be critically coupled to the incident wave. More importantly, it is demonstrated that this total absorption can be obtained with an antenna occupying only a few percent of the period area. At critical coupling, the excited mode in the IMI structure is a dipole-like mode with a maximum electric field at the ends of the metal bar, and a maximum magnetic field at the sides, at mid-length. This arrangement is similar to that observed for 2D MIM structures in similar conditions. Here again, the coupling is provided by the electric field of the standing wave and the magnetic field of the mode.

The study of the influence of the geometric parameters of the antenna on the optical response of the array shows that its width and its thickness allow to control the effective index of the mode and the non-radiative losses, while the length of the metal antenna is associated with the resonance wavelength. In the case of the rectangular shaped antenna, as with MIM structures, the current distribution in the metal bar can be presented as the sum of four exponential terms. Thus, the optical losses can be calculated for different sections of the metal bar, and the critical coupling conditions can thus be found.

It is demonstrated that due to the specificity of the coupling process, IMI arrays have a large manufacturing tolerance on the distance between them and the mirror, having high absorption values in a band approximately one micrometer wide near $\lambda_{res}/4$. Antennas of the rectangular geometry are extremely sensitive to the polarisation of incident radiation, showing a high absorption tolerance for the incident angle when the electric field is along the antenna but having no absorption when the electric field is orthogonal to that direction.

The simplicity of IMI arrays allows them to be modeled as a point dipole array. The dipole

model makes it possible to predict their absorbance spectra, thus avoiding time consuming full-wave simulations as soon as the polarizability of the single antenna is extracted.

Finally, the experimental validation of the IMI array as an efficient absorber was provided in MWIR. It was proved that the IMI array, composed of a layer of SiO_2 of thickness $\lambda_{res}/(4n)$, deposited on a thick gold layer and of a periodic array of rectangular antennas of 10 nm of gold, shows quasi-total absorption. A good agreement is demonstrated with the numerical simulations (peak position and absorption value) when the electron relaxation time in the metal is much smaller than the literature value. These higher losses in metal are caused by the very small thinness (i.e. smaller than the electron mean free path) and imperfections of the gold layer.

The volume density of IMI arrays at the critical coupling is found to be the smallest among the considered structures in this study, less than 1 nm for all studied structures. In this way, the IMI antenna is the simplest plasmonic structure which, being placed at a quarter of the resonant wavelength above the mirror, has the smallest possible volume and efficiently absorbs incoming radiation.

In this way, the purpose of this work is fulfilled. Numerically for LWIR region and experimentally for MWIR region, it is shown that an array of ultra-small plasmonic antennas placed above the mirror and occupying only a few percent of the surface area, can provide total (or almost total) absorption of the incident radiation. The use of these antennas as absorbent presents a very high potential for future bolometric structures. As an antenna occupies only a few percent of the period area, multiple antennas can be placed in the same period, allowing a response at multiple wavelengths to be provided. Besides, the structure miniaturisation also allows to decrease the mechanical support of the absorber, and offers more space for the isolation system.

The present work can be developed in the following directions:

1. First of all, the second stage for a new concept of bolometer – a new kind of thermometer – will have to be developed. This could involve adapting the method currently used in microbolometers – the variation in resistance with temperature – to the case of IMI antennas. To avoid the inevitable disturbances of the electromagnetic field by the metallic measurement lines, it could also be designed on the basis of plasmonics and photonics for remote temperature measurement, providing in this way outstanding characteristics of the device working at room temperature.

2. The experimental validation in the LWIR domain of the antennas on a membrane are a necessary step on the way to the creation of new devices.

3. A more precise approximation of the behaviour of the IMI antenna could be useful to increase the precision of the result while maintaining a short computation time. This is of great importance for massive 3D numerical simulations. It could be done by replacing the point dipole approximation used in this work by a modeling of the current distribution in the metal.

Appendices

A. Legal and natural units

Nowadays, the legal SI units kg, meter, ampere, are defined from the physical constants $h = 6.626\,070\,15 \times 10^{-34} \text{ J s}$ ($\text{kg m}^2 \text{s}^{-1}$), $c = 299\,792\,458 \text{ m s}^{-1}$, and $e = 1.602\,176\,634 \times 10^{-19} \text{ C}$ (s A) [160].

Here we give some conversion formulas between legal (L) and natural (N) units.

For a length value, legally measured as $L_{(L)}$ in meters, and naturally measured as $L_{(N)}$ in seconds:

$$\left[\frac{L_{(L)}}{\text{m}} \right] = c \left[\frac{L_{(N)}}{\text{s}} \right] \quad (1)$$

For a speed $\mathbf{v}$:

$$\left[\frac{\mathbf{v}_{(L)}}{\text{m s}^{-1}} \right] = c \left[\frac{\mathbf{v}_{(N)}}{1} \right] \quad (2)$$

and specially for speed of light with $c_{(N)} = 1$:

$$\left[\frac{c_{(L)}}{\text{m s}^{-1}} \right] = c \left[\frac{c_{(N)}}{1} \right] \quad (3)$$

For an electric charge Q :

$$\left[\frac{Q_{(L)}}{\text{C} = \text{A s}} \right] = e \left[\frac{Q_{(N)}}{1} \right] \quad (4)$$

same conversion for an electric flux Φ_e

$$\left[\frac{\Phi_{e(L)}}{\text{C} = \text{s A}} \right] = e \left[\frac{\Phi_{e(N)}}{1} \right] \quad (5)$$

The electric flux density $\mathbf{D}$ is the density of electric flux on a space-space surface:

$$\left[\frac{\mathbf{D}_{(L)}}{C \text{ m}^{-2} = \text{m}^{-2} \text{ s A}} \right] = \frac{e}{c^2} \left[\frac{\mathbf{D}_{(N)}}{\text{s}^{-2}} \right] \quad (6)$$

The magnetic field $\mathbf{H}$ is the density of electric flux on a time-space surface:

$$\left[\frac{\mathbf{H}_{(L)}}{C \text{ m}^{-1} \text{ s}^{-1} = \text{m}^{-1} \text{ A}} \right] = \frac{e}{c} \left[\frac{\mathbf{H}_{(N)}}{\text{s}^{-2}} \right] \quad (7)$$

The magnetic flux Φ_m with natural unit being the Dirac magnetic monopole $q_m = \frac{h}{e}$

$$\left[\frac{\Phi_{m(L)}}{\text{Wb} = \text{kg m}^2 \text{ s}^{-2} \text{ A}^{-1}} \right] = \frac{h}{e} \left[\frac{\Phi_{m(N)}}{1} \right] \quad (8)$$

The magnetic flux density $\mathbf{B}$ is the density of magnetic flux on a space-space surface:

$$\left[\frac{\mathbf{B}_{(L)}}{\text{T} = \text{Wb m}^{-2} = \text{kg s}^{-2} \text{ A}^{-1}} \right] = \frac{h}{ec^2} \left[\frac{\mathbf{B}_{(N)}}{\text{s}^{-2}} \right] \quad (9)$$

The electric field $\mathbf{E}$ is the density of magnetic flux on a space-time surface:

$$\left[\frac{\mathbf{E}_{(L)}}{\text{V m}^{-1} = \text{kg m s}^{-3} \text{ A}^{-1}} \right] = \frac{h}{ec} \left[\frac{\mathbf{E}_{(N)}}{\text{s}^{-2}} \right] \quad (10)$$

The vacuum magnetic permeability μ_0 which is no more defined exactly as $4\pi \times 10^{-7} \text{ N A}^{-2} (\text{kg m s}^{-2} \text{ A}^{-2})$ but from fine structure experimental determination

$$\mu_0 = 2\alpha \frac{h}{e^2 c} \quad (11)$$

The more recent measure of the fine structure constant is $\alpha = 1/137.035999206(11)$ [161] giving $\frac{\mu_0}{4\pi \times 10^{-7}} = 0.9999999965(8)$ The characteristic impedance of vacuum $Z_0 = \mu_0 c$ gives the ratio of $\mathbf{B} \cdot \mathbf{n}$ the magnetic flux density on a space-space surface orthogonal to $\mathbf{n}$ over $\mathbf{H} \cdot \mathbf{n}/c$ the electric flux density on the fully orthogonal c-time-space surface $c\mathbf{t} \wedge \mathbf{n}$. As $\mathbf{n}$ is arbitrary, we can write:

$$\mu_0 c = \frac{\mathbf{B}}{\mathbf{H}/c} \quad (12)$$

Z_0 is also the ratio of $\mathbf{E} \cdot \mathbf{n}/c$ the magnetic flux density on a space-c-time surface $\mathbf{n} \wedge c\mathbf{t}$ over $\mathbf{D} \cdot \mathbf{n}$ the electric flux density on the fully orthogonal space-space surface orthogonal to $\mathbf{n}$.

$$\mu_0 c = \frac{1}{\varepsilon_0 c} = \frac{\mathbf{E}/c}{\mathbf{D}} \quad (13)$$

$$\left[\frac{Z_{0(L)}}{\Omega = \text{kg m}^{-3} \text{ A}^{-2}} \right] = 2\alpha \frac{h}{e^2} = \frac{h}{e^2} \left[\frac{Z_{0(N)}}{1} \right] \quad (14)$$

with

$$Z_{0(N)} = 2\alpha. \quad (15)$$

B.
Different languages of electromagnetism

There are many different languages to describe classical electromagnetism. Vector and tensor approaches[3, 17], are common and well-known. Besides them, differential forms[17, 22, 47, 23, 24] and geometrical algebra[48] present the electromagnetic laws in an elegant way.

In this appendix we briefly bridge the gap between the different languages and the symbols they use, and we specify our sign conventions. The same unit is used for space and time ($c = 1$), and metric tensor $\mathbf{g}$ has signature $(+, -, -, -)$.

An inertial frame of reference is an orthonormal vector basis $\{\mathbf{e}_i\}$ ($\mathbf{e}_0 = \mathbf{t}$, $\mathbf{e}_1 = \mathbf{x}$, $\mathbf{e}_2 = \mathbf{y}$, $\mathbf{e}_3 = \mathbf{z}$), with

$$\mathbf{t}^2 = 1 \tag{16}$$

$$\mathbf{x}^2 = \mathbf{y}^2 = \mathbf{z}^2 = -1. \tag{17}$$

The orthonormal 1-form basis corresponding to this basis is $\{\mathbf{e}^i\}$ ($\mathbf{e}^0 = \tilde{\mathbf{t}}$, $\mathbf{e}^1 = \tilde{\mathbf{x}}$, $\mathbf{e}^2 = \tilde{\mathbf{y}}$, $\mathbf{e}^3 = \tilde{\mathbf{z}}$)¹, with

$$\tilde{\mathbf{t}}^2 = 1 \tag{18}$$

$$\tilde{\mathbf{x}}^2 = \tilde{\mathbf{y}}^2 = \tilde{\mathbf{z}}^2 = -1. \tag{19}$$

We also have:

$$\mathbf{e}^i \mathbf{e}_j = \delta_j^i, \tag{20}$$

with δ_j^i the Kronecker symbol, equal to 1 if $i = j$ and equal to 0 if $i \neq j$. This equation is also valid for an arbitrary, not orthonormal basis.

The field vectors are $\mathbf{E}$, $\mathbf{B}$, $\mathbf{H}$, $\mathbf{D}$. In order to make explicit the symmetry between the two

¹The 1-form basis is also commonly written $(dt, dx, dy, dz) = (dx^0, dx^1, dx^2, dx^3)$, with some drawbacks

groups of Maxwell's equations the fields $\mathbf{H}$ and $\mathbf{D}$ are considered:

$$\nabla \cdot \mathbf{B} = 0 \quad (21)$$

$$\nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0 \quad (22)$$

$$\nabla \cdot \mathbf{D} = \rho_e \quad (23)$$

$$\nabla \times \mathbf{H} - \partial_t \mathbf{D} = \mathbf{j}_e, \quad (24)$$

the vacuum constitutive equations being

$$\mathbf{B} = \mu_0 \mathbf{H} \quad (25)$$

$$\mathbf{E} = \mu_0 \mathbf{D}. \quad (26)$$

In natural units (see Appendix A), the flux of $\mathbf{D}$ on a space-space surface and the flux of $\mathbf{H}$ on a time-space surface (the electric flux) are measured in units of the elementary electric charge $q_e = e$; the flux of $\mathbf{B}$ on a space-space surface and the flux of $\mathbf{E}$ on a space-time surface (the magnetic flux) are measured in units of the elementary magnetic charge $q_m = h/e$ with h the unit of action quantum. With these units, $\mu_0 = 2\alpha$.

Defining the antisymmetric tensors with components

$$\mathcal{F}^{ij} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{pmatrix} \quad (27)$$

and

$$G^{ij} = \begin{pmatrix} 0 & D_x & D_y & D_z \\ -D_x & 0 & H_z & -H_y \\ -D_y & -H_z & 0 & H_x \\ -D_z & H_y & -H_x & 0 \end{pmatrix}, \quad (28)$$

Maxwell's and constitutive equations become[3]

$$\partial_j \mathcal{F}^{ij} = \mathcal{F}^{ij}_{;j} = 0 \quad (29)$$

$$\partial_j G^{ij} = G^{ij}_{;j} = J^i \quad (30)$$

$$F_{ij} = \mu_0 G_{ij} \quad (31)$$

with

$$F_{ij} = \frac{1}{2} \epsilon_{ijkl} \mathcal{F}^{kl} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix} \quad (32)$$

and

$$G_{ij} = g_{ik} g_{jl} G^{kl} = \begin{pmatrix} 0 & -D_x & -D_y & -D_z \\ D_x & 0 & H_z & -H_y \\ D_y & -H_z & 0 & H_x \\ D_z & H_y & -H_x & 0 \end{pmatrix}. \quad (33)$$

The $\mathcal{G}$ tensor dual to G has components

$$\mathcal{G}_{ij} = \frac{1}{2} \epsilon_{ijkl} G^{kl} = \begin{pmatrix} 0 & H_x & H_y & H_z \\ -H_x & 0 & D_z & -D_y \\ -H_y & -D_z & 0 & D_x \\ -H_z & D_y & -D_x & 0 \end{pmatrix} \quad (34)$$

The underlying objects represented by antisymmetrical tensors F_{ij} and $\mathcal{G}_{ij}$ are 2-forms. The magnetic and electric 2-forms (also called by MTW Faraday's and Maxwell's 2-forms[17], and by Deschamps as force and source 2-forms[22]) are

$$\tilde{\Phi}_m = \frac{1}{2} F_{ij} \mathbf{e}^i \wedge \mathbf{e}^j = E_x \mathbf{e}^{10} + E_y \mathbf{e}^{20} + E_z \mathbf{e}^{30} + B_x \mathbf{e}^{23} + B_y \mathbf{e}^{31} + B_z \mathbf{e}^{12} \quad (35)$$

$$\tilde{\Phi}_e = \frac{1}{2} \mathcal{G}_{ij} \mathbf{e}^i \wedge \mathbf{e}^j = H_x \mathbf{e}^{01} + H_y \mathbf{e}^{02} + H_z \mathbf{e}^{03} + D_x \mathbf{e}^{23} + D_y \mathbf{e}^{31} + D_z \mathbf{e}^{12} \quad (36)$$

with

$$-\mathbf{e}^{10} = \mathbf{e}^{01} = \tilde{\mathbf{t}} \wedge \tilde{\mathbf{x}} \quad (37)$$

$$-\mathbf{e}^{20} = \mathbf{e}^{02} = \tilde{\mathbf{t}} \wedge \tilde{\mathbf{y}} \quad (38)$$

$$-\mathbf{e}^{30} = \mathbf{e}^{03} = \tilde{\mathbf{t}} \wedge \tilde{\mathbf{z}} \quad (39)$$

$$\mathbf{e}^{23} = \tilde{\mathbf{y}} \wedge \tilde{\mathbf{z}} \quad (40)$$

$$\mathbf{e}^{31} = \tilde{\mathbf{z}} \wedge \tilde{\mathbf{x}} \quad (41)$$

$$\mathbf{e}^{12} = \tilde{\mathbf{x}} \wedge \tilde{\mathbf{y}}. \quad (42)$$

Taking into account that $\mathbf{e}^{01}(\mathbf{t} \wedge \mathbf{x}) = 1$, $\mathbf{e}^{01}(\mathbf{t} \wedge \mathbf{y}) = 0$, etc. the density of magnetic flux on space-space surfaces are

$$B_x = \tilde{\Phi}_m(\mathbf{y} \wedge \mathbf{z}) \quad (43)$$

$$B_y = \tilde{\Phi}_m(\mathbf{z} \wedge \mathbf{x}) \quad (44)$$

$$B_z = \tilde{\Phi}_m(\mathbf{x} \wedge \mathbf{y}). \quad (45)$$

Indeed, $\mathbf{B}$ is called the magnetic flux density.

The density of electric flux on space-space surfaces are

$$D_x = \tilde{\Phi}_e(\mathbf{y} \wedge \mathbf{z}) \quad (46)$$

$$D_y = \tilde{\Phi}_e(\mathbf{z} \wedge \mathbf{x}) \quad (47)$$

$$D_z = \tilde{\Phi}_e(\mathbf{x} \wedge \mathbf{y}). \quad (48)$$

Indeed, $\mathbf{D}$ which was called the electric displacement is now called the electric flux density.

The density of magnetic flux on space-time surfaces are

$$E_x = \tilde{\Phi}_m(\mathbf{x} \wedge \mathbf{t}) \quad (49)$$

$$E_y = \tilde{\Phi}_m(\mathbf{y} \wedge \mathbf{t}) \quad (50)$$

$$E_z = \tilde{\Phi}_m(\mathbf{z} \wedge \mathbf{t}). \quad (51)$$

The common name of $\mathbf{E}$ is the electric field strength, but as $\mathbf{B}$, it is a magnetic flux density!

The density of electric flux on time-space surfaces are

$$H_x = \tilde{\Phi}_e(\mathbf{t} \wedge \mathbf{x}) \quad (52)$$

$$H_y = \tilde{\Phi}_e(\mathbf{t} \wedge \mathbf{y}) \quad (53)$$

$$H_z = \tilde{\Phi}_e(\mathbf{t} \wedge \mathbf{z}). \quad (54)$$

The common name of $\mathbf{H}$ is the magnetic field strength, but as $\mathbf{D}$, it is an electric flux density! It should be noticed that the orientations of space-time surfaces (for $\mathbf{E}$) and time-space surfaces (for $\mathbf{H}$) are opposite.

The infinitesimal magnetic and electric fluxes on an infinitesimal oriented parallelogram $dS = d\mathbf{u} \wedge d\mathbf{v}$ with $d\mathbf{u} = du^i \mathbf{e}_i$ and $d\mathbf{v} = dv^j \mathbf{e}_j$ are

$$d\Phi_m = \tilde{\Phi}_m(dS) = F_{ij} du^i dv^j \quad (55)$$

$$d\Phi_e = \tilde{\Phi}_e(dS) = \mathcal{G}_{ij} du^i dv^j \quad (56)$$

Take care to avoid the confusion with the common notation $\tilde{\Phi}_m = \frac{1}{2} F_{ij} dx^i dx^j$. Our letter d is not the exterior derivative $\tilde{d}$, it is just used here to mark a quantity as infinitesimal (sufficiently small to compute a model with high accuracy).

The infinitesimal fluxes can be integrated on any oriented 2D surface Σ of the 4D spacetime defining the magnetic and electric fluxes

$$\Phi_m(\Sigma) = \int_{\Sigma} \tilde{\Phi}_m(dS) \quad (57)$$

$$\Phi_e(\Sigma) = \int_{\Sigma} \tilde{\Phi}_e(dS) \quad (58)$$

In the language of electromagnetic fluxes, Maxwell's equations are

$$\Phi_m(\Sigma_{\text{closed}}) = 0 \quad (59)$$

$$\Phi_e(\Sigma_{\text{closed}}) = n_e \quad (60)$$

where n_e is the net number of electric charges in any 3D volume enclosed by the 2D surface Σ_{closed} in the 4D spacetime. As pointed out by Cartan[31], this integral formulation doesn't need to suppose that partial derivatives of the field do exist, and they are valid in curved spacetime, with no assumption concerning its affine connexion.

In the language of differential forms, the Maxwell's and constitutive equations are

$$\tilde{d}\tilde{\Phi}_m = 0 \quad (61)$$

$$\tilde{d}\tilde{\Phi}_e = \mathcal{J} \quad (62)$$

$$\star\tilde{\Phi}_m = \mu_0\tilde{\Phi}_e \quad (63)$$

where $\tilde{d}$ is the exterior derivative, $\mathcal{J}$ is the 3-form current, and $\star$ is the Hodge operator. The exterior derivative of each term $F_{ij}e^i \wedge e^j$ (35) is equal to $\partial_k F_{ij}e^k \wedge e^i \wedge e^j$. Considering that the only non-zero terms are (notwithstanding a permutation):

$$e^{123} = e^1 \wedge e^2 \wedge e^3 = -e^1 \wedge e^3 \wedge e^2 = \dots \quad (64)$$

$$e^{023} = e^0 \wedge e^2 \wedge e^3 \quad (65)$$

$$e^{031} = e^0 \wedge e^3 \wedge e^1 \quad (66)$$

$$e^{012} = e^0 \wedge e^1 \wedge e^2 \quad (67)$$

the first equation $\tilde{d}\tilde{\Phi}_m = 0$ computes as

$$\begin{aligned} \tilde{d}\tilde{\Phi}_m &= \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z} \right) e^{123} \\ &+ \left(\frac{\partial B_x}{\partial t} + \frac{\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z} \right) e^{023} \\ &+ \left(\frac{\partial B_y}{\partial t} + \frac{\partial E_x}{\partial z} + \frac{\partial E_z}{\partial x} \right) e^{031} \\ &+ \left(\frac{\partial B_z}{\partial t} + \frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} \right) e^{012} = 0 \end{aligned}$$

which corresponds to first group of Maxwell's equation $\nabla \cdot \mathbf{B} = 0$ and $\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$. Similarly, the second equation corresponds to second group of Maxwell's equation.

Advantages over conventional vector calculus make exterior algebra an elegant tool for working with electromagnetic problems, as well as for illustrating essential principles of electromagnetism [23]. An example of its effectiveness would be the direct construction of $\tau_S = H_y \mathbf{e}_0 - D_z \mathbf{e}_1 + D_x \mathbf{e}_3$ tangent vector from differential form (Eq. (2.3)) $\tilde{\phi}_e = H_y e^0 + D_z e^1 + D_x e^3$ that results directly from $\tau^i = g^{ij} \phi_j$.

From the historical perspective, the idea behind the metric-free approach to electrodynamics (based on integral conservation laws) was proposed in Refs. [162, 31, 42]. Withal, the corre-

sponding approach is presented as a discipline of mathematical physics in [163, 164, 165], as well as a branch of electrical engineering [166, 167, 50]. Unfortunately, very few textbooks employ the exterior calculus in electrodynamics [47], and it is not widely used despite its many advantages [47, 48, 40, 168, 169, 170, 49, 171, 39]. Differential form notation markedly simplifies well-known formulae and calculations of electrodynamics, improving problem solving in a number of cases, and establishing a direct connection to geometric images [50, 23]. Unfortunately, time-space splitting is almost universal in the literature and geometric constructions are never absolute. One of the few exceptions to this 1-3 splitting was made by Riesz [38].

C.

Metric free approach

C.1 Introduction

The foundation of the metric-free approach to electrodynamics, based on integral conservation laws was laid down by Friedrich Kottler [162], Elie Cartan [172], and David van Dantzig [42]. And the basis for it was created by Hermann Grassman more than fifty years before [173]. In his book *"Die lineale Ausdehnungslehre, ein neuer Zweig der Mathematik"* (1844) he developed the conception of an algebra which represent geometric entities, such as points, lines, and planes, and manage them using certain rules [174]. Its main element is the wedge (exterior) product [174].

Unfortunately, initially Grassman's algebra did not receive much attention [174]. Lawvere [175] has noted the following reasons. At first, Grassmann's writing style was found very complicated even for native German-speaking mathematics students. Secondly, the first half of study was devoted unclear philosophical discussions on mathematical issues. And thirdly, Grassmann's mathematical misconceptions and unusual terminology [174]. Withal, in those days William Rowan Hamilton presented his quaternion formulation. By the time of publication, he was well-known and had supporters [176]. These reasons led to the fact that the Grassmann algebra was not received proper attention.

Only in the beginning of 1900 Elie Cartan implemented Grassmann's algebra to the theory of differential forms in his work *"Leçons sur les invariants intégraux"* (paragraph 55). After more than a half-century of neglecting the advantages of the differential forms have become much appreciated [174]. It is worth to mention Adhémar Saint-Venant, who also published similar ideas of exterior calculus (one year later than Grassman) [177]. But only in early 1900s Cartan finally developed the differential forms theory, based on Grassman work [40].

The exterior algebra is an elegant tool for operation with electromagnetic problems as well as for illustration the fundamental principles of electromagnetism [23]. Application of differential forms notation in electromagnetic quantities is quite natural [22]. It significantly simplifies well-known formulas and calculations, provide improved problem solving, and allow to build a direct geometrical interpretation of electromagnetism, that can provide additional physical insights [23, 50]. Exterior algebra unifies the electric with the magnetic field, and presents the EM field as one object of a single grade. Thus, description of Maxwell's equations in terms of exterior algebra on arbitrary spacetime represent them in a elegant formulation [23], making

their form is unchanged with change of coordinates (including curvilinear ones) [22]. Many scientists have found differential forms to be a powerful means for electrodynamics [22, 17, 47, 40, 48, 50, 23, 24, 168, 169, 178, 170, 49, 171]. Withal, exterior algebra approach is presented as a discipline of mathematical physics at [163, 164, 165] and at a branch of electrical engineering [166, 167].

Differential forms provide a unique way to visualize fields and sources [23, 17], that allows to look deeper into physical processes and more clearly understand some of the electromagnetic effects. All of the above make the exterior differential forms a great framework for teaching basic laws and the underlying principles of electromagnetism [23]. Nevertheless, although the exterior algebra formulation of the basis of the classical electrodynamics has been described by many authors, it has not become widely known between physicists. No significant progress was observed besides a very few modern paper with simple images.

In this Appendix, the brief mathematical introduction to the differential forms theory is provided.

C.2 1-forms and 2-forms

Let us imagine that a special function, which gives electromagnetic flux value on any surface, exists. It works like a special machine, which "eats" a piece of surface in spacetime, and gives one the flux value on it instead. This kind of "function" really exists. It is called the differential two-form. The detailed mathematical review on differential forms is presented in this Section. Using differential forms approach, it is possible to describe the generalize case of spaghetti – lasagna. Lasagna is a special 2D surface in 4D spacetime map, which construction is based on the electromagnetic field in the 4D volume. Electric and magnetic flux on its surface, being the result of action of a two-form on a bivector, are scalars. Two families of lasagne surfaces in 4D Minkowski spacetime build the absolute electromagnetic map.

To present differential forms, at first we consider some simple example to introduce to the subject. However, understanding of these simplest forms of lower degrees is the most important and largely sufficient for the further theory. Therefore, in this review, they are given maximum attention. Hereinafter the bold font and Latin letters are used for the vectors, while bold letters with tilde (often Greek) are used for forms, unless otherwise specified.

Exterior differential forms can be divided by their degree, which is an integer between 0 and n on an n -dimensional manifold [24]. While null-form is just a mathematical function [17], the

most interesting in the context of this work are one- and two-forms.

One-form

The combination of differentials represents the one-form [23]. One-form in 3D space occurs under integral sign [41]:

$$\tilde{\alpha} = A_1 d\tilde{x} + A_2 d\tilde{y} + A_3 d\tilde{z}. \quad (68)$$

In this work it is enough to understand one-form as a linear operation (linear functional) which, acting on a vector, returns a number² [17]. In other words, one-forms can be considered as covectors which are dual to ordinary contravariant vectors. Thus, they create a linear space, and a basis can be constructed. Using the metric of the space, one may make a contraction of an one-form and a vector using the inner product (also called the bar product or the duality product) [46]. For example, in 4D the bar product of a vector $\mathbf{a} = a_1 \mathbf{e}_1 + a_2 \mathbf{e}_2 + a_3 \mathbf{e}_3 + a_4 \mathbf{e}_4$ and an one-form $\tilde{\alpha} = \alpha^1 \tilde{\epsilon}^1 + \alpha^2 \tilde{\epsilon}^2 + \alpha^3 \tilde{\epsilon}^3 + \alpha^4 \tilde{\epsilon}^4$

$$\mathbf{a}|\tilde{\alpha} = \sum_{i=1}^4 \sum_{j=1}^4 a_i \alpha^j \mathbf{e}_i | \tilde{\epsilon}^j, \quad (69)$$

where $\{\mathbf{e}_i\}$ and $\{\tilde{\epsilon}^i\}$ are the basis of vectors and one-forms [46]. If bases are reciprocal (or dual [17]), i.e. they satisfy $\mathbf{e}_i | \tilde{\epsilon}^j = \delta_i^j$ with $\delta_i^i = 1$ and $\delta_i^j = 0$ when $i \neq j$, then [46]

$$a_j = \tilde{\epsilon}^j | \mathbf{a}, \quad \alpha^i = \mathbf{e}_i | \tilde{\alpha}, \quad (70)$$

and the bar product is expanded as

$$\mathbf{a}|\tilde{\alpha} = a_1 \alpha^1 + a_2 \alpha^2 + a_3 \alpha^3 + a_4 \alpha^4. \quad (71)$$

For example, the product of the one-form $\tilde{x}$ with the unit basis vector $\mathbf{x}$ is 1, etc. The product (in the orthonormal basis) of the one-form $\tilde{x}$ with the vector $\mathbf{b} = 3\mathbf{x} + 4\mathbf{y}$ equals 3. In literature (for example, in [17]) one can also find the other terminology, having the sense of the product form Eq. 69. One-form is a linear functional, thus it act on a vector and return a scalar. Thus the mapping of a vector $\mathbf{a}$ with one-form $\tilde{\alpha}$ produce $a_i \alpha_i$. Also the property of linearity is important. One of the example of one-form is the gradient of a function, but not all one-forms are gradients [17, 180].

²In general, one-form should be understood as mapping of the tangent bundle of some point of a manifold to real number [179].

In classical literature, for example, *The Gravitation* [17], an one-form is visualized as a set of equally spaced planes in 3D, or hyperplanes in higher dimensions. In 3D spacetime dual vector and one-form bases can be presented as shown in Fig. 1 (a). While basis vectors are arrows, the one-form basis is a set of planes. Each plane is orthogonal to one of the basis vectors, and the other two lie in its plane. Then the scalar value, which is returned by an one-form acting on a vector $\mathbf{a}$, is nothing more than the number of planes which $\mathbf{a}$ pierces (and the orientation define a sign of the scalar), or the number of segments into which the vector $\mathbf{a}$ was cut by these planes.

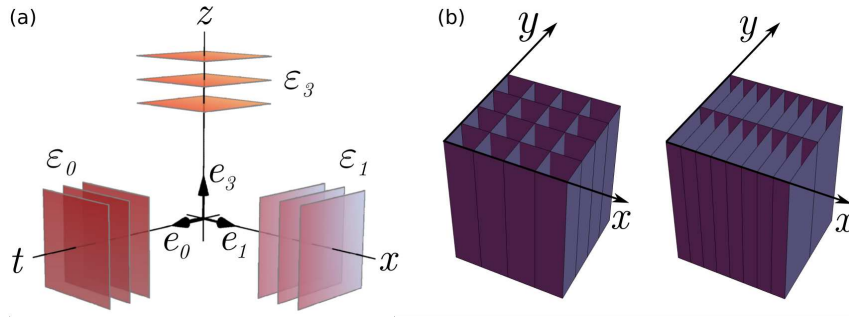


Figure 1: One-forms representation in 3D. (a) The presentation of vectors (arrows) and one-forms (planes) bases of a Lorentz coordinate frame in the case $\mathbf{e}_i|\tilde{\mathbf{e}}_i = \delta_{i,j}$. (b) A two-form representation in 3D. In this representation two-forms should only be considered as the tubes direction and their density, but not their walls positions. Both the left and right representations give the same two-form, which can be written as $B_1 d\tilde{\mathbf{x}} \wedge d\tilde{\mathbf{y}}$ (left) or $B_1(2d\tilde{\mathbf{x}}) \wedge (\frac{1}{2}d\tilde{\mathbf{y}})$ (right). The pictures are inspired by [17].

Any vector $\mathbf{a}$ has corresponding unique one-form $\tilde{\mathbf{a}}$, dual to it. The components α^i of $\tilde{\mathbf{a}}$ can be received using the coefficients of the metric tensor

$$\alpha^i = g^{ij}a_j, \quad (72)$$

and, oppositely, components of $\mathbf{a}$ can be obtained from components of $\tilde{\mathbf{a}}$.

One-forms are independent of any choice of coordinates [17]. But what is more important for this work, that is the fact that the result of the operation, when an one-form act on a vector, is a scalar. This value does not depends on chose of reference frame, or coordinates of a particular frame.

Two-form

The special bilinear combination based on exterior product of one-forms is called two-form [17]

$$\beta = B_1 d\tilde{\mathbf{x}} \wedge d\tilde{\mathbf{y}} + B_2 d\tilde{\mathbf{y}} \wedge d\tilde{\mathbf{z}} + B_3 d\tilde{\mathbf{z}} \wedge d\tilde{\mathbf{x}}. \quad (73)$$

Two-forms occur under a surface integral [41]. Here a new element, the wedge $\wedge$ appears. It is used to a new type of product, the exterior product. There are three main axioms for operation with the wedge product [41]. Anticommutativity

$$(A1) \quad \tilde{\lambda} \wedge \tilde{\mu} = -\tilde{\mu} \wedge \tilde{\lambda}.$$

Also exterior product obeys the distributive property

$$(A2) \quad \tilde{\lambda} \wedge (\tilde{\mu} + \tilde{\nu}) = \tilde{\lambda} \wedge \tilde{\mu} + \tilde{\lambda} \wedge \tilde{\nu},$$

and the associative law

$$(A3) \quad \tilde{\lambda} \wedge (\tilde{\mu} \wedge \tilde{\nu}) = (\tilde{\lambda} \wedge \tilde{\mu}) \wedge \tilde{\nu}.$$

There is one more useful operation in differential forms, called the Hodge star dual operator $\star$. In n -dimensional space it connects p -form and Hodge dual $n - p$ -form, constructed with use of the "remain" part of a basis. For example, in 4D the Hodge dual to $d\tilde{t} \wedge d\tilde{x}$ two-form is also two-form $d\tilde{y} \wedge d\tilde{z}$.

In literature one may meet that a two-form is visualize in 3D as a set of tubes, organized by two intersecting sets of planes [17]. At first, this visualisation should not mislead one: the positions of the tubes walls can be different for the same forms, for example, $B_1 d\tilde{x} \wedge d\tilde{y}$ and $B_1(2d\tilde{x}) \wedge (\frac{1}{2}d\tilde{y})$, etc., is considering as the same two-form (see Fig. 1 (b)). Therefore, when visualizing a bundle of such tubes, it is better to think not about the walls, but about its general guidance. Secondly, it is worth to note that tube presentation can be applied only in 3D, in 4D it becomes not clear. As it is discussed in Section 3.4, in work [24] two-forms are presented as 2D surfaces. As it is described in Section C.3, an exterior product built from two vectors $\mathbf{a}$ and $\mathbf{b}$, produces the bivector $\mathbf{a} \wedge \mathbf{b}$. This is an element of another space than vectors [40]. The set of bivectors creates a linear space, which often designated as $\wedge^2 \mathbb{R}^n$ (n is the dimension of a vector space). This space has the basis, the oriented segments of the coordinate planes [39]. Then, a wedge product $\mathbf{a} \wedge \mathbf{b}$ can be presented as [40]

$$\mathbf{a} \wedge \mathbf{b} = \sum_{i < j} (a_i b_j - a_j b_i) \mathbf{e}_i \wedge \mathbf{e}_j, \quad (74)$$

where $\mathbf{a} = a_i \mathbf{e}_i$ and $\mathbf{b} = b_i \mathbf{e}_i$, $\{\mathbf{e}_i\}$ is an n vector space basis. Nevertheless, in fact, the basis of bivectors can be built without any vector basis. It is enough to have a set of linearly independent bivectors $\{\mathbf{e}_i \wedge \mathbf{e}_j\}$ [46].

In terms of spaces, bivectors are dual to two-forms [17]. Then, their bar product can be

constructed as follows (here is a simple bivector and a simple two-form, see definition below)

$$(\tilde{\alpha} \wedge \tilde{\beta})|(\mathbf{a} \wedge \mathbf{b}) = (\tilde{\alpha}|\mathbf{a})(\tilde{\beta}|\mathbf{b}) - (\tilde{\beta}|\mathbf{a})(\tilde{\alpha}|\mathbf{b}) = \det \begin{pmatrix} \tilde{\alpha}|\mathbf{a} & \tilde{\beta}|\mathbf{a} \\ \tilde{\alpha}|\mathbf{b} & \tilde{\beta}|\mathbf{b} \end{pmatrix}, \quad (75)$$

which follows from the asymmetry of the wedge product and orthogonality of the basis bivectors and two-forms [46]. For example, the product of the two-form $\tilde{\mathbf{t}} \wedge \tilde{\mathbf{x}}$ and the bivector $\mathbf{t} \wedge \mathbf{x}$ can be computed as follows

$$(\tilde{\mathbf{t}} \wedge \tilde{\mathbf{x}})|(\mathbf{t} \wedge \mathbf{x}) = (\tilde{\mathbf{t}}|\mathbf{t})(\tilde{\mathbf{x}}|\mathbf{x}) - (\tilde{\mathbf{x}}|\mathbf{t})(\tilde{\mathbf{t}}|\mathbf{x}) = 1, \quad (76)$$

because $(\tilde{\mathbf{t}}|\mathbf{t}) = 1$, $(\tilde{\mathbf{x}}|\mathbf{x}) = 1$, $(\tilde{\mathbf{x}}|\mathbf{t}) = 0$, $(\tilde{\mathbf{t}}|\mathbf{x}) = 0$.

A bivector can be represented as a linear combination of other bivectors $\mathbf{A} = \mathbf{a} \wedge \mathbf{b} + \mathbf{c} \wedge \mathbf{d} + \dots$, if a bivector can be expressed as a exterior product of two vectors, for example $\mathbf{a} \wedge \mathbf{b}$, it is called a *simple bivector* [40]. The same is true for the two-forms [46]. In 3D all bivectors are simple, but it is no more true in 4D [39]. In 4D a bivector cannot be presented as the exterior product of two vectors [39] but can be written as a sum of two simple bivectors $\mathbf{t} \wedge \mathbf{x} + \mathbf{y} \wedge \mathbf{z}$ [40]. In spacetime this sum includes temporal (the first term) and spatial (the second term) parts, which are completely orthogonal [46, 17]. In this way, such a bivector in 4D does not present a plane anymore. This decomposition is unique [38].

C.3 Bivector

This section is based on Chapter 1 of [49], Chapter 1 of [40], and the book [39]. The discussion about the difference between the cross product and a bivector is built on the basis of [180] and [39]. The description starts with a simple bivector in 2D and 3D. After that, this concept is extended to the more complex 4D case.

The wedge product of two vectors $\mathbf{a}$ and $\mathbf{b}$, written as

$$\mathbf{A} = \mathbf{a} \wedge \mathbf{b} \quad (77)$$

is called a *bivector*. It is constructed with the special multiplication sign $\wedge$, called "wedge". Thus, in literature one may meet a name "wedge product" (also outer or exterior product).

From geometrical point of view, a simple bivector corresponds to an oriented parallelogram formed by the two vectors (Fig. 2, (a)). Importantly, opposite to the cross product, this product

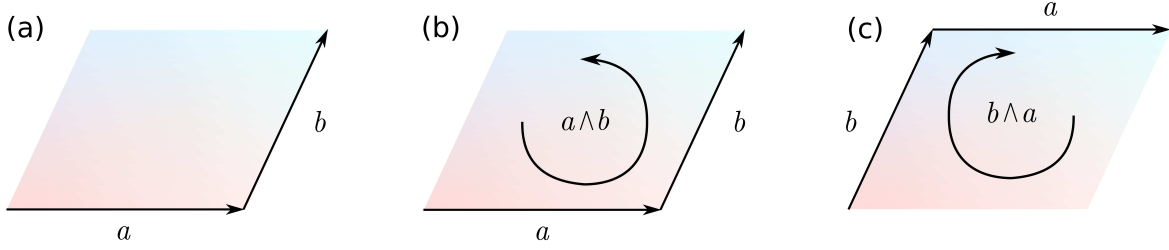


Figure 2: Geometrical interpretation of the bivector $\mathbf{a} \wedge \mathbf{b}$. (a) Construction of a bivector. (b) and (c): the orientation of a bivector. The change of the vectors order in the wedge product changes the orientation [49].

does not define a vector perpendicular to the direction of the segment, but a surface element. This surface element is assigned an orientation. It can be defined with simple right-hand rule, from the first vector in the wedge product to the second one (see Fig. 2, (b) and (c)). Change in the order of the vectors flips the orientation of a bivector:

$$\mathbf{a} \wedge \mathbf{b} = -\mathbf{b} \wedge \mathbf{a}, \quad (78)$$

but saves the magnitude the same. The consequences of this anticommutative property is that the wedge product of the bivector with itself is zero:

$$\mathbf{a} \wedge \mathbf{a} = \mathbf{0}. \quad (79)$$

As a result, the wedge product can only be constructed from linearly independent vectors. Also, bivectors can be added, and be multiplied by scalars. The last operation simply scales the area proportionally to this scalar.

Like vectors, bivectors form a linear space, and they can be decomposed into a bivector basis. If $\{\mathbf{e}_i\}$ is an orthonormal basis in the vector space, $\{\mathbf{e}_{ij}\}$ with $\mathbf{e}_{ij} = \mathbf{e}_i \wedge \mathbf{e}_j$ as an orthonormal basis for bivectors in their space. In the simplest 2D case there is only one basis bivector $\mathbf{e}_{12} = \mathbf{e}_1 \wedge \mathbf{e}_2$, and any bivector $\mathbf{a} \wedge \mathbf{b}$ can be expressed as (Fig. 3 (a))

$$\mathbf{a} \wedge \mathbf{b} = (a_1 b_2 - a_2 b_1) \mathbf{e}_{12}, \quad (80)$$

where $\mathbf{a} = a_1 \mathbf{e}_1 + a_2 \mathbf{e}_2$ and $\mathbf{b} = b_1 \mathbf{e}_1 + b_2 \mathbf{e}_2$.

In the 3D case the basis consists of three linear independent basis bivectors (Fig. 3, (b)).

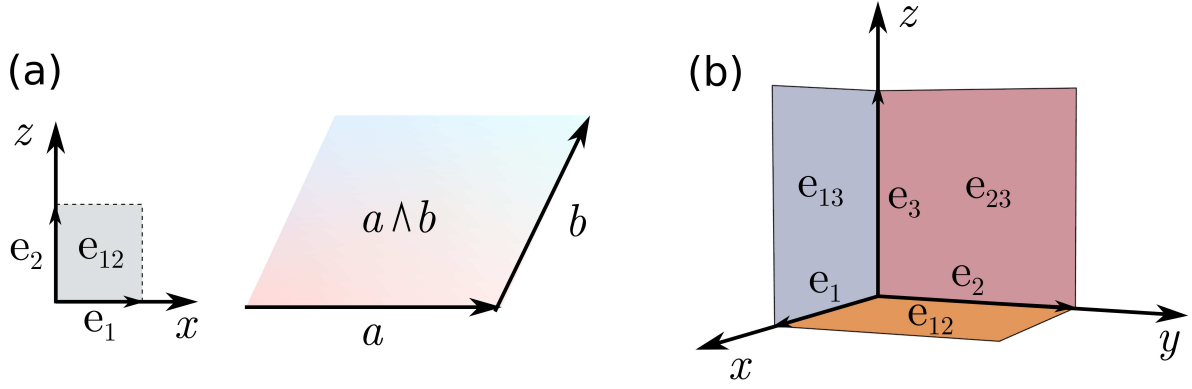


Figure 3: Demonstrations for a bivector basis (a) in 2D case and (b) in 3D case.

Then a bivector $\mathbf{a} \wedge \mathbf{b}$ can be expressed as "determinant"

$$\mathbf{a} \wedge \mathbf{b} = \begin{vmatrix} \mathbf{e}_{23} & \mathbf{e}_{31} & \mathbf{e}_{12} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}. \quad (81)$$

In any dimension, the $\mathbf{e}_{ij}$ component of the bivector $\mathbf{a} \wedge \mathbf{b}$ is $a_i b_j - a_j b_i$. In 3D a bivector can be seemed as something similar to the cross vector product (besides they have the orientations defined in different ways). The cross vector product on two 3-dimensional vectors $\mathbf{x}$ and $\mathbf{y}$ yields in result 3-dimensional (axial) vector $\mathbf{z}$. This vector is orthogonal to the piece of surface, formed by vectors $\mathbf{x}$ and $\mathbf{y}$, which can be considered as the bivector $\mathbf{x} \wedge \mathbf{y}$. Thus, a bivector and the vector product seem to be connected in 3D. But let us note, that the vector product as operation exist only in 3D space. In higher (and lower) dimensions there is no direct analogue of the binary cross product, while a bivector can be built for every dimensions with keeping its geometrical meaning.³

The distinctive property of bivectors in 2D or 3D is that any bivector (or sum of bivectors) can be presented as the wedge product of two vectors. Such a bivector is called a *simple* bivector. All bivectors in 2D and 3D are simple. In 4D this is not the case. A bivector basis consists of 6 bivectors in 4D. The example of decomposition of a general bivector into basis bivectors in 4D

$$\mathbf{A} = A_{01}\mathbf{e}_{01} + A_{02}\mathbf{e}_{02} + A_{03}\mathbf{e}_{03} + A_{12}\mathbf{e}_{12} + A_{13}\mathbf{e}_{13} + A_{23}\mathbf{e}_{23}. \quad (82)$$

The difference of 4D space is that a general bivector can not be presented as a simple bivector.

³The another distinction that the cross product requires a metric, while the exterior product does not.

There is no way to present $\mathbf{e}_1 \wedge \mathbf{e}_2 + \mathbf{e}_3 \wedge \mathbf{e}_4$ as $\mathbf{a} \wedge \mathbf{b}$. The bivector $\mathbf{A}$ from Eq. 82 can be simplified to the sum of minimum *two* terms. Moreover, in 4D spacetime this decomposition is unique, and it is the basis of the idea of the spacetime map for electromagnetic world. Thus, this property is important for this work, and it is discussed in more detail in following chapters.

Describing the vector basis above, it was assumed that the basis bivector $\mathbf{e}_{ij}$ is the product of two basis vectors $\mathbf{e}_i \wedge \mathbf{e}_j$. Nevertheless, the basis of bivectors can be built without any vector basis. One of the interesting properties of such representation is that it is enough to have a set of linearly independent bivectors $\{\mathbf{e}_{ij}\}$. The presentation with vectors is provided here for a simplified explanation: a bivector itself does not need vectors, it is just a surface with an orientation. If needed, it can be decompose in any basis, but the advantage of exterior algebra is that one can operate with bivectors without a traditional vector basis.

C.4 General case of n -form

In general case, on the base of the exterior product, in n -dimensional space we can construct a n -form as a wedge product of n 1-forms [47]

$$\tilde{\omega} = \omega_{1\dots n}(x) \tilde{\mathbf{x}}_1 \wedge \dots \wedge \tilde{\mathbf{x}}_n. \quad (83)$$

It defines a volume element on manifold, and can be integrated over n -dimensional domain [41]. In the same like in Eq. 83 way, the n -dimensional multivector can be constructed.

C.5 Differential forms and tensors

Intrinsically, the differential form is a skew-symmetric tensor field. The completely skew-symmetric covariant tensor of the rank $(0, p)$ corresponds to the p -form, and $(p, 0)$ to p -vector (multivector) [17, 181]. Hence, the multivectors and forms are dual [17]. A general p -form can be expanded using tensor $\{\alpha_{ij}\}$ with components [17]

$$\tilde{\alpha} = \frac{1}{p!} \alpha_{i_1 \dots i_p} \tilde{\mathbf{x}}^{i_1} \dots \tilde{\mathbf{x}}^{i_p}. \quad (84)$$

In n -dimensional space a bivector can be represented as a $n \times n$ skew-symmetric matrix. Some of problems expressed in matrices can be adapted in terms of bivectors, providing advanced geometric meaning [39].

Being only the calculus technique, tensors alone are devoid of clarity or illustration com-

ponent in many situation, while exterior calculus (often combined with moving Cartan frame) produces easier solution ⁴. In addition, difficulties often arise with mappings of tensor fields, and because a large number of indexes one may miss the difference between various types of quantities. The tensors are attached to a basis, which can change arbitrarily with transformation to new coordinates [182]. Meanwhile, differential forms provide a coordinate-free solution for a bunch of problems [181, 183, 47]. Another their advantage is easy and clear interpretation [41].

The technique of differential forms and multivectors is not intended to completely replace tensor calculus in physics, since in various problems each of these methods can show great efficiency [23, 41]. For the set of particular problems, the differential forms is more natural tool due to their inner simplicity [41]. Also they can provide an additional physical insight in to the common picture [23]. And they are there wherever integrals occur [41].

C.6 Hodge dual operator

Here we discuss in more the Hodge dual star operator $\star$. Using the metric on the vectoral space, one can set up linear isomorphism between vector and bivector linear spaces of the same dimension [39]. For example, in 3D the star operator convert a vector to a bivector with missing coordinates [39, 23]

$$\star \mathbf{x} = \mathbf{y} \wedge \mathbf{z},$$

and oppositely. The Hodge dual depends also on the choice of orientation. Normally, a right-handed and orthormormal basis is meant [39]. The general definition of Hodge duality in n -dimensional space connects p -form and dual $n - p$ -form with components

$$(\star \tilde{\alpha})_{k_1 \dots k_{n-p}} = \frac{1}{p!} (\tilde{\alpha})^{i_1 \dots i_p} \varepsilon_{i_1 \dots i_p k_1 \dots k_{n-p}} \quad (85)$$

includes Levi-Cevita tensor [17].

In spacetime for Hodge dual [17]:

$$\star \star \tilde{\alpha} = (-1)^{p-1} \tilde{\alpha}. \quad (86)$$

The easiest way to compute the Hodge dual is to write the product of the form and it is dual

⁴For example, working on problems with Riemannian geometry by tensor methods could be overly complicated [41].

on the one side and the full volume product on the another (4D case)

$$\tilde{\alpha} \wedge (\star \tilde{\alpha}) = C \tilde{t} \wedge \tilde{x} \wedge \tilde{y} \wedge \tilde{z}. \quad (87)$$

Besides bivectors, there are many ways to set an area [48]. As it was mentioned above, the geometrical sense of a bivector is a surface of a parallelogram, built on two vectors, and from this point of view bivector is similar to the vector product (besides the orientation). But let us note, that the vector product as operation exist only in 3-dimension space as a binary bilinear antisymmetric non-degenerate map, i.e. the operation on the two 3-dimensional vectors yielding in result 3-dimensional (axial) vector. In higher (and lower) dimensions there is no direct analogue of the binary cross product, while the bivector can be built for every dimensions with keeping geometrical meaning [180]. In 4D the Hodge dual to a bivector is another bivector.

C.7 Differentiation and integration

Finally, it is important to mention the exterior derivative and integration. Exterior derivative $\tilde{d}$ produce a $p + 1$ -form from p -form in n -space ($p < n$):

$$\tilde{\alpha} = \sum \alpha_{j_1 \dots j_p} \tilde{e}^{j_1} \wedge \dots \wedge \tilde{e}^{j_p}. \quad (88)$$

we obtain

$$\tilde{d}\tilde{\alpha} = \sum_i (\partial_i \alpha_{j_1 \dots j_p}) \tilde{e}^i \wedge \tilde{e}^{j_1} \wedge \dots \wedge \tilde{e}^{j_p} \quad (89)$$

with

$$\partial_i f = \frac{\partial f}{\partial x^i}.$$

Using well-known derivative (1-form) of a scalar function (0-form) [17, 22]

$$df(x^1, \dots, x^n) = (\partial_i f) e^i$$

and adding a new factor e^i in the wedge product. Exterior derivative has the next properties

$$\begin{aligned} \tilde{d}(\tilde{\alpha} \wedge \tilde{\beta}) &= \tilde{d}\tilde{\alpha} \wedge \tilde{\beta} + (-1)^p \tilde{\alpha} \wedge \tilde{d}\tilde{\beta}, \\ \tilde{d}^2 &= \tilde{d}\tilde{d} = 0, \end{aligned} \quad (90)$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are p -form and q -form. Thus $\tilde{d}(\tilde{\alpha} \wedge d\tilde{\beta}) = \tilde{d}\tilde{\alpha} \wedge d\tilde{\beta}$.

In 4D exterior derivative can be obtain from 3D exterior derivative as [22]

$$\tilde{d} = d + dt \frac{\partial}{\partial t}, \quad (91)$$

which is useful for the Maxwell' equations.

As differential form is stand under the sign of the integral, it can be integrated. Integration decrease the degree of the form. There is geometrical interpretation for integration of differential forms [17](p.94):

- the integral of one-form $\tilde{\lambda}$ along a specific curve is interpreted as a "number of λ -surfaces pierced along this curve",
- the integral of two-form $\tilde{\mu}$ over a piece of surface which has an orientation is interpreted as a "number of tubes cut through by that surface", etc.

To integrate p -form $\tilde{\alpha} = \frac{1}{p!} \alpha_{i_1 \dots i_p} (x^1, \dots, x^n) \tilde{x}^{i_1} \dots \tilde{x}^{i_p}$ over a p -dimensional surface is needed of substitution of a parametric equation of the surface $x^k(\xi^1, \dots, \xi^p)$ into $\tilde{\alpha}$:

$$\tilde{\alpha} = a(\xi^j) \tilde{d}\xi^1 \wedge \dots \wedge \tilde{d}\xi^p. \quad (92)$$

And than, integrate $\tilde{\alpha}$ using elementary integral definition [17]

$$\int \tilde{\alpha} = \int a(\xi^j) d\xi^1 \dots d\xi^p. \quad (93)$$

The integral relation for generalized Stokes theorem then

$$\int_{\mathcal{V}} \tilde{d}\tilde{\sigma} = \int_{\partial\mathcal{V}} \tilde{\sigma}, \quad (94)$$

where $\tilde{\sigma}$ is p -form, and $\partial\mathcal{V}$ is a border of an interior $\mathcal{V}$ [17]. This theorem and its geometrical presentation is discussed in more details in the Section 3.4.

C.8 Clifford algebra

It is also worth mentioning that some works on electrodynamics [171, 50, 184, 185, 186, 48] are based on an extended version of the Grassmann approach, the Clifford algebra. Being familiar with both Hamilton's and Grassmann's approaches, Clifford successfully combined them, with a bias towards Grassmann's ideas [174]. Clifford's geometric algebra generalize an idea of the

vectors multiplication by introducing the geometrical product for vectors, applicable in any dimension and signature [187]. The geometrical product is written as ab , and it is the sum of a scalar and a bivector [49]

$$ab = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \wedge \mathbf{b}. \quad (95)$$

This sum should be viewed in the same way as a complex number: it is a mixture of two different objects [49]. This rewriting has effects that are more than just notation. The combination of inner and outer multiplication into one operation has the great advantage: it makes multiplication associative [171].

Similar to the differential forms approach, in Clifford algebra div and rot operations are united, and this united operator is invertible, while separately they are not. This leads, for example, to novel methods for solution of EM scattering problems [187, 49]. As Eq. 95 includes a scalar product and a bivector, it stays simple to use even in the case of curvilinear coordinates.

D.

Electromagnetism as 4D-static magnetic and electric flux equations

This Appendix begins with consideration of the traditional conception for the flux in 3D. To expand the flux conception for the general 4D case, which requires the introduction of the bivector concept. After that the connection of vectorial Maxwell's equations with two integral ones for fluxes is proved.

D.1 Conventional flux in 3D

The electric and magnetic fluxes on 2D surface in 4D spacetime is the key concept of the presented theory. In the integral formulation of Maxwell's equations only fluxes are used to describe the electromagnetic fields. Below it is shown that the flux is a convenient tool to operate with electromagnetism. The flux conception in electromagnetism, starting from classical approach, is discussed, and then this idea is transformed for general case of 4D spacetime. For this, the geometric algebra tools are applied which advantage is a frame-free approach.

The traditional concept of electric or magnetic flux in electrodynamics implies the computation of the field through the surface(Fig. 4), i.e. the flux is a scalar product of field and surface [3]. For example, in the case of infinitesimal area dS the electric flux

$$d\Phi_e = \mathbf{D} \cdot d\mathbf{S}, \quad (96)$$

where $d\mathbf{S} = dS\mathbf{n}$, $\mathbf{n}$ is the normal to the surface dS . Then the integral formulation is

$$\Phi_e = \iint_{\Sigma} \mathbf{D} \cdot d\mathbf{S}, \quad (97)$$

where Σ is the surface through which the electric flux is computed. In the case of magnetic flux Φ_m , $\mathbf{D}$ is replaced by $\mathbf{B}$.

For 4D spacetime, this traditional definition for 3D should be revised, because there are two normals, corresponding to a piece of surface. In 4D spacetime there is another way to present the flux: an algebraic value on a surface. To make the transition to these calculations, the conception of a *bivector* should be seen. It is presented in Appendix C, Section C.3.

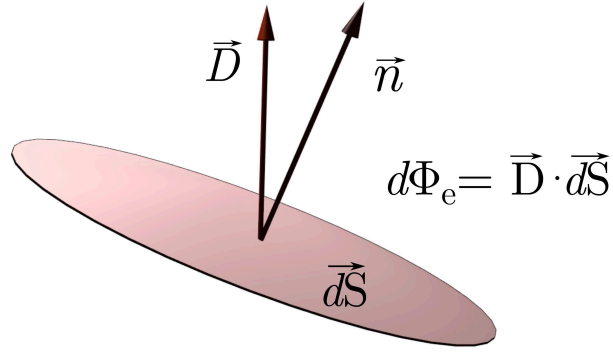


Figure 4: Definition of the electric flux in the case of 3D space.

D.2 Integral formulation of electromagnetism

As it is discussed in Section 1.3, Maxwell's equations can be formulated in terms of the electric and magnetic flux as the following postulates:

1. Any 2D oriented surface of 4D spacetime can be labelled with two scalar quantities, its magnetic flux Φ_m and its electric flux Φ_e .
2. The magnetic flux on any closed surface is zero

$$\Phi_m(\text{closed surface}) = 0. \quad (98)$$

3. The electric flux on any closed surface is equal to the number of charges in any 3D volume enclosed by this surface

$$\Phi_e(\text{closed surface}) = n. \quad (99)$$

Below connection of the integral formulation with traditional form of curl Maxwell's equations is demonstrated.

D.3 Maxwell's equations from 4D static flux equations

In this work we consider the flux as the product of a field component on an oriented spatial area. The electric flux on the basis surfaces

$$\begin{aligned} \tilde{\Phi}_e(\mathbf{e}_y \wedge \mathbf{e}_z) &= D_x, \\ \tilde{\Phi}_e(\mathbf{e}_x \wedge \mathbf{e}_z) &= D_y, \\ \tilde{\Phi}_e(\mathbf{e}_x \wedge \mathbf{e}_y) &= D_z, \end{aligned} \quad (100)$$

and magnetic flux on the basis surfaces

$$\begin{aligned}\tilde{\Phi}_m(\mathbf{e}_y \wedge \mathbf{e}_z) &= B_x, \\ \tilde{\Phi}_m(\mathbf{e}_x \wedge \mathbf{e}_z) &= B_y, \\ \tilde{\Phi}_m(\mathbf{e}_x \wedge \mathbf{e}_y) &= B_z.\end{aligned}\tag{101}$$

It is important to consider attentively the orientation of the surface, as the change of the orientation of the bivector change the sign of the flux on it.

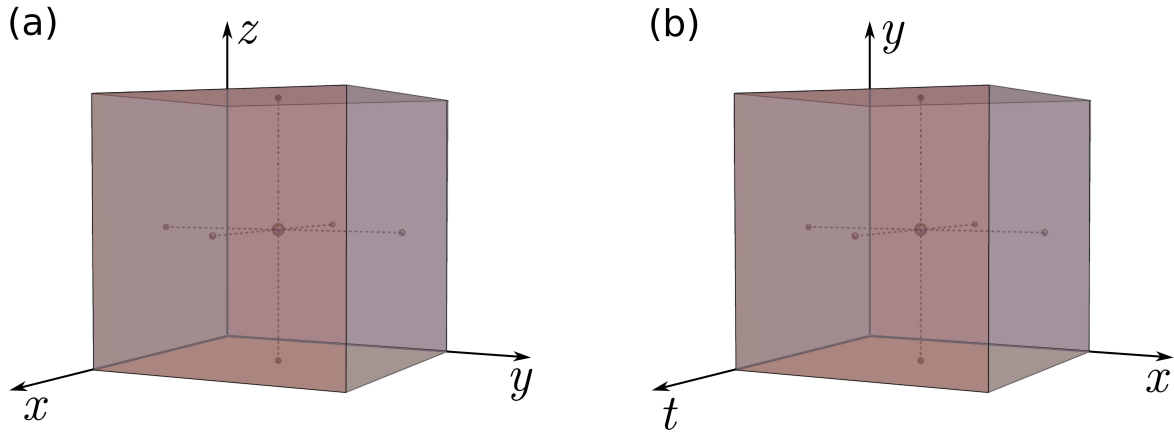


Figure 5: Infinitesimal volume in (a) 3D space and (b) 3D spacetime.

For example, let us compute the magnetic flux on an infinitesimal parallelepiped (Fig 5 (a)) with sides dx , dy , dz , centred at (x, y, z) . Here and below, the unbold letter "d" do not mean the differentiation, but has sense of infinitesimal value, if other is not specified. For computation the flux value on an infinitesimal cube face, the field in the centre of this face is considered. The magnetic flux on each surface is presented in Table 1. The orthonormal basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ correspond to the axes x , y , and z , correspondingly.

The sign of $d\Phi_m$ in Table 1 is determined depending on the external orientation of the face of the parallelepiped. All the surfaces of the parallelepiped should be oriented in the same way. It is worth to discuss the orientation of surfaces in more details.

At first, let us consider a surface in Fig. 6 (a). To set the orientation of this surface, it is enough to add only one arrow, which shows the direction of the traversal. In case when this surface is divided into several parts, each part keeps the same orientation, as it is presented in Fig. 6 (b). One may see, that in this case the orientation of the common border is opposite for neighboring cells (the direction of the black arrows near the border). This follows the principle

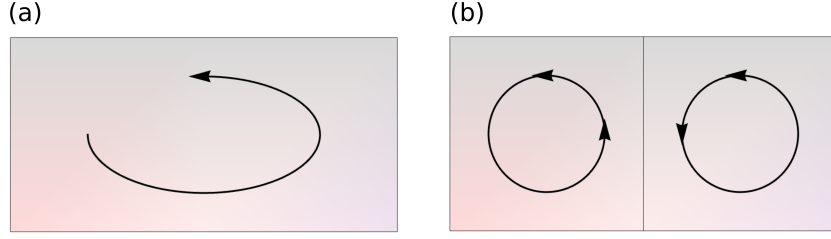


Figure 6: To discussion about the orientation. (a) The orientation of the surface: a black arrow shows the orientation. (b) If the surface is divided into two cells, they should have the same orientation as the surface. Then the common border has the opposite orientation for these two cells: for the left cell arrow is pointed up, and for the right cell the arrow is pointed down.

of the unified orientation: merged together cells will have one orientation for all the surface, as if the common border disappears.

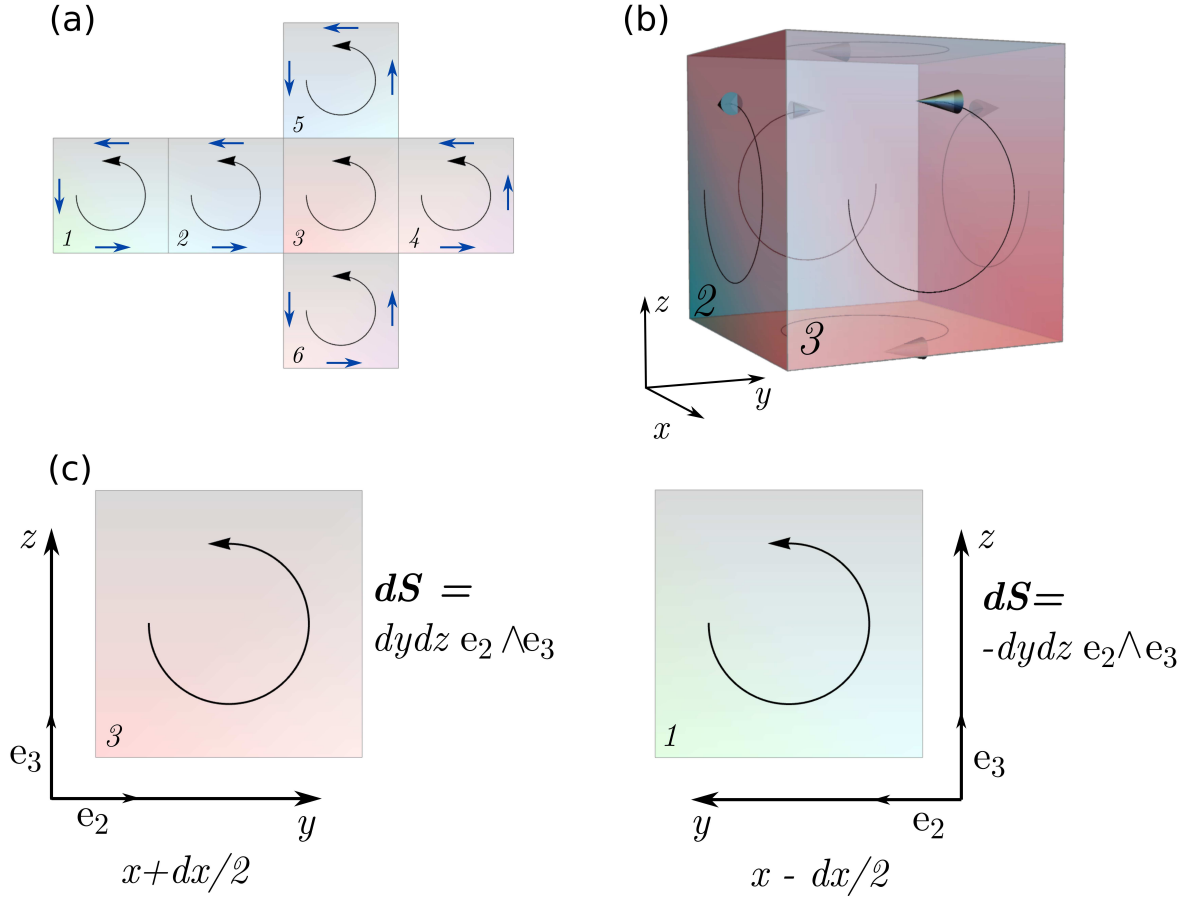


Figure 7: To discussion about the orientation. (a) A surface can be presented as a set of cells with the same orientation (presented with black and blue arrows). (b) The surface is folded into a parallelepiped, keeping the same external orientation. (c) After the surface was folded into the parallelepiped, the same orientation for two opposite faces is written as $d\mathbf{y} \wedge d\mathbf{z}$ for the face 3, and $d\mathbf{z} \wedge d\mathbf{y}$ for the face 1.

The surface of considered infinitesimal parallelepiped is present in Fig. 7 (a). It has the common for all faces orientation, shown with blue arrows on the border, and black arrows in

the centre of cells. When this surface is folded into a parallelepiped, all faces keep the same orientation (Fig. 7 (b)). All connecting edges (for example, left edge of the face 1 and right edge of the face 4, or the top edge of the face 4 and the right edge of the face 5) have the opposite direction for the same orientation. That means that the orientation for cells is chosen in right way. Again, it is important to consider the external orientation at all the parallelepiped surfaces. Also, the same rule for neighboring cells keeps working: if this parallelepiped is surrounded by the same ones, then the orientation on the adjacent faces of parallelepipeds are opposite. In this case, if two such parallelepipeds are joined, the common border disappears.

The orientation affect the sign of the flux. For example, let us consider the cells 1 and 3 (Fig. 7 (c)) of the surface in Fig. 7 (a). They are the opposite faces of the parallelepiped when it is folded (Fig. 7 (b)). However, the same orientation of this two cells for the folded parallelepiped is written as $\mathbf{z} \wedge \mathbf{y} = -\mathbf{y} \wedge \mathbf{z}$ for the face 1 and $\mathbf{y} \wedge \mathbf{z}$ for the face 3. In this way, the magnetic flux on the surface $d\mathbf{y} \wedge d\mathbf{z}$: at $x - dx/2$ is $d\Phi_m = -B_x(x - dx/2, y, z)dydz$, at $x + dx/2$ is $d\Phi_m = B_x(x + dx/2, y, z)dydz$. The same sign difference is true for all parallel pairs of faces for the parallelepiped.

Table 1: Magnetic flux on infinitesimal faces of the parallelepiped.

Surface	$d\Phi_m$ on the surface
$-dy dz \mathbf{e}_2 \wedge \mathbf{e}_3$ at $(x - dx/2)$	$-B_x(x - dx/2, y, z) dy dz$
$dy dz \mathbf{e}_2 \wedge \mathbf{e}_3$ at $(x + dx/2)$	$B_x(x + dx/2, y, z) dy dz$
$-dx dz \mathbf{e}_1 \wedge \mathbf{e}_3$ at $(y - dy/2)$	$-B_y(x, y - dy/2, z) dx dz$
$dx dz \mathbf{e}_1 \wedge \mathbf{e}_3$ at $(y + dy/2)$	$B_y(x, y + dy/2, z) dx dz$
$-dx dy \mathbf{e}_1 \wedge \mathbf{e}_2$ at $(z - dz/2)$	$-B_z(x, y, z - dz/2) dx dy$
$dx dy \mathbf{e}_1 \wedge \mathbf{e}_2$ at $(z + dz/2)$	$B_z(x, y, z + dz/2) dz dy$

Then, the total magnetic flux on the surface of this parallelepiped is

$$\begin{aligned}
\Phi_{m total} = & (B_x(x + dx/2, y, z) - B_x(x - dx/2, y, z)) dy dz \\
& + (B_y(x, y + dy/2, z) - B_y(x, y - dy/2, z)) dx dz \\
& + (B_z(x, y, z + dz/2) - B_z(x, y, z - dz/2)) dz dy.
\end{aligned} \tag{102}$$

From other side, according to the Gauss's law for magnetism 1.34 $\Phi_{m\,total} = 0$. Then

$$\begin{aligned} \frac{\Phi_{m\,total}}{dx\,dy\,dz} &= \frac{(B_x(x+dx/2, y, z) - B_x(x-dx/2, y, z))}{dx} \\ &+ \frac{(B_y(x, y+dy/2, z) - B_y(x, y-dy/2, z))}{dy} \\ &+ \frac{(B_z(x, y, z+dz/2) - B_z(x, y, z-dz/2))}{dz} = 0, \end{aligned} \quad (103)$$

or, passing to limit case $dx\,dy\,dz \rightarrow 0$

$$\frac{\partial B_x(x, y, z)}{\partial x} + \frac{\partial B_y(x, y, z)}{\partial y} + \frac{\partial B_z(x, y, z)}{\partial z} = 0. \quad (104)$$

In this way, using the flux on a surface approach, the differential formulation of the Gauss's law $\text{div}\mathbf{B} = 0$ in 3D is obtained.

In a lot of textbooks, there is a habit to separate space and time, but in this work we do not do this separation. Then there is an unified way, where the $\mathbf{E}$ and $\mathbf{B}$ fields have a common origin. In this case, the $\mathbf{E}$ field components are the density of the magnetic flux on time-space surfaces. There are different names the Φ_m from Eq. 1.34 and Φ_e from Eq. 1.35. For example, Deschamps in [22] calls the objects, associated with Φ_m and Φ_e the *EM Force-Field* Φ and the *EM Source-Field* Ψ correspondingly. In Misner's book [17] they are called as *Maxwell* and *Faraday*. Nevertheless, as one already has seen, Φ_m and Φ_e are the magnetic and electric flux for the case spatial (space-space) surfaces. Then, it is a consequence, which can be seen as paradoxical, that the density of the magnetic flux on space-time surfaces is the electric field strength $\mathbf{E}$. And, oppositely, the density of the electric flux on space-time surfaces is the magnetic field strength $\mathbf{H}$ (Fig. 8). The full list of the correspondence of a surface and the flux density on it is presented in Table 2.

Table 2: Magnetic and electric flux on the basis surfaces in spacetime.

Surface	$x \wedge y$	$x \wedge z$	$y \wedge z$	$x \wedge t$	$y \wedge t$	$z \wedge t$
$\Phi_m(\text{surface})$	B_z	B_y	B_x	E_x	E_y	E_z
Surface	$x \wedge y$	$x \wedge z$	$y \wedge z$	$t \wedge x$	$t \wedge y$	$t \wedge z$
$\Phi_e(\text{surface})$	D_z	D_y	D_x	H_x	H_y	H_z

To demonstrate the way how Eq. 1.34 produce Faraday's law of induction, let us compute the magnetic flux for a time-space-space parallelepiped when, for example, the coordinate z is fixed (Fig. 5 (b)) The similar procedure, which was done for cube in 3D space, can be done for

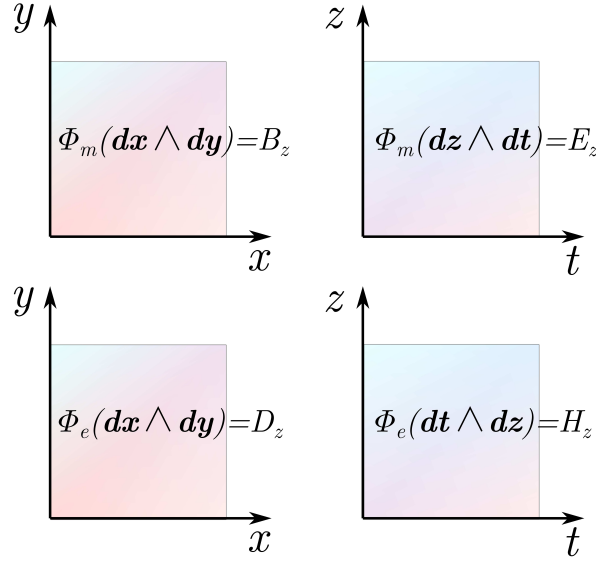


Figure 8: On the spatial surfaces the magnetic flux density is $\mathbf{B}$ field, and the electric flux density is the $\mathbf{D}$ field. But on space-time surfaces the magnetic flux density is the $\mathbf{E}$ field, and the electric flux density is $\mathbf{H}$ field.

this parallelepiped with sides dt , dx , and dy .

The magnetic flux values on the faces of an infinitesimal parallelepiped in 3D spacetime, is presented in Table 3. The basis vector $\mathbf{e}_0$ corresponds to the time direction. Again, the surface of the parallelepiped is oriented in the same way, and it affects on the sign of the flux. Then,

Table 3: Magnetic flux on infinitesimal surfaces of the space-time parallelepiped.

Surface	$d\Phi_m$ on the surface
$-dt dx \mathbf{e}_0 \wedge \mathbf{e}_1$ at $(y - dy/2)$	$-E_x(t, x, y - dy/2) dt dx$
$dt dx \mathbf{e}_0 \wedge \mathbf{e}_1$ at $(y + dy/2)$	$E_x(t, x, y + dy/2) dt dx$
$-dt dy \mathbf{e}_0 \wedge \mathbf{e}_2$ at $(x - dx/2)$	$-E_y(t, x - dx/2, y) dt dy$
$dt dy \mathbf{e}_0 \wedge \mathbf{e}_2$ at $(x + dx/2)$	$E_y(t, x + dx/2, y) dt dy$
$-dx dy \mathbf{e}_1 \wedge \mathbf{e}_2$ at $(t - dt/2)$	$-B_z(t - dt/2, x, y) dx dy$
$dx dy \mathbf{e}_1 \wedge \mathbf{e}_2$ at $(t + dt/2)$	$B_z(t + dt/2, x, y) dx dy$

the total magnetic flux can be written as

$$\begin{aligned}
 \Phi_{m total} = & (E_x(t, x, y + dy/2) - E_x(t, x, y - dy/2)) dt dx \\
 & + (E_y(t, x + dx/2, y) - E_y(t, x - dx/2, y)) dt dy \\
 & + (B_z(t + dt/2, x, y) - B_z(t - dt/2, x, y)) dx dz.
 \end{aligned} \tag{105}$$

Again, using the Gauss's law from Eq. 1.34 for the magnetic flux, and dividing by the infinitesimal

volume $dt dx dy$, one obtains

$$\begin{aligned} \frac{(E_x(t, x, y + dy/2) - E_x(t, x, y - dy/2))}{dy} + \frac{(E_y(t, x + dx/2, y) - E_y(t, x - dx/2, y))}{dx} \\ + \frac{(B_z(t + dt/2, x, y) - B_z(t - dt/2, x, y))}{dt} = 0. \end{aligned} \quad (106)$$

In the limiting case $dt dx dy \rightarrow 0$ Eq. 106 becomes

$$\frac{\partial E_x(t, x, y)}{\partial y} + \frac{\partial E_y(t, x, y)}{\partial x} + \frac{\partial B_z(t, x, y)}{\partial t} = 0. \quad (107)$$

One easily can recognize the z -component of the Faraday's law of induction

$$\text{rot} \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}. \quad (108)$$

Fixing x and y coordinates of 4D spacetime and doing the same procedure, x and y components of this equation can be obtained. It is also the proof that there is no contradiction to consider the electric field strength $\mathbf{E}$ as the density of the magnetic flux on space-time surfaces.

Doing the same procedures for the electric flux in 4D space-time, the second pair of Maxwell's equations (Gauss's law for the electric field and Ampère's circulation law) can be obtained. In this way, it was proved, that, the classical Maxwell's equations can be obtained from postulates Eq. 1.34 and Eq. 1.35. Thus, the electromagnetic world can be seen as an absolute map of spacetime, in which some 2D surfaces are constructed with the magnetic and electric flux on them.

E.

Riesz demonstration

A demonstration in all generality of the uniqueness of the decomposition of a 2-form

$$\Phi = \Phi_{01}e^{01} + \Phi_{02}e^{02} + \Phi_{03}e^{03} + \Phi_{23}e^{23} + \Phi_{31}e^{31} + \Phi_{12}e^{12} \quad (109)$$

of 4D space-time in sum of two 2-forms

$$\Phi = aA + bB \quad (110)$$

may seem difficult. Especially since this uniqueness does not exist in the Euclidean four-dimensional case (as demonstrated in Chapter 3, Section 3.3). And yet there is an algebra that allows us to demonstrate it in a few lines, it is Clifford's algebra. This demonstration was given by the mathematician Marcel Riesz, three times in Swedish and once in English, during a private lesson given on April 9 and 10, 1959 at Indiana University.

In Clifford's algebra, we allow ourselves to add potatoes and carrots (see Appendix C, Section C.8), in contradiction to what we learn in the early grades. More precisely, we add simple numbers, scalars, numbers which correspond to small oriented 1D segments (vectors), numbers which correspond to small oriented surfaces (bivectors), numbers which correspond to small three-dimensional volumes and finally numbers which correspond to small four-dimensional volumes.

Bivectors can be simple, for example $\mathbf{x} \wedge \mathbf{y}$. The sum of two simple bivectors can provide a single bivector, for example $\mathbf{x} \wedge \mathbf{y} + \mathbf{x} \wedge \mathbf{z} = \mathbf{x} \wedge (\mathbf{y} + \mathbf{z})$. But it can also provide a bivector which is not reducible to a simple bivector, for example $\mathbf{x} \wedge \mathbf{y} + \mathbf{t} \wedge \mathbf{z}$.

Clifford's algebra like any algebra is not only endowed with addition, but also with multiplication. For two vectors, we know the scalar product $\mathbf{x} \cdot \mathbf{y}$ which as its name suggests provides a simple number, a scalar, and the external product which provides a bivector $\mathbf{x} \wedge \mathbf{y}$. And the Clifford's product is simply the sum of these two products:

$$\mathbf{xy} = \mathbf{x} \cdot \mathbf{y} + \mathbf{x} \wedge \mathbf{y} \quad (111)$$

The dot product is commutative: $\mathbf{x} \cdot \mathbf{y} = \mathbf{y} \cdot \mathbf{x}$, while the external product is anticommutative: $\mathbf{x} \wedge \mathbf{y} = -\mathbf{y} \wedge \mathbf{x}$. Thus, the Clifford's product $\mathbf{xy}$ usually has none of these properties. It

is easy to express the dot product and the outer product in terms of the Clifford product:

$$\mathbf{x} \cdot \mathbf{y} = \frac{1}{2} (\mathbf{xy} + \mathbf{yx}), \quad \mathbf{x} \wedge \mathbf{y} = \frac{1}{2} (\mathbf{xy} - \mathbf{yx}).$$

Clifford's product is not commutative but it is associative $(\mathbf{ab})\mathbf{c} = \mathbf{a}(\mathbf{bc})$.

If $\mathbf{t}, \mathbf{x}, \mathbf{y}, \mathbf{z}$ are orthogonal unit vectors, then $\mathbf{tt} = 1$, $\mathbf{xx} = \mathbf{yy} = \mathbf{zz} = -1$, $\mathbf{tx} = \mathbf{t} \wedge \mathbf{x}$, $\mathbf{ty} = \mathbf{t} \wedge \mathbf{y}$, $\mathbf{tz} = \mathbf{t} \wedge \mathbf{z}$, $\mathbf{xy} = \mathbf{x} \wedge \mathbf{y}$, $\mathbf{yz} = \mathbf{y} \wedge \mathbf{z}$ et $\mathbf{zx} = \mathbf{z} \wedge \mathbf{x}$. And since the outer product is anti-commutative, we have $\mathbf{tx} = -\mathbf{xt}$, $\mathbf{ty} = -\mathbf{yt}$, $\mathbf{tz} = -\mathbf{zt}$, $\mathbf{xy} = -\mathbf{yx}$, $\mathbf{yz} = -\mathbf{zy}$, $\mathbf{zx} = -\mathbf{xz}$.

The product of the 4 unit vectors $I = \mathbf{txyz}$ is called the pseudo-scalar unit. It anti-commutes with any vector and commutes with a bivector. for example $I\mathbf{x} = \mathbf{txyzx} = -\mathbf{txyxx} = \mathbf{txxyz} = -\mathbf{xtxyz} = -\mathbf{xI}$.

On the other hand, the product of I by itself is -1 . Indeed, $\mathbf{txyztxyz} = -\mathbf{txyztxyz} = \mathbf{txtyzxyz} = -\mathbf{ttxyzxyz} = \mathbf{ttxyxyz} = -\mathbf{ttxyxyz} = \mathbf{ttxyxyz} = (1)(-1)(-1)(-1) = -1$. If α and β are real numbers, $\alpha + \beta I$ is called a quasi-scalar. All non-zero quasi-scalars $\alpha + \beta I$ have an inverse that can be calculated explicitly $(\alpha - \beta I) / (\alpha^2 + \beta^2)$, following the same rules as for complex numbers $\alpha + \beta i$.

This possibility of finding inverses is specific to space-time with one dimension of time and three of space (or three dimensions of time and one of space). But a four-dimensional system with four dimensions of space, or two of time and two of space would be much poorer, because the possibility of inverting quasi-scalars disappears. For example, $1 + I$ does not have an inverse with two dimensions of time. This is because we would have $I^2 = 1$ instead of $I^2 = -1$. Let us suppose that the inverse of $1 + I$ exists, and write it in general terms as $a + bI$. Thus, we should have $a + bI^2 + (a + b)I = 1 + 0I$, and therefore $b = -a$ and $(1 - I^2)a = 1$. Now in this case $I^2 = 1$, which makes it impossible to find a that satisfies the equation.

First theorem, if $\mathbf{F}$ is a simple bivector, its square $\mathbf{F}^2$ is a real scalar number. It can be demonstrated in the following way. Let us chose two vectors two vectors $\mathbf{x}$ and $\mathbf{y}$ which make it possible to define the simple bivector $\mathbf{F} = \mathbf{x} \wedge \mathbf{y}$. The two Clifford's products based on them are $\mathbf{xy} = \mathbf{x} \cdot \mathbf{y} + \mathbf{x} \wedge \mathbf{y}$ and $\mathbf{yx} = \mathbf{x} \cdot \mathbf{y} - \mathbf{x} \wedge \mathbf{y}$. Then, $\mathbf{xyyx} = (\mathbf{xy})(\mathbf{yx}) = (\mathbf{x} \cdot \mathbf{y})^2 - (\mathbf{x} \wedge \mathbf{y})^2$. Hence, we have $\mathbf{F}^2 = (\mathbf{x} \wedge \mathbf{y})^2 = \mathbf{xyyx} - (\mathbf{x} \cdot \mathbf{y})^2$. Also, $\mathbf{xyyx} = \mathbf{x}(\mathbf{yy})\mathbf{x}$. As $\mathbf{y} \wedge \mathbf{y} = -\mathbf{y} \wedge \mathbf{y} = 0$, $\mathbf{yy}$ is the scalar $\mathbf{y} \cdot \mathbf{y}$. It commutes with the vector $\mathbf{x}$, thus $\mathbf{xyyx} = (\mathbf{y} \cdot \mathbf{y})(\mathbf{x} \cdot \mathbf{x})$. Thereby, $\mathbf{F}^2 = (\mathbf{y} \cdot \mathbf{y})(\mathbf{x} \cdot \mathbf{x}) - (\mathbf{x} \cdot \mathbf{y})^2$, and it is clearly a real scalar. Q.E.D.

The reverse is also valid. And more precisely: 1) if it is positive, like

$$(\mathbf{t} \wedge \mathbf{x})^2 = -\mathbf{txxt} = \mathbf{tt} = 1 \tag{112}$$

it is a time-space bivector; 2) if it is negative, like

$$(\mathbf{y} \wedge \mathbf{y})^2 = -\mathbf{y}zzy = \mathbf{y}\mathbf{y} = -1 \quad (113)$$

it is a space-space bivector; 3) if it is zero, like

$$((\mathbf{t} + \mathbf{z}) \wedge \mathbf{x})^2 = 0 \quad (114)$$

it is a null-space bivector that contains a light-line.

All these computations made in a vector space, with a basis $(\mathbf{t}, \mathbf{x}, \mathbf{y}, \mathbf{z}) = (\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ can be made in the dual space, which is a form-space, with basis $(\mathbf{e}^0, \mathbf{e}^1, \mathbf{e}^2, \mathbf{e}^3)$.

Starting from decompositions

$$\tilde{\Phi}_m = \frac{1}{2} F_{ij} \mathbf{e}^i \wedge \mathbf{e}^j = E_x \mathbf{e}^{10} + E_y \mathbf{e}^{20} + E_z \mathbf{e}^{30} + B_x \mathbf{e}^{23} + B_y \mathbf{e}^{31} + B_z \mathbf{e}^{12} \quad (115)$$

$$\tilde{\Phi}_e = \frac{1}{2} G_{ij} \mathbf{e}^i \wedge \mathbf{e}^j = H_x \mathbf{e}^{01} + H_y \mathbf{e}^{02} + H_z \mathbf{e}^{03} + D_x \mathbf{e}^{23} + D_y \mathbf{e}^{31} + D_z \mathbf{e}^{12} \quad (116)$$

we can write

$$\tilde{\Phi}_m^2 = (\mathbf{E}^2 - \mathbf{B}^2) - (\mathbf{E} \cdot \mathbf{B}) I \quad (117)$$

$$\tilde{\Phi}_e^2 = (\mathbf{H}^2 - \mathbf{D}^2) + (\mathbf{H} \cdot \mathbf{D}) I \quad (118)$$

with $I = \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^2 \mathbf{e}^3$, and

$$\mathbf{e}^{01} \mathbf{e}^{01} = \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^0 \mathbf{e}^1 = -\mathbf{e}^0 \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^1 = -(1)(-1) = 1 \quad (119)$$

$$\mathbf{e}^{23} \mathbf{e}^{23} = \mathbf{e}^2 \mathbf{e}^3 \mathbf{e}^2 \mathbf{e}^3 = -\mathbf{e}^2 \mathbf{e}^2 \mathbf{e}^3 \mathbf{e}^3 = -(-1)(-1) = -1 \quad (120)$$

$$\mathbf{e}^{01} \mathbf{e}^{23} = \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^2 \mathbf{e}^3 = I \quad (121)$$

$$\mathbf{e}^{23} \mathbf{e}^{01} = \mathbf{e}^2 \mathbf{e}^3 \mathbf{e}^0 \mathbf{e}^1 = \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^2 \mathbf{e}^3 = I \quad (122)$$

$$\mathbf{e}^{01} \mathbf{e}^{02} = \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^0 \mathbf{e}^2 = -\mathbf{e}^0 \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^2 = -(\mathbf{e}^1 \wedge \mathbf{e}^2) \quad (123)$$

$$\mathbf{e}^{02} \mathbf{e}^{01} = \mathbf{e}^0 \mathbf{e}^1 \mathbf{e}^0 \mathbf{e}^2 = -\mathbf{e}^0 \mathbf{e}^0 \mathbf{e}^2 \mathbf{e}^1 = -\mathbf{e}^{01} \mathbf{e}^{02} \quad (124)$$

$$\dots \quad (125)$$

the last two terms and their conjugate annihilate each other.

The expressions Eq. 117 and Eq. 118 are of the form $\Phi = a + bI$, they can be seen as two

complex numbers, since $I^2 = -1$.

Except the case when $\Phi^2 = 0$ (the electric and magnetic fields are perpendicular and equal), the two-form $\mathbf{T}$ can be defined

$$\mathbf{T} = \frac{\Phi}{\sqrt{\Phi^2}} \quad (126)$$

We obviously can find that $\mathbf{T}^2 = 1$. It is a real positive number, thus $\mathbf{T}$ is a unit time-space two-form. From this formula it can be find that $\Phi = \sqrt{\Phi^2}\mathbf{T}$. Since $I^2 = -1$, the computation with using of complex algebra $\sqrt{\Phi^2} = a + bI$ can be done. Thus, we have

$$\Phi = a\mathbf{T} + b\mathbf{S} \quad (127)$$

with

$$\mathbf{S} = I\mathbf{T}, \quad (128)$$

the space-space unit two-form completely orthogonal to $\mathbf{T}$.

The last step is to prove the uniqueness of this decomposition. Indeed, if we consider another decomposition $\Phi = a'\mathbf{T}' + b'\mathbf{S}'$, we obtain $\Phi^2 = a'^2 - b'^2 + 2a'b'I = a'^2 - b'^2 + 2a'b'I$, and therefore both $a^2 - b^2 = a'^2 - b'^2$ and $ab = a'b'$. We can deduce that a' is solution of the equation $a'^4 - (a^2 - b^2)a'^2 - a^2b^2 = 0$. Thus, $a' = \pm a$ and $b' = \pm b$. For $a' + b'I = a + bI$, we have $\mathbf{T}' = \Phi/(a+bI) = \mathbf{T}$, and also $\mathbf{S}' = I\mathbf{T}' = \mathbf{S}$. For $a' + b'I = -a - bI$, we have $\mathbf{T}' = \Phi/(-a-bI) = -\mathbf{T}$, and also $\mathbf{S}' = -\mathbf{S}$. Two decompositions $\Phi = a\mathbf{T} + b\mathbf{S}$ and $\Phi = (-a)(-\mathbf{T}) + (-b)(-\mathbf{S})$ are the same, the decomposition is unique. Q.E.D

F.**S-parameters computation**

In this Appendix, definitions of the scattering parameters in COMSOL Multiphysics are given and a way these parameters can be used in analytical models to extrapolate simulation results are presented. For instance, if the scattering parameters of a periodic array of optical resonators (e.g., MIM antennas) in a vacuum are calculated, then the response of the same resonators placed above the mirror can be calculated analytically and no additional numerical simulations are required as it is shown in what follows.

F.1 S -matrix

The complex amplitudes of the field scattered by a regular array of optical resonators are related to the amplitudes of the incident waves through the S -matrix also known as the scattering matrix. The components of the S -matrix are called S - or scattering parameters. In the context of this thesis, the S -matrix can be a 4×4 matrix in a general case. It allows one to describe cross-polarization effects but does not account for the appearance of diffraction orders. If a resonator does not possess a prominent cross-polarization response, the scattering matrix can be represented by two 2×2 matrices, one for each polarization. These 2×2 scattering matrices relate the amplitudes r_1 and r_2 of the scattered waves with the amplitudes of the incident waves i_1 and i_2 as follows

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}. \quad (129)$$

It is assumed that the two waves i_1 and i_2 impinge the array from two opposite sides at the same time. Then r_1 (r_2) has the physical meaning of the sum of the reflected wave i_1 (i_2) and the transmitted wave i_2 (i_1). The S -matrix completely determines the response of a given system in a given scattering problem. Once the S -matrix is known, the array of scatterers can be considered as a black box characterized by its S -matrix, see a schematics in Fig. 9.

Since only reciprocal and passive optical resonators are considered in this work, the S -matrix is a symmetric matrix. If additionally a resonator under consideration is symmetric with respect to the array plane, then $S_{11} = S_{22}$.

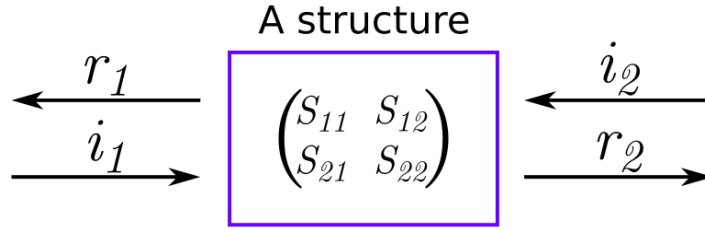


Figure 9: A regular array of optical resonators as a black box characterized by the S -matrix.

F.2 S-parameters in COMSOL Multiphysics

At high frequencies the scattering parameters are defined through the distribution of the electric field along the surface of wave ports [143]. By default, in COMSOL Multiphysics and other commercial electromagnetic simulators, all wave ports are excited one by one. Thus, the complex-valued reflection coefficient S_{11} (S_{22} is defined in the same manner) is computed as

$$S_{11} = \left(\int_{\text{Port 1}} ((\mathbf{E}_{c1} - \mathbf{E}_1) \cdot \mathbf{E}_1^*) dA_1 \right) / \left(\int_{\text{Port 1}} (\mathbf{E}_1 \cdot \mathbf{E}_1^*) dA_1 \right), \quad (130)$$

where $\mathbf{E}_{c1}$ is the total electric field computed on the surface of Port 1, $\mathbf{E}_1$ is the electric field distribution of the fundamental mode for this port. It corresponds the incident wave launched by Port 1, all other ports in the model (if there are any) are switched off and do not launch any waves. The integration is performed over the surface of Port 1 and the differential element of this surface is denoted as dA_1 . According to this definition, the amount of the incident power reflected by a periodic array of optical resonators can be calculated as $R = |S_{11}|^2$.

The complex-valued transmission coefficient S_{21} from Port 1 to Port 2 (it is assumed that only Port 1 is on) is calculated from the total electric field $\mathbf{E}_{c2}$ computed on the surface of Port 2 as

$$S_{21} = \left(\int_{\text{Port 2}} (\mathbf{E}_{c2} \cdot \mathbf{E}_2^*) dA_2 \right) / \left(\int_{\text{Port 2}} (\mathbf{E}_2 \cdot \mathbf{E}_2^*) dA_2 \right), \quad (131)$$

where $\mathbf{E}_2$ is the electric field pattern of the fundamental mode of this port, and dA_2 is the differential element of the surface of Port 2. The transmitted power is calculated as $T = |S_{21}|^2$. For reciprocal resonators $S_{12} = S_{21}$.

F.3 Analytical extrapolation of simulation results

In optics, S -matrix formalism describes the response of any array of optical resonators [188]. While a resonator can have any internal structure, it is enough to know only the amplitudes of the incident and scattered waves to fully describe the far-field response of the array.

In the context of the thesis, S-parameters are calculated assuming that the incident wave is excited by Port 1 as schematically shown in Fig. 10 (a). Since only the case of the period smaller than the wavelength and of the impinging wave at normal incidence is mainly considered in this work⁵, it is sufficient to take into account only the reflected and the transmitted plane waves. The amplitude of the reflected wave $r_1 = S_{11}i_1$ is calculated at Port 1 and the amplitude of the transmitted wave $t_1 = S_{21}i_1$ is calculated at Port 2, placed under the structure. For simplicity, the two ports are located at the same distance h from the structure.

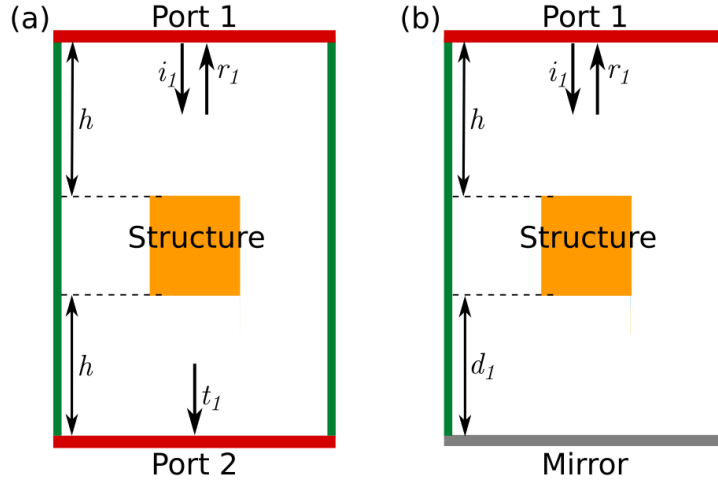


Figure 10: Schematics of a simulation model of a periodic array of optical resonators in vacuum (a) and above the mirror (b).

To compute analytically the optical response of an array of complex resonators placed at the distance d_1 above the mirror (Fig. 10 (b)), the intrinsic response of the array (determined by its S-matrix) and multiple reflections between the array and the mirror should be considered. A similar case with multiple reflections taken into account was covered in [189]. For a given wavelength λ , for which S_{11} and S_{21} of the array in vacuum are known, the reflection coefficient $S_{11}^{(w/mirror)}$ of the array above the mirror can be recalculated analytically as:

$$S_{11}^{(w/mirror)} = S_{11} - \exp\left[i\frac{4\pi(h-d_1)}{\lambda}\right] S_{21}^2 \left(1 + \exp\left[i\frac{4\pi(h-d_1)}{\lambda}\right] S_{11}\right)^{-1}, \quad (132)$$

where h is the distance from the top of the structure to Port 1, and from the bottom of the structure to Port 2, and d_1 is the distance from the bottom of the MIM antennas to the mirror.

When the array is very close to the mirror ($d_1 \ll \lambda$), evanescent modes (which are higher diffraction orders) have large amplitudes and can change the response of the structure after being reflected from the mirror. In this case, a similar analytical calculation can be done in the

⁵If not, the formula below should be modified.

presence of evanescent modes as well. It would involve computing amplitudes of more diffraction orders and scalar equations would transform into matrix ones. The effect of evanescent modes is described, for example, in Section 6.4.4 (see Fig. 6.32). In the case of oblique incidence or when the period is larger than the wavelength, diffraction orders must be taken into account in the same way as the evanescent modes discussed above.

The presented model allows to calculate, in a very fast way, the optical response of a complex resonator for various distances between the array and the mirror, once the intrinsic optical response of the structure is calculated numerically (by means of COMSOL Multiphysics, for example). It is used and validated in the main text by means of comparison with the fully numerical results obtained with COMSOL Multiphysics (see Fig. 6.8).

G. The critical coupling for MIM structures

G.1 Critical coupling parameters for 1D MIM placed at $\lambda/4$ above the mirror

Table 4: Critical coupling for an array of 1D MIM placed at $\lambda/4$ above the mirror with the 15 nm metal thickness

Metal thickness t, nm	Dielectric thickness h, nm	Length $l, \mu m$	Period $p, \mu m$	l/p
15	365	1.202	1.285	0.935409
15	380	1.216	1.320	0.921212
15	400	1.238	1.400	0.884286
15	425	1.254	1.482	0.846154
15	450	1.264	1.550	0.815484
15	475	1.276	1.655	0.770997
15	500	1.282	1.730	0.74104
15	530	1.29	1.825	0.706849
15	536	1.291	1.843	0.700488
15	600	1.303	2.030	0.641872
15	700	1.317	2.300	0.572609

Table 5: Critical coupling for an array of 1D MIM placed at $\lambda/4$ above the mirror with the 45 nm metal thickness

Metal thickness t, nm	Dielectric thickness h, nm	Length $l, \mu m$	Period $p, \mu m$	l/p
45	190	1.207	1.250	0.9656
45	200	1.241	1.323	0.93802
45	210	1.261	1.400	0.900714
45	230	1.278	1.490	0.857718
45	250	1.292	1.600	0.8075
45	267	1.300	1.669	0.77891
45	280	1.305	1.720	0.758721
45	300	1.312	1.820	0.720879
45	314	1.317	1.883	0.699416
45	320	1.320	1.910	0.691099
45	350	1.326	2.020	0.656436
45	400	1.336	2.214	0.603433
45	450	1.345	2.420	0.555785
45	500	1.351	2.640	0.511742
45	550	1.356	2.840	0.477465
45	600	1.359	3.020	0.45
45	700	1.365	3.400	0.401471

Table 6: Critical coupling for an array of 1D MIM placed at $\lambda/4$ above the mirror with the 100 nm metal thickness

Metal thickness t, nm	Dielectric thickness h, nm	Length $l, \mu m$	Period $p, \mu m$	l/p
100	166	1.2167	1.3	0.935923
100	200	1.2625	1.458	0.865912
100	250	1.299	1.780	0.729775
100	265	1.306	1.870	0.698396
100	269	1.3078	1.897	0.689404
100	300	1.3176	2.010	0.655522
100	350	1.3318	2.307	0.577287
100	400	1.3412	2.521	0.532011
100	450	1.3483	2.750	0.490291
100	500	1.3537	2.980	0.454262
100	550	1.358	3.210	0.423053
100	600	1.3608	3.420	0.397895
100	700	1.3648	4.020	0.339502

Table 7: The critical coupling parameters for the 1D MIM array with the constant ratio l/p , placed at the distance $\lambda_{res}/4$ above the perfect mirror, and the normalized power for the Joule losses per one MIM.

t, nm	h, nm	$l, \mu m$	$p, \mu m$	l/p	The normalized power losses
15	536	1.291	1.843	0.7005	0.973
22	460	1.312	1.875	0.6995	0.994
30	403	1.320	1.887	0.6996	1.0
38	354	1.321	1.887	0.7001	1.0
45	314	1.317	1.883	0.6994	0.994
65	285	1.313	1.878	0.6991	0.994
100	265	1.306	1.870	0.6984	0.990

The right column of Table 7 showing the power of the Joule losses in MIM structures arrays demonstrates close values for all thicknesses. The losses are computed numerically as $\mathbf{E} \cdot \mathbf{j}^*$ for both metal layers and are normalized by the maximum value (for the structure with the metal thickness 30 nm). Here $\mathbf{j}$ is the electric current density and $\mathbf{E}$ is the total electric field.

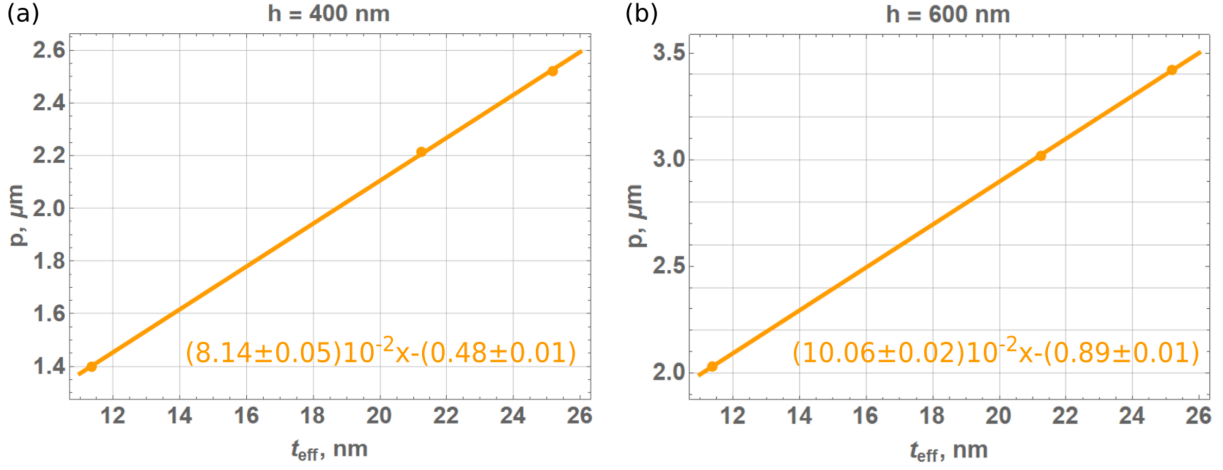


Figure 11: The dependence of the period p on the effective thickness of the metal layer t_{eff} for the vertical movement in Fig. 6.13, i.e. $h = \text{const.}$ Each point corresponds to the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror with a fixed value of the dielectric thickness (a) $h = 400 \text{ nm}$ and (b) $h = 600 \text{ nm}$, other parameters are indicated in Table 4 – Table 6. Solid lines are interpolation functions, their parameters are indicated in the plots.

G.2 The Poynting vector behaviour in the case of the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror

In this section, some aspects of the behaviour of the Poynting vector for the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror are considered. As it is shown in Section 6.3.4, at the critical coupling, all the incident energy funnels into the MIM through the air-silicon side interfaces. The funneling process is due to the electric and magnetic field of the mode. In the case of 1D periodic MIM the component of the time-average Poynting vector P_x directs the energy inside the structure. It is built from the components of the complex amplitudes of the electric and magnetic field of the mode E_{mz} and H_{my}

$$P_x = 0.5(\text{Re}(E_{mz})\text{Re}(H_{my}) + \text{Im}(E_{mz})\text{Im}(H_{my})). \quad (133)$$

As fields of the mode have complex decomposition along the air-silicon side interfaces, a more precise result is provided by integrated values over this interface. The integrated value of both terms of P_x for the resonators at the critical coupling (Table 5) is presented in Fig. 12 by points. The points are fitted in both cases by the quadratic function (its parameters are shown in the plot and in the figure caption). It can be seen that the coefficients of the square terms are opposite (although not perfectly) for $r = \int_0^h \text{Re}(E_{mz})\text{Re}(H_{my})dz$ and $i = \int_0^h \text{Im}(E_{mz})\text{Im}(H_{my})dz$, thus their sum, providing the total funneled energy $P_{\text{in}} = \int_0^h P_x dz$, is a linear function, as it can be

seen in Fig. 13.

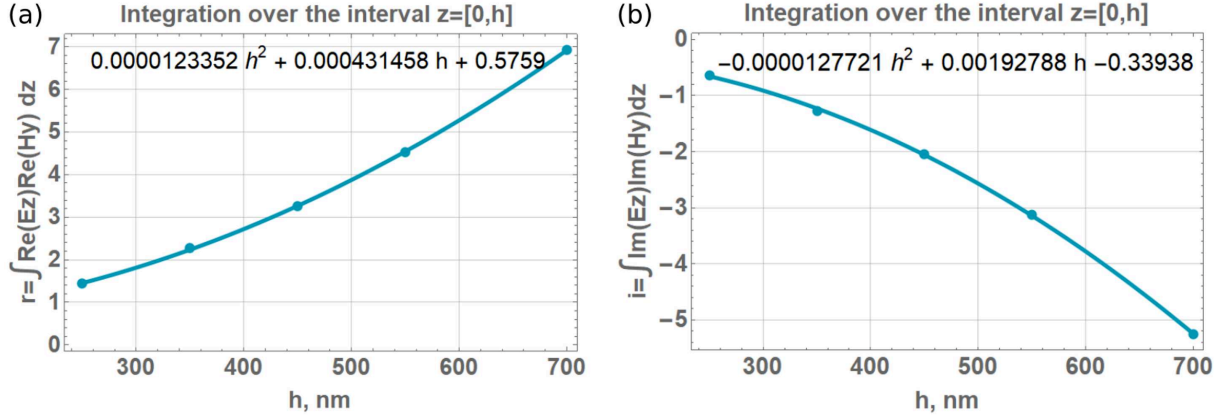


Figure 12: The numerically integrated values of (a) the first ($\text{Re}(E_{mz})\text{Re}(H_{my})$) and (b) the second ($\text{Im}(E_{mz})\text{Im}(H_{my})$) terms of P_x for the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror. Each point corresponds to the critical coupled array with a given value of the dielectric thickness h , while the metal thickness for all structures is the same, $t = 45 \text{ nm}$. Solid lines are interpolation functions, $r = (1.23 \pm 0.04)10^{-5}h^2 + (4.31 \pm 3.6)10^{-4}h + (0.58 \pm 0.08)$ and $i = (-1.28 \pm 0.04)10^{-5}h^2 + (1.93 \pm 0.37)10^{-3}h + (-0.34 \pm 0.08)$.

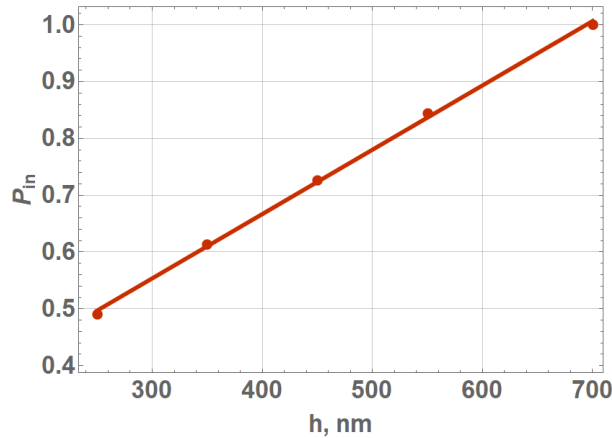


Figure 13: The numerically integrated value of P_x over the interval $z = [0, h]$ for the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror. Each point corresponds to the critical coupled array with a given value of the dielectric thickness h , while the metal thickness for all structures is the same, $t = 45 \text{ nm}$. Points are normalized by the highest value. The solid line is an interpolation function $(1.13 \pm 0.01)10^{-3}h + (0.21 \pm 0.01)$.

Both terms of P_x can be considered at the middle point of the side air-silicon interface of a MIM resonator (in the point $h/2$). They are shown in Fig. 14 for the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror, which parameters are given in Table 5. It can be seen that both of them depend linearly on h and have opposite signs of the coefficients of the linear term. Thus, indeed, after integration over the range $[0, h]$ both of them provide the quadratic function with opposite signs of the coefficients of the square terms. As fields of the mode are proportional to each other, indeed, both the electric and magnetic fields of the mode

are proportional to $\sqrt{h}$.

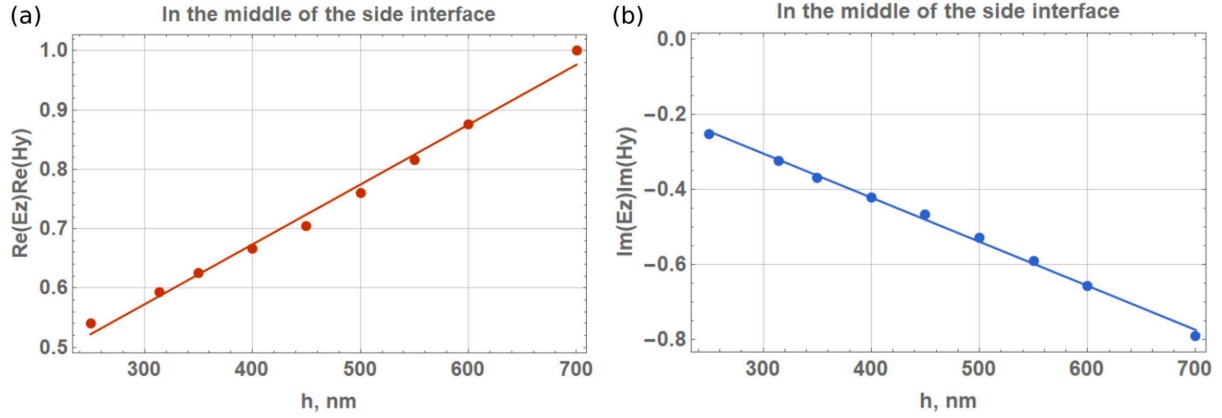


Figure 14: Both terms of P_x at the middle point of the air-silicon side interface of the resonator (the point $h/2$) for the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror, with the constant thickness of the metal layer $t = 45$ nm. Values of both plots are normalized by the same value. Each point corresponds to the critical coupled array with a given value of the dielectric thickness h . Each array is normalized by its maximum value. The solid lines are interpolation curves, the red line is $(1.01 \pm 0.02)10^{-3}h + (0.27 \pm 0.01)$, and the blue line is $(-1.18 \pm 0.01)10^{-3}h + (0.049 \pm 0.006)$.

Oppositely to the case of the constant thickness of the metal layer, in the other case, considered in Section 6.3.4, when the period is constant while the metal and dielectric thicknesses change (keeping the critical coupling), P_{in} does not change. As it is shown in Fig. 15, in this case both terms of P_x integrated over z are independent of the dielectric thickness h in the first approximation (resonators parameters are from Table 7).

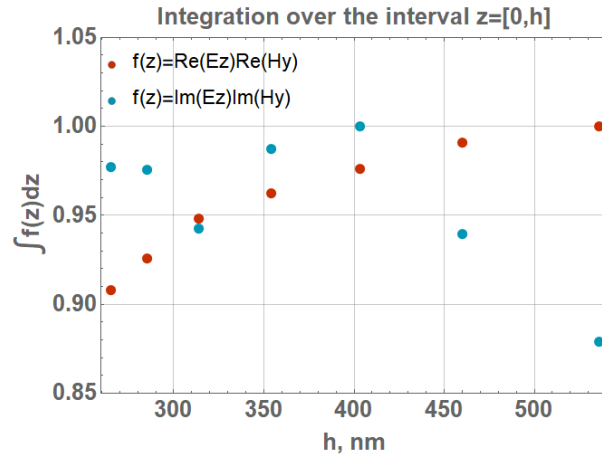


Figure 15: The numerically integrated values for both terms of P_x for the critical coupled 1D periodic MIM arrays placed at $\lambda/4$ above the mirror. Each point corresponds to the critical coupled array with a given value of the dielectric thickness h , while the period values are almost constant, and metal thickness changes.

G.3 The volume density of 1D periodic MIM resonators placed at $\lambda/4$ above the mirror

The total volume density of 1D periodic MIM resonators can be presented as

$$\omega_V = l \cdot (2t + h)/p. \quad (134)$$

As the critical coupling curves have the form $l/p(h) = a/(b + h) + c$, the total volume density can be presented as

$$\omega_V = a(2t + h)/(b + h) + c(2t + h). \quad (135)$$

It can be presented as a sum of three terms

$$\omega_V = 2at/(b + h) + a/(1 + b/h) + c(2t + h). \quad (136)$$

Every of these terms for three curves from Fig. 6.17, Chapter 6 ($t = 15 \text{ nm}$, $t = 45 \text{ nm}$, $t = 100 \text{ nm}$) are presented in Fig. 16 in dependence on the dielectric thickness h .

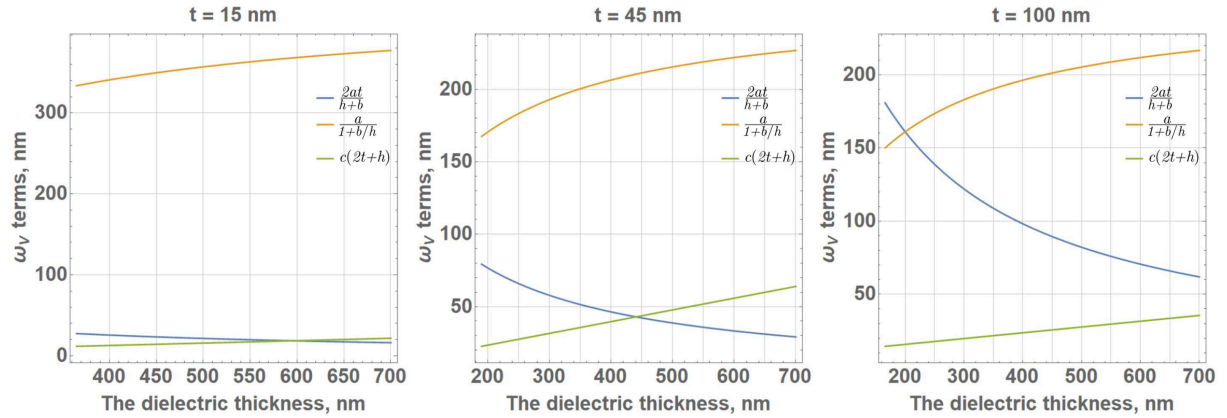


Figure 16: The terms of ω_V for 1D periodic MIM structures of different metal thicknesses placed at $\lambda/4$ above the mirror at the critical coupling.

G.4 Critical coupling data for different geometries for 2D MIM

The tables present the critical coupling conditions at the resonant wavelength $10\ \mu m$ for half cut structures with different geometries on the mirror.

Table 8: Critical coupling for gold disk half-structures

Period p , μm	Diameter d , μm	d/p	Dielectric thickness, nm	Gold thickness, nm
1.52104	1.48101	0.973684	87.0594	35.0239
1.74671	1.5361	0.879429	96.8174	34.9341
2.13831	1.58475	0.741121	112.911	34.9724
2.24789	1.5985	0.711111	119.887	34.9671
2.66769	1.64533	0.61676	151.02	34.7744
3.3414	1.67869	0.502392	177.861	34.9727
4.08696	1.7164	0.419971	224.053	35.0082
4.69385	1.7369	0.370036	256.907	35.2618
5.30903	1.75267	0.33013	291.111	35.0134
7.14794	1.78724	0.250035	404.28	35.0242
8.0600	1.793	0.22283	450.00	35.000
8.97817	1.79583	0.200022	509.52	35.0358
9.62981	1.78244	0.185096	545.973	35.127

Table 9: Critical coupling for gold square half-structures

Period p , μm	Side length, l_1 , μm	l_1/p	Dielectric thickness, nm	Gold thickness, nm
1.38027	1.25391	0.908451	82.6219	34.0208
1.61322	1.30641	0.809816	94.0223	34.6398
2.1158	1.38379	0.654028	130.357	35.0961
2.39366	1.40689	0.587755	146.551	34.1952
3.49405	1.46551	0.419429	201.657	34.9405
4.57645	1.50149	0.32809	257.104	35.9945
5.40271	1.52076	0.281481	304.153	35.0176
6.18573	1.53688	0.248455	352.033	35.2033
7.10912	1.54821	0.217778	401.611	35.141
8.1700	1.5550	0.19033	450.00	35.00
8.89764	1.55964	0.175287	505.72	35.4004

Table 10: Critical coupling for gold rectangle half-structures

Period p , μm	Side length, l_1 , μm	Side length, w , μm	l_1/p	Dielectric thickness, nm	Gold thickness, nm
2.12159	1.96792	0.212159	0.92757	269.66	34.6989
2.63628	2.07895	0.224535	0.788593	314.749	35.0835
3.24601	2.15328	0.231997	0.663363	379.189	34.1172
3.89563	2.21953	0.239581	0.56975	450.919	34.0867
3.9000	2.24800	0.22500	0.57641	450.00	35.00
4.39933	2.27299	0.246363	0.516667	531.831	34.217
5.26063	2.32904	0.25237	0.44273	641.684	34.2362
5.92888	2.369	0.256241	0.399569	737.306	34.3618
6.78023	2.39933	0.259468	0.353871	832.501	35.0632
7.44137	2.41629	0.261681	0.32471	915.383	35.0914
8.09742	2.43013	0.261984	0.300112	997.748	35.132

Table 11: Critical coupling for gold cross half-structures

Period p , μm	Side length, l_1 , μm	Side length, w , μm	l_1/p	Dielectric thickness, nm	Gold thickness, nm
2.10922	1.98959	0.215145	0.943279	268.428	35.1872
2.66584	2.13407	0.230467	0.800524	330.237	34.9193
3.33087	2.20936	0.237566	0.663299	389.014	34.645
3.9000	2.29400	0.23000	0.58821	450.00	35.0000
4.12285	2.27685	0.247371	0.55225	463.821	36.075
4.84828	2.32519	0.252308	0.479592	544.194	34.6306
5.95248	2.38099	0.258277	0.4	655.782	35.3113
6.90763	2.41267	0.261289	0.349275	750.829	35.0387
7.30844	2.4248	0.262303	0.331781	793.917	35.0405
7.60157	2.44161	0.263714	0.321198	857.32	35.095
8.91492	2.46412	0.266446	0.276404	1047.75	35.0587

G.5 A rectangular half-MIM structure on the perfect mirror

An example of the half-MIM structure with the rectangular-shaped patches at the critical coupling is presented here. The parameters of the structures: the period $p = 3.9 \mu m$, the length $l_1 = 2.248 \mu m$, the width $l_2 = 225 nm$, the dielectric thickness $h = 450 nm$, and the metal thickness $t = 35 nm$. The resonant wavelength $\lambda_{res} = 10 \mu m$. The half-structure configuration means that the metal patch is separated by the dielectric patch of the same shape from the perfect mirror (Fig. 6.22 (a)). Thus, excitation, similar to the classical MIM mode is expected.

However, for 2D periodic patches the decomposition of the fields and the electric current is

much more complex compared to the 1D case (Fig. 17). However, the excited mode is similar to the MIM mode in the x direction, as it is shown in Fig. 17 (c) and (d), but with some magnetic field above the structure. A significant disturbance at sides in the y direction is caused by edge effects due to the H_z component of the mode (Fig. 17 (a) and (b) and (e) and (f)). It can be noted that the magnetic field in the yz cross-section behaves similarly to the magnetic field around a wire with the electric current (in the x direction for the considered case), and field in the dielectric is due to excited MIM mode. For the considered case the electric field of the normally incident plane wave is directed along the x axis, as the longer side of the rectangle. Other considered geometries have similar behaviour, although for the square and disk geometries the magnetic field is not excited significantly above and below the structure (discussed in Section G.6).

Due to the limitation in the y direction, the way, how energy enters to the structure, is more complex too. It can be seen from Fig. 18, that in the x direction it is similar to the MIM mode funneling. The S_x component of the time-average Poynting vector at the edges of the structure in this case is induced due to the H_y component of the magnetic field of the standing wave, and E_z component of the electric field of the mode. In the same time, the energy in the y direction is provided due to the E_x component of the electric field of the standing wave, and the H_z component of the magnetic field of the mode. The magnetic field of the mode leads to the more complex electric current distribution in the metal bar (Section G.8). Thus, in the yz section incident energy comes both at the top and bottom of the metal layer. However, as in the discussion below only proportionality is discussed, the integration of the S_y component along the air-dielectric interface does not contradict the Poynting vector behaviour.

For this mode the electric field in the gold-silicon interface increase linearly with the growth of the dielectric thickness. Indeed, let us start with the classic 1D periodic MIM structure (thick homogeneous bottom metal layer and top metal layer is infinite periodic ribbons). For this structure, the incoming power is proportional to the period p . In the same time, dissipated power

$$P_d \propto l E_m^2. \quad (137)$$

Here E_m is the electric field of the excited mode, and l is the width of the patch (infinite in another direction). In the same time, all incoming power is come through sides of MIM inside the structure, because all absorption is on the inner side of metal inside the structure. Incoming inside structure energy can be counted as the integral of the time-average Poynting vector S .

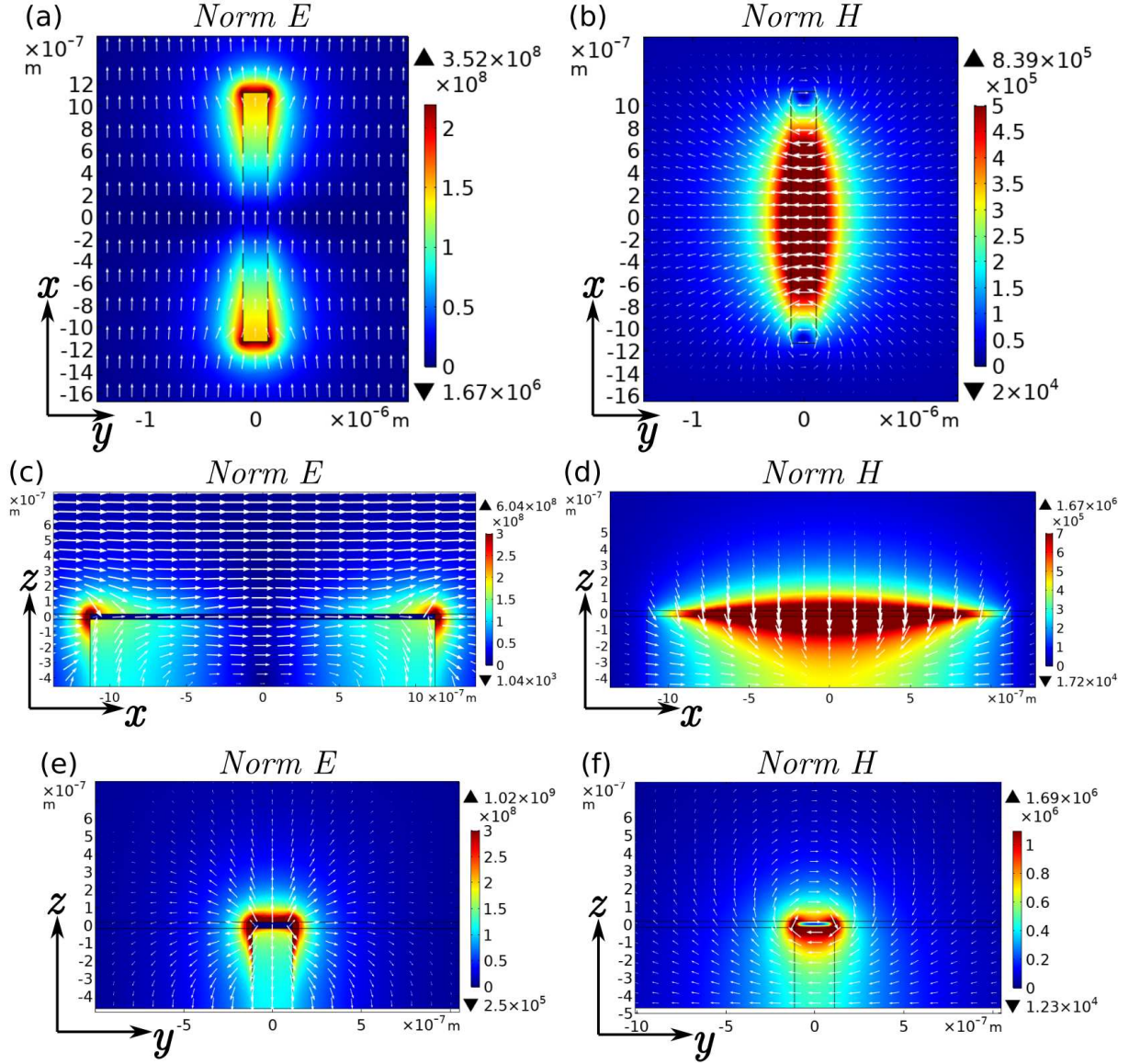


Figure 17: The field norms and directions in different cross-sections for a rectangular half-MIM (the parameters are indicated in the beginning of the section). Color maps shows the norm of the field, while white arrows indicate its direction in the considered plane: (a) the electric and (b) the magnetic field for xy plane located 100 nm lower than the metal-dielectric interface inside MIM. (c) the electric field in xz plane at the middle of the structure ($l_2/2$), (d) the magnetic field in the xz plane at the side edge of the structure, (e) the electric field in the yz plane at the edge of the structure, (f) the magnetic field in the yz plane in the middle of the structure. The values near triangles in the top and bottom of the color legends indicate the maximum and minimum values correspondingly.

For the classical MIM mode, this vector is composed with the electric field of the mode $\mathbf{E}_m$ and the magnetic field of the incident wave $\mathbf{H}_i$, which, being reflected from the thick bottom metal forms a standing wave.

$$\int_0^h S dz \propto E_m(h) H_i h. \quad (138)$$

As at the critical coupling all incident power is absorbed, the entered energy is connected with

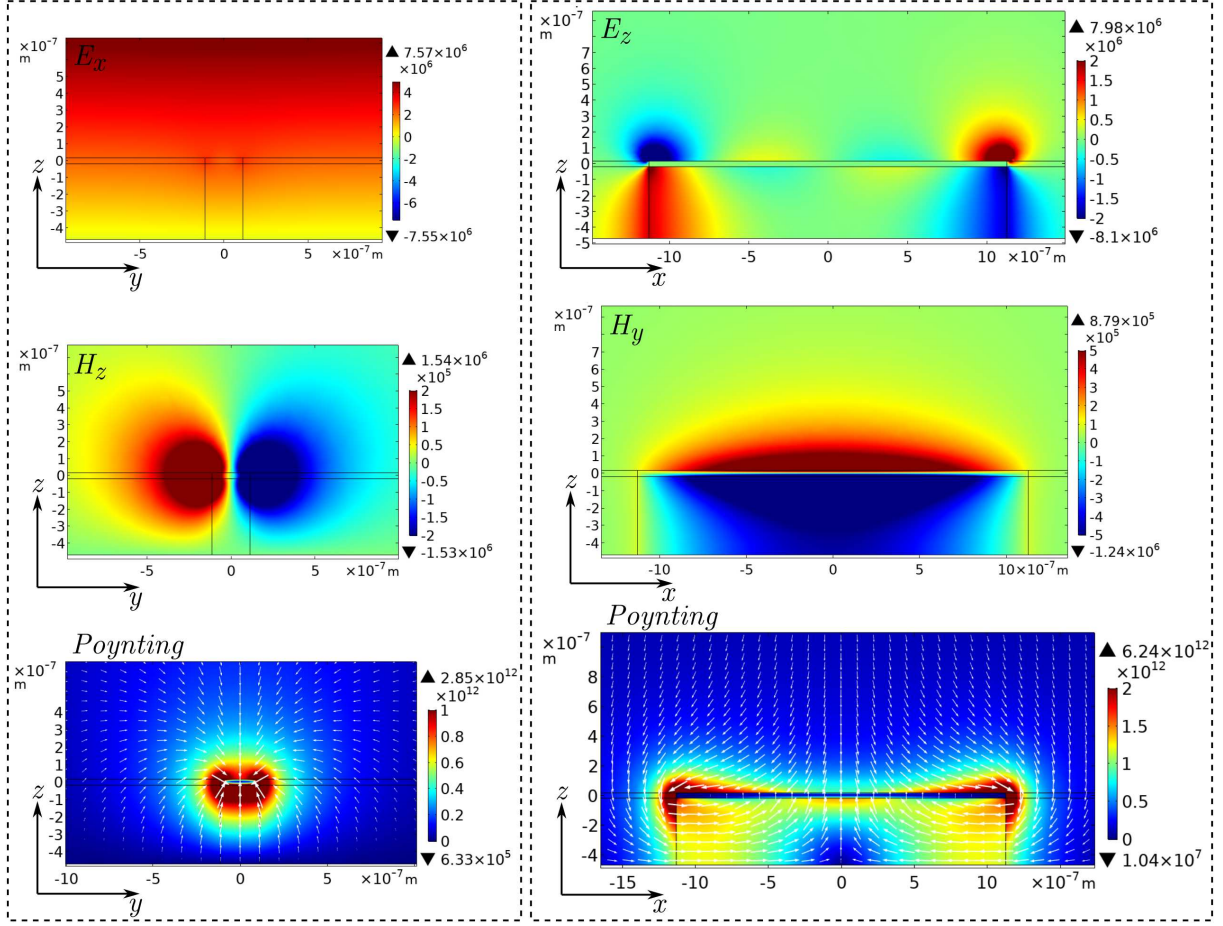


Figure 18: The Poynting vector for the rectangular half-MIM structure on the perfect mirror. The left column is for yz plane at the middle of the structure ($l_1/2$), and the right one is for xz plane at the middle of the structure ($l_2/2$). The top and middle row show field maps of the real part of the corresponding component of the total fields. The color maps at the bottom row present the norm of the Poynting vector, and arrows show the Poynting vector in the presented plane.

the dissipation in the metal layers:

$$lE_m^2 \propto E_m H_i h. \quad (139)$$

As the amplitude of the incident wave H_i is constant (with the change of the period, the energy density is kept constant),

$$E_m \propto h/l. \quad (140)$$

Thus, for the classical 1D MIM at the critical coupling the increase of the thickness (keeping the critical coupling) leads to increase of the amplitude of the electric field. In the same time, period increase too. Thus, as incident power increases, the absorbed electrical current increases (proportionally to the field of the mode), as the length of the structure changed slightly.

Now, let us consider the 2D MIM. In this case the energy comes trough four sides, not two,

as it was for 1D MIM. In this case, the total Poynting vector components, responsible to the energy entrance inside the structure, are S_x and S_y . In this way, to compute the energy, entered into the structure, both S_x and S_y should be integrated in the z direction in the range $(0, h)$.

As the structure is orientated along x , the electric field of the incident wave is orientated along x , and the magnetic field of the incident wave is along y . In this case, the S_x component of the Poynting vector, providing entrance the energy in the x direction, is composed by the electric field of the mode E_{mx} and the magnetic field of the incident wave H_{iy} (similar to 1D MIM).

In the 3D model the field decomposition is more complex compare to 2D model, and, due to limitation, new components of the field of the mode are excited. Consequently, the S_y component of the Poynting vector, providing entrance the energy in the y direction, can be composed in two different ways. At first, it is composition of the electric field of the incident wave E_{ix} and the magnetic field of the mode H_{mz} . Secondly, it also can be composed by the mode fields E_{mx} and H_{mz} . To evaluate their impact, the integral trough dielectric surface in the xz planes at the edges of the structure was computed in numerical model. For both sides computation shows that $S_{yim} = \frac{1}{2}E_{ix}(H_{mz})^*$ is responsible for the funneling the energy inside the structure, while $S_{ymm} = \frac{1}{2}E_{mx}(H_{mz})^*$ induces the leakage of the energy from the structure, but with a smaller rate. In this way, the physical behavior of 2D MIM is more complex. However, as the energy flux inside the structure is provided by the S_{yim} component, only this one is taken into account in this model.

As in the both components of the Poynting vector include one component of the incident wave, which does not depend on h , and one component of the field of the mode, which changes proportionally to h . Thus, the total Poynting vector through all four interfaces S_{sum} is still proportional ⁶ to the $E_m(h)h$. In the same time, the dissipated energy in the metal bar is proportional to the $\sigma l A E_m^2$. As the change of the metal parameters is small, in the first approximation they are considered as constant for one geometry. For simplicity, they are included in one coefficient $C = \sigma l A$. Then,

$$E_m(h)h \propto C E_m(h)^2, \quad (141)$$

or

$$E_m(h) \propto h/C. \quad (142)$$

⁶Here an assumption that the H_{mz} and E_{mz} are proportional is made. But, as it shown below, the conclusions drawn from this assumption (Eq. 142) is in fine agreement with numerical result.

It also should be noted that in the last case the energy is absorbed in the top part of the metal (due to the magnetic field of the mode), thus only a part of the energy in the y direction enters the dielectric. However, as it is included as the proportional relation, and the agreement of this model with the numerical results is fine, this calculation can be used.

The electric field norm values were extracted from the numerical model in the center of the metal bar for the half-MIM with rectangular geometry at the critical coupling. Their dependence on the dielectric thickness is presented in Fig. 19. It can be seen, that, indeed, the points, corresponding to the numerical results are fitted fine by the linear function.

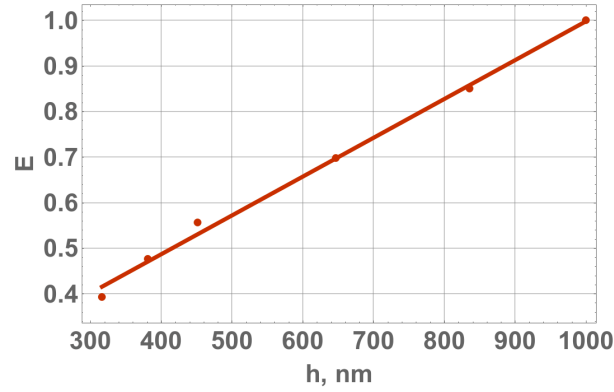


Figure 19: The dependence of the electric field norm (normalized) on the dielectric thickness. The points correspond to the values, extracted from the numerical model. The solid line is linear fitting function $ax + b$, with $a = (8.5 \pm 0.2)10^{-4}$ and $b = (0.15 \pm 0.01)$.

G.6 Field maps for complete 2D MIM with different geometries

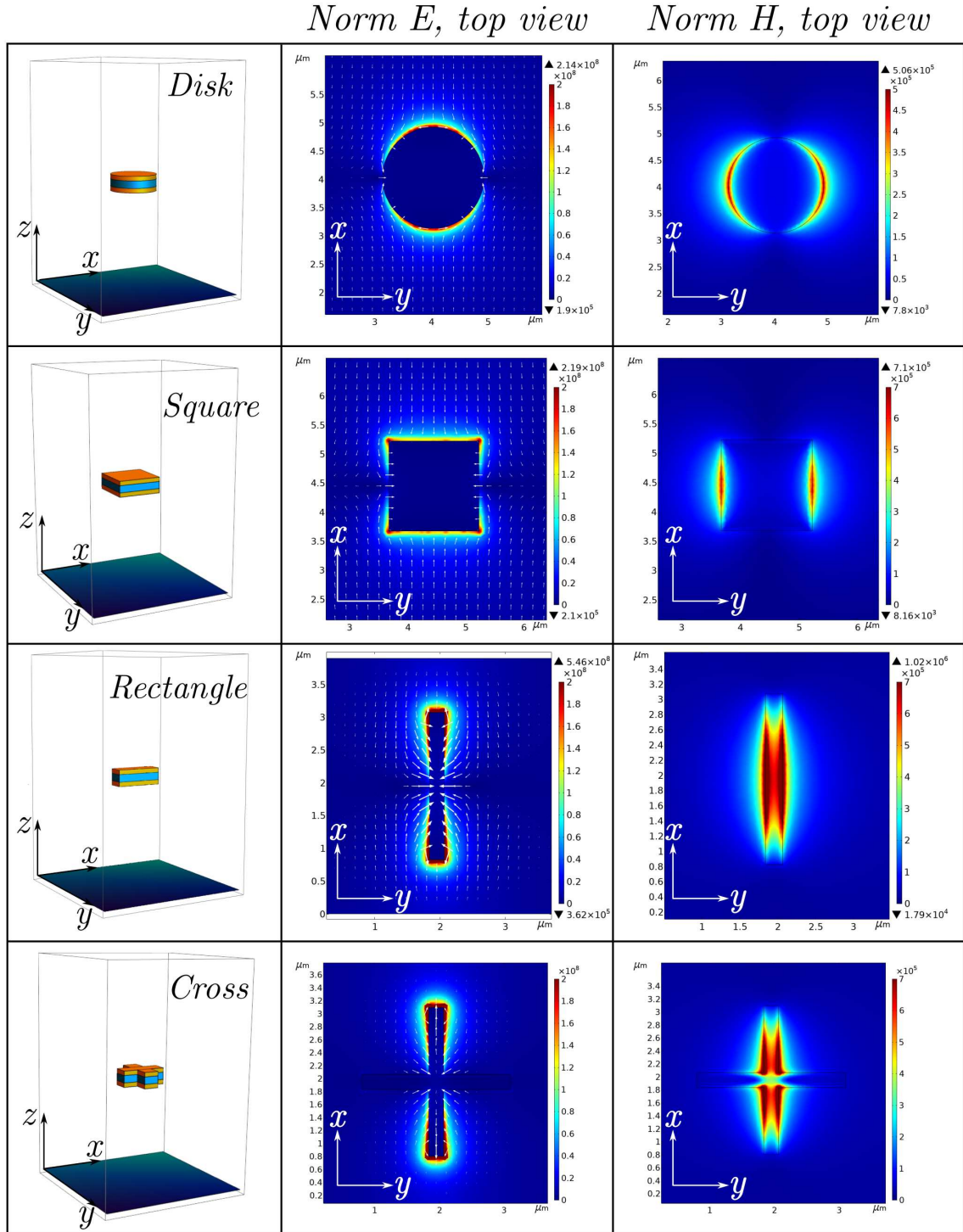


Figure 20: Maps of the norm of the electric and magnetic fields for the structure of four different geometries. The same color legend is used for the electric field for all geometries, and the same for the magnetic field. The white arrows present The Poynting vector (logarithmic scale), the scale is the same for one geometry, but differs for different ones. The metal thickness is 45 nm and the dielectric thickness is 900 nm for all the structures. Other parameters: the diameter is $d = 1.793 \mu\text{m}$, and the period $p = 8.06 \mu\text{m}$ for the disk geometry; the side length $l = 1.555 \mu\text{m}$, and the period $p = 8.17 \mu\text{m}$ for the square geometry; the length $l_1 = 2.248 \mu\text{m}$, the width $w = 225 \text{ nm}$, and the period $p = 3.9 \mu\text{m}$ for the rectangular geometry; the length $l_1 = 2.294 \mu\text{m}$, the width $w = 230 \text{ nm}$, and the period $p = 3.9 \mu\text{m}$ for the cross geometry.

Here, the excited mode for arrays of structures with different geometry, critically coupled at $\lambda/4$ above the perfect mirror, are considered. The norms of the total electric and the total magnetic fields, and the Poynting vector are presented in Fig. 20 in top views (the level of the top air-metal interface) and in Fig. 22, in side views. The electric field of the incident wave is oriented along x direction, the magnetic field is along y direction. The square, rectangle and cross shaped antennas have a maximum of electric field at their corners. Moreover, the side view for the electric field in the central (i.e. at the center of the structure) xOz plane is similar to that of the 1D MIM structure.

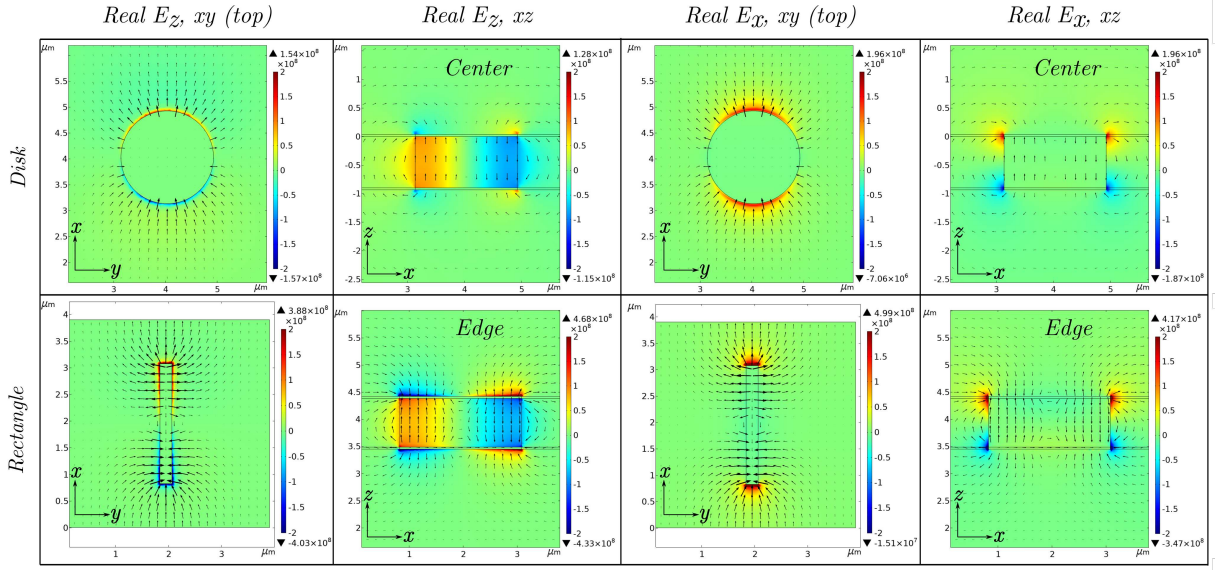


Figure 21: Maps of the real part of the E_x and E_z for the disk and rectangular geometries in the xOy plane at the top of the structure and in the xOz plane at the middle of the structure ($l_2/2$). The same color legend is used for all maps. The black arrows present the Poynting vector (logarithmic scale), the scale is the same for one geometry, but differs for different ones. The metal thickness is 45 nm and the dielectric thickness is 900 nm for all the structures. Other parameters: the diameter is $d = 1.793 \mu\text{m}$, and the period $p = 8.06 \mu\text{m}$ for the disk geometry; the length $l_1 = 2.248 \mu\text{m}$, the width $w = 225 \text{ nm}$, and the period $p = 3.9 \mu\text{m}$ for the rectangular geometry.

The maps of the components of the electric fields separately for the disk and the rectangle geometries are presented in Fig. 21. The fields maps for the square and cross geometries are similar to ones for the rectangle, therefore, it is enough to restrict consideration to the latter. Color maps present the field amplitude, and arrows – its direction.

However, due to the limitation in the y direction, the behavior of the magnetic field is different from that of 1D MIM. For all geometries, the excitation of the magnetic field is observed at the edges of the structure in the y direction (Fig. 20). The side view in Fig. 22 for magnetic field in the central plane xOz shows that the distribution of the field for the disk and the square

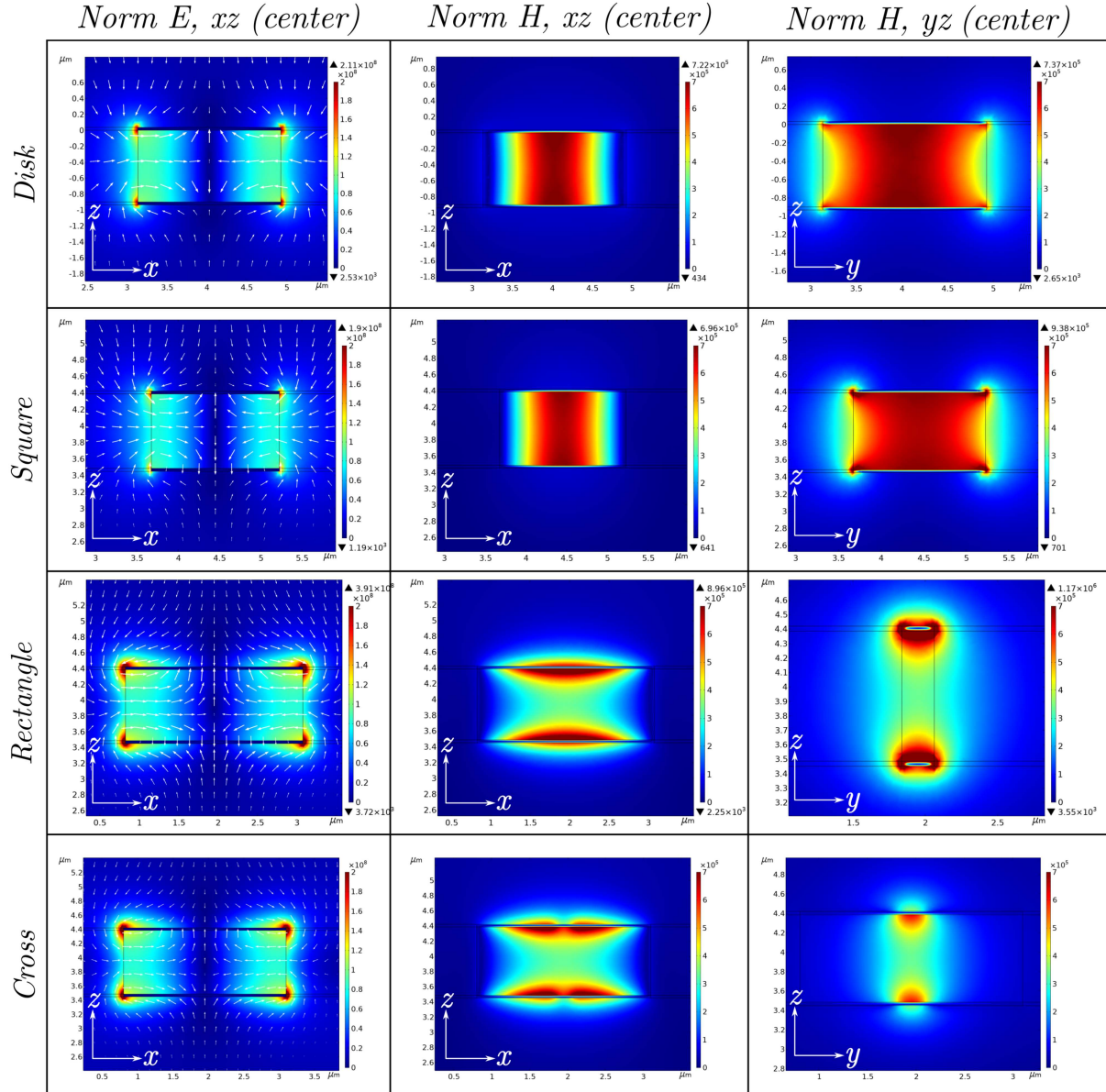


Figure 22: Maps of the norm of the magnetic fields for the structure of four different geometries in the xOz plane at the middle of the structure ($l_2/2$). The same color legend is used for all maps. The white arrows present The Poynting vector (logarithmic scale), the scale is the same for one geometry, but differs for different ones. The metal thickness is 45 nm and the dielectric thickness is 900 nm for all the structures. Other parameters: the diameter is $d = 1.793 \mu\text{m}$, and the period $p = 8.06 \mu\text{m}$ for the disk geometry; the side length $l = 1.555 \mu\text{m}$, and the period $p = 8.17 \mu\text{m}$ for the square geometry; the length $l_1 = 2.248 \mu\text{m}$, the width $w = 225 \text{ nm}$, and the period $p = 3.9 \mu\text{m}$ for the rectangular geometry; the length $l_1 = 2.294 \mu\text{m}$, the width $w = 230 \text{ nm}$, and the period $p = 3.9 \mu\text{m}$ for the cross geometry.

geometries is similar to the 1D MIM magnetic field distribution, the stronger magnetic field is concentrated in the center of the structure, and value does not change significantly along the entire dielectric thickness.

For the rectangle and cross geometries the magnetic field distribution looks different. A region of stronger amplitudes is located inside the structure close to the metal, while in the

center of dielectric the norm of the magnetic field is about two times smaller than near the metal. This change appears as the strong magnetic field, excited at the edges, does not decay significantly until the middle of the structure (in the y direction) due to small width l_2 . Thus, the magnetic field does not look homogeneous, and it also appears on external metal-dielectric interfaces. In the case of the disk and square geometries, the magnetic field also excited above and below the structure (yOz planes), but, due to significant width of the structures, they are located only at the edges.

The Poynting vector distributions for different cut planes demonstrate that, similar to the classic MIM mode, the energy flow is directed inside the structure. In the same way as for 1D MIM mode, there is the energy funneling in the x direction. In the same time, there is some energy comes directly from above and below the structure, and the energy funneling in the y direction. Inside the structures the arrows are directed to the center of the top and bottom metal layers.

In this way, as the distribution of the fields of the mode is more complex, the Poynting vector and the electric current is also distributed in a more complex way. In the case of 2D MIM, oppositely to 1D MIM, the energy comes to the metal from all sides. Consequently, the electric current is exited on all sides of each metal layer, as it shown in Fig. 23. It also can be seen that the electrical current distribution is non-linear and similar to the magnetic field distribution.

As the current indicates the areas with high absorption, it is seemed clearly that for the disk and square geometries, alike for the classical MIM mode, the current gradient is observed over the thickness of the structure, with the strongest absorption on the inner side of the structure. However, the strongest electric current density is exited at the edges for all geometries.

Although the higher absorption occurs at the edges for all structures, for the disk and square geometries its intensity changes twice or three in the central part of the structure, for the rectangle it almost not changed. For the cross geometry, the current flow is slightly weaker in the center, but occupies the second crossbar, having a hyperbolic-like behavior. The area surface plays the major role for the currents decomposition: the larger area of the current with less intensity can provide higher integral value, thus, the higher optical losses.

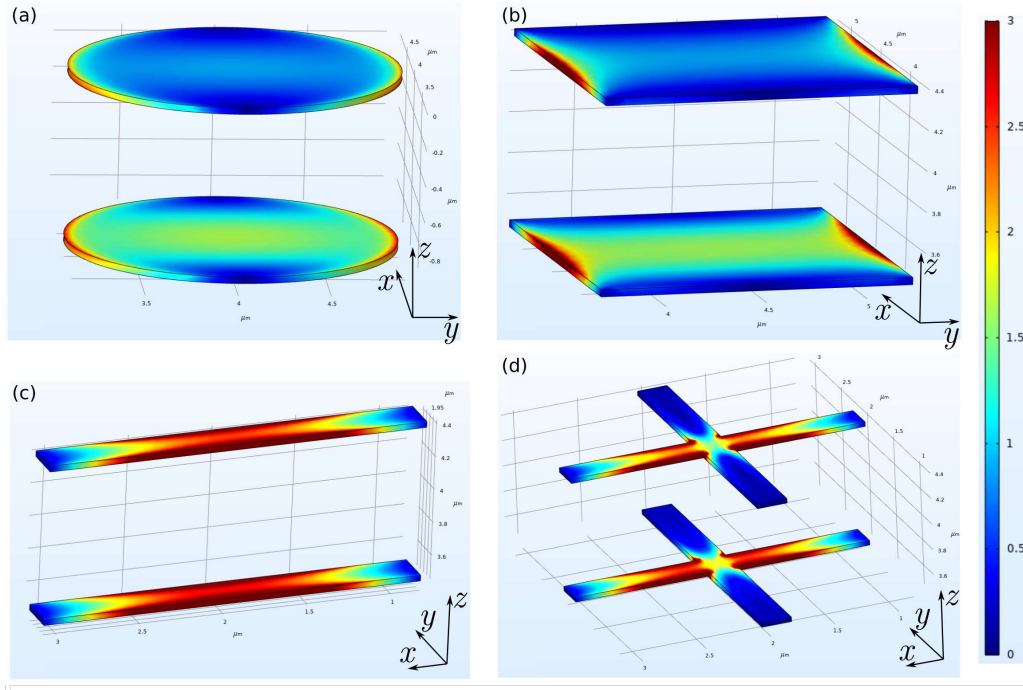


Figure 23: Maps of the norm of the electric current density on the metal parts (other parts are invisible) of the 2D MIM of four different geometries. The same color legend is used for all maps. The metal thickness is 45 nm and the dielectric thickness is 900 nm for all the structures. Other parameters: the diameter is $d = 1.793 \mu\text{m}$, and the period $p = 8.06 \mu\text{m}$ for the disk geometry; the side length $l = 1.555 \mu\text{m}$, and the period $p = 8.17 \mu\text{m}$ for the square geometry; the length $l_1 = 2.248 \mu\text{m}$, the width $w = 225 \text{ nm}$, and the period $p = 3.9 \mu\text{m}$ for the rectangular geometry; the length $l_1 = 2.294 \mu\text{m}$, the width $w = 230 \text{ nm}$, and the period $p = 3.9 \mu\text{m}$ for the cross geometry.

G.7 Critical coupling data for 2D MIM made of different metals

The tables present the critical coupling data for different half cut 2D MIM structures on the mirror at the resonant wavelength $10 \mu\text{m}$.

Table 12: Critical coupling for silver cross half-structures

Period $p, \mu\text{m}$	Side length, $l_1, \mu\text{m}$	Side length, $w, \mu\text{m}$	l_1/p	Dielectric thickness, nm	Gold thickness, nm
2.72592	2.18054	0.235734	0.799927	337.618	34.9605
3.36748	2.25231	0.243817	0.668843	396.703	34.9738
3.80263	2.30159	0.250173	0.605263	450.312	35.0242
4.79317	2.36663	0.256634	0.49375	554.211	34.9502
7.40522	2.47975	0.268189	0.334865	870.614	35.0247

Table 13: Critical coupling for aluminum cross half-structures

Period p , μm	Side length, l_1 , μm	Side length, w , μm	l_1/p	Dielectric thickness, nm	Gold thickness, nm
2.32324	2.16001	0.233325	0.929741	309.431	35.0489
3.03889	2.30258	0.248093	0.757705	387.582	34.8725
3.4105	2.34741	0.25326	0.688289	421.432	35.0359
3.9231	2.40406	0.259007	0.612796	498.013	35.0009
5.0565	2.47571	0.268586	0.48961	632.037	35.0242
6.5045	2.53575	0.273189	0.389846	823.57	35.0242

Table 14: Critical coupling for copper cross half-structures

Period p , μm	Side length, l_1 , μm	Side length, w , μm	l_1/p	Dielectric thickness, nm	Gold thickness, nm
2.69035	2.16131	0.21583	0.803358	288.108	35.1351
3.7951	2.26008	0.243685	0.595526	398.486	34.9549
4.42577	2.31579	0.25076	0.523251	470.55	34.9666
7.18861	2.44812	0.264581	0.340556	760.794	34.9446
7.9302	2.47706	0.268007	0.312358	849.021	35.0009

G.8 The fitting of the electric current density in 2D MIM made of different metals

The electric current density, exited in the metal bar of a 2D MIM (half-structure) can be described $j(x, z) = a(\exp(-x/\delta) + \exp(x/\delta)) + b(\exp(-z/\delta) + \exp(z/\delta))$ (the electric field is along y).

Fitting parameters a , b were found for the normalized electric current densities, which were extracted from numerical simulations for different metals at the critical coupling (the resonant wavelength is $\lambda_{res} = 10 \mu m$) in the middle section of the metal bar. Parameters are presented in Table 15.

Table 15: Fitting parameters for the normalized electric current density in different metal bars

Metal	a	b	j_{max}
Gold	0.0038658	0.20482	0.81
Silver	0.0023424	0.18213	0.87
Aluminum	0.0006813	0.13683	0.96
Copper	0.0022299	0.18014	1.0

The fitting was performed as following.

- The electric current density in the middle xOz plane of the metal bar was extracted. The obtained array of data consists of elements as $\{x_i, z_i, j_i\}$.
- The maximum electric current density j_m is identify for the array. The current density values of the array is normalized, thus a new array with elements $\{x_i, z_i, j_i/j_m\}$ is computed. To simplify computations, a shift was made along the x and z axes, so that the center of the structure is at point $(0, 0)$.
- The fitting for the array with normalized current density is done in according to Eq. 6.12.
- To compare losses for different metals, the maximum values for every structure j_m were normalized on the highest value among them. As a result, j_{max} is obtained for every metal (Table 15).
- The dissipated power is proportional to square of the current though the cross-section I_{cr}^2 . The last is computed as $I_{cr}^2 = j_{max}^2 \iint_{x=-l_2/2, z=-t/2}^{x=l_2/2, z=t/2} j^2(x, z) dx dz$, $I_{cr}^2 = j_{max}^2 (2((a^2 + b^2)tw + b^2w\delta \sinh[t/\delta] + a\delta(16b\delta \sinh[t/(2\delta)] \sinh[w/(2\delta)] + at \sinh[w/\delta])))$.

G.9 The critical coupling for 2D MIM arrays of rectangular geometry, placed at $\lambda/4$ above the mirror

Table 16: Critical coupling parameters at the resonance wavelength $10\ \mu\text{m}$ for 2D MIM rectangle structures with the gold thickness $20\ \text{nm}$.

Period $p, \mu\text{m}$	Side length $l_1, \mu\text{m}$	Side length l_2, nm	Dielectric thickness h, nm	l_1/p
5.5	2.88	280	2700	0.52
6.75	2.85	285	2500	0.42
8.02	2.811	281	2300	0.35
8.7	2.78	278	2200	0.32
9.24	2.751	275	2100	0.30
9.6	2.726	273	1998	0.28

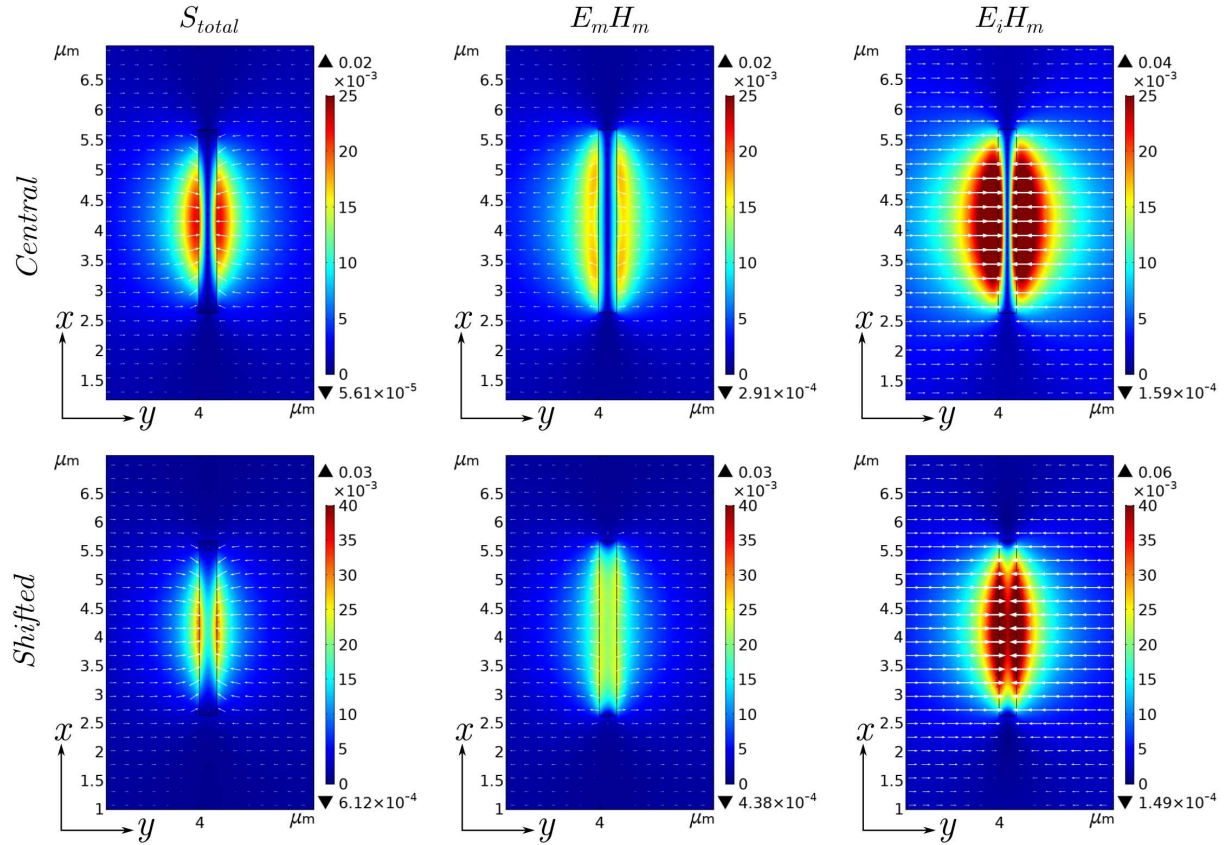


Figure 24: The time-average total Poynting vector S_{tot} and two its main components $E_m H_m^*$ and $E_i H_m^*$ in xOy planes. Color maps show the norm of the amplitude, and white arrows – the direction (also the amplitude in the logarithmic scale). The central plane is at the middle of the dielectric layer ($h/2$), and the shifted one at the level of $h/6$.

The analysis of the way how the energy coming to the 2D MIM structure at the distance $\lambda/4$ above the mirror can be made with separation of the total fields to the field of the standing

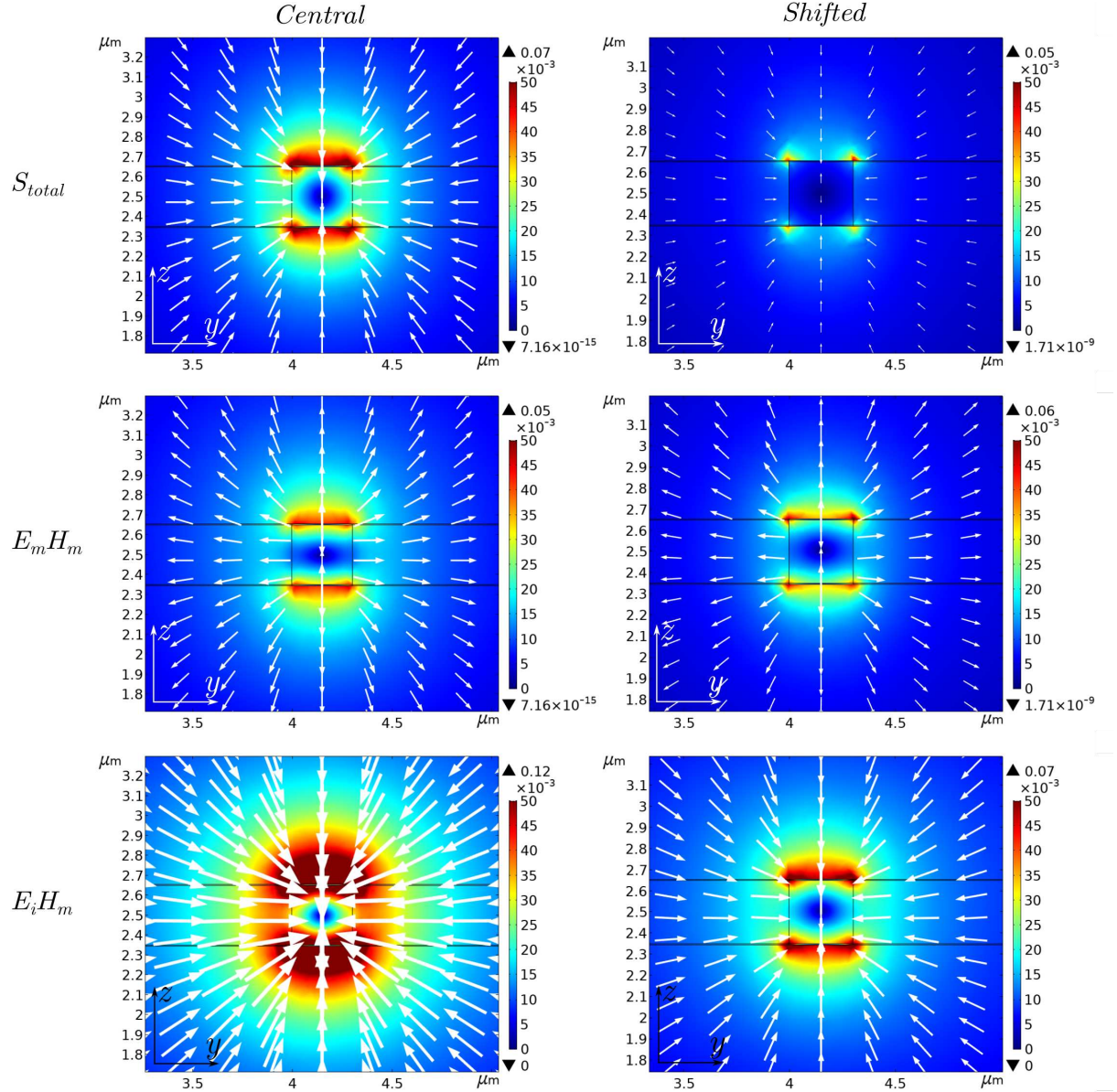


Figure 25: The time-average total Poynting vector S_{tot} and two its main components $E_m H_m^*$ and $E_i H_m^*$ in yOz planes. Color maps shows the norm of the amplitude, and white arrows – the direction (also the amplitude in the logarithmic scale). The central plane is at the middle of the longer side ($l_1/2$), and the shifted on at the distance $l_1/4$ from the edge of the structure (l_1 is along x and l_2 is along y).

way and the field of the mode. Similar analyses is presented in Section 6.3.2 and Section 6.3.4 of Chapter 6. Then, in according with the Eq. 6.3, the total Poynting vector can be presented as a sum of four terms: the all combinations created by the components of the standing wave (here the index i) and the mode (the index m). As the standing wave do not transfer energy, this term is zero. Also, as the structure is placed into the node of the magnetic field, the terms, connected with the magnetic field of the mode is much smaller than two others.

The geometric parameters of the considered structure: $p = 8.3 \mu m$, $l_1 = 3.05 \mu m$, $l_2 = 305 nm$, $t = 5 nm$, $h = 300 nm$.

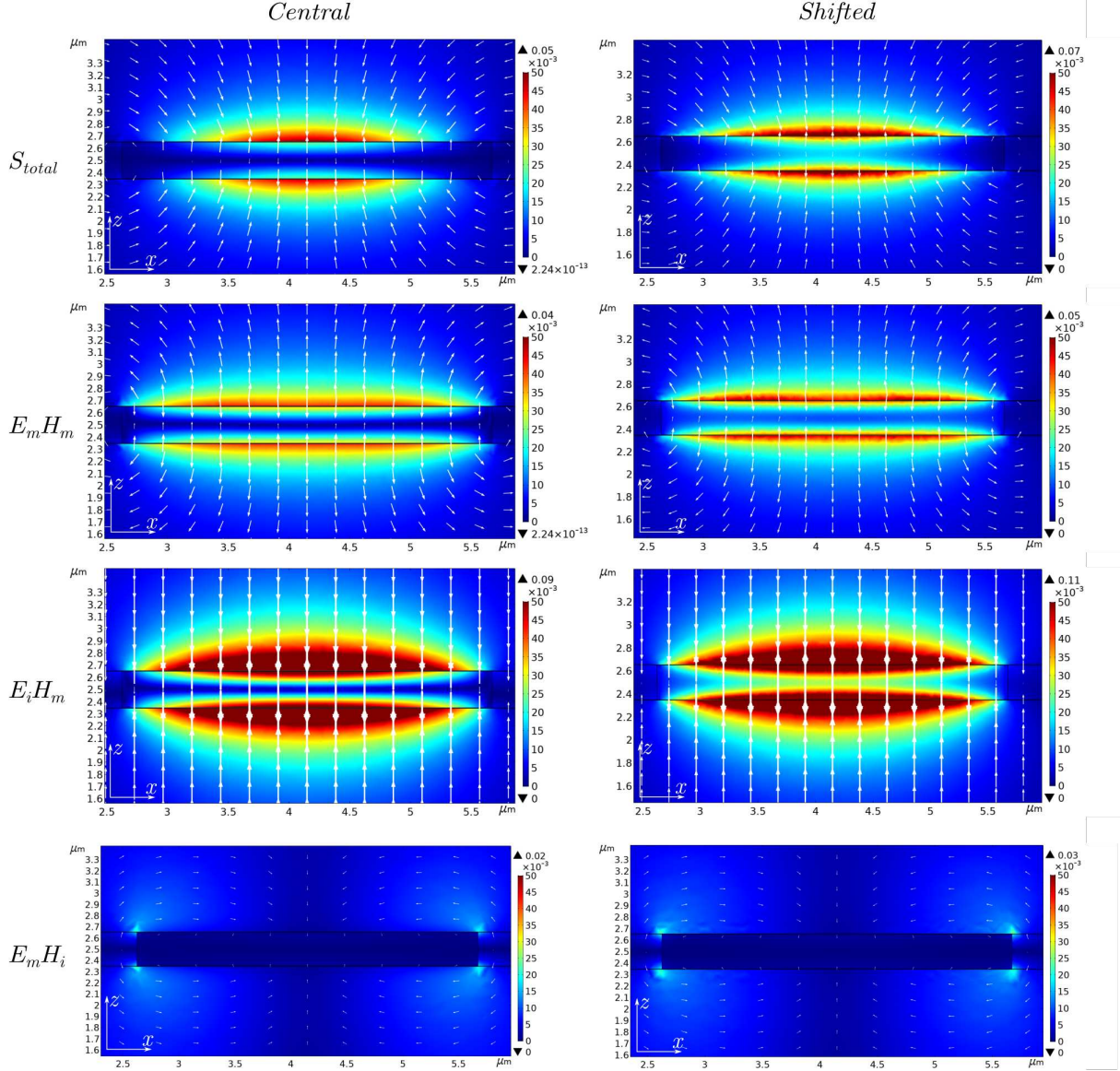


Figure 26: The time-average total Poynting vector S_{tot} and three its non-zero components $E_m H_m^*$, $E_i H_m^*$, and $E_m H_i^*$ in xOz planes. Color maps shows the norm of the amplitude, and white arrows – the direction (also the amplitude in the logarithmic scale). The central plane is at the middle of the shorter side ($l_2/2$), and the shifted on at the distance $l_2/4$ from the edge of the structure (l_1 is along x and l_2 is along y).

The xOy sections for the total Poynting vector and two its main terms are presented in Fig. 24. It can be seen that the funneling of the energy into the structure in the y direction (S_y) is mainly provided by the component of the Poynting vector, constructed by the electric field of the standing wave ($E_i = E_{ix}$) and the magnetic field of the mode ($H_m = H_{mz}$). In the same time, the term built from both fields of the mode $E_m H_m^*$ causes energy to leak out of the structure.

The yOz sections for the total Poynting vector and two its main terms are presented in Fig. 25. It can be seen, that again, the energy funneling in the y direction, and also energy

fluxes above and below the structure are provided by the component of the Poynting vector, constructed by the electric field of the standing wave and the magnetic field of the mode. For the z direction, S_{zim} is built by the E_{ix} and H_{my} . Again, the term $E_m H_m^*$ draws the energy outside the structure.

Finally, the xOz sections for the total Poynting vector and three its non-zero terms are presented in Fig. 26. It can be seen, that the term $E_m H_i^*$ provide much smaller energy flux compare to other two components. Also, for this structure there is almost no energy funneling in the x direction.

Oppositely, for the structure with 35 nm metal thickness, there is some energy flux in the x direction. The geometric parameters of the structure $p = 2.8\text{ }\mu\text{m}$, $l_1 = 2.255\text{ }\mu\text{m}$, $l_2 = 226\text{ nm}$, $t = 35\text{ nm}$, $h = 1200\text{ nm}$. The total Poynting vector and its non-zero components are presented in Fig. 27. The funneling in this direction is provided by the $E_m H_m^*$ term, and also the $E_m H_i^*$ in the top part of the structure. But, similar to the 1D MIM, the last also lead to leak of some energy outside the structure, so does not make the significant impact in total. In the same time, according to computations, the energy funneling in the y direction is provided in the same way as it was done for the 5 nm -metal structure.

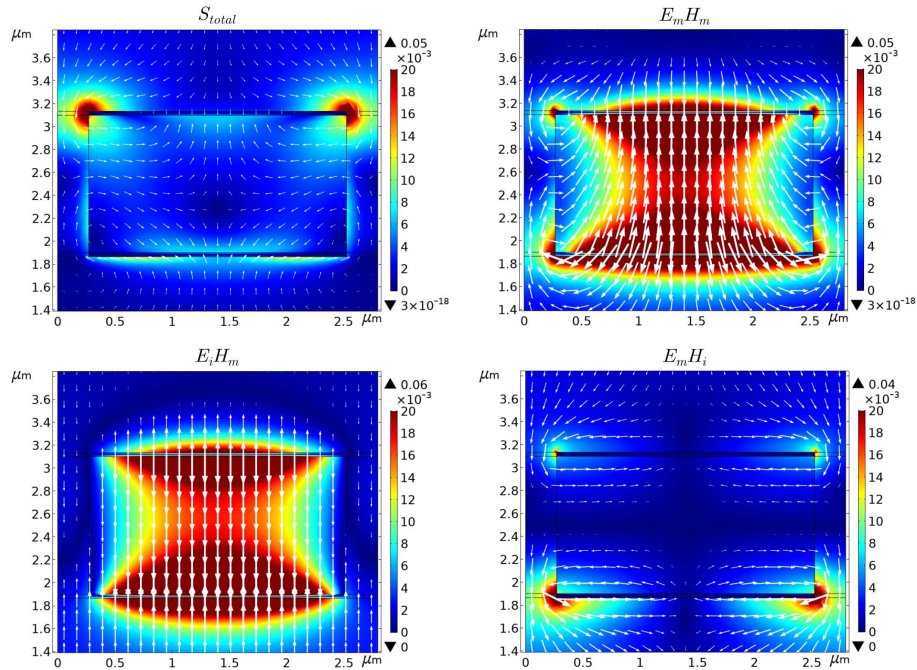


Figure 27: The time-average total Poynting vector S_{tot} and three its non-zero components $E_m H_m^*$, $E_i H_m^*$, and $E_m H_i^*$ in xOz plane at the middle ($l_2/2$) of the 35 nm -metal structure. Color maps shows the norm of the amplitude, and white arrows – the direction (also the amplitude in the logarithmic scale). l_1 is along x and l_2 is along y .

H.

The critical coupling for IMI structures

H.1 Coupling with the incident wave

Here, the coupling with the incident wave is considered in the similar way, as it was presented in Section 6.3.2 of Chapter 6. Instead of the mirror, the reflection of the system above the mirror is presented in the model. It allows imitating the system above the mirror, exited by the standing wave, and separating the field of the standing wave and the scatted field (COMSOL). Then, the Poynting vector, as it was described in Eq. 6.3, can be presented as a sum of four component, corresponding to the Poynting vector of the mode ($\mathbf{E}_m \times \mathbf{H}_m^*$), the Poynting vector of the standing wave ($\mathbf{E}_i \times \mathbf{H}_i^* = 0$), and two mixed components ($\mathbf{E}_i \times \mathbf{H}_m^*$ and $\mathbf{E}_m \times \mathbf{H}_i^*$). In Fig. 28 some of components of the total Poynting vector $\mathbf{S}_{total}$ are presented in different planes. Each plane is the cut by the corresponding plane at the middle of the metal bar: xy at $t/2$, yz at $l/2$, and xz at $w/2$.

Meanwhile, a small asymmetry in the absorption by the top and bottom xy metal interfaces connected with $\mathbf{E}_m \times \mathbf{H}_i^*$ component of the Poynting vector (Fig. 29). Although its amplitude is a few times smaller than one for other components, it can provide an observed few percent difference.

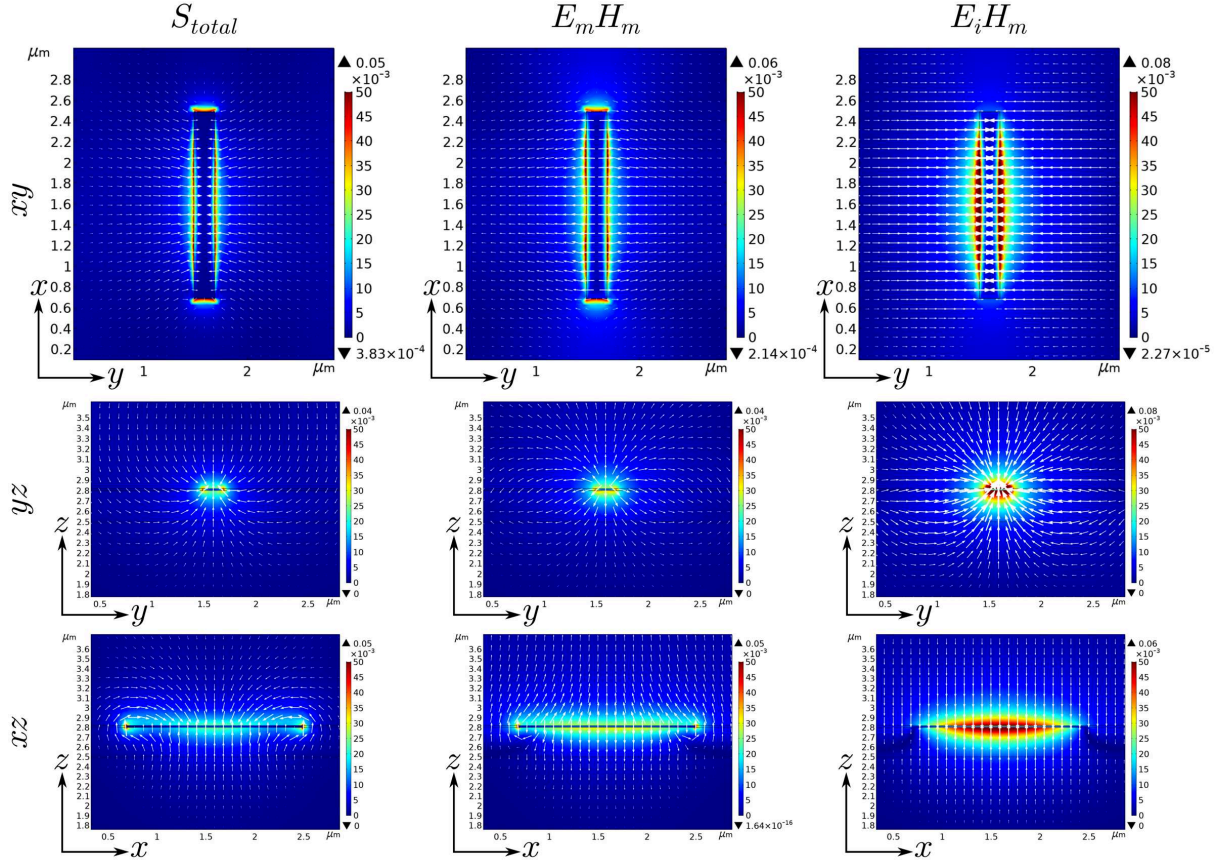


Figure 28: The time-average total Poynting vector and its terms $\mathbf{E}_m \times \mathbf{H}_m^*$ and $\mathbf{E}_i \times \mathbf{H}_m^*$ for the critical coupled the IMI array. The geometrical parameters of the array: $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, and $t = 20 \text{ nm}$. The color of the map present the absolute value of the amplitude of the corresponding Poynting vectors. Arrows maps present its direction, while their lengths are proportional to the corresponding Poynting vector amplitude (in the logarithmic scale). For the xz and yz maps the arrow scale is the same, and for the xy plane the same arrow length correspond to the 25 % smaller amplitude compare to two other planes.

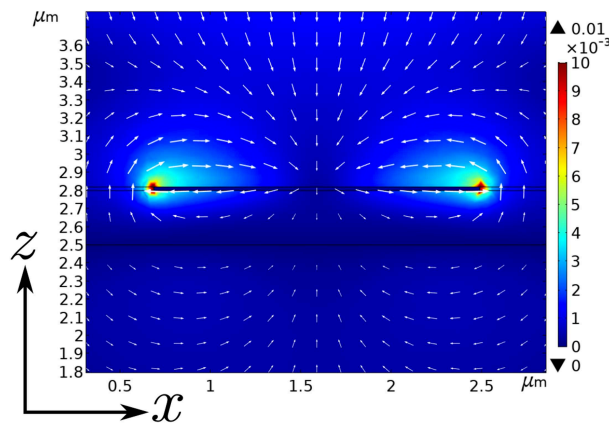


Figure 29: The map of $\mathbf{E}_m \times \mathbf{H}_i^*$ term of the Poynting vector. The geometrical parameters of the structure: $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, and $t = 20 \text{ nm}$. The color of the map present the absolute value of the vector amplitude, and arrows present the vector direction, while their lengths are proportional to the vector amplitude (in the logarithmic scale). For presented arrow map the same length as in the yz map of Fig. 28 corresponds to the five times smaller amplitude.

H.2 The fitting of the electric current density in IMI structures

Similar to Section G.8 for one metal bar of 2D MIM, the electric current density, exited in the metal bar of a IMI structure can be written as $j(y, z) = a(\exp(-y/\delta) + \exp(y/\delta)) + b(\exp(-z/\delta) + \exp(z/\delta))$ (the electric field is along x). The fitting procedure is the same, and it is also performed for the medium plane in respect to the metal bar length.

Fitting parameters a , b were found for the normalized electric current densities, which were extracted from numerical simulations in Comsol Multiphysics at the critical coupling (the resonant wavelength is $\lambda_{res} = 10 \mu m$). Parameters of structures together with a , b , and the maximum electric current density j_{max} are presented in Table 17.

Table 17: Fitting parameters for the normalized electric current density in metal bars of IMI structures on the 300 nm silicon membrane, and normalized values of the maximums of the electric current density j_{max} and I_{cr}^2

$p, \mu m$	$l, \mu m$	w, nm	t, nm	a	b	j_{max}	I_{cr}^2
2.8	1.77	150	20	0.0180	0.3205	0.952	0.72
3.18	1.8	200	20	0.007964	0.3078	0.965	0.90
3.18	1.878	100	40	0.060	0.2333	0.965	0.88
3.2	2.02	50	80	0.19078	0.0960	0.947	0.85
3.4	1.839	250	20	0.003343	0.2807	1.0	1.0

An example of the fitting of $j(y, z)$ for one of IMI structures considered in the main text is presented in Fig. 30. The procedure of computation of I_{cr}^2 is the same as in Section G.8:

$$I_{cr}^2 = j_{max}^2 \iint_{y=-l_2/2, z=-t/2}^{y=l_2/2, z=t/2} j^2(y, z) dy dz.$$

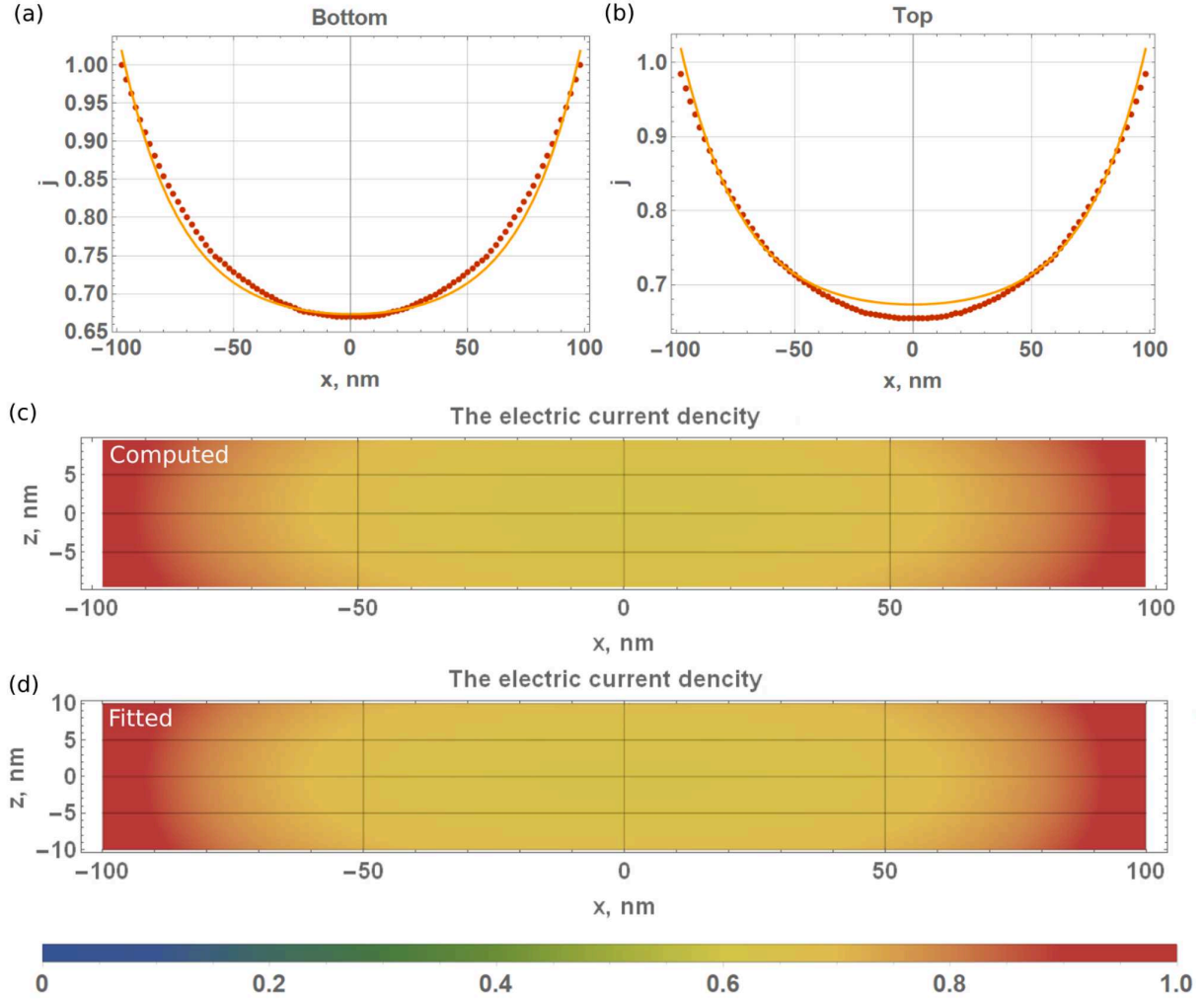


Figure 30: The distribution of the electric current density in the metal part of the IMI structure above the mirror. The geometrical parameters of the structure: $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, and $t = 20 \text{ nm}$. The silicon membrane thickness is 300 nm , the distance between the perfect mirror and the bottom of the membrane is $2.5 \mu\text{m}$. (a) Extracted (points) and fitted (solid line) normalized distribution of the electric current density in the middle ($l/2$) cut plane at the bottom of the metal ($z = -t/2$). (b) Extracted (points) and fitted (solid line) normalized distribution of the electric current density in the middle ($l/2$) cut plane at the top of the metal ($z = t/2$). (c) The normalized electric current density in the middle xOz plane ($l/2$), extracted from numerical simulations. (d) The fitted map of the normalized electric current density for the same plane.

H.3 Angular dependence IMI

Two additional angular dependencies, which are not presented in the main text, are shown in Fig. 31. The incidence planes are the same, but polarisations are different compare to Fig. 7.10.

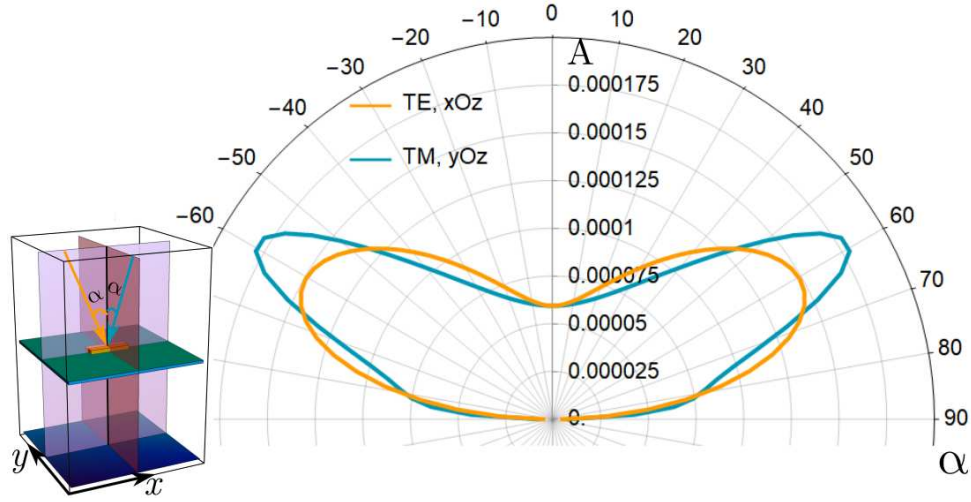


Figure 31: The dependence of absorption of IMI structure on the incidence angle for two polarisation and incident planes. The polarisation and the incidence plane for the angular dependence are indicate in the legend. Parameters of the structure $p = 3.18 \mu\text{m}$, $l = 1.8 \mu\text{m}$, $w = 200 \text{ nm}$, $t = 20 \text{ nm}$, $h = 300 \text{ nm}$. The distance to the mirror is $2.5 \mu\text{m}$.

H.4 The volume density of IMI structures

Table 18: The volume density values for IMI structures at the critical coupling at $\lambda_{\text{res}} = 10 \mu\text{m}$ placed at $\lambda_{\text{res}}/4$ above teh perfect mirror

$p, \mu\text{m}$	$l, \mu\text{m}$	w, nm	t, nm	ω_V, nm
2.8	1.77	150	20	0.68
3.18	1.8	200	20	0.71
3.18	1.878	100	40	0.74
3.2	2.02	50	80	0.79
3.4	1.839	250	20	0.80

Bibliography

- [1] Alesandro De Angelis and Catarina Espirito Santo. The contribution of Giordano Bruno to the principle of relativity. *J. Astro. Hist. Herit.*, pages 241 – 248, 2015.
- [2] James Clerk Maxwell. A dynamical theory of the electromagnetic field. *Philosophical transactions of the Royal Society of London*, 155:459–512, 1865.
- [3] John David Jackson. *Classical electrodynamics*, 3rd ed. John Wiley & Sons, 1999.
- [4] Hermann Minkowski. Die grundgleichungen für die elektromagnetischen vorgänge in bewegten körpern. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse*, pages 53–111, 1908.
- [5] Arthur I. Miller. The physics of Einsteins relativity paper of 1905 and the electromagnetic world picture of 1905. *Am. J. Phys.*, 45(11):1040–1048, 1977.
- [6] Henri Poincaré. Sur la dynamique de l’électron. *Rend. Circ. Matem. Palermo*, 21:129–175, 1906.
- [7] Albert Einstein. Zur elektrodynamik bewegter körpe. *Ann. Phys.*, 322:891–921, 1905.
- [8] Hermann Minkowski. Raum und zeit. *Phys. Z.*, 10:104–111, 1909.
- [9] Vesselin Petkov, editor. *Minkowski Spacetime: A Hundred Years Later*, volume 165 of *Fundamental Theories of Physics*. Springer Science & Business Media, 2009.
- [10] H Minkowski. Space and time: Minkowskis papers on relativity, translated by f lewertoff and v petkov, edited by v petkov, 2012.
- [11] Olivia Levrini. The substantialist view of spacetime proposed by Minkowski and its educational implications. *Science & Education*, 11:601–617, 2002.
- [12] Peter Louis Galison. Minkowski’s space-time: From visual thinking to the absolute world. *Historical Studies in the Physical Sciences*, 10:85–121, 1979.
- [13] Edwin F. Taylor and John Archibald Wheeler. *Spacetime Physics, Second Edition*. W. H. Freeman and Co, New York, 1992.
- [14] Kip S. Thorne. *Black Holes and Time Warps*. W. W. Norton & Company, 1994.
- [15] Leonard Susskind. *Special Relativity and Classical Field Theory*. Harmondsworth, 2017.
- [16] Hermann Weyl. *Raum, Zeit, Materie*. Springer, Berlin, 1919.
- [17] Charles W. Misner, Kip S. Thorne, and John A. Wheeler. *Gravitation*. W. H. Freeman, Princeton University Press, 1973.
- [18] Fabrice Pardo. L’électrodynamique sous le fil du rasoir d’Occam, séminaire général du département de physique de l’École Polytechnique. URL: <https://youtu.be/PAnEON6PtI8>, 2016. [Online; accessed 1-September-2021].
- [19] James Clerk Maxwell. On physical lines of force. *Philos. Mag.*, 25:161, 1861.
- [20] Richard P Feynman, Robert B Leighton, and Matthew Sands. *The Feynman Lectures on Physics: Mainly Electromagnetism and Matter, Vol. 2*. Addison-Wesley, Reading.

- reprinted, 1977.
- [21] F.J. Dyson. Why is Maxwell's theory so hard to understand? In *The Second European Conference on Antennas and Propagation, EuCAP 2007*, pages 1–6, 11 2007.
 - [22] Georges A Deschamps. Electromagnetics and differential forms. *Proceedings of the IEEE*, 69(6):676–696, 1981.
 - [23] Karl F Warnick and Peter H Russer. Differential forms and electromagnetic field theory. *Progress In Electromagnetics Research*, 148:83–112, 2014.
 - [24] Jonathan Gratus. A pictorial introduction to differential geometry, leading to maxwell's equations as three pictures. *arXiv preprint arXiv:1709.08492*, 2017.
 - [25] C. N. Yang and R. L. Mills. Conservation of isotopic spin and isotopic gauge invariance. *Phys. Rev.*, 96:191–195, Oct 1954.
 - [26] BIPM. *SI Brochure, 9th edition*. BIPM, 2019.
 - [27] James Clerk Maxwell. *A treatise on electricity and magnetism*, volume 1. Oxford: Clarendon Press, 1873.
 - [28] Bruce J Hunt. Oliver heaviside: A first-rate oddity. *Phys. Today*, 65(11):48–54, 2012.
 - [29] Oliver Heaviside. *Electromagnetic theory. 1 (1893)*. Electrician Print. and Publishing Company, 1893.
 - [30] Lifshitz E.M. Landau L.D. *The Classical Theory of Fields : Volume 2*, volume 4th Edition, revised. Elsevier Science & Technology, 1995.
 - [31] Elie Joseph Cartan. *On manifolds with an affine connection and the theory of general relativity*. Bibliopolis, 1986.
 - [32] Thomas Weiland. A discretization model for the solution of maxwell's equations for six-component fields. *Archiv Elektronik und Uebertragungstechnik*, 31:116–120, 1977.
 - [33] Markus Clemens and Thomas Weiland. Discrete electromagnetism with the finite integration technique. *Progress In Electromagnetics Research*, 32:65–87, 2001.
 - [34] Enzo Tonti. Finite formulation of the electromagnetic field. *Progress in electromagnetics research*, 32:1–44, 2001.
 - [35] Benjamin Portier, Fabrice Pardo, Patrick Bouchon, Riad Haïdar, and Jean-Luc Pelouard. Fast modal method for crossed grating computation, combining finite formulation of maxwell equations with polynomial approximated constitutive relations. *JOSA A*, 30(4):573–581, 2013.
 - [36] Paul N Stavrinou and László Solymár. The propagation of electromagnetic power through subwavelength slits in a metallic grating. *Optics communications*, 206(4-6):217–223, 2002.
 - [37] Fabrice Pardo, Patrick Bouchon, Riad Haïdar, and Jean-Luc Pelouard. Light funneling mechanism explained by magnetoelectric interference. *Physical review letters*, 107(9):093902, 2011.
 - [38] Marcel Riesz. *Clifford Numbers and Spinors, edited by E. Folke Bolinder and Pertti Lounesto*. Springer Science + Business Media Dordrecht, 1993.
 - [39] Pertti Lounesto. *Clifford algebras and spinors*, volume 286. Cambridge university press, 2001.
 - [40] Ismo V Lindell. *Differential forms in electromagnetics*, volume 22. John Wiley & Sons, 2004.
 - [41] Harley Flanders. *Differential Forms with Applications to the Physical Sciences by Harley Flanders*. Elsevier, 1963.
 - [42] David van Dantzig. The fundamental equations of electromagnetism, independent of metrical geometry. *Mathematical Proceedings of the Cambridge Philosophical Society*, 30(4):421–427, 1934.
 - [43] John Archibald Wheeler. *Geometrodynamics*. Academic Press, 1962.

-
- [44] Mark Buchanan. Geometric intuition. *Nature Physics*, 7(6):442–442, 2011.
 - [45] David Hestenes. The genesis of geometric algebra: a personal retrospective. *Advances in Applied Clifford Algebras*, 27(1):351–379, 2017.
 - [46] Ismo V Lindell. *Multiforms, Dyadics, and Electromagnetic Media*. John Wiley & Sons, 2015.
 - [47] Friedrich W Hehl and Yuri N Obukhov. *Foundations of classical electrodynamics: Charge, flux, and metric*, volume 33. Springer Science & Business Media, 2012.
 - [48] John W Arthur. *Understanding geometric algebra for electromagnetic theory*, volume 38. John Wiley & Sons, 2011.
 - [49] Chris Doran, Anthony Lasenby, and Joan Lasenby. *Geometric algebra for physicists*. Cambridge University Press, 2003.
 - [50] James M Chappell, Samuel P Drake, Cameron L Seidel, Lachlan J Gunn, Azhar Iqbal, Andrew Allison, and Derek Abbott. Geometric algebra for electrical and electronic engineers. *Proceedings of the IEEE*, 102(9):1340–1363, 2014.
 - [51] Karl F Warnick and Peter Russer. Two, three and four-dimensional electromagnetics using differential forms. *Turkish Journal Of Electrical Engineering & Computer Sciences*, 14(1):153–172, 2006.
 - [52] A Sommerfeld. *Elektrodynamik. Vorlesungen über Theoretische Physik, Band 3. Dieterich’sche Verlagsbuchhandlung, Wiesbaden (1948). English translation: Sommerfeld, A., Electrodynamics, vol. 3 of Lectures in Theoretical Physics*. Academic Press, New York, 1952.
 - [53] Giorgio Ferrarese and Donato Bini. *Introduction to relativistic continuum mechanics*, volume 727. Springer, 2007.
 - [54] Luigi Stazi. On the canonical form of the electromagnetic field. *Annali dell’Universita’ di Ferrara*, 52(1):127–135, 2006.
 - [55] Rufus H Ritchie. Plasma losses by fast electrons in thin films. *Physical review*, 106(5):874, 1957.
 - [56] JA Dionne, LA Sweatlock, HA Atwater, and A Polman. Planar metal plasmon waveguides: frequency-dependent dispersion, propagation, localization, and loss beyond the free electron model. *Physical Review B*, 72(7):075405, 2005.
 - [57] Masanobu Iwanaga. *Plasmonic Resonators: Fundamentals, Advances, and Applications*. CRC Press, 2016.
 - [58] Stefan Alexander Maier. *Plasmonics: fundamentals and applications*. Springer Science & Business Media, 2007.
 - [59] Jeremy J Baumberg, Javier Aizpurua, Maiken H Mikkelsen, and David R Smith. Extreme nanophotonics from ultrathin metallic gaps. *Nature materials*, 18(7):668–678, 2019.
 - [60] Kuiru Li, Mark I Stockman, and David J Bergman. Self-similar chain of metal nanospheres as an efficient nanolens. *Physical review letters*, 91(22):227402, 2003.
 - [61] Jon A Schuller, Edward S Barnard, Wenshan Cai, Young Chul Jun, Justin S White, and Mark L Brongersma. Plasmonics for extreme light concentration and manipulation. *Nature materials*, 9(3):193–204, 2010.
 - [62] Alexander Cuadrado, Javier Alda, and Francisco Javier González. Multiphysics simulation for the optimization of optical nanoantennas working as distributed bolometers in the infrared. *Journal of Nanophotonics*, 7(1):073093, 2013.
 - [63] Shinpei Ogawa and Masafumi Kimata. Metal-insulator-metal-based plasmonic metamaterial absorbers at visible and infrared wavelengths: a review. *Materials*, 11(3):458, 2018.
 - [64] Na Liu, Martin Mesch, Thomas Weiss, Mario Hentschel, and Harald Giessen. Infrared perfect absorber and its application as plasmonic sensor. *Nano letters*, 10(7):2342–2348, 2010.

- [65] Xingjie Ni, Zi Jing Wong, Michael Mrejen, Yuan Wang, and Xiang Zhang. An ultrathin invisibility skin cloak for visible light. *Science*, 349(6254):1310–1314, 2015.
- [66] Sander A Mann and Erik C Garnett. Resonant nanophotonic spectrum splitting for ultrathin multijunction solar cells. *Acs Photonics*, 2(7):816–821, 2015.
- [67] Jeffrey N Anker, W Paige Hall, Olga Lyandres, Nilam C Shah, Jing Zhao, and Richard P Van Duyne. Biosensing with plasmonic nanosensors. In *Nanoscience and Technology: A Collection of Reviews from Nature Journals*, pages 308–319. World Scientific, 2010.
- [68] Gang Lu, Herlinde De Keersmaecker, Liang Su, Bart Kenens, Susana Rocha, Eduard Fron, Chang Chen, Pol Van Dorpe, Hideaki Mizuno, Johan Hofkens, et al. Live-cell sers endoscopy using plasmonic nanowire waveguides. *Advanced Materials*, 26(30):5124–5128, 2014.
- [69] Barbora Špačková, Piotr Wrobel, Markéta Bocková, and Jiří Homola. Optical biosensors based on plasmonic nanostructures: a review. *Proceedings of the IEEE*, 104(12):2380–2408, 2016.
- [70] Martti Kauranen and Anatoly V Zayats. Nonlinear plasmonics. *Nature photonics*, 6(11):737–748, 2012.
- [71] Mark W Knight, Heidar Sobhani, Peter Nordlander, and Naomi J Halas. Photodetection with active optical antennas. *Science*, 332(6030):702–704, 2011.
- [72] Hamidreza Chalabi, David Schoen, and Mark L Brongersma. Hot-electron photodetection with a plasmonic nanostripe antenna. *Nano letters*, 14(3):1374–1380, 2014.
- [73] Thomas Siegfried, Yasin Ekinci, Olivier JF Martin, and Hans Sigg. Gap plasmons and near-field enhancement in closely packed sub-10 nm gap resonators. *Nano letters*, 13(11):5449–5453, 2013.
- [74] Zhi Wei Seh, Shuhua Liu, Michelle Low, Shuang-Yuan Zhang, Zhaolin Liu, Adnen Mlayah, and Ming-Yong Han. Janus au-tio₂ photocatalysts with strong localization of plasmonic near-fields for efficient visible-light hydrogen generation. *Advanced materials*, 24(17):2310–2314, 2012.
- [75] Syed Mubeen, Joun Lee, Nirala Singh, Stephan Krämer, Galen D Stucky, and Martin Moskovits. An autonomous photosynthetic device in which all charge carriers derive from surface plasmons. *Nature nanotechnology*, 8(4):247–251, 2013.
- [76] César Clavero. Plasmon-induced hot-electron generation at nanoparticle/metal-oxide interfaces for photovoltaic and photocatalytic devices. *Nature Photonics*, 8(2):95–103, 2014.
- [77] Yu Hui, Juan Sebastian Gomez-Diaz, Zhenyun Qian, Andrea Alu, and Matteo Rinaldi. Plasmonic piezoelectric nanomechanical resonator for spectrally selective infrared sensing. *Nature communications*, 7(1):1–9, 2016.
- [78] HT Miyazaki, T Kasaya, M Iwanaga, B Choi, Y Sugimoto, and K Sakoda. Dual-band infrared metasurface thermal emitter for co₂ sensing. *Applied Physics Letters*, 105(12):121107, 2014.
- [79] Jianji Yang, Christophe Sauvan, Anthony Jouanin, Stéphane Collin, J-L Pelouard, and Philippe Lalanne. Ultrasmall metal-insulator-metal nanoresonators: impact of slow-wave effects on the quality factor. *Optics Express*, 20(15):16880–16891, 2012.
- [80] A Rogalski. Next decade in infrared detectors. In *Electro-Optical and Infrared Systems: Technology and Applications XIV*, volume 10433, page 104330L. International Society for Optics and Photonics, 2017.
- [81] Antoni Rogalski. *Infrared and Terahertz Detectors*. CRC Press, 2019.
- [82] Heinz Raether. Surface plasmons on smooth surfaces. In *Surface plasmons on smooth and rough surfaces and on gratings*, pages 4–39. Springer, 1988.
- [83] Joel Henzie, Jeunghoon Lee, Min Hyung Lee, Warefta Hasan, and Teri W Odom. Nanofabrication of plasmonic structures. *Annual review of physical chemistry*, 60, 2009.

-
- [84] Yujun Zhong, Shyamala Devi Malagari, Travis Hamilton, and Daniel M Wasserman. Review of mid-infrared plasmonic materials. *Journal of Nanophotonics*, 9(1):093791, 2015.
 - [85] Kostya S Novoselov, Andre K Geim, Sergei V Morozov, D Jiang, Y Zhang, Sergey V Dubonos, Irina V Grigorieva, and Alexandr A Firsov. Electric field effect in atomically thin carbon films. *science*, 306(5696):666–669, 2004.
 - [86] Kostya S Novoselov, Andre K Geim, Sergei Vladimirovich Morozov, Da Jiang, Michail I Katsnelson, IVa Grigorieva, SVb Dubonos, and AA Firsov. Two-dimensional gas of massless dirac fermions in graphene. *nature*, 438(7065):197–200, 2005.
 - [87] Jessica R Piper and Shanhui Fan. Total absorption in a graphene monolayer in the optical regime by critical coupling with a photonic crystal guided resonance. *Acs Photonics*, 1(4):347–353, 2014.
 - [88] JJ Mock, M Barbic, DR Smith, DA Schultz, and Sh Schultz. Shape effects in plasmon resonance of individual colloidal silver nanoparticles. *The Journal of Chemical Physics*, 116(15):6755–6759, 2002.
 - [89] Patrick Bouchon, Charlie Koechlin, Fabrice Pardo, Riad Haïdar, and Jean-Luc Pelouard. Wideband omnidirectional infrared absorber with a patchwork of plasmonic nanoantennas. *Optics letters*, 37(6):1038–1040, 2012.
 - [90] Erdem Aslan, Ekin Aslan, Mustafa Turkmen, and Omer Galip Saracoglu. Experimental and numerical characterization of a mid-infrared plasmonic perfect absorber for dual-band enhanced vibrational spectroscopy. *Optical Materials*, 73:213–222, 2017.
 - [91] Bingxin Zhang, Yanhui Zhao, Qingzhen Hao, Brian Kiraly, Iam-Choon Khoo, Shufen Chen, and Tony Jun Huang. Polarization-independent dual-band infrared perfect absorber based on a metal-dielectric-metal elliptical nanodisk array. *Optics express*, 19(16):15221–15228, 2011.
 - [92] Antoine Moreau, Cristian Ciraci, Jack J Mock, Ryan T Hill, Qiang Wang, Benjamin J Wiley, Ashutosh Chilkoti, and David R Smith. Controlled-reflectance surfaces with film-coupled colloidal nanoantennas. *Nature*, 492(7427):86–89, 2012.
 - [93] Stéphane Collin, Fabrice Pardo, and Jean-Luc Pelouard. Waveguiding in nanoscale metallic apertures. *Optics Express*, 15(7):4310–4320, 2007.
 - [94] Ludivine Emeric, Claire Deeb, Fabrice Pardo, and Jean-Luc Pelouard. Critical coupling and extreme confinement in nanogap antennas. *Optics Letters*, 44(19):4761–4764, 2019.
 - [95] Pierre Berini. Plasmon-polariton waves guided by thin lossy metal films of finite width: Bound modes of symmetric structures. *Physical Review B*, 61(15):10484, 2000.
 - [96] Yurui Fang, Zhipeng Li, Yingzhou Huang, Shunping Zhang, Peter Nordlander, Naomi J Halas, and Hongxing Xu. Branched silver nanowires as controllable plasmon routers. *Nano letters*, 10(5):1950–1954, 2010.
 - [97] Hai Zhu, Fei Yi, and Ertugrul Cubukcu. Nanoantenna absorbers for thermal detectors. *IEEE Photonics Technology Letters*, 24(14):1194–1196, 2012.
 - [98] Alexandra Boltasseva, Sergey I Bozhevolnyi, Thomas Nikolajsen, and Kristjan Leosson. Compact bragg gratings for long-range surface plasmon polaritons. *Journal of Lightwave Technology*, 24(2):912, 2006.
 - [99] Jacek Gosciniak, Tobias Holmgaard, and Sergey I Bozhevolnyi. Theoretical analysis of long-range dielectric-loaded surface plasmon polariton waveguides. *Journal of Lightwave Technology*, 29(10):1473–1481, 2011.
 - [100] Rinu Abraham Maniyara, Daniel Rodrigo, Renwen Yu, Josep Canet-Ferrer, Dhriti Sundar Ghosh, Ruchirej Yongsunthon, David E Baker, Aram Rezikyan, F Javier García de Abajo, and Valerio Pruneri. Tunable plasmons in ultrathin metal films. *Nature Photonics*, 13(5):328–333, 2019.
 - [101] Arif E Cetin, Sabri Kaya, Alket Mertiri, Ekin Aslan, Shyamsunder Erramilli, Hatice Altug, and Mustafa Turkmen. Dual-band plasmonic resonator based on jerusalem cross-shaped

- nanoapertures. *Photonics and Nanostructures-Fundamentals and Applications*, 15:73–80, 2015.
- [102] Gemini Observatory. Ir transmission spectra. <https://www.gemini.edu/?q=node/10789>, 2021. [Online; accessed 06-July-2021].
- [103] Charles Greeley Abbot. Samuel pierpont langley. *The Astrophysical Journal*, 23:271, 1906.
- [104] PL Richards. Bolometers for infrared and millimeter waves. *Journal of Applied Physics*, 76(1):1–24, 1994.
- [105] Robert E Flannery and James E Miller. Status of uncooled infrared imagers. In *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing III*, volume 1689, pages 379–395. International Society for Optics and Photonics, 1992.
- [106] RA Wood, CJ Han, and PW Kruse. Integrated uncooled infrared detector imaging arrays. In *Technical Digest IEEE Solid-State Sensor and Actuator Workshop*, pages 132–135. IEEE, 1992.
- [107] Michael A Kinch. *State-of-the-art infrared detector technology*. SPIE, 2014.
- [108] Aapo Varpula, Kirsi Tappura, Jonna Tiira, Kestutis Grigorasi, Kai Viherkanto, Jouni Ahopelto, and Mika Prunnila. High-performance infrared thermoelectric bolometers based on nanomembranes. In *Photonic and Phononic Properties of Engineered Nanostructures X*, volume 11289, page 112891O. International Society for Optics and Photonics, 2020.
- [109] M Abdel-Rahman, N Al-Khali, MF Zia, M Alduraibi, B Ilahi, E Awad, and N Debbar. Fabrication and design of vanadium oxide microbolometer. *AIP Conference Proceedings*, 1809(1):020001, 2017.
- [110] Alexander B Christiansen, Gideon P Caringal, Jeppe S Clausen, Meir Grajower, Hesham Taha, Uriel Levy, N Asger Mortensen, and Anders Kristensen. Black metal thin films by deposition on dielectric antireflective moth-eye nanostructures. *Scientific reports*, 5(1):1–9, 2015.
- [111] Jon W Stewart, Nathaniel C Wilson, and Maiken H Mikkelsen. Nanophotonic engineering: A new paradigm for spectrally sensitive thermal photodetectors. *ACS Photonics*, 2020.
- [112] Elham Mohammadi and Nader Behdad. A wide dynamic range polarization sensing long wave infrared detector. *Scientific reports*, 7(1):1–9, 2017.
- [113] Thang Duy Dao, Anh Tung Doan, Satoshi Ishii, Takahiro Yokoyama, Handegård Sele Ørjan, Dang Hai Ngo, Tomoko Ohki, Akihiko Ohi, Yoshiki Wada, Chisato Niikura, et al. Mems-based wavelength-selective bolometers. *Micromachines*, 10(6):416, 2019.
- [114] Omar Alkorjia, Amjed Abdullah, Akshay Koppula, Cameron Warder, Tao Liu, Chuang Qu, Chen Zhu, Edward Kinzel, and Mahmoud Almasri. Metasurface based uncooled microbolometer with high fill factor. In *2019 20th International Conference on Solid-State Sensors, Actuators and Microsystems & Eurosensors XXXIII (Transducers & Eurosensors XXXIII)*, pages 2126–2129. IEEE, 2019.
- [115] Edgar Briones, Riemann Ruiz-Cruz, Joel Briones, Natalia Gonzalez, Jorge Simon, Mayela Arreola, and Gregorio Alvarez-Alvarez. Particle swarm optimization of nanoantenna-based infrared detectors. *Optics express*, 26(22):28484–28496, 2018.
- [116] Ben A Munk. *Frequency selective surfaces: theory and design*. John Wiley & Sons, 2005.
- [117] George Duffett, Ralph Wirth, Mathieu Rayer, Emiliano R Martins, and Thomas F Krauss. Metal-insulator-metal nanoresonators—strongly confined modes for high surface sensitivity. *Nanophotonics*, 1(ahead-of-print), 2020.
- [118] Ricky Gibson, Ivan Avrutsky, Shivashankar Vangala, Dennis E Walker, and Joshua R Hendrickson. Pole-based analysis of coupled modes in metal–insulator–metal plasmonic structures. *JOSA B*, 38(3):776–782, 2021.
- [119] Jianji Yang, Christophe Sauvan, Anthony Jouanin, Stéphane Collin, J-L Pelouard, and Philippe Lalanne. Ultrasmall metal-insulator-metal nanoresonators: impact of slow-wave effects on the quality factor. *Optics Express*, 20(15):16880–16891, 2012.

-
- [120] Na Liu and Harald Giessen. Coupling effects in optical metamaterials. *Angewandte Chemie International Edition*, 49(51):9838–9852, 2010.
 - [121] G Dolling, C Enkrich, M Wegener, JF Zhou, Costas M Soukoulis, and S Linden. Cut-wire pairs and plate pairs as magnetic atoms for optical metamaterials. *Optics letters*, 30(23):3198–3200, 2005.
 - [122] Seied Ali Safiabadi Tali and Wei Zhou. Multiresonant plasmonics with spatial mode overlap: overview and outlook. *Nanophotonics*, 8(7):1199–1225, 2019.
 - [123] Hideki T Miyazaki and Yoichi Kurokawa. Squeezing visible light waves into a 3-nm-thick and 55-nm-long plasmon cavity. *Physical review letters*, 96(9):097401, 2006.
 - [124] Martin Kuttge, F Javier Garcia de Abajo, and Albert Polman. Ultrasmall mode volume plasmonic nanodisk resonators. *Nano letters*, 10(5):1537–1541, 2010.
 - [125] Nuttawut Kongsuwan, Angela Demetriadou, Rohit Chikkaraddy, Felix Benz, Vladimir A Turek, Ulrich F Keyser, Jeremy J Baumberg, and Ortwin Hess. Suppressed quenching and strong-coupling of purcell-enhanced single-molecule emission in plasmonic nanocavities. *Acs Photonics*, 5(1):186–191, 2018.
 - [126] Jiaming Hao, Jing Wang, Xianliang Liu, Willie J Padilla, Lei Zhou, and Min Qiu. High performance optical absorber based on a plasmonic metamaterial. *Applied Physics Letters*, 96(25):251104, 2010.
 - [127] Chihhui Wu, Burton Neuner III, Gennady Shvets, Jeremy John, Andrew Milder, Byron Zollars, and Steve Savoy. Large-area wide-angle spectrally selective plasmonic absorber. *Physical Review B*, 84(7):075102, 2011.
 - [128] Charlie Koechlin, Patrick Bouchon, Fabrice Pardo, Julien Jaeck, Xavier Lafosse, Jean-Luc Pelouard, and Riad Haïdar. Total routing and absorption of photons in dual color plasmonic antennas. *Applied Physics Letters*, 99(24):241104, 2011.
 - [129] Thomas Maier and Hubert Brueckl. Multispectral microbolometers for the midinfrared. *Optics letters*, 35(22):3766–3768, 2010.
 - [130] Shinpei Ogawa and Masafumi Kimata. Wavelength-or polarization-selective thermal infrared detectors for multi-color or polarimetric imaging using plasmonics and metamaterials. *Materials*, 10(5):493, 2017.
 - [131] Andrea Cattoni, Petru Ghenuche, Anne-Marie Haghiri-Gosnet, Dominique Decanini, Jing Chen, Jean-Luc Pelouard, and Stéphane Collin. $\lambda/1000$ plasmonic nanocavities for biosensing fabricated by soft uv nanoimprint lithography. *Nano letters*, 11(9):3557–3563, 2011.
 - [132] Zhenyun Qian, Sungho Kang, Vageeswar Rajaram, Cristian Cassella, Nicol E McGruer, and Matteo Rinaldi. Zero-power infrared digitizers based on plasmonically enhanced micro-mechanical photoswitches. *Nature nanotechnology*, 12(10):969, 2017.
 - [133] Masashi Miyata, Hideaki Hatada, and Junichi Takahara. Full-color subwavelength printing with gap-plasmonic optical antennas. *Nano letters*, 16(5):3166–3172, 2016.
 - [134] Atsushi Ishikawa and Takuo Tanaka. Metamaterial absorbers for infrared detection of molecular self-assembled monolayers. *Scientific reports*, 5(1):1–7, 2015.
 - [135] Alexander Lochbaum, Yuriy Fedoryshyn, Alexander Dorodnyy, Ueli Koch, Christian Hafner, and Juerg Leuthold. On-chip narrowband thermal emitter for mid-ir optical gas sensing. *ACS photonics*, 4(6):1371–1380, 2017.
 - [136] Y. Ra’di, C. R. Simovski, and S. A. Tretyakov. Thin perfect absorbers for electromagnetic waves: Theory, design, and realizations. *Phys. Rev. Applied*, 3:037001, Mar 2015.
 - [137] Paul Chevalier, Patrick Bouchon, Julien Jaeck, Diane Lauwick, Nathalie Bardou, Alain Kattnig, Fabrice Pardo, and Riad Haïdar. Absorbing metasurface created by diffractionless disordered arrays of nanoantennas. *Applied Physics Letters*, 107(25):251108, 2015.
 - [138] Shaista Babar and JH Weaver. Optical constants of cu, ag, and au revisited. *Applied Optics*, 54(3):477–481, 2015.

- [139] Deane Chandler-Horowitz and Paul M Amirtharaj. High-accuracy, midinfrared ($450\text{ cm}^{-1} \leq \omega \leq 4000\text{ cm}^{-1}$) refractive index values of silicon. *Journal of Applied physics*, 97(12):123526, 2005.
- [140] Uwe Kreibig and Michael Vollmer. *Optical properties of metal clusters*, volume 25. Springer Science & Business Media, 2013.
- [141] Alexander Cuadrado, Francisco J González, Jordi Agustí, and Javier Alda. Material dependence of the distributed bolometric effect in resonant metallic nanostructures. In *Plasmonics: Metallic Nanostructures and Their Optical Properties X*, volume 8457, page 845724. International Society for Optics and Photonics, 2012.
- [142] Stavroula Foteinopoulou, Ganga Chinna Rao Devarapu, Ganapathi S Subramania, Sanjay Krishna, and Daniel Wasserman. Phonon-polaritons: enabling powerful capabilities for infrared photonics. *Nanophotonics*, 8(12):2129–2175, 2019.
- [143] Documentation for version Comsol 5.4. *RF Module Users Guide*. COMSOL, 2018.
- [144] Stéphane Collin. Nanostructure arrays in free-space: optical properties and applications. *Reports on Progress in Physics*, 77(12):126402, 2014.
- [145] Borislav Vasić and Goran Isić. Refractive index sensing with hollow metal–insulator–metal metasurfaces. *Journal of Physics D: Applied Physics*, 54(28):285106, 2021.
- [146] Paul Chevalier, Patrick Bouchon, Emilie Sakat, Jean-Luc Pelouard, Fabrice Pardo, and Riad Haïdar. Extraordinary transmission in optical helmholtz resonators. *Optics letters*, 40(12):2735–2738, 2015.
- [147] George W Hanson. On the applicability of the surface impedance integral equation for optical and near infrared copper dipole antennas. *IEEE Transactions on Antennas and Propagation*, 54(12):3677–3685, 2006.
- [148] ZY Dang, M Motapothula, YS Ow, T Venkatesan, MBH Breese, MA Rana, and A Osman. Fabrication of large-area ultra-thin single crystal silicon membranes. *Applied Physics Letters*, 99(22):223105, 2011.
- [149] Giovanni Brucoli, Patrick Bouchon, Riad Haïdar, Mondher Besbes, Henri Benisty, and Jean-Jacques Greffet. High efficiency quasi-monochromatic infrared emitter. *Applied Physics Letters*, 104(8):081101, 2014.
- [150] B. T. Draine and P. J. Flatau. Discrete-dipole approximation for scattering calculations. *J. Opt. Soc. Am. A*, 11(4):1491–1499, Apr 1994.
- [151] R. Alaei, M. Albooyeh, and C. Rockstuhl. Theory of metasurface based perfect absorbers. *Journal of Physics D: Applied Physics*, 50(50):503002, nov 2017.
- [152] C. Gasquet and P. Witomski. Fourier analysis and applications, 1999.
- [153] Pavel A. Belov and Constantin R. Simovski. Homogenization of electromagnetic crystals formed by uniaxial resonant scatterers. *Phys. Rev. E*, 72:026615, Aug 2005.
- [154] D. Pozar and D. Schaubert. Scan blindness in infinite phased arrays of printed dipoles. *IEEE Transactions on Antennas and Propagation*, 32(6):602–610, 1984.
- [155] Jan Kischkat, Sven Peters, Bernd Gruska, Mykhaylo Semtsiv, Mikaela Chashnikova, Matthias Klinkmüller, Oliana Fedosenko, Stephan Machulik, Anna Aleksandrova, Gregorii Monastyrskiy, et al. Mid-infrared optical properties of thin films of aluminum oxide, titanium dioxide, silicon dioxide, aluminum nitride, and silicon nitride. *Applied optics*, 51(28):6789–6798, 2012.
- [156] Robert L Olmon, Brian Slovick, Timothy W Johnson, David Shelton, Sang-Hyun Oh, Glenn D Boreman, and Markus B Raschke. Optical dielectric function of gold. *Physical Review B*, 86(23):235147, 2012.
- [157] Wenqi Zhang, Sywert H Brongersma, Olivier Richard, Bert Brijs, Roger Palmans, Ludo Froyen, and Karen Maex. Influence of the electron mean free path on the resistivity of thin metal films. *Microelectronic engineering*, 76(1-4):146–152, 2004.

- [158] Eduardo A Coronado and George C Schatz. Surface plasmon broadening for arbitrary shape nanoparticles: A geometrical probability approach. *The Journal of chemical physics*, 119(7):3926–3934, 2003.
- [159] David M Pozar. *Microwave engineering*. John wiley & sons, 2011.
- [160] Bureau international des poids et mesures. <http://www.bipm.org/>, 2021. [Online; accessed 07-July-2021].
- [161] Léo Morel, Zhibin Yao, Pierre Cladé, and Guellati-Khélifa Saïda. Determination of the fine-structure constant with an accuracy of 81 parts per trillion. *nature*, 588:61–65, 2020.
- [162] Friedrich Kottler. *Maxwell’sche Gleichungen und Metrik*. Österreichischer Staatsdr., 1922.
- [163] Paul Bamberg and Shlomo Sternberg. *A course in mathematics for students of physics*, volume 2. Cambridge University Press, 1991.
- [164] Walter Thirring. *Classical mathematical physics: dynamical systems and field theories*. Springer Science & Business Media, 2013.
- [165] Constantin Piron, Etienne Pittet, and Université de Genève. Département de physique théorique. *Electrodynamique et optique: partie I*. Université de Genève Section de physique DPT, 1975.
- [166] Alain Bossavit. Differential geometry for the student of numerical methods in electromagnetism. *Electricite de France, Etudes et Recherchers*, 1991.
- [167] Daniel Baldomir and Percy Hammond. *Geometry of electromagnetic systems*. Clarendon press, 1996.
- [168] Roman S Ingarden and Andrzej Jamiolkowski. *Classical electrodynamics*, volume 12. Elsevier Science Ltd, 1985.
- [169] Kurt Meetz and Walter L Engl. *Elektromagnetische Felder: Mathematische und physikalische Grundlagen/Anwendungen in Physik und Technik*. Springer-Verlag, 2013.
- [170] Jancewicz Bernard. *Multivectors and Clifford algebra in electrodynamics*. World Scientific, 1989.
- [171] David Hestenes. *Space-Time Algebra*. Springer International Publishing Switzerland, 2015.
- [172] Elie Cartan. *Leçons sur les invariants intégraux*. Paris, 1922.
- [173] Hermann Grassmann. *Die lineale Ausdehnungslehre ein neuer Zweig der Mathematik: dargestellt und durch Anwendungen auf die übrigen Zweige der Mathematik, wie auch auf die Statik, Mechanik, die Lehre vom Magnetismus und die Krystallonomie erläutert*. Cambridge University Press, 2012.
- [174] Asim Egemen Yilmaz. Grassmann and his contributions to electromagnetics [historical corner]. *IEEE Antennas and Propagation Magazine*, 52(4):186–193, 2010.
- [175] F William Lawvere. History of mathematics. *Historia mathematica*, 32(1):99–106, 2005.
- [176] Michael J Crowe. *A history of vector analysis: The evolution of the idea of a vectorial system*. Courier Corporation, 1994.
- [177] Jean Itard. Biography in dictionary of scientific biography. *New York*, 1990, 1970.
- [178] MR Zirnbauer. *Elektrodynamik*. tex-script july 1998.
- [179] Werner Hildbert Greub, Stephen Halperin, and Ray Vanstone. *De Rham cohomology of manifolds and vector bundles*, volume 47. Academic Press, 1972.
- [180] Bernard F Schutz and Director Bernard F Schutz. *Geometrical methods of mathematical physics*. Cambridge university press, 1980.
- [181] Raoul Bott and Loring W Tu. *Differential forms in algebraic topology*, volume 82. Springer Science & Business Media, 2013.
- [182] L Petrova. *Properties of conservation laws and the mechanism of occurrence of physical structures*. MAX Press M, 2001. in Russian.

- [183] Ismo Veikko Lindell. Electromagnetic wave equation in differential-form representation. *Progress In Electromagnetics Research*, 54:321–333, 2005.
- [184] George Soliman, David Yevick, and Paul Jessop. Geometric algebra description of polarization mode dispersion, polarization-dependent loss, and stokes tensor transformations. *JOSA A*, 31(9):1956–1962, 2014.
- [185] Stephen Gull, Anthony Lasenby, and Chris Doran. Imaginary numbers are not real?the geometric algebra of spacetime. *Foundations of Physics*, 23(9):1175–1201, 1993.
- [186] Tomislav Ivezic. Invariant relativistic electrodynamics. clifford algebra approach. *arXiv preprint hep-th/0207250*, 2002.
- [187] AN Lasenby. Grassmann, geometric algebra and cosmology. *Annalen der Physik*, 19(3-5):161–176, 2010.
- [188] Lifeng Li. Formulation and comparison of two recursive matrix algorithms for modeling layered diffraction gratings. *JOSA A*, 13(5):1024–1035, 1996.
- [189] Thomas Estruch, Fabrice Pardo, Benjamin Portier, Julien Jaeck, Sophie Derelle, and Riad Haidar. Masons rule and signal flow graphs applied to subwavelength resonant structures. *Optics express*, 20(24):27155–27162, 2012.

List of publications

Publication in scientific journals in the framework of the PhD work

1. **M. Yakovleva**, J-L. Pelouard, F. Pardo. *Absolute description of electromagnetic world in spacetime: the y -invariant case*, in the submission process to PR Research journal.
2. **M. Yakovleva**, J-L. Pelouard, F. Pardo. *Electromagnetic field on the space-time map*. Proceedings of 12th International Congress on Artificial Materials for Novel Wave Phenomena *Metamaterials 2018* held 27 August - 1 September 2018, Espoo, Finland, VI-368 – VI-370, ISBN 978-88-941141-3-3.
3. V. Popov, **M. Yakovleva**, F. Boust, J-L. Pelouard, F. Pardo, and S. N. Burokur. *Designing metagratings via local periodic approximation: From microwaves to infrared*. Phys. Rev. Applied, 11:044054, Apr 2019.

Publication in scientific journals outside the framework of the PhD work

1. **M. A. Yakovleva**, K. G. Batrakov. Two layer graphene heterostructures for waves slowing down: operator approach to waveguide problem. Journal of the Belarusian State University. Physics, Vol. 1: 73-82, 2020.

International conferences in the framework of the PhD work

1. **M. Yakovleva**, J-L. Pelouard, F. Pardo. *Electromagnetic Field on the Space-Time Map*. Metamaterials 2018, Espoo, Finland, Aug 27 – Sep 1, 2018.
2. **F. Pardo**, M. Yakovleva, J-L. Pelouard. *Lasagna and Spaghetti in the Space-time Map: an Absolute Metric-less Picture of Electromagnetic Fields*. The 41st PhotonIcs and Electromagnetics Research Symposium (PIERS), Rome, Italy, June 17 – 20, 2019.

French conferences & Scientific days

1. **M. Yakovleva**, J-L. Pelouard, F. Pardo. *Electromagnetic spaghetti on the space-time map*. Journées Scientifiques 2019 de l'EDOM, Institute of Optics, Palaiseau, the 28th of February – the 1st of March 2019.

2. **M. Yakovleva**, J-L. Pelouard, F. Pardo. *Electric spaghetti in the space-time map*, C2N PhD students days, C2N, Palaiseau, the 4th – 5th of April, 2019.
3. **M. Yakovleva**, J-L. Pelouard, F. Pardo. *Electromagnetic Spaghetti in the Spacetime Map*, La journée "Optronique et photonique à Paris-Saclay", l'Institut d'Optique Graduate School, Palaiseau, the 27th of May, 2019.