The neurocognitive bases of fraction processing in the learning brain

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Bases cérébrales et comportementales de l'apprentissage des fractions

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To my daadu (always in my heart) and daadima,

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"It takes a village to raise a child" - African Proverb

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ABSTRACT

Understanding fractions is critical for the holistic development of mathematical knowledge. While fraction instruction typically begins in elementary school, children often encounter relational numerical concepts much earlier in their environment (e.g. sharing candies, varying the sweetness of a drink, baking muffins). A recently proposed theory, the Ratio Processing System (RPS), posits that the understanding of non-symbolic relational magnitudes and symbolic fractions are fundamentally intertwined. However, research on the RPS theory and fraction learning interventions in the classroom are limited. In this dissertation, we examine both symbolic and non-symbolic relational magnitude processing from the perspectives of both cognitive neuroscience and educational game-based approaches. First, performance accuracy on a match-to-sample task reveals individuals with varied mathematics skills to be perceptually sensitive to non-symbolic ratios but not to symbolic fractions. Second, univariate and multivariate analyses of neural activity patterns using an fMRI-adaptation paradigm suggest an absence of overlapping neural activations in all participants for absolute (represented as lines and as numbers) and relational magnitudes (represented as line ratio and as fractions). Third, analyses of fifth grader's fraction knowledge after playing a fraction educational game developed in the context of this thesis (*Math Mathews Fractions*) revealed that the game did not improve overall fraction skills above traditional classroom instruction. However, it was successful at improving decimal knowledge. The results of this thesis lead us to argue for fraction instruction focused on both perceptual methods as well as building connections between the multiple constructs of fractions. Future research holds great potential for

examining fraction games that support teachers in building a holistic fraction understanding, rooted in the percept-concept links.

Keywords*:* ratio processing system; fractions; fMRI; game-based learning; math cognition

RÉSUMÉ

La comprehension des fractions est essentielle au développement général des connaissances mathématiques. Bien que l'enseignement des fractions commence généralement à l'école primaire, les enfants rencontrent souvent des concepts liés aux fractions bien plus tôt dans leur environnement (par exemple, en partageant des bonbons, en variant le goût sucré d'une boisson, en préparant des muffins). Une théorie récemment proposée < *the ratio processing system theory* > postule que la compréhension des fractions et des rapports de grandeurs (i.e. ratio) sous forme non symbolique est fondamentalement liée. Cependant, les recherches sur la théorie RPS et les interventions d'apprentissage des fractions en classe sont limitées. Dans cette thèse, nous examinons le traitement de ratios, sous forme symbolique et non symbolique, du point de vue des neurosciences cognitives et de l'approche pédagogique basée sur le jeu. Tout d'abord, la précision de la performance dans une tâche d'appariement à l'échantillon révèle que des personnes ayant des compétences variées en mathématiques sont perceptivement sensibles aux ratios non symboliques mais pas aux ratios symboliques (i.e. les fractions). Deuxièmement, des analyses univariées et multivariées des schémas d'activité neuronale à l'aide d'un paradigme d'adaptation par IRMf n'ont trouvé aucune preuve d'activations communes dans le sulcus intrapariétal de tous les participants pour les grandeurs absolues (représentées sous forme de lignes et sous forme de nombres) et relationnelles (représentées sous forme de rapport de lignes et sous forme de fractions). Troisièmement, des analyses de la compréhension des fractions chez les élèves de cinquième année (CM2), soit après l'utilisation d'un jeu en classe sur les fractions (*Math Mathews Fractions*), soit après un enseignement classique (sans jeu), ont révélé que le jeu n'a pas amélioré les compétences générales sur les fractions par rapport à l'enseignement classique

(sans jeu). En revanche, il a amélioré la compréhension des nombres décimaux. Les résultats de cette thèse nous amènent à plaider en faveur d'un enseignement des fractions axé, d'une part sur des méthodes perceptives et d'autre part, sur la construction de liens entre les différents aspects des fractions. Les recherches futures offrent un grand potentiel pour l'étude de jeux sur les fractions qui aident les enseignants à développer, chez leurs élèves, une compréhension générale des fractions, enracinée dans les liens percept-concept.

Mots clés : ratio processing system ; fractions ; IRMf ; apprentissage par le jeu; la cognition mathématique

DESCRIPTION DU PROJET

La compréhension des fractions est essentielle pour un développement général des compétences mathématiques. L'importance de la compréhension des fractions ne se limite pas au milieu scolaire, mais il est également démontré qu'elle est essentielle pour la santé et l'emploi (Handel, 2016 ; Chakkalakal et al., 2017 ; Osborn et al., 2013). Bien que l'enseignement des fractions commence à l'école primaire, les enfants et les adultes ont montré des difficultés persistantes à comprendre les fractions (Chan, Leu, & Chen, 2007 ; Ni, 2001 ; Yoshida & Sawano, 2002). Les difficultés persistantes dans la compréhension des fractions ont conduit certains à postuler que l'architecture neurocognitive humaine n'est pas conçue pour traiter les fractions (Gallistel & Gelman 1992 ; Feigenson, Dehaene, & Spelke, 2004). D'autres ont adopté une approche moins pessimiste, attribuant les difficultés liées aux fractions à un enseignement qui ne tire pas parti de l'ensemble des architectures neurocognitives pouvant servir à soutenir les compétences en fractions (Lewis et al., 2016). Les théories contradictoires concernant l'existence d'un code primitif et abstrait pour les grandeurs relationnelles exigent des recherches supplémentaires dans ce domaine théorique.

Par conséquent, l'objectif principal de cette thèse était de faire la lumière sur l'ensemble des architectures neurocognitives (i.e. Ratio Processing System, RPS) qui pourraient sous-tendre le traitement des grandeurs relationnelles, représentées sous forme non symboliques et symboliques. Dans l'étude I, nous avons utilisé un paradigme dit de « match to sample » pour mesurer la sensibilité au ratio chez des individus ayant différents niveaux de compétences en mathématiques. Nous avons constaté qu'indépendamment de leurs compétences en mathématiques, tous les participants ont montré une sensibilité perceptive au ratio de deux grandeurs non

symboliques (i.e. représenté par un rapport de lignes) mais pas au ratio de deux grandeurs symboliques (i.e. ratio représenté par une fraction). Nous avons également constaté que les adultes présentaient cette sensibilité lorsqu'ils devaient estimer l'ordre de grandeur d'une fraction, révélant ainsi un rôle limité du RPS (Bhatia et al., 2020). Alors que l'étude I utilisait un paradigme comportemental, dans l'étude II, nous avons utilisé l'imagerie par résonance magnétique fonctionnelle pour examiner les activations cérébrales associées aux représentations symboliques et non symboliques en termes de grandeurs absolues et relatives. Dans l'étude II, nous avons utilisé un paradigme d'adaptation passive pour examiner les effets de l'adaptation neuronale pour les grandeurs absolues (représentées sous forme de lignes et sous forme de nombres) et relationnelles (représentées sous forme de rapport de lignes et sous forme de fractions). Contrairement à notre hypothèse, nous n'avons pas trouvé de preuve d'activations communes dans le sulcus intra-pariétal pour ces deux représentations. Cependant, les adultes ayant une plus grande maîtrise des mathématiques ont montré des activations dans le sulcus intra-pariétal gauche pour les grandeurs absolues, ce qui indique que les participants ayant une plus grande maîtrise des mathématiques pourraient être en mesure de recruter des régions cérébrales similaires pour le traitement des grandeurs absolues symboliques et non symboliques. Il est intéressant de noter que les analyses multivariées ont également révélé des modèles d'activité neuronale dissociables entre les grandeurs absolues et relationnelles dans le cortex préfrontal rostrolatéral (RLPFC) et les zones visuelles du cerveau. Étant donné le rôle du RLPFC dans le raisonnement relationnel, ces résultats pourraient suggérer les différentes manières dont les grandeurs absolues et relationnelles sont traitées au niveau neuronal. De plus, les participants adultes ne présentent pas de preuve de « recyclage neuronal » pour les grandeurs relationnelles

telles que les fractions et les rapports de lignes. Par conséquent, les résultats des études I et II nous amènent à penser que la sensibilité perceptive aux grandeurs relationnelles non symboliques pourrait dépendre de la tâche. Il est important de noter que des recherches futures utilisant différents paradigmes et outils pour examiner le RPS pourraient aider les chercheurs et les éducateurs à concevoir des interventions centrées sur l'apprentissage perceptif pour favoriser la compréhension des fractions.

Alors qu'une méthode pour favoriser la compréhension des fractions pourrait consister à exploiter la sensibilité perceptive aux grandeurs relationnelles non symboliques (Lewis et al., 2016), une autre méthode complémentaire pourrait se concentrer sur l'établissement de liens entre les différentes interprétations des fractions (Pitkethly & Hunting, 1996 ; Misquitta, 2011). Sans aucun doute, ces différentes interprétations rendent le concept de fraction difficile à comprendre. Par conséquent, l'objectif secondaire de cette thèse était d'évaluer l'efficacité d'un jeu (*Math Mathews Fractions*) qui aide à établir des liens entre les différentes interprétations des fractions. Dans l'étude III, nous avons réparti au hasard des élèves de CM2 entre un groupe de contrôle et un groupe de jeu. L'étude a comporté un total de huit sessions et s'est déroulée sur quatre semaines. Au cours de ces sessions, les élèves du groupe expérimental ont travaillé les fractions avec *Math Mathews Fractions* en classe, avec une interaction limitée de l'enseignant. En revanche, les élèves du groupe témoin ont travaillé les mêmes notions sur les fractions, sous forme papier-crayon et avec leur enseignant. Les résultats indiquent que le jeu n’a pas eu d’effets significativement positifs par rapport à l'apprentissage traditionnel pour les performances globales sur les fractions. Cependant, le jeu a eu un effet positif sur l'apprentissage des nombres décimaux. Nous avons également constaté une relation positive entre les paramètres du jeu et les scores de connaissances globales des

fractions. Cela suggère que des jeux tels que *Math Mathews Fractions* pourraient jouer un rôle dans l'enseignement traditionnel en classe en aidant les élèves à apprendre des représentations spécifiques des fractions et en aidant les enseignants à établir des liens entre les fractions et les nombres décimaux.

Enfin, la portée plus large de ces résultats dans le domaine des neurosciences cognitives et de la recherche en éducation est discutée. Un modèle conceptuel d'utilisation des résultats de cette étude pour l'enseignement des fractions est également proposé dans la discussion générale. Nous espérons que les efforts de recherche futurs utiliseront des paradigmes variés pour examiner la théorie RPS qui pourrait aider l'enseignement des fractions en classe. De plus, les recherches futures qui aideront les enseignants à utiliser des jeux tels que *Math Mathews Fractions* dans le cadre du programme scolaire pourraient faire la lumière sur les méthodes qui aident les élèves à développer une compréhension générale des fractions.

LIST OF PUBLICATIONS

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LIST OF ABBREVIATIONS

ANS	Approximate Number System
ASER	Annual Status of Education Report
BOLD	Blood Oxygen Level Dependent
DGBL	Digital Game-based Learning
fMRI	Functional Magnetic Resonance Imaging
IPS	Intra-Parietal Sulcus
MMF	Math Mathews Fractions
MVPA	Multivariate Pattern Analysis
NCTM	National Council of Teachers of Mathematics
OECD	Organisation for Economic Co-operation and Development
PFC	Pre-frontal Cortex
RLPFC	Rostrolateral- Prefrontal Cortex
RPS	Ratio Processing System
RSA	Representational Similarity Analysis
WHO	World Health Organisation

INTRODUCTION

Numerical information is ubiquitous and so numeracy plays an invaluable role in today's contemporary society. According to the OECD, numeracy is defined as, "*the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life*" (OECD, 2016, p.49). Several studies in the past have evaluated the short-term and long-term implications of low numeracy skills for both society and individuals (Gross et al., 2009; Parsons & Bynner, 2005). Particularly, for individuals, poor numeracy skills are not only associated with lower financial outcomes but also impact physical and mental well-being (Bynner & Parsons, 1997; Rivera-batiz, 1992; Rothman et al., 2008; Parsons & Bynner, 2005). Consequently, there has been considerable interest in examining the difficulties individuals encounter with numeracy, both in terms of brain and behavior.

Among the varied mathematics skills, rational numbers are shown to be notoriously challenging for both children and adults globally (National Mathematics Advisory Panel, 2008; Siegler, Fazio, Bailey, & Zhou, 2013). They form a crucial component of mathematical proficiency by acting as a bridge between middle school and high school mathematics development (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012). Less acknowledged is the role they play in domains such as health and employment (Handel, 2016; Chakkalakal et al., 2017). For example, studies on health numeracy show rational numbers to be strong predictors of health outcomes (Osborn et al., 2013). In fact, the Covid-19 crisis is an unfortunate example of how people struggle to understand proportional data related to the pandemic (such as the fatality and mortality rates) and ignore critical health guidelines (Roozenbeek et al.,

2020). Though these factors indicate the far-reaching effects of rational number difficulties, they also highlight the potential impact of research on understanding the brain-behavior basis of these difficulties in learners.

Accordingly, in this thesis, we were interested to investigate the neurocognitive correlates of fraction processing and learning. Just like any other mathematical domain, learning fractions ($numbers\ \frac{a}{b}\ where\ b\ \neq 0$) requires individuals to understand new mathematical notations (numerator and denominator) and concepts (part-whole, measure, operator, etc.). These new concepts can be built upon pre-existing knowledge, but can also contradict it (Rosenberg-Lee, 2021). For example, prior knowledge of whole numbers can help learners understand that all numbers have magnitudes, however, some whole number properties like having unique successors and a unique symbol do not apply to fractions (Rosenberg-Lee, 2021; Siegler, Fazio, Bailey, & Zhou, 2013). Therefore, the pre-existing knowledge about whole numbers is not as easily applicable to fractions and requires some form of conceptual change (Ni & Zhou, 2005). Undoubtedly, fractions pose severe difficulties for both learners and teachers (Newton, 2008; Post, Harel, Behr, & Lesh, 1991; Reyna & Brainerd, 2008; Stigler, Givvin, & Thompson, 2010). Two divergent hypotheses that explain possible reasons for fraction difficulties exist. On the one hand, researchers claim that problems with fraction learning exist due to their incompatibility with the core human neurocognitive systems (Gallistel & Gelman, 1992; Feigenson, Dehaene, & Spelke, 2004; Ni & Zhou, 2005). On the other hand, recent evidence suggests the presence of neurocognitive architectures (the Ratio Processing System) that are specifically suited for supporting fractions and relational magnitudes (Lewis, Matthews, & Hubbard, 2016). The authors of the latter account claim the pervasive difficulties with fraction learning to arise as a result of ineffective teaching methods that do not

leverage the pre-existing abilities. To provide evidence for this relatively novel account, in this thesis, we tested the hypothesis for the presence of a Ratio Processing System in individuals with varying levels of mathematics abilities. We also examined for a neural basis of this system in adults using functional magnetic resonance imaging. While the abovementioned part focuses on exploring the neurocognitive bases of fraction representation and processing, a more practical, on-ground work of this thesis involves investigating the effectiveness of a game-based intervention on fraction learning of fifth-graders. Particularly, the game, *Math Mathews Fractions* (https://kiupe.com/games/math-mathews-fractions-en/) focuses on the multifaceted, complex representations of fractions and aims to provide support to teachers during fraction instruction. Thus, in a pre-registered randomized controlled study, we investigate whether *Math Mathews Fractions* may serve as a useful tool to enhance students' fraction knowledge.

Therefore, the thesis is divided into three introductory chapters that review the literature on the current knowledge about fraction representation and learning. The thesis begins with the *what* and *why* of fractions. The first chapter describes the construct of fractions and some of the major difficulties reported in the literature. The second chapter puts forward the two major accounts that hypothesize the reasons for difficulties in fraction learning. Here, we also provide emerging evidence in support of the RPS theory and its hypothesized role in fraction learning. The third chapter reviews the potential role of game-based learning in supporting fraction understanding. These three chapters are then followed by the experimental part which includes three studies. Study I, presented in chapter 4, uses a match-to-sample paradigm to investigate the existence of the RPS and its role in symbolic fraction processing. Chapter 5 includes Study II that examines the neural bases of the RPS. In this study, we test whether the

culturally developed ability to represent numbers and fractions relies on the neural representations of lines and line ratios respectively. Lastly, in efforts to understand the role of game-based learning in supporting fraction instruction, Study III uses a pre-registered randomized controlled design to assess the effectiveness of *Math Mathews Fractions* on fraction learning outcomes of fifth-graders. The thesis concludes with a general discussion that speculates on the wider scope of the experimental results on fraction understanding and learning. In this section, we also comment on the potential role of the RPS in symbolic fraction understanding and recommend future research directions that could potentially explore the links between the *Ratio Processing System*, *Math Mathews Fractions*, and fraction instruction in the classroom.

CHAPTER I

FRACTIONS: THE WHAT AND WHY?

"Covid-19 is less fatal than the seasonal flu" was a key narrative in the United States and some other parts of the world around March 2020. This fallacious belief may have stemmed from a basic lack of rational number knowledge, specifically fractions (Thompson et al., 2021). While the number of people infected with Covid-19 worldwide was far less than the seasonal flu, the trend for the number of deaths was inverse (Mielicki et al., 2021). The relation between these two data points was difficult for many to decipher (e.g. case fatality rate, infection rate, etc). The case fatality rate (CFR), used as a measure to test disease severity, is calculated by dividing the total number of deaths by the total number of cases during a definite period multiplied by 100 (WHO, 2020; Rajgor et al., 2020). While the estimates are variable due to the ongoing pandemic, the case fatality rate was much higher for Covid-19 (2.1 %) as compared to the seasonal flu (0.1 %) (Rajgor et al., 2020; Piroth et al., 2021). Most people, however, focused exclusively on either the total number of cases or the number of deaths but not their relational magnitude (CFR), which led many to disregard critical health guidelines. This is just one example of how fraction knowledge taught at the middle school level is extremely crucial for making informed health choices in society. Additionally, from a practical perspective, fraction knowledge is important for making sound financial decisions and laying the foundation for careers in science, technology, engineering, and mathematics (Lortie-Forgues, Tian, & Siegler, 2015). From an educational perspective, fractions form a central part of the mathematics curriculum at the middle school level and are the basic building blocks for algebra and calculus (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton,

2012). Indeed, a large-scale, longitudinal study on a nationally representative population in both the USA and UK revealed fifth grader's fraction knowledge to predict their high school mathematics achievement and algebra skills even after controlling for IQ, reading, working memory, and whole number knowledge (Siegler et al., 2012). Though fraction instruction begins early in school, individuals continue to struggle with fractions throughout life (Stigler, Givvin, & Thompson, 2010; Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013). This chapter delves deeper into what exactly fractions are, the different ways in which they can be represented, and the difficulties associated with fraction understanding in both children and adults.

1.1 What is a Fraction?

1.1.1 The terminology

The term *fraction* is derived from the Latin word *frangere* which means 'to break' and is most often associated with the 'part of a whole' or 'equal number of parts' concept. Precisely, a fraction is a notation of a rational number, where the rational number is defined as a number to the solution of an equation, $b \times x = a$, where a and b are integers and b can be any non-zero quantity. Thus, fraction notation refers to a three-part notation (the numerator, the denominator, and the line that separates the two) for writing any real number. In this particular notation $\frac{a}{b}$, *a* and *b* are real numbers and *b* is non-zero. Thus, all rational numbers can be written in the fraction notation (called specifically fraction). However, not every fraction notation represents a rational number. For example, $\frac{\sqrt{2}}{2}$ is an irrational number (Lamon, 2012). Despite this conceptual distinction, the terminology used in mathematics education has not been quite consistent. Several researchers use different terms such as 'rational numbers', 'fractional numbers', and 'fraction symbols' interchangeably (Payne, 1976; Kieren,

1995). To add to the complexity, rational numbers and fractions have also been used interchangeably in elementary schools (Lamon, 2012). This interchangeable terminology made the already difficult concept of fractions more complex. Therefore, researchers in the past stressed the need to clearly define the term 'fraction' and its meaning (Lamon, 2012). Simply put, 'fractions' can now be interpreted in a three-fold manner; as the bipartite structure or *fractional notation*, as a *positive rational number* (Lamon, 2007; 2012), and as a broader *fraction concept* (Kieren, 1976; Behr et al., 1993). While the original terminology of fractions focuses on the bipartite notation, particularly the part-whole concept, the *fraction concept* is a broad, multifaceted construct.

1.1.2 The construct

The holistic framework used to understand the broad concept of fractions was conceptualized independently by Kieren, Vergnaud, and Freudenthal in the '70s and '80s. They recognized the different ways in which the *fraction concept* could be interpreted. Kieren (1976) developed the theoretical framework that included multiple, inter-related sub-constructs such as the ratio, the operator, the quotient, and the measure. During the same period, Vergnaud (1983) and Freudenthal (1983) also proposed similar aspects to fraction concepts focussing on the multiplicative relations and partitioning scheme respectively. Acknowledging the complexity of fractions, Freudenthal (1983) further notes, "*In spite of the many-sided classification and the wealth of possible examples, the approach to fractions from the point of view of "part-whole" is much too restricted not only phenomenologically but also mathematically*" (Freudenthal, 1983, p. 144). Kieren (1976) also emphasized that a good comprehension of *fraction concepts* requires an understanding of all the sub-constructs and their inter-relatedness. While Kieren's theoretical model is the most

widely accepted framework (Behr et al., 1983; Charalambous & Pitta-Pantazi, 2007; Tsai & Li, 2017), it is not the only way to understand the construct of *fraction*. For instance, Hecht and colleagues (2003) considered the part-whole and measurement interpretation to be meaningful for conceptual fraction knowledge. Another framework focuses on the alternate notations of decimals and fractions and the way they capture continuous and discrete entities (Rapp et al., 2015). In this framework, fraction best represents countable discrete objects (6/8 of the balls are blue) such that the ratio formed between the two sets of integers; the numerator and the denominator capture the value of the object. On the other hand, decimal is better at capturing the uncountable, continuous entities (i.e. easier to understand 0.4 feet than 2/5 of a foot). So, fraction represents a two-dimensional relation (**a**/**b**) whereas decimal is a one-dimensional magnitude (a/b = **c**) (Rapp et al., 2015). Thus, the abovementioned literature reveals the multiple ways in which the construct *fraction* and its associated framework are conceived. While no particular framework on the interpretation of fractions is universal, Kieren's model captures the complex, multifaceted concept of fractions and mostly includes all the other frameworks. Therefore, to further comprehend the complexity associated with each of the different subconstructs of fractions, Kieren's theoretical model will be discussed in detail.

1.1.2.1 The Five Subconstructs of Fractions

Earlier conceptions of the model included the four subconstructs: measure, ratio, quotient, and operator where the part-whole concept was implied to be embedded in each of the subconstructs (Kieren, 1976). Later, Behr and colleagues (1983) developed this model by including the part-whole concept as a distinct sub-construct. They further expanded the model by connecting the different interpretations of fractions to the basic operations like equivalence, multiplication, addition, and

problem solving (**Figure 1**). The following sub-sections describe the different subconstructs of fractions and their importance in understanding the broader construct of *fraction concepts*.

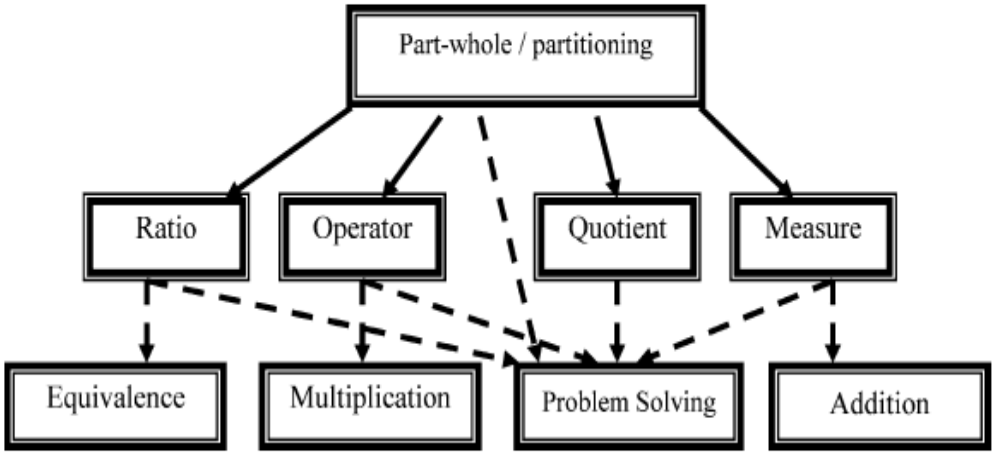

Figure 1: The different subconstructs of fraction and their inter-relationships (Figure from Charalambous & Pitta-Pantazi, 2007)

1.1.2.1.1 Part-Whole / Partitioning

The part-whole subconstruct of fractions is based on the partitioning scheme where the learner has to partition a continuous or discrete quantity into equal-sized parts or sets (Behr et al., 1983; Lamon, 1999). Here, the learner needs to develop mastery of partitioning the total number of parts or the whole into equal-sized parts. Also, it is important to understand the relationship between the parts and the whole such that (a) the more parts the whole is divided into the smaller the size of individual parts become (b) the parts are components of the whole and must exhaust the whole (Lamon 1999; Charalambous & Pitta-Pantazi, 2007). Failure to understand this relationship leads to misconceptions when counting the parts. For instance, students' misidentified the fraction 2/3 as 2/5 as they counted the parts twice, once for the numerator and another time for the denominator (Charalambous & Pitta-Pantazi,

2007). Despite the first concept to be introduced to students to explain fractions (Fuchs et al., 2013), the part-whole concept is found to be challenging for learners across a wide range of age groups (Ciosek & Samborska, 2016).

1.1.2.1.2 Ratio

The ratio subconstruct of a fraction is the comparative relationship between two quantities (a numerator and a denominator) that elucidates the notion of a relative magnitude (Behr et al., 1983; Kieren, 1993). Here, the learners must understand the covariance-invariance property of ratios which means that the two quantities must change together such that the relationship remains invariant (Charalambous & Pitta-Pantazi, 2007). For instance, if the ratio is one-half, then the numerator '2' and the denominator '4' of a fraction ($\frac{2}{4}$) both have to change together ($\frac{4}{8}$) to maintain the relative magnitude as one-half. In other words, the proportional relation between the numerator and the denominator will hold the ratio constant, i.e. a change in the numerator will lead to an expected change in the denominator (Charalambous & Pitta-Pantazi, 2007; Marshall, 1993). Thus, this subconstruct plays a critical role in building a better understanding of the concept of equivalence (Kieren, 1976; Marshall, 1993). Its importance is also emphasized in later mathematical concepts such as calculus (Tsai & Li, 2017). Recently, it has also been shown to play a key role in building an intuitive understanding of fractions (Lewis, Matthew, & Hubbard, 2016). The ratio subconstruct is an important part of this thesis and will be present in study I and II.

1.1.2.1.3 Quotient

This interpretation refers to the way the notation of the fraction $\frac{a}{b}$ can be seen as a mathematical operation of division (a ÷ b) (Kieren, 1993). Unlike the part-whole

subconstruct, this subconstruct has no constraint on the size of the numerator. Thus, the quantity that results from equal partitioning or division can be less, equal, or greater than the whole unit (Streefland, 1993). Thus, it can help students understand that fraction magnitude can be both greater and smaller than 1 (Charalambous & Pitta-Pantazi, 2007). For the understanding of this particular subconstruct learners need to understand the concepts of division, particularly the roles of the dividend and the divisor (Kieren, 1993). So, the quotient subconstruct can also be thought of as a process that begins with two entities; the numerator (divisor) and the denominator (dividend), and results in a single quantity through partitive or quantitative division (e.g. $\frac{8}{4}$ = 8 ÷ 4 = 2) (Behr et al., 1993). Another interpretation of quotient could be a number such that $\frac{a}{b}$ when multiplied by 'b' results in 'a'. Thus, the quotient subconstruct aids students in both the understanding of the whole number operation as well as its connection with the fraction concept. In fact, prior studies have shown proficiency in division tasks to support student's fraction understanding (Siegler & Pyke, 2013; Ye et al., 2016).

1.1.2.1.4 Operator

Another one of the five subconstructs of fractions, the operator works as an operation or function that is to be performed on a number or sets of objects (Behr et al, 1993; Kieren, 1976; Tsai & Li, 2017). To master the concept of the operator, students should be able to understand the different ways in which the operator could function (e.g. 2/4 can be 2 x [¼ of a unit] or ¼ x 2 units) (Charalambous & Pitta-Pantazi, 2007). An example of a question that includes the operator concept could be where a student is asked to solve $\frac{3}{4}$ of 12 meters. Here, the operator first stretches the original quantity and then shrinks it (e.g. $\frac{3}{4}$ of 12 is stretched to-12 multiplied by 3 which is 36,

then divided by 4 which shrinks to- 9) (Behr et al., 1993). This example shows how a sound understanding of the operator concept can help comprehend the concept of fraction multiplication (Behr et al., 1993).

1.1.2.1.5 Measure

This subconstruct is particularly used with the number lines where the unit fraction is used to determine the distance from a starting point (Lamon, 2001). It also contains two interpretations; first, the fraction holds an inherent magnitude (Siegler et al., 2011) and second, the fraction can be used as a measure such as a distance or a size (Charalambous & Pitta-Pantazi, 2007). Here, the unit fraction can be used to repeatedly measure the distance between a certain point from the starting point (Behr et al., 1993; Marshall, 1993). A graphical representation of the measure subconstruct is shown in **Figure 2**. Additionally, the iteration of unit fractions can be used to understand the concept of improper fractions (e.g. iteration of 1/4 can result in 2/4, 3/4, 4/4, 5/4, 6/4, and so on). The measure concept also aids in building an understanding of the property of density of rational numbers (Behr et al., 1993; Pitkethly & Hunting, 1996). This property is novel and complex for students who have learned the counting sequence in the past as it implies an infinite number of fractions exist between any two fractions (Lamon, 1999). To develop a comprehensive understanding of the measure construct of fractions, the student should be able to locate the fraction on the number line as well as identify it on a specific point on the number line (Smith, 2002). Intervention studies also show that practicing number line tasks may lead to improved knowledge of fractions (Barbieri et al., 2019; Fuchs et al., 2016; Saxe et al., 2013).

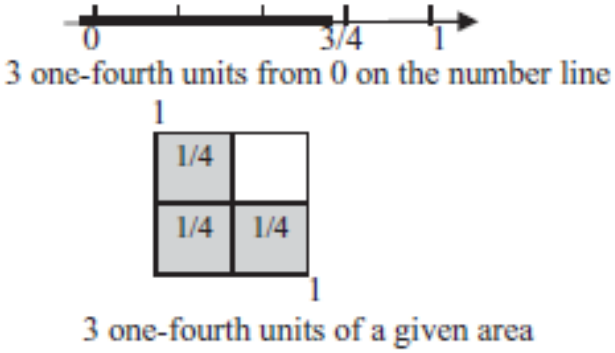

Figure 2: Graphical representation of the measure subconstruct of fraction showing a whole partitioned into 4 equal parts and ¾ is measuring 3 units of size ¼ (Figure from Tsai & Li, 2017).

Conclusively, the above-mentioned five subconstructs focus on the different properties of fractions that are required to attain mastery in this domain. Each subconstruct is required for a holistic understanding of fractions (Pitkethly & Hunting, 1996; Tsai & Li., 2017). However, curriculum and pedagogy worldwide have focussed disproportionately on the part-whole concept (Behr, Lesh, Post, & Silver, 1983; NMAP, 2008; Fuchs et al., 2013). Predictably, fractions are difficult to learn and mathematics educators and researchers worldwide have noted a pervasive lack of fraction understanding among students (Chan, Leu, & Chen, 2007; Ni, 2001; Yoshida & Sawano, 2002). Nonetheless, the disproportionate focus on the subconstructs of fractions is just one of the many reasons for difficulties encountered during fraction learning. In the section below, we will describe some reasons for the difficulties associated with fraction understanding.

1.2 Why are Fractions Difficult?

Several researchers and educators have examined why children and adults struggle to learn fractions (Hiebert, 1985; Mack, 1995; Ni & Zhou, 2005; Pitkethly &

Hunting, 1996; Behr et al., 1983; Lortie-Forgues et al., 2015). Three main reasons that contribute to fraction difficulties are discussed below.

1.2.1 Multiple subconstructs of fractions

As mentioned above, fractions comprise multiple subconstructs or interpretations. These subconstructs are required to build a coherent understanding of fractions (Behr et al., 1983). For instance, the part-whole subconstruct helps understand the concept of equipartitioning, the measure subconstruct focuses on the property of density of rational numbers, the operator interpretation aids in fraction multiplication, and the ratio builds the foundation for fraction equivalence (Charalambous & Pitta-Pantazi, 2007). Over-reliance on any one subconstruct leads to constraints on understanding fractions (Kieren, 1993). For example, a disproportionate focus on the part-whole construct in schools may lead to difficulties in understanding improper fractions, but also to difficulties in grasping properties of equivalence, infinite rational numbers between any two natural numbers, and fair shares (Misquitta, 2011). Therefore, proficiency in fraction knowledge requires developing a balanced understanding of all the subconstructs and their inter-relationships.

1.2.2 The natural-number bias

Both natural numbers and rational numbers can be placed and ordered on a number line by the virtue of their inherent property of magnitude (Siegler & Lortie-Forgues, 2017). However, they differ widely in other properties (**Figure 3**). While natural numbers can be counted, have predecessors and successors, and unique symbols (e.g. 7), between any two fractions there is an infinite number of other numbers and infinite ways to represent the same magnitude (e.g. ½ = 2/4 = 0.5)

(Obersteiner et al., 2019). Natural number schemes can therefore be inhibiting when it comes to generating new fraction knowledge (Pitkethly & Hunting, 1996). Indeed, research indicates children's prior whole number knowledge impedes the acquisition of fraction concepts (Gallistel & Gelman, 1992). The phenomenon, known as the *whole-number/ natural number bias*[1], often leads to errors in problems that require holistic processing of fractions (Ni, 2001; Ni & Zhou, 2005; Vamvakoussi et al., 2012; Van Hoof et al., 2013).

Additionally, the bipartite structure of fractions (two natural numbers separated by a horizontal line, $\frac{a}{b}$) adds to the complexity of learning overall fraction magnitudes (Hiebert, 1985). It often leads learners to overly rely on fraction components (*a*- numerator and *b*- denominator) to estimate fraction magnitudes. Fraction notation is difficult to process and even though learning occurs gradually, it requires more cognitive resources such as working memory when solving a fraction problem (23/66 + 34/78) than a whole number problem (78 + 67) (Hecht & Vagi, 2010). During the early stages of fraction learning, children often misconceive a fraction as *familiar distinct* whole numbers or arithmetic operations (e.g. 2/3 as 2 and 3 or 2+3) (Hartnett & Gelman, 1998). Thus, the physical notation of fractions also contributes to the natural-number bias.

Further, the erroneous assumption that properties of natural numbers are similar to that of fractions causes difficulties when processing fraction arithmetic (Siegler, Fazio, Bailey, & Zhou, 2013). For instance, when 13-year-old students are asked to solve the addition problem 12/13 +7/8, most of them choose 19 or 21. This

[1] Both terms have been used to describe the phenomenon.

indicates that they have summed either the numerator or the denominator respectively, instead of approximating the overall magnitude (Carpenter et al., 1980; Lortie-Forgues et al., 2015). Response patterns on a fraction magnitude comparison task, where participants are asked to judge the relative fraction magnitudes (e.g. is 7/9 more or less than 2/8), also indicates that sometimes participants rely on the components of fraction rather than the overall magnitude of the fraction (Dewolf, Grounds, Bassok, & Holyoak, 2014; Schneider & Siegler, 2010). In another study, community college students also showed the natural number bias by adding numerators and denominators separately across the fractions (e.g. 1/2 + 2/3 = 3/5) (Stigler et al., 2010). The natural number bias is not only restricted to children but is also observed in adults, even expert mathematicians (Meert, Gregoire, & Noel, 2010; DeWolf & Vosniadou, 2015; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Obersteiner et al., 2013). Therefore, successful learning of fractions demands an understanding of the different properties of fractions and natural numbers and inhibition of the 'intuitive' natural number bias when processing fractions (Van Hoof, Verschaffel, De Neys, Van Dooren, 2020).

	Natural numbers	Fractions
1. Representation of Magnitude	Base-10 place-value structure More digits—larger number $123 > 45$	Quotient of two numbers Neither number of digits nor natural number magnitudes as such determine fraction magnitudes $\frac{2}{3} > \frac{5}{19}$
2. Symbolic Representation	Unique for each number 2 as unique representation	Multiple (infinitely many) fractions can represent the same number $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \text{etc.}$
3. Density	Unique successors and predecessors Finite number of numbers between two natural numbers 1, 2, 3, 4, 5, etc.	No unique successors and predecessors Infinite number of numbers between two fractions $\frac{3}{5}$ is not the successor of $\frac{2}{5}$
4. Operation	Multiplication as repeated addition $3 \cdot 4 = 4 + 4 + 4$ Multiplication makes bigger, division smaller $2 \cdot 4 = 8,\ 15 \div 3 = 5$	Multiplication as repeated addition insufficient, more abstract definition required Multiplication and division can make bigger or smaller $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8},\ \frac{1}{2} \div \frac{1}{4} = 2$

Figure 3: Differences between the properties of natural numbers and fractions (Figure from Obersteiner et al., 2019).

1.2.3 Varied and distinct fraction arithmetic procedures

The importance of fraction arithmetic extends beyond mathematics achievement to other domains such as physics, chemistry, statistics, economics, and many more (Lortie-Forgues, Tian, & Siegler, 2015). It further extends to daily skills such as banking, baking, and medical dosage calculation. The concepts required to master fraction arithmetic are complex and ambiguous. For instance, both fraction addition and subtraction require equal denominators but this is not the case for fraction multiplication and division. Further, fraction division is a procedurally complex two-step process that requires the inversion of the denominator and multiplication of the fractions (e.g. 3/6 ÷ 4/8 = 3/6 x 8/4 = 24/24). Thus, the number of distinct procedures required to master fraction arithmetic is far more than what elementary students have ever encountered before (Lortie-Forgues, Tian, & Siegler, 2015).

A prior study on 6th and 8th graders recruited from three public school districts in the US noted a success rate of only 41% and 57% on fraction arithmetic problems (Siegler & Pyke, 2013). Further, these students showed better performance on fraction addition and subtraction problems (60 and 68% correct respectively) as compared to fraction multiplication and division (48% and 20% correct respectively). In general, students and adults are also less accurate on fraction multiplication and division problems than fraction addition and subtraction (Givvin et al., 2011; Stigler et al., 2010; Braithwaite et al., 2017; Siegler & Lortie-Forgues, 2017).

A variety of factors contribute to these statistics. A limited understanding of fraction arithmetic procedures by teachers, rote memorization of procedures by students, minimal practice and instruction on fraction division, and limited conceptual understanding are some of the factors that exacerbate students' difficulties with fraction arithmetic (Lortie-Forgues, Tian, & Siegler, 2015). Thus, mastery of fraction arithmetic entails a multi-pronged approach that builds a strong conceptual understanding of fraction magnitudes, that explicitly focuses on the variety of procedures, and involves a thorough and balanced practice of all types of fraction arithmetic problems.

1.3 Summary

Learning and teaching fractions is undeniably challenging. The initial conceptualization of number theory by children that focuses on the counting sequence results in *resistance* to learning fractions (Lamon, 1999). This *resistance* leads to misconceptions in fraction concepts for many adults. Thus, a reconceptualization of number theory is a prerequisite for building proficiency in rational number knowledge (Stafylidou and Vosniadou, 2004). The current chapter highlighted some studies that

examined the source of fraction difficulties in children and adults. The forthcoming chapters will shed light on the neurocognitive correlates of fraction understanding and recent game-based interventions to overcome the struggles associated with fraction learning.

CHAPTER II

THE NEUROCOGNITIVE BASIS OF FRACTION LEARNING

"Intuition is important to build mathematical knowledge"

- Beth & Piaget 1966

Human cognition is postulated to be built on specific innate intuitions, also known as the *core systems of knowledge*[2](Spelke, 2000; Spelke & Kinzler, 2007). The core systems of knowledge, namely, object representation, agent and goal-directed action, the geometry of the environment, and, the number representations[3] support evolutionarily relevant functions in most species (Spelke, 2004; Spelke & Kinzler, 2007). Present from infancy, these systems serve as foundational blocks for building complex knowledge systems required to advance human cognitive development (Spelke & Kinzler, 2007). Specifically, the *number sense*, or the ability to perceive numerical quantity, found in animals, pre-verbal infants, and human adults (Dehaene, 1997, 2001; Nieder, 2019; but see: Leibovich et al., 2017; Wilkey & Ansari, 2019) is hypothesized to support the development of symbolic number knowledge in humans (Bailey, Hoard, Nugent, & Geary, 2012; Dehaene & Cohen, 2007; Feigenson et al., 2004; Gallistel & Gelman, 1992; Piazza, 2010; Neider, 2005, 2017; for a review see De Smedt, Noel, Gilmore, & Ansari, 2013). While these systems help guide the acquisition of new knowledge, they can also pose limitations and constraints when the

[2]Developmental theories find their roots in Nativist (Kant, Chomsky), Constructionist (Piaget), and/or Empiricist (Skinner) approaches. In addition to the core knowledge theory mentioned above, some other cognitive development theories include connectionism, theory theory, modularity, and dynamic systems theory (detailed review in Newcombe, 2013).

[3] A fifth core system for social interactions has also been proposed (Spelke & Kinzler, 2007).

new knowledge violates the principles of the core system (Geary, 2006). Proponents of this view, the Innate Constraint account, argue that pervasive difficulties in fraction learning may stem from the inability of the core systems to represent and process relational quantities, mainly because these systems are exclusively evolved to deal with discrete numerosities (Bonato et al., 2007; Dehaene, 1997; Gallistel & Gelman, 1992; Geary, 2007; Piazza, 2010). Contrarily, the Cognitive Primitive approach posits the presence of core perceptual systems or intuitions that are tuned to process relational quantities (Lewis, Matthews, & Hubbard, 2016; Matthews & Ziols, 2019). Researchers supporting this account believe fraction difficulties arise as a result of teaching practices and curriculum that does not leverage the existing intuitive abilities to process ratios and proportions (Ni & Zhou, 2005; Lewis, Matthews, & Hubbard, 2016). Thus, whether the human cognitive architecture can or cannot support fractional magnitudes remains elusive. This question has inspired a significant amount of work in this thesis and so the objective of the current chapter is to provide a review of the literature on the neural and behavioral correlates of fraction learning.

1.1 Fraction Learning: Innate constraint or cognitive primitive?

The question of why fractions are difficult to master is a matter of current debate (Lewis et al., 2015; Mohring et al., 2016). Two contrasting accounts have been proposed, namely, the innate constraint and the cognitive primitive approach. The innate constraint account is supported by two main sources of evidence (Ni & Zhou, 2005). First, there are severe and pervasive difficulties encountered by individuals globally when learning fractions and rational numbers (Bialystok & Codd, 2000; Chan, Leu, & Chen, 2007; Hartnett & Gelman, 1998; Ni, 2001; Yoshida & Sawano, 2002), and second, there is an observed early competence with discrete quantities in animals and human infants (Brannon & Terrace, 1998; Cantlon & Brannon, 2006; Dehaene,

1992, 1997; Koechlin, Dehaene, & Mehler, 1997; Starkey, Spelke, & Gelman, 1990). Dehaene (2011) captures this account cogently in his book:

I would like to suggest that these mathematical entities (negative integers, irrational numbers, and fractions except ½ and 1/4) are so difficult for us to accept, and so defy intuition, because they do not correspond to any pre-existing category in our brain. To function in an intuitive mode, our brain needs images- and as far as number theory is concerned, evolution has endowed us with an intuitive picture only of positive integers (p. 76).

According to the innate constraint theorists, neurocognitive architecture is endowed with an innate ability to process only *discrete magnitudes* (Gallistel & Gelman 1992; Wynn 1992). One of the core systems that support this ability is known as the Approximate Number System (ANS)[4], which estimates the non-symbolic quantities approximately (Dehaene, 1997). The ANS is shown to be present in many animal species, including rats, pigeons, monkeys, chimpanzees (Agrillo et al., 2011; Beran, 2001; Dehaene, Dehaene-Lambertz, & Cohen, 1998a; Ditz & Nieder, 2015; Nieder et al., 2002; Scarf, Hayne, & Colombo, 2011; Xia, Emmerton, Siemann, & Delius, 2001; for a review see: Boysen & Capaldi, 1993 and Nieder, 2019). Studies also suggest that human infants possess an ANS. For instance, a seminal study using the habituation paradigm on five to six-month-old infants showed that infants who were habituated to two dots looked longer at displays containing three dots, and vice versa (Starkey & Cooper, 1980), thus indicating that infants can discriminate between these two non-symbolic quantities (Hyde & Spelke, 2011; Wood & Spelke, 2005; Xu &

[4] The other system, the object tracking system (OTS) which is relatively more precise but only for processing smaller number of items (about one to four) (Dehaene, 2011; Nieder, 2019).

Arriaga, 2007). Crucially, this innate ability to represent magnitudes mentally is approximate, imprecise, and follows Weber's law (Halberda, 2011). First discovered for the sensation of weights, Weber's law is found to apply to other sensory intensities and importantly to numerical discrimination (Weber, 1850; Moyer & Landauer, 1967). Weber's Law for numerical discrimination refers to the ability to discriminate numerosities as a function of their ratio, i.e., as the ratio between the numerosities increases so does the ease of discriminability (Halberda, 2011). In other words, Weber's law captures both the distance (e.g., 2 and 6 is easier to compare than 2 and 3) and the size effects (e.g., 10 and 14 is easier to discriminate than 40 and 44). The precision with which the infants can discriminate between the numerical stimuli (i.e., Weber fraction) improves with age (**Figure 4**). While newborns can discriminate numerosity arrays with a 1:3 ratio, six-month-old infants can discriminate 1:2 and ten-month-old infants can discriminate a 2:3 ratio (Lipton & Spelke, 2003; Wood & Spelke, 2005; Xu & Arriaga, 2007). An infant's ability to perceive numerical stimuli in the environment is not only limited to perceptual discrimination between numerosities but also extends to rudimentary arithmetic calculations like addition and subtraction (Wynn, 1992, 2004).

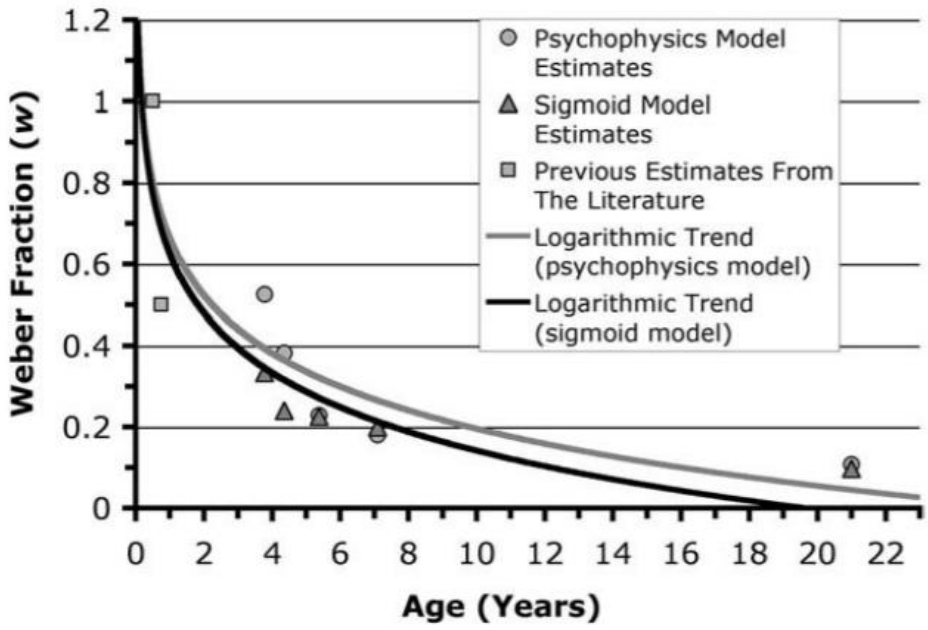

Figure 4: Figure depicting the developmental change in the estimated Weber fraction for each age group from different models (decreasing Weber fraction = increase of ANS acuity) (Figure from Halberda & Feigenson, 2008)

Studies comparing children's ease to acquire whole number skills as compared to rational numbers further add to the argument for the innate constraint account. For example, several studies have shown that the whole number strategy exerts a strong and persistent interference during fraction learning (Mack, 1995; Ni, 2001; Streefland, 1993; Zhang, Fang, Gabriel, & Szücs, 2014), also known as the whole-number bias (this was discussed in detail in Chapter I). These studies support the idea that the innate mechanisms responsible for supporting the representation of discrete quantities interfere and impede the acquisition of fraction and rational numbers (Feigenson, Dehaene, & Spelke, 2004).

In his evolutionary theory, Geary (2006) also proposed a distinction between whole numbers and rational numbers. Specifically, while whole numbers were proposed to be biologically primary, fractions and other number types were considered to be biologically secondary, indicating that a lack of innate ability might be the reason

for the difficulty in acquiring fraction concepts (Siegler, Thompson, & Schneider, 2011). The innate constraint proponents suggest that "*when humans push number representations further to embrace fractions, square roots, negative numbers, and complex numbers, they move even further from the intuitive sense of number provided by the core systems*." (Feigenson et al., 2004, p. 313). Therefore, theorists of this account suggest that the innate mechanisms assumed to help children learn discrete number concepts are not designed to deal with a relative, continuous quantities and so acquiring fraction and rational number concepts is challenging. However, evidence from animal and infant studies that show discrimination for discrete numerosities (Brannon & Terrace, 1998; Cantlon & Brannon, 2006; Dehaene, 1992, 1997; Koechlin, Dehaene, & Mehler, 1997; Starkey, Spelke, & Gelman, 1990) does not automatically suggest the absence to comprehend continuous, relational magnitudes. Unless proven, it is incorrect to assume the non-existence of mental representations for continuous numerosities.

Indeed, emerging evidence from both animal and infant studies falsifies the proposal of the innate constraint account, which privileges discrete quantities. Some have even suggested perceptions of relative quantities to be more primitive than absolute, discrete quantities (Ni & Zhou, 2005), arguing for the *number sense* to represent rational numbers (Clarke & Beck, 2021). Recently, a similar system to the ANS, the *ratio processing system* (RPS) has been proposed (Lewis, Matthews, & Hubbard, 2016). Studies indicate that the RPS might be a phylogenetically ancient system that processes non-symbolic ratios such as relative lengths of two lines or areas of two figures or dot arrays (Jacob, Vallentin, & Nieder, 2012; Matthews & Chesney, 2015). According to the cognitive primitive hypothesis, the RPS might serve as a neurocognitive startup tool (Piazza, 2010) to build robust fraction knowledge

(Lewis et al., 2016). Following the relation between ANS and symbolic math achievement (Chen & Li, 2014; Bugden et al., 2016; Piazza et al., 2004; for contrary views Reynvoet & Sasanguie, 2016, Leibovich et al., 2017), proponents of the RPS believe that leveraging this innate perceptual sensitivity can help develop a better understanding of symbolic fraction magnitudes. Since a major part of this thesis involves exploring the behavioral and neural correlates of the RPS, the following sections will provide a detailed account of the literature so far.

1.2 The Ratio Processing System and the Neurocognitive Correlates of Fraction Learning

Recent evidence suggests the presence of a perceptual-cognitive system for processing relational non-symbolic magnitudes. Indeed, perceptual sensitivity to non-symbolic relational magnitudes has been shown in non-human primates (Drucker et al., 2016; Eckert et al., 2018; Tecwyn et al., 2017; Vallentin & Nieder, 2008, 2010; Woodruff & Premack, 1981), infants (Denison & Xu, 2014; McCrink & Wynn., 2007), school-aged children (Bhatia et al., 2020; Boyer, Levine, & Huttenlocher, 2008; Meert, Grégoire, Seron, & Noël, 2013; Sophian, 2000; Spinillo & Bryant, 1999; Szkudlarek & Brannon, 2021), and both typically achieving adults (Bhatia et al., 2020; Hollands & Dyre, 2000; Jacob & Nieder, 2009 a, b; Matthews & Chesney, 2015; Meert, Grégoire, Seron, & Noël, 2011) and adults with mathematics difficulties (Bhatia et al., 2020). This widespread ability to process non-symbolic relational magnitudes - among nonhuman animals and individuals with varying levels of mathematics skills – demonstrates the evidence for a long-standing *ratio processing system* (Jacob, Vallentin, & Nieder, 2012). Focusing on both human and nonhuman animals, the next sections will highlight the important behavioral, neuroimaging, and neurophysiological experiments that reveal evidence for the RPS.

1.2.1 Behavioral experiments

Assessing relational magnitudes provides animals the evolutionary advantage in behaviors like social interactions, hunting, and mating. For instance, estimating the ratio between the size of the opponent invader and their group helps determine female lions and chimpanzees whether or not to attack (McComb et al., 1994; Wilson et al., 2002). Foraging behavior in ducks also shows preference to proportional magnitudes (Harper, 1982). Thus, the understanding of proportionality is required for key life skills. However, these behavioral patterns observed in field studies could very well be explained by low-level sensory processes. To this end, several controlled laboratory experiments have examined whether or not the ability to discriminate between ratios extends beyond these low-level sensory-driven processes.

Using the operant conditioning paradigm, researchers trained pigeons to peck arrays that consisted of an equal number of blue and red dots (Honig & Stewart, 1989; Emmerton, 2001). The pigeons were also trained to learn that one specific color was not rewarded (e.g. blue). Thereafter, the birds were tested with different ratios of the two discrete dot colors. A similar experiment was also performed with continuous horizontal bars of colors (Emmerton, 2001). Results revealed that irrespective of the absolute number of dots, pigeons responded more frequently to arrays that contained a greater proportion of rewarded color (e.g. red). In another study, Vallentin & Nieder (2008) tested both monkeys and human adults on a proportion discrimination task (**Figure 5**). They presented a delayed match to sample task where target non-symbolic line proportions had to be matched with their correct match proportion (1:4, 2:4, 3:4, 4:4). The presentation time was short to prevent calculation or verbalization of the stimuli by human adult participants. Also, the individual line lengths differed so that the participants could not rely on absolute line lengths. Results showed a similar level of

performance for both human adults and monkeys (**Figure 5**). These studies also revealed the signature for an analog magnitude representation, i.e., the distance effect (which reveals an abstract understanding of proportionality).

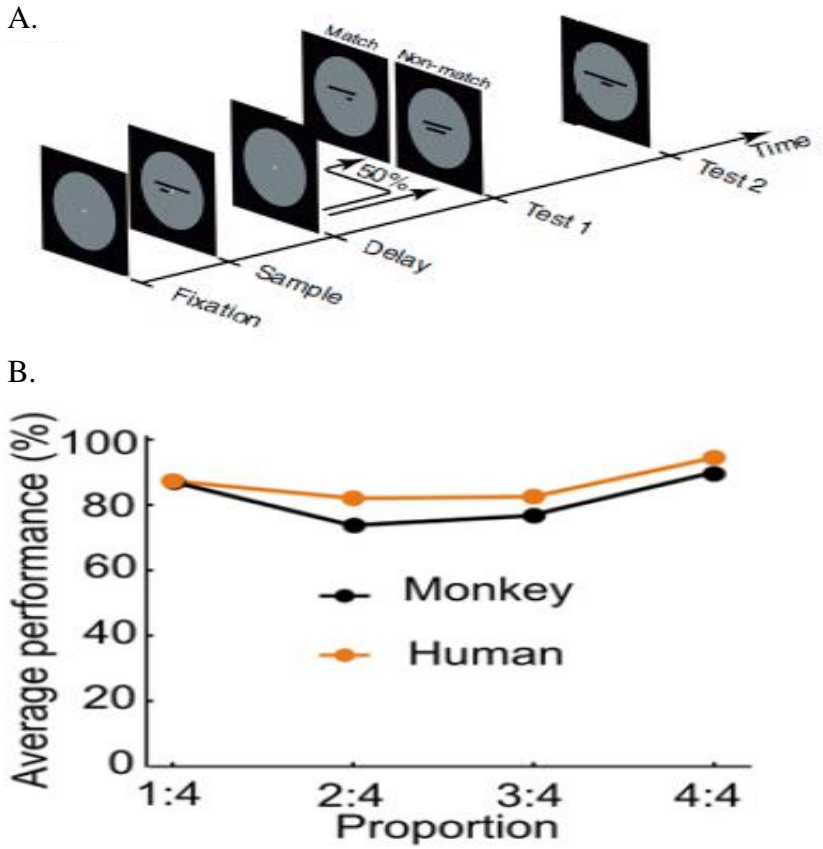

Figure 5: A. Delayed match to sample task using line proportions. B. The average performance of 18 human adults and two monkeys on the task (Figure from Vallentin & Nieder, 2008)

Similar to animals, pre-verbal infants have also shown sensitivity to proportionality. For instance, McCrink & Wynn, 2007 observed that five to six months old infants habituated to specific non-symbolic ratios looked longer at novel ratios (differing by a factor of two), indicating that infants as young as six months can process differences between non-symbolic ratios. By the age of four, children can also accurately perform addition and subtraction on non-symbolic part-whole representations of fractions (Mix, Levine, & Huttenlocher, 1999). Taken together, these

studies raise the possibility of an evolutionarily old system, the RPS, involved in processing non-symbolic relational magnitudes. It has also been argued that the RPS might provide a scaffold to build symbolic fraction knowledge in humans (Lewis, Matthews, & Hubbard, 2016). Indeed, in a recent study 85 first and second graders (~ 6-8 years old) performed non-symbolic (dot arrays) and symbolic (numerals) ratio comparison tasks where they were asked to select the machine which would give them the best chance of giving a specific color (blue/white) of gumball. Practice trials were conducted to ensure that children did not rely on absolute numbers of gumballs. Results revealed an above-chance performance on both symbolic and non-symbolic ratio comparison tasks (Szkudlarek & Brannon, 2021). Importantly, the findings show that children who performed the non-symbolic ratio task first were much better at symbolic ratio reasoning than the other way around (Szkudlarek & Brannon, 2021). This result might indicate that the non-symbolic ratio reasoning provides a basis for symbolic ratio understanding.

This idea of the RPS acting as a building block for symbolic relational magnitude understanding is also supported by other recent studies. For instance, individual differences in non-symbolic ratio processing have been shown to relate to individual differences in symbolic fraction processing in school-aged children (Möhring, Newcombe, Levine, & Frick, 2016) and adults (Matthews, Lewis, & Hubbard, 2016). Further, when adults compare the magnitude of symbolic fractions as well as the magnitude of non-symbolic ratios (e.g., pairs of lines), their performance depends on the ratio between the magnitudes (i.e., a Ratio of Ratios, RoR) (Hurst & Cordes, 2016; Matthews & Ziols, 2019) For example, participants are faster and more accurate at comparing symbolic fractions and non-symbolic ratios when the RoR is large (e.g., 2/6 versus 1/9 = 3:1) than when it is small (e.g., 4/5 versus 2/3 = 6:5) (Hurst & Cordes,

2016). This effect – also observed with whole numbers – is generally thought to indicate that numerical magnitudes are ordered along a mental number line (Moyer & Landauer, 1967). Accordingly, representations of magnitudes are more difficult to distinguish when the distance between them decreases (i.e., a distance effect) and when the overall magnitude increases (i.e., a size effect). Another study has found that adults are faster at comparing symbolic fractions to non-symbolic fractions (i.e., a mixed-format comparison) than symbolic fractions to symbolic fractions (i.e., a within-format comparison) (Matthews & Chesney, 2015), suggesting a shared magnitude code that might be accessed intuitively when performing mixed-format comparisons (Matthews & Chesney, 2015; Matthews, Chesney, & McNeil, 2014). Together, these studies provide important evidence for the presence of the RPS and support for the cognitive primitive approach.

1.2.2 Neuroimaging experiments

While much is known about the neural correlates of absolute magnitude, until recently there was no data on the neural representations of relative magnitudes. This lack of evidence might have contributed towards strengthening the innate constraint theorists' account. Jacob & Nieder (2009a) were the first to conduct an fMRI study with human adults using an adaptation paradigm to explore the cortical areas responsible for processing non-symbolic ratios (line lengths and dot arrays). The adaptation paradigm (Grill-Spector and Malach, 2001) relies on the phenomenon of repetition-suppression whereby repeated presentation of a particular stimulus leads to suppression in the neuronal activity (adaptation effect). Presentation of novel stimuli will then lead to an increase in neuronal activity (or recovery from the adapted state). In the study, adult participants were repeatedly presented with a specific non-symbolic ratio (e.g. 1:5) which led to a decrease in blood oxygen level-dependent (BOLD)

activity in the neuronal populations sensitive to that ratio. Presentation of close (2:5 and 3:5) and far (5:5 and 4:5) deviants then activated a different neuronal population to recover from the adaptation with a stronger BOLD activity. Particularly, the adaptation effect was observed in the bilateral IPS, the precentral, and the prefrontal cortex. Additionally, the difference in the strength of BOLD activity was modulated by how far the novel stimulus ratio was from the adapted ratio. This distance effect was found to be invariant to changes in the representation of the stimuli (line ratios or dot arrays). Further, an overlap of the distance effect with that observed for only numerosities revealed that both absolute and relative magnitudes were processed by the same brain regions i.e. bilateral intra-parietal sulcus and the lateral prefrontal cortex (Jacob & Nieder, 2009a). Since this was a passive viewing paradigm, participants were not asked to perform any task (e.g. magnitude comparison), further supporting the idea that representation of non-symbolic proportions is implicit and automatic in the human brain. Rhesus monkeys trained on a delayed match to sample task (described above) with non-symbolic ratios also showed selective neural tuning for preferred ratios in the prefrontal and parietal regions (Vallentin & Nieder, 2010). Specific populations of neurons displayed the signature distance effect responses, i.e., neurons tuned to 1/4 activated more strongly in response to 1/4, less strongly for ratios closer in distance, and weakly for distant ratios. Therefore, these results indicate that both humans and monkeys demonstrate an abstract understanding of proportionality. Supporting the behavioral data discussed above, the abovementioned neuroimaging study along with the others discussed below hints at the possibility for the presence of the ratio processing system.

Evidence for the use of an analog code for processing symbolic proportions comes from another study by Jacob & Nieder (2009 b). In this study, adults were

adapted to symbolic fractions (approximately 1:6) and then presented with deviants in two formats, fraction numerals ($\frac{4}{24}$) and fraction words (one-half). Similar to non-symbolic ratios, participants passively viewed symbolic fractions and showed an adaptation effect in the bilateral IPS. Importantly, the effect was independent of the notation of the deviants (numerals and words) (**Figure 6**). Therefore, the invariance to the notation of deviants and the observed distance effect in this cross-format notation hints at an amodal representation of magnitude (i.e. a mental number line; Siegler et al., 2011). Remarkably, another study on symbolic fractions reveals that the right IPS *does not* respond as a function of the distance between the components (numerator and denominator), but specifically gets modulated by the total distance between the fraction magnitudes (Ischebeck et al., 2010), indicating that the human brain does represent the real numerical value of the fractions.

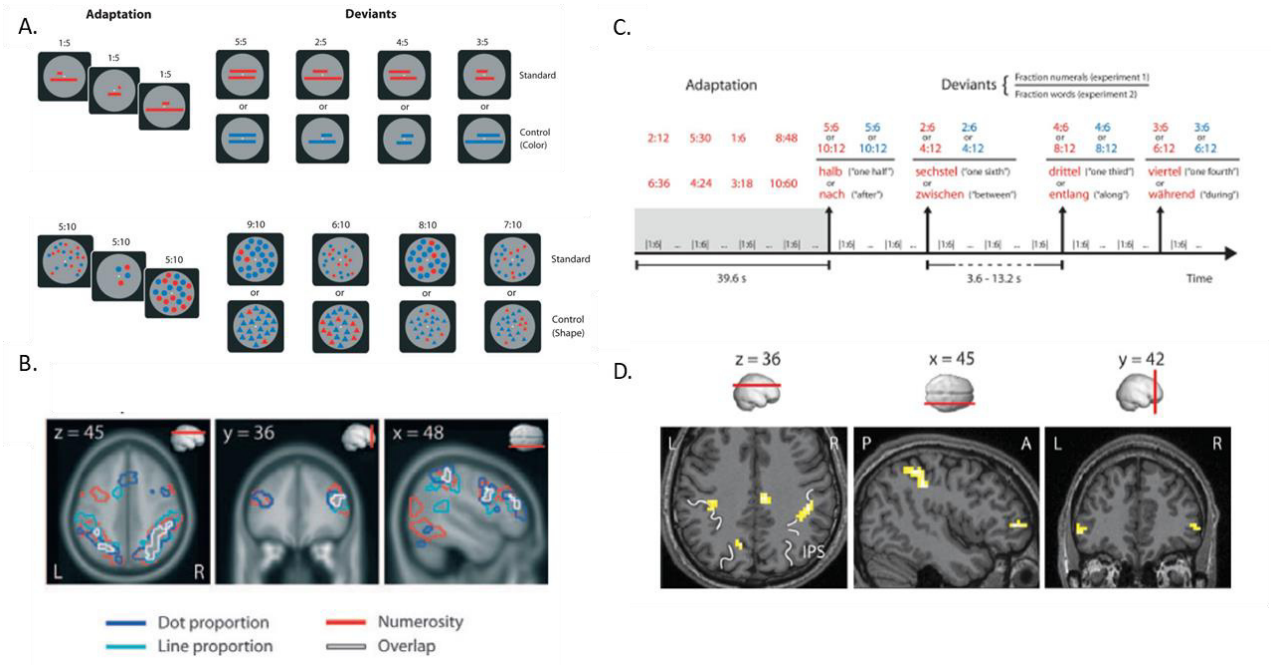

Figure 6: Functional MRI adaptation experiments for non-symbolic ratios and fractions A. Non-symbolic stimuli representing proportions in both line ratios and dot arrays. B. Overlapping brain regions for absolute (red: numerosity) and relative (blue: dot and line proportion) magnitudes C. In another experiment, participants were adapted to 1:6, and

deviants were presented in both numerals and fraction words D. Overlapping brain activation in the bilateral IPS and prefrontal cortex for both fraction notations (fraction numerals and fraction words) (Figures from Jacob & Nieder, 2009 a, b)

While the evidence above suggests that both non-symbolic ratios and symbolic fractions are encoded holistically at the neural level and hint at the existence of the RPS, not much research exists on identifying a *common neural substrate* for relative magnitude processing (independent of the symbolic or non-symbolic format). To this end, Mock and colleagues (2018) designed a magnitude comparison task with both symbolic (fractions and decimals) and non-symbolic proportions (dot patterns and pie charts). Participants had to perform a timed magnitude comparison task in all four formats in the MRI scanner. The fractions were chosen specifically to avoid componential processing (i.e. participants could not rely on the numerical values of the numerator and/ or denominator alone to arrive at the correct answer e.g. $5/9 < 2/3$, but $5 > 2$ and $9 > 3$). Therefore, the task aimed at eliciting the holistic magnitude of the fraction. Conjunction analysis revealed activations in the occipito-parietal network including the right IPS (**Figure 7**). Notwithstanding the relatively limited sample size ($n = 24$), this work provides the first evidence for a shared neural correlate for relative magnitude processing irrespective of the presentation format.

Overall, the abovementioned behavioral and neuroimaging evidence hints at the presence of an evolutionarily ancient system for relative magnitudes, known as the RPS. While research is still in its infancy, if the proposition holds, RPS could be leveraged to help foster a deeper, intuitive understanding of symbolic fraction magnitudes (Lewis et al., 2016).

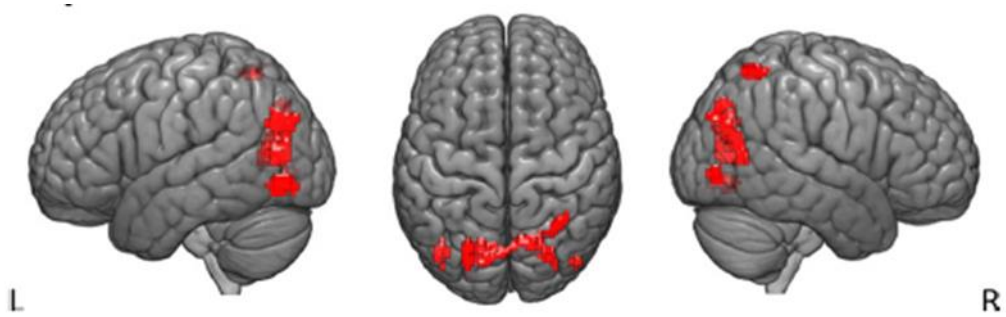

Figure 7: Joint activation of brain regions (red) for both symbolic and non-symbolic formats (e.g. fractions, decimals, dot patterns, pie charts) (Figure from Mock et al., 2018)

1.3 Can the RPS help in developing an understanding of symbolic fractions?

To inform fraction intervention and instruction, several questions on the RPS model of learning still need to be examined. For instance, some of the most compelling questions include (but are not limited to)- what is the developmental trajectory of the RPS? Where is the brain's ratio processing system and how is it connected to other systems? At what age is the RPS most important in symbolic fraction instruction? How does formal education impact RPS acuity? What is the role of the RPS in mathematics learning difficulty?

However, a preliminary hypothesis on how RPS can be leveraged to support fraction learning has been put forward by Lewis, Matthews, & Hubbard (2016). Given the pervasive difficulties in fraction learning, there is no doubt that conventional fraction instruction might not be as efficient. Indeed, as mentioned earlier a disproportionate focus on the partitioning construct (Pitkethly & Hunting, 1996; Siegler et al., 2010) leads students to overgeneralize the whole number strategies to fractions (Ni & Zhou, 2005). For instance, early fraction instruction often relies on children to identify the shaded fractions for which they *count* the number of shaded parts and the number of total parts. This count-based strategy could lead students to apply whole

number schemas to fractions (Mack, 1995). According to the hypothesized model (Lewis et al., 2016), such instruction might not be able to take advantage of the innate capabilities of the RPS. Therefore, the authors emphasize fraction teaching that uses pie-charts and number lines (continuous, uncountable representations) that helps in the holistic magnitude understanding of fractions and aids in building the symbol-percept links implicitly (Lewis et al., 2016). They argue for instruction that helps in building the symbol-percept links, or links between symbolic fractions and their non-symbolic instantiations (e.g. line ratios, circle areas, etc) which can aid in an intuitive understanding of holistic fraction magnitudes. The importance of understanding the holistic magnitude of fractions has also been emphasized by Siegler and colleagues (2011) in their integrated theory of numerical development. Particularly, fraction magnitude knowledge is shown to predict proficiency in fraction arithmetic (Siegler et al., 2011) which further predicts success on high school mathematics concepts like algebra (Booth & Newton, 2012). Additionally, the RPS acuity might differ between individuals and future research on individual differences in RPS acuity and their relation to symbolic fraction knowledge might be able to provide a better understanding of the role of RPS in fraction learning.

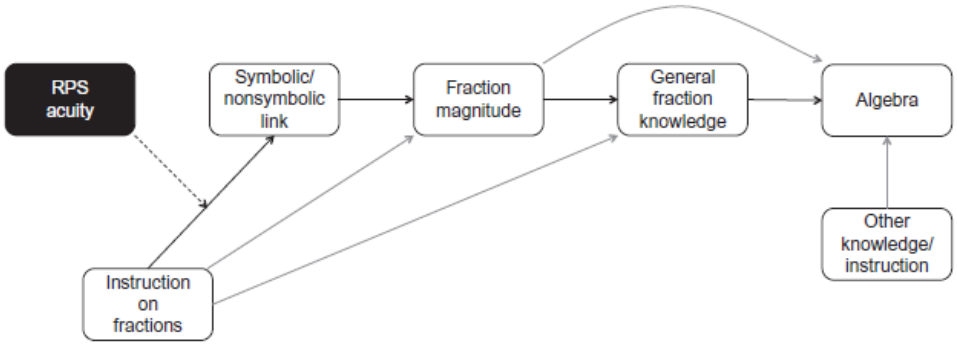

Figure 8: Conceptual model of the role of RPS in fraction learning (Figure from Lewis et al., 2016)

1.4 Summary

The notoriously difficult concept of fractions, interference by whole number strategies, and paucity of research on relative magnitudes might have led the innate constraint theorists to relegate fractions to a biologically secondary concept. On the contrary, emerging evidence has revealed specific cortical regions to be tuned to non-symbolic ratios and symbolic fractions. Proponents of the cognitive primitive or the RPS theory believe that the challenges encountered during fraction learning arise from the current teaching methodologies that do not leverage the existing perceptual abilities (Ni & Zhou, 2005; Lewis, Matthews, & Hubbard, 2016). In their hypothesized model, Lewis and colleagues (2016) further suggest leveraging the RPS to improve fraction instruction in the classroom. They urge educational experiences to focus on building symbol-percept links to strengthen the pre-existing RPS abilities. Specifically, they support pedagogical practices that build connections between the non-symbolic (pie charts, number lines) and symbolic (fractions, decimals) formats and resist instructional practices that rely solely on counting and partitioning schemes that may activate the incompatible ANS structures. To shed light on the relatively novel RPS theory, Study I and II examined the presence of the RPS using both behavioral and neuroimaging methods.

CHAPTER III

THE ROLE OF GAME-BASED LEARNING IN FRACTION UNDERSTANDING

Play has long been acknowledged as an important tool for the cognitive development of children. Piaget (1962), for instance, describes play as an integral tool to build connections between prior knowledge and new information. Vygotsky (1980) considers play to create a *zone of proximal development*, or scaffolded learning, which helps children to comprehend and grasp complex content. Csikszentmihalyi (1990) also finds the state of *flow*[5] during play to result in deeper engagement with the task and, thus, a deeper form of learning. Over the years, researchers and psychologists have studied the significance of play in learning (e.g., Gee 2003; Kafai and Ching 1996; Malone 1981; Prensky 2001; Squire 2002). This has led national educational institutions and international organizations to recommend playful learning as part of the curriculum (ASER, 2020; NCTM, 2014; NMAP 2008; OECD, 2019). With technological advancement, the role of play has informed the principles of *game-based learning* and hence, an increased interest in the adoption of serious video games in educational spaces (Plass, Homer, & Kinzer, 2015). Research on *digital game-based learning* has shown the potential for learning abstract concepts, supporting classroom instruction, and presenting content engagingly and innovatively (Al-Azawi, Al-Faliti, & Al-Blushi, 2016; Prensky, 2001). Despite their potential, some notable drawbacks of game-based learning include lack of acceptance by educators as educational tools, the difficulty of integration into formal schooling, and the lack of

[5] "a state in which people are so involved in an activity that nothing else seems to matter; the experience is so enjoyable that people will continue to do it even at great cost, for the sheer sake of doing it." (Cskikszentmihalyi, 1990, p.4).

transfer of knowledge gained in the game to the real world (Egenfeldt-Nielsen 2006; Ferdig 2007; Gros, 2015). The objective of this chapter, therefore, is to provide an overview of game-based learning in the domain of mathematics. The current landscape of education technology for fraction understanding will also be detailed in this chapter. Lastly, the design of the fraction game *Math Mathews Fractions*, which was designed in our laboratory by Prof. Marie-Line Gardes (my co-advisor on this thesis) and used in Study III, will be discussed in detail.

1.1 Game-based Learning: Definition and Terminology

To date, there is no consensus on the definition of game-based learning. This is mainly due to the variety of games that target different disciplines, learning design principles, and a wide variety of learning outcomes (skills, motivation, engagement, behavior, etc) (Boyle et al., 2016; Connolly et al., 2012). For instance, Salen & Zimmerman (2004) define it as “a system in which players engage in an artificial conflict, defined by rules, that results in a quantifiable outcome (p.5)”; Hays (2005) provides the following definition “game is an artificially constructed, competitive activity with a specific goal, a set of rules and constraints that is located in a specific context (p.15)”; Chen & Michael (2005) state “games that do not have entertainment, enjoyment or fun as their primary purpose”; and Van Eck (2015) defines it as “the use of games within an existing lesson, classroom, or other instructional contexts where the intent is at least as much to learn rather than to (exclusively) have fun (p. 144)”. Most definitions, therefore, focus on specific features of a game-based learning environment. These include quantifiable learning outcomes (Shaffer, Halverson, Squire, & Gee, 2005), the balance between content and play (Plass, Perlin, & Nordlinger, 2010), feedback (Prensky, 2001), and a set of rules and conflicts to engage the players (Salen & Zimmerman, 2004). While the debate on the definition of game-

based learning is hard to reconcile (Plass, Homer, & Kinzer, 2015), the abovementioned features are necessary to consider when referring to or designing game-based learning environments. The games discussed in this chapter mostly include these features and so does the game that is designed in our lab (*Math Mathews Fractions*).

Similar to the issues related to the definition of game-based learning, extensive debate exists around the usage of terminology. Some common terms include serious games, instructional games, learning games, game-based learning, and computer games (Boyle et al., 2016; Connolly et al., 2012; Hays, 2005; Tobias & Fletcher, 2012; Wouters & van Oostendorp, 2013). In educational research, the terms serious games, educational video games, and game-based learning (Boyle et al., 2016; Connolly et al., 2012) are most commonly used. Here, in this dissertation, the term *game-based learning* will be used to refer to all digital games intended for educational purposes (for both cognitive and affective outcomes).

1.2 Game-based Learning in Mathematics Instruction

While the use of game-based learning has gathered interest for improving learning outcomes across all disciplines, this trend is particularly strong for the domain of science and mathematics (Hainey et al., 2016). The use of technology for mathematics teaching and learning is strongly emphasized in many curricula (ASER, 2020; NCTM, 2014). Mostly, this emphasis on game-based learning might have emerged due to outdated methods of teaching that are disproportionately focused on rote memorization of procedures and facts (Hoyles, 2016; Schoenfeld, 2004). This type of pedagogy is practiced in most mathematics classrooms in many countries (Albert & Kim, 2013; Ayinde, 2014). To date, similar methods, where the teacher is the

authority and procedural content is disproportionately favored over problem-solving skills, prevail (Conway & Sloane, 2005; MaaB & Artigue, 2013). Predictably, mathematics learning that disproportionately focuses on rote memorization of procedures has led to issues with the motivation and engagement of learners (Hoyles, 2016; Star et al., 2014). As mentioned above, this has led educational organizations to press for the need for curricular reform in mathematics instruction, especially focusing on game-based learning environments (ASER, 2020; NCTM, 2014). It is imperative to note that while game-based learning might aid in the motivation and problem-solving skills of learners, mastery of procedural content and facts is equally important for mathematical literacy (Kadijevich, 2018).

Game-based learning has the capability to transform mathematics instruction by helping students construct and engage with complex mathematical knowledge (Bray & Tangney, 2017). It also has a positive impact on student's motivation and attitude towards mathematics (Rosas et al., 2003; Ke & Grabowski, 2007; Ke, 2008). For instance, the game Beyond Nintendo was designed in line with the curriculum on basic mathematics and reading comprehension for first and second graders in Chile. The study reviewing the game Beyond Nintendo in 1274 students showed a positive influence of the game on student's motivation and engagement (Rosas et al., 2003). Besides motivation, a game-based mathematics summer program also found improvement in students' attitudes towards mathematics (Ke, 2008). Additionally, a meta-analysis by Dvijak and Tomic (2011) found pedagogically designed game-based learning to enhance positive attitudes of students towards mathematics. Both positive attitudes towards mathematics and higher motivation are associated with mathematics achievement (Schiefele & Csikzentmihalyi, 1995; Singh, Granville, & Dika, 2002; Chen et al., 2018). Though many studies reveal positive affective outcomes (motivation,

engagement, interest, and attitude towards math) of game-based learning, data on the cognitive outcomes (achievement in mathematics) is still inconclusive.

A recent meta-analysis examining the effectiveness of game-based learning in mathematics notes a staggeringly low percentage (11%) of studies assessing the empirical effectiveness of games on learning (Byun & Joung, 2018). Of the studies that did examine the effectiveness of games, the authors found a small to moderate effect of the game intervention for a majority of studies (9 out of 17), with an overall effect size of $d = 0.37$[6]. This was higher than the effect size found in a previous meta-analysis $d = 0.16$ (Cheung & Slavin, 2013). These effect sizes are likely inflated because of publication bias. Indeed, it has been found that effect sizes reported in studies that are published in peer-reviewed journals are higher than effect sizes reported in unpublished studies and/or studies in proceedings (journals: $d = 0.44$, unpublished: $d = 0.14$, proceedings*:* $d = 0.08$) (Wouters & van Oostendorp, 2013). The small to moderate effect sizes observed in prior studies were mostly due to a lack of randomized studies (Cheung & Slavin, 2013; Li & Ma, 2010; Slavin & Lake, 2008), small sample sizes (Cheung & Slavin, 2013), study's methodological quality (Young, 2017), and issues with aligning the game outcome measures to outside-game measures of learning outcomes (Slavin & Lake, 2008). Additionally, Byun & Jeong (2018) point out that most mathematics games reviewed in the meta-analysis were predominantly "*drill and practice games*" that focus on procedural fluency and hence, posed limitations for building more complex problem-solving skills. Further, factors such as game design, transfer of knowledge and skills, and adequate teacher support were often overlooked in the game-based learning environments (Barnett & Ceci,

[6] $d < 0.2$ – small, $d \sim 0.5$ – moderate, and $d > 0.8$ – large effect, Cohen (1998).

2002; Linderoth, 2012; Tobias et al., 2014; Rick & Weber, 2010). As Drijvers (2015) rightly states " *The three factors (design, teacher, and educational context) identified above may seem very trivial, and to a certain extent they are quite straightforward indeed; however, their importance, I believe, can hardly be overestimated and to take them into account in educational practice is far from trivial* (p.14)". Therefore, technology-enhanced mathematics instruction may be of great advantage to learners specifically in developing robust, complex, and flexible mathematical knowledge, only if it considers a strong game design, adequate support of the teacher, and transfer of skills to real-world problems.

1.3 The Role of Game-based Learning in Fraction Understanding

As reviewed in chapter I, the fraction concept is a complex construct composed of multiple subconstructs or interpretations. These different interpretations make mastery in fractions challenging. Besides, most often the part-whole concept of fractions takes the major share in classroom instruction (Fuchs et al., 2013) as well as in textbooks (de Souza & Powell, 2021). This over-representation of the part-whole subconstruct hinders the acquisition of other fraction subconstructs and concepts like improper fractions and the infinite divisibility of fractions (Pitkethly & Hunting, 1996 as in Misquitta, 2011) leading students to err on basic fraction concepts. Predictably, researchers and educators demand a more holistic understanding of all interpretations of fractions (Kieren, 1993; Lamon, 2001 as in Charalambous & Pitta-Pantazi, 2007). Game-based learning, in particular, offers unique affordances (e.g. visualizations, manipulations, the introduction of complex scenarios, experiential and immersive integration of curricula; Geiger et al., 2010; Olive et al., 2010 as in Bray & Tangney, 2017, Vandercruysse et al., 2017) that can be utilized to build a better understanding of the complex construct of fractions. To this end, studies on the impact of game-based

learning on fraction outcomes have been conducted. Here, we will discuss five serious games in the domain of fractions; Refraction (Martin, Petrick-Smith, Forsgren, Aghababyan, & Janisiewicz, & Baker, 2015), Semideus (Ninaus, Kiili, Mcmullen, Moeller, 2017; Kiili et al., 2018), Motion math (Riconscente, 2013), Abydos (Masek, Boston, Lam, & Corcoron, 2017), and Slice fractions (Cyr et al., 2019). These game-based learning environments on fractions have typically focused on specific subconstructs, for example, the game Refraction and Slice fractions focus on the partition subconstruct (Martin et al., 2015; Cyr et al., 2019), whereas the game Semideus and Motion math focus on the measurement interpretation (Ninaus et al., 2017; Riconscente, 2013). While it is important to develop a deeper understanding of each of these subconstructs, to date, no game exists on addressing the multiple representations of fractions. Additionally, the transfer of knowledge to real-world outcomes (assessments) is something that is largely missing in these games. To this end, *Math Mathews Fractions* was designed to support teachers in building connections between the different subconstructs of fractions and each level of the game was specifically aligned to the learning outcomes in the french school curriculum (Zarpas & Gardes, 2019). The game, *Math Mathews Fractions* is designed for students who already have prior knowledge of fractions. A detailed introduction to *Math Mathews Fractions* will be provided in the next section. Meanwhile, an exhaustive overview of games that have helped improve fraction learning outcomes is provided here. The games in the domain of fraction learning discussed here include Refraction (Martin et al., 2015), Semideus (Ninaus et al., 2017; Kiili et al., 2018), Motion math (Riconscente, 2013), Abydos (Masek et al., 2017), and Slice fractions (Cyr et al., 2019). We will describe each of them below.

Refraction (Martin et al., 2015) is based on the partitioning subconstruct of fractions. The study is designed to examine how different ways of splitting or partitioning can influence students' (third graders) understanding of fractions. In this game, players save animals in the spaceship by powering the ships with the correct amount of laser beam. The amount of fraction required is displayed on the spaceship (e.g. 1/12) and the students have to split the laser beam (using 1/2 or 1/3 splitters) to create the correct fractional amounts (**Figure 9**). Feedback is provided through a message- warning if the player is incorrect.

To assess the impact of the game Refraction on third grader's partitioning construct, the authors made use of the log data in the game to cluster students according to their splitting strategies and frequency. The log data was also used to analyze the performance on transfer tests of fraction knowledge. Thus, the log data helped to group the students according to their splitting strategies and frequencies and was also used to assess the impact of splitting on students' overall fraction knowledge. Findings reveal that the game improves students' fraction understanding on the transfer tests. Additionally, the more the students explored the different ways of splitting the laser beam the better their performance was on the transfer test. Interestingly, the study highlights the importance of learning complex mathematical concepts (like fractions) through means of exploration. In terms of assessing the effectiveness of the intervention, the major drawback of the study was the lack of a control group and standardized assessments (Bertram, 2020). Therefore, this makes it difficult to attribute the learning gains exclusively to the game. Also, the game exclusively focuses on the partitioning construct of fractions which is already practiced more in the classroom as compared to the other constructs (Fuchs et al., 2013). This

disproportionate focus on partitioning could essentially impede the learning of other concepts and thus impact the transfer of learning outside the game.

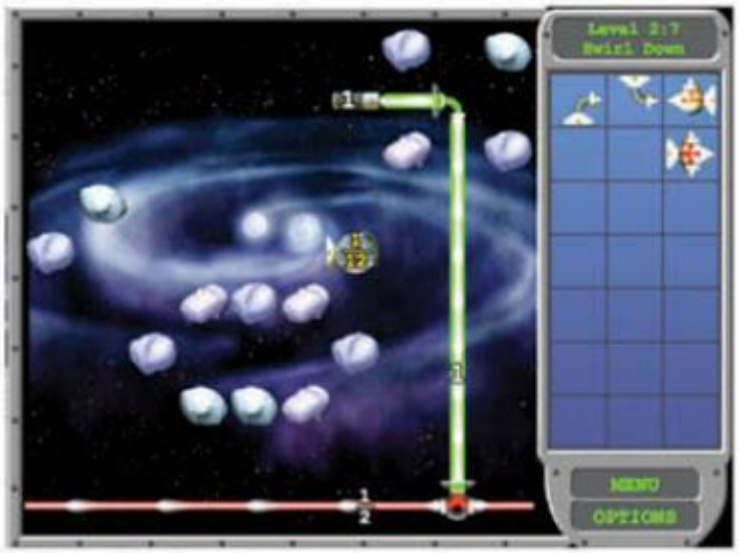

Figure 9: Example of the spaceship and the laser beam that needs to be split to generate the correct fraction amount (1/12) (Figure from Martin et al., 2015)

Similar to Refraction, the game Slice fractions (Cyr et al., 2019) begins with a visual representation of the concept of splitting fractions (part-whole representation of fractions) but gradually advances to concepts like fraction comparison and equivalent fractions. The players are required to remove obstacles that are blocking the path of a mammoth. To remove the obstacles, they have to split pieces of ice and lava. As the game progresses, the students advance from visual representations to symbolic representations of fractions (**Figure 10**). Using a quasi-experimental design, the impact of the game was assessed on third graders' fraction understanding (Cyr et al., 2019). For the fraction knowledge test, the authors selected 13 items from the Trends in International Mathematics and Science Study (TIMMS). These items mostly consisted of part-whole visual representations and symbolic fraction magnitude comparison type of questions. Notwithstanding the relatively small sample size (n = ~ 18 per group) and the lack of structured feedback, the study shows a positive impact

of the game on fraction learning which indicates that the game group performed better than the group with only traditional instruction.

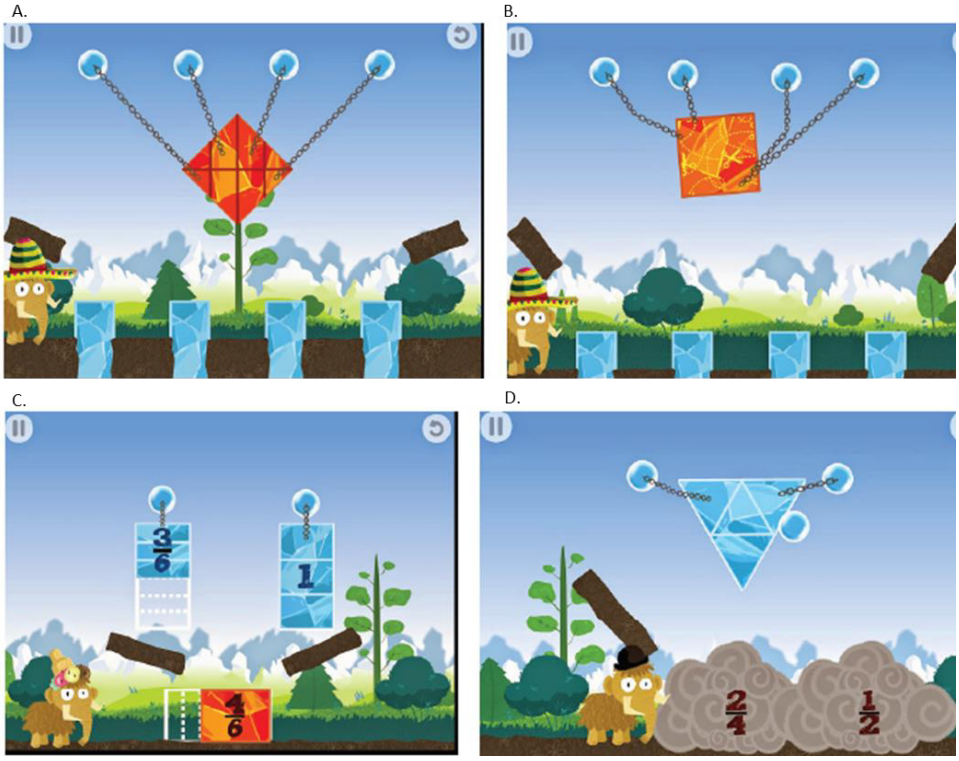

Figure 10: Examples of the different levels in the game Slice Fractions (Figure from Cyr et al., 2019). Panels A and B show levels with visual representations of fractions and Panels C and D present symbolic notations of fractions

The third game, Motion math (Riconscente, 2013) was designed based on the measure interpretation of fractions (detailed in Chapter I, the measurement interpretation signifies the inherent property of magnitudes of fraction) utilizing number lines to teach fraction concepts. Here, the player has to tilt the device (iPad) to direct the falling fraction (percentage and decimal are also presented) to the correct place on the number line (**Figure 11**). Feedback in the form of audiovisual responses is generated for both correct and incorrect answers. Interestingly, incorrect answers also generate instructional hints in the form of arrows in the direction of the right answer. Fractions are presented both in symbolic (numerals) and non-symbolic (pie-charts)

representations. To examine the effectiveness of the game on fraction learning of fifth graders, 122 students participated in this study (Riconscente, 2013). Findings of the study evaluating Motion math reveal significant improvement in fraction knowledge (as measured by an adapted test that consisted of both NAEP and TIMMS items) in the game group as compared to the comparison control. Additionally, the findings also show a positive influence of the game on students' self-efficacy and attitude towards fraction learning. However, one drawback in terms of assessing the effectiveness of the game was that teachers were explicitly asked to not teach fractions in the comparison control group, therefore, it is hard to comment on the effectiveness of the game as compared to other modes of instruction.

Figure 11: Examples of the different level's that players encounter during the game Motion Math (Figure from Riconscente, 2013)

Semideus (Kiili et al., 2015) was also designed based on the measure construct of fractions to address the understanding of rational numbers as magnitudes (using number lines). The game's foundation is based on the integrated theory of numerical development (Siegler et al., 2011). According to the theory, all types of real numbers have numerical magnitudes and, hence, can be mapped onto number lines.

Importantly, the number line can be used to teach students the unique property of density of rational numbers (Behr et al., 1993). Indeed, intervention studies show that practicing number line tasks lead to improved fraction knowledge (Barbieri et al., 2019; Fuchs et al., 2016; Saxe et al., 2013). Therefore, the game includes four different task types that make use of number lines. These are magnitude estimation, magnitude ordering, magnitude comparison, and density tasks. The task of the player is to collect coins while solving these four tasks. In the study assessing the effectiveness of Semideus, 54 fourth graders were assigned to either a game group or a control group (Kiili et al., 2018). Findings show significantly better performance of the game group on magnitude estimation and ordering tasks (**Figure 12**) as compared to the control group. The study also analyzed the in-game performance and showed the overall game performance to be associated with student's post-test rational number knowledge. Thus, in addition to testing for the effectiveness of game-based learning, the study also supports the use of in-game metrics as learning indicators that might help teachers personalize instruction in the classroom.

Figure 12: An example of magnitude estimation and ordering tasks in the Semideus game (Figure from Kiili et al., 2018)

Finally, Abydos (Masek et al., 2017) includes three mini-games based on real-world problems. These mini-games focused on the concepts of equivalent fractions,

identifying least common multiples, addition, and subtraction of fractions (**Figure 13**). Within each game, the level of difficulty increases as the player advances. Students in Australia (aged 10-11) participated in this study over four weeks. The intervention and control groups both practiced fraction concepts during this time. Since the study was based on a Blended Learning model (a system of instruction where both technology and teacher-mediated support is present during learning; Graham, 2006), the teacher played an active role in the game group by debriefing and discussing the fraction concepts practiced in the game. The students were administered a pretest and posttest based on a modified version of standard assessments and country-specific curriculum. These questions particularly addressed the competencies practiced in the game such as equivalence of fractions, finding the least common multiple, the addition of proper and improper fractions with both like and unlike denominators. The results revealed a significant increase in the post-test scores for the game group as compared to the control group.

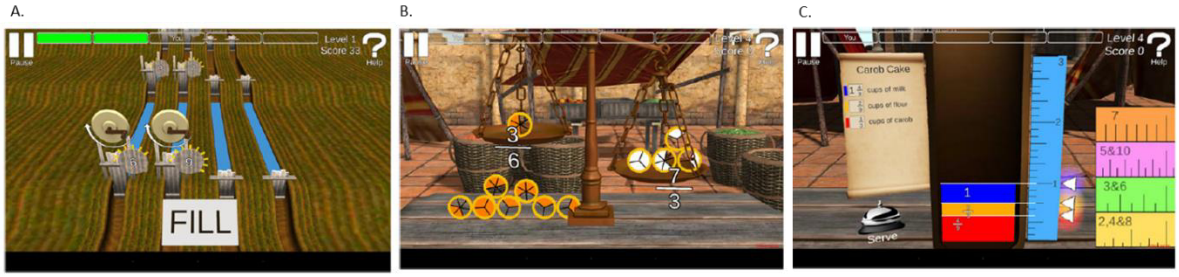

Figure 13: Example of the three mini-games in Abydos. A. The task depicted here is to determine the least common multiple of the set of numbers. B. The equivalence task C. The addition and subtraction task

The aforementioned studies demonstrate the potential of game-based learning in supporting fraction knowledge. They also provide a model for other researchers and game developers to consider when designing future technology on fraction learning. Likewise, depending on their students' needs teachers can use existing game-based

platforms to design classroom instruction. However, as mentioned above, one of the major limitations of prior game-based fraction learning platforms is the lack of support for teachers and students to build a holistic understanding of all subconstructs of fractions and the transfer of knowledge to real-world outcomes or assessments. Despite researchers and educators' demand for a more holistic understanding of all interpretations or sub-constructs of fractions (Kieren, 1993; Lamon, 2001 as in Charalambous & Pitta-Pantazi, 2007), prior games all focus on a specific interpretation/sub-construct of fractions (Behr et al., 1993) (**Figure 14**). To this end, *Math Mathews Fractions* was developed to provide support to educators to build connections between multiple representations of fractions.

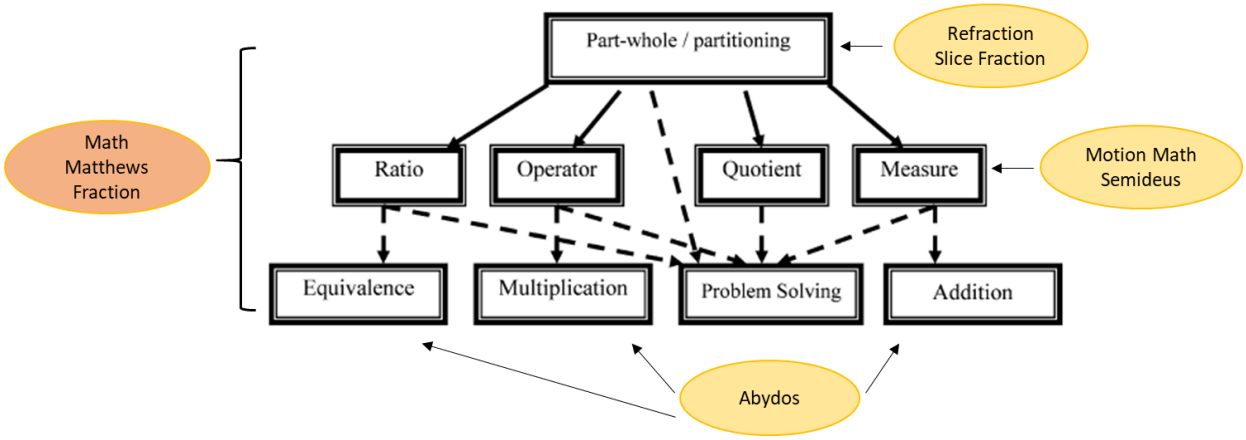

Figure 14: The different game-based learning environments and their underlying fraction constructs

1.4 Math Mathews Fractions

Math Mathews Fractions is an educational video game developed by the studio Kiupe in collaboration with my co-advisor. The game is about the adventures of a pirate who has to collect gems (treasure) by solving different challenges (i.e., modules). The game progression is in line with the objectives and curriculum standards of the French school system for children aged 9-12. Thus, the play situations (i) increase in difficulty

throughout the game and (ii) remain appropriate for children aged 9-12. The modules are typically different types of questions involving rational numbers. Players must choose or construct the answer to proceed further. For example, in the Dragon module, students are asked to feed the dragon the specified amount of fraction (e.g. 2/6). For gaining maximum gems on this module, the students must slice the rectangle into six parts and feed two parts to the dragon in their first attempt. If the students err on any of the modules they cannot proceed to the next stage. There are 13 modules that are based on specific curriculum standards in the French school system. Each module is presented ten times throughout the game and can be presented several times during a level. The modules include specific fraction competencies like fraction concepts, arithmetic, word problems, number lines, and decimals. Given below is a detailed description of each module and the associated learning competency.

1.4.1 Hungry Dragon

The hungry dragon module focuses on the part-whole subconstruct by making connections between the symbolic form of the fraction to the unit surface provided for constructing the fraction. In this module, the student has to feed the dragon by partitioning the object/ surface according to the fraction displayed on the screen (**Figure 15**). As the level increases, the nature of the fraction progresses from less than 1 to greater than 1, and the written form of fraction changes from simple ($\frac{a}{b}$) to mixed ($a + \frac{1}{b}$) fractions.

Figure 15: The Hungry Dragon module where the student has to cut the object in 6 parts and feed 2 parts to the dragon

1.4.2 Broken Cogs

In this module, the part-whole subconstruct of fractions is emphasized. Here, the student has to reconstruct a disk by using different sized parts provided to them (**Figure 16**). The task here is to advance the learner's knowledge of unit fractions and understand the property of equal and unequal shares. As the levels increase, the complexity of the task increases by increasing the number of disks to be reconstructed, the number of different sized parts to choose from, and the size of the fractions ranging from smaller than 1 (levels 1 to 6) to greater than 1 (level 7 onwards).

Figure 16: Example of the module Broken Cogs where the student has to reconstruct the entire disk/cog by the two equal disks provided below. An example of a more complex problem on this module would include extra parts to choose from when constructing the entire disk.

1.4.3 Warrior

This module is based on associating the partitioned surface that is provided in different shapes (rectangle, pentagon, hexagon, trapezoid) with simple and mixed types of fractions. The students have the option to select the number of unit surfaces (by clicking on '+') required to construct the fractions (**Figure 17**). The units are indicated on the small panel next to the figures. For example, if the learner has to make the fraction $\frac{8}{5}$, they can select two equally partitioned pentagons and then select all 5 parts in one pentagon and 3 parts in the second pentagon to proceed. Additionally, the concept of equivalence and addition of fractions is also included in the higher levels of this module. For instance, if the student has to construct the fraction $\frac{2}{3}$ with each unit area sliced into 6 equal parts, for the correct answer in this module the student would be required to understand equivalence and select 4 out of the 6 equal parts.

Figure 17: Example of the Warrior Module A. The student has to select the unit surface to make the fraction $\frac{3}{4}$. For this, they can use one-unit surface and select three parts of the four parts on the rectangle to proceed forward. B. Here, the student is tested for the property of equivalence, they have to select 7 triangular parts of the three equally sized parts on the three-unit surfaces to generate a fraction equivalent to $\frac{14}{6}$ (e.g. $\frac{7}{3}$).

1.4.4 Trapped passage

The trapped passage module is focused on the measure construct of fractions. In this module, the student is required to place the fraction on the graduated number line (**Figure 18**). This module is aimed at developing an understanding of fraction magnitudes. The module begins from level 6 of the game and the complexity of the fractions to be placed on the number line increases with the levels. The simple fractions (e.g. halves and thirds) are presented in levels 5 and 6 and the more difficult fractions (e.g. fifths', sevenths', eighths) are presented in levels 8, 9, 10, and 11. Additionally, the concept of equivalence is also tested by asking the learner to place the fraction $\frac{a}{b}$ on the line that is graduated into *2b* equal parts. Fraction addition with like denominators is also tested on this module at higher levels.

Figure 18: In this example of Trapped Passage, the student has to place the fraction 4/2 on the number line

1.4.5 Door with weights

This module requires the student to make connections between the symbolic fractions and the corresponding colored surface (**Figure 19**). The competency practiced in this module is using fractions to measure the quantities and making connections between the symbolic form of fractions and the pictorial representations. The increased difficulty of the module is based on the type of fraction (simple, mixed) and the type of surface (rectangle, hexagon, pentagon) presented. Additionally, the students also practice the concept of equivalence and addition of fractions by making connections between the symbolic and pictorial representations. For instance, the student will see the question $\frac{2}{6} + \frac{8}{6}$ written on the weights on the left side. They are required to solve the addition task and then place the resultant fraction on the correct pictorial representation.

Figure 19: An example of Door with Weights module. Here, the students have to place the three fractions corresponding to the colored surface on the right. For example, $\frac{10}{6}$ needs to be placed on the first surface where the hexagon is cut into six parts, each part corresponding to $\frac{10}{6}$

1.4.6 Graduated Bridge

In this module, students have to select the fraction corresponding to the marked point on the number line. In addition to understanding the measure subconstruct, the students also practice equivalence concepts. Fractions are presented with a blocked numerator or denominator to enforce students to utilize the concepts of equivalence on the number line. For example, if the number line is segmented into three parts and each part corresponds to one third, by blocking the denominator of the fraction to 6 the student must explore the relationship between $\frac{1}{6}$ and $\frac{1}{3}$ to successfully solve the problem. The module increases in difficulty by modulating the relationship between the denominator of the fraction and the graduated number line.

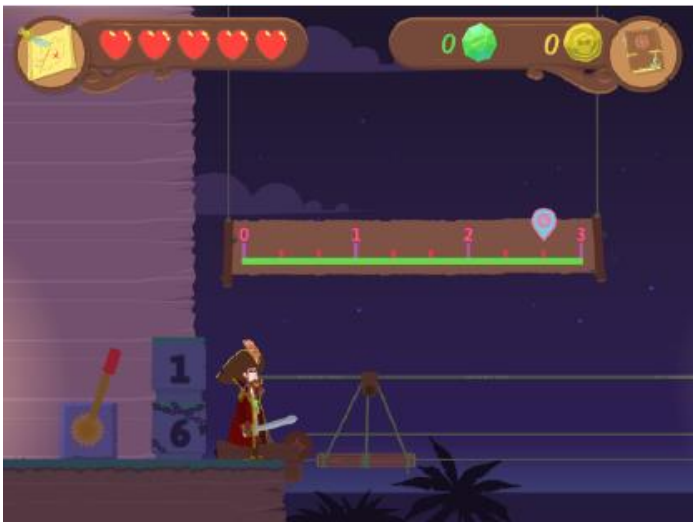

Figure 20: The Graduated Bridge Module. Here, the denominator is locked to 6 and the point corresponds to $\frac{8}{3}$, the student must understand the relation between one-sixth and one-third and then find the numerator of the fraction equivalent to $\frac{8}{3}$

1.4.7 Totem

Here, the student has to construct the totem of a given length by using multiple units of the totems provided. The unit is represented by the totem with '1' written below it. This module also aids in understanding the operator construct of the fraction. For instance, in the given example (**Figure 21**), the student must construct the totem of value $\frac{1}{2}$ with an option to choose multiple totems of values one-fourth, one-half, or one-third. To solve this problem with one-fourth, the student must understand the operation; $a \times \frac{1}{4} = \frac{1}{2}$, to be able to use two totems of one-fourth value. Similar to other modules, the complexity of this module increases with the type of fraction to be constructed and the types of fractions that are provided for the same. This module is also present in decimal form and helps students to build connections between the magnitudes of decimals and fractions (**Figure 14. B**).

Figure 21: Example of the Totem module. A. Here the learner can use either one half or two one-fourths to construct the one-half. B. This is a complex version of the same module where the learner has to create a totem of length $\frac{12}{10}$ (or 1.2) by using the totems 0.5 and 0.1

1.4.8 Organs

The concept practiced in this module includes the comparison of fractions and using the fractions to measure length. The unit is represented by the red tube with '1' marked on the top. The student has to choose the length of the segment that corresponds to the given fraction of unit length. The module is present in levels 1, 2, 3, 4, 7, and 8. The complexity of the module increases by introducing both proper and improper fractions to the students. Additionally, higher levels also include some unnecessary tubes that may not have any corresponding fractions.

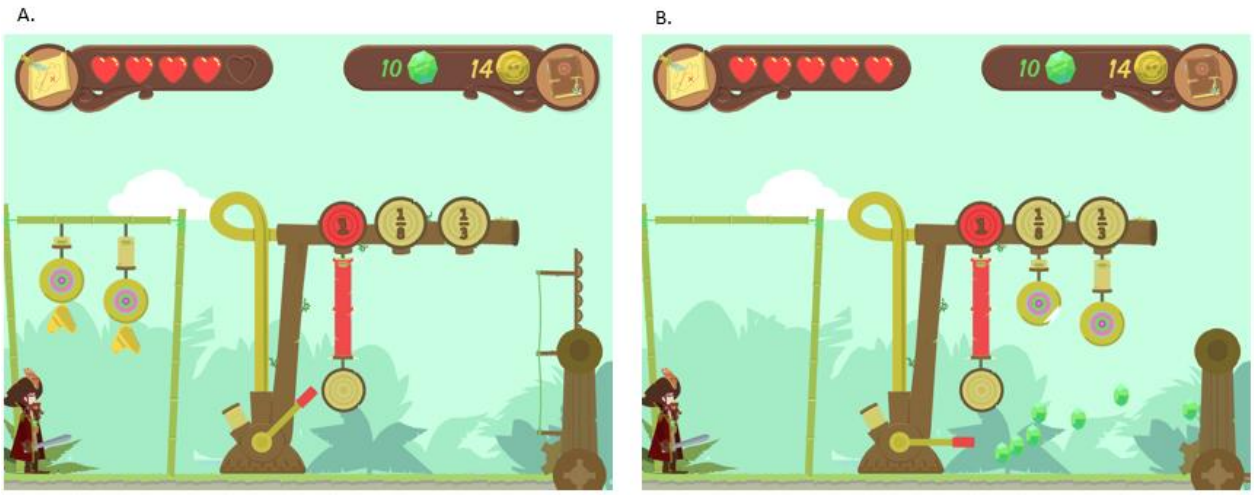

Figure 22: Example of the module Organs. A. The learner has to place the tubes with the arrows shown in the bottom to the correct fractions on the right. B. The learner has placed the tubes correctly by placing the smaller tube on the one-eighth fraction and the larger tube on the fraction marked as one-third

1.4.9 Skull

In this module, the students have to arrange the fractions in an increasing order. The focus of this module is to understand the concept of fraction as a number with a specific magnitude to be able to arrange the fractions in increasing or decreasing order. To increase the complexity of the module, the fractions are presented with common multiples of the denominator (**Figure 23**) and different denominators. Additionally, the fractions to be compared increase from two to three and finally to four at the higher levels of the game.

Figure 23: An example of the module Skull. Here, two fractions have denominators that are multiples of each other. One of the concepts the students might use is to reduce the fraction $\frac{12}{16}$ to $\frac{6}{8}$ and then compare that magnitude with $\frac{9}{8}$

1.4.10 Pit, Trapped Chest, and Spider

All these modules are based on the concept of finding an equivalent fraction or association of two equivalent fractions (**Figure 24**). The numbers to be compared are presented in the form of a simple fraction, mixed fraction, and decimal. While the module Pit and Trapped Chest are present in lower levels of the game, the module spider is present after level 8 and includes the addition of fractions with unlike denominators.

Figure 24: Example of the module Pit, Trapped Chest, and Spider. A. In Pit, students must choose from the two fractions in yellow on the left to match the fraction in red. B. Trapped Chest module requires students to find the correct equivalent fraction to match the decimal presented on the banner on the left side. C. In the module Spider, the students have to associate two equivalent fractions, sometimes this module also has addition problems that have to be associated with the correct answer

1.4.11 Riddles

This module comprises word problems. The complexity of the word problems is linked to the number of steps and operations required to arrive at the correct answer. The word problems are related to the following expression a/b × b = a, and the three types of questions include 1. What is the quantity 'a'? (**Figure 25 A**) 2. What is the quantity 'b'? (**Figure 25 B**) and 3. What is the fraction 'a/b'? (**Figure 25 C**).

Figure 25: Example of a word problem in the module Riddles. A. Surprised by the bravery and strength of the children, a third of the 66 pirates jumped overboard, and the rest quickly capitulated. How many pirates jumped over the board? B. One last Ptot hits another mango tree with his club and knocks down a third of its fruit. He puts 20 fruits in his bag. How many fruits were there in the tree? C. So, admire my collection of rare pearls. I have 60 of them if you take 20- I'll give you a good price. The Ptot shook his head again and this time he got angry. What fraction does it represent? (Translated to English)

The abovementioned 13 modules focus on the different representations of fractions. These modules help students understand fractions as a measure, operator, quotient, part-whole subconstructs as well as focus on important concepts included in assessments like equivalence, fraction arithmetic, and problem-solving. The game includes 12 levels and increases in difficulty to provide a scaffolded learning environment to the students.

The game was played through an application pre-installed on the tablets. Each student had to create a profile with a pseudonym before starting the game. The first level was preceded by a small video to familiarize players with the basic controls and rules of the game as well as to guide them about the objective of the game. The game

was configured in a way that each player had to correctly perform in all the modules that were visible to them in the game to finish the levels and only then could they proceed to the next level. The interface of the game also consists of a *journal* and a *calculator*. The journal was used to teach the player about the rules of each module and the fraction concepts involved in the module. Students could consult the journal anytime during the game by tapping on the icon. Additionally, the in-game performance of each of the students could also be measured by the different variables. These variables include the maximum level achieved, the number of attempts on each minigame, and the overall game performance. Study III of this dissertation is focused on analyzing the effectiveness of *Math Mathews Fractions* on fraction learning outcomes of fifth graders.

1.5 Summary

Traditional methods of fraction instruction are disproportionately focused on either a specific concept of fraction or on developing rote memorization of rules for procedural knowledge. Naturally, the difficulties encountered by students during fraction learning are persistent. In the era of technology, game-based learning, if designed and utilized appropriately can provide an efficient support system to educators. Game-based learning might support the understanding of complex concepts like fractions through visualization, manipulation, and building immersive environments. When coupled with adequate teacher support, students might be able to overcome misconceptions as well as enhance their motivation towards learning. To this end, we pre-registered a randomized controlled study to assess the effectiveness of *Math Mathews Fractions* on fraction learning outcomes of students in the fifth grade (study III).

RESEARCH AGENDA AND HYPOTHESES

Rational numbers play a critical role in bridging early natural number skills to domains such as algebra, geometry, and calculus. The importance of rational number computation is not only restricted to mathematics but also necessary for biology, physics, statistics, economics, sociology, and many other disciplines (Lortie-Forgues, Tian, & Siegler, 2015). Indeed, 68% of participants in a nationally representative sample of the United States workforce reported using fractions at work (Handel, 2016). Despite their importance, rational numbers, specifically fractions are difficult to understand. In 1978, for the National Assessment of Educational Progress (NAEP), 8th graders (13-14-year old's) from all over the US were asked to choose the correct answer to the sum of 12/13 +7/8. The most common answer was '19' and only 24% of the students were able to choose the correct answer (Carpenter et al., 1980). Recently, this item (12/13 + 7/8) was administered again to 48 eighth-graders to assess the impact of change in fraction instruction over the years. It was noted that the performance remained largely unchanged, i.e., 27% solved it correctly as compared to 24% in 1978 (Lortie-Forgues et al., 2015).

To this end, several researchers and educators sought to investigate the reasons for the challenges encountered during fraction learning (see Chapter I). In the ensuing years, several researchers in the field of numerical cognition postulated the *innate constraint account* (Ni & Zhou, 2005). The innate constraint account holds fractions to be biologically secondary and thus, not supported by the human neurocognitive architecture (Dehaene, 1997; Gallistel & Gelman, 1992; Geary, 2007). Recent studies, however, revealed processing of non-symbolic relational magnitudes by non-human primates, infants, and human adults (Jacob, Vallentin, & Nieder, 2012),

indicating the presence of a Ratio Processing System (RPS) (see Chapter II). These two conflicting propositions (innate constraint and the ratio processing system) demand further investigation in this domain. For this, the first part of the thesis is focused on using both neuroimaging and behavioral tools to examine the perceptual processing of non-symbolic relational magnitudes and their relation to symbolic fraction understanding thereby, shedding light on the RPS (Study I and II). Understanding the RPS might potentially help in designing perceptual learning interventions for symbolic fraction learning. Further, given the complex, multifaceted construct of fractions, both learning and teaching fractions might be challenging. Indeed, many adults and teachers have shown a limited understanding of fraction concepts (Ma, 1999; Newton, 2008; Siegler & Lortie-Forgues, 2015). To this end, the *Math Mathews Fractions* game was designed in the lab to support teachers during fraction instruction in the classroom. Specifically, the game focuses on building connections between the multiple representations of fractions thereby, building a better understanding of the mega concept of fractions. Thus, the second part of the thesis assesses the impact of the game on fraction learning outcomes of students in the fifth grade (Study III).

Experimental Hypotheses

The overarching aim of the thesis was twofold. The first aim was to shed light on the neurocognitive bases of fractions and add evidence to the ongoing debate on the innate constraint versus the RPS account. This objective might help cognitive psychologists and mathematics educators better understand the core cognitive systems that are involved in fraction skills which might further help design instruction and educational interventions. The second objective was to assess the effectiveness of a game-based intervention in the classroom for supporting complex fraction

concepts. The research data in this thesis is drawn from three separate studies, Study I and II examined the neurocognitive bases of fraction processing at both the behavioral and cerebral levels and Study III examined the effectiveness of a specific game (designed in the lab, *Math Mathews Fractions*) on fraction learning outcomes of fifth graders. Given below is a brief introduction of each study, highlighting the aim, hypotheses, as well as research design employed.

In study I, we aimed to investigate the presence of the ratio processing system in individuals with varying levels of math skills. We adapted a match to sample task (Matthews, 2015) in which participants were asked to match the stimuli (non-symbolic line ratio or symbolic fraction) on the left side of the screen (i.e. *target*) to one of the correct stimuli on the right side of the screen (i.e. *match and distractor*). The ratio was varied between the match and the distractor (i.e Ratio of Ratios, RoR) such that it was small in half of the trials and large in the other half. We hypothesized that if participants' accuracy was sensitive to the ratio between the match and the distractor (a ratio of ratios, RoR), then this would serve as evidence for the existence of the RPS. Indeed, representations of magnitudes are more difficult to distinguish when the distance between them decreases (i.e., a distance effect) and when the overall magnitude increases (i.e. a size effect). This combined effect of distance and size results in performance that varies with the ratio between the magnitudes (in the case of fractions a RoR) and is generally taken to indicate an intuitive representation of magnitude on an internal mental number line (Moyer & Landauer, 1967). Also, participants in the study varied in their mathematics skills from those who might be proficient in fraction processing (i.e., adults) to elementary school children who are in the process of acquiring fraction skills. By examining participants with varying levels of mathematical knowledge, we could also measure if the sensitivity to ratio magnitudes was affected

by the level of their mathematics skills. Thus, if there is an intuitive representation of ratio magnitudes (RPS) as posited in prior studies (McCrink & Wynn, 2007; Kalra et al, 2020; Jacob & Nieder, 2009; Woodruff & Premack, 1981), then irrespective of participants' level of symbolic mathematical knowledge or fraction competence, all participants would show the RoR effect. To this end, we also tested whether adults with dyscalculia (i.e., a learning disability that severely affects the acquisition of math skills; Iuculano, 2016; Price & Ansari, 2013) exhibit an RoR effect in that task. Additionally, to test for a single amodal representation of magnitudes (as hypothesized by Matthews & Chesney, 2015), we also tested another group of typically achieving adults on cross-format comparisons (i.e. matching a non-symbolic line ratio with the correct symbolic fraction). *If* these participants showed RoR effects for cross-format comparisons, *then* this might suggest intuitive access to abstract fractional magnitudes.

In study II, we aimed to test whether the culturally developed ability to represent both absolute and relative magnitudes symbolically (e.g., using natural numbers and fractions) relies on the neural representations of absolute and relative magnitudes in a non-symbolic format (as was posited by the neuronal recycling hypothesis, Nieder, 2016). To this aim, we adapted an fMRI adaptation paradigm used in Girard et al., 2021, and Perrachione et al., 2016 to investigate the neural representations of absolute and relative magnitudes in different formats. FMRI adaptation refers to the idea that repeatedly presenting a series of visual stimuli with a common property leads to a decrease in the activity of neurons that are sensitive to that property (Grill-Spector and Malach, 2001). This sensitivity is captured by the *neural adaptation effect*, measured by comparing blocks of stimuli that differ from one another with respect to the property (i.e., no-adaptation blocks) to blocks of stimuli that do not (i.e., adaptation

blocks). We assessed whether the processing of symbolic stimuli (i.e., numbers and fractions) relies on neural mechanisms supporting non-symbolic stimuli (i.e., lines and ratios) by examining the neural adaptation effects in these regions. It was hypothesized that *if* symbolic processing of magnitudes relies on mechanisms subserving non-symbolic processing, *then* the neural adaptation effect for non-symbolic (lines and line ratios) and symbolic (numbers and fractions) magnitudes will be observed in overlapping regions. Additionally, we used multivariate methods to explore the relationship between the patterns of activation associated with symbolic and non-symbolic stimuli across magnitude types. Here, we hypothesized that *if* the patterns of activity are similar between symbolic and non-symbolic stimuli (and may only depend on the type of numerical magnitudes, i.e., absolute versus relative), *then* this evidence would provide further support for the idea that the processing of symbolic magnitudes relies on the processing of non-symbolic magnitudes.

In study III, we aimed to evaluate the effectiveness of a game-based training of rational number skills on fraction knowledge of children in 5th grade. The game, i.e., *Math Mathews Fractions*, was designed to complement fraction learning in the classroom with a focus on understanding and building connections with the multiple representations of fractions. The study involved a total of eight sessions and was conducted over four weeks. During these sessions, students from the experimental group played with *Math Mathews Fractions* in the classroom (each student had a tablet) with limited teacher interaction. In contrast, students from the control group practiced similar rational number concepts with their teacher. Fraction knowledge of both groups was assessed pre and post-intervention using paper-based tests. We pre-registered three hypotheses. First, *if* the overall score on the fraction knowledge test is higher in the experimental group as compared to the control group, *then Math*

Mathews Fractions is a more effective tool than classroom instruction alone (which was matched on rigor and competency). Second, since the game is specifically designed for fraction learning, no difference between control and experimental groups on arithmetic fluency was hypothesized. Third, *if* the game influences affective outcomes, *then* students in the experimental group will show lower math anxiety scores at post-test as compared to the control group.

CHAPTER IV: STUDY I

Bhatia, P., Delem, M., Léone, J., Boisin, E., Cheylus, A., Gardes, M. L., & Prado, J. (2020). The ratio processing system and its role in fraction understanding: Evidence from a match-to-sample task in children and adults with and without dyscalculia. *Quarterly Journal of Experimental Psychology*, *73*(12), 2158-2176.

Abstract

It has been hypothesized that the human neurocognitive architecture may include a perceptual Ratio Processing System (RPS) that supports symbolic fraction understanding. In the present study, we aimed to provide further evidence for the existence of the RPS by exploring whether individuals with a range of math skills are indeed perceptually sensitive to non-symbolic ratio magnitudes. We also aimed to test to what extent the RPS may underlie symbolic fraction processing in those individuals. In a match-to-sample task, typical adults, elementary school children, and adults with dyscalculia were asked to match a non-symbolic ratio (i.e., target) to one of two non-symbolic ratios (i.e., the match and distractor). We found that all groups of participants were sensitive to the ratio between the match and the distractor, suggesting a common reliance on the RPS. This ratio sensitivity was also observed in another group of typical adults who had to choose which of two symbolic fractions match a non-symbolic ratio, indicating that the RPS may also contribute to symbolic fraction understanding. However, no ratio dependence was observed when participants had to choose which of two symbolic fractions match another symbolic fraction, suggesting that reliance on the RPS in symbolic fraction processing is limited and may not support exact fraction processing.

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The ratio processing system and its role in fraction understanding: Evidence from a match-to-sample task in children and adults with and without dyscalculia

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Abstract

It has been hypothesised that the human neurocognitive architecture may include a perceptual ratio processing system (RPS) that supports symbolic fraction understanding. In the present study, we aimed to provide further evidence for the existence of the RPS by exploring whether individuals with a range of math skills are indeed perceptually sensitive to non-symbolic ratio magnitudes. We also aimed to test to what extent the RPS may underlie symbolic fraction processing in those individuals. In a match-to-sample task, typical adults, elementary school children, and adults with dyscalculia were asked to match a non-symbolic ratio (i.e., target) to one of two non-symbolic ratios (i.e., the match and distractor). We found that all groups of participants were sensitive to the ratio between the match and the distractor, suggesting a common reliance on the RPS. This ratio sensitivity was also observed in another group of typical adults who had to choose which of two symbolic fractions match a non-symbolic ratio, indicating that the RPS may also contribute to symbolic fraction understanding. However, no ratio dependence was observed when participants had to choose which of two symbolic fractions match another symbolic fraction, suggesting that reliance on the RPS in symbolic fraction processing is limited and may not support exact fraction processing.

Keywords

Ratio processing system; symbolic fractions; match-to-sample task; dyscalculia

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Introduction

Fractions have a key role to play in the holistic development of numerical concepts. From a life-skills perspective, they are crucial for calculating discounts, managing medical dosage, cooking, banking, and in laying the foundation for careers in science, technology, engineering, and mathematics. From an educational perspective, they are central in the mathematics curriculum at both the elementary and secondary school levels and have a strong predictive relationship with algebra and mathematics achievement later in life (Bailey et al., 2012; Booth & Newton, 2012; Siegler et al., 2012). Although children receive fraction instruction beginning around 8 years of age, they continue to struggle with fractions in high school and even after college

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(Kloosterman, 2010; Stigler et al., 2010; Van Hoof et al., 2013). Attesting to the problem, mathematic educators and researchers worldwide have noted a pervasive lack of fraction understanding among students (Chan et al., 2007; Ni, 2001; Yoshida & Sawano, 2002).

Why are fractions difficult?

Research in mathematics education and cognitive science has documented two main reasons that may explain why adults and children across countries face difficulties with fractions. The first reason is that accessing the holistic magnitude of symbolic fractions appears to be difficult for most individuals (Kallai & Tzelgov, 2009, 2012; Stafylidou & Vosniadou, 2004). Being able to estimate the magnitude of fractions is critical in the development of conceptual knowledge of fractions and predicts success in fraction computation and arithmetic (Siegler & Pyke, 2013; Siegler et al., 2011). Unfortunately, the challenges to access and process the holistic magnitude of fractions are all too common even for expert mathematicians (Obersteiner et al., 2013).

Limited understanding of fraction magnitude is quite evident in children. For instance, a study of sixth and eighth graders noted that one third of the students solved fraction arithmetic problems by handling numerators and denominators separately (Siegler & Pyke, 2013). In a one-to-one instructional session, seven out of eight sixth graders responded 1/8 to be greater than 1/6 (Mack, 1990). It has also been observed that students are challenged by the concepts of equivalence because they fail to understand that different symbolic fractions can represent the same magnitude (e.g., 4/8 and 2/4) (Clarke & Roche, 2009). In addition, studies of adult participants suggest that the holistic magnitude of fractions can be accessed when componential strategies relying on the separate processing of numerators and denominators fail (Meert et al., 2009, 2010, 2012).

A second reason for the difficulty individuals face with fractions is that they often assume that properties of whole numbers apply to all numbers (Siegler et al., 2013; Vamvakoussi & Vosniadou, 2010). This so-called whole-number bias (Braithwaite & Siegler, 2017; Ni & Zhou, 2005) leads to the misapplication of whole-number concepts and procedures on fractions. For instance, children might erroneously think that fractions, like whole numbers, have unique successors, can be counted, can be represented by a single symbol, or increase in magnitude if they are multiplied (Siegler et al., 2012). The whole-number bias often leads to erroneous conclusions, such as those reached when performing arithmetic operations separately on the numerators and denominators of the operands (e.g., $1/8 + 1/8 = 2/16$; Mack, 1995; Stigler et al., 2010). For example, when estimating the answer to a problem such as $12/13 + 7/8$, 76% of 13-year-olds choose either 19 or 21 (i.e., the addition of numerators and denominators separately; Carpenter et al., 1980). Importantly, this bias is difficult to overcome. A qualitative study on 52 seventh graders demonstrated that students were persistent in their way of reasoning about fractions. This indicates that the most frequent errors were not random but rooted in deep misconceptions (González-Forte et al., 2019).

Finally, other sources of difficulties with fractions have been noted by researchers on math education. For example, these may stem from a limited understanding of part–whole concepts (Pitkethly & Hunting, 1996; Steffe & Olive, 2010; Streefland, 1991), the multiple ways in which rational numbers can be represented (Kieren, 1981), as well as the variety of procedures that are involved in fraction arithmetic (Siegler et al., 2013). For example, adding fractions with common denominators (e.g., $2/6 + 3/6$) requires one to maintain the denominator constant and add the numerator, whereas multiplying the same fractions (e.g., $2/6 \times 3/6$) requires one to multiply both the numerator and the denominator. Thus, the procedures used in fraction arithmetic are varied and complex.

The innate constraint account

Owing to the severity of challenges faced, some researchers have argued that the human cognitive architecture is endowed with a foundational system that is exclusively designed to deal with discrete quantities (Gelman & Meck, 1983; Leslie et al., 2008). This approximate number system (ANS) is thought to be inherited from evolution because early competence with discrete quantities can be observed in many animal species, including rats, pigeons, monkeys, chimpanzees, and several infant studies (Rumbaugh, Washburn, Boysen , & Capaldi, 1993; Dehaene et al., 1998). In humans, the ANS is also thought to serve as a foundation for symbolic, whole-number concepts (Bailey et al., 2012; Dehaene & Cohen, 2007; Dehaene et al., 1998; Feigenson et al., 2004; Piazza, 2010; but see De Smedt et al., 2013; Leibovich et al., 2017). As this system is not designed to deal with the physical features of fractional and rational number quantities, it has been theorised that it may pose constraints on acquiring rational number concepts (Bonato et al., 2007; Feigenson et al., 2004). According to this innate constraint account, it is not surprising that acquiring elementary level skills in whole numbers is more efficient and easier compared with fractions and rational numbers. Therefore, the innate mechanism that is assumed to help children learn discrete number concepts may impede and/or interfere with children's acquisition of fractional numbers.

The RPS

However, a growing body of evidence is difficult to reconcile with the innate constraint account. For instance, McCrink and Wynn (2007) observed that 6-month-old

infants, when habituated to specific non-symbolic ratios, tend to look longer at novel ratios (differing by a factor of two), indicating that infants as young as 6 months can process differences between non-symbolic ratios. By the age of 4, children can also accurately perform addition and subtraction on part–whole representations of fractions (Mix et al., 1999). Moreover, studies on adults have revealed that non-symbolic ratios are encoded by specific neuronal populations in the parietal cortex and lateral prefrontal cortex (Jacob & Nieder, 2009a, 2009b). Research on non-human primates also shows that trained monkeys can match non-symbolic ratios accurately (Vallentin & Nieder, 2010). Taken together, these studies raise the possibility that there exists an evolutionary old perceptual system for representing and processing non-symbolic ratios in the human brain. This system, which has been termed the ratio processing system (RPS), is thus also likely to be present relatively early in children's development (Lewis, Matthews, & Hubbard, 2016).

Critically, some have argued that the RPS might provide a basis for processing symbolic fraction magnitudes in humans (Lewis et al., 2016). This is suggested by four lines of evidence. First, individual differences in non-symbolic ratio processing relate to individual differences in symbolic fraction processing in school-aged children (Möhring et al., 2016) and adults (Matthews et al., 2016). Second, when adults compare the magnitude of symbolic fractions with the magnitude of non-symbolic ratios (e.g., pairs of lines), their performance depends on the ratio between the magnitudes (i.e., a ratio of ratios [RoR]) (Hurst & Cordes, 2016; Matthews & Ziols, 2019). For example, participants are faster and more accurate at comparing symbolic fractions with non-symbolic ratios when the RoR is large (e.g., 2/6 vs. 1/9=3:1) than when it is small (e.g., 4/5 vs. 2/3=6:5; Hurst & Cordes, 2016). This effect—also observed with whole numbers—is generally thought to indicate that numerical magnitudes are ordered along a mental number line (Moyer & Landauer, 1967). Accordingly, representations of magnitudes are more difficult to distinguish when the distance between them decreases (i.e., a distance effect) and when the overall magnitude increases (i.e., a size effect). This combined effect of distance and size results in performance that varies with the ratio between magnitudes (in the case of RoR fractions). Third, a study has found that adults are faster at comparing symbolic fractions with non-symbolic fractions (i.e., a mixed-format comparison) than symbolic fractions with symbolic fractions (i.e., a within-format comparison; Matthews & Chesney, 2015). This also suggests that a shared magnitude code might be accessed intuitively when performing mixed-format comparisons (Matthews & Chesney, 2015; Matthews et al., 2014). Fourth, functional magnetic resonance imaging (fMRI) studies have shown that processing both symbolic fractions and non-symbolic ratios rely on similar brain regions in and around the intraparietal sulcus (Ischebeck et al., 2009; Jacob & Nieder, 2009a, 2009b; Mock et al., 2018, 2019). Therefore, both behavioural and neural evidence raise the possibility that a perceptual sense of ratio might be involved in processing symbolic fractions.

The current study

The goal of the current study is twofold. First, we aimed to provide further evidence for the universality of the RPS by exploring to what extent individuals with a range of math skills are perceptually sensitive to non-symbolic ratio magnitudes. To this end, we used a match-to-sample task in which participants were asked to match a non-symbolic ratio (a pair of lines) presented on the left side of a screen (i.e., target) to one of two non-symbolic ratios presented on the right side (i.e., the match and distractor; see Figure 1a). Critically, the ratio between the match and the distractor (i.e., the RoR) was systematically varied, such that it was small in some trials and large in others. This task was adapted from a preliminary study by Matthews (2015), in which an RoR effect (i.e., more accurate performance for large than small RoR) was found in both adults and 16 children from kindergarten. This suggests an early developing sensitivity to differences in non-symbolic ratios, consistent with the RPS. In the present study, we aimed to replicate these preliminary findings by testing individuals who are proficient in fraction processing (i.e., adults) and individuals who are not (i.e., elementary-school children). We also aimed to test whether adults with dyscalculia (i.e., a learning disability that severely affects the acquisition of math skills; Iuculano, 2016; Price & Ansari, 2013) exhibit an RoR effect in that task. Indeed, it has been argued that a main cause of dyscalculia is a deficit in processing approximate non-symbolic numerosities (Mazzocco et al., 2011; Piazza et al., 2010). A major question, however, is whether this deficit may extend to the RPS or whether individuals with dyscalculia have a relatively intact RPS despite poor calculation skills.

A second goal of the present study was to examine to what extent the RPS remains involved when participants have to match symbolic fractions (rather than non-symbolic ratios) in this match-to-sample task. In other words, we aimed to examine whether participants remain sensitive to ratio magnitudes when these are expressed as symbols. Although it has been argued that the RPS may contribute to symbolic fraction understanding (Lewis et al., 2016), prior studies have largely relied on magnitude comparison tasks that only require participants to access and estimate the approximate value of fractions. In contrast, a match-to-sample task would involve asking participants to find which of two fractions is equivalent to a target fraction, which may encourage them to access the exact values of the fractions (though strategies based on approximation may still be involved to some extent). On one hand,

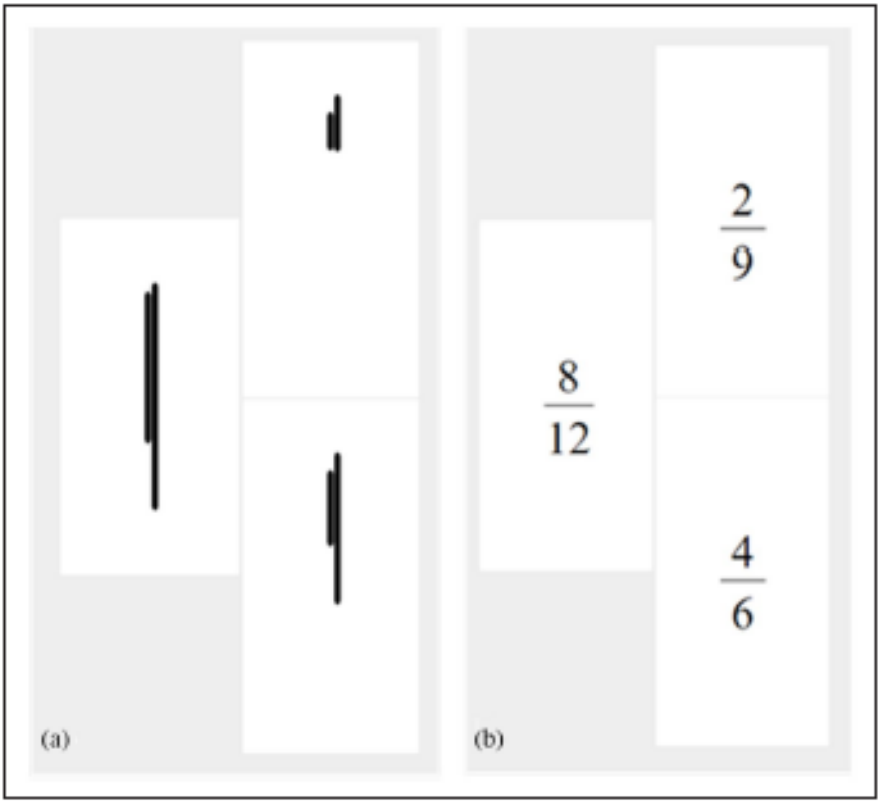

Figure 1. Match-to-sample task. (a) In each non-symbolic trial, participants had to match the pair of line segments on the left of the screen to the equivalent pair of line segments on the right of the screen. (b) In each symbolic trial, participants had to match the fraction on the left of the screen to the equivalent fraction on the right of the screen.

this task might trigger explicit calculation strategies that rely on components of fractions (i.e., numerator and denominator) and have little to do with the RPS. On the other hand, if symbolic fraction understanding is grounded in the RPS, participants may still intuitively access the RPS even in such match-to-sample tasks. To test between these possibilities, we adapted the match-to-sample task described above and designed a condition in which the pairs of lines were replaced by symbolic fractions representing the same ratios (see Figure 1b). We reasoned that, if the RPS contributes to the understanding of symbolic fractions in this task, there should be an RoR effect similar to that obtained with non-symbolic ratios. In contrast, if the RPS does not contribute to the understanding of symbolic fractions, performance should not depend on RoR.

Experiment 1

Method

Participants. It has been argued that an effect size of $d=0.4$ is a good estimate of the smallest effect size of interest in experimental psychology (Brysbaert, 2019). Therefore, to be able to detect an effect of that size with 80% power in a within-subject comparison, we planned to recruit about 50 participants.

In all, 48 individuals from the University of Lyon volunteered to participate in this experiment. Data collection for the participants was carried out in 2 days. Six participants were excluded because of technological issues ($n=2$) and non-compliance with the instructions ($n=4$). The remaining 42 participants ($M=20.7$years, range=18–54, 31 females) were included in the analyses, for an achieved power of 72% (based on a hypothesised effect size of $d=0.4$). As compensation, participants received a book or chocolate. The experiment was performed in accordance with the ethical standards established by the Declaration of Helsinki.

Materials. Participants completed a match-to-sample task in which they were asked to match a ratio presented on the left of the screen to one of two ratios presented on the right of the screen (one of which was equivalent to the ratio on the left). The task was adapted from Matthews (2015). In two separate blocks, ratios were presented either as symbolic fractions or as non-symbolic ratios (Figure 1).

Symbolic fractions. Symbolic fractions were composed of numerators and denominators ranging from 1 to 15 and were presented in a font size of 20 point in Times New Roman. In each trial, three fractions (target, match, and distractor) were presented simultaneously. The target was

presented on the left of the screen and the match and distractor were presented on the right (see Figure 1). The ratio between the fractions presented as match and distractor (i.e., an *RoR*) was varied such that it was large in half of the trials and small in the other half. Large RoR were of 3:1 and 2:1. Small RoR were of 4:3 and 6:5. For instance, for a 4:3 RoR, a 2/3 match could be paired with a 4/8 distractor. These ratios were chosen based on Matthews (2015). Specifically, they corresponded to the two largest and the two smallest ratios from this preliminary study. For each RoR, 40 unique fraction pairs were generated (see Supplemental Appendix). Following previous studies (Meert et al., 2010; Schneider & Siegler, 2010), we also generated an equal mixture of fractions where numerators of target and distractor were either the same or different. Critically, because there is often a relation between overall fraction magnitude and size of the numerator, we found the ratio between numerators of match and distractor to increase with that of the RoR. That is, small and large RoRs were associated with average ratios between numerators of match and distractor of 1.44 and 2.37 (respectively). This raises the possibility that participants might just compare the numerators of match with distractor to complete the task. To evaluate whether this was the case, fraction pairs within small and large RoRs were further broken down into pairs for which the ratio between numerators of match and distractor was small (i.e., smaller than 1.44 for small RoRs and smaller than 2.37 for large RoRs) and pairs for which the ratio between numerators of match and distractor was relatively larger (i.e., larger than 1.44 for small RoRs and larger than 2.37 for large RoRs). This allowed us to explore the effects of RoR and effects of ratio between numerators of match and distractor independently.

Non-symbolic ratios. Non-symbolic ratios were composed of pairs of line segments representing the same ratios as those used in symbolic fractions. Individual line lengths ranged between 10 and 160 px approximately on an 800 px × 994 px full screen. This corresponded to a size between 0.2 and 3.2 cm (visual angle 0.19°–3.05°). As for symbolic fractions, the ratio between the non-symbolic ratios presented as match and distractor was varied, such that it was large (i.e., 3:1 and 2:1) in half of the trials and small (4:3 and 6:5) in the other half. For each RoR, 40 non-symbolic ratio pairs were generated. Since the ratio size determined the magnitude of the line lengths, participants could not rely on individual line lengths and had to process the relationship between the two lines. For instance, for line ratios such as those shown in Figure 1, all the stimuli (target, match, and distractor) had different sizes of individual components. Furthermore, a ratio of 2/4 when matched to a ratio of 5/10 was composed of line segments of 2/4 and 5/10 units of length, respectively, where a single unit length of line segment was fixed in size. Because of this, the individual components of the line ratios differed in size but the relationship between the ratios stayed the same. In the following analyses, we consider the "numerator" of each pair of line to be the shortest line of the pair.

Procedure. During the entire experiment, participants were seated approximately 60 cm from a 14 in. screen. All instructions were presented on the laptop screen in French. Participants could ask questions about the task after reading the instructions and receive additional instructions if they needed to. Participants were told that they would see in each trial three ratios (one on the left side of the screen and two on the right side) presented in either symbolic or non-symbolic format. For both notations, they were instructed to match the ratio on the left (i.e., the target) to the equivalent ratio on the right (i.e., the match) as accurately and quickly as possible. They could do so by pressing the button "H" if the upper stimulus was equivalent to the target and the button "B" if the lower stimulus was equivalent to the target (on a QWERTY keyboard). The three ratios were presented for a duration of 3,000 ms. Participants could respond from the onset of stimuli until the timeout at 6,000 ms.

Participants completed one block of non-symbolic ratios and one block of symbolic fractions. Each block included 4 practice trials (with feedback) that were followed by 40 experimental trials. Each block was made of 20 trials with a large RoR (i.e., 10 trials with a RoR of 3:1 and 10 trials with a RoR of 2:1) and 20 trials with a small RoR (i.e., 10 trials with a RoR of 4:3 and 10 trials with a RoR of 6:5). These trials were selected randomly from the triplets of fractions generated for each RoR. The practice trials were constructed of easier RoR (4:1) and participants had to reach 100% accuracy to be presented with experimental trials. Almost all participants were able to complete the practice trials with 100% accuracy. Those who did not were given additional instructions and were successful in completing the practice trials. Block order was counterbalanced and trial order was randomised within blocks. The side of the presentation of the match was counterbalanced with half of the match ratios on the top and the other half at the bottom. The experiment was designed using JavaScript and HTML. The entire experiment lasted between 10 and 15 min.

Results

The proportion correct was analysed in a repeated-measures analysis of variance (ANOVA) with the within-subject factors RoR (large, small), Notation (line, fraction), Ratio between numerators of match and distractor (large, small), and Numerator of the distractor (identical to the target, different from the target). All analyses were carried out in Jamovi (www.jamovi.org/). The main effects and interactions relevant to our hypotheses are presented below.

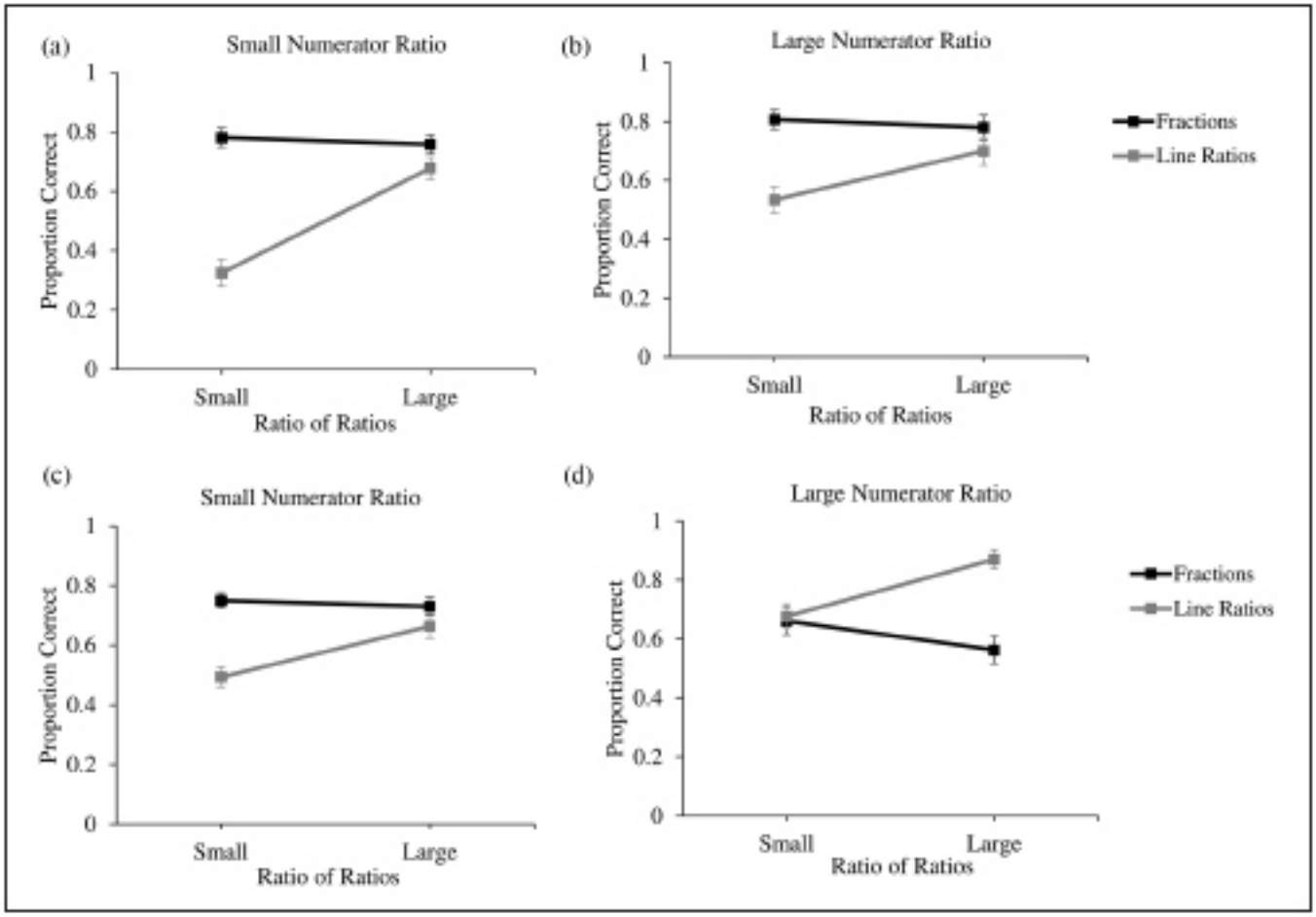

Figure 2. Results of Experiment 1 (typical adults). (a) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *identical* to the numerator of the distractor and in which the ratio between numerators of match and distractor was small. (b) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *identical* from the numerator of the distractor and in which the ratio between numerators of match and distractor was large. (c) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *different* from the numerator of the distractor and in which the ratio between numerators of match and distractor was small. (d) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *different* from the numerator of the distractor and in which the ratio between numerators of match and distractor was large. The bars here represent standard errors.

First, this ANOVA revealed a main effect of Notation ($F(1, 41) = 16.36$, $p < .001$, $\eta_p^2 = .285$) and a main effect RoR ($F(1, 41) = 22.96$, $p < .001$, $\eta_p^2 = .359$), which were qualified by an interaction between Notation and RoR ($F(1, 41) = 49.79$, $p < .001$, $\eta_p^2 = .548$). Bonferroni corrected *t*-tests revealed that participants were more accurate for large than small RoR when choosing between non-symbolic ratios ($p < .001$). However, there was no accuracy difference between large and small RoR when choosing between symbolic fractions ($p = .701$). This interaction between notation and RoR was neither affected by numerator of the distractor ($F(1, 41) = 0.45$, $p = .505$, $\eta_p^2 = .011$) nor by ratio between numerators of match and distractor ($F(1, 41) = 0.33$, $p = .567$, $\eta_p^2 = .008$). Thus, RoR differently affected performance on non-symbolic and symbolic stimuli across all different trial types (see Figure 2).

Second, there was a significant interaction between Notation and Numerator of the distractor ($F(1, 41) = 32.62$, $p < .001$, $\eta_p^2 = .443$), revealing that participants were most accurate when the numerator of the distractor was different from the target for non-symbolic ratios ($p < .001$), whereas they were most accurate when numerators were identical across target and distractor for symbolic fractions ($p < .001$).

Third, the ANOVA revealed a main effect of Ratio between numerators of match and distractor ($F(1, 41) = 5.53$, $p = .024$, $\eta_p^2 = .119$), which was qualified by an interaction with Notation ($F(1, 41) = 40.26$, $p < .001$, $\eta_p^2 = .495$). For non-symbolic ratios, participants were more accurate when the ratio between numerators of match and distractor was large than small ($p < .001$). For symbolic fractions, however, performance was not affected by the ratio between numerators of match and distractor ($p = .378$).

Because frequentist statistics do not provide evidence for a null hypothesis, we turned to Bayesian statistics (Lee & Wagenmakers, 2013; Morey et al., 2016) to estimate the strength of evidence (i.e., the Bayes factor [BF]) for a lack of difference between large and small RoR (H0) versus a higher accuracy for large than small RoR (H1) in both symbolic and non-symbolic trials. Following Jeffreys (1961), a $BF < 3$ was considered anecdotal evidence, a $3 < BF < 10$ was considered substantial evidence, a $10 < BF < 30$ was considered strong evidence, a $30 < BF < 100$ was considered very strong evidence, and a $BF > 100$ was considered extreme evidence that our data are more likely under the alternate than the null hypothesis (i.e., $BF+_0$) or under the null hypothesis than the alternate hypothesis (i.e., BF_0+). Bayesian paired *t*-tests with default priors (0.707) revealed strong evidence for H0 versus H1 in the case of symbolic fractions ($BF_0+ = 14.4$). In contrast, there was extreme evidence for H1 versus H0 in the case of non-symbolic ratios ($BF+_0 > 100$). Therefore, while there was evidence that large RoR was more accurate than small RoR for non-symbolic ratios, there was also evidence that it was not the case for symbolic fractions.

Discussion

In this first experiment, we explored the extent to which typical adults are sensitive to ratio magnitudes when they match either non-symbolic ratios or symbolic fractions. The findings demonstrate a clear RoR effect for non-symbolic line ratios, with a higher accuracy for large RoR than for small RoR. These findings replicate the preliminary results described in Matthews (2015). They also suggest that adults are perceptually sensitive to ratio magnitudes, even when they have to match (and not only compare, as in magnitude comparison tasks) non-symbolic stimuli. This supports the claim that adults possess an RPS that may allow them to access a perceptual sense of ratios, akin to the ANS for whole numerosities (Dehaene, 1997; Feigenson et al., 2004). Interestingly, our study indicates that some componential strategies may also be involved in this task. For example, in addition to the RoR effect, accuracy depended on whether numerators of target and distractor (i.e., the shortest line of each pair) were identical or different. It also depended on the ratio between numerators of match and distractor. This suggests that participants may have attended to components of each pair of lines in addition to their overall ratio. This is broadly consistent with previous findings that have shown that participants may rely on both holistic and componential strategies when comparing fractions (though these studies mainly focused on symbolic fractions; Bonato et al., 2007; Faulkenberry & Pierce, 2011; Meert et al., 2010, 2012; Obersteiner et al., 2013; Schneider & Siegler, 2010).

A novel aspect of our study is that we asked participants to match symbolic fractions in addition to non-symbolic ratios. In sharp contrast to the results obtained with non-symbolic ratios, our results revealed an absence of RoR effect for symbolic fractions. This suggests that participants do not access a representation of fractional magnitudes when they have to match symbolic fractions in this task. Rather, they are likely to use componential strategies, focusing on the numerators and denominators separately. This is suggested by the fact that participants' accuracy depended on whether the numerator of the target was identical or different from the numerator of the distractor. Clearly, focusing on the numerator is an efficient strategy when the numerator of the target is the same as the distractor, because it indicates that these fractions are not equivalent. However, the effectiveness of such a componential strategy is less obvious when numerators differ between target and distractor. Yet, our results clearly indicate that performance does not depend on RoR, suggesting that participants do not rely on fractional magnitudes to complete the task.

Other strategies that participants are likely to rely on in Experiment 1 may involve calculating (e.g., reducing a fraction to its simplest form) and their mastery may emerge after years of practice in school. Therefore, reliance on these strategies might depend on the degree of familiarity with fractions as well as proficiency with arithmetic. Such strategies may also tax working memory and therefore depend on the working memory resources of participants. In other words, it is possible that participants with limited practice with manipulating symbolic fractions, limited arithmetic fluency, or limited working memory resources may rely on an intuitive sense of fractional magnitudes to a greater extent than competent adults. To test this hypothesis, we presented the same task to children who are beginning to learn about symbolic fractions in the fourth and fifth grades. These children should, therefore, have more limited experience with the strategies used to solve fraction problems than adults. They should also have lower arithmetic fluency and working memory ability than adults (Cowan, 2016; De Brauwer et al., 2006; Imbo & Vandierendonck, 2008). None of these factors may influence the RPS, because it is thought to be an evolutionarily old system that requires few cognitive resources and is early developing. However, these factors may affect the extent to which children use strategies in the symbolic version of the task. Therefore, we predicted that these children should rely (at least to some extent) on the RPS to match symbolic fractions as well as non-symbolic ratios (i.e., they should exhibit a sensitivity to ratio magnitudes when matching non-symbolic ratios and symbolic fractions).

Experiment 2

Method

Participants. Children are introduced to symbolic fractions during the fourth grade in France. Thus, we recruited 40

fourth and fifth graders from a private school in Lyon, France. The difference between large and small RoR for non-symbolic stimuli in Experiment 1 was very large (Cohen's $d = 1.43$). Therefore, this sample size, which would yield more than 99% power to detect an effect size of $d = 0.80$ in a within-subject comparison, was more than adequately powered to detect an effect of that size. Two children were excluded because of non-compliance with instructions. The remaining 38 children ($M = 9.92$ years, range: 9–11, 26 females), 21 fourth graders and 17 fifth graders, were included in the study. Data collection was done during one school day. The experiment was performed in accordance with the ethical standards established by the Declaration of Helsinki.

Materials and procedure. Materials and procedures remained the same as in Experiment 1. However, children were given oral rather than written instructions about the task and were tested in groups of 4.

Results

Proportion correct was analysed in a repeated-measures ANOVA with the factors RoR (large, small), Notation (line, fraction), Ratio between numerators of match and distractor (large, small), and Numerator of the distractor (identical to the target, different from the target).

First, the ANOVA revealed a main effect of RoR ($F(1, 37) = 14.74$, $p < .001$, $\eta_p^2 = .285$), which was qualified by an interaction between RoR and Notation ($F(1, 37) = 39.76$, $p < .001$, $\eta_p^2 = .518$). Bonferroni-corrected post hoc tests showed that children were more accurate for large than small RoR for non-symbolic ratios ($p < .001$). In contrast, there was no difference in accuracy between large and small RoR for symbolic fractions ($p = 1.000$). The interaction between Notation and RoR was neither affected by Numerator of the distractor ($F(1, 37) = 0.56$, $p = .460$, $\eta_p^2 = .015$) nor by Ratio between numerators of match and distractor ($F(1, 37) = 0.55$, $p = .462$, $\eta_p^2 = .015$). Thus, much like in typical adults, RoR differently affected performance on non-symbolic and symbolic stimuli across all different trial types in children (see Figure 3).

Second, the ANOVA revealed a main effect of Ratio between numerators of match and distractor ($F(1, 37) = 11.42$, $p = .002$, $\eta_p^2 = .236$). Although the interaction with Notation did not reach significance ($F(1, 37) = 1.51$, $p = .227$, $\eta_p^2 = .039$), follow-up Bonferroni tests indicated that the main effect of Ratio between numerators of match and distractor was driven by non-symbolic stimuli. For non-symbolic ratios, children were more accurate when the ratio between numerators of match and distractor was large than small ($p = .011$). For symbolic fractions, performance was not affected by the ratio between numerators of match and distractor ($p = .892$).

Bayesian paired t-tests with default priors (0.707) revealed extreme evidence for H1 (i.e., large RoR is associated with higher proportion correct than small RoR) versus H0 (i.e., no difference between large and small RoR) when children had to choose between non-symbolic ratios ($BF+_0 > 100$). There was, however, moderate evidence for H0 versus H1 when children had to choose between symbolic fractions ($BF_0+ = 6.92$). Therefore, much like adults in Experiment 1, there was evidence with children that large RoR was more accurate than small RoR for non-symbolic ratios. However, there was also evidence that this was not the case for symbolic fractions.

Discussion

In Experiment 2, we presented the symbolic and non-symbolic versions of the match-to-sample task to children in the fourth and fifth grades. Because children are introduced to symbolic fractions in the fourth grade in France, we reasoned that these participants may have less experience with manipulating symbolic fractions than adults and lack practice with different strategies. Accordingly, they may rely to a lesser extent on these strategies when matching symbolic fractions (and might instead rely on a more intuitive sense of ratio magnitudes). Consistent with the fact that these participants are less proficient with symbolic fractions than adults, children matched symbolic fractions with a relatively low accuracy of about 54.6% (as opposed to 73.8% on average for adults in Experiment 1). Accuracy was also comparable in trials in which the numerator of the target fraction was similar to that of the distractor fraction (54.1%) and in trials in which the numerator of the target fraction was different from that of the distractor fraction (55.2%). This suggests that children struggled to perform this symbolic version of the task and focused less systematically on numerators than adults did in Experiment 2 (similar to Zhang et al., 2014). Yet, the pattern of results is largely similar to Experiment 1.

First, children exhibited RoR effects when matching non-symbolic ratios, indicating that children around the age of 10 are perceptually sensitive to ratio magnitudes. This is in line with several prior studies also showing sensitivity to non-symbolic ratio magnitudes in children. For example, in the preliminary study by Matthews (2015). 16 preschoolers showed RoR effects when matching non-symbolic ratios. In another study, 6-month-old infants show similar discrimination acuity for non-symbolic ratios as they do for non-symbolic numerosities in magnitude discrimination tasks (McCrink & Wynn, 2007). Finally, studies of non-symbolic proportional reasoning tasks employing continuous quantities (in which participants have to focus on the relationship between the quantities because individual units cannot be counted) have shown that children as young as 6 years old are successful in processing proportional information across a wide range of non-symbolic formats (Boyer et al., 2008; Hurst & Cordes, 2018; Jeong et al., 2007; Meert et al., 2013). Therefore,

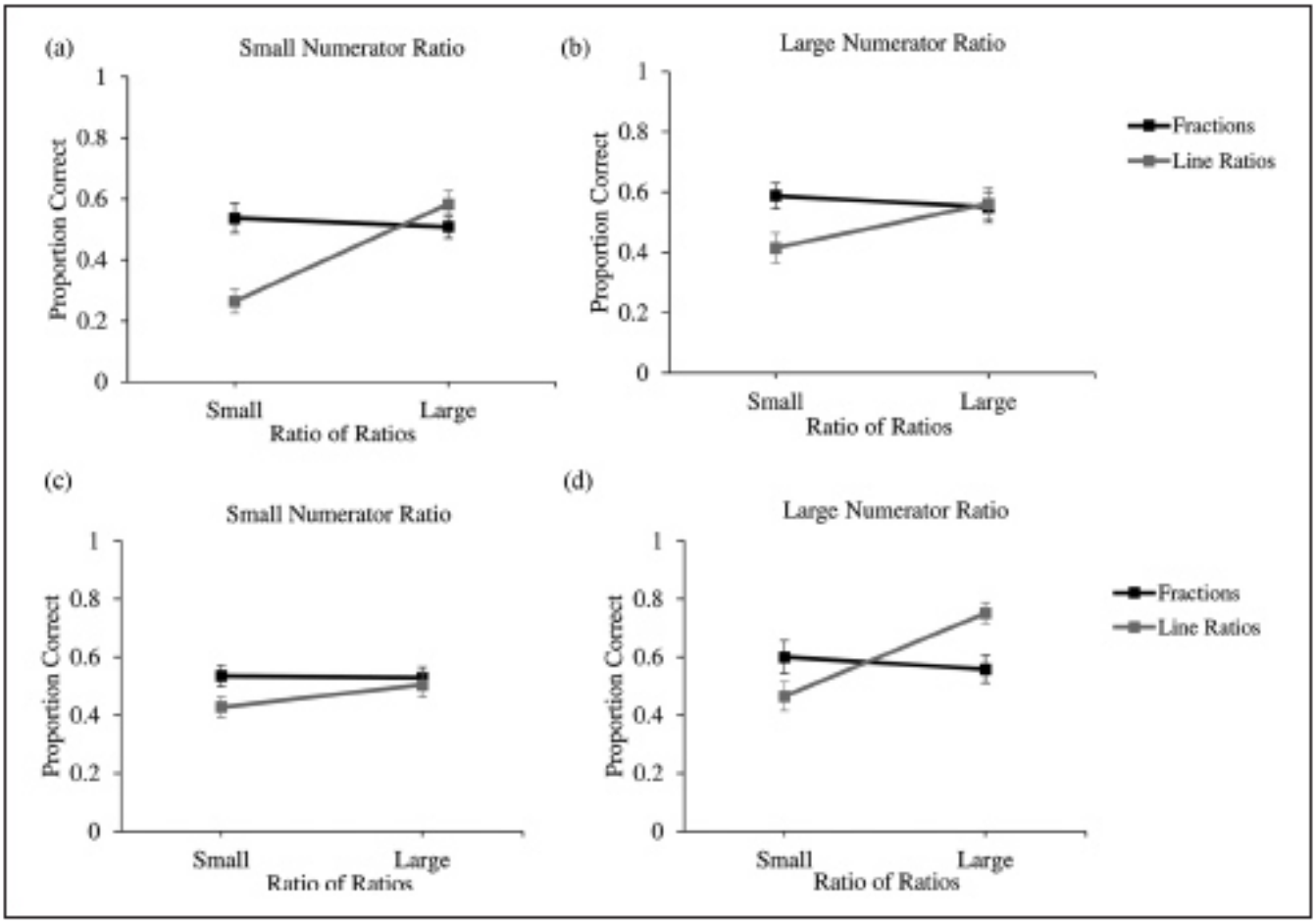

Figure 3. Results of Experiment 2 (children). (a) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *identical* to the numerator of the distractor and in which the ratio between numerators of match and distractor was small. (b) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *identical* from the numerator of the distractor and in which the ratio between numerators of match and distractor was large. (c) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *different* from the numerator of the distractor and in which the ratio between numerators of match and distractor was small. (d) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *different* from the numerator of the distractor and in which the ratio between numerators of match and distractor was large. The bars here represent standard errors.

together with these studies, our data are consistent with the idea that the RPS is relatively early developing.

Second, despite a level of accuracy relatively similar between the non-symbolic ratios and the symbolic fractions (49.4% vs. 54.6%), children's accuracy in the symbolic task was not dependent on the RoR. Therefore, contrary to our prediction, even children with a lack of extensive experience with componential and other strategies do not appear to access representations of magnitude when matching symbolic fractions. Of course, children are beginning to learn about fractions in the fourth grade, as indicated by the relatively high error rates. It is thus possible that they were not exposed to enough instruction on rational numbers to be able to access the underlying ratio magnitudes. To test this possibility, we turned to adults with dyscalculia.

Dyscalculia is a specific learning disability affecting the acquisition of numerical skills, despite normal intelligence and educational opportunities. Research has shown that individuals with developmental dyscalculia possess an impaired ANS as well as relatively poor working memory ability (Geary, 2011; Iuculano, 2016; Mazzocco et al., 2011; Piazza et al., 2010). This, in turn, is thought to significantly affect arithmetic learning. Clearly, adults with dyscalculia have been exposed to extensive instruction on rational numbers in school. Yet, their impaired calculation skills may prevent them from effectively using componential strategies when matching symbolic fractions. Although this may translate into greater use of the RPS, it is also possible that ANS impairments in individuals with dyscalculia may extend to the RPS. Therefore, Experiment 3 investigated the sensitivity to ratio magnitudes of adults with dyscalculia who were asked to match both non-symbolic ratios and symbolic fractions.

Experiment 3

Method

Participants. Seventeen adult participants were recruited using advertisements in social media, the university "mission-handicap," and by word of mouth. Thirteen participants ($M = 23.3$ years, range: 19–27, 12 females) met the inclusion criteria based on clinical diagnosis and standardised tests (described below), yielding 80% power to detect an effect size of $d = 0.85$ in a within-subject comparison. Because the effect sizes associated with the difference in accuracy between large and small RoR in Experiments 1 and 2 were larger than $d = 0.85$ (i.e., $d = 1.43$ and $d = 1.08$, respectively), we considered this sample size to be adequate. The experiment was performed in accordance with the ethical standards established by the Declaration of Helsinki.

Standardised tests. Adults were administered various tests to assess cognitive, numerical, and reading skills. Using the Wechsler Adult Intelligence Scale–Fourth edition (WAIS-IV; Wechsler, 2008), verbal IQ was estimated using the verbal reasoning subtest, while spatial IQ was estimated using the matrix reasoning sub-test. Math skills were assessed using two sub-tests of the Woodcock-Johnson Test of Achievement (WJ-III) (Woodcock et al., 2001): Math Fluency and Applied Problems. The Math Fluency sub-test is a timed test in which the participants have to solve as many single-digit additions, subtraction, multiplication, and division problems as they can within 3 min. The Applied Problems sub-test is untimed and measures the ability to analyse basic numerical concepts and oral word problems. This test stops after six consecutive errors or when the last item is reached. Reading fluency was assessed with the Alouette-R test (Lefavrais, 1967), which is frequently used to test reading skills in French-speaking countries. This test requires participants to read a 265-word text aloud in 3 min and measures the number of words read, the time required, and the number of pronunciation errors to evaluate the reading speed and efficiency, respectively. Because the test is standardised for children and adolescents but not for adults, we used the guidelines of Cavalli et al. (2018) to identify the cut-off criteria for dyslexia (i.e., reading speed score above 8.7 and/or reading efficiency score above 402.26)

Psychometric measures are shown in Table 1. To be classified as having dyscalculia, participants had to have been diagnosed by a clinical specialist, complained of mathematics difficulties since school, and perform at or below the 25th percentile on at least one of the math sub-tests of fluency and applied problems (Jordan et al., 2013). In addition, all 13 participants had to have a normal verbal IQ, as reflected by a score above the 30th percentile (i.e., 92) on the verbal reasoning subtest of the WAIS-IV. All but three participants had reading scores above the cut-off criterion for dyslexia (Cavalli et al., 2018).

Table 1. Psychometric measures of adults with dyscalculia.

Measure	*M* (range)
WAIS-IV	
Verbal reasoning[a]	110 (95–130)
Matrix reasoning[a]	90 (75–100)
WJ-III	
Math fluency[a]	70.9 (54–82)
Applied problems[a]	84.0 (74–102)
Alouette-R	
Reading efficiency[b]	437 (282–532)
Reading speed[b]	9.28 (6.02–11.2)

WAIS-IV: Wechsler Adult Intelligence Scale–Fourth edition; WJIII: Woodcock-Johnson Test of Achievement.
[a]Standardised score ($M = 100$, $SD = 15$).
[b]Raw score.

Materials and procedures. Materials and procedures remained the same as in Experiment 1 with one exception. Because the sample size was smaller than in Experiment 1, we presented participants in Experiment 3 with twice more trials to improve individual estimates of performance. That is, after completing 4 practice trials in each block, participants evaluated 80 experimental trials. Therefore, there was a total of 160 trials (i.e., 80 non-symbolic ratios and 80 symbolic fractions). The whole experiment lasted about 20 min.

Results

As for Experiments 1 and 2, the proportion correct was analysed in a repeated-measures ANOVA with RoR (large, small), Notation (line, fraction), Ratio between numerators of match and distractor (large, small), and Numerator of the distractor (identical to the target, different from the target) as within-subject factors.

First, this ANOVA revealed an interaction between Notation and RoR ($F(1, 12) = 35.04$, $p < .001$, $\eta_p^2 = .745$). Bonferroni corrected *t*-tests revealed that participants were more accurate for large than small RoR when choosing between non-symbolic ratios ($p < .001$). However, there was no accuracy difference between large and small RoR when choosing between symbolic fractions ($p = .106$). This interaction between Notation and RoR was neither affected by Numerator of the distractor ($F(1, 12) = 0.36$, $p = .557$, $\eta_p^2 = .029$) nor by Ratio between numerators of match and distractor ($F(1, 12) = 1.47$, $p = .248$, $\eta_p^2 = .109$). Thus, as in Experiments 1 and 2, RoR differently affected performance on non-symbolic and symbolic stimuli across all different trial types in adults with dyscalculia (see Figure 4).

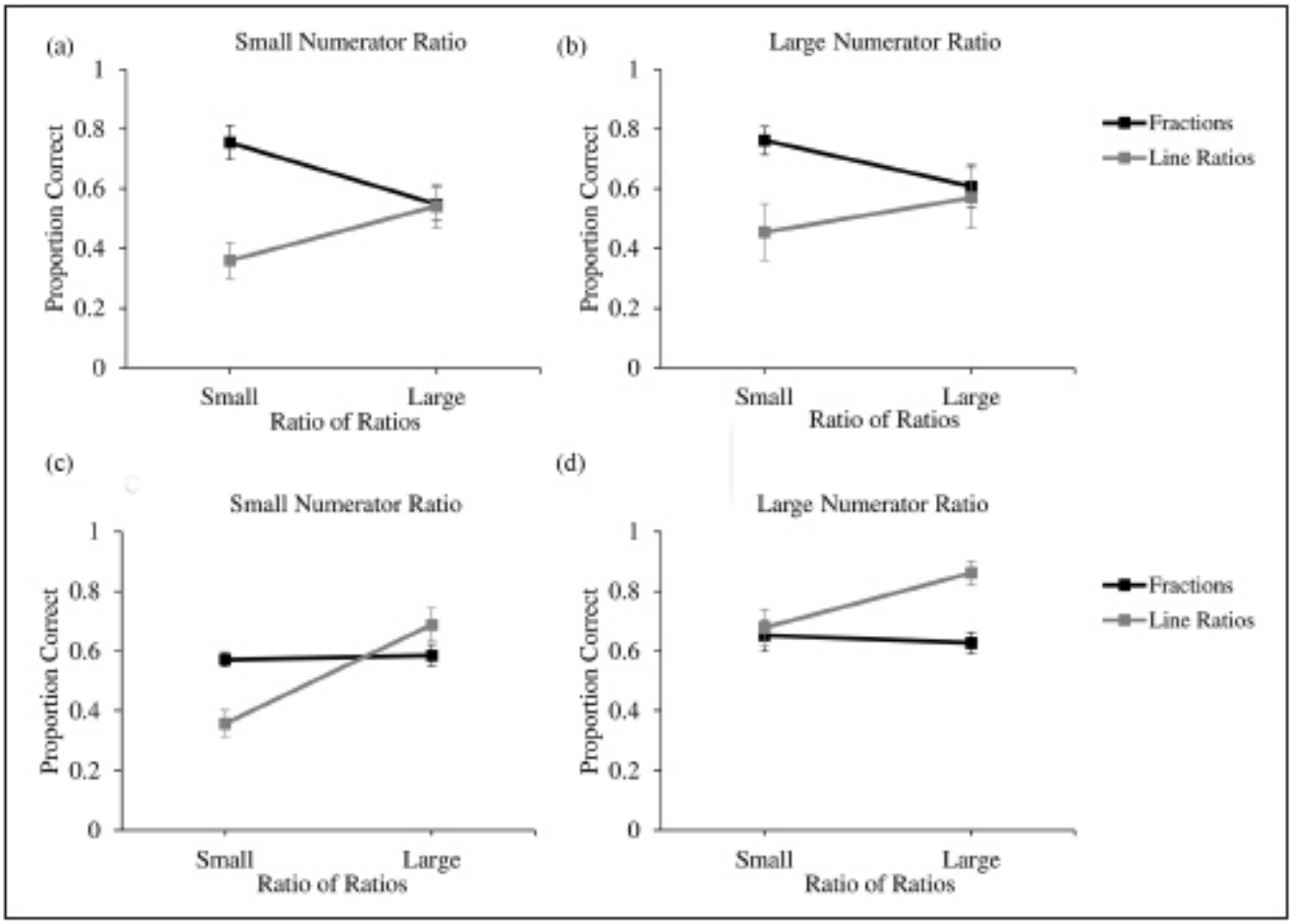

Figure 4. Results of Experiment 3 (adults with dyscalculia). (a) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *identical* to the numerator of the distractor and in which the ratio between numerators of match and distractor was small. (b) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *identical* from the numerator of the distractor and in which the ratio between numerators of match and distractor was large. (c) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *different* from the numerator of the distractor and in which the ratio between numerators of match and distractor was small. (d) Proportion correct as a function of Notation and RoR for trials in which the numerator of the target was *different* from the numerator of the distractor and in which the ratio between numerators of match and distractor was large. The bars here represent standard errors.

Second, there was a significant interaction between Notation and Numerator of the distractor ($F(1, 12) = 20.09$, $p < .001$, $\eta_p^2 = .626$), revealing that participants were most accurate when the numerator of the distractor was different from the target for non-symbolic ratios ($p = .017$), whereas this factor was no affecting accuracy for symbolic fractions ($p = 1.000$).

Third, the ANOVA revealed a main effect of Ratio between numerators of match and distractor ($F(1, 12) = 22.17$, $p < .001$, $\eta_p^2 = .649$), which was qualified by an interaction with Notation ($F(1, 12) = 11.47$, $p = .005$, $\eta_p^2 = .489$). For non-symbolic ratios, participants were more accurate when the ratio between numerators of match and distractor was large than small ($p < .001$). For symbolic fractions, however, performance was not affected by the ratio between numerators of match and distractor ($p = .522$).

Bayesian paired *t*-tests with default priors (0.707) revealed extreme evidence for H1 (i.e., large RoR is associated with higher proportion correct than small RoR) versus H0 (i.e., no difference between large and small RoR) when participants had to choose between non-symbolic ratios ($BF+_0 > 100$). There was moderate evidence for H0 versus H1 when participants had to choose between symbolic fractions ($BF_0+ = 10.89$). Thus, adults with dyscalculia displayed a higher accuracy for large than small RoR for non-symbolic ratios, but not for symbolic fractions.

Discussion

The results of Experiment 3 show that adults with dyscalculia exhibit a pattern of responses very similar to that of children in Experiment 2. First, overall accuracy was low on both non-symbolic ratios (54%) and symbolic fractions

(62.6%), suggesting that participants completed the tasks with some difficulty. Second, despite this relatively low accuracy, there was a clear RoR effect with non-symbolic ratios. This suggests that adults with dyscalculia have a perceptual sensitivity to ratio magnitudes, much like normal adults in Experiment 1 and children in Experiment 2. This is noteworthy because it presents evidence that adults with dyscalculia may have access to a relatively intact RPS. Interestingly, prior research on number acuity has revealed an impaired ANS in individuals with dyscalculia (Mazzocco et al., 2011; Piazza et al., 2010; for a contrasting view see Iuculano et al., 2008). To our knowledge, the present study is the first to examine the intuitive, perceptual ability to process non-symbolic ratios in this population. Our findings raise the possibility that the RPS may be relatively independent from the ANS and spared in dyscalculia. Note, however, that these results come from a relatively small sample size (despite moderate to large effect size, $\eta_p^2 = .346$) and that future studies would need to confirm this claim by gathering concomitant measures of both ANS and RPS in individuals with dyscalculia.

Third, we expected to observe an RoR effect for symbolic fractions, due to less automatised calculative abilities and lower working memory capacity in adults with dyscalculia. However, performance on symbolic fractions was not affected by RoR, indicating that adults with dyscalculia do not access a representation of ratio magnitudes when they match symbolic fractions.

Overall, the results of Experiment 3 are similar to the results of Experiments 1 and 2 in that participants do not naturally appear to use the RPS to match symbolic fractions in this task, independently of their level of math skills. This, however, may not mean that they *cannot* use the RPS when processing symbolic fractions. For example, accessing the RPS may be necessary if participants are forced to estimate the ratio that underlie a symbolic fraction. To explore this hypothesis, we designed a mixed-format version of the match-to-sample task in which another sample of adults were asked to choose which of two symbolic fractions corresponds to a non-symbolic ratio (see Figure 5). In such a task, participants are forced to *estimate* (rather than precisely determine) which symbolic fraction is the best match for the non-symbolic ratio. We expected that such a mixed-format task would require participants to access fractional magnitudes in a way that previous symbolic versions of the task did not.

Experiment 4

Participants

Thirty-three participants (M=20.5years, range: 18–25, 25 females) from the University of Lyon volunteered to participate in the study. All of them were included in the analyses. This yields 80% power to detect an effect size of d=0.50 (which would be much smaller than the effect

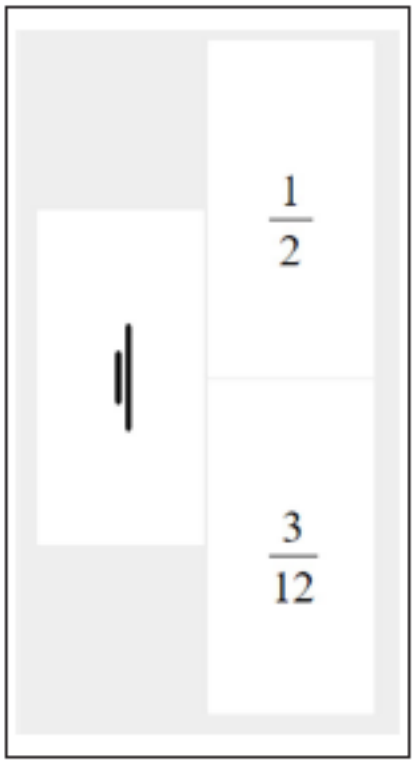

Figure 5. Match-to-sample task in mixed-notation condition. In each trial, participants had to match the pair of line segments on the left of the screen to the equivalent fraction on the right side of the screen.

reported for the difference between large and small RoR in non-symbolic stimuli in the previous experiments). The experiment was performed in accordance with the ethical standards established by the Declaration of Helsinki.

Materials and procedures. The procedure remained the same as in the experiments above. However, stimuli for the match-to-sample task were modified. For this task, non-symbolic ratios were presented on the left side of the screen and symbolic fractions were presented on the right side of the screen (see Figure 5). Participants were asked to match the non-symbolic ratio on the left with the correct symbolic fraction on the right. Participants completed one block with 4 practice trials (with feedback) followed by 80 experimental trials (consisting of 20 trials per RoR). The entire experiment lasted for about 10 min.

Results

Proportion correct was analysed in a repeated-measures ANOVA with the factor RoR (large, small). The analysis revealed a main effect of RoR ($F(1, 32) = 63.9$, $p < .001$, $\eta_p^2 = .666$), indicating that participants were more accurate for large RoR than small RoR (see Figure 6).

In line with the results above, a Bayesian paired t-test with default priors (0.707) revealed extreme evidence for H1 (i.e., large RoR are associated with higher proportion

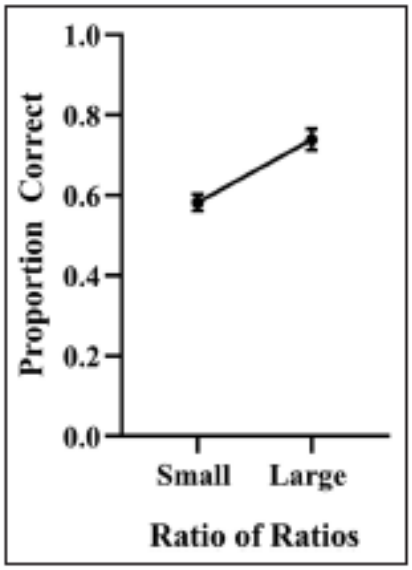

Figure 6. Results of Experiment 4 (adults, mixed-notation paradigm). Proportion correct as a function of RoR. The bars here represent standard errors.

correct than small RoR) versus H0 (i.e., no difference between large and small RoR) across all trials of the experiment ($BF+_0 > 100$). Therefore, there was evidence for a difference in accuracy between large and small RoR.

Discussion

The results of Experiment 4 clearly show an RoR effect in this mixed-format version of the match-to-sample task, indicating that adult participants do access fractional magnitudes to complete this task. This demonstrates that the lack of RoR effect in symbolic fractions in Experiments 1–3 may not necessarily reflect an inability to use the RPS to process symbolic fractions. Rather, it suggests that participants do not naturally access this non-symbolic representation when they are asked to match symbolic fractions and are most likely reverting to componential strategies. Yet, it is interesting to note that overall accuracy in Experiment 4 is reliably above chance (66%, $t(32) = 7.48$, $p < .001$, $d = 1.30$), suggesting that strategies relying on an intuitive understanding of fractional magnitudes may be relatively efficient. The results of the four experiments are discussed in detail below.

General discussion

In the above experiments, we aimed to test the hypothesis that participants are sensitive to ratio magnitudes when they match non-symbolic ratios as well as symbolic fractions, thereby providing further evidence for the existence of the RPS and its potential role in symbolic fraction processing. Using a version of a match-to-sample task developed by Matthews (2015), we asked participants to choose which of two non-symbolic ratios or symbolic fractions corresponded to a given non-symbolic ratio or fraction. Our results indicate that adults have a perceptual sensitivity to ratios when these are presented in a non-symbolic format. This sensitivity does not appear to be affected by the level of math skill, because it was also largely present in individuals with lower math abilities (i.e., children in fourth and fifth grades and adults with dyscalculia). Furthermore, this sensitivity was observed when symbolic fractions had to be matched with a non-symbolic ratio, suggesting that the RPS may contribute to symbolic fraction processing in some situations. However, performance did not depend on the RoR when symbolic fractions had to be matched with other symbolic fractions.

A perceptual route to process the approximate magnitude of fractions

Over our first three experiments, the results clearly indicate that participants' performance is affected by the RoR when they match non-symbolic ratios. Therefore, our results add to the growing evidence that the human cognitive architecture provides a perceptual route to process approximate magnitudes of fractions when represented by non-symbolic ratios (Fazio et al., 2014; Matthews & Chesney, 2015; Matthews & Lewis, 2017). For instance, Matthews (2015) noticed that preschoolers may already possess some perceptual sensitivity towards non-symbolic ratios and that the accuracy of some high-achieving preschoolers might match adult levels (Matthews, 2015; Matthews & Ziols, 2019). Furthermore, this sensitivity has been shown to be present across species, age groups, and even in societies that lack formal mathematic education (Jacob et al., 2012; Jeong et al., 2007; Kalra et al., 2020; McCrink et al., 2013; McCrink & Wynn, 2007; Meert et al., 2012, 2013; Sophian, 2000). Thus, this evidence points towards the RPS being an early developing, evolutionarily ancient system (Lewis et al., 2016).

We also observed that performance of fourth and fifth graders on the non-symbolic ratio comparison task in Experiment 2 was much lower than that of adults in Experiment 1 (contrary to the preliminary study by Matthews, 2015). This might be due to the fact that the RPS might still be developing in children and/or that formal mathematical experience might influence RPS acuity. This would be relatively consistent with research on the ANS, which shows that formal mathematical experience typically improves ANS acuity (Halberda et al., 2008; Pica, 2004). Similarly, even though the RPS might be an evolutionarily ancient system present early on in developmental years, its acuity might be dependent on both the quantity and quality of mathematical experience and the age.

Finally, we found clear RoR effects for non-symbolic line ratios in adults with dyscalculia. Dyscalculia is a

complex, multifaceted neurodevelopmental disorder that hinders acquisition of mathematical concepts despite normal intelligence (Iuculano, 2016). Much of the research on dyscalculia has focused on whether and how the disability affects the processing of whole-number magnitudes. Some researchers have identified ANS deficits to be a potential source of dyscalculia (Butterworth, 2005, 2011; Mazzocco et al., 2011; for a contrasting view, see De Smedt & Gilmore, 2011; Iuculano et al., 2008; Rousselle & Noel, 2007). To our knowledge, no research on perceptual sensitivity of relational magnitudes in adults with dyscalculia exists. Our results in adults with dyscalculia highlight two important points. First, the fact that performance of adults with dyscalculia depends on RoR in our non-symbolic task suggests that these individuals possess the perceptual ability to process ratios. This naturally suggests a spare RPS in adults with dyscalculia. Second, performance of adults with dyscalculia in this task was much lower than that of typical adults, which could be attributed either to the quality of mathematical experience or a slower development of the RPS system. Nonetheless, our findings raise the intriguing possibility that even though adults with dyscalculia may experience difficulties with whole-number concepts, they might still be able to leverage the RPS to learn fractions. One can speculate that this ability to process relational magnitudes between numbers might even be utilised to teach whole-number concepts (Lewis et al., 2016). Future studies on larger sample size and across different age groups will further help develop a comprehensive picture of the development of the RPS in individuals with dyscalculia. This knowledge might aid in current efforts to develop identification criteria and screeners for dyscalculia in children (Rodrigues & Jordan, 2019) and adults.

A limited role for the RPS in symbolic fraction processing

The results of Experiment 4 suggest RPS involvement in symbolic fraction processing. Specifically, we found that typical adults exhibit the signature RoR effect when they have to choose between two symbolic fractions to match a non-symbolic ratio. Our results are in line with a prior study done on cross-format comparisons revealing distance effects across different formats (dot arrays, circle areas, symbolic fractions) indicating magnitude abstraction (Matthews & Chesney, 2015). It is noteworthy to mention that several other studies on fraction magnitude comparison tasks have also demonstrated a link between perceptual sensitivity to non-symbolic ratios and symbolic fraction processing (Matthews et al., 2014). Furthermore, it has been argued that the cross-format comparison tasks are key to understanding a shared system for non-symbolic and symbolic magnitudes, as they rely on the abstract nature of magnitude between the different notations (Matthews & Chesney, 2015).

However, we did not find any RoR effect when participants had to choose which of two symbolic fractions matched another symbolic fraction. This was true for adults (Experiment 1), children from fourth and fifth grades (Experiment 2), and adults with dyscalculia (Experiment 3). Thus, participants do not appear to access ratio magnitudes when performing this matching task. On one hand, these findings are consistent with a body of literature showing that accessing the holistic magnitude of fractions is challenging for typical adults (Kallai & Tzelgov, 2009, 2012; Stafylidou & Vosniadou, 2004). On the other hand, they somewhat conflict with prior studies that suggest that the RPS contributes to symbolic fraction processing. For example, in contrast to our findings, studies have found that performance on fraction comparison tasks is influenced by linear or RoR distance (Kalra et al., 2020; Matthews & Chesney, 2015; Schneider & Siegler, 2010).

Interestingly, this difference between our results and those prior findings does not seem to stem from a clear difference in the timing of response. For example, the average response time was 2,284 ms for adults in Experiment 1, which is only slightly higher than the average response time of 1,995 ms found in the fraction comparison task designed by Matthews & Chesney (2015); consider also that three fractions are displayed simultaneously in a match-to-sample task, compared with only two fractions in a comparison task. It is also unlikely that our participants may have chosen not to rely on fractional magnitude because they used another analogical strategy involving a comparison of the size of components of fractions. Indeed, in contrast to what was found for non-symbolic ratios (see below), the ratio between numerators of match and distractor did not influence performance on symbolic fractions. Thus, differences between our findings and that of previous studies are more likely due to differences in the processes involved in magnitude comparison versus match-to-sample tasks. In magnitude comparison tasks, participants are asked to compare the relative sizes of two fractions (e.g., assess which fraction is larger). This task can arguably be done without accessing the exact value of each fraction. Thus, magnitude comparison tasks are likely to encourage participants to simply estimate approximate magnitudes (which might promote strategies relying on analogical processing). In our match-to-sample task, however, we asked participants to find which of two fractions corresponded to a target fraction. Although we acknowledge that it is possible to complete this task by approximating ratios, we speculate that such an equivalence task more likely prompts participants to access the exact rather than the approximate value of fractions. Therefore, although children, typical adults, and adults with dyscalculia may rely on an intuitive sense of ratios to process non-symbolic ratio magnitudes, our findings raise the possibility that this sense may only drive performance in tasks that encourage subjects to approximate ratios.

The role of componential strategies in symbolic fraction and non-symbolic ratio processing

Overall, our study clearly highlights an influence of components of fractions in typical adults. Interestingly, this influence was not limited to symbolic fractions but was also observed in non-symbolic line ratios. For example, in addition to RoR, accuracy in the non-symbolic task depended upon (a) whether the numerator (i.e., smallest of the lines) of the target was the same as the numerator of the distractor and (b) whether the ratio between the numerators of the match and distractor was small or large. For symbolic fractions, accuracy was also higher when numerators were identical between match and distractor, indicating that participants might have used common components to process magnitudes. This influence of components on fraction processing is consistent with previous reports. For instance, experts have been shown to make use of componential strategies when they are efficient to solve fraction problems (Obersteiner et al., 2013). Drawing on concepts from the *Dynamic Strategy Choice* account (Alibali & Sidney, 2015), we argue that the specific task feature of a common numerator might have led participants to utilise a simple and effective "compare the denominator" strategy. It is also possible that the use of such a strategy with symbolic fractions might have influenced the adults to utilise a similar strategy for non-symbolic formats (in addition to the RPS). Note that it is also possible that the use of trials in which the numerator of the target was identical to that of the distractor might have also encouraged participants to use componential strategies, even on other trials in which the numerator of the target was different than that of the distractor. Although this equal mixture of trials was chosen to present participants with a variety of fractions, future studies might investigate to what extent the use of componential strategies may depend on different types of fractions on such match-to-sample tasks.

This ability to utilise and form new strategies to solve problems may also develop with age and mathematical experience, consistent with the fact that the identity of the numerator (i.e., same or different across target and distractor) influenced performance in adults (Experiment 1) but not in children (Experiment 2). These results are similar to a study done by Zhang and colleagues (2014), whereby based on accuracy rates the authors noted that adults are better equipped to adopt componential processing strategies than children. Finally, an important question regarding the role of formal education in shaping perceptual abilities was raised by Lewis et al. (2015) in their chapter. We believe our results might have shed some light on this phenomenon. The finding that processing non-symbolic line ratios may rely on componential strategies might suggest that formal education and years of experience could be responsible for the transfer of these efficient strategies from symbolic fractions to non-symbolic line ratios, thus playing a role in shaping perceptual abilities.

Symbolic fraction knowledge in adults with dyscalculia

A couple of prior studies have investigated fraction knowledge in children with dyscalculia. These studies point to the difficulties encountered by children with dyscalculia when processing symbolic fractions. For example, it has been shown that children with dyscalculia fail to order fractions, estimate the sum of two fractions, and identify equivalent fractions (Hansen et al., 2017; Mazzocco & Devlin, 2008). In a study by Siegler and Pyke (2013) and Bailey et al. (2015), understanding magnitudes of fractions and performing arithmetic computations were shown to be challenging for children with dyscalculia. Another study showed that children with dyscalculia also persist to view visual models of fractions as concrete representations until Grade 8, which misguides them on Arabic fraction representations (Mazzocco et al., 2013). To our knowledge, no studies to date have examined the difficulties and strategies used by adults with dyscalculia when performing fraction tasks. Exploring how symbolic fractions are processed by adults with dyscalculia might aid in understanding the misconceptions and challenges faced by them on fraction tasks. Our results show that the performance of adults with dyscalculia was lower for both symbolic and non-symbolic tasks compared with typical adults. Furthermore, effective effortful strategies like componential processing were not utilised by adults with dyscalculia, which might have been one of the reasons for a lower accuracy score. This indicates that adults with dyscalculia struggle with equivalent fractions and are unable to utilise componential processing strategies effectively compared with typical adults.

Conclusion

The aim of this research was twofold; first, to examine perceptual sensitivity towards non-symbolic ratio magnitudes in participants with varying levels of math skills and, second, to investigate the extent to which the perceptual ability is utilised for symbolic fraction processing. We found evidence for perceptual sensitivity towards non-symbolic ratio magnitudes in all groups of participants, irrespective of their age or level of math skill. A clear pattern that also emerged in our study was that children and adults with and without dyscalculia do not rely on the RPS when they have to find equivalent symbolic fractions (Experiments 1–3). However, they do rely on the RPS when the ratios behind symbolic fractions have to be estimated (Experiment 4). Together these findings add to growing evidence for an intuitive, perceptual sensitivity for non-symbolic relational magnitudes. They also indicate that the role of the RPS in symbolic fraction processing is limited to tasks in which participants have to approximate the value of a fraction.

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Supplementary Material

The Supplementary Material is available at qjep.sagepub.com

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CHAPTER V: STUDY II

Bhatia, P., Longo, L., Chesnokova, H., & Prado, J. (submitted). Neural representations of absolute and relative magnitudes in symbolic and non-symbolic formats

Abstract

Although many animal species can represent magnitudes non-symbolically (e.g., ••), humans are unique in their use of symbols to represent numerical information (e.g., two or 2). This symbolic representation of numerical magnitudes has long been thought to emerge from the 'neural recycling' of brain mechanisms processing non-symbolic magnitudes in the intraparietal sulcus (IPS), a hypothesis that has been applied to both absolute (i.e., numbers) and relative (i.e., fractions) magnitudes. Yet, evidence for the neuronal recycling hypothesis is inconsistent for absolute magnitudes and scarce for relative magnitudes. Here, we investigated to what extent the neural representations of symbolic absolute and relative magnitudes overlap with the neural representations of non-symbolic absolute and relative magnitudes in the IPS. In an fMRI adaptation design, adult participants were presented with blocks of (1) non-symbolic absolute magnitudes (lines), (2) symbolic absolute magnitudes (numbers), (3) non-symbolic relative magnitudes (line ratios), and (4) symbolic relative magnitudes (fractions). Univariate analyses provided limited evidence for the neuronal recycling hypothesis, with an overlap between symbolic and non-symbolic representations in the IPS that was restricted to absolute magnitudes and depended upon participants' fluency in symbolic math. Multivariate analyses did not provide any evidence that similar IPS brain regions support both non-symbolic and symbolic magnitudes across all participants. Instead, a region of the right IPS encoded differences in format (non-symbolic versus symbolic) across both absolute and relative

magnitudes. Therefore, our study suggests that IPS activity during numerical tasks may depend on the presentation format (non-symbolic versus symbolic) more than it depends on the type of magnitude (absolute versus relative), at least for most adult participants.

Introduction

Humans possess the ability to represent magnitudes both non-symbolically (e.g., ••) and symbolically (e.g., two or 2). This ability is both shared and unique among other animals. On the one hand, infants and many animal species can estimate and discriminate non-symbolic absolute magnitudes, suggesting that the human brain may be endowed with a non-symbolic Approximate Number System (ANS) that is innate and evolutionarily old (Barth, La Mont, Lipton, & Spelke, 2005; Boysen & Capaldi, 1993; Brannon, 2005; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Dehaene, 1997; Pica, Lemer, Izard, & Dehaene, 2004; Xu, Spelke, & Goddard, 2005). On the other hand, the ability to represent absolute magnitudes as symbolic natural numbers is only found in humans and is largely believed to be a product of culture and language (Ansari, 2008). Yet, it has long been proposed that this culturally developed ability is grounded in the evolutionarily old capacity to process non-symbolic magnitudes (Dehaene et al., 2003; Ansari, 2008). For instance, the 'neuronal recycling' hypothesis argues that learning symbolic natural numbers relies on the co-option of brain mechanisms supporting non-symbolic magnitude processing, which are largely thought to be located in the intra-parietal sulcus (IPS) (Nieder, 2016). In other words, it has been claimed that the same mechanisms of the IPS may represent both non-symbolic and symbolic magnitudes at an abstract level in adults (Dehaene & Cohen, 2007), such that symbolic natural numbers may automatically activate the neural

representations of absolute magnitudes in that region (Eger, Sterzer, Russ, Giraud, & Kleinschmidt, 2003).

Symbolic mathematical skills, however, go largely beyond the ability to represent absolute magnitudes in humans. They also involve the ability to represent magnitudes in relation with one another, for instance using fractions or decimals (i.e., rational numbers). Although the neuronal recycling theory was initially developed in the context of natural numbers, a similar proposal has recently emerged to explain the cultural acquisition of symbolic rational numbers (Lewis, Matthews, & Hubbard, 2016). Indeed, a growing body of evidence shows that infants and non-human primates are sensitive to ratios and relational quantities (Drucker et al., 2016; Eckert et al., 2018; Tecwyn et al., 2017; Vallentin & Nieder, 2008, 2010; Woodruff & Premack, 1981; Denison & Xu, 2014; McCrink & Wynn., 2007). This suggests the existence of an evolutionary old non-symbolic Ratio Processing System (RPS) akin to the ANS but tuned exclusively to relative quantities (Lewis, Matthews, & Hubbard, 2016). This cognitive system might provide the foundation for the acquisition of symbolic rational numbers (Lewis, Matthews, & Hubbard, 2016). Though the neural basis of this RPS is less clear than that of the ANS, this line of thought suggests that overlapping brain regions may represent both non-symbolic and symbolic ratios abstractly. In other words, symbolic rational numbers may automatically activate the neural representations of relative magnitudes.

To date, evidence that the acquisition of either natural or rational numbers relies on the recycling of brain pathways dedicated to the processing of non-symbolic magnitudes remains equivocal. Overall, neuroimaging studies focusing on the processing of natural numbers have consistently found involvement of the IPS in both symbolic (Arabic digits or number words) and non-symbolic (dot patterns) tasks

(Neider, 2016; Sokolowski, Fias, Mousa, & Ansari, 2017). However, studies that directly compared the neural substrates of symbolic and non-symbolic natural number processing within the same participants show inconsistent results (Cohen Kadosh, Cohen Kadosh, Kaas, Henik, & Goebel, 2007; Cohen Kadosh, Bahrami, Walsh, Butterworth, Popescu, & Price, 2011; Damarla & Just, 2013; Bulthé, De Smedt, & Op de Beeck, 2014; Eger, Michel, Thirion, Amadon, Dehaene, Kleinsch-midth, 2009; Piazza et al., 2007). For instance, in a seminal study using fMRI adaptation, Piazza et al. (2007) found that Arabic digits and dot patterns were represented in the same region of the IPS, supporting the neuronal recycling hypothesis. Some studies using multivariate analysis have also shown some degree of overlap between the brain mechanisms supporting symbolic and non-symbolic magnitude processing (Eger et al., 2009). However, other studies concluded that Arabic numerals and dot patterns are supported by different neural populations in the IPS and surrounding brain regions (Bulthe, De Smedt, & Op de Beeck, 2013; Cohen Kadosh, Cohen Kadosh, Kaas, Henik, & Goebel, 2007; Cohen Kadosh, Bahrami, Walsh, Butterworth, Popescu, & Price, 2011). Overall, neuroimaging evidence is inconsistent regarding whether a natural number is represented abstractly or in a format-dependent manner in the human brain (Ansari, 2016; Damarla & Just, 2013; Wilkey & Ansari, 2019).

Compared to the neuroimaging literature on the representations of natural numbers, few studies have investigated the neural representations and processing of rational numbers. Therefore, support for the idea that the neural substrates of the RPS may be ‘recycled’ for the processing of symbolic fractions is scarce (Lewis, Matthews, & Hubbard, 2016; Mock et al., 2018). Nonetheless, the available studies suggest that the IPS is involved in the processing of both symbolic fractions (DeWolf et al., 2016; Jacob & Neider, 2009b; Ischebeck, Schocke, & Delazer, 2009) and non-symbolic line

ratios (Jacob & Neider, 2009a) in adults. To our knowledge, there is only one study comparing the neural bases of symbolic and non-symbolic relative magnitude processing within the same participants (Mock et al., 2018, 2019). In that study, fMRI activity was measured while adult participants performed a magnitude comparison task in four formats (fractions, dot patterns, decimals, and pie charts) (Mock et al., 2018). Results point to overlapping activation between symbolic (e.g., fractions and decimals) and non-symbolic (e.g., dot patterns and pie charts) proportions in the IPS, but also to format-dependent activity in other brain regions. To some extent, the format-independent activity found in the IPS supports the idea that there might be an abstract representation of relative magnitudes in the human brain. However, because this study used active tasks, it is unclear whether any overlapping neural activation is due to common processing of relative magnitudes or to a common reliance on response selection processes that also rely on the IPS (Göbel, Johansen-Berg, Behrens, & Rushworth, 2004).

Here, we aimed to test whether the culturally developed ability to represent both absolute and relative magnitudes symbolically (e.g., using natural numbers and fractions) relies on the neural representations of absolute and relative magnitudes in a non-symbolic format. To this aim, we adapted a passive blocked fMRI adaptation paradigm used in Girard et al., 2021 and Perrachione et al., 2016 to investigate the neural representations of absolute and relative magnitudes in different formats, while avoiding confounds due to active tasks. FMRI adaptation refers to the idea that repeatedly presenting a series of visual stimuli with a common property leads to a decrease in the activity of neurons that are sensitive to that property (Grill-Spector and Malach, 2001). This sensitivity is captured by the *neural adaptation effect*, measured by comparing blocks of stimuli that differ from one another with respect to the property

(i.e., no-adaptation blocks) to blocks of stimuli that do not (i.e., adaptation blocks). In the present study, participants were presented with adaptation and no-adaptation blocks of (1) non-symbolic absolute magnitudes (lines), (2) symbolic absolute magnitudes (numbers), (3) non-symbolic relative magnitudes (line ratios), and (4) symbolic relative magnitudes (fractions) (**Figure 1**). Adaptation and no-adaptation blocks differed with respect to the numerical distance separating lines, numbers, line ratios, or fractions within a block, such that stimuli were close in magnitude from one another in adaptation blocks and further apart in no-adaptation blocks.

We tested the neuronal recycling hypothesis of absolute and relative magnitudes using both univariate and multivariate methods. First, using univariate analyses, we aimed to identify the neural regions that may be sensitive to a change in numerical distance between stimuli across all participants, either with respect to their absolute magnitude (for lines and numbers) or relative magnitude (for line ratios and fractions). This should translate into a decrease of activity in adaptation compared to no-adaptation blocks in these regions (i.e., a neural adaptation effect). The neuronal recycling hypothesis assumes that processing symbolic stimuli (i.e., numbers and fractions) relies on neural mechanisms supporting non-symbolic stimuli (i.e., lines and ratios). Thus, this hypothesis predicts that overlapping regions of the IPS may be associated with a neural adaptation effect for (1) numbers and lines and (2) fractions and ratios. Second, using multivariate analyses, we aimed to explore the relations between the patterns of activation associated with symbolic and non-symbolic stimuli across magnitude types. Specifically, because the neuronal recycling hypothesis assumes that similar IPS mechanisms process symbolic and non-symbolic stimuli, it predicts that patterns of IPS activity may be similar between symbolic and non-

symbolic stimuli and are more likely to depend on the type of numerical magnitude (i.e., absolute versus relative).

Material and methods

Participants

Fifty-three right-handed adults participated in the experiment. Participants were contacted through advertisements on social media. Five participants were excluded from the study because of technical errors in the experimentation set-up (n=4) and contraindications to the MRI (n=1). Therefore, 48 adults *(Mean age* = 22.09*,* 34 *females)* were included in the main analyses. All adults were native French speakers and with no history of neurological or psychiatric disorders. Participants gave written informed consent and were paid 80 euros for their participation. The study was approved by a national ethics committee (CPP- Strasbourg Est IV).

Psychometric testing

Verbal IQ and spatial IQ were estimated using the verbal reasoning and matrix reasoning subtest of the WAIS-IV (Wechsler, 2008). Fluency in symbolic math was assessed using the Math Fluency of the Woodcock-Johnson Test of Achievement (WJ-III) (Woodcock, Mather, McGrew, & Wendling, 2001). In this test, participants have to solve as many single-digit addition, subtraction, multiplication, and division problems as they can within 3 min. Participants also completed the Applied Problems subtest of the WJ-III. Unlike the Math Fluency subtest, this test is un-timed and measures the ability to analyze basic numerical concepts and oral word problems. The test stops after 6 consecutive errors or when the last item is reached. Reading fluency was assessed with the Alouette-R test (Lefavrais, 1967). This test requires participants

to read a 265-word text aloud in 3 minutes and measures the number of words read correctly to evaluate the reading precision and speed.

In-scanner task

Participants were presented with a passive blocked adaptation paradigm adapted from Girard et al., 2021 and Perrachione et al., 2016. In this paradigm, participants are passively presented with blocks of stimuli at the center of the screen. Here we presented four types of stimuli in four different runs of approximately 5 min: lines, numbers, line ratios, and fractions (**Figure 1**.). Numbers ranged from 1 to 62 (~ 1.72° of visual angle), fractions ranged from $\frac{1}{24}$ to 1 in magnitude (~ 3.45° of visual angle), and lines ranged from 0.98 to 17.8 cm in length on a 37 cm screen (corresponded to ~ 0.69 - 12.24° of visual angle). All stimuli were shown in white on a black background. Within each run, participants were presented with adaptation and no-adaptation blocks (**Figure 1**). Adaptation and no-adaptation blocks differed with respect to the numerical distance between the stimuli. Specifically, adaptation blocks consisted in the sequential presentation of 8 quantities in a total of which 4 quantities were the same in magnitude and the other 4 quantities were relatively close. For instance, in the number adaptation block of 23, 26, 25, 26, 24, 26, 21, 26, four stimuli (e.g., 26) are identical and the other four stimuli have a minimum distance of 1 and a maximum distance of 5 between each other. Similarly, for fractions, adaptation to 1:4 was composed of four exact equivalent fractions 2/8, 1/4, 4/16, 7/28 and the rest of the stimuli had the denominator changed by adding or subtracting 1 to the original fractions (i.e., 3/11, 5/19, 8/31,6/23). Half of the stimuli for the adaptation block of fractions were constructed by small changes to the denominator (+1 or -1) to prevent the participant from reducing the fraction to its lowest form, thereby avoiding

confounds due to calculation. No-adaptation blocks consisted in the sequential presentation of 8 quantities that were relatively far from one another in magnitude (e.g., a minimum distance of 2 and a maximum distance of 55 between consecutive numbers in a block, and minimum magnitude of 1/24 to maximum magnitude of 11/12 for a block of fractions). The size of the individual line lengths and line ratios corresponded to those used for numbers and fractions. So, for line ratios, the length of the smaller line was calculated as the ratio of the longer line length such that a fraction corresponding to $\frac{3}{15}$ would be a line ratio where the smaller line length is $\frac{3}{15}$ as long as the longer line length. Thus, the absolute line lengths did not vary with proportion. The shorter line was always on the left (i.e., corresponding to the numerator of a proper fraction) while the longer line was always on the right (i.e., corresponding to the denominator of a proper fraction). The complete list of stimuli can be found in **Appendix B.**

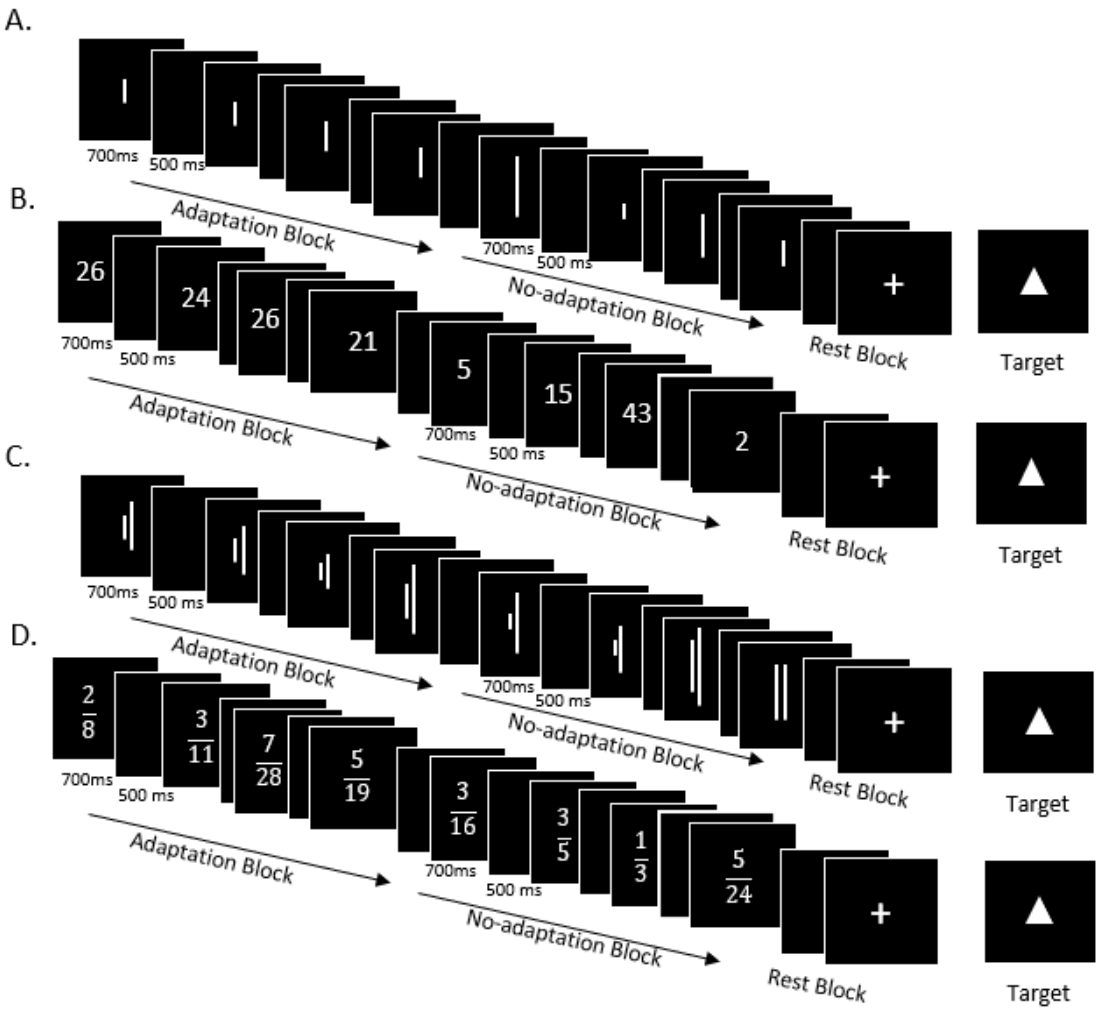

Figure 1: Experimental design. Participants were adapted to the sequential presentation of four types of stimuli that varied in format (non-symbolic versus symbolic) and magnitude type (absolute versus relative). A. Adaptation to lines. B. Adaptation to numbers. C. Adaptation to line ratios. D. Adaptation to fractions.

Experimental timeline

In each adaptation and no-adaptation block, stimuli remained on the screen for 700 ms, with a 500 ms inter-stimulus interval (for a total block duration of 9.6 seconds). Ten adaptation blocks and ten no-adaptation blocks were presented along with ten blocks of visual fixation (duration = 9.6 seconds) in each run. Block presentation was pseudo-randomized such that 2 blocks of the same type could not follow each other. Finally, 10 target stimuli (a triangle) randomly appeared in each run (outside of the

blocks) to ensure that participants paid attention to the stimuli. Participants were asked to press a button every time this target appeared. The task was presented using Psychopy (Peirce et al., 2019).

fMRI Data Acquisition

Images were collected with a Siemens Prisma 3T MRI scanner (Siemens Healthcare, Erlangen, Germany) at the CERMEP Imagerie du vivant in Lyon, France. The BOLD signal was measured with a susceptibility-weighted single-shot EPI sequence. Imaging parameters were as follows: TR = 2000 ms, TE = 24 ms, flip angle = 80°, matrix size = 128 × 120, field of view = 220 × 206 mm, slice thickness = 3 mm (0.48 mm gap), number of slices = 32. A high-resolution T1-weighted whole-brain anatomical volume was also collected for each participant. Parameters were as follows: TR = 3500 ms, TE = 2.24 ms, flip angle = 8°, matrix size = 256 × 256, field of view = 224 × 224 mm, slice thickness = 0.9 mm, number of slices = 192.

fMRI data preprocessing

fMRI data analysis was performed using SPM12 (http://www.fil.ion.ucl.ac.uk/spm, Welcome department of Cognitive Neurology, London, UK). The first 3 images of each run were discarded to allow for T1 equilibration effects. Functional images were corrected for slice acquisition delays and spatially realigned to the first image of the first run to correct for head movements. Realigned images were smoothed with a Gaussian filter (4 × 4 × 7 mm full-width at half maximum). Using ArtRepair (https://www.nitrc.org/projects/art_repair/), functional volumes with a global mean intensity greater than 3 standard deviations from the average of the run or a volume-to-volume motion greater than 2 mm were identified as outliers and substituted by the interpolation of the 2 nearest non-repaired volumes

(Romeo et al., 2018). Finally, functional images were normalized into the standard Montreal Neurological Institute (MNI) space. This was done in two steps. First, after coregistration with the functional data, the structural image was segmented into grey matter, white matter, and cerebrospinal fluid by using a unified segmentation algorithm (Ashburner and Friston, 2005). Second, the functional data were normalized to the MNI space by using the normalization parameters estimated during unified segmentation (normalized voxel size, $2 \times 2 \times 3.5$ mm^3).

Univariate analyses

For each participant and each run, a general linear model analysis was conducted on brain activity associated with adaptation and no-adaptation blocks. Blocks were modeled as epochs with onsets time-locked to the beginning of each block and a duration of 9.6 sec per block. All epochs were convolved with a canonical hemodynamic response function. The time-series data were high-pass filtered (1/128Hz), and serial correlations were corrected using an auto-regressive AR (1) model. The neural adaptation effect was measured by subtracting activity associated with adaptation blocks from activity associated with no-adaptation blocks. These subject-specific contrasts were then submitted to the second level for group-level random effect analyses. Clusters were considered significant at a FWE-corrected threshold of $p < .05$ (using a cluster-defining threshold of $p < .005$, uncorrected).

Multivariate analyses

In addition to the main univariate analysis, we also used the CosmoMVPA toolbox (https://www.cosmomvpa.org/) to perform an exploratory RSA analysis assessing the similarity and dissimilarity of neural activation patterns associated with different magnitude types (absolute versus relative) and presentation formats

(symbolic versus non-symbolic). This analysis was conducted on the four beta maps corresponding to the contrasts of lines versus fixation, numbers versus fixation, line ratios versus fixation, and fractions versus fixation (collapsing across adaptation and no-adaptation blocks). First, we created two 4x4 theoretical representation dissimilarity matrices (RDMs), corresponding to (1) the expected dissimilarity between absolute and relative magnitudes (and expected similarity between symbolic and non-symbolic stimuli) and (2) the expected dissimilarity between symbolic and non-symbolic stimuli (and expected similarity between absolute and relative magnitudes). In the *absolute versus relative* RDM (see **Figure. 4A**), all stimuli of the same magnitude type (numbers - numbers, numbers - lines, fractions - fractions, fractions - line ratios) had a dissimilarity coefficient of 0, whereas all stimuli of a different magnitude type (numbers - fractions, numbers – line ratios, fractions - lines, lines – line ratios) had a dissimilarity coefficient of 1. In the *symbolic versus non-symbolic* RDM (see **Figure. 5A**), all stimuli of the same format (numbers - numbers, numbers - fractions, lines - lines, lines - line ratios) had a dissimilarity coefficient of 0, whereas all stimuli of a different format (numbers - lines, numbers – line ratios, lines - fractions, line ratios - fractions) had a dissimilarity coefficient of 1. Second, we extracted brain activity from the four contrasts (i.e., lines versus fixation, numbers versus fixation, line ratios versus fixation, and fractions versus fixation) using spherical searchlights (1.4 cc, i.e., 100 voxels) at every voxel in the brain. A 4x4 neural DSM was constructed for each searchlight, which each cell representing 1 minus the Pearson correlation between the voxel-wise beta value for each pair of contrasts. The Pearson correlation between the neural DSM and each theoretical DSM was then calculated for each searchlight and converted to a z value using a Fisher transform. The Fisher-transformed correlation coefficient for each searchlight was systematically associated

with the central voxel of that searchlight. Fisher-transformed correlation maps were then submitted to second-level one-sample t-tests across all participants to identify voxels for which the correlation between the theoretical and neural DSMs was greater than 0. Clusters were considered significant at a FWE-corrected threshold of $p < .05$ (using a cluster-defining threshold of $p < .005$, uncorrected).

Data and software availability

For each participant and each task, whole-brain unthresholded maps of adaptation effects will be publicly available via NeuroVault. The general and custom scripts used to analyze fMRI data are available on https://github.com/BBL-lab/BBL-batch-system.

Results

Psychometric testing and in-scanner performance

Standardized verbal IQ ranged from 85 to 140 (*mean* = 117.29), while standardized spatial IQ ranged from 70 to 120 (*mean* = 94.68). Thus, participants' IQ was in the normal to the superior range. The number of arithmetic problems correctly solved in 3 min in the Math fluency subtest ranged from 47 to 160 (*mean* = 114.25), suggesting a substantial variability in arithmetic fluency among participants. The untimed Applied problems subtest indicated less variability, with scores ranging from 39 to 61 (*mean* = 49.06). Finally, participants' reading precision scores ranged from 90.18 to 100 (mean = 98.28), and reading speed ranged from 336.69 to 787.11 (*mean* = 551.198) (the optimal cut-off for dyslexia is a reading precision score above 87 or reading speed above 402.26; Cavalli et al., 2018).

To make sure participants were attentive to the stimuli in the scanner, a target detection task was inserted in all four tasks. Participants had to press a button when they saw a triangle during the task. Average performance on detection of the target for the different runs was 95.3% (*SD* = 0.152) for fraction, 92.7% (*SD* = 0.186) for numbers, 91.4% (*SD* = 0.208) for line ratios, and 92.4% (*SD* = 0.194) for lines. There was no difference in target detection between the four tasks (*all t's < 1.9, all p's >* 0.06), indicating that participants paid equal attention to the stimuli in the tasks. The response to target stimuli was not correlated with math fluency and applied problem skills (*all r's* >- 0.24, *all p's* > 0.10).

Univariate analyses

For each stimulus type (lines, numbers, line ratios, and fractions), brain activity associated with adaptation blocks was subtracted from activity associated with no-adaptation blocks to identify brain regions showing a neural adaptation effect across all participants. For lines, a neural adaptation effect was observed in the bilateral IPS as well as in a wider network of brain regions encompassing the precentral and occipital cortices (see **Table 1** and **Figure 2A**). For numbers, the only region showing a significant neural adaptation effect was located in the left fusiform gyrus (see **Table 1** and **Figure 2B**). No significant adaptation effect was observed in any brain region for either fractions or line ratios.

Contrary to our assumptions, lines were the only stimuli associated with a significant neural adaptation effect in the IPS across all participants. Therefore, we did not find any evidence that common neural mechanisms in the IPS may process both symbolic and non-symbolic stimuli. However, there was relatively large variability in participants' fluency with symbolic math (as suggested by the Math fluency subtest,

see above). It is thus possible that the neural adaptation effect for symbolic stimuli may depend on participants' levels of fluency. This would be consistent with the results of our previous study using a similar paradigm, in which we found a positive correlation between neural adaptation to numbers and Math fluency scores (Girard et al., 2021). In other words, because our paradigm is passive, magnitudes of symbolic stimuli such as numbers and fractions may only be automatically processed by the most fluent participants. To examine this possibility, we performed an exploratory analysis in which we regressed neural adaptation effects on participants' Math fluency scores across the whole brain. We did not find any positive relation between math fluency and neural adaptation effect for fractions, lines, or line ratios. For numbers, however, the neural adaptation effect increased with math fluency in a region of the left IPS (see **Figure. 3A**). Critically, a conjunction analysis revealed that this region overlapped with the region showing an overall neural adaptation effect across all participants for lines (center of mass: x=-28, y=-42, z=55, the volume of overlap = 70mm^3) (see **Figure. 3B**). Thus, increased math fluency was linked to an enhanced neural adaptation effect for numbers in the same left IPS region that exhibited a neural adaptation effect across all participants for lines.

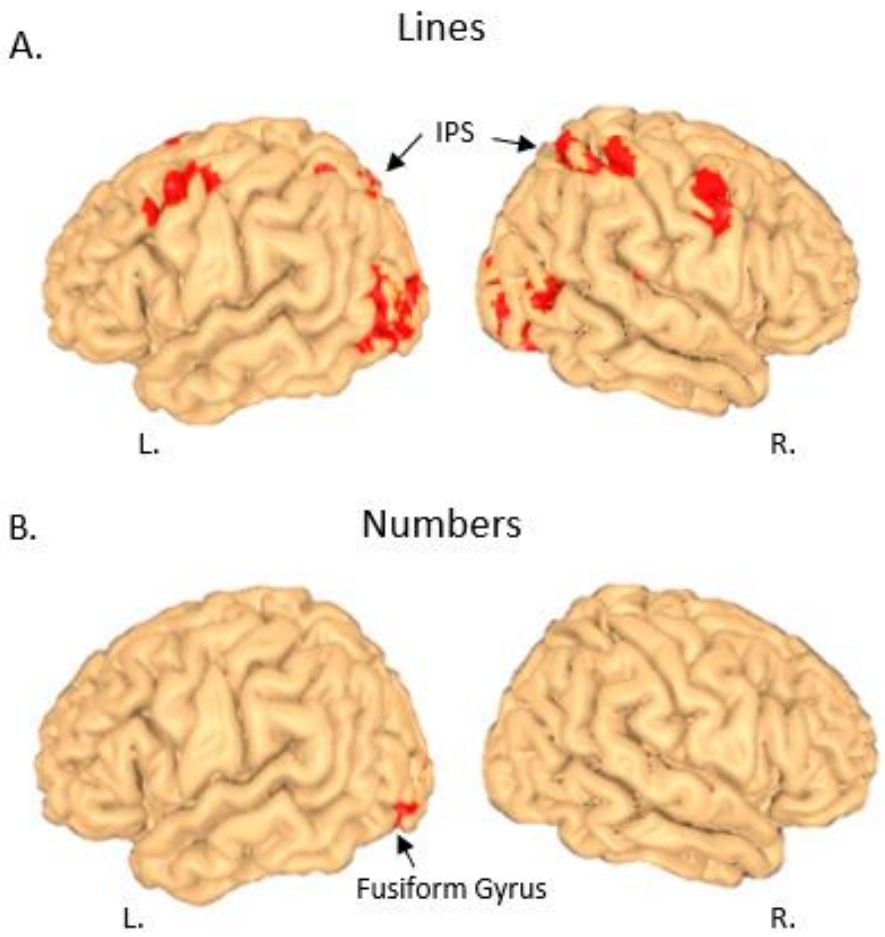

Figure 2: Neural adaptation effects across all participants (univariate analyses). A. Brain regions showing a neural adaptation effect for lines. B. Brain regions showing a neural adaptation effect for numbers.

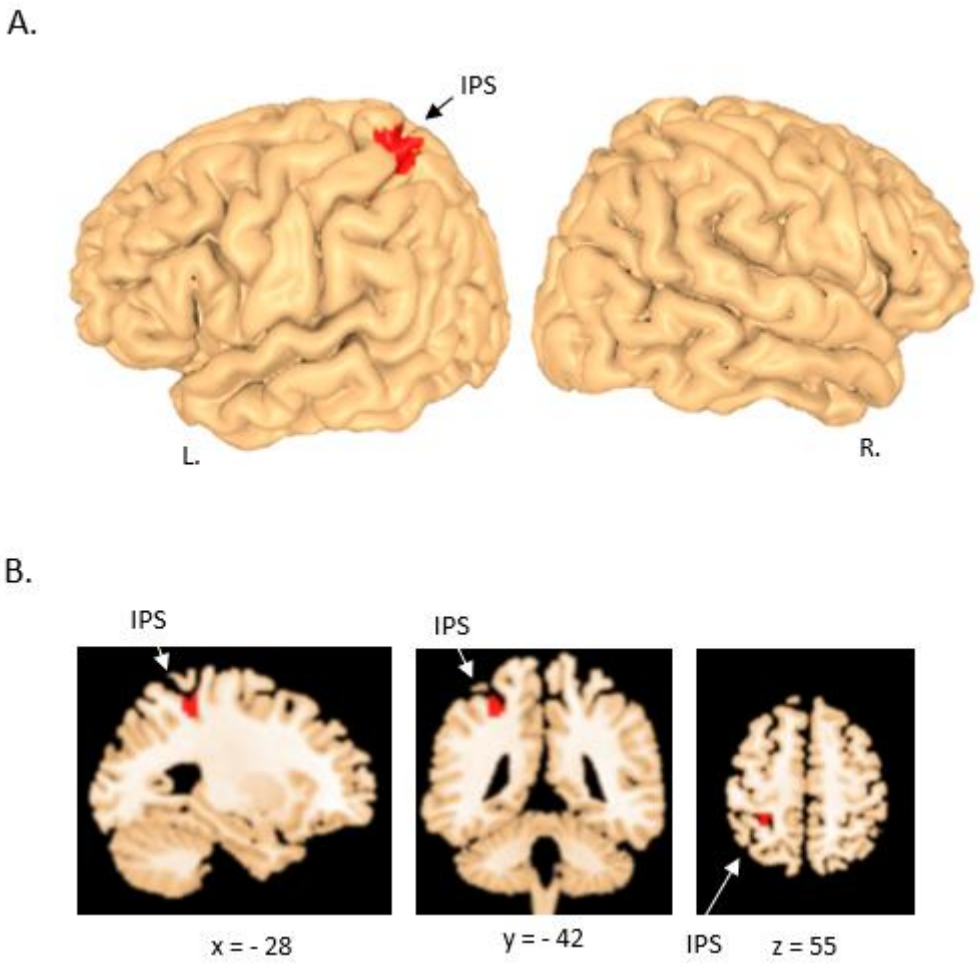

Figure 3: Relation between arithmetic fluency and neural adaptation effects (univariate analyses). A. Brain region showing a positive relation between arithmetic fluency and neural adaptation effect for numbers. B. Brain region showing both a positive relation between arithmetic fluency and neural adaptation effect for numbers and a neural adaptation effect for lines across all participants (conjunction analysis).

Table 1: Brain regions showing an effect of neural adaptation across all participants (univariate analyses).

Anatomical Location	Cluster level $P_{FWE\text{-}corrected}$	Cluster size (cc)	MNI coordinates			t-score
			x	y	z	
Line adaptation task						
L. Inferior Occipital Gyrus	.000	28.55	-32	-90	-4	5.89
L. Intraparietal Sulcus	-	-	-34	-50	51	3.33
R. Inferior Occipital Gyrus	.000	21.01	40	-84	-12	5.48
R. Intraparietal Sulcus	-	-	30	-58	55	3.99
R. Thalamus	.019	2.13	10	-16	13	5.44
R. Supplementary Motor area	.000	6.27	2	12	66	5.16
R. Hippocampus	.004	2.74	22	-20	-12	4.81
L. Caudate	.026	2.0	-14	-2	13	4.37
R. Precentral Gyrus	.033	1.9	-44	4	52	4.33
R. Precentral Gyrus	.003	2.85	54	-2	44	4.31
L. Posterior Cingulate	.025	2.01	-6	-42	16	4.31
Number adaptation task						
L. Occipital Fusiform Gyrus	.004	2.81	-26	-90	-12	4.46
Line Ratio adaptation task						
No suprathreshold cluster						
Fraction adaptation task						
No suprathreshold cluster						

Notes. L = left; R = right; MNI: Montreal Neurological Institute; FWE-corr: Family-wise error corrected.

Multivariate analyses

As described above, the univariate analyses only provided limited evidence for common brain mechanisms in the IPS processing non-symbolic and symbolic magnitudes. We then turned to multivariate analyses (RSA) to test whether patterns of IPS activity may depend on the type of numerical magnitude (and be similar between non-symbolic and symbolic stimuli) or on the presentation format (and be similar between absolute and relative magnitudes). This was done by evaluating the degree of (1) dissimilarity between patterns of activation associated with absolute and relative magnitudes (and similarity between non-symbolic and symbolic stimuli) (see **Figure 4A**) and (2) dissimilarity between patterns of activation associated with symbolic and non-symbolic stimuli (and similarity between absolute and relative magnitudes) (see **Figure 5A**). On the one hand, as shown in **Figure 4B** and **Table 2**, a limited brain system distinguished between absolute and relative magnitudes while representing similarly non-symbolic and symbolic magnitudes. This system was composed of the right occipital cortex and left rostro-lateral prefrontal cortex. Critically, this system did not include the IPS. On the other hand, as shown in **Figure 5B** and **Table 2**, a larger brain system distinguished between symbolic and non-symbolic format while representing similarly absolute and relative magnitudes. This system encompassed the bilateral occipital and middle temporal cortices, but also included the right IPS. Overall, these results suggest that patterns of IPS activity depend on the presentation format (non-symbolic versus symbolic) more so than they depend on the type of magnitudes (absolute versus relative).

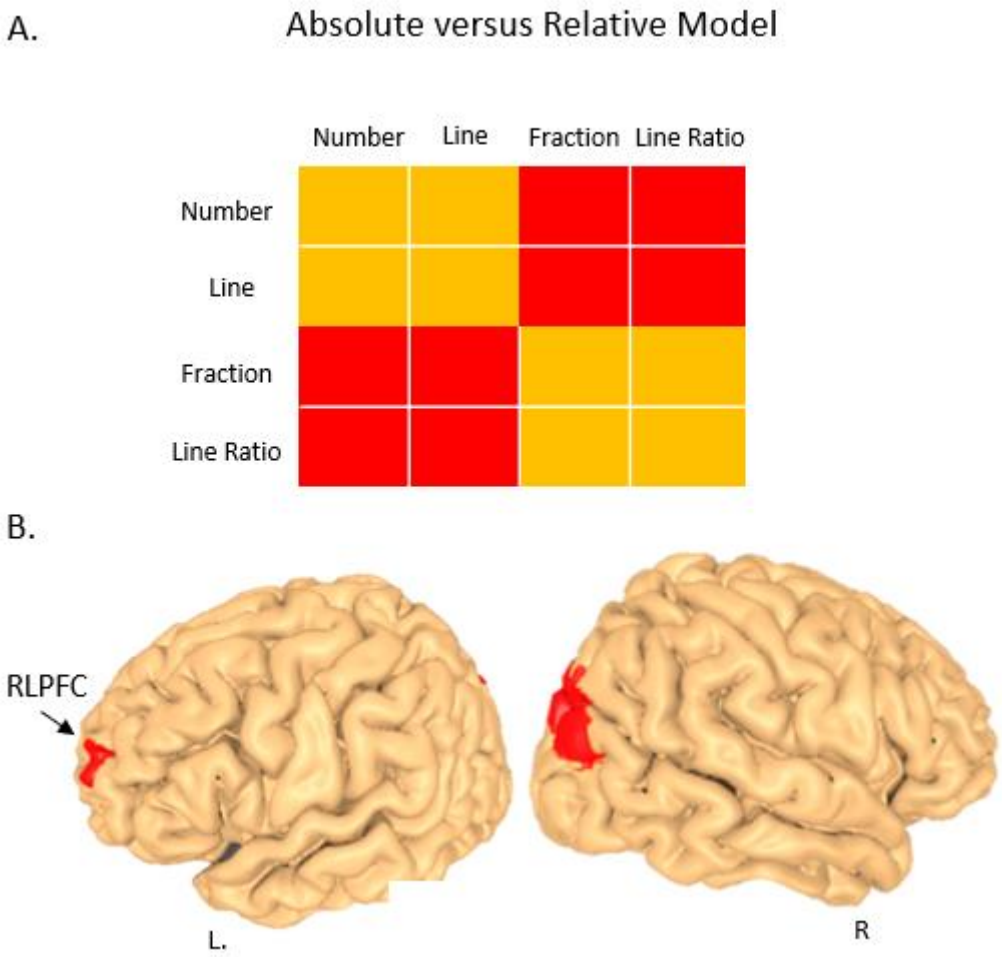

Figure 4: Results of the Representational Similarity Analysis for the Absolute versus Relative model (multivariate analysis). A. Hypothesized Model for the RSA, the matrix represents a dissimilarity matrix where red denotes dissimilar items (0) and yellow denotes similar items (1). B. Brain regions representing differently absolute and relative magnitudes while representing similarly non-symbolic and symbolic magnitudes.

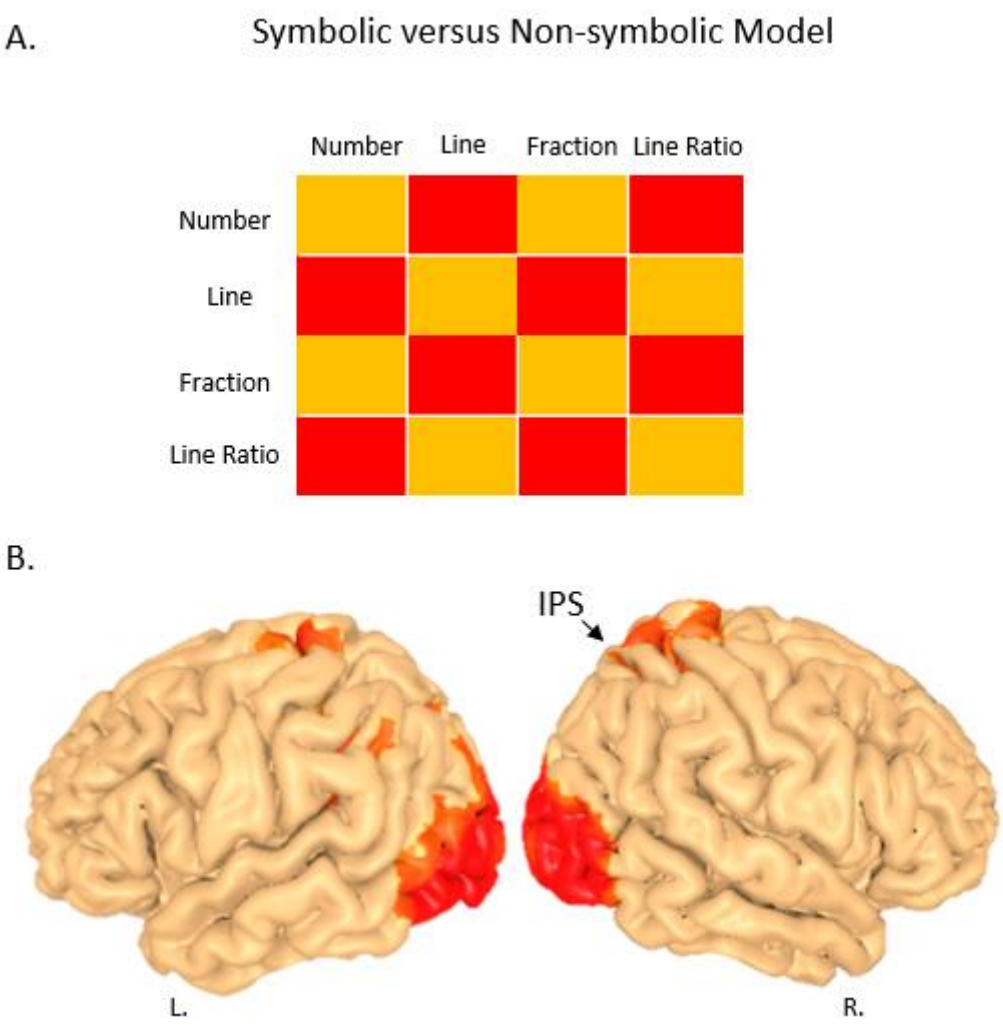

Figure 5: Results of the Representational Similarity Analysis for the Symbolic versus Non-symbolic model (multivariate analysis). A. Hypothesized Model for the RSA, the matrix represents a dissimilarity matrix where red denotes dissimilar items (0) and yellow denotes similar items (1). B. Brain regions representing differently non-symbolic and symbolic magnitudes while representing similarly absolute and relative magnitudes.

Table 2: Brain regions identified in Representational Similarity Analysis (multivariate analyses)

Anatomical Location	Cluster level $P_{FWE\text{-}corrected}$	Cluster size (cc)	MNI coordinates			t-score
			x	y	z	
Symbolic versus Non-symbolic						
L. Inferior Occipital Gyrus	.00	91.44	-34	-84	-4	8.18
L. Supramarginal Gyrus	.012	1.51	-50	-38	34	4.76
L. Postcentral Gyrus	.00	4.11	-30	-30	66	4.72
R. Superior Parietal Lobule	.00	7.91	26	-46	58	4.60
R. Intra-parietal sulcus	-	-	30	-58	55	2.75
Absolute versus Relative						
L. Superior Occipital Gyrus	.00	3.09	-10	-90	27	4.74
L. Lingual Gyrus	.009	1.61	-28	-62	-1	4.36
L. Superior Frontal Gyrus	.046	1.21	-24	60	13	3.74
L. Middle Frontal Gyrus	-	-	-28	48	13	3.74
R. Middle Occipital Gyrus	.00	7.15	38	-84	20	5.29

Notes. L = left; R = right; MNI: Montreal Neurological Institute; FWE-corr: Family-wise error corrected.

Discussion

In the present study, we used univariate and multivariate analyses to test the neuronal recycling hypothesis of absolute and relative magnitudes. Participants passively attended to numerical stimuli in the scanner, presented as symbolic fractions, non-symbolic line ratios, symbolic numbers, and non-symbolic lines. Each

of these stimuli was presented in adaptation and no-adaptation blocks, wherein the numerical quantity presented was near and far in magnitude (respectively). In the following, we will first discuss the result of the univariate analyses (comparing the difference in activity between no-adaptation and adaptation blocks, or neural adaptation effect, for the four stimuli). We will then discuss the findings of multivariate analyses.

Univariate analyses provide limited evidence for neuronal recycling of absolute magnitudes

In line with the neuronal recycling hypothesis of both absolute and relative magnitudes, we predicted that symbolic and non-symbolic magnitudes would rely on overlapping brain mechanisms in the IPS. This would have translated into neural adaptation effects in similar regions of the IPS for symbolic and non-symbolic magnitudes, suggesting an abstract representation of magnitudes in the IPS. Across all participants, we found a neural adaptation effect for non-symbolic absolute magnitudes (i.e., lines) in a relatively wide network of brain areas encompassing the bilateral IPS, the occipital, the supplementary motor area, and the precentral cortices. These results (particularly concerning the recruitment of occipito-parietal areas) are in line with prior passive viewing paradigms investigating the representation of non-symbolic absolute magnitudes (Ansari & Dhital., 2006; Demeyere, Rotshtein, & Humphreys, 2014; Pinel et al., 2004; Roggeman et al., 2011). In fact, a recent meta-analysis revealed that non-symbolic magnitude processing was associated with consistent activations in the bilateral parietal cortex and occipital gyri across studies (Sokolowski et al., 2017). A recent study using a magnitude comparison task involving both dot patterns and lines of different lengths also revealed overlapping activations for these conditions in the bilateral parietal and occipital cortices (Borghesani et al.,

2019). Overall, the fact that the IPS exhibits a neural adaptation effect for lines in our study is consistent with a long line of studies pointing to the IPS as a major locus for the representation of non-symbolic absolute magnitudes in the human brain (Nieder, 2016).

In contrast to our predictions, however, we failed to find any neural adaptation effect for symbolic absolute magnitudes (i.e., numbers) in the IPS. Instead, a neural adaptation effect was only found in the left fusiform gyrus, which may reflect the visual processing of numerals (e.g., Holloway et al., 2013). At first glance, the lack of IPS activation is in contrast to the majority of literature on the role of left-lateralized IPS in the development of symbolic magnitude processing (Vogel, Goffin, & Ansari, 2014). However, it is important to note that, in contrast to most previous studies, our adaptation paradigm is passive and therefore captures an automatic representation of numerical magnitude from the viewing of symbolic stimuli. This is critical because the IPS is also involved in response selection (Cappelletti et al., 2010; Göbel et al., 2004). Previous studies using active tasks (e.g., number comparison tasks, in which participants select the largest number; Ansari et al., 2005; Cohen Kadosh et al., 2005; Lyons & Ansari, 2009; Holloway & Ansari, 2010) may have thus confounded magnitude-related activity in the IPS with response demands. In other words, access to magnitudes from symbolic stimuli may not be as automatic as often argued. In fact, our study provides some evidence that it may depend on participants' fluency with symbolic math. Indeed, we found that neural adaptation for numbers in the IPS increased with arithmetic fluency. This result was similar to a prior study performed on children using digits (Girard et al., 2021). Similar to findings reported here, Girard and colleagues (2021) did not find a digit adaptation effect in the IPS but did report IPS activity in children with higher arithmetic fluency. Together with that study, our findings

suggest that participants with higher levels of mathematics fluency might be more able to automatically access numerical magnitudes than participants with lower levels of mathematics fluency. Interestingly, the IPS cluster in which this relation was found overlapped with the cluster showing a neural adaptation for lines across all participants. Therefore, it might be that individuals with higher levels of math fluency are able to better recruit and recycle the IPS pathways involved in non-symbolic magnitude processing for symbolic magnitude tasks, thereby creating stronger links between the two magnitude formats (but see Schwartz et al., 2021).

Univariate analyses fail to capture automatic processing of relative magnitudes

Contrary to our expectations, we did not observe any neural adaptation effect for fractions and line ratios. While research on relative magnitudes is limited, these findings conflict with prior studies that also used adaptation tasks (Jacob & Nieder, 2009 a, b). A major difference between our study and that of Jacob & Nieder (2009) is that the stimuli used here were more complex, mostly because all of the ten adaptation blocks corresponded to different ratios (e.g. 1:5, 2:3, 2:5, 1:4, 3:5, 2:9). Contrarily, Jacob & Nieder (2009) used only one simple adapting ratio of 1:6 for symbolic fractions with a higher repetition frequency of the stimuli (Jacob & Nieder, 2009). Therefore, there is a possibility that during that task participants were able to explicitly compute the magnitude of these simple fractions. However, this was near to impossible in the task used here because each adaptation block for a specific adapting ratio (there were 10 adapting ratios in total) included only eight fraction stimuli. Therefore, as compared to the prior study, the task used in the current study was better controlled for confounds related to the calculation of the magnitude. In any case, the lack of neural adaptation effect for fractions in the current study suggests a lack of automatic processing of the relative magnitudes of symbolic fractions. The lack of adaptation effect for line ratios

also highlights the absence of automatic processing for non-symbolic relative magnitudes. While behavioral studies in children, typically achieving adults and adults with mathematics difficulty indicate access to proportional information when comparing and estimating non-symbolic line ratios (Matthews, Lewis, & Hubbard, 2015; Bhatia et al., 2020), research on the neural representation of line ratios is scarce (Jacob & Nieder, 2009a). It is possible that the contradictory results may have been due to the differences in the task design. For example, the task in the current study used a greater variety of ratios (e.g., 1:3, 1:4, 1:5, 2:3, 2:5, 3:5, 2:9, 3:7, 1:6, 2:7) than in prior studies (e.g., Jacob & Nieder, 2009b). Future experiments varying the complexity of ratios while controlling for calculation and estimation strategies are needed to identify the source of inconsistencies between studies. In any case, it is difficult from the lack of neural adaptation effect for fractions and line ratios in the present study to evaluate the neuronal recycling hypothesis of relative magnitudes.

Multivariate analyses do not provide evidence for neuronal recycling of magnitudes in the IPS

To provide further evidence for the neuronal recycling hypothesis, we complemented univariate analyses with searchlight RSA. This allowed us to explore the relations between the patterns of activation associated with symbolic and non-symbolic stimuli across magnitude types. Specifically, if similar IPS mechanisms process symbolic and non-symbolic stimuli, we reasoned that patterns of IPS activity may depend on the type of numerical magnitude (i.e., absolute versus relative) more so than they may depend on the presentation format (i.e., non-symbolic versus symbolic). In contrast to this hypothesis, RSA revealed differences between neural representations of absolute and relative magnitudes (across presentation formats) in the left rostro lateral prefrontal cortex (RLPFC) and the right occipital cortices, but not

in the IPS. That is, we did not find any evidence that patterns of activity were similar between symbolic and non-symbolic stimuli in the IPS (and only depended on the type of numerical magnitude). Interestingly, several studies have suggested that the RLPFC may support relational comparisons and integrating relational information (Krawczyk, 2012). Specifically, relative magnitudes such as fractions cannot be understood without relating the two components (numerator and denominator) to each other. Similarly, for line ratios, the correct magnitude cannot be determined unless the magnitude of the two lines are thought in relation to each other. In line with this claim, recent studies have highlighted the role of relational thinking in processing fractions and rational numbers (Dewolf et al., 2015; Kalra et al., 2020). Therefore, our finding might provide initial evidence linking relational reasoning and relative magnitude processing at the neural level.

Not only did we not find evidence that the IPS represented similarly non-symbolic and symbolic magnitudes (while distinguishing between absolute and relative magnitudes), we found evidence that a cluster of the right IPS represented differently non-symbolic and symbolic magnitudes (while representing similarly absolute and relative magnitudes). This cluster was part of a larger occipital-parieto-temporal network distinguishing between non-symbolic and symbolic magnitudes. Therefore, multivariate results suggest that patterns of activity in several brain regions depend on the presentation format (non-symbolic versus symbolic) more so than they depend on the type of magnitudes (absolute versus relative). Although some studies have found evidence for overlapping activity between non-symbolic and symbolic stimuli, these findings are consistent with a stream of recent evidence suggesting that non-symbolic and symbolic magnitudes rely on separate neural resources (Cohen Kadosh and Walsh, 2009; Cohen Kadosh et al., 2011 ; Roi Cohen Kadosh et al., 2007).

A recent study using MVPA decoding also found distinguishable neural patterns of dots and digits in occipital, parietal, frontal, and temporal areas (Bluthé et al., 2014). A growing body of evidence on hemispheric specialization within the parietal lobes also challenges the idea that a single system processes numbers abstractly. That is, the left IPS is often shown to be involved in processing symbolic numbers (Vogel et al., 2014) while the right IPS is more often found to be activated during non-symbolic number processing, indicating different regions within the parietal lobe for both notations (Cantlon et al., 2006; Holloway & Ansari, 2010). Furthermore, a developmental meta-analysis focused on symbolic and non-symbolic number processing in children also showed the influence of the notation of numbers on the neural activation patterns within and outside the parietal areas (Kaufman et al., 2011). Therefore, the multivariate results reaffirm the growing body of literature suggesting that separate neural regions process both symbolic and non-symbolic magnitudes.

Conclusion

In conclusion, the current study shows limited support for the neuronal recycling hypothesis. On the one hand, consistent with the hypothesis, univariate analyses do show some overlap between the brain regions supporting non-symbolic and symbolic absolute magnitudes. However, this overlap was limited to absolute (not relative) magnitudes and dependent upon the degree of symbolic math fluency of participants. That is, we found an increase in the adaptation effect for numbers (not fractions) as a function of math fluency in a region of the left IPS that supports the representation of non-symbolic absolute magnitudes. Thus, individuals with higher levels of math fluency might be able to better recruit and recycle the IPS pathways involved in non-symbolic magnitude processing for symbolic tasks. On the other hand, inconsistent with the neuronal recycling hypothesis, univariate and multivariate analyses do not

provide any evidence that similar IPS brain regions support both non-symbolic and symbolic magnitudes across all participants. Instead, we found a region of the right IPS encoding differences in format (non-symbolic versus symbolic) across both absolute and relative magnitudes. Therefore, our study suggests that IPS activity depends on the presentation format (non-symbolic versus symbolic) more than it depends on the type of magnitude (absolute versus relative) for most participants.

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CHAPTER VI: STUDY III

Bhatia, P., Le Diagon, S., Langlois, E., William, M., Prado, J., & Gardes, M-L. (submitted). Impact of a game-based intervention on fraction learning for fifth-grade students: A pre-registered randomized controlled study.

Abstract

Digital game-based learning is gaining increased attention from both researchers and educators for improving mathematics instruction. However, the evidence for game-based learning is mixed and research with rigorous research design and analyses are limited. Here, in a pre-registered randomized controlled study, we investigated whether a fraction game designed collaboratively by educational experts and professional game developers may serve as a useful tool to enhance students' fraction knowledge. We assigned French fifth-graders to either an experimental group who used the game (n=110) or a control group (n=78) who received traditional instruction on fractions. Fraction knowledge was assessed pre- and post-intervention using a curriculum-based fraction test. Results show students in the experimental group did not have superior overall fraction performance than students in the control group at the end of the intervention. However, the game had a positive effect on decimal learning. We also found a positive relation between game performance and overall fraction knowledge scores at post-test. The study highlights the critical role of instructional support during game-based learning and the importance of game metrics as indicators of personalized assessment tools. Given the increased usage of games in learning mathematics, our results may inform rational number instruction.

Introduction

Mathematical proficiency is critical for success in later life outcomes, including professional and personal prospects (Parsons & Bynner, 2006). Among the range of math skills, fraction knowledge forms a crucial component of mathematical proficiency because it acts as a bridge between middle school and high school mathematical development (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012). Indeed, fraction knowledge at 5th grade predicts gains in algebra, calculus, and overall mathematics achievement (Bailey et al., 2012; Siegler, Duncan, Davis-Kean, Duckworth, Claessens, Engel, Susperreguy, & Chen, 2012). Moreover, mastery of fraction computations is also associated with success in other domains like biology, chemistry, physics, and many others (Lortie-Forgues, Tian, & Siegler, 2015). Thus, fraction knowledge is not only limited to success in mathematics but also central to many other domains. However, as detailed below, fractions are particularly difficult to learn and teach (Chan, Leu, & Chen, 2007; Ni, 2001; Yoshida & Sawano, 2002).

Difficulties with fractions

Difficulties with fraction learning can be attributed to at least two broad reasons. First, students often struggle to make connections between the various sub-constructs of fractions. Behr and colleagues (1983) suggest that there are six sub-constructs or ways to represent fractions: Part-whole, Decimals, Ratios, Quotient, Operators, and Measurements. Over-reliance on any one of these sub-constructs leads to constraints on understanding fractions (Kieren, 1993). For example, a disproportionate focus on the part-whole construct in schools may lead to difficulties in understanding improper fractions, but also to difficulties in grasping properties of equivalence, infinite rational numbers between any two natural numbers, and fair shares (Behr, Lesh, Post, &

Silver, 1983; Pitkethly & Hunting, 1996 as in Misquitta, 2011). Additionally, common struggles include the inability to comprehend the infinite ways in which rational number magnitudes can be represented (e.g., 2/4 = 1/2 = 0.50) (Vamvakoussi & Vosniadou, 2010) and the complex procedures involved when solving fraction arithmetic (Lortie-Forgues et al., 2015; Siegler, Fazio, Bailey, & Zhou, 2013). Thus, successful learning of fractions involves making adequate connections between the various ways fractions can be represented.

Second, students often experience difficulties in understanding the magnitude of fractions (Van Hoof et al., 2013), which may lead to errors in fraction arithmetic and fraction comparison. These difficulties mainly arise due to the phenomenon known as the whole number bias. The bias leads individuals to process the components of fractions (numerator and denominator) separately, usually because students over-generalize natural number properties when processing rational numbers. For example, a recent study found that eighth-graders were found to choose 19 or 21 as the correct answer when solving fraction addition problems 12/13 + 7/8, indicating that they summed the numerators or the denominators separately (Lortie-Forgues et al., 2015). Other studies have shown that even undergraduates excessively rely on the magnitude of the components (numerator and denominator) to compare fractions (Bonato, Fabbri, Umilta, & Zorzi, 2007; Schneider & Siegler, 2010). Therefore, the whole number bias is not restricted to children but is also observed in adults, even expert mathematicians (Alibali & Sidney, 2015; Meert, Gregoire, & Noel, 2010; DeWolf & Vosniadou, 2015; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Obersteiner et al., 2013). Successful learning of fractions involves being able to go beyond component magnitudes to grasp the holistic magnitude of fractions.

Digital game-based learning: A promising way to teach mathematics with conflicting evidence to date

Research on digital game-based learning (DGBL) has shown potential for learning abstract concepts, supporting classroom instruction, and presenting content engagingly and innovatively (Al-Azawi, Al-Faliti, & Al-Blushi, 2016; Prensky, 2001). Educational researchers have also advocated for using DGBL in curriculum delivery due to improvements in student engagement and pedagogical outcomes (Lumsden et al., 2016; Prensky, 2001; Squire, 2003; Tompson & Dass, 2000). In keeping with the observations above, prior research has shown that DGBL may support mathematics learning (Kiili, Devlin, & Multisilta, 2015; Li & Ma, 2010; Gaggi, Ciraulo, & Casagrande, 2018; Gaggi & Petenazzi, 2019; Riconscente, 2013). In their exhaustive review of the literature, Dvijak & Tomic (2011) note that DGBL may contribute "to a more efficient and quicker realization of educational goals at all levels of education" (p. 27, Dvijak & Tomic, 2011).

However, evidence for the impact of DGBL on learning is very contrasted. A recent meta-analysis examining the effectiveness of DGBL in mathematics learning notes a staggeringly low percentage (11%) of studies assessing the empirical effectiveness of games (Byun & Joung, 2018). Of the studies that did examine the effectiveness of games, the authors found a small effect of the game intervention for a majority of studies (9 out of 17), with an overall effect size of d=0.37[7]. This was higher than the effect size found in a previous meta-analysis d = 0.16 (Cheung & Slavin, 2013). However, these effect sizes are likely inflated because of publication bias. Indeed, effect sizes reported in studies that are published in peer-reviewed

[7] $d < 0.2$ – small, $d \sim 0.5$ – moderate, and $d > 0.8$ – large effect, Cohen (1998)

journals are higher than effect sizes reported in unpublished studies and/or studies in proceedings (journals: d = 0.44, unpublished: d = 0.14, proceedings*:* d = 0.08) (Wouters & van Oostendorp, 2013). The high variability of effect sizes observed in the meta-analysis of Byun & Joung (2018) are also indicative of differences in the methods, design, and, content of the games. Approximately 50% of these empirical studies had a modest sample size (n ≤ 50) (Ke & Grabowski, 2007; Lin et al., 2013; Plass et al., 2013; Sedig, 2007, 2008; Shin, Sutherland, Norris, & Soloway, 2012; Yang & Chen, 2010). This is problematic because small sample sizes are biased to produce large effect sizes when an effect is found (Cheung & Slavin, 2013). Critically, only three studies (out of the 17 analyzed in the meta-analysis) employed a randomized controlled design (Plass et al., 2013; Ke, 2008; Kebritchi, Hirumi, & Bai, 2010), which provides the most robust evidence in educational studies (Bertram, 2020). Further, most of the games that are evaluated for effectiveness are either designed for research or include an educational expert (Gresalfi, Rittle-Johnson, Loehr, & Nichols 2018). This leaves a large proportion of games that self-identify as 'educational' unevaluated for their support in learning. Therefore, previous studies paint a mixed picture of the effectiveness of DGBL in mathematics learning.

DGBL may complement traditional classroom instruction

The mixed findings on the effectiveness of DGBL require more rigorous experimental studies, with random assignment of participants in experimental and control groups and pre-registration of hypotheses, research design, and methods to limit analytic flexibility. It also requires a critical examination of the limitations of game-based learning. A balanced discourse around educational games and their role in learning can help increase learning outcomes by improving the design of the intervention and the development of future games. Here we review some difficulties

reported in the literature on digital games and their recommendations to enhance game-based learning.

The first difficulty pertains to the pedagogical validity of the games, i.e., the game design and transfer of knowledge and skills (Linderoth, 2012; Tobias, Fletcher, & Wind, 2014). Not all games are inherently educational and have the potential to facilitate learning (Linderoth, 2012). Thus, it is important to design games while focusing on the learning content and an in-depth performance and exploration of the content using game-play (Linderoth, 2012). Indeed, a collaboration between the game designers and educational experts might be the best way forward when designing games for teaching and learning. Most technological tools focused on mathematics learning and teaching are generally designed by technologists or game experts (Gaggi & Petenazzi, 2019). Since these commercial games do not use insights from teachers or mathematics education researchers, they tend to focus primarily on procedural knowledge rather than concepts (Kiili, Moeller, & Ninaus, 2018). Contrarily, serious video games that have been shown to influence learning are often developed by concerted efforts between mathematics education researchers and game designers (Cyr, Charland, Riopel, & Bruyere, 2019). Thus, the design of the game is critical when supporting learning outcomes.

A second difficulty is the ability to transfer the skills learned in the game to real-world problems or assessments (Barnett & Ceci, 2002; Rick & Weber, 2010). Indeed, even if games might improve some sets of skills, it is often not clear that this could be transferred to other related tasks. Therefore, it is important to use pre and post-test standardized instruments or country-based assessments to evaluate the effects of interventions (Bertram, 2020).

A third difficulty is the ability to contextualize the learning content of game (Tobias et al., 2014). This can often be improved by debriefing and discussions with the teacher and/or peers. The use of game-based learning along with traditional instruction might be helpful to support classroom instruction and allow for a more personalized learning experience (Osguthorpe & Graham, 2003). Instructional support has also been shown to enhance game-based learning environments by focusing the cognitive resources of learners on content rather than the narrative of the game (Wouters & van Oostendorp, 2013). For instance, students can benefit from their peers and teachers when solving problems matched at their competency and skill levels. Overall, while the impact of DGBL alone on learning is debatable, incorporating DGBL with classroom instruction may help teachers to teach abstract and difficult concepts and also aid in student learning and engagement.

Fractions and DGBL

As reviewed above, fractions are notoriously difficult to teach and learn. Integrating thoroughly designed educational games to aid in fraction instruction might support teachers and learners when learning fraction concepts and procedures. To our knowledge, there are five serious video games in the domain of fraction learning mentioned in the literature. These include Refraction (Martin, Petrick-Smith, Forsgren, Aghababyan, & Janisiewicz, & Baker, 2015), Semideus (Ninaus, Kiili, Mcmullen, Moeller, 2017; Kiili et al., 2018), Motion math (Riconscente, 2013), Abydos (Masek, Boston, Lam, & Corcoron, 2017), and Slice fractions (Cyr et al., 2019). The games were developed by either the researchers themselves or educators and aimed at enhancing conceptual knowledge of fractions. These games focus on specific fraction concepts.

Refraction (Martin et al., 2015) and Slice fractions (Cyr et al., 2019) are based on the concept of splitting fractions (part-whole representation of fractions). Both of these games have been found effective in previous studies (Martin et al., 2015; Cyr et al., 2019), with gains in the post-test assessment of fraction knowledge. Motion math (Riconscente, 2013) and Semideus (Kiili et al., 2018) were designed based on the measurement interpretation of fractions and used number lines to teach fraction concepts. The study evaluating Motion math did not include a comparison control group but showed significant improvement in fraction knowledge in the game group (Riconscente, 2013). On the other hand, the study evaluating Semideus had a comparison control group and showed significantly better performance in post-test fraction knowledge in the game group (Kiili et al., 2018). While both the games showed learning gains in fraction knowledge, it is difficult to comment on the relative effectiveness of the game as compared to other fraction learning approaches (Riconscente, 2013; Kiili et al., 2018). Lastly, Abydos (Masek et al., 2017) included high-level fraction concepts such as equivalent fractions, identifying least common multiples, addition, and subtraction of fractions. The study showed a significant increase in post-test scores in the game group as compared to the control group.

While the aforementioned studies indicate some effectiveness of previously developed fraction games, these games and the associated studies have several important limitations. First, despite researchers and educators' demand for a more holistic understanding of all interpretations or sub-constructs of fractions (Kieren, 1993; Lamon, 2001 as in Charalambous & Pitta-Pantazi, 2007), the games all focus on a specific interpretation/sub-construct of fractions (Behr et al., 1993). For instance, Refraction and Slice fraction focus on the concept of splitting or equipartitioning, whereas Semideus and motion math makes exclusive use of fractions number lines.

While each of these aspects is supported by previous research (Hackenberg & Lee, 2016; Fuchs, Schumacher, Long, Namkung, Hamlett et al., 2013), over-reliance on any one sub-construct of the fraction may lead to misconceptions and constraints in understanding (Kieren, 1993; Pitkethly & Hunting, 1996). Second, some of the studies evaluating the effectiveness are these games tend to have modest sample sizes (e.g., Semideus: n = 54, Slice fractions: n = 18 to 25 per group), which raises power issues (Brysbaert, 2019). Third, studies evaluating Semideus (Ninaus et al., 2017), Refraction (Martin et al., 2015), and Motion math (Riconscente, 2013) did not include a control group. This makes it impossible to (1) know whether learning gains can be attributed solely to the game and (2) conclude about the effectiveness of the game relative to other methods of rational number instruction (including traditional classroom instruction). Fourth, studies on fraction games that did include a control group (i.e., Abydos (Masek et al., 2017), Slice fraction (Cyr et al., 2019), Semideus (Kiili et al., 2018)) exclusively employed frequentist statistics, making it difficult to know if any lack of difference between the experimental and control groups supports the null hypothesis or if it may reflect a lack of power (Wagenmakers et al., 2008). Finally, studies have not always used an exhaustive, standardized, or country-based assessment to test for fraction skills (Martin et al., 2015; Kiili et al., 2018). The studies that did use fraction test items from standardized assessments also vary in their selection of the test items, raising the possibility that the results are dependent upon that selection (Riconscente, 2013; Masek et al., 2017). This highlights the need for pre-registering hypotheses and methods when assessing the impact of an intervention (Bertram, 2020; Nosek, Ebersole, Dehaven, & Mellor, 2018). However, to our knowledge, there has not been any preregistered study on DGBL and fraction learning.

The current study

In the current pre-registered randomized controlled study, we aimed to evaluate the effectiveness of a game-based training of rational number skills on fraction knowledge of children in 5th grade. The game, i.e. *Math Mathews Fractions*, was designed to complement fraction learning in the classroom with a focus on understanding and building connections with the multiple representations of fractions. Fraction instruction in the classroom is often disproportionately focused on a specific representation of fractions, such as part-whole (Fuchs et al., 2013). This over-representation may hinder the acquisition of other fraction concepts like improper fractions and the infinite divisibility of fractions (Pitkethly & Hunting, 1996 as in Misquitta, 2011) leading students to err on basic fraction concepts. Based on bridging the multiple representations of fractions (e.g., part-whole, measurement, and operations), we expected that *Math Mathews Fractions* might serve as a tool to help teachers teach the various representations of fractions in the classroom. In addition, the game includes elements of a personalized learning environment such as self-regulated learning (increased complexity of levels, repetition of tasks) and real-time feedback (wrong attempts decrease total points) (Basham, Hall, Carter, & Stahl, 2016).

Participants were recruited from five public schools (i.e., 10 classrooms) in Lyon, France. Classrooms were randomly assigned either to the experimental group (n=110) or the control group (n=78). The study involved a total of eight sessions and was conducted over four weeks. During these sessions, students from the experimental group played with *Math Mathews Fractions* in the classroom (each student had a tablet) with limited teacher interaction. In contrast, students from the control group practiced similar rational number concepts with their teacher. Fraction

knowledge of both groups was assessed pre and post-intervention using paper-based tests. The tests, hypotheses (based on prior research on serious video games and mathematics learning), and analytic strategy were pre-registered.

Methods

Participants

Participants were 193 fifth graders recruited from five public schools in France. Data were collected for two years at the middle of their school year (typically around the time when fraction instruction was practiced in class), i.e., December 2018, February 2019 for year 1, January 2020, and February 2020 for year 2. The experiment was approved by the school board (who designated the schools) and was performed following the ethical standards established by the Declaration of Helsinki. Parents gave their written informed consent and children gave their assent to participate in the experiment. Three schools were located in a neighborhood in which the median equivalized disposable income is above the national median equivalized income of € 20,809 (i.e., € 26,190), whereas two schools were located in a neighborhood in which the median equivalized disposable income is below that national median equivalized income (i.e., €19,032) (https://www.insee.fr/fr/statistiques). Enrollment in all public schools in France is mostly based on the neighborhood in which the children live and is free of charge for parents. Therefore, the sample enrolled was most likely representative of the population of the district.

From the original sample of 193 students, children were excluded if they had a diagnosed disability (n=3) or if their parents did not give consent (n=2). Therefore, our final sample consisted of 188 students (mean age = 10.5; SD = 0.32; 91 males). The

classes were randomly assigned to the two conditions (control and experimental) by lottery. The control group had 78 students and the experimental group consisted of 110 students. Participants from the two groups did not differ in age (BF_{01} = 4.57, F (1,182) = 0.642, p < 0.424) and gender (χ^2 (1, N = 188) = 1.58, p = 0.209). Note that we also presented a sample of fourth-graders with the game. At the end of the first year of testing, however, we realized that the game (particularly in its highest levels) was not appropriate for children in 4th grade. Indeed, most children could not reach the highest levels, teachers reported several difficulties encountered by students, and we observed several instances of students guessing and skipping levels during a classroom observation. Therefore, we decided to stop collecting data at that level. The only exception was for mixed-grade classrooms (i.e., classrooms with both fourth and fifth graders), in which the game was presented to all children for practical reasons (only data on 5th graders were analyzed).

Pre-registration

The study was pre-registered using the AsPredicted.org template via the Open Science Framework at OSF link. There were four main differences with the pre-registration. First, frequentist analyses are presented along with the pre-registered Bayesian analyses. Second, a delayed post-test could not be conducted in the second year due to Covid-19 and school closures in France. Third, the total number of children that were analyzed (n = 188) was less than those pre-registered (n = 240). This was because of absenteeism, lack of parental consent, and diagnosed disability. This was also because we realized that the game was not appropriate for 4th graders and had to reorient our recruiting strategy towards 5th graders. Finally, the mathematics anxiety test used in the first year (Carey, Hill, Devine, & Szűcs, 2017) was replaced in the second year by a more detailed test (Henschel & Roick, 2018) due to the difficulties

encountered by students when filling the questionnaire. Particularly, students had difficulties understanding several sentences in the French translation of the original questionnaire.

Measures

Both groups were tested for their (1) arithmetic fluency, (2) mathematics anxiety, and (3) fraction knowledge at two separate time points (before and after the intervention).

First, arithmetic fluency was measured using the Math Fluency subtest of the Woodcock-Johnson III battery. The Math Fluency sub-test is a timed test in which participants have to solve as many single-digit additions, subtraction, multiplication, and division problems as they can within 3 min (Woodcock et al., 2001). The test was administered in the classroom by the researcher with the support of the teacher. The items were marked and coded by two research assistants and verified independently by another researcher. Raw scores range from 0 to 160. The score is calculated based on the total number of correctly solved items in 3 minutes.

Second, mathematics anxiety was measured using two different tests. The modified abbreviated mathematics anxiety scale (Carey et al., 2017) was used in 2019 whereas the affective and cognitive mathematics anxiety test was used in 2020 (Henschel & Roick, 2018). For both the tests, items were read aloud by the researcher or the teacher, and children were given extra time, in the end, to fill the questionnaire or clarify their doubts. The modified mathematics anxiety (Carey et al., 2017) score was calculated by the addition of all responses on the 5-point Likert scale for all the 9-items (score ranged from 9 to 45). The affective and cognitive mathematics anxiety

test had 36 items measured by a 4-point Likert scale. The score is calculated based on the average of the score on each sub-test (scores range from 1 to 4).

Third, fraction knowledge was assessed using a test that was designed to assess fraction skills according to the French national curriculum standards. The test consisted of 24 questions with different items. Specifically, it assessed 6 competencies, some based on prior research (Rodrigues, Jordan, & Hansen, 2019): fraction concepts, fraction arithmetic, symbolic representation, number line, word problems, and decimals. *Fraction concepts* were measured using a total of 10 questions (question no. 1, 2, 3, 4, 5,10, 11, 16, 17, 18). The items assessed part-whole understanding of area models, set models, equivalence, comparing fractions, ordering fractions, and, mixed fractions. *Fraction arithmetic* skills were measured using 4 questions (Q.12, 13, 19, 20). Each question had 3 to 5 items and participants were presented with addition and subtraction problems written in symbolic form. *Symbolic representation of fractions* was tested using 2 types of questions (Q. 6, 7) consisting of 4 items each. The first type was identifying the verbal representations of fractions (e.g. three halves) and writing the symbolic form ($\frac{3}{2}$). The second type was identifying the symbolic form and writing verbal representation. *Fraction number line* was assessed using two questions (Q. 8, 9). The questions involved placing four fractions on the number line (e.g. $\frac{8}{5}$, $\frac{4}{5}$, $\frac{16}{5}$, $\frac{10}{5}$ Q.8a), and the other type involved finding the fractions marked on the number line. *Word problem-solving* skills were measured using four-word problems (Q. 21, 22, 23, 24). Lastly, *decimal* skills were measured using two questions requiring conversion of the fraction to decimals and vice-versa (Q.14,15). Cronbach's α ranged from 0.719 to 0.832 across all 6 measures (fraction concepts: 0.815, arithmetic skills: 0.830, symbolic representation: 0.771, number line:

0.832, word problems: 0.719, decimals: 0.822), indicating acceptable to very good internal consistency. The inter-rater reliability for the categorization of questions by three independent researchers for all the above measures was very good (Cohen's kappa = 0.84).

The fraction achievement test was scored using a template with correct answers by two independent research assistants and a researcher. The data entry was checked independently by two other research assistants to identify incorrect entries as well as correction errors on the paper-based tests. Any discrepancy in scoring or data entry was discussed among the three coders and if one of the coders was not convinced the item was marked for rechecking by a researcher in mathematics education in the lab. The inter-rater reliability between the final two researchers was very strong (Cohen's kappa = 1). For each item, the correct response was scored 1 and the incorrect/ no response (marked as 'do not know/?' by the participant) was scored 0. The percentage correct was calculated for each of the six competencies.

Fraction game

Math Mathews Fractions is an educational video game developed by the studio Kiupe in collaboration with a mathematics education researcher (the last author of the current paper). The game is about the adventures of a pirate who has to collect gems (treasure) by solving different challenges (i.e., modules). The game progression is in line with the objectives and curriculum standards of the French school system for children aged 9-12. Thus, the play situations (i) increase in difficulty throughout the game and (ii) remain appropriate for children aged 9-12. The modules are typically different types of questions involving rational numbers. Players must choose or

construct the answer to proceed further. For example, in the Dragon module, students are asked to feed the dragon the specified amount of fraction (e.g. 2/6). For gaining maximum gems on this module, the students must slice the rectangle into six parts and feed two parts to the dragon in their first attempt. There are 13 modules based on the curriculum standards in the French school system (**Table 1 and Fig. 1**). Each module is presented ten times throughout the game and can be presented several times during a level. The modules include specific fraction competencies like fraction concepts, arithmetic, word problems, number lines, and decimals (**Table 1**). The game consists of 12 levels. The game was played through an application pre-installed on the tablets. Each student had to create a profile with a pseudonym before starting the game. The first level was preceded by a small video to familiarize players with the basic controls and rules of the game as well as to guide them about the objective of the game. The game was configured in a way that each player had to correctly perform in all the modules that were visible to them in the game to finish the levels and only then could they proceed to the next level. The interface of the game also consists of a *journal* and a *calculator*. The journal was used to teach the player about the rules of each module and the fraction concepts involved in the module. Students could consult the journal anytime during the game by tapping on the icon.

Figure 1. Examples of the interface of the game showing the different modules that include different representations of fractions. A. Here the player has to split the block to feed the dragon $\frac{2}{6}$. B. Player must select $\frac{3}{4}$ of the rectangular surfaces to proceed. Here, the player has to select 6 blocks from the total of 8 blocks. C. Player must associate each fraction ($\frac{3}{2}$, $\frac{10}{6}$, $\frac{6}{3}$) to the shaded hexagons, trapezoids, and, rectangles. D. Player has to place the fraction $\frac{4}{2}$ on the 0-2 number line. E. Player must move the square number blocks to indicate the fraction marked on the number line. F. Player must find the segment that measures $\frac{1}{3}$ and $\frac{1}{8}$ of the red segment. On the picture the student has already placed the $\frac{1}{3}$ segment correctly

Table 1: Description of the game modules and objectives in the French national curriculum

Curriculum Objective	Modules	Fraction Competencies
Make connections between different representations of fractions Use fractions to divide quantities	Broken Wheel Dragon Warrior Weight Door	Fraction Concept (part-whole, area=circle)
Place fractions on a graduated number line	Trapped Passage	Fraction Number Line
Identify fractions on a graduated number line	Graduated bridge	Fraction Number Line
Use fractions to measure quantities Make connections between different representations of fractions Compare two simple fractions	Totem Organ	Fraction Concept - measure, Length
Sorting fractions in ascending / descending order	Skull	Fraction Concept - Ordering
Establish equality between two simple fractions/ equivalence Compare two simple fractions Add fractions with the same denominator	Pit Trapped Chest Spider	Fraction Concept- Equivalence Fraction Arithmetic (Level 7 to 9, 11)
Solve Word Problems Using Simple Fractions	Riddles	Word Problem

Game metrics

The *Math Mathews* game recorded the player's individual scores on each level. The data logged as per the pseudonym data profiles included:

i. Maximum Level Achieved: the highest level that a player successfully completed
ii. Number of attempts on each mini-game: number of times a player re-tried a module in a level before they moved on to the next module.
iii. Maximum points on a module: number of gems that could be obtained when the player correctly solved the module in a level.
iv. Obtained points on a module: number of gems that player collected on each module within a level.
v. Game Performance: (obtained points/ maximum points) *100.

Procedure

The study was conducted in eight sessions and varied in duration between the two years depending on teacher availability (4 to 5 weeks). Before the study, teachers were presented with the objectives of the game and the practice book that could be used in the control group. Teachers were also given tablets to play and understand the game before the sessions started. They were free to use the game either as part of instruction in the classroom or as independent work time for students. Thereafter, in the first week of experimentation, all students completed the pre-tests on two separate days. The first day included the arithmetic fluency and mathematics anxiety test and the second day assessed the untimed fraction achievement test. The following four weeks included paper-based practice sessions for the control group and individual game sessions for the experimental group (two 45 min pre-tests – four 45 min game-play and paper-based practice sessions – two 45 min post-test sessions). The control group started with an introduction to the fraction concepts and then solved problems either individually or in groups depending on the teacher's mode of instruction. The problems practiced during the session were matched on competency

and rigor to the experimental group (see **Figure 2**). All teachers were asked to select the problems from a specific book (Anselmo & Zucchetta, 2018). The experimental group had students build their profiles on the tablet during the first session and thereafter the students played the game individually on the tablets. The students were allowed to help each other during the playing sessions and seek guidance from the teacher and the experimenter. Even though the game included sound effects, the students had to keep the game muted in the classroom. Students in both groups could review and discuss fraction concepts encountered during the paper and game-based sessions with the teacher or the experimenter. After the fourth session, the experimenter administered the post-tests in two sessions similar to the pre-test.

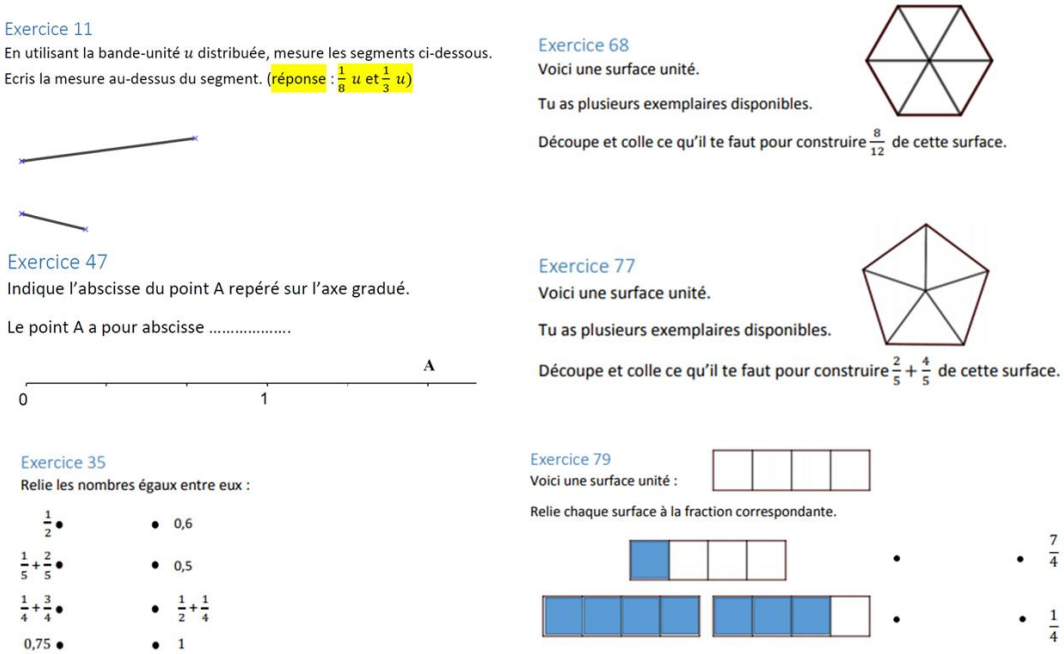

Figure 2. Examples of exercises practiced in the control group (in the original French). In Exercise 11, students have to using the unity band to measure the length of segments. In Exercise 47, students have to indicate the value of the marked point A on the number line. In Exercise 35, students have to match the equal numbers in the two columns. In Exercise 68, students are given different cut-outs of shapes. They have to cut and paste the shapes that correspond to 8/12 of the surface shown. In Exercise 77, students are given different cut-outs of shapes. They have to cut and paste the shapes that correspond to the sum of

2/5 + 4/5 of the surface shown. In Exercise 79, students are shown with a total surface. For each colored rectangle on the left, they have to choose the correct fraction on the right.

Analyses

Data were analyzed using both frequentist and Bayesian statistics. Due to absenteeism, missing data were removed listwise for the specific tests analyzed (less than 15% of the data on each test). Post-test arithmetic fluency scores, post-test mathematics anxiety scores (separately for each year), and post-test fraction achievement scores were entered in frequentist ANCOVAs with the between-subject factor Group (control, experimental). Pre-test scores were entered as a covariate to control for potential differences in baseline scores.

Additionally, Bayesian statistics were used to estimate the strength of evidence for both the null (no difference between groups, H0) and alternate hypothesis (difference between groups, H1). Post-test scores were entered as outcome variables of Bayesian ANCOVAs with the between-subject factor Group (control, experimental) and pre-test scores as covariate. Following Jeffreys (1961), a BF< 3 was considered anecdotal evidence, a 3<BF< 10 was considered substantial evidence, a 10 < BF < 30 was considered strong evidence, a 30< BF< 100 was considered very strong evidence, and BF > 100 was extreme evidence that our data are more likely under the alternate than the null hypothesis (i.e., BF_{10}) or under the null hypothesis than the alternate hypothesis (i.e., BF_{01}).

Finally, we calculated frequentist and Bayesian bivariate correlations between the game metrics and the fraction competency scores. All analyses were performed with the JAMOVI software (The Jamovi project, 2019).

Data Availability

All tests (translated and original versions) and anonymized scored data for each participant are available via the Open Science Foundation (OSF) at OSF Link. A brief demonstration of the Math Mathews Fractions game is available online

Results

Pre-registered hypotheses

We made 3 pre-registered predictions. First, we expected a higher overall score on the fraction knowledge test for the experimental group as compared to the control group at the end of the intervention. Second, we expected this effect to be specific to fraction learning, with no post-test difference in performance between the control and the experimental group on arithmetic fluency. Third, because it involves learning math with a video game, we expected that the intervention might have an effect on children's mathematics anxiety levels (with higher post-test mathematics anxiety scores in the control group as compared to the experimental group).

Confirmatory findings

Post-test scores for each group are shown in **Figure 3**. First, in contrast to our prediction, fraction post-test scores were not higher in the experimental than in the control group. Frequentist analyses even showed that fraction post-test scores were lower in the experimental than in the control group after controlling for pre-test scores (F (1,162) = 5.66, p = 0.019, η^2_p = 0.034), though Bayesian analyses only indicated anecdotal evidence for this difference (BF_{10} = 2.16). Second, in line with our predictions, there was no significant main effect of group on arithmetic fluency post-test scores after controlling for pre-test scores (F (1,158) = 0.53, p = 0.468, η^2_p =

0.003). Bayesian statistics also indicated substantial evidence for a lack of difference between groups (BF_{01}= 4.56). Third, in contrast to our predictions, no significant main effect of the group was observed for post-test mathematics anxiety scores after controlling for pre-test anxiety scores (2020: F (1,107) = 0.027, p = 0.871, η^2_p = 0.000, 2019: F (1,58) = 1.13, p = 0.293, η^2_p = 0.019). Bayesian statistics indicated substantial and anecdotal evidence for a lack of difference between groups (2020: BF_{01} = 4.73; 2019: BF_{01} = 2.46). Thus, there was no evidence, across all children and skills practiced in the game, that children who experienced the video game learned more about fractions than children who did not experience the game.

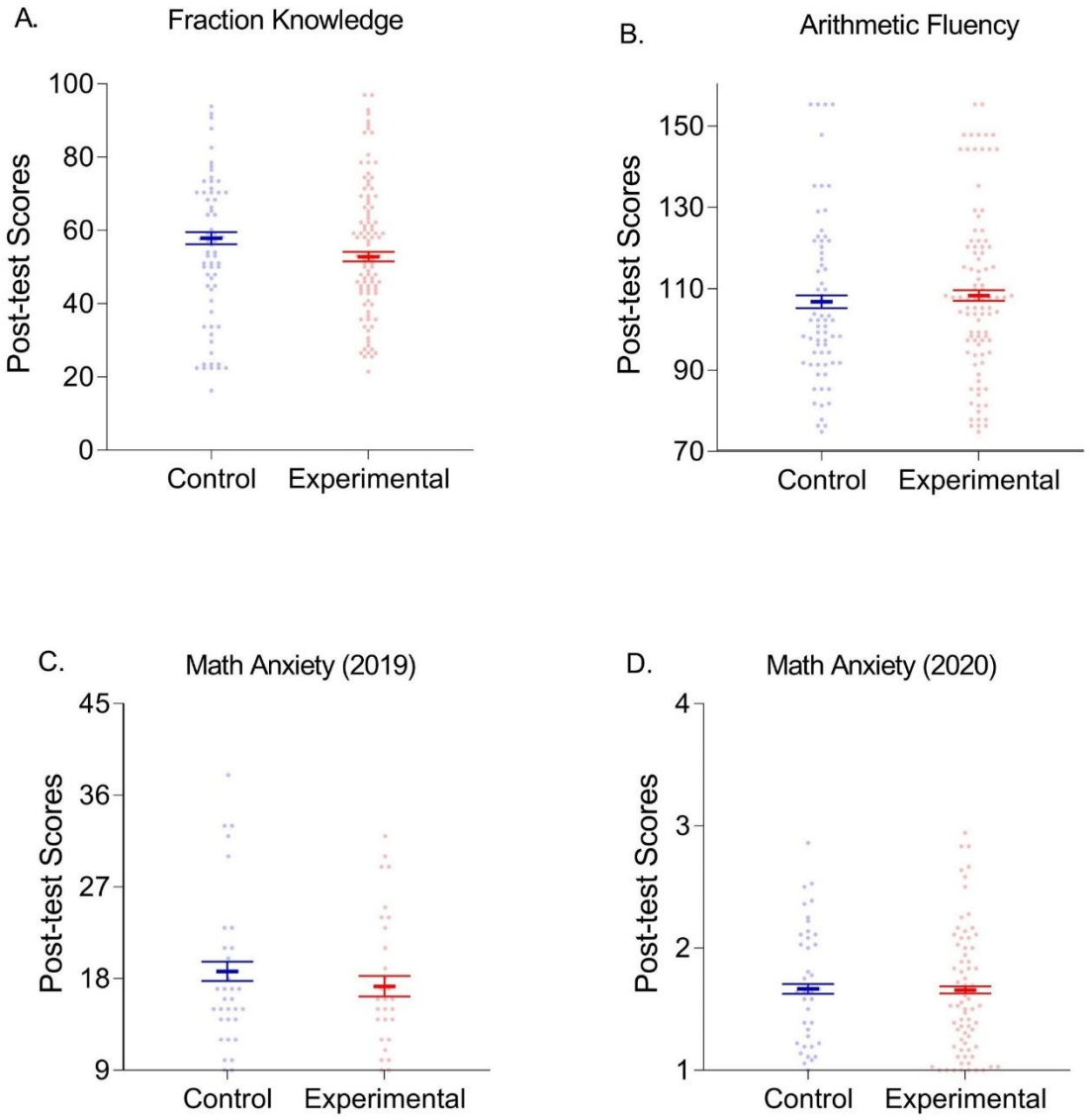

Figure 3. Student performance in the post-test (after controlling for baseline scores) for the control and experimental group. A. Fraction Knowledge. B. Arithmetic Fluency. C. Mathematics Anxiety Year 2019 D. Mathematics Anxiety Year 2020. Each dot represents the score of a student. Error bars depict the standard error of the mean.

Exploratory findings: Are effects dependent on competency?

The pre-registered analyses above show that the intervention does not positively affect overall fraction scores in children. However, the fraction knowledge test assessed 6 major competencies: fraction concepts, fraction arithmetic, symbolic

representation, fraction word problems, fraction number lines, and decimals. Therefore, it is possible that the intervention may affect some competencies more than others. To test for this possibility, frequentist and Bayesian ANCOVAs were conducted on each of the 6 competency sub-scores (controlling for their specific baseline scores) (**Figure 4**). First, frequentist ANCOVAs revealed a main effect of group on fraction concepts (F (1,162) = 6.19, p = 0.014, η^2_p = 0.037) and fraction arithmetic (F (1,161) = 14.52, p < 0.001, η^2_p = 0.083), with lower post-test scores in the experimental as compared to the control group (**Fig. 4A and Fig. 4B**). Bayesian statistics indicated anecdotal evidence for a difference between groups on fraction concepts (BF_{10} = 2.74) and strong evidence for a difference between groups on fraction arithmetic (BF_{10} = 99.41). Second, there was no main effect of group (all Fs < 1.95, all ps > 0.164) on symbolic representation (**Fig. 4C**), fraction word problems (**Fig. 4D**), fraction number lines (**Fig. 4E**). Bayesian statistics indicates a substantial evidence for a lack of difference between groups on symbolic representation (BF_{01} = 5.40) and fraction number line (BF_{01} = 3.01) and an anecdotal evidence for word problems (BF_{01} = 2.44). Third, frequentist ANCOVAs revealed a main effect of group on decimals (F (1,161) = 7.23, p = 0.008, η^2_p = 0.043), with higher post-test scores in the experimental as compared to the control group (**Fig. 4F**). Bayesian statistics also indicated substantial evidence for a difference between groups on decimals (BF_{10}= 4.81). Therefore, the only fraction competency for which we measured a positive effect of the game was the understanding of decimals.

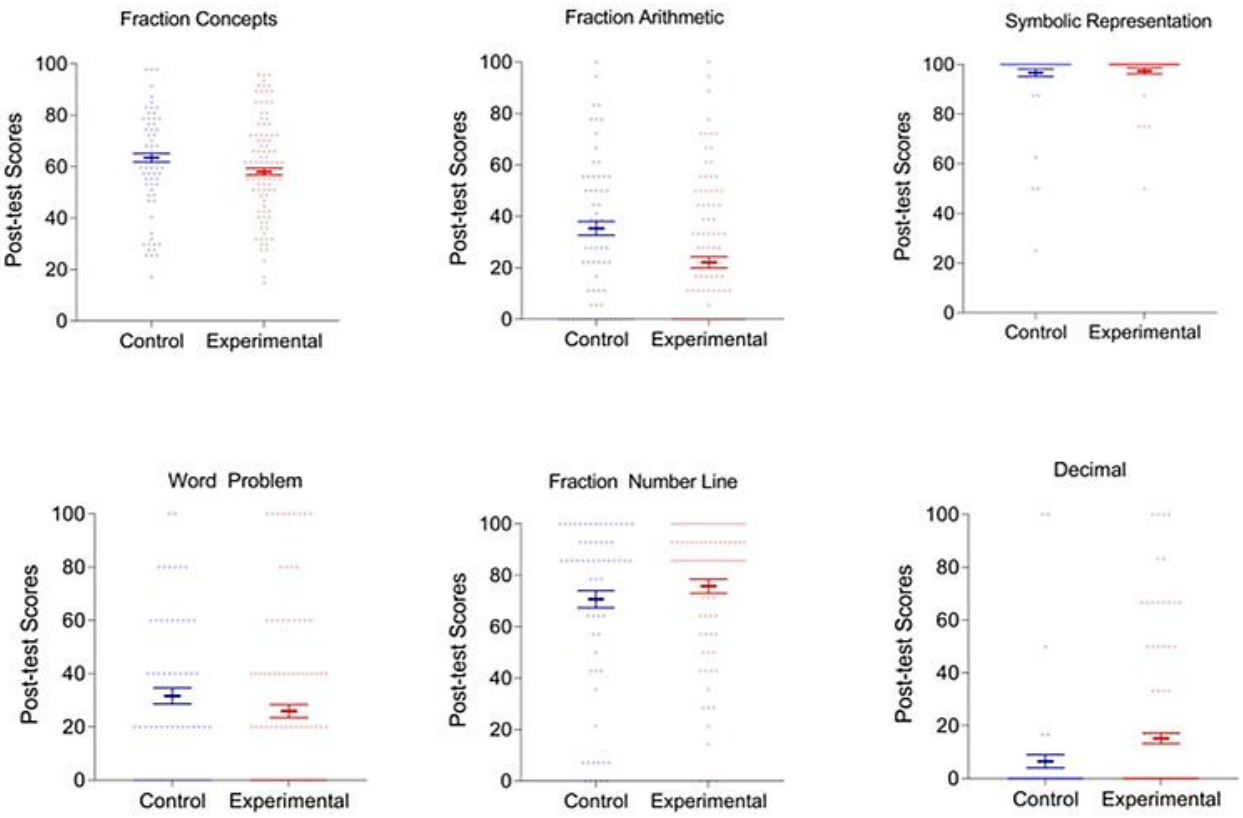

Figure 4: Student performance on different fraction competencies in the paper-based post-tests (after controlling for baseline scores) for the control and experimental group. A. Fraction Concepts. B. Fraction Arithmetic. C. Symbolic Representation D. Word Problem. E. Fraction Number Line F. Decimal. Each dot represents the score of a student. Error bars depict standard error of the mean.

Exploratory Findings: Are effects dependent on individual differences in-game usage?

It is possible that the intervention may only affect the competencies of children who progressed the most at the game, thereby benefiting from its content. To test for this possibility, we used frequentist and Bayesian correlation analyses to identify relations between game metrics and fraction knowledge while controlling for pre-test scores (see **Table 2** and **Figure 5**). Frequentist analysis revealed a significant positive correlation between overall game performance and fraction post-test scores (r (92) = 0.292; p = 0.005), indicating that greater overall in-game performance (obtained

points/maximum points*100) was associated with better fraction knowledge at post-test. Bayesian analyses also indicated substantial evidence for this correlation (BF_{10} = 6.57). Therefore, the more children gain points in the game, the greater their score at post-test. However, maximum level attained (r (92) = 0.182; p = 0.083) and number of attempts (r (92) = -0.038; p = 0.718) did not correlate significantly with the fraction post-test scores. Bayesian analyses indicated anecdotal (BF_{01} = 1.74) and substantial (BF_{01} = 7.198) evidence for no association between the variables respectively.

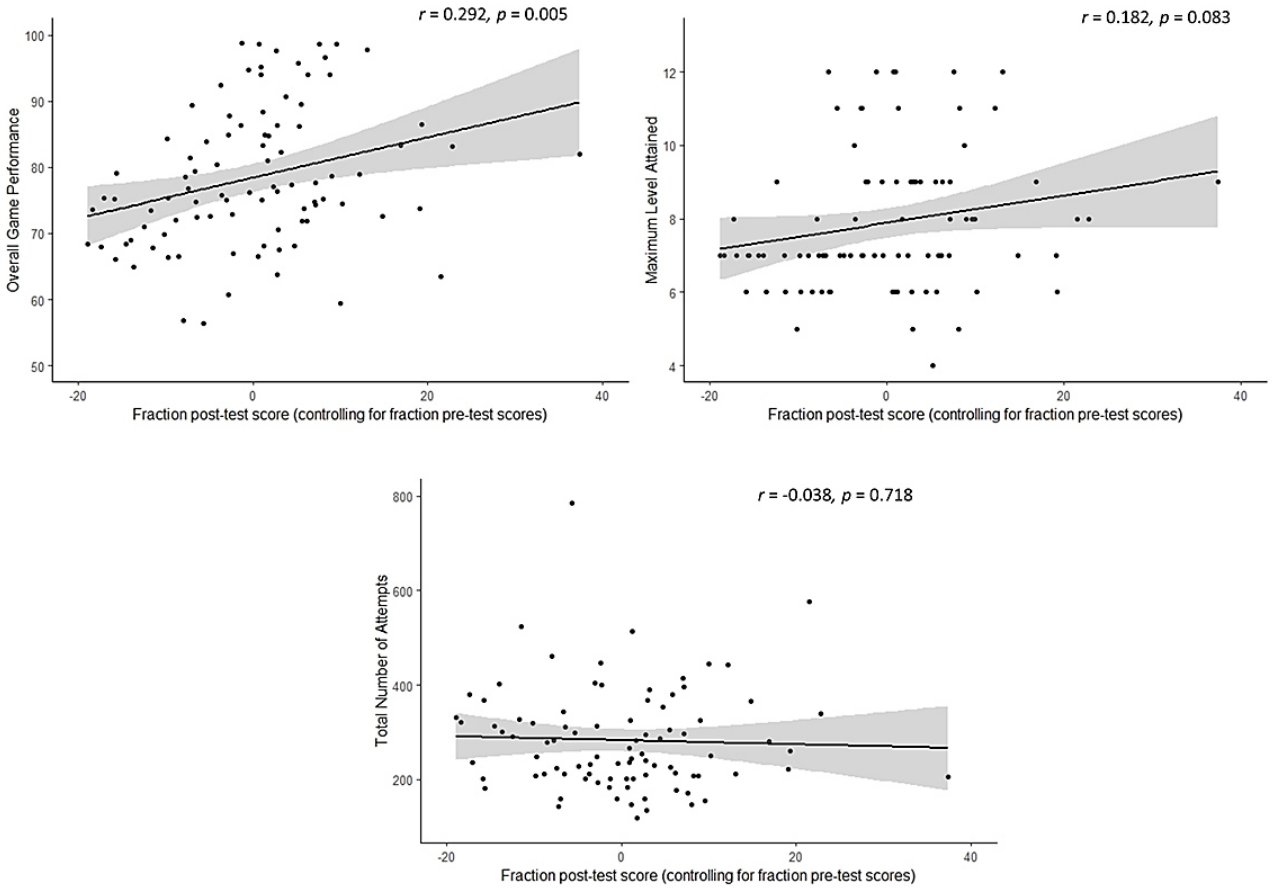

Figure 5: Correlation of fraction post-test scores and game variables. Grey shaded area indicates 95% confidence region for the correlation.

Table 2: Semipartial Correlations between fraction post-test scores and game metrics for the experimental group

			1.	2.	3.	4.
1.	Fraction post-test scores	r	-	-	-	-
		BF_{10}				
2.	Game Performance	r	0.292**	-	-	-
		BF_{10}	6.57			
3.	Maximum Level Attained	r	0.182	0.356***	-	-
		BF_{10}	0.573	50.093		
4.	Number of Attempts	r	-0.038	-0.601***	0.294**	-
		BF_{10}	0.139	5.012e+7	6.999	

Notes. * $p < 0.05$ ** $p < 0.01$, *** $p < 0.001$.

BF_{10} indicates the strength of the evidence for the alternative (there is an association between the variables). Bayes factors BF < 3 are considered anecdotal; 3 < BFs < 10 are considered substantial; 10 < BFs < 30 are considered strong; 30 < BFs < 100 are considered very strong and BFs > 100 are considered decisive. BFs > 3 are indicated in bold. All variables are controlled for baseline fraction pre-test scores.

Discussion

To our knowledge, this is the first pre-registered, randomized controlled study to examine the impact of a game-based intervention on fraction knowledge of fifth graders. In the following, we first discuss the findings of the impact of the game on fraction learning and then elaborate on the game metrics to better understand the game-based intervention and its impact on fraction learning.

Math Mathews Fractions does not promote overall fraction knowledge

Contrary to our preregistered hypothesis, the students in the experimental group did not outperform the control group on fraction knowledge. These results are inconsistent with the nascent literature on game-based interventions and fraction knowledge development. However, as mentioned in the introduction, prior literature has several limitations that might partially account for these conflicting results. For example, unlike several other studies on fraction learning and game-based intervention (Kiili et al., 2018; Riconscente, 2013; Masek et al., 2017; Cyr et al., 2019), our study included an active control group, meaning that the teachers were given a sample book (Anselmo & Zucchetta, 2018) to practice fraction concepts matched in rigor and competency to the game. Control groups in prior studies included either regular math instruction or regular fraction-based instruction. Hence, the two groups are not often matched on rigor or the competencies practiced, which makes it difficult to compare game-based intervention and traditional instruction. Our findings are consistent with game-based interventions that have utilized an active and well-matched control group (Carr, 2012; Singer, 2015). Unlike several other studies (Riconcente, 2013; Cyr et al., 2019; Kiili et al., 2018), our study also included a relatively large sample size and both frequentist and Bayesian analyses thereby providing robust evidence for the findings without compromising on the power.

What, then, could explain the lack of benefits from the game across all participants? We can mention two potential reasons for this lack of positive difference between the two groups. First, it might be attributed to the limited instructional support in the experimental group. Indeed, the essential role of instructional support during gameplay has been highlighted often in literature (Wouters & van Oostendorp, 2013). The use of well-designed instructional support during DGBL can help learners focus

on relevant information in the game that contributes to learning (e.g. modeling, reflection, context integration) (Wouters et al., 2008; Wouters & van Oostendorp, 2013, 2017). Though the teachers were given two training sessions before the study began, these were limited to understanding the interface and objectives of the game. Our observations in the classroom also indicated that the teacher and student interactions were relatively limited in the experimental group (mostly when students asked questions about the game interface or a specific concept). Thus, students were essentially playing the game individually without much debriefing or intermittent instructional sessions by the teachers. Therefore, future research designed to promote the active involvement of teachers during the game-based intervention might promote fraction learning.

Second, in efforts to match the rigor and competency in both groups, we might have introduced a solid method to teach and practice fraction curriculum to the teachers in the control group. Classrooms in the control group included group-based learning with peer-to-peer interactions and also other concrete activities that were provided in the book (see **Figure 2**). Teachers who used the exercises from the book systematically could have inadvertently led the instruction using the Concrete Pictorial Abstract (CPA) method. The CPA method is a learning approach to teach mathematical concepts that have shown improvement in mathematical learning outcomes in elementary and high school students (Salingay & Tan, 2018; M Salimi et al., 2020). The method is based on reconstructing knowledge using a three-stage approach through manipulation of concrete objects, representation of images, and, finally, abstract notation or symbols (Witzel, 2005). Indeed, the book provided tools for physically manipulating concrete objects and learning through images (Anselmo & Zucchetta, 2018). Thus, it might be possible that in our effort to match the two groups

on competencies, we created a particularly efficient control group. This might explain the fact that students from that group showed better overall fraction knowledge at the post-test than the experimental group.

Math Mathews Fractions promotes decimal knowledge

Though the game did not show any impact on the overall fraction learning scores, we examined the impact of the game on different competencies in the pre and post-paper-based tests (fraction concepts, fraction arithmetic, symbolic representation, fraction word problems, fraction number lines, and decimals). These exploratory analyses indicated that the game had no or limited effect on the symbolic representation, fraction word problems, and fraction concepts (i.e., which only showed anecdotal evidence for a difference). It did, however, have an effect (with at least substantial evidence in Bayesian terms) for decimals and fraction arithmetic.

First, students who experienced the game scored higher on decimal knowledge at post-test than the control group. The difference in performance could be attributed to the structure of the typical instructional sequence for rational numbers in traditional classrooms (i.e. fraction first, decimals next, and percentages last) as compared to the game which focused on building connections between the two notations (fractions and decimals). Understanding and translating between multiple interpretations of rational numbers and the three notations (decimals, fractions, and percentages) is a requisite skill for mastery of rational number knowledge (Tian & Siegler, 2017). Studies have shown that children and adults face difficulties when translating between decimals and fractions, thus revealing their poor understanding of these alternate numerical notations (Stigler et al., 2010; Vamvakoussi & Vosniadou, 2010). A curriculum intervention study on fourth graders aimed at building these connections noted a

higher performance on translation tasks (fractions to decimals) in the experimental group as compared to the control group (typical teaching condition: fractions first, decimals, and percentages later) (Moss & Case, 1999). In the current study, despite the control classrooms practicing decimal concepts in the latter part of the sessions, the students did not perform better than the experimental group on decimals (level 9 onwards). Thus, this might highlight the importance of instruction based on making connections between multiple notations of rational numbers rather than teaching them as distinct number types. Another reason could be the type of decimal problems in the pre and post-test. These were limited to conversion of decimals to fractions and vice versa (e.g. 0.25 =? , $\frac{7}{10}$ = ?). Interestingly, this is the key skill that was practiced by students in the experimental group. An example of the decimal games practiced is shown in **Figure 6**. Here, the students have to select or choose the fraction that is equal to the given decimal, similar to the paper-based tests. Therefore, this along with the mode of instruction might explain the better performance of students in the experimental group as compared to the control group on decimals.

Figure 6: Examples of the core decimal concepts that were present in the game.

Second, fraction arithmetic performance was lower in the experimental than in the control group after the intervention. One can speculate that the control group practiced more problems with the teacher than the experimental group. This is mainly because in the game the problems on fraction arithmetic are at higher levels (above level 7, see Table 1.). Thus, the game may not allow for enough training to perform these procedural concepts. Prior literature has also shown fraction arithmetic rules to be varied and complex (Siegler & Lortie-Forgues, 2017). The complexity and the procedural nature of these problems might then benefit from a more instructional, teacher-led practice approach. Again, this highlights the importance of instructional support during DGBL (Wouters & van Oostendorp, 2013, 2017), as well as the presence of an active control group when comparing game-based interventions to traditional classroom instruction.

Taken together, these exploratory findings indicate two interesting aspects of game-based training. First, our game-based training was more effective than traditional classroom instruction for making connections between fraction and decimal magnitude representations. Second, given the complexity of fraction arithmetic procedures, a rigorous teacher-led practice approach might be more helpful than game-based training. These two results extend the nascent literature on fraction game-based learning and highlight the importance of different instructional methods for different representations of fractions. This is relevant as not all game-based interventions designed by experts are empirically assessed in the educational setting (Gresalfi et al., 2018). Nevertheless, evidence-based pedagogy is critical for teachers and educators to understand the specific, beneficial aspects of the game so that they can choose appropriate interventions for their students.

Game metrics as a potential tool for personalized instruction and assessment

Prior studies note the importance of correlational research designs for understanding various aspects of game variables such as player behavior, engagement, and training success (Boyle, Hainey, Connolly, Gray, Earp, Ott et al., 2016). The game metrics in the current study reflect different aspects of student learning such as accuracy (higher overall game performance), increased guessing (higher number of attempts), and progression on task (maximum level attained). Through our exploratory analyses, we observed that overall game performance was positively associated with students' fraction knowledge at post-test. This indicates that better game performance was related to higher performance on post-training fraction learning outcomes. These results are consistent with another game-based study where the overall game performance notes positive associations with both math grades and paper-based post-test scores (Kiili et al., 2018). Thus, in-game metrics

might be useful for teachers to assess learning outcomes in real time (Serrano-Laguna, Martínez-Ortiz, Haag, Regan, Johnson, & Fernández-Manjón, 2017; Kiili et al., 2018; Zaki, Zain, Noor, & Hashim 2020).

Additionally, we did not find any significant relationship between the level attained on the game and fraction post-test scores. This is not in line with findings from prior studies on another game on rational number knowledge (Kiili et al., 2018). One interpretation of this finding could be based on the fact that not all students who attain higher levels in the game do so by learning the content. Some students could have used other techniques such as guessing or choosing answers similar to their peers. This indicates that individual differences in the level attained may not explain differential learning effects in the game group. Further support for this speculation comes from the significant positive correlation between the number of attempts and the level attained in the game. Students who are unsure about their answers attempt the modules often by guessing thereby reaching higher levels. Importantly, no association between the number of attempts and fraction post-test scores was observed. This finding is consistent with another study using the semideus game (Kiili et al., 2018). Again, this could be connected to students guessing the answers and not necessarily focusing on the content. Observations in the classroom corroborate this speculation. Despite the *journal* feature of the game and encouraging students to ask peers, teachers, and researchers in the classroom, some students continued to either guess or ask their partners for answers without explanations. Thus, the game metrics not only provide important insights on game-based learning but also act as personalized assessment tools to understand individual-level performance.

Future studies with more information on these variables might serve as evidence for understanding student's problem-solving strategies during learning. For

instance, the number of attempts could be detailed to provide the answers generated in each attempt by the students highlighting core misconceptions. Conclusively, the game metrics provide critical information on the game design and the intervention making it an invaluable tool as a predictor of training success in future studies (Kiili et al., 2018) and also a real-time assessment indicator for teachers and educators.

Limitations

There are at least three potential limitations concerning the results of this study. The first limitation concerns the lack of qualitative data. A mixed-methods study is informative to determine the mechanisms involved in learning as well as to better understand the methods (Bertram, 2020). Classroom discussions, student interactions, and the type of questions asked during the game-based training would have enriched our quantitative measures. A second potential limitation is the passive role of teachers in the game-based group. The role of instructional support during DGBL cannot be undermined (Wouters & van Oostendorp, 2017). Despite conducting sessions for teachers to understand the objectives of the game and its interface, we did not provide a structured, rigorous training session on teaching with the game in the classroom. As a result, all teachers in the experimental group played a passive role in student learning, which might affect the outcome of the intervention. Finally, because of school closures during the COVID-19 outbreak, we could not investigate the possible long-term effects of game-based training.

Conclusion

In the present pre-registered, randomized controlled intervention study, we evaluated the impact of game-based training on rational number concepts. The game, *Math Mathews Fractions* was designed according to the French National Curriculum

by a mathematics education researcher and game developers. By including an active control group that practiced fraction concepts matched on rigor and competency, we aimed to assess the effectiveness of the game with respect to traditional learning of fractions. Our results indicate that the game was not superior to traditional learning for overall fraction performance. However, the game had a positive effect on the learning of some rational number concepts. We also found a relation between the game metrics and overall fraction knowledge scores. This suggests that games such as *Math Mathews Fractions* might play a role in traditional classroom instruction by helping students learn specific fraction representations and supporting teachers to help build connections between fraction and decimal representations.

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CHAPTER VII: DISCUSSION

Growing evidence suggests the presence of a Ratio Processing System and its likely role in the acquisition of symbolic fraction concepts. Prior work includes both behavioral and brain imaging studies (review in Chapter II). However, research on the RPS is relatively new and limited in its use of the tasks and paradigms employed. Therefore, in addition to shedding light on the RPS theory, this work may offer new insights both theoretically and experimentally to assess and examine the presence of the RPS. Further, the RPS could potentially serve as an interesting concept in the design of learning environments to support fraction understanding. Additionally, there is a decent scope of improvement in the literature on learning environments for fraction instruction. Given their multifaceted nature, fractions are typically hard for both learners and teachers. To this end, we examined a game-based intervention specifically designed to build connections between the multiple representations of fractions, thereby supporting teachers in classroom instruction. The overarching aim of the thesis, therefore, was to examine the neurocognitive bases of fraction processing and learning in children and adults. In this general discussion, we first summarize the findings for all the three studies presented. Thereafter, we speculate on the implications of these findings in the wider context of fraction learning and teaching.

1.1 Summary of the experimental results

In Study I, we tested for the existence of the Ratio Processing System in individuals with varying levels of mathematics skills. It was hypothesized that *if* the performance accuracy of participants were sensitive to the ratio magnitudes*, then* this would serve as evidence for the existence of the RPS. Here, we adapted a match-to-

sample task developed by Matthews (2015), where participants were asked to choose the correct *match* from two non-symbolic ratios or symbolic fractions (both these formats were presented in separate blocks) that corresponded to the *target* fraction or line ratio. The results show that all participants (irrespective of their mathematics skills) have perceptual sensitivity when processing non-symbolic line ratios. This sensitivity is also present in typically achieving adults when they perform cross notation tasks where the symbolic fractions have to be matched with a non-symbolic ratio. However, the perceptual sensitivity is not observed when participants matched only the symbolic fractions. These findings indicate a limited role of the RPS, such that sensitivity to non-symbolic relational magnitudes is present only in tasks that require participants to approximate/estimate the ratio magnitudes. Taken together, these findings add to the growing evidence for an intuitive, perceptual sensitivity for non-symbolic relational magnitudes, but challenges the current literature that claims for perceptual sensitivity to symbolic fractions.

While study I provided initial evidence for the RPS using a behavioral paradigm, in Study II we investigated the neural correlates of both absolute and relative magnitudes to examine the presence of overlapping neural regions for both these representations. To this end, a passive-viewing fMRI adaptation paradigm (Girard et al., 2021; Perrachione et al., 2016) was used to determine the neural activity of both symbolic and non-symbolic absolute and relative magnitudes. We also examined whether the culturally developed ability to represent symbolic numbers and fractions relies on the neural representations of their non-symbolic counterparts (the neuronal recycling hypothesis; Nieder, 2016). The findings reveal the absence of an adaptation effect for the relative magnitudes, line ratios and symbolic fractions. For the non-symbolic and symbolic absolute magnitudes, the neural adaptation effect was shown

to be dependent on the symbolic mathematics fluency skills of the participants. This means that individuals with higher mathematics fluency were better able to recruit and recycle neural pathways for lines and numbers in the left intra-parietal sulcus. Contrary to prior studies (Jacob & Nieder, 2009 a, b; Mock et al., 2018), the absence of the neural adaptation effect for relative magnitudes indicates that it might not be possible to automatically access the magnitudes of the line ratios and fractions. Interestingly, the multivariate analyses revealed a region in the right intra-parietal sulcus that showed differentiation in the activity patterns for non-symbolic and symbolic formats across both absolute and relative magnitudes. This indicates that the IPS activity may depend on the presentation format (non-symbolic versus symbolic) more than it may depend on the type of magnitude (absolute versus relative). While Study I provides some behavioral support for the RPS account, Study II provides neuroimaging evidence contradicting the RPS account by demonstrating a lack of adaptation effect for relative magnitudes. Some probable explanations for these contradictory findings will be discussed in the sections below.

Further, the complex, multifaceted construct of fractions might pose limitations on fraction teaching and learning. Therefore, study III was focused on examining *how* the complex fraction representations might be accessed in formal educational environments. Game-based learning environments offer unique affordances such as manipulations and visualizations that might help teachers during fraction instruction to link the multiple representations of fractions. To this end, *Math Mathews Fractions* was designed collaboratively by professional game developers and an educational expert to bridge the multiple representations of fractions. In a pre-registered randomized controlled study, we assigned fifth-graders to either an experimental group that used the game or a control group that received traditional instruction on fractions. Fraction

knowledge was assessed pre-and post-intervention using a curriculum-based fraction test. Contrary to our predictions, the results show that students in the experimental group did not have higher overall fraction performance than students in the control group. However, the game did have a positive effect on decimal learning. We also found a positive relation between game performance and overall fraction knowledge scores at post-test. Taken together, these findings highlight the importance of adequate teacher support along with game-based learning for the acquisition of complex rational number concepts.

1.2 Evidence for the Ratio Processing System and its potential role in symbolic fraction understanding

Can individuals perceptually access the meaning of symbolic fractions and non-symbolic ratios? Why are magnitudes of symbolic fractions particularly difficult to grasp? How can the perceptual ability to access non-symbolic ratio aid in symbolic fraction understanding? These were some of the questions that initiated study I and II in this thesis. The findings reviewed earlier suggest that the RPS may exist as a system that supports the processing of non-symbolic relative magnitudes, though its role in symbolic fraction processing is limited. In line with prior studies, study I presented here shows evidence for a perceptual route to process approximate magnitudes of fractions when presented as non-symbolic ratios. Importantly, this perceptual sensitivity was also present in individuals with lower math skills like children in fourth and fifth grade as well as adults with dyscalculia. On the other hand, it was somewhat surprising to note the absence of neural adaptation effects for line ratios in study II. This finding is contrary to prior research that suggests automatic access to non-symbolic proportional magnitudes in the parietal cortex (Jacob & Nieder, 2009; Jacob et al., 2012). Furthermore, the absence of both a perceptual sensitivity to

symbolic fractions in study I and the neural adaptation effects for symbolic fractions in study II indicate the inaccessibility to implicitly access the fraction magnitudes and thus, hints at the complexity of symbolic fraction representations. On the one hand, these findings are consistent with research that indicates difficulty in accessing the holistic magnitudes of fractions for both children and adults (Kallai & Tzelgov, 2009, 2012; Stafylidou & Vosniadou, 2004). On the other hand, they conflict with prior studies that suggest the RPS contributes to symbolic fraction processing (Kalra et al., 2020; Matthews & Chesney, 2015; Schneider & Siegler, 2010). The abovementioned findings will be discussed in the sections below.

1.2.1 Is perceptual sensitivity to relational magnitudes abstract and automatic?

As mentioned in Chapter II, research on the RPS is relatively recent. The studies that do investigate the perceptual sensitivity to non-symbolic ratios mostly rely on tasks that encourage participants to estimate approximate magnitudes (Kalra et al., 2020; Park et al., 2020; Matthews & Chesney, 2015). In these tasks, participants are asked to compare the relative size of the two ratios. These tasks explicitly focus participants' attention on the numerical aspect of the stimuli (by using number words like “greater than” or “smaller than”). Study I in this thesis uses a match-to-sample paradigm in which participants are asked to match the *target* ratio to the correct *match*. Here, participants must access the relational meanings between the individual line lengths and the three-line ratios, thereby focusing on the relative magnitudes. Overall, these behavioral studies provide converging evidence for the presence of perceptual sensitivity to non-symbolic ratios in school-age children, adults, and adults with dyscalculia (Bhatia et al., 2020; Kalra et al., 2020; Matthews & Chesney, 2015). Additionally, performance in the cross-notation task (where adults had to match the non-symbolic line ratios with the correct symbolic fraction) was also ratio-dependent.

This finding is in line with prior studies using cross-format notations (Kalra et al., 2020; Matthews et al., 2014; Park et al., 2020). Further, the cross-format comparison tasks are argued to provide evidence for a shared system for non-symbolic and symbolic magnitudes as they rely on the abstract nature of magnitude between the different notations (Matthews & Chesney, 2015). While these results suggest an *abstract nature of relational magnitudes*, the absence of ratio effect in the symbolic fraction task, as well as the neuroimaging study (Study II), show otherwise. Both these results are in contrast to prior studies. For instance, studies have found that performance on symbolic fraction comparison tasks is influenced by linear or RoR distance (Kalra et al., 2020; Matthews & Chesney, 2015; Schneider & Siegler, 2010). The possible reason for this discrepancy could be the *task design*. For instance, magnitude comparison tasks might encourage participants to estimate approximate magnitudes. While there is a possibility to approximate ratios in the match to sample task, the equivalence task may prompt participants' attention to use exact values of fractions. Therefore, while children, typical adults, and adults with dyscalculia may rely on an intuitive sense of ratios to process non-symbolic ratio magnitudes, our findings raise the possibility that this sense may only drive performance in tasks that encourage subjects to approximate ratios. Indeed, the results of study II using a passive viewing design add further evidence to this claim. Our findings, therefore, suggest that the RPS might be *task-dependent*.

The absence of neural adaptation effects for line ratios and fractions in study II suggests a lack of automatic access to relational magnitudes. Contrarily, prior studies show automatic access for both non-symbolic and symbolic relative magnitudes (Jacob & Nieder, 2009 a, b; Mock et al., 2018). The discrepancy between the results of prior neuroimaging studies and study II might also be attributed to the difference in

task design. For instance, in the study by Mock & colleagues (2018), the task required participants to compare the magnitudes of the non-symbolic ratios (pie charts and dot patterns). Participants were therefore required to attend to the numerical stimuli in the task and revealed activations in the occipito-parietal areas. Another study that examined the neural representations of non-symbolic ratios and symbolic fractions used an adaptation paradigm (Jacob & Nieder, 2009). One possible explanation for the observed activations in the IPS in this particular study could be due to the simple ratios that were presented to adult participants. Adult participants with years of formal education likely are leveraging a *heuristic* for these simple ratios that are most frequently encountered in their environment (Siegler, Thompson, & Schneider, 2011). These heuristics might lead them to automatically access the numerical magnitudes of these ratios. On the contrary, the stimuli used in study II might be better controlled than the abovementioned study (Jacob & Nieder, 2009 a, b). The stimuli was composed of a complex variety of non-symbolic line ratios and symbolic fractions. These varied and complex stimuli might have made it more difficult for participants to apply the *heuristics* to access their magnitudes automatically, especially when the task did not require them to do so. While future work is required to establish the properties of the RPS as an abstract system for relational magnitudes, the present results might be a significant first step in revealing that the RPS might be a *task-dependent* system. Additionally, the presence of perceptual sensitivity for non-symbolic ratios in individuals with varying levels of math skills indicates that participants with lower math abilities can access the relational magnitudes of non-symbolic ratios; these abilities might be harnessed early on in development to help individuals utilize them as a visual aid for perceptual learning of fractions.

1.2.2 Implications for the role of RPS in fraction learning

The RPS theory posits that understanding of rational number concepts acquired during formal math instruction is rooted in the perceptions of proportions and ratios (Jacob et al., 2012; Lewis et al., 2016). The theory further argues that the observed difficulties and misconceptions in symbolic fraction understanding arise due to the instructional methods that do not leverage the existing perceptual abilities (Lewis et al., 2016). Given the recently developing literature on whether or not the RPS is utilized for processing symbolic fractions and its abstract nature, the implications must be interpreted with caution. The combination of findings in the current study might provide some conceptual support for the potential role of the RPS in symbolic fraction learning. Two major assumptions will be addressed here: First, we will assume the existence of the RPS as an abstract code for relational magnitudes, thereby serving as a core system of representation (Feigenson et al., 2004) for relative magnitudes, and second, we will depict that even in the absence of such a system the core ideas might be utilized for a better understanding of fraction concepts in the classroom (see **Figure 27**).

If we assume the RPS to be a universal, primitive, core cognitive system for the representation of abstract relative magnitudes, leveraging the RPS might help re-imagine conventional fraction instruction. Note, however, that this does not mean a complete reinvention of fraction instruction. Indeed, current classroom practices do utilize non-symbolic tools to teach fractions (e.g. sharing a pizza), particularly for the part-whole subconstruct of fractions, to build an informal understanding of sharing (Siegler et al., 2010). Yet, these approaches aim towards building an informal understanding using visual representations (Mack, 1990; Siegler et al., 2010). With the RPS as a pre-existing ability, instruction could be approached by utilizing the

foundational non-symbolic abilities to build symbolic fraction concepts (Lewis et al., 2016). For young children, fraction instruction in the classroom that uses conventional discrete, non-symbolic pie-charts (that use whole number counting procedures; Boyer et al., 2008) might shift to more continuous, uncountable non-symbolic ratios like lines and blobs (Park et al., 2020). This would help children to utilize their inherent perceptual sensitivities of non-symbolic ratios to understand relative magnitudes with unique properties and without the need to use whole-number strategies like counting. For instance, using non-symbolic line ratios as a teaching tool to help children make judgments of the relative value of the line lengths may help children in understanding relational magnitudes. These could then be used to make associations with symbolic fraction concepts which could further help students at estimating the holistic magnitudes of fractions.

Another speculation could be in the design of targeted interventions for adults with dyscalculia. Preliminary findings suggest that adults with dyscalculia possess sensitivity to discriminate between non-symbolic relational magnitudes but not their symbolic counterparts (Bhatia et al., 2020). Strategies to build links between percept-concept of relational magnitudes might aid these individuals to access underlying representations to support symbolic fraction understanding. For instance, a training intervention using both within and cross-format comparisons could include both implicit, perceptual fluency building processes and explicit verbally mediated connectional understandings between representations (**Figure 26**) (Rau & Matthews, 2017). The first step in such a targeted intervention could focus on building perceptual fluency on within-format and cross-format tasks. Once individuals can link the cross-format representations of relative magnitudes, instruction could then focus on helping them verbalize their sense-making processes (but see Rau, Aleven, & Rummel, 2017).

The difficulty of the tasks could also be modulated so that each individual is in their *zone of proximal development*[8]. Gradually increasing practice and reflection of the heuristics used by individuals could then further lead to fluency in understanding the holistic symbolic relational magnitudes. This instruction could also make use of blended learning systems where the role of the technology might be to help educators personalize this intervention (perhaps depending on each individual's RPS acuity score).

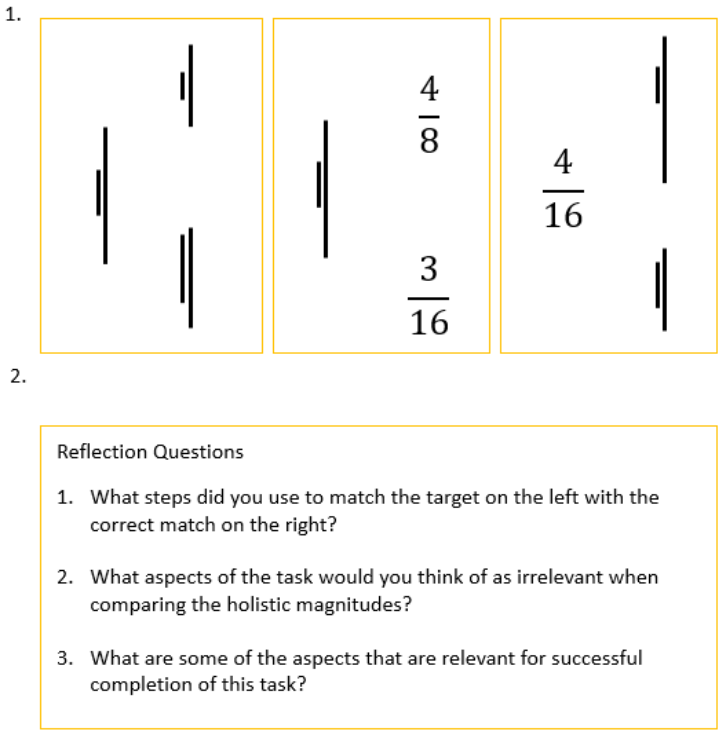

Figure 26: Example of an intervention using the RPS framework.1. Building perceptual fluency: Here, students could start from within format tasks and progress to cross-format tasks. The difficulty could also be modulated within each task by presenting conditions with the same and different components. 2. Building connectional understanding using

[8] Defined by Vygotsky (1978) as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86)

reflection questions (exemplars). This step focuses on verbalizing the process and making sense of the implicit perceptual sensitivity

Even if the RPS is found to be a *task-dependent* system that does not automatically activate relative magnitudes, non-symbolic relational magnitudes using continuous representations might serve as better instructional tools than the discrete or discretized representations that are currently used in the classroom (e.g. count shaded parts) (Jeong, Levine, & Huttenlocher, 2007; Ni & Zhou, 2005). Indeed, perceptual learning interventions on fractions and algebra have shown considerable gains that go beyond classroom instruction alone (Kellman et al., 2008). So, the prototype of the targeted intervention on perceptual learning detailed above (**Figure 26**) could perhaps improve the learner's rational number understanding. However, given the debate on sense-making first or fluency-first hypothesis (Rau, Aleven, & Rummel, 2017), this hypothetical account must be approached with caution.

Perceptual learning theory majorly involves discovery and fluency effects (Kellman, 2002). The discovery effect refers to learners finding relevant information while ignoring irrelevant information to form and process higher-level concepts (Kellman et al., 2008). The practice of similar concepts leads to an increased attentional selectivity which helps learners to extract invariant information quickly and categorize the new information (Petrov, Dosher, and Lu, 2005). In the context of the role of RPS in fraction learning, discovery effects may help learners by selectively ignoring componential processing during a comparison fraction task with different components and attend to the relative value of the components, thereby processing the holistic magnitudes. With increased practice and reflection, the fluency effects (which refer to the efficiency to extract relevant information) could ultimately lead to automatic processing of the symbolic magnitudes (Kellman et al., 2008). An important

design learning principle of the perceptual learning approach essentially involves presenting the learner with a large number of classification trials to make learners interact with a varied set of problem types (Kellman et al., 2008). Therefore, employing a similar design to study I but with more varied trials might possibly be used as a perceptual learning intervention. While much about the RPS remains speculative and needs more research both at the theoretical and intervention levels, the findings presented here, the different paradigms used (match-to-sample and adaptation), and the speculative recommendations for fraction interventions might aid future research efforts on the RPS.

1.3 Neuronal recycling hypothesis for absolute and relative magnitudes

Prior studies using functional magnetic resonance imaging adaptation (fMRI-A) paradigms reveal convergent findings on the passive processing of symbolic numbers in the left intra-parietal lobule and passive processing of non-symbolic magnitudes in the bilateral parietal lobes (Sokolowski et al., 2017). This might indicate that both overlapping, as well as distinct brain regions, are responsible for the automatic processing of magnitudes (in the absence of tasks). However, most studies included in the meta-analysis (passive viewing paradigms) included either a symbolic (Cohen Kadosh et al., 2007; Notebaert et al., 2011; Price & Ansari, 2011; Holloway et al., 2013; Vogel et al., 2017) or non-symbolic (Piazza et al., 2004; Ansari et al., 2006; Cantlon et al., 2006; Jacob & Nieder, 2009; Roggeman et al., 2011; Demeyere et al., 2014) stimuli but not both. Some passive viewing studies that have used both conditions within the same sample of participants (Piazza et al., 2007; Roggeman et al., 2007; Damarla & Just, 2013) show contrasting results of either overlapping (Piazza et al., 2007) or distinct (Cohen Kadosh et al., 2011; Sokolowski, Hawes, Peters, & Ansari, 2021) brain regions supporting the different magnitude formats. The

fundamental question of whether the culturally acquired symbolic abilities co-opt or re-use the primitive neural pathways for non-symbolic quantities remains unanswered. The findings of our study mostly support the argument that the human adult brain processes symbolic and non-symbolic magnitudes using distinct pathways (except for participants with high mathematics fluency and only for absolute magnitudes). In fact, similar to a recent study (Sokolowski et al, 2021), the multivariate analyses in Study II reveal that the right intra-parietal sulcus (among the larger cluster of occipito-parieto-temporal network) represents the two formats (symbolic and non-symbolic) differently.

While the present results cannot conclusively answer the question of whether there is an abstract neural code for magnitudes (and thereby the neuronal recycling hypothesis), they do reveal some interesting findings that might guide future research efforts. First, for absolute magnitudes, participants with higher mathematics fluency showed an overlap for both lines and numbers in the left intra-parietal sulcus. This might indicate that individuals with higher math fluency are able to recruit similar neural pathways for processing both non-symbolic and symbolic absolute magnitudes. Second, irrespective of the format (symbolic or non-symbolic) absolute and relative magnitudes show distinct patterns of brain activity in the RLPFC and the visual cortex. These distinct brain activity patterns between absolute and relative magnitudes seem to suggest diverse ways in which these two magnitudes could be neurally processed. Both these findings are discussed in detail below.

1.3.1 Is neuronal recycling related to arithmetic skills?

The answer to this question as well as the direction of the relation is presently difficult to explicate. The result of study II (for absolute magnitudes) is in contrast to a recent study that reports a developmental shift in the relationship between non-

symbolic and symbolic overlapping representation patterns in the brain and arithmetic skills (Schwartz et al., 2021). The authors in that study report that while children who are less proficient in arithmetic skills might rely on common mechanisms for both symbolic and non-symbolic magnitude processing, young adults do not (Schwartz et al., 2021). Notwithstanding the difference in experimental design, the studies might suggest completely opposing ideas: 1) participants with better math fluency might be able to recruit primitive brain regions when processing symbolic numbers, and 2) participants with a lower arithmetic fluency might need to rely on common neural mechanisms for symbolic and non-symbolic magnitude processing. Interestingly, a recent study on children revealed similar results to study II where the children with higher arithmetic fluency showed overlapping activation in the intra-parietal sulcus for both dots and digits (Girard et al., 2021). An additional question in this domain might be with regards to the direction of these phenomena. So, does better symbolic arithmetic fluency result from the ability to recruit primitive brain regions of non-symbolic processing for symbolic magnitude processing? or Does the presence of overlapping brain regions between symbolic and non-symbolic magnitudes determine the acquisition of symbolic mathematics skill? There is also a possibility that neuronal recycling might not be a universal phenomenon governing all symbolic mathematics skills but is dependent on the task or the paradigm used (Marinova et al., 2018). This means that depending on the task given to the participants as well as their math abilities, the co-option of brain pathways for symbolic and non-symbolic magnitudes might differ.

The *symbol-grounding problem* (i.e. how symbols acquire their meaning) has long been a question of great interest in the field of numerical cognition. In addition to the neural recycling account that posits perception of symbolic numbers to be

grounded in the same neural areas as their non-symbolic counterparts, alternative accounts on the relationship between symbolic and non-symbolic number processing have been proposed. While the *cardinal principle* hypothesis focuses on other core systems of representation (object tracking or parallel individuation) than the approximate number system, the *symbolic estrangement* account provides a developmental account of the link between the symbolic and non-symbolic magnitudes (Carey, 2001; Carey & Barner, 2019; Sasanguie & Reynvoet, 2017; Lyons, Ansari, & Beilock, 2012). According to the *cardinal principle* hypothesis, learning symbolic numbers cannot rely on approximations alone as it requires an understanding of specific properties of numbers like cardinality (Carey & Barner, 2019). Therefore, symbolic number processing may be associated with the more precise object tracking system (i.e. grasping a small set of objects about 1 to 4 in a fast but precise manner) which might initially help children acquire the idea of counting and thus, the property of cardinality (Carey & Barner, 2019). On the other hand, the *symbolic estrangement* account emphasizes the role of development in symbol-non-symbol links. The authors of this account posit that during the early stages of numerical development symbolic and non-symbolic numerical abilities might be linked, however, over the years and with the increasing complexity of symbolic number skills these two representations might become estranged (Lyons, Ansari, & Beilock, 2012). Indeed, a recent study on the development of these two representations over three different time points for Grade I students indicates both non-symbolic and symbolic skills to be related to each other at the beginning of the academic year, and later the symbolic magnitude processing skills predict non-symbolic skills but not vice-versa (Matjeko & Ansari 2016). Additionally, the absence of overlapping activation patterns for non-symbolic and symbolic magnitudes using RSA and MVPA studies on adult participants could also

be argued to be evidence of the symbolic estrangement (Wilkey & Ansari, 2019) or the cardinal principle account. Given the current inconsistent evidence, further research is required to shed light on these accounts, which in turn may help in understanding the development and acquisition of symbolic mathematics skills.

1.3.2 Differential processing of absolute and relative magnitudes

An important unexpected finding in study II (multivariate analyses) revealed differences between neural representations of absolute and relative magnitudes in the left rostro-lateral prefrontal cortex (RLPFC) and the right occipital cortices. Prior studies have suggested that the RLPFC may support relational comparisons as well as integrating relational information (Krawczyk, 2012). Recent studies have also highlighted the role of relational thinking in processing fractions (Dewolf et al., 2015; Kalra et al., 2020). However, to our knowledge, this is the first neural study that suggests the relationship between relative magnitudes and RLPFC. While future research aimed at examining the similarities and differences in processing absolute and relative magnitudes are required, initial speculation on these results seems to suggest the role of relational reasoning in fraction understanding. Understanding the relation between the two numbers for a fraction (numerator and denominator) and the two-line lengths for line ratios is essential to access their holistic magnitudes. The same strategy, however, does not apply for accessing absolute magnitudes. Therefore, the results on the dissociation of neural representation patterns for absolute and relative magnitudes suggest that access to these two magnitude types might engage different neural areas. Given these findings, there is a possibility that fraction learning that makes use of relational reasoning might aid learners in fraction understanding. Additionally, a recent study on 194-second grade and 145 fifth grade students showed relational reasoning to be a significant predictor of fraction

knowledge, even after controlling for non-verbal IQ, overall mathematics knowledge, and fraction magnitude processing (Kalra et al., 2020). Thus, we can imagine that the perceptual learning approach described above (**Figure 26**) might potentially help link relational reasoning and symbolic fraction understanding. Furthermore, the dissociable activity patterns for the two magnitudes might highlight the importance of building an explicit understanding of the differences and similarities in the properties of the two number types (whole numbers and fractions) (Rosenberg-Lee, 2021).

1.4 Can game-based learning help fraction instruction?

The findings of study III reviewed above show that the students in the game group did not outperform the control group on the fraction knowledge test. However, the game group showed higher performance at decimals than the control group. Further, a positive association was found between the overall game performance and student's fraction knowledge indicating that better game performance was related to higher scores on fraction post-test learning outcomes. Therefore, the answer to the question, 'can game-based learning help fraction instruction?' might not be a simple *yes* or *no*. Indeed, game-based learning environments offer unique affordances such as manipulations and visualization of complex mathematical concepts. Additionally, they also show positive affective outcomes for students towards mathematics learning. Students are more engaged and show positive attitudes towards mathematics during game-based learning (Rosas et al., 2003; Ke & Grabowski, 2007; Ke, 2008). However, as reviewed in Chapter III, there is mixed evidence on the effectiveness of game-based learning for mathematics learning outcomes. Though game-based learning environments do have their benefits yet inadequate game design, teacher support, and transfer of knowledge (Tobias et al., 2014; Wouters & van Oostendorp, 2013) might limit their potential. The game design and transfer of knowledge were already

well established in the game *Math Mathews Fractions*. The conceptual model presented here provides details on these factors, with a specific focus on what teacher support in the classroom might look like when using the game *Math Mathews Fractions*.

1.4.1 How might the game Math Mathews Fractions support the development of fraction knowledge in the classroom?

To support the development of fraction knowledge in the classroom, we propose three aspects of game-based learning environments that need to be considered. These include the game design, transfer of learning, and teacher support. First, the design of the game, *Math Mathews Fractions* was based on addressing the complexity of the multifaceted construct of fractions. Prior research notes fraction instruction in the classroom to be disproportionately focused on the part-whole construct of fractions (Fuchs et al., 2013). This might hinder the acquisition of other constructs as well as limit the development of holistic knowledge of fractions (Pitkethly & Hunting, 1996). Indeed, as noted by Rau & Matthews (2017), “no single visual representation can convey the multiplicity of *related but only partially overlapping ideas* that constitute the fractions mega-concept” p.3. In this vein, a mathematics education expert and professional game designer collaboratively developed *Math Mathews Fractions* to complement fraction learning in the classroom with a focus on building connections with the multiple representations of fractions. The game progression was designed in line with the objectives and curriculum standards of the French school system for children aged 9-12. Thus, the play situations (i) increase in difficulty throughout the game and (ii) remain appropriate for children aged 9-12. Additionally, the game includes elements of a personalized learning environment such as self-regulated learning (increased complexity of levels, repetition of tasks) and real-time

feedback (wrong attempts decrease total points) (Basham, Hall, Carter, & Stahl, 2016). Therefore, *Math Mathews Fractions* addressed the elements of game design by including the content aligned with national curriculum standards as well as engaged play situations for the specific age group. Through multiple modules focused on the visual understanding of multiple subconstructs and building connections between different representations of fractions, *Math Mathews Fractions* aims to build a holistic understanding of the fraction construct.

Second, the different modules in the game were designed similar to the fraction tasks students might encounter on real-world assessments, possibly supporting the transfer of knowledge. For instance, the module focused on the measure interpretation of fractions includes a number line task where the pirate has to place the fraction by jumping on the number line and find the fraction on the number line corresponding to a point by rotating a panel. Though the representations in the game modules were similar to real-world fraction tasks which might aid in the near transfer of knowledge, the role of the teacher to leverage the game modules for far transfer effects must not be underestimated.

Finally, it has been shown that adequate teacher support complements game-based learning (Wouters & Oostendorp, 2012) and helps with learning outcomes. *Math Mathews Fractions* is designed for students with prior knowledge of fractions. So, teachers were given a short training before the experiment in study III to help utilize the game to build connections between the fraction constructs. Varying levels of teacher support in the game and control group might have possibly influenced the results of the pre-registered study. Indeed, observations in the classroom revealed minimal teacher support in the game group. While possible reasons for the findings are discussed in detail in study III, in this section we speculate on an exemplar lesson

in the classroom with *Math Mathews Fractions* (inspired by the blended learning approach; Graham, 2006).

The classroom instruction could possibly be divided into three phases: Before the game, between the session, and after the game. Specific questions to initiate reflection and review of concepts could be practiced in these phases. For instance, before the game is provided to the students, the teacher could review one or two constructs of fractions with the entire class. The game metrics could potentially guide this discussion (especially if the game has been used in the classroom before). So, if the game metrics reveal the average level of the class to be around 4, the teacher could review specific curriculum objectives from level 4 before the game. This session could be solely teacher-led. During the game session, the teacher might be able to gauge some of the most challenging tasks or concepts that students encounter in the game. These could generate active whole group discussions in the classroom where the challenging concepts are re-visited. Finally, a debrief session after the game can help students to synthesize their knowledge. This session could be related to fraction concepts or more general reflection questions aimed at a new concept acquired, a misconception realized, or different problem-solving techniques that students used during the session. Adequate teacher instruction with *Math Mathews Fractions* could potentially lead to higher fraction learning outcomes. However, the lack of acceptance of serious games as educational tools by teachers and the difficulty of integration into formal schooling has been shown to limit the potential of games (Egenfeldt-Nielsen 2006; Ferdig 2007; Gros, 2015). Besides, the learning potential of technology has also been shown to be directly related to the teacher's ability to leverage it to make connections with the curriculum (Hutchison et al., 2012). Therefore, future investigations with *Math Mathews Fractions* might aim at providing teachers an

exhaustive training to develop such lesson plans or to provide them with pre-made lessons to help them visualize classroom instruction with the game. Additionally, these future studies might also help in establishing the validity of the game *Math Mathews Fractions* as an instruction and personalization tool in the classroom.

1.5 Integrating RPS and Math Mathews Fraction: implications for fraction learning

A complex concept such as fractions possibly needs a multipronged approach during instruction. The current approaches to fraction instruction mostly involve a combination of declarative and procedural learning. Declarative knowledge deals with facts and concepts whereas procedural knowledge focuses on specific rules and sequence of steps to be enacted (Kellman et al., 2008). An approach that is typically missing from fraction instruction in the classroom is the perceptual learning approach – an experience-based improvement in the learner's ability to extract patterns and relationships from the environment (Goldstone, 1998; Kellman, 2002). Abstract concepts such as fractions which might require a conceptual change about numbers in young learners (Ni & Zhou, 2005) as well as pose high cognitive demands due to their physical structure (Cognitive Load Theory; Mayer, 2002), might benefit from a balanced approach using declarative, procedural, and perceptual learning approaches. One potential example of using these three learning approaches by utilizing the perceptual sensitivity to non-symbolic ratios and *Math Mathews Fractions* is discussed below.

While we do acknowledge that the RPS literature is nascent and needs further exploration, humans have been shown to possess the ability to extract visual information from the environment and make sense of it. Also, the ubiquitous perceptual

ability to process non-symbolic magnitudes (length, numerosity, area, etc.) (Leibovich, Kallai, & Itamar, 2016) might be foundational for developing perceptual learning interventions as well as instruction on rational numbers. Here, we speculate a conceptual model of how the knowledge that we gained in the past three years on RPS and game-based learning might help teachers in the classroom. Of course, this is just a hypothetical model and further research is required to make any claims on the validity of this approach.

As shown in the Figure below, fraction instruction might require a three-pronged approach. Beginning from the perceptual learning perspective, learners could be introduced to fraction learning by making use of non-symbolic continuous magnitudes. Technology can serve an important function here by incorporating the principles of perceptual learning. Specifically, a large number of trials with varied stimuli can aid learners to assess the invariant and variant properties that are required to make correct judgments on relative magnitudes (as described in section 1.2.2). The relational reasoning skills of learners could also be harnessed and developed in this approach. Eventually, when learners have built fluency to extract the right information from both within and cross-format trials they could be introduced to symbolic fractions. At this stage, they must have sound declarative and procedural knowledge to complement their perceptual learnings. This could potentially be achieved with the help of *Math Mathews Fractions* (see section 1.4). Teachers could integrate the game during instruction to help build connectional understandings between the complex subconstructs of fractions and provide for a more holistic understanding of fractions. As mentioned before, it is still too early to be definitively providing answers to complex questions about fraction instruction and interventions. Future studies using robust

methods like randomized controlled trials should be undertaken to investigate the validity of these speculations.

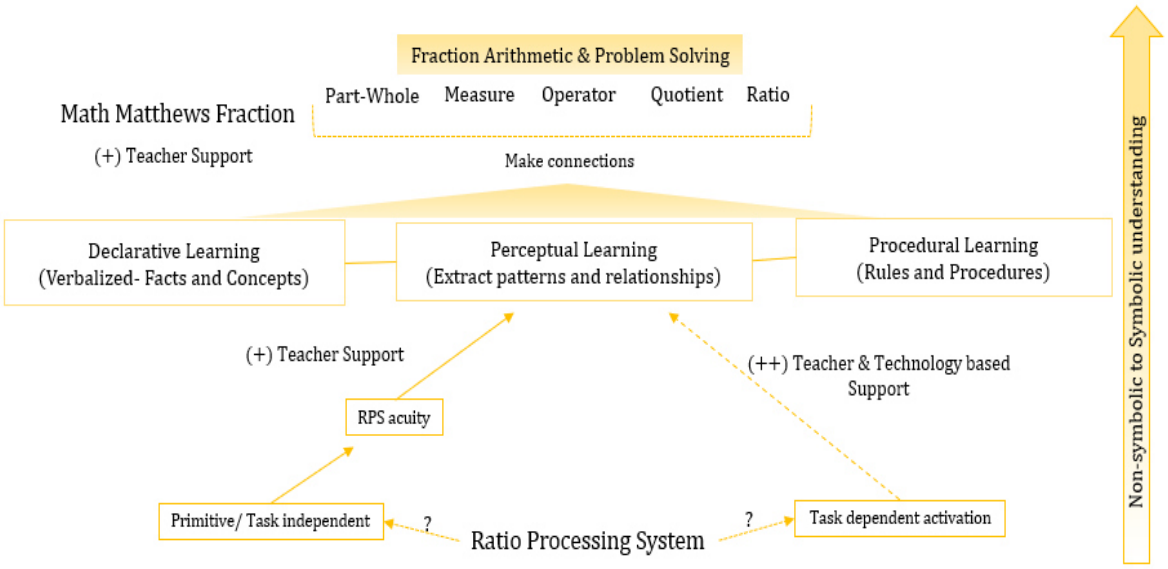

Figure 27: A conceptual model of fraction learning using declarative, procedural, and perceptual learning approaches

CONCLUSION

In this dissertation, we examined the neurocognitive correlates of fraction processing and learning. The thesis is divided into two main parts. The first part tests a relatively recent theory, the Ratio Processing System for processing relational magnitudes and the second part focuses on the effectiveness of a game-based intervention on fraction learning.

First, our results show a limited role of the RPS in symbolic fraction processing. On the one hand, the findings support the idea that the RPS underlies non-symbolic ratio processing in children, typically achieving adults, and adults with developmental dyscalculia. Interestingly, typically achieving adults also show perceptual sensitivity in a cross-format task. On the other hand, the absence of perceptual sensitivity to process symbolic fraction magnitudes indicates a relatively limited role of the RPS. Taken together, these findings indicate that the RPS may be utilized mainly when participants are required to approximate or estimate the magnitudes of ratios. Second, the fMRA study shows that in a passive viewing task with varied fractions and line ratios, adults did not show any neural adaptation effects to either fractions or line ratios in the intra-parietal regions. Additionally, we found non-symbolic and symbolic absolute magnitudes to share overlapping neural regions in the left IPS in participants with higher mathematics fluency. Further, multivariate analyses revealed a dissociation of neural activation patterns in the right intra-parietal region for non-symbolic and symbolic formats across magnitude types. Integrating these results, we speculate that the RPS might be utilized when approximation or estimation of relational magnitudes is required. Third, the results of the game-based intervention, *Math Mathews Fractions*, indicate the game to be effective for building links between fraction

and decimal representations, but not overall fraction knowledge. In addition to that, performance in the game is positively related to overall fraction knowledge at post-test. This study highlights the critical role of instructional support during game-based learning and the importance of game metrics as indicators for personalized assessment tools.

The insights gained from the abovementioned studies may help in re-imagining symbolic fraction instruction. However, future research is required to establish both the RPS and the game *Math Mathews Fractions'* role in fraction instruction. While a natural progression of this work will be to analyze the cross-format tasks in children (before and after fraction instruction) as well as adults with dyscalculia, more broadly, to establish the role of the RPS in symbolic fraction understanding, research focussed on utilizing different paradigms in participants with differing levels of fraction knowledge might be required. It might also be interesting to test the effectiveness of an RPS based intervention on symbolic fraction learning of individuals. Specifically, testing interventions that can aid in explicit sense-making processes for cross-format tasks that might aid in improving magnitude understanding and thus, connectional fluency between different representations (Rau et al., 2017). Additionally, future studies could also test the strategies participants use when solving both magnitude comparison tasks and match to sample tasks in the symbolic fraction format. This could provide some insights into how individuals might be processing these two tasks distinctly, but more importantly could aid in understanding the development of symbolic fraction skills (Obersteiner et al., 2020; Sidney & Alibali, 2017). Continued efforts are also needed to understand the neural mechanisms behind the RPS as well as the distinct neural regions that might be involved in processing absolute and relative magnitudes. Furthermore, investigations using randomized controlled trials where

teachers are provided with specific training or lesson plans to adequately integrate *Math Mathews Fractions* in the classroom could provide definitive evidence for the role of game-based tools in fraction learning. Lastly, the pursuit of understanding the RPS using both neuroimaging and behavioral tools could potentially help enhance teacher training, support individuals with disabilities, and help design better educational games on fractions.

CHAPTER IX: REFERENCES

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APPENDIX A

STUDY I

Table 1: Fractions generated for 3:1 RoR (Note: Nt = numerator of target, Nd = numerator of distractor)

Nt = Nd				Nt ≠ Nd			
	target	match	distractor		target	match	distractor
	2/4	5/10	2/12		9/12	3/4	2/8
	2/4	4/8	2/12		6/8	3/4	1/4
	1/3	3/9	1/9		3/4	6/8	1/4
	2/3	6/9	2/9		3/12	1/4	1/12
	3/4	9/12	3/12		6/8	3/4	3/12
	1/3	2/6	1/9		3/5	6/10	2/10
	2/4	3/6	2/12		8/12	6/9	2/9
	1/2	2/4	1/6		4/12	1/3	1/9
	2/4	6/12	2/12		3/9	1/3	1/9
	2/3	4/6	2/9		6/9	8/12	2/9
	2/4	1/2	2/12		4/6	8/12	2/9
	1/2	5/10	1/6		6/12	1/2	1/6
	1/3	4/12	1/9		3/6	6/12	2/12
	1/2	6/12	1/6		8/12	4/6	2/9
	1/2	4/8	1/6		8/12	2/3	2/9
	1/4	2/8	1/12		4/8	2/4	1/6
	1/4	3/12	1/12		5/10	6/12	2/12
	3/4	6/8	3/12		3/6	1/2	2/12
	2/3	8/12	2/9		6/12	4/8	1/6
	1/2	3/6	1/6		3/4	6/8	2/8

Table 2: Fractions generated for 4:3 RoR

Nt = Nd				Nt ≠ Nd			
	target	match	distractor		target	match	distractor
	3/9	1/3	3/12		8/12	4/6	5/10
	4/6	6/9	4/8		2/3	6/9	5/10
	1/3	3/9	1/4		4/12	3/9	2/8
	3/6	1/2	3/8		4/12	3/9	1/4
	6/9	2/3	6/12		6/9	8/12	5/10
	2/3	4/6	2/4		8/12	2/3	4/8

	2/6	1/3	2/8		1/3	4/12	2/8
	3/6	4/8	3/8		4/8	1/2	3/8
	4/6	8/12	4/8		6/9	8/12	3/6
	1/3	4/12	1/4		4/6	6/9	5/10
	4/6	2/3	4/8		8/12	4/6	3/6
	3/6	2/4	3/8		1/3	4/12	3/12
	5/6	10/12	5/8		6/9	4/6	1/2
	3/9	4/12	3/12		2/3	4/6	1/2
	6/9	8/12	6/12		4/12	1/3	2/8
	2/6	4/12	2/8		2/5	4/10	3/10
	1/3	2/6	1/4		2/3	8/12	3/6
	6/9	4/6	6/12		1/2	6/12	3/8
	3/6	5/10	3/8		4/5	8/10	6/10
	2/6	3/9	2/8		4/12	1/3	1/4

Table 3: Fractions generated for 2:1 RoR

Nt = Nd				Nt ≠ Nd			
	target	match	distractor		target	match	distractor
	1/5	2/10	1/10		8/12	6/9	3/9
	4/6	6/9	4/12		8/10	4/5	2/5
	1/2	5/10	1/4		4/8	3/6	1/4
	3/4	9/12	3/8		3/9	2/6	2/12
	2/5	4/10	2/10		3/12	1/4	1/8
	2/4	3/6	2/8		4/6	6/9	2/6
	2/4	4/8	2/8		6/12	5/10	1/4
	2/6	1/3	2/12		5/10	6/12	3/12
	3/6	1/2	3/12		6/12	4/8	2/8
	2/3	4/6	2/6		6/8	3/4	3/8
	2/4	1/2	2/8		8/12	6/9	1/3
	1/3	2/6	1/6		4/6	2/3	1/3
	1/4	2/8	1/8		5/10	4/8	1/4
	1/3	3/9	1/6		2/3	4/6	3/9
	3/6	4/8	3/12		5/10	4/8	2/8
	3/6	5/10	3/12		2/8	1/4	1/8
	4/5	8/10	4/10		8/12	4/6	1/3
	3/6	6/12	3/12		8/12	6/9	4/12
	2/6	4/12	2/12		4/8	3/6	3/12
	1/2	3/6	1/4		1/2	5/10	3/12

Table 4: Fractions generated for 6:5 RoR

Nt = Nd				Nt ≠ Nd			
	target	match	distractor		target	match	distractor
	4/10	2/5	4/12		3/6	4/8	5/12
	5/10	2/4	5/12		10/15	6/9	5/9
	6/10	3/5	6/12		6/9	2/3	5/9
	5/10	4/8	5/12		12/14	6/7	10/14
	5/10	3/6	5/12		6/15	2/5	3/9
	2/10	1/5	2/12		4/8	6/12	5/12
	2/5	6/15	2/6		12/15	8/10	6/9
	1/5	3/15	1/6		4/8	7/14	5/12
	4/10	6/15	4/12		4/8	1/2	5/12
	6/10	9/15	6/12		12/15	4/5	2/3
	4/5	12/15	4/6		3/15	1/5	1/6
	8/10	4/5	8/12		6/10	3/5	3/6
	5/10	1/2	5/12		12/15	4/5	10/15
	8/10	12/15	8/12		3/5	9/15	2/4
	3/5	9/15	3/6		9/15	3/5	2/4
	2/5	4/10	2/6		2/5	4/10	3/9
	1/5	2/10	1/6		4/6	8/12	5/9
	3/5	6/10	3/6		6/12	2/4	5/12
	5/10	6/12	5/12		6/10	3/5	4/8
	2/10	3/15	2/12		2/4	4/8	5/12

APPENDIX B

STUDY II

Table 1: Stimuli used for fractions and line ratios.

Block	Numerator	Denominator
No-Adapt	2	6
	3	7
	11	32
	9	17
	7	9
	2	7
	11	70
	5	48
No-Adapt	2	4
	12	15
	6	17
	16	26
	2	15
	16	54
	13	28
	5	6
Adapt_1:3	1	3
	4	12
	6	19
	3	9
	8	25
	2	7

	5	15
	7	22
No Adapt	14	15
	11	32
	1	8
	13	27
	34	62
	7	26
	12	39
	7	11
No Adapt	15	19
	1	4
	14	35
	8	13
	12	31
	5	18
	3	23
	8	10
Adapt_1:6	2	12
	1	6
	8	48
	6	36
	4	24
	3	17
	5	29
	7	41
Adapt_1:5	1	5
	3	15

	5	26
	8	40
	2	11
	6	30
	4	21
	7	35
No Adapt	3	5
	15	19
	7	14
	8	62
	4	21
	9	19
	45	68
	4	37
Adapt_2:3	8	12
	6	9
	2	3
	10	16
	4	7
	12	18
	14	22
	16	24
Adapt_2:5	4	10
	8	19
	14	35
	6	15
	12	29
	10	25

	16	39
	2	5
Adapt_3:5	3	5
	6	10
	15	25
	18	31
	9	16
	12	21
	24	40
	21	35
No Adapt	1	3
	21	67
	5	24
	3	16
	24	54
	5	8
	3	5
	1	8
Adapt_2:9	2	9
	4	18
	8	35
	6	26
	10	45
	12	54
	14	63
	16	71
No Adapt	8	9
	11	14

	3	8
	14	41
	2	5
	21	33
	2	3
	8	19
Adapt_3:7	18	41
	6	14
	3	7
	9	20
	12	28
	15	34
	21	49
	24	55
Adapt_1:4	2	8
	3	11
	5	19
	8	31
	1	4
	6	23
	4	16
	7	28
No Adapt	4	9
	16	18
	15	35
	2	6
	13	31
	8	9

	12	34
	11	27
Adapt_2:7	2	7
	4	14
	6	22
	8	28
	10	35
	12	43
	14	49
	16	57
No Adapt	21	66
	3	4
	2	26
	13	23
	5	5
	9	11
	7	17
	14	36
No Adapt	5	9
	15	29
	14	31
	3	8
	2	16
	34	62
	11	12
	1	24

Table 2: Stimuli used for lines and numbers

Block	Magnitude
No Adapt	5
	14
	33
	2
	9
	17
	6
	10
Adapt	8
	6
	8
	10
	8
	7
	8
	9
No-Adapt	32
	6
	18
	3
	17
	7
	22
	11
No-Adapt	4
	18

	22
	5
	34
	9
	7
	62
No Adapt	19
	4
	35
	44
	6
	18
	3
	10
Adapt	23
	26
	25
	26
	24
	26
	21
	26
No-Adapt	1
	14
	7
	28
	4
	40

	16
	37
Adapt	11
	14
	16
	11
	12
	11
	17
	11
Adapt	33
	31
	36
	31
	32
	31
	35
	31
Adapt	6
	5
	4
	6
	3
	6
	7
	6
No Adapt	3
	10

	32
	6
	15
	8
	25
	9
Adapt	23
	25
	26
	23
	23
	24
	23
	27
No Adapt	2
	13
	7
	14
	20
	11
	25
	39
Adapt	53
	52
	55
	52
	54
	52

	56
	52
Adapt	9
	8
	5
	9
	7
	9
	11
	9
No-Adapt	36
	18
	2
	6
	16
	35
	8
	21
Adapt	3
	5
	3
	4
	3
	6
	3
	2
No Adapt	8
	29

	17
	54
	16
	9
	36
	5
Adapt	16
	17
	15
	16
	16
	13
	16
	11
No-Adapt	1
	16
	25
	18
	9
	4
	33
	2