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# Optimization of Reverse Logistics for the Recovery of End-of-life Products

Meisam Pour Massahian Tafti

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Thèse  
de doctorat  
de l'UTT

**Meisam POUR MASSAHIAN TAFTI**

# Optimization of Reverse Logistics for the Recovery of End-of-life Products



**Champ disciplinaire :**  
Sciences pour l'Ingénieur

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# THESE

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## DOCTEUR

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**Spécialité : OPTIMISATION ET SURETE DES SYSTEMES**

*présentée et soutenue par*

**Meisam POUR MASSAHIAN TAFTI**

*le 7 janvier 2021*

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**Optimization of Reverse Logistics for the Recovery  
of End-of-life Products**

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## JURY

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I would like to dedicate this thesis to my loving Parents and my Family members ...



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## Abstract

This thesis is dedicated to inventory models for disassembly planning problem. We first deal with three cases of disassembly lot sizing problem with considering disposal to handle surplus inventory accumulations due to disassembly operations. Three cases are considered. The first case is single-product with two-level product structure and we after propose a procedure to calculate surplus inventory and the problem is modeled by three new MIP formulations, which have different lower bound qualities. The addition of valid inequalities and two efficient heuristics with different inventory strategies are also developed. The second case is multi-product with two-level product structure and parts commonality. Two new MIP formulations are proposed to model the problem. LP-relaxation of the formulations can improve lower bound of the problem and a two-phase heuristic is also adapted for large-sized problems. The third case is multi-product with multi-level product structure, parts commonality, and capacity restrictions. A new MIP formulation is proposed. For large-sized problems, a Fix-and-Optimize algorithm solves the problem and we improved it by considering interrelatedness characteristics between linked variables.

We also deal with the disassembly EOQ model which integrate price-sensitive demands and disposal decisions to maximize the profit of disassembly systems. Three models are developed to obtain solution approaches that give prices, the replenishment cycle time, and the disposal quantity. The inventory policy integrating both pricing and disposal decisions allows higher profits to be achieved.

**Keywords:** Reverse logistics, Planning, Inventory control, Operation research, Heuristic.



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# Chapter 1

## Introduction

### 1.1 Context

Nowadays, the supply chains are no longer limited to one-way flows of product from producers (raw materials) to customer (finished goods). In recent years, Reverse Logistics (RL) has gain remarkable attention among both academia and business worlds ([Dekker et al. \(2013\)](#)). There are several reasons for the companies, more specifically, in the manufacturing sector, to get involved in RL. These reasons represent at the same time the economical, environmental, and social aspects: (i) Environmental concerns have become more serious and complex around the world. (ii) Legislation pressures such as eco-design requirements for energy using products (EuP) and restrictions on the use of certain hazardous substance (RoHS) in electrical and electronic equipment are forcing companies to take into account laws regulating the treatment of used or end-of-life (EOL) products and consider possibilities of product recovery or even disposal. More severely in Europe, some directives of the European Union (EU) such as those relating to EOL vehicles (ELVs) and waste electrical and electronic equipment (WEEE) will put greater pressure on companies to find the solution for the problem of waste management by recycling and reuse. (iii) Companies are become progressively aware of the profitability of the reusing valuable parts or materials from EOL products. (iv) Consumers now understand the opportunities to buy environmentally friendly items ([Kim et al. \(2016\)](#)), this can be a competitive factor for nowadays companies. In such context, new decision-making methods should be developed which aim at balancing environmental and social impacts against the economic aspect ([García-Alvarado et al. \(2015\)](#)).

RL consists of a variety of activities appropriate for collecting and reprocessing used or EOL products to obtain their residual value or dispose of them in a conscious environmental way. It also has become popular for businesses participating in the conventional forward logistics to also collect and reprocess EOL products ([Gupta \(2013\)](#)). The border between forward and reverse logistics is not clearly specified in modern supply chains so that a new view on supply chain combining both forward and reverse logistics is emerged as closed-loop supply chain concept ([Dekker et al. \(2013\)](#)). For example, a recycled glass can be used as a new raw material for the production of new glass. Figure 1.1 is adapted form [Dekker et al. \(2004\)](#), [Bloemhof-Ruwaard et al. \(2004\)](#), and [Kim et al. \(2016\)](#). It represents importance role of reverse logistics in a generic closed loop supply chain to have an efficient business. The return flow starts from the end costumers, then collecting /inspecting returned products. The returned products will be allocated to the relevant recycling channel. The obtained recovered materials or components and

new material might be used in manufacturing. This schematic also refers that first objective of reverse logistics is to find out some ways to create potential benefits from the returned products, which can support both economical and environmental aspects and guarantees the business circular economy.



Fig. 1.1 Schematic of both forward and reverse logistics in a closed loop supply chain.

RL has the role of complementary in modern supply chains activities, but its operations are more complex than forward logistics. The reason is there are several aspects than RL differs from forward logistics. Table 1.1 presents some of the differences. These differences make managing of RL more complicate that forward logistics, therefore, new models and methods are necessary (Tibben-Lembke and Rogers (2002); Gupta (2013)).

Forced by legislation limits or attracted by the obtainable value in EOL products, RL has become an importance key in nowadays modern supply chain and growing number of researches are devoted to it. reverse logistics can refer to a variety of domain regarding its application and it can be defined with regarding the related domain. There exists several definitions of RL in the literature. The definitions differ from each other but they are agree on many points. We provide the most frequently used definition which also corresponds the works developed in this thesis. Rogers et al. (1999) defined RL with regarding the definition of Logistics by the Council of Logistics Management (Stock (1992)) as follows: "The process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal."

Table 1.1 Differences between forward and reverse logistics (Rogers et al. (1999); Gupta (2013)).

Forward	Reverse
Based on profit and cost optimization	Environmental aspect as well as profit and cost optimization
Easier forecasting for demands	More difficult forecasting for demands
Product quality with less variation	Product with highly stochastic quality
Processing times are well defined	Processing times depends on the condition of the returned product
Product are delivered from one location to many other locations	Product are collected from many locations to one center
Cost estimation is easier	Cost estimation is complicated
Consistent inventory management	Inconsistent inventory management

There exist five general types of product return in RL (Gupta (2013)): Customer returns, repair/service returns, EOL returns, reusable container returns, and leased product returns. In this research we deal with EOL products returns which means that the returned product can be used no longer by the customer. There are several return reasons for the customer:

- Used products returned due to the dissatisfaction of customers and they are mainly damaged because of misusing or improper handling,
- Returned products for customers which cannot be repaired,
- Returned products which have been recalled or reach their prematurely due to rapid technology development,
- Leased products returned which are nonfunctional at all,
- Returned container or packaging which is severely damaged and nonrepairable.

These reasons represent that there is an important flow of return for the EOL products which should be considered and managed. Moreover, large variety of today's products makes the return planning more complicated. Figure 1.2 provides the product recovery operations for each of these mentioned above product return types. All returned product types will first be collected, sorted, and inspected. Then, if the returned product is regarded as an EOL product, disassembly is an important step in product recovery of returned products: It reduces the environmental impacts of EOL products by removing dangerous materials or parts and provides economic balance by obtaining valuable materials or parts for reusing, re-manufacturing, material recycling or energy recovering (Kim and Xirouchakis (2010); Godichaud and Amodeo (2018)). It can also contribute to economic benefits through efficient management and strategy in industrial cases. The cost reductions provide the companies with economic opportunities in the recovery of EOL products, while the world often benefits from both less waste and the reduction of the new material used for new production (Inderfurth and Langella (2006)). Therefore, a growing number of researches are emerged to make disassembly systems profitable (Liu and Zhang (2018); Tian and Zhang (2019)).

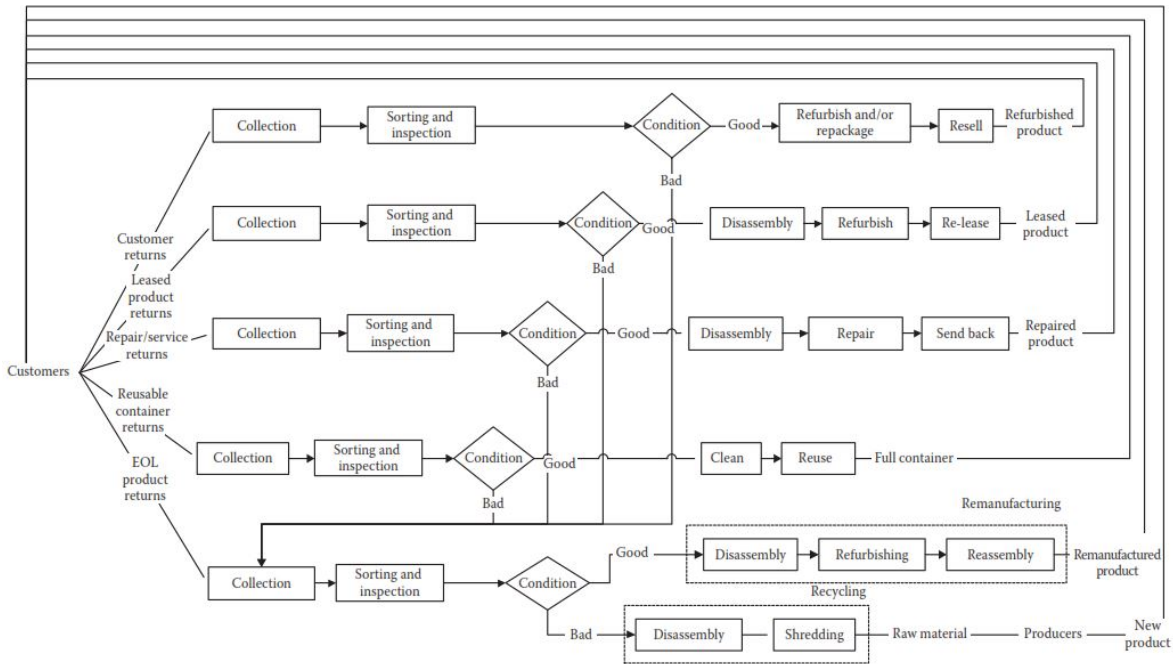


Fig. 1.2 Reverse logistics activities (Gupta (2013)).

Figure 1.3 presents associated activities related to product recovery of EOL products in RL, which shows that EOL products will be disassembled before being refurbished, remanufactured, repaired, reused, recycled, or being disposed of (Kim and Xirouchakis (2010); Kim et al. (2016)). The disassembly processes consist of generating components from the EOL products that can be allocated to a recovery channel which generates demands and revenues, or to other channels which reduce environmental impacts. However, the economic gain can be small between revenues and disassembly costs. Efficient planning of disassembly systems can increase opportunities for cost saving and make them more profitable.

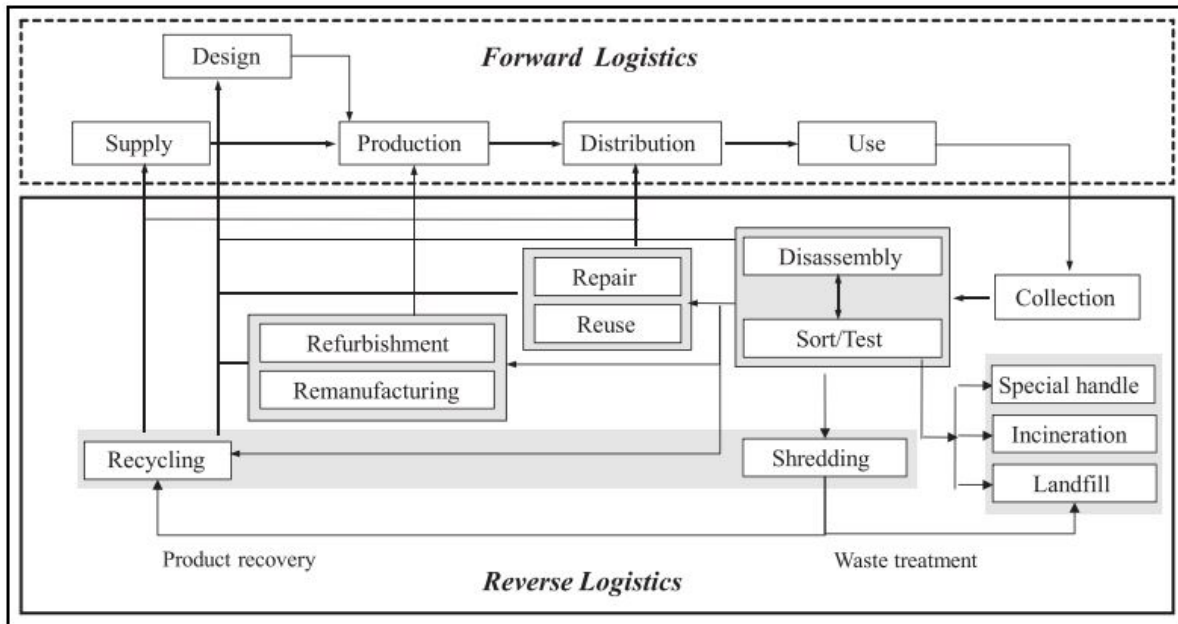


Fig. 1.3 Reverse logistics process (Kim et al. (2016)).

In this thesis we deal with EOL products and we suppose that EOL products are collected, inspected, and sorted before being disassembled in order to obtain valuable or reusable components. Disassembly operations have special characteristics that make them challenging for planning decisions:

- The product is decomposed to meet multiple demands for parts or components,
- The demands for components are independent and not necessarily well balanced,
- The disassembly operation generates all the parts or components simultaneously.

These features imply that the quantity of product to be disassembled is not necessarily equal to the number of requested components and an unnecessary surplus inventory is likely to be generated after each disassembly operation. It can be held to satisfy future demands or be disposed of in a conscious environmental way. Managers have several solutions to handle this surplus inventory such as managing with:

- Lost sales, which means that certain demands for the components are not met. This results in a cost of lost sales. The disassembly problem with lost sales is mainly studied in Hrouga (2016). The lost sales model is interesting in practice to limit the surplus. The demands for the components can be lost, if the costs are high in relation to profits by balancing three types of costs: setup, inventory holding, and lost sales.
- Disposal, which means that the non-needed items will be removed from the disassembly system. They might be sold to a secondary market, be transported to landfilling, or be forwarded to a recycling center in order to material recycling or energy recovering (Inderfurth and Langella (2006); Kongar and Gupta (2006)). This option is not applied specifically in the literature to handle surplus inventory. In this thesis, we will consider disposal options to handle surplus inventory and provide economic balance in disassembly centers.

- Pricing, which means the demands are considered as decision variables according to the prices. This price sensitive demands is defined as a function. We mention that lost sales and disposal are considered as stockouts options to manage the surplus inventory (Godichaud and Amodeo (2019b)). But, the models including pricing decisions can be applied to balance the demands of the components and to handle the surplus inventory issue in disassembly systems (Godichaud and Amodeo (2018)).

Inventory policy for disassembly systems can be critical within specific industries, more specifically the one with a high return rate. For example, every year, End-of-Life Vehicles (ELVs) generate between 8 and 9 million tonnes of waste in the EU, which need to be managed appropriately. In an ELVs recycling sector, the EOL vehicles will be disassembled into their parts and materials, such as engines, doors, seats, tyres, plastics, metals, etc. All obtained parts and materials are checked to ensure that they are in good working order, but they are also identified, catalogued and stored for better management and traceability in order to guarantee optimal safety at the best price. According to the EU, all ELVs would need a rate of reuse and recovery of not less than 95% by no later than 1 January 2015 (the European Directive 2000/53/EC). This implies that one of the principal aims of this directive is to increase the level of recycling and recovery. Obtaining high level of recovery leads to increase significantly the level of inventory of obtained parts and materials. Several costs are involved such as purchasing, disassembly operation, inventory holding, disposal and disassembly order costs. The companies must provide optimal policies so that the ELVs can be efficiently dismantled. They can sell the obtained parts on the secondary market in order to make the disassembly system profitable.

## 1.2 The contribution of the thesis

The objectives of this thesis are, on one hand, to investigate inventory control models in order to manage the excess inventory of the obtained components /materials in disassembly systems, on the other hand to develop approaches to solve the proposed models. Different methods and models are developed that can support decision-makers to cope with several optimization problems facing in real life such as defining optimum disassembly plan. They also can select inventory management policies which lead to economic balance and more profits in disassembly systems. To achieve these objectives, we first consider the disassembly lot sizing problem (DLSP) which is to determine when and how much to disassemble the EOL product in order to meet the demands for the components over a finite planning horizon. Due to some specific characteristics of disassembly systems, surplus inventory can be generated while satisfying the demand for the components. Disposal decisions are considered here to avoid inventory accumulations throughout the planning horizon. Three cases of the problem are considered: (i) Single product with two-level disassembly structure. (ii) Multi-product with two-level disassembly structure and parts commonality. (iii) Multi-product with multi-level disassembly structure, parts commonality, and capacity restrictions. We also develop several approaches for each case to solve the proposed models. In the second inventory control model, we consider EOQ models which are an interesting first step to optimize the inventory policy of a disassembly centers when no inventory model is available. The EOQ models are often used where there are few parameters to characterize the system and they are an approximation model when modeling stochastic environment. But, there is a need to develop limited assumptions of basic EOQ model in order to improve its applications in other contexts like disassembly systems. We aim to integrate price-sensitive demands and disposal decisions to maximize the profit of



disassembly systems without inventory accumulations. For each problem we propose different methods to solve. The main contributions of this research are:

- A procedure is presented to compute surplus inventory for the disassembly lot-sizing problem without using optimization methods. This is the issue of surplus inventory of components accumulating through a finite planning horizon. The procedure compute both parts of surplus inventory i.e., the part that will be used in future periods and the part that will be accumulated at the end of planning horizon.
- Different MIP models are developed to formulate disassembly lot-sizing problems which consider disposal decisions to avoid surplus inventory accumulations over the planning horizon. The formulations differ from each other concerning the quality of the lower bound provided by their linear relaxation, which is an important issue in MIP resolution methods.
- Two efficient heuristics which consider different inventory strategies in real case applications are developed for the case of single product with two-level structure. A two-phase heuristic is adapted for the case of multi-product with two-level structure and parts commonality. A Fix-and-Optimize algorithm with considering interrelatedness characteristics between linked variables is proposed for the case of multi-product with multi-level structure and parts commonality and capacity restrictions.
- A comparative study of the formulations and the heuristics is presented for each problem case based on new benchmark instances for disassembly lot-sizing problems.
- For disassembly systems with continues demand, EOQ models which integrate price-sensitive demands and disposal decisions in disassembly economic order quantity problems are developed to maximize the profit of disassembly systems without inventory accumulations. A numerical experiment is provided which shows efficiency of the developed models and highlights their potential implementations in practical cases.

Overall, the developed models and methods can contribute to increase the use of disassembly operations in reverse logistics, to provide the opportunity of inventory cost reduction and to create more profitable as well as environmental-friendly disassembly systems.

### 1.3 The structure the thesis

This section clarifies the problems considered in this thesis by a brief description of the problems and resolution methods. This thesis focuses mainly on the problem of inventory accumulation in disassembly systems and considers different inventory management strategies bases on the disassembly lot-sizing (DLS) and economic order quantity (EOQ) settings. For the DLS problems, three cases are considered: the case of single-product with two-level disassembly structure; the case of multi-product with two-level disassembly structure with parts commonality; and the case of multi-product with multi-level disassembly structure with part commonality and capacity constraints. For the EOQ problem, the case of single-product with two-level disassembly structure is considered. The objective of the problems is minimize the related costs in disassembly systems by including decisions to handle inherent surplus inventory in disassembly operations.

A literature review of contributions on related works on disassembly lot sizing problem and on disassembly economic order quantity problems is presented in Chapter 2. An overview of the recently

published paper disassembly lot sizing problems, EOQ models, and cases studies is included. The research existed are classified according to their characteristics and resolution methods proposed.

Chapter 3 focuses on the single product disassembly lot sizing problem with two-level disassembly structure. The issue of inventory accumulation in disassembly systems is highlighted and a procedure is proposed to compute different surplus inventories. Three different MIP formulations with considering disposal decisions to handle surplus inventory are provided. They are compared with regarding their linear relaxation lower bounds, which is as important issue for the efficiency of MIP resolution methods. Also two efficient heuristics to solve real case applications are proposed, when MIP approaches are not relevant. The performance of the models and methods are compared by a different cost settings to generate the problem instances and a sensivity analysis is presented to highlight their effects.

In Chapter 4, we focus on the multi-product disassembly lot sizing problem with parts commonality. Two cases are considered. First a two-level disassembly structure is considered and two different MIP formulations are developed which consider disposal decisions to avoid surplus inventory accumulations. A two-phase heuristic is adapted for the problem with parts commonality, which constructs an initial solution by using LP relaxation of the aggregate model, and improves the solutions by changing them and considering cost trade-offs, iteratively. For the second case, multi-level disassembly structure with capacity restrictions is considered and a MIP formulation with considering disposal decisions is proposed, which provides potential cost savings. A Fix-and-Optimize (FO) heuristic is suggested to solve large-sized problems efficiently. It consists of solving a series of simple sub-problems in an iterative algorithm that can be solved in very short computational times. To improve the quality of solution obtained, the FO algorithm is improved by considering interrelatedness characteristics between variables. Finally, Computational experiments are performed on new randomly generated problem instances and the results are presented.

The last contributions on this thesis, Chapter 5, focuses on the Disassembly Economic Order Quantity problem (DEOQ), which is to determine the quantities of a product to be disassembled at different times over an infinite planning horizon by considering ordering, operation and inventory costs. The demands for the components are independent, which can lead to accumulations of unnecessary inventories over time. The new models which integrate price sensitive demands and disposal decisions in DEOQ problems to maximize the profit of disassembly systems without inventory accumulations. Three models are developed and analyzed to obtain solution approaches that give prices, replenishment cycle time (or, equivalently, order quantity) and disposal quantity. The inventory policy integrating both pricing and disposal decisions allows higher profits to be achieved. A numerical experiment is performed which shows efficiency of the proposed inventory policy and highlights its potential implementation in practical cases.

Chapter 6 presents a general conclusion of the thesis with the perspectives for future works.

# Chapter 2

## Literature Review

This Chapter provides a state-of-art on the problems studied in this thesis. Three main research fields are identified to position our contributions: disassembly lot sizing, disassembly economic order quantity (DEOQ) and EOQ models with price sensitive demand. The first two fields highlight inventory issues in disassembly systems, and the last one considers the opportunity of considering pricing in EOQ models. We note that there are few works on EOQ models adapted for disassembly systems, despite their advantages for real-case applications. Overall, we point out the differences between our problem and the works in the three mentioned research fields.

### 2.1 Disassembly planning

In the last decades, reverse logistics has gained growing attention among various supply chain stakeholders. The reason is that environmental concerns have become more serious and complex around the world. Also, legislation pressures are forcing companies to take into account laws regulating the treatment of used or End-Of-Life (EOL) products and consider possibilities of product recovery or even disposal. In addition, rapid development of technologies and increasing desire of improvement of products have significantly increased the flow of returned products. The management of this return flow is challenging. Disassembly is one the domain which is becoming increasingly important because of its role in saving resources and minimizing the impact of EOL product on the environment [Liu and Zhang \(2018\)](#); [Godichaud and Amodeo \(2019a\)](#). Disassembly aims to separate systematically reusable parts and materials from EOL products or isolating hazardous substances. Disassembly can be selective which means one or more components from EOL product are separated to obtain valuables parts or materials. It can be Complete which means EOL product will be disassembled to all of its components in order to recover them ([Lee et al. \(2001\)](#)). In this thesis we focus on the complete disassembly. Also, Various disassembly problems have been addressed in the literature ([Lee et al. \(2001\)](#); [Kim et al. \(2007, 2016\)](#)). They can be classified as:

- Disassembly leveling, which is to make decision on whether additional disassembly operations are done or not at each stage of disassembling the EOL product ([Kim and Lee \(2011\)](#); [Kang et al. \(2016\)](#)),
- Disassembly sequencing problems, which is to determine the sequence of disassembly operations ([Zhang and Kuo \(1997\)](#); [Gungor and Gupta \(1998\)](#); [Andrés et al. \(2007\)](#); [Smith et al. \(2012\)](#)),

- Disassembly line balancing problems, which is the problem of assigning the set of tasks to each workstation for each EOL product to be disassembled ([Gungor and Gupta \(2001\)](#); [Bentaha et al. \(2015\)](#); [Hezer and Kara \(2015\)](#); [Özceylan et al. \(2019\)](#)),
- Disassembly lot sizing, which is to determine the quantity and timing of disassembling EOL products in order to satisfy the demands for their components over a finite planning horizon ([Kim et al. \(2007\)](#)). Only disassembly lot-sizing problem considers significantly inventory aspects. We next review this problem and its variants.

## 2.2 Disassembly Lot Sizing

### 2.2.1 Lot sizing problem

This section focuses on the problem of lot-sizing in production, including models and methods of resolution. The objective is to investigate the existed models and methods and analyze the possibility to be used for the lot sizing problem in disassembly. There exists comprehensive studies about the characteristics of the lot-sizing problems and related solution methods. First, [Brahimi et al. \(2006\)](#) review a particular planning problem: the single item lot sizing problem and present both capacitated and uncapacitated versions. In [Brahimi et al. \(2017\)](#), the previous state-of-art has extended and provide a comparative survey about the characteristics of the lot sizing problems and existed solution methods.

The problem of lot-sizing in production is one of the most important problems in production planning. It requires specific methods of resolution according to the complexity. In this problem, the objective is to determine the optimal size of production batches. Indeed, it is often too costly to produce a product every period due to launch times associated with each product (i.e. setup cost). On the other hand, producing a large quantity of products over a long period of time can result in significant storage costs. The objective of lot sizing problems is therefore to find a trade-off between the different costs in order to calculate an optimal production size ([Phouratsamay and Cheng \(2019\)](#)). This topic has been extensively studied in the literature.

The first research on lot-sizing in production dates from the 50s. The first model to our knowledge is that of [Wagner and Whitin \(1958\)](#) which was proposed in 1958. Starting in the 1990s, there was a significant evolution until now. This is because its basic models are extended to be applied in various areas. For example, [Adler and Nanda \(1974\)](#) propose a single product model based on the properties of Wagner and Whitin to be applied in real case. Different types of the problem are studied. [Brahimi et al. \(2017\)](#) classifies lot-sizing problem with regarding different aspect, such as deterministic or stochastic, finite or infinite, discrete or continuous, single-item or multi-item, and single-level or multi-level, uncapacitated or capacitated. A lot of researches have addressed single item lot-sizing problem and its extensions. This is interesting because single item lot sizing problem appears as a subproblems in the solution method of various complex lot-sizing problems. [Wolsey \(1995\)](#), [Brahimi et al. \(2006\)](#), and [Brahimi et al. \(2017\)](#) have proposed a state of the art on the single product lot-sizing problem. [Karimi et al. \(2003\)](#) also proposed a state of the art for the single-level problem with capacity constraints, and then proposed different methods of resolution (exact and approximate methods). In [Glock et al. \(2014\)](#), the authors proposed a state of the art allowing researchers to identify their work in relation to the literature. A wide variety of solution techniques in operation research field has been allocated to solve different lot sizing problems ([Jans and Degraeve \(2007\)](#); [Buschkühl et al. \(2010\)](#)). A lot of researches deal with deal with exact solution approaches ([Brahimi et al. \(2017\)](#)). They aim at

providing the guaranteed optimal solutions for the problems. Most of these techniques are based on mathematical modeling as integer programming (IP) or mixed-integer programming (MIP), which rely on Branch&Bound (B&B) type procedures based on the research tree to solve the problem (Pochet and Wolsey (2006); Gicquel and Minoux (2015)). However, the efficiency of such a method strictly depends on the quality of the lower bounds used in the procedure (Gicquel and Minoux (2015)). Different MIP formulations are developed in the literature which differ from each other concerning the quality of the lower bound (LB) provided by their linear relaxations, which is an important issue in several resolution methods such as B&B method. For example Brahimi et al. (2006) discuss strong formulations for the single item uncapacitated lot sizing problem. They mention FAL as a strong reformulation which is based on the facility location problem concept. It is considered as a tight formulation because its LP relaxation can provide an optimal solution with integer setup variables (Cornuéjols et al. (1983); Maes et al. (1991); Krarup and Bilde (1977)). Much researches devoted to study the polyhedral description of the problem. It involves to obtain the tight linear relaxations of the problem and improve the LB. In particular, adding valid inequalities which reduce significantly the solution spaces by cutting off irrelevant solutions (Barany et al. (1984); Gicquel and Minoux (2015); Brahimi et al. (2017)). In the following, we present the literature on the solution techniques on the variants of lot sizing problems. Lot sizing problem in production systems has several similarities with lot sizing problem in disassembly systems. Our objective is to analyze different models and solution methods existed in the literature of ordinary lot sizing problems to find out the possible applications in the context of disassembly.

For the case of single item and single-level without capacity constraints, Wagner and Whitin (1958) proposed a first exact algorithm. It is a forward dynamic programming algorithm that runs in  $O(T^2)$ , where  $T$  represents the number of periods. This motivated some authors to propose the dynamic programming method based on Wagner and Whitin (1958) (Federgruen and Tzur (1991)). Several works propose different dynamic programming of Wagner and Whitin (1958) with reducing CPU time of algorithm. For the problem with capacity constraint, Chen et al. (1994) and Beltrán and Krass (2002) propose a dynamic programming algorithm. The computational efficiency of dynamic programming algorithm reduce when solving large problems.

For more complex problems i.e. the case of multi-item with capacity constraint, researchers propose different resolution methods such as Lagrangian relaxation (Manne (1958)), Branch and Cut (Absi and Kedad-Sidhoum (2008)), heuristics (González-Ramírez et al. (2011)), and meta-heuristics (Genetic algorithm as in Wu et al. (2011), Simulated Annealing as in Tang (2004)). Buschkühl et al. (2010) also proposed a state-of-the-art on the problem of lot-sizing with capacity constraints, they presented several lot-sizing problems and different resolution methods. For the case of multi-level, the methods based on Lagrangian relaxation are mostly used. Afentakis and Gavish (1986) proposed a Lagrangian relaxation generating sub-problems consisting of multi-level problems with serial structure. Tempelmeier and Derstroff (1996) proposed a Lagrangian heuristic where capacity constraints and flow balancing equations were relaxed. other researches applied the methods based on heuristic to solve the problem. Cattrysse et al. (1990), Jans and Degraeve (2004), and Cunha et al. (2019) have proposed a heuristic based on column generation procedure. Maes et al. (1991) proposed a heuristic based on linear programming to solve the problem of lot-sizing with limited capacity and several production levels. The result show that the heuristic methods are more efficient than exact methods for the complex problems according to computational times.

Recently, the researchers have proposed new heuristic methods to solve more complex lot-sizing problems. For example, Helber and Sahling (2010) and Levi and Shi (2013) proposed two models

of integer linear programming that are solved using a Fix-and-Optimize (FO) heuristic. They then compared the heuristic with the solutions obtained by using CPLEX solver. [Chen \(2015\)](#) also proposed a FO heuristic to solve the problem with multiple products and capacity constraints. He generated an initial solution with a setup cost for each period by considering interrelatedness characteristics between linked variables, then applied a local search to minimize the setup cost and then minimize the total cost. [Karimi-Nasab and Modarres \(2015\)](#) and [Gicquel and Minoux \(2015\)](#) have also proposed a new family of valid inequalities to represent the multi-product aspect in the formulation of the problem. Then, they proposed a Branch and Bound algorithm to accelerate the research process using common solvers.

Although lot-sizing problem in production is different from the lot-sizing in disassembly, but they exist several similarities. Different models and methods can be applied or have been applied for disassembly lot-sizing such as dynamic programming as in [Kim et al. \(2005b\)](#) and B&B as in [Kim et al. \(2009\)](#). But the methods existed in lot-sizing in production cannot be applied directly in disassembly lot-sizing, and they require specific adaptations to solve these types of problems. To this order, in this thesis we are interesting to investigate the methods and models applied in the literature of lot-sizing problem in production or other problems close to disassembly lot-sizing in order to find the possibility of adaptation. In the following section, we investigate different models and methods applied in disassembly lot-sizing problem. We propose to develop new models existed in the literature of ordinary lot-sizing problem. For example, possibility of adaptation FAL formulation for the disassembly lot-sizing problems. Also, new heuristics bases on the dynamic programming and strong formulation can be proposed.

### 2.2.2 Disassembly lot sizing problem

Disassembly problems have gained increasing attention in studies concerning product recovery during the last decade. There are several classes of problems arising in disassembly systems identified in the literature ([Özceylan et al. \(2019\)](#)). Inventory issues are mainly considered in disassembly scheduling problems, which consist of determining the overall timing and quantities of EOL products to disassemble over a planning horizon under various assumptions (on demands, returns and costs). Most of the works in this disassembly research field consider a planning decision with time varying demands over a discrete and finite planning horizon. In this context, the seminal paper of [Gupta and Taleb \(1994\)](#) presents MRP-like (Material Requirement Planning) procedure. It determines the quantity of a product to disassemble in order to satisfy the demands for its components based on its bill-of-material. The general specificities of disassembly scheduling are highlighted in Lee et al. [Lee et al. \(2002\)](#) and several extensions of the MRP-like procedure have been made to consider multiple products with part commonalities ([Taleb and Gupta \(1997\)](#)), lot-sizing heuristics ([Barba-Gutiérrez et al. \(2008\)](#)) and demand uncertainties ([Barba-Gutiérrez and Adenso-Díaz \(2009\)](#)). These papers do not consider the problem of inventory accumulation mentioned in Chapter 1. When considering various types of planning costs, mathematical programming approaches are another approach to find optimal planning. If set-up or fixed ordering cost is not considered, on-the-shelf solvers can find optimal solutions as in [Lee et al. \(2002\)](#), [Kongar and Gupta \(2002\)](#), and [Langella \(2007\)](#). Uncertainties make the problem more difficult to solve ([Inderfurth et al. \(2015\)](#), [Inderfurth and Langella \(2006\)](#), [Kongar and Gupta \(2006\)](#)). Disposal decisions, which are an option to handle inventory accumulation, are considered in [Kongar and Gupta \(2002\)](#), [Langella \(2007\)](#), [Inderfurth and Langella \(2006\)](#) and [Kongar and Gupta \(2006\)](#), but only for a one period planning horizon. The authors mention that the disposal cost includes transportation cost and landfilling of unnecessary items or selling them in a secondary market, which results in an economic gain or very

low cost. Different costs of EOL treatments are discussed in [Favi et al. \(2017\)](#). The integration of disposal decisions in the DLSP has not been investigated, as the authors know. When considering a multi-period model with set-up cost, the inventory accumulation has to be handled period to period. The main features and properties of the problem are reviewed in [Kim et al. \(2007\)](#), and its different variants are presented in [Slama et al. \(2019a\)](#). [Kim et al. \(2007\)](#) point out the inventory surplus inherent to disassembly scheduling but, few papers consider this issue. Inventory accumulation in disassembly lot sizing problem can be indirectly reduced by allowing lost sales or external purchasing ([Godichaud et al. \(2015\)](#), [Hrouga et al. \(2016b\)](#), [Hrouga et al. \(2016a\)](#), [Ji et al. \(2016\)](#)), but they don't provide explicit decisions for managing the surplus inventory in each period. For the capacitated problem, [Ji et al. \(2016\)](#) consider a multi-product problem with part commonality and propose a Lagrangian heuristic algorithm to solve the problem. The problem with random parameters is addressed in [Liu and Zhang \(2018\)](#) with a mixed-integer nonlinear program. [Slama et al. \(2019b\)](#) propose a scenario-based stochastic LP model to deal with lead time uncertainty. They extend this work to a capacitated problem and propose a Mont-carlo-Mixed-Integer programming model (MC-MIP) to formulate the stochastic optimization problem ([Slama et al. \(2020b\)](#)). They solve MC-MIP model by using a sample average approximation (SAA) approach. [Tian and Zhang \(2019\)](#) considered a capacitated disassembly scheduling problem with a pricing decision on returned products. The authors aim to minimize the sum of setup, acquisition, inventory holding, disposal cost while determining optimal acquisition price and disassembly quantity and timing. Recently, [Slama et al. \(2020a\)](#) propose a profit maximization model to optimize the disassembly planning of a capacitated disassembly lot-sizing problem including external purchase, defective, and barebacked items.

Among the researches that have been done, few articles studied inventory management issue in disassembly systems and compared different relevant models and methods.

Since the first paper addressing disassembly lot sizing problem, the number of contributions in this field of research increases rapidly. But there is no much works done on addressing disassembly lot sizing problem in the literature unlike the huge number of papers in the ordinary lot sizing problem ([Brahimi et al. \(2017\)](#)). [Figure 2.1](#) presents the number of articles published per year in disassembly lot sizing problem ([Hrouga \(2016\)](#)). The researcher are studied different variants of the disassembly lot-sizing problems. [Table 2.1](#) summarizes and classifies these researches with respect to product structure, capacity, decisions on the management of surplus inventory, models and solution approaches. To the best of authors knowledge, there is no work that study the problem of disassembly lot sizing with disposal decisions in order to manage the surplus inventory. DLSP solutions with high inventory surplus provided by the methods in literature are not relevant for real industrial applications and we propose to address this issue. The research questions are to highlight the potential cost saving by allowing disposing of surplus inventory and to model the problem in order to provide efficient solutions.

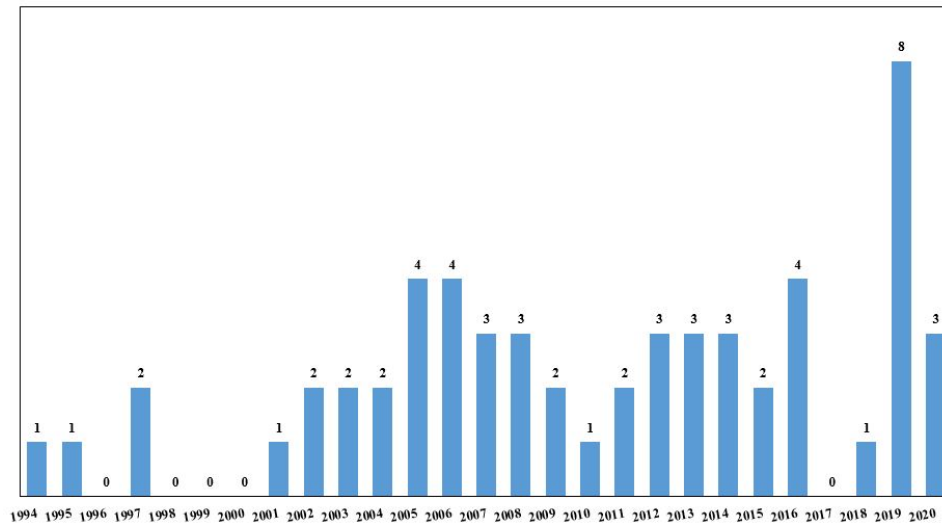


Fig. 2.1 Annual number of publications in disassembly lot sizing problem.

Disassembly Lot-Sizing Problem (DLSP) has some similarities with other problems such as the Lot-Sizing Problem in assembly (LSP), joint replenishment, and distribution problems. There is a possibility to apply the existed methods for the DLSP. The zero inventory property proposed in the LSP needs to be adapted for the basic DLSP (Kim et al. (2007)), and it is not optimal for other variants. In these cases, different MIP formulations can be investigated as for the LSP (Brahimi et al. (2017), and Brahimi et al. (2006)). The material flows in disassembly systems are divergent as in the distributions systems (Herer and Roundy (1997)). The resolution methods are however not transposable since in the DLSP, the material flows are connected concerning the product structure (it requires additional constraints). In the joint replenishment problem (Boctor et al. (2004)), a major setup (or order) cost is incurred to replenish several item inventories simultaneously as for the DLSP. The advantages of different MIP formulations for the joint replenishment problem is discussed in Gao et al. (2008) and several heuristics are proposed in (Boctor et al. (2004)). These approaches are not directly applicable to the DLSP: There are several independent demand streams for components, but they are linked according to the product structure (Brennan et al. (1994); Kim et al. (2007)). We contribute to extend and investigate them for the DLSP with disposal decisions.

Several researches in the literature present practical applications of the DLSP in various industries: valve company, where EOL valves are disassembled to meet the demands from their parts (Ji et al. (2016); Liu and Zhang (2018)), inkjet printers of a company in Switzerland, where used printers will be disassembled into their parts (Lee et al. (2002); Kim et al. (2003)), an automobile manufacturer that disassembles used engines and re-manufactures them to be reusable (Inderfurth et al. (2015)), a meat cut industry to decompose a product into multiple sub-products with different demand sources of each component (Piewthongngam et al. (2019)). We note that the models and methods proposed in this research not only can be applied to the mentioned above examples but also provide a significant reduction in inventory holding cost by considering different strategies. The End-of-Life Vehicles (ELVs) recycling industries can be a specific example of the application of models and methods proposed in this research. The ELVs will be disassembled into their parts and materials, such as plastics, engines, doors, tires, lamps, mirrors, and pistons, in order to satisfy their demand channels (such as repair services, second-hand markets, and energy recovery). All obtained parts are checked and re-manufactured to



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ensure their good conditions before satisfying their demand. The European Union (EU) directives required all ELVs to have a rate of reuse and recovery of no less than 95% by no later than January 1, 2015 (Directive 2000/53/EC). France officially has about 1 million vehicles to be recycled every year in the ELVs sector ([Indra automobile recycling \(1985\)](#)). Managing this huge amount of material and meeting the EU's target is a real challenge facing the automotive industry. This is because achieving a high recovery rate necessitates more efficient disassembly systems, but it increases disassembly operations as well as inventory holding costs. The proposed models and methods in this thesis are aimed especially at helping decision-makers to achieve their objectives more efficiently. In the following, we investigate EOQ model as another inventory control model with simple assumptions than lot-sizing models.

Table 2.1 Classification of researches in the DLSP according to modeling and algorithm

Literature	Structure				Product			Capacity			Uncertainty		SI decision	Models / Methods
	TL	ML	PC	NPC	SP	MP	U	C	D	S				
	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓				
Gupta and Taleb (1994)	✓			✓	✓		✓		✓			-	R-MRP	
Gupta and Taleb (1995)	✓		✓	✓	✓		✓		✓			-	R-MRP	
Taleb et al. (1997)	✓		✓		✓		✓		✓			-	R-MRP	
Taleb and Gupta (1997)	✓		✓			✓	✓		✓			-	R-MRP	
Lee and Xirouchakis (2002)	✓		✓	✓	✓		✓		✓			-	AGG, IP, H	
Lee et al. (2002)	✓		✓	✓	✓			✓	✓			-	AGG, IP	
Lee et al. (2003)	✓		✓	✓	✓		✓		✓			-	AGG, IP	
Kim et al. (2003)		✓	✓	✓	✓		✓		✓			-	AGG, IP, LPRH	
Lee et al. (2004)	✓		✓	✓	✓		✓		✓			-	AGG, IP	
Lee and Xirouchakis (2004)	✓		✓	✓	✓		✓		✓			-	AGG, IP, H	
Lee (2005)	✓		✓	✓	✓		✓		✓			-	AGG, IP	
Kim et al. (2005a)	✓		✓	✓	✓			✓	✓			-	AGG, NIF, IP, LRH	
Kim et al. (2005b)	✓		✓	✓	✓		✓		✓			-	AGG, IP, B&B, LRH	
Kim et al. (2005c)	✓		✓	✓	✓			✓	✓			-	AGG, IP, LRH	
Jeon et al. (2006)	✓		✓			✓	✓		✓			-	AGG, IP, H	
Kim et al. (2006a)	✓		✓		✓		✓		✓			-	AGG, IP, LRH	
Kim et al. (2006b)	✓		✓			✓	✓		✓			-	AGG, IP, H	
Kim et al. (2006c)	✓		✓			✓	✓		✓			-	AGG, IP, H	
Kim et al. (2008)	✓			✓	✓		✓		✓			-	AGG, IP, DP	
Gao and Chen (2008)		✓		✓	✓		✓		✓			-	AGG, IP, MH	
Barba-Gutiérrez et al. (2008)	✓		✓	✓	✓		✓		✓			-	R-MRP	
Barba-Gutiérrez and Adenso-Díaz (2009)	✓		✓	✓	✓		✓		✓		✓	-	R-MRP, F-RMRP	
Kim et al. (2009)	✓		✓	✓	✓		✓		✓		✓	-	AGG, IP, B&B, LRH	

Literature	Structure			Product			Capacity			Uncertainty			SI decision	Models /Methods
	TL	ML	PC	NPC	SP	MP	U	C	D	S	D	S		
	Kim and Xirouchakis (2010)	✓			✓		✓	✓				✓		
Kim and Lee (2011)		✓				✓	✓			✓			-	AGG, IP, H
Prakash et al. (2012)		✓				✓	✓				✓		-	AGG, IP, MH
Godichaud et al. (2015)	✓			✓		✓	✓			✓			Lost sales	AGG, IP, MH
Ji et al. (2016)		✓				✓	✓			✓			Purchasing	AGG, IP, LRH
Hrouga et al. (2016a)	✓			✓		✓	✓			✓			Lost sales	AGG, IP, H, MH
Hrouga et al. (2016b)	✓			✓		✓	✓			✓			Lost sales	AGG, IP, MH
Liu and Zhang (2018)	✓			✓		✓	✓			✓			-	MINLP, OA
Liu et al. (2019)	✓		✓			✓	✓			✓			-	MINLP, OA
Tian and Zhang (2019)	✓			✓		✓	✓			✓			-	NCMIP, MIP, MH
Slama et al. (2019b)	✓			✓		✓	✓						-	AGG, SBSP
Slama et al. (2020a)		✓				✓	✓			✓			Purchasing	AGG, MIP
Slama et al. (2020b)	✓			✓		✓	✓			✓			-	AGG, MC-MIP, SAA
<b>This study</b>	✓		✓			✓	✓			✓			Disposal	AGG, AGG-VIs, NIF, FAL, MIP, LPRH, H

Notes: **TL**: Two Levels; **ML**: Multi Levels; **PC**: Parts Commonality; **NPC**: No parts Commonality; **SP**: Single Product; **MP**: Multi products; **U**: Uncapacitated; **C**: Capacitated; **D**: Deterministic; **S**: Stochastic; **SI**: Surplus Inventory **AGG**: Aggregate formulation; **AGG-VIs**: AGG with Valid Inequalities; **FAL**: Facility Location-based formulation; **NIF**: AGG without inventory variable; **R-MRP**: Modified Material Requirements Planning; **MIP**: Mixed-Integer Programming; **MINLP**: Mixed-Integer-Nonlinear Programming; **NCMIP**: Non-Convex MIP; **IP**: Integer Programming; **DP**: Dynamic Programming; **B&B**: Branch and Bound; **LPRH**: Linear Programming Relaxation Heuristic; **LRH**: Lagrangian Relaxation Heuristic; **H**: Heuristics; **OA**: Outer Approximation; **MH**: Meta-Heuristics; **SBSP**: Scenario-Based Stochastic programming; **F-RMRP**: Fuzzy logic R-MRP; ; **MC-MIP**: Monte-Carlo-Mixed-Integer-Programming; **SAA**: Sample Average Approximation approach.

## 2.3 Disassembly economic order quantity

### 2.3.1 EOQ model

#### Basic EOQ model

Economic order quantity (EOQ) model is a classical inventory control model that can be considered as a simple heuristic for the uncapacitated lot sizing problem (Brahimi et al. (2017); Tomotani and de Mesquita (2018)). It is calculated based on average of setup and inventory holding costs over the planning horizon. EOQ models are often used as an interesting first step to obtain an optimal inventory policy of different systems where no inventory model is available (Godichaud and Amodeo (2018)). Classical EQO models are limited with assumptions and despite its wide applications in various contexts, there is a need for developing the new models and methods for other contexts as context of disassembly. For this reason, we investigate the existed EOQ models in the literature to find out the possibilities to develop for the context of disassembly with concentrating on the inventory holding costs aspect and making profit benefits in disassembly system.

Due to the simplicity of EOQ model and its adaptability to various applications, the well-known Economic Order Quantity (EOQ) is one of the most common inventory control model. However, the assumptions of the EOQ model are often constrained, and need to be expanded to improve its applicability to the current market climate. Studies with dealing with inventory consideration are mainly extended EOQ models in several directions. EOQ model as one of the oldest classical production scheduling models was first introduced and published by Ford Whitman Harris in 1913, but has been attributed to R. H. Wilson for its in-depth analysis (Erlenkotter (1989); Harris (1990); Kumar (2016)). It aims at determining the optimal quantity of the order allowing to minimize their frequency and to maximize the possible cost savings. Since its first appearance, different researchers have studied the EOQ models in different concepts. Many of them, specify EOQ model as the optimum quantity of order that balances minimal inventory holding costs with ordering costs. This means that the EOQ makes a simple inventory planning model with a trade-off between fixed ordering and inventory holding costs. The EOQ model can also be used when parameters of systems are uncertain which is often the case of reverse logistics field (Gou et al. (2008); Maddah and Noueihed (2017)). Godichaud and Amodeo (2018) mentioned that the EOQ model can be applied as an interesting first step to optimize the systems where no inventory model is available such as disassembly sector. Although the EOQ is a useful basis inventory model, its assumption such as uniform, constant and continuous demand, constant lead time, unlimited order size, placing cost order independent of the order size, and holding cost of a unit of stock independent of stock quantity, limit its application in real industrial cases. In this regard, various research are addressed the modifications and extensions with specified features to consider real cases issues (Khan et al. (2011); García-Alvarado et al. (2015); Godichaud and Amodeo (2018); Thinakaran et al. (2019)). The EOQ model is used in different area in the literature. In distribution systems where there are several stages supplied by only one upstream stage, Schwarz (1973) provides an optimal policy which is not necessary stationary and practicable. Joint replenishment problem is a multi-item EOQ problem with the same assumptions as the single-item EOQ. The only difference is there are two fixed ordering cost i.e., one specified to each item and one major fixed order cost associated with each order (for more details, see the review of Khouja and Goyal (2008)). When considering assembly system, several stages converge to the final product associated with one demand. Three optimal properties for

optimal policies are provided, stationary, nested, on order when zero inventory reached (Muckstadt and Roundy (1993); Godichaud and Amodeo (2018)).

Despite the widespread use of EOQ model and its extensions to model different problems in different areas, there is no much works done on remanufacturing systems specially when considering disassembly systems and their characteristics. There exist a research gap regarding inventory management models addressing specific characteristics of disassembly systems, in terms of handling excess inventory due to disassembly operations. Moreover, although recently limited number of works in the literature has been addressed EOQ model in disassembly systems, but the proposed optimal inventory policy cannot maximize profit. In this regard, it is necessary to investigate considering different strategies such as pricing and to develop inventory management models which guarantee higher profit while minimizing inventory costs. Next section reviews pricing decisions in ordinary EOQ models and evaluates its application for EOQ model in disassembly systems.

### EOQ pricing

In many real cases, item demands are price sensitive according to a mathematical function containing market potential and price elasticity as parameters. EOQ models with price sensitive demands have been considered in several papers, but we aim to show that, due to disassembly specificity, new models have to be proposed in the case of disassembly systems. The basic model, integrating pricing and inventory decisions under EOQ assumptions for a single item, is presented in Kunreuther and Richard (1971). The model is a maximization of mean profit per unit of time function, including revenues (function of price and demand), unit purchasing cost, inventory holding and order costs. Compared to sequential optimization, i.e. a pricing decision based on revenues and unit purchasing cost first, and then an inventory decision based on holding and order costs, the joint optimization results in higher profit. The pricing part of the problem is developed in Lau and Lau (2003) by considering several price-demand functions. The authors derive the optimal price for each function in the cases of single and serial stage systems. The model with pricing and inventory decision is analyzed in detail in Ray et al. (2005) with two widespread demand functions: the negative power of the price (also called iso-elastic function) and a linear function. The main result, based on the first and second order derivative of the profit function, is that the optimal price has the smallest value when the first derivative of the profit function is zero, assuming there is a contiguous price range where the profit function is positive. Mathematical sensitivity analysis is performed on all the parameters of the model to give managerial insights. A similar result is presented in Abad (1988) for the same demand function and the author integrates a quantity discount mechanism for the unit purchasing cost. Teksan and Geunes (2016) perform the same analytical approach as in Ray et al. (2005) for systems where the supply is made according to a price sensitive rate. The supply rate is an increasing function of the supply price, which is directly related to the sales price, as the supply rate must be equal to the demand rate. Recently, Adeinat and Ventura (2018) have discussed the problem of integrating pricing and lot-sizing decisions by considering price-dependent deterministic demand in order to maximize the profit per time unit in a serial supply chain. In disassembly systems, the prices of several items have to be considered simultaneously in the profit function with several additional constraints, and the single item models reviewed do not apply.

The single item EOQ model with pricing decision has been extended for multi-item and multi-stage problems. In multi-item problems, the item EOQ decisions are subject to linked constraints. In Cheng (1990) and Chen and Min (1994), they are linked according to an inventory limit constraint (investment

in storage space), and the authors derived the optimal decision. [Pal et al. \(2012\)](#) develops multi-item models with a price break level that links all the items. The first order condition gives a closed form for the optimal decision, but the second order condition can only be checked numerically on given examples. [Salvietti and Smith \(2008\)](#) propose a solution method for the multi-item economic lot-scheduling problem (EOQ like assumption except that all the items are produced at a given rate on a single facility), with pricing decisions and capacity constraint. In multi-stage problems, one upstream stage supplies one or many downstream stages which deal with the demands. The supplier sells the item to the retailers at a wholesale price, which is a decision variable like the sales price and the order quantity. In [Abad \(1994\)](#), the problem with one supplier and one retailer is analyzed to compare different bargaining schemes. The problem is also studied in [Weng \(1995\)](#) by considering quantity discount on the wholesale price. These works consider, as an example, the negative power of price demand function on the retailer. The problem is extended in [Viswanathan and Wang \(2003\)](#) by considering different discount schemes (on quantity or on volume). [Bernstein and Federgruen \(2003\)](#) consider the problem with one supplier and several retailers, where the retailers are in competition (the demand volume of one retailer affects the demand of the others). The demand function of each retailer is a linear function of his own price, but it is also a function of other retailers' prices (if the price of one retailer increases, the demands of other retailers increase). Different decision strategies are analyzed: centralized (all the decisions are optimized together) and decentralized (each retailer optimizes his profit function). For the decentralized problem, the results are not the same when considering the prices or the demands (Bertrand or Cournot competition) as decision variables. The conditions guaranteeing a unique equilibrium (optimal price or demand) are proposed. The disassembly case differs from the ones mentioned here due to the different coordination between items. In the disassembly problem addressed in this thesis, all the component inventories are replenished simultaneously from a single source at each operation, while the demands remain independent.

The model with EOQ assumptions and pricing decision has been extended by taking into account perishable products (or subject to deterioration) and partial backorders (customers agree to wait with respect to a function of the waiting time). The model is presented assuming general assumptions for the demand function of the price in [Abad \(1996\)](#) and [Abad \(2008\)](#) and, in [Dye et al. \(2007\)](#) and [Papachristos and Skouri \(2003\)](#), for the same model with quantity discount. Linear and iso-elastic demand functions are used in examples. A different demand function for this model is proposed in [Sana \(2010\)](#). We note that the most used demand functions are the linear and iso-elastic functions, which are good compromise between real case representations and computational efficiency (ease of finding optimal solutions). Some researchers consider both disposal rate and pricing decision, as in [El Saadany and Jaber \(2010\)](#) and [El Saadany and Jaber \(2011\)](#). The authors analyse a production and remanufacturing system with the return rate as a function of EOL product price and quality level. They propose new models to help firms find an optimal policy between pure manufacturing, pure remanufacturing and mixed strategy. However, in the problem we address in this thesis, the demands for components are independent and can be sold to different recovery channels (material recycling, spare part markets as well as remanufacturing processes).

Table 2.2 summarizes the researches which consider EOQ models in different area as inventory control policy and it classifies different strategies applied in order to make inventory systems cost saving and more profitable.

Table 2.2 Classification of the researches considering EOQ models

Literature	System				Product			Level (Stage)			Echelon			Model	ICD
	Asse	Disa	P-Re	Dist	JR	S	M	T	M	O	T	M			
Kunreuther and Richard (1971)				✓		✓						✓		PM	Pri
Abad (1988)				✓		✓						✓		PM	Pri
Cheng (1990)							✓						✓	PM	Pri
Richter (1996)			✓			✓								CM	Disp
Lau and Lau (2003)				✓		✓						✓		PM	Pri
Ray et al. (2005)				✓		✓						✓		PM	Pri
Jaber and El Saadany (2009)			✓			✓								CM	Disp, LS
El Saadany and Jaber (2010)			✓			✓								CM	Disp, Pri
Teksan and Geunes (2016)	✓					✓			✓					CM	Disp, Pri
Adeinat and Ventura (2018)						✓								PM	Pri
Godichaud and Amodeo (2018)						✓							✓	CM	Disp
Godichaud and Amodeo (2019a)						✓							✓	CM	Disp, LS
Godichaud and Amodeo (2019b)						✓							✓	CM	Disp, LS
This study						✓							✓	PM	Disp, Pri

**Asse:** Assembly; **Disa:** Disassembly; **P-Re:** Production and Repair or Remanufacturing; **Dist:** Distribution; **JR:** Joint Replenishment; **S:** Single; **M:** Multi; **T:** Two; **ML:** O: One; **PM:** Profit Maximization; **CM:** Cost Minimization; **Disp:** Disposal; **LS:** Lost Sales; **Pri:** Pricing; **ICD:** Inventory Cost-saving Decision.

### 2.3.2 Disassembly EOQ model with disposal

Disassembly scheduling with inventory consideration can be addressed under EOQ-like assumptions. It comes down to a lot-sizing problem but with continuous time, infinite planning horizon and constant parameters. Historically, EOQ seems to be the oldest and the most widespread lot-sizing problem, but in disassembly it came after the variant with discrete and time varying demands. The main specificity of disassembly economic order quantity (DEOQ) is presented in [Godichaud and Amodeo \(2018\)](#). The problem is to determine an optimal inventory policy for disassembly systems by considering disassembly and inventory costs. Due to specific characteristics of disassembly systems, stationary policy cannot be obtained. The reason is that disassembly operations of EOL products leads to an excess inventory of components which accumulates over continue periods. The disposal decision is applied to obtain a stationary policy which corresponds zero-inventory property. The authors show that if no decision is considered for handling surplus inventory, stationary policies cannot be found and the model is difficult to apply in practice. Disposal decisions are then considered in the model, and the authors propose solution approaches to find the optimal disassembly reorder interval.

They consider a single stage disassembly systems where an EOL product will be disassembled to their components with different yields. The problem is modeled as the total cost per unit time to determine the reorder interval between two disassembly operations of EOL product while satisfying the demands for components (Let  $i = 1 \cdots N$  be the set of components for a given EOL product (0)) and minimizing the sum of setup and inventory holding costs, and The excess inventory of components will be disposed of. Let  $\bar{R}_i$  be the quantity of excess inventory of components  $i$  accumulating over cycles and  $G$  is the component with maximum ratio of demand/yield among all the components.  $\bar{R}_i$  for  $i \neq G$  is equal to  $T_0 \left( a_i \frac{d_G}{a_G} - d_i \right)$ . Where  $T_0$ ,  $a_i$ ,  $d_i$  represent reorder interval for disassembling EOL product 0, yield of component  $i$ , and demand of component  $i$ , respectively. It cleat that the mean disposal quantity ( $R_i = \frac{\bar{R}_i}{T_0}$ ) in independent from  $T_0$ . So, the model can be presented as (2.1) which represents the mean cost per unit time and has the same form as the basic EOQ model:

$$C(T_0) = \frac{k_0}{T_0} + \sum_{i=1}^N \frac{1}{2} h_i d_i T_0 \quad (2.1)$$

We can see that by derivation of equation (2.1) we can obtain the optimal reorder interval  $T_0^* = \left( \frac{2k_0}{\sum_{i=1}^N h_i d_i} \right)^{1/2}$ . So optimal quantity of EOL product to be disassembled is  $Q^* = \left( \frac{d_G}{a_G} \right) T^*$  with the optimal total cost ( $TC^*$ ) which can be calculated by adding the other costs which are independent from  $T^*$  as follow:

$$TC^* = c_0 \left( \frac{d_G}{a_G} \right) + \sum_{i=1}^N r_i \left( a_i \frac{d_G}{a_G} - d_i \right) + \sqrt{2k_0 \sum_{i=1}^N h_i d_i} \quad (2.2)$$

Where  $c$  and  $r_i$  represent disassembly operations cost and disposal cost of component  $i$ , respectively. The work is extended in [Godichaud and Amodeo \(2019a\)](#) for considering both disposal and lost sales decisions. The results show that the proposed models can determine inventory policies which allow surplus inventories to be avoided. Recently, [Godichaud and Amodeo \(2019b\)](#) have proposed several DEOQ models for the problem with variant stockouts policies, such as full backorders, full-lost sales and partial backorders. In this thesis, we propose another variant to handle surplus inventory in EOQ



models. In fact, the equation (2.2) cannot guarantee profit maximization of an inventory policy. If the demands can be varied with respect to the price, they can be adjusted to avoid the surplus inventory, and help firms determine profit-maximizing policies integrating both disposal and pricing decisions.

## 2.4 Conclusion

In this chapter a brief review on disassembly problem, different inventory management models to be applied in disassembly systems (lot sizing and EOQ) and solution methods and approaches are presented. In this chapter a brief review on disassembly lot-sizing problems, disassembly EOQ models, their variant, and solution methods is presented. According to the literature done, the number of papers considered disassembly assumptions in the existed inventory models are limited. In addition, application of inventory management models is a complex task because of existing assumptions. This particularly true for the case of inventory policy in disassembly systems. At first, lot sizing model and its variants are reviewed, and the possibility of application of existed models and solutions methods in disassembly systems to handle surplus inventory is reported. Secondly, as a basic inventory model, EOQ models with considering different inventory policy are presented. Next chapter focuses on the problem of single product disassembly lot sizing with considering decisions on surplus inventory and proposes the relevant models and resolutions methods.



## Chapter 3

# Single-Product Disassembly Lot Sizing Problem With Two-Level Disassembly Structure

### 3.1 Introduction

The disassembly process can be defined as a methodical separation of valuable or hazardous materials/components from EOL products to be allocated to a recovery channel such as recycling, repair services, second-hand markets, and energy recovery. It aims to reduce environmental impacts and energy consumption. The disassembly process has attracted significant interest due to its important role in recovery processes of used or end-of-life (EOL) products. Disassembly process planning is very challenging for the companies because of existing different costs associated to disassembly operations. It is necessary to develop new optimal methods and models in order to require economic justification, especially for practitioners. We consider demand-driven disassembly systems where a specific quantity of EOL products will be disassembled into their components in order to satisfy their demands ([Langella \(2007\)](#)). These disassembly systems have some specific characteristics, unlike assembly systems, so that make optimal policies cannot be found if no optimal inventory policy is applied and this is complicated to apply in practice; 1) The product diverges into multiple demand channels of components 2) The demands for the components are independent for each others and that they are not necessarily well balanced, 3) In addition, disassembly operation generates all the components, simultaneously. These characteristics of disassembly lead to generate an excess inventory over the planning horizon, we call the surplus inventory. Several inventory management models have been suggested in the literature to manage the inventory towards minimizing their associated costs. This chapter focuses on lot sizing models in disassembly systems. New models and methods need to be developed to handle the surplus inventory, and it is necessary to extend the existed models in the related literature towards increasing their applicability to the modern-day business environment. Among different problem studied in the literature of disassembly planning, we consider Disassembly lot-sizing problem (DLSP) which consider inventory aspects. Several strategies are possible to reduce surplus inventory in the DLSP: allowing lost sales, external purchasing, disposal or balancing the demands by applying pricing. We apply disposal

decisions in the DLSP in order to make opportunities of inventory holding cost reduction in disassembly systems.

Disassembly Lot Sizing Problem (DLSP) can be defined as the problem of determining the timing and the quantity of disassembling EOL products in order to satisfy the demand for their components. This disassembly planning can be described as how many, and when to disassemble EOL products in order to fulfill the demand channels for the components. According to the classification scheme provided in Chapter 2, the previous literature on disassembly lot sizing can be classified with respect to different aspects: product structure, resource capacity restrictions, decisions on the management of surplus inventory, models and solution approaches. This chapter studies the problem of disassembly lot sizing with disposal decisions (DLSPD) to manage surplus inventory and to make opportunities of cost saving in disassembly systems. We focus on the case of single product type without capacity constraint and a two-level product structure is considered. We start by considering the basic case of the problem i.e., single product type without capacity constraint and a two-level product structure. It can be considered as a problem in itself but also as a sub-problem in the solution procedures of many complex DLSP (such as capacitated multi-product problems). New models are developed and analyzed to solve the problem efficiently specially for the practitioners. We highlight the contributions of this chapter as follows:

- A procedure is presented to compute surplus inventory for the disassembly lot-sizing problem without disposal decisions.
- Three different MIP formulations are proposed for the disassembly lot-sizing problem which consider disposal decisions. The formulations differ from each other concerning the quality of the lower bound provided by their linear relaxation, which is an important issue in MIP resolution methods.
- Two efficient heuristics are developed with different inventory management strategies to solve real case applications when MIP approaches are not relevant.
- A comparative study of the formulations and the heuristics is presented based on new benchmark instances for the disassembly lot-sizing problems.

Section 3.2 presents the problem considered in this chapter. First, the issue of surplus inventory accumulations in disassembly planning is analyzed and a new procedure is proposed to compute different types of surplus inventories. We also highlight the potential inventory cost savings. By using the disassembly lot-sizing models that do not consider any decision for the surplus inventory, we show that when a surplus inventory is generated in one period, it can be either consumed in later periods or stored until the end of the planning horizon. Section 3.3 provides different formulations for the single product Disassembly lot sizing problem with disposal decisions to handle surplus inventory. Two efficient heuristics with different strategies are proposed to handle surplus inventory accumulation, which can be applied especially in practice for large-sized problems. The test results on new randomly generated instances are reported in Section 3.4. A sensitivity analysis of the important parameters and their impacts on the models and methods are presented and managerial insights are discussed. Finally, Section 3.6 concludes this chapter with a summary and an overview of future research.

## 3.2 Problem description

Two-level product disassembly structure is considered in this chapter. This is a basic product structure in disassembly which means there is a direct connection between the EOL product to be disassembled and its components after being disassembled without any intermediate sub-assembly. An example of the structure is given in Fig. 3.1. The number in parenthesis is the yield of related leaf item when one unit of root item (0) is disassembled. The first level represents leaf items (1, 2, 3, and 4), while the second level represents a single root item or EOL product (0).

A new single product disassembly lot sizing problem with disposal decisions (DLSPD) on surplus inventory is studied in this section. The DLSPD can be defined as follows: For a single-type EOL product, determining the quantity and timing of disassembling the EOL product in order to satisfy the demand for the components over a finite planning horizon, while the unnecessary surplus inventory of components can be disposed of the disassembly system. The objective is to minimize the overall cost, which consists of setup, disassembly operations, inventory holding costs.

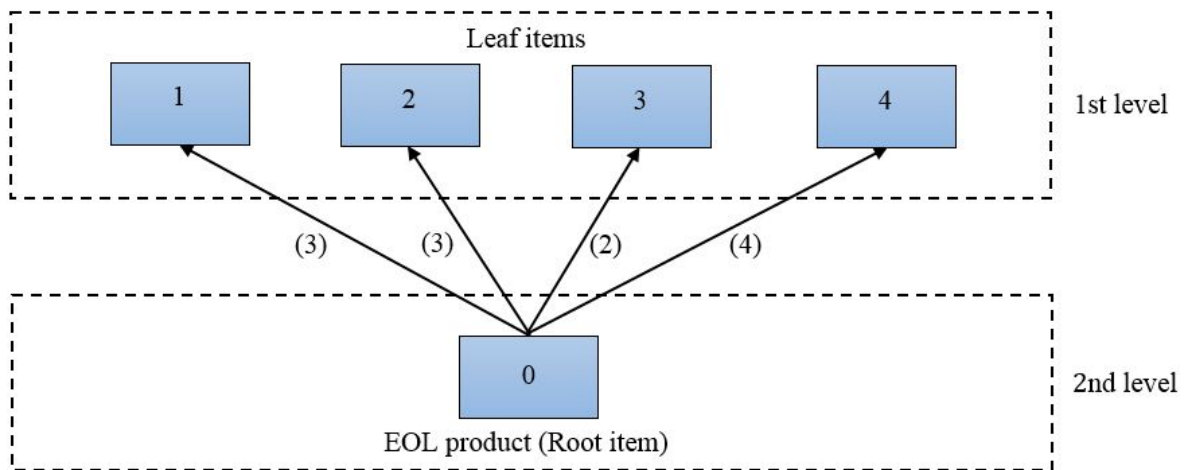


Fig. 3.1 Example of a two-level product structure.

The assumptions made in this chapter are summarized as follows:

- a) Disassembly structure of the EOL product is given.
- b) The EOL product can be obtained whenever ordered and there is no holding cost for it.
- c) Disassembly lead times are not considered, they are assumed to be negligible.
- d) Backlogging and lost sales are not allowed and demands should be satisfied on time.
- e) The demand for the components are given and deterministic.
- f) The disassembled components are of equal quality.
- g) The initial stock of the root and leaf items are considered zero.
- h) There is no disposal cost for the unnecessary leaf items.

The following notations and parameters are used in this chapter:

- $i$ : index for the leaf items,  $i = 1, 2, \dots, N$ .
- $t$ : index for the periods,  $t = 1, 2, \dots, T$  ( $T$  is the planning horizon).
- $M_t$ : arbitrary large number considered in period  $t$ .
- $s_t$ : setup cost of the root item (EOL product) in period  $t$ .
- $p_t$ : disassembly operation cost of the root item (EOL product) in period  $t$ .
- $a_i$ : number of units of leaf item  $i$  obtained from disassembling one unit of the root item (EOL product).
- $I_{i0}$ : initial inventory of leaf item  $i$ .
- $h_{it}$ : inventory holding cost of leaf item  $i$  in period  $t$ .
- $d_{it}$ : demand of leaf item  $i$  in period  $t$ .
- $D_{ikt}$ : cumulative demand of leaf item  $i$  from period  $k$  to  $t$ .

The decision variables for the disassembly lot sizing problem are defined as follows:

- $Y_t$ : 1 if there is a setup in period  $t$ , and 0 otherwise.
- $X_t$ : disassembly quantity of the root item (EOL product) in period  $t$ .
- $I_{it}$ : inventory level of leaf item  $i$  at the end of period  $t$ .

**Example data:** Table 3.1 illustrates a numerical example of the parameters for an EOL product such as an EOL vehicle with the disassembly structure presented in Fig. 3.1. Note that  $a_1 = 3$ ,  $a_2 = 3$ ,  $a_3 = 2$ , and  $a_4 = 4$  are considered as the yield of leaf items (e.g., lamps, tires, mirrors, and pistons, respectively) from the EOL vehicle. The objective is to determine the quantity and timing of disassembling EOL vehicles in the ELVs recycling center, while satisfying the demand for the components lamps, tires, mirrors, and pistons over the ten-period planning horizon (N.B., The parameter setting of our benchmark presented in Section 3.4.1 is used to generate data example for the single-product DLSP).

Table 3.1 A data set for the single-product DLSP

	Planning period (t)									
	1	2	3	4	5	6	7	8	9	10
$d_{1t}$	191	161	86	114	160	196	121	147	191	191
$d_{2t}$	136	119	124	129	183	180	66	150	175	194
$d_{3t}$	144	82	91	136	123	81	169	83	166	149
$d_{4t}$	117	132	141	160	135	170	185	62	62	90
$s_t$	24422	21547	21774	23773	23321	23852	21154	23717	23377	22960
$p_t$	84	64	93	76	57	55	96	85	70	67
$h_{1t}$	6	5	9	8	9	10	7	5	9	8
$h_{2t}$	6	9	10	7	8	7	7	6	8	9
$h_{3t}$	10	6	5	6	8	7	7	6	5	8
$h_{4t}$	8	6	8	6	10	8	5	6	6	5

### 3.2.1 Problem without surplus inventory decisions

Most of previous research on disassembly scheduling have been modeled the problem as an Integer Programming (IP) formulation. Their objective is either to find probably optimal solutions or to obtain near-optimal solutions with a measurable performance guarantee (i.e., percentage deviation from optimal value). Two different formulations which consider the costs associated with disassembly process are existed in the literature for the single-product disassembly lot sizing problem as follows:

#### Aggregate formulation (AGG)

A classical formulation which is called aggregate formulation (AGG) is given blow:

$$[\mathbf{P}_a] \text{ Min } \left\{ \sum_{t=1}^T s_t \cdot Y_t + \sum_{t=1}^T p_t \cdot X_t + \sum_{i=1}^N \sum_{t=1}^T h_{it} \cdot I_{it} \right\} \quad (3.1)$$

Subject to

$$I_{it} = I_{it-1} + a_i \cdot X_t - d_{it} \quad \forall i = 1, \dots, N \ \& \ t = 1, \dots, T \ (I_{i0} = 0) \quad (3.2)$$

$$X_t \leq M_t \cdot Y_t \quad \forall t = 1, \dots, T \quad (3.3)$$

$$X_t \geq 0 \ \& \ \text{integer} \quad \forall t = 1, \dots, T \quad (3.4)$$

$$I_{it} \geq 0 \ \& \ \text{integer} \quad \forall i = 1, \dots, N \ \& \ t = 1, \dots, T \quad (3.5)$$

$$Y_t = 0 \ \text{or} \ 1 \quad \forall t = 1, \dots, T \quad (3.6)$$

In this formulation, objective function (3.1) minimizes the sum of setup, disassembly operation, and inventory holding costs over the planning horizon. Constraints (3.2) are the inventory balance equations for the leaf items. They express that the entering stock ( $I_{it-1}$ ) added to the current period disassembly quantity ( $a_i X_t$ ) are used to fulfill the demand of current period ( $d_{it}$ ). The remains are kept in stock at end of period ( $I_{it}$ ). Note that the inventory balance constraint for the root item is not considered since its excess inventory leads to an unnecessary cost increase (Kim et al. (2007)). Constraints (3.3) guarantee that a setup cost is performed in period  $t$  if any disassembly operation is done in that period. Constraints (3.4–3.6) impose the non-negativity and binary restrictions on the variables.

#### Aggregate formulation without inventory variable (NIF)

Above original formulation can be reformulated as another integer program that can be applied in solution methods such as the Lagrangean relaxation heuristic algorithm. By replacing the inventory variables of the original formulation using equations (3.7), we can obtain AGG formulation without the inventory variables (NIF formulation):

$$I_{it} = \sum_{k=1}^t a_i \cdot X_k - D_{i1t} \quad \forall i = 1, \dots, N \ \& \ t = 1, \dots, T \ (I_{i0} = 0) \quad (3.7)$$

Let  $c_t = \sum_{k=t}^T \sum_{i=1}^N h_{ik} a_i$  and  $C = \sum_{t=1}^T \sum_{i=1}^N h_{it} D_{i1t}$ . The NIF formulation for the single DLSP can be defined as follows:

$$[\text{P}_b] \quad \text{Min} \left\{ \sum_{t=1}^T s_t \cdot Y_t + \sum_{t=1}^T (p_t + c_t) \cdot X_t - C \right\} \quad (3.8)$$

Subject to 3.3, 3.4, 3.6, and

$$\sum_{k=1}^t a_i \cdot X_k \geq D_{i1t} \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.9)$$

### Dynamic programming algorithm

Some studies in the literature of the disassembly lot sizing transformed the AGG model into an exact algorithm based on the dynamic programming model which can obtain the optimal solutions in polynomial time. To determine the optimal solution of the entire problem, the problem is decomposed into  $T$  sub-problem from 1 to  $T$ . Each sub-problem from the decomposition is solved recursively, starting from the 1-period and ending with the  $T$ -period. Kim (2005) represents the dynamic program as follow:

$$[\text{DP1}] \quad F(t) = \min_{1 \leq j \leq t} \left\{ \sum_{i=1}^N \sum_{k=j}^t h_{ik} \cdot I_{ik} + p_j \cdot \left( \max_{i=1 \dots N} \left\{ \left\lceil \frac{\sum_{k=j}^t d_{ik} - I_{ij-1}}{a_i} \right\rceil \right\} \right) + s_j + F(j-1) \right\} \quad (3.10)$$

Where  $F(0) = 0$  and  $F(t)$  represents the optimal total cost function for period 1 through  $t$ . This is a recursive cost function which consist of the inventory holding costs of all leaf items from period setup period  $j$  to period  $t$ , the setup cost of the root when the setup occurred in period  $j$ , and the optimal cost function for period 1 through  $j-1$ . item.

As mentioned before, specific characteristics of disassembly systems result in surplus inventories of the leaf items. The existed models and methods cannot handle this issue. We obtain the optimal solution of the data example presented in Table 3.1 by solving model  $P_a$  (AGG without decisions on the surplus inventory) by using CPLEX solver. The results are provided in Table 3.2. It requires that 159, 171, 126, and 157 units of EOL product (root item) should be disassembled in periods 1, 4, 7, and 9, receptively. Unlike the lot sizing problem for assembly systems, the well-know zero inventory property of Wagner and Whitin (1958) does not hold in the disassembly lot sizing. This property in assembly systems implies that the optimal solution satisfies  $I_{t-1} \cdot X_t = 0$  for all  $t$ . As can be seen in Table 3.2, in disassembly systems, there may exist an optimal solution such that  $I_{t-1} \cdot X_t > 0$  so that the zero inventory property (in assembly) does not hold. This disassembly planning leads to a huge surplus inventory of 278, 380, and 1194 units of leaf item 1, 2, and 4, respectively, at the end of the planning horizon (period 10). These unnecessary inventories can be disposed of the disassembly center to be recycled as a material source or an energy recovery. If no decisions are considered to handle the surplus inventory, optimal inventory policies cannot be found, and it will be difficult to apply in practice.



Table 3.2 Optimal solutions obtained by model  $P_a$  (without surplus inventory decisions)

	Planning period (t)									
	1	2	3	4	5	6	7	8	9	10
$X_t$	159	0	0	171	0	0	126	0	157	0
$I_{1t}$	286	125	39	435	275	79	336	189	469	<b>278</b>
$I_{2t}$	341	222	98	479	296	116	428	278	574	<b>380</b>
$I_{3t}$	174	92	1	205	82	1	84	1	149	<b>0</b>
$I_{4t}$	519	387	246	766	631	461	780	718	1284	<b>1194</b>
Optimal objective function = <b>235947</b>										

In this chapter, disposal decisions are included in the disassembly lot sizing problem to handle the surplus inventory. Also, the new efficient models and methods are suggested which make opportunities for cost saving, especially in real-industrial cases. In the following, the issue of surplus inventory accumulation in disassembly systems is analyzed and a procedure is proposed to compute different surplus inventories.

### 3.2.2 Surplus inventory analysis

In demand-driven disassembly systems, EOL product will be disassembled into their components to satisfy their demands. But, one disassembly operation on an EOL product simultaneously generates all its components with respect to their yield. The demands for the components are independent, and an unnecessary surplus inventory can be generated. This surplus inventory is identical for all solutions (i.e., whatever the lot size) and can be computed in advance. One part of this surplus inventory, generated in one period, is consumed in future periods while the other part is stored until the end of the planning horizon. For the first part, the decision is either to store it until its consumption or to dispose of it while increasing disassembly quantities in a next period. For the second part, it can be disposed of as soon as it is obtained to save inventory holding costs. We aim to calculate this non-used surplus inventory of leaf items and provide a disassembly planning with disposal decision and economic balance.

Several additional notations are used to calculate surplus inventory:

- $b_t$ : quantity of the root item (EOL product) to satisfy the demand of leaf items in period  $t$
- $B_t$ : required quantity of the root item (EOL product) to satisfy the cumulative demand for all leaf items from period 1 to  $t$
- $R_{it}$ : amount of obtained leaf item  $i$  in period  $t$  after satisfying its demand by disassembling  $b_t$  units of the root item (EOL product)
- $w_{ijt}$ : quantity of leaf item  $i$  generated in period  $j$  to be consumed in period  $t$  ( $j \leq t$ )
- $S_{it}^I$ : surplus inventory of leaf item  $i$  in period  $t$
- $S_{it}^c$ : consumed surplus inventory of leaf item  $i$  in period  $t$
- $S_{it}^e$ : non-used surplus inventory of leaf item  $i$  in period  $t$

Equations (3.11) state the quantity of the root item (EOL product) in period  $t$  needed to satisfy the cumulative demand of all leaf items from period 1 to  $t$ :

$$B_t = \max_{i=1, \dots, N} \left\{ \left\lceil \frac{\sum_{k=1}^t d_{ik}}{a_i} \right\rceil \right\} \quad \forall t = 1, \dots, T \quad (3.11)$$

Where  $\lceil \bullet \rceil$  denotes to the smallest integer no less than  $\bullet$ . The amount of surplus inventory of leaf items at the end of period  $t$  can be calculated using equations (3.12):

$$S_{it}^I = a_i \cdot B_t - \sum_{k=1}^t d_{ik} \quad \forall i = 1, \dots, N, t = 1, \dots, T \quad (3.12)$$

This surplus inventory consists of two categories: consumed and non-used surplus inventory. Consumed surplus inventory is the part of the surplus inventory that will be used to satisfy the demands of future periods. Non-used surplus inventory will be held and accumulated during planning periods. Consumed surplus inventory can be computed by using the following original procedure:

**Step 1:** Compute the quantity of root item (EOL product) to fulfill the demand of all leaf items in period  $t$  by using equations (3.13) (note that  $B_0 = 0$ ):

$$b_t = B_t - B_{t-1} \quad \forall t = 1, \dots, T \quad (3.13)$$

**Step 2:** Calculate the amount of obtained leaf item  $i$  in period  $t$  by using equations (3.14):

$$R_{it} = a_i \cdot b_t - d_{it} \quad \forall i = 1, \dots, N, t = 1 \dots T \quad (3.14)$$

Note that  $R_{it} < 0$  means that we obtained the leaf items from the last periods with  $R_{ij} > 0$  ( $j < t$ ).

**Step 3:** Compute the values of  $w_{ijt}$  for all  $i = 1, \dots, N$ ,  $t = 1, \dots, T$ , and  $j \leq t$  using Algorithm 1, by inspection of each period starting from period  $T$  to 1. The objective is to find the first period with  $R_{it} < 0$ , then find the first  $R_{ij} > 0$  ( $j < t$ ) to satisfy the negative one (note that  $R_{i1} \geq 0$ ;  $\forall i = 1, \dots, N$ ).

**Step 4:** Calculate the amount of the consumed surplus inventory of leaf item  $i$  in period  $t$  by using equations (3.15):

$$S_{it}^c = \sum_{j=1}^t \sum_{k=t+1}^T w_{ijk} \quad \forall i = 1, \dots, N, t = 1, \dots, T \quad (3.15)$$

Then, we can calculate the amount of the non-used surplus inventory of leaf item  $i$  by using equations (3.16). This implies that this part of surplus inventory will be held while it is unnecessary and may incur considerable inventory holding costs:

$$S_{it}^e = S_{it}^I - S_{it}^c \quad \forall i = 1, \dots, N, t = 1, \dots, T \quad (3.16)$$

---

**Algorithm 1** Computing  $w_{ijt}$  for all  $i = 1, \dots, N, t = 1, \dots, T, j \leq t$

---

```

1: Input:  $i$ : item,  $t$ : period,  $R_{it}$ : obtained by Step 2
   Output: the value of  $w_{ijk}$ , the updated value of  $R_{it}$ 
2:  $i = 1$ 
3: while  $i \leq N$  do
4:    $t = T$ 
5:   while  $t > 1$  do
6:      $j = t - 1$ 
7:     while  $R_{it} < 0$  do
8:       if  $R_{ij} \leq 0$  then
9:          $j = j - 1$ 
10:      else
11:         $w_{ijt} = \min\{R_{ij}, -R_{it}\}$ 
12:         $R_{it} \leftarrow (R_{it} + w_{ijt})$  &  $R_{ij} \leftarrow (R_{ij} - w_{ijt})$ 
13:      end if
14:    end while
15:     $t = t - 1$ 
16:  end while
17:   $i = i + 1$ 
18: end while

```

---

Data example presented in Table 3.1 is used to illustrate an application of the procedure. Table 3.3 provides the obtained value of  $b_t$ ,  $R_{it}$ , and  $w_{ijt}$  for leaf item 1 of the tested example by using equations (3.13), (3.14), and Step 3 (Algorithm 1), respectively.  $w_{ijt}$  expresses quantity of leaf items  $i$  generated in period  $j$  to be consumed in period  $t$  ( $j \leq t$ ).

We start from the period  $T$ . We find first  $R_{it} < 0$ , then we find first period  $k$  ( $k < t$ ) with  $R_{ik} > 0$ . For example, for leaf item 1,  $R_{1,8}$  is negative (-21) and  $R_{1,7}$  is positive (131) as the first period before 8. We set  $w_{1,7,8} = 21$ , Then we update the values ( $R_{1,8} = 0$  and  $R_{1,7} = 131 - 21 = 110$ ). We apply this procedure for all leaf items until either there is no more  $R_{it} > 0$  or we attained period 1. It is clear that the sum of  $w_{1,j,t}$  for all  $j = 1, \dots, 10$  for a given period  $t$  is equal to  $d_{1,t}$ . For example,  $w_{1,4,6} + w_{1,5,6} + w_{1,6,6} = 50 + 23 + 123 = 196$  i.e.,  $d_{1,6}$ . This implies all the demands for components should be satisfied.

Table 3.4 presents the value of surplus, consumed, and non-used inventories calculated by equations (3.12), (3.15), and (3.16), respectively. It is clear that the sum of consumed and non-used is equal to the surplus inventory. Figure 3.2 represents visually the surplus inventory ( $S_{it}^I$ ) accumulation (Table 3.4) and the optimal inventory of leaf items (Table 3.2) during ten-period planning horizon. As an example, as can be seen in Fig. 3.2, for leaf item 2, it is possible to dispose of 80 units of inventory from period 1 to the end of the ten-period planning horizon (or 116 units from period 4 to the end). This quantity is a non-used surplus inventory ( $S_{it}^e$ ) and its accumulation results in an excess amount of inventory at the end of planning ( i.e., 380 units). There is an opportunity of cost saving by disposing of all these non-used surplus inventories:  $\sum_{i=1}^N \sum_{t=1}^T h_{it} S_{it}^e = 58351$  i.e., at least around 25% cost saving in total cost obtained in Table 3.2 (i.e.,  $\frac{58351}{235947} * 100 = 24.73\%$ ) in this example. We also observe that surplus inventories are identical in all solutions and it is precomputed. The existed models and methods in the literature do not consider this problem of disassembly lot sizing and they are not applicable. Disposal decisions can be applied to manage this surplus inventory.

Table 3.4 Surplus inventories values obtained by using equations (1-6) and Algorithm 1.

		T									
		1	2	3	4	5	6	7	8	9	10
$B_t$		72	118	159	227	288	329	413	455	538	612
Surplus( $S_{it}^I$ )	$S_{1t}^I$	25	2	39	129	152	79	210	189	247	<b>278</b>
	$S_{2t}^I$	80	99	98	173	173	116	302	278	352	<b>380</b>
	$S_{3t}^I$	0	10	1	1	0	1	0	1	1	<b>0</b>
	$S_{4t}^I$	171	223	246	358	467	461	612	718	988	<b>1194</b>
Consumed( $S_{it}^c$ )	$S_{1t}^c$	23	0	0	50	73	0	21	0	0	0
	$S_{2t}^c$	0	1	0	57	57	0	24	0	0	0
	$S_{3t}^c$	0	10	1	1	0	1	0	1	1	0
	$S_{4t}^c$	0	0	0	0	6	0	0	0	0	0
Non-used( $S_{it}^e$ )	$S_{1t}^e$	<b>2</b>	2	39	79	79	79	189	189	247	<b>278</b>
	$S_{2t}^e$	<b>80</b>	98	98	116	116	116	278	278	352	<b>380</b>
	$S_{3t}^e$	<b>0</b>	0	0	0	0	0	0	0	0	<b>0</b>
	$S_{4t}^e$	<b>171</b>	223	246	358	461	461	612	718	988	<b>1194</b>

Table 3.3 Value of  $b_t$ ,  $R_{it}$ , and  $w_{ijt}$  for leaf item 1 of the example data

	Planning period (t)									
	1	2	3	4	5	6	7	8	9	10
$B_t$	72	118	159	227	288	329	413	455	538	612
$b_t$	72	46	41	68	61	41	84	42	83	74
$R_{1,t}$	25	-23	37	90	23	-73	<b>131</b>	<b>-21</b>	58	31
$w_{1,1,t}$	191	23	0	0	0	0	0	0	0	0
$w_{1,2,t}$		138	0	0	0	0	0	0	0	0
$w_{1,3,t}$			86	0	0	0	0	0	0	0
$w_{1,4,t}$				114	0	<b>50</b>	0	0	0	0
$w_{1,5,t}$					160	<b>23</b>	0	0	0	0
$w_{1,6,t}$						<b>123</b>	0	0	0	0
$w_{1,7,t}$							121	21	0	0
$w_{1,8,t}$								126	0	0
$w_{1,9,t}$									191	0
$w_{1,10,t}$										191
$d_{1,t}$	<b>191</b>	<b>161</b>	<b>86</b>	<b>114</b>	<b>160</b>	<b>196</b>	<b>121</b>	<b>147</b>	<b>191</b>	<b>191</b>

The removal of non-used surplus inventory saves inventory holding costs. For the consumed surplus inventory, it can be stored until its consumption or disposed of but it changes the disassembly quantities of future periods and, hence, lot sizes. To determine the best decisions, new models and methods are proposed in the following sections.

We note that the value of surplus inventory ( $S_{it}^I$ ) (which is precomputed) can be used to calculate a more appropriate value of big M in constraints (3.13) of model  $P_a$  by using the following formula (3.17). This improves the lower bound of the problem without disposal:

$$M_t = \max_{i=1 \dots N} \left\{ \left\lceil \frac{\sum_{k=t}^T d_{ik} - S_{it-1}^I}{a_i} \right\rceil \right\} \quad \forall t = 1 \dots T \quad (3.17)$$

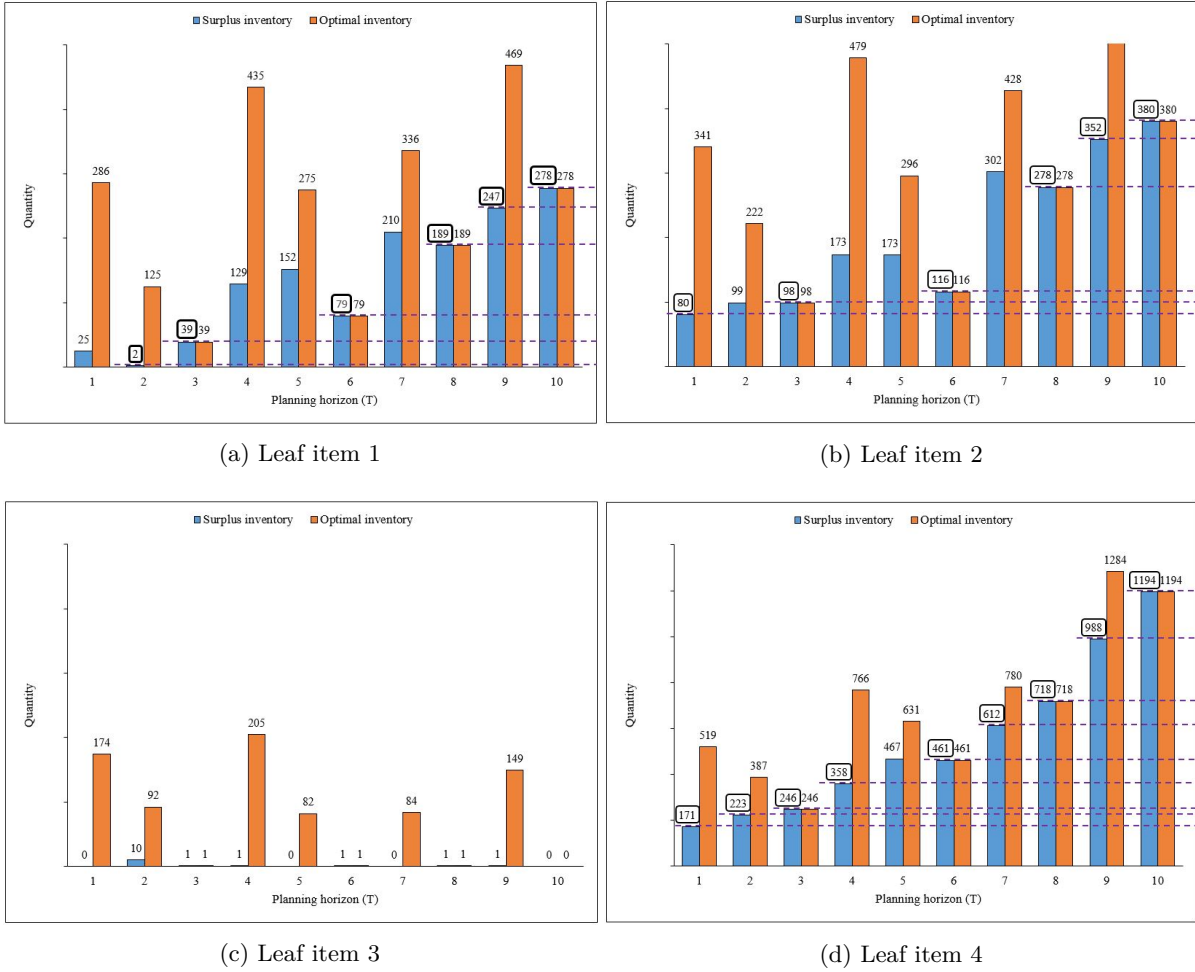


Fig. 3.2 Comparison of surplus inventory accumulation (Table 3.4) and optimal inventory (Table 3.2).

### 3.3 Models for disassembly lot sizing with disposal

#### 3.3.1 Aggregate formulation (AGG)

This model is based on inventory variables and flow conversations constraints, which is used traditionally in the literature of disassembly lot sizing. The disposal decisions are considered in this model by an additional decision variable:

- $E_{it}$ : disposed quantity of leaf item  $i$  in period  $t$

The single product disassembly lot sizing problem with disposal (DLSPD) can be formulated as a Mixed-Integer Programming (MIP) model as follow:

$$[\mathbf{P1}] \quad \text{Min} \left\{ \sum_{t=1}^T s_t \cdot Y_t + \sum_{t=1}^T p_t \cdot X_t + \sum_{i=1}^N \sum_{t=1}^T h_{it} \cdot I_{it} \right\} \quad (3.18)$$

Subject to

$$I_{it} = I_{it-1} + a_i \cdot X_t - E_{it} - d_{it} \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \ (I_{i0} = 0) \quad (3.19)$$

$$X_t \leq M_t \cdot Y_t \quad \forall t = 1 \dots T \quad (3.20)$$

$$X_t \geq 0 \ \& \ \text{integer} \quad \forall t = 1 \dots T \quad (3.21)$$

$$E_{it} \geq 0 \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.22)$$

$$I_{it} \geq 0 \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.23)$$

$$Y_t = 0 \ \text{or} \ 1 \quad \forall t = 1 \dots T \quad (3.24)$$

In this formulation, objective function (3.18) minimizes the sum of setup, disassembly operation, and inventory holding costs over the period horizon. Constraints (3.19) are the inventory balance equations for the leaf items. The disposal decision is considered into the ordinary disassembly lot sizing problem by using a new variable of disposed quantity of leaf item  $i$  in period  $t$  ( $E_{it}$ ). They express that the entering stock ( $I_{it-1}$ ) added to the current period disassembly quantity ( $a_i X_t$ ) are used to fulfill the demand of current period ( $d_{it}$ ), then surplus obtained quantity of leaf items will be disposed of ( $E_{it}$ ). The remains are kept in stock at end of period ( $I_{it}$ ). Disposal variables ( $E_{it}$ ) allow to manage surplus inventory of leaf items by balancing between inventory holding and disassembly operation costs. A graphical description of inventory conservation constraints is shown in Fig. 3.3. Constraints (3.20) guarantee that a setup cost is performed in period  $t$  if any disassembly operation is done in that period. Constraints (3.21–3.24) impose the non-negativity and binary restrictions on the variables.

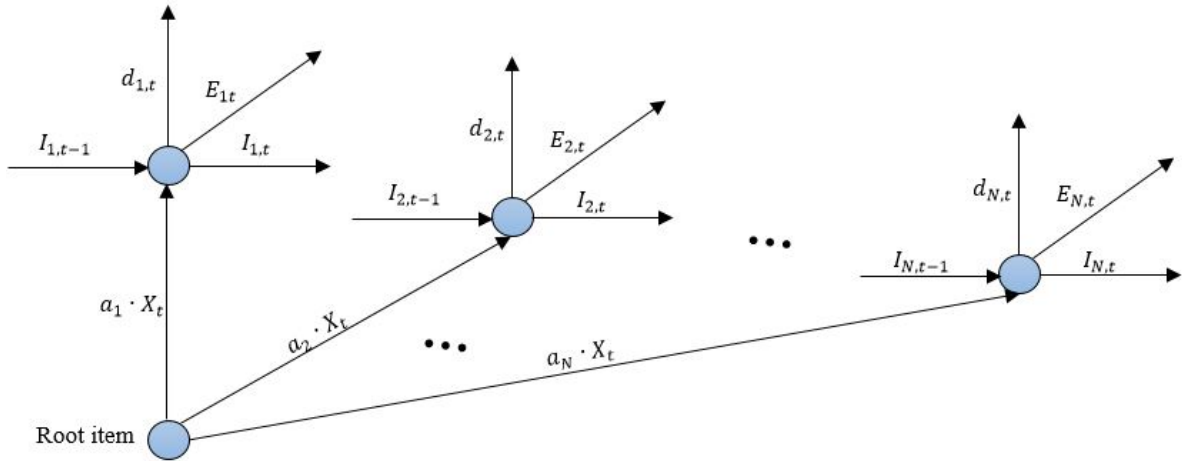


Fig. 3.3 Inventory flow conservation in a given period  $t$ .

The big M values are used widely in many classes of models (MIPs) and usually have the effect of making the model much difficult to solve. Equation (3.25) can be used to calculate an appropriate value of big M in each period which leads to improve LB of the problem (Alfieri et al. (2002)):

$$M_t = \max_{i=1 \dots N} \left\{ \left\lceil \frac{\sum_{k=t}^T d_{ik}}{a_i} \right\rceil \right\} \quad \forall t = 1 \dots T \quad (3.25)$$

To demonstrate the usefulness of disposal decisions on handling surplus inventory to provide inventory holding cost saving, we use the example presented in Table 3.1, and then we solve the DLSPD optimally by model P1. Table 3.5 shows the results when allowing disposal decisions, which can be led to a significant cost-saving in total disassembly system cost when not allowing disposal (Table 3.2) (about 30% i.e.,  $\frac{235947-165538}{235947} * 100 = 29.84\%$ ). Also disassembly planning with disposal decisions leads to a significant reduction in the inventory level of obtained parts over the planning horizon. For example, the total cumulative inventory of leaf item 4 held thorough the ten-period planning horizon is reduced from 6986 units (without disposal decisions) to 1710 units (with disposal decisions), and there is no inventory at the end of the planning horizon. This implies an important cost reduction in inventory holding costs as well as opportunities to disassembly more EOL products, which also leads to more environmentally-friendly benefits.

Table 3.5 Optimal solutions obtained by model P1 (with disposal decision)

	Planning period (t)									
	1	2	3	4	5	6	7	8	9	10
$X_t$	227	0	0	0	228	0	0	0	157	0
$I_{1t}$	361	200	114	0	464	268	147	0	191	0
$I_{2t}$	372	253	129	0	396	216	150	0	194	0
$I_{3t}$	310	228	137	1	334	253	84	1	149	0
$I_{4t}$	433	301	160	0	417	247	62	0	90	0
$E_{1t}$	129	0	0	0	60	0	0	0	89	0
$E_{2t}$	173	0	0	0	105	0	0	0	102	0
$E_{3t}$	0	0	0	0	0	0	0	0	0	0
$E_{4t}$	358	0	0	0	360	0	0	0	476	0
Optimal objective function = <b>165538</b>										

By comparing optimal solutions presented in Table 3.2 for model  $P_a$  (the model without disposal decisions) with optimal solutions in Table 3.5 for model P1 (the model with disposal decisions):

1. In optimal solutions of the model with disposal decisions, there is no surplus inventory of leaf items at the end of the planning horizon. This means that a significant reduction in inventory holding costs can be obtained (around 25% cost saving).
2. The inventory of the period before the setup period is not zero for all the periods. This shows that zero inventory of lot-sizing does not work in disassembly systems.
3. This inventory of the period before each setup period is used in the future periods, so that leads to zero inventory of leaf items at the end of the planning horizon.

LP relaxation of MIP models has an important role in various methods such as Branch-and-Bound algorithm, obtaining initial solutions in a two-phase or LP-based heuristics or, etc. (Alferi et al. (2002); Lee and Xirouchakis (2004); Pochet and Wolsey (2006)). We define [P2] as the LP relaxation of P1 by removing the integrality restrictions on the  $X_t$  variables.

It is well known that the single product disassembly lot sizing is a sub-problem in the solution procedures of many complex disassembly lot sizing problems (such as capacitated multi-product problem).

For this reason, it is interesting to study the different IP formulations of the single product DLSPD which can be applied to solve other extension of the problem. We propose to adapt different formulations existed in the literature of the similar problem such as ordinary lot sizing and to analyze their advantage/drawbacks for the studied problem.

### 3.3.2 Aggregate formulation without inventory variable (NIF)

The aggregate model can be modified by removing inventory variables, which helps us to reduce the number of constraints and variables. This reformulation of model AGG is applied in the literature so that the solution methods such as the Lagrangian relaxation techniques and heuristics can be applied more efficiently (Kim et al. (2005a)). We adapt it for the DLSPD. Equation (3.26) is an alternative way to write the inventory conservation flow for the DLSPD:

$$I_{it} = \sum_{k=1}^t a_i \cdot X_k - \sum_{k=1}^t E_{ik} - D_{i1t} \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \ (I_{i0} = 0) \quad (3.26)$$

Let  $c_t = \sum_{k=t}^T \sum_{i=1}^N h_{ik} a_i$ ,  $c'_{it} = \sum_{k=t}^T h_{ik}$  and  $C = \sum_{t=1}^T \sum_{i=1}^N h_{it} D_{i1t}$ . By using equation 3.26 and combining it with the constraints (3.23) and the inventory balance constraints (3.19), formulation without inventory variables for the DLSPD can be driven as **[P3]** (for more details, please see Appendix C):

$$[\mathbf{P3}] \text{ Min} \left\{ \sum_{t=1}^T s_t \cdot Y_t + \sum_{t=1}^T (p_t + c_t) X_t - \sum_{t=1}^T \sum_{i=1}^N c'_{it} \cdot E_{it} - C \right\} \quad (3.27)$$

Subject to

$$\sum_{k=1}^t (a_i \cdot X_k - E_{ik}) \geq D_{i1t} \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.28)$$

$$X_t \leq M_t \cdot Y_t \quad \forall t = 1 \dots T \quad (3.29)$$

$$X_t \geq 0 \ \& \ \text{integer} \quad \forall t = 1 \dots T \quad (3.30)$$

$$E_{it} \geq 0 \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.31)$$

$$Y_t = 0 \text{ or } 1 \quad \forall t = 1 \dots T \quad (3.32)$$

In this formulation, objective function (3.27) minimizes the sum of setup, disassembly operation, and inventory holding costs over the period horizon. Constraints (3.28) ensure that the demand for each leaf item in each period will be satisfied. Constraints (3.29) indicate a setup cost is incurred in every period in which at least one product is disassembled. Constraints (3.30–3.32) represent the conditions on the decision variables. Note that the value of  $C$  in the objective function is constant and hence can be omitted during optimization.

We define **[P4]** as the LP relaxation of P3 by removing the integrality restrictions on the  $X_t$  variables (constraints (3.30)).



### 3.3.3 AGG formulation with additional constraints

Several researches in the literature of lot sizing problems in production have been devoted to obtain the tight LP relaxations and improve corresponding Lower Bound (LB). In particular, Valid inequalities (VIs) which reduce the volume of the linear relaxation solution space by cutting off irrelevant parts. There is no research in the literature of the DLSP who studies tight formulation by adding VIs. We adapt the expression of the inequalities proposed for ordinary lot sizing problem by [Pochet and Wolsey \(2006\)](#) for our problem (the DLSPD). The constraints (3.33) define the more general class of so-called  $(l, s)$  inequalities: For any  $1 \leq l \leq T$ ,  $L = \{1 \cdots l\}$ , and  $S \subseteq L$ , the following inequalities are valid inequalities for the LP relaxation of the DLSPD:

$$\sum_{s \in S} a_i \cdot X_s - \sum_{s \in S} E_{is} \leq \sum_{s \in L \setminus S} D_{isl} \cdot Y_s + I_{il} \quad \forall i = 1 \dots N \ \& \ 1 \leq l \leq T, S \subseteq L \quad (3.33)$$

By adding additional inequalities or constraints to the LP relaxation of AGG model (here model P2), it is possible to obtain tight LB of the problem. This is because these additional constraints will cut off the irrelevant parts of the solution space. The idea underlying constraints (3.33) is to compute a LB on the inventory level of a leaf item  $i$  at the end of a period  $l$  (for more details please see [Appendix C](#)).

We define **[P5]** (AGG-VIs) as the P2 with considering equations 3.33. It generates optimal solutions or tight LB in small computational times. We apply a cutting-plane generation algorithm to add most violated inequalities of family of equations (3.33) at each iteration. Since in practice the number of such inequalities is limited but it is not possible to consider all them together, especially for the large problem. It is necessary to apply a separation algorithm which performs by enumeration. [Table 3.6](#) represents the recursive adding of the  $(l, s)$  inequalities to the initial formulation (i.e. model P2) and the related improvement of gap (%) in each iteration for the example data provided in [Table 3.1](#). We note that here the default settings of CPLEX solver is considered to solve the model P5 (i.e., model P2 with added most violated inequalities iteratively). The optimal solution obtained by model P1 using CPLEX is presented in [Table 3.5](#) (Optimal obj. is 165538). As can be seen in [Table 3.6](#), the addition of VIs are very efficient to obtain the tighten formulation for the problem. For the example data the improvement gap attain 21.20 % ( $21.57 \rightarrow 0.37$ ), which implies a strong lower bound for the problem. The cutting-plane generation algorithm adds most violated inequalities at each iteration and it is stopped after seven iterations which means there is no more inequalities improving the obtained solution. For the example data, after 7 iterations cutting-plane generation algorithm found no more violated inequalities. In the sensitivity analysis section we analyse the impact of considering the default settings (i.e. default cuts and presolved processes) of CPLEX solver for the model P5.

Table 3.6 Iterations of cutting-plane algorithm to add most violated valid inequalities to the initial model (i.e. model P2).

Itr.	1	2	3	4	5	6	7
Obj.	<b>129827.08</b>	139156.09	152733.73	162060.48	163757.44	164898.29	<b>164925.70</b>
Gap(%)	<b>21.57</b>	15.94	7.73	2.10	1.08	0.39	<b>0.37</b>

### 3.3.4 Disaggregate or facility-location based formulation (FAL)

The lot-sizing problem (LSP) can be modeled based on the Facility Location Problem (FLP). It consists in selecting (or positioning) the facilities from a given set of potential facilities (or places) in order to satisfy the demands for the costumers so that the related costs are minimized. It include fixed costs for selecting (or positioning) the facilities (or places) and associated costs for distributing the products to the costumers. A facility could represent a production machine or disassembly operation with a fixed setup cost and related production or disassembly operations costs (Cornuéjols et al. (1983); Maes et al. (1991); Krarup and Bilde (1977)). LSP can be formulated based on the FLP concept and is considered as a tight formulation because its LP relaxation can provide an optimal solution with integer setup variables (Brahimi et al. (2006)). We note that there is no study in the literature of DLSP who proposes FAL formulation.

We adapt the facility location model for the disassembly system. Additional variables are introduced:

- $Z_{ijt}$ : fraction of the demand of leaf item  $i$  in period  $t$  that is obtained in period  $j$ .

They are associated with inventory holding costs:

- $H_{ijt}$ : total inventory holding cost of leaf item  $i$  obtained in period  $j$  to satisfy the demand of period  $t$  ( $H_{ijt} = \sum_{k=j}^{t-1} h_{ik} \cdot d_{it}$ ).

The purpose of this formulation is to obtain stronger LB. This improves the efficiency of the MIP solution procedure for the problem at hand and also for other extensions as a sub-problem. The product disassembly variable  $X_t$  cannot, however, be removed since each component can be received after a disassembly operation in a period. The disagreeable formulation for the single-product DLSPD (when considering zero disposal cost) is presented in the following:

$$[\mathbf{P6}] \quad \text{Min} \left\{ \sum_{t=1}^T s_t \cdot Y_t + \sum_{t=1}^T p_t \cdot X_t + \sum_{i=1}^N \sum_{t=1}^T \sum_{j=1}^t H_{ijt} \cdot Z_{ijt} \right\} \quad (3.34)$$

Subject to

$$\sum_{j=1}^t Z_{ijt} = 1 \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.35)$$

$$Z_{ijt} \leq Y_j \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \ \& \ j \leq t \quad (3.36)$$

$$a_i \cdot X_t \geq \sum_{k=t}^T Z_{itk} \cdot d_{ik} \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.37)$$

$$X_t \geq 0 \ \& \ \text{integer} \quad \forall t = 1 \dots T \quad (3.38)$$

$$Z_{ijt} \geq 0 \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \ \& \ j \leq t \quad (3.39)$$

$$Y_t = 0 \ \text{or} \ 1 \quad \forall t = 1 \dots T \quad (3.40)$$

In this formulation, objective function (3.34) minimizes the sum of setup, disassembly operation, and inventory holding costs over the whole T-period planning horizon. Constraints (3.35) guarantee that the demands of leaf items in each period will be satisfied. Constraints (3.36) relate the variable  $Z_{ijt}$  to the binary setup variable  $Y_t$ . Constraints (3.37) express that the total quantity of leaf item  $i$  obtained

in period  $t$ , after disassembly of product, will be delivered to satisfy the demand or will be disposed of. Surplus quantity obtained in period  $t$  is  $a_i X_t - \sum_{k=t}^T Z_{itk} d_{ik}$  since  $a_i X_t$  is the total quantity of leaf items  $i$  received in period  $t$  and  $\sum_{k=t}^T Z_{itk} d_{ik}$  is the quantity of leaf item  $i$  delivered to the periods from  $t$  to  $T$  to fulfill their demand. This surplus inventory is supposed to be disposed of in period  $t$  which is equal to the variable  $E_{it}$ , as mentioned in the model P1. Constraints (3.38–3.40) define the domains of decision variables. We define [P7] as the LP relaxation of P6 by removing the integrality restrictions on the  $X_t$  variables, and we expect it obtains the optimal or near optimal solutions of the problem in small computational times.

### General form of FAL model

In this section, we present the equations for the model FAL with or without disposal and with or without disposal cost. As mentioned before as the assumption, we consider that the non-used surplus inventory will be disposed of as soon as possible after disassembly operation. In following, first of all, the general form of FAL formulation is presented. Afterwards, we develop the FAL model for the disassembly lot sizing problem without and with disposing of the non-used surplus inventory. Let  $a_i X_t - \sum_{k=t}^T Z_{itk} d_{ik}$  be the quantity of non-used surplus inventory of leaf item  $i$  generated in period  $t$ . Additional parameters are defined:

- $g_{it}$ : unit disposal cost of leaf item  $i$  in period  $t$
- $G_{it}$ : disposal cost of one unit of leaf item  $i$  generated in period  $t$  and disposed of in period  $k$  ( $G_{it} = \sum_{m=t}^{k-1} h_{im} + g_{ik}$ )

The optimal disposal cost of one unit of leaf item  $i$  generated in period  $t$  can be calculated by equations 3.41:

$$G_{it} = \min_{t \leq k \leq T+1} \left\{ \sum_{m=t}^{k-1} h_{im} + g_{ik} \right\} \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.41)$$

The total disposal cost for the non-used surplus inventory of leaf item  $i$  in period  $t$  can be calculated by equations 3.42:

$$\left( a_i \cdot X_t - \sum_{j=t}^T Z_{itk} \cdot d_{ij} \right) G_{it} \quad \forall i = 1 \dots N \ \& \ t = 1 \dots T \quad (3.42)$$

Following model represents the general form of FAL model for disassembly lot sizing problem (with or without disposal):

$$\text{Min} \left\{ \sum_{t=1}^T s_t \cdot Y_t + \sum_{t=1}^T p_t \cdot X_t + \sum_{t=1}^T \sum_{i=1}^N G_{it} \cdot a_i \cdot X_t + \sum_{i=1}^N \sum_{t=1}^T \sum_{j=1}^t (H_{ijt} - G_{ij} \cdot d_{it}) Z_{ijt} \right\} \quad (3.43)$$

Subject to equations (3.35–3.40). For the problem without disposal, following values (3.44 & 3.45) can be used (note that  $g_{iT+1} = 0$  and  $k = T + 1$ ):

$$G_{it} = \sum_{m=t}^T h_{im} \quad (3.44)$$

$$H'_{ijt} = \left( \sum_{k=j}^{t-1} h_{ik} - \sum_{k=j}^T h_{ik} \right) \cdot d_{it} = - \sum_{k=t}^T h_{ik} \cdot d_{it} \quad (3.45)$$

It is clear that the value of  $H'_{ijt}$  is independent of  $j$ . So objective function 3.43 for the problem without disposal can be rewritten as follow:

$$\begin{aligned} \text{Min} \{ & \sum_{t=1}^T s_t \cdot Y_t + \sum_{t=1}^T p_t \cdot X_t + \sum_{t=1}^T \sum_{i=1}^N G_{it} \cdot a_i \cdot X_t + C \} \\ C = & \sum_{i=1}^N \sum_{t=1}^T \left( \sum_{k=1}^T Z_{ikt} \right) \left( - \sum_{j=1}^T h_{ij} d_{it} \right) = - \sum_{t=1}^T \sum_{k=1}^T \sum_{j=1}^T h_{ij} d_{it} \end{aligned} \quad (3.46)$$

It is necessary to mentioned that  $C$  is constant and hence can be omitted. So there is no more variable of  $Z_{ikt}$  in the objective function.

For the problem with disposal, where the non-used surplus inventories will be disposed of as soon as they generated, the following values (3.47-3.49) can be used:

$$G_{it} = g_{it} \quad (3.47)$$

$$H'_{ijt} = \left( \sum_{m=j}^{t-1} h_{im} - g_{ij} \right) d_{it} \quad (3.48)$$

$$p'_t = p_t + \sum_{i=1}^N a_i \cdot g_{it} \quad (3.49)$$

It is clear that for the zero disposal cost ( $g_{it} = 0$ ), objective function 3.34 will be obtained.

### 3.3.5 Exact resolution

Since, the model P1, P4 and P6 are MIP, they can be solved by using CPLEX or another on-the-shelf solver to arrive at optimal solution. Today's modern solvers such as CPLEX use different cuts and include presolved processes when solving MIPs. We expect also to analysis the impact of allowing or not default cuts and presolved processes when solving the problem by using CPLEX solver. It is necessary to mention that when solving more realistic size of the problem by using CPLEX solver, the computational time will increase with increasingly higher number of leaf items as well as longer time horizons.

### 3.3.6 Heuristics for disassembly lot sizing problem with disposal

The heuristic algorithms have the advantage that they can be coded and executed via simple applications, which facilitates their application especially in real-industrial cases. It is necessary to mention that for a more realistic size of the problem, the computational time increases quickly for MIP formulations with an increasingly higher number of leaf items and longer time horizons. In this situation, the heuristics can be used to obtain a solution in a very short computational time. Two heuristics which consider disassembly planning with disposal are proposed for the DLSPD. They rely on two principles: disposing

of all surplus inventories or disposing of the non-used surplus inventory. Both heuristics can obtain solution of the tested problem in a very short computational time.

As stated before in Section 3.2.1, problem without disposal decisions can be formulated as a dynamic programming model (DP1). By using the surplus inventory ( $S_{it}^I$ ) which is a precomputed value, model DP1 can be reformulated to reduce the computation time. This means that the amount of inventory at the end of last  $j - 1$  periods sub-problem (i.e.  $I_{ij-1}$ ) can be calculated in advance (which is  $S_{ij-1}^I$ ). This dynamic program (DP2) can be represented as follow (note that  $j$  is setup period):

$$\begin{aligned}
 \text{[DP2]} \quad F(t) = \min_{1 \leq j \leq t} \{ & \sum_{i=1}^N \sum_{k=j+1}^t \sum_{l=j}^{k-1} h_{il} \cdot d_{ik} + \sum_{i=1}^N \sum_{k=j}^t h_{ik} \cdot S_{it}^I \\
 & + p_j \cdot \left( \max_{i=1 \dots N} \left\{ \left\lceil \frac{\sum_{k=j}^t d_{ik} - S_{ij-1}^I}{a_i} \right\rceil \right\} \right) + s_j + F(j-1) \} \quad (3.50)
 \end{aligned}$$

Note that  $S_{i0}^I$  and  $F(0) = 0$ . Despite the efficiency of model DP2, it is not suitable for the disassembly lot sizing problem with disposal. Model DP2 does not consider disposal decisions. This is because it considers the amount of optimal inventory of the last subproblem (i.e.,  $I_{ij-1}$  which is equal to  $S_{ij-1}^I$ ) to calculate the disassembly quantity of the EOL product (i.e.  $X_j$ ) in period  $j$ . If disposal decisions are considered, the value of  $I_{ij-1}$  will change (i.e. it will not be equal to the precalculated value  $S_{ij-1}^I$ ), so that a quantity of leaf item  $i$  should be disposed of in the period  $j - 1$  ( $E_{ij-1}$ ). As an option, the surplus inventory at the end of the last sub-problem (i.e. the value  $S_{ij-1}^I$ ) can be modified. We will have a maximum number of  $S_{ij-1}^I$  alternatives for disposing of leaf item  $i$ , i.e., from 1 to  $S_{ij-1}^I$  units. One solution is to consider all of the combinations and solving the dynamic program for each possible combination. In this case, the number of possible combinations will explode, and it depends on the value of the parameter  $S_{ij-1}^I$ . So model DP2 is not efficient for the problem with disposal decisions (DLSPD). We propose two heuristics based on model DP2 which consider disposal decisions by changing the value of  $S_{ij-1}^I$  in the equation 3.50. It means that a part or all of this value will be disposed of.

### Heuristics method 1 (H1)

The first heuristic method (H1) is a dynamic programming approach where the surplus inventory generated by one setup is disposed of. Only the quantities of components necessary to satisfy the demands until the next setup are kept in stock (i.e.  $S_{it}^I = 0$ ). This leads to the dynamic program presented in (3.51) (note that  $F(0)=0$ ).

$$\begin{aligned}
 \text{[H1]} \quad F(t) = \min_{1 \leq j \leq t} \{ & \sum_{i=1}^N \sum_{k=j+1}^t \sum_{l=j}^{k-1} h_{il} \cdot d_{ik} \\
 & + p_j \cdot \left( \max_{i=1 \dots N} \left\{ \left\lceil \frac{\sum_{k=j}^t d_{ik}}{a_i} \right\rceil \right\} \right) + s_j + F(j-1) \} \quad (3.51)
 \end{aligned}$$

### Heuristics method 2 (H2)

The second heuristic (H2) is also a dynamic programming approach but, compared to (H1), the consumed inventory is kept in stock even between several setup. The consumed inventory ( $S_{it}^c$ ) is precomputed with the procedure proposed in section 3.2.2 and  $S_{it}^I = S_{it}^c$ . The purpose is to disassemble less products by

keeping some items in stock longer (decrease of disassembly costs). The method H2 can be represented as follow (note that  $S_{i0}^c$  and  $F(0) = 0$ ):

$$[\mathbf{H2}] \quad F(t) = \min_{1 \leq j \leq t} \left\{ \sum_{i=1}^N \sum_{k=j+1}^t \sum_{l=j}^{k-1} h_{il} \cdot d_{ik} + \sum_{i=1}^N \sum_{k=j}^t h_{ik} \cdot S_{it}^c \right. \\ \left. + p_j \cdot \left( \max_{i=1 \dots N} \left\{ \left[ \frac{\sum_{k=j}^t d_{ik} - S_{ij-1}^c}{a_i} \right] \right\} \right) + s_j + F(j-1) \right\} \quad (3.52)$$

Tables 3.7 and 3.8 present the result of heuristic H1 and H2 for the two-level product structure example (e.g., an EOL vehicle) presented in Fig. 3.1 and Table 3.1. We note that the value of variable  $E_{it}$  can be easily calculated by using the inventory conservation equation. As shown in Table 3.7, heuristic H1 can obtain solutions near-optimal for the example so that its gap is only 0.01%. Table 3.7 shows that Heuristic H2 is also efficient and its gap is 1.95%. Both heuristic are efficient in managing the surplus inventory for the data example, especially at the end of the planning horizon. The decision-maker can choose the strategy of disposing of all surplus inventories (heuristic H1), which leads to an essential reduction in inventory holding costs (total number of all leaf items which are held over the planning horizon is 6654 for H1 versus 7090 for H2). Also, it allows for disassembly more EOL products, which also leads to more environmental-friendly benefits.

Table 3.7 The solutions of the example obtained by H1.

	Planning period (t)									
	1	2	3	4	5	6	7	8	9	10
$X_t$	<b>227</b>	0	0	0	<b>228</b>	0	0	0	<b>158</b>	0
$I_{1t}$	361	200	114	0	464	268	147	0	191	<b>0</b>
$I_{2t}$	372	253	129	0	396	216	150	0	194	<b>0</b>
$I_{3t}$	309	227	136	0	333	252	83	0	149	<b>0</b>
$I_{4t}$	433	301	160	0	417	247	62	0	90	<b>0</b>
Objective function obtained by H1 = <b>165553</b>										

Table 3.8 The solutions of the example obtained by H2.

	Planning period (t)									
	1	2	3	4	5	6	7	8	9	10
$X_t$	<b>227</b>	0	0	0	<b>228</b>	0	0	0	<b>157</b>	0
$I_{1t}$	411	250	164	50	464	268	147	0	191	<b>0</b>
$I_{2t}$	429	310	186	57	396	216	150	0	194	<b>0</b>
$I_{3t}$	310	228	137	1	334	253	84	1	149	<b>0</b>
$I_{4t}$	433	301	160	0	417	247	62	0	90	<b>0</b>
Objective function obtained by H2 = <b>168762</b>										

### 3.4 Computational experiments

This section presents the test results on the proposed models and methods in this research. Computational tests are performed on a number of new randomly generated problems. We use three performance measures in the test: the percentage deviation from the optimal solution (gap) (or best upper bound if an optimal solution could not be obtained within limited time); the number of optimal solutions obtained by each model and method; and computational time (CPU seconds). New benchmark for the DLSPD are also proposed to highlight the effect of different cost settings and problem sizes. The models and algorithms have been implemented using Java programming language. Optimal solutions of the problem are obtained by solving the MIP models using CPLEX solver 12.8. Also, the tests are coded on a system with an Intel Core i7-7700T, 2.9 GHz, and 16 Go RAM on windows 10. Note that we use the following formula to calculate the gap percentage of models and methods:

$$Gap(\%) = \frac{(Solution\ obtained) - (Best\ solution\ obtained)}{(Best\ solution\ obtained)} * 100 \quad (3.53)$$

#### 3.4.1 New Benchmark for the single-product DLSPD

To demonstrate the performance of the proposed models and methods, the generation of the various instances is performed on different problem sizes and different values of parameters. The existed benchmarks in the literature such as [Kim and Xirouchakis \(2010\)](#) are not relevant for the disassembly lot sizing problem studied in this research. They required setup operations in all periods. The benchmark proposed in [Kim and Xirouchakis \(2010\)](#) is adapted to generate instances with different cost settings. The ratio of setup cost on holding cost is set according to an expected Time Between Order (TBO). Equation (3.54) presents the TBO formula by using average values of the problem parameters, which is adapted from the TBO formula presented in [Godichaud and Amodeo \(2018, 2019a\)](#). We define  $N$  as the number of leaf items and  $\bar{s}$ ,  $\bar{a}$ ,  $\bar{h}$ , and  $\bar{d}$  as the mean of the discrete uniform distribution used to randomly generate setup, yield, holding inventory, and demand, respectively.

$$TBO = \sqrt{\frac{2\bar{s}}{N \cdot \bar{a} \cdot \bar{h} \cdot \bar{d}}} \quad (3.54)$$

For the test, we generated 150 instances with different sizes, i.e. 10 instances for each combination of three levels of number of leaf items ( $N=10, 100, 1000$ ; low (**L**), Medium (**M**), High (**H**), respectively), and five levels of number of periods ( $T=10, 20, 30, 40, 50$ ). Table 3.9 provides the parameter setting on our benchmark and benchmark of [Kim and Xirouchakis \(2010\)](#). Note that  $DU(b1, b2)$  means the discrete uniform distribution with a range of  $[b1, b2]$  and it is supposed that there is no disposal cost. As can be seen in Table 3.9, the benchmark of [Kim and Xirouchakis \(2010\)](#) provides a TBO less than 1. This implies that their generated problem instances by using this benchmark settings require setup in each period, which is not really a lot sizing problem.

Table 3.9 Parameters value of our benchmark and benchmark of [Kim and Xirouchakis \(2010\)](#)

Parameters	Benchmark	
	<a href="#">Kim and Xirouchakis (2010)</a>	Chapter 3
Root items ( $R$ )	(10, 20, 30)	1
Leaf items ( $N$ )	Low (L) $\Rightarrow$ DU(1, 10) Medium (M) $\Rightarrow$ DU(10, 100) High (H) $\Rightarrow$ DU(100, 1000)	Low (L) $\Rightarrow$ 10 Medium (M) $\Rightarrow$ 100 High (H) $\Rightarrow$ 1000
Periods ( $T$ )	(10, 20, 30)	(10, 20, 30, 40, 50)
Holding cost ( $h_{it}$ )	DU(5, 10)	$N=10\Rightarrow$ DU(0.3, 0.5) $N=100\Rightarrow$ DU(0.03, 0.05) $N=1000\Rightarrow$ DU(0.003, 0.005)
Demand ( $d_{it}$ )	DU(50, 200)	DU(50, 250)
Yield ( $a_i$ )	DU(1, 4)	DU(1, 4)
Disassembly cost ( $p_t$ )	-	DU(38, 62)
Setup cost ( $s_t$ )	DU(500, 1000)	DU(2500, 3500)
TBO ( $N=Low$ )	0.34	2
TBO ( $N=Medium$ )	0.11	2
TBO ( $N=High$ )	0.03	2

### 3.4.2 Numerical results

The results are reported in Table 3.10 and 3.11. 150 problem instances are solved directly on CPLEX solver by the formulations P1, P3, and P6 (within 3600s time limit) to obtain the optimal solution. As can be seen in Table 3.5, only MIP model P1 can obtain optimal solution of all tested problem instances within 3600 seconds limited times. But, MIP models P3 and P6 cannot obtain optimal solution of all tested instances. For example, the average of gap of P3 for the problem instances with high number of leaf items and 50 number of periods can attain 0.58%. We observe that MIP models are not efficient to solve the very large size of the problem (especially in real industrial cases): the CPU time increases quickly as the problem size increases, variation of computational times is significant for the different problem sizes. For example, P3 cannot obtain optimal solutions of several problem instances with 50 periods when high number of leaf items (1000). We observe that P1 indicates the superiority over both of models P3 and P6 in terms of CPU time.

Table 3.10 also shows CPU times for solving P2 and P4 as the LP relaxation of P1 and P3, respectively. We note that for the LP relaxation models P2, P4, and P7, we solved the models P1, P3, and P6 by removing all of their integrality constraints. The average of CPU times for P2 and P4 are only 0.37 and 0.93, respectively. However, as shown in Table 3.11, they have a weak lower bound (LB) (with an average of gap 11.98% and 12.01%, respectively). The addition of certain valid inequalities (model P5) can improve the LB of the problem. The results indicate that model P5 can obtain the solution of the problem with very small gap (%) in a reasonable CPU time. Its average of gap is only 0.01% and, it can obtain optimal solutions of about 80% of the tested problems, and for the others, the average of gap is only 1.19%. Also, Table 3.10 shows the efficiency of adding certain valid inequalities (VIs) to model P2 (without VIs), e.g., the average of gap is significantly improved from 11.98% (the model without VIs (P2)) to 0.01% (the model with VIs (P5)). This improvement of gap can even reach about 27%



for the tested instances (from 27.45% to 0.16%). The number of valid inequalities can explode for the large-sized problem. LP relaxation of a MIP reformulation can be suitable as P7. It can obtain optimal solutions in 84% of the tested problems, and for the others, the average of gap is only 0.94%. These results indicate that they can be used in the more complex variant of the DLSPD or LP-based heuristics.

The result of the heuristics for the different problem instances shows that H2 obtains the solution with better quality than H1 for the problem instances. This implies that applying disposal decisions of all surplus inventories (consumed and non-used) is not more economical. It is more economic to hold the consumed surplus inventory and disposing of the non-used surplus inventory. The two heuristics are very fast and simple to apply; this is particularly interesting for real industrial cases. These heuristics can easily help decision makers to choose a suitable disassembly planning to handle surplus inventory; either holding consumed surplus inventory to satisfy future demands (H2) or disposing of all surplus inventory (H1) in an environmental conscious way.

Table 3.10 CPU seconds for P1, P2, P3, P4, P5, P6, P7, H1, and H2

N	T	AGG			NIF		FAL		Heuristics	
		P1	P2	P5	P3	P4	P6	P7	H1	H2
L	10	0.05*	0.01	<b>0.00</b>	0.05	0.01	0.02	0.01	<b>0.00</b>	<b>0.00</b>
	20	0.14	0.01	0.02	0.36	0.01	0.07	0.02	<b>0.00</b>	<b>0.00</b>
	30	0.55	0.01	0.06	2.01	0.01	0.19	0.06	<b>0.00</b>	<b>0.00</b>
	40	0.65	0.01	0.12	4.22	0.02	0.22	0.08	<b>0.00</b>	<b>0.00</b>
	50	0.57	0.01	0.14	5.60	0.02	0.24	0.13	<b>0.00</b>	<b>0.00</b>
M	10	0.11	0.01	0.41	0.25	0.02	0.22	0.08	<b>0.00</b>	<b>0.00</b>
	20	0.86	0.03	4.05	5.46	0.06	1.14	0.71	<b>0.00</b>	<b>0.00</b>
	30	2.84	0.04	2.15	22.05	0.18	3.34	1.97	<b>0.00</b>	<b>0.00</b>
	40	6.81	0.04	3.05	37.94	0.30	9.85	5.07	<b>0.00</b>	<b>0.00</b>
	50	6.55	0.06	6.43	59.08	0.46	18.13	7.49	<b>0.00</b>	<b>0.00</b>
H	10	1.99	0.17	5.96	6.11	0.29	5.04	2.41	<b>0.00</b>	0.01
	20	27.21	0.55	21.12	177.08	0.82	38.41	18.25	<b>0.00</b>	0.01
	30	129.00	1.00	59.71	459.63	1.86	129.49	58.13	<b>0.01</b>	<b>0.01</b>
	40	277.15	1.82	96.50	1223.00	3.61	319.08	174.48	0.02	<b>0.01</b>
	50	517.25	1.98	293.93	2463.74	6.25	1242.21	447.10	<b>0.01</b>	0.02
Min.		0.03	0.01	<b>0.00</b>	0.03	0.01	0.02	0.01	<b>0.00</b>	<b>0.00</b>
Avg.		53.43	0.37	25.63	297.77	0.93	75.09	39.63	<b>0.00</b>	<b>0.00</b>
Max.		759.88	2.16	439.87	<b>3600.00</b>	6.43	<b>3600.00</b>	807.91	0.07	<b>0.03</b>

(\*): Average of gap (%) for ten different problem instances.

0.00 implies that the computational time was less than 0.005

Table 3.11 Gap (%) and number of optimal solutions obtained for P1, P2, P3, P4, P5, P6, P7, H1, and H2

N	T	AGG			NIF		FAL		Heuristics	
		P1	P2	P5	P3	P4	P6	P7	H1	H2
L	10	<b>0.00*</b>	8.73	<b>0.00</b>	<b>0.00</b>	8.73	<b>0.00</b>	<b>0.00</b>	1.63	0.66
	20	<b>0.00</b>	12.10	<b>0.00</b>	<b>0.00</b>	12.10	<b>0.00</b>	0.02	4.28	0.60
	30	<b>0.00</b>	13.17	0.02	<b>0.00</b>	13.71	<b>0.00</b>	0.01	5.48	0.92
	40	<b>0.00</b>	16.35	0.02	<b>0.00</b>	16.35	<b>0.00</b>	0.01	5.87	0.96
	50	<b>0.00</b>	13.96	<b>0.00</b>	<b>0.00</b>	13.96	<b>0.00</b>	<b>0.00</b>	16.16	1.13
M	10	<b>0.00</b>	9.09	0.03	<b>0.00</b>	9.09	<b>0.00</b>	0.03	7.22	0.32
	20	<b>0.00</b>	11.27	<b>0.00</b>	<b>0.00</b>	11.11	<b>0.00</b>	<b>0.00</b>	11.47	0.52
	30	<b>0.00</b>	11.91	0.01	<b>0.00</b>	11.91	<b>0.00</b>	0.01	13.51	0.66
	40	<b>0.00</b>	13.05	<b>0.00</b>	<b>0.00</b>	13.05	<b>0.00</b>	<b>0.00</b>	13.77	1.06
	50	<b>0.00</b>	13.10	<b>0.00</b>	<b>0.00</b>	13.10	<b>0.00</b>	<b>0.00</b>	15.69	1.23
H	10	<b>0.00</b>	9.05	<b>0.00</b>	<b>0.00</b>	9.05	<b>0.00</b>	<b>0.00</b>	10.17	0.24
	20	<b>0.00</b>	10.92	0.01	<b>0.00</b>	10.92	<b>0.00</b>	0.01	14.94	0.73
	30	<b>0.00</b>	11.93	<b>0.00</b>	<b>0.00</b>	11.93	<b>0.00</b>	<b>0.00</b>	17.08	0.77
	40	<b>0.00</b>	12.42	<b>0.00</b>	<b>0.00</b>	12.43	<b>0.00</b>	<b>0.00</b>	18.54	0.98
	50	<b>0.00</b>	12.69	<b>0.00</b>	0.08	12.77	<b>0.00</b>	<b>0.00</b>	19.46	1.16
Min.		<b>0.00</b>	5.91	<b>0.00</b>	<b>0.00</b>	5.91	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
Avg.		<b>0.00</b>	11.98	0.01	0.01	12.01	<b>0.00</b>	0.01	11.68	0.80
Max.		<b>0.00</b>	27.45	0.16	0.58	27.45	<b>0.00</b>	0.16	22.80	3.64
No. Opt.		<b>150</b>	0	<b>119</b>	<b>148</b>	0	<b>149</b>	<b>126</b>	2	1

(\*): Average of gap (%) for ten different problem instances.

0.00 implies that the gap was less than 0.005.

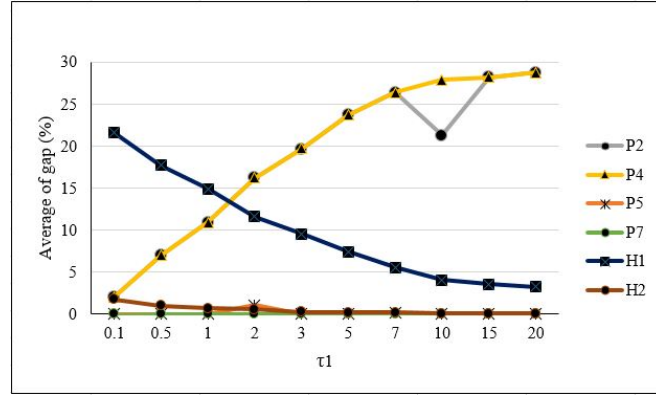
No. Opt.: Number of optimal solutions obtained among 150 instances.

### 3.4.3 Sensivity analysis

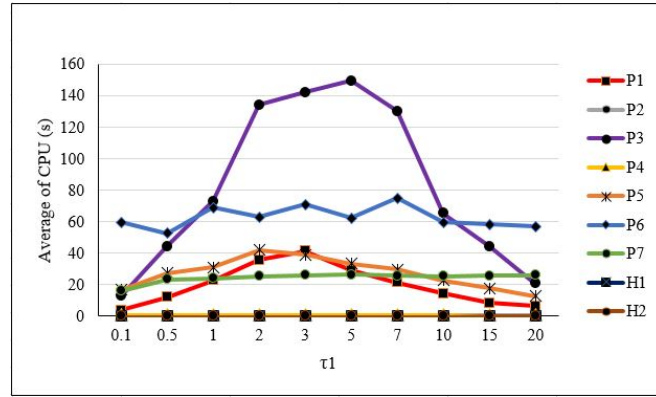
The generated parameters are experimental but they can correspond to any type of real case product that can be disassembled into several components with different ratios. A sensitivity analysis is proposed to examine the effect of a change in important parameters in disassembly systems ( $s_t$ ,  $h_{it}$ ,  $d_{it}$ , and  $a_i$ ) and how are their impacts on the performance of the proposed models and methods. Managerial insights are also highlight based on these results. Sensitivity analysis is generated when  $N=1000$ ,  $T=20$ ; unvaried parameters are also generated in the same manner as Section 3.4.1.

Setup costs are of great importance and usually high especially in practical disassembly processes (Kang et al. (2002, 2003); Kim et al. (2006b)). This requires evaluation of the effects of the amount of setup costs on the performance of the models and methods. The analysis of the effects of setup costs is done on 80 randomly problem instances: ten different problems for each eight different amounts of setup cost parameters. The setup costs change between  $(2500\tau_1, 3500\tau_1)$ , where  $\tau_1$  is the setup factor ( $\tau_1 = 0.1, 0.5, 1, 2, 5, 10, 15, 20$ ). The test results are represented in Fig. 3.4. The change in setup cost has a little effect on CPU running time of P2, P4, P7, H1, and H2. But CPU time of P1, P5, and P4 decrease for high setup costs. Among the MIP formulations, P6 (P5) is more stable (more variable) in terms of CUP time. Setup cost variation does not also affect the gap (%) of H2, P5, and P7. The performance of H2 is always better than H1, and they become better when increasing the setup factor. This implied

that in disassembly systems with low setup costs, the strategy of disposing of all surplus inventories is more economical and holding consumed surplus inventory to satisfy future demands can provide optimal inventory holding strategy. We also observe that the gap (%) of P2 gets worse as the amount of setup cost increases.



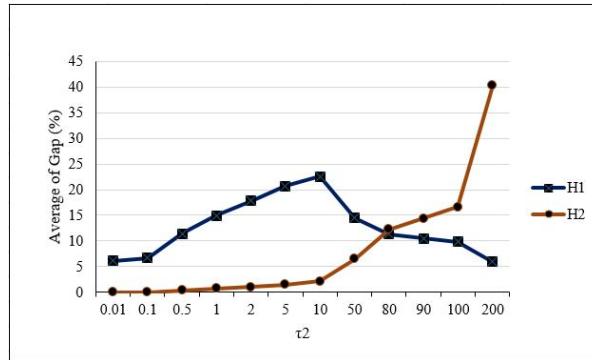
(a) Average of gap (%) variations



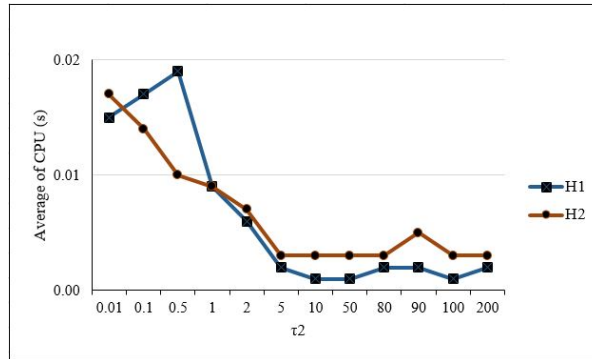
(b) Average of CPU (s) variations

Fig. 3.4 Performance of the proposed methods for different amount of setup costs.

The analysis of the effect of inventory cost is performed on 120 test problems (ten different problems for each 12 different amounts of inventory holding cost). The inventory holding costs are generated between  $(0.003\tau_2, 0.005\tau_2)$ , where  $\tau_2$  is the inventory holding factor ( $\tau_2 = 0.01, 0.1, 0.5, 1, 2, 5, 10, 50, 80, 90, 100, 200$ ). Figure 3.5 shows the performance of H1 and H2 when changing the inventory holding cost factor. The gap (%) of H2 gets worse as holding cost increases. In fact, its performance is the best when holding cost factor  $\tau_2 < 80$ . When holding cost is extremely high, holding consumed surplus inventory ( $S_{it}^c$ ) is not an economic strategy, and all the surplus inventory ( $S_{it}^I$ ) should be disposed of in order to reduce overall disassembly system costs. As shown in Figure 3.5, unlike H2, the performance of H1 improves when increasing inventory holding costs. The improvement in the gap for H1 is started from  $\tau_2=10$ , which means the inventory holding cost parameter is generated randomly from  $DU(0.03, 0.05)$  for the case with number of items 1000 and 20 periods. This means that by increasing inventory holding cost, the strategy of disposing of all the surplus inventory (i.e.,  $S_{ij-1}^I$ ), towards an optimal decision. Note that CPU (s) running time of the heuristics is very short for all the tested instances ( $\leq 0.02$  seconds), and it decreases when increasing the inventory holding cost for both the proposed heuristics.



(a) Average of gap (%) variations



(b) Average of CPU (s) variations

Fig. 3.5 Performance of the heuristics for different amount of inventory holding costs.

The analysis of the effect of yield and demand on the total cost is performed on 490 test problems (i.e., ten different problem for each seven different amounts of yield and seven different amounts of demand). The demand and yield are generated between  $(50\tau_3, 250\tau_3)$  and  $(1\tau_4, 4\tau_4)$ , respectively.  $\tau_3$  and  $\tau_4$  are demand and yield factor, respectively. Table 3.12 illustrates the results. The main contribution is that increasing yield ratio for a given demand leads to more cost saving. The reason is disposal decisions can provide a significant economic balance for EOL products, particularly for the ones with high amount of yield.

Table 3.12 Impact of variation of yield and demand on the total cost

		$\tau_3$						
		1	2	3	4	5	6	10
$\tau_4$	1	194512	371020	544328	716781	888558	1059885	1743391
	2	115456	210169	302366	393448	483637	572877	928155
	3	88352	155756	220432	283826	346113	407632	650414
	4	74643	128121	179049	228271	234455	323919	509443
	5	66346	111411	153970	194701	234455	273246	423758
	6	60760	100192	137151	172142	206188	239245	365999
	10	49457	77596	103071	126488	149092	170546	248506

Potential cost reduction of considering disposal decisions is examined for the different sizes of instance. We solve the generated instances by using the model of Kim et al. (2008) (AGG model without disposal). The solutions (objective function values) are compared with those obtained by P1 (AGG model with disposal). As can be seen in Table 3.13, disposal decisions can result in significant cost reduction in disassembly systems (for example average of cost saving can attain 78.06 % for the problems with high number fo leaf items and 50 periods). We also observe that the amount of cost saving increases with larger problem sizes (higher number of leaf items and longer planning horizon). This shows that applying decisions on surplus inventory like disposal is important to guarantee economic balance, particularly for the large size of real industrial problems.

Table 3.13 Cost reduction average (%) by considering disposal decisions

Leaf item case	Low				
Periods	10	20	30	40	50
Cost reduction (%)	42.94	58.30	65.67	68.70	75.66
Leaf item case	Medium				
Periods	10	20	30	40	50
Cost reduction (%)	45.81	60.52	68.77	73.29	76.18
Leaf item case	High				
Periods	10	20	30	40	50
Cost reduction (%)	46.93	65.13	69.92	74.12	<b>78.06</b>

We also analyze using CPLEX solver with or without cuts and presolved processes on the performance of model P1 and P5 (i.e. model P2 with additional constraints (VIs)). Most of modern solvers such as CPLEX solver use also additional methods such as cutting planes and include presolved processes to reduce the size of the problem instance at hand and to obtain a tighten formulation (Achterberg and Wunderling (2013)). We analyze the impact on the performance of the default CPLEX solver against turning off the new techniques (i.e. default cuts and presolved processes). Table 3.9 provides the results of ten problem instances with N=1000 leaf items and T=40 periods. We mention that we apply the following formula to calculate the Gap:

$$Gap(\%) = \frac{UB - LB}{LB} * 100 \quad (3.55)$$

Our main objective is to analyse the performance of the proposed Valid Inequalities when using CPLEX solver without allowing default cuts and presolved processes. Note that model P5 is P2 with additional constraints (i.e. VIs).  $P5^{(**)}$  means solving model P5 by CPLEX when not allowing default cuts and presolved processes. The results show that allowing default cuts, including presolved processes, are of major importance to solve MIPs. As can be seen in Table 3.14, the computational times required to obtain optimal solution of model P1 when disabling default cuts and presolved processes increase exponentially for the considered problem instances. we note that we consider default MIPs gap when solving models  $P1^{(**)}$  and  $P5^{(**)}$ . CPLEX solver can only obtain the solution of two instances within limited 3600s computational time when not allowing default cuts and presolved processes for model P1 (model  $P1^{(**)}$ ), and for the others, maximum gap is 2.10 %. As when enabling default cuts and presolved process, CPLEX solver can obtain optimal solution of all instances in very short computational times. Table 3.14 shows the efficiency of the proposed valid inequalities (VIs) to solve the problem. Model

$P5^{(**)}$  (which includes VIs when not allowing default cuts and presolved processes for the CPLEX solver) can obtain the same gaps as model  $P5^{(*)}$  (which includes VIs when allowing default cuts and presolved processes). Although computational times increase slightly when not allowing default cuts and presolved processes (model  $P5^{(**)}$ ). This implied that considering valid inequalities are useful to obtain tighten formulation of the problem especially when using simple CPLEX solver (i.e. when not allowing default settings).

Table 3.14 Analysis of default settings of CPLEX on the performance of models P1 and P3.

Instance	$P1^{(*)}$		$P1^{(**)}$		$P5^{(*)}$		$P5^{(**)}$	
	Gap(%)	CPU(s)	Gap(%)	CPU(s)	Gap(%)	CPU(s)	Gap(%)	CPU(s)
1	0.00	92.29	1.86	3600.00	<b>0.00</b>	159.41	<b>0.00</b>	597.35
2	0.00	64.05	1.66	3600.00	<b>0.01</b>	130.63	<b>0.01</b>	435.28
3	0.00	78.45	2.05	3600.00	<b>0.00</b>	125.37	<b>0.00</b>	454.44
4	0.00	82.83	<b>2.10</b>	3600.00	<b>0.00</b>	130.68	<b>0.00</b>	534.92
5	0.00	52.73	0.01	<b>858.49</b>	<b>0.00</b>	160.79	<b>0.00</b>	301.59
6	0.00	55.26	0.92	3600.00	<b>0.00</b>	97.78	<b>0.00</b>	427.56
7	0.00	88.68	1.00	3600.00	<b>0.02</b>	137.60	<b>0.02</b>	423.61
8	0.00	75.23	0.01	3600.00	<b>0.00</b>	134.03	<b>0.00</b>	586.02
9	0.00	45.31	0.50	3600.00	<b>0.00</b>	111.90	<b>0.00</b>	376.70
10	0.00	59.85	0.01	<b>1844.01</b>	<b>0.00</b>	116.03	<b>0.00</b>	461.81
Min	0.00	45.31	<b>0.01</b>	858.49	<b>0.00</b>	97.78	<b>0.00</b>	301.59
Avg.	0.00	69.47	<b>1.01</b>	3150.25	<b>0.00</b>	130.42	<b>0.00</b>	459.93
Max	0.00	92.29	<b>2.10</b>	3600.00	<b>0.02</b>	160.79	<b>0.02</b>	597.35

(\*) solving model by CPLEX when allowing default cuts and presolved processes.

(\*\*) Solving model by CPLEX not when allowing default cuts and presolved processes.

### 3.5 Applications and managerial insights

This chapter provided different models and methods for the single-product disassembly lot-sizing problem, which consider the decisions to manage surplus inventory accumulations in disassembly systems. The researchers can apply them in several ways, such as be used in the algorithms for more complex disassembly product structures. Also, the generated parameters are experimental, but they can correspond to the different real cases where EOL products will be disassembled into several components and materials with different ratios to satisfy demand channels, such as repair services, second-hand markets, and energy recovery channels. Our results can be applied to the several industries such as EOL electrical and electronic equipment (EEE) recycling (e.g., [Environnement Recycling \(2003\)](#)), for the low number of items, EOL vehicles recycling (e.g., [Indra automobile recycling \(1985\)](#)), for the medium number of items, and EOL Aircraft recycling (e.g., [Tarmac Aerosave \(2007\)](#)), for the high number of items. The methods and models provided are efficient enough to be applied in real-life cases, which can help managers find the strategy providing economic balance.

The results reveal that inventory management decisions can be critical in disassembly systems because of their specific characteristics that have a significant impact on the economic balance. Different efficient models and methods are proposed, which can demonstrate how decision-makers might consider different inventory management strategies of a given disassembly system to make it more profitable. For example,

an average of 5500 vehicles per year are disassembled through 5 stages of recycling by [Indra automobile recycling \(1985\)](#) and about 20 to 30 different parts (depending on the type of vehicle) are obtained with different ratios, then stored to satisfy the second-hand market demands. Providing the recovery rate of 95% required by the ELVs directive have a significant impact on the economic balance of the sector. Surplus inventories of these obtained parts will lead to a considerable inventory holding cost. But they can be held to satisfy future demands or disposed of to minimize the inventory holding cost. The models and methods can be applied by decision-makers to determine which part, when, and how many should be disposed of the system while satisfying the demands and minimizing total disassembly system costs. Different strategies to manage surplus inventories are proposed with relevant advantages. All the methods and models can provide inventory policies that avoid surplus inventories at the end of the planning horizon. Besides, decision-makers can improve the recovery rate of their disassembly system by optimizing disassembly operations using proposed models and methods. The recommended tools also are varied, fast, and efficient enough so that decision-makers can make the decision quickly and easily.

However, data acquisition will be challenging for decision-makers in disassembly systems. The reason is that disassembly systems are not usually digitized as much as the modern production systems. In some cases, they might need the interface to facilitate the application of the models and methods. In practice, to apply the models and methods efficiently, we will need to collect large amounts of data. Also, a correct estimation of the parameters used is of major importance to guarantee the performance of models and methods. Concerning the relevant industry, decision-makers are recommended to establish a data acquisition and analysis system, which collects and evaluates the required parameters continuously. In addition, applying interface modules will facilitate the application of models and methods. By using different proposed strategies and methods which correspond strategy of the industry, the decision-maker can choose an appropriate decision that makes more economical as well as environmental benefits for the system. The optimal decision may vary from different industries or products depending on the data.

## 3.6 Conclusion

This chapter presents a new disassembly lot sizing problem, which is an important challenge for industrial companies. We explore the decisions on the management of surplus inventory in disassembly systems. This surplus inventory can appear after disassembly of returned products and increases inventory holding costs. The detailed analysis of the inventory accumulation shows that the disposal decision allows important cost saving (e.g. average of cost saving can attain 78% for the problem instances with 50 periods when high number of leaf items (1000)).

Three different mathematical formulations have been developed. They provide different linear relaxation lower bounds and the effect on the efficiency of MIP resolution methods has been outlined. CPU times increase quickly for the large problem sizes and they require more complex MIP procedures or commercial solvers. LP relaxation of AGG model is fast and easy to apply, but the quality of its solutions is too weak to be used within an exact or heuristic procedure. The addition of certain valid inequalities can improve its lower bound. Also, LP relaxation of FAL model as a strong reformulation of the problem is proposed. It can obtain optimal or near optimal solutions in a reasonable CPU time. In addition, For the real industrial case, two heuristic algorithms considering different policies of inventory management are proposed. The advantage is that their CPU time is very short and they can be programmed into code via simple spreadsheet applications.

In the current chapter, the basic form of product structure is considered i.e., single product type with the two-level disassembly structure without parts commonality. In Chapter ?? more complex structures are considered i.e., multi-product with two/multi-level disassembly structure, and with parts commonality.

### Contributions

Preliminary results of this chapter were presented at:

- Tafti, M. P. M., Godichaud, M., and Amodeo, L. (2019b). Single product disassembly lot sizing problem with disposal. In 2019 IEEE 6th International Conference on Industrial Engineering and Applications (ICIEA), pages 135–140. IEEE (DOI: 10.1109/IEA.2019.8715056).
- Tafti, M. P. M., Godichaud, M., and Amodeo, L. (2019a). Models for the single product disassembly lot sizing problem with disposal. IFAC-PapersOnLine, 52(13):547–552 (DOI: 10.1016/j.ifacol.2019.11.215).
- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2019). Models for disassembly lot sizing problem with decisions on surplus inventory. In Advances in Optimization and Decision Science for Society, Services and Enterprises, pages 423–432. Springer (DOI: 10.1007/978-3-030-34960-8\_37).
- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2020). New models and efficient methods for disassembly lot sizing problem with surplus inventory decisions, International Journal of Production Research (10.1080/00207543.2020.1829148).



## Chapter 4

# Multi-Product Disassembly Lot Sizing Problem With Two/Multi-Level Disassembly Structure

### 4.1 Introduction

This chapter focuses on the multiple product types disassembly lot sizing problem with parts commonality as a more general case among different disassembly structures. Parts commonality is one of the prevalent characteristics of product design nowadays, which implies that end-of-life products or subassemblies have common parts/ materials. This includes some benefits but it makes disassembly lot sizing problem more complicated, since it incurs interdependencies between different products or subassemblies. In addition, The issue of surplus inventory is more challenging for the multi-level products structure. This because there will exist surplus inventory of subassemblies in addition to surplus inventory of leaf items. Managing these surplus inventories is an important challenge for companies because it has a significant impact on inventory holding costs. The disposal decisions are considered to handle this surplus inventory, especially for the more complicated disassembly structure, i.e., multi level multi-product with parts commonality.

The solution of the problem determines the quantity and timing of disassembling end-of-life (EOL) products to satisfy their demand over a finite planning horizon, while surplus subassemblies and leaf items will be disposed of. This can provide economic balance in disassembly systems. In this chapter, two cases of the problem are considered: (i) Multi-product two-level disassembly structure with parts commonality. (2) Multi-level disassembly structure with capacity restrictions. Capacity restriction means that there is a resource limitations in disassembly systems.

This chapter is organized as follows. Section 4.2 introduces the problem for the two-level EOL product disassembly structure. Two new MIP formulations are presented to model the problem. A two-phase heuristics method is proposed especially for large-sized problems in real applications. A numerical

experiment on randomly generated problem instances compares the proposed models and methods with respect to criteria for performance evaluation (deviation from optimal solution and computational time). In Section 4.3, we extend the problem to multi-level product disassembly structure with capacity restrictions. A new MIP formulation is presented to model the problem. Then, a classical FO algorithm is presented for the large problems. We improve the quality of its solution by considering the relationship between linked variables. Numerical result demonstrates the performance of the models and methods (i.e., CPU times, and the gap percentage) on randomly generated problem instances. It shows that new FO heuristic are efficient to improve significantly the quality of the solutions. Finally, Section 4.4 concludes this chapter by providing the summary of the works and future perspectives.

## 4.2 Multi-product disassembly lot sizing problem with disposal for the two-level products disassembly structure with parts commonality

### 4.2.1 Problem description

A two-level products disassembly structure is considered in this section. The first level represents EOL products and the second level corresponds to their leaf items or components. An example of this structure is given in Fig. 4.1. The number in parenthesis is the yield of component when one unit of root item (items 1, 2, and 3) is disassembled. The first level represents leaf items, while the second level represents root items (EOL Products). The parts 6 and 7 can be obtained by root items 1 and 3 with related yield which implies parts commonality.

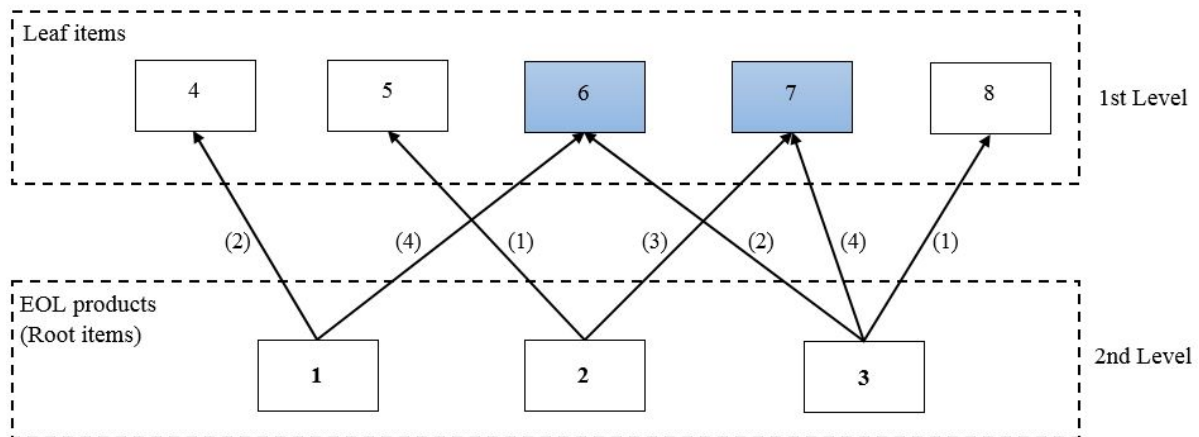


Fig. 4.1 Example of multi-product disassembly structure for the two-level products structure with parts commonality.

The assumptions for the problem are: *a)* disassembly structure of EOL products is given. *b)* EOL products can be obtained whenever they are needed and there is no holding cost for them; *c)* backlogging and lost sales are not allowed, and all the demands should be satisfied on time; *d)* demand of parts are given and deterministic; *e)* we assume that the stock of the root and leaf items at the beginning of the planning horizon are zero. *f)* Surplus leaf items will be disposed of as soon as the disassembly

operation is done. In the proposed models, without loss of generality, all items (N) are numbered with integers:  $1, 2, \dots, i_r, i_l, \dots, N$ , where  $i_r$  indicates the index for the root items and the numbers that are greater than or equal to  $i_l$  represent the leaf items. The following notations and parameters are used to formulate the multi-product disassembly lot sizing problem:

- $i$ : Index for the items,  $1, 2, \dots, i_r, i_l, \dots, N$ .
- $t$ : Index for the periods,  $1, 2, \dots, T$ .
- $M_{it}$ : Arbitrary large number considered for the root item (EOL product)  $i$  in period  $t$ .
- $s_{it}$ : Setup cost of disassembling root item (EOL product)  $i$  in period  $t$ .
- $p_{it}$ : Disassembly operation cost of root item (EOL product)  $i$  in period  $t$ .
- $a_{ij}$ : number of units of the leaf item  $j$  obtained by disassembly of one unit of the root item (EOL product)  $i$ .
- $h_{it}$ : inventory holding cost of leaf item  $i$  in period  $t$ .
- $d_{it}$ : demand of leaf item  $i$  in period  $t$ .
- $\Phi(i)$ : parents of leaf item  $i$ .
- $\eta(i)$ : children of root item (EOL product)  $i$ .

The decision variables for the multi-product disassembly lot sizing problem are defined:

- $Y_{it}$ : 1 if there is a setup in period  $t$  for root item (EOL product)  $i$ , and 0 otherwise.
- $X_{it}$ : disassembly quantity of root item (EOL product)  $i$  in period  $t$ .
- $I_{it}$ : inventory level of leaf item  $i$  at the end of period  $t$ .
- $E_{it}$ : disposed quantity of leaf item  $i$  in period  $t$ .

#### 4.2.2 Aggregate formulation (AGG)

A natural formulation of the problem is presented in this section. Aggregate model seems to be the most used model in the literature. We derive it directly from the aggregate formulation of the single-product disassembly lot sizing with disposal (Section 3.3.1):

$$[\mathbf{P1}] \quad \text{Min} \left\{ \sum_{i=1}^{i_r} \sum_{t=1}^T s_{it} \cdot Y_{it} + \sum_{i=1}^{i_r} \sum_{t=1}^T p_{it} \cdot X_{it} + \sum_{i=i_l}^N \sum_{t=1}^T h_{it} \cdot I_{it} \right\} \quad (4.1)$$

Subject to

$$I_{it} = I_{it-1} + \sum_{r \in \Phi(i)} a_{ri} \cdot X_{rt} - E_{it} - d_{it} \quad \forall i = i_l \dots N \quad \& \quad t = 1 \dots T \quad (4.2)$$

$$X_{it} \leq M_{it} \cdot Y_{it} \quad \forall i = 1, 2 \dots i_r \quad \& \quad t = 1 \dots T \quad (4.3)$$

$$X_{it} \geq 0 \ \& \ \text{integer} \quad \forall i = 1, 2 \dots i_r \quad \& \quad t = 1 \dots T \quad (4.4)$$

$$I_{it} \geq 0 \quad \forall i = i_l \dots N \quad \& \quad t = 1 \dots T \quad (4.5)$$

$$E_{it} \geq 0 \quad \forall i = i_l \dots N \ \& \ t = 1 \dots T \quad (4.6)$$

$$Y_{it} = 0 \text{ or } 1 \quad \forall i = 1, 2 \dots i_r \ \& \ t = 1 \dots T \quad (4.7)$$

Objective function (4.1) is to minimize the sum of setup, disassembly operation, and inventory holding costs over a  $T$ -period horizon planning. Constraints (4.2) express the inventory balance equations for the leaf items. Constraints (4.3) guarantee that a setup cost is performed in period  $t$  if any disassembly operation is done in that period. Constraints (4.4–4.7) impose the non-negativity and binary restrictions on the variables.

The big  $M$  constraints are used widely in many classes of MIP models and usually have the effect of making the model much difficult to solve. One solution is to put a very large number, but it can be resulted in much longer solving times. Also, it may lead to a weak LP-based lower bounds (Alfieri et al. (2002)). The value of  $M_{it}$  in equations (4.3) can be replaced by the equations (4.8) which can improve Lower Bound (LB) of the problem.

$$M_{it} = \max_{j \in \eta(i)} \left\{ \left\lceil \frac{\sum_{k=t}^T d_{jk}}{a_{ij}} \right\rceil \right\} \quad \forall i = 1 \dots i_r \ \& \ t = 1 \dots T \quad (4.8)$$

LP relaxation of model P1 can be used in different resolution method such Branch-and-Bound (B&B), LP-based rounding heuristics or Two-phase heuristic. We define  $[\mathbf{P1r}]$  as the LP relaxation of P1 by removing the integrality constraints (4.4).

### 4.2.3 Facility-location based formulation (FAL)

Facility Location-Bases Formulations (FAL) were initially developed by Bilde and Krarup (1977) for the uncapacitated single-item lot sizing problem (Brahimi (2004)). They show than the LP-relaxation of this formulations are better and stronger than the LP-relaxation of the AGG formulation. Also, there is no more big  $M$  constraints limitations as in AGG formulation. Exploration of FAL formulations seems to be limited in the literature of disassembly lot sizing. We extend the FAL formulation of the single-product disassembly lot sizing with disposal (Section 3.3.4) for the multi-product case with parts commonality. In the disaggregate or Facility-Location based formulation (FAL), additional variables are introduced:

- $Z_{irjt}$ : fraction of the demand of leaf item  $i$  in period  $t$  that is obtained in period  $j$  from disassembly of the root item  $r$ .

They are associated with inventory holding costs defined as follow:

- $H_{ijt}$ : total inventory holding cost of the leaf item  $i$  obtained in period  $j$  to satisfy demand of the period  $t$  ( $H_{ijt} = \sum_{k=j}^{t-1} h_{ik} \cdot d_{it}$ ).

The below formulation is called the disaggregate formulation or facility location-based formulation (FAL). In this model, the variables  $X_{it}$  need to be integer but the variables  $Z_{irjt}$  and  $I_{it}$  can be set as real. The disaggregate formulation for the multi-product with part commonality DLSPD is presented in the following:

$$[\mathbf{P2}] \quad Min \left\{ \sum_{i=1}^{i_r} \sum_{t=1}^T (s_{it} \cdot Y_{it} + p_{it} \cdot X_{it}) + \sum_{i=i_l}^N \sum_{r \in \Phi(i)} \sum_{t=1}^T \sum_{j=1}^t H_{ijt} \cdot Z_{irjt} \right\} \quad (4.9)$$

Subject to

$$\sum_{j=1}^t \sum_{r \in \Phi(i)} Z_{irjt} = 1 \quad \forall i = i_l \dots N \ \& \ t = 1 \dots T \quad (4.10)$$

$$Z_{irjt} \leq Y_{rj} \quad \forall i = i_l \dots N \ \& \ r \in \Phi(i) \ \& \ t = 1 \dots T \ \& \ j \leq t \quad (4.11)$$

$$a_{ri} \cdot X_{rt} \geq \sum_{j=t}^T Z_{irtj} \cdot d_{ij} \quad \forall i = i_l \dots N \ \& \ r \in \Phi(i) \ \& \ t = 1 \dots T \quad (4.12)$$

$$X_{it} \geq 0 \ \& \ \text{integer} \quad \forall i = 1, 2 \dots i_r \ \& \ t = 1 \dots T \quad (4.13)$$

$$Z_{irjt} \geq 0 \quad \forall i = i_l \dots N \ \& \ r \in \Phi(i) \ \& \ t = 1 \dots T \ \& \ j \leq t \quad (4.14)$$

$$Y_{it} = 0 \ \text{or} \ 1 \quad \forall i = 1, 2 \dots i_r \ \& \ t = 1 \dots T \quad (4.15)$$

Objective function (4.9) is to minimize the sum of setup, disassembly operation, and inventory holding costs over the whole  $T$ -period horizon. Constraints (4.10) represent that the demands of leaf items should be satisfied. Constraints (4.11) relate the variable  $Z_{irjt}$  to the binary setup variable  $Y_{rj}$ . Constraints (4.12) express that the total quantity of leaf item  $i$  obtained by root item  $r$  in period  $t$ , after disassembly operation, will be delivered to satisfy the demand or will be disposed of. Constraints (4.13–4.15) define the domains of decision variables.

LP relaxation of MIP models can be an idea to solve the problem specially when using a tight formulation. We define [P2r] as the LP relaxation of P2 by removing the integrality constraints (4.13). We suppose that the solution of model P2r is optimal or it has a very strong lower bound (LB) of the problem. This is interesting specially for the LP-based rounding heuristics where the quality of initial solution (or LB) has an important role on the quality of their final solutions. In numerical results, we also analyze the LP-relaxation of model FAL to demonstrate its performance to obtain a very strong lower bounds of the problem which can be useful especially for LP-bases heuristics.

#### 4.2.4 Exact resolution

Since the proposed models are MIPs, they can be used to obtain the optimal solution using CPLEX solver for the problem and CPLEX solver can obtain optimal solution in a reasonable computation time for the generated problems. But, we cannot guarantee that CPLEX solver will be efficient for all the problem sizes, especially for a very large sizes of the problem. We propose a two-phase heuristic in which an initial solution is constructed, and then improved by changing the initial solution using a dynamic program algorithm with forward-looking check.

#### 4.2.5 Two-phase heuristic (TPH)

Heuristics methods are efficient for the real environment cases. They have the advantage that can be applied to solve the problem in very short computational times (Brahimi (2004)). In this section, we adapt the two-phase heuristic method proposed by Kim et al. (2006b) for the multiple product types

with parts commonality disassembly lot sizing problem with disposal. The heuristic algorithm proposed consists of first phase in which the initial solution obtained from LP relaxation heuristic is constructed. Then, it is improved by a forward-looking algorithm based on dynamic programming approach.

**Phase 1. Solution construction:** The solution obtained by solving LP relaxation of model AGG (P1r) using CPLEX solver will be rounded down. The rounded-down solution is modified so that all the constrains in model P1 are satisfied. Balanced quantities ( $BL_{it}$ ) defined by equation (4.16) are used to check the feasibility of the rounded-down solution.

$$BL_{it} = I_{it} - I_{it-1} - \sum_{r \in \Phi(i)} a_{ri} \cdot X_{rt} + E_{it} + d_{it} \quad \forall i = i_l \dots N \ \& \ t = 1 \dots T \quad (4.16)$$

For a given  $i$  and  $t$ , if  $BL_{it} \neq 0$ , the corresponding rounded-down solution should be modified by increasing or decreasing the decision variables, while considering cost changes. For the case with  $BL_{it} > 0$ , if  $I_{it} \geq BL_{it}$ , we calculate new inventory level by equation (4.17). Otherwise, if  $E_{it} \geq BL_{it}$ , we update the disposed quantity by equation (4.18):

$$I'_{it} = I_{it} - BL_{it} \quad (4.17)$$

$$E'_{it} = E_{it} - BL_{it} \quad (4.18)$$

Where  $I'_{it}$  and  $E'_{it}$  are the changed inventory and disposed quantity of a given leaf item in period  $t$ , respectively. If both cases above result in infeasible solutions, we consider the case of increasing root item  $r$ ;  $X_{rt}$ ,  $r \in \Phi(i)$  so that  $X'_{rt} = X_{rt} + \Delta_r$ . The amount of  $\Delta_r$  for a root item  $r \in \eta(i)$  is  $\lceil BL_{it}/a_{ri} \rceil$ . Where  $\lceil \bullet \rceil$  represents the smallest integer value greater than or equal to  $\bullet$ . This change can be resulted in the violation of constraints 4.2 of child items  $k \in \eta(r)$ ,  $k \neq i$ . The modification of inventory level of child items can be done as follows:

$$\begin{aligned} I'_{kt} &= I_{kt} + a_{rk} \cdot \Delta_r & \forall k \in \eta(r), k \neq i, \text{ and} \\ I'_{it} &= I_{it} + (a_{ri} \cdot \Delta_r - BL_{it}) \end{aligned} \quad (4.19)$$

This increment of the disassembly quantity of a given root item  $r$  results in cost increasing.  $A_r$  represents the cost change when increasing the disassembly quantity of the root item  $r \in \Phi(i)$ :

$$A_r = p_{rt} \cdot \Delta_r + \sum_{k \in \eta(r), k \neq i} h_{kt} \cdot a_{rk} \cdot \Delta_r + s_{rt} \cdot \{1 - \delta(X_{rt})\} + h_{it} \cdot (a_{ri} \cdot \Delta_r - BL_{it}) \quad (4.20)$$

where  $\delta(*) = 1$  if and only if  $* > 0$ , and 0 otherwise. Equation 4.20 represents the increase in: disassembly operation cost of the root item  $r$ , inventory holding cost of child items ( $\neq i$ ), setup cost, and excess inventory holding cost incurred by the leaf item  $i$  after satisfying the balance  $BL_{it}$ . Then, we choose the best candidate root item which minimizes increasing cost. For the case with  $BL_{it} < 0$ , we consider increasing the quantity of disposed  $E_{it}$  as equations (4.21), so that there is no change in cost (N.B., Disposal cost is considered zero):

$$E'_{it} = E_{it} + |BL_{it}| \quad (4.21)$$

**Phase 2. Solution improvement:** The initial solution obtained by Phase 1 is improved by using dynamic programming based principles presented by Kim et al. (2006b). It is applied to each root item with a sequential application. This algorithm improves the initial solution by shifting disassembly lots around. The problem for each root item is decomposed into  $T$  sub-problems and generates a new disassembly planning by solving each sub-problem recursively. If last setup occurs in period  $j$  ( $1 \leq j \leq t$ ) for a  $t$ -period sub-problem of a given root item  $r$ , the change in the current disassembly lot sizing is calculated in equations (4.22):

$$\begin{aligned} X'_{ru} &= \sum_{k=j}^t X_{rk} & \forall u = j \\ X'_{ru} &= 0 & \forall u = j + 1 \dots t \end{aligned} \quad (4.22)$$

Where  $X_{ru}$  and  $X'_{ru}$  represent the current and new disassembly lot sizing for a given root item  $r$ , respectively. This change can decrease setup cost while increasing inventory holding cost. In the following, a method is proposed to update the inventory levels of leaf items after changing current disassembly lot sizing:

$$\begin{aligned} I'_{iu} &= I_{iu} - (X'_{rj} - \sum_{k=j}^u X_{rk}) & \forall u = j \dots v - 1 \\ I'_{iu} &= I_{iu} & \forall u = 1, 2 \dots j - 1, t, t + 1 \dots T \end{aligned} \quad (4.23)$$

We note that we do not consider change in disposed quantity ( $E_{iu}$ ) of leaf items in two-phase heuristic. This means that when changing the current disassembly lot sizing using the equation (4.22), the disposed quantity of leaf item  $i$  obtained by the root item  $r$  will be held to be disposed as the current disassembly lot sizing. This can be improved in the future work.  $B(j, t)$  represents the decrease in the total cost for a given root item  $r$  when last setup occurs in period  $j$  in the  $t$ -period sub-problem:

$$B(j, t) = \max\{0, \sum_{k=j}^t p_{rk} \cdot X_{rk} - p_{rj} \cdot X'_{rj}\} + \sum_{u=j+1}^t s_{ru} \cdot \delta(X_{ru}) \quad (4.24)$$

Where  $\delta(*) = 1$ , if  $X_{ru} > 0$ , otherwise,  $\delta(*) = 0$ .  $C(j, t)$  represents the increase in the total cost as follow:

$$\begin{aligned} C(j, t) &= \max\{0, p_{rj} \cdot X'_{rj} - \sum_{k=j}^t p_{rk} \cdot X_{rk}\} + \sum_{u=j}^{t-1} \sum_{i \in \eta(i)} h_{iu} \cdot a_{ri} \cdot (X'_{rj} - \sum_{k=j}^u X_{rk}) \\ &\quad + s_{rj} \cdot (1 - \delta(X_{rj})) \end{aligned} \quad (4.25)$$

Formulation (4.26) represents recursive cost saving function for the  $t$ -period sub-problem of the root item  $r$  (Where  $F_r(0) = 0$ ):

$$F_r(t) = \max_{1 \leq j \leq t} \{\max(0, B(j, t) - C(j, t) + F_r(j - 1))\} \quad (4.26)$$

**Example data:** An example of the problem is solved to show the applicability of the TH. Table 4.1 provides an example data for the multi-product disassembly structure presented in Fig. 4.1.

We solve the example data by using model AGG with and without disposal decision (model AGG without disposal presented in Kim (2005)). Tables 4.2 and 4.3 provides the results. The disposal decisions consideration results in cost reduction in total associated costs for disassembly planning. For the example this reduction is about 7% (i.e.,  $\frac{127166-118687.50}{127166} * 100 = 6.7\%$ ). In addition, there is no more surplus inventory of leaf items at the end of planning horizon. The number of total leaf items in stock through the planning horizon is reduced significantly from 22632 units to 3136 units.

Table 4.4 presents the result of two-phase heuristics (first and second phases) for the example data. TPH heuristic is efficient to solve the example data and to improve initial solution obtained by the first Phase. The gap of TPH for the example data is only 0.27% (i.e.,  $\frac{119012-118687.50}{118687.50} * 100 = 0.27\%$ ), which is significantly improve from the gap of the first phase i.e., 11.61% ( $\frac{132472.20-118687.50}{118687.50} * 100 = 11.61\%$ ).

Table 4.1 Example data for the structure represented in Fig. 4.1.

Item (N)	(Setup ( $s_{it}$ ), Disassembly operation ( $p_{it}$ ), Holding ( $h_{it}$ ), Demand ( $d_{it}$ ))				
	Period (t)				
	1	2	3	4	5
1	(3427, 62, -, -)	(3296, 57, -, -)	(2544, 49, -, -)	(2628, 46, -, -)	(3330, 43, -, -)
2	(2904, 62, -, -)	(3202, 60, -, -)	(2568, 44, -, -)	(2555, 38, -, -)	(3146, 55, -, -)
3	(3486, 60, -, -)	(3390, 55, -, -)	(2998, 40, -, -)	(2510, 49, -, -)	(3066, 49, -, -)
4	(-, -, 0.4, 237)	(-, -, 0.3, 145)	(-, -, 0.3, 219)	(-, -, 0.5, 229)	(-, -, 0.3, 250)
5	(-, -, 0.5, 189)	(-, -, 0.5, 161)	(-, -, 0.4, 246)	(-, -, 0.5, 233)	(-, -, 0.4, 248)
6	(-, -, 0.5, 101)	(-, -, 0.4, 94)	(-, -, 0.5, 249)	(-, -, 0.5, 171)	(-, -, 0.4, 181)
7	(-, -, 0.3, 60)	(-, -, 0.4, 186)	(-, -, 0.5, 250)	(-, -, 0.5, 197)	(-, -, 0.3, 219)
8	(-, -, 0.5, 215)	(-, -, 0.5, 180)	(-, -, 0.5, 243)	(-, -, 0.5, 239)	(-, -, 0.3, 201)

Table 4.2 Solution for the problem without disposal

	Period (t)				
	1	2	3	4	5
$X_{1t}$	191	0	349	0	0
$X_{2t}$	350	0	246	481	0
$X_{3t}$	132	0	228	0	0
$I_{4t}$	145	0	479	250	0
$I_{5t}$	161	0	0	248	0
$I_{6t}$	1059	965	2796	2625	2444
$I_{7t}$	1254	1068	2012	3258	3039
$I_{8t}$	181	1	442	203	2
Optimal Obj. = <b>127166</b>					



Table 4.3 Solution for the problem with disposal

	Period (t)				
	1	2	3	4	5
$X_{1t}$	191	0	349	0	0
$X_{2t}$	350	0	246	481	0
$X_{3t}$	132	0	228	0	0
$I_{4t}$	145	0	479	250	0
$I_{5t}$	161	0	0	248	0
$I_{6t}$	94	0	352	181	0
$I_{7t}$	186	0	0	219	0
$I_{8t}$	180	0	440	201	0
$E_{4t}$	0	0	0	0	0
$E_{5t}$	0	0	0	0	0
$E_{6t}$	965	0	1479	0	0
$E_{7t}$	1068	0	944	1027	0
$E_{8t}$	1	0	1	0	0
Optimal Obj. = <b>118687.50</b>					

Table 4.4 Solution of Phase 1 and Phase 2 for the example data.

	Phase 1					Phase 2				
	Period (t)					Period (t)				
	1	2	3	4	5	1	2	3	4	5
$X_{1t}$	119	72	261	88	0	191	0	349	0	0
$X_{2t}$	189	161	246	481	0	350	0	246	481	0
$X_{3t}$	72	60	227	0	1	132	0	228	0	0
$I_{4t}$	1	0	303	250	0	145	0	479	250	0
$I_{5t}$	0	0	0	248	0	161	0	0	248	0
$I_{6t}$	7	0	0	181	3	475	0	355	184	3
$I_{7t}$	1	0	0	219	2	604	0	2	221	2
$I_{8t}$	1	0	438	199	1	181	0	441	202	1
$E_{4t}$	0	0	0	0	0	0	0	0	0	0
$E_{5t}$	0	0	0	0	0	0	0	0	0	0
$E_{6t}$	584	381	1476	0	0	584	381	1476	0	0
$E_{7t}$	650	418	942	1027	0	650	418	942	1027	0
$E_{8t}$	0	1	0	0	0	0	1	0	0	0
<b>Obj. function phase 1 = 132472.20</b>						<b>Obj. function phase 1 = 119012</b>				

#### 4.2.6 Computational experiments

Computational tests are performed on a number of randomly generated problems. The proposed models and methods are compared regarding to the gap (%) and computational times. The tests are done on a laptop with an Intel Core i5-3210M 2.5 GHz and 8 Go RAM on windows 7 and the optimal solution are obtained by solving the MIP models directly using CPLEX 12.8.

## Benchmark

The generation of the various instances is performed on different problem sizes and different values of parameters. The existed benchmarks in the literature are not relevant for the disassembly lot sizing problem studied in this research, because they required setup operations in all periods. The benchmark proposed in Kim (2005) is adapted to generate instances with different cost settings. We use the same equation as (3.54) to obtain an estimation of Time Between Order (TBO). We have modified the inventory holding cost parameter to obtain  $TBO \geq 2$ . We generate 225 problems i.e. 25 problems for each combination of three levels of number of items ( $N = 10, 20, \text{ and } 30$ ) and three levels of number of periods ( $T = 10, 20, \text{ and } 30$ ). For each levels of the number of items, 5 different disassembly structures are randomly generated. The number of leaf items for each root item are generated from  $DU(2, 5)$ ,  $DU(5, 10)$  and  $DU(10, 15)$  for each level of the number of items. In the generated disassembly structure, the number of root items are generated from  $DU(2, N/5)$ . Also, the number of common leaf items are generated from  $DU(1, \lfloor N/3 \rfloor)$ . For each disassembly structure, 5 problems with different data are generated for each level of the number of periods. Table 4.5 provides the generated parameters. Here,  $DU(b1, b2)$  means the discrete uniform distribution with a range of  $[b1, b2]$ .

Table 4.5 Parameters value of benchmark

Parameters	Benchmark	
	Kim (2005)	Chapter 4
Items ( $T$ )	(10, 20, 30)	(10, 20, 30)
Periods ( $T$ )	(10, 15, 20)	(10, 20, 30)
Holding cost ( $h_{it}$ )	$DU(5, 10)$	$N=10 \Rightarrow DU(0.3, 0.5)$ $N=100 \Rightarrow DU(0.1, 0.3)$ $N=1000 \Rightarrow DU(0.1, 0.16)$
Demand ( $d_{it}$ )	$DU(50, 200)$	$DU(50, 250)$
Yield ( $a_i$ )	$DU(1, 3)$	$DU(1, 4)$
Disassembly cost ( $p_t$ )	$DU(50, 100)$	$DU(38, 62)$
Setup cost ( $s_t$ )	$DU(500, 1000)$	$DU(2500, 3500)$

## Numerical results

Tables 4.6 and 4.7 summarize the test results of the proposed MIP models and their LP relaxation. The result shows that both models P1 and P2 can solve all tested instances in a short computational time. But we cannot guarantee it can solve very large problem instances. Also, the overall average CPU(s) of model P1 is lower than model P2. This means that original formulation AGG is more efficient when using CPLEX solver to solve MIP models. But, FAL formulation is more efficient to obtain very strong lower bound of the problem which can be applied in the procedures of various heuristics methods.

Table 4.7 summarizes the result of LP relaxation of the proposed models. It shows that model P2r can obtain optimal solutions of the problem in 46.6% and for the others, the overall average of gap is only 0.06%. It implies that FAL model is efficient as a tighter formulation to obtain strong lower bound. Although Model P1r has a weak lower bound (its overall average Gap is 8.94% and its gap can attain 28.59%), but it is faster than model P2r with the overall average CPU of 0.11 seconds. The significant performance of model P2 is that its LP relaxation can obtain a very strong lower bound of the problem, especially for large-sized problems.

The results of two-phase heuristic are summarized in Table 4.8. Two-phase heuristic terminates in very short computational times (overall average of CPU (s) is only 0.13). It can be seen from the table that second phase can improve significantly the initial solution obtained from the Phase 1. In the case with 30 items and 30 periods, second phase can improve the overall average of gap from 20.55% to 3.21%, and it only requires 0.13 seconds solving time. The overall computational time of the two-phase heuristic is significantly shorter than models P1, P2, and P2r when solving by CPLEX solver (0.13, 0.57, 0.71, and 0.35 seconds for TPH, P1, P2, and P2r, respectively).

Table 4.6 CPU (s) of model P1, P1r, P2, and P2r

N	T	AGG		FAL	
		P1	P1r	P2	P2r
		Mean (Min, Max)	Mean (Min, Max)	Mean (Min, Max)	Mean (Min, Max)
10	10	0.19(0.15, 0.27)	<b>0.13</b> (0.12, 0.14)	0.18(0.15, 0.23)	0.17(0.16, 0.18)
	20	0.30(0.21, 0.76)	<b>0.14</b> (0.13, 0.14)	0.29(0.21, 0.73)	0.20(0.18, 0.22)
	30	0.67(0.28, 1.88)	<b>0.14</b> (0.14, 0.15)	0.66(0.27, 1.89)	0.29(0.33, 0.41)
20	10	0.23(0.20, 0.28)	<b>0.09</b> (0.08, 0.10)	0.23(0.19, 0.23)	0.17(0.17, 0.19)
	20	0.50(0.33, 0.77)	<b>0.10</b> (0.09, 0.10)	0.57(0.34, 0.83)	0.28(0.24, 0.32)
	30	0.94(0.54, 1.91)	<b>0.10</b> (0.10, 0.12)	1.25(0.66, 2.60)	0.50(0.40, 0.72)
30	10	0.29(0.20, 0.46)	<b>0.09</b> (0.09, 0.10)	0.32(0.22, 0.51)	0.21(0.18, 0.28)
	20	0.65(0.35, 1.22)	<b>0.10</b> (0.09, 0.11)	0.82(0.46, 1.36)	0.44(0.31, 0.65)
	30	1.38(0.62, 2.63)	<b>0.12</b> (0.11, 0.19)	2.07(0.94, 5.18)	0.89(0.52, 1.66)
Avg.		0.57(0.32, 1.13)	<b>0.11(0.11, 0.13)</b>	0.71(0.38, 1.51)	0.35(0.28, 0.51)

Table 4.7 Gap (%) for P1r and P2r.

N	T	AGG		FAL	
		P1r		P2r	
		Mean (Min, Max)	N.O.	Mean (Min, Max)	N.O.
10	10	10.75(3.57, 25.38)	0	<b>0.25</b> (0.00, <b>2.52</b> )	<b>12</b>
	20	11.40(5.59, <b>28.59</b> )	0	<b>0.11</b> (0.00, 1.50)	<b>16</b>
	30	12.99(6.58, 21.15)	0	<b>0.14</b> (0.00, 0.97)	<b>8</b>
20	10	8.18(3.41, 16.81)	0	<b>0.02</b> (0.00, 0.12)	<b>8</b>
	20	9.22(5.29, 14.67)	0	<b>0.01</b> (0.00, 0.14)	<b>9</b>
	30	8.50(4.14, 14.63)	0	<b>0.02</b> (0.00, 0.14)	<b>13</b>
30	10	6.76(3.90, 12.66)	0	<b>0.02</b> (0.00, 0.23)	<b>7</b>
	20	6.45(4.27, 9.36)	0	<b>0.01</b> (0.00, 0.03)	<b>14</b>
	30	6.24(4.83, 9.34)	0	<b>0.00</b> (0.00, 0.02)	<b>18</b>
Avg.		8.94(4.62, 16.95)	0%	<b>0.06(0.00, 0.63)</b>	<b>46.6%</b>

N.O.: Number of optimal solutions obtained in 25 problems for each level of problem instances

Table 4.9 Impact of disposal decisions on reducing total cost in disassembly planing (maximum cost reduction (%)).

N=30, T=30	Structures				
	1	2	3	4	5
Cost reduction (%)	<b>57.66</b>	49.19	39.93	51.37	50.59

Table 4.8 CPU (s) of two-phase heuristic and Gap (%) of first and second phase of two-phase heuristic

N	T	Phase 1 (gap (%))		Phase 2 (gap (%))		Two-phase (CPU (s))
		Mean (Min, Max)	N.O.	Mean (Min, Max)	N.O.	Mean (Min, Max)
10	10	19.56(7.08, <b>38.82</b> )	0	<b>2.71</b> (0.21, 6.73)	0	0.10(0.09, 0.10)
	20	18.55(8.83, 38.70)	0	<b>3.00</b> (0.71, 7.35)	0	0.12(0.11, 0.13)
	30	<b>20.55</b> (11.20, 33.99)	0	<b>3.21</b> (1.50, 5.57)	0	0.13(0.12, 0.14)
20	10	13.25(5.95, 26.30)	0	<b>1.81</b> (0.18, 5.45)	0	0.11(0.11, 0.13)
	20	16.96(9.06, 28.10)	0	<b>2.71</b> (0.90, 4.89)	0	0.13(0.13, 0.14)
	30	16.11(10.00, 28.53)	0	<b>2.82</b> (1.04, 6.43)	0	0.14(0.13, 0.15)
30	10	10.29(4.13, 20.53)	0	<b>2.28</b> (0.15, <b>9.17</b> )	0	0.13(0.12, 0.14)
	20	12.72(5.87, 21.96)	0	<b>2.32</b> (0.08, 3.94)	0	0.15(0.14, 0.17)
	30	12.40(8.34, 19.82)	0	<b>2.78</b> (1.06, 5.98)	0	0.17(0.15, <b>0.18</b> )
Avg.		13.60(7.83, 28.53)	<b>0%</b>	<b>2.63(0.65, 6.17)</b>	<b>0%</b>	<b>0.13(0.12, 0.14)</b>

N.O.: Number of optimal solutions obtained in 25 problems for each level of problem instances.

The proposed two-phase heuristic is useful to solve the problem especially for real industrial problem size. It can be improved by applying stronger lower bound of the problem. The two-phase heuristic always obtains better solutions than model P1r and its second phase significantly improves the initial solution obtained by the first phase. FAL formulation has an important advantage than standard formulation (AGG), its LP relaxation provides a very strong lower bound of the problem. In practice, there are EOL products with multi-level disassembly structure. In addition, there are industries with limited disassembly capacity.

The impact of considering disposal decision on the total cost of disassembly planning is analyzed for the 25 problem instances with  $N = 30$  number of items and  $T = 30$  periods. Five different problem instances for each five different product disassembly structures are used as a benchmark for comparison. Table 4.9 provides the results of maximum cost reduction for each structure. The cost reduction can attain around 58% by considering disposal in disassembly planning, this makes the opportunity to make disassembly system more profitable. In the following, we extend the problem studied in this section by considering multi-level products structure with capacity constraints, and new models and methods are proposed.

### 4.3 Capacitated multi-product disassembly lot sizing problem with disposal for the multi-level products disassembly structure with parts commonality

#### 4.3.1 Problem description

Multi-level products disassembly structure is considered in this section. In this case, surplus inventory issue is more challenging because EOL products will be disassembled into their subassembly parts before obtaining leaf items to satisfy the demands for the leaf items. Disposal decisions are considered to handle this surplus inventory of both subassembly parts and leaf items to make opportunities for cost saving in setup, disassembly operations, and inventory holding costs. In addition, capacity restrictions are considered for the real environment. As in production planning problems, resource capacity constraint is an important consideration in disassembly planning problems. The capacity limitation in a period is considered on the time of disassembly carried out during this period. In other words, there is an upper limit on the amount of time available at each period of the planning horizon so that each disassembly operation affected to a period consumes a portion of the time available at that period (Hrouga (2016)).

The root items are the EOL products to be disassembled, and leaf items are the demanded parts and cannot be disassembled further. A child item expresses an item with at least one parent; similarly, a parent item has one or more child items. An example of the structure with three levels is given in Fig. 4.2. Items 1–3 are root items (parents or EOL products). The number in parenthesis is the yield of the item when one unit of its parent (parents) is disassembled. The third (lowest) level represents leaf items (8–12). The parts commonality implies that a given item may have more than one parents. This is one of change in product design nowadays which bring several benefits such as cost saving in production costs. In the example, the shades boxed represent common items (item 4, 6, 9 and 10).

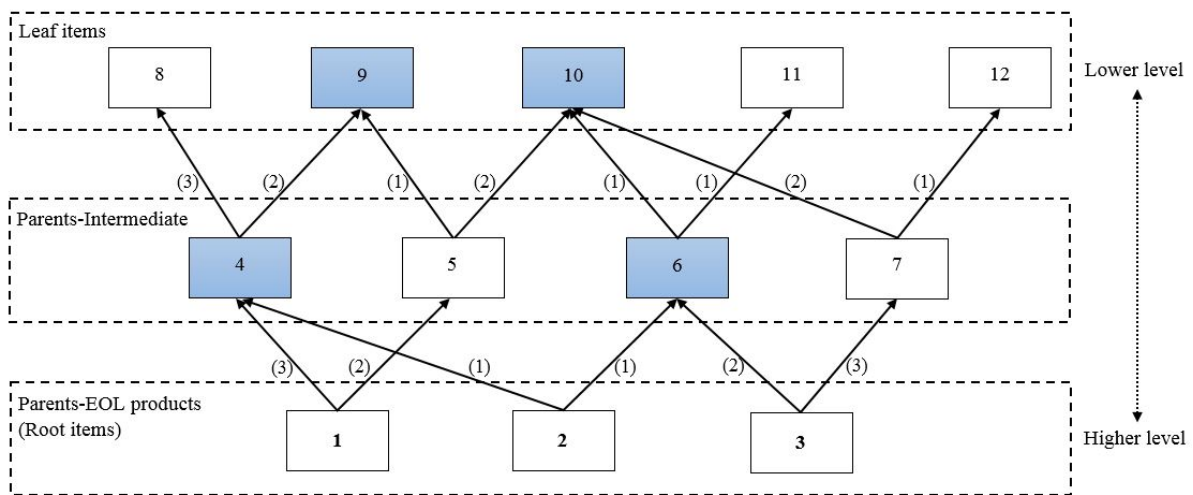


Fig. 4.2 Example of multi-product disassembly structure for the multi-level products structure with parts commonality.

The problem can be defined as determining the quantity and timing of disassembling EOL products and their subassemblies in order to fulfill the demand of leaf items over a finite planning horizon, while

respecting the capacity limitations in each period. The unnecessary subassemblies and leaf items will be disposed of the disassembly system.

In the proposed models, without loss of generality, all items ( $N$ ) are numbered with integers:  $1, 2, \dots, i_r, i_l, \dots, N$ , where  $i_r$  is the index for the root items and the index that are  $\geq i_l$  represent leaf items. The following notations and parameters are used in this section:

- $i$ : Index for items ( $1, 2 \dots i_r, i_{r+1} \dots i_{l-1}, i_l \dots N$ )
- $t$ : Index for periods ( $1, 2 \dots T$ )
- $Mit$ : Arbitrary big number considered for parent item  $i$  in period  $t$
- $s_{it}$ : Setup cost of disassembling root item  $i$  in period  $t$
- $p_{it}$ : Disassembly operation cost of root item  $i$  in period  $t$
- $a_{ij}$ : Number of unit of item  $j$  obtained by disassembly of one unit of item  $i$
- $h_{it}$ : Inventory holding cost of item  $i$  in period  $t$
- $d_{it}$ : Demand of leaf item  $i$  in period  $t$
- $g_{it}$ : Disassembly time for one unit of item  $i$  in period  $t$
- $G_t$ : Available capacity (time) in period  $t$
- $\Phi(i)$ : Parents of item  $i$
- $\eta(i)$ : Children of parent item  $i$

The decision variables for the capacitated disassembly lot sizing problem with disposal and multi-level multi-product structure with parts commonality are defined as follows:

- $Y_{it}$ : 1 if there is a setup in period  $t$  for item  $i$ , and 0 otherwise
- $X_{it}$ : Disassembly quantity of item  $i$  in period  $t$
- $E_{it}$ : Disposed quantity of item  $i$  in period  $t$
- $I_{it}$ : Inventory level of item  $i$  at the end of period  $t$

We next develop the standard formulation AGG for the problem. FAL formulation cannot be obtained because of high complex relationship between root items, subassemblies, and leaf items.

### 4.3.2 Aggregate formulation (AGG)

$$[\mathbf{P3}] \text{ Min} \left\{ \sum_{i=1}^{i_l-1} \sum_{t=1}^T s_{it} \cdot Y_{it} + \sum_{i=1}^{i_l-1} \sum_{t=1}^T p_{it} \cdot X_{it} + \sum_{i=i_r+1}^N \sum_{t=1}^T h_{it} \cdot I_{it} \right\} \quad (4.27)$$

Subject to

$$I_{it} = I_{it-1} + \sum_{k \in \Phi(i)} a_{ki} \cdot X_{kt} - E_{it} - X_{it} \quad \forall i = i_r + 1 \dots i_l - 1 \ \& \ t = 1 \dots T \quad (4.28)$$

$$I_{it} = I_{it-1} + \sum_{k \in \Phi(i)} a_{ki} \cdot X_{kt} - E_{it} - d_{it} \quad \forall i = i_l, i_l + 1 \dots N \text{ \& } t = 1 \dots T \quad (4.29)$$

$$\sum_{i=1}^{i_l-1} g_{it} \cdot X_{it} \leq G_t \quad \forall t = 1 \dots T \quad (4.30)$$

$$X_{it} \leq M_{it} \cdot Y_{it} \quad \forall i = 1, 2 \dots i_l - 1 \text{ \& } t = 1 \dots T \quad (4.31)$$

$$X_{it} \geq 0 \text{ \& } \textit{integer} \quad \forall i = 1, 2 \dots i_l - 1 \text{ \& } t = 1 \dots T \quad (4.32)$$

$$I_{it} \geq 0 \quad \forall i = i_r + 1 \dots N \text{ \& } t = 1 \dots T \quad (4.33)$$

$$E_{it} \geq 0 \quad \forall i = i_r + 1 \dots N \text{ \& } t = 1 \dots T \quad (4.34)$$

$$Y_{it} = 0 \textit{ or } 1 \quad \forall i = 1, 2 \dots i_l - 1 \text{ \& } t = 1 \dots T \quad (4.35)$$

Objective function (4.27) is to minimize the sum of setup, disassembly operation, and inventory holding costs over the  $T$ -period horizon. Constraints (4.28) and (4.29) are the inventory balance equations for the parent items and leaf items, respectively (Note that  $I_{i0}=0$  for all items). Constraints (4.30) respect capacity limits on available time in each period. Constraints (4.31) guarantee that a setup cost for item  $i$  is performed in period  $t$  if any disassembly operation of item  $i$  is done in that period. Constraints (4.32–4.35) impose the non-negativity and binary restrictions on the variables. Note that in the model P3, the variables  $I_{it}$  and  $E_{it}$  can be set as real. We define [P3r] as the LP relaxation of P3 by removing the integrality constraints (4.32).

### 4.3.3 Exact resolution

Model P3 is MIP so it can be directly used to obtain optimal solutions by using CPLEX solver, but its application is limited to the small-sized problems. Also, Kim and Xirouchakis (2010) showed that multi-product disassembly lot sizing problem with capacity constraints is NP-hard. Therefore, by analogy, the problem considered in this section is NP-hard. CPLEX solver involves several complex techniques to solve hard MIP problems. Applying efficient strategies to reduce the number of integer variables can help CPLEX solver to improve computational time for solving (Achterberg and Wunderling (2013); Manual (1987)). Decomposition heuristics such as Fix-and-optimize (FO) are efficient, because of the advantage that they can solve the complex problem in short computational times by successively solving sub-problems with limited number of integer variables. In the following, Fix-and-Optimize (FO) heuristic is proposed to solve large-sized and more complex problems efficiently.

### 4.3.4 Fix-and-Optimize heuristic (FO)

The Fix-and-Optimize heuristic has been widely used in the literature to solve the lot sizing problem. Sahling et al. (2009) first propose a FO to solve the multi-level lot sizing problem with capacity constraints. They present three principles for determining sub-problems: product, resource, and process-oriented. In Helber and Sahling (2010), the authors reapply the same FO approach for the multi-level case and they obtain a good solution quality in reasonable CPU time by solving a series of sub-problems with a set of fixed binary variables.

FO is a decomposition heuristic which consists of successively solving sub-problems using exact integer linear program solving techniques. A sub-problem is defined by separating the set of variables of

the general problem (multi-product, multi-level capacitated disassembly lot sizing with part commonality and disposal) into two subsets. The first sub-set consists of variables to be optimized (depending on the function to be minimized and the constraints of the general problem) and the second sub-set consists of the other variables whose values are fixed by adding constraints in the general problem. In this study, variables are partitioned according to:

- **Period-oriented decomposition:** Each subproblem is decomposed according to periods and a subset of variables are optimized in a forward or backward sequence, from the first to the last or from the last to the first.
- **Product-oriented decomposition:** Each subproblem is decomposed according to items and a subset of variables are optimized by starting from lowest or highest level, from root items to leaf items or from leaf items to root items.

The subproblem is thus modeled by a MIP, identical to the one modeling the general problem with the additional constraints allowing to fix the variables of the second subproblem. The heuristic works by iteration by modifying at each iteration the sub-set of the fixed variables and by integrating the values of the variables obtained at the previous iteration. It is therefore necessary to generate an initial solution to launch the heuristic. In this research, an initial solution is generated by setting the disassembly operation to be performed in every period and we consider a period-oriented problem decomposition strategy.

A pseudo-code of FO period-oriented from the first period to the last period (1- $T$ ) is presented in Algorithm 2. Let  $E = \{X_{it}, Y_{it}, I_{it}, \text{ and } E_{it}\}$  be the set of solution of model P3,  $TW_e^s = \{s, e\}$  be a time window between two given periods  $s$  and  $e$  ( $1 \leq s \leq e \leq T$ ). We define  $SP3_e^s$  as the solution of a sub-problem with  $TW_e^s$ : all variables for all  $i, t < s$ , and  $t > e$  are fixed to the related values of current best solution of model P3 ( $\bar{E}$  = current best solution) and the remaining variables are optimized. We note that an initial solution is obtained by fixing all setup variables within 120 seconds computational times and we set CPLEX solver gap as zero.

---

**Algorithm 2** Fix-and-Optimize (FO) period-oriented algorithm

---

- 1: **Input:** initial solution ( $E^I = \{X_{it}, Y_{it}, I_{it}, E_{it}\}$ ) obtained by solving the model P3 when setting  $Y_{it} = 1$  for all  $i$  and  $t$ , period decomposition started from 1 to  $T$ .  
**Output:** final solution.
  - 2: Set the initial solution as the current best solution of the model P3 ( $\bar{E} = E^I$ )
  - 3:  $s \leftarrow 1$  &  $e \leftarrow 1$
  - 4: **while**  $s$  &  $e \leq T$  **do**
  - 5:     Solve sub-problem  $SP3_e^s$ :
  - 6:     All variables for all  $i$  and  $t \in \{s, e\}$  are optimized,
  - 7:     Other variables are fixed to their value in  $\bar{E}$ .
  - 8:     **if** the solution of  $SP3_e^s$  is better than  $\bar{E}$  **then**
  - 9:         Set  $\bar{E} \leftarrow SP3_e^s$
  - 10:          $s \leftarrow e + 1$  &  $e \leftarrow s$
  - 11:     **else**
  - 12:          $e \leftarrow e + 1$
  - 13:     **end if**
  - 14: **end while**
-



**Interrelatedness between variables**

We propose another FO approach for the problem by considering interrelationship characteristic between variables. Various researches propose different methods to control the number of variables to be optimized in each MIP subproblem of FO approach (e.g., Seeanner et al. (2013); Chen (2015)). Similar to the FO approach of Chen (2015), we consider interrelationship characteristic to define variables to be optimized in each MIP subproblem of FO approach. In model P3, the binary setup variables are interrelated: if the value of  $Y_{it}$  changes, the value of its disassembly quantity  $X_{it}$  can also change according to coupling setup costs constraints (4.31). The change of  $X_{it}$  may result in a change of  $X_{it-1}$  and  $X_{it+1}$  due to the inventory balance connection between two linked periods imposed by constraints (4.28) and (4.29). This change can cause the change of  $X_{i't}$ ,  $i' \in \Phi(i)$  and  $i' \in \eta(i)$  due to the connection of  $X_{i't}$  with  $X_{it}$  in the disassembly structure and capacity constraints (4.30). As a consequence, if there is a change in  $Y_{it}$ , there will be a change in  $Y_{it-1}$ ,  $Y_{it+1}$ , and  $Y_{i't}$ ,  $i' \in \Phi(i)$  and  $i' \in \eta(i)$ . In the same manner there is a linkage between other variables such as  $I_{it}$  and  $E_{it}$ .

Above linkage between variables is called Interrelatedness (Chen (2015)) which can be defined as: two variables  $Y_{it}$  and  $Y_{i't}$  are directly or 1-step interrelatedness if  $i = i'$  and  $t' \in t - 1, t, t + 1$  or  $i' \in \Phi(i) \cup \eta(i)$ . We define  $\Omega_{it}^l(Y_{it})$  as the set of  $l$ -step Interrelatedness of setup variable  $Y_{it}$ , which implies there is a series of setup variable of items which are interrelated with setup variables of item  $i$  (i.e.,  $Y_{it-1}$ ,  $Y_{it}$ , and  $Y_{it+1}$ ). Figure 4.3 represents 1, 2, 3, and 4-step interrelatedness for leaf item 1 of the example structure shown in Fig. 4.2. For example, item 1 has a 2-step interrelatedness with items 2, 4, 5, 8, 9, 10, so its 2-step interrelatedness set of setup variables can be found as  $\Omega_{1t}^2(Y_{1t}) = \{Y_{1t-1}, Y_{1t}, Y_{1t+1}, Y_{2t-1}, Y_{2t}, Y_{2t+1}, Y_{4t-1}, Y_{4t}, Y_{4t+1}, Y_{5t-1}, Y_{5t}, Y_{5t+1}, Y_{8t-1}, Y_{8t}, Y_{8t+1}, Y_{9t-1}, Y_{9t}, Y_{9t+1}\}$ . We note that maximum  $l$ -step interrelatedness is obtained when the set of  $l$ -step and  $l + 1$ -step interrelatedness are identical for each item in the product disassembly structure.

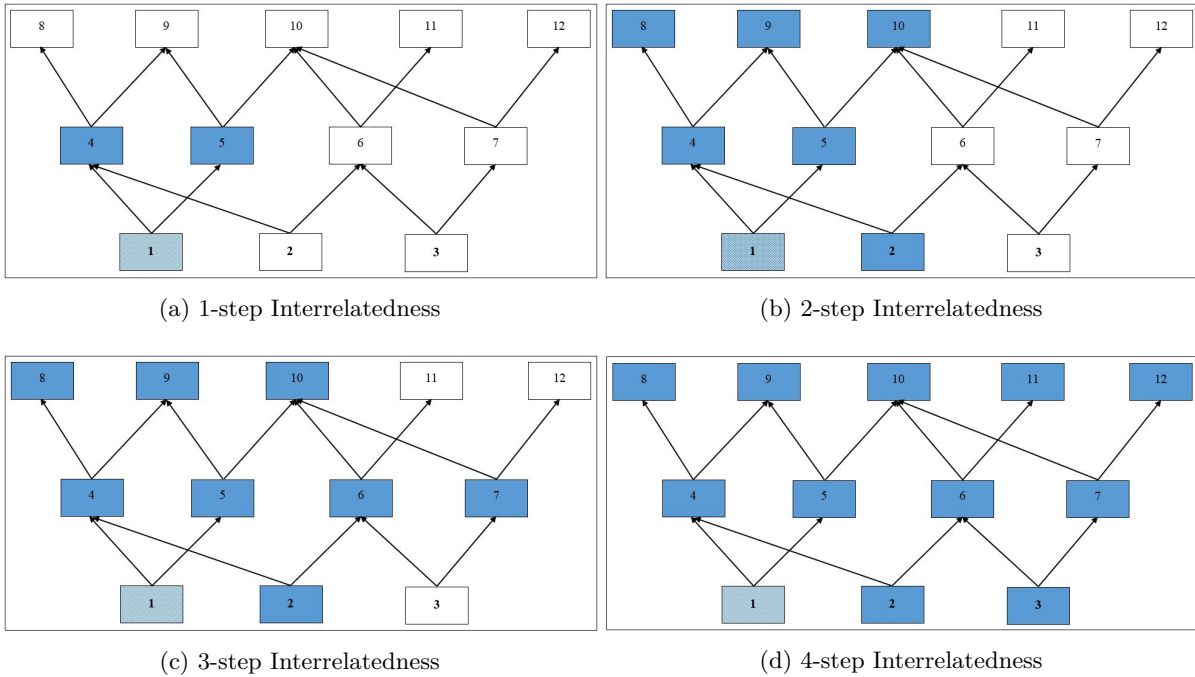


Fig. 4.3 Different  $l$ -step Interrelatedness for leaf item 1 of the example structure in Fig. 4.2.

From above discussion of interrelatedness characteristic between variables, we can define the subproblems for the new FO approach for the problem. We define  $\Omega'_{it}(Y_{it})$  as  $\{Y_{it}, i \in N \text{ and } t \in T\} \setminus \Omega^l_{it}(Y_{it})$ . For a given  $l$ -step interrelatedness ( $l \geq 1$ ), the subproblem of P3 with  $l$ -step associated with setup variable  $Y_{it}$  is defined by  $SP3^l_{i,t}$ , which consists in fixing all setup variables in  $\Omega'_{it}(Y_{it})$  and re-optimizing setup variables in  $\Omega^l_{it}(Y_{it})$ . It is obvious that increasing parameter  $l$  leads to increase computational time due to the larger set  $\Omega'_{it}(Y_{it})$  which means more setup variables to be re-optimized. In our numerical results, as an example, we consider  $l=1$  (1-step) and 5 (5-step). Pseudo-code of FO approach is presented in Algorithm 3:

---

**Algorithm 3** FO  $l$ -Interrelatedness with Maximum iteration  $N \times T$

---

- 1: **Input:** initial solution ( $E^I = \{X_{it}, Y_{it}, I_{it}, E_{it}\}$ ) obtained by solving the model P3 when setting  $Y_{it} = 1$  for all  $i$  and  $t$ , set of  $l$ -step interrelatedness of item  $i$ .
  - Output:** final solution.
  - 2: Set the initial solution as the current best solution of the model P3 ( $\bar{E} = E^I$ );
  - 3:  $Iteration \leftarrow 1$ ;
  - 4: **while**  $Iteration \leq N \times T$  **do**
  - 5:     Select randomly a pair  $(i, t)$  from  $N \times T$  possible choices with the same probability;
  - 6:     Solve sub-problem  $SP3^l_{i,t}$ ;
  - 7:     All  $l$ -step Interrelatedness variables for  $i$  and  $t$  are optimized i.e., Set of  $\Omega^l_{it}(Y_{it}, X_{it}, I_{it}, E_{it})$ ;
  - 8:     Other variables are fixed to their value in  $\bar{E}$ ;
  - 9:     **if** the solution of  $SP3^l_{i,t}$  is better than  $\bar{E}$  **then**
  - 10:         Set  $\bar{E} \leftarrow SP3^l_{i,t}$
  - 11:     **end if**
  - 12:      $Iteration \leftarrow Iteration + 1$
  - 13: **end while**
- 

We note that an initial solution is obtained within 120 seconds computational times and we set CPLEX solver gap as the default value settings. As an expectation, Algorithm 3 may obtain different solutions (solution qualities) in each execution. The reason is that a pair  $(i, t)$  is chosen randomly. To guarantee obtaining the same solution in each execution of Algorithm 3, We choose a pair  $(i, t)$  in lexicography order by starting  $i$  from 1 to  $N$  and  $t$  from 1 to  $T$ . We have tested four variants to choose a pair  $(i, t)$  based on lexicography order: (1) from 1 to  $N$  and from 1 to  $T$ , (2) from 1 to  $N$  and from  $T$  to 1, (3) from  $N$  to 1 and from 1 to  $T$ , (4) from  $N$  to 1 and from  $T$  to 1. In the numerical results we report only the first one as it obtains a lower overall average of gap (%).

### 4.3.5 Computational experiments

The model and methods are implemented by using Java programming language. All tests are run on a system with an Intel Core i7-7700T, 2.9 GHz, and 16 Go RAM on Windows 10. We use CPLEX solver 12.8 to solve the problems. We note that CPLEX solver with default parameter settings could not obtain the solution of several problem instances (in particular, large-sized ones) because of insufficient available memory. We set 1024 megabytes memory available for working storage of CPLEX to solve the problem instances. We note that CPLEX still could not obtain the optimal solution because of out of memory but we could obtain its best solution obtained within 3600s limited time. We set other CPLEX

solver settings as default (such as allowing cuts and presolved processes). CPLEX is terminated when CPU time reached 3600 seconds. The percentage deviations from the optimal solution obtained (lower bound, for the cases that CPLEX cannot obtain the optimal solution within limited 3600 seconds), and CPU seconds are used to evaluate the performance of the model and methods. We mention that we use the following formula to calculate the Gap (%):

$$Gap(\%) = \frac{UB - LB}{LB} * 100 \quad (4.36)$$

### Benchmark

The generation of the test instances for the considered problem in section 4.3 is presented in this section. The parameter setting presented in Section 4.2 is used to randomly generate problem instances. We have modified the inventory holding cost with regard to setup cost by using the formula (3.54) to obtain TBO=2. We generate 225 problem instances, i.e. 25 problem instances for each combination of three levels of the number of items (N) (10, 20, and 30) and three levels of number of periods (T) (10, 20, and 30). Five disassembly structures are generated for each level of the number of items and for each disassembly structure, five different problems are generated. The number of root items and common items are generated from DU(2, N/5) and DU(1,  $\lfloor N/3 \rfloor$ ), respectively, where  $N$  is the number of items and  $\lfloor \bullet \rfloor$  means the largest integer value less than or equal to  $\bullet$ . Finally, The number of leaf items is generated from DU(2, 5), DU(5, 10), and DU(10, 15) for the problems with 10, 20, and 30 items, respectively. Here, DU( $b_1$ ,  $b_2$ ) means the discrete uniform distribution with a range of [ $b_1$ ,  $b_2$ ].

Different methods are proposed in the literature to generate the problem instances for the problem with capacity constraints. We use the method proposed by Kim and Xirouchakis (2010) for the disassembly lot sizing problem in order to generate feasible problem instances with capacity restrictions: **Step 1:** Initial available capacity per period is generated as 400, 480, and 540 with probabilities 0.2, 0.5, and 0.3, respectively.

**Step 2:** The uncapacitated problem is solved by performing the disassembly operation in every period, then the disassembly quantity variables ( $X_{it}$ ) are obtained.

**Step 3:** The total available capacities (TC) and the overall used capacity (UC) are calculated for the solution obtained in Step 2 as follows:

$$TC = \sum_{t=1}^T G_t$$

$$UC = \sum_{i=1}^{i_{t-1}} \sum_{t=1}^T g_{it} \cdot X_{it}$$

Note that  $G_t$  is the initial capacity generated by Step 1.

**Step 4:** The initial capacity is modified by using  $G'_t = \alpha \cdot CU/TC \cdot G_t$ . Note that  $\alpha$  is selected to be equal to 1.25, but it will be increased by 0.05 if feasibility is not obtained .

## Numerical results

We first evaluate the performance of FO approach (four variants: 1 to  $T$ ,  $T$  to 1, 1 to  $N$ , and  $N$  to 1) and compare with models P3 and P3r. Tables 4.10 and 4.11 summarizes the result of gap percentages and computational times, respectively. Model P3 by using CPLEX solver cannot obtain the optimal solution of all problem instances within 3600 seconds limited times and its overall average gap is 0.74%. The gap of CPLEX solver when solving model P3 ( i.e., MIP model) can attain a maximum of 13.33% within 3600s limited computational time. Computational time of CPLEX solver increases significantly when increasing the size of the problem (e.g., average of CPU times for the problem instances with  $N = 30$  and  $T = 30$  is 3584.97, with minimum and maximum of 3224.14 and 3600.00 seconds, respectively. This implies that model P3 is not efficient for the large-sized problems, and we cannot guarantee that CPLEX solver can solve very large-sized of the problem.) LP relaxation of model P3 is very fast but it can obtain a very weak lower bound of the problem so that its Gap can attain 95.49%. We observe that FO approaches (all four variants) obtain smaller average of gap when solving large-sized problem instances i.e.,  $N = 30$  number of items. FO approaches are very fast and it can solve all the problem instances. Among the four variants of FO approach, both FO approaches with period-oriented (1 to  $T$  and  $T$  to 1) work better than product-oriented. For the tested problem instances, FO period-oriented with forward sequence (1 to  $T$ ) obtained an overall average of 11.27%. FO approaches product-oriented are faster than period-oriented with overall maximum CPU of 5.51 and 12.73 seconds, respectively. FO approaches always gives better solutions than LP relaxation approaches and they are efficient to improve the initial solution: overall maximum improvement of gap (%) is 14.97, 15.32, 11.56, and 8.32 for the FO approaches with lexicography order of 1 to  $T$ ,  $T$  to 1, 1 to  $N$ , and  $N$  to 1, respectively. But, the gap percentage can be significant for some tested problems instances (e.g, 45.30% for the problem with  $N = 20$  and  $T = 20$ ). We proposed considering interrelatedness characteristic between variables to improve the gap performance of FO approaches.

We next evaluate the performance of our new FO approach for the multi-product capacitated disassembly lot sizing product with disposal for the multi-level disassembly structure with parts commonality. We solve 45 problem instances i.e., we consider a given disassembly structure for each level of number of items and five different problem instances are generated for each level of number of periods. Tables 4.12, 4.13, and 4.14 show the result of the comparison of model P3 and its LP relaxation (P3r) solved by using CPLEX solver, initial solution, FO approach period-oriented (1 to  $T$ ) (FO1), new FO approach for 1-step and 5-step interrelatedness using Algorithm 3 (FO2), FO2 approach with lexicography order (1 to  $T$  and 1 to  $N$ ) for 1-step and and 5-step interrelatedness (FO3).

Tables 4.12, 4.13, and 4.14 presents the results for problem instances with  $N = 10$ ,  $N = 20$ , and  $N = 30$  number of items, respectively. The results show that considering interrelatedness characteristic between variables can improve significantly the gap of classic FO approach, especially for the larger size of the problem instances. For example, for the problem instances with  $N = 10$ ,  $N = 20$ , and  $N = 30$  number of items, maximum improvement of gap (from classic FO approach (1 to  $T$ )) by considering interrelatedness between variables can attain 15.33 (i.e., 25.73  $\rightarrow$ 10.40), 13.36 (i.e., 16.14  $\rightarrow$  2.78), and 10.53 (i.e., 20.81  $\rightarrow$  10.28), respectively. Also, increasing the level of interrelatedness between variables can significantly improve the gap percentage. For example, the gap improvement from 1-step to 5-step interrelatedness for the problem instance with  $N = 10$  and  $T = 10$  can attain 17.21 (i.e., 25.82  $\rightarrow$  8.61). This implies the importance of considering interrelatedness characteristic when applying FO approach. Both FO2 and FO3 approaches are efficient to improve the gap of FO approach without considering

interrelatedness (FO1). FO approaches with interrelatedness are fast to solve the tested problem instances, maximum CPU time is 25.28 seconds. As expected, the computational time of FO approaches with interrelatedness increases when increasing higher level of interrelatedness between variables. The reason is that higher number of connected variables will be re-optimized in each subproblem of the FO algorithm. Model P3 by using CPLEX solver cannot obtain optimal solution for large-sized problem instances within 3600 seconds. FO approaches with interrelatedness (FO2 and FO3) can improve the gap of solution obtain by CPLEX solver (model P3). For example, for the problem instance with  $N = 30$  and  $T = 30$  (instance 3), FO3 (5-step) obtain an improvement of gap 5.51% (i.e.,  $11.17 \rightarrow 7.66$ ) in a reasonable CPU time (20.29 seconds). This means FO approaches are efficient for the large-sized real problem instances, unlike the CPLEX solver.

Table 4.10 Test results of FO approach, period-oriented and product-oriented (Gap (%)).

$N$	$T$	P3	P3r	IS	FO (1- $T$ )	FO ( $T-1$ )	FO (1- $N$ )	FO ( $N-1$ )
10	10	<b>0.01</b> (0.00, 0.01)	44.43 (13.24, 85.53)	13.61 (5.31, 31.02)	7.57 (2.05, 16.05)	8.12 (2.63, 20.98)	10.75 (0.00, 26.78)	11.16 (2.71, 26.78)
	20	<b>0.01</b> (0.01, 0.01)	39.93 (22.65, 83.56)	17.55 (6.48, 36.55)	11.86 (3.45, 27.45)	12.29 (4.64, 24.97)	14.47 (3.41, 35.80)	15.20 (5.14, 35.80)
	30	<b>0.07</b> (0.01, 1.34)	34.61 (17.50, 62.24)	17.68 (7.82, 28.85)	12.47 (5.95, 20.71)	12.45 (5.89, 20.02)	13.69 (6.63, 21.72)	14.57 (6.16, 26.96)
20	10	<b>0.01</b> (0.00, 0.01)	54.59 (20.38, 95.49)	18.43 (7.73, 42.31)	12.03 (4.31, 35.87)	12.33 (3.22, 35.11)	16.00 (6.08, 40.02)	16.30 (6.34, 38.84)
	20	<b>0.53</b> (0.01, 3.08)	39.16 (21.30, 77.21)	19.48 (9.67, 48.16)	14.33 (6.32, 36.42)	14.17 (5.96, 36.42)	17.04 (8.35, 45.13)	17.45 (8.67, 45.30)
	30	<b>2.28</b> (0.01, 13.33)	36.65 (19.89, 75.25)	21.19 (11.76, 32.82)	17.02 (8.67, 26.30)	16.99 (7.51, 28.22)	18.60 (10.23, 31.41)	19.06 (10.17, 31.27)
30	10	<b>0.01</b> (0.00, 0.01)	36.13 (19.10, 57.15)	10.75 (5.97, 17.18)	7.04 (3.19, 12.00)	7.13 (3.12, 13.33)	9.71 (3.63, 16.17)	9.77 (4.25, 17.14)
	20	<b>0.93</b> (0.01, 5.10)	29.27 (16.78, 55.92)	11.51 (6.06, 26.51)	8.59 (4.54, 20.81)	8.70 (4.16, 21.24)	10.67 (5.37, 23.68)	10.57 (4.10, 24.53)
	30	<b>2.80</b> (0.01, 11.17)	25.70 (17.23, 40.51)	13.24 (8.18, 22.17)	10.52 (6.92, 17.89)	10.52 (6.67, 18.43)	12.29 (7.36, 21.35)	12.38 (7.37, 21.55)
Overall		<b>0.74</b> (0.00, 13.33)	37.83 (13.24, 95.49)	15.93 (5.31, 48.16)	11.27 (2.05, 36.42)	11.41 (2.63, 36.42)	13.66 (0.00, 45.13)	14.05 (2.71, 45.30)

P3(MIP): Mixed-Integer Programming; P3r: LP relaxation of P3; IS: Initial Solution; FO (1- $T$ ): Fix-and-Optimize period-oriented from 1 to  $T$ ; FO ( $T-1$ ): Fix-and-Optimize period-oriented from  $T$  to 1; FO (1- $N$ ): Fix-and-Optimize product-oriented from 1 to  $N$ ; FO ( $N-1$ ): Fix-and-Optimize product-oriented from  $N$  to 1; (\*):  $\frac{Gap(\%):Mean}{Gap(\%):(Min,Max)}$

Table 4.11 Test results of FO approach, period-oriented and product-oriented (CPU (s)).

$N$	$T$	P3	P3r	IS	FO (1- $T$ )	FO ( $T$ -1)	FO (1- $N$ )	FO ( $N$ -1)
10	10	0.72 (0.16, 1.57)	<b>0.01</b> (0.01, 0.01)	0.41 (0.12, 1.68)	0.20 (0.09, 0.47)	0.15 (0.08, 0.26)	0.09 (0.04, 0.27)	0.09 (0.03, 0.36)
	20	8.80 (1.49, 32.94)	<b>0.01</b> (0.01, 0.02)	11.06 (0.17, 120.00)	0.72 (0.27, 1.67)	0.48 (0.19, 1.72)	0.21 (0.05, 1.43)	0.22 (0.06, 1.40)
	30	393.03 (2.40, 3600.00)	<b>0.01</b> (0.01, 0.02)	18.96 (0.19, 120.00)	1.07 (0.44, 2.52)	0.67 (0.32, 1.62)	0.37 (0.06, 2.02)	0.34 (0.07, 1.39)
20	10	18.43 (0.82, 112.44)	<b>0.01</b> (0.01, 0.02)	57.43 (0.21, 120.00)	0.60 (0.12, 1.56)	0.56 (0.11, 1.55)	0.61 (0.11, 1.56)	0.62 (0.10, 2.28)
	20	1619.49 (17.59, 3600.00)	<b>0.01</b> (0.01, 0.01)	26.01 (0.18, 120.00)	0.63 (0.28, 1.53)	0.66 (0.30, 1.80)	0.37 (0.13, 1.26)	0.37 (0.14, 1.24)
	30	3480.94 (623.43, 3600.00)	<b>0.02</b> (0.01, 0.02)	30.02 (0.34, 120.00)	0.91 (0.53, 1.80)	0.86 (0.50, 1.84)	0.45 (0.18, 1.15)	0.48 (0.19, 1.22)
30	10	47.97 (2.03, 255.61)	<b>0.01</b> (0.01, 0.01)	15.57 (0.16, 120.00)	0.40 (0.14, 1.36)	0.36 (0.14, 1.26)	0.38 (0.15, 1.32)	0.35 (0.14, 1.29)
	20	3063.24 (13.96, 3600.00)	<b>0.02</b> (0.01, 0.02)	25.22 (0.18, 120.00)	0.68 (0.39, 1.39)	0.71 (0.35, 1.40)	0.54 (0.22, 1.47)	0.49 (0.26, 1.16)
	30	3584.97 (3224.14, 3600.00)	<b>0.03</b> (0.02, 0.06)	2.83 (0.29, 11.21)	2.13 (0.64, 12.73)	2.44 (0.62, 10.81)	0.97 (0.30, 5.51)	0.73 (0.31, 5.23)
Overall		1357.51 (0.16, 3600.00)	<b>0.01</b> (0.01, 0.06)	20.83 (0.12, 120.00)	0.82 (0.09, 12.73)	0.76 (0.08, 10.81)	0.44 (0.04, 5.51)	0.41 (0.03, 5.23)

P3(MIP): Mixed-Integer Programming; P3r: LP relaxation of P3; IS: Initial Solution; FO (1- $T$ ): Fix-and-Optimize period-oriented from 1 to  $T$ ; FO ( $T$ -1): Fix-and-Optimize period-oriented from  $T$  to 1; FO (1- $N$ ): Fix-and-Optimize product-oriented from 1 to  $N$ ; FO ( $N$ -1): Fix-and-Optimize product-oriented from  $N$  to 1; (\*):  $\frac{CPU(s):Mean}{CPU(s):(Min,Max)}$

Table 4.12 Test results of new FO approach for N=10 items and a disassembly structure with three levels which consists of two root items, three leaf items, and three items in common.

T	Instance	P3	P3r	IS	FO1	FO2		FO3	
						1-step	5-step	1step	5-step
10	1	<b>0.01</b> *	<u>27.55</u>	<u>8.61</u>	<u>4.55</u>	<u>8.61</u>	<u>4.26</u>	<u>8.61</u>	<u>4.26</u>
		0.22	<b>0.01</b>	0.23	0.11	0.52	1.56	0.34	2.38
	2	<b>0.01</b>	<u>24.66</u>	<u>5.31</u>	<u>3.39</u>	<u>3.11</u>	<u>1.97</u>	<u>3.11</u>	<u>2.51</u>
		0.53	<b>0.01</b>	0.22	0.20	0.72	1.63	0.39	1.62
	3	<b>0.01</b>	<u>47.19</u>	<u>8.14</u>	<u>2.05</u>	<u>7.21</u>	<u>1.91</u>	<u>4.72</u>	<u>0.85</u>
	0.29	<b>0.01</b>	0.20	0.19	0.48	2.31	0.34	2.21	
	4	<b>0.01</b>	<u>39.35</u>	<u>19.41</u>	<u>9.79</u>	<u>11.63</u>	<u>3.81</u>	<u>11.60</u>	<u>3.11</u>
		0.69	<b>0.01</b>	0.17	0.16	0.65	2.29	0.47	2.10
	5	<b>0.01</b>	<u>71.67</u>	<u>13.80</u>	<u>13.80</u>	<u>12.11</u>	<u>3.29</u>	<u>12.11</u>	<u>2.12</u>
		0.69	<b>0.01</b>	0.38	0.38	0.55	1.99	0.41	2.46
20	1	<b>0.01</b>	<u>83.56</u>	<u>26.45</u>	<u>14.43</u>	<u>21.59</u>	<u>12.55</u>	<u>19.03</u>	<u>11.64</u>
		7.08	<b>0.01</b>	0.52	0.29	1.14	3.32	1.08	3.23
	2	<b>0.01</b>	<u>50.58</u>	<u>36.55</u>	<u>25.73</u>	<u>25.82</u>	<u>8.61</u>	<u>23.94</u>	<u>10.40</u>
		3.27	<b>0.01</b>	0.21	0.31	1.49	3.98	1.35	3.26
	3	<b>0.01</b>	<u>31.00</u>	<u>19.64</u>	<u>10.77</u>	<u>12.35</u>	<u>7.87</u>	<u>12.47</u>	<u>8.80</u>
	1.68	<b>0.01</b>	0.20	0.29	1.50	3.57	1.02	3.31	
	4	<b>0.01</b>	<u>56.60</u>	<u>35.75</u>	<u>27.45</u>	<u>26.58</u>	<u>16.31</u>	<u>26.93</u>	<u>16.05</u>
		11.73	<b>0.01</b>	120.00	1.54	2.56	4.98	2.31	4.48
	5	<b>0.01</b>	<u>45.77</u>	<u>17.78</u>	<u>11.92</u>	<u>12.44</u>	<u>5.96</u>	<u>14.61</u>	<u>6.04</u>
		3.88	<b>0.01</b>	0.38	0.27	2.23	3.63	1.22	2.99
30	1	<b>0.01</b>	<u>33.79</u>	<u>15.59</u>	<u>10.34</u>	<u>13.39</u>	<u>6.25</u>	<u>12.56</u>	<u>6.96</u>
		95.40	<b>0.01</b>	0.19	0.53	2.17	6.13	1.56	5.48
	2	<b>0.01</b>	<u>30.02</u>	<u>15.18</u>	<u>9.51</u>	<u>11.88</u>	<u>6.07</u>	<u>11.68</u>	<u>5.99</u>
		3.91	<b>0.01</b>	0.20	0.56	1.79	4.86	1.65	4.73
	3	<b>0.01</b>	<u>49.64</u>	<u>28.85</u>	<u>20.12</u>	<u>22.26</u>	<u>10.95</u>	<u>22.44</u>	<u>10.80</u>
	136.96	<b>0.01</b>	0.29	0.57	2.79	6.23	2.28	8.05	
	4	<b>1.34</b>	<u>56.91</u>	<u>20.63</u>	<u>13.35</u>	<u>14.75</u>	<u>9.72</u>	<u>13.92</u>	<u>10.24</u>
		3600.00	<b>0.01</b>	3.33	0.44	1.97	6.10	1.70	9.74
	5	<b>0.01</b>	<u>39.09</u>	<u>24.21</u>	<u>19.65</u>	<u>18.13</u>	<u>9.94</u>	<u>18.62</u>	<u>9.17</u>
		173.17	<b>0.01</b>	0.23	0.56	2.35	5.06	2.08	5.67
Overall Min.		<b>0.01</b>	<u>24.66</u>	<u>5.31</u>	<u>2.05</u>	<u>3.11</u>	<u>1.91</u>	<u>3.11</u>	<u>0.85</u>
		0.22	<b>0.01</b>	0.19	0.11	0.48	1.56	0.34	1.62
Overall Avg.		<b>0.10</b>	<u>45.83</u>	<u>19.73</u>	<u>13.12</u>	<u>14.79</u>	<u>7.30</u>	<u>14.42</u>	<u>7.26</u>
		269.31	<b>0.01</b>	8.45	1.54	1.53	3.84	1.21	4.12
Overall Max.		<b>1.34</b>	<u>83.56</u>	<u>36.55</u>	<u>27.45</u>	<u>26.58</u>	<u>16.31</u>	<u>26.93</u>	<u>16.05</u>
		3600.00	<b>0.01</b>	120.00	1.54	2.79	6.23	2.31	9.74

**P3**: Mixed-Integer-Programming(MIP); **P3r**: LP relaxation of P3; **IS**: initial solution; **FO1**: Fix-and-Optimize period-oriented; **FO2**: Fix-and-Optimize *l*-step interrelatedness when random selection; **FO3**: Fix-and-Optimize *l*-step interrelatedness when lexicography order selection; (\*):  $\frac{Gap(\%)}{CPU(s)}$



Table 4.13 Test results of new FO approach for N=20 items and a disassembly structure with five levels which consists of four root items, six leaf items, and five items in common.

T	Instance	P3	P3r	IS	FO1	FO2		FO3	
						1-step	5-step	1step	5-step
10	1	<b>0.01</b> *	95.49	24.06	10.75	17.76	3.65	17.08	2.27
		5.19	<b>0.01</b>	0.54	0.15	1.36	5.06	0.88	4.34
	2	<b>0.01</b>	78.77	25.49	14.33	22.69	5.93	22.40	7.11
		118.19	<b>0.01</b>	120.00	0.18	1.80	7.42	2.20	6.68
	3	<b>0.01</b>	78.57	19.15	16.14	14.65	2.91	14.23	2.78
	9.53	<b>0.01</b>	120.00	0.13	1.98	6.82	1.95	5.66	
	4	<b>0.01</b>	58.82	21.88	13.93	18.86	3.39	17.09	4.66
		16.61	<b>0.01</b>	0.53	0.15	2.05	5.14	0.83	4.72
	5	<b>0.01</b>	59.20	16.49	9.78	14.68	3.52	14.68	3.52
		17.00	<b>0.01</b>	120.00	0.13	1.56	8.57	2.10	8.77
20	1	<b>1.66</b>	31.18	12.61	10.53	9.52	5.51	10.65	5.38
		3600.00	<b>0.01</b>	2.79	0.44	2.35	8.65	2.25	9.01
	2	<b>0.01</b>	24.37	12.53	10.25	8.96	7.28	8.91	7.43
		22.71	<b>0.01</b>	0.24	0.34	2.78	10.02	2.47	9.38
	3	<b>0.01</b>	29.23	11.89	8.46	7.91	6.24	5.45	4.64
	1123.77	<b>0.01</b>	0.23	0.58	2.46	5.64	8.41	9.21	
	4	<b>1.76</b>	51.32	19.93	15.45	15.52	10.96	17.14	11.20
		3600.00	<b>0.01</b>	2.15	<b>0.38</b>	2.45	8.83	2.03	8.82
	5	<b>0.01</b>	22.47	12.83	9.58	10.93	4.59	11.08	5.31
		426.21	<b>0.01</b>	0.43	0.44	3.06	9.14	2.67	9.72
30	1	<b>1.18</b>	29.10	17.20	14.42	13.48	8.81	13.78	8.19
		3600.00	<b>0.02</b>	0.78	<b>0.62</b>	4.21	12.65	3.94	14.30
	2	<b>0.76</b>	31.33	19.76	19.76	15.40	7.46	16.26	8.19
		3600.00	<b>0.02</b>	1.53	1.53	5.05	12.44	3.77	11.35
	3	<b>1.53</b>	27.41	12.12	9.95	10.98	7.76	11.11	7.39
	3600.00	<b>0.01</b>	0.54	0.79	4.09	15.68	3.71	14.25	
	4	<b>0.93</b>	23.99	13.38	10.90	10.85	7.23	10.84	7.22
		3600.00	<b>0.02</b>	0.40	0.58	4.15	11.63	3.87	13.06
	5	<b>1.06</b>	21.64	15.07	10.68	12.81	7.50	13.41	6.85
		3600.00	<b>0.02</b>	0.36	0.86	4.76	14.62	4.06	13.62
Overall Min.		<b>0.01</b>	21.64	11.89	8.46	7.91	2.91	8.40	2.27
		519	<b>0.01</b>	0.23	0.13	1.36	5.06	0.83	4.34
Overall Avg.		<b>0.60</b>	44.19	16.96	12.33	13.67	6.13	13.80	6.14
		1795.95	<b>0.01</b>	24.70	0.49	2.94	9.67	2.62	9.53
Overall Max.		<b>1.76</b>	95.49	25.49	19.76	22.69	10.96	22.40	11.20
		3600.00	<b>0.02</b>	120.00	1.53	5.05	15.68	4.06	14.30

**P3r**: LP relaxation of P3; **P3**: Mixed-Integer-Programming(MIP); **IS**: initial solution; **FO1**: Fix-and-Optimize period-oriented; **FO2**: Fix-and-Optimize  $l$ -step interrelatedness when random selection; **FO3**: Fix-and-Optimize  $l$ -step interrelatedness when lexicography order selection; (\*):  $\frac{Gap(\%)}{CPU(s)}$

Table 4.14 Test results of new FO approach for N=30 items and a disassembly structure with five levels which consists of three root items, 11 leaf items, and five items in common.

T	Instance	P3	P3r	IS	FO1	FO2		FO3	
						1-step	5-step	1-step	5-step
10	1	<b>0.01</b> * 105.26	46.66 <b>0.01</b>	13.49 0.67	7.79 0.39	8.48 1.62	0.92 7.38	8.52 1.36	0.39 7.05
	2	<b>0.01</b> 6.05	26.90 <b>0.01</b>	7.08 0.20	4.56 0.16	5.06 1.44	1.74 6.89	4.67 1.40	2.32 7.34
	3	<b>0.01</b> 165.67	28.92 0.01	11.40 0.47	8.28 0.17	7.73 1.62	2.26 8.75	8.21 1.61	2.40 8.81
	4	<b>0.01</b> 255.61	39.44 <b>0.01</b>	11.38 71.70	8.48 1.32	10.02 2.01	4.09 9.46	10.02 1.75	3.86 8.14
	5	<b>0.01</b> 125.81	50.65 <b>0.01</b>	17.18 120.00	10.55 <b>1.36</b>	15.54 2.49	4.55 10.79	15.54 2.38	3.09 10.20
20	1	<b>0.01</b> 13.96	20.68 <b>0.01</b>	10.71 0.26	9.41 0.47	8.38 4.08	6.46 11.40	8.45 4.35	5.52 11.45
	2	<b>5.10</b> 3600.00	55.92 <b>0.02</b>	26.51 120.00	20.81 1.07	21.83 4.54	10.28 13.05	21.11 4.30	11.78 13.02
	3	<b>0.65</b> 3600.00	25.41 <b>0.02</b>	12.45 2.05	9.16 0.50	11.31 3.52	3.64 13.45	11.33 4.27	4.38 14.91
	4	<b>0.61</b> 3600.00	34.36 <b>0.02</b>	10.74 0.98	7.41 0.56	9.47 3.81	4.96 14.21	9.62 3.83	4.22 13.77
	5	<b>1.23</b> 3600.00	27.04 <b>0.02</b>	8.93 7.60	7.22 0.53	8.02 3.59	4.98 14.24	8.02 3.65	4.69 12.03
30	1	<b>2.77</b> <b>3600.00</b>	21.20 <b>0.06</b>	11.67 2.08	9.69 12.73	9.92 43.08	7.32 25.28	9.93 18.53	7.35 22.27
	2	<b>2.90</b> 3600.00	25.35 <b>0.04</b>	17.05 1.39	14.52 5.65	15.16 33.29	9.09 21.00	14.87 16.66	9.15 19.81
	3	<b>11.17</b> 3600.00	40.51 <b>0.03</b>	15.29 7.06	12.65 7.84	12.88 34.35	7.69 19.95	12.83 14.68	7.66 20.29
	4	<b>4.32</b> 3600.00	30.59 <b>0.06</b>	11.80 3.15	8.34 6.02	10.44 16.88	5.65 18.05	10.28 13.59	5.95 18.51
	5	<b>4.05</b> 3600.00	24.10 <b>0.03</b>	15.37 6.78	13.16 3.81	13.63 23.19	9.43 19.10	13.56 15.28	9.30 19.38
Overall Min.	<b>0.01</b> 6.05	20.68 <b>0.01</b>	7.08 0.20	4.56 0.16	5.06 1.44	0.92 6.89	4.67 1.36	0.39 7.05	
Overall Avg.	<b>2.19</b> 2204.82	33.18 <b>0.02</b>	13.40 22.96	10.14 2.84	11.17 11.97	5.54 14.20	11.13 7.18	5.47 13.80	
Overall Max.	<b>11.17</b> 3600.00	55.92 <b>0.06</b>	26.51 120.00	20.81 12.73	21.83 43.08	10.28 25.28	21.11 18.53	11.78 22.27	

**P3r**: LP relaxation of P3; **P3**: Mixed-Integer-Programming(MIP); **IS**: initial solution; **FO1**: Fix-and-Optimize period-oriented; **FO2**: Fix-and-Optimize *l*-step interrelatedness when random selection; **FO3**: Fix-and-Optimize *l*-step interrelatedness when lexicography order selection; (\*):  $\frac{Gap(\%)}{CPU(s)}$

The impact of considering disposal decision on the total cost is analyzed for the problem instances with 30 numbers of items and 10 periods (25 problem instances i.e., five different problem instances for each five different product disassembly structures). We solve the problem instances for the problem without disposal decisions by adapting the model proposed by Kim et al. in Kim and Xirouchakis (2010) (model without disposal). We use model P3 to solve the problem instances for the problem with disposal decisions. Table 4.15 provides the results of maximum cost reduction when considering disposal decisions in disassembly planning. The cost reduction by considering disposal can attain 10.49 %, there is an important opportunity to make disassembly systems more profitable.

Table 4.15 The result of cost reduction (maximum) when considering disposal decisions.

N=30, T=10	Structures				
	1	2	3	4	5
Cost reduction (%)	<b>10.49</b>	2.87	2.54	4.92	3.12

## 4.4 Conclusion

In this chapter, we consider the disassembly lot sizing problem with disposal for multiple product types disassembly structure with parts commonality. Parts commonality have been created by new products design nowadays due to its benefits, which means different products can be assembly by the same parts or components. First the problem for the two-level product disassembly structure is considered and disposal decisions are applied to handle the issue of surplus inventory in disassembly systems. According to the results, it can attain a significant cost reduction of about 58%, for the tested problem instances. Two new MIP formulations (AGG and FAL) which consider disposal decisions are proposed. For large-sized problems (especially, in real industrial cases), a two-phase heuristic is suggested in which an initial solution is obtained by using LP-relaxation approach, and it is improved by using a forward-looking check dynamic programming based algorithm. The improvement is made by changing the current solution and considering cost changes efficiently. The two-phase heuristic gives always the solution better than LP-relaxation approach in short computational times and it has an efficient performance to improve the initial solution obtained by the first phase. Also, we apply LP-relaxation approach for the FAL model which can obtain optimal solution of 46.6% of the problem instances and it has a very strong lower bound for the others (the overall average of gap is only 0.06%).

We then consider the problem with multi-level product structure and capacity restrictions. Disposal decision to handle surplus inventory, can make a significant cost reduction of around 10%, for the tested instances. A new MIP model is presented to formulate this problem. Exact method using CPLEX solver for the original problem is not efficient for the large-sized problems, with the disadvantage that CPU time increases significantly. A Fix-and-Optimize (FO) heuristic is proposed, which can solve the problems (especially, large-sized problems) in shorter computational times with good solution quality. We develop FO heuristic by considering interrelatedness characteristic between variables which can improve the quality of the solutions obtained by FO classic.

We consider one of the most popular inventory management model i.e., EOQ model, in disassembly system in next chapter. EOQ model is usually used in the literature as a first step to provide an optimal inventory level in industries with no inventory model. Disassembly EOQ model is studied with disposal decision to investigate the opportunity of inventory cost reduction in a disassembly system. We expect that we can make more cost saving and improve disassembly profits by integrating pricing decisions into disassembly EOQ model with disposal. In Chapter 5, we propose the models which integrate price-sensitive demands and disposal decisions in disassembly economic order quantity problems to maximize the profit of disassembly systems without inventory accumulations.

### Contributions

Preliminary results of this chapter were presented at:

- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2020). Disassembly Lot Sizing Problem with Disposal Decisions for Multiple Product Types with Parts Commonality (IFAC 2020-Accepted).



## Chapter 5

# Disassembly EOQ Models With Disposal And Price-Sensitive Demand

### 5.1 Introduction

Several inventory management policies and models have been studied in the literature to obtain an optimal and efficient inventory system so as the companies will have the opportunities to make profit and to enhance the performance of their system. Previous chapters have discussed the lot sizing problem in disassembly systems and are provided the models and methods to handle to make disassembly systems more profitable. The well-known Economic Order Quantity (EOQ) is one of the most popular inventory management policy, due to its simplicity and adaptability to various applications. It considers known, continuous, and constant demand over a continuous planning horizon. The EOQ model assumptions are limited and it is necessary to extend it to increase its applicability in specific real cases. EOQ model in disassembly system and its main specificities are investigated in [Godichaud and Amodeo \(2018\)](#). They show that disassembly operations can generate unnecessary inventory of components which accumulate over continuous planning horizons and if no decision is considered to handle surplus inventory of components, stationary policies cannot be found and the model is difficult to apply in practice. The disposal decisions are considered in the literature to avoid surplus inventory, but it cannot guarantee profits. The surplus stock in disassembly can also be managed by varying the demands (or prices with demands function of price). The EOQ model in disassembly with pricing decisions are not considered in the literature. Several costs are involved in disassembly such as purchasing, disassembly operation, inventory holding, disposal and disassembly order costs. The companies must provide optimal policies so that end-of-life (EOL) products can be efficiently dismantled. They can sell the obtained parts on the channels, such as the secondary market, material recycling or energy recovering, in order to make the disassembly system more profitable. This chapter considers the effect of both inventory and pricing policies on the profitability of disassembly systems. The use of pricing associated with demand and surplus inventory decisions help companies manage their disassembly operations and optimize inventory policies.

This chapter is organized as follows: Section 5.2 describes the problem, followed by the model assumptions and notations, and develops the profit function with constraints for the disassembly system. In Section 5.4, we develop a solution approach to determine prices (or alternatively demands) for the components, the reorder interval of component inventories and order quantities. The solution approach is developed gradually by considering inventory costs, disposal decisions and upper bounds on the demands. The results of a numerical study that compares the models for the profit maximization problem (with and without a disposal option) are presented in Section 5.5. Section 5.6 provides a summary of the chapter with some concluding remarks.

## 5.2 Problem description

This section begins with introduction of the two-level product disassembly structure considered in this chapter. The first level represents the leaf items obtained directly by one disassembly operation and the second level represents the EOL product (root item) to be disassembled. A product disassembly structure can contain spare parts, remanufacturing or requested parts as well as material fractions that can be recycled. Disassembly operations are on the product and there is no inventory for this item. The leaf items (or components) are associated with demand functions, and there is an inventory for each of them. All the inventories of leaf items are replenished by one disassembly operation on the product, simultaneously. The disassembly yield of each leaf item is the number of units of leaf item obtained at each disassembly operation of one unit of EOL product. Fig. 5.1 represents an example of a disassembly system for an EOL product with three leaf items. The disassembly operation is on the EOL product (item 0), and it generates all the leaf items (items 1, 2, and 3) in the quantity noted on the edge (disassembly yields).

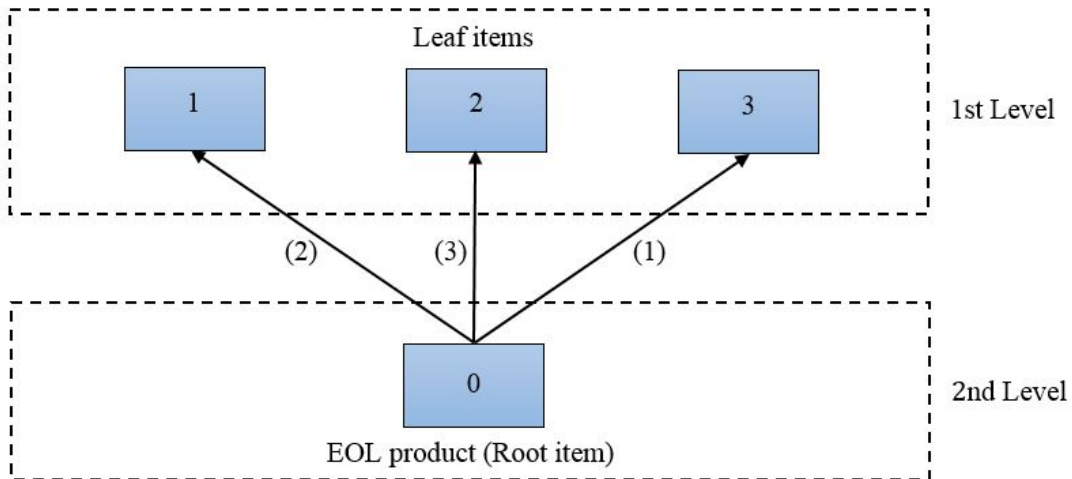


Fig. 5.1 An example of a two-level EOL product disassembly structure.

The problem is to determine the sale prices (or alternatively the demands) of leaf items simultaneously with the disassembly policy, which sets the timing of disassembly orders and the associated quantities. The objective is to maximize the profit function. This profit function includes both the revenues

generated by the sales of the leaf items and also the costs of the unit disassembly, inventory holding and order.

The following assumptions are made for all the models in the chapter (these are basic EOQ settings):

- demands of leaf items are independent, constant, and continuous,
- each demand is characterized by a constant rate in units per unit time (per year for example),
- the planning horizon is considered as infinite,
- the replenishment of inventories (by disassembly operations) are instantaneous,
- the disposal of a quantity of items in inventory is also instantaneous,
- the disassembly yields are known and constant,
- there is a fixed cost for each disassembly operation incurred whenever an order is placed,
- there is an inventory holding cost for each unit of an inventory per unit time.

One of the characteristics of disassembly systems is that the disassembly of one unit of EOL product generates one or more units of each of its components. It implies that, for each leaf item, the quantity of product disassembled does not necessarily give exactly the requested quantity of each component, and unnecessary stocks can be generated at each disassembly operation. By considering stationary demands, these surplus inventories accumulate with time. As can be seen in the Fig. 5.2, surplus inventory of leaf item  $i$  ( $\bar{R}_i$ ) will be accumulated over the continues replenishment cycle times which leads to a significant accumulation of inventory holding costs. This surplus inventory can be handle by allowing lots sales and disposal (Godichaud and Amodeo (2019a)) to reduce total inventory holding costs.

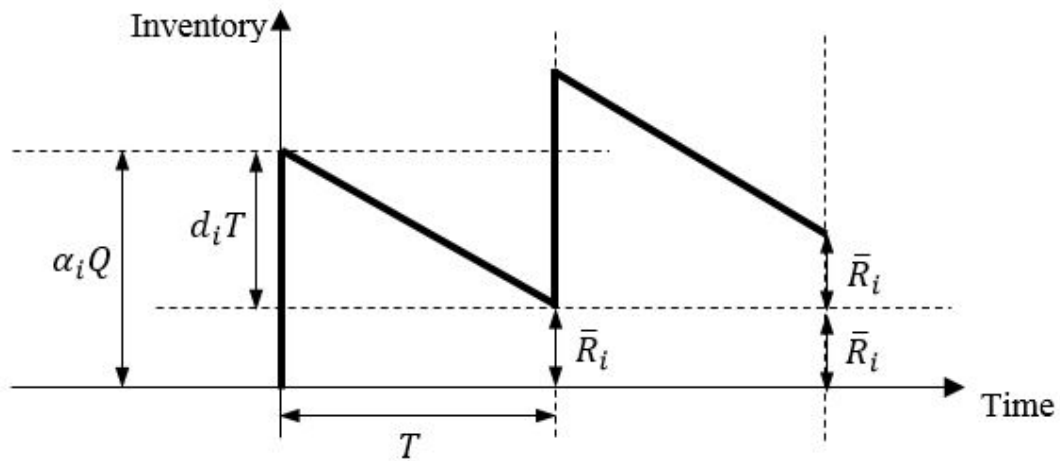


Fig. 5.2 Surplus inventory accumulation in disassembly systems with a EOQ inventory policy (Godichaud and Amodeo (2018)).

Godichaud and Amodeo (2018) mentioned that disposal is an option for handling surplus inventory accumulation to obtain inventory cost saving, but it can incur additional costs. When the demands can be varied with respect to the sale price of the items (price sensitive demands), they can be optimized to maximize the profit of disassembly systems including disposal costs. A disassembly order launches disassembly operations on products and replenishes the inventories of each leaf item. The price can be

defined as an increasing function of demand (or conversely the demands can be defined by using the reverse function). By integrating both disposal and pricing policies, firms can investigate the impact of price on consumer demands and examine the optimal pricing and disposal decisions.

We restrict our attention to the policies which are stationary and have the zero-inventory property. In a stationary policy, the orders are repeated according to a time cycle with a constant length. The zero-inventory property means that an inventory can be replenished only when it is zero (excluding surplus). This type of policy is commonly used in practice and studied in inventory systems (Muckstadt and Roundy (1993)).

### 5.3 Model statement

The decision variables and functions used in the mathematical models are as follows:

- $Q$ : Disassembly quantity per order (so that  $\alpha_i Q$  units of the leaf item  $i$  will be received after disassembly);
- $T$ : Cycle time (time between two disassembly orders);
- $X = Q/T$ : Virtual demand (used as decision variable);
- $D = \{d_1 \dots d_N\}$ : Set of demands for each component;
- $D_i(p_i)$ ,  $P_i(d_i)$ : Demand function of price and the price function of demand for the leaf item  $i$ , respectively;
- $\Pi(T, X, D)$ : Total mean profit per unit time;
- $R(X, D)$ : Total mean profit per unit time without ordering and inventory holding costs;
- $C(T, D)$ : Total mean ordering and inventory holding costs per unit time.

The following sets and parameters are used in this chapter:

- $i = 0, 1 \dots N$ : Index for the leaf items ( $i = 0$  is used for the EOL product);
- $\alpha_i$ : Yield of the leaf item  $i$  (number of units leaf item  $i$  in the EOL product);
- $h_i$ : Inventory holding cost of one unit of the leaf item  $i$  per unit time;
- $r_i$ : Disposal cost for one unit of the leaf item  $i$ ;
- $c$ : Disassembly operations cost of one unit of the EOL product;
- $k$ : Ordering cost of the EOL product;
- $d_i$ : Constant demand per unit time for the leaf item  $i$ ;
- $p_i$ : Constant price for one unit of the leaf item  $i$ ;
- $a_i, b_i$ : Parameters of the price-demand function of the leaf item  $i$ .



We consider the demand of items to be deterministic with price-sensitive function. This means that the demands are defined by a decreasing function of price. In the literature, several demand functions of price are commonly studied, such as linear demand, iso-elastic, exponential and algebraic curves. Among them, linear and iso-elastic demand function of the price are the two most commonly-used mathematical functions in inventory modeling (Huang et al. (2013), Ray et al. (2005)). In this chapter, we use an iso-elastic model, also called constant elasticity model, written as follows:

$$D(p) = ap^{-b} \quad \text{where } a, b > 0 \quad (5.1)$$

An example of iso-elastic function is presented in Fig. 5.3. This curve has a constant demand elasticity of  $a$ , which can be interpreted as the market potential, and  $b$  is the price elasticity (Lau and Lau (2003); Ray et al. (2005)). We note that this function is not mathematically bounded (the demand tends to infinity when the price tends to zero). We also note that the function can be easily reversed to use demand as a decision variable instead of the price. We note  $P(d)$  price function of demand, written as  $P(d) = (a/d)^{1/b}$  where  $a, b > 0$ .

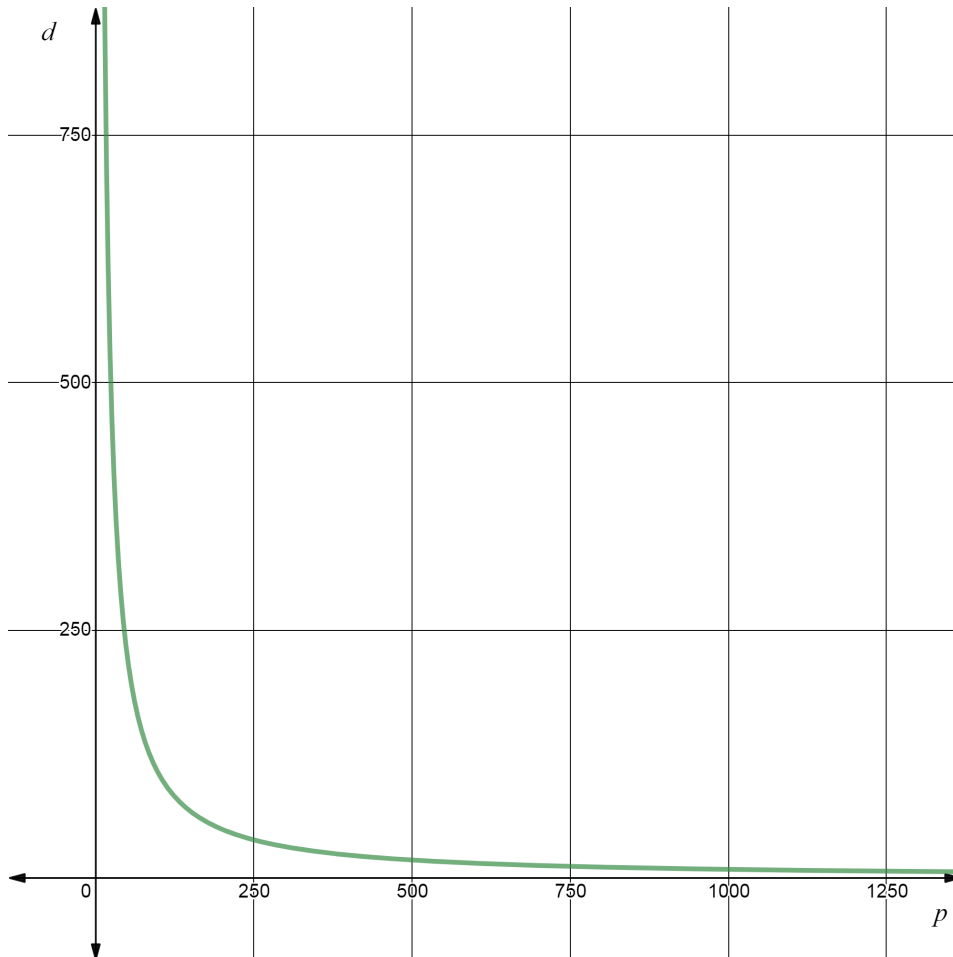


Fig. 5.3 Example of an iso-elastic function with  $a = 16592$  and  $b = 1.1$ .

In disassembly planning, pricing decisions, when they are possible, are also advantageous to reduce surplus inventory by balancing the demands for the components. Fig. 5.4 represents the effect of considering both disposal decisions and price sensitive demand on a given leaf item inventory with two different policies. The quantity received for item  $i$  is represented by  $\alpha_i Q$ . The first policy is represented by  $D1_i(p1_i)$  which is less than  $\alpha_i Q/T$  and a quantity  $\bar{R}_i$  has to be disposed of. In the second policy, by increasing the demand of item  $i$  to  $D2_i(p2_i)$  (or reducing the price), the disposed quantity is reduced but the inventory is increased if  $T$  is fixed. The problem is to find the values of  $T$  and  $d_i$  for all the items to obtain the maximum profit.

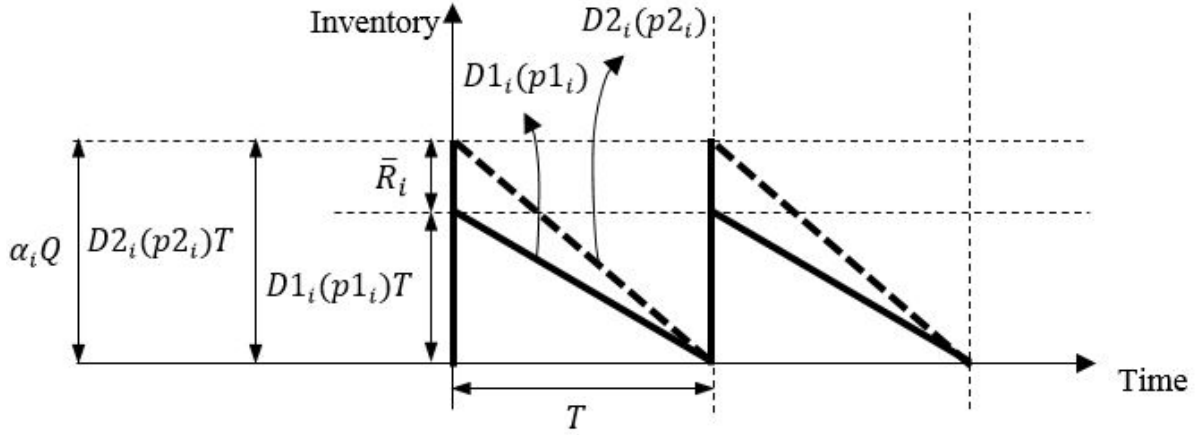


Fig. 5.4 Inventory evolution of a given leaf item  $i$  with respect to time by varying its demand

The problem is defined in (5.2) as a non-linear problem with constraints. The decision variables are  $T$ ,  $X$ , and  $D$  with:

- $D = \{d_1 \cdots d_N\}$  is the set of demands for each component as decision variables (the functions  $P_i(d_i)$  determine the associated price),
- $X = Q/T$  is the quantity of EOL product disassembled per unit time such that all component demands can be served, which indicates virtually demands.

The objective function is the total mean profit per unit time. The constraints ensure that the disassembly quantity for each order is enough to serve all the demands.

$$\begin{aligned} \text{Max} \quad \Pi(T, X, D) &= \sum_{i=1}^N d_i (P_i(d_i) + r_i) - X \left( c + \sum_{i=1}^N r_i \alpha_i \right) - \frac{k}{T} - T \left( \sum_{i=1}^N \frac{h_i d_i}{2} \right) \\ \text{s.t.} \quad \alpha_i X &\geq d_i \quad \forall i = 1 \dots N \end{aligned} \quad (5.2)$$

The profit function is a difference between a profit function  $R(X, D)$ , and a DEOQ cost function  $C(T, D)$  including ordering and inventory holding costs, which are studied separately below to analyze the problem:

$$\Pi(T, X, D) = R(X, D) - C(T, D) \quad (5.3)$$

with

$$R(X, D) = \sum_{i=1}^N P_i(d_i) d_i - cX - \sum_{i=1}^N r_i (\alpha_i X - d_i) \quad (5.4)$$

and

$$C(T, D) = \frac{k}{T} + T \left( \sum_{i=1}^N \frac{h_i d_i}{2} \right) \quad (5.5)$$

$R(X, D)$  consists of the sum of the revenues per unit of time for each leaf item, the disassembly cost per unit of time and the sum of the disposal costs per leaf item, respectively. We note that  $\alpha_i X - d_i$  is the quantity of leaf item  $i$  disposed of per unit of time.  $C(T, D)$  is the DEOQ cost function if  $D$  is fixed. In this chapter, we formulate the problem with a non-linear model with constraints to obtain optimal inventory policy while minimizing the associated costs. The maximization of  $R(X, D)$  is a pricing problem without inventory costs and we highlight its properties in the following section as a sub-problem of (5.2).

## 5.4 Solution approach

The Non-Linear Programming (NLP) model with constraints (equation (5.2)) is analyzed in this section. The solution method is gradually developed with respect to the different problems, including inventory, disposal option, and demand limits considerations. The problem without inventory consideration is studied as a sub-problem, but it can be applied in a relevant context. We analyze the problem (defined as 5.2) for the both cases with and without disposal option. The first one can be used when the disposal option is effectively not allowed, but it is also used as a sub-problem to solve the second one. The problem with disposal options and limits on demands is also studied.

### 5.4.1 Problem without inventory holding costs

By decomposing  $\Pi(T, X, D)$  into the difference between  $R(X, D)$  and  $C(T, D)$ , the pricing problem without inventory costs defined in (5.6) is studied as a sub-problem of (5.2). The two propositions in this section state optimal values for  $X$  and  $D$  and the shape of  $R(X, D)$  that can be used to solve the problem (5.2) efficiently.

$$\begin{aligned} \text{Max} \quad & R(X, D) = \sum_{i=1}^N d_i (P_i(d_i) + r_i) - X \left( c + \sum_{i=1}^N r_i \alpha_i \right) \\ \text{s.t.} \quad & \alpha_i X \geq d_i \quad \forall i = 1 \dots N \end{aligned} \quad (5.6)$$

**Proposition 5.1.** *The problem (5.6) can be written as follows:*

$$R(X) = \left( \left( \sum_{i=1}^N \alpha_i P_i(\alpha_i X) \right) - c \right) X$$

with only  $X$  as a decision variable by setting  $d_i = \alpha_i X$  and  $P_i(\alpha_i X) = (a_i / (\alpha_i X))^{(1/b_i)}$  for all  $i = 1 \dots N$  for the case with  $b_i > 1$ .

*Proof.* If  $X$  is fixed, the problem is to maximize  $V(D) = \sum_{i=1}^N V_i(d_i)$  with  $V_i(d_i) = (P_i(d_i) + r_i)d_i$  subject to  $\alpha_i X \geq d_i$  (for all  $i = 1 \dots N$ ). The function  $V(D)$  can be decomposed into  $N$  independent sub-functions of  $V_i(d_i)$  to set the value of the  $d_i$ s. Based on its first derivative with respect to  $d_i$ , denoted by  $V_i'(d_i)$ , a stationary point of  $V_i(d_i)$  must satisfy  $P_i(d_i) + r_i + P_i'(d_i)d_i = P_i(d_i)((b_i - 1)/b_i) + r_i = 0$ .

In the case of  $b_i < 1$ , the function is convex (the second derivative is  $V_i''(d_i) = P_i'(d_i)((b_i - 1)/b_i) > 0$ ) and, since we must have  $\alpha_i X \geq d_i$ ,  $d_i$  must be set as small as possible, which is an uninteresting scenario. In the following we consider that  $b_i > 1$  for all  $i = 1 \cdots N$ . In this case, the equation  $V_i'(d_i) = 0$  has no solution and  $V_i(d_i)$  increases. Each  $d_i$  is then chosen as large as possible, subject to the constraints  $\alpha_i X \geq d_i$  which lead to  $d_i = \alpha_i X$ . ■

Proposition 5.1 states that the optimal solution of (5.6) is to set the demands so that there is no surplus inventory for each leaf item and, therefore, no disposal. As a result, the problem has only one decision variable, which can be found according to Proposition 5.2.

**Proposition 5.2.** For  $b_i > 1$  (for all  $i = 1 \cdots N$ ),  $R(X)$  is concave and attains its maximum at  $X_0$  the unique solution of  $(R'(X) = 0)$ :

$$\sum_{i=1}^N \alpha_i \left( \frac{a_i}{\alpha_i X} \right)^{1/b_i} \left( \frac{b_i - 1}{b_i} \right) - c = 0$$

*Proof.* For  $P_i(\alpha_i X) = (a_i/(\alpha_i X))^{(1/b_i)}$ , the first derivative is  $P_i'(\alpha_i X) = -P_i(\alpha_i X)/(b_i X)$ . The first derivative of  $R(X)$  is then:

$$\begin{aligned} R'(X) &= \sum_{i=1}^N \alpha_i P_i(\alpha_i X) - c + \left( \sum_{i=1}^N \alpha_i P_i'(\alpha_i X) \right) X \\ &= \sum_{i=1}^N \alpha_i P_i(\alpha_i X) \left( \frac{b_i - 1}{b_i} \right) - c \end{aligned}$$

For  $b_i > 1$  (for all  $i = 1 \cdots N$ ), the function  $\alpha_i P_i(\alpha_i X)(b_i - 1)/b_i$  then strictly decreases from  $+\infty$  to 0 as  $X$  varies from 0 to  $+\infty$ . The sum function  $\sum_{i=1}^N \alpha_i P_i(\alpha_i X)(b_i - 1)/b_i$  is then strictly decreasing from  $+\infty$  to 0 and  $R'(X) = 0$  has one solution. We obtain second derivative of function  $R(X)$  as follows:

$$R''(X) = \sum_{i=1}^N \left( \frac{-\alpha_i P_i(\alpha_i X)}{b_i X} \right) \left( \frac{b_i - 1}{b_i} \right) < 0$$

The second derivative of  $R(X)$  is strictly negative so  $R(X)$  is concave. ■

In Proposition 5.2, the assumption  $b_i > 1$  has been discussed in the proof of Proposition 5.1. Based on this analysis, a simple line search method can be used to find  $X_0$ .

#### 5.4.2 Problem with inventory holding costs and no disposal decisions

In the problem without disposal options, each demand is set to  $d_i = \alpha_i X$  in order to have no surplus inventory to dispose of at the end of each replenishment cycle. The constraints in (5.2) are not necessary and the problem is to determine the values of  $T$  and  $X$  which maximize  $\Pi_1(T, X)$ , as defined in (5.7). The problem is always mathematically feasible with an iso-elastic price demand function, as the function is not bounded ( $P_i(\alpha_i X)$  is always defined for  $X > 0$ ). The case with maximum potential demand is discussed later.

$$\Pi_1(T, X) = \left( \left( \sum_{i=1}^N \alpha_i P_i(\alpha_i X) \right) - c \right) X - \frac{k}{T} - (TX) \left( \sum_{i=1}^N \frac{h_i \alpha_i}{2} \right) \quad (5.7)$$

For a fixed value of  $X$ , the optimal reorder interval  $T^*(X)$  is the minimum of the DEOQ cost function. With  $T^*(X) = \sqrt{2k/(X(\sum_{i=1}^N h_i \alpha_i))}$ , the problem is equivalent to (5.8):

$$\Pi_1(X) = R(X) - C(X) = \left( \left( \sum_{i=1}^N \alpha_i P_i(\alpha_i X) \right) - c \right) X - \sqrt{2kX \left( \sum_{i=1}^N h_i \alpha_i \right)} \quad (5.8)$$

The analysis of  $\Pi_1(X)$  gives Proposition 5.3. It shows that a simple non-linear search method can find the optimal solution (global maximum). The proof of the proposition also highlights the shape of the function on which the search method is based. We note that a maximum profit associated with  $X^*$ , if it exists, is  $X^*$  less than  $X_0$ . The search method can be started efficiently at  $X_0$  and  $X^*$  is the first point such that  $\Pi_1'(X) \geq 0$  by decreasing  $X$  from  $X_0$ .

**Proposition 5.3.** *For  $b_i > 1$ ,  $\Pi_1(X)$  attains its maximum at the largest value of  $X$  such that  $\Pi_1'(X) = 0$  or at  $X = 0$  if  $\Pi_1(X)$  is always negative.*

*Proof.* After analyzing the derivatives we found that there is no closed-form for the first and second derivatives.

$$\Pi_1'(X) = R'(X) - C'(X) = \sum_{i=1}^N \alpha_i \left( \frac{b_i - 1}{b_i} \right) P_i(\alpha_i X) - c - \sqrt{\frac{k(\sum_{i=1}^N h_i \alpha_i)}{2X}}$$

We analyze  $\Pi_1(X)$  directly based on the shape of  $R(X)$  and  $C(X)$ . Figure 5.5 represent visually different possibilities of the shape of  $R(X)$  and  $C(X)$ . We obtain the following results:

1.  $R(X)$  is concave with maximum  $X = X_0$  and  $R(0) = 0$ .
2.  $C(X)$  is strictly increasing with no stationary point.
3. Based on (1) and (2), there are 0, 1 or 2 intersection points between  $R(X)$  and  $C(X)$  functions:
  - 0 intersection point:  $\Pi_1$  is always negative (which means  $C(X)$  always lies above  $R(X)$  and  $C(X) > R(X)$ ) and the solution is to disassemble nothing with  $X = 0$ ,
  - 1 intersection point  $X_{max}$ :  $\Pi_1$  is positive from  $X = 0$  to  $X_{max}$  and then negative (which means  $R(X)$  first sits above  $C(X)$  and then below),
  - 2 intersection points  $X_{min}$  and  $X_{max}$ :  $\Pi_1$  is positive only between  $X_{min}$  and  $X_{max}$  (which means  $R(X)$  first sits below  $C(X)$ , then above, and finally below again).

■

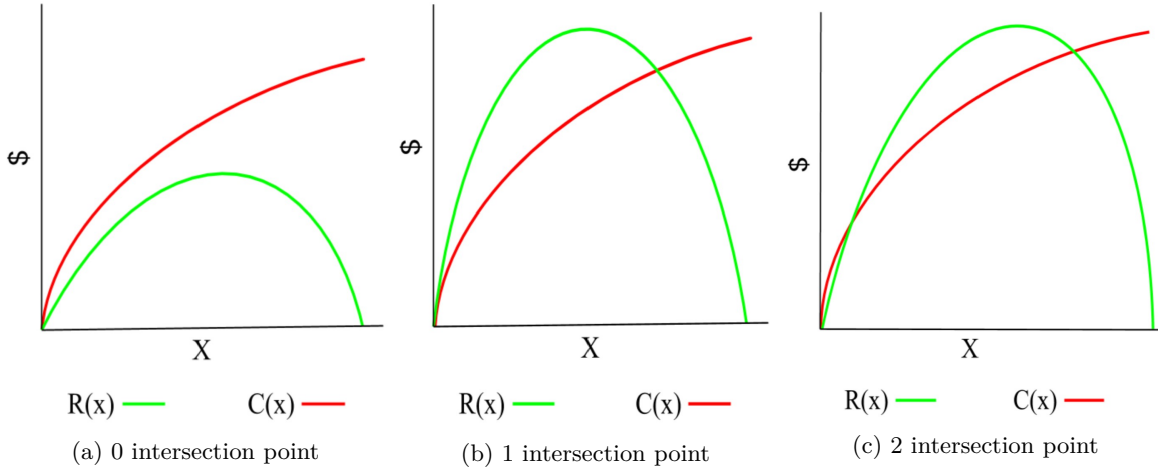


Fig. 5.5 Analysis of  $\Pi_1(X)$  based on the shape of  $R(X)$  and  $C(X)$ .

In real cases, upper bounds on the demands  $d_i$  can be imposed according to the context. A maximum demand  $D_i^M$  is then defined for each item  $i = 1 \dots N$ . We must have  $d_i \leq D_i^M$  for all  $i = 1 \dots N$ , and as  $d_i = \alpha_i X$  for the problem without disposal, there is an additional constraint:  $X \leq \min_i \{D_i^M / \alpha_i\}$ . This is an upper bound for  $X$  and, if  $X^* > \min_i \{D_i^M / \alpha_i\}$ , then the solution of the problem is  $X = \min_i \{D_i^M / \alpha_i\}$ .

### 5.4.3 Problem with disposal decisions

Based on the results obtained in sections 5.4.1 and 5.4.2, a solution approach is proposed to solve the initial problem (5.2). This is a non-linear programming problem with constraints and an optimal solution satisfies the Karush-Kuhn-Tucker (KKT) conditions. The proposed search procedure uses these conditions to solve the problem by iteratively changing a solution until they are achieved. We first state these conditions before describing the resolution procedure.

By denoting the Lagrangian multipliers  $\theta_i$  associated with each constraint  $\alpha_i X \geq d_i$ , the Lagrange function of the problem is defined as:

$$L_{\Pi}(T, X, D, \theta) = \sum_{i=1}^N d_i (P_i(d_i) + r_i) - X \left( c + \sum_{i=1}^N r_i \alpha_i \right) - \frac{k}{T} - \sum_{i=1}^N \frac{h_i d_i}{2} T - \sum_{i=1}^N \theta_i (d_i - \alpha_i X)$$

The KKT conditions are as follows:

$$\frac{k}{T^2} - \sum_{i=1}^N \frac{h_i d_i}{2} = 0 \quad (5.9a)$$

$$P_i(d_i) \left( \frac{b_i - 1}{b_i} \right) + r_i - \frac{h_i T}{2} - \theta_i = 0 \quad \forall i = 1 \dots N \quad (5.9b)$$

$$- \left( c + \sum_{i=1}^N \alpha_i r_i \right) + \sum_{i=1}^N \alpha_i \theta_i = 0 \quad (5.9c)$$

$$\theta_i (d_i - \alpha_i X) = 0, \theta_i \geq 0 \quad \forall i = 1 \dots N \quad (5.9d)$$

A first solution is found by solving the problem with  $d_i = \alpha_i X$  for all  $i = 1 \dots N$ . This solution is feasible for the problem (5.2) and easy to find, as shown in section 5.4.1. By setting  $\theta_i$  with respect to (5.9b), the initial solution is optimal if  $\theta_i \geq 0$  for all  $i = 1 \dots N$  (the other conditions (5.9a) to (5.9c) are satisfied based on proposition 5.3). If there are some  $\theta_i < 0$ , the initial solution can be improved by setting  $\theta_i = 0$  and changing the related  $d_i$  with respect to (5.9b). At this point, the following steps are repeated iteratively until the KKT conditions (5.9a) to (5.9d) are satisfied at a given threshold:

- **Step 1.** The  $d_{iS}$  with  $d_i < \alpha_i X$  ( $\theta_i = 0$ ) and  $T$  are fixed. The change in  $X$  is made to satisfy (5.9c) (the values of  $\theta_i$  are set at the previous iteration).
- **Step 2.**  $X$  and  $T$  are fixed. With respect to (5.9b): update the  $d_i$  with  $d_i < \alpha_i X$ , compute  $\theta_i$  for the  $d_i$  with  $d_i = \alpha_i X$  and change the  $d_i$  if  $\theta_i < 0$  ( $\theta_i$  is changed to  $\theta_i = 0$ ).
- **Step 3.** The  $d_i$  is fixed,  $T$  is set with respect to (5.9a).

The procedure optimizes one variable, considering the others fixed at each step with respect to KKT conditions. It improves the profit function iteratively from the initial solution without disposal. At Step 1, only  $X$  is changed (and implicitly the  $d_i$ s with  $d_i = \alpha_i X$ ). If the subset of items such that  $d_i = \alpha_i X$  is denoted by  $S$ , solving (5.9c) is equivalent to solving the following equation:

$$\sum_{i \in S} \alpha_i P_i(\alpha_i X) \left( \frac{b_i - 1}{b_i} \right) = c + \sum_{i \notin S} \alpha_i r_i + T \sum_{i \in S} \alpha_i h_i / 2$$

The right side is constant at Step 1, and the left side is convex in  $X$ . Starting with the value of  $X$  from the previous iteration, a new value satisfying the equality is found by simple line search. At Step 2,  $d_i$  is directly found with (5.9b) if  $\theta_i = 0$  and  $T$  is fixed at the value from the previous iteration (Step 3). If  $d_i = \alpha_i X$ , the value of  $\theta_i$  is found with (5.9b) and the value of  $X$  found in Step 1. However, if  $\theta_i < 0$ , the condition (5.9d) is not satisfied and the related  $d_i$  can be decreased to improve the profit. Once all the  $d_i$ s have been set in previous steps,  $T$  is easily computed in Step 3 with respect to (5.9a).

We note that with an iso-elastic demand function, the demand is not bounded and a huge value can be optimal. This could not be practicable in real cases. The problem then is extended by imposing upper bounds for the demands which is studied in the next section.

#### 5.4.4 Problem with disposal decisions and limits on demands

Potential demands of items cannot be unbounded. Without loss of generality, we use  $a_i$  as an upper bound on the demand of item  $i$ . This assumption is modeled with the following set of constraints added in (5.2):

$$d_i \leq a_i \quad \forall i = 1 \dots N \quad (5.10)$$

The Lagrangian multipliers associated with each constraint (5.10) are denoted by  $\lambda_i$ . The constraints (5.10) change the conditions (5.9b) into (5e) and add the condition (5f).

$$P_i(d_i) \left( \frac{b_i - 1}{b_i} \right) + r_i - \frac{h_i T}{2} - \theta_i - \lambda_i = 0 \quad \forall i = 1 \dots N \quad (5e)$$

$$\lambda_i (d_i - a_i) = 0, \lambda_i \geq 0 \quad \forall i = 1 \dots N \quad (5f)$$

The previous search method is modified by integrating these new conditions. The solution without bounds on the demands is used as an initial solution. The demands are changed to satisfy the constraints (5.10) if necessary. If  $d_i > a_i$  in the previous solution then set  $d_i = a_i$  (all other constraints are satisfied since  $d_i$  is decreased and  $X$  is fixed, i.e.  $\alpha_i X > d_i$ ) and  $\theta_i = 0$  since  $d_i < \alpha_i X$ .  $T$  is iteratively updated according to the changes in  $d_i$ s with (5.9a), (5e) and  $X$  fixed.

The changes in  $X$  are then made according to the condition (5.9c) considering  $T$  and  $d_i$  with  $d_i < \alpha_i X$  as fixed. The changes in some  $d_i$ s by setting  $\theta_i = 0$  result in a change in the left side of condition (5.9c). If in the current solution  $-\left(c + \sum_{i=1}^N \alpha_i r_i\right) + \sum_{i=1}^N \alpha_i \theta_i > 0$  (resp.  $< 0$ ),  $X$  must be increased (resp. decreased) to improve the solution. We note that after changing the initial solution to satisfy (5.10), if some  $d_i$ s are changed to  $d_i = a_i$  then  $\theta_i = 0$  and the right side of (5.9c) becomes negative.  $X$  can be decreased to improve the initial solution.

Starting from the corrected initial solution and by decreasing  $X$ , the search is positioned on an interval  $[X_L X_U]$  for  $X$  as follows. At first,  $X_U$  is set to the solution of the problem without an upper bound on the demands and  $X_L$  is set to the maximum value of  $a_i/\alpha_i$  with  $i$  such that  $\alpha_i X > a_i$  in the current solution. Then, while condition (5.9c) is positive in  $X_L$  and negative in  $X_U$ ,  $X_U$  is set to  $X_L$  and  $X_L$  is set to the next maximum value of  $a_i/\alpha_i$  such that  $\alpha_i X > a_i$ . After each change in  $X$ ,  $D$  and  $T$  are changed according to (5.9a) and (5e) while considering  $X$  fixed. We note that, if for one item  $d_i = \alpha_i X = a_i$ ,  $\theta_i$  and  $\lambda_i$  can be positive while satisfying (5e). In this case,  $\theta_i$  is set to satisfy (5.9c) if  $\theta_i + \lambda_i$  is enough with respect to (5e).

After finding the search interval  $[X_L X_U]$  for  $X$ , a bisection approach on  $X$  is used to find the best solution. Each value of  $X$  is tested with respect to (5.9c) after having changed  $D$  and  $T$  iteratively according to (5.9a) and (5e) while considering  $X$  fixed.

## 5.5 Computational experiments

This section presents a numerical results and analysis based on several data sets (all data sets are provided in Appendix D). The data sets are experimental, but they can correspond to any type of real case product with several components that can be sold in different markets. For example, disassembly centers of the end-of-life vehicles (ELVs) have several types of glass, metal, and plastic as material outputs and headlights and engine components as spare parts outputs. These outputs correspond to the component items  $i = 1$  to 10 in the data sets. The results on the experimental data sets show that the solution method is efficient enough to be applied to any real case data. The solution provided indicates if a disassembly operation is profitable, at which level and with which options.

The models and methods have been implemented on eclipse using Java programming language. Also, the tests are coded on a system with an Intel Core i7-7700T, 2.9 GHz, and 16 Go RAM on windows 10.

### 5.5.1 Numerical results

The proposed models and methods are applied to the example data presented in Table 5.1. It is a product with ten components. The results are presented in Table 5.2, with the initial solution and the last solution obtained by the procedure. Note that the disassembly quantity of EOL product per order ( $Q$ ) can be calculated by  $Q = X \cdot T$ . The precision on the KKT conditions (stopping criteria) is fixed at 0.001. The computational time is negligible and the last solution is obtained after 20 iterations. The gap between the initial and last solutions is important for the tested instances, but it depends mainly on the



disparity of the demand function parameters between leaf items. It is more economical to have disposal rather than more sales for items 1, 2, 4, 5 and 9 for this instance. As managerial insights, this example illustrates the data required to apply the proposed policy and shows the efficiency of the solution method. Decision makers can rapidly have some indications as to the profitability of the disassembly of a product, and on the effect of changing one model parameter (a solution is easily restarted with alternative data).

Table 5.1 An illustrative instance with 10 items

$c = 10, k = 9407$										
$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	1.0	5.0	1.0	3.0	3.0	1.0	3.0	1.0	1.0	3.0
$h_i$	0.92	0.17	0.85	0.88	0.41	0.48	0.26	0.2	0.56	0.14
$a_i$	897.81	110.40	16592.38	486.58	1048.51	2541.04	5926.08	2714.32	1143.93	948.5
$b_i$	2.3	1.8	2.6	2.9	2.6	2.4	2.1	1.8	1.5	3.0
$r_i$	0.092	0.059	0.092	0.078	0.024	0.034	0.058	0.045	0.011	0.031

Table 5.2 Result for the illustrative example

	X	T	Q	Profit		Solution without disposal				
	42.1	7.074	297.815	<b>207.008</b>						
$i$	1	2	3	4	5	6	7	8	9	10
$d_i$	42.1	210.5	42.1	126.3	126.3	42.1	126.3	42.1	42.1	126.3
$p_i$	3.78	1.88	3.78	2.35	2.35	3.78	2.35	3.78	3.78	2.35
	X	T	Q	Profit		Solution with disposal				
	75.062	7.828	587.585	<b>579.278</b>		(disposal per unit time, for an item $i$ : $\alpha_i X - d_i$ )				
$i$	1	2	3	4	5	6	7	8	9	10
$d_i$	13.474	63.124	75.062	4.226	90.25	75.062	225.186	75.062	68.368	225.186
$p_i$	6.207	3.172	2.942	10.277	2.715	2.942	1.825	2.942	3.064	1.825

Fig. 5.6 illustrates the shape of  $\Pi_1(X)$ ,  $R(X)$  and  $C(X)$  for this instance and brings additional justifications of the solution approach. We note that  $C(X)$  passes above  $R(X)$  before it reaches its maximum. As shown in proposition 5.3,  $\Pi_1(X)$  is concave while  $R(X)$  is above  $C(X)$ . We also note that the solver Excel (which used a generalized reduced gradient method) finds the solution for the problem without disposal (only  $X$  as decision variable), but not for the problem with disposal (with  $X$ ,  $T$  and  $D$  as decision variables) starting with the initial solution. Furthermore, if the last solution is loaded in the solver, it does not improve it.

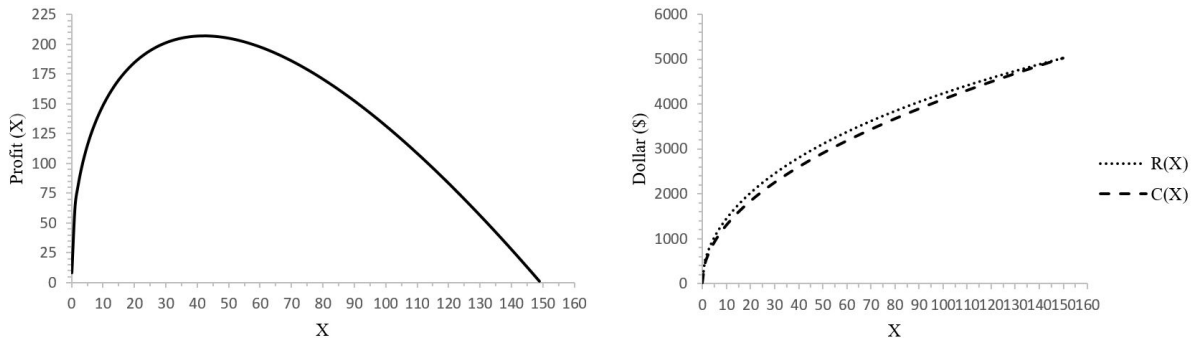


Fig. 5.6 Shape of the curves for  $\Pi_1(X)$ ,  $R(X)$  and  $C(X)$ .

### 5.5.2 Numerical analysis

This section provides the analysis of the results obtained by the models and methods. We note from all the experiments that the proposed models and methods find the solution efficiently for all the cases with or without disposal option, and with and without limit on demands. If the disposal is allowed and the demands are limited, an optimal solution can be without disposal and all the optimal demands obtained are under the limit, but the solution approach finds it.

#### 1) Effect of the disposal option and limits on the demand

We note in Table 5.2 that the disposal option is profitable in this instance. Another effect of allowing disposal is that more products are disassembled. It can be considered as negative or positive depending on whether recovery channels with proper environmental conditions are considered for the disposed items (however this is not considered in the model). Table 5.3 illustrates the improvement of the profit that can be achieved over the other nine examples by considering disposal option (data sets are provided in Appendix D). We can note that the solution without disposal can be optimal, as in example 8. It is detected in the procedure after having found the initial solution by testing the KKT conditions. For all the instances, as previously, the solver Excel finds the solution for the problem without disposal but not for the problem with disposal. The solution procedure starts with the solution without disposal option and then tries to improve it by considering disposal. We note that the solution determines whether disposal is profitable and what quantity of each item to dispose of.

Table 5.3 Illustration of the gap between initial and last solutions (without and with disposal)

Inst.	1	2	3	4	5	6	7	8	9
Initial	11653.403	<b>147.504</b>	2694.759	1224.661	2925.528	549.553	<b>478.126</b>	2364.457	3141.517
Opt.	11656.177	<b>596.561</b>	2722.754	1370.793	2941.938	620.709	<b>1232.796</b>	2364.457	3141.779
Gap	0.024%	<b>304%</b>	1.039%	11.93%	0.561%	12.9%	<b>157.8%</b>	0%	0.008%

As mentioned in previous sections, the limits on the demands are an additional consideration for using power iso-elastic function. In real cases, the markets are limited. By considering  $a_i$  as the maximum demand for each component in all examples, the solutions of examples 3, 8 and 9 are not feasible at this step of the procedure, as can be seen in Table 5.4. The procedure can be continued, as presented in section 5.4.4, to improve the solution within the constraints. The results are presented in Table 5.4. As

previously, the computational time is negligible and all the KKT conditions are satisfied (with a given threshold) for the last solution found.

Table 5.4 Solution for the problem with upper bound on demands (Instances 3, 8 & 9)

<b>Instance 3</b>										
	X	T	Q	Profit		Solution without limit on demands				
	<b>224.052</b>	<b>2.022</b>	<b>453.033</b>	<b>2722.754</b>						
<i>i</i>	1	2	3	4	5	6	7	8	9	10
<i>d<sub>i</sub></i>	448.104	224.052	448.104	224.052	448.104	458.860	448.104	442.347	672.156	672.156
<i>p<sub>i</sub></i>	2.723	8.350	4.409	2.453	1.129	2.258	2.569	1.504	1.966	0.726
<i>a<sub>i</sub></i>	4058.3	8263.83	4147.93	3023.47	614.31	1832.65	2956.57	922.46	3898.07	<b>311.85</b>
	X	T	Q	Profit		Solution with limit on demands				
	<b>207.95</b>	<b>2.17</b>	<b>451.251</b>	<b>2694.596</b>		(the demand of an item must be $d_i \leq a_i$ )				
<i>i</i>	1	2	3	4	5	6	7	8	9	10
<i>d<sub>i</sub></i>	415.9	207.95	415.9	207.95	415.9	403.37	415.9	387.47	623.85	<b>311.85</b>
<i>p<sub>i</sub></i>	2.816	8.724	4.633	2.517	1.162	2.436	2.666	1.619	2.023	1

<b>Instance 8</b>										
	X	T	Q	Profit		Solution without limit on demands				
	<b>469.64</b>	<b>1.324</b>	<b>621.803</b>	<b>2364.457</b>						
<i>i</i>	1	2	3	4	5	6	7	8	9	10
<i>d<sub>i</sub></i>	1878.56	939.28	1408.92	1878.56	939.28	469.64	469.64	1878.56	939.28	1408.92
<i>p<sub>i</sub></i>	0.885	2.101	1.268	1.036	0.957	1.966	6.423	0.861	0.938	1.003
<i>a<sub>i</sub></i>	<b>1336.24</b>	7514.05	2548.69	2066.2	838.94	3333.54	13355.88	1413.39	832.28	1419.53
	X	T	Q	Profit		Solution with limit on demands				
	<b>334.06</b>	<b>1.628</b>	<b>543.849</b>	<b>2293.275</b>		(the demand of an item must be $d_i \leq a_i$ )				
<i>i</i>	1	2	3	4	5	6	7	8	9	10
<i>d<sub>i</sub></i>	<b>1336.24</b>	668.12	1002.18	1199.15	668.12	334.06	334.06	1336.24	668.12	793.338
<i>p<sub>i</sub></i>	1	2.373	1.453	1.223	1.092	2.211	7.761	1.030	1.123	1.231

<b>Instance 9</b>										
	X	T	Q	Profit		Solution without limit on demands				
	<b>367.68</b>	<b>1.412</b>	<b>519.164</b>	<b>3141.779</b>						
<i>i</i>	1	2	3	4	5	6	7	8	9	10
<i>d<sub>i</sub></i>	735.36	1470.72	367.68	1470.72	1470.72	735.36	735.36	367.68	1398.89	367.68
<i>p<sub>i</sub></i>	2.175	0.518	3.262	1.250	1.374	2.193	2.040	5.893	0.708	2.538
<i>a<sub>i</sub></i>	2978.69	<b>548.3</b>	6278.22	2198.47	3812.5	5238.36	4070.88	10692.41	726.1	1965.56
	X	T	Q	Profit		Solution with limit on demands				
	<b>315.35</b>	<b>1.609</b>	<b>507.398</b>	<b>3037.924</b>		(the demand of an item must be $d_i \leq a_i$ )				
<i>i</i>	1	2	3	4	5	6	7	8	9	10
<i>d<sub>i</sub></i>	630.7	<b>548.3</b>	315.35	1261.4	1261.4	630.7	630.7	315.35	726.1	315.35
<i>p<sub>i</sub></i>	2.369	1	3.478	1.362	1.446	2.332	2.175	6.389	1	2.764

## 2) Sensitivity analysis on disposal cost and the disassembly yield

We analyze the sensitivity of the model with respect to disassembly yield and disposal cost. One of the characteristics of disassembly systems is the yield attached to each component. Insights of the effect of the variation of this parameter on decisions and profit is interested. We test our sensitivity analysis on the instance 1 as an example. Table 5.5 reports the results of a sensitivity analysis with respect to disassembly yield for the instance 1 (data on Table 5.1 in the chapter). The experiment is based on the variation of each yield by one percentage point between  $\pm 25\%$  of the initial value in 0.05% steps. The effect on the decisions ( $X$ ,  $T$  and  $D$ ) and the profit are evaluated on average in percentage variation from the initial values (for instance, the first line indicates that a gap of -25% on one yield parameters leads to variation of -3.5% of the profit).

From Table 5.5, we can assume that if one yield increases, the reorder interval  $T$  is decreased, the quantity of product disassembled per unit of time  $X$  is increased and the item demands are increased. We note that the variation is contained within  $\pm 4\%$  for  $X$ ,  $T$  and in  $\pm 6\%$  for the demand of items whose yield is not varied, while the variation is significantly more important for the demand of the item whose yield is varied. However, this variation has little effect on the profit.

Table 5.5 Impact of disassembly yield changes on optimal decisions

One yield variation (%)	Average profit variation (%)	Average X-value variation (%)	Average T-value variation (%)	Average $d_i$ -value variation (*) (%)	Average $d_i$ -value variation (**) (%)
-25	-3.5	-4.05	3.87	-17.06	-5.99
-20	-2.75	-3.21	3.00	-13.61	-4.78
-15	-2.03	-2.39	2.18	-10.08	-3.58
-10	-1.33	-1.58	1.38	-6.46	-2.35
-5	-0.66	-0.77	0.66	-3.19	-1.16
5	0.64	0.79	-0.66	3.32	1.22
10	1.26	1.58	-1.29	6.73	2.46
15	1.87	2.33	-1.88	10.16	3.66
20	2.47	3.09	-2.46	13.71	4.91
25	3.05	3.81	-3.00	17.27	6.13

(\*) for the item whose yield is varied, (\*\*) for the item whose yield is not varied

One specificity of the proposed model is the disposal option and we conducted a sensitivity analysis with respect to disposal costs in instance 0. As for the analysis of the disassembly yield, each initial value of  $r_i$  is varied by one percentage point between  $\pm 25\%$ . The results are reported in Table 5.6. We note that small variations of the disposal cost have a limited effect on the decision. It has no effect for the component with no disposal options included in the initial decision (the variation of  $\pm 25\%$  does not generate disposal for these components). In a second experiment, we vary each disposal cost more widely until it becomes unprofitable to dispose of the related component.

Table 5.6 Impact of disposal cost changes on optimal decisions

One dispo. cost variation (%)	Average profit variation (%)	Average X-value variation (%)	Average T-value variation (%)	Average $d_i$ -value variation (*) (%)	Average $d_i$ -value variation (**) (%)
-25	0.19	0.39	-0.17	-0.44	0.39
-20	0.16	0.31	-0.14	-0.35	0.32
-15	0.12	0.22	-0.10	-0.28	0.22
-10	0.08	0.15	-0.06	-0.19	0.15
-5	0.04	0.07	-0.03	-0.10	0.07
5	-0.04	-0.07	0.03	0.10	-0.07
10	-0.08	-0.15	0.06	0.19	-0.15
15	-0.11	-0.22	0.10	0.29	-0.22
20	-0.15	-0.29	0.13	0.39	-0.29
25	-0.19	-0.36	0.16	0.49	-0.36

(\*) for the item whose disposal cost is varied, (\*\*) for the item whose disposal cost is not varied

In this second experiment, each  $r_i$  is varied from 0 up to the value at which there is no more disposal for  $i$ . Fig. 5.7 presents the variation of the profit with respect to disposal cost for each item that has disposal quantity in the initial solution (items 1, 2, 4, 5 and 9). The values at which there is no more disposal for each  $i$  are highlighted in Fig. 5.7. We note that, for each item, these values exceed the inventory holding cost ( $h_i$  is a cost per unit of item per unit of time while  $r_i$  is cost per unit of item). We also note that the profit decreases with respect to the increase in disposal cost, and the variation of the profit is not the same for each item: it is more important for item 4 than for item 9. Finally, an important managerial insight is that the optimization finds solutions efficiently whatever the data. Decision makers can use it to have an idea of the profitability of the disassembly process of any EOL product.

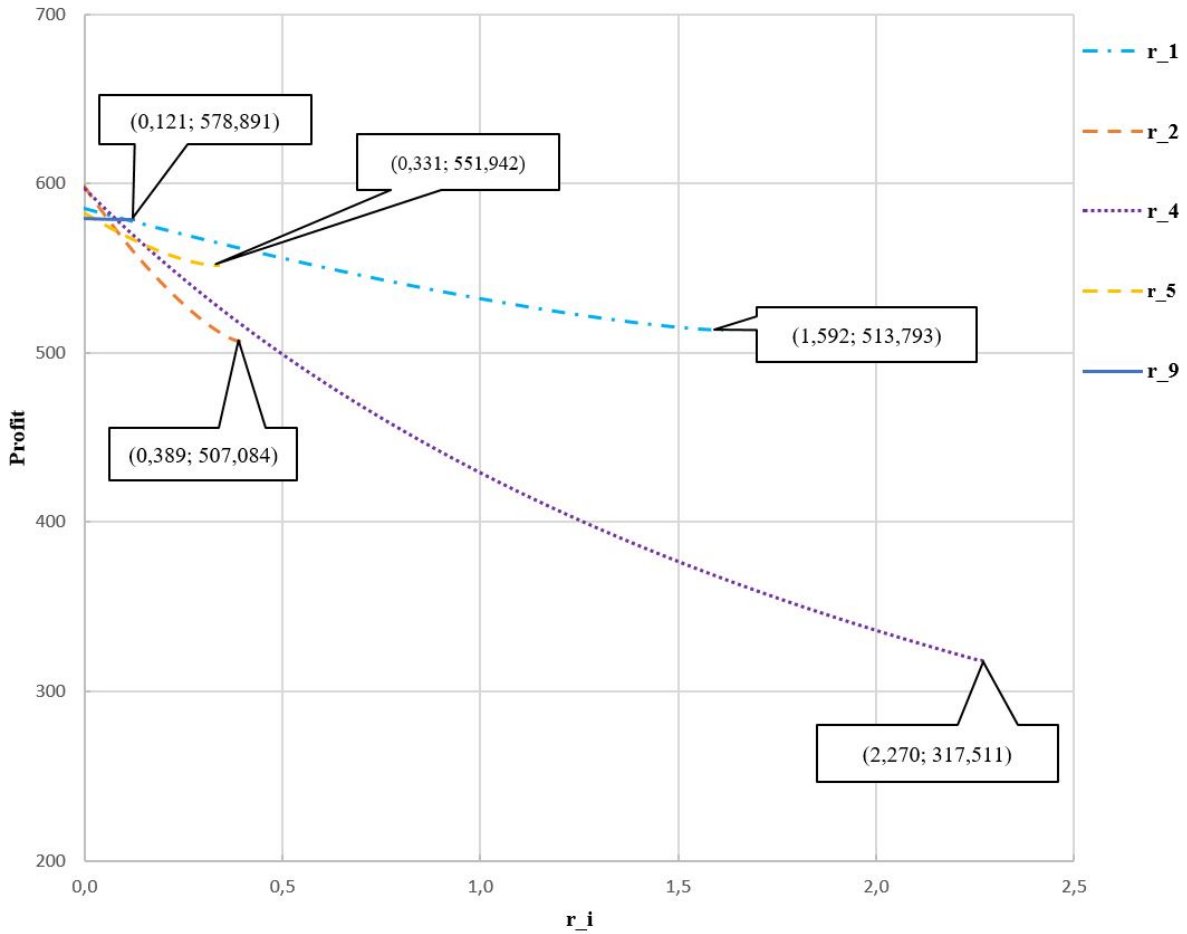


Fig. 5.7 Variation of the profit with respect to the disposal cost (from 0 to the value at which there is no more disposal for item  $i$ ).

### 3) Effect of uncertain disassembly yields

We analyse the application of the proposed model with uncertain disassembly yields: we compare three policies that can be applied in this situations. The policies are based on the model proposed in the chapter. The comparison is made according to a simulation study (i.e. simulation of uncertain yields) over a planning horizon with 1000 orders (or cycles).

For the three policies, we assume that the yields are estimated and the values used for the decision optimization are expected values. We assume that the components that cannot be disassembled (or recycled) are detected at the receipt of the order. The quantity received in each component inventory can be above or below the nominal quantity (the repartition over the nominal value can be represented by a probability distribution). We also note that the three policies are stationary to maintain EOQ-like conditions: each cycle is independent and all the inventories are zero at the beginning of a cycle. We apply the model without limits on maximum demand rates (the results can be easily extended).

Based on these assumptions, the yields are simulated at each order with respect to a probability distribution. We use the data of the previous instance where  $\alpha_i$  is used as the mean of uniform distribution with a range of 0.1 around  $\alpha_i$  for the support of the distribution (i.e.  $\hat{\alpha}_i = U(0.95\alpha_i, 1.05\alpha_i)$ ).  $\hat{\alpha}_i$  is the

observed (simulated) value of the yield of component  $i$  at the beginning of a cycle, after having received the order.

**Policy P1.** The decisions  $Q$ ,  $T$  and  $D$  are fixed for all orders. The quantity received in each inventory is  $\hat{\alpha}_i Q$ . If  $d_i T > \hat{\alpha}_i Q$  then  $\hat{\alpha}_i Q$  items  $i$  are sold during the cycle and  $d_i T - \hat{\alpha}_i Q$  sales are lost. If  $d_i T \leq \hat{\alpha}_i Q$  then  $\hat{\alpha}_i Q - d_i T$  items  $i$  are disposed of and  $d_i T$  are sold. The sale revenue, the disposal cost and the average inventory cost for each item  $i$  during the cycle are then, respectively:  $p_i \min\{d_i T, \hat{\alpha}_i Q\}$ ,  $r_i \max\{0, \hat{\alpha}_i Q - d_i T\}$  and  $(h_i/2) \min\{d_i T^2, (\hat{\alpha}_i Q)^2/d_i\}$ .

In this policy, stockouts are allowed and are considered as lost sales without additional costs. The only effect is that fewer items can be sold during one cycle. The two other policies avoid lost sales by changing more variables at each cycle.

**Policy P2.** The decisions  $Q$  and  $D$  are fixed for all orders but  $T$  is changed every cycle to prevent lost sales. After having observed the value of the yields, the next order will arrive after  $T = \min_i \{\hat{\alpha}_i Q/d_i\}$  units of time, with  $T^*$  the reorder interval obtained by optimization. It means that  $T$  is adjusted with respect to the first inventory that reaches zero during the cycle. The surplus inventories for other items are disposed of. The sales revenue, the disposal cost and the average inventory cost for each leaf item  $i$  during the cycle are then, respectively,  $p_i d_i T$ ,  $r_i (\hat{\alpha}_i Q - d_i T)$  and  $(h_i/2) d_i T^2$ .

In this policy, stockouts are avoided in contrast to policy P1, but variation of the reorder interval must be allowed. We also note that the variation of the cycle times, while keeping the order quantity fixed, is the same principle as for the policy  $(s, Q)$  applied to the single item inventory model with uncertain demand and non-zero lead time.

**Policy P3.** The decisions  $Q$  and  $T$  are fixed for all orders but the demand-price decisions  $D$  and the disposal quantity are optimized at each cycle, after having observed the value of the yields, with respect to equation (5.2) considering  $T$  and  $X$  fixed (NB  $X = Q/T$ ). The first derivative of (5.2) with respect to  $d_i$  gives the stationary point:

$$P_i(d_i) = p_i = \left( \frac{h_i T}{2} - r_i \right) \left( \frac{b_i}{b_i - 1} \right) \text{ and } d_i = \max\{p_i, \alpha_i X\}$$

The profit for each cycle is computed according to (1) with the modified value of  $d_i$ . This policy prevents stockouts as in policy P2, but it necessitates accepting price variation at each cycle.

The results of the simulation are presented in Table 5.7. We compared the policies according to the average profit per unit of time. The Table presents the profit found by the solution approach for all the instances studied in this chapter. The gaps are calculated with respect to the profit found by the solution approach proposed in the chapter (static profit).

Table 5.7 Simulation study under yield uncertainty

Inst.	Profit (per unit time)				Gap (%)		
	Static	P1	P2	P3	P1	P2	P3
0	579.28	562.39	549.30	578.93	2.9146	5.1749	0.0593
1	11656.18	11392.97	11223.36	11655.22	2.2581	3.7132	0.0082
2	596.56	574.61	553.92	595.87	3.6792	7.1468	0.1148
3	2722.75	2657.19	2618.95	2721.70	2.4080	3.8124	0.0385
4	1370.79	1324.79	1280.18	1370.42	3.3558	6.6103	0.0269
5	2941.94	2869.33	2818.59	2941.42	2.4680	4.1928	0.0177
6	620.71	597.31	567.08	620.27	3.7699	8.6403	0.0706
7	1232.80	1198.15	1179.01	1232.59	2.8100	4.3630	0.0169
8	2364.46	2268.11	2138.96	2365.10	4.0749	9.5371	-0.0272
9	3141.78	3045.93	2963.29	3140.87	3.0509	5.6810	0.0289
				Min	2.2581	3.7132	-0.0272
				Avg.	3.0789	5.8872	0.0355
				Max	4.0749	9.5371	0.1148

We note that, based on the assumption of the simulation study, policy P3 is always better than policies P1 and P2. It is also not surprising that policy P3 can slightly improve the profit: average yield is used for optimization and more items can be received in the simulation. This would not be the case if a maximal value were used for simulation. The drawback of P3 is that it requires a modification of the prices at each cycle, which is not always possible, and which can have a negative effect not taken into account in the simulation. Policy P1 is better than policy P2 on all instances, but it requires the possibility of stockouts. We finally note that the proposed model, initially dedicated to a deterministic context, is sufficiently efficient to be adapted to situations with uncertain quantity yields.

## 5.6 Conclusion

This chapter addresses a disassembly EOQ model which integrates decisions to provide profits in disassembly systems. Due to specific characteristics of disassembly systems surplus inventory can be generated. This surplus inventory can lead to inappropriate inventory decisions that are not environmentally sound. Disposal decisions can be applied to handle it. However, when it is possible, it can be both economically and environmentally advantageous to vary the demands if they are price sensitive according to a profit function.

A model has been developed in this chapter to set the prices of components in disassembly systems in order to optimize a profit function with different cost structures. When considering only the disassembly cost or the disassembly with inventory costs (order and holding cost under EOQ assumptions), the optimality conditions have been derived showing that simple non-linear search methods can be used. Disposal decisions and costs have been considered subsequently, which has led to a non-linear problem with constraints. The KKT conditions have been derived to propose a solution approach. It shows that keeping the disposal option can lead to higher profit margins if the demands are price sensitive, depending on the data instances. The numerical examples show that the procedure provides solutions in short computational times.



In practical applications, estimation of the required data for the proposed model is a real challenge. However, based on the numerical analysis, the model seems to be robust enough to be used, in a first attempt to manage the process, with rough data. The quality or ratio of obtained parts and materials may be different after the disassembly operation. The solution obtained from the model can, however, be used in an uncertain context as shown in the numerical experiment. Further research can extend the results in several ways. In disassembly centers, the return of products can be limited and a price sensitive return function can be added to the model. Shortages can also be taken into account with different strategies and associated costs. Additionally, our model considered EOQ-like assumptions, but it could be interesting to study the effect of pricing decisions with other inventory assumptions such as dynamic lot-sizing problems, or stochastic models.

### Contributions

Preliminary results of this chapter were presented at:

- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2020). Disassembly EOQ models with Price-Sensitive Demands, *Applied Mathematical Modeling* (<https://doi.org/10.1016/j.apm.2020.06.011>).



## Chapter 6

# Conclusions And Perspectives

### 6.1 Conclusions

Disassembly problems, as an important step in reverse logistics has attained a lot of research interests. This is because of the growing environmental concerns and legislation obligations such as End-of-Life Vehicles (ELVs) directive (2000/53/EC). Disassembly aims to decrease environmental impacts of End-Of-Life (EOL) products by separating dangerous or valuable materials and parts. Disassembly problems have specific characteristics which lead to the issue of surplus inventory accumulations in disassembly systems. There are different inventory management models in the literature, but optimal inventory policies can not be found if no decisions are considered to manage the surplus inventory, and this is impossible to apply in practice. based on this fact, the objective of this thesis is to propose new inventory management models and efficient solution approaches which consider the decisions on surplus inventory in disassembly systems and investigate the opportunities of cost saving.

Chapter 2 reviews the related literature on different inventory management models and affirms that the existed inventory control models and their assumptions cannot be applied to provide an optimal inventory policy in disassembly systems. New models and methods are needed. We evaluate two popular inventory management models in the literature i.e., lot-sizing and EOQ, and their applicability in disassembly systems are investigated. Section 2.2 investigate the studies on the lots-sizing models and it is been seen that disposal decisions have never been integrated to lot sizing models in disassembly systems to handle surplus inventory. In section 2.3, the EOQ model as a most used inventory model in the literature is reviewed, but there is limited number of researches who proposed EOQ models in disassembly systems and integrate disposal decisions to handle surplus inventory due to disassembly operations. To maximize the profit of disassembly systems integration of both pricing and disposal decisions in the inventory policy is proposed for the real life practices.

Chapter 3 considers the single-product disassembly lot-sizing problem with disposal decisions. First, the issue of inventory accumulation in disassembly systems is highlighted. By using the disassembly lot-sizing methods that do not consider any decision for the surplus inventory, we show that when a surplus inventory is generated in one period, it can be either consumed in later periods or stored until the end of the planning horizon. A procedure is proposed to compute the two types of surplus inventory and we highlight the potential inventory cost saving. Three new MIP formulations are presented provide different linear relaxation lower bounds (LB). Two efficient heuristics are proposed that find feasible

solutions in short computational times based on different rules for ending inventories between periods. They can also be implemented via spreadsheet applications, which can facilitate their application in real industrial cases. To compared the proposed models and methods, different cost settings are used to generate the instances. Finally a sensitivity analysis is proposed to highlight their effects, and managerial insights are also highlighted.

In Chapter 4, the problem is developed for the multi-product with parts commonality. For the case of problem with two-level product disassembly structure, two new MIP formulations are proposed which differ from quality of lower bound. A two-phase heuristic is proposed for large-sized problems, which obtains an initial solution constructed from the solution of LP relaxation approach, then improves it by using a dynamic programming based algorithm. The problem is extended for the cases of multi-level with capacity restrictions. MIP formulation cannot solve optimally large-sized problem instances. A Fixed-and-optimize (FO) heuristic is proposed, which can solve the problems in a short computational time. Its quality of solution is improved by considering interrelatedness characteristics between linked variables. The proposed models and methods for each problem case are compared by using randomly generated problems instances.

Chapter 5 is devoted to DEOQ model that permits the effect of pricing to be used on the demands, with consideration of disposal decisions in order to handle unnecessary accumulations of components in a recovery system optimally. This surplus inventory can lead to inappropriate inventory decisions that are not environmentally sound, but disposal decisions can be applied to handle this. However, when possible, it can be both economically and environmentally advantageous to vary the demands if they are price-sensitive according to a profit function. A model is developed to set the prices of components in disassembly systems in order to optimize a profit function with different cost structures. When only the disassembly cost or the disassembly and inventory costs (order and holding costs under EOQ assumptions) are considered, the optimality conditions are derived and showed that simple nonlinear search methods can be used. Disposal decisions and costs are then considered subsequently, which lead to a nonlinear problem with constraints. The KKT conditions are derived to propose a solution approach. This shows that keeping the disposal option can lead to higher profit margins if the demands are price-sensitive, depending on the data instances. The numerical examples show that the procedure provides solutions in short computational times.

## 6.2 Perspectives

- Future work of Chapter 3 must be dedicated to explore heuristic based algorithm based on initial solution constructed by the LP relaxation of FAL formulation. This is interesting because FAL formulation obtain a very strong lower bound of the problem. As a first attempt, an initial solution for the problem can be obtained by rounding up or down the obtained solution of the LP relaxation of FAL formulation. The initial solution then can be improved. Another extension can be taken into account demand balancing by using pricing problem, to handle the surplus inventory problem and to provide profitable decisions in disassembly systems. Also, The number of available EOL product can be limited in real industrial cases which can be considered in the proposed models. It should be also interesting to consider stochastic parameters like demand and yield in the proposed models to model more realistic problem.

- One of first future work of Chapter 4 is to improve two-phase heuristic proposed so that it can compute surplus inventory obtained from each root item and applying it during the phase of improvement. Then Two-phase heuristic can be extended to consider the problem with multi-level product structure and capacity constraints. The result can be compared by the one obtained by the proposed FO. Another future work can focus on developing FAL formulation for the multi-level product disassembly structure. Then the heuristic algorithm based on LP relaxation of FAL formulation can be considered to compare with other solution methods such as Two-phase heuristics based on LP relaxation of AGG formulation. Also the solution obtained by LP relaxation of FAL formulation can be applied in a B&B algorithm. Algorithm FO is based on quality of initial solution. Applying new methods to obtain more stronger lower bound of the problem and obtaining initial solutions with higher quality can improve significantly the solution quality of the proposed FO. So it may be interesting to develop FO base on FAL formulation which has a strong lower bound.
- Future works on Chapter 5 can extend the results in several ways. In disassembly centers, the returned of products can be limited, and a price-sensitive return function can be added to the model. Also, a more complex disassembly product structure can be considering such as multi-product, multi-level with parts commonality. In real industrial cases, the returned product are not in same quality and obtained parts from disassembly operations can be varied. Considering stochastic parameters such as demand and yield can help to model a more realistic problem.



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# Appendix A

## List of publications

The research conducted in this thesis has been presented in several international journals and conferences as follows:

### International journals:

- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2020). New models and efficient methods for disassembly lot sizing problem with surplus inventory decisions. *International Journal of Production Research* (DOI: 10.1080/00207543.2020.1829148).
- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2020). "Disassembly EOQ models with price-sensitive demands" *Applied Mathematical Modelling* 88: 810–826 (<https://doi.org/10.1016/j.apm.2020.06.011>).

### International conferences:

- Tafti, M. P. M., Godichaud, M., and Amodeo, L. (2019). Single product disassembly lot sizing problem with disposal. In *2019 IEEE 6th International Conference on Industrial Engineering and Applications (ICIEA)*, pages 135–140. IEEE (DOI: 10.1109/IEA.2019.8715056).
- Tafti, M. P. M., Godichaud, M., and Amodeo, L. (2019). Models for the single product disassembly lot sizing problem with disposal. *IFAC-PapersOnLine*, 52(13):547–552 (DOI: 10.1016/j.ifacol.2019.11.215).
- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2019). Models for disassembly lot sizing problem with decisions on surplus inventory. In *Advances in Optimization and Decision Science for Society, Services and Enterprises*, pages 423–432. Springer (DOI: 10.1007/978-3-030-34960-8\_37).
- Pour-Massahian-Tafti, M., Godichaud, M., and Amodeo, L. (2020). Disassembly Lot Sizing Problem with Disposal Decisions for Multiple Product Types with Parts Commonality. (IFAC 2020-Accepted).



# Appendix B

## Resumé en français

### Optimisation de la logistique inverse pour la revalorisation des produits en fin de vie

**Résumé :** Cette thèse étudie des modèles de gestion de stocks pour la planification du désassemblage. Nous traitons d'abord trois cas du problème de lot-sizing en désassemblage avec l'évacuation pour gérer le surplus de stocks. Le premier cas est mono-produit avec une structure de produit à deux niveaux pour lequel nous proposons une procédure pour calculer les surplus de stocks et trois modélisations selon des formulations MIP avec différentes qualités de borne inférieure. L'ajout d'inégalités valides et de deux heuristiques sont aussi proposés. Le deuxième cas est multi-produit avec des structures de produit à deux niveaux et des pièces communes et deux nouveaux MIP sont proposés pour modéliser le problème. Une relaxation des formulations est proposé pour améliorer la borne inférieure du problème et une heuristique à deux phases est pour une résolution plus rapide des problèmes de grande taille. Le troisième cas est multi-produit avec des structures de produits à plusieurs niveaux, des pièces communes et des contraintes de capacité pour lequel une nouvelle formulation MIP est proposée. Pour les problèmes de grande taille, Fix-and-Optimize est développés comme méthode de résolution avec les caractéristiques d'interrelation entre des variables liées. On traite aussi un modèle EOQ en désassemblage avec des demandes fonction des prix et l'évacuation pour maximiser le profit. Trois modèles sont développés pour obtenir des solutions qui proposent les prix, la durée du cycle de réapprovisionnement et la quantité à évacuer. La politique de gestion de stocks intégrant les décisions de prix et de la disposition permet d'obtenir des profits plus élevés.

**Mots clés :** Logistique inverse, Planification, Gestion des stocks, Recherche opérationnelle, Heuristique.

### B.1 Introduction

Au cours de la dernière décennie, de nombreuses entreprises qui ne consacraient pas beaucoup de temps à l'étude et à la mise en œuvre de la logistique inverse ont commencé à y accorder une attention considérable. Cela s'explique par les préoccupations croissantes concernant les impacts environnementaux des produits en fin de vie (EOL) et la valeur résiduelle de leurs composants. En outre, les pressions législatives forcent les pays à adopter des lois réglementant le traitement des produits en fin de vie, et les entreprises doivent reprendre leurs produits usagés en respectant l'environnement. Par exemple, la

directive sur les véhicules en fin de vie (directive 2000/53/CE) pousse les entreprises à envisager des méthodes de réutilisation, de recyclage ou de récupération d'énergie pour obtenir un taux de récupération d'au moins 95%. Le volume potentiel des activités de logistique inverse dans l'économie est important et continue de croître, et le taux de retour des produits peut être considérable. Il peut s'agir de sources potentielles de matières premières, de pièces détachées ou d'énergie à un coût moindre et, le traitement optimal des retours de produits constitue un avantage concurrentiel ((Rogers et al. (1999); Min et al. (2006); Kim et al. (2007)). Au début de l'apparition de la logistique inverse, elle était principalement axée sur le recyclage de certains produits contenant des substances dangereuses pour l'environnement, ou la récupération des matières premières réutilisables comme l'or des composants électroniques. Au cours des dernières décennies, la logistique inverse est devenue un marché à multiples facettes, elle a pris en compte les cas de retour des produits en fin de vie (ou des produits défectueux), les retours d'emballages, et les produits usagés ou non désirés par les clients. Les entreprises peuvent aussi tirer des bénéfices de la réutilisation de composants et/ou de matériaux de valeur ou de redonner une nouvelle vie aux produits en fin de vie en les remettant en bon état. En raison de son impact économique et environnemental, le contrôle adéquat des flux inversés de produits et de matériaux retournés est d'une importance considérable dans de nombreuses industries. C'est particulièrement vrai pour les processus de récupération des produits dans les systèmes de reconditionnement où les produits retournés sont retraités afin d'obtenir leurs pièces de valeur pour être réintégrés dans le flux logistique ou la récupération d'énergie des matériaux.

Le contrôle des stocks est l'un des défis à relever dans la gestion du flux de retour des produits en fin de vie. En effet, la gestion des stocks en présence de produits retournés entraîne une plus grande complexité (de Brito and Dekker (2003); Fleischmann and Minner (2004); Kleber (2007). Le flux de retour entrant des produits en fin de vie doit être intégré de manière appropriée dans la planification traditionnelle des matériaux (Fleischmann and Kuik (2003)). Il existe différents modèles de gestion de stock dans les recherches sur la planification de la production traditionnelle, mais ils nécessitent une adaptation pour garantir l'exploitation réussie des pièces et des matériaux des produits retournés (Fleischmann et al. (1997)). Dans cette thèse, nous considérons les questions de gestion des stocks liées aux opérations de récupération dans la logistique inverse afin de fournir des stratégies de gestion de stock optimales pour que les décideurs puissent réagir au mieux dans ce domaine complexe. Le désassemblage est considéré comme une étape importante dans la récupération des produits en fin de vie dans la logistique inverse, la plupart des produits retournés sont désassemblés avant d'être remanufacturés, recyclés ou éliminés Kim and Xirouchakis (2010). Les processus de désassemblage consistent à générer des composants à partir des produits en fin de vie qui peuvent être affectés à une filière de valorisation, qui génère des demandes et des revenus, ou à d'autres filières, qui réduisent les impacts environnementaux. Toutefois, le gain économique peut être faible entre les revenus et les coûts de démontage. Une planification efficace des systèmes de désassemblage peut accroître les possibilités de réduction des coûts et les rendre plus rentables. Les systèmes de désassemblage présentent des caractéristiques particulières qui les rendent difficiles à planifier : le produit est décomposé pour répondre à de multiples demandes de pièces ou de composants ; les demandes de composants sont indépendantes et pas nécessairement bien équilibrées ; l'opération de démontage génère toutes les pièces ou composants simultanément.

Ces caractéristiques impliquent que la quantité d'un produit à désassembler n'est pas nécessairement égale au nombre de composants demandés, et qu'un surplus inutile de stock est susceptible d'être généré après chaque opération de désassemblage. Il peut être tenu pour satisfaire des demandes futures ou être éliminé de manière écologique dans des cas industriels réels. Plusieurs stratégies peuvent être proposées

pour aider les gestionnaires à gérer ce stock excédentaire, telles que l'acceptation des ventes perdues, l'élimination ou l'équilibrage de la demande par la tarification. Nous considérons les deux modèles de stocks les plus utilisés dans la littérature : le lot sizing et la quantité économique de commande (EOQ). Nous définissons ces deux modèles pour le désassemblage et nous étudions de nouvelles méthodes et de nouveaux modèles pour traiter les problèmes de stocks excédentaires dans les opérations de désassemblage.

Cette thèse est organisée comme suit : un état de l'art est présenté sur deux modèles de gestion des stocks et planification en désassemblage, lot sizing en désassemblage et EOQ en désassemblage. Nous présentons une étude bibliographique sur le problème de lot sizing en production (différentes techniques de modélisations et d'optimisation). Ensuite nous présentons les travaux réalisés sur les problèmes de désassemblage notamment le problème de lot sizing en désassemblage. Nous nous intéressons aux différents types de planification pour gérer les stock surplus en désassemblage. Dans la deuxième partie, nous commençons par présenter le modèle classique EOQ et ses différentes applications dans les différents problèmes. Nous présentons différentes stratégies pour gérer le stock dans un modèle EOQ. Nous analysons les applicabilités des hypothèses et stratégies dans les systèmes de désassemblage.

Dans la section B.2.4 nous traitons le problème de lot sizing en désassemblage pour mono produit avec une structure de produit à deux niveaux. Nous proposons d'abord une procédure pour calculer les accumulations de stocks sur l'horizon de planification, puis trois nouvelles formulations de programmation mixte (MIP) sont proposées pour modéliser le problème. Les formulations diffèrent les unes des autres en ce qui concerne la qualité de la limite inférieure fournie par leur relaxation linéaire, ce qui est une question importante dans les méthodes de résolution des MIP. L'ajout d'inégalités valides est proposé pour améliorer la limite inférieure du problème. Deux heuristiques efficaces avec des stratégies de gestion de stocks différentes sont également étudiées pour des applications réelles lorsque les algorithmes MIP ne sont pas pertinents.

Dans la section B.2.5 nous traitons le problème de lot sizing en désassemblage pour plusieurs produits avec la structure de produit à deux niveaux et avec des pièces communes. Deux nouvelles formulations de MIP sont développées pour le problème qui tiennent en compte de la disposition du surplus de stocks. Une formulation utilisant l'approche de LP-relaxation est proposée pour améliorer la limite inférieure du problème. Pour les cas industriels réels, une heuristique à deux phases est proposée, qui construit une solution initiale en utilisant l'approche de LP-relaxation et l'améliore ensuite par une heuristique basée sur la programmation dynamique.

Dans la section B.2.6 nous traitons le problème de lot sizing en désassemblage pour plusieurs produits avec la structure de produits à plusieurs niveaux et avec des pièces communes, ainsi que l'existence des contraintes de capacité. Une nouvelle formulation de MIP est proposée pour résoudre ce problème. La méthode exacte en utilisant le solveur CPLEX peut être appliquée pour obtenir des solutions optimales pour le problème de petite taille. Pour les cas industriels plus réalistes, une heuristique Fix-and-Optimize (FO) qui résout successivement une série de sous-problèmes où un sous-ensemble restreint de variables est fixé tandis que les autres sont optimisées en utilisant une méthode exacte. Ensuite, nous améliorons la qualité de la solution de l'heuristique FO en considérant les caractéristiques d'interdépendance entre les variables connectées.

Dans la section B.3, pour le deuxième modèle de gestion de stock, nous traitons le problème de la quantité de commande économique en désassemblage (EOQ en désassemblage (DEOQ)). Nous proposons les modèles qui intègrent les demandes sensibles au prix et les décisions de la disposition de surplus de stocks dans le DEOQ pour maximiser le profit des systèmes de désassemblage sans l'accumulation de

stocks. Trois modèles sont développés et analysés pour obtenir des approches de solution qui donnent les prix, le temps du cycle de réapprovisionnement (ou, de manière équivalente, la quantité de commande) et la quantité de disposition. La politique d'inventaire intégrant à la fois les décisions de prix et de la disposition permet d'atteindre un profit plus élevé.

Finalement, nous terminons par une conclusion générale pour résumer l'ensemble de nos travaux réalisés dans cette thèse. Les principaux résultats obtenus pour chaque section sont discutés. Nous présentons enfin des perspectives de nos travaux à réaliser dans la suite de la thèse.

## B.2 Problème de lot sizing en désassemblage avec l'évacuation

### B.2.1 Introduction

Parmi les diverses activités de la logistique inverse, le désassemblage est devenu une activité centrale de plus en plus importante. Les activités de désassemblage relient la collecte des produits en fin de vie chez le client aux centres de recyclage des composants afin de récupérer une valeur résiduelle pour la réutilisation, la re-fabrication, le recyclage ou même l'évacuation appropriée ([Godichaud and Amodeo \(2018\)](#)). Par exemple, des constructeurs automobiles tels que BMW, Volkswagen et Audi effectuent des opérations de désassemblage. Par ailleurs, plusieurs entreprises de désassemblage en Europe (par exemple en France et Suisse) réalisent des bénéfices en vendant des pièces démontées et des composants de produits retournés ([Kim et al. \(2009\)](#)). Les processus de désassemblage consistent à générer systématiquement des composants à partir de produits en fin de vie qui peuvent être affectés à un circuit de récupération, ce qui génère des demandes et des revenus, ou être affectés à d'autres circuits, ce qui réduit les impacts environnementaux. Malgré l'impact positif des activités de désassemblage sur l'environnement, le gain économique entre les revenus et les coûts de désassemblage peut être faible. Cela a incité certains chercheurs à formuler des modèles plus efficaces et plus appropriés pour accroître les possibilités de réduction des coûts et rendre les opérations de désassemblage plus rentables. Divers problèmes de décision en désassemblage apparaissent dans la littérature, nous nous intéressons au problème du lot sizing en désassemblage qui prend en compte l'aspect de la gestion des stocks. Le but du problème est de déterminer la quantité de produits en fin de vie à désassembler sur chaque période de l'horizon de planification afin de satisfaire les demandes des différents composants désassemblés. Dans ce chapitre, nous considérons le problème de lot sizing en désassemblage avec les décisions pour gérer des stocks. Trois cas sont traités dans ce chapitre : mono produit à deux niveaux, multi-produit à multiniveaux avec des pièces communes, multi-produit à multiniveaux avec des pièces communes et des contraintes de capacité. La section suivante étudie le problème d'accumulation de stocks en désassemblage.

### B.2.2 l'analyse d'accumulation du surplus de stocks

Dans les systèmes de désassemblage à la demande, une seule opération de désassemblage sur un produit en fin de vie génère simultanément tous ses composants en ce qui concerne leur rendement. Les demandes de composants sont indépendantes et un surplus de stocks peut être généré. Dans le cas du désassemblage par lots, ce stock supplémentaire est identique pour toutes les solutions (c'est-à-dire quelle que soit la taille du lot) et peut être calculé à l'avance. Une partie de ce surplus, généré dans une période, est consommée dans les périodes futures tandis que l'autre partie est stockée jusqu'à la fin de l'horizon de planification. Pour la première partie, la décision est soit de stocker jusqu'à sa consommation, soit de

l'évacuer, ce qui pourrait augmenter les quantités de désassemblage dans une période suivante. Pour la deuxième partie, elle est évacuée pour économiser les coûts de stockage. Notre objectif est de calculer ce surplus de stock non utilisé et de trouver une stratégie permettant de les évacuer systématiquement, ce qui offre une possibilité d'économie de coûts. Dans les sections suivantes, de nouveaux modèles et méthodes sont proposés pour déterminer les meilleures décisions dans un système du désassemblage.

### B.2.3 Problème sans les décisions sur le surplus de stocks

La plupart des recherches précédentes sur le problème de lot sizing en désassemblage ont modélisé le problème sous la forme d'une formulation de programmation en nombres entiers (IP). Leur objectif est soit de trouver des solutions probablement optimales, soit d'obtenir des solutions quasi optimales avec une garantie de performance mesurable (c'est-à-dire un écart en pourcentage par rapport à la valeur optimale). Deux formulations différentes qui tiennent compte des coûts associés au processus de désassemblage existent dans la littérature pour le problème de lot sizing en désassemblage de mono-produit : Formulation agrégée (AGG) et AGG sans les variables de stocks (NIF) (Section 3.2.1). Certaines études dans la littérature sur le problème de lot sizing en désassemblage ont proposé un algorithme exact basé sur le modèle de programmation dynamique qui peut obtenir les solutions optimales en temps polynomial. Pour déterminer la solution optimale de l'ensemble du problème, le problème est décomposé en sous-problème  $T$  de 1 à  $T$ . Chaque sous-problème de la décomposition est résolu récursivement, en commençant par la période 1 et en terminant par la période  $T$ . Le programme dynamique est représenté comme équation (3.10) (Kim (2005)). La question de stocks surplus de composants conduit à son accumulation tout au long de l'horizon de planification, surtout la quantité accumulée à la fin de l'horizon qui ne sert à satisfaire aucune demande. Ces stocks supplémentaires peuvent augmenter pour certains composants en raison des écarts importants de demandes entre les composants. Le rendement peuvent être aussi différents d'un composant à un autre. Si aucune décision n'est envisagée pour traiter les surplus, comme dans les modèles et méthodes existants dans la littérature, les coûts de stockage sont plus importants et cela est difficile à appliquer dans la pratique au milieu industriels. Dans ce chapitre, nous considérons l'évacuation comme une option pour gérer des stocks de surplus et nous développons différentes formulations pour modéliser le problème de lot sizing en désassemblage avec l'évacuation pour gérer les stocks. Trois cas de problème sont traités et nous développons de différentes approches et techniques d'optimisation permettant de résoudre les modèles proposés.

### B.2.4 Problème mono produit à deux niveaux

Nous traitons le problème de lot sizing en désassemblage avec l'évacuation qui vise à déterminer la quantité et le timing de désassemblage du produit en fin de vie afin de satisfaire les demandes de composants sur un horizon de planification limité, tandis que les stocks surplus de composants peuvent être redirigés vers d'autres filières de valorisation du système de désassemblage. L'objectif est de minimiser le coût global, qui comprend le coût de setup relatif au produit en fin de vie, les coûts des opérations de désassemblage et le stockage des composants désassemblés. Dans cette section nous étudions le problème avec une structure à deux niveaux et un seul produit. Un exemple de cette structure est donné à la figure 3.1 qui illustre le désassemblage d'un produit en fin de vie (produit A) ou root item en anglais. Le nombre entre parenthèses est le rendement du composant ou yield en anglais, lorsqu'une unité de produit A est désassemblée. Le premier niveau représente les composants ou leaf items en anglais (composant 1, 2, 3, et 4), tandis que le deuxième niveau représente un seul produit A. Le désassemblage du produit

A permet d'obtenir trois composants de 1, trois composants de 2, deux composants de 3, et quatre composants de 4. Nous développons ensuite trois formulations qui considèrent l'évacuation pour gérer l'accumulation de stocks. Nos modèles sont soit des extensions du modèle classique du lot sizing en désassemblage ou des adaptations des modèles du lot sizing en production.

**Modele AGG (P1)** Le problème peut être formulé sous la forme d'une programmation mixte en nombres entiers (MIP en anglais). Ce modèle est basé sur des variables de stocks et des contraintes de conservations de flux, ce qui est traditionnellement utilisé dans la littérature sur le lot sizing en désassemblage et production. Les décisions d'évacuation sont prises en compte dans ce modèle par une variable de décision supplémentaire  $E_{it}$  qui peut être définie comme la quantité de composants  $i$  évacuées dans la période  $t$ . La formulation mathématique (modèle [P1]) du problème est présentée dans les équations (3.18–3.24). La fonction objectif (3.18) vise à minimiser la somme des coûts de setup, des opérations de désassemblage, et de stockage. Les contraintes (3.19) représentent la conservation des flux de stockage pour les composants. Elles expriment que le stock entrant ( $I_{it-1}$ ) ajouté à la quantité de désassemblage de la période en cours ( $a_i \cdot X_t$ ) est utilisé pour satisfaire la demande ( $d_{it}$ ), puis la quantité désassemblée supplémentaire sera éliminée ( $E_{it}$ ). La quantité restante est conservée en stock à la fin de la période ( $I_{it}$ ). Il convient de noter que nous n'avons pas tenu compte de la contrainte de flux de stock pour les produits en fin de vie et qu'aucun coût d'évacuation n'est pris en compte dans la fonction objectif. Cependant, les variables d'évacuation ( $E_{it}$ ) permettent de gérer les stocks de surplus de composants en équilibrant les coûts des opérations de stockage et de désassemblage. Les contraintes (3.20) garantissent qu'un coût de setup est effectué au cours de la période  $t$  si une opération de désassemblage est effectuée au cours de cette période. Les contraintes (3.21–3.24) imposent la non-négativité et des restrictions binaires sur les variables. Nous avons utilisé la formule (3.25) pour calculer une valeur appropriée du paramètre  $M_t$ , qui nous permet d'améliorer la borne inférieure du problème. La relaxation continue des modèles MIP joue un rôle important dans diverses méthodes telles que l'algorithme Branch-and-Bound, l'obtention de solutions initiales dans une heuristique à deux phases, les heuristiques basée sur la programmation linéaire ou, etc. (Alfieri et al. (2002); Lee and Xirouchakis (2004); Pochet and Wolsey (2006)). Nous définissons [P2] comme la relaxation continue de [P1] en supprimant les restrictions d'intégralité sur les variables  $X_t$ .

**Modele AGG sans variable se stocks (NIF)(P3)** Le modèle agrégé peut être modifié pour obtenir une formulation compacte sans variables d'inventaire. L'équation (3.26) est une autre façon d'écrire le flux de conservation de l'inventaire pour le problème. La formulation mathématique du modèle NIF (modèle [P3]) du problème est présentée dans les équations (3.27–3.32). Nous définissons [P4] comme la relaxation continue de [P3] en supprimant les restrictions d'intégralité sur les variables  $X_t$ .

**Modele AGG avec les contraintes supplémentaires (P5)** De nombreuses recherches ont été consacrées à l'obtention la relaxation de programmation linéaire plus serrée et à l'amélioration des bornes inférieures correspondantes. En particulier, les inégalités valides (Valid Inequalities (VIS) en anglais) qui réduisent le volume de l'espace de solution de relaxation continue de la programmation linéaire en coupant les parties non pertinentes. Nous adaptons l'expression des inégalités ( $l, S$ ) proposées par Pochet and Wolsey (2006) pour notre problème. Les contraintes définissent la classe plus générale des inégalités dites ( $l, S$ ). En ajoutant ces contraintes à la formulation de modèle [P2], nous pouvons obtenir une description linéaire plus serrée du problème. Les contraintes (3.33) permettent de calculer une



limite inférieure sur le niveau de stocks de chaque composant à la fin d'une période. Nous définissons le modèle [P5] comme le modèle [P2] en ajoutant les équations (3.33). Dans les résultats expérimentaux, nous utilisons un algorithme de génération de plan de coupe utilisant l'algorithme de séparation pour ajouter les inégalités les plus isolées de la famille d'équation (3.33). Comme en pratique le nombre de ces inégalités est limité, l'algorithme de séparation dans ce cas est effectué par dénombrement. Notez que les coupes VIs ne peuvent pas être utilisées pour la formulation FAL and NIF parce qu'elle exige une équation de la conservation des flux de stockage. Le tableau 3.6 représente les résultats de l'addition récursive des inégalités ( $l$ ,  $S$ ) à la formulation initiale ([P2]) et l'ajout de VIs est très efficace pour obtenir améliorer la borne inférieure du problème. Pour les données de l'exemple, cette amélioration atteint 21,20%, ce qui est important.

**Modele FAL (P6)** Cette section présente la formulation désagrégée ou Facility-Location based (FAL) en anglais pour le problème (modèle [P6]). Cette formulation est le plus souvent utilisée pour le problème de lot sizing en production, car sa relaxation linéaire continue offre une solution optimale dans laquelle les variables de binaires sont entières et elle a des bornes inférieures plus fortes pour le problème de lot sizing en production. Une variable de décision ( $Z_{ikt}$ ) est considérée, elles correspondent à la quantité de composants  $i$  désassemblés dans la période  $t$  pour satisfaire la demande de la période  $k$ . La variable de quantité de désassemblage des produits en fin de vie à la période  $t$  ( $X_t$ ) ne peut cependant pas être supprimée car chaque composant peut être reçu après une opération de désassemblage au cours d'une période. Dans ce modèle, les variables  $X_t$  doivent être entières mais les variables  $Z_{ikt}$  peuvent être définies comme réelles. La formulation désagrégée pour le problème de lot sizing en désassemblage avec l'évacuation pour des problèmes de lot sizing en désassemblage pour un seul produit avec une structure de produit à désassembler à deux niveaux est présentée dans les équations (3.34-3.40). La fonction objectif (3.34) consiste à minimiser la somme des coûts de setup, d'opérations de désassemblage associée au désassemblage des produits en fin de vie et de stockage associée aux composants désassemblés, sur l'ensemble de l'horizon de la période  $T$ . Les contraintes (3.35) garantissent la satisfaction de demandes de composants pour à chaque période  $t$ . Les contraintes (3.36) relient la variable de quantité de composants désassemblés ( $Z_{ikt}$ ) à la variable binaire de setup ( $Y_t$ ). Les contraintes (3.37) expriment que la quantité totale de composants  $i$  obtenue dans la période  $t$ , après désassemblage du produit en fin de vie, sera livrée pour satisfaire la demande ou sera évoquée. Les contraintes (3.38-3.40) définissent le domaine des variables de décision. Nous définissons [P7] comme la relaxation continue de la programmation linière de modèle [P6] en supprimant les restrictions d'intégralité sur les variables  $X_t$ , et nous pensons qu'elle permet d'obtenir les solutions optimales ou quasi-optimales du problème dans des temps de calculs assez courts. Dans la formulation FAL, en utilisant le fait que la quantité de stocks de surplus de composants  $i$  dans la période  $t$  est  $a_i \cdot X_t - \sum_{j=t}^T Z_{itj}$  puisque  $a_i \cdot X_t$  est la quantité totale de composant  $i$  reçus dans la période  $t$  et  $\sum_{j=t}^T Z_{itj}$  est la quantité de composant  $i$  livrées de la période  $t$  à  $T$ . Cette quantité supplémentaire peut être évacuée de la période  $t$ , ce qui est égal à la nouvelle variable  $E_{it}$ . Dans la section (3.3.4), nous présentons également la forme générale de la formulation FAL et nous développons le modèle FAL pour le problème du lot sizing en désassemblage sans et avec évacuation des stocks surplus non utilisés.

**Résolution exacte** Les modèles P1, P4 et P6 sont des MIP, ils peuvent être résolus de façon exacte en utilisant le solveur CPLEX pour arriver à une solution optimale. Les solveurs de programmation linéaire moderne tels que CPLEX utilisent différentes coupes et incluent des processus pré-résolus lors

de la résolution des MIP. Nous analysons l'impact de l'autorisation ou non des coupes par défaut et des processus de pré-résolutions lors de la résolution du problème en utilisant le solveur CPLEX. Il est nécessaire de mentionner que lorsque l'on résout un problème de taille plus réaliste en utilisant le solveur CPLEX, le temps de calcul augmentera avec un nombre de plus en plus élevé de composants ainsi qu'avec des horizons de planification de temps plus longs.

**Heuristiques pour le problème de lot sizing avec l'évacuation** Les algorithmes heuristiques ont l'avantage de pouvoir être codés et exécutés via de simples applications de tableur, ce qui facilite leur application. Il est nécessaire de mentionner que pour une taille plus réaliste du problème, le temps de calcul augmente rapidement pour les formulations MIP avec un nombre de plus en plus élevé de composants et des horizons de planification beaucoup plus longs. Dans cette situation, il est possible d'utiliser les heuristiques, qui peuvent résoudre les problèmes en un temps de calcul très court. Dans ce qui suit, deux heuristiques permettant d'envisager différentes politiques de gestion des stocks sont proposées pour le problème du lot sizing en désassemblage avec évacuation pour un seul produit avec une structure du produit à deux niveaux :

- Heuristique 1 [**H1**] : La première heuristique est une approche basée sur la programmation dynamique où l'on se débarrasse de tous les stocks supplémentaires générés par des opérations de désassemblage des produits en fin de vie. Seules les quantités de composants nécessaires pour satisfaire la demande jusqu'au prochain setup sont conservées en stock (c'est-à-dire, dans la formulation  $S_{it}^I = 0$ ). Cela conduit au programme dynamique présenté dans l'équation (3.51) (notez que  $S_{i0}^c$  et  $F(0) = 0$ ).
- Heuristique 2 [**H2**] : La deuxième heuristique est également une approche basée sur la programmation dynamique, mais par rapport à H1, le stock consommé est maintenu en stock même entre plusieurs étapes. Le stock consommé (c'est-à-dire,  $S_{it}^c$ ) est précalculé avec la procédure proposée dans la section 3.2.2, donc dans la formulation  $S_{it}^I = S_{it}^c$ . L'objectif est de désassembler moins de produits en fin de vie en gardant certains composants désassemblés en stock plus longtemps (diminution des coûts de désassemblage et stockage). La méthode H2 peut être représentée comme l'équation (3.52) (notez que  $S_{i0}^c$  et  $F(0) = 0$ ).

Les tableaux 3.7 et 3.8 présentent le résultat des heuristiques H1 et H2 pour l'exemple de structure de produit à deux niveaux (par exemple, un véhicule en fin de vie) présenté dans la figure 3.1 et le tableau 3.1. Comme le montre le tableau 3.7, l'heuristique H1 peut obtenir des solutions quasi optimales pour l'exemple et son écart en pourcentage (gap) de la solution obtenue n'est que de 0,01%. L'heuristique H2 est également efficace et son gap est de 1,95%. Les deux heuristiques sont efficaces pour gérer les stocks surplus, en particulier à la fin de l'horizon de planification. Le décideur peut choisir la stratégie d'évacuation de tous les stocks surplus (heuristique H1), ce qui conduit à une réduction essentielle des coûts de stockage (le nombre total de tous les composants qui sont détenus sur l'horizon de planification est de 6654 pour H1 contre 7090 pour H2). En outre, cela permet de démonter plus de produits en fin de vie (par exemple des véhicules en fin de vie), ce qui entraîne également des avantages plus écologiques.

**Expérimentations numériques** Dans cette section nous générons des instances pour le problème étudié afin de tester les performances des modèles et des méthodes proposées. Un nouveau benchmark pour le problème du lot sizing en désassemblage est également proposé afin de mettre en évidence l'effet des différents paramètres de coût et de la taille des problèmes. Les modèles et les algorithmes ont été mis en œuvre sur eclipse en utilisant le langage de programmation Java. Les solutions optimales du

problème sont obtenues en résolvant les modèles MIP à l'aide du solveur CPLEX 12.8. De plus, les tests sont codés sur une machine équipée d'un processeur Intel Core i7-7700T avec une vitesse de 2.9 GHz, et 16 Go de RAM sur Windows 10. Nous avons adapté les paramètres des instances de [Kim and Xirouchakis \(2010\)](#) pour obtenir les nouvelles instances pour le problème de lot sizing en désassemblage. Nous avons modifié les paramètres des instances pour obtenir le ratio TBO (en anglais Time Between Order) plus adapté au problème de lot sizing en désassemblage. Nous avons adapté l'équation de TBO présentée par [Godichaud and Amodeo \(2018\)](#) pour modifier les paramètres de génération des instances. Tableau 3.9 présente les valeurs de paramètres utilisées pour générer les instances. 150 instances de tailles différentes ont été générées, c'est-à-dire 10 instances pour chaque combinaison de trois niveaux de nombre de composant (c'est-à-dire,  $\mathbf{N}=10, 100, 1000$  ; bas (**L**), moyen (**M**), haut (**H**)), et cinq niveaux de nombre de périodes ( $\mathbf{T} = 10, 20, 30, 40, 50$ ). Le tableau 3.9 compare aussi le TBO de notre benchmark avec celui de [Kim and Xirouchakis \(2010\)](#). Les résultats sont présentés dans les tableaux 3.10 et 3.11 qui fournissent le gap et les temps de calcul moyen pour chaque niveau de nombre de composants ( $N$ ) et de horizons de planification ( $T$ ). Ils représentent également le gap et les temps de calcul minimal, moyen et maximal pour chaque méthode et modèle sur toutes les instances. D'après ces résultats, les MIP P1, P3, P6 sont plus performants avec les instances de tailles petites. Le temps de calcul augmente rapidement à mesure que la taille du problème augmente et la variation des temps de calcul est significative pour les différentes tailles de problème. Contrairement aux MIP P3 et P6, P1 peut obtenir une solution optimale de tous les cas de problèmes testés dans un temps limité de 3600 secondes. Par exemple, le gap de P3 pour les instances avec un grand nombre de composants et un nombre de périodes de 50 peut atteindre 0,58%. Les modèles P2 et P4 peuvent résoudre les instances dans un temps de calcul très court. Cependant, comme le montre le tableau 3.11, ils ont une borne inférieure faible (avec un gap moyen de 11,98% et 12,01%, respectivement). L'ajout des inégalités valides est proposé pour améliorer la borne inférieure du problème, c'est-à-dire, le modèle P3. Il permet d'obtenir des solutions optimales ou quasi-optimales dans un temps de calcul raisonnable. Son gap n'est que de 0,01% et il peut obtenir des solutions optimales pour environ 80% des instances, et pour les autres, le gap moyen n'est que de 1,19%. Nous constatons également que la relaxation continue d'une reformulation MIP peut convenir comme le modèle P7 ce qui permet d'obtenir des solutions optimales dans 84% des instances, et pour les autres, le gap moyen n'est que de 0,94%. Ces résultats indiquent qu'ils peuvent être utilisés dans la variante plus complexe du lot sizing en désassemblage avec l'évacuation ou des heuristique basée sur la programmation linéaire. Dans cette étude, nous avons proposé deux heuristiques efficaces avec des stratégies de gestion de stocks différentes qui sont également étudiées pour des applications réelles lorsque les algorithmes MIP ne sont pas pertinents. Les résultats des tableaux 3.10 et 3.11 présentent des performances des heuristiques proposées. Nous constatons que H2 obtient des solutions avec le gap plus petit que H1. Cela montre que la stratégie d'évacuation des surplus non utilisés et conserver des stock consommables est plus économique que la stratégie d'évacuation de tous les surplus (c'est-à-dire, H1). Il est à noter que les heuristiques proposées ici sont aussi très performantes pour les grandes instances générées pour le problème. Le décideur peut utiliser les heuristiques facilement pour choisir sa stratégie de stockage car elles sont très rapides; cela est particulièrement intéressant pour les cas industriels. De plus, une analyse de sensibilité est proposée pour examiner l'effet d'une variation des paramètres importants dans les systèmes de désassemblage (c'est-à-dire,  $s_t, h_{it}, d_{it}$  et  $a_i$ ). Nous remarquons leurs impacts sur la performance des modèles et méthodes proposés. Ces résultats permettent également d'aider le décideur pour prendre une décision plus stable dans un système de désassemblage. Les résultats de

cette analyse de sensibilité sont présentés dans la section 3.4.3. Nous résumons ci-dessous certains des résultats importants :

- La variation du coût de setup a un peu d'effet sur le temps de calcul pour P2, P4, P7, H1 et H2. Mais le temps de calcul P1, P5 et P4 diminue quand les coûts de setup sont élevés. Parmi les formulations de MIP, P6 (P5) est plus stable (plus variable) en termes du CPU. La variation des du coût de setup n'affecte pas également le gap (%) de H2, P5 et P7. La performance de H2 est toujours meilleure que celle de H1. Le gap moyen de H1 est significatif pour les coûts de setup faible et il s'améliore lorsqu'on augmente le coût de setup. Cela signifie que dans les systèmes de désassemblage à faible coût de setup, la stratégie d'évacuation des surplus non utilisés peut constituer une stratégie optimale de stockage.
- la variation du coût de stockage a un impact sur la performance des heuristiques proposées. Le gap (%) de H2 s'aggrave à mesure que le coût de stockage augmente. Cela signifie que la stratégie d'évacuation des surplus utilisés n'est plus économique dans un système de désassemblage avec des coûts de stockage assez élevés. C'est plus économique d'évacuer tous les stocks surplus (consumable et non-utilisé)
- Nous analysons également l'effet du rendement et de la demande sur le coût total. Les résultats nous montrent qu'il y a un problème d'accumulation de stocks surplus qui peut influencer directement le coût total d'un système de désassemblage. Cette accumulation peut être importante pour les instances avec les demandes importantes et l'écart important entre les rendements de désassemblage de composants, surtout à la fin de l'horizon de planification. Nous constatons que les modèles avec l'évacuation sont efficaces pour gérer ces stocks et minimiser le coût total de désassemblage. Nous constatons également que l'augmentation du taux de rendement pour une demande donnée entraîne une plus grande économie de coûts. La raison en est que les décisions d'évacuation peuvent fournir un équilibre économique important pour les produits en fin de vie, en particulier pour ceux qui ont un rendement élevé.
- Comme on peut le voir dans le tableau 3.13, les décisions d'évaluation peuvent entraîner une réduction significative des coûts des systèmes de désassemblage (par exemple, la moyenne des économies peut atteindre 78,06% pour les problèmes liés au nombre élevé composant et à 50 périodes). Nous observons également que le montant des économies réalisées augmente avec la taille des problèmes (nombre élevé de composants et d'horizon de planification plus long). Cela montre que l'application des décisions relatives aux stocks surplus, comme l'évacuation, est importante pour garantir l'équilibre économique, en particulier pour les problèmes industriels réels de grande taille.
- Le tableau 3.14 représente les résultats d'analyser l'impact de l'intégration des nouvelles techniques dans le solveur CPLEX en comparant les performances du solveur CPLEX par défaut à la désactivation des techniques (coupes par défaut et processus pré-résolution). Cela peut avoir un impact (positif ou négatif) sur les performances des modèles et méthodes proposées.

**Conclusions** Un nouveau problème de lot sizing en désassemblage avec une structure de produits à désassembler à deux niveaux est présenté, ce qui est un défi essentiel pour les entreprises industrielles. Les décisions relatives à la gestion des stocks surplus dans les systèmes de désassemblage sont étudiées afin d'établir un équilibre économique. Ce stock de surplus peut apparaître après le désassemblage des

produits en fin de vie et entraîne un coût de stockage considérable. L'analyse détaillée de l'accumulation des stocks montre que les décisions d'évacuation permettent une économie de coûts significative (par exemple, la moyenne des économies de coûts peut atteindre environs 78% pour les cas problématiques avec 50 périodes où le nombre de composant est élevé (c'est-à-dire 1000 composants)). Trois formulations mathématiques différentes ont été développées. Elles fournissent différentes bornes inférieures du problème, et leur effet sur l'efficacité des méthodes de résolution MIP a été décrit. Les temps de calcul augmentent rapidement pour les problèmes de grande taille, et ils nécessitent des procédures MIP plus complexes ou des solveurs commerciaux. La relaxation continue du modèle AGG est rapide et facile à appliquer, mais la qualité de ses solutions est trop faible pour être utilisée dans une procédure exacte ou heuristique. L'ajout de certaines inégalités valables peut améliorer sa borne inférieure. La relaxation continue du modèle FAL est également proposée comme une formulation forte du problème. Il permet d'obtenir des solutions optimales ou quasi-optimales dans un temps de calcul raisonnable. De plus, pour les cas industriels, deux heuristiques considérant différentes politiques de gestion des surplus sont proposées. L'avantage est que leur temps de calcul est très court, et qu'ils peuvent être programmés plus facilement. Dans la suite, nous développons le problème en considérant la structure de produits à désassembler à deux et multi niveaux avec des pièces communes et nous considérons plusieurs produits avec contraintes de capacité.

### B.2.5 Problème multi-produit à deux niveaux avec des pièces communes

Le problème de lot sizing en désassemblage de plusieurs produits avec une structure de produits à désassembler à deux niveaux et des pièces communes est traité dans cette section. L'utilisation de pièces communes (standardisation) est très fréquente dans la conception des produits de nos jours, ce qui implique qu'un produit ou un sous-ensemble a des pièces ou des matériaux communs. Cela comporte certains avantages mais complique plus le problème de lot sizing en désassemblage, car il y a des interdépendances entre les différents produits ou sous-ensembles. Nous avons proposé dans la section précédente, l'évacuation pour gérer les surplus inhérent au désassemblage et minimiser le coût total des opérations de désassemblage. Dans la suite, nous considérons l'évacuation pour le problème de lot sizing en désassemblage pour plusieurs produits avec des structures de produits à désassembler à deux niveaux avec des pièces communes. La figure 4.1 présente un exemple de cette structure. Le nombre entre parenthèses est le rendement d'une pièce donnée lorsqu'une unité du produit en fin de vie (c'est-à-dire, 1, 2, et 3) est désassemblée. Le premier niveau représente les composants, tandis que le deuxième niveau représente les produits de fin de vie. Les pièces 6 et 7 peuvent être obtenues par plusieurs produits en fin de vie, ce qui implique que les composants sont communs. L'objectif du problème est de déterminer les quantités et les dates de désassemblage des produits en fin de vie pour satisfaire des demandes de leurs composants sur un horizon de planification avec des périodes en temps discret, et les stocks surplus seront évacués pour établir diminuer les coûts de désassemblage dans les systèmes de désassemblage. Ensuite, nous développons deux nouvelles formulations MIP pour le problème qui prennent en compte l'évacuation pour gérer des stocks de surplus. Les performances des modèles proposées sont comparées dans la section Expérimentations numériques.

**Modele AGG (P1)** Une formulation mathématique naturelle du problème est présentée dans les équations (4.1-4.7). Il s'agit de la formulation agrégé (AGG en anglais) qui est un modèle de programmation en nombres entiers mixtes (MIP en anglais) avec les contraintes de capacité. La fonction

objectif (4.1) est la somme des coûts de setup, des opérations de désassemblage et de stockage sur un horizon de planification avec  $T$  périodes. Les contraintes (4.2) expriment la conservation des flux pour les composants. Les contraintes (4.3) garantissent qu'un coût de setup est effectué en période  $t$  pour le lancement de désassemblage. Contraintes (4.4-4.5) imposent la non-négativité et les restrictions binaires sur les variables. L'équation (4.8) est utilisée pour calculer la valeur de  $M_{it}$ . Nous définissons  $[\mathbf{P1r}]$  comme la relaxation continue de la programmation linéaire de modèle  $[\mathbf{P1}]$  en supprimant les restrictions d'intégralité sur les variables  $X_{it}$ , ce qui peut être utilisé dans les heuristiques basées sur la programmation linéaire.

**Modele FAL (P2)** Dans cette section, nous développons la formulation FAL proposée pour le problème à un seul produit à deux niveaux, pour le problème avec plusieurs produits avec l'existence des pièces communes et avec des contraintes de capacité. Il s'agit également d'un modèle de programmation en nombres entiers mixtes (MIP en anglais) avec les contraintes de capacité. Nous considérons une variable supplémentaire qui correspond à la fraction de la demande de composant  $i$  dans la période  $t$  qui est obtenue dans la période  $j$  à partir du désassemblage du produit en fin de vie  $r$ . La formulation mathématique du modèle est présentée dans les équations (4.9-4.15). La fonction objectif (4.9) consiste à minimiser la somme des coûts de setup, des opérations de désassemblage, et de stockage sur l'ensemble de l'horizon de planification à  $T$  périodes. Les contraintes (4.10) représentent le fait que les demandes de composants doivent être satisfaites. Les contraintes (4.11) relient la quantité de composants désassemblés aux variables binaires de setup. Les contraintes (4.12) indiquent que la quantité totale de composant  $i$  obtenue par produit en fin de vie  $k$  au cours de la période  $t$ , après les opérations de désassemblage, sera livrée pour satisfaire la demande ou sera évacuée. Les contraintes (4.13-4.15) définissent les domaines des variables de décision. Nous définissons  $[\mathbf{P2r}]$  comme la relaxation continue de la programmation linéaire de modèle  $[\mathbf{P2}]$  en supprimant les restrictions d'intégralité sur les variables  $X_{it}$ , et nous pensons qu'elle permet d'obtenir les solutions optimales ou quasi-optimales du problème dans le temps de calcul assez court.

**Methode exacte** Comme les modèles proposés sont des MIP, ils peuvent être appliqués pour obtenir la solution optimale des instances de problèmes générées en utilisant le solveur CPLEX. Mais nous ne pouvons pas garantir que le solveur CPLEX sera capable de résoudre tous les problèmes. Nous proposons une heuristique à deux phases avec l'avantage qu'elle peut être programmée dans un code via des applications simples dans des cas industriels réels.

**Heuristique en deux phases** Étant donné que les méthodes exactes sont essentiellement développées pour résoudre de manière optimale un problème simple (par exemple, un produit unique sans l'existence des contraintes de capacité). Les méthodes heuristiques sont efficaces pour les grandes tailles du problème surtout dans les environnements réels. Elles présentent l'avantage de pouvoir être appliquées par simple application et dans le temps de calcul très rapides (Brahimi (2004)). Dans cette section, nous adaptons la méthode heuristique à deux phases proposées par Kim et al. (2006b) pour le problème étudié. L'heuristique proposée consiste en une première phase dans laquelle la solution initiale obtenue à partir de l'heuristique de relaxation continue est construite. Ensuite, elle est améliorée par un algorithme prospectif basé sur une approche de programmation dynamique. Nous présentons ensuite l'heuristique en deux phases :

**[Phase 1. Construction de la solution :]** la solution obtenue en résolvant le modèle P1r (c'est-à-dire, la relaxation continue du modelé AGG (P1)) à l'aide du solveur CPLEX sera arrondie à l'unité inférieure. La solution arrondie à l'inférieur est ensuite modifiée de manière à ce que toutes les contraintes du modèle P1 soient satisfaites. La quantité d'équilibre (BL) définie par l'équation (4.16) est utilisée pour vérifier la faisabilité de la solution arrondie. Si  $BL_{it} = 0$ , la solution obtenue est aussi faisable pour le problème. Si  $BL_{it} \neq 0$ , la solution obtenue doit être modifiée en augmentant ou en diminuant les variables de décision, tout en tenant compte des changements de coûts. Pour le cas où  $BL_{it} > 0$ , si  $I_{it} \geq BL_{it}$ , nous calculons le nouveau niveau de stocks par l'équation 4.17. Sinon, si  $E_{it} \geq BL_{it}$ , nous mettons à jour la quantité éliminée par l'équation (4.18). Si les deux cas ci-dessus aboutissent à des solutions infaisables, nous considérons le cas de l'augmentation du produit en fin de vie  $r$  ;  $X_{rt}$ ,  $r \in \Phi(i)$ . La quantité de cette augmentation pour le produit en fin de vie (root item en anglais)  $r \in \eta(i)$  est  $\lceil BL_{it}/a_{ri} \rceil$ . Les équations 4.19 et 4.20 calculent respectivement les modifications des variables de stocks et la variation de coût en cas d'augmentation de la quantité à désassembler de root item  $r$ . Pour le cas où  $BL_{it} < 0$ , nous considérons l'augmentation de la quantité d'évacuation ( $E_{it}$ ) comme des équations (4.21), de sorte qu'il n'y a pas de changement de coût car le coût d'évacuation est considéré comme nul.

**[Phase 2. Amélioration de la solution :]** La solution obtenue en phase 1 est améliorée par l'utilisation d'un algorithme basé sur la programmation dynamique qui est appliqué à chaque root item, c'est-à-dire en partant du premier élément root item jusqu'au dernier. Si le dernier setup est à la période  $j$  ( $1 \leq j \leq t$ ) pour un sous-problème à la période  $t$  d'un root item  $r$  donné, le changement dans le dimensionnement du lot de désassemblage actuel est dans l'équation (4.22). Ce changement peut réduire le coût de setup tout en augmentant le coût de stockage. L'équation (4.23) est proposée pour mettre à jour les niveaux de stocks des composants après avoir modifié la taille actuelle du lot de désassemblage. La formulation (4.26) représente une fonction d'économie récursive pour le sous-problème de la période  $t$  de root item  $r$  (où  $F_r(0) = 0$ ).  $B(j, t)$  et représente respectivement la diminution et l'augmentation du coût total pour un root item donné  $r$  lorsque le dernier setup a lieu à la période  $j$  pour un sous-problème de la période  $t$ .

Un exemple du problème est résolu pour démontrer l'applicabilité du TH. Le tableau 4.1 fournit un exemple de données pour le problème a plusieurs produit avec la structure de produits à désassembler à deux niveaux avec des pièces communes présentée dans la figure 4.1. Les résultats sont présentés dans les tableaux 4.2, 4.3, et 4.4. Nous constatons que l'évacuation entraîne une réduction du coût total associé à la planification du désassemblage et cette réduction est d'environ 7% pour l'exemple testé. En plus, il n'y a plus de stocks de surplus de composant à la fin de l'horizon de planification ce qui est pratique en cas industriel. D'après le tableau 4.4, l'heuristique TH est efficace pour résoudre cet exemple et pour améliorer la solution initiale obtenue par la première phase. Le gap du TH pour l'exemple testé n'est que de 0,27%, ce qui représente une amélioration significative par rapport au gap de la première phase, soit 11,61%.

**Expérimentations numériques** Les tests sont effectués sur les exemples de problèmes et nous avons comparé les modèles et méthodes par rapport au pourcentage d'écart par rapport à la solution optimale (ou meilleure solutions obtenue). Les modèles, méthodes et les données générées peuvent être appliquées dans des cas industriels. Un exemple peut être le secteur du recyclage des véhicules en fin de vie (EOL vehicles en anglais), où les véhicules en fin de vie seront désassemblés en leurs pièces telles que comme les moteurs, les portes, les sièges, les pneus, etc. Les pièces obtenues sont utilisées pour satisfaire leurs demandes et les dangereux, inutilisables, et les pièces ou matériaux supplémentaires seront évacués à la

des filières spécialisées telles que le recyclage, les sources de matériaux pour la récupération d'énergie. Les tests sont effectués sur un ordinateur portable équipé d'un Intel Core i5-3210M avec une vitesse de 2,5 GHz et 8 Go de RAM sur le Windows 7. Les résultats de MIP modèles sont obtenues en résolvant directement sur le solveur CPLEX v.12.8. La génération des différentes instances est effectuée sur des problèmes de taille différente et des valeurs de paramètres différentes. Les recherches existantes dans la littérature ne sont pas pertinentes pour le problème de lot sizing en désassemblage étudié dans cette section. Nous avons suivi la même méthode que dans la section B.2.5 pour générer les instances. Le benchmark proposé dans Kim (2005) est adapté pour générer des instances avec des paramètres de coût différents. Le tableau 4.5 présente les paramètres utilisés pour générer les instances. Les tableaux 4.6 et 4.7 représentent respectivement le temps de calcul pour les modèles P1, P2, P1r, et P2r, le gap pour la relaxation continue des modèles P1 et P2. Nous constatons que les modèles MIP P1 et P2 peuvent obtenir les solutions optimales des instances générées, mais on ne peut pas assurer pour les tailles très grandes des instances. Modèle originale P1 est plus rapide que le modèle P2 mais le modèle P2 peut obtenir les solutions optimales de 46.6% des instances et pour les autres, son gap moyen globale n'est que 0.06%, ceci montre que la formulation FAL est efficace pour fournir une borne inférieure de bonne qualité du problème, en particulier pour les problèmes de grande taille. Le tableau 4.8 présente le résultat du TH. Nous observons du tableau que la deuxième phase apporte des améliorations très efficaces par rapport à la solution initiale obtenue lors de la première phase. Cela signifie que l'approche basée sur le programme dynamique suggérée est très efficace pour améliorer les solutions initiales construites. Par exemple, dans le cas de 30 composants et 30 périodes, la deuxième phase peut améliorer la moyenne globale du gap de 20,55 % à 3,21 %, et elle ne nécessite que 0,13 seconde de temps de résolution. Nous analysons également l'impact de décisions d'évacuation sur le coût total des opérations de désassemblage avec 30 composants et 30 périodes. Les résultats montrent qu'une réduction maximale des coûts d'environ 58% peut être obtenue en autorisant l'évacuation, ce qui est une opportunité très importante pour rendre un système de désassemblage profitable.

**Conclusions** Cette section aborde le problème de lot sizing en désassemblage pour la structure des produits à désassembler à deux niveaux et plusieurs produits avec des pièces communes. L'objectif est de minimiser la somme des coûts de setup, des opérations de désassemblage et de stockage. Les décisions d'évacuation sont appliquées pour gérer des stocks de surplus dans les systèmes de désassemblage, ce qui peut permettre une réduction maximale des coûts d'environ 58%, pour les cas problématiques testés. Deux nouvelles formulations MIP (AGG et FAL) sont proposées avec prise en compte des décisions d'évacuation. Pour les cas industriels réels, une heuristique à deux phases est suggérée, dans laquelle une solution initiale est obtenue en utilisant l'approche de relaxation continue, et elle est améliorée en utilisant un algorithme basé sur la programmation dynamique. L'amélioration est réalisée en modifiant la solution actuelle et en tenant compte des changements de coûts. L'heuristique à deux phases donne des solutions quasi-optimales en très peu de temps de calcul. L'heuristique proposée à une performance à améliorer la solution initiale obtenue par la première phase. De plus, nous appliquons l'approche de relaxation continue pour le modèle FAL (la reformulation du problème) pour améliorer la borne inférieure du problème. En tant que le travail de la prochaine section, nous visons à développer le modèle et la méthode proposés pour une structure de produit plus complexe avec plusieurs niveaux et avec des pièces communes. Il est également nécessaire d'examiner le problème avec une contrainte de capacité de ressources pour les cas industriels.



### B.2.6 Problème multi-produit à multiniveaux avec des pièces communes et contraintes de capacité

Dans cette section, nous traitons le problème de lot sizing en désassemblage pour plusieurs produits avec une structure de produits à désassembler à multiniveaux avec des pièces communes, ainsi que l'existence des contraintes de capacité. La contrainte de capacité des ressources est un challenge important dans les problèmes de planification de production ainsi que dans la planification du désassemblage. Cette contrainte signifie qu'il existe une limite supérieure sur le temps disponible à chaque période de l'horizon de planification dans un système de désassemblage. D'une autre façon, les opérations de désassemblage affectées à une période ne doivent pas dépasser ce temps disponible à cette période (Hrouga (2016)). La structure de produits à désassembler dans cette section est plus complexe que dans la dernière section. En outre, la structure de produits à désassembler a plusieurs niveaux rend le problème de stocks surplus dans les systèmes du désassemblage plus difficiles, car les produits en fin de vie seront désassemblés en sous-ensembles ou les éléments intermédiaires (c'est-à-dire, les composants obtenue directement par l'opérations de désassemblage à partir des produits en fin de vie) avant d'obtenir des composants enfants (c'est-à-dire, les éléments obtenue directement par l'opération de désassemblage à partir des éléments intermédiaires) pour satisfaire les demandes de ces derniers. En plus, certains composants peuvent être obtenus par les mêmes parents (c'est-à-dire, les éléments intermédiaires ou les produits en fin de vie), ce qui complique en plus la planification des opérations de désassemblage. Pour faire face au problème de stocks surplus, nous intégrons l'évacuation dans les modèles. Un exemple de la structure de produits à désassembler traité dans cette section avec trois niveaux est donné dans la figure 4.2. Les composants dans le premier niveau sont les produits de fin de vie qui doivent être désassemblés, et les composants enfants sont les pièces demandées et ne peuvent pas être désassemblés. Un composant enfant est obtenu en désassemblant un composant intermédiaire. Le désassemblage d'un élément parent (élément intermédiaire ou le produit en fin de vie) génère un ou plusieurs composants (éléments enfants ou intermédiaires). L'exemple comporte trois éléments root item (produits de fin de vie), c'est-à-dire les éléments 1–3. Le nombre entre parenthèses est le rendement de l'élément lorsqu'une unité de son parent (ou ses parents) est désassemblée. Le troisième (dernier) niveau représente les composants enfants (c'est-à-dire, les éléments 8-12). Les pièces communes sont autorisées ici, ce qui signifie qu'un composant (intermédiaire ou enfant) peut avoir plus d'un parent (intermédiaire ou produit en fin de vie). Dans l'exemple, les éléments 4, 6, 9 et 10 représentent des éléments communs. Le problème est de déterminer la quantité et le timing du désassemblage de tous les éléments parents (les éléments intermédiaire ou produits en fin de vie) pour satisfaire la demande d'éléments enfants sur l'horizon de planification en respectant les contraintes de capacité, tandis que les éléments obtenus surplus seront évacués après l'opération de désassemblage. Nous développons un modèle MIP pour le problème qui prend en compte les décisions d'évacuation pour gérer les stocks de surplus. Le modèle et les méthodes de résolution proposés seront ensuite comparés dans la section d'expérimentations numériques et les résultats sont présents.

**Modele AGG (P3)** Dans cette section, nous développons la formulation standard AGG pour le problème. Notez que la formulation FAL ne peut pas être obtenue en raison de la grande complexité des relations entre les éléments parents, les intermédiaires et les éléments enfants. La formulation mathématique est présentée dans les équations (4.27-4.35). La fonction objectif (4.27) consiste à minimiser la somme des coûts de setup, des opérations de désassemblage et de stockage sur l'ensemble

de l'horizon de planification. Les contraintes (4.28) et (4.29) représentent respectivement la conservation des flux de stockage pour les éléments intermédiaires et les éléments enfants (Notez que  $I_{i0}=0$  pour tous les éléments). Les contraintes (4.30) respectent les limites de capacité sur le temps disponible dans chaque période. Les contraintes (4.31) garantissent qu'un coût de si une opération de désassemblage est effectuée au cours de cette période. Les contraintes (4.32-4.35) imposent la non-négativité et des restrictions binaires sur les variables. Notez que dans le modèle P3, les variables  $I_{it}$  et  $E_{it}$  peuvent être définies comme réelles. Nous définissons [P3r] comme la relaxation continue de la programmation linéaire de modèle [P3] en supprimant les restrictions d'intégralité sur les variables  $X_{it}$ , et nous pensons qu'elle permet d'obtenir les solutions avec le temps de calcul très courts, qui peuvent utiliser dans les heuristiques basées sur la programmation linéaire.

**Methods exacte** Le problème de lot sizing en désassemblage avec des contraintes de capacité est considéré NP-difficile ((Hrouga (2016))). Le modèle P3 est MIP et il est possible donc être directement utilisé pour obtenir des solutions optimales en utilisant le solveur CPLEX. Le solveur CPLEX fait appel à plusieurs techniques complexes pour résoudre les problèmes de MIP difficiles, mais ses performances diminuent lorsqu'il s'agit de résoudre des problèmes de grande taille. L'application de stratégies efficaces pour réduire le nombre de variables entières peut aider le solveur CPLEX à améliorer le temps de calcul pour la résolution (Achterberg and Wunderling (2013); Manual (1987)). Fix-and-Optimize est une méthode heuristique avec différentes stratégies de décompositions qui nous permet de résoudre le problème complexe dans le temps de calcul courts en résolvant successivement des sous-problèmes avec un nombre limité de variables entières.

**Heuristique Fix-and-optimize (FO)** FO est une heuristique de décomposition qui consiste à résoudre successivement des sous-problèmes en utilisant des techniques de résolution de programmes linéaires à nombres entiers. Les variables peuvent être partitionnées selon différentes stratégies. Un sous-problème est défini en séparant l'ensemble des variables du problème général (c'est-à-dire, le problème de lot sizing en désassemblage avec l'évacuation pour les plusieurs produits à multiniveaux avec des pièces communes et avec des contraintes de capacité) en deux sous-ensembles. Le premier sous-ensemble est constitué des variables à optimiser (la fonction objectif à minimiser et des contraintes du problème général) et le second sous-ensemble est constitué des autres variables dont les valeurs sont fixées par l'ajout de contraintes dans le problème général. Le sous-problème est donc modélisé par un MIP, identique à celui modélisant le problème général avec les contraintes supplémentaires permettant de fixer les variables du second sous-ensemble. L'heuristique fonctionne par itération en modifiant à chaque itération le sous-ensemble des variables fixées et en intégrant les valeurs des variables obtenues à l'itération précédente. Il est donc nécessaire de générer une solution initiale pour lancer l'heuristique. Dans cette étude, une solution initiale est générée en fixant l'opération de désassemblage à effectuer à chaque période et nous considérons une stratégie de décomposition orientée à la fois sur la période et sur le produit. L'algorithme 2 de la section 4.3.4 présente un pseudo-code de cet algorithme pour la stratégie orientée sur la période en commençant de 1 à T.

**FO avec l'interdépendance entre les variables liées** Nous améliorons la méthodes FO présentée dans la section précédente. Diverses recherches proposent différentes méthodes pour contrôler le nombre de variables à optimiser dans chaque sous-problème MIP de l'approche FO (par exemple, Seeanner et al. (2013); Chen (2015)). Comme pour l'approche FO de Chen (2015), nous considérons la caractéristique

d'interrelation pour définir les variables à optimiser dans chaque sous-problème MIP de l'approche FO. Nous définissons les caractéristiques d'interrelation en les variables liées pour le modèle P3 comme suit :

- les variables binaires de configuration sont liées entre elles : si la valeur de  $Y_{it}$  change, la valeur de sa quantité de désassemblage  $X_{it}$  peut également changer en fonction des contraintes de coûts de configuration du couplage (4.31)
- La variation de  $X_{it}$  peut entraîner une variation de  $X_{it-1}$  et de  $X_{it+1}$  en raison du lien entre le bilan d'inventaire de deux périodes liées imposées par des contraintes (4.28) et (4.29)
- Ce changement peut entraîner la modification de  $X_{i't}$ ,  $i' \in \Phi(i)$  et  $i' \in \eta(i)$  en raison de la connexion de  $X_{i't}$  avec  $X_{it}$  dans la structure de désassemblage et des contraintes de capacité (4.30).

Le lien ci-dessus entre les variables est appelé Interdépendance, en anglais Interrelatedness between the linked variables (Chen (2015)) qui peut être défini comme suit : deux variables  $Y_{it}$  et  $Y_{i't}$  sont directement liées ou une interdépendance en une étape si  $i = i'$  et  $t' \in t - 1, t, t + 1$  ou  $i' \in \Phi(i) \cup \eta(i)$ . La figure 4.3 représente l'interdépendance à 1, 2, 3 et 4 étapes pour le composant 1 de l'exemple de structure présenté dans la figure 4.2. Par exemple, le composant 1 présente une interdépendance en deux étapes avec les composants 2, 4, 5, 8, 9 et 10.

À partir de la discussion ci-dessus sur les caractéristiques d'interdépendance entre les variables, nous pouvons définir les sous-problèmes de la nouvelle approche FO pour ce problème. Nous définissons  $\Omega_{it}^l(Y_{it})$  comme  $\{Y_{it}, i \in N \text{ et } t \in T\} \setminus \Omega_{it}^l(Y_{it})$ . Pour une interdépendance donnée de  $l$ -étape ( $l \geq 1$ ), le sous-problème de P3 avec  $l$ -étape associé à la variable de configuration  $Y_{it}$  est défini par  $SP3_{i,t}^l$ , qui consiste à fixer toutes les variables de setup dans  $\Omega_{it}^l(Y_{it})$  et à réoptimiser les variables de setup dans  $\Omega_{it}^l(Y_{it})$ . Il est évident que l'augmentation du paramètre  $l$  entraîne une augmentation du temps de calcul en raison de l'ensemble plus important  $\Omega_{it}^l(Y_{it})$ , c'est-à-dire plus il y a de variables de setup à réoptimiser. Dans nos résultats numériques, nous considérons que  $l = 1$  (1 étape) et 5 (5 étapes). L'algorithme 3 fournit un pseudo-code de la nouvelle approche FO considérant les caractéristiques d'interrelation entre les variables liées.

Nous constatons qu'une solution initiale est obtenue en 120 secondes de temps de calcul et nous avons fixé l'écart du solveur CPLEX comme valeur par défaut. Comme prévu, l'algorithme 3 peut obtenir différentes solutions (qualités de solution) dans chaque exécution. La raison en est qu'une paire  $(i, t)$  est choisie au hasard. Pour garantir l'obtention de la même solution dans chaque exécution de l'algorithme 3, nous choisissons une paire  $(i, t)$  dans l'ordre lexicographique en commençant par  $i$  de 1 à  $N$  et  $t$  de 1 à  $T$ . Nous avons quatre variantes de tests pour choisir une paire  $(i, t)$  dans l'ordre lexicographique : (1) de 1 à  $N$  et de 1 à  $T$ , (2) de 1 à  $N$  et de  $T$  à 1, (3) de  $N$  à 1 et de 1 à  $T$ , (4) de  $N$  à 1 et de  $T$  à 1. Dans les résultats numériques, nous ne présentons que la première car elle nous permet d'obtenir une moyenne globale du gap (%) inférieure.

**Expérimentations numériques** Cette section présente les résultats des tests sur les modèles P3 et P3r et les méthodes proposées. Les modèles et les méthodes sont implémentés sur Eclipse en utilisant le langage de programmation Java. De plus, tous les tests sont effectués sur un système avec un Intel Core i7-7700T avec une vitesse de 2,9 GHz, et 16 Go de RAM sur Windows 10. Nous utilisons le solveur CPLEX 12.8 pour résoudre les modèles. Nous constatons que le solveur CPLEX avec des paramètres par défaut n'a pas pu obtenir la solution de plusieurs problèmes MIP (en particulier, les problèmes de grande taille) en raison d'un manque de mémoire disponible. Nous avons donc mis 1024 mégaoctets de

mémoire disponible pour le solveur CPLEX afin de résoudre les cas problématiques. Nous constatons que CPLEX n'a toujours pas pu obtenir la solution optimale en raison d'un manque de mémoire, mais nous avons pu obtenir sa meilleure solution dans un délai limité de 3600s. Le paramétrage présenté dans la section 4.2 est utilisé pour générer de manière aléatoire les instances du problème. Nous avons modifié le coût de détention des stocks par rapport au coût de réglage en utilisant la formule (3.54) pour obtenir  $TBO \leq 2$ . Nous générons 225 cas de problèmes (les instances), c'est-à-dire 25 cas de problèmes pour chaque combinaison de trois niveaux de nombre d'éléments ( $N = 10, 20$  et  $30$ ) et trois niveaux de nombre de périodes ( $T = 10, 20$  et  $30$ ). Cinq structures de désassemblage sont générées pour chaque niveau du nombre de composants et pour chaque structure de désassemblage, cinq problèmes différents sont générés. Notez que la méthode proposée par Kim and Xirouchakis (2010) pour le problème de lot sizing en désassemblage sans l'évacuation est utilisée afin de générer des cas de problèmes réalisables avec des restrictions de capacité. Les tableaux 4.10 et 4.11 présentent les résultats des modèles P3 et P3r, la solution initiale, et la méthode FO. Quatre différents FO sont considérés : FO orientée sur la période 1 à  $T$  et  $T$  à 1, FO orientée sur le produit 1 à  $N$  et  $N$  à 1. Nous constatons que le solveur CPLEX ne peut pas obtenir la solution optimale de toutes les instances, en particulier, les instances de grandes tailles, son gap peut atteindre 13.33% dans le temps de calcul limité de 3600s. Cela signifie que la méthode exacte en utilisant directement le solveur CPLEX n'est pas efficace pour les problèmes de grandes tailles. En revanche, la relaxation continue du problème (c'est-à-dire, le modèle P3r) est très rapide mais il obtient les solutions avec des mauvaises qualités de bornes inférieures. Les approches FO sont rapides et ils obtiennent des meilleures solutions que le modèle P3r pour toutes les instances testées, et ils sont efficaces pour améliorer la solution initiale obtenue. Parmi les quatre variantes de l'approche FO, les deux approches FO orientées sur la période (c'est-à-dire, 1 à  $T$  et  $T$  à 1) fonctionnent mieux que celles orientées sur le produit. Pour les cas problématiques testés, l'approche FO orientée sur la période avec séquence prospective (c'est-à-dire, 1 à  $T$ ) a obtenu une moyenne globale du gap de 11,27 %. Les approches FO orientées sur les produits sont plus rapides que celles orientées sur les périodes, avec un CPU global maximum de 5.51 et 12.73 secondes, respectivement. Nous évaluons ensuite les performances de l'algorithme FO pour le problème en tenant en compte les interrelations entre les variables liées. Les tableaux 4.12, 4.13, et 4.14 présentent respectivement les résultats pour les instances avec des nombres de 10, 20, et 30 composants. D'après les résultats, nous constatons que le nouveau FO en tenant en compte les interrelations entre les variables liées est efficace et peut améliorer significativement la qualité des solutions obtenue par FO classique (sans la considération des interrelations entre les variables liées). Nous observons que l'amélioration maximale du gap pour FO avec considération des interrelations entre les variables peut atteindre 15.33, 13.36, 10.53 %, respectivement pour les instances avec des nombres de 10, 20, et 30 composants. Nous constatons également que l'augmentation de niveaux d'interrelations entre des variables liées peut améliorer la qualité des solutions obtenues. Par exemple pour les instances avec un nombre de 10 composants et 10 périodes, l'amélioration du gap de 1-etape à 5-etape interrelation entre les variables liées, peut atteindre 17.21%. Nous avons analysé aussi l'impact de la considération l'évacuation sur le coût total pour les instances avec un nombre de 30 composants et 10 périodes. Nous observons que la réduction des coûts en considérant l'évacuation peut atteindre environ 10 %, il y a une opportunité importante de rendre les systèmes de désassemblage plus rentables.

**Conclusions** Nous avons traité le problème de lot sizing en désassemblage pour les plusieurs produits avec la structure de produits à multiniveaux et avec des pièces communes, ainsi que l'existence des contraintes de capacité. La décision d'évacuer les stocks surplus peut permettre une réduction significative

des coûts d'environ 10% pour les cas testés. Un nouveau modèle MIP est présenté pour ce problème. La méthode exacte utilisant le solveur CPLEX pour le problème original n'est pas efficace pour le problème de grande taille, avec l'inconvénient que le temps CPU augmente de manière significative. Une heuristique Fix-and-Optimize (FO) est proposée, qui permet de résoudre les problèmes (en particulier les problèmes de grande taille) en un temps de calcul plus court et avec une bonne qualité de solution. Ensuite, nous améliorons la qualité de la solution de l'heuristique FO en considérant les caractéristiques d'interdépendance entre les variables liées, ce qui nous permet d'atteindre une amélioration maximale du gap de 15.33% par rapport à l'heuristique FO classique.

## B.3 EOQ en désassemblage avec l'évacuation et les demandes sensibles au prix

Différentes politiques et différents modèles de gestion de stocks sont mis en place par les entreprises afin d'obtenir un système de gestion des stocks efficace, de manière à améliorer les performances de leur système. Les sections précédentes traitent du problème de lot sizing en désassemblage et fournissent les modèles et les méthodes permettant de rendre les systèmes de désassemblage plus rentables en optimisant leur manière de gestion de stocks. La quantité économique de commande (EOQ) est une autre politique de contrôle des stocks qui considère les stocks connus, continus et constants sur un horizon de planification continue. Les hypothèses du modèle EOQ sont souvent limitées et c'est nécessaire de l'étendre pour faciliter son application aux différents environnements réels. Les assumptions du modèle EOQ en désassemblage ont été adaptées par [Godichaud and Amodeo \(2018\)](#). Ils montrent que les opérations de désassemblage peuvent générer des stocks de surplus inutiles des composants qui s'accumulent au fil de l'horizon de planification continue. La raison s'explique par le fait que les demandes pour les composants sont différentes et mal équilibrées ainsi que les rendements de désassemblage peuvent également varier d'un composant à un autre. La figure 5.2 montre cette accumulation du stock de surplus pour un composant donné. Nous nous intéressons aux décisions pour gérer ces stocks surplus des composants après des opérations de désassemblage. Ces surplus de stocks peuvent entraîner un coût de stockage important dans un système de désassemblage. [Godichaud and Amodeo \(2018\)](#) propose des solutions pour la gestion de ce surplus comme l'évacuation. Cette décision ne peut assurer la rentabilité d'un système de désassemblage. Dans cette section, nous proposons les modèles qui intègrent les demandes sensibles au prix et les décisions de l'évacuation des stocks surplus dans le DEOQ pour maximiser le profit des systèmes de désassemblage sans l'accumulation de stocks de surplus.

### B.3.1 Définition du problème

Dans la planification du désassemblage, les décisions de prix (pricing decisions en anglais), lorsqu'elles sont possibles, sont également avantageuses pour réduire les stocks surplus en équilibrant les demandes de composants. La figure 5.5 montre l'effet de pricing en désassemblage. Nous considérons que la demande de composants est déterministe avec une fonction sensible au prix. Cela signifie que les demandes sont définies par une fonction décroissante du prix. Dans cette étude, nous supposons que la fonction de demande est linéaire et iso-élastique du prix. Nous remarquons que cette fonction peut être facilement inversée en utilisant la demande comme variable de décision au lieu du prix. Nous considérons un système de désassemblage pour un seul produit en fin de vie avec une structure de produits à désassembler à deux niveaux. Le premier niveau représente les éléments enfants obtenus directement par une opération de

désassemblage et le deuxième niveau représente le produit de fin de vie (élément parent) à désassembler. Le rendement de désassemblage de chaque composant est le nombre d'unités de composant obtenues à chaque opération de désassemblage d'une unité de produit de fin de vie. La Fig. 5.1 représente un exemple de système de désassemblage pour un produit de fin de vie avec trois composants. Le problème est de déterminer les prix de vente (ou les demandes) des composants en même temps que la politique de désassemblage, qui détermine le temps de désassembler des produits en fin de vie et les quantités associées. L'objectif est de maximiser la fonction de profit. Cette fonction de profit comprend à la fois les recettes générées par les ventes des articles en feuilles et les coûts du démontage de l'unité, de la tenue des stocks et de la commande. Le problème peut être modélisé comme un problème d'optimisation non linéaire avec contraintes, présenté dans les équations (5.2). La fonction objectif qui est une fonction de profit est une différence entre une fonction de profit  $R(X, D)$ , et une fonction de coût DEOQ  $C(T, D)$  incluant les coûts de setup et de stockage, qui sont étudiés séparément pour analyser le problème. Les contraintes garantissent que toutes les demandes des composants doivent être satisfaites à partir du désassemblage des produits en fin de vie.

### B.3.2 Approches de résolution

Nous analysons le modèle 5.2 et nous présentons des solutions qui sont élaborées progressivement en fonction des différents problèmes, y compris les considérations relatives aux stocks, aux décisions de l'évacuation et aux limites des demandes.

**Problème sans coût de stockage** Le modèle 5.6 peut être étudié en sous-problème comme dans l'équation (5.2). Nous proposons 2 propositions qui déterminent les valeurs optimales de quantité à désassembler ( $X$ ) et les demandes ( $D$ ). La proposition 1 stipule que la solution optimale de (5.6) consiste à fixer les demandes de manière à ce que pour chaque composant il n'y ait pas de stocks surplus et, par conséquent, pas d'évacuation. Dans ce cas, le problème n'a qu'une seule variable de décision (c'est-à-dire,  $X$ ), qui peut être trouvée efficacement selon la proposition 2. Selon la proposition 2, la fonction de revenu ( $R(X)$ ) est concave et une simple méthode de recherche par ligne peut être utilisée pour trouver la valeur de  $X$  ou  $R(X)$  atteint son maximum.

**Problème avec coût de stockage et sans l'évacuation** Pour le problème sans l'évacuation, chaque demande est fixée à  $d_i = \alpha_i X$  afin de ne pas avoir de stock surplus à évacuer à la fin de chaque cycle de réapprovisionnement ( $T$ ). Equation (5.7) présente la formulation du problème. Pour une valeur fixe de  $X$ , le problème est équivalent à (5.8). Une analyse d'équation (5.8) (la proposition 3) montre qu'une simple méthode de recherche non linéaire permet de trouver la solution optimale (maximum global). Nous analysons (5.8) directement sur la base de la forme de  $R(X)$  et de  $C(X)$ .

**Problème avec l'évacuation** Sur la base des résultats obtenus dans les sections précédentes, une approche de solution est proposée pour résoudre le problème initial (5.2). Il s'agit d'un problème de programmation non linéaire avec des contraintes, et une solution optimale satisfait aux conditions de Karush-Kuhn-Tucker (KKT). La procédure de recherche proposée utilise ces conditions (les équations (5.9a-5.9d) pour résoudre le problème en modifiant itérativement une solution jusqu'à ce qu'elles soient atteintes. Une procédure est proposée qui optimise une variable, en considérant les autres comme fixées à chaque étape par rapport aux conditions de KKT. Elle améliore la fonction de profit de manière itérative à partir de la solution initiale sans élimination.

**Problème avec l'évacuation et limit en demandes** Avec une fonction de demande iso-élastique, la demande n'est pas limitée et une valeur énorme peut être optimale. Cela n'est pas réalisable dans des cas réels. Le problème est alors étendu en imposant des limites supérieures pour les demandes. Dans ce cas, l'équation 5.10 est ajoutée dans le modèle 5.2 et les conditions (5.9b) passent à (5e) et ajoutent les équation (5f). La méthode présentée pour le problème avec l'évacuation sera modifiée en tenant en compte les nouvelles équations.

### B.3.3 Expérimentations numériques

Une analyse numérique est présentée dans cette section, basée sur plusieurs ensembles de données et les ensembles de données sont expérimentaux (les 10 exemples sont présentés en annexe du manuscrit). La solution fournie indique si une opération de désassemblage est rentable, à quel niveau et avec quelles options. Nous présentons les résultats des modèles et méthodes appliqués sur les données présentés en tableaux 5.1. D'après les résultats présentés dans le tableau 5.2, il est plus économique d'évacuer les composants 1, 2, 4, 5 et 9 plutôt que d'augmenter les ventes. Nous constatons que la décision d'évacuation est profitable pour cette instance. Le tableau 5.3 montre les résultats de l'augmentation du bénéfice qui peut être réalisé par rapport aux neuf autres exemples en considérant la décision de l'évacuation. Nous constatons que la solution sans évacuation peut être optimale, comme dans l'exemple 8. En considérant une limite ( $a_i$ ) sur la demande maximale pour chaque composant dans tous les exemples, nous constatons que les solutions des exemples 3, 8 et 9 ne sont pas réalisables. La procédure peut continuer pour trouver la solution qui respecte les contraintes de capacité. Les résultats sont présentés en tableau 5.4. Enfin, nous terminons les Expérimentations numériques avec une analyse de sensibilité sur le rendement de désassemblage et le coût d'évacuation. Nous présentons trois politiques pour les cas industriels ou il y a l'incertitude sur le rendement de désassemblage.

### B.3.4 Conclusions

Dans cette section, nous avons traité des modèles EOQ pour la planification du désassemblage des produits en fin de vie. Le modèle DEOQ permet d'utiliser l'effet de pricing sur les demandes, en tenant compte des décisions d'évacuation afin de traiter de manière optimale les accumulations des stocks de surplus de composants dans un système de désassemblage. Ce surplus de composants peut conduire à des décisions de gestion de stocks inappropriées qui ne sont pas respectueuses de l'environnement, mais les décisions d'évacuation ainsi que le pricing peuvent être appliquées pour y faire face. Toutefois, lorsque cela est possible, il peut être avantageux, tant sur le plan économique qu'environnemental, de faire varier les demandes si elles sont sensibles au prix en fonction d'une fonction de profit.

Un modèle est développé pour déterminer la quantité et timing du désassemblage du produit en fin de vie, les prix des composants dans les systèmes de désassemblage, et la quantité des composants à évacuer afin d'optimiser une fonction de profit avec différentes structures de coûts. Lorsque seuls les coûts de désassemblage ou les coûts de désassemblage et de stockage sont pris en compte, les conditions d'optimalité sont dérivées et montrent que des méthodes de recherche non linéaires simples peuvent être utilisées. Les décisions et les coûts de désassemblage sont ensuite pris en compte, ce qui conduit à un problème de contraintes non linéaires. Les conditions de KKT sont dérivées pour proposer une approche de solution. Cela montre que le maintien de l'option d'évacuation peut entraîner des marges bénéficiaires plus élevées si les demandes sont sensibles au prix, selon les cas testés. Les exemples numériques montrent que la procédure proposée fournit des solutions dans des temps de calcul courts.

## B.4 Conclusion générale

Cette thèse est dédiée aux différents modèles de gestion de stock pour gérer des stock surplus des systèmes de désassemblage. Des nouvelles stratégies et des outils de solution pour la planification de désassemblage dans un contexte de logistique inverse sont proposés. L'objectif est de développer un système de planification du désassemblage efficace permettant la revalorisation des produits en fin de vie pour la possibilité de réutilisation du produit ou de ses composants, de refabrication, ou la préparation des pièces au recyclage. Une analyse de la littérature est présentée sur les différents modèles et stratégies de gestion de stock en désassemblage. Nous avons commencé d'abord par le problème de lot sizing en production, leurs modélisations et les méthodes de résolution utilisée. Ensuite nous avons étudié les travaux de recherche sur le problème de lot sizing en désassemblage. D'après une analyse de bibliographique sur les problèmes traités en lot sizing en désassemblage, nous nous sommes aperçus qu'il n'y a peu de travail qui considère la gestion des stocks dans le cadre du système de désassemblage. Notre objectif est de développer les modèles et méthodes tenant en compte les décisions pour gérer les stocks surplus des systèmes de désassemblage tout en satisfaisant les demandes en vue de minimiser le coût total associée. En plus, nous avons réalisé une analyse de bibliographique sur le modèle EOQ et ses variants afin de trouver les possibilités de les développer dans des systèmes de désassemblage. Différents problèmes avec des décisions associées pour gestion de stocks sont analysés. Nous proposons les modèles qui intègrent les demandes sensibles au prix et les décisions de la disposition de surplus de stocks dans le DEOQ pour maximiser le profit des systèmes de désassemblage sans l'accumulation de stocks. Tout d'abord, le problème du lot-sizing en désassemblage avec l'évacuation est présenté. Nous considérons à la fois les cas de base et les cas complexes du problème. d'abord, problème mono produit pour des structures de produit à deux niveaux : nous proposons d'abord une procédure pour calculer les accumulations de surplus de stocks sur l'horizon de planification, puis trois nouvelles formulations de programmation mixte (MIP) sont proposées pour modéliser le problème. Les formulations diffèrent les unes des autres en ce qui concerne la qualité de la limite inférieure fournie par leur relaxation linéaire, ce qui est une question importante dans les méthodes de résolution des MIP. L'ajout des inégalités valides est proposé pour améliorer la limite inférieure du problème. Deux heuristiques efficaces avec des stratégies de gestion de stocks différentes sont également étudiées pour des applications réelles lorsque les algorithmes MIP ne sont pas pertinents. Deuxième, problème plusieurs produits pour des structures de produits à deux niveaux avec pièces communes : deux nouvelles formulations de MIP sont développées pour le problème en tenant compte de la disposition du surplus de stocks. Une formulation forte utilisant l'approche de LP-relaxation est proposée pour améliorer la limite inférieure du problème. Pour les cas industriels réels, une heuristique à deux phases est proposée, qui construit une solution initiale en utilisant l'approche de LP-relaxation et l'améliore ensuite par une heuristique basée sur la programmation dynamique. Troisième, problème plusieurs produits avec contraintes de capacité pour des structures de produits à plusieurs niveaux avec pièces communes : une nouvelle formulation de MIP est proposée pour le problème. Les solutions exactes sont obtenues en utilisant le solveur CPLEX pour les problèmes de petite taille. Pour les cas de plus grande taille, une heuristique Fix-and-Optimize (FO) qui résout successivement une série de sous-problèmes où un petit sous-ensemble de variables est fixé tandis que les autres sont optimisées en utilisant une méthode exacte. Ensuite, nous améliorons la qualité de la solution de l'heuristique FO en considérant les caractéristiques d'interdépendance entre les variables connectées. Enfin, nous avons étudié des modèles EOQ en désassemblage afin de développer les modèles efficaces pour gérer des stocks surplus. Trois modèles sont développés et analysés pour



obtenir des approches de solution qui donnent les prix, le temps du cycle de réapprovisionnement (ou, de manière équivalente, la quantité de commande) et la quantité de disposition. La politique d'inventaire intégrant à la fois les décisions de prix et de la disposition permet d'atteindre un profit plus élevé.

Les futurs travaux du chapitre 3 doivent être consacrés à l'exploration d'un algorithme heuristique basé sur la solution initiale construite par la relaxation LP de la formulation FAL. Ceci est intéressant car la formulation FAL permet d'obtenir une limite inférieure très forte du problème. De plus, l'intégration des décisions de prix et d'élimination dans le problème du dimensionnement du lot de désassemblage d'un seul produit peut aider à obtenir une politique de stock optimale qui est plus réaliste.

L'un des premiers futurs travaux du chapitre 4 est d'améliorer l'heuristique à deux phases proposée afin qu'elle puisse calculer le surplus de stock obtenu à partir de chaque article racine et l'appliquer pendant la phase d'amélioration. Ensuite, l'heuristique à deux phases peut être étendue pour considérer le problème avec une structure de produits à plusieurs niveaux et des contraintes de capacité. Un autre travail futur peut se concentrer sur le développement de la formulation FAL pour la structure de désassemblage de produits à plusieurs niveaux. Ensuite, l'algorithme heuristique basé sur la relaxation LP de la formulation FAL peut être comparé à d'autres méthodes de solution telles que l'heuristique à deux phases basée sur la relaxation LP de la formulation AGG. De plus, la solution obtenue par la relaxation LP de la formulation FAL peut être appliquée dans un algorithme de B&B. L'algorithme FO est basé sur la qualité de la solution initiale. L'application de nouvelles méthodes pour obtenir une limite inférieure plus forte du problème et l'obtention de solutions initiales de meilleure qualité peuvent améliorer considérablement la qualité de la solution de l'algorithme FO proposé.

Les futurs travaux sur le chapitre 5 peuvent étendre les résultats de plusieurs façons. Dans les centres de désassemblage, le retour des produits peut être limité, et une fonction de retour sensible au prix peut être ajoutée au modèle. De plus, une structure de produit de désassemblage plus complexe peut être envisagée, telle que multi-produits, multi-niveaux avec des pièces communes. Dans les cas industriels réels, les produits retournés ne sont pas de la même qualité et les pièces obtenues lors des opérations de désassemblage peuvent varier. La prise en compte de paramètres stochastiques tels que la demande et le rendement peut aider à modéliser un problème plus réaliste.



## Appendix C

# Valid Inequalities (VIs) For The Single-Product DLSPD

Some researchers looked for a complete linear description of model for assembly lot sizing problem. (1, s) inequalities are one of the valid inequalities which can be identified from the fractional solution of the linear relaxation of the initial formulation. Consider a data example of a ten-period single-product disassembly lot sizing problem with disposal decision (DLSPD) presented in Table 3.1. The problem is to determine the quantity of EOL product ( $X_t$ ) to be disassembled at period  $t$  to satisfy demand of all leaf items ( $d_{it}$ ), and the excess quantity of leaf item  $i$  will be disposed of ( $E_{it}$ ). The values of  $Y_t$  are minimized because of objective function (3.18) and constraints (3.20). Note that for all  $t = 1 \dots T$ ,  $M_t = \max_{i=1 \dots N} \left\{ \frac{\sum_{k=t}^T d_{ik}}{a_i} \right\}$  is used for the linear programming relaxation of model P1. So equation (3.7) can be rewritten as follow:

$$X_t \leq \left( \max_{i=1 \dots N} \left\{ \frac{\sum_{k=t}^T d_{ik}}{a_i} \right\} \right) \cdot Y_t \quad \forall t = 1 \dots T \quad (\text{C.1})$$

For the disassembly lot sizing problem with disposal, inventory quantity of leaf items at the end of horizon planning ( $I_{iT}$ ) will be zero (with disposal cost equal to zero or less than inventory holding cost). In this case, if  $X_{10} > 0$ , a valid upper bound on the obtained quantity of leaf item  $i$  can be represented as follow:

$$a_i \cdot X_{10} \leq d_{i10} + E_{i10} \quad \forall i = 1 \dots N \quad (\text{C.2})$$

This means that the obtained quantity of leaf item  $i$  should satisfy demand of item  $i$  at period 10, then  $E_{i10}$  quantity will be disposed of. Note that if  $X_{10} = 0$ , above inequality is also satisfied. We could write equation (C.2) for period 4 only if period 4 would the last period or if there is no inventory at the end of period 4. Hence equation (C.2) can be rewritten for period 4 as follows:

$$a_i \cdot X_4 \leq (=) d_{i4} \cdot Y_4 + E_{i4} + I_{i4} \quad \forall i = 1 \dots N \quad (\text{C.3})$$

The above valid inequality can be generalized as follow:

$$a_i \cdot X_t \leq D_{itl} \cdot Y_t + E_{it} + I_{il} \quad \forall i = 1 \dots N \& 1 \leq t \leq l \leq T \quad (\text{C.4})$$

The class (C.4) of valid inequalities are not effective enough to obtain tighter linear relaxation of model. In this case, a more general class of so-called  $(l,s)$  inequalities for the DLSPD are defined as follow:

$$\sum_{s \in S} a_i \cdot X_s - \sum_{s \in S} E_{is} \leq \sum_{s \in S} D_{isl} \cdot Y_s + I_{il} \quad \forall i = 1 \dots N \& 1 \leq l \leq T, S \subseteq L \quad (\text{C.5})$$

For a given set of  $S$ , if  $\sum_{s \in S} Y_s = 0$ , then as  $X_s = 0$  (also  $E_{is}$ ) for  $s \in S$  and  $I_{i,l} \leq$ , the inequality is satisfied. Otherwise let  $t = \min s \in S : Y_s > 0$ . Then following inequalities for all  $i = 1, \dots, N$  can be defined:

$$\sum_{s \in S} a_i \cdot X_s \leq \sum_{s=t}^l a_i \cdot X_s \leq D_{itl} + \sum_{s=t}^l E_{is} + I_{il} \leq \sum_{s \in S} D_{isl} \cdot Y_s + \sum_{s=t}^l E_{is} + I_{il} \quad (\text{C.6})$$

Unlike the  $(l,s)$  inequalities for lot sizing problem, the above class of inequalities cannot obtain a complete convex hull but it can improve a very strong LB. Note that if  $d_{i1} > 0$ , the  $(l,s)$  inequality  $a_i \cdot X_1 \leq d_{i1} \cdot Y_1 + E_{i1} + I_{i1}$  together with the equation  $a_i \cdot X_1 = d_{i1} + E_{i1} + I_{i1}$  and  $Y_1 \leq 1$  entail the equality  $Y_1 = 1$ . Numerical result showed that these inequalities are valid for any feasible solution of initial linear relaxation model of P1 and they can improve LB of the problem. As mentioned in Section 3.3.3, we used a cutting-plane generation algorithm, which uses a separation algorithm to find the most violated inequalities of family of (C.5). provide a data example for a ten-period disassembly planning of a single EOL product. To demonstrate the efficiency of the addition of VIs and the cutting-plane generation algorithm, we use a data example for a ten-period disassembly planning of a single EOL product provided in Table C.1.

Table C.1 Data example to demonstrate the application of VIs

	Planning period (t)						
	1	2	3	4	5	6	7
$d_{1t}$	190	178	145	170	116	187	209
$d_{2t}$	113	206	199	233	77	239	224
$d_{3t}$	198	224	64	177	159	106	120
$d_{4t}$	176	109	236	76	110	82	242
$s_t$	2996	3312	2923	2736	2759	3405	2534
$p_t$	45	44	44	52	57	57	39
$h_{1t}$	0.5	0.3	0.5	0.5	0.5	0.5	0.3
$h_{2t}$	0.4	0.4	0.5	0.5	0.4	0.4	0.3
$h_{3t}$	0.5	0.5	0.5	0.3	0.5	0.5	0.3
$h_{4t}$	0.3	0.5	0.3	0.3	0.5	0.4	0.3

Table C.2 provides the result of the iterations of the cutting-plane algorithm to add the most violated inequalities into the initial model (P2). We note that the iteration zero implied the initial solution obtained by model P2 before adding VIs. The algorithm can obtain the optimal solution only after two iterations. The result of Tables 3.6 and C.2 confirm that the addition of VIs can obtain optimal or near optimal solutions of the problem.

Table C.2 Iterations of the cutting-plane generation algorithm for the data example presented in Table C.1

Iteration	Variable	Periods							$\frac{Obj.}{Gap(\%)}$
		1	2	3	4	5	6	7	
0	$X_t$	189.00	59.33	219.67	0.00	0.00	0.00	56.00	$\frac{30379.43}{6.80}$
	$Y_t$	0.36	0.14	0.70	0.00	0.00	0.00	0.80	
1	$X_t$	337.50	0.00	186.50	0.00	0.00	0.00	0.00	$\frac{32198.57}{0.76}$
	$Y_t$	1.00	0.00	0.60	0.00	0.00	0.00	0.00	
2	$X_t$	211.00	0.00	313	0.00	0.00	0.00	0.00	$\frac{32444.40}{0.00}$
	$Y_t$	1.00	0.00	1.00	0.00	0.00	0.00	0.00	



## Appendix D

# Example Data Sets For Chapter 5 (Example 1 to 9)

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**Instance 1** ( $c = 10, k = 9407$ )

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$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	1.0	5.0	1.0	3.0	3.0	1.0	3.0	1.0	1.0	3.0
$h_i$	0.92	0.17	0.85	0.88	0.41	0.48	0.26	0.20	0.56	0.14
$a_i$	897.81	110.40	16592.23	486.58	1048.51	2541.04	5926.08	2714.32	1143.93	948.50
$b_i$	2.3	1.8	2.6	2.9	2.6	2.4	2.1	1.8	1.5	3.0
$r_i$	0.092	0.059	0.092	0.078	0.024	0.034	0.058	0.045	0.011	0.031

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---

**Instance 2** ( $c = 10, k = 7753$ )

---

$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	3.0	3.0	3.0	3.0	4.0	3.0	3.0	1.0	2.0	3.0
$h_i$	0.48	0.78	0.16	0.97	0.85	0.87	0.34	0.18	0.21	0.21
$a_i$	3983.73	182.9	1020.66	2660.12	704.53	1415.28	2230.57	4287.82	530.48	2996.21
$b_i$	2.9	1.9	2.0	1.9	3.0	2.6	2.4	2.2	2.0	2.0
$r_i$	0.027	0.016	0.019	0.064	0.054	0.089	0.052	0.099	0.064	0.057

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---

**Instance 3** ( $c = 10, k = 5863$ )

---

$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	2.0	1.0	2.0	1.0	2.0	3.0	2.0	3.0	3.0	3.0
$h_i$	0.98	0.67	0.87	0.75	0.4	0.99	0.44	0.69	0.44	0.43
$a_i$	4058.3	8263.83	4147.93	3023.47	614.31	1832.65	2956.57	922.46	3898.07	311.85
$b_i$	2.2	1.7	1.5	2.9	2.6	1.7	2.0	1.8	2.6	2.4
$r_i$	0.085	0.069	0.022	0.051	0.056	0.071	0.027	0.029	0.055	0.041

---

---

**Instance 4** ( $c = 10, k = 9517$ )

---

$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	1.0	1.0	2.0	2.0	1.0	1.0	4.0	2.0	3.0	1.0
$h_i$	0.98	0.67	0.87	0.75	0.4	0.99	0.44	0.69	0.44	0.43
$a_i$	1922.3	1008.89	6770.25	569.75	8160.06	3820.94	2014.76	4528.52	1154.33	1416.09
$b_i$	2.0	2.3	2.9	2.4	2.3	1.8	2.4	1.9	2.4	1.9
$r_i$	0.018	0.059	0.099	0.029	0.038	0.095	0.072	0.021	0.019	0.055

---



---

**Instance 5** ( $c = 10, k = 6623$ )

---

$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	1.0	2.0	1.0	2.0	2.0	1.0	1.0	3.0	2.0	1.0
$h_i$	0.63	0.44	0.98	0.37	0.95	0.82	0.14	0.56	0.38	0.12
$a_i$	929.47	3734.65	6285.2	3405.1	1995.09	1400.92	7022.46	4053.93	4290.27	534.52
$b_i$	2.8	1.7	2.7	2.2	1.9	2.0	2.2	1.6	2.0	2.5
$r_i$	0.056	0.02	0.063	0.083	0.018	0.06	0.043	0.085	0.03	0.072

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**Instance 6** ( $c = 10, k = 5595$ )

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$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	2.0	4.0	4.0	4.0	1.0	3.0	4.0	2.0	3.0	2.0
$h_i$	0.68	0.41	0.23	0.82	0.7	0.48	0.44	0.84	0.18	0.32
$a_i$	2900.67	1381.02	628.43	3293.88	3289.64	3363.23	234.75	613.37	937.7	550.6
$b_i$	1.8	2.6	2.1	2.7	2.7	2.4	2.0	1.8	1.9	2.0
$r_i$	0.094	0.036	0.022	0.041	0.084	0.067	0.021	0.042	0.021	0.03

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**Instance 7** ( $c = 10, k = 9725$ )

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$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	1.0	2.0	4.0	2.0	1.0	1.0	4.0	3.0	3.0	2.0
$h_i$	0.24	0.23	0.94	0.47	0.75	0.5	0.39	0.48	0.58	0.18
$a_i$	3202.42	3777.52	344.13	842.73	2894.51	7123.17	595.35	7025.1	178.04	1469.85
$b_i$	1.8	2.1	1.7	3.0	2.0	2.5	2.8	1.8	2.3	2.2
$r_i$	0.069	0.067	0.088	0.053	0.011	0.081	0.035	0.079	0.012	0.059

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**Instance 8** ( $c = 10, k = 5996$ )

---

$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	4.0	2.0	3.0	4.0	2.0	1.0	1.0	4.0	2.0	3.0
$h_i$	0.35	0.9	0.28	0.98	0.35	0.31	0.11	0.37	0.52	0.99
$a_i$	1336.24	7514.05	2548.69	2066.2	838.94	3333.54	13355.88	1413.39	832.28	1419.53
$b_i$	2.8	2.8	2.5	2.7	2.6	2.9	1.8	1.9	1.9	2.8
$r_i$	0.019	0.096	0.03	0.027	0.033	0.092	0.014	0.041	0.048	0.014

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<b>Instance 9</b> ( $c = 10, k = 5505$ )										
$i$	1	2	3	4	5	6	7	8	9	10
$\alpha_i$	2.0	4.0	1.0	4.0	4.0	2.0	2.0	1.0	4.0	1.0
$h_i$	0.96	0.29	0.15	0.66	0.92	0.4	0.86	0.24	0.59	0.47
$a_i$	2978.69	548.3	6278.22	2198.47	3812.5	5238.36	4070.88	10692.41	726.1	1965.56
$b_i$	1.8	1.5	2.4	1.8	3.0	2.5	2.4	1.9	1.9	1.8
$r_i$	0.017	0.082	0.042	0.026	0.06	0.084	0.014	0.054	0.081	0.064

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# Meisam POUR MASSAHIAN TAFTI

## Doctorat : Optimisation et Sûreté des Systèmes

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### Optimisation de la logistique inverse pour la revalorisation des produits en fin de vie

Cette thèse étudie des modèles de gestion de stocks pour la planification du désassemblage. Nous traitons d'abord trois cas du problème de lot-sizing en désassemblage avec l'évacuation pour gérer le surplus de stocks. Le premier cas est mono-produit avec structure à deux niveaux et on propose une procédure pour calculer les surplus de stocks et le problème est modélisé par trois nouvelles formulations MIP avec différentes qualités de borne inférieure. L'ajout des inégalités valides et deux heuristiques avec différentes stratégies sont aussi proposés. Le deuxième cas est multi-produit avec structure à deux niveaux et des pièces communes et deux nouveaux MIP sont proposés pour modéliser le problème. Une relaxation des formulations peut améliorer la borne inférieure du problème et une heuristique à deux phases est également adaptée aux problèmes de grande taille. Le troisième cas est multi-produit avec structure à plusieurs niveaux, des pièces communes et contraintes de capacité et une nouvelle formulation MIP est proposée. Pour les problèmes de grande taille, *Fix-and-Optimize* est la méthode de résolution et on l'améliore avec les caractéristiques d'interrelation entre des variables liées. On traite aussi le modèle EOQ en désassemblage avec les demandes fonction des prix et l'évacuation pour maximiser le profit. Trois modèles sont développés pour obtenir des solutions qui proposent les prix, la durée du cycle de réapprovisionnement et la quantité à évacuer. La politique de gestion de stocks intégrant les décisions de prix et de la disposition permet d'obtenir des profits plus élevés.

Mots clés : logistique inverse – planification – gestion des stocks – recherche opérationnelle – heuristique.

### Optimization of Reverse Logistics for the Recovery of End-of-life Products

This thesis is dedicated to inventory models for disassembly planning problem. We first deal with three cases of disassembly lot sizing problem with considering disposal to handle surplus inventory accumulations due to disassembly operations. Three cases are considered. The first case is single-product with two-level product structure and we after propose a procedure to calculate surplus inventory and the problem is modeled by three new MIP formulations, which have different lower bound qualities. The addition of valid inequalities and two efficient heuristics with different inventory strategies are also developed. The second case is multi-product with two-level product structure and parts commonality. Two new MIP formulations are proposed to model the problem. LP-relaxation of the formulations can improve lower bound of the problem and a two-phase heuristic is also adapted for large-sized problems. The third case is multi-product with multi-level product structure, parts commonality, and capacity restrictions. A new MIP formulation is proposed. For large-sized problems, a *Fix-and-Optimize* algorithm solves the problem and we improved it by considering interrelatedness characteristics between linked variables. We also deal with the disassembly EOQ model which integrate price-sensitive demands and disposal decisions to maximize the profit of disassembly systems. Three models are developed to obtain solution approaches that give prices, the replenishment cycle time, and the disposal quantity. The inventory policy integrating both pricing and disposal decisions allows higher profits to be achieved.

Keywords: reverse logistics – planning – inventory control – operation research – heuristic.

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